Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/99-1.2.1.3-e2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**507**]. This is test number [99].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.61 (505)	0.39 (2)
Mathematica	99.21 (503)	0.79 (4)
Maple	96.45 (489)	3.55 (18)
Fricas	89.94 (456)	10.06 (51)
Giac	78.30 (397)	21.70 (110)
Reduce	76.73 (389)	23.27 (118)
Maxima	70.02 (355)	29.98 (152)
Mupad	55.82 (283)	44.18 (224)
Sympy	47.14 (239)	52.86 (268)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

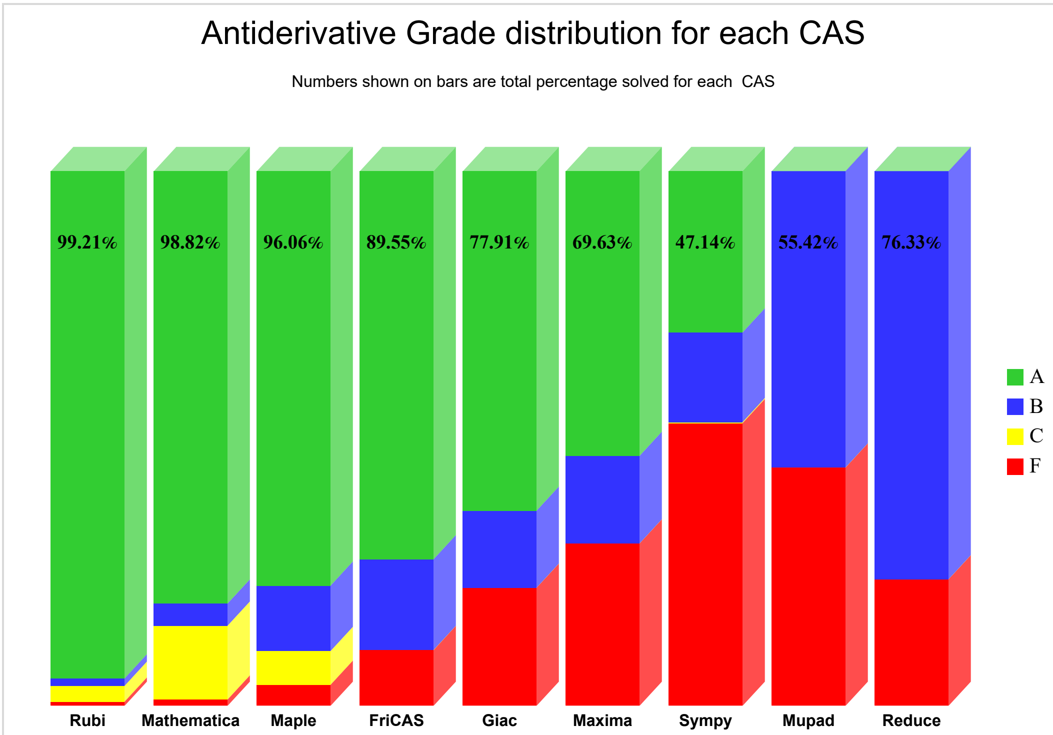
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

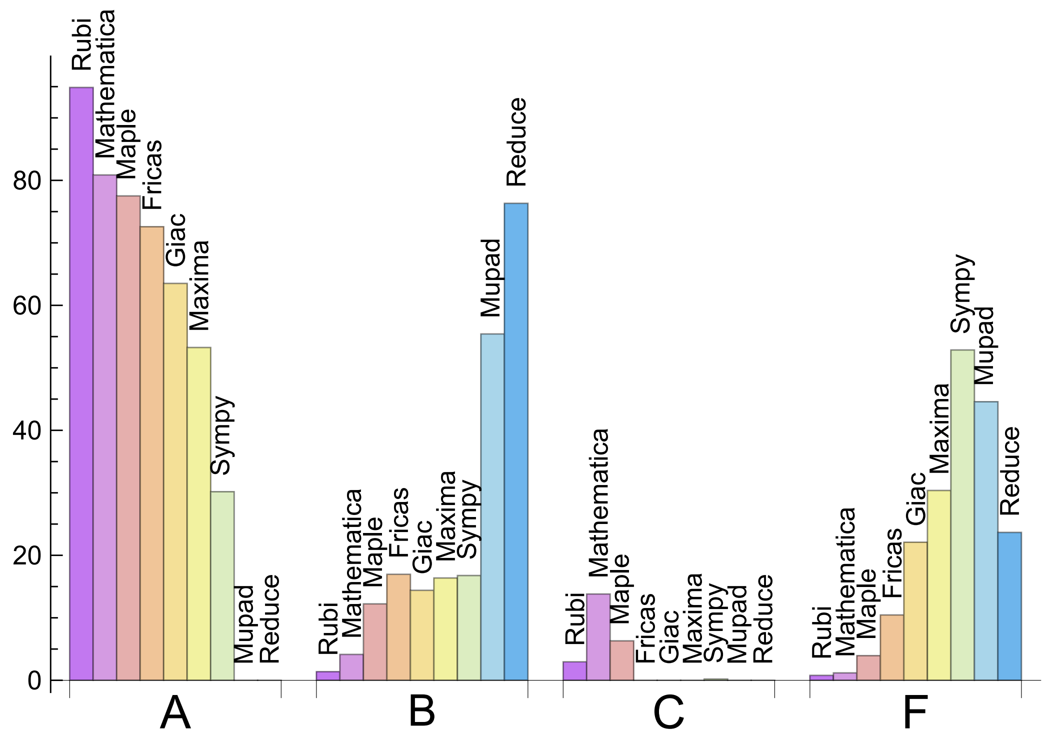
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.872	1.381	2.959	0.789
Mathematica	80.868	4.142	13.807	1.183
Maple	77.515	12.229	6.312	3.945
Fricas	72.584	16.963	0.000	10.454
Giac	63.511	14.398	0.000	22.091
Maxima	53.254	16.371	0.000	30.375
Sympy	30.178	16.765	0.197	52.860
Mupad	0.000	55.424	0.000	44.576
Reduce	0.000	76.331	0.000	23.669

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	18	100.00	0.00	0.00
Fricas	51	56.86	43.14	0.00
Giac	110	78.18	0.91	20.91
Reduce	118	100.00	0.00	0.00
Maxima	152	75.66	0.00	24.34
Mupad	224	0.00	100.00	0.00
Sympy	268	80.22	19.03	0.75

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Giac	0.30
Rubi	0.56
Reduce	0.82
Maple	1.36
Fricas	2.79
Mathematica	3.89
Sympy	4.14
Mupad	6.10

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	168.88	1.30	129.00	1.05
Rubi	174.45	0.94	124.00	1.00
Mathematica	213.60	1.07	126.00	0.96
Maple	329.83	1.37	124.00	0.93
Giac	464.31	1.91	128.00	0.93
Reduce	481.61	3.58	67.00	0.64
Fricas	641.73	2.48	119.00	1.00
Sympy	1046.97	5.99	131.00	1.33
Mupad	1174.36	3.71	118.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

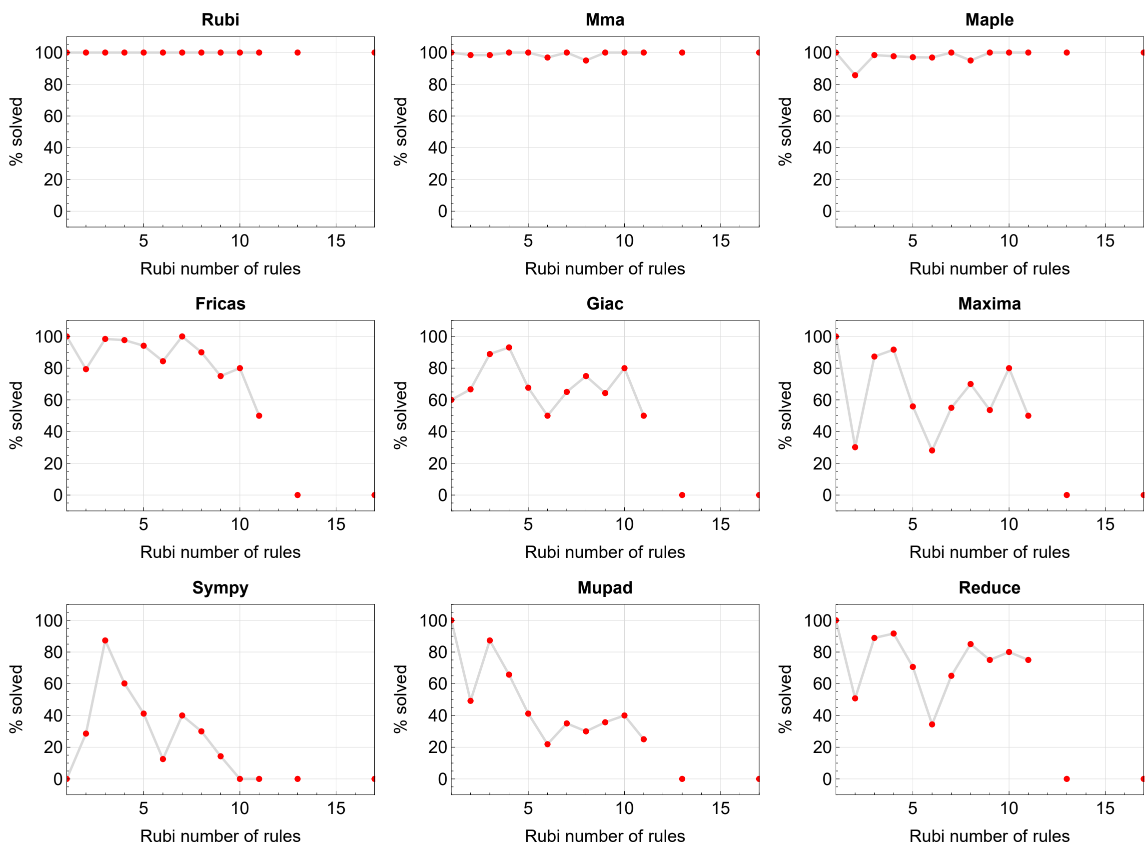


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

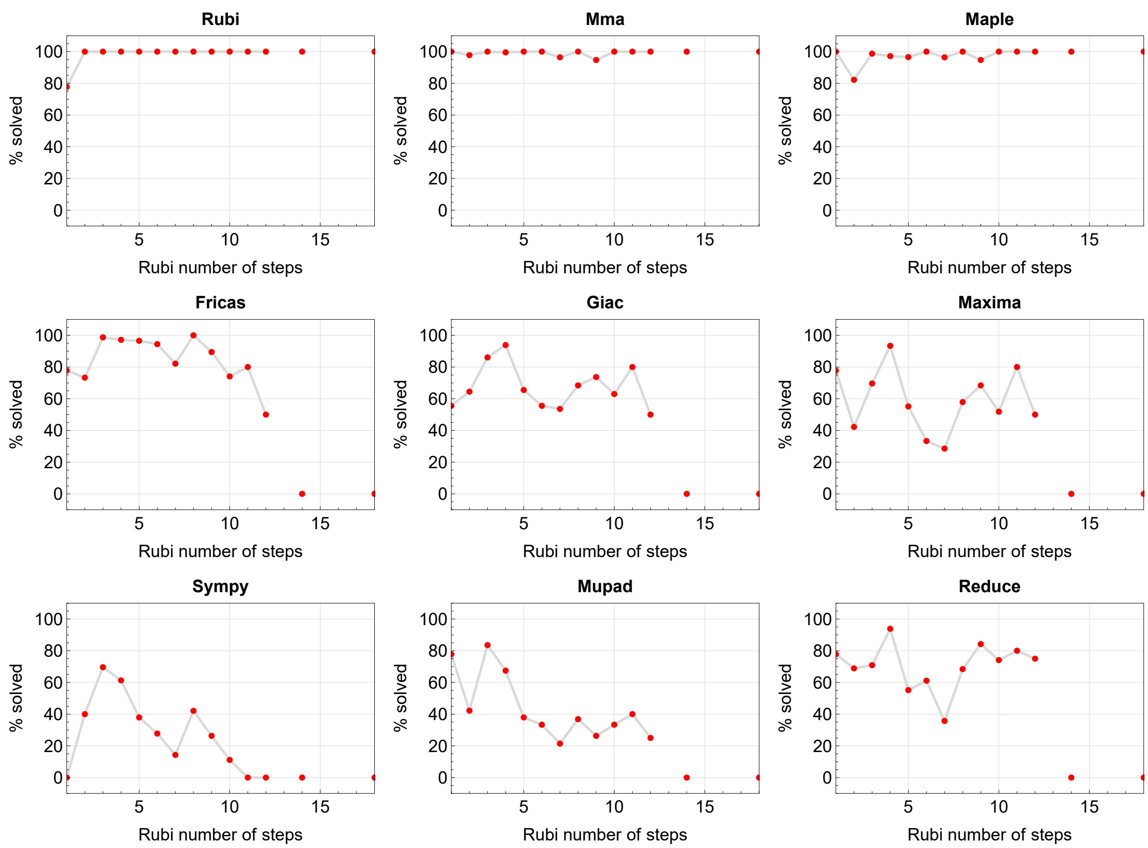


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

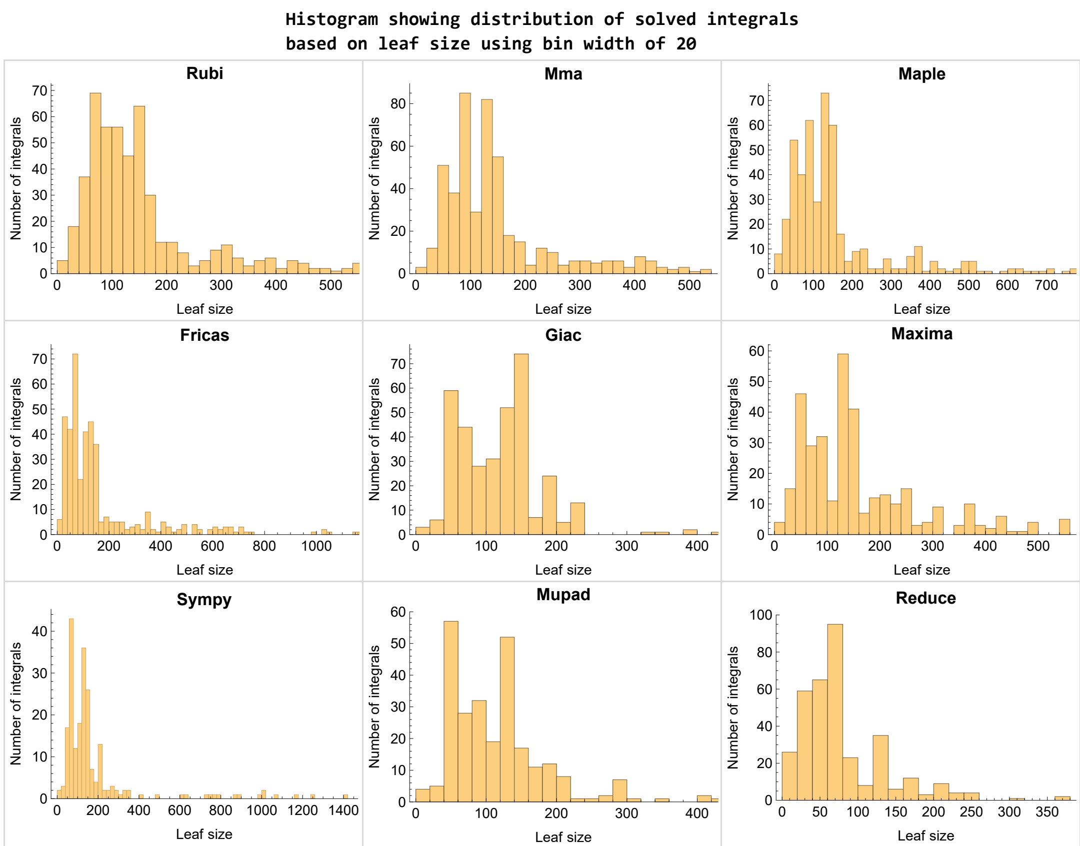


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

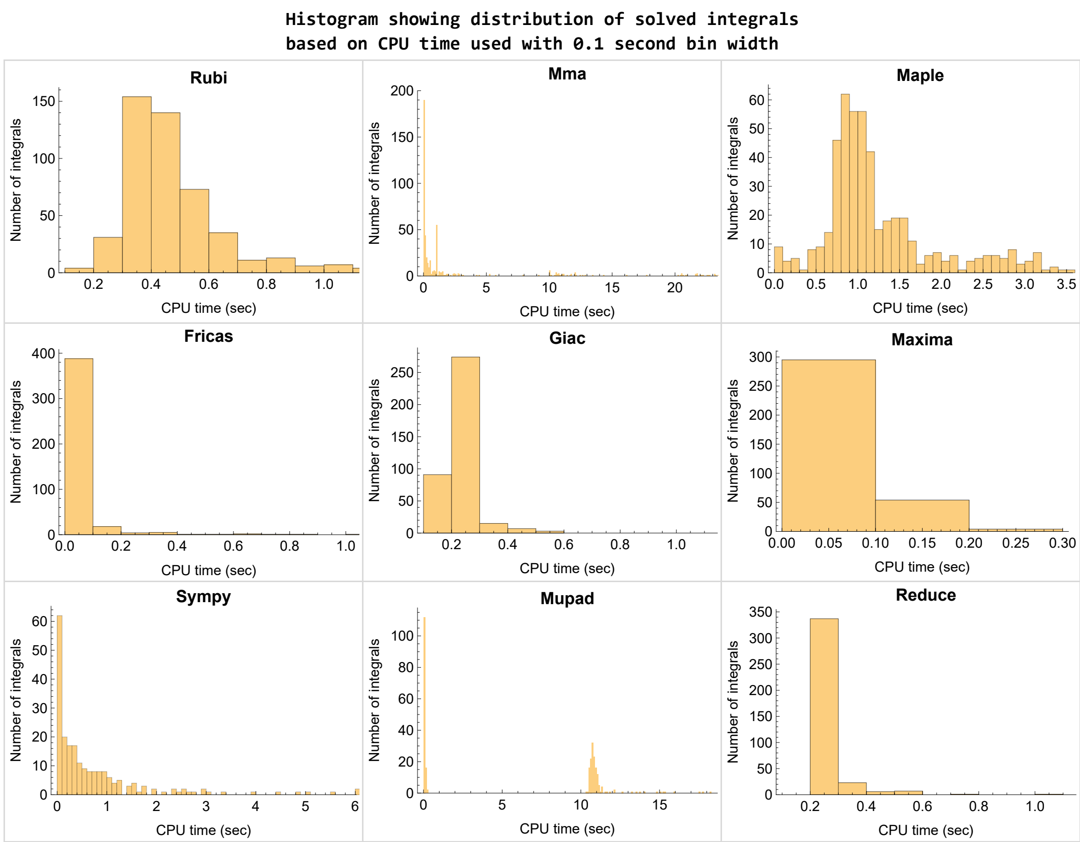


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

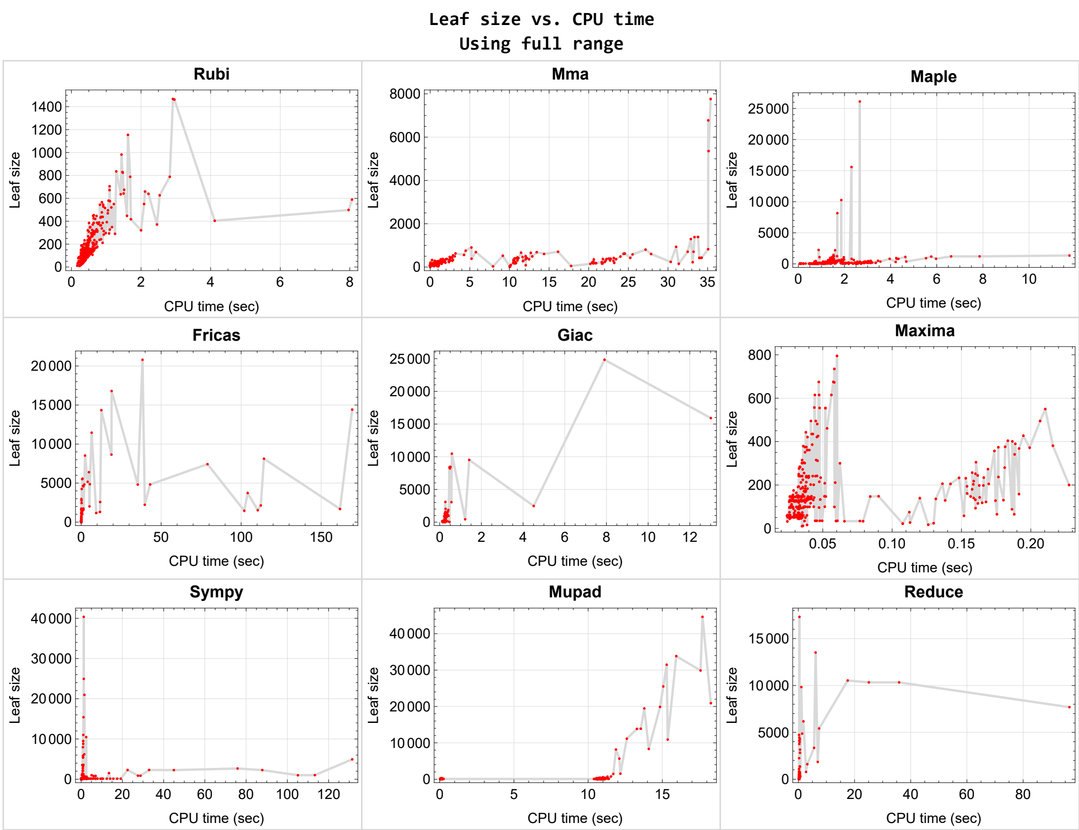


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{137, 138}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 474, 477, 484, 488, 491, 495, 498, 502, 505}

Mathematica {52, 481, 484}

Maple {144, 145, 146, 147, 148, 149, 150, 151, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 286, 287, 288, 289, 290, 291, 292, 293, 411, 412, 413, 414, 415, 416, 417, 418, 419}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

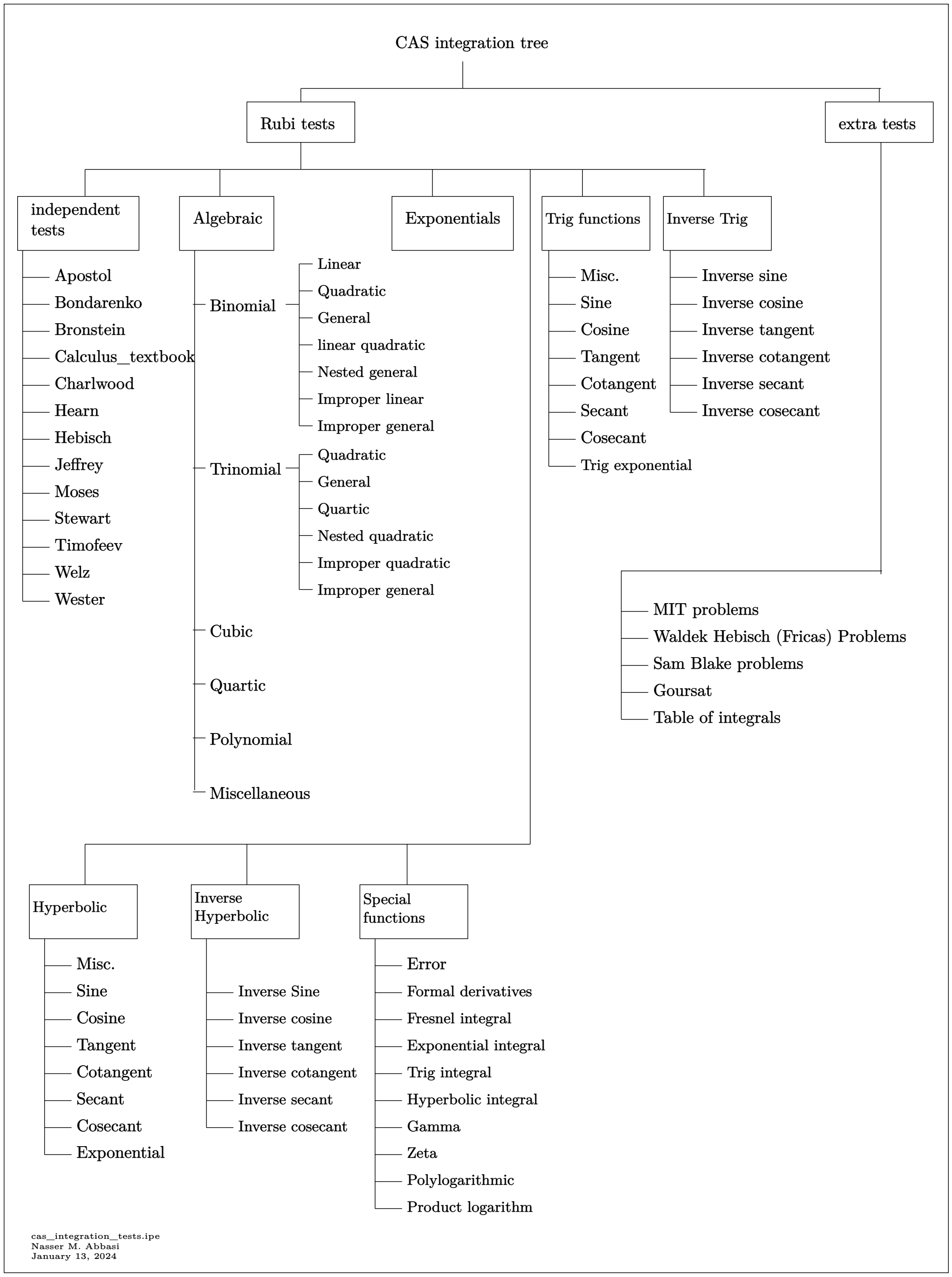
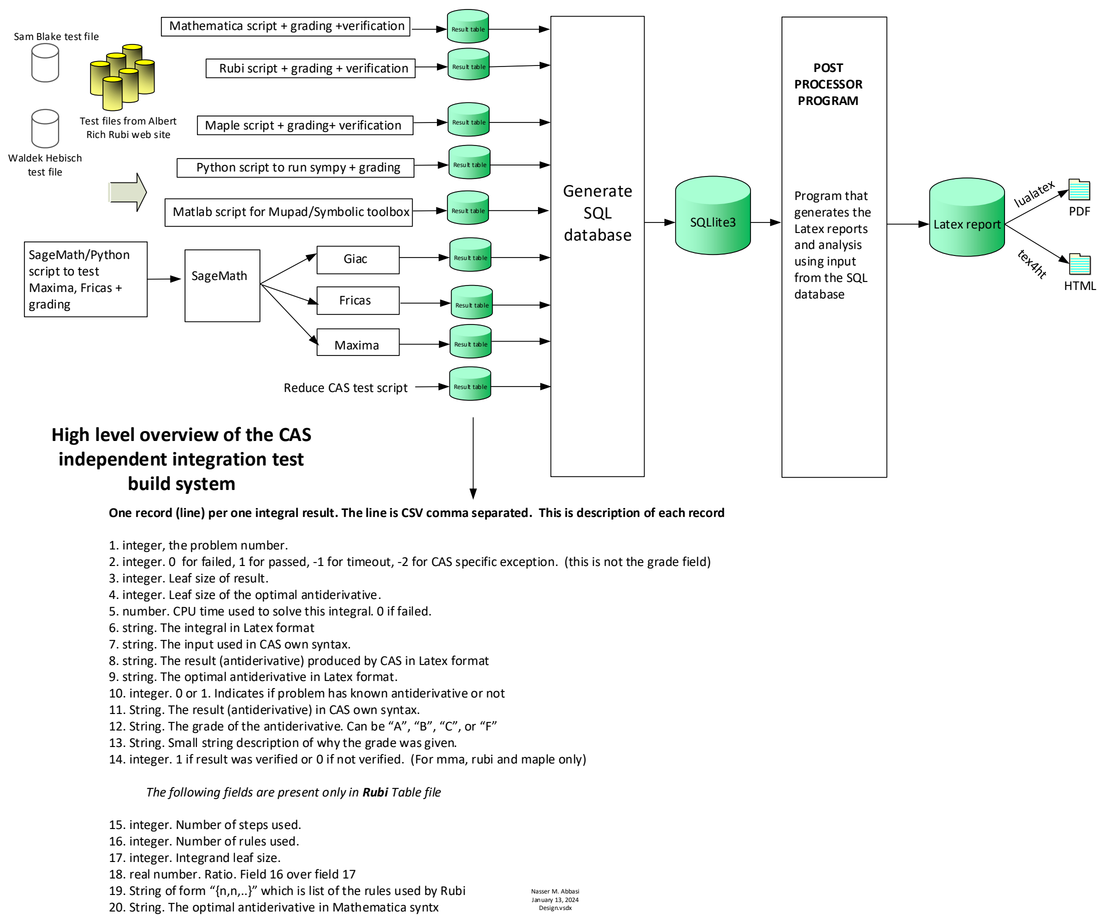


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507 }

B grade { 17, 25, 26, 27, 28, 94, 101 }

C grade { 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

F normal fail { 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 59, 60, 63, 64, 65, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 476, 480, 483, 487, 490, 494, 497, 501, 504 }

B grade { 156, 172, 173, 182, 192, 193, 194, 195, 208, 209, 210, 225, 226, 227, 241, 242, 243, 289, 337, 347, 354 }

C grade { 53, 54, 55, 58, 61, 62, 66, 67, 69, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 472, 474, 475, 477, 478, 479, 481, 482, 484, 485, 486, 488, 489, 491, 492, 493, 495, 496, 498, 499, 500, 502, 503, 505, 506, 507 }

F normal fail { 132, 133, 134, 135 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 50, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 92, 93, 94, 97, 98, 99, 100, 101, 120, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507 }

B grade { 16, 24, 25, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 55, 74, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 117, 118, 119, 156, 163, 172, 173, 182, 192, 193, 194, 195, 208, 209, 225, 226, 227, 241, 242, 318, 326, 458, 459, 460, 461, 470, 471 }

C grade { 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 286, 287, 288, 289, 290, 291, 292, 293, 411, 412, 413, 414, 415, 416, 417, 418, 419 }

F normal fail { 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 468, 469 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 23, 27, 28, 29, 56, 57, 74, 75, 76, 77, 79, 88, 89, 90, 95, 96, 97, 98, 102, 103, 104, 105, 110, 111, 112, 113, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506 }

B grade { 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 83, 84, 91, 117, 118, 119, 120, 156, 163, 172, 173, 182, 183, 192, 193, 194, 195, 208, 209, 210, 225, 226, 227, 228, 241, 242, 264, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 318, 403, 404, 405, 409, 465, 466, 467, 470, 471 }

C grade { }

F normal fail { 93, 94, 101, 106, 107, 108, 109, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 468, 469, 507 }

F(-1) timedout fail { 12, 20, 21, 22, 24, 25, 26, 30, 38, 52, 53, 54, 55, 78, 81, 82, 85, 86, 87, 92, 99, 100 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 56, 57, 118, 119, 120, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 279, 280, 281, 287, 297, 300, 313, 314, 319, 320, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 447, 448, 449, 455, 456, 457, 465, 466, 467, 470, 471, 473, 480, 487, 494, 501 }

B grade { 117, 156, 163, 172, 173, 182, 183, 192, 193, 194, 195, 208, 209, 210, 225, 226, 227, 241, 242, 275, 278, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 353, 442, 443, 444, 445, 446, 450, 451, 452, 453, 454, 458, 459, 460, 461, 462, 463, 464 }

C grade { }

F normal fail { 17, 18, 19, 25, 26, 27, 28, 29, 35, 36, 37, 43, 44, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 468, 469, 472, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499, 500, 502, 503, 504, 505, 506, 507 }

F(-1) timedout fail { }

F(-2) exception fail { 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 17, 34, 36, 56, 57, 74, 75, 76, 77, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 319, 320, 321, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 487 }

B grade { 18, 19, 27, 28, 29, 37, 40, 41, 42, 44, 45, 46, 47, 48, 49, 51, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 81, 83, 84, 85, 117, 118, 119, 120, 156, 163, 172, 173, 182, 183, 192, 193, 194, 195, 208, 209, 210, 225, 226, 227, 228, 241, 242, 299, 305, 317, 318, 326, 327, 465, 466, 467, 470, 471, 473, 480 }

C grade { }

F normal fail { 52, 53, 54, 55, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 468, 469, 472, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507 }

F(-1) timedout fail { 79 }

F(-2) exception fail { 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 35, 38, 39, 43, 50, 78, 82 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 117, 118, 119, 120, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 305, 306, 307, 308, 309, 310, 311, 326, 327, 328, 329, 330, 331, 332, 337, 338, 339, 347, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 425, 434, 465, 466, 467, 470, 471, 473, 480, 487, 494, 501 }

C grade { }

F normal fail { }

F(-1) timedout fail { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 333, 334, 335, 336, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 356, 357, 411, 412, 413, 414, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 472, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499, 500, 502, 503, 504, 505, 506, 507 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 10, 11, 56, 57, 121, 122, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 271, 272, 273, 274, 276, 277, 279, 281, 284, 285, 286, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { 5, 6, 7, 8, 9, 74, 75, 76, 77, 117, 118, 119, 120, 123, 124, 155, 156, 163, 171, 172, 173, 182, 183, 192, 193, 194, 195, 208, 209, 210, 224, 225, 226, 227, 228, 241, 242, 243, 258, 259, 260, 269, 270, 275, 278, 280, 282, 283, 294, 295, 296, 297, 298, 299, 312, 313, 314, 315, 316, 317, 318, 333, 334, 335, 336, 337, 386, 387, 388, 389, 390, 391, 392, 393, 396, 397, 398, 399, 400, 407, 465, 466, 467, 470, 471 }

C grade { 131 }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 70, 71, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 126, 127, 128, 129, 287, 288, 289, 290, 291, 292, 293, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 414, 415, 416, 421, 422, 423, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 444, 449, 450, 451, 452, 453, 468, 469, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507 }

F(-1) timedout fail { 66, 67, 68, 69, 72, 73, 130, 132, 133, 134, 135, 136, 137, 394, 395, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 417, 418, 419, 420, 429, 430, 431, 438, 439, 445, 446, 447, 448, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464 }

F(-2) exception fail { 125, 138 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 50, 51, 56, 57, 61, 62, 70, 76, 77, 78, 79, 80, 81, 82, 83, 117, 118, 119, 120, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 470, 471, 473, 480, 487, 494, 501 }

C grade { }

F normal fail { 12, 13, 19, 20, 21, 27, 28, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 468, 469, 472, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499, 500, 502, 503, 504, 505, 506, 507 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	132	130	143	144	139	156	167	157
N.S.	1	1.00	0.96	0.95	1.04	1.05	1.01	1.14	1.22	1.15
time (sec)	N/A	0.338	0.068	0.690	0.027	0.066	0.182	0.230	0.206	0.060

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	103	111	113	105	119	131	121
N.S.	1	1.00	0.99	0.95	1.03	1.05	0.97	1.10	1.21	1.12
time (sec)	N/A	0.292	0.041	0.724	0.028	0.067	0.147	0.227	0.229	0.046

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	76	81	82	71	84	95	85
N.S.	1	1.00	0.94	0.96	1.03	1.04	0.90	1.06	1.20	1.08
time (sec)	N/A	0.250	0.028	0.788	0.030	0.068	0.128	0.235	0.232	0.050

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	49	50	52	44	50	62	51
N.S.	1	1.00	0.92	0.94	0.96	1.00	0.85	0.96	1.19	0.98
time (sec)	N/A	0.212	0.016	0.644	0.033	0.071	0.101	0.242	0.225	10.781

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	45	44	44	75	46	54	40
N.S.	1	1.00	1.00	1.02	1.00	1.00	1.70	1.05	1.23	0.91
time (sec)	N/A	0.211	0.019	0.740	0.026	0.074	0.451	0.213	0.215	0.129

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	54	53	57	126	55	68	50
N.S.	1	1.00	1.04	1.06	1.04	1.12	2.47	1.08	1.33	0.98
time (sec)	N/A	0.224	0.023	0.648	0.033	0.074	0.410	0.241	0.206	11.080

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	75	73	78	185	86	104	58
N.S.	1	1.00	0.92	0.97	0.95	1.01	2.40	1.12	1.35	0.75
time (sec)	N/A	0.243	0.056	0.694	0.035	0.074	0.345	0.213	0.206	0.100

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	99	102	106	111	233	118	141	121
N.S.	1	1.00	0.94	0.97	1.01	1.06	2.22	1.12	1.34	1.15
time (sec)	N/A	0.274	0.083	0.757	0.027	0.075	0.412	0.233	0.212	10.694

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	130	139	144	275	149	179	153
N.S.	1	1.00	0.99	0.96	1.03	1.07	2.04	1.10	1.33	1.13
time (sec)	N/A	0.307	0.086	0.774	0.028	0.072	0.472	0.204	0.224	10.877

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	12	10
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	1.00	0.83
time (sec)	N/A	0.168	0.005	0.656	0.035	0.068	0.037	0.232	0.219	10.613

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	18	14	18	18	16
N.S.	1	1.00	1.00	0.94	0.89	1.00	0.78	1.00	1.00	0.89
time (sec)	N/A	0.181	0.005	0.748	0.039	0.073	0.046	0.468	0.214	0.043

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	392	317	467	0	0	0	0	25	0
N.S.	1	1.08	0.88	1.29	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.017	2.501	1.493	0.000	0.000	0.000	0.000	200.034	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	278	242	353	0	1459	0	0	25	0
N.S.	1	1.09	0.95	1.38	0.00	5.72	0.00	0.00	0.10	0.00
time (sec)	N/A	0.590	1.398	1.454	0.000	101.975	0.000	0.000	200.044	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	203	184	280	0	1165	0	0	4268	0
N.S.	1	1.06	0.96	1.47	0.00	6.10	0.00	0.00	22.35	0.00
time (sec)	N/A	0.443	0.944	1.366	0.000	9.429	0.000	0.000	0.517	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	175	236	0	992	0	0	3976	0
N.S.	1	1.04	1.15	1.55	0.00	6.53	0.00	0.00	26.16	0.00
time (sec)	N/A	0.354	1.217	1.582	0.000	0.680	0.000	0.000	0.447	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	170	157	413	0	2218	0	0	163	0
N.S.	1	1.02	0.94	2.47	0.00	13.28	0.00	0.00	0.98	0.00
time (sec)	N/A	0.416	0.747	1.404	0.000	39.785	0.000	0.000	0.493	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	324	142	249	0	1020	0	197	203	0
N.S.	1	2.08	0.91	1.60	0.00	6.54	0.00	1.26	1.30	0.00
time (sec)	N/A	0.597	0.854	1.526	0.000	0.268	0.000	0.203	0.272	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	420	223	288	0	1217	0	432	377	0
N.S.	1	1.94	1.03	1.33	0.00	5.63	0.00	2.00	1.75	0.00
time (sec)	N/A	0.682	1.595	1.537	0.000	0.478	0.000	1.220	0.505	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	544	293	368	0	1507	0	844	25	0
N.S.	1	1.83	0.98	1.23	0.00	5.06	0.00	2.83	0.08	0.00
time (sec)	N/A	0.819	2.798	1.842	0.000	1.088	0.000	0.277	200.028	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	675	662	890	0	0	0	0	25	0
N.S.	1	1.06	1.04	1.40	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.516	11.466	1.480	0.000	0.000	0.000	0.000	200.039	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	495	443	664	0	0	0	0	25	0
N.S.	1	1.07	0.96	1.44	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.049	11.012	1.424	0.000	0.000	0.000	0.000	200.044	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	364	324	500	0	0	0	0	10332	0
N.S.	1	1.07	0.96	1.47	0.00	0.00	0.00	0.00	30.48	0.00
time (sec)	N/A	0.826	2.869	1.504	0.000	0.000	0.000	0.000	25.089	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	271	247	383	0	1523	0	0	9833	0
N.S.	1	1.08	0.98	1.52	0.00	6.04	0.00	0.00	39.02	0.00
time (sec)	N/A	0.601	2.338	1.388	0.000	110.405	0.000	0.000	1.058	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	318	223	793	0	0	0	0	562	0
N.S.	1	1.26	0.88	3.15	0.00	0.00	0.00	0.00	2.23	0.00
time (sec)	N/A	0.744	1.511	1.434	0.000	0.000	0.000	0.000	0.522	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	516	210	406	0	0	0	0	513	0
N.S.	1	2.23	0.91	1.76	0.00	0.00	0.00	0.00	2.22	0.00
time (sec)	N/A	0.966	1.412	1.537	0.000	0.000	0.000	0.000	0.263	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	672	253	356	0	0	0	0	649	0
N.S.	1	2.61	0.98	1.39	0.00	0.00	0.00	0.00	2.53	0.00
time (sec)	N/A	1.095	1.717	1.605	0.000	0.000	0.000	0.000	0.533	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	835	289	402	0	1591	0	916	25	0
N.S.	1	2.99	1.04	1.44	0.00	5.70	0.00	3.28	0.09	0.00
time (sec)	N/A	1.290	2.497	1.546	0.000	1.484	0.000	0.335	200.026	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	982	384	526	0	2011	0	1853	25	0
N.S.	1	2.35	0.92	1.26	0.00	4.82	0.00	4.44	0.06	0.00
time (sec)	N/A	1.439	5.220	1.691	0.000	5.220	0.000	0.411	200.033	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	1154	522	702	0	2575	0	3046	1597	0
N.S.	1	2.00	0.90	1.22	0.00	4.46	0.00	5.28	2.77	0.00
time (sec)	N/A	1.624	9.165	1.984	0.000	11.921	0.000	0.560	3.181	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	327	254	337	0	0	0	0	5414	0
N.S.	1	1.11	0.86	1.15	0.00	0.00	0.00	0.00	18.41	0.00
time (sec)	N/A	1.104	1.686	1.681	0.000	0.000	0.000	0.000	7.401	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	235	198	269	0	1685	0	0	4867	0
N.S.	1	1.07	0.90	1.23	0.00	7.69	0.00	0.00	22.22	0.00
time (sec)	N/A	0.659	1.928	1.536	0.000	161.776	0.000	0.000	1.347	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	167	147	230	0	1306	0	0	4437	0
N.S.	1	1.06	0.94	1.46	0.00	8.32	0.00	0.00	28.26	0.00
time (sec)	N/A	0.364	1.445	1.434	0.000	11.804	0.000	0.000	0.359	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	134	190	0	1034	0	0	4099	0
N.S.	1	1.00	1.08	1.53	0.00	8.34	0.00	0.00	33.06	0.00
time (sec)	N/A	0.272	0.698	1.407	0.000	0.748	0.000	0.000	0.341	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	92	157	0	343	0	71	3755	0
N.S.	1	1.00	1.16	1.99	0.00	4.34	0.00	0.90	47.53	0.00
time (sec)	N/A	0.199	0.408	1.455	0.000	0.125	0.000	0.238	0.326	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	135	194	0	1047	0	0	243	0
N.S.	1	1.00	1.07	1.54	0.00	8.31	0.00	0.00	1.93	0.00
time (sec)	N/A	0.321	0.713	1.371	0.000	0.227	0.000	0.000	0.477	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	196	192	237	0	1340	0	191	431	0
N.S.	1	1.21	1.19	1.46	0.00	8.27	0.00	1.18	2.66	0.00
time (sec)	N/A	0.389	1.291	1.599	0.000	0.505	0.000	0.259	0.280	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	309	232	278	0	1737	0	399	756	0
N.S.	1	1.39	1.04	1.25	0.00	7.79	0.00	1.79	3.39	0.00
time (sec)	N/A	0.499	2.369	1.558	0.000	1.422	0.000	0.260	0.497	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	314	288	815	0	0	0	0	25	0
N.S.	1	1.15	1.05	2.97	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.927	11.740	1.539	0.000	0.000	0.000	0.000	200.025	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	253	229	615	0	4811	0	0	25	0
N.S.	1	1.15	1.04	2.80	0.00	21.87	0.00	0.00	0.11	0.00
time (sec)	N/A	0.539	1.223	1.493	0.000	35.399	0.000	0.000	200.047	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	191	498	0	1351	0	461	25	0
N.S.	1	1.00	1.22	3.19	0.00	8.66	0.00	2.96	0.16	0.00
time (sec)	N/A	0.343	0.995	1.412	0.000	0.342	0.000	0.264	200.058	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	180	438	0	1320	0	444	23	0
N.S.	1	1.00	1.20	2.92	0.00	8.80	0.00	2.96	0.15	0.00
time (sec)	N/A	0.320	0.946	1.356	0.000	0.358	0.000	0.278	200.054	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	190	400	0	1349	0	461	22	0
N.S.	1	1.00	1.23	2.58	0.00	8.70	0.00	2.97	0.14	0.00
time (sec)	N/A	0.300	1.229	1.353	0.000	0.359	0.000	0.274	200.057	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	248	230	493	0	4819	0	0	3348	0
N.S.	1	1.14	1.06	2.26	0.00	22.11	0.00	0.00	15.36	0.00
time (sec)	N/A	0.510	1.871	1.411	0.000	2.012	0.000	0.000	5.604	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	386	332	649	0	6396	0	757	4729	0
N.S.	1	1.39	1.20	2.34	0.00	23.09	0.00	2.73	17.07	0.00
time (sec)	N/A	0.642	2.412	1.517	0.000	4.883	0.000	0.307	0.252	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	520	478	1448	0	4618	0	8448	25	0
N.S.	1	1.04	0.96	2.91	0.00	9.27	0.00	16.96	0.05	0.00
time (sec)	N/A	1.147	12.059	1.522	0.000	1.208	0.000	0.517	200.040	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	446	412	1131	0	4600	0	8382	25	0
N.S.	1	1.07	0.99	2.71	0.00	11.03	0.00	20.10	0.06	0.00
time (sec)	N/A	0.722	12.200	1.467	0.000	1.301	0.000	0.480	200.063	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	371	356	936	0	4636	0	8210	25	0
N.S.	1	1.06	1.02	2.68	0.00	13.28	0.00	23.52	0.07	0.00
time (sec)	N/A	0.611	12.042	1.494	0.000	1.248	0.000	0.489	200.034	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	345	304	839	0	4596	0	8378	23	0
N.S.	1	1.13	0.99	2.74	0.00	15.02	0.00	27.38	0.08	0.00
time (sec)	N/A	0.534	10.501	1.381	0.000	1.403	0.000	0.467	200.045	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	350	309	766	0	4612	0	8447	22	0
N.S.	1	1.14	1.01	2.50	0.00	15.02	0.00	27.51	0.07	0.00
time (sec)	N/A	0.544	10.523	1.444	0.000	1.362	0.000	0.497	200.050	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	477	459	953	0	16795	0	0	13522	0
N.S.	1	0.93	0.89	1.85	0.00	32.61	0.00	0.00	26.26	0.00
time (sec)	N/A	0.873	11.658	1.417	0.000	19.054	0.000	0.000	6.129	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	705	654	1239	0	20807	0	10475	17328	0
N.S.	1	1.12	1.04	1.97	0.00	33.13	0.00	16.68	27.59	0.00
time (sec)	N/A	1.096	12.692	1.678	0.000	38.371	0.000	0.580	0.383	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	635	488	1071	0	0	0	0	27	0
N.S.	1	1.15	0.89	1.94	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.417	12.879	1.970	0.000	0.000	0.000	0.000	200.027	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	337	298	638	0	0	0	0	27	0
N.S.	1	0.88	0.78	1.66	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.875	22.143	1.942	0.000	0.000	0.000	0.000	200.034	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	212	284	238	0	0	0	0	27	0
N.S.	1	0.98	1.31	1.10	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.607	23.112	2.803	0.000	0.000	0.000	0.000	200.032	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	550	424	819	0	0	0	0	27	0
N.S.	1	1.31	1.01	1.95	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.217	25.239	3.957	0.000	0.000	0.000	0.000	200.050	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	102	78	68	75	134	88	71	238	84
N.S.	1	1.13	0.87	0.76	0.83	1.49	0.98	0.79	2.64	0.93
time (sec)	N/A	0.275	0.074	2.589	0.112	0.077	0.107	0.256	0.298	0.137

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	33	22	20	32	32	31	20	33	20
N.S.	1	1.10	0.73	0.67	1.07	1.07	1.03	0.67	1.10	0.67
time (sec)	N/A	0.180	0.013	1.392	0.024	0.072	0.051	0.207	0.281	11.088

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	498	627	594	0	5507	0	1200	25	13879
N.S.	1	1.02	1.28	1.21	0.00	11.24	0.00	2.45	0.05	28.32
time (sec)	N/A	7.965	3.195	1.763	0.000	0.652	0.000	0.334	200.041	13.549

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	404	465	497	0	4245	0	1074	25	11143
N.S.	1	1.02	1.17	1.25	0.00	10.69	0.00	2.71	0.06	28.07
time (sec)	N/A	4.118	2.467	1.677	0.000	0.360	0.000	0.328	200.033	12.606

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	321	375	425	0	2966	0	897	25	8171
N.S.	1	1.02	1.19	1.34	0.00	9.39	0.00	2.84	0.08	25.86
time (sec)	N/A	2.001	1.579	1.577	0.000	0.173	0.000	0.296	200.033	11.873

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	293	341	325	0	1721	0	758	1844	5664
N.S.	1	1.02	1.19	1.13	0.00	6.00	0.00	2.64	6.43	19.74
time (sec)	N/A	1.082	1.540	1.552	0.000	0.113	0.000	0.303	6.896	12.103

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	233	252	222	0	715	0	445	1369	709
N.S.	1	1.18	1.27	1.12	0.00	3.61	0.00	2.25	6.91	3.58
time (sec)	N/A	0.378	0.988	1.415	0.000	0.086	0.000	0.282	0.482	11.306

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	283	266	291	0	2443	0	719	25	10894
N.S.	1	1.03	0.97	1.06	0.00	8.88	0.00	2.61	0.09	39.61
time (sec)	N/A	0.930	1.549	1.522	0.000	0.324	0.000	0.290	200.041	15.362

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	371	337	373	0	4857	0	776	25	19887
N.S.	1	1.12	1.02	1.13	0.00	14.72	0.00	2.35	0.08	60.26
time (sec)	N/A	2.459	1.886	1.659	0.000	5.452	0.000	0.328	200.040	14.847

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	550	433	489	0	7422	0	1043	25	33838
N.S.	1	1.19	0.94	1.06	0.00	16.06	0.00	2.26	0.05	73.24
time (sec)	N/A	2.088	2.763	1.746	0.000	78.988	0.000	0.329	200.024	15.940

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	659	901	762	0	14340	0	1612	25	31485
N.S.	1	1.01	1.39	1.17	0.00	22.06	0.00	2.48	0.04	48.44
time (sec)	N/A	2.114	5.201	1.951	0.000	12.640	0.000	0.392	200.036	15.293

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	589	755	680	0	11459	0	1397	25	25497
N.S.	1	1.01	1.30	1.17	0.00	19.72	0.00	2.40	0.04	43.88
time (sec)	N/A	8.063	4.477	1.869	0.000	6.579	0.000	0.352	200.043	15.063

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	447	537	541	0	8530	0	1195	25	19465
N.S.	1	1.01	1.22	1.23	0.00	19.34	0.00	2.71	0.06	44.14
time (sec)	N/A	1.598	2.883	1.760	0.000	2.450	0.000	0.371	200.039	13.785

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	386	493	485	0	5572	0	986	23	13841
N.S.	1	1.03	1.31	1.29	0.00	14.82	0.00	2.62	0.06	36.81
time (sec)	N/A	0.899	3.096	1.610	0.000	0.883	0.000	0.306	200.035	13.286

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	300	364	345	0	2770	0	797	2819	8334
N.S.	1	0.93	1.13	1.07	0.00	8.60	0.00	2.48	8.75	25.88
time (sec)	N/A	0.607	1.996	1.556	0.000	0.266	0.000	0.290	0.567	14.088

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	348	371	370	0	5164	0	833	25	20897
N.S.	1	1.02	1.09	1.09	0.00	15.19	0.00	2.45	0.07	61.46
time (sec)	N/A	1.166	2.113	1.931	0.000	4.070	0.000	0.327	200.029	18.268

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	417	416	409	0	8650	0	890	25	29890
N.S.	1	1.11	1.11	1.09	0.00	23.01	0.00	2.37	0.07	79.49
time (sec)	N/A	1.708	2.560	1.662	0.000	18.955	0.000	0.358	200.043	17.574

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	626	560	509	0	14414	0	1129	25	44649
N.S.	1	1.18	1.06	0.96	0.00	27.25	0.00	2.13	0.05	84.40
time (sec)	N/A	2.536	4.271	1.855	0.000	169.419	0.000	0.332	200.032	17.706

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	521	389	689	988	0	1719	5372	948	23	0
N.S.	1	0.75	1.32	1.90	0.00	3.30	10.31	1.82	0.04	0.00
time (sec)	N/A	0.795	5.765	1.383	0.000	0.243	0.923	0.273	200.038	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	268	446	610	0	1151	2742	602	23	0
N.S.	1	0.81	1.35	1.84	0.00	3.48	8.28	1.82	0.07	0.00
time (sec)	N/A	0.479	2.706	1.287	0.000	0.185	0.862	0.269	200.053	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	178	254	316	0	689	1175	329	775	0
N.S.	1	0.90	1.28	1.60	0.00	3.48	5.93	1.66	3.91	0.00
time (sec)	N/A	0.318	1.376	1.137	0.000	0.127	0.808	0.235	2.739	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	148	128	128	0	341	343	156	317	199
N.S.	1	1.09	0.94	0.94	0.00	2.51	2.52	1.15	2.33	1.46
time (sec)	N/A	0.273	0.565	1.185	0.000	0.149	0.394	0.223	0.728	11.313

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	364	324	500	0	0	0	0	10332	0
N.S.	1	1.07	0.96	1.47	0.00	0.00	0.00	0.00	30.48	0.00
time (sec)	N/A	0.848	0.237	1.372	0.000	0.000	0.000	0.000	35.858	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	311	304	709	0	2135	0	0	1343	0
N.S.	1	1.05	1.03	2.40	0.00	7.21	0.00	0.00	4.54	0.00
time (sec)	N/A	0.672	11.011	1.451	0.000	112.282	0.000	0.000	0.594	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	294	507	1235	0	3719	0	973	3154	0
N.S.	1	1.02	1.76	4.29	0.00	12.91	0.00	3.38	10.95	0.00
time (sec)	N/A	0.565	12.040	1.541	0.000	104.095	0.000	0.426	0.595	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	432	685	2195	0	0	0	2470	6174	0
N.S.	1	1.18	1.87	5.98	0.00	0.00	0.00	6.73	16.82	0.00
time (sec)	N/A	0.835	13.440	1.593	0.000	0.000	0.000	4.508	1.797	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	580	605	8161	0	0	0	0	10533	0
N.S.	1	1.32	1.38	18.55	0.00	0.00	0.00	0.00	23.94	0.00
time (sec)	N/A	1.066	14.364	1.690	0.000	0.000	0.000	0.000	17.555	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	321	274	10265	0	4824	0	9517	7687	0
N.S.	1	1.10	0.94	35.03	0.00	16.46	0.00	32.48	26.24	0.00
time (sec)	N/A	0.470	11.705	1.861	0.000	43.121	0.000	1.413	96.498	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	425	357	15588	0	8120	0	15909	23	0
N.S.	1	1.03	0.86	37.65	0.00	19.61	0.00	38.43	0.06	0.00
time (sec)	N/A	0.641	12.474	2.304	0.000	114.285	0.000	13.009	200.041	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	566	705	26119	0	0	0	24842	23	0
N.S.	1	0.93	1.16	43.10	0.00	0.00	0.00	40.99	0.04	0.00
time (sec)	N/A	0.875	16.129	2.663	0.000	0.000	0.000	7.907	200.044	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	0	1390	1177	0	0	0	0	25	0
N.S.	1	0.00	2.13	1.81	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	33.792	5.758	0.000	0.000	0.000	0.000	200.039	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	0	7762	1344	0	0	0	0	25	0
N.S.	1	0.00	10.74	1.86	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	35.376	11.750	0.000	0.000	0.000	0.000	200.033	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	642	5357	1125	0	588	0	0	25	0
N.S.	1	1.07	8.94	1.88	0.00	0.98	0.00	0.00	0.04	0.00
time (sec)	N/A	1.499	35.119	4.637	0.000	0.095	0.000	0.000	200.052	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	528	707	925	0	486	0	0	25	0
N.S.	1	1.06	1.43	1.86	0.00	0.98	0.00	0.00	0.05	0.00
time (sec)	N/A	0.916	32.477	4.244	0.000	0.100	0.000	0.000	200.060	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	447	602	807	0	418	0	0	23	0
N.S.	1	1.06	1.42	1.91	0.00	0.99	0.00	0.00	0.05	0.00
time (sec)	N/A	0.624	27.869	2.306	0.000	0.090	0.000	0.000	200.049	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	188	365	746	0	359	0	0	22	0
N.S.	1	1.06	2.05	4.19	0.00	2.02	0.00	0.00	0.12	0.00
time (sec)	N/A	0.286	21.793	1.370	0.000	0.087	0.000	0.000	200.037	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	575	363	638	0	0	0	0	25	0
N.S.	1	1.42	0.90	1.58	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.108	23.448	2.401	0.000	0.000	0.000	0.000	200.033	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	824	1292	820	0	0	0	0	25	0
N.S.	1	1.32	2.07	1.32	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.477	32.881	4.276	0.000	0.000	0.000	0.000	200.034	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	1462	6774	1205	0	0	0	0	25	0
N.S.	1	2.03	9.42	1.68	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.959	35.077	6.621	0.000	0.000	0.000	0.000	200.042	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	545	707	913	0	485	0	0	25	0
N.S.	1	1.07	1.39	1.79	0.00	0.95	0.00	0.00	0.05	0.00
time (sec)	N/A	0.913	33.222	5.533	0.000	0.088	0.000	0.000	200.045	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	453	934	812	0	415	0	0	25	0
N.S.	1	1.06	2.19	1.91	0.00	0.97	0.00	0.00	0.06	0.00
time (sec)	N/A	0.642	31.027	4.338	0.000	0.078	0.000	0.000	200.024	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	385	804	459	0	356	0	0	23	0
N.S.	1	1.05	2.20	1.26	0.00	0.98	0.00	0.00	0.06	0.00
time (sec)	N/A	0.506	27.169	3.026	0.000	0.073	0.000	0.000	200.027	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	189	308	287	0	128	0	0	22	0
N.S.	1	1.06	1.72	1.60	0.00	0.72	0.00	0.00	0.12	0.00
time (sec)	N/A	0.312	22.689	2.838	0.000	0.070	0.000	0.000	200.029	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	387	482	313	0	0	0	0	25	0
N.S.	1	1.74	2.17	1.41	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.738	24.000	4.217	0.000	0.000	0.000	0.000	200.034	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	829	1381	827	0	0	0	0	25	0
N.S.	1	1.32	2.19	1.31	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.465	33.337	5.971	0.000	0.000	0.000	0.000	200.032	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	722	1467	823	1210	0	0	0	0	25	0
N.S.	1	2.03	1.14	1.68	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.917	35.061	7.855	0.000	0.000	0.000	0.000	200.037	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	266	430	373	0	55	0	0	133	0
N.S.	1	0.78	1.25	1.09	0.00	0.16	0.00	0.00	0.39	0.00
time (sec)	N/A	0.571	34.246	4.677	0.000	0.071	0.000	0.000	0.575	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	234	425	368	0	50	0	0	113	0
N.S.	1	0.74	1.34	1.16	0.00	0.16	0.00	0.00	0.36	0.00
time (sec)	N/A	0.464	33.948	2.667	0.000	0.080	0.000	0.000	0.571	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	203	419	362	0	45	0	0	95	0
N.S.	1	0.69	1.43	1.24	0.00	0.15	0.00	0.00	0.32	0.00
time (sec)	N/A	0.365	23.162	2.926	0.000	0.071	0.000	0.000	0.492	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	87	238	287	0	26	0	0	30	0
N.S.	1	0.35	0.94	1.14	0.00	0.10	0.00	0.00	0.12	0.00
time (sec)	N/A	0.215	30.340	2.843	0.000	0.069	0.000	0.000	0.406	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	313	244	201	0	0	0	0	23	0
N.S.	1	1.09	0.85	0.70	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.644	20.572	2.606	0.000	0.000	0.000	0.000	200.044	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	438	615	373	0	0	0	0	23	0
N.S.	1	1.29	1.81	1.10	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.898	24.449	3.556	0.000	0.000	0.000	0.000	200.045	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	788	620	504	0	0	0	0	23	0
N.S.	1	1.58	1.24	1.01	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.689	24.551	3.451	0.000	0.000	0.000	0.000	200.042	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	309	385	225	0	0	0	0	57	0
N.S.	1	0.47	0.59	0.34	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.744	22.779	3.145	0.000	0.000	0.000	0.000	0.339	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	234	425	368	0	50	0	0	113	0
N.S.	1	0.72	1.31	1.14	0.00	0.15	0.00	0.00	0.35	0.00
time (sec)	N/A	0.482	34.165	2.866	0.000	0.075	0.000	0.000	0.539	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	203	420	362	0	45	0	0	38	0
N.S.	1	0.68	1.40	1.21	0.00	0.15	0.00	0.00	0.13	0.00
time (sec)	N/A	0.413	33.972	2.556	0.000	0.082	0.000	0.000	0.449	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	173	391	216	0	26	0	0	36	0
N.S.	1	0.65	1.46	0.81	0.00	0.10	0.00	0.00	0.13	0.00
time (sec)	N/A	0.337	22.543	2.953	0.000	0.071	0.000	0.000	0.439	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	85	153	150	0	11	0	0	35	0
N.S.	1	0.84	1.51	1.49	0.00	0.11	0.00	0.00	0.35	0.00
time (sec)	N/A	0.235	31.359	2.314	0.000	0.071	0.000	0.000	0.462	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	229	228	158	0	0	0	0	23	0
N.S.	1	0.76	0.76	0.52	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.502	21.649	2.765	0.000	0.000	0.000	0.000	200.042	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	438	615	373	0	0	0	0	23	0
N.S.	1	0.95	1.33	0.81	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.856	24.402	3.085	0.000	0.000	0.000	0.000	200.046	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	788	585	504	0	0	0	0	23	0
N.S.	1	1.59	1.18	1.02	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.826	25.529	3.135	0.000	0.000	0.000	0.000	200.040	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	172	469	426	709	10479	1304	836	553
N.S.	1	1.00	0.85	2.32	2.11	3.51	51.88	6.46	4.14	2.74
time (sec)	N/A	0.713	0.324	0.755	0.047	0.086	2.471	0.238	0.247	10.991

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	374	312	498	6195	921	582	419
N.S.	1	1.00	0.85	2.32	1.94	3.09	38.48	5.72	3.61	2.60
time (sec)	N/A	0.602	0.231	0.698	0.046	0.083	1.516	0.211	0.260	10.818

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	142	281	215	342	3267	604	373	300
N.S.	1	1.00	1.17	2.32	1.78	2.83	27.00	4.99	3.08	2.48
time (sec)	N/A	0.505	0.334	0.743	0.047	0.082	0.977	0.234	0.256	10.757

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	135	132	201	1416	350	206	201
N.S.	1	1.00	1.01	1.65	1.61	2.45	17.27	4.27	2.51	2.45
time (sec)	N/A	0.425	0.145	0.682	0.034	0.086	0.610	0.242	0.259	10.653

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	308	0	271	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.71	0.00	3.27	0.00
time (sec)	N/A	0.396	0.143	0.000	0.000	0.000	3.062	0.000	0.264	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	87	69	0	0	0	250	0	207	0
N.S.	1	1.07	0.85	0.00	0.00	0.00	3.09	0.00	2.56	0.00
time (sec)	N/A	0.372	0.127	0.000	0.000	0.000	2.939	0.000	0.256	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	119	83	0	0	0	1006	0	194	0
N.S.	1	1.13	0.79	0.00	0.00	0.00	9.58	0.00	1.85	0.00
time (sec)	N/A	0.425	0.143	0.000	0.000	0.000	5.067	0.000	0.266	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	123	77	0	0	0	1527	0	572	0
N.S.	1	1.03	0.65	0.00	0.00	0.00	12.83	0.00	4.81	0.00
time (sec)	N/A	0.449	0.196	0.000	0.000	0.000	13.504	0.000	0.282	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	320	0	0	0	0	0	1239	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	4.27	0.00
time (sec)	N/A	1.253	1.474	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	289	0	0	0	0	0	487	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	2.05	0.00
time (sec)	N/A	0.808	0.577	0.000	0.000	0.000	0.000	0.000	0.308	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	198	183	0	0	0	0	0	170	0
N.S.	1	0.91	0.84	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.624	0.380	0.000	0.000	0.000	0.000	0.000	0.281	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0	22	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.664	0.298	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	242	207	0	0	0	0	0	26	0
N.S.	1	0.92	0.78	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.849	0.550	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	296	246	0	0	0	0	0	28	0
N.S.	1	0.96	0.80	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.987	0.640	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	176	122	0	0	0	121	0	0	0
N.S.	1	1.11	0.77	0.00	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.614	0.180	0.000	0.000	0.000	9.392	0.000	0.310	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	29	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.895	0.000	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	650	639	0	0	0	0	0	0	25	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.215	0.000	0.000	0.000	0.000	0.000	0.000	200.042	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	471	469	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.044	0.000	0.000	0.000	0.000	0.000	0.000	200.039	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	0	0	0	0	0	23	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.768	0.000	0.000	0.000	0.000	0.000	0.000	200.034	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	0.052	0.000	0.000	0.000	0.000	0.000	7.697	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	4722	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	205.30	1.09
time (sec)	N/A	0.266	3.098	0.873	0.076	0.092	0.000	0.215	0.547	10.879

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	8766	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	381.13	1.09
time (sec)	N/A	0.271	2.735	0.893	0.076	0.164	0.000	0.203	0.753	10.961

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	35	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.396	0.011	0.214	0.032	0.066	0.030	0.216	0.300	0.060

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	35	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.372	0.010	0.218	0.028	0.064	0.020	0.203	0.270	0.046

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	51	51	54	53	35	51
N.S.	1	1.00	0.91	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.351	0.013	0.211	0.025	0.063	0.022	0.225	0.270	0.049

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	51	51	54	53	35	51
N.S.	1	1.00	0.91	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.354	0.012	0.217	0.026	0.062	0.020	0.203	0.270	0.047

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	48	48	48	49	49	32	47
N.S.	1	1.00	1.21	1.26	1.26	1.26	1.29	1.29	0.84	1.24
time (sec)	N/A	0.326	0.011	0.207	0.026	0.063	0.021	0.219	0.278	0.045

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	46	46	46	46	46	31	45
N.S.	1	1.00	1.08	1.15	1.15	1.15	1.15	1.15	0.78	1.12
time (sec)	N/A	0.312	0.018	0.072	0.034	0.068	0.056	0.198	0.292	0.041

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	44	46	52	42	46	36	46
N.S.	1	1.00	0.98	1.00	1.05	1.18	0.95	1.05	0.82	1.05
time (sec)	N/A	0.330	0.026	0.081	0.034	0.065	0.088	0.190	0.267	0.046

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	43	46	53	46	47	37	46
N.S.	1	1.00	0.98	0.98	1.05	1.20	1.05	1.07	0.84	1.05
time (sec)	N/A	0.324	0.028	0.090	0.032	0.067	0.169	0.231	0.281	0.058

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	46	50	53	54	51	37	48
N.S.	1	1.00	0.98	0.94	1.02	1.08	1.10	1.04	0.76	0.98
time (sec)	N/A	0.333	0.029	0.065	0.026	0.070	0.281	0.204	0.280	0.054

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	48	51	51	56	51	35	49
N.S.	1	1.00	1.07	1.09	1.16	1.16	1.27	1.16	0.80	1.11
time (sec)	N/A	0.291	0.019	0.060	0.026	0.063	0.371	0.233	0.283	0.037

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.336	0.018	0.064	0.029	0.064	0.527	0.212	0.291	0.037

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.333	0.019	0.062	0.030	0.064	0.545	0.227	0.280	0.036

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.330	0.018	0.073	0.035	0.065	0.654	0.203	0.270	0.037

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	98	99	99	109	101	57	91
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.10	1.02	0.58	0.92
time (sec)	N/A	0.505	0.017	0.967	0.028	0.064	0.024	0.203	0.265	0.048

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	97	99	99	105	100	57	91
N.S.	1	1.00	1.00	0.98	1.00	1.00	1.06	1.01	0.58	0.92
time (sec)	N/A	0.461	0.014	0.959	0.027	0.073	0.027	0.188	0.287	0.035

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	96	99	99	104	99	57	90
N.S.	1	1.00	1.01	1.10	1.14	1.14	1.20	1.14	0.66	1.03
time (sec)	N/A	0.438	0.029	0.998	0.028	0.062	0.025	0.199	0.291	0.036

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	88	97	99	99	107	100	57	91
N.S.	1	1.00	1.44	1.59	1.62	1.62	1.75	1.64	0.93	1.49
time (sec)	N/A	0.385	0.026	0.916	0.034	0.067	0.026	0.212	0.289	0.035

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	84	94	96	96	100	97	54	88
N.S.	1	1.00	2.21	2.47	2.53	2.53	2.63	2.55	1.42	2.32
time (sec)	N/A	0.316	0.017	0.852	0.047	0.064	0.027	0.191	0.289	0.036

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	83	92	93	93	95	94	53	84
N.S.	1	0.97	1.26	1.39	1.41	1.41	1.44	1.42	0.80	1.27
time (sec)	N/A	0.352	0.026	0.872	0.032	0.066	0.091	0.208	0.277	0.040

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	93	94	101	94	95	59	86
N.S.	1	1.00	0.99	1.08	1.09	1.17	1.09	1.10	0.69	1.00
time (sec)	N/A	0.420	0.040	0.934	0.034	0.064	0.118	0.192	0.286	0.044

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	90	96	101	97	96	59	91
N.S.	1	1.00	0.96	1.00	1.07	1.12	1.08	1.07	0.66	1.01
time (sec)	N/A	0.415	0.037	0.842	0.027	0.065	0.212	0.201	0.300	10.734

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	85	96	101	99	96	59	94
N.S.	1	1.00	0.97	0.96	1.08	1.13	1.11	1.08	0.66	1.06
time (sec)	N/A	0.437	0.038	0.911	0.032	0.063	0.393	0.213	0.286	10.623

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	83	95	101	99	96	59	93
N.S.	1	1.00	0.99	0.97	1.10	1.17	1.15	1.12	0.69	1.08
time (sec)	N/A	0.413	0.041	0.868	0.036	0.067	0.648	0.165	0.280	0.076

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	87	86	98	101	105	99	59	94
N.S.	1	0.97	1.23	1.21	1.38	1.42	1.48	1.39	0.83	1.32
time (sec)	N/A	0.356	0.055	0.921	0.036	0.066	0.975	0.228	0.278	10.608

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	85	88	99	99	107	99	57	95
N.S.	1	1.00	1.93	2.00	2.25	2.25	2.43	2.25	1.30	2.16
time (sec)	N/A	0.293	0.029	0.860	0.033	0.064	1.225	0.215	0.295	0.052

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	88	99	99	107	99	57	96
N.S.	1	1.00	0.89	0.89	1.00	1.00	1.08	1.00	0.58	0.97
time (sec)	N/A	0.420	0.027	0.852	0.032	0.060	1.565	0.192	0.302	10.586

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	88	99	99	107	99	57	95
N.S.	1	1.00	0.89	0.89	1.00	1.00	1.08	1.00	0.58	0.96
time (sec)	N/A	0.426	0.028	0.900	0.033	0.060	1.938	0.242	0.278	10.661

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	88	99	99	107	99	57	96
N.S.	1	1.00	0.89	0.89	1.00	1.00	1.08	1.00	0.58	0.97
time (sec)	N/A	0.418	0.028	0.858	0.037	0.062	2.394	0.205	0.298	10.646

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	88	99	99	107	99	57	97
N.S.	1	1.00	0.89	0.89	1.00	1.00	1.08	1.00	0.58	0.98
time (sec)	N/A	0.411	0.028	0.907	0.037	0.062	2.790	0.235	0.291	10.568

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	144	147	147	162	149	79	131
N.S.	1	1.00	1.00	1.01	1.03	1.03	1.13	1.04	0.55	0.92
time (sec)	N/A	0.602	0.025	0.998	0.032	0.062	0.029	0.243	0.279	0.066

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	143	143	147	147	162	148	79	130
N.S.	1	1.00	1.03	1.03	1.06	1.06	1.17	1.06	0.57	0.94
time (sec)	N/A	0.569	0.020	0.992	0.026	0.064	0.035	0.176	0.274	10.517

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	143	143	147	147	162	148	79	131
N.S.	1	1.00	1.28	1.28	1.31	1.31	1.45	1.32	0.71	1.17
time (sec)	N/A	0.510	0.021	0.937	0.037	0.063	0.029	0.209	0.274	0.050

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	143	144	147	147	163	149	79	131
N.S.	1	1.00	1.64	1.66	1.69	1.69	1.87	1.71	0.91	1.51
time (sec)	N/A	0.452	0.021	0.931	0.031	0.062	0.031	0.274	0.305	0.050

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	140	144	146	146	160	149	79	130
N.S.	1	1.00	2.30	2.36	2.39	2.39	2.62	2.44	1.30	2.13
time (sec)	N/A	0.378	0.019	1.007	0.026	0.064	0.031	0.283	0.289	0.050

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	122	140	142	142	148	145	76	126
N.S.	1	1.00	3.21	3.68	3.74	3.74	3.89	3.82	2.00	3.32
time (sec)	N/A	0.325	0.038	0.937	0.035	0.064	0.032	0.196	0.282	0.049

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	96	92	128	138	142	142	148	142	75	125
N.S.	1	0.96	1.33	1.44	1.48	1.48	1.54	1.48	0.78	1.30
time (sec)	N/A	0.399	0.037	0.995	0.044	0.066	0.131	0.210	0.273	0.056

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	141	143	149	148	143	81	127
N.S.	1	1.00	0.97	1.06	1.08	1.12	1.11	1.08	0.61	0.95
time (sec)	N/A	0.511	0.065	0.944	0.032	0.066	0.144	0.197	0.257	10.511

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	138	143	149	148	144	81	130
N.S.	1	1.00	0.98	1.05	1.09	1.14	1.13	1.10	0.62	0.99
time (sec)	N/A	0.503	0.069	0.924	0.028	0.065	0.254	0.209	0.257	10.514

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	127	134	145	149	150	145	81	135
N.S.	1	1.00	0.95	1.00	1.08	1.11	1.12	1.08	0.60	1.01
time (sec)	N/A	0.515	0.232	0.873	0.030	0.066	0.434	0.189	0.264	0.057

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	128	130	145	149	150	145	81	138
N.S.	1	1.00	0.96	0.97	1.08	1.11	1.12	1.08	0.60	1.03
time (sec)	N/A	0.507	0.089	0.920	0.041	0.070	0.795	0.228	0.265	10.560

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	125	144	149	150	144	81	140
N.S.	1	1.00	0.98	0.95	1.10	1.14	1.15	1.10	0.62	1.07
time (sec)	N/A	0.511	0.062	0.892	0.033	0.068	1.187	0.188	0.287	0.070

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	125	123	143	149	150	144	81	139
N.S.	1	1.00	0.95	0.93	1.08	1.13	1.14	1.09	0.61	1.05
time (sec)	N/A	0.491	0.072	0.943	0.027	0.067	1.752	0.186	0.291	10.634

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	101	97	132	126	146	149	156	147	81	140
N.S.	1	0.96	1.31	1.25	1.45	1.48	1.54	1.46	0.80	1.39
time (sec)	N/A	0.401	0.064	0.876	0.027	0.066	2.379	0.126	0.292	10.768

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	123	128	147	147	158	147	79	141
N.S.	1	1.00	2.80	2.91	3.34	3.34	3.59	3.34	1.80	3.20
time (sec)	N/A	0.303	0.038	0.903	0.034	0.065	2.972	0.189	0.299	10.786

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	71	126	128	147	147	158	147	79	143
N.S.	1	0.99	1.75	1.78	2.04	2.04	2.19	2.04	1.10	1.99
time (sec)	N/A	0.324	0.037	0.894	0.027	0.077	3.909	0.179	0.281	0.075

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	128	147	147	158	147	79	143
N.S.	1	1.00	0.88	0.90	1.03	1.03	1.10	1.03	0.55	1.00
time (sec)	N/A	0.515	0.038	0.837	0.035	0.064	4.808	0.159	0.292	0.075

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	128	147	147	158	147	79	142
N.S.	1	1.00	0.88	0.90	1.03	1.03	1.10	1.03	0.55	0.99
time (sec)	N/A	0.494	0.040	0.929	0.039	0.065	6.008	0.150	0.302	10.861

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	128	147	147	158	147	79	142
N.S.	1	1.00	0.88	0.90	1.03	1.03	1.10	1.03	0.55	0.99
time (sec)	N/A	0.496	0.038	0.852	0.028	0.067	7.887	0.140	0.306	0.076

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	128	147	147	158	147	79	143
N.S.	1	1.00	0.88	0.90	1.03	1.03	1.10	1.03	0.55	1.00
time (sec)	N/A	0.506	0.039	0.932	0.035	0.064	10.150	0.152	0.302	10.843

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	149	122	129	129	133	132	131	121
N.S.	1	1.00	1.14	0.93	0.98	0.98	1.02	1.01	1.00	0.92
time (sec)	N/A	0.572	0.031	0.815	0.028	0.061	0.042	0.124	0.291	10.960

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	150	124	128	128	134	133	131	123
N.S.	1	1.00	1.26	1.04	1.08	1.08	1.13	1.12	1.10	1.03
time (sec)	N/A	0.499	0.021	0.862	0.032	0.065	0.040	0.188	0.284	0.095

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	151	124	129	129	136	133	131	123
N.S.	1	1.00	1.53	1.25	1.30	1.30	1.37	1.34	1.32	1.24
time (sec)	N/A	0.480	0.021	0.886	0.026	0.079	0.036	0.199	0.263	0.095

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	153	124	129	129	139	133	131	123
N.S.	1	1.00	1.76	1.43	1.48	1.48	1.60	1.53	1.51	1.41
time (sec)	N/A	0.458	0.020	0.807	0.029	0.063	0.035	0.139	0.284	0.098

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	153	124	129	129	136	133	131	123
N.S.	1	1.00	2.22	1.80	1.87	1.87	1.97	1.93	1.90	1.78
time (sec)	N/A	0.396	0.020	0.873	0.029	0.063	0.037	0.162	0.297	0.095

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	148	124	128	128	133	133	131	123
N.S.	1	1.00	2.69	2.25	2.33	2.33	2.42	2.42	2.38	2.24
time (sec)	N/A	0.372	0.022	0.803	0.028	0.063	0.036	0.142	0.316	0.094

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	147	124	129	129	133	133	131	123
N.S.	1	1.00	3.77	3.18	3.31	3.31	3.41	3.41	3.36	3.15
time (sec)	N/A	0.343	0.018	0.885	0.031	0.063	0.034	0.180	0.281	0.095

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	113	119	126	126	119	129	129	118
N.S.	1	1.00	4.52	4.76	5.04	5.04	4.76	5.16	5.16	4.72
time (sec)	N/A	0.285	0.024	0.807	0.038	0.065	0.042	0.217	0.282	0.096

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	78	85	116	124	124	117	126	125	115
N.S.	1	0.90	0.98	1.33	1.43	1.43	1.34	1.45	1.44	1.32
time (sec)	N/A	0.333	0.038	0.903	0.030	0.075	0.123	0.186	0.278	0.103

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	123	125	131	121	126	135	118
N.S.	1	1.00	1.00	0.88	0.90	0.94	0.87	0.91	0.97	0.85
time (sec)	N/A	0.506	0.044	0.786	0.031	0.065	0.135	0.192	0.282	0.105

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	123	125	131	122	126	135	119
N.S.	1	1.00	1.01	0.89	0.91	0.95	0.88	0.91	0.98	0.86
time (sec)	N/A	0.514	0.047	0.852	0.029	0.064	0.210	0.166	0.272	10.984

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	123	127	131	124	128	135	121
N.S.	1	1.00	1.01	0.89	0.92	0.95	0.90	0.93	0.98	0.88
time (sec)	N/A	0.514	0.039	0.868	0.036	0.066	0.329	0.204	0.271	0.085

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	139	123	126	131	122	128	135	121
N.S.	1	1.00	1.01	0.90	0.92	0.96	0.89	0.93	0.99	0.88
time (sec)	N/A	0.502	0.046	0.856	0.027	0.066	0.525	0.156	0.303	0.066

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	123	127	131	124	128	135	121
N.S.	1	1.00	1.01	0.88	0.91	0.94	0.89	0.91	0.96	0.86
time (sec)	N/A	0.522	0.041	0.795	0.035	0.065	0.856	0.212	0.274	10.841

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	123	127	131	128	128	135	121
N.S.	1	1.00	1.01	0.88	0.91	0.94	0.91	0.91	0.96	0.86
time (sec)	N/A	0.514	0.041	0.784	0.035	0.064	1.234	0.180	0.271	10.853

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	122	127	131	128	128	135	121
N.S.	1	1.00	1.01	0.88	0.92	0.95	0.93	0.93	0.98	0.88
time (sec)	N/A	0.529	0.043	0.807	0.026	0.066	1.760	0.184	0.278	0.062

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	121	127	131	126	128	135	121
N.S.	1	1.00	1.01	0.88	0.92	0.95	0.91	0.93	0.98	0.88
time (sec)	N/A	0.515	0.044	0.896	0.026	0.064	2.525	0.375	0.285	10.827

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	139	119	126	131	126	127	135	119
N.S.	1	1.00	1.01	0.87	0.92	0.96	0.92	0.93	0.99	0.87
time (sec)	N/A	0.519	0.049	0.790	0.026	0.069	3.331	0.180	0.279	0.072

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	119	125	131	124	126	135	118
N.S.	1	1.00	1.01	0.86	0.91	0.95	0.90	0.91	0.98	0.86
time (sec)	N/A	0.496	0.051	0.799	0.032	0.068	4.483	0.190	0.280	10.873

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	83	143	122	128	131	129	129	133	118
N.S.	1	0.90	1.55	1.33	1.39	1.42	1.40	1.40	1.45	1.28
time (sec)	N/A	0.378	0.055	0.839	0.032	0.067	5.566	0.144	0.275	10.841

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	114	123	129	129	131	131	131	120
N.S.	1	1.00	3.68	3.97	4.16	4.16	4.23	4.23	4.23	3.87
time (sec)	N/A	0.268	0.031	0.773	0.027	0.063	6.909	0.173	0.289	0.131

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	115	123	129	129	131	131	131	123
N.S.	1	0.96	2.21	2.37	2.48	2.48	2.52	2.52	2.52	2.37
time (sec)	N/A	0.292	0.029	0.862	0.033	0.064	7.934	0.159	0.284	10.700

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	67	149	123	129	129	131	131	131	123
N.S.	1	0.94	2.10	1.73	1.82	1.82	1.85	1.85	1.85	1.73
time (sec)	N/A	0.300	0.043	0.770	0.033	0.073	9.486	0.190	0.277	10.632

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	84	153	123	129	129	131	131	131	123
N.S.	1	0.93	1.70	1.37	1.43	1.43	1.46	1.46	1.46	1.37
time (sec)	N/A	0.319	0.042	0.825	0.027	0.069	10.519	0.231	0.281	0.130

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	101	153	123	129	129	131	131	131	123
N.S.	1	0.93	1.40	1.13	1.18	1.18	1.20	1.20	1.20	1.13
time (sec)	N/A	0.338	0.042	0.764	0.028	0.067	12.493	0.166	0.287	10.607

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	118	151	123	129	129	131	131	131	123
N.S.	1	0.92	1.18	0.96	1.01	1.01	1.02	1.02	1.02	0.96
time (sec)	N/A	0.363	0.049	0.761	0.026	0.062	14.073	0.160	0.264	0.124

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	123	129	129	131	131	131	123
N.S.	1	1.00	1.00	0.81	0.85	0.85	0.87	0.87	0.87	0.81
time (sec)	N/A	0.522	0.054	0.774	0.028	0.062	15.620	0.171	0.278	0.128

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	123	129	129	131	131	131	121
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.88	0.88	0.88	0.81
time (sec)	N/A	0.511	0.047	0.813	0.036	0.061	17.479	0.192	0.273	0.124

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	123	129	129	131	131	131	121
N.S.	1	1.00	1.00	0.81	0.85	0.85	0.87	0.87	0.87	0.80
time (sec)	N/A	0.515	0.050	0.751	0.026	0.064	19.227	0.182	0.283	0.132

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	61	61	61	73	61	60	61
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.88	0.73	0.72	0.73
time (sec)	N/A	0.378	0.003	0.931	0.026	0.062	0.020	0.162	0.299	0.077

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	61	61	61	75	61	60	61
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.90	0.73	0.72	0.73
time (sec)	N/A	0.378	0.003	0.823	0.032	0.070	0.024	0.195	0.283	0.076

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	81	60	59	59	73	59	60	59
N.S.	1	1.00	0.89	0.66	0.65	0.65	0.80	0.65	0.66	0.65
time (sec)	N/A	0.401	0.003	0.924	0.027	0.060	0.026	0.182	0.275	0.075

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	61	61	61	73	61	60	61
N.S.	1	1.00	1.01	0.76	0.76	0.76	0.91	0.76	0.75	0.76
time (sec)	N/A	0.383	0.003	0.816	0.024	0.059	0.021	0.178	0.275	0.075

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	79	61	61	61	71	61	60	61
N.S.	1	1.00	1.08	0.84	0.84	0.84	0.97	0.84	0.82	0.84
time (sec)	N/A	0.369	0.003	0.938	0.035	0.062	0.021	0.156	0.269	0.076

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	61	61	61	73	61	60	61
N.S.	1	1.00	1.27	0.95	0.95	0.95	1.14	0.95	0.94	0.95
time (sec)	N/A	0.352	0.003	0.817	0.029	0.064	0.028	0.184	0.282	0.075

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	81	61	61	61	73	61	60	61
N.S.	1	1.00	1.47	1.11	1.11	1.11	1.33	1.11	1.09	1.11
time (sec)	N/A	0.352	0.003	0.883	0.030	0.059	0.024	0.148	0.270	0.076

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	83	61	61	61	75	61	60	61
N.S.	1	1.00	1.80	1.33	1.33	1.33	1.63	1.33	1.30	1.33
time (sec)	N/A	0.325	0.003	0.817	0.031	0.069	0.021	0.174	0.270	0.075

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	83	61	61	61	75	61	60	61
N.S.	1	1.00	2.24	1.65	1.65	1.65	2.03	1.65	1.62	1.65
time (sec)	N/A	0.319	0.003	0.876	0.025	0.061	0.022	0.180	0.282	0.075

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	79	61	61	61	71	61	60	61
N.S.	1	1.00	2.82	2.18	2.18	2.18	2.54	2.18	2.14	2.18
time (sec)	N/A	0.312	0.003	0.802	0.034	0.058	0.025	0.179	0.284	0.075

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	77	61	61	61	70	61	60	61
N.S.	1	1.00	4.05	3.21	3.21	3.21	3.68	3.21	3.16	3.21
time (sec)	N/A	0.279	0.003	0.786	0.030	0.062	0.022	0.161	0.269	0.074

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	12	55	65	62	56	55
N.S.	1	1.00	1.00	1.44	1.33	6.11	7.22	6.89	6.22	6.11
time (sec)	N/A	0.227	0.002	0.832	0.036	0.060	0.027	0.183	0.281	0.072

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	96	57	56	56	68	57	56	56
N.S.	1	1.00	1.33	0.79	0.78	0.78	0.94	0.79	0.78	0.78
time (sec)	N/A	0.323	0.013	0.755	0.031	0.061	0.041	0.191	0.268	0.075

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	59	58	62	66	59	62	58
N.S.	1	1.00	1.00	0.82	0.81	0.86	0.92	0.82	0.86	0.81
time (sec)	N/A	0.348	0.005	0.815	0.025	0.060	0.043	0.145	0.289	0.075

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	66	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.94	0.84	0.89	0.83
time (sec)	N/A	0.345	0.004	0.780	0.030	0.063	0.058	0.196	0.295	0.064

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	65	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.93	0.84	0.89	0.83
time (sec)	N/A	0.344	0.005	0.835	0.035	0.067	0.053	0.125	0.276	0.056

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	63	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.90	0.84	0.89	0.83
time (sec)	N/A	0.347	0.004	0.764	0.031	0.067	0.054	0.127	0.272	0.038

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	58	58	62	63	59	62	58
N.S.	1	1.00	1.00	0.81	0.81	0.86	0.88	0.82	0.86	0.81
time (sec)	N/A	0.346	0.005	0.787	0.031	0.071	0.057	0.145	0.271	0.033

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	58	58	62	65	59	62	58
N.S.	1	1.00	1.00	0.81	0.81	0.86	0.90	0.82	0.86	0.81
time (sec)	N/A	0.352	0.004	0.812	0.033	0.060	0.061	0.157	0.283	0.029

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	63	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.90	0.84	0.89	0.83
time (sec)	N/A	0.367	0.004	0.760	0.024	0.063	0.058	0.142	0.268	0.027

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	61	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.87	0.84	0.89	0.83
time (sec)	N/A	0.358	0.004	0.772	0.032	0.062	0.076	0.178	0.291	0.026

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	58	62	60	59	62	58
N.S.	1	1.00	1.00	0.83	0.83	0.89	0.86	0.84	0.89	0.83
time (sec)	N/A	0.354	0.004	0.803	0.032	0.062	0.072	0.144	0.275	0.033

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	56	56	62	58	57	62	62
N.S.	1	1.00	1.00	0.80	0.80	0.89	0.83	0.81	0.89	0.89
time (sec)	N/A	0.353	0.005	0.851	0.031	0.063	0.070	0.232	0.271	10.556

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	58	62	60	59	62	58
N.S.	1	1.00	1.00	0.78	0.78	0.84	0.81	0.80	0.84	0.78
time (sec)	N/A	0.355	0.004	0.799	0.026	0.064	0.074	0.191	0.266	0.036

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	75	60	60	60	61	60	60	56
N.S.	1	1.00	6.25	5.00	5.00	5.00	5.08	5.00	5.00	4.67
time (sec)	N/A	0.245	0.004	0.745	0.026	0.060	0.073	0.180	0.271	0.031

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	77	60	60	60	61	60	60	60
N.S.	1	1.00	3.08	2.40	2.40	2.40	2.44	2.40	2.40	2.40
time (sec)	N/A	0.265	0.004	0.746	0.029	0.061	0.104	0.200	0.320	10.549

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	79	60	60	60	61	60	60	60
N.S.	1	1.14	2.14	1.62	1.62	1.62	1.65	1.62	1.62	1.62
time (sec)	N/A	0.264	0.004	0.815	0.039	0.061	0.115	0.212	0.280	0.033

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	83	60	60	60	61	60	60	60
N.S.	1	1.20	1.69	1.22	1.22	1.22	1.24	1.22	1.22	1.22
time (sec)	N/A	0.283	0.004	0.749	0.026	0.061	0.089	0.178	0.289	10.523

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	76	83	60	60	60	61	60	60	60
N.S.	1	1.25	1.36	0.98	0.98	0.98	1.00	0.98	0.98	0.98
time (sec)	N/A	0.296	0.004	0.808	0.025	0.065	0.084	0.220	0.271	0.033

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	93	81	60	60	60	61	60	60	60
N.S.	1	1.27	1.11	0.82	0.82	0.82	0.84	0.82	0.82	0.82
time (sec)	N/A	0.312	0.004	0.829	0.036	0.062	0.089	0.169	0.283	10.665

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	110	81	60	60	60	61	60	60	60
N.S.	1	1.29	0.95	0.71	0.71	0.71	0.72	0.71	0.71	0.71
time (sec)	N/A	0.321	0.004	0.762	0.029	0.060	0.085	0.189	0.289	10.637

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	127	79	60	60	60	61	60	60	60
N.S.	1	1.31	0.81	0.62	0.62	0.62	0.63	0.62	0.62	0.62
time (sec)	N/A	0.343	0.004	0.739	0.031	0.065	0.096	0.186	0.278	0.034

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	60	60	60	61	60	60	60
N.S.	1	1.00	1.00	0.74	0.74	0.74	0.75	0.74	0.74	0.74
time (sec)	N/A	0.345	0.004	0.856	0.031	0.066	0.101	0.198	0.265	10.695

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	60	60	60	61	60	60	58
N.S.	1	1.00	1.00	0.72	0.72	0.72	0.73	0.72	0.72	0.70
time (sec)	N/A	0.349	0.007	0.762	0.025	0.066	0.097	0.189	0.290	0.037

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	127	136	149	188	143	152	63	279
N.S.	1	1.00	0.95	1.01	1.11	1.40	1.07	1.13	0.47	2.08
time (sec)	N/A	0.615	0.071	1.096	0.031	0.072	0.288	0.210	0.279	0.075

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	116	123	164	119	126	52	173
N.S.	1	1.00	0.95	1.03	1.09	1.45	1.05	1.12	0.46	1.53
time (sec)	N/A	0.542	0.065	1.025	0.026	0.072	0.250	0.208	0.285	10.843

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	90	101	140	92	104	41	115
N.S.	1	1.00	0.97	1.00	1.12	1.56	1.02	1.16	0.46	1.28
time (sec)	N/A	0.472	0.057	1.129	0.027	0.079	0.236	0.197	0.297	0.055

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	67	74	113	68	75	29	77
N.S.	1	1.00	0.96	0.97	1.07	1.64	0.99	1.09	0.42	1.12
time (sec)	N/A	0.418	0.065	1.025	0.026	0.071	0.197	0.168	0.282	0.063

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	46	53	72	44	51	17	54
N.S.	1	1.00	0.91	1.02	1.18	1.60	0.98	1.13	0.38	1.20
time (sec)	N/A	0.355	0.027	1.111	0.034	0.067	0.147	0.240	0.280	11.142

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	34	37	27	32	10	32
N.S.	1	1.00	0.97	1.03	1.06	1.16	0.84	1.00	0.31	1.00
time (sec)	N/A	0.322	0.013	1.004	0.037	0.071	0.085	0.204	0.280	0.050

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	42	44	62	32	48	15	39
N.S.	1	1.00	0.90	1.00	1.05	1.48	0.76	1.14	0.36	0.93
time (sec)	N/A	0.354	0.029	1.089	0.033	0.074	0.140	0.230	0.301	10.972

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	64	67	107	128	71	26	58
N.S.	1	1.00	0.86	0.98	1.03	1.65	1.97	1.09	0.40	0.89
time (sec)	N/A	0.414	0.042	1.095	0.033	0.079	0.253	0.200	0.268	11.018

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	84	99	150	184	106	43	104
N.S.	1	1.00	1.00	0.99	1.16	1.76	2.16	1.25	0.51	1.22
time (sec)	N/A	0.468	0.080	1.064	0.031	0.074	0.299	0.189	0.264	10.975

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	109	128	179	219	133	54	131
N.S.	1	1.00	0.94	0.96	1.13	1.58	1.94	1.18	0.48	1.16
time (sec)	N/A	0.519	0.104	1.237	0.038	0.074	0.381	0.193	0.272	0.114

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	127	152	203	243	157	65	150
N.S.	1	1.00	0.97	0.95	1.14	1.53	1.83	1.18	0.49	1.13
time (sec)	N/A	0.566	0.097	1.066	0.032	0.073	0.369	0.205	0.270	10.972

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	147	140	168	257	168	149	109	180
N.S.	1	1.00	1.03	0.98	1.17	1.80	1.17	1.04	0.76	1.26
time (sec)	N/A	0.636	0.057	1.093	0.037	0.075	0.766	0.216	0.287	11.011

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	129	115	143	231	144	124	97	141
N.S.	1	1.00	1.07	0.95	1.18	1.91	1.19	1.02	0.80	1.17
time (sec)	N/A	0.564	0.048	1.096	0.040	0.078	0.732	0.194	0.275	0.087

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	89	120	193	119	96	85	118
N.S.	1	1.00	1.00	0.92	1.24	1.99	1.23	0.99	0.88	1.22
time (sec)	N/A	0.488	0.039	1.052	0.037	0.070	0.613	0.231	0.273	10.663

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	75	73	71	100	128	100	76	71	96
N.S.	1	1.04	1.01	0.99	1.39	1.78	1.39	1.06	0.99	1.33
time (sec)	N/A	0.396	0.028	1.026	0.037	0.075	0.342	0.203	0.274	0.082

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	47	71	71	75	45	26	68
N.S.	1	1.00	0.71	0.80	1.20	1.20	1.27	0.76	0.44	1.15
time (sec)	N/A	0.380	0.018	1.060	0.029	0.065	0.263	0.230	0.304	10.393

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	33	50	50	53	25	23	52
N.S.	1	1.00	0.71	0.87	1.32	1.32	1.39	0.66	0.61	1.37
time (sec)	N/A	0.332	0.013	1.103	0.037	0.066	0.195	0.212	0.294	0.031

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	69	91	156	90	71	97	84
N.S.	1	1.00	0.90	0.96	1.26	2.17	1.25	0.99	1.35	1.17
time (sec)	N/A	0.405	0.050	1.027	0.029	0.072	0.276	0.235	0.297	10.457

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	102	106	134	267	204	122	119	118
N.S.	1	1.00	0.92	0.95	1.21	2.41	1.84	1.10	1.07	1.06
time (sec)	N/A	0.516	0.069	1.052	0.039	0.077	0.391	0.200	0.274	10.503

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	130	172	318	264	157	138	168
N.S.	1	1.00	0.91	0.96	1.27	2.36	1.96	1.16	1.02	1.24
time (sec)	N/A	0.574	0.138	1.064	0.038	0.077	0.515	0.211	0.264	0.137

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	148	158	193	333	291	175	149	195
N.S.	1	1.00	0.89	0.95	1.16	2.01	1.75	1.05	0.90	1.17
time (sec)	N/A	0.686	0.150	1.132	0.033	0.076	0.501	0.190	0.305	10.563

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	151	163	213	349	216	172	172	210
N.S.	1	1.00	0.88	0.95	1.25	2.04	1.26	1.01	1.01	1.23
time (sec)	N/A	0.732	0.106	1.055	0.038	0.081	1.659	0.234	0.285	10.586

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	129	190	311	190	144	159	187
N.S.	1	1.00	0.89	0.88	1.30	2.13	1.30	0.99	1.09	1.28
time (sec)	N/A	0.651	0.091	1.102	0.036	0.078	1.415	0.203	0.295	0.128

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	113	111	170	222	172	124	147	161
N.S.	1	1.01	1.07	1.05	1.60	2.09	1.62	1.17	1.39	1.52
time (sec)	N/A	0.475	0.049	1.033	0.037	0.076	1.055	0.202	0.267	10.727

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	85	139	139	150	93	48	128
N.S.	1	1.00	1.33	1.49	2.44	2.44	2.63	1.63	0.84	2.25
time (sec)	N/A	0.313	0.038	1.062	0.039	0.068	0.775	0.190	0.265	0.052

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	78	119	119	126	70	64	113
N.S.	1	1.00	0.72	0.90	1.37	1.37	1.45	0.80	0.74	1.30
time (sec)	N/A	0.447	0.028	1.033	0.033	0.068	0.527	0.200	0.287	0.046

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	56	95	95	100	46	53	93
N.S.	1	1.00	0.75	0.92	1.56	1.56	1.64	0.75	0.87	1.52
time (sec)	N/A	0.384	0.021	1.085	0.033	0.066	0.360	0.179	0.267	10.735

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	72	72	76	25	45	74
N.S.	1	1.00	0.71	0.92	1.89	1.89	2.00	0.66	1.18	1.95
time (sec)	N/A	0.330	0.013	1.118	0.033	0.080	0.279	0.218	0.276	10.613

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	95	137	250	141	95	197	130
N.S.	1	1.00	0.87	0.93	1.34	2.45	1.38	0.93	1.93	1.27
time (sec)	N/A	0.497	0.073	1.049	0.039	0.076	0.410	0.181	0.288	0.100

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	142	148	202	427	275	168	223	180
N.S.	1	1.00	0.90	0.94	1.29	2.72	1.75	1.07	1.42	1.15
time (sec)	N/A	0.660	0.103	1.144	0.041	0.079	0.623	0.202	0.287	10.543

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	162	172	242	484	335	205	242	230
N.S.	1	1.00	0.92	0.97	1.37	2.73	1.89	1.16	1.37	1.30
time (sec)	N/A	0.744	0.193	1.214	0.046	0.084	0.607	0.175	0.284	10.558

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	49	44	361	27	418	78	24	431
N.S.	1	0.54	0.43	0.39	3.17	0.24	3.67	0.68	0.21	3.78
time (sec)	N/A	0.394	1.026	0.592	0.039	0.063	2.569	0.199	0.278	11.272

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	49	44	301	27	357	78	24	340
N.S.	1	0.54	0.43	0.39	2.64	0.24	3.13	0.68	0.21	2.98
time (sec)	N/A	0.366	1.023	0.588	0.035	0.066	1.902	0.233	0.283	10.730

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	49	44	241	27	292	78	24	271
N.S.	1	0.54	0.43	0.39	2.11	0.24	2.56	0.68	0.21	2.38
time (sec)	N/A	0.365	1.025	0.569	0.034	0.069	1.721	0.264	0.278	10.649

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	95	44	183	27	228	77	24	176
N.S.	1	0.54	0.83	0.39	1.61	0.24	2.00	0.68	0.21	1.54
time (sec)	N/A	0.350	0.851	0.545	0.034	0.063	1.420	0.253	0.280	10.594

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	91	33	125	24	156	74	21	77
N.S.	1	1.00	1.32	0.48	1.81	0.35	2.26	1.07	0.30	1.12
time (sec)	N/A	0.323	0.902	0.484	0.034	0.067	0.935	0.237	0.283	10.687

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	105	52	44	54	133	22	0	46	20	122
N.S.	1	0.50	0.42	0.51	1.27	0.21	0.00	0.44	0.19	1.16
time (sec)	N/A	0.341	1.024	0.512	0.037	0.068	0.000	0.242	0.291	10.937

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	103	50	44	44	175	26	0	47	24	207
N.S.	1	0.49	0.43	0.43	1.70	0.25	0.00	0.46	0.23	2.01
time (sec)	N/A	0.348	1.024	0.513	0.029	0.065	0.000	0.246	0.284	11.086

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	108	55	225	38	172	29	0	50	26	134
N.S.	1	0.51	2.08	0.35	1.59	0.27	0.00	0.46	0.24	1.24
time (sec)	N/A	0.342	0.547	0.506	0.036	0.066	0.000	0.239	0.293	11.022

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	34	195	27	0	77	24	43
N.S.	1	1.00	0.61	0.45	2.60	0.36	0.00	1.03	0.32	0.57
time (sec)	N/A	0.357	0.478	0.594	0.038	0.069	0.000	0.259	0.295	10.813

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	47	34	255	27	0	77	24	43
N.S.	1	0.54	0.41	0.30	2.24	0.24	0.00	0.68	0.21	0.38
time (sec)	N/A	0.352	0.402	0.631	0.037	0.067	0.000	0.188	0.273	10.726

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	49	34	315	27	0	77	24	43
N.S.	1	0.54	0.43	0.30	2.76	0.24	0.00	0.68	0.21	0.38
time (sec)	N/A	0.355	0.503	0.698	0.035	0.069	0.000	0.284	0.259	10.772

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	61	49	34	375	27	0	77	24	43
N.S.	1	0.54	0.43	0.30	3.29	0.24	0.00	0.68	0.21	0.38
time (sec)	N/A	0.355	0.529	0.787	0.037	0.072	0.000	0.223	0.271	10.902

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	92	421	73	8803	150	46	0
N.S.	1	0.49	0.41	0.44	2.00	0.35	41.92	0.71	0.22	0.00
time (sec)	N/A	0.472	1.041	1.122	0.039	0.066	1.015	0.232	0.279	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	92	361	73	5448	150	46	0
N.S.	1	0.49	0.41	0.44	1.72	0.35	25.94	0.71	0.22	0.00
time (sec)	N/A	0.452	1.043	1.076	0.039	0.065	0.937	0.221	0.282	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	92	301	73	3364	149	46	0
N.S.	1	0.49	0.41	0.44	1.43	0.35	16.02	0.71	0.22	0.00
time (sec)	N/A	0.438	1.043	1.007	0.037	0.064	0.872	0.260	0.279	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	92	241	73	2071	150	46	0
N.S.	1	0.49	0.41	0.44	1.15	0.35	9.86	0.71	0.22	0.00
time (sec)	N/A	0.447	1.038	1.096	0.035	0.067	0.831	0.265	0.262	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	89	87	92	183	73	1258	148	46	0
N.S.	1	0.74	0.72	0.76	1.51	0.60	10.40	1.22	0.38	0.00
time (sec)	N/A	0.419	1.039	0.977	0.037	0.068	1.113	0.209	0.278	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	83	90	125	69	745	144	43	42
N.S.	1	1.00	1.17	1.27	1.76	0.97	10.49	2.03	0.61	0.59
time (sec)	N/A	0.327	1.042	0.975	0.035	0.067	0.769	0.251	0.297	10.852

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	81	83	91	186	68	0	118	42	0
N.S.	1	0.45	0.46	0.50	1.02	0.37	0.00	0.65	0.23	0.00
time (sec)	N/A	0.375	1.047	1.026	0.036	0.067	0.000	0.231	0.268	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	93	89	96	283	75	0	119	47	0
N.S.	1	0.46	0.44	0.48	1.42	0.38	0.00	0.60	0.24	0.00
time (sec)	N/A	0.427	1.048	1.036	0.034	0.068	0.000	0.204	0.274	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	93	85	95	351	74	0	117	47	0
N.S.	1	0.46	0.42	0.48	1.76	0.37	0.00	0.58	0.24	0.00
time (sec)	N/A	0.427	1.051	1.008	0.037	0.069	0.000	0.243	0.275	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	92	88	96	443	75	0	118	48	0
N.S.	1	0.46	0.44	0.48	2.23	0.38	0.00	0.59	0.24	0.00
time (sec)	N/A	0.425	1.047	1.112	0.038	0.073	0.000	0.278	0.279	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	86	278	94	379	75	0	121	48	0
N.S.	1	0.46	1.49	0.50	2.03	0.40	0.00	0.65	0.26	0.00
time (sec)	N/A	0.396	1.047	1.097	0.036	0.071	0.000	0.241	0.283	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	43	90	315	73	0	149	46	196
N.S.	1	1.03	0.57	1.20	4.20	0.97	0.00	1.99	0.61	2.61
time (sec)	N/A	0.366	0.863	1.217	0.045	0.074	0.000	0.263	0.281	10.771

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	103	87	90	375	73	0	149	46	195
N.S.	1	0.90	0.76	0.78	3.26	0.63	0.00	1.30	0.40	1.70
time (sec)	N/A	0.429	0.886	1.190	0.038	0.067	0.000	0.268	0.291	10.855

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	103	87	90	435	73	0	149	46	196
N.S.	1	0.66	0.55	0.57	2.77	0.46	0.00	0.95	0.29	1.25
time (sec)	N/A	0.433	1.006	1.276	0.042	0.066	0.000	0.238	0.275	10.844

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	90	495	73	0	149	46	196
N.S.	1	0.49	0.41	0.43	2.36	0.35	0.00	0.71	0.22	0.93
time (sec)	N/A	0.429	1.012	1.448	0.041	0.070	0.000	0.251	0.299	10.760

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	90	555	73	0	149	46	196
N.S.	1	0.49	0.41	0.43	2.64	0.35	0.00	0.71	0.22	0.93
time (sec)	N/A	0.435	1.037	1.563	0.048	0.067	0.000	0.202	0.283	10.806

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	90	615	73	0	149	46	196
N.S.	1	0.49	0.41	0.43	2.93	0.35	0.00	0.71	0.22	0.93
time (sec)	N/A	0.422	1.041	1.614	0.044	0.067	0.000	0.270	0.261	10.800

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	103	87	90	675	73	0	149	46	196
N.S.	1	0.49	0.41	0.43	3.21	0.35	0.00	0.71	0.22	0.93
time (sec)	N/A	0.426	1.034	1.869	0.047	0.067	0.000	0.277	0.280	10.849

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	142	125	140	481	118	40368	220	68	0
N.S.	1	0.47	0.41	0.46	1.59	0.39	133.23	0.73	0.22	0.00
time (sec)	N/A	0.573	1.054	1.184	0.046	0.065	1.252	0.186	0.283	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	145	125	140	421	119	24941	220	68	0
N.S.	1	0.47	0.41	0.46	1.38	0.39	81.51	0.72	0.22	0.00
time (sec)	N/A	0.540	1.053	1.277	0.046	0.066	1.273	0.241	0.305	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	145	125	140	361	119	15400	222	68	0
N.S.	1	0.47	0.41	0.46	1.18	0.39	50.33	0.73	0.22	0.00
time (sec)	N/A	0.527	1.053	1.086	0.040	0.066	1.144	0.275	0.253	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	140	125	140	301	119	9493	221	68	0
N.S.	1	0.66	0.59	0.66	1.42	0.56	44.78	1.04	0.32	0.00
time (sec)	N/A	0.529	1.052	1.117	0.035	0.068	0.985	0.249	0.267	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	115	125	140	241	118	5836	220	68	0
N.S.	1	0.69	0.75	0.84	1.44	0.71	34.95	1.32	0.41	0.00
time (sec)	N/A	0.494	1.055	1.050	0.033	0.069	0.905	0.254	0.263	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	89	125	140	183	119	3563	221	68	0
N.S.	1	0.74	1.03	1.16	1.51	0.98	29.45	1.83	0.56	0.00
time (sec)	N/A	0.424	1.055	1.056	0.039	0.067	1.121	0.235	0.278	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	121	138	125	115	2147	217	65	0
N.S.	1	1.00	1.70	1.94	1.76	1.62	30.24	3.06	0.92	0.00
time (sec)	N/A	0.327	1.052	1.010	0.038	0.069	0.857	0.277	0.273	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	105	122	139	236	114	0	190	64	0
N.S.	1	0.40	0.47	0.53	0.90	0.44	0.00	0.73	0.24	0.00
time (sec)	N/A	0.453	1.069	0.993	0.039	0.072	0.000	0.205	0.284	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	133	128	144	386	121	0	191	70	0
N.S.	1	0.45	0.44	0.49	1.31	0.41	0.00	0.65	0.24	0.00
time (sec)	N/A	0.516	1.055	1.031	0.042	0.072	0.000	0.231	0.265	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	136	126	144	461	121	0	191	69	0
N.S.	1	0.46	0.42	0.48	1.55	0.41	0.00	0.64	0.23	0.00
time (sec)	N/A	0.502	1.073	1.017	0.053	0.071	0.000	0.260	0.262	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	136	127	144	557	121	0	190	69	0
N.S.	1	0.46	0.43	0.48	1.88	0.41	0.00	0.64	0.23	0.00
time (sec)	N/A	0.519	1.067	1.118	0.044	0.076	0.000	0.264	0.258	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	135	126	144	615	121	0	188	70	0
N.S.	1	0.46	0.43	0.49	2.08	0.41	0.00	0.64	0.24	0.00
time (sec)	N/A	0.506	1.061	1.164	0.047	0.069	0.000	0.233	0.265	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	132	127	144	673	121	0	188	70	0
N.S.	1	0.45	0.43	0.49	2.30	0.41	0.00	0.64	0.24	0.00
time (sec)	N/A	0.508	1.065	1.279	0.058	0.066	0.000	0.254	0.289	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	110	354	142	554	121	0	191	70	0
N.S.	1	0.41	1.33	0.53	2.07	0.45	0.00	0.72	0.26	0.00
time (sec)	N/A	0.437	1.340	1.228	0.052	0.067	0.000	0.226	0.285	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	122	136	435	119	0	221	68	284
N.S.	1	1.03	1.63	1.81	5.80	1.59	0.00	2.95	0.91	3.79
time (sec)	N/A	0.377	1.050	1.374	0.043	0.068	0.000	0.220	0.288	10.801

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	98	125	136	495	119	0	221	68	284
N.S.	1	0.85	1.09	1.18	4.30	1.03	0.00	1.92	0.59	2.47
time (sec)	N/A	0.371	1.049	1.381	0.045	0.067	0.000	0.229	0.267	10.794

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	143	125	136	555	119	0	221	68	284
N.S.	1	0.91	0.80	0.87	3.54	0.76	0.00	1.41	0.43	1.81
time (sec)	N/A	0.501	1.049	1.628	0.052	0.070	0.000	0.226	0.311	10.942

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	145	125	136	615	119	0	221	68	283
N.S.	1	0.73	0.63	0.68	3.09	0.60	0.00	1.11	0.34	1.42
time (sec)	N/A	0.507	1.056	1.831	0.056	0.069	0.000	0.243	0.275	11.059

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	145	125	136	675	119	0	221	68	284
N.S.	1	0.60	0.52	0.56	2.80	0.49	0.00	0.92	0.28	1.18
time (sec)	N/A	0.510	1.064	2.049	0.058	0.066	0.000	0.251	0.273	10.916

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	145	125	136	735	119	0	221	68	284
N.S.	1	0.47	0.41	0.44	2.40	0.39	0.00	0.72	0.22	0.93
time (sec)	N/A	0.501	1.056	2.115	0.058	0.066	0.000	0.227	0.280	11.005

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	143	125	136	795	119	0	221	68	284
N.S.	1	0.47	0.41	0.45	2.62	0.39	0.00	0.73	0.22	0.93
time (sec)	N/A	0.506	1.052	2.488	0.060	0.071	0.000	0.248	0.266	10.724

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	134	116	138	272	117	726	185	5	0
N.S.	1	0.52	0.45	0.53	1.05	0.45	2.81	0.72	0.02	0.00
time (sec)	N/A	0.542	1.072	0.938	0.035	0.068	1.592	0.213	0.277	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	113	96	114	212	94	488	148	5	0
N.S.	1	0.53	0.45	0.54	1.00	0.44	2.30	0.70	0.02	0.00
time (sec)	N/A	0.494	1.055	0.898	0.034	0.070	1.489	0.253	0.274	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	92	77	90	125	71	330	113	5	0
N.S.	1	0.55	0.46	0.54	0.75	0.43	1.99	0.68	0.03	0.00
time (sec)	N/A	0.440	1.054	0.797	0.032	0.071	1.209	0.243	0.266	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	71	57	66	72	47	219	75	5	0
N.S.	1	0.59	0.48	0.55	0.60	0.39	1.82	0.62	0.04	0.00
time (sec)	N/A	0.396	1.031	0.780	0.032	0.068	1.048	0.215	0.276	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	177	43	52	25	134	45	1	79
N.S.	1	1.00	2.57	0.62	0.75	0.36	1.94	0.65	0.01	1.14
time (sec)	N/A	0.330	0.704	0.829	0.028	0.070	0.820	0.243	0.283	10.997

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	56	111	49	53	28	0	49	2	68
N.S.	1	0.70	1.39	0.61	0.66	0.35	0.00	0.61	0.02	0.85
time (sec)	N/A	0.362	0.510	0.766	0.030	0.074	0.000	0.250	0.292	10.965

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	147	61	106	41	0	81	5	117
N.S.	1	1.10	1.88	0.78	1.36	0.53	0.00	1.04	0.06	1.50
time (sec)	N/A	0.385	0.524	0.792	0.033	0.072	0.000	0.234	0.296	11.097

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	88	180	92	164	69	0	117	5	0
N.S.	1	0.54	1.11	0.57	1.01	0.43	0.00	0.72	0.03	0.00
time (sec)	N/A	0.431	0.579	0.855	0.031	0.069	0.000	0.206	0.273	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	112	230	119	224	94	0	153	5	0
N.S.	1	0.53	1.09	0.56	1.06	0.45	0.00	0.73	0.02	0.00
time (sec)	N/A	0.475	0.716	0.869	0.036	0.081	0.000	0.234	0.280	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	132	263	143	284	117	0	188	5	0
N.S.	1	0.52	1.03	0.56	1.11	0.46	0.00	0.73	0.02	0.00
time (sec)	N/A	0.520	0.852	1.000	0.033	0.086	0.000	0.259	0.256	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	142	140	155	303	197	0	149	71	0
N.S.	1	0.57	0.56	0.62	1.22	0.79	0.00	0.60	0.29	0.00
time (sec)	N/A	0.585	1.080	1.197	0.036	0.074	0.000	0.265	0.251	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	120	117	129	242	171	0	134	60	0
N.S.	1	0.59	0.58	0.64	1.20	0.85	0.00	0.66	0.30	0.00
time (sec)	N/A	0.519	1.065	1.197	0.033	0.077	0.000	0.266	0.283	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	97	233	110	154	134	0	96	46	0
N.S.	1	0.63	1.51	0.71	1.00	0.87	0.00	0.62	0.30	0.00
time (sec)	N/A	0.464	1.449	1.145	0.028	0.074	0.000	0.249	0.291	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	81	187	75	89	81	0	70	33	0
N.S.	1	0.72	1.65	0.66	0.79	0.72	0.00	0.62	0.29	0.00
time (sec)	N/A	0.416	1.253	1.098	0.033	0.072	0.000	0.271	0.282	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	176	32	56	38	0	32	12	42
N.S.	1	1.00	2.55	0.46	0.81	0.55	0.00	0.46	0.17	0.61
time (sec)	N/A	0.325	0.861	1.144	0.034	0.076	0.000	0.223	0.273	10.684

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	83	80	97	96	109	0	83	44	0
N.S.	1	0.59	0.57	0.69	0.69	0.78	0.00	0.59	0.31	0.00
time (sec)	N/A	0.417	1.044	1.070	0.026	0.075	0.000	0.249	0.267	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	114	110	132	191	187	0	123	70	0
N.S.	1	0.58	0.56	0.67	0.97	0.95	0.00	0.63	0.36	0.00
time (sec)	N/A	0.509	1.085	1.348	0.032	0.078	0.000	0.269	0.262	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	136	133	153	250	225	0	148	86	0
N.S.	1	0.56	0.55	0.63	1.03	0.93	0.00	0.61	0.35	0.00
time (sec)	N/A	0.562	1.094	3.126	0.033	0.077	0.000	0.256	0.276	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	148	127	150	211	252	0	144	120	0
N.S.	1	0.60	0.52	0.61	0.86	1.03	0.00	0.59	0.49	0.00
time (sec)	N/A	0.603	1.077	1.311	0.059	0.074	0.000	0.222	0.274	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	118	103	115	201	175	0	116	108	0
N.S.	1	0.63	0.55	0.61	1.07	0.93	0.00	0.62	0.57	0.00
time (sec)	N/A	0.461	1.064	1.237	0.043	0.074	0.000	0.218	0.321	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	73	75	156	105	0	77	37	201
N.S.	1	1.03	0.97	1.00	2.08	1.40	0.00	1.03	0.49	2.68
time (sec)	N/A	0.385	1.042	1.365	0.036	0.076	0.000	0.230	0.269	10.821

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	87	319	52	90	80	0	52	42	62
N.S.	1	0.72	2.64	0.43	0.74	0.66	0.00	0.43	0.35	0.51
time (sec)	N/A	0.421	1.307	1.220	0.032	0.068	0.000	0.208	0.281	10.799

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	39	33	56	61	0	33	34	43
N.S.	1	1.00	0.55	0.46	0.79	0.86	0.00	0.46	0.48	0.61
time (sec)	N/A	0.324	1.025	1.106	0.026	0.066	0.000	0.243	0.270	10.780

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	113	104	121	138	203	0	107	145	0
N.S.	1	0.54	0.50	0.58	0.66	0.97	0.00	0.51	0.69	0.00
time (sec)	N/A	0.500	1.065	1.171	0.041	0.073	0.000	0.255	0.258	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	160	148	172	276	347	0	169	171	0
N.S.	1	0.57	0.52	0.61	0.98	1.23	0.00	0.60	0.61	0.00
time (sec)	N/A	0.617	1.114	1.167	0.033	0.078	0.000	0.245	0.261	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.342	0.055	0.481	0.033	0.068	0.495	0.243	0.251	0.064

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.345	0.052	0.486	0.027	0.070	0.316	0.253	0.287	0.049

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.336	0.049	0.488	0.034	0.070	0.195	0.223	0.279	0.049

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	54	66	53	35	51
N.S.	1	1.00	0.83	0.83	0.81	0.86	1.05	0.84	0.56	0.81
time (sec)	N/A	0.337	0.056	0.463	0.029	0.068	0.667	0.258	0.263	0.053

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	51	51	51	51	78	53	34	51
N.S.	1	1.00	0.84	0.84	0.84	0.84	1.28	0.87	0.56	0.84
time (sec)	N/A	0.337	0.050	0.316	0.027	0.067	0.142	0.263	0.265	0.049

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	52	51	51	75	53	35	51
N.S.	1	1.00	0.83	0.88	0.86	0.86	1.27	0.90	0.59	0.86
time (sec)	N/A	0.328	0.060	0.113	0.032	0.069	0.245	0.271	0.264	0.053

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	51	51	50	73	51	38	51
N.S.	1	1.00	0.80	0.86	0.86	0.85	1.24	0.86	0.64	0.86
time (sec)	N/A	0.340	0.066	0.125	0.034	0.075	0.218	0.282	0.265	10.538

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	48	52	51	75	52	39	52
N.S.	1	1.00	0.80	0.81	0.88	0.86	1.27	0.88	0.66	0.88
time (sec)	N/A	0.333	0.063	0.124	0.030	0.071	0.270	0.238	0.264	0.059

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	48	51	51	80	51	39	51
N.S.	1	1.00	0.82	0.79	0.84	0.84	1.31	0.84	0.64	0.84
time (sec)	N/A	0.339	0.060	0.121	0.032	0.073	0.394	0.261	0.271	10.633

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	100	99	104	148	101	59	91
N.S.	1	1.00	0.81	0.90	0.89	0.94	1.33	0.91	0.53	0.82
time (sec)	N/A	0.433	0.087	0.920	0.039	0.070	0.712	0.221	0.286	10.756

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	100	99	104	148	101	59	91
N.S.	1	1.00	0.81	0.90	0.89	0.94	1.33	0.91	0.53	0.82
time (sec)	N/A	0.439	0.077	0.895	0.026	0.070	0.489	0.235	0.266	0.040

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	100	99	104	148	101	59	91
N.S.	1	1.00	0.81	0.90	0.89	0.94	1.33	0.91	0.53	0.82
time (sec)	N/A	0.425	0.077	0.961	0.036	0.069	0.319	0.255	0.284	0.039

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	100	99	102	124	101	57	91
N.S.	1	1.00	0.81	0.90	0.89	0.92	1.12	0.91	0.51	0.82
time (sec)	N/A	0.405	0.076	0.882	0.026	0.073	0.985	0.252	0.279	0.040

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	89	99	99	99	146	101	56	91
N.S.	1	1.00	0.82	0.91	0.91	0.91	1.34	0.93	0.51	0.83
time (sec)	N/A	0.409	0.076	0.898	0.033	0.067	0.211	0.241	0.271	0.039

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	87	100	99	99	141	101	58	91
N.S.	1	1.00	0.81	0.93	0.93	0.93	1.32	0.94	0.54	0.85
time (sec)	N/A	0.413	0.089	0.922	0.040	0.071	0.292	0.243	0.262	0.042

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	99	99	99	139	99	61	92
N.S.	1	1.00	0.80	0.93	0.93	0.93	1.30	0.93	0.57	0.86
time (sec)	N/A	0.408	0.110	0.878	0.058	0.074	0.324	0.229	0.282	0.043

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	85	95	100	99	141	100	61	95
N.S.	1	1.00	0.79	0.89	0.93	0.93	1.32	0.93	0.57	0.89
time (sec)	N/A	0.408	0.083	0.928	0.051	0.070	0.390	0.261	0.290	0.069

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	85	91	100	99	139	100	61	98
N.S.	1	1.00	0.79	0.85	0.93	0.93	1.30	0.93	0.57	0.92
time (sec)	N/A	0.426	0.105	0.878	0.043	0.069	0.452	0.237	0.268	0.074

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	152	148	147	152	214	149	81	131
N.S.	1	1.00	0.96	0.93	0.92	0.96	1.35	0.94	0.51	0.82
time (sec)	N/A	0.514	0.122	0.981	0.033	0.074	1.044	0.235	0.281	0.070

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	128	148	147	152	214	149	81	131
N.S.	1	1.00	0.81	0.93	0.92	0.96	1.35	0.94	0.51	0.82
time (sec)	N/A	0.501	0.140	0.962	0.035	0.069	0.793	0.241	0.255	10.786

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	152	148	147	152	214	149	81	131
N.S.	1	1.00	0.96	0.93	0.92	0.96	1.35	0.94	0.51	0.82
time (sec)	N/A	0.520	0.118	0.987	0.028	0.082	0.494	0.256	0.252	0.052

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	128	148	147	150	182	149	79	131
N.S.	1	1.00	0.81	0.93	0.92	0.94	1.14	0.94	0.50	0.82
time (sec)	N/A	0.504	0.121	0.875	0.034	0.066	1.033	0.244	0.269	0.052

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	127	147	147	147	211	149	78	131
N.S.	1	1.00	0.81	0.94	0.94	0.94	1.34	0.95	0.50	0.83
time (sec)	N/A	0.485	0.103	0.968	0.052	0.067	0.332	0.252	0.279	0.052

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	125	148	147	147	204	149	80	131
N.S.	1	1.00	0.83	0.98	0.97	0.97	1.35	0.99	0.53	0.87
time (sec)	N/A	0.485	0.118	0.894	0.036	0.076	0.408	0.263	0.309	0.056

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	124	147	147	147	204	147	83	132
N.S.	1	1.00	0.81	0.96	0.96	0.96	1.33	0.96	0.54	0.86
time (sec)	N/A	0.493	0.122	1.000	0.084	0.072	0.570	0.215	0.270	0.055

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	124	143	148	147	204	148	83	135
N.S.	1	1.00	0.80	0.92	0.95	0.95	1.32	0.95	0.54	0.87
time (sec)	N/A	0.513	0.116	0.878	0.042	0.071	0.550	0.257	0.281	0.054

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	123	139	148	147	202	148	83	138
N.S.	1	1.00	0.81	0.92	0.98	0.97	1.34	0.98	0.55	0.91
time (sec)	N/A	0.496	0.108	0.902	0.090	0.069	0.687	0.222	0.276	0.056

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	135	148	147	204	148	83	141
N.S.	1	1.00	0.79	0.87	0.95	0.95	1.32	0.95	0.54	0.91
time (sec)	N/A	0.489	0.137	0.876	0.051	0.070	0.799	0.218	0.277	10.681

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	144	128	124	139	341	986	146	68	209
N.S.	1	1.02	0.91	0.88	0.99	2.42	6.99	1.04	0.48	1.48
time (sec)	N/A	0.431	0.260	1.072	0.120	0.087	105.121	0.282	0.275	0.075

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	110	99	115	290	877	122	55	146
N.S.	1	1.07	0.94	0.85	0.98	2.48	7.50	1.04	0.47	1.25
time (sec)	N/A	0.407	0.222	1.119	0.162	0.086	28.786	0.261	0.277	0.065

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	106	88	77	88	231	762	95	39	107
N.S.	1	1.12	0.93	0.81	0.93	2.43	8.02	1.00	0.41	1.13
time (sec)	N/A	0.374	0.182	1.069	0.186	0.082	6.067	0.265	0.282	0.085

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	86	67	62	65	198	634	65	29	62
N.S.	1	1.18	0.92	0.85	0.89	2.71	8.68	0.89	0.40	0.85
time (sec)	N/A	0.354	0.129	1.165	0.176	0.081	2.126	0.240	0.271	0.099

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	57	58	177	615	60	25	51
N.S.	1	1.00	1.02	0.90	0.92	2.81	9.76	0.95	0.40	0.81
time (sec)	N/A	0.324	0.130	1.030	0.152	0.079	2.643	0.259	0.275	10.795

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	87	67	64	65	215	794	60	33	65
N.S.	1	1.16	0.89	0.85	0.87	2.87	10.59	0.80	0.44	0.87
time (sec)	N/A	0.354	0.125	1.198	0.188	0.084	7.032	0.254	0.259	10.734

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	106	90	77	93	257	882	85	46	81
N.S.	1	1.12	0.95	0.81	0.98	2.71	9.28	0.89	0.48	0.85
time (sec)	N/A	0.379	0.180	1.171	0.160	0.088	27.553	0.260	0.261	10.794

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	112	101	118	316	1017	110	61	103
N.S.	1	1.03	0.93	0.83	0.98	2.61	8.40	0.91	0.50	0.85
time (sec)	N/A	0.396	0.229	1.105	0.165	0.086	113.336	0.265	0.268	10.791

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	131	121	143	367	0	136	72	121
N.S.	1	1.01	0.92	0.85	1.00	2.57	0.00	0.95	0.50	0.85
time (sec)	N/A	0.419	0.254	1.221	0.163	0.082	0.000	0.234	0.263	10.867

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	160	128	118	161	467	0	143	136	176
N.S.	1	1.03	0.82	0.76	1.03	2.99	0.00	0.92	0.87	1.13
time (sec)	N/A	0.420	0.364	1.309	0.154	0.094	0.000	0.272	0.257	10.750

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	141	110	104	136	437	2271	111	120	131
N.S.	1	1.05	0.82	0.78	1.01	3.26	16.95	0.83	0.90	0.98
time (sec)	N/A	0.427	0.311	1.270	0.131	0.086	87.834	0.214	0.274	10.780

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	122	105	96	130	411	2261	107	113	112
N.S.	1	0.98	0.85	0.77	1.05	3.31	18.23	0.86	0.91	0.90
time (sec)	N/A	0.392	0.292	1.107	0.180	0.088	44.966	0.258	0.273	11.396

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	120	105	92	128	399	2302	106	110	107
N.S.	1	0.96	0.84	0.74	1.02	3.19	18.42	0.85	0.88	0.86
time (sec)	N/A	0.384	0.285	1.053	0.175	0.103	22.530	0.255	0.277	11.354

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	121	105	97	129	409	2300	107	113	112
N.S.	1	0.93	0.81	0.75	0.99	3.15	17.69	0.82	0.87	0.86
time (sec)	N/A	0.388	0.256	1.050	0.138	0.093	32.980	0.272	0.267	10.861

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	141	112	105	132	445	2660	110	124	147
N.S.	1	1.04	0.83	0.78	0.98	3.30	19.70	0.81	0.92	1.09
time (sec)	N/A	0.416	0.295	1.152	0.160	0.095	75.858	0.274	0.300	10.708

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	160	130	119	158	477	0	136	144	145
N.S.	1	1.01	0.82	0.75	0.99	3.00	0.00	0.86	0.91	0.91
time (sec)	N/A	0.417	0.357	1.070	0.191	0.089	0.000	0.255	0.281	10.682

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	215	169	165	231	703	0	191	240	246
N.S.	1	1.00	0.79	0.77	1.08	3.29	0.00	0.89	1.12	1.15
time (sec)	N/A	0.468	0.535	2.152	0.153	0.105	0.000	0.249	0.291	0.180

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	195	150	150	206	673	0	159	224	200
N.S.	1	1.03	0.79	0.79	1.08	3.54	0.00	0.84	1.18	1.05
time (sec)	N/A	0.468	0.520	2.141	0.136	0.103	0.000	0.276	0.271	10.686

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	176	143	135	198	639	0	155	217	173
N.S.	1	0.97	0.79	0.74	1.09	3.51	0.00	0.85	1.19	0.95
time (sec)	N/A	0.432	0.466	2.032	0.163	0.104	0.000	0.251	0.284	10.702

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	176	145	137	205	657	0	156	217	175
N.S.	1	0.95	0.78	0.74	1.11	3.55	0.00	0.84	1.17	0.95
time (sec)	N/A	0.445	0.750	2.029	0.170	0.105	0.000	0.282	0.279	10.749

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	174	144	124	194	619	0	154	217	161
N.S.	1	0.96	0.79	0.68	1.07	3.40	0.00	0.85	1.19	0.88
time (sec)	N/A	0.443	0.451	1.099	0.154	0.106	0.000	0.221	0.273	10.810

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	176	144	137	205	657	4933	156	217	174
N.S.	1	0.92	0.75	0.72	1.07	3.44	25.83	0.82	1.14	0.91
time (sec)	N/A	0.446	0.380	1.047	0.142	0.098	131.444	0.258	0.291	10.983

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	175	144	135	198	637	0	155	217	172
N.S.	1	0.92	0.76	0.71	1.04	3.35	0.00	0.82	1.14	0.91
time (sec)	N/A	0.452	0.336	1.039	0.166	0.102	0.000	0.231	0.262	10.969

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	195	152	151	200	673	0	158	228	209
N.S.	1	1.02	0.80	0.79	1.05	3.52	0.00	0.83	1.19	1.09
time (sec)	N/A	0.450	0.510	1.178	0.228	0.104	0.000	0.225	0.266	11.094

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	172	165	233	729	0	180	248	207
N.S.	1	0.99	0.79	0.76	1.07	3.36	0.00	0.83	1.14	0.95
time (sec)	N/A	0.499	0.547	1.095	0.148	0.119	0.000	0.217	0.267	11.042

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	49	34	35	32	0	53	26	0
N.S.	1	0.56	0.41	0.28	0.29	0.27	0.00	0.44	0.22	0.00
time (sec)	N/A	0.357	0.047	0.651	0.044	0.074	0.000	0.284	0.278	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	49	34	35	32	0	53	26	0
N.S.	1	0.56	0.41	0.28	0.29	0.27	0.00	0.44	0.22	0.00
time (sec)	N/A	0.345	0.039	0.645	0.050	0.074	0.000	0.263	0.285	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	51	34	35	32	0	53	26	0
N.S.	1	0.56	0.42	0.28	0.29	0.27	0.00	0.44	0.22	0.00
time (sec)	N/A	0.350	0.038	0.649	0.048	0.074	0.000	0.258	0.267	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	51	34	35	30	0	53	24	0
N.S.	1	0.56	0.42	0.28	0.29	0.25	0.00	0.44	0.20	0.00
time (sec)	N/A	0.348	0.038	0.648	0.046	0.070	0.000	0.247	0.259	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	65	49	34	34	27	0	53	23	56
N.S.	1	0.55	0.42	0.29	0.29	0.23	0.00	0.45	0.19	0.47
time (sec)	N/A	0.345	0.043	0.651	0.060	0.075	0.000	0.212	0.255	11.041

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	63	49	34	33	26	0	53	24	53
N.S.	1	0.54	0.42	0.29	0.28	0.22	0.00	0.46	0.21	0.46
time (sec)	N/A	0.343	0.041	0.473	0.066	0.071	0.000	0.235	0.277	10.914

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	63	46	33	33	27	0	51	28	54
N.S.	1	0.54	0.40	0.28	0.28	0.23	0.00	0.44	0.24	0.47
time (sec)	N/A	0.347	0.048	0.470	0.079	0.070	0.000	0.228	0.267	10.799

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	65	49	34	34	27	0	51	28	54
N.S.	1	0.55	0.42	0.29	0.29	0.23	0.00	0.43	0.24	0.46
time (sec)	N/A	0.354	0.044	0.516	0.077	0.069	0.000	0.261	0.275	10.797

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	49	34	35	27	0	51	28	54
N.S.	1	0.56	0.41	0.28	0.29	0.22	0.00	0.42	0.23	0.45
time (sec)	N/A	0.354	0.046	0.484	0.061	0.068	0.000	0.278	0.269	10.762

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	113	89	92	137	78	0	125	48	0
N.S.	1	0.51	0.40	0.42	0.62	0.35	0.00	0.57	0.22	0.00
time (sec)	N/A	0.429	0.088	0.961	0.042	0.067	0.000	0.265	0.271	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	113	89	92	137	78	0	125	48	0
N.S.	1	0.51	0.40	0.42	0.62	0.35	0.00	0.57	0.22	0.00
time (sec)	N/A	0.434	0.078	0.904	0.040	0.071	0.000	0.259	0.269	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	113	89	92	137	78	0	125	48	0
N.S.	1	0.51	0.40	0.42	0.62	0.35	0.00	0.57	0.22	0.00
time (sec)	N/A	0.434	0.074	0.944	0.041	0.071	0.000	0.259	0.273	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	113	89	92	137	76	0	125	46	0
N.S.	1	0.51	0.40	0.42	0.62	0.35	0.00	0.57	0.21	0.00
time (sec)	N/A	0.415	0.076	0.983	0.039	0.070	0.000	0.236	0.268	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	111	88	92	136	73	0	125	45	0
N.S.	1	0.51	0.40	0.42	0.62	0.33	0.00	0.57	0.21	0.00
time (sec)	N/A	0.422	0.084	0.922	0.041	0.071	0.000	0.208	0.258	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	107	85	92	133	73	0	125	47	107
N.S.	1	0.50	0.40	0.43	0.62	0.34	0.00	0.58	0.22	0.50
time (sec)	N/A	0.433	0.097	0.963	0.048	0.072	0.000	0.234	0.270	11.115

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	109	84	92	130	73	0	123	50	0
N.S.	1	0.50	0.39	0.43	0.60	0.34	0.00	0.57	0.23	0.00
time (sec)	N/A	0.415	0.102	0.986	0.039	0.076	0.000	0.246	0.273	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	109	83	92	131	73	0	124	50	0
N.S.	1	0.50	0.38	0.43	0.61	0.34	0.00	0.57	0.23	0.00
time (sec)	N/A	0.417	0.083	0.911	0.043	0.080	0.000	0.251	0.266	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	107	84	92	134	73	0	124	50	0
N.S.	1	0.50	0.39	0.43	0.63	0.34	0.00	0.58	0.23	0.00
time (sec)	N/A	0.415	0.083	0.996	0.040	0.073	0.000	0.258	0.252	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	159	127	140	241	124	0	197	70	0
N.S.	1	0.50	0.40	0.44	0.75	0.39	0.00	0.62	0.22	0.00
time (sec)	N/A	0.510	0.116	0.915	0.043	0.076	0.000	0.234	0.268	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	159	127	140	241	124	0	197	70	0
N.S.	1	0.50	0.40	0.44	0.75	0.39	0.00	0.62	0.22	0.00
time (sec)	N/A	0.493	0.107	0.961	0.038	0.077	0.000	0.224	0.264	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	159	127	140	241	124	0	197	70	0
N.S.	1	0.50	0.40	0.44	0.75	0.39	0.00	0.62	0.22	0.00
time (sec)	N/A	0.504	0.106	0.921	0.045	0.070	0.000	0.246	0.241	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	159	127	140	241	122	0	197	68	0
N.S.	1	0.50	0.40	0.44	0.75	0.38	0.00	0.62	0.21	0.00
time (sec)	N/A	0.511	0.112	1.045	0.040	0.069	0.000	0.243	0.250	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	155	126	140	240	119	0	197	67	0
N.S.	1	0.49	0.40	0.44	0.76	0.38	0.00	0.62	0.21	0.00
time (sec)	N/A	0.491	0.111	0.921	0.040	0.071	0.000	0.263	0.248	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	153	124	140	238	119	0	197	69	140
N.S.	1	0.49	0.39	0.45	0.76	0.38	0.00	0.63	0.22	0.45
time (sec)	N/A	0.497	0.123	1.033	0.039	0.076	0.000	0.249	0.249	11.193

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	153	122	140	236	119	0	195	72	0
N.S.	1	0.49	0.39	0.45	0.75	0.38	0.00	0.62	0.23	0.00
time (sec)	N/A	0.505	0.098	0.911	0.038	0.072	0.000	0.214	0.277	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	155	122	140	234	119	0	196	72	0
N.S.	1	0.49	0.39	0.44	0.74	0.38	0.00	0.62	0.23	0.00
time (sec)	N/A	0.497	0.118	0.949	0.048	0.070	0.000	0.270	0.279	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	155	122	140	234	119	0	196	72	0
N.S.	1	0.49	0.39	0.44	0.74	0.38	0.00	0.62	0.23	0.00
time (sec)	N/A	0.488	0.115	0.937	0.040	0.073	0.000	0.257	0.284	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	149	139	156	257	276	0	205	7	0
N.S.	1	0.52	0.49	0.55	0.90	0.97	0.00	0.72	0.02	0.00
time (sec)	N/A	0.420	0.225	0.759	0.158	0.093	0.000	0.264	0.294	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	130	120	132	203	229	0	169	7	0
N.S.	1	0.55	0.50	0.55	0.85	0.96	0.00	0.71	0.03	0.00
time (sec)	N/A	0.396	0.178	0.747	0.159	0.088	0.000	0.208	0.270	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	111	100	108	147	180	0	133	7	0
N.S.	1	0.58	0.53	0.57	0.77	0.95	0.00	0.70	0.04	0.00
time (sec)	N/A	0.370	0.154	0.829	0.157	0.082	0.000	0.268	0.276	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	92	82	85	122	129	0	94	5	0
N.S.	1	0.64	0.57	0.59	0.85	0.90	0.00	0.65	0.03	0.00
time (sec)	N/A	0.346	0.099	0.725	0.168	0.082	0.000	0.226	0.271	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	75	72	65	140	102	0	57	4	0
N.S.	1	0.76	0.73	0.66	1.41	1.03	0.00	0.58	0.04	0.00
time (sec)	N/A	0.346	0.093	0.783	0.159	0.083	0.000	0.268	0.262	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	75	81	71	180	112	0	57	6	0
N.S.	1	0.76	0.82	0.72	1.82	1.13	0.00	0.58	0.06	0.00
time (sec)	N/A	0.342	0.079	0.783	0.158	0.077	0.000	0.262	0.270	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	93	88	86	244	141	0	85	9	0
N.S.	1	0.65	0.61	0.60	1.69	0.98	0.00	0.59	0.06	0.00
time (sec)	N/A	0.349	0.131	0.734	0.162	0.082	0.000	0.272	0.260	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	112	107	111	305	192	0	122	9	0
N.S.	1	0.59	0.56	0.58	1.61	1.01	0.00	0.64	0.05	0.00
time (sec)	N/A	0.373	0.159	0.739	0.160	0.083	0.000	0.275	0.248	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	131	127	135	357	241	0	158	9	0
N.S.	1	0.55	0.53	0.57	1.50	1.01	0.00	0.66	0.04	0.00
time (sec)	N/A	0.402	0.186	0.796	0.174	0.103	0.000	0.248	0.267	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	179	148	152	273	408	0	170	99	0
N.S.	1	0.63	0.52	0.53	0.96	1.43	0.00	0.60	0.35	0.00
time (sec)	N/A	0.460	0.309	1.367	0.169	0.090	0.000	0.259	0.255	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	160	127	130	252	349	0	143	84	0
N.S.	1	0.67	0.53	0.55	1.06	1.47	0.00	0.60	0.35	0.00
time (sec)	N/A	0.436	0.268	1.112	0.160	0.090	0.000	0.222	0.261	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	140	108	115	237	319	0	111	67	0
N.S.	1	0.74	0.57	0.61	1.25	1.69	0.00	0.59	0.35	0.00
time (sec)	N/A	0.414	0.234	1.184	0.177	0.090	0.000	0.256	0.259	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	121	103	194	218	291	0	98	60	0
N.S.	1	0.78	0.66	1.25	1.41	1.88	0.00	0.63	0.39	0.00
time (sec)	N/A	0.400	0.230	1.080	0.159	0.081	0.000	0.241	0.256	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	120	104	194	234	291	0	98	59	0
N.S.	1	0.76	0.66	1.23	1.48	1.84	0.00	0.62	0.37	0.00
time (sec)	N/A	0.391	0.207	1.302	0.166	0.092	0.000	0.252	0.249	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	140	110	117	280	331	0	110	71	0
N.S.	1	0.74	0.58	0.62	1.47	1.74	0.00	0.58	0.37	0.00
time (sec)	N/A	0.419	0.239	1.148	0.181	0.082	0.000	0.268	0.258	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	159	130	130	341	375	0	132	92	0
N.S.	1	0.67	0.55	0.55	1.43	1.58	0.00	0.55	0.39	0.00
time (sec)	N/A	0.449	0.279	1.216	0.188	0.085	0.000	0.232	0.258	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	178	157	156	404	434	0	159	107	0
N.S.	1	0.62	0.54	0.54	1.40	1.50	0.00	0.55	0.37	0.00
time (sec)	N/A	0.463	0.303	1.224	0.183	0.087	0.000	0.279	0.268	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	233	188	196	401	644	0	218	203	0
N.S.	1	0.61	0.49	0.51	1.05	1.69	0.00	0.57	0.53	0.00
time (sec)	N/A	0.522	0.456	1.139	0.187	0.100	0.000	0.268	0.267	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	214	167	174	381	585	0	191	188	0
N.S.	1	0.64	0.50	0.52	1.14	1.75	0.00	0.57	0.56	0.00
time (sec)	N/A	0.501	0.418	1.174	0.216	0.096	0.000	0.258	0.261	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	194	146	159	374	555	0	159	172	0
N.S.	1	0.68	0.51	0.56	1.31	1.95	0.00	0.56	0.60	0.00
time (sec)	N/A	0.468	0.393	1.145	0.177	0.099	0.000	0.259	0.259	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	175	146	357	376	525	0	147	165	0
N.S.	1	0.69	0.58	1.42	1.49	2.08	0.00	0.58	0.65	0.00
time (sec)	N/A	0.462	0.374	1.240	0.182	0.093	0.000	0.237	0.260	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	175	148	357	372	537	0	148	165	0
N.S.	1	0.68	0.58	1.39	1.45	2.10	0.00	0.58	0.64	0.00
time (sec)	N/A	0.455	0.375	1.089	0.199	0.093	0.000	0.187	0.264	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	175	146	357	368	537	0	148	165	0
N.S.	1	0.68	0.57	1.38	1.43	2.08	0.00	0.57	0.64	0.00
time (sec)	N/A	0.445	0.350	1.168	0.192	0.093	0.000	0.270	0.255	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	174	145	357	390	523	0	147	165	0
N.S.	1	0.67	0.56	1.38	1.51	2.03	0.00	0.57	0.64	0.00
time (sec)	N/A	0.441	0.432	1.065	0.189	0.090	0.000	0.226	0.253	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	194	149	161	427	559	0	158	176	0
N.S.	1	0.68	0.52	0.56	1.49	1.95	0.00	0.55	0.62	0.00
time (sec)	N/A	0.482	0.413	1.171	0.195	0.095	0.000	0.257	0.253	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	213	170	174	495	611	0	180	196	0
N.S.	1	0.63	0.51	0.52	1.47	1.82	0.00	0.54	0.58	0.00
time (sec)	N/A	0.498	0.502	1.185	0.207	0.100	0.000	0.235	0.257	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	232	191	200	550	670	0	207	211	0
N.S.	1	0.59	0.49	0.51	1.41	1.71	0.00	0.53	0.54	0.00
time (sec)	N/A	0.539	0.500	1.146	0.210	0.107	0.000	0.278	0.248	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	137	218	300	1193	7961	2032	783	745
N.S.	1	1.00	0.63	1.00	1.37	5.45	36.35	9.28	3.58	3.40
time (sec)	N/A	0.721	0.529	1.126	0.062	0.094	0.872	0.281	0.256	11.538

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	105	154	208	609	3538	1046	421	405
N.S.	1	1.00	0.68	0.99	1.34	3.93	22.83	6.75	2.72	2.61
time (sec)	N/A	0.572	0.220	1.004	0.051	0.083	0.538	0.275	0.244	11.131

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	90	116	217	1073	380	171	179
N.S.	1	1.00	0.80	0.99	1.27	2.38	11.79	4.18	1.88	1.97
time (sec)	N/A	0.425	0.139	0.093	0.041	0.075	0.326	0.256	0.241	11.131

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	83	64	0	0	0	0	0	35	0
N.S.	1	1.06	0.82	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.375	0.095	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	72	0	0	0	0	0	555	0
N.S.	1	1.12	0.89	0.00	0.00	0.00	0.00	0.00	6.85	0.00
time (sec)	N/A	0.373	0.089	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	1095	143	757	11008	1560	1094	1459
N.S.	1	1.00	0.83	7.66	1.00	5.29	76.98	10.91	7.65	10.20
time (sec)	N/A	0.473	0.095	0.916	0.032	0.087	1.037	0.270	0.263	11.697

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	135	2245	283	1569	20971	3092	2244	1515
N.S.	1	1.00	0.65	10.74	1.35	7.51	100.34	14.79	10.74	7.25
time (sec)	N/A	0.636	0.554	0.878	0.034	0.090	1.594	0.271	0.281	12.173

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	221	169	0	33	0	0	60	0
N.S.	1	1.00	1.30	0.99	0.00	0.19	0.00	0.00	0.35	0.00
time (sec)	N/A	0.453	33.111	3.353	0.000	0.070	0.000	0.000	0.362	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	22	0	67	20	22
N.S.	1	1.00	1.00	0.78	0.96	0.96	0.00	2.91	0.87	0.96
time (sec)	N/A	0.258	10.038	2.181	0.108	0.064	0.000	0.263	0.280	10.878

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	289	347	216	0	30	0	0	45	0
N.S.	1	0.98	1.18	0.73	0.00	0.10	0.00	0.00	0.15	0.00
time (sec)	N/A	0.628	20.641	3.202	0.000	0.067	0.000	0.000	0.336	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	149	169	157	0	25	0	0	42	0
N.S.	1	1.03	1.17	1.09	0.00	0.17	0.00	0.00	0.29	0.00
time (sec)	N/A	0.387	20.563	2.997	0.000	0.070	0.000	0.000	0.314	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	56	48	43	0	60	0	0	21	0
N.S.	1	0.85	0.73	0.65	0.00	0.91	0.00	0.00	0.32	0.00
time (sec)	N/A	0.309	17.778	2.365	0.000	0.070	0.000	0.000	200.026	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	349	216	0	32	0	0	48	0
N.S.	1	1.00	1.22	0.75	0.00	0.11	0.00	0.00	0.17	0.00
time (sec)	N/A	0.630	10.541	2.646	0.000	0.074	0.000	0.000	0.331	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	151	185	159	0	32	0	0	50	0
N.S.	1	1.03	1.27	1.09	0.00	0.22	0.00	0.00	0.34	0.00
time (sec)	N/A	0.393	20.503	2.869	0.000	0.068	0.000	0.000	0.389	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	191	235	176	0	38	0	0	78	0
N.S.	1	0.95	1.17	0.88	0.00	0.19	0.00	0.00	0.39	0.00
time (sec)	N/A	0.470	22.025	2.622	0.000	0.074	0.000	0.000	0.448	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	27	27	0	173	25	25
N.S.	1	1.00	1.00	0.78	1.17	1.17	0.00	7.52	1.09	1.09
time (sec)	N/A	0.261	10.047	2.506	0.113	0.066	0.000	0.276	0.297	10.780

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	310	244	230	0	38	0	0	63	0
N.S.	1	0.95	0.75	0.71	0.00	0.12	0.00	0.00	0.19	0.00
time (sec)	N/A	0.687	10.701	2.454	0.000	0.073	0.000	0.000	0.450	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	168	176	171	0	33	0	0	60	0
N.S.	1	0.97	1.02	0.99	0.00	0.19	0.00	0.00	0.35	0.00
time (sec)	N/A	0.416	20.830	2.168	0.000	0.069	0.000	0.000	0.409	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	69	53	57	0	65	0	0	21	0
N.S.	1	0.73	0.56	0.61	0.00	0.69	0.00	0.00	0.22	0.00
time (sec)	N/A	0.349	10.083	2.051	0.000	0.075	0.000	0.000	200.032	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	308	244	230	0	39	0	0	67	0
N.S.	1	0.97	0.77	0.72	0.00	0.12	0.00	0.00	0.21	0.00
time (sec)	N/A	0.685	10.609	2.846	0.000	0.072	0.000	0.000	0.631	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	192	173	0	38	0	0	69	0
N.S.	1	1.00	1.13	1.02	0.00	0.22	0.00	0.00	0.41	0.00
time (sec)	N/A	0.449	10.477	2.529	0.000	0.070	0.000	0.000	0.531	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	149	169	157	0	25	0	0	42	0
N.S.	1	1.05	1.19	1.11	0.00	0.18	0.00	0.00	0.30	0.00
time (sec)	N/A	0.410	21.752	3.022	0.000	0.070	0.000	0.000	0.376	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	17	0	18	15	9
N.S.	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.65	0.39
time (sec)	N/A	0.267	10.027	1.931	0.108	0.068	0.000	0.251	0.306	10.919

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	268	375	202	0	9	0	0	24	0
N.S.	1	1.06	1.48	0.80	0.00	0.04	0.00	0.00	0.09	0.00
time (sec)	N/A	0.600	22.250	2.741	0.000	0.068	0.000	0.000	0.305	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	6	0	0	23	0
N.S.	1	1.00	1.35	1.25	0.00	0.05	0.00	0.00	0.21	0.00
time (sec)	N/A	0.342	20.205	1.911	0.000	0.071	0.000	0.000	0.292	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	33	0	43	0	0	21	0
N.S.	1	1.00	0.69	0.79	0.00	1.02	0.00	0.00	0.50	0.00
time (sec)	N/A	0.309	7.929	2.717	0.000	0.072	0.000	0.000	200.028	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	287	400	215	0	31	0	0	25	0
N.S.	1	1.02	1.42	0.76	0.00	0.11	0.00	0.00	0.09	0.00
time (sec)	N/A	0.658	10.658	2.169	0.000	0.076	0.000	0.000	0.337	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	151	171	159	0	30	0	0	25	0
N.S.	1	1.03	1.17	1.09	0.00	0.21	0.00	0.00	0.17	0.00
time (sec)	N/A	0.405	10.745	2.443	0.000	0.071	0.000	0.000	0.326	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	149	161	157	0	38	0	0	86	0
N.S.	1	1.09	1.18	1.15	0.00	0.28	0.00	0.00	0.63	0.00
time (sec)	N/A	0.396	21.675	3.121	0.000	0.078	0.000	0.000	0.517	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	24	0	0	27	17
N.S.	1	1.00	1.00	0.78	0.74	1.04	0.00	0.00	1.17	0.74
time (sec)	N/A	0.265	10.037	2.495	0.126	0.072	0.000	0.000	0.336	0.208

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	288	402	216	0	42	0	0	29	0
N.S.	1	1.02	1.43	0.77	0.00	0.15	0.00	0.00	0.10	0.00
time (sec)	N/A	0.649	10.767	3.185	0.000	0.075	0.000	0.000	0.324	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	149	216	157	0	37	0	0	28	0
N.S.	1	1.09	1.58	1.15	0.00	0.27	0.00	0.00	0.20	0.00
time (sec)	N/A	0.382	20.442	2.671	0.000	0.069	0.000	0.000	0.338	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	56	81	43	0	78	0	0	21	0
N.S.	1	0.85	1.23	0.65	0.00	1.18	0.00	0.00	0.32	0.00
time (sec)	N/A	0.362	12.055	2.550	0.000	0.067	0.000	0.000	200.040	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	308	409	228	0	47	0	0	30	0
N.S.	1	0.97	1.29	0.72	0.00	0.15	0.00	0.00	0.09	0.00
time (sec)	N/A	0.689	11.113	3.031	0.000	0.072	0.000	0.000	0.314	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	170	169	0	48	0	0	30	0
N.S.	1	1.01	1.00	0.99	0.00	0.28	0.00	0.00	0.18	0.00
time (sec)	N/A	0.440	10.584	2.871	0.000	0.078	0.000	0.000	0.327	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	141	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.84	0.00
time (sec)	N/A	0.423	21.729	2.820	0.000	0.076	0.000	0.000	0.455	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	24	29	0	0	38	82
N.S.	1	1.00	1.00	0.78	1.04	1.26	0.00	0.00	1.65	3.57
time (sec)	N/A	0.271	10.089	2.528	0.130	0.084	0.000	0.000	0.403	0.263

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	309	409	228	0	61	0	0	34	0
N.S.	1	0.97	1.29	0.72	0.00	0.19	0.00	0.00	0.11	0.00
time (sec)	N/A	0.671	11.128	3.182	0.000	0.070	0.000	0.000	0.434	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	167	0	56	0	0	33	0
N.S.	1	1.00	1.06	0.99	0.00	0.33	0.00	0.00	0.20	0.00
time (sec)	N/A	0.431	20.939	2.793	0.000	0.072	0.000	0.000	0.356	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	69	95	60	0	101	0	0	21	0
N.S.	1	0.72	0.99	0.62	0.00	1.05	0.00	0.00	0.22	0.00
time (sec)	N/A	0.331	10.168	2.705	0.000	0.077	0.000	0.000	200.035	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	329	414	240	0	62	0	0	35	0
N.S.	1	0.94	1.19	0.69	0.00	0.18	0.00	0.00	0.10	0.00
time (sec)	N/A	0.720	10.976	3.165	0.000	0.093	0.000	0.000	0.456	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	193	183	179	0	63	0	0	35	0
N.S.	1	0.95	0.90	0.88	0.00	0.31	0.00	0.00	0.17	0.00
time (sec)	N/A	0.488	10.826	3.346	0.000	0.070	0.000	0.000	0.299	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	107	188	148	0	0	0	0	50	0
N.S.	1	1.20	2.11	1.66	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.605	23.219	2.224	0.000	0.000	0.000	0.000	8.579	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [108] had the largest ratio of [.68000000000000049]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	21	0.095
2	A	2	2	1.00	21	0.095
3	A	2	2	1.00	19	0.105
4	A	2	2	1.00	18	0.111
5	A	2	2	1.00	21	0.095
6	A	2	2	1.00	21	0.095
7	A	2	2	1.00	21	0.095
8	A	2	2	1.00	21	0.095
9	A	2	2	1.00	21	0.095
10	A	2	2	1.00	15	0.133
11	A	2	2	1.00	19	0.105
12	A	12	11	1.08	25	0.440
13	A	10	9	1.09	25	0.360
14	A	8	7	1.06	23	0.304
15	A	7	6	1.04	22	0.273
16	A	10	9	1.02	25	0.360
17	B	2	2	2.08	25	0.080
18	A	2	2	1.94	25	0.080
19	A	2	2	1.83	25	0.080
20	A	14	13	1.06	25	0.520
21	A	12	11	1.07	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	10	9	1.07	23	0.391
23	A	9	8	1.08	22	0.364
24	A	12	11	1.26	25	0.440
25	B	2	2	2.23	25	0.080
26	B	2	2	2.61	25	0.080
27	B	2	2	2.99	25	0.080
28	B	2	2	2.35	25	0.080
29	A	2	2	2.00	25	0.080
30	A	12	11	1.11	25	0.440
31	A	10	9	1.07	25	0.360
32	A	8	7	1.06	25	0.280
33	A	6	5	1.00	23	0.217
34	A	3	2	1.00	22	0.091
35	A	2	2	1.00	25	0.080
36	A	2	2	1.21	25	0.080
37	A	2	2	1.39	25	0.080
38	A	10	9	1.15	25	0.360
39	A	8	7	1.15	25	0.280
40	A	5	4	1.00	25	0.160
41	A	5	4	1.00	23	0.174
42	A	5	4	1.00	22	0.182
43	A	2	2	1.14	25	0.080
44	A	2	2	1.39	25	0.080
45	A	7	6	1.04	25	0.240
46	A	7	6	1.07	25	0.240
47	A	7	6	1.06	25	0.240
48	A	7	6	1.13	23	0.261
49	A	7	6	1.14	22	0.273
50	A	2	2	0.93	25	0.080
51	A	2	2	1.12	25	0.080
52	A	2	2	1.15	29	0.069
53	A	11	10	0.88	29	0.345

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	6	0.98	29	0.207
55	A	2	2	1.31	29	0.069
56	A	4	4	1.13	19	0.211
57	A	3	3	1.10	15	0.200
58	A	3	2	1.02	25	0.080
59	A	3	2	1.02	25	0.080
60	A	3	2	1.02	25	0.080
61	A	6	5	1.02	23	0.217
62	A	4	3	1.18	22	0.136
63	A	3	2	1.03	25	0.080
64	A	3	2	1.12	25	0.080
65	A	3	2	1.19	25	0.080
66	A	3	2	1.01	25	0.080
67	A	3	2	1.01	25	0.080
68	A	3	2	1.01	25	0.080
69	A	7	6	1.03	23	0.261
70	A	7	6	0.93	22	0.273
71	A	3	2	1.02	25	0.080
72	A	3	2	1.11	25	0.080
73	A	3	2	1.18	25	0.080
74	A	10	9	0.75	23	0.391
75	A	8	7	0.81	23	0.304
76	A	6	5	0.90	21	0.238
77	A	6	5	1.09	16	0.312
78	A	10	9	1.07	23	0.391
79	A	9	8	1.05	23	0.348
80	A	9	8	1.02	23	0.348
81	A	10	9	1.18	23	0.391
82	A	10	9	1.32	23	0.391
83	A	6	5	1.10	23	0.217
84	A	8	7	1.03	23	0.304
85	A	10	9	0.93	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	F	0	0	N/A	0.000	N/A
87	F	0	0	N/A	0.000	N/A
88	A	10	9	1.07	27	0.333
89	A	8	7	1.06	27	0.259
90	A	7	6	1.06	25	0.240
91	A	3	2	1.06	24	0.083
92	A	9	8	1.42	27	0.296
93	A	14	13	1.32	27	0.481
94	B	18	17	2.03	27	0.630
95	A	8	7	1.07	27	0.259
96	A	7	6	1.06	27	0.222
97	A	5	4	1.05	25	0.160
98	A	3	2	1.06	24	0.083
99	A	6	5	1.74	27	0.185
100	A	14	13	1.32	27	0.481
101	B	18	17	2.03	27	0.630
102	C	10	9	0.78	25	0.360
103	C	9	8	0.74	25	0.320
104	C	7	6	0.69	23	0.261
105	C	3	2	0.35	22	0.091
106	C	10	9	1.09	25	0.360
107	C	14	13	1.29	25	0.520
108	C	18	17	1.58	25	0.680
109	C	7	6	0.47	27	0.222
110	C	8	7	0.72	25	0.280
111	C	7	6	0.68	25	0.240
112	C	5	4	0.65	23	0.174
113	C	3	2	0.84	22	0.091
114	C	7	6	0.76	25	0.240
115	C	14	13	0.95	25	0.520
116	C	18	17	1.59	25	0.680
117	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	19	0.105
120	A	2	2	1.00	18	0.111
121	A	2	2	1.00	21	0.095
122	A	4	4	1.07	21	0.190
123	A	4	4	1.13	21	0.190
124	A	4	4	1.03	23	0.174
125	A	2	2	1.00	23	0.087
126	A	2	2	1.00	23	0.087
127	A	2	2	0.91	21	0.095
128	A	2	2	1.00	20	0.100
129	A	2	2	0.92	23	0.087
130	A	2	2	0.96	23	0.087
131	A	5	5	1.11	23	0.217
132	A	2	2	1.00	25	0.080
133	A	9	8	0.98	23	0.348
134	A	7	6	1.00	23	0.261
135	A	4	3	1.00	21	0.143
136	A	3	2	1.00	20	0.100
137	N/A	1	0	1.00	23	0.000
138	N/A	1	0	1.00	23	0.000
139	A	4	4	1.00	25	0.160
140	A	4	4	1.00	25	0.160
141	A	4	4	1.00	25	0.160
142	A	4	4	1.00	23	0.174
143	A	4	4	1.00	22	0.182
144	A	5	5	1.00	25	0.200
145	A	4	4	1.00	25	0.160
146	A	4	4	1.00	25	0.160
147	A	4	4	1.00	25	0.160
148	A	4	4	1.00	25	0.160
149	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	4	4	1.00	27	0.148
153	A	4	4	1.00	27	0.148
154	A	4	4	1.00	27	0.148
155	A	4	4	1.00	25	0.160
156	A	4	4	1.00	24	0.167
157	A	5	5	0.97	27	0.185
158	A	4	4	1.00	27	0.148
159	A	4	4	1.00	27	0.148
160	A	4	4	1.00	27	0.148
161	A	4	4	1.00	27	0.148
162	A	5	5	0.97	27	0.185
163	A	4	4	1.00	27	0.148
164	A	4	4	1.00	27	0.148
165	A	4	4	1.00	27	0.148
166	A	4	4	1.00	27	0.148
167	A	4	4	1.00	27	0.148
168	A	4	4	1.00	27	0.148
169	A	4	4	1.00	27	0.148
170	A	4	4	1.00	27	0.148
171	A	4	4	1.00	27	0.148
172	A	4	4	1.00	25	0.160
173	A	4	4	1.00	24	0.167
174	A	5	5	0.96	27	0.185
175	A	4	4	1.00	27	0.148
176	A	4	4	1.00	27	0.148
177	A	4	4	1.00	27	0.148
178	A	4	4	1.00	27	0.148
179	A	4	4	1.00	27	0.148
180	A	4	4	1.00	27	0.148
181	A	5	5	0.96	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	1.00	27	0.148
183	A	5	5	0.99	27	0.185
184	A	4	4	1.00	27	0.148
185	A	4	4	1.00	27	0.148
186	A	4	4	1.00	27	0.148
187	A	4	4	1.00	27	0.148
188	A	3	3	1.00	19	0.158
189	A	3	3	1.00	19	0.158
190	A	3	3	1.00	19	0.158
191	A	3	3	1.00	19	0.158
192	A	3	3	1.00	19	0.158
193	A	3	3	1.00	19	0.158
194	A	3	3	1.00	17	0.176
195	A	3	3	1.00	16	0.188
196	A	4	4	0.90	19	0.211
197	A	3	3	1.00	19	0.158
198	A	3	3	1.00	19	0.158
199	A	3	3	1.00	19	0.158
200	A	3	3	1.00	19	0.158
201	A	3	3	1.00	19	0.158
202	A	3	3	1.00	19	0.158
203	A	3	3	1.00	19	0.158
204	A	3	3	1.00	19	0.158
205	A	3	3	1.00	19	0.158
206	A	3	3	1.00	19	0.158
207	A	4	4	0.90	19	0.211
208	A	3	3	1.00	19	0.158
209	A	4	4	0.96	19	0.211
210	A	5	5	0.94	19	0.263
211	A	6	6	0.93	19	0.316
212	A	7	7	0.93	19	0.368
213	A	8	8	0.92	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	3	3	1.00	19	0.158
215	A	3	3	1.00	19	0.158
216	A	3	3	1.00	19	0.158
217	A	3	3	1.00	17	0.176
218	A	3	3	1.00	17	0.176
219	A	3	3	1.00	17	0.176
220	A	3	3	1.00	17	0.176
221	A	3	3	1.00	17	0.176
222	A	3	3	1.00	17	0.176
223	A	3	3	1.00	17	0.176
224	A	3	3	1.00	17	0.176
225	A	3	3	1.00	17	0.176
226	A	3	3	1.00	17	0.176
227	A	3	3	1.00	15	0.200
228	A	2	2	1.00	14	0.143
229	A	3	3	1.00	17	0.176
230	A	3	3	1.00	17	0.176
231	A	3	3	1.00	17	0.176
232	A	3	3	1.00	17	0.176
233	A	3	3	1.00	17	0.176
234	A	3	3	1.00	17	0.176
235	A	3	3	1.00	17	0.176
236	A	3	3	1.00	17	0.176
237	A	3	3	1.00	17	0.176
238	A	3	3	1.00	17	0.176
239	A	3	3	1.00	17	0.176
240	A	3	3	1.00	17	0.176
241	A	2	2	1.00	17	0.118
242	A	3	3	1.00	17	0.176
243	A	4	4	1.14	17	0.235
244	A	5	5	1.20	17	0.294
245	A	6	6	1.25	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	7	7	1.27	17	0.412
247	A	8	8	1.29	17	0.471
248	A	9	9	1.31	17	0.529
249	A	3	3	1.00	17	0.176
250	A	3	3	1.00	17	0.176
251	A	4	4	1.00	27	0.148
252	A	4	4	1.00	27	0.148
253	A	4	4	1.00	27	0.148
254	A	4	4	1.00	27	0.148
255	A	4	4	1.00	25	0.160
256	A	4	4	1.00	24	0.167
257	A	4	4	1.00	27	0.148
258	A	4	4	1.00	27	0.148
259	A	4	4	1.00	27	0.148
260	A	4	4	1.00	27	0.148
261	A	4	4	1.00	27	0.148
262	A	4	4	1.00	27	0.148
263	A	4	4	1.00	27	0.148
264	A	4	4	1.00	27	0.148
265	A	5	5	1.04	27	0.185
266	A	4	4	1.00	25	0.160
267	A	4	4	1.00	24	0.167
268	A	4	4	1.00	27	0.148
269	A	4	4	1.00	27	0.148
270	A	4	4	1.00	27	0.148
271	A	4	4	1.00	27	0.148
272	A	4	4	1.00	27	0.148
273	A	4	4	1.00	27	0.148
274	A	5	5	1.01	27	0.185
275	A	4	4	1.00	27	0.148
276	A	4	4	1.00	27	0.148
277	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	4	4	1.00	24	0.167
279	A	4	4	1.00	27	0.148
280	A	4	4	1.00	27	0.148
281	A	4	4	1.00	27	0.148
282	A	4	4	0.54	29	0.138
283	A	4	4	0.54	29	0.138
284	A	4	4	0.54	29	0.138
285	A	4	4	0.54	27	0.148
286	A	3	3	1.00	26	0.115
287	A	4	4	0.50	29	0.138
288	A	4	4	0.49	29	0.138
289	A	4	4	0.51	29	0.138
290	A	4	4	1.00	29	0.138
291	A	4	4	0.54	29	0.138
292	A	4	4	0.54	29	0.138
293	A	4	4	0.54	29	0.138
294	A	4	4	0.49	29	0.138
295	A	4	4	0.49	29	0.138
296	A	4	4	0.49	29	0.138
297	A	4	4	0.49	29	0.138
298	A	4	4	0.74	27	0.148
299	A	3	3	1.00	26	0.115
300	A	5	5	0.45	29	0.172
301	A	4	4	0.46	29	0.138
302	A	4	4	0.46	29	0.138
303	A	4	4	0.46	29	0.138
304	A	5	5	0.46	29	0.172
305	A	4	4	1.03	29	0.138
306	A	4	4	0.90	29	0.138
307	A	4	4	0.66	29	0.138
308	A	4	4	0.49	29	0.138
309	A	4	4	0.49	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	4	4	0.49	29	0.138
311	A	4	4	0.49	29	0.138
312	A	4	4	0.47	29	0.138
313	A	4	4	0.47	29	0.138
314	A	4	4	0.47	29	0.138
315	A	4	4	0.66	29	0.138
316	A	4	4	0.69	29	0.138
317	A	4	4	0.74	27	0.148
318	A	3	3	1.00	26	0.115
319	A	5	5	0.40	29	0.172
320	A	4	4	0.45	29	0.138
321	A	4	4	0.46	29	0.138
322	A	4	4	0.46	29	0.138
323	A	4	4	0.46	29	0.138
324	A	4	4	0.45	29	0.138
325	A	5	5	0.41	29	0.172
326	A	4	4	1.03	29	0.138
327	A	5	5	0.85	29	0.172
328	A	4	4	0.91	29	0.138
329	A	4	4	0.73	29	0.138
330	A	4	4	0.60	29	0.138
331	A	4	4	0.47	29	0.138
332	A	4	4	0.47	29	0.138
333	A	4	4	0.52	29	0.138
334	A	4	4	0.53	29	0.138
335	A	4	4	0.55	29	0.138
336	A	4	4	0.59	27	0.148
337	A	3	3	1.00	26	0.115
338	A	4	4	0.70	29	0.138
339	A	6	6	1.10	29	0.207
340	A	4	4	0.54	29	0.138
341	A	4	4	0.53	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	4	4	0.52	29	0.138
343	A	4	4	0.57	29	0.138
344	A	4	4	0.59	29	0.138
345	A	4	4	0.63	29	0.138
346	A	4	4	0.72	27	0.148
347	A	2	2	1.00	26	0.077
348	A	4	4	0.59	29	0.138
349	A	4	4	0.58	29	0.138
350	A	4	4	0.56	29	0.138
351	A	4	4	0.60	29	0.138
352	A	5	5	0.63	29	0.172
353	A	4	4	1.03	29	0.138
354	A	4	4	0.72	27	0.148
355	A	2	2	1.00	26	0.077
356	A	4	4	0.54	29	0.138
357	A	4	4	0.57	29	0.138
358	A	4	4	1.00	27	0.148
359	A	4	4	1.00	27	0.148
360	A	4	4	1.00	27	0.148
361	A	4	4	1.00	27	0.148
362	A	4	4	1.00	27	0.148
363	A	4	4	1.00	27	0.148
364	A	4	4	1.00	27	0.148
365	A	4	4	1.00	27	0.148
366	A	4	4	1.00	27	0.148
367	A	4	4	1.00	29	0.138
368	A	4	4	1.00	29	0.138
369	A	4	4	1.00	29	0.138
370	A	4	4	1.00	29	0.138
371	A	4	4	1.00	29	0.138
372	A	4	4	1.00	29	0.138
373	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	4	4	1.00	29	0.138
375	A	4	4	1.00	29	0.138
376	A	4	4	1.00	29	0.138
377	A	4	4	1.00	29	0.138
378	A	4	4	1.00	29	0.138
379	A	4	4	1.00	29	0.138
380	A	4	4	1.00	29	0.138
381	A	4	4	1.00	29	0.138
382	A	4	4	1.00	29	0.138
383	A	4	4	1.00	29	0.138
384	A	4	4	1.00	29	0.138
385	A	4	4	1.00	29	0.138
386	A	10	9	1.02	29	0.310
387	A	9	8	1.07	29	0.276
388	A	8	7	1.12	29	0.241
389	A	7	6	1.18	29	0.207
390	A	6	5	1.00	29	0.172
391	A	7	6	1.16	29	0.207
392	A	8	7	1.12	29	0.241
393	A	9	8	1.03	29	0.276
394	A	10	9	1.01	29	0.310
395	A	10	9	1.03	29	0.310
396	A	9	8	1.05	29	0.276
397	A	8	7	0.98	29	0.241
398	A	8	7	0.96	29	0.241
399	A	8	7	0.93	29	0.241
400	A	9	8	1.04	29	0.276
401	A	10	9	1.01	29	0.310
402	A	12	11	1.00	29	0.379
403	A	11	10	1.03	29	0.345
404	A	10	9	0.97	29	0.310
405	A	10	9	0.95	29	0.310

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	10	9	0.96	29	0.310
407	A	10	9	0.92	29	0.310
408	A	10	9	0.92	29	0.310
409	A	11	10	1.02	29	0.345
410	A	12	11	0.99	29	0.379
411	A	4	4	0.56	31	0.129
412	A	4	4	0.56	31	0.129
413	A	4	4	0.56	31	0.129
414	A	4	4	0.56	31	0.129
415	A	4	4	0.55	31	0.129
416	A	4	4	0.54	31	0.129
417	A	4	4	0.54	31	0.129
418	A	4	4	0.55	31	0.129
419	A	4	4	0.56	31	0.129
420	A	4	4	0.51	31	0.129
421	A	4	4	0.51	31	0.129
422	A	4	4	0.51	31	0.129
423	A	4	4	0.51	31	0.129
424	A	4	4	0.51	31	0.129
425	A	4	4	0.50	31	0.129
426	A	4	4	0.50	31	0.129
427	A	4	4	0.50	31	0.129
428	A	4	4	0.50	31	0.129
429	A	4	4	0.50	31	0.129
430	A	4	4	0.50	31	0.129
431	A	4	4	0.50	31	0.129
432	A	4	4	0.50	31	0.129
433	A	4	4	0.49	31	0.129
434	A	4	4	0.49	31	0.129
435	A	4	4	0.49	31	0.129
436	A	4	4	0.49	31	0.129
437	A	4	4	0.49	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	10	9	0.52	31	0.290
439	A	9	8	0.55	31	0.258
440	A	8	7	0.58	31	0.226
441	A	7	6	0.64	31	0.194
442	A	6	5	0.76	31	0.161
443	A	6	5	0.76	31	0.161
444	A	7	6	0.65	31	0.194
445	A	8	7	0.59	31	0.226
446	A	9	8	0.55	31	0.258
447	A	10	9	0.63	31	0.290
448	A	9	8	0.67	31	0.258
449	A	8	7	0.74	31	0.226
450	A	7	6	0.78	31	0.194
451	A	7	6	0.76	31	0.194
452	A	8	7	0.74	31	0.226
453	A	9	8	0.67	31	0.258
454	A	10	9	0.62	31	0.290
455	A	12	11	0.61	31	0.355
456	A	11	10	0.64	31	0.323
457	A	10	9	0.68	31	0.290
458	A	9	8	0.69	31	0.258
459	A	9	8	0.68	31	0.258
460	A	9	8	0.68	31	0.258
461	A	9	8	0.67	31	0.258
462	A	10	9	0.68	31	0.290
463	A	11	10	0.63	31	0.323
464	A	12	11	0.59	31	0.355
465	A	4	4	1.00	29	0.138
466	A	4	4	1.00	29	0.138
467	A	4	4	1.00	27	0.148
468	A	4	4	1.06	29	0.138
469	A	4	4	1.12	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	3	3	1.00	17	0.176
471	A	3	3	1.00	19	0.158
472	A	4	4	1.00	23	0.174
473	A	1	1	1.00	23	0.043
474	A	5	5	0.98	21	0.238
475	A	3	3	1.03	20	0.150
476	A	6	5	0.85	23	0.217
477	A	5	5	1.00	23	0.217
478	A	3	3	1.03	23	0.130
479	A	5	5	0.95	23	0.217
480	A	1	1	1.00	23	0.043
481	A	6	6	0.95	21	0.286
482	A	4	4	0.97	20	0.200
483	A	7	6	0.73	23	0.261
484	A	6	6	0.97	23	0.261
485	A	4	4	1.00	23	0.174
486	A	3	3	1.05	23	0.130
487	A	1	1	1.00	23	0.043
488	A	4	4	1.06	21	0.190
489	A	2	2	1.00	20	0.100
490	A	5	4	1.00	23	0.174
491	A	5	5	1.02	23	0.217
492	A	3	3	1.03	23	0.130
493	A	3	3	1.09	23	0.130
494	A	1	1	1.00	23	0.043
495	A	5	5	1.02	21	0.238
496	A	3	3	1.09	20	0.150
497	A	6	5	0.85	23	0.217
498	A	6	6	0.97	23	0.261
499	A	4	4	1.01	23	0.174
500	A	4	4	1.00	23	0.174
501	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	6	6	0.97	21	0.286
503	A	4	4	1.00	20	0.200
504	A	7	6	0.72	23	0.261
505	A	7	7	0.94	23	0.304
506	A	5	5	0.95	23	0.217
507	A	7	6	1.20	27	0.222

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{d^2(cd^2-bde+ae^2)\,x}{e^5} - \frac{d(cd^2-bde+ae^2)\,x^2}{2e^4} + \frac{(cd^2-bde+ae^2)\,x^3}{3e^3} - \frac{(cd-be)x^4}{4e^2} + \frac{cx^5}{5e} - \frac{d^3(cd^2-bde+ae^2)\log(d+ex)}{e^6}$$

output
```
d^2*(a*e^2-b*d*e+c*d^2)*x/e^5-1/2*d*(a*e^2-b*d*e+c*d^2)*x^2/e^4+1/3*(a*e^2
-b*d*e+c*d^2)*x^3/e^3-1/4*(-b*e+c*d)*x^4/e^2+1/5*c*x^5/e-d^3*(a*e^2-b*d*e+
c*d^2)*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{60d^2e(cd^2+e(-bd+ae))\,x - 30de^2(cd^2+e(-bd+ae))\,x^2 + 20e^3(cd^2+e(-bd+ae))\,x^3 + 15e^4(-cd+}{60e^6}$$

input
```
Integrate[(x^3*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
(60*d^2*e*(c*d^2 + e*(-(b*d) + a*e))*x - 30*d*e^2*(c*d^2 + e*(-(b*d) + a*e
))*x^2 + 20*e^3*(c*d^2 + e*(-(b*d) + a*e))*x^3 + 15*e^4*(-(c*d) + b*e)*x^4
 + 12*c*e^5*x^5 - 60*(c*d^5 + d^3*e*(-(b*d) + a*e))*Log[d + e*x])/(60*e^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{d^2(ae^2-bde+cd^2)}{e^5}-\frac{dx(ae^2-bde+cd^2)}{e^4}+\frac{x^2(ae^2-bde+cd^2)}{e^3}-\frac{d^3(ae^2-bde+cd^2)}{e^5(d+ex)}+\frac{x^3(be-cd)}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2x(ae^2-bde+cd^2)}{e^5}-\frac{dx^2(ae^2-bde+cd^2)}{2e^4}+\frac{x^3(ae^2-bde+cd^2)}{3e^3}-\frac{d^3\log(d+ex)(ae^2-bde+cd^2)}{e^6}-\frac{x^4(cd-be)}{4e^2}+\frac{cx^5}{5e}$$

input
```
Int[(x^3*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
(d^2*(c*d^2 - b*d*e + a*e^2)*x)/e^5 - (d*(c*d^2 - b*d*e + a*e^2)*x^2)/(2*e
^4) + ((c*d^2 - b*d*e + a*e^2)*x^3)/(3*e^3) - ((c*d - b*e)*x^4)/(4*e^2) +
(c*x^5)/(5*e) - (d^3*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/e^6
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

method	result
norman	$\frac{d^2(a\,e^2-bde+c\,d^2)x}{e^5}+\frac{c\,x^5}{5e}+\frac{(be-cd)x^4}{4e^2}+\frac{(a\,e^2-bde+c\,d^2)x^3}{3e^3}-\frac{d(a\,e^2-bde+c\,d^2)x^2}{2e^4}-\frac{d^3(a\,e^2-bde+c\,d^2)\ln(ex\cdot}{e^6}$
default	$\frac{\frac{1}{5}c\,x^5e^4+\frac{1}{4}b\,e^4x^4-\frac{1}{4}cd\,e^3x^4+\frac{1}{3}a\,e^4x^3-\frac{1}{3}bd\,e^3x^3+\frac{1}{3}c\,d^2e^2x^3-\frac{1}{2}ad\,e^3x^2+\frac{1}{2}b\,d^2e^2x^2-\frac{1}{2}c\,d^3e\,x^2+a\,d^2e^2x-b\,d^3ex+c\,d^4x}{e^5}-$
risch	$\frac{c\,x^5}{5e}+\frac{b\,x^4}{4e}-\frac{cd\,x^4}{4e^2}+\frac{a\,x^3}{3e}-\frac{bd\,x^3}{3e^2}+\frac{c\,d^2x^3}{3e^3}-\frac{ad\,x^2}{2e^2}+\frac{b\,d^2x^2}{2e^3}-\frac{c\,d^3x^2}{2e^4}+\frac{a\,d^2x}{e^3}-\frac{b\,d^3x}{e^4}+\frac{c\,d^4x}{e^5}-\frac{d^3\ln(}{}$
parallelrisch	$-\frac{-12c\,x^5e^5-15b\,e^5x^4+15x^4cd\,e^4-20x^3a\,e^5+20x^3bd\,e^4-20x^3c\,d^2e^3+30x^2ad\,e^4-30x^2b\,d^2e^3+30x^2c\,d^3e^2+60\ln(ex+d)a\,d}{60e^6}$

input
```
int(x^3*(c*x^2+b*x+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d^2*(a*e^2-b*d*e+c*d^2)*x/e^5+1/5*c*x^5/e+1/4/e^2*(b*e-c*d)*x^4+1/3*(a*e^2
-b*d*e+c*d^2)*x^3/e^3-1/2*d*(a*e^2-b*d*e+c*d^2)*x^2/e^4-d^3*(a*e^2-b*d*e+c
*d^2)*ln(e*x+d)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{12\,ce^5x^5-15\,(cde^4-be^5)x^4+20\,(cd^2e^3-bde^4+ae^5)x^3-30\,(cd^3e^2-bd^2e^3+ade^4)x^2+60\,(cd^4e-bd}{60\,e^6}$$

input
```
integrate(x^3*(c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/60*(12*c*e^5*x^5 - 15*(c*d*e^4 - b*e^5)*x^4 + 20*(c*d^2*e^3 - b*d*e^4 +
a*e^5)*x^3 - 30*(c*d^3*e^2 - b*d^2*e^3 + a*d*e^4)*x^2 + 60*(c*d^4*e - b*d^
3*e^2 + a*d^2*e^3)*x - 60*(c*d^5 - b*d^4*e + a*d^3*e^2)*log(e*x + d))/e^6
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{cx^5}{5e} - \frac{d^3(ae^2-bde+cd^2)\log(d+ex)}{e^6} + x^4\left(\frac{b}{4e}-\frac{cd}{4e^2}\right) + x^3\left(\frac{a}{3e}-\frac{bd}{3e^2}+\frac{cd^2}{3e^3}\right) + x^2\left(-\frac{ad}{2e^2}+\frac{bd^2}{2e^3}-\frac{cd^3}{2e^4}\right) + x\left(\frac{ad^2}{e^3}-\frac{bd^3}{e^4}+\frac{cd^4}{e^5}\right)$$

input
```
integrate(x**3*(c*x**2+b*x+a)/(e*x+d),x)
```

output
```
c*x**5/(5*e) - d**3*(a*e**2 - b*d*e + c*d**2)*log(d + e*x)/e**6 + x**4*(b/
(4*e) - c*d/(4*e**2)) + x**3*(a/(3*e) - b*d/(3*e**2) + c*d**2/(3*e**3)) +
x**2*(-a*d/(2*e**2) + b*d**2/(2*e**3) - c*d**3/(2*e**4)) + x*(a*d**2/e**3
- b*d**3/e**4 + c*d**4/e**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{12\,ce^4x^5 - 15\,(cde^3-be^4)x^4 + 20\,(cd^2e^2-bde^3+ae^4)x^3 - 30\,(cd^3e-bd^2e^2+ade^3)x^2 + 60\,(cd^4-bd^3e}{60\,e^5} - \frac{(cd^5-bd^4e+ad^3e^2)\log(ex+d)}{e^6}$$

input
```
integrate(x^3*(c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/60*(12*c*e^4*x^5 - 15*(c*d*e^3 - b*e^4)*x^4 + 20*(c*d^2*e^2 - b*d*e^3 +
a*e^4)*x^3 - 30*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*x^2 + 60*(c*d^4 - b*d^3*e
+ a*d^2*e^2)*x)/e^5 - (c*d^5 - b*d^4*e + a*d^3*e^2)*log(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.14

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = \frac{12\,ce^4x^5 - 15\,cde^3x^4 + 15\,be^4x^4 + 20\,cd^2e^2x^3 - 20\,bde^3x^3 + 20\,ae^4x^3 - 30\,cd^3ex^2 + 30\,bd^2e^2x^2 - 30\,ade^3}{60\,e^5} - \frac{(cd^5 - bd^4e + ad^3e^2)\log(|ex+d|)}{e^6}$$

input
```
integrate(x^3*(c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/60*(12*c*e^4*x^5 - 15*c*d*e^3*x^4 + 15*b*e^4*x^4 + 20*c*d^2*e^2*x^3 - 20
*b*d*e^3*x^3 + 20*a*e^4*x^3 - 30*c*d^3*e*x^2 + 30*b*d^2*e^2*x^2 - 30*a*d*e
^3*x^2 + 60*c*d^4*x - 60*b*d^3*e*x + 60*a*d^2*e^2*x)/e^5 - (c*d^5 - b*d^4*
e + a*d^3*e^2)*log(abs(e*x + d))/e^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

$$\int \frac{x^3(a+bx+cx^2)}{d+ex}\,dx = x^4\left(\frac{b}{4\,e}-\frac{c\,d}{4\,e^2}\right)+x^3\left(\frac{a}{3\,e}-\frac{d\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)}{3\,e}\right) + \frac{c\,x^5}{5\,e}-\frac{\ln(d+e\,x)\,\left(c\,d^5-b\,d^4\,e+a\,d^3\,e^2\right)}{e^6} - \frac{d\,x^2\left(\frac{a}{e}-\frac{d\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)}{e}\right)}{2\,e}+\frac{d^2\,x\left(\frac{a}{e}-\frac{d\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)}{e}\right)}{e^2}$$

input
```
int((x^3*(a + b*x + c*x^2))/(d + e*x),x)
```

output
```
x^4*(b/(4*e) - (c*d)/(4*e^2)) + x^3*(a/(3*e) - (d*(b/e - (c*d)/e^2))/(3*e)
) + (c*x^5)/(5*e) - (log(d + e*x)*(c*d^5 + a*d^3*e^2 - b*d^4*e))/e^6 - (d*
x^2*(a/e - (d*(b/e - (c*d)/e^2))/e))/(2*e) + (d^2*x*(a/e - (d*(b/e - (c*d)
/e^2))/e))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a + bx + cx^2)}{d + ex} \, dx$$
$$= \frac{-60 \log(ex + d) \, a \, d^3e^2 + 60 \log(ex + d) \, b \, d^4e - 60 \log(ex + d) \, c \, d^5 + 60a \, d^2e^3x - 30ad \, e^4x^2 + 20a \, e^5x^3 -}{}$$

input
```
int(x^3*(c*x^2+b*x+a)/(e*x+d),x)
```

output
```
( - 60*log(d + e*x)*a*d**3*e**2 + 60*log(d + e*x)*b*d**4*e - 60*log(d + e*
x)*c*d**5 + 60*a*d**2*e**3*x - 30*a*d*e**4*x**2 + 20*a*e**5*x**3 - 60*b*d*
*3*e**2*x + 30*b*d**2*e**3*x**2 - 20*b*d*e**4*x**3 + 15*b*e**5*x**4 + 60*c
*d**4*e*x - 30*c*d**3*e**2*x**2 + 20*c*d**2*e**3*x**3 - 15*c*d*e**4*x**4 +
 12*c*e**5*x**5)/(60*e**6)
```

3.2 $\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = -\frac{d(cd^2-bde+ae^2)\,x}{e^4}+\frac{(cd^2-bde+ae^2)\,x^2}{2e^3} -\frac{(cd-be)x^3}{3e^2}+\frac{cx^4}{4e}+\frac{d^2(cd^2-bde+ae^2)\log(d+ex)}{e^5}$$

output
```
-d*(a*e^2-b*d*e+c*d^2)*x/e^4+1/2*(a*e^2-b*d*e+c*d^2)*x^2/e^3-1/3*(-b*e+c*d
)*x^3/e^2+1/4*c*x^4/e+d^2*(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = \frac{ex(c(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3ae(-2d+ex)+b(6d^2-3dex+2e^2x^2)))+12(cd^4+d^2e}{12e^5}$$

input
```
Integrate[(x^2*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
(e*x*(c*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*a*e*(-2*d
 + e*x) + b*(6*d^2 - 3*d*e*x + 2*e^2*x^2))) + 12*(c*d^4 + d^2*e*(-(b*d) +
a*e))*Log[d + e*x])/(12*e^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{d^2(ae^2-bde+cd^2)}{e^4(d+ex)}-\frac{d(ae^2-bde+cd^2)}{e^4}+\frac{x(ae^2-bde+cd^2)}{e^3}+\frac{x^2(be-cd)}{e^2}+\frac{cx^3}{e}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{d^2\log(d+ex)(ae^2-bde+cd^2)}{e^5}-\frac{dx(ae^2-bde+cd^2)}{e^4}+\frac{x^2(ae^2-bde+cd^2)}{2e^3}-\frac{x^3(cd-be)}{3e^2}+\frac{cx^4}{4e}$$

input
```
Int[(x^2*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
-((d*(c*d^2 - b*d*e + a*e^2)*x)/e^4) + ((c*d^2 - b*d*e + a*e^2)*x^2)/(2*e^
3) - ((c*d - b*e)*x^3)/(3*e^2) + (c*x^4)/(4*e) + (d^2*(c*d^2 - b*d*e + a*e
^2)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

method	result
norman	$\frac{c\,x^4}{4e}+\frac{(be-cd)x^3}{3e^2}+\frac{(a\,e^2-bde+c\,d^2)x^2}{2e^3}-\frac{d(a\,e^2-bde+c\,d^2)x}{e^4}+\frac{d^2(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^5}$
default	$-\frac{-\frac{1}{4}c\,x^4e^3-\frac{1}{3}b\,e^3x^3+\frac{1}{3}cd\,x^3e^2-\frac{1}{2}a\,e^3x^2+\frac{1}{2}bd\,e^2x^2-\frac{1}{2}c\,d^2e\,x^2+d\,e^2ax-b\,d^2ex+d^3cx}{e^4}+\frac{d^2(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^5}$
risch	$\frac{c\,x^4}{4e}+\frac{b\,x^3}{3e}-\frac{cd\,x^3}{3e^2}+\frac{a\,x^2}{2e}-\frac{bd\,x^2}{2e^2}+\frac{c\,d^2x^2}{2e^3}-\frac{dax}{e^2}+\frac{b\,d^2x}{e^3}-\frac{d^3cx}{e^4}+\frac{d^2\ln(ex+d)a}{e^3}-\frac{d^3\ln(ex+d)b}{e^4}+\frac{d^4\ln}{}$
parallelrisch	$\frac{3c\,x^4e^4+4x^3b\,e^4-4x^3cd\,e^3+6x^2a\,e^4-6x^2bd\,e^3+6x^2c\,d^2e^2+12\ln(ex+d)a\,d^2e^2-12\ln(ex+d)b\,d^3e+12\ln(ex+d)c\,d^4-12xad\,e}{12e^5}$

input
```
int(x^2*(c*x^2+b*x+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*x^4/e+1/3/e^2*(b*e-c*d)*x^3+1/2*(a*e^2-b*d*e+c*d^2)*x^2/e^3-d*(a*e^2
-b*d*e+c*d^2)*x/e^4+d^2*(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx$$
$$=\frac{3\,ce^4x^4-4\,(cde^3-be^4)x^3+6\,(cd^2e^2-bde^3+ae^4)x^2-12\,(cd^3e-bd^2e^2+ade^3)x+12\,(cd^4-bd^3e+a}{12\,e^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*c*e^4*x^4 - 4*(c*d*e^3 - b*e^4)*x^3 + 6*(c*d^2*e^2 - b*d*e^3 + a*e
^4)*x^2 - 12*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*x + 12*(c*d^4 - b*d^3*e + a*d
^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = \frac{cx^4}{4e} + \frac{d^2(ae^2-bde+cd^2)\log{(d+ex)}}{e^5} + x^3\left(\frac{b}{3e}-\frac{cd}{3e^2}\right) + x^2\left(\frac{a}{2e}-\frac{bd}{2e^2}+\frac{cd^2}{2e^3}\right) + x\left(-\frac{ad}{e^2}+\frac{bd^2}{e^3}-\frac{cd^3}{e^4}\right)$$

input
```
integrate(x**2*(c*x**2+b*x+a)/(e*x+d),x)
```

output
```
c*x**4/(4*e) + d**2*(a*e**2 - b*d*e + c*d**2)*log(d + e*x)/e**5 + x**3*(b/
(3*e) - c*d/(3*e**2)) + x**2*(a/(2*e) - b*d/(2*e**2) + c*d**2/(2*e**3)) +
x*(-a*d/e**2 + b*d**2/e**3 - c*d**3/e**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,(cde^2-be^3)x^3+6\,(cd^2e-bde^2+ae^3)x^2-12\,(cd^3-bd^2e+ade^2)x}{12\,e^4} + \frac{(cd^4-bd^3e+ad^2e^2)\log{(ex+d)}}{e^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*c*e^3*x^4 - 4*(c*d*e^2 - b*e^3)*x^3 + 6*(c*d^2*e - b*d*e^2 + a*e^3
)*x^2 - 12*(c*d^3 - b*d^2*e + a*d*e^2)*x)/e^4 + (c*d^4 - b*d^3*e + a*d^2*e
^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,cde^2x^3+4\,be^3x^3+6\,cd^2ex^2-6\,bde^2x^2+6\,ae^3x^2-12\,cd^3x+12\,bd^2ex-12\,ade^2x}{12\,e^4} + \frac{(cd^4-bd^3e+ad^2e^2)\log(|ex+d|)}{e^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*c*e^3*x^4 - 4*c*d*e^2*x^3 + 4*b*e^3*x^3 + 6*c*d^2*e*x^2 - 6*b*d*e^
2*x^2 + 6*a*e^3*x^2 - 12*c*d^3*x + 12*b*d^2*e*x - 12*a*d*e^2*x)/e^4 + (c*d
^4 - b*d^3*e + a*d^2*e^2)*log(abs(e*x + d))/e^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = x^3\left(\frac{b}{3\,e}-\frac{c\,d}{3\,e^2}\right)+x^2\left(\frac{a}{2\,e}-\frac{d\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)}{2\,e}\right)+\frac{c\,x^4}{4\,e} + \frac{\ln(d+e\,x)\,(c\,d^4-b\,d^3\,e+a\,d^2\,e^2)}{e^5}-\frac{d\,x\left(\frac{a}{e}-\frac{d\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)}{e}\right)}{e}$$

input
```
int((x^2*(a + b*x + c*x^2))/(d + e*x),x)
```

output
```
x^3*(b/(3*e) - (c*d)/(3*e^2)) + x^2*(a/(2*e) - (d*(b/e - (c*d)/e^2))/(2*e)
) + (c*x^4)/(4*e) + (log(d + e*x)*(c*d^4 + a*d^2*e^2 - b*d^3*e))/e^5 - (d*
x*(a/e - (d*(b/e - (c*d)/e^2))/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

$$\int \frac{x^2(a+bx+cx^2)}{d+ex}\,dx = \frac{12\log(ex+d)\,a\,d^2e^2 - 12\log(ex+d)\,b\,d^3e + 12\log(ex+d)\,c\,d^4 - 12ad\,e^3x + 6a\,e^4x^2 + 12b\,d^2e^2x - 6bd}{12e^5}$$

input
```
int(x^2*(c*x^2+b*x+a)/(e*x+d),x)
```

output
```
(12*log(d + e*x)*a*d**2*e**2 - 12*log(d + e*x)*b*d**3*e + 12*log(d + e*x)*
c*d**4 - 12*a*d*e**3*x + 6*a*e**4*x**2 + 12*b*d**2*e**2*x - 6*b*d*e**3*x**
2 + 4*b*e**4*x**3 - 12*c*d**3*e*x + 6*c*d**2*e**2*x**2 - 4*c*d*e**3*x**3 +
 3*c*e**4*x**4)/(12*e**5)
```

3.3 $\int \frac{x(a+bx+cx^2)}{d+ex} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{x(a+bx+cx^2)}{d+ex} \, dx = \frac{(cd^2-bde+ae^2)\,x}{e^3} - \frac{(cd-be)x^2}{2e^2} + \frac{cx^3}{3e} - \frac{d(cd^2-bde+ae^2)\log(d+ex)}{e^4}$$

output
```
(a*e^2-b*d*e+c*d^2)*x/e^3-1/2*(-b*e+c*d)*x^2/e^2+1/3*c*x^3/e-d*(a*e^2-b*d*
e+c*d^2)*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx+cx^2)}{d+ex} \, dx = \frac{ex(3e(-2bd+2ae+bex)+c(6d^2-3dex+2e^2x^2))-6(cd^3+de(-bd+ae))\log(d+ex)}{6e^4}$$

input
```
Integrate[(x*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
(e*x*(3*e*(-2*b*d + 2*a*e + b*e*x) + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - 6*
(c*d^3 + d*e*(-(b*d) + a*e))*Log[d + e*x])/(6*e^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(-\frac{d(ae^2-bde+cd^2)}{e^3(d+ex)}+\frac{ae^2-bde+cd^2}{e^3}+\frac{x(be-cd)}{e^2}+\frac{cx^2}{e}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)}{e^3}-\frac{x^2(cd-be)}{2e^2}+\frac{cx^3}{3e}$$

input
```
Int[(x*(a + b*x + c*x^2))/(d + e*x),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/e^3 - ((c*d - b*e)*x^2)/(2*e^2) + (c*x^3)/(3*e
) - (d*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

method	result	size
norman	$\frac{(a\,e^2-bde+c\,d^2)x}{e^3}+\frac{c\,x^3}{3e}+\frac{(be-cd)x^2}{2e^2}-\frac{d(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^4}$	76
default	$\frac{\frac{1}{3}c\,e^2x^3+\frac{1}{2}b\,e^2x^2-\frac{1}{2}cd\,x^2e+a\,e^2x-bdxe+c\,d^2x}{e^3}-\frac{d(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^4}$	79
risch	$\frac{c\,x^3}{3e}+\frac{b\,x^2}{2e}-\frac{cd\,x^2}{2e^2}+\frac{ax}{e}-\frac{bdx}{e^2}+\frac{c\,d^2x}{e^3}-\frac{d\ln(ex+d)a}{e^2}+\frac{d^2\ln(ex+d)b}{e^3}-\frac{d^3\ln(ex+d)c}{e^4}$	95
parallelrisch	$-\frac{-2c\,e^3x^3-3x^2b\,e^3+3cd\,e^2x^2+6\ln(ex+d)ad\,e^2-6\ln(ex+d)b\,d^2e+6\ln(ex+d)c\,d^3-6a\,e^3x+6dxb\,e^2-6c\,d^2ex}{6e^4}$	96

input
```
int(x*(c*x^2+b*x+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(a*e^2-b*d*e+c*d^2)*x/e^3+1/3*c*x^3/e+1/2/e^2*(b*e-c*d)*x^2-d*(a*e^2-b*d*e
+c*d^2)*ln(e*x+d)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx = \frac{2\,ce^3x^3-3\,(cde^2-be^3)x^2+6\,(cd^2e-bde^2+ae^3)x-6\,(cd^3-bd^2e+ade^2)\log\left(ex+d\right)}{6\,e^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/6*(2*c*e^3*x^3 - 3*(c*d*e^2 - b*e^3)*x^2 + 6*(c*d^2*e - b*d*e^2 + a*e^3)
*x - 6*(c*d^3 - b*d^2*e + a*d*e^2)*log(e*x + d))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx = \frac{cx^3}{3e} - \frac{d(ae^2-bde+cd^2)\log(d+ex)}{e^4} + x^2\left(\frac{b}{2e}-\frac{cd}{2e^2}\right) + x\left(\frac{a}{e}-\frac{bd}{e^2}+\frac{cd^2}{e^3}\right)$$

input
```
integrate(x*(c*x**2+b*x+a)/(e*x+d),x)
```

output
```
c*x**3/(3*e) - d*(a*e**2 - b*d*e + c*d**2)*log(d + e*x)/e**4 + x**2*(b/(2*
e) - c*d/(2*e**2)) + x*(a/e - b*d/e**2 + c*d**2/e**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx = \frac{2\,ce^2x^3 - 3\,(cde-be^2)x^2 + 6\,(cd^2-bde+ae^2)x}{6\,e^3} - \frac{(cd^3-bd^2e+ade^2)\log(ex+d)}{e^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/6*(2*c*e^2*x^3 - 3*(c*d*e - b*e^2)*x^2 + 6*(c*d^2 - b*d*e + a*e^2)*x)/e^
3 - (c*d^3 - b*d^2*e + a*d*e^2)*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx = \frac{2\,ce^2x^3 - 3\,cdex^2 + 3\,be^2x^2 + 6\,cd^2x - 6\,bdex + 6\,ae^2x}{6\,e^3} - \frac{(cd^3 - bd^2e + ade^2)\log(|ex+d|)}{e^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/6*(2*c*e^2*x^3 - 3*c*d*e*x^2 + 3*b*e^2*x^2 + 6*c*d^2*x - 6*b*d*e*x + 6*a
*e^2*x)/e^3 - (c*d^3 - b*d^2*e + a*d*e^2)*log(abs(e*x + d))/e^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx = x^2\left(\frac{b}{2\,e} - \frac{c\,d}{2\,e^2}\right) + x\left(\frac{a}{e} - \frac{d\left(\frac{b}{e} - \frac{c\,d}{e^2}\right)}{e}\right) - \frac{\ln(d+e\,x)\,\left(c\,d^3 - b\,d^2\,e + a\,d\,e^2\right)}{e^4} + \frac{c\,x^3}{3\,e}$$

input
```
int((x*(a + b*x + c*x^2))/(d + e*x),x)
```

output
```
x^2*(b/(2*e) - (c*d)/(2*e^2)) + x*(a/e - (d*(b/e - (c*d)/e^2))/e) - (log(d
 + e*x)*(c*d^3 + a*d*e^2 - b*d^2*e))/e^4 + (c*x^3)/(3*e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

$$\int \frac{x(a+bx+cx^2)}{d+ex}\,dx$$
$$= \frac{-6\log(ex+d)\,ad\,e^2+6\log(ex+d)\,b\,d^2e-6\log(ex+d)\,c\,d^3+6a\,e^3x-6bd\,e^2x+3b\,e^3x^2+6c\,d^2ex-}{6e^4}$$

input
```
int(x*(c*x^2+b*x+a)/(e*x+d),x)
```

output
```
( - 6*log(d + e*x)*a*d*e**2 + 6*log(d + e*x)*b*d**2*e - 6*log(d + e*x)*c*d
**3 + 6*a*e**3*x - 6*b*d*e**2*x + 3*b*e**3*x**2 + 6*c*d**2*e*x - 3*c*d*e**
2*x**2 + 2*c*e**3*x**3)/(6*e**4)
```

3.4 $\int \frac{a+bx+cx^2}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{a+bx+cx^2}{d+ex}\,dx=-\frac{(cd-be)x}{e^2}+\frac{cx^2}{2e}+\frac{(cd^2-bde+ae^2)\log(d+ex)}{e^3}$$

output
```
-(-b*e+c*d)*x/e^2+1/2*c*x^2/e+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{d+ex}\,dx=\frac{ex(-2cd+2be+cex)+2(cd^2+e(-bd+ae))\log(d+ex)}{2e^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(d + e*x),x]
```

output
```
(e*x*(-2*c*d + 2*b*e + c*e*x) + 2*(c*d^2 + e*(-(b*d) + a*e))*Log[d + e*x])
/(2*e^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{d+ex} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{ae^2-bde+cd^2}{e^2(d+ex)}+\frac{be-cd}{e^2}+\frac{cx}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^3}-\frac{x(cd-be)}{e^2}+\frac{cx^2}{2e}$$

input
```
Int[(a + b*x + c*x^2)/(d + e*x),x]
```

output
```
-(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + ((c*d^2 - b*d*e + a*e^2)*Log[d +
e*x])/e^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{1}{2}ce\,x^2+bex-cdx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^3}$	49
norman	$\frac{(be-cd)x}{e^2}+\frac{c\,x^2}{2e}+\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{e^3}$	50
risch	$\frac{c\,x^2}{2e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{\ln(ex+d)a}{e}-\frac{\ln(ex+d)bd}{e^2}+\frac{\ln(ex+d)c\,d^2}{e^3}$	63
parallelrisch	$\frac{x^2c\,e^2+2\ln(ex+d)a\,e^2-2\ln(ex+d)bde+2\ln(ex+d)c\,d^2+2xb\,e^2-2cdxe}{2e^3}$	63

input
```
int((c*x^2+b*x+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/2*c*e*x^2+b*e*x-c*d*x)+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{d+ex}\,dx=\frac{ce^2x^2-2\,(cde-be^2)x+2\,(cd^2-bde+ae^2)\log\left(ex+d\right)}{2\,e^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*(c*e^2*x^2 - 2*(c*d*e - b*e^2)*x + 2*(c*d^2 - b*d*e + a*e^2)*log(e*x +
 d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{d+ex}\,dx = \frac{cx^2}{2e}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)+\frac{(ae^2-bde+cd^2)\log(d+ex)}{e^3}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d),x)
```

output
```
c*x**2/(2*e) + x*(b/e - c*d/e**2) + (a*e**2 - b*d*e + c*d**2)*log(d + e*x)
/e**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{a+bx+cx^2}{d+ex}\,dx = \frac{cex^2-2\,(cd-be)x}{2\,e^2}+\frac{(cd^2-bde+ae^2)\log(ex+d)}{e^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*(c*e*x^2 - 2*(c*d - b*e)*x)/e^2 + (c*d^2 - b*d*e + a*e^2)*log(e*x + d)
/e^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{a+bx+cx^2}{d+ex}\,dx = \frac{cex^2-2\,cdx+2\,bex}{2\,e^2}+\frac{(cd^2-bde+ae^2)\log(|ex+d|)}{e^3}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/2*(c*e*x^2 - 2*c*d*x + 2*b*e*x)/e^2 + (c*d^2 - b*d*e + a*e^2)*log(abs(e*
x + d))/e^3
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{a+bx+cx^2}{d+ex}\,dx = x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^2}{2\,e}+\frac{\ln\left(d+e\,x\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^3}$$

input
```
int((a + b*x + c*x^2)/(d + e*x),x)
```

output
```
x*(b/e - (c*d)/e^2) + (c*x^2)/(2*e) + (log(d + e*x)*(a*e^2 + c*d^2 - b*d*e
))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{d+ex}\,dx = \frac{2\log(ex+d)\,a\,e^2-2\log(ex+d)\,bde+2\log(ex+d)\,c\,d^2+2b\,e^2x-2cdex+c\,e^2x^2}{2e^3}$$

input
```
int((c*x^2+b*x+a)/(e*x+d),x)
```

output
```
(2*log(d + e*x)*a*e**2 - 2*log(d + e*x)*b*d*e + 2*log(d + e*x)*c*d**2 + 2*
b*e**2*x - 2*c*d*e*x + c*e**2*x**2)/(2*e**3)
```

3.5 $\int \frac{a+bx+cx^2}{x(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx=\frac{cx}{e}+\frac{a\log(x)}{d}-\frac{(cd^2-bde+ae^2)\log(d+ex)}{de^2}$$

output
```
c*x/e+a*ln(x)/d-(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/d/e^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx=\frac{cx}{e}+\frac{a\log(x)}{d}+\frac{(-cd^2+bde-ae^2)\log(d+ex)}{de^2}$$

input
```
Integrate[(a + b*x + c*x^2)/(x*(d + e*x)),x]
```

output
```
(c*x)/e + (a*Log[x])/d + ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(d*e^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x(d+ex)} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{-ae^2+bde-cd^2}{de(d+ex)}+\frac{a}{dx}+\frac{c}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{de^2}+\frac{a\log(x)}{d}+\frac{cx}{e}$$

input
```
Int[(a + b*x + c*x^2)/(x*(d + e*x)),x]
```

output
```
(c*x)/e + (a*Log[x])/d - ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(d*e^2)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{cx}{e}+\frac{(-a\,e^{2}+bde-c\,d^{2})\ln(ex+d)}{d\,e^{2}}+\frac{a\ln(x)}{d}$	45
norman	$\frac{cx}{e}+\frac{a\ln(x)}{d}-\frac{(a\,e^{2}-bde+c\,d^{2})\ln(ex+d)}{d\,e^{2}}$	45
risch	$\frac{cx}{e}-\frac{\ln(ex+d)a}{d}+\frac{\ln(ex+d)b}{e}-\frac{cd\ln(ex+d)}{e^{2}}+\frac{a\ln(-x)}{d}$	53
parallelrisch	$\frac{a\ln(x)e^{2}-\ln(ex+d)a\,e^{2}+\ln(ex+d)bde-\ln(ex+d)c\,d^{2}+cdxe}{d\,e^{2}}$	55

input
```
int((c*x^2+b*x+a)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
c*x/e+(-a*e^2+b*d*e-c*d^2)/d/e^2*ln(e*x+d)+a*ln(x)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx=\frac{cdex+ae^2\log(x)-(cd^2-bde+ae^2)\log(ex+d)}{de^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(e*x+d),x, algorithm="fricas")
```

output
```
(c*d*e*x + a*e^2*log(x) - (c*d^2 - b*d*e + a*e^2)*log(e*x + d))/(d*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx=\frac{a\log(x)}{d}+\frac{cx}{e}-\frac{(ae^2-bde+cd^2)\log\left(x+\frac{ade+\frac{d(ae^2-bde+cd^2)}{e}}{2ae^2-bde+cd^2}\right)}{de^2}$$

input
```
integrate((c*x**2+b*x+a)/x/(e*x+d),x)
```

output
```
a*log(x)/d + c*x/e - (a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e + d*(a*e**2
- b*d*e + c*d**2)/e)/(2*a*e**2 - b*d*e + c*d**2))/(d*e**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx = \frac{cx}{e} + \frac{a\log(x)}{d} - \frac{(cd^2-bde+ae^2)\log(ex+d)}{de^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(e*x+d),x, algorithm="maxima")
```

output
```
c*x/e + a*log(x)/d - (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(d*e^2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx = \frac{cx}{e} + \frac{a\log(|x|)}{d} - \frac{(cd^2-bde+ae^2)\log(|ex+d|)}{de^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(e*x+d),x, algorithm="giac")
```

output
```
c*x/e + a*log(abs(x))/d - (c*d^2 - b*d*e + a*e^2)*log(abs(e*x + d))/(d*e^2
)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx = \frac{a\,\ln(x)}{d} - \ln(d+e\,x)\left(\frac{a}{d}-\frac{b}{e}+\frac{c\,d}{e^2}\right)+\frac{c\,x}{e}$$

input
```
int((a + b*x + c*x^2)/(x*(d + e*x)),x)
```

output
```
(a*log(x))/d - log(d + e*x)*(a/d - b/e + (c*d)/e^2) + (c*x)/e
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{a+bx+cx^2}{x(d+ex)}\,dx = \frac{-\log(ex+d)\,a\,e^2+\log(ex+d)\,bde-\log(ex+d)\,c\,d^2+\log(x)\,a\,e^2+cdex}{d\,e^2}$$

input
```
int((c*x^2+b*x+a)/x/(e*x+d),x)
```

output
```
( - log(d + e*x)*a*e**2 + log(d + e*x)*b*d*e - log(d + e*x)*c*d**2 + log(x
)*a*e**2 + c*d*e*x)/(d*e**2)
```

3.6 $\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx=-\frac{a}{dx}+\frac{(bd-ae)\log(x)}{d^2}-\left(\frac{b}{d}-\frac{c}{e}-\frac{ae}{d^2}\right)\log(d+ex)$$

output
```
-a/d/x+(-a*e+b*d)*ln(x)/d^2-(b/d-c/e-a*e/d^2)*ln(e*x+d)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx=-\frac{a}{dx}+\frac{(bd-ae)\log(x)}{d^2}+\frac{(cd^2-bde+ae^2)\log(d+ex)}{d^2e}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^2*(d + e*x)),x]
```

output
```
-(a/(d*x)) + ((b*d - a*e)*Log[x])/d^2 + ((c*d^2 - b*d*e + a*e^2)*Log[d + e
*x])/(d^2*e)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2(d+ex)} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{ae^2-bde+cd^2}{d^2(d+ex)}+\frac{bd-ae}{d^2x}+\frac{a}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\log(d+ex)\left(-\frac{ae}{d^2}+\frac{b}{d}-\frac{c}{e}\right)+\frac{\log(x)(bd-ae)}{d^2}-\frac{a}{dx}$$

input
```
Int[(a + b*x + c*x^2)/(x^2*(d + e*x)),x]
```

output
```
-(a/(d*x)) + ((b*d - a*e)*Log[x])/d^2 - (b/d - c/e - (a*e)/d^2)*Log[d + e*
x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{d^2e}-\frac{a}{dx}+\frac{(-ae+bd)\ln(x)}{d^2}$	54
norman	$-\frac{a}{dx}+\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{d^2e}-\frac{(ae-bd)\ln(x)}{d^2}$	55
parallelrisch	$-\frac{\ln(x)xa\,e^2-\ln(x)xbde-\ln(ex+d)xa\,e^2+\ln(ex+d)xbde-\ln(ex+d)xc\,d^2+ade}{x\,d^2e}$	70
risch	$-\frac{a}{dx}+\frac{e\ln(-ex-d)a}{d^2}-\frac{\ln(-ex-d)b}{d}+\frac{\ln(-ex-d)c}{e}-\frac{\ln(x)ae}{d^2}+\frac{\ln(x)b}{d}$	71

input
```
int((c*x^2+b*x+a)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(a*e^2-b*d*e+c*d^2)/d^2/e*ln(e*x+d)-a/d/x+(-a*e+b*d)*ln(x)/d^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx=-\frac{ade-(cd^2-bde+ae^2)x\log(ex+d)-(bde-ae^2)x\log(x)}{d^2ex}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
-(a*d*e - (c*d^2 - b*d*e + a*e^2)*x*log(e*x + d) - (b*d*e - a*e^2)*x*log(x
))/(d^2*e*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(37) = 74.

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.47

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx = -\frac{a}{dx} - \frac{(ae-bd)\log\left(x+\frac{ade-bd^2-d(ae-bd)}{2ae^2-2bde+cd^2}\right)}{d^2} + \frac{(ae^2-bde+cd^2)\log\left(x+\frac{ade-bd^2+\frac{d(ae^2-bde+cd^2)}{e}}{2ae^2-2bde+cd^2}\right)}{d^2e}$$

input
```
integrate((c*x**2+b*x+a)/x**2/(e*x+d),x)
```

output
```
-a/(d*x) - (a*e - b*d)*log(x + (a*d*e - b*d**2 - d*(a*e - b*d))/(2*a*e**2
- 2*b*d*e + c*d**2))/d**2 + (a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e - b*d
**2 + d*(a*e**2 - b*d*e + c*d**2)/e)/(2*a*e**2 - 2*b*d*e + c*d**2))/(d**2*
e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx = \frac{(bd-ae)\log(x)}{d^2} - \frac{a}{dx} + \frac{(cd^2-bde+ae^2)\log(ex+d)}{d^2e}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
(b*d - a*e)*log(x)/d^2 - a/(d*x) + (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(d
^2*e)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx=\frac{(bd-ae)\log\left(|x|\right)}{d^2}-\frac{a}{dx}+\frac{(cd^2-bde+ae^2)\log\left(|ex+d|\right)}{d^2e}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
(b*d - a*e)*log(abs(x))/d^2 - a/(d*x) + (c*d^2 - b*d*e + a*e^2)*log(abs(e*
x + d))/(d^2*e)
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx=\ln\left(d+e\,x\right)\,\left(\frac{c}{e}-\frac{b}{d}+\frac{a\,e}{d^2}\right)-\frac{a}{d\,x}-\frac{\ln\left(x\right)\,\left(a\,e-b\,d\right)}{d^2}$$

input
```
int((a + b*x + c*x^2)/(x^2*(d + e*x)),x)
```

output
```
log(d + e*x)*(c/e - b/d + (a*e)/d^2) - a/(d*x) - (log(x)*(a*e - b*d))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x^2(d+ex)}\,dx$$
$$=\frac{\log(ex+d)\,a\,e^2x-\log(ex+d)\,bdex+\log(ex+d)\,c\,d^2x-\log(x)\,a\,e^2x+\log(x)\,bdex-ade}{d^2ex}$$

input
```
int((c*x^2+b*x+a)/x^2/(e*x+d),x)
```

output
```
(log(d + e*x)*a*e**2*x - log(d + e*x)*b*d*e*x + log(d + e*x)*c*d**2*x - lo
g(x)*a*e**2*x + log(x)*b*d*e*x - a*d*e)/(d**2*e*x)
```

3.7 $\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = -\frac{a}{2dx^2} - \frac{bd-ae}{d^2x} + \frac{(cd^2-bde+ae^2)\log(x)}{d^3} - \frac{(cd^2-bde+ae^2)\log(d+ex)}{d^3}$$

output
```
-1/2*a/d/x^2-(-a*e+b*d)/d^2/x+(a*e^2-b*d*e+c*d^2)*ln(x)/d^3-(a*e^2-b*d*e+c
*d^2)*ln(e*x+d)/d^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = -\frac{\frac{d(ad+2bdx-2aex)}{x^2} - 2(cd^2+e(-bd+ae))\log(x) + 2(cd^2-bde+ae^2)\log(d+ex)}{2d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^3*(d + e*x)),x]
```

output
```
-1/2*((d*(a*d + 2*b*d*x - 2*a*e*x))/x^2 - 2*(c*d^2 + e*(-(b*d) + a*e))*Log
[x] + 2*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{ae^2-bde+cd^2}{d^3x}-\frac{e\left(ae^2-bde+cd^2\right)}{d^3(d+ex)}+\frac{bd-ae}{d^2x^2}+\frac{a}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x)\left(ae^2-bde+cd^2\right)}{d^3}-\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{d^3}-\frac{bd-ae}{d^2x}-\frac{a}{2dx^2}$$

input
```
Int[(a + b*x + c*x^2)/(x^3*(d + e*x)),x]
```

output
```
-1/2*a/(d*x^2) - (b*d - a*e)/(d^2*x) + ((c*d^2 - b*d*e + a*e^2)*Log[x])/d^
3 - ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/d^3
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{(ae-bd)x}{d^2}-\frac{a}{2d}}{x^2}+\frac{(a\,e^2-bde+c\,d^2)\ln(x)}{d^3}-\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{d^3}$
default	$-\frac{a}{2d\,x^2}-\frac{-ae+bd}{d^2x}+\frac{(a\,e^2-bde+c\,d^2)\ln(x)}{d^3}-\frac{(a\,e^2-bde+c\,d^2)\ln(ex+d)}{d^3}$
risch	$\frac{\frac{(ae-bd)x}{d^2}-\frac{a}{2d}}{x^2}+\frac{\ln(-x)a\,e^2}{d^3}-\frac{\ln(-x)be}{d^2}+\frac{\ln(-x)c}{d}-\frac{\ln(ex+d)a\,e^2}{d^3}+\frac{\ln(ex+d)be}{d^2}-\frac{\ln(ex+d)c}{d}$
parallelrisch	$\frac{2\ln(x)x^2a\,e^2-2\ln(x)x^2bde+2\ln(x)x^2c\,d^2-2\ln(ex+d)x^2a\,e^2+2\ln(ex+d)x^2bde-2\ln(ex+d)x^2c\,d^2+2adex-2bx\,d^2-a\,d^2}{2x^2d^3}$

input
```
int((c*x^2+b*x+a)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
((a*e-b*d)/d^2*x-1/2*a/d)/x^2+(a*e^2-b*d*e+c*d^2)*ln(x)/d^3-(a*e^2-b*d*e+c
*d^2)*ln(e*x+d)/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = -\frac{2\,(cd^2-bde+ae^2)x^2\log(ex+d)-2\,(cd^2-bde+ae^2)x^2\log(x)+ad^2+2\,(bd^2-ade)x}{2\,d^3x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*(2*(c*d^2 - b*d*e + a*e^2)*x^2*log(e*x + d) - 2*(c*d^2 - b*d*e + a*e^
2)*x^2*log(x) + a*d^2 + 2*(b*d^2 - a*d*e)*x)/(d^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(66) = 132$.

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.40

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = \frac{-ad+x(2ae-2bd)}{2d^2x^2} + \frac{(ae^2-bde+cd^2)\log\left(x+\frac{ade^2-bd^2e+cd^3-d(ae^2-bde+cd^2)}{2ae^3-2bde^2+2cd^2e}\right)}{d^3} - \frac{(ae^2-bde+cd^2)\log\left(x+\frac{ade^2-bd^2e+cd^3+d(ae^2-bde+cd^2)}{2ae^3-2bde^2+2cd^2e}\right)}{d^3}$$

input
```
integrate((c*x**2+b*x+a)/x**3/(e*x+d),x)
```

output
```
(-a*d + x*(2*a*e - 2*b*d))/(2*d**2*x**2) + (a*e**2 - b*d*e + c*d**2)*log(x
 + (a*d*e**2 - b*d**2*e + c*d**3 - d*(a*e**2 - b*d*e + c*d**2))/(2*a*e**3
- 2*b*d*e**2 + 2*c*d**2*e))/d**3 - (a*e**2 - b*d*e + c*d**2)*log(x + (a*d*
e**2 - b*d**2*e + c*d**3 + d*(a*e**2 - b*d*e + c*d**2))/(2*a*e**3 - 2*b*d*
e**2 + 2*c*d**2*e))/d**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = -\frac{(cd^2-bde+ae^2)\log(ex+d)}{d^3} + \frac{(cd^2-bde+ae^2)\log(x)}{d^3} - \frac{ad+2(bd-ae)x}{2d^2x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-(c*d^2 - b*d*e + a*e^2)*log(e*x + d)/d^3 + (c*d^2 - b*d*e + a*e^2)*log(x)
/d^3 - 1/2*(a*d + 2*(b*d - a*e)*x)/(d^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = \frac{(cd^2-bde+ae^2)\log(|x|)}{d^3} - \frac{(cd^2e-bde^2+ae^3)\log(|ex+d|)}{d^3e} - \frac{ad^2+2(bd^2-ade)x}{2d^3x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
(c*d^2 - b*d*e + a*e^2)*log(abs(x))/d^3 - (c*d^2*e - b*d*e^2 + a*e^3)*log(
abs(e*x + d))/(d^3*e) - 1/2*(a*d^2 + 2*(b*d^2 - a*d*e)*x)/(d^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = -\frac{\frac{a}{2d}-\frac{x(ae-bd)}{d^2}}{x^2} - \frac{2\,\mathrm{atanh}\left(\frac{2ex}{d}+1\right)(c\,d^2-b\,d\,e+a\,e^2)}{d^3}$$

input
```
int((a + b*x + c*x^2)/(x^3*(d + e*x)),x)
```

output
```
- (a/(2*d) - (x*(a*e - b*d))/d^2)/x^2 - (2*atanh((2*e*x)/d + 1)*(a*e^2 + c
*d^2 - b*d*e))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{a+bx+cx^2}{x^3(d+ex)}\,dx = \frac{-2\log(ex+d)\,a\,e^2x^2+2\log(ex+d)\,bde\,x^2-2\log(ex+d)\,c\,d^2x^2+2\log(x)\,a\,e^2x^2-2\log(x)\,bde\,x^2+2}{2d^3x^2}$$

input
```
int((c*x^2+b*x+a)/x^3/(e*x+d),x)
```

output
```
( - 2*log(d + e*x)*a*e**2*x**2 + 2*log(d + e*x)*b*d*e*x**2 - 2*log(d + e*x
)*c*d**2*x**2 + 2*log(x)*a*e**2*x**2 - 2*log(x)*b*d*e*x**2 + 2*log(x)*c*d*
*2*x**2 - a*d**2 + 2*a*d*e*x - 2*b*d**2*x)/(2*d**3*x**2)
```

3.8 $\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = -\frac{a}{3dx^3}-\frac{bd-ae}{2d^2x^2}-\frac{cd^2-bde+ae^2}{d^3x} -\frac{e(cd^2-bde+ae^2)\log(x)}{d^4}+\frac{e(cd^2-bde+ae^2)\log(d+ex)}{d^4}$$

output
```
-1/3*a/d/x^3-1/2*(-a*e+b*d)/d^2/x^2-(a*e^2-b*d*e+c*d^2)/d^3/x-e*(a*e^2-b*d
*e+c*d^2)*ln(x)/d^4+e*(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/d^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = -\frac{\frac{d(3dx(bd+2cdx-2bex)+a(2d^2-3dex+6e^2x^2))}{x^3}+6e(cd^2+e(-bd+ae))\log(x)-6e(cd^2+e(-bd+ae))\log(d+e}{6d^4}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^4*(d + e*x)),x]
```

output
```
-1/6*((d*(3*d*x*(b*d + 2*c*d*x - 2*b*e*x) + a*(2*d^2 - 3*d*e*x + 6*e^2*x^2
)))/x^3 + 6*e*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - 6*e*(c*d^2 + e*(-(b*d) +
 a*e))*Log[d + e*x])/d^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{e^2\left(ae^2-bde+cd^2\right)}{d^4(d+ex)}-\frac{e\left(ae^2-bde+cd^2\right)}{d^4x}+\frac{ae^2-bde+cd^2}{d^3x^2}+\frac{bd-ae}{d^2x^3}+\frac{a}{dx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\log(x)\left(ae^2-bde+cd^2\right)}{d^4}+\frac{e\log(d+ex)\left(ae^2-bde+cd^2\right)}{d^4}-\frac{ae^2-bde+cd^2}{d^3x}-\frac{bd-ae}{2d^2x^2}-\frac{a}{3dx^3}$$

input
```
Int[(a + b*x + c*x^2)/(x^4*(d + e*x)),x]
```

output
```
-1/3*a/(d*x^3) - (b*d - a*e)/(2*d^2*x^2) - (c*d^2 - b*d*e + a*e^2)/(d^3*x)
 - (e*(c*d^2 - b*d*e + a*e^2)*Log[x])/d^4 + (e*(c*d^2 - b*d*e + a*e^2)*Log
[d + e*x])/d^4
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

method	result
default	$-\frac{a}{3d\,x^3}-\frac{-ae+bd}{2d^2x^2}-\frac{a\,e^2-bde+c\,d^2}{d^3x}-\frac{e\left(a\,e^2-bde+c\,d^2\right)\ln(x)}{d^4}+\frac{e\left(a\,e^2-bde+c\,d^2\right)\ln(ex+d)}{d^4}$
norman	$\frac{-\frac{a}{3d}+\frac{(ae-bd)x}{2d^2}-\frac{\left(a\,e^2-bde+c\,d^2\right)x^2}{d^3}}{x^3}+\frac{e\left(a\,e^2-bde+c\,d^2\right)\ln(ex+d)}{d^4}-\frac{e\left(a\,e^2-bde+c\,d^2\right)\ln(x)}{d^4}$
risch	$\frac{-\frac{a}{3d}+\frac{(ae-bd)x}{2d^2}-\frac{\left(a\,e^2-bde+c\,d^2\right)x^2}{d^3}}{x^3}+\frac{e^3\ln(-ex-d)a}{d^4}-\frac{e^2\ln(-ex-d)b}{d^3}+\frac{e\ln(-ex-d)c}{d^2}-\frac{e^3\ln(x)a}{d^4}+\frac{e^2\ln(x)b}{d^3}-$
parallelrisch	$-\frac{6\ln(x)x^3a\,e^3-6\ln(x)x^3bd\,e^2+6\ln(x)x^3c\,d^2e-6\ln(ex+d)x^3a\,e^3+6\ln(ex+d)x^3bd\,e^2-6\ln(ex+d)x^3c\,d^2e+6ad\,e^2x^2-6b\,d^2}{6d^4x^3}$

input
```
int((c*x^2+b*x+a)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3-1/2*(-a*e+b*d)/d^2/x^2-(a*e^2-b*d*e+c*d^2)/d^3/x-e*(a*e^2-b*d
*e+c*d^2)*ln(x)/d^4+e*(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx$$
$$=\frac{6\,(cd^2e-bde^2+ae^3)x^3\log\left(ex+d\right)-6\,(cd^2e-bde^2+ae^3)x^3\log\left(x\right)-2\,ad^3-6\,(cd^3-bd^2e+ade^2)x^2}{6\,d^4x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
1/6*(6*(c*d^2*e - b*d*e^2 + a*e^3)*x^3*log(e*x + d) - 6*(c*d^2*e - b*d*e^2
 + a*e^3)*x^3*log(x) - 2*a*d^3 - 6*(c*d^3 - b*d^2*e + a*d*e^2)*x^2 - 3*(b*
d^3 - a*d^2*e)*x)/(d^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(95) = 190$.

Time = 0.41 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.22

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = \frac{-2ad^2+x^2(-6ae^2+6bde-6cd^2)+x(3ade-3bd^2)}{6d^3x^3} - \frac{e(ae^2-bde+cd^2)\log\left(x+\frac{ade^3-bd^2e^2+cd^3e-de(ae^2-bde+cd^2)}{2ae^4-2bde^3+2cd^2e^2}\right)}{d^4} + \frac{e(ae^2-bde+cd^2)\log\left(x+\frac{ade^3-bd^2e^2+cd^3e+de(ae^2-bde+cd^2)}{2ae^4-2bde^3+2cd^2e^2}\right)}{d^4}$$

input
```
integrate((c*x**2+b*x+a)/x**4/(e*x+d),x)
```

output
```
(-2*a*d**2 + x**2*(-6*a*e**2 + 6*b*d*e - 6*c*d**2) + x*(3*a*d*e - 3*b*d**2
))/(6*d**3*x**3) - e*(a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e**3 - b*d**2*
e**2 + c*d**3*e - d*e*(a*e**2 - b*d*e + c*d**2))/(2*a*e**4 - 2*b*d*e**3 +
2*c*d**2*e**2))/d**4 + e*(a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e**3 - b*d
**2*e**2 + c*d**3*e + d*e*(a*e**2 - b*d*e + c*d**2))/(2*a*e**4 - 2*b*d*e**
3 + 2*c*d**2*e**2))/d**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = \frac{(cd^2e-bde^2+ae^3)\log(ex+d)}{d^4} - \frac{(cd^2e-bde^2+ae^3)\log(x)}{d^4} - \frac{2\,ad^2+6\,(cd^2-bde+ae^2)x^2+3\,(bd^2-ade)x}{6\,d^3x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
(c*d^2*e - b*d*e^2 + a*e^3)*log(e*x + d)/d^4 - (c*d^2*e - b*d*e^2 + a*e^3)
*log(x)/d^4 - 1/6*(2*a*d^2 + 6*(c*d^2 - b*d*e + a*e^2)*x^2 + 3*(b*d^2 - a*
d*e)*x)/(d^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = -\frac{(cd^2e-bde^2+ae^3)\log(|x|)}{d^4} + \frac{(cd^2e^2-bde^3+ae^4)\log(|ex+d|)}{d^4e} - \frac{2\,ad^3+6\,(cd^3-bd^2e+ade^2)x^2+3\,(bd^3-ad^2e)x}{6\,d^4x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
-(c*d^2*e - b*d*e^2 + a*e^3)*log(abs(x))/d^4 + (c*d^2*e^2 - b*d*e^3 + a*e^
4)*log(abs(e*x + d))/(d^4*e) - 1/6*(2*a*d^3 + 6*(c*d^3 - b*d^2*e + a*d*e^2
)*x^2 + 3*(b*d^3 - a*d^2*e)*x)/(d^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = \frac{2\,e\,\mathrm{atanh}\left(\frac{e\,(d+2\,e\,x)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{d\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{d^4} - \frac{\frac{a}{3\,d}-\frac{x\,(a\,e-b\,d)}{2\,d^2}+\frac{x^2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{d^3}}{x^3}$$

input
```
int((a + b*x + c*x^2)/(x^4*(d + e*x)),x)
```

output
```
(2*e*atanh((e*(d + 2*e*x)*(a*e^2 + c*d^2 - b*d*e))/(d*(a*e^3 - b*d*e^2 + c
*d^2*e)))*(a*e^2 + c*d^2 - b*d*e))/d^4 - (a/(3*d) - (x*(a*e - b*d))/(2*d^2
) + (x^2*(a*e^2 + c*d^2 - b*d*e))/d^3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

$$\int \frac{a+bx+cx^2}{x^4(d+ex)}\,dx = \frac{6\log(ex+d)\,a\,e^3x^3-6\log(ex+d)\,bd\,e^2x^3+6\log(ex+d)\,c\,d^2e\,x^3-6\log(x)\,a\,e^3x^3+6\log(x)\,bd\,e^2x^3-}{6d^4x^3}$$

input
```
int((c*x^2+b*x+a)/x^4/(e*x+d),x)
```

output
```
(6*log(d + e*x)*a*e**3*x**3 - 6*log(d + e*x)*b*d*e**2*x**3 + 6*log(d + e*x
)*c*d**2*e*x**3 - 6*log(x)*a*e**3*x**3 + 6*log(x)*b*d*e**2*x**3 - 6*log(x)
*c*d**2*e*x**3 - 2*a*d**3 + 3*a*d**2*e*x - 6*a*d*e**2*x**2 - 3*b*d**3*x +
6*b*d**2*e*x**2 - 6*c*d**3*x**2)/(6*d**4*x**3)
```

3.9 $\int \frac{a+bx+cx^2}{x^5(d+ex)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{a+bx+cx^2}{x^5(d+ex)} \, dx = -\frac{a}{4dx^4} - \frac{bd-ae}{3d^2x^3} - \frac{cd^2-bde+ae^2}{2d^3x^2} + \frac{e(cd^2-bde+ae^2)}{d^4x} + \frac{e^2(cd^2-bde+ae^2)\log(x)}{d^5} - \frac{e^2(cd^2-bde+ae^2)\log(d+ex)}{d^5}$$

output
```
-1/4*a/d/x^4-1/3*(-a*e+b*d)/d^2/x^3-1/2*(a*e^2-b*d*e+c*d^2)/d^3/x^2+e*(a*e
^2-b*d*e+c*d^2)/d^4/x+e^2*(a*e^2-b*d*e+c*d^2)*ln(x)/d^5-e^2*(a*e^2-b*d*e+c
*d^2)*ln(e*x+d)/d^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{x^5(d+ex)} \, dx = \frac{\frac{d(a(-3d^3+4d^2ex-6de^2x^2+12e^3x^3)-2dx(3cdx(d-2ex)+b(2d^2-3dex+6e^2x^2)))}{x^4} + 12e^2(cd^2+e(-bd+ae))\log(x) - 12e^2(c}{12d^5}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^5*(d + e*x)),x]
```

output
```
((d*(a*(-3*d^3 + 4*d^2*e*x - 6*d*e^2*x^2 + 12*e^3*x^3) - 2*d*x*(3*c*d*x*(d
 - 2*e*x) + b*(2*d^2 - 3*d*e*x + 6*e^2*x^2))))/x^4 + 12*e^2*(c*d^2 + e*(-(
b*d) + a*e))*Log[x] - 12*e^2*(c*d^2 + e*(-(b*d) + a*e))*Log[d + e*x])/(12*
d^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{e^2(ae^2-bde+cd^2)}{d^5x}-\frac{e^3(ae^2-bde+cd^2)}{d^5(d+ex)}-\frac{e(ae^2-bde+cd^2)}{d^4x^2}+\frac{ae^2-bde+cd^2}{d^3x^3}+\frac{bd-ae}{d^2x^4}+\frac{a}{dx^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\log(x)\left(ae^2-bde+cd^2\right)}{d^5}-\frac{e^2\log(d+ex)\left(ae^2-bde+cd^2\right)}{d^5}+\frac{e(ae^2-bde+cd^2)}{d^4x}-\frac{ae^2-bde+cd^2}{2d^3x^2}-\frac{bd-ae}{3d^2x^3}-\frac{a}{4dx^4}$$

input
```
Int[(a + b*x + c*x^2)/(x^5*(d + e*x)),x]
```

output
```
-1/4*a/(d*x^4) - (b*d - a*e)/(3*d^2*x^3) - (c*d^2 - b*d*e + a*e^2)/(2*d^3*
x^2) + (e*(c*d^2 - b*d*e + a*e^2))/(d^4*x) + (e^2*(c*d^2 - b*d*e + a*e^2)*
Log[x])/d^5 - (e^2*(c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/d^5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a}{4d\,x^4}-\frac{-ae+bd}{3d^2x^3}-\frac{a\,e^2-bde+c\,d^2}{2d^3x^2}+\frac{e\left(a\,e^2-bde+c\,d^2\right)}{d^4x}+\frac{e^2\left(a\,e^2-bde+c\,d^2\right)\ln\left(x\right)}{d^5}-\frac{e^2\left(a\,e^2-bde+c\,d^2\right)\ln(ex+}{d^5}$
norman	$\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)e\,x^3}{d^4}-\frac{a}{4d}+\frac{(ae-bd)x}{3d^2}-\frac{\left(a\,e^2-bde+c\,d^2\right)x^2}{2d^3}}{x^4}+\frac{e^2\left(a\,e^2-bde+c\,d^2\right)\ln\left(x\right)}{d^5}-\frac{e^2\left(a\,e^2-bde+c\,d^2\right)\ln\left(ex+d\right)}{d^5}$
risch	$\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)e\,x^3}{d^4}-\frac{a}{4d}+\frac{(ae-bd)x}{3d^2}-\frac{\left(a\,e^2-bde+c\,d^2\right)x^2}{2d^3}}{x^4}-\frac{e^4\ln\left(ex+d\right)a}{d^5}+\frac{e^3\ln\left(ex+d\right)b}{d^4}-\frac{e^2\ln\left(ex+d\right)c}{d^3}+\frac{e^4\ln\left(-x\right)}{d^5}$
parallelrisch	$-\frac{12\ln\left(ex+d\right)x^4a\,e^4-12\ln\left(ex+d\right)x^4bd\,e^3+12\ln\left(ex+d\right)x^4c\,d^2e^2-12\ln\left(x\right)x^4a\,e^4+12\ln\left(x\right)x^4bd\,e^3-12\ln\left(x\right)x^4c\,d^2e^2-12x^3a}{12d^5x^4}$

input
```
int((c*x^2+b*x+a)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4-1/3*(-a*e+b*d)/d^2/x^3-1/2*(a*e^2-b*d*e+c*d^2)/d^3/x^2+e*(a*e
^2-b*d*e+c*d^2)/d^4/x+e^2*(a*e^2-b*d*e+c*d^2)*ln(x)/d^5-e^2*(a*e^2-b*d*e+c
*d^2)*ln(e*x+d)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = -\frac{12\,(cd^2e^2-bde^3+ae^4)x^4\log(ex+d)-12\,(cd^2e^2-bde^3+ae^4)x^4\log(x)+3\,ad^4-12\,(cd^3e-bd^2e^2}{12\,d^5x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
-1/12*(12*(c*d^2*e^2 - b*d*e^3 + a*e^4)*x^4*log(e*x + d) - 12*(c*d^2*e^2 -
 b*d*e^3 + a*e^4)*x^4*log(x) + 3*a*d^4 - 12*(c*d^3*e - b*d^2*e^2 + a*d*e^3
)*x^3 + 6*(c*d^4 - b*d^3*e + a*d^2*e^2)*x^2 + 4*(b*d^4 - a*d^3*e)*x)/(d^5*
x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(128) = 256.

Time = 0.47 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.04

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = \frac{-3ad^3+x^3\cdot(12ae^3-12bde^2+12cd^2e)+x^2(-6ade^2+6bd^2e-6cd^3)+x(4ad^2e-4bd^3)}{12d^4x^4} + \frac{e^2(ae^2-bde+cd^2)\log\left(x+\frac{ade^4-bd^2e^3+cd^3e^2-de^2(ae^2-bde+cd^2)}{2ae^5-2bde^4+2cd^2e^3}\right)}{d^5} - \frac{e^2(ae^2-bde+cd^2)\log\left(x+\frac{ade^4-bd^2e^3+cd^3e^2+de^2(ae^2-bde+cd^2)}{2ae^5-2bde^4+2cd^2e^3}\right)}{d^5}$$

input
```
integrate((c*x**2+b*x+a)/x**5/(e*x+d),x)
```

output
```
(-3*a*d**3 + x**3*(12*a*e**3 - 12*b*d*e**2 + 12*c*d**2*e) + x**2*(-6*a*d*e
**2 + 6*b*d**2*e - 6*c*d**3) + x*(4*a*d**2*e - 4*b*d**3))/(12*d**4*x**4) +
 e**2*(a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e**4 - b*d**2*e**3 + c*d**3*e
**2 - d*e**2*(a*e**2 - b*d*e + c*d**2))/(2*a*e**5 - 2*b*d*e**4 + 2*c*d**2*
e**3))/d**5 - e**2*(a*e**2 - b*d*e + c*d**2)*log(x + (a*d*e**4 - b*d**2*e*
*3 + c*d**3*e**2 + d*e**2*(a*e**2 - b*d*e + c*d**2))/(2*a*e**5 - 2*b*d*e**
4 + 2*c*d**2*e**3))/d**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = -\frac{(cd^2e^2-bde^3+ae^4)\log(ex+d)}{d^5} + \frac{(cd^2e^2-bde^3+ae^4)\log(x)}{d^5} - \frac{3\,ad^3-12\,(cd^2e-bde^2+ae^3)x^3+6\,(cd^3-bd^2e+ade^2)x^2+4\,(bd^3-ad^2e)x}{12\,d^4x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
-(c*d^2*e^2 - b*d*e^3 + a*e^4)*log(e*x + d)/d^5 + (c*d^2*e^2 - b*d*e^3 + a
*e^4)*log(x)/d^5 - 1/12*(3*a*d^3 - 12*(c*d^2*e - b*d*e^2 + a*e^3)*x^3 + 6*
(c*d^3 - b*d^2*e + a*d*e^2)*x^2 + 4*(b*d^3 - a*d^2*e)*x)/(d^4*x^4)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = \frac{(cd^2e^2-bde^3+ae^4)\log(|x|)}{d^5} - \frac{(cd^2e^3-bde^4+ae^5)\log(|ex+d|)}{d^5e} - \frac{3\,ad^4-12\,(cd^3e-bd^2e^2+ade^3)x^3+6\,(cd^4-bd^3e+ad^2e^2)x^2+4\,(bd^4-ad^3e)x}{12\,d^5x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
(c*d^2*e^2 - b*d*e^3 + a*e^4)*log(abs(x))/d^5 - (c*d^2*e^3 - b*d*e^4 + a*e
^5)*log(abs(e*x + d))/(d^5*e) - 1/12*(3*a*d^4 - 12*(c*d^3*e - b*d^2*e^2 +
a*d*e^3)*x^3 + 6*(c*d^4 - b*d^3*e + a*d^2*e^2)*x^2 + 4*(b*d^4 - a*d^3*e)*x
)/(d^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = -\frac{\frac{a}{4\,d}-\frac{x\,(a\,e-b\,d)}{3\,d^2}+\frac{x^2\,(c\,d^2-b\,d\,e+a\,e^2)}{2\,d^3}-\frac{e\,x^3\,(c\,d^2-b\,d\,e+a\,e^2)}{d^4}}{x^4} - \frac{2\,e^2\,\mathrm{atanh}\left(\frac{e^2\,(d+2\,e\,x)\,(c\,d^2-b\,d\,e+a\,e^2)}{d\,(c\,d^2\,e^2-b\,d\,e^3+a\,e^4)}\right)\,(c\,d^2-b\,d\,e+a\,e^2)}{d^5}$$

input
```
int((a + b*x + c*x^2)/(x^5*(d + e*x)),x)
```

output
```
- (a/(4*d) - (x*(a*e - b*d))/(3*d^2) + (x^2*(a*e^2 + c*d^2 - b*d*e))/(2*d^
3) - (e*x^3*(a*e^2 + c*d^2 - b*d*e))/d^4)/x^4 - (2*e^2*atanh((e^2*(d + 2*e
*x)*(a*e^2 + c*d^2 - b*d*e))/(d*(a*e^4 + c*d^2*e^2 - b*d*e^3)))*(a*e^2 + c
*d^2 - b*d*e))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x^5(d+ex)}\,dx = \frac{-12\log(ex+d)\,a\,e^4x^4+12\log(ex+d)\,bd\,e^3x^4-12\log(ex+d)\,c\,d^2e^2x^4+12\log(x)\,a\,e^4x^4-12\log(x)\,b}{}$$

input
```
int((c*x^2+b*x+a)/x^5/(e*x+d),x)
```

output

```
( - 12*log(d + e*x)*a*e**4*x**4 + 12*log(d + e*x)*b*d*e**3*x**4 - 12*log(d
 + e*x)*c*d**2*e**2*x**4 + 12*log(x)*a*e**4*x**4 - 12*log(x)*b*d*e**3*x**4
 + 12*log(x)*c*d**2*e**2*x**4 - 3*a*d**4 + 4*a*d**3*e*x - 6*a*d**2*e**2*x*
*2 + 12*a*d*e**3*x**3 - 4*b*d**4*x + 6*b*d**3*e*x**2 - 12*b*d**2*e**2*x**3
 - 6*c*d**4*x**2 + 12*c*d**3*e*x**3)/(12*d**5*x**4)
```

3.10 $\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx = -\frac{1}{x} + \log(3-x)$$

output
```
-1/x+ln(3-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx = -\frac{1}{x} + \log(3-x)$$

input
```
Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

output
```
-x^(-1) + Log[3 - x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-3}{(x-3)x^2} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{1}{x^2}+\frac{1}{x-3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(3-x)-\frac{1}{x}$$

input
```
Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

output
```
-x^(-1) + Log[3 - x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\ln\left(-3+x\right)-\frac{1}{x}$	11
norman	$\ln\left(-3+x\right)-\frac{1}{x}$	11
risch	$\ln\left(-3+x\right)-\frac{1}{x}$	11
meijerg	$\ln\left(1-\frac{x}{3}\right)-\frac{1}{x}$	13
parallelrisch	$\frac{\ln(-3+x)x-1}{x}$	13

input
```
int((x^2+x-3)/(-3+x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
ln(-3+x)-1/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx=\frac{x\log\left(x-3\right)-1}{x}$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
```

output
```
(x*log(x - 3) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \log{(x-3)} - \frac{1}{x}$$

input
```
integrate((x**2+x-3)/(-3+x)/x**2,x)
```

output
```
log(x - 3) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log{(x-3)}$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
```

output
```
-1/x + log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log{(|x-3|)}$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
```

output
```
-1/x + log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \ln(x-3) - \frac{1}{x}$$

input
```
int((x + x^2 - 3)/(x^2*(x - 3)),x)
```

output
```
log(x - 3) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \frac{\log(x-3)\,x-1}{x}$$

input
```
int((x^2+x-3)/(-3+x)/x^2,x)
```

output
```
(log(x - 3)*x - 1)/x
```

3.11 $\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx = -\frac{1}{x}+3\log(2-x)+2\log(x)$$

output
```
-1/x+3*ln(2-x)+2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} \, dx = -\frac{1}{x}+3\log(2-x)+2\log(x)$$

input
```
Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

output
```
-x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2-3x-2}{(x-2)x^2} \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{1}{x^2}+\frac{2}{x}+\frac{3}{x-2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

input
```
Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

output
```
-x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$3\ln\left(x-2\right)-\frac{1}{x}+2\ln\left(x\right)$	17
norman	$3\ln\left(x-2\right)-\frac{1}{x}+2\ln\left(x\right)$	17
risch	$3\ln\left(x-2\right)-\frac{1}{x}+2\ln\left(x\right)$	17
parallelrisch	$\frac{2\ln(x)x+3\ln(x-2)x-1}{x}$	19
meijerg	$3\ln\left(-\frac{x}{2}+1\right)+2\ln\left(x\right)-2\ln\left(2\right)+2i\pi-\frac{1}{x}$	27

input
```
int((5*x^2-3*x-2)/(x-2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
3*ln(x-2)-1/x+2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx=\frac{3\,x\log\left(x-2\right)+2\,x\log\left(x\right)-1}{x}$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")
```

output
```
(3*x*log(x - 2) + 2*x*log(x) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = 2\log(x) + 3\log(x-2) - \frac{1}{x}$$

input
```
integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
```

output
```
2*log(x) + 3*log(x - 2) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = -\frac{1}{x} + 3\log(x-2) + 2\log(x)$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")
```

output
```
-1/x + 3*log(x - 2) + 2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = -\frac{1}{x} + 3\log(|x-2|) + 2\log(|x|)$$

input
```
integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")
```

output
```
-1/x + 3*log(abs(x - 2)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = 3\,\ln(x-2)+2\,\ln(x)-\frac{1}{x}$$

input
```
int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)
```

output
```
3*log(x - 2) + 2*log(x) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx = \frac{3\log(x-2)\,x+2\log(x)\,x-1}{x}$$

input
```
int((5*x^2-3*x-2)/(-2+x)/x^2,x)
```

output
```
(3*log(x - 2)*x + 2*log(x)*x - 1)/x
```

3.12 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 362

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = -\frac{(64c^3d^3-16bc^2d^2e-5b^3e^3-4bce^2(2bd-ae)-2ce(16c^2d^2+5b^2e^2+4ce(2bd-ae))\,x)\sqrt{a+bx+cx^2}}{64c^3e^4} - \frac{(14cd+5be)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{(128c^4d^4-5b^4e^4-8b^2ce^3(bd-3ae)-64c^3d^2e(bd-ae)-16c^2e^2(bd-ae)^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}e^5} - \frac{d^3\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

output
```
-1/64*(64*c^3*d^3-16*b*c^2*d^2*e-5*b^3*e^3-4*b*c*e^2*(-a*e+2*b*d)-2*c*e*(1
6*c^2*d^2+5*b^2*e^2+4*c*e*(-a*e+2*b*d))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^4-1/2
4*(5*b*e+14*c*d)*(c*x^2+b*x+a)^(3/2)/c^2/e^2+1/4*(e*x+d)*(c*x^2+b*x+a)^(3/
2)/c/e^2+1/128*(128*c^4*d^4-5*b^4*e^4-8*b^2*c*e^3*(-3*a*e+b*d)-64*c^3*d^2*
e*(-a*e+b*d)-16*c^2*e^2*(-a*e+b*d)^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(7/2)/e^5-d^3*(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-
2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.88

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{2\sqrt{c}e\sqrt{a+x(b+cx)}(15b^3e^3-2bce^2(-12bd+26ae+5bex)-16c^3(12d^3-6d^2ex+4de^2x^2-3e^3x^3)+}{}$$

input
```
Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3 - 2*b*c*e^2*(-12*b*d + 26*a
*e + 5*b*e*x) - 16*c^3*(12*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 - 3*e^3*x^3) + 8*
c^2*e*(a*e*(-8*d + 3*e*x) + b*(6*d^2 - 2*d*e*x + e^2*x^2))) - 768*c^(7/2)*
d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - 3*(128*c^4*d^4 - 5*b^4*e^4
 - 8*b^2*c*e^3*(b*d - 3*a*e) - 64*c^3*d^2*e*(b*d - a*e) - 16*c^2*e^2*(b*d
- a*e)^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7/2)*e
^5)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1267, 27, 2184, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$\Big\downarrow \ 1267$$
$$\frac{\int -\frac{\sqrt{cx^2+bx+a}\left(e^2(14cd+5be)x^2+2e\left(3cd^2+e(4bd+ae)\right)x+de(3bd+2ae)\right)}{2(d+ex)}dx}{4ce^3}+\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}$$
$$\Big\downarrow \ 27$$

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\int \frac{\sqrt{cx^2+bx+a}\left(e^2(14cd+5be)x^2+2e\left(3cd^2+e(4bd+ae)\right)x+de(3bd+2ae)\right)}{d+ex}dx}{8ce^3}$$

$\big\downarrow$ 2184

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\frac{\int -\frac{3e^3\left(d\left(5eb^2+8cdb-4ace\right)+\left(16c^2d^2+5b^2e^2+4ce(2bd-ae)\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{3ce^2}+\frac{e\left(a+bx+cx^2\right)^{3/2}(5be+14cd)}{3c}}{8ce^3}$$

$\big\downarrow$ 27

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{3/2}(5be+14cd)}{3c}-\frac{e\int \frac{\left(d\left(5eb^2+8cdb-4ace\right)+\left(16c^2d^2+5b^2e^2+4ce(2bd-ae)\right)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{2c}}{8ce^3}$$

$\big\downarrow$ 1231

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{3/2}(5be+14cd)}{3c}-\frac{e\left(-\frac{\int \frac{d\left(5e^3b^4+8cde^2b^3+8ce\left(2cd^2-3ae^2\right)b^2-32c^2d\left(2cd^2+ae^2\right)b+16ac^2e\left(4cd^2+ae^2\right)\right)-\left(128c^4d^4-64c^3e(bd-ae)d^2-5\cdots\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{4ce^2}\cdots\right.}{\cdots}}{8ce^3}$$

$\big\downarrow$ 27

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{3/2}(5be+14cd)}{3c}-\frac{e\left(-\frac{\int \frac{d\left(5e^3b^4+8cde^2b^3+8ce\left(2cd^2-3ae^2\right)b^2-32c^2d\left(2cd^2+ae^2\right)b+16ac^2e\left(4cd^2+ae^2\right)\right)-\left(128c^4d^4-64c^3e(bd-ae)d^2-5\cdots\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{8ce^2}\cdots\right.}{\cdots}}{8ce^3}$$

$\big\downarrow$ 1269

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{3/2}(5be+14cd)}{3c}-\frac{e\left(-\frac{\frac{128c^3d^3\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(-8b^2ce^3(bd-3ae)-64c^3d^2e(bd-ae)-16c^2e^2(bd-ae)^2-5b^4e^4+\cdots\right.}{e}}{8ce^2}\cdots\right.}{\cdots}}{8ce^3}$$

$\big\downarrow$ 1092

$$\frac{\frac{e(a+bx+cx^2)^{3/2}(5be+14cd)}{3c}-\frac{\frac{(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}-\frac{e\left(-\frac{\frac{128c^3d^3(ae^2-bde+cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(-8b^2ce^3(bd-3ae)-64c^3d^2e(bd-ae)-16c^2e^2(bd-ae)^2-5b^4e^4\cdots\right.}{e}}{8ce^2}\right.}{\cdots}}{\cdots}}{8ce^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{e(a+bx+cx^2)^{3/2}(5be+14cd)}{3c}-\frac{\frac{(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}-\frac{e\left(-\frac{\frac{128c^3d^3(ae^2-bde+cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8b^2ce^3(bd-3ae)-64c^3d^2e(bd\cdots\right.}{\sqrt{c}e}}{8ce^2}\right.}{\cdots}}{\cdots}}{8ce^3}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{e(a+bx+cx^2)^{3/2}(5be+14cd)}{3c}-\frac{\frac{(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}-\frac{e\left(-\frac{-\frac{256c^3d^3(ae^2-bde+cd^2)\int\frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{\operatorname{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\right)\cdots}{\cdots}}{8ce^2}\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{e(a+bx+cx^2)^{3/2}(5be+14cd)}{3c}-\frac{\frac{(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}-\frac{e\left(-\frac{\frac{128c^3d^3\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8b^2ce^3(bd\cdots\right.}{\cdots}}{8ce^2}\right.}{\cdots}}{\cdots}}{8\cdots}$$

input
```
Int[(x^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
((d + e*x)*(a + b*x + c*x^2)^(3/2))/(4*c*e^2) - ((e*(14*c*d + 5*b*e)*(a +
b*x + c*x^2)^(3/2))/(3*c) - (e*(-1/4*((64*c^3*d^3 - 16*b*c^2*d^2*e - 5*b^3
*e^3 - 4*b*c*e^2*(2*b*d - a*e) - 2*c*e*(16*c^2*d^2 + 5*b^2*e^2 + 4*c*e*(2*
b*d - a*e))*x)*Sqrt[a + b*x + c*x^2])/(c*e^2) - (-(((128*c^4*d^4 - 5*b^4*e
^4 - 8*b^2*c*e^3*(b*d - 3*a*e) - 64*c^3*d^2*e*(b*d - a*e) - 16*c^2*e^2*(b*
d - a*e)^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
c]*e)) + (128*c^3*d^3*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (
2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)
/(8*c*e^2)))/(2*c))/(8*c*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{(-48c^3x^3e^3-8b\,c^2e^3x^2+64c^3d\,e^2x^2-24a\,c^2e^3x+10x\,b^2c\,e^3+16b\,c^2d\,e^2x-96d^2e\,c^3x+52abc\,e^3+64d\,e^2a\,c^2-15b^3e^3-24d\,e^2b^2c-\cdots}{192c^3e^4}$
default	$\frac{d^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{e^3}+\frac{\frac{x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\cdots\right)}{\cdots}\right)}{8c}\cdots}{\cdots}$

input
```
int(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*c^3*e^3*x^3-8*b*c^2*e^3*x^2+64*c^3*d*e^2*x^2-24*a*c^2*e^3*x+10
*b^2*c*e^3*x+16*b*c^2*d*e^2*x-96*c^3*d^2*e*x+52*a*b*c*e^3+64*a*c^2*d*e^2-1
5*b^3*e^3-24*b^2*c*d*e^2-48*b*c^2*d^2*e+192*c^3*d^3)/c^3*(c*x^2+b*x+a)^(1/
2)/e^4-1/128/e^4/c^3*((16*a^2*c^2*e^4-24*a*b^2*c*e^4-32*a*b*c^2*d*e^3-64*a
*c^3*d^2*e^2+5*b^4*e^4+8*b^3*c*d*e^3+16*b^2*c^2*d^2*e^2+64*b*c^3*d^3*e-128
*c^4*d^4)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-128*d^3*(a
*e^2-b*d*e+c*d^2)*c^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d
*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(
x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate(x**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^3\sqrt{c\,x^2+bx+a}}{ex+d}dx$$

input
```
int(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

output
```
int(x^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

3.13 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 255

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx &= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x)\sqrt{a+bx+cx^2}}{8c^2e^3}+\frac{(a+bx+cx^2)^{3/2}}{3ce}\\ &\quad-\frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}e^4}\\ &\quad+\frac{d^2\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}\end{aligned}$$

output
```
1/8*((-b*e+2*c*d)*(b*e+4*c*d)-2*c*e*(b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c^2
/e^3+1/3*(c*x^2+b*x+a)^(3/2)/c/e-1/16*(16*c^3*d^3-b^3*e^3-2*b*c*e^2*(-2*a*
e+b*d)-8*c^2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)/e^4+d^2*(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-2*a*e+(-b
*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{2\sqrt{c}e\sqrt{a+x(b+cx)}(-3b^2e^2+2ce(-3bd+4ae+bex)+4c^2(6d^2-3dex+2e^2x^2))+96c^{5/2}d^2\sqrt{-cd^2+}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[a + x*(b + c*x)]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e + b
*e*x) + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) + 96*c^(5/2)*d^2*Sqrt[-(c*d^2
) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sq
rt[-(c*d^2) + e*(b*d - a*e)]] + 3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d -
 2*a*e) + 8*c^2*d*e*(-(b*d) + a*e))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)]])/(48*c^(5/2)*e^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1267, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$\Big\downarrow \ 1267$$
$$\frac{\int -\frac{3e(bd+(2cd+be)x)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{3ce^2}+\frac{(a+bx+cx^2)^{3/2}}{3ce}$$
$$\Big\downarrow \ 27$$
$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\int \frac{(bd+(2cd+be)x)\sqrt{cx^2+bx+a}}{d+ex}dx}{2ce}$$

$\Big\downarrow$ 1231

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{-\frac{\int -\frac{d(2cd-be)\left(eb^2+4cdb-4ace\right)+\left(16c^3d^3-8c^2e(bd-ae)d-b^3e^3-2bce^2(bd-2ae)\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}((2cd-be)(be+4cd)-2cex(be+2cd))}{4ce^2}}{2ce}$$

$\Big\downarrow$ 27

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\frac{\int \frac{d(2cd-be)\left(eb^2+4cdb-4ace\right)+\left(16c^3d^3-8c^2e(bd-ae)d-b^3e^3-2bce^2(bd-2ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2}((2cd-be)(be+4cd)-2cex(be+2cd))}{4ce^2}}{2ce}$$

$\Big\downarrow$ 1269

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\frac{\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}((2cd-be)(be+4cd\cdots}{4ce^2}}{2ce}$$

$\Big\downarrow$ 1092

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\frac{\frac{2\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\sqrt{a+bx+cx^2}((2c\cdots}{2ce}$$

$\Big\downarrow$ 219

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)}{\sqrt{ce}}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\sqrt{a+bx+cx^2}((2cd-\cdots}{2ce}$$

$\Big\downarrow$ 1154

$$\frac{(a+bx+cx^2)^{3/2}}{3ce}-\frac{\frac{\frac{32c^2d^2\left(ae^2-bde+cd^2\right)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2de(bd-ae)-2bce^2(bd-\cdots\right.}{\sqrt{ce}}}{8ce^2}\cdots}{2ce}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+bx+cx^2)^{3/2}}{3ce} - \frac{\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)}{\sqrt{c}e} - \frac{16c^2d^2\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8ce^2} - \frac{\sqrt{}}{}}{2ce}$$

input
```
Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(a + b*x + c*x^2)^(3/2)/(3*c*e) - (-1/4*(((2*c*d - b*e)*(4*c*d + b*e) - 2*
c*e*(2*c*d + b*e)*x)*Sqrt[a + b*x + c*x^2])/(c*e^2) + (((16*c^3*d^3 - b^3*
e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (16*c^2*d^2*Sqrt[c*d^2
- b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(2*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(8c^2e^2x^2+2e^2xbc-12c^2dex+8ace^2-3b^2e^2-6bcde+24c^2d^2)\sqrt{cx^2+bx+a}}{24c^2e^3} - \frac{\frac{(4abce^3+8de^2ac^2-b^3e^3-2de^2b^2c-8d^2ebc^2+16d^3c^3)\,\mathrm{l}}{e\sqrt{c}}}{}$
default	$\frac{\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}}{e} + \frac{d^2\left(\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2-bde+cd}{e^2}}\right.}{}$

input
```
int(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*e^2*x^2+2*b*c*e^2*x-12*c^2*d*e*x+8*a*c*e^2-3*b^2*e^2-6*b*c*d*e
+24*c^2*d^2)/c^2*(c*x^2+b*x+a)^(1/2)/e^3-1/16/e^3/c^2*((4*a*b*c*e^3+8*a*c^
2*d*e^2-b^3*e^3-2*b^2*c*d*e^2-8*b*c^2*d^2*e+16*c^3*d^3)/e*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+16*d^2*(a*e^2-b*d*e+c*d^2)*c^2/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 101.97 (sec) , antiderivative size = 1459, normalized size of antiderivative = 5.72

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/96*(48*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log((8*a*b*d*e - 8*a^2*e^2 -
 (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*s
qrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b
*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d
*e*x + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 -
 (b^3 - 4*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^
2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^2 + 24*c^3*d^2*
e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c^2*e^3)*
x)*sqrt(c*x^2 + b*x + a))/(c^3*e^4), 1/96*(96*sqrt(-c*d^2 + b*d*e - a*e^2)
*c^3*d^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b
*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*
c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + 3*(16*c^3*d^3 -
 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(c)*
log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
c) - 4*a*c) + 4*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 6*b*c^2*d*e^2 - (3*b^2*c -
 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^3
*e^4), 1/48*(24*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log((8*a*b*d*e - 8*a^2
*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2
 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c
*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*...
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x^2\sqrt{c\,x^2+bx+a}}{ex+d}dx$$

input
```
int(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

output
```
int(x^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

3.14 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = -\frac{(4cd-be-2cex)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{((b^2-4ac)\,e^2-4cd(2cd-be))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}e^3} - \frac{d\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3}$$

output
```
-1/4*(-2*c*e*x-b*e+4*c*d)*(c*x^2+b*x+a)^(1/2)/c/e^2-1/8*((-4*a*c+b^2)*e^2-
4*c*d*(-b*e+2*c*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
3/2)/e^3-d*(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x
)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$= \frac{2\sqrt{c}\left(e(-4cd+be+2cex)\sqrt{a+x(b+cx)}-8cd\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)\right)+(}{8c^{3/2}e^3}$$

input
```
Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x)*Sqrt[a + x*(b + c*x)] - 8*c*d*Sqrt[
-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*
x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) + (8*c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*
d) + a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3
/2)*e^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx$$
$$\Big\downarrow \ 1231$$
$$-\frac{\int -\frac{d(-eb^2+4cdb-4ace)+(8c^2d^2-b^2e^2-4ce(bd-ae))x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{d(-eb^2+4cdb-4ace)+(8c^2d^2-b^2e^2-4ce(bd-ae))x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{(-4ce(bd-ae)-b^2e^2+8c^2d^2)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{2(-4ce(bd-ae)-b^2e^2+8c^2d^2)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{16cd(ae^2-bde+cd^2)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(bd-ae)-b^2e^2+8}{\sqrt{c}e}}{8ce^2}$$
$$\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}-\frac{8cd\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}(-be+4cd-2cex)}{4ce^2}$$

input
```
Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

output
```
-1/4*((4*c*d - b*e - 2*c*e*x)*Sqrt[a + b*x + c*x^2])/(c*e^2) + (((8*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*e) - (8*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b
*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
+ c*x^2])])/e)/(8*c*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.47

method	result
risch	$\frac{(2cex+be-4cd)\sqrt{c\,x^2+bx+a}}{4c\,e^2}+\frac{\frac{\left(4ac\,e^2-b^2e^2-4bcde+8c^2d^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{8d\left(a\,e^2-bde+c\,d^2\right)c\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\cdots}{\cdots}\right.}{\cdots}}{8e^2c}$
default	$\frac{\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}}{e}-\frac{d\left(\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*e*x+b*e-4*c*d)/c*(c*x^2+b*x+a)^(1/2)/e^2+1/8/e^2/c*((4*a*c*e^2-b^
2*e^2-4*b*c*d*e+8*c^2*d^2)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c
^(1/2)+8*d*(a*e^2-b*d*e+c*d^2)*c/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 1165, normalized size of antiderivative = 6.10

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log((8*a*b*d*e - 8*a^2*e^2 - (b
^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt
(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)
*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*
x + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2
*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c
) + 4*(2*c^2*e^2*x - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^2 + b*x + a))/(c^2*e^3)
, -1/16*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt(-c*d^2 + b
*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d
^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^
2*d*e + a*b*e^2)*x)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)
*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(c) - 4*a*c) - 4*(2*c^2*e^2*x - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^2 + b*x + a)
)/(c^2*e^3), 1/8*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log((8*a*b*d*e - 8*a
^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x
^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x
^2 + 2*d*e*x + d^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c
)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + 2*(2*c^2*e^2*x - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^2 + b*x + a))/(...
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 4268, normalized size of antiderivative = 22.35

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

output
```
(8*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2
)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
 b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2))*b*c**2*d*e - 16*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**
2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e
**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(
c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 -
b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*
e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**3*d**2 + 16*sqrt(c)*sqrt(4
*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e
 + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2
*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e
**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d -
4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**2*d*e**2 - 16*sqrt
(c)*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e*
*2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**
2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c
)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c...
```

3.15 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2}$$

output
```
(c*x^2+b*x+a)^(1/2)/e-1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^
2+b*x+a)^(1/2))/c^(1/2)/e^2+(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-2*a
*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

$$= \frac{e\sqrt{a+x(b+cx)} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}\,x}{\sqrt{a}(d+ex)-d\sqrt{a+x(b+cx)}}\right) + 2\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{e^2}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

output
```
(e*Sqrt[a + x*(b + c*x)] + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[-
(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])]
 + 2*Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] + (b
*e*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c])/e^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1162, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{2(2cd-be)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{4(ae^2-bde+cd^2)\int\frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{2e}}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{2e}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

output
```
Sqrt[a + b*x + c*x^2]/e - (((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTa
nh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x + c*x^2])])/e)/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.55

method	result
risch	$\frac{\sqrt{c x^2+bx+a}}{e}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{e\sqrt{c}}-\frac{2\left(a e^2-bde+c d^2\right)\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c(\cdots}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}}{2e}$
default	$\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}\right)}{2e\sqrt{c}}-\frac{\left(a e^2 \cdots\right.}{\cdots}}{e}$

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1/2)/e+1/2/e*((b*e-2*c*d)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))/c^(1/2)-2*(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*
sqrt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d
^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*
e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c
*e^2), 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e
+ a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*
d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e
^2), 1/4*(4*sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*a
rctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e
 + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*
c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log
(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c))/(c*e^2), 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*
e - a*e^2)*c*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a
)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 3976, normalized size of antiderivative = 26.16

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

output

```
( - 2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*b*c*e + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*
b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2
 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**
2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**2*d - 4*sqrt(c)*sqrt(4*sqrt(c
)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sqrt(c
)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b
*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e
**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c*e**2 + 4*sqrt(c)*sqrt(4*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e +
c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2
 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - ...
```

3.16 $\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)} \, dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e} - \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{de}$$

output
```
-a^(1/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d+c^(1/2)*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e-(a*e^2-b*d*e+c*d^2)^(1/2)
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/d/e
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)} \, dx = -\frac{2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+bde-ae^2}}\right) - 2\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) + \sqrt{c}d\log\left(e\left(b\right.\right.}{de}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x)),x]
```

output
```
-((2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 x*(b + c*x)])/Sqrt[-(c*d^2) + b*d*e - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[(Sqrt
[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Sqrt[c]*d*Log[e*(b + 2*c*x - 2*S
qrt[c]*Sqrt[a + x*(b + c*x)])])/(d*e))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1270, 25, 1154, 219, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}dx$$

$$\Big\downarrow \text{1270}$$

$$\frac{a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{d}-\frac{\int -\frac{bd+cxd-ae}{(d+ex)\sqrt{cx^2+bx+a}}dx}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bd+cxd-ae}{(d+ex)\sqrt{cx^2+bx+a}}dx}{d}+\frac{a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{d}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\int \frac{bd+cxd-ae}{(d+ex)\sqrt{cx^2+bx+a}}dx}{d}-\frac{2a\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{bd+cxd-ae}{(d+ex)\sqrt{cx^2+bx+a}}dx}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{cd\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{2cd\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e}-\frac{(ae^2-bde+cd^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2(ae^2-bde+cd^2)\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e}-\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x)),x]
```

output
```
-((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) + ((
Sqrt[c]*d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/e - (Sqr
t[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c
*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

Time = 1.40 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.47

method	result
default	$\frac{\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d}-\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}+\frac{(b}{}}{}$

input
```
int((c*x^2+b*x+a)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
/c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/d*((c*(x
+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*
d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 39.79 (sec) , antiderivative size = 2218, normalized size of antiderivative = 13.28

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c)*d*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2
- 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + sqrt(c*d^2 -
 b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^
2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqr
t(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*
e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*s
qrt(-c)*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 +
 b*c*x + a*c)) - sqrt(a)*e*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^
2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - sqrt(c*d^2 - b*d*e + a*e^
2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e
 + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2
+ 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(c)*d*log(-8*
c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*
a*c) + sqrt(a)*e*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arct
an(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e +
(2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e
^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/(d*e), -1/2*(2*sqrt(-c)*d*...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{x\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx$$
$$= \frac{\sqrt{a\,e^2-bde+c\,d^2}\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)-\sqrt{a\,e^2-bde+}}{}$$

input
```
int((c*x^2+b*x+a)^(1/2)/x/(e*x+d),x)
```

output
```
(sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x) - sqrt(a*e**2 - b*d*e
+ c*d**2)*log(d + e*x) + sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*
a - b*x)*e - sqrt(a)*log(x)*e + sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*
x**2) - b - 2*c*x)*d)/(d*e)
```

3.17 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d^2} + \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2}$$

output
```
-(c*x^2+b*x+a)^(1/2)/d/x-1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c
*x^2+b*x+a)^(1/2))/a^(1/2)/d^2+(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-
2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \frac{-\frac{d\sqrt{a+x(b+cx)}}{x} + 2\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right) + \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x)),x]
```

output
```
(-((d*Sqrt[a + x*(b + c*x)])/x) + 2*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[
(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e
)]] + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])
/Sqrt[a])/d^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. 2(156) = 312.

Time = 0.60 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)} \, dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{\sqrt{a+bx+cx^2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}+\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}}{dx}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
+ b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) + (Sqrt[c*d^2 - b*d*e + a*e^
2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*
Sqrt[a + b*x + c*x^2])])/d^2
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{dx}-\frac{-\frac{(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{2\left(a\,e^2-bde+c\,d^2\right)\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde}{e^2}}}{x}\right)}{de\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}}{2d}$
default	$\frac{-\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\right)}{2a}+\frac{2c\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b}{}\right)}{a}}{d}$

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/d/x-1/2/d*(-(2*a*e-b*d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)+2*(a*e^2-b*d*e+c*d^2)/d/e/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1020, normalized size of antiderivative = 6.54

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x*log((8*a*b*d*e - 8*a^2*e^2 - (b^2
+ 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*
d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)
 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x +
 d^2)) - (b*d - 2*a*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqr
t(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x
+ a)*a*d)/(a*d^2*x), 1/4*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x*arctan(-1/2*s
qrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 +
 (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (b*d - 2*a*e)*sqrt(a)*x*log(-(8*a*b*x
 + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2
)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a*d)/(a*d^2*x), 1/2*((b*d - 2*a*e)*sqrt(-
a)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*
x + a^2)) + sqrt(c*d^2 - b*d*e + a*e^2)*a*x*log((8*a*b*d*e - 8*a^2*e^2 - (
b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqr
t(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e
*x + d^2)) - 2*sqrt(c*x^2 + b*x + a)*a*d)/(a*d^2*x), 1/2*(2*sqrt(-c*d^2 +
b*d*e - a*e^2)*a*x*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b
*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + ...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**2/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \frac{2\,(cd^2-bde+ae^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2} + \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^2} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b+2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)d}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
2*(c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e +
 sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^
2) + (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(
sqrt(-a)*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b + 2*a*sqrt(c))/(((s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx = \frac{2\sqrt{a\,e^2-bde+c\,d^2}\,\log\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)ax-2\sqrt{a\,e^2-bd}}{}$$

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d),x)
```

output
```
(2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*x - 2*sqrt(a*e**2 - b
*d*e + c*d**2)*log(d + e*x)*a*x - 2*sqrt(a + b*x + c*x**2)*a*d + 2*sqrt(a)
*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*e*x - sqrt(a)*log(
 - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*d*x - 2*sqrt(a)*log(x)*
a*e*x + sqrt(a)*log(x)*b*d*x)/(2*a*d**2*x)
```

3.18 $\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx =& -\frac{\sqrt{a+bx+cx^2}}{2dx^2} - \frac{(bd-4ae)\sqrt{a+bx+cx^2}}{4ad^2x} \\ &+ \frac{(b^2d^2+4abde-4a(cd^2+2ae^2))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d^3} \\ &- \frac{e\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^3}\end{aligned}$$

output

```
-1/2*(c*x^2+b*x+a)^(1/2)/d/x^2-1/4*(-4*a*e+b*d)*(c*x^2+b*x+a)^(1/2)/a/d^2/
x+1/8*(b^2*d^2+4*a*b*d*e-4*a*(2*a*e^2+c*d^2))*arctanh(1/2*(b*x+2*a)/a^(1/2
)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d^3-e*(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2
*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
/d^3
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = \frac{\sqrt{a}\left(d(-2ad-bdx+4aex)\sqrt{a+x(b+cx)}-8ae\sqrt{-cd^2+bde-ae^2}x^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)\right)}{4a^{3/2}d^3x^2}$$

input

```
Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d + e*x)),x]
```

output

```
(Sqrt[a]*(d*(-2*a*d - b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)] - 8*a*e*Sqrt[
-(c*d^2) + b*d*e - a*e^2]*x^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b
+ c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) + 8*a^2*e^2*x^2*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + d*(b^2*d - 4*a*c*d + 4*a*b*e)*x^2*A
rcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(3/2)*d^3*x^2
)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(-\frac{e^3\sqrt{a+bx+cx^2}}{d^3(d+ex)}+\frac{e^2\sqrt{a+bx+cx^2}}{d^3x}-\frac{e\sqrt{a+bx+cx^2}}{d^2x^2}+\frac{\sqrt{a+bx+cx^2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}e^2\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^3}+\frac{be^2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^3}+\frac{e(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^3}+\frac{b\text{earctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{c}\text{earctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{e\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^3}+\frac{e\sqrt{a+bx+cx^2}}{d^2x}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^3*(d + e*x)),x]
```

output
```
(e*Sqrt[a + b*x + c*x^2])/(d^2*x) - ((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4
*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c
*x^2])])/(8*a^(3/2)*d) + (b*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[a]*d^2) - (Sqrt[a]*e^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^3 - (Sqrt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/d^2 + (b*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(2*Sqrt[c]*d^3) + (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^3) - (e*Sqrt[c*d^2 - b*d*
e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x + c*x^2])])/d^3
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{c x^2+bx+a}\,(-4aex+bdx+2ad)}{4a d^2 x^2}+\frac{-\frac{(8e^2a^2-4abde+4a d^2c-b^2d^2)\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{c x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{8a(a e^2-bde+c d^2)\ln\left(\frac{2a e^2-\cdots}{\cdots}\right)}{\cdots}}{8a\cdots}$
default	$-\frac{(c x^2+bx+a)^{\frac{3}{2}}}{2a x^2}-\frac{b\left(-\frac{(c x^2+bx+a)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{c x^2+bx+a}}{x}\right)\right)}{2a}+\frac{2c\left(\frac{(2cx+b)\cdots}{\cdots}\right)}{\cdots}\right)}{4a}\cdots$

input
```
int((c*x^2+b*x+a)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-4*a*e*x+b*d*x+2*a*d)/a/d^2/x^2+1/8/a/d^2*(-(8*a
^2*e^2-4*a*b*d*e+4*a*c*d^2-b^2*d^2)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x)+8*a*(a*e^2-b*d*e+c*d^2)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1217, normalized size of antiderivative = 5.63

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e*x^2*log((8*a*b*d*e - 8*a^2*e^2
- (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*
sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*
d*e*x + d^2)) - (4*a*b*d*e - 8*a^2*e^2 + (b^2 - 4*a*c)*d^2)*sqrt(a)*x^2*lo
g(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt
(a) + 8*a^2)/x^2) - 4*(2*a^2*d^2 + (a*b*d^2 - 4*a^2*d*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^2*d^3*x^2), -1/16*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e*x^2*a
rctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e
 + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*
c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (4*a*b*d*e - 8*a^2*e^2 +
(b^2 - 4*a*c)*d^2)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(
c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*a^2*d^2 + (a*b*d
^2 - 4*a^2*d*e)*x)*sqrt(c*x^2 + b*x + a))/(a^2*d^3*x^2), 1/8*(4*sqrt(c*d^2
 - b*d*e + a*e^2)*a^2*e*x^2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e
+ a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*
d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - (4*
a*b*d*e - 8*a^2*e^2 + (b^2 - 4*a*c)*d^2)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^
2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*d...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx=\int \frac{\sqrt{a+bx+cx^2}}{x^3\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**3/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(190) = 380.

Time = 1.22 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = -\frac{2\,(cd^2e-bde^2+ae^3)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^3} - \frac{(b^2d^2-4\,acd^2+4\,abde-8\,a^2e^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}ad^3} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe+8\,(\sqrt{}}{}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
-2*(c*d^2*e - b*d*e^2 + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^
2)*d^3) - 1/4*(b^2*d^2 - 4*a*c*d^2 + 4*a*b*d*e - 8*a^2*e^2)*arctan(-(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*d^3) + 1/4*((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b^2*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a*c*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d + 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^2*b*e + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 -
 a)^2*a*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d+ex)}\,dx = \frac{8\sqrt{a\,e^2-bde+c\,d^2}\,\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)a^2e\,x^2-8\sqrt{a\,e^2}}{}$$

input
```
int((c*x^2+b*x+a)^(1/2)/x^3/(e*x+d),x)
```

output
```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*e*x**2 - 8*sqrt
(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**2*e*x**2 - 4*sqrt(a + b*x + c*x*
*2)*a**2*d**2 + 8*sqrt(a + b*x + c*x**2)*a**2*d*e*x - 2*sqrt(a + b*x + c*x
**2)*a*b*d**2*x + 8*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*a**2*e**2*x**2 - 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a*b*d*e*x**2 + 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*a*c*d**2*x**2 - sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*b**2*d**2*x**2 - 8*sqrt(a)*log(x)*a**2*e**2*x**2 + 4*sqrt(a)*log(x
)*a*b*d*e*x**2 - 4*sqrt(a)*log(x)*a*c*d**2*x**2 + sqrt(a)*log(x)*b**2*d**2
*x**2)/(8*a**2*d**3*x**2)
```

3.19 $\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 298

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)} \, dx &= -\frac{\sqrt{a+bx+cx^2}}{3dx^3} - \frac{(bd-6ae)\sqrt{a+bx+cx^2}}{12ad^2x^2} \\ &+ \frac{(3b^2d^2+6abde-8a(cd^2+3ae^2))\sqrt{a+bx+cx^2}}{24a^2d^3x} \\ &- \frac{(b^3d^3+2ab^2d^2e-4abd(cd^2-2ae^2)-8a^2e(cd^2+2ae^2))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{5/2}d^4} \\ &+ \frac{e^2\sqrt{cd^2-bde+ae^2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^4}\end{aligned}$$

output

```
-1/3*(c*x^2+b*x+a)^(1/2)/d/x^3-1/12*(-6*a*e+b*d)*(c*x^2+b*x+a)^(1/2)/a/d^2
/x^2+1/24*(3*b^2*d^2+6*a*b*d*e-8*a*(3*a*e^2+c*d^2))*(c*x^2+b*x+a)^(1/2)/a^
2/d^3/x-1/16*(b^3*d^3+2*a*b^2*d^2*e-4*a*b*d*(-2*a*e^2+c*d^2)-8*a^2*e*(2*a*
e^2+c*d^2))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d^4
+e^2*(a*e^2-b*d*e+c*d^2)^(1/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 2.80 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)} \, dx$$
$$= \frac{\sqrt{a}\Big(d\sqrt{a+x(b+cx)}(3b^2d^2x^2-2adx(bd+4cdx-3bex)-4a^2(2d^2-3dex+6e^2x^2))+48a^2e^2\sqrt{-cd^2}}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^4*(d + e*x)),x]
```

output
```
(Sqrt[a]*(d*Sqrt[a + x*(b + c*x)]*(3*b^2*d^2*x^2 - 2*a*d*x*(b*d + 4*c*d*x
- 3*b*e*x) - 4*a^2*(2*d^2 - 3*d*e*x + 6*e^2*x^2)) + 48*a^2*e^2*Sqrt[-(c*d^
2) + b*d*e - a*e^2]*x^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)
])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) - 48*a^3*e^3*x^3*ArcTanh[(Sqrt[c]*x -
Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 3*d*(b^3*d^2 + 2*a*b^2*d*e - 8*a^2*c*d*e
 + 4*a*b*(-(c*d^2) + 2*a*e^2))*x^3*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b +
 c*x)])/Sqrt[a]])/(24*a^(5/2)*d^4*x^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)} \, dx$$
$$\Big\downarrow \text{ 1289}$$
$$\int \left(\frac{e^4\sqrt{a+bx+cx^2}}{d^4(d+ex)} - \frac{e^3\sqrt{a+bx+cx^2}}{d^4x} + \frac{e^2\sqrt{a+bx+cx^2}}{d^3x^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x^3} + \frac{\sqrt{a+bx+cx^2}}{dx^4}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{e(b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d^2}-\frac{b(b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{5/2}d}+\frac{b(2a+bx)\sqrt{a+bx+cx^2}}{8a^2dx^2}+\frac{\sqrt{a}e^3\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^4}-\frac{be^3\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^4}-\frac{e^2(2cd-be)\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^4}-\frac{be^2\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d^3}+\frac{\sqrt{c}e^2\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d^3}+\frac{e^2\sqrt{ae^2-bde+cd^2}\mathrm{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^4}-\frac{e^2\sqrt{a+bx+cx^2}}{d^3x}+\frac{e(2a+bx)\sqrt{a+bx+cx^2}}{4ad^2x^2}-\frac{(a+bx+cx^2)^{3/2}}{3adx^3}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^4*(d + e*x)),x]
```

output
```
-((e^2*Sqrt[a + b*x + c*x^2])/(d^3*x)) + (b*(2*a + b*x)*Sqrt[a + b*x + c*x
^2])/(8*a^2*d*x^2) + (e*(2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d^2*x^2) -
 (a + b*x + c*x^2)^(3/2)/(3*a*d*x^3) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x
)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(16*a^(5/2)*d) - ((b^2 - 4*a*c)*e*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d^2) - (b
*e^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d^
3) + (Sqrt[a]*e^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
d^4 + (Sqrt[c]*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/d^3 - (b*e^3*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*S
qrt[c]*d^4) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*Sqrt[c]*d^4) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x + c*x^2])])/d^4
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(24a^2e^2x^2-6abde\,x^2+8a\,d^2x^2c-3b^2d^2x^2-12a^2dex+2ab\,d^2x+8a^2d^2)}{24a^2d^3x^3}-\frac{-\frac{(16e^3a^3-8a^2bd\,e^2+8a^2c\,d^2e-2a\,b^2d^2e+}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(24*a^2*e^2*x^2-6*a*b*d*e*x^2+8*a*c*d^2*x^2-3*b^
2*d^2*x^2-12*a^2*d*e*x+2*a*b*d^2*x+8*a^2*d^2)/a^2/d^3/x^3-1/16/a^2/d^3*(-(
16*a^3*e^3-8*a^2*b*d*e^2+8*a^2*c*d^2*e-2*a*b^2*d^2*e+4*a*b*c*d^3-b^3*d^3)/
d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+16*a^2*e*(a*e^2-b*
d*e+c*d^2)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1507, normalized size of antiderivative = 5.06

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
[1/96*(48*sqrt(c*d^2 - b*d*e + a*e^2)*a^3*e^2*x^3*log((8*a*b*d*e - 8*a^2*e
^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 -
 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
 - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 +
 2*d*e*x + d^2)) - 3*(8*a^2*b*d*e^2 - 16*a^3*e^3 + (b^3 - 4*a*b*c)*d^3 + 2
*(a*b^2 - 4*a^2*c)*d^2*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 +
4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(8*a^3*d^3 -
 (6*a^2*b*d^2*e - 24*a^3*d*e^2 + (3*a*b^2 - 8*a^2*c)*d^3)*x^2 + 2*(a^2*b*d
^3 - 6*a^3*d^2*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*d^4*x^3), 1/96*(96*sqrt(-
c*d^2 + b*d*e - a*e^2)*a^3*e^2*x^3*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2
)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e
 + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*
e^2)*x)) - 3*(8*a^2*b*d*e^2 - 16*a^3*e^3 + (b^3 - 4*a*b*c)*d^3 + 2*(a*b^2
- 4*a^2*c)*d^2*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(8*a^3*d^3 - (6*a^2*
b*d^2*e - 24*a^3*d*e^2 + (3*a*b^2 - 8*a^2*c)*d^3)*x^2 + 2*(a^2*b*d^3 - 6*a
^3*d^2*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*d^4*x^3), 1/48*(24*sqrt(c*d^2 - b
*d*e + a*e^2)*a^3*e^2*x^3*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 -
 (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{x^4\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**4/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)x^4}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(268) = 536$.

Time = 0.28 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
2*(c*d^2*e^2 - b*d*e^3 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e
^2)*d^4) + 1/8*(b^3*d^3 - 4*a*b*c*d^3 + 2*a*b^2*d^2*e - 8*a^2*c*d^2*e + 8*
a^2*b*d*e^2 - 16*a^3*e^3)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt
(-a))/(sqrt(-a)*a^2*d^4) - 1/24*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b
^3*d^2 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d^2 + 6*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a*b^2*d*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^2*c*d*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*e^2 - 48
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*d^2 + 48*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*d*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a^3*sqrt(c)*e^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3
*d^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d^2 + 48*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^3*b*e^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^2*b^2*sqrt(c)*d^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3
*b*sqrt(c)*d*e + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*sqrt(c)*e^2
- 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d^2 - 36*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^3*b*c*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3
*b^2*d*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*d*e - 24*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^4*b*e^2 - 16*a^4*c^(3/2)*d^2 - 48*a^5*sqrt(c)
*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3*a^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^4\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^4(d+ex)}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}}{x^4\,(ex+d)}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)/x^4/(e*x+d),x)
```

3.20 $\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 635

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx =$$

$$-\frac{(512c^5d^5+7b^5e^5+4b^3ce^4(3bd-8ae)-128c^4d^3e(5bd-4ae)+32bc^3d^2e^2(2bd-3ae)+8bc^2e^3(3b^2d^2-6\ldots}{\ldots}$$

$$-\frac{(64c^3d^3-24bc^2d^2e-7b^3e^3-4bce^2(3bd-ae)-2ce(24c^2d^2+7b^2e^2+4ce(3bd-ae))\,x)\,(a+bx+cx^2)^{3\ldots}}{192c^3e^4}$$

$$-\frac{(22cd+7be)\,(a+bx+cx^2)^{5/2}}{60c^2e^2}+\frac{(d+ex)\,(a+bx+cx^2)^{5/2}}{6ce^2}$$

$$+\frac{(1024c^6d^6+7b^6e^6+12b^4ce^5(bd-5ae)-1536c^5d^4e(bd-ae)+384c^4d^2e^2(bd-ae)^2+64c^3e^3(bd-ae)^3\ldots}{1024c^{9/2}e^7}$$

$$-\frac{d^3(cd^2-bde+ae^2)^{3/2}\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^7}$$

output
```
-1/512*(512*c^5*d^5+7*b^5*e^5+4*b^3*c*e^4*(-8*a*e+3*b*d)-128*c^4*d^3*e*(-4
*a*e+5*b*d)+32*b*c^3*d^2*e^2*(-3*a*e+2*b*d)+8*b*c^2*e^3*(2*a^2*e^2-6*a*b*d
*e+3*b^2*d^2)-2*c*e*(128*c^4*d^4-7*b^4*e^4-4*b^2*c*e^3*(-8*a*e+3*b*d)-32*c
^3*d^2*e*(-3*a*e+2*b*d)-8*c^2*e^2*(2*a^2*e^2-6*a*b*d*e+3*b^2*d^2))*x)*(c*x
^2+b*x+a)^(1/2)/c^4/e^6-1/192*(64*c^3*d^3-24*b*c^2*d^2*e-7*b^3*e^3-4*b*c*e
^2*(-a*e+3*b*d)-2*c*e*(24*c^2*d^2+7*b^2*e^2+4*c*e*(-a*e+3*b*d))*x)*(c*x^2+
b*x+a)^(3/2)/c^3/e^4-1/60*(7*b*e+22*c*d)*(c*x^2+b*x+a)^(5/2)/c^2/e^2+1/6*(
e*x+d)*(c*x^2+b*x+a)^(5/2)/c/e^2+1/1024*(1024*c^6*d^6+7*b^6*e^6+12*b^4*c*e
^5*(-5*a*e+b*d)-1536*c^5*d^4*e*(-a*e+b*d)+384*c^4*d^2*e^2*(-a*e+b*d)^2+64*
c^3*e^3*(-a*e+b*d)^3+24*b^2*c^2*e^4*(6*a^2*e^2-4*a*b*d*e+b^2*d^2))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)/e^7-d^3*(a*e^2-b*d*e+c
*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/e^7
```

Mathematica [A] (verified)

Time = 11.47 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.04

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{-5120d^3(a+x(b+cx))^{3/2}+\frac{1920d^2e(b+2cx)(a+x(b+cx))^{3/2}}{c}-\frac{3072de^2(a+x(b+cx))^{5/2}}{c}+$$

input
```
Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(-5120*d^3*(a + x*(b + c*x))^(3/2) + (1920*d^2*e*(b + 2*c*x)*(a + x*(b + c
*x))^(3/2))/c - (3072*d*e^2*(a + x*(b + c*x))^(5/2))/c + (2560*e^3*x*(a +
x*(b + c*x))^(5/2))/c + (360*(b^2 - 4*a*c)*d^2*e*(-2*Sqrt[c]*(b + 2*c*x)*S
qrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])]))/c^(5/2) + (60*b*d*e^2*((16*(b + 2*c*x)*(a + x*(b + c*x
))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x
)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/c^(5/2)))/c + (e^3*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*
((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]
*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c^2 + (960*d^3*((2*c*d - b*e)
*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e
^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2
+ e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sq
rt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))))/(c^(3/2)*e^3))/(15
360*e^4)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1267, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx+cx^2)^{3/2}}{d+ex} \, dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int -\frac{(cx^2+bx+a)^{3/2}(e^2(22cd+7be)x^2+2e(5cd^2+e(6bd+ae))x+de(5bd+2ae))}{2(d+ex)}dx}{6ce^3}+\frac{(d+ex)(a+bx+cx^2)^{5/2}}{6ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)(a+bx+cx^2)^{5/2}}{6ce^2}-\frac{\int \frac{(cx^2+bx+a)^{3/2}(e^2(22cd+7be)x^2+2e(5cd^2+e(6bd+ae))x+de(5bd+2ae))}{d+ex}dx}{12ce^3}$$

$$\Big\downarrow \text{2184}$$

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}-\frac{\frac{\int -\frac{5e^3\left(d\left(7eb^2+12cdb-4ace\right)+\left(24c^2d^2+7b^2e^2+4ce(3bd-ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)}dx}{5ce^2}+\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c}}{12ce^3}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c}-\frac{e\int \frac{\left(d\left(7eb^2+12cdb-4ace\right)+\left(24c^2d^2+7b^2e^2+4ce(3bd-ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{d+ex}dx}{2c}}{12ce^3}$$

$\Big\downarrow$ 1231

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c}-\frac{e\left(-\frac{\int \frac{3\left(d\left(7e^3b^4+12cde^2b^3+8ce\left(3cd^2-4ae^2\right)b^2-16c^2d\left(4cd^2+3ae^2\right)b+16ac^2e\left(2cd^2+ae^2\right)\right)-\left(128c^4d^4-32c^3e(2bd-3ae\right.}{2(d+ex)}}{8ce^2}\right.}{}}{}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c}-\frac{e\left(-\frac{3\int \frac{\left(d\left(7e^3b^4+12cde^2b^3+8ce\left(3cd^2-4ae^2\right)b^2-16c^2d\left(4cd^2+3ae^2\right)b+16ac^2e\left(2cd^2+ae^2\right)\right)-\left(128c^4d^4-32c^3e(2bd-3ae\right.}{d+ex}}{16ce^2}\right.}{}}{}$$

$\Big\downarrow$ 1231

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2}-\frac{\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c}-\frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\right.\right.}{4ce^2}\right.}{}\right.}{}}{}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

$$\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\cdots\right.\right.}{4ce^2\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 1269

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

$$\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\cdots\right.\right.}{4ce^2\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 1092

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

$$\frac{e\left(a+bx+cx^2\right)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\cdots\right.\right.}{4ce^2\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 219

$$\frac{\frac{e(a+bx+cx^2)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\right.\right.}{4ce^2}\right.}{}\right.}{}}{}$$

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

$\downarrow$ 1154

$$\frac{\frac{e(a+bx+cx^2)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\right.\right.}{4ce^2}\right.}{}\right.}{}}{}$$

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

$\downarrow$ 219

$$\frac{\frac{e(a+bx+cx^2)^{5/2}(7be+22cd)}{5c} - \frac{e\left(-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(8bc^2e^3\left(2a^2e^2-6abde+3b^2d^2\right)-2cex\left(-8c^2e^2\left(2a^2e^2-6abde+3b^2d^2\right)-4b^2ce^3(3bd-8ae)-32c^3d^2e(2\right.\right.}{4ce^2}\right.}{}\right.}{}}{}$$

$$\frac{(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} -$$

input
```
Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
((d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) - ((e*(22*c*d + 7*b*e)*(a +
b*x + c*x^2)^(5/2))/(5*c) - (e*(-1/8*((64*c^3*d^3 - 24*b*c^2*d^2*e - 7*b^3
*e^3 - 4*b*c*e^2*(3*b*d - a*e) - 2*c*e*(24*c^2*d^2 + 7*b^2*e^2 + 4*c*e*(3*
b*d - a*e))*x)*(a + b*x + c*x^2)^(3/2))/(c*e^2) - (3*(((512*c^5*d^5 + 7*b^
5*e^5 + 4*b^3*c*e^4*(3*b*d - 8*a*e) - 128*c^4*d^3*e*(5*b*d - 4*a*e) + 32*b
*c^3*d^2*e^2*(2*b*d - 3*a*e) + 8*b*c^2*e^3*(3*b^2*d^2 - 6*a*b*d*e + 2*a^2*
e^2) - 2*c*e*(128*c^4*d^4 - 7*b^4*e^4 - 4*b^2*c*e^3*(3*b*d - 8*a*e) - 32*c
^3*d^2*e*(2*b*d - 3*a*e) - 8*c^2*e^2*(3*b^2*d^2 - 6*a*b*d*e + 2*a^2*e^2))*
x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) - (((1024*c^6*d^6 + 7*b^6*e^6 + 12*b^4
*c*e^5*(b*d - 5*a*e) - 1536*c^5*d^4*e*(b*d - a*e) + 384*c^4*d^2*e^2*(b*d -
 a*e)^2 + 64*c^3*e^3*(b*d - a*e)^3 + 24*b^2*c^2*e^4*(b^2*d^2 - 4*a*b*d*e +
 6*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt
[c]*e) - (1024*c^4*d^3*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])
/e)/(8*c*e^2)))/(16*c*e^2)))/(2*c))/(12*c*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{(-1280c^5e^5x^5-1664b\,c^4e^5x^4+1536c^5d\,e^4x^4-2240a\,c^4e^5x^3-48b^2c^3e^5x^3+2112b\,c^4d\,e^4x^3-1920c^5d^2e^3x^3-288ab\,c^3e^5x^2+3072a}{}$
default	Expression too large to display

input
```
int(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^4*(-1280*c^5*e^5*x^5-1664*b*c^4*e^5*x^4+1536*c^5*d*e^4*x^4-2240*
a*c^4*e^5*x^3-48*b^2*c^3*e^5*x^3+2112*b*c^4*d*e^4*x^3-1920*c^5*d^2*e^3*x^3
-288*a*b*c^3*e^5*x^2+3072*a*c^4*d*e^4*x^2+56*b^3*c^2*e^5*x^2+96*b^2*c^3*d*
e^4*x^2-2880*b*c^4*d^2*e^3*x^2+2560*c^5*d^3*e^2*x^2-480*a^2*c^3*e^5*x+432*
a*b^2*c^2*e^5*x+672*a*b*c^3*d*e^4*x-4800*a*c^4*d^2*e^3*x-70*b^4*c*e^5*x-12
0*b^3*c^2*d*e^4*x-240*b^2*c^3*d^2*e^3*x+4480*b*c^4*d^3*e^2*x-3840*c^5*d^4*
e*x+1296*a^2*b*c^2*e^5+1536*a^2*c^3*d*e^4-760*a*b^3*c*e^5-1200*a*b^2*c^2*d
*e^4-2400*a*b*c^3*d^2*e^3+10240*a*c^4*d^3*e^2+105*b^5*e^5+180*b^4*c*d*e^4+
360*b^3*c^2*d^2*e^3+960*b^2*c^3*d^3*e^2-9600*b*c^4*d^4*e+7680*c^5*d^5)*(c*
x^2+b*x+a)^(1/2)/e^6-1/1024/e^6/c^4*((64*a^3*c^3*e^6-144*a^2*b^2*c^2*e^6-1
92*a^2*b*c^3*d*e^5-384*a^2*c^4*d^2*e^4+60*a*b^4*c*e^6+96*a*b^3*c^2*d*e^5+1
92*a*b^2*c^3*d^2*e^4+768*a*b*c^4*d^3*e^3-1536*a*c^5*d^4*e^2-7*b^6*e^6-12*b
^5*c*d*e^5-24*b^4*c^2*d^2*e^4-64*b^3*c^3*d^3*e^3-384*b^2*c^4*d^4*e^2+1536*
b*c^5*d^5*e-1024*c^6*d^6)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^
(1/2)-1024*d^3*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+
c^2*d^4)*c^4/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^3(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate(x**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^3\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^3(c\,x^2+bx+a)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

output
```
int(x^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

3.21 $\int \frac{x^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 461

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2}{1}$$
$$+\frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x)\left(a+bx+cx^2\right)^{3/2}}{48c^2e^3}+\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}$$
$$-\frac{(256c^5d^5+3b^5e^5+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+16bc^2e^3(b^2d^2-3abde+}{256c^{7/2}e^6}$$
$$+\frac{d^2(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^6}$$

output
```
1/128*(128*c^4*d^4+3*b^4*e^4-32*c^3*d^2*e*(-4*a*e+5*b*d)+8*b*c^2*d*e^2*(-3
*a*e+2*b*d)+6*b^2*c*e^3*(-2*a*e+b*d)-2*c*e*(32*c^3*d^3-3*b^3*e^3-8*c^2*d*e
*(-3*a*e+2*b*d)-6*b*c*e^2*(-2*a*e+b*d))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^5+1/4
8*(16*c^2*d^2-6*b*c*d*e-3*b^2*e^2-6*c*e*(b*e+2*c*d)*x)*(c*x^2+b*x+a)^(3/2)
/c^2/e^3+1/5*(c*x^2+b*x+a)^(5/2)/c/e-1/256*(256*c^5*d^5+3*b^5*e^5+6*b^3*c*
e^4*(-4*a*e+b*d)-384*c^4*d^3*e*(-a*e+b*d)+96*c^3*d*e^2*(-a*e+b*d)^2+16*b*c
^2*e^3*(3*a^2*e^2-3*a*b*d*e+b^2*d^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(7/2)/e^6+d^2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-
2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^6
```

Mathematica [A] (verified)

Time = 11.01 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{-15(256c^5d^5+3b^5e^5+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-}{}$$

input
```
Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(-15*(256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^4*d^3*e*
(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d^2 - 3*a*b*d
*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] +
2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(45*b^4*e^4 - 30*b^2*c*e^3*(-3*b*d + 10
*a*e + b*e*x) + 32*c^4*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^
3 + 12*e^4*x^4) + 12*c^2*e^2*(32*a^2*e^2 + 2*a*b*e*(-25*d + 7*e*x) + b^2*(
20*d^2 - 5*d*e*x + 2*e^2*x^2)) + 16*c^3*e*(a*e*(160*d^2 - 75*d*e*x + 48*e^
2*x^2) + b*(-150*d^3 + 70*d^2*e*x - 45*d*e^2*x^2 + 33*e^3*x^3))) - 1920*c^
3*d^2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x +
 b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(3840*
c^(7/2)*e^6)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1267, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int -\frac{5e(bd+(2cd+be)x)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)}dx}{5ce^2}+\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}$$

$\Big\downarrow$ 27

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-\frac{\int \frac{(bd+(2cd+be)x)(cx^2+bx+a)^{3/2}}{d+ex}dx}{2ce}$$

$\Big\downarrow$ 1231

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-\frac{-\frac{\int \frac{\left(d\left(3e^2b^3+6cdeb^2-4c\left(4cd^2+3ae^2\right)b+8ac^2de\right)-\left(32c^3d^3-8c^2e(2bd-3ae)d-3b^3e^3-6bce^2(bd-2ae)\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{8ce^2}-\frac{(a+bx+cx^2)^{3/2}\left(-3b^2e^2-6c\right.}{24}}{2ce}$$

$\Big\downarrow$ 27

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-\frac{-\frac{\int \frac{\left(d\left(3e^2b^3+6cdeb^2-4c\left(4cd^2+3ae^2\right)b+8ac^2de\right)-\left(32c^3d^3-8c^2e(2bd-3ae)d-3b^3e^3-6bce^2(bd-2ae)\right)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{16ce^2}-\frac{(a+bx+cx^2)^{3/2}\left(-3b^2e^2-6c\right.}{24}}{2ce}$$

$\Big\downarrow$ 1231

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-$$
$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{\int \frac{d}{-}}{}}{}$$

$\Big\downarrow$ 27

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-$$
$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{\int \frac{d}{-}}{}}{}$$

$\Big\downarrow$ 1269

$$\frac{(a+bx+cx^2)^{5/2}}{5ce}-$$
$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{(16}{}}{}$$

$\Big\downarrow$ 1092

$$\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}-$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{2(1}{}}{}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}-$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{\text{arc}}{}}{}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}-$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{512}{}}{}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}}{5ce}-$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(-2cex\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{4ce^2}-\frac{\text{arc}}{}}{}$$

input
```
Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
(a + b*x + c*x^2)^(5/2)/(5*c*e) - (-1/24*((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*
e^2 - 6*c*e*(2*c*d + b*e)*x)*(a + b*x + c*x^2)^(3/2))/(c*e^2) - (((128*c^4
*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2*b*d - 3
*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 - 8*c^2*
d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x)*Sqrt[a + b*x + c*x^2])/(
4*c*e^2) - (((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^
4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d^2 -
 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[c]*e) - (256*c^3*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b
*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
+ c*x^2])])/e)/(8*c*e^2))/(16*c*e^2))/(2*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.44

method	result
risch	$\frac{(384c^4e^4x^4+528b\,c^3e^4x^3-480c^4d\,e^3x^3+768a\,c^3e^4x^2+24b^2c^2e^4x^2-720b\,c^3d\,e^3x^2+640c^4d^2e^2x^2+168ab\,c^2e^4x-1200a\,c^3d\,e^3x-30\ldots}{\ldots}$
default	$\frac{\frac{(c\,x^2+bx+a)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(c\,x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}}{e}+\frac{d^2\left(\frac{c\left(x+\ldots\right.}{\ldots}\right.}{\ldots}$

input

```
int(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output

```
1/1920/c^3*(384*c^4*e^4*x^4+528*b*c^3*e^4*x^3-480*c^4*d*e^3*x^3+768*a*c^3*
e^4*x^2+24*b^2*c^2*e^4*x^2-720*b*c^3*d*e^3*x^2+640*c^4*d^2*e^2*x^2+168*a*b
*c^2*e^4*x-1200*a*c^3*d*e^3*x-30*b^3*c*e^4*x-60*b^2*c^2*d*e^3*x+1120*b*c^3
*d^2*e^2*x-960*c^4*d^3*e*x+384*a^2*c^2*e^4-300*a*b^2*c*e^4-600*a*b*c^2*d*e
^3+2560*a*c^3*d^2*e^2+45*b^4*e^4+90*b^3*c*d*e^3+240*b^2*c^2*d^2*e^2-2400*b
*c^3*d^3*e+1920*c^4*d^4)*(c*x^2+b*x+a)^(1/2)/e^5-1/256/e^5/c^3*((48*a^2*b*
c^2*e^5+96*a^2*c^3*d*e^4-24*a*b^3*c*e^5-48*a*b^2*c^2*d*e^4-192*a*b*c^3*d^2
*e^3+384*a*c^4*d^3*e^2+3*b^5*e^5+6*b^4*c*d*e^4+16*b^3*c^2*d^2*e^3+96*b^2*c
^3*d^3*e^2-384*b*c^4*d^4*e+256*c^5*d^5)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))/c^(1/2)+256*d^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2
-2*b*c*d^3*e+c^2*d^4)*c^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2(c\,x^2+bx+a)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

output
```
int(x^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

3.22 $\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 339

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx =$$
$$-\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+2ce(3(b^2-4ac)e^2-8cd(2cd-be))x)\sqrt{a+}}{64c^2e^4}$$
$$-\frac{(8cd-3be-6cex)(a+bx+cx^2)^{3/2}}{24ce^2}$$
$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}e^5}$$
$$-\frac{d(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

output
```
-1/64*(64*c^3*d^3+3*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*
b*d)+2*c*e*(3*(-4*a*c+b^2)*e^2-8*c*d*(-b*e+2*c*d))*x)*(c*x^2+b*x+a)^(1/2)/
c^2/e^4-1/24*(-6*c*e*x-3*b*e+8*c*d)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(128*c
^4*d^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*
e^2*(-a*e+b*d)^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/
2)/e^5-d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{\frac{2e\sqrt{a+x(b+cx)}\left(-9b^3e^3+6bce^2(-4bd+10ae+bex)-16c^3\left(12d^3-6d^2ex+4de^2x^2-3e^3x^3\right)+8c^2e\left(ae(-32d+1\ldots\right.\right.}{c^2}}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b*e
*x) - 16*c^3*(12*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 - 3*e^3*x^3) + 8*c^2*e*(a*e
*(-32*d + 15*e*x) + b*(30*d^2 - 14*d*e*x + 9*e^2*x^2))))/c^2 - 768*d*Sqrt[
-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d +
e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(128*
c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e
) + 48*c^2*e^2*(b*d - a*e)^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/c^(5/2))/(384*e^5)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \; 1231$$

$$-\frac{\int -\frac{\left(d\left(-3eb^2+8cdb-4ace\right)+\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{8ce^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d(-3eb^2+8cdb-4ace)+(16c^2d^2-3b^2e^2-4ce(2bd-3ae))x)\sqrt{cx^2+bx+a}}{d+ex}dx}{16ce^2}-$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\frac{\int -\frac{d\left(3e^3b^4+8cde^2b^3-8ce\left(10cd^2+3ae^2\right)b^2+32c^2d\left(2cd^2+5ae^2\right)b-16ac^2e\left(4cd^2+5ae^2\right)\right)+\left(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(b\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{4ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{d\left(3e^3b^4+8cde^2b^3-8ce\left(10cd^2+3ae^2\right)b^2+32c^2d\left(2cd^2+5ae^2\right)b-16ac^2e\left(4cd^2+5ae^2\right)\right)+\left(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(bd-3a\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+}}{}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\frac{2\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)}{\sqrt{ce}}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{256c^2d\left(ae^2-bde+cd^2\right)^2\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e\right.}{\sqrt{ce}}}{8ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)}{\sqrt{ce}}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+}{2\sqrt{a+bx+c}}\right.}{e}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
-1/24*((8*c*d - 3*b*e - 6*c*e*x)*(a + b*x + c*x^2)^(3/2))/(c*e^2) + (-1/4*
((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x)*Sqrt[a
 + b*x + c*x^2])/(c*e^2) + (((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d -
 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (128*c^2*d*(c*
d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt
[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(16*c*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.47

method	result
risch	$\frac{(48c^3x^3e^3+72b\,c^2e^3x^2-64c^3d\,e^2x^2+120a\,c^2e^3x+6x\,b^2c\,e^3-112b\,c^2d\,e^2x+96d^2e\,c^3x+60abc\,e^3-256d\,e^2a\,c^2-9b^3e^3-24d\,e^2b^2c+24}{192c^2e^4}$
default	$\frac{\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}}{e}-\frac{d\left(\frac{\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\right.}{3}\right.}{}$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/192/c^2*(48*c^3*e^3*x^3+72*b*c^2*e^3*x^2-64*c^3*d*e^2*x^2+120*a*c^2*e^3*
x+6*b^2*c*e^3*x-112*b*c^2*d*e^2*x+96*c^3*d^2*e*x+60*a*b*c*e^3-256*a*c^2*d*
e^2-9*b^3*e^3-24*b^2*c*d*e^2+240*b*c^2*d^2*e-192*c^3*d^3)*(c*x^2+b*x+a)^(1
/2)/e^4+1/128/e^4/c^2*((48*a^2*c^2*e^4-24*a*b^2*c*e^4-96*a*b*c^2*d*e^3+192
*a*c^3*d^2*e^2+3*b^4*e^4+8*b^3*c*d*e^3+48*b^2*c^2*d^2*e^2-192*b*c^3*d^3*e+
128*c^4*d^4)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+128*d*(
a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)*c^2/e^2
/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)
/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx=\int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 25.09 (sec) , antiderivative size = 10332, normalized size of antiderivative = 30.48

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

output
```
(384*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e
**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d*
*2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e
 + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqr
t(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*
e - 8*c**2*d**2))*a*b*c**3*d*e**3 - 768*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
 + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2
- b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan(
(2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a
*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d
- 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**4*d**2*e**2 - 38
4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2
 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)
*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c
)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e -
 8*c**2*d**2))*b**2*c**3*d**2*e**2 + 1152*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**
2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*ata
n((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*s...
```

3.23 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4}$$

output
```
1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b
*x+a)^(1/2)/c/e^3+1/3*(c*x^2+b*x+a)^(3/2)/e-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b
^2*e^2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(3/2)/e^4+(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2
*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{\frac{e\sqrt{a+x(b+cx)}(3b^2e^2+2ce(-15bd+16ae+7bex)+4c^2(6d^2-3dex+2e^2x^2))}{c} + 48\sqrt{-cd^2+bde-ae^2}(}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
((e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x)
+ 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c + 48*Sqrt[-(c*d^2) + b*d*e - a*e
^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(
Sqrt[a]*(d + e*x) - d*Sqrt[a + x*(b + c*x)])] - (3*(2*c*d - b*e)*(8*c^2*d^
2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt
[a + x*(b + c*x)])])/c^(3/2))/(24*e^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1162, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{cx^2+bx+a}}{d+ex}dx}{2e}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+\right.}{4ce^2}}{2e}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+\right.}{4ce^2}}{2e}$$

$$\Big\downarrow \; 1269$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2c\right.}{4ce^2}}{2e}$$

$$\Big\downarrow \; 1092$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-\right.}{}}{2e}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)}{\sqrt{c}e}}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4\right.}{}}{2e}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{-\frac{32c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(2bd-3ae)-\right.}{\sqrt{c}e}}{8ce^2}}{2e}$$

$$\Big\downarrow \; 219$$

$$\frac{(a+bx+cx^2)^{3/2}}{3e}-\frac{-\frac{\frac{16c(ae^2-bde+cd^2)^{3/2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}}{8ce^2}-\frac{\sqrt{a+bx}}{}}{2e}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
(a + b*x + c*x^2)^(3/2)/(3*e) - (-1/4*((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d
 - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(c*e^2) - (-(((2
*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e)) + (16*c*(c*d^2 - b*d*
e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(2*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(8c^2e^2x^2+14e^2xbc-12c^2dex+32ac\,e^2+3b^2e^2-30bcde+24c^2d^2)\sqrt{c\,x^2+bx+a}}{24c\,e^3}+\frac{\frac{\left(12abc\,e^3-24d\,e^2a\,c^2-b^3e^3-6d\,e^2b^2c+24d^2eb\,c^2-16\right.}{e\sqrt{c}}}{}$
default	$\frac{\frac{\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}\right)^{\frac{3}{2}}}{3}+\frac{(be-2cd)\left(\frac{\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{4c}+\frac{\left(\frac{4c(}{}\right.}{}\right.}{}}{}$

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*c^2*e^2*x^2+14*b*c*e^2*x-12*c^2*d*e*x+32*a*c*e^2+3*b^2*e^2-30*b*
c*d*e+24*c^2*d^2)*(c*x^2+b*x+a)^(1/2)/e^3+1/16/c/e^3*((12*a*b*c*e^3-24*a*c
^2*d*e^2-b^3*e^3-6*b^2*c*d*e^2+24*b*c^2*d^2*e-16*c^3*d^3)/e*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*c*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*
e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 110.40 (sec) , antiderivative size = 1523, normalized size of antiderivative = 6.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
- 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2
)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d
^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*
e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4
*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3
 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/48
*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a
*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/
(c^2*x^2 + b*c*x + a*c)) + 24*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(c*d^2
 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*
b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^3*e^3*
x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*
d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^4), 1/96*(96*(c^3*d^
2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-
c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*...
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 9833, normalized size of antiderivative = 39.02

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

output
```
( - 48*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a
*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*
d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)
*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*s
qrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*
d*e - 8*c**2*d**2))*a*b*c**2*e**3 + 96*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
+ c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 -
 b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((
2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d -
 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**3*d*e**2 + 48*sqr
t(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*
d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt
(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e +
 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c*
*2*d**2))*b**2*c**2*d*e**2 - 144*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*
e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt
(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**...
```

3.24 $\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)} dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)} dx = -\frac{(4cd-5be)\sqrt{a+bx+cx^2}}{4e^2} + \frac{cx\sqrt{a+bx+cx^2}}{2e} - \frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(8c^2d^2+3b^2e^2-12ce(bd-ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}e^3} - \frac{(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{de^3}$$

output
```
-1/4*(-5*b*e+4*c*d)*(c*x^2+b*x+a)^(1/2)/e^2+1/2*c*x*(c*x^2+b*x+a)^(1/2)/e-
a^(3/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d+1/8*(8*c^2*d^
2+3*b^2*e^2-12*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))/c^(1/2)/e^3-(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e
+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d/e^3
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \frac{(-4cd+5be+2cex)\sqrt{a+x(b+cx)}}{4e^2} + \frac{2(-cd^2+e(bd-ae))^{3/2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{de^3} + \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{d} - \frac{(8c^2d^2+3b^2e^2+12ce(-bd+ae))\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{8\sqrt{c}e^3}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d + e*x)),x]
```

output
```
((-4*c*d + 5*b*e + 2*c*e*x)*Sqrt[a + x*(b + c*x)])/(4*e^2) + (2*(-(c*d^2)
+ e*(b*d - a*e))^(3/2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)]
)/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*e^3) + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x
 - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/d - ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(
-(b*d) + a*e))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*Sqrt[c
]*e^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1270, 25, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{ 1270}$$

$$\frac{a\int \frac{\sqrt{cx^2+bx+a}}{x}dx}{d}-\frac{\int -\frac{(bd+cxd-ae)\sqrt{cx^2+bx+a}}{d+ex}dx}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bd+cxd-ae)\sqrt{cx^2+bx+a}}{d+ex}dx}{d}+\frac{a\int \frac{\sqrt{cx^2+bx+a}}{x}dx}{d}$$

$$\Big\downarrow \; 1162$$

$$\frac{\int \frac{(bd+cxd-ae)\sqrt{cx^2+bx+a}}{d+ex}dx}{d}+\frac{a\left(\sqrt{a+bx+cx^2}-\frac{1}{2}\int -\frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bd+cxd-ae)\sqrt{cx^2+bx+a}}{d+ex}dx}{d}+\frac{a\left(\frac{1}{2}\int \frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx+\sqrt{a+bx+cx^2}\right)}{d}$$

$$\Big\downarrow \; 1231$$

$$\frac{-\frac{\int -\frac{c\left(4bcd^3-5b^2ed^2-4aced^2+12abe^2d-8a^2e^3+\left(8c^2d^3-12ce(bd-ae)d+be^2(3bd-4ae)\right)x\right)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-e(5bd-4ae)+4cd^2-2cdex\right)}{4e^2}}{d}+\frac{a\left(\frac{1}{2}\int \frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx+\sqrt{a+bx+cx^2}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{4bcd^3-5b^2ed^2-4aced^2+12abe^2d-8a^2e^3+\left(8c^2d^3-12ce(bd-ae)d+be^2(3bd-4ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8e^2}-\frac{\sqrt{a+bx+cx^2}\left(-e(5bd-4ae)+4cd^2-2cdex\right)}{4e^2}}{d}+\frac{a\left(\frac{1}{2}\int \frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx+\sqrt{a+bx+cx^2}\right)}{d}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8e^2}-\frac{\sqrt{a+bx+cx^2}\left(-e(5bd-4ae)+4cd^2-2cdex\right)}{4e^2}}{d}+\frac{a\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{cx^2+bx+a}}dx+2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)+\sqrt{a+bx+cx^2}\right)}{d}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\frac{2\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8e^2}-\frac{\sqrt{a+bx+cx^2}(-e(5bd-4ae)+}{4e^2}}{d}$$

$$\frac{a\left(\frac{1}{2}\left(2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2b\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)+\sqrt{a+bx+cx^2}\right)}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)}{\sqrt{c}e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8e^2}-\frac{\sqrt{a+bx+cx^2}(-e(5bd-4ae)+4c}{4e^2}}{d}$$

$$\frac{a\left(\frac{1}{2}\left(2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx+cx^2}\right)}{d}$$

$\downarrow$ 1154

$$\frac{\frac{\frac{16\left(ae^2-bde+cd^2\right)^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)\right.}{\sqrt{c}e}}{8e^2}}{d}$$

$$\frac{a\left(\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-4a\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)+\sqrt{a+bx+cx^2}\right)}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)}{\sqrt{c}e}-\frac{8\left(ae^2-bde+cd^2\right)^{3/2}\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8e^2}-\sqrt{a+bx+cx}}{d}$$

$$\frac{a\left(\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\right)+\sqrt{a+bx+cx^2}\right)}{d}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x*(d + e*x)),x]
```

output
```
(a*(Sqrt[a + b*x + c*x^2] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqr
t[a + b*x + c*x^2])] + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/Sqrt[c])/2))/d + (-1/4*((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e*x)*
Sqrt[a + b*x + c*x^2])/e^2 + (((8*c^2*d^3 + b*e^2*(3*b*d - 4*a*e) - 12*c*d
*e*(b*d - a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(S
qrt[c]*e) - (8*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d
 - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*e
^2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(218) = 436$.

Time = 1.43 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.15

method	result
default	$\frac{\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2}+a\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\right.}{d}$

input
```
int((c*x^2+b*x+a)^(3/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*(c*x^2+b*x+a)^(3/2)+1/2*b*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+a*((c*x
^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-
a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-1/d*(1/3*(c*(x+d/e
)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e
*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2
/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*(
(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e
-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^
2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{x\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{x\,\left(d+e\,x\right)} \,d x$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx = \frac{8\sqrt{a\,e^2-bde+c\,d^2}\,\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex\right.}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/x/(e*x+d),x)
```

output

```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c*e**2 - 8*sqrt(a*
e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e
 + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b*c*d*e + 8*sqrt(a*e**2 - b*d*
e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2)
- 2*a*e + b*d - b*e*x + 2*c*d*x)*c**2*d**2 - 8*sqrt(a*e**2 - b*d*e + c*d**
2)*log(d + e*x)*a*c*e**2 + 8*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*
c*d*e - 8*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*c**2*d**2 + 10*sqrt(a
 + b*x + c*x**2)*b*c*d*e**2 - 8*sqrt(a + b*x + c*x**2)*c**2*d**2*e + 4*sqr
t(a + b*x + c*x**2)*c**2*d*e**2*x + 8*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x +
 c*x**2) - 2*a - b*x)*a*c*e**3 - 8*sqrt(a)*log(x)*a*c*e**3 + 12*sqrt(c)*lo
g( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*c*d*e**2 + 3*sqrt(c)*
log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b**2*d*e**2 - 12*sqrt
(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b*c*d**2*e + 8*sq
rt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*c**2*d**3)/(8*c
*d*e**3)
```

3.25 $\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 231

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \frac{c\sqrt{a+bx+cx^2}}{e} - \frac{a\sqrt{a+bx+cx^2}}{dx} - \frac{\sqrt{a}(3bd-2ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d^2} - \frac{\sqrt{c}(2cd-3be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2e^2} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2e^2}$$

output

```
c*(c*x^2+b*x+a)^(1/2)/e-a*(c*x^2+b*x+a)^(1/2)/d/x-1/2*a^(1/2)*(-2*a*e+3*b*
d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2-1/2*c^(1/2)*(-3*
b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2+(a*e^2-b
*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/e^2
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \frac{-4(-cd^2+e(bd-ae))^{3/2}\,x\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)+2\sqrt{a}e^2(-3bd+2ae$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

output
```
(-4*(-(c*d^2) + e*(b*d - a*e))^(3/2)*x*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[
a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + 2*Sqrt[a]*e^2*(-3*b*d
+ 2*a*e)*x*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]] + d*(2*
e*(-(a*e) + c*d*x)*Sqrt[a + x*(b + c*x)] + Sqrt[c]*d*(2*c*d - 3*b*e)*x*Log
[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*d^2*e^2*x)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 516 vs. 2(231) = 462.

Time = 0.97 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \ 1289$$

$$\int \left(\frac{e^2(a+bx+cx^2)^{3/2}}{d^2(d+ex)} - \frac{e(a+bx+cx^2)^{3/2}}{d^2x} + \frac{(a+bx+cx^2)^{3/2}}{dx^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^{3/2}e\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}+\frac{be(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{16c^{3/2}d^2e^2}+\frac{3(4ac+b^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2e^2}-\frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}+\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8cd^2e}-\frac{e(8ac+b^2+2bcx)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{(a+bx+cx^2)^{3/2}}{dx}+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

output
```
(3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - (e*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d -
 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2*e) - (a +
 b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sq
rt[a + b*x + c*x^2])])/(2*d) + (a^(3/2)*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + b*x + c*x^2])])/d^2 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) + (b*(b^2 - 12*a*c)*e*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((2*c*d
- b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e
+ a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d
*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d^2*e^2)
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(199) = 398$.

Time = 1.54 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.76

method	result
risch	$-\frac{a\sqrt{c\,x^2+bx+a}}{dx}+\frac{-\frac{2\left(a^2e^4-2d\,e^3ab+2ac\,d^2e^2+d^2e^2b^2-2bc\,d^3e+c^2d^4\right)\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{}}{x+\frac{d}{e}}\right.}{e^3d\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-a*(c*x^2+b*x+a)^(1/2)/d/x+1/2/d*(-2/e^3*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^
2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x+d/e))+a^(1/2)*(2*a*e-3*b*d)/d*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)+2*c*d/e^2*(c*e*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2)))+2*b*e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))/c^(1/2)-c^(1/2)*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{x^2\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**2/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx=\frac{2\sqrt{a\,e^2-bde+c\,d^2}\log\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+\right.}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/x^2/(e*x+d),x)
```

output
```
(2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*e**2*x - 2*sqrt(a*e**
2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d
**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b*d*e*x + 2*sqrt(a*e**2 - b*d*e + c*
d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e +
 b*d - b*e*x + 2*c*d*x)*c*d**2*x - 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d +
 e*x)*a*e**2*x + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*d*e*x - 2*
sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*c*d**2*x - 2*sqrt(a + b*x + c*x
**2)*a*d*e**2 + 2*sqrt(a + b*x + c*x**2)*c*d**2*e*x + 2*sqrt(a)*log( - 2*s
qrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*e**3*x - 3*sqrt(a)*log( - 2*s
qrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*d*e**2*x - 2*sqrt(a)*log(x)*a
*e**3*x + 3*sqrt(a)*log(x)*b*d*e**2*x + 3*sqrt(c)*log( - 2*sqrt(c)*sqrt(a
+ b*x + c*x**2) - b - 2*c*x)*b*d**2*e*x - 2*sqrt(c)*log( - 2*sqrt(c)*sqrt(
a + b*x + c*x**2) - b - 2*c*x)*c*d**3*x)/(2*d**2*e**2*x)
```

3.26 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)} dx$

Optimal result

Integrand size = 25, antiderivative size = 257

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)} dx = -\frac{a\sqrt{a+bx+cx^2}}{2dx^2} - \frac{(5bd-4ae)\sqrt{a+bx+cx^2}}{4d^2x} - \frac{(3b^2d^2+12acd^2-12abde+8a^2e^2)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d^3} + \frac{c^{3/2}\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e} - \frac{(cd^2-bde+ae^2)^{3/2}\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^3e}$$

output
```
-1/2*a*(c*x^2+b*x+a)^(1/2)/d/x^2-1/4*(-4*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/d^
2/x-1/8*(8*a^2*e^2-12*a*b*d*e+12*a*c*d^2+3*b^2*d^2)*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)/d^3+c^(3/2)*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/e-(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a
*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/e
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)}\,dx = \frac{\frac{d(-2ad-5bdx+4aex)\sqrt{a+x(b+cx)}}{x^2}+\frac{8(-cd^2+e(bd-ae))^{3/2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e}+8a^{3/2}}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

output
```
((d*(-2*a*d - 5*b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)])/x^2 + (8*(-(c*d^2)
 + e*(b*d - a*e))^(3/2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)
])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/e + 8*a^(3/2)*e^2*ArcTanh[(Sqrt[c]*x -
 Sqrt[a + x*(b + c*x)])/Sqrt[a]] - (3*d*(b^2*d + 4*a*c*d - 4*a*b*e)*ArcTan
h[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - (4*c^(3/2)*d^
3*Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/e)/(4*d^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 672 vs. 2(257) = 514.

Time = 1.10 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(-\frac{e^3(a+bx+cx^2)^{3/2}}{d^3(d+ex)}+\frac{e^2(a+bx+cx^2)^{3/2}}{d^3x}-\frac{e(a+bx+cx^2)^{3/2}}{d^2x^2}+\frac{(a+bx+cx^2)^{3/2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}e^2\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^3}-\frac{be^2(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^3}+\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{16c^{3/2}d^3e}-\frac{3e(4ac+b^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d^2}-\frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}+\frac{3\sqrt{a}be\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d^2}-\frac{(ae^2-bde+cd^2)^{3/2}\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^3e}+\frac{3b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8cd^3}+\frac{e^2(8ac+b^2+2bcx)\sqrt{a+bx+cx^2}}{8cd^3}+\frac{e(a+bx+cx^2)^{3/2}}{d^2x}-\frac{3e(3b+2cx)\sqrt{a+bx+cx^2}}{4d^2}-\frac{(a+bx+cx^2)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

output
```
(-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) - (3*e*(3*b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(4*d^2) + (e^2*(b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x
^2])/(8*c*d^3) - ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*
c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^3) - (a + b*x + c*x^2)^(3/2)/(
2*d*x^2) + (e*(a + b*x + c*x^2)^(3/2))/(d^2*x) - (3*(b^2 + 4*a*c)*ArcTanh[
(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d) + (3*Sqrt[a]
*b*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d^2) - (a^
(3/2)*e^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^3 + (3
*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*d) -
 (3*(b^2 + 4*a*c)*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*Sqrt[c]*d^2) - (b*(b^2 - 12*a*c)*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^3) + ((2*c*d - b*e)*(8*c^2*d^2 - b^2
*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2])])/(16*c^(3/2)*d^3*e) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b
*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
+ c*x^2])])/(d^3*e)
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{\sqrt{c x^2+bx+a}\,(-4aex+5bdx+2ad)}{4d^2x^2}+\frac{-\frac{\left(8e^2a^2-12abde+12ad^2c+3b^2d^2\right)\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{c x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{8\left(a^2e^4-2de^3ab+2acd^2e^2 \cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-4*a*e*x+5*b*d*x+2*a*d)/d^2/x^2+1/8/d^2*(-(8*a^2
*e^2-12*a*b*d*e+12*a*c*d^2+3*b^2*d^2)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x
^2+b*x+a)^(1/2))/x)+8/e^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2
*b*c*d^3*e+c^2*d^4)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c
*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+8*c^(3
/2)*d^2/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{x^3(d + ex)} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a + bx + cx^2)^{3/2}}{x^3(d + ex)} \, dx = \int \frac{(a + bx + cx^2)^{\frac{3}{2}}}{x^3 \, (d + ex)} \, dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**3/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a + bx + cx^2)^{3/2}}{x^3(d + ex)} \, dx = \int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(ex + d)x^3} \, dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^3*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d+ex)}\,dx = \frac{8\sqrt{a\,e^2-bde+c\,d^2}\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex\right.}{}$$

input
```
int((c*x^2+b*x+a)^(3/2)/x^3/(e*x+d),x)
```

output
```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*e**2*x**2 - 8*s
qrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*d*e*x**2 + 8*sqrt(a*
e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e
 + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c*d**2*x**2 - 8*sqrt(a*e**2
- b*d*e + c*d**2)*log(d + e*x)*a**2*e**2*x**2 + 8*sqrt(a*e**2 - b*d*e + c*
d**2)*log(d + e*x)*a*b*d*e*x**2 - 8*sqrt(a*e**2 - b*d*e + c*d**2)*log(d +
e*x)*a*c*d**2*x**2 - 4*sqrt(a + b*x + c*x**2)*a**2*d**2*e + 8*sqrt(a + b*x
 + c*x**2)*a**2*d*e**2*x - 10*sqrt(a + b*x + c*x**2)*a*b*d**2*e*x + 8*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*e**3*x**2 - 12*
sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*d*e**2*x**2
+ 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*d**2*e*
x**2 + 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*d*
*2*e*x**2 - 8*sqrt(a)*log(x)*a**2*e**3*x**2 + 12*sqrt(a)*log(x)*a*b*d*e**2
*x**2 - 12*sqrt(a)*log(x)*a*c*d**2*e*x**2 - 3*sqrt(a)*log(x)*b**2*d**2*e*x
**2 + 8*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*c*d
**3*x**2)/(8*a*d**3*e*x**2)
```

3.27 $\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = -\frac{a\sqrt{a+bx+cx^2}}{3dx^3}-\frac{(7bd-6ae)\sqrt{a+bx+cx^2}}{12d^2x^2} - \frac{(3b^2d^2+32acd^2-30abde+24a^2e^2)\sqrt{a+bx+cx^2}}{24ad^3x} + \frac{(bd-2ae)\left(b^2d^2+8abde-4a(3cd^2+2ae^2)\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}d^4} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^4}$$

output
```
-1/3*a*(c*x^2+b*x+a)^(1/2)/d/x^3-1/12*(-6*a*e+7*b*d)*(c*x^2+b*x+a)^(1/2)/d
^2/x^2-1/24*(24*a^2*e^2-30*a*b*d*e+32*a*c*d^2+3*b^2*d^2)*(c*x^2+b*x+a)^(1/
2)/a/d^3/x+1/16*(-2*a*e+b*d)*(b^2*d^2+8*a*b*d*e-4*a*(2*a*e^2+3*c*d^2))*arc
tanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d^4+(a*e^2-b*d*e+c
*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \frac{-\sqrt{a}\left(d\sqrt{a+x(b+cx)}(3b^2d^2x^2+2adx(7bd+16cdx-15bex)+4a^2(2d^2-3de\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
(-(Sqrt[a]*(d*Sqrt[a + x*(b + c*x)]*(3*b^2*d^2*x^2 + 2*a*d*x*(7*b*d + 16*c
*d*x - 15*b*e*x) + 4*a^2*(2*d^2 - 3*d*e*x + 6*e^2*x^2)) + 48*a*(-(c*d^2) +
 e*(b*d - a*e))^(3/2)*x^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*
x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])) - 48*a^3*e^3*x^3*ArcTanh[(Sqrt[c]*x
 - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 3*d*(b^3*d^2 + 6*a*b^2*d*e + 24*a^2*c
*d*e - 12*a*b*(c*d^2 + 2*a*e^2))*x^3*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b
 + c*x)])/Sqrt[a]])/(24*a^(3/2)*d^4*x^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 835 vs. 2(279) = 558.

Time = 1.29 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^4(a+bx+cx^2)^{3/2}}{d^4(d+ex)}-\frac{e^3(a+bx+cx^2)^{3/2}}{d^4x}+\frac{e^2(a+bx+cx^2)^{3/2}}{d^3x^2}-\frac{e(a+bx+cx^2)^{3/2}}{d^2x^3}+\frac{(a+bx+cx^2)^{3/2}}{dx^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^3}{d^4}+\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{16c^{3/2}d^4}-\frac{\left(b^2+2cxb+8ac\right)\sqrt{cx^2+bx+a}e^3}{8cd^4}-\frac{\left(cx^2+bx+a\right)^{3/2}e^2}{d^3x}-\frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^2}{2d^3}+\frac{3\left(b^2+4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{8\sqrt{c}d^3}+\frac{3(3b+2cx)\sqrt{cx^2+bx+a}e^2}{4d^3}+\frac{\left(cx^2+bx+a\right)^{3/2}e}{2d^2x^2}+\frac{3\left(b^2+4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e}{8\sqrt{a}d^2}-\frac{3b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e}{2d^2}+\frac{3(b-2cx)\sqrt{cx^2+bx+a}e}{4d^2x}+\frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{cx^2+bx+a}e}{8cd^4}-\frac{\left(cx^2+bx+a\right)^{3/2}}{3dx^3}+\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{16a^{3/2}d}-\frac{(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{16c^{3/2}d^4}+\frac{c^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{d}+\frac{\left(cd^2-bed+ae^2\right)^{3/2}\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{d^4}-\frac{\left(2ab+\left(b^2+8ac\right)x\right)\sqrt{cx^2+bx+a}}{8adx^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
(3*e*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d^2*x) + (3*e^2*(3*b + 2*c*x)*S
qrt[a + b*x + c*x^2])/(4*d^3) - (e^3*(b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x
+ c*x^2])/(8*c*d^4) - ((2*a*b + (b^2 + 8*a*c)*x)*Sqrt[a + b*x + c*x^2])/(8
*a*d*x^2) + (e*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d
 - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^4) - (a + b*x + c*x^2)^(3/2)/(3*d
*x^3) + (e*(a + b*x + c*x^2)^(3/2))/(2*d^2*x^2) - (e^2*(a + b*x + c*x^2)^(
3/2))/(d^3*x) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
b*x + c*x^2])])/(16*a^(3/2)*d) + (3*(b^2 + 4*a*c)*e*ArcTanh[(2*a + b*x)/(2
*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d^2) - (3*Sqrt[a]*b*e^2*ArcTa
nh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d^3) + (a^(3/2)*e^3*
ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^4 + (c^(3/2)*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d - (3*b*Sqrt[c]*e*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*d^2) + (3*(b^2 +
4*a*c)*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt
[c]*d^3) + (b*(b^2 - 12*a*c)*e^3*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(16*c^(3/2)*d^4) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*
c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(16*c^(3/2)*d^4) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])
/d^4
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.44

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(24a^2e^2x^2-30abde\,x^2+32a\,d^2x^2c+3b^2d^2x^2-12a^2dex+14ab\,d^2x+8a^2d^2\right)}{24a\,d^3x^3}-\frac{-\frac{\left(16e^3a^3-24a^2bd\,e^2+24a^2c\,d^2e+6a\,b\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(24*a^2*e^2*x^2-30*a*b*d*e*x^2+32*a*c*d^2*x^2+3*
b^2*d^2*x^2-12*a^2*d*e*x+14*a*b*d^2*x+8*a^2*d^2)/a/d^3/x^3-1/16/d^3/a*(-1/
d*(16*a^3*e^3-24*a^2*b*d*e^2+24*a^2*c*d^2*e+6*a*b^2*d^2*e-12*a*b*c*d^3+b^3
*d^3)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+16*a*(a^2*e^4-
2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d/e/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 1591, normalized size of antiderivative = 5.70

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
[1/96*(48*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*x^
3*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e
+ (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
+ a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(24*a^2*b*d*e^2 - 16*a^3*e^
3 - (b^3 - 12*a*b*c)*d^3 - 6*(a*b^2 + 4*a^2*c)*d^2*e)*sqrt(a)*x^3*log(-(8*
a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
8*a^2)/x^2) - 4*(8*a^3*d^3 - (30*a^2*b*d^2*e - 24*a^3*d*e^2 - (3*a*b^2 + 3
2*a^2*c)*d^3)*x^2 + 2*(7*a^2*b*d^3 - 6*a^3*d^2*e)*x)*sqrt(c*x^2 + b*x + a)
)/(a^2*d^4*x^3), 1/96*(96*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-c*d^2 +
b*d*e - a*e^2)*x^3*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b
*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^
2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 3*(24
*a^2*b*d*e^2 - 16*a^3*e^3 - (b^3 - 12*a*b*c)*d^3 - 6*(a*b^2 + 4*a^2*c)*d^2
*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(8*a^3*d^3 - (30*a^2*b*d^2*e - 24*
a^3*d*e^2 - (3*a*b^2 + 32*a^2*c)*d^3)*x^2 + 2*(7*a^2*b*d^3 - 6*a^3*d^2*e)*
x)*sqrt(c*x^2 + b*x + a))/(a^2*d^4*x^3), 1/48*(3*(24*a^2*b*d*e^2 - 16*a^3*
e^3 - (b^3 - 12*a*b*c)*d^3 - 6*(a*b^2 + 4*a^2*c)*d^2*e)*sqrt(-a)*x^3*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2...
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{x^4\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**4/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)x^4}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(249) = 498$.

Time = 0.34 (sec) , antiderivative size = 916, normalized size of antiderivative = 3.28

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2
*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^4) - 1/8*(b^3*d^3 -
12*a*b*c*d^3 + 6*a*b^2*d^2*e + 24*a^2*c*d^2*e - 24*a^2*b*d*e^2 + 16*a^3*e^
3)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*d^4)
+ 1/24*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d^2 + 60*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a*b*c*d^2 - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a*b^2*d*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*d*e + 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*e^2 + 48*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a*b^2*sqrt(c)*d^2 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^2*c^(3/2)*d^2 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)
*d*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*sqrt(c)*e^2 + 8*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^2*b^2*d*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*e^
2 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d^2 + 144*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*d*e - 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^2*b^3*d^2 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d^2 - 18*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*d*e + 24*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a^4*c*d*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*e^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^4\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^4(d+ex)}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{x^4\,(ex+d)}dx$$

input

```
int((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x)
```

output

```
int((c*x^2+b*x+a)^(3/2)/x^4/(e*x+d),x)
```

3.28 $\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)} dx$

Optimal result

Integrand size = 25, antiderivative size = 417

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)} dx = -\frac{a\sqrt{a+bx+cx^2}}{4dx^4} - \frac{(9bd-8ae)\sqrt{a+bx+cx^2}}{24d^2x^3}$$
$$-\frac{(3b^2d^2+60acd^2-56abde+48a^2e^2)\sqrt{a+bx+cx^2}}{96ad^3x^2}$$
$$+\frac{(9b^3d^3+24ab^2d^2e+64a^2e(4cd^2+3ae^2)-60abd(cd^2+4ae^2))\sqrt{a+bx+cx^2}}{192a^2d^4x}$$
$$-\frac{(3b^4d^4+8ab^3d^3e-24ab^2d^2(cd^2-2ae^2)-96a^2bde(cd^2+2ae^2)+16a^2(3c^2d^4+12acd^2e^2+8a^2e^4))\,\text{arcta}}{128a^{5/2}d^5}$$
$$-\frac{e(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^5}$$

output
```
-1/4*a*(c*x^2+b*x+a)^(1/2)/d/x^4-1/24*(-8*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/d
^2/x^3-1/96*(48*a^2*e^2-56*a*b*d*e+60*a*c*d^2+3*b^2*d^2)*(c*x^2+b*x+a)^(1/
2)/a/d^3/x^2+1/192*(9*b^3*d^3+24*a*b^2*d^2*e+64*a^2*e*(3*a*e^2+4*c*d^2)-60
*a*b*d*(4*a*e^2+c*d^2))*(c*x^2+b*x+a)^(1/2)/a^2/d^4/x-1/128*(3*b^4*d^4+8*a
*b^3*d^3*e-24*a*b^2*d^2*(-2*a*e^2+c*d^2)-96*a^2*b*d*e*(2*a*e^2+c*d^2)+16*a
^2*(8*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*
x^2+b*x+a)^(1/2))/a^(5/2)/d^5-e*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d
-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{a}\Big(d\sqrt{a+x(b+cx)}(9b^3d^3x^3-16a^3(3d^3-4d^2ex+6de^2x^2-12e^3x^3)-6abd^2}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[a]*(d*Sqrt[a + x*(b + c*x)]*(9*b^3*d^3*x^3 - 16*a^3*(3*d^3 - 4*d^2*e
*x + 6*d*e^2*x^2 - 12*e^3*x^3) - 6*a*b*d^2*x^2*(10*c*d*x + b*(d - 4*e*x))
- 8*a^2*d*x*(c*d*x*(15*d - 32*e*x) + b*(9*d^2 - 14*d*e*x + 30*e^2*x^2))) +
 384*a^2*e*(-(c*d^2) + e*(b*d - a*e))^(3/2)*x^4*ArcTan[(Sqrt[c]*(d + e*x)
- e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]) + 384*a^4*e^4*
x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 3*d*(3*b^4*d^3
+ 8*a*b^3*d^2*e + 24*a*b^2*d*(-(c*d^2) + 2*a*e^2) - 96*a^2*b*e*(c*d^2 + 2*
a*e^2) + 48*a^2*c*d*(c*d^2 + 4*a*e^2))*x^4*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a
+ x*(b + c*x)])/Sqrt[a]])/(192*a^(5/2)*d^5*x^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 982 vs. 2(417) = 834.

Time = 1.44 (sec) , antiderivative size = 982, normalized size of antiderivative = 2.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \Bigg(-\frac{e^5(a+bx+cx^2)^{3/2}}{d^5(d+ex)}+\frac{e^4(a+bx+cx^2)^{3/2}}{d^5x}-\frac{e^3(a+bx+cx^2)^{3/2}}{d^4x^2}+\frac{e^2(a+bx+cx^2)^{3/2}}{d^3x^3}-\frac{e(a+bx+c}{d^2x^4}$$

$\downarrow$ 2009

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^4}{d^5}-\frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^4}{16c^{3/2}d^5}+\frac{(b^2+2cxb+8ac)\sqrt{cx^2+bx+a}e^4}{8cd^5}+\frac{(cx^2+bx+a)^{3/2}e^3}{d^4x}+\frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^3}{2d^4}-\frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{8\sqrt{c}d^4}-\frac{3(3b+2cx)\sqrt{cx^2+bx+a}e^3}{4d^4}-\frac{(cx^2+bx+a)^{3/2}e^2}{2d^3x^2}-\frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^2}{8\sqrt{a}d^3}+\frac{3b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{2d^3}-\frac{3(b-2cx)\sqrt{cx^2+bx+a}e^2}{4d^3x}-\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{cx^2+bx+a}e^2}{8cd^5}+\frac{(cx^2+bx+a)^{3/2}e}{3d^2x^3}-\frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e}{16a^{3/2}d^2}+\frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e}{16c^{3/2}d^5}-\frac{c^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e}{d^2}-\frac{(cd^2-bed+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e}{d^5}+\frac{(2ab+(b^2+8ac)x)\sqrt{cx^2+bx+a}}{8ad^2x^2}-\frac{(2a+bx)(cx^2+bx+a)^{3/2}}{8adx^4}-\frac{3(b^2-4ac)^2\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{128a^{5/2}d}+\frac{3(b^2-4ac)(2a+bx)\sqrt{cx^2+bx+a}}{64a^2dx^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

output
```
(3*(b^2 - 4*a*c)*(2*a + b*x)*Sqrt[a + b*x + c*x^2])/(64*a^2*d*x^2) - (3*e^
2*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d^3*x) - (3*e^3*(3*b + 2*c*x)*Sqrt
[a + b*x + c*x^2])/(4*d^4) + (e^4*(b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c
*x^2])/(8*c*d^5) + (e*(2*a*b + (b^2 + 8*a*c)*x)*Sqrt[a + b*x + c*x^2])/(8*
a*d^2*x^2) - (e^2*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*
c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^5) + (e*(a + b*x + c*x^2)^(3/2
))/(3*d^2*x^3) - (e^2*(a + b*x + c*x^2)^(3/2))/(2*d^3*x^2) + (e^3*(a + b*x
 + c*x^2)^(3/2))/(d^4*x) - ((2*a + b*x)*(a + b*x + c*x^2)^(3/2))/(8*a*d*x^
4) - (3*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^
2])])/(128*a^(5/2)*d) - (b*(b^2 - 12*a*c)*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*d^2) - (3*(b^2 + 4*a*c)*e^2*ArcTanh[
(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d^3) + (3*Sqrt[
a]*b*e^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d^4) -
 (a^(3/2)*e^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^5
- (c^(3/2)*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d^2 +
 (3*b*Sqrt[c]*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(2*d^3) - (3*(b^2 + 4*a*c)*e^3*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(8*Sqrt[c]*d^4) - (b*(b^2 - 12*a*c)*e^4*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^5) + (e*(2*c*d - b*e)*(8*
c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[...
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(-192 x^{3} e^{3} a^{3}+240 x^{3} a^{2} b d\,e^{2}-256 a^{2} c\,d^{2} e\,x^{3}-24 x^{3} a\,b^{2} d^{2} e+60 a b c\,d^{3} x^{3}-9 x^{3} b^{3} d^{3}+96 a^{3} d\,e^{2} x^{2}-112 a^{2} b\,d^{2} e\,x^{2}+12\right.}{192 a^{2} d^{4} x^{4}}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(-192*a^3*e^3*x^3+240*a^2*b*d*e^2*x^3-256*a^2*c
*d^2*e*x^3-24*a*b^2*d^2*e*x^3+60*a*b*c*d^3*x^3-9*b^3*d^3*x^3+96*a^3*d*e^2*
x^2-112*a^2*b*d^2*e*x^2+120*a^2*c*d^3*x^2+6*a*b^2*d^3*x^2-64*a^3*d^2*e*x+7
2*a^2*b*d^3*x+48*a^3*d^3)/a^2/d^4/x^4+1/128/a^2/d^4*(-(128*a^4*e^4-192*a^3
*b*d*e^3+192*a^3*c*d^2*e^2+48*a^2*b^2*d^2*e^2-96*a^2*b*c*d^3*e+48*a^2*c^2*
d^4+8*a*b^3*d^3*e-24*a*b^2*c*d^4+3*b^4*d^4)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)+128*a^2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d
^2*e^2-2*b*c*d^3*e+c^2*d^4)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)
))
```

Fricas [A] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 2011, normalized size of antiderivative = 4.82

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
[1/768*(384*(a^3*c*d^2*e - a^3*b*d*e^2 + a^4*e^3)*sqrt(c*d^2 - b*d*e + a*e
^2)*x^4*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2
+ b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3
*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(192*a^3*b*d*e^3 - 12
8*a^4*e^4 - 3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*d^4 - 8*(a*b^3 - 12*a^2*b*c)*
d^3*e - 48*(a^2*b^2 + 4*a^3*c)*d^2*e^2)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 +
 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) -
4*(48*a^4*d^4 + (240*a^3*b*d^2*e^2 - 192*a^4*d*e^3 - 3*(3*a*b^3 - 20*a^2*b
*c)*d^4 - 8*(3*a^2*b^2 + 32*a^3*c)*d^3*e)*x^3 - 2*(56*a^3*b*d^3*e - 48*a^4
*d^2*e^2 - 3*(a^2*b^2 + 20*a^3*c)*d^4)*x^2 + 8*(9*a^3*b*d^4 - 8*a^4*d^3*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^3*d^5*x^4), -1/768*(768*(a^3*c*d^2*e - a^3*b
*d*e^2 + a^4*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*x^4*arctan(-1/2*sqrt(-c*d^2
 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a
*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2
- b^2*d*e + a*b*e^2)*x)) + 3*(192*a^3*b*d*e^3 - 128*a^4*e^4 - 3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*d^4 - 8*(a*b^3 - 12*a^2*b*c)*d^3*e - 48*(a^2*b^2 + 4*
a^3*c)*d^2*e^2)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x
^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(48*a^4*d^4 + (240*a^3
*b*d^2*e^2 - 192*a^4*d*e^3 - 3*(3*a*b^3 - 20*a^2*b*c)*d^4 - 8*(3*a^2*b^...
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{x^5\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**5/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x**5*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)x^5}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1853 vs. $2(383) = 766$.

Time = 0.41 (sec) , antiderivative size = 1853, normalized size of antiderivative = 4.44

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
-2*(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4
+ a^2*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqr
t(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^5) + 1/64*(3*b^
4*d^4 - 24*a*b^2*c*d^4 + 48*a^2*c^2*d^4 + 8*a*b^3*d^3*e - 96*a^2*b*c*d^3*e
 + 48*a^2*b^2*d^2*e^2 + 192*a^3*c*d^2*e^2 - 192*a^3*b*d*e^3 + 128*a^4*e^4)
*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2*d^5)
- 1/192*(9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*d^3 - 72*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d^3 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*a^2*c^2*d^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*d^2*e
 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c*d^2*e - 240*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*d*e^2 - 192*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^3*c*d*e^2 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b
*e^3 - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(3/2)*d^3 + 384*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*sqrt(c)*d^2*e + 768*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(3/2)*d^2*e - 768*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^3*b*sqrt(c)*d*e^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^4*sqrt(c)*e^3 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d^
3 - 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c*d^3 - 144*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^2*d^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^2*b^3*d^2*e - 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^5\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^5*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^5(d+ex)}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{x^5\,(ex+d)}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x)
```

output
```
int((c*x^2+b*x+a)^(3/2)/x^5/(e*x+d),x)
```

3.29 $\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 577

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = -\frac{a\sqrt{a+bx+cx^2}}{5dx^5} - \frac{(11bd-10ae)\sqrt{a+bx+cx^2}}{40d^2x^4}$$
$$-\frac{(3b^2d^2+96acd^2-90abde+80a^2e^2)\sqrt{a+bx+cx^2}}{240ad^3x^3}$$
$$+\frac{(15b^3d^3-84abcd^3+30ab^2d^2e+600a^2cd^2e-560a^2bde^2+480a^3e^3)\sqrt{a+bx+cx^2}}{960a^2d^4x^2}$$
$$-\frac{(45b^4d^4+90ab^3d^3e-60ab^2d^2(5cd^2-4ae^2)-600a^2bde(cd^2+4ae^2)+128a^2(3c^2d^4+20acd^2e^2+15a^2e^4}{1920a^3d^5x}$$
$$+\frac{(3b^5d^5+6ab^4d^4e-48a^2b^2d^2e(cd^2-2ae^2)-8ab^3d^3(3cd^2-2ae^2)+48a^2bd(c^2d^4-4acd^2e^2-8a^2e^4)+32}{256a^{7/2}d^6}$$
$$+\frac{e^2(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^6}$$

output
```
-1/5*a*(c*x^2+b*x+a)^(1/2)/d/x^5-1/40*(-10*a*e+11*b*d)*(c*x^2+b*x+a)^(1/2)
/d^2/x^4-1/240*(80*a^2*e^2-90*a*b*d*e+96*a*c*d^2+3*b^2*d^2)*(c*x^2+b*x+a)^
(1/2)/a/d^3/x^3+1/960*(480*a^3*e^3-560*a^2*b*d*e^2+600*a^2*c*d^2*e+30*a*b^
2*d^2*e-84*a*b*c*d^3+15*b^3*d^3)*(c*x^2+b*x+a)^(1/2)/a^2/d^4/x^2-1/1920*(4
5*b^4*d^4+90*a*b^3*d^3*e-60*a*b^2*d^2*(-4*a*e^2+5*c*d^2)-600*a^2*b*d*e*(4*
a*e^2+c*d^2)+128*a^2*(15*a^2*e^4+20*a*c*d^2*e^2+3*c^2*d^4))*(c*x^2+b*x+a)^
(1/2)/a^3/d^5/x+1/256*(3*b^5*d^5+6*a*b^4*d^4*e-48*a^2*b^2*d^2*e*(-2*a*e^2+
c*d^2)-8*a*b^3*d^3*(-2*a*e^2+3*c*d^2)+48*a^2*b*d*(-8*a^2*e^4-4*a*c*d^2*e^2
+c^2*d^4)+32*a^3*e*(8*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4))*arctanh(1/2*(b*x+
2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)/d^6+e^2*(a*e^2-b*d*e+c*d^2)^(3/2
)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 9.17 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \frac{-\sqrt{a}\Big(d\sqrt{a+x(b+cx)}(45b^4d^4x^4+32a^4(12d^4-15d^3ex+20d^2e^2x^2-30de^3x^3}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output
```
(-(Sqrt[a]*(d*Sqrt[a + x*(b + c*x)]*(45*b^4*d^4*x^4 + 32*a^4*(12*d^4 - 15*
d^3*e*x + 20*d^2*e^2*x^2 - 30*d*e^3*x^3 + 60*e^4*x^4) - 30*a*b^2*d^3*x^3*(
10*c*d*x + b*(d - 3*e*x)) + 12*a^2*d^2*x^2*(32*c^2*d^2*x^2 + 2*b*c*d*x*(7*
d - 25*e*x) + b^2*(2*d^2 - 5*d*e*x + 20*e^2*x^2)) + 16*a^3*d*x*(c*d*x*(48*
d^2 - 75*d*e*x + 160*e^2*x^2) + b*(33*d^3 - 45*d^2*e*x + 70*d*e^2*x^2 - 15
0*e^3*x^3))) + 3840*a^3*e^2*(-(c*d^2) + e*(b*d - a*e))^(3/2)*x^5*ArcTan[(S
qrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]
])) - 3840*a^5*e^5*x^5*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]
] + 15*d*(3*b^5*d^4 + 6*a*b^4*d^3*e + 8*a*b^3*d^2*(-3*c*d^2 + 2*a*e^2) + 4
8*a^2*b^2*d*e*(-(c*d^2) + 2*a*e^2) + 96*a^3*c*d*e*(c*d^2 + 4*a*e^2) - 48*a
^2*b*(-(c^2*d^4) + 4*a*c*d^2*e^2 + 8*a^2*e^4))*x^5*ArcTanh[(-(Sqrt[c]*x) +
 Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(1920*a^(7/2)*d^6*x^5)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx$$

$$\Big\downarrow \ 1289$$

$$\int \left(\frac{e^6(a+bx+cx^2)^{3/2}}{d^6(d+ex)}-\frac{e^5(a+bx+cx^2)^{3/2}}{d^6x}+\frac{e^4(a+bx+cx^2)^{3/2}}{d^5x^2}-\frac{e^3(a+bx+cx^2)^{3/2}}{d^4x^3}+\frac{e^2(a+bx+cx^2)^{3/2}}{d^3x^4}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^5}{d^6}+\frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^5}{16c^{3/2}d^6}-\frac{(b^2+2cxb+8ac)\sqrt{cx^2+bx+a}e^5}{8cd^6}-\frac{(cx^2+bx+a)^{3/2}e^4}{d^5x}-\frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^4}{2d^5}+\frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^4}{8\sqrt{c}d^5}+\frac{3(3b+2cx)\sqrt{cx^2+bx+a}e^4}{4d^5}+\frac{(cx^2+bx+a)^{3/2}e^3}{2d^4x^2}+\frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^3}{8\sqrt{a}d^4}-\frac{3b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2d^4}+\frac{3(b-2cx)\sqrt{cx^2+bx+a}e^3}{4d^4x}+\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{cx^2+bx+a}e^3}{8cd^6}-\frac{(cx^2+bx+a)^{3/2}e^2}{3d^3x^3}+\frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e^2}{16a^{3/2}d^3}-\frac{(2cd-be)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}d^6}+\frac{c^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{d^3}+\frac{(cd^2-bed+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^2}{d^6}-\frac{(2ab+(b^2+8ac)\,x)\sqrt{cx^2+bx+a}e^2}{8ad^3x^2}+\frac{(2a+bx)\,(cx^2+bx+a)^{3/2}e}{8ad^2x^4}+\frac{3(b^2-4ac)^2\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)e}{128a^{5/2}d^2}-\frac{3(b^2-4ac)\,(2a+bx)\sqrt{cx^2+bx+a}e}{64a^2d^2x^2}-\frac{(cx^2+bx+a)^{5/2}}{5adx^5}+\frac{b(2a+bx)\,(cx^2+bx+a)^{3/2}}{16a^2dx^4}+\frac{3b(b^2-4ac)^2\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{256a^{7/2}d}-\frac{3b(b^2-4ac)\,(2a+bx)\sqrt{cx^2+bx+a}}{128a^3dx^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output
```
(-3*b*(b^2 - 4*a*c)*(2*a + b*x)*Sqrt[a + b*x + c*x^2])/(128*a^3*d*x^2) - (
3*(b^2 - 4*a*c)*e*(2*a + b*x)*Sqrt[a + b*x + c*x^2])/(64*a^2*d^2*x^2) + (3
*e^3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d^4*x) + (3*e^4*(3*b + 2*c*x)*S
qrt[a + b*x + c*x^2])/(4*d^5) - (e^5*(b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x
+ c*x^2])/(8*c*d^6) - (e^2*(2*a*b + (b^2 + 8*a*c)*x)*Sqrt[a + b*x + c*x^2]
)/(8*a*d^3*x^2) + (e^3*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*
e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^6) - (e^2*(a + b*x + c*x^
2)^(3/2))/(3*d^3*x^3) + (e^3*(a + b*x + c*x^2)^(3/2))/(2*d^4*x^2) - (e^4*(
a + b*x + c*x^2)^(3/2))/(d^5*x) + (b*(2*a + b*x)*(a + b*x + c*x^2)^(3/2))/
(16*a^2*d*x^4) + (e*(2*a + b*x)*(a + b*x + c*x^2)^(3/2))/(8*a*d^2*x^4) - (
a + b*x + c*x^2)^(5/2)/(5*a*d*x^5) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(7/2)*d) + (3*(b^2 - 4*a*c)
^2*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(128*a^(5/2)*
d^2) + (b*(b^2 - 12*a*c)*e^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x +
 c*x^2])])/(16*a^(3/2)*d^3) + (3*(b^2 + 4*a*c)*e^3*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*d^4) - (3*Sqrt[a]*b*e^4*ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d^5) + (a^(3/2)*e^5*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^6 + (c^(3/2)*e^2*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d^3 - (3*b*Sqrt[c]
*e^3*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*d^4) + ...
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(1920a^4e^4x^4-2400a^3bd\,e^3x^4+2560a^3c\,d^2e^2x^4+240a^2b^2d^2e^2x^4-600a^2bc\,d^3e\,x^4+384a^2c^2d^4x^4+90a\,b^3d^3e\,x^4-30}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^6/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x^2+b*x+a)^(1/2)*(1920*a^4*e^4*x^4-2400*a^3*b*d*e^3*x^4+2560*a^
3*c*d^2*e^2*x^4+240*a^2*b^2*d^2*e^2*x^4-600*a^2*b*c*d^3*e*x^4+384*a^2*c^2*
d^4*x^4+90*a*b^3*d^3*e*x^4-300*a*b^2*c*d^4*x^4+45*b^4*d^4*x^4-960*a^4*d*e^
3*x^3+1120*a^3*b*d^2*e^2*x^3-1200*a^3*c*d^3*e*x^3-60*a^2*b^2*d^3*e*x^3+168
*a^2*b*c*d^4*x^3-30*a*b^3*d^4*x^3+640*a^4*d^2*e^2*x^2-720*a^3*b*d^3*e*x^2+
768*a^3*c*d^4*x^2+24*a^2*b^2*d^4*x^2-480*a^4*d^3*e*x+528*a^3*b*d^4*x+384*a
^4*d^4)/a^3/d^5/x^5-1/256/a^3/d^5*(-(256*a^5*e^5-384*a^4*b*d*e^4+384*a^4*c
*d^2*e^3+96*a^3*b^2*d^2*e^3-192*a^3*b*c*d^3*e^2+96*a^3*c^2*d^4*e+16*a^2*b^
3*d^3*e^2-48*a^2*b^2*c*d^4*e+48*a^2*b*c^2*d^5+6*a*b^4*d^4*e-24*a*b^3*c*d^5
+3*b^5*d^5)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+256*a^
3*e*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/
e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 2575, normalized size of antiderivative = 4.46

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^6/(e*x+d),x, algorithm="fricas")
```

output
```
[1/7680*(3840*(a^4*c*d^2*e^2 - a^4*b*d*e^3 + a^5*e^4)*sqrt(c*d^2 - b*d*e +
 a*e^2)*x^5*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 -
8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*
x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2
- (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(384*a^4*b*d*e^4
 - 256*a^5*e^5 - 3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^5 - 6*(a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)*d^4*e - 16*(a^2*b^3 - 12*a^3*b*c)*d^3*e^2 - 96*(a^3*b
^2 + 4*a^4*c)*d^2*e^3)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d^5 -
(2400*a^4*b*d^2*e^3 - 1920*a^5*d*e^4 - 3*(15*a*b^4 - 100*a^2*b^2*c + 128*a
^3*c^2)*d^5 - 30*(3*a^2*b^3 - 20*a^3*b*c)*d^4*e - 80*(3*a^3*b^2 + 32*a^4*c
)*d^3*e^2)*x^4 + 2*(560*a^4*b*d^3*e^2 - 480*a^5*d^2*e^3 - 3*(5*a^2*b^3 - 2
8*a^3*b*c)*d^5 - 30*(a^3*b^2 + 20*a^4*c)*d^4*e)*x^3 - 8*(90*a^4*b*d^4*e -
80*a^5*d^3*e^2 - 3*(a^3*b^2 + 32*a^4*c)*d^5)*x^2 + 48*(11*a^4*b*d^5 - 10*a
^5*d^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^4*d^6*x^5), 1/7680*(7680*(a^4*c*d^2
*e^2 - a^4*b*d*e^3 + a^5*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)*x^5*arctan(-1/2
*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 15*(384*a^4*b*d*e^4 - 256*a^5*e^5 -
 3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^5 - 6*(a*b^4 - 8*a^2*b^2*c + 16*a...
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{x^6\,(d+ex)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**6/(e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(x**6*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(ex+d)x^6}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^6/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3046 vs. $2(539) = 1078$.

Time $= 0.56$ (sec) , antiderivative size $= 3046$, normalized size of antiderivative $= 5.28$

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^6/(e*x+d),x, algorithm="giac")
```

output
```
2*(c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4 + 2*a*c*d^2*e^4 - 2*a*b*d*e^5
 + a^2*e^6)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sq
rt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^6) - 1/128*(3*
b^5*d^5 - 24*a*b^3*c*d^5 + 48*a^2*b*c^2*d^5 + 6*a*b^4*d^4*e - 48*a^2*b^2*c
*d^4*e + 96*a^3*c^2*d^4*e + 16*a^2*b^3*d^3*e^2 - 192*a^3*b*c*d^3*e^2 + 96*
a^3*b^2*d^2*e^3 + 384*a^4*c*d^2*e^3 - 384*a^4*b*d*e^4 + 256*a^5*e^5)*arcta
n(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3*d^6) + 1/19
20*(45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d^4 - 360*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^9*a*b^3*c*d^4 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^2*b*c^2*d^4 + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*d^3*e
- 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*d^3*e - 2400*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^2*d^3*e + 240*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b^3*d^2*e^2 + 4800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*a^3*b*c*d^2*e^2 - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*d*e
^3 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*c*d*e^3 + 1920*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*e^4 + 3840*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*a^3*c^(5/2)*d^4 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
^3*b*c^(3/2)*d^3*e + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^2*sq
rt(c)*d^2*e^2 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(3/2)*d^2
*e^2 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*sqrt(c)*d*e^3 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^6\,(d+e\,x)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^6*(d + e*x)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 1597, normalized size of antiderivative = 2.77

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^6(d+ex)}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)/x^6/(e*x+d),x)
```

output
```
(3840*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*e**4*x**5 - 384
0*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*b*d*e**3*x**5 + 384
0*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*c*d**2*e**2*x**5 -
3840*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**5*e**4*x**5 + 3840*sqrt
(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**4*b*d*e**3*x**5 - 3840*sqrt(a*e*
*2 - b*d*e + c*d**2)*log(d + e*x)*a**4*c*d**2*e**2*x**5 - 768*sqrt(a + b*x
 + c*x**2)*a**5*d**5 + 960*sqrt(a + b*x + c*x**2)*a**5*d**4*e*x - 1280*sqr
t(a + b*x + c*x**2)*a**5*d**3*e**2*x**2 + 1920*sqrt(a + b*x + c*x**2)*a**5
*d**2*e**3*x**3 - 3840*sqrt(a + b*x + c*x**2)*a**5*d*e**4*x**4 - 1056*sqrt
(a + b*x + c*x**2)*a**4*b*d**5*x + 1440*sqrt(a + b*x + c*x**2)*a**4*b*d**4
*e*x**2 - 2240*sqrt(a + b*x + c*x**2)*a**4*b*d**3*e**2*x**3 + 4800*sqrt(a
+ b*x + c*x**2)*a**4*b*d**2*e**3*x**4 - 1536*sqrt(a + b*x + c*x**2)*a**4*c
*d**5*x**2 + 2400*sqrt(a + b*x + c*x**2)*a**4*c*d**4*e*x**3 - 5120*sqrt(a
+ b*x + c*x**2)*a**4*c*d**3*e**2*x**4 - 48*sqrt(a + b*x + c*x**2)*a**3*b**
2*d**5*x**2 + 120*sqrt(a + b*x + c*x**2)*a**3*b**2*d**4*e*x**3 - 480*sqrt(
a + b*x + c*x**2)*a**3*b**2*d**3*e**2*x**4 - 336*sqrt(a + b*x + c*x**2)*a*
*3*b*c*d**5*x**3 + 1200*sqrt(a + b*x + c*x**2)*a**3*b*c*d**4*e*x**4 - 7...
```

3.30 $\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\begin{aligned}\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{(16c^2d^2+15b^2e^2+2ce(9bd-8ae))\sqrt{a+bx+cx^2}}{24c^3e^3} \\ &- \frac{(14cd+5be)x\sqrt{a+bx+cx^2}}{12c^2e^2} + \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} \\ &- \frac{(16c^3d^3+5b^3e^3+6bce^2(bd-2ae)+8c^2de(bd-ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}e^4} \\ &+ \frac{d^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

output
```
1/24*(16*c^2*d^2+15*b^2*e^2+2*c*e*(-8*a*e+9*b*d))*(c*x^2+b*x+a)^(1/2)/c^3/
e^3-1/12*(5*b*e+14*c*d)*x*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/3*(e*x+d)^2*(c*x^2
+b*x+a)^(1/2)/c/e^3-1/16*(16*c^3*d^3+5*b^3*e^3+6*b*c*e^2*(-2*a*e+b*d)+8*c^
2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/
2)/e^4+d^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2
)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{2e\sqrt{a+x(b+cx)}\left(15b^2e^2-2ce(-9bd+8ae+5bex)+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)}{c^3}+\frac{96d^4\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{3(\cdots}{\cdots}}{48e^4}$$

input
```
Integrate[x^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(15*b^2*e^2 - 2*c*e*(-9*b*d + 8*a*e + 5*b*e*x)
 + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c^3 + (96*d^4*Sqrt[-(c*d^2) + b*d
*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*
d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (3*(16*c^3*d^3 + 5*b^
3*e^3 + 6*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(b*d - a*e))*Log[b + 2*c*x - 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(7/2))/(48*e^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1267, 27, 2184, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int -\frac{e^3(14cd+5be)x^3+e^2\left(10cd^2+e(11bd+4ae)\right)x^2+de\left(2cd^2+e(7bd+8ae)\right)x+d^2e(bd+4ae)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{3ce^4}+\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\int \frac{e^3(14cd+5be)x^3+e^2\left(10cd^2+e(11bd+4ae)\right)x^2+de\left(2cd^2+e(7bd+8ae)\right)x+d^2e(bd+4ae)}{(d+ex)\sqrt{cx^2+bx+a}}dx}{6ce^4}$$

$\Big\downarrow$ 2184

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\frac{\int -\frac{\left(44c^2d^2+15b^2e^2+4ce(7bd-4ae)\right)x^2e^5+d\left(5deb^2+10\left(cd^2+ae^2\right)b+12acde\right)e^4+2\left(10c^2d^3+ce(19bd-2ae)d+5be^2(2bd+ae)\right)xe^4}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce^3}+\frac{e(d+ex)\sqrt{a+bx+cx}}{2c}}{6ce^4}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c}-\frac{\int \frac{\left(44c^2d^2+15b^2e^2+4ce(7bd-4ae)\right)x^2e^5+d\left(5deb^2+10\left(cd^2+ae^2\right)b+12acde\right)e^4+2\left(10c^2d^3+ce(19bd-2ae)d+5be^2(2bd+\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{4ce^3}}{6ce^4}$$

$\Big\downarrow$ 2184

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c}-\frac{\frac{\int -\frac{3e^6\left(d\left(5e^2b^3+6cdeb^2+4c\left(2cd^2-3ae^2\right)b-8ac^2de\right)+\left(16c^3d^3+8c^2e(bd-ae)d+5b^3e^3+6bce^2(bd-2ae)\right)x\right)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{e^4\sqrt{a}}{}}{4ce^3}}{6ce^4}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c}-\frac{\frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c}-\frac{3e^4\int \frac{d\left(5e^2b^3+6cdeb^2+4c\left(2cd^2-3ae^2\right)b-8ac^2de\right)+\left(16c^3d^3+8c^2\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{2c}}{4ce^3}}{6ce^4}$$

$\Big\downarrow$ 1269

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-\frac{\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c}-\frac{\frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c}-\frac{3e^4\left(\frac{\left(8c^2de(bd-ae)+6bce^2(bd-2ae)+5b^3e^3+16c^3d^3\right)\int \frac{1}{\sqrt{cx^2+bx}}}{e}\right.}{2c}}{4ce^3}}{6ce^4}$$

$\Big\downarrow$ 1092

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-$$

$$\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c} - \frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c} - 3e^4\left(\frac{2\left(8c^2de(bd-ae)+6bce^2(bd-2ae)+5b^3e^3+16c^3d^3\right)\int \frac{1}{4c-\frac{(b+ \cdots}{cx^2 \cdots}}}{e} \cdots \right.$$
$$\cdots \quad 2c \quad \cdots \quad 4ce^3 \quad \cdots \quad 6ce^4$$

$\downarrow$ 219

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-$$

$$\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c} - \frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c} - 3e^4\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2de(bd-ae)+6bce^2(bd-2a \cdots\right.}{\sqrt{c}e} \cdots \right.$$
$$\cdots \quad 2c \quad \cdots \quad 4ce^3 \quad \cdots \quad 6ce^4$$

$\downarrow$ 1154

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-$$

$$\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c} - \frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c} - 3e^4\left(\frac{32c^3d^4\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd- \cdots}{\cdots}\right.}{e} \cdots \right.$$
$$\cdots \quad 4ce^3 \quad \cdots \quad 6ce^4$$

$\downarrow$ 219

$$\frac{(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}-$$

$$\frac{e(d+ex)\sqrt{a+bx+cx^2}(5be+14cd)}{2c} - \frac{e^4\sqrt{a+bx+cx^2}\left(4ce(7bd-4ae)+15b^2e^2+44c^2d^2\right)}{c} - 3e^4\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2de(bd-ae)+6bce^2(bd-2a \cdots\right.}{\sqrt{c}e} \cdots \right.$$
$$\cdots \quad 4ce^3 \quad \cdots \quad 6ce^4$$

input
```
Int[x^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - ((e*(14*c*d + 5*b*e)*(d +
e*x)*Sqrt[a + b*x + c*x^2])/(2*c) - ((e^4*(44*c^2*d^2 + 15*b^2*e^2 + 4*c*e
*(7*b*d - 4*a*e))*Sqrt[a + b*x + c*x^2])/c - (3*e^4*(((16*c^3*d^3 + 5*b^3*
e^3 + 6*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (16*c^3*d^4*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x +
 c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])))/(2*c))/(4*c*e^3))/(6*c*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{(-8c^2e^2x^2+10e^2xbc+12c^2dex+16ace^2-15b^2e^2-18bcde-24c^2d^2)\sqrt{cx^2+bx+a}}{24c^3e^3}+\frac{\frac{(12abce^3+8de^2ac^2-5b^3e^3-6de^2b^2c-8d^2ebc\cdots}{e\sqrt{c}}}{\cdots}$
default	$\frac{\frac{x^2\sqrt{cx^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{cx^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{6c}-\frac{2a\left(\frac{\sqrt{cx^2+bx+a}}{c}\cdots\right)}{\cdots}}{e}$

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*c^2*e^2*x^2+10*b*c*e^2*x+12*c^2*d*e*x+16*a*c*e^2-15*b^2*e^2-18*b
*c*d*e-24*c^2*d^2)/c^3*(c*x^2+b*x+a)^(1/2)/e^3+1/16/e^3/c^3*((12*a*b*c*e^3
+8*a*c^2*d*e^2-5*b^3*e^3-6*b^2*c*d*e^2-8*b*c^2*d^2*e-16*c^3*d^3)/e*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*d^4*c^3/e^2/((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^4}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^4}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 5414, normalized size of antiderivative = 18.41

$$\int \frac{x^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 48*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a
*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*
d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)
*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*s
qrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*
d*e - 8*c**2*d**2))*b*c**4*d**4*e + 96*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
+ c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 -
 b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((
2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d -
 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**5*d**5 - 96*sqrt(c)
*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2
- b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*
atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*s
qrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)
*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**4*d**4*e**2
 + 96*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)
*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e -
8*c**2*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqr
t(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 -...
```

3.31 $\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\frac{3(2cd+be)\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}\\ &\quad+\frac{(8c^2d^2+3b^2e^2+4ce(bd-ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^3}\\ &\quad-\frac{d^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

output
```
-3/4*(b*e+2*c*d)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*(e*x+d)*(c*x^2+b*x+a)^(1/
2)/c/e^2+1/8*(8*c^2*d^2+3*b^2*e^2+4*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3-d^3*arctanh(1/2*(b*d-2*a*e+(-b*e+
2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+
c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{2e(4cd+3be-2cex)\sqrt{a+x(b+cx)}}{c^2}+\frac{16d^3\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{(8c^2d^2+3b^2e^2+4ce(bd-ae))\log\left(b+2c\right.}{c^{5/2}}}{8e^3}$$

input
```
Integrate[x^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/8*((2*e*(4*c*d + 3*b*e - 2*c*e*x)*Sqrt[a + x*(b + c*x)])/c^2 + (16*d^3*
Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b
 + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + ((
8*c^2*d^2 + 3*b^2*e^2 + 4*c*e*(b*d - a*e))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)]])/c^(5/2))/e^3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1267, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1267$$

$$\frac{\int -\frac{3e^2(2cd+be)x^2+2e\left(cd^2+e(2bd+ae)\right)x+de(bd+2ae)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce^3}+\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\int \frac{3e^2(2cd+be)x^2+2e\left(cd^2+e(2bd+ae)\right)x+de(bd+2ae)}{(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^3}$$

$\Big\downarrow$ 2184

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{\int -\frac{e^3\left(d\left(3eb^2+4cdb-4ace\right)+\left(8c^2d^2+3b^2e^2+4ce(bd-ae)\right)x\right)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}}{4ce^3}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\int \frac{d\left(3eb^2+4cdb-4ace\right)+\left(8c^2d^2+3b^2e^2+4ce(bd-ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2c}}{4ce^3}$$

$\Big\downarrow$ 1269

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\left(\frac{\left(4ce(bd-ae)+3b^2e^2+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{8c^2d^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c}}{4ce^3}$$

$\Big\downarrow$ 1092

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\left(\frac{2\left(4ce(bd-ae)+3b^2e^2+8c^2d^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{8c^2d^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c}}{4ce^3}$$

$\Big\downarrow$ 219

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(bd-ae)+3b^2e^2+8c^2d^2\right)}{\sqrt{c}e}-\frac{8c^2d^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}\right)}{2c}}{4ce^3}$$

$\Big\downarrow$ 1154

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\left(\frac{16c^2d^3\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(bd-a\cdots\right.}{\sqrt{c}e}\cdots\right.}{2c}}{4ce^3}$$

$\Big\downarrow$ 219

$$\frac{(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}-\frac{\frac{3e\sqrt{a+bx+cx^2}(be+2cd)}{c}-\frac{e\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(4ce(bd-ae)+3b^2e^2+8c^2d^2)}{\sqrt{c}e}-\frac{8c^2d^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}\right)}{2c}}{4ce^3}$$

input
```
Int[x^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) - ((3*e*(2*c*d + b*e)*Sqrt[a +
 b*x + c*x^2])/c - (e*(((8*c^2*d^2 + 3*b^2*e^2 + 4*c*e*(b*d - a*e))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (8*c^2*d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])))/(2*c))/(4*c*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{(-2cex+3be+4cd)\sqrt{cx^2+bx+a}}{4c^2e^2}-\frac{\frac{(4ace^2-3b^2e^2-4bcde-8c^2d^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{e\sqrt{c}}-\frac{8c^2d^3\ln\left(\frac{\frac{2ae^2-2bde+2cd^2}{e^2}+\frac{(be-2cd}{}}{}\right.}{}}{8c^2e^2}$
default	$\frac{d^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{e^3\sqrt{c}}+\frac{\frac{x\sqrt{cx^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{e}-\frac{d\left(\right.}{}$

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*c*e*x+3*b*e+4*c*d)/c^2*(c*x^2+b*x+a)^(1/2)/e^2-1/8/c^2/e^2*((4*a*
c*e^2-3*b^2*e^2-4*b*c*d*e-8*c^2*d^2)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))/c^(1/2)-8*c^2*d^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^
2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(193) = 386.

Time = 161.78 (sec) , antiderivative size = 1685, normalized size of antiderivative = 7.69

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^3*log((8*a*b*d*e - 8*a^2*e^2 -
(b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sq
rt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*
e*x + d^2)) - (8*c^3*d^4 - 4*b*c^2*d^3*e - (b^2*c - 4*a*c^2)*d^2*e^2 - (3*
b^3 - 8*a*b*c)*d*e^3 + (3*a*b^2 - 4*a^2*c)*e^4)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(4
*c^3*d^3*e - b*c^2*d^2*e^2 + 3*a*b*c*e^4 - (3*b^2*c - 4*a*c^2)*d*e^3 - 2*(
c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c^4*d^2*
e^3 - b*c^3*d*e^4 + a*c^3*e^5), -1/16*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^3
*d^3*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d -
 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*
e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (8*c^3*d^4 - 4*b*c^
2*d^3*e - (b^2*c - 4*a*c^2)*d^2*e^2 - (3*b^3 - 8*a*b*c)*d*e^3 + (3*a*b^2 -
 4*a^2*c)*e^4)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(4*c^3*d^3*e - b*c^2*d^2*e^2 + 3*a*
b*c*e^4 - (3*b^2*c - 4*a*c^2)*d*e^3 - 2*(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2
*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c^4*d^2*e^3 - b*c^3*d*e^4 + a*c^3*e^5), 1
/8*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^3*log((8*a*b*d*e - 8*a^2*e^2 - (b^
2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sq...
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 4867, normalized size of antiderivative = 22.22

$$\int \frac{x^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2
)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
 b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2))*b*c**3*d**3*e - 16*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*
d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**
2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2
 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a
*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**4*d**4 + 16*sqrt(c)*sqr
t(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*
d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan
((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(
a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d
 - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**3*d**3*e**2 - 1
6*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c*
*2*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*
sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d...
```

3.32 $\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{a+bx+cx^2}}{ce} - \frac{(2cd+be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{d^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

output
```
(c*x^2+b*x+a)^(1/2)/c/e-1/2*(b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))/c^(3/2)/e^2+d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a
*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{-\frac{2e\sqrt{a+x(b+cx)}}{c} + \frac{4d^2\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\sqrt{-cd^2+e(bd-ae)}} + \frac{(2cd+be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}}{2e^2}$$

input
```
Integrate[x^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((-2*e*Sqrt[a + x*(b + c*x)])/c + (4*d^2*ArcTan[(Sqrt[c]*(d + e*x) -
e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/Sqrt[-(c*d^2) +
e*(b*d - a*e)] + ((2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*
(b + c*x)])])/c^(3/2))/e^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1267, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1267}$$

$$\frac{\int -\frac{e(bd+(2cd+be)x)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{\sqrt{a+bx+cx^2}}{ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\int \frac{bd+(2cd+be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{(be+2cd)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2cd^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{2(be+2cd)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2cd^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}}-\frac{2cd^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{4cd^2\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}}}{2ce}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}}-\frac{2cd^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}}{2ce}$$

input
```
Int[x^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Sqrt[a + b*x + c*x^2]/(c*e) - (((2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*c*d^2*ArcTanh[(b*d - 2*a*e +
(2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(
e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(2*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

method	result
risch	$\frac{\sqrt{c\,x^{2}+bx+a}}{ce}-\frac{\frac{(be+2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{e\sqrt{c}}+\frac{2c\,d^{2}\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be \cdots}{\cdots}}}{x+\frac{d}{e}}\cdots\right)}{e^{2}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}}{2ce}\cdots$
default	$\frac{\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}}{e}-\frac{d^{2}\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\cdots\right)}{e}\cdots}}{x+\frac{d}{e}}\cdots\right)}{e^{3}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}\cdots$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1/2)/c/e-1/2/c/e*((b*e+2*c*d)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))/c^(1/2)+2*c*d^2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(137) = 274$.

Time = 11.80 (sec) , antiderivative size = 1306, normalized size of antiderivative = 8.32

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log((8*a*b*d*e - 8*a^2*e^2 - (
b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqr
t(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e
*x + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(
c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^2 + b*x + a)
)/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(4*sqrt(-c*d^2 + b*d*e - a*
e^2)*c^2*d^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a
)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2
- b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c^2*d^3
- b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(c^2*
d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^2 + b*x + a))/(c^3*d^2*e^2 - b*c^2*d
*e^3 + a*c^2*e^4), 1/2*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log((8*a*b*d*e
 - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*
e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/
(e^2*x^2 + 2*d*e*x + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a
*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-...
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 4437, normalized size of antiderivative = 28.26

$$\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output

```
( - 2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*b*c**2*d**2*e + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e +
c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b
**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*
sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e*
*2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4
*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**3*d**3 - 4*sqrt(c)*sq
rt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b
*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*ata
n((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt
(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*
d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**2*d**2*e**2 +
4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c*
*2*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*
sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d...
```

3.33 $\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}} - \frac{d\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

output
```
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/e-d*arctanh(1/2
*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{2d\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{\log\left(e\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}}{e}$$

input
```
Integrate[x/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((2*d*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[
a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*
e)) + Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/Sqrt[c])/e)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{d\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2d\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{d\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[x/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*e) - (d*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[
a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

method	result
default	$\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c}{e^2}}}{x+\frac{d}{e}}\right.}{e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(108) = 216.

Time = 0.75 (sec) , antiderivative size = 1034, normalized size of antiderivative = 8.34

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 +
4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^
2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) -
 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d
^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2
 + a*c*e^3), -1/2*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(-c*
d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)
/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d
^2 - b^2*d*e + a*b*e^2)*x)) - (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x
^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))
/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/2*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*l
og((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a
)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*
a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-
c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
+ a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -(sqrt(-c*d^2 + b*d*e - a*e^2)
*c*d*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d -
 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c...
```

Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 4099, normalized size of antiderivative = 33.06

$$\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2
)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
 b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2))*b*c*d*e - 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b
*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2
+ 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*s
qrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2
 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**2*d**2 + 4*sqrt(c)*sqrt(4*sqrt
(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*
d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sqrt
(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 -
 b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c
*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c*d*e**2 - 4*sqrt(c)*sqrt(
4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((
2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*...
```

3.34 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}$$

output
```
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\frac{2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x
*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.

Time = 1.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$	157

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/
e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.34

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-(b^2+4\,ac)d^2-(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^2-4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+a}(bd-2\,ae+(2\,cd-be)x)-2\,(4\,bcd^2+4\,ab}{e^2x^2+2\,dex+d^2}\right.}{2\,\sqrt{cd^2-bde+ae^2}}\right.$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d
*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b
*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^
2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2),
 sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqr
t(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^
2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*
x))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2
+ b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 3755, normalized size of antiderivative = 47.53

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output

```
( - 2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*b*e + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*
e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 +
 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
 + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2
- b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c*d - 4*sqrt(c)*sqrt(4*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c
*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
+ c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 -
 b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*e**2 + 4*sqrt(c)*sqrt(4*sqrt(c)*s
qrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)
*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sqrt(c)*s
qrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e...
```

3.35 $\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{e\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d\sqrt{cd^2-bde+ae^2}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)/d-e*arctanh(1/
2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(-\frac{e\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

input
```
Integrate[1/(x*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(-((e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt
[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a
*e))) + ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]/Sqrt[a]))/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}}-\frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d\sqrt{ae^2-bde+cd^2}}-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(x*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (e
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x + c*x^2])])/(d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/d/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(110) = 220.

Time = 0.23 (sec) , antiderivative size = 1047, normalized size of antiderivative = 8.31

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 +
4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^
2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) -
 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d
^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 -
 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*c*d^3 - a*b
*d^2*e + a^2*d*e^2), -1/2*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
 b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
+ (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(
-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a
) + 8*a^2)/x^2))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/2*(sqrt(c*d^2 - b*d*
e + a*e^2)*a*e*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2
 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt
(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e
^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 - b*d*e
 + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(
a*c*x^2 + a*b*x + a^2)))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), -(sqrt(-c*d^2
+ b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 ...
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.93

$$\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{a\,e^2-bde+c\,d^2}\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)ae-\sqrt{a\,e^2-bd}}{}$$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*e - sqrt(a*e**2 - b*
d*e + c*d**2)*log(d + e*x)*a*e + sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*a*e**2 - sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2
*a - b*x)*b*d*e + sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x
)*c*d**2 - sqrt(a)*log(x)*a*e**2 + sqrt(a)*log(x)*b*d*e - sqrt(a)*log(x)*c
*d**2)/(a*d*(a*e**2 - b*d*e + c*d**2))
```

3.36 $\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx=-\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{(bd+2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2\sqrt{cd^2-bde+ae^2}}$$

output

```
-(c*x^2+b*x+a)^(1/2)/a/d/x+1/2*(2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(
c*x^2+b*x+a)^(1/2))/a^(3/2)/d^2+e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$=\frac{-\frac{2d\sqrt{a+x(b+cx)}}{ax}+\frac{4e^2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x}{\sqrt{a}(d+ex)-d\sqrt{a+x(b+cx)}}\right)}{cd^2+e(-bd+ae)}+\frac{(bd+2ae)\log(x)}{a^{3/2}}-\frac{(bd+2ae)\log\left(ad^2\left(2a+bx-2\sqrt{a}\sqrt{a+
}{a^{3/2}}}{2d^2}$$

input
```
Integrate[1/(x^2*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-2*d*Sqrt[a + x*(b + c*x)])/(a*x) + (4*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2
]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt[a
+ x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e)) + ((b*d + 2*a*e)*Log[x])/a^(3
/2) - ((b*d + 2*a*e)*Log[a*d^2*(2*a + b*x - 2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])])/a^(3/2))/(2*d^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1289$$

$$\int \left(\frac{e^2}{d^2(d+ex)\sqrt{a+bx+cx^2}}-\frac{e}{d^2x\sqrt{a+bx+cx^2}}+\frac{1}{dx^2\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{e^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

input
```
Int[1/(x^2*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d^2*Sqrt[
c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{adx}-\frac{-\frac{(2ae+bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{2ea\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right.}}{x+\frac{d}{e}}\right.}{d\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}}{2ad}$
default	$\frac{-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{e\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd}{e}}}{x+\frac{d}{e}}\right.}{d^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/a/d*(-(2*a*e+b*d)/d/a^(1/2)*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+2*e/d*a/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(142) = 284$.

Time = 0.50 (sec) , antiderivative size = 1340, normalized size of antiderivative = 8.27

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x*log((8*a*b*d*e - 8*a^2*e^2 -
 (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*s
qrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b
*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d
*e*x + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqr
t(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^
2 + b*x + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x), 1/4*(4*sqrt(-c*
d^2 + b*d*e - a*e^2)*a^2*e^2*x*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a
^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)
*x)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt(a)*x*l
og(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqr
t(a) + 8*a^2)/x^2) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^2 + b*x
+ a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x), 1/2*(sqrt(c*d^2 - b*d*e
 + a*e^2)*a^2*e^2*x*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^
2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)
*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*
a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - (b*c*d^3 -
a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt(-a)*x*arctan(1/2*sqrt...
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^2\left(d+ex\right)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x**2*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\,e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}d^2} - \frac{(bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ad^2} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b+2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)ad}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - (b*d + 2*a*e)*a
rctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*d^2) + ((
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2 - a)*a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{a\,e^2-bde+c\,d^2}\,\log\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)a^2e^2x-2\sqrt{a\,e^2-}$$

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*e**2*x - 2*sqrt(a*
e**2 - b*d*e + c*d**2)*log(d + e*x)*a**2*e**2*x - 2*sqrt(a + b*x + c*x**2)
*a**2*d*e**2 + 2*sqrt(a + b*x + c*x**2)*a*b*d**2*e - 2*sqrt(a + b*x + c*x*
*2)*a*c*d**3 + 2*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*a**2*e**3*x - sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
b*x)*a*b*d*e**2*x + 2*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*
a - b*x)*a*c*d**2*e*x - sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) -
2*a - b*x)*b**2*d**2*e*x + sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*b*c*d**3*x - 2*sqrt(a)*log(x)*a**2*e**3*x + sqrt(a)*log(x)*a
*b*d*e**2*x - 2*sqrt(a)*log(x)*a*c*d**2*e*x + sqrt(a)*log(x)*b**2*d**2*e*x
 - sqrt(a)*log(x)*b*c*d**3*x)/(2*a**2*d**2*x*(a*e**2 - b*d*e + c*d**2))
```

3.37 $\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\begin{aligned}\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{(3bd+4ae)\sqrt{a+bx+cx^2}}{4a^2d^2x}\\ &\quad-\frac{(3b^2d^2-4acd^2+4abde+8a^2e^2)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d^3}\\ &\quad-\frac{e^3\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^3\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

output
```
-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+1/4*(4*a*e+3*b*d)*(c*x^2+b*x+a)^(1/2)/a^2
/d^2/x-1/8*(8*a^2*e^2+4*a*b*d*e-4*a*c*d^2+3*b^2*d^2)*arctanh(1/2*(b*x+2*a)
/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d^3-e^3*arctanh(1/2*(b*d-2*a*e+(-b*e
+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/(a*e^2-b*d*e
+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{d(-2ad+3bdx+4aex)\sqrt{a+x(b+cx)}}{a^2x^2} - \frac{8e^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{8e^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{d}{\cdots}}{4d^3}$$

input
```
Integrate[1/(x^3*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d*(-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) - (8*e^3
*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(
b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (
8*e^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + (d*(
-3*b^2*d + 4*a*c*d - 4*a*b*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)
])/Sqrt[a]])/a^(5/2))/(4*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{ 1289}$$
$$\int \left(-\frac{e^3}{d^3(d+ex)\sqrt{a+bx+cx^2}} + \frac{e^2}{d^3x\sqrt{a+bx+cx^2}} - \frac{e}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{1}{dx^3\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{e^2\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}-\frac{e^3\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^3\sqrt{ae^2-bde+cd^2}}+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

input
```
Int[1/(x^3*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*Sqrt[a + b*x + c*x^2]/(a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*
d*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - (e^2*ArcT
anh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) - (e^3*A
rcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])])/(d^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(-4 a e x-3 b d x+2 a d\right)}{4 a^{2} d^{2} x^{2}}+\frac{-\frac{\left(8 e^{2} a^{2}+4 a b d e-4 a\,d^{2} c+3 b^{2} d^{2}\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{d \sqrt{a}}+8 e^{2} a^{2} \ln\left(\frac{\frac{2 a\,e^{2}-2 b d e+2 c\,d^{2}}{e^{2}}}{}\right.}{8 a^{2} d^{2}}$
default	$\frac{-\frac{\sqrt{c\,x^{2}+b x+a}}{2 a\,x^{2}}-\frac{3 b\left(-\frac{\sqrt{c\,x^{2}+b x+a}}{a x}+\frac{b \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}+\frac{c \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{2 a^{\frac{3}{2}}}}{d}-\frac{e^{2} \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+}}{x}\right.}{d^{3} \sqrt{a}}$

input
```
int(1/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-4*a*e*x-3*b*d*x+2*a*d)/a^2/d^2/x^2+1/8/a^2/d^2*
(-(8*a^2*e^2+4*a*b*d*e-4*a*c*d^2+3*b^2*d^2)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)+8*e^2*a^2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(197) = 394.

Time = 1.42 (sec) , antiderivative size = 1737, normalized size of antiderivative = 7.79

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(c*d^2 - b*d*e + a*e^2)*a^3*e^3*x^2*log((8*a*b*d*e - 8*a^2*e^
2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 +
4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 +
2*d*e*x + d^2)) - (4*a^2*b*d*e^3 - 8*a^3*e^4 - (3*b^2*c - 4*a*c^2)*d^4 + (
3*b^3 - 8*a*b*c)*d^3*e + (a*b^2 - 4*a^2*c)*d^2*e^2)*sqrt(a)*x^2*log(-(8*a*
b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*
a^2)/x^2) - 4*(2*a^2*c*d^4 - 2*a^2*b*d^3*e + 2*a^3*d^2*e^2 - (3*a*b*c*d^4
- a^2*b*d^2*e^2 + 4*a^3*d*e^3 - (3*a*b^2 - 4*a^2*c)*d^3*e)*x)*sqrt(c*x^2 +
 b*x + a))/((a^3*c*d^5 - a^3*b*d^4*e + a^4*d^3*e^2)*x^2), -1/16*(16*sqrt(-
c*d^2 + b*d*e - a*e^2)*a^3*e^3*x^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2
)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e
 + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*
e^2)*x)) + (4*a^2*b*d*e^3 - 8*a^3*e^4 - (3*b^2*c - 4*a*c^2)*d^4 + (3*b^3 -
 8*a*b*c)*d^3*e + (a*b^2 - 4*a^2*c)*d^2*e^2)*sqrt(a)*x^2*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^
2) + 4*(2*a^2*c*d^4 - 2*a^2*b*d^3*e + 2*a^3*d^2*e^2 - (3*a*b*c*d^4 - a^2*b
*d^2*e^2 + 4*a^3*d*e^3 - (3*a*b^2 - 4*a^2*c)*d^3*e)*x)*sqrt(c*x^2 + b*x +
a))/((a^3*c*d^5 - a^3*b*d^4*e + a^4*d^3*e^2)*x^2), 1/8*(4*sqrt(c*d^2 - b*d
*e + a*e^2)*a^3*e^3*x^2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 ...
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^3\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x**3*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(197) = 394$.

Time = 0.26 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{2\,e^3\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}d^3} + \frac{\left(3\,b^2d^2-4\,acd^2+4\,abde+8\,a^2e^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2d^3} - \frac{3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe+8\,(}{}$$

input
```
integrate(1/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*e^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^3) + 1/4*(3*b^2*d^2
 - 4*a*c*d^2 + 4*a*b*d*e + 8*a^2*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))/sqrt(-a))/(sqrt(-a)*a^2*d^3) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^2*b*e - 8*a^2*b*sqrt(c)*d - 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2 - a)^2*a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^3\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.39

$$\int \frac{1}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*e**3*x**2 - 8*s
qrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**3*e**3*x**2 - 4*sqrt(a + b*x
 + c*x**2)*a**3*d**2*e**2 + 8*sqrt(a + b*x + c*x**2)*a**3*d*e**3*x + 4*sqrt
(a + b*x + c*x**2)*a**2*b*d**3*e - 2*sqrt(a + b*x + c*x**2)*a**2*b*d**2*e*
*2*x - 4*sqrt(a + b*x + c*x**2)*a**2*c*d**4 + 8*sqrt(a + b*x + c*x**2)*a**
2*c*d**3*e*x - 6*sqrt(a + b*x + c*x**2)*a*b**2*d**3*e*x + 6*sqrt(a + b*x +
 c*x**2)*a*b*c*d**4*x + 8*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2
*a - b*x)*a**3*e**4*x**2 - 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*a**2*b*d*e**3*x**2 + 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c
*x**2) - 2*a - b*x)*a**2*c*d**2*e**2*x**2 - sqrt(a)*log(2*sqrt(a)*sqrt(a +
 b*x + c*x**2) - 2*a - b*x)*a*b**2*d**2*e**2*x**2 + 8*sqrt(a)*log(2*sqrt(a
)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*d**3*e*x**2 - 4*sqrt(a)*log(2*
sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c**2*d**4*x**2 - 3*sqrt(a)*l
og(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*d**3*e*x**2 + 3*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*c*d**4*x**2 - 8
*sqrt(a)*log(x)*a**3*e**4*x**2 + 4*sqrt(a)*log(x)*a**2*b*d*e**3*x**2 - 4*s
qrt(a)*log(x)*a**2*c*d**2*e**2*x**2 + sqrt(a)*log(x)*a*b**2*d**2*e**2*x**2
 - 8*sqrt(a)*log(x)*a*b*c*d**3*e*x**2 + 4*sqrt(a)*log(x)*a*c**2*d**4*x**2
+ 3*sqrt(a)*log(x)*b**3*d**3*e*x**2 - 3*sqrt(a)*log(x)*b**2*c*d**4*x**2...
```

3.38 $\int \frac{x^4}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{x^4}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = -\frac{2(a(b^3d-3abcd-ab^2e+2a^2ce)+(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,x)}{c^2(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} + \frac{\sqrt{a+bx+cx^2}}{c^2e} - \frac{(2cd+3be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}e^2} + \frac{d^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2(cd^2-bde+ae^2)^{3/2}}$$

output
```
(-2*a*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)-2*(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3
*e-4*a*b^2*c*d+b^4*d)*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a
)^(1/2)+(c*x^2+b*x+a)^(1/2)/c^2/e-1/2*(3*b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^2+d^4*arctanh(1/2*(b*d-2*a*e+(-b*e+
2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+
c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 11.74 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\sqrt{a+x(b+cx)}\left(\frac{1}{e}+\frac{2(2a^3ce+b^4dx+ab^2(-4cdx+b(d-ex))+a^2(-b^2e+2c^2dx-3bc(d-ex}{(b^2-4ac)(-cd^2+e(bd-ae))(a+x(b+cx))}\right.}{c^2}$$
$$+\frac{d^4\log(d+ex)}{e^2\left(cd^2+e(-bd+ae)\right)^{3/2}}-\frac{(2cd+3be)\log\left(b+2cx+2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{2c^{5/2}e^2}$$
$$-\frac{d^4\log\left(-bd+2ae-2cdx+bex+2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}\right)}{e^2\left(cd^2+e(-bd+ae)\right)^{3/2}}$$

input
```
Integrate[x^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(e^(-1) + (2*(2*a^3*c*e + b^4*d*x + a*b^2*(-4*c*d*x
 + b*(d - e*x)) + a^2*(-(b^2*e) + 2*c^2*d*x - 3*b*c*(d - e*x))))/((b^2 - 4
*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x)))))/c^2 + (d^4*Log[d + e
*x])/(e^2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) - ((2*c*d + 3*b*e)*Log[b + 2*c
*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(2*c^(5/2)*e^2) - (d^4*Log[-(b*d) +
 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b
+ c*x)]])/(e^2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1264, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \; 1264$$

$$-\frac{2\int -\frac{\frac{\left(b^2-4ac\right)x^2}{c}-\frac{b\left(b^2-4ac\right)x}{c^2}+\frac{\left(b^2-4ac\right)d\left(db^2-aeb-acd\right)}{c^2\left(cd^2-bed+ae^2\right)}}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\frac{\left(b^2-4ac\right)x^2}{c}-\frac{b\left(b^2-4ac\right)x}{c^2}+\frac{\left(b^2-4ac\right)d\left(db^2-aeb-acd\right)}{c^2\left(cd^2-bed+ae^2\right)}}{(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 2184$$

$$\frac{\frac{\int -\frac{\left(b^2-4ac\right)e\left(\frac{d\left(-3deb^2+cd^2b+3ae^2b+2acde\right)}{cd^2-bed+ae^2}+(2cd+3be)x\right)}{2c(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{c^2e}-\frac{\left(b^2-4ac\right)\int \frac{\frac{d\left(-3deb^2+cd^2b+3ae^2b+2acde\right)}{cd^2-bed+ae^2}+(2cd+3be)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{c^2e}-\frac{\left(b^2-4ac\right)\left(\frac{(3be+2cd)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2c^2d^4\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}\right)}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}-\frac{(b^2-4ac)\left(\frac{2(3be+2cd)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2c^2d^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}\right)}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}-\frac{(b^2-4ac)\left(\frac{(3be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2c^2d^4\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}\right)}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}-\frac{(b^2-4ac)\left(\frac{4c^2d^4\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e\left(ae^2-bde+cd^2\right)}+\frac{(3be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right)}{\sqrt{c}e}\right)}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(b^2-4ac)\sqrt{a+bx+cx^2}}{c^2e}-\frac{(b^2-4ac)\left(\frac{(3be+2cd)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{2c^2d^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)^{3/2}}\right)}{2c^2e}}{b^2-4ac}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

input

```
Int[x^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(a*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e) + (b^4*d - 4*a*b^2*c*d +
2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*x))/(c^2*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])/
(c^2*e) - ((b^2 - 4*a*c)*(((2*c*d + 3*b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*c^2*d^4*ArcTanh[(b*d - 2*a*e + (
2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e
*(c*d^2 - b*d*e + a*e^2)^(3/2))))/(2*c^2*e))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(253) = 506$.

Time = 1.54 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.97

method	result
default	$\frac{\frac{x^{2}}{c\sqrt{c\,x^{2}+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{c^{\frac{3}{2}}}\right)}{2c}-\frac{2a\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\right.}{}}{e}$
risch	Expression too large to display

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(
-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/
c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*
x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+d^2/e^3*(-1/c/(
c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+d^4/e^5*
(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c
*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-2*d^3/e^4*(2*c*x+b)/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)-d/e^2*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^4}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^4}{(ex+d)\,(c\,x^2+bx+a)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.39 $\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2(a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x)}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}+\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}-\frac{d^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
2*(a*(-a*b*e-2*a*c*d+b^2*d)+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x)/c/(-4*a
*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)+arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e-d^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)
*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(
3/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \frac{2(-b^3dx+ab(-bd+3cdx+bex)+a^2(be+2c(d-ex)))}{c\,(-b^2+4ac)\,(cd^2+e(-bd+ae))\,\sqrt{a+x(b+cx)}} - \frac{2d^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\,(cd^2+e(-bd+ae))^2} - \frac{\log\left(ce\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}e}$$

input
```
Integrate[x^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(-(b^3*d*x) + a*b*(-(b*d) + 3*c*d*x + b*e*x) + a^2*(b*e + 2*c*(d - e*x)
)))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]) -
(2*d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
 + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*(c*d^2 + e*(-(b*d) +
a*e))^2) - Log[c*e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/(c^(3/2)
*e)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1264, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1264$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)} - \frac{2\int \frac{\frac{(b^2-4ac)d(bd-ae)}{cd^2-bed+ae^2}-(b^2-4ac)x}{2c(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{\frac{\left(b^2-4ac\right)d(bd-ae)}{cd^2-bed+ae^2}-\left(b^2-4ac\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{2\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e(ae^2-bde+cd^2)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{-\frac{2cd^3\left(b^2-4ac\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e(ae^2-bde+cd^2)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2-bde+cd^2)^{3/2}}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}}{c\left(b^2-4ac\right)}$$

input
```
Int[x^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e
)*x))/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (-
(((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(S
qrt[c]*e)) + (c*(b^2 - 4*a*c)*d^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/
(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(c*d^2 - b*d*e
+ a*e^2)^(3/2)))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(202) = 404$.

Time = 1.49 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.80

method	result
default	$\frac{2d^2(2cx+b)}{e^3(4ac-b^2)\sqrt{c x^2+bx+a}}+\frac{-\frac{x}{c\sqrt{c x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{c^{\frac{3}{2}}}}{e}-\frac{d\left(-\frac{}{c\sqrt{c}}\right.}{}$

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d^2/e^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/e*(-x/c/(c*x^2+b*x+a
)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-d/e^
2*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
-d^3/e^4*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)
+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)
^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d
^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(202) = 404.

Time = 35.40 (sec) , antiderivative size = 4811, normalized size of antiderivative = 21.87

$$\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^3}{(d+ex)(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{x^3}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.40 $\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(a(bd-2ae)+(b^2d-2acd-abe)\,x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} + \frac{d^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}}$$

output

```
(-2*a*(-2*a*e+b*d)-2*(-a*b*e-2*a*c*d+b^2*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c
*d^2)/(c*x^2+b*x+a)^(1/2)+d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \frac{2(cd^2+e(-bd+ae))\,(2a^2e-b^2dx+a(-bd+2cdx+bex))-2(-b^2+\cdots}{(b^2-4ac)\,(cd^2+e(-b\cdots}$$

input

```
Integrate[x^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(c*d^2 + e*(-(b*d) + a*e))*(2*a^2*e - b^2*d*x + a*(-(b*d) + 2*c*d*x + b
*e*x)) - 2*(-b^2 + 4*a*c)*d^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b
 + c*x)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2
) + e*(b*d - a*e)]])/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a +
x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1264, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1264}$$

$$-\frac{2\int -\frac{(b^2-4ac)d^2}{2(cd^2-bed+ae^2)(d+ex)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2d^2\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{d^2\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[x^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x))/((b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d
^2 - b*d*e + a*e^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(147) = 294$.

Time = 1.41 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.19

method	result
default	$\frac{-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}}{e}+\frac{d^2\left(\frac{e^2}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}-\frac{\quad}{\left(a\,e^2-bde+c\,d^2\right)\left(\frac{4c}{\quad}\right.}\right.}{\quad}$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
))+d^2/e^3*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/
e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c
*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-2*d/e^2*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(146) = 292$.

Time = 0.34 (sec) , antiderivative size = 1351, normalized size of antiderivative = 8.66

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*
c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*
a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2
- b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2
*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2
)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e -
(a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*
c)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^
3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(
a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4
)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*
d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x
^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((b^2*c - 4*a*c^2)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x
 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c
*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x
)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*
d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 -
(a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a...
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(146) = 292$.

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.96

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}} - \frac{2\left(\frac{\left(b^2cd^3-2\,ac^2d^3-b^3d^2e+abcd^2e+2\,ab^2de^2-2\,a^2cde^2-a^2be^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd}\right.}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*d^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - 2*((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2
 - 2*a^2*c*d*e^2 - a^2*b*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e
 + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 -
 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3
 - a*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 -
 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d
^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4
 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx=\int \frac{x^2}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx=\int \frac{x^2}{(ex+d)\,(c\,x^2+bx+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.41 $\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(a(2cd-be)+c(bd-2ae)x)}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} - \frac{de\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}}$$

output
```
2*(a*(-b*e+2*c*d)+c*(-2*a*e+b*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^
2+b*x+a)^(1/2)-d*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.20

$$\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left((cd^2+e(-bd+ae))(-abe+bcdx+2ac(d-ex))+(-b^2+4ac)\,de\cdots\right.}{(b^2-4ac)(cd^2+e(-bd+\cdots}$$

input
```
Integrate[x/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*((c*d^2 + e*(-(b*d) + a*e))*(-(a*b*e) + b*c*d*x + 2*a*c*(d - e*x)) + (-
b^2 + 4*a*c)*d*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b + c*x)]*ArcT
an[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d -
a*e)]]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x)]
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{1235}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{2\int \frac{\left(b^2-4ac\right)de}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$
$$\Big\downarrow \text{27}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{de\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}$$
$$\Big\downarrow \text{1154}$$
$$\frac{2de\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}+\frac{2(cx(bd-2ae)+a(2cd-be))}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$
$$\Big\downarrow \text{219}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}-\frac{de\,\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}$$

input
```
Int[x/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e +
a*e^2)*Sqrt[a + b*x + c*x^2]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*
x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e
+ a*e^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(140) = 280$.

Time = 1.36 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.92

method	result
default	$\frac{4cx+2b}{e(4ac-b^2)\sqrt{c\,x^2+bx+a}} - \frac{d\left(\frac{e^2}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}} - \frac{(be-2}{\left(a\,e^2-bde+c\,d^2\right)\left(\frac{4c\left(a\,e^2-bde+c\,d^2\right)}{e^2}-\right.}\right.}{}$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-d/e^2*(1/(a*e^2-b*d*e+c*d^2)
*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*
e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d
*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(140) = 280$.

Time = 0.36 (sec) , antiderivative size = 1320, normalized size of antiderivative = 8.80

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*d*e*x^2 + (b^3 - 4*a*b*c)*d*e*x + (a*b^2 - 4*a^2*
c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*
a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2
- b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2
*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2
)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2
+ (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*x)*s
qrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*
c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^
3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3
*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a
*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2
 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a
^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), -(((b^2*c - 4*a*c^2)*d*e*x^2 + (b^3 - 4*a*b*c)*d*e*x + (a*b^2 - 4
*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e
- a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 -
a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*
a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*...
```

Sympy [F]

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(140) = 280$.

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.96

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\,de\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}} + \frac{2\left(\frac{\left(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3}\right.}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*d*e*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + 2*((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*
c*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^
4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*
b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*
b^2*d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*
c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^
2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt
(c*x^2 + b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x}{(ex+d)\,(c\,x^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.42 $\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
(-2*b*c*d+2*b^2*e-4*a*c*e-2*c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
d^2)/(c*x^2+b*x+a)^(1/2)+e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\Big((cd^2+e(-bd+ae))\left(-b^2e+2c(ae+cdx)+bc(d-ex)\right)+(-b^2+4ac)\,e^2\sqrt{-cd^2+bde-ae^2}\sqrt{a+x}}{(b^2-4ac)\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*((c*d^2 + e*(-(b*d) + a*e))*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e
*x)) + (-b^2 + 4*a*c)*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b + c
*x)]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x)/(Sqrt[a]*(d + e*x) - d*Sqrt
[a + x*(b + c*x)])]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a +
 x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2e^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{e^2\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2
 - b*d*e + a*e^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(146) = 292.

Time = 1.35 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.58

method	result
default	$\frac{\frac{e^{2}}{\left(a\,e^{2}-bde+c\,d^{2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}-\frac{(be-2cd)e\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)}{\left(a\,e^{2}-bde+c\,d^{2}\right)\left(\frac{4c\left(a\,e^{2}-bde+c\,d^{2}\right)}{e^{2}}-\frac{(be-2cd)^{2}}{e^{2}}\right)\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd}{}}}}{}$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e
-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e
^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(145) = 290.

Time = 0.36 (sec) , antiderivative size = 1349, normalized size of antiderivative = 8.70

$$\int \frac{1}{(d+ex)\left(a+bx+cx^{2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*
c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*
a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2
- b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2
*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2
)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2
 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^
2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^
3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(
a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4
)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*
d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x
^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x
 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c
*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x
)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*
d^2 - b^2*d*e + a*b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b
^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e ...
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.97

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\,e^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}} - \frac{2\left(\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd\cdots}\right.}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - 2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e
^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d
^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2
*d^2*e + b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 -
 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d
^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4
 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,dx$$

input
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(ex+d\right)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.43 $\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2\left(c(b^2-2ac)\,d-b^3e+3abce+c(bcd-b^2e+2ace)\,x\right)}{a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{e^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
2*(c*(-2*a*c+b^2)*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x)/a/(-4*a*c+b
^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)-arctanh(1/2*(b*x+2*a)/a^(1/2)/
(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2b^3e-2bc(3ae+cdx)-2b^2c(d-ex)+4ac^2(d-ex)}{a\left(-b^2+4ac\right)\left(cd^2+e(-bd+ae)\right)\sqrt{a+x(b+cx)}}-\frac{2e^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{d\left(cd^2+e(-bd+ae)\right)^2}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

input
```
Integrate[1/(x*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*b^3*e - 2*b*c*(3*a*e + c*d*x) - 2*b^2*c*(d - e*x) + 4*a*c^2*(d - e*x))/
(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]) - (2*e
^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x
*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*(c*d^2 + e*(-(b*d) + a*e)
)^2) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(3/2)*d
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)(a+bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} - \frac{e}{d(d+ex)(a+bx+cx^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{e^3\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d\left(ae^2-bde+cd^2\right)^{3/2}} + \frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)} + \frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(x*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*e
*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*d*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqr
t[a + b*x + c*x^2])]/(a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2]])/(d*(c*d^2 -
b*d*e + a*e^2)^(3/2))
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(200) = 400$.

Time = 1.41 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.26

method	result
default	$\frac{\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}}{d}-\frac{\frac{e^2}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c}{e^2}}}}{}$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/d*(1/(a*e^2-b*
d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*
(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(201) = 402$.

Time = 2.01 (sec) , antiderivative size = 4819, normalized size of antiderivative = 22.11

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/(x*(d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 3348, normalized size of antiderivative = 15.36

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(4*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*c*e**3 - sqrt(a
*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*
e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*b**2*e**3 + 4*sqrt(a*e**
2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e +
c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*b*c*e**3*x + 4*sqrt(a*e**2 -
 b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d
**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**2*e**3*x**2 - sqrt(a*e**2 -
b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d*
*2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b**3*e**3*x - sqrt(a*e**2 - b*d*
e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2)
- 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b**2*c*e**3*x**2 - 4*sqrt(a*e**2 - b
*d*e + c*d**2)*log(d + e*x)*a**4*c*e**3 + sqrt(a*e**2 - b*d*e + c*d**2)*lo
g(d + e*x)*a**3*b**2*e**3 - 4*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a
**3*b*c*e**3*x - 4*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**3*c**2*e*
*3*x**2 + sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**2*b**3*e**3*x + sq
rt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**2*b**2*c*e**3*x**2 - 6*sqrt(a
+ b*x + c*x**2)*a**3*b*c*d*e**3 + 4*sqrt(a + b*x + c*x**2)*a**3*c**2*d**2*
e**2 - 4*sqrt(a + b*x + c*x**2)*a**3*c**2*d*e**3*x + 2*sqrt(a + b*x + c*x*
*2)*a**2*b**3*d*e**3 + 4*sqrt(a + b*x + c*x**2)*a**2*b**2*c*d**2*e**2 +...
```

3.44 $\int \frac{1}{x^2(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{1}{x^2(d+ex)(a+bx+cx^2)^{3/2}}\,dx = -\frac{2(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e+c(b^2cd-2ac^2d-b^3e+3abce)x)}{a^2(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}} - \frac{\sqrt{a+bx+cx^2}}{a^2dx} + \frac{(3bd+2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d^2} + \frac{e^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2(cd^2-bde+ae^2)^{3/2}}$$

output
```
(-2*b^3*c*d+6*a*b*c^2*d+2*b^4*e-8*a*b^2*c*e+4*a^2*c^2*e-2*c*(3*a*b*c*e-2*a
*c^2*d-b^3*e+b^2*c*d)*x)/a^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a
)^(1/2)-(c*x^2+b*x+a)^(1/2)/a^2/d/x+1/2*(2*a*e+3*b*d)*arctanh(1/2*(b*x+2*a
)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d^2+e^4*arctanh(1/2*(b*d-2*a*e+(-b*
e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/(a*e^2-b*d*
e+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{d\left(4a^3ce^2+3b^2d(-cd+be)x(b+cx)+a^2\left(-b^2e^2+4bce(-d+ex)+4c^2\left(d^2+dex+e^2x^2\right)\right)+a\left(8c^3d^2x^2+b^3e(d-ex)+10bc^2dx(d-ex)-b^2c\left(d^2+12dex+\right.\right.\right.}{a^2(b^2-4ac)(-cd^2+e(bd-ae))x\sqrt{a+x(b+cx)}}}{d^2}$$

input
```
Integrate[1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
-(((d*(4*a^3*c*e^2 + 3*b^2*d*(-(c*d) + b*e)*x*(b + c*x) + a^2*(-(b^2*e^2)
+ 4*b*c*e*(-d + e*x) + 4*c^2*(d^2 + d*e*x + e^2*x^2)) + a*(8*c^3*d^2*x^2 +
 b^3*e*(d - e*x) + 10*b*c^2*d*x*(d - e*x) - b^2*c*(d^2 + 12*d*e*x + e^2*x^
2))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x*Sqrt[a + x*(b + c*x)
]) - (2*e^4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*S
qrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d)
+ a*e))^2 + ((3*b*d + 2*a*e)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/S
qrt[a]])/a^(5/2))/d^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1289$$

$$\int \left(\frac{e^2}{d^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}-\frac{e}{d^2x\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d^2}+\frac{3b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)}+\frac{e^4\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2(ae^2-bde+cd^2)^{3/2}}-\frac{2e^2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{d^2(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{2e(-2ac+b^2+bcx)}{ad^2(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{2(-2ac+b^2+bcx)}{adx(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*e*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^2]) +
 (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (
2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*d^2*(c
*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c)*Sqrt[a + b
*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/(a^(3/2)*d^2) + (e^4*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d^2
*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(256) = 512$.

Time = 1.52 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.34

method	result
default	$\frac{-\frac{1}{ax\sqrt{c\,x^2+bx+a}}-\frac{3b\left(\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}-\frac{4c(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}}{d}+\frac{e\left(\frac{\cdots}{\left(a\,e^2\cdots\right.}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/x/(c*x^2+b*x+a)^(1/2)-3/2*b/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x))-4*c/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+e/d^2
*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*
c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/
e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-e/d^2*(1/a/(c*x^2+b*x+a)^(1
/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(255) = 510$.

Time = 4.88 (sec) , antiderivative size = 6396, normalized size of antiderivative = 23.09

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(1/(x**2*(d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\,e^4\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^4-bd^3e+ad^2e^2)\sqrt{-cd^2+bde-ae^2}} - \frac{2\left(\frac{(a^2b^2c^3d^3-2\,a^3c^4d^3-2\,a^2b^3c^2d^2e+5\,a^3bc^3d^2e+a^2b^4cde^2-2\,a^3b^2c^2de^2-2\,a^4c^3de^2-a^3b^3ce^3+3\,a^4bc^2e^3)x}{a^4b^2c^2d^4-4\,a^5c^3d^4-2\,a^4b^3cd^3e+8\,a^5bc^2d^3e+a^4b^4d^2e^2-2\,a^5b^2cd^2e^2-8\,a^6c^2d^2e^2-2\,a^5b^3de^3+8\,a^6bcde^3+a^6b^2e^4-4\,a^7ce^4} + \frac{a^2b^3c^2d^3-3}{a^4b^2c^2d^4-4}\right.}{\sqrt{cx^2+bx+a}} - \frac{(3\,bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2d^2} + \frac{(\sqrt{c}x-\sqrt{cx^2+bx+a})b+2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a^2d}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*e^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e -
 a*e^2)) - 2*((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a
^3*b*c^3*d^2*e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 -
 a^3*b^3*c*e^3 + 3*a^4*b*c^2*e^3)*x/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a
^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2
 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 - 4
*a^7*c*e^4) + (a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7*a
^3*b^2*c^2*d^2*e - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 - a
^4*b*c^2*d*e^2 - a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2*c
^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d
^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6
*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*c*e^4))/sqrt(c*x^2 + b*x + a) - (3*b*d +
2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2
*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2 - a)*a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4729, normalized size of antiderivative = 17.07

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output

```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*c*e**4*x - 2*sqrt(
a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e
+ c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*b**2*e**4*x + 8*sqrt(a*e**
2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d
**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*b*c*e**4*x**2 + 8*sqrt(a*e**2 -
 b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2
) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*c**2*e**4*x**3 - 2*sqrt(a*e**2 - b
*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2)
- 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*b**3*e**4*x**2 - 2*sqrt(a*e**2 - b*d
*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) -
2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*b**2*c*e**4*x**3 - 8*sqrt(a*e**2 - b*d
*e + c*d**2)*log(d + e*x)*a**5*c*e**4*x + 2*sqrt(a*e**2 - b*d*e + c*d**2)*
log(d + e*x)*a**4*b**2*e**4*x - 8*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*
x)*a**4*b*c*e**4*x**2 - 8*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**4*
c**2*e**4*x**3 + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**3*b**3*e*
*4*x**2 + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**3*b**2*c*e**4*x*
*3 - 8*sqrt(a + b*x + c*x**2)*a**5*c*d*e**4 + 2*sqrt(a + b*x + c*x**2)*a**
4*b**2*d*e**4 + 16*sqrt(a + b*x + c*x**2)*a**4*b*c*d**2*e**3 - 8*sqrt(a +
b*x + c*x**2)*a**4*b*c*d*e**4*x - 16*sqrt(a + b*x + c*x**2)*a**4*c**2*d...
```

3.45 $\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 498

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(a(b^3d-3abcd-ab^2e+2a^2ce)+(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,x)}{3c^2\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2(b^6d^2e+2a^2b^2ce(4cd^2-3ae^2)-ab^4e(6cd^2-ae^2)+24a^3c^2e(2cd^2+ae^2)-4a^2bc^2d(8cd^2+11ae^2)+ab}{}$$
$$+\frac{d^4\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{5/2}}$$

output

```
1/3*(-2*a*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)-2*(3*a^2*b*c*e+2*a^2*c^2*d-a
*b^3*e-4*a*b^2*c*d+b^4*d)*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b
*x+a)^(3/2)-2/3*(b^6*d^2*e+2*a^2*b^2*c*e*(-3*a*e^2+4*c*d^2)-a*b^4*e*(-a*e^
2+6*c*d^2)+24*a^3*c^2*e*(a*e^2+2*c*d^2)-4*a^2*b*c^2*d*(11*a*e^2+8*c*d^2)+a
*b^3*c*d*(13*a*e^2+10*c*d^2)-b^5*(2*a*d*e^2+c*d^3)-c*(b^5*d^2*e-a*b^3*e*(-
a*e^2+6*c*d^2)-8*a^2*c^2*d*(a*e^2+4*c*d^2)-4*a^2*b*c*e*(3*a*e^2+4*c*d^2)+2
*a*b^2*c*d*(11*a*e^2+14*c*d^2)-2*b^4*(a*d*e^2+2*c*d^3))*x)/c^2/(-4*a*c+b^2
)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)+d^4*arctanh(1/2*(b*d-2*a*e+(
-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e
+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 12.06 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(2a^3ce+b^4dx+ab^2(-4cdx+b(d-ex))+a^2(-b^2e+2c^2dx-3bc(d-}{3c^2\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(a+x(b+cx))^{3/2}}$$
$$-\frac{2(b^6d^2e+b^4(a^2e^3+4c^2d^3x+2acde(-3d+ex))-b^5d(2ae^2+cd(d+ex))+8a^2c^2(3a^2e^3+4c^2d^3x+ac}{3c^2}$$
$$+\frac{d^4\log(d+ex)}{(cd^2+e(-bd+ae))^{5/2}}$$
$$-\frac{d^4\log\left(-bd+2ae-2cdx+bex+2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}\right)}{(cd^2+e(-bd+ae))^{5/2}}$$

input
```
Integrate[x^4/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(2*a^3*c*e + b^4*d*x + a*b^2*(-4*c*d*x + b*(d - e*x)) + a^2*(-(b^2*e) +
 2*c^2*d*x - 3*b*c*(d - e*x))))/(3*c^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d -
a*e))*(a + x*(b + c*x))^(3/2)) - (2*(b^6*d^2*e + b^4*(a^2*e^3 + 4*c^2*d^3*
x + 2*a*c*d*e*(-3*d + e*x)) - b^5*d*(2*a*e^2 + c*d*(d + e*x)) + 8*a^2*c^2*
(3*a^2*e^3 + 4*c^2*d^3*x + a*c*d*e*(6*d + e*x)) + 4*a^2*b*c^2*(4*c*d^2*(-2
*d + e*x) + a*e^2*(-11*d + 3*e*x)) + a*b^3*c*(a*e^2*(13*d - e*x) + 2*c*d^2
*(5*d + 3*e*x)) - 2*a*b^2*c*(3*a^2*e^3 + 14*c^2*d^3*x + a*c*d*e*(-4*d + 11
*e*x))))/(3*c^2*(b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b
 + c*x)]) + (d^4*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(5/2) - (d^4*Log
[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[
a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(5/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1264, 27, 2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1264

$$-\frac{2\int \frac{-\frac{3\left(b^2-4ac\right)x^2}{c}-\frac{\left(-deb^4-\left(3cd^2-ae^2\right)b^3+4acdeb^2+12ac^2d^2b-8a^2c^2de\right)x}{c^2\left(cd^2-bed+ae^2\right)}+\frac{d\left(db^4-aeb^3-acdb^2-4a^2c^2d\right)}{c^2\left(cd^2-bed+ae^2\right)}}{2(d+ex)\left(cx^2+bx+a\right)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{-\frac{3\left(b^2-4ac\right)x^2}{c}-\frac{\left(-deb^4-\left(3cd^2-ae^2\right)b^3+4acdeb^2+12ac^2d^2b-8a^2c^2de\right)x}{c^2\left(cd^2-bed+ae^2\right)}+\frac{d\left(db^4-aeb^3-acdb^2-4a^2c^2d\right)}{c^2\left(cd^2-bed+ae^2\right)}}{(d+ex)\left(cx^2+bx+a\right)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 2177

$$-\frac{\frac{2\left(24a^3c^2e\left(ae^2+2cd^2\right)+2a^2b^2ce\left(4cd^2-3ae^2\right)-cx\left(-4a^2bce\left(3ae^2+4cd^2\right)-8a^2c^2d\left(ae^2+4cd^2\right)-2b^4\left(ade^2+2cd^3\right)-ab^3e\left(6cd^2-ae^2\right)+2ab^2cd(11\right.\right.}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+\right.}}{}$$

$$-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2\left(24a^3c^2e\left(ae^2+2cd^2\right)+2a^2b^2ce\left(4cd^2-3ae^2\right)-cx\left(-4a^2bce\left(3ae^2+4cd^2\right)-8a^2c^2d\left(ae^2+4cd^2\right)-2b^4\left(ade^2+2cd^3\right)-ab^3e\left(6cd^2-ae^2\right)+2ab^2cd(11\right.\right.}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+\right.}}{3}$$

$$-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1154

$$-\frac{\frac{6d^4\left(b^2-4ac\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{\left(ae^2-bde+cd^2\right)^2}+\frac{2\left(24a^3c^2e\left(ae^2+2cd^2\right)+2a^2b^2ce\left(4cd^2-3ae^2\right)-cx\left(-4a^2bc\right.\right.}{}}{}$$

$$-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 219

$$-\frac{2\left(a\left(2a^2ce-ab^2e-3abcd+b^3d\right)+x\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}-$$

$$\frac{\frac{2\left(24a^3c^2e\left(ae^2+2cd^2\right)+2a^2b^2ce\left(4cd^2-3ae^2\right)-cx\left(-4a^2bce\left(3ae^2+4cd^2\right)-8a^2c^2d\left(ae^2+4cd^2\right)-2b^4\left(ade^2+2cd^3\right)-ab^3e\left(6cd^2-ae^2\right)+2ab^2cd\left(11ae\right.\right.\right.}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd\right.}}{}$$

input
```
Int[x^4/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(a*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e) + (b^4*d - 4*a*b^2*c*d +
2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*x))/(3*c^2*(b^2 - 4*a*c)*(c*d^2 - b*d
*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - ((2*(b^6*d^2*e + 2*a^2*b^2*c*e*(4*c
*d^2 - 3*a*e^2) - a*b^4*e*(6*c*d^2 - a*e^2) + 24*a^3*c^2*e*(2*c*d^2 + a*e^
2) - 4*a^2*b*c^2*d*(8*c*d^2 + 11*a*e^2) + a*b^3*c*d*(10*c*d^2 + 13*a*e^2)
- b^5*(c*d^3 + 2*a*d*e^2) - c*(b^5*d^2*e - a*b^3*e*(6*c*d^2 - a*e^2) - 8*a
^2*c^2*d*(4*c*d^2 + a*e^2) - 4*a^2*b*c*e*(4*c*d^2 + 3*a*e^2) + 2*a*b^2*c*d
*(14*c*d^2 + 11*a*e^2) - 2*b^4*(2*c*d^3 + a*d*e^2))*x))/(c^2*(b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*c)*d^4*A
rcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(5/2))/(3*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(484) = 968$.

Time = 1.52 (sec) , antiderivative size = 1448, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	1448

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*
b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*
(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x
+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c
*x^2+b*x+a)^(1/2))))+d^2/e^3*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c
*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^
2+b*x+a)^(1/2)))+d^4/e^5*(1/3/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)-1/2*(b*e-2*c*d)*e/(a*e^2-b*d
*e+c*d^2)*(2/3*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b
*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)+16/3*c/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)^2*(2*c*(x+
d/e)+(b*e-2*c*d)/e)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))+1/(a*e^2-b*d*e+c*d^2)*e^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a
*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2
-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2288 vs. $2(482) = 964$.

Time = 1.21 (sec) , antiderivative size = 4618, normalized size of antiderivative = 9.27

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x**4/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8448 vs. $2(482) = 964$.

Time = 0.52 (sec) , antiderivative size = 8448, normalized size of antiderivative = 16.96

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 2/3*((((4*b^4*c
^7*d^15 - 28*a*b^2*c^8*d^15 + 32*a^2*c^9*d^15 - 25*b^5*c^6*d^14*e + 174*a*
b^3*c^7*d^14*e - 176*a^2*b*c^8*d^14*e + 66*b^6*c^5*d^13*e^2 - 430*a*b^4*c^
6*d^13*e^2 + 194*a^2*b^2*c^7*d^13*e^2 + 200*a^3*c^8*d^13*e^2 - 95*b^7*c^4*
d^12*e^3 + 512*a*b^5*c^5*d^12*e^3 + 607*a^2*b^3*c^6*d^12*e^3 - 900*a^3*b*c
^7*d^12*e^3 + 80*b^8*c^3*d^11*e^4 - 240*a*b^6*c^4*d^11*e^4 - 1952*a^2*b^4*
c^5*d^11*e^4 + 936*a^3*b^2*c^6*d^11*e^4 + 528*a^4*c^7*d^11*e^4 - 39*b^9*c^
2*d^10*e^5 - 82*a*b^7*c^3*d^10*e^5 + 2198*a^2*b^5*c^4*d^10*e^5 + 1484*a^3*
b^3*c^5*d^10*e^5 - 1848*a^4*b*c^6*d^10*e^5 + 10*b^10*c*d^9*e^6 + 146*a*b^8
*c^2*d^9*e^6 - 1034*a^2*b^6*c^3*d^9*e^6 - 4180*a^3*b^4*c^4*d^9*e^6 + 1150*
a^4*b^2*c^5*d^9*e^6 + 760*a^5*c^6*d^9*e^6 - b^11*d^8*e^7 - 60*a*b^9*c*d^8*
e^7 + 31*a^2*b^7*c^2*d^8*e^7 + 3520*a^3*b^5*c^3*d^8*e^7 + 2865*a^4*b^3*c^4
*d^8*e^7 - 1900*a^5*b*c^5*d^8*e^7 + 8*a*b^10*d^7*e^8 + 128*a^2*b^8*c*d^7*e
^8 - 1060*a^3*b^6*c^2*d^7*e^8 - 4820*a^4*b^4*c^3*d^7*e^8 + 100*a^5*b^2*c^4
*d^7*e^8 + 640*a^6*c^5*d^7*e^8 - 28*a^2*b^9*d^6*e^9 - 56*a^3*b^7*c*d^6*e^9
 + 2447*a^4*b^5*c^2*d^6*e^9 + 3150*a^5*b^3*c^3*d^6*e^9 - 960*a^6*b*c^4*d^6
*e^9 + 56*a^3*b^8*d^5*e^10 - 252*a^4*b^6*c*d^5*e^10 - 2726*a^5*b^4*c^2*d^5
*e^10 - 738*a^6*b^2*c^3*d^5*e^10 + 312*a^7*c^4*d^5*e^10 - 70*a^4*b^7*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^4}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(x^4/((d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{x^4}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^4/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

3.46 $\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 417

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(a(b^2d-2acd-abe)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x\right)}{3c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$-\frac{2(24a^2c^3d^3-b^5d^2e-2ab^2cd(3cd^2-2ae^2)+ab^3e(2cd^2-ae^2)-4a^2bce(4cd^2+ae^2)+b^4(cd^3+2ade^2)-}{3c\left(b^2-4ac\right)^2\left(cd^2-bde+\right.}$$
$$-\frac{d^3e\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{5/2}}$$

output
```
2/3*(a*(-a*b*e-2*a*c*d+b^2*d)+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x)/c/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(3/2)-2/3*(24*a^2*c^3*d^3-b^5*
d^2*e-2*a*b^2*c*d*(-2*a*e^2+3*c*d^2)+a*b^3*e*(-a*e^2+2*c*d^2)-4*a^2*b*c*e*
(a*e^2+4*c*d^2)+b^4*(2*a*d*e^2+c*d^3)-c*(2*b^4*d^2*e-2*a*b^2*e*(-a*e^2+2*c
*d^2)+8*a^2*c*e*(a*e^2+4*c*d^2)-4*a*b*c*d*(2*a*e^2+3*c*d^2)+b^3*(-4*a*d*e^
2+c*d^3))*x)/c/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)-d^
3*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 12.20 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(-b^3dx+ab(-bd+3cdx+bex)+a^2(be+2c(d-ex)))}{3c\left(-b^2+4ac\right)\left(cd^2+e(-bd+ae)\right)(a+x(b+cx))^{3/2}}$$
$$+\frac{2(b^5d^2e-b^4d(2ae^2+cd(d-2ex))+4abc(a^2e^3-3c^2d^3x+2acde(2d-ex))+2ab^2c(cd^2(3d-2ex)+ae}{3c\left(b^2-4ac\right)^2\left(cd^2+e(-bd+ae)\right)^2}$$
$$-\frac{d^3e\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$
$$+\frac{d^3e\log\left(-bd+2ae-2cdx+bex+2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}\right)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$

input
```
Integrate[x^3/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(-(b^3*d*x) + a*b*(-(b*d) + 3*c*d*x + b*e*x) + a^2*(b*e + 2*c*(d - e*x)
)))/(3*c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(3/2)
) + (2*(b^5*d^2*e - b^4*d*(2*a*e^2 + c*d*(d - 2*e*x)) + 4*a*b*c*(a^2*e^3 -
 3*c^2*d^3*x + 2*a*c*d*e*(2*d - e*x)) + 2*a*b^2*c*(c*d^2*(3*d - 2*e*x) + a
*e^2*(-2*d + e*x)) + b^3*(a^2*e^3 + c^2*d^3*x - 2*a*c*d*e*(d + 2*e*x)) + 8
*a^2*c^2*(a*e^3*x + c*d^2*(-3*d + 4*e*x))))/(3*c*(b^2 - 4*a*c)^2*(c*d^2 +
e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x)]) - (d^3*e*Log[d + e*x])/(c*d^2 +
 e*(-(b*d) + a*e))^(5/2) + (d^3*e*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d)
 + a*e))^(5/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1264, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}dx$$

$\downarrow$ 1264

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{2\int -\frac{d\left(db^3-aeb^2-4a^2ce\right)+\left(deb^3+\left(3cd^2-ae^2\right)b^2-4ac\left(3cd^2+ae^2\right)\right)x}{2c\left(cd^2-bed+ae^2\right)(d+ex)\left(cx^2+bx+a\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{d\left(db^3-aeb^2-4a^2ce\right)+\left(deb^3+\left(3cd^2-ae^2\right)b^2-4ac\left(3cd^2+ae^2\right)\right)x}{(d+ex)\left(cx^2+bx+a\right)^{3/2}}dx}{3c\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}+\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1235

$$\frac{-\frac{2\int \frac{3c\left(b^2-4ac\right)^2d^3e}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}-\frac{2\left(cx\left(d(2cd-be)\left(-4a^2ce-ab^2e+b^3d\right)-(bd-2ae)\left(b^2\left(3cd^2-ae^2\right)-4ac\left(ae^2+3cd^2\right)+b^3de\right)\right)+d\left(2ace+b^2(-e)+bcd\cdots\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\cdots\right.}}{3c\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{3cd^3e\left(b^2-4ac\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\left(cx\left(d(2cd-be)\left(-4a^2ce-ab^2e+b^3d\right)-(bd-2ae)\left(b^2\left(3cd^2-ae^2\right)-4ac\left(ae^2+3cd^2\right)+b^3de\right)\right)+d\left(2ace\cdots\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(\cdots\right.}}{3c\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1154

$$\frac{\frac{6cd^3e\left(b^2-4ac\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}-\frac{2\left(cx\left(d(2cd-be)\left(-4a^2ce-ab^2e+b^3d\right)-(bd-2ae)\left(b^2\left(3cd^2\cdots\right.\right.\right.\right.}{\cdots}}{3c\left(b^2-4ac\right)\left(ae^2-\cdots\right.}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 219

$$-\frac{\frac{2\left(cx\left(d(2cd-be)\left(-4a^2ce-ab^2e+b^3d\right)-(bd-2ae)\left(b^2\left(3cd^2-ae^2\right)-4ac\left(ae^2+3cd^2\right)+b^3de\right)\right)+d\left(2ace+b^2(-e)+bcd\right)\left(-4a^2ce-ab^2e+b^3d\right)-a(2ca}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}}{3c\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right.}$$

$$\frac{2\left(x\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[x^3/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e
)*x))/(3*c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))
+ ((-2*(d*(b*c*d - b^2*e + 2*a*c*e)*(b^3*d - a*b^2*e - 4*a^2*c*e) - a*(2*c
*d - b*e)*(b^3*d*e + b^2*(3*c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2)) + c*
(d*(2*c*d - b*e)*(b^3*d - a*b^2*e - 4*a^2*c*e) - (b*d - 2*a*e)*(b^3*d*e +
b^2*(3*c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2)))*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (3*c*(b^2 - 4*a*c)*d^3*e*ArcTa
nh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2))/(3*c*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(401) = 802$.

Time = 1.47 (sec) , antiderivative size = 1131, normalized size of antiderivative = 2.71

method	result	size
default	Expression too large to display	1131

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^2/e^3*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^
2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))+1/e*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c
*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/
3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)
/(c*x^2+b*x+a)^(1/2)))-d/e^2*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c
*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^
2+b*x+a)^(1/2)))-d^3/e^4*(1/3/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)-1/2*(b*e-2*c*d)*e/(a*e^2-b*d
*e+c*d^2)*(2/3*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b
*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)+16/3*c/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)^2*(2*c*(x+
d/e)+(b*e-2*c*d)/e)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))+1/(a*e^2-b*d*e+c*d^2)*e^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a
*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2
-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(401) = 802$.

Time = 1.30 (sec) , antiderivative size = 4600, normalized size of antiderivative = 11.03

$$\int \frac{x^3}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x**3/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8382 vs. $2(401) = 802$.

Time = 0.48 (sec) , antiderivative size = 8382, normalized size of antiderivative = 20.10

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2*d^3*e*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 + b*d*e - a*e^2))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2
*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 2/3*((((b^3*
c^8*d^15 - 12*a*b*c^9*d^15 - 4*b^4*c^7*d^14*e + 68*a*b^2*c^8*d^14*e + 32*a
^2*c^9*d^14*e + 3*b^5*c^6*d^13*e^2 - 154*a*b^3*c^7*d^13*e^2 - 272*a^2*b*c^
8*d^13*e^2 + 10*b^6*c^5*d^12*e^3 + 186*a*b^4*c^6*d^12*e^3 + 866*a^2*b^2*c^
7*d^12*e^3 + 200*a^3*c^8*d^12*e^3 - 25*b^7*c^4*d^11*e^4 - 160*a*b^5*c^5*d^
11*e^4 - 1381*a^2*b^3*c^6*d^11*e^4 - 1236*a^3*b*c^7*d^11*e^4 + 24*b^8*c^3*
d^10*e^5 + 152*a*b^6*c^4*d^10*e^5 + 1240*a^2*b^4*c^5*d^10*e^5 + 2952*a^3*b
^2*c^6*d^10*e^5 + 528*a^4*c^7*d^10*e^5 - 11*b^9*c^2*d^9*e^6 - 138*a*b^7*c^
3*d^9*e^6 - 742*a^2*b^5*c^4*d^9*e^6 - 3500*a^3*b^3*c^5*d^9*e^6 - 2520*a^4*
b*c^6*d^9*e^6 + 2*b^10*c*d^8*e^7 + 74*a*b^8*c^2*d^8*e^7 + 422*a^2*b^6*c^3*
d^8*e^7 + 2260*a^3*b^4*c^4*d^8*e^7 + 4510*a^4*b^2*c^5*d^8*e^7 + 760*a^5*c^
6*d^8*e^7 - 16*a*b^9*c*d^7*e^8 - 221*a^2*b^7*c^2*d^7*e^8 - 960*a^3*b^5*c^3
*d^7*e^8 - 3785*a^4*b^3*c^4*d^7*e^8 - 2740*a^5*b*c^5*d^7*e^8 + 56*a^2*b^8*
c*d^6*e^9 + 396*a^3*b^6*c^2*d^6*e^9 + 1620*a^4*b^4*c^3*d^6*e^9 + 3460*a^5*
b^2*c^4*d^6*e^9 + 640*a^6*c^5*d^6*e^9 - 112*a^3*b^7*c*d^5*e^10 - 493*a^4*b
^5*c^2*d^5*e^10 - 1834*a^5*b^3*c^3*d^5*e^10 - 1632*a^6*b*c^4*d^5*e^10 + 14
0*a^4*b^6*c*d^4*e^11 + 466*a^5*b^4*c^2*d^4*e^11 + 1278*a^6*b^2*c^3*d^4*e^1
1 + 312*a^7*c^4*d^4*e^11 - 112*a^5*b^5*c*d^3*e^12 - 339*a^6*b^3*c^2*d^3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(x^3/((d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^3}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^3/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

3.47 $\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 349

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(a(bd-2ae)+(b^2d-2acd-abe)\,x)}{3\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2(b^4d^2e-24a^2c^2d^2e-4abcd(cd^2-2ae^2)+2ab^2e(7cd^2+2ae^2)-b^3(cd^3+8ade^2)+c(5b^3d^2e-8acd(cd}{3\,(b^2-4ac)^2\,(cd^2-bde+ae^2)^2\,\sqrt{a+bx+cx^2}}$$
$$+\frac{d^2e^2\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{5/2}}$$

output
```
1/3*(-2*a*(-2*a*e+b*d)-2*(-a*b*e-2*a*c*d+b^2*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d
*e+c*d^2)/(c*x^2+b*x+a)^(3/2)-2/3*(b^4*d^2*e-24*a^2*c^2*d^2*e-4*a*b*c*d*(-
2*a*e^2+c*d^2)+2*a*b^2*e*(2*a*e^2+7*c*d^2)-b^3*(8*a*d*e^2+c*d^3)+c*(5*b^3*
d^2*e-8*a*c*d*(-2*a*e^2+c*d^2)+4*a*b*e*(2*a*e^2+c*d^2)-2*b^2*(8*a*d*e^2+c*
d^3))*x)/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)+d^2*e^2*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 12.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(-2a^2e+b^2dx-2acdx+ab(d-ex))}{3\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(a+x(b+cx))^{3/2}}$$
$$-\frac{2(b^4d^2e+2b^2(2a^2e^3-c^2d^3x+acde(7d-8ex))-b^3d(8ae^2+cd(d-5ex))-8ac^2d(cd^2x+ae(3d-2ex}{3\left(b^2-4ac\right)^2\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$
$$+\frac{d^2e^2\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$
$$-\frac{d^2e^2\log\left(-bd+2ae-2cdx+bex+2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}\right)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$

input
```
Integrate[x^2/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(-2*a^2*e + b^2*d*x - 2*a*c*d*x + a*b*(d - e*x)))/(3*(b^2 - 4*a*c)*(-(c
*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))^(3/2)) - (2*(b^4*d^2*e + 2*b^2*(2
*a^2*e^3 - c^2*d^3*x + a*c*d*e*(7*d - 8*e*x)) - b^3*d*(8*a*e^2 + c*d*(d -
5*e*x)) - 8*a*c^2*d*(c*d^2*x + a*e*(3*d - 2*e*x)) + 4*a*b*c*(c*d^2*(-d + e
*x) + 2*a*e^2*(d + e*x))))/(3*(b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2
*Sqrt[a + x*(b + c*x)]) + (d^2*e^2*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e)
)^(5/2) - (d^2*e^2*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e
*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(5/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1264, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1264

$$-\frac{2\int \frac{d(db^2-4aeb+4acd)+4e(db^2-aeb-2acd)x}{2(cd^2-bed+ae^2)(d+ex)(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{d(db^2-4aeb+4acd)+4e(db^2-aeb-2acd)x}{(d+ex)(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)(ae^2-bde+cd^2)}-\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 1235

$$-\frac{\frac{2\left(-cx\left(d(2cd-be)\left(-4abe+4acd+b^2d\right)-4e(bd-2ae)\left(-abe-2acd+b^2d\right)\right)+4ae(2cd-be)\left(-abe-2acd+b^2d\right)-d\left(-4abe+4acd+b^2d\right)\left(2ace+b^2(-e\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2\left(-cx\left(d(2cd-be)\left(-4abe+4acd+b^2d\right)-4e(bd-2ae)\left(-abe-2acd+b^2d\right)\right)+4ae(2cd-be)\left(-abe-2acd+b^2d\right)-d\left(-4abe+4acd+b^2d\right)\left(2ace+b^2(-e\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 1154

$$-\frac{\frac{6d^2e^2(b^2-4ac)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}+\frac{2\left(-cx\left(d(2cd-be)\left(-4abe+4acd+b^2d\right)-4e(bd-2ae)\left(-a\right.\right.\right.}{}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 219

$$-\frac{\frac{2\left(-cx\left(d(2cd-be)\left(-4abe+4acd+b^2d\right)-4e(bd-2ae)\left(-abe-2acd+b^2d\right)\right)+4ae(2cd-be)\left(-abe-2acd+b^2d\right)-d\left(-4abe+4acd+b^2d\right)\left(2ace+b^2(-e\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2\left(x\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

input
```
Int[x^2/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x))/(3*(b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - ((2*(4*a*e*(2*c*d - b*e)*(b
^2*d - 2*a*c*d - a*b*e) - d*(b^2*d + 4*a*c*d - 4*a*b*e)*(b*c*d - b^2*e + 2
*a*c*e) - c*(d*(2*c*d - b*e)*(b^2*d + 4*a*c*d - 4*a*b*e) - 4*e*(b*d - 2*a*
e)*(b^2*d - 2*a*c*d - a*b*e))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*S
qrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*c)*d^2*e^2*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*
d^2 - b*d*e + a*e^2)^(3/2))/(3*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(335) = 670$.

Time = 1.49 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.68

method	result
default	$\frac{-\frac{1}{3c\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^{2}\right)^{2}\sqrt{c\,x^{2}+bx+a}}\right)}{2c}}{e}+\frac{d^{2}\left(\frac{e^{2}}{3\left(a\,e^{2}-bde+c\,d^{2}\right)\left(c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\cdots\right.}\right.}{}$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+d^2/e^3*
(1/3/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(3/2)-1/2*(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2/3*(2*c*(x+d/
e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+16/3*c/(4*c*(a*e
^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)^2*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(c*(x
+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))+1/(a*e^2-b*d
*e+c*d^2)*e^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x
+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x
+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*
e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))-d/e^2*(2/3*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*
x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(333) = 666$.

Time = 1.25 (sec) , antiderivative size = 4636, normalized size of antiderivative = 13.28

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x**2/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8210 vs. $2(333) = 666$.

Time = 0.49 (sec) , antiderivative size = 8210, normalized size of antiderivative = 23.52

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*d^2*e^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 2/3*((((2*b
^2*c^9*d^15 + 8*a*c^10*d^15 - 17*b^3*c^8*d^14*e - 52*a*b*c^9*d^14*e + 60*b
^4*c^7*d^13*e^2 + 172*a*b^2*c^8*d^13*e^2 + 32*a^2*c^9*d^13*e^2 - 115*b^5*c
^6*d^12*e^3 - 406*a*b^3*c^7*d^12*e^3 - 176*a^2*b*c^8*d^12*e^3 + 130*b^6*c^
5*d^11*e^4 + 710*a*b^4*c^6*d^11*e^4 + 534*a^2*b^2*c^7*d^11*e^4 + 24*a^3*c^
8*d^11*e^4 - 87*b^7*c^4*d^10*e^5 - 848*a*b^5*c^5*d^10*e^5 - 1195*a^2*b^3*c
^6*d^10*e^5 - 108*a^3*b*c^7*d^10*e^5 + 32*b^8*c^3*d^9*e^6 + 632*a*b^6*c^4*
d^9*e^6 + 1840*a^2*b^4*c^5*d^9*e^6 + 520*a^3*b^2*c^6*d^9*e^6 - 80*a^4*c^7*
d^9*e^6 - 5*b^9*c^2*d^8*e^7 - 262*a*b^7*c^3*d^8*e^7 - 1722*a^2*b^5*c^4*d^8
*e^7 - 1540*a^3*b^3*c^5*d^8*e^7 + 280*a^4*b*c^6*d^8*e^7 + 46*a*b^8*c^2*d^7
*e^8 + 866*a^2*b^6*c^3*d^7*e^8 + 2220*a^3*b^4*c^4*d^7*e^8 + 110*a^4*b^2*c^
5*d^7*e^8 - 200*a^5*c^6*d^7*e^8 - 179*a^2*b^7*c^2*d^6*e^9 - 1504*a^3*b^5*c
^3*d^6*e^9 - 1255*a^4*b^3*c^4*d^6*e^9 + 500*a^5*b*c^5*d^6*e^9 + 388*a^3*b^
6*c^2*d^5*e^10 + 1460*a^4*b^4*c^3*d^5*e^10 + 12*a^5*b^2*c^4*d^5*e^10 - 192
*a^6*c^5*d^5*e^10 - 515*a^4*b^5*c^2*d^4*e^11 - 742*a^5*b^3*c^3*d^4*e^11 +
288*a^6*b*c^4*d^4*e^11 + 430*a^5*b^4*c^2*d^3*e^12 + 122*a^6*b^2*c^3*d^3*e^
12 - 88*a^7*c^4*d^3*e^12 - 221*a^6*b^3*c^2*d^2*e^13 + 44*a^7*b*c^3*d^2*e^1
3 + 64*a^7*b^2*c^2*d*e^14 - 16*a^8*c^3*d*e^14 - 8*a^8*b*c^2*e^15)*x/(b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^2}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(x^2/((d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x^2}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

3.48 $\int \frac{x}{(d+ex)(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 306

$$\int \frac{x}{(d+ex)(a+bx+cx^2)^{5/2}}\,dx = \frac{2(a(2cd-be)+c(bd-2ae)x)}{3(b^2-4ac)(cd^2-bde+ae^2)(a+bx+cx^2)^{3/2}}$$
$$+\frac{2(4ace(bd-2ae)(2cd-be)-d(bcd-b^2e+2ace)(4bcd-3b^2e+4ace)+c(4ce(bd-2ae)^2-d(2cd-be}{3(b^2-4ac)^2(cd^2-bde+ae^2)^2\sqrt{a+bx+cx^2}}$$
$$-\frac{de^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{5/2}}$$

output
```
2/3*(a*(-b*e+2*c*d)+c*(-2*a*e+b*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
x^2+b*x+a)^(3/2)+2/3*(4*a*c*e*(-2*a*e+b*d)*(-b*e+2*c*d)-d*(2*a*c*e-b^2*e+b
*c*d)*(4*a*c*e-3*b^2*e+4*b*c*d)+c*(4*c*e*(-2*a*e+b*d)^2-d*(-b*e+2*c*d)*(4*
a*c*e-3*b^2*e+4*b*c*d))*x)/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x
+a)^(1/2)-d*e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)
^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 10.50 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.99

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{-2bcdx+a(-4cd+2be+4cex)}{3\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(a+x(b+cx))^{3/2}}$$
$$-\frac{2(3b^4de^2+b^3cde(-7d+3ex)+2b^2cd(-3ae^2+cd(2d-7ex))+8ac^2e(cd^2x+ae(3d-2ex))+4bc(-2a}{3\left(b^2-4ac\right)^2\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$
$$+\frac{de^3\operatorname{arctanh}\left(\frac{-bd+2ae-2cdx+bex}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$

input
```
Integrate[x/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*b*c*d*x + a*(-4*c*d + 2*b*e + 4*c*e*x))/(3*(b^2 - 4*a*c)*(-(c*d^2) + e
*(b*d - a*e))*(a + x*(b + c*x))^(3/2)) - (2*(3*b^4*d*e^2 + b^3*c*d*e*(-7*d
 + 3*e*x) + 2*b^2*c*d*(-3*a*e^2 + c*d*(2*d - 7*e*x)) + 8*a*c^2*e*(c*d^2*x
+ a*e*(3*d - 2*e*x)) + 4*b*c*(-2*a^2*e^3 + 2*c^2*d^3*x + a*c*d*e*(d + 3*e*
x))))/(3*(b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x)
]) + (d*e^3*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(
-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(5/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1235, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \; 1235$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{2\int -\frac{d\left(-3eb^2+4cdb+4ace\right)+4ce(bd-2ae)x}{2(d+ex)\left(cx^2+bx+a\right)^{3/2}}dx}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{d(-3eb^2+4cdb+4ace)+4ce(bd-2ae)x}{(d+ex)(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)(ae^2-bde+cd^2)}+\frac{2(cx(bd-2ae)+a(2cd-be))}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\downarrow$ 1235

$$\frac{\frac{2(cx(4ce(bd-2ae)^2-d(2cd-be)(4ace-3b^2e+4bcd))-d(2ace+b^2(-e)+bcd)(4ace-3b^2e+4bcd)+4ace(bd-2ae)(2cd-be))}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{2\int \frac{3(b^2-4ac)^2d\ldots}{2(d+ex)\sqrt{cx^2+b\ldots}}}{(b^2-4ac)(ae^2-bde\ldots}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\downarrow$ 27

$$\frac{\frac{2(cx(4ce(bd-2ae)^2-d(2cd-be)(4ace-3b^2e+4bcd))-d(2ace+b^2(-e)+bcd)(4ace-3b^2e+4bcd)+4ace(bd-2ae)(2cd-be))}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{3de^3(b^2-4ac)\int \frac{\ldots}{(d\ldots}}{ae^2-b\ldots}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\downarrow$ 1154

$$\frac{\frac{6de^3(b^2-4ac)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}+\frac{2(cx(4ce(bd-2ae)^2-d(2cd-be)(4ace-3b^2e+4bcd))-d(2ac\ldots}{(b^2-4ac)\sqrt{a+bx+\ldots}}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

$\downarrow$ 219

$$\frac{\frac{2(cx(4ce(bd-2ae)^2-d(2cd-be)(4ace-3b^2e+4bcd))-d(2ace+b^2(-e)+bcd)(4ace-3b^2e+4bcd)+4ace(bd-2ae)(2cd-be))}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{3de^3(b^2-4ac)\text{arct}\ldots}{(\ldots}}{3(b^2-4ac)(ae^2-bde+cd^2)}$$
$$\frac{2(cx(bd-2ae)+a(2cd-be))}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(ae^2-bde+cd^2)}$$

input
```
Int[x/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2 - b*d*e
+ a*e^2)*(a + b*x + c*x^2)^(3/2)) + ((2*(4*a*c*e*(b*d - 2*a*e)*(2*c*d - b*
e) - d*(b*c*d - b^2*e + 2*a*c*e)*(4*b*c*d - 3*b^2*e + 4*a*c*e) + c*(4*c*e*
(b*d - 2*a*e)^2 - d*(2*c*d - b*e)*(4*b*c*d - 3*b^2*e + 4*a*c*e))*x))/((b^2
 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*c
)*d*e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*
e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2))/(3*(b^2 - 4*a
*c)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(290) = 580$.

Time = 1.38 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.74

method	result
default	$\frac{\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}}{e} - \frac{d\left(\frac{e^2}{3\left(a\,e^2-bde+c\,d^2\right)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}\right)^{\frac{3}{2}}} - \frac{(be-2cd \cdots}{\cdots}\right.}{\cdots}$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2))-d/e^2*(1/3/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)
^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)-1/2*(b*e-2*c*d)*e/
(a*e^2-b*d*e+c*d^2)*(2/3*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d
^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(3/2)+16/3*c/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)^
2*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))+1/(a*e^2-b*d*e+c*d^2)*e^2*(1/(a*e^2-b*d*e+c*d^2)*e^
2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2
*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+
c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. $2(290) = 580$.

Time = 1.40 (sec) , antiderivative size = 4596, normalized size of antiderivative = 15.02

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8378 vs. $2(290) = 580$.

Time = 0.47 (sec) , antiderivative size = 8378, normalized size of antiderivative = 27.38

$$\int \frac{x}{(d+ex)\,(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2*d*e^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 + b*d*e - a*e^2))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2
*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 2/3*((((8*b*
c^10*d^15 - 62*b^2*c^9*d^14*e + 8*a*c^10*d^14*e + 207*b^3*c^8*d^13*e^2 + 1
2*a*b*c^9*d^13*e^2 - 388*b^4*c^7*d^12*e^3 - 276*a*b^2*c^8*d^12*e^3 + 32*a^
2*c^9*d^12*e^3 + 445*b^5*c^6*d^11*e^4 + 938*a*b^3*c^7*d^11*e^4 + 48*a^2*b*
c^8*d^11*e^4 - 318*b^6*c^5*d^10*e^5 - 1530*a*b^4*c^6*d^10*e^5 - 810*a^2*b^
2*c^7*d^10*e^5 + 24*a^3*c^8*d^10*e^5 + 137*b^7*c^4*d^9*e^6 + 1392*a*b^5*c^
5*d^9*e^6 + 2165*a^2*b^3*c^6*d^9*e^6 + 340*a^3*b*c^7*d^9*e^6 - 32*b^8*c^3*
d^8*e^7 - 712*a*b^6*c^4*d^8*e^7 - 2640*a^2*b^4*c^5*d^8*e^7 - 1720*a^3*b^2*
c^6*d^8*e^7 - 80*a^4*c^7*d^8*e^7 + 3*b^9*c^2*d^7*e^8 + 186*a*b^7*c^3*d^7*e
^8 + 1638*a^2*b^5*c^4*d^7*e^8 + 2940*a^3*b^3*c^5*d^7*e^8 + 840*a^4*b*c^6*d
^7*e^8 - 18*a*b^8*c^2*d^6*e^9 - 478*a^2*b^6*c^3*d^6*e^9 - 2260*a^3*b^4*c^4
*d^6*e^9 - 2130*a^4*b^2*c^5*d^6*e^9 - 200*a^5*c^6*d^6*e^9 + 45*a^2*b^7*c^2
*d^5*e^10 + 736*a^3*b^5*c^3*d^5*e^10 + 2105*a^4*b^3*c^4*d^5*e^10 + 948*a^5
*b*c^5*d^5*e^10 - 60*a^3*b^6*c^2*d^4*e^11 - 780*a^4*b^4*c^3*d^4*e^11 - 133
2*a^5*b^2*c^4*d^4*e^11 - 192*a^6*c^5*d^4*e^11 + 45*a^4*b^5*c^2*d^3*e^12 +
602*a^5*b^3*c^3*d^3*e^12 + 512*a^6*b*c^4*d^3*e^12 - 18*a^5*b^4*c^2*d^2*e^1
3 - 326*a^6*b^2*c^3*d^2*e^13 - 88*a^7*c^4*d^2*e^13 + 3*a^6*b^3*c^2*d*e^14
+ 108*a^7*b*c^3*d*e^14 - 16*a^8*c^3*e^15)*x/(b^4*c^8*d^16 - 8*a*b^2*c^9...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(x/((d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{x}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

3.49 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 307

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{3\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$-\frac{2(4ace(2cd-be)^2+\left(bcd-b^2e+2ace\right)\left(3\left(b^2-4ac\right)e^2-4cd(2cd-be)\right)-c(2cd-be)\left(8c^2d^2-3b^2e^2-\right.}{3\left(b^2-4ac\right)^2\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{e^4\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{5/2}}$$

output
```
1/3*(-2*b*c*d+2*b^2*e-4*a*c*e-2*c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*
e+c*d^2)/(c*x^2+b*x+a)^(3/2)-2/3*(4*a*c*e*(-b*e+2*c*d)^2+(2*a*c*e-b^2*e+b*
c*d)*(3*(-4*a*c+b^2)*e^2-4*c*d*(-b*e+2*c*d))-c*(-b*e+2*c*d)*(8*c^2*d^2-3*b
^2*e^2-4*c*e*(-5*a*e+2*b*d))*x)/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^
2+b*x+a)^(1/2)+e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 10.52 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2(-b^2e+2c(ae+cdx)+bc(d-ex))}{3\left(b^2-4ac\right)\left(cd^2+e(-bd+ae)\right)(a+x(b+cx))^{3/2}}$$
$$+\frac{2(3b^4e^3+b^3ce^2(d+3ex)+8c^2(3a^2e^3+2c^2d^3x+5acde^2x)+4bc^2(2cd^2(d-3ex)+5ae^2(d-ex))+2b^2c}{3\left(b^2-4ac\right)^2\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$
$$-\frac{e^4\operatorname{arctanh}\left(\frac{-bd+2ae-2cdx+bex}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\left(cd^2+e(-bd+ae)\right)^{5/2}}$$

input
```
Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x)))/(3*(b^2 - 4*a*c)*(c*d^
2 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(3/2)) + (2*(3*b^4*e^3 + b^3*c*e^2
*(d + 3*e*x) + 8*c^2*(3*a^2*e^3 + 2*c^2*d^3*x + 5*a*c*d*e^2*x) + 4*b*c^2*(
2*c*d^2*(d - 3*e*x) + 5*a*e^2*(d - e*x)) + 2*b^2*c*e*(-11*a*e^2 + c*d*(-6*
d + e*x))))/(3*(b^2 - 4*a*c)^2*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b
+ c*x)]) - (e^4*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 +
 e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(5/
2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1165, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}dx$$
$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{8c^2d^2-3b^2e^2-4ce(bd-3ae)+4ce(2cd-be)x}{2(d+ex)(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{8c^2d^2-3b^2e^2-4ce(bd-3ae)+4ce(2cd-be)x}{(d+ex)(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 1235

$$-\frac{\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d^2\right)+4ace(2cd-be)^2\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{2\int\frac{3\left(b^2-4ac\right)\cdots}{2(d+ex)\sqrt{cx^2\cdots}}}{(b^2-4ac)(ae^2-b\cdots}}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 27

$$-\frac{\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d^2\right)+4ace(2cd-be)^2\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{3e^4\left(b^2-4ac\right)\int\frac{\cdots}{(\cdots}}{ae^2-\cdots}}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 1154

$$-\frac{\frac{6e^4\left(b^2-4ac\right)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{ae^2-bde+cd^2}+\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+\cdots\right)\right)}{(b^2-4ac)\sqrt{a+bx\cdots}}}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 219

$$-\frac{\frac{2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d^2\right)+4ace(2cd-be)^2\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}-\frac{3e^4\left(b^2-4ac\right)\text{arc}\cdots}{\cdots}}{3\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[1/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) - ((2*(4*a*c*e*(2*c*d - b*e)^2
- (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(b*d - 3*a*e))
- c*(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*x))/((b^
2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*
c)*e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2))/(3*(b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(293) = 586$.

Time = 1.44 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.50

method	result
default	$\frac{e^2}{3\left(a\,e^2-bde+c\,d^2\right)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}\right)^{\frac{3}{2}}}-\frac{(be-2cd)e\left(\frac{\frac{4c\left(x+\frac{d}{e}\right)}{3}+\frac{2(be-2cd)}{3e}}{\left(\frac{4c\left(a\,e^2-bde+c\,d^2\right)}{e^2}-\frac{(be-2cd)^2}{e^2}\right)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)}{e}\right.}\right.}{}$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(1/3/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(3/2)-1/2*(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2/3*(2*c*(
x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(
x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+16/3*c/(4*c*
(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)^2*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(
c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))+1/(a*e^2
-b*d*e+c*d^2)*e^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*
c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(
c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-
b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(292) = 584$.

Time = 1.36 (sec) , antiderivative size = 4612, normalized size of antiderivative = 15.02

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(1/((d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8447 vs. $2(292) = 584$.

Time $= 0.50$ (sec) , antiderivative size $= 8447$, normalized size of antiderivative $= 27.51$

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*e^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 2/3*((((16*c^11
*d^15 - 120*b*c^10*d^14*e + 386*b^2*c^9*d^13*e^2 + 136*a*c^10*d^13*e^2 - 6
89*b^3*c^8*d^12*e^3 - 884*a*b*c^9*d^12*e^3 + 732*b^4*c^7*d^11*e^4 + 2412*a
*b^2*c^8*d^11*e^4 + 480*a^2*c^9*d^11*e^4 - 451*b^5*c^6*d^10*e^5 - 3542*a*b
^3*c^7*d^10*e^5 - 2640*a^2*b*c^8*d^10*e^5 + 130*b^6*c^5*d^9*e^6 + 2950*a*b
^4*c^6*d^9*e^6 + 5910*a^2*b^2*c^7*d^9*e^6 + 920*a^3*c^8*d^9*e^6 + 9*b^7*c^
4*d^8*e^7 - 1296*a*b^5*c^5*d^8*e^7 - 6795*a^2*b^3*c^6*d^8*e^7 - 4140*a^3*b
*c^7*d^8*e^7 - 16*b^8*c^3*d^7*e^8 + 184*a*b^6*c^4*d^7*e^8 + 4080*a^2*b^4*c
^5*d^7*e^8 + 7240*a^3*b^2*c^6*d^7*e^8 + 1040*a^4*c^7*d^7*e^8 + 3*b^9*c^2*d
^6*e^9 + 58*a*b^7*c^3*d^6*e^9 - 1050*a^2*b^5*c^4*d^6*e^9 - 6020*a^3*b^3*c^
5*d^6*e^9 - 3640*a^4*b*c^6*d^6*e^9 - 18*a*b^8*c^2*d^5*e^10 - 30*a^2*b^6*c^
3*d^5*e^10 + 2220*a^3*b^4*c^4*d^5*e^10 + 4590*a^4*b^2*c^5*d^5*e^10 + 696*a
^5*c^6*d^5*e^10 + 45*a^2*b^7*c^2*d^4*e^11 - 160*a^3*b^5*c^3*d^4*e^11 - 237
5*a^4*b^3*c^4*d^4*e^11 - 1740*a^5*b*c^5*d^4*e^11 - 60*a^3*b^6*c^2*d^3*e^12
 + 340*a^4*b^4*c^3*d^3*e^12 + 1356*a^5*b^2*c^4*d^3*e^12 + 256*a^6*c^5*d^3*
e^12 + 45*a^4*b^5*c^2*d^2*e^13 - 294*a^5*b^3*c^3*d^2*e^13 - 384*a^6*b*c^4*
d^2*e^13 - 18*a^5*b^4*c^2*d*e^14 + 122*a^6*b^2*c^3*d*e^14 + 40*a^7*c^4*d*e
^14 + 3*a^6*b^3*c^2*e^15 - 20*a^7*b*c^3*e^15)*x/(b^4*c^8*d^16 - 8*a*b^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{1}{(ex+d)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(1/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

3.50 $\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 515

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(c(b^2-2ac)\,d-b^3e+3abce+c(bcd-b^2e+2ace)\,x)}{3a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$+\,2(3b^6de^2+b^4cd(3cd^2-16ae^2)+24a^2c^3d(cd^2+2ae^2)-2ab^2c^2d(11cd^2+4ae^2)-4a^2bc^2e(14cd^2+17ae^2$$

$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{5/2}d}-\frac{e^5\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d\left(cd^2-bde+ae^2\right)^{5/2}}$$

output
```
2/3*(c*(-2*a*c+b^2)*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x)/a/(-4*a*c
+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(3/2)+2/3*(3*b^6*d*e^2+b^4*c*d*(-1
6*a*e^2+3*c*d^2)+24*a^2*c^3*d*(2*a*e^2+c*d^2)-2*a*b^2*c^2*d*(4*a*e^2+11*c*
d^2)-4*a^2*b*c^2*e*(17*a*e^2+14*c*d^2)+a*b^3*c*e*(43*a*e^2+44*c*d^2)-6*b^5
*(a*e^3+c*d^2*e)+c*(3*b^5*d*e^2+b^3*c*d*(-14*a*e^2+3*c*d^2)-4*a*b*c^2*d*(4
*a*e^2+5*c*d^2)-8*a^2*c^2*e*(5*a*e^2+2*c*d^2)+2*a*b^2*c*e*(19*a*e^2+20*c*d
^2)-6*b^4*(a*e^3+c*d^2*e))*x)/a^2/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*
x^2+b*x+a)^(1/2)-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2
)/d-e^5*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 11.66 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{\frac{2(b^2-2ac+bcx)}{a(a+x(b+cx))^{3/2}}+\frac{2((b^2-6ac)(3b^2-4ac)+bc(3b^2-20ac)x)}{a^2(b^2-4ac)\sqrt{a+x(b+cx)}}+\frac{2e(-b^2e+2c(ae+cdx)+bc(\cdots}{(cd^2+e(-bd+ae))(a+x(b+\cdots}}{\cdots}$$

input
```
Integrate[1/(x*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x))/(a*(a + x*(b + c*x))^(3/2)) + (2*((b^2 - 6*a*c)
*(3*b^2 - 4*a*c) + b*c*(3*b^2 - 20*a*c)*x))/(a^2*(b^2 - 4*a*c)*Sqrt[a + x*
(b + c*x)]) + (2*e*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x)))/((c*d^2
 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(3/2)) - (2*e*(3*b^4*e^3 + b^3*c*e^
2*(d + 3*e*x) + 8*c^2*(3*a^2*e^3 + 2*c^2*d^3*x + 5*a*c*d*e^2*x) + 4*b*c^2*
(2*c*d^2*(d - 3*e*x) + 5*a*e^2*(d - e*x)) + 2*b^2*c*e*(-11*a*e^2 + c*d*(-6
*d + e*x))))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c
*x)]) - (3*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*
x)])])/a^(5/2) + (3*(b^2 - 4*a*c)*e^5*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x +
b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 +
 e*(-(b*d) + a*e))^(5/2))/(3*(b^2 - 4*a*c)*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{5/2}}-\frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{5/2}}\right)dx$$

$$\big\downarrow \; 2009$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{5/2}d}+\frac{2\left(bcx\left(3b^2-20ac\right)+\left(b^2-6ac\right)\left(3b^2-4ac\right)\right)}{3a^2d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{e^5\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d\left(ae^2-bde+cd^2\right)^{5/2}}+$$

$$\frac{2e\big(-cx(2cd-be)\big(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\big)-\big(2ace+b^2(-e)+bcd\big)\big(-4ce(bd-3ae)-3b^2e^2+8c^2d^2}{3d\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}$$

$$\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{3ad\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[1/(x*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(3*a*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^(3/2)) +
(2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a*c)*d*(c*
d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) + (2*((b^2 - 6*a*c)*(3*b^2 -
 4*a*c) + b*c*(3*b^2 - 20*a*c)*x))/(3*a^2*(b^2 - 4*a*c)^2*d*Sqrt[a + b*x +
 c*x^2]) + (2*e*(4*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^
2*d^2 - 3*b^2*e^2 - 4*c*e*(b*d - 3*a*e)) - c*(2*c*d - b*e)*(8*c^2*d^2 - 3*
b^2*e^2 - 4*c*e*(2*b*d - 5*a*e))*x))/(3*(b^2 - 4*a*c)^2*d*(c*d^2 - b*d*e +
 a*e^2)^2*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])]/(a^(5/2)*d) - (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d*(c*d^2 - b*d*e
 + a*e^2)^(5/2))
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.85

method	result
default	$\frac{\frac{1}{3a\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^{2}\right)^{2}\sqrt{c\,x^{2}+bx+a}}\right)}{2a}+\frac{\frac{1}{a\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\cdots}{x}\right)}{a^{\frac{3}{2}}}}{a}}{d}$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))+1/a*(1/a/(
c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)
*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-1/d*(1/3/(a*e^2-b*d*e+c*d
^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)-
1/2*(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2/3*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*
c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+16/3*c/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(
b*e-2*c*d)^2/e^2)^2*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(c*(x+d/e)^2+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))+1/(a*e^2-b*d*e+c*d^2)*e^2*(1/(a*e
^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/
(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4174 vs. $2(492) = 984$.

Time = 19.05 (sec) , antiderivative size = 16795, normalized size of antiderivative = 32.61

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(1/(x*(d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(5/2)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{x\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(1/(x*(d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 13522, normalized size of antiderivative = 26.26

$$\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(48*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e
**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**7*c**2*e**5 - 24
*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*b**2*c*e**5 + 96*
sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*b*c**2*e**5*x + 96
*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*c**3*e**5*x**2 +
3*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**
2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*b**4*e**5 - 48*s
qrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*b**3*c*e**5*x + 96*
sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*b*c**3*e**5*x**3 +
 48*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e
**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*c**4*e**5*x**4
 + 6*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*
e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*b**5*e**5*x -
 18*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e
**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**4*b**4*c*e**5...
```

3.51 $\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 628

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx =$$

$$-\frac{2(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e+c(b^2cd-2ac^2d-b^3e+3abce)\,x)}{3a^2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$-\frac{2(6b^7de^2+b^5cd(6cd^2-37ae^2)-ab^3c^2d(43cd^2-26ae^2)+24a^3c^3e(cd^2+2ae^2)+4a^2bc^3d(17cd^2+20ae^2}{}$$

$$-\frac{\sqrt{a+bx+cx^2}}{a^3dx}+\frac{(5bd+2ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{7/2}d^2}$$

$$+\frac{e^6\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2\left(cd^2-bde+ae^2\right)^{5/2}}$$

output
```
1/3*(-2*b^3*c*d+6*a*b*c^2*d+2*b^4*e-8*a*b^2*c*e+4*a^2*c^2*e-2*c*(3*a*b*c*e
-2*a*c^2*d-b^3*e+b^2*c*d)*x)/a^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b
*x+a)^(3/2)-2/3*(6*b^7*d*e^2+b^5*c*d*(-37*a*e^2+6*c*d^2)-a*b^3*c^2*d*(-26*
a*e^2+43*c*d^2)+24*a^3*c^3*e*(2*a*e^2+c*d^2)+4*a^2*b*c^3*d*(20*a*e^2+17*c*
d^2)+a*b^4*c*e*(70*a*e^2+89*c*d^2)-2*a^2*b^2*c^2*e*(72*a*e^2+79*c*d^2)-3*b
^6*(3*a*e^3+4*c*d^2*e)+c*(6*b^6*d*e^2+2*b^4*c*d*(-16*a*e^2+3*c*d^2)-2*a*b^
2*c^2*d*(-2*a*e^2+19*c*d^2)+8*a^2*c^3*d*(8*a*e^2+5*c*d^2)-4*a^2*b*c^2*e*(2
4*a*e^2+25*c*d^2)+a*b^3*c*e*(62*a*e^2+79*c*d^2)-3*b^5*(3*a*e^3+4*c*d^2*e))
*x)/a^3/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)-(c*x^2+b*
x+a)^(1/2)/a^3/d/x+1/2*(2*a*e+5*b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+
b*x+a)^(1/2))/a^(7/2)/d^2+e^6*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 12.69 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{-\frac{4e(b^2-2ac+bcx)}{a(a+x(b+cx))^{3/2}}+\frac{4d(b^2-2ac+bcx)}{ax(a+x(b+cx))^{3/2}}-\frac{4e((b^2-6ac)(3b^2-4ac)+bc(3b^2-20ac)x)}{a^2(b^2-4ac)\sqrt{a+x(b+cx)}}+}{}$$

input
```
Integrate[1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
((-4*e*(b^2 - 2*a*c + b*c*x))/(a*(a + x*(b + c*x))^(3/2)) + (4*d*(b^2 - 2*
a*c + b*c*x))/(a*x*(a + x*(b + c*x))^(3/2)) - (4*e*((b^2 - 6*a*c)*(3*b^2 -
 4*a*c) + b*c*(3*b^2 - 20*a*c)*x))/(a^2*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)
]) + (4*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((c*d^2 + e*(-(b
*d) + a*e))*(a + x*(b + c*x))^(3/2)) + (4*e^2*(3*b^4*e^3 + b^3*c*e^2*(d +
3*e*x) + 8*c^2*(3*a^2*e^3 + 2*c^2*d^3*x + 5*a*c*d*e^2*x) + 4*b*c^2*(2*c*d^
2*(d - 3*e*x) + 5*a*e^2*(d - e*x)) + 2*b^2*c*e*(-11*a*e^2 + c*d*(-6*d + e*
x))))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b + c*x)]) +
 (6*(b^2 - 4*a*c)*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]
)/a^(5/2) + (d*((2*Sqrt[a]*(64*a^3*c^2 + 15*b^4*x*(b + c*x) + 5*a*b^2*(b^2
 - 22*b*c*x - 20*c^2*x^2) + 4*a^2*c*(-9*b^2 + 46*b*c*x + 32*c^2*x^2)))/(x*
Sqrt[a + x*(b + c*x)]) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x)/(2*Sqrt[
a]*Sqrt[a + x*(b + c*x)])]))/(a^(7/2)*(-b^2 + 4*a*c)) - (6*(b^2 - 4*a*c)*e
^6*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(5/2))/(6*(b^2 -
 4*a*c)*d^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1289}$$

$$\int \left(\frac{e^2}{d^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}-\frac{e}{d^2x\left(a+bx+cx^2\right)^{5/2}}+\frac{1}{dx^2\left(a+bx+cx^2\right)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{5/2}d^2}+\frac{5b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{7/2}d}-\frac{2e\left(bcx\left(3b^2-20ac\right)+\left(b^2-6ac\right)\left(3b^2-4ac\right)\right)}{3a^2d^2\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}+\frac{2\left(32a^2c^2+bcx\left(5b^2-28ac\right)-32ab^2c+5b^4\right)}{3a^2dx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+bx+cx^2}}{3a^3dx\left(b^2-4ac\right)^2}+\frac{e^6\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2\left(ae^2-bde+cd^2\right)^{5/2}}-\frac{2e^2\left(-cx(2cd-be)\left(-4ce(2bd-5ae)-3b^2e^2+8c^2d^2\right)-\left(2ace+b^2(-e)+bcd\right)\left(-4ce(bd-3ae)-3b^2e^2+8c^2d\right.\right.}{3d^2\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{3d^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{2e\left(-2ac+b^2+bcx\right)}{3ad^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(-2ac+b^2+bcx\right)}{3adx\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*e*(b^2 - 2*a*c + b*c*x))/(3*a*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^(3/2
)) + (2*(b^2 - 2*a*c + b*c*x))/(3*a*(b^2 - 4*a*c)*d*x*(a + b*x + c*x^2)^(3
/2)) - (2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/(3*(b^2 - 4*a
*c)*d^2*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) + (2*(5*b^4 - 32*
a*b^2*c + 32*a^2*c^2 + b*c*(5*b^2 - 28*a*c)*x))/(3*a^2*(b^2 - 4*a*c)^2*d*x
*Sqrt[a + b*x + c*x^2]) - (2*e*((b^2 - 6*a*c)*(3*b^2 - 4*a*c) + b*c*(3*b^2
 - 20*a*c)*x))/(3*a^2*(b^2 - 4*a*c)^2*d^2*Sqrt[a + b*x + c*x^2]) - (2*e^2*
(4*a*c*e*(2*c*d - b*e)^2 - (b*c*d - b^2*e + 2*a*c*e)*(8*c^2*d^2 - 3*b^2*e^
2 - 4*c*e*(b*d - 3*a*e)) - c*(2*c*d - b*e)*(8*c^2*d^2 - 3*b^2*e^2 - 4*c*e*
(2*b*d - 5*a*e))*x))/(3*(b^2 - 4*a*c)^2*d^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt
[a + b*x + c*x^2]) - ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + b*x +
c*x^2])/(3*a^3*(b^2 - 4*a*c)^2*d*x) + (5*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(2*a^(7/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + b*x + c*x^2])])/(a^(5/2)*d^2) + (e^6*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d^2*
(c*d^2 - b*d*e + a*e^2)^(5/2))
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(602) = 1204$.

Time = 1.68 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.97

method	result	size
default	Expression too large to display	1239
risch	Expression too large to display	12685

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/x/(c*x^2+b*x+a)^(3/2)-5/2*b/a*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a
*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*
x+b)/(c*x^2+b*x+a)^(1/2))+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
1/2))/x)))-4*c/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*
a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+e/d^2*(1/3/(a*e^2-b*d*e+c*d^2)*
e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)-1/2*
(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2/3*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a
*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+16/3*c/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-
2*c*d)^2/e^2)^2*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))+1/(a*e^2-b*d*e+c*d^2)*e^2*(1/(a*e^2-b
*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c
*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))-e/d^2*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b
/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5177 vs. $2(600) = 1200$.

Time = 38.37 (sec) , antiderivative size = 20807, normalized size of antiderivative = 33.13

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(d+ex\right)\left(a+bx+cx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(1/(x**2*(d + e*x)*(a + b*x + c*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(cx^2+bx+a\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(5/2)*(e*x + d)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10475 vs. 2(600) = 1200.

Time = 0.58 (sec) , antiderivative size = 10475, normalized size of antiderivative = 16.68

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*e^6*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c^2*d^6 - 2*b*c*d^5*e + b^2*d^4*e^2 + 2*a*c*d^4*e^
2 - 2*a*b*d^3*e^3 + a^2*d^2*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 2/3*((((6
*a^8*b^4*c^10*d^15 - 38*a^9*b^2*c^11*d^15 + 40*a^10*c^12*d^15 - 48*a^8*b^5
*c^9*d^14*e + 307*a^9*b^3*c^10*d^14*e - 340*a^10*b*c^11*d^14*e + 168*a^8*b
^6*c^8*d^13*e^2 - 1040*a^9*b^4*c^9*d^13*e^2 + 976*a^10*b^2*c^10*d^13*e^2 +
 304*a^11*c^11*d^13*e^2 - 336*a^8*b^7*c^7*d^12*e^3 + 1876*a^9*b^5*c^8*d^12
*e^3 - 648*a^10*b^3*c^9*d^12*e^3 - 2280*a^11*b*c^10*d^12*e^3 + 420*a^8*b^8
*c^6*d^11*e^4 - 1820*a^9*b^6*c^7*d^11*e^4 - 2464*a^10*b^4*c^8*d^11*e^4 + 6
390*a^11*b^2*c^9*d^11*e^4 + 984*a^12*c^10*d^11*e^4 - 336*a^8*b^9*c^5*d^10*
e^5 + 658*a^9*b^7*c^6*d^10*e^5 + 6496*a^10*b^5*c^7*d^10*e^5 - 7403*a^11*b^
3*c^8*d^10*e^5 - 6396*a^12*b*c^9*d^10*e^5 + 168*a^8*b^10*c^4*d^9*e^6 + 448
*a^9*b^8*c^5*d^9*e^6 - 6720*a^10*b^6*c^6*d^9*e^6 - 60*a^11*b^4*c^7*d^9*e^6
 + 15620*a^12*b^2*c^8*d^9*e^6 + 1760*a^13*c^9*d^9*e^6 - 48*a^8*b^11*c^3*d^
8*e^7 - 620*a^9*b^9*c^4*d^8*e^7 + 2984*a^10*b^7*c^5*d^8*e^7 + 8991*a^11*b^
5*c^6*d^8*e^7 - 16450*a^12*b^3*c^7*d^8*e^7 - 9680*a^13*b*c^8*d^8*e^7 + 6*a
^8*b^12*c^2*d^7*e^8 + 274*a^9*b^10*c^3*d^7*e^8 + 88*a^10*b^8*c^4*d^7*e^8 -
 8810*a^11*b^6*c^5*d^7*e^8 + 3350*a^12*b^4*c^6*d^7*e^8 + 19430*a^13*b^2*c^
7*d^7*e^8 + 1880*a^14*c^8*d^7*e^8 - 45*a^9*b^11*c^2*d^6*e^9 - 556*a^10*b^9
*c^3*d^6*e^9 + 2835*a^11*b^7*c^4*d^6*e^9 + 7456*a^12*b^5*c^5*d^6*e^9 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17328, normalized size of antiderivative = 27.59

$$\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(96*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**8*c**2*e**6*x - 48*
sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b
*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**7*b**2*c*e**6*x + 192*s
qrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*
d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**7*b*c**2*e**6*x**2 + 192
*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**7*c**3*e**6*x**3 + 6*s
qrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*
d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*b**4*e**6*x - 96*sqrt(
a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e
+ c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*b**3*c*e**6*x**2 + 192*sqr
t(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*
e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*b*c**3*e**6*x**4 + 96*sq
rt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d
*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**6*c**4*e**6*x**5 + 12*sqr
t(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*
e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*b**5*e**6*x**2 - 36*sqrt
(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e
 + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**5*b**4*c*e**6*x**3 - 96*...
```

3.52 $\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 551

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{ce\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,dg^2\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{ce^2\sqrt{gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}\,d^2g^2\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{b+\sqrt{}}\right.}{e^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{gx}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*g*(g*x)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*
((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c/e/(-c*x/(b+(-4*a*c+b^
2)^(1/2)))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a*c+b^2)^(1/2)*d*g^2*(-
c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*El
lipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*
c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/c/e^2/(g*x)^(1/2)/(c*x^2+b*x+a
)^(1/2)+4*2^(1/2)*(-4*a*c+b^2)^(1/2)*d^2*g^2*(-c*x/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c
+b^2)^(1/2))*e),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))
/e^2/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 12.88 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.89

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}(gx)^{3/2}\sqrt{\frac{c(a+x(b+cx))}{-b^2+4ac}}\left(e\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)xE\left(\mathrm{arc}\right.\right.}{}$$

input
```
Integrate[(g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*(g*x)^(3/2)*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 +
4*a*c)]*(e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x*EllipticE[ArcSin[Sqrt[(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 -
4*a*c])/(-b + Sqrt[b^2 - 4*a*c])] + 2*d*Sqrt[(c*x)/(-b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[-((c*x)/(b + Sqrt[b^2 - 4*a*c]))]*((-2*c*d + (b + Sqrt[b^2 - 4*a*c
])*e)*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] + 2*c*d*Elli
pticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e), ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqr
t[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])))/(c*e^2*(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)*x^2*Sqrt[(c*x)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + x*(b + c*x)
])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{d^2g^2}{e^2\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}-\frac{dg^2}{e^2\sqrt{gx}\sqrt{a+bx+cx^2}}+\frac{g\sqrt{gx}}{e\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{2}dg^{3/2}\sqrt{\sqrt{b^2-4ac}-b}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{gx}}{\sqrt{\sqrt{b^2-4ac}-b}\sqrt{g}}\right),\frac{b-}{b+}\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}}$$
$$\frac{\sqrt[4]{a}g\sqrt{gx}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}e\sqrt{x}\sqrt{a+bx+cx^2}}-$$
$$\frac{2\sqrt[4]{a}g\sqrt{gx}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}e\sqrt{x}\sqrt{a+bx+cx^2}}-$$
$$\frac{dg^2\sqrt{x}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt[4]{c}e^2\sqrt{gx}\sqrt{a+bx+cx^2}}+$$
$$\frac{2g\sqrt{gx}\sqrt{a+bx+cx^2}}{\sqrt{c}e\left(\sqrt{a}+\sqrt{c}x\right)}$$

input
```
Int[(g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*g*Sqrt[g*x]*Sqrt[a + b*x + c*x^2])/(Sqrt[c]*e*(Sqrt[a] + Sqrt[c]*x)) -
(2*a^(1/4)*g*Sqrt[g*x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[
a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*e*Sqrt[x]*Sqrt[a + b*x + c*x^2]) - (d*g^2*S
qrt[x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(a^(1/4)*c^(1/4)*e^2*Sqrt[g*x]*Sqrt[a + b*x + c*x^2]) + (a^(1/4)*g*S
qrt[g*x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(c^(3/4)*e*Sqrt[x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[-b + Sqr
t[b^2 - 4*a*c]]*d*g^(3/2)*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*EllipticPi[((b - Sqrt[b^2 - 4*a*c])*e)
/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[g*x])/(Sqrt[-b + Sqrt[b^2 - 4*a*c]]
*Sqrt[g])], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[c]*e^2
*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(488) = 976$.

Time = 1.97 (sec) , antiderivative size = 1071, normalized size of antiderivative = 1.94

method	result	size
elliptic	Expression too large to display	1071
default	Expression too large to display	1419

input
```
int((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/g/x*(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(x*g*(c*x^2+b*x+a))^(1/2)*(-d/e^2*g^
2*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4
*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(c*g*x^3+b*g*x^2+a*g*x)^(1/2)*EllipticF(2^(1/2)*((x+1
/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-
4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2))+g^2/e*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^
(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*g*x^3+b*g*x^2+a*g*x)^(1/2)*((-
1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*
(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(
b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2))+d^2*g^2/e^3*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-
4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx)^{\frac{3}{2}}}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((g*x)**(3/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(g\,x)^{3/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx)^{\frac{3}{2}}}{(ex+d)\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.53 $\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 384

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{ce\sqrt{gx}\sqrt{a+bx+cx^2}} - \frac{4\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\ldots\right)}{e\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{gx}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*(-4*a*c+b^2)^(1/2)*g*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2
))/c/e/(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*(-4*a*c+b^2)^(1/2)*d*g*(-
c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*El
lipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2
)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+
(-4*a*c+b^2)^(1/2)))^(1/2))/e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(g*x)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.14 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{i\sqrt{2}\sqrt{gx}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x}\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{El}\right.}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e\sqrt{x}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[g*x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d)
, I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*
Sqrt[x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1284, 1171, 1170, 1279, 187, 25, 413, 413, 412, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1284}$$

$$\frac{g\int \frac{1}{\sqrt{gx}\sqrt{cx^2+bx+a}}dx}{e} - \frac{dg\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1171}$$

$$\frac{g\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{e\sqrt{gx}}-\frac{dg\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1170}$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{dg\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1279}$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{dg\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{\sqrt{gx}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)}dx}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{187}$$

$$\frac{2dg\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(dg+exg)}d\sqrt{gx}}{e\sqrt{a+bx+cx^2}}+\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{2dg\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(dg+exg)}d\sqrt{gx}}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{2dg\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\int \frac{1}{\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}(dg+exg)}d\sqrt{gx}}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{2dg\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}(dg+exg)}d\sqrt{gx}}{e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 412$$

$$\frac{2g\sqrt{x}\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{e\sqrt{gx}}-\frac{\sqrt{2}\sqrt{g}\sqrt{\sqrt{b^2-4ac}-b}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{gx}}{\sqrt{\sqrt{b^2-4ac}-b}\sqrt{g}}\right),\frac{b-\sqrt{}}{b+\sqrt{}}\right.}{\sqrt{c}e\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{g\sqrt{x}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt[4]{c}e\sqrt{gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{g}\sqrt{\sqrt{b^2-4ac}-b}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{gx}}{\sqrt{\sqrt{b^2-4ac}-b}\sqrt{g}}\right),\frac{b-\sqrt{}}{b+\sqrt{}}\right.}{\sqrt{c}e\sqrt{a+bx+cx^2}}$$

input
```
Int[Sqrt[g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(a^(1/4)*c^(1/4)*e*Sqrt[g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*S
qrt[-b + Sqrt[b^2 - 4*a*c]]*Sqrt[g]*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*EllipticPi[((b - Sqrt[b^2 -
4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[g*x])/(Sqrt[-b + Sqrt[b^2
 - 4*a*c]]*Sqrt[g])], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(S
qrt[c]*e*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1170
```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2
 Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, b, c}, x] && EqQ[m^2, 1/4]
```

rule 1171
```
Int[((e_)*(x_))^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> S
imp[(e*x)^m/x^m   Int[x^m/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c,
e}, x] && EqQ[m^2, 1/4]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1284
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]), x], x] + Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{gx}\,\sqrt{xg(c\,x^2+bx+a)}\left(\frac{g\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\operatorname{EllipticF}\left(\sqrt{\cdots}\right.}{ec\sqrt{cg\,x^3+bg\,x^2+agx}}\right.}{\cdots}$
default	$-\frac{2\left(2\operatorname{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ace-\operatorname{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)b^2e+\text{Ellipti}\cdots\right.}{\cdots}$

input
```
int((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/g/x*(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)*(x*g*(c*x^2+b*x+a))^(1/2)*(g/e*(b+(-
4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^
2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(c*g*x^3+b*g*x^2+a*g*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b
^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2))-d*g/e^2*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-
2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*g*x^3+b*g*x^2+a*g*x)^(1/2)/(d/e-1/2
*(b+(-4*a*c+b^2)^(1/2))/c)*EllipticPi(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4
*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

input
```
integrate((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{g\,x}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{gx}}{(ex+d)\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.54 $\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4}}{b+\sqrt{b^2-}}\right.}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{gx}\sqrt{a+bx+cx^2}}$$

output
```
4*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*x^
2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e),
2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/(2*c*d-(b+(-4*a
*c+b^2)^(1/2))*e)/(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{3/2}\sqrt{2+\frac{4a}{bx-\sqrt{b^2-4ac}x}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticP}\right.}{\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\,d\sqrt{gx}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/(Sqrt[g*x]*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[2 + (4*a)/(b*x
 - Sqrt[b^2 - 4*a*c]*x)]*(EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^
2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] -
 EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[(Sqrt[2]*Sqrt[a
/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]))/(Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[g*x]*Sqrt[a + x*(b +
 c*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{1279}$$
$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{\sqrt{gx}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(d+ex)}dx}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 187$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(dg+exg)}d\sqrt{gx}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}(dg+exg)}d\sqrt{gx}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 413$$

$$\frac{2\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\int \frac{1}{\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}(dg+exg)}d\sqrt{gx}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 413$$

$$\frac{2\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}(dg+exg)}d\sqrt{gx}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 412$$

$$\frac{\sqrt{2}\sqrt{\sqrt{b^2-4ac}-b}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{gx}}{\sqrt{\sqrt{b^2-4ac}-b}\sqrt{g}}\right),\frac{b-\sqrt{b^2-\ldots}}{b+\sqrt{b^2-\ldots}}\right.}{\sqrt{c}d\sqrt{g}\sqrt{a+bx+cx^2}}$$

input

```
Int[1/(Sqrt[g*x]*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output

```
(Sqrt[2]*Sqrt[-b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*EllipticPi[((b - Sqrt[b^2
- 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[g*x])/(Sqrt[-b + Sqrt[b
^2 - 4*a*c]]*Sqrt[g])], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/
(Sqrt[c]*d*Sqrt[g]*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.10

method	result
default	$-\frac{2\left(b+\sqrt{-4ac+b^2}\right)\operatorname{EllipticPi}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\left(b+\sqrt{-4ac+b^2}\right)e}{\sqrt{-4ac+b^2}\,e+be-2cd},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+\cdots}}{\sqrt{-4ac+b^{\cdots}}}}\cdots}{\sqrt{c\,x^2+bx+a}\left(\sqrt{-4ac+b^2}\,e+be-2cd\right)\sqrt{gx}}$
elliptic	$\frac{\sqrt{xg(c\,x^2+bx+a)}\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\operatorname{EllipticPi}\left(\sqrt{2}\cdots\right)}{\sqrt{gx}\sqrt{c\,x^2+bx+a}\,ec\sqrt{cg\,x^3+bg\,x^2+agx}\left(\frac{d}{e}-\frac{b+\sqrt{\cdots}}{\cdots}\cdots\right)}$

input
```
int(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b+(-4*a*c+b^2)^(1/2))*EllipticPi(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*
a*c+b^2)^(1/2)))^(1/2),(b+(-4*a*c+b^2)^(1/2))*e/((-4*a*c+b^2)^(1/2)*e+b*e-
2*c*d),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*(-c*
x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2
)^(1/2))^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
/(c*x^2+b*x+a)^(1/2)/((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d)/(g*x)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{gx}\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(g*x)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(g*x)*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx}}\,dx$$

input
```
integrate(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx}}\,dx$$

input
```
integrate(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{g\,x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{gx}\,\left(ex+d\right)\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/(g*x)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.55 $\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 419

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{2\sqrt{a+bx+cx^2}}{adg\sqrt{gx}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{adg^2\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx+cx^2}}$$
$$-\frac{4\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-}}{b+\sqrt{b^2-}}\right.}{d\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)g\sqrt{gx}\sqrt{a+bx+cx^2}}$$

output
```
-2*(c*x^2+b*x+a)^(1/2)/a/d/g/(g*x)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)
^(1/2)))^(1/2))/a/d/g^2/(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^2+b*x+a)^
(1/2)-4*2^(1/2)*(-4*a*c+b^2)^(1/2)*e*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1
/2))*e),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))/d/(2*c*
d-(b+(-4*a*c+b^2)^(1/2))*e)/g/(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.24 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.01

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x^{5/2}\sqrt{\frac{4a+2bx-2\sqrt{b^2-4ac}x}{bx-\sqrt{b^2-4ac}x}}\left(\left(-b+\sqrt{b^2-4ac}\right)dE\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right.}{}$$

input
```
Integrate[1/((g*x)^(3/2)*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-1/2*I)*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(5/2)*Sqrt[(4*a +
2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*((-b + Sqrt[b^
2 - 4*a*c])*d*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]
)/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*d - Sqrt
[b^2 - 4*a*c]*d + 2*a*e)*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2
 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] -
2*a*e*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[(Sqrt[2]*S
qrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])]))/(a*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*d^2*(g*x)^(3/2)*Sqrt
[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1288}$$

$$\int \left(\frac{1}{d(gx)^{3/2}\sqrt{a+bx+cx^2}}-\frac{e}{dg\sqrt{gx}(d+ex)\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{c}\sqrt{gx}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}dg^2\sqrt{x}\sqrt{a+bx+cx^2}}-\frac{2\sqrt[4]{c}\sqrt{gx}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}dg^2\sqrt{x}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}e\sqrt{\sqrt{b^2-4ac}-b}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\operatorname{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{gx}}{\sqrt{\sqrt{b^2-4ac}-b}\sqrt{g}}\right),\frac{b-\sqrt{b^2-}}{b+\sqrt{b^2-}}\right.}{\sqrt{c}d^2g^{3/2}\sqrt{a+bx+cx^2}}+\frac{2\sqrt{c}\sqrt{gx}\sqrt{a+bx+cx^2}}{adg^2\left(\sqrt{a}+\sqrt{c}x\right)}-\frac{2\sqrt{a+bx+cx^2}}{adg\sqrt{gx}}$$

input
```
Int[1/((g*x)^(3/2)*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x + c*x^2])/(a*d*g*Sqrt[g*x]) + (2*Sqrt[c]*Sqrt[g*x]*Sqrt[a
 + b*x + c*x^2])/(a*d*g^2*(Sqrt[a] + Sqrt[c]*x)) - (2*c^(1/4)*Sqrt[g*x]*(S
qrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^
(3/4)*d*g^2*Sqrt[x]*Sqrt[a + b*x + c*x^2]) + (c^(1/4)*Sqrt[g*x]*(Sqrt[a] +
 Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*d*
g^2*Sqrt[x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[-b + Sqrt[b^2 - 4*a*c]]
*e*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x)/(b + Sqrt[b^
2 - 4*a*c])]*EllipticPi[((b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[
2]*Sqrt[c]*Sqrt[g*x])/(Sqrt[-b + Sqrt[b^2 - 4*a*c]]*Sqrt[g])], (b - Sqrt[b
^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[c]*d^2*g^(3/2)*Sqrt[a + b*x +
 c*x^2])
```

Defintions of rubi rules used

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(371) = 742$.

Time = 3.96 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{2\sqrt{c\,x^{2}+bx+a}}{adg\sqrt{gx}}+\frac{\left(\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)c}{b+\sqrt{-4ac+b^{2}}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{b+\sqrt{-4ac+b^{2}}}{2c}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^{2}}}}\left(-\frac{b+\sqrt{-4ac}\cdots}{2c}\right.\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{xg(c\,x^{2}+bx+a)}\left(-\frac{2\left(cg\,x^{2}+bgx+ag\right)}{g^{2}ad\sqrt{x\left(cg\,x^{2}+bgx+ag\right)}}+\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)c}{b+\sqrt{-4ac+b^{2}}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{b+\sqrt{-4ac+b^{2}}}{2c}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{-\frac{\cdots}{b+\cdots}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*x^2+b*x+a)^(1/2)/a/d/g/(g*x)^(1/2)+1/d/a*((b+(-4*a*c+b^2)^(1/2))*2^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*g*x^3+
b*g*x^2+a*g*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2
)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b
^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)
^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1
/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(
1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2
))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-a*(b+(-4*a*c+b^2)^(1/2))/c*2^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*g*x^3
+b*g*x^2+a*g*x)^(1/2)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*EllipticPi(2^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),-1/2*(
b+(-4*a*c+b^2)^(1/2))/c/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c),1/2*(-2*(b+(-4*
a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)))/g*(x*g*(c*x^2+b*x+a))^(1/2)/(g*x)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(gx)^{\frac{3}{2}}\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(g*x)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((g*x)**(3/2)*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\,(gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\,(gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(g\,x)^{3/2}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(gx)^{3/2}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(gx)^{\frac{3}{2}}\,(ex+d)\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/(g*x)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.56 $\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = -\frac{1}{288(1-x)^2} - \frac{7}{864(1-x)} + \frac{975+2204x}{39744(3+5x+4x^2)} + \frac{6023\arctan\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}} + \frac{11\log(1-x)}{2304} - \frac{11\log(3+5x+4x^2)}{4608}$$

output
```
-1/288/(1-x)^2-7/(864-864*x)+(975+2204*x)/(158976*x^2+198720*x+119232)+602
3/1218816*arctan(1/23*(5+8*x)*23^(1/2))*23^(1/2)+11/2304*ln(1-x)-11/4608*l
n(4*x^2+5*x+3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{-\frac{25392}{(-1+x)^2} + \frac{59248}{-1+x} + \frac{184(975+2204x)}{3+5x+4x^2} + 36138\sqrt{23}\arctan\left(\frac{5+8x}{\sqrt{23}}\right) + 34914\log(1-x) - 17457\log(3+5x+4}{7312896}$$

input
```
Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

output
```
(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^
2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*
Log[3 + 5*x + 4*x^2])/7312896
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-1)^3 (4x^2+5x+3)^2} \, dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{276} \int -\frac{3(44x+19)}{(1-x)^3 (4x^2+5x+3)} dx + \frac{44x+39}{276(1-x)^2 (4x^2+5x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{44x+39}{276(1-x)^2 (4x^2+5x+3)} - \frac{1}{92} \int \frac{44x+19}{(1-x)^3 (4x^2+5x+3)} dx$$

$$\Big\downarrow \text{1200}$$

$$\frac{44x+39}{276(1-x)^2 (4x^2+5x+3)} - \frac{1}{92} \int \left(\frac{1012x-2379}{576 (4x^2+5x+3)} - \frac{253}{576(x-1)} + \frac{97}{48(x-1)^2} - \frac{21}{4(x-1)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{92} \left(\frac{6023 \arctan\left(\frac{8x+5}{\sqrt{23}}\right)}{576\sqrt{23}} - \frac{253 \log\left(4x^2+5x+3\right)}{1152} - \frac{97}{48(1-x)} - \frac{21}{8(1-x)^2} + \frac{253}{576} \log(1-x)\right) + \frac{44x+39}{276(1-x)^2 (4x^2+5x+3)}$$

input
```
Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

output
```
(39 + 44*x)/(276*(1 - x)^2*(3 + 5*x + 4*x^2)) + (-21/(8*(1 - x)^2) - 97/(4
8*(1 - x)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(576*Sqrt[23]) + (253*Log[1
 - x])/576 - (253*Log[3 + 5*x + 4*x^2])/1152)/92
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	siz
default	$-\frac{-\frac{2204x}{23}-\frac{975}{23}}{6912\left(x^2+\frac{5}{4}x+\frac{3}{4}\right)}-\frac{11\ln\left(4x^2+5x+3\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}-\frac{1}{288(x-1)^2}+\frac{7}{864(x-1)}+\frac{11\ln(x-1)}{2304}$	68
risch	$\frac{\frac{97}{1104}x^3-\frac{407}{4416}x^2-\frac{5}{184}x-\frac{15}{1472}}{(x-1)^2\left(4x^2+5x+3\right)}-\frac{11\ln\left(64x^2+80x+48\right)}{4608}+\frac{6023\arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816}+\frac{11\ln(x-1)}{2304}$	71

input
```
int(x/(x-1)^3/(4*x^2+5*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6912*(-2204/23*x-975/23)/(x^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/1
218816*arctan(1/23*(5+8*x)*23^(1/2))*23^(1/2)-1/288/(x-1)^2+7/864/(x-1)+11
/2304*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$$
$$= \frac{214176\,x^3+12046\,\sqrt{23}(4\,x^4-3\,x^3-3\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(8\,x+5)\right)-224664\,x^2-5819\,(4\,x^4-}{2437632\,(4\,x^4-}$$

input
```
integrate(x/(x-1)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

output
```
1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arc
tan(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 -
x + 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x
- 1) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{388x^3-407x^2-120x-45}{17664x^4-13248x^3-13248x^2-4416x+13248} + \frac{11\log(x-1)}{2304} - \frac{11\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608} + \frac{6023\sqrt{23}\,\mathrm{atan}\left(\frac{8\sqrt{23}x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

input
```
integrate(x/(x-1)**3/(4*x**2+5*x+3)**2,x)
```

output
```
(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
 4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 +
6023*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2}\,dx = \frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right) + \frac{388x^3-407x^2-120x-45}{4416(4x^4-3x^3-3x^2-x+3)} - \frac{11}{4608}\log\left(4x^2+5x+3\right) + \frac{11}{2304}\log(x-1)$$

input
```
integrate(x/(x-1)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

output
```
6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 -
407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2
+ 5*x + 3) + 11/2304*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = \frac{6023}{1218816}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right) + \frac{388\,x^3-407\,x^2-120\,x-45}{4416\,(4\,x^2+5\,x+3)(x-1)^2} - \frac{11}{4608}\,\log\left(4\,x^2+5\,x+3\right) + \frac{11}{2304}\,\log\left(|x-1|\right)$$

input
```
integrate(x/(x-1)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

output
```
6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 -
407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = \frac{11\,\ln(x-1)}{2304} + \frac{-\frac{97\,x^3}{4416}+\frac{407\,x^2}{17664}+\frac{5\,x}{736}+\frac{15}{5888}}{-x^4+\frac{3\,x^3}{4}+\frac{3\,x^2}{4}+\frac{x}{4}-\frac{3}{4}} - \ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(\frac{11}{4608}+\frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right) + \ln\left(x+\frac{5}{8}+\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(-\frac{11}{4608}+\frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right)$$

input
```
int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)
```

output
```
(11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/5
888)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 +
 5/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8
)*((23^(1/2)*6023i)/2437632 - 11/4608)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.64

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx$$
$$= \frac{48184\sqrt{23}\, atan\left(\frac{8x+5}{\sqrt{23}}\right) x^4 - 36138\sqrt{23}\, atan\left(\frac{8x+5}{\sqrt{23}}\right) x^3 - 36138\sqrt{23}\, atan\left(\frac{8x+5}{\sqrt{23}}\right) x^2 - 12046\sqrt{23}\, atan\left(\frac{8x}{\sqrt{}}\right.}{}$$

input
```
int(x/(x-1)^3/(4*x^2+5*x+3)^2,x)
```

output
```
(48184*sqrt(23)*atan((8*x + 5)/sqrt(23))*x**4 - 36138*sqrt(23)*atan((8*x +
 5)/sqrt(23))*x**3 - 36138*sqrt(23)*atan((8*x + 5)/sqrt(23))*x**2 - 12046*
sqrt(23)*atan((8*x + 5)/sqrt(23))*x + 36138*sqrt(23)*atan((8*x + 5)/sqrt(2
3)) - 23276*log(4*x**2 + 5*x + 3)*x**4 + 17457*log(4*x**2 + 5*x + 3)*x**3
+ 17457*log(4*x**2 + 5*x + 3)*x**2 + 5819*log(4*x**2 + 5*x + 3)*x - 17457*
log(4*x**2 + 5*x + 3) + 46552*log(x - 1)*x**4 - 34914*log(x - 1)*x**3 - 34
914*log(x - 1)*x**2 - 11638*log(x - 1)*x + 34914*log(x - 1) + 285568*x**4
- 438840*x**2 - 137632*x + 189336)/(2437632*(4*x**4 - 3*x**3 - 3*x**2 - x
+ 3))
```

3.57 $\int \frac{x(1+x)^2}{\left(1+x+x^2\right)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x(1+x)^2}{\left(1+x+x^2\right)^3} \, dx = \frac{1-x}{6\left(1+x+x^2\right)^2} - \frac{1}{2\left(1+x+x^2\right)}$$

output
```
1/6*(1-x)/(x^2+x+1)^2-1/(2*x^2+2*x+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x(1+x)^2}{\left(1+x+x^2\right)^3} \, dx = -\frac{2+4x+3x^2}{6\left(1+x+x^2\right)^2}$$

input
```
Integrate[(x*(1 + x)^2)/(1 + x + x^2)^3,x]
```

output
```
-1/6*(2 + 4*x + 3*x^2)/(1 + x + x^2)^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1233, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x+1)^2}{(x^2+x+1)^3}\,dx$$

$$\Big\downarrow \ 1233$$

$$-\frac{1}{6}\int -\frac{2x+1}{(x^2+x+1)^2}dx-\frac{(x+1)(2x+1)}{6\,(x^2+x+1)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\int \frac{2x+1}{(x^2+x+1)^2}dx-\frac{(x+1)(2x+1)}{6\,(x^2+x+1)^2}$$

$$\Big\downarrow \ 1104$$

$$-\frac{(x+1)(2x+1)}{6\,(x^2+x+1)^2}-\frac{1}{6\,(x^2+x+1)}$$

input
```
Int[(x*(1 + x)^2)/(1 + x + x^2)^3,x]
```

output
```
-1/6*((1 + x)*(1 + 2*x))/(1 + x + x^2)^2 - 1/(6*(1 + x + x^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{-\frac{1}{2}x^2-\frac{2}{3}x-\frac{1}{3}}{(x^2+x+1)^2}$	20
norman	$\frac{-\frac{1}{2}x^2-\frac{2}{3}x-\frac{1}{3}}{(x^2+x+1)^2}$	20
risch	$\frac{-\frac{1}{2}x^2-\frac{2}{3}x-\frac{1}{3}}{(x^2+x+1)^2}$	20
gosper	$-\frac{3x^2+4x+2}{6(x^2+x+1)^2}$	21
parallelrisch	$\frac{-3x^2-4x-2}{6(x^2+x+1)^2}$	21
orering	$-\frac{3x^2+4x+2}{6(x^2+x+1)^2}$	21

input
```
int(x*(x+1)^2/(x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+x+1)^2*(-1/2*x^2-2/3*x-1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = -\frac{3\,x^2+4\,x+2}{6\,(x^4+2\,x^3+3\,x^2+2\,x+1)}$$

input
```
integrate(x*(1+x)^2/(x^2+x+1)^3,x, algorithm="fricas")
```

output
```
-1/6*(3*x^2 + 4*x + 2)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = \frac{-3x^2-4x-2}{6x^4+12x^3+18x^2+12x+6}$$

input
```
integrate(x*(1+x)**2/(x**2+x+1)**3,x)
```

output
```
(-3*x**2 - 4*x - 2)/(6*x**4 + 12*x**3 + 18*x**2 + 12*x + 6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = -\frac{3\,x^2+4\,x+2}{6\,(x^4+2\,x^3+3\,x^2+2\,x+1)}$$

input
```
integrate(x*(1+x)^2/(x^2+x+1)^3,x, algorithm="maxima")
```

output
```
-1/6*(3*x^2 + 4*x + 2)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = -\frac{3\,x^2+4\,x+2}{6\,(x^2+x+1)^2}$$

input
```
integrate(x*(1+x)^2/(x^2+x+1)^3,x, algorithm="giac")
```

output
```
-1/6*(3*x^2 + 4*x + 2)/(x^2 + x + 1)^2
```

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = -\frac{3\,x^2+4\,x+2}{6\,(x^2+x+1)^2}$$

input
```
int((x*(x + 1)^2)/(x + x^2 + 1)^3,x)
```

output
```
-(4*x + 3*x^2 + 2)/(6*(x + x^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{x(1+x)^2}{(1+x+x^2)^3}\,dx = \frac{-3x^2-4x-2}{6x^4+12x^3+18x^2+12x+6}$$

input
```
int(x*(1+x)^2/(x^2+x+1)^3,x)
```

output
```
( - 3*x**2 - 4*x - 2)/(6*(x**4 + 2*x**3 + 3*x**2 + 2*x + 1))
```

3.58 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 490

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}$$

$$+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))\,(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3}$$

$$+\frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e-\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\cdots}{\sqrt{2cd-\left(b-\sqrt{\cdots}\right.}}\cdots\right.}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{\sqrt{2}\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e+\frac{b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\cdots}{\sqrt{2cd-\left(b+\sqrt{\cdots}\right.}}\cdots\right.}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*b*(-2*a*c+b^2)*(e*x+d)^(1/2)/c^4+2/3*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(
e*x+d)^(3/2)/c^3/e^3-2/5*(b*e+2*c*d)*(e*x+d)^(5/2)/c^2/e^3+2/7*(e*x+d)^(7/
2)/c/e^3+2^(1/2)*(b^3*c*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e-(-5*a^2*
b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^
(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))
*e)^(1/2))/c^(9/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*(b^3*c*d
-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b
^3*c*e-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(9/2)/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.28

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{2\sqrt{d+ex}(-105b^3e^3-7c^2e(d+ex)(-2bd+5ae+3bex)+c^3(8d^3-4d^2ex+3de^2x^2+15e^3x^3)+35bce^2}{105c^4e^3}$$

$$+\frac{\left(ib^5e-b^3c\left(\sqrt{-b^2+4ac}d+5iae\right)+abc^2\left(2\sqrt{-b^2+4ac}d+5iae\right)+ab^2c\left(4icd-3\sqrt{-b^2+4ac}e\right)+b^4(}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+(b-}}$$

$$+\frac{\left(-ib^5e+abc^2\left(2\sqrt{-b^2+4ac}d-5iae\right)+b^3c\left(-\sqrt{-b^2+4ac}d+5iae\right)+ab^2c\left(-4icd-3\sqrt{-b^2+4ac}e\right)}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+(b+}}$$

input
```
Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e
*x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a
*e + b*(d + e*x))))/(105*c^4*e^3) + ((I*b^5*e - b^3*c*(Sqrt[-b^2 + 4*a*c]*
d + (5*I)*a*e) + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e) + a*b^2*c*((
4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*((-I)*c*d + Sqrt[-b^2 + 4*a*c]*e)
 + a^2*c^2*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-
1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)*b^5
*e + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e) + b^3*c*(-(Sqrt[-b^2 + 4
*a*c]*d) + (5*I)*a*e) + a*b^2*c*((-4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^
4*(I*c*d + Sqrt[-b^2 + 4*a*c]*e) + a^2*c^2*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]
*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^
2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[
-b^2 + 4*a*c])*e])
```

Rubi [A] (verified)

Time = 7.96 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^3}{ce^2}-\frac{(2cd+be)(d+ex)^2}{c^2e^2}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)}{c^3e^2}-\frac{b\left(b^2-2ac\right)e}{c^4}+\frac{b\left(b^2-2ac\right)\left(cd^2-bed+ae^2\right)-\left(-eb^4+cdb^3+3aceb^2\right.}{c^4e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\right.}\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left(-5\right.}{}\right.$$

input
```
Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(-((b*(b^2 - 2*a*c)*e*Sqrt[d + e*x])/c^4) + ((c^2*d^2 + b^2*e^2 + c*e*(
b*d - a*e))*(d + e*x)^(3/2))/(3*c^3*e^2) - ((2*c*d + b*e)*(d + e*x)^(5/2))
/(5*c^2*e^2) + (d + e*x)^(7/2)/(7*c*e^2) + (e*(b^3*c*d - 2*a*b*c^2*d - b^4
*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^
5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(
9/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
 - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2
]*c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-e^{3}\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,\sqrt{2}\left(\left(\left(a^{2}c^{2}-3ca\,b^{2}+b^{4}\right)e+2ab\,c^{2}d-b^{3}cd\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}-5e\left(b\left(a^{2}c^{2}-ca\,b^{2}\right.\right.\right.}{}$
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex}\right.}{c^{4}}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex}\right.}{c^{4}}$
risch	$\frac{2\left(15c^{3}x^{3}e^{3}-21b\,c^{2}e^{3}x^{2}+3c^{3}d\,e^{2}x^{2}-35a\,c^{2}e^{3}x+35x\,b^{2}c\,e^{3}-7b\,c^{2}d\,e^{2}x-4d^{2}e\,c^{3}x+210abc\,e^{3}-35d\,e^{2}a\,c^{2}-105b^{3}e^{3}+3\right.}{105e^{3}c^{4}}$

input
```
int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-e^3*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*(((a^2*c^
2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*c-1/4*b^2))^(1/2)-5*e*(
b*(a^2*c^2-c*a*b^2+1/5*b^4)*e-2/5*d*(a^2*c^2-2*c*a*b^2+1/2*b^4)*c))*arctan
h((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1
/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e^3*2^(1/2)*(((a
^2*c^2-3*a*b^2*c+b^4)*e+2*a*b*c^2*d-b^3*c*d)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+
5*e*(b*(a^2*c^2-c*a*b^2+1/5*b^4)*e-2/5*d*(a^2*c^2-2*c*a*b^2+1/2*b^4)*c))*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2))+4*(e*x+d)^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)
*((1/14*c^3*x^3-1/6*x*(3/5*b*x+a)*c^2+b*(1/6*b*x+a)*c-1/2*b^3)*e^3-1/6*(-3
/35*c^2*x^2+(1/5*b*x+a)*c-b^2)*d*c*e^2+1/15*d^2*(-2/7*c*x+b)*c^2*e+4/105*d
^3*c^3)*(-4*e^2*(a*c-1/4*b^2))^(1/2)))/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^
(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^
2*(a*c-1/4*b^2))^(1/2)/c^4/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5507 vs. $2(436) = 872$.

Time = 0.65 (sec) , antiderivative size = 5507, normalized size of antiderivative = 11.24

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**4*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^4}{cx^2+bx+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(436) = 872$.

Time = 0.33 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.45

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c
^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e)*c^
2*e^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^
2*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)
*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*
(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c^3 - 14*a*b^3*c^4 + 12
*a^2*b*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/
sqrt(-(2*c^8*d*e^24 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 +
a*c^7*e^26)*c^8*e^24 + (2*c^8*d*e^24 - b*c^7*e^25)^2))/(c^8*e^24)))/((sqrt
(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*
c^6*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*
e)*((b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*
c^2 - 4*a^3*c^3)*e)*c^2*e^2 + 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d
^2 - (b^4*c^2 - 2*a*b^2*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*
c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*
e)*abs(c)*abs(e) + (2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2*e - (3*b^5*c
^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e^2 + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b
^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sq
rt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^8*d*e^24 - b*c^7*e^25 - sqrt(-4*(c^8*d...
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 13879, normalized size of antiderivative = 28.32

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^
2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c
*d*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^
5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^
3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d +
e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10
*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7
*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)
^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d
*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^
2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*
c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e
^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)
^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^
5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e
*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e
 - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^...
```

Reduce [F]

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{x^4\sqrt{ex+d}}{c\,x^2+bx+a}dx$$

input
```
int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

output
```
int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

3.59 $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 397

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}$$
$$-\frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*(-a*c+b^2)*(e*x+d)^(1/2)/c^3-2/3*(b*e+c*d)*(e*x+d)^(3/2)/c^2/e^2+2/5*(e*
x+d)^(5/2)/c/e^2-2^(1/2)*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e-(-2*a^2*c^2*e+4*
a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c
^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(7/2)/(2*c*
d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-2^(1/2)*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e
+(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+b^2)^(1/2))*
arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/
2))/c^(7/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.17

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex}(15b^2e^2+c^2(-2d^2+dex+3e^2x^2)-5ce(3ae+b(d+ex)))}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd+\sqrt{\right.\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+}}}{}$$

input
```
Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) -
 5*c*e*(3*a*e + b*(d + e*x))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b
^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d
 + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*
(b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d
+ 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2
- 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e]))/(15*c^(7/2))
```

Rubi [A] (verified)

Time = 4.12 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1199}$$

$$\frac{2\int \left(\frac{(d+ex)^2}{ce}-\frac{(cd+be)(d+ex)}{c^2e}+\frac{(b^2-ac)e}{c^3}-\frac{(b^2-ac)\left(cd^2-bed+ae^2\right)-\left(-eb^3+cdb^2+2aceb-ac^2d\right)(d+ex)}{c^3e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)}\right)d\sqrt{d+ex}}{e}$$

$\Big\downarrow$ 2009

$$\frac{2\left(-\frac{e\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{e\left(\frac{-2a^2c^2e+4ab^2ce-3abc}{\sqrt{b^2-4a}}\right.}{}\right.}{e}$$

input
```
Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(((b^2 - a*c)*e*Sqrt[d + e*x])/c^3 - ((c*d + b*e)*(d + e*x)^(3/2))/(3*c
^2*e) + (d + e*x)^(5/2)/(5*c*e) - (e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
 - (e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/
2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{a c^2 d}{2}+b\left(ae+\frac{bd}{2}\right)c-\frac{e b^3}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(a\left(\frac{3bd}{2}+ae\right)c^2+\left(-2eab^2-\frac{1}{2}b^3 d\right)c+\frac{b^4 e}{2}\right)\right)e^2\sqrt{2}\sqrt{\left(be-2c\right.}}{}$
risch	$-\frac{2\left(-3c^2e^2x^2+5e^2xbc-c^2dex+15ace^2-15b^2e^2+5bcde+2c^2d^2\right)\sqrt{ex+d}}{15e^2c^3}+\frac{-\left(2a^2c^2e^2-4ab^2ce^2+3abc^2de+b^4e^2-b^3c\right.}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4ab^2ce^2+3abc^2de+b^4e^2-b^3\right.}{}\right.}{}}{}$
default	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(-\frac{\left(2a^2c^2e^2-4ab^2ce^2+3abc^2de+b^4e^2-b^3\right.}{}\right.}{}}{}$

input
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*(((-1/2*a*c^2*d+b*(a*e+1/2*b*d)*c-1/2*e*b^3)*(-4*e^2*(a*c-1/4*b^2))^(1/
2)+e*(a*(3/2*b*d+a*e)*c^2+(-2*e*a*b^2-1/2*b^3*d)*c+1/2*b^4*e))*e^2*2^(1/2)
*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*
c*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(((1/2*a*c^
2*d+(-a*b*e-1/2*b^2*d)*c+1/2*e*b^3)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(3/2
*b*d+a*e)*c^2+(-2*e*a*b^2-1/2*b^3*d)*c+1/2*b^4*e))*e^2*2^(1/2)*arctan((e*x
+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(e
*x+d)^(1/2)*(2/15*(-3/2*e*x+d)*(e*x+d)*c^2+e*((1/3*b*x+a)*e+1/3*b*d)*c-b^2
*e^2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-4*e^2*(a*c-1/4*
b^2))^(1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((b*e-2*
c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b
^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/e^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4245 vs. $2(347) = 694$.

Time = 0.36 (sec) , antiderivative size = 4245, normalized size of antiderivative = 10.69

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**3*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(347) = 694$.

Time = 0.33 (sec) , antiderivative size = 1074, normalized size of antiderivative = 2.71

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^
2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e)*c^2*e^2 - 2*((b^2*c^
3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d
*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c -
 sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^4 - 3*a*b*c^5)*d^2*e -
(3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*
a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2
*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d
^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13
)^2))/(c^6*e^12)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d
*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 -
6*a*b^3*c + 8*a^2*b*c^2)*e)*c^2*e^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*
c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)
*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a
bs(c)*abs(e) + (2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4
+ 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/s
qrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a
*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c^6*e^12)))/((sq...
```

Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 11143, normalized size of antiderivative = 28.07

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*
e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^
6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*
c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a
^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c
^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6
*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*
e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c
*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c
^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d
+ 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(1
6*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 +
 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e
^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e
^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28...
```

Reduce [F]

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{x^3\sqrt{ex+d}}{c\,x^2+bx+a}dx$$

input
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

output
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

3.60 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce} + \frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*b*(e*x+d)^(1/2)/c^2+2/3*(e*x+d)^(3/2)/c/e+2^(1/2)*(b*c*d-b^2*e+a*c*e-(3
*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1
/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(5/2)/(2*c*d-(
b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c
^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1
/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(5/2)/(2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.19

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex}(-3be+c(d+ex))}{e} + \frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4a}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{3c^{5/2}}$$

input
```
Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/e + (3*Sqrt[2]*(-(b^3*e)
 + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)
- a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2
 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a
*c])*e]))/(3*c^(5/2))
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1199$$

$$\frac{2\int\left(-\frac{be}{c^2}+\frac{d+ex}{c}+\frac{b(cd^2-bed+ae^2)-(-eb^2+cdb+ace)(d+ex)}{c^2\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)e}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{e\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right.}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2\right.}}}\right.}{e}$$

input
```
Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*(-((b*e*Sqrt[d + e*x])/c^2) + (d + e*x)^(3/2)/(3*c) + (e*(b*c*d - b^2*e
 + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))
/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{2(-cex+3be-cd)\sqrt{ex+d}}{3e\,c^{2}}-\frac{8\left(-\frac{\left(-3abc\,e^{2}+2a\,c^{2}de+e^{2}b^{3}-b^{2}cde+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e+\sqrt{-e^{2}\left(4a\right.}}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\right.}}}\right.}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+\sqrt{ex+d}\,be\right)}{c^{2}}+\frac{8e\left(\frac{\left(-3abc\,e^{2}+2a\,c^{2}de+e^{2}b^{3}-b^{2}cde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e-\sqrt{-e^{2}\left(4ac-\right.}}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-\right.}}}\right.}{}}{}$
default	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+\sqrt{ex+d}\,be\right)}{c^{2}}+\frac{8e\left(\frac{\left(-3abc\,e^{2}+2a\,c^{2}de+e^{2}b^{3}-b^{2}cde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ace+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,b^{2}e-\sqrt{-e^{2}\left(4ac-\right.}}{8c\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-\right.}}}\right.}{}}{}$
pseudoelliptic	$-\frac{2\left(-\frac{e\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,\sqrt{2}\left(\left((ae+bd)c-e\,b^{2}\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}+2a\,c^{2}de+\left(-3ab\,e^{2}-de\,b^{2}\right)c+e^{2}b^{3}\right)\operatorname{arctanh}\left(\right.}{2}\right.}{}$

input
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-c*e*x+3*b*e-c*d)*(e*x+d)^(1/2)/e/c^2-8/c*(-1/8*(-3*a*b*c*e^2+2*a*c^
2*d*e+e^2*b^3-b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a*c-b^2))^
(1/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(3*a*b*c*e
^2-2*a*c^2*d*e-e^2*b^3+b^2*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*e-(-e^2*(4*a
*c-b^2))^(1/2)*b^2*e+(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(270) = 540$.

Time = 0.17 (sec) , antiderivative size = 2966, normalized size of antiderivative = 9.39

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*
a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^
3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*
b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3)*e - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 +
 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/
(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4
*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3
*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3
*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2
)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2
*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
- 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 +...
```

Sympy [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x**2*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(270) = 540$.

Time = 0.30 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.84

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2
)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*c^4
- 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*c^3)
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)
*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c
^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/
((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a
*c)*a*c^4*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2
 + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b
^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*
abs(c)*abs(e) + (2*(b^2*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e
^2 + (b^4*c^2 - 3*a*b^2*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5
- sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 -
 b*c^3*e^5)^2))/(c^4*e^4)))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c
)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2*abs(e)) + 2/3*((e*x + d)^(3
/2)*c^2*e^2 - 3*sqrt(e*x + d)*b*c*e^3)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 8171, normalized size of antiderivative = 25.86

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4
- b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3)
)/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^
3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b
^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))
^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^
3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1
/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4
*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2
*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(
-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*
b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^...
```

Reduce [F]

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x^2\sqrt{ex+d}}{c\,x^2+bx+a}dx$$

input
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

output
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

3.61 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx &= \frac{2\sqrt{d+ex}}{c}\\ &+ \frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\\ &- \frac{\sqrt{2}\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\end{aligned}$$

output
```
2*(e*x+d)^(1/2)/c+2^(1/2)*(b*c*d-b^2*e+2*a*c*e-(-4*a*c+b^2)^(1/2)*(-b*e+c*
d))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)
^(1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-
2^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-4*a*c+b^2)^(1/2)*(-b*e+c*d))*arctanh(2^(1/2
)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(3/2)/(-
4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.19

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{d+ex} - \frac{\left(-ibcd-c\sqrt{-b^2+4ac}d+ib^2e-2iace+b\sqrt{-b^2+4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} - \frac{\left(ibcd-c\sqrt{-b^2+4ac}d-ib^2e+2\ldots}{\sqrt{-\frac{b^2}{2}\ldots}}}{c^{3/2}}$$

input

```
Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output

```
(2*Sqrt[c]*Sqrt[d + e*x] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*e
 - (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c
]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 +
 4*a*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1196, 25, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1196}$$

$$\frac{\int -\frac{ae-(cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c} + \frac{2\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{d+ex}}{c}-\frac{\int \frac{ae-(cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c}$$

$$\Big\downarrow \text{1197}$$

$$\frac{2\sqrt{d+ex}}{c}-\frac{2\int \frac{cd^2-bed+ae^2-(cd-be)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2\sqrt{d+ex}}{c}-$$

$$\frac{2\left(\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}}{2\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\int \frac{1}{\frac{1}{2}\left(\left(b+\right.\right.}}{2\sqrt{b^2-4ac}}\right.}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d+ex}}{c}-$$

$$\frac{2\left(\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\text{arctanh}\left(\frac{}{\sqrt{2cd}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.}{c}$$

input
```
Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[d + e*x])/c - (2*(-(((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*
(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e])) + ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c
*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}+\frac{\left(2ac\,e^2-b^2e^2+bcde-\sqrt{-e^2\left(4ac-b^2\right)}\,be+\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(2ac\,e \cdots\right.}{\cdots}}{c}$
derivativedivides	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}} \cdots$
default	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde-\sqrt{-e^2(4ac-b^2)}\,be+\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}} \cdots$
risch	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde+\sqrt{-e^2(4ac-b^2)}\,be-\sqrt{-e^2(4ac-b^2)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right. \cdots}}\right. \cdots}{c\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}$

input
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+b*c*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b
*e+(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(2*a*c*e^2-b^2*e^2+b*c*d*e+(-e
^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))
^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*
x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(241) = 482$.

Time = 0.11 (sec) , antiderivative size = 1721, normalized size of antiderivative = 6.00

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3
*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4
- 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4
*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2
*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c
 - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*s
qrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b
^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)
*sqrt(e*x + d)) + sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
- (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 -
 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l...
```

Sympy [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(x*sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(241) = 482$.

Time = 0.30 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.64

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c
^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*c^4*d^2*e
- (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4*c^2*d +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-
(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d -
b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e
 + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^
2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2
- 4*a*c)*a*c^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c
)*abs(e) + (2*b*c^4*d^2*e - (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b
*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt
(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e +
a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - s
qrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 5664, normalized size of antiderivative = 19.74

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^
4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d
+ e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*
c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5
*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c
*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d
^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7
*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2)*1i - (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^
3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*(
(8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)...
```

Reduce [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 1844, normalized size of antiderivative = 6.43

$$\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*sqrt(a*e**2
 - b*d*e + c*d**2)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*
e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e +
 c*d**2) + b*e - 2*c*d))*b*c + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 -
 b*d*e + c*d**2) + b*e - 2*c*d))*a*c*e - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e
**2 - b*d*e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*
d*e + c*d**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*b**2*e + 2*sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(
a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*
sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*b*c*d - 2*sqrt(2*sqr
t(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*sqrt(a*e**2 - b*d*e + c*
d**2)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) +
2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*
e - 2*c*d))*b*c - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) +
 b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2
*c*d) + 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2) + b*e - 2*c*d))*a*c*e + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*...
```

3.62 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-2^(1/2)*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*arctanh(2^(1/2)*c^(1/2)*(e
*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(1/2)/(-4*a*c+b^2)^(
1/2)+2^(1/2)*(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)*arctanh(2^(1/2)*c^(1/2
)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(1/2)/(-4*a*c+b^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{\sqrt{2}\left(\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}}$$

input
```
Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(Sqrt[2]*((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*
c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 +
4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sq
rt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(Sqrt[
c]*Sqrt[-b^2 + 4*a*c])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1148, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1148}$$

$$2e\int \frac{d+ex}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}$$

$$\downarrow \text{1450}$$

$$2e\left(\frac{1}{2}\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}+\frac{1}{2}\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{1}{2}\left(\left(\right.\right.}\right.$$

$$\downarrow \text{221}$$

$$2e\left(-\frac{\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)$$

input
```
Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
2*e*(-(((1 + (2*c*d - b*e)/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((1 - (2*c*d - b*e)/(Sqrt[b^2
 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{e\sqrt{2}\left(-\frac{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}+\frac{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\cdots\right.}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4a\cdots\right.\right.}}\right.}{\sqrt{-e^{2}\left(4ac-b^{2}\right)}}$
derivativedivides	$8ec\left(-\frac{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}+\frac{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\cdots}{8c\sqrt{-e^{2}(4ac-\cdots}}\right.$
default	$8ec\left(-\frac{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}+\frac{\left(be-2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)\cdots}{8c\sqrt{-e^{2}(4ac-\cdots}}\right.$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((
e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.61

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+(b^2c}{b^2c}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c-}{b^2c-}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\sqrt{2}(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be-(b^2c}{b^2c}}\right.$$

$$\left.+2\sqrt{ex+d}e\right)$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d
- b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)
)*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d
 - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2
)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2
)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4
*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2
)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) -
 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a
*c^3)))/(b^2*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^
2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx=\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \int \frac{\sqrt{ex+d}}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(158) = 316.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$$

$$= \frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\right.}}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-\right.}}\right.}{4\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right.}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 -
(4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e + sqrt((
2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2
*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e
)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e
^3 - (4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c*d - b*e - s
qrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c
)*c^2*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*
abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)+\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,c\cdots\right.}{\cdots}\right.}{16\,c^2\,d^2\,e^3-1\cdots}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)-\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,c\cdots\right.}{\cdots}\right.}{16\,c^2\,d^2\,e^3-16\cdots}\right.$$

input
```
int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)
```

output
```
- 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
 + 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 -
32*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*c^2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(
-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/
(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^
5 - 16*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b
^2*c*d - 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atan
h((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*
c^2*d*e^3) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c
^3*e^3 + 64*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d +
 2*b^2*c*d + 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*
a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*
c*d*e^4))*((e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4
*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1369, normalized size of antiderivative = 6.91

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

output

```
( - 4*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*sqrt(a*e
**2 - b*d*e + c*d**2)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) -
 b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2) + b*e - 2*c*d))*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d
**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2
- b*d*e + c*d**2) + b*e - 2*c*d))*b*e - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e*
*2 - b*d*e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt
(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*c*d + 4*sqrt(2*sqrt(c)*sqrt(a*e*
*2 - b*d*e + c*d**2) + b*e - 2*c*d)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((sq
rt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) + 2*sqrt(c)*sqrt
(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*c
- 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*at
an((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) + 2*sqrt(c
)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*
d))*b*e + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2
*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) +
2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*
e - 2*c*d))*c*d + 2*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e ...
```

3.63 $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 275

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = -\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{c}\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a+2^(1/2)*c^(1/2)*(b*d+(-4*a*c+b
^2)^(1/2)*d-2*a*e)*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c
+b^2)^(1/2))*e)^(1/2))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*
e)^(1/2)-2^(1/2)*c^(1/2)*(b*d-(-4*a*c+b^2)^(1/2)*d-2*a*e)*arctanh(2^(1/2)*
c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/a/(-4*a*c+b^
2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx = -\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{}}{a}$$

input
```
Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

output
```
-(((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2
- 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b
*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d
+ (b + Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
a)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{d}{ax}+\frac{e\left(cd^2-bed-c(d+ex)d+ae^2\right)}{a\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\sqrt{c}e\left(d\sqrt{b^2-4ac}-2ae+bd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}e\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{de}\cdots}{}\right.}{e}$$

input
```
Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

output
```
(2*(-((Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a) + (Sqrt[c]*e*(b*d + Sq
rt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*e*(b*d - Sqrt[b^2 - 4*a*c]*d - 2
*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

method	result
derivativedivides	$2e^{2}\left(-\frac{\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{a\,e^{2}}+\frac{4c\left(\frac{\left(-2a\,e^{2}+bde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}-\right.}{a\,e^{2}}\right.$
default	$2e^{2}\left(-\frac{\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{a\,e^{2}}+\frac{4c\left(\frac{\left(-2a\,e^{2}+bde-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,d\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{8\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}-\right.}{a\,e^{2}}\right.$
pseudoelliptic	$-\frac{2\left(\left(-\frac{\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\,d}{2}+e\left(ae-\frac{bd}{2}\right)\right)\sqrt{2}\,c\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-4e^{2}\left(ac-\right.}\right.}}\right.\right.}{}$

input

```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^2*(-d^(1/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2))+4/a/e^2*c*(1/8*(-2*a*
e^2+b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b
*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2*a*e^2-b*d*e-(-e^2*
(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(225) = 450$.

Time = 0.32 (sec) , antiderivative size = 2443, normalized size of antiderivative = 8.88

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))
*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b
*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*
e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*
e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e
*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^
3*c))*log(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4
*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(
-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*
sqrt(e*x + d)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a
^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2
- 4*a^3*c))*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^
3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*
sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*
c*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2
- 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx=\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(225) = 450$.

Time = 0.29 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{d+ex}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*d*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 - 2*(sqrt(b^2 - 4*a*
c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*
e + 2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b
*e + sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*
c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^
2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c
*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e + 2*
a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - s
qrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/(
(sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*
a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 15.36 (sec) , antiderivative size = 10894, normalized size of antiderivative = 39.61

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)
```

output
```
- atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) -
b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-
(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^
2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*
((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b
^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c
^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*
d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8
 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10)
 - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c
^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 +
64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*
a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3
*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a
*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e
^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 -...
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{ex+d}}{x\left(c\,x^2+bx+a\right)}dx$$

input
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x)
```

3.64 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 330

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = -\frac{\sqrt{d+ex}}{ax}+\frac{(2bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-(e*x+d)^(1/2)/a/x+(-a*e+2*b*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/a^2/d^(1/2)
-2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d-a*b*e+(-4*a*c+b^2)^(1/2)*(-a*e+b*d))*arcta
nh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/a
^2/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*c^(1/
2)*(b^2*d-2*a*c*d-a*b*e-(-4*a*c+b^2)^(1/2)*(-a*e+b*d))*arctanh(2^(1/2)*c^(
1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^2/(-4*a*c+b^2
)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{-\frac{a\sqrt{d+ex}}{x} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-\cdots}{\sqrt{b^2-4ac}\sqrt{-\cdots}}}{a^2}$$

input
```
Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(-((a*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^2 -
 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a*c
*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/a^2
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \text{ 1199}$$

$$\frac{2\int \left(\frac{d}{ax^2}-\frac{bd-ae}{a^2x}-\frac{e\left(b\left(cd^2-bed+ae^2\right)-c(bd-ae)(d+ex)\right)}{a^2\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{\sqrt{c}e\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\text{arctanh}\left(\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\right.}}\right.}{e}$$

input
```
Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/2*(e*Sqrt[d + e*x])/(a*x) + (e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(2
*a*Sqrt[d]) + (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d])
 - (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) + (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.13

method	result
derivativedivides	$2e^3\left(\frac{4c\left(\frac{\left(ab\,e^2+2acde-de\,b^2-\sqrt{-e^2\left(4ac-b^2\right)}\,ae+\sqrt{-e^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-a \cdots\right.}{\cdots}\right.}{a^2e^3}\cdots\right.$
default	$2e^3\left(\frac{4c\left(\frac{\left(ab\,e^2+2acde-de\,b^2-\sqrt{-e^2\left(4ac-b^2\right)}\,ae+\sqrt{-e^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-a \cdots\right.}{\cdots}\right.}{a^2e^3}\cdots\right.$
risch	$-\frac{\sqrt{ex+d}}{ax}-\frac{e\left(-\frac{\left(-ae+2bd\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{8c\left(-\frac{\left(ab\,e^2+2acde-de\,b^2+\sqrt{-e^2\left(4ac-b^2\right)}\,ae-\sqrt{-e^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\,\mathrm{ar}\cdots}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+2cd+\sqrt{-e^2\cdots}\right.}}\cdots\right.}{\cdots}\right.}{\cdots}$
pseudoelliptic	$\frac{2\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x\sqrt{2}\left(\frac{\left(\sqrt{d}\,ae-b\,d^{\frac{3}{2}}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}+\frac{b\sqrt{d}\,ae}{2}\right)\right)c\,\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{\left(\cdots\right.}}\right.\cdots}{\cdots}$

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(4/a^2/e^3*c*(1/8*(a*b*e^2+2*a*c*d*e-d*e*b^2-(-e^2*(4*a*c-b^2))^(1/2
)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-a*b*e^2-2*a*c*d*e+d*e
*b^2-(-e^2*(4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)))+1/a^2/e^3*(-1/2*a*(e*x+d)^(1/2)/x-1/2*(a*e-2*b*d)/d^(1/2)*arctanh((e*
x+d)^(1/2)/d^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2424 vs. $2(278) = 556$.

Time = 5.45 (sec) , antiderivative size = 4857, normalized size of antiderivative = 14.72

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(278) = 556.

Time = 0.33 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d+ex}}{x^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-(2*b*d - a*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/4*(sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4
*a^2*c)*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^2*d*e
+ sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)
*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/
sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2
*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - s
qrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1
/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d -
(a*b^2 - 4*a^2*c)*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c
)*b^2*d*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)
*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt
(e*x + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^
3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2
*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*a
bs(e)) - sqrt(e*x + d)/(a*x)
```

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 19887, normalized size of antiderivative = 60.26

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
(atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*
e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 1
8*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((
a*e - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12
 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a
^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84
*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^
7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*
c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e -
 2*b*d)*((8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4
*b^5*c^2*e^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d
^3*e^8 + 8*a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d
^2*e^9 - 108*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2
*d*e^10 + 87*a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d
*e^10))/a^4 - (((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*
e^11 + 32*a^7*c^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9
+ 16*a^6*b^2*c^4*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32
*a^6*b*c^5*d^3*e^8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e
*x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96
*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112...
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{x^2(c\,x^2+bx+a)}dx$$

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x)
```

3.65 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 462

$$\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$$

$$= -\frac{\sqrt{d+ex}}{2ax^2} + \frac{(4bd-ae)\sqrt{d+ex}}{4a^2dx} - \frac{(8b^2d^2-4abde-a(8cd^2+ae^2))\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a^3d^{3/2}}$$

$$+ \frac{\sqrt{2}\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\text{arctanh}\left(\frac{\sqrt{\cdots}}{\sqrt{2cd\cdots}}\cdots\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}\,e\right)\right)\text{arctanh}\left(\frac{\sqrt{\cdots}}{\sqrt{2cd\cdots}}\cdots\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/2*(e*x+d)^(1/2)/a/x^2+1/4*(-a*e+4*b*d)*(e*x+d)^(1/2)/a^2/d/x-1/4*(8*b^2
*d^2-4*a*b*d*e-a*(a*e^2+8*c*d^2))*arctanh((e*x+d)^(1/2)/d^(1/2))/a^3/d^(3/
2)+2^(1/2)*c^(1/2)*(b^3*d-a*c*((-4*a*c+b^2)^(1/2)*d-2*a*e)+b^2*((-4*a*c+b^
2)^(1/2)*d-a*e)-a*b*(3*c*d+(-4*a*c+b^2)^(1/2)*e))*arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^3/(-4*a*c+b^2)^(1/
2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-2^(1/2)*c^(1/2)*(b^3*d-b^2*((-4*
a*c+b^2)^(1/2)*d+a*e)+a*c*((-4*a*c+b^2)^(1/2)*d+2*a*e)-a*b*(3*c*d-(-4*a*c+
b^2)^(1/2)*e))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2
)^(1/2))*e)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
^(1/2)
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{d+ex}}{x^3\,(a+bx+cx^2)}\,dx$$

$$= \frac{\frac{a\sqrt{d+ex}(4bdx-a(2d+ex))}{dx^2} + \frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{}$$

input
```
Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
((a*Sqrt[d + e*x]*(4*b*d*x - a*(2*d + e*x)))/(d*x^2) + (4*Sqrt[2]*Sqrt[c]*
(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d
) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3
*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-
2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b +
 Sqrt[b^2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*
ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2))/(4*a^3)
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1199$$

$$\frac{2\int \left(\frac{d}{ax^3}+\frac{db^2-aeb-acd}{a^3x}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bed+ae^2\right)-c\left(db^2-aeb-acd\right)(d+ex)\right)}{a^3\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}-\frac{bd-ae}{a^2x^2}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \ 2009$$

$$2\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}e\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\cdots}{\sqrt{2c\cdots}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.$$

input
```
Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/4*(e*Sqrt[d + e*x])/(a*x^2) + (3*e^2*Sqrt[d + e*x])/(8*a*d*x) + (e*
(b*d - a*e)*Sqrt[d + e*x])/(2*a^2*d*x) - (3*e^3*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(8*a*d^(3/2)) - (e^2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(2*
a^2*d^(3/2)) - (e*(b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
(a^3*Sqrt[d]) + (Sqrt[c]*e*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^
2*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) -
 (Sqrt[c]*e*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a
*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{ex+d}\,(aex-4bdx+2ad)}{4d\,a^2x^2}-\frac{e\left(-\frac{\left(e^2a^2+4abde+8a\,d^2c-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea\sqrt{d}}-\frac{32dc\left(-\frac{\left(-2a^2c\,e^2+a\,b^2e^2+3abcde\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8a\,d^2c-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{e^4a^3}+\frac{4c\left(-\frac{\left(-2a\right.}{}\right.}{}\right.$
default	$2e^4\left(-\frac{\frac{\frac{ae(ae-4bd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{2}abde+\frac{1}{8}e^2a^2\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(e^2a^2+4abde+8a\,d^2c-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{e^4a^3}+\frac{4c\left(-\frac{\left(-2a\right.}{}\right.}{}\right.$
pseudoelliptic	$-\frac{-8\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\frac{\left(-a\,d^{\frac{3}{2}}be-d^{\frac{5}{2}}\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}-\frac{3d^{\frac{5}{2}}b\left(ac-\frac{b^2}{3}\right)}{2}\right.\right.}{}$

input
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(a*e*x-4*b*d*x+2*a*d)/d/a^2/x^2-1/4/a^2/d*e*(-1/e*(a^2*
e^2+4*a*b*d*e+8*a*c*d^2-8*b^2*d^2)/a/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2)
)-32*d/a/e*c*(-1/8*(-2*a^2*c*e^2+a*b^2*e^2+3*a*b*c*d*e-b^3*d*e+(-e^2*(4*a*
c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(1/2
)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))+1/8*(2*a^2*c*e^2-a*b^2*e^2-3*a*b*c*d*e+b^3*d*e+(-e^
2*(4*a*c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2
))^(1/2)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3707 vs. $2(395) = 790$.

Time = 78.99 (sec) , antiderivative size = 7422, normalized size of antiderivative = 16.06

$$\int \frac{\sqrt{d+ex}}{x^3 (a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^3 (a+bx+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{x^3 (a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

output
```
Integral(sqrt(d + e*x)/(x**3*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^3\,(a+bx+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx+a)x^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(395) = 790.

Time = 0.33 (sec) , antiderivative size = 1043, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{d+ex}}{x^3\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 - 2*((a*b^2*c - a^2*c^2)*sq
rt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 -
 a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*
b^2*c - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^
3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3
*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4
*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2
*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2
 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c
)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*
abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c
- 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^3*c*d
- a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d -
 a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*
a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(8*b...
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 33838, normalized size of antiderivative = 73.24

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
atan(((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e
^10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^
3*e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c
^3*d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a
^10*b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 38
4*a^11*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(
1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33
*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*
b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*
c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(
2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 10
24*a^13*c^4*d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9
 - 896*a^11*b^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d
^2*e^10 - 512*a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2
))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a
^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^
5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^
2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/...
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx=\int \frac{\sqrt{ex+d}}{x^3\left(c\,x^2+bx+a\right)}dx$$

input
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x)
```

3.66 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 650

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} \, dx = -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5}$$
$$-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{5/2}}{5c^3e^3}$$
$$-\frac{2(2cd+be)(d+ex)^{7/2}}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$
$$+\frac{\sqrt{2}\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce)+\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-}{\sqrt{b^2-4ac}}\right.}{c^{11/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{2}\left((bcd-b^2e+ace)(b^2cd-2ac^2d-b^3e+3abce)-\frac{2b^5cde-10ab^3c^2de+10a^2bc^3de-b^6e^2+ab^2c^2(4cd^2-9ae^2)-b^4c(cd^2-}{\sqrt{b^2-4ac}}\right.}{c^{11/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*(e*x+d)^(1/2)/c^5-2/
3*b*(-2*a*c+b^2)*(e*x+d)^(3/2)/c^4+2/5*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(e
*x+d)^(5/2)/c^3/e^3-2/7*(b*e+2*c*d)*(e*x+d)^(7/2)/c^2/e^3+2/9*(e*x+d)^(9/2
)/c/e^3+2^(1/2)*((a*c*e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(
2*b^5*c*d*e-10*a*b^3*c^2*d*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*e^2+
4*c*d^2)-b^4*c*(-6*a*e^2+c*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/
2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)
^(1/2))/c^(11/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*((a*c*e-b^
2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)-(2*b^5*c*d*e-10*a*b^3*c^2*d
*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*e^2+4*c*d^2)-b^4*c*(-6*a*e^2+c
*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2
)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(11/2)/(2*c*d-(b
+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.20 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.39

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{d+ex}\left(315b^4e^4-105b^2ce^3(4bd+9ae+bex)-9c^3e(d+ex)^2(-2bd+7ae+5be\right.}{}$$

$$-\frac{\left(ib^6e^2+b^5e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ib^4c\left(cd^2+2i\sqrt{-b^2+4ac}de-6ae^2\right)+ab^2c^2\left(-4icd^2+6\sqrt{-b^2+\right.\right.}{}$$

$$-\frac{\left(-ib^6e^2+b^5e\left(2icd+\sqrt{-b^2+4ac}e\right)+b^4c\left(-icd^2-2\sqrt{-b^2+4ac}de+6iae^2\right)+ab^2c^2\left(4icd^2+6\sqrt{-b^2+\right.\right.}{}$$

input
```
Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*
c^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20
*d*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b
*e*(4*d + e*x))))/(315*c^5*e^3) - ((I*b^6*e^2 + b^5*e*((-2*I)*c*d + Sqrt[-
b^2 + 4*a*c]*e) + I*b^4*c*(c*d^2 + (2*I)*Sqrt[-b^2 + 4*a*c]*d*e - 6*a*e^2)
 + a*b^2*c^2*((-4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e + (9*I)*a*e^2) + a*b
*c^2*(3*a*Sqrt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e
)) + b^3*c*(-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (10*
I)*a*e)) - (2*I)*a^2*c^3*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d + a*e)))
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 +
4*a*c]*e]])/(c^(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^
2 + 4*a*c])*e]) - (((-I)*b^6*e^2 + b^5*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e
) + b^4*c*((-I)*c*d^2 - 2*Sqrt[-b^2 + 4*a*c]*d*e + (6*I)*a*e^2) + a*b^2*c^
2*((4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e - (9*I)*a*e^2) + a*b*c^2*(3*a*Sq
rt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*(
-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (10*I)*a*e)) + (
2*I)*a^2*c^3*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^
(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^4}{ce^2}-\frac{(2cd+be)(d+ex)^3}{c^2e^2}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^2}{c^3e^2}-\frac{b\left(b^2-2ac\right)e(d+ex)}{c^4}-\frac{e\left(-eb^4+cdb^3+3aceb^2-2ac^2db-a^2c^2e\right)}{c^5}\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)-b^4c\left(cd^2-6ae^2\right)-10ab^3c^2de+ab^2c^2\left(4cd^2-9ae^2\right)+b^6\left(-e^2\right)+2b^5cde}{\sqrt{b^2-4ac}}+(ace+b^2(-e)+bcd)\left(3abce-2ac^2d+b^3(-e)+\right.\right.}{\sqrt{2}c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.$$

input
```
Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(-((e*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d
+ e*x])/c^5) - (b*(b^2 - 2*a*c)*e*(d + e*x)^(3/2))/(3*c^4) + ((c^2*d^2 + b
^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^2) - ((2*c*d + b*e)*(d
 + e*x)^(7/2))/(7*c^2*e^2) + (d + e*x)^(9/2)/(9*c*e^2) + (e*((b*c*d - b^2*
e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 10*a
*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2)
- b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(Sqrt[2]*c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*((
b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*
c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2
 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^
2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{2e^3\left(\frac{3\left(\left(ac-b^2\right)e+dbc\right)\left(\left(abc-\frac{1}{3}b^3\right)e-\frac{2d\left(ac-\frac{b^2}{2}\right)c}{3}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(-\frac{1}{2}b^6-\frac{9}{2}a^2b^2c^2+3a\,b^4c+a^3c^3\right)e^2+5\left(a^2c^2-\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e^3*(3/2*((a*c-b^2)*e+d*b*c)*((a*b*c-1/3*b
^3)*e-2/3*d*(a*c-1/2*b^2)*c)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((-1/2*b^6-9/2
*a^2*b^2*c^2+3*a*b^4*c+a^3*c^3)*e^2+5*(a^2*c^2-c*a*b^2+1/5*b^4)*d*b*c*e-d^
2*(a^2*c^2-2*c*a*b^2+1/2*b^4)*c^2))*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/
2))*c)^(1/2)*2^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)*(e^3*(-3/2*((a*c-b^2)*e+d*b*c)*((a*b*c-1/3*b^3)*e-2/3*d*(a*c-1/2
*b^2)*c)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((-1/2*b^6-9/2*a^2*b^2*c^2+3*a*b^4
*c+a^3*c^3)*e^2+5*(a^2*c^2-c*a*b^2+1/5*b^4)*d*b*c*e-d^2*(a^2*c^2-2*c*a*b^2
+1/2*b^4)*c^2))*2^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(e*x+d)^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/
4*b^2))^(1/2))*c)^(1/2)*((1/9*c^4*x^4-1/5*(5/7*b*x+a)*x^2*c^3+(1/5*b^2*x^2
+a^2+2/3*a*b*x)*c^2+(-3*a*b^2-1/3*b^3*x)*c+b^4)*e^4+8/3*d*(5/84*c^3*x^3-3/
20*x*(4/7*b*x+a)*c^2+b*(3/20*b*x+a)*c-1/2*b^3)*c*e^3-1/5*d^2*(-1/21*c^2*x^
2+(1/7*b*x+a)*c-b^2)*c^2*e^2+2/35*d^3*(-2/9*c*x+b)*c^3*e+8/315*d^4*c^4)*(-
4*e^2*(a*c-1/4*b^2))^(1/2)))/(-4*e^2*(a*c-1/4*b^2))^(1/2)/c^5/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14340 vs. 2(592) = 1184.

Time = 12.64 (sec) , antiderivative size = 14340, normalized size of antiderivative = 22.06

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a}\,dx$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(592) = 1184$.

Time = 0.39 (sec) , antiderivative size = 1612, normalized size of antiderivative = 2.48

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c^2 - 6*a*b^3
*c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^6*c - 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*
c^4)*d*e + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 - 12*a^3*b*c^3)*e^2)*c^2*e^2
- 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^
4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)
*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2
- 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(
e) + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3*e - (5*b^5*c^4 - 24*a*b^3*
c^5 + 22*a^2*b*c^6)*d^2*e^2 + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5
 - 2*a^3*c^6)*d*e^3 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5
)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2
)*sqrt(e*x + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30
 - b*c^9*d*e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))
/(c^10*e^30)))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e +
 sqrt(b^2 - 4*a*c)*a*c^7*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2 - 2*(b^
6*c - 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e + (b^7 - 8*a*b^5*c + 1
9*a^2*b^3*c^2 - 12*a^3*b*c^3)*e^2)*c^2*e^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt
(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*
d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (...
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 31485, normalized size of antiderivative = 48.44

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2
)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3)
+ (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3
) + ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 -
2*c*d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 -
2*c*d*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6)
 + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3
))/(c*e^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^
5 + b^7*c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^
2*e^3 + b^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a
*b^5*c^7*d*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^
2*e^3 + 8*a^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1
/2)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11
*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3
+ 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5
*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35
*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c...
```

Reduce [F]

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{x^4(ex+d)^{\frac{3}{2}}}{c\,x^2+bx+a}dx$$

input
```
int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

output
```
int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

3.67 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}$$
$$+\frac{2(b^2-ac)\,(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$
$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)-\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)}{\sqrt{b^2-4ac}} \cdots}{c^{9/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{2}\Big(2b^3cde-4abc^2de-b^4e^2-b^2c(cd^2-3ae^2)+ac^2(cd^2-ae^2)+\frac{2b^4cde-8ab^2c^2de+4a^2c^3de-b^5e^2-b^3c(cd^2-5ae^2)}{\sqrt{b^2-4ac}} \cdots}{c^{9/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(e*x+d)^(1/2)/c^4+2/3*(-a*c+b^2)*(e*x+
d)^(3/2)/c^3-2/5*(b*e+c*d)*(e*x+d)^(5/2)/c^2/e^2+2/7*(e*x+d)^(7/2)/c/e^2+2
^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a
*e^2+c*d^2)-(2*b^4*c*d*e-8*a*b^2*c^2*d*e+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a
*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2
)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(9/2)/(2
*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^
4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a*e^2+c*d^2)+(2*b^4*c*d*e-8*a*b^2*c^2
*d*e+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3*c*d^
2))/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-
4*a*c+b^2)^(1/2))*e)^(1/2))/c^(9/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.48 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.30

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx =$$

$$-\frac{2\sqrt{d+ex}(105b^3e^3+3c^3(2d-5ex)(d+ex)^2-35bce^2(4bd+6ae+bex)+7c^2e(3b(d+ex)^2+5ae(4d+}{105c^4e^2}$$

$$+\frac{\left(ib^5e^2+b^4e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ib^3c\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-5ae\right)\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2-cd\right.}{}$$

$$+\frac{\left(-ib^5e^2+b^4e\left(2icd+\sqrt{-b^2+4ac}e\right)+ac^2\left(a\sqrt{-b^2+4ac}e^2+cd\left(-\sqrt{-b^2+4ac}d+4iae\right)\right)+abc^2\left(3icd^2\right.}{}$$

input
```
Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*
e^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)
)))/(105*c^4*e^2) + ((I*b^5*e^2 + b^4*e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e
) + I*b^3*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sq
rt[-b^2 + 4*a*c]*e^2 - c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)) + a*b*c^2*(
(-3*I)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[
-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (8*I)*a*e)))*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c
^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e])
 + (((-I)*b^5*e^2 + b^4*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a*c^2*(a*Sq
rt[-b^2 + 4*a*c]*e^2 + c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + a*b*c^
2*((3*I)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*((-I)*c*d
^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 + 4*
a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Rubi [A] (verified)

Time = 8.06 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{(d+ex)^3}{ce} - \frac{(cd+be)(d+ex)^2}{c^2e} + \frac{(b^2-ac)e(d+ex)}{c^3} + \frac{e\left(-eb^3+cdb^2+2aceb-ac^2d\right)}{c^4} - \frac{\left(-eb^3+cdb^2+2aceb-ac^2d\right)\left(cd^2-bed+ae^2\right)+\left(-e\right.}{c^4e\left(\frac{c(d+ex)^2}{e^2}-\right.}\right.}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e\left(-\frac{4a^2c^3de-b^3c\left(cd^2-5ae^2\right)-8ab^2c^2de+abc^2\left(3cd^2-5ae^2\right)+b^5\left(-e^2\right)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc^2de+ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right)+2b^3cde\right)\text{ar}}{\sqrt{2}c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right.$$

input
```
Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*((e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + ((b^2
 - a*c)*e*(d + e*x)^(3/2))/(3*c^3) - ((c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e
) + (d + e*x)^(7/2)/(7*c*e) + (e*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*
d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])
/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(9/2)*Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]) + (e*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 - b
^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c^
2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(9/2)*Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-e^2\left(\left(\left(a^2c^2-3ca\,b^2+b^4\right)e^2+2d\left(2ab\,c^2-b^3c\right)e-c^2d^2\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-5e\left(b\left(a^2c^2-ca\,b^2+\frac{1}{5}b^4\right)e^2-\frac{4d\left(a^2c\right.}{}\right.\right.}{}$
risch	$\frac{2\left(15c^3x^3e^3-21b\,c^2e^3x^2+24c^3d\,e^2x^2-35a\,c^2e^3x+35x\,b^2c\,e^3-42b\,c^2d\,e^2x+3d^2e\,c^3x+210abc\,e^3-140d\,e^2a\,c^2-105b^3e^3\right.}{105e^2c^4}$
derivativedivides	$\frac{2\left(\frac{\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+d}-a\,c^2d\,e^2\sqrt{ex+d}-b^3e^3\sqrt{ex+d}+}{c^4}\right.}{}$
default	$\frac{2\left(\frac{\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{c^3d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^2e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{b^2c\,e^2(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^3\sqrt{ex+d}-a\,c^2d\,e^2\sqrt{ex+d}-b^3e^3\sqrt{ex+d}+}{c^4}\right.}{}$

input
```
int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*(((a^2*c^2-3*a*b^2*c+b^4)*e^2+2*d*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b
^2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)-5*e*(b*(a^2*c^2-c*a*b^2+1/5*b^4)*e^2-4/5
*d*(a^2*c^2-2*c*a*b^2+1/2*b^4)*c*e-3/5*d^2*b*(a*c-1/3*b^2)*c^2))*((b*e-2*c
*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*arctanh((e*x+d)^(1/2)*c*
2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(e^2*(((a^2*c
^2-3*a*b^2*c+b^4)*e^2+2*d*(2*a*b*c^2-b^3*c)*e-c^2*d^2*(a*c-b^2))*(-4*e^2*(
a*c-1/4*b^2))^(1/2)+5*e*(b*(a^2*c^2-c*a*b^2+1/5*b^4)*e^2-4/5*d*(a^2*c^2-2*
c*a*b^2+1/2*b^4)*c*e-3/5*d^2*b*(a*c-1/3*b^2)*c^2))*2^(1/2)*arctan((e*x+d)^
(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+4*(e*x
+d)^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/14*c^3*x^
3-1/6*x*(3/5*b*x+a)*c^2+b*(1/6*b*x+a)*c-1/2*b^3)*e^3-2/3*d*c*(-6/35*c^2*x^
2+(3/10*b*x+a)*c-b^2)*e^2-1/10*d^2*(-1/7*c*x+b)*c^2*e-1/35*d^3*c^3)*(-4*e^
2*(a*c-1/4*b^2))^(1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2
))/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/c^4/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. $2(527) = 1054$.

Time = 6.58 (sec) , antiderivative size = 11459, normalized size of antiderivative = 19.72

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(527) = 1054$.

Time $= 0.35$ (sec) , antiderivative size $= 1397$, normalized size of antiderivative $= 2.40$

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c^2 - 5*a*b^2*
c^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e + (b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 - 2*((b^2*c^4 - a*c^5
)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e
+ (b^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2
*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e - (5*b^4*c^4 - 19
*a*b^2*c^5 + 8*a^2*c^6)*d^2*e^2 + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5
)*d*e^3 - (b^6*c^2 - 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^4)*sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c
^8*d*e^16 - b*c^7*e^17 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18
)*c^8*e^16 + (2*c^8*d*e^16 - b*c^7*e^17)^2))/(c^8*e^16)))/((sqrt(b^2 - 4*a
*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c
^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c
^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
d*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2*e^2 + 2*((b^
2*c^4 - a*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 -
4*a*c)*d^2*e + (b^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (
a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3*e -...
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 25497, normalized size of antiderivative = 43.88

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*
e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*
d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 -
 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(
b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e
 - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*
b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 +
15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^
2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*
(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 -
 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3
*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4
*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)...
```

Reduce [F]

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{x^3(ex+d)^{\frac{3}{2}}}{c\,x^2+bx+a}dx$$

input
```
int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

output
```
int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

3.68 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 441

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$

$$+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)+\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right.}}\right.}{c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)-\frac{2b^3cde-6abc^2de-b^4e^2-b^2c(cd^2-4ae^2)+2ac^2(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right.}}\right.}{c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-2*(a*c*e-b^2*e+b*c*d)*(e*x+d)^(1/2)/c^3-2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+
d)^(5/2)/c/e+2^(1/2)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*a*b*
c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2
)^(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2
))*e)^(1/2))/c^(7/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*((-b*e
+c*d)*(2*a*c*e-b^2*e+b*c*d)-(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a
*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(7/2)/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{\frac{2\sqrt{c}\sqrt{d+ex}(15b^2e^2+3c^2(d+ex)^2-5ce(4bd+3ae+bex))}{e} - \frac{15\sqrt{2}\left(-b^4e^2+b^3e\left(2cd+\sqrt{b^2-4ac}e\right)+bc\left(-2a\sqrt{b^2-4ac}\right.\right.}{}}{}$$

input
```
Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d +
 3*a*e + b*e*x)))/e - (15*Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 -
4*a*c]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6
*a*e)) - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^
2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])
/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d
 + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) +
b^2*c*(c*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4
*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]))/(15*c^(7/2))
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{(d+ex)^2}{c}-\frac{be(d+ex)}{c^2}-\frac{e(-eb^2+cdb+ace)}{c^3}+\frac{(-eb^2+cdb+ace)(cd^2-bed+ae^2)-(cd-be)(-eb^2+cdb+2ace)(d+ex)}{c^3e\left(\frac{c(d+ex)^2}{e^2}-\frac{(2cd-be)(d+ex)}{e^2}+a+\frac{d(cd-be)}{e^2}\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{e\left((cd-be)(2ace+b^2(-e)+bcd)+\frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left((c\right.}{}\right.$$

input
```
Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(-((e*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3) - (b*e*(d + e*x)^(3/2
))/(3*c^2) + (d + e*x)^(5/2)/(5*c) + (e*((c*d - b*e)*(b*c*d - b^2*e + 2*a*
c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[
2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*((c*d - b*e)*(b*c*d - b^2*e + 2*a
*c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{2\left(e\left((be-cd)\left(\left(ae+\frac{bd}{2}\right)c-\frac{e\,b^2}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+\left(-a\,c^3d^2+\left(e^2a^2+3abde+\frac{1}{2}b^2d^2\right)c^2+\left(-2a\,b^2e^2-b^3de\right)c+\frac{b^4e^{\cdots}}{2}\cdots\right.\right.\right.}{\cdots}$
risch	$-\frac{2\left(-3c^2e^2x^2+5e^2xbc-6c^2dex+15ac\,e^2-15b^2e^2+20bcde-3c^2d^2\right)\sqrt{ex+d}}{15e\,c^3}+\frac{-\frac{\left(2a^2c^2e^3-4a\,b^2c\,e^3+6ab\,c^2d\,e^2-2a\,c^3\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^3}+\frac{8e\left(-\frac{\left(2a^2c^2e^3-4a\,b^2c\,e^3+6ab\,c^2d\,e^2-2a\,c^3d^2e+\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ac\,e^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}+bcde\sqrt{ex+d}\right)}{c^3}+\frac{8e\left(-\frac{\left(2a^2c^2e^3-4a\,b^2c\,e^3+6ab\,c^2d\,e^2-2a\,c^3d^2e+\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$

input
```
int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)*(e*((b*e-c*d)
*((a*e+1/2*b*d)*c-1/2*e*b^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(-a*c^3*d^2+(e^2
*a^2+3*a*b*d*e+1/2*b^2*d^2)*c^2+(-2*a*b^2*e^2-b^3*d*e)*c+1/2*b^4*e^2)*e)*(
(b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*arctanh((e*x+d)^
(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*
e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(e*2^(1/2)*(-(b*e-c*d)*((a*
e+1/2*b*d)*c-1/2*e*b^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(-a*c^3*d^2+(e^2*a^2+
3*a*b*d*e+1/2*b^2*d^2)*c^2+(-2*a*b^2*e^2-b^3*d*e)*c+1/2*b^4*e^2)*e)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2
))+(e*x+d)^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-1/5*
(e*x+d)^2*c^2+e*((1/3*b*x+a)*e+4/3*b*d)*c-b^2*e^2)*(-4*e^2*(a*c-1/4*b^2))^
(1/2)))/e/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8530 vs. $2(391) = 782$.

Time = 2.45 (sec) , antiderivative size = 8530, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(391) = 782$.

Time = 0.37 (sec) , antiderivative size = 1195, normalized size of antiderivative = 2.71

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c
^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a
^2*b*c^2)*e^2)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c
)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2
 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e)*abs(c)*abs(e) + (2*(b^2*c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a
*b*c^5)*d^2*e^2 + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2
 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7
 + sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6
- b*c^5*e^7)^2))/(c^6*e^6)))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*
c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2*abs(e)) + 1/4*(sqrt(-4*c^2
*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*
c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^
2*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 -
 (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(
b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*
abs(e) + (2*(b^2*c^5 - 2*a*c^6)*d^3*e - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e^2 +
 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^3 - (b^5*c^2 - 4*a*b^3*c^3 +
2*a^2*b*c^4)*e^4)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arc...
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 19465, normalized size of antiderivative = 44.14

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output

```
atan(((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*
e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*
d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/
2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*
c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2
) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e +
 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^
4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^
8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2)
)/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c
^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 +
a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(...
```

Reduce [F]

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx=\int \frac{x^2(ex+d)^{\frac{3}{2}}}{c\,x^2+bx+a}dx$$

input

```
int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

output

```
int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

3.69 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 376

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(cd-be)\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3c} - \frac{\sqrt{2}\left(c^2d^2+b^2e^2-ce(2bd+ae)+\frac{2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(c^2d^2+b^2e^2-ce(2bd+ae)-\frac{2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae^2)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*(-b*e+c*d)*(e*x+d)^(1/2)/c^2+2/3*(e*x+d)^(3/2)/c-2^(1/2)*(c^2*d^2+b^2*e^
2-c*e*(a*e+2*b*d)+(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))/(
-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c
+b^2)^(1/2))*e)^(1/2))/c^(5/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-2^(1
/2)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d)-(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*
(-3*a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(5/2)/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.31

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{d+ex}(4cd-3be+cex)+\frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-3ae\right.\right.\right.}{\sqrt{-\frac{b^2}{2}+2ac}}}{}$$

input

```
Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output

```
(2*Sqrt[c]*Sqrt[d + e*x]*(4*c*d - 3*b*e + c*e*x) + (3*(I*b^3*e^2 + b^2*e*(
(-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4
*a*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*
c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b
*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b -
I*Sqrt[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^
2 + 4*a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e))
 + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)
))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2
+ 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a
*c])*e]))/(3*c^(5/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1196, 25, 1196, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1196$$

$$\frac{\int -\frac{\sqrt{d+ex}(ae-(cd-be)x)}{cx^2+bx+a}dx}{c}+\frac{2(d+ex)^{3/2}}{3c}$$

$\Big\downarrow$ 25

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\int \frac{\sqrt{d+ex}(ae-(cd-be)x)}{cx^2+bx+a}dx}{c}$$

$\Big\downarrow$ 1196

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{\int \frac{ae(2cd-be)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(cx^2+bx+a\right)}dx}{c}-\frac{2\sqrt{d+ex}(cd-be)}{c}}{c}$$

$\Big\downarrow$ 1197

$$\frac{2(d+ex)^{3/2}}{3c}-\frac{\frac{2\int \frac{(cd-be)\left(cd^2-bed+ae^2\right)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}-\frac{2\sqrt{d+ex}(cd-be)}{c}}{c}$$

$\Big\downarrow$ 1480

$$\frac{2(d+ex)^{3/2}}{3c}-$$

$$\frac{\frac{2\left(-\frac{1}{2}\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}-\frac{1}{2}\left(-\frac{-bc\left(cd^2-3ae^2\right)-\ldots}{\ldots}\right.\ldots}{c}}{c}$$

$\Big\downarrow$ 221

$$\frac{2(d+ex)^{3/2}}{3c}-$$

$$\frac{\frac{2\left(\frac{\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)\ldots}{\sqrt{b^2-4ac}}\right.\ldots}{\ldots}\right.\ldots}{c}}{c}$$

input
```
Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(3/2))/(3*c) - ((-2*(c*d - b*e)*Sqrt[d + e*x])/c + (2*(((c^2*
d^2 + b^2*e^2 - c*e*(2*b*d + a*e) + (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 -
 b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*
e) - (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^
2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{2\left(-\frac{\left(\left(-c^{2}d^{2}+\left(a\,e^{2}+2bde\right)c-b^{2}e^{2}\right)\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}+\left(4ad\,e^{2}+b\,d^{2}e\right)c^{2}+\left(-3a\,e^{3}b-2d\,e^{2}b^{2}\right)c+b^{3}e^{3}\right)\sqrt{\left(be-2cd+\sqrt{-4e^{2}\cdots}\right.}}{2}\cdots\right)}{\cdots}$
risch	$-\frac{2(-cex+3be-4cd)\sqrt{ex+d}}{3c^{2}}-\frac{8\left(-\frac{\left(-3abc\,e^{3}+4d\,e^{2}a\,c^{2}+b^{3}e^{3}-2d\,e^{2}b^{2}c+d^{2}eb\,c^{2}+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}-\sqrt{-e^{2}\left(4ac-b^{2}\cdots\right.}\cdots\right)}{8c\sqrt{-e^{2}\left(4ac-b^{2}\cdots\right.}}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+\sqrt{ex+d}\,be-cd\sqrt{ex+d}\right)}{c^{2}}+\frac{\frac{\left(-3abc\,e^{3}+4d\,e^{2}a\,c^{2}+b^{3}e^{3}-2d\,e^{2}b^{2}c+d^{2}eb\,c^{2}-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}+\sqrt{-e^{2}\cdots}\cdots\right)}{c\sqrt{-e^{2}\left(\cdots\right.}}\cdots}{\cdots}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+\sqrt{ex+d}\,be-cd\sqrt{ex+d}\right)}{c^{2}}+\frac{\frac{\left(-3abc\,e^{3}+4d\,e^{2}a\,c^{2}+b^{3}e^{3}-2d\,e^{2}b^{2}c+d^{2}eb\,c^{2}-\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,ac\,e^{2}+\sqrt{-e^{2}\cdots}\cdots\right)}{c\sqrt{-e^{2}\left(\cdots\right.}}\cdots}{\cdots}$

input
```
int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-1/2*((-c^2*d^2+(a*
e^2+2*b*d*e)*c-b^2*e^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(4*a*d*e^2+b*d^2*e)*c
^2+(-3*a*b*e^3-2*b^2*d*e^2)*c+b^3*e^3)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^
(1/2))*c)^(1/2)*2^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-4*e
^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(1/2*2^(1/2)*((-c^2*d^2+(a*e^2+2*b*d*e)
*c-b^2*e^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(-4*a*d*e^2-b*d^2*e)*c^2+(3*a*b*e
^3+2*b^2*d*e^2)*c-b^3*e^3)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*
e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+(1/3*(-e*x-4*d)*c+b*e)*(e*x+d)^(1/2)*(
(b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-4*e^2*(a*c-1/4*b^2))^(
1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((-b*e+2*c*d+(-
4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5572 vs. $2(330) = 660$.

Time = 0.88 (sec) , antiderivative size = 5572, normalized size of antiderivative = 14.82

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(330) = 660.

Time = 0.31 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.62

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/3*((e*x + d)^(3/2)*c^2 + 3*sqrt(e*x + d)*c^2*d - 3*sqrt(e*x + d)*b*c*e)/
c^3 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c^2 - 4*
a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2
)*c^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e
 - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e
^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) + (2*b*
c^5*d^3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^
3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c
)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(
-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/(
(sqrt(b^2 - 4*a*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*
c)*a*c^4*e^2)*c^2*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c
)*c)*e)*((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^
2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)
*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)
*abs(c)*abs(e) + (2*b*c^5*d^3*e - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3
*c^3 - 5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4)*sqrt(-4*c^2*d + 2*(
b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*c
^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4...
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 13841, normalized size of antiderivative = 36.81

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1
/2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^
2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*
c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*
a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4
*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*
d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e -
3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d
*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 +
a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a
*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*
e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2...
```

Reduce [F]

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{x(ex+d)^{\frac{3}{2}}}{c\,x^2+bx+a}dx$$

input
```
int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

output
```
int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

3.70 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 322

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*e*(e*x+d)^(1/2)/c-2^(1/2)*(2*c^2*d^2+b*(b-(-4*a*c+b^2)^(1/2))*e^2-2*c*e*
(b*d-(-4*a*c+b^2)^(1/2)*d+a*e))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c
*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*(2*c^2*d^2+b*(b+(-4*a*c+b^2)^(1/2))*e^
2-2*c*e*(b*d+(-4*a*c+b^2)^(1/2)*d+a*e))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1
/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*
c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex} + \frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}d+iae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+}}}\right.}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{c^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[d + e*x] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*c
])*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1
/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2
*d^2 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 +
 4*a*c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b
*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b +
I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1146, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}dx$$

$$\Big\downarrow \ 1146$$

$$\frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx+a)}dx}{c} + \frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \ 1197$$

$$\frac{2\int -\frac{e(cd^2-bed+ae^2-(2cd-be)(d+ex))}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \ 25$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2\int \frac{e(cd^2-bed+ae^2-(2cd-be)(d+ex))}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\int \frac{cd^2-bed+ae^2-(2cd-be)(d+ex)}{cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \ 1480$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(-\frac{1}{2}\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)+c(d+ex)}d\sqrt{d+ex}-\frac{1}{2}\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}\right.\right.}{c}$$

$$\Big\downarrow \ 221$$

$$\frac{2e\sqrt{d+ex}}{c}-$$

$$\frac{2e\left(\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-e(}}\right.}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right.}{c}$$

input
```
Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x])/c - (2*e*(((2*c*d - b*e + (2*c^2*d^2 + b^2*e^2 - 2*c*e
*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c*d - b*e - (2*c^2*d^2 + b^2*e^2 - 2*c*
e*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{e\left(2\sqrt{ex+d}+\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,be-2\sqrt{-e^{2}\left(4ac-b^{2}\right)}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b\right.}\right.}}\right.}{\sqrt{-e^{2}\left(4ac-b^{2}\right)}\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right.}{}$
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^{2}+b^{2}e^{2}-2bcde+2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{\sqrt{\left(-be+2cd\right.}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
default	$2e\left(\frac{\sqrt{ex+d}}{c}-\frac{\left(-2ac\,e^{2}+b^{2}e^{2}-2bcde+2c^{2}d^{2}-\sqrt{-e^{2}(4ac-b^{2})}\,be+2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{\sqrt{\left(-be+2cd\right.}}\right.}{2c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$
risch	$\frac{2e\sqrt{ex+d}}{c}-8e\left(-\frac{\left(2ac\,e^{2}-b^{2}e^{2}+2bcde-2c^{2}d^{2}+\sqrt{-e^{2}(4ac-b^{2})}\,be-2\sqrt{-e^{2}(4ac-b^{2})}\,cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{}{\sqrt{\left(-be+\right.}}\right.}{8c\sqrt{-e^{2}(4ac-b^{2})}\sqrt{\left(-be+2cd+\sqrt{-e^{2}(4ac-b^{2})}\right)c}}\right.$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e/c*(2*(e*x+d)^(1/2)+(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2+(-e^2*(4*a*c-b
^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2))^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(2*a*c*e^2-b^
2*e^2+2*b*c*d*e-2*c^2*d^2-(-e^2*(4*a*c-b^2))^(1/2)*b*e+2*(-e^2*(4*a*c-b^2)
)^(1/2)*c*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(272) = 544$.

Time = 0.27 (sec) , antiderivative size = 2770, normalized size of antiderivative = 8.60

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output

```
-1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^
2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c
^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^
3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 -
2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)
*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2
*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*
e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*
c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^
2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt
(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 -
 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3
 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sq
rt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*...
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**(3/2)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(272) = 544$.

Time = 0.29 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*(2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 - 2*(sqrt(b^2 -
4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*
c^5*d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b
*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt
(1/2)*sqrt(e*x + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e +
a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - s
qrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/
4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d
*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(
b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*
(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*d^3*e - 6*b*c^4*d^2*
e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)*e^4)*sqrt(-4*c^2*d
 + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt
(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d
- b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d
*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 8334, normalized size of antiderivative = 25.88

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

output
```
(2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*
c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(
d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2)
)/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*
d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*
e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^
3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*...
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 2819, normalized size of antiderivative = 8.75

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*sqrt(a*e**2
 - b*d*e + c*d**2)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*
e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e +
 c*d**2) + b*e - 2*c*d))*b*c*e - 4*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*
d**2) + b*e - 2*c*d)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((sqrt(2*sqrt(c)*sq
rt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt
(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*c**2*d + 4*sqrt(c
)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2
*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d +
 e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d))*a*c*e*
*2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2*c*d)
*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e + 2*c*d) - 2*sqr
t(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) + b*e - 2
*c*d))*b**2*e**2 + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)
+ b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2) - b*e +
2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d
**2) + b*e - 2*c*d))*b*c*d*e - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2) + b*e - 2*c*d)*atan((sqrt(2*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2) - b*e + 2*c*d) - 2*sqrt(c)*sqrt(d + e*x))/sqrt(2*sqrt(c)*sqrt(a*e**2 -
 b*d*e + c*d**2) + b*e - 2*c*d))*c**2*d**2 - 2*sqrt(2*sqrt(c)*sqrt(a*e*...
```

3.71 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 340

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = -\frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b(cd^2+ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b(cd^2+ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output

```
-2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a-2^(1/2)*(a*(-4*a*c+b^2)^(1/2)*
e^2-c*d*((-4*a*c+b^2)^(1/2)*d-4*a*e)-b*(a*e^2+c*d^2))*arctanh(2^(1/2)*c^(1
/2)*(e*x+d)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/a/c^(1/2)/(-4*a*
c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-2^(1/2)*(a*(-4*a*c+b^2
)^(1/2)*e^2-c*d*((-4*a*c+b^2)^(1/2)*d+4*a*e)+b*(a*e^2+c*d^2))*arctanh(2^(1
/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/a/c^(1/2
)/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx =$$

$$-\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d+4iae\right)-ib(cd^2+ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d-\cdots\right.\right.}{\sqrt{c}\sqrt{-b^2+4ac\cdots}}}{a}$$

input
```
Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

output
```
-(((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*
I)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt
[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c
]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*
c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]) + 2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int\left(\frac{d^2}{ax}+\frac{e(d(cd^2-bed+ae^2)-(cd^2-ae^2)(d+ex))}{a(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex))}\right)d\sqrt{d+ex}}{e}$$

$\big\downarrow$ 2009

$$\frac{2\left(-\frac{e\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{e\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}}\right.}{e}$$

input
```
Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

output
```
(2*(-((d^(3/2)*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a) - (e*(a*Sqrt[b^2 - 4*a
*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (e*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(
c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.09

method	result
derivativedivides	$2e^2 \left(4c \left(-\frac{\left(-a\,e^3 b+4ad\,e^2 c-d^2 ebc+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.\right.$
default	$2e^2 \left(4c \left(-\frac{\left(-a\,e^3 b+4ad\,e^2 c-d^2 ebc+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.\right.$
pseudoelliptic	$-\frac{2\left(\frac{\left(\left(a\,e^2-c\,d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+\left(4ad\,e^2-b\,d^2 e\right)c-a\,e^3 b\right)\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+}}{\sqrt{\left(-be+2cd+\sqrt{-}\right.}}\right.}{2}\right.}{}$

input
```
int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^2*(4/a/e^2*c*(-1/8*(-a*e^3*b+4*a*d*e^2*c-d^2*e*b*c+(-e^2*(4*a*c-b^2))^
(1/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/8*(a*e^3*b-4
*a*d*e^2*c+d^2*e*b*c+(-e^2*(4*a*c-b^2))^(1/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/
2)*c*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)))-d^(3/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. $2(286) = 572$.

Time = 4.07 (sec) , antiderivative size = 5164, normalized size of antiderivative = 15.19

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{x(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**(3/2)/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(286) = 572.

Time = 0.33 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*d^2*arctan(sqrt(e*x + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2
*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)
*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d
^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^
2*e^4 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2
 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e +
 sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))
/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^
2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2)*
a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e +
 sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)
*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4
- (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(
-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqr
t(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*
a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 20897, normalized size of antiderivative = 61.46

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

output
```
atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a
^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a
^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*
d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10
 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8
- 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 38
4*a^3*c^5*d^4*e^8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^
2*b^3*c^3*d^3*e^9 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 12
8*a^4*b*c^3*d*e^11 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d +
 e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^1...
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{x\,(c\,x^2+bx+a)}dx$$

input
```
int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x)
```

3.72 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 376

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = -\frac{d\sqrt{d+ex}}{ax}+\frac{\sqrt{d}(2bd-3ae)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(b^2d^2-2acd^2+bd\left(\sqrt{b^2-4ac}\,d-2ae\right)-2ae\left(\sqrt{b^2-4ac}\,d-ae\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(b^2d^2-2acd^2+2ae\left(\sqrt{b^2-4ac}\,d+ae\right)-bd\left(\sqrt{b^2-4ac}\,d+2ae\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-d*(e*x+d)^(1/2)/a/x+d^(1/2)*(-3*a*e+2*b*d)*arctanh((e*x+d)^(1/2)/d^(1/2))
/a^2-2^(1/2)*c^(1/2)*(b^2*d^2-2*a*c*d^2+b*d*((-4*a*c+b^2)^(1/2)*d-2*a*e)-2
*a*e*((-4*a*c+b^2)^(1/2)*d-a*e))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*
c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*
a*c+b^2)^(1/2))*e)^(1/2)+2^(1/2)*c^(1/2)*(b^2*d^2-2*a*c*d^2+2*a*e*((-4*a*c
+b^2)^(1/2)*d+a*e)-b*d*((-4*a*c+b^2)^(1/2)*d+2*a*e))*arctanh(2^(1/2)*c^(1/
2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^2/(-4*a*c+b^2)^
(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \frac{-\frac{ad\sqrt{d+ex}}{x}+\frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}d+ae\right)\right)\right)\arctan\left(\frac{\ }{\sqrt{-}}\right.}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{}$$

input
```
Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(-((a*d*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[-b^
2 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]
*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*S
qrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2
+ 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d -
(2*I)*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*
e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sq
rt[d]*(2*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \; 1199$$

$$\frac{2\int \left(\frac{d^2}{ax^2}-\frac{(bd-2ae)d}{a^2x}-\frac{e\left((bd-ae)\left(cd^2-bed+ae^2\right)-cd(bd-2ae)(d+ex)\right)}{a^2\left(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex)\right)}\right)d\sqrt{d+ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\sqrt{c}e\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+\right.}{}\right.$$

input
```
Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/2*(d*e*Sqrt[d + e*x])/(a*x) + (Sqrt[d]*e^2*ArcTanh[Sqrt[d + e*x]/Sq
rt[d]])/(2*a) + (Sqrt[d]*e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a
^2 - (Sqrt[c]*e*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*(c*d^2
+ e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*e*(b^2*d^2 - 2*a*c*d^2
+ 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.09

method	result
derivativedivides	$2e^3 \left(\frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2d^2eac+d^2e\,b^2-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{}}{\sqrt{\left(-be+2cd+\sqrt{-e^2}\right.}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{} \right.$
default	$2e^3 \left(\frac{4c\left(-\frac{\left(2a^2e^3-2abd\,e^2-2d^2eac+d^2e\,b^2-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}\,c\sqrt{}}{\sqrt{\left(-be+2cd+\sqrt{-e^2}\right.}}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{} \right.$
risch	$-\frac{d\sqrt{ex+d}}{ax}-\frac{e\left(\frac{\sqrt{d}\,(3ae-2bd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ea}+\frac{8c\left(-\frac{\left(-2a^2e^3+2abd\,e^2+2d^2eac-d^2e\,b^2+2\sqrt{-e^2\left(4ac-b^2\right)}\,ade-\sqrt{-e^2\left(\right.}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(-be+\right.}}\right.}{}\right.}{}$
pseudoelliptic	$\frac{2x\left(d\left(ae-\frac{bd}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-\left(e^2a^2-abde-a\,d^2c+\frac{1}{2}b^2d^2\right)e\right)\sqrt{d}\,\sqrt{2}\,c\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\mathrm{arctanh}\left(\right.}{}$

input
```
int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(4/a^2/e^3*c*(-1/8*(2*a^2*e^3-2*a*b*d*e^2-2*d^2*e*a*c+d^2*e*b^2-2*(-
e^2*(4*a*c-b^2))^(1/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
+1/8*(-2*a^2*e^3+2*a*b*d*e^2+2*d^2*e*a*c-d^2*e*b^2-2*(-e^2*(4*a*c-b^2))^(1
/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(
1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d/a^2/e^3*(1/2*a*(e*
x+d)^(1/2)/x+1/2*(3*a*e-2*b*d)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4321 vs. $2(322) = 644$.

Time = 18.95 (sec) , antiderivative size = 8650, normalized size of antiderivative = 23.01

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(322) = 644.

Time = 0.36 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-sqrt(e*x + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(e*x + d)/sqrt(-d)
)/(a^2*sqrt(-d)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*
b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2
 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c
)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c
)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d)/sqrt(-(2*a^2*c*d - a^2*b*e
 + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)
^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e
 + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c
 + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)
*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt
(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d
 + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a
*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4
*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x + d
)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a
^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 -
 sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)...
```

Mupad [B] (verification not implemented)

Time = 17.57 (sec) , antiderivative size = 29890, normalized size of antiderivative = 79.49

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
(d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^
8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4
*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*
a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 +
 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*
a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*
b*c^4*d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e
^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e
^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d
^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2
*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a
^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*
c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d
^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*(
(8*(d + e*x)^(1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*
e^13 - 40*a^5*c^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 -
 8*a^2*b^7*c^2*d^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 +
 28*a^3*b^6*c^2*d^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^
9 - 179*a^4*b^4*c^3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d
^3*e^10 + 215*a^5*b^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4...
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{x^2\,(c\,x^2+bx+a)}dx$$

input
```
int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x)
```

3.73 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 529

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(4bd-5ae)\sqrt{d+ex}}{4a^2x} - \frac{(8b^2d^2-12abde-a(8cd^2-3ae^2))\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a^3\sqrt{d}}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(b^3d^2+b^2d\left(\sqrt{b^2-4ac}\,d-2ae\right)+a\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)\right)-ab\left(3cd^2+e\left(2\sqrt{b^2\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$-\frac{\sqrt{2}\sqrt{c}\left(b^3d^2-b^2d\left(\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd^2-e\left(2\sqrt{b^2-4ac}\,d+ae\right)\right)-a\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2\right.\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/2*d*(e*x+d)^(1/2)/a/x^2+1/4*(-5*a*e+4*b*d)*(e*x+d)^(1/2)/a^2/x-1/4*(8*b
^2*d^2-12*a*b*d*e-a*(-3*a*e^2+8*c*d^2))*arctanh((e*x+d)^(1/2)/d^(1/2))/a^3
/d^(1/2)+2^(1/2)*c^(1/2)*(b^3*d^2+b^2*d*((-4*a*c+b^2)^(1/2)*d-2*a*e)+a*(a*
(-4*a*c+b^2)^(1/2)*e^2-c*d*((-4*a*c+b^2)^(1/2)*d-4*a*e))-a*b*(3*c*d^2+e*(2
*(-4*a*c+b^2)^(1/2)*d-a*e)))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+
b^2)^(1/2))*e)^(1/2)-2^(1/2)*c^(1/2)*(b^3*d^2-b^2*d*((-4*a*c+b^2)^(1/2)*d+
2*a*e)-a*b*(3*c*d^2-e*(2*(-4*a*c+b^2)^(1/2)*d+a*e))-a*(a*(-4*a*c+b^2)^(1/2
)*e^2-c*d*((-4*a*c+b^2)^(1/2)*d+4*a*e)))*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(
1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.27 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx=\frac{\frac{a\sqrt{d+ex}(-2ad+4bdx-5aex)}{x^2}+\frac{4\sqrt{2}\sqrt{c}\left(ib^3d^2-b^2d\left(\sqrt{-b^2+4ac}d+2iae\right)+ab\left(-3icd^2+e\left(2\sqrt{-b^2+4ac}d+iae\right.\right.\right.}{\sqrt{-b^2+4ac}\sqrt{\ }}}{}$$

input
```
Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
((a*Sqrt[d + e*x]*(-2*a*d + 4*b*d*x - 5*a*e*x))/x^2 + (4*Sqrt[2]*Sqrt[c]*(
I*b^3*d^2 - b^2*d*(Sqrt[-b^2 + 4*a*c]*d + (2*I)*a*e) + a*b*((-3*I)*c*d^2 +
 e*(2*Sqrt[-b^2 + 4*a*c]*d + I*a*e)) + a*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*
d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt
[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(I*b^3*d^2 +
 b^2*d*(Sqrt[-b^2 + 4*a*c]*d - (2*I)*a*e) + a*(a*Sqrt[-b^2 + 4*a*c]*e^2 +
c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + I*a*b*(-3*c*d^2 + e*((2*I)*Sq
rt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2
*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (
b + I*Sqrt[-b^2 + 4*a*c])*e]) + ((-8*b^2*d^2 + 12*a*b*d*e + a*(8*c*d^2 - 3
*a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/(4*a^3)
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\int \left(\frac{d^2}{ax^3}-\frac{(bd-2ae)d}{a^2x^2}+\frac{b^2d^2-2abed-a(cd^2-ae^2)}{a^3x}+\frac{e((db^2-aeb-acd)(cd^2-bed+ae^2)-c(b^2d^2-2abed-a(cd^2-ae^2))(d+ex))}{a^3(cd^2-bed+ae^2+c(d+ex)^2-(2cd-be)(d+ex))}\right)d\sqrt{d+}}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(-2abde-a(cd^2-ae^2)+b^2d^2)}{a^3\sqrt{d}}+\frac{\sqrt{c}e\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)+\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\right.}}\right.$$

input
```
Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(2*(-1/4*(d*e*Sqrt[d + e*x])/(a*x^2) + (3*e^2*Sqrt[d + e*x])/(8*a*x) + (e*
(b*d - 2*a*e)*Sqrt[d + e*x])/(2*a^2*x) - (3*e^3*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(8*a*Sqrt[d]) - (e^2*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(
2*a^2*Sqrt[d]) - (e*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt
[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*e*(b^3*d^2 + b^2*d*(Sqrt[b^2
- 4*a*c]*d - 2*a*e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*
d - 4*a*e)) - a*b*(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sq
rt[c]*e*(b^3*d^2 - b^2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*
(2*Sqrt[b^2 - 4*a*c]*d + a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^
2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e])))/e
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x^2\sqrt{d}\,\sqrt{2}\left(\left(\left(a\,d^2c-(ae-bd)^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-e\left(\left(4de\,a^2-3ab\,d^2\right)c+b(ae-bd)\right.\right.\right.}{}$
risch	$-\frac{\sqrt{ex+d}\,(5aex-4bdx+2ad)}{4a^2x^2}-\frac{e\left(-\frac{\left(-3e^2a^2+12abde+8a\,d^2c-8b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{ae\sqrt{d}}+\frac{32c\left(-\frac{\left(a^2b\,e^3+4a^2cd\,e^2-2a\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^4\left(\frac{4c\left(\frac{\left(a^2b\,e^3+4a^2cd\,e^2-2a\,b^2d\,e^2-3abc\,d^2e+b^3d^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,a^2e^2+2\sqrt{-e^2\left(4ac-b^2\right)}\,abde+\sqrt{-e^2\left(4ac-b^2\right)}\,ac\,d^2\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}\right.$
default	$2e^4\left(\frac{4c\left(\frac{\left(a^2b\,e^3+4a^2cd\,e^2-2a\,b^2d\,e^2-3abc\,d^2e+b^3d^2e-\sqrt{-e^2\left(4ac-b^2\right)}\,a^2e^2+2\sqrt{-e^2\left(4ac-b^2\right)}\,abde+\sqrt{-e^2\left(4ac-b^2\right)}\,ac\,d^2\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}\right.$

input
```
int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/d^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((-b*e+2*c*d
+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2)
)^(1/2))*c)^(1/2)*x^2*d^(1/2)*2^(1/2)*((a*d^2*c-(a*e-b*d)^2)*(-4*e^2*(a*c-
1/4*b^2))^(1/2)-e*((4*a^2*d*e-3*a*b*d^2)*c+b*(a*e-b*d)^2))*c*arctanh((e*x+
d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))-((
-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(x^2*d^(1/2)*2^(1/2)*c*(
(a*d^2*c-(a*e-b*d)^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*((4*a^2*d*e-3*a*b*d^2
)*c+b*(a*e-b*d)^2))*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*
c-1/4*b^2))^(1/2))*c)^(1/2))-1/2*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)*(3/2*x^2*(e^2*a^2-4*a*b*d*e-8/3*a*d^2*c+8/3*b^2*d^2)*arctanh((e*
x+d)^(1/2)/d^(1/2))+((-2*b*x+a)*d+5/2*a*e*x)*d^(1/2)*a*(e*x+d)^(1/2))*(-4*
e^2*(a*c-1/4*b^2))^(1/2)))/(-4*e^2*(a*c-1/4*b^2))^(1/2)/x^2/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7202 vs. $2(459) = 918$.

Time = 169.42 (sec) , antiderivative size = 14414, normalized size of antiderivative = 27.25

$$\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)x^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(459) = 918$.

Time = 0.33 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(e*x + d)/
sqrt(-d))/(a^3*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a
^2*b^2 - 4*a^3*c)*e^2)*a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b
^2 - 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2
*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
 4*a*c)*c)*e)*abs(a)*abs(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d
^3*e + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)
*sqrt(e*x + d)/sqrt(-(2*a^3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c
)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs
(a)*abs(c)*abs(e)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)
*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2
 - 4*a^3*c)*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4
*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 -
 a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c
)*c)*e)*abs(a)*abs(e) + (a^4*b^2*e^4 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3*e +
 (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - a^4*b*c)*d*e^3)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sq...
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 44649, normalized size of antiderivative = 84.40

$$\int \frac{(d+ex)^{3/2}}{x^3\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
(((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*
(d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan(((((
(192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^
10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*
b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*
a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^1
0 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^
9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^
5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3
 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2
 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a
^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*
a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(
1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a
^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 -...
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{x^3 (a+bx+cx^2)} \, dx = \int \frac{(ex+d)^{\frac{3}{2}}}{x^3 (c\,x^2+bx+a)} dx$$

input
```
int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)
```

3.74 $\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 521

$$\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} \, dx = \frac{3(b^2-4ac)\left(144b^3cde^2-33b^4e^3-8b^2ce(28cd^2-9ae^2)+64bc^2d(2cd^2-3ae^2)+16ac^2e(8cd\right.}{16384c^6}$$
$$-\frac{\left(144b^3cde^2-33b^4e^3-8b^2ce(28cd^2-9ae^2)+64bc^2d(2cd^2-3ae^2)+16ac^2e(8cd^2-ae^2)\right)(b+2cx)\left(a+\right.}{2048c^5}$$
$$+\frac{(6cd-11be)(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2}+\frac{(d+ex)^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$
$$+\frac{\left(3(32c^3d^3-77b^3e^3+4bce^2(84bd+31ae)-8c^2de(47bd+32ae))+10ce(8c^2d^2+33b^2e^2-28ce(2bd+ae\right.}{4480c^4}$$
$$-\frac{3(b^2-4ac)^2\left(144b^3cde^2-33b^4e^3-8b^2ce(28cd^2-9ae^2)+64bc^2d(2cd^2-3ae^2)+16ac^2e(8cd^2-ae^2)\right)\text{ar}}{32768c^{13/2}}$$

output
```
3/16384*(-4*a*c+b^2)*(144*b^3*c*d*e^2-33*b^4*e^3-8*b^2*c*e*(-9*a*e^2+28*c*
d^2)+64*b*c^2*d*(-3*a*e^2+2*c*d^2)+16*a*c^2*e*(-a*e^2+8*c*d^2))*(2*c*x+b)*
(c*x^2+b*x+a)^(1/2)/c^6-1/2048*(144*b^3*c*d*e^2-33*b^4*e^3-8*b^2*c*e*(-9*a
*e^2+28*c*d^2)+64*b*c^2*d*(-3*a*e^2+2*c*d^2)+16*a*c^2*e*(-a*e^2+8*c*d^2))*
(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^5+1/112*(-11*b*e+6*c*d)*(e*x+d)^2*(c*x^2+b
*x+a)^(5/2)/c^2+1/8*(e*x+d)^3*(c*x^2+b*x+a)^(5/2)/c+1/4480*(96*c^3*d^3-231
*b^3*e^3+12*b*c*e^2*(31*a*e+84*b*d)-24*c^2*d*e*(32*a*e+47*b*d)+10*c*e*(8*c
^2*d^2+33*b^2*e^2-28*c*e*(a*e+2*b*d))*x)*(c*x^2+b*x+a)^(5/2)/c^4-3/32768*(
-4*a*c+b^2)^2*(144*b^3*c*d*e^2-33*b^4*e^3-8*b^2*c*e*(-9*a*e^2+28*c*d^2)+64
*b*c^2*d*(-3*a*e^2+2*c*d^2)+16*a*c^2*e*(-a*e^2+8*c*d^2))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.32

$$\int x(d+ex)^3\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^7e^3+210b^6ce^2(72d+11ex)-84b^5ce(-365ae^2+2c(140d^2+6}{}$$

input
```
Integrate[x*(d + e*x)^3*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7*e^3 + 210*b^6*c*e^2*(72*d + 11
*e*x) - 84*b^5*c*e*(-365*a*e^2 + 2*c*(140*d^2 + 60*d*e*x + 11*e^2*x^2)) -
16*b^3*c^2*(5103*a^2*e^3 - 4*a*c*e*(2660*d^2 + 1092*d*e*x + 195*e^2*x^2) +
 8*c^2*x*(70*d^3 + 98*d^2*e*x + 54*d*e^2*x^2 + 11*e^3*x^3)) + 32*b^2*c^3*(
a^2*e^2*(8232*d + 1181*e*x) + 8*c^2*x^2*(28*d^3 + 42*d^2*e*x + 24*d*e^2*x^
2 + 5*e^3*x^3) - 4*a*c*(700*d^3 + 756*d^2*e*x + 372*d*e^2*x^2 + 71*e^3*x^3
)) + 64*b*c^3*(919*a^3*e^3 - 2*a^2*c*e*(2268*d^2 + 876*d*e*x + 151*e^2*x^2
) + 8*a*c^2*x*(98*d^3 + 126*d^2*e*x + 66*d*e^2*x^2 + 13*e^3*x^3) + 16*c^3*
x^3*(154*d^3 + 364*d^2*e*x + 300*d*e^2*x^2 + 85*e^3*x^3)) + 8*b^4*c^2*(-21
*a*e^2*(720*d + 107*e*x) + 2*c*(840*d^3 + 980*d^2*e*x + 504*d*e^2*x^2 + 99
*e^3*x^3)) + 128*c^4*(-3*a^3*e^2*(256*d + 35*e*x) + 16*c^3*x^4*(56*d^3 + 1
40*d^2*e*x + 120*d*e^2*x^2 + 35*e^3*x^3) + 2*a^2*c*(448*d^3 + 420*d^2*e*x
+ 192*d*e^2*x^2 + 35*e^3*x^3) + 8*a*c^2*x^2*(224*d^3 + 490*d^2*e*x + 384*d
*e^2*x^2 + 105*e^3*x^3))) - 105*(b^2 - 4*a*c)^2*(-144*b^3*c*d*e^2 + 33*b^4
*e^3 + 8*b^2*c*e*(28*c*d^2 - 9*a*e^2) - 64*b*c^2*d*(2*c*d^2 - 3*a*e^2) + 1
6*a*c^2*e*(-8*c*d^2 + a*e^2))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/(1146880*c^(13/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.75, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1267, 27, 2184, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int \frac{1}{2}x\left(cx^2+bx+a\right)^{3/2}\left(16cd^3+e^2(48cd-11be)x^2+6e\left(8cd^2-ae^2\right)x\right)dx}{8c}+\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\left(cx^2+bx+a\right)^{3/2}\left(16cd^3+e^2(48cd-11be)x^2+6e\left(8cd^2-ae^2\right)x\right)dx}{16c}+\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 2184

$$\frac{\frac{\int \frac{1}{2}x\left(4\left(56c^2d^3-48ace^2d+11abe^3\right)+3e\left(224c^2d^2+33b^2e^2-4ce(36bd+7ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{7c}+\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}(48cd-11be)}{7c}}{16c}+\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 27

$$\frac{\frac{\int x\left(4\left(56c^2d^3-48ace^2d+11abe^3\right)+3e\left(224c^2d^2+33b^2e^2-4ce(36bd+7ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{14c}+\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}(48cd-11be)}{7c}}{16c}+\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 1225

$$\frac{\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(10cex\left(-4ce(7ae+36bd)+33b^2e^2+224c^2d^2\right)-32c^2de(24ae+49bd)+12bce^2(31ae+84bd)-231b^3e^3+896c^3d^3\right)}{20c^2}-\frac{7\left(-8b^2ce\left(28cd^2-9ae^2\right)+64bc^2\cdots\right.}{\cdots}}{14c}\cdots}{16c}$$

$$\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 1087

$$\frac{\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(10cex\left(-4ce(7ae+36bd)+33b^2e^2+224c^2d^2\right)-32c^2de(24ae+49bd)+12bce^2(31ae+84bd)-231b^3e^3+896c^3d^3\right)}{20c^2}-\frac{7\left(-8b^2ce\left(28cd^2-9ae^2\right)+64bc^2\cdots\right.}{\cdots}}{14c}\cdots}{16\cdots}$$

$$\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 1087

$$\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(10cex\left(-4ce(7ae+36bd)+33b^2e^2+224c^2d^2\right)-32c^2de(24ae+49bd)+12bce^2(31ae+84bd)-231b^3e^3+896c^3d^3\right)}{20c^2}-\frac{7\left(-8b^2ce\left(28cd^2-9ae^2\right)+64bc^2 \cdots\right.}{\cdots}}{14c \cdots}$$

$$\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 1092

$$\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(10cex\left(-4ce(7ae+36bd)+33b^2e^2+224c^2d^2\right)-32c^2de(24ae+49bd)+12bce^2(31ae+84bd)-231b^3e^3+896c^3d^3\right)}{20c^2}-\frac{7\left(-8b^2ce\left(28cd^2-9ae^2\right)+64bc^2 \cdots\right.}{\cdots}}{\cdots}$$

$$\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 219

$$\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(10cex\left(-4ce(7ae+36bd)+33b^2e^2+224c^2d^2\right)-32c^2de(24ae+49bd)+12bce^2(31ae+84bd)-231b^3e^3+896c^3d^3\right)}{20c^2}-\frac{7\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3(\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{e^3x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

input

```
Int[x*(d + e*x)^3*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(e^3*x^3*(a + b*x + c*x^2)^(5/2))/(8*c) + ((e^2*(48*c*d - 11*b*e)*x^2*(a +
 b*x + c*x^2)^(5/2))/(7*c) + (((896*c^3*d^3 - 231*b^3*e^3 - 32*c^2*d*e*(49
*b*d + 24*a*e) + 12*b*c*e^2*(84*b*d + 31*a*e) + 10*c*e*(224*c^2*d^2 + 33*b
^2*e^2 - 4*c*e*(36*b*d + 7*a*e))*x)*(a + b*x + c*x^2)^(5/2))/(20*c^2) - (7
*(144*b^3*c*d*e^2 - 33*b^4*e^3 - 8*b^2*c*e*(28*c*d^2 - 9*a*e^2) + 64*b*c^2
*d*(2*c*d^2 - 3*a*e^2) + 16*a*c^2*e*(8*c*d^2 - a*e^2))*(((b + 2*c*x)*(a +
b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x +
c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(14*c))/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(488) = 976.

Time = 1.38 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.90

method	result
risch	$\frac{(71680e^{3}c^{7}x^{7}+87040b\,c^{6}e^{3}x^{6}+245760c^{7}d\,e^{2}x^{6}+107520a\,c^{6}e^{3}x^{5}+1280b^{2}c^{5}e^{3}x^{5}+307200b\,c^{6}d\,e^{2}x^{5}+286720c^{7}d^{2}e\,x^{5}+6656ab\,c^{5}e^{3}}{}$
default	Expression too large to display

input
```
int(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/573440/c^6*(71680*c^7*e^3*x^7+87040*b*c^6*e^3*x^6+245760*c^7*d*e^2*x^6+1
07520*a*c^6*e^3*x^5+1280*b^2*c^5*e^3*x^5+307200*b*c^6*d*e^2*x^5+286720*c^7
*d^2*e*x^5+6656*a*b*c^5*e^3*x^4+393216*a*c^6*d*e^2*x^4-1408*b^3*c^4*e^3*x^
4+6144*b^2*c^5*d*e^2*x^4+372736*b*c^6*d^2*e*x^4+114688*c^7*d^3*x^4+8960*a^
2*c^5*e^3*x^3-9088*a*b^2*c^4*e^3*x^3+33792*a*b*c^5*d*e^2*x^3+501760*a*c^6*
d^2*e*x^3+1584*b^4*c^3*e^3*x^3-6912*b^3*c^4*d*e^2*x^3+10752*b^2*c^5*d^2*e*
x^3+157696*b*c^6*d^3*x^3-19328*a^2*b*c^4*e^3*x^2+49152*a^2*c^5*d*e^2*x^2+1
2480*a*b^3*c^3*e^3*x^2-47616*a*b^2*c^4*d*e^2*x^2+64512*a*b*c^5*d^2*e*x^2+2
29376*a*c^6*d^3*x^2-1848*b^5*c^2*e^3*x^2+8064*b^4*c^3*d*e^2*x^2-12544*b^3*
c^4*d^2*e*x^2+7168*b^2*c^5*d^3*x^2-13440*a^3*c^4*e^3*x+37792*a^2*b^2*c^3*e
^3*x-112128*a^2*b*c^4*d*e^2*x+107520*a^2*c^5*d^2*e*x-17976*a*b^4*c^2*e^3*x
+69888*a*b^3*c^3*d*e^2*x-96768*a*b^2*c^4*d^2*e*x+50176*a*b*c^5*d^3*x+2310*
b^6*c*e^3*x-10080*b^5*c^2*d*e^2*x+15680*b^4*c^3*d^2*e*x-8960*b^3*c^4*d^3*x
+58816*a^3*b*c^3*e^3-98304*a^3*c^4*d*e^2-81648*a^2*b^3*c^2*e^3+263424*a^2*
b^2*c^3*d*e^2-290304*a^2*b*c^4*d^2*e+114688*a^2*c^5*d^3+30660*a*b^5*c*e^3-
120960*a*b^4*c^2*d*e^2+170240*a*b^3*c^3*d^2*e-89600*a*b^2*c^4*d^3-3465*b^7
*e^3+15120*b^6*c*d*e^2-23520*b^5*c^2*d^2*e+13440*b^4*c^3*d^3)*(c*x^2+b*x+a
)^(1/2)+3/32768*(256*a^4*c^4*e^3-1280*a^3*b^2*c^3*e^3+3072*a^3*b*c^4*d*e^2
-2048*a^3*c^5*d^2*e+1120*a^2*b^4*c^2*e^3-3840*a^2*b^3*c^3*d*e^2+4608*a^2*b
^2*c^4*d^2*e-2048*a^2*b*c^5*d^3-336*a*b^6*c*e^3+1344*a*b^5*c^2*d*e^2-19...
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1719, normalized size of antiderivative = 3.30

$$\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2293760*(105*(128*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d^3 - 32*(7*b
^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*e + 48*(3*b^7*c
- 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*e^2 - (33*b^8 - 336*a*b^
6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*e^3)*sqrt(c)*log(
-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) -
 4*a*c) - 4*(71680*c^8*e^3*x^7 + 5120*(48*c^8*d*e^2 + 17*b*c^7*e^3)*x^6 +
1280*(224*c^8*d^2*e + 240*b*c^7*d*e^2 + (b^2*c^6 + 84*a*c^7)*e^3)*x^5 + 12
8*(896*c^8*d^3 + 2912*b*c^7*d^2*e + 48*(b^2*c^6 + 64*a*c^7)*d*e^2 - (11*b^
3*c^5 - 52*a*b*c^6)*e^3)*x^4 + 896*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c
^6)*d^3 - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d^2*e + 48*(3
15*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*d*e^2 - (34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*e^3 + 16
*(9856*b*c^7*d^3 + 224*(3*b^2*c^6 + 140*a*c^7)*d^2*e - 48*(9*b^3*c^5 - 44*
a*b*c^6)*d*e^2 + (99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*e^3)*x^3 + 8*(
896*(b^2*c^6 + 32*a*c^7)*d^3 - 224*(7*b^3*c^5 - 36*a*b*c^6)*d^2*e + 48*(21
*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*d*e^2 - (231*b^5*c^3 - 1560*a*b^3*
c^4 + 2416*a^2*b*c^5)*e^3)*x^2 - 2*(896*(5*b^3*c^5 - 28*a*b*c^6)*d^3 - 224
*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d^2*e + 48*(105*b^5*c^3 - 728*
a*b^3*c^4 + 1168*a^2*b*c^5)*d*e^2 - (1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896
*a^2*b^2*c^4 - 6720*a^3*c^5)*e^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/1146...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5372 vs. $2(541) = 1082$.

Time = 0.92 (sec) , antiderivative size = 5372, normalized size of antiderivative = 10.31

$$\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)**3*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(3*a**2*d**2*e + 2*a*b*d**3 - 3*a*(a**2*e**3 + 6*a*b*d*e**2
 + 6*a*c*d**2*e - 5*a*(9*a*c*e**3/8 + b**2*e**3 + 6*b*c*d*e**2 - 13*b*(17*
b*c*e**3/16 + 3*c**2*d*e**2)/(14*c) + 3*c**2*d**2*e)/(6*c) + 3*b**2*d**2*e
 + 2*b*c*d**3 - 9*b*(2*a*b*e**3 + 6*a*c*d*e**2 - 6*a*(17*b*c*e**3/16 + 3*c
**2*d*e**2)/(7*c) + 3*b**2*d*e**2 + 6*b*c*d**2*e - 11*b*(9*a*c*e**3/8 + b*
*2*e**3 + 6*b*c*d*e**2 - 13*b*(17*b*c*e**3/16 + 3*c**2*d*e**2)/(14*c) + 3*
c**2*d**2*e)/(12*c) + c**2*d**3)/(10*c))/(4*c) - 5*b*(3*a**2*d*e**2 + 6*a*
b*d**2*e + 2*a*c*d**3 - 4*a*(2*a*b*e**3 + 6*a*c*d*e**2 - 6*a*(17*b*c*e**3/
16 + 3*c**2*d*e**2)/(7*c) + 3*b**2*d*e**2 + 6*b*c*d**2*e - 11*b*(9*a*c*e**
3/8 + b**2*e**3 + 6*b*c*d*e**2 - 13*b*(17*b*c*e**3/16 + 3*c**2*d*e**2)/(14
*c) + 3*c**2*d**2*e)/(12*c) + c**2*d**3)/(5*c) + b**2*d**3 - 7*b*(a**2*e**
3 + 6*a*b*d*e**2 + 6*a*c*d**2*e - 5*a*(9*a*c*e**3/8 + b**2*e**3 + 6*b*c*d*
e**2 - 13*b*(17*b*c*e**3/16 + 3*c**2*d*e**2)/(14*c) + 3*c**2*d**2*e)/(6*c)
 + 3*b**2*d**2*e + 2*b*c*d**3 - 9*b*(2*a*b*e**3 + 6*a*c*d*e**2 - 6*a*(17*b
*c*e**3/16 + 3*c**2*d*e**2)/(7*c) + 3*b**2*d*e**2 + 6*b*c*d**2*e - 11*b*(9
*a*c*e**3/8 + b**2*e**3 + 6*b*c*d*e**2 - 13*b*(17*b*c*e**3/16 + 3*c**2*d*e
**2)/(14*c) + 3*c**2*d**2*e)/(12*c) + c**2*d**3)/(10*c))/(8*c))/(6*c))/(2*
c) - b*(a**2*d**3 - 2*a*(3*a**2*d*e**2 + 6*a*b*d**2*e + 2*a*c*d**3 - 4*a*(
2*a*b*e**3 + 6*a*c*d*e**2 - 6*a*(17*b*c*e**3/16 + 3*c**2*d*e**2)/(7*c) + 3
*b**2*d*e**2 + 6*b*c*d**2*e - 11*b*(9*a*c*e**3/8 + b**2*e**3 + 6*b*c*d*...
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+ex)^3\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 948, normalized size of antiderivative = 1.82

$$\int x(d+ex)^3\left(a+bx+cx^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*e^3*x + (48*c^8*d*
e^2 + 17*b*c^7*e^3)/c^7)*x + (224*c^8*d^2*e + 240*b*c^7*d*e^2 + b^2*c^6*e^
3 + 84*a*c^7*e^3)/c^7)*x + (896*c^8*d^3 + 2912*b*c^7*d^2*e + 48*b^2*c^6*d*
e^2 + 3072*a*c^7*d*e^2 - 11*b^3*c^5*e^3 + 52*a*b*c^6*e^3)/c^7)*x + (9856*b
*c^7*d^3 + 672*b^2*c^6*d^2*e + 31360*a*c^7*d^2*e - 432*b^3*c^5*d*e^2 + 211
2*a*b*c^6*d*e^2 + 99*b^4*c^4*e^3 - 568*a*b^2*c^5*e^3 + 560*a^2*c^6*e^3)/c^
7)*x + (896*b^2*c^6*d^3 + 28672*a*c^7*d^3 - 1568*b^3*c^5*d^2*e + 8064*a*b*
c^6*d^2*e + 1008*b^4*c^4*d*e^2 - 5952*a*b^2*c^5*d*e^2 + 6144*a^2*c^6*d*e^2
 - 231*b^5*c^3*e^3 + 1560*a*b^3*c^4*e^3 - 2416*a^2*b*c^5*e^3)/c^7)*x - (44
80*b^3*c^5*d^3 - 25088*a*b*c^6*d^3 - 7840*b^4*c^4*d^2*e + 48384*a*b^2*c^5*
d^2*e - 53760*a^2*c^6*d^2*e + 5040*b^5*c^3*d*e^2 - 34944*a*b^3*c^4*d*e^2 +
 56064*a^2*b*c^5*d*e^2 - 1155*b^6*c^2*e^3 + 8988*a*b^4*c^3*e^3 - 18896*a^2
*b^2*c^4*e^3 + 6720*a^3*c^5*e^3)/c^7)*x + (13440*b^4*c^4*d^3 - 89600*a*b^2
*c^5*d^3 + 114688*a^2*c^6*d^3 - 23520*b^5*c^3*d^2*e + 170240*a*b^3*c^4*d^2
*e - 290304*a^2*b*c^5*d^2*e + 15120*b^6*c^2*d*e^2 - 120960*a*b^4*c^3*d*e^2
 + 263424*a^2*b^2*c^4*d*e^2 - 98304*a^3*c^5*d*e^2 - 3465*b^7*c*e^3 + 30660
*a*b^5*c^2*e^3 - 81648*a^2*b^3*c^3*e^3 + 58816*a^3*b*c^4*e^3)/c^7) + 3/327
68*(128*b^5*c^3*d^3 - 1024*a*b^3*c^4*d^3 + 2048*a^2*b*c^5*d^3 - 224*b^6*c^
2*d^2*e + 1920*a*b^4*c^3*d^2*e - 4608*a^2*b^2*c^4*d^2*e + 2048*a^3*c^5*d^2
*e + 144*b^7*c*d*e^2 - 1344*a*b^5*c^2*d*e^2 + 3840*a^2*b^3*c^3*d*e^2 - ...
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^3\left(a+bx+cx^2\right)^{3/2}\,dx=\int x\,{\left(d+e\,x\right)}^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,dx$$

input
```
int(x*(d + e*x)^3*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x*(d + e*x)^3*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d+ex)^3 \left(a+bx+cx^2\right)^{3/2} dx = \int x(ex+d)^3 \left(c\,x^2+bx+a\right)^{\frac{3}{2}} dx$$

input
```
int(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x*(e*x+d)^3*(c*x^2+b*x+a)^(3/2),x)
```

3.75 $\int x(d+ex)^2 \left(a+bx+cx^2\right)^{3/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 331

$$\begin{aligned}
&\int x(d+ex)^2 \left(a+bx+cx^2\right)^{3/2} dx = \\
&-\frac{(b^2-4ac)\left(28b^2cde-16ac^2de-9b^3e^2-12bc(2cd^2-ae^2)\right)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^5} \\
&+\frac{\left(28b^2cde-16ac^2de-9b^3e^2-12bc(2cd^2-ae^2)\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{384c^4} \\
&+\frac{(d+ex)^2\left(a+bx+cx^2\right)^{5/2}}{7c} \\
&+\frac{\left(48c^2d^2+63b^2e^2-4ce(49bd+12ae)+10ce(4cd-9be)x\right)\left(a+bx+cx^2\right)^{5/2}}{840c^3} \\
&+\frac{(b^2-4ac)^2\left(28b^2cde-16ac^2de-9b^3e^2-12bc(2cd^2-ae^2)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}}
\end{aligned}$$

output
```
-1/1024*(-4*a*c+b^2)*(28*b^2*c*d*e-16*a*c^2*d*e-9*b^3*e^2-12*b*c*(-a*e^2+2
*c*d^2))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5+1/384*(28*b^2*c*d*e-16*a*c^2*d*
e-9*b^3*e^2-12*b*c*(-a*e^2+2*c*d^2))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7
*(e*x+d)^2*(c*x^2+b*x+a)^(5/2)/c+1/840*(48*c^2*d^2+63*b^2*e^2-4*c*e*(12*a*
e+49*b*d)+10*c*e*(-9*b*e+4*c*d)*x)*(c*x^2+b*x+a)^(5/2)/c^3+1/2048*(-4*a*c+
b^2)^2*(28*b^2*c*d*e-16*a*c^2*d*e-9*b^3*e^2-12*b*c*(-a*e^2+2*c*d^2))*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.35

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(945b^6e^2-210b^5ce(14d+3ex)+56b^4c(-135ae^2+c(45d^2+35dex+9\right.}{}$$

input
```
Integrate[x*(d + e*x)^2*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^6*e^2 - 210*b^5*c*e*(14*d + 3*e*x)
 + 56*b^4*c*(-135*a*e^2 + c*(45*d^2 + 35*d*e*x + 9*e^2*x^2)) + 16*b^3*c^2*
(7*a*e*(190*d + 39*e*x) - c*x*(105*d^2 + 98*d*e*x + 27*e^2*x^2)) + 48*b^2*
c^2*(343*a^2*e^2 + 4*c^2*x^2*(7*d^2 + 7*d*e*x + 2*e^2*x^2) - 2*a*c*(175*d^
2 + 126*d*e*x + 31*e^2*x^2)) + 128*c^3*(-48*a^3*e^2 + 3*a^2*c*(56*d^2 + 35
*d*e*x + 8*e^2*x^2) + 8*c^3*x^4*(21*d^2 + 35*d*e*x + 15*e^2*x^2) + 2*a*c^2
*x^2*(168*d^2 + 245*d*e*x + 96*e^2*x^2)) + 32*b*c^3*(-3*a^2*e*(378*d + 73*
e*x) + 6*a*c*x*(49*d^2 + 42*d*e*x + 11*e^2*x^2) + 4*c^2*x^3*(231*d^2 + 364
*d*e*x + 150*e^2*x^2))) + 105*(b^2 - 4*a*c)^2*(-28*b^2*c*d*e + 16*a*c^2*d*
e + 9*b^3*e^2 + 12*b*c*(2*c*d^2 - a*e^2))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)]])/(215040*c^(11/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1267, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1267}$$

$$\frac{\int \frac{1}{2}x\left(14cd^2-4ae^2+e(28cd-9be)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{7c}+\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\left(2\left(7cd^2-2ae^2\right)+e(28cd-9be)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{14c}+\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{7\left(-12bc\left(2cd^2-ae^2\right)-16ac^2de-9b^3e^2+28b^2cde\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-24c\left(7cd^2-2ae^2\right)-10cex(28cd-9be)+7be(28cd-9be\right.}{60c^2}}{14c}$$

$$\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{7\left(-12bc\left(2cd^2-ae^2\right)-16ac^2de-9b^3e^2+28b^2cde\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^2+bx+a}dx}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-24c\left(7cd^2-2ae^2\right)\right.}{6\ldots}}{14c}$$

$$\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{7\left(-12bc\left(2cd^2-ae^2\right)-16ac^2de-9b^3e^2+28b^2cde\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{24c^2}-\ldots}{14c}$$

$$\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{7\left(-12bc\left(2cd^2-ae^2\right)-16ac^2de-9b^3e^2+28b^2cde\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx}\ldots}}{4c}\ldots\right)}{16c}\ldots\right)}{24c^2}\ldots}{14c}$$

$$\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{7\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-12bc\left(2cd^2-ae^2\right)-16ac^2de-9b^3e^2+28\right.}{24c^2}}{14c}$$

$$\frac{e^2x^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

input
```
Int[x*(d + e*x)^2*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(e^2*x^2*(a + b*x + c*x^2)^(5/2))/(7*c) + (-1/60*((7*b*e*(28*c*d - 9*b*e)
- 24*c*(7*c*d^2 - 2*a*e^2) - 10*c*e*(28*c*d - 9*b*e)*x)*(a + b*x + c*x^2)^
(5/2))/c^2 + (7*(28*b^2*c*d*e - 16*a*c^2*d*e - 9*b^3*e^2 - 12*b*c*(2*c*d^2
 - a*e^2))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)
*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24*c^2)
)/(14*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{\left(-15360c^6e^2x^6-19200b\,c^5e^2x^5-35840c^6de\,x^5-24576a\,c^5e^2x^4-384b^2c^4e^2x^4-46592b\,c^5de\,x^4-21504c^6d^2x^4-2112ab\,c^4e^2x^3-6\right.}{}$
default	$d^2\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}\right)+$

input
```
int(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/107520/c^5*(-15360*c^6*e^2*x^6-19200*b*c^5*e^2*x^5-35840*c^6*d*e*x^5-24
576*a*c^5*e^2*x^4-384*b^2*c^4*e^2*x^4-46592*b*c^5*d*e*x^4-21504*c^6*d^2*x^
4-2112*a*b*c^4*e^2*x^3-62720*a*c^5*d*e*x^3+432*b^3*c^3*e^2*x^3-1344*b^2*c^
4*d*e*x^3-29568*b*c^5*d^2*x^3-3072*a^2*c^4*e^2*x^2+2976*a*b^2*c^3*e^2*x^2-
8064*a*b*c^4*d*e*x^2-43008*a*c^5*d^2*x^2-504*b^4*c^2*e^2*x^2+1568*b^3*c^3*
d*e*x^2-1344*b^2*c^4*d^2*x^2+7008*a^2*b*c^3*e^2*x-13440*a^2*c^4*d*e*x-4368
*a*b^3*c^2*e^2*x+12096*a*b^2*c^3*d*e*x-9408*a*b*c^4*d^2*x+630*b^5*c*e^2*x-
1960*b^4*c^2*d*e*x+1680*b^3*c^3*d^2*x+6144*a^3*c^3*e^2-16464*a^2*b^2*c^2*e
^2+36288*a^2*b*c^3*d*e-21504*a^2*c^4*d^2+7560*a*b^4*c*e^2-21280*a*b^3*c^2*
d*e+16800*a*b^2*c^3*d^2-945*b^6*e^2+2940*b^5*c*d*e-2520*b^4*c^2*d^2)*(c*x^
2+b*x+a)^(1/2)+1/2048*(192*a^3*b*c^3*e^2-256*a^3*c^4*d*e-240*a^2*b^3*c^2*e
^2+576*a^2*b^2*c^3*d*e-384*a^2*b*c^4*d^2+84*a*b^5*c*e^2-240*a*b^4*c^2*d*e+
192*a*b^3*c^3*d^2-9*b^7*e^2+28*b^6*c*d*e-24*b^5*c^2*d^2)/c^(11/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.48

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/430080*(105*(24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d^2 - 4*(7*b^6*
c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*d*e + 3*(3*b^7 - 28*a*b^5
*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(15360*c^
7*e^2*x^6 + 1280*(28*c^7*d*e + 15*b*c^6*e^2)*x^5 + 128*(168*c^7*d^2 + 364*
b*c^6*d*e + 3*(b^2*c^5 + 64*a*c^6)*e^2)*x^4 + 16*(1848*b*c^6*d^2 + 28*(3*b
^2*c^5 + 140*a*c^6)*d*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*e^2)*x^3 + 168*(15*b^
4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*d^2 - 28*(105*b^5*c^2 - 760*a*b^3*c^3
 + 1296*a^2*b*c^4)*d*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3
- 2048*a^3*c^4)*e^2 + 8*(168*(b^2*c^5 + 32*a*c^6)*d^2 - 28*(7*b^3*c^4 - 36
*a*b*c^5)*d*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*e^2)*x^2 - 2*
(168*(5*b^3*c^4 - 28*a*b*c^5)*d^2 - 28*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a
^2*c^5)*d*e + 3*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*e^2)*x)*sqr
t(c*x^2 + b*x + a))/c^6, 1/215040*(105*(24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*d^2 - 4*(7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*d*
e + 3*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*e^2)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*
c)) + 2*(15360*c^7*e^2*x^6 + 1280*(28*c^7*d*e + 15*b*c^6*e^2)*x^5 + 128*(1
68*c^7*d^2 + 364*b*c^6*d*e + 3*(b^2*c^5 + 64*a*c^6)*e^2)*x^4 + 16*(1848*b*
c^6*d^2 + 28*(3*b^2*c^5 + 140*a*c^6)*d*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*e...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2742 vs. $2(335) = 670$.

Time = 0.86 (sec) , antiderivative size = 2742, normalized size of antiderivative = 8.28

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)**2*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(2*a**2*d*e + 2*a*b*d**2 - 3*a*(2*a*b*e**2 + 4*a*c*d*e - 5*
a*(15*b*c*e**2/14 + 2*c**2*d*e)/(6*c) + 2*b**2*d*e + 2*b*c*d**2 - 9*b*(8*a
*c*e**2/7 + b**2*e**2 + 4*b*c*d*e - 11*b*(15*b*c*e**2/14 + 2*c**2*d*e)/(12
*c) + c**2*d**2)/(10*c))/(4*c) - 5*b*(a**2*e**2 + 4*a*b*d*e + 2*a*c*d**2 -
 4*a*(8*a*c*e**2/7 + b**2*e**2 + 4*b*c*d*e - 11*b*(15*b*c*e**2/14 + 2*c**2
*d*e)/(12*c) + c**2*d**2)/(5*c) + b**2*d**2 - 7*b*(2*a*b*e**2 + 4*a*c*d*e
- 5*a*(15*b*c*e**2/14 + 2*c**2*d*e)/(6*c) + 2*b**2*d*e + 2*b*c*d**2 - 9*b*
(8*a*c*e**2/7 + b**2*e**2 + 4*b*c*d*e - 11*b*(15*b*c*e**2/14 + 2*c**2*d*e)
/(12*c) + c**2*d**2)/(10*c))/(8*c))/(6*c))/(2*c) - b*(a**2*d**2 - 2*a*(a**
2*e**2 + 4*a*b*d*e + 2*a*c*d**2 - 4*a*(8*a*c*e**2/7 + b**2*e**2 + 4*b*c*d*
e - 11*b*(15*b*c*e**2/14 + 2*c**2*d*e)/(12*c) + c**2*d**2)/(5*c) + b**2*d*
*2 - 7*b*(2*a*b*e**2 + 4*a*c*d*e - 5*a*(15*b*c*e**2/14 + 2*c**2*d*e)/(6*c)
 + 2*b**2*d*e + 2*b*c*d**2 - 9*b*(8*a*c*e**2/7 + b**2*e**2 + 4*b*c*d*e - 1
1*b*(15*b*c*e**2/14 + 2*c**2*d*e)/(12*c) + c**2*d**2)/(10*c))/(8*c))/(3*c)
 - 3*b*(2*a**2*d*e + 2*a*b*d**2 - 3*a*(2*a*b*e**2 + 4*a*c*d*e - 5*a*(15*b*
c*e**2/14 + 2*c**2*d*e)/(6*c) + 2*b**2*d*e + 2*b*c*d**2 - 9*b*(8*a*c*e**2/
7 + b**2*e**2 + 4*b*c*d*e - 11*b*(15*b*c*e**2/14 + 2*c**2*d*e)/(12*c) + c*
*2*d**2)/(10*c))/(4*c) - 5*b*(a**2*e**2 + 4*a*b*d*e + 2*a*c*d**2 - 4*a*(8*
a*c*e**2/7 + b**2*e**2 + 4*b*c*d*e - 11*b*(15*b*c*e**2/14 + 2*c**2*d*e)/(1
2*c) + c**2*d**2)/(5*c) + b**2*d**2 - 7*b*(2*a*b*e**2 + 4*a*c*d*e - 5*a...
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.82

$$\int x(d+ex)^2 \left(a+bx+cx^2\right)^{3/2} dx = \frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,ce^2x+\frac{28\,c^7de+15\,bc^6e^2}{c^6}\right)x+\frac{168\,c^7d^2+364\right.\right.\right.\right.\right.\right.$$
$$+\frac{(24\,b^5c^2d^2-192\,ab^3c^3d^2+384\,a^2bc^4d^2-28\,b^6cde+240\,ab^4c^2de-576\,a^2b^2c^3de+256\,a^3c^4de+9\,b^7e^2-8}{2048\,c^{\frac{11}{2}}}$$

input
```
integrate(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*e^2*x + (28*c^7*d*e +
 15*b*c^6*e^2)/c^6)*x + (168*c^7*d^2 + 364*b*c^6*d*e + 3*b^2*c^5*e^2 + 192
*a*c^6*e^2)/c^6)*x + (1848*b*c^6*d^2 + 84*b^2*c^5*d*e + 3920*a*c^6*d*e - 2
7*b^3*c^4*e^2 + 132*a*b*c^5*e^2)/c^6)*x + (168*b^2*c^5*d^2 + 5376*a*c^6*d^
2 - 196*b^3*c^4*d*e + 1008*a*b*c^5*d*e + 63*b^4*c^3*e^2 - 372*a*b^2*c^4*e^
2 + 384*a^2*c^5*e^2)/c^6)*x - (840*b^3*c^4*d^2 - 4704*a*b*c^5*d^2 - 980*b^
4*c^3*d*e + 6048*a*b^2*c^4*d*e - 6720*a^2*c^5*d*e + 315*b^5*c^2*e^2 - 2184
*a*b^3*c^3*e^2 + 3504*a^2*b*c^4*e^2)/c^6)*x + (2520*b^4*c^3*d^2 - 16800*a*
b^2*c^4*d^2 + 21504*a^2*c^5*d^2 - 2940*b^5*c^2*d*e + 21280*a*b^3*c^3*d*e -
 36288*a^2*b*c^4*d*e + 945*b^6*c*e^2 - 7560*a*b^4*c^2*e^2 + 16464*a^2*b^2*
c^3*e^2 - 6144*a^3*c^4*e^2)/c^6) + 1/2048*(24*b^5*c^2*d^2 - 192*a*b^3*c^3*
d^2 + 384*a^2*b*c^4*d^2 - 28*b^6*c*d*e + 240*a*b^4*c^2*d*e - 576*a^2*b^2*c
^3*d*e + 256*a^3*c^4*d*e + 9*b^7*e^2 - 84*a*b^5*c*e^2 + 240*a^2*b^3*c^2*e^
2 - 192*a^3*b*c^3*e^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(
c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^2 \left(a+bx+cx^2\right)^{3/2} dx = \int x\,{\left(d+e\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2} \,d x$$

input
```
int(x*(d + e*x)^2*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x*(d + e*x)^2*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d+ex)^2\left(a+bx+cx^2\right)^{3/2}\,dx=\int x(ex+d)^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x*(e*x+d)^2*(c*x^2+b*x+a)^(3/2),x)
```

3.76 $\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{(b^2-4ac)\left(12bcd-7b^2e+4ace\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}-\frac{\left(12bcd-7b^2e+4ace\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12cd-7be+10cex)\left(a+bx+cx^2\right)^{5/2}}{60c^2}-\frac{(b^2-4ac)^2\left(12bcd-7b^2e+4ace\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}$$

output
```
1/512*(-4*a*c+b^2)*(4*a*c*e-7*b^2*e+12*b*c*d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2
)/c^4-1/192*(4*a*c*e-7*b^2*e+12*b*c*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3+1
/60*(10*c*e*x-7*b*e+12*c*d)*(c*x^2+b*x+a)^(5/2)/c^2-1/1024*(-4*a*c+b^2)^2*
(4*a*c*e-7*b^2*e+12*b*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/
2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5e+10b^4c(18d+7ex)+8b^3c(95ae-cx(15d+7ex))+48b^2c^2(c$$

input
```
Integrate[x*(d + e*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*e + 10*b^4*c*(18*d + 7*e*x) + 8
*b^3*c*(95*a*e - c*x*(15*d + 7*e*x)) + 48*b^2*c^2*(c*x^2*(2*d + e*x) - a*(
25*d + 9*e*x)) + 16*b*c^2*(-81*a^2*e + 6*a*c*x*(7*d + 3*e*x) + 4*c^2*x^3*(
33*d + 26*e*x)) + 32*c^3*(8*c^2*x^4*(6*d + 5*e*x) + 3*a^2*(16*d + 5*e*x) +
 2*a*c*x^2*(48*d + 35*e*x))) - 15*(b^2 - 4*a*c)^2*(-12*b*c*d + 7*b^2*e - 4
*a*c*e)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(15360*c^(9/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(-7be+12cd+10cex)}{60c^2}-\frac{\left(4ace-7b^2e+12bcd\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+bx+cx^2)^{5/2}(-7be+12cd+10cex)}{60c^2}-\frac{(4ace-7b^2e+12bcd)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\int\sqrt{cx^2+bx+a}dx}{16c}\right)}{24c^2}$$

$\downarrow$ 1087

$$\frac{(a+bx+cx^2)^{5/2}(-7be+12cd+10cex)}{60c^2}-\frac{(4ace-7b^2e+12bcd)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{24c^2}$$

$\downarrow$ 1092

$$\frac{(a+bx+cx^2)^{5/2}(-7be+12cd+10cex)}{60c^2}-\frac{(4ace-7b^2e+12bcd)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{24c^2}$$

$\downarrow$ 219

$$\frac{(a+bx+cx^2)^{5/2}(-7be+12cd+10cex)}{60c^2}-\frac{\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)(4ace-7b^2e+12bcd)}{24c^2}$$

input
```
Int[x*(d + e*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
((12*c*d - 7*b*e + 10*c*e*x)*(a + b*x + c*x^2)^(5/2))/(60*c^2) - ((12*b*c*
d - 7*b^2*e + 4*a*c*e)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(
b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c
)))/(24*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{\left(-1280c^5 e\,x^5-1664 b\,c^4 e\,x^4-1536 c^5 d\,x^4-2240 a\,c^4 e\,x^3-48 b^2 c^3 e\,x^3-2112 b\,c^4 d\,x^3-288 b\,c^3 a e\,x^2-3072 a\,c^4 d\,x^2+56 b^3 c^2 e\,x^2-96 b\ldots\right.}{}$
default	$d\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}\right)+$

input
```
int(x*(e*x+d)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^4*(-1280*c^5*e*x^5-1664*b*c^4*e*x^4-1536*c^5*d*x^4-2240*a*c^4*e*
x^3-48*b^2*c^3*e*x^3-2112*b*c^4*d*x^3-288*a*b*c^3*e*x^2-3072*a*c^4*d*x^2+5
6*b^3*c^2*e*x^2-96*b^2*c^3*d*x^2-480*a^2*c^3*e*x+432*a*b^2*c^2*e*x-672*a*b
*c^3*d*x-70*b^4*c*e*x+120*b^3*c^2*d*x+1296*a^2*b*c^2*e-1536*a^2*c^3*d-760*
a*b^3*c*e+1200*a*b^2*c^2*d+105*b^5*e-180*b^4*c*d)*(c*x^2+b*x+a)^(1/2)-1/10
24*(64*a^3*c^3*e-144*a^2*b^2*c^2*e+192*a^2*b*c^3*d+60*a*b^4*c*e-96*a*b^3*c
^2*d-7*b^6*e+12*b^5*c*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.48

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/30720*(15*(12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (7*b^6 - 60*a*b^
4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*c^6*e
*x^5 + 128*(12*c^6*d + 13*b*c^5*e)*x^4 + 16*(132*b*c^5*d + (3*b^2*c^4 + 14
0*a*c^5)*e)*x^3 + 8*(12*(b^2*c^4 + 32*a*c^5)*d - (7*b^3*c^3 - 36*a*b*c^4)*
e)*x^2 + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*d - (105*b^5*c - 76
0*a*b^3*c^2 + 1296*a^2*b*c^3)*e - 2*(12*(5*b^3*c^3 - 28*a*b*c^4)*d - (35*b
^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*e)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/
15360*(15*(12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (7*b^6 - 60*a*b^4*c
 + 144*a^2*b^2*c^2 - 64*a^3*c^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*e*x^5 + 12
8*(12*c^6*d + 13*b*c^5*e)*x^4 + 16*(132*b*c^5*d + (3*b^2*c^4 + 140*a*c^5)*
e)*x^3 + 8*(12*(b^2*c^4 + 32*a*c^5)*d - (7*b^3*c^3 - 36*a*b*c^4)*e)*x^2 +
12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*d - (105*b^5*c - 760*a*b^3*c
^2 + 1296*a^2*b*c^3)*e - 2*(12*(5*b^3*c^3 - 28*a*b*c^4)*d - (35*b^4*c^2 -
216*a*b^2*c^3 + 240*a^2*c^4)*e)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(199) = 398.

Time = 0.81 (sec) , antiderivative size = 1175, normalized size of antiderivative = 5.93

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(a**2*e + 2*a*b*d - 3*a*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b
*(13*b*c*e/12 + c**2*d)/(10*c))/(4*c) - 5*b*(2*a*b*e + 2*a*c*d - 4*a*(13*b
*c*e/12 + c**2*d)/(5*c) + b**2*d - 7*b*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b
*(13*b*c*e/12 + c**2*d)/(10*c))/(8*c))/(6*c))/(2*c) - b*(a**2*d - 2*a*(2*a
*b*e + 2*a*c*d - 4*a*(13*b*c*e/12 + c**2*d)/(5*c) + b**2*d - 7*b*(7*a*c*e/
6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/12 + c**2*d)/(10*c))/(8*c))/(3*c) - 3
*b*(a**2*e + 2*a*b*d - 3*a*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/1
2 + c**2*d)/(10*c))/(4*c) - 5*b*(2*a*b*e + 2*a*c*d - 4*a*(13*b*c*e/12 + c*
*2*d)/(5*c) + b**2*d - 7*b*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/1
2 + c**2*d)/(10*c))/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x
+ c*x**2)*(c*e*x**5/6 + x**4*(13*b*c*e/12 + c**2*d)/(5*c) + x**3*(7*a*c*e/
6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/12 + c**2*d)/(10*c))/(4*c) + x**2*(2*
a*b*e + 2*a*c*d - 4*a*(13*b*c*e/12 + c**2*d)/(5*c) + b**2*d - 7*b*(7*a*c*e
/6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/12 + c**2*d)/(10*c))/(8*c))/(3*c) +
x*(a**2*e + 2*a*b*d - 3*a*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/12
 + c**2*d)/(10*c))/(4*c) - 5*b*(2*a*b*e + 2*a*c*d - 4*a*(13*b*c*e/12 + c**
2*d)/(5*c) + b**2*d - 7*b*(7*a*c*e/6 + b**2*e + 2*b*c*d - 9*b*(13*b*c*e/12
 + c**2*d)/(10*c))/(8*c))/(6*c))/(2*c) + (a**2*d - 2*a*(2*a*b*e + 2*a*c...
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.66

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cex+\frac{12\,c^6d+13\,bc^5e}{c^5}\right)x+\frac{132\,bc^5d+3\,b^2c^4e+140}{c^5}\right.\right.\right.\right.$$
$$+\frac{\left(12\,b^5cd-96\,ab^3c^2d+192\,a^2bc^3d-7\,b^6e+60\,ab^4ce-144\,a^2b^2c^2e+64\,a^3c^3e\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right.\right.\right.}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate(x*(e*x+d)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*e*x + (12*c^6*d + 13*b*c^5*
e)/c^5)*x + (132*b*c^5*d + 3*b^2*c^4*e + 140*a*c^5*e)/c^5)*x + (12*b^2*c^4
*d + 384*a*c^5*d - 7*b^3*c^3*e + 36*a*b*c^4*e)/c^5)*x - (60*b^3*c^3*d - 33
6*a*b*c^4*d - 35*b^4*c^2*e + 216*a*b^2*c^3*e - 240*a^2*c^4*e)/c^5)*x + (18
0*b^4*c^2*d - 1200*a*b^2*c^3*d + 1536*a^2*c^4*d - 105*b^5*c*e + 760*a*b^3*
c^2*e - 1296*a^2*b*c^3*e)/c^5) + 1/1024*(12*b^5*c*d - 96*a*b^3*c^2*d + 192
*a^2*b*c^3*d - 7*b^6*e + 60*a*b^4*c*e - 144*a^2*b^2*c^2*e + 64*a^3*c^3*e)*
log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x\,(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int(x*(d + e*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x*(d + e*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 775, normalized size of antiderivative = 3.91

$$\int x(d+ex)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{3072\sqrt{c\,x^2+bx+a}\,a^2c^4d-210\sqrt{c\,x^2+bx+a}\,b^5ce+360\sqrt{c\,x^2+bx+a}\,b^4c^2d+3072\sqrt{c\,x}}{}$$

input
```
int(x*(e*x+d)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
( - 2592*sqrt(a + b*x + c*x**2)*a**2*b*c**3*e + 3072*sqrt(a + b*x + c*x**2
)*a**2*c**4*d + 960*sqrt(a + b*x + c*x**2)*a**2*c**4*e*x + 1520*sqrt(a + b
*x + c*x**2)*a*b**3*c**2*e - 2400*sqrt(a + b*x + c*x**2)*a*b**2*c**3*d - 8
64*sqrt(a + b*x + c*x**2)*a*b**2*c**3*e*x + 1344*sqrt(a + b*x + c*x**2)*a*
b*c**4*d*x + 576*sqrt(a + b*x + c*x**2)*a*b*c**4*e*x**2 + 6144*sqrt(a + b*
x + c*x**2)*a*c**5*d*x**2 + 4480*sqrt(a + b*x + c*x**2)*a*c**5*e*x**3 - 21
0*sqrt(a + b*x + c*x**2)*b**5*c*e + 360*sqrt(a + b*x + c*x**2)*b**4*c**2*d
 + 140*sqrt(a + b*x + c*x**2)*b**4*c**2*e*x - 240*sqrt(a + b*x + c*x**2)*b
**3*c**3*d*x - 112*sqrt(a + b*x + c*x**2)*b**3*c**3*e*x**2 + 192*sqrt(a +
b*x + c*x**2)*b**2*c**4*d*x**2 + 96*sqrt(a + b*x + c*x**2)*b**2*c**4*e*x**
3 + 4224*sqrt(a + b*x + c*x**2)*b*c**5*d*x**3 + 3328*sqrt(a + b*x + c*x**2
)*b*c**5*e*x**4 + 3072*sqrt(a + b*x + c*x**2)*c**6*d*x**4 + 2560*sqrt(a +
b*x + c*x**2)*c**6*e*x**5 - 960*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c**3*e + 2160*sqrt(c)*log((2*sqr
t(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**
2*e - 2880*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt
(4*a*c - b**2))*a**2*b*c**3*d - 900*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*e + 1440*sqrt(c)*log((2*
sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**
2*d + 105*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sq...
```

3.77 $\int x(a+bx+cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int x\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{3b(b^2-4ac)\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^3}-\frac{b(b+2cx)\,(a+bx+cx^2)^{3/2}}{16c^2}+\frac{(a+bx+cx^2)^{5/2}}{5c}-\frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}}$$

output
```
3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3-1/16*b*(2*c*x+b)*(c
*x^2+b*x+a)^(3/2)/c^2+1/5*(c*x^2+b*x+a)^(5/2)/c-3/256*b*(-4*a*c+b^2)^2*arc
tanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int x(a+bx+cx^2)^{3/2}\,dx = \frac{\sqrt{a+x(b+cx)}\left(15b^4-10b^3cx+128c^2(a+cx^2)^2+4b^2c(-25a+2cx^2)+8bc^2x(7a+22cx\right.}{640c^3} + \frac{3b(b^2-4ac)^2\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{256c^{7/2}}$$

input
```
Integrate[x*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^
2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a + 22*c*x^2)))/(640*c^3) + (3*b*(b^2
 - 4*a*c)^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(256*c^(7/2)
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx+cx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(a+bx+cx^2)^{5/2}}{5c} - \frac{b\int (cx^2+bx+a)^{3/2}\,dx}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(a+bx+cx^2)^{5/2}}{5c} - \frac{b\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c} - \frac{3(b^2-4ac)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}$$

$$\Big\downarrow \; 1087$$

$$\frac{(a+bx+cx^2)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \; 1092$$

$$\frac{(a+bx+cx^2)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \; 219$$

$$\frac{(a+bx+cx^2)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}$$

input
```
Int[x*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(a + b*x + c*x^2)^(5/2)/(5*c) - (b*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/
(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^
2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3
/2))))/(16*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(c\,x^{2}+b x+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^{2}+b x+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+b x+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}$
risch	$\frac{\left(128c^{4}x^{4}+176b\,c^{3}x^{3}+256a\,c^{3}x^{2}+8b^{2}c^{2}x^{2}+56ab\,c^{2}x-10b^{3}cx+128a^{2}c^{2}-100ca\,b^{2}+15b^{4}\right)\sqrt{c\,x^{2}+b x+a}}{640c^{3}}-\frac{3b\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}{25}$

input
```
int(x*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.51

$$\int x(a+bx+cx^2)^{3/2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-\right.}{}\right.$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(128*
c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*
c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))
/c^4, 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(128*c
^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c
^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x^2 + b*x + a))/
c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(128) = 256$.

Time = 0.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.52

$$\int x(a+bx+cx^2)^{3/2}\,dx = \begin{cases} \left(-\frac{a\left(\frac{47ab}{40}-\frac{5b\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{6c}\right)}{2c}-\frac{b\left(a^2-\frac{2a\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{3c}-\frac{3b\left(\frac{47ab}{40}-\frac{5b\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}\right.}{\sqrt{c}} \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}}\end{cases}\right. \\ \frac{2\left(-\frac{a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^2} \\ \frac{a^{\frac{3}{2}}x^2}{2} \end{cases}$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(47*a*b/40 - 5*b*(6*a*c/5 + 3*b**2/80)/(6*c))/(2*c) - b*(a*
*2 - 2*a*(6*a*c/5 + 3*b**2/80)/(3*c) - 3*b*(47*a*b/40 - 5*b*(6*a*c/5 + 3*b
**2/80)/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c
*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c
) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(11*b*x**
3/40 + c*x**4/5 + x**2*(6*a*c/5 + 3*b**2/80)/(3*c) + x*(47*a*b/40 - 5*b*(6
*a*c/5 + 3*b**2/80)/(6*c))/(2*c) + (a**2 - 2*a*(6*a*c/5 + 3*b**2/80)/(3*c)
 - 3*b*(47*a*b/40 - 5*b*(6*a*c/5 + 3*b**2/80)/(6*c))/(4*c))/c), Ne(c, 0)),
 (2*(-a*(a + b*x)**(5/2)/5 + (a + b*x)**(7/2)/7)/b**2, Ne(b, 0)), (a**(3/2
)*x**2/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x(a+bx+cx^2)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.15

$$\int x(a+bx+cx^2)^{3/2}\,dx = \frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,(8\,cx+11\,b)x+\frac{b^2c^3+32\,ac^4}{c^4}\right)x-\frac{5\,b^3c^2-28\,abc^3}{c^4}\right)x+\frac{15\,b\right. + \frac{3\,(b^5-8\,ab^3c+16\,a^2bc^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x + 11*b)*x + (b^2*c^3 + 32*a*c^
4)/c^4)*x - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x + (15*b^4*c - 100*a*b^2*c^2 +
128*a^2*c^3)/c^4) + 3/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int x(a+bx+cx^2)^{3/2}\,dx = \frac{(c\,x^2+b\,x+a)^{5/2}}{5\,c} - \frac{b\left(\frac{3\,a\left(\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{(b+2\,c\,x)\,\sqrt{c\,x^2+b\,x+a}}{4\,c}\right)}{4}+\frac{x\,(c\,x^2+b\,x+a)^{3/2}}{4}+\frac{b\,(c\,x^2+b\,x+a)^{3/2}}{8\,c}-\frac{3\,b^2\left(\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.\right.}{}\right.}{2\,c}$$

input
```
int(x*(a + b*x + c*x^2)^(3/2),x)
```

output
```
(a + b*x + c*x^2)^(5/2)/(5*c) - (b*((3*a*(log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x)*(a + b
*x + c*x^2)^(1/2))/(4*c)))/4 + (x*(a + b*x + c*x^2)^(3/2))/4 + (b*(a + b*x
 + c*x^2)^(3/2))/(8*c) - (3*b^2*(log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x)*(a + b*x + c*x^
2)^(1/2))/(4*c)))/(16*c)))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.33

$$\int x(a+bx+cx^2)^{3/2}\,dx = \frac{256\sqrt{c\,x^2+bx+a}\,a^2c^3-200\sqrt{c\,x^2+bx+a}\,a\,b^2c^2+112\sqrt{c\,x^2+bx+a}\,ab\,c^3x+512\sqrt{c\,x^2}}{}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2),x)
```

output
```
(256*sqrt(a + b*x + c*x**2)*a**2*c**3 - 200*sqrt(a + b*x + c*x**2)*a*b**2*
c**2 + 112*sqrt(a + b*x + c*x**2)*a*b*c**3*x + 512*sqrt(a + b*x + c*x**2)*
a*c**4*x**2 + 30*sqrt(a + b*x + c*x**2)*b**4*c - 20*sqrt(a + b*x + c*x**2)
*b**3*c**2*x + 16*sqrt(a + b*x + c*x**2)*b**2*c**3*x**2 + 352*sqrt(a + b*x
 + c*x**2)*b*c**4*x**3 + 256*sqrt(a + b*x + c*x**2)*c**5*x**4 - 240*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
**2*b*c**2 + 120*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)/sqrt(4*a*c - b**2))*a*b**3*c - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**5)/(1280*c**4)
```

3.78 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 339

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+2ce(3(b^2-4ac)e^2-8cd(2cd-be))x)\sqrt{a+}}{64c^2e^4}$$
$$-\frac{(8cd-3be-6cex)\left(a+bx+cx^2\right)^{3/2}}{24ce^2}$$
$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}e^5}$$
$$-\frac{d(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^5}$$

output
```
-1/64*(64*c^3*d^3+3*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*
b*d)+2*c*e*(3*(-4*a*c+b^2)*e^2-8*c*d*(-b*e+2*c*d))*x)*(c*x^2+b*x+a)^(1/2)/
c^2/e^4-1/24*(-6*c*e*x-3*b*e+8*c*d)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(128*c
^4*d^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*
e^2*(-a*e+b*d)^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/
2)/e^5-d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{\frac{2e\sqrt{a+x(b+cx)}\left(-9b^3e^3+6bce^2(-4bd+10ae+bex)-16c^3\left(12d^3-6d^2ex+4de^2x^2-3e^3x^3\right)+8c^2e\left(ae(-32d+15\right.\right.}{c^2}}{}$$

input

```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output

```
((2*e*Sqrt[a + x*(b + c*x)]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b*e
*x) - 16*c^3*(12*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 - 3*e^3*x^3) + 8*c^2*e*(a*e
*(-32*d + 15*e*x) + b*(30*d^2 - 14*d*e*x + 9*e^2*x^2))))/c^2 - 768*d*Sqrt[
-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d +
e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(128*
c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e
) + 48*c^2*e^2*(b*d - a*e)^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/c^(5/2))/(384*e^5)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\int -\frac{\left(d\left(-3eb^2+8cdb-4ace\right)+\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)}dx}{8ce^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(d\left(-3eb^2+8cdb-4ace\right)+\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right)x\right)\sqrt{cx^2+bx+a}}{d+ex}dx}{16ce^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 1231$$

$$-\frac{\int -\frac{d\left(3e^3b^4+8cde^2b^3-8ce\left(10cd^2+3ae^2\right)b^2+32c^2d\left(2cd^2+5ae^2\right)b-16ac^2e\left(4cd^2+5ae^2\right)\right)+\left(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(b\right.}{2(d+ex)\sqrt{cx^2+bx+a}}}{4ce^2}$$

$$16ce^2$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{d\left(3e^3b^4+8cde^2b^3-8ce\left(10cd^2+3ae^2\right)b^2+32c^2d\left(2cd^2+5ae^2\right)b-16ac^2e\left(4cd^2+5ae^2\right)\right)+\left(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(bd-3a\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{8ce^2}$$

$$16ce^2$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\sqrt{a+bx+}$$

$$16ce^2$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{2\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}$$

$$16ce^2$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)}{\sqrt{ce}}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\frac{256c^2d\left(ae^2-bde+cd^2\right)^2\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e\right.}{\sqrt{ce}}}{8ce^2}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)}{\sqrt{ce}}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^{3/2}\text{arctanh}\left(\frac{-2ae+}{2\sqrt{a+bx+c}}\right.}{e}}{8ce^2}}{16ce^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-3be+8cd-6cex)}{24ce^2}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

output
```
-1/24*((8*c*d - 3*b*e - 6*c*e*x)*(a + b*x + c*x^2)^(3/2))/(c*e^2) + (-1/4*
((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x)*Sqrt[a
 + b*x + c*x^2])/(c*e^2) + (((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d -
 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (128*c^2*d*(c*
d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt
[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(16*c*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.47

method	result
risch	$\frac{(48c^3x^3e^3+72b\,c^2e^3x^2-64c^3d\,e^2x^2+120a\,c^2e^3x+6x\,b^2c\,e^3-112b\,c^2d\,e^2x+96d^2e\,c^3x+60abc\,e^3-256d\,e^2a\,c^2-9b^3e^3-24d\,e^2b^2c+24}{192c^2e^4}$
default	$\frac{\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}}{e}-\frac{d\left(\frac{\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\right.}{3}\right.}{}$

input

```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output

```
1/192/c^2*(48*c^3*e^3*x^3+72*b*c^2*e^3*x^2-64*c^3*d*e^2*x^2+120*a*c^2*e^3*
x+6*b^2*c*e^3*x-112*b*c^2*d*e^2*x+96*c^3*d^2*e*x+60*a*b*c*e^3-256*a*c^2*d*
e^2-9*b^3*e^3-24*b^2*c*d*e^2+240*b*c^2*d^2*e-192*c^3*d^3)*(c*x^2+b*x+a)^(1
/2)/e^4+1/128/e^4/c^2*((48*a^2*c^2*e^4-24*a*b^2*c*e^4-96*a*b*c^2*d*e^3+192
*a*c^3*d^2*e^2+3*b^4*e^4+8*b^3*c*d*e^3+48*b^2*c^2*d^2*e^2-192*b*c^3*d^3*e+
128*c^4*d^4)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+128*d*(
a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)*c^2/e^2
/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)
/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 35.86 (sec) , antiderivative size = 10332, normalized size of antiderivative = 30.48

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

output

```
(384*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e
**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d*
*2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e
 + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqr
t(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*
e - 8*c**2*d**2))*a*b*c**3*d*e**3 - 768*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e
 + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2
- b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan(
(2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a
*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d
- 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**4*d**2*e**2 - 38
4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2
 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)
*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c
)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e -
 8*c**2*d**2))*b**2*c**3*d**2*e**2 + 1152*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**
2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*ata
n((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*s...
```

3.79 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 296

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2} dx = \frac{(32c^2d^2+b^2e^2-4ce(7bd-2ae)-2ce(8cd-be)x)\sqrt{a+bx+cx^2}}{8ce^4} + \frac{(4d+ex)(a+bx+cx^2)^{3/2}}{3e^2(d+ex)} - \frac{(64c^3d^3+b^3e^3-24c^2de(3bd-2ae)+12bce^2(bd-ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^5} + \frac{\sqrt{cd^2-bde+ae^2}(8cd^2-e(5bd-2ae))\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{2e^5}$$

output
```
1/8*(32*c^2*d^2+b^2*e^2-4*c*e*(-2*a*e+7*b*d)-2*c*e*(-b*e+8*c*d)*x)*(c*x^2+
b*x+a)^(1/2)/c/e^4+1/3*(e*x+4*d)*(c*x^2+b*x+a)^(3/2)/e^2/(e*x+d)-1/16*(64*
c^3*d^3+b^3*e^3-24*c^2*d*e*(-2*a*e+3*b*d)+12*b*c*e^2*(-a*e+b*d))*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^5+1/2*(a*e^2-b*d*e+c*d
^2)^(1/2)*(8*c*d^2-e*(-2*a*e+5*b*d))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x
)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5
```

Mathematica [A] (verified)

Time = 11.01 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.03

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \frac{-3(64c^3d^3+b^3e^3+12bce^2(bd-ae)+24c^2de(-3bd+2ae))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(}}\right.}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^2,x]
```

output
```
(-3*(64*c^3*d^3 + b^3*e^3 + 12*b*c*e^2*(b*d - a*e) + 24*c^2*d*e*(-3*b*d +
2*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]
*((e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2*(d + e*x) + 8*c^2*(12*d^3 + 6*d^2*e*
x - 2*d*e^2*x^2 + e^3*x^3) + 2*c*e*(4*a*e*(7*d + 4*e*x) + b*(-42*d^2 - 23*
d*e*x + 7*e^2*x^2))))/(d + e*x) - 12*c*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(8*c
*d^2 + e*(-5*b*d + 2*a*e))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(48*c^(3/2)*e^5)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1230, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1230}$$

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{\int \frac{(2(2bd-ae)+(8cd-be)x)\sqrt{cx^2+bx+a}}{d+ex}dx}{2e^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\int \frac{8ce(bd-2ae)(2bd-ae)-d(8cd-be)\left(-eb^2+4cdb-4ace\right)-\left(64c^3d^3-24c^2e(3bd-2ae)d+b^3e^3+12bce^2(bd-ae)\right)x}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{4ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-4ce(7bd-2ae)+b^2e\cdots\right.}{4ce^2}\cdots}{2e^2}$$

$\Big\downarrow$ 27

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\int \frac{8ce(bd-2ae)(2bd-ae)-d(8cd-be)\left(-eb^2+4cdb-4ace\right)-\left(64c^3d^3-24c^2e(3bd-2ae)d+b^3e^3+12bce^2(bd-ae)\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8ce^2}-\frac{\sqrt{a+bx+cx^2}\left(-4ce(7bd-2ae)+b^2e\cdots\right.}{4ce^2}\cdots}{2e^2}$$

$\Big\downarrow$ 1269

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\frac{8c\left(ae^2-bde+cd^2\right)\left(2ae^2-5bde+8cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(-24c^2de(3bd-2ae)+12bce^2(bd-ae)+b^3e^3+64c^3d^3\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{8ce^2}-\sqrt{a+bx+cx^2}\cdots}{2e^2}$$

$\Big\downarrow$ 1092

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\frac{8c\left(ae^2-bde+cd^2\right)\left(2ae^2-5bde+8cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(-24c^2de(3bd-2ae)+12bce^2(bd-ae)+b^3e^3+64c^3d^3\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}}{8ce^2}-\cdots}{2e^2}$$

$\Big\downarrow$ 219

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\frac{8c\left(ae^2-bde+cd^2\right)\left(2ae^2-5bde+8cd^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24c^2de(3bd-2ae)+12bce^2(bd-ae)+b^3e^3+64c^3d^3\right)}{\sqrt{c}e}}{8ce^2}-\cdots}{2e^2}$$

$\Big\downarrow$ 1154

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{-\frac{16c\left(ae^2-bde+cd^2\right)\left(2ae^2-5bde+8cd^2\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24c^2d\cdots\right.}{\cdots}}{8ce^2}\cdots}{2e^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4d+ex)\left(a+bx+cx^2\right)^{3/2}}{3e^2(d+ex)}-\frac{-\frac{\frac{8c\sqrt{ae^2-bde+cd^2}\left(2ae^2-5bde+8cd^2\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24c^2de(3bd-2ae)+12bce^2(bd-ae)+b\cdots\right.}{\sqrt{c}e}}{8ce^2}}{2e^2}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^2,x]
```

output
```
((4*d + e*x)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(d + e*x)) - (-1/4*((32*c^2*d
^2 + b^2*e^2 - 4*c*e*(7*b*d - 2*a*e) - 2*c*e*(8*c*d - b*e)*x)*Sqrt[a + b*x
 + c*x^2])/(c*e^2) - (-(((64*c^3*d^3 + b^3*e^3 - 24*c^2*d*e*(3*b*d - 2*a*e
) + 12*b*c*e^2*(b*d - a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[c]*e)) + (8*c*Sqrt[c*d^2 - b*d*e + a*e^2]*(8*c*d^2 - 5*b*d
*e + 2*a*e^2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*
e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e)/(8*c*e^2))/(2*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(268) = 536$.

Time = 1.45 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.40

method	result
risch	$\frac{(8c^2e^2x^2+14e^2xbc-24c^2dex+32ac\,e^2+3b^2e^2-60bcde+72c^2d^2)\sqrt{c\,x^2+bx+a}}{24c\,e^4}+\frac{\frac{(12abc\,e^3-48d\,e^2a\,c^2-b^3e^3-12d\,e^2b^2c+72d^2eb\,c^2-6}{e\sqrt{c}}}{}$
default	Expression too large to display

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*c^2*e^2*x^2+14*b*c*e^2*x-24*c^2*d*e*x+32*a*c*e^2+3*b^2*e^2-60*b*
c*d*e+72*c^2*d^2)*(c*x^2+b*x+a)^(1/2)/e^4+1/16/e^4/c*((12*a*b*c*e^3-48*a*c
^2*d*e^2-b^3*e^3-12*b^2*c*d*e^2+72*b*c^2*d^2*e-64*c^3*d^3)/e*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*c/e^2*(a^2*e^4-4*a*b*d*e^3+6*a*c
*d^2*e^2+3*b^2*d^2*e^2-8*b*c*d^3*e+5*c^2*d^4)/((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x+d/e))-16*c*d*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*
b*c*d^3*e+c^2*d^4)/e^3*(-1/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)*e/(a*e^
2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Fricas [A] (verification not implemented)

Time = 112.28 (sec) , antiderivative size = 2135, normalized size of antiderivative = 7.21

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/96*(3*(64*c^3*d^4 - 72*b*c^2*d^3*e + 12*(b^2*c + 4*a*c^2)*d^2*e^2 + (b
^3 - 12*a*b*c)*d*e^3 + (64*c^3*d^3*e - 72*b*c^2*d^2*e^2 + 12*(b^2*c + 4*a*
c^2)*d*e^3 + (b^3 - 12*a*b*c)*e^4)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 24*(8*c^3*d^3
- 5*b*c^2*d^2*e + 2*a*c^2*d*e^2 + (8*c^3*d^2*e - 5*b*c^2*d*e^2 + 2*a*c^2*e
^3)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a
*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 -
 b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)
) - 4*(8*c^3*e^4*x^3 + 96*c^3*d^3*e - 84*b*c^2*d^2*e^2 + (3*b^2*c + 56*a*c
^2)*d*e^3 - 2*(8*c^3*d*e^3 - 7*b*c^2*e^4)*x^2 + (48*c^3*d^2*e^2 - 46*b*c^2
*d*e^3 + (3*b^2*c + 32*a*c^2)*e^4)*x)*sqrt(c*x^2 + b*x + a))/(c^2*e^6*x +
c^2*d*e^5), 1/48*(3*(64*c^3*d^4 - 72*b*c^2*d^3*e + 12*(b^2*c + 4*a*c^2)*d^
2*e^2 + (b^3 - 12*a*b*c)*d*e^3 + (64*c^3*d^3*e - 72*b*c^2*d^2*e^2 + 12*(b^
2*c + 4*a*c^2)*d*e^3 + (b^3 - 12*a*b*c)*e^4)*x)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 12*(8*c^3*
d^3 - 5*b*c^2*d^2*e + 2*a*c^2*d*e^2 + (8*c^3*d^2*e - 5*b*c^2*d*e^2 + 2*a*c
^2*e^3)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 +
 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d
^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*...
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^2}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**2,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^2}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^2,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 1343, normalized size of antiderivative = 4.54

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^2,x)
```

output

```
(48*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c**2*d*e**2 + 48*sqr
t(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*
e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c**2*e**3*x - 120*sqrt(a*e*
*2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*
d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b*c**2*d**2*e - 120*sqrt(a*e**2 - b
*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2)
- 2*a*e + b*d - b*e*x + 2*c*d*x)*b*c**2*d*e**2*x + 192*sqrt(a*e**2 - b*d*e
 + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*
a*e + b*d - b*e*x + 2*c*d*x)*c**3*d**3 + 192*sqrt(a*e**2 - b*d*e + c*d**2)
*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d
- b*e*x + 2*c*d*x)*c**3*d**2*e*x - 48*sqrt(a*e**2 - b*d*e + c*d**2)*log(d
+ e*x)*a*c**2*d*e**2 - 48*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a*c**
2*e**3*x + 120*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*c**2*d**2*e +
120*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*c**2*d*e**2*x - 192*sqrt(
a*e**2 - b*d*e + c*d**2)*log(d + e*x)*c**3*d**3 - 192*sqrt(a*e**2 - b*d*e
+ c*d**2)*log(d + e*x)*c**3*d**2*e*x + 112*sqrt(a + b*x + c*x**2)*a*c**2*d
*e**3 + 64*sqrt(a + b*x + c*x**2)*a*c**2*e**4*x + 6*sqrt(a + b*x + c*x**2)
*b**2*c*d*e**3 + 6*sqrt(a + b*x + c*x**2)*b**2*c*e**4*x - 168*sqrt(a + b*x
 + c*x**2)*b*c**2*d**2*e**2 - 92*sqrt(a + b*x + c*x**2)*b*c**2*d*e**3*x...
```

3.80 $\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 288

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3} dx = -\frac{3(2(4cd^2-e(2bd-ae))+e(4cd-be)x)\sqrt{a+bx+cx^2}}{4e^4(d+ex)} + \frac{(2d+ex)(a+bx+cx^2)^{3/2}}{2e^2(d+ex)^2} - \frac{3(4ce(bd-ae)-(4cd-be)^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}e^5} - \frac{3(16c^2d^3+be^2(5bd-4ae)-4cde(5bd-3ae))\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{8e^5\sqrt{cd^2-bde+ae^2}}$$

output
```
-3/4*(8*c*d^2-2*e*(-a*e+2*b*d)+e*(-b*e+4*c*d)*x)*(c*x^2+b*x+a)^(1/2)/e^4/(
e*x+d)+1/2*(e*x+2*d)*(c*x^2+b*x+a)^(3/2)/e^2/(e*x+d)^2-3/8*(4*c*e*(-a*e+b*
d)-(-b*e+4*c*d)^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1
/2)/e^5-3/8*(16*c^2*d^3+b*e^2*(-4*a*e+5*b*d)-4*c*d*e*(-3*a*e+5*b*d))*arcta
nh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/e^5/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 12.04 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.76

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \frac{\frac{d(a+x(b+cx))^{5/2}}{(d+ex)^2} - \frac{(6cd^2+e(-5bd+4ae))(a+x(b+cx))^{5/2}}{2(cd^2+e(-bd+ae))(d+ex)} - \frac{(a+x(b+cx))^{3/2}\left(be^2(-5bd+4ae)+c^2\left(-8d^3+6\right.\right.}{2e^2}}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
((d*(a + x*(b + c*x))^(5/2))/(d + e*x)^2 - ((6*c*d^2 + e*(-5*b*d + 4*a*e))
*(a + x*(b + c*x))^(5/2))/(2*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) - (((a
+ x*(b + c*x))^(3/2)*(b*e^2*(-5*b*d + 4*a*e) + c^2*(-8*d^3 + 6*d^2*e*x) +
c*e*(b*d*(13*d - 5*e*x) + 2*a*e*(-3*d + 2*e*x))))/(2*e^2) + (3*(-2*c^2*e*(
c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]*(b*e^2*(4*b*d - 3*a*e) + 4
*c^2*d^2*(2*d - e*x) + c*e*(3*b*d*(-4*d + e*x) - 2*a*e*(-3*d + e*x))) + c^
(3/2)*(16*c^2*d^2 + b^2*e^2 + 4*c*e*(-3*b*d + a*e))*(c*d^2 + e*(-(b*d) + a
*e))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c^2*(16*c^
2*d^3 + b*e^2*(5*b*d - 4*a*e) - 4*c*d*e*(5*b*d - 3*a*e))*(c*d^2 + e*(-(b*d
) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 +
 e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^2*e^5))/(-(c*d^2) + e*(b
*d - a*e)))/(2*(c*d^2 + e*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1230, 27, 1230, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1230}$$

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-\frac{3\int\frac{2(2(bd-ae)+(4cd-be)x)\sqrt{cx^2+bx+a}}{(d+ex)^2}dx}{8e^2}$$

$\big\downarrow$ 27

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-\frac{3\int\frac{(2(bd-ae)+(4cd-be)x)\sqrt{cx^2+bx+a}}{(d+ex)^2}dx}{4e^2}$$

$\big\downarrow$ 1230

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{\int\frac{4(bd-ae)(2cd-be)-\left(4ce(bd-ae)-(be-4cd)^2\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e^2}\right)}{4e^2}$$

$\big\downarrow$ 1269

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-$$
$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{-\frac{\left(-4cde(5bd-3ae)+be^2(5bd-4ae)+16c^2d^3\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(4ce(bd-ae)-(be-4cd)\right.}{e}}{2e^2}\right.}{4e^2}$$

$\big\downarrow$ 1092

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-$$
$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{-\frac{\left(-4cde(5bd-3ae)+be^2(5bd-4ae)+16c^2d^3\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{2\left(4ce(bd-ae)-(be-4cd\right.}{}}{2e^2}\right.}{4e^2}$$

$\big\downarrow$ 219

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-$$
$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{-\frac{\left(-4cde(5bd-3ae)+be^2(5bd-4ae)+16c^2d^3\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{\text{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{}}{2e^2}\right.}{4e^2}$$

$\big\downarrow$ 1154

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-$$
$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{\frac{2\left(-4cde(5bd-3ae)+be^2(5bd-4ae)+16c^2d^3\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd}{\cdots}\right.}{e}}{2e^2}\cdots\right.}{4e^2}$$

$\big\downarrow$ 219

$$\frac{(2d+ex)\left(a+bx+cx^2\right)^{3/2}}{2e^2(d+ex)^2}-$$
$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2\left(ae^2-2bde+4cd^2\right)+ex(4cd-be)\right)}{e^2(d+ex)}-\frac{-\frac{\left(-4cde(5bd-3ae)+be^2(5bd-4ae)+16c^2d^3\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}-\frac{\text{arc}\cdots}{\cdots}}{2e^2}\cdots\right.}{4e^2}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
((2*d + e*x)*(a + b*x + c*x^2)^(3/2))/(2*e^2*(d + e*x)^2) - (3*(((2*(4*c*d
^2 - 2*b*d*e + a*e^2) + e*(4*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(e^2*(d
+ e*x)) - (-(((4*c*e*(b*d - a*e) - (-4*c*d + b*e)^2)*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e)) - ((16*c^2*d^3 + b*e^2*(5*
b*d - 4*a*e) - 4*c*d*e*(5*b*d - 3*a*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^
2 - b*d*e + a*e^2]))/(2*e^2)))/(4*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. $2(257) = 514$.

Time = 1.54 (sec) , antiderivative size = 1235, normalized size of antiderivative = 4.29

method	result	size
risch	Expression too large to display	1235
default	Expression too large to display	2953

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*e*x+5*b*e-12*c*d)*(c*x^2+b*x+a)^(1/2)/e^4+1/8/e^4*(-8/e^2*(2*a*b*
e^3-6*a*c*d*e^2-3*b^2*d*e^2+12*b*c*d^2*e-10*c^2*d^3)/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))/(x+d/e))+8/e^3*(a^2*e^4-4*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^
2*e^2-8*b*c*d^3*e+5*c^2*d^4)*(-1/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)*(c*(x+d/e
)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)*e
/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e
)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+3*(4*a
*c*e^2+b^2*e^2-12*b*c*d*e+16*c^2*d^2)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))/c^(1/2)-8*d*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c
*d^3*e+c^2*d^4)/e^4*(-1/2/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)^2*(c*(x+d/e)^2+(
b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-3/4*(b*e-2*c*d)*e/(a*e
^2-b*d*e+c*d^2)*(-1/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)*e/(a*e^2-b*d*e
+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/2*c/(a*e^2-b*d*e+
c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(257) = 514$.

Time = 104.09 (sec) , antiderivative size = 3719, normalized size of antiderivative = 12.91

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^3}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**3,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(257) = 514.

Time = 0.43 (sec) , antiderivative size = 973, normalized size of antiderivative = 3.38

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*c*x/e^3 - (12*c^2*d*e^8 - 5*b*c*e^9)/(c*e^12)
) - 3/4*(16*c^2*d^3 - 20*b*c*d^2*e + 5*b^2*d*e^2 + 12*a*c*d*e^2 - 4*a*b*e^
3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*e^5) - 3/8*(16*c^2*d^2 -
12*b*c*d*e + b^2*e^2 + 4*a*c*e^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*sqrt(c) + b))/(sqrt(c)*e^5) - 1/4*(32*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*c^2*d^3*e - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d^2*e^2 +
 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*e^3 + 12*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*c*d*e^3 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*
b*e^4 + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d^4 - 44*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d^3*e + 3*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*b^2*sqrt(c)*d^2*e^2 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a*c^(3/2)*d^2*e^2 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c
)*d*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*e^4 + 56*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d^4 - 48*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*b^2*c*d^3*e - 80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d^3*e
 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d^2*e^2 + 80*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*d^2*e^2 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*b^2*d*e^3 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*e^3 + 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*e^4 + 14*b^2*c^(3/2)*d^4 - 9*b^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3}\,dx=\int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^3}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^3,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 3154, normalized size of antiderivative = 10.95

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^3} \, dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^3,x)
```

output
```
( - 12*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(
a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*c*d**2*e**3
- 24*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*
e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*c*d*e**4*x - 1
2*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**
2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*c*e**5*x**2 + 36*
sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c**2*d**3*e**2 + 72*s
qrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c**2*d**2*e**3*x + 36*
sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*c**2*d*e**4*x**2 + 15
*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b**2*c*d**3*e**2 + 30*
sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b**2*c*d**2*e**3*x + 15
*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b**2*c*d*e**4*x**2 - 6
0*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**
2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b*c**2*d**4*e - 12...
```

3.81 $\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 367

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4} dx = \frac{(8cd-3be)\sqrt{a+bx+cx^2}}{2e^4(d+ex)}$$
$$+ \frac{(8cd^2-e(7bd-6ae))\,(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{8e^3\,(cd^2-bde+ae^2)\,(d+ex)^2}$$
$$+ \frac{(4d+3ex)\,(a+bx+cx^2)^{3/2}}{3e^2(d+ex)^3} - \frac{\sqrt{c}(8cd-3be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2e^5}$$
$$+ \frac{(64c^3d^4-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+12ce^2(5b^2d^2-7abde+2a^2e^2))\ \text{arctanh}\left(\frac{bd-2ae+(2c}{2\sqrt{cd^2-bde+ae^2}}\right.}{16e^5\,(cd^2-bde+ae^2)^{3/2}}$$

output
```
1/2*(-3*b*e+8*c*d)*(c*x^2+b*x+a)^(1/2)/e^4/(e*x+d)+1/8*(8*c*d^2-e*(-6*a*e+
7*b*d))*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*
d^2)/(e*x+d)^2+1/3*(3*e*x+4*d)*(c*x^2+b*x+a)^(3/2)/e^2/(e*x+d)^3-1/2*c^(1/
2)*(-3*b*e+8*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5+1
/16*(64*c^3*d^4-b^2*e^3*(-6*a*e+5*b*d)-24*c^2*d^2*e*(-4*a*e+5*b*d)+12*c*e^
2*(2*a^2*e^2-7*a*b*d*e+5*b^2*d^2))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5/(a*e^2-b*d*e+c*d^2)^(3/
2)
```

Mathematica [A] (verified)

Time = 13.44 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.87

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \frac{\frac{2d(cd^2+e(-bd+ae))(a+x(b+cx))^{5/2}}{(d+ex)^3} - \frac{(4cd^2+e(-5bd+6ae))(a+x(b+cx))^{5/2}}{2(d+ex)^2} + \frac{(24c^2d^3+be^2(5bd-6ae}{4(cd}}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^4,x]
```

output
```
((2*d*(c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x))^(5/2))/(d + e*x)^3 - ((
4*c*d^2 + e*(-5*b*d + 6*a*e))*(a + x*(b + c*x))^(5/2))/(2*(d + e*x)^2) + (
(24*c^2*d^3 + b*e^2*(5*b*d - 6*a*e) + 2*c*d*e*(-15*b*d + 14*a*e))*(a + x*(
b + c*x))^(5/2))/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)) + (((a + x*(b +
c*x))^(3/2)*(b^2*e^3*(5*b*d - 6*a*e) - 8*c^3*d^3*(4*d - 3*e*x) + c*e^2*(-1
2*a^2*e^2 + 2*a*b*e*(28*d - 3*e*x) + 5*b^2*d*(-9*d + e*x)) + 2*c^2*d*e*(3*
b*d*(12*d - 5*e*x) + 2*a*e*(-12*d + 7*e*x))))/(4*e^2) + (3*(-2*e*(c*d^2 +
e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]*(b^2*e^3*(-5*b*d + 6*a*e) + 16*c^3
*d^3*(2*d - e*x) + c*e^2*(12*a^2*e^2 + b^2*d*(41*d - 5*e*x) + 2*a*b*e*(-26
*d + 3*e*x)) + 2*c^2*d*e*(2*a*e*(12*d - 5*e*x) + b*d*(-34*d + 11*e*x))) +
8*Sqrt[c]*(8*c*d - 3*b*e)*(c*d^2 + e*(-(b*d) + a*e))^3*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (c*d^2 + e*(-(b*d) + a*e))^(3/2)*(64
*c^3*d^4 - 24*c^2*d^2*e*(5*b*d - 4*a*e) + b^2*e^3*(-5*b*d + 6*a*e) + 12*c*
e^2*(5*b^2*d^2 - 7*a*b*d*e + 2*a^2*e^2))*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x
 + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(8*e
^5))/(-(c*d^2) + e*(b*d - a*e)))/(6*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \ 1229$$

$$-\frac{\int -\frac{\left(-5deb^2+8cd^2b+6ae^2b-8acde+2c\left(8cd^2-e(7bd-6ae)\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)^2}dx}{4e^2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(-5deb^2+8cd^2b+6ae^2b-8acde+2c\left(8cd^2-e(7bd-6ae)\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^2}dx}{8e^2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1230$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2cex\left(8cd^2-e(7bd-6ae)\right)-4cde(9bd-8ae)+be^2(5bd-6ae)+32c^2d^3\right)}{e^2(d+ex)}-\frac{\int -\frac{be\left(-5deb^2+8cd^2b+6ae^2b-8acde\right)-4c(bd-ae)\left(8cd^2-e(7bd-\right.}{(d+ex)\sqrt{cx^2+bx+a}}}{2e^2}}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{be\left(-5deb^2+8cd^2b+6ae^2b-8acde\right)-4c(bd-ae)\left(8cd^2-e(7bd-6ae)\right)-8c(8cd-3be)\left(cd^2-bed+ae^2\right)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2e^2}+\frac{\sqrt{a+bx+cx^2}\left(2cex\left(8cd^2-e(7bd-6ae)\right)-4cde(\right.}{e^2(d+ex)}}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{\frac{\left(12ce^2\left(2a^2e^2-7abde+5b^2d^2\right)-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+64c^3d^4\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{8c(8cd-3be)\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}}{2e^2}+\frac{\sqrt{}}{}}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{\frac{\left(12ce^2\left(2a^2e^2-7abde+5b^2d^2\right)-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+64c^3d^4\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{16c(8cd-3be)\left(ae^2-bde+cd^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b\cdots}{\sqrt{cx\cdots}}}{e}}{2e^2}\cdots}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(12ce^2\left(2a^2e^2-7abde+5b^2d^2\right)-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+64c^3d^4\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}-\frac{8\sqrt{c}(8cd-3be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ae^2-b\cdots\right.}{e}}{2e^2}\cdots}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{-\frac{2\left(12ce^2\left(2a^2e^2-7abde+5b^2d^2\right)-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+64c^3d^4\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}-\frac{8\sqrt{c}\cdots}{\cdots}}{2e^2}\cdots}{8e^2\left(ae^2-bd\cdots\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(12ce^2\left(2a^2e^2-7abde+5b^2d^2\right)-b^2e^3(5bd-6ae)-24c^2d^2e(5bd-4ae)+64c^3d^4\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}-\frac{8\sqrt{c}(8cd-3be)\text{arctanh}\left(\frac{b\cdots}{2\sqrt{c}\sqrt{a\cdots}}\right.}{e}}{2e^2}\cdots}{8e^2\left(ae^2-bde+cd^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(3ex\left(4cd^2-e(3bd-2ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{12e^2(d+ex)^3\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^4,x]
```

output
```
-1/12*((d*(8*c*d^2 - e*(5*b*d - 2*a*e)) + 3*e*(4*c*d^2 - e*(3*b*d - 2*a*e)
)*x)*(a + b*x + c*x^2)^(3/2))/(e^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) +
(((32*c^2*d^3 - 4*c*d*e*(9*b*d - 8*a*e) + b*e^2*(5*b*d - 6*a*e) + 2*c*e*(8
*c*d^2 - e*(7*b*d - 6*a*e))*x)*Sqrt[a + b*x + c*x^2])/(e^2*(d + e*x)) + ((
-8*Sqrt[c]*(8*c*d - 3*b*e)*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/e + ((64*c^3*d^4 - b^2*e^3*(5*b*d - 6*a*e
) - 24*c^2*d^2*e*(5*b*d - 4*a*e) + 12*c*e^2*(5*b^2*d^2 - 7*a*b*d*e + 2*a^2
*e^2))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(2*e^2))/(8*
e^2*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2194 vs. $2(335) = 670$.

Time = 1.59 (sec) , antiderivative size = 2195, normalized size of antiderivative = 5.98

method	result	size
risch	Expression too large to display	2195
default	Expression too large to display	4942

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*c*(c*x^2+b*x+a)^(1/2)+1/2/e^4*(c^(1/2)*(3*b*e-8*c*d)/e*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/e^2*(2*a*c*e^2+b^2*e^2-8*b*c*d*e+10*c^2*d
^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+2/e^3*(2*a*b*e^3-6*a*c*d
*e^2-3*b^2*d*e^2+12*b*c*d^2*e-10*c^2*d^3)*(-1/(a*e^2-b*d*e+c*d^2)*e^2/(x+d
/e)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*
(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+
d/e)))+2/e^4*(a^2*e^4-4*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2-8*b*c*d^3*e+
5*c^2*d^4)*(-1/2/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)^2*(c*(x+d/e)^2+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-3/4*(b*e-2*c*d)*e/(a*e^2-b*d*e+
c*d^2)*(-1/(a*e^2-b*d*e+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/2*c/(a*e^2-b*d*e+c*d^2)*e^
2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**4,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2470 vs. $2(335) = 670$.

Time = 4.51 (sec) , antiderivative size = 2470, normalized size of antiderivative = 6.73

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(64*c^3*d^4 - 120*b*c^2*d^3*e + 60*b^2*c*d^2*e^2 + 96*a*c^2*d^2*e^2 -
5*b^3*d*e^3 - 84*a*b*c*d*e^3 + 6*a*b^2*e^4 + 24*a^2*c*e^4)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))
/((c*d^2*e^5 - b*d*e^6 + a*e^7)*sqrt(-c*d^2 + b*d*e - a*e^2)) + sqrt(c*x^2
 + b*x + a)*c/e^4 + 1/24*(288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*d^
4*e^2 - 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d^3*e^3 + 252*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d^2*e^4 + 288*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*a*c^2*d^2*e^4 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*b^3*d*e^5 - 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d*e^5 + 30*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e^6 + 24*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*c*e^6 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/
2)*d^5*e - 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d^4*e^2 +
540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^3*e^3 + 672*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d^3*e^3 - 21*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*b^3*sqrt(c)*d^2*e^4 - 180*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^4*a*b*c^(3/2)*d^2*e^4 - 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*
b^2*sqrt(c)*d*e^5 - 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*
d*e^5 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*e^6 + 832*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d^6 - 400*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*b*c^3*d^5*e - 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx=\int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^4}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^4,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 6174, normalized size of antiderivative = 16.82

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input

```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^4,x)
```

output

```
(72*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*c*d**3*e**4 + 216
*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*c*d**2*e**5*x + 216*
sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b
*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*c*d*e**6*x**2 + 72*sq
rt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d
*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*c*e**7*x**3 + 18*sqrt(a
*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e +
 c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b**2*d**3*e**4 + 54*sqrt(a*e**
2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d
**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b**2*d**2*e**5*x + 54*sqrt(a*e**2
- b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**
2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b**2*d*e**6*x**2 + 18*sqrt(a*e**2 -
b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2)
 - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b**2*e**7*x**3 - 252*sqrt(a*e**2 - b*d
*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) -
2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*c*d**4*e**3 - 756*sqrt(a*e**2 - b*d*e +
 c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*
e + b*d - b*e*x + 2*c*d*x)*a*b*c*d**3*e**4*x - 756*sqrt(a*e**2 - b*d*e ...
```

3.82 $\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 440

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = -\frac{c\sqrt{a+bx+cx^2}}{e^4(d+ex)}$$
$$-\frac{(16c^2d^3+be^2(5bd-8ae)-4cde(6bd-7ae))\,(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{64e^3\,(cd^2-bde+ae^2)^2\,(d+ex)^2}$$
$$-\frac{(a+bx+cx^2)^{3/2}}{3e^2(d+ex)^3}-\frac{d(bd-2ae+(2cd-be)x)\,(a+bx+cx^2)^{3/2}}{8e\,(cd^2-bde+ae^2)\,(d+ex)^4}$$
$$+\frac{c^{3/2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{e^5}$$
$$-\frac{(128c^4d^5-b^3e^4(5bd-8ae)-320c^3d^3e(bd-ae)+240c^2de^2(bd-ae)^2-8bce^3(5b^2d^2-15abde+12a^2e^2}{128e^5\,(cd^2-bde+ae^2)^{5/2}}$$

output
```
-c*(c*x^2+b*x+a)^(1/2)/e^4/(e*x+d)-1/64*(16*c^2*d^3+b*e^2*(-8*a*e+5*b*d)-4
*c*d*e*(-7*a*e+6*b*d))*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/e^3/
(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2-1/3*(c*x^2+b*x+a)^(3/2)/e^2/(e*x+d)^3-1/8*
d*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*
x+d)^4+c^(3/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5-1/12
8*(128*c^4*d^5-b^3*e^4*(-8*a*e+5*b*d)-320*c^3*d^3*e*(-a*e+b*d)+240*c^2*d*e
^2*(-a*e+b*d)^2-8*b*c*e^3*(12*a^2*e^2-15*a*b*d*e+5*b^2*d^2))*arctanh(1/2*(
b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e
^5/(a*e^2-b*d*e+c*d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 14.36 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.38

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \frac{-\frac{2e\sqrt{a+x(b+cx)}(16c^3d^4(12d^3+42d^2ex+52de^2x^2+25e^3x^3)+e^3(16a^3e^3(d+4ex)-8a^2be^2(d^2+5dex-14e^2}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^5,x]
```

output
```
((-2*e*Sqrt[a + x*(b + c*x)]*(16*c^3*d^4*(12*d^3 + 42*d^2*e*x + 52*d*e^2*x
^2 + 25*e^3*x^3) + e^3*(16*a^3*e^3*(d + 4*e*x) - 8*a^2*b*e^2*(d^2 + 5*d*e*
x - 14*e^2*x^2) + b^3*d*(15*d^3 + 55*d^2*e*x + 73*d*e^2*x^2 - 15*e^3*x^3)
- 2*a*b^2*e*(7*d^3 + 26*d^2*e*x + 79*d*e^2*x^2 - 12*e^3*x^3)) - 8*c^2*d^2*
e*(-(a*e*(44*d^3 + 155*d^2*e*x + 184*d*e^2*x^2 + 91*e^3*x^3)) + b*d*(42*d^
3 + 148*d^2*e*x + 185*d*e^2*x^2 + 97*e^3*x^3)) + 2*c*e^2*(4*a^2*e^2*(13*d^
3 + 52*d^2*e*x + 53*d*e^2*x^2 + 32*e^3*x^3) - 2*a*b*d*e*(65*d^3 + 237*d^2*
e*x + 267*d*e^2*x^2 + 167*e^3*x^3) + b^2*d^2*(60*d^3 + 215*d^2*e*x + 274*d
*e^2*x^2 + 191*e^3*x^3))))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)^4) + 38
4*c^(3/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (3*(128
*c^4*d^5 - 320*c^3*d^3*e*(b*d - a*e) + 240*c^2*d*e^2*(b*d - a*e)^2 + b^3*e
^4*(-5*b*d + 8*a*e) - 8*b*c*e^3*(5*b^2*d^2 - 15*a*b*d*e + 12*a^2*e^2))*Arc
Tanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(5/2))/(384*e^5)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^5}\,dx$$

$$\Big\downarrow \ 1229$$

$$-\frac{\int \frac{\left(5deb^2-8\left(cd^2+ae^2\right)b+12acde-16c\left(cd^2-bed+ae^2\right)x\right)\sqrt{cx^2+bx+a}}{2(d+ex)^3}dx}{8e^2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\left(5deb^2-8\left(cd^2+ae^2\right)b+12acde-16c\left(cd^2-bed+ae^2\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^3}dx}{16e^2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1229$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\cdots\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}\cdots}{\cdots}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\cdots\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}\cdots}{\cdots}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1269

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1092

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 219

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1154

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 219

$$-\frac{\frac{\sqrt{a+bx+cx^2}\left(4cde^2\left(2a^2e^2-13abde+10b^2d^2\right)+be^3\left(16a^2e^2-18abde+5b^2d^2\right)-16c^2d^3e(7bd-6ae)+ex\left((2cd-be)\left(-4cde(6bd-7ae)+be^2(5bd-8\right.\right.\right.}{4e^2(d+ex)^2\left(ae^2-bde+cd^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(ex\left(14cd^2-e(11bd-8ae)\right)+d\left(8cd^2-e(5bd-2ae)\right)\right)}{24e^2(d+ex)^4\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^5,x]
```

output
```
-1/24*((d*(8*c*d^2 - e*(5*b*d - 2*a*e)) + e*(14*c*d^2 - e*(11*b*d - 8*a*e)
)*x)*(a + b*x + c*x^2)^(3/2))/(e^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) -
(((64*c^3*d^5 - 16*c^2*d^3*e*(7*b*d - 6*a*e) + 4*c*d*e^2*(10*b^2*d^2 - 13*
a*b*d*e + 2*a^2*e^2) + b*e^3*(5*b^2*d^2 - 18*a*b*d*e + 16*a^2*e^2) + e*((2
*c*d - b*e)*(16*c^2*d^3 + b*e^2*(5*b*d - 8*a*e) - 4*c*d*e*(6*b*d - 7*a*e))
 + 64*c*(c*d^2 - e*(b*d - a*e))^2)*x)*Sqrt[a + b*x + c*x^2])/(4*e^2*(c*d^2
 - b*d*e + a*e^2)*(d + e*x)^2) - ((128*c^(3/2)*(c*d^2 - b*d*e + a*e^2)^2*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/e - ((128*c^4*d^5 -
 b^3*e^4*(5*b*d - 8*a*e) - 320*c^3*d^3*e*(b*d - a*e) + 240*c^2*d*e^2*(b*d
- a*e)^2 - 8*b*c*e^3*(5*b^2*d^2 - 15*a*b*d*e + 12*a^2*e^2))*ArcTanh[(b*d -
 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(8*e^2*(c*d^2 - b*d*e + a*e^2)))/
(16*e^2*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8160 vs. 2(408) = 816.

Time = 1.69 (sec) , antiderivative size = 8161, normalized size of antiderivative = 18.55

method	result	size
default	Expression too large to display	8161

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^5}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**5,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^5}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^5,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 17.56 (sec) , antiderivative size = 10533, normalized size of antiderivative = 23.94

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^5,x)
```

output
```
( - 288*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt
(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b*c*d**4*e
**5 - 1152*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*s
qrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b*c*d**
3*e**6*x - 1728*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x*
*2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b*
c*d**2*e**7*x**2 - 1152*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*
x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)
*a**2*b*c*d*e**8*x**3 - 288*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a
+ b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*
d*x)*a**2*b*c*e**9*x**4 + 720*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(
a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*
c*d*x)*a**2*c**2*d**5*e**4 + 2880*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*s
qrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x
+ 2*c*d*x)*a**2*c**2*d**4*e**5*x + 4320*sqrt(a*e**2 - b*d*e + c*d**2)*log(
 - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d -
b*e*x + 2*c*d*x)*a**2*c**2*d**3*e**6*x**2 + 2880*sqrt(a*e**2 - b*d*e + c*d
**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e
 + b*d - b*e*x + 2*c*d*x)*a**2*c**2*d**2*e**7*x**3 + 720*sqrt(a*e**2 - b*d
*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d*...
```

3.83 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 293

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \frac{3(b^2-4ac)\,(bd-2ae)(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{128\,(cd^2-bde+ae^2)^3\,(d+ex)^2} - \frac{(bd-2ae)(bd-2ae+(2cd-be)x)\,(a+bx+cx^2)^{3/2}}{16\,(cd^2-bde+ae^2)^2\,(d+ex)^4} + \frac{d(a+bx+cx^2)^{5/2}}{5\,(cd^2-bde+ae^2)\,(d+ex)^5} - \frac{3(b^2-4ac)^2\,(bd-2ae)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{256\,(cd^2-bde+ae^2)^{7/2}}$$

output
```
3/128*(-4*a*c+b^2)*(-2*a*e+b*d)*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(
1/2)/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^2-1/16*(-2*a*e+b*d)*(b*d-2*a*e+(-b*e+2*
c*d)*x)*(c*x^2+b*x+a)^(3/2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^4+1/5*d*(c*x^2+b
*x+a)^(5/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^5-3/256*(-4*a*c+b^2)^2*(-2*a*e+b*d
)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(7/2)
```

Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \frac{d(a+x(b+cx))^{5/2}}{5\,(cd^2+e(-bd+ae))\,(d+ex)^5}$$

$$+\frac{(bd-2ae)\left(\frac{2(-bd+2ae-2cdx+bex)(a+x(b+cx))^{3/2}}{(d+ex)^4}+3(b^2-4ac)\left(\frac{\sqrt{a+x(b+cx)}(-2ae+2cdx+b(d-ex))}{4(cd^2+e(-bd+ae))(d+ex)^2}+\frac{(b^2-4ac)\text{arctan}}{8(c}\right.\right.}{32\,(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^6,x]
```

output
```
(d*(a + x*(b + c*x))^(5/2))/(5*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^5) + (
(b*d - 2*a*e)*((2*(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)*(a + x*(b + c*x))^(3/
2))/(d + e*x)^4 + 3*(b^2 - 4*a*c)*((Sqrt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*
x + b*(d - e*x)))/(4*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + ((b^2 - 4*a
*c)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])])/(8*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))))/(3
2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \; 1228$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}-\frac{(bd-2ae)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^5}dx}{2\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1152

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}-\frac{(bd-2ae)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3\left(b^2-4ac\right)\int\frac{\sqrt{cx^2+bx+a}}{(d+ex)^3}dx}{16(ae^2-bde+cd^2)}\right)}{2\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1152

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}-\frac{(bd-2ae)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\left(b^2-4ac\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{8\left(ae^2-bde+cd^2\right)}\right)}{16(ae^2-bde+cd^2)}\right)}{2\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1154

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}-\frac{(bd-2ae)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be}{\sqrt{cx^2+bx+a}}\right.}{4\left(ae^2-bde+cd^2\right)}\right.}{16(ae^2-bde+cd^2)}\right.}{2\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 219

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}-\frac{(bd-2ae)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{-2ae+x(2c}{2\sqrt{a+bx+cx^2}\sqrt{}}\right.}{8\left(ae^2-bde+cd^2\right)^{3/2}}\right.}{16(ae^2-bde+cd^2)}\right.}{2\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^6,x]
```

output
```
(d*(a + b*x + c*x^2)^(5/2))/(5*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^5) - ((b*
d - 2*a*e)*(((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(8*(
c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) - (3*(b^2 - 4*a*c)*(((b*d - 2*a*e + (2
*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)
^2) - ((b^2 - 4*a*c)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*e + a*e^2)^(3/2
))))/(16*(c*d^2 - b*d*e + a*e^2))))/(2*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10264 vs. 2(271) = 542.

Time = 1.86 (sec) , antiderivative size = 10265, normalized size of antiderivative = 35.03

method	result	size
default	Expression too large to display	10265

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2391 vs. 2(271) = 542.

Time = 43.12 (sec) , antiderivative size = 4824, normalized size of antiderivative = 16.46

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^6}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**6,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9517 vs. $2(271) = 542$.

Time = 1.41 (sec) , antiderivative size = 9517, normalized size of antiderivative = 32.48

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x, algorithm="giac")
```

output
```
-3/128*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d - 2*a*b^4*e + 16*a^2*b^2*c*e
- 32*a^3*c^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*
a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1
/640*(1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*c^5*d^6*e^4 - 3840*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^4*d^5*e^5 + 3840*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^9*b^2*c^3*d^4*e^6 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^9*a*c^4*d^4*e^6 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*d^3
*e^7 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*d^3*e^7 + 3840*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^2*d^2*e^8 + 3840*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^9*a^2*c^3*d^2*e^8 + 15*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^9*b^5*d*e^9 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d
*e^9 - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*d*e^9 - 30*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e^10 + 240*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a^2*b^2*c*e^10 + 800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*a^3*c^2*e^10 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(11/2)*d^7*e^
3 - 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^(9/2)*d^6*e^4 + 7680*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*d^5*e^5 + 15360*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*a*c^(9/2)*d^5*e^5 + 2560*(sqrt(c)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^6}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^6,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 96.50 (sec) , antiderivative size = 7687, normalized size of antiderivative = 26.24

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^6} \, dx = \text{Too large to display}$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^6,x)
```

output
```
(480*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**
2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**2*d**5*e + 24
00*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**2*d**4*e**2*x +
 4800*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**2*d**3*e**3*
x**2 + 4800*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqr
t(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**2*d**2
*e**4*x**3 + 2400*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**
2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c**
2*d*e**5*x**4 + 480*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x
**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*c
**2*e**6*x**5 - 240*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x
**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b
**2*c*d**5*e - 1200*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x
**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b
**2*c*d**4*e**2*x - 2400*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x
+ c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a
**2*b**2*c*d**3*e**3*x**2 - 2400*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(
a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x +...
```

3.84 $\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7} dx$

Optimal result

Integrand size = 23, antiderivative size = 414

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7} dx = -\frac{(b^2-4ac)\left(5b^2de+28acde-12b(cd^2+ae^2)\right)\left(bd-2ae+(2cd-be)x\right)\sqrt{a+bx+cx^2}}{512\left(cd^2-bde+ae^2\right)^4(d+ex)^2}$$
$$+\frac{\left(5b^2de+28acde-12b(cd^2+ae^2)\right)\left(bd-2ae+(2cd-be)x\right)\left(a+bx+cx^2\right)^{3/2}}{192\left(cd^2-bde+ae^2\right)^3(d+ex)^4}$$
$$+\frac{d(a+bx+cx^2)^{5/2}}{6\left(cd^2-bde+ae^2\right)(d+ex)^6}+\frac{\left(2cd^2+e(5bd-12ae)\right)\left(a+bx+cx^2\right)^{5/2}}{60\left(cd^2-bde+ae^2\right)^2(d+ex)^5}$$
$$+\frac{(b^2-4ac)^2\left(5b^2de+28acde-12b(cd^2+ae^2)\right)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{1024\left(cd^2-bde+ae^2\right)^{9/2}}$$

output
```
-1/512*(-4*a*c+b^2)*(5*b^2*d*e+28*a*c*d*e-12*b*(a*e^2+c*d^2))*(b*d-2*a*e+(
-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^2+1/192*(
5*b^2*d*e+28*a*c*d*e-12*b*(a*e^2+c*d^2))*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2
+b*x+a)^(3/2)/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^4+1/6*d*(c*x^2+b*x+a)^(5/2)/(a
*e^2-b*d*e+c*d^2)/(e*x+d)^6+1/60*(2*c*d^2+e*(-12*a*e+5*b*d))*(c*x^2+b*x+a)
^(5/2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^5+1/1024*(-4*a*c+b^2)^2*(5*b^2*d*e+28
*a*c*d*e-12*b*(a*e^2+c*d^2))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 12.47 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.86

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \frac{\frac{d(a+x(b+cx))^{5/2}}{(d+ex)^6}+\frac{(2cd^2+e(5bd-12ae))(a+x(b+cx))^{5/2}}{10(cd^2+e(-bd+ae))(d+ex)^5}-\frac{\left(\frac{5}{2}b^2de+14acde-6b(cd^2+ae^2)\right)\left(\frac{2(-bd+2a\cdots}{\cdots}\right.}{\cdots}}{6\cdots}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^7,x]
```

output
```
((d*(a + x*(b + c*x))^(5/2))/(d + e*x)^6 + ((2*c*d^2 + e*(5*b*d - 12*a*e))
*(a + x*(b + c*x))^(5/2))/(10*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^5) - ((
(5*b^2*d*e)/2 + 14*a*c*d*e - 6*b*(c*d^2 + a*e^2))*((2*(-(b*d) + 2*a*e - 2*
c*d*x + b*e*x)*(a + x*(b + c*x))^(3/2))/(d + e*x)^4 + 3*(b^2 - 4*a*c)*((Sq
rt[a + x*(b + c*x)]*(-2*a*e + 2*c*d*x + b*(d - e*x)))/(4*(c*d^2 + e*(-(b*d
) + a*e))*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x
+ b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(8*(c*
d^2 + e*(-(b*d) + a*e))^(3/2)))))/(32*(c*d^2 + e*(-(b*d) + a*e))^2))/(6*(c
*d^2 + e*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{(5bd-2cxd-12ae)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)^6}dx}{6\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{(5bd-2cxd-12ae)\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^6}dx}{12\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-\frac{-\frac{\left(-12b\left(ae^2+cd^2\right)+28acde+5b^2de\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^5}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(e(5bd-12ae)+2cd^2\right)}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}}{12\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-\frac{-\frac{\left(-12b\left(ae^2+cd^2\right)+28acde+5b^2de\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{(d+ex)^3}dx}{16\left(ae^2-bde+cd^2\right)}\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(e(5bd-12ae)+2\right.}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}}{12\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-$$

$$-\frac{\left(-12b\left(ae^2+cd^2\right)+28acde+5b^2de\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\left(b^2-4ac\right)\int\frac{}{(d+ex)\sqrt{c}}}{8\left(ae^2-bde+c\right.}}{16\left(ae^2-bde+cd^2\right)}}{2(ae^2-bde+cd^2)}}{12\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1154

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-$$

$$-\frac{\left(-12b\left(ae^2+cd^2\right)+28acde+5b^2de\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\right.}{4\left(ae^2-bde+cd^2\right)}\right.}{16\left(ae^2-bde+c\right.}}{2(ae^2-bde+cd^2)}}{12\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 219

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{6(d+ex)^6\left(ae^2-bde+cd^2\right)}-$$

$$-\frac{\left(-12b\left(ae^2+cd^2\right)+28acde+5b^2de\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-be)+bd)}{4(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\right.}{8\left(ae^2\right.}\right.}{16\left(ae^2-bde+cd^2\right)}}{2(ae^2-bde+cd^2)}}{12\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^7,x]
```

output
```
(d*(a + b*x + c*x^2)^(5/2))/(6*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^6) - (-1/
5*((2*c*d^2 + e*(5*b*d - 12*a*e))*(a + b*x + c*x^2)^(5/2))/((c*d^2 - b*d*e
 + a*e^2)*(d + e*x)^5) - ((5*b^2*d*e + 28*a*c*d*e - 12*b*(c*d^2 + a*e^2))*
(((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*d^2 - b*d
*e + a*e^2)*(d + e*x)^4) - (3*(b^2 - 4*a*c)*(((b*d - 2*a*e + (2*c*d - b*e)
*x)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - ((b^2
 - 4*a*c)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*d^2 - b*d*e + a*e^2)^(3/2))))/(16*(c
*d^2 - b*d*e + a*e^2))))/(2*(c*d^2 - b*d*e + a*e^2)))/(12*(c*d^2 - b*d*e +
 a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15587 vs. $2(388) = 776$.

Time = 2.30 (sec) , antiderivative size = 15588, normalized size of antiderivative = 37.65

method	result	size
default	Expression too large to display	15588

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4039 vs. $2(388) = 776$.

Time = 114.28 (sec) , antiderivative size = 8120, normalized size of antiderivative = 19.61

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^7}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**7,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15909 vs. $2(388) = 776$.

Time = 13.01 (sec) , antiderivative size = 15909, normalized size of antiderivative = 38.43

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

output
```
-1/512*(12*b^5*c*d^2 - 96*a*b^3*c^2*d^2 + 192*a^2*b*c^3*d^2 - 5*b^6*d*e +
12*a*b^4*c*d*e + 144*a^2*b^2*c^2*d*e - 448*a^3*c^3*d*e + 12*a*b^5*e^2 - 96
*a^2*b^3*c*e^2 + 192*a^3*b*c^2*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c^4*d^8 - 4*b*c^3*d^
7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 - 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d
^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 + 6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^
3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^6 + 4*a^3*c*d^2*e^6 - 4*a^3*b
*d*e^7 + a^4*e^8)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/7680*(180*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^11*b^5*c*d^2*e^10 - 1440*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^11*a*b^3*c^2*d^2*e^10 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^11*a^2*b*c^3*d^2*e^10 - 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*d*
e^11 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c*d*e^11 + 2160*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^2*c^2*d*e^11 - 6720*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^11*a^3*c^3*d*e^11 + 180*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^11*a*b^5*e^12 - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2
*b^3*c*e^12 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*c^2*e^12 +
 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(13/2)*d^8*e^4 - 61440*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*b*c^(11/2)*d^7*e^5 + 92160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^10*b^2*c^(9/2)*d^6*e^6 + 61440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^10*a*c^(11/2)*d^6*e^6 - 61440*(sqrt(c)*x - sqrt(c*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^7}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^7,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^7, x)
```

Reduce [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^7}\,dx = \int \frac{x(c\,x^2+bx+a)^{\frac{3}{2}}}{(ex+d)^7}dx$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x)
```

output
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^7,x)
```

3.85 $\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 606

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8} dx = \frac{(b^2-4ac)\left(5b^3de^2-8ace(8cd^2-ae^2)+4bcd(6cd^2+13ae^2)-2b^2(10cd^2e+7a\right.}{1024\left(cd^2-bde+ae^2\right)^5(d+ex)^2}$$
$$-\frac{(5b^3de^2-8ace(8cd^2-ae^2)+4bcd(6cd^2+13ae^2)-2b^2(10cd^2e+7ae^3))\,(bd-2ae+(2cd-be)x)\,(a+b}{384\left(cd^2-bde+ae^2\right)^4(d+ex)^4}$$
$$+\frac{d(a+bx+cx^2)^{5/2}}{7\left(cd^2-bde+ae^2\right)(d+ex)^7}+\frac{(4cd^2+e(5bd-14ae))\,(a+bx+cx^2)^{5/2}}{84\left(cd^2-bde+ae^2\right)^2(d+ex)^6}$$
$$+\frac{(8c^2d^3+2cde(45bd-122ae)-7be^2(5bd-14ae))\,(a+bx+cx^2)^{5/2}}{840\left(cd^2-bde+ae^2\right)^3(d+ex)^5}$$
$$-\frac{(b^2-4ac)^2\,(5b^3de^2-8ace(8cd^2-ae^2)+4bcd(6cd^2+13ae^2)-2b^2(10cd^2e+7ae^3))\operatorname{arctanh}\left(\frac{bd-2ae+}{2\sqrt{cd^2-bde+}}\right.}{2048\left(cd^2-bde+ae^2\right)^{11/2}}$$

output
```
1/1024*(-4*a*c+b^2)*(5*b^3*d*e^2-8*a*c*e*(-a*e^2+8*c*d^2)+4*b*c*d*(13*a*e^
2+6*c*d^2)-2*b^2*(7*a*e^3+10*c*d^2*e))*(b*d-2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b
*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^5/(e*x+d)^2-1/384*(5*b^3*d*e^2-8*a*c*e*(-a
*e^2+8*c*d^2)+4*b*c*d*(13*a*e^2+6*c*d^2)-2*b^2*(7*a*e^3+10*c*d^2*e))*(b*d-
2*a*e+(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(3/2)/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^4+
1/7*d*(c*x^2+b*x+a)^(5/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^7+1/84*(4*c*d^2+e*(-
14*a*e+5*b*d))*(c*x^2+b*x+a)^(5/2)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^6+1/840*(
8*c^2*d^3+2*c*d*e*(-122*a*e+45*b*d)-7*b*e^2*(-14*a*e+5*b*d))*(c*x^2+b*x+a)
^(5/2)/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^5-1/2048*(-4*a*c+b^2)^2*(5*b^3*d*e^2-
8*a*c*e*(-a*e^2+8*c*d^2)+4*b*c*d*(13*a*e^2+6*c*d^2)-2*b^2*(7*a*e^3+10*c*d^
2*e))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*
x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(11/2)
```

Mathematica [A] (verified)

Time = 16.13 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.16

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx = \frac{d(a+bx+cx^2)\,(a+x(b+cx))^{3/2}}{7\,(cd^2-bde+ae^2)\,(d+ex)^7}$$

$$-\frac{(a+x(b+cx))^{3/2}\left(-\frac{\left(2cd^2+\frac{1}{2}e(5bd-14ae)\right)\left(a+bx+cx^2\right)^{5/2}}{6(cd^2-bde+ae^2)(d+ex)^6}-\frac{\frac{\left(-\frac{1}{4}e\left(-80bcd^2+35b^2de+216acde-98abe^2\right)+\frac{1}{2}cd\left(4cd^2+e(5bd-14ae)\right)\right)\left(a\right.}{5\left(cd^2-bde+ae^2\right)(d+ex)^5}}{}\right.}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^8,x]
```

output
```
(d*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(7*(c*d^2 - b*d*e + a*e^2)*(
d + e*x)^7) - ((a + x*(b + c*x))^(3/2)*(-1/6*((2*c*d^2 + (e*(5*b*d - 14*a*
e))/2)*(a + b*x + c*x^2)^(5/2))/((c*d^2 - b*d*e + a*e^2)*(d + e*x)^6) - ((
(-1/4*(e*(-80*b*c*d^2 + 35*b^2*d*e + 216*a*c*d*e - 98*a*b*e^2)) + (c*d*(4*
c*d^2 + e*(5*b*d - 14*a*e)))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*d^2 - b*d*e
 + a*e^2)*(d + e*x)^5) - ((b*((e*(-80*b*c*d^2 + 35*b^2*d*e + 216*a*c*d*e -
 98*a*b*e^2))/4 + (c*d*(4*c*d^2 + e*(5*b*d - 14*a*e)))/2) - 2*((c*d*(-80*b
*c*d^2 + 35*b^2*d*e + 216*a*c*d*e - 98*a*b*e^2))/4 + (a*c*e*(4*c*d^2 + e*(
5*b*d - 14*a*e)))/2))*(((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^
(3/2))/(8*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4) - (3*(b^2 - 4*a*c)*(((b*d -
 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2
)*(d + e*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*d) + 2*a*e - (2*c*d - b*e)*x)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*(4*c*d^2 - 4*b*d*e + 4*a*e^2))))/(16*(c*d^2 - b*d*e + a*e^2)
)))/(2*(c*d^2 - b*d*e + a*e^2)))/(6*(c*d^2 - b*d*e + a*e^2))))/(7*(c*d^2 -
 b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{(d+ex)^8}\,dx$$

$$\Big\downarrow \text{1237}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{(5bd-4cxd-14ae)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)^7}dx}{7\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{(5bd-4cxd-14ae)\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^7}dx}{14\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1237}$$

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{-\frac{\int -\frac{\left(-35deb^2+80cd^2b+98ae^2b-216acde-2c\left(4cd^2+e(5bd-14ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(d+ex)^6}dx}{6(ae^2-bde+cd^2)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(e(5bd-14ae)+4cd^2\right)}{6(d+ex)^6(ae^2-bde+cd^2)}}{14\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\frac{\int \frac{\left(-35deb^2+80cd^2b+98ae^2b-216acde-2c\left(4cd^2+e(5bd-14ae)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^6}dx}{12(ae^2-bde+cd^2)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(e(5bd-14ae)+4cd^2\right)}{6(d+ex)^6(ae^2-bde+cd^2)}}{14\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1228

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\frac{\frac{7\left(-2b^2\left(7ae^3+10cd^2e\right)+4bcd\left(13ae^2+6cd^2\right)-8ace\left(8cd^2-ae^2\right)+5b^3de^2\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{(d+ex)^5}dx}{2\left(ae^2-bde+cd^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(2cde(45bd-122ae)-7be^2(5bd-14ae)+8c^2d^3\cdots\right.}{5(d+ex)^5\left(ae^2-bde+cd^2\right)}}{12(ae^2-bde+cd^2)}}{14\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1152

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\frac{\frac{7\left(-2b^2\left(7ae^3+10cd^2e\right)+4bcd\left(13ae^2+6cd^2\right)-8ace\left(8cd^2-ae^2\right)+5b^3de^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{(d+ex)^3}dx}{16\left(ae^2-bde+cd^2\right)}\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{(a+bx+\cdots}{\cdots}}{12(ae^2-bde+cd^2)}}{14\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1152

$$\frac{d\left(a+bx+cx^2\right)^{5/2}}{7(d+ex)^7\left(ae^2-bde+cd^2\right)}-\frac{\frac{\frac{7\left(-2b^2\left(7ae^3+10cd^2e\right)+4bcd\left(13ae^2+6cd^2\right)-8ace\left(8cd^2-ae^2\right)+5b^3de^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4\left(ae^2-bde+cd^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-\cdots}{4(d+ex)^2\left(ae^2-bde+cd\cdots\right.}\cdots\right.}{16\left(ae^2\cdots\right.}\cdots\right.}{2\left(ae^2-bde+cd^2\right)}\cdots}{12(ae^2-bde+cd^2)}}{14\left(ae^2-\cdots\right.}$$

$\downarrow$ 1154

$$\frac{d(a+bx+cx^2)^{5/2}}{7(d+ex)^7(ae^2-bde+cd^2)} - \frac{\frac{7\left(-2b^2\left(7ae^3+10cd^2e\right)+4bcd\left(13ae^2+6cd^2\right)-8ace\left(8cd^2-ae^2\right)+5b^3de^2\right)\left(\frac{(a+bx+cx^2)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int \frac{}{4(cd^2-bed+ae^2}}{}\right.}{}\right.}{2(ae^2-bde+cd^2)}}{12(ae^2-}$$

$\downarrow$ 219

$$\frac{d(a+bx+cx^2)^{5/2}}{7(d+ex)^7(ae^2-bde+cd^2)} - \frac{\frac{7\left(-2b^2\left(7ae^3+10cd^2e\right)+4bcd\left(13ae^2+6cd^2\right)-8ace\left(8cd^2-ae^2\right)+5b^3de^2\right)\left(\frac{(a+bx+cx^2)^{3/2}(-2ae+x(2cd-be)+bd)}{8(d+ex)^4(ae^2-bde+cd^2)}-\frac{3(b^2-4ac)\left(\frac{\sqrt{a+bx+cx^2}(-2ae+x(2cd-b}{4(d+ex)^2(ae^2-bde+cd}\right.}{}\right.}{2(ae^2-bde+cd^2)}}{12(ae^2-bde+cd^2)} \quad 14$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^8,x]
```

output
```
(d*(a + b*x + c*x^2)^(5/2))/(7*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^7) - (-1/
6*((4*c*d^2 + e*(5*b*d - 14*a*e))*(a + b*x + c*x^2)^(5/2))/((c*d^2 - b*d*e
 + a*e^2)*(d + e*x)^6) + (-1/5*((8*c^2*d^3 + 2*c*d*e*(45*b*d - 122*a*e) -
7*b*e^2*(5*b*d - 14*a*e))*(a + b*x + c*x^2)^(5/2))/((c*d^2 - b*d*e + a*e^2
)*(d + e*x)^5) + (7*(5*b^3*d*e^2 - 8*a*c*e*(8*c*d^2 - a*e^2) + 4*b*c*d*(6*
c*d^2 + 13*a*e^2) - 2*b^2*(10*c*d^2*e + 7*a*e^3))*(((b*d - 2*a*e + (2*c*d
- b*e)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^4)
 - (3*(b^2 - 4*a*c)*(((b*d - 2*a*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2
])/(4*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*d -
 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(8*(c*d^2 - b*d*e + a*e^2)^(3/2))))/(16*(c*d^2 - b*d*e + a*e^2))))
/(2*(c*d^2 - b*d*e + a*e^2)))/(12*(c*d^2 - b*d*e + a*e^2)))/(14*(c*d^2 - b
*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26118 vs. 2(576) = 1152.

Time = 2.66 (sec) , antiderivative size = 26119, normalized size of antiderivative = 43.10

method	result	size
default	Expression too large to display	26119

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx=\text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx=\int \frac{x(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)^8}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(e*x+d)**8,x)
```

output
```
Integral(x*(a + b*x + c*x**2)**(3/2)/(d + e*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24842 vs. $2(576) = 1152$.

Time = 7.91 (sec) , antiderivative size = 24842, normalized size of antiderivative = 40.99

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/1024*(24*b^5*c^2*d^3 - 192*a*b^3*c^3*d^3 + 384*a^2*b*c^4*d^3 - 20*b^6*c
*d^2*e + 96*a*b^4*c^2*d^2*e + 192*a^2*b^2*c^3*d^2*e - 1024*a^3*c^4*d^2*e +
 5*b^7*d*e^2 + 12*a*b^5*c*d*e^2 - 336*a^2*b^3*c^2*d*e^2 + 832*a^3*b*c^3*d*
e^2 - 14*a*b^6*e^3 + 120*a^2*b^4*c*e^3 - 288*a^3*b^2*c^2*e^3 + 128*a^4*c^3
*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))/((c^5*d^10 - 5*b*c^4*d^9*e + 10*b^2*c^3*d^8*e^2 + 5*
a*c^4*d^8*e^2 - 10*b^3*c^2*d^7*e^3 - 20*a*b*c^3*d^7*e^3 + 5*b^4*c*d^6*e^4
+ 30*a*b^2*c^2*d^6*e^4 + 10*a^2*c^3*d^6*e^4 - b^5*d^5*e^5 - 20*a*b^3*c*d^5
*e^5 - 30*a^2*b*c^2*d^5*e^5 + 5*a*b^4*d^4*e^6 + 30*a^2*b^2*c*d^4*e^6 + 10*
a^3*c^2*d^4*e^6 - 10*a^2*b^3*d^3*e^7 - 20*a^3*b*c*d^3*e^7 + 10*a^3*b^2*d^2
*e^8 + 5*a^4*c*d^2*e^8 - 5*a^4*b*d*e^9 + a^5*e^10)*sqrt(-c*d^2 + b*d*e - a
*e^2)) + 1/107520*(2520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^5*c^2*d^3
*e^11 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^3*c^3*d^3*e^11 +
40320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b*c^4*d^3*e^11 - 2100*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^6*c*d^2*e^12 + 10080*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^13*a*b^4*c^2*d^2*e^12 + 20160*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^13*a^2*b^2*c^3*d^2*e^12 - 107520*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^13*a^3*c^4*d^2*e^12 + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b
^7*d*e^13 + 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^5*c*d*e^13 - 3
5280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^3*c^2*d*e^13 + 87360*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{(d+e\,x)^8}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^8,x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d + e*x)^8, x)
```

Reduce [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{(d+ex)^8}\,dx = \int \frac{x(c\,x^2+bx+a)^{\frac{3}{2}}}{(ex+d)^8}dx$$

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x)
```

output
```
int(x*(c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x)
```

3.86 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 652

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = -\frac{3e\sqrt{a+bx+cx^2}}{d^2\sqrt{d+ex}}-\frac{\sqrt{a+bx+cx^2}}{dx\sqrt{d+ex}}$$
$$+\frac{3\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}d^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(bd-3ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\right.}{\left(b+\sqrt{b^2-4ac}\right)d^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-3*e*(c*x^2+b*x+a)^(1/2)/d^2/(e*x+d)^(1/2)-(c*x^2+b*x+a)^(1/2)/d/x/(e*x+d)
^(1/2)+3/2*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(
-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/d^2/(
c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1
/2)*(-4*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2))/d/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(-3*a*e+b*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),(-2*(-
4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/(b+(-4*a*c+b^2
)^(1/2))/d^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.79 (sec) , antiderivative size = 1390, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x)^(3/2)),x]
```

output
```
Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]*(-(1/(d^2*x)) - (2*e)/(d^2*(d + e*x)))
 + ((d + e*x)^(3/2)*Sqrt[a + x*(b + c*x)]*(12*d*Sqrt[(c*d^2 + e*(-(b*d) +
a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 + d/(d + e*x))^2 +
(e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) - ((3*I)*Sqrt[2]*d*
(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2
*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))
/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] +
(2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x))
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*S
qrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(4*c*d^2 - 5*b*d*e + 6
*a*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a
*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(
2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2
*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/
(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[
d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - ((2*I)*Sqrt[2]*e*(-(b*d) + 3*a*e...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \; 1292$$

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}dx$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(572) = 1144$.

Time = 5.76 (sec) , antiderivative size = 1177, normalized size of antiderivative = 1.81

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	1672
default	Expression too large to display	3114

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/d^2/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)-1/d^2*(c*e*x^3+b*e*x
^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/x+2*(-(b*e-c*d)/d^2+b*e/d^2)*(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+3*c*e/d^2*(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/d^3*(3*a*e-b*d)*(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**2/(e*x+d)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x**2*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}}{x^2\,(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)/x^2/(e*x+d)^(3/2),x)
```

3.87 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 723

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)\sqrt{d+ex}}{a\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}}$$
$$-\frac{(3b^2-8ac)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{a^2\left(b^2-4ac\right)x}$$
$$+\frac{(3b^2-8ac)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}(3b^2d-8acd-2abe)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{a^2\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(3bd-ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{a^2\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2*(b*c*x-2*a*c+b^2)*(e*x+d)^(1/2)/a/(-4*a*c+b^2)/x/(c*x^2+b*x+a)^(1/2)-(-8
*a*c+3*b^2)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a^2/(-4*a*c+b^2)/x+1/2*(-8*a
*c+3*b^2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/
2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/
(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(1/2)/(c
*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1/
2)*(-2*a*b*e-8*a*c*d+3*b^2*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c
+b^2)^(1/2))*e))^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)
^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-a*e+3*b*d)*(c*(e*x+d)/(2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi
(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e))^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.38 (sec) , antiderivative size = 7762, normalized size of antiderivative = 10.74

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1292}$$
$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}dx$$

input
```
Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1343 vs. $2(644) = 1288$.

Time = 11.75 (sec) , antiderivative size = 1344, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1344
risch	Expression too large to display	1731
default	Expression too large to display	5827

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/a^2*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/x-2*(c*e*x+c*d)*((2*a*c-b^2)
/a^2/(4*a*c-b^2)*x+(3*a*c-b^2)*b/a^2/(4*a*c-b^2)/c)/((a/c+b/c*x+x^2)*(c*e*
x+c*d))^(1/2)+2*(-(4*a*b*c*e+4*a*c^2*d-b^3*e-2*b^2*c*d)/a^2/(4*a*c-b^2)+e*
(3*a*c-b^2)*b/a^2/(4*a*c-b^2)+2*c*d*(2*a*c-b^2)/a^2/(4*a*c-b^2))*(d/e-1/2*
(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(1/2*c*e/a^2+(2*a*
c-b^2)*c*e/a^2/(4*a*c-b^2))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a
*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+
d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b
^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((
-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x**2*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{d+e\,x}}{x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,dx$$

input
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{ex+d}}{x^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(3/2),x)
```

3.88 $\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 599

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2(8c^2d^2+24b^2e^2+ce(13bd-25ae))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e^2}$$
$$-\frac{6(3cd+2be)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e^2}+\frac{2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^2}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8c^3d^3-48b^3e^3+c^2de(9bd-19ae)+8bce^2(2bd+13ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\Big(\mathrm{arc}}{105c^4e^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(8c^2d^2+24b^2e^2+ce(13bd-25ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{105c^4e^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/105*(8*c^2*d^2+24*b^2*e^2+c*e*(-25*a*e+13*b*d))*(e*x+d)^(1/2)*(c*x^2+b*x
+a)^(1/2)/c^3/e^2-6/35*(2*b*e+3*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2
/e^2+2/7*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/105*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(8*c^3*d^3-48*b^3*e^3+c^2*d*e*(-19*a*e+9*b*d)+8*b*c*e^2*(13*a*e+2*
b*d))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1
+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c
*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^3/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+
b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/105*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2+24*b^2*e^2+c*e*(-25*a*e+13*b*d))*(c*(e*x+d
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^3/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.12 (sec) , antiderivative size = 5357, normalized size of antiderivative = 8.94

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^3*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1283, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$\Big\downarrow$ 1283

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\int \frac{x\left(-\left((cd-6be)x^2\right)+5(bd+ae)x+4ad\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{7c}$$

$\Big\downarrow$ 2184

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\frac{2\int \frac{e^2\left(7c^2d^2-24b^2e^2-ce(13bd-25ae)\right)x^2+e\left(2c^2d^3-ce(7bd-23ae)d-6be^2(5bd+3ae)\right)x+de(bd+3ae)(cd-6be)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}-\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(cd-6be)}{5ce^2}}{7c}$$

$\Big\downarrow$ 27

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\frac{\int \frac{e^2\left(7c^2d^2-24b^2e^2-ce(13bd-25ae)\right)x^2+e\left(2c^2d^3-ce(7bd-23ae)d-6be^2(5bd+3ae)\right)x+de(bd+3ae)(cd-6be)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}-\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(cd-6be)}{5ce^2}}{7c}$$

$\Big\downarrow$ 2184

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\frac{\frac{2\int \frac{e^3\left(24de^2b^3-\left(5cd^2e-24ae^3\right)b^2-2cd\left(2cd^2+33ae^2\right)b+ace\left(2cd^2-25ae^2\right)-\left(8c^3d^3+c^2e(9bd-19ae)d-48b^3e^3+8bce^2(2bd+13ae)\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2e\sqrt{d+ex}\sqrt{a+bx+c\cdots}}{\cdots}}{5ce^3}\cdots}{7c}$$

$\Big\downarrow$ 27

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\frac{\frac{e\int \frac{24de^2b^3-\left(5cd^2e-24ae^3\right)b^2-2cd\left(2cd^2+33ae^2\right)b+ace\left(2cd^2-25ae^2\right)-\left(8c^3d^3+c^2e(9bd-19ae)d-48b^3e^3+8bce^2(2bd+13ae)\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-\cdots\right)}{\cdots}}{5ce^3}\cdots}{7c}$$

$\Big\downarrow$ 1269

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-\frac{\frac{\frac{e\left(\frac{\left(ae^2-bde+cd^2\right)\left(-25ace^2+24b^2e^2+13bcde+8c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(c^2de(9bd-19ae)+8bce^2(13ae+2bd)-48b^3e^3+8c^3d^3\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}\right)}{3c}+\cdots}{5ce^3}\cdots}{7c}$$

$\downarrow$ 1172

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-$$

$$e\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-25ace^2+24b^2e^2+13bcde+8c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int\frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4a\right.}}{2cd-\left(b+\sqrt{b^2-4ac}\right.}}}}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\right.$$

$\downarrow$ 321

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-$$

$$e\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-25ace^2+24b^2e^2+13bcde+8c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2cd}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\right.$$

$\downarrow$ 327

$$\frac{2x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7c}-$$

$$e\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\left(-25ace^2+24b^2e^2+13bcde+8c^2d^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2cd}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\right.$$

input
```
Int[(x^3*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*x^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(7*c) - ((-2*(c*d - 6*b*e)*(d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e^2) + ((2*e*(7*c^2*d^2 - 24*b^2*
e^2 - c*e*(13*b*d - 25*a*e))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) +
(e*(-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*c^3*d^3 - 48*b^3*e^3 + c^2*d*e*(9*b*d
- 19*a*e) + 8*b*c*e^2*(2*b*d + 13*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 -
b*d*e + a*e^2)*(8*c^2*d^2 + 13*b*c*d*e + 24*b^2*e^2 - 25*a*c*e^2)*Sqrt[(c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])))/(3*c))/(5*c*e
^3))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

rule 2184

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(533) = 1066$.

Time = 4.64 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1125
risch	Expression too large to display	2640
default	Expression too large to display	6517

input
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7/c*x^2
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(d-2/7/c*(3*b*e+3*c*d
))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(-2/7/c*(5/2*
a*e+5/2*b*d)-2/5*(d-2/7/c*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(-2/5*(d-2/7/c*(3*b*e+3*c*d))/c/e*
a*d-2/3*(-2/7/c*(5/2*a*e+5/2*b*d)-2/5*(d-2/7/c*(3*b*e+3*c*d))/c/e*(2*b*e+2
*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/
(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2))+2*(-4/7*a/c*d-2/5*(d-2/7/c*(3*b*e+3*c*d))/c/e*(3/2*a*e+
3/2*b*d)-2/3*(-2/7/c*(5/2*a*e+5/2*b*d)-2/5*(d-2/7/c*(3*b*e+3*c*d))/c/e*(2*
b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/
(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*
e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.98

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left(\left(8\,c^4d^4+5\,bc^3d^3e+(10\,b^2c^2-13\,ac^3)d^2e^2+(40\,b^3c-113\,abc^2)de^3-(48\,b^4-176\,ab^2c+75\,a^2c^2)e\right.\right.}{}$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*c^4*d^4 + 5*b*c^3*d^3*e + (10*b^2*c^2 - 13*a*c^3)*d^2*e^2 + (40
*b^3*c - 113*a*b*c^2)*d*e^3 - (48*b^4 - 176*a*b^2*c + 75*a^2*c^2)*e^4)*sqr
t(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^
2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*
b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(8*c^4
*d^3*e + 9*b*c^3*d^2*e^2 + (16*b^2*c^2 - 19*a*c^3)*d*e^3 - 8*(6*b^3*c - 13
*a*b*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(
3*c*e*x + c*d + b*e)/(c*e))) - 3*(15*c^4*e^4*x^2 - 4*c^4*d^2*e^2 - 5*b*c^3
*d*e^3 + (24*b^2*c^2 - 25*a*c^3)*e^4 + 3*(c^4*d*e^3 - 6*b*c^3*e^4)*x)*sqrt
(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^5*e^4)
```

Sympy [F]

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**3*sqrt(d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x^3}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^3/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x^3}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*x^3/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3\,\sqrt{d+e\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3\sqrt{ex+d}}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.89 $\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 496

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{4(cd+2be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e}+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^2d^2-8b^2e^2+3ce(bd+3ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2cd-}\right.}{15c^3e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(cd+2be)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+}}{}\right.\right.}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-4/15*(2*b*e+c*d)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2/5*(e*x+d)^(3/2
)*(c*x^2+b*x+a)^(1/2)/c/e-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c^2*d^2-8*b^2
*e^2+3*c*e*(3*a*e+b*d))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c
+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^2/(c*(e*x+d)/
(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+4/15*2^(1/2)*(
-4*a*c+b^2)^(1/2)*(2*b*e+c*d)*(a*e^2-b*d*e+c*d^2)*(c*(e*x+d)/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF
(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)
*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^2/(e*x+d)^(1/2)/(c*x^2+b
*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.48 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.43

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{-\frac{4e^2(2c^2d^2-8b^2e^2+3ce(bd+3ae))(a+x(b+cx))}{\sqrt{d+ex}} + 4ce^2\sqrt{d+ex}(a+x(b+cx))(-4be+c(d+3ex)) + \frac{i(d+ex)\sqrt{1-\frac{\cdots}{(2c\cdots}}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^2*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output

```
((-4*e^2*(2*c^2*d^2 - 8*b^2*e^2 + 3*c*e*(b*d + 3*a*e))*(a + x*(b + c*x)))/
Sqrt[d + e*x] + 4*c*e^2*Sqrt[d + e*x]*(a + x*(b + c*x))*(-4*b*e + c*(d + 3
*e*x)) + (I*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*
e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) +
a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*(-((2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2])*(-2*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(b*d + 3*a*e
))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]) + (-8*b^3*e^3 +
b^2*e^2*(11*c*d + 8*Sqrt[(b^2 - 4*a*c)*e^2]) + b*c*e*(17*a*e^2 - 3*d*Sqrt[
(b^2 - 4*a*c)*e^2]) - c*(2*c*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + a*e^2*(14*c*d +
 9*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b
*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -
((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2]))]))/Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])])/(30*c^3*e^3*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1283, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1283}$$

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\int \frac{-\left((cd-4be)x^2\right)+3(bd+ae)x+2ad}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5c}$$

$$\Big\downarrow \text{2184}$$

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{2\int \frac{e\left(-4deb^2+cd^2b-4ae^2b+7acde+\left(2c^2d^2-8b^2e^2+3ce(bd+3ae)\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(cd-4be)}{3ce}}{5c}$$

$\downarrow$ 27

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{\int\frac{-4deb^2+cd^2b-4ae^2b+7acde+\left(2c^2d^2-8b^2e^2+3ce(bd+3ae)\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(cd-4be)}{3ce}}{5c}$$

$\downarrow$ 1269

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{\frac{\left(3ce(3ae+bd)-8b^2e^2+2c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2(2be+cd)\left(ae^2-bde+cd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(cd-4be)}{3ce}}{5c}$$

$\downarrow$ 1172

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(3ce(3ae+bd)-8b^2e^2+2c^2d^2\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2b\cdots}{\cdots}}{3ce}\cdots}{5\cdots}$$

$\downarrow$ 321

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(3ce(3ae+bd)-8b^2e^2+2c^2d^2\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2b\cdots}{\cdots}}{3ce}\cdots}{5c}$$

$\downarrow$ 327

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5c}-\frac{\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(3ce(3ae+bd)-8b^2e^2+2c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+c\cdots\right)}{b^2-4a\cdots}}\cdots}{\cdots}}{3ce}\cdots}{5\cdots}$$

input
```
Int[(x^2*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(5*c) - ((-2*(c*d - 4*b*e)*Sqrt[
d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c
^2*d^2 - 8*b^2*e^2 + 3*c*e*(b*d + 3*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)]])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d + 2
*b*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]])/(c*e*Sqrt[d + e*x]*
Sqrt[a + b*x + c*x^2]))/(3*c*e))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(436) = 872.

Time = 4.24 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2x\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{5c}+\frac{2\left(d-\frac{2(2be+2cd)}{5c}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ce}+\frac{2\left(-\frac{2ad}{5c}-\frac{2\left(d-\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*x/c*(
c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(d-2/5/c*(2*b*e+2*c*d))
/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(-2/5*a/c*d-2/3*(d-
2/5/c*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(-2/5/c*(3/2*a*e+3/2*b*d)-2/3*(d-2/5/c*
(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/
(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((
x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c
+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.98

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\left((2\,c^3d^3+2\,bc^2d^2e+(7\,b^2c-12\,ac^2)de^2-(8\,b^3-21\,abc)e^3)\sqrt{ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+(b^2-3}{3\,c^2e^2}\right.\right.}{}$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*c^3*d^3 + 2*b*c^2*d^2*e + (7*b^2*c - 12*a*c^2)*d*e^2 - (8*b^3 - 2
1*a*b*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2
- 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a
*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/
(c*e)) + 3*(2*c^3*d^2*e + 3*b*c^2*d*e^2 - (8*b^2*c - 9*a*c^2)*e^3)*sqrt(c*
e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*
c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(
c*e))) + 3*(3*c^3*e^3*x + c^3*d*e^2 - 4*b*c^2*e^3)*sqrt(c*x^2 + b*x + a)*s
qrt(e*x + d))/(c^4*e^3)
```

Sympy [F]

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x^2}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x^2/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x^2}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*x^2/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\,\sqrt{d+e\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\sqrt{ex+d}}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.90 $\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 423

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cd-2be)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right), -\right.}{3c^2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-2
*b*e+c*d)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/
2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/
(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e/(c*(e*x+d)/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/3*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(a*e^2-b*d*e+c*d^2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/
2))*e))^(1/2))/c^2/e/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.87 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.42

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{d+ex}\left(c(a+x(b+cx)) + \frac{(d+ex)\left(\frac{e^2(cd-2be)(a+x(b+cx))}{(d+ex)^2} + \frac{i\sqrt{1-\frac{2\left(cd^2+e(-bd+ae)\right)}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{1+\frac{2\left(cd^2+e(-bd+ae)\right)}{\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right.}}}{}\right.}{}\right.}{}$$

input
```
Integrate[(x*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[d + e*x]*(c*(a + x*(b + c*x)) + ((d + e*x)*((e^2*(c*d - 2*b*e)*(a
+ x*(b + c*x)))/(d + e*x)^2 + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)
))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2
 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))
]*((-(c*d) + 2*b*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*EllipticE[I*Ar
cSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2
*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (2*b^2*e^2 - 2*a*c*e^2 + c*d*Sqr
t[(b^2 - 4*a*c)*e^2] - b*e*(3*c*d + 2*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[
I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(
b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/e^
2))/(3*c^2*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2\int -\frac{bd+ae-(cd-2be)x}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3c}+\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}-\frac{\int \frac{bd+ae-(cd-2be)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{(ae^2-bde+cd^2)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{(cd-2be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}}{3c}$$

$$\Big\downarrow \text{1172}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}-$$

$$\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\cdots}{3c}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\sqrt{2}\sqrt{\cdots}}{3c}$$

$\downarrow$ 327

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\sqrt{2}\sqrt{\cdots}}{3c}$$

input
```
Int[(x*Sqrt[d + e*x])/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (-((Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(c*d - 2*b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
 + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(369) = 738$.

Time = 2.31 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.91

method	result
elliptic	$\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3c}-\frac{4\left(\frac{ae}{2}+\frac{bd}{2}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+}}{2c}}}}{3c\sqrt{ce\,x^3+be}}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/c*(c*
e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)-4/3/c*(1/2*a*e+1/2*b*d)*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(d-2/3/c*(b*e+
c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*
((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.99

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}c^2e^2-(c^2d^2+2\,bcde-(2\,b^2-3\,ac)e^2)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+}{3\,c^2}\right.\right.}{}$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*c^2*e^2 - (c^2*d^2 + 2*b*c*d*e
- (2*b^2 - 3*a*c)*e^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*
e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^
2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*
d + b*e)/(c*e)) - 3*(c^2*d*e - 2*b*c*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -
4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a
*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^2)
```

Sympy [F]

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*x/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}x}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*x/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\,\sqrt{d+e\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\sqrt{ex+d}}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.91 $\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*
a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/(c*(e*x+d)/(2*
c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.79 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \frac{i\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cd+(b+\sqrt{}}}\right.\right.\right.}{\sqrt{2}ce\sqrt{\frac{}{-2cd+}}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(I*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c
*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d + e*x]], (2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)] - EllipticF[I*ArcSinh[Sq
rt[2]*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d + e*x]], (2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt
[2]*c*e*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + x*(b + c*x)]
)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1172, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1172$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2]))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(156) = 312$.

Time = 1.37 (sec) , antiderivative size = 746, normalized size of antiderivative = 4.19

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2d\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x}{\frac{d}{e}-\frac{b+\sqrt{}}{}}}\right.}{\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}\right.}{}$
default	$\frac{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}\left(\sqrt{-4ac+b^2}\,e+be-2cd\right)\sqrt{2}\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be-2cd}}\sqrt{\frac{\left(-2cx+\sqrt{-4ac+b^2}-b\right)e}{2cd-be+\sqrt{-4ac+b^2}\,e}}\sqrt{\frac{\left(2cx+\sqrt{-4ac+b^2}+b\right)e}{\sqrt{-4ac+b^2}\,e+be-2cd}}\left(\mathrm{Ellip}\right.}{}$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*d*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*e*(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(160) = 320$.

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{ce}ce\text{weierstrassZeta}\left(\frac{4\left(c^2d^2-bcde+(b^2-3ac)e^2\right)}{3c^2e^2},-\frac{4\left(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3\right)}{27c^3e^3},\text{weierst}\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*e)*c*e*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*
c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*
d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e))) - (2*c*d - b*e)*sqrt(c*e)*weierstrassPInverse(4/
3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*
b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3)
, 1/3*(3*c*e*x + c*d + b*e)/(c*e)))/(c^2*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.92 $\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 404

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4\sqrt{2}\sqrt{b^2-4ac}\,d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\cdots}{2cd-(\cdots}\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*(-4*a*c+b^2)^(1/2)*e*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2))/c/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*(-4
*a*c+b^2)^(1/2)*d*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(
c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a
*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/(b+(-4*a*c+b^2)^(
1/2))/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.45 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{i\sqrt{2}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{d+ex}\right.\right.\right.}{\sqrt{-2cd+}}$$

input
```
Integrate[Sqrt[d + e*x]/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[d + e*x]], (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)] - EllipticPi[1 - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*
d), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[d
+ e*x]], (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e)]))/(Sqrt[c/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + x*(b + c
*x)])
```

Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1284, 1172, 321, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1284}$$

$$e\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+d\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\Big\downarrow$ 1172

$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}+$$
$$d\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\Big\downarrow$ 321

$$d\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+$$
$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1279

$$\frac{d\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}}+$$
$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 187

$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$
$$\frac{2d\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+}}{\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 413

$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$

$$\frac{2d\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{}{2cd-}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$\big\downarrow$ 413

$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$

$$\frac{2d\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{}{ex\sqrt{1-\frac{}{2cd-}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}}}$$

$\big\downarrow$ 412

$$\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$

$$\frac{\sqrt{2}\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2cd-}}}{\sqrt{c}\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

input

```
Int[Sqrt[d + e*x]/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[d + e*x]*S
qrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqr
t[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(
d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e + S
qrt[b^2 - 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*Sqrt[a + b*x + c*x^2]*Sqrt[b
- Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4
*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e])
```

Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1284
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]), x], x] + Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2e\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\ \text{EllipticF}\left(\sqrt{\frac{x \cdots}{\frac{d}{e}-\frac{b+\sqrt{\cdots}}{\cdots}}}\cdots\right.}{\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}\cdots\right.}{\cdots}$
default	$\frac{\left(-\text{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be-2cd}},\sqrt{-\frac{\sqrt{-4ac+b^2}\,e+be-2cd}{2cd-be+\sqrt{-4ac+b^2}\,e}}\right)e\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\ \text{EllipticPi}\left(\sqrt{2}\,\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be\cdots}}\cdots\right.\right.}{\cdots}$

input
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*e*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*e*EllipticPi(((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),-(-d/e+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/d*e,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\,x}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\,x}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{x\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)/x/(c*x^2+b*x+a)^(1/2),x)
```

3.93 $\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 623

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax} + \frac{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}a\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{d+ex}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(bd-ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{a\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/x+1/2*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2
)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e))^(1/2))*2^(1/2)/a/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*d*(c*(e*x+d)/(2*c*d-(
b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*Elli
pticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^
(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/a/(e*x+d)^(1/2)/(c*x^2+b*
x+a)^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-a*e+b*d)*(c*(e*x+d)/(2*c*d-(b+(-
4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*Elliptic
Pi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)
/(b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1
/2))*e))^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.88 (sec) , antiderivative size = 1292, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[d + e*x]/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*((-4*(a + x*(b + c*x)))/x + ((d + e*x)*((4*d*e^2*Sqrt[(c*d^
2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(a + x*(b
+ c*x)))/(d + e*x)^2 - (I*Sqrt[2]*d*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4
*a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d
+ e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b
^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)
/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sq
rt[d + e*x] + (I*Sqrt[2]*(b*d*e - 2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqr
t[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)
*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 -
4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*
(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*
c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + S
qrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d
+ e*x] + ((2*I)*Sqrt[2]*e*(-(b*d) + a*e)*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*
a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((...
```

Rubi [A] (warning: unable to verify)

Time = 1.48 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.32, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1285, 2154, 25, 27, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1285}$$

$$-\frac{\int \frac{-cex^2+bd-ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{2154}$$

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int -\frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 25

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 27

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 1269

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}\right)}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 1172

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{\cdots}}{\cdots}\right)}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 321

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{\cdots}}{\cdots}\right)}{2a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 327

$$-\frac{(bd-ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx - ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{2a}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 1279

$$-\frac{\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(bd-ae)\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}} - ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+c\right.}{b^2-4a}}}{ce\sqrt{}}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 187

$$-\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(bd-ae)\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+ex}}{\sqrt{a+bx+cx^2}} - ce\left(\sqrt{2}\sqrt{}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\downarrow$ 413

$$-\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(bd-ae)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}}d\sqrt{}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}$$

$\Big\downarrow$ 413

$$-\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(bd-ae)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{1}{ex\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}}{\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{ax}}$$

$\Big\downarrow$ 412

$$-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}cd\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right.}}}{}\right.}{\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{ax}}$$

input
```
Int[Sqrt[d + e*x]/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(a*x)) - (-(c*e*((Sqrt[2]*Sqrt[b^2
 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2
]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2]))) - (Sqrt[2]*(b*d - a*e)*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2
- 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Ellipt
icPi[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a
 + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e
]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{\left(-\frac{2(ae-bd)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\,e\,\mathrm{Ellipt}}{\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+aex+bdx}}\right.}{}$
elliptic	$\frac{\sqrt{(ex+d)(c\,x^{2}+bx+a)}\left(-\frac{\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+aex+bdx+ad}}{ax}+\frac{ce\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\sqrt{\frac{x}{-}}}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/x+1/2/a*(-2*(a*e-b*d)*(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/d*e*EllipticPi(((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),-(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/d*e,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2))+2*c*e*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(
-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e
*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2
)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d
/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2))))*((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+ax^2}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*x^4 + b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}x^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{x^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{x^2\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

3.94 $\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 719

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-ae)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{4a^2dx}$$

$$-\frac{\sqrt{b^2-4ac}(3bd-ae)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{4\sqrt{2}a^2d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{3\sqrt{b^2-4ac}(bd-ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{2\sqrt{2}a^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{b^2-4ac}(3b^2d^2-2abde-a(4cd^2+ae^2))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\mathrm{ar}\right.}{\sqrt{2}a^2\left(b+\sqrt{b^2-4ac}\right)d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-1/2*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-a*e+3*b*d)*(e*x+d)^(1/2
)*(c*x^2+b*x+a)^(1/2)/a^2/d/x-1/8*(-4*a*c+b^2)^(1/2)*(-a*e+3*b*d)*(e*x+d)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+
b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/d/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/
2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+3/4*(-4*a*c+b^2)^(1/2)*(-a*e+b*d)*(c*(e*
x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/
(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-1/2*(-4*a*c+b^2)^(1/2)*(3*b^2*d^2-2*a*b*
d*e-a*(a*e^2+4*c*d^2))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),(-2*
(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/
(b+(-4*a*c+b^2)^(1/2))/d/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.08 (sec) , antiderivative size = 6774, normalized size of antiderivative = 9.42

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x]/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1462 vs. 2(719) = 1438.

Time = 2.96 (sec) , antiderivative size = 1462, normalized size of antiderivative = 2.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {1285, 2154, 1282, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1285}$$

$$-\frac{\int \frac{cex^2+2(cd+be)x+3bd-ae}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 2154}$$

$$-\frac{(3bd-ae)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 1282}$$

$$-\frac{\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{\int \frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)}{4a}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 2154}$$

$$-\frac{\int \frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int -\frac{c\ldots}{\sqrt{d+ex}\sqrt{\ldots}}}{2ad}\ldots\right.}{4a}\ldots$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx\ldots}}}{2ad}\ldots\right.}{4a}\ldots$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{ce\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{}{\sqrt{d+e}}}{2ad}\right.}{4a}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$\Big\downarrow$ 1172

$$-\frac{\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}+2(be+cd)\int}{4a}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$\Big\downarrow$ 321

$$-\frac{2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+b}}{adx}\right.}{4a}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$\Big\downarrow$ 1269

$$-\frac{2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{e}\right.}{2ad}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$\Big\downarrow$ 1172

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\int \frac{}{x}$$

$$-\frac{\quad}{\quad}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$\downarrow$ 321

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\int \frac{}{x}$$

$$-\frac{\quad}{\quad}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$\downarrow$ 327

$$-\frac{(3bd-ae)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4a\right.}}\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\right.}{2\ldots}\right.}{\ldots}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2ax^2}$$

$\downarrow$ 1279

$$-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ace}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+\frac{2(cd+be)\sqrt{b+2cx\ldots}}{\ldots}}{\ldots}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$\downarrow$ 187

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}} - \frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$$\downarrow 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}} - \frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$$\downarrow 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

$$\Big\downarrow \; 412$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}(cd+be)\sqrt{2cd}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2ax^2}$$

input
```
Int[Sqrt[d + e*x]/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(a*x^2) - ((2*Sqrt[2]*Sqrt[b^2
- 4*a*c]*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
- (2*Sqrt[2]*(c*d + b*e)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b -
Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2
*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e + Sqrt[b^2
- 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt
[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c]
- (2*c*d)/e + (2*c*(d + e*x))/e]) + (3*b*d - a*e)*(-((Sqrt[d + e*x]*Sqrt[a
 + b*x + c*x^2])/(a*d*x)) - (-(c*e*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*
x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[
b^2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1285

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154

```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1205
risch	Expression too large to display	1577
default	Expression too large to display	5158

input
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2/a/x^
2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)-1/4/a^2/d*(a*e-3*b*d)*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/x-1/2*c/a*e*(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/4*c*e*(a*e-3*b*d)/a^2/d*(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+1/4*(a^2*e^2+2*a*b*d*e
+4*a*c*d^2-3*b^2*d^2)/a^2/d^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/
(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1...
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+ax^3}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*x^5 + b*x^4 + a*x^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+ax^3}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}x^3}\,dx$$

input
```
integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{d+e\,x}}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{ex+d}}{x^3\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

3.95 $\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 509

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(7cd+4be)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e^2}+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8c^2d^2+8b^2e^2+ce(7bd-9ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2cd-\left(\ldots\right.}\right.}{15c^3e^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8c^2d^3+cde(3bd-7ae)+4be^2(bd-ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\ldots\right.}{15c^3e^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2/15*(4*b*e+7*c*d)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+2/5*(e*x+d)^
(3/2)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(8*c^2*d^2
+8*b^2*e^2+c*e*(-9*a*e+7*b*d))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^3/(c*(
e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/15*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(8*c^2*d^3+c*d*e*(-7*a*e+3*b*d)+4*b*e^2*(-a*e+b*d
))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3
/e^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.22 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{4e^2(8c^2d^2+8b^2e^2+ce(7bd-9ae))(a+x(b+cx))}{\sqrt{d+ex}} + 4ce^2\sqrt{d+ex}(-4cd-4be+3cex)(a+x(b+cx)) - \frac{i(d+ex)\sqrt{1-\frac{\cdots}{\left(2cd-\cdots\right.}}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^3/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((4*e^2*(8*c^2*d^2 + 8*b^2*e^2 + c*e*(7*b*d - 9*a*e))*(a + x*(b + c*x)))/S
qrt[d + e*x] + 4*c*e^2*Sqrt[d + e*x]*(-4*c*d - 4*b*e + 3*c*e*x)*(a + x*(b
+ c*x)) - (I*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) +
 a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*
e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*c^2*d^2 + 8*b^2*e^2 + c*e*(7*b*d - 9*a*e))
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (-8*b^3*e^3 + b^2
*e^2*(c*d + 8*Sqrt[(b^2 - 4*a*c)*e^2]) + b*c*e*(17*a*e^2 + 7*d*Sqrt[(b^2 -
 4*a*c)*e^2]) + c*(8*c*d^2*Sqrt[(b^2 - 4*a*c)*e^2] - a*e^2*(4*c*d + 9*Sqrt
[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e +
a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
)]))/Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/(30*c^3*e^4*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1278, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1278}$$

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-\frac{\int \frac{4(cd+be)x^2+3(bd+ae)x+2ad}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce}$$

$$\Big\downarrow \text{2184}$$

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\int -\frac{e\left(2\left(2deb^2+2\left(cd^2+ae^2\right)b-acde\right)+\left(8c^2d^2+8b^2e^2+ce(7bd-9ae)\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}}{5ce}$$

$\Big\downarrow$ 27

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}-\frac{\int\frac{2\left(2deb^2+2\left(cd^2+ae^2\right)b-acde\right)+\left(8c^2d^2+8b^2e^2+ce(7bd-9ae)\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}}{5ce}$$

$\Big\downarrow$ 1269

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}-\frac{\frac{\left(ce(7bd-9ae)+8b^2e^2+8c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(cde(3bd-7ae)+4be^2(bd-ae)+8c^2d^3\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}}{5ce}$$

$\Big\downarrow$ 1172

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-$$

$$\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ce(7bd-9ae)+8b^2e^2+8c^2d^2\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\cdots$$

$\Big\downarrow$ 321

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-$$

$$\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ce(7bd-9ae)+8b^2e^2+8c^2d^2\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\cdots$$

$\Big\downarrow$ 327

$$\frac{2x\sqrt{d+ex}\sqrt{a+bx+cx^2}}{5ce}-$$

$$\frac{8\sqrt{d+ex}\sqrt{a+bx+cx^2}(be+cd)}{3ce}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ce(7bd-9ae)+8b^2e^2+8c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4\cdots}\right)}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\cdots$$

input
```
Int[x^3/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(5*c*e) - ((8*(c*d + b*e)*Sqrt[d
 + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*c^
2*d^2 + 8*b^2*e^2 + c*e*(7*b*d - 9*a*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
+ c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*c^2*d^
3 + c*d*e*(3*b*d - 7*a*e) + 4*b*e^2*(b*d - a*e))*Sqrt[(c*(d + e*x))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e))/(5*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(449) = 898.

Time = 5.53 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.79

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2x\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{5ec}-\frac{4(2be+2cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{15c^2e^2}+\frac{2\left(-\frac{2ad}{5ce}+\frac{4(2be+2cd)\left(\frac{a}{2}\right.}{15c^2e^2}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5/e/c*x
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)-4/15/c^2/e^2*(2*b*e+2*c*d
)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(-2/5*a/c*d/e+4/15/c^2
/e^2*(2*b*e+2*c*d)*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^
2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2
+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2))+2*(-2/5/e/c*(3/2*a*e+3/2*b*d)+4/15/c^2/e^2*(2*b
*e+2*c*d)*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-
d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*
d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/
2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left((8\,c^3d^3+3\,bc^2d^2e+3\,(b^2c-ac^2)de^2+(8\,b^3-21\,abc)e^3)\sqrt{ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+(b^2-3}{3\,c^2e^2}\right.\right.}{}$$

input
```
integrate(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*((8*c^3*d^3 + 3*b*c^2*d^2*e + 3*(b^2*c - a*c^2)*d*e^2 + (8*b^3 - 21*
a*b*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c
^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c
*e)) + 3*(8*c^3*d^2*e + 7*b*c^2*d*e^2 + (8*b^2*c - 9*a*c^2)*e^3)*sqrt(c*e)
*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4
/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*
b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*
e))) - 3*(3*c^3*e^3*x - 4*c^3*d*e^2 - 4*b*c^2*e^3)*sqrt(c*x^2 + b*x + a)*s
qrt(e*x + d))/(c^4*e^4)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**3/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^3}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^3/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^3/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.96 $\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 426

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(cd+be)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cd^2+e(bd-ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
b*e+c*d)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2
*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(
2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(c*(e*x+d)/(2*c*d-(b+(-4*a
*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/3*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(2*c*d^2+e*(-a*e+b*d))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2
)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e))^(1/2))/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.03 (sec) , antiderivative size = 934, normalized size of antiderivative = 2.19

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{d+ex}\left(2ce^2(a+x(b+cx)) + \frac{(d+ex)\left(-\frac{4e^2(cd+be)\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(a+x(b+cx))}{(d+ex)^2} + \frac{i\sqrt{2}(cd+be)\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^2/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(2*c*e^2*(a + x*(b + c*x)) + ((d + e*x)*((-4*e^2*(c*d + b*e
)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
]*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(c*d + b*e)*(2*c*d - b*e + S
qrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d
*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] -
 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(
c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d +
 e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b^2*e^2 - b*e*Sqrt[(b^2 -
4*a*c)*e^2] - c*(a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*a*e^2 + d*Sq
rt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d -
 e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2
+ d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*
e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/Sqrt[(c*d
^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]))/(3*c...
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1278, 9, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1278}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{2(cd+be)x^2+(bd+ae)x}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \text{9}$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{bd+ae+2(cd+be)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \; 1269$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{2(be+cd)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(e(bd-ae)+2cd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}$$

$$\Big\downarrow \; 1172$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(be+cd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(e(bd-ae)+2c\cdots\right.}{\cdots}}{3ce}$$

$$\Big\downarrow \; 321$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(be+cd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(e(bd-ae)+2c\cdots\right.}{\cdots}}{3ce}$$

$$\Big\downarrow \; 327$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(be+cd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(e(b\cdots\right.}{\cdots}}{3ce}$$

input
```
Int[x^2/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((2*Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(c*d + b*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
 + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*d^2 + e*(b*d - a*e))*Sqrt[(
c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(372) = 744$.

Time = 4.34 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.91

method	result
elliptic	$\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ec}-\frac{4\left(\frac{ae}{2}+\frac{bd}{2}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+}}{2c}}}}{3ec\sqrt{ce\,x^3+be}}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3/e/c*(
c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)-4/3/e/c*(1/2*a*e+1/2*b*d)*(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ellipti
cF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-4/3*(b*e+c*d
)/c/e*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*
((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}c^2e^2+(2\,c^2d^2+bcde+(2\,b^2-3\,ac)e^2)\sqrt{ce}\,\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+}{3\,c^2}\right.\right.}{}$$

input
```
integrate(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*c^2*e^2 + (2*c^2*d^2 + b*c*d*e
+ (2*b^2 - 3*a*c)*e^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*
e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^
2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*
d + b*e)/(c*e)) + 6*(c^2*d*e + b*c*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2
*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*
d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/
27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b
*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^2/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^2/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.97 $\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 365

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*
a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e/(c*(e*x+d)/(
2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a
*c+b^2)^(1/2)*d*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e
))^(1/2))/c/e/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.17 (sec) , antiderivative size = 804, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{(d+ex)^{3/2}\left(\frac{4e^2\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(a+x(b+cx))}{(d+ex)^2} - \frac{i\sqrt{2}\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ae^2+2cdex+be(d-ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}}{}\right.}{}$$

input
```
Integrate[x/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d + e*x)^(3/2)*((4*e^2*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + S
qrt[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 - (I*Sqrt[2]*(2*c*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + 2*c*d*e*x + b*e*(d - e
*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2
 - 4*a*c)*e^2]*(d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e
*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]
 + (I*Sqrt[2]*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + 2*c*d*e*
x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 - 2*c*d*e*x + b*e*(-d + e*
x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e
^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x]))/(2*c*e^2*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \ 1269$$

$$\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \ 1172$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}d\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}d\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}d\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

input
```
Int[x/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{2\sqrt{(ex+d)(cx^2+bx+a)}\,\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\Big(\Big(-\frac{d}{e}-\frac{-b+\sqrt{-4ac}}{2c}}{\sqrt{ex+d}\,\sqrt{cx^2+bx+a}\,\sqrt{ce}}$
default	$\Big(\sqrt{-4ac+b^2}\,\mathrm{EllipticF}\Big(\sqrt{2}\,\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be-2cd}},\sqrt{-\frac{\sqrt{-4ac+b^2}\,e+be-2cd}{2cd-be+\sqrt{-4ac+b^2}\,e}}\Big)de+2\,\mathrm{EllipticF}\Big(\sqrt{2}\,\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be-2cd}},\sqrt{-$

input
```
int(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(d/e-1/2
*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(
-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{ce}ce\mathrm{weierstrassZeta}\left(\frac{4\left(c^2d^2-bcde+(b^2-3ac)e^2\right)}{3c^2e^2}, -\frac{4\left(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3\right)}{27c^3e^3}, \mathrm{weierst}\right.\right.}{}$$

input
```
integrate(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*e)*c*e*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*
c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*
d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e))) + (c*d + b*e)*sqrt(c*e)*weierstrassPInverse(4/3*
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
1/3*(3*c*e*x + c*d + b*e)/(c*e)))/(c^2*e^2)
```

Sympy [F]

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.98 $\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+
b^2)^(1/2))*e))^(1/2))/c/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.69 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i(d+ex)\sqrt{2-\frac{4(cd^2+e(-bd+ae))}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{1+\frac{2(cd^2+e(-bd+ae))}{\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{c}{-2cd+}}}{\sqrt{}}\right.\right.}{e\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}\sqrt{a+x(b+cx)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*(d + e*x)*Sqrt[2 - (4*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(S
qrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/(e*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1172, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 1172$$
$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[d + e*x]*Sqr
t[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.60

method	result
default	$\frac{\left(-\sqrt{-4ac+b^2}\,e-be+2cd\right)\operatorname{EllipticF}\left(\sqrt{2}\sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\,e+be-2cd}},\sqrt{-\frac{\sqrt{-4ac+b^2}\,e+be-2cd}{2cd-be+\sqrt{-4ac+b^2}\,e}}\right)\sqrt{\frac{\left(2cx+\sqrt{-4ac+b^2}+b\right)e}{\sqrt{-4ac+b^2}\,e+be-2cd}}\sqrt{\frac{\left(-2cx+\sqrt{-\right.}}{2cd-be+\sqrt{}}}}{ce\left(ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad\right)}$
elliptic	$\frac{2\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\operatorname{EllipticF}\left(\sqrt{\frac{}{\frac{d}{e}-b+}}\right.}{\sqrt{ex+d}\sqrt{c\,x^2+bx+a}\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)/c*EllipticF(2^(1/2)*(-c*(e*x+d)/((-4*a*c
+b^2)^(1/2)*e+b*e-2*c*d))^(1/2),(-((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d)/(2*c*d-
b*e+(-4*a*c+b^2)^(1/2)*e))^(1/2))*((2*c*x+(-4*a*c+b^2)^(1/2)+b)*e/((-4*a*c
+b^2)^(1/2)*e+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*c*d-b*
e+(-4*a*c+b^2)^(1/2)*e))^(1/2)*2^(1/2)*(-c*(e*x+d)/((-4*a*c+b^2)^(1/2)*e+b
*e-2*c*d))^(1/2)/e*(c*x^2+b*x+a)^(1/2)*(e*x+d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*
x^2+a*e*x+b*d*x+a*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{ce}\,\text{weierstrassPInverse}\left(\frac{4\left(c^2d^2-bcde+(b^2-3\,ac)e^2\right)}{3\,c^2e^2},-\frac{4\left(2\,c^3d^3-3\,bc^2d^2e-3\,(b^2c-6\,ac^2)de^2+(2\,b^3-9\,abc)e^3\right)}{27\,c^3e^3},\frac{3\,cex+cd+be}{3\,ce}\right)}{ce}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2
+ (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))/(c*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.99 $\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 222

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{\ldots}}{2cd-\left(b\ldots\right.}\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-4*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2
)),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/(b+(-
4*a*c+b^2)^(1/2))/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.00 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.17

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{i(d+ex)\sqrt{1-\frac{2(cd^2+e(-bd+ae))}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{2+\frac{4(cd^2+e(-bd+ae))}{\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

input
```
Integrate[1/(x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-I)*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sq
rt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e)))
/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*(EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - EllipticPi[(d*(2*c*d - b*e - Sqrt[(b
^2 - 4*a*c)*e^2]))/(2*(c*d^2 + e*(-(b*d) + a*e))), I*ArcSinh[(Sqrt[2]*Sqrt
[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d
 + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2]))]))/(d*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 1279$$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 187$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 413$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{}{2cd-}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$$\Big\downarrow \; 413$$

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{}{ex\sqrt{1-\frac{}{2cd-}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+ex}{e}}}$$

$$\Big\downarrow \; 412$$

$$-\frac{\sqrt{2}\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-}}$$

input
```
Int[1/(x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a
*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2
*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c]
- (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e +
(2*c*(d + e*x))/e]))
```

Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{-4ac+b^2}\, e+be-2cd\right) \mathrm{EllipticPi}\left(\sqrt{2}\, \sqrt{-\frac{c(ex+d)}{\sqrt{-4ac+b^2}\, e+be-2cd}}, -\frac{\sqrt{-4ac+b^2}\, e+be-2cd}{2dc}, \sqrt{-\frac{\sqrt{-4ac+b^2}\, e+be-2cd}{2cd-be+\sqrt{-4ac+b^2}\, e}}\right) \sqrt{\frac{\left(2cx+\sqrt{-4ac+\cdots}\right.}{\sqrt{-4ac+b^2}\, e+\cdots}}}{cd(ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad)}$
elliptic	$-\frac{2\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\,\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\, e\,\mathrm{EllipticPi}\left(\sqrt{\frac{\cdots}{\frac{d}{e}\cdots}}\right.}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}\,\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}\,d}$

input
```
int(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d)*EllipticPi(2^(1/2)*(-c*(e*x+d)/((-4*a*c+b
^2)^(1/2)*e+b*e-2*c*d))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d)/d/c,(-
((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d)/(2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e))^(1/2))*
((2*c*x+(-4*a*c+b^2)^(1/2)+b)*e/((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d))^(1/2)*((
-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e))^(1/2)*2^(
1/2)*(-c*(e*x+d)/((-4*a*c+b^2)^(1/2)*e+b*e-2*c*d))^(1/2)/c*(c*x^2+b*x+a)^(
1/2)*(e*x+d)^(1/2)/d/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x}\,dx$$

input
```
integrate(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x}\,dx$$

input
```
integrate(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{x\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{1}{x\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.100 $\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 630

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$
$$+\frac{\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}ad\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(bd+ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{a\left(b+\sqrt{b^2-4ac}\right)d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x+1/2*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^
2)^(1/2))*e))^(1/2))*2^(1/2)/a/d/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*
e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*El
lipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2
)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/a/(e*x+d)^(1/2)/(c*x^2+
b*x+a)^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e+b*d)*(c*(e*x+d)/(2*c*d-(b+(
-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*Ellipti
cPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))/d/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.34 (sec) , antiderivative size = 1381, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[d + e*x]*(a + b*x + c*x^2))/(a*d*x*Sqrt[a + x*(b + c*x)])) + ((d +
 e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(4*d*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 + d/(d + e*x))^2 + (e*(b - (
b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) - (I*Sqrt[2]*d*(2*c*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d +
e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d
+ e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 -
b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]],
-((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b*d*e + 2*a*e^2 + d*Sqrt[(b^2 - 4*
a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1
 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(
-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt
[d + e*x] - ((2*I)*Sqrt[2]*e*(b*d + a*e)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] ...
```

Rubi [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.32, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1282, 2154, 25, 27, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1282}$$

$$-\frac{\int \frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$$\Big\downarrow \text{ 2154}$$

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int -\frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 25

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 27

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 1269

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}\right)}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 1172

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\text{—}\ 2\sqrt{\ldots}\right.}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 321

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\text{—}\ 2\sqrt{\ldots}\right.}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\downarrow$ 327

$$-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\right.}{2ad}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\big\downarrow$ 1279

$$-\frac{\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ae+bd)\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}}-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+c\right.}{b^2-4a}}}{ce\sqrt{}}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\big\downarrow$ 187

$$-\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ae+bd)\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}-ce\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\big\downarrow$ 413

$$-\frac{-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ae+bd)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}}d\sqrt{}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\Big\downarrow$ 413

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}(ae+bd)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{1}{ex\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$$-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$

$\Big\downarrow$ 412

$$-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right.}}}{}\right.$$

$$-\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{adx}$$

input
```
Int[1/(x^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(a*d*x)) - (-(c*e*((Sqrt[2]*Sqrt[b
^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*
Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x
^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d
 + e*x]*Sqrt[a + b*x + c*x^2]))) - (Sqrt[2]*(b*d + a*e)*Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^
2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Elli
pticPi[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt
[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))
/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]))/(2*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 827, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{c\,x^{2}+bx+a}}{adx}-\frac{\left(\frac{2(-ae-bd)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\,e\,\mathrm{Ellipt}}{\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+aex+bdx+}}\right.}{}$
elliptic	$\frac{\sqrt{(ex+d)(c\,x^{2}+bx+a)}\left(-\frac{\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+aex+bdx+ad}}{dax}+\frac{ce\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}}}\,\sqrt{\frac{x}{-}}}{}\right.}{}$
default	Expression too large to display

input

```
int(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/d/a*(2*(-a*e-b*d)*(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/d*e*EllipticPi(((
x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),-(-d/e+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/d*e,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2))-2*c*e*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/
(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((
x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2
)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c
+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))))*((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+dx^2}}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+dx^2}}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^2\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.101 $\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 722

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3(bd+ae)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{4a^2d^2x}$$

$$- \frac{3\sqrt{b^2-4ac}(bd+ae)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{4\sqrt{2}a^2d^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{b^2-4ac}(3bd+ae)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{2\sqrt{2}a^2d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{b^2-4ac}(3b^2d^2+2abde-a(4cd^2-3ae^2))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},a\right.}{\sqrt{2}a^2\left(b+\sqrt{b^2-4ac}\right)d^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-1/2*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*(a*e+b*d)*(e*x+d)^(1/2)
*(c*x^2+b*x+a)^(1/2)/a^2/d^2/x-3/8*(-4*a*c+b^2)^(1/2)*(a*e+b*d)*(e*x+d)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^
2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/d^2/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/
2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+1/4*(-4*a*c+b^2)^(1/2)*(a*e+3*b*d)*(c*(e
*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^
2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2
/d/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-1/2*(-4*a*c+b^2)^(1/2)*(3*b^2*d^2+2*a
*b*d*e-a*(-3*a*e^2+4*c*d^2))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2
)/a^2/(b+(-4*a*c+b^2)^(1/2))/d^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.06 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{-12de(bd+ae)(a+x(b+cx)) + \frac{4d(d+ex)(-2ad+3bdx+3aex)(a+x(b+cx))}{x^2} + \frac{i(d+ex)^{3/2}\sqrt{1-\frac{2\left(cd^2+e(-bd+ae)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}}{}}{}$$

input
```
Integrate[1/(x^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-12*d*e*(b*d + a*e)*(a + x*(b + c*x)) + (4*d*(d + e*x)*(-2*a*d + 3*b*d*x
+ 3*a*e*x)*(a + x*(b + c*x)))/x^2 + (I*(d + e*x)^(3/2)*Sqrt[1 - (2*(c*d^2
+ e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*
Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])*(d + e*x))]*(3*d*(b*d + a*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
])*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (3*b^2*d^2*e +
b*d*(a*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(-4*c*d^2 + 6*a*e^2 + 3*d*
Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e
 + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^
2]))] + 2*e*(3*b^2*d^2 + 2*a*b*d*e + a*(-4*c*d^2 + 3*a*e^2))*EllipticPi[(d
*(2*c*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(c*d^2 + e*(-(b*d) + a*e))),
I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(e*Sqrt[(c*d^2 + e*(-(b*d) +
 a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]))/(16*a^2*d^3*Sqrt[d + e*
x]*Sqrt[a + x*(b + c*x)])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1467 vs. 2(722) = 1444.

Time = 2.92 (sec) , antiderivative size = 1467, normalized size of antiderivative = 2.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {1282, 2154, 1282, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1282}$$

$$-\frac{\int \frac{cex^2+2(cd+be)x+3(bd+ae)}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$$\Big\downarrow \text{2154}$$

$$-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 1282

$$-\frac{\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{\int \frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 2154

$$-\frac{\int \frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int -\frac{c}{\sqrt{d+ex}\sqrt{\cdots}}}{2ad}\cdots\right.}{4ad}\cdots\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 25

$$-\frac{\int \frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx\cdots}}}{2ad}\cdots\right.}{4ad}\cdots\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 27

$$-\frac{ce\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{\cdots}{\sqrt{d+\cdots}}}{2ad}\cdots\right.}{4ad}\cdots\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 1172

$$-\frac{\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}+2(be+cd)\int\cdots}{4ad}\cdots\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 321

$$-\frac{2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+b}}{adx}\right.}{4ad}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 1269

$$-\frac{2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{e}\right.}{2ad}\right.}{}$$

$$\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}$$

$\downarrow$ 1172

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\ \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\int \frac{}{x}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

$\downarrow$ 321

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\int\frac{}{x}$$

$$-\frac{\qquad}{\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}}$$

$\downarrow$ 327

$$3(ae+bd)\left(-\frac{(ae+bd)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\right.}{2}\right.$$

$$-\frac{\qquad}{\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}}$$

$\downarrow$ 1279

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+\frac{2(cd+be)\sqrt{b+2cx}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

$$\Big\downarrow \; 187$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-\frac{}{}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

$$\Big\downarrow \; 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}} - \frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

$\downarrow$ 413

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}} - \frac{4(cd+be)\sqrt{b+2cx}}{}$$

$$-$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

$\downarrow$ 412

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}(cd+be)\sqrt{2cd}\cdots}{\cdots}$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{2adx^2}$$

input
```
Int[1/(x^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(a*d*x^2) - ((2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]
) - (2*Sqrt[2]*(c*d + b*e)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b
 - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 -
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e + Sqrt[b^
2 - 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sq
rt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c
] - (2*c*d)/e + (2*c*(d + e*x))/e]) + 3*(b*d + a*e)*(-((Sqrt[d + e*x]*Sqrt
[a + b*x + c*x^2])/(a*d*x)) - (-(c*e*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d +
e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqr
t[b^2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 1210, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1210
risch	Expression too large to display	1578
default	Expression too large to display	5202

input
```
int(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/2/d/a/
x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+3/4*(a*e+b*d)*(c*e*x^3
+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/a^2/d^2/x-1/2*c*e/d/a*(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-3/4*c*e*(a*e+b*d)/a^2/d
^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-
d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+
d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-1/4/a^2/d^3*(3*a^2
*e^2+2*a*b*d*e-4*a*c*d^2+3*b^2*d^2)*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x
+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b...
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e*x^6 + (c*d + b*e)*x^5 +
a*d*x^3 + (b*d + a*e)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**3/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^3\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^3*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x^3\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.102 $\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 343

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{6954\sqrt{7+x}\sqrt{3+2x+5x^2}}{4375} - \frac{654}{875}(7+x)^{3/2}\sqrt{3+2x+5x^2} + \frac{2}{35}(7+x)^{5/2}\sqrt{3+2x+5x^2} + \frac{240034\sqrt{7+x}\sqrt{3+2x+5x^2}}{4375\left(3\sqrt{130}+5(7+x)\right)} - \frac{240034\sqrt{3}\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right.}{4375\ 5^{3/4}\sqrt{3+2x+5x^2}} + \frac{\sqrt{3}\sqrt[4]{26}\left(120017-10431\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+}\right.\right.}{4375\ 5^{3/4}\sqrt{3+2x+5x^2}}$$

output
```
6954/4375*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)-654/875*(7+x)^(3/2)*(5*x^2+2*x+3
)^(1/2)+2/35*(7+x)^(5/2)*(5*x^2+2*x+3)^(1/2)+240034*(7+x)^(1/2)*(5*x^2+2*x
+3)^(1/2)/(13125*130^(1/2)+153125+21875*x)-240034/21875*3^(1/2)*26^(1/4)*(
(5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticE
(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+663
0*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)+1/21875*3^(1/2)*26^(1/4)*(
120017-10431*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+1
30^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)
*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.25 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.25

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2\sqrt{7+x}\sqrt{3+2x+5x^2}(-1843+115x+125x^2)}{4375} - \frac{2(7+x)^{3/2}\left(-\frac{4680663\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2}+\frac{120017i\sqrt{13}\left(17\sqrt{2}+i\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\operatorname{arcsinh}\left(\cdots\right.\right.}{\sqrt{7+x}}\cdots\right)}{853125\sqrt{-\cdots}}$$

input
```
Integrate[(x^3*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]*(-1843 + 115*x + 125*x^2))/4375 - (2*
(7 + x)^(3/2)*((-4680663*Sqrt[(-I)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(
7 + x)^2 + ((120017*I)*Sqrt[13]*(17*Sqrt[2] + I*Sqrt[7])*Sqrt[(34*I + Sqrt
[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I
)/(7 + x))/(-34*I + Sqrt[14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I +
 Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 +
x] + (Sqrt[13]*((-6244*I)*Sqrt[2] + 120017*Sqrt[7])*Sqrt[(34*I + Sqrt[14]
- (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7
+ x))/(-34*I + Sqrt[14])]*EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt
[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x]))/
(853125*Sqrt[(-I)/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1283, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1283}$$

$$\frac{2}{35}x^2\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{35}\int \frac{x\left(-23x^2+85x+84\right)}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{35}\left(\frac{46}{25}(x+7)^{3/2}\sqrt{5x^2+2x+3}-\frac{2}{25}\int \frac{7944x^2+15187x+3703}{2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)+\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}\left(\frac{46}{25}(x+7)^{3/2}\sqrt{5x^2+2x+3}-\frac{1}{25}\int \frac{7944x^2+15187x+3703}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)+\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{35}\left(\frac{1}{25}\left(-\frac{2}{15}\int -\frac{3(120017x+26501)}{2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)+\frac{46}{25}\sqrt{5x^2+2x+3}(x+7)^{3/2}\right)+\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}\left(\frac{1}{25}\left(\frac{1}{5}\int \frac{120017x+26501}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)+\frac{46}{25}\sqrt{5x^2+2x+3}(x+7)^{3/2}\right)+\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{35}\left(\frac{1}{25}\left(\frac{1}{5}\left(120017\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-813618\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right.\right.$$
$$\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2$$

$$\Big\downarrow \text{1172}$$

$$\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2+$$
$$\frac{1}{35}\left(\frac{46}{25}\sqrt{5x^2+2x+3}(x+7)^{3/2}+\frac{1}{25}\left(-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{240034i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.\right.\right.$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2+$$
$$\frac{1}{35}\left(\frac{46}{25}\sqrt{5x^2+2x+3}(x+7)^{3/2}+\frac{1}{25}\left(-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{240034i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.\right.\right.$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{35}\sqrt{x+7}\sqrt{5x^2+2x+3}x^2+$$
$$\frac{1}{35}\left(\frac{46}{25}\sqrt{5x^2+2x+3}(x+7)^{3/2}+\frac{1}{25}\left(-\frac{5296}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{240034i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x\right.}}{2^{3/}}\right.\right.}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.\right.\right.$$

input
```
Int[(x^3*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*x^2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/35 + ((46*(7 + x)^(3/2)*Sqrt[3 +
 2*x + 5*x^2])/25 + ((-5296*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/5 + ((((240
034*I)/5)*Sqrt[7 + x]*EllipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(
2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*S
qrt[14])] - ((1627236*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[
Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I +
 Sqrt[14])])/Sqrt[7 + x])/5)/25)/35
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.09

method	result
risch	$\frac{2(125x^2+115x-1843)\sqrt{x+7}\sqrt{5x^2+2x+3}}{4375}+\frac{\left(\frac{53002\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\right.}{4375\sqrt{5x^3+37x^2+17x+21}}\right.}{}$
default	$\frac{2\sqrt{x+7}\sqrt{5x^2+2x+3}\left(813618i\sqrt{14}\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\operatorname{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)+3125x^5\right.}{}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{46x\sqrt{5x^3+37x^2+17x+21}}{875}-\frac{3686\sqrt{5x^3+37x^2+17x+21}}{4375}+\frac{53002\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}}{4375\sqrt{5x^3+37x^2+17}}\right.}{}$

input
```
int(x^3*(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/4375*(125*x^2+115*x-1843)*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)+(53002/4375*(3
4/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(
1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*1
4^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticF(((x+7)/(34/5-1/5*I*
14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+24
0034/4375*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/
5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-
34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((-34/5-1/5*I*14^
(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2
))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*EllipticF(((x+7)/(
34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2))
)^(1/2))))*((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.16

$$\begin{aligned}\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx &= \frac{2}{4375}\left(125\,x^2+115\,x-1843\right)\sqrt{5\,x^2+2\,x+3}\sqrt{x+7} \\ &\quad - \frac{8086228}{328125}\,\sqrt{5}\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right) \\ &\quad - \frac{240034}{21875}\,\sqrt{5}\mathrm{weierstrassZeta}\left(\frac{4456}{75},\right. \\ &\qquad \left.-\frac{348704}{3375},\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)\right)\end{aligned}$$

input
```
integrate(x^3*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
2/4375*(125*x^2 + 115*x - 1843)*sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7) - 808622
8/328125*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) - 2
40034/21875*sqrt(5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInv
erse(4456/75, -348704/3375, x + 37/15))
```

Sympy [F]

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^3\sqrt{x+7}}{\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x**3*(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x + 7)/sqrt(5*x**2 + 2*x + 3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x^3}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x^3*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)*x^3/sqrt(5*x^2 + 2*x + 3), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x^3}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x^3*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)*x^3/sqrt(5*x^2 + 2*x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx=\int \frac{x^3\,\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x^3*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int((x^3*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^3\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx&=\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{35}+\frac{46\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x}{875}\\&\quad-\frac{3273\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{32375}-\frac{360051\left(\int\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{5x^3+37x^2+17x+21}dx\right)}{64750}\\&\quad-\frac{15843\left(\int\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^3+37x^2+17x+21}dx\right)}{64750}\end{aligned}$$

input
```
int(x^3*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
(3700*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x**2 + 3404*sqrt(x + 7)*sqrt(5*x*
*2 + 2*x + 3)*x - 6546*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3) - 360051*int((sq
rt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x**2)/(5*x**3 + 37*x**2 + 17*x + 21),x) -
 15843*int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3))/(5*x**3 + 37*x**2 + 17*x +
 21),x))/64750
```

3.103 $\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 318

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = -\frac{52}{125}\sqrt{7+x}\sqrt{3+2x+5x^2} + \frac{2}{25}(7+x)^{3/2}\sqrt{3+2x+5x^2} - \frac{1842\sqrt{7+x}\sqrt{3+2x+5x^2}}{125\left(3\sqrt{130}+5(7+x)\right)} + \frac{1842\sqrt{3}\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{125\ 5^{3/4}\sqrt{3+2x+5x^2}} - \frac{3\sqrt{3}\sqrt[4]{26}\left(307-26\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right), 3\right.}{125\ 5^{3/4}\sqrt{3+2x+5x^2}}$$

output
```
-52/125*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)+2/25*(7+x)^(3/2)*(5*x^2+2*x+3)^(1/
2)-1842*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(375*130^(1/2)+4375+625*x)+1842/62
5*3^(1/2)*26^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1
/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2
))),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)-3/625*
3^(1/2)*26^(1/4)*(307-26*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)
^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)
*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2
*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.95 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2}{125}\sqrt{7+x}(9+5x)\sqrt{3+2x+5x^2} + \frac{2(7+x)^{3/2}\left(-\frac{11973\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2}+\frac{307i\sqrt{13}\left(17\sqrt{2}+i\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\text{arcsinh}\left(\frac{3\sqrt{-\ldots}}{\sqrt{\ldots}}\right.\right.}{\sqrt{7+x}}\ldots\right.}{8125\sqrt{-\frac{i}{34i+\sqrt{1\ldots}}}\ldots}$$

input
```
Integrate[(x^2*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*Sqrt[7 + x]*(9 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/125 + (2*(7 + x)^(3/2)*((-
11973*Sqrt[(-I)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x)^2 + ((307*I)
*Sqrt[13]*(17*Sqrt[2] + I*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x)
)/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sq
rt[14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 +
x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] + (Sqrt[13]*((-149*
I)*Sqrt[2] + 307*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I +
 Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*
EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*
I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x]))/(8125*Sqrt[(-I)/(34*I + Sq
rt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1283, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1283}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{25}\int \frac{3\left(-9x^2+17x+14\right)}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\int \frac{-9x^2+17x+14}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(\frac{2}{15}\int \frac{3(307x+121)}{2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(\frac{1}{5}\int \frac{307x+121}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(\frac{1}{5}\left(307\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-2028\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)$$

$$\Big\downarrow \text{1172}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{614i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{4056i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int\frac{}{\sqrt{\frac{i(5x}{}}}}{}\right.\right.$$

$\downarrow$ 321

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{614i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{4056i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{Elliptic}}{}\right.\right.$$

$\downarrow$ 327

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{3}{25}\left(-\frac{6}{5}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{5}\left(\frac{614i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{4056i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{E}}{}\right.\right.$$

input
```
Int[(x^2*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*x*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/25 - (3*((-6*Sqrt[7 + x]*Sqrt[3 +
2*x + 5*x^2])/5 + ((((614*I)/5)*Sqrt[7 + x]*EllipticE[ArcSin[Sqrt[(-I)*(1
+ I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/
Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((4056*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])
]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (
2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x])/5))/25
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1283
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((
d + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(
d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f
*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*
f + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
ntegerQ[2*m] && GtQ[m, 1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec), antiderivative size = 368, normalized size of antiderivative = 1.16

method	result
risch	$\frac{2(9+5x)\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{125}+\frac{\left(-\frac{726\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{125\sqrt{5x^3+37x^2+17x+21}}\right.}{}$
default	$-\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(6084i\sqrt{14}\,\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\operatorname{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)+8658\sqrt{\right.}{}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{2x\sqrt{5x^3+37x^2+17x+21}}{25}+\frac{18\sqrt{5x^3+37x^2+17x+21}}{125}-\frac{726\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\ \mathrm{El}}{125\sqrt{5x^3+37x^2+17x+21}}\right.}{}$

input
```
int(x^2*(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/125*(9+5*x)*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)+(-726/125*(34/5-1/5*I*14^(1/
2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5
*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/
(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)
,((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))-1842/125*(34/5-1/5
*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(
-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)
))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((-34/5-1/5*I*14^(1/2))*EllipticE(((
x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^
(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)
))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))))*((x+7)*(
5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.16

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2}{125}\sqrt{5x^2+2x+3}(5x+9)\sqrt{x+7} + \frac{19088}{3125}\sqrt{5}\text{weierstrassPInverse}\left(\frac{4456}{75}, -\frac{348704}{3375}, x+\frac{37}{15}\right) + \frac{1842}{625}\sqrt{5}\text{weierstrassZeta}\left(\frac{4456}{75}, -\frac{348704}{3375}, \text{weierstrassPInverse}\left(\frac{4456}{75}, -\frac{348704}{3375}, x+\frac{37}{15}\right)\right)$$

input
```
integrate(x^2*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
2/125*sqrt(5*x^2 + 2*x + 3)*(5*x + 9)*sqrt(x + 7) + 19088/3125*sqrt(5)*wei
erstrassPInverse(4456/75, -348704/3375, x + 37/15) + 1842/625*sqrt(5)*weie
rstrassZeta(4456/75, -348704/3375, weierstrassPInverse(4456/75, -348704/33
75, x + 37/15))
```

Sympy [F]

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2\sqrt{x+7}}{\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x**2*(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x + 7)/sqrt(5*x**2 + 2*x + 3), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x^2*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)*x^2/sqrt(5*x^2 + 2*x + 3), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x^2*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)*x^2/sqrt(5*x^2 + 2*x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2\,\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x^2*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int((x^2*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x}{25} - \frac{51\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{925} + \frac{2763\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{5x^3+37x^2+17x+21}dx\right)}{1850} - \frac{2241\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^3+37x^2+17x+21}dx\right)}{1850}$$

input
```
int(x^2*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
(148*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x - 102*sqrt(x + 7)*sqrt(5*x**2 +
2*x + 3) + 2763*int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x**2)/(5*x**3 + 37
*x**2 + 17*x + 21),x) - 2241*int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3))/(5*x
**3 + 37*x**2 + 17*x + 21),x))/1850
```

3.104 $\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 293

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2}{15}\sqrt{7+x}\sqrt{3+2x+5x^2} + \frac{62\sqrt{7+x}\sqrt{3+2x+5x^2}}{15\left(3\sqrt{130}+5(7+x)\right)}$$
$$- \frac{62\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{5\sqrt{3}5^{3/4}\sqrt{3+2x+5x^2}}$$
$$+ \frac{\sqrt[4]{26}\left(31-3\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}(195\right.}{5\sqrt{3}5^{3/4}\sqrt{3+2x+5x^2}}$$

output
```
2/15*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)+62*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(4
5*130^(1/2)+525+75*x)-62/75*3^(1/2)*26^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*
(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*2
6^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4)/
(5*x^2+2*x+3)^(1/2)+1/75*26^(1/4)*(31-3*130^(1/2))*((5*x^2+2*x+3)/(78+130^
(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5
^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*3
^(1/2)*5^(1/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.16 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.43

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2}{15}\sqrt{7+x}\sqrt{3+2x+5x^2} + \frac{(7+x)^{3/2}\left(\frac{2418\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2} + \frac{62\sqrt{13}\left(-17i\sqrt{2}+\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\operatorname{arcsinh}\left(\frac{3\sqrt{-\frac{26i}{34i+\sqrt{\cdots}}}}{\sqrt{7+x}}\cdots\right.\right.}{\sqrt{7+x}}\cdots\right.}{2925\sqrt{-\frac{i}{34i+\sqrt{14}}}\sqrt{\cdots}}$$

input
```
Integrate[(x*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/15 + ((7 + x)^(3/2)*((2418*Sqrt[(-I)
/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x)^2 + (62*Sqrt[13]*((-17*I)*S
qrt[2] + Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14
])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*Elliptic
E[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt
[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] - ((2*I)*Sqrt[13]*(58*Sqrt[2] - (31*
I)*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sq
rt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticF[I*Ar
cSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/
(34*I - Sqrt[14])])/Sqrt[7 + x]))/(2925*Sqrt[(-I)/(34*I + Sqrt[14])]*Sqrt[
3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{15}\int -\frac{17-31x}{2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{15}\int \frac{17-31x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{15}\left(31\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-234\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)+\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{1172}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+$$

$$\frac{1}{15}\left(\frac{62i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{468i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{1}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+\right.}}{2^{3/4}}}{\sqrt{x+7}}\right.$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{62i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{468i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2}{34i}\right.}{\sqrt{x+7}}\right.$$

$$\Big\downarrow \; 327$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{62i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{468i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{\sqrt{x+7}}\right.$$

input
```
Int[(x*Sqrt[7 + x])/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/15 + ((((62*I)/5)*Sqrt[7 + x]*Ellipt
icE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[1
4])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((468*I)*Sqrt[(7
 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)
]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x])/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{2\sqrt{5x^3+37x^2+17x+21}}{15}-\frac{34\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\text{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\cdots}\right.}{15\sqrt{5x^3+37x^2+17x+21}}\cdots\right.}{\cdots}$
risch	$\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{15}+\frac{\left(-\frac{34\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\text{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{15\sqrt{5x^3+37x^2+17x+21}}+\frac{62\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(234i\sqrt{14}\,\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\text{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)-702\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x*(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)*(2/15*(5*x^3+3
7*x^2+17*x+21)^(1/2)-34/15*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/
2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1
/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*
EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34
/5-1/5*I*14^(1/2)))^(1/2))+62/15*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*
14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x
+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^
(1/2)*((-34/5-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2
),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1
/2))*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))
/(-34/5-1/5*I*14^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.15

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = -\frac{2804}{1125}\sqrt{5}\text{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right) - \frac{62}{75}\sqrt{5}\text{weierstrassZeta}\left(\frac{4456}{75},-\frac{348704}{3375},\text{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)\right) + \frac{2}{15}\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}$$

input
```
integrate(x*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
-2804/1125*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) -
 62/75*sqrt(5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInverse(
4456/75, -348704/3375, x + 37/15)) + 2/15*sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7
)
```

Sympy [F]

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{x\sqrt{x+7}}{\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x*(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x*sqrt(x + 7)/sqrt(5*x**2 + 2*x + 3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)*x/sqrt(5*x^2 + 2*x + 3), x)
```

Giac [F]

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}x}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate(x*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)*x/sqrt(5*x^2 + 2*x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{x\,\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int((x*(x + 7)^(1/2))/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{7\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{37} - \frac{31\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{5x^3+37x^2+17x+21}dx\right)}{74} - \frac{119\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^3+37x^2+17x+21}dx\right)}{74}$$

input

```
int(x*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output

```
(14*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3) - 31*int((sqrt(x + 7)*sqrt(5*x**2 +
 2*x + 3)*x**2)/(5*x**3 + 37*x**2 + 17*x + 21),x) - 119*int((sqrt(x + 7)*s
qrt(5*x**2 + 2*x + 3))/(5*x**3 + 37*x**2 + 17*x + 21),x))/74
```

3.105 $\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{2\sqrt{7+x}\sqrt{3+2x+5x^2}}{3\sqrt{130}+5(7+x)} - \frac{2\sqrt{3}\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{5^{3/4}\sqrt{3+2x+5x^2}} + \frac{\sqrt{3}\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{5^{3/4}\sqrt{3+2x+5x^2}}$$

output

```
2*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(5*x+35+3*130^(1/2))-2/5*3^(1/2)*26^(1/4
)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*Ellipt
icE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+
6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)+1/5*3^(1/2)*26^(1/4)*((
5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJac
obiAM(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+663
0*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = -\frac{2\left(-34i+\sqrt{14}\right)\sqrt{\frac{-i+\sqrt{14}-5ix}{34i+\sqrt{14}}}\sqrt{\frac{i+\sqrt{14}+5ix}{-34i+\sqrt{14}}}\sqrt{7+x}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{5}\sqrt{-\frac{i(7+x)}{34i+\sqrt{14}}}\right)|\frac{34i+\sqrt{14}}{34i-\sqrt{14}}\right)-\text{EllipticF}\right.}{5\sqrt{5}\sqrt{-\frac{i(7+x)}{34i+\sqrt{14}}}\sqrt{3+2x+5x^2}}$$

input
```
Integrate[Sqrt[7 + x]/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-2*(-34*I + Sqrt[14])*Sqrt[(-I + Sqrt[14] - (5*I)*x)/(34*I + Sqrt[14])]*S
qrt[(I + Sqrt[14] + (5*I)*x)/(-34*I + Sqrt[14])]*Sqrt[7 + x]*(EllipticE[I*
ArcSinh[Sqrt[5]*Sqrt[((-I)*(7 + x))/(34*I + Sqrt[14])]], (34*I + Sqrt[14])
/(34*I - Sqrt[14])] - EllipticF[I*ArcSinh[Sqrt[5]*Sqrt[((-I)*(7 + x))/(34*
I + Sqrt[14])]], (34*I + Sqrt[14])/(34*I - Sqrt[14])]))/(5*Sqrt[5]*Sqrt[((
-I)*(7 + x))/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.35, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1172, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx$$
$$\Big\downarrow \; 1172$$

$$\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}$$

input
```
Int[Sqrt[7 + x]/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(((2*I)/5)*Sqrt[7 + x]*EllipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/
(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*
Sqrt[14])]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,\left(-34+i\sqrt{14}\right)\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)\sqrt{14}\right.}{25(}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{14\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{\sqrt{5x^3+37x^2+17x+21}}+\frac{2\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right.}{}\right.}{}$

input
```
int((x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/25*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(-34+I*14^(1/2))*(-5*(x+7)/(-34+I*14
^(1/2)))^(1/2)*((I*14^(1/2)-5*x-1)/(I*14^(1/2)+34))^(1/2)*((I*14^(1/2)+5*x
+1)/(-34+I*14^(1/2)))^(1/2)*(I*EllipticF((-5*(x+7)/(-34+I*14^(1/2)))^(1/2)
,(-(-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2))*14^(1/2)-I*EllipticE((-5*(x+7)
/(-34+I*14^(1/2)))^(1/2),(-(-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2))*14^(1/
2)+34*EllipticF((-5*(x+7)/(-34+I*14^(1/2)))^(1/2),(-(-34+I*14^(1/2))/(I*14
^(1/2)+34))^(1/2))-34*EllipticE((-5*(x+7)/(-34+I*14^(1/2)))^(1/2),(-(-34+I
*14^(1/2))/(I*14^(1/2)+34))^(1/2)))/(5*x^3+37*x^2+17*x+21)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \frac{136}{75}\sqrt{5}\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right) - \frac{2}{5}\sqrt{5}\mathrm{weierstrassZeta}\left(\frac{4456}{75},-\frac{348704}{3375},\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)\right)$$

input
```
integrate((7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
136/75*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) - 2/5
*sqrt(5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInverse(4456/7
5, -348704/3375, x + 37/15))
```

Sympy [F]

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate((7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(sqrt(x + 7)/sqrt(5*x**2 + 2*x + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)/sqrt(5*x^2 + 2*x + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)/sqrt(5*x^2 + 2*x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x + 7)^(1/2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int((x + 7)^(1/2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^2+2x+3}dx$$

input
```
int((7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3))/(5*x**2 + 2*x + 3),x)
```

3.106 $\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 287

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = -\sqrt{\frac{7}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right)$$

$$+\frac{\sqrt{3}\sqrt[4]{\frac{26}{5}}\left(7\sqrt{5}-3\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\right.}{11\sqrt{3+2x+5x^2}}$$

$$+\frac{\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right.\right.}{22\sqrt{3}\sqrt[4]{130}\sqrt{3+2x+5x^2}}$$

output
```
-1/3*21^(1/2)*arctanh(1/7*21^(1/2)*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2))+1/55*3
^(1/2)*26^(1/4)*5^(3/4)*(7*5^(1/2)-3*26^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2
)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/
4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))/(5*x^
2+2*x+3)^(1/2)+1/8580*(479-42*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x
))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticPi(sin(2*arctan(1/78*5^(1/4)*26^(
3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*130^(1/2),1/390*(76050+6630*130^(
1/2))^(1/2))*3^(1/2)*130^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = -\frac{2i\sqrt{\frac{-i+\sqrt{14}-5ix}{34i+\sqrt{14}}}\sqrt{\frac{i+\sqrt{14}+5ix}{-34i+\sqrt{14}}}\sqrt{7+x}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{5}\sqrt{-\frac{i(7+x)}{34i+\sqrt{14}}}\right),\frac{34i+\sqrt{14}}{34i-\sqrt{14}}\right)-\text{EllipticPi}\left(\frac{1}{35}(3\right.\right.}{\sqrt{5}\sqrt{-\frac{i(7+x)}{34i+\sqrt{14}}}\sqrt{3+2x+5x^2}}$$

input
```
Integrate[Sqrt[7 + x]/(x*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((-2*I)*Sqrt[(-I + Sqrt[14] - (5*I)*x)/(34*I + Sqrt[14])]*Sqrt[(I + Sqrt[1
4] + (5*I)*x)/(-34*I + Sqrt[14])]*Sqrt[7 + x]*(EllipticF[I*ArcSinh[Sqrt[5]
*Sqrt[((-I)*(7 + x))/(34*I + Sqrt[14])]], (34*I + Sqrt[14])/(34*I - Sqrt[1
4])] - EllipticPi[(34 - I*Sqrt[14])/35, I*ArcSinh[Sqrt[5]*Sqrt[((-I)*(7 +
x))/(34*I + Sqrt[14])]], (34*I + Sqrt[14])/(34*I - Sqrt[14])]))/(Sqrt[5]*S
qrt[((-I)*(7 + x))/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1284, 1172, 321, 1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+7}}{x\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1284}$$

$$\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+7\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1172}$$

$$7\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{1}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{\sqrt{x+7}}$$

$$\Big\downarrow \text{321}$$

$$7\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

$$\Big\downarrow \text{1279}$$

$$\frac{14\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{2x\sqrt{x+7}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}}dx}{\sqrt{5x^2+2x+3}}+$$
$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{x\sqrt{x+7}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}}dx}{\sqrt{5x^2+2x+3}}+\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

$$\Big\downarrow \text{187}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-\frac{14\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int -\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-\frac{14\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{5(x+7)+i\sqrt{14}-34}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-\frac{14\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-$$

$$\frac{2\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\,\text{EllipticPi}\left(\frac{1}{35}\left(34-i\sqrt{14}\right),\text{arcs}\right.}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}$$

input
```
Int[Sqrt[7 + x]/(x*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((2*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*S
qrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[
7 + x] - (2*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 +
I*Sqrt[14] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 +
 x))/(34 + I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*S
qrt[7 + x])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/
(Sqrt[3 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqr
t[14] + 5*(7 + x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1284
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]), x], x] + Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(-34+i\sqrt{14}\right)\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\left(\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)-\mathrm{Elli}\right.}{5(5x^3+37x^2+17x+21)}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{2\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{\sqrt{5x^3+37x^2+17x+21}}-\frac{2\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right.}{}\right.}{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}$

input
```
int((x+7)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(-34+I*14^(1/2))*(-5*(x+7)/(-34+I*14^
(1/2)))^(1/2)*((I*14^(1/2)-5*x-1)/(I*14^(1/2)+34))^(1/2)*((I*14^(1/2)+5*x+
1)/(-34+I*14^(1/2)))^(1/2)*(EllipticF((-5*(x+7)/(-34+I*14^(1/2)))^(1/2),(-
(-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2))-EllipticPi((-5*(x+7)/(-34+I*14^(1
/2)))^(1/2),34/35-1/35*I*14^(1/2),(-(-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2
)))/(5*x^3+37*x^2+17*x+21)
```

Fricas [F]

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx=\int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}\,x}\,dx$$

input
```
integrate((7+x)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^3 + 2*x^2 + 3*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate((7+x)**(1/2)/x/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(sqrt(x + 7)/(x*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x}\,dx$$

input
```
integrate((7+x)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x}\,dx$$

input
```
integrate((7+x)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x + 7)^(1/2)/(x*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int((x + 7)^(1/2)/(x*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{7+x}}{x\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x\sqrt{5x^2+2x+3}}dx$$

input
```
int((7+x)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((7+x)^(1/2)/x/(5*x^2+2*x+3)^(1/2),x)
```

3.107 $\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 339

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{3x}+\frac{\sqrt{130}\sqrt{7+x}\sqrt{3+2x+5x^2}}{3\left(78+\sqrt{130}(7+x)\right)}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right)}{6\sqrt{21}}$$

$$-\frac{\sqrt[4]{130}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{\sqrt{3}\sqrt{3+2x+5x^2}}$$

$$-\frac{\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right.\right.}{84\sqrt{3}\sqrt[4]{130}\sqrt{3+2x+5x^2}}$$

output
```
-1/3*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/x+130^(1/2)*(7+x)^(1/2)*(5*x^2+2*x+3)
^(1/2)/(234+3*130^(1/2)*(7+x))+11/126*21^(1/2)*arctanh(1/7*21^(1/2)*(7+x)^
(1/2)/(5*x^2+2*x+3)^(1/2))-1/3*130^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x
))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*26^(3
/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*3^(1/2)/(5*x
^2+2*x+3)^(1/2)-1/32760*(479-42*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7
+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticPi(sin(2*arctan(1/78*5^(1/4)*26
^(3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*130^(1/2),1/390*(76050+6630*130
^(1/2))^(1/2))*3^(1/2)*130^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.45 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{3x} + \frac{(7+x)^{3/2}\left(2730\sqrt{-\frac{i}{34i+\sqrt{14}}}+\frac{127764\sqrt{-\frac{i}{34i+\sqrt{14}}}}{(7+x)^2}-\frac{37128\sqrt{-\frac{i}{34i+\sqrt{14}}}}{7+x}+\frac{14\sqrt{13}\left(-17i\sqrt{2}+\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\cdots}{-3\cdots}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[7 + x]/(x^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/3*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + ((7 + x)^(3/2)*(2730*Sqrt[(-I
)/(34*I + Sqrt[14])] + (127764*Sqrt[(-I)/(34*I + Sqrt[14])])/(7 + x)^2 - (
37128*Sqrt[(-I)/(34*I + Sqrt[14])])/(7 + x) + (14*Sqrt[13]*((-17*I)*Sqrt[2
] + Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*S
qrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticE[I*A
rcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])
/(34*I - Sqrt[14])])/Sqrt[7 + x] + ((2*I)*Sqrt[13]*(2*Sqrt[2] + (7*I)*Sqrt
[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34
*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticF[I*ArcSinh[(
3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I -
 Sqrt[14])])/Sqrt[7 + x] - ((11*I)*Sqrt[26]*Sqrt[(34*I + Sqrt[14] - (234*I
)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-
34*I + Sqrt[14])]*EllipticPi[(7*(34 - I*Sqrt[14]))/234, I*ArcSinh[(3*Sqrt[
(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[1
4])])/Sqrt[7 + x]))/(1638*Sqrt[(-I)/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^
2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1285, 2154, 27, 1269, 1172, 321, 327, 1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+7}}{x^2\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1285}$$

$$-\frac{1}{6}\int \frac{11-5x^2}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}$$

$$\Big\downarrow \text{2154}$$

$$\frac{1}{6}\left(-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\int -\frac{5x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}$$

$$\downarrow 27$$

$$\frac{1}{6}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}$$

$$\downarrow 1269$$

$$\frac{1}{6}\left(5\left(\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-7\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}$$

$$\downarrow 1172$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+\frac{1}{6}\left(-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \cdots}{\cdots}\right.\right.$$

$$\downarrow 321$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+\frac{1}{6}\left(-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{Ellipt}\cdots}{\cdots}\right.\right.$$

$$\downarrow 327$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+\frac{1}{6}\left(-11\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\cdots}{\cdots}\right.\right.$$

$$\downarrow 1279$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$
$$\frac{1}{6}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{\sqrt{x+7}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$
$$\frac{1}{6}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{\sqrt{x+7}}\right.\right.$$

$$\Big\downarrow \text{187}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$
$$\frac{1}{6}\left(\frac{22\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int -\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}}+5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\right.}{}\right.\right.$$

$$\Big\downarrow \text{413}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$
$$\frac{1}{6}\left(\frac{22\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{5(x+7)+i\sqrt{14}-34}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}+5\left(\frac{2i\sqrt{x+7}}{}\right.\right.$$

$$\Big\downarrow \text{413}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$

$$\frac{1}{6}\left(\frac{22\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}+5\left(\right.\right.$$

$$\Big\downarrow \text{412}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{3x}+$$

$$\frac{1}{6}\left(\frac{22\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\text{EllipticPi}\left(\frac{1}{35}\left(34-i\sqrt{14}\right)\right.}{7\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}\right.$$

input
```
Int[Sqrt[7 + x]/(x^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/3*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + (5*((((2*I)/5)*Sqrt[7 + x]*El
lipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sq
rt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((14*I)*Sqrt
[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5
*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x]) + (
22*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*Sqrt[14
] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x))/(34
+ I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqrt[7 + x
])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7*Sqrt[3
 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqrt[14] +
 5*(7 + x)]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(-\frac{\sqrt{5x^3+37x^2+17x+21}}{3x}+\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\left(\left(-\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\mathrm{EllipticE}\left(\sqrt{\cdots}\right.\right.}{3\sqrt{5x^3+37x^2\cdots}}\right.}{\cdots}$
risch	$-\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{3x}+\frac{\left(\frac{11\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\frac{34}{35}-\frac{i\sqrt{14}}{35},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\cdots\right.}{21\sqrt{5x^3+37x^2+17x+21}}\right.}{\cdots}$
default	$\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(245i\sqrt{14}\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)x-11i\sqrt{14}\sqrt{\cdots}\right.}{\cdots}$

input

```
int((x+7)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)*(-1/3/x*(5*x^3
+37*x^2+17*x+21)^(1/2)+5/3*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/
2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1
/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*
((-34/5-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-3
4/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*E
llipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/
5-1/5*I*14^(1/2)))^(1/2)))+11/21*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*
14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x
+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^
(1/2)*EllipticPi(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),34/35-1/35*I*14^(1/2)
,((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^2}\,dx$$

input
```
integrate((7+x)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^4 + 2*x^3 + 3*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^2\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate((7+x)**(1/2)/x**2/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(sqrt(x + 7)/(x**2*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^2}\,dx$$

input
```
integrate((7+x)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^2}\,dx$$

input
```
integrate((7+x)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^2\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x + 7)^(1/2)/(x^2*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int((x + 7)^(1/2)/(x^2*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{7+x}}{x^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^2\sqrt{5x^2+2x+3}}dx$$

input
```
int((7+x)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((7+x)^(1/2)/x^2/(5*x^2+2*x+3)^(1/2),x)
```

3.108 $\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 499

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{6x^2}+\frac{13\sqrt{7+x}\sqrt{3+2x+5x^2}}{84x}$$
$$-\frac{13\sqrt{\frac{65}{2}}\sqrt{7+x}\sqrt{3+2x+5x^2}}{42\left(78+\sqrt{130}(7+x)\right)}+\frac{815\text{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right)}{168\sqrt{21}}$$
$$+\frac{13\sqrt[4]{65}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{14\ 2^{3/4}\sqrt{3}\sqrt{3+2x+5x^2}}$$
$$-\frac{\sqrt[4]{65}\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}(1\cdots\right.}{154\ 2^{3/4}\sqrt{3}\sqrt{3+2x+5x^2}}$$
$$-\frac{163\ 5^{3/4}\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\cdots\right.\right.}{25872\sqrt{3}\sqrt[4]{26}\sqrt{3+2x+5x^2}}$$

output
```
-1/6*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/x^2+13/84*(7+x)^(1/2)*(5*x^2+2*x+3)^(
1/2)/x-13/2*130^(1/2)*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(3276+42*130^(1/2)*(
7+x))+815/3528*21^(1/2)*arctanh(1/7*21^(1/2)*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/
2))+13/84*65^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1
/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2
))),1/390*(76050+6630*130^(1/2))^(1/2))*2^(1/4)*3^(1/2)/(5*x^2+2*x+3)^(1/2
)-1/924*65^(1/4)*(479-42*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)
^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)
*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*2^(1/4)*3^(1/2)/
(5*x^2+2*x+3)^(1/2)-163/2018016*5^(3/4)*(479-42*130^(1/2))*((5*x^2+2*x+3)/
(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticPi(sin(2*arctan
(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*130^(1/2),1/390
*(76050+6630*130^(1/2))^(1/2))*3^(1/2)*26^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.55 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \frac{\sqrt{7+x}(-14+13x)\sqrt{3+2x+5x^2}}{84x^2}$$

$$-\frac{(7+x)^{3/2}\left(35490\sqrt{-\frac{i}{34i+\sqrt{14}}}+\frac{1660932\sqrt{-\frac{i}{34i+\sqrt{14}}}}{(7+x)^2}-\frac{482664\sqrt{-\frac{i}{34i+\sqrt{14}}}}{7+x}+\frac{182\sqrt{13}\left(-17i\sqrt{2}+\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[7 + x]/(x^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(Sqrt[7 + x]*(-14 + 13*x)*Sqrt[3 + 2*x + 5*x^2])/(84*x^2) - ((7 + x)^(3/2)
*(35490*Sqrt[(-I)/(34*I + Sqrt[14])] + (1660932*Sqrt[(-I)/(34*I + Sqrt[14]
)])/(7 + x)^2 - (482664*Sqrt[(-I)/(34*I + Sqrt[14])])/(7 + x) + (182*Sqrt[
13]*((-17*I)*Sqrt[2] + Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(
34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[
14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]]
, (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] - ((26*I)*Sqrt[13]*(16
*Sqrt[2] - (7*I)*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I +
 Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*
EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*
I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] + ((815*I)*Sqrt[26]*Sqrt[(34
*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14]
 + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticPi[(7*(34 - I*Sqrt[14]))/2
34, I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sq
rt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x]))/(45864*Sqrt[(-I)/(34*I + Sqrt[14
])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.58, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {1285, 2154, 1282, 2154, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+7}}{x^3\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1285}$$

$$-\frac{1}{12}\int \frac{5x^2+74x+39}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}$$

$$\Big\downarrow \text{2154}$$

$$\frac{1}{12}\left(-39\int \frac{1}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\int \frac{5x+74}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}$$

$$\Big\downarrow \text{ 1282}$$

$$\frac{1}{12}\left(-\int \frac{5x+74}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(-\frac{1}{42}\int \frac{17-5x^2}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}\right)\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{1}{12}\left(-\int \frac{5}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\right.\right.\right.$$
$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{12}\left(-5\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right.\right.\right.$$
$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}$$

$$\Big\downarrow \text{ 1172}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$
$$\frac{1}{12}\left(-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-74\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(\frac{1}{42}\left(5\int\frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$$\Big\downarrow \text{1269}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-74\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(\frac{1}{42}\left(5\left(\int\frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-7\int\frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-1\right.\right.\right.$$

$$\Big\downarrow \text{1172}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-74\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right.\right.\right.$$

$$\Big\downarrow \text{321}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{327}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$
$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i(5\cdots}}{2^{3\cdots}}\right.\right.}{5\sqrt{\frac{x+\cdots}{34-i\cdots}}}\right.\right.\right.\right.$$

$\downarrow$ 1279

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$
$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{\cdots}}}}{\cdots}\right.\right.\right.\right.$$

$\downarrow$ 27

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$
$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{\cdots}}}}{\cdots}\right.\right.\right.\right.$$

$\downarrow$ 187

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$
$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int-\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\cdots}}}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$\downarrow$ 413

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{}{x\sqrt{5(x+7)+i\sqrt{14}}}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}\right.\right.\right.$$

$\downarrow$ 413

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(-39\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)}}\right.\right.\right.$$

$\downarrow$ 412

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{6x^2}+$$

$$\frac{1}{12}\left(\frac{148\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\text{EllipticPi}\left(\frac{1}{35}\left(34-i\sqrt{1}\right.\right.}{7\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}\right.$$

input
```
Int[Sqrt[7 + x]/(x^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/6*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x^2 + (((-10*I)*Sqrt[(7 + x)/(34
- I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)
*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x] + (148*Sqrt[(34 -
 I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*Sqrt[14] + 5*x]*Sqrt
[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x))/(34 + I*Sqrt[14])
]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqrt[7 + x])/Sqrt[34 -
I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7*Sqrt[3 + 2*x + 5*x^
2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqrt[14] + 5*(7 + x)])
- 39*(-1/21*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + (5*((((2*I)/5)*Sqrt[7
+ x]*EllipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))]
, (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((14*
I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[
14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 +
x]) + (34*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*
Sqrt[14] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x
))/(34 + I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqr
t[7 + x])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7
*Sqrt[3 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqr
t[14] + 5*(7 + x)]))/42))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,(13x-14)}{84x^2}+\frac{\left(-\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{6\sqrt{5x^3+37x^2+17x+21}}\right.}{}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(-\frac{\sqrt{5x^3+37x^2+17x+21}}{6x^2}+\frac{13\sqrt{5x^3+37x^2+17x+21}}{84x}-\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{Ellip}}{6\sqrt{5x^3+37x^2+17x+21}}\right.}{}$
default	$-\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(539i\sqrt{14}\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)x^2+163i\sqrt{}\right.}{}$

input

```
int((x+7)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/84*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(13*x-14)/x^2+(-5/6*(34/5-1/5*I*14^(1
/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/
5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)
/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2
),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+815/588*(34/5-1/5
*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(
-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)
))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticPi(((x+7)/(34/5-1/5*I*14^(1/
2)))^(1/2),34/35-1/35*I*14^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(
1/2)))^(1/2))-65/84*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1
/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14
^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((-34/5
-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5
*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*Elliptic
F(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I
*14^(1/2)))^(1/2))))*((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)
^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^3}\,dx$$

input
```
integrate((7+x)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^5 + 2*x^4 + 3*x^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^3\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate((7+x)**(1/2)/x**3/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(sqrt(x + 7)/(x**3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^3}\,dx$$

input
```
integrate((7+x)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{\sqrt{5\,x^2+2\,x+3}x^3}\,dx$$

input
```
integrate((7+x)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 7)/(sqrt(5*x^2 + 2*x + 3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^3\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int((x + 7)^(1/2)/(x^3*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int((x + 7)^(1/2)/(x^3*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{7+x}}{x^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}}{x^3\sqrt{5x^2+2x+3}}dx$$

input
```
int((7+x)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((7+x)^(1/2)/x^3/(5*x^2+2*x+3)^(1/2),x)
```

3.109 $\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 653

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = -\frac{\left(\sqrt{5}f+\sqrt{5f^2-2fg+3g^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{f+gx}}{\sqrt{f}\sqrt{3+2x+5x^2}}\right)}{\sqrt{15}\sqrt{f}\left(f+\sqrt{f^2-\frac{2fg}{5}+\frac{3g^2}{5}}\right)}$$

$$-\frac{\sqrt[4]{5f^2-2fg+3g^2}\sqrt{\frac{g^2(3+2x+5x^2)}{(5f^2-2fg+3g^2)\left(1+\frac{\sqrt{5}(f+gx)}{\sqrt{5f^2-2fg+3g^2}}\right)^2}}\left(1+\frac{\sqrt{5}(f+gx)}{\sqrt{5f^2-2fg+3g^2}}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}\ldots}{\sqrt[4]{5f^2-\ldots}}\right.\right.}{\sqrt[4]{5}\left(f+\sqrt{f^2-\frac{2fg}{5}+\frac{3g^2}{5}}\right)\sqrt{3+2x+5x^2}}$$

$$+\frac{\left(\sqrt{5}f-\sqrt{5f^2-2fg+3g^2}\right)\sqrt{\frac{g^2(3+2x+5x^2)}{\left(\sqrt{5f^2-2fg+3g^2}+\sqrt{5}(f+gx)\right)^2}}\left(\sqrt{5f^2-2fg+3g^2}+\sqrt{5}(f+gx)\right)\operatorname{EllipticPi}\ldots}{2\,5^{3/4}f\sqrt[4]{5f^2-2fg+3g^2}\left(f+\sqrt{f^2-\frac{2fg}{5}\ldots}\right.}$$

output
```
-1/15*(5^(1/2)*f+(5*f^2-2*f*g+3*g^2)^(1/2))*arctanh(3^(1/2)*(g*x+f)^(1/2)/
f^(1/2)/(5*x^2+2*x+3)^(1/2))*15^(1/2)/f^(1/2)/(f+1/5*(25*f^2-10*f*g+15*g^2
)^(1/2))-1/5*(5*f^2-2*f*g+3*g^2)^(1/4)*(g^2*(5*x^2+2*x+3)/(5*f^2-2*f*g+3*g
^2)/(1+5^(1/2)*(g*x+f)/(5*f^2-2*f*g+3*g^2)^(1/2))^2)^(1/2)*(1+5^(1/2)*(g*x
+f)/(5*f^2-2*f*g+3*g^2)^(1/2))*InverseJacobiAM(2*arctan(5^(1/4)*(g*x+f)^(1
/2)/(5*f^2-2*f*g+3*g^2)^(1/4)),1/2*(2+2*(5*f-g)/(25*f^2-10*f*g+15*g^2)^(1/
2))^(1/2))*5^(3/4)/(f+1/5*(25*f^2-10*f*g+15*g^2)^(1/2))/(5*x^2+2*x+3)^(1/2
)+1/10*(5^(1/2)*f-(5*f^2-2*f*g+3*g^2)^(1/2))*(g^2*(5*x^2+2*x+3)/((5*f^2-2*
f*g+3*g^2)^(1/2)+5^(1/2)*(g*x+f))^2)^(1/2)*((5*f^2-2*f*g+3*g^2)^(1/2)+5^(1
/2)*(g*x+f))*EllipticPi(sin(2*arctan(5^(1/4)*(g*x+f)^(1/2)/(5*f^2-2*f*g+3*
g^2)^(1/4))),1/20*(5^(1/2)*f+(5*f^2-2*f*g+3*g^2)^(1/2))^2*5^(1/2)/f/(5*f^2
-2*f*g+3*g^2)^(1/2),1/2*(2+2*(5*f-g)/(25*f^2-10*f*g+15*g^2)^(1/2))^(1/2))*
5^(1/4)/f/(5*f^2-2*f*g+3*g^2)^(1/4)/(f+1/5*(25*f^2-10*f*g+15*g^2)^(1/2))/(
5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.78 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{2i(f+gx)\sqrt{1+\frac{-5f^2+2fg-3g^2}{\left(5f+\left(-1-i\sqrt{14}\right)g\right)(f+gx)}}\sqrt{1+\frac{-5f^2+2fg-3g^2}{\left(5f+i\left(i+\sqrt{14}\right)g\right)(f+gx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{-5f^2+2fg-3g}{5f+\left(-1-i\sqrt{14}\right.}}}{\sqrt{f+gx}}\right.\right.\right.}{f\sqrt{\frac{-5f^2+2fg-3g^2}{5f+\left(-1-i\sqrt{14}\right)g}}\sqrt{3}}$$

input
```
Integrate[1/(x*Sqrt[f + g*x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((-2*I)*(f + g*x)*Sqrt[1 + (-5*f^2 + 2*f*g - 3*g^2)/((5*f + (-1 - I*Sqrt[1
4])*g)*(f + g*x))]*Sqrt[1 + (-5*f^2 + 2*f*g - 3*g^2)/((5*f + I*(I + Sqrt[1
4])*g)*(f + g*x))]*(EllipticF[I*ArcSinh[Sqrt[(-5*f^2 + 2*f*g - 3*g^2)/(5*f
 + (-1 - I*Sqrt[14])*g)]/Sqrt[f + g*x]], (5*f - g - I*Sqrt[14]*g)/(5*f - g
 + I*Sqrt[14]*g)] - EllipticPi[(f*(5*f + (-1 - I*Sqrt[14])*g))/(5*f^2 - 2*
f*g + 3*g^2), I*ArcSinh[Sqrt[(-5*f^2 + 2*f*g - 3*g^2)/(5*f + (-1 - I*Sqrt[
14])*g)]/Sqrt[f + g*x]], (5*f - g - I*Sqrt[14]*g)/(5*f - g + I*Sqrt[14]*g)
]))/(f*Sqrt[(-5*f^2 + 2*f*g - 3*g^2)/(5*f + (-1 - I*Sqrt[14])*g)]*Sqrt[3 +
 2*x + 5*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.47, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{5x^2+2x+3}\sqrt{f+gx}}dx$$

$$\Big\downarrow \textbf{1279}$$

$$\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{2x\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{f+gx}}dx}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{x\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{f+gx}}dx}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{187}$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int -\frac{1}{gx\sqrt{-\frac{5f}{g}+\frac{5(f+gx)}{g}-i\sqrt{14}+1}\sqrt{-\frac{5f}{g}+\frac{5(f+gx)}{g}+i\sqrt{14}+1}}d\sqrt{f+gx}}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{413}$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(f+gx)}{5f+i\sqrt{14}g-g}}\int -\frac{1}{gx\sqrt{-\frac{5f}{g}+\frac{5(f+gx)}{g}+i\sqrt{14}+1}\sqrt{1-\frac{5(f+gx)}{5f+i\sqrt{14}g-g}}}d\sqrt{f+gx}}{\sqrt{5x^2+2x+3}\sqrt{\frac{5(f+gx)}{g}-\frac{5f}{g}-i\sqrt{14}+1}}$$

$$\Big\downarrow 413$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(f+gx)}{5f+i\sqrt{14}g-g}}\sqrt{1-\frac{5(f+gx)}{5f-\left(1+i\sqrt{14}\right)g}}\int -\frac{1}{gx\sqrt{1-\frac{5(f+gx)}{5f+i\sqrt{14}g-g}}\sqrt{1-\frac{5(f+gx)}{5f-\left(1+i\sqrt{14}\right)g}}}d}{\sqrt{5x^2+2x+3}\sqrt{\frac{5(f+gx)}{g}-\frac{5f}{g}-i\sqrt{14}+1}\sqrt{\frac{5(f+gx)}{g}-\frac{5f}{g}+i\sqrt{14}+1}}$$

$$\Big\downarrow 412$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{5f+i\left(\sqrt{14}+i\right)g}\sqrt{1-\frac{5(f+gx)}{5f+i\sqrt{14}g-g}}\sqrt{1-\frac{5(f+gx)}{5f-\left(1+i\sqrt{14}\right)g}}\text{EllipticPi}\left(\frac{5f+i}{}\right.}{\sqrt{5}f\sqrt{5x^2+2x+3}\sqrt{\frac{5(f+gx)}{g}-\frac{5f}{g}-i\sqrt{14}+1}\sqrt{\frac{5(f+gx)}{g}-\frac{5f}{g}+i\sqrt{1}}}$$

input

```
Int[1/(x*Sqrt[f + g*x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output

```
(-2*Sqrt[5*f + I*(I + Sqrt[14])*g]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*S
qrt[14] + 5*x]*Sqrt[1 - (5*(f + g*x))/(5*f - g + I*Sqrt[14]*g)]*Sqrt[1 - (
5*(f + g*x))/(5*f - (1 + I*Sqrt[14])*g)]*EllipticPi[(5*f - g + I*Sqrt[14]*
g)/(5*f), ArcSin[(Sqrt[5]*Sqrt[f + g*x])/Sqrt[5*f + I*(I + Sqrt[14])*g]],
(5*f - g + I*Sqrt[14]*g)/(5*f - g - I*Sqrt[14]*g)])/(Sqrt[5]*f*Sqrt[3 + 2*
x + 5*x^2]*Sqrt[1 - I*Sqrt[14] - (5*f)/g + (5*(f + g*x))/g]*Sqrt[1 + I*Sqr
t[14] - (5*f)/g + (5*(f + g*x))/g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.34

method	result
default	$\frac{2\left(i\sqrt{14}\,g-5f+g\right)\operatorname{EllipticPi}\left(\sqrt{-\frac{5(gx+f)}{i\sqrt{14}\,g-5f+g}},-\frac{i\sqrt{14}\,g-5f+g}{5f},\sqrt{-\frac{i\sqrt{14}\,g-5f+g}{i\sqrt{14}\,g+5f-g}}\right)\sqrt{\frac{g\left(i\sqrt{14}+5x+1\right)}{i\sqrt{14}\,g-5f+g}}\sqrt{\frac{\left(i\sqrt{14}-5x-1\right)g}{i\sqrt{14}\,g+5f-g}}\sqrt{-\frac{5(gx+}{i\sqrt{14}\,g-5}}}{5(5g\,x^3+5f\,x^2+2g\,x^2+2fx+3gx+3f)f}$
elliptic	$-\frac{2\sqrt{(gx+f)(5x^2+2x+3)}\left(\frac{f}{g}-\frac{1}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{1}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{f}{g}+\frac{1}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{f}{g}+\frac{1}{5}+\frac{i\sqrt{14}}{5}}}\,g\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{1}{5}-\frac{i\sqrt{14}}{5}}},-\frac{\left(-\frac{f}{g}+\frac{1}{5}\right.}{}\right.}{\sqrt{gx+f}\sqrt{5x^2+2x+3}\sqrt{5g\,x^3+5f\,x^2+2g\,x^2+2fx+3gx+3f}\,f}$

input
```
int(1/x/(g*x+f)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(I*14^(1/2)*g-5*f+g)*EllipticPi((-5*(g*x+f)/(I*14^(1/2)*g-5*f+g))^(1/2
),-1/5*(I*14^(1/2)*g-5*f+g)/f,(-(I*14^(1/2)*g-5*f+g)/(I*14^(1/2)*g+5*f-g))
^(1/2))*(g*(I*14^(1/2)+5*x+1)/(I*14^(1/2)*g-5*f+g))^(1/2)*((I*14^(1/2)-5*x
-1)*g/(I*14^(1/2)*g+5*f-g))^(1/2)*(-5*(g*x+f)/(I*14^(1/2)*g-5*f+g))^(1/2)*
(5*x^2+2*x+3)^(1/2)*(g*x+f)^(1/2)/(5*g*x^3+5*f*x^2+2*g*x^2+2*f*x+3*g*x+3*f
)/f
```

Fricas [F]

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{gx+f}\sqrt{5\,x^2+2\,x+3}x}\,dx$$

input
```
integrate(1/x/(g*x+f)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*sqrt(5*x^2 + 2*x + 3)/(5*g*x^4 + (5*f + 2*g)*x^3 +
(2*f + 3*g)*x^2 + 3*f*x), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x\sqrt{f+gx}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(1/x/(g*x+f)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(f + g*x)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{gx+f}\sqrt{5\,x^2+2\,x+3}x}\,dx$$

input
```
integrate(1/x/(g*x+f)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(g*x + f)*sqrt(5*x^2 + 2*x + 3)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{gx+f}\sqrt{5\,x^2+2\,x+3}x}\,dx$$

input
```
integrate(1/x/(g*x+f)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x\,\sqrt{f+g\,x}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(1/(x*(f + g*x)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(1/(x*(f + g*x)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{f+gx}\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{5x^2+2x+3}}{5g\,x^4+5f\,x^3+2g\,x^3+2f\,x^2+3g\,x^2+3fx}dx$$

input
```
int(1/x/(g*x+f)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(5*x**2 + 2*x + 3))/(5*f*x**3 + 2*f*x**2 + 3*f*x +
5*g*x**4 + 2*g*x**3 + 3*g*x**2),x)
```

3.110 $\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 324

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = -\frac{506}{375}\sqrt{7+x}\sqrt{3+2x+5x^2} + \frac{2}{25}(7+x)^{3/2}\sqrt{3+2x+5x^2} + \frac{20374\sqrt{7+x}\sqrt{3+2x+5x^2}}{375\left(3\sqrt{130}+5(7+x)\right)} - \frac{20374\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{125\sqrt{3}5^{3/4}\sqrt{3+2x+5x^2}} - \frac{\left(23141\sqrt{5}-10187\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{39}\ldots\right)}{125\sqrt{3}5^{3/4}\sqrt[4]{26}\sqrt{3+2x+5x^2}}$$

output
```
-506/375*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)+2/25*(7+x)^(3/2)*(5*x^2+2*x+3)^(1
/2)+20374*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(1125*130^(1/2)+13125+1875*x)-20
374/1875*3^(1/2)*26^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78
+130^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)
*3^(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)
-1/48750*(23141*5^(1/2)-10187*26^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x)
)^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/4)*26^(
3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*3^(1/2)*5^(1
/4)*26^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.17 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2}{375}\sqrt{7+x}(-148+15x)\sqrt{3+2x+5x^2} - \frac{(7+x)^{3/2}\left(-\frac{794586\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2} + \frac{20374i\sqrt{13}\left(17\sqrt{2}+i\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\text{arcsinh}\left(\frac{3\sqrt{-}\cdots}{\cdots}\right.\right.}{\sqrt{7+x}} \cdots\right.}{73125\sqrt{-\frac{i}{34i+\cdots}}\cdots}$$

input
```
Integrate[x^3/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(2*Sqrt[7 + x]*(-148 + 15*x)*Sqrt[3 + 2*x + 5*x^2])/375 - ((7 + x)^(3/2)*(
(-794586*Sqrt[(-I)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x)^2 + ((203
74*I)*Sqrt[13]*(17*Sqrt[2] + I*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7
 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I
 + Sqrt[14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt
[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])]])/Sqrt[7 + x] + (Sqrt[13]*((
757*I)*Sqrt[2] + 20374*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(
34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[
14])]*EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]]
, (34*I + Sqrt[14])/(34*I - Sqrt[14])]])/Sqrt[7 + x]))/(73125*Sqrt[(-I)/(34
*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1278, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 1278}$$

$$\frac{2}{25}x\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{25}\int \frac{148x^2+51x+42}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{25}\left(-\frac{2}{15}\int -\frac{10187x+1886}{2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)+\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{25}\left(\frac{1}{15}\int \frac{10187x+1886}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)+\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{25}\left(\frac{1}{15}\left(10187\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-69423\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}\right)+\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x$$

$$\Big\downarrow \text{ 1172}$$

$$\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x+\frac{1}{25}\left(-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{20374i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{138846i\sqrt{\frac{x+7}{34-i\sqrt{14}}}}{\right.\right.$$

$\downarrow$ 321

$$\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x+\frac{1}{25}\left(-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{20374i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{138846i\sqrt{\frac{x+7}{34-i\sqrt{14}}}}{\right.\right.$$

$\downarrow$ 327

$$\frac{2}{25}\sqrt{x+7}\sqrt{5x^2+2x+3}x+\frac{1}{25}\left(-\frac{296}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{20374i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{138846i\sqrt{\frac{}{3}}}{\right.\right.$$

input
```
Int[x^3/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(2*x*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/25 + ((-296*Sqrt[7 + x]*Sqrt[3 + 2
*x + 5*x^2])/15 + ((((20374*I)/5)*Sqrt[7 + x]*EllipticE[ArcSin[Sqrt[(-I)*(
1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])]
)/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((138846*I)*Sqrt[(7 + x)/(34 - I*Sqrt[
14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))
], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x])/15)/25
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.14

method	result
risch	$\frac{2(-148+15x)\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{375}+\frac{\left(\frac{3772\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\ldots}{-\frac{34}{5}-\ldots}}\right.\ldots}{375\sqrt{5x^3+37x^2+17x+21}}\ldots}{\ldots}$
default	$\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(69423i\sqrt{14}\,\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)+23376\sqrt{\ldots}\right.}{\ldots}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{2x\sqrt{5x^3+37x^2+17x+21}}{25}-\frac{296\sqrt{5x^3+37x^2+17x+21}}{375}+\frac{3772\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\ldots}{375\sqrt{5x^3+37x^2+17x+\ldots}}\right.}{\ldots}$

input
```
int(x^3/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/375*(-148+15*x)*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)+(3772/375*(34/5-1/5*I*14
^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5
-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1
/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(
1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+20374/375*(34/
5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/
2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^
(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((-34/5-1/5*I*14^(1/2))*Ellipti
cE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*
I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*EllipticF(((x+7)/(34/5-1/5*I*14^
(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))))*((x
+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.15

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2}{375}\,\sqrt{5\,x^2+2\,x+3}(15\,x-148)\sqrt{x+7} - \frac{697258}{28125}\,\sqrt{5}\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right) - \frac{20374}{1875}\,\sqrt{5}\mathrm{weierstrassZeta}\left(\frac{4456}{75},-\frac{348704}{3375},\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)\right)$$

input
```
integrate(x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
2/375*sqrt(5*x^2 + 2*x + 3)*(15*x - 148)*sqrt(x + 7) - 697258/28125*sqrt(5
)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) - 20374/1875*sqrt(
5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInverse(4456/75, -34
8704/3375, x + 37/15))
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^3}{\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x**3/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^3}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^3}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^3}{\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(x^3/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(x^3/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x}{25} - \frac{51\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{925} - \frac{10187\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{5x^3+37x^2+17x+21}dx\right)}{1850} - \frac{2241\left(\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^3+37x^2+17x+21}dx\right)}{1850}$$

input
```
int(x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
(148*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x - 102*sqrt(x + 7)*sqrt(5*x**2 +
2*x + 3) - 10187*int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x**2)/(5*x**3 + 3
7*x**2 + 17*x + 21),x) - 2241*int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3))/(5*
x**3 + 37*x**2 + 17*x + 21),x))/1850
```

3.111 $\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 299

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2}{15}\sqrt{7+x}\sqrt{3+2x+5x^2}-\frac{148\sqrt{7+x}\sqrt{3+2x+5x^2}}{15\left(3\sqrt{130}+5(7+x)\right)}$$
$$+\frac{148\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{5\sqrt{3}5^{3/4}\sqrt{3+2x+5x^2}}$$
$$+\frac{\left(167\sqrt{5}-74\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\left(195\right.\right.}{5\sqrt{3}5^{3/4}\sqrt[4]{26}\sqrt{3+2x+5x^2}}$$

output
```
2/15*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)-148*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(
45*130^(1/2)+525+75*x)+148/75*3^(1/2)*26^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2
)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)
*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*5^(1/4
)/(5*x^2+2*x+3)^(1/2)+1/1950*(167*5^(1/2)-74*26^(1/2))*((5*x^2+2*x+3)/(78+
130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/
78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2
))*3^(1/2)*5^(1/4)*26^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.97 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2}{15}\sqrt{7+x}\sqrt{3+2x+5x^2} + \frac{(7+x)^{3/2}\left(-\frac{5772\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2} + \frac{148i\sqrt{13}\left(17\sqrt{2}+i\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\operatorname{arcsinh}\left(\frac{3\sqrt{-\frac{2}{34i+}}}{\sqrt{7+}}\right.\right.}{\sqrt{7+x}}\right.}{2925\sqrt{-\frac{i}{34i+\sqrt{14}}}}$$

input
```
Integrate[x^2/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/15 + ((7 + x)^(3/2)*((-5772*Sqrt[(-I
)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x)^2 + ((148*I)*Sqrt[13]*(17*
Sqrt[2] + I*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt
[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*Ellip
ticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + S
qrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] + (Sqrt[13]*((-11*I)*Sqrt[2] + 14
8*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqr
t[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticF[I*Arc
Sinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(
34*I - Sqrt[14])])/Sqrt[7 + x]))/(2925*Sqrt[(-I)/(34*I + Sqrt[14])]*Sqrt[3
 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1278, 9, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 1278}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{15}\int \frac{74x^2+17x}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}-\frac{1}{15}\int \frac{74x+17}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{15}\left(501\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-74\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx\right)+\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{ 1172}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+$$
$$\frac{1}{15}\left(\frac{1002i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{1}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{\sqrt{x+7}}-\frac{148i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i(5\dots}}{2\dots}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.$$

$$\Big\downarrow \text{ 321}$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{1002i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-\frac{148i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x\right.}}{2^{3/}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.$$

$$\Big\downarrow \; 327$$

$$\frac{2}{15}\sqrt{x+7}\sqrt{5x^2+2x+3}+\frac{1}{15}\left(\frac{1002i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}-\frac{148i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+\right.}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.$$

input
```
Int[x^2/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/15 + ((((-148*I)/5)*Sqrt[7 + x]*Elli
pticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt
[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] + ((1002*I)*Sqrt
[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5
*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x])/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1278
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))
Int[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*
f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*
f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c
*d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{2\sqrt{5x^3+37x^2+17x+21}}{15}-\frac{34\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\ldots}\right.}{15\sqrt{5x^3+37x^2+17x+21}}\ldots\right.}{\ldots}$
risch	$\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{15}+\frac{\left(-\frac{34\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{15\sqrt{5x^3+37x^2+17x+21}}-\frac{148\left(\ldots\right.}{\ldots}\ldots\right.}{\ldots}$
default	$-\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(501i\sqrt{14}\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)+282\sqrt{-\ldots}\right.}{\ldots}$

input
```
int(x^2/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)*(2/15*(5*x^3+3
7*x^2+17*x+21)^(1/2)-34/15*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/
2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1
/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*
EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34
/5-1/5*I*14^(1/2)))^(1/2))-148/15*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I
*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((
x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)
^(1/2)*((-34/5-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/
2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(
1/2))*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2)
)/(-34/5-1/5*I*14^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{4966}{1125}\sqrt{5}\text{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right) + \frac{148}{75}\sqrt{5}\text{weierstrassZeta}\left(\frac{4456}{75},-\frac{348704}{3375},\text{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)\right) + \frac{2}{15}\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}$$

input
```
integrate(x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
4966/1125*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) +
148/75*sqrt(5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInverse(
4456/75, -348704/3375, x + 37/15)) + 2/15*sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7
)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2}{\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x**2/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2}{\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(x^2/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(x^2/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x^2}{5x^3+37x^2+17x+21}dx$$

input
```
int(x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x**2)/(5*x**3 + 37*x**2 + 17*x + 2
1),x)
```

3.112 $\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 268

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2\sqrt{7+x}\sqrt{3+2x+5x^2}}{3\sqrt{130}+5(7+x)} - \frac{2\sqrt{3}\sqrt[4]{26}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{5^{3/4}\sqrt{3+2x+5x^2}} - \frac{\left(7\sqrt{5}-3\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\left(195+1\right.\right.}{\sqrt{3}5^{3/4}\sqrt[4]{26}\sqrt{3+2x+5x^2}}$$

output
```
2*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(5*x+35+3*130^(1/2))-2/5*3^(1/2)*26^(1/4
)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*Ellipt
icE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+
6630*130^(1/2))^(1/2))*5^(1/4)/(5*x^2+2*x+3)^(1/2)-1/390*(7*5^(1/2)-3*26^(
1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*In
verseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(7
6050+6630*130^(1/2))^(1/2))*3^(1/2)*5^(1/4)*26^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.54 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.46

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{(7+x)^{3/2}\left(\frac{78\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2}+\frac{2\sqrt{13}\left(-17i\sqrt{2}+\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\text{arcsinh}\left(\frac{3\sqrt{-\frac{26i}{34i+\sqrt{14}}}}{\sqrt{7+x}}\right)\middle|\cdots\right)}{\sqrt{7+x}}\cdots\right)}{195\sqrt{-\frac{i}{34i+\sqrt{14}}}\sqrt{3+2\cdots}}$$

input
```
Integrate[x/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((7 + x)^(3/2)*((78*Sqrt[(-I)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x
)^2 + (2*Sqrt[13]*((-17*I)*Sqrt[2] + Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234
*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/
(-34*I + Sqrt[14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])]
)/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] - (I*Sqr
t[13]*(Sqrt[2] - (2*I)*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(
34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[
14])]*EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]]
, (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x]))/(195*Sqrt[(-I)/(34*I
 + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$\big\downarrow$ 1269

$$\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-7\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$\big\downarrow$ 1172

$$\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{1}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{\sqrt{x+7}}$$

$\big\downarrow$ 321

$$\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

$\big\downarrow$ 327

$$\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

input
```
Int[x/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(((2*I)/5)*Sqrt[7 + x]*EllipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/
(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*
Sqrt[14])] - ((14*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt
[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqr
t[14])])/Sqrt[7 + x]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.81

method	result
elliptic	$\frac{2\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\left(\left(-\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\operatorname{EllipticE}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\cdots}{-\frac{34}{5}-\cdots}}\cdots\right.\right.}{\sqrt{x+7}\sqrt{5x^2+2x+3}\sqrt{5x^3+37x^2+17x+21}}$
default	$\frac{2\sqrt{x+7}\sqrt{5x^2+2x+3}\left(-34+i\sqrt{14}\right)\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\left(i\operatorname{EllipticE}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)\sqrt{14}-\cdots\right.}{25(5x^3\cdots}$

input
```
int(x/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)*(34/5-1/5*I*
14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34
/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^
(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((-34/5-1/5*I*14^(1/2))*EllipticE(((x+7
)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/
2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^
(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = -\frac{74}{75}\sqrt{5}\text{weierstrassPInverse}\left(\frac{4456}{75}, -\frac{348704}{3375}, x+\frac{37}{15}\right) - \frac{2}{5}\sqrt{5}\text{weierstrassZeta}\left(\frac{4456}{75}, -\frac{348704}{3375}, \text{weierstrassPInverse}\left(\frac{4456}{75}, -\frac{348704}{3375}, x+\frac{37}{15}\right)\right)$$

input
```
integrate(x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
-74/75*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15) - 2/5
*sqrt(5)*weierstrassZeta(4456/75, -348704/3375, weierstrassPInverse(4456/7
5, -348704/3375, x + 37/15))
```

Sympy [F]

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(x/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(x/(sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{x}{\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(x/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(x/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,x}{5x^3+37x^2+17x+21}dx$$

input
```
int(x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)*x)/(5*x**3 + 37*x**2 + 17*x + 21),
x)
```

3.113 $\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{\sqrt{3}\sqrt[4]{130}\sqrt{3+2x+5x^2}}$$

output
```
1/390*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*In
verseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(7
6050+6630*130^(1/2))^(1/2))*3^(1/2)*130^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{i(7+x)\sqrt{1+\frac{234i}{\left(-34i+\sqrt{14}\right)(7+x)}}\sqrt{\frac{2}{13}-\frac{36i}{\left(34i+\sqrt{14}\right)(7+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{3\sqrt{-\frac{26i}{34i+\sqrt{14}}}}{\sqrt{7+x}}\right),\frac{34i+\sqrt{14}}{34i-\sqrt{14}}\right)}{3\sqrt{-\frac{i}{34i+\sqrt{14}}}\sqrt{3+2x+5x^2}}$$

input
```
Integrate[1/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((I/3)*(7 + x)*Sqrt[1 + (234*I)/((-34*I + Sqrt[14])*(7 + x))]*Sqrt[2/13 -
(36*I)/((34*I + Sqrt[14])*(7 + x))]*EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(3
4*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(Sqr
t[(-I)/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1172, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \text{1172}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{1}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{\sqrt{x+7}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right),\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{\sqrt{x+7}}$$

input
```
Int[1/(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((2*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*S
qrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[
7 + x]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

method	result	si
default	$-\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,\left(-34+i\sqrt{14}\right)\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)}{5(5x^3+37x^2+17x+21)}$	15
elliptic	$\frac{2\sqrt{(x+7)(5x^2+2x+3)}\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\right)}{\sqrt{x+7}\sqrt{5x^2+2x+3}\sqrt{5x^3+37x^2+17x+21}}$	15

input
```
int(1/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(-34+I*14^(1/2))*(-5*(x+7)/(-34+I*14^
(1/2)))^(1/2)*((I*14^(1/2)-5*x-1)/(I*14^(1/2)+34))^(1/2)*((I*14^(1/2)+5*x+
1)/(-34+I*14^(1/2)))^(1/2)*EllipticF((-5*(x+7)/(-34+I*14^(1/2)))^(1/2),(-(
-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2))/(5*x^3+37*x^2+17*x+21)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{2}{5}\,\sqrt{5}\,\mathrm{weierstrassPInverse}\left(\frac{4456}{75},-\frac{348704}{3375},x+\frac{37}{15}\right)$$

input
```
integrate(1/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(5)*weierstrassPInverse(4456/75, -348704/3375, x + 37/15)
```

Sympy [F]

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(1/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(1/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}}\,dx$$

input
```
integrate(1/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx=\int \frac{1}{\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(1/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(1/((x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx=\int \frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{5x^3+37x^2+17x+21}dx$$

input
```
int(1/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3))/(5*x**3 + 37*x**2 + 17*x + 21),x)
```

3.114 $\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 301

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right)}{\sqrt{21}} - \frac{5^{3/4}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{\sqrt{3}\sqrt[4]{26}\left(35+3\sqrt{130}\right)\sqrt{3+2x+5x^2}} + \frac{\sqrt[4]{\frac{5}{26}}\left(7\sqrt{5}-3\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\ldots\right)}{14\sqrt{3}\left(35+3\sqrt{130}\right)\sqrt{3+2x+5x^2}}$$

output
```
-1/21*21^(1/2)*arctanh(1/7*21^(1/2)*(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2))-1/78*
5^(3/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*
InverseJacobiAM(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*
(76050+6630*130^(1/2))^(1/2))*3^(1/2)*26^(3/4)/(35+3*130^(1/2))/(5*x^2+2*x
+3)^(1/2)+1/1092*5^(1/4)*26^(3/4)*(7*5^(1/2)-3*26^(1/2))*((5*x^2+2*x+3)/(7
8+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticPi(sin(2*arctan(1
/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*130^(1/2),1/390*(
76050+6630*130^(1/2))^(1/2))*3^(1/2)/(35+3*130^(1/2))/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.65 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = -\frac{i(7+x)\sqrt{\frac{2}{13}+\frac{36i}{\left(-34i+\sqrt{14}\right)(7+x)}}\sqrt{1-\frac{234i}{\left(34i+\sqrt{14}\right)(7+x)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{3\sqrt{-\frac{26i}{34i+\sqrt{14}}}}{\sqrt{7+x}}\right),\frac{34i+\sqrt{14}}{34i-\sqrt{14}}\right)-\text{E}\right.}{21\sqrt{-\frac{i}{34i+\sqrt{14}}}\sqrt{3+2x+5x^2}}$$

input
```
Integrate[1/(x*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((-1/21*I)*(7 + x)*Sqrt[2/13 + (36*I)/((-34*I + Sqrt[14])*(7 + x))]*Sqrt[1
 - (234*I)/((34*I + Sqrt[14])*(7 + x))]*(EllipticF[I*ArcSinh[(3*Sqrt[(-26*
I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])]
- EllipticPi[(7*(34 - I*Sqrt[14]))/234, I*ArcSinh[(3*Sqrt[(-26*I)/(34*I +
Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])]))/(Sqrt[(-I
)/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 1279}$$

$$\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{2x\sqrt{x+7}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}}dx}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int \frac{1}{x\sqrt{x+7}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}}dx}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{ 187}$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int -\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{5(x+7)+i\sqrt{14}-34}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{2\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\text{EllipticPi}\left(\frac{1}{35}\left(34-i\sqrt{14}\right),\text{ar}\right.}{7\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}$$

input
```
Int[1/(x*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(-2*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*Sqrt[1
4] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x))/(34
 + I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqrt[7 +
x])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7*Sqrt[
3 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqrt[14]
+ 5*(7 + x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.52

method	result
default	$\frac{2\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,\left(-34+i\sqrt{14}\right)\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\operatorname{EllipticPi}\Big(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\frac{34}{35}-\frac{i\sqrt{14}}{35},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}}{35(5x^3+37x^2+17x+21)}$
elliptic	$-\frac{2\sqrt{(x+7)(5x^2+2x+3)}\,\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\operatorname{EllipticPi}\Big(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\frac{34}{35}-\frac{i\sqrt{14}}{35},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\Big)}{7\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,\sqrt{5x^3+37x^2+17x+21}}$

input
```
int(1/x/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(-34+I*14^(1/2))*(-5*(x+7)/(-34+I*14^
(1/2)))^(1/2)*((I*14^(1/2)-5*x-1)/(I*14^(1/2)+34))^(1/2)*((I*14^(1/2)+5*x+
1)/(-34+I*14^(1/2)))^(1/2)*EllipticPi((-5*(x+7)/(-34+I*14^(1/2)))^(1/2),34
/35-1/35*I*14^(1/2),(-(-34+I*14^(1/2))/(I*14^(1/2)+34))^(1/2))/(5*x^3+37*x
^2+17*x+21)
```

Fricas [F]

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x}\,dx$$

input
```
integrate(1/x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^4 + 37*x^3 + 17*x^2 + 21*x
), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(1/x/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x}\,dx$$

input
```
integrate(1/x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x}\,dx$$

input
```
integrate(1/x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x\,\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(1/(x*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(1/(x*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x\sqrt{x+7}\,\sqrt{5x^2+2x+3}}dx$$

input
```
int(1/x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int(1/x/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

3.115 $\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 462

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{21x} + \frac{\sqrt{130}\sqrt{7+x}\sqrt{3+2x+5x^2}}{21\left(78+\sqrt{130}(7+x)\right)} + \frac{17\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right)}{42\sqrt{21}}$$

$$- \frac{\sqrt[4]{130}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{7\sqrt{3}\sqrt{3+2x+5x^2}}$$

$$+ \frac{\sqrt[4]{\frac{5}{26}}\left(7\sqrt{5}-3\sqrt{26}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}(19\right.}{77\sqrt{3}\sqrt{3+2x+5x^2}}$$

$$- \frac{17\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\text{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7\right.}{\sqrt{3}}\right.}{6468\sqrt{3}\sqrt[4]{130}\sqrt{3+2x+5x^2}}$$

output
```
-1/21*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/x+130^(1/2)*(7+x)^(1/2)*(5*x^2+2*x+3
)^(1/2)/(1638+21*130^(1/2)*(7+x))+17/882*21^(1/2)*arctanh(1/7*21^(1/2)*(7+
x)^(1/2)/(5*x^2+2*x+3)^(1/2))-1/21*130^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*
(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*2
6^(3/4)*(7+x)^(1/2)*3^(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*3^(1/2)/
(5*x^2+2*x+3)^(1/2)+1/6006*(7*5^(1/2)-3*26^(1/2))*((5*x^2+2*x+3)/(78+130^(
1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^
(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*3^
(1/2)*5^(1/4)*26^(3/4)/(5*x^2+2*x+3)^(1/2)-17/2522520*(479-42*130^(1/2))*(
(5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticP
i(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*1
30^(1/2),1/390*(76050+6630*130^(1/2))^(1/2))*3^(1/2)*130^(3/4)/(5*x^2+2*x+
3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.40 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{21x}$$

$$+\frac{(7+x)^{3/2}\left(2730\sqrt{-\frac{i}{34i+\sqrt{14}}}+\frac{127764\sqrt{-\frac{i}{34i+\sqrt{14}}}}{(7+x)^2}-\frac{37128\sqrt{-\frac{i}{34i+\sqrt{14}}}}{7+x}+\frac{14\sqrt{13}\left(-17i\sqrt{2}+\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\cdots}{-3\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[1/(x^2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/21*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + ((7 + x)^(3/2)*(2730*Sqrt[(-
I)/(34*I + Sqrt[14])] + (127764*Sqrt[(-I)/(34*I + Sqrt[14])])/(7 + x)^2 -
(37128*Sqrt[(-I)/(34*I + Sqrt[14])])/(7 + x) + (14*Sqrt[13]*((-17*I)*Sqrt[
2] + Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*
Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticE[I*
ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14]
)/(34*I - Sqrt[14])])/Sqrt[7 + x] + ((2*I)*Sqrt[13]*(5*Sqrt[2] + (7*I)*Sqr
t[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-3
4*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticF[I*ArcSinh[
(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I
- Sqrt[14])])/Sqrt[7 + x] - ((17*I)*Sqrt[26]*Sqrt[(34*I + Sqrt[14] - (234*
I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(
-34*I + Sqrt[14])]*EllipticPi[(7*(34 - I*Sqrt[14]))/234, I*ArcSinh[(3*Sqrt
[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[
14])])/Sqrt[7 + x]))/(11466*Sqrt[(-I)/(34*I + Sqrt[14])]*Sqrt[3 + 2*x + 5*
x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1282, 2154, 27, 1269, 1172, 321, 327, 1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{ 1282}$$

$$-\frac{1}{42}\int \frac{17-5x^2}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\int -\frac{5x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{42}\left(5\left(\int \frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-7\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}$$

$$\Big\downarrow \ 1172$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\int \frac{}{\sqrt{\ldots}}}{\ldots}\right.\right.$$

$$\Big\downarrow \ 321$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int \frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\,\text{Ellip}\ldots}{\ldots}\right.\right.$$

$$\Big\downarrow \ 327$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{1\ldots}}}}{\ldots}\right.\right.$$

$$\Big\downarrow \text{1279}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{\sqrt{x+7}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{14}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right.\right.}{\sqrt{x+7}}\right.\right.$$

$$\Big\downarrow \text{187}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int -\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}}+5\left(\frac{2i\sqrt{x+7}E\left(\text{arcs}\right.}{}\right.\right.$$

$$\Big\downarrow \text{413}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{5(x+7)+i\sqrt{14}-34}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}+5\left(\frac{2i\sqrt{x+\cdots}}{}\right.\right.$$

$$\Big\downarrow \text{413}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+$$

$$\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -\frac{1}{x\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}}d\sqrt{x+7}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}+5\right.$$

$$\Big\downarrow 412$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+$$

$$\frac{1}{42}\left(\frac{34\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\text{EllipticPi}\left(\frac{1}{35}(34-i\sqrt{14}\right.}{7\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}\right.$$

input
```
Int[1/(x^2*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/21*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + (5*((((2*I)/5)*Sqrt[7 + x]*E
llipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*S
qrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((14*I)*Sqr
t[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] +
5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x]) +
(34*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*Sqrt[1
4] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x))/(34
 + I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqrt[7 +
x])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7*Sqrt[
3 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqrt[14]
+ 5*(7 + x)]))/42
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.81

method	result
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(-\frac{\sqrt{5x^3+37x^2+17x+21}}{21x}+\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\left(\left(-\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\mathrm{EllipticE}\left(\sqrt{\cdots}\cdots\right)\cdots\right)}{21\sqrt{5x^3+37x^2\cdots}}\cdots\right)}{\cdots}$
risch	$-\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}}{21x}+\frac{\left(\frac{17\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\frac{34}{35}-\frac{i\sqrt{14}}{35},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\cdots\right)}{147\sqrt{5x^3+37x^2+17x+21}}\cdots\right)}{\cdots}$
default	$\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(245i\sqrt{14}\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)x-17i\sqrt{14}\sqrt{\cdots}\cdots\right)}{\cdots}$

input
```
int(1/x^2/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2)*(-1/21/x*(5*x^
3+37*x^2+17*x+21)^(1/2)+5/21*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(
1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5
+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2
)*((-34/5-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((
-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))
*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-3
4/5-1/5*I*14^(1/2)))^(1/2)))+17/147*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5
*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*
((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+2
1)^(1/2)*EllipticPi(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),34/35-1/35*I*14^(1
/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^2}\,dx$$

input
```
integrate(1/x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^5 + 37*x^4 + 17*x^3 + 21*x
^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(1/x**2/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^2}\,dx$$

input
```
integrate(1/x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^2}\,dx$$

input
```
integrate(1/x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(1/(x^2*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(1/(x^2*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^2\sqrt{x+7}\,\sqrt{5x^2+2x+3}}dx$$

input
```
int(1/x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int(1/x^2/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

3.116 $\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 496

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx =& -\frac{\sqrt{7+x}\sqrt{3+2x+5x^2}}{42x^2}+\frac{17\sqrt{7+x}\sqrt{3+2x+5x^2}}{588x} \\
&-\frac{17\sqrt{\frac{65}{2}}\sqrt{7+x}\sqrt{3+2x+5x^2}}{294\left(78+\sqrt{130}(7+x)\right)}+\frac{83}{392}\sqrt{\frac{3}{7}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{7}}\sqrt{7+x}}{\sqrt{3+2x+5x^2}}\right) \\
&+\frac{17\sqrt[4]{65}\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)E\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right)\middle|\frac{1}{390}\left(195+17\sqrt{130}\right)\right)}{98\, 2^{3/4}\sqrt{3}\sqrt{3+2x+5x^2}} \\
&-\frac{\sqrt[4]{5}\left(6071-532\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\sqrt{7+x}}{\sqrt{3}}\right),\frac{1}{390}(\cdots}{1078\sqrt{3}26^{3/4}\sqrt{3+2x+5x^2}} \\
&-\frac{83\sqrt{3}\left(479-42\sqrt{130}\right)\sqrt{\frac{3+2x+5x^2}{\left(78+\sqrt{130}(7+x)\right)^2}}\left(78+\sqrt{130}(7+x)\right)\operatorname{EllipticPi}\left(\frac{5460+479\sqrt{130}}{10920},2\arctan\left(\frac{\sqrt[4]{\frac{5}{26}}\cdots}{\cdots}\right.\right.}{60368\sqrt[4]{130}\sqrt{3+2x+5x^2}}
\end{aligned}
$$

output
```
-1/42*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/x^2+17/588*(7+x)^(1/2)*(5*x^2+2*x+3)
^(1/2)/x-17/2*130^(1/2)*(7+x)^(1/2)*(5*x^2+2*x+3)^(1/2)/(22932+294*130^(1/
2)*(7+x))+83/2744*21^(1/2)*arctanh(1/7*21^(1/2)*(7+x)^(1/2)/(5*x^2+2*x+3)^
(1/2))+17/588*65^(1/4)*((5*x^2+2*x+3)/(78+130^(1/2)*(7+x))^2)^(1/2)*(78+13
0^(1/2)*(7+x))*EllipticE(sin(2*arctan(1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^
(1/2))),1/390*(76050+6630*130^(1/2))^(1/2))*2^(1/4)*3^(1/2)/(5*x^2+2*x+3)^
(1/2)-1/84084*5^(1/4)*(6071-532*130^(1/2))*((5*x^2+2*x+3)/(78+130^(1/2)*(7
+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*InverseJacobiAM(2*arctan(1/78*5^(1/4)*2
6^(3/4)*(7+x)^(1/2)*3^(1/2)),1/390*(76050+6630*130^(1/2))^(1/2))*3^(1/2)*2
6^(1/4)/(5*x^2+2*x+3)^(1/2)-83/7847840*(479-42*130^(1/2))*((5*x^2+2*x+3)/(
78+130^(1/2)*(7+x))^2)^(1/2)*(78+130^(1/2)*(7+x))*EllipticPi(sin(2*arctan(
1/78*5^(1/4)*26^(3/4)*(7+x)^(1/2)*3^(1/2))),1/2+479/10920*130^(1/2),1/390*
(76050+6630*130^(1/2))^(1/2))*3^(1/2)*130^(3/4)/(5*x^2+2*x+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.53 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \frac{\sqrt{7+x}(-14+17x)\sqrt{3+2x+5x^2}}{588x^2}$$

$$+ \frac{(7+x)^{3/2}\left(-\frac{9282\sqrt{-\frac{i}{34i+\sqrt{14}}}(3+2x+5x^2)}{(7+x)^2} + \frac{238i\sqrt{13}\left(17\sqrt{2}+i\sqrt{7}\right)\sqrt{\frac{34i+\sqrt{14}-\frac{234i}{7+x}}{34i+\sqrt{14}}}\sqrt{\frac{-34i+\sqrt{14}+\frac{234i}{7+x}}{-34i+\sqrt{14}}}E\left(i\text{arcsinh}\left(\frac{3\sqrt{-\frac{2}{34i+}}}{\sqrt{7+}}\right.\right.}{\sqrt{7+x}}\right.}{}$$

input
```
Integrate[1/(x^3*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(Sqrt[7 + x]*(-14 + 17*x)*Sqrt[3 + 2*x + 5*x^2])/(588*x^2) + ((7 + x)^(3/2
)*((-9282*Sqrt[(-I)/(34*I + Sqrt[14])]*(3 + 2*x + 5*x^2))/(7 + x)^2 + ((23
8*I)*Sqrt[13]*(17*Sqrt[2] + I*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7
+ x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I
+ Sqrt[14])]*EllipticE[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[
7 + x]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] + (2*Sqrt[13]*(
(188*I)*Sqrt[2] + 119*Sqrt[7])*Sqrt[(34*I + Sqrt[14] - (234*I)/(7 + x))/(3
4*I + Sqrt[14])]*Sqrt[(-34*I + Sqrt[14] + (234*I)/(7 + x))/(-34*I + Sqrt[1
4])]*EllipticF[I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]],
 (34*I + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x] - ((747*I)*Sqrt[26]*Sqr
t[(34*I + Sqrt[14] - (234*I)/(7 + x))/(34*I + Sqrt[14])]*Sqrt[(-34*I + Sqr
t[14] + (234*I)/(7 + x))/(-34*I + Sqrt[14])]*EllipticPi[(7*(34 - I*Sqrt[14
]))/234, I*ArcSinh[(3*Sqrt[(-26*I)/(34*I + Sqrt[14])])/Sqrt[7 + x]], (34*I
 + Sqrt[14])/(34*I - Sqrt[14])])/Sqrt[7 + x]))/(321048*Sqrt[(-I)/(34*I + S
qrt[14])]*Sqrt[3 + 2*x + 5*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.59, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {1282, 2154, 1282, 2154, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 27, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x+7}\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1282}$$

$$-\frac{1}{84}\int \frac{5x^2+74x+51}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}$$

$$\Big\downarrow \text{2154}$$

$$\frac{1}{84}\left(-51\int \frac{1}{x^2\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\int \frac{5x+74}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}$$

$$\Big\downarrow \; 1282$$

$$\frac{1}{84}\left(-\int \frac{5x+74}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(-\frac{1}{42}\int \frac{17-5x^2}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}\right)\right)-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}$$

$$\Big\downarrow \; 2154$$

$$\frac{1}{84}\left(-\int \frac{5}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(\frac{1}{42}\left(-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+\right.\right.$$

$$\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{84}\left(-5\int \frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right.\right.\right.$$

$$\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}$$

$$\Big\downarrow \; 1172$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$$\Big\downarrow \; 321$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-74\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(\frac{1}{42}\left(5\int \frac{x}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-17\int \frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$$\Big\downarrow \; 1269$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-74\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(\frac{1}{42}\left(5\left(\int\frac{\sqrt{x+7}}{\sqrt{5x^2+2x+3}}dx-7\int\frac{1}{\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right)-1\right.\right.\right.$$

$\downarrow$ 1172

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-74\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx\right.\right.\right.$$

$\downarrow$ 321

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}\int\frac{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{34i+\sqrt{14}}+1}}{\sqrt{\frac{i\left(5x+i\sqrt{14}+1\right)}{2\sqrt{14}}+1}}d}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}\right.\right.\right.\right.$$

$\downarrow$ 327

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(-17\int\frac{1}{x\sqrt{x+7}\sqrt{5x^2+2x+3}}dx+5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i(5}}{2^3}\right.\right.}{5\sqrt{\frac{x+}{34-i}}}\right.\right.\right.\right.$$

$\downarrow$ 1279

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$
$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{}}}}{}\right.\right.\right.\right.$$

$\downarrow$ 27

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$
$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(5\left(\frac{2i\sqrt{x+7}E\left(\arcsin\left(\frac{\sqrt{-i\left(5x+i\sqrt{14}+1\right)}}{2^{3/4}\sqrt[4]{7}}\right)|\frac{2\sqrt{14}}{34i+\sqrt{14}}\right)}{5\sqrt{\frac{x+7}{34-i\sqrt{14}}}}-\frac{14i\sqrt{\frac{x+7}{34-i\sqrt{}}}}{}\right.\right.\right.\right.$$

$\downarrow$ 187

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$
$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\int-\frac{1}{x\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{}}}}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$\downarrow$ 413

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$
$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int-\frac{}{x\sqrt{5(x+7)+i\sqrt{14}}}}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}}\right.\right.\right.$$

$\downarrow$ 413

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(-51\left(-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{21x}+\frac{1}{42}\left(\frac{34\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\int -}{\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)}}\right.\right.\right.$$

$$\Big\downarrow \color{blue}{412}$$

$$-\frac{\sqrt{x+7}\sqrt{5x^2+2x+3}}{42x^2}+$$

$$\frac{1}{84}\left(\frac{148\sqrt{\frac{1}{5}\left(34-i\sqrt{14}\right)}\sqrt{5x-i\sqrt{14}+1}\sqrt{5x+i\sqrt{14}+1}\sqrt{1-\frac{5(x+7)}{34-i\sqrt{14}}}\sqrt{1-\frac{5(x+7)}{34+i\sqrt{14}}}\text{EllipticPi}\left(\frac{1}{35}\left(34-i\sqrt{1}\right.\right.}{7\sqrt{5x^2+2x+3}\sqrt{5(x+7)-i\sqrt{14}-34}\sqrt{5(x+7)+i\sqrt{14}-34}}\right.$$

input
```
Int[1/(x^3*Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/42*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x^2 + (((-10*I)*Sqrt[(7 + x)/(34
 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4
)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 + x] + (148*Sqrt[(34
- I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I*Sqrt[14] + 5*x]*Sqr
t[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 + x))/(34 + I*Sqrt[14]
)]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sqrt[7 + x])/Sqrt[34 -
 I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(7*Sqrt[3 + 2*x + 5*x
^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sqrt[14] + 5*(7 + x)])
 - 51*(-1/21*(Sqrt[7 + x]*Sqrt[3 + 2*x + 5*x^2])/x + (5*((((2*I)/5)*Sqrt[7
 + x]*EllipticE[ArcSin[Sqrt[(-I)*(1 + I*Sqrt[14] + 5*x)]/(2^(3/4)*7^(1/4))
], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[(7 + x)/(34 - I*Sqrt[14])] - ((14
*I)*Sqrt[(7 + x)/(34 - I*Sqrt[14])]*EllipticF[ArcSin[Sqrt[(-I)*(1 + I*Sqrt
[14] + 5*x)]/(2^(3/4)*7^(1/4))], (2*Sqrt[14])/(34*I + Sqrt[14])])/Sqrt[7 +
 x]) + (34*Sqrt[(34 - I*Sqrt[14])/5]*Sqrt[1 - I*Sqrt[14] + 5*x]*Sqrt[1 + I
*Sqrt[14] + 5*x]*Sqrt[1 - (5*(7 + x))/(34 - I*Sqrt[14])]*Sqrt[1 - (5*(7 +
x))/(34 + I*Sqrt[14])]*EllipticPi[(34 - I*Sqrt[14])/35, ArcSin[(Sqrt[5]*Sq
rt[7 + x])/Sqrt[34 - I*Sqrt[14]]], (34*I + Sqrt[14])/(34*I - Sqrt[14])])/(
7*Sqrt[3 + 2*x + 5*x^2]*Sqrt[-34 - I*Sqrt[14] + 5*(7 + x)]*Sqrt[-34 + I*Sq
rt[14] + 5*(7 + x)]))/42))/84
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.02

method	result
risch	$\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\,(-14+17x)}{588x^2}+\frac{\left(-\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}},\sqrt{\frac{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\cdots\right.}{42\sqrt{5x^3+37x^2+17x+21}}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(x+7)(5x^2+2x+3)}\left(-\frac{\sqrt{5x^3+37x^2+17x+21}}{42x^2}+\frac{17\sqrt{5x^3+37x^2+17x+21}}{588x}-\frac{5\left(\frac{34}{5}-\frac{i\sqrt{14}}{5}\right)\sqrt{\frac{x+7}{\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}-\frac{i\sqrt{14}}{5}}{-\frac{34}{5}-\frac{i\sqrt{14}}{5}}}\,\sqrt{\frac{x+\frac{1}{5}+\frac{i\sqrt{14}}{5}}{-\frac{34}{5}+\frac{i\sqrt{14}}{5}}}\,\mathrm{Ellip}\cdots}{42\sqrt{5x^3+37x^2+17x+21}}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{x+7}\,\sqrt{5x^2+2x+3}\left(3675i\sqrt{14}\,\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}}\,\sqrt{\frac{i\sqrt{14}-5x-1}{i\sqrt{14}+34}}\,\sqrt{\frac{i\sqrt{14}+5x+1}{-34+i\sqrt{14}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{5(x+7)}{-34+i\sqrt{14}}},\sqrt{-\frac{-34+i\sqrt{14}}{i\sqrt{14}+34}}\right)x^2+747i\cdots\right.}{\cdots}$

input
```
int(1/x^3/(x+7)^(1/2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/588*(x+7)^(1/2)*(5*x^2+2*x+3)^(1/2)*(-14+17*x)/x^2+(-5/42*(34/5-1/5*I*14
^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5
-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1
/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(
1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+249/1372*(34/5
-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5-1/5*I*14^(1/2
))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5*I*14^(1/2))/(-34/5+1/5*I*14^(
1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*EllipticPi(((x+7)/(34/5-1/5*I*14
^(1/2)))^(1/2),34/35-1/35*I*14^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-1/5*I*
14^(1/2)))^(1/2))-85/588*(34/5-1/5*I*14^(1/2))*((x+7)/(34/5-1/5*I*14^(1/2)
))^(1/2)*((x+1/5-1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2)*((x+1/5+1/5
*I*14^(1/2))/(-34/5+1/5*I*14^(1/2)))^(1/2)/(5*x^3+37*x^2+17*x+21)^(1/2)*((
-34/5-1/5*I*14^(1/2))*EllipticE(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/
5+1/5*I*14^(1/2))/(-34/5-1/5*I*14^(1/2)))^(1/2))+(-1/5+1/5*I*14^(1/2))*Ell
ipticF(((x+7)/(34/5-1/5*I*14^(1/2)))^(1/2),((-34/5+1/5*I*14^(1/2))/(-34/5-
1/5*I*14^(1/2)))^(1/2))))*((x+7)*(5*x^2+2*x+3))^(1/2)/(x+7)^(1/2)/(5*x^2+2
*x+3)^(1/2)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^3}\,dx$$

input
```
integrate(1/x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)/(5*x^6 + 37*x^5 + 17*x^4 + 21*x
^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^3\sqrt{x+7}\sqrt{5x^2+2x+3}}\,dx$$

input
```
integrate(1/x**3/(7+x)**(1/2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x + 7)*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^3}\,dx$$

input
```
integrate(1/x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x+3}\sqrt{x+7}x^3}\,dx$$

input
```
integrate(1/x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 2*x + 3)*sqrt(x + 7)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x+7}\,\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(1/(x^3*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)),x)
```

output
```
int(1/(x^3*(x + 7)^(1/2)*(2*x + 5*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{7+x}\sqrt{3+2x+5x^2}}\,dx = \int \frac{1}{x^3\sqrt{x+7}\,\sqrt{5x^2+2x+3}}dx$$

input
```
int(1/x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int(1/x^3/(7+x)^(1/2)/(5*x^2+2*x+3)^(1/2),x)
```

3.117 $\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx = &-\frac{d^3(cd^2-bde+ae^2)\,(d+ex)^{1+m}}{e^6(1+m)}\\&+\frac{d^2(5cd^2-e(4bd-3ae))\,(d+ex)^{2+m}}{e^6(2+m)}\\&-\frac{d(10cd^2-3e(2bd-ae))\,(d+ex)^{3+m}}{e^6(3+m)}\\&+\frac{(10cd^2-e(4bd-ae))\,(d+ex)^{4+m}}{e^6(4+m)}\\&-\frac{(5cd-be)(d+ex)^{5+m}}{e^6(5+m)}+\frac{c(d+ex)^{6+m}}{e^6(6+m)}\end{aligned}$$

output

```
-d^3*(a*e^2-b*d*e+c*d^2)*(e*x+d)^(1+m)/e^6/(1+m)+d^2*(5*c*d^2-e*(-3*a*e+4*
b*d))*(e*x+d)^(2+m)/e^6/(2+m)-d*(10*c*d^2-3*e*(-a*e+2*b*d))*(e*x+d)^(3+m)/
e^6/(3+m)+(10*c*d^2-e*(-a*e+4*b*d))*(e*x+d)^(4+m)/e^6/(4+m)-(-b*e+5*c*d)*(
e*x+d)^(5+m)/e^6/(5+m)+c*(e*x+d)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$=\frac{(d+ex)^{1+m}\left(-\frac{d^3(cd^2+e(-bd+ae))}{1+m}+\frac{d^2(5cd^2+e(-4bd+3ae))(d+ex)}{2+m}-\frac{d(10cd^2+3e(-2bd+ae))(d+ex)^2}{3+m}+\frac{(10cd^2+e(-4bd+ae)}{4+m}\right.}{e^6}$$

input
```
Integrate[x^3*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(-((d^3*(c*d^2 + e*(-(b*d) + a*e)))/(1 + m)) + (d^2*(5*
c*d^2 + e*(-4*b*d + 3*a*e))*(d + e*x))/(2 + m) - (d*(10*c*d^2 + 3*e*(-2*b*
d + a*e))*(d + e*x)^2)/(3 + m) + ((10*c*d^2 + e*(-4*b*d + a*e))*(d + e*x)^
3)/(4 + m) - ((5*c*d - b*e)*(d + e*x)^4)/(5 + m) + (c*(d + e*x)^5)/(6 + m)
))/e^6
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx+cx^2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int\left(\frac{d^2(d+ex)^{m+1}\left(5cd^2-e(4bd-3ae)\right)}{e^5}+\frac{d(d+ex)^{m+2}\left(3e(2bd-ae)-10cd^2\right)}{e^5}+\frac{(d+ex)^{m+3}\left(10cd^2-e(4\right.}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(d+ex)^{m+2}\left(5cd^2-e(4bd-3ae)\right)}{e^6(m+2)}-\frac{d(d+ex)^{m+3}\left(10cd^2-3e(2bd-ae)\right)}{e^6(m+3)}+\frac{(d+ex)^{m+4}\left(10cd^2-e(4bd-ae)\right)}{e^6(m+4)}-\frac{d^3(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^6(m+1)}-\frac{(5cd-be)(d+ex)^{m+5}}{e^6(m+5)}+\frac{c(d+ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[x^3*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
-((d^3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^6*(1 + m))) + (d^2*(5
*c*d^2 - e*(4*b*d - 3*a*e))*(d + e*x)^(2 + m))/(e^6*(2 + m)) - (d*(10*c*d^
2 - 3*e*(2*b*d - a*e))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((10*c*d^2 - e*(
4*b*d - a*e))*(d + e*x)^(4 + m))/(e^6*(4 + m)) - ((5*c*d - b*e)*(d + e*x)^
(5 + m))/(e^6*(5 + m)) + (c*(d + e*x)^(6 + m))/(e^6*(6 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(202) = 404$.

Time = 0.76 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.32

method	result
norman	$\frac{c\,x^{6} {\mathrm e}^{m \ln \left(e x +d \right)}}{6+m}+\frac{\left(b e m +c d m +6 b e \right) x^{5} {\mathrm e}^{m \ln \left(e x +d \right)}}{e \left(m^{2}+11 m +30\right)}+\frac{\left(a\,e^{2} m^{2}+b d e\,m^{2}+11 a\,e^{2} m +6 b d e m -5 c\,d^{2} m +30 a\,e^{2}\right) x^{4} {\mathrm e}^{m \ln \left(e x +d \right)}}{e^{2} \left(m^{3}+15 m^{2}+74 m +120\right)}$
gosper	$-\frac{\left(e x +d \right)^{1+m} \left(-c\,e^{5} m^{5} x^{5}-b\,e^{5} m^{5} x^{4}-15 c\,e^{5} m^{4} x^{5}-a\,e^{5} m^{5} x^{3}-16 b\,e^{5} m^{4} x^{4}+5 c d\,e^{4} m^{4} x^{4}-85 c\,e^{5} m^{3} x^{5}-17 a\,e^{5} m^{4} x^{3}+4 b d\,e\right.}{}$
orering	$-\frac{\left(e x +d \right)^{m} \left(-c\,e^{5} m^{5} x^{5}-b\,e^{5} m^{5} x^{4}-15 c\,e^{5} m^{4} x^{5}-a\,e^{5} m^{5} x^{3}-16 b\,e^{5} m^{4} x^{4}+5 c d\,e^{4} m^{4} x^{4}-85 c\,e^{5} m^{3} x^{5}-17 a\,e^{5} m^{4} x^{3}+4 b d\,e^{4}\right.}{}$
risch	$-\frac{\left(-c\,e^{6} m^{5} x^{6}-b\,e^{6} m^{5} x^{5}-c d\,e^{5} m^{5} x^{5}-15 c\,e^{6} m^{4} x^{6}-a\,e^{6} m^{5} x^{4}-b d\,e^{5} m^{5} x^{4}-16 b\,e^{6} m^{4} x^{5}-10 c d\,e^{5} m^{4} x^{5}-85 c\,e^{6} m^{3} x^{6}-a d\,e\right.}{}$
parallelrisch	Expression too large to display

input

```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output

```
c/(6+m)*x^6*exp(m*ln(e*x+d))+(b*e*m+c*d*m+6*b*e)/e/(m^2+11*m+30)*x^5*exp(m
*ln(e*x+d))+(a*e^2*m^2+b*d*e*m^2+11*a*e^2*m+6*b*d*e*m-5*c*d^2*m+30*a*e^2)/
e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+m*d*(a*e^2*m^2+11*a*e^2*m-4
*b*d*e*m+30*a*e^2-24*b*d*e+20*c*d^2)/e^3/(m^4+18*m^3+119*m^2+342*m+360)*x^
3*exp(m*ln(e*x+d))-6*d^4*(a*e^2*m^2+11*a*e^2*m-4*b*d*e*m+30*a*e^2-24*b*d*e
+20*c*d^2)/e^6/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*exp(m*ln(e
*x+d))+6/e^5*m*d^3*(a*e^2*m^2+11*a*e^2*m-4*b*d*e*m+30*a*e^2-24*b*d*e+20*c*
d^2)/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*x*exp(m*ln(e*x+d))-3
*(a*e^2*m^2+11*a*e^2*m-4*b*d*e*m+30*a*e^2-24*b*d*e+20*c*d^2)*d^2/e^4*m/(m^
5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(202) = 404$.

Time = 0.09 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.51

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx=\text{Too large to display}$$

input

```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-(6*a*d^4*e^2*m^2 + 120*c*d^6 - 144*b*d^5*e + 180*a*d^4*e^2 - (c*e^6*m^5 +
 15*c*e^6*m^4 + 85*c*e^6*m^3 + 225*c*e^6*m^2 + 274*c*e^6*m + 120*c*e^6)*x^
6 - (144*b*e^6 + (c*d*e^5 + b*e^6)*m^5 + 2*(5*c*d*e^5 + 8*b*e^6)*m^4 + 5*(
7*c*d*e^5 + 19*b*e^6)*m^3 + 10*(5*c*d*e^5 + 26*b*e^6)*m^2 + 12*(2*c*d*e^5
+ 27*b*e^6)*m)*x^5 - (180*a*e^6 + (b*d*e^5 + a*e^6)*m^5 - (5*c*d^2*e^4 - 1
2*b*d*e^5 - 17*a*e^6)*m^4 - (30*c*d^2*e^4 - 47*b*d*e^5 - 107*a*e^6)*m^3 -
(55*c*d^2*e^4 - 72*b*d*e^5 - 307*a*e^6)*m^2 - 6*(5*c*d^2*e^4 - 6*b*d*e^5 -
 66*a*e^6)*m)*x^4 - (a*d*e^5*m^5 - 2*(2*b*d^2*e^4 - 7*a*d*e^5)*m^4 + (20*c
*d^3*e^3 - 36*b*d^2*e^4 + 65*a*d*e^5)*m^3 + 4*(15*c*d^3*e^3 - 20*b*d^2*e^4
 + 28*a*d*e^5)*m^2 + 4*(10*c*d^3*e^3 - 12*b*d^2*e^4 + 15*a*d*e^5)*m)*x^3 +
 3*(a*d^2*e^4*m^4 - 4*(b*d^3*e^3 - 3*a*d^2*e^4)*m^3 + (20*c*d^4*e^2 - 28*b
*d^3*e^3 + 41*a*d^2*e^4)*m^2 + 2*(10*c*d^4*e^2 - 12*b*d^3*e^3 + 15*a*d^2*e
^4)*m)*x^2 - 6*(4*b*d^5*e - 11*a*d^4*e^2)*m - 6*(a*d^3*e^3*m^3 - (4*b*d^4*
e^2 - 11*a*d^3*e^3)*m^2 + 2*(10*c*d^5*e - 12*b*d^4*e^2 + 15*a*d^3*e^3)*m)*
x)*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 735*e^6*m^3 + 1624*e^
6*m^2 + 1764*e^6*m + 720*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10479 vs. $2(178) = 356$.

Time = 2.47 (sec) , antiderivative size = 10479, normalized size of antiderivative = 51.88

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)**m*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*x**4/4 + b*x**5/5 + c*x**6/6), Eq(e, 0)), (-3*a*d**3*e*
*2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x*
*3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 15*a*d**2*e**3*x/(60*d**5*e**6 +
300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x*
*4 + 60*e**11*x**5) - 30*a*d*e**4*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 6
00*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5)
 - 30*a*e**5*x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 6
00*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*b*d**4*e/(60*d*
*5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*
d*e**10*x**4 + 60*e**11*x**5) - 60*b*d**3*e**2*x/(60*d**5*e**6 + 300*d**4*
e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) - 120*b*d**2*e**3*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d*
*3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12
0*b*d*e**4*x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600
*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*b*e**5*x**4/(60*d
**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300
*d*e**10*x**4 + 60*e**11*x**5) + 60*c*d**5*log(d/e + x)/(60*d**5*e**6 + 30
0*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4
 + 60*e**11*x**5) + 137*c*d**5/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*
e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 30...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(202) = 404$.

Time = 0.05 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.11

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$= \frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex+d}{(m^4+10m^3+35m^2+50m+24)e^4}$$
$$+\frac{((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4(m^3+3m^2+2m)d}{(m^5+15m^4+85m^3+225m^2+274m+120}$$
$$+\frac{((m^5+15m^4+85m^3+225m^2+274m+120)e^6x^6+(m^5+10m^4+35m^3+50m^2+24m)de^5x^5-}{(m^6+21m^5+175}$$

input
```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a/((m^4 + 10*m^3 + 35
*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m
^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 +
 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((m^5 + 15*m^4 + 85*m^3 + 2
25*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*
e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2
*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x
 + d)^m*c/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^
6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(202) = 404$.

Time = 0.24 (sec) , antiderivative size = 1304, normalized size of antiderivative = 6.46

$$\int x^3(d+ex)^m \left(a+bx+cx^2\right) dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^6*m^5*x^6 + (e*x + d)^m*c*d*e^5*m^5*x^5 + (e*x + d)^m*b*e
^6*m^5*x^5 + 15*(e*x + d)^m*c*e^6*m^4*x^6 + (e*x + d)^m*b*d*e^5*m^5*x^4 +
(e*x + d)^m*a*e^6*m^5*x^4 + 10*(e*x + d)^m*c*d*e^5*m^4*x^5 + 16*(e*x + d)^
m*b*e^6*m^4*x^5 + 85*(e*x + d)^m*c*e^6*m^3*x^6 + (e*x + d)^m*a*d*e^5*m^5*x
^3 - 5*(e*x + d)^m*c*d^2*e^4*m^4*x^4 + 12*(e*x + d)^m*b*d*e^5*m^4*x^4 + 17
*(e*x + d)^m*a*e^6*m^4*x^4 + 35*(e*x + d)^m*c*d*e^5*m^3*x^5 + 95*(e*x + d)
^m*b*e^6*m^3*x^5 + 225*(e*x + d)^m*c*e^6*m^2*x^6 - 4*(e*x + d)^m*b*d^2*e^4
*m^4*x^3 + 14*(e*x + d)^m*a*d*e^5*m^4*x^3 - 30*(e*x + d)^m*c*d^2*e^4*m^3*x
^4 + 47*(e*x + d)^m*b*d*e^5*m^3*x^4 + 107*(e*x + d)^m*a*e^6*m^3*x^4 + 50*(
e*x + d)^m*c*d*e^5*m^2*x^5 + 260*(e*x + d)^m*b*e^6*m^2*x^5 + 274*(e*x + d)
^m*c*e^6*m*x^6 - 3*(e*x + d)^m*a*d^2*e^4*m^4*x^2 + 20*(e*x + d)^m*c*d^3*e^
3*m^3*x^3 - 36*(e*x + d)^m*b*d^2*e^4*m^3*x^3 + 65*(e*x + d)^m*a*d*e^5*m^3*
x^3 - 55*(e*x + d)^m*c*d^2*e^4*m^2*x^4 + 72*(e*x + d)^m*b*d*e^5*m^2*x^4 +
307*(e*x + d)^m*a*e^6*m^2*x^4 + 24*(e*x + d)^m*c*d*e^5*m*x^5 + 324*(e*x +
d)^m*b*e^6*m*x^5 + 120*(e*x + d)^m*c*e^6*x^6 + 12*(e*x + d)^m*b*d^3*e^3*m^
3*x^2 - 36*(e*x + d)^m*a*d^2*e^4*m^3*x^2 + 60*(e*x + d)^m*c*d^3*e^3*m^2*x^
3 - 80*(e*x + d)^m*b*d^2*e^4*m^2*x^3 + 112*(e*x + d)^m*a*d*e^5*m^2*x^3 - 3
0*(e*x + d)^m*c*d^2*e^4*m*x^4 + 36*(e*x + d)^m*b*d*e^5*m*x^4 + 396*(e*x +
d)^m*a*e^6*m*x^4 + 144*(e*x + d)^m*b*e^6*x^5 + 6*(e*x + d)^m*a*d^3*e^3*m^3
*x - 60*(e*x + d)^m*c*d^4*e^2*m^2*x^2 + 84*(e*x + d)^m*b*d^3*e^3*m^2*x^...
```

Mupad [B] (verification not implemented)

Time = 10.99 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.74

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$= (d+e\,x)^m\left(\frac{c\,x^6\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}\right.$$
$$-\frac{6\,d^4\,(20\,c\,d^2-4\,b\,d\,e\,m-24\,b\,d\,e+a\,e^2\,m^2+11\,a\,e^2\,m+30\,a\,e^2)}{e^6\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}$$
$$+\frac{x^4\,(m^3+6\,m^2+11\,m+6)\,(-5\,c\,d^2\,m+b\,d\,e\,m^2+6\,b\,d\,e\,m+a\,e^2\,m^2+11\,a\,e^2\,m+30\,a\,e^2)}{e^2\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}$$
$$+\frac{x^5\,(6\,b\,e+b\,e\,m+c\,d\,m)\,(m^4+10\,m^3+35\,m^2+50\,m+24)}{e\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}$$
$$+\frac{6\,d^3\,m\,x\,(20\,c\,d^2-4\,b\,d\,e\,m-24\,b\,d\,e+a\,e^2\,m^2+11\,a\,e^2\,m+30\,a\,e^2)}{e^5\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}$$
$$-\frac{3\,d^2\,m\,x^2\,(m+1)\,(20\,c\,d^2-4\,b\,d\,e\,m-24\,b\,d\,e+a\,e^2\,m^2+11\,a\,e^2\,m+30\,a\,e^2)}{e^4\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}$$
$$\left.+\frac{d\,m\,x^3\,(m^2+3\,m+2)\,(20\,c\,d^2-4\,b\,d\,e\,m-24\,b\,d\,e+a\,e^2\,m^2+11\,a\,e^2\,m+30\,a\,e^2)}{e^3\,(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)}\right)$$

input
```
int(x^3*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^m*((c*x^6*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764
*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) - (6*d^4*(30*a*e^2
 + 20*c*d^2 + a*e^2*m^2 - 24*b*d*e + 11*a*e^2*m - 4*b*d*e*m))/(e^6*(1764*m
 + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x^4*(11*m + 6*m^
2 + m^3 + 6)*(30*a*e^2 + a*e^2*m^2 + 11*a*e^2*m - 5*c*d^2*m + b*d*e*m^2 +
6*b*d*e*m))/(e^2*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 7
20)) + (x^5*(6*b*e + b*e*m + c*d*m)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(
e*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (6*d^3*m
*x*(30*a*e^2 + 20*c*d^2 + a*e^2*m^2 - 24*b*d*e + 11*a*e^2*m - 4*b*d*e*m))/
(e^5*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) - (3*d^
2*m*x^2*(m + 1)*(30*a*e^2 + 20*c*d^2 + a*e^2*m^2 - 24*b*d*e + 11*a*e^2*m -
 4*b*d*e*m))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 +
720)) + (d*m*x^3*(3*m + m^2 + 2)*(30*a*e^2 + 20*c*d^2 + a*e^2*m^2 - 24*b*d
*e + 11*a*e^2*m - 4*b*d*e*m))/(e^3*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4
+ 21*m^5 + m^6 + 720)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.14

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$=\frac{(ex+d)^m\left(c\,e^6m^5x^6+b\,e^6m^5x^5+cd\,e^5m^5x^5+15c\,e^6m^4x^6+a\,e^6m^5x^4+bd\,e^5m^5x^4+16b\,e^6m^4x^5+10c\right.}{}$$

input
```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a),x)
```

output
```
((d + e*x)**m*( - 6*a*d**4*e**2*m**2 - 66*a*d**4*e**2*m - 180*a*d**4*e**2
+ 6*a*d**3*e**3*m**3*x + 66*a*d**3*e**3*m**2*x + 180*a*d**3*e**3*m*x - 3*a
*d**2*e**4*m**4*x**2 - 36*a*d**2*e**4*m**3*x**2 - 123*a*d**2*e**4*m**2*x**
2 - 90*a*d**2*e**4*m*x**2 + a*d*e**5*m**5*x**3 + 14*a*d*e**5*m**4*x**3 + 6
5*a*d*e**5*m**3*x**3 + 112*a*d*e**5*m**2*x**3 + 60*a*d*e**5*m*x**3 + a*e**
6*m**5*x**4 + 17*a*e**6*m**4*x**4 + 107*a*e**6*m**3*x**4 + 307*a*e**6*m**2
*x**4 + 396*a*e**6*m*x**4 + 180*a*e**6*x**4 + 24*b*d**5*e*m + 144*b*d**5*e
 - 24*b*d**4*e**2*m**2*x - 144*b*d**4*e**2*m*x + 12*b*d**3*e**3*m**3*x**2
+ 84*b*d**3*e**3*m**2*x**2 + 72*b*d**3*e**3*m*x**2 - 4*b*d**2*e**4*m**4*x*
*3 - 36*b*d**2*e**4*m**3*x**3 - 80*b*d**2*e**4*m**2*x**3 - 48*b*d**2*e**4*
m*x**3 + b*d*e**5*m**5*x**4 + 12*b*d*e**5*m**4*x**4 + 47*b*d*e**5*m**3*x**
4 + 72*b*d*e**5*m**2*x**4 + 36*b*d*e**5*m*x**4 + b*e**6*m**5*x**5 + 16*b*e
**6*m**4*x**5 + 95*b*e**6*m**3*x**5 + 260*b*e**6*m**2*x**5 + 324*b*e**6*m*
x**5 + 144*b*e**6*x**5 - 120*c*d**6 + 120*c*d**5*e*m*x - 60*c*d**4*e**2*m*
*2*x**2 - 60*c*d**4*e**2*m*x**2 + 20*c*d**3*e**3*m**3*x**3 + 60*c*d**3*e**
3*m**2*x**3 + 40*c*d**3*e**3*m*x**3 - 5*c*d**2*e**4*m**4*x**4 - 30*c*d**2*
e**4*m**3*x**4 - 55*c*d**2*e**4*m**2*x**4 - 30*c*d**2*e**4*m*x**4 + c*d*e*
*5*m**5*x**5 + 10*c*d*e**5*m**4*x**5 + 35*c*d*e**5*m**3*x**5 + 50*c*d*e**5
*m**2*x**5 + 24*c*d*e**5*m*x**5 + c*e**6*m**5*x**6 + 15*c*e**6*m**4*x**6 +
 85*c*e**6*m**3*x**6 + 225*c*e**6*m**2*x**6 + 274*c*e**6*m*x**6 + 120*c...
```

3.118 $\int x^2(d+ex)^m \left(a+bx+cx^2\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int x^2(d+ex)^m \left(a+bx+cx^2\right) \, dx = \frac{d^2(cd^2-bde+ae^2)\,(d+ex)^{1+m}}{e^5(1+m)} - \frac{d(4cd^2-e(3bd-2ae))\,(d+ex)^{2+m}}{e^5(2+m)} + \frac{(6cd^2-e(3bd-ae))\,(d+ex)^{3+m}}{e^5(3+m)} - \frac{(4cd-be)(d+ex)^{4+m}}{e^5(4+m)} + \frac{c(d+ex)^{5+m}}{e^5(5+m)}$$

output
```
d^2*(a*e^2-b*d*e+c*d^2)*(e*x+d)^(1+m)/e^5/(1+m)-d*(4*c*d^2-e*(-2*a*e+3*b*d
))*(e*x+d)^(2+m)/e^5/(2+m)+(6*c*d^2-e*(-a*e+3*b*d))*(e*x+d)^(3+m)/e^5/(3+m
)-(-b*e+4*c*d)*(e*x+d)^(4+m)/e^5/(4+m)+c*(e*x+d)^(5+m)/e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx = \frac{(d+ex)^{1+m}\left(\frac{d^2(cd^2+e(-bd+ae))}{1+m}-\frac{d(4cd^2+e(-3bd+2ae))(d+ex)}{2+m}+\frac{(6cd^2+e(-3bd+ae))(d+ex)^2}{3+m}-\frac{(4cd-be)(d+ex)^3}{4+m}+\frac{c(d+e\ldots}{5+\ldots}\right.}{e^5}$$

input
```
Integrate[x^2*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*((d^2*(c*d^2 + e*(-(b*d) + a*e)))/(1 + m) - (d*(4*c*d^2
 + e*(-3*b*d + 2*a*e))*(d + e*x))/(2 + m) + ((6*c*d^2 + e*(-3*b*d + a*e))*
(d + e*x)^2)/(3 + m) - ((4*c*d - b*e)*(d + e*x)^3)/(4 + m) + (c*(d + e*x)^
4)/(5 + m)))/e^5
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx+cx^2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{d(d+ex)^{m+1}\left(e(3bd-2ae)-4cd^2\right)}{e^4}+\frac{(d+ex)^{m+2}\left(6cd^2-e(3bd-ae)\right)}{e^4}+\frac{d^2(d+ex)^m\left(ae^2-bde+cd^2\right)}{e^4}\ldots\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(d+ex)^{m+2}\left(4cd^2-e(3bd-2ae)\right)}{e^5(m+2)}+\frac{(d+ex)^{m+3}\left(6cd^2-e(3bd-ae)\right)}{e^5(m+3)}+\frac{d^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(4cd-be)(d+ex)^{m+4}}{e^5(m+4)}+\frac{c(d+ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[x^2*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
(d^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (d*(4*c*d^
2 - e*(3*b*d - 2*a*e))*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c*d^2 - e*(3
*b*d - a*e))*(d + e*x)^(3 + m))/(e^5*(3 + m)) - ((4*c*d - b*e)*(d + e*x)^(
4 + m))/(e^5*(4 + m)) + (c*(d + e*x)^(5 + m))/(e^5*(5 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(161) = 322$.

Time = 0.70 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.32

method	result
norman	$\frac{c\,x^5 \mathrm{e}^{m \ln(ex+d)}}{5+m}+\frac{(bem+cdm+5be)x^4 \mathrm{e}^{m \ln(ex+d)}}{e(m^2+9m+20)}+\frac{(a\,e^2m^2+bde\,m^2+9a\,e^2m+5bdem-4c\,d^2m+20a\,e^2)x^3 \mathrm{e}^{m \ln(ex+d)}}{e^2(m^3+12m^2+47m+60)}$
gosper	$(ex+d)^{1+m}(c\,e^4m^4x^4+b\,e^4m^4x^3+10c\,e^4m^3x^4+a\,e^4m^4x^2+11b\,e^4m^3x^3-4cd\,e^3m^3x^3+35c\,e^4m^2x^4+12a\,e^4m^3x^2-3bd\,e^3m^3$
orering	$(ex+d)(c\,e^4m^4x^4+b\,e^4m^4x^3+10c\,e^4m^3x^4+a\,e^4m^4x^2+11b\,e^4m^3x^3-4cd\,e^3m^3x^3+35c\,e^4m^2x^4+12a\,e^4m^3x^2-3bd\,e^3m^3x^2+$
risch	$(c\,e^5m^4x^5+b\,e^5m^4x^4+cd\,e^4m^4x^4+10c\,e^5m^3x^5+a\,e^5m^4x^3+bd\,e^4m^4x^3+11b\,e^5m^3x^4+6cd\,e^4m^3x^4+35c\,e^5m^2x^5+ad\,e^4m^4x$
parallelrisch	$29x^2(ex+d)^m ad\,e^4m^2-18x^2(ex+d)^m b\,d^2e^3m^2+12x^2(ex+d)^m c\,d^3e^2m^2-2x(ex+d)^m a\,d^2e^3m^3+20x^2(ex+d)^m ad\,e^4m-15$

input
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
c/(5+m)*x^5*exp(m*ln(e*x+d))+(b*e*m+c*d*m+5*b*e)/e/(m^2+9*m+20)*x^4*exp(m*
ln(e*x+d))+(a*e^2*m^2+b*d*e*m^2+9*a*e^2*m+5*b*d*e*m-4*c*d^2*m+20*a*e^2)/e^
2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+(a*e^2*m^2+9*a*e^2*m-3*b*d*e*m
+20*a*e^2-15*b*d*e+12*c*d^2)*d/e^3*m/(m^4+14*m^3+71*m^2+154*m+120)*x^2*exp
(m*ln(e*x+d))+2*d^3*(a*e^2*m^2+9*a*e^2*m-3*b*d*e*m+20*a*e^2-15*b*d*e+12*c*
d^2)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)*exp(m*ln(e*x+d))-2/e^4*m*d^
2*(a*e^2*m^2+9*a*e^2*m-3*b*d*e*m+20*a*e^2-15*b*d*e+12*c*d^2)/(m^5+15*m^4+8
5*m^3+225*m^2+274*m+120)*x*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(161) = 322$.

Time = 0.08 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.09

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx = \frac{(2\,ad^3e^2m^2+24\,cd^5-30\,bd^4e+40\,ad^3e^2+(ce^5m^4+10\,ce^5m^3+35\,ce^5m^2+50\,ce^5m+24\,ce^5)x^5+(30}{}$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
(2*a*d^3*e^2*m^2 + 24*c*d^5 - 30*b*d^4*e + 40*a*d^3*e^2 + (c*e^5*m^4 + 10*
c*e^5*m^3 + 35*c*e^5*m^2 + 50*c*e^5*m + 24*c*e^5)*x^5 + (30*b*e^5 + (c*d*e
^4 + b*e^5)*m^4 + (6*c*d*e^4 + 11*b*e^5)*m^3 + (11*c*d*e^4 + 41*b*e^5)*m^2
 + (6*c*d*e^4 + 61*b*e^5)*m)*x^4 + (40*a*e^5 + (b*d*e^4 + a*e^5)*m^4 - 4*(
c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*m^3 - (12*c*d^2*e^3 - 17*b*d*e^4 - 49*a*e
^5)*m^2 - 2*(4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*m)*x^3 + (a*d*e^4*m^4 - (
3*b*d^2*e^3 - 10*a*d*e^4)*m^3 + (12*c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)
*m^2 + (12*c*d^3*e^2 - 15*b*d^2*e^3 + 20*a*d*e^4)*m)*x^2 - 6*(b*d^4*e - 3*
a*d^3*e^2)*m - 2*(a*d^2*e^3*m^3 - 3*(b*d^3*e^2 - 3*a*d^2*e^3)*m^2 + (12*c*
d^4*e - 15*b*d^3*e^2 + 20*a*d^2*e^3)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m
^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6195 vs. $2(141) = 282$.

Time = 1.52 (sec) , antiderivative size = 6195, normalized size of antiderivative = 38.48

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)**m*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*x**3/3 + b*x**4/4 + c*x**5/5), Eq(e, 0)), (-a*d**2*e**2
/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*
e**9*x**4) - 4*a*d*e**3*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x*
*2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*a*e**4*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e
/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*
e**9*x**4) - 12*b*d**2*e**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**
7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 18*b*d*e**3*x**2/(12*d**4*e**5 +
 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*
b*e**4*x**3/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) + 12*c*d**4*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**
6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4/(12*d
**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) + 48*c*d**3*e*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2
*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*x/(12*d**4*e**5
+ 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 72
*c*d**2*e**2*x**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e*
*7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 108*c*d**2*e**2*x**2/(12*d**4*e
**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
+ 48*c*d*e**3*x**3*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.94

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$=\frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m a}{(m^3+6m^2+11m+6)e^3}$$
$$+\frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex -}{(m^4+10m^3+35m^2+50m+24)e^4}$$
$$+\frac{((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4(m^3+3m^2+2m)d}{(m^5+15m^4+85m^3+225m^2+274m+12}$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
 + d)^m*a/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^4*x
^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x
 - 6*d^4)*(e*x + d)^m*b/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4
+ 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^
4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*
d^4*e*m*x + 24*d^5)*(e*x + d)^m*c/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*
m + 120)*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. 2(161) = 322.

Time = 0.21 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.72

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^5*m^4*x^5 + (e*x + d)^m*c*d*e^4*m^4*x^4 + (e*x + d)^m*b*e
^5*m^4*x^4 + 10*(e*x + d)^m*c*e^5*m^3*x^5 + (e*x + d)^m*b*d*e^4*m^4*x^3 +
(e*x + d)^m*a*e^5*m^4*x^3 + 6*(e*x + d)^m*c*d*e^4*m^3*x^4 + 11*(e*x + d)^m
*b*e^5*m^3*x^4 + 35*(e*x + d)^m*c*e^5*m^2*x^5 + (e*x + d)^m*a*d*e^4*m^4*x^
2 - 4*(e*x + d)^m*c*d^2*e^3*m^3*x^3 + 8*(e*x + d)^m*b*d*e^4*m^3*x^3 + 12*(
e*x + d)^m*a*e^5*m^3*x^3 + 11*(e*x + d)^m*c*d*e^4*m^2*x^4 + 41*(e*x + d)^m
*b*e^5*m^2*x^4 + 50*(e*x + d)^m*c*e^5*m*x^5 - 3*(e*x + d)^m*b*d^2*e^3*m^3*
x^2 + 10*(e*x + d)^m*a*d*e^4*m^3*x^2 - 12*(e*x + d)^m*c*d^2*e^3*m^2*x^3 +
17*(e*x + d)^m*b*d*e^4*m^2*x^3 + 49*(e*x + d)^m*a*e^5*m^2*x^3 + 6*(e*x + d
)^m*c*d*e^4*m*x^4 + 61*(e*x + d)^m*b*e^5*m*x^4 + 24*(e*x + d)^m*c*e^5*x^5
- 2*(e*x + d)^m*a*d^2*e^3*m^3*x + 12*(e*x + d)^m*c*d^3*e^2*m^2*x^2 - 18*(e
*x + d)^m*b*d^2*e^3*m^2*x^2 + 29*(e*x + d)^m*a*d*e^4*m^2*x^2 - 8*(e*x + d)
^m*c*d^2*e^3*m*x^3 + 10*(e*x + d)^m*b*d*e^4*m*x^3 + 78*(e*x + d)^m*a*e^5*m
*x^3 + 30*(e*x + d)^m*b*e^5*x^4 + 6*(e*x + d)^m*b*d^3*e^2*m^2*x - 18*(e*x
+ d)^m*a*d^2*e^3*m^2*x + 12*(e*x + d)^m*c*d^3*e^2*m*x^2 - 15*(e*x + d)^m*b
*d^2*e^3*m*x^2 + 20*(e*x + d)^m*a*d*e^4*m*x^2 + 40*(e*x + d)^m*a*e^5*x^3 +
 2*(e*x + d)^m*a*d^3*e^2*m^2 - 24*(e*x + d)^m*c*d^4*e*m*x + 30*(e*x + d)^m
*b*d^3*e^2*m*x - 40*(e*x + d)^m*a*d^2*e^3*m*x - 6*(e*x + d)^m*b*d^4*e*m +
18*(e*x + d)^m*a*d^3*e^2*m + 24*(e*x + d)^m*c*d^5 - 30*(e*x + d)^m*b*d^4*e
 + 40*(e*x + d)^m*a*d^3*e^2)/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e...
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.60

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx = (d+e\,x)^m\left(\frac{c\,x^5\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{2\,d^3\,\left(12\,c\,d^2-3\,b\,d\,e\,m-15\,b\,d\,e+a\,e^2\,m^2+9\,a\,e^2\,m+20\,a\,e^2\right)}{e^5\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{x^3\,\left(m^2+3\,m+2\right)\,\left(-4\,c\,d^2\,m+b\,d\,e\,m^2+5\,b\,d\,e\,m+a\,e^2\,m^2+9\,a\,e^2\,m+20\,a\,e^2\right)}{e^2\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{x^4\,\left(5\,b\,e+b\,e\,m+c\,d\,m\right)\,\left(m^3+6\,m^2+11\,m+6\right)}{e\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} - \frac{2\,d^2\,m\,x\,\left(12\,c\,d^2-3\,b\,d\,e\,m-15\,b\,d\,e+a\,e^2\,m^2+9\,a\,e^2\,m+20\,a\,e^2\right)}{e^4\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{d\,m\,x^2\,\left(m+1\right)\,\left(12\,c\,d^2-3\,b\,d\,e\,m-15\,b\,d\,e+a\,e^2\,m^2+9\,a\,e^2\,m+20\,a\,e^2\right)}{e^3\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)}\right)$$

input
```
int(x^2*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^m*((c*x^5*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2
+ 85*m^3 + 15*m^4 + m^5 + 120) + (2*d^3*(20*a*e^2 + 12*c*d^2 + a*e^2*m^2 -
 15*b*d*e + 9*a*e^2*m - 3*b*d*e*m))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^
4 + m^5 + 120)) + (x^3*(3*m + m^2 + 2)*(20*a*e^2 + a*e^2*m^2 + 9*a*e^2*m -
 4*c*d^2*m + b*d*e*m^2 + 5*b*d*e*m))/(e^2*(274*m + 225*m^2 + 85*m^3 + 15*m
^4 + m^5 + 120)) + (x^4*(5*b*e + b*e*m + c*d*m)*(11*m + 6*m^2 + m^3 + 6))/
(e*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) - (2*d^2*m*x*(20*a*e^2
 + 12*c*d^2 + a*e^2*m^2 - 15*b*d*e + 9*a*e^2*m - 3*b*d*e*m))/(e^4*(274*m +
 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (d*m*x^2*(m + 1)*(20*a*e^2 + 12
*c*d^2 + a*e^2*m^2 - 15*b*d*e + 9*a*e^2*m - 3*b*d*e*m))/(e^3*(274*m + 225*
m^2 + 85*m^3 + 15*m^4 + m^5 + 120)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 582, normalized size of antiderivative = 3.61

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(ex+d)^m\left(c\,e^5m^4x^5+b\,e^5m^4x^4+cd\,e^4m^4x^4+10c\,e^5m^3x^5+a\,e^5m^4x^3+bd\,e^4m^4x^3+11b\,e^5m^3x^4+6ca\right.}{}$$

input
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a),x)
```

output
```
((d + e*x)**m*(2*a*d**3*e**2*m**2 + 18*a*d**3*e**2*m + 40*a*d**3*e**2 - 2*
a*d**2*e**3*m**3*x - 18*a*d**2*e**3*m**2*x - 40*a*d**2*e**3*m*x + a*d*e**4
*m**4*x**2 + 10*a*d*e**4*m**3*x**2 + 29*a*d*e**4*m**2*x**2 + 20*a*d*e**4*m
*x**2 + a*e**5*m**4*x**3 + 12*a*e**5*m**3*x**3 + 49*a*e**5*m**2*x**3 + 78*
a*e**5*m*x**3 + 40*a*e**5*x**3 - 6*b*d**4*e*m - 30*b*d**4*e + 6*b*d**3*e**
2*m**2*x + 30*b*d**3*e**2*m*x - 3*b*d**2*e**3*m**3*x**2 - 18*b*d**2*e**3*m
**2*x**2 - 15*b*d**2*e**3*m*x**2 + b*d*e**4*m**4*x**3 + 8*b*d*e**4*m**3*x*
*3 + 17*b*d*e**4*m**2*x**3 + 10*b*d*e**4*m*x**3 + b*e**5*m**4*x**4 + 11*b*
e**5*m**3*x**4 + 41*b*e**5*m**2*x**4 + 61*b*e**5*m*x**4 + 30*b*e**5*x**4 +
 24*c*d**5 - 24*c*d**4*e*m*x + 12*c*d**3*e**2*m**2*x**2 + 12*c*d**3*e**2*m
*x**2 - 4*c*d**2*e**3*m**3*x**3 - 12*c*d**2*e**3*m**2*x**3 - 8*c*d**2*e**3
*m*x**3 + c*d*e**4*m**4*x**4 + 6*c*d*e**4*m**3*x**4 + 11*c*d*e**4*m**2*x**
4 + 6*c*d*e**4*m*x**4 + c*e**5*m**4*x**5 + 10*c*e**5*m**3*x**5 + 35*c*e**5
*m**2*x**5 + 50*c*e**5*m*x**5 + 24*c*e**5*x**5))/(e**5*(m**5 + 15*m**4 + 8
5*m**3 + 225*m**2 + 274*m + 120))
```

3.119 $\int x(d+ex)^m \left(a+bx+cx^2\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int x(d+ex)^m \left(a+bx+cx^2\right) \, dx = -\frac{d(cd^2-bde+ae^2)\,(d+ex)^{1+m}}{e^4(1+m)} + \frac{(3cd^2-e(2bd-ae))\,(d+ex)^{2+m}}{e^4(2+m)} - \frac{(3cd-be)(d+ex)^{3+m}}{e^4(3+m)} + \frac{c(d+ex)^{4+m}}{e^4(4+m)}$$

output
```
-d*(a*e^2-b*d*e+c*d^2)*(e*x+d)^(1+m)/e^4/(1+m)+(3*c*d^2-e*(-a*e+2*b*d))*(e
*x+d)^(2+m)/e^4/(2+m)-(-b*e+3*c*d)*(e*x+d)^(3+m)/e^4/(3+m)+c*(e*x+d)^(4+m)
/e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.17

$$\int x(d+ex)^m \left(a+bx+cx^2\right) \, dx = \frac{(d+ex)^{1+m}\left(c(-6d^3+6d^2e(1+m)x-3de^2(2+3m+m^2)\,x^2+e^3(6+11m+6m^2+m^3)\,x^3)+e(4+\right.}{e^4(1+m)(2+m)(3+m)(4}$$

input
```
Integrate[x*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(c*(-6*d^3 + 6*d^2*e*(1 + m)*x - 3*d*e^2*(2 + 3*m + m^2
)*x^2 + e^3*(6 + 11*m + 6*m^2 + m^3)*x^3) + e*(4 + m)*(a*e*(3 + m)*(-d + e
*(1 + m)*x) + b*(2*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2))))/(e^
4*(1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx+cx^2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(d+ex)^{m+1}\left(3cd^2-e(2bd-ae)\right)}{e^3}-\frac{d(d+ex)^m\left(ae^2-bde+cd^2\right)}{e^3}+\frac{(be-3cd)(d+ex)^{m+2}}{e^3}+\frac{c(d+ex)^m}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+2}\left(3cd^2-e(2bd-ae)\right)}{e^4(m+2)}-\frac{d(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(3cd-be)(d+ex)^{m+3}}{e^4(m+3)}+\frac{c(d+ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[x*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
-((d*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^4*(1 + m))) + ((3*c*d^2
 - e*(2*b*d - a*e))*(d + e*x)^(2 + m))/(e^4*(2 + m)) - ((3*c*d - b*e)*(d +
 e*x)^(3 + m))/(e^4*(3 + m)) + (c*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(121) = 242$.

Time = 0.74 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.32

method	result
gosper	$-\frac{(ex+d)^{1+m}(-c\,e^3m^3x^3-b\,e^3m^3x^2-6c\,e^3m^2x^3-a\,e^3m^3x-7b\,e^3m^2x^2+3cd\,e^2m^2x^2-11c\,e^3m\,x^3-8a\,e^3m^2x+2bd\,e^2m^2x}{}$
norman	$\frac{c\,x^4\mathrm{e}^{m\ln(ex+d)}}{4+m}+\frac{(bem+cdm+4be)x^3\mathrm{e}^{m\ln(ex+d)}}{e(m^2+7m+12)}+\frac{(a\,e^2m^2+bde\,m^2+7a\,e^2m+4bdem-3c\,d^2m+12a\,e^2)x^2\mathrm{e}^{m\ln(ex+d)}}{e^2(m^3+9m^2+26m+24)}$
orering	$-\frac{(ex+d)^m(-c\,e^3m^3x^3-b\,e^3m^3x^2-6c\,e^3m^2x^3-a\,e^3m^3x-7b\,e^3m^2x^2+3cd\,e^2m^2x^2-11c\,e^3m\,x^3-8a\,e^3m^2x+2bd\,e^2m^2x-1}{}$
risch	$-\frac{(-c\,e^4m^3x^4-b\,e^4m^3x^3-cd\,e^3m^3x^3-6c\,e^4m^2x^4-a\,e^4m^3x^2-bd\,e^3m^3x^2-7b\,e^4m^2x^3-3cd\,e^3m^2x^3-11c\,e^4m\,x^4-ad\,e^3m^3}{}$
parallelrisch	$\frac{8x^2(ex+d)^m ad\,e^4m^2+5x^2(ex+d)^m b\,d^2e^3m^2-3x^2(ex+d)^m c\,d^3e^2m^2+x(ex+d)^m a\,d^2e^3m^3+19x^2(ex+d)^m ad\,e^4m+4x^2(ex}{}$

input
```
int(x*(e*x+d)^m*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/e^4*(e*x+d)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-c*e^3*m^3*x^3-b*e^3*m^3
*x^2-6*c*e^3*m^2*x^3-a*e^3*m^3*x-7*b*e^3*m^2*x^2+3*c*d*e^2*m^2*x^2-11*c*e^
3*m*x^3-8*a*e^3*m^2*x+2*b*d*e^2*m^2*x-14*b*e^3*m*x^2+9*c*d*e^2*m*x^2-6*c*e
^3*x^3+a*d*e^2*m^2-19*a*e^3*m*x+10*b*d*e^2*m*x-8*b*e^3*x^2-6*c*d^2*e*m*x+6
*c*d*e^2*x^2+7*a*d*e^2*m-12*a*e^3*x-2*b*d^2*e*m+8*b*d*e^2*x-6*c*d^3*e*x+12
*a*d*e^2-8*b*d^2*e+6*c*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(121) = 242$.

Time = 0.08 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.83

$$\int x(d+ex)^m \left(a+bx+cx^2\right) dx = -\frac{(ad^2e^2m^2 + 6\,cd^4 - 8\,bd^3e + 12\,ad^2e^2 - (ce^4m^3 + 6\,ce^4m^2 + 11\,ce^4m + 6\,ce^4)x^4 - (8\,be^4 + (cde^3 + be^4}{}$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-(a*d^2*e^2*m^2 + 6*c*d^4 - 8*b*d^3*e + 12*a*d^2*e^2 - (c*e^4*m^3 + 6*c*e^
4*m^2 + 11*c*e^4*m + 6*c*e^4)*x^4 - (8*b*e^4 + (c*d*e^3 + b*e^4)*m^3 + (3*
c*d*e^3 + 7*b*e^4)*m^2 + 2*(c*d*e^3 + 7*b*e^4)*m)*x^3 - (12*a*e^4 + (b*d*e
^3 + a*e^4)*m^3 - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*m^2 - (3*c*d^2*e^2 -
 4*b*d*e^3 - 19*a*e^4)*m)*x^2 - (2*b*d^3*e - 7*a*d^2*e^2)*m - (a*d*e^3*m^3
 - (2*b*d^2*e^2 - 7*a*d*e^3)*m^2 + 2*(3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)
*m)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3267 vs. $2(104) = 208$.

Time = 0.98 (sec) , antiderivative size = 3267, normalized size of antiderivative = 27.00

$$\int x(d+ex)^m \left(a+bx+cx^2\right) dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)**m*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*x**2/2 + b*x**3/3 + c*x**4/4), Eq(e, 0)), (-a*d*e**2/(6
*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*x/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e/
(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2
*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**
3*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c
*d**3*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7
*x**3) + 11*c*d**3/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7
*x**3) + 18*c*d**2*e*x*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e
**6*x**2 + 6*e**7*x**3) + 27*c*d**2*e*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*x**2*log(d/e + x)/(6*d**3*e**4 +
 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*x**2/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**3*x**3*log
(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), E
q(m, -4)), (-a*d*e**2/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*
x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*b*d**2*e*log(d/e + x)/(2*d*
*2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 3*b*d**2*e/(2*d**2*e**4 + 4*d*e**5*x
 + 2*e**6*x**2) + 4*b*d*e**2*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*
e**6*x**2) + 4*b*d*e**2*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*b*e
**3*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.78

$$\int x(d+ex)^m\left(a+bx+cx^2\right)\,dx = \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m a}{(m^2+3\,m+2)e^2} + \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m b}{(m^3+6\,m^2+11\,m+6)e^3} + \frac{((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex-}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4}$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a/((m^2 + 3*m + 2)*e^2) + ((
m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x +
 d)^m*b/((m^3 + 6*m^2 + 11*m + 6)*e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4
 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x -
 6*d^4)*(e*x + d)^m*c/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(121) = 242.

Time = 0.23 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.99

$$\int x(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$= \frac{(ex+d)^m ce^4m^3x^4 + (ex+d)^m cde^3m^3x^3 + (ex+d)^m be^4m^3x^3 + 6\,(ex+d)^m ce^4m^2x^4 + (ex+d)^m bde^3n}{}$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^4*m^3*x^4 + (e*x + d)^m*c*d*e^3*m^3*x^3 + (e*x + d)^m*b*e
^4*m^3*x^3 + 6*(e*x + d)^m*c*e^4*m^2*x^4 + (e*x + d)^m*b*d*e^3*m^3*x^2 + (
e*x + d)^m*a*e^4*m^3*x^2 + 3*(e*x + d)^m*c*d*e^3*m^2*x^3 + 7*(e*x + d)^m*b
*e^4*m^2*x^3 + 11*(e*x + d)^m*c*e^4*m*x^4 + (e*x + d)^m*a*d*e^3*m^3*x - 3*
(e*x + d)^m*c*d^2*e^2*m^2*x^2 + 5*(e*x + d)^m*b*d*e^3*m^2*x^2 + 8*(e*x + d
)^m*a*e^4*m^2*x^2 + 2*(e*x + d)^m*c*d*e^3*m*x^3 + 14*(e*x + d)^m*b*e^4*m*x
^3 + 6*(e*x + d)^m*c*e^4*x^4 - 2*(e*x + d)^m*b*d^2*e^2*m^2*x + 7*(e*x + d)
^m*a*d*e^3*m^2*x - 3*(e*x + d)^m*c*d^2*e^2*m*x^2 + 4*(e*x + d)^m*b*d*e^3*m
*x^2 + 19*(e*x + d)^m*a*e^4*m*x^2 + 8*(e*x + d)^m*b*e^4*x^3 - (e*x + d)^m*
a*d^2*e^2*m^2 + 6*(e*x + d)^m*c*d^3*e*m*x - 8*(e*x + d)^m*b*d^2*e^2*m*x +
12*(e*x + d)^m*a*d*e^3*m*x + 12*(e*x + d)^m*a*e^4*x^2 + 2*(e*x + d)^m*b*d^
3*e*m - 7*(e*x + d)^m*a*d^2*e^2*m - 6*(e*x + d)^m*c*d^4 + 8*(e*x + d)^m*b*
d^3*e - 12*(e*x + d)^m*a*d^2*e^2)/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*
e^4*m + 24*e^4)
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.48

$$\int x(d+ex)^m \left(a+bx+cx^2\right)\,dx = (d+e\,x)^m\left(\frac{c\,x^4\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24} - \frac{d^2\,(6\,c\,d^2-2\,b\,d\,e\,m-8\,b\,d\,e+a\,e^2\,m^2+7\,a\,e^2\,m+12\,a\,e^2)}{e^4\,(m^4+10\,m^3+35\,m^2+50\,m+24)} + \frac{x^3\,(4\,b\,e+b\,e\,m+c\,d\,m)\,(m^2+3\,m+2)}{e\,(m^4+10\,m^3+35\,m^2+50\,m+24)} + \frac{x^2\,(m+1)\,(-3\,c\,d^2\,m+b\,d\,e\,m^2+4\,b\,d\,e\,m+a\,e^2\,m^2+7\,a\,e^2\,m+12\,a\,e^2)}{e^2\,(m^4+10\,m^3+35\,m^2+50\,m+24)} + \frac{d\,m\,x\,(6\,c\,d^2-2\,b\,d\,e\,m-8\,b\,d\,e+a\,e^2\,m^2+7\,a\,e^2\,m+12\,a\,e^2)}{e^3\,(m^4+10\,m^3+35\,m^2+50\,m+24)}\right)$$

input
```
int(x*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^m*((c*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) - (d^2*(12*a*e^2 + 6*c*d^2 + a*e^2*m^2 - 8*b*d*e + 7*a*e^2*m - 2*b
*d*e*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(4*b*e + b*e*m +
 c*d*m)*(3*m + m^2 + 2))/(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m
 + 1)*(12*a*e^2 + a*e^2*m^2 + 7*a*e^2*m - 3*c*d^2*m + b*d*e*m^2 + 4*b*d*e*
m))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (d*m*x*(12*a*e^2 + 6*c*d^2
 + a*e^2*m^2 - 8*b*d*e + 7*a*e^2*m - 2*b*d*e*m))/(e^3*(50*m + 35*m^2 + 10*
m^3 + m^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.08

$$\int x(d+ex)^m \left(a+bx+cx^2\right)\,dx = \frac{(ex+d)^m\,(c\,e^4m^3x^4+b\,e^4m^3x^3+cd\,e^3m^3x^3+6c\,e^4m^2x^4+a\,e^4m^3x^2+bd\,e^3m^3x^2+7b\,e^4m^2x^3+3cd\,e^3}{}$$

input
```
int(x*(e*x+d)^m*(c*x^2+b*x+a),x)
```

output

```
((d + e*x)**m*( - a*d**2*e**2*m**2 - 7*a*d**2*e**2*m - 12*a*d**2*e**2 + a*
d*e**3*m**3*x + 7*a*d*e**3*m**2*x + 12*a*d*e**3*m*x + a*e**4*m**3*x**2 + 8
*a*e**4*m**2*x**2 + 19*a*e**4*m*x**2 + 12*a*e**4*x**2 + 2*b*d**3*e*m + 8*b
*d**3*e - 2*b*d**2*e**2*m**2*x - 8*b*d**2*e**2*m*x + b*d*e**3*m**3*x**2 +
5*b*d*e**3*m**2*x**2 + 4*b*d*e**3*m*x**2 + b*e**4*m**3*x**3 + 7*b*e**4*m**
2*x**3 + 14*b*e**4*m*x**3 + 8*b*e**4*x**3 - 6*c*d**4 + 6*c*d**3*e*m*x - 3*
c*d**2*e**2*m**2*x**2 - 3*c*d**2*e**2*m*x**2 + c*d*e**3*m**3*x**3 + 3*c*d*
e**3*m**2*x**3 + 2*c*d*e**3*m*x**3 + c*e**4*m**3*x**4 + 6*c*e**4*m**2*x**4
 + 11*c*e**4*m*x**4 + 6*c*e**4*x**4))/(e**4*(m**4 + 10*m**3 + 35*m**2 + 50
*m + 24))
```

3.120 $\int (d+ex)^m (a+bx+cx^2) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int (d+ex)^m (a+bx+cx^2) \, dx = \frac{(cd^2-bde+ae^2)(d+ex)^{1+m}}{e^3(1+m)} - \frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)} + \frac{c(d+ex)^{3+m}}{e^3(3+m)}$$

output
```
(a*e^2-b*d*e+c*d^2)*(e*x+d)^(1+m)/e^3/(1+m)-(-b*e+2*c*d)*(e*x+d)^(2+m)/e^3
/(2+m)+c*(e*x+d)^(3+m)/e^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int (d+ex)^m (a+bx+cx^2) \, dx = \frac{(cd^2-e(bd-ae))(d+ex)^{1+m}}{e^3(1+m)} - \frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)} + \frac{c(d+ex)^{3+m}}{e^3(3+m)}$$

input
```
Integrate[(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
((c*d^2 - e*(b*d - a*e))*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)
*(d + e*x)^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)(d+ex)^m \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^m \left(ae^2-bde+cd^2\right)}{e^2}+\frac{(be-2cd)(d+ex)^{m+1}}{e^2}+\frac{c(d+ex)^{m+2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+1} \left(ae^2-bde+cd^2\right)}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

input
```
Int[(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)
*(d + e*x)^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

method	result
gosper	$\frac{(ex+d)^{1+m}(c\,e^2m^2x^2+b\,e^2m^2x+3c\,e^2m\,x^2+a\,e^2m^2+4b\,e^2mx-2cdemx+2x^2c\,e^2+5a\,e^2m-bdem+3xb\,e^2-2cdxe+6a\,e^2-3}{e^3(m^3+6m^2+11m+6)}$
orering	$\frac{(ex+d)(c\,e^2m^2x^2+b\,e^2m^2x+3c\,e^2m\,x^2+a\,e^2m^2+4b\,e^2mx-2cdemx+2x^2c\,e^2+5a\,e^2m-bdem+3xb\,e^2-2cdxe+6a\,e^2-3bde+}{e^3(m^3+6m^2+11m+6)}$
norman	$\frac{c\,x^3\mathrm{e}^{m\ln(ex+d)}}{3+m}+\frac{d(a\,e^2m^2+5a\,e^2m-bdem+6a\,e^2-3bde+2c\,d^2)\mathrm{e}^{m\ln(ex+d)}}{e^3(m^3+6m^2+11m+6)}+\frac{(bem+cdm+3be)x^2\mathrm{e}^{m\ln(ex+d)}}{e(m^2+5m+6)}+\frac{(a\,e^2}{}$
risch	$\frac{(c\,e^3m^2x^3+b\,e^3m^2x^2+cd\,e^2m^2x^2+3c\,e^3m\,x^3+a\,e^3m^2x+bd\,e^2m^2x+4b\,e^3m\,x^2+cd\,e^2m\,x^2+2c\,e^3x^3+ad\,e^2m^2+5a\,e^3mx+3b}{(2+m)(3+m)(1+m)e^3}$
parallelrisch	$\frac{x^3(ex+d)^mc\,e^3m^2+3x^3(ex+d)^mc\,e^3m+x^2(ex+d)^mb\,e^3m^2+x^2(ex+d)^mcd\,e^2m^2+2x^3(ex+d)^mc\,e^3+4x^2(ex+d)^mb\,e^3m+x}{}$

input
```
int((e*x+d)^m*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(e*x+d)^(1+m)/(m^3+6*m^2+11*m+6)*(c*e^2*m^2*x^2+b*e^2*m^2*x+3*c*e^2*
m*x^2+a*e^2*m^2+4*b*e^2*m*x-2*c*d*e*m*x+2*c*e^2*x^2+5*a*e^2*m-b*d*e*m+3*b*
e^2*x-2*c*d*e*x+6*a*e^2-3*b*d*e+2*c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.45

$$\int (d+ex)^m \left(a+bx+cx^2\right)\, dx$$
$$= \frac{(ade^2m^2 + 2\,cd^3 - 3\,bd^2e + 6\,ade^2 + (ce^3m^2 + 3\,ce^3m + 2\,ce^3)x^3 + (3\,be^3 + (cde^2 + be^3)m^2 + (cde^2 + 4}{e^3m^3 + 6\,e^3m^2 +}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
(a*d*e^2*m^2 + 2*c*d^3 - 3*b*d^2*e + 6*a*d*e^2 + (c*e^3*m^2 + 3*c*e^3*m +
2*c*e^3)*x^3 + (3*b*e^3 + (c*d*e^2 + b*e^3)*m^2 + (c*d*e^2 + 4*b*e^3)*m)*x
^2 - (b*d^2*e - 5*a*d*e^2)*m + (6*a*e^3 + (b*d*e^2 + a*e^3)*m^2 - (2*c*d^2
*e - 3*b*d*e^2 - 5*a*e^3)*m)*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*
m + 6*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(70) = 140$.

Time = 0.61 (sec) , antiderivative size = 1416, normalized size of antiderivative = 17.27

$$\int (d+ex)^m \left(a+bx+cx^2\right)\, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a),x)
```

output
```
Piecewise((d**m*(a*x + b*x**2/2 + c*x**3/3), Eq(e, 0)), (-a*e**2/(2*d**2*e
**3 + 4*d*e**4*x + 2*e**5*x**2) - b*d*e/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5
*x**2) - 2*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 2*c*d**2*lo
g(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*c*d**2/(2*d**2*e**
3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x*log(d/e + x)/(2*d**2*e**3 + 4*d*
e**4*x + 2*e**5*x**2) + 4*c*d*e*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2)
 + 2*c*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2), Eq
(m, -3)), (-a*e**2/(d*e**3 + e**4*x) + b*d*e*log(d/e + x)/(d*e**3 + e**4*x
) + b*d*e/(d*e**3 + e**4*x) + b*e**2*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*
c*d**2*log(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2/(d*e**3 + e**4*x) - 2*c*d
*e*x*log(d/e + x)/(d*e**3 + e**4*x) + c*e**2*x**2/(d*e**3 + e**4*x), Eq(m,
 -2)), (a*log(d/e + x)/e - b*d*log(d/e + x)/e**2 + b*x/e + c*d**2*log(d/e
+ x)/e**3 - c*d*x/e**2 + c*x**2/(2*e), Eq(m, -1)), (a*d*e**2*m**2*(d + e*x
)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a*d*e**2*m*(d + e*
x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a*d*e**2*(d + e*x
)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + a*e**3*m**2*x*(d + e
*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 5*a*e**3*m*x*(d +
e*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 6*a*e**3*x*(d + e
*x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) - b*d**2*e*m*(d + e*
x)**m/(e**3*m**3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) - 3*b*d**2*e*(d + ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int (d+ex)^m\left(a+bx+cx^2\right)\,dx = \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m b}{(m^2+3\,m+2)e^2} + \frac{(ex+d)^{m+1}a}{e(m+1)} + \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m c}{(m^3+6\,m^2+11\,m+6)e^3}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b/((m^2 + 3*m + 2)*e^2) + (e
*x + d)^(m + 1)*a/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2
*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c/((m^3 + 6*m^2 + 11*m + 6)*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(82) = 164$.

Time = 0.24 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.27

$$\int (d+ex)^m (a+bx+cx^2)\, dx = \frac{(ex+d)^m ce^3m^2x^3 + (ex+d)^m cde^2m^2x^2 + (ex+d)^m be^3m^2x^2 + 3(ex+d)^m ce^3mx^3 + (ex+d)^m bde^2m}{}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^3*m^2*x^3 + (e*x + d)^m*c*d*e^2*m^2*x^2 + (e*x + d)^m*b*e
^3*m^2*x^2 + 3*(e*x + d)^m*c*e^3*m*x^3 + (e*x + d)^m*b*d*e^2*m^2*x + (e*x
+ d)^m*a*e^3*m^2*x + (e*x + d)^m*c*d*e^2*m*x^2 + 4*(e*x + d)^m*b*e^3*m*x^2
 + 2*(e*x + d)^m*c*e^3*x^3 + (e*x + d)^m*a*d*e^2*m^2 - 2*(e*x + d)^m*c*d^2
*e*m*x + 3*(e*x + d)^m*b*d*e^2*m*x + 5*(e*x + d)^m*a*e^3*m*x + 3*(e*x + d)
^m*b*e^3*x^2 - (e*x + d)^m*b*d^2*e*m + 5*(e*x + d)^m*a*d*e^2*m + 6*(e*x +
d)^m*a*e^3*x + 2*(e*x + d)^m*c*d^3 - 3*(e*x + d)^m*b*d^2*e + 6*(e*x + d)^m
*a*d*e^2)/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m + 6*e^3)
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.45

$$\int (d+ex)^m (a+bx+cx^2)\, dx = (d+e\,x)^m \left(\frac{c\,x^3\,(m^2+3\,m+2)}{m^3+6\,m^2+11\,m+6} + \frac{d\,(2\,c\,d^2-b\,d\,e\,m-3\,b\,d\,e+a\,e^2\,m^2+5\,a\,e^2\,m+6\,a\,e^2)}{e^3\,(m^3+6\,m^2+11\,m+6)} + \frac{x\,(-2\,c\,d^2\,e\,m+b\,d\,e^2\,m^2+3\,b\,d\,e^2\,m+a\,e^3\,m^2+5\,a\,e^3\,m+6\,a\,e^3)}{e^3\,(m^3+6\,m^2+11\,m+6)} + \frac{x^2\,(m+1)\,(3\,b\,e+b\,e\,m+c\,d\,m)}{e\,(m^3+6\,m^2+11\,m+6)}\right)$$

input
```
int((d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
(d + e*x)^m*((c*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (d*(6*a*e^
2 + 2*c*d^2 + a*e^2*m^2 - 3*b*d*e + 5*a*e^2*m - b*d*e*m))/(e^3*(11*m + 6*m
^2 + m^3 + 6)) + (x*(6*a*e^3 + a*e^3*m^2 + 5*a*e^3*m + 3*b*d*e^2*m - 2*c*d
^2*e*m + b*d*e^2*m^2))/(e^3*(11*m + 6*m^2 + m^3 + 6)) + (x^2*(m + 1)*(3*b*
e + b*e*m + c*d*m))/(e*(11*m + 6*m^2 + m^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.51

$$\int (d+ex)^m \left(a+bx+cx^2\right) dx$$
$$= \frac{(ex+d)^m \left(c e^3m^2x^3 + b e^3m^2x^2 + cd e^2m^2x^2 + 3c e^3m x^3 + a e^3m^2x + bd e^2m^2x + 4b e^3m x^2 + cd e^2m x\right.}{e^3 (}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a),x)
```

output
```
((d + e*x)**m*(a*d*e**2*m**2 + 5*a*d*e**2*m + 6*a*d*e**2 + a*e**3*m**2*x +
 5*a*e**3*m*x + 6*a*e**3*x - b*d**2*e*m - 3*b*d**2*e + b*d*e**2*m**2*x + 3
*b*d*e**2*m*x + b*e**3*m**2*x**2 + 4*b*e**3*m*x**2 + 3*b*e**3*x**2 + 2*c*d
**3 - 2*c*d**2*e*m*x + c*d*e**2*m**2*x**2 + c*d*e**2*m*x**2 + c*e**3*m**2*
x**3 + 3*c*e**3*m*x**3 + 2*c*e**3*x**3))/(e**3*(m**3 + 6*m**2 + 11*m + 6))
```

3.121 $\int \frac{(d+ex)^m(a+bx+cx^2)}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\begin{aligned}\int \frac{(d+ex)^m(a+bx+cx^2)}{x}\,dx &= -\frac{(cd-be)(d+ex)^{1+m}}{e^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2(2+m)}\\ &\quad -\frac{a(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{d(1+m)}\end{aligned}$$

output

```
-(-b*e+c*d)*(e*x+d)^(1+m)/e^2/(1+m)+c*(e*x+d)^(2+m)/e^2/(2+m)-a*(e*x+d)^(1
+m)*hypergeom([1, 1+m],[2+m],1+e*x/d)/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{x}\,dx = -\frac{(d+ex)^{1+m}\left(-bde(2+m)+cd(d-e(1+m)x)+ae^2(2+m)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\right.\right.}{de^2(1+m)(2+m)}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/x,x]
```

output
```
-(((d + e*x)^(1 + m)*(-(b*d*e*(2 + m)) + c*d*(d - e*(1 + m)*x) + a*e^2*(2
+ m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d]))/(d*e^2*(1 + m)*(2 +
 m)))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{x}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a(d+ex)^m}{x}+\frac{(be-cd)(d+ex)^m}{e}+\frac{c(d+ex)^{m+1}}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)}{d(m+1)}-\frac{(cd-be)(d+ex)^{m+1}}{e^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2(m+2)}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/x,x]
```

output
```
-(((c*d - b*e)*(d + e*x)^(1 + m))/(e^2*(1 + m))) + (c*(d + e*x)^(2 + m))/(
e^2*(2 + m)) - (a*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 +
 (e*x)/d])/(d*(1 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)}{x}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/x,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/x,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/x, x)
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.71

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx = -\frac{ae^{m+1}m\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d\Gamma\left(m+2\right)} - \frac{ae^{m+1}\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d\Gamma\left(m+2\right)} + b\left(\begin{cases} d^m x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(d+ex)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log\left(d+ex\right) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} \frac{d^m x^2}{2} & \text{for } e=0 \\ \frac{d\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} + \frac{d}{de^2+e^3x} + \frac{ex\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} & \text{for } m=-2 \\ -\frac{d\log\left(\frac{d}{e}+x\right)}{e^2} + \frac{x}{e} & \text{for } m=-1 \\ -\frac{d^2(d+ex)^m}{e^2m^2+3e^2m+2e^2} + \frac{demx(d+ex)^m}{e^2m^2+3e^2m+2e^2} + \frac{e^2mx^2(d+ex)^m}{e^2m^2+3e^2m+2e^2} + \frac{e^2x^2(d+ex)^m}{e^2m^2+3e^2m+2e^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/x,x)
```

output
```
-a*e**(m + 1)*m*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma(m +
 1)/(d*gamma(m + 2)) - a*e**(m + 1)*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d,
 1, m + 1)*gamma(m + 1)/(d*gamma(m + 2)) + b*Piecewise((d**m*x, Eq(e, 0)),
 (Piecewise(((d + e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(d + e*x), True))
/e, True)) + c*Piecewise((d**m*x**2/2, Eq(e, 0)), (d*log(d/e + x)/(d*e**2
+ e**3*x) + d/(d*e**2 + e**3*x) + e*x*log(d/e + x)/(d*e**2 + e**3*x), Eq(m
, -2)), (-d*log(d/e + x)/e**2 + x/e, Eq(m, -1)), (-d**2*(d + e*x)**m/(e**2
*m**2 + 3*e**2*m + 2*e**2) + d*e*m*x*(d + e*x)**m/(e**2*m**2 + 3*e**2*m +
2*e**2) + e**2*m*x**2*(d + e*x)**m/(e**2*m**2 + 3*e**2*m + 2*e**2) + e**2*
x**2*(d + e*x)**m/(e**2*m**2 + 3*e**2*m + 2*e**2), True))
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x,x, algorithm="maxima")
```

output
```
(e*x + d)^(m + 1)*b/(e*(m + 1)) + integrate((c*x^2 + a)*(e*x + d)^m/x, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{x}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x}\,dx$$
$$= \frac{(ex+d)^m a\,e^2m^2 + 3(ex+d)^m a\,e^2m + 2(ex+d)^m a\,e^2 + (ex+d)^m bde\,m^2 + 2(ex+d)^m bdem + (ex+$$

input

```
int((e*x+d)^m*(c*x^2+b*x+a)/x,x)
```

output

```
((d + e*x)**m*a*e**2*m**2 + 3*(d + e*x)**m*a*e**2*m + 2*(d + e*x)**m*a*e**
2 + (d + e*x)**m*b*d*e*m**2 + 2*(d + e*x)**m*b*d*e*m + (d + e*x)**m*b*e**2
*m**2*x + 2*(d + e*x)**m*b*e**2*m*x - (d + e*x)**m*c*d**2*m + (d + e*x)**m
*c*d*e*m**2*x + (d + e*x)**m*c*e**2*m**2*x**2 + (d + e*x)**m*c*e**2*m*x**2
 + int((d + e*x)**m/(d*x + e*x**2),x)*a*d*e**2*m**3 + 3*int((d + e*x)**m/(
d*x + e*x**2),x)*a*d*e**2*m**2 + 2*int((d + e*x)**m/(d*x + e*x**2),x)*a*d*
e**2*m)/(e**2*m*(m**2 + 3*m + 2))
```

3.122 $\int \frac{(d+ex)^m(a+bx+cx^2)}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{x^2}\,dx = \frac{c(d+ex)^{1+m}}{e(1+m)} - \frac{a(d+ex)^{1+m}}{dx} - \frac{(bd+aem)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{d^2(1+m)}$$

output
```
c*(e*x+d)^(1+m)/e/(1+m)-a*(e*x+d)^(1+m)/d/x-(a*e*m+b*d)*(e*x+d)^(1+m)*hype
rgeom([1, 1+m],[2+m],1+e*x/d)/d^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{x^2}\,dx = \frac{(d+ex)^{1+m}\left(cd^2 - bde\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right) + ae^2\,\text{Hypergeometric2F1}\left(2,1+\right.\right.}{d^2e(1+m)}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/x^2,x]
```

output
```
((d + e*x)^(1 + m)*(c*d^2 - b*d*e*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (
e*x)/d] + a*e^2*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + (e*x)/d]))/(d^2*e*(
1 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1193, 25, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{x^2}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{\int -\frac{(bd+cxd+aem)(d+ex)^m}{x}dx}{d}-\frac{a(d+ex)^{m+1}}{dx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bd+cxd+aem)(d+ex)^m}{x}dx}{d}-\frac{a(d+ex)^{m+1}}{dx}$$

$$\Big\downarrow \text{90}$$

$$\frac{(aem+bd)\int \frac{(d+ex)^m}{x}dx+\frac{cd(d+ex)^{m+1}}{e(m+1)}}{d}-\frac{a(d+ex)^{m+1}}{dx}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{cd(d+ex)^{m+1}}{e(m+1)}-\frac{(d+ex)^{m+1}(aem+bd)\text{ Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)}{d(m+1)}}{d}-\frac{a(d+ex)^{m+1}}{dx}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/x^2,x]
```

output
```
-((a*(d + e*x)^(1 + m))/(d*x)) + ((c*d*(d + e*x)^(1 + m))/(e*(1 + m)) - ((
b*d + a*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x
)/d])/(d*(1 + m)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

Maple [F]

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{x^2}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{x^2}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^2} dx = -\frac{ae^{m+2}m\left(\frac{d}{e}+x\right)^{m+1}\Gamma(m+1)}{dex\Gamma(m+2)} - \frac{ae^{m+2}\left(\frac{d}{e}+x\right)^{m+1}\Gamma(m+1)}{dex\Gamma(m+2)} - \frac{ae^{m+2}m^2\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d^2\Gamma(m+2)} - \frac{ae^{m+2}m\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d^2\Gamma(m+2)} - \frac{be^{m+1}m\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d\Gamma(m+2)} - \frac{be^{m+1}\left(\frac{d}{e}+x\right)^{m+1}\Phi\left(1+\frac{ex}{d},1,m+1\right)\Gamma(m+1)}{d\Gamma(m+2)} + c\left(\begin{cases} d^m x & \text{for } e=0 \\ \frac{\begin{cases} \frac{(d+ex)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(d+ex) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/x**2,x)
```

output
```
-a*e**(m + 2)*m*(d/e + x)**(m + 1)*gamma(m + 1)/(d*e*x*gamma(m + 2)) - a*e
**(m + 2)*(d/e + x)**(m + 1)*gamma(m + 1)/(d*e*x*gamma(m + 2)) - a*e**(m +
 2)*m**2*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(d*
*2*gamma(m + 2)) - a*e**(m + 2)*m*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1
, m + 1)*gamma(m + 1)/(d**2*gamma(m + 2)) - b*e**(m + 1)*m*(d/e + x)**(m +
 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(d*gamma(m + 2)) - b*e**(m
+ 1)*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(d*gamm
a(m + 2)) + c*Piecewise((d**m*x, Eq(e, 0)), (Piecewise(((d + e*x)**(m + 1)
/(m + 1), Ne(m, -1)), (log(d + e*x), True))/e, True))
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^2} dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^2,x, algorithm="maxima")
```

output
```
(e*x + d)^(m + 1)*c/(e*(m + 1)) + integrate((b*x + a)*(e*x + d)^m/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^2} dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^2} dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{x^2} dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x^2,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^2}\,dx$$
$$= \frac{-(ex+d)^m ae\,m^2 - (ex+d)^m aem + (ex+d)^m bemx + (ex+d)^m bex + (ex+d)^m cdmx + (ex+d)^m}{em}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^2,x)
```

output
```
( - (d + e*x)**m*a*e*m**2 - (d + e*x)**m*a*e*m + (d + e*x)**m*b*e*m*x + (d
 + e*x)**m*b*e*x + (d + e*x)**m*c*d*m*x + (d + e*x)**m*c*e*m*x**2 + int((d
 + e*x)**m/(d*x + e*x**2),x)*a*e**2*m**3*x + int((d + e*x)**m/(d*x + e*x**
2),x)*a*e**2*m**2*x + int((d + e*x)**m/(d*x + e*x**2),x)*b*d*e*m**2*x + in
t((d + e*x)**m/(d*x + e*x**2),x)*b*d*e*m*x)/(e*m*x*(m + 1))
```

3.123 $\int \frac{(d+ex)^m(a+bx+cx^2)}{x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{x^3} dx = -\frac{a(d+ex)^{1+m}}{2dx^2} + \frac{c(d+ex)^{1+m}}{emx} + \frac{(2cd^2+e(2bd-ae(1-m))m)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{ex}{d}\right)}{2d^3m(1+m)}$$

output
```
-1/2*a*(e*x+d)^(1+m)/d/x^2+c*(e*x+d)^(1+m)/e/m/x+1/2*(2*c*d^2+e*(2*b*d-a*e
*(1-m))*m)*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],1+e*x/d)/d^3/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^m(a+bx+cx^2)}{x^3} dx = -\frac{(d+ex)^{1+m}\left(cd^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)+e\left(-bd\,\text{Hypergeometric2F1}\left(2,1+\right.\right.\right.}{d^3(1+m)}$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/x^3,x]
```

output
```
-(((d + e*x)^(1 + m)*(c*d^2*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d
] + e*(-(b*d*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + (e*x)/d]) + a*e*Hyperg
eometric2F1[3, 1 + m, 2 + m, 1 + (e*x)/d])))/(d^3*(1 + m)))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1193, 25, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^m}{x^3}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{\int -\frac{(2bd+2cxd-ae(1-m))(d+ex)^m}{x^2}dx}{2d}-\frac{a(d+ex)^{m+1}}{2dx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(2bd+2cxd-ae(1-m))(d+ex)^m}{x^2}dx}{2d}-\frac{a(d+ex)^{m+1}}{2dx^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(em(2bd-ae(1-m))+2cd^2)\int \frac{(d+ex)^m}{x}dx}{d}-\frac{(d+ex)^{m+1}(2bd-ae(1-m))}{dx}}{2d}-\frac{a(d+ex)^{m+1}}{2dx^2}$$

$$\Big\downarrow \text{75}$$

$$\frac{-\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)(em(2bd-ae(1-m))+2cd^2)}{d^2(m+1)}-\frac{(d+ex)^{m+1}(2bd-ae(1-m))}{dx}}{2d}-\frac{a(d+ex)^{m+1}}{2dx^2}$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2))/x^3,x]
```

output
```
-1/2*(a*(d + e*x)^(1 + m))/(d*x^2) + (-(((2*b*d - a*e*(1 - m))*(d + e*x)^(
1 + m))/(d*x)) - ((2*c*d^2 + e*(2*b*d - a*e*(1 - m))*m)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/(d^2*(1 + m)))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

Maple [F]

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{x^3}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^3,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{x^3}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m/x^3, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(82) = 164$.

Time $= 5.07$ (sec) , antiderivative size $= 1006$, normalized size of antiderivative $= 9.58$

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)/x**3,x)
```

output
```
a*d**2*e**(m + 3)*m**3*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*ga
mma(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gamma(m + 2) + 2*d**3*e**2*(
d/e + x)**2*gamma(m + 2)) - a*d**2*e**(m + 3)*m*(d/e + x)**(m + 1)*lerchph
i(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gam
ma(m + 2) + 2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) - a*d**2*e**(m + 3)*m*(
d/e + x)**(m + 1)*gamma(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gamma(m
+ 2) + 2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) - a*d**2*e**(m + 3)*(d/e + x
)**(m + 1)*gamma(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gamma(m + 2) +
2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) + 2*a*d*e*e**(m + 3)*m**3*x*(d/e +
x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(-2*d**5*gamma(m +
2) - 4*d**4*e*x*gamma(m + 2) + 2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) - a*
d*e*e**(m + 3)*m**2*x*(d/e + x)**(m + 1)*gamma(m + 1)/(-2*d**5*gamma(m + 2
) - 4*d**4*e*x*gamma(m + 2) + 2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) - 2*a
*d*e*e**(m + 3)*m*x*(d/e + x)**(m + 1)*lerchphi(1 + e*x/d, 1, m + 1)*gamma
(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gamma(m + 2) + 2*d**3*e**2*(d/e
 + x)**2*gamma(m + 2)) + a*d*e*e**(m + 3)*x*(d/e + x)**(m + 1)*gamma(m + 1
)/(-2*d**5*gamma(m + 2) - 4*d**4*e*x*gamma(m + 2) + 2*d**3*e**2*(d/e + x)*
*2*gamma(m + 2)) - a*e**2*e**(m + 3)*m**3*(d/e + x)**2*(d/e + x)**(m + 1)*
lerchphi(1 + e*x/d, 1, m + 1)*gamma(m + 1)/(-2*d**5*gamma(m + 2) - 4*d**4*
e*x*gamma(m + 2) + 2*d**3*e**2*(d/e + x)**2*gamma(m + 2)) + a*e**2*e**(...
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^3} \, dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^3} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^3} dx = \int \frac{(cx^2+bx+a)(ex+d)^m}{x^3} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^3} dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{x^3} dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x^3,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{x^3} dx$$
$$= \frac{-(ex+d)^m adm-(ex+d)^m ae\,m^2x-2(ex+d)^m bdmx+2(ex+d)^m cd\,x^2+\left(\int \frac{(ex+d)^m}{e\,x^2+dx}dx\right) a\,e^2m^3x^2}{2dm\,x^2}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)/x^3,x)
```

output
```
( - (d + e*x)**m*a*d*m - (d + e*x)**m*a*e*m**2*x - 2*(d + e*x)**m*b*d*m*x
+ 2*(d + e*x)**m*c*d*x**2 + int((d + e*x)**m/(d*x + e*x**2),x)*a*e**2*m**3
*x**2 - int((d + e*x)**m/(d*x + e*x**2),x)*a*e**2*m**2*x**2 + 2*int((d + e
*x)**m/(d*x + e*x**2),x)*b*d*e*m**2*x**2 + 2*int((d + e*x)**m/(d*x + e*x**
2),x)*c*d**2*m*x**2)/(2*d*m*x**2)
```

3.124 $\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3} \, dx = -\frac{a(d+ex)^{-1+m}}{2dx^2} - \frac{c(d+ex)^{-1+m}}{e(2-m)x} + \frac{(2cd^2 - e(2bd - ae(3-m))(2-m))(d+ex)^{-1+m} \text{Hypergeometric2F1}\left(2, -1+m, m, 1+\frac{ex}{d}\right)}{2d^3(1-m)(2-m)}$$

output
```
-1/2*a*(e*x+d)^(-1+m)/d/x^2-c*(e*x+d)^(-1+m)/e/(2-m)/x+1/2*(2*c*d^2-e*(2*b
*d-a*e*(3-m))*(2-m))*(e*x+d)^(-1+m)*hypergeom([2, -1+m],[m],1+e*x/d)/d^3/(
1-m)/(2-m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3} \, dx = -\frac{(d+ex)^{-1+m}\left(cd^2 \text{Hypergeometric2F1}\left(1, -1+m, m, 1+\frac{ex}{d}\right) + e\left(-bd \text{Hypergeometric2F1}\left(2, -1+\right.\right.\right.}{d^3(-1+m)}$$

input
```
Integrate[((d + e*x)^(-2 + m)*(a + b*x + c*x^2))/x^3,x]
```

output
```
-(((d + e*x)^(-1 + m)*(c*d^2*Hypergeometric2F1[1, -1 + m, m, 1 + (e*x)/d]
+ e*(-(b*d*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d]) + a*e*Hypergeomet
ric2F1[3, -1 + m, m, 1 + (e*x)/d])))/(d^3*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1193, 25, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)(d+ex)^{m-2}}{x^3}dx$$

$$\Big\downarrow \text{1193}$$

$$-\frac{\int -\frac{(2bd+2cxd-ae(3-m))(d+ex)^{m-2}}{x^2}dx}{2d}-\frac{a(d+ex)^{m-1}}{2dx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(2bd+2cxd-ae(3-m))(d+ex)^{m-2}}{x^2}dx}{2d}-\frac{a(d+ex)^{m-1}}{2dx^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(2cd^2-e(2-m)(2bd-ae(3-m)))\int \frac{(d+ex)^{m-2}}{x}dx}{d}-\frac{(d+ex)^{m-1}(2bd-ae(3-m))}{dx}}{2d}-\frac{a(d+ex)^{m-1}}{2dx^2}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{(d+ex)^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{ex}{d}+1\right)(2cd^2-e(2-m)(2bd-ae(3-m)))}{d^2(1-m)}-\frac{(d+ex)^{m-1}(2bd-ae(3-m))}{dx}}{2d}-\frac{a(d+ex)^{m-1}}{2dx^2}$$

input
```
Int[((d + e*x)^(-2 + m)*(a + b*x + c*x^2))/x^3,x]
```

output
```
-1/2*(a*(d + e*x)^(-1 + m))/(d*x^2) + (-(((2*b*d - a*e*(3 - m))*(d + e*x)^
(-1 + m))/(d*x)) + ((2*c*d^2 - e*(2*b*d - a*e*(3 - m))*(2 - m))*(d + e*x)^
(-1 + m)*Hypergeometric2F1[1, -1 + m, m, 1 + (e*x)/d])/(d^2*(1 - m)))/(2*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

Maple [F]

$$\int \frac{(ex+d)^{-2+m}(c\,x^2+bx+a)}{x^3}dx$$

input
```
int((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x)
```

output
```
int((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^{m-2}}{x^3}\,dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^(m - 2)/x^3, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1527 vs. $2(85) = 170$.

Time = 13.50 (sec) , antiderivative size = 1527, normalized size of antiderivative = 12.83

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**(-2+m)*(c*x**2+b*x+a)/x**3,x)
```

output
```
a*d**2*e**(m + 1)*m**3*(d/e + x)**(m - 1)*lerchphi(1 + e*x/d, 1, m - 1)*ga
mma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*gamma(m) + 2*d**3*e**2*(d/e + x)
**2*gamma(m)) - 6*a*d**2*e**(m + 1)*m**2*(d/e + x)**(m - 1)*lerchphi(1 + e
*x/d, 1, m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*gamma(m) + 2*d
**3*e**2*(d/e + x)**2*gamma(m)) + 11*a*d**2*e**(m + 1)*m*(d/e + x)**(m - 1
)*lerchphi(1 + e*x/d, 1, m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*
x*gamma(m) + 2*d**3*e**2*(d/e + x)**2*gamma(m)) - a*d**2*e**(m + 1)*m*(d/e
 + x)**(m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*gamma(m) + 2*d*
*3*e**2*(d/e + x)**2*gamma(m)) - 6*a*d**2*e**(m + 1)*(d/e + x)**(m - 1)*le
rchphi(1 + e*x/d, 1, m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*ga
mma(m) + 2*d**3*e**2*(d/e + x)**2*gamma(m)) + a*d**2*e**(m + 1)*(d/e + x)*
*(m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*gamma(m) + 2*d**3*e**
2*(d/e + x)**2*gamma(m)) + 2*a*d*e*e**(m + 1)*m**3*x*(d/e + x)**(m - 1)*le
rchphi(1 + e*x/d, 1, m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4*e*x*ga
mma(m) + 2*d**3*e**2*(d/e + x)**2*gamma(m)) - 12*a*d*e*e**(m + 1)*m**2*x*(
d/e + x)**(m - 1)*lerchphi(1 + e*x/d, 1, m - 1)*gamma(m - 1)/(-2*d**5*gamm
a(m) - 4*d**4*e*x*gamma(m) + 2*d**3*e**2*(d/e + x)**2*gamma(m)) - a*d*e*e*
*(m + 1)*m**2*x*(d/e + x)**(m - 1)*gamma(m - 1)/(-2*d**5*gamma(m) - 4*d**4
*e*x*gamma(m) + 2*d**3*e**2*(d/e + x)**2*gamma(m)) + 22*a*d*e*e**(m + 1)*m
*x*(d/e + x)**(m - 1)*lerchphi(1 + e*x/d, 1, m - 1)*gamma(m - 1)/(-2*d*...
```

Maxima [F]

$$\int \frac{(d+ex)^{-2+m}\,(a+bx+cx^2)}{x^3}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^{m-2}}{x^3}\,dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^(m - 2)/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3}\,dx = \int \frac{(cx^2+bx+a)(ex+d)^{m-2}}{x^3}\,dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^(m - 2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3}\,dx = \int \frac{(d+e\,x)^{m-2}\,(c\,x^2+b\,x+a)}{x^3}\,dx$$

input
```
int(((d + e*x)^(m - 2)*(a + b*x + c*x^2))/x^3,x)
```

output
```
int(((d + e*x)^(m - 2)*(a + b*x + c*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^{-2+m}(a+bx+cx^2)}{x^3}\,dx$$
$$= \frac{-(ex+d)^m ad-(ex+d)^m aemx+3(ex+d)^m aex-2(ex+d)^m bdx+\left(\int \frac{(ex+d)^m}{e^2x^3+2de\,x^2+d^2x}dx\right) ad\,e^2m^2x}{}$$

input
```
int((e*x+d)^(-2+m)*(c*x^2+b*x+a)/x^3,x)
```

output
```
( - (d + e*x)**m*a*d - (d + e*x)**m*a*e*m*x + 3*(d + e*x)**m*a*e*x - 2*(d
+ e*x)**m*b*d*x + int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*a*
d*e**2*m**2*x**2 - 5*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)
*a*d*e**2*m*x**2 + 6*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)
*a*d*e**2*x**2 + int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*a*e
**3*m**2*x**3 - 5*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*a*
e**3*m*x**3 + 6*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*a*e*
*3*x**3 + 2*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*b*d**2*e
*m*x**2 - 4*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*b*d**2*e
*x**2 + 2*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*b*d*e**2*m
*x**3 - 4*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*b*d*e**2*x
**3 + 2*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*c*d**3*x**2
+ 2*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*c*d**2*e*x**3)/(
2*d**2*x**2*(d + e*x))
```

3.125 $\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 290

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx = -\frac{(cd+be)(d+ex)^{1+m}}{c^2e^2(1+m)}+\frac{(d+ex)^{2+m}}{ce^2(2+m)}$$
$$+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)}$$
$$+\frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)}$$

output
```
-(b*e+c*d)*(e*x+d)^(1+m)/c^2/e^2/(1+m)+(e*x+d)^(2+m)/c/e^2/(2+m)+(a-b^2/c+
b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m
],2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/c/(2*c*d-(b-(-4*a*c+b^2)^(
1/2))*e)/(1+m)+(a-b^2/c-b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)
*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/c/
(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(1+m)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.10

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{\left(b^3-3abc-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{-\left(\left(b\left(b+\sqrt{b^2-4ac}\right)\right.\right.}{}\right.}{c^2\cdots}$$

input
```
Integrate[(x^3*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2
 - 4*a*c])*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d + (-b
 + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + (-((b*(b
 + Sqrt[b^2 - 4*a*c])*e - 2*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*(b*e*(2 + m)
+ c*(d - e*(1 + m)*x))) + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqr
t[b^2 - 4*a*c])*e^2*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(-2*c*d + (b + Sqrt[b^2 -
4*a*c])*e)*(2 + m))))/(c^2*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1200$$

$$\int \left(\frac{\left(-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}+\frac{b^2}{c^2}\right)(d+ex)^m}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}+\frac{b^2}{c^2}\right)(d+ex)^m}{\sqrt{b^2-4ac}+b+2cx}+\frac{(-be-cd)(d+ex)^m}{c^2e}+\frac{(d+ex)^m}{ce}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c(m+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(be+cd)(d+ex)^{m+1}}{c^2e^2(m+1)}+\frac{(d+ex)^{m+2}}{ce^2(m+2)}$$

input
```
Int[(x^3*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
-(((c*d + b*e)*(d + e*x)^(1 + m))/(c^2*e^2*(1 + m))) + (d + e*x)^(2 + m)/(
c*e^2*(2 + m)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d
 + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 +
 m)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d + e*x)^(1
 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^3(ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int(x^3*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int(x^3*(e*x+d)^m/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*x^3/(c*x^2 + b*x + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**3*(e*x+d)**m/(c*x**2+b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{(ex+d)^m x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m*x^3/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{(ex+d)^m x^3}{cx^2+bx+a}\,dx$$

input
```
integrate(x^3*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*x^3/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{x^3\,(d+e\,x)^m}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x^3*(d + e*x)^m)/(a + b*x + c*x^2),x)
```

output
```
int((x^3*(d + e*x)^m)/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(d+ex)^m}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int(x^3*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
((d + e*x)**m*a*b*e**2*m**2 + 3*(d + e*x)**m*a*b*e**2*m + 2*(d + e*x)**m*a
*b*e**2 - (d + e*x)**m*a*c*d*e*m**2 - 3*(d + e*x)**m*a*c*d*e*m - 2*(d + e*
x)**m*a*c*d*e + (d + e*x)**m*b**2*d*e*m + 2*(d + e*x)**m*b**2*d*e - (d + e
*x)**m*b**2*e**2*m**2*x - 2*(d + e*x)**m*b**2*e**2*m*x - (d + e*x)**m*b*c*
d**2*m + (d + e*x)**m*b*c*d*e*m**2*x + (d + e*x)**m*b*c*e**2*m**2*x**2 + (
d + e*x)**m*b*c*e**2*m*x**2 - int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*
x**2 + c*d*x**2 + c*e*x**3),x)*a**2*b*e**3*m**3 - 3*int((d + e*x)**m/(a*d
 + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a**2*b*e**3*m**2 - 2*
int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)
*a**2*b*e**3*m + int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x*
*2 + c*e*x**3),x)*a**2*c*d*e**2*m**3 + 3*int((d + e*x)**m/(a*d + a*e*x + b
*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a**2*c*d*e**2*m**2 + 2*int((d +
e*x)**m/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a**2*c*d
*e**2*m - 2*int(((d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*
x**2 + c*e*x**3),x)*a*b*c*e**3*m**3 - 6*int(((d + e*x)**m*x**2)/(a*d + a*e
*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a*b*c*e**3*m**2 - 4*int(((
d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x
)*a*b*c*e**3*m + int(((d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 +
 c*d*x**2 + c*e*x**3),x)*a*c**2*d*e**2*m**3 + 3*int(((d + e*x)**m*x**2)/(a
*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a*c**2*d*e**2*m...
```

3.126 $\int \frac{x^2(d+ex)^m}{a+bx+cx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2} dx = \frac{(d+ex)^{1+m}}{ce(1+m)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)}$$

output
```
(e*x+d)^(1+m)/c/e/(1+m)+(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*
hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/c/(
2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(1+m)+(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*
(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^
(1/2))*e))/c/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(1+m)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx =$$
$$-\frac{2(d+ex)^{1+m}\left(2\sqrt{b^2-4ac}(cd^2+e(-bd+ae))+e\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\sqrt{b^2-4ac}\right.\right.}{\sqrt{}}$$

input
```
Integrate[(x^2*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
(-2*(d + e*x)^(1 + m)*(2*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e)) + e*
(-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)] + e*(b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a
*Sqrt[b^2 - 4*a*c]*e)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[b^2 - 4*a*c]*e*(2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e)*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \ 1200$$
$$\int \left(\frac{\left(\frac{b^2-2ac}{c\sqrt{b^2-4ac}}-\frac{b}{c}\right)(d+ex)^m}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}-\frac{b}{c}\right)(d+ex)^m}{\sqrt{b^2-4ac}+b+2cx}+\frac{(d+ex)^m}{c}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c(m+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{(d+ex)^{m+1}}{ce(m+1)}$$

input
```
Int[(x^2*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
(d + e*x)^(1 + m)/(c*e*(1 + m)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(
d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1
+ m)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x)^(1 + m)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^2(ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int(x^2*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int(x^2*(e*x+d)^m/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*x^2/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx$$

input
```
integrate(x**2*(e*x+d)**m/(c*x**2+b*x+a),x)
```

output
```
Integral(x**2*(d + e*x)**m/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{(ex+d)^m x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m*x^2/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{(ex+d)^m x^2}{cx^2+bx+a}\,dx$$

input
```
integrate(x^2*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*x^2/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{x^2\,(d+e\,x)^m}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x^2*(d + e*x)^m)/(a + b*x + c*x^2),x)
```

output
```
int((x^2*(d + e*x)^m)/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(d+ex)^m}{a+bx+cx^2}\,dx$$
$$= \frac{-(ex+d)^m aem - (ex+d)^m ae - (ex+d)^m bd + (ex+d)^m bemx + \left(\int \frac{(ex+d)^m}{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}dx\right) a}{}$$

input
```
int(x^2*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
( - (d + e*x)**m*a*e*m - (d + e*x)**m*a*e - (d + e*x)**m*b*d + (d + e*x)**
m*b*e*m*x + int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 +
c*e*x**3),x)*a**2*e**2*m**2 + int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*
x**2 + c*d*x**2 + c*e*x**3),x)*a**2*e**2*m + int(((d + e*x)**m*x**2)/(a*d
+ a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*a*c*e**2*m**2 + int((
(d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),
x)*a*c*e**2*m - int(((d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 +
c*d*x**2 + c*e*x**3),x)*b**2*e**2*m**2 - int(((d + e*x)**m*x**2)/(a*d + a*
e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*b**2*e**2*m + int(((d + e
*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*b*c
*d*e*m**2 + int(((d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*
x**2 + c*e*x**3),x)*b*c*d*e*m)/(b*c*e*m*(m + 1))
```

3.127 $\int \frac{x(d+ex)^m}{a+bx+cx^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{x(d+ex)^m}{a+bx+cx^2} dx = \frac{\left(b-\sqrt{b^2-4ac}\right)(d+ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} - \frac{\left(b+\sqrt{b^2-4ac}\right)(d+ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)}$$

output

```
(b-(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/
(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2
)^(1/2))*e)/(1+m)-(b+(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],
[2+m],2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/(-4*a*c+b^2)^(1/2)/(2*
c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(1+m)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.84

$$\int \frac{x(d+ex)^m}{a+bx+cx^2}\,dx = \frac{(d+ex)^{1+m}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1\right.}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{1+m}$$

input
```
Integrate[(x*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
m, 2 + m, (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F
1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^m}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^m}{-\sqrt{b^2-4ac}+b+2cx}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex)^m}{\sqrt{b^2-4ac}+b+2cx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

input
```
Int[(x*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
-(((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e)*(1 + m))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x)^
(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x(ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int(x*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int(x*(e*x+d)^m/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{x(d+ex)^m}{a+bx+cx^2} dx = \int \frac{(ex+d)^m x}{cx^2+bx+a} dx$$

input
```
integrate(x*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*x/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{x(d+ex)^m}{a+bx+cx^2} dx = \int \frac{x(d+ex)^m}{a+bx+cx^2} dx$$

input
```
integrate(x*(e*x+d)**m/(c*x**2+b*x+a),x)
```

output
```
Integral(x*(d + e*x)**m/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{x(d+ex)^m}{a+bx+cx^2} dx = \int \frac{(ex+d)^m x}{cx^2+bx+a} dx$$

input
```
integrate(x*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m*x/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{x(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m x}{cx^2+bx+a}\,dx$$

input
```
integrate(x*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*x/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{x\,(d+e\,x)^m}{c\,x^2+b\,x+a}\,dx$$

input
```
int((x*(d + e*x)^m)/(a + b*x + c*x^2),x)
```

output
```
int((x*(d + e*x)^m)/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{x(d+ex)^m}{a+bx+cx^2}\,dx = \frac{(ex+d)^m d-\left(\int \frac{(ex+d)^m}{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}dx\right)adem+\left(\int \frac{(ex+d)^m x^2}{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}dx\right)b\,e^2m-\left(\int \frac{}{ce\,x}\right.}{bem}$$

input
```
int(x*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
((d + e*x)**m*d - int((d + e*x)**m/(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x
**2 + c*e*x**3),x)*a*d*e*m + int(((d + e*x)**m*x**2)/(a*d + a*e*x + b*d*x
+ b*e*x**2 + c*d*x**2 + c*e*x**3),x)*b*e**2*m - int(((d + e*x)**m*x**2)/(a
*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*c*d*e*m)/(b*e*m)
```

3.128 $\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = -\frac{2c(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} + \frac{2c(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)}$$

output

```
-2*c*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+
b^2)^(1/2))*e))/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(1+m)+
2*c*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b
^2)^(1/2))*e))/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(1+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx$$
$$= \frac{2c(d+ex)^{1+m}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}(1+m)}$$

input
```
Integrate[(d + e*x)^m/(a + b*x + c*x^2),x]
```

output
```
(2*c*(d + e*x)^(1 + m)*(-(Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x
))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)) + Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a
*c]*(1 + m))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1150$$
$$\int \left(\frac{2c(d+ex)^m}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c(d+ex)^m}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2c(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(m+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

input
```
Int[(d + e*x)^m/(a + b*x + c*x^2),x]
```

output
```
(-2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))
/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e)*(1 + m)) + (2*c*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt
[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
```

Defintions of rubi rules used

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] &&  !IntegerQ[2*m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(d+ex)^m}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**m/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(d+e\,x)^m}{c\,x^2+b\,x+a}\,dx$$

input
```
int((d + e*x)^m/(a + b*x + c*x^2),x)
```

output
```
int((d + e*x)^m/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int((d + e*x)**m/(a + b*x + c*x**2),x)
```

3.129 $\int \frac{(d+ex)^m}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 264

$$\int \frac{(d+ex)^m}{x(a+bx+cx^2)}\,dx = \frac{c\left(b+\sqrt{b^2-4ac}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} - \frac{c\left(b-\sqrt{b^2-4ac}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)} - \frac{(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{ad(1+m)}$$

output
```
c*(b+(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d
)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c
+b^2)^(1/2))*e)/(1+m)-c*(b-(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1,
 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/a/(-4*a*c+b^2)^(
1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(1+m)-(e*x+d)^(1+m)*hypergeom([1, 1+
m],[2+m],1+e*x/d)/a/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^m}{x\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1\right.}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{a(1+m)}$$

input
```
Integrate[(d + e*x)^m/(x*(a + b*x + c*x^2)),x]
```

output
```
((d + e*x)^(1 + m)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
m, 2 + m, (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2
F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + m, 2 + m, 1
 + (e*x)/d]/d))/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{x\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{(-b-cx)(d+ex)^m}{a\left(a+bx+cx^2\right)}+\frac{(d+ex)^m}{ax}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a(m+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)}{ad(m+1)}$$

input
```
Int[(d + e*x)^m/(x*(a + b*x + c*x^2)),x]
```

output
```
(c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e)*(1 + m)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d + e*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + m))
- ((d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/(a*d
*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{x\,(c\,x^2+bx+a)}dx$$

input
```
int((e*x+d)^m/x/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^m/x/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^m/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c*x^3 + b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx$$

input
```
integrate((e*x+d)**m/x/(c*x**2+b*x+a),x)
```

output
```
Integral((d + e*x)**m/(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^m/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/((c*x^2 + b*x + a)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx+a)x}\,dx$$

input
```
integrate((e*x+d)^m/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/((c*x^2 + b*x + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(d+e\,x)^m}{x\,(c\,x^2+b\,x+a)}\,dx$$

input
```
int((d + e*x)^m/(x*(a + b*x + c*x^2)),x)
```

output
```
int((d + e*x)^m/(x*(a + b*x + c*x^2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{x\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^m}{c\,x^3+b\,x^2+ax}dx$$

input
```
int((e*x+d)^m/x/(c*x^2+b*x+a),x)
```

output
```
int((d + e*x)**m/(a*x + b*x**2 + c*x**3),x)
```

3.130 $\int \frac{(d+ex)^m}{x^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 307

$$\begin{aligned}\int \frac{(d+ex)^m}{x^2(a+bx+cx^2)}\,dx = & -\frac{(d+ex)^{1+m}}{adx} \\ & -\frac{c\left(b^2-2ac+b\sqrt{b^2-4ac}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a^2\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+m)} \\ & +\frac{c\left(b^2-2ac-b\sqrt{b^2-4ac}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a^2\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+m)} \\ & +\frac{(bd-aem)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{a^2d^2(1+m)}\end{aligned}$$

output
```
-(e*x+d)^(1+m)/a/d/x-c*(b^2-2*a*c+b*(-4*a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hype
rgeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))/a^2/(-4
*a*c+b^2)^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(1+m)+c*(b^2-2*a*c-b*(-4*
a*c+b^2)^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],2*c*(e*x+d)/(2*c*d-
(b+(-4*a*c+b^2)^(1/2))*e))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e)/(1+m)+(-a*e*m+b*d)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],1+e*x/d
)/a^2/d^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^m}{x^2(a+bx+cx^2)}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}-\frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(\right.}{2cd-\left(b+\sqrt{b^2\right.}}\right.}{a^2(1+m}$$

input
```
Integrate[(d + e*x)^m/(x^2*(a + b*x + c*x^2)),x]
```

output
```
((d + e*x)^(1 + m)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[b
^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b*Hyper
geometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/d + (a*e*Hypergeometric2F1[2,
1 + m, 2 + m, 1 + (e*x)/d])/d^2))/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{x^2(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \ 1200$$
$$\int \left(\frac{(-ac+b^2+bcx)(d+ex)^m}{a^2(a+bx+cx^2)}-\frac{b(d+ex)^m}{a^2x}+\frac{(d+ex)^m}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a^2(m+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a^2(m+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)}{a^2d(m+1)}+\frac{e(d+ex)^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{ex}{d}+1\right)}{ad^2(m+1)}$$

input
```
Int[(d + e*x)^m/(x^2*(a + b*x + c*x^2)),x]
```

output
```
-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x)^(1 + m)*Hypergeometri
c2F1[1, 1 + m, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]
)/(a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + m))) - (c*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a^2*(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)*(1 + m)) + (b*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, 1 + (e*x)/d])/(a^2*d*(1 + m)) + (e*(d + e*x)^(1 + m)*Hyperg
eometric2F1[2, 1 + m, 2 + m, 1 + (e*x)/d])/(a*d^2*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{x^2\,(c\,x^2+bx+a)}dx$$

input
```
int((e*x+d)^m/x^2/(c*x^2+b*x+a),x)
```

output
```
int((e*x+d)^m/x^2/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{x^2\,(a+bx+cx^2)}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^m/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c*x^4 + b*x^3 + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{x^2\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(ex+d)^m}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^m/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/((c*x^2 + b*x + a)*x^2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(ex+d)^m}{(cx^2+bx+a)x^2}\,dx$$

input
```
integrate((e*x+d)^m/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/((c*x^2 + b*x + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{x^2\left(a+bx+cx^2\right)}\,dx=\int \frac{(d+e\,x)^m}{x^2\,\left(c\,x^2+b\,x+a\right)}\,dx$$

input
```
int((d + e*x)^m/(x^2*(a + b*x + c*x^2)),x)
```

output
```
int((d + e*x)^m/(x^2*(a + b*x + c*x^2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{x^2 (a+bx+cx^2)} \, dx = \int \frac{(ex+d)^m}{c\,x^4+b\,x^3+a\,x^2} dx$$

input
```
int((e*x+d)^m/x^2/(c*x^2+b*x+a),x)
```

output
```
int((d + e*x)**m/(a*x**2 + b*x**3 + c*x**4),x)
```

3.131 $\int (gx)^n(d+ex)^m \left(a+bx+cx^2\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int (gx)^n(d+ex)^m \left(a+bx+cx^2\right) dx = -\frac{(cd(2+n)-be(3+m+n))(gx)^{1+n}(d+ex)^{1+m}}{e^2g(2+m+n)(3+m+n)}+\frac{c(gx)^{2+n}(d+ex)^{1+m}}{eg^2(3+m+n)} +\frac{\left(\frac{a}{d+dn}+\frac{cd(2+n)-be(3+m+n)}{e^2(2+m+n)(3+m+n)}\right)(gx)^{1+n}(d+ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,2+m+n,2+n,-\frac{ex}{d}\right)}{g}$$

output
```
-(c*d*(2+n)-b*e*(3+m+n))*(g*x)^(1+n)*(e*x+d)^(1+m)/e^2/g/(2+m+n)/(3+m+n)+c
*(g*x)^(2+n)*(e*x+d)^(1+m)/e/g^2/(3+m+n)+(a/(d*n+d)+(c*d*(2+n)-b*e*(3+m+n)
)/e^2/(2+m+n)/(3+m+n))*(g*x)^(1+n)*(e*x+d)^(1+m)*hypergeom([1, 2+m+n],[2+n
],-e*x/d)/g
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx$$
$$=\frac{x(gx)^n(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\left(cd^2\,\text{Hypergeometric2F1}\left(-2-m,1+n,2+n,-\frac{ex}{d}\right)+d(-2cd+be)\,\text{Hyp}\right.}{}$$

input
```
Integrate[(g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
(x*(g*x)^n*(d + e*x)^m*(c*d^2*Hypergeometric2F1[-2 - m, 1 + n, 2 + n, -((e
*x)/d)] + d*(-2*c*d + b*e)*Hypergeometric2F1[-1 - m, 1 + n, 2 + n, -((e*x)
/d)] + (c*d^2 - b*d*e + a*e^2)*Hypergeometric2F1[-m, 1 + n, 2 + n, -((e*x)
/d)]))/(e^2*(1 + n)*(1 + (e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1194, 27, 90, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^n\left(a+bx+cx^2\right)(d+ex)^m\,dx$$
$$\Big\downarrow \; 1194$$
$$\frac{\int g^2(gx)^n(d+ex)^m(ae(m+n+3)-(cd(n+2)-be(m+n+3))x)dx}{eg^2(m+n+3)}+\frac{c(gx)^{n+2}(d+ex)^{m+1}}{eg^2(m+n+3)}$$
$$\Big\downarrow \; 27$$
$$\frac{\int (gx)^n(d+ex)^m(ae(m+n+3)-(cd(n+2)-be(m+n+3))x)dx}{e(m+n+3)}+\frac{c(gx)^{n+2}(d+ex)^{m+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \; 90$$

$$\frac{\left(ae(m+n+3)+\frac{d(n+1)(cd(n+2)-be(m+n+3))}{e(m+n+2)}\right)\int (gx)^n(d+ex)^m dx-\frac{(gx)^{n+1}(d+ex)^{m+1}(cd(n+2)-be(m+n+3))}{eg(m+n+2)}}{e(m+n+3)}+\frac{c(gx)^{n+2}(d+ex)^{m+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \; 76$$

$$\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}\left(ae(m+n+3)+\frac{d(n+1)(cd(n+2)-be(m+n+3))}{e(m+n+2)}\right)\int (gx)^n\left(\frac{ex}{d}+1\right)^m dx-\frac{(gx)^{n+1}(d+ex)^{m+1}(cd(n}{eg(m+n}}{e(m+n+3)}$$
$$\frac{c(gx)^{n+2}(d+ex)^{m+1}}{eg^2(m+n+3)}$$

$$\Big\downarrow \; 74$$

$$\frac{\frac{(gx)^{n+1}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}\text{Hypergeometric2F1}\left(-m,n+1,n+2,-\frac{ex}{d}\right)\left(ae(m+n+3)+\frac{d(n+1)(cd(n+2)-be(m+n+3))}{e(m+n+2)}\right)}{g(n+1)}-\frac{(gx)^{n+1}(d+ex)^{m+1}(c}{eg(m+}}{e(m+n+3)}$$
$$\frac{c(gx)^{n+2}(d+ex)^{m+1}}{eg^2(m+n+3)}$$

input
```
Int[(g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x]
```

output
```
(c*(g*x)^(2 + n)*(d + e*x)^(1 + m))/(e*g^2*(3 + m + n)) + (-((((c*d*(2 + n)
 - b*e*(3 + m + n))*(g*x)^(1 + n)*(d + e*x)^(1 + m))/(e*g*(2 + m + n))) +
((a*e*(3 + m + n) + (d*(1 + n)*(c*d*(2 + n) - b*e*(3 + m + n)))/(e*(2 + m
+ n)))*(g*x)^(1 + n)*(d + e*x)^m*Hypergeometric2F1[-m, 1 + n, 2 + n, -((e*
x)/d)])/(g*(1 + n)*(1 + (e*x)/d)^m))/(e*(3 + m + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] && !IntegerQ[m] && !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int (gx)^n (ex+d)^m (c\,x^2+bx+a)\,dx$$

input
```
int((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x)
```

output
```
int((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx=\int \left(cx^2+bx+a\right)(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x)^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx=\frac{ad^m g^n x^{n+1}\Gamma(n+1)\,{}_2F_1\left(\begin{matrix}-m,n+1\\ n+2\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(n+2)}+\frac{bd^m g^n x^{n+2}\Gamma(n+2)\,{}_2F_1\left(\begin{matrix}-m,n+2\\ n+3\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(n+3)}+\frac{cd^m g^n x^{n+3}\Gamma(n+3)\,{}_2F_1\left(\begin{matrix}-m,n+3\\ n+4\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(n+4)}$$

input
```
integrate((g*x)**n*(e*x+d)**m*(c*x**2+b*x+a),x)
```

output
```
a*d**m*g**n*x**(n + 1)*gamma(n + 1)*hyper((-m, n + 1), (n + 2,), e*x*exp_p
olar(I*pi)/d)/gamma(n + 2) + b*d**m*g**n*x**(n + 2)*gamma(n + 2)*hyper((-m
, n + 2), (n + 3,), e*x*exp_polar(I*pi)/d)/gamma(n + 3) + c*d**m*g**n*x**(
n + 3)*gamma(n + 3)*hyper((-m, n + 3), (n + 4,), e*x*exp_polar(I*pi)/d)/ga
mma(n + 4)
```

Maxima [F]

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx=\int (cx^2+bx+a)(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x)^n, x)
```

Giac [F]

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx=\int (cx^2+bx+a)(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)^m\left(a+bx+cx^2\right)\,dx=\int (g\,x)^n\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int((g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x)
```

output
```
int((g*x)^n*(d + e*x)^m*(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int (gx)^n (d+ex)^m \left(a+bx+cx^2\right) \, dx = \text{too large to display}$$

input
```
int((g*x)^n*(e*x+d)^m*(c*x^2+b*x+a),x)
```

output
```
(g**n*(x**n*(d + e*x)**m*a*d*e**2*m**3 + 2*x**n*(d + e*x)**m*a*d*e**2*m**2
*n + 5*x**n*(d + e*x)**m*a*d*e**2*m**2 + x**n*(d + e*x)**m*a*d*e**2*m*n**2
 + 5*x**n*(d + e*x)**m*a*d*e**2*m*n + 6*x**n*(d + e*x)**m*a*d*e**2*m + x**
n*(d + e*x)**m*a*e**3*m**3*x + 3*x**n*(d + e*x)**m*a*e**3*m**2*n*x + 5*x**
n*(d + e*x)**m*a*e**3*m**2*x + 3*x**n*(d + e*x)**m*a*e**3*m*n**2*x + 10*x*
*n*(d + e*x)**m*a*e**3*m*n*x + 6*x**n*(d + e*x)**m*a*e**3*m*x + x**n*(d +
e*x)**m*a*e**3*n**3*x + 5*x**n*(d + e*x)**m*a*e**3*n**2*x + 6*x**n*(d + e*
x)**m*a*e**3*n*x - x**n*(d + e*x)**m*b*d**2*e*m**2*n - x**n*(d + e*x)**m*b
*d**2*e*m**2 - x**n*(d + e*x)**m*b*d**2*e*m*n**2 - 4*x**n*(d + e*x)**m*b*d
**2*e*m*n - 3*x**n*(d + e*x)**m*b*d**2*e*m + x**n*(d + e*x)**m*b*d*e**2*m*
*3*x + 2*x**n*(d + e*x)**m*b*d*e**2*m**2*n*x + 3*x**n*(d + e*x)**m*b*d*e**
2*m**2*x + x**n*(d + e*x)**m*b*d*e**2*m*n**2*x + 3*x**n*(d + e*x)**m*b*d*e
**2*m*n*x + x**n*(d + e*x)**m*b*e**3*m**3*x**2 + 3*x**n*(d + e*x)**m*b*e**
3*m**2*n*x**2 + 4*x**n*(d + e*x)**m*b*e**3*m**2*x**2 + 3*x**n*(d + e*x)**m
*b*e**3*m*n**2*x**2 + 8*x**n*(d + e*x)**m*b*e**3*m*n*x**2 + 3*x**n*(d + e*
x)**m*b*e**3*m*x**2 + x**n*(d + e*x)**m*b*e**3*n**3*x**2 + 4*x**n*(d + e*x
)**m*b*e**3*n**2*x**2 + 3*x**n*(d + e*x)**m*b*e**3*n*x**2 + x**n*(d + e*x)
**m*c*d**3*m*n**2 + 3*x**n*(d + e*x)**m*c*d**3*m*n + 2*x**n*(d + e*x)**m*c
*d**3*m - x**n*(d + e*x)**m*c*d**2*e*m**2*n*x - 2*x**n*(d + e*x)**m*c*d**2
*e*m**2*x - x**n*(d + e*x)**m*c*d**2*e*m*n**2*x - 2*x**n*(d + e*x)**m*c...
```

3.132 $\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2} dx = \frac{2c(gx)^{1+n}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \operatorname{AppellF1}\left(1+n,-m,1,2+n,-\frac{ex}{d},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)g(1+n)} - \frac{2c(gx)^{1+n}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \operatorname{AppellF1}\left(1+n,-m,1,2+n,-\frac{ex}{d},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)g(1+n)}$$

output
```
2*c*(g*x)^(1+n)*(e*x+d)^m*AppellF1(1+n,1,-m,2+n,-2*c*x/(b-(-4*a*c+b^2)^(1/
2)),-e*x/d)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/g/(1+n)/((1+e*x/d)^m
)-2*c*(g*x)^(1+n)*(e*x+d)^m*AppellF1(1+n,1,-m,2+n,-2*c*x/(b+(-4*a*c+b^2)^(
1/2)),-e*x/d)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/g/(1+n)/((1+e*x/d)
^m)
```

Mathematica [F]

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx$$

input
```
Integrate[((g*x)^n*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
Integrate[((g*x)^n*(d + e*x)^m)/(a + b*x + c*x^2), x]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1205$$

$$\int \left(\frac{2c(gx)^n(d+ex)^m}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c(gx)^n(d+ex)^m}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2c(gx)^{n+1}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}\text{AppellF1}\left(n+1,-m,1,n+2,-\frac{ex}{d},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{g(n+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(gx)^{n+1}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}\text{AppellF1}\left(n+1,-m,1,n+2,-\frac{ex}{d},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{g(n+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((g*x)^n*(d + e*x)^m)/(a + b*x + c*x^2),x]
```

output
```
(2*c*(g*x)^(1 + n)*(d + e*x)^m*AppellF1[1 + n, -m, 1, 2 + n, -((e*x)/d), (
-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c
])*g*(1 + n)*(1 + (e*x)/d)^m) - (2*c*(g*x)^(1 + n)*(d + e*x)^m*AppellF1[1
+ n, -m, 1, 2 + n, -((e*x)/d), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^
2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*g*(1 + n)*(1 + (e*x)/d)^m)
```

Defintions of rubi rules used

rule 1205
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(gx)^n (ex+d)^m}{c\,x^2+bx+a}dx$$

input
```
int((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
int((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m(gx)^n}{cx^2+bx+a}\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*(g*x)^n/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**n*(e*x+d)**m/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m(gx)^n}{cx^2+bx+a}\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m*(g*x)^n/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx = \int \frac{(ex+d)^m(gx)^n}{cx^2+bx+a}\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*(g*x)^n/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx=\int \frac{(g\,x)^n\,(d+e\,x)^m}{c\,x^2+b\,x+a}\,dx$$

input
```
int(((g*x)^n*(d + e*x)^m)/(a + b*x + c*x^2),x)
```

output
```
int(((g*x)^n*(d + e*x)^m)/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{(gx)^n(d+ex)^m}{a+bx+cx^2}\,dx=g^n\left(\int \frac{x^n(ex+d)^m}{c\,x^2+bx+a}dx\right)$$

input
```
int((g*x)^n*(e*x+d)^m/(c*x^2+b*x+a),x)
```

output
```
g**n*int((x**n*(d + e*x)**m)/(a + b*x + c*x**2),x)
```

3.133 $\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 650

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$
$$= -\frac{(be(3+m+p)+cd(6+m+4p))(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{c^2e^2(3+m+2p)(4+m+2p)}$$
$$+\frac{(d+ex)^{2+m}\left(a+bx+cx^2\right)^{1+p}}{ce^2(4+m+2p)}$$
$$-\frac{\left(be(bd-ae)(3+m+p)+\frac{2c^2d^3\left(3+5p+2p^2\right)}{e(1+m)}+cd(3bd(1+p)-ae(6+m+4p))\right)(d+ex)^{1+m}\left(a+bx+\cdots\right.}{c^{\cdots}}$$
$$+\frac{\left(2c^2d^2\left(3+5p+2p^2\right)+b^2e^2\left(6+m^2+5p+p^2+m(5+2p)\right)-ce(ae(2+m)(3+m+2p)-bd(1+p\cdots\right.}{\cdots}$$

output
```
-(b*e*(3+m+p)+c*d*(6+m+4*p))*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(p+1)/c^2/e^2/(3+
m+2*p)/(4+m+2*p)+(e*x+d)^(2+m)*(c*x^2+b*x+a)^(p+1)/c/e^2/(4+m+2*p)-(b*e*(-
a*e+b*d)*(3+m+p)+2*c^2*d^3*(2*p^2+5*p+3)/e/(1+m)+c*d*(3*b*d*(p+1)-a*e*(6+m
+4*p)))*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(
2*c*d-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*
e))/c^2/e^3/(3+m+2*p)/(4+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/
2))*e))^p)/((1-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)+(2*c^2*d^2
*(2*p^2+5*p+3)+b^2*e^2*(6+m^2+5*p+p^2+m*(5+2*p))-c*e*(a*e*(2+m)*(3+m+2*p)-
b*d*(p+1)*(6+3*m+2*p)))*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3
+m,2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e))/c^2/e^4/(2+m)/(3+m+2*p)/(4+m+2*p)/((1-2*c*(e*x+d)/(2*c
*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/((1-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1
/2))*e))^p)
```

Mathematica [F]

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$

input
```
Integrate[x^3*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
Integrate[x^3*(d + e*x)^m*(a + b*x + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1291, 25, 2184, 25, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{1291}$$

$$\int -(d+ex)^m \left(cx^2+bx+a\right)^p \left(e^2(be(m+p+3)+cd(m+4p+6))x^2+e\left(2c(p+1)d^2+e(ae(m+2)+bd(m+\right.\right.$$

$$ce^3(m+2p+4)$$

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}$$

$\downarrow$ 25

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\int (d+ex)^m \left(cx^2+bx+a\right)^p \left(e^2(be(m+p+3)+cd(m+4p+6))x^2+e\left(2c(p+1)d^2+e(ae(m+2)+bd(m+2\right.\right.$$

$$ce^3(m+2p+4)$$

$\downarrow$ 2184

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\int -e^3(d+ex)^m\left(de(p+1)(m+p+3)b^2+ae^2(m+1)(m+p+3)b+cd^2\left(2p^2+5p+3\right)b+2acdem(p+1)+\left(2c^2\left(2p^2+5p+3\right)d^2+b^2e^2\left(m^2+(2p+5)m+p^2+\right.\right.\right.$$

$$ce^2(m+2p+3)$$

$$ce^3(m+2$$

$\downarrow$ 25

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\frac{e(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}(be(m+p+3)+cd(m+4p+6))}{c(m+2p+3)}-\int e^3(d+ex)^m\left(de(p+1)(m+p+3)b^2+ae^2(m+1)(m+p+3)b+cd^2\left(2p^2+5p+3\right)b+2a\right.$$

$$ce^3(m+2p$$

$\downarrow$ 27

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\frac{e(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}(be(m+p+3)+cd(m+4p+6))}{c(m+2p+3)}-e\int(d+ex)^m\left(de(p+1)(m+p+3)b^2+ae^2(m+1)(m+p+3)b+cd^2\left(2p^2+5p+3\right)b+2ac\right.$$

$$ce^3(m+2p$$

$\downarrow$ 1269

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\frac{e(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}(be(m+p+3)+cd(m+4p+6))}{c(m+2p+3)}-e\left(\frac{\left(-ce(ae(m+2)(m+2p+3)-bd(p+1)(3m+2p+6))+b^2e^2\left(m^2+m(2p+5)+p^2+5p+6\right)+2c\right.}{e}\right.$$

$$ce^3($$

$\downarrow$ 1179

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\frac{e(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}(be(m+p+3)+cd(m+4p+6))}{c(m+2p+3)} - e\left(\frac{\left(a+bx+cx^2\right)^p\left(-ce(ae(m+2)(m+2p+3)-bd(p+1)(3m+2p+6))+b^2e^2\left(m^2+m(2p+5)+\right.\right.}{}\right.$$

$\downarrow$ 150

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^{p+1}}{ce^2(m+2p+4)}-$$

$$\frac{e(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}(be(m+p+3)+cd(m+4p+6))}{c(m+2p+3)} - e\left(\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(-ce(ae(m+2)(m+2p+3)-bd(p+1)(3m+2p+6))+b^2e^2\left(m^2\right.\right.}{}\right.$$

input
```
Int[x^3*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(2 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e^2*(4 + m + 2*p)) - ((e*
(b*e*(3 + m + p) + c*d*(6 + m + 4*p))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^
(1 + p))/(c*(3 + m + 2*p)) - (e*(-(((b*e^2*(b*d - a*e)*(1 + m)*(3 + m + p)
 + 2*c^2*d^3*(3 + 5*p + 2*p^2) + c*d*e*(1 + m)*(3*b*d*(1 + p) - a*e*(6 + m
 + 4*p)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2
+ m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e))^p)) + ((2*c^2*d^2*(3 + 5*p + 2*p^2) + b^2*e^2*(6 + m^
2 + 5*p + p^2 + m*(5 + 2*p)) - c*e*(a*e*(2 + m)*(3 + m + 2*p) - b*d*(1 + p
)*(6 + 3*m + 2*p)))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m,
-p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(
d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e))^p)))/(c*(3 + m + 2*p)))/(c*e^3*(4 + m + 2*p
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int x^3(ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x^3(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p(ex+d)^m x^3 \, dx$$

input
```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (ex+d)^m x^3 \, dx$$

input
```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x^3, x)
```

Giac [F]

$$\int x^3(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (cx^2+bx+a)^p (ex+d)^m x^3 \, dx$$

input
```
integrate(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x^3\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int(x^3*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int(x^3*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x^3(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x^3(ex+d)^m\left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^3*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

3.134 $\int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 471

$$\int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}$$
$$+ \frac{\left(bd-ae+\frac{2cd^2(1+p)}{e(1+m)}\right)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-}}{ce^2(3+m+2p)}$$
$$- \frac{(2cd(1+p)+be(2+m+p))(d+ex)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex}{2cd-\left(b+\sqrt{b^2-}\right.}\right.}{ce^3(2+m)(3+m+2p}$$

output

```
(e*x+d)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/e/(3+m+2*p)+(b*d-a*e+2*c*d^2*(p+1)/e/(
1+m))*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*
c*d-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)/c/e^2/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/((1
-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)-(2*c*d*(p+1)+b*e*(2+m+p)
)*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/c/
e^3/(2+m)/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/(
(1-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)
```

Mathematica [F]

$$\int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$

input
```
Integrate[x^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
Integrate[x^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1291, 25, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1291}$$

$$\frac{\int -e(d+ex)^m(ae(m+1)+bd(p+1)+(2cd(p+1)+be(m+p+2))x) \left(cx^2+bx+a\right)^p dx}{ce^2(m+2p+3)}+\frac{(d+ex)^{m+1} \left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d+ex)^{m+1} \left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}-\frac{\int e(d+ex)^m(ae(m+1)+bd(p+1)+(2cd(p+1)+be(m+p+2))x) \left(cx^2+bx+a\right)^p dx}{ce^2(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}-\frac{\int (d+ex)^m(ae(m+1)+bd(p+1)+(2cd(p+1)+be(m+p+2))x)\left(cx^2+bx+a\right)^p dx}{ce(m+2p+3)}$$

$\big\downarrow$ 1269

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}-\frac{\frac{(be(m+p+2)+2cd(p+1))\int (d+ex)^{m+1}\left(cx^2+bx+a\right)^p dx}{e}-\frac{\left(e(m+1)(bd-ae)+2cd^2(p+1)\right)\int (d+ex)^m\left(cx^2+bx+a\right)^p dx}{e}}{ce(m+2p+3)}$$

$\big\downarrow$ 1179

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}-\frac{\frac{(a+bx+cx^2)^p(be(m+p+2)+2cd(p+1))\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\int (d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p\left(1-\right.}{e^2}}{}$$

$\big\downarrow$ 150

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}-\frac{\frac{(d+ex)^{m+2}(a+bx+cx^2)^p(be(m+p+2)+2cd(p+1))\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+2,-p,-p,m+3,\frac{}{2cd}\right.}{e^2(m+2)}}{}$$

input
```
Int[x^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output

```
((d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) - (-(((e
*(b*d - a*e)*(1 + m) + 2*c*d^2*(1 + p))*(d + e*x)^(1 + m)*(a + b*x + c*x^2
)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2
- 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(
1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2
*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)) + ((2*c*d*(1 + p) +
 b*e*(2 + m + p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p
, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d
+ e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*c*(d +
e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e))^p))/(c*e*(3 + m + 2*p))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1291
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int x^{2}(ex+d)^{m} \left(c\,x^{2}+bx+a\right)^{p} dx$$

input
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x^2(d+ex)^m \left(a+bx+cx^2\right)^p \, dx=\int (cx^2+bx+a)^p(ex+d)^m x^2 \, dx$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx = \int \left(cx^2+bx+a\right)^p(ex+d)^m x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x^2, x)
```

Giac [F]

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx = \int \left(cx^2+bx+a\right)^p(ex+d)^m x^2\,dx$$

input
```
integrate(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x^2\left(d+e\,x\right)^m\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int(x^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int(x^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x^2(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x^2(ex+d)^m\left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^2*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

3.135 $\int x(d+ex)^m (a+bx+cx^2)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 377

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = -\frac{d(d+ex)^{1+m}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \text{AppellF1}\left(1+m, \ldots\right)}{e^2(1+m)} + \frac{(d+ex)^{2+m}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \text{AppellF1}\left(2+m, \ldots\right)}{e^2(2+m)}$$

output
```
-d*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d
-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/e
^2/(1+m)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/((1-2*c*(e*x
+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)+(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*App
ellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+
d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/e^2/(2+m)/((1-2*c*(e*x+d)/(2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e))^p)/((1-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)^p)
```

Mathematica [F]

$$\int x(d+ex)^m\left(a+bx+cx^2\right)^p\,dx=\int x(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$

input
```
Integrate[x*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
Integrate[x*(d + e*x)^m*(a + b*x + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)^m\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{\int (d+ex)^{m+1}\left(cx^2+bx+a\right)^p\,dx}{e}-\frac{d\int (d+ex)^m\left(cx^2+bx+a\right)^p\,dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\int (d+ex)^{m+1}\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p}{e^2}$$

$$\frac{d\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\int (d+ex)^m\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+2,-p,-p,\right.}{e^2(m+2)}$$

$$\frac{d(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(m+1,-p,-p\right.}{e^2(m+1)}$$

input
```
Int[x*(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
-((d*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m,
(2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e))^p)) + ((d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 +
m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*
c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int x(ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m x \, dx$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x*(e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m x \, dx$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x, x)
```

Giac [F]

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m x \, dx$$

input
```
integrate(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int x\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p \, dx$$

input
```
int(x*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int(x*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x(d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int x(ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int(x*(e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

3.136 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 187

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{(d+ex)^{1+m}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\operatorname{AppellF1}\left(1+m,-p,\right.}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-(b
-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/e/(1
+m)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/((1-2*c*(e*x+d)/(
2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}(d+ex)^{1+m}(a+x(b+cx))^p\operatorname{AppellF1}\left(1+m,-p,-p,\right.}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \int (d+ex)^m \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^p \left(\right.}{e}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(d+ex)^{m+1} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,\right.}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c
*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^m \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int \left(cx^2+bx+a\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \int (d+e\,x)^m \left(c\,x^2+b\,x+a\right)^p \, dx$$

input
```
int((d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

output
```
int((d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

output
```
(2*(d + e*x)**m*(a + b*x + c*x**2)**p*a*e*p + (d + e*x)**m*(a + b*x + c*x*
*2)**p*b*d*m + (d + e*x)**m*(a + b*x + c*x**2)**p*b*d*p + (d + e*x)**m*(a
+ b*x + c*x**2)**p*b*e*m*x + (d + e*x)**m*(a + b*x + c*x**2)**p*b*e*p*x +
2*(d + e*x)**m*(a + b*x + c*x**2)**p*c*d*p*x - 2*int(((d + e*x)**m*(a + b*
x + c*x**2)**p*x**2)/(a*b*d*e*m**2 + 3*a*b*d*e*m*p + a*b*d*e*m + 2*a*b*d*e
*p**2 + a*b*d*e*p + a*b*e**2*m**2*x + 3*a*b*e**2*m*p*x + a*b*e**2*m*x + 2*
a*b*e**2*p**2*x + a*b*e**2*p*x + 2*a*c*d**2*m*p + 4*a*c*d**2*p**2 + 2*a*c*
d**2*p + 2*a*c*d*e*m*p*x + 4*a*c*d*e*p**2*x + 2*a*c*d*e*p*x + b**2*d*e*m**
2*x + 3*b**2*d*e*m*p*x + b**2*d*e*m*x + 2*b**2*d*e*p**2*x + b**2*d*e*p*x +
 b**2*e**2*m**2*x**2 + 3*b**2*e**2*m*p*x**2 + b**2*e**2*m*x**2 + 2*b**2*e*
*2*p**2*x**2 + b**2*e**2*p*x**2 + 2*b*c*d**2*m*p*x + 4*b*c*d**2*p**2*x + 2
*b*c*d**2*p*x + b*c*d*e*m**2*x**2 + 5*b*c*d*e*m*p*x**2 + b*c*d*e*m*x**2 +
6*b*c*d*e*p**2*x**2 + 3*b*c*d*e*p*x**2 + b*c*e**2*m**2*x**3 + 3*b*c*e**2*m
*p*x**3 + b*c*e**2*m*x**3 + 2*b*c*e**2*p**2*x**3 + b*c*e**2*p*x**3 + 2*c**
2*d**2*m*p*x**2 + 4*c**2*d**2*p**2*x**2 + 2*c**2*d**2*p*x**2 + 2*c**2*d*e*
m*p*x**3 + 4*c**2*d*e*p**2*x**3 + 2*c**2*d*e*p*x**3),x)*a*b*c*e**3*m**3*p
- 10*int(((d + e*x)**m*(a + b*x + c*x**2)**p*x**2)/(a*b*d*e*m**2 + 3*a*b*d
*e*m*p + a*b*d*e*m + 2*a*b*d*e*p**2 + a*b*d*e*p + a*b*e**2*m**2*x + 3*a*b*
e**2*m*p*x + a*b*e**2*m*x + 2*a*b*e**2*p**2*x + a*b*e**2*p*x + 2*a*c*d**2*
m*p + 4*a*c*d**2*p**2 + 2*a*c*d**2*p + 2*a*c*d*e*m*p*x + 4*a*c*d*e*p**2...
```

3.137 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x} \, dx = \text{Int}\left(\frac{(d+ex)^m (a+bx+cx^2)^p}{x}, x\right)$$

output
```
Defer(Int)((e*x+d)^m*(c*x^2+b*x+a)^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x} \, dx = \int \frac{(d+ex)^m (a+bx+cx^2)^p}{x} \, dx$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/x,x]
```

output
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x} \, dx$$

$$\Big\downarrow \text{ 1292}$$

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x}dx$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)^p}{x}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x}\,dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x}\,dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/x, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{x}\,dx = \int \frac{(cx^2+bx+a)^p(ex+d)^m}{x}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/x, x)
```

Mupad [N/A]

Not integrable

Time = 10.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{x}\,dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^p}{x}\,dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/x,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 4722, normalized size of antiderivative = 205.30

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x,x)
```

output
```
((d + e*x)**m*(a + b*x + c*x**2)**p*b*e + (d + e*x)**m*(a + b*x + c*x**2)*
*p*c*d - int(((d + e*x)**m*(a + b*x + c*x**2)**p*x**2)/(a*b*d*e*m + a*b*d*
e*p + a*b*e**2*m*x + a*b*e**2*p*x + 2*a*c*d**2*p + 2*a*c*d*e*p*x + b**2*d*
e*m*x + b**2*d*e*p*x + b**2*e**2*m*x**2 + b**2*e**2*p*x**2 + 2*b*c*d**2*p*
x + b*c*d*e*m*x**2 + 3*b*c*d*e*p*x**2 + b*c*e**2*m*x**3 + b*c*e**2*p*x**3
+ 2*c**2*d**2*p*x**2 + 2*c**2*d*e*p*x**3),x)*b**2*c*e**3*m*p - int(((d + e
*x)**m*(a + b*x + c*x**2)**p*x**2)/(a*b*d*e*m + a*b*d*e*p + a*b*e**2*m*x +
 a*b*e**2*p*x + 2*a*c*d**2*p + 2*a*c*d*e*p*x + b**2*d*e*m*x + b**2*d*e*p*x
 + b**2*e**2*m*x**2 + b**2*e**2*p*x**2 + 2*b*c*d**2*p*x + b*c*d*e*m*x**2 +
 3*b*c*d*e*p*x**2 + b*c*e**2*m*x**3 + b*c*e**2*p*x**3 + 2*c**2*d**2*p*x**2
 + 2*c**2*d*e*p*x**3),x)*b**2*c*e**3*p**2 - int(((d + e*x)**m*(a + b*x + c
*x**2)**p*x**2)/(a*b*d*e*m + a*b*d*e*p + a*b*e**2*m*x + a*b*e**2*p*x + 2*a
*c*d**2*p + 2*a*c*d*e*p*x + b**2*d*e*m*x + b**2*d*e*p*x + b**2*e**2*m*x**2
 + b**2*e**2*p*x**2 + 2*b*c*d**2*p*x + b*c*d*e*m*x**2 + 3*b*c*d*e*p*x**2 +
 b*c*e**2*m*x**3 + b*c*e**2*p*x**3 + 2*c**2*d**2*p*x**2 + 2*c**2*d*e*p*x**
3),x)*b*c**2*d*e**2*m**2 - int(((d + e*x)**m*(a + b*x + c*x**2)**p*x**2)/(
a*b*d*e*m + a*b*d*e*p + a*b*e**2*m*x + a*b*e**2*p*x + 2*a*c*d**2*p + 2*a*c
*d*e*p*x + b**2*d*e*m*x + b**2*d*e*p*x + b**2*e**2*m*x**2 + b**2*e**2*p*x*
*2 + 2*b*c*d**2*p*x + b*c*d*e*m*x**2 + 3*b*c*d*e*p*x**2 + b*c*e**2*m*x**3
+ b*c*e**2*p*x**3 + 2*c**2*d**2*p*x**2 + 2*c**2*d*e*p*x**3),x)*b*c**2*d...
```

3.138 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{x^2}\,dx = \operatorname{Int}\left(\frac{(d+ex)^m\,(a+bx+cx^2)^p}{x^2},x\right)$$

output
```
Defer(Int)((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{x^2}\,dx = \int \frac{(d+ex)^m\,(a+bx+cx^2)^p}{x^2}\,dx$$

input
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/x^2,x]
```

output
```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1292}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^p}{x^2} \, dx$$

$$\Big\downarrow \text{1292}$$

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^p}{x^2}dx$$

input
```
Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m \left(c\,x^2+bx+a\right)^p}{x^2}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{x^2} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p*(e*x + d)^m/x^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x+a)**p/x**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{x^2} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} \, dx = \int \frac{(cx^2+bx+a)^p (ex+d)^m}{x^2} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 10.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} \, dx = \int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^p}{x^2} \, dx$$

input
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/x^2,x)
```

output
```
int(((d + e*x)^m*(a + b*x + c*x^2)^p)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 8766, normalized size of antiderivative = 381.13

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{x^2} dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+b*x+a)^p/x^2,x)
```

output
```
( - (d + e*x)**m*(a + b*x + c*x**2)**p + int(((d + e*x)**m*(a + b*x + c*x*
*2)**p*x)/(a*b*d*e*m + a*b*d*e*p - a*b*d*e + a*b*e**2*m*x + a*b*e**2*p*x -
 a*b*e**2*x + 2*a*c*d**2*p - a*c*d**2 + 2*a*c*d*e*p*x - a*c*d*e*x + b**2*d
*e*m*x + b**2*d*e*p*x - b**2*d*e*x + b**2*e**2*m*x**2 + b**2*e**2*p*x**2 -
 b**2*e**2*x**2 + 2*b*c*d**2*p*x - b*c*d**2*x + b*c*d*e*m*x**2 + 3*b*c*d*e
*p*x**2 - 2*b*c*d*e*x**2 + b*c*e**2*m*x**3 + b*c*e**2*p*x**3 - b*c*e**2*x*
*3 + 2*c**2*d**2*p*x**2 - c**2*d**2*x**2 + 2*c**2*d*e*p*x**3 - c**2*d*e*x*
*3),x)*b*c*e**2*m**2*x + 3*int(((d + e*x)**m*(a + b*x + c*x**2)**p*x)/(a*b
*d*e*m + a*b*d*e*p - a*b*d*e + a*b*e**2*m*x + a*b*e**2*p*x - a*b*e**2*x +
2*a*c*d**2*p - a*c*d**2 + 2*a*c*d*e*p*x - a*c*d*e*x + b**2*d*e*m*x + b**2*
d*e*p*x - b**2*d*e*x + b**2*e**2*m*x**2 + b**2*e**2*p*x**2 - b**2*e**2*x**
2 + 2*b*c*d**2*p*x - b*c*d**2*x + b*c*d*e*m*x**2 + 3*b*c*d*e*p*x**2 - 2*b*
c*d*e*x**2 + b*c*e**2*m*x**3 + b*c*e**2*p*x**3 - b*c*e**2*x**3 + 2*c**2*d*
*2*p*x**2 - c**2*d**2*x**2 + 2*c**2*d*e*p*x**3 - c**2*d*e*x**3),x)*b*c*e**
2*m*p*x - int(((d + e*x)**m*(a + b*x + c*x**2)**p*x)/(a*b*d*e*m + a*b*d*e*
p - a*b*d*e + a*b*e**2*m*x + a*b*e**2*p*x - a*b*e**2*x + 2*a*c*d**2*p - a*
c*d**2 + 2*a*c*d*e*p*x - a*c*d*e*x + b**2*d*e*m*x + b**2*d*e*p*x - b**2*d*
e*x + b**2*e**2*m*x**2 + b**2*e**2*p*x**2 - b**2*e**2*x**2 + 2*b*c*d**2*p*
x - b*c*d**2*x + b*c*d*e*m*x**2 + 3*b*c*d*e*p*x**2 - 2*b*c*d*e*x**2 + b*c*
e**2*m*x**3 + b*c*e**2*p*x**3 - b*c*e**2*x**3 + 2*c**2*d**2*p*x**2 - c*...
```

3.139 $\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{6}a(2Ab+aB)x^6+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{8}b^2Bx^8$$

output
```
1/5*a^2*A*x^5+1/6*a*(2*A*b+B*a)*x^6+1/7*b*(A*b+2*B*a)*x^7+1/8*b^2*B*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{6}a(2Ab+aB)x^6+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{8}b^2Bx^8$$

input
```
Integrate[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^6)/6 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^8)/8
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx+b^2x^2\right) (A+Bx) \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^2x^4(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^4(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax^4+bx^6(2aB+Ab)+ax^5(aB+2Ab)+b^2Bx^7\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}a^2Ax^5+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{6}ax^6(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

input
```
Int[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^6)/6 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^8)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{5} \left(105 x^{3} B \,b^{2}+120 x^{2} b^{2} A +240 B a \,x^{2} b +280 a b A x +140 a^{2} B x +168 a^{2} A \right)}{840}$	52
default	$\frac{b^{2} B \,x^{8}}{8}+\frac{\left(b^{2} A +2 a b B \right) x^{7}}{7}+\frac{\left(2 a b A +a^{2} B \right) x^{6}}{6}+\frac{a^{2} A \,x^{5}}{5}$	52
norman	$\frac{b^{2} B \,x^{8}}{8}+\left(\frac{1}{7} b^{2} A +\frac{2}{7} a b B \right) x^{7}+\left(\frac{1}{3} a b A +\frac{1}{6} a^{2} B \right) x^{6}+\frac{a^{2} A \,x^{5}}{5}$	52
risch	$\frac{1}{8} b^{2} B \,x^{8}+\frac{1}{7} A \,b^{2} x^{7}+\frac{2}{7} x^{7} a b B +\frac{1}{3} x^{6} a b A +\frac{1}{6} x^{6} a^{2} B +\frac{1}{5} a^{2} A \,x^{5}$	54
parallelrisch	$\frac{1}{8} b^{2} B \,x^{8}+\frac{1}{7} A \,b^{2} x^{7}+\frac{2}{7} x^{7} a b B +\frac{1}{3} x^{6} a b A +\frac{1}{6} x^{6} a^{2} B +\frac{1}{5} a^{2} A \,x^{5}$	54
orering	$\frac{x^{5} \left(105 x^{3} B \,b^{2}+120 x^{2} b^{2} A +240 B a \,x^{2} b +280 a b A x +140 a^{2} B x +168 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{840 \left(b x +a \right)^{2}}$	75

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/840*x^5*(105*B*b^2*x^3+120*A*b^2*x^2+240*B*a*b*x^2+280*A*a*b*x+140*B*a^2
*x+168*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{8}\,Bb^2x^8+\frac{1}{5}\,Aa^2x^5+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{6}\,(Ba^2+2\,Aab)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/8*B*b^2*x^8 + 1/5*A*a^2*x^5 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/6*(B*a^2 + 2
*A*a*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{Aa^2x^5}{5}+\frac{Bb^2x^8}{8}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

input
```
integrate(x**4*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*a**2*x**5/5 + B*b**2*x**8/8 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**6*(A*a*b/
3 + B*a**2/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{8}\,Bb^2x^8+\frac{1}{5}\,Aa^2x^5+\frac{1}{7}\,\left(2\,Bab+Ab^2\right)x^7+\frac{1}{6}\,\left(Ba^2+2\,Aab\right)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/8*B*b^2*x^8 + 1/5*A*a^2*x^5 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/6*(B*a^2 + 2
*A*a*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{8}\,Bb^2x^8+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{6}\,Ba^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{5}\,Aa^2x^5$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/8*B*b^2*x^8 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/6*B*a^2*x^6 + 1/3*A*a*b*
x^6 + 1/5*A*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=x^6\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{A\,a^2\,x^5}{5}+\frac{B\,b^2\,x^8}{8}$$

input
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^6*((B*a^2)/6 + (A*a*b)/3) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (A*a^2*x^5)/
5 + (B*b^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x^5(35b^3x^3+120a\,b^2x^2+140a^2bx+56a^3)}{280}$$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**5*(56*a**3 + 140*a**2*b*x + 120*a*b**2*x**2 + 35*b**3*x**3))/280
```

3.140 $\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{7}b^2Bx^7$$

output
```
1/4*a^2*A*x^4+1/5*a*(2*A*b+B*a)*x^5+1/6*b*(A*b+2*B*a)*x^6+1/7*b^2*B*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{7}b^2Bx^7$$

input
```
Integrate[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^4)/4 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B
*x^7)/7
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx+b^2x^2\right) (A+Bx) \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^2x^3(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^3(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax^3+bx^5(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^6\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^2Ax^4+\frac{1}{6}bx^6(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input
```
Int[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^4)/4 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B
*x^7)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{4} \left(60 x^{3} B \,b^{2}+70 x^{2} b^{2} A +140 B a \,x^{2} b +168 a b A x +84 a^{2} B x +105 a^{2} A \right)}{420}$	52
default	$\frac{b^{2} B \,x^{7}}{7}+\frac{\left(b^{2} A +2 a b B \right) x^{6}}{6}+\frac{\left(2 a b A +a^{2} B \right) x^{5}}{5}+\frac{a^{2} A \,x^{4}}{4}$	52
norman	$\frac{b^{2} B \,x^{7}}{7}+\left(\frac{1}{6} b^{2} A +\frac{1}{3} a b B \right) x^{6}+\left(\frac{2}{5} a b A +\frac{1}{5} a^{2} B \right) x^{5}+\frac{a^{2} A \,x^{4}}{4}$	52
risch	$\frac{1}{7} b^{2} B \,x^{7}+\frac{1}{6} x^{6} b^{2} A +\frac{1}{3} B b \,x^{6} a +\frac{2}{5} x^{5} a b A +\frac{1}{5} x^{5} a^{2} B +\frac{1}{4} a^{2} A \,x^{4}$	54
parallelrisch	$\frac{1}{7} b^{2} B \,x^{7}+\frac{1}{6} x^{6} b^{2} A +\frac{1}{3} B b \,x^{6} a +\frac{2}{5} x^{5} a b A +\frac{1}{5} x^{5} a^{2} B +\frac{1}{4} a^{2} A \,x^{4}$	54
orering	$\frac{x^{4} \left(60 x^{3} B \,b^{2}+70 x^{2} b^{2} A +140 B a \,x^{2} b +168 a b A x +84 a^{2} B x +105 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{420 \left(b x +a \right)^{2}}$	75

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/420*x^4*(60*B*b^2*x^3+70*A*b^2*x^2+140*B*a*b*x^2+168*A*a*b*x+84*B*a^2*x+
105*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(2\,Bab+Ab^2)x^6+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/7*B*b^2*x^7 + 1/4*A*a^2*x^4 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{Aa^2x^4}{4}+\frac{Bb^2x^7}{7}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate(x**3*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*a**2*x**4/4 + B*b**2*x**7/7 + x**6*(A*b**2/6 + B*a*b/3) + x**5*(2*A*a*b/
5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{1}{7}\,Bb^2x^7 + \frac{1}{4}\,Aa^2x^4 + \frac{1}{6}\,(2\,Bab + Ab^2)x^6 + \frac{1}{5}\,(Ba^2 + 2\,Aab)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/4*A*a^2*x^4 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{1}{7}\,Bb^2x^7 + \frac{1}{3}\,Babx^6 + \frac{1}{6}\,Ab^2x^6 + \frac{1}{5}\,Ba^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{4}\,Aa^2x^4$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/5*B*a^2*x^5 + 2/5*A*a*b*
x^5 + 1/4*A*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right) + x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right) + \frac{A\,a^2\,x^4}{4} + \frac{B\,b^2\,x^7}{7}$$

input
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/6 + (B*a*b)/3) + (A*a^2*x^4)/
4 + (B*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x^4(20b^3x^3+70a\,b^2x^2+84a^2bx+35a^3)}{140}$$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**4*(35*a**3 + 84*a**2*b*x + 70*a*b**2*x**2 + 20*b**3*x**3))/140
```

3.141 $\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{6}b^2Bx^6$$

output
```
1/3*a^2*A*x^3+1/4*a*(2*A*b+B*a)*x^4+1/5*b*(A*b+2*B*a)*x^5+1/6*b^2*B*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{60}x^3\left(5a^2(4A+3Bx)+6abx(5A+4Bx)+2b^2x^2(6A+5Bx)\right)$$

input
```
Integrate[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(x^3*(5*a^2*(4*A + 3*B*x) + 6*a*b*x*(5*A + 4*B*x) + 2*b^2*x^2*(6*A + 5*B*x
)))/60
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right) (A+Bx) \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^2x^2(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int x^2(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax^2+bx^4(2aB+Ab)+ax^3(aB+2Ab)+b^2Bx^5\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

input
```
Int[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B
*x^6)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{3} \left(10 x^{3} B \,b^{2}+12 x^{2} b^{2} A +24 B a \,x^{2} b +30 a b A x +15 a^{2} B x +20 a^{2} A \right)}{60}$	52
default	$\frac{B \,b^{2} x^{6}}{6}+\frac{\left(b^{2} A +2 a b B \right) x^{5}}{5}+\frac{\left(2 a b A +a^{2} B \right) x^{4}}{4}+\frac{a^{2} A \,x^{3}}{3}$	52
norman	$\frac{B \,b^{2} x^{6}}{6}+\left(\frac{1}{5} b^{2} A +\frac{2}{5} a b B \right) x^{5}+\left(\frac{1}{2} a b A +\frac{1}{4} a^{2} B \right) x^{4}+\frac{a^{2} A \,x^{3}}{3}$	52
risch	$\frac{1}{6} B \,b^{2} x^{6}+\frac{1}{5} A \,b^{2} x^{5}+\frac{2}{5} B a b \,x^{5}+\frac{1}{2} A a b \,x^{4}+\frac{1}{4} a^{2} B \,x^{4}+\frac{1}{3} a^{2} A \,x^{3}$	54
parallelrisch	$\frac{1}{6} B \,b^{2} x^{6}+\frac{1}{5} A \,b^{2} x^{5}+\frac{2}{5} B a b \,x^{5}+\frac{1}{2} A a b \,x^{4}+\frac{1}{4} a^{2} B \,x^{4}+\frac{1}{3} a^{2} A \,x^{3}$	54
orering	$\frac{x^{3} \left(10 x^{3} B \,b^{2}+12 x^{2} b^{2} A +24 B a \,x^{2} b +30 a b A x +15 a^{2} B x +20 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{60 \left(b x +a \right)^{2}}$	75

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*B*b^2*x^3+12*A*b^2*x^2+24*B*a*b*x^2+30*A*a*b*x+15*B*a^2*x+20*
A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(2\,Bab+Ab^2)x^5+\frac{1}{4}\,(Ba^2+2\,Aab)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/6*B*b^2*x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{Aa^2x^3}{3}+\frac{Bb^2x^6}{6}+x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**6/6 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**4*(A*a*b/
2 + B*a**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{1}{6}\,Bb^2x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,\left(2\,Bab+Ab^2\right)x^5+\frac{1}{4}\,\left(Ba^2+2\,Aab\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{1}{6}\,Bb^2x^6+\frac{2}{5}\,Babx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{4}\,Ba^2x^4+\frac{1}{2}\,Aabx^4+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/4*B*a^2*x^4 + 1/2*A*a*b*
x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = x^4\,\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^5\,\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^6}{6}$$

input
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (A*a^2*x^3)/
3 + (B*b^2*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x^3(10b^3x^3+36a\,b^2x^2+45a^2bx+20a^3)}{60}$$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**3*(20*a**3 + 45*a**2*b*x + 36*a*b**2*x**2 + 10*b**3*x**3))/60
```

3.142 $\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{5}b^2Bx^5$$

output
```
1/2*a^2*A*x^2+1/3*a*(2*A*b+B*a)*x^3+1/4*b*(A*b+2*B*a)*x^4+1/5*b^2*B*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{60}x^2\left(10a^2(3A+2Bx)+10abx(4A+3Bx)+3b^2x^2(5A+4Bx)\right)$$

input
```
Integrate[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(x^2*(10*a^2*(3*A + 2*B*x) + 10*a*b*x*(4*A + 3*B*x) + 3*b^2*x^2*(5*A + 4*B
*x)))/60
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^2x(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int x(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax+bx^3(2aB+Ab)+ax^2(aB+2Ab)+b^2Bx^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{4}bx^4(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

input
```
Int[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B
*x^5)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{2} \left(12 x^{3} B \,b^{2}+15 x^{2} b^{2} A +30 B a \,x^{2} b +40 a b A x +20 a^{2} B x +30 a^{2} A \right)}{60}$	52
default	$\frac{B \,b^{2} x^{5}}{5}+\frac{\left(b^{2} A +2 a b B \right) x^{4}}{4}+\frac{\left(2 a b A +a^{2} B \right) x^{3}}{3}+\frac{a^{2} A \,x^{2}}{2}$	52
norman	$\frac{B \,b^{2} x^{5}}{5}+\left(\frac{1}{4} b^{2} A +\frac{1}{2} a b B \right) x^{4}+\left(\frac{2}{3} a b A +\frac{1}{3} a^{2} B \right) x^{3}+\frac{a^{2} A \,x^{2}}{2}$	52
risch	$\frac{1}{5} B \,b^{2} x^{5}+\frac{1}{4} b^{2} A \,x^{4}+\frac{1}{2} x^{4} a b B +\frac{2}{3} A b \,x^{3} a +\frac{1}{3} a^{2} B \,x^{3}+\frac{1}{2} a^{2} A \,x^{2}$	54
parallelrisch	$\frac{1}{5} B \,b^{2} x^{5}+\frac{1}{4} b^{2} A \,x^{4}+\frac{1}{2} x^{4} a b B +\frac{2}{3} A b \,x^{3} a +\frac{1}{3} a^{2} B \,x^{3}+\frac{1}{2} a^{2} A \,x^{2}$	54
orering	$\frac{x^{2} \left(12 x^{3} B \,b^{2}+15 x^{2} b^{2} A +30 B a \,x^{2} b +40 a b A x +20 a^{2} B x +30 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{60 \left(b x +a \right)^{2}}$	75

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^2*(12*B*b^2*x^3+15*A*b^2*x^2+30*B*a*b*x^2+40*A*a*b*x+20*B*a^2*x+30*
A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,\left(2\,Bab+Ab^2\right)x^4+\frac{1}{3}\,\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/5*B*b^2*x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{Aa^2x^2}{2}+\frac{Bb^2x^5}{5}+x^4\left(\frac{Ab^2}{4}+\frac{Bab}{2}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate(x*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*a**2*x**2/2 + B*b**2*x**5/5 + x**4*(A*b**2/4 + B*a*b/2) + x**3*(2*A*a*b/
3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(2\,Bab+Ab^2\right)x^4+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/5*B*b^2*x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Babx^4+\frac{1}{4}\,Ab^2x^4+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/5*B*b^2*x^5 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^5}{5}$$

input
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^4*((A*b^2)/4 + (B*a*b)/2) + (A*a^2*x^2)/
2 + (B*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x^2(4b^3x^3+15a\,b^2x^2+20a^2bx+10a^3)}{20}$$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**2*(10*a**3 + 20*a**2*b*x + 15*a*b**2*x**2 + 4*b**3*x**3))/20
```

3.143 $\int (A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = \frac{(Ab-aB)(a+bx)^3}{3b^2}+\frac{B(a+bx)^4}{4b^2}$$

output
```
1/3*(A*b-B*a)*(b*x+a)^3/b^2+1/4*B*(b*x+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = \frac{1}{12}x\left(6a^2(2A+Bx)+4abx(3A+2Bx)+b^2x^2(4A+3Bx)\right)$$

input
```
Integrate[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(x*(6*a^2*(2*A + B*x) + 4*a*b*x*(3*A + 2*B*x) + b^2*x^2*(4*A + 3*B*x)))/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1098}$$

$$\frac{\int b^2(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int (a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{(a+bx)^2(Ab-aB)}{b}+\frac{B(a+bx)^3}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^3(Ab-aB)}{3b^2}+\frac{B(a+bx)^4}{4b^2}$$

input
```
Int[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*(a + b*x)^3)/(3*b^2) + (B*(a + b*x)^4)/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{x^4 B\,b^2}{4}+\left(\frac{1}{3}b^2 A+\frac{2}{3}abB\right)x^3+\left(abA+\frac{1}{2}a^2 B\right)x^2+a^2 Ax$	48
default	$\frac{x^4 B\,b^2}{4}+\frac{(b^2 A+2abB)x^3}{3}+\frac{(2abA+a^2 B)x^2}{2}+a^2 Ax$	49
gosper	$\frac{x(3x^3 B\,b^2+4x^2 b^2 A+8Ba\,x^2 b+12abAx+6a^2 Bx+12a^2 A)}{12}$	50
risch	$\frac{1}{4}x^4 B\,b^2+\frac{1}{3}A\,x^3 b^2+\frac{2}{3}abB\,x^3+abA\,x^2+\frac{1}{2}B\,a^2 x^2+a^2 Ax$	50
parallelrisch	$\frac{1}{4}x^4 B\,b^2+\frac{1}{3}A\,x^3 b^2+\frac{2}{3}abB\,x^3+abA\,x^2+\frac{1}{2}B\,a^2 x^2+a^2 Ax$	50
orering	$\frac{x(3x^3 B\,b^2+4x^2 b^2 A+8Ba\,x^2 b+12abAx+6a^2 Bx+12a^2 A)(b^2 x^2+2abx+a^2)}{12(bx+a)^2}$	73

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*B*b^2+(1/3*b^2*A+2/3*a*b*B)*x^3+(a*b*A+1/2*a^2*B)*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{4}\,Bb^2x^4+Aa^2x+\frac{1}{3}\left(2\,Bab+Ab^2\right)x^3+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/4*B*b^2*x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=Aa^2x+\frac{Bb^2x^4}{4}+x^3\left(\frac{Ab^2}{3}+\frac{2Bab}{3}\right)+x^2\left(Aab+\frac{Ba^2}{2}\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*a**2*x + B*b**2*x**4/4 + x**3*(A*b**2/3 + 2*B*a*b/3) + x**2*(A*a*b + B*a
**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{1}{4}\,Bb^2x^4+Aa^2x+\frac{1}{3}\left(2\,Bab+Ab^2\right)x^3+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
1/4*B*b^2*x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{1}{4}Bb^2x^4+\frac{2}{3}Babx^3+\frac{1}{3}Ab^2x^3+\frac{1}{2}Ba^2x^2+Aabx^2+Aa^2x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
1/4*B*b^2*x^4 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + 1/2*B*a^2*x^2 + A*a*b*x^2
+ A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = x^2\left(\frac{B\,a^2}{2}+A\,b\,a\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{B\,b^2\,x^4}{4}+A\,a^2\,x$$

input
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^3*((A*b^2)/3 + (2*B*a*b)/3) + (B*b^2*x^4)/4 +
A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4
```

3.144 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx=2aAbx+\frac{1}{2}Ab^2x^2+\frac{B(a+bx)^3}{3b}+a^2A\log(x)$$

output
```
2*a*A*b*x+1/2*A*b^2*x^2+1/3*B*(b*x+a)^3/b+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx=a^2Bx+abx(2A+Bx)+\frac{1}{6}b^2x^2(3A+2Bx)+a^2A\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x,x]
```

output
```
a^2*B*x + a*b*x*(2*A + B*x) + (b^2*x^2*(3*A + 2*B*x))/6 + a^2*A*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1184, 27, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx + b^2x^2)(A + Bx)}{x} dx$$

$$\Big\downarrow \ 1184$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x} dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a + bx)^2(A + Bx)}{x} dx$$

$$\Big\downarrow \ 90$$

$$A \int \frac{(a + bx)^2}{x} dx + \frac{B(a + bx)^3}{3b}$$

$$\Big\downarrow \ 49$$

$$A \int \left(\frac{a^2}{x} + 2ba + b^2x\right) dx + \frac{B(a + bx)^3}{3b}$$

$$\Big\downarrow \ 2009$$

$$A\left(a^2 \log(x) + 2abx + \frac{b^2x^2}{2}\right) + \frac{B(a + bx)^3}{3b}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x,x]
```

output
```
(B*(a + b*x)^3)/(3*b) + A*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{x^{3} B \,b^{2}}{3}+\frac{x^{2} b^{2} A}{2}+B a \,x^{2} b +2 a b A x +a^{2} B x +a^{2} A \ln \left(x \right)$	46
norman	$\left(\frac{1}{2} b^{2} A +a b B \right) x^{2}+\left(2 a b A +a^{2} B \right) x +\frac{x^{3} B \,b^{2}}{3}+a^{2} A \ln \left(x \right)$	46
risch	$\frac{x^{3} B \,b^{2}}{3}+\frac{x^{2} b^{2} A}{2}+B a \,x^{2} b +2 a b A x +a^{2} B x +a^{2} A \ln \left(x \right)$	46
parallelrisch	$\frac{x^{3} B \,b^{2}}{3}+\frac{x^{2} b^{2} A}{2}+B a \,x^{2} b +2 a b A x +a^{2} B x +a^{2} A \ln \left(x \right)$	46

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*B*b^2+1/2*x^2*b^2*A+B*a*x^2*b+2*a*b*A*x+a^2*B*x+a^2*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx = \frac{1}{3}\,Bb^2x^3 + Aa^2\log\left(x\right) + \frac{1}{2}\,\left(2\,Bab + Ab^2\right)x^2 + \left(Ba^2 + 2\,Aab\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="fricas")
```

output
```
1/3*B*b^2*x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx = Aa^2\log\left(x\right) + \frac{Bb^2x^3}{3} + x^2\left(\frac{Ab^2}{2} + Bab\right) + x\left(2Aab + Ba^2\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x,x)
```

output
```
A*a**2*log(x) + B*b**2*x**3/3 + x**2*(A*b**2/2 + B*a*b) + x*(2*A*a*b + B*a
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x}\,dx = \frac{1}{3}\,Bb^2x^3 + Aa^2\log\left(x\right) + \frac{1}{2}\,\left(2\,Bab + Ab^2\right)x^2 + \left(Ba^2 + 2\,Aab\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="maxima")
```

output
```
1/3*B*b^2*x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x}\,dx = \frac{1}{3}\,Bb^2x^3 + Babx^2 + \frac{1}{2}\,Ab^2x^2 + Ba^2x + 2\,Aabx + Aa^2\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x,x, algorithm="giac")
```

output
```
1/3*B*b^2*x^3 + B*a*b*x^2 + 1/2*A*b^2*x^2 + B*a^2*x + 2*A*a*b*x + A*a^2*lo
g(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x}\,dx = x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) + x\,\left(B\,a^2 + 2\,A\,b\,a\right) + \frac{B\,b^2\,x^3}{3} + A\,a^2\,\ln\left(x\right)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x,x)
```

output
```
x^2*((A*b^2)/2 + B*a*b) + x*(B*a^2 + 2*A*a*b) + (B*b^2*x^3)/3 + A*a^2*log(
x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x}\,dx = \log(x)\,a^3+3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x,x)
```

output
```
(6*log(x)*a**3 + 18*a**2*b*x + 9*a*b**2*x**2 + 2*b**3*x**3)/6
```

3.145 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^2} dx = -\frac{a^2A}{x}+b(Ab+2aB)x+\frac{1}{2}b^2Bx^2+a(2Ab+aB)\log(x)$$

output
```
-a^2*A/x+b*(A*b+2*B*a)*x+1/2*b^2*B*x^2+a*(2*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^2} dx = -\frac{a^2A}{x}+2abBx+\frac{1}{2}b^2x(2A+Bx)+a(2Ab+aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^2,x]
```

output
```
-((a^2*A)/x) + 2*a*b*B*x + (b^2*x*(2*A + B*x))/2 + a*(2*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^2}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^2}+\frac{a(aB+2Ab)}{x}+b(2aB+Ab)+b^2Bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+bx(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{2}b^2Bx^2$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^2,x]
```

output
```
-((a^2*A)/x) + b*(A*b + 2*a*B)*x + (b^2*B*x^2)/2 + a*(2*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x^2 B\, b^2}{2}+x\, b^2 A+2xabB+a(2Ab+Ba)\ln\left(x\right)-\frac{a^2 A}{x}$	44
risch	$\frac{x^2 B\, b^2}{2}+x\, b^2 A+2xabB-\frac{a^2 A}{x}+2A\ln\left(x\right)ab+B\ln\left(x\right)a^2$	46
norman	$\frac{\left(b^2 A+2abB\right)x^2-a^2 A+\frac{x^3 B\, b^2}{2}}{x}+\left(2abA+a^2 B\right)\ln\left(x\right)$	51
parallelrisch	$\frac{x^3 B\, b^2+4A\ln\left(x\right)xab+2x^2 b^2 A+2B\ln\left(x\right)x\, a^2+4Ba\, x^2 b-2a^2 A}{2x}$	55

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*B*b^2+x*b^2*A+2*x*a*b*B+a*(2*A*b+B*a)*ln(x)-a^2*A/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx = \frac{Bb^2x^3-2\,Aa^2+2\,(2\,Bab+Ab^2)x^2+2\,(Ba^2+2\,Aab)x\log{(x)}}{2\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(B*b^2*x^3 - 2*A*a^2 + 2*(2*B*a*b + A*b^2)*x^2 + 2*(B*a^2 + 2*A*a*b)*x
*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx = -\frac{Aa^2}{x}+\frac{Bb^2x^2}{2}+a(2Ab+Ba)\log{(x)} + x\left(Ab^2+2Bab\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**2/2 + a*(2*A*b + B*a)*log(x) + x*(A*b**2 + 2*B*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx = \frac{1}{2}\,Bb^2x^2-\frac{Aa^2}{x}+\left(2\,Bab+Ab^2\right)x + \left(Ba^2+2\,Aab\right)\log{(x)}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="maxima")
```

output
```
1/2*B*b^2*x^2 - A*a^2/x + (2*B*a*b + A*b^2)*x + (B*a^2 + 2*A*a*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^2}\,dx = \frac{1}{2}Bb^2x^2 + 2\,Babx + Ab^2x - \frac{Aa^2}{x} + \left(Ba^2 + 2\,Aab\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^2,x, algorithm="giac")
```

output
```
1/2*B*b^2*x^2 + 2*B*a*b*x + A*b^2*x - A*a^2/x + (B*a^2 + 2*A*a*b)*log(abs(
x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^2}\,dx = \ln(x)\,\left(B\,a^2 + 2\,A\,b\,a\right) + x\,\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^2}{2}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^2,x)
```

output
```
log(x)*(B*a^2 + 2*A*a*b) + x*(A*b^2 + 2*B*a*b) - (A*a^2)/x + (B*b^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^2}\,dx = \frac{6\log(x)\,a^2bx-2a^3+6a\,b^2x^2+b^3x^3}{2x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^2,x)
```

output
```
(6*log(x)*a**2*b*x - 2*a**3 + 6*a*b**2*x**2 + b**3*x**3)/(2*x)
```

3.146 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^3} \, dx = -\frac{a^2A}{2x^2} - \frac{a(2Ab+aB)}{x} + b^2Bx + b(Ab+2aB)\log(x)$$

output
```
-1/2*a^2*A/x^2-a*(2*A*b+B*a)/x+b^2*B*x+b*(A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^3} \, dx = -\frac{2aAb}{x} + b^2Bx - \frac{a^2(A+2Bx)}{2x^2} + b(Ab+2aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^3,x]
```

output
```
(-2*a*A*b)/x + b^2*B*x - (a^2*(A + 2*B*x))/(2*x^2) + b*(A*b + 2*a*B)*Log[x
]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^3}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^3}dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a(aB+2Ab)}{x^2}+\frac{b(2aB+Ab)}{x}+b^2B\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{2x^2}-\frac{a(aB+2Ab)}{x}+b\log(x)(2aB+Ab)+b^2Bx$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a*(2*A*b + a*B))/x + b^2*B*x + b*(A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a^{2} A}{2 x^{2}}-\frac{a \left(2 A b +B a \right)}{x}+x B \,b^{2}+b \left(A b +2 B a \right) \ln \left(x \right)$	43
risch	$x B \,b^{2}+\frac{\left(-2 a b A -a^{2} B \right) x -\frac{a^{2} A}{2}}{x^{2}}+A \,b^{2} \ln \left(x \right)+2 B \ln \left(x \right) a b$	47
norman	$\frac{\left(-2 a b A -a^{2} B \right) x +x^{3} B \,b^{2}-\frac{a^{2} A}{2}}{x^{2}}+\left(b^{2} A +2 a b B \right) \ln \left(x \right)$	49
parallelrisch	$\frac{2 A \ln \left(x \right) x^{2} b^{2}+4 B \ln \left(x \right) x^{2} a b +2 x^{3} B \,b^{2}-4 a b A x -2 a^{2} B x -a^{2} A}{2 x^{2}}$	56

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*A/x^2-a*(2*A*b+B*a)/x+x*B*b^2+b*(A*b+2*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = \frac{2\,Bb^2x^3+2\,(2\,Bab+Ab^2)x^2\log\left(x\right)-Aa^2-2\,(Ba^2+2\,Aab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*b^2*x^3 + 2*(2*B*a*b + A*b^2)*x^2*log(x) - A*a^2 - 2*(B*a^2 + 2*A
*a*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = Bb^2x+b(Ab+2Ba)\log\left(x\right) + \frac{-Aa^2+x(-4Aab-2Ba^2)}{2x^2}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**3,x)
```

output
```
B*b**2*x + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x*(-4*A*a*b - 2*B*a**2))/(2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = Bb^2x+\left(2\,Bab+Ab^2\right)\log\left(x\right) - \frac{Aa^2+2\,(Ba^2+2\,Aab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="maxima")
```

output
```
B*b^2*x + (2*B*a*b + A*b^2)*log(x) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*x)/x
^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = Bb^2x+\left(2\,Bab+Ab^2\right)\log\left(|x|\right) - \frac{Aa^2+2\,(Ba^2+2\,Aab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^3,x, algorithm="giac")
```

output
```
B*b^2*x + (2*B*a*b + A*b^2)*log(abs(x)) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)
*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = \ln(x)\ \left(A\,b^2+2\,B\,a\,b\right) - \frac{\frac{A\,a^2}{2}+x\,\left(B\,a^2+2\,A\,b\,a\right)}{x^2}+B\,b^2\,x$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^3,x)
```

output
```
log(x)*(A*b^2 + 2*B*a*b) - ((A*a^2)/2 + x*(B*a^2 + 2*A*a*b))/x^2 + B*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^3}\,dx = \frac{6\log(x)\,a\,b^2x^2-a^3-6a^2bx+2b^3x^3}{2x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^3,x)
```

output
```
(6*log(x)*a*b**2*x**2 - a**3 - 6*a**2*b*x + 2*b**3*x**3)/(2*x**2)
```

3.147 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^4} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^4} \, dx = -\frac{a^2A}{3x^3} - \frac{a(2Ab+aB)}{2x^2} - \frac{b(Ab+2aB)}{x} + b^2B\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2-b*(A*b+2*B*a)/x+b^2*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^4} \, dx = -\frac{6Ab^2x^2+6abx(A+2Bx)+a^2(2A+3Bx)}{6x^3} + b^2B\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^4,x]
```

output
```
-1/6*(6*A*b^2*x^2 + 6*a*b*x*(A + 2*B*x) + a^2*(2*A + 3*B*x))/x^3 + b^2*B*L
og[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^4}dx$$
$$\Big\downarrow \text{1184}$$
$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^4}dx}{b^2}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+bx)^2(A+Bx)}{x^4}dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{a^2A}{x^4}+\frac{a(aB+2Ab)}{x^3}+\frac{b(2aB+Ab)}{x^2}+\frac{b^2B}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2A}{3x^3}-\frac{a(aB+2Ab)}{2x^2}-\frac{b(2aB+Ab)}{x}+b^2B\log(x)$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/(2*x^2) - (b*(A*b + 2*a*B))/x + b^2*B
*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{2} A}{3 x^{3}}-\frac{a \left(2 A b+B a\right)}{2 x^{2}}-\frac{b \left(A b+2 B a\right)}{x}+b^{2} B \ln \left(x \right)$	46
norman	$\frac{\left(-a b A-\frac{1}{2} a^{2} B\right) x+\left(-b^{2} A-2 a b B\right) x^{2}-\frac{a^{2} A}{3}}{x^{3}}+b^{2} B \ln \left(x \right)$	50
risch	$\frac{\left(-a b A-\frac{1}{2} a^{2} B\right) x+\left(-b^{2} A-2 a b B\right) x^{2}-\frac{a^{2} A}{3}}{x^{3}}+b^{2} B \ln \left(x \right)$	50
parallelrisch	$-\frac{-6 B \,b^{2} \ln \left(x \right) x^{3}+6 x^{2} b^{2} A+12 B a \,x^{2} b+6 a b A x+3 a^{2} B x+2 a^{2} A}{6 x^{3}}$	54

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2-b*(A*b+2*B*a)/x+b^2*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = \frac{6\,Bb^2x^3\log\left(x\right)-2\,Aa^2-6\,(2\,Bab+Ab^2)x^2-3\,(Ba^2+2\,Aab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*B*b^2*x^3*log(x) - 2*A*a^2 - 6*(2*B*a*b + A*b^2)*x^2 - 3*(B*a^2 + 2
*A*a*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = Bb^2\log\left(x\right)+\frac{-2Aa^2+x^2(-6Ab^2-12Bab)+x(-6Aab-3Ba^2)}{6x^3}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**4,x)
```

output
```
B*b**2*log(x) + (-2*A*a**2 + x**2*(-6*A*b**2 - 12*B*a*b) + x*(-6*A*a*b - 3
*B*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = Bb^2\log\left(x\right) - \frac{2\,Aa^2+6\,(2\,Bab+Ab^2)x^2+3\,(Ba^2+2\,Aab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="maxima")
```

output
```
B*b^2*log(x) - 1/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 3*(B*a^2 + 2*A*a*b
)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = Bb^2\log\left(|x|\right) - \frac{2\,Aa^2+6\,(2\,Bab+Ab^2)x^2+3\,(Ba^2+2\,Aab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^4,x, algorithm="giac")
```

output
```
B*b^2*log(abs(x)) - 1/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 3*(B*a^2 + 2*
A*a*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = B\,b^2\,\ln(x) - \frac{x^2\,\left(A\,b^2+2\,B\,a\,b\right)+\frac{A\,a^2}{3}+x\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)}{x^3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^4,x)
```

output
```
B*b^2*log(x) - (x^2*(A*b^2 + 2*B*a*b) + (A*a^2)/3 + x*((B*a^2)/2 + A*a*b))
/x^3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^4}\,dx = \frac{6\log(x)\,b^3x^3-2a^3-9a^2bx-18a\,b^2x^2}{6x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^4,x)
```

output
```
(6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

3.148 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^5}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = -\frac{A(a+bx)^3}{4ax^4}+\frac{(Ab-4aB)(a+bx)^3}{12a^2x^3}$$

output
```
-1/4*A*(b*x+a)^3/a/x^4+1/12*(A*b-4*B*a)*(b*x+a)^3/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = -\frac{6b^2x^2(A+2Bx)+4abx(2A+3Bx)+a^2(3A+4Bx)}{12x^4}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^5,x]
```

output
```
-1/12*(6*b^2*x^2*(A + 2*B*x) + 4*a*b*x*(2*A + 3*B*x) + a^2*(3*A + 4*B*x))/
x^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^5}dx$$

$$\Big\downarrow \; 1184$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^5}dx}{b^2}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(Ab-4aB)\int \frac{(a+bx)^2}{x^4}dx}{4a}-\frac{A(a+bx)^3}{4ax^4}$$

$$\Big\downarrow \; 48$$

$$\frac{(a+bx)^3(Ab-4aB)}{12a^2x^3}-\frac{A(a+bx)^3}{4ax^4}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^3)/(a*x^4) + ((A*b - 4*a*B)*(a + b*x)^3)/(12*a^2*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{a(2Ab+Ba)}{3x^3}-\frac{b(Ab+2Ba)}{2x^2}-\frac{a^2A}{4x^4}-\frac{B\,b^2}{x}$	48
norman	$\frac{-x^3B\,b^2+(-\frac{1}{2}b^2A-abB)x^2+(-\frac{2}{3}abA-\frac{1}{3}a^2B)x-\frac{a^2A}{4}}{x^4}$	51
risch	$\frac{-x^3B\,b^2+(-\frac{1}{2}b^2A-abB)x^2+(-\frac{2}{3}abA-\frac{1}{3}a^2B)x-\frac{a^2A}{4}}{x^4}$	51
gosper	$-\frac{12x^3B\,b^2+6x^2b^2A+12Ba\,x^2b+8abAx+4a^2Bx+3a^2A}{12x^4}$	52
parallelrisch	$-\frac{12x^3B\,b^2+6x^2b^2A+12Ba\,x^2b+8abAx+4a^2Bx+3a^2A}{12x^4}$	52
orering	$-\frac{(12x^3B\,b^2+6x^2b^2A+12Ba\,x^2b+8abAx+4a^2Bx+3a^2A)(b^2x^2+2abx+a^2)}{12x^4(bx+a)^2}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*(2*A*b+B*a)/x^3-1/2*b*(A*b+2*B*a)/x^2-1/4*a^2*A/x^4-B*b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = -\frac{12\,Bb^2x^3+3\,Aa^2+6\,(2\,Bab+Ab^2)x^2+4\,(Ba^2+2\,Aab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(12*B*b^2*x^3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 4*(B*a^2 + 2*A*a
*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = \frac{-3Aa^2-12Bb^2x^3+x^2(-6Ab^2-12Bab)+x(-8Aab-4Ba^2)}{12x^4}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**5,x)
```

output
```
(-3*A*a**2 - 12*B*b**2*x**3 + x**2*(-6*A*b**2 - 12*B*a*b) + x*(-8*A*a*b -
4*B*a**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = -\frac{12\,Bb^2x^3+3\,Aa^2+6\,(2\,Bab+Ab^2)x^2+4\,(Ba^2+2\,Aab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(12*B*b^2*x^3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 4*(B*a^2 + 2*A*a
*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = -\frac{12\,Bb^2x^3+12\,Babx^2+6\,Ab^2x^2+4\,Ba^2x+8\,Aabx+3\,Aa^2}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^5,x, algorithm="giac")
```

output
```
-1/12*(12*B*b^2*x^3 + 12*B*a*b*x^2 + 6*A*b^2*x^2 + 4*B*a^2*x + 8*A*a*b*x +
 3*A*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx$$
$$= -\frac{x^2\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\frac{A\,a^2}{4}+x\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+B\,b^2\,x^3}{x^4}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^5,x)
```

output
```
-(x^2*((A*b^2)/2 + B*a*b) + (A*a^2)/4 + x*((B*a^2)/3 + (2*A*a*b)/3) + B*b^
2*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^5}\,dx = \frac{-4b^3x^3-6a\,b^2x^2-4a^2bx-a^3}{4x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^5,x)
```

output
```
( - a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

3.149 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^6} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6} \, dx = -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{3x^3}-\frac{b^2B}{2x^2}$$

output
```
-1/5*a^2*A/x^5-1/4*a*(2*A*b+B*a)/x^4-1/3*b*(A*b+2*B*a)/x^3-1/2*b^2*B/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6} \, dx = -\frac{10b^2x^2(2A+3Bx)+10abx(3A+4Bx)+3a^2(4A+5Bx)}{60x^5}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^6,x]
```

output
```
-1/60*(10*b^2*x^2*(2*A + 3*B*x) + 10*a*b*x*(3*A + 4*B*x) + 3*a^2*(4*A + 5*
B*x))/x^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^6}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a(aB+2Ab)}{x^5}+\frac{b(2aB+Ab)}{x^4}+\frac{b^2B}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{2x^2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(3*x^3) -
 (b^2*B)/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{2} A}{5 x^{5}}-\frac{a \left(2 A b+B a\right)}{4 x^{4}}-\frac{b \left(A b+2 B a\right)}{3 x^{3}}-\frac{b^{2} B}{2 x^{2}}$	48
norman	$\frac{-\frac{x^{3} B \,b^{2}}{2}+\left(-\frac{1}{3} b^{2} A-\frac{2}{3} a b B\right) x^{2}+\left(-\frac{1}{2} a b A-\frac{1}{4} a^{2} B\right) x-\frac{a^{2} A}{5}}{x^{5}}$	51
risch	$\frac{-\frac{x^{3} B \,b^{2}}{2}+\left(-\frac{1}{3} b^{2} A-\frac{2}{3} a b B\right) x^{2}+\left(-\frac{1}{2} a b A-\frac{1}{4} a^{2} B\right) x-\frac{a^{2} A}{5}}{x^{5}}$	51
gosper	$-\frac{30 x^{3} B \,b^{2}+20 x^{2} b^{2} A+40 B a \,x^{2} b+30 a b A x+15 a^{2} B x+12 a^{2} A}{60 x^{5}}$	52
parallelrisch	$-\frac{30 x^{3} B \,b^{2}+20 x^{2} b^{2} A+40 B a \,x^{2} b+30 a b A x+15 a^{2} B x+12 a^{2} A}{60 x^{5}}$	52
orering	$-\frac{\left(30 x^{3} B \,b^{2}+20 x^{2} b^{2} A+40 B a \,x^{2} b+30 a b A x+15 a^{2} B x+12 a^{2} A \right) \left(b^{2} x^{2}+2 a b x+a^{2}\right)}{60 x^{5} \left(b x+a \right)^{2}}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2*A/x^5-1/4*a*(2*A*b+B*a)/x^4-1/3*b*(A*b+2*B*a)/x^3-1/2*b^2*B/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = -\frac{30\,Bb^2x^3+12\,Aa^2+20\,(2\,Bab+Ab^2)x^2+15\,(Ba^2+2\,Aab)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(30*B*b^2*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*
A*a*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = \frac{-12Aa^2-30Bb^2x^3+x^2(-20Ab^2-40Bab)+x(-30Aab-15Ba^2)}{60x^5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**6,x)
```

output
```
(-12*A*a**2 - 30*B*b**2*x**3 + x**2*(-20*A*b**2 - 40*B*a*b) + x*(-30*A*a*b
 - 15*B*a**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = -\frac{30\,Bb^2x^3+12\,Aa^2+20\,(2\,Bab+Ab^2)x^2+15\,(Ba^2+2\,Aab)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(30*B*b^2*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*
A*a*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = -\frac{30\,Bb^2x^3+40\,Babx^2+20\,Ab^2x^2+15\,Ba^2x+30\,Aabx+12\,Aa^2}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^6,x, algorithm="giac")
```

output
```
-1/60*(30*B*b^2*x^3 + 40*B*a*b*x^2 + 20*A*b^2*x^2 + 15*B*a^2*x + 30*A*a*b*
x + 12*A*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = -\frac{x^2\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{A\,a^2}{5}+x\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+\frac{B\,b^2\,x^3}{2}}{x^5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^6,x)
```

output
```
-(x^2*((A*b^2)/3 + (2*B*a*b)/3) + (A*a^2)/5 + x*((B*a^2)/4 + (A*a*b)/2) +
(B*b^2*x^3)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^6}\,dx = \frac{-10b^3x^3-20a\,b^2x^2-15a^2bx-4a^3}{20x^5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^6,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

3.150 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^7} dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{4x^4} - \frac{b^2B}{3x^3}$$

output
```
-1/6*a^2*A/x^6-1/5*a*(2*A*b+B*a)/x^5-1/4*b*(A*b+2*B*a)/x^4-1/3*b^2*B/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^7} dx = -\frac{5b^2x^2(3A+4Bx)+6abx(4A+5Bx)+2a^2(5A+6Bx)}{60x^6}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^7,x]
```

output
```
-1/60*(5*b^2*x^2*(3*A + 4*B*x) + 6*a*b*x*(4*A + 5*B*x) + 2*a^2*(5*A + 6*B*
x))/x^6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^7}dx$$

$$\Big\downarrow \; 1184$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^7}dx}{b^2}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^7}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^2A}{x^7}+\frac{a(aB+2Ab)}{x^6}+\frac{b(2aB+Ab)}{x^5}+\frac{b^2B}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{4x^4}-\frac{b^2B}{3x^3}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^7,x]
```

output
```
-1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(4*x^4) -
 (b^2*B)/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{2} A}{6 x^{6}}-\frac{a \left(2 A b +B a \right)}{5 x^{5}}-\frac{b \left(A b +2 B a \right)}{4 x^{4}}-\frac{b^{2} B}{3 x^{3}}$	48
norman	$\frac{-\frac{x^{3} B \,b^{2}}{3}+\left(-\frac{1}{4} b^{2} A -\frac{1}{2} a b B \right) x^{2}+\left(-\frac{2}{5} a b A -\frac{1}{5} a^{2} B \right) x -\frac{a^{2} A}{6}}{x^{6}}$	51
risch	$\frac{-\frac{x^{3} B \,b^{2}}{3}+\left(-\frac{1}{4} b^{2} A -\frac{1}{2} a b B \right) x^{2}+\left(-\frac{2}{5} a b A -\frac{1}{5} a^{2} B \right) x -\frac{a^{2} A}{6}}{x^{6}}$	51
gosper	$-\frac{20 x^{3} B \,b^{2}+15 x^{2} b^{2} A +30 B a \,x^{2} b +24 a b A x +12 a^{2} B x +10 a^{2} A}{60 x^{6}}$	52
parallelrisch	$-\frac{20 x^{3} B \,b^{2}+15 x^{2} b^{2} A +30 B a \,x^{2} b +24 a b A x +12 a^{2} B x +10 a^{2} A}{60 x^{6}}$	52
orering	$-\frac{\left(20 x^{3} B \,b^{2}+15 x^{2} b^{2} A +30 B a \,x^{2} b +24 a b A x +12 a^{2} B x +10 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{60 x^{6} \left(b x +a \right)^{2}}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*A/x^6-1/5*a*(2*A*b+B*a)/x^5-1/4*b*(A*b+2*B*a)/x^4-1/3*b^2*B/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^7}\,dx = -\frac{20\,Bb^2x^3+10\,Aa^2+15\,(2\,Bab+Ab^2)x^2+12\,(Ba^2+2\,Aab)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(20*B*b^2*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 12*(B*a^2 + 2*
A*a*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^7}\,dx = \frac{-10Aa^2-20Bb^2x^3+x^2(-15Ab^2-30Bab)+x(-24Aab-12Ba^2)}{60x^6}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**7,x)
```

output
```
(-10*A*a**2 - 20*B*b**2*x**3 + x**2*(-15*A*b**2 - 30*B*a*b) + x*(-24*A*a*b
 - 12*B*a**2))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^7}\,dx = -\frac{20\,Bb^2x^3+10\,Aa^2+15\,(2\,Bab+Ab^2)x^2+12\,(Ba^2+2\,Aab)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^7,x, algorithm="maxima")
```

output
```
-1/60*(20*B*b^2*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 12*(B*a^2 + 2*
A*a*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^7}\,dx = -\frac{20\,Bb^2x^3+30\,Babx^2+15\,Ab^2x^2+12\,Ba^2x+24\,Aabx+10\,Aa^2}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^7,x, algorithm="giac")
```

output
```
-1/60*(20*B*b^2*x^3 + 30*B*a*b*x^2 + 15*A*b^2*x^2 + 12*B*a^2*x + 24*A*a*b*
x + 10*A*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^7}\,dx$$
$$=-\frac{x^2\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2}{6}+x\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+\frac{B\,b^2\,x^3}{3}}{x^6}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^7,x)
```

output
```
-(x^2*((A*b^2)/4 + (B*a*b)/2) + (A*a^2)/6 + x*((B*a^2)/5 + (2*A*a*b)/5) +
(B*b^2*x^3)/3)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^7}\,dx=\frac{-20b^3x^3-45a\,b^2x^2-36a^2bx-10a^3}{60x^6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^7,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

3.151 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^8} dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^8} dx = -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{6x^6} - \frac{b(Ab+2aB)}{5x^5} - \frac{b^2B}{4x^4}$$

output
```
-1/7*a^2*A/x^7-1/6*a*(2*A*b+B*a)/x^6-1/5*b*(A*b+2*B*a)/x^5-1/4*b^2*B/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^8} dx = -\frac{21b^2x^2(4A+5Bx)+28abx(5A+6Bx)+10a^2(6A+7Bx)}{420x^7}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^8,x]
```

output
```
-1/420*(21*b^2*x^2*(4*A + 5*B*x) + 28*a*b*x*(5*A + 6*B*x) + 10*a^2*(6*A + 
7*B*x))/x^7
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^8}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^8}+\frac{a(aB+2Ab)}{x^7}+\frac{b(2aB+Ab)}{x^6}+\frac{b^2B}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{6x^6}-\frac{b(2aB+Ab)}{5x^5}-\frac{b^2B}{4x^4}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(6*x^6) - (b*(A*b + 2*a*B))/(5*x^5) -
 (b^2*B)/(4*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{2} A}{7 x^{7}}-\frac{a \left(2 A b +B a \right)}{6 x^{6}}-\frac{b \left(A b +2 B a \right)}{5 x^{5}}-\frac{b^{2} B}{4 x^{4}}$	48
norman	$\frac{-\frac{x^{3} B \,b^{2}}{4}+\left(-\frac{1}{5} b^{2} A -\frac{2}{5} a b B \right) x^{2}+\left(-\frac{1}{3} a b A -\frac{1}{6} a^{2} B \right) x -\frac{a^{2} A}{7}}{x^{7}}$	51
risch	$\frac{-\frac{x^{3} B \,b^{2}}{4}+\left(-\frac{1}{5} b^{2} A -\frac{2}{5} a b B \right) x^{2}+\left(-\frac{1}{3} a b A -\frac{1}{6} a^{2} B \right) x -\frac{a^{2} A}{7}}{x^{7}}$	51
gosper	$-\frac{105 x^{3} B \,b^{2}+84 x^{2} b^{2} A +168 B a \,x^{2} b +140 a b A x +70 a^{2} B x +60 a^{2} A}{420 x^{7}}$	52
parallelrisch	$-\frac{105 x^{3} B \,b^{2}+84 x^{2} b^{2} A +168 B a \,x^{2} b +140 a b A x +70 a^{2} B x +60 a^{2} A}{420 x^{7}}$	52
orering	$-\frac{\left(105 x^{3} B \,b^{2}+84 x^{2} b^{2} A +168 B a \,x^{2} b +140 a b A x +70 a^{2} B x +60 a^{2} A \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}{420 x^{7} \left(b x +a \right)^{2}}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*A/x^7-1/6*a*(2*A*b+B*a)/x^6-1/5*b*(A*b+2*B*a)/x^5-1/4*b^2*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^8}\,dx = -\frac{105\,Bb^2x^3+60\,Aa^2+84\,(2\,Bab+Ab^2)x^2+70\,(Ba^2+2\,Aab)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^8,x, algorithm="fricas")
```

output
```
-1/420*(105*B*b^2*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^8}\,dx = \frac{-60Aa^2-105Bb^2x^3+x^2(-84Ab^2-168Bab)+x(-140Aab-70Ba^2)}{420x^7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**8,x)
```

output
```
(-60*A*a**2 - 105*B*b**2*x**3 + x**2*(-84*A*b**2 - 168*B*a*b) + x*(-140*A*
a*b - 70*B*a**2))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^8}\,dx = -\frac{105\,Bb^2x^3+60\,Aa^2+84\,(2\,Bab+Ab^2)x^2+70\,(Ba^2+2\,Aab)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^8,x, algorithm="maxima")
```

output
```
-1/420*(105*B*b^2*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^8}\,dx = -\frac{105\,Bb^2x^3+168\,Babx^2+84\,Ab^2x^2+70\,Ba^2x+140\,Aabx+60\,Aa^2}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^8,x, algorithm="giac")
```

output
```
-1/420*(105*B*b^2*x^3 + 168*B*a*b*x^2 + 84*A*b^2*x^2 + 70*B*a^2*x + 140*A*
a*b*x + 60*A*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^8}\,dx = -\frac{x^2\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{A\,a^2}{7}+x\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+\frac{B\,b^2\,x^3}{4}}{x^7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^8,x)
```

output
```
-(x^2*((A*b^2)/5 + (2*B*a*b)/5) + (A*a^2)/7 + x*((B*a^2)/6 + (A*a*b)/3) +
(B*b^2*x^3)/4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^8}\,dx = \frac{-35b^3x^3-84a\,b^2x^2-70a^2bx-20a^3}{140x^7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^8,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

3.152 $\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{5}a^4Ax^5+\frac{1}{6}a^3(4Ab+aB)x^6+\frac{2}{7}a^2b(3Ab+2aB)x^7+\frac{1}{4}ab^2(2Ab+3aB)x^8+\frac{1}{9}b^3(Ab+4aB)x^9+\frac{1}{10}b^4Bx^{10}$$

output
```
1/5*a^4*A*x^5+1/6*a^3*(4*A*b+B*a)*x^6+2/7*a^2*b*(3*A*b+2*B*a)*x^7+1/4*a*b^
2*(2*A*b+3*B*a)*x^8+1/9*b^3*(A*b+4*B*a)*x^9+1/10*b^4*B*x^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{5}a^4Ax^5+\frac{1}{6}a^3(4Ab+aB)x^6+\frac{2}{7}a^2b(3Ab+2aB)x^7+\frac{1}{4}ab^2(2Ab+3aB)x^8+\frac{1}{9}b^3(Ab+4aB)x^9+\frac{1}{10}b^4Bx^{10}$$

input
```
Integrate[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*A*x^5)/5 + (a^3*(4*A*b + a*B)*x^6)/6 + (2*a^2*b*(3*A*b + 2*a*B)*x^7)/
7 + (a*b^2*(2*A*b + 3*a*B)*x^8)/4 + (b^3*(A*b + 4*a*B)*x^9)/9 + (b^4*B*x^1
0)/10
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx+b^2x^2\right)^2 (A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^4x^4(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^4(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^4Ax^4+a^3x^5(aB+4Ab)+2a^2bx^6(2aB+3Ab)+b^3x^8(4aB+Ab)+2ab^2x^7(3aB+2Ab)+b^4Bx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^4Ax^5+\frac{1}{6}a^3x^6(aB+4Ab)+\frac{2}{7}a^2bx^7(2aB+3Ab)+\frac{1}{9}b^3x^9(4aB+Ab)+\frac{1}{4}ab^2x^8(3aB+2Ab)+\frac{1}{10}b^4Bx^{10}$$

input
```
Int[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*A*x^5)/5 + (a^3*(4*A*b + a*B)*x^6)/6 + (2*a^2*b*(3*A*b + 2*a*B)*x^7)/
7 + (a*b^2*(2*A*b + 3*a*B)*x^8)/4 + (b^3*(A*b + 4*a*B)*x^9)/9 + (b^4*B*x^1
0)/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^4 B\,x^{10}}{10}+\left(\frac{1}{9}A\,b^4+\frac{4}{9}Ba\,b^3\right)x^9+\left(\frac{1}{2}Aa\,b^3+\frac{3}{4}B\,a^2b^2\right)x^8+\left(\frac{6}{7}a^2A\,b^2+\frac{4}{7}B\,a^3b\right)x^7+\left(\frac{2}{3}A\,a^3\right.$
gosper	$\frac{x^5\left(126b^4B\,x^5+140A\,b^4x^4+560Ba\,b^3x^4+630Aa\,b^3x^3+945B\,a^2b^2x^3+1080A\,a^2b^2x^2+720B\,a^3b\,x^2+840A\,a^3bx+210a^4Bx+2\right.}{1260}$
default	$\frac{b^4B\,x^{10}}{10}+\frac{(A\,b^4+4Ba\,b^3)x^9}{9}+\frac{(4Aa\,b^3+6B\,a^2b^2)x^8}{8}+\frac{(6a^2A\,b^2+4B\,a^3b)x^7}{7}+\frac{(4A\,a^3b+a^4B)x^6}{6}+\frac{a^4A\,x^5}{5}$
risch	$\frac{1}{10}b^4B\,x^{10}+\frac{1}{9}x^9A\,b^4+\frac{4}{9}x^9Ba\,b^3+\frac{1}{2}x^8Aa\,b^3+\frac{3}{4}x^8B\,a^2b^2+\frac{6}{7}x^7a^2A\,b^2+\frac{4}{7}x^7B\,a^3b+\frac{2}{3}x^6A$
parallelrisch	$\frac{1}{10}b^4B\,x^{10}+\frac{1}{9}x^9A\,b^4+\frac{4}{9}x^9Ba\,b^3+\frac{1}{2}x^8Aa\,b^3+\frac{3}{4}x^8B\,a^2b^2+\frac{6}{7}x^7a^2A\,b^2+\frac{4}{7}x^7B\,a^3b+\frac{2}{3}x^6A$
orering	$\frac{x^5\left(126b^4B\,x^5+140A\,b^4x^4+560Ba\,b^3x^4+630Aa\,b^3x^3+945B\,a^2b^2x^3+1080A\,a^2b^2x^2+720B\,a^3b\,x^2+840A\,a^3bx+210a^4Bx+2\right.}{1260(bx+a)^4}$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^4*B*x^10+(1/9*A*b^4+4/9*B*a*b^3)*x^9+(1/2*A*a*b^3+3/4*B*a^2*b^2)*x^
8+(6/7*a^2*A*b^2+4/7*B*a^3*b)*x^7+(2/3*A*a^3*b+1/6*a^4*B)*x^6+1/5*a^4*A*x^
5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=&\ \frac{1}{10}\,Bb^4x^{10}+\frac{1}{5}\,Aa^4x^5+\frac{1}{9}\,\left(4\,Bab^3+Ab^4\right)x^9\\&+\frac{1}{4}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^8\\&+\frac{2}{7}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^7\\&+\frac{1}{6}\,\left(Ba^4+4\,Aa^3b\right)x^6\end{aligned}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/10*B*b^4*x^10 + 1/5*A*a^4*x^5 + 1/9*(4*B*a*b^3 + A*b^4)*x^9 + 1/4*(3*B*a
^2*b^2 + 2*A*a*b^3)*x^8 + 2/7*(2*B*a^3*b + 3*A*a^2*b^2)*x^7 + 1/6*(B*a^4 +
 4*A*a^3*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{Aa^4x^5}{5}+\frac{Bb^4x^{10}}{10}+x^9\left(\frac{Ab^4}{9}+\frac{4Bab^3}{9}\right)+x^8\left(\frac{Aab^3}{2}+\frac{3Ba^2b^2}{4}\right)+x^7\cdot\left(\frac{6Aa^2b^2}{7}+\frac{4Ba^3b}{7}\right)+x^6\cdot\left(\frac{2Aa^3b}{3}+\frac{Ba^4}{6}\right)$$

input
```
integrate(x**4*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*a**4*x**5/5 + B*b**4*x**10/10 + x**9*(A*b**4/9 + 4*B*a*b**3/9) + x**8*(A
*a*b**3/2 + 3*B*a**2*b**2/4) + x**7*(6*A*a**2*b**2/7 + 4*B*a**3*b/7) + x**
6*(2*A*a**3*b/3 + B*a**4/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{10}\,Bb^4x^{10}+\frac{1}{5}\,Aa^4x^5+\frac{1}{9}\,\left(4\,Bab^3+Ab^4\right)x^9+\frac{1}{4}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^8+\frac{2}{7}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^7+\frac{1}{6}\,\left(Ba^4+4\,Aa^3b\right)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/10*B*b^4*x^10 + 1/5*A*a^4*x^5 + 1/9*(4*B*a*b^3 + A*b^4)*x^9 + 1/4*(3*B*a
^2*b^2 + 2*A*a*b^3)*x^8 + 2/7*(2*B*a^3*b + 3*A*a^2*b^2)*x^7 + 1/6*(B*a^4 +
 4*A*a^3*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{10}\,Bb^4x^{10}+\frac{4}{9}\,Bab^3x^9+\frac{1}{9}\,Ab^4x^9+\frac{3}{4}\,Ba^2b^2x^8+\frac{1}{2}\,Aab^3x^8+\frac{4}{7}\,Ba^3bx^7+\frac{6}{7}\,Aa^2b^2x^7+\frac{1}{6}\,Ba^4x^6+\frac{2}{3}\,Aa^3bx^6+\frac{1}{5}\,Aa^4x^5$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/10*B*b^4*x^10 + 4/9*B*a*b^3*x^9 + 1/9*A*b^4*x^9 + 3/4*B*a^2*b^2*x^8 + 1/
2*A*a*b^3*x^8 + 4/7*B*a^3*b*x^7 + 6/7*A*a^2*b^2*x^7 + 1/6*B*a^4*x^6 + 2/3*
A*a^3*b*x^6 + 1/5*A*a^4*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^6\left(\frac{B\,a^4}{6}+\frac{2\,A\,b\,a^3}{3}\right)+x^9\left(\frac{A\,b^4}{9}+\frac{4\,B\,a\,b^3}{9}\right)+\frac{A\,a^4\,x^5}{5}+\frac{B\,b^4\,x^{10}}{10}+\frac{2\,a^2\,b\,x^7\,\left(3\,A\,b+2\,B\,a\right)}{7}+\frac{a\,b^2\,x^8\,\left(2\,A\,b+3\,B\,a\right)}{4}$$

input
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^6*((B*a^4)/6 + (2*A*a^3*b)/3) + x^9*((A*b^4)/9 + (4*B*a*b^3)/9) + (A*a^4
*x^5)/5 + (B*b^4*x^10)/10 + (2*a^2*b*x^7*(3*A*b + 2*B*a))/7 + (a*b^2*x^8*(
2*A*b + 3*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{x^5(126b^5x^5+700a\,b^4x^4+1575a^2b^3x^3+1800a^3b^2x^2+1050a^4bx+252a^5)}{1260}$$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**5*(252*a**5 + 1050*a**4*b*x + 1800*a**3*b**2*x**2 + 1575*a**2*b**3*x**
3 + 700*a*b**4*x**4 + 126*b**5*x**5))/1260
```

3.153 $\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{4}a^4Ax^4+\frac{1}{5}a^3(4Ab+aB)x^5+\frac{1}{3}a^2b(3Ab+2aB)x^6+\frac{2}{7}ab^2(2Ab+3aB)x^7+\frac{1}{8}b^3(Ab+4aB)x^8+\frac{1}{9}b^4Bx^9$$

output
```
1/4*a^4*A*x^4+1/5*a^3*(4*A*b+B*a)*x^5+1/3*a^2*b*(3*A*b+2*B*a)*x^6+2/7*a*b^
2*(2*A*b+3*B*a)*x^7+1/8*b^3*(A*b+4*B*a)*x^8+1/9*b^4*B*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{4}a^4Ax^4+\frac{1}{5}a^3(4Ab+aB)x^5+\frac{1}{3}a^2b(3Ab+2aB)x^6+\frac{2}{7}ab^2(2Ab+3aB)x^7+\frac{1}{8}b^3(Ab+4aB)x^8+\frac{1}{9}b^4Bx^9$$

input
```
Integrate[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*A*x^4)/4 + (a^3*(4*A*b + a*B)*x^5)/5 + (a^2*b*(3*A*b + 2*a*B)*x^6)/3
+ (2*a*b^2*(2*A*b + 3*a*B)*x^7)/7 + (b^3*(A*b + 4*a*B)*x^8)/8 + (b^4*B*x^9
)/9
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx+b^2x^2\right)^2 (A+Bx) \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^4x^3(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^3(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^4Ax^3+a^3x^4(aB+4Ab)+2a^2bx^5(2aB+3Ab)+b^3x^7(4aB+Ab)+2ab^2x^6(3aB+2Ab)+b^4Bx^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^4Ax^4+\frac{1}{5}a^3x^5(aB+4Ab)+\frac{1}{3}a^2bx^6(2aB+3Ab)+\frac{1}{8}b^3x^8(4aB+Ab)+\frac{2}{7}ab^2x^7(3aB+2Ab)+\frac{1}{9}b^4Bx^9$$

input
```
Int[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*A*x^4)/4 + (a^3*(4*A*b + a*B)*x^5)/5 + (a^2*b*(3*A*b + 2*a*B)*x^6)/3
+ (2*a*b^2*(2*A*b + 3*a*B)*x^7)/7 + (b^3*(A*b + 4*a*B)*x^8)/8 + (b^4*B*x^9
)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

method	result
norman	$\frac{B b^4 x^9}{9}+\left(\frac{1}{8}A b^4+\frac{1}{2}B a b^3\right)x^8+\left(\frac{4}{7}A a b^3+\frac{6}{7}B a^2 b^2\right)x^7+\left(a^2 A b^2+\frac{2}{3}B a^3 b\right)x^6+\left(\frac{4}{5}A a^3 b+\right.$
gosper	$\frac{x^4\left(280 b^4 B x^5+315 A b^4 x^4+1260 B a b^3 x^4+1440 A a b^3 x^3+2160 B a^2 b^2 x^3+2520 A a^2 b^2 x^2+1680 B a^3 b x^2+2016 A a^3 b x+504 a^4\right.}{2520}$
default	$\frac{B b^4 x^9}{9}+\frac{(A b^4+4 B a b^3)x^8}{8}+\frac{(4 A a b^3+6 B a^2 b^2)x^7}{7}+\frac{(6 a^2 A b^2+4 B a^3 b)x^6}{6}+\frac{(4 A a^3 b+a^4 B)x^5}{5}+\frac{a^4 A x^4}{4}$
risch	$\frac{1}{9}B b^4 x^9+\frac{1}{8}b^4 A x^8+\frac{1}{2}x^8 B a b^3+\frac{4}{7}x^7 A a b^3+\frac{6}{7}x^7 B a^2 b^2+x^6 a^2 A b^2+\frac{2}{3}x^6 B a^3 b+\frac{4}{5}x^5 A a^3$
parallelrisch	$\frac{1}{9}B b^4 x^9+\frac{1}{8}b^4 A x^8+\frac{1}{2}x^8 B a b^3+\frac{4}{7}x^7 A a b^3+\frac{6}{7}x^7 B a^2 b^2+x^6 a^2 A b^2+\frac{2}{3}x^6 B a^3 b+\frac{4}{5}x^5 A a^3$
orering	$\frac{x^4\left(280 b^4 B x^5+315 A b^4 x^4+1260 B a b^3 x^4+1440 A a b^3 x^3+2160 B a^2 b^2 x^3+2520 A a^2 b^2 x^2+1680 B a^3 b x^2+2016 A a^3 b x+504 a^4\right.}{2520(b x+a)^4}$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*b^4*x^9+(1/8*A*b^4+1/2*B*a*b^3)*x^8+(4/7*A*a*b^3+6/7*B*a^2*b^2)*x^7+
(a^2*A*b^2+2/3*B*a^3*b)*x^6+(4/5*A*a^3*b+1/5*a^4*B)*x^5+1/4*a^4*A*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{9}\,Bb^4x^9+\frac{1}{4}\,Aa^4x^4+\frac{1}{8}\,(4\,Bab^3+Ab^4)x^8+\frac{2}{7}\,(3\,Ba^2b^2+2\,Aab^3)x^7+\frac{1}{3}\,(2\,Ba^3b+3\,Aa^2b^2)x^6+\frac{1}{5}\,(Ba^4+4\,Aa^3b)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/9*B*b^4*x^9 + 1/4*A*a^4*x^4 + 1/8*(4*B*a*b^3 + A*b^4)*x^8 + 2/7*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^7 + 1/3*(2*B*a^3*b + 3*A*a^2*b^2)*x^6 + 1/5*(B*a^4 + 4
*A*a^3*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{Aa^4x^4}{4}+\frac{Bb^4x^9}{9}+x^8\left(\frac{Ab^4}{8}+\frac{Bab^3}{2}\right)+x^7\cdot\left(\frac{4Aab^3}{7}+\frac{6Ba^2b^2}{7}\right)+x^6\left(Aa^2b^2+\frac{2Ba^3b}{3}\right)+x^5\cdot\left(\frac{4Aa^3b}{5}+\frac{Ba^4}{5}\right)$$

input
```
integrate(x**3*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*a**4*x**4/4 + B*b**4*x**9/9 + x**8*(A*b**4/8 + B*a*b**3/2) + x**7*(4*A*a
*b**3/7 + 6*B*a**2*b**2/7) + x**6*(A*a**2*b**2 + 2*B*a**3*b/3) + x**5*(4*A
*a**3*b/5 + B*a**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{9}\,Bb^4x^9+\frac{1}{4}\,Aa^4x^4+\frac{1}{8}\,\left(4\,Bab^3+Ab^4\right)x^8+\frac{2}{7}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^7+\frac{1}{3}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^6+\frac{1}{5}\,\left(Ba^4+4\,Aa^3b\right)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/9*B*b^4*x^9 + 1/4*A*a^4*x^4 + 1/8*(4*B*a*b^3 + A*b^4)*x^8 + 2/7*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^7 + 1/3*(2*B*a^3*b + 3*A*a^2*b^2)*x^6 + 1/5*(B*a^4 + 4
*A*a^3*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{9}Bb^4x^9+\frac{1}{2}Bab^3x^8+\frac{1}{8}Ab^4x^8+\frac{6}{7}Ba^2b^2x^7+\frac{4}{7}Aab^3x^7+\frac{2}{3}Ba^3bx^6+Aa^2b^2x^6+\frac{1}{5}Ba^4x^5+\frac{4}{5}Aa^3bx^5+\frac{1}{4}Aa^4x^4$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/9*B*b^4*x^9 + 1/2*B*a*b^3*x^8 + 1/8*A*b^4*x^8 + 6/7*B*a^2*b^2*x^7 + 4/7*
A*a*b^3*x^7 + 2/3*B*a^3*b*x^6 + A*a^2*b^2*x^6 + 1/5*B*a^4*x^5 + 4/5*A*a^3*
b*x^5 + 1/4*A*a^4*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^5\left(\frac{B\,a^4}{5}+\frac{4\,A\,b\,a^3}{5}\right)+x^8\left(\frac{A\,b^4}{8}+\frac{B\,a\,b^3}{2}\right)+\frac{A\,a^4\,x^4}{4}+\frac{B\,b^4\,x^9}{9}+\frac{a^2\,b\,x^6\,(3\,A\,b+2\,B\,a)}{3}+\frac{2\,a\,b^2\,x^7\,(2\,A\,b+3\,B\,a)}{7}$$

input
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^5*((B*a^4)/5 + (4*A*a^3*b)/5) + x^8*((A*b^4)/8 + (B*a*b^3)/2) + (A*a^4*x
^4)/4 + (B*b^4*x^9)/9 + (a^2*b*x^6*(3*A*b + 2*B*a))/3 + (2*a*b^2*x^7*(2*A*
b + 3*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$=\frac{x^4(56b^5x^5+315a\,b^4x^4+720a^2b^3x^3+840a^3b^2x^2+504a^4bx+126a^5)}{504}$$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**4*(126*a**5 + 504*a**4*b*x + 840*a**3*b**2*x**2 + 720*a**2*b**3*x**3 +
 315*a*b**4*x**4 + 56*b**5*x**5))/504
```

3.154 $\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^2(Ab-aB)(a+bx)^5}{5b^4}-\frac{a(2Ab-3aB)(a+bx)^6}{6b^4}+\frac{(Ab-3aB)(a+bx)^7}{7b^4}+\frac{B(a+bx)^8}{8b^4}$$

output
```
1/5*a^2*(A*b-B*a)*(b*x+a)^5/b^4-1/6*a*(2*A*b-3*B*a)*(b*x+a)^6/b^4+1/7*(A*b
-3*B*a)*(b*x+a)^7/b^4+1/8*B*(b*x+a)^8/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{840}x^3\left(70a^4(4A+3Bx)+168a^3bx(5A+4Bx)+168a^2b^2x^2(6A+5Bx)+80ab^3x^3(7A+6Bx)+15b^4x^4(8A+7Bx)\right)$$

input
```
Integrate[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(x^3*(70*a^4*(4*A + 3*B*x) + 168*a^3*b*x*(5*A + 4*B*x) + 168*a^2*b^2*x^2*(
6*A + 5*B*x) + 80*a*b^3*x^3*(7*A + 6*B*x) + 15*b^4*x^4*(8*A + 7*B*x)))/840
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^2 (A+Bx) \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^4x^2(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^2(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(-\frac{a^2(a+bx)^4(aB-Ab)}{b^3}+\frac{(a+bx)^6(Ab-3aB)}{b^3}+\frac{a(a+bx)^5(3aB-2Ab)}{b^3}+\frac{B(a+bx)^7}{b^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^5(Ab-aB)}{5b^4}+\frac{(a+bx)^7(Ab-3aB)}{7b^4}-\frac{a(a+bx)^6(2Ab-3aB)}{6b^4}+\frac{B(a+bx)^8}{8b^4}$$

input
```
Int[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^2*(A*b - a*B)*(a + b*x)^5)/(5*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x)^6)/(6
*b^4) + ((A*b - 3*a*B)*(a + b*x)^7)/(7*b^4) + (B*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

method	result
norman	$\frac{B\,b^4x^8}{8}+\left(\frac{1}{7}A\,b^4+\frac{4}{7}Ba\,b^3\right)x^7+\left(\frac{2}{3}Aa\,b^3+B\,a^2b^2\right)x^6+\left(\frac{6}{5}a^2A\,b^2+\frac{4}{5}B\,a^3b\right)x^5+\left(A\,a^3b+\right.$
gosper	$\frac{x^3\left(105b^4B\,x^5+120A\,b^4x^4+480Ba\,b^3x^4+560Aa\,b^3x^3+840B\,a^2b^2x^3+1008A\,a^2b^2x^2+672B\,a^3b\,x^2+840A\,a^3bx+210a^4Bx+2\right.}{840}$
default	$\frac{B\,b^4x^8}{8}+\frac{(A\,b^4+4Ba\,b^3)x^7}{7}+\frac{(4Aa\,b^3+6B\,a^2b^2)x^6}{6}+\frac{(6a^2A\,b^2+4B\,a^3b)x^5}{5}+\frac{(4A\,a^3b+a^4B)x^4}{4}+\frac{a^4A\,x^3}{3}$
risch	$\frac{1}{8}B\,b^4x^8+\frac{1}{7}A\,b^4x^7+\frac{4}{7}a\,b^3B\,x^7+\frac{2}{3}a\,b^3A\,x^6+x^6B\,a^2b^2+\frac{6}{5}x^5a^2A\,b^2+\frac{4}{5}x^5B\,a^3b+x^4A\,a^3$
parallelrisch	$\frac{1}{8}B\,b^4x^8+\frac{1}{7}A\,b^4x^7+\frac{4}{7}a\,b^3B\,x^7+\frac{2}{3}a\,b^3A\,x^6+x^6B\,a^2b^2+\frac{6}{5}x^5a^2A\,b^2+\frac{4}{5}x^5B\,a^3b+x^4A\,a^3$
orering	$\frac{x^3\left(105b^4B\,x^5+120A\,b^4x^4+480Ba\,b^3x^4+560Aa\,b^3x^3+840B\,a^2b^2x^3+1008A\,a^2b^2x^2+672B\,a^3b\,x^2+840A\,a^3bx+210a^4Bx+2\right.}{840(bx+a)^4}$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*b^4*x^8+(1/7*A*b^4+4/7*B*a*b^3)*x^7+(2/3*A*a*b^3+B*a^2*b^2)*x^6+(6/5
*a^2*A*b^2+4/5*B*a^3*b)*x^5+(A*a^3*b+1/4*a^4*B)*x^4+1/3*a^4*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{8}\,Bb^4x^8+\frac{1}{3}\,Aa^4x^3+\frac{1}{7}\left(4\,Bab^3+Ab^4\right)x^7+\frac{1}{3}\left(3\,Ba^2b^2+2\,Aab^3\right)x^6+\frac{2}{5}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^5+\frac{1}{4}\left(Ba^4+4\,Aa^3b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/8*B*b^4*x^8 + 1/3*A*a^4*x^3 + 1/7*(4*B*a*b^3 + A*b^4)*x^7 + 1/3*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^6 + 2/5*(2*B*a^3*b + 3*A*a^2*b^2)*x^5 + 1/4*(B*a^4 + 4
*A*a^3*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{Aa^4x^3}{3}+\frac{Bb^4x^8}{8}+x^7\left(\frac{Ab^4}{7}+\frac{4Bab^3}{7}\right)+x^6\cdot\left(\frac{2Aab^3}{3}+Ba^2b^2\right)+x^5\cdot\left(\frac{6Aa^2b^2}{5}+\frac{4Ba^3b}{5}\right)+x^4\left(Aa^3b+\frac{Ba^4}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*a**4*x**3/3 + B*b**4*x**8/8 + x**7*(A*b**4/7 + 4*B*a*b**3/7) + x**6*(2*A
*a*b**3/3 + B*a**2*b**2) + x**5*(6*A*a**2*b**2/5 + 4*B*a**3*b/5) + x**4*(A
*a**3*b + B*a**4/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{8}\,Bb^4x^8+\frac{1}{3}\,Aa^4x^3+\frac{1}{7}\left(4\,Bab^3+Ab^4\right)x^7+\frac{1}{3}\left(3\,Ba^2b^2+2\,Aab^3\right)x^6+\frac{2}{5}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^5+\frac{1}{4}\left(Ba^4+4\,Aa^3b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/8*B*b^4*x^8 + 1/3*A*a^4*x^3 + 1/7*(4*B*a*b^3 + A*b^4)*x^7 + 1/3*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^6 + 2/5*(2*B*a^3*b + 3*A*a^2*b^2)*x^5 + 1/4*(B*a^4 + 4
*A*a^3*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{8}\,Bb^4x^8+\frac{4}{7}\,Bab^3x^7+\frac{1}{7}\,Ab^4x^7+Ba^2b^2x^6+\frac{2}{3}\,Aab^3x^6+\frac{4}{5}\,Ba^3bx^5+\frac{6}{5}\,Aa^2b^2x^5+\frac{1}{4}\,Ba^4x^4+Aa^3bx^4+\frac{1}{3}\,Aa^4x^3$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/8*B*b^4*x^8 + 4/7*B*a*b^3*x^7 + 1/7*A*b^4*x^7 + B*a^2*b^2*x^6 + 2/3*A*a*
b^3*x^6 + 4/5*B*a^3*b*x^5 + 6/5*A*a^2*b^2*x^5 + 1/4*B*a^4*x^4 + A*a^3*b*x^
4 + 1/3*A*a^4*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^4\left(\frac{B\,a^4}{4}+A\,b\,a^3\right)+x^7\left(\frac{A\,b^4}{7}+\frac{4\,B\,a\,b^3}{7}\right)+\frac{A\,a^4\,x^3}{3}+\frac{B\,b^4\,x^8}{8}+\frac{2\,a^2\,b\,x^5\,\left(3\,A\,b+2\,B\,a\right)}{5}+\frac{a\,b^2\,x^6\,\left(2\,A\,b+3\,B\,a\right)}{3}$$

input
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^4*((B*a^4)/4 + A*a^3*b) + x^7*((A*b^4)/7 + (4*B*a*b^3)/7) + (A*a^4*x^3)/
3 + (B*b^4*x^8)/8 + (2*a^2*b*x^5*(3*A*b + 2*B*a))/5 + (a*b^2*x^6*(2*A*b +
3*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{x^3(21b^5x^5+120a\,b^4x^4+280a^2b^3x^3+336a^3b^2x^2+210a^4bx+56a^5)}{168}$$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**3*(56*a**5 + 210*a**4*b*x + 336*a**3*b**2*x**2 + 280*a**2*b**3*x**3 +
120*a*b**4*x**4 + 21*b**5*x**5))/168
```

3.155 $\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=-\frac{a(Ab-aB)(a+bx)^5}{5b^3}+\frac{(Ab-2aB)(a+bx)^6}{6b^3}+\frac{B(a+bx)^7}{7b^3}$$

output
```
-1/5*a*(A*b-B*a)*(b*x+a)^5/b^3+1/6*(A*b-2*B*a)*(b*x+a)^6/b^3+1/7*B*(b*x+a)
^7/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{210}x^2\left(35a^4(3A+2Bx)+70a^3bx(4A+3Bx)+63a^2b^2x^2(5A+4Bx)+28ab^3x^3(6A+5Bx)+5b^4x^4(7A+6Bx)\right)$$

input
```
Integrate[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(x^2*(35*a^4*(3*A + 2*B*x) + 70*a^3*b*x*(4*A + 3*B*x) + 63*a^2*b^2*x^2*(5*
A + 4*B*x) + 28*a*b^3*x^3*(6*A + 5*B*x) + 5*b^4*x^4*(7*A + 6*B*x)))/210
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^4x(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int x(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{(a+bx)^5(Ab-2aB)}{b^2}+\frac{a(a+bx)^4(aB-Ab)}{b^2}+\frac{B(a+bx)^6}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^6(Ab-2aB)}{6b^3}-\frac{a(a+bx)^5(Ab-aB)}{5b^3}+\frac{B(a+bx)^7}{7b^3}$$

input
```
Int[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/5*(a*(A*b - a*B)*(a + b*x)^5)/b^3 + ((A*b - 2*a*B)*(a + b*x)^6)/(6*b^3)
 + (B*(a + b*x)^7)/(7*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result
norman	$\frac{B\,b^{4}x^{7}}{7}+\left(\frac{1}{6}A\,b^{4}+\frac{2}{3}Ba\,b^{3}\right)x^{6}+\left(\frac{4}{5}Aa\,b^{3}+\frac{6}{5}B\,a^{2}b^{2}\right)x^{5}+\left(\frac{3}{2}a^{2}A\,b^{2}+B\,a^{3}b\right)x^{4}+\left(\frac{4}{3}A\,a^{3}b+\right.$
gosper	$\frac{x^{2}(30b^{4}B\,x^{5}+35A\,b^{4}x^{4}+140Ba\,b^{3}x^{4}+168Aa\,b^{3}x^{3}+252B\,a^{2}b^{2}x^{3}+315A\,a^{2}b^{2}x^{2}+210B\,a^{3}b\,x^{2}+280A\,a^{3}bx+70a^{4}Bx+105a^{4}}{210}$
default	$\frac{B\,b^{4}x^{7}}{7}+\frac{(A\,b^{4}+4Ba\,b^{3})x^{6}}{6}+\frac{(4Aa\,b^{3}+6B\,a^{2}b^{2})x^{5}}{5}+\frac{(6a^{2}A\,b^{2}+4B\,a^{3}b)x^{4}}{4}+\frac{(4A\,a^{3}b+a^{4}B)x^{3}}{3}+\frac{a^{4}A\,x^{2}}{2}$
risch	$\frac{1}{7}B\,b^{4}x^{7}+\frac{1}{6}A\,b^{4}x^{6}+\frac{2}{3}Ba\,b^{3}x^{6}+\frac{4}{5}Aa\,b^{3}x^{5}+\frac{6}{5}a^{2}B\,b^{2}x^{5}+\frac{3}{2}a^{2}A\,b^{2}x^{4}+x^{4}B\,a^{3}b+\frac{4}{3}x^{3}A\,a^{3}$
parallelrisch	$\frac{1}{7}B\,b^{4}x^{7}+\frac{1}{6}A\,b^{4}x^{6}+\frac{2}{3}Ba\,b^{3}x^{6}+\frac{4}{5}Aa\,b^{3}x^{5}+\frac{6}{5}a^{2}B\,b^{2}x^{5}+\frac{3}{2}a^{2}A\,b^{2}x^{4}+x^{4}B\,a^{3}b+\frac{4}{3}x^{3}A\,a^{3}$
orering	$\frac{x^{2}(30b^{4}B\,x^{5}+35A\,b^{4}x^{4}+140Ba\,b^{3}x^{4}+168Aa\,b^{3}x^{3}+252B\,a^{2}b^{2}x^{3}+315A\,a^{2}b^{2}x^{2}+210B\,a^{3}b\,x^{2}+280A\,a^{3}bx+70a^{4}Bx+105a^{4}}{210(bx+a)^{4}}$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*b^4*x^7+(1/6*A*b^4+2/3*B*a*b^3)*x^6+(4/5*A*a*b^3+6/5*B*a^2*b^2)*x^5+
(3/2*a^2*A*b^2+B*a^3*b)*x^4+(4/3*A*a^3*b+1/3*a^4*B)*x^3+1/2*a^4*A*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.62

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{7}\,Bb^4x^7+\frac{1}{2}\,Aa^4x^2+\frac{1}{6}\,\left(4\,Bab^3+Ab^4\right)x^6+\frac{2}{5}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^5+\frac{1}{2}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^4+\frac{1}{3}\,\left(Ba^4+4\,Aa^3b\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/7*B*b^4*x^7 + 1/2*A*a^4*x^2 + 1/6*(4*B*a*b^3 + A*b^4)*x^6 + 2/5*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^5 + 1/2*(2*B*a^3*b + 3*A*a^2*b^2)*x^4 + 1/3*(B*a^4 + 4
*A*a^3*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{Aa^4x^2}{2}+\frac{Bb^4x^7}{7}+x^6\left(\frac{Ab^4}{6}+\frac{2Bab^3}{3}\right)+x^5\cdot\left(\frac{4Aab^3}{5}+\frac{6Ba^2b^2}{5}\right)+x^4\cdot\left(\frac{3Aa^2b^2}{2}+Ba^3b\right)+x^3\cdot\left(\frac{4Aa^3b}{3}+\frac{Ba^4}{3}\right)$$

input
```
integrate(x*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*a**4*x**2/2 + B*b**4*x**7/7 + x**6*(A*b**4/6 + 2*B*a*b**3/3) + x**5*(4*A
*a*b**3/5 + 6*B*a**2*b**2/5) + x**4*(3*A*a**2*b**2/2 + B*a**3*b) + x**3*(4
*A*a**3*b/3 + B*a**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.62

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{7}\,Bb^4x^7+\frac{1}{2}\,Aa^4x^2+\frac{1}{6}\left(4\,Bab^3+Ab^4\right)x^6+\frac{2}{5}\left(3\,Ba^2b^2+2\,Aab^3\right)x^5+\frac{1}{2}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^4+\frac{1}{3}\left(Ba^4+4\,Aa^3b\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/7*B*b^4*x^7 + 1/2*A*a^4*x^2 + 1/6*(4*B*a*b^3 + A*b^4)*x^6 + 2/5*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^5 + 1/2*(2*B*a^3*b + 3*A*a^2*b^2)*x^4 + 1/3*(B*a^4 + 4
*A*a^3*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{1}{7}\,Bb^4x^7+\frac{2}{3}\,Bab^3x^6+\frac{1}{6}\,Ab^4x^6+\frac{6}{5}\,Ba^2b^2x^5+\frac{4}{5}\,Aab^3x^5+Ba^3bx^4+\frac{3}{2}\,Aa^2b^2x^4+\frac{1}{3}\,Ba^4x^3+\frac{4}{3}\,Aa^3bx^3+\frac{1}{2}\,Aa^4x^2$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/7*B*b^4*x^7 + 2/3*B*a*b^3*x^6 + 1/6*A*b^4*x^6 + 6/5*B*a^2*b^2*x^5 + 4/5*
A*a*b^3*x^5 + B*a^3*b*x^4 + 3/2*A*a^2*b^2*x^4 + 1/3*B*a^4*x^3 + 4/3*A*a^3*
b*x^3 + 1/2*A*a^4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^3\left(\frac{B\,a^4}{3}+\frac{4\,A\,b\,a^3}{3}\right)+x^6\left(\frac{A\,b^4}{6}+\frac{2\,B\,a\,b^3}{3}\right)+\frac{A\,a^4\,x^2}{2}+\frac{B\,b^4\,x^7}{7}+\frac{a^2\,b\,x^4\,(3\,A\,b+2\,B\,a)}{2}+\frac{2\,a\,b^2\,x^5\,(2\,A\,b+3\,B\,a)}{5}$$

input
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^3*((B*a^4)/3 + (4*A*a^3*b)/3) + x^6*((A*b^4)/6 + (2*B*a*b^3)/3) + (A*a^4
*x^2)/2 + (B*b^4*x^7)/7 + (a^2*b*x^4*(3*A*b + 2*B*a))/2 + (2*a*b^2*x^5*(2*
A*b + 3*B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{x^2(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5)}{42}$$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**2*(21*a**5 + 70*a**4*b*x + 105*a**3*b**2*x**2 + 84*a**2*b**3*x**3 + 35
*a*b**4*x**4 + 6*b**5*x**5))/42
```

3.156 $\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{(Ab-aB)(a+bx)^5}{5b^2}+\frac{B(a+bx)^6}{6b^2}$$

output
```
1/5*(A*b-B*a)*(b*x+a)^5/b^2+1/6*B*(b*x+a)^6/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{1}{30}x\left(15a^4(2A+Bx)+20a^3bx(3A+2Bx) + 15a^2b^2x^2(4A+3Bx)+6ab^3x^3(5A+4Bx) + b^4x^4(6A+5Bx)\right)$$

input
```
Integrate[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(x*(15*a^4*(2*A + B*x) + 20*a^3*b*x*(3*A + 2*B*x) + 15*a^2*b^2*x^2*(4*A +
3*B*x) + 6*a*b^3*x^3*(5*A + 4*B*x) + b^4*x^4*(6*A + 5*B*x)))/30
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^2 (A+Bx) \, dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int b^4(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int (a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^4(Ab-aB)}{b}+\frac{B(a+bx)^5}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^5(Ab-aB)}{5b^2}+\frac{B(a+bx)^6}{6b^2}$$

input
```
Int[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*(a + b*x)^5)/(5*b^2) + (B*(a + b*x)^6)/(6*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.

Time = 0.85 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

method	result
norman	$\frac{B\,b^4x^6}{6}+\left(\frac{1}{5}A\,b^4+\frac{4}{5}Ba\,b^3\right)x^5+\left(Aa\,b^3+\frac{3}{2}B\,a^2b^2\right)x^4+\left(2a^2A\,b^2+\frac{4}{3}B\,a^3b\right)x^3+\left(2A\,a^3b+\right.$
default	$\frac{B\,b^4x^6}{6}+\frac{(A\,b^4+4Ba\,b^3)x^5}{5}+\frac{(4Aa\,b^3+6B\,a^2b^2)x^4}{4}+\frac{(6a^2A\,b^2+4B\,a^3b)x^3}{3}+\frac{(4A\,a^3b+a^4B)x^2}{2}+a^4Ax$
gosper	$\frac{x(5b^4B\,x^5+6A\,b^4x^4+24Ba\,b^3x^4+30Aa\,b^3x^3+45B\,a^2b^2x^3+60A\,a^2b^2x^2+40B\,a^3b\,x^2+60A\,a^3bx+15a^4Bx+30a^4A)}{30}$
risch	$\frac{1}{6}B\,b^4x^6+\frac{1}{5}A\,b^4x^5+\frac{4}{5}Ba\,b^3x^5+Aa\,b^3x^4+\frac{3}{2}a^2B\,x^4b^2+2A\,a^2b^2x^3+\frac{4}{3}B\,a^3b\,x^3+2a^3Ab\,x$
parallelrisch	$\frac{1}{6}B\,b^4x^6+\frac{1}{5}A\,b^4x^5+\frac{4}{5}Ba\,b^3x^5+Aa\,b^3x^4+\frac{3}{2}a^2B\,x^4b^2+2A\,a^2b^2x^3+\frac{4}{3}B\,a^3b\,x^3+2a^3Ab\,x$
orering	$\frac{x(5b^4B\,x^5+6A\,b^4x^4+24Ba\,b^3x^4+30Aa\,b^3x^3+45B\,a^2b^2x^3+60A\,a^2b^2x^2+40B\,a^3b\,x^2+60A\,a^3bx+15a^4Bx+30a^4A)(b^2x^2+2}{30(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*b^4*x^6+(1/5*A*b^4+4/5*B*a*b^3)*x^5+(A*a*b^3+3/2*B*a^2*b^2)*x^4+(2*a
^2*A*b^2+4/3*B*a^3*b)*x^3+(2*A*a^3*b+1/2*a^4*B)*x^2+a^4*A*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{6}\,Bb^4x^6 + Aa^4x + \frac{1}{5}\left(4\,Bab^3 + Ab^4\right)x^5 + \frac{1}{2}\left(3\,Ba^2b^2 + 2\,Aab^3\right)x^4 + \frac{2}{3}\left(2\,Ba^3b + 3\,Aa^2b^2\right)x^3 + \frac{1}{2}\left(Ba^4 + 4\,Aa^3b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*B*b^4*x^6 + A*a^4*x + 1/5*(4*B*a*b^3 + A*b^4)*x^5 + 1/2*(3*B*a^2*b^2 +
 2*A*a*b^3)*x^4 + 2/3*(2*B*a^3*b + 3*A*a^2*b^2)*x^3 + 1/2*(B*a^4 + 4*A*a^3
*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = Aa^4x + \frac{Bb^4x^6}{6} + x^5\left(\frac{Ab^4}{5} + \frac{4Bab^3}{5}\right) + x^4\left(Aab^3 + \frac{3Ba^2b^2}{2}\right) + x^3 \cdot \left(2Aa^2b^2 + \frac{4Ba^3b}{3}\right) + x^2 \cdot \left(2Aa^3b + \frac{Ba^4}{2}\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*a**4*x + B*b**4*x**6/6 + x**5*(A*b**4/5 + 4*B*a*b**3/5) + x**4*(A*a*b**3
 + 3*B*a**2*b**2/2) + x**3*(2*A*a**2*b**2 + 4*B*a**3*b/3) + x**2*(2*A*a**3
*b + B*a**4/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{6}\,Bb^4x^6 + Aa^4x + \frac{1}{5}\left(4\,Bab^3 + Ab^4\right)x^5 + \frac{1}{2}\left(3\,Ba^2b^2 + 2\,Aab^3\right)x^4 + \frac{2}{3}\left(2\,Ba^3b + 3\,Aa^2b^2\right)x^3 + \frac{1}{2}\left(Ba^4 + 4\,Aa^3b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*B*b^4*x^6 + A*a^4*x + 1/5*(4*B*a*b^3 + A*b^4)*x^5 + 1/2*(3*B*a^2*b^2 +
 2*A*a*b^3)*x^4 + 2/3*(2*B*a^3*b + 3*A*a^2*b^2)*x^3 + 1/2*(B*a^4 + 4*A*a^3
*b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{1}{6}\,Bb^4x^6 + \frac{4}{5}\,Bab^3x^5 + \frac{1}{5}\,Ab^4x^5 + \frac{3}{2}\,Ba^2b^2x^4 + Aab^3x^4 + \frac{4}{3}\,Ba^3bx^3 + 2\,Aa^2b^2x^3 + \frac{1}{2}\,Ba^4x^2 + 2\,Aa^3bx^2 + Aa^4x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/6*B*b^4*x^6 + 4/5*B*a*b^3*x^5 + 1/5*A*b^4*x^5 + 3/2*B*a^2*b^2*x^4 + A*a*
b^3*x^4 + 4/3*B*a^3*b*x^3 + 2*A*a^2*b^2*x^3 + 1/2*B*a^4*x^2 + 2*A*a^3*b*x^
2 + A*a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = x^2\left(\frac{B\,a^4}{2}+2\,A\,b\,a^3\right)+x^5\left(\frac{A\,b^4}{5}+\frac{4\,B\,a\,b^3}{5}\right)+\frac{B\,b^4\,x^6}{6}+A\,a^4\,x+\frac{2\,a^2\,b\,x^3\,(3\,A\,b+2\,B\,a)}{3}+\frac{a\,b^2\,x^4\,(2\,A\,b+3\,B\,a)}{2}$$

input
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^2*((B*a^4)/2 + 2*A*a^3*b) + x^5*((A*b^4)/5 + (4*B*a*b^3)/5) + (B*b^4*x^6
)/6 + A*a^4*x + (2*a^2*b*x^3*(3*A*b + 2*B*a))/3 + (a*b^2*x^4*(2*A*b + 3*B*
a))/2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)}{6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 + 6*a*b**
4*x**4 + b**5*x**5))/6
```

3.157 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = 4a^3Abx+3a^2Ab^2x^2+\frac{4}{3}aAb^3x^3 + \frac{1}{4}Ab^4x^4+\frac{B(a+bx)^5}{5b}+a^4A\log(x)$$

output
```
4*a^3*A*b*x+3*a^2*A*b^2*x^2+4/3*a*A*b^3*x^3+1/4*A*b^4*x^4+1/5*B*(b*x+a)^5/
b+a^4*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = a^4Bx+2a^3bx(2A+Bx) + a^2b^2x^2(3A+2Bx)+\frac{1}{3}ab^3x^3(4A+3Bx) + \frac{1}{20}b^4x^4(5A+4Bx)+a^4A\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x,x]
```

output
```
a^4*B*x + 2*a^3*b*x*(2*A + B*x) + a^2*b^2*x^2*(3*A + 2*B*x) + (a*b^3*x^3*(
4*A + 3*B*x))/3 + (b^4*x^4*(5*A + 4*B*x))/20 + a^4*A*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1184, 27, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2(A+Bx)}{x}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x}dx$$

$$\Big\downarrow \text{90}$$

$$A\int \frac{(a+bx)^4}{x}dx+\frac{B(a+bx)^5}{5b}$$

$$\Big\downarrow \text{49}$$

$$A\int \left(\frac{a^4}{x}+4ba^3+6b^2xa^2+4b^3x^2a+b^4x^3\right)dx+\frac{B(a+bx)^5}{5b}$$

$$\Big\downarrow \text{2009}$$

$$A\left(a^4\log(x)+4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}\right)+\frac{B(a+bx)^5}{5b}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x,x]
```

output
```
(B*(a + b*x)^5)/(5*b) + A*(4*a^3*b*x + 3*a^2*b^2*x^2 + (4*a*b^3*x^3)/3 + (
b^4*x^4)/4 + a^4*Log[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result
norman	$\left(\frac{1}{4}A\,b^{4}+B a\,b^{3}\right)x^{4}+\left(\frac{4}{3}Aa\,b^{3}+2B\,a^{2}b^{2}\right)x^{3}+\left(3a^{2}A\,b^{2}+2B\,a^{3}b\right)x^{2}+\left(4A\,a^{3}b+a^{4}B\right)x$ -
default	$\frac{b^{4}B\,x^{5}}{5}+\frac{A\,b^{4}x^{4}}{4}+Ba\,b^{3}x^{4}+\frac{4Aa\,b^{3}x^{3}}{3}+2B\,a^{2}b^{2}x^{3}+3A\,a^{2}b^{2}x^{2}+2B\,a^{3}b\,x^{2}+4A\,a^{3}bx+a^{4}B$
risch	$\frac{b^{4}B\,x^{5}}{5}+\frac{A\,b^{4}x^{4}}{4}+Ba\,b^{3}x^{4}+\frac{4Aa\,b^{3}x^{3}}{3}+2B\,a^{2}b^{2}x^{3}+3A\,a^{2}b^{2}x^{2}+2B\,a^{3}b\,x^{2}+4A\,a^{3}bx+a^{4}B$
parallelrisch	$\frac{b^{4}B\,x^{5}}{5}+\frac{A\,b^{4}x^{4}}{4}+Ba\,b^{3}x^{4}+\frac{4Aa\,b^{3}x^{3}}{3}+2B\,a^{2}b^{2}x^{3}+3A\,a^{2}b^{2}x^{2}+2B\,a^{3}b\,x^{2}+4A\,a^{3}bx+a^{4}B$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/4*A*b^4+B*a*b^3)*x^4+(4/3*A*a*b^3+2*B*a^2*b^2)*x^3+(3*A*a^2*b^2+2*B*a^3
*b)*x^2+(4*A*a^3*b+B*a^4)*x+1/5*b^4*B*x^5+a^4*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a^{2}+2abx+b^{2}x^{2}\right)^{2}}{x}\,dx=\frac{1}{5}\,Bb^{4}x^{5}+Aa^{4}\log\left(x\right)+\frac{1}{4}\left(4\,Bab^{3}+Ab^{4}\right)x^{4}+\frac{2}{3}\left(3\,Ba^{2}b^{2}+2\,Aab^{3}\right)x^{3}+\left(2\,Ba^{3}b+3\,Aa^{2}b^{2}\right)x^{2}+\left(Ba^{4}+4\,Aa^{3}b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="fricas")
```

output
```
1/5*B*b^4*x^5 + A*a^4*log(x) + 1/4*(4*B*a*b^3 + A*b^4)*x^4 + 2/3*(3*B*a^2*
b^2 + 2*A*a*b^3)*x^3 + (2*B*a^3*b + 3*A*a^2*b^2)*x^2 + (B*a^4 + 4*A*a^3*b)
*x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = Aa^4\log{(x)} + \frac{Bb^4x^5}{5} + x^4\left(\frac{Ab^4}{4} + Bab^3\right) + x^3\cdot\left(\frac{4Aab^3}{3} + 2Ba^2b^2\right) + x^2\cdot\left(3Aa^2b^2 + 2Ba^3b\right) + x\left(4Aa^3b + Ba^4\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x,x)
```

output
```
A*a**4*log(x) + B*b**4*x**5/5 + x**4*(A*b**4/4 + B*a*b**3) + x**3*(4*A*a*b
**3/3 + 2*B*a**2*b**2) + x**2*(3*A*a**2*b**2 + 2*B*a**3*b) + x*(4*A*a**3*b
 + B*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = \frac{1}{5}\,Bb^4x^5 + Aa^4\log{(x)} + \frac{1}{4}\left(4\,Bab^3 + Ab^4\right)x^4 + \frac{2}{3}\left(3\,Ba^2b^2 + 2\,Aab^3\right)x^3 + \left(2\,Ba^3b + 3\,Aa^2b^2\right)x^2 + \left(Ba^4 + 4\,Aa^3b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="maxima")
```

output
```
1/5*B*b^4*x^5 + A*a^4*log(x) + 1/4*(4*B*a*b^3 + A*b^4)*x^4 + 2/3*(3*B*a^2*
b^2 + 2*A*a*b^3)*x^3 + (2*B*a^3*b + 3*A*a^2*b^2)*x^2 + (B*a^4 + 4*A*a^3*b)
*x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = \frac{1}{5}Bb^4x^5 + Bab^3x^4 + \frac{1}{4}Ab^4x^4 + 2Ba^2b^2x^3 + \frac{4}{3}Aab^3x^3 + 2Ba^3bx^2 + 3Aa^2b^2x^2 + Ba^4x + 4Aa^3bx + Aa^4\log(|x|)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x,x, algorithm="giac")
```

output
```
1/5*B*b^4*x^5 + B*a*b^3*x^4 + 1/4*A*b^4*x^4 + 2*B*a^2*b^2*x^3 + 4/3*A*a*b^
3*x^3 + 2*B*a^3*b*x^2 + 3*A*a^2*b^2*x^2 + B*a^4*x + 4*A*a^3*b*x + A*a^4*lo
g(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = x\left(B\,a^4+4\,A\,b\,a^3\right) + x^4\left(\frac{A\,b^4}{4}+B\,a\,b^3\right) + \frac{B\,b^4\,x^5}{5} + A\,a^4\,\ln(x) + a^2\,b\,x^2\,(3\,A\,b+2\,B\,a) + \frac{2\,a\,b^2\,x^3\,(2\,A\,b+3\,B\,a)}{3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x,x)
```

output
```
x*(B*a^4 + 4*A*a^3*b) + x^4*((A*b^4)/4 + B*a*b^3) + (B*b^4*x^5)/5 + A*a^4*
log(x) + a^2*b*x^2*(3*A*b + 2*B*a) + (2*a*b^2*x^3*(2*A*b + 3*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x}\,dx = \log(x)\,a^5+5a^4bx+5a^3b^2x^2 + \frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x,x)
```

output
```
(60*log(x)*a**5 + 300*a**4*b*x + 300*a**3*b**2*x**2 + 200*a**2*b**3*x**3 +
 75*a*b**4*x**4 + 12*b**5*x**5)/60
```

3.158 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = -\frac{a^4A}{x}+2a^2b(3Ab+2aB)x+ab^2(2Ab+3aB)x^2+\frac{1}{3}b^3(Ab+4aB)x^3+\frac{1}{4}b^4Bx^4+a^3(4Ab+aB)\log(x)$$

output
```
-a^4*A/x+2*a^2*b*(3*A*b+2*B*a)*x+a*b^2*(2*A*b+3*B*a)*x^2+1/3*b^3*(A*b+4*B*
a)*x^3+1/4*b^4*B*x^4+a^3*(4*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = -\frac{a^4A}{x}+4a^3bBx+3a^2b^2x(2A+Bx)+\frac{2}{3}ab^3x^2(3A+2Bx)+\frac{1}{12}b^4x^3(4A+3Bx)+a^3(4Ab+aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^2,x]
```

output
```
-((a^4*A)/x) + 4*a^3*b*B*x + 3*a^2*b^2*x*(2*A + B*x) + (2*a*b^3*x^2*(3*A +
 2*B*x))/3 + (b^4*x^3*(4*A + 3*B*x))/12 + a^3*(4*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2 (A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^2}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^2}+\frac{a^3(aB+4Ab)}{x}+2a^2b(2aB+3Ab)+b^3x^2(4aB+Ab)+2ab^2x(3aB+2Ab)+b^4Bx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{x}+a^3\log(x)(aB+4Ab)+2a^2bx(2aB+3Ab)+\frac{1}{3}b^3x^3(4aB+Ab)+ab^2x^2(3aB+2Ab)+\frac{1}{4}b^4Bx^4$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^2,x]
```

output
```
-((a^4*A)/x) + 2*a^2*b*(3*A*b + 2*a*B)*x + a*b^2*(2*A*b + 3*a*B)*x^2 + (b^
3*(A*b + 4*a*B)*x^3)/3 + (b^4*B*x^4)/4 + a^3*(4*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

method	result
default	$\frac{b^4 B x^4}{4}+\frac{A b^4 x^3}{3}+\frac{4 B a b^3 x^3}{3}+2 A a b^3 x^2+3 B a^2 b^2 x^2+6 A a^2 b^2 x+4 B a^3 b x+a^3 (4 A b+B a) \ln$
risch	$\frac{b^4 B x^4}{4}+\frac{A b^4 x^3}{3}+\frac{4 B a b^3 x^3}{3}+2 A a b^3 x^2+3 B a^2 b^2 x^2+6 A a^2 b^2 x+4 B a^3 b x-\frac{a^4 A}{x}+4 A \ln (x)$
norman	$\frac{\left(\frac{1}{3} A b^4+\frac{4}{3} B a b^3\right) x^4+\left(2 A a b^3+3 B a^2 b^2\right) x^3+\left(6 a^2 A b^2+4 B a^3 b\right) x^2-a^4 A+\frac{b^4 B x^5}{4}}{x}+\left(4 A a^3 b+a^4 B\right) \ln (x)$
parallelrisch	$\frac{3 b^4 B x^5+4 A b^4 x^4+16 B a b^3 x^4+24 A a b^3 x^3+36 B a^2 b^2 x^3+48 A \ln (x) x a^3 b+72 A a^2 b^2 x^2+12 B \ln (x) x a^4+48 B a^3 b x^2-12 a^4 A}{12 x}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^4*B*x^4+1/3*A*b^4*x^3+4/3*B*a*b^3*x^3+2*A*a*b^3*x^2+3*B*a^2*b^2*x^2+
6*A*a^2*b^2*x+4*B*a^3*b*x+a^3*(4*A*b+B*a)*ln(x)-a^4*A/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx$$
$$= \frac{3\,Bb^4x^5 - 12\,Aa^4 + 4\,(4\,Bab^3 + Ab^4)x^4 + 12\,(3\,Ba^2b^2 + 2\,Aab^3)x^3 + 24\,(2\,Ba^3b + 3\,Aa^2b^2)x^2 + 12\,(Ba^4}{12\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*B*b^4*x^5 - 12*A*a^4 + 4*(4*B*a*b^3 + A*b^4)*x^4 + 12*(3*B*a^2*b^2
 + 2*A*a*b^3)*x^3 + 24*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 12*(B*a^4 + 4*A*a^3
*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = -\frac{Aa^4}{x} + \frac{Bb^4x^4}{4} + a^3\cdot(4Ab+Ba)\log{(x)} + x^3\left(\frac{Ab^4}{3}+\frac{4Bab^3}{3}\right) + x^2 \cdot\left(2Aab^3+3Ba^2b^2\right) + x\left(6Aa^2b^2+4Ba^3b\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**2,x)
```

output
```
-A*a**4/x + B*b**4*x**4/4 + a**3*(4*A*b + B*a)*log(x) + x**3*(A*b**4/3 + 4
*B*a*b**3/3) + x**2*(2*A*a*b**3 + 3*B*a**2*b**2) + x*(6*A*a**2*b**2 + 4*B*
a**3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = \frac{1}{4}\,Bb^4x^4 - \frac{Aa^4}{x} + \frac{1}{3}\left(4\,Bab^3 + Ab^4\right)x^3 + \left(3\,Ba^2b^2 + 2\,Aab^3\right)x^2 + 2\left(2\,Ba^3b + 3\,Aa^2b^2\right)x + \left(Ba^4 + 4\,Aa^3b\right)\log\left(x\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/4*B*b^4*x^4 - A*a^4/x + 1/3*(4*B*a*b^3 + A*b^4)*x^3 + (3*B*a^2*b^2 + 2*A
*a*b^3)*x^2 + 2*(2*B*a^3*b + 3*A*a^2*b^2)*x + (B*a^4 + 4*A*a^3*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = \frac{1}{4}\,Bb^4x^4 + \frac{4}{3}\,Bab^3x^3 + \frac{1}{3}\,Ab^4x^3 + 3\,Ba^2b^2x^2 + 2\,Aab^3x^2 + 4\,Ba^3bx + 6\,Aa^2b^2x - \frac{Aa^4}{x} + \left(Ba^4 + 4\,Aa^3b\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^2,x, algorithm="giac")
```

output
```
1/4*B*b^4*x^4 + 4/3*B*a*b^3*x^3 + 1/3*A*b^4*x^3 + 3*B*a^2*b^2*x^2 + 2*A*a*
b^3*x^2 + 4*B*a^3*b*x + 6*A*a^2*b^2*x - A*a^4/x + (B*a^4 + 4*A*a^3*b)*log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = x^3\left(\frac{A\,b^4}{3}+\frac{4\,B\,a\,b^3}{3}\right)+\ln\left(x\right)\,\left(B\,a^4+4\,A\,b\,a^3\right) - \frac{A\,a^4}{x}+\frac{B\,b^4\,x^4}{4}+2\,a^2\,b\,x\,(3\,A\,b+2\,B\,a) + a\,b^2\,x^2\,(2\,A\,b+3\,B\,a)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^2,x)
```

output
```
x^3*((A*b^4)/3 + (4*B*a*b^3)/3) + log(x)*(B*a^4 + 4*A*a^3*b) - (A*a^4)/x +
 (B*b^4*x^4)/4 + 2*a^2*b*x*(3*A*b + 2*B*a) + a*b^2*x^2*(2*A*b + 3*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^2}\,dx = \frac{60\log(x)\,a^4bx-12a^5+120a^3b^2x^2+60a^2b^3x^3+20a\,b^4x^4+3b^5x^5}{12x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^2,x)
```

output
```
(60*log(x)*a**4*b*x - 12*a**5 + 120*a**3*b**2*x**2 + 60*a**2*b**3*x**3 + 2
0*a*b**4*x**4 + 3*b**5*x**5)/(12*x)
```

3.159 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3} \, dx = -\frac{a^4A}{2x^2}-\frac{a^3(4Ab+aB)}{x}+2ab^2(2Ab+3aB)x+\frac{1}{2}b^3(Ab+4aB)x^2+\frac{1}{3}b^4Bx^3+2a^2b(3Ab+2aB)\log(x)$$

output
```
-1/2*a^4*A/x^2-a^3*(4*A*b+B*a)/x+2*a*b^2*(2*A*b+3*B*a)*x+1/2*b^3*(A*b+4*B*
a)*x^2+1/3*b^4*B*x^3+2*a^2*b*(3*A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3} \, dx = -\frac{4a^3Ab}{x}+6a^2b^2Bx+2ab^3x(2A+Bx)-\frac{a^4(A+2Bx)}{2x^2}+\frac{1}{6}b^4x^2(3A+2Bx)+2a^2b(3Ab+2aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^3,x]
```

output
```
(-4*a^3*A*b)/x + 6*a^2*b^2*B*x + 2*a*b^3*x*(2*A + B*x) - (a^4*(A + 2*B*x))
/(2*x^2) + (b^4*x^2*(3*A + 2*B*x))/6 + 2*a^2*b*(3*A*b + 2*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \ 1184$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^3}dx}{b^4}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^3}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^4A}{x^3}+\frac{a^3(aB+4Ab)}{x^2}+\frac{2a^2b(2aB+3Ab)}{x}+b^3x(4aB+Ab)+2ab^2(3aB+2Ab)+b^4Bx^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^4A}{2x^2}-\frac{a^3(aB+4Ab)}{x}+2a^2b\log(x)(2aB+3Ab)+\frac{1}{2}b^3x^2(4aB+Ab)+2ab^2x(3aB+2Ab)+\frac{1}{3}b^4Bx^3$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^3,x]
```

output
```
-1/2*(a^4*A)/x^2 - (a^3*(4*A*b + a*B))/x + 2*a*b^2*(2*A*b + 3*a*B)*x + (b^
3*(A*b + 4*a*B)*x^2)/2 + (b^4*B*x^3)/3 + 2*a^2*b*(3*A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^4 B\,x^3}{3}+\frac{A\,b^4x^2}{2}+2Ba\,b^3x^2+4aA\,b^3x+6B\,a^2b^2x-\frac{a^4A}{2x^2}+2a^2b(3Ab+2Ba)\ln\left(x\right)-\frac{a^3(4Ab}{x}$
risch	$\frac{b^4 B\,x^3}{3}+\frac{A\,b^4x^2}{2}+2Ba\,b^3x^2+4aA\,b^3x+6B\,a^2b^2x+\frac{(-4A\,a^3b-a^4B)x-\frac{a^4A}{2}}{x^2}+6A\ln\left(x\right)a^2b^2+4$
norman	$\frac{(\frac{1}{2}A\,b^4+2Ba\,b^3)x^4+(4Aa\,b^3+6B\,a^2b^2)x^3+(-4A\,a^3b-a^4B)x-\frac{a^4A}{2}+\frac{b^4B\,x^5}{3}}{x^2}+(6a^2A\,b^2+4B\,a^3b)\ln\left(x\right)$
parallelrisch	$\frac{2b^4B\,x^5+3A\,b^4x^4+12Ba\,b^3x^4+36A\ln(x)x^2a^2b^2+24Aa\,b^3x^3+24B\ln(x)x^2a^3b+36B\,a^2b^2x^3-24A\,a^3bx-6a^4Bx-3a^4A}{6x^2}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^4*B*x^3+1/2*A*b^4*x^2+2*B*a*b^3*x^2+4*a*A*b^3*x+6*B*a^2*b^2*x-1/2*a^
4*A/x^2+2*a^2*b*(3*A*b+2*B*a)*ln(x)-a^3*(4*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx = \frac{2\,Bb^4x^5-3\,Aa^4+3\,(4\,Bab^3+Ab^4)x^4+12\,(3\,Ba^2b^2+2\,Aab^3)x^3+12\,(2\,Ba^3b+3\,Aa^2b^2)x^2\log\left(x\right)-6}{6\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^4*x^5 - 3*A*a^4 + 3*(4*B*a*b^3 + A*b^4)*x^4 + 12*(3*B*a^2*b^2 +
 2*A*a*b^3)*x^3 + 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2*log(x) - 6*(B*a^4 + 4*A
*a^3*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx &= \frac{Bb^4x^3}{3}+2a^2b(3Ab+2Ba)\log\left(x\right)\\ &\quad+x^2\left(\frac{Ab^4}{2}+2Bab^3\right)+x\left(4Aab^3+6Ba^2b^2\right)\\ &\quad+\frac{-Aa^4+x(-8Aa^3b-2Ba^4)}{2x^2}\end{aligned}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**3,x)
```

output
```
B*b**4*x**3/3 + 2*a**2*b*(3*A*b + 2*B*a)*log(x) + x**2*(A*b**4/2 + 2*B*a*b
**3) + x*(4*A*a*b**3 + 6*B*a**2*b**2) + (-A*a**4 + x*(-8*A*a**3*b - 2*B*a*
*4))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx = \frac{1}{3}\,Bb^4x^3 + \frac{1}{2}\left(4\,Bab^3 + Ab^4\right)x^2 + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)x + 2\left(2\,Ba^3b + 3\,Aa^2b^2\right)\log\left(x\right) - \frac{Aa^4 + 2\,(Ba^4 + 4\,Aa^3b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/3*B*b^4*x^3 + 1/2*(4*B*a*b^3 + A*b^4)*x^2 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*
x + 2*(2*B*a^3*b + 3*A*a^2*b^2)*log(x) - 1/2*(A*a^4 + 2*(B*a^4 + 4*A*a^3*b
)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx = \frac{1}{3}\,Bb^4x^3 + 2\,Bab^3x^2 + \frac{1}{2}\,Ab^4x^2 + 6\,Ba^2b^2x + 4\,Aab^3x + 2\left(2\,Ba^3b + 3\,Aa^2b^2\right)\log\left(|x|\right) - \frac{Aa^4 + 2\,(Ba^4 + 4\,Aa^3b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^3,x, algorithm="giac")
```

output
```
1/3*B*b^4*x^3 + 2*B*a*b^3*x^2 + 1/2*A*b^4*x^2 + 6*B*a^2*b^2*x + 4*A*a*b^3*
x + 2*(2*B*a^3*b + 3*A*a^2*b^2)*log(abs(x)) - 1/2*(A*a^4 + 2*(B*a^4 + 4*A*
a^3*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx = \ln(x)\ \left(4\,B\,a^3\,b+6\,A\,a^2\,b^2\right) - \frac{x\,\left(B\,a^4+4\,A\,b\,a^3\right)+\frac{A\,a^4}{2}}{x^2} + x^2\,\left(\frac{A\,b^4}{2}+2\,B\,a\,b^3\right)+\frac{B\,b^4\,x^3}{3} + 2\,a\,b^2\,x\,\left(2\,A\,b+3\,B\,a\right)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^3,x)
```

output
```
log(x)*(6*A*a^2*b^2 + 4*B*a^3*b) - (x*(B*a^4 + 4*A*a^3*b) + (A*a^4)/2)/x^2
 + x^2*((A*b^4)/2 + 2*B*a*b^3) + (B*b^4*x^3)/3 + 2*a*b^2*x*(2*A*b + 3*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^3}\,dx = \frac{60\log(x)\,a^3b^2x^2-3a^5-30a^4bx+60a^2b^3x^3+15a\,b^4x^4+2b^5x^5}{6x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^3,x)
```

output
```
(60*log(x)*a**3*b**2*x**2 - 3*a**5 - 30*a**4*b*x + 60*a**2*b**3*x**3 + 15*
a*b**4*x**4 + 2*b**5*x**5)/(6*x**2)
```

3.160 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = -\frac{a^4A}{3x^3}-\frac{a^3(4Ab+aB)}{2x^2}-\frac{2a^2b(3Ab+2aB)}{x}+b^3(Ab+4aB)x+\frac{1}{2}b^4Bx^2+2ab^2(2Ab+3aB)\log(x)$$

output
```
-1/3*a^4*A/x^3-1/2*a^3*(4*A*b+B*a)/x^2-2*a^2*b*(3*A*b+2*B*a)/x+b^3*(A*b+4*
B*a)*x+1/2*b^4*B*x^2+2*a*b^2*(2*A*b+3*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = -\frac{6a^2Ab^2}{x}+4ab^3Bx+\frac{1}{2}b^4x(2A+Bx)-\frac{2a^3b(A+2Bx)}{x^2}-\frac{a^4(2A+3Bx)}{6x^3}+2ab^2(2Ab+3aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^4,x]
```

output
```
(-6*a^2*A*b^2)/x + 4*a*b^3*B*x + (b^4*x*(2*A + B*x))/2 - (2*a^3*b*(A + 2*B
*x))/x^2 - (a^4*(2*A + 3*B*x))/(6*x^3) + 2*a*b^2*(2*A*b + 3*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2 (A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^4}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^4}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^4}+\frac{a^3(aB+4Ab)}{x^3}+\frac{2a^2b(2aB+3Ab)}{x^2}+b^3(4aB+Ab)+\frac{2ab^2(3aB+2Ab)}{x}+b^4Bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{3x^3}-\frac{a^3(aB+4Ab)}{2x^2}-\frac{2a^2b(2aB+3Ab)}{x}+b^3x(4aB+Ab)+2ab^2\log(x)(3aB+2Ab)+\frac{1}{2}b^4Bx^2$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^4,x]
```

output
```
-1/3*(a^4*A)/x^3 - (a^3*(4*A*b + a*B))/(2*x^2) - (2*a^2*b*(3*A*b + 2*a*B))
/x + b^3*(A*b + 4*a*B)*x + (b^4*B*x^2)/2 + 2*a*b^2*(2*A*b + 3*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$\frac{B\,b^4x^2}{2}+A\,b^4x+4Ba\,b^3x-\frac{a^4A}{3x^3}-\frac{a^3(4Ab+Ba)}{2x^2}+2a\,b^2(2Ab+3Ba)\ln\left(x\right)-\frac{2a^2b(3Ab+2Ba)}{x}$
risch	$\frac{B\,b^4x^2}{2}+A\,b^4x+4Ba\,b^3x+\frac{(-6a^2A\,b^2-4B\,a^3b)x^2+(-2A\,a^3b-\frac{1}{2}a^4B)x-\frac{a^4A}{3}}{x^3}+4A\ln\left(x\right)a\,b^3+6B\ln$
norman	$\frac{(-2A\,a^3b-\frac{1}{2}a^4B)x+(A\,b^4+4Ba\,b^3)x^4+(-6a^2A\,b^2-4B\,a^3b)x^2-\frac{a^4A}{3}+\frac{b^4B\,x^5}{2}}{x^3}+\left(4Aa\,b^3+6B\,a^2b^2\right)\ln\left(x\right)$
parallelrisch	$\frac{3b^4B\,x^5+24A\ln(x)x^3a\,b^3+6A\,b^4x^4+36B\ln(x)x^3a^2b^2+24Ba\,b^3x^4-36A\,a^2b^2x^2-24B\,a^3b\,x^2-12A\,a^3bx-3a^4Bx-2a^4A}{6x^3}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*b^4*x^2+A*b^4*x+4*B*a*b^3*x-1/3*a^4*A/x^3-1/2*a^3*(4*A*b+B*a)/x^2+2*
a*b^2*(2*A*b+3*B*a)*ln(x)-2*a^2*b*(3*A*b+2*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = \frac{3\,Bb^4x^5-2\,Aa^4+6\,(4\,Bab^3+Ab^4)x^4+12\,(3\,Ba^2b^2+2\,Aab^3)x^3\log\left(x\right)-12\,(2\,Ba^3b+3\,Aa^2b^2)x^2-3}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*B*b^4*x^5 - 2*A*a^4 + 6*(4*B*a*b^3 + A*b^4)*x^4 + 12*(3*B*a^2*b^2 +
 2*A*a*b^3)*x^3*log(x) - 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 3*(B*a^4 + 4*A
*a^3*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = \frac{Bb^4x^2}{2}+2ab^2\cdot(2Ab+3Ba)\log\left(x\right)+x\left(Ab^4+4Bab^3\right) + \frac{-2Aa^4+x^2(-36Aa^2b^2-24Ba^3b)+x(-12Aa^3b-3Ba^4)}{6x^3}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**4,x)
```

output
```
B*b**4*x**2/2 + 2*a*b**2*(2*A*b + 3*B*a)*log(x) + x*(A*b**4 + 4*B*a*b**3)
+ (-2*A*a**4 + x**2*(-36*A*a**2*b**2 - 24*B*a**3*b) + x*(-12*A*a**3*b - 3*
B*a**4))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = \frac{1}{2}\,Bb^4x^2 + \left(4\,Bab^3 + Ab^4\right)x + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)\log\left(x\right) - \frac{2\,Aa^4 + 12\,(2\,Ba^3b + 3\,Aa^2b^2)x^2 + 3\,(Ba^4 + 4\,Aa^3b)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/2*B*b^4*x^2 + (4*B*a*b^3 + A*b^4)*x + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*log(x)
 - 1/6*(2*A*a^4 + 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 3*(B*a^4 + 4*A*a^3*b)
*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = \frac{1}{2}\,Bb^4x^2 + 4\,Bab^3x + Ab^4x + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)\log\left(|x|\right) - \frac{2\,Aa^4 + 12\,(2\,Ba^3b + 3\,Aa^2b^2)x^2 + 3\,(Ba^4 + 4\,Aa^3b)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^4,x, algorithm="giac")
```

output
```
1/2*B*b^4*x^2 + 4*B*a*b^3*x + A*b^4*x + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*log(ab
s(x)) - 1/6*(2*A*a^4 + 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 3*(B*a^4 + 4*A*a
^3*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = x\left(A\,b^4+4\,B\,a\,b^3\right)+\ln\left(x\right)\,\left(6\,B\,a^2\,b^2+4\,A\,a\,b^3\right) - \frac{x\left(\frac{B\,a^4}{2}+2\,A\,b\,a^3\right)+\frac{A\,a^4}{3}+x^2\left(4\,B\,a^3\,b+6\,A\,a^2\,b^2\right)}{x^3}+\frac{B\,b^4\,x^2}{2}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^4,x)
```

output
```
x*(A*b^4 + 4*B*a*b^3) + log(x)*(6*B*a^2*b^2 + 4*A*a*b^3) - (x*((B*a^4)/2 +
 2*A*a^3*b) + (A*a^4)/3 + x^2*(6*A*a^2*b^2 + 4*B*a^3*b))/x^3 + (B*b^4*x^2)
/2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^4}\,dx = \frac{60\log(x)\,a^2b^3x^3-2a^5-15a^4bx-60a^3b^2x^2+30a\,b^4x^4+3b^5x^5}{6x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^4,x)
```

output
```
(60*log(x)*a**2*b**3*x**3 - 2*a**5 - 15*a**4*b*x - 60*a**3*b**2*x**2 + 30*
a*b**4*x**4 + 3*b**5*x**5)/(6*x**3)
```

3.161 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = -\frac{a^4A}{4x^4}-\frac{a^3(4Ab+aB)}{3x^3}-\frac{a^2b(3Ab+2aB)}{x^2}-\frac{2ab^2(2Ab+3aB)}{x}+b^4Bx+b^3(Ab+4aB)\log(x)$$

output
```
-1/4*a^4*A/x^4-1/3*a^3*(4*A*b+B*a)/x^3-a^2*b*(3*A*b+2*B*a)/x^2-2*a*b^2*(2*
A*b+3*B*a)/x+b^4*B*x+b^3*(A*b+4*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = -\frac{4aAb^3}{x}+b^4Bx-\frac{3a^2b^2(A+2Bx)}{x^2}-\frac{2a^3b(2A+3Bx)}{3x^3}-\frac{a^4(3A+4Bx)}{12x^4}+b^3(Ab+4aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^5,x]
```

output
```
(-4*a*A*b^3)/x + b^4*B*x - (3*a^2*b^2*(A + 2*B*x))/x^2 - (2*a^3*b*(2*A + 3
*B*x))/(3*x^3) - (a^4*(3*A + 4*B*x))/(12*x^4) + b^3*(A*b + 4*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^5} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^5}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^5}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^5}+\frac{a^3(aB+4Ab)}{x^4}+\frac{2a^2b(2aB+3Ab)}{x^3}+\frac{b^3(4aB+Ab)}{x}+\frac{2ab^2(3aB+2Ab)}{x^2}+b^4B\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{4x^4}-\frac{a^3(aB+4Ab)}{3x^3}-\frac{a^2b(2aB+3Ab)}{x^2}+b^3\log(x)(4aB+Ab)-\frac{2ab^2(3aB+2Ab)}{x}+b^4Bx$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^5,x]
```

output
```
-1/4*(a^4*A)/x^4 - (a^3*(4*A*b + a*B))/(3*x^3) - (a^2*b*(3*A*b + 2*a*B))/x
^2 - (2*a*b^2*(2*A*b + 3*a*B))/x + b^4*B*x + b^3*(A*b + 4*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

method	result
default	$-\frac{a^4 A}{4x^4}-\frac{a^3(4Ab+Ba)}{3x^3}-\frac{a^2 b(3Ab+2Ba)}{x^2}-\frac{2a\,b^2(2Ab+3Ba)}{x}+B\,b^4 x+b^3(Ab+4Ba)\ln(x)$
risch	$B\,b^4 x+\frac{(-4Aa\,b^3-6B\,a^2b^2)x^3+(-3a^2A\,b^2-2B\,a^3b)x^2+(-\frac{4}{3}A\,a^3b-\frac{1}{3}a^4B)x-\frac{a^4A}{4}}{x^4}+A\ln(x)\,b^4+4B\ln(x)$
norman	$\frac{(-\frac{4}{3}A\,a^3b-\frac{1}{3}a^4B)x+(-4Aa\,b^3-6B\,a^2b^2)x^3+(-3a^2A\,b^2-2B\,a^3b)x^2+b^4B\,x^5-\frac{a^4A}{4}}{x^4}+(A\,b^4+4Ba\,b^3)\ln(x)$
parallelrisch	$\frac{12A\ln(x)x^4b^4+48B\ln(x)x^4a\,b^3+12b^4B\,x^5-48Aa\,b^3x^3-72B\,a^2b^2x^3-36A\,a^2b^2x^2-24B\,a^3b\,x^2-16A\,a^3bx-4a^4Bx-3a^4A}{12x^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^4*A/x^4-1/3*a^3*(4*A*b+B*a)/x^3-a^2*b*(3*A*b+2*B*a)/x^2-2*a*b^2*(2*
A*b+3*B*a)/x+B*b^4*x+b^3*(A*b+4*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = \frac{12\,Bb^4x^5+12\,(4\,Bab^3+Ab^4)x^4\log\left(x\right)-3\,Aa^4-24\,(3\,Ba^2b^2+2\,Aab^3)x^3-12\,(2\,Ba^3b+3\,Aa^2b^2)x^2-}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*B*b^4*x^5 + 12*(4*B*a*b^3 + A*b^4)*x^4*log(x) - 3*A*a^4 - 24*(3*B
*a^2*b^2 + 2*A*a*b^3)*x^3 - 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 4*(B*a^4 +
4*A*a^3*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = Bb^4x+b^3(Ab+4Ba)\log\left(x\right) + \frac{-3Aa^4+x^3(-48Aab^3-72Ba^2b^2)+x^2(-36Aa^2b^2-24Ba^3b)+x(-16Aa^3b-4Ba^4)}{12x^4}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**5,x)
```

output
```
B*b**4*x + b**3*(A*b + 4*B*a)*log(x) + (-3*A*a**4 + x**3*(-48*A*a*b**3 - 7
2*B*a**2*b**2) + x**2*(-36*A*a**2*b**2 - 24*B*a**3*b) + x*(-16*A*a**3*b -
4*B*a**4))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = Bb^4x+\left(4\,Bab^3+Ab^4\right)\log\left(x\right) - \frac{3\,Aa^4+24\,(3\,Ba^2b^2+2\,Aab^3)x^3+12\,(2\,Ba^3b+3\,Aa^2b^2)x^2+4\,(Ba^4+4\,Aa^3b)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^5,x, algorithm="maxima")
```

output
```
B*b^4*x + (4*B*a*b^3 + A*b^4)*log(x) - 1/12*(3*A*a^4 + 24*(3*B*a^2*b^2 + 2
*A*a*b^3)*x^3 + 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 4*(B*a^4 + 4*A*a^3*b)*x
)/x^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^5}\,dx = Bb^4x+\left(4\,Bab^3+Ab^4\right)\log\left(|x|\right) - \frac{3\,Aa^4+24\,(3\,Ba^2b^2+2\,Aab^3)x^3+12\,(2\,Ba^3b+3\,Aa^2b^2)x^2+4\,(Ba^4+4\,Aa^3b)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^5,x, algorithm="giac")
```

output
```
B*b^4*x + (4*B*a*b^3 + A*b^4)*log(abs(x)) - 1/12*(3*A*a^4 + 24*(3*B*a^2*b^
2 + 2*A*a*b^3)*x^3 + 12*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 4*(B*a^4 + 4*A*a^3
*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^5}\,dx$$
$$= \ln(x)\,\left(A\,b^4+4\,B\,a\,b^3\right) - \frac{x\,\left(\frac{B\,a^4}{3}+\frac{4\,A\,b\,a^3}{3}\right)+\frac{A\,a^4}{4}+x^2\,\left(2\,B\,a^3\,b+3\,A\,a^2\,b^2\right)+x^3\,\left(6\,B\,a^2\,b^2+4\,A\,a\,b^3\right)}{x^4} + B\,b^4\,x$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^5,x)
```

output
```
log(x)*(A*b^4 + 4*B*a*b^3) - (x*((B*a^4)/3 + (4*A*a^3*b)/3) + (A*a^4)/4 +
x^2*(3*A*a^2*b^2 + 2*B*a^3*b) + x^3*(6*B*a^2*b^2 + 4*A*a*b^3))/x^4 + B*b^4
*x
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^5}\,dx = \frac{60\log(x)\,a\,b^4x^4-3a^5-20a^4bx-60a^3b^2x^2-120a^2b^3x^3+12b^5x^5}{12x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^5,x)
```

output
```
(60*log(x)*a*b**4*x**4 - 3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120*a*
*2*b**3*x**3 + 12*b**5*x**5)/(12*x**4)
```

3.162 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = -\frac{a^4B}{4x^4}-\frac{4a^3bB}{3x^3}-\frac{3a^2b^2B}{x^2}-\frac{4ab^3B}{x}-\frac{A(a+bx)^5}{5ax^5}+b^4B\log(x)$$

output
```
-1/4*a^4*B/x^4-4/3*a^3*b*B/x^3-3*a^2*b^2*B/x^2-4*a*b^3*B/x-1/5*A*(b*x+a)^5
/a/x^5+b^4*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = -\frac{60Ab^4x^4+120ab^3x^3(A+2Bx)+60a^2b^2x^2(2A+3Bx)+20a^3bx(3A+4Bx)+3a^4(4A+5Bx)}{60x^5}+b^4B\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^6,x]
```

output
```
-1/60*(60*A*b^4*x^4 + 120*a*b^3*x^3*(A + 2*B*x) + 60*a^2*b^2*x^2*(2*A + 3*
B*x) + 20*a^3*b*x*(3*A + 4*B*x) + 3*a^4*(4*A + 5*B*x))/x^5 + b^4*B*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1184, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^6} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^6}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^4}{x^5}dx-\frac{A(a+bx)^5}{5ax^5}$$

$$\Big\downarrow \text{49}$$

$$B\int \left(\frac{a^4}{x^5}+\frac{4ba^3}{x^4}+\frac{6b^2a^2}{x^3}+\frac{4b^3a}{x^2}+\frac{b^4}{x}\right)dx-\frac{A(a+bx)^5}{5ax^5}$$

$$\Big\downarrow \text{2009}$$

$$B\left(-\frac{a^4}{4x^4}-\frac{4a^3b}{3x^3}-\frac{3a^2b^2}{x^2}-\frac{4ab^3}{x}+b^4\log(x)\right)-\frac{A(a+bx)^5}{5ax^5}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^6,x]
```

output
```
-1/5*(A*(a + b*x)^5)/(a*x^5) + B*(-1/4*a^4/x^4 - (4*a^3*b)/(3*x^3) - (3*a^
2*b^2)/x^2 - (4*a*b^3)/x + b^4*Log[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a^4 A}{5x^5}-\frac{2a^2 b(3Ab+2Ba)}{3x^3}-\frac{a\,b^2(2Ab+3Ba)}{x^2}-\frac{a^3(4Ab+Ba)}{4x^4}+b^4 B\ln(x)-\frac{b^3(Ab+4Ba)}{x}$
norman	$\frac{(-A\,a^3 b-\frac{1}{4}a^4 B)x+(-2a^2 A\,b^2-\frac{4}{3}B\,a^3 b)x^2+(-A\,b^4-4Ba\,b^3)x^4+(-2Aa\,b^3-3B\,a^2 b^2)x^3-\frac{a^4 A}{5}}{x^5}+b^4 B\ln(x)$
risch	$\frac{(-A\,a^3 b-\frac{1}{4}a^4 B)x+(-2a^2 A\,b^2-\frac{4}{3}B\,a^3 b)x^2+(-A\,b^4-4Ba\,b^3)x^4+(-2Aa\,b^3-3B\,a^2 b^2)x^3-\frac{a^4 A}{5}}{x^5}+b^4 B\ln(x)$
parallelrisch	$-\frac{-60B\,b^4\ln(x)x^5+60A\,b^4x^4+240Ba\,b^3x^4+120Aa\,b^3x^3+180B\,a^2b^2x^3+120A\,a^2b^2x^2+80B\,a^3b\,x^2+60A\,a^3bx+15a^4Bx+12}{60x^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^4*A/x^5-2/3*a^2*b*(3*A*b+2*B*a)/x^3-a*b^2*(2*A*b+3*B*a)/x^2-1/4*a^3
*(4*A*b+B*a)/x^4+b^4*B*ln(x)-b^3*(A*b+4*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx$$
$$=\frac{60\,Bb^4x^5\log(x)-12\,Aa^4-60\,(4\,Bab^3+Ab^4)x^4-60\,(3\,Ba^2b^2+2\,Aab^3)x^3-40\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^4*x^5*log(x) - 12*A*a^4 - 60*(4*B*a*b^3 + A*b^4)*x^4 - 60*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 - 40*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 15*(B*a^4
+ 4*A*a^3*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = Bb^4\log{(x)} + \frac{-12Aa^4 + x^4(-60Ab^4 - 240Bab^3) + x^3(-120Aab^3 - 180Ba^2b^2) + x^2(-120Aa^2b^2 - 80Ba^3b) + x(-6}{60x^5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**6,x)
```

output
```
B*b**4*log(x) + (-12*A*a**4 + x**4*(-60*A*b**4 - 240*B*a*b**3) + x**3*(-12
0*A*a*b**3 - 180*B*a**2*b**2) + x**2*(-120*A*a**2*b**2 - 80*B*a**3*b) + x*
(-60*A*a**3*b - 15*B*a**4))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = Bb^4\log{(x)} - \frac{12\,Aa^4 + 60\,(4\,Bab^3 + Ab^4)x^4 + 60\,(3\,Ba^2b^2 + 2\,Aab^3)x^3 + 40\,(2\,Ba^3b + 3\,Aa^2b^2)x^2 + 15\,(Ba^4 + 4\,Aa}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^6,x, algorithm="maxima")
```

output
```
B*b^4*log(x) - 1/60*(12*A*a^4 + 60*(4*B*a*b^3 + A*b^4)*x^4 + 60*(3*B*a^2*b
^2 + 2*A*a*b^3)*x^3 + 40*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 15*(B*a^4 + 4*A*a
^3*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = Bb^4\log\left(|x|\right) - \frac{12\,Aa^4+60\,(4\,Bab^3+Ab^4)x^4+60\,(3\,Ba^2b^2+2\,Aab^3)x^3+40\,(2\,Ba^3b+3\,Aa^2b^2)x^2+15\,(Ba^4+4\,Aa}{60\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^6,x, algorithm="giac")
```

output
```
B*b^4*log(abs(x)) - 1/60*(12*A*a^4 + 60*(4*B*a*b^3 + A*b^4)*x^4 + 60*(3*B*
a^2*b^2 + 2*A*a*b^3)*x^3 + 40*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 15*(B*a^4 +
4*A*a^3*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = B\,b^4\,\ln\left(x\right) - \frac{x\left(\frac{B\,a^4}{4}+A\,b\,a^3\right)+\frac{A\,a^4}{5}+x^3\left(3\,B\,a^2\,b^2+2\,A\,a\,b^3\right)+x^2\left(\frac{4\,B\,a^3\,b}{3}+2\,A\,a^2\,b^2\right)+x^4\left(A\,b^4+4\,B\,a\,b^3\right)}{x^5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^6,x)
```

output
```
B*b^4*log(x) - (x*((B*a^4)/4 + A*a^3*b) + (A*a^4)/5 + x^3*(3*B*a^2*b^2 + 2
*A*a*b^3) + x^2*(2*A*a^2*b^2 + (4*B*a^3*b)/3) + x^4*(A*b^4 + 4*B*a*b^3))/x
^5
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^6}\,dx = \frac{60\log(x)\,b^5x^5-12a^5-75a^4bx-200a^3b^2x^2-300a^2b^3x^3-300a\,b^4x^4}{60x^5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^6,x)
```

output
```
(60*log(x)*b**5*x**5 - 12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a*
*2*b**3*x**3 - 300*a*b**4*x**4)/(60*x**5)
```

3.163 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx = -\frac{A(a+bx)^5}{6ax^6}+\frac{(Ab-6aB)(a+bx)^5}{30a^2x^5}$$

output
```
-1/6*A*(b*x+a)^5/a/x^6+1/30*(A*b-6*B*a)*(b*x+a)^5/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx = -\frac{15b^4x^4(A+2Bx)+20ab^3x^3(2A+3Bx)+15a^2b^2x^2(3A+4Bx)+6a^3bx(4A+5Bx)+a^4(5A+6Bx}{30x^6}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^7,x]
```

output
```
-1/30*(15*b^4*x^4*(A + 2*B*x) + 20*a*b^3*x^3*(2*A + 3*B*x) + 15*a^2*b^2*x^
2*(3*A + 4*B*x) + 6*a^3*b*x*(4*A + 5*B*x) + a^4*(5*A + 6*B*x))/x^6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^7} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^7}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^7}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-6aB)\int \frac{(a+bx)^4}{x^6}dx}{6a}-\frac{A(a+bx)^5}{6ax^6}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^5(Ab-6aB)}{30a^2x^5}-\frac{A(a+bx)^5}{6ax^6}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^7,x]
```

output
```
-1/6*(A*(a + b*x)^5)/(a*x^6) + ((A*b - 6*a*B)*(a + b*x)^5)/(30*a^2*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(40) = 80$.

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

method	result
default	$-\frac{a^{3}(4Ab+Ba)}{5x^{5}}-\frac{a^{4}A}{6x^{6}}-\frac{2a\,b^{2}(2Ab+3Ba)}{3x^{3}}-\frac{b^{3}(Ab+4Ba)}{2x^{2}}-\frac{a^{2}b(3Ab+2Ba)}{2x^{4}}-\frac{B\,b^{4}}{x}$
norman	$\frac{-b^{4}B\,x^{5}+\left(-\frac{1}{2}A\,b^{4}-2Ba\,b^{3}\right)x^{4}+\left(-\frac{4}{3}Aa\,b^{3}-2B\,a^{2}b^{2}\right)x^{3}+\left(-\frac{3}{2}a^{2}A\,b^{2}-B\,a^{3}b\right)x^{2}+\left(-\frac{4}{5}A\,a^{3}b-\frac{1}{5}a^{4}B\right)x-\frac{a^{4}A}{6}}{x^{6}}$
risch	$\frac{-b^{4}B\,x^{5}+\left(-\frac{1}{2}A\,b^{4}-2Ba\,b^{3}\right)x^{4}+\left(-\frac{4}{3}Aa\,b^{3}-2B\,a^{2}b^{2}\right)x^{3}+\left(-\frac{3}{2}a^{2}A\,b^{2}-B\,a^{3}b\right)x^{2}+\left(-\frac{4}{5}A\,a^{3}b-\frac{1}{5}a^{4}B\right)x-\frac{a^{4}A}{6}}{x^{6}}$
gosper	$-\frac{30b^{4}B\,x^{5}+15A\,b^{4}x^{4}+60Ba\,b^{3}x^{4}+40Aa\,b^{3}x^{3}+60B\,a^{2}b^{2}x^{3}+45A\,a^{2}b^{2}x^{2}+30B\,a^{3}b\,x^{2}+24A\,a^{3}bx+6a^{4}Bx+5a^{4}A}{30x^{6}}$
parallelrisch	$-\frac{30b^{4}B\,x^{5}+15A\,b^{4}x^{4}+60Ba\,b^{3}x^{4}+40Aa\,b^{3}x^{3}+60B\,a^{2}b^{2}x^{3}+45A\,a^{2}b^{2}x^{2}+30B\,a^{3}b\,x^{2}+24A\,a^{3}bx+6a^{4}Bx+5a^{4}A}{30x^{6}}$
orering	$-\frac{\left(30b^{4}B\,x^{5}+15A\,b^{4}x^{4}+60Ba\,b^{3}x^{4}+40Aa\,b^{3}x^{3}+60B\,a^{2}b^{2}x^{3}+45A\,a^{2}b^{2}x^{2}+30B\,a^{3}b\,x^{2}+24A\,a^{3}bx+6a^{4}Bx+5a^{4}A\right)\left(b^{2}x^{2}+\right.}{30x^{6}(bx+a)^{4}}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3*(4*A*b+B*a)/x^5-1/6*a^4*A/x^6-2/3*a*b^2*(2*A*b+3*B*a)/x^3-1/2*b^3
*(A*b+4*B*a)/x^2-1/2*a^2*b*(3*A*b+2*B*a)/x^4-B*b^4/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx = -\frac{30\,Bb^4x^5+5\,Aa^4+15\,(4\,Bab^3+Ab^4)x^4+20\,(3\,Ba^2b^2+2\,Aab^3)x^3+15\,(2\,Ba^3b+3\,Aa^2b^2)x^2+6\,(B}{30\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^7,x, algorithm="fricas")
```

output
```
-1/30*(30*B*b^4*x^5 + 5*A*a^4 + 15*(4*B*a*b^3 + A*b^4)*x^4 + 20*(3*B*a^2*b
^2 + 2*A*a*b^3)*x^3 + 15*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 6*(B*a^4 + 4*A*a^
3*b)*x)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(37) = 74$.

Time = 1.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^7}\,dx = \frac{-5Aa^4-30Bb^4x^5+x^4(-15Ab^4-60Bab^3)+x^3(-40Aab^3-60Ba^2b^2)+x^2(-45Aa^2b^2-30Ba^3b)+x}{30x^6}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**7,x)
```

output
```
(-5*A*a**4 - 30*B*b**4*x**5 + x**4*(-15*A*b**4 - 60*B*a*b**3) + x**3*(-40*
A*a*b**3 - 60*B*a**2*b**2) + x**2*(-45*A*a**2*b**2 - 30*B*a**3*b) + x*(-24
*A*a**3*b - 6*B*a**4))/(30*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(41) = 82$.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^7}\,dx = -\frac{30\,Bb^4x^5+5\,Aa^4+15\,(4\,Bab^3+Ab^4)x^4+20\,(3\,Ba^2b^2+2\,Aab^3)x^3+15\,(2\,Ba^3b+3\,Aa^2b^2)x^2+6\,(B}{30\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^7,x, algorithm="maxima")
```

output
```
-1/30*(30*B*b^4*x^5 + 5*A*a^4 + 15*(4*B*a*b^3 + A*b^4)*x^4 + 20*(3*B*a^2*b
^2 + 2*A*a*b^3)*x^3 + 15*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 6*(B*a^4 + 4*A*a^
3*b)*x)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(41) = 82$.

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx =$$
$$-\frac{30\,Bb^4x^5+60\,Bab^3x^4+15\,Ab^4x^4+60\,Ba^2b^2x^3+40\,Aab^3x^3+30\,Ba^3bx^2+45\,Aa^2b^2x^2+6\,Ba^4x+2}{30\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^7,x, algorithm="giac")
```

output
```
-1/30*(30*B*b^4*x^5 + 60*B*a*b^3*x^4 + 15*A*b^4*x^4 + 60*B*a^2*b^2*x^3 + 4
0*A*a*b^3*x^3 + 30*B*a^3*b*x^2 + 45*A*a^2*b^2*x^2 + 6*B*a^4*x + 24*A*a^3*b
*x + 5*A*a^4)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx =$$
$$-\frac{x\left(\frac{B\,a^4}{5}+\frac{4\,A\,b\,a^3}{5}\right)+\frac{A\,a^4}{6}+x^2\left(B\,a^3\,b+\frac{3\,A\,a^2\,b^2}{2}\right)+x^3\left(2\,B\,a^2\,b^2+\frac{4\,A\,a\,b^3}{3}\right)+x^4\left(\frac{A\,b^4}{2}+2\,B\,a\,b^3\right)+B}{x^6}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^7,x)
```

output
```
-(x*((B*a^4)/5 + (4*A*a^3*b)/5) + (A*a^4)/6 + x^2*((3*A*a^2*b^2)/2 + B*a^3
*b) + x^3*(2*B*a^2*b^2 + (4*A*a*b^3)/3) + x^4*((A*b^4)/2 + 2*B*a*b^3) + B*
b^4*x^5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^7}\,dx = \frac{-6b^5x^5-15a\,b^4x^4-20a^2b^3x^3-15a^3b^2x^2-6a^4bx-a^5}{6x^6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^7,x)
```

output
```
( - a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*
x**4 - 6*b**5*x**5)/(6*x**6)
```

3.164 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{a^4A}{7x^7}-\frac{a^3(4Ab+aB)}{6x^6}-\frac{2a^2b(3Ab+2aB)}{5x^5} -\frac{ab^2(2Ab+3aB)}{2x^4}-\frac{b^3(Ab+4aB)}{3x^3}-\frac{b^4B}{2x^2}$$

output
```
-1/7*a^4*A/x^7-1/6*a^3*(4*A*b+B*a)/x^6-2/5*a^2*b*(3*A*b+2*B*a)/x^5-1/2*a*b
^2*(2*A*b+3*B*a)/x^4-1/3*b^3*(A*b+4*B*a)/x^3-1/2*b^4*B/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{35b^4x^4(2A+3Bx)+70ab^3x^3(3A+4Bx)+63a^2b^2x^2(4A+5Bx)+28a^3bx(5A+6Bx)+5a^4(6A+7}{210x^7}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^8,x]
```

output
```
-1/210*(35*b^4*x^4*(2*A + 3*B*x) + 70*a*b^3*x^3*(3*A + 4*B*x) + 63*a^2*b^2
*x^2*(4*A + 5*B*x) + 28*a^3*b*x*(5*A + 6*B*x) + 5*a^4*(6*A + 7*B*x))/x^7
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^8} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^8}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^8}+\frac{a^3(aB+4Ab)}{x^7}+\frac{2a^2b(2aB+3Ab)}{x^6}+\frac{b^3(4aB+Ab)}{x^4}+\frac{2ab^2(3aB+2Ab)}{x^5}+\frac{b^4B}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{7x^7}-\frac{a^3(aB+4Ab)}{6x^6}-\frac{2a^2b(2aB+3Ab)}{5x^5}-\frac{b^3(4aB+Ab)}{3x^3}-\frac{ab^2(3aB+2Ab)}{2x^4}-\frac{b^4B}{2x^2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^8,x]
```

output
```
-1/7*(a^4*A)/x^7 - (a^3*(4*A*b + a*B))/(6*x^6) - (2*a^2*b*(3*A*b + 2*a*B))
/(5*x^5) - (a*b^2*(2*A*b + 3*a*B))/(2*x^4) - (b^3*(A*b + 4*a*B))/(3*x^3) -
 (b^4*B)/(2*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{4} A}{7 x^{7}}-\frac{a^{3} \left(4 A b +B a \right)}{6 x^{6}}-\frac{2 a^{2} b \left(3 A b +2 B a \right)}{5 x^{5}}-\frac{a \,b^{2} \left(2 A b +3 B a \right)}{2 x^{4}}-\frac{b^{3} \left(A b +4 B a \right)}{3 x^{3}}-\frac{b^{4} B}{2 x^{2}}$
norman	$\frac{-\frac{b^{4} B \,x^{5}}{2}+\left(-\frac{1}{3} A \,b^{4}-\frac{4}{3} B a \,b^{3}\right) x^{4}+\left(-A a \,b^{3}-\frac{3}{2} B \,a^{2} b^{2}\right) x^{3}+\left(-\frac{6}{5} a^{2} A \,b^{2}-\frac{4}{5} B \,a^{3} b \right) x^{2}+\left(-\frac{2}{3} A \,a^{3} b -\frac{1}{6} a^{4} B \right) x -\frac{a^{4} A}{7}}{x^{7}}$
risch	$\frac{-\frac{b^{4} B \,x^{5}}{2}+\left(-\frac{1}{3} A \,b^{4}-\frac{4}{3} B a \,b^{3}\right) x^{4}+\left(-A a \,b^{3}-\frac{3}{2} B \,a^{2} b^{2}\right) x^{3}+\left(-\frac{6}{5} a^{2} A \,b^{2}-\frac{4}{5} B \,a^{3} b \right) x^{2}+\left(-\frac{2}{3} A \,a^{3} b -\frac{1}{6} a^{4} B \right) x -\frac{a^{4} A}{7}}{x^{7}}$
gosper	$-\frac{105 b^{4} B \,x^{5}+70 A \,b^{4} x^{4}+280 B a \,b^{3} x^{4}+210 A a \,b^{3} x^{3}+315 B \,a^{2} b^{2} x^{3}+252 A \,a^{2} b^{2} x^{2}+168 B \,a^{3} b \,x^{2}+140 A \,a^{3} b x +35 a^{4} B x +30 a^{4} A}{210 x^{7}}$
parallelrisch	$-\frac{105 b^{4} B \,x^{5}+70 A \,b^{4} x^{4}+280 B a \,b^{3} x^{4}+210 A a \,b^{3} x^{3}+315 B \,a^{2} b^{2} x^{3}+252 A \,a^{2} b^{2} x^{2}+168 B \,a^{3} b \,x^{2}+140 A \,a^{3} b x +35 a^{4} B x +30 a^{4} A}{210 x^{7}}$
orering	$-\frac{\left(105 b^{4} B \,x^{5}+70 A \,b^{4} x^{4}+280 B a \,b^{3} x^{4}+210 A a \,b^{3} x^{3}+315 B \,a^{2} b^{2} x^{3}+252 A \,a^{2} b^{2} x^{2}+168 B \,a^{3} b \,x^{2}+140 A \,a^{3} b x +35 a^{4} B x +30 a^{4}}{210 x^{7} \left(b x +a \right)^{4}}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^4*A/x^7-1/6*a^3*(4*A*b+B*a)/x^6-2/5*a^2*b*(3*A*b+2*B*a)/x^5-1/2*a*b
^2*(2*A*b+3*B*a)/x^4-1/3*b^3*(A*b+4*B*a)/x^3-1/2*b^4*B/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{105\,Bb^4x^5+30\,Aa^4+70\,(4\,Bab^3+Ab^4)x^4+105\,(3\,Ba^2b^2+2\,Aab^3)x^3+84\,(2\,Ba^3b+3\,Aa^2b^2)x^2+3}{210\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^8,x, algorithm="fricas")
```

output
```
-1/210*(105*B*b^4*x^5 + 30*A*a^4 + 70*(4*B*a*b^3 + A*b^4)*x^4 + 105*(3*B*a
^2*b^2 + 2*A*a*b^3)*x^3 + 84*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 35*(B*a^4 + 4
*A*a^3*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = \frac{-30Aa^4-105Bb^4x^5+x^4(-70Ab^4-280Bab^3)+x^3(-210Aab^3-315Ba^2b^2)+x^2(-252Aa^2b^2-168B}{210x^7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**8,x)
```

output
```
(-30*A*a**4 - 105*B*b**4*x**5 + x**4*(-70*A*b**4 - 280*B*a*b**3) + x**3*(-
210*A*a*b**3 - 315*B*a**2*b**2) + x**2*(-252*A*a**2*b**2 - 168*B*a**3*b) +
 x*(-140*A*a**3*b - 35*B*a**4))/(210*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{105\,Bb^4x^5+30\,Aa^4+70\,(4\,Bab^3+Ab^4)x^4+105\,(3\,Ba^2b^2+2\,Aab^3)x^3+84\,(2\,Ba^3b+3\,Aa^2b^2)x^2+3}{210\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^8,x, algorithm="maxima")
```

output
```
-1/210*(105*B*b^4*x^5 + 30*A*a^4 + 70*(4*B*a*b^3 + A*b^4)*x^4 + 105*(3*B*a
^2*b^2 + 2*A*a*b^3)*x^3 + 84*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 35*(B*a^4 + 4
*A*a^3*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{105\,Bb^4x^5+280\,Bab^3x^4+70\,Ab^4x^4+315\,Ba^2b^2x^3+210\,Aab^3x^3+168\,Ba^3bx^2+252\,Aa^2b^2x^2+35\,B}{210\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^8,x, algorithm="giac")
```

output
```
-1/210*(105*B*b^4*x^5 + 280*B*a*b^3*x^4 + 70*A*b^4*x^4 + 315*B*a^2*b^2*x^3
 + 210*A*a*b^3*x^3 + 168*B*a^3*b*x^2 + 252*A*a^2*b^2*x^2 + 35*B*a^4*x + 14
0*A*a^3*b*x + 30*A*a^4)/x^7
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = -\frac{x\left(\frac{B\,a^4}{6}+\frac{2\,A\,b\,a^3}{3}\right)+\frac{A\,a^4}{7}+x^3\left(\frac{3\,B\,a^2\,b^2}{2}+A\,a\,b^3\right)+x^2\left(\frac{4\,B\,a^3\,b}{5}+\frac{6\,A\,a^2\,b^2}{5}\right)+x^4\left(\frac{A\,b^4}{3}+\frac{4\,B\,a\,b^3}{3}\right)+\frac{B\,b^4\,x^5}{2}}{x^7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^8,x)
```

output
```
-(x*((B*a^4)/6 + (2*A*a^3*b)/3) + (A*a^4)/7 + x^3*((3*B*a^2*b^2)/2 + A*a*b
^3) + x^2*((6*A*a^2*b^2)/5 + (4*B*a^3*b)/5) + x^4*((A*b^4)/3 + (4*B*a*b^3)
/3) + (B*b^4*x^5)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^8}\,dx = \frac{-21b^5x^5-70a\,b^4x^4-105a^2b^3x^3-84a^3b^2x^2-35a^4bx-6a^5}{42x^7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^8,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b
**4*x**4 - 21*b**5*x**5)/(42*x**7)
```

3.165 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = -\frac{a^4A}{8x^8}-\frac{a^3(4Ab+aB)}{7x^7}-\frac{a^2b(3Ab+2aB)}{3x^6} -\frac{2ab^2(2Ab+3aB)}{5x^5}-\frac{b^3(Ab+4aB)}{4x^4}-\frac{b^4B}{3x^3}$$

output
```
-1/8*a^4*A/x^8-1/7*a^3*(4*A*b+B*a)/x^7-1/3*a^2*b*(3*A*b+2*B*a)/x^6-2/5*a*b
^2*(2*A*b+3*B*a)/x^5-1/4*b^3*(A*b+4*B*a)/x^4-1/3*b^4*B/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = -\frac{70b^4x^4(3A+4Bx)+168ab^3x^3(4A+5Bx)+168a^2b^2x^2(5A+6Bx)+80a^3bx(6A+7Bx)+15a^4(7A}{840x^8}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^9,x]
```

output
```
-1/840*(70*b^4*x^4*(3*A + 4*B*x) + 168*a*b^3*x^3*(4*A + 5*B*x) + 168*a^2*b
^2*x^2*(5*A + 6*B*x) + 80*a^3*b*x*(6*A + 7*B*x) + 15*a^4*(7*A + 8*B*x))/x^
8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2(A+Bx)}{x^9}dx$$
$$\Big\downarrow \text{1184}$$
$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^9}dx}{b^4}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+bx)^4(A+Bx)}{x^9}dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{a^4A}{x^9}+\frac{a^3(aB+4Ab)}{x^8}+\frac{2a^2b(2aB+3Ab)}{x^7}+\frac{b^3(4aB+Ab)}{x^5}+\frac{2ab^2(3aB+2Ab)}{x^6}+\frac{b^4B}{x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^4A}{8x^8}-\frac{a^3(aB+4Ab)}{7x^7}-\frac{a^2b(2aB+3Ab)}{3x^6}-\frac{b^3(4aB+Ab)}{4x^4}-\frac{2ab^2(3aB+2Ab)}{5x^5}-\frac{b^4B}{3x^3}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^9,x]
```

output
```
-1/8*(a^4*A)/x^8 - (a^3*(4*A*b + a*B))/(7*x^7) - (a^2*b*(3*A*b + 2*a*B))/(
3*x^6) - (2*a*b^2*(2*A*b + 3*a*B))/(5*x^5) - (b^3*(A*b + 4*a*B))/(4*x^4) -
 (b^4*B)/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^4A}{8x^8}-\frac{a^3(4Ab+Ba)}{7x^7}-\frac{a^2b(3Ab+2Ba)}{3x^6}-\frac{2a\,b^2(2Ab+3Ba)}{5x^5}-\frac{b^3(Ab+4Ba)}{4x^4}-\frac{b^4B}{3x^3}$
norman	$\frac{-\frac{b^4B\,x^5}{3}+(-\frac{1}{4}A\,b^4-Ba\,b^3)x^4+(-\frac{4}{5}Aa\,b^3-\frac{6}{5}B\,a^2b^2)x^3+(-a^2A\,b^2-\frac{2}{3}B\,a^3b)x^2+(-\frac{4}{7}A\,a^3b-\frac{1}{7}a^4B)x-\frac{a^4A}{8}}{x^8}$
risch	$\frac{-\frac{b^4B\,x^5}{3}+(-\frac{1}{4}A\,b^4-Ba\,b^3)x^4+(-\frac{4}{5}Aa\,b^3-\frac{6}{5}B\,a^2b^2)x^3+(-a^2A\,b^2-\frac{2}{3}B\,a^3b)x^2+(-\frac{4}{7}A\,a^3b-\frac{1}{7}a^4B)x-\frac{a^4A}{8}}{x^8}$
gosper	$-\frac{280b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+672Aa\,b^3x^3+1008B\,a^2b^2x^3+840A\,a^2b^2x^2+560B\,a^3b\,x^2+480A\,a^3bx+120a^4Bx+10}{840x^8}$
parallelrisch	$-\frac{280b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+672Aa\,b^3x^3+1008B\,a^2b^2x^3+840A\,a^2b^2x^2+560B\,a^3b\,x^2+480A\,a^3bx+120a^4Bx+10}{840x^8}$
orering	$-\frac{(280b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+672Aa\,b^3x^3+1008B\,a^2b^2x^3+840A\,a^2b^2x^2+560B\,a^3b\,x^2+480A\,a^3bx+120a^4Bx+1}{840x^8(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^4*A/x^8-1/7*a^3*(4*A*b+B*a)/x^7-1/3*a^2*b*(3*A*b+2*B*a)/x^6-2/5*a*b
^2*(2*A*b+3*B*a)/x^5-1/4*b^3*(A*b+4*B*a)/x^4-1/3*b^4*B/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = -\frac{280\,Bb^4x^5+105\,Aa^4+210\,(4\,Bab^3+Ab^4)x^4+336\,(3\,Ba^2b^2+2\,Aab^3)x^3+280\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{840\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^9,x, algorithm="fricas")
```

output
```
-1/840*(280*B*b^4*x^5 + 105*A*a^4 + 210*(4*B*a*b^3 + A*b^4)*x^4 + 336*(3*B
*a^2*b^2 + 2*A*a*b^3)*x^3 + 280*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 120*(B*a^4
 + 4*A*a^3*b)*x)/x^8
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx$$
$$= \frac{-105Aa^4-280Bb^4x^5+x^4(-210Ab^4-840Bab^3)+x^3(-672Aab^3-1008Ba^2b^2)+x^2(-840Aa^2b^2-56}{840x^8}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**9,x)
```

output
```
(-105*A*a**4 - 280*B*b**4*x**5 + x**4*(-210*A*b**4 - 840*B*a*b**3) + x**3*
(-672*A*a*b**3 - 1008*B*a**2*b**2) + x**2*(-840*A*a**2*b**2 - 560*B*a**3*b
) + x*(-480*A*a**3*b - 120*B*a**4))/(840*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx =$$
$$-\frac{280\,Bb^4x^5+105\,Aa^4+210\,(4\,Bab^3+Ab^4)x^4+336\,(3\,Ba^2b^2+2\,Aab^3)x^3+280\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{840\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^9,x, algorithm="maxima")
```

output
```
-1/840*(280*B*b^4*x^5 + 105*A*a^4 + 210*(4*B*a*b^3 + A*b^4)*x^4 + 336*(3*B
*a^2*b^2 + 2*A*a*b^3)*x^3 + 280*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 120*(B*a^4
 + 4*A*a^3*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = -\frac{280\,Bb^4x^5+840\,Bab^3x^4+210\,Ab^4x^4+1008\,Ba^2b^2x^3+672\,Aab^3x^3+560\,Ba^3bx^2+840\,Aa^2b^2x^2+1}{840\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^9,x, algorithm="giac")
```

output
```
-1/840*(280*B*b^4*x^5 + 840*B*a*b^3*x^4 + 210*A*b^4*x^4 + 1008*B*a^2*b^2*x
^3 + 672*A*a*b^3*x^3 + 560*B*a^3*b*x^2 + 840*A*a^2*b^2*x^2 + 120*B*a^4*x +
 480*A*a^3*b*x + 105*A*a^4)/x^8
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = -\frac{x\left(\frac{B\,a^4}{7}+\frac{4\,A\,b\,a^3}{7}\right)+\frac{A\,a^4}{8}+x^2\left(\frac{2\,B\,a^3\,b}{3}+A\,a^2\,b^2\right)+x^3\left(\frac{6\,B\,a^2\,b^2}{5}+\frac{4\,A\,a\,b^3}{5}\right)+x^4\left(\frac{A\,b^4}{4}+B\,a\,b^3\right)+\frac{B\,b^4\,x}{3}}{x^8}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^9,x)
```

output
```
-(x*((B*a^4)/7 + (4*A*a^3*b)/7) + (A*a^4)/8 + x^2*(A*a^2*b^2 + (2*B*a^3*b)
/3) + x^3*((6*B*a^2*b^2)/5 + (4*A*a*b^3)/5) + x^4*((A*b^4)/4 + B*a*b^3) +
(B*b^4*x^5)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^9}\,dx = \frac{-56b^5x^5-210a\,b^4x^4-336a^2b^3x^3-280a^3b^2x^2-120a^4bx-21a^5}{168x^8}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^9,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210
*a*b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

3.166 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = -\frac{a^4A}{9x^9}-\frac{a^3(4Ab+aB)}{8x^8}-\frac{2a^2b(3Ab+2aB)}{7x^7}-\frac{ab^2(2Ab+3aB)}{3x^6}-\frac{b^3(Ab+4aB)}{5x^5}-\frac{b^4B}{4x^4}$$

output
```
-1/9*a^4*A/x^9-1/8*a^3*(4*A*b+B*a)/x^8-2/7*a^2*b*(3*A*b+2*B*a)/x^7-1/3*a*b
^2*(2*A*b+3*B*a)/x^6-1/5*b^3*(A*b+4*B*a)/x^5-1/4*b^4*B/x^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = -\frac{126b^4x^4(4A+5Bx)+336ab^3x^3(5A+6Bx)+360a^2b^2x^2(6A+7Bx)+180a^3bx(7A+8Bx)+35a^4(8}{2520x^9}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^10,x]
```

output
```
-1/2520*(126*b^4*x^4*(4*A + 5*B*x) + 336*a*b^3*x^3*(5*A + 6*B*x) + 360*a^2
*b^2*x^2*(6*A + 7*B*x) + 180*a^3*b*x*(7*A + 8*B*x) + 35*a^4*(8*A + 9*B*x))
/x^9
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{10}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^{10}}+\frac{a^3(aB+4Ab)}{x^9}+\frac{2a^2b(2aB+3Ab)}{x^8}+\frac{b^3(4aB+Ab)}{x^6}+\frac{2ab^2(3aB+2Ab)}{x^7}+\frac{b^4B}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{9x^9}-\frac{a^3(aB+4Ab)}{8x^8}-\frac{2a^2b(2aB+3Ab)}{7x^7}-\frac{b^3(4aB+Ab)}{5x^5}-\frac{ab^2(3aB+2Ab)}{3x^6}-\frac{b^4B}{4x^4}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^10,x]
```

output
```
-1/9*(a^4*A)/x^9 - (a^3*(4*A*b + a*B))/(8*x^8) - (2*a^2*b*(3*A*b + 2*a*B))
/(7*x^7) - (a*b^2*(2*A*b + 3*a*B))/(3*x^6) - (b^3*(A*b + 4*a*B))/(5*x^5) -
 (b^4*B)/(4*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^4 A}{9x^9}-\frac{a^3(4Ab+Ba)}{8x^8}-\frac{2a^2b(3Ab+2Ba)}{7x^7}-\frac{a\,b^2(2Ab+3Ba)}{3x^6}-\frac{b^3(Ab+4Ba)}{5x^5}-\frac{b^4B}{4x^4}$
norman	$\frac{-\frac{b^4 B\,x^5}{4}+(-\frac{1}{5}A\,b^4-\frac{4}{5}Ba\,b^3)x^4+(-\frac{2}{3}Aa\,b^3-B\,a^2b^2)x^3+(-\frac{6}{7}a^2A\,b^2-\frac{4}{7}B\,a^3b)x^2+(-\frac{1}{2}A\,a^3b-\frac{1}{8}a^4B)x-\frac{a^4A}{9}}{x^9}$
risch	$\frac{-\frac{b^4 B\,x^5}{4}+(-\frac{1}{5}A\,b^4-\frac{4}{5}Ba\,b^3)x^4+(-\frac{2}{3}Aa\,b^3-B\,a^2b^2)x^3+(-\frac{6}{7}a^2A\,b^2-\frac{4}{7}B\,a^3b)x^2+(-\frac{1}{2}A\,a^3b-\frac{1}{8}a^4B)x-\frac{a^4A}{9}}{x^9}$
gosper	$-\frac{630b^4B\,x^5+504A\,b^4x^4+2016Ba\,b^3x^4+1680Aa\,b^3x^3+2520B\,a^2b^2x^3+2160A\,a^2b^2x^2+1440B\,a^3b\,x^2+1260A\,a^3bx+315a^4B}{2520x^9}$
parallelrisch	$-\frac{630b^4B\,x^5+504A\,b^4x^4+2016Ba\,b^3x^4+1680Aa\,b^3x^3+2520B\,a^2b^2x^3+2160A\,a^2b^2x^2+1440B\,a^3b\,x^2+1260A\,a^3bx+315a^4B}{2520x^9}$
orering	$-\frac{(630b^4B\,x^5+504A\,b^4x^4+2016Ba\,b^3x^4+1680Aa\,b^3x^3+2520B\,a^2b^2x^3+2160A\,a^2b^2x^2+1440B\,a^3b\,x^2+1260A\,a^3bx+315a^4}{2520x^9(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^4*A/x^9-1/8*a^3*(4*A*b+B*a)/x^8-2/7*a^2*b*(3*A*b+2*B*a)/x^7-1/3*a*b
^2*(2*A*b+3*B*a)/x^6-1/5*b^3*(A*b+4*B*a)/x^5-1/4*b^4*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = -\frac{630\,Bb^4x^5+280\,Aa^4+504\,(4\,Bab^3+Ab^4)x^4+840\,(3\,Ba^2b^2+2\,Aab^3)x^3+720\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{2520\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^10,x, algorithm="fricas")
```

output
```
-1/2520*(630*B*b^4*x^5 + 280*A*a^4 + 504*(4*B*a*b^3 + A*b^4)*x^4 + 840*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 + 720*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 315*(B*a^
4 + 4*A*a^3*b)*x)/x^9
```

Sympy [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx$$
$$= \frac{-280Aa^4-630Bb^4x^5+x^4(-504Ab^4-2016Bab^3)+x^3(-1680Aab^3-2520Ba^2b^2)+x^2(-2160Aa^2b^2 -}{2520x^9}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**10,x)
```

output
```
(-280*A*a**4 - 630*B*b**4*x**5 + x**4*(-504*A*b**4 - 2016*B*a*b**3) + x**3
*(-1680*A*a*b**3 - 2520*B*a**2*b**2) + x**2*(-2160*A*a**2*b**2 - 1440*B*a*
*3*b) + x*(-1260*A*a**3*b - 315*B*a**4))/(2520*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx =$$
$$-\frac{630\,Bb^4x^5+280\,Aa^4+504\,(4\,Bab^3+Ab^4)x^4+840\,(3\,Ba^2b^2+2\,Aab^3)x^3+720\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{2520\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^10,x, algorithm="maxima")
```

output
```
-1/2520*(630*B*b^4*x^5 + 280*A*a^4 + 504*(4*B*a*b^3 + A*b^4)*x^4 + 840*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 + 720*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 315*(B*a^
4 + 4*A*a^3*b)*x)/x^9
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = -\frac{630\,Bb^4x^5+2016\,Bab^3x^4+504\,Ab^4x^4+2520\,Ba^2b^2x^3+1680\,Aab^3x^3+1440\,Ba^3bx^2+2160\,Aa^2b^2x^2 \cdots}{2520\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^10,x, algorithm="giac")
```

output
```
-1/2520*(630*B*b^4*x^5 + 2016*B*a*b^3*x^4 + 504*A*b^4*x^4 + 2520*B*a^2*b^2
*x^3 + 1680*A*a*b^3*x^3 + 1440*B*a^3*b*x^2 + 2160*A*a^2*b^2*x^2 + 315*B*a^
4*x + 1260*A*a^3*b*x + 280*A*a^4)/x^9
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = -\frac{x\left(\frac{B\,a^4}{8}+\frac{A\,b\,a^3}{2}\right)+\frac{A\,a^4}{9}+x^3\left(B\,a^2\,b^2+\frac{2\,A\,a\,b^3}{3}\right)+x^2\left(\frac{4\,B\,a^3\,b}{7}+\frac{6\,A\,a^2\,b^2}{7}\right)+x^4\left(\frac{A\,b^4}{5}+\frac{4\,B\,a\,b^3}{5}\right)+\frac{B\,b^4\,x^5}{4}}{x^9}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^10,x)
```

output
```
-(x*((B*a^4)/8 + (A*a^3*b)/2) + (A*a^4)/9 + x^3*(B*a^2*b^2 + (2*A*a*b^3)/3
) + x^2*((6*A*a^2*b^2)/7 + (4*B*a^3*b)/7) + x^4*((A*b^4)/5 + (4*B*a*b^3)/5
) + (B*b^4*x^5)/4)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{10}}\,dx = \frac{-126b^5x^5-504a\,b^4x^4-840a^2b^3x^3-720a^3b^2x^2-315a^4bx-56a^5}{504x^9}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^10,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504
*a*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.167 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx = -\frac{a^4A}{10x^{10}}-\frac{a^3(4Ab+aB)}{9x^9}-\frac{a^2b(3Ab+2aB)}{4x^8} -\frac{2ab^2(2Ab+3aB)}{7x^7}-\frac{b^3(Ab+4aB)}{6x^6}-\frac{b^4B}{5x^5}$$

output
```
-1/10*a^4*A/x^10-1/9*a^3*(4*A*b+B*a)/x^9-1/4*a^2*b*(3*A*b+2*B*a)/x^8-2/7*a
*b^2*(2*A*b+3*B*a)/x^7-1/6*b^3*(A*b+4*B*a)/x^6-1/5*b^4*B/x^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx = -\frac{42b^4x^4(5A+6Bx)+120ab^3x^3(6A+7Bx)+135a^2b^2x^2(7A+8Bx)+70a^3bx(8A+9Bx)+14a^4(9A}{1260x^{10}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^11,x]
```

output
```
-1/1260*(42*b^4*x^4*(5*A + 6*B*x) + 120*a*b^3*x^3*(6*A + 7*B*x) + 135*a^2*
b^2*x^2*(7*A + 8*B*x) + 70*a^3*b*x*(8*A + 9*B*x) + 14*a^4*(9*A + 10*B*x))/
x^10
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^{11}} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{11}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{11}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^{11}}+\frac{a^3(aB+4Ab)}{x^{10}}+\frac{2a^2b(2aB+3Ab)}{x^9}+\frac{b^3(4aB+Ab)}{x^7}+\frac{2ab^2(3aB+2Ab)}{x^8}+\frac{b^4B}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4A}{10x^{10}}-\frac{a^3(aB+4Ab)}{9x^9}-\frac{a^2b(2aB+3Ab)}{4x^8}-\frac{b^3(4aB+Ab)}{6x^6}-\frac{2ab^2(3aB+2Ab)}{7x^7}-\frac{b^4B}{5x^5}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^11,x]
```

output
```
-1/10*(a^4*A)/x^10 - (a^3*(4*A*b + a*B))/(9*x^9) - (a^2*b*(3*A*b + 2*a*B))
/(4*x^8) - (2*a*b^2*(2*A*b + 3*a*B))/(7*x^7) - (b^3*(A*b + 4*a*B))/(6*x^6)
 - (b^4*B)/(5*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^4 A}{10x^{10}}-\frac{a^3(4Ab+Ba)}{9x^9}-\frac{a^2b(3Ab+2Ba)}{4x^8}-\frac{2a\,b^2(2Ab+3Ba)}{7x^7}-\frac{b^3(Ab+4Ba)}{6x^6}-\frac{b^4B}{5x^5}$
norman	$\frac{-\frac{b^4 B\,x^5}{5}+\left(-\frac{1}{6}A\,b^4-\frac{2}{3}Ba\,b^3\right)x^4+\left(-\frac{4}{7}Aa\,b^3-\frac{6}{7}B\,a^2b^2\right)x^3+\left(-\frac{3}{4}a^2A\,b^2-\frac{1}{2}B\,a^3b\right)x^2+\left(-\frac{4}{9}A\,a^3b-\frac{1}{9}a^4B\right)x-\frac{a^4A}{10}}{x^{10}}$
risch	$\frac{-\frac{b^4 B\,x^5}{5}+\left(-\frac{1}{6}A\,b^4-\frac{2}{3}Ba\,b^3\right)x^4+\left(-\frac{4}{7}Aa\,b^3-\frac{6}{7}B\,a^2b^2\right)x^3+\left(-\frac{3}{4}a^2A\,b^2-\frac{1}{2}B\,a^3b\right)x^2+\left(-\frac{4}{9}A\,a^3b-\frac{1}{9}a^4B\right)x-\frac{a^4A}{10}}{x^{10}}$
gosper	$-\frac{252b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+720Aa\,b^3x^3+1080B\,a^2b^2x^3+945A\,a^2b^2x^2+630B\,a^3b\,x^2+560A\,a^3bx+140a^4Bx+12}{1260x^{10}}$
parallelrisch	$-\frac{252b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+720Aa\,b^3x^3+1080B\,a^2b^2x^3+945A\,a^2b^2x^2+630B\,a^3b\,x^2+560A\,a^3bx+140a^4Bx+12}{1260x^{10}}$
orering	$-\frac{(252b^4B\,x^5+210A\,b^4x^4+840Ba\,b^3x^4+720Aa\,b^3x^3+1080B\,a^2b^2x^3+945A\,a^2b^2x^2+630B\,a^3b\,x^2+560A\,a^3bx+140a^4Bx+1}{1260x^{10}(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^4*A/x^10-1/9*a^3*(4*A*b+B*a)/x^9-1/4*a^2*b*(3*A*b+2*B*a)/x^8-2/7*a
*b^2*(2*A*b+3*B*a)/x^7-1/6*b^3*(A*b+4*B*a)/x^6-1/5*b^4*B/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx = -\frac{252\,Bb^4x^5+126\,Aa^4+210\,(4\,Bab^3+Ab^4)x^4+360\,(3\,Ba^2b^2+2\,Aab^3)x^3+315\,(2\,Ba^3b+3\,Aa^2b^2)x^2}{1260\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^11,x, algorithm="fricas")
```

output
```
-1/1260*(252*B*b^4*x^5 + 126*A*a^4 + 210*(4*B*a*b^3 + A*b^4)*x^4 + 360*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 + 315*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 140*(B*a^
4 + 4*A*a^3*b)*x)/x^10
```

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx$$
$$= \frac{-126Aa^4 - 252Bb^4x^5 + x^4(-210Ab^4 - 840Bab^3) + x^3(-720Aab^3 - 1080Ba^2b^2) + x^2(-945Aa^2b^2 - 63}{1260x^{10}}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**11,x)
```

output
```
(-126*A*a**4 - 252*B*b**4*x**5 + x**4*(-210*A*b**4 - 840*B*a*b**3) + x**3*
(-720*A*a*b**3 - 1080*B*a**2*b**2) + x**2*(-945*A*a**2*b**2 - 630*B*a**3*b
) + x*(-560*A*a**3*b - 140*B*a**4))/(1260*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx =$$
$$-\frac{252\,Bb^4x^5 + 126\,Aa^4 + 210\,(4\,Bab^3 + Ab^4)x^4 + 360\,(3\,Ba^2b^2 + 2\,Aab^3)x^3 + 315\,(2\,Ba^3b + 3\,Aa^2b^2)x^2}{1260\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^11,x, algorithm="maxima")
```

output
```
-1/1260*(252*B*b^4*x^5 + 126*A*a^4 + 210*(4*B*a*b^3 + A*b^4)*x^4 + 360*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 + 315*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 140*(B*a^
4 + 4*A*a^3*b)*x)/x^10
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx =$$
$$-\frac{252\,Bb^4x^5+840\,Bab^3x^4+210\,Ab^4x^4+1080\,Ba^2b^2x^3+720\,Aab^3x^3+630\,Ba^3bx^2+945\,Aa^2b^2x^2+1}{1260\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^11,x, algorithm="giac")
```

output
```
-1/1260*(252*B*b^4*x^5 + 840*B*a*b^3*x^4 + 210*A*b^4*x^4 + 1080*B*a^2*b^2*
x^3 + 720*A*a*b^3*x^3 + 630*B*a^3*b*x^2 + 945*A*a^2*b^2*x^2 + 140*B*a^4*x
+ 560*A*a^3*b*x + 126*A*a^4)/x^10
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx =$$
$$-\frac{x\left(\frac{B\,a^4}{9}+\frac{4\,A\,b\,a^3}{9}\right)+\frac{A\,a^4}{10}+x^2\left(\frac{B\,a^3\,b}{2}+\frac{3\,A\,a^2\,b^2}{4}\right)+x^3\left(\frac{6\,B\,a^2\,b^2}{7}+\frac{4\,A\,a\,b^3}{7}\right)+x^4\left(\frac{A\,b^4}{6}+\frac{2\,B\,a\,b^3}{3}\right)+\frac{B\,b^4\,x^5}{5}}{x^{10}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^11,x)
```

output
```
-(x*((B*a^4)/9 + (4*A*a^3*b)/9) + (A*a^4)/10 + x^2*((3*A*a^2*b^2)/4 + (B*a
^3*b)/2) + x^3*((6*B*a^2*b^2)/7 + (4*A*a*b^3)/7) + x^4*((A*b^4)/6 + (2*B*a
*b^3)/3) + (B*b^4*x^5)/5)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{11}}\,dx = \frac{-252b^5x^5-1050a\,b^4x^4-1800a^2b^3x^3-1575a^3b^2x^2-700a^4bx-126a^5}{1260x^{10}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^11,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 -
1050*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

3.168 $\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{6}a^6Ax^6+\frac{1}{7}a^5(6Ab+aB)x^7+\frac{3}{8}a^4b(5Ab+2aB)x^8+\frac{5}{9}a^3b^2(4Ab+3aB)x^9+\frac{1}{2}a^2b^3(3Ab+4aB)x^{10}+\frac{3}{11}ab^4(2Ab+5aB)x^{11}+\frac{1}{12}b^5(Ab+6aB)x^{12}+\frac{1}{13}b^6Bx^{13}$$

output
```
1/6*a^6*A*x^6+1/7*a^5*(6*A*b+B*a)*x^7+3/8*a^4*b*(5*A*b+2*B*a)*x^8+5/9*a^3*
b^2*(4*A*b+3*B*a)*x^9+1/2*a^2*b^3*(3*A*b+4*B*a)*x^10+3/11*a*b^4*(2*A*b+5*B
*a)*x^11+1/12*b^5*(A*b+6*B*a)*x^12+1/13*b^6*B*x^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{6}a^6Ax^6+\frac{1}{7}a^5(6Ab+aB)x^7+\frac{3}{8}a^4b(5Ab+2aB)x^8+\frac{5}{9}a^3b^2(4Ab+3aB)x^9+\frac{1}{2}a^2b^3(3Ab+4aB)x^{10}+\frac{3}{11}ab^4(2Ab+5aB)x^{11}+\frac{1}{12}b^5(Ab+6aB)x^{12}+\frac{1}{13}b^6Bx^{13}$$

input
```
Integrate[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^6)/6 + (a^5*(6*A*b + a*B)*x^7)/7 + (3*a^4*b*(5*A*b + 2*a*B)*x^8)/
8 + (5*a^3*b^2*(4*A*b + 3*a*B)*x^9)/9 + (a^2*b^3*(3*A*b + 4*a*B)*x^10)/2 +
 (3*a*b^4*(2*A*b + 5*a*B)*x^11)/11 + (b^5*(A*b + 6*a*B)*x^12)/12 + (b^6*B*
x^13)/13
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^5(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^5(a+bx)^6(A+Bx)dx$$

$$\downarrow 85$$

$$\int \left(a^6 A x^5 + a^5 x^6 (aB + 6Ab) + 3a^4 b x^7 (2aB + 5Ab) + 5a^3 b^2 x^8 (3aB + 4Ab) + 5a^2 b^3 x^9 (4aB + 3Ab) + b^5 x^{11} (6aB$$

$$\downarrow 2009$$

$$\frac{1}{6}a^6 A x^6 + \frac{1}{7}a^5 x^7 (aB + 6Ab) + \frac{3}{8}a^4 b x^8 (2aB + 5Ab) + \frac{5}{9}a^3 b^2 x^9 (3aB + 4Ab) + \frac{1}{2}a^2 b^3 x^{10} (4aB + 3Ab) + \frac{1}{12}b^5 x^{12} (6aB + Ab) + \frac{3}{11}a b^4 x^{11} (5aB + 2Ab) + \frac{1}{13}b^6 B x^{13}$$

input
```
Int[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^6)/6 + (a^5*(6*A*b + a*B)*x^7)/7 + (3*a^4*b*(5*A*b + 2*a*B)*x^8)/
8 + (5*a^3*b^2*(4*A*b + 3*a*B)*x^9)/9 + (a^2*b^3*(3*A*b + 4*a*B)*x^10)/2 +
 (3*a*b^4*(2*A*b + 5*a*B)*x^11)/11 + (b^5*(A*b + 6*a*B)*x^12)/12 + (b^6*B*
x^13)/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

method	result
norman	$\frac{b^6 B x^{13}}{13}+\left(\frac{1}{12}A b^6+\frac{1}{2}Ba b^5\right)x^{12}+\left(\frac{6}{11}Aa b^5+\frac{15}{11}B a^2b^4\right)x^{11}+\left(\frac{3}{2}A a^2b^4+2B a^3b^3\right)x^{10}+($
gosper	$\frac{x^6(5544b^6Bx^7+6006Ab^6x^6+36036Bab^5x^6+39312Aab^5x^5+98280Ba^2b^4x^5+108108Aa^2b^4x^4+144144Ba^3b^3x^4+160160A}{72072}$
default	$\frac{b^6 B x^{13}}{13}+\frac{(A b^6+6Ba b^5)x^{12}}{12}+\frac{(6Aa b^5+15B a^2b^4)x^{11}}{11}+\frac{(15A a^2b^4+20B a^3b^3)x^{10}}{10}+\frac{(20A a^3b^3+15B a^4b^2)x^9}{9}+$
risch	$\frac{1}{13}b^6B x^{13}+\frac{1}{12}x^{12}A b^6+\frac{1}{2}x^{12}Ba b^5+\frac{6}{11}x^{11}Aa b^5+\frac{15}{11}x^{11}B a^2b^4+\frac{3}{2}x^{10}A a^2b^4+2x^{10}B a^3b$
parallelrisch	$\frac{1}{13}b^6B x^{13}+\frac{1}{12}x^{12}A b^6+\frac{1}{2}x^{12}Ba b^5+\frac{6}{11}x^{11}Aa b^5+\frac{15}{11}x^{11}B a^2b^4+\frac{3}{2}x^{10}A a^2b^4+2x^{10}B a^3b$
orering	$\frac{x^6(5544b^6Bx^7+6006Ab^6x^6+36036Bab^5x^6+39312Aab^5x^5+98280Ba^2b^4x^5+108108Aa^2b^4x^4+144144Ba^3b^3x^4+160160A}{72072(b}$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^6*B*x^13+(1/12*A*b^6+1/2*B*a*b^5)*x^12+(6/11*A*a*b^5+15/11*B*a^2*b^
4)*x^11+(3/2*A*a^2*b^4+2*B*a^3*b^3)*x^10+(20/9*A*a^3*b^3+5/3*B*a^4*b^2)*x^
9+(15/8*A*a^4*b^2+3/4*B*a^5*b)*x^8+(6/7*A*a^5*b+1/7*B*a^6)*x^7+1/6*a^6*A*x
^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{1}{13}\,Bb^6x^{13}+\frac{1}{6}\,Aa^6x^6+\frac{1}{12}\,(6\,Bab^5+Ab^6)x^{12}+\frac{3}{11}\,(5\,Ba^2b^4+2\,Aab^5)x^{11}$$
$$+\frac{1}{2}\,(4\,Ba^3b^3+3\,Aa^2b^4)x^{10}+\frac{5}{9}\,(3\,Ba^4b^2+4\,Aa^3b^3)x^9$$
$$+\frac{3}{8}\,(2\,Ba^5b+5\,Aa^4b^2)x^8+\frac{1}{7}\,(Ba^6+6\,Aa^5b)x^7$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/13*B*b^6*x^13 + 1/6*A*a^6*x^6 + 1/12*(6*B*a*b^5 + A*b^6)*x^12 + 3/11*(5*
B*a^2*b^4 + 2*A*a*b^5)*x^11 + 1/2*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^10 + 5/9*(
3*B*a^4*b^2 + 4*A*a^3*b^3)*x^9 + 3/8*(2*B*a^5*b + 5*A*a^4*b^2)*x^8 + 1/7*(
B*a^6 + 6*A*a^5*b)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.13

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{Aa^6x^6}{6}+\frac{Bb^6x^{13}}{13}+x^{12}\left(\frac{Ab^6}{12}+\frac{Bab^5}{2}\right)+x^{11}\cdot\left(\frac{6Aab^5}{11}+\frac{15Ba^2b^4}{11}\right)+x^{10}\cdot\left(\frac{3Aa^2b^4}{2}+2Ba^3b^3\right)+x^9\cdot\left(\frac{20Aa^3b^3}{9}+\frac{5Ba^4b^2}{3}\right)+x^8\cdot\left(\frac{15Aa^4b^2}{8}+\frac{3Ba^5b}{4}\right)+x^7\cdot\left(\frac{6Aa^5b}{7}+\frac{Ba^6}{7}\right)$$

input
```
integrate(x**5*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x**6/6 + B*b**6*x**13/13 + x**12*(A*b**6/12 + B*a*b**5/2) + x**11*(
6*A*a*b**5/11 + 15*B*a**2*b**4/11) + x**10*(3*A*a**2*b**4/2 + 2*B*a**3*b**
3) + x**9*(20*A*a**3*b**3/9 + 5*B*a**4*b**2/3) + x**8*(15*A*a**4*b**2/8 +
3*B*a**5*b/4) + x**7*(6*A*a**5*b/7 + B*a**6/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{13}\,Bb^6x^{13}+\frac{1}{6}\,Aa^6x^6+\frac{1}{12}\,\left(6\,Bab^5+Ab^6\right)x^{12}+\frac{3}{11}\,\left(5\,Ba^2b^4+2\,Aab^5\right)x^{11}+\frac{1}{2}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{10}+\frac{5}{9}\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^9+\frac{3}{8}\,\left(2\,Ba^5b+5\,Aa^4b^2\right)x^8+\frac{1}{7}\,\left(Ba^6+6\,Aa^5b\right)x^7$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/13*B*b^6*x^13 + 1/6*A*a^6*x^6 + 1/12*(6*B*a*b^5 + A*b^6)*x^12 + 3/11*(5*
B*a^2*b^4 + 2*A*a*b^5)*x^11 + 1/2*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^10 + 5/9*(
3*B*a^4*b^2 + 4*A*a^3*b^3)*x^9 + 3/8*(2*B*a^5*b + 5*A*a^4*b^2)*x^8 + 1/7*(
B*a^6 + 6*A*a^5*b)*x^7
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{13}\,Bb^6x^{13}+\frac{1}{2}\,Bab^5x^{12}+\frac{1}{12}\,Ab^6x^{12}+\frac{15}{11}\,Ba^2b^4x^{11}+\frac{6}{11}\,Aab^5x^{11}+2\,Ba^3b^3x^{10}+\frac{3}{2}\,Aa^2b^4x^{10}+\frac{5}{3}\,Ba^4b^2x^9+\frac{20}{9}\,Aa^3b^3x^9+\frac{3}{4}\,Ba^5bx^8+\frac{15}{8}\,Aa^4b^2x^8+\frac{1}{7}\,Ba^6x^7+\frac{6}{7}\,Aa^5bx^7+\frac{1}{6}\,Aa^6x^6$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/13*B*b^6*x^13 + 1/2*B*a*b^5*x^12 + 1/12*A*b^6*x^12 + 15/11*B*a^2*b^4*x^1
1 + 6/11*A*a*b^5*x^11 + 2*B*a^3*b^3*x^10 + 3/2*A*a^2*b^4*x^10 + 5/3*B*a^4*
b^2*x^9 + 20/9*A*a^3*b^3*x^9 + 3/4*B*a^5*b*x^8 + 15/8*A*a^4*b^2*x^8 + 1/7*
B*a^6*x^7 + 6/7*A*a^5*b*x^7 + 1/6*A*a^6*x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=x^7\left(\frac{B\,a^6}{7}+\frac{6\,A\,b\,a^5}{7}\right)+x^{12}\left(\frac{A\,b^6}{12}+\frac{B\,a\,b^5}{2}\right)+\frac{A\,a^6\,x^6}{6}+\frac{B\,b^6\,x^{13}}{13}+\frac{5\,a^3\,b^2\,x^9\,(4\,A\,b+3\,B\,a)}{9}+\frac{a^2\,b^3\,x^{10}\,(3\,A\,b+4\,B\,a)}{2}+\frac{3\,a^4\,b\,x^8\,(5\,A\,b+2\,B\,a)}{8}+\frac{3\,a\,b^4\,x^{11}\,(2\,A\,b+5\,B\,a)}{11}$$

input
```
int(x^5*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^7*((B*a^6)/7 + (6*A*a^5*b)/7) + x^12*((A*b^6)/12 + (B*a*b^5)/2) + (A*a^6
*x^6)/6 + (B*b^6*x^13)/13 + (5*a^3*b^2*x^9*(4*A*b + 3*B*a))/9 + (a^2*b^3*x
^10*(3*A*b + 4*B*a))/2 + (3*a^4*b*x^8*(5*A*b + 2*B*a))/8 + (3*a*b^4*x^11*(
2*A*b + 5*B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{x^6(792b^7x^7+6006a\,b^6x^6+19656a^2b^5x^5+36036a^3b^4x^4+40040a^4b^3x^3+27027a^5b^2x^2+10296a^6bx+17}{10296}$$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**6*(1716*a**7 + 10296*a**6*b*x + 27027*a**5*b**2*x**2 + 40040*a**4*b**3
*x**3 + 36036*a**3*b**4*x**4 + 19656*a**2*b**5*x**5 + 6006*a*b**6*x**6 + 7
92*b**7*x**7))/10296
```

3.169 $\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 139

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^4(Ab-aB)(a+bx)^7}{7b^6}-\frac{a^3(4Ab-5aB)(a+bx)^8}{8b^6}+\frac{2a^2(3Ab-5aB)(a+bx)^9}{9b^6}-\frac{a(2Ab-5aB)(a+bx)^{10}}{5b^6}+\frac{(Ab-5aB)(a+bx)^{11}}{11b^6}+\frac{B(a+bx)^{12}}{12b^6}$$

output
```
1/7*a^4*(A*b-B*a)*(b*x+a)^7/b^6-1/8*a^3*(4*A*b-5*B*a)*(b*x+a)^8/b^6+2/9*a^
2*(3*A*b-5*B*a)*(b*x+a)^9/b^6-1/5*a*(2*A*b-5*B*a)*(b*x+a)^10/b^6+1/11*(A*b
-5*B*a)*(b*x+a)^11/b^6+1/12*B*(b*x+a)^12/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{5}a^6Ax^5+\frac{1}{6}a^5(6Ab+aB)x^6+\frac{3}{7}a^4b(5Ab+2aB)x^7+\frac{5}{8}a^3b^2(4Ab+3aB)x^8+\frac{5}{9}a^2b^3(3Ab+4aB)x^9+\frac{3}{10}ab^4(2Ab+5aB)x^{10}+\frac{1}{11}b^5(Ab+6aB)x^{11}+\frac{1}{12}b^6Bx^{12}$$

input
```
Integrate[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^5)/5 + (a^5*(6*A*b + a*B)*x^6)/6 + (3*a^4*b*(5*A*b + 2*a*B)*x^7)/
7 + (5*a^3*b^2*(4*A*b + 3*a*B)*x^8)/8 + (5*a^2*b^3*(3*A*b + 4*a*B)*x^9)/9
+ (3*a*b^4*(2*A*b + 5*a*B)*x^10)/10 + (b^5*(A*b + 6*a*B)*x^11)/11 + (b^6*B
*x^12)/12
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^4(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^4(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(-\frac{a^4(a+bx)^6(aB-Ab)}{b^5}+\frac{a^3(a+bx)^7(5aB-4Ab)}{b^5}-\frac{2a^2(a+bx)^8(5aB-3Ab)}{b^5}+\frac{(a+bx)^{10}(Ab-5aB)}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4(a+bx)^7(Ab-aB)}{7b^6}-\frac{a^3(a+bx)^8(4Ab-5aB)}{8b^6}+\frac{2a^2(a+bx)^9(3Ab-5aB)}{9b^6}+\frac{(a+bx)^{11}(Ab-5aB)}{11b^6}-\frac{a(a+bx)^{10}(2Ab-5aB)}{5b^6}+\frac{B(a+bx)^{12}}{12b^6}$$

input
```
Int[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^4*(A*b - a*B)*(a + b*x)^7)/(7*b^6) - (a^3*(4*A*b - 5*a*B)*(a + b*x)^8)/
(8*b^6) + (2*a^2*(3*A*b - 5*a*B)*(a + b*x)^9)/(9*b^6) - (a*(2*A*b - 5*a*B)
*(a + b*x)^10)/(5*b^6) + ((A*b - 5*a*B)*(a + b*x)^11)/(11*b^6) + (B*(a + b
*x)^12)/(12*b^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_^2))^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

method	result
norman	$\frac{B\,b^{6}x^{12}}{12}+\left(\frac{1}{11}A\,b^{6}+\frac{6}{11}Ba\,b^{5}\right)x^{11}+\left(\frac{3}{5}Aa\,b^{5}+\frac{3}{2}B\,a^{2}b^{4}\right)x^{10}+\left(\frac{5}{3}A\,a^{2}b^{4}+\frac{20}{9}B\,a^{3}b^{3}\right)x^{9}+\left(\frac{5}{2}\right.$
gosper	$\frac{x^{5}\left(2310b^{6}B\,x^{7}+2520A\,b^{6}x^{6}+15120Ba\,b^{5}x^{6}+16632Aa\,b^{5}x^{5}+41580B\,a^{2}b^{4}x^{5}+46200A\,a^{2}b^{4}x^{4}+61600B\,a^{3}b^{3}x^{4}+69300A\,a^{3}\right.}{27720}$
default	$\frac{B\,b^{6}x^{12}}{12}+\frac{\left(A\,b^{6}+6Ba\,b^{5}\right)x^{11}}{11}+\frac{\left(6Aa\,b^{5}+15B\,a^{2}b^{4}\right)x^{10}}{10}+\frac{\left(15A\,a^{2}b^{4}+20B\,a^{3}b^{3}\right)x^{9}}{9}+\frac{\left(20A\,a^{3}b^{3}+15B\,a^{4}b^{2}\right)x^{8}}{8}+$
risch	$\frac{1}{12}B\,b^{6}x^{12}+\frac{1}{11}x^{11}A\,b^{6}+\frac{6}{11}x^{11}Ba\,b^{5}+\frac{3}{5}x^{10}Aa\,b^{5}+\frac{3}{2}x^{10}B\,a^{2}b^{4}+\frac{5}{3}x^{9}A\,a^{2}b^{4}+\frac{20}{9}x^{9}B\,a^{3}b^{3}$
parallelrisch	$\frac{1}{12}B\,b^{6}x^{12}+\frac{1}{11}x^{11}A\,b^{6}+\frac{6}{11}x^{11}Ba\,b^{5}+\frac{3}{5}x^{10}Aa\,b^{5}+\frac{3}{2}x^{10}B\,a^{2}b^{4}+\frac{5}{3}x^{9}A\,a^{2}b^{4}+\frac{20}{9}x^{9}B\,a^{3}b^{3}$
orering	$\frac{x^{5}\left(2310b^{6}B\,x^{7}+2520A\,b^{6}x^{6}+15120Ba\,b^{5}x^{6}+16632Aa\,b^{5}x^{5}+41580B\,a^{2}b^{4}x^{5}+46200A\,a^{2}b^{4}x^{4}+61600B\,a^{3}b^{3}x^{4}+69300A\,a^{3}\right.}{27720(bx+a)}$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*B*b^6*x^12+(1/11*A*b^6+6/11*B*a*b^5)*x^11+(3/5*A*a*b^5+3/2*B*a^2*b^4)
*x^10+(5/3*A*a^2*b^4+20/9*B*a^3*b^3)*x^9+(5/2*A*a^3*b^3+15/8*B*a^4*b^2)*x^
8+(15/7*A*a^4*b^2+6/7*B*a^5*b)*x^7+(A*a^5*b+1/6*B*a^6)*x^6+1/5*A*a^6*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx &= \frac{1}{12}\,Bb^6x^{12}+\frac{1}{5}\,Aa^6x^5+\frac{1}{11}\,\left(6\,Bab^5+Ab^6\right)x^{11}+\frac{3}{10}\,\left(5\,Ba^2b^4+2\,Aab^5\right)x^{10}\\ &\quad+\frac{5}{9}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^9+\frac{5}{8}\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^8\\ &\quad+\frac{3}{7}\,\left(2\,Ba^5b+5\,Aa^4b^2\right)x^7+\frac{1}{6}\,\left(Ba^6+6\,Aa^5b\right)x^6\end{aligned}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/12*B*b^6*x^12 + 1/5*A*a^6*x^5 + 1/11*(6*B*a*b^5 + A*b^6)*x^11 + 3/10*(5*
B*a^2*b^4 + 2*A*a*b^5)*x^10 + 5/9*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^9 + 5/8*(3
*B*a^4*b^2 + 4*A*a^3*b^3)*x^8 + 3/7*(2*B*a^5*b + 5*A*a^4*b^2)*x^7 + 1/6*(B
*a^6 + 6*A*a^5*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{Aa^6x^5}{5}+\frac{Bb^6x^{12}}{12}+x^{11}\left(\frac{Ab^6}{11}+\frac{6Bab^5}{11}\right)+x^{10}\cdot\left(\frac{3Aab^5}{5}+\frac{3Ba^2b^4}{2}\right)+x^9\cdot\left(\frac{5Aa^2b^4}{3}+\frac{20Ba^3b^3}{9}\right)+x^8\cdot\left(\frac{5Aa^3b^3}{2}+\frac{15Ba^4b^2}{8}\right)+x^7\cdot\left(\frac{15Aa^4b^2}{7}+\frac{6Ba^5b}{7}\right)+x^6\left(Aa^5b+\frac{Ba^6}{6}\right)$$

input
```
integrate(x**4*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x**5/5 + B*b**6*x**12/12 + x**11*(A*b**6/11 + 6*B*a*b**5/11) + x**1
0*(3*A*a*b**5/5 + 3*B*a**2*b**4/2) + x**9*(5*A*a**2*b**4/3 + 20*B*a**3*b**
3/9) + x**8*(5*A*a**3*b**3/2 + 15*B*a**4*b**2/8) + x**7*(15*A*a**4*b**2/7
+ 6*B*a**5*b/7) + x**6*(A*a**5*b + B*a**6/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{1}{12}\,Bb^6x^{12}+\frac{1}{5}\,Aa^6x^5+\frac{1}{11}\left(6\,Bab^5+Ab^6\right)x^{11}+\frac{3}{10}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{10}+\frac{5}{9}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^9+\frac{5}{8}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^8+\frac{3}{7}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^7+\frac{1}{6}\left(Ba^6+6\,Aa^5b\right)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/12*B*b^6*x^12 + 1/5*A*a^6*x^5 + 1/11*(6*B*a*b^5 + A*b^6)*x^11 + 3/10*(5*
B*a^2*b^4 + 2*A*a*b^5)*x^10 + 5/9*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^9 + 5/8*(3
*B*a^4*b^2 + 4*A*a^3*b^3)*x^8 + 3/7*(2*B*a^5*b + 5*A*a^4*b^2)*x^7 + 1/6*(B
*a^6 + 6*A*a^5*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{12}\,Bb^6x^{12}+\frac{6}{11}\,Bab^5x^{11}+\frac{1}{11}\,Ab^6x^{11}+\frac{3}{2}\,Ba^2b^4x^{10}+\frac{3}{5}\,Aab^5x^{10}+\frac{20}{9}\,Ba^3b^3x^9+\frac{5}{3}\,Aa^2b^4x^9+\frac{15}{8}\,Ba^4b^2x^8+\frac{5}{2}\,Aa^3b^3x^8+\frac{6}{7}\,Ba^5bx^7+\frac{15}{7}\,Aa^4b^2x^7+\frac{1}{6}\,Ba^6x^6+Aa^5bx^6+\frac{1}{5}\,Aa^6x^5$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/12*B*b^6*x^12 + 6/11*B*a*b^5*x^11 + 1/11*A*b^6*x^11 + 3/2*B*a^2*b^4*x^10
 + 3/5*A*a*b^5*x^10 + 20/9*B*a^3*b^3*x^9 + 5/3*A*a^2*b^4*x^9 + 15/8*B*a^4*
b^2*x^8 + 5/2*A*a^3*b^3*x^8 + 6/7*B*a^5*b*x^7 + 15/7*A*a^4*b^2*x^7 + 1/6*B
*a^6*x^6 + A*a^5*b*x^6 + 1/5*A*a^6*x^5
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^6\left(\frac{B\,a^6}{6}+A\,b\,a^5\right)+x^{11}\left(\frac{A\,b^6}{11}+\frac{6\,B\,a\,b^5}{11}\right)+\frac{A\,a^6\,x^5}{5}+\frac{B\,b^6\,x^{12}}{12}+\frac{5\,a^3\,b^2\,x^8\,(4\,A\,b+3\,B\,a)}{8}+\frac{5\,a^2\,b^3\,x^9\,(3\,A\,b+4\,B\,a)}{9}+\frac{3\,a^4\,b\,x^7\,(5\,A\,b+2\,B\,a)}{7}+\frac{3\,a\,b^4\,x^{10}\,(2\,A\,b+5\,B\,a)}{10}$$

input
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^6*((B*a^6)/6 + A*a^5*b) + x^11*((A*b^6)/11 + (6*B*a*b^5)/11) + (A*a^6*x^
5)/5 + (B*b^6*x^12)/12 + (5*a^3*b^2*x^8*(4*A*b + 3*B*a))/8 + (5*a^2*b^3*x^
9*(3*A*b + 4*B*a))/9 + (3*a^4*b*x^7*(5*A*b + 2*B*a))/7 + (3*a*b^4*x^10*(2*
A*b + 5*B*a))/10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{x^5(330b^7x^7+2520a\,b^6x^6+8316a^2b^5x^5+15400a^3b^4x^4+17325a^4b^3x^3+11880a^5b^2x^2+4620a^6bx+792a}{3960}$$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**5*(792*a**7 + 4620*a**6*b*x + 11880*a**5*b**2*x**2 + 17325*a**4*b**3*x
**3 + 15400*a**3*b**4*x**4 + 8316*a**2*b**5*x**5 + 2520*a*b**6*x**6 + 330*
b**7*x**7))/3960
```

3.170 $\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=-\frac{a^3(Ab-aB)(a+bx)^7}{7b^5}+\frac{a^2(3Ab-4aB)(a+bx)^8}{8b^5}-\frac{a(Ab-2aB)(a+bx)^9}{3b^5}+\frac{(Ab-4aB)(a+bx)^{10}}{10b^5}+\frac{B(a+bx)^{11}}{11b^5}$$

output
```
-1/7*a^3*(A*b-B*a)*(b*x+a)^7/b^5+1/8*a^2*(3*A*b-4*B*a)*(b*x+a)^8/b^5-1/3*a
*(A*b-2*B*a)*(b*x+a)^9/b^5+1/10*(A*b-4*B*a)*(b*x+a)^10/b^5+1/11*B*(b*x+a)^
11/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{4}a^6Ax^4+\frac{1}{5}a^5(6Ab+aB)x^5 + \frac{1}{2}a^4b(5Ab+2aB)x^6+\frac{5}{7}a^3b^2(4Ab+3aB)x^7 + \frac{5}{8}a^2b^3(3Ab+4aB)x^8+\frac{1}{3}ab^4(2Ab+5aB)x^9 + \frac{1}{10}b^5(Ab+6aB)x^{10}+\frac{1}{11}b^6Bx^{11}$$

input
```
Integrate[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^4)/4 + (a^5*(6*A*b + a*B)*x^5)/5 + (a^4*b*(5*A*b + 2*a*B)*x^6)/2
+ (5*a^3*b^2*(4*A*b + 3*a*B)*x^7)/7 + (5*a^2*b^3*(3*A*b + 4*a*B)*x^8)/8 +
(a*b^4*(2*A*b + 5*a*B)*x^9)/3 + (b^5*(A*b + 6*a*B)*x^10)/10 + (b^6*B*x^11)
/11
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^3(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^3(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3(a+bx)^6(aB-Ab)}{b^4} - \frac{a^2(a+bx)^7(4aB-3Ab)}{b^4} + \frac{(a+bx)^9(Ab-4aB)}{b^4} + \frac{3a(a+bx)^8(2aB-Ab)}{b^4} + \frac{B(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(a+bx)^7(Ab-aB)}{7b^5} + \frac{a^2(a+bx)^8(3Ab-4aB)}{8b^5} + \frac{(a+bx)^{10}(Ab-4aB)}{10b^5} - \frac{a(a+bx)^9(Ab-2aB)}{3b^5} + \frac{B(a+bx)^{11}}{11b^5}$$

input
```
Int[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/7*(a^3*(A*b - a*B)*(a + b*x)^7)/b^5 + (a^2*(3*A*b - 4*a*B)*(a + b*x)^8)
/(8*b^5) - (a*(A*b - 2*a*B)*(a + b*x)^9)/(3*b^5) + ((A*b - 4*a*B)*(a + b*x
)^10)/(10*b^5) + (B*(a + b*x)^11)/(11*b^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

method	result
norman	$\frac{B\,b^{6}x^{11}}{11}+\left(\frac{1}{10}A\,b^{6}+\frac{3}{5}Ba\,b^{5}\right)x^{10}+\left(\frac{2}{3}Aa\,b^{5}+\frac{5}{3}B\,a^{2}b^{4}\right)x^{9}+\left(\frac{15}{8}A\,a^{2}b^{4}+\frac{5}{2}B\,a^{3}b^{3}\right)x^{8}+\left(\frac{20}{7}\right.$
gosper	$\frac{x^{4}\left(840b^{6}B\,x^{7}+924A\,b^{6}x^{6}+5544Ba\,b^{5}x^{6}+6160Aa\,b^{5}x^{5}+15400B\,a^{2}b^{4}x^{5}+17325A\,a^{2}b^{4}x^{4}+23100B\,a^{3}b^{3}x^{4}+26400A\,a^{3}b^{3}x^{3}\right.}{9240}$
default	$\frac{B\,b^{6}x^{11}}{11}+\frac{\left(A\,b^{6}+6Ba\,b^{5}\right)x^{10}}{10}+\frac{\left(6Aa\,b^{5}+15B\,a^{2}b^{4}\right)x^{9}}{9}+\frac{\left(15A\,a^{2}b^{4}+20B\,a^{3}b^{3}\right)x^{8}}{8}+\frac{\left(20A\,a^{3}b^{3}+15B\,a^{4}b^{2}\right)x^{7}}{7}+$
risch	$\frac{1}{11}B\,b^{6}x^{11}+\frac{1}{10}x^{10}A\,b^{6}+\frac{3}{5}x^{10}Ba\,b^{5}+\frac{2}{3}x^{9}Aa\,b^{5}+\frac{5}{3}x^{9}B\,a^{2}b^{4}+\frac{15}{8}x^{8}A\,a^{2}b^{4}+\frac{5}{2}x^{8}B\,a^{3}b^{3}+$
parallelrisch	$\frac{1}{11}B\,b^{6}x^{11}+\frac{1}{10}x^{10}A\,b^{6}+\frac{3}{5}x^{10}Ba\,b^{5}+\frac{2}{3}x^{9}Aa\,b^{5}+\frac{5}{3}x^{9}B\,a^{2}b^{4}+\frac{15}{8}x^{8}A\,a^{2}b^{4}+\frac{5}{2}x^{8}B\,a^{3}b^{3}+$
orering	$\frac{x^{4}\left(840b^{6}B\,x^{7}+924A\,b^{6}x^{6}+5544Ba\,b^{5}x^{6}+6160Aa\,b^{5}x^{5}+15400B\,a^{2}b^{4}x^{5}+17325A\,a^{2}b^{4}x^{4}+23100B\,a^{3}b^{3}x^{4}+26400A\,a^{3}b^{3}x^{3}\right.}{9240(bx+a)^{6}}$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*B*b^6*x^11+(1/10*A*b^6+3/5*B*a*b^5)*x^10+(2/3*A*a*b^5+5/3*B*a^2*b^4)*
x^9+(15/8*A*a^2*b^4+5/2*B*a^3*b^3)*x^8+(20/7*A*a^3*b^3+15/7*B*a^4*b^2)*x^7
+(5/2*A*a^4*b^2+B*a^5*b)*x^6+(6/5*A*a^5*b+1/5*B*a^6)*x^5+1/4*A*a^6*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{1}{11}Bb^6x^{11}+\frac{1}{4}Aa^6x^4+\frac{1}{10}\left(6\,Bab^5+Ab^6\right)x^{10}+\frac{1}{3}\left(5\,Ba^2b^4+2\,Aab^5\right)x^9$$
$$+\frac{5}{8}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^8+\frac{5}{7}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^7$$
$$+\frac{1}{2}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^6+\frac{1}{5}\left(Ba^6+6\,Aa^5b\right)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/11*B*b^6*x^11 + 1/4*A*a^6*x^4 + 1/10*(6*B*a*b^5 + A*b^6)*x^10 + 1/3*(5*B
*a^2*b^4 + 2*A*a*b^5)*x^9 + 5/8*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^8 + 5/7*(3*B
*a^4*b^2 + 4*A*a^3*b^3)*x^7 + 1/2*(2*B*a^5*b + 5*A*a^4*b^2)*x^6 + 1/5*(B*a
^6 + 6*A*a^5*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.45

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{Aa^6x^4}{4}+\frac{Bb^6x^{11}}{11}+x^{10}\left(\frac{Ab^6}{10}+\frac{3Bab^5}{5}\right)+x^9\cdot\left(\frac{2Aab^5}{3}+\frac{5Ba^2b^4}{3}\right)+x^8\cdot\left(\frac{15Aa^2b^4}{8}+\frac{5Ba^3b^3}{2}\right)+x^7\cdot\left(\frac{20Aa^3b^3}{7}+\frac{15Ba^4b^2}{7}\right)+x^6\cdot\left(\frac{5Aa^4b^2}{2}+Ba^5b\right)+x^5\cdot\left(\frac{6Aa^5b}{5}+\frac{Ba^6}{5}\right)$$

input
```
integrate(x**3*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x**4/4 + B*b**6*x**11/11 + x**10*(A*b**6/10 + 3*B*a*b**5/5) + x**9*
(2*A*a*b**5/3 + 5*B*a**2*b**4/3) + x**8*(15*A*a**2*b**4/8 + 5*B*a**3*b**3/
2) + x**7*(20*A*a**3*b**3/7 + 15*B*a**4*b**2/7) + x**6*(5*A*a**4*b**2/2 +
B*a**5*b) + x**5*(6*A*a**5*b/5 + B*a**6/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{11}\,Bb^6x^{11}+\frac{1}{4}\,Aa^6x^4+\frac{1}{10}\left(6\,Bab^5+Ab^6\right)x^{10}+\frac{1}{3}\left(5\,Ba^2b^4+2\,Aab^5\right)x^9 + \frac{5}{8}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^8+\frac{5}{7}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^7 + \frac{1}{2}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^6+\frac{1}{5}\left(Ba^6+6\,Aa^5b\right)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/11*B*b^6*x^11 + 1/4*A*a^6*x^4 + 1/10*(6*B*a*b^5 + A*b^6)*x^10 + 1/3*(5*B
*a^2*b^4 + 2*A*a*b^5)*x^9 + 5/8*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^8 + 5/7*(3*B
*a^4*b^2 + 4*A*a^3*b^3)*x^7 + 1/2*(2*B*a^5*b + 5*A*a^4*b^2)*x^6 + 1/5*(B*a
^6 + 6*A*a^5*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{11}\,Bb^6x^{11}+\frac{3}{5}\,Bab^5x^{10}+\frac{1}{10}\,Ab^6x^{10}+\frac{5}{3}\,Ba^2b^4x^9 + \frac{2}{3}\,Aab^5x^9+\frac{5}{2}\,Ba^3b^3x^8+\frac{15}{8}\,Aa^2b^4x^8 + \frac{15}{7}\,Ba^4b^2x^7+\frac{20}{7}\,Aa^3b^3x^7+Ba^5bx^6 + \frac{5}{2}\,Aa^4b^2x^6+\frac{1}{5}\,Ba^6x^5+\frac{6}{5}\,Aa^5bx^5+\frac{1}{4}\,Aa^6x^4$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/11*B*b^6*x^11 + 3/5*B*a*b^5*x^10 + 1/10*A*b^6*x^10 + 5/3*B*a^2*b^4*x^9 +
 2/3*A*a*b^5*x^9 + 5/2*B*a^3*b^3*x^8 + 15/8*A*a^2*b^4*x^8 + 15/7*B*a^4*b^2
*x^7 + 20/7*A*a^3*b^3*x^7 + B*a^5*b*x^6 + 5/2*A*a^4*b^2*x^6 + 1/5*B*a^6*x^
5 + 6/5*A*a^5*b*x^5 + 1/4*A*a^6*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.17

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^5\left(\frac{B\,a^6}{5}+\frac{6\,A\,b\,a^5}{5}\right)+x^{10}\left(\frac{A\,b^6}{10}+\frac{3\,B\,a\,b^5}{5}\right)+\frac{A\,a^6\,x^4}{4}+\frac{B\,b^6\,x^{11}}{11}+\frac{5\,a^3\,b^2\,x^7\,(4\,A\,b+3\,B\,a)}{7}+\frac{5\,a^2\,b^3\,x^8\,(3\,A\,b+4\,B\,a)}{8}+\frac{a^4\,b\,x^6\,(5\,A\,b+2\,B\,a)}{2}+\frac{a\,b^4\,x^9\,(2\,A\,b+5\,B\,a)}{3}$$

input
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^5*((B*a^6)/5 + (6*A*a^5*b)/5) + x^10*((A*b^6)/10 + (3*B*a*b^5)/5) + (A*a
^6*x^4)/4 + (B*b^6*x^11)/11 + (5*a^3*b^2*x^7*(4*A*b + 3*B*a))/7 + (5*a^2*b
^3*x^8*(3*A*b + 4*B*a))/8 + (a^4*b*x^6*(5*A*b + 2*B*a))/2 + (a*b^4*x^9*(2*
A*b + 5*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{x^4(120b^7x^7+924a\,b^6x^6+3080a^2b^5x^5+5775a^3b^4x^4+6600a^4b^3x^3+4620a^5b^2x^2+1848a^6bx+330a^7)}{1320}$$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**4*(330*a**7 + 1848*a**6*b*x + 4620*a**5*b**2*x**2 + 6600*a**4*b**3*x**
3 + 5775*a**3*b**4*x**4 + 3080*a**2*b**5*x**5 + 924*a*b**6*x**6 + 120*b**7
*x**7))/1320
```

3.171 $\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{a^2(Ab-aB)(a+bx)^7}{7b^4}-\frac{a(2Ab-3aB)(a+bx)^8}{8b^4}+\frac{(Ab-3aB)(a+bx)^9}{9b^4}+\frac{B(a+bx)^{10}}{10b^4}$$

output
```
1/7*a^2*(A*b-B*a)*(b*x+a)^7/b^4-1/8*a*(2*A*b-3*B*a)*(b*x+a)^8/b^4+1/9*(A*b
-3*B*a)*(b*x+a)^9/b^4+1/10*B*(b*x+a)^10/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{3}a^6Ax^3+\frac{1}{4}a^5(6Ab+aB)x^4+\frac{3}{5}a^4b(5Ab+2aB)x^5+\frac{5}{6}a^3b^2(4Ab+3aB)x^6+\frac{5}{7}a^2b^3(3Ab+4aB)x^7+\frac{3}{8}ab^4(2Ab+5aB)x^8+\frac{1}{9}b^5(Ab+6aB)x^9+\frac{1}{10}b^6Bx^{10}$$

input
```
Integrate[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^3)/3 + (a^5*(6*A*b + a*B)*x^4)/4 + (3*a^4*b*(5*A*b + 2*a*B)*x^5)/
5 + (5*a^3*b^2*(4*A*b + 3*a*B)*x^6)/6 + (5*a^2*b^3*(3*A*b + 4*a*B)*x^7)/7
+ (3*a*b^4*(2*A*b + 5*a*B)*x^8)/8 + (b^5*(A*b + 6*a*B)*x^9)/9 + (b^6*B*x^1
0)/10
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx+b^2x^2\right)^3 (A+Bx) \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^2(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^2(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(-\frac{a^2(a+bx)^6(aB-Ab)}{b^3}+\frac{(a+bx)^8(Ab-3aB)}{b^3}+\frac{a(a+bx)^7(3aB-2Ab)}{b^3}+\frac{B(a+bx)^9}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^7(Ab-aB)}{7b^4}+\frac{(a+bx)^9(Ab-3aB)}{9b^4}-\frac{a(a+bx)^8(2Ab-3aB)}{8b^4}+\frac{B(a+bx)^{10}}{10b^4}$$

input
```
Int[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^2*(A*b - a*B)*(a + b*x)^7)/(7*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x)^8)/(8
*b^4) + ((A*b - 3*a*B)*(a + b*x)^9)/(9*b^4) + (B*(a + b*x)^10)/(10*b^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

method	result
norman	$\frac{B\,b^{6}x^{10}}{10}+\left(\frac{1}{9}A\,b^{6}+\frac{2}{3}Ba\,b^{5}\right)x^{9}+\left(\frac{3}{4}Aa\,b^{5}+\frac{15}{8}B\,a^{2}b^{4}\right)x^{8}+\left(\frac{15}{7}A\,a^{2}b^{4}+\frac{20}{7}B\,a^{3}b^{3}\right)x^{7}+\left(\frac{10}{3}\right.$
gosper	$\frac{x^{3}\left(252b^{6}B\,x^{7}+280A\,b^{6}x^{6}+1680Ba\,b^{5}x^{6}+1890Aa\,b^{5}x^{5}+4725B\,a^{2}b^{4}x^{5}+5400A\,a^{2}b^{4}x^{4}+7200B\,a^{3}b^{3}x^{4}+8400A\,a^{3}b^{3}x^{3}+630\right.}{2520}$
default	$\frac{B\,b^{6}x^{10}}{10}+\frac{\left(A\,b^{6}+6Ba\,b^{5}\right)x^{9}}{9}+\frac{\left(6Aa\,b^{5}+15B\,a^{2}b^{4}\right)x^{8}}{8}+\frac{\left(15A\,a^{2}b^{4}+20B\,a^{3}b^{3}\right)x^{7}}{7}+\frac{\left(20A\,a^{3}b^{3}+15B\,a^{4}b^{2}\right)x^{6}}{6}+\frac{(15}{}$
risch	$\frac{1}{10}B\,b^{6}x^{10}+\frac{1}{9}x^{9}A\,b^{6}+\frac{2}{3}x^{9}Ba\,b^{5}+\frac{3}{4}x^{8}Aa\,b^{5}+\frac{15}{8}x^{8}B\,a^{2}b^{4}+\frac{15}{7}x^{7}A\,a^{2}b^{4}+\frac{20}{7}x^{7}B\,a^{3}b^{3}+\frac{10}{3}$
parallelrisch	$\frac{1}{10}B\,b^{6}x^{10}+\frac{1}{9}x^{9}A\,b^{6}+\frac{2}{3}x^{9}Ba\,b^{5}+\frac{3}{4}x^{8}Aa\,b^{5}+\frac{15}{8}x^{8}B\,a^{2}b^{4}+\frac{15}{7}x^{7}A\,a^{2}b^{4}+\frac{20}{7}x^{7}B\,a^{3}b^{3}+\frac{10}{3}$
orering	$\frac{x^{3}\left(252b^{6}B\,x^{7}+280A\,b^{6}x^{6}+1680Ba\,b^{5}x^{6}+1890Aa\,b^{5}x^{5}+4725B\,a^{2}b^{4}x^{5}+5400A\,a^{2}b^{4}x^{4}+7200B\,a^{3}b^{3}x^{4}+8400A\,a^{3}b^{3}x^{3}+630\right.}{2520(bx+a)^{6}}$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*b^6*x^10+(1/9*A*b^6+2/3*B*a*b^5)*x^9+(3/4*A*a*b^5+15/8*B*a^2*b^4)*x
^8+(15/7*A*a^2*b^4+20/7*B*a^3*b^3)*x^7+(10/3*A*a^3*b^3+5/2*B*a^4*b^2)*x^6+
(3*A*a^4*b^2+6/5*B*a^5*b)*x^5+(3/2*A*a^5*b+1/4*B*a^6)*x^4+1/3*A*a^6*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\begin{aligned}\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx &= \frac{1}{10}Bb^6x^{10}+\frac{1}{3}Aa^6x^3+\frac{1}{9}\left(6\,Bab^5+Ab^6\right)x^9+\frac{3}{8}\left(5\,Ba^2b^4+2\,Aab^5\right)x^8\\&\quad+\frac{5}{7}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^7+\frac{5}{6}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^6\\&\quad+\frac{3}{5}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^5+\frac{1}{4}\left(Ba^6+6\,Aa^5b\right)x^4\end{aligned}$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/10*B*b^6*x^10 + 1/3*A*a^6*x^3 + 1/9*(6*B*a*b^5 + A*b^6)*x^9 + 3/8*(5*B*a
^2*b^4 + 2*A*a*b^5)*x^8 + 5/7*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^7 + 5/6*(3*B*a
^4*b^2 + 4*A*a^3*b^3)*x^6 + 3/5*(2*B*a^5*b + 5*A*a^4*b^2)*x^5 + 1/4*(B*a^6
 + 6*A*a^5*b)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(78) = 156$.

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{Aa^6x^3}{3}+\frac{Bb^6x^{10}}{10}+x^9\left(\frac{Ab^6}{9}+\frac{2Bab^5}{3}\right)+x^8\cdot\left(\frac{3Aab^5}{4}+\frac{15Ba^2b^4}{8}\right)+x^7\cdot\left(\frac{15Aa^2b^4}{7}+\frac{20Ba^3b^3}{7}\right)+x^6\cdot\left(\frac{10Aa^3b^3}{3}+\frac{5Ba^4b^2}{2}\right)+x^5\cdot\left(3Aa^4b^2+\frac{6Ba^5b}{5}\right)+x^4\cdot\left(\frac{3Aa^5b}{2}+\frac{Ba^6}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x**3/3 + B*b**6*x**10/10 + x**9*(A*b**6/9 + 2*B*a*b**5/3) + x**8*(3
*A*a*b**5/4 + 15*B*a**2*b**4/8) + x**7*(15*A*a**2*b**4/7 + 20*B*a**3*b**3/
7) + x**6*(10*A*a**3*b**3/3 + 5*B*a**4*b**2/2) + x**5*(3*A*a**4*b**2 + 6*B
*a**5*b/5) + x**4*(3*A*a**5*b/2 + B*a**6/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{10}\,Bb^6x^{10}+\frac{1}{3}\,Aa^6x^3+\frac{1}{9}\,\left(6\,Bab^5+Ab^6\right)x^9+\frac{3}{8}\,\left(5\,Ba^2b^4+2\,Aab^5\right)x^8+\frac{5}{7}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^7+\frac{5}{6}\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^6+\frac{3}{5}\,\left(2\,Ba^5b+5\,Aa^4b^2\right)x^5+\frac{1}{4}\,\left(Ba^6+6\,Aa^5b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/10*B*b^6*x^10 + 1/3*A*a^6*x^3 + 1/9*(6*B*a*b^5 + A*b^6)*x^9 + 3/8*(5*B*a
^2*b^4 + 2*A*a*b^5)*x^8 + 5/7*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^7 + 5/6*(3*B*a
^4*b^2 + 4*A*a^3*b^3)*x^6 + 3/5*(2*B*a^5*b + 5*A*a^4*b^2)*x^5 + 1/4*(B*a^6
 + 6*A*a^5*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{10}\,Bb^6x^{10}+\frac{2}{3}\,Bab^5x^9+\frac{1}{9}\,Ab^6x^9+\frac{15}{8}\,Ba^2b^4x^8+\frac{3}{4}\,Aab^5x^8+\frac{20}{7}\,Ba^3b^3x^7+\frac{15}{7}\,Aa^2b^4x^7+\frac{5}{2}\,Ba^4b^2x^6+\frac{10}{3}\,Aa^3b^3x^6+\frac{6}{5}\,Ba^5bx^5+3\,Aa^4b^2x^5+\frac{1}{4}\,Ba^6x^4+\frac{3}{2}\,Aa^5bx^4+\frac{1}{3}\,Aa^6x^3$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/10*B*b^6*x^10 + 2/3*B*a*b^5*x^9 + 1/9*A*b^6*x^9 + 15/8*B*a^2*b^4*x^8 + 3
/4*A*a*b^5*x^8 + 20/7*B*a^3*b^3*x^7 + 15/7*A*a^2*b^4*x^7 + 5/2*B*a^4*b^2*x
^6 + 10/3*A*a^3*b^3*x^6 + 6/5*B*a^5*b*x^5 + 3*A*a^4*b^2*x^5 + 1/4*B*a^6*x^
4 + 3/2*A*a^5*b*x^4 + 1/3*A*a^6*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=x^4\left(\frac{B\,a^6}{4}+\frac{3\,A\,b\,a^5}{2}\right)+x^9\left(\frac{A\,b^6}{9}+\frac{2\,B\,a\,b^5}{3}\right)+\frac{A\,a^6\,x^3}{3}+\frac{B\,b^6\,x^{10}}{10}+\frac{5\,a^3\,b^2\,x^6\,\left(4\,A\,b+3\,B\,a\right)}{6}+\frac{5\,a^2\,b^3\,x^7\,\left(3\,A\,b+4\,B\,a\right)}{7}+\frac{3\,a^4\,b\,x^5\,\left(5\,A\,b+2\,B\,a\right)}{5}+\frac{3\,a\,b^4\,x^8\,\left(2\,A\,b+5\,B\,a\right)}{8}$$

input
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^4*((B*a^6)/4 + (3*A*a^5*b)/2) + x^9*((A*b^6)/9 + (2*B*a*b^5)/3) + (A*a^6
*x^3)/3 + (B*b^6*x^10)/10 + (5*a^3*b^2*x^6*(4*A*b + 3*B*a))/6 + (5*a^2*b^3
*x^7*(3*A*b + 4*B*a))/7 + (3*a^4*b*x^5*(5*A*b + 2*B*a))/5 + (3*a*b^4*x^8*(
2*A*b + 5*B*a))/8
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{x^3(36b^7x^7+280a\,b^6x^6+945a^2b^5x^5+1800a^3b^4x^4+2100a^4b^3x^3+1512a^5b^2x^2+630a^6bx+120a^7)}{360}$$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**3*(120*a**7 + 630*a**6*b*x + 1512*a**5*b**2*x**2 + 2100*a**4*b**3*x**3
 + 1800*a**3*b**4*x**4 + 945*a**2*b**5*x**5 + 280*a*b**6*x**6 + 36*b**7*x*
*7))/360
```

3.172 $\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=-\frac{a(Ab-aB)(a+bx)^7}{7b^3}+\frac{(Ab-2aB)(a+bx)^8}{8b^3}+\frac{B(a+bx)^9}{9b^3}$$

output
```
-1/7*a*(A*b-B*a)*(b*x+a)^7/b^3+1/8*(A*b-2*B*a)*(b*x+a)^8/b^3+1/9*B*(b*x+a)
^9/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(61) = 122.

Time = 0.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.30

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{2}a^6Ax^2+\frac{1}{3}a^5(6Ab+aB)x^3+\frac{3}{4}a^4b(5Ab+2aB)x^4+a^3b^2(4Ab+3aB)x^5+\frac{5}{6}a^2b^3(3Ab+4aB)x^6+\frac{3}{7}ab^4(2Ab+5aB)x^7+\frac{1}{8}b^5(Ab+6aB)x^8+\frac{1}{9}b^6Bx^9$$

input
```
Integrate[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*x^2)/2 + (a^5*(6*A*b + a*B)*x^3)/3 + (3*a^4*b*(5*A*b + 2*a*B)*x^4)/
4 + a^3*b^2*(4*A*b + 3*a*B)*x^5 + (5*a^2*b^3*(3*A*b + 4*a*B)*x^6)/6 + (3*a
*b^4*(2*A*b + 5*a*B)*x^7)/7 + (b^5*(A*b + 6*a*B)*x^8)/8 + (b^6*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{(a+bx)^7(Ab-2aB)}{b^2}+\frac{a(a+bx)^6(aB-Ab)}{b^2}+\frac{B(a+bx)^8}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^8(Ab-2aB)}{8b^3}-\frac{a(a+bx)^7(Ab-aB)}{7b^3}+\frac{B(a+bx)^9}{9b^3}$$

input
```
Int[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/7*(a*(A*b - a*B)*(a + b*x)^7)/b^3 + ((A*b - 2*a*B)*(a + b*x)^8)/(8*b^3)
 + (B*(a + b*x)^9)/(9*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(55) = 110$.

Time = 1.01 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

method	result
norman	$\frac{B\,b^6x^9}{9}+\left(\frac{1}{8}A\,b^6+\frac{3}{4}Ba\,b^5\right)x^8+\left(\frac{6}{7}Aa\,b^5+\frac{15}{7}B\,a^2b^4\right)x^7+\left(\frac{5}{2}A\,a^2b^4+\frac{10}{3}B\,a^3b^3\right)x^6+(4A\,a$
gosper	$\frac{x^2(56b^6B\,x^7+63A\,b^6x^6+378Ba\,b^5x^6+432Aa\,b^5x^5+1080B\,a^2b^4x^5+1260A\,a^2b^4x^4+1680B\,a^3b^3x^4+2016A\,a^3b^3x^3+1512B\,a}{504}$
default	$\frac{B\,b^6x^9}{9}+\frac{(A\,b^6+6Ba\,b^5)x^8}{8}+\frac{(6Aa\,b^5+15B\,a^2b^4)x^7}{7}+\frac{(15A\,a^2b^4+20B\,a^3b^3)x^6}{6}+\frac{(20A\,a^3b^3+15B\,a^4b^2)x^5}{5}+(15$
risch	$\frac{1}{9}B\,b^6x^9+\frac{1}{8}x^8A\,b^6+\frac{3}{4}x^8Ba\,b^5+\frac{6}{7}x^7Aa\,b^5+\frac{15}{7}x^7B\,a^2b^4+\frac{5}{2}x^6A\,a^2b^4+\frac{10}{3}x^6B\,a^3b^3+4A\,a$
parallelrisch	$\frac{1}{9}B\,b^6x^9+\frac{1}{8}x^8A\,b^6+\frac{3}{4}x^8Ba\,b^5+\frac{6}{7}x^7Aa\,b^5+\frac{15}{7}x^7B\,a^2b^4+\frac{5}{2}x^6A\,a^2b^4+\frac{10}{3}x^6B\,a^3b^3+4A\,a$
orering	$\frac{x^2(56b^6B\,x^7+63A\,b^6x^6+378Ba\,b^5x^6+432Aa\,b^5x^5+1080B\,a^2b^4x^5+1260A\,a^2b^4x^4+1680B\,a^3b^3x^4+2016A\,a^3b^3x^3+1512B\,a}{504(bx+a)^6}$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*b^6*x^9+(1/8*A*b^6+3/4*B*a*b^5)*x^8+(6/7*A*a*b^5+15/7*B*a^2*b^4)*x^7
+(5/2*A*a^2*b^4+10/3*B*a^3*b^3)*x^6+(4*A*a^3*b^3+3*B*a^4*b^2)*x^5+(15/4*A*
a^4*b^2+3/2*B*a^5*b)*x^4+(2*A*a^5*b+1/3*B*a^6)*x^3+1/2*A*a^6*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.39

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{9}\,Bb^6x^9+\frac{1}{2}\,Aa^6x^2+\frac{1}{8}\left(6\,Bab^5+Ab^6\right)x^8 + \frac{3}{7}\left(5\,Ba^2b^4+2\,Aab^5\right)x^7 + \frac{5}{6}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^6 + \left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^5 + \frac{3}{4}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^4 + \frac{1}{3}\left(Ba^6+6\,Aa^5b\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/9*B*b^6*x^9 + 1/2*A*a^6*x^2 + 1/8*(6*B*a*b^5 + A*b^6)*x^8 + 3/7*(5*B*a^2
*b^4 + 2*A*a*b^5)*x^7 + 5/6*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^6 + (3*B*a^4*b^2
 + 4*A*a^3*b^3)*x^5 + 3/4*(2*B*a^5*b + 5*A*a^4*b^2)*x^4 + 1/3*(B*a^6 + 6*A
*a^5*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{Aa^6x^2}{2}+\frac{Bb^6x^9}{9}+x^8\left(\frac{Ab^6}{8}+\frac{3Bab^5}{4}\right)+x^7\cdot\left(\frac{6Aab^5}{7}+\frac{15Ba^2b^4}{7}\right)+x^6\cdot\left(\frac{5Aa^2b^4}{2}+\frac{10Ba^3b^3}{3}\right)+x^5\cdot\left(4Aa^3b^3+3Ba^4b^2\right)+x^4\cdot\left(\frac{15Aa^4b^2}{4}+\frac{3Ba^5b}{2}\right)+x^3\cdot\left(2Aa^5b+\frac{Ba^6}{3}\right)$$

input
```
integrate(x*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x**2/2 + B*b**6*x**9/9 + x**8*(A*b**6/8 + 3*B*a*b**5/4) + x**7*(6*A
*a*b**5/7 + 15*B*a**2*b**4/7) + x**6*(5*A*a**2*b**4/2 + 10*B*a**3*b**3/3)
+ x**5*(4*A*a**3*b**3 + 3*B*a**4*b**2) + x**4*(15*A*a**4*b**2/4 + 3*B*a**5
*b/2) + x**3*(2*A*a**5*b + B*a**6/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.39

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{9}Bb^6x^9+\frac{1}{2}Aa^6x^2+\frac{1}{8}\left(6\,Bab^5+Ab^6\right)x^8+\frac{3}{7}\left(5\,Ba^2b^4+2\,Aab^5\right)x^7+\frac{5}{6}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^6+\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^5+\frac{3}{4}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^4+\frac{1}{3}\left(Ba^6+6\,Aa^5b\right)x^3$$

input

```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output

```
1/9*B*b^6*x^9 + 1/2*A*a^6*x^2 + 1/8*(6*B*a*b^5 + A*b^6)*x^8 + 3/7*(5*B*a^2
*b^4 + 2*A*a*b^5)*x^7 + 5/6*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^6 + (3*B*a^4*b^2
 + 4*A*a^3*b^3)*x^5 + 3/4*(2*B*a^5*b + 5*A*a^4*b^2)*x^4 + 1/3*(B*a^6 + 6*A
*a^5*b)*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.44

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{1}{9}Bb^6x^9+\frac{3}{4}Bab^5x^8+\frac{1}{8}Ab^6x^8+\frac{15}{7}Ba^2b^4x^7+\frac{6}{7}Aab^5x^7+\frac{10}{3}Ba^3b^3x^6+\frac{5}{2}Aa^2b^4x^6+3\,Ba^4b^2x^5+4\,Aa^3b^3x^5+\frac{3}{2}Ba^5bx^4+\frac{15}{4}Aa^4b^2x^4+\frac{1}{3}Ba^6x^3+2\,Aa^5bx^3+\frac{1}{2}Aa^6x^2$$

input

```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/9*B*b^6*x^9 + 3/4*B*a*b^5*x^8 + 1/8*A*b^6*x^8 + 15/7*B*a^2*b^4*x^7 + 6/7
*A*a*b^5*x^7 + 10/3*B*a^3*b^3*x^6 + 5/2*A*a^2*b^4*x^6 + 3*B*a^4*b^2*x^5 +
4*A*a^3*b^3*x^5 + 3/2*B*a^5*b*x^4 + 15/4*A*a^4*b^2*x^4 + 1/3*B*a^6*x^3 + 2
*A*a^5*b*x^3 + 1/2*A*a^6*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.13

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^3\left(\frac{B\,a^6}{3}+2\,A\,b\,a^5\right)+x^8\left(\frac{A\,b^6}{8}+\frac{3\,B\,a\,b^5}{4}\right)+\frac{A\,a^6\,x^2}{2}+\frac{B\,b^6\,x^9}{9}+a^3\,b^2\,x^5\,(4\,A\,b+3\,B\,a)+\frac{5\,a^2\,b^3\,x^6\,(3\,A\,b+4\,B\,a)}{6}+\frac{3\,a^4\,b\,x^4\,(5\,A\,b+2\,B\,a)}{4}+\frac{3\,a\,b^4\,x^7\,(2\,A\,b+5\,B\,a)}{7}$$

input
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^3*((B*a^6)/3 + 2*A*a^5*b) + x^8*((A*b^6)/8 + (3*B*a*b^5)/4) + (A*a^6*x^2
)/2 + (B*b^6*x^9)/9 + a^3*b^2*x^5*(4*A*b + 3*B*a) + (5*a^2*b^3*x^6*(3*A*b
+ 4*B*a))/6 + (3*a^4*b*x^4*(5*A*b + 2*B*a))/4 + (3*a*b^4*x^7*(2*A*b + 5*B*
a))/7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{x^2(8b^7x^7+63a\,b^6x^6+216a^2b^5x^5+420a^3b^4x^4+504a^4b^3x^3+378a^5b^2x^2+168a^6bx+36a^7)}{72}$$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x**2*(36*a**7 + 168*a**6*b*x + 378*a**5*b**2*x**2 + 504*a**4*b**3*x**3 +
420*a**3*b**4*x**4 + 216*a**2*b**5*x**5 + 63*a*b**6*x**6 + 8*b**7*x**7))/7
2
```

3.173 $\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{(Ab-aB)(a+bx)^7}{7b^2}+\frac{B(a+bx)^8}{8b^2}$$

output
```
1/7*(A*b-B*a)*(b*x+a)^7/b^2+1/8*B*(b*x+a)^8/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 122 vs. 2(38) = 76.

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.21

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{1}{56}x\big(28a^6(2A+Bx)+56a^5bx(3A+2Bx) + 70a^4b^2x^2(4A+3Bx)+56a^3b^3x^3(5A+4Bx) + 28a^2b^4x^4(6A+5Bx)+8ab^5x^5(7A+6Bx) + b^6x^6(8A+7Bx)\big)$$

input
```
Integrate[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(x*(28*a^6*(2*A + B*x) + 56*a^5*b*x*(3*A + 2*B*x) + 70*a^4*b^2*x^2*(4*A +
3*B*x) + 56*a^3*b^3*x^3*(5*A + 4*B*x) + 28*a^2*b^4*x^4*(6*A + 5*B*x) + 8*a
*b^5*x^5*(7*A + 6*B*x) + b^6*x^6*(8*A + 7*B*x)))/56
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^3 (A+Bx)\,dx$$

$$\Big\downarrow \text{1098}$$

$$\frac{\int b^6(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int (a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^6(Ab-aB)}{b}+\frac{B(a+bx)^7}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^7(Ab-aB)}{7b^2}+\frac{B(a+bx)^8}{8b^2}$$

input
```
Int[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*(a + b*x)^7)/(7*b^2) + (B*(a + b*x)^8)/(8*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(34) = 68$.

Time = 0.94 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.68

method	result
norman	$\frac{B\,b^6x^8}{8}+\left(\frac{1}{7}A\,b^6+\frac{6}{7}Ba\,b^5\right)x^7+\left(Aa\,b^5+\frac{5}{2}B\,a^2b^4\right)x^6+\left(3A\,a^2b^4+4B\,a^3b^3\right)x^5+\left(5A\,a^3b^3\right.$
default	$\frac{B\,b^6x^8}{8}+\frac{\left(A\,b^6+6Ba\,b^5\right)x^7}{7}+\frac{\left(6Aa\,b^5+15B\,a^2b^4\right)x^6}{6}+\frac{\left(15A\,a^2b^4+20B\,a^3b^3\right)x^5}{5}+\frac{\left(20A\,a^3b^3+15B\,a^4b^2\right)x^4}{4}+\frac{(15}{}$
gosper	$\frac{x\left(7b^6B\,x^7+8A\,b^6x^6+48Ba\,b^5x^6+56Aa\,b^5x^5+140B\,a^2b^4x^5+168A\,a^2b^4x^4+224B\,a^3b^3x^4+280A\,a^3b^3x^3+210B\,a^4b^2x^3+280\right.}{56}$
risch	$\frac{1}{8}B\,b^6x^8+\frac{1}{7}x^7A\,b^6+\frac{6}{7}x^7Ba\,b^5+x^6Aa\,b^5+\frac{5}{2}x^6B\,a^2b^4+3A\,a^2b^4x^5+4B\,a^3b^3x^5+5x^4A\,a$
parallelrisch	$\frac{1}{8}B\,b^6x^8+\frac{1}{7}x^7A\,b^6+\frac{6}{7}x^7Ba\,b^5+x^6Aa\,b^5+\frac{5}{2}x^6B\,a^2b^4+3A\,a^2b^4x^5+4B\,a^3b^3x^5+5x^4A\,a$
orering	$\frac{x\left(7b^6B\,x^7+8A\,b^6x^6+48Ba\,b^5x^6+56Aa\,b^5x^5+140B\,a^2b^4x^5+168A\,a^2b^4x^4+224B\,a^3b^3x^4+280A\,a^3b^3x^3+210B\,a^4b^2x^3+280\right.}{56(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*b^6*x^8+(1/7*A*b^6+6/7*B*a*b^5)*x^7+(A*a*b^5+5/2*B*a^2*b^4)*x^6+(3*A
*a^2*b^4+4*B*a^3*b^3)*x^5+(5*A*a^3*b^3+15/4*B*a^4*b^2)*x^4+(5*A*a^4*b^2+2*
B*a^5*b)*x^3+(3*A*a^5*b+1/2*B*a^6)*x^2+A*a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(34) = 68$.

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{8}\,Bb^6x^8+Aa^6x+\frac{1}{7}\left(6\,Bab^5+Ab^6\right)x^7 + \frac{1}{2}\left(5\,Ba^2b^4+2\,Aab^5\right)x^6 + \left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^5 + \frac{5}{4}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^4 + \left(2\,Ba^5b+5\,Aa^4b^2\right)x^3+\frac{1}{2}\left(Ba^6+6\,Aa^5b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/8*B*b^6*x^8 + A*a^6*x + 1/7*(6*B*a*b^5 + A*b^6)*x^7 + 1/2*(5*B*a^2*b^4 +
 2*A*a*b^5)*x^6 + (4*B*a^3*b^3 + 3*A*a^2*b^4)*x^5 + 5/4*(3*B*a^4*b^2 + 4*A
*a^3*b^3)*x^4 + (2*B*a^5*b + 5*A*a^4*b^2)*x^3 + 1/2*(B*a^6 + 6*A*a^5*b)*x^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.89

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = Aa^6x+\frac{Bb^6x^8}{8}+x^7\left(\frac{Ab^6}{7}+\frac{6Bab^5}{7}\right)+x^6\left(Aab^5+\frac{5Ba^2b^4}{2}\right)+x^5\cdot\left(3Aa^2b^4+4Ba^3b^3\right)+x^4\cdot\left(5Aa^3b^3+\frac{15Ba^4b^2}{4}\right)+x^3\cdot\left(5Aa^4b^2+2Ba^5b\right)+x^2\cdot\left(3Aa^5b+\frac{Ba^6}{2}\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*a**6*x + B*b**6*x**8/8 + x**7*(A*b**6/7 + 6*B*a*b**5/7) + x**6*(A*a*b**5
 + 5*B*a**2*b**4/2) + x**5*(3*A*a**2*b**4 + 4*B*a**3*b**3) + x**4*(5*A*a**
3*b**3 + 15*B*a**4*b**2/4) + x**3*(5*A*a**4*b**2 + 2*B*a**5*b) + x**2*(3*A
*a**5*b + B*a**6/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{8}\,Bb^6x^8+Aa^6x+\frac{1}{7}\left(6\,Bab^5+Ab^6\right)x^7+\frac{1}{2}\left(5\,Ba^2b^4+2\,Aab^5\right)x^6+\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^5+\frac{5}{4}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^4+\left(2\,Ba^5b+5\,Aa^4b^2\right)x^3+\frac{1}{2}\left(Ba^6+6\,Aa^5b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/8*B*b^6*x^8 + A*a^6*x + 1/7*(6*B*a*b^5 + A*b^6)*x^7 + 1/2*(5*B*a^2*b^4 +
 2*A*a*b^5)*x^6 + (4*B*a^3*b^3 + 3*A*a^2*b^4)*x^5 + 5/4*(3*B*a^4*b^2 + 4*A
*a^3*b^3)*x^4 + (2*B*a^5*b + 5*A*a^4*b^2)*x^3 + 1/2*(B*a^6 + 6*A*a^5*b)*x^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(34) = 68$.

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.82

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{1}{8}Bb^6x^8+\frac{6}{7}Bab^5x^7+\frac{1}{7}Ab^6x^7+\frac{5}{2}Ba^2b^4x^6+Aab^5x^6+4Ba^3b^3x^5+3Aa^2b^4x^5+\frac{15}{4}Ba^4b^2x^4+5Aa^3b^3x^4+2Ba^5bx^3+5Aa^4b^2x^3+\frac{1}{2}Ba^6x^2+3Aa^5bx^2+Aa^6x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/8*B*b^6*x^8 + 6/7*B*a*b^5*x^7 + 1/7*A*b^6*x^7 + 5/2*B*a^2*b^4*x^6 + A*a*
b^5*x^6 + 4*B*a^3*b^3*x^5 + 3*A*a^2*b^4*x^5 + 15/4*B*a^4*b^2*x^4 + 5*A*a^3
*b^3*x^4 + 2*B*a^5*b*x^3 + 5*A*a^4*b^2*x^3 + 1/2*B*a^6*x^2 + 3*A*a^5*b*x^2
 + A*a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.32

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^2\left(\frac{B\,a^6}{2}+3\,A\,b\,a^5\right)+x^7\left(\frac{A\,b^6}{7}+\frac{6\,B\,a\,b^5}{7}\right)+\frac{B\,b^6\,x^8}{8}+A\,a^6\,x+\frac{5\,a^3\,b^2\,x^4\,(4\,A\,b+3\,B\,a)}{4}+a^2\,b^3\,x^5\,(3\,A\,b+4\,B\,a)+a^4\,b\,x^3\,(5\,A\,b+2\,B\,a)+\frac{a\,b^4\,x^6\,(2\,A\,b+5\,B\,a)}{2}$$

input
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^2*((B*a^6)/2 + 3*A*a^5*b) + x^7*((A*b^6)/7 + (6*B*a*b^5)/7) + (B*b^6*x^8
)/8 + A*a^6*x + (5*a^3*b^2*x^4*(4*A*b + 3*B*a))/4 + a^2*b^3*x^5*(3*A*b + 4
*B*a) + a^4*b*x^3*(5*A*b + 2*B*a) + (a*b^4*x^6*(2*A*b + 5*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^3 \, dx = \frac{x(b^7x^7+8a\,b^6x^6+28a^2b^5x^5+56a^3b^4x^4+70a^4b^3x^3+56a^5b^2x^2+28a^6bx+8a^7)}{8}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(x*(8*a**7 + 28*a**6*b*x + 56*a**5*b**2*x**2 + 70*a**4*b**3*x**3 + 56*a**3
*b**4*x**4 + 28*a**2*b**5*x**5 + 8*a*b**6*x**6 + b**7*x**7))/8
```

3.174 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = 6a^5Abx+\frac{15}{2}a^4Ab^2x^2+\frac{20}{3}a^3Ab^3x^3+\frac{15}{4}a^2Ab^4x^4+\frac{6}{5}aAb^5x^5+\frac{1}{6}Ab^6x^6+\frac{B(a+bx)^7}{7b}+a^6A\log(x)$$

output
```
6*a^5*A*b*x+15/2*a^4*A*b^2*x^2+20/3*a^3*A*b^3*x^3+15/4*a^2*A*b^4*x^4+6/5*a
*A*b^5*x^5+1/6*A*b^6*x^6+1/7*B*(b*x+a)^7/b+a^6*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = a^6Bx+3a^5bx(2A+Bx)+\frac{5}{2}a^4b^2x^2(3A+2Bx)+\frac{5}{3}a^3b^3x^3(4A+3Bx)+\frac{3}{4}a^2b^4x^4(5A+4Bx)+\frac{1}{5}ab^5x^5(6A+5Bx)+\frac{1}{42}b^6x^6(7A+6Bx)+a^6A\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x,x]
```

output
```
a^6*B*x + 3*a^5*b*x*(2*A + B*x) + (5*a^4*b^2*x^2*(3*A + 2*B*x))/2 + (5*a^3
*b^3*x^3*(4*A + 3*B*x))/3 + (3*a^2*b^4*x^4*(5*A + 4*B*x))/4 + (a*b^5*x^5*(
6*A + 5*B*x))/5 + (b^6*x^6*(7*A + 6*B*x))/42 + a^6*A*Log[x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1184, 27, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3 (A+Bx)}{x} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x}dx$$

$$\Big\downarrow \text{90}$$

$$A\int \frac{(a+bx)^6}{x}dx+\frac{B(a+bx)^7}{7b}$$

$$\Big\downarrow \text{49}$$

$$A\int \left(\frac{a^6}{x}+6ba^5+15b^2xa^4+20b^3x^2a^3+15b^4x^3a^2+6b^5x^4a+b^6x^5\right)dx+\frac{B(a+bx)^7}{7b}$$

$$\Big\downarrow \text{2009}$$

$$A\left(a^6\log(x)+6a^5bx+\frac{15}{2}a^4b^2x^2+\frac{20}{3}a^3b^3x^3+\frac{15}{4}a^2b^4x^4+\frac{6}{5}ab^5x^5+\frac{b^6x^6}{6}\right)+\frac{B(a+bx)^7}{7b}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x,x]
```

output
```
(B*(a + b*x)^7)/(7*b) + A*(6*a^5*b*x + (15*a^4*b^2*x^2)/2 + (20*a^3*b^3*x^
3)/3 + (15*a^2*b^4*x^4)/4 + (6*a*b^5*x^5)/5 + (b^6*x^6)/6 + a^6*Log[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

method	result
norman	$\left(\frac{1}{6}A\,b^{6}+B a\,b^{5}\right)x^{6}+\left(\frac{6}{5}A a\,b^{5}+3B\,a^{2}b^{4}\right)x^{5}+\left(\frac{15}{4}A\,a^{2}b^{4}+5B\,a^{3}b^{3}\right)x^{4}+\left(\frac{20}{3}A\,a^{3}b^{3}+5B$
default	$\frac{b^{6}B\,x^{7}}{7}+\frac{A\,b^{6}x^{6}}{6}+B a\,b^{5}x^{6}+\frac{6A a\,b^{5}x^{5}}{5}+3B\,a^{2}b^{4}x^{5}+\frac{15A\,a^{2}b^{4}x^{4}}{4}+5B\,a^{3}b^{3}x^{4}+\frac{20A\,a^{3}b^{3}x^{3}}{3}+5B$
risch	$\frac{b^{6}B\,x^{7}}{7}+\frac{A\,b^{6}x^{6}}{6}+B a\,b^{5}x^{6}+\frac{6A a\,b^{5}x^{5}}{5}+3B\,a^{2}b^{4}x^{5}+\frac{15A\,a^{2}b^{4}x^{4}}{4}+5B\,a^{3}b^{3}x^{4}+\frac{20A\,a^{3}b^{3}x^{3}}{3}+5B$
parallelrisch	$\frac{b^{6}B\,x^{7}}{7}+\frac{A\,b^{6}x^{6}}{6}+B a\,b^{5}x^{6}+\frac{6A a\,b^{5}x^{5}}{5}+3B\,a^{2}b^{4}x^{5}+\frac{15A\,a^{2}b^{4}x^{4}}{4}+5B\,a^{3}b^{3}x^{4}+\frac{20A\,a^{3}b^{3}x^{3}}{3}+5B$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/6*A*b^6+B*a*b^5)*x^6+(6/5*A*a*b^5+3*B*a^2*b^4)*x^5+(15/4*A*a^2*b^4+5*B*
a^3*b^3)*x^4+(20/3*A*a^3*b^3+5*B*a^4*b^2)*x^3+(15/2*A*a^4*b^2+3*B*a^5*b)*x
^2+(6*A*a^5*b+B*a^6)*x+1/7*b^6*B*x^7+a^6*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = \frac{1}{7}\,Bb^6x^7 + Aa^6\log\left(x\right) + \frac{1}{6}\left(6\,Bab^5 + Ab^6\right)x^6 + \frac{3}{5}\left(5\,Ba^2b^4 + 2\,Aab^5\right)x^5 + \frac{5}{4}\left(4\,Ba^3b^3 + 3\,Aa^2b^4\right)x^4 + \frac{5}{3}\left(3\,Ba^4b^2 + 4\,Aa^3b^3\right)x^3 + \frac{3}{2}\left(2\,Ba^5b + 5\,Aa^4b^2\right)x^2 + \left(Ba^6 + 6\,Aa^5b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="fricas")
```

output
```
1/7*B*b^6*x^7 + A*a^6*log(x) + 1/6*(6*B*a*b^5 + A*b^6)*x^6 + 3/5*(5*B*a^2*
b^4 + 2*A*a*b^5)*x^5 + 5/4*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 5/3*(3*B*a^4*
b^2 + 4*A*a^3*b^3)*x^3 + 3/2*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + (B*a^6 + 6*A*
a^5*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = Aa^6\log(x)+\frac{Bb^6x^7}{7}+x^6\left(\frac{Ab^6}{6}+Bab^5\right)+x^5\cdot\left(\frac{6Aab^5}{5}+3Ba^2b^4\right)+x^4\cdot\left(\frac{15Aa^2b^4}{4}+5Ba^3b^3\right)+x^3\cdot\left(\frac{20Aa^3b^3}{3}+5Ba^4b^2\right)+x^2\cdot\left(\frac{15Aa^4b^2}{2}+3Ba^5b\right)+x\left(6Aa^5b+Ba^6\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x,x)
```

output
```
A*a**6*log(x) + B*b**6*x**7/7 + x**6*(A*b**6/6 + B*a*b**5) + x**5*(6*A*a*b
**5/5 + 3*B*a**2*b**4) + x**4*(15*A*a**2*b**4/4 + 5*B*a**3*b**3) + x**3*(2
0*A*a**3*b**3/3 + 5*B*a**4*b**2) + x**2*(15*A*a**4*b**2/2 + 3*B*a**5*b) +
x*(6*A*a**5*b + B*a**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = \frac{1}{7}\,Bb^6x^7 + Aa^6\log\left(x\right) + \frac{1}{6}\left(6\,Bab^5 + Ab^6\right)x^6 + \frac{3}{5}\left(5\,Ba^2b^4 + 2\,Aab^5\right)x^5 + \frac{5}{4}\left(4\,Ba^3b^3 + 3\,Aa^2b^4\right)x^4 + \frac{5}{3}\left(3\,Ba^4b^2 + 4\,Aa^3b^3\right)x^3 + \frac{3}{2}\left(2\,Ba^5b + 5\,Aa^4b^2\right)x^2 + \left(Ba^6 + 6\,Aa^5b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="maxima")
```

output
```
1/7*B*b^6*x^7 + A*a^6*log(x) + 1/6*(6*B*a*b^5 + A*b^6)*x^6 + 3/5*(5*B*a^2*
b^4 + 2*A*a*b^5)*x^5 + 5/4*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 5/3*(3*B*a^4*
b^2 + 4*A*a^3*b^3)*x^3 + 3/2*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + (B*a^6 + 6*A*
a^5*b)*x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = \frac{1}{7}\,Bb^6x^7 + Bab^5x^6 + \frac{1}{6}\,Ab^6x^6 + 3\,Ba^2b^4x^5 + \frac{6}{5}\,Aab^5x^5 + 5\,Ba^3b^3x^4 + \frac{15}{4}\,Aa^2b^4x^4 + 5\,Ba^4b^2x^3 + \frac{20}{3}\,Aa^3b^3x^3 + 3\,Ba^5bx^2 + \frac{15}{2}\,Aa^4b^2x^2 + Ba^6x + 6\,Aa^5bx + Aa^6\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x,x, algorithm="giac")
```

output
```
1/7*B*b^6*x^7 + B*a*b^5*x^6 + 1/6*A*b^6*x^6 + 3*B*a^2*b^4*x^5 + 6/5*A*a*b^
5*x^5 + 5*B*a^3*b^3*x^4 + 15/4*A*a^2*b^4*x^4 + 5*B*a^4*b^2*x^3 + 20/3*A*a^
3*b^3*x^3 + 3*B*a^5*b*x^2 + 15/2*A*a^4*b^2*x^2 + B*a^6*x + 6*A*a^5*b*x + A
*a^6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = x\left(B\,a^6+6\,A\,b\,a^5\right)+x^6\left(\frac{A\,b^6}{6}+B\,a\,b^5\right)+\frac{B\,b^6\,x^7}{7} + A\,a^6\,\ln\left(x\right)+\frac{5\,a^3\,b^2\,x^3\,\left(4\,A\,b+3\,B\,a\right)}{3} + \frac{5\,a^2\,b^3\,x^4\,\left(3\,A\,b+4\,B\,a\right)}{4} + \frac{3\,a^4\,b\,x^2\,\left(5\,A\,b+2\,B\,a\right)}{2} + \frac{3\,a\,b^4\,x^5\,\left(2\,A\,b+5\,B\,a\right)}{5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x,x)
```

output
```
x*(B*a^6 + 6*A*a^5*b) + x^6*((A*b^6)/6 + B*a*b^5) + (B*b^6*x^7)/7 + A*a^6*
log(x) + (5*a^3*b^2*x^3*(4*A*b + 3*B*a))/3 + (5*a^2*b^3*x^4*(3*A*b + 4*B*a
))/4 + (3*a^4*b*x^2*(5*A*b + 2*B*a))/2 + (3*a*b^4*x^5*(2*A*b + 5*B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x}\,dx = \log(x)\,a^7+7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x,x)
```

output
```
(420*log(x)*a**7 + 2940*a**6*b*x + 4410*a**5*b**2*x**2 + 4900*a**4*b**3*x*
*3 + 3675*a**3*b**4*x**4 + 1764*a**2*b**5*x**5 + 490*a*b**6*x**6 + 60*b**7
*x**7)/420
```

3.175 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2} \, dx = -\frac{a^6A}{x} + 3a^4b(5Ab+2aB)x + \frac{5}{2}a^3b^2(4Ab+3aB)x^2 + \frac{5}{3}a^2b^3(3Ab+4aB)x^3 + \frac{3}{4}ab^4(2Ab+5aB)x^4 + \frac{1}{5}b^5(Ab+6aB)x^5 + \frac{1}{6}b^6Bx^6 + a^5(6Ab+aB)\log(x)$$

output
```
-a^6*A/x+3*a^4*b*(5*A*b+2*B*a)*x+5/2*a^3*b^2*(4*A*b+3*B*a)*x^2+5/3*a^2*b^3
*(3*A*b+4*B*a)*x^3+3/4*a*b^4*(2*A*b+5*B*a)*x^4+1/5*b^5*(A*b+6*B*a)*x^5+1/6
*b^6*B*x^6+a^5*(6*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = -\frac{a^6A}{x}+6a^5bBx+\frac{15}{2}a^4b^2x(2A+Bx)+\frac{10}{3}a^3b^3x^2(3A+2Bx)+\frac{5}{4}a^2b^4x^3(4A+3Bx)+\frac{3}{10}ab^5x^4(5A+4Bx)+\frac{1}{30}b^6x^5(6A+5Bx)+a^5(6Ab+aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^2,x]
```

output
```
-((a^6*A)/x) + 6*a^5*b*B*x + (15*a^4*b^2*x*(2*A + B*x))/2 + (10*a^3*b^3*x^
2*(3*A + 2*B*x))/3 + (5*a^2*b^4*x^3*(4*A + 3*B*x))/4 + (3*a*b^5*x^4*(5*A +
 4*B*x))/10 + (b^6*x^5*(6*A + 5*B*x))/30 + a^5*(6*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^2}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^2}+\frac{a^5(aB+6Ab)}{x}+3a^4b(2aB+5Ab)+5a^3b^2x(3aB+4Ab)+5a^2b^3x^2(4aB+3Ab)+b^5x^4(6aB+Ab)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{x}+a^5\log(x)(aB+6Ab)+3a^4bx(2aB+5Ab)+\frac{5}{2}a^3b^2x^2(3aB+4Ab)+\frac{5}{3}a^2b^3x^3(4aB+3Ab)+\frac{1}{5}b^5x^5(6aB+Ab)+\frac{3}{4}ab^4x^4(5aB+2Ab)+\frac{1}{6}b^6Bx^6$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^2,x]
```

output
```
-((a^6*A)/x) + 3*a^4*b*(5*A*b + 2*a*B)*x + (5*a^3*b^2*(4*A*b + 3*a*B)*x^2)
/2 + (5*a^2*b^3*(3*A*b + 4*a*B)*x^3)/3 + (3*a*b^4*(2*A*b + 5*a*B)*x^4)/4 +
 (b^5*(A*b + 6*a*B)*x^5)/5 + (b^6*B*x^6)/6 + a^5*(6*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^6 B\,x^6}{6}+\frac{A\,b^6 x^5}{5}+\frac{6 B a\,b^5 x^5}{5}+\frac{3 A a\,b^5 x^4}{2}+\frac{15 B\,a^2 b^4 x^4}{4}+5 A\,a^2 b^4 x^3+\frac{20 B\,a^3 b^3 x^3}{3}+10 A\,a^3 b^3 x^2+\frac{15 B}{}$
risch	$\frac{b^6 B\,x^6}{6}+\frac{A\,b^6 x^5}{5}+\frac{6 B a\,b^5 x^5}{5}+\frac{3 A a\,b^5 x^4}{2}+\frac{15 B\,a^2 b^4 x^4}{4}+5 A\,a^2 b^4 x^3+\frac{20 B\,a^3 b^3 x^3}{3}+10 A\,a^3 b^3 x^2+\frac{15 B}{}$
norman	$\frac{\left(\frac{1}{5} A\,b^6+\frac{6}{5} B a\,b^5\right) x^6+\left(\frac{3}{2} A a\,b^5+\frac{15}{4} B\,a^2 b^4\right) x^5+\left(5 A\,a^2 b^4+\frac{20}{3} B\,a^3 b^3\right) x^4+\left(10 A\,a^3 b^3+\frac{15}{2} B\,a^4 b^2\right) x^3+\left(15 A\,a^4 b^2+6 B\,a^5 b\right) x^2-}{x}$
parallelrisch	$\frac{10 b^6 B\,x^7+12 A\,b^6 x^6+72 B a\,b^5 x^6+90 A a\,b^5 x^5+225 B\,a^2 b^4 x^5+300 A\,a^2 b^4 x^4+400 B\,a^3 b^3 x^4+600 A\,a^3 b^3 x^3+450 B\,a^4 b^2 x^3+360}{60 x}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^6*B*x^6+1/5*A*b^6*x^5+6/5*B*a*b^5*x^5+3/2*A*a*b^5*x^4+15/4*B*a^2*b^4
*x^4+5*A*a^2*b^4*x^3+20/3*B*a^3*b^3*x^3+10*A*a^3*b^3*x^2+15/2*B*a^4*b^2*x^
2+15*A*a^4*b^2*x+6*B*a^5*b*x+a^5*(6*A*b+B*a)*ln(x)-a^6*A/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx$$
$$=\frac{10\,Bb^6x^7-60\,Aa^6+12\,(6\,Bab^5+Ab^6)x^6+45\,(5\,Ba^2b^4+2\,Aab^5)x^5+100\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+150}{60\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*B*b^6*x^7 - 60*A*a^6 + 12*(6*B*a*b^5 + A*b^6)*x^6 + 45*(5*B*a^2*b
^4 + 2*A*a*b^5)*x^5 + 100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 150*(3*B*a^4*b
^2 + 4*A*a^3*b^3)*x^3 + 180*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 60*(B*a^6 + 6*
A*a^5*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = -\frac{Aa^6}{x}+\frac{Bb^6x^6}{6}+a^5\cdot(6Ab+Ba)\log(x)+x^5\left(\frac{Ab^6}{5}+\frac{6Bab^5}{5}\right)+x^4\cdot\left(\frac{3Aab^5}{2}+\frac{15Ba^2b^4}{4}\right)+x^3\cdot\left(5Aa^2b^4+\frac{20Ba^3b^3}{3}\right)+x^2\cdot\left(10Aa^3b^3+\frac{15Ba^4b^2}{2}\right)+x\left(15Aa^4b^2+6Ba^5b\right)$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**2,x)
```

output
```
-A*a**6/x + B*b**6*x**6/6 + a**5*(6*A*b + B*a)*log(x) + x**5*(A*b**6/5 + 6
*B*a*b**5/5) + x**4*(3*A*a*b**5/2 + 15*B*a**2*b**4/4) + x**3*(5*A*a**2*b**
4 + 20*B*a**3*b**3/3) + x**2*(10*A*a**3*b**3 + 15*B*a**4*b**2/2) + x*(15*A
*a**4*b**2 + 6*B*a**5*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = \frac{1}{6}Bb^6x^6-\frac{Aa^6}{x}+\frac{1}{5}\left(6\,Bab^5+Ab^6\right)x^5+\frac{3}{4}\left(5\,Ba^2b^4+2\,Aab^5\right)x^4+\frac{5}{3}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^3+\frac{5}{2}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^2+3\left(2\,Ba^5b+5\,Aa^4b^2\right)x+\left(Ba^6+6\,Aa^5b\right)\log(x)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="maxima")
```

output
```
1/6*B*b^6*x^6 - A*a^6/x + 1/5*(6*B*a*b^5 + A*b^6)*x^5 + 3/4*(5*B*a^2*b^4 +
 2*A*a*b^5)*x^4 + 5/3*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^3 + 5/2*(3*B*a^4*b^2 +
 4*A*a^3*b^3)*x^2 + 3*(2*B*a^5*b + 5*A*a^4*b^2)*x + (B*a^6 + 6*A*a^5*b)*lo
g(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bb^6x^6+\frac{6}{5}\,Bab^5x^5+\frac{1}{5}\,Ab^6x^5+\frac{15}{4}\,Ba^2b^4x^4 + \frac{3}{2}\,Aab^5x^4+\frac{20}{3}\,Ba^3b^3x^3+5\,Aa^2b^4x^3 + \frac{15}{2}\,Ba^4b^2x^2+10\,Aa^3b^3x^2+6\,Ba^5bx + 15\,Aa^4b^2x-\frac{Aa^6}{x}+\left(Ba^6+6\,Aa^5b\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^2,x, algorithm="giac")
```

output
```
1/6*B*b^6*x^6 + 6/5*B*a*b^5*x^5 + 1/5*A*b^6*x^5 + 15/4*B*a^2*b^4*x^4 + 3/2
*A*a*b^5*x^4 + 20/3*B*a^3*b^3*x^3 + 5*A*a^2*b^4*x^3 + 15/2*B*a^4*b^2*x^2 +
 10*A*a^3*b^3*x^2 + 6*B*a^5*b*x + 15*A*a^4*b^2*x - A*a^6/x + (B*a^6 + 6*A*
a^5*b)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = x^5\left(\frac{A\,b^6}{5}+\frac{6\,B\,a\,b^5}{5}\right)+\ln\left(x\right)\,\left(B\,a^6+6\,A\,b\,a^5\right) - \frac{A\,a^6}{x}+\frac{B\,b^6\,x^6}{6}+\frac{5\,a^3\,b^2\,x^2\,\left(4\,A\,b+3\,B\,a\right)}{2} + \frac{5\,a^2\,b^3\,x^3\,\left(3\,A\,b+4\,B\,a\right)}{3} +3\,a^4\,b\,x\,\left(5\,A\,b+2\,B\,a\right)+\frac{3\,a\,b^4\,x^4\,\left(2\,A\,b+5\,B\,a\right)}{4}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^2,x)
```

output
```
x^5*((A*b^6)/5 + (6*B*a*b^5)/5) + log(x)*(B*a^6 + 6*A*a^5*b) - (A*a^6)/x +
 (B*b^6*x^6)/6 + (5*a^3*b^2*x^2*(4*A*b + 3*B*a))/2 + (5*a^2*b^3*x^3*(3*A*b
 + 4*B*a))/3 + 3*a^4*b*x*(5*A*b + 2*B*a) + (3*a*b^4*x^4*(2*A*b + 5*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^2}\,dx = \frac{420\log(x)\,a^6bx-60a^7+1260a^5b^2x^2+1050a^4b^3x^3+700a^3b^4x^4+315a^2b^5x^5+84a\,b^6x^6+10b^7x^7}{60x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^2,x)
```

output
```
(420*log(x)*a**6*b*x - 60*a**7 + 1260*a**5*b**2*x**2 + 1050*a**4*b**3*x**3
 + 700*a**3*b**4*x**4 + 315*a**2*b**5*x**5 + 84*a*b**6*x**6 + 10*b**7*x**7
)/(60*x)
```

3.176 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = {} & -\frac{a^6A}{2x^2}-\frac{a^5(6Ab+aB)}{x}+5a^3b^2(4Ab+3aB)x \\ & +\frac{5}{2}a^2b^3(3Ab+4aB)x^2+ab^4(2Ab+5aB)x^3 \\ & +\frac{1}{4}b^5(Ab+6aB)x^4+\frac{1}{5}b^6Bx^5 \\ & +3a^4b(5Ab+2aB)\log(x)\end{aligned}$$

output
```
-1/2*a^6*A/x^2-a^5*(6*A*b+B*a)/x+5*a^3*b^2*(4*A*b+3*B*a)*x+5/2*a^2*b^3*(3*
A*b+4*B*a)*x^2+a*b^4*(2*A*b+5*B*a)*x^3+1/4*b^5*(A*b+6*B*a)*x^4+1/5*b^6*B*x
^5+3*a^4*b*(5*A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = -\frac{6a^5Ab}{x}+15a^4b^2Bx+10a^3b^3x(2A+Bx) - \frac{a^6(A+2Bx)}{2x^2}+\frac{5}{2}a^2b^4x^2(3A+2Bx) + \frac{1}{2}ab^5x^3(4A+3Bx)+\frac{1}{20}b^6x^4(5A+4Bx) + 3a^4b(5Ab+2aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^3,x]
```

output
```
(-6*a^5*A*b)/x + 15*a^4*b^2*B*x + 10*a^3*b^3*x*(2*A + B*x) - (a^6*(A + 2*B
*x))/(2*x^2) + (5*a^2*b^4*x^2*(3*A + 2*B*x))/2 + (a*b^5*x^3*(4*A + 3*B*x))
/2 + (b^6*x^4*(5*A + 4*B*x))/20 + 3*a^4*b*(5*A*b + 2*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^3}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^3}dx$$

$$\downarrow 85$$

$$\int \left(\frac{a^6 A}{x^3} + \frac{a^5(aB + 6Ab)}{x^2} + \frac{3a^4 b(2aB + 5Ab)}{x} + 5a^3b^2(3aB + 4Ab) + 5a^2b^3x(4aB + 3Ab) + b^5x^3(6aB + Ab) +$$

$$\downarrow 2009$$

$$-\frac{a^6 A}{2x^2} - \frac{a^5(aB + 6Ab)}{x} + 3a^4 b\log(x)(2aB + 5Ab) + 5a^3b^2x(3aB + 4Ab) + \frac{5}{2}a^2b^3x^2(4aB + 3Ab) + \frac{1}{4}b^5x^4(6aB + Ab) + ab^4x^3(5aB + 2Ab) + \frac{1}{5}b^6Bx^5$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^3,x]
```

output
```
-1/2*(a^6*A)/x^2 - (a^5*(6*A*b + a*B))/x + 5*a^3*b^2*(4*A*b + 3*a*B)*x + (
5*a^2*b^3*(3*A*b + 4*a*B)*x^2)/2 + a*b^4*(2*A*b + 5*a*B)*x^3 + (b^5*(A*b +
 6*a*B)*x^4)/4 + (b^6*B*x^5)/5 + 3*a^4*b*(5*A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^6 B x^5}{5}+\frac{A b^6 x^4}{4}+\frac{3 B a b^5 x^4}{2}+2 A a b^5 x^3+5 B a^2 b^4 x^3+\frac{15 A a^2 b^4 x^2}{2}+10 B a^3 b^3 x^2+20 A a^3 b^3 x+$
norman	$\frac{\left(\frac{1}{4} A b^6+\frac{3}{2} B a b^5\right) x^6+\left(\frac{15}{2} A a^2 b^4+10 B a^3 b^3\right) x^4+\left(2 A a b^5+5 B a^2 b^4\right) x^5+\left(20 A a^3 b^3+15 B a^4 b^2\right) x^3+\left(-6 A a^5 b-B a^6\right) x-\frac{A a^6}{2}}{x^2}$
risch	$\frac{b^6 B x^5}{5}+\frac{A b^6 x^4}{4}+\frac{3 B a b^5 x^4}{2}+2 A a b^5 x^3+5 B a^2 b^4 x^3+\frac{15 A a^2 b^4 x^2}{2}+10 B a^3 b^3 x^2+20 A a^3 b^3 x+$
parallelrisch	$\frac{4 b^6 B x^7+5 A b^6 x^6+30 B a b^5 x^6+40 A a b^5 x^5+100 B a^2 b^4 x^5+150 A a^2 b^4 x^4+200 B a^3 b^3 x^4+300 A \ln(x) x^2 a^4 b^2+400 A a^3 b^3 x^3+1}{20 x^2}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^6*B*x^5+1/4*A*b^6*x^4+3/2*B*a*b^5*x^4+2*A*a*b^5*x^3+5*B*a^2*b^4*x^3+
15/2*A*a^2*b^4*x^2+10*B*a^3*b^3*x^2+20*A*a^3*b^3*x+15*B*a^4*b^2*x-1/2*a^6*
A/x^2+3*a^4*b*(5*A*b+2*B*a)*ln(x)-a^5*(6*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx$$
$$=\frac{4\,Bb^6x^7-10\,Aa^6+5\,(6\,Bab^5+Ab^6)x^6+20\,(5\,Ba^2b^4+2\,Aab^5)x^5+50\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+100\,(3}{20\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="fricas")
```

output
```
1/20*(4*B*b^6*x^7 - 10*A*a^6 + 5*(6*B*a*b^5 + A*b^6)*x^6 + 20*(5*B*a^2*b^4
 + 2*A*a*b^5)*x^5 + 50*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 100*(3*B*a^4*b^2
+ 4*A*a^3*b^3)*x^3 + 60*(2*B*a^5*b + 5*A*a^4*b^2)*x^2*log(x) - 20*(B*a^6 +
 6*A*a^5*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = \frac{Bb^6x^5}{5}+3a^4b(5Ab+2Ba)\log{(x)} +x^4\left(\frac{Ab^6}{4}+\frac{3Bab^5}{2}\right)+x^3\cdot\left(2Aab^5+5Ba^2b^4\right)+x^2 \cdot\left(\frac{15Aa^2b^4}{2}+10Ba^3b^3\right)+x\left(20Aa^3b^3+15Ba^4b^2\right) +\frac{-Aa^6+x(-12Aa^5b-2Ba^6)}{2x^2}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**3,x)
```

output
```
B*b**6*x**5/5 + 3*a**4*b*(5*A*b + 2*B*a)*log(x) + x**4*(A*b**6/4 + 3*B*a*b
**5/2) + x**3*(2*A*a*b**5 + 5*B*a**2*b**4) + x**2*(15*A*a**2*b**4/2 + 10*B
*a**3*b**3) + x*(20*A*a**3*b**3 + 15*B*a**4*b**2) + (-A*a**6 + x*(-12*A*a*
*5*b - 2*B*a**6))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bb^6x^5+\frac{1}{4}\,\left(6\,Bab^5+Ab^6\right)x^4 +\left(5\,Ba^2b^4+2\,Aab^5\right)x^3 +\frac{5}{2}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^2 +5\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x +3\,\left(2\,Ba^5b+5\,Aa^4b^2\right)\log\left(x\right) -\frac{Aa^6+2\,(Ba^6+6\,Aa^5b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/5*B*b^6*x^5 + 1/4*(6*B*a*b^5 + A*b^6)*x^4 + (5*B*a^2*b^4 + 2*A*a*b^5)*x^
3 + 5/2*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^2 + 5*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x
+ 3*(2*B*a^5*b + 5*A*a^4*b^2)*log(x) - 1/2*(A*a^6 + 2*(B*a^6 + 6*A*a^5*b)*
x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bb^6x^5+\frac{3}{2}\,Bab^5x^4+\frac{1}{4}\,Ab^6x^4+5\,Ba^2b^4x^3+2\,Aab^5x^3+10\,Ba^3b^3x^2+\frac{15}{2}\,Aa^2b^4x^2+15\,Ba^4b^2x+20\,Aa^3b^3x+3\left(2\,Ba^5b+5\,Aa^4b^2\right)\log\left(|x|\right)-\frac{Aa^6+2\,(Ba^6+6\,Aa^5b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^3,x, algorithm="giac")
```

output
```
1/5*B*b^6*x^5 + 3/2*B*a*b^5*x^4 + 1/4*A*b^6*x^4 + 5*B*a^2*b^4*x^3 + 2*A*a*
b^5*x^3 + 10*B*a^3*b^3*x^2 + 15/2*A*a^2*b^4*x^2 + 15*B*a^4*b^2*x + 20*A*a^
3*b^3*x + 3*(2*B*a^5*b + 5*A*a^4*b^2)*log(abs(x)) - 1/2*(A*a^6 + 2*(B*a^6
+ 6*A*a^5*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = \ln\left(x\right)\,\left(6\,B\,a^5\,b+15\,A\,a^4\,b^2\right)-\frac{x\,\left(B\,a^6+6\,A\,b\,a^5\right)+\frac{A\,a^6}{2}}{x^2}+x^4\,\left(\frac{A\,b^6}{4}+\frac{3\,B\,a\,b^5}{2}\right)+\frac{B\,b^6\,x^5}{5}+\frac{5\,a^2\,b^3\,x^2\,\left(3\,A\,b+4\,B\,a\right)}{2}+5\,a^3\,b^2\,x\,\left(4\,A\,b+3\,B\,a\right)+a\,b^4\,x^3\,\left(2\,A\,b+5\,B\,a\right)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^3,x)
```

output
```
log(x)*(15*A*a^4*b^2 + 6*B*a^5*b) - (x*(B*a^6 + 6*A*a^5*b) + (A*a^6)/2)/x^
2 + x^4*((A*b^6)/4 + (3*B*a*b^5)/2) + (B*b^6*x^5)/5 + (5*a^2*b^3*x^2*(3*A*
b + 4*B*a))/2 + 5*a^3*b^2*x*(4*A*b + 3*B*a) + a*b^4*x^3*(2*A*b + 5*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^3}\,dx = \frac{420\log(x)\,a^5b^2x^2-10a^7-140a^6bx+700a^4b^3x^3+350a^3b^4x^4+140a^2b^5x^5+35a\,b^6x^6+4b^7x^7}{20x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^3,x)
```

output
```
(420*log(x)*a**5*b**2*x**2 - 10*a**7 - 140*a**6*b*x + 700*a**4*b**3*x**3 +
 350*a**3*b**4*x**4 + 140*a**2*b**5*x**5 + 35*a*b**6*x**6 + 4*b**7*x**7)/(
20*x**2)
```

3.177 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx = & -\frac{a^6A}{3x^3}-\frac{a^5(6Ab+aB)}{2x^2}-\frac{3a^4b(5Ab+2aB)}{x} \\ & +5a^2b^3(3Ab+4aB)x+\frac{3}{2}ab^4(2Ab+5aB)x^2 \\ & +\frac{1}{3}b^5(Ab+6aB)x^3+\frac{1}{4}b^6Bx^4 \\ & +5a^3b^2(4Ab+3aB)\log(x)\end{aligned}$$

output
```
-1/3*a^6*A/x^3-1/2*a^5*(6*A*b+B*a)/x^2-3*a^4*b*(5*A*b+2*B*a)/x+5*a^2*b^3*(
3*A*b+4*B*a)*x+3/2*a*b^4*(2*A*b+5*B*a)*x^2+1/3*b^5*(A*b+6*B*a)*x^3+1/4*b^6
*B*x^4+5*a^3*b^2*(4*A*b+3*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx = -\frac{15a^4Ab^2}{x}+20a^3b^3Bx+\frac{15}{2}a^2b^4x(2A+Bx) - \frac{3a^5b(A+2Bx)}{x^2}+ab^5x^2(3A+2Bx) - \frac{a^6(2A+3Bx)}{6x^3}+\frac{1}{12}b^6x^3(4A+3Bx) + 5a^3b^2(4Ab+3aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^4,x]
```

output
```
(-15*a^4*A*b^2)/x + 20*a^3*b^3*B*x + (15*a^2*b^4*x*(2*A + B*x))/2 - (3*a^5
*b*(A + 2*B*x))/x^2 + a*b^5*x^2*(3*A + 2*B*x) - (a^6*(2*A + 3*B*x))/(6*x^3
) + (b^6*x^3*(4*A + 3*B*x))/12 + 5*a^3*b^2*(4*A*b + 3*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^4}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^4}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^4}dx$$

$$\downarrow \text{85}$$

$$\int \left(\frac{a^6 A}{x^4}+\frac{a^5(aB+6Ab)}{x^3}+\frac{3a^4b(2aB+5Ab)}{x^2}+\frac{5a^3b^2(3aB+4Ab)}{x}+5a^2b^3(4aB+3Ab)+b^5x^2(6aB+Ab)+\right.$$

$$\downarrow \text{2009}$$

$$-\frac{a^6A}{3x^3}-\frac{a^5(aB+6Ab)}{2x^2}-\frac{3a^4b(2aB+5Ab)}{x}+5a^3b^2\log(x)(3aB+4Ab)+5a^2b^3x(4aB+3Ab)+\frac{1}{3}b^5x^3(6aB+Ab)+\frac{3}{2}ab^4x^2(5aB+2Ab)+\frac{1}{4}b^6Bx^4$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^4,x]
```

output
```
-1/3*(a^6*A)/x^3 - (a^5*(6*A*b + a*B))/(2*x^2) - (3*a^4*b*(5*A*b + 2*a*B))
/x + 5*a^2*b^3*(3*A*b + 4*a*B)*x + (3*a*b^4*(2*A*b + 5*a*B)*x^2)/2 + (b^5*
(A*b + 6*a*B)*x^3)/3 + (b^6*B*x^4)/4 + 5*a^3*b^2*(4*A*b + 3*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^{6}B\,x^{4}}{4}+\frac{A\,b^{6}x^{3}}{3}+2Ba\,b^{5}x^{3}+3Aa\,b^{5}x^{2}+\frac{15B\,a^{2}b^{4}x^{2}}{2}+15A\,a^{2}b^{4}x+20B\,a^{3}b^{3}x-\frac{a^{6}A}{3x^{3}}-\frac{a^{5}(6Ab}{2x}$
risch	$\frac{b^{6}B\,x^{4}}{4}+\frac{A\,b^{6}x^{3}}{3}+2Ba\,b^{5}x^{3}+3Aa\,b^{5}x^{2}+\frac{15B\,a^{2}b^{4}x^{2}}{2}+15A\,a^{2}b^{4}x+20B\,a^{3}b^{3}x+\frac{(-15A\,a^{4}b^{2}-6B}{}$
norman	$\frac{\left(\frac{1}{3}A\,b^{6}+2Ba\,b^{5}\right)x^{6}+\left(3Aa\,b^{5}+\frac{15}{2}B\,a^{2}b^{4}\right)x^{5}+\left(-3A\,a^{5}b-\frac{1}{2}B\,a^{6}\right)x+\left(15A\,a^{2}b^{4}+20B\,a^{3}b^{3}\right)x^{4}+\left(-15A\,a^{4}b^{2}-6B\,a^{5}b\right)x^{2}-\frac{A\,a}{3}}{x^{3}}$
parallelrisch	$\frac{3b^{6}B\,x^{7}+4A\,b^{6}x^{6}+24Ba\,b^{5}x^{6}+36Aa\,b^{5}x^{5}+90B\,a^{2}b^{4}x^{5}+240A\ln(x)x^{3}a^{3}b^{3}+180A\,a^{2}b^{4}x^{4}+180B\ln(x)x^{3}a^{4}b^{2}+240B\,a^{3}b^{3}x}{12x^{3}}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^6*B*x^4+1/3*A*b^6*x^3+2*B*a*b^5*x^3+3*A*a*b^5*x^2+15/2*B*a^2*b^4*x^2
+15*A*a^2*b^4*x+20*B*a^3*b^3*x-1/3*a^6*A/x^3-1/2*a^5*(6*A*b+B*a)/x^2+5*a^3
*b^2*(4*A*b+3*B*a)*ln(x)-3*a^4*b*(5*A*b+2*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx$$
$$=\frac{3\,Bb^6x^7-4\,Aa^6+4\,(6\,Bab^5+Ab^6)x^6+18\,(5\,Ba^2b^4+2\,Aab^5)x^5+60\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+60\,(3\,Ba}{12\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*B*b^6*x^7 - 4*A*a^6 + 4*(6*B*a*b^5 + A*b^6)*x^6 + 18*(5*B*a^2*b^4
+ 2*A*a*b^5)*x^5 + 60*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 60*(3*B*a^4*b^2 +
4*A*a^3*b^3)*x^3*log(x) - 36*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 6*(B*a^6 + 6*
A*a^5*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx = \frac{Bb^6x^4}{4} + 5a^3b^2\cdot(4Ab+3Ba)\log(x) + x^3\left(\frac{Ab^6}{3}+2Bab^5\right) + x^2\cdot\left(3Aab^5+\frac{15Ba^2b^4}{2}\right) + x\left(15Aa^2b^4+20Ba^3b^3\right) + \frac{-2Aa^6+x^2(-90Aa^4b^2-36Ba^5b)+x(-18Aa^5b-3Ba^6)}{6x^3}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**4,x)
```

output
```
B*b**6*x**4/4 + 5*a**3*b**2*(4*A*b + 3*B*a)*log(x) + x**3*(A*b**6/3 + 2*B*
a*b**5) + x**2*(3*A*a*b**5 + 15*B*a**2*b**4/2) + x*(15*A*a**2*b**4 + 20*B*
a**3*b**3) + (-2*A*a**6 + x**2*(-90*A*a**4*b**2 - 36*B*a**5*b) + x*(-18*A*
a**5*b - 3*B*a**6))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx = \frac{1}{4}\,Bb^6x^4 + \frac{1}{3}\left(6\,Bab^5+Ab^6\right)x^3 + \frac{3}{2}\left(5\,Ba^2b^4+2\,Aab^5\right)x^2 + 5\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x + 5\left(3\,Ba^4b^2+4\,Aa^3b^3\right)\log(x) - \frac{2\,Aa^6+18\,(2\,Ba^5b+5\,Aa^4b^2)x^2+3\,(Ba^6+6\,Aa^5b)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^4,x, algorithm="maxima")
```

output
```
1/4*B*b^6*x^4 + 1/3*(6*B*a*b^5 + A*b^6)*x^3 + 3/2*(5*B*a^2*b^4 + 2*A*a*b^5
)*x^2 + 5*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x + 5*(3*B*a^4*b^2 + 4*A*a^3*b^3)*lo
g(x) - 1/6*(2*A*a^6 + 18*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 3*(B*a^6 + 6*A*a^
5*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx &= \frac{1}{4}\,Bb^6x^4+2\,Bab^5x^3+\frac{1}{3}\,Ab^6x^3+\frac{15}{2}\,Ba^2b^4x^2+3\,Aab^5x^2 \\ &\quad+20\,Ba^3b^3x+15\,Aa^2b^4x+5\left(3\,Ba^4b^2+4\,Aa^3b^3\right)\log\left(|x|\right) \\ &\quad-\frac{2\,Aa^6+18\,(2\,Ba^5b+5\,Aa^4b^2)x^2+3\,(Ba^6+6\,Aa^5b)x}{6\,x^3}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^4,x, algorithm="giac")
```

output
```
1/4*B*b^6*x^4 + 2*B*a*b^5*x^3 + 1/3*A*b^6*x^3 + 15/2*B*a^2*b^4*x^2 + 3*A*a
*b^5*x^2 + 20*B*a^3*b^3*x + 15*A*a^2*b^4*x + 5*(3*B*a^4*b^2 + 4*A*a^3*b^3)
*log(abs(x)) - 1/6*(2*A*a^6 + 18*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 3*(B*a^6
+ 6*A*a^5*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx &= x^3\left(\frac{A\,b^6}{3}+2\,B\,a\,b^5\right)-\frac{x\left(\frac{B\,a^6}{2}+3\,A\,b\,a^5\right)+\frac{A\,a^6}{3}+x^2\,(6\,B\,a^5\,b+15\,A\,a^4\,b^2)}{x^3} \\ &\quad+\ln\left(x\right)\,\left(15\,B\,a^4\,b^2+20\,A\,a^3\,b^3\right)+\frac{B\,b^6\,x^4}{4} \\ &\quad+5\,a^2\,b^3\,x\,(3\,A\,b+4\,B\,a)+\frac{3\,a\,b^4\,x^2\,(2\,A\,b+5\,B\,a)}{2}\end{aligned}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^4,x)
```

output
```
x^3*((A*b^6)/3 + 2*B*a*b^5) - (x*((B*a^6)/2 + 3*A*a^5*b) + (A*a^6)/3 + x^2
*(15*A*a^4*b^2 + 6*B*a^5*b))/x^3 + log(x)*(20*A*a^3*b^3 + 15*B*a^4*b^2) +
(B*b^6*x^4)/4 + 5*a^2*b^3*x*(3*A*b + 4*B*a) + (3*a*b^4*x^2*(2*A*b + 5*B*a)
)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^4}\,dx$$
$$=\frac{420\log(x)\,a^4b^3x^3-4a^7-42a^6bx-252a^5b^2x^2+420a^3b^4x^4+126a^2b^5x^5+28a\,b^6x^6+3b^7x^7}{12x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^4,x)
```

output
```
(420*log(x)*a**4*b**3*x**3 - 4*a**7 - 42*a**6*b*x - 252*a**5*b**2*x**2 + 4
20*a**3*b**4*x**4 + 126*a**2*b**5*x**5 + 28*a*b**6*x**6 + 3*b**7*x**7)/(12
*x**3)
```

3.178 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx = -\frac{a^6A}{4x^4}-\frac{a^5(6Ab+aB)}{3x^3}-\frac{3a^4b(5Ab+2aB)}{2x^2}-\frac{5a^3b^2(4Ab+3aB)}{x}+3ab^4(2Ab+5aB)x+\frac{1}{2}b^5(Ab+6aB)x^2+\frac{1}{3}b^6Bx^3+5a^2b^3(3Ab+4aB)\log(x)$$

output
```
-1/4*a^6*A/x^4-1/3*a^5*(6*A*b+B*a)/x^3-3/2*a^4*b*(5*A*b+2*B*a)/x^2-5*a^3*b
^2*(4*A*b+3*B*a)/x+3*a*b^4*(2*A*b+5*B*a)*x+1/2*b^5*(A*b+6*B*a)*x^2+1/3*b^6
*B*x^3+5*a^2*b^3*(3*A*b+4*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx = -\frac{20a^3Ab^3}{x}+15a^2b^4Bx+3ab^5x(2A+Bx) - \frac{15a^4b^2(A+2Bx)}{2x^2}+\frac{1}{6}b^6x^2(3A+2Bx) - \frac{a^5b(2A+3Bx)}{x^3}-\frac{a^6(3A+4Bx)}{12x^4} + 5a^2b^3(3Ab+4aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^5,x]
```

output
```
(-20*a^3*A*b^3)/x + 15*a^2*b^4*B*x + 3*a*b^5*x*(2*A + B*x) - (15*a^4*b^2*(
A + 2*B*x))/(2*x^2) + (b^6*x^2*(3*A + 2*B*x))/6 - (a^5*b*(2*A + 3*B*x))/x^
3 - (a^6*(3*A + 4*B*x))/(12*x^4) + 5*a^2*b^3*(3*A*b + 4*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^5}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^5}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^5}dx$$

$\downarrow$ 85

$$\int \left(\frac{a^6 A}{x^5}+\frac{a^5(aB+6Ab)}{x^4}+\frac{3a^4b(2aB+5Ab)}{x^3}+\frac{5a^3b^2(3aB+4Ab)}{x^2}+\frac{5a^2b^3(4aB+3Ab)}{x}+b^5x(6aB+Ab)+$$

$\downarrow$ 2009

$$-\frac{a^6 A}{4x^4}-\frac{a^5(aB+6Ab)}{3x^3}-\frac{3a^4b(2aB+5Ab)}{2x^2}-\frac{5a^3b^2(3aB+4Ab)}{x}+5a^2b^3\log(x)(4aB+3Ab)+\frac{1}{2}b^5x^2(6aB+Ab)+3ab^4x(5aB+2Ab)+\frac{1}{3}b^6Bx^3$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^5,x]
```

output
```
-1/4*(a^6*A)/x^4 - (a^5*(6*A*b + a*B))/(3*x^3) - (3*a^4*b*(5*A*b + 2*a*B))
/(2*x^2) - (5*a^3*b^2*(4*A*b + 3*a*B))/x + 3*a*b^4*(2*A*b + 5*a*B)*x + (b^
5*(A*b + 6*a*B)*x^2)/2 + (b^6*B*x^3)/3 + 5*a^2*b^3*(3*A*b + 4*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^6 B x^3}{3}+\frac{A b^6 x^2}{2}+3 B a b^5 x^2+6 A a b^5 x+15 B a^2 b^4 x-\frac{a^5(6 A b+B a)}{3 x^3}-\frac{3 a^4 b(5 A b+2 B a)}{2 x^2}-\frac{a^6 A}{4 x^4}+5 a$
risch	$\frac{b^6 B x^3}{3}+\frac{A b^6 x^2}{2}+3 B a b^5 x^2+6 A a b^5 x+15 B a^2 b^4 x+\frac{\left(-20 A a^3 b^3-15 B a^4 b^2\right) x^3+\left(-\frac{15}{2} A a^4 b^2-3 B a^5 b\right) x}{x^4}$
norman	$\frac{\left(\frac{1}{2} A b^6+3 B a b^5\right) x^6+\left(-\frac{15}{2} A a^4 b^2-3 B a^5 b\right) x^2+\left(-2 A a^5 b-\frac{1}{3} B a^6\right) x+\left(6 A a b^5+15 B a^2 b^4\right) x^5+\left(-20 A a^3 b^3-15 B a^4 b^2\right) x^3-}{x^4}$
parallelrisch	$\frac{4 b^6 B x^7+6 A b^6 x^6+36 B a b^5 x^6+180 A \ln (x) x^4 a^2 b^4+72 A a b^5 x^5+240 B \ln (x) x^4 a^3 b^3+180 B a^2 b^4 x^5-240 A a^3 b^3 x^3-180 B a^4 b^2}{12 x^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^6*B*x^3+1/2*A*b^6*x^2+3*B*a*b^5*x^2+6*A*a*b^5*x+15*B*a^2*b^4*x-1/3*a
^5*(6*A*b+B*a)/x^3-3/2*a^4*b*(5*A*b+2*B*a)/x^2-1/4*a^6*A/x^4+5*a^2*b^3*(3*
A*b+4*B*a)*ln(x)-5*a^3*b^2*(4*A*b+3*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx$$
$$=\frac{4\,Bb^6x^7-3\,Aa^6+6\,(6\,Bab^5+Ab^6)x^6+36\,(5\,Ba^2b^4+2\,Aab^5)x^5+60\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4\log{(x)}-6}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*B*b^6*x^7 - 3*A*a^6 + 6*(6*B*a*b^5 + A*b^6)*x^6 + 36*(5*B*a^2*b^4
+ 2*A*a*b^5)*x^5 + 60*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4*log(x) - 60*(3*B*a^4
*b^2 + 4*A*a^3*b^3)*x^3 - 18*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 4*(B*a^6 + 6*
A*a^5*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^3}{x^5}\,dx$$
$$= \frac{Bb^6x^3}{3} + 5a^2b^3\cdot(3Ab+4Ba)\log(x) + x^2\left(\frac{Ab^6}{2}+3Bab^5\right) + x(6Aab^5+15Ba^2b^4) + \frac{-3Aa^6+x^3(-240Aa^3b^3-180Ba^4b^2)+x^2(-90Aa^4b^2-36Ba^5b)+x(-24Aa^5b-4Ba^6)}{12x^4}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**5,x)
```

output
```
B*b**6*x**3/3 + 5*a**2*b**3*(3*A*b + 4*B*a)*log(x) + x**2*(A*b**6/2 + 3*B*
a*b**5) + x*(6*A*a*b**5 + 15*B*a**2*b**4) + (-3*A*a**6 + x**3*(-240*A*a**3
*b**3 - 180*B*a**4*b**2) + x**2*(-90*A*a**4*b**2 - 36*B*a**5*b) + x*(-24*A
*a**5*b - 4*B*a**6))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^3}{x^5}\,dx$$
$$= \frac{1}{3}Bb^6x^3 + \frac{1}{2}(6Bab^5+Ab^6)x^2 + 3(5Ba^2b^4+2Aab^5)x + 5(4Ba^3b^3+3Aa^2b^4)\log(x) - \frac{3Aa^6+60(3Ba^4b^2+4Aa^3b^3)x^3+18(2Ba^5b+5Aa^4b^2)x^2+4(Ba^6+6Aa^5b)x}{12x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^5,x, algorithm="maxima")
```

output
```
1/3*B*b^6*x^3 + 1/2*(6*B*a*b^5 + A*b^6)*x^2 + 3*(5*B*a^2*b^4 + 2*A*a*b^5)*
x + 5*(4*B*a^3*b^3 + 3*A*a^2*b^4)*log(x) - 1/12*(3*A*a^6 + 60*(3*B*a^4*b^2
 + 4*A*a^3*b^3)*x^3 + 18*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 4*(B*a^6 + 6*A*a^
5*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx$$
$$=\frac{1}{3}Bb^6x^3+3Bab^5x^2+\frac{1}{2}Ab^6x^2+15Ba^2b^4x+6Aab^5x+5\left(4Ba^3b^3+3Aa^2b^4\right)\log\left(|x|\right)$$
$$-\frac{3Aa^6+60(3Ba^4b^2+4Aa^3b^3)x^3+18(2Ba^5b+5Aa^4b^2)x^2+4(Ba^6+6Aa^5b)x}{12x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^5,x, algorithm="giac")
```

output
```
1/3*B*b^6*x^3 + 3*B*a*b^5*x^2 + 1/2*A*b^6*x^2 + 15*B*a^2*b^4*x + 6*A*a*b^5
*x + 5*(4*B*a^3*b^3 + 3*A*a^2*b^4)*log(abs(x)) - 1/12*(3*A*a^6 + 60*(3*B*a
^4*b^2 + 4*A*a^3*b^3)*x^3 + 18*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 4*(B*a^6 +
6*A*a^5*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx$$
$$=x^2\left(\frac{A\,b^6}{2}+3\,B\,a\,b^5\right)$$
$$-\frac{x\left(\frac{B\,a^6}{3}+2\,A\,b\,a^5\right)+\frac{A\,a^6}{4}+x^2\left(3\,B\,a^5\,b+\frac{15\,A\,a^4\,b^2}{2}\right)+x^3\left(15\,B\,a^4\,b^2+20\,A\,a^3\,b^3\right)}{x^4}$$
$$+\ln\left(x\right)\left(20\,B\,a^3\,b^3+15\,A\,a^2\,b^4\right)+\frac{B\,b^6\,x^3}{3}+3\,a\,b^4\,x\,(2\,A\,b+5\,B\,a)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^5,x)
```

output
```
x^2*((A*b^6)/2 + 3*B*a*b^5) - (x*((B*a^6)/3 + 2*A*a^5*b) + (A*a^6)/4 + x^2
*((15*A*a^4*b^2)/2 + 3*B*a^5*b) + x^3*(20*A*a^3*b^3 + 15*B*a^4*b^2))/x^4 +
 log(x)*(15*A*a^2*b^4 + 20*B*a^3*b^3) + (B*b^6*x^3)/3 + 3*a*b^4*x*(2*A*b +
 5*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^5}\,dx = \frac{420\log(x)\,a^3b^4x^4-3a^7-28a^6bx-126a^5b^2x^2-420a^4b^3x^3+252a^2b^5x^5+42a\,b^6x^6+4b^7x^7}{12x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^5,x)
```

output
```
(420*log(x)*a**3*b**4*x**4 - 3*a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 4
20*a**4*b**3*x**3 + 252*a**2*b**5*x**5 + 42*a*b**6*x**6 + 4*b**7*x**7)/(12
*x**4)
```

3.179 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx = -\frac{a^6A}{5x^5}-\frac{a^5(6Ab+aB)}{4x^4}-\frac{a^4b(5Ab+2aB)}{x^3} -\frac{5a^3b^2(4Ab+3aB)}{2x^2}-\frac{5a^2b^3(3Ab+4aB)}{x} +b^5(Ab+6aB)x+\frac{1}{2}b^6Bx^2 +3ab^4(2Ab+5aB)\log(x)$$

output

```
-1/5*a^6*A/x^5-1/4*a^5*(6*A*b+B*a)/x^4-a^4*b*(5*A*b+2*B*a)/x^3-5/2*a^3*b^2
*(4*A*b+3*B*a)/x^2-5*a^2*b^3*(3*A*b+4*B*a)/x+b^5*(A*b+6*B*a)*x+1/2*b^6*B*x
^2+3*a*b^4*(2*A*b+5*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx = -\frac{15a^2Ab^4}{x}+6ab^5Bx+\frac{1}{2}b^6x(2A+Bx) - \frac{10a^3b^3(A+2Bx)}{x^2}-\frac{5a^4b^2(2A+3Bx)}{2x^3} - \frac{a^5b(3A+4Bx)}{2x^4}-\frac{a^6(4A+5Bx)}{20x^5} + 3ab^4(2Ab+5aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^6,x]
```

output
```
(-15*a^2*A*b^4)/x + 6*a*b^5*B*x + (b^6*x*(2*A + B*x))/2 - (10*a^3*b^3*(A +
 2*B*x))/x^2 - (5*a^4*b^2*(2*A + 3*B*x))/(2*x^3) - (a^5*b*(3*A + 4*B*x))/(
2*x^4) - (a^6*(4*A + 5*B*x))/(20*x^5) + 3*a*b^4*(2*A*b + 5*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^6}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^6}+\frac{a^5(aB+6Ab)}{x^5}+\frac{3a^4b(2aB+5Ab)}{x^4}+\frac{5a^3b^2(3aB+4Ab)}{x^3}+\frac{5a^2b^3(4aB+3Ab)}{x^2}+b^5(6aB+Ab)+\frac{3}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{5x^5}-\frac{a^5(aB+6Ab)}{4x^4}-\frac{a^4b(2aB+5Ab)}{x^3}-\frac{5a^3b^2(3aB+4Ab)}{2x^2}-\frac{5a^2b^3(4aB+3Ab)}{x}+ b^5x(6aB+Ab)+3ab^4\log(x)(5aB+2Ab)+\frac{1}{2}b^6Bx^2$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^6,x]
```

output
```
-1/5*(a^6*A)/x^5 - (a^5*(6*A*b + a*B))/(4*x^4) - (a^4*b*(5*A*b + 2*a*B))/x
^3 - (5*a^3*b^2*(4*A*b + 3*a*B))/(2*x^2) - (5*a^2*b^3*(3*A*b + 4*a*B))/x +
 b^5*(A*b + 6*a*B)*x + (b^6*B*x^2)/2 + 3*a*b^4*(2*A*b + 5*a*B)*Log[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^6 B x^2}{2} + A b^6 x + 6 B a b^5 x - \frac{a^6 A}{5x^5} - \frac{a^4 b(5Ab+2Ba)}{x^3} - \frac{5a^3 b^2(4Ab+3Ba)}{2x^2} - \frac{a^5(6Ab+Ba)}{4x^4} + 3a b^4(2Ab +$
risch	$\frac{b^6 B x^2}{2} + A b^6 x + 6 B a b^5 x + \frac{(-15 A a^2 b^4 - 20 B a^3 b^3) x^4 + (-10 A a^3 b^3 - \frac{15}{2} B a^4 b^2) x^3 + (-5 A a^4 b^2 - 2 B a^5 b) x^2 + (-}{x^5}$
norman	$\frac{(-10 A a^3 b^3 - \frac{15}{2} B a^4 b^2) x^3 + (-\frac{3}{2} A a^5 b - \frac{1}{4} B a^6) x + (A b^6 + 6 B a b^5) x^6 + (-15 A a^2 b^4 - 20 B a^3 b^3) x^4 + (-5 A a^4 b^2 - 2 B a^5 b) x^2 -}{x^5}$
parallelrisch	$\frac{10 b^6 B x^7 + 120 A \ln(x) x^5 a b^5 + 20 A b^6 x^6 + 300 B \ln(x) x^5 a^2 b^4 + 120 B a b^5 x^6 - 300 A a^2 b^4 x^4 - 400 B a^3 b^3 x^4 - 200 A a^3 b^3 x^3 - 150 B}{20 x^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^6*B*x^2+A*b^6*x+6*B*a*b^5*x-1/5*a^6*A/x^5-a^4*b*(5*A*b+2*B*a)/x^3-5/
2*a^3*b^2*(4*A*b+3*B*a)/x^2-1/4*a^5*(6*A*b+B*a)/x^4+3*a*b^4*(2*A*b+5*B*a)*
ln(x)-5*a^2*b^3*(3*A*b+4*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx$$
$$= \frac{10\,Bb^6x^7 - 4\,Aa^6 + 20\,(6\,Bab^5 + Ab^6)x^6 + 60\,(5\,Ba^2b^4 + 2\,Aab^5)x^5\log\left(x\right) - 100\,(4\,Ba^3b^3 + 3\,Aa^2b^4)x^4}{20\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^6,x, algorithm="fricas")
```

output
```
1/20*(10*B*b^6*x^7 - 4*A*a^6 + 20*(6*B*a*b^5 + A*b^6)*x^6 + 60*(5*B*a^2*b^
4 + 2*A*a*b^5)*x^5*log(x) - 100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 - 50*(3*B*
a^4*b^2 + 4*A*a^3*b^3)*x^3 - 20*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 5*(B*a^6 +
 6*A*a^5*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx$$
$$= \frac{Bb^6x^2}{2} + 3ab^4\cdot(2Ab+5Ba)\log(x) + x\left(Ab^6+6Bab^5\right)$$
$$+ \frac{-4Aa^6+x^4(-300Aa^2b^4-400Ba^3b^3)+x^3(-200Aa^3b^3-150Ba^4b^2)+x^2(-100Aa^4b^2-40Ba^5b)+x}{20x^5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**6,x)
```

output
```
B*b**6*x**2/2 + 3*a*b**4*(2*A*b + 5*B*a)*log(x) + x*(A*b**6 + 6*B*a*b**5)
+ (-4*A*a**6 + x**4*(-300*A*a**2*b**4 - 400*B*a**3*b**3) + x**3*(-200*A*a*
*3*b**3 - 150*B*a**4*b**2) + x**2*(-100*A*a**4*b**2 - 40*B*a**5*b) + x*(-3
0*A*a**5*b - 5*B*a**6))/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx$$
$$= \frac{1}{2}\,Bb^6x^2 + \left(6\,Bab^5+Ab^6\right)x + 3\left(5\,Ba^2b^4+2\,Aab^5\right)\log(x)$$
$$- \frac{4\,Aa^6+100\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+50\,(3\,Ba^4b^2+4\,Aa^3b^3)x^3+20\,(2\,Ba^5b+5\,Aa^4b^2)x^2+5\,(Ba^6+}{20\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^6,x, algorithm="maxima")
```

output
```
1/2*B*b^6*x^2 + (6*B*a*b^5 + A*b^6)*x + 3*(5*B*a^2*b^4 + 2*A*a*b^5)*log(x)
 - 1/20*(4*A*a^6 + 100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 50*(3*B*a^4*b^2 +
 4*A*a^3*b^3)*x^3 + 20*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 5*(B*a^6 + 6*A*a^5*
b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx$$
$$=\frac{1}{2}\,Bb^6x^2+6\,Bab^5x+Ab^6x+3\left(5\,Ba^2b^4+2\,Aab^5\right)\log\left(|x|\right)$$
$$-\frac{4\,Aa^6+100\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+50\,(3\,Ba^4b^2+4\,Aa^3b^3)x^3+20\,(2\,Ba^5b+5\,Aa^4b^2)x^2+5\,(Ba^6+}{20\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^6,x, algorithm="giac")
```

output
```
1/2*B*b^6*x^2 + 6*B*a*b^5*x + A*b^6*x + 3*(5*B*a^2*b^4 + 2*A*a*b^5)*log(ab
s(x)) - 1/20*(4*A*a^6 + 100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 50*(3*B*a^4*
b^2 + 4*A*a^3*b^3)*x^3 + 20*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 5*(B*a^6 + 6*A
*a^5*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx=x\left(A\,b^6+6\,B\,a\,b^5\right)$$
$$-\frac{x\left(\frac{B\,a^6}{4}+\frac{3\,A\,b\,a^5}{2}\right)+\frac{A\,a^6}{5}+x^2\left(2\,B\,a^5\,b+5\,A\,a^4\,b^2\right)+x^3\left(\frac{15\,B\,a^4\,b^2}{2}+10\,A\,a^3\,b^3\right)+x^4\left(20\,B\,a^3\,b^3+15\right.}{x^5}$$
$$+\ln\left(x\right)\,\left(15\,B\,a^2\,b^4+6\,A\,a\,b^5\right)+\frac{B\,b^6\,x^2}{2}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^6,x)
```

output
```
x*(A*b^6 + 6*B*a*b^5) - (x*((B*a^6)/4 + (3*A*a^5*b)/2) + (A*a^6)/5 + x^2*(
5*A*a^4*b^2 + 2*B*a^5*b) + x^3*(10*A*a^3*b^3 + (15*B*a^4*b^2)/2) + x^4*(15
*A*a^2*b^4 + 20*B*a^3*b^3))/x^5 + log(x)*(15*B*a^2*b^4 + 6*A*a*b^5) + (B*b
^6*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^6}\,dx = \frac{420\log(x)\,a^2b^5x^5-4a^7-35a^6bx-140a^5b^2x^2-350a^4b^3x^3-700a^3b^4x^4+140a\,b^6x^6+10b^7x^7}{20x^5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^6,x)
```

output
```
(420*log(x)*a**2*b**5*x**5 - 4*a**7 - 35*a**6*b*x - 140*a**5*b**2*x**2 - 3
50*a**4*b**3*x**3 - 700*a**3*b**4*x**4 + 140*a*b**6*x**6 + 10*b**7*x**7)/(
20*x**5)
```

3.180 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^3}{x^7}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = -\frac{a^6A}{6x^6}-\frac{a^5(6Ab+aB)}{5x^5}-\frac{3a^4b(5Ab+2aB)}{4x^4}-\frac{5a^3b^2(4Ab+3aB)}{3x^3}-\frac{5a^2b^3(3Ab+4aB)}{2x^2}-\frac{3ab^4(2Ab+5aB)}{x}+b^6Bx+b^5(Ab+6aB)\log(x)$$

output
```
-1/6*a^6*A/x^6-1/5*a^5*(6*A*b+B*a)/x^5-3/4*a^4*b*(5*A*b+2*B*a)/x^4-5/3*a^3
*b^2*(4*A*b+3*B*a)/x^3-5/2*a^2*b^3*(3*A*b+4*B*a)/x^2-3*a*b^4*(2*A*b+5*B*a)
/x+b^6*B*x+b^5*(A*b+6*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = -\frac{360aAb^5x^5-60b^6Bx^7+450a^2b^4x^4(A+2Bx)+200a^3b^3x^3(2A+3Bx)+75a^4b^2x^2(3A+4Bx)+18a}{60x^6}+b^5(Ab+6aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^7,x]
```

output
```
-1/60*(360*a*A*b^5*x^5 - 60*b^6*B*x^7 + 450*a^2*b^4*x^4*(A + 2*B*x) + 200*
a^3*b^3*x^3*(2*A + 3*B*x) + 75*a^4*b^2*x^2*(3*A + 4*B*x) + 18*a^5*b*x*(4*A
 + 5*B*x) + 2*a^6*(5*A + 6*B*x))/x^6 + b^5*(A*b + 6*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^7} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^7}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^7}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^7}+\frac{a^5(aB+6Ab)}{x^6}+\frac{3a^4b(2aB+5Ab)}{x^5}+\frac{5a^3b^2(3aB+4Ab)}{x^4}+\frac{5a^2b^3(4aB+3Ab)}{x^3}+\frac{b^5(6aB+Ab)}{x}+\frac{3}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{6x^6}-\frac{a^5(aB+6Ab)}{5x^5}-\frac{3a^4b(2aB+5Ab)}{4x^4}-\frac{5a^3b^2(3aB+4Ab)}{3x^3}-\frac{5a^2b^3(4aB+3Ab)}{2x^2}+ b^5\log(x)(6aB+Ab)-\frac{3ab^4(5aB+2Ab)}{x}+b^6Bx$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^7,x]
```

output
```
-1/6*(a^6*A)/x^6 - (a^5*(6*A*b + a*B))/(5*x^5) - (3*a^4*b*(5*A*b + 2*a*B))
/(4*x^4) - (5*a^3*b^2*(4*A*b + 3*a*B))/(3*x^3) - (5*a^2*b^3*(3*A*b + 4*a*B
))/(2*x^2) - (3*a*b^4*(2*A*b + 5*a*B))/x + b^6*B*x + b^5*(A*b + 6*a*B)*Log
[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

method	result
default	$-\frac{a^6 A}{6x^6}-\frac{a^5(6Ab+Ba)}{5x^5}-\frac{3a^4b(5Ab+2Ba)}{4x^4}-\frac{5a^3b^2(4Ab+3Ba)}{3x^3}-\frac{5a^2b^3(3Ab+4Ba)}{2x^2}-\frac{3a\,b^4(2Ab+5Ba)}{x}+b^6Bx$
risch	$b^6Bx+\frac{(-6Aa\,b^5-15B\,a^2b^4)x^5+(-\frac{15}{2}A\,a^2b^4-10B\,a^3b^3)x^4+(-\frac{20}{3}A\,a^3b^3-5B\,a^4b^2)x^3+(-\frac{15}{4}A\,a^4b^2-\frac{3}{2}B\,a^5b)x^2+(-}{x^6}$
norman	$\frac{(-\frac{15}{2}A\,a^2b^4-10B\,a^3b^3)x^4+(-\frac{20}{3}A\,a^3b^3-5B\,a^4b^2)x^3+(-\frac{15}{4}A\,a^4b^2-\frac{3}{2}B\,a^5b)x^2+(-\frac{6}{5}A\,a^5b-\frac{1}{5}B\,a^6)x+(-6Aa\,b^5-15B\,a}{x^6}$
parallelrisch	$\frac{60A\ln(x)x^6b^6+360B\ln(x)x^6a\,b^5+60b^6B\,x^7-360Aa\,b^5x^5-900B\,a^2b^4x^5-450A\,a^2b^4x^4-600B\,a^3b^3x^4-400A\,a^3b^3x^3-300B}{60x^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^6*A/x^6-1/5*a^5*(6*A*b+B*a)/x^5-3/4*a^4*b*(5*A*b+2*B*a)/x^4-5/3*a^3
*b^2*(4*A*b+3*B*a)/x^3-5/2*a^2*b^3*(3*A*b+4*B*a)/x^2-3*a*b^4*(2*A*b+5*B*a)
/x+b^6*B*x+b^5*(A*b+6*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx$$
$$=\frac{60\,Bb^6x^7+60\,(6\,Bab^5+Ab^6)x^6\log{(x)}-10\,Aa^6-180\,(5\,Ba^2b^4+2\,Aab^5)x^5-150\,(4\,Ba^3b^3+3\,Aa^2b^4)}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^6*x^7 + 60*(6*B*a*b^5 + A*b^6)*x^6*log(x) - 10*A*a^6 - 180*(5
*B*a^2*b^4 + 2*A*a*b^5)*x^5 - 150*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 - 100*(3
*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 - 45*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 12*(B*a
^6 + 6*A*a^5*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = Bb^6x + b^5(Ab+6Ba)\log{(x)} + \frac{-10Aa^6 + x^5(-360Aab^5 - 900Ba^2b^4) + x^4(-450Aa^2b^4 - 600Ba^3b^3) + x^3(-400Aa^3b^3 - 300Ba^4b^2)}{60x^6}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**7,x)
```

output
```
B*b**6*x + b**5*(A*b + 6*B*a)*log(x) + (-10*A*a**6 + x**5*(-360*A*a*b**5 -
 900*B*a**2*b**4) + x**4*(-450*A*a**2*b**4 - 600*B*a**3*b**3) + x**3*(-400
*A*a**3*b**3 - 300*B*a**4*b**2) + x**2*(-225*A*a**4*b**2 - 90*B*a**5*b) +
x*(-72*A*a**5*b - 12*B*a**6))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = Bb^6x + \left(6\,Bab^5 + Ab^6\right)\log{(x)} - \frac{10\,Aa^6 + 180\,(5\,Ba^2b^4 + 2\,Aab^5)x^5 + 150\,(4\,Ba^3b^3 + 3\,Aa^2b^4)x^4 + 100\,(3\,Ba^4b^2 + 4\,Aa^3b^3)x^3 + 45\,(2}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^7,x, algorithm="maxima")
```

output
```
B*b^6*x + (6*B*a*b^5 + A*b^6)*log(x) - 1/60*(10*A*a^6 + 180*(5*B*a^2*b^4 +
 2*A*a*b^5)*x^5 + 150*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 100*(3*B*a^4*b^2 +
 4*A*a^3*b^3)*x^3 + 45*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 12*(B*a^6 + 6*A*a^5
*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = Bb^6x + \left(6\,Bab^5 + Ab^6\right)\log\left(|x|\right)$$
$$-\frac{10\,Aa^6 + 180\,(5\,Ba^2b^4 + 2\,Aab^5)x^5 + 150\,(4\,Ba^3b^3 + 3\,Aa^2b^4)x^4 + 100\,(3\,Ba^4b^2 + 4\,Aa^3b^3)x^3 + 45\,(2}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^7,x, algorithm="giac")
```

output
```
B*b^6*x + (6*B*a*b^5 + A*b^6)*log(abs(x)) - 1/60*(10*A*a^6 + 180*(5*B*a^2*
b^4 + 2*A*a*b^5)*x^5 + 150*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 100*(3*B*a^4*
b^2 + 4*A*a^3*b^3)*x^3 + 45*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 12*(B*a^6 + 6*
A*a^5*b)*x)/x^6
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = \ln(x)\ \left(A\,b^6 + 6\,B\,a\,b^5\right)$$
$$-\frac{x\left(\frac{B\,a^6}{5} + \frac{6\,A\,b\,a^5}{5}\right) + \frac{A\,a^6}{6} + x^2\left(\frac{3\,B\,a^5\,b}{2} + \frac{15\,A\,a^4\,b^2}{4}\right) + x^5\left(15\,B\,a^2\,b^4 + 6\,A\,a\,b^5\right) + x^3\left(5\,B\,a^4\,b^2 + \frac{20\,A\,a^3\,b}{3}\right.}{x^6}$$
$$+ B\,b^6\,x$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^7,x)
```

output
```
log(x)*(A*b^6 + 6*B*a*b^5) - (x*((B*a^6)/5 + (6*A*a^5*b)/5) + (A*a^6)/6 +
x^2*((15*A*a^4*b^2)/4 + (3*B*a^5*b)/2) + x^5*(15*B*a^2*b^4 + 6*A*a*b^5) +
x^3*((20*A*a^3*b^3)/3 + 5*B*a^4*b^2) + x^4*((15*A*a^2*b^4)/2 + 10*B*a^3*b^
3))/x^6 + B*b^6*x
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^7}\,dx = \frac{420\log(x)\,a\,b^6x^6-10a^7-84a^6bx-315a^5b^2x^2-700a^4b^3x^3-1050a^3b^4x^4-1260a^2b^5x^5+60b^7x^7}{60x^6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^7,x)
```

output
```
(420*log(x)*a*b**6*x**6 - 10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 700
*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5 + 60*b**7*x**7
)/(60*x**6)
```

3.181 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx=-\frac{a^6B}{6x^6}-\frac{6a^5bB}{5x^5}-\frac{15a^4b^2B}{4x^4}-\frac{20a^3b^3B}{3x^3}-\frac{15a^2b^4B}{2x^2}-\frac{6ab^5B}{x}-\frac{A(a+bx)^7}{7ax^7}+b^6B\log(x)$$

output
```
-1/6*a^6*B/x^6-6/5*a^5*b*B/x^5-15/4*a^4*b^2*B/x^4-20/3*a^3*b^3*B/x^3-15/2*
a^2*b^4*B/x^2-6*a*b^5*B/x-1/7*A*(b*x+a)^7/a/x^7+b^6*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx=-\frac{Ab^6}{x}-\frac{3ab^5(A+2Bx)}{x^2}-\frac{5a^2b^4(2A+3Bx)}{2x^3}-\frac{5a^3b^3(3A+4Bx)}{3x^4}-\frac{3a^4b^2(4A+5Bx)}{4x^5}-\frac{a^5b(5A+6Bx)}{5x^6}-\frac{a^6(6A+7Bx)}{42x^7}+b^6B\log(x)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^8,x]
```

output
```
-((A*b^6)/x) - (3*a*b^5*(A + 2*B*x))/x^2 - (5*a^2*b^4*(2*A + 3*B*x))/(2*x^
3) - (5*a^3*b^3*(3*A + 4*B*x))/(3*x^4) - (3*a^4*b^2*(4*A + 5*B*x))/(4*x^5)
 - (a^5*b*(5*A + 6*B*x))/(5*x^6) - (a^6*(6*A + 7*B*x))/(42*x^7) + b^6*B*Lo
g[x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1184, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^8} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^8}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^6}{x^7}dx - \frac{A(a+bx)^7}{7ax^7}$$

$$\Big\downarrow \text{49}$$

$$B\int \left(\frac{a^6}{x^7}+\frac{6ba^5}{x^6}+\frac{15b^2a^4}{x^5}+\frac{20b^3a^3}{x^4}+\frac{15b^4a^2}{x^3}+\frac{6b^5a}{x^2}+\frac{b^6}{x}\right)dx - \frac{A(a+bx)^7}{7ax^7}$$

$$\Big\downarrow \text{2009}$$

$$B\left(-\frac{a^6}{6x^6}-\frac{6a^5b}{5x^5}-\frac{15a^4b^2}{4x^4}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{2x^2}-\frac{6ab^5}{x}+b^6\log(x)\right)-\frac{A(a+bx)^7}{7ax^7}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^8,x]
```

output
```
-1/7*(A*(a + b*x)^7)/(a*x^7) + B*(-1/6*a^6/x^6 - (6*a^5*b)/(5*x^5) - (15*a
^4*b^2)/(4*x^4) - (20*a^3*b^3)/(3*x^3) - (15*a^2*b^4)/(2*x^2) - (6*a*b^5)/
x + b^6*Log[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

method	result
default	$-\frac{3a^4b(5Ab+2Ba)}{5x^5}-\frac{a^5(6Ab+Ba)}{6x^6}-\frac{5a^2b^3(3Ab+4Ba)}{3x^3}-\frac{3a\,b^4(2Ab+5Ba)}{2x^2}-\frac{5a^3b^2(4Ab+3Ba)}{4x^4}+b^6B\ln(x)-$
norman	$\frac{(-3Aa\,b^5-\frac{15}{2}B\,a^2b^4)x^5+(-5A\,a^2b^4-\frac{20}{3}B\,a^3b^3)x^4+(-5A\,a^3b^3-\frac{15}{4}B\,a^4b^2)x^3+(-3A\,a^4b^2-\frac{6}{5}B\,a^5b)x^2+(-A\,a^5b-\frac{1}{6}B\,a^6}{x^7}$
risch	$\frac{(-3Aa\,b^5-\frac{15}{2}B\,a^2b^4)x^5+(-5A\,a^2b^4-\frac{20}{3}B\,a^3b^3)x^4+(-5A\,a^3b^3-\frac{15}{4}B\,a^4b^2)x^3+(-3A\,a^4b^2-\frac{6}{5}B\,a^5b)x^2+(-A\,a^5b-\frac{1}{6}B\,a^6}{x^7}$
parallelrisch	$-\frac{-420B\,b^6\ln(x)x^7+420A\,b^6x^6+2520Ba\,b^5x^6+1260Aa\,b^5x^5+3150B\,a^2b^4x^5+2100A\,a^2b^4x^4+2800B\,a^3b^3x^4+2100A\,a^3b^3x}{420x^7}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-3/5*a^4*b*(5*A*b+2*B*a)/x^5-1/6*a^5*(6*A*b+B*a)/x^6-5/3*a^2*b^3*(3*A*b+4*
B*a)/x^3-3/2*a*b^4*(2*A*b+5*B*a)/x^2-5/4*a^3*b^2*(4*A*b+3*B*a)/x^4+b^6*B*l
n(x)-1/7*A*a^6/x^7-b^5*(A*b+6*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx = \frac{420\,Bb^6x^7\log(x)-60\,Aa^6-420\,(6\,Bab^5+Ab^6)x^6-630\,(5\,Ba^2b^4+2\,Aab^5)x^5-700\,(4\,Ba^3b^3+3\,Aa^2b}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^8,x, algorithm="fricas")
```

output
```
1/420*(420*B*b^6*x^7*log(x) - 60*A*a^6 - 420*(6*B*a*b^5 + A*b^6)*x^6 - 630
*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 - 700*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 - 525
*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 - 252*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 70*
(B*a^6 + 6*A*a^5*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx = Bb^6\log\left(x\right) + \frac{-60Aa^6 + x^6(-420Ab^6 - 2520Bab^5) + x^5(-1260Aab^5 - 3150Ba^2b^4) + x^4(-2100Aa^2b^4 - 2800Ba^3b^3}{420x^7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**8,x)
```

output
```
B*b**6*log(x) + (-60*A*a**6 + x**6*(-420*A*b**6 - 2520*B*a*b**5) + x**5*(-
1260*A*a*b**5 - 3150*B*a**2*b**4) + x**4*(-2100*A*a**2*b**4 - 2800*B*a**3*
b**3) + x**3*(-2100*A*a**3*b**3 - 1575*B*a**4*b**2) + x**2*(-1260*A*a**4*b
**2 - 504*B*a**5*b) + x*(-420*A*a**5*b - 70*B*a**6))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx = Bb^6\log\left(x\right) - \frac{60\,Aa^6 + 420\,(6\,Bab^5 + Ab^6)x^6 + 630\,(5\,Ba^2b^4 + 2\,Aab^5)x^5 + 700\,(4\,Ba^3b^3 + 3\,Aa^2b^4)x^4 + 525\,(3\,Ba^4}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^8,x, algorithm="maxima")
```

output
```
B*b^6*log(x) - 1/420*(60*A*a^6 + 420*(6*B*a*b^5 + A*b^6)*x^6 + 630*(5*B*a^
2*b^4 + 2*A*a*b^5)*x^5 + 700*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 525*(3*B*a^
4*b^2 + 4*A*a^3*b^3)*x^3 + 252*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 70*(B*a^6 +
 6*A*a^5*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx = Bb^6\log\left(|x|\right) - \frac{60\,Aa^6+420\,(6\,Bab^5+Ab^6)x^6+630\,(5\,Ba^2b^4+2\,Aab^5)x^5+700\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+525\,(3\,Ba^4}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^8,x, algorithm="giac")
```

output
```
B*b^6*log(abs(x)) - 1/420*(60*A*a^6 + 420*(6*B*a*b^5 + A*b^6)*x^6 + 630*(5
*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 700*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 525*(3
*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 252*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 70*(B*
a^6 + 6*A*a^5*b)*x)/x^7
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx = B\,b^6\,\ln\left(x\right) - \frac{x\left(\frac{B\,a^6}{6}+A\,b\,a^5\right)+\frac{A\,a^6}{7}+x^2\left(\frac{6\,B\,a^5\,b}{5}+3\,A\,a^4\,b^2\right)+x^5\left(\frac{15\,B\,a^2\,b^4}{2}+3\,A\,a\,b^5\right)+x^6\left(A\,b^6+6\,B\,a\,b^5\right)+}{x^7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^8,x)
```

output
```
B*b^6*log(x) - (x*((B*a^6)/6 + A*a^5*b) + (A*a^6)/7 + x^2*(3*A*a^4*b^2 + (
6*B*a^5*b)/5) + x^5*((15*B*a^2*b^4)/2 + 3*A*a*b^5) + x^6*(A*b^6 + 6*B*a*b^
5) + x^3*(5*A*a^3*b^3 + (15*B*a^4*b^2)/4) + x^4*(5*A*a^2*b^4 + (20*B*a^3*b
^3)/3))/x^7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^8}\,dx$$
$$=\frac{420\log(x)\,b^7x^7-60a^7-490a^6bx-1764a^5b^2x^2-3675a^4b^3x^3-4900a^3b^4x^4-4410a^2b^5x^5-2940a\,b^6x^6}{420x^7}$$

input

```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^8,x)
```

output

```
(420*log(x)*b**7*x**7 - 60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 367
5*a**4*b**3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6
*x**6)/(420*x**7)
```

3.182 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{A(a+bx)^7}{8ax^8}+\frac{(Ab-8aB)(a+bx)^7}{56a^2x^7}$$

output
```
-1/8*A*(b*x+a)^7/a/x^8+1/56*(A*b-8*B*a)*(b*x+a)^7/a^2/x^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(44) = 88.

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{28b^6x^6(A+2Bx)+56ab^5x^5(2A+3Bx)+70a^2b^4x^4(3A+4Bx)+56a^3b^3x^3(4A+5Bx)+28a^4b^2x^2(5}{56x^8}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^9,x]
```

output
```
-1/56*(28*b^6*x^6*(A + 2*B*x) + 56*a*b^5*x^5*(2*A + 3*B*x) + 70*a^2*b^4*x^
4*(3*A + 4*B*x) + 56*a^3*b^3*x^3*(4*A + 5*B*x) + 28*a^4*b^2*x^2*(5*A + 6*B
*x) + 8*a^5*b*x*(6*A + 7*B*x) + a^6*(7*A + 8*B*x))/x^8
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^9} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^9}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^9}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-8aB)\int \frac{(a+bx)^6}{x^8}dx}{8a}-\frac{A(a+bx)^7}{8ax^8}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^7(Ab-8aB)}{56a^2x^7}-\frac{A(a+bx)^7}{8ax^8}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^9,x]
```

output
```
-1/8*(A*(a + b*x)^7)/(a*x^8) + ((A*b - 8*a*B)*(a + b*x)^7)/(56*a^2*x^7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(40) = 80$.

Time = 0.90 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.91

method	result
default	$-\frac{a^3b^2(4Ab+3Ba)}{x^5}-\frac{a^4b(5Ab+2Ba)}{2x^6}-\frac{a\,b^4(2Ab+5Ba)}{x^3}-\frac{b^5(Ab+6Ba)}{2x^2}-\frac{A\,a^6}{8x^8}-\frac{5a^2b^3(3Ab+4Ba)}{4x^4}-\frac{a^5(6Ab+}{7x^7}$
norman	$\frac{-b^6B\,x^7+(-\frac{1}{2}A\,b^6-3Ba\,b^5)x^6+(-2Aa\,b^5-5B\,a^2b^4)x^5+(-\frac{15}{4}A\,a^2b^4-5B\,a^3b^3)x^4+(-4A\,a^3b^3-3B\,a^4b^2)x^3+(-\frac{5}{2}A\,a^4b^2}{x^8}$
risch	$\frac{-b^6B\,x^7+(-\frac{1}{2}A\,b^6-3Ba\,b^5)x^6+(-2Aa\,b^5-5B\,a^2b^4)x^5+(-\frac{15}{4}A\,a^2b^4-5B\,a^3b^3)x^4+(-4A\,a^3b^3-3B\,a^4b^2)x^3+(-\frac{5}{2}A\,a^4b^2}{x^8}$
gosper	$-\frac{56b^6B\,x^7+28A\,b^6x^6+168Ba\,b^5x^6+112Aa\,b^5x^5+280B\,a^2b^4x^5+210A\,a^2b^4x^4+280B\,a^3b^3x^4+224A\,a^3b^3x^3+168B\,a^4b^2x^3}{56x^8}$
parallelrisch	$-\frac{56b^6B\,x^7+28A\,b^6x^6+168Ba\,b^5x^6+112Aa\,b^5x^5+280B\,a^2b^4x^5+210A\,a^2b^4x^4+280B\,a^3b^3x^4+224A\,a^3b^3x^3+168B\,a^4b^2x^3}{56x^8}$
orering	$-\frac{(56b^6B\,x^7+28A\,b^6x^6+168Ba\,b^5x^6+112Aa\,b^5x^5+280B\,a^2b^4x^5+210A\,a^2b^4x^4+280B\,a^3b^3x^4+224A\,a^3b^3x^3+168B\,a^4b^2x^3}{56x^8(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-a^3*b^2*(4*A*b+3*B*a)/x^5-1/2*a^4*b*(5*A*b+2*B*a)/x^6-a*b^4*(2*A*b+5*B*a)
/x^3-1/2*b^5*(A*b+6*B*a)/x^2-1/8*A*a^6/x^8-5/4*a^2*b^3*(3*A*b+4*B*a)/x^4-1
/7*a^5*(6*A*b+B*a)/x^7-B*b^6/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(41) = 82.

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{56\,Bb^6x^7+7\,Aa^6+28\,(6\,Bab^5+Ab^6)x^6+56\,(5\,Ba^2b^4+2\,Aab^5)x^5+70\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+56}{56\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^9,x, algorithm="fricas")
```

output
```
-1/56*(56*B*b^6*x^7 + 7*A*a^6 + 28*(6*B*a*b^5 + A*b^6)*x^6 + 56*(5*B*a^2*b
^4 + 2*A*a*b^5)*x^5 + 70*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 56*(3*B*a^4*b^2
 + 4*A*a^3*b^3)*x^3 + 28*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 8*(B*a^6 + 6*A*a^
5*b)*x)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(37) = 74$.

Time = 2.97 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.59

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = \frac{-7Aa^6-56Bb^6x^7+x^6(-28Ab^6-168Bab^5)+x^5(-112Aab^5-280Ba^2b^4)+x^4(-210Aa^2b^4-280Ba^3b}{56x^8}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**9,x)
```

output
```
(-7*A*a**6 - 56*B*b**6*x**7 + x**6*(-28*A*b**6 - 168*B*a*b**5) + x**5*(-11
2*A*a*b**5 - 280*B*a**2*b**4) + x**4*(-210*A*a**2*b**4 - 280*B*a**3*b**3)
+ x**3*(-224*A*a**3*b**3 - 168*B*a**4*b**2) + x**2*(-140*A*a**4*b**2 - 56*
B*a**5*b) + x*(-48*A*a**5*b - 8*B*a**6))/(56*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(41) = 82$.

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{56\,Bb^6x^7+7\,Aa^6+28\,(6\,Bab^5+Ab^6)x^6+56\,(5\,Ba^2b^4+2\,Aab^5)x^5+70\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4+56}{56\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^9,x, algorithm="maxima")
```

output
```
-1/56*(56*B*b^6*x^7 + 7*A*a^6 + 28*(6*B*a*b^5 + A*b^6)*x^6 + 56*(5*B*a^2*b
^4 + 2*A*a*b^5)*x^5 + 70*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 56*(3*B*a^4*b^2
 + 4*A*a^3*b^3)*x^3 + 28*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 8*(B*a^6 + 6*A*a^
5*b)*x)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(41) = 82$.

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{56\,Bb^6x^7+168\,Bab^5x^6+28\,Ab^6x^6+280\,Ba^2b^4x^5+112\,Aab^5x^5+280\,Ba^3b^3x^4+210\,Aa^2b^4x^4+168}{56\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^9,x, algorithm="giac")
```

output
```
-1/56*(56*B*b^6*x^7 + 168*B*a*b^5*x^6 + 28*A*b^6*x^6 + 280*B*a^2*b^4*x^5 +
 112*A*a*b^5*x^5 + 280*B*a^3*b^3*x^4 + 210*A*a^2*b^4*x^4 + 168*B*a^4*b^2*x
^3 + 224*A*a^3*b^3*x^3 + 56*B*a^5*b*x^2 + 140*A*a^4*b^2*x^2 + 8*B*a^6*x +
48*A*a^5*b*x + 7*A*a^6)/x^8
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.20

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^9}\,dx = -\frac{x\left(\frac{B\,a^6}{7}+\frac{6\,A\,b\,a^5}{7}\right)+\frac{A\,a^6}{8}+x^2\left(B\,a^5\,b+\frac{5\,A\,a^4\,b^2}{2}\right)+x^5\left(5\,B\,a^2\,b^4+2\,A\,a\,b^5\right)+x^6\left(\frac{A\,b^6}{2}+3\,B\,a\,b^5\right)+x}{x^8}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^9,x)
```

output
```
-(x*((B*a^6)/7 + (6*A*a^5*b)/7) + (A*a^6)/8 + x^2*((5*A*a^4*b^2)/2 + B*a^5
*b) + x^5*(5*B*a^2*b^4 + 2*A*a*b^5) + x^6*((A*b^6)/2 + 3*B*a*b^5) + x^3*(4
*A*a^3*b^3 + 3*B*a^4*b^2) + x^4*((15*A*a^2*b^4)/4 + 5*B*a^3*b^3) + B*b^6*x
^7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^3}{x^9}\,dx$$
$$= \frac{-8b^7x^7-28a\,b^6x^6-56a^2b^5x^5-70a^3b^4x^4-56a^4b^3x^3-28a^5b^2x^2-8a^6bx-a^7}{8x^8}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^9,x)
```

output
```
( - a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b*
*4*x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

3.183 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx=-\frac{A(a+bx)^7}{9ax^9}+\frac{(2Ab-9aB)(a+bx)^7}{72a^2x^8}-\frac{b(2Ab-9aB)(a+bx)^7}{504a^3x^7}$$

output
```
-1/9*A*(b*x+a)^7/a/x^9+1/72*(2*A*b-9*B*a)*(b*x+a)^7/a^2/x^8-1/504*b*(2*A*b
-9*B*a)*(b*x+a)^7/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.75

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx=-\frac{84b^6x^6(2A+3Bx)+252ab^5x^5(3A+4Bx)+378a^2b^4x^4(4A+5Bx)+336a^3b^3x^3(5A+6Bx)+180a^4}{504x^9}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^10,x]
```

output
```
-1/504*(84*b^6*x^6*(2*A + 3*B*x) + 252*a*b^5*x^5*(3*A + 4*B*x) + 378*a^2*b
^4*x^4*(4*A + 5*B*x) + 336*a^3*b^3*x^3*(5*A + 6*B*x) + 180*a^4*b^2*x^2*(6*
A + 7*B*x) + 54*a^5*b*x*(7*A + 8*B*x) + 7*a^6*(8*A + 9*B*x))/x^9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1184, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{10}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(2Ab-9aB)\int \frac{(a+bx)^6}{x^9}dx}{9a}-\frac{A(a+bx)^7}{9ax^9}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(2Ab-9aB)\left(-\frac{b\int \frac{(a+bx)^6}{x^8}dx}{8a}-\frac{(a+bx)^7}{8ax^8}\right)}{9a}-\frac{A(a+bx)^7}{9ax^9}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{b(a+bx)^7}{56a^2x^7}-\frac{(a+bx)^7}{8ax^8}\right)(2Ab-9aB)}{9a}-\frac{A(a+bx)^7}{9ax^9}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^10,x]
```

output
```
-1/9*(A*(a + b*x)^7)/(a*x^9) - ((2*A*b - 9*a*B)*(-1/8*(a + b*x)^7/(a*x^8)
+ (b*(a + b*x)^7)/(56*a^2*x^7)))/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

method	result
default	$-\frac{a^2b^3(3Ab+4Ba)}{x^5}-\frac{5a^3b^2(4Ab+3Ba)}{6x^6}-\frac{b^5(Ab+6Ba)}{3x^3}-\frac{B\,b^6}{2x^2}-\frac{a^5(6Ab+Ba)}{8x^8}-\frac{3a\,b^4(2Ab+5Ba)}{4x^4}-\frac{A\,a^6}{9x^9}-3a$
norman	$\frac{-\frac{b^6Bx^7}{2}+(-\frac{1}{3}A\,b^6-2Ba\,b^5)x^6+(-\frac{3}{2}Aa\,b^5-\frac{15}{4}B\,a^2b^4)x^5+(-3A\,a^2b^4-4B\,a^3b^3)x^4+(-\frac{10}{3}A\,a^3b^3-\frac{5}{2}B\,a^4b^2)x^3+(-\frac{15}{7}A\,a}{x^9}$
risch	$\frac{-\frac{b^6Bx^7}{2}+(-\frac{1}{3}A\,b^6-2Ba\,b^5)x^6+(-\frac{3}{2}Aa\,b^5-\frac{15}{4}B\,a^2b^4)x^5+(-3A\,a^2b^4-4B\,a^3b^3)x^4+(-\frac{10}{3}A\,a^3b^3-\frac{5}{2}B\,a^4b^2)x^3+(-\frac{15}{7}A\,a}{x^9}$
gosper	$-\frac{252b^6Bx^7+168A\,b^6x^6+1008Ba\,b^5x^6+756Aa\,b^5x^5+1890B\,a^2b^4x^5+1512A\,a^2b^4x^4+2016B\,a^3b^3x^4+1680A\,a^3b^3x^3+1260A}{504x^9}$
parallelrisch	$-\frac{252b^6Bx^7+168A\,b^6x^6+1008Ba\,b^5x^6+756Aa\,b^5x^5+1890B\,a^2b^4x^5+1512A\,a^2b^4x^4+2016B\,a^3b^3x^4+1680A\,a^3b^3x^3+1260A}{504x^9}$
orering	$-\frac{(252b^6Bx^7+168A\,b^6x^6+1008Ba\,b^5x^6+756Aa\,b^5x^5+1890B\,a^2b^4x^5+1512A\,a^2b^4x^4+2016B\,a^3b^3x^4+1680A\,a^3b^3x^3+1260}{504x^9(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-a^2*b^3*(3*A*b+4*B*a)/x^5-5/6*a^3*b^2*(4*A*b+3*B*a)/x^6-1/3*b^5*(A*b+6*B*
a)/x^3-1/2*B*b^6/x^2-1/8*a^5*(6*A*b+B*a)/x^8-3/4*a*b^4*(2*A*b+5*B*a)/x^4-1
/9*A*a^6/x^9-3/7*a^4*b*(5*A*b+2*B*a)/x^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx = -\frac{252\,Bb^6x^7+56\,Aa^6+168\,(6\,Bab^5+Ab^6)x^6+378\,(5\,Ba^2b^4+2\,Aab^5)x^5+504\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4}{504\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^10,x, algorithm="fricas")
```

output
```
-1/504*(252*B*b^6*x^7 + 56*A*a^6 + 168*(6*B*a*b^5 + A*b^6)*x^6 + 378*(5*B*
a^2*b^4 + 2*A*a*b^5)*x^5 + 504*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 420*(3*B*
a^4*b^2 + 4*A*a^3*b^3)*x^3 + 216*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 63*(B*a^6
 + 6*A*a^5*b)*x)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.

Time = 3.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx = \frac{-56Aa^6-252Bb^6x^7+x^6(-168Ab^6-1008Bab^5)+x^5(-756Aab^5-1890Ba^2b^4)+x^4(-1512Aa^2b^4-2}{504x}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**10,x)
```

output
```
(-56*A*a**6 - 252*B*b**6*x**7 + x**6*(-168*A*b**6 - 1008*B*a*b**5) + x**5*
(-756*A*a*b**5 - 1890*B*a**2*b**4) + x**4*(-1512*A*a**2*b**4 - 2016*B*a**3
*b**3) + x**3*(-1680*A*a**3*b**3 - 1260*B*a**4*b**2) + x**2*(-1080*A*a**4*
b**2 - 432*B*a**5*b) + x*(-378*A*a**5*b - 63*B*a**6))/(504*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(66) = 132$.

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx = -\frac{252\,Bb^6x^7+56\,Aa^6+168\,(6\,Bab^5+Ab^6)x^6+378\,(5\,Ba^2b^4+2\,Aab^5)x^5+504\,(4\,Ba^3b^3+3\,Aa^2b^4)x^4}{504\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^10,x, algorithm="maxima")
```

output
```
-1/504*(252*B*b^6*x^7 + 56*A*a^6 + 168*(6*B*a*b^5 + A*b^6)*x^6 + 378*(5*B*
a^2*b^4 + 2*A*a*b^5)*x^5 + 504*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 420*(3*B*
a^4*b^2 + 4*A*a^3*b^3)*x^3 + 216*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 63*(B*a^6
 + 6*A*a^5*b)*x)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(66) = 132$.

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^3}{x^{10}}\,dx = -\frac{252\,Bb^6x^7+1008\,Bab^5x^6+168\,Ab^6x^6+1890\,Ba^2b^4x^5+756\,Aab^5x^5+2016\,Ba^3b^3x^4+1512\,Aa^2b^4x^4}{504\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^10,x, algorithm="giac")
```

output
```
-1/504*(252*B*b^6*x^7 + 1008*B*a*b^5*x^6 + 168*A*b^6*x^6 + 1890*B*a^2*b^4*
x^5 + 756*A*a*b^5*x^5 + 2016*B*a^3*b^3*x^4 + 1512*A*a^2*b^4*x^4 + 1260*B*a
^4*b^2*x^3 + 1680*A*a^3*b^3*x^3 + 432*B*a^5*b*x^2 + 1080*A*a^4*b^2*x^2 + 6
3*B*a^6*x + 378*A*a^5*b*x + 56*A*a^6)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^3}{x^{10}}\,dx = -\frac{x\left(\frac{B\,a^6}{8}+\frac{3\,A\,b\,a^5}{4}\right)+\frac{A\,a^6}{9}+x^5\left(\frac{15\,B\,a^2\,b^4}{4}+\frac{3\,A\,a\,b^5}{2}\right)+x^2\left(\frac{6\,B\,a^5\,b}{7}+\frac{15\,A\,a^4\,b^2}{7}\right)+x^6\left(\frac{A\,b^6}{3}+2\,B\,a\,b^5\right)+x}{x^9}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^10,x)
```

output
```
-(x*((B*a^6)/8 + (3*A*a^5*b)/4) + (A*a^6)/9 + x^5*((15*B*a^2*b^4)/4 + (3*A
*a*b^5)/2) + x^2*((15*A*a^4*b^2)/7 + (6*B*a^5*b)/7) + x^6*((A*b^6)/3 + 2*B
*a*b^5) + x^4*(3*A*a^2*b^4 + 4*B*a^3*b^3) + x^3*((10*A*a^3*b^3)/3 + (5*B*a
^4*b^2)/2) + (B*b^6*x^7)/2)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{10}}\,dx = \frac{-36b^7x^7-168a\,b^6x^6-378a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3-216a^5b^2x^2-63a^6bx-8a^7}{72x^9}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^10,x)
```

output
```
( - 8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a
**3*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x
**9)
```

3.184 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx = -\frac{a^6A}{10x^{10}}-\frac{a^5(6Ab+aB)}{9x^9}-\frac{3a^4b(5Ab+2aB)}{8x^8} -\frac{5a^3b^2(4Ab+3aB)}{7x^7}-\frac{5a^2b^3(3Ab+4aB)}{6x^6} -\frac{3ab^4(2Ab+5aB)}{5x^5}-\frac{b^5(Ab+6aB)}{4x^4}-\frac{b^6B}{3x^3}$$

output
```
-1/10*a^6*A/x^10-1/9*a^5*(6*A*b+B*a)/x^9-3/8*a^4*b*(5*A*b+2*B*a)/x^8-5/7*a
^3*b^2*(4*A*b+3*B*a)/x^7-5/6*a^2*b^3*(3*A*b+4*B*a)/x^6-3/5*a*b^4*(2*A*b+5*
B*a)/x^5-1/4*b^5*(A*b+6*B*a)/x^4-1/3*b^6*B/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx = -\frac{210b^6x^6(3A+4Bx)+756ab^5x^5(4A+5Bx)+1260a^2b^4x^4(5A+6Bx)+1200a^3b^3x^3(6A+7Bx)+67}{2520x^{10}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^11,x]
```

output
```
-1/2520*(210*b^6*x^6*(3*A + 4*B*x) + 756*a*b^5*x^5*(4*A + 5*B*x) + 1260*a^
2*b^4*x^4*(5*A + 6*B*x) + 1200*a^3*b^3*x^3*(6*A + 7*B*x) + 675*a^4*b^2*x^2
*(7*A + 8*B*x) + 210*a^5*b*x*(8*A + 9*B*x) + 28*a^6*(9*A + 10*B*x))/x^10
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^{11}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{11}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{11}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{11}}+\frac{a^5(aB+6Ab)}{x^{10}}+\frac{3a^4b(2aB+5Ab)}{x^9}+\frac{5a^3b^2(3aB+4Ab)}{x^8}+\frac{5a^2b^3(4aB+3Ab)}{x^7}+\frac{b^5(6aB+Ab)}{x^5}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{10x^{10}}-\frac{a^5(aB+6Ab)}{9x^9}-\frac{3a^4b(2aB+5Ab)}{8x^8}-\frac{5a^3b^2(3aB+4Ab)}{7x^7}-\frac{5a^2b^3(4aB+3Ab)}{6x^6}-\frac{b^5(6aB+Ab)}{4x^4}-\frac{3ab^4(5aB+2Ab)}{5x^5}-\frac{b^6B}{3x^3}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^11,x]
```

output
```
-1/10*(a^6*A)/x^10 - (a^5*(6*A*b + a*B))/(9*x^9) - (3*a^4*b*(5*A*b + 2*a*B
))/(8*x^8) - (5*a^3*b^2*(4*A*b + 3*a*B))/(7*x^7) - (5*a^2*b^3*(3*A*b + 4*a
*B))/(6*x^6) - (3*a*b^4*(2*A*b + 5*a*B))/(5*x^5) - (b^5*(A*b + 6*a*B))/(4*
x^4) - (b^6*B)/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^6 A}{10x^{10}}-\frac{a^5(6Ab+Ba)}{9x^9}-\frac{3a^4b(5Ab+2Ba)}{8x^8}-\frac{5a^3b^2(4Ab+3Ba)}{7x^7}-\frac{5a^2b^3(3Ab+4Ba)}{6x^6}-\frac{3a\,b^4(2Ab+5Ba)}{5x^5}-\frac{b^5(A$
norman	$\frac{-\frac{b^6 B\,x^7}{3}+(-\frac{1}{4}A\,b^6-\frac{3}{2}Ba\,b^5)x^6+(-\frac{6}{5}Aa\,b^5-3B\,a^2b^4)x^5+(-\frac{5}{2}A\,a^2b^4-\frac{10}{3}B\,a^3b^3)x^4+(-\frac{20}{7}A\,a^3b^3-\frac{15}{7}B\,a^4b^2)x^3+(-\frac{15}{8}A}{x^{10}}$
risch	$\frac{-\frac{b^6 B\,x^7}{3}+(-\frac{1}{4}A\,b^6-\frac{3}{2}Ba\,b^5)x^6+(-\frac{6}{5}Aa\,b^5-3B\,a^2b^4)x^5+(-\frac{5}{2}A\,a^2b^4-\frac{10}{3}B\,a^3b^3)x^4+(-\frac{20}{7}A\,a^3b^3-\frac{15}{7}B\,a^4b^2)x^3+(-\frac{15}{8}A}{x^{10}}$
gosper	$-\frac{840b^6B\,x^7+630A\,b^6x^6+3780Ba\,b^5x^6+3024Aa\,b^5x^5+7560B\,a^2b^4x^5+6300A\,a^2b^4x^4+8400B\,a^3b^3x^4+7200A\,a^3b^3x^3+5400}{2520x^{10}}$
parallelrisch	$-\frac{840b^6B\,x^7+630A\,b^6x^6+3780Ba\,b^5x^6+3024Aa\,b^5x^5+7560B\,a^2b^4x^5+6300A\,a^2b^4x^4+8400B\,a^3b^3x^4+7200A\,a^3b^3x^3+5400}{2520x^{10}}$
orering	$-\frac{(840b^6B\,x^7+630A\,b^6x^6+3780Ba\,b^5x^6+3024Aa\,b^5x^5+7560B\,a^2b^4x^5+6300A\,a^2b^4x^4+8400B\,a^3b^3x^4+7200A\,a^3b^3x^3+540}{2520x^{10}(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^6*A/x^10-1/9*a^5*(6*A*b+B*a)/x^9-3/8*a^4*b*(5*A*b+2*B*a)/x^8-5/7*a
^3*b^2*(4*A*b+3*B*a)/x^7-5/6*a^2*b^3*(3*A*b+4*B*a)/x^6-3/5*a*b^4*(2*A*b+5*
B*a)/x^5-1/4*b^5*(A*b+6*B*a)/x^4-1/3*b^6*B/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx = -\frac{840\,Bb^6x^7+252\,Aa^6+630\,(6\,Bab^5+Ab^6)x^6+1512\,(5\,Ba^2b^4+2\,Aab^5)x^5+2100\,(4\,Ba^3b^3+3\,Aa^2b^4}{2520\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(840*B*b^6*x^7 + 252*A*a^6 + 630*(6*B*a*b^5 + A*b^6)*x^6 + 1512*(5
*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 2100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 1800*
(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 945*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 280*
(B*a^6 + 6*A*a^5*b)*x)/x^10
```

Sympy [A] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx$$
$$= \frac{-252Aa^6 - 840Bb^6x^7 + x^6(-630Ab^6 - 3780Bab^5) + x^5(-3024Aab^5 - 7560Ba^2b^4) + x^4(-6300Aa^2b^4 -}{25}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**11,x)
```

output
```
(-252*A*a**6 - 840*B*b**6*x**7 + x**6*(-630*A*b**6 - 3780*B*a*b**5) + x**5
*(-3024*A*a*b**5 - 7560*B*a**2*b**4) + x**4*(-6300*A*a**2*b**4 - 8400*B*a*
*3*b**3) + x**3*(-7200*A*a**3*b**3 - 5400*B*a**4*b**2) + x**2*(-4725*A*a**
4*b**2 - 1890*B*a**5*b) + x*(-1680*A*a**5*b - 280*B*a**6))/(2520*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx =$$
$$-\frac{840\,Bb^6x^7 + 252\,Aa^6 + 630\,(6\,Bab^5 + Ab^6)x^6 + 1512\,(5\,Ba^2b^4 + 2\,Aab^5)x^5 + 2100\,(4\,Ba^3b^3 + 3\,Aa^2b^4}{2520\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^11,x, algorithm="maxima")
```

output
```
-1/2520*(840*B*b^6*x^7 + 252*A*a^6 + 630*(6*B*a*b^5 + A*b^6)*x^6 + 1512*(5
*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 2100*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 1800*
(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 945*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 280*
(B*a^6 + 6*A*a^5*b)*x)/x^10
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx =$$
$$-\frac{840\,Bb^6x^7+3780\,Bab^5x^6+630\,Ab^6x^6+7560\,Ba^2b^4x^5+3024\,Aab^5x^5+8400\,Ba^3b^3x^4+6300\,Aa^2b^4x}{252}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^11,x, algorithm="giac")
```

output
```
-1/2520*(840*B*b^6*x^7 + 3780*B*a*b^5*x^6 + 630*A*b^6*x^6 + 7560*B*a^2*b^4
*x^5 + 3024*A*a*b^5*x^5 + 8400*B*a^3*b^3*x^4 + 6300*A*a^2*b^4*x^4 + 5400*B
*a^4*b^2*x^3 + 7200*A*a^3*b^3*x^3 + 1890*B*a^5*b*x^2 + 4725*A*a^4*b^2*x^2
+ 280*B*a^6*x + 1680*A*a^5*b*x + 252*A*a^6)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx =$$
$$-\frac{x\left(\frac{B\,a^6}{9}+\frac{2\,A\,b\,a^5}{3}\right)+\frac{A\,a^6}{10}+x^5\left(3\,B\,a^2\,b^4+\frac{6\,A\,a\,b^5}{5}\right)+x^2\left(\frac{3\,B\,a^5\,b}{4}+\frac{15\,A\,a^4\,b^2}{8}\right)+x^6\left(\frac{A\,b^6}{4}+\frac{3\,B\,a\,b^5}{2}\right)+x^4}{x^{10}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^11,x)
```

output
```
-(x*((B*a^6)/9 + (2*A*a^5*b)/3) + (A*a^6)/10 + x^5*(3*B*a^2*b^4 + (6*A*a*b
^5)/5) + x^2*((15*A*a^4*b^2)/8 + (3*B*a^5*b)/4) + x^6*((A*b^6)/4 + (3*B*a*
b^5)/2) + x^4*((5*A*a^2*b^4)/2 + (10*B*a^3*b^3)/3) + x^3*((20*A*a^3*b^3)/7
 + (15*B*a^4*b^2)/7) + (B*b^6*x^7)/3)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11}}\,dx = \frac{-120b^7x^7-630a\,b^6x^6-1512a^2b^5x^5-2100a^3b^4x^4-1800a^4b^3x^3-945a^5b^2x^2-280a^6bx-36a^7}{360x^{10}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^11,x)
```

output
```
( - 36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 21
00*a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)
/(360*x**10)
```

3.185 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx = -\frac{a^6A}{11x^{11}}-\frac{a^5(6Ab+aB)}{10x^{10}}-\frac{a^4b(5Ab+2aB)}{3x^9} -\frac{5a^3b^2(4Ab+3aB)}{8x^8}-\frac{5a^2b^3(3Ab+4aB)}{7x^7} -\frac{ab^4(2Ab+5aB)}{2x^6}-\frac{b^5(Ab+6aB)}{5x^5}-\frac{b^6B}{4x^4}$$

output
```
-1/11*a^6*A/x^11-1/10*a^5*(6*A*b+B*a)/x^10-1/3*a^4*b*(5*A*b+2*B*a)/x^9-5/8
*a^3*b^2*(4*A*b+3*B*a)/x^8-5/7*a^2*b^3*(3*A*b+4*B*a)/x^7-1/2*a*b^4*(2*A*b+
5*B*a)/x^6-1/5*b^5*(A*b+6*B*a)/x^5-1/4*b^6*B/x^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx = -\frac{462b^6x^6(4A+5Bx)+1848ab^5x^5(5A+6Bx)+3300a^2b^4x^4(6A+7Bx)+3300a^3b^3x^3(7A+8Bx)+1}{9240x^{11}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^12,x]
```

output
```
-1/9240*(462*b^6*x^6*(4*A + 5*B*x) + 1848*a*b^5*x^5*(5*A + 6*B*x) + 3300*a
^2*b^4*x^4*(6*A + 7*B*x) + 3300*a^3*b^3*x^3*(7*A + 8*B*x) + 1925*a^4*b^2*x
^2*(8*A + 9*B*x) + 616*a^5*b*x*(9*A + 10*B*x) + 84*a^6*(10*A + 11*B*x))/x^
11
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^{12}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{12}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{12}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{12}}+\frac{a^5(aB+6Ab)}{x^{11}}+\frac{3a^4b(2aB+5Ab)}{x^{10}}+\frac{5a^3b^2(3aB+4Ab)}{x^9}+\frac{5a^2b^3(4aB+3Ab)}{x^8}+\frac{b^5(6aB+Ab)}{x^6}+\frac{3\cdots}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{11x^{11}}-\frac{a^5(aB+6Ab)}{10x^{10}}-\frac{a^4b(2aB+5Ab)}{3x^9}-\frac{5a^3b^2(3aB+4Ab)}{8x^8}-\frac{5a^2b^3(4aB+3Ab)}{7x^7}-\frac{b^5(6aB+Ab)}{5x^5}-\frac{ab^4(5aB+2Ab)}{2x^6}-\frac{b^6B}{4x^4}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^12,x]
```

output
```
-1/11*(a^6*A)/x^11 - (a^5*(6*A*b + a*B))/(10*x^10) - (a^4*b*(5*A*b + 2*a*B
))/(3*x^9) - (5*a^3*b^2*(4*A*b + 3*a*B))/(8*x^8) - (5*a^2*b^3*(3*A*b + 4*a
*B))/(7*x^7) - (a*b^4*(2*A*b + 5*a*B))/(2*x^6) - (b^5*(A*b + 6*a*B))/(5*x^
5) - (b^6*B)/(4*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^6 A}{11x^{11}}-\frac{a^5(6Ab+Ba)}{10x^{10}}-\frac{a^4 b(5Ab+2Ba)}{3x^9}-\frac{5a^3b^2(4Ab+3Ba)}{8x^8}-\frac{5a^2b^3(3Ab+4Ba)}{7x^7}-\frac{a\,b^4(2Ab+5Ba)}{2x^6}-\frac{b^5(Ab+}{5x}$
norman	$\frac{-\frac{b^6 B\,x^7}{4}+\left(-\frac{1}{5}A\,b^6-\frac{6}{5}Ba\,b^5\right)x^6+\left(-Aa\,b^5-\frac{5}{2}B\,a^2b^4\right)x^5+\left(-\frac{15}{7}A\,a^2b^4-\frac{20}{7}B\,a^3b^3\right)x^4+\left(-\frac{5}{2}A\,a^3b^3-\frac{15}{8}B\,a^4b^2\right)x^3+(-\frac{5}{3}A\,a}{x^{11}}$
risch	$\frac{-\frac{b^6 B\,x^7}{4}+\left(-\frac{1}{5}A\,b^6-\frac{6}{5}Ba\,b^5\right)x^6+\left(-Aa\,b^5-\frac{5}{2}B\,a^2b^4\right)x^5+\left(-\frac{15}{7}A\,a^2b^4-\frac{20}{7}B\,a^3b^3\right)x^4+\left(-\frac{5}{2}A\,a^3b^3-\frac{15}{8}B\,a^4b^2\right)x^3+(-\frac{5}{3}A\,a}{x^{11}}$
gosper	$-\frac{2310b^6B\,x^7+1848A\,b^6x^6+11088Ba\,b^5x^6+9240Aa\,b^5x^5+23100B\,a^2b^4x^5+19800A\,a^2b^4x^4+26400B\,a^3b^3x^4+23100A\,a^3b^3}{9240x^{11}}$
parallelrisch	$-\frac{2310b^6B\,x^7+1848A\,b^6x^6+11088Ba\,b^5x^6+9240Aa\,b^5x^5+23100B\,a^2b^4x^5+19800A\,a^2b^4x^4+26400B\,a^3b^3x^4+23100A\,a^3b^3}{9240x^{11}}$
orering	$-\frac{(2310b^6B\,x^7+1848A\,b^6x^6+11088Ba\,b^5x^6+9240Aa\,b^5x^5+23100B\,a^2b^4x^5+19800A\,a^2b^4x^4+26400B\,a^3b^3x^4+23100A\,a^3b}{9240x^{11}(bx+a)}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^6*A/x^11-1/10*a^5*(6*A*b+B*a)/x^10-1/3*a^4*b*(5*A*b+2*B*a)/x^9-5/8
*a^3*b^2*(4*A*b+3*B*a)/x^8-5/7*a^2*b^3*(3*A*b+4*B*a)/x^7-1/2*a*b^4*(2*A*b+
5*B*a)/x^6-1/5*b^5*(A*b+6*B*a)/x^5-1/4*b^6*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx = -\frac{2310\,Bb^6x^7+840\,Aa^6+1848\,(6\,Bab^5+Ab^6)x^6+4620\,(5\,Ba^2b^4+2\,Aab^5)x^5+6600\,(4\,Ba^3b^3+3\,Aa}{9240\,x^{11}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^12,x, algorithm="fricas")
```

output
```
-1/9240*(2310*B*b^6*x^7 + 840*A*a^6 + 1848*(6*B*a*b^5 + A*b^6)*x^6 + 4620*
(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 6600*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 577
5*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 3080*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 9
24*(B*a^6 + 6*A*a^5*b)*x)/x^11
```

Sympy [A] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx$$
$$= \frac{-840Aa^6-2310Bb^6x^7+x^6(-1848Ab^6-11088Bab^5)+x^5(-9240Aab^5-23100Ba^2b^4)+x^4(-19800A}{}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**12,x)
```

output
```
(-840*A*a**6 - 2310*B*b**6*x**7 + x**6*(-1848*A*b**6 - 11088*B*a*b**5) + x
**5*(-9240*A*a*b**5 - 23100*B*a**2*b**4) + x**4*(-19800*A*a**2*b**4 - 2640
0*B*a**3*b**3) + x**3*(-23100*A*a**3*b**3 - 17325*B*a**4*b**2) + x**2*(-15
400*A*a**4*b**2 - 6160*B*a**5*b) + x*(-5544*A*a**5*b - 924*B*a**6))/(9240*
x**11)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx =$$
$$-\frac{2310\,Bb^6x^7+840\,Aa^6+1848\,(6\,Bab^5+Ab^6)x^6+4620\,(5\,Ba^2b^4+2\,Aab^5)x^5+6600\,(4\,Ba^3b^3+3\,Aa}{9240\,x^{11}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^12,x, algorithm="maxima")
```

output
```
-1/9240*(2310*B*b^6*x^7 + 840*A*a^6 + 1848*(6*B*a*b^5 + A*b^6)*x^6 + 4620*
(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 6600*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 577
5*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 3080*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 9
24*(B*a^6 + 6*A*a^5*b)*x)/x^11
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx = -\frac{2310\,Bb^6x^7 + 11088\,Bab^5x^6 + 1848\,Ab^6x^6 + 23100\,Ba^2b^4x^5 + 9240\,Aab^5x^5 + 26400\,Ba^3b^3x^4 + 19800}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^12,x, algorithm="giac")
```

output
```
-1/9240*(2310*B*b^6*x^7 + 11088*B*a*b^5*x^6 + 1848*A*b^6*x^6 + 23100*B*a^2
*b^4*x^5 + 9240*A*a*b^5*x^5 + 26400*B*a^3*b^3*x^4 + 19800*A*a^2*b^4*x^4 +
17325*B*a^4*b^2*x^3 + 23100*A*a^3*b^3*x^3 + 6160*B*a^5*b*x^2 + 15400*A*a^4
*b^2*x^2 + 924*B*a^6*x + 5544*A*a^5*b*x + 840*A*a^6)/x^11
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx = -\frac{x\left(\frac{B\,a^6}{10}+\frac{3\,A\,b\,a^5}{5}\right)+\frac{A\,a^6}{11}+x^5\left(\frac{5\,B\,a^2\,b^4}{2}+A\,a\,b^5\right)+x^2\left(\frac{2\,B\,a^5\,b}{3}+\frac{5\,A\,a^4\,b^2}{3}\right)+x^6\left(\frac{A\,b^6}{5}+\frac{6\,B\,a\,b^5}{5}\right)+x^3\left(\right.}{x^{11}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^12,x)
```

output
```
-(x*((B*a^6)/10 + (3*A*a^5*b)/5) + (A*a^6)/11 + x^5*((5*B*a^2*b^4)/2 + A*a
*b^5) + x^2*((5*A*a^4*b^2)/3 + (2*B*a^5*b)/3) + x^6*((A*b^6)/5 + (6*B*a*b^
5)/5) + x^3*((5*A*a^3*b^3)/2 + (15*B*a^4*b^2)/8) + x^4*((15*A*a^2*b^4)/7 +
 (20*B*a^3*b^3)/7) + (B*b^6*x^7)/4)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{12}}\,dx$$
$$= \frac{-330b^7x^7-1848a\,b^6x^6-4620a^2b^5x^5-6600a^3b^4x^4-5775a^4b^3x^3-3080a^5b^2x^2-924a^6bx-120a^7}{1320x^{11}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^12,x)
```

output
```
( - 120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 -
6600*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x*
*7)/(1320*x**11)
```

3.186 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{a^6A}{12x^{12}}-\frac{a^5(6Ab+aB)}{11x^{11}}-\frac{3a^4b(5Ab+2aB)}{10x^{10}}-\frac{5a^3b^2(4Ab+3aB)}{9x^9}-\frac{5a^2b^3(3Ab+4aB)}{8x^8}-\frac{3ab^4(2Ab+5aB)}{7x^7}-\frac{b^5(Ab+6aB)}{6x^6}-\frac{b^6B}{5x^5}$$

output
```
-1/12*a^6*A/x^12-1/11*a^5*(6*A*b+B*a)/x^11-3/10*a^4*b*(5*A*b+2*B*a)/x^10-5
/9*a^3*b^2*(4*A*b+3*B*a)/x^9-5/8*a^2*b^3*(3*A*b+4*B*a)/x^8-3/7*a*b^4*(2*A*
b+5*B*a)/x^7-1/6*b^5*(A*b+6*B*a)/x^6-1/5*b^6*B/x^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{924b^6x^6(5A+6Bx)+3960ab^5x^5(6A+7Bx)+7425a^2b^4x^4(7A+8Bx)+7700a^3b^3x^3(8A+9Bx)+4}{27720x^{12}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^13,x]
```

output
```
-1/27720*(924*b^6*x^6*(5*A + 6*B*x) + 3960*a*b^5*x^5*(6*A + 7*B*x) + 7425*
a^2*b^4*x^4*(7*A + 8*B*x) + 7700*a^3*b^3*x^3*(8*A + 9*B*x) + 4620*a^4*b^2*
x^2*(9*A + 10*B*x) + 1512*a^5*b*x*(10*A + 11*B*x) + 210*a^6*(11*A + 12*B*x
))/x^12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3 (A+Bx)}{x^{13}} \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{13}}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{13}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^6A}{x^{13}}+\frac{a^5(aB+6Ab)}{x^{12}}+\frac{3a^4b(2aB+5Ab)}{x^{11}}+\frac{5a^3b^2(3aB+4Ab)}{x^{10}}+\frac{5a^2b^3(4aB+3Ab)}{x^9}+\frac{b^5(6aB+Ab)}{x^7}+\frac{3}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^6A}{12x^{12}}-\frac{a^5(aB+6Ab)}{11x^{11}}-\frac{3a^4b(2aB+5Ab)}{10x^{10}}-\frac{5a^3b^2(3aB+4Ab)}{9x^9}-\frac{5a^2b^3(4aB+3Ab)}{8x^8}-\frac{b^5(6aB+Ab)}{6x^6}-\frac{3ab^4(5aB+2Ab)}{7x^7}-\frac{b^6B}{5x^5}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^13,x]
```

output
```
-1/12*(a^6*A)/x^12 - (a^5*(6*A*b + a*B))/(11*x^11) - (3*a^4*b*(5*A*b + 2*a
*B))/(10*x^10) - (5*a^3*b^2*(4*A*b + 3*a*B))/(9*x^9) - (5*a^2*b^3*(3*A*b +
 4*a*B))/(8*x^8) - (3*a*b^4*(2*A*b + 5*a*B))/(7*x^7) - (b^5*(A*b + 6*a*B))
/(6*x^6) - (b^6*B)/(5*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^6 A}{12x^{12}}-\frac{a^5(6Ab+Ba)}{11x^{11}}-\frac{3a^4b(5Ab+2Ba)}{10x^{10}}-\frac{5a^3b^2(4Ab+3Ba)}{9x^9}-\frac{5a^2b^3(3Ab+4Ba)}{8x^8}-\frac{3a\,b^4(2Ab+5Ba)}{7x^7}-\frac{b^5(A}{}$
norman	$\frac{-\frac{b^6Bx^7}{5}+(-\frac{1}{6}A\,b^6-Ba\,b^5)x^6+(-\frac{6}{7}Aa\,b^5-\frac{15}{7}B\,a^2b^4)x^5+(-\frac{15}{8}A\,a^2b^4-\frac{5}{2}B\,a^3b^3)x^4+(-\frac{20}{9}A\,a^3b^3-\frac{5}{3}B\,a^4b^2)x^3+(-\frac{3}{2}A\,a}{x^{12}}$
risch	$\frac{-\frac{b^6Bx^7}{5}+(-\frac{1}{6}A\,b^6-Ba\,b^5)x^6+(-\frac{6}{7}Aa\,b^5-\frac{15}{7}B\,a^2b^4)x^5+(-\frac{15}{8}A\,a^2b^4-\frac{5}{2}B\,a^3b^3)x^4+(-\frac{20}{9}A\,a^3b^3-\frac{5}{3}B\,a^4b^2)x^3+(-\frac{3}{2}A\,a}{x^{12}}$
gosper	$-\frac{5544b^6Bx^7+4620A\,b^6x^6+27720Ba\,b^5x^6+23760Aa\,b^5x^5+59400B\,a^2b^4x^5+51975A\,a^2b^4x^4+69300B\,a^3b^3x^4+61600A\,a^3b}{27720x^{12}}$
parallelrisch	$-\frac{5544b^6Bx^7+4620A\,b^6x^6+27720Ba\,b^5x^6+23760Aa\,b^5x^5+59400B\,a^2b^4x^5+51975A\,a^2b^4x^4+69300B\,a^3b^3x^4+61600A\,a^3b}{27720x^{12}}$
orering	$-\frac{(5544b^6Bx^7+4620A\,b^6x^6+27720Ba\,b^5x^6+23760Aa\,b^5x^5+59400B\,a^2b^4x^5+51975A\,a^2b^4x^4+69300B\,a^3b^3x^4+61600A\,a^3}{27720x^{12}(bx}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^6*A/x^12-1/11*a^5*(6*A*b+B*a)/x^11-3/10*a^4*b*(5*A*b+2*B*a)/x^10-5
/9*a^3*b^2*(4*A*b+3*B*a)/x^9-5/8*a^2*b^3*(3*A*b+4*B*a)/x^8-3/7*a*b^4*(2*A*
b+5*B*a)/x^7-1/6*b^5*(A*b+6*B*a)/x^6-1/5*b^6*B/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{5544\,Bb^6x^7+2310\,Aa^6+4620\,(6\,Bab^5+Ab^6)x^6+11880\,(5\,Ba^2b^4+2\,Aab^5)x^5+17325\,(4\,Ba^3b^3+3}{27720\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^13,x, algorithm="fricas")
```

output
```
-1/27720*(5544*B*b^6*x^7 + 2310*A*a^6 + 4620*(6*B*a*b^5 + A*b^6)*x^6 + 118
80*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 17325*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 +
 15400*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 8316*(2*B*a^5*b + 5*A*a^4*b^2)*x^
2 + 2520*(B*a^6 + 6*A*a^5*b)*x)/x^12
```

Sympy [A] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = \frac{-2310Aa^6-5544Bb^6x^7+x^6(-4620Ab^6-27720Bab^5)+x^5(-23760Aab^5-59400Ba^2b^4)+x^4(-51975}{}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**13,x)
```

output
```
(-2310*A*a**6 - 5544*B*b**6*x**7 + x**6*(-4620*A*b**6 - 27720*B*a*b**5) +
x**5*(-23760*A*a*b**5 - 59400*B*a**2*b**4) + x**4*(-51975*A*a**2*b**4 - 69
300*B*a**3*b**3) + x**3*(-61600*A*a**3*b**3 - 46200*B*a**4*b**2) + x**2*(-
41580*A*a**4*b**2 - 16632*B*a**5*b) + x*(-15120*A*a**5*b - 2520*B*a**6))/(
27720*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{5544\,Bb^6x^7+2310\,Aa^6+4620\,(6\,Bab^5+Ab^6)x^6+11880\,(5\,Ba^2b^4+2\,Aab^5)x^5+17325\,(4\,Ba^3b^3+3}{27720\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^13,x, algorithm="maxima")
```

output
```
-1/27720*(5544*B*b^6*x^7 + 2310*A*a^6 + 4620*(6*B*a*b^5 + A*b^6)*x^6 + 118
80*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 17325*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 +
 15400*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 8316*(2*B*a^5*b + 5*A*a^4*b^2)*x^
2 + 2520*(B*a^6 + 6*A*a^5*b)*x)/x^12
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{5544\,Bb^6x^7+27720\,Bab^5x^6+4620\,Ab^6x^6+59400\,Ba^2b^4x^5+23760\,Aab^5x^5+69300\,Ba^3b^3x^4+5197}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^13,x, algorithm="giac")
```

output
```
-1/27720*(5544*B*b^6*x^7 + 27720*B*a*b^5*x^6 + 4620*A*b^6*x^6 + 59400*B*a^
2*b^4*x^5 + 23760*A*a*b^5*x^5 + 69300*B*a^3*b^3*x^4 + 51975*A*a^2*b^4*x^4
+ 46200*B*a^4*b^2*x^3 + 61600*A*a^3*b^3*x^3 + 16632*B*a^5*b*x^2 + 41580*A*
a^4*b^2*x^2 + 2520*B*a^6*x + 15120*A*a^5*b*x + 2310*A*a^6)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx = -\frac{x\left(\frac{B\,a^6}{11}+\frac{6\,A\,b\,a^5}{11}\right)+\frac{A\,a^6}{12}+x^2\left(\frac{3\,B\,a^5\,b}{5}+\frac{3\,A\,a^4\,b^2}{2}\right)+x^5\left(\frac{15\,B\,a^2\,b^4}{7}+\frac{6\,A\,a\,b^5}{7}\right)+x^6\left(\frac{A\,b^6}{6}+B\,a\,b^5\right)+x^4\left(\right.}{x^{12}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^13,x)
```

output
```
-(x*((B*a^6)/11 + (6*A*a^5*b)/11) + (A*a^6)/12 + x^2*((3*A*a^4*b^2)/2 + (3
*B*a^5*b)/5) + x^5*((15*B*a^2*b^4)/7 + (6*A*a*b^5)/7) + x^6*((A*b^6)/6 + B
*a*b^5) + x^4*((15*A*a^2*b^4)/8 + (5*B*a^3*b^3)/2) + x^3*((20*A*a^3*b^3)/9
 + (5*B*a^4*b^2)/3) + (B*b^6*x^7)/5)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{13}}\,dx$$
$$= \frac{-792b^7x^7-4620a\,b^6x^6-11880a^2b^5x^5-17325a^3b^4x^4-15400a^4b^3x^3-8316a^5b^2x^2-2520a^6bx-330a^7}{3960x^{12}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^13,x)
```

output
```
( - 330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3
- 17325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**
7*x**7)/(3960*x**12)
```

3.187 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx = -\frac{a^6A}{13x^{13}}-\frac{a^5(6Ab+aB)}{12x^{12}}-\frac{3a^4b(5Ab+2aB)}{11x^{11}}-\frac{a^3b^2(4Ab+3aB)}{2x^{10}}-\frac{5a^2b^3(3Ab+4aB)}{9x^9}-\frac{3ab^4(2Ab+5aB)}{8x^8}-\frac{b^5(Ab+6aB)}{7x^7}-\frac{b^6B}{6x^6}$$

output
```
-1/13*a^6*A/x^13-1/12*a^5*(6*A*b+B*a)/x^12-3/11*a^4*b*(5*A*b+2*B*a)/x^11-1
/2*a^3*b^2*(4*A*b+3*B*a)/x^10-5/9*a^2*b^3*(3*A*b+4*B*a)/x^9-3/8*a*b^4*(2*A
*b+5*B*a)/x^8-1/7*b^5*(A*b+6*B*a)/x^7-1/6*b^6*B/x^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx = -\frac{1716b^6x^6(6A+7Bx)+7722ab^5x^5(7A+8Bx)+15015a^2b^4x^4(8A+9Bx)+16016a^3b^3x^3(9A+10Bx}{72072x^{13}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^14,x]
```

output
```
-1/72072*(1716*b^6*x^6*(6*A + 7*B*x) + 7722*a*b^5*x^5*(7*A + 8*B*x) + 1501
5*a^2*b^4*x^4*(8*A + 9*B*x) + 16016*a^3*b^3*x^3*(9*A + 10*B*x) + 9828*a^4*
b^2*x^2*(10*A + 11*B*x) + 3276*a^5*b*x*(11*A + 12*B*x) + 462*a^6*(12*A + 1
3*B*x))/x^13
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^{14}} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{14}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{14}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{14}}+\frac{a^5(aB+6Ab)}{x^{13}}+\frac{3a^4b(2aB+5Ab)}{x^{12}}+\frac{5a^3b^2(3aB+4Ab)}{x^{11}}+\frac{5a^2b^3(4aB+3Ab)}{x^{10}}+\frac{b^5(6aB+Ab)}{x^8}+\frac{3}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6A}{13x^{13}}-\frac{a^5(aB+6Ab)}{12x^{12}}-\frac{3a^4b(2aB+5Ab)}{11x^{11}}-\frac{a^3b^2(3aB+4Ab)}{2x^{10}}-\frac{5a^2b^3(4aB+3Ab)}{9x^9}-\frac{b^5(6aB+Ab)}{7x^7}-\frac{3ab^4(5aB+2Ab)}{8x^8}-\frac{b^6B}{6x^6}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^14,x]
```

output
```
-1/13*(a^6*A)/x^13 - (a^5*(6*A*b + a*B))/(12*x^12) - (3*a^4*b*(5*A*b + 2*a
*B))/(11*x^11) - (a^3*b^2*(4*A*b + 3*a*B))/(2*x^10) - (5*a^2*b^3*(3*A*b +
4*a*B))/(9*x^9) - (3*a*b^4*(2*A*b + 5*a*B))/(8*x^8) - (b^5*(A*b + 6*a*B))/
(7*x^7) - (b^6*B)/(6*x^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^6 A}{13x^{13}}-\frac{a^5(6Ab+Ba)}{12x^{12}}-\frac{3a^4b(5Ab+2Ba)}{11x^{11}}-\frac{a^3b^2(4Ab+3Ba)}{2x^{10}}-\frac{5a^2b^3(3Ab+4Ba)}{9x^9}-\frac{3a\,b^4(2Ab+5Ba)}{8x^8}-\frac{b^5(Ab}{7}$
norman	$\frac{-\frac{b^6 B\,x^7}{6}+(-\frac{1}{7}A\,b^6-\frac{6}{7}Ba\,b^5)x^6+(-\frac{3}{4}Aa\,b^5-\frac{15}{8}B\,a^2b^4)x^5+(-\frac{5}{3}A\,a^2b^4-\frac{20}{9}B\,a^3b^3)x^4+(-2A\,a^3b^3-\frac{3}{2}B\,a^4b^2)x^3+(-\frac{15}{11}A}{x^{13}}$
risch	$\frac{-\frac{b^6 B\,x^7}{6}+(-\frac{1}{7}A\,b^6-\frac{6}{7}Ba\,b^5)x^6+(-\frac{3}{4}Aa\,b^5-\frac{15}{8}B\,a^2b^4)x^5+(-\frac{5}{3}A\,a^2b^4-\frac{20}{9}B\,a^3b^3)x^4+(-2A\,a^3b^3-\frac{3}{2}B\,a^4b^2)x^3+(-\frac{15}{11}A}{x^{13}}$
gosper	$-\frac{12012b^6B\,x^7+10296A\,b^6x^6+61776Ba\,b^5x^6+54054Aa\,b^5x^5+135135B\,a^2b^4x^5+120120A\,a^2b^4x^4+160160B\,a^3b^3x^4+14414}{72072x^{13}}$
parallelrisch	$-\frac{12012b^6B\,x^7+10296A\,b^6x^6+61776Ba\,b^5x^6+54054Aa\,b^5x^5+135135B\,a^2b^4x^5+120120A\,a^2b^4x^4+160160B\,a^3b^3x^4+14414}{72072x^{13}}$
orering	$-\frac{(12012b^6B\,x^7+10296A\,b^6x^6+61776Ba\,b^5x^6+54054Aa\,b^5x^5+135135B\,a^2b^4x^5+120120A\,a^2b^4x^4+160160B\,a^3b^3x^4+1441}{72072x^{13}}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^6*A/x^13-1/12*a^5*(6*A*b+B*a)/x^12-3/11*a^4*b*(5*A*b+2*B*a)/x^11-1
/2*a^3*b^2*(4*A*b+3*B*a)/x^10-5/9*a^2*b^3*(3*A*b+4*B*a)/x^9-3/8*a*b^4*(2*A
*b+5*B*a)/x^8-1/7*b^5*(A*b+6*B*a)/x^7-1/6*b^6*B/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx = -\frac{12012\,Bb^6x^7+5544\,Aa^6+10296\,(6\,Bab^5+Ab^6)x^6+27027\,(5\,Ba^2b^4+2\,Aab^5)x^5+40040\,(4\,Ba^3b^3+}{72072}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^14,x, algorithm="fricas")
```

output
```
-1/72072*(12012*B*b^6*x^7 + 5544*A*a^6 + 10296*(6*B*a*b^5 + A*b^6)*x^6 + 2
7027*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 40040*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4
 + 36036*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 19656*(2*B*a^5*b + 5*A*a^4*b^2)
*x^2 + 6006*(B*a^6 + 6*A*a^5*b)*x)/x^13
```

Sympy [A] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx$$
$$= \frac{-5544Aa^6-12012Bb^6x^7+x^6(-10296Ab^6-61776Bab^5)+x^5(-54054Aab^5-135135Ba^2b^4)+x^4(-12}{}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**14,x)
```

output
```
(-5544*A*a**6 - 12012*B*b**6*x**7 + x**6*(-10296*A*b**6 - 61776*B*a*b**5)
+ x**5*(-54054*A*a*b**5 - 135135*B*a**2*b**4) + x**4*(-120120*A*a**2*b**4
- 160160*B*a**3*b**3) + x**3*(-144144*A*a**3*b**3 - 108108*B*a**4*b**2) +
x**2*(-98280*A*a**4*b**2 - 39312*B*a**5*b) + x*(-36036*A*a**5*b - 6006*B*a
**6))/(72072*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx =$$
$$-\frac{12012\,Bb^6x^7+5544\,Aa^6+10296\,(6\,Bab^5+Ab^6)x^6+27027\,(5\,Ba^2b^4+2\,Aab^5)x^5+40040\,(4\,Ba^3b^3+}{72072}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^14,x, algorithm="maxima")
```

output
```
-1/72072*(12012*B*b^6*x^7 + 5544*A*a^6 + 10296*(6*B*a*b^5 + A*b^6)*x^6 + 2
7027*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 40040*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4
 + 36036*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 19656*(2*B*a^5*b + 5*A*a^4*b^2)
*x^2 + 6006*(B*a^6 + 6*A*a^5*b)*x)/x^13
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx = -\frac{12012\,Bb^6x^7+61776\,Bab^5x^6+10296\,Ab^6x^6+135135\,Ba^2b^4x^5+54054\,Aab^5x^5+160160\,Ba^3b^3x^4+}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^14,x, algorithm="giac")
```

output
```
-1/72072*(12012*B*b^6*x^7 + 61776*B*a*b^5*x^6 + 10296*A*b^6*x^6 + 135135*B
*a^2*b^4*x^5 + 54054*A*a*b^5*x^5 + 160160*B*a^3*b^3*x^4 + 120120*A*a^2*b^4
*x^4 + 108108*B*a^4*b^2*x^3 + 144144*A*a^3*b^3*x^3 + 39312*B*a^5*b*x^2 + 9
8280*A*a^4*b^2*x^2 + 6006*B*a^6*x + 36036*A*a^5*b*x + 5544*A*a^6)/x^13
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx = -\frac{x\left(\frac{B\,a^6}{12}+\frac{A\,b\,a^5}{2}\right)+\frac{A\,a^6}{13}+x^5\left(\frac{15\,B\,a^2\,b^4}{8}+\frac{3\,A\,a\,b^5}{4}\right)+x^2\left(\frac{6\,B\,a^5\,b}{11}+\frac{15\,A\,a^4\,b^2}{11}\right)+x^6\left(\frac{A\,b^6}{7}+\frac{6\,B\,a\,b^5}{7}\right)+x^3\left(\right.}{x^{13}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^14,x)
```

output
```
-(x*((B*a^6)/12 + (A*a^5*b)/2) + (A*a^6)/13 + x^5*((15*B*a^2*b^4)/8 + (3*A
*a*b^5)/4) + x^2*((15*A*a^4*b^2)/11 + (6*B*a^5*b)/11) + x^6*((A*b^6)/7 + (
6*B*a*b^5)/7) + x^3*(2*A*a^3*b^3 + (3*B*a^4*b^2)/2) + x^4*((5*A*a^2*b^4)/3
 + (20*B*a^3*b^3)/9) + (B*b^6*x^7)/6)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{14}}\,dx$$
$$=\frac{-1716b^7x^7-10296a\,b^6x^6-27027a^2b^5x^5-40040a^3b^4x^4-36036a^4b^3x^3-19656a^5b^2x^2-6006a^6bx-79}{10296x^{13}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^14,x)
```

output
```
( - 792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3
 - 40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*
b**7*x**7)/(10296*x**13)
```

3.188 $\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\begin{aligned}\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx =& -\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(7d-8e)(1+x)^{12}\\&-\frac{7}{13}(3d-4e)(1+x)^{13}\\&+\frac{1}{2}(5d-8e)(1+x)^{14}-\frac{7}{3}(d-2e)(1+x)^{15}\\&+\frac{7}{16}(3d-8e)(1+x)^{16}-\frac{7}{17}(d-4e)(1+x)^{17}\\&+\frac{1}{18}(d-8e)(1+x)^{18}+\frac{1}{19}e(1+x)^{19}\end{aligned}$$

output

```
-1/11*(d-e)*(1+x)^11+1/12*(7*d-8*e)*(1+x)^12-7/13*(3*d-4*e)*(1+x)^13+1/2*(
5*d-8*e)*(1+x)^14-7/3*(d-2*e)*(1+x)^15+7/16*(3*d-8*e)*(1+x)^16-7/17*(d-4*e
)*(1+x)^17+1/18*(d-8*e)*(1+x)^18+1/19*e*(1+x)^19
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^8}{8}+\frac{1}{9}(10d+e)x^9+\frac{1}{2}(9d+2e)x^{10}+\frac{15}{11}(8d+3e)x^{11}+\frac{5}{2}(7d+4e)x^{12}+\frac{42}{13}(6d+5e)x^{13}+3(5d+6e)x^{14}+2(4d+7e)x^{15}+\frac{15}{16}(3d+8e)x^{16}+\frac{5}{17}(2d+9e)x^{17}+\frac{1}{18}(d+10e)x^{18}+\frac{ex^{19}}{19}$$

input
```
Integrate[x^7*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^8)/8 + ((10*d + e)*x^9)/9 + ((9*d + 2*e)*x^10)/2 + (15*(8*d + 3*e)*x^
11)/11 + (5*(7*d + 4*e)*x^12)/2 + (42*(6*d + 5*e)*x^13)/13 + 3*(5*d + 6*e)
*x^14 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^1
7)/17 + ((d + 10*e)*x^18)/18 + (e*x^19)/19
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(x^2+2x+1\right)^5(d+ex)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^7(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left((x+1)^{17}(d-8e) - 7(x+1)^{16}(d-4e) + 7(x+1)^{15}(3d-8e) - 35(x+1)^{14}(d-2e) + 7(x+1)^{13}(5d-8e) - \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{18}(x+1)^{18}(d-8e) - \frac{7}{17}(x+1)^{17}(d-4e) + \frac{7}{16}(x+1)^{16}(3d-8e) - \frac{7}{3}(x+1)^{15}(d-2e) + \frac{1}{2}(x+1)^{14}(5d-8e) - \frac{7}{13}(x+1)^{13}(3d-4e) + \frac{1}{12}(x+1)^{12}(7d-8e) - \frac{1}{11}(x+1)^{11}(d-e) + \frac{1}{19}e(x+1)^{19}$$

input
```
Int[x^7*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/11*((d - e)*(1 + x)^11) + ((7*d - 8*e)*(1 + x)^12)/12 - (7*(3*d - 4*e)*
(1 + x)^13)/13 + ((5*d - 8*e)*(1 + x)^14)/2 - (7*(d - 2*e)*(1 + x)^15)/3 +
 (7*(3*d - 8*e)*(1 + x)^16)/16 - (7*(d - 4*e)*(1 + x)^17)/17 + ((d - 8*e)*
(1 + x)^18)/18 + (e*(1 + x)^19)/19
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

method	result
norman	$\frac{d\,x^{8}}{8}+\left(\frac{10d}{9}+\frac{e}{9}\right)x^{9}+\left(\frac{9d}{2}+e\right)x^{10}+\left(\frac{120d}{11}+\frac{45e}{11}\right)x^{11}+\left(\frac{35d}{2}+10e\right)x^{12}+\left(\frac{252d}{13}+\frac{210e}{13}\right)x^{13}$
default	$\frac{e\,x^{19}}{19}+\frac{(d+10e)x^{18}}{18}+\frac{(10d+45e)x^{17}}{17}+\frac{(45d+120e)x^{16}}{16}+\frac{(120d+210e)x^{15}}{15}+\frac{(210d+252e)x^{14}}{14}+\frac{(252d+210e)x^{13}}{13}$
gosper	$x^{8}\left(350064e\,x^{11}+369512d\,x^{10}+3695120e\,x^{10}+3912480d\,x^{9}+17606160e\,x^{9}+18706545d\,x^{8}+49884120e\,x^{8}+53209728d\,x^{7}+93\right.$
risch	$\frac{1}{19}e\,x^{19}+\frac{1}{18}x^{18}d+\frac{5}{9}e\,x^{18}+\frac{10}{17}d\,x^{17}+\frac{45}{17}x^{17}e+\frac{45}{16}d\,x^{16}+\frac{15}{2}x^{16}e+8x^{15}d+14x^{15}e+15x^{14}$
parallelrisch	$\frac{1}{19}e\,x^{19}+\frac{1}{18}x^{18}d+\frac{5}{9}e\,x^{18}+\frac{10}{17}d\,x^{17}+\frac{45}{17}x^{17}e+\frac{45}{16}d\,x^{16}+\frac{15}{2}x^{16}e+8x^{15}d+14x^{15}e+15x^{14}$
orering	$x^{8}\left(350064e\,x^{11}+369512d\,x^{10}+3695120e\,x^{10}+3912480d\,x^{9}+17606160e\,x^{9}+18706545d\,x^{8}+49884120e\,x^{8}+53209728d\,x^{7}+93\right.$

input
```
int(x^7*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*d*x^8+(10/9*d+1/9*e)*x^9+(9/2*d+e)*x^10+(120/11*d+45/11*e)*x^11+(35/2*
d+10*e)*x^12+(252/13*d+210/13*e)*x^13+(15*d+18*e)*x^14+(8*d+14*e)*x^15+(45
/16*d+15/2*e)*x^16+(10/17*d+45/17*e)*x^17+(1/18*d+5/9*e)*x^18+1/19*e*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{19}\,ex^{19}+\frac{1}{18}\,(d+10\,e)x^{18}+\frac{5}{17}\,(2\,d+9\,e)x^{17}+\frac{15}{16}\,(3\,d+8\,e)x^{16}+2\,(4\,d+7\,e)x^{15}+3\,(5\,d+6\,e)x^{14}+\frac{42}{13}\,(6\,d+5\,e)x^{13}+\frac{5}{2}\,(7\,d+4\,e)x^{12}+\frac{15}{11}\,(8\,d+3\,e)x^{11}+\frac{1}{2}\,(9\,d+2\,e)x^{10}+\frac{1}{9}\,(10\,d+e)x^{9}+\frac{1}{8}\,dx^{8}$$

input
```
integrate(x^7*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/19*e*x^19 + 1/18*(d + 10*e)*x^18 + 5/17*(2*d + 9*e)*x^17 + 15/16*(3*d +
8*e)*x^16 + 2*(4*d + 7*e)*x^15 + 3*(5*d + 6*e)*x^14 + 42/13*(6*d + 5*e)*x^
13 + 5/2*(7*d + 4*e)*x^12 + 15/11*(8*d + 3*e)*x^11 + 1/2*(9*d + 2*e)*x^10
+ 1/9*(10*d + e)*x^9 + 1/8*d*x^8
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^8}{8}+\frac{ex^{19}}{19}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{17}\cdot\left(\frac{10d}{17}+\frac{45e}{17}\right)+x^{16}\cdot\left(\frac{45d}{16}+\frac{15e}{2}\right)+x^{15}\cdot(8d+14e)+x^{14}\cdot(15d+18e)+x^{13}\cdot\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{12}\cdot\left(\frac{35d}{2}+10e\right)+x^{11}\cdot\left(\frac{120d}{11}+\frac{45e}{11}\right)+x^{10}\cdot\left(\frac{9d}{2}+e\right)+x^9\cdot\left(\frac{10d}{9}+\frac{e}{9}\right)$$

input
```
integrate(x**7*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**8/8 + e*x**19/19 + x**18*(d/18 + 5*e/9) + x**17*(10*d/17 + 45*e/17) +
 x**16*(45*d/16 + 15*e/2) + x**15*(8*d + 14*e) + x**14*(15*d + 18*e) + x**
13*(252*d/13 + 210*e/13) + x**12*(35*d/2 + 10*e) + x**11*(120*d/11 + 45*e/
11) + x**10*(9*d/2 + e) + x**9*(10*d/9 + e/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{19}\,ex^{19}+\frac{1}{18}\,(d+10\,e)x^{18}+\frac{5}{17}\,(2\,d+9\,e)x^{17}+\frac{15}{16}\,(3\,d+8\,e)x^{16}+2\,(4\,d+7\,e)x^{15}+3\,(5\,d+6\,e)x^{14}+\frac{42}{13}\,(6\,d+5\,e)x^{13}+\frac{5}{2}\,(7\,d+4\,e)x^{12}+\frac{15}{11}\,(8\,d+3\,e)x^{11}+\frac{1}{2}\,(9\,d+2\,e)x^{10}+\frac{1}{9}\,(10\,d+e)x^9+\frac{1}{8}\,dx^8$$

input
```
integrate(x^7*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/19*e*x^19 + 1/18*(d + 10*e)*x^18 + 5/17*(2*d + 9*e)*x^17 + 15/16*(3*d +
8*e)*x^16 + 2*(4*d + 7*e)*x^15 + 3*(5*d + 6*e)*x^14 + 42/13*(6*d + 5*e)*x^
13 + 5/2*(7*d + 4*e)*x^12 + 15/11*(8*d + 3*e)*x^11 + 1/2*(9*d + 2*e)*x^10
+ 1/9*(10*d + e)*x^9 + 1/8*d*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{19}ex^{19}+\frac{1}{18}dx^{18}+\frac{5}{9}ex^{18}+\frac{10}{17}dx^{17}+\frac{45}{17}ex^{17}+\frac{45}{16}dx^{16} + \frac{15}{2}ex^{16}+8\,dx^{15}+14\,ex^{15}+15\,dx^{14}+18\,ex^{14} + \frac{252}{13}dx^{13}+\frac{210}{13}ex^{13}+\frac{35}{2}dx^{12}+10\,ex^{12}+\frac{120}{11}dx^{11} + \frac{45}{11}ex^{11}+\frac{9}{2}dx^{10}+ex^{10}+\frac{10}{9}dx^{9}+\frac{1}{9}ex^{9}+\frac{1}{8}dx^{8}$$

input
```
integrate(x^7*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/19*e*x^19 + 1/18*d*x^18 + 5/9*e*x^18 + 10/17*d*x^17 + 45/17*e*x^17 + 45/
16*d*x^16 + 15/2*e*x^16 + 8*d*x^15 + 14*e*x^15 + 15*d*x^14 + 18*e*x^14 + 2
52/13*d*x^13 + 210/13*e*x^13 + 35/2*d*x^12 + 10*e*x^12 + 120/11*d*x^11 + 4
5/11*e*x^11 + 9/2*d*x^10 + e*x^10 + 10/9*d*x^9 + 1/9*e*x^9 + 1/8*d*x^8
```

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{e\,x^{19}}{19}+\left(\frac{d}{18}+\frac{5\,e}{9}\right)\,x^{18}+\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)\,x^{17}+\left(\frac{45\,d}{16}+\frac{15\,e}{2}\right)\,x^{16}+\left(8\,d+14\,e\right)\,x^{15}+\left(15\,d+18\,e\right)\,x^{14}+\left(\frac{252\,d}{13}+\frac{210\,e}{13}\right)\,x^{13}+\left(\frac{35\,d}{2}+10\,e\right)\,x^{12}+\left(\frac{120\,d}{11}+\frac{45\,e}{11}\right)\,x^{11}+\left(\frac{9\,d}{2}+e\right)\,x^{10}+\left(\frac{10\,d}{9}+\frac{e}{9}\right)\,x^9+\frac{d\,x^8}{8}$$

input
```
int(x^7*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^15*(8*d + 14*e) + x^9*((10*d)/9 + e/9) + x^14*(15*d + 18*e) + x^18*(d/18
 + (5*e)/9) + x^12*((35*d)/2 + 10*e) + x^16*((45*d)/16 + (15*e)/2) + x^17*
((10*d)/17 + (45*e)/17) + x^11*((120*d)/11 + (45*e)/11) + x^13*((252*d)/13
 + (210*e)/13) + (d*x^8)/8 + (e*x^19)/19 + x^10*((9*d)/2 + e)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int x^7(d+ex)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^8(350064e\,x^{11}+369512d\,x^{10}+3695120e\,x^{10}+3912480d\,x^9+17606160e\,x^9+18706545d\,x^8+498841}{}$$

input
```
int(x^7*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**8*(369512*d*x**10 + 3912480*d*x**9 + 18706545*d*x**8 + 53209728*d*x**7
 + 99768240*d*x**6 + 128931264*d*x**5 + 116396280*d*x**4 + 72558720*d*x**3
 + 29930472*d*x**2 + 7390240*d*x + 831402*d + 350064*e*x**11 + 3695120*e*x
**10 + 17606160*e*x**9 + 49884120*e*x**8 + 93117024*e*x**7 + 119721888*e*x
**6 + 107442720*e*x**5 + 66512160*e*x**4 + 27209520*e*x**3 + 6651216*e*x**
2 + 739024*e*x))/6651216
```

3.189 $\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\begin{aligned}\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{11}(d-e)(1+x)^{11}-\frac{1}{12}(6d-7e)(1+x)^{12}\\ &\quad+\frac{3}{13}(5d-7e)(1+x)^{13}-\frac{5}{14}(4d-7e)(1+x)^{14}\\ &\quad+\frac{1}{3}(3d-7e)(1+x)^{15}-\frac{3}{16}(2d-7e)(1+x)^{16}\\ &\quad+\frac{1}{17}(d-7e)(1+x)^{17}+\frac{1}{18}e(1+x)^{18}\end{aligned}$$

output
```
1/11*(d-e)*(1+x)^11-1/12*(6*d-7*e)*(1+x)^12+3/13*(5*d-7*e)*(1+x)^13-5/14*(
4*d-7*e)*(1+x)^14+1/3*(3*d-7*e)*(1+x)^15-3/16*(2*d-7*e)*(1+x)^16+1/17*(d-7
*e)*(1+x)^17+1/18*e*(1+x)^18
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^7}{7}+\frac{1}{8}(10d+e)x^8+\frac{5}{9}(9d+2e)x^9+\frac{3}{2}(8d+3e)x^{10}+\frac{30}{11}(7d+4e)x^{11}+\frac{7}{2}(6d+5e)x^{12}+\frac{42}{13}(5d+6e)x^{13}+\frac{15}{7}(4d+7e)x^{14}+(3d+8e)x^{15}+\frac{5}{16}(2d+9e)x^{16}+\frac{1}{17}(d+10e)x^{17}+\frac{ex^{18}}{18}$$

input
```
Integrate[x^6*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^7)/7 + ((10*d + e)*x^8)/8 + (5*(9*d + 2*e)*x^9)/9 + (3*(8*d + 3*e)*x^
10)/2 + (30*(7*d + 4*e)*x^11)/11 + (7*(6*d + 5*e)*x^12)/2 + (42*(5*d + 6*e
)*x^13)/13 + (15*(4*d + 7*e)*x^14)/7 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x
^16)/16 + ((d + 10*e)*x^17)/17 + (e*x^18)/18
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(x^2+2x+1\right)^5(d+ex)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^6(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left((x+1)^{16}(d-7e)-3(x+1)^{15}(2d-7e)+5(x+1)^{14}(3d-7e)-5(x+1)^{13}(4d-7e)+3(x+1)^{12}(5d-7e)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{17}(x+1)^{17}(d-7e)-\frac{3}{16}(x+1)^{16}(2d-7e)+\frac{1}{3}(x+1)^{15}(3d-7e)-\frac{5}{14}(x+1)^{14}(4d-7e)+\frac{3}{13}(x+1)^{13}(5d-7e)-\frac{1}{12}(x+1)^{12}(6d-7e)+\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{18}e(x+1)^{18}$$

input
```
Int[x^6*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
((d - e)*(1 + x)^11)/11 - ((6*d - 7*e)*(1 + x)^12)/12 + (3*(5*d - 7*e)*(1
+ x)^13)/13 - (5*(4*d - 7*e)*(1 + x)^14)/14 + ((3*d - 7*e)*(1 + x)^15)/3 -
 (3*(2*d - 7*e)*(1 + x)^16)/16 + ((d - 7*e)*(1 + x)^17)/17 + (e*(1 + x)^18
)/18
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

method	result
norman	$\frac{d\,x^{7}}{7}+\left(\frac{5d}{4}+\frac{e}{8}\right)x^{8}+\left(5d+\frac{10e}{9}\right)x^{9}+\left(12d+\frac{9e}{2}\right)x^{10}+\left(\frac{210d}{11}+\frac{120e}{11}\right)x^{11}+\left(21d+\frac{35e}{2}\right)x^{12}$ -
default	$\frac{e\,x^{18}}{18}+\frac{(d+10e)x^{17}}{17}+\frac{(10d+45e)x^{16}}{16}+\frac{(45d+120e)x^{15}}{15}+\frac{(120d+210e)x^{14}}{14}+\frac{(210d+252e)x^{13}}{13}+\frac{(252d+210e)x^{12}}{12}$
gosper	$x^{7}\left(136136e\,x^{11}+144144d\,x^{10}+1441440e\,x^{10}+1531530d\,x^{9}+6891885e\,x^{9}+7351344d\,x^{8}+19603584e\,x^{8}+21003840d\,x^{7}+3675\right.$
risch	$\frac{1}{18}e\,x^{18}+\frac{1}{17}d\,x^{17}+\frac{10}{17}x^{17}e+\frac{5}{8}d\,x^{16}+\frac{45}{16}x^{16}e+3x^{15}d+8x^{15}e+\frac{60}{7}x^{14}d+15x^{14}e+\frac{210}{13}d\,x^{1}$
parallelrisch	$\frac{1}{18}e\,x^{18}+\frac{1}{17}d\,x^{17}+\frac{10}{17}x^{17}e+\frac{5}{8}d\,x^{16}+\frac{45}{16}x^{16}e+3x^{15}d+8x^{15}e+\frac{60}{7}x^{14}d+15x^{14}e+\frac{210}{13}d\,x^{1}$
orering	$x^{7}\left(136136e\,x^{11}+144144d\,x^{10}+1441440e\,x^{10}+1531530d\,x^{9}+6891885e\,x^{9}+7351344d\,x^{8}+19603584e\,x^{8}+21003840d\,x^{7}+3675\right.$

input
```
int(x^6*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*d*x^7+(5/4*d+1/8*e)*x^8+(5*d+10/9*e)*x^9+(12*d+9/2*e)*x^10+(210/11*d+1
20/11*e)*x^11+(21*d+35/2*e)*x^12+(210/13*d+252/13*e)*x^13+(60/7*d+15*e)*x^
14+(3*d+8*e)*x^15+(5/8*d+45/16*e)*x^16+(1/17*d+10/17*e)*x^17+1/18*e*x^18
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\begin{aligned}\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{18}\,ex^{18}+\frac{1}{17}\,(d+10\,e)x^{17}+\frac{5}{16}\,(2\,d+9\,e)x^{16}\\&\quad+(3\,d+8\,e)x^{15}+\frac{15}{7}\,(4\,d+7\,e)x^{14}\\&\quad+\frac{42}{13}\,(5\,d+6\,e)x^{13}+\frac{7}{2}\,(6\,d+5\,e)x^{12}\\&\quad+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{3}{2}\,(8\,d+3\,e)x^{10}\\&\quad+\frac{5}{9}\,(9\,d+2\,e)x^{9}+\frac{1}{8}\,(10\,d+e)x^{8}+\frac{1}{7}\,dx^{7}\end{aligned}$$

input
```
integrate(x^6*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/18*e*x^18 + 1/17*(d + 10*e)*x^17 + 5/16*(2*d + 9*e)*x^16 + (3*d + 8*e)*x
^15 + 15/7*(4*d + 7*e)*x^14 + 42/13*(5*d + 6*e)*x^13 + 7/2*(6*d + 5*e)*x^1
2 + 30/11*(7*d + 4*e)*x^11 + 3/2*(8*d + 3*e)*x^10 + 5/9*(9*d + 2*e)*x^9 +
1/8*(10*d + e)*x^8 + 1/7*d*x^7
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^7}{7}+\frac{ex^{18}}{18}+x^{17}\left(\frac{d}{17}+\frac{10e}{17}\right)+x^{16}\cdot\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{15}\cdot(3d+8e)+x^{14}\cdot\left(\frac{60d}{7}+15e\right)+x^{13}\cdot\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{12}\cdot\left(21d+\frac{35e}{2}\right)+x^{11}\cdot\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{10}\cdot\left(12d+\frac{9e}{2}\right)+x^{9}\cdot\left(5d+\frac{10e}{9}\right)+x^{8}\cdot\left(\frac{5d}{4}+\frac{e}{8}\right)$$

input
```
integrate(x**6*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**7/7 + e*x**18/18 + x**17*(d/17 + 10*e/17) + x**16*(5*d/8 + 45*e/16) +
 x**15*(3*d + 8*e) + x**14*(60*d/7 + 15*e) + x**13*(210*d/13 + 252*e/13) +
 x**12*(21*d + 35*e/2) + x**11*(210*d/11 + 120*e/11) + x**10*(12*d + 9*e/2
) + x**9*(5*d + 10*e/9) + x**8*(5*d/4 + e/8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{18}\,ex^{18}+\frac{1}{17}\,(d+10\,e)x^{17}+\frac{5}{16}\,(2\,d+9\,e)x^{16}+(3\,d+8\,e)x^{15}+\frac{15}{7}\,(4\,d+7\,e)x^{14}+\frac{42}{13}\,(5\,d+6\,e)x^{13}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{3}{2}\,(8\,d+3\,e)x^{10}+\frac{5}{9}\,(9\,d+2\,e)x^{9}+\frac{1}{8}\,(10\,d+e)x^{8}+\frac{1}{7}\,dx^{7}$$

input
```
integrate(x^6*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/18*e*x^18 + 1/17*(d + 10*e)*x^17 + 5/16*(2*d + 9*e)*x^16 + (3*d + 8*e)*x
^15 + 15/7*(4*d + 7*e)*x^14 + 42/13*(5*d + 6*e)*x^13 + 7/2*(6*d + 5*e)*x^1
2 + 30/11*(7*d + 4*e)*x^11 + 3/2*(8*d + 3*e)*x^10 + 5/9*(9*d + 2*e)*x^9 +
1/8*(10*d + e)*x^8 + 1/7*d*x^7
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{18}ex^{18}+\frac{1}{17}dx^{17}+\frac{10}{17}ex^{17}+\frac{5}{8}dx^{16}+\frac{45}{16}ex^{16}+3\,dx^{15}+8\,ex^{15}+\frac{60}{7}dx^{14}+15\,ex^{14}+\frac{210}{13}dx^{13}+\frac{252}{13}ex^{13}+21\,dx^{12}+\frac{35}{2}ex^{12}+\frac{210}{11}dx^{11}+\frac{120}{11}ex^{11}+12\,dx^{10}+\frac{9}{2}ex^{10}+5\,dx^9+\frac{10}{9}ex^9+\frac{5}{4}dx^8+\frac{1}{8}ex^8+\frac{1}{7}dx^7$$

input
```
integrate(x^6*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/18*e*x^18 + 1/17*d*x^17 + 10/17*e*x^17 + 5/8*d*x^16 + 45/16*e*x^16 + 3*d
*x^15 + 8*e*x^15 + 60/7*d*x^14 + 15*e*x^14 + 210/13*d*x^13 + 252/13*e*x^13
 + 21*d*x^12 + 35/2*e*x^12 + 210/11*d*x^11 + 120/11*e*x^11 + 12*d*x^10 + 9
/2*e*x^10 + 5*d*x^9 + 10/9*e*x^9 + 5/4*d*x^8 + 1/8*e*x^8 + 1/7*d*x^7
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{18}}{18}+\left(\frac{d}{17}+\frac{10\,e}{17}\right)\,x^{17}+\left(\frac{5\,d}{8}+\frac{45\,e}{16}\right)\,x^{16} +\left(3\,d+8\,e\right)\,x^{15}+\left(\frac{60\,d}{7}+15\,e\right)\,x^{14} +\left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)\,x^{13}+\left(21\,d+\frac{35\,e}{2}\right)\,x^{12} +\left(\frac{210\,d}{11}+\frac{120\,e}{11}\right)\,x^{11}+\left(12\,d+\frac{9\,e}{2}\right)\,x^{10} +\left(5\,d+\frac{10\,e}{9}\right)\,x^9+\left(\frac{5\,d}{4}+\frac{e}{8}\right)\,x^8+\frac{d\,x^7}{7}$$

input
```
int(x^6*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^8*((5*d)/4 + e/8) + x^15*(3*d + 8*e) + x^9*(5*d + (10*e)/9) + x^10*(12*d
 + (9*e)/2) + x^17*(d/17 + (10*e)/17) + x^12*(21*d + (35*e)/2) + x^16*((5*
d)/8 + (45*e)/16) + x^14*((60*d)/7 + 15*e) + x^11*((210*d)/11 + (120*e)/11
) + x^13*((210*d)/13 + (252*e)/13) + (d*x^7)/7 + (e*x^18)/18
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int x^6(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{x^7(136136e\,x^{11}+144144d\,x^{10}+1441440e\,x^{10}+1531530d\,x^9+6891885e\,x^9+7351344d\,x^8+19603584e}{}$$

input
```
int(x^6*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**7*(144144*d*x**10 + 1531530*d*x**9 + 7351344*d*x**8 + 21003840*d*x**7
+ 39584160*d*x**6 + 51459408*d*x**5 + 46781280*d*x**4 + 29405376*d*x**3 +
12252240*d*x**2 + 3063060*d*x + 350064*d + 136136*e*x**11 + 1441440*e*x**1
0 + 6891885*e*x**9 + 19603584*e*x**8 + 36756720*e*x**7 + 47500992*e*x**6 +
 42882840*e*x**5 + 26732160*e*x**4 + 11027016*e*x**3 + 2722720*e*x**2 + 30
6306*e*x))/2450448
```

3.190 $\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(5d-6e)(1+x)^{12}-\frac{5}{13}(2d-3e)(1+x)^{13}+\frac{5}{7}(d-2e)(1+x)^{14}-\frac{1}{3}(d-3e)(1+x)^{15}+\frac{1}{16}(d-6e)(1+x)^{16}+\frac{1}{17}e(1+x)^{17}$$

output
```
-1/11*(d-e)*(1+x)^11+1/12*(5*d-6*e)*(1+x)^12-5/13*(2*d-3*e)*(1+x)^13+5/7*(
d-2*e)*(1+x)^14-1/3*(d-3*e)*(1+x)^15+1/16*(d-6*e)*(1+x)^16+1/17*e*(1+x)^17
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^6}{6}+\frac{1}{7}(10d+e)x^7+\frac{5}{8}(9d+2e)x^8+\frac{5}{3}(8d+3e)x^9+3(7d+4e)x^{10}+\frac{42}{11}(6d+5e)x^{11}+\frac{7}{2}(5d+6e)x^{12}+\frac{30}{13}(4d+7e)x^{13}+\frac{15}{14}(3d+8e)x^{14}+\frac{1}{3}(2d+9e)x^{15}+\frac{1}{16}(d+10e)x^{16}+\frac{ex^{17}}{17}$$

input
```
Integrate[x^5*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^6)/6 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^8)/8 + (5*(8*d + 3*e)*x^
9)/3 + 3*(7*d + 4*e)*x^10 + (42*(6*d + 5*e)*x^11)/11 + (7*(5*d + 6*e)*x^12
)/2 + (30*(4*d + 7*e)*x^13)/13 + (15*(3*d + 8*e)*x^14)/14 + ((2*d + 9*e)*x
^15)/3 + ((d + 10*e)*x^16)/16 + (e*x^17)/17
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(x^2+2x+1\right)^5(d+ex)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^5(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left((x+1)^{15}(d-6e)-5(x+1)^{14}(d-3e)+10(x+1)^{13}(d-2e)-5(x+1)^{12}(2d-3e)+(x+1)^{11}(5d-6e)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}(x+1)^{16}(d-6e)-\frac{1}{3}(x+1)^{15}(d-3e)+\frac{5}{7}(x+1)^{14}(d-2e)-\frac{5}{13}(x+1)^{13}(2d-3e)+\frac{1}{12}(x+1)^{12}(5d-6e)-\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{17}e(x+1)^{17}$$

input
```
Int[x^5*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/11*((d - e)*(1 + x)^11) + ((5*d - 6*e)*(1 + x)^12)/12 - (5*(2*d - 3*e)*
(1 + x)^13)/13 + (5*(d - 2*e)*(1 + x)^14)/7 - ((d - 3*e)*(1 + x)^15)/3 + (
(d - 6*e)*(1 + x)^16)/16 + (e*(1 + x)^17)/17
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

method	result
norman	$\frac{d\,x^{6}}{6}+\left(\frac{10d}{7}+\frac{e}{7}\right)x^{7}+\left(\frac{45d}{8}+\frac{5e}{4}\right)x^{8}+\left(\frac{40d}{3}+5e\right)x^{9}+\left(21d+12e\right)x^{10}+\left(\frac{252d}{11}+\frac{210e}{11}\right)x^{11}+$
default	$\frac{x^{17}e}{17}+\frac{(d+10e)x^{16}}{16}+\frac{(10d+45e)x^{15}}{15}+\frac{(45d+120e)x^{14}}{14}+\frac{(120d+210e)x^{13}}{13}+\frac{(210d+252e)x^{12}}{12}+\frac{(252d+210e)x^{11}}{11}$
gosper	$x^{6}(48048e\,x^{11}+51051d\,x^{10}+510510e\,x^{10}+544544d\,x^{9}+2450448e\,x^{9}+2625480d\,x^{8}+7001280e\,x^{8}+7539840d\,x^{7}+13194720e$
risch	$\frac{1}{17}x^{17}e+\frac{1}{16}d\,x^{16}+\frac{5}{8}x^{16}e+\frac{2}{3}x^{15}d+3x^{15}e+\frac{45}{14}x^{14}d+\frac{60}{7}x^{14}e+\frac{120}{13}d\,x^{13}+\frac{210}{13}x^{13}e+\frac{35}{2}d\,x$
parallelrisch	$\frac{1}{17}x^{17}e+\frac{1}{16}d\,x^{16}+\frac{5}{8}x^{16}e+\frac{2}{3}x^{15}d+3x^{15}e+\frac{45}{14}x^{14}d+\frac{60}{7}x^{14}e+\frac{120}{13}d\,x^{13}+\frac{210}{13}x^{13}e+\frac{35}{2}d\,x$
orering	$x^{6}(48048e\,x^{11}+51051d\,x^{10}+510510e\,x^{10}+544544d\,x^{9}+2450448e\,x^{9}+2625480d\,x^{8}+7001280e\,x^{8}+7539840d\,x^{7}+13194720e$

input
```
int(x^5*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*d*x^6+(10/7*d+1/7*e)*x^7+(45/8*d+5/4*e)*x^8+(40/3*d+5*e)*x^9+(21*d+12*
e)*x^10+(252/11*d+210/11*e)*x^11+(35/2*d+21*e)*x^12+(120/13*d+210/13*e)*x^
13+(45/14*d+60/7*e)*x^14+(2/3*d+3*e)*x^15+(1/16*d+5/8*e)*x^16+1/17*x^17*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\begin{aligned}\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{17}\,ex^{17}+\frac{1}{16}\,(d+10\,e)x^{16}+\frac{1}{3}\,(2\,d+9\,e)x^{15}\\&\quad+\frac{15}{14}\,(3\,d+8\,e)x^{14}+\frac{30}{13}\,(4\,d+7\,e)x^{13}\\&\quad+\frac{7}{2}\,(5\,d+6\,e)x^{12}+\frac{42}{11}\,(6\,d+5\,e)x^{11}\\&\quad+3\,(7\,d+4\,e)x^{10}+\frac{5}{3}\,(8\,d+3\,e)x^{9}\\&\quad+\frac{5}{8}\,(9\,d+2\,e)x^{8}+\frac{1}{7}\,(10\,d+e)x^{7}+\frac{1}{6}\,dx^{6}\end{aligned}$$

input
```
integrate(x^5*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/17*e*x^17 + 1/16*(d + 10*e)*x^16 + 1/3*(2*d + 9*e)*x^15 + 15/14*(3*d + 8
*e)*x^14 + 30/13*(4*d + 7*e)*x^13 + 7/2*(5*d + 6*e)*x^12 + 42/11*(6*d + 5*
e)*x^11 + 3*(7*d + 4*e)*x^10 + 5/3*(8*d + 3*e)*x^9 + 5/8*(9*d + 2*e)*x^8 +
 1/7*(10*d + e)*x^7 + 1/6*d*x^6
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^6}{6}+\frac{ex^{17}}{17}+x^{16}\left(\frac{d}{16}+\frac{5e}{8}\right)+x^{15}\cdot\left(\frac{2d}{3}+3e\right)+x^{14}\cdot\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{13}\cdot\left(\frac{120d}{13}+\frac{210e}{13}\right)+x^{12}\cdot\left(\frac{35d}{2}+21e\right)+x^{11}\cdot\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^{10}\cdot(21d+12e)+x^9\cdot\left(\frac{40d}{3}+5e\right)+x^8\cdot\left(\frac{45d}{8}+\frac{5e}{4}\right)+x^7\cdot\left(\frac{10d}{7}+\frac{e}{7}\right)$$

input
```
integrate(x**5*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**6/6 + e*x**17/17 + x**16*(d/16 + 5*e/8) + x**15*(2*d/3 + 3*e) + x**14
*(45*d/14 + 60*e/7) + x**13*(120*d/13 + 210*e/13) + x**12*(35*d/2 + 21*e)
+ x**11*(252*d/11 + 210*e/11) + x**10*(21*d + 12*e) + x**9*(40*d/3 + 5*e)
+ x**8*(45*d/8 + 5*e/4) + x**7*(10*d/7 + e/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{17}\,ex^{17}+\frac{1}{16}\,(d+10\,e)x^{16}+\frac{1}{3}\,(2\,d+9\,e)x^{15}+\frac{15}{14}\,(3\,d+8\,e)x^{14}+\frac{30}{13}\,(4\,d+7\,e)x^{13}+\frac{7}{2}\,(5\,d+6\,e)x^{12}+\frac{42}{11}\,(6\,d+5\,e)x^{11}+3\,(7\,d+4\,e)x^{10}+\frac{5}{3}\,(8\,d+3\,e)x^9+\frac{5}{8}\,(9\,d+2\,e)x^8+\frac{1}{7}\,(10\,d+e)x^7+\frac{1}{6}\,dx^6$$

input
```
integrate(x^5*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/17*e*x^17 + 1/16*(d + 10*e)*x^16 + 1/3*(2*d + 9*e)*x^15 + 15/14*(3*d + 8
*e)*x^14 + 30/13*(4*d + 7*e)*x^13 + 7/2*(5*d + 6*e)*x^12 + 42/11*(6*d + 5*
e)*x^11 + 3*(7*d + 4*e)*x^10 + 5/3*(8*d + 3*e)*x^9 + 5/8*(9*d + 2*e)*x^8 +
 1/7*(10*d + e)*x^7 + 1/6*d*x^6
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.34

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{17}ex^{17}+\frac{1}{16}dx^{16}+\frac{5}{8}ex^{16}+\frac{2}{3}dx^{15}+3\,ex^{15}+\frac{45}{14}dx^{14}+\frac{60}{7}ex^{14}+\frac{120}{13}dx^{13}+\frac{210}{13}ex^{13}+\frac{35}{2}dx^{12}+21\,ex^{12}+\frac{252}{11}dx^{11}+\frac{210}{11}ex^{11}+21\,dx^{10}+12\,ex^{10}+\frac{40}{3}dx^9+5\,ex^9+\frac{45}{8}dx^8+\frac{5}{4}ex^8+\frac{10}{7}dx^7+\frac{1}{7}ex^7+\frac{1}{6}dx^6$$

input
```
integrate(x^5*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/17*e*x^17 + 1/16*d*x^16 + 5/8*e*x^16 + 2/3*d*x^15 + 3*e*x^15 + 45/14*d*x
^14 + 60/7*e*x^14 + 120/13*d*x^13 + 210/13*e*x^13 + 35/2*d*x^12 + 21*e*x^1
2 + 252/11*d*x^11 + 210/11*e*x^11 + 21*d*x^10 + 12*e*x^10 + 40/3*d*x^9 + 5
*e*x^9 + 45/8*d*x^8 + 5/4*e*x^8 + 10/7*d*x^7 + 1/7*e*x^7 + 1/6*d*x^6
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{17}}{17}+\left(\frac{d}{16}+\frac{5\,e}{8}\right)\,x^{16}+\left(\frac{2\,d}{3}+3\,e\right)\,x^{15} +\left(\frac{45\,d}{14}+\frac{60\,e}{7}\right)\,x^{14}+\left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)\,x^{13} +\left(\frac{35\,d}{2}+21\,e\right)\,x^{12}+\left(\frac{252\,d}{11}+\frac{210\,e}{11}\right)\,x^{11} +\left(21\,d+12\,e\right)\,x^{10}+\left(\frac{40\,d}{3}+5\,e\right)\,x^9 +\left(\frac{45\,d}{8}+\frac{5\,e}{4}\right)\,x^8+\left(\frac{10\,d}{7}+\frac{e}{7}\right)\,x^7+\frac{d\,x^6}{6}$$

input
```
int(x^5*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^15*((2*d)/3 + 3*e) + x^7*((10*d)/7 + e/7) + x^10*(21*d + 12*e) + x^16*(d
/16 + (5*e)/8) + x^9*((40*d)/3 + 5*e) + x^8*((45*d)/8 + (5*e)/4) + x^12*((
35*d)/2 + 21*e) + x^14*((45*d)/14 + (60*e)/7) + x^13*((120*d)/13 + (210*e)
/13) + x^11*((252*d)/11 + (210*e)/11) + (d*x^6)/6 + (e*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int x^5(d+ex)\left(1+2x+x^2\right)^5\,dx$$
$$= \frac{x^6(48048e\,x^{11}+51051d\,x^{10}+510510e\,x^{10}+544544d\,x^9+2450448e\,x^9+2625480d\,x^8+7001280e\,x^8+}{}$$

input
```
int(x^5*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**6*(51051*d*x**10 + 544544*d*x**9 + 2625480*d*x**8 + 7539840*d*x**7 + 1
4294280*d*x**6 + 18712512*d*x**5 + 17153136*d*x**4 + 10890880*d*x**3 + 459
4590*d*x**2 + 1166880*d*x + 136136*d + 48048*e*x**11 + 510510*e*x**10 + 24
50448*e*x**9 + 7001280*e*x**8 + 13194720*e*x**7 + 17153136*e*x**6 + 155937
60*e*x**5 + 9801792*e*x**4 + 4084080*e*x**3 + 1021020*e*x**2 + 116688*e*x)
)/816816
```

3.191 $\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{11}(d-e)(1+x)^{11}-\frac{1}{12}(4d-5e)(1+x)^{12}+\frac{2}{13}(3d-5e)(1+x)^{13}-\frac{1}{7}(2d-5e)(1+x)^{14}+\frac{1}{15}(d-5e)(1+x)^{15}+\frac{1}{16}e(1+x)^{16}$$

output
```
1/11*(d-e)*(1+x)^11-1/12*(4*d-5*e)*(1+x)^12+2/13*(3*d-5*e)*(1+x)^13-1/7*(2
*d-5*e)*(1+x)^14+1/15*(d-5*e)*(1+x)^15+1/16*e*(1+x)^16
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^5}{5}+\frac{1}{6}(10d+e)x^6+\frac{5}{7}(9d+2e)x^7+\frac{15}{8}(8d+3e)x^8+\frac{10}{3}(7d+4e)x^9+\frac{21}{5}(6d+5e)x^{10}+\frac{42}{11}(5d+6e)x^{11}+\frac{5}{2}(4d+7e)x^{12}+\frac{15}{13}(3d+8e)x^{13}+\frac{5}{14}(2d+9e)x^{14}+\frac{1}{15}(d+10e)x^{15}+\frac{ex^{16}}{16}$$

input
```
Integrate[x^4*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^5)/5 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^7)/7 + (15*(8*d + 3*e)*x
^8)/8 + (10*(7*d + 4*e)*x^9)/3 + (21*(6*d + 5*e)*x^10)/5 + (42*(5*d + 6*e)
*x^11)/11 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^13)/13 + (5*(2*d +
9*e)*x^14)/14 + ((d + 10*e)*x^15)/15 + (e*x^16)/16
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(x^2+2x+1\right)^5 (d+ex) \, dx$$

$$\Big\downarrow \; 1184$$

$$\int x^4 (x+1)^{10} (d+ex) dx$$

$$\Big\downarrow \; 85$$

$$\int ((x+1)^{14}(d-5e) - 2(x+1)^{13}(2d-5e) + 2(x+1)^{12}(3d-5e) + (x+1)^{11}(5e-4d) + (x+1)^{10}(d-e) + e($$

$$\Big\downarrow \; 2009$$

$$\frac{1}{15}(x+1)^{15}(d-5e) - \frac{1}{7}(x+1)^{14}(2d-5e) + \frac{2}{13}(x+1)^{13}(3d-5e) - \frac{1}{12}(x+1)^{12}(4d-5e) + \frac{1}{11}(x+1)^{11}(d-e) + \frac{1}{16}e(x+1)^{16}$$

input
```
Int[x^4*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
((d - e)*(1 + x)^11)/11 - ((4*d - 5*e)*(1 + x)^12)/12 + (2*(3*d - 5*e)*(1
+ x)^13)/13 - ((2*d - 5*e)*(1 + x)^14)/7 + ((d - 5*e)*(1 + x)^15)/15 + (e*
(1 + x)^16)/16
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

method	result
norman	$\frac{d\,x^5}{5}+\left(\frac{5d}{3}+\frac{e}{6}\right)x^6+\left(\frac{45d}{7}+\frac{10e}{7}\right)x^7+\left(15d+\frac{45e}{8}\right)x^8+\left(\frac{70d}{3}+\frac{40e}{3}\right)x^9+\left(\frac{126d}{5}+21e\right)x^{10}+$
default	$\frac{x^{16}e}{16}+\frac{(d+10e)x^{15}}{15}+\frac{(10d+45e)x^{14}}{14}+\frac{(45d+120e)x^{13}}{13}+\frac{(120d+210e)x^{12}}{12}+\frac{(210d+252e)x^{11}}{11}+\frac{(252d+210e)x^{10}}{10}$
gosper	$\frac{x^5\left(15015e\,x^{11}+16016d\,x^{10}+160160e\,x^{10}+171600d\,x^9+772200e\,x^9+831600d\,x^8+2217600e\,x^8+2402400d\,x^7+4204200e\,x^7+\right.}{}$
risch	$\frac{1}{16}x^{16}e+\frac{1}{15}x^{15}d+\frac{2}{3}x^{15}e+\frac{5}{7}x^{14}d+\frac{45}{14}x^{14}e+\frac{45}{13}d\,x^{13}+\frac{120}{13}x^{13}e+10d\,x^{12}+\frac{35}{2}x^{12}e+\frac{210}{11}x^{1}$
parallelrisch	$\frac{1}{16}x^{16}e+\frac{1}{15}x^{15}d+\frac{2}{3}x^{15}e+\frac{5}{7}x^{14}d+\frac{45}{14}x^{14}e+\frac{45}{13}d\,x^{13}+\frac{120}{13}x^{13}e+10d\,x^{12}+\frac{35}{2}x^{12}e+\frac{210}{11}x^{1}$
orering	$\frac{x^5\left(15015e\,x^{11}+16016d\,x^{10}+160160e\,x^{10}+171600d\,x^9+772200e\,x^9+831600d\,x^8+2217600e\,x^8+2402400d\,x^7+4204200e\,x^7+\right.}{}$

input
```
int(x^4*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*d*x^5+(5/3*d+1/6*e)*x^6+(45/7*d+10/7*e)*x^7+(15*d+45/8*e)*x^8+(70/3*d+
40/3*e)*x^9+(126/5*d+21*e)*x^10+(210/11*d+252/11*e)*x^11+(10*d+35/2*e)*x^1
2+(45/13*d+120/13*e)*x^13+(5/7*d+45/14*e)*x^14+(1/15*d+2/3*e)*x^15+1/16*x^
16*e
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{16}\,ex^{16} + \frac{1}{15}\,(d+10\,e)x^{15} + \frac{5}{14}\,(2\,d+9\,e)x^{14} + \frac{15}{13}\,(3\,d+8\,e)x^{13} + \frac{5}{2}\,(4\,d+7\,e)x^{12} + \frac{42}{11}\,(5\,d+6\,e)x^{11} + \frac{21}{5}\,(6\,d+5\,e)x^{10} + \frac{10}{3}\,(7\,d+4\,e)x^9 + \frac{15}{8}\,(8\,d+3\,e)x^8 + \frac{5}{7}\,(9\,d+2\,e)x^7 + \frac{1}{6}\,(10\,d+e)x^6 + \frac{1}{5}\,dx^5$$

input
```
integrate(x^4*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/16*e*x^16 + 1/15*(d + 10*e)*x^15 + 5/14*(2*d + 9*e)*x^14 + 15/13*(3*d +
8*e)*x^13 + 5/2*(4*d + 7*e)*x^12 + 42/11*(5*d + 6*e)*x^11 + 21/5*(6*d + 5*
e)*x^10 + 10/3*(7*d + 4*e)*x^9 + 15/8*(8*d + 3*e)*x^8 + 5/7*(9*d + 2*e)*x^
7 + 1/6*(10*d + e)*x^6 + 1/5*d*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.60

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^5}{5}+\frac{ex^{16}}{16}+x^{15}\left(\frac{d}{15}+\frac{2e}{3}\right)+x^{14}\cdot\left(\frac{5d}{7}+\frac{45e}{14}\right)+x^{13}\cdot\left(\frac{45d}{13}+\frac{120e}{13}\right)+x^{12}\cdot\left(10d+\frac{35e}{2}\right)+x^{11}\cdot\left(\frac{210d}{11}+\frac{252e}{11}\right)+x^{10}\cdot\left(\frac{126d}{5}+21e\right)+x^9\cdot\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^8\cdot\left(15d+\frac{45e}{8}\right)+x^7\cdot\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^6\cdot\left(\frac{5d}{3}+\frac{e}{6}\right)$$

input
```
integrate(x**4*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**5/5 + e*x**16/16 + x**15*(d/15 + 2*e/3) + x**14*(5*d/7 + 45*e/14) + x
**13*(45*d/13 + 120*e/13) + x**12*(10*d + 35*e/2) + x**11*(210*d/11 + 252*
e/11) + x**10*(126*d/5 + 21*e) + x**9*(70*d/3 + 40*e/3) + x**8*(15*d + 45*
e/8) + x**7*(45*d/7 + 10*e/7) + x**6*(5*d/3 + e/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{16}\,ex^{16}+\frac{1}{15}\,(d+10\,e)x^{15}+\frac{5}{14}\,(2\,d+9\,e)x^{14}+\frac{15}{13}\,(3\,d+8\,e)x^{13}+\frac{5}{2}\,(4\,d+7\,e)x^{12}+\frac{42}{11}\,(5\,d+6\,e)x^{11}+\frac{21}{5}\,(6\,d+5\,e)x^{10}+\frac{10}{3}\,(7\,d+4\,e)x^9+\frac{15}{8}\,(8\,d+3\,e)x^8+\frac{5}{7}\,(9\,d+2\,e)x^7+\frac{1}{6}\,(10\,d+e)x^6+\frac{1}{5}\,dx^5$$

input
```
integrate(x^4*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/16*e*x^16 + 1/15*(d + 10*e)*x^15 + 5/14*(2*d + 9*e)*x^14 + 15/13*(3*d +
8*e)*x^13 + 5/2*(4*d + 7*e)*x^12 + 42/11*(5*d + 6*e)*x^11 + 21/5*(6*d + 5*
e)*x^10 + 10/3*(7*d + 4*e)*x^9 + 15/8*(8*d + 3*e)*x^8 + 5/7*(9*d + 2*e)*x^
7 + 1/6*(10*d + e)*x^6 + 1/5*d*x^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.53

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{16}\,ex^{16}+\frac{1}{15}\,dx^{15}+\frac{2}{3}\,ex^{15}+\frac{5}{7}\,dx^{14}+\frac{45}{14}\,ex^{14}+\frac{45}{13}\,dx^{13}+\frac{120}{13}\,ex^{13}+10\,dx^{12}+\frac{35}{2}\,ex^{12}+\frac{210}{11}\,dx^{11}+\frac{252}{11}\,ex^{11}+\frac{126}{5}\,dx^{10}+21\,ex^{10}+\frac{70}{3}\,dx^9+\frac{40}{3}\,ex^9+15\,dx^8+\frac{45}{8}\,ex^8+\frac{45}{7}\,dx^7+\frac{10}{7}\,ex^7+\frac{5}{3}\,dx^6+\frac{1}{6}\,ex^6+\frac{1}{5}\,dx^5$$

input
```
integrate(x^4*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/16*e*x^16 + 1/15*d*x^15 + 2/3*e*x^15 + 5/7*d*x^14 + 45/14*e*x^14 + 45/13
*d*x^13 + 120/13*e*x^13 + 10*d*x^12 + 35/2*e*x^12 + 210/11*d*x^11 + 252/11
*e*x^11 + 126/5*d*x^10 + 21*e*x^10 + 70/3*d*x^9 + 40/3*e*x^9 + 15*d*x^8 +
45/8*e*x^8 + 45/7*d*x^7 + 10/7*e*x^7 + 5/3*d*x^6 + 1/6*e*x^6 + 1/5*d*x^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{16}}{16}+\left(\frac{d}{15}+\frac{2\,e}{3}\right)\,x^{15}+\left(\frac{5\,d}{7}+\frac{45\,e}{14}\right)\,x^{14}+\left(\frac{45\,d}{13}+\frac{120\,e}{13}\right)\,x^{13}+\left(10\,d+\frac{35\,e}{2}\right)\,x^{12}+\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)\,x^{11}+\left(\frac{126\,d}{5}+21\,e\right)\,x^{10}+\left(\frac{70\,d}{3}+\frac{40\,e}{3}\right)\,x^9+\left(15\,d+\frac{45\,e}{8}\right)\,x^8+\left(\frac{45\,d}{7}+\frac{10\,e}{7}\right)\,x^7+\left(\frac{5\,d}{3}+\frac{e}{6}\right)\,x^6+\frac{d\,x^5}{5}$$

input
```
int(x^4*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^6*((5*d)/3 + e/6) + x^15*(d/15 + (2*e)/3) + x^12*(10*d + (35*e)/2) + x^7
*((45*d)/7 + (10*e)/7) + x^8*(15*d + (45*e)/8) + x^14*((5*d)/7 + (45*e)/14
) + x^9*((70*d)/3 + (40*e)/3) + x^10*((126*d)/5 + 21*e) + x^13*((45*d)/13
+ (120*e)/13) + x^11*((210*d)/11 + (252*e)/11) + (d*x^5)/5 + (e*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\int x^4(d+ex)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^5(15015e\,x^{11}+16016d\,x^{10}+160160e\,x^{10}+171600d\,x^9+772200e\,x^9+831600d\,x^8+2217600e\,x^8+24}{}$$

input
```
int(x^4*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**5*(16016*d*x**10 + 171600*d*x**9 + 831600*d*x**8 + 2402400*d*x**7 + 45
86400*d*x**6 + 6054048*d*x**5 + 5605600*d*x**4 + 3603600*d*x**3 + 1544400*
d*x**2 + 400400*d*x + 48048*d + 15015*e*x**11 + 160160*e*x**10 + 772200*e*
x**9 + 2217600*e*x**8 + 4204200*e*x**7 + 5503680*e*x**6 + 5045040*e*x**5 +
 3203200*e*x**4 + 1351350*e*x**3 + 343200*e*x**2 + 40040*e*x))/240240
```

3.192 $\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(3d-4e)(1+x)^{12}-\frac{3}{13}(d-2e)(1+x)^{13}+\frac{1}{14}(d-4e)(1+x)^{14}+\frac{1}{15}e(1+x)^{15}$$

output
```
-1/11*(d-e)*(1+x)^11+1/12*(3*d-4*e)*(1+x)^12-3/13*(d-2*e)*(1+x)^13+1/14*(d
-4*e)*(1+x)^14+1/15*e*(1+x)^15
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. $2(69) = 138$.

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^4}{4}+\frac{1}{5}(10d+e)x^5+\frac{5}{6}(9d+2e)x^6+\frac{15}{7}(8d+3e)x^7+\frac{15}{4}(7d+4e)x^8+\frac{14}{3}(6d+5e)x^9+\frac{21}{5}(5d+6e)x^{10}+\frac{30}{11}(4d+7e)x^{11}+\frac{5}{4}(3d+8e)x^{12}+\frac{5}{13}(2d+9e)x^{13}+\frac{1}{14}(d+10e)x^{14}+\frac{ex^{15}}{15}$$

input
```
Integrate[x^3*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^4)/4 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x
^7)/7 + (15*(7*d + 4*e)*x^8)/4 + (14*(6*d + 5*e)*x^9)/3 + (21*(5*d + 6*e)*
x^10)/5 + (30*(4*d + 7*e)*x^11)/11 + (5*(3*d + 8*e)*x^12)/4 + (5*(2*d + 9*
e)*x^13)/13 + ((d + 10*e)*x^14)/14 + (e*x^15)/15
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(x^2+2x+1\right)^5(d+ex)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^3(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left((x+1)^{13}(d-4e)-3(x+1)^{12}(d-2e)+(x+1)^{11}(3d-4e)+(x+1)^{10}(e-d)+e(x+1)^{14}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{14}(x+1)^{14}(d-4e)-\frac{3}{13}(x+1)^{13}(d-2e)+\frac{1}{12}(x+1)^{12}(3d-4e)-\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{15}e(x+1)^{15}$$

input
```
Int[x^3*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/11*((d - e)*(1 + x)^11) + ((3*d - 4*e)*(1 + x)^12)/12 - (3*(d - 2*e)*(1
 + x)^13)/13 + ((d - 4*e)*(1 + x)^14)/14 + (e*(1 + x)^15)/15
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 0.87 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

method	result
norman	$\frac{d x^4}{4}+\left(2d+\frac{e}{5}\right)x^5+\left(\frac{15d}{2}+\frac{5e}{3}\right)x^6+\left(\frac{120d}{7}+\frac{45e}{7}\right)x^7+\left(\frac{105d}{4}+15e\right)x^8+\left(28d+\frac{70e}{3}\right)x^9+$
default	$\frac{x^{15}e}{15}+\frac{(d+10e)x^{14}}{14}+\frac{(10d+45e)x^{13}}{13}+\frac{(45d+120e)x^{12}}{12}+\frac{(120d+210e)x^{11}}{11}+\frac{(210d+252e)x^{10}}{10}+\frac{(252d+210e)x^9}{9}$
gosper	$x^4(4004e\,x^{11}+4290d\,x^{10}+42900e\,x^{10}+46200d\,x^9+207900e\,x^9+225225d\,x^8+600600e\,x^8+655200d\,x^7+1146600e\,x^7+126126$
risch	$\frac{1}{15}x^{15}e+\frac{1}{14}x^{14}d+\frac{5}{7}x^{14}e+\frac{10}{13}d\,x^{13}+\frac{45}{13}x^{13}e+\frac{15}{4}d\,x^{12}+10x^{12}e+\frac{120}{11}x^{11}d+\frac{210}{11}e\,x^{11}+21d$
parallelrisch	$\frac{1}{15}x^{15}e+\frac{1}{14}x^{14}d+\frac{5}{7}x^{14}e+\frac{10}{13}d\,x^{13}+\frac{45}{13}x^{13}e+\frac{15}{4}d\,x^{12}+10x^{12}e+\frac{120}{11}x^{11}d+\frac{210}{11}e\,x^{11}+21d$
orering	$x^4(4004e\,x^{11}+4290d\,x^{10}+42900e\,x^{10}+46200d\,x^9+207900e\,x^9+225225d\,x^8+600600e\,x^8+655200d\,x^7+1146600e\,x^7+126126$

input
```
int(x^3*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*d*x^4+(2*d+1/5*e)*x^5+(15/2*d+5/3*e)*x^6+(120/7*d+45/7*e)*x^7+(105/4*d
+15*e)*x^8+(28*d+70/3*e)*x^9+(21*d+126/5*e)*x^10+(120/11*d+210/11*e)*x^11+
(15/4*d+10*e)*x^12+(10/13*d+45/13*e)*x^13+(1/14*d+5/7*e)*x^14+1/15*x^15*e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(59) = 118$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\begin{aligned}\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{15}ex^{15}+\frac{1}{14}(d+10e)x^{14}+\frac{5}{13}(2d+9e)x^{13}\\&\quad+\frac{5}{4}(3d+8e)x^{12}+\frac{30}{11}(4d+7e)x^{11}\\&\quad+\frac{21}{5}(5d+6e)x^{10}+\frac{14}{3}(6d+5e)x^9\\&\quad+\frac{15}{4}(7d+4e)x^8+\frac{15}{7}(8d+3e)x^7\\&\quad+\frac{5}{6}(9d+2e)x^6+\frac{1}{5}(10d+e)x^5+\frac{1}{4}dx^4\end{aligned}$$

input
```
integrate(x^3*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/15*e*x^15 + 1/14*(d + 10*e)*x^14 + 5/13*(2*d + 9*e)*x^13 + 5/4*(3*d + 8*
e)*x^12 + 30/11*(4*d + 7*e)*x^11 + 21/5*(5*d + 6*e)*x^10 + 14/3*(6*d + 5*e
)*x^9 + 15/4*(7*d + 4*e)*x^8 + 15/7*(8*d + 3*e)*x^7 + 5/6*(9*d + 2*e)*x^6
+ 1/5*(10*d + e)*x^5 + 1/4*d*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(60) = 120$.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^4}{4}+\frac{ex^{15}}{15}+x^{14}\left(\frac{d}{14}+\frac{5e}{7}\right)+x^{13}\cdot\left(\frac{10d}{13}+\frac{45e}{13}\right)+x^{12}\cdot\left(\frac{15d}{4}+10e\right)+x^{11}\cdot\left(\frac{120d}{11}+\frac{210e}{11}\right)+x^{10}\cdot\left(21d+\frac{126e}{5}\right)+x^9\cdot\left(28d+\frac{70e}{3}\right)+x^8\cdot\left(\frac{105d}{4}+15e\right)+x^7\cdot\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^6\cdot\left(\frac{15d}{2}+\frac{5e}{3}\right)+x^5\cdot\left(2d+\frac{e}{5}\right)$$

input
```
integrate(x**3*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**4/4 + e*x**15/15 + x**14*(d/14 + 5*e/7) + x**13*(10*d/13 + 45*e/13) +
 x**12*(15*d/4 + 10*e) + x**11*(120*d/11 + 210*e/11) + x**10*(21*d + 126*e
/5) + x**9*(28*d + 70*e/3) + x**8*(105*d/4 + 15*e) + x**7*(120*d/7 + 45*e/
7) + x**6*(15*d/2 + 5*e/3) + x**5*(2*d + e/5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(59) = 118.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{15}\,ex^{15}+\frac{1}{14}\,(d+10\,e)x^{14}+\frac{5}{13}\,(2\,d+9\,e)x^{13}+\frac{5}{4}\,(3\,d+8\,e)x^{12}+\frac{30}{11}\,(4\,d+7\,e)x^{11}+\frac{21}{5}\,(5\,d+6\,e)x^{10}+\frac{14}{3}\,(6\,d+5\,e)x^{9}+\frac{15}{4}\,(7\,d+4\,e)x^{8}+\frac{15}{7}\,(8\,d+3\,e)x^{7}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{5}\,(10\,d+e)x^{5}+\frac{1}{4}\,dx^4$$

input
```
integrate(x^3*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/15*e*x^15 + 1/14*(d + 10*e)*x^14 + 5/13*(2*d + 9*e)*x^13 + 5/4*(3*d + 8*
e)*x^12 + 30/11*(4*d + 7*e)*x^11 + 21/5*(5*d + 6*e)*x^10 + 14/3*(6*d + 5*e
)*x^9 + 15/4*(7*d + 4*e)*x^8 + 15/7*(8*d + 3*e)*x^7 + 5/6*(9*d + 2*e)*x^6
+ 1/5*(10*d + e)*x^5 + 1/4*d*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(59) = 118.

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{15}\,ex^{15}+\frac{1}{14}\,dx^{14}+\frac{5}{7}\,ex^{14}+\frac{10}{13}\,dx^{13}+\frac{45}{13}\,ex^{13}+\frac{15}{4}\,dx^{12}+10\,ex^{12}+\frac{120}{11}\,dx^{11}+\frac{210}{11}\,ex^{11}+21\,dx^{10}+\frac{126}{5}\,ex^{10}+28\,dx^9+\frac{70}{3}\,ex^9+\frac{105}{4}\,dx^8+15\,ex^8+\frac{120}{7}\,dx^7+\frac{45}{7}\,ex^7+\frac{15}{2}\,dx^6+\frac{5}{3}\,ex^6+2\,dx^5+\frac{1}{5}\,ex^5+\frac{1}{4}\,dx^4$$

input
```
integrate(x^3*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/15*e*x^15 + 1/14*d*x^14 + 5/7*e*x^14 + 10/13*d*x^13 + 45/13*e*x^13 + 15/
4*d*x^12 + 10*e*x^12 + 120/11*d*x^11 + 210/11*e*x^11 + 21*d*x^10 + 126/5*e
*x^10 + 28*d*x^9 + 70/3*e*x^9 + 105/4*d*x^8 + 15*e*x^8 + 120/7*d*x^7 + 45/
7*e*x^7 + 15/2*d*x^6 + 5/3*e*x^6 + 2*d*x^5 + 1/5*e*x^5 + 1/4*d*x^4
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{15}}{15}+\left(\frac{d}{14}+\frac{5\,e}{7}\right)\,x^{14}+\left(\frac{10\,d}{13}+\frac{45\,e}{13}\right)\,x^{13} +\left(\frac{15\,d}{4}+10\,e\right)\,x^{12}+\left(\frac{120\,d}{11}+\frac{210\,e}{11}\right)\,x^{11} +\left(21\,d+\frac{126\,e}{5}\right)\,x^{10}+\left(28\,d+\frac{70\,e}{3}\right)\,x^9 +\left(\frac{105\,d}{4}+15\,e\right)\,x^8+\left(\frac{120\,d}{7}+\frac{45\,e}{7}\right)\,x^7 +\left(\frac{15\,d}{2}+\frac{5\,e}{3}\right)\,x^6+\left(2\,d+\frac{e}{5}\right)\,x^5+\frac{d\,x^4}{4}$$

input
```
int(x^3*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^5*(2*d + e/5) + x^6*((15*d)/2 + (5*e)/3) + x^12*((15*d)/4 + 10*e) + x^14
*(d/14 + (5*e)/7) + x^13*((10*d)/13 + (45*e)/13) + x^9*(28*d + (70*e)/3) +
 x^8*((105*d)/4 + 15*e) + x^10*(21*d + (126*e)/5) + x^7*((120*d)/7 + (45*e
)/7) + x^11*((120*d)/11 + (210*e)/11) + (d*x^4)/4 + (e*x^15)/15
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

$$\int x^3(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{x^4(4004e\,x^{11}+4290d\,x^{10}+42900e\,x^{10}+46200d\,x^9+207900e\,x^9+225225d\,x^8+600600e\,x^8+655200d}{}$$

input
```
int(x^3*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**4*(4290*d*x**10 + 46200*d*x**9 + 225225*d*x**8 + 655200*d*x**7 + 12612
60*d*x**6 + 1681680*d*x**5 + 1576575*d*x**4 + 1029600*d*x**3 + 450450*d*x*
*2 + 120120*d*x + 15015*d + 4004*e*x**11 + 42900*e*x**10 + 207900*e*x**9 +
 600600*e*x**8 + 1146600*e*x**7 + 1513512*e*x**6 + 1401400*e*x**5 + 900900
*e*x**4 + 386100*e*x**3 + 100100*e*x**2 + 12012*e*x))/60060
```

3.193 $\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{11}(d-e)(1+x)^{11}-\frac{1}{12}(2d-3e)(1+x)^{12}+\frac{1}{13}(d-3e)(1+x)^{13}+\frac{1}{14}e(1+x)^{14}$$

output
```
1/11*(d-e)*(1+x)^11-1/12*(2*d-3*e)*(1+x)^12+1/13*(d-3*e)*(1+x)^13+1/14*e*(
1+x)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(55) = 110.

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.69

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^3}{3}+\frac{1}{4}(10d+e)x^4+(9d+2e)x^5+\frac{5}{2}(8d+3e)x^6+\frac{30}{7}(7d+4e)x^7+\frac{21}{4}(6d+5e)x^8+\frac{14}{3}(5d+6e)x^9+3(4d+7e)x^{10}+\frac{15}{11}(3d+8e)x^{11}+\frac{5}{12}(2d+9e)x^{12}+\frac{1}{13}(d+10e)x^{13}+\frac{ex^{14}}{14}$$

input
```
Integrate[x^2*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^3)/3 + ((10*d + e)*x^4)/4 + (9*d + 2*e)*x^5 + (5*(8*d + 3*e)*x^6)/2 +
 (30*(7*d + 4*e)*x^7)/7 + (21*(6*d + 5*e)*x^8)/4 + (14*(5*d + 6*e)*x^9)/3
+ 3*(4*d + 7*e)*x^10 + (15*(3*d + 8*e)*x^11)/11 + (5*(2*d + 9*e)*x^12)/12
+ ((d + 10*e)*x^13)/13 + (e*x^14)/14
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(x^2+2x+1\right)^5 (d+ex)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^2(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left((x+1)^{12}(d-3e)+(x+1)^{11}(3e-2d)+(x+1)^{10}(d-e)+e(x+1)^{13}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{13}(x+1)^{13}(d-3e)-\frac{1}{12}(x+1)^{12}(2d-3e)+\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{14}e(x+1)^{14}$$

input
```
Int[x^2*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
((d - e)*(1 + x)^11)/11 - ((2*d - 3*e)*(1 + x)^12)/12 + ((d - 3*e)*(1 + x)
^13)/13 + (e*(1 + x)^14)/14
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(47) = 94$.

Time = 0.80 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

method	result
norman	$\frac{d\,x^{3}}{3}+\left(\frac{5d}{2}+\frac{e}{4}\right)x^{4}+\left(9d+2e\right)x^{5}+\left(20d+\frac{15e}{2}\right)x^{6}+\left(30d+\frac{120e}{7}\right)x^{7}+\left(\frac{63d}{2}+\frac{105e}{4}\right)x^{8}+($
default	$\frac{x^{14}e}{14}+\frac{(d+10e)x^{13}}{13}+\frac{(10d+45e)x^{12}}{12}+\frac{(45d+120e)x^{11}}{11}+\frac{(120d+210e)x^{10}}{10}+\frac{(210d+252e)x^{9}}{9}+\frac{(252d+210e)x^{8}}{8}$
gosper	$x^{3}\left(858e\,x^{11}+924d\,x^{10}+9240e\,x^{10}+10010d\,x^{9}+45045e\,x^{9}+49140d\,x^{8}+131040e\,x^{8}+144144d\,x^{7}+252252e\,x^{7}+280280d\,x^{6}+3\right.$
risch	$\frac{1}{14}x^{14}e+\frac{1}{13}d\,x^{13}+\frac{10}{13}x^{13}e+\frac{5}{6}d\,x^{12}+\frac{15}{4}x^{12}e+\frac{45}{11}x^{11}d+\frac{120}{11}e\,x^{11}+12d\,x^{10}+21e\,x^{10}+\frac{70}{3}c$
parallelrisch	$\frac{1}{14}x^{14}e+\frac{1}{13}d\,x^{13}+\frac{10}{13}x^{13}e+\frac{5}{6}d\,x^{12}+\frac{15}{4}x^{12}e+\frac{45}{11}x^{11}d+\frac{120}{11}e\,x^{11}+12d\,x^{10}+21e\,x^{10}+\frac{70}{3}c$
orering	$x^{3}\left(858e\,x^{11}+924d\,x^{10}+9240e\,x^{10}+10010d\,x^{9}+45045e\,x^{9}+49140d\,x^{8}+131040e\,x^{8}+144144d\,x^{7}+252252e\,x^{7}+280280d\,x^{6}+3\right.$

input
```
int(x^2*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x^3+(5/2*d+1/4*e)*x^4+(9*d+2*e)*x^5+(20*d+15/2*e)*x^6+(30*d+120/7*e)
*x^7+(63/2*d+105/4*e)*x^8+(70/3*d+28*e)*x^9+(12*d+21*e)*x^10+(45/11*d+120/
11*e)*x^11+(5/6*d+15/4*e)*x^12+(1/13*d+10/13*e)*x^13+1/14*x^14*e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(47) = 94$.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.33

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{14}\,ex^{14}+\frac{1}{13}\,(d+10\,e)x^{13}+\frac{5}{12}\,(2\,d+9\,e)x^{12}+\frac{15}{11}\,(3\,d+8\,e)x^{11}+3\,(4\,d+7\,e)x^{10}+\frac{14}{3}\,(5\,d+6\,e)x^9+\frac{21}{4}\,(6\,d+5\,e)x^8+\frac{30}{7}\,(7\,d+4\,e)x^7+\frac{5}{2}\,(8\,d+3\,e)x^6+(9\,d+2\,e)x^5+\frac{1}{4}\,(10\,d+e)x^4+\frac{1}{3}\,dx^3$$

input
```
integrate(x^2*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/14*e*x^14 + 1/13*(d + 10*e)*x^13 + 5/12*(2*d + 9*e)*x^12 + 15/11*(3*d +
8*e)*x^11 + 3*(4*d + 7*e)*x^10 + 14/3*(5*d + 6*e)*x^9 + 21/4*(6*d + 5*e)*x
^8 + 30/7*(7*d + 4*e)*x^7 + 5/2*(8*d + 3*e)*x^6 + (9*d + 2*e)*x^5 + 1/4*(1
0*d + e)*x^4 + 1/3*d*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^3}{3}+\frac{ex^{14}}{14}+x^{13}\left(\frac{d}{13}+\frac{10e}{13}\right)+x^{12}\cdot\left(\frac{5d}{6}+\frac{15e}{4}\right)+x^{11}\cdot\left(\frac{45d}{11}+\frac{120e}{11}\right)+x^{10}\cdot(12d+21e)+x^9\cdot\left(\frac{70d}{3}+28e\right)+x^8\cdot\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^7\cdot\left(30d+\frac{120e}{7}\right)+x^6\cdot\left(20d+\frac{15e}{2}\right)+x^5\cdot(9d+2e)+x^4\cdot\left(\frac{5d}{2}+\frac{e}{4}\right)$$

input
```
integrate(x**2*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**3/3 + e*x**14/14 + x**13*(d/13 + 10*e/13) + x**12*(5*d/6 + 15*e/4) +
x**11*(45*d/11 + 120*e/11) + x**10*(12*d + 21*e) + x**9*(70*d/3 + 28*e) +
x**8*(63*d/2 + 105*e/4) + x**7*(30*d + 120*e/7) + x**6*(20*d + 15*e/2) + x
**5*(9*d + 2*e) + x**4*(5*d/2 + e/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(47) = 94$.

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.33

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{14}\,ex^{14}+\frac{1}{13}\,(d+10\,e)x^{13}+\frac{5}{12}\,(2\,d+9\,e)x^{12}+\frac{15}{11}\,(3\,d+8\,e)x^{11}+3\,(4\,d+7\,e)x^{10}+\frac{14}{3}\,(5\,d+6\,e)x^9+\frac{21}{4}\,(6\,d+5\,e)x^8+\frac{30}{7}\,(7\,d+4\,e)x^7+\frac{5}{2}\,(8\,d+3\,e)x^6+(9\,d+2\,e)x^5+\frac{1}{4}\,(10\,d+e)x^4+\frac{1}{3}\,dx^3$$

input
```
integrate(x^2*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/14*e*x^14 + 1/13*(d + 10*e)*x^13 + 5/12*(2*d + 9*e)*x^12 + 15/11*(3*d +
8*e)*x^11 + 3*(4*d + 7*e)*x^10 + 14/3*(5*d + 6*e)*x^9 + 21/4*(6*d + 5*e)*x
^8 + 30/7*(7*d + 4*e)*x^7 + 5/2*(8*d + 3*e)*x^6 + (9*d + 2*e)*x^5 + 1/4*(1
0*d + e)*x^4 + 1/3*d*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{14}ex^{14}+\frac{1}{13}dx^{13}+\frac{10}{13}ex^{13}+\frac{5}{6}dx^{12}+\frac{15}{4}ex^{12}+\frac{45}{11}dx^{11} + \frac{120}{11}ex^{11}+12\,dx^{10}+21\,ex^{10}+\frac{70}{3}dx^9+28\,ex^9 + \frac{63}{2}dx^8+\frac{105}{4}ex^8+30\,dx^7+\frac{120}{7}ex^7+20\,dx^6 + \frac{15}{2}ex^6+9\,dx^5+2\,ex^5+\frac{5}{2}dx^4+\frac{1}{4}ex^4+\frac{1}{3}dx^3$$

input
```
integrate(x^2*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/14*e*x^14 + 1/13*d*x^13 + 10/13*e*x^13 + 5/6*d*x^12 + 15/4*e*x^12 + 45/1
1*d*x^11 + 120/11*e*x^11 + 12*d*x^10 + 21*e*x^10 + 70/3*d*x^9 + 28*e*x^9 +
 63/2*d*x^8 + 105/4*e*x^8 + 30*d*x^7 + 120/7*e*x^7 + 20*d*x^6 + 15/2*e*x^6
 + 9*d*x^5 + 2*e*x^5 + 5/2*d*x^4 + 1/4*e*x^4 + 1/3*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.24

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{14}}{14}+\left(\frac{d}{13}+\frac{10\,e}{13}\right)x^{13}+\left(\frac{5\,d}{6}+\frac{15\,e}{4}\right)x^{12}+\left(\frac{45\,d}{11}+\frac{120\,e}{11}\right)x^{11}+\left(12\,d+21\,e\right)x^{10}+\left(\frac{70\,d}{3}+28\,e\right)x^9+\left(\frac{63\,d}{2}+\frac{105\,e}{4}\right)x^8+\left(30\,d+\frac{120\,e}{7}\right)x^7+\left(20\,d+\frac{15\,e}{2}\right)x^6+\left(9\,d+2\,e\right)x^5+\left(\frac{5\,d}{2}+\frac{e}{4}\right)x^4+\frac{d\,x^3}{3}$$

input
```
int(x^2*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^4*((5*d)/2 + e/4) + x^5*(9*d + 2*e) + x^12*((5*d)/6 + (15*e)/4) + x^6*(2
0*d + (15*e)/2) + x^10*(12*d + 21*e) + x^13*(d/13 + (10*e)/13) + x^9*((70*
d)/3 + 28*e) + x^7*(30*d + (120*e)/7) + x^8*((63*d)/2 + (105*e)/4) + x^11*
((45*d)/11 + (120*e)/11) + (d*x^3)/3 + (e*x^14)/14
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int x^2(d+ex)\left(1+2x+x^2\right)^5\,dx$$

$$= \frac{x^3(858e\,x^{11}+924d\,x^{10}+9240e\,x^{10}+10010d\,x^9+45045e\,x^9+49140d\,x^8+131040e\,x^8+144144d\,x^7+}{}$$

input
```
int(x^2*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**3*(924*d*x**10 + 10010*d*x**9 + 49140*d*x**8 + 144144*d*x**7 + 280280*
d*x**6 + 378378*d*x**5 + 360360*d*x**4 + 240240*d*x**3 + 108108*d*x**2 + 3
0030*d*x + 4004*d + 858*e*x**11 + 9240*e*x**10 + 45045*e*x**9 + 131040*e*x
**8 + 252252*e*x**7 + 336336*e*x**6 + 315315*e*x**5 + 205920*e*x**4 + 9009
0*e*x**3 + 24024*e*x**2 + 3003*e*x))/12012
```

3.194 $\int x(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int x(d+ex)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{11}(d-e)(1+x)^{11}+\frac{1}{12}(d-2e)(1+x)^{12}+\frac{1}{13}e(1+x)^{13}$$

output
```
-1/11*(d-e)*(1+x)^11+1/12*(d-2*e)*(1+x)^12+1/13*e*(1+x)^13
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(39) = 78.

Time = 0.02 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.77

$$\begin{aligned}\int x(d+ex)\left(1+2x+x^2\right)^5\,dx&=\frac{dx^2}{2}+\frac{1}{3}(10d+e)x^3+\frac{5}{4}(9d+2e)x^4\\&+3(8d+3e)x^5+5(7d+4e)x^6+6(6d+5e)x^7\\&+\frac{21}{4}(5d+6e)x^8+\frac{10}{3}(4d+7e)x^9+\frac{3}{2}(3d+8e)x^{10}\\&+\frac{5}{11}(2d+9e)x^{11}+\frac{1}{12}(d+10e)x^{12}+\frac{ex^{13}}{13}\end{aligned}$$

input
```
Integrate[x*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^2)/2 + ((10*d + e)*x^3)/3 + (5*(9*d + 2*e)*x^4)/4 + 3*(8*d + 3*e)*x^5
 + 5*(7*d + 4*e)*x^6 + 6*(6*d + 5*e)*x^7 + (21*(5*d + 6*e)*x^8)/4 + (10*(4
*d + 7*e)*x^9)/3 + (3*(3*d + 8*e)*x^10)/2 + (5*(2*d + 9*e)*x^11)/11 + ((d
+ 10*e)*x^12)/12 + (e*x^13)/13
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(x^2+2x+1\right)^5(d+ex)\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int x(x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left((x+1)^{11}(d-2e)+(x+1)^{10}(e-d)+e(x+1)^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{12}(x+1)^{12}(d-2e)-\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{13}e(x+1)^{13}$$

input
```
Int[x*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/11*((d - e)*(1 + x)^11) + ((d - 2*e)*(1 + x)^12)/12 + (e*(1 + x)^13)/13
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(33) = 66$.

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.18

method	result
norman	$\frac{x^{13} e}{13}+\left(\frac{d}{12}+\frac{5 e}{6}\right) x^{12}+\left(\frac{10 d}{11}+\frac{45 e}{11}\right) x^{11}+\left(\frac{9 d}{2}+12 e\right) x^{10}+\left(\frac{40 d}{3}+\frac{70 e}{3}\right) x^{9}+\left(\frac{105 d}{4}+\frac{63 e}{2}\right) x^{8}$
default	$\frac{x^{13} e}{13}+\frac{(d+10 e) x^{12}}{12}+\frac{(10 d+45 e) x^{11}}{11}+\frac{(45 d+120 e) x^{10}}{10}+\frac{(120 d+210 e) x^{9}}{9}+\frac{(210 d+252 e) x^{8}}{8}+\frac{(252 d+210 e) x^{7}}{7}+$
gosper	$\frac{x^{2}\left(132 e\,x^{11}+143 d\,x^{10}+1430 e\,x^{10}+1560 d\,x^{9}+7020 e\,x^{9}+7722 d\,x^{8}+20592 e\,x^{8}+22880 d\,x^{7}+40040 e\,x^{7}+45045 d\,x^{6}+54054 e\,x\right.}{1716}$
risch	$\frac{1}{13} x^{13} e+\frac{1}{12} d\,x^{12}+\frac{5}{6} x^{12} e+\frac{10}{11} x^{11} d+\frac{45}{11} e\,x^{11}+\frac{9}{2} d\,x^{10}+12 e\,x^{10}+\frac{40}{3} d\,x^{9}+\frac{70}{3} e\,x^{9}+\frac{105}{4} d\,x$
parallelrisch	$\frac{1}{13} x^{13} e+\frac{1}{12} d\,x^{12}+\frac{5}{6} x^{12} e+\frac{10}{11} x^{11} d+\frac{45}{11} e\,x^{11}+\frac{9}{2} d\,x^{10}+12 e\,x^{10}+\frac{40}{3} d\,x^{9}+\frac{70}{3} e\,x^{9}+\frac{105}{4} d\,x$
orering	$\frac{x^{2}\left(132 e\,x^{11}+143 d\,x^{10}+1430 e\,x^{10}+1560 d\,x^{9}+7020 e\,x^{9}+7722 d\,x^{8}+20592 e\,x^{8}+22880 d\,x^{7}+40040 e\,x^{7}+45045 d\,x^{6}+54054 e\,x\right.}{171}$

input
```
int(x*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/13*x^13*e+(1/12*d+5/6*e)*x^12+(10/11*d+45/11*e)*x^11+(9/2*d+12*e)*x^10+(
40/3*d+70/3*e)*x^9+(105/4*d+63/2*e)*x^8+(36*d+30*e)*x^7+(35*d+20*e)*x^6+(2
4*d+9*e)*x^5+(45/4*d+5/2*e)*x^4+(10/3*d+1/3*e)*x^3+1/2*d*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(33) = 66.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.31

$$\int x(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{13}\,ex^{13} + \frac{1}{12}\,(d+10\,e)x^{12} + \frac{5}{11}\,(2\,d+9\,e)x^{11} + \frac{3}{2}\,(3\,d+8\,e)x^{10} + \frac{10}{3}\,(4\,d+7\,e)x^9 + \frac{21}{4}\,(5\,d+6\,e)x^8 + 6\,(6\,d+5\,e)x^7 + 5\,(7\,d+4\,e)x^6 + 3\,(8\,d+3\,e)x^5 + \frac{5}{4}\,(9\,d+2\,e)x^4 + \frac{1}{3}\,(10\,d+e)x^3 + \frac{1}{2}\,dx^2$$

input
```
integrate(x*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/13*e*x^13 + 1/12*(d + 10*e)*x^12 + 5/11*(2*d + 9*e)*x^11 + 3/2*(3*d + 8*
e)*x^10 + 10/3*(4*d + 7*e)*x^9 + 21/4*(5*d + 6*e)*x^8 + 6*(6*d + 5*e)*x^7
+ 5*(7*d + 4*e)*x^6 + 3*(8*d + 3*e)*x^5 + 5/4*(9*d + 2*e)*x^4 + 1/3*(10*d
+ e)*x^3 + 1/2*d*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int x(d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{dx^2}{2} + \frac{ex^{13}}{13} + x^{12}\left(\frac{d}{12}+\frac{5e}{6}\right) + x^{11}\cdot\left(\frac{10d}{11}+\frac{45e}{11}\right) + x^{10}\cdot\left(\frac{9d}{2}+12e\right) + x^9\cdot\left(\frac{40d}{3}+\frac{70e}{3}\right) + x^8\cdot\left(\frac{105d}{4}+\frac{63e}{2}\right) + x^7\cdot(36d+30e) + x^6\cdot(35d+20e) + x^5\cdot(24d+9e) + x^4\cdot\left(\frac{45d}{4}+\frac{5e}{2}\right) + x^3\cdot\left(\frac{10d}{3}+\frac{e}{3}\right)$$

input
```
integrate(x*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x**2/2 + e*x**13/13 + x**12*(d/12 + 5*e/6) + x**11*(10*d/11 + 45*e/11) +
 x**10*(9*d/2 + 12*e) + x**9*(40*d/3 + 70*e/3) + x**8*(105*d/4 + 63*e/2) +
 x**7*(36*d + 30*e) + x**6*(35*d + 20*e) + x**5*(24*d + 9*e) + x**4*(45*d/
4 + 5*e/2) + x**3*(10*d/3 + e/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(33) = 66$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.31

$$\int x(d+ex)\left(1+2x+x^2\right)^5 dx = \frac{1}{13}ex^{13} + \frac{1}{12}(d+10e)x^{12} + \frac{5}{11}(2d+9e)x^{11} + \frac{3}{2}(3d+8e)x^{10} + \frac{10}{3}(4d+7e)x^9 + \frac{21}{4}(5d+6e)x^8 + 6(6d+5e)x^7 + 5(7d+4e)x^6 + 3(8d+3e)x^5 + \frac{5}{4}(9d+2e)x^4 + \frac{1}{3}(10d+e)x^3 + \frac{1}{2}dx^2$$

input
```
integrate(x*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/13*e*x^13 + 1/12*(d + 10*e)*x^12 + 5/11*(2*d + 9*e)*x^11 + 3/2*(3*d + 8*
e)*x^10 + 10/3*(4*d + 7*e)*x^9 + 21/4*(5*d + 6*e)*x^8 + 6*(6*d + 5*e)*x^7
+ 5*(7*d + 4*e)*x^6 + 3*(8*d + 3*e)*x^5 + 5/4*(9*d + 2*e)*x^4 + 1/3*(10*d
+ e)*x^3 + 1/2*d*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(33) = 66$.

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int x(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{1}{13}ex^{13}+\frac{1}{12}dx^{12}+\frac{5}{6}ex^{12}+\frac{10}{11}dx^{11}+\frac{45}{11}ex^{11}+\frac{9}{2}dx^{10}+12\,ex^{10}+\frac{40}{3}dx^9+\frac{70}{3}ex^9+\frac{105}{4}dx^8+\frac{63}{2}ex^8+36\,dx^7+30\,ex^7+35\,dx^6+20\,ex^6+24\,dx^5+9\,ex^5+\frac{45}{4}dx^4+\frac{5}{2}ex^4+\frac{10}{3}dx^3+\frac{1}{3}ex^3+\frac{1}{2}dx^2$$

input
```
integrate(x*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/13*e*x^13 + 1/12*d*x^12 + 5/6*e*x^12 + 10/11*d*x^11 + 45/11*e*x^11 + 9/2
*d*x^10 + 12*e*x^10 + 40/3*d*x^9 + 70/3*e*x^9 + 105/4*d*x^8 + 63/2*e*x^8 +
 36*d*x^7 + 30*e*x^7 + 35*d*x^6 + 20*e*x^6 + 24*d*x^5 + 9*e*x^5 + 45/4*d*x
^4 + 5/2*e*x^4 + 10/3*d*x^3 + 1/3*e*x^3 + 1/2*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.15

$$\int x(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{e\,x^{13}}{13}+\left(\frac{d}{12}+\frac{5\,e}{6}\right)x^{12}+\left(\frac{10\,d}{11}+\frac{45\,e}{11}\right)x^{11}+\left(\frac{9\,d}{2}+12\,e\right)x^{10}+\left(\frac{40\,d}{3}+\frac{70\,e}{3}\right)x^9+\left(\frac{105\,d}{4}+\frac{63\,e}{2}\right)x^8+(36\,d+30\,e)\,x^7+(35\,d+20\,e)\,x^6+(24\,d+9\,e)\,x^5+\left(\frac{45\,d}{4}+\frac{5\,e}{2}\right)x^4+\left(\frac{10\,d}{3}+\frac{e}{3}\right)x^3+\frac{d\,x^2}{2}$$

input
```
int(x*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^3*((10*d)/3 + e/3) + x^10*((9*d)/2 + 12*e) + x^12*(d/12 + (5*e)/6) + x^5
*(24*d + 9*e) + x^4*((45*d)/4 + (5*e)/2) + x^6*(35*d + 20*e) + x^7*(36*d +
 30*e) + x^11*((10*d)/11 + (45*e)/11) + x^9*((40*d)/3 + (70*e)/3) + x^8*((
105*d)/4 + (63*e)/2) + (d*x^2)/2 + (e*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

$$\int x(d+ex)\left(1+2x+x^2\right)^5 dx$$
$$= \frac{x^2(132e\,x^{11}+143d\,x^{10}+1430e\,x^{10}+1560d\,x^9+7020e\,x^9+7722d\,x^8+20592e\,x^8+22880d\,x^7+40040}{}$$

input
```
int(x*(e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x**2*(143*d*x**10 + 1560*d*x**9 + 7722*d*x**8 + 22880*d*x**7 + 45045*d*x*
*6 + 61776*d*x**5 + 60060*d*x**4 + 41184*d*x**3 + 19305*d*x**2 + 5720*d*x
+ 858*d + 132*e*x**11 + 1430*e*x**10 + 7020*e*x**9 + 20592*e*x**8 + 40040*
e*x**7 + 54054*e*x**6 + 51480*e*x**5 + 34320*e*x**4 + 15444*e*x**3 + 4290*
e*x**2 + 572*e*x))/1716
```

3.195 $\int (d+ex) (1+2x+x^2)^5 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int (d+ex) (1+2x+x^2)^5 \, dx = \frac{1}{11}(d-e)(1+x)^{11} + \frac{1}{12}e(1+x)^{12}$$

output
```
1/11*(d-e)*(1+x)^11+1/12*e*(1+x)^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(25) = 50.

Time = 0.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int (d+ex) (1+2x+x^2)^5 \, dx = \frac{1}{132}ex^2(66 + 440x + 1485x^2 + 3168x^3 + 4620x^4 + 4752x^5 + 3465x^6 + 1760x^7 + 594x^8 + 120x^9 + 11x^{10}) + d\left(x + 5x^2 + 15x^3 + 30x^4 + 42x^5 + 42x^6 + 30x^7 + 15x^8 + 5x^9 + x^{10} + \frac{x^{11}}{11}\right)$$

input
```
Integrate[(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(e*x^2*(66 + 440*x + 1485*x^2 + 3168*x^3 + 4620*x^4 + 4752*x^5 + 3465*x^6
+ 1760*x^7 + 594*x^8 + 120*x^9 + 11*x^10))/132 + d*(x + 5*x^2 + 15*x^3 + 3
0*x^4 + 42*x^5 + 42*x^6 + 30*x^7 + 15*x^8 + 5*x^9 + x^10 + x^11/11)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+2x+1\right)^5 (d+ex)\,dx$$

$$\Big\downarrow \text{1098}$$

$$\int (x+1)^{10}(d+ex)dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{10}(d-e)+e(x+1)^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{11}(x+1)^{11}(d-e)+\frac{1}{12}e(x+1)^{12}$$

input
```
Int[(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
((d - e)*(1 + x)^11)/11 + (e*(1 + x)^12)/12
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(21) = 42$.

Time = 0.81 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.76

method	result
norman	$\frac{x^{12}e}{12}+\left(\frac{d}{11}+\frac{10e}{11}\right)x^{11}+\left(d+\frac{9e}{2}\right)x^{10}+\left(5d+\frac{40e}{3}\right)x^{9}+\left(15d+\frac{105e}{4}\right)x^{8}+\left(30d+36e\right)x^{7}+$
default	$\frac{x^{12}e}{12}+\frac{(d+10e)x^{11}}{11}+\frac{(10d+45e)x^{10}}{10}+\frac{(45d+120e)x^{9}}{9}+\frac{(120d+210e)x^{8}}{8}+\frac{(210d+252e)x^{7}}{7}+\frac{(252d+210e)x^{6}}{6}+$
gosper	$\frac{x\left(11e\,x^{11}+12d\,x^{10}+120e\,x^{10}+132d\,x^{9}+594e\,x^{9}+660d\,x^{8}+1760e\,x^{8}+1980d\,x^{7}+3465e\,x^{7}+3960d\,x^{6}+4752e\,x^{6}+5544d\,x^{5}+4\right.}{132}$
risch	$\frac{1}{12}x^{12}e+\frac{1}{11}x^{11}d+\frac{10}{11}e\,x^{11}+d\,x^{10}+\frac{9}{2}e\,x^{10}+5d\,x^{9}+\frac{40}{3}e\,x^{9}+15d\,x^{8}+\frac{105}{4}e\,x^{8}+30d\,x^{7}+$
parallelrisch	$\frac{1}{12}x^{12}e+\frac{1}{11}x^{11}d+\frac{10}{11}e\,x^{11}+d\,x^{10}+\frac{9}{2}e\,x^{10}+5d\,x^{9}+\frac{40}{3}e\,x^{9}+15d\,x^{8}+\frac{105}{4}e\,x^{8}+30d\,x^{7}+$
orering	$\frac{x\left(11e\,x^{11}+12d\,x^{10}+120e\,x^{10}+132d\,x^{9}+594e\,x^{9}+660d\,x^{8}+1760e\,x^{8}+1980d\,x^{7}+3465e\,x^{7}+3960d\,x^{6}+4752e\,x^{6}+5544d\,x^{5}+4\right.}{132(x+1)^{10}}$

input
```
int((e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^12*e+(1/11*d+10/11*e)*x^11+(d+9/2*e)*x^10+(5*d+40/3*e)*x^9+(15*d+10
5/4*e)*x^8+(30*d+36*e)*x^7+(42*d+35*e)*x^6+(42*d+24*e)*x^5+(30*d+45/4*e)*x
^4+(15*d+10/3*e)*x^3+(5*d+1/2*e)*x^2+d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(21) = 42$.

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.04

$$\int (d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{1}{12}\,ex^{12} + \frac{1}{11}\,(d+10\,e)x^{11} + \frac{1}{2}\,(2\,d+9\,e)x^{10} + \frac{5}{3}\,(3\,d+8\,e)x^9 + \frac{15}{4}\,(4\,d+7\,e)x^8 + 6\,(5\,d+6\,e)x^7 + 7\,(6\,d+5\,e)x^6 + 6\,(7\,d+4\,e)x^5 + \frac{15}{4}\,(8\,d+3\,e)x^4 + \frac{5}{3}\,(9\,d+2\,e)x^3 + \frac{1}{2}\,(10\,d+e)x^2 + dx$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/12*e*x^12 + 1/11*(d + 10*e)*x^11 + 1/2*(2*d + 9*e)*x^10 + 5/3*(3*d + 8*e
)*x^9 + 15/4*(4*d + 7*e)*x^8 + 6*(5*d + 6*e)*x^7 + 7*(6*d + 5*e)*x^6 + 6*(
7*d + 4*e)*x^5 + 15/4*(8*d + 3*e)*x^4 + 5/3*(9*d + 2*e)*x^3 + 1/2*(10*d +
e)*x^2 + d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.76

$$\int (d+ex)\left(1+2x+x^2\right)^5\,dx = dx + \frac{ex^{12}}{12} + x^{11}\left(\frac{d}{11}+\frac{10e}{11}\right) + x^{10}\left(d+\frac{9e}{2}\right) + x^9\cdot\left(5d+\frac{40e}{3}\right) + x^8\cdot\left(15d+\frac{105e}{4}\right) + x^7\cdot(30d+36e) + x^6\cdot(42d+35e) + x^5\cdot(42d+24e) + x^4\cdot\left(30d+\frac{45e}{4}\right) + x^3\cdot\left(15d+\frac{10e}{3}\right) + x^2\cdot\left(5d+\frac{e}{2}\right)$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
d*x + e*x**12/12 + x**11*(d/11 + 10*e/11) + x**10*(d + 9*e/2) + x**9*(5*d
+ 40*e/3) + x**8*(15*d + 105*e/4) + x**7*(30*d + 36*e) + x**6*(42*d + 35*e
) + x**5*(42*d + 24*e) + x**4*(30*d + 45*e/4) + x**3*(15*d + 10*e/3) + x**
2*(5*d + e/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.04

$$\begin{aligned}\int (d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{12}\,ex^{12}+\frac{1}{11}\,(d+10\,e)x^{11}+\frac{1}{2}\,(2\,d+9\,e)x^{10}\\&\quad+\frac{5}{3}\,(3\,d+8\,e)x^9+\frac{15}{4}\,(4\,d+7\,e)x^8+6\,(5\,d+6\,e)x^7\\&\quad+7\,(6\,d+5\,e)x^6+6\,(7\,d+4\,e)x^5+\frac{15}{4}\,(8\,d+3\,e)x^4\\&\quad+\frac{5}{3}\,(9\,d+2\,e)x^3+\frac{1}{2}\,(10\,d+e)x^2+dx\end{aligned}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/12*e*x^12 + 1/11*(d + 10*e)*x^11 + 1/2*(2*d + 9*e)*x^10 + 5/3*(3*d + 8*e
)*x^9 + 15/4*(4*d + 7*e)*x^8 + 6*(5*d + 6*e)*x^7 + 7*(6*d + 5*e)*x^6 + 6*(
7*d + 4*e)*x^5 + 15/4*(8*d + 3*e)*x^4 + 5/3*(9*d + 2*e)*x^3 + 1/2*(10*d +
e)*x^2 + d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(21) = 42$.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.16

$$\begin{aligned}\int (d+ex)\left(1+2x+x^2\right)^5\,dx &= \frac{1}{12}\,ex^{12}+\frac{1}{11}\,dx^{11}+\frac{10}{11}\,ex^{11}+dx^{10}+\frac{9}{2}\,ex^{10}+5\,dx^9\\&\quad+\frac{40}{3}\,ex^9+15\,dx^8+\frac{105}{4}\,ex^8+30\,dx^7+36\,ex^7\\&\quad+42\,dx^6+35\,ex^6+42\,dx^5+24\,ex^5+30\,dx^4\\&\quad+\frac{45}{4}\,ex^4+15\,dx^3+\frac{10}{3}\,ex^3+5\,dx^2+\frac{1}{2}\,ex^2+dx\end{aligned}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/12*e*x^12 + 1/11*d*x^11 + 10/11*e*x^11 + d*x^10 + 9/2*e*x^10 + 5*d*x^9 +
 40/3*e*x^9 + 15*d*x^8 + 105/4*e*x^8 + 30*d*x^7 + 36*e*x^7 + 42*d*x^6 + 35
*e*x^6 + 42*d*x^5 + 24*e*x^5 + 30*d*x^4 + 45/4*e*x^4 + 15*d*x^3 + 10/3*e*x
^3 + 5*d*x^2 + 1/2*e*x^2 + d*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.72

$$\int (d+ex)\left(1+2x+x^2\right)^5\,dx = \frac{e\,x^{12}}{12}+\left(\frac{d}{11}+\frac{10\,e}{11}\right)\,x^{11}+\left(d+\frac{9\,e}{2}\right)\,x^{10}+\left(5\,d+\frac{40\,e}{3}\right)\,x^9+\left(15\,d+\frac{105\,e}{4}\right)\,x^8+\left(30\,d+36\,e\right)\,x^7+\left(42\,d+35\,e\right)\,x^6+\left(42\,d+24\,e\right)\,x^5+\left(30\,d+\frac{45\,e}{4}\right)\,x^4+\left(15\,d+\frac{10\,e}{3}\right)\,x^3+\left(5\,d+\frac{e}{2}\right)\,x^2+d\,x$$

input
```
int((d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
x^2*(5*d + e/2) + x^3*(15*d + (10*e)/3) + x^11*(d/11 + (10*e)/11) + x^9*(5
*d + (40*e)/3) + x^5*(42*d + 24*e) + x^7*(30*d + 36*e) + x^4*(30*d + (45*e
)/4) + x^6*(42*d + 35*e) + x^8*(15*d + (105*e)/4) + d*x + (e*x^12)/12 + x^
10*(d + (9*e)/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.16

$$\int (d+ex)\left(1+2x+x^2\right)^5\,dx$$
$$= \frac{x(11e\,x^{11}+12d\,x^{10}+120e\,x^{10}+132d\,x^9+594e\,x^9+660d\,x^8+1760e\,x^8+1980d\,x^7+3465e\,x^7+3960}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5,x)
```

output
```
(x*(12*d*x**10 + 132*d*x**9 + 660*d*x**8 + 1980*d*x**7 + 3960*d*x**6 + 554
4*d*x**5 + 5544*d*x**4 + 3960*d*x**3 + 1980*d*x**2 + 660*d*x + 132*d + 11*
e*x**11 + 120*e*x**10 + 594*e*x**9 + 1760*e*x**8 + 3465*e*x**7 + 4752*e*x*
*6 + 4620*e*x**5 + 3168*e*x**4 + 1485*e*x**3 + 440*e*x**2 + 66*e*x))/132
```

3.196 $\int \frac{(d+ex)(1+2x+x^2)^5}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x}\,dx = 10dx + \frac{45dx^2}{2} + 40dx^3 + \frac{105dx^4}{2} + \frac{252dx^5}{5} + 35dx^6 + \frac{120dx^7}{7} + \frac{45dx^8}{8} + \frac{10dx^9}{9} + \frac{dx^{10}}{10} + \frac{1}{11}e(1+x)^{11} + d\log(x)$$

output
```
10*d*x+45/2*d*x^2+40*d*x^3+105/2*d*x^4+252/5*d*x^5+35*d*x^6+120/7*d*x^7+45
/8*d*x^8+10/9*d*x^9+1/10*d*x^10+1/11*e*(1+x)^11+d*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x}\,dx = \frac{1}{11}e(1+x)^{11} + d\left(\frac{7381}{2520} + 10x + \frac{45x^2}{2} + 40x^3 + \frac{105x^4}{2} + \frac{252x^5}{5} + 35x^6 + \frac{120x^7}{7} + \frac{45x^8}{8} + \frac{10x^9}{9} + \frac{x^{10}}{10}\right) + d\log(-x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x,x]
```

output
```
(e*(1 + x)^11)/11 + d*(7381/2520 + 10*x + (45*x^2)/2 + 40*x^3 + (105*x^4)/
2 + (252*x^5)/5 + 35*x^6 + (120*x^7)/7 + (45*x^8)/8 + (10*x^9)/9 + x^10/10
) + d*Log[-x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1184, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x}dx$$

$$\Big\downarrow \text{90}$$

$$d\int \frac{(x+1)^{10}}{x}dx+\frac{1}{11}e(x+1)^{11}$$

$$\Big\downarrow \text{49}$$

$$d\int \left(x^9+10x^8+45x^7+120x^6+210x^5+252x^4+210x^3+120x^2+45x+10+\frac{1}{x}\right)dx+\frac{1}{11}e(x+1)^{11}$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{x^{10}}{10}+\frac{10x^9}{9}+\frac{45x^8}{8}+\frac{120x^7}{7}+35x^6+\frac{252x^5}{5}+\frac{105x^4}{2}+40x^3+\frac{45x^2}{2}+10x+\log(x)\right)+\frac{1}{11}e(x+1)^{11}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x,x]
```

output
```
(e*(1 + x)^11)/11 + d*(10*x + (45*x^2)/2 + 40*x^3 + (105*x^4)/2 + (252*x^5
)/5 + 35*x^6 + (120*x^7)/7 + (45*x^8)/8 + (10*x^9)/9 + x^10/10 + Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result
norman	$(10d+e)\,x+(35d+42e)\,x^{6}+(40d+15e)\,x^{3}+\left(\frac{d}{10}+e\right)x^{10}+\left(\frac{10d}{9}+5e\right)x^{9}+\left(\frac{45d}{2}+5e\right)$
default	$\frac{e\,x^{11}}{11}+\frac{d\,x^{10}}{10}+e\,x^{10}+\frac{10d\,x^{9}}{9}+5e\,x^{9}+\frac{45d\,x^{8}}{8}+15e\,x^{8}+\frac{120d\,x^{7}}{7}+30e\,x^{7}+35d\,x^{6}+42e\,x^{6}+$
risch	$\frac{e\,x^{11}}{11}+\frac{d\,x^{10}}{10}+e\,x^{10}+\frac{10d\,x^{9}}{9}+5e\,x^{9}+\frac{45d\,x^{8}}{8}+15e\,x^{8}+\frac{120d\,x^{7}}{7}+30e\,x^{7}+35d\,x^{6}+42e\,x^{6}+$
parallelrisch	$\frac{e\,x^{11}}{11}+\frac{d\,x^{10}}{10}+e\,x^{10}+\frac{10d\,x^{9}}{9}+5e\,x^{9}+\frac{45d\,x^{8}}{8}+15e\,x^{8}+\frac{120d\,x^{7}}{7}+30e\,x^{7}+35d\,x^{6}+42e\,x^{6}+$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x,x,method=_RETURNVERBOSE)
```

output
```
(10*d+e)*x+(35*d+42*e)*x^6+(40*d+15*e)*x^3+(1/10*d+e)*x^10+(10/9*d+5*e)*x^
9+(45/2*d+5*e)*x^2+(45/8*d+15*e)*x^8+(105/2*d+30*e)*x^4+(120/7*d+30*e)*x^7
+(252/5*d+42*e)*x^5+1/11*e*x^11+d*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x}\,dx = \frac{1}{11}\,ex^{11}+\frac{1}{10}\,(d+10\,e)x^{10}+\frac{5}{9}\,(2\,d+9\,e)x^{9} + \frac{15}{8}\,(3\,d+8\,e)x^{8}+\frac{30}{7}\,(4\,d+7\,e)x^{7}+7\,(5\,d+6\,e)x^{6} + \frac{42}{5}\,(6\,d+5\,e)x^{5}+\frac{15}{2}\,(7\,d+4\,e)x^{4}+5\,(8\,d+3\,e)x^{3} + \frac{5}{2}\,(9\,d+2\,e)x^{2}+(10\,d+e)x+d\log\left(x\right)$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x,x, algorithm="fricas")
```

output
```
1/11*e*x^11 + 1/10*(d + 10*e)*x^10 + 5/9*(2*d + 9*e)*x^9 + 15/8*(3*d + 8*e
)*x^8 + 30/7*(4*d + 7*e)*x^7 + 7*(5*d + 6*e)*x^6 + 42/5*(6*d + 5*e)*x^5 +
15/2*(7*d + 4*e)*x^4 + 5*(8*d + 3*e)*x^3 + 5/2*(9*d + 2*e)*x^2 + (10*d + e
)*x + d*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x}\,dx = d\log(x)+\frac{ex^{11}}{11}+x^{10}\left(\frac{d}{10}+e\right)+x^9\cdot\left(\frac{10d}{9}+5e\right)+x^8\cdot\left(\frac{45d}{8}+15e\right)+x^7\cdot\left(\frac{120d}{7}+30e\right)+x^6\cdot(35d+42e)+x^5\cdot\left(\frac{252d}{5}+42e\right)+x^4\cdot\left(\frac{105d}{2}+30e\right)+x^3\cdot(40d+15e)+x^2\cdot\left(\frac{45d}{2}+5e\right)+x(10d+e)$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x,x)
```

output
```
d*log(x) + e*x**11/11 + x**10*(d/10 + e) + x**9*(10*d/9 + 5*e) + x**8*(45*
d/8 + 15*e) + x**7*(120*d/7 + 30*e) + x**6*(35*d + 42*e) + x**5*(252*d/5 +
 42*e) + x**4*(105*d/2 + 30*e) + x**3*(40*d + 15*e) + x**2*(45*d/2 + 5*e)
+ x*(10*d + e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x}\,dx = \frac{1}{11}ex^{11}+\frac{1}{10}(d+10e)x^{10}+\frac{5}{9}(2d+9e)x^9+\frac{15}{8}(3d+8e)x^8+\frac{30}{7}(4d+7e)x^7+7(5d+6e)x^6+\frac{42}{5}(6d+5e)x^5+\frac{15}{2}(7d+4e)x^4+5(8d+3e)x^3+\frac{5}{2}(9d+2e)x^2+(10d+e)x+d\log(x)$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x,x, algorithm="maxima")
```

output
```
1/11*e*x^11 + 1/10*(d + 10*e)*x^10 + 5/9*(2*d + 9*e)*x^9 + 15/8*(3*d + 8*e
)*x^8 + 30/7*(4*d + 7*e)*x^7 + 7*(5*d + 6*e)*x^6 + 42/5*(6*d + 5*e)*x^5 +
15/2*(7*d + 4*e)*x^4 + 5*(8*d + 3*e)*x^3 + 5/2*(9*d + 2*e)*x^2 + (10*d + e
)*x + d*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x}\,dx = \frac{1}{11}\,ex^{11} + \frac{1}{10}\,dx^{10} + ex^{10} + \frac{10}{9}\,dx^9 + 5\,ex^9 + \frac{45}{8}\,dx^8 + 15\,ex^8 + \frac{120}{7}\,dx^7 + 30\,ex^7 + 35\,dx^6 + 42\,ex^6 + \frac{252}{5}\,dx^5 + 42\,ex^5 + \frac{105}{2}\,dx^4 + 30\,ex^4 + 40\,dx^3 + 15\,ex^3 + \frac{45}{2}\,dx^2 + 5\,ex^2 + 10\,dx + ex + d\log(|x|)$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x,x, algorithm="giac")
```

output
```
1/11*e*x^11 + 1/10*d*x^10 + e*x^10 + 10/9*d*x^9 + 5*e*x^9 + 45/8*d*x^8 + 1
5*e*x^8 + 120/7*d*x^7 + 30*e*x^7 + 35*d*x^6 + 42*e*x^6 + 252/5*d*x^5 + 42*
e*x^5 + 105/2*d*x^4 + 30*e*x^4 + 40*d*x^3 + 15*e*x^3 + 45/2*d*x^2 + 5*e*x^
2 + 10*d*x + e*x + d*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x}\,dx = x^9\left(\frac{10\,d}{9}+5\,e\right) + x^2\left(\frac{45\,d}{2}+5\,e\right) + x^3\,(40\,d+15\,e) + x^8\left(\frac{45\,d}{8}+15\,e\right) + x^6\,(35\,d+42\,e) + x^4\left(\frac{105\,d}{2}+30\,e\right) + x^7\left(\frac{120\,d}{7}+30\,e\right) + x^5\left(\frac{252\,d}{5}+42\,e\right) + x\,(10\,d+e) + \frac{e\,x^{11}}{11} + d\,\ln(x) + x^{10}\left(\frac{d}{10}+e\right)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x,x)
```

output
```
x^9*((10*d)/9 + 5*e) + x^2*((45*d)/2 + 5*e) + x^3*(40*d + 15*e) + x^8*((45
*d)/8 + 15*e) + x^6*(35*d + 42*e) + x^4*((105*d)/2 + 30*e) + x^7*((120*d)/
7 + 30*e) + x^5*((252*d)/5 + 42*e) + x*(10*d + e) + (e*x^11)/11 + d*log(x)
 + x^10*(d/10 + e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x}\,dx = \log(x)\,d + \frac{d\,x^{10}}{10} + \frac{10d\,x^9}{9} + \frac{45d\,x^8}{8} + \frac{120d\,x^7}{7} + 35d\,x^6 + \frac{252d\,x^5}{5} + \frac{105d\,x^4}{2} + 40d\,x^3 + \frac{45d\,x^2}{2} + 10dx + \frac{e\,x^{11}}{11} + e\,x^{10} + 5e\,x^9 + 15e\,x^8 + 30e\,x^7 + 42e\,x^6 + 42e\,x^5 + 30e\,x^4 + 15e\,x^3 + 5e\,x^2 + ex$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x,x)
```

output
```
(27720*log(x)*d + 2772*d*x**10 + 30800*d*x**9 + 155925*d*x**8 + 475200*d*x
**7 + 970200*d*x**6 + 1397088*d*x**5 + 1455300*d*x**4 + 1108800*d*x**3 + 6
23700*d*x**2 + 277200*d*x + 2520*e*x**11 + 27720*e*x**10 + 138600*e*x**9 +
 415800*e*x**8 + 831600*e*x**7 + 1164240*e*x**6 + 1164240*e*x**5 + 831600*
e*x**4 + 415800*e*x**3 + 138600*e*x**2 + 27720*e*x)/27720
```

3.197 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\begin{aligned}\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx = &-\frac{d}{x}+5(9d+2e)x+\frac{15}{2}(8d+3e)x^2\\ &+10(7d+4e)x^3+\frac{21}{2}(6d+5e)x^4+\frac{42}{5}(5d+6e)x^5\\ &+5(4d+7e)x^6+\frac{15}{7}(3d+8e)x^7+\frac{5}{8}(2d+9e)x^8\\ &+\frac{1}{9}(d+10e)x^9+\frac{ex^{10}}{10}+(10d+e)\log(x)\end{aligned}$$

output
```
-d/x+5*(9*d+2*e)*x+15/2*(8*d+3*e)*x^2+10*(7*d+4*e)*x^3+21/2*(6*d+5*e)*x^4+
42/5*(5*d+6*e)*x^5+5*(4*d+7*e)*x^6+15/7*(3*d+8*e)*x^7+5/8*(2*d+9*e)*x^8+1/
9*(d+10*e)*x^9+1/10*e*x^10+(10*d+e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx=-\frac{d}{x}+5(9d+2e)x+\frac{15}{2}(8d+3e)x^2+10(7d+4e)x^3+\frac{21}{2}(6d+5e)x^4+\frac{42}{5}(5d+6e)x^5+5(4d+7e)x^6+\frac{15}{7}(3d+8e)x^7+\frac{5}{8}(2d+9e)x^8+\frac{1}{9}(d+10e)x^9+\frac{ex^{10}}{10}+(10d+e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^2,x]
```

output
```
-(d/x) + 5*(9*d + 2*e)*x + (15*(8*d + 3*e)*x^2)/2 + 10*(7*d + 4*e)*x^3 + (
21*(6*d + 5*e)*x^4)/2 + (42*(5*d + 6*e)*x^5)/5 + 5*(4*d + 7*e)*x^6 + (15*(
3*d + 8*e)*x^7)/7 + (5*(2*d + 9*e)*x^8)/8 + ((d + 10*e)*x^9)/9 + (e*x^10)/
10 + (10*d + e)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5(d+ex)}{x^2}\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^2}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^8(d+10e)+5x^7(2d+9e)+15x^6(3d+8e)+30x^5(4d+7e)+42x^4(5d+6e)+42x^3(6d+5e)+30x^2(7d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}x^9(d+10e)+\frac{5}{8}x^8(2d+9e)+\frac{15}{7}x^7(3d+8e)+5x^6(4d+7e)+\frac{42}{5}x^5(5d+6e)+\frac{21}{2}x^4(6d+5e)+10x^3(7d+4e)+\frac{15}{2}x^2(8d+3e)+5x(9d+2e)+(10d+e)\log(x)-\frac{d}{x}+\frac{ex^{10}}{10}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^2,x]
```

output
```
-(d/x) + 5*(9*d + 2*e)*x + (15*(8*d + 3*e)*x^2)/2 + 10*(7*d + 4*e)*x^3 + (
21*(6*d + 5*e)*x^4)/2 + (42*(5*d + 6*e)*x^5)/5 + 5*(4*d + 7*e)*x^6 + (15*(
3*d + 8*e)*x^7)/7 + (5*(2*d + 9*e)*x^8)/8 + ((d + 10*e)*x^9)/9 + (e*x^10)/
10 + (10*d + e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

method	result
norman	$\frac{(20d+35e)x^7+(42d+\frac{252e}{5})x^6+(45d+10e)x^2+(60d+\frac{45e}{2})x^3+(63d+\frac{105e}{2})x^5+(70d+40e)x^4+\left(\frac{d}{9}+\frac{10e}{9}\right)x^{10}+\left(\frac{5d}{4}+\frac{45e}{8}\right)x^9+}{x}$
default	$\frac{e\,x^{10}}{10}+\frac{d\,x^9}{9}+\frac{10e\,x^9}{9}+\frac{5d\,x^8}{4}+\frac{45e\,x^8}{8}+\frac{45d\,x^7}{7}+\frac{120e\,x^7}{7}+20d\,x^6+35e\,x^6+42d\,x^5+\frac{252x^5e}{5}+6$
risch	$\frac{e\,x^{10}}{10}+\frac{d\,x^9}{9}+\frac{10e\,x^9}{9}+\frac{5d\,x^8}{4}+\frac{45e\,x^8}{8}+\frac{45d\,x^7}{7}+\frac{120e\,x^7}{7}+20d\,x^6+35e\,x^6+42d\,x^5+\frac{252x^5e}{5}+6$
parallelrisch	$252e\,x^{11}+280d\,x^{10}+2800e\,x^{10}+3150d\,x^9+14175e\,x^9+16200d\,x^8+43200e\,x^8+50400d\,x^7+88200e\,x^7+105840d\,x^6+127008e$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
((20*d+35*e)*x^7+(42*d+252/5*e)*x^6+(45*d+10*e)*x^2+(60*d+45/2*e)*x^3+(63*
d+105/2*e)*x^5+(70*d+40*e)*x^4+(1/9*d+10/9*e)*x^10+(5/4*d+45/8*e)*x^9+(45/
7*d+120/7*e)*x^8-d+1/10*e*x^11)/x+(10*d+e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{252\,ex^{11}+280\,(d+10\,e)x^{10}+1575\,(2\,d+9\,e)x^9+5400\,(3\,d+8\,e)x^8+12600\,(4\,d+7\,e)x^7+21168\,(5\,d}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^2,x, algorithm="fricas")
```

output
```
1/2520*(252*e*x^11 + 280*(d + 10*e)*x^10 + 1575*(2*d + 9*e)*x^9 + 5400*(3*
d + 8*e)*x^8 + 12600*(4*d + 7*e)*x^7 + 21168*(5*d + 6*e)*x^6 + 26460*(6*d
+ 5*e)*x^5 + 25200*(7*d + 4*e)*x^4 + 18900*(8*d + 3*e)*x^3 + 12600*(9*d +
2*e)*x^2 + 2520*(10*d + e)*x*log(x) - 2520*d)/x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx = -\frac{d}{x} + \frac{ex^{10}}{10} + x^9\left(\frac{d}{9} + \frac{10e}{9}\right) + x^8\cdot\left(\frac{5d}{4} + \frac{45e}{8}\right) + x^7\cdot\left(\frac{45d}{7} + \frac{120e}{7}\right) + x^6\cdot(20d+35e) + x^5\cdot\left(42d + \frac{252e}{5}\right) + x^4\cdot\left(63d + \frac{105e}{2}\right) + x^3\cdot(70d+40e) + x^2\cdot\left(60d + \frac{45e}{2}\right) + x(45d+10e) + (10d+e)\log(x)$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**2,x)
```

output
```
-d/x + e*x**10/10 + x**9*(d/9 + 10*e/9) + x**8*(5*d/4 + 45*e/8) + x**7*(45
*d/7 + 120*e/7) + x**6*(20*d + 35*e) + x**5*(42*d + 252*e/5) + x**4*(63*d
+ 105*e/2) + x**3*(70*d + 40*e) + x**2*(60*d + 45*e/2) + x*(45*d + 10*e) +
 (10*d + e)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{1}{10}\,ex^{10} + \frac{1}{9}\,(d+10\,e)x^9 + \frac{5}{8}\,(2\,d+9\,e)x^8 + \frac{15}{7}\,(3\,d+8\,e)x^7 + 5\,(4\,d+7\,e)x^6 + \frac{42}{5}\,(5\,d+6\,e)x^5 + \frac{21}{2}\,(6\,d+5\,e)x^4 + 10\,(7\,d+4\,e)x^3 + \frac{15}{2}\,(8\,d+3\,e)x^2 + 5\,(9\,d+2\,e)x + (10\,d+e)\log(x) - \frac{d}{x}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^2,x, algorithm="maxima")
```

output
```
1/10*e*x^10 + 1/9*(d + 10*e)*x^9 + 5/8*(2*d + 9*e)*x^8 + 15/7*(3*d + 8*e)*
x^7 + 5*(4*d + 7*e)*x^6 + 42/5*(5*d + 6*e)*x^5 + 21/2*(6*d + 5*e)*x^4 + 10
*(7*d + 4*e)*x^3 + 15/2*(8*d + 3*e)*x^2 + 5*(9*d + 2*e)*x + (10*d + e)*log
(x) - d/x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^2}\,dx = \frac{1}{10}\,ex^{10} + \frac{1}{9}\,dx^9 + \frac{10}{9}\,ex^9 + \frac{5}{4}\,dx^8 + \frac{45}{8}\,ex^8 + \frac{45}{7}\,dx^7 + \frac{120}{7}\,ex^7 + 20\,dx^6 + 35\,ex^6 + 42\,dx^5 + \frac{252}{5}\,ex^5 + 63\,dx^4 + \frac{105}{2}\,ex^4 + 70\,dx^3 + 40\,ex^3 + 60\,dx^2 + \frac{45}{2}\,ex^2 + 45\,dx + 10\,ex + (10\,d+e)\log(|x|) - \frac{d}{x}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^2,x, algorithm="giac")
```

output
```
1/10*e*x^10 + 1/9*d*x^9 + 10/9*e*x^9 + 5/4*d*x^8 + 45/8*e*x^8 + 45/7*d*x^7
 + 120/7*e*x^7 + 20*d*x^6 + 35*e*x^6 + 42*d*x^5 + 252/5*e*x^5 + 63*d*x^4 +
 105/2*e*x^4 + 70*d*x^3 + 40*e*x^3 + 60*d*x^2 + 45/2*e*x^2 + 45*d*x + 10*e
*x + (10*d + e)*log(abs(x)) - d/x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^2}\,dx = x^9\left(\frac{d}{9}+\frac{10\,e}{9}\right) + x^6\,(20\,d+35\,e) + x^8\left(\frac{5\,d}{4}+\frac{45\,e}{8}\right) + x^2\left(60\,d+\frac{45\,e}{2}\right) + x^3\,(70\,d+40\,e) + x^4\left(63\,d+\frac{105\,e}{2}\right) + x^7\left(\frac{45\,d}{7}+\frac{120\,e}{7}\right) + x^5\left(42\,d+\frac{252\,e}{5}\right) - \frac{d}{x} + \frac{e\,x^{10}}{10} + x\,(45\,d+10\,e) + \ln(x)\,(10\,d+e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^2,x)
```

output
```
x^9*(d/9 + (10*e)/9) + x^6*(20*d + 35*e) + x^8*((5*d)/4 + (45*e)/8) + x^2*
(60*d + (45*e)/2) + x^3*(70*d + 40*e) + x^4*(63*d + (105*e)/2) + x^7*((45*
d)/7 + (120*e)/7) + x^5*(42*d + (252*e)/5) - d/x + (e*x^10)/10 + x*(45*d +
 10*e) + log(x)*(10*d + e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^2}\,dx$$
$$= \frac{25200\log(x)\,dx + 2520\log(x)\,ex + 280d\,x^{10} + 3150d\,x^9 + 16200d\,x^8 + 50400d\,x^7 + 105840d\,x^6 + 15876}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^2,x)
```

output
```
(25200*log(x)*d*x + 2520*log(x)*e*x + 280*d*x**10 + 3150*d*x**9 + 16200*d*
x**8 + 50400*d*x**7 + 105840*d*x**6 + 158760*d*x**5 + 176400*d*x**4 + 1512
00*d*x**3 + 113400*d*x**2 - 2520*d + 252*e*x**11 + 2800*e*x**10 + 14175*e*
x**9 + 43200*e*x**8 + 88200*e*x**7 + 127008*e*x**6 + 132300*e*x**5 + 10080
0*e*x**4 + 56700*e*x**3 + 25200*e*x**2)/(2520*x)
```

3.198 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\begin{aligned}\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = &-\frac{d}{2x^2}-\frac{10d+e}{x}+15(8d+3e)x+15(7d+4e)x^2\\ &+14(6d+5e)x^3+\frac{21}{2}(5d+6e)x^4\\ &+6(4d+7e)x^5+\frac{5}{2}(3d+8e)x^6+\frac{5}{7}(2d+9e)x^7\\ &+\frac{1}{8}(d+10e)x^8+\frac{ex^9}{9}+5(9d+2e)\log(x)\end{aligned}$$

output
```
-1/2*d/x^2-(10*d+e)/x+15*(8*d+3*e)*x+15*(7*d+4*e)*x^2+14*(6*d+5*e)*x^3+21/
2*(5*d+6*e)*x^4+6*(4*d+7*e)*x^5+5/2*(3*d+8*e)*x^6+5/7*(2*d+9*e)*x^7+1/8*(d
+10*e)*x^8+1/9*e*x^9+5*(9*d+2*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = -\frac{d}{2x^2}+\frac{-10d-e}{x}+15(8d+3e)x + 15(7d+4e)x^2+14(6d+5e)x^3+\frac{21}{2}(5d+6e)x^4 + 6(4d+7e)x^5+\frac{5}{2}(3d+8e)x^6+\frac{5}{7}(2d+9e)x^7 + \frac{1}{8}(d+10e)x^8+\frac{ex^9}{9}+5(9d+2e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^3,x]
```

output
```
-1/2*d/x^2 + (-10*d - e)/x + 15*(8*d + 3*e)*x + 15*(7*d + 4*e)*x^2 + 14*(6
*d + 5*e)*x^3 + (21*(5*d + 6*e)*x^4)/2 + 6*(4*d + 7*e)*x^5 + (5*(3*d + 8*e
)*x^6)/2 + (5*(2*d + 9*e)*x^7)/7 + ((d + 10*e)*x^8)/8 + (e*x^9)/9 + 5*(9*d
 + 2*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^3}\,dx$$

$$\Big\downarrow \; 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^3}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(x^7(d+10e)+5x^6(2d+9e)+15x^5(3d+8e)+30x^4(4d+7e)+42x^3(5d+6e)+42x^2(6d+5e)+\frac{10d+e}{x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}x^8(d+10e)+\frac{5}{7}x^7(2d+9e)+\frac{5}{2}x^6(3d+8e)+6x^5(4d+7e)+\frac{21}{2}x^4(5d+6e)+14x^3(6d+5e)+15x^2(7d+4e)+15x(8d+3e)-\frac{10d+e}{x}+5(9d+2e)\log(x)-\frac{d}{2x^2}+\frac{ex^9}{9}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^3,x]
```

output
```
-1/2*d/x^2 - (10*d + e)/x + 15*(8*d + 3*e)*x + 15*(7*d + 4*e)*x^2 + 14*(6*
d + 5*e)*x^3 + (21*(5*d + 6*e)*x^4)/2 + 6*(4*d + 7*e)*x^5 + (5*(3*d + 8*e)
*x^6)/2 + (5*(2*d + 9*e)*x^7)/7 + ((d + 10*e)*x^8)/8 + (e*x^9)/9 + 5*(9*d
+ 2*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

method	result
norman	$\frac{(-10d-e)x+(24d+42e)x^7+(84d+70e)x^5+(105d+60e)x^4+(120d+45e)x^3+\left(\frac{d}{8}+\frac{5e}{4}\right)x^{10}+\left(\frac{10d}{7}+\frac{45e}{7}\right)x^9+\left(\frac{15d}{2}+20e\right)x^8+(}{x^2}$
default	$\frac{e\,x^9}{9}+\frac{d\,x^8}{8}+\frac{5e\,x^8}{4}+\frac{10d\,x^7}{7}+\frac{45e\,x^7}{7}+\frac{15d\,x^6}{2}+20e\,x^6+24d\,x^5+42x^5e+\frac{105d\,x^4}{2}+63x^4e+8$
risch	$\frac{e\,x^9}{9}+\frac{d\,x^8}{8}+\frac{5e\,x^8}{4}+\frac{10d\,x^7}{7}+\frac{45e\,x^7}{7}+\frac{15d\,x^6}{2}+20e\,x^6+24d\,x^5+42x^5e+\frac{105d\,x^4}{2}+63x^4e+8$
parallelrisch	$\frac{56e\,x^{11}+63d\,x^{10}+630e\,x^{10}+720d\,x^9+3240e\,x^9+3780d\,x^8+10080e\,x^8+12096d\,x^7+21168e\,x^7+26460d\,x^6+31752e\,x^6+42336}{504x^2}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
((-10*d-e)*x+(24*d+42*e)*x^7+(84*d+70*e)*x^5+(105*d+60*e)*x^4+(120*d+45*e)
*x^3+(1/8*d+5/4*e)*x^10+(10/7*d+45/7*e)*x^9+(15/2*d+20*e)*x^8+(105/2*d+63*
e)*x^6-1/2*d+1/9*e*x^11)/x^2+(45*d+10*e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx$$
$$=\frac{56\,ex^{11}+63\,(d+10\,e)x^{10}+360\,(2\,d+9\,e)x^9+1260\,(3\,d+8\,e)x^8+3024\,(4\,d+7\,e)x^7+5292\,(5\,d+6\,e}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^3,x, algorithm="fricas")
```

output
```
1/504*(56*e*x^11 + 63*(d + 10*e)*x^10 + 360*(2*d + 9*e)*x^9 + 1260*(3*d +
8*e)*x^8 + 3024*(4*d + 7*e)*x^7 + 5292*(5*d + 6*e)*x^6 + 7056*(6*d + 5*e)*
x^5 + 7560*(7*d + 4*e)*x^4 + 7560*(8*d + 3*e)*x^3 + 2520*(9*d + 2*e)*x^2*l
og(x) - 504*(10*d + e)*x - 252*d)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{ex^9}{9} + x^8\left(\frac{d}{8}+\frac{5e}{4}\right) + x^7\cdot\left(\frac{10d}{7}+\frac{45e}{7}\right) + x^6 \cdot\left(\frac{15d}{2}+20e\right) + x^5\cdot(24d+42e) + x^4\cdot\left(\frac{105d}{2}+63e\right) + x^3\cdot(84d+70e) + x^2\cdot(105d+60e) + x(120d+45e) + 5\cdot(9d+2e)\log(x) + \frac{-d+x(-20d-2e)}{2x^2}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**3,x)
```

output
```
e*x**9/9 + x**8*(d/8 + 5*e/4) + x**7*(10*d/7 + 45*e/7) + x**6*(15*d/2 + 20
*e) + x**5*(24*d + 42*e) + x**4*(105*d/2 + 63*e) + x**3*(84*d + 70*e) + x*
*2*(105*d + 60*e) + x*(120*d + 45*e) + 5*(9*d + 2*e)*log(x) + (-d + x*(-20
*d - 2*e))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{1}{9}\,ex^9 + \frac{1}{8}\,(d+10\,e)x^8 + \frac{5}{7}\,(2\,d+9\,e)x^7 + \frac{5}{2}\,(3\,d+8\,e)x^6 + 6\,(4\,d+7\,e)x^5 + \frac{21}{2}\,(5\,d+6\,e)x^4 + 14\,(6\,d+5\,e)x^3 + 15\,(7\,d+4\,e)x^2 + 15\,(8\,d+3\,e)x + 5\,(9\,d+2\,e)\log(x) - \frac{2\,(10\,d+e)x+d}{2\,x^2}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^3,x, algorithm="maxima")
```

output
```
1/9*e*x^9 + 1/8*(d + 10*e)*x^8 + 5/7*(2*d + 9*e)*x^7 + 5/2*(3*d + 8*e)*x^6
 + 6*(4*d + 7*e)*x^5 + 21/2*(5*d + 6*e)*x^4 + 14*(6*d + 5*e)*x^3 + 15*(7*d
 + 4*e)*x^2 + 15*(8*d + 3*e)*x + 5*(9*d + 2*e)*log(x) - 1/2*(2*(10*d + e)*
x + d)/x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{1}{9}\,ex^9 + \frac{1}{8}\,dx^8 + \frac{5}{4}\,ex^8 + \frac{10}{7}\,dx^7 + \frac{45}{7}\,ex^7 + \frac{15}{2}\,dx^6 + 20\,ex^6 + 24\,dx^5 + 42\,ex^5 + \frac{105}{2}\,dx^4 + 63\,ex^4 + 84\,dx^3 + 70\,ex^3 + 105\,dx^2 + 60\,ex^2 + 120\,dx + 45\,ex + 5\,(9\,d+2\,e)\log\left(|x|\right) - \frac{2\,(10\,d+e)x+d}{2\,x^2}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^3,x, algorithm="giac")
```

output
```
1/9*e*x^9 + 1/8*d*x^8 + 5/4*e*x^8 + 10/7*d*x^7 + 45/7*e*x^7 + 15/2*d*x^6 +
 20*e*x^6 + 24*d*x^5 + 42*e*x^5 + 105/2*d*x^4 + 63*e*x^4 + 84*d*x^3 + 70*e
*x^3 + 105*d*x^2 + 60*e*x^2 + 120*d*x + 45*e*x + 5*(9*d + 2*e)*log(abs(x))
 - 1/2*(2*(10*d + e)*x + d)/x^2
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^3}\,dx = x^8\left(\frac{d}{8}+\frac{5\,e}{4}\right) + x^6\left(\frac{15\,d}{2}+20\,e\right) + x^5\,(24\,d+42\,e) + x^7\left(\frac{10\,d}{7}+\frac{45\,e}{7}\right) + x^3\,(84\,d+70\,e) + x^2\,(105\,d+60\,e) + x^4\left(\frac{105\,d}{2}+63\,e\right) + \ln\left(x\right)\,(45\,d+10\,e) + \frac{e\,x^9}{9} - \frac{\frac{d}{2}+x\,(10\,d+e)}{x^2} + x\,(120\,d+45\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^3,x)
```

output
```
x^8*(d/8 + (5*e)/4) + x^6*((15*d)/2 + 20*e) + x^5*(24*d + 42*e) + x^7*((10
*d)/7 + (45*e)/7) + x^3*(84*d + 70*e) + x^2*(105*d + 60*e) + x^4*((105*d)/
2 + 63*e) + log(x)*(45*d + 10*e) + (e*x^9)/9 - (d/2 + x*(10*d + e))/x^2 +
x*(120*d + 45*e)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^3}\,dx = \frac{22680\log(x)\,d\,x^2+5040\log(x)\,e\,x^2+63d\,x^{10}+720d\,x^9+3780d\,x^8+12096d\,x^7+26460d\,x^6+42336d\,x^5 \cdots}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^3,x)
```

output
```
(22680*log(x)*d*x**2 + 5040*log(x)*e*x**2 + 63*d*x**10 + 720*d*x**9 + 3780
*d*x**8 + 12096*d*x**7 + 26460*d*x**6 + 42336*d*x**5 + 52920*d*x**4 + 6048
0*d*x**3 - 5040*d*x - 252*d + 56*e*x**11 + 630*e*x**10 + 3240*e*x**9 + 100
80*e*x**8 + 21168*e*x**7 + 31752*e*x**6 + 35280*e*x**5 + 30240*e*x**4 + 22
680*e*x**3 - 504*e*x)/(504*x**2)
```

3.199 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\begin{aligned}\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4} dx = &-\frac{d}{3x^3} - \frac{10d+e}{2x^2} - \frac{5(9d+2e)}{x} + 30(7d+4e)x \\ &+ 21(6d+5e)x^2 + 14(5d+6e)x^3 \\ &+ \frac{15}{2}(4d+7e)x^4 + 3(3d+8e)x^5 + \frac{5}{6}(2d+9e)x^6 \\ &+ \frac{1}{7}(d+10e)x^7 + \frac{ex^8}{8} + 15(8d+3e)\log(x)\end{aligned}$$

output
```
-1/3*d/x^3-1/2*(10*d+e)/x^2-5*(9*d+2*e)/x+30*(7*d+4*e)*x+21*(6*d+5*e)*x^2+
14*(5*d+6*e)*x^3+15/2*(4*d+7*e)*x^4+3*(3*d+8*e)*x^5+5/6*(2*d+9*e)*x^6+1/7*
(d+10*e)*x^7+1/8*e*x^8+15*(8*d+3*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^4}\,dx = -\frac{d}{3x^3}+\frac{-10d-e}{2x^2}-\frac{5(9d+2e)}{x}+30(7d+4e)x+21(6d+5e)x^2+14(5d+6e)x^3+\frac{15}{2}(4d+7e)x^4+3(3d+8e)x^5+\frac{5}{6}(2d+9e)x^6+\frac{1}{7}(d+10e)x^7+\frac{ex^8}{8}+15(8d+3e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^4,x]
```

output
```
-1/3*d/x^3 + (-10*d - e)/(2*x^2) - (5*(9*d + 2*e))/x + 30*(7*d + 4*e)*x +
21*(6*d + 5*e)*x^2 + 14*(5*d + 6*e)*x^3 + (15*(4*d + 7*e)*x^4)/2 + 3*(3*d
+ 8*e)*x^5 + (5*(2*d + 9*e)*x^6)/6 + ((d + 10*e)*x^7)/7 + (e*x^8)/8 + 15*(
8*d + 3*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^4}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^4}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^6(d+10e)+5x^5(2d+9e)+15x^4(3d+8e)+30x^3(4d+7e)+\frac{10d+e}{x^3}+42x^2(5d+6e)+\frac{5(9d+2e)}{x^2}+4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(d+10e)+\frac{5}{6}x^6(2d+9e)+3x^5(3d+8e)+\frac{15}{2}x^4(4d+7e)+14x^3(5d+6e)+21x^2(6d+5e)-\frac{10d+e}{2x^2}+30x(7d+4e)-\frac{5(9d+2e)}{x}+15(8d+3e)\log(x)-\frac{d}{3x^3}+\frac{ex^8}{8}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^4,x]
```

output
```
-1/3*d/x^3 - (10*d + e)/(2*x^2) - (5*(9*d + 2*e))/x + 30*(7*d + 4*e)*x + 2
1*(6*d + 5*e)*x^2 + 14*(5*d + 6*e)*x^3 + (15*(4*d + 7*e)*x^4)/2 + 3*(3*d +
 8*e)*x^5 + (5*(2*d + 9*e)*x^6)/6 + ((d + 10*e)*x^7)/7 + (e*x^8)/8 + 15*(8
*d + 3*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

method	result
norman	$\frac{(-45d-10e)x^2+\left(-5d-\frac{e}{2}\right)x+(9d+24e)x^8+\left(30d+\frac{105e}{2}\right)x^7+(70d+84e)x^6+(126d+105e)x^5+(210d+120e)x^4+\left(\frac{d}{7}+\frac{10e}{7}\right)x^{10}}{x^3}$
default	$\frac{e\,x^8}{8}+\frac{d\,x^7}{7}+\frac{10e\,x^7}{7}+\frac{5d\,x^6}{3}+\frac{15e\,x^6}{2}+9d\,x^5+24x^5e+30d\,x^4+\frac{105x^4e}{2}+70d\,x^3+84x^3e+12$
risch	$\frac{e\,x^8}{8}+\frac{d\,x^7}{7}+\frac{10e\,x^7}{7}+\frac{5d\,x^6}{3}+\frac{15e\,x^6}{2}+9d\,x^5+24x^5e+30d\,x^4+\frac{105x^4e}{2}+70d\,x^3+84x^3e+12$
parallelrisch	$\frac{21e\,x^{11}+24d\,x^{10}+240e\,x^{10}+280d\,x^9+1260e\,x^9+1512d\,x^8+4032e\,x^8+5040d\,x^7+8820e\,x^7+11760d\,x^6+14112e\,x^6+21168d\,x}{168x^3}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
((-45*d-10*e)*x^2+(-5*d-1/2*e)*x+(9*d+24*e)*x^8+(30*d+105/2*e)*x^7+(70*d+8
4*e)*x^6+(126*d+105*e)*x^5+(210*d+120*e)*x^4+(1/7*d+10/7*e)*x^10+(5/3*d+15
/2*e)*x^9-1/3*d+1/8*e*x^11)/x^3+(120*d+45*e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{21\,ex^{11}+24\,(d+10\,e)x^{10}+140\,(2\,d+9\,e)x^9+504\,(3\,d+8\,e)x^8+1260\,(4\,d+7\,e)x^7+2352\,(5\,d+6\,e)}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^4,x, algorithm="fricas")
```

output
```
1/168*(21*e*x^11 + 24*(d + 10*e)*x^10 + 140*(2*d + 9*e)*x^9 + 504*(3*d + 8
*e)*x^8 + 1260*(4*d + 7*e)*x^7 + 2352*(5*d + 6*e)*x^6 + 3528*(6*d + 5*e)*x
^5 + 5040*(7*d + 4*e)*x^4 + 2520*(8*d + 3*e)*x^3*log(x) - 840*(9*d + 2*e)*
x^2 - 84*(10*d + e)*x - 56*d)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{ex^8}{8} + x^7\left(\frac{d}{7}+\frac{10e}{7}\right) + x^6\cdot\left(\frac{5d}{3}+\frac{15e}{2}\right) + x^5 \cdot (9d+24e) + x^4\cdot\left(30d+\frac{105e}{2}\right) + x^3\cdot(70d+84e) + x^2 \cdot (126d+105e) + x(210d+120e) + 15\cdot(8d+3e)\log(x) + \frac{-2d+x^2(-270d-60e)+x(-30d-3e)}{6x^3}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**4,x)
```

output
```
e*x**8/8 + x**7*(d/7 + 10*e/7) + x**6*(5*d/3 + 15*e/2) + x**5*(9*d + 24*e)
 + x**4*(30*d + 105*e/2) + x**3*(70*d + 84*e) + x**2*(126*d + 105*e) + x*(
210*d + 120*e) + 15*(8*d + 3*e)*log(x) + (-2*d + x**2*(-270*d - 60*e) + x*
(-30*d - 3*e))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{1}{8}\,ex^8 + \frac{1}{7}\,(d+10\,e)x^7 + \frac{5}{6}\,(2\,d+9\,e)x^6 + 3\,(3\,d+8\,e)x^5 + \frac{15}{2}\,(4\,d+7\,e)x^4 + 14\,(5\,d+6\,e)x^3 + 21\,(6\,d+5\,e)x^2 + 30\,(7\,d+4\,e)x + 15\,(8\,d+3\,e)\log(x) - \frac{30\,(9\,d+2\,e)x^2 + 3\,(10\,d+e)x + 2\,d}{6\,x^3}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^4,x, algorithm="maxima")
```

output
```
1/8*e*x^8 + 1/7*(d + 10*e)*x^7 + 5/6*(2*d + 9*e)*x^6 + 3*(3*d + 8*e)*x^5 +
 15/2*(4*d + 7*e)*x^4 + 14*(5*d + 6*e)*x^3 + 21*(6*d + 5*e)*x^2 + 30*(7*d
+ 4*e)*x + 15*(8*d + 3*e)*log(x) - 1/6*(30*(9*d + 2*e)*x^2 + 3*(10*d + e)*
x + 2*d)/x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{1}{8}\,ex^8 + \frac{1}{7}\,dx^7 + \frac{10}{7}\,ex^7 + \frac{5}{3}\,dx^6 + \frac{15}{2}\,ex^6 + 9\,dx^5 + 24\,ex^5 + 30\,dx^4 + \frac{105}{2}\,ex^4 + 70\,dx^3 + 84\,ex^3 + 126\,dx^2 + 105\,ex^2 + 210\,dx + 120\,ex + 15\,(8\,d+3\,e)\log\left(|x|\right) - \frac{30\,(9\,d+2\,e)x^2 + 3\,(10\,d+e)x + 2\,d}{6\,x^3}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^4,x, algorithm="giac")
```

output
```
1/8*e*x^8 + 1/7*d*x^7 + 10/7*e*x^7 + 5/3*d*x^6 + 15/2*e*x^6 + 9*d*x^5 + 24
*e*x^5 + 30*d*x^4 + 105/2*e*x^4 + 70*d*x^3 + 84*e*x^3 + 126*d*x^2 + 105*e*
x^2 + 210*d*x + 120*e*x + 15*(8*d + 3*e)*log(abs(x)) - 1/6*(30*(9*d + 2*e)
*x^2 + 3*(10*d + e)*x + 2*d)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = x^6\left(\frac{5\,d}{3}+\frac{15\,e}{2}\right) + x^7\left(\frac{d}{7}+\frac{10\,e}{7}\right) + x^5\,(9\,d+24\,e) + x^4\left(30\,d+\frac{105\,e}{2}\right) + x^3\,(70\,d+84\,e) + x^2\,(126\,d+105\,e) + \ln(x)\,(120\,d+45\,e) - \frac{(45\,d+10\,e)\,x^2 + \left(5\,d+\frac{e}{2}\right)\,x + \frac{d}{3}}{x^3} + \frac{e\,x^8}{8} + x\,(210\,d+120\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^4,x)
```

output
```
x^6*((5*d)/3 + (15*e)/2) + x^7*(d/7 + (10*e)/7) + x^5*(9*d + 24*e) + x^4*(
30*d + (105*e)/2) + x^3*(70*d + 84*e) + x^2*(126*d + 105*e) + log(x)*(120*
d + 45*e) - (d/3 + x^2*(45*d + 10*e) + x*(5*d + e/2))/x^3 + (e*x^8)/8 + x*
(210*d + 120*e)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{20160\log(x)\,d\,x^3+7560\log(x)\,e\,x^3+24d\,x^{10}+280d\,x^9+1512d\,x^8+5040d\,x^7+11760d\,x^6+21168d\,x^5}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^4,x)
```

output
```
(20160*log(x)*d*x**3 + 7560*log(x)*e*x**3 + 24*d*x**10 + 280*d*x**9 + 1512
*d*x**8 + 5040*d*x**7 + 11760*d*x**6 + 21168*d*x**5 + 35280*d*x**4 - 7560*
d*x**2 - 840*d*x - 56*d + 21*e*x**11 + 240*e*x**10 + 1260*e*x**9 + 4032*e*
x**8 + 8820*e*x**7 + 14112*e*x**6 + 17640*e*x**5 + 20160*e*x**4 - 1680*e*x
**2 - 84*e*x)/(168*x**3)
```

3.200 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^5} \, dx = & -\frac{d}{4x^4}-\frac{10d+e}{3x^3}-\frac{5(9d+2e)}{2x^2}-\frac{15(8d+3e)}{x} \\ & +42(6d+5e)x+21(5d+6e)x^2 \\ & +10(4d+7e)x^3+\frac{15}{4}(3d+8e)x^4+(2d+9e)x^5 \\ & +\frac{1}{6}(d+10e)x^6+\frac{ex^7}{7}+30(7d+4e)\log(x)\end{aligned}$$

output
```
-1/4*d/x^4-1/3*(10*d+e)/x^3-5/2*(9*d+2*e)/x^2-15*(8*d+3*e)/x+42*(6*d+5*e)*
x+21*(5*d+6*e)*x^2+10*(4*d+7*e)*x^3+15/4*(3*d+8*e)*x^4+(2*d+9*e)*x^5+1/6*(
d+10*e)*x^6+1/7*e*x^7+30*(7*d+4*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^5}\,dx = -\frac{d}{4x^4} + \frac{-10d-e}{3x^3} - \frac{5(9d+2e)}{2x^2} - \frac{15(8d+3e)}{x} + 42(6d+5e)x + 21(5d+6e)x^2 + 10(4d+7e)x^3 + \frac{15}{4}(3d+8e)x^4 + (2d+9e)x^5 + \frac{1}{6}(d+10e)x^6 + \frac{ex^7}{7} + 30(7d+4e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^5,x]
```

output
```
-1/4*d/x^4 + (-10*d - e)/(3*x^3) - (5*(9*d + 2*e))/(2*x^2) - (15*(8*d + 3*
e))/x + 42*(6*d + 5*e)*x + 21*(5*d + 6*e)*x^2 + 10*(4*d + 7*e)*x^3 + (15*(
3*d + 8*e)*x^4)/4 + (2*d + 9*e)*x^5 + ((d + 10*e)*x^6)/6 + (e*x^7)/7 + 30*
(7*d + 4*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^5}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^5}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^5(d+10e)+5x^4(2d+9e)+\frac{10d+e}{x^4}+15x^3(3d+8e)+\frac{5(9d+2e)}{x^3}+30x^2(4d+7e)+\frac{15(8d+3e)}{x^2}+42x\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(d+10e)+x^5(2d+9e)+\frac{15}{4}x^4(3d+8e)+10x^3(4d+7e)-\frac{10d+e}{3x^3}+21x^2(5d+6e)-\frac{5(9d+2e)}{2x^2}+42x(6d+5e)-\frac{15(8d+3e)}{x}+30(7d+4e)\log(x)-\frac{d}{4x^4}+\frac{ex^7}{7}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^5,x]
```

output
```
-1/4*d/x^4 - (10*d + e)/(3*x^3) - (5*(9*d + 2*e))/(2*x^2) - (15*(8*d + 3*e
))/x + 42*(6*d + 5*e)*x + 21*(5*d + 6*e)*x^2 + 10*(4*d + 7*e)*x^3 + (15*(3
*d + 8*e)*x^4)/4 + (2*d + 9*e)*x^5 + ((d + 10*e)*x^6)/6 + (e*x^7)/7 + 30*(
7*d + 4*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result
norman	$\frac{(-120d-45e)x^3+(2d+9e)x^9+(40d+70e)x^7+(105d+126e)x^6+(252d+210e)x^5+\left(-\frac{45d}{2}-5e\right)x^2+\left(-\frac{10d}{3}-\frac{e}{3}\right)x+\left(\frac{d}{6}+\frac{5e}{3}\right)x^1}{x^4}$
risch	$\frac{e\,x^7}{7}+\frac{d\,x^6}{6}+\frac{5e\,x^6}{3}+2d\,x^5+9x^5e+\frac{45d\,x^4}{4}+30x^4e+40d\,x^3+70x^3e+105d\,x^2+126e\,x^2+$
default	$\frac{e\,x^7}{7}+\frac{d\,x^6}{6}+\frac{5e\,x^6}{3}+2d\,x^5+9x^5e+\frac{45d\,x^4}{4}+30x^4e+40d\,x^3+70x^3e+105d\,x^2+126e\,x^2+$
parallelrisch	$\frac{12e\,x^{11}+14d\,x^{10}+140e\,x^{10}+168d\,x^9+756e\,x^9+945d\,x^8+2520e\,x^8+3360d\,x^7+5880e\,x^7+8820d\,x^6+10584e\,x^6+17640\ln(x)x}{84x^4}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
((-120*d-45*e)*x^3+(2*d+9*e)*x^9+(40*d+70*e)*x^7+(105*d+126*e)*x^6+(252*d+
210*e)*x^5+(-45/2*d-5*e)*x^2+(-10/3*d-1/3*e)*x+(1/6*d+5/3*e)*x^10+(45/4*d+
30*e)*x^8-1/4*d+1/7*e*x^11)/x^4+(210*d+120*e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^5}\,dx = \frac{12\,ex^{11}+14\,(d+10\,e)x^{10}+84\,(2\,d+9\,e)x^9+315\,(3\,d+8\,e)x^8+840\,(4\,d+7\,e)x^7+1764\,(5\,d+6\,e)x^6}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^5,x, algorithm="fricas")
```

output
```
1/84*(12*e*x^11 + 14*(d + 10*e)*x^10 + 84*(2*d + 9*e)*x^9 + 315*(3*d + 8*e
)*x^8 + 840*(4*d + 7*e)*x^7 + 1764*(5*d + 6*e)*x^6 + 3528*(6*d + 5*e)*x^5
+ 2520*(7*d + 4*e)*x^4*log(x) - 1260*(8*d + 3*e)*x^3 - 210*(9*d + 2*e)*x^2
 - 28*(10*d + e)*x - 21*d)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^5}\,dx$$
$$= \frac{ex^7}{7} + x^6\left(\frac{d}{6}+\frac{5e}{3}\right) + x^5\cdot(2d+9e) + x^4\cdot\left(\frac{45d}{4}+30e\right) + x^3\cdot(40d+70e) + x^2\cdot(105d+126e) + x(252d+210e) + 30\cdot(7d+4e)\log(x) + \frac{-3d+x^3(-1440d-540e)+x^2(-270d-60e)+x(-40d-4e)}{12x^4}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**5,x)
```

output
```
e*x**7/7 + x**6*(d/6 + 5*e/3) + x**5*(2*d + 9*e) + x**4*(45*d/4 + 30*e) +
x**3*(40*d + 70*e) + x**2*(105*d + 126*e) + x*(252*d + 210*e) + 30*(7*d +
4*e)*log(x) + (-3*d + x**3*(-1440*d - 540*e) + x**2*(-270*d - 60*e) + x*(-
40*d - 4*e))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^5}\,dx$$
$$= \frac{1}{7}ex^7 + \frac{1}{6}(d+10e)x^6 + (2d+9e)x^5 + \frac{15}{4}(3d+8e)x^4 + 10(4d+7e)x^3 + 21(5d+6e)x^2 + 42(6d+5e)x + 30(7d+4e)\log(x) - \frac{180(8d+3e)x^3+30(9d+2e)x^2+4(10d+e)x+3d}{12x^4}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^5,x, algorithm="maxima")
```

output
```
1/7*e*x^7 + 1/6*(d + 10*e)*x^6 + (2*d + 9*e)*x^5 + 15/4*(3*d + 8*e)*x^4 +
10*(4*d + 7*e)*x^3 + 21*(5*d + 6*e)*x^2 + 42*(6*d + 5*e)*x + 30*(7*d + 4*e
)*log(x) - 1/12*(180*(8*d + 3*e)*x^3 + 30*(9*d + 2*e)*x^2 + 4*(10*d + e)*x
 + 3*d)/x^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^5}\,dx$$
$$= \frac{1}{7}\,ex^7 + \frac{1}{6}\,dx^6 + \frac{5}{3}\,ex^6 + 2\,dx^5 + 9\,ex^5 + \frac{45}{4}\,dx^4 + 30\,ex^4 + 40\,dx^3 + 70\,ex^3 + 105\,dx^2 + 126\,ex^2 + 252\,dx + 210\,ex + 30\,(7\,d + 4\,e)\log(|x|) - \frac{180\,(8\,d + 3\,e)x^3 + 30\,(9\,d + 2\,e)x^2 + 4\,(10\,d + e)x + 3\,d}{12\,x^4}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^5,x, algorithm="giac")
```

output
```
1/7*e*x^7 + 1/6*d*x^6 + 5/3*e*x^6 + 2*d*x^5 + 9*e*x^5 + 45/4*d*x^4 + 30*e*
x^4 + 40*d*x^3 + 70*e*x^3 + 105*d*x^2 + 126*e*x^2 + 252*d*x + 210*e*x + 30
*(7*d + 4*e)*log(abs(x)) - 1/12*(180*(8*d + 3*e)*x^3 + 30*(9*d + 2*e)*x^2
+ 4*(10*d + e)*x + 3*d)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^5}\,dx = x^5\,(2\,d + 9\,e) + x^6\left(\frac{d}{6} + \frac{5\,e}{3}\right) + x^4\left(\frac{45\,d}{4} + 30\,e\right) + x^3\,(40\,d + 70\,e) + x^2\,(105\,d + 126\,e) + \ln(x)\,\,(210\,d + 120\,e) - \frac{(120\,d + 45\,e)\,\,x^3 + \left(\frac{45\,d}{2} + 5\,e\right)\,x^2 + \left(\frac{10\,d}{3} + \frac{e}{3}\right)\,x + \frac{d}{4}}{x^4} + \frac{e\,x^7}{7} + x\,(252\,d + 210\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^5,x)
```

output
```
x^5*(2*d + 9*e) + x^6*(d/6 + (5*e)/3) + x^4*((45*d)/4 + 30*e) + x^3*(40*d
+ 70*e) + x^2*(105*d + 126*e) + log(x)*(210*d + 120*e) - (d/4 + x^2*((45*d
)/2 + 5*e) + x^3*(120*d + 45*e) + x*((10*d)/3 + e/3))/x^4 + (e*x^7)/7 + x*
(252*d + 210*e)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^5}\,dx$$
$$= \frac{17640\log(x)\,d\,x^4 + 10080\log(x)\,e\,x^4 + 14d\,x^{10} + 168d\,x^9 + 945d\,x^8 + 3360d\,x^7 + 8820d\,x^6 + 21168d\,x^5}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^5,x)
```

output
```
(17640*log(x)*d*x**4 + 10080*log(x)*e*x**4 + 14*d*x**10 + 168*d*x**9 + 945
*d*x**8 + 3360*d*x**7 + 8820*d*x**6 + 21168*d*x**5 - 10080*d*x**3 - 1890*d
*x**2 - 280*d*x - 21*d + 12*e*x**11 + 140*e*x**10 + 756*e*x**9 + 2520*e*x*
*8 + 5880*e*x**7 + 10584*e*x**6 + 17640*e*x**5 - 3780*e*x**3 - 420*e*x**2
- 28*e*x)/(84*x**4)
```

3.201 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\begin{aligned}\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx = & -\frac{d}{5x^5}-\frac{10d+e}{4x^4}-\frac{5(9d+2e)}{3x^3}-\frac{15(8d+3e)}{2x^2} \\ & -\frac{30(7d+4e)}{x}+42(5d+6e)x+15(4d+7e)x^2 \\ & +5(3d+8e)x^3+\frac{5}{4}(2d+9e)x^4 \\ & +\frac{1}{5}(d+10e)x^5+\frac{ex^6}{6}+42(6d+5e)\log(x)\end{aligned}$$

output
```
-1/5*d/x^5-1/4*(10*d+e)/x^4-5/3*(9*d+2*e)/x^3-15/2*(8*d+3*e)/x^2-30*(7*d+4
*e)/x+42*(5*d+6*e)*x+15*(4*d+7*e)*x^2+5*(3*d+8*e)*x^3+5/4*(2*d+9*e)*x^4+1/
5*(d+10*e)*x^5+1/6*e*x^6+42*(6*d+5*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx = -\frac{d}{5x^5}+\frac{-10d-e}{4x^4}-\frac{5(9d+2e)}{3x^3}-\frac{15(8d+3e)}{2x^2}-\frac{30(7d+4e)}{x}+42(5d+6e)x+15(4d+7e)x^2+5(3d+8e)x^3+\frac{5}{4}(2d+9e)x^4+\frac{1}{5}(d+10e)x^5+\frac{ex^6}{6}+42(6d+5e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^6,x]
```

output
```
-1/5*d/x^5 + (-10*d - e)/(4*x^4) - (5*(9*d + 2*e))/(3*x^3) - (15*(8*d + 3*
e))/(2*x^2) - (30*(7*d + 4*e))/x + 42*(5*d + 6*e)*x + 15*(4*d + 7*e)*x^2 +
 5*(3*d + 8*e)*x^3 + (5*(2*d + 9*e)*x^4)/4 + ((d + 10*e)*x^5)/5 + (e*x^6)/
6 + 42*(6*d + 5*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5(d+ex)}{x^6}\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^6}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{10d+e}{x^5} + x^4(d+10e) + \frac{5(9d+2e)}{x^4} + 5x^3(2d+9e) + \frac{15(8d+3e)}{x^3} + 15x^2(3d+8e) + \frac{30(7d+4e)}{x^2} + 30x(4\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(d+10e) + \frac{5}{4}x^4(2d+9e) - \frac{10d+e}{4x^4} + 5x^3(3d+8e) - \frac{5(9d+2e)}{3x^3} + 15x^2(4d+7e) - \frac{15(8d+3e)}{2x^2} + 42x(5d+6e) - \frac{30(7d+4e)}{x} + 42(6d+5e)\log(x) - \frac{d}{5x^5} + \frac{ex^6}{6}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^6,x]
```

output
```
-1/5*d/x^5 - (10*d + e)/(4*x^4) - (5*(9*d + 2*e))/(3*x^3) - (15*(8*d + 3*e
))/(2*x^2) - (30*(7*d + 4*e))/x + 42*(5*d + 6*e)*x + 15*(4*d + 7*e)*x^2 +
5*(3*d + 8*e)*x^3 + (5*(2*d + 9*e)*x^4)/4 + ((d + 10*e)*x^5)/5 + (e*x^6)/6
 + 42*(6*d + 5*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

method	result
norman	$\frac{(-210d-120e)x^4+\left(-60d-\frac{45e}{2}\right)x^3+\left(-15d-\frac{10e}{3}\right)x^2+(15d+40e)x^8+(60d+105e)x^7+(210d+252e)x^6+\left(-\frac{5d}{2}-\frac{e}{4}\right)x+\left(\frac{d}{5}+2e\right.}{x^5}$
risch	$\frac{e\,x^6}{6}+\frac{d\,x^5}{5}+2x^5e+\frac{5d\,x^4}{2}+\frac{45x^4e}{4}+15d\,x^3+40x^3e+60d\,x^2+105e\,x^2+210dx+252ex+$
default	$\frac{e\,x^6}{6}+\frac{d\,x^5}{5}+2x^5e+\frac{5d\,x^4}{2}+\frac{45x^4e}{4}+15d\,x^3+40x^3e+60d\,x^2+105e\,x^2+210dx+252ex-$
parallelrisch	$\frac{10e\,x^{11}+12d\,x^{10}+120e\,x^{10}+150d\,x^9+675e\,x^9+900d\,x^8+2400e\,x^8+3600d\,x^7+6300e\,x^7+15120\ln(x)x^5d+12600\ln(x)x^5e+1}{60x^5}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
((-210*d-120*e)*x^4+(-60*d-45/2*e)*x^3+(-15*d-10/3*e)*x^2+(15*d+40*e)*x^8+
(60*d+105*e)*x^7+(210*d+252*e)*x^6+(-5/2*d-1/4*e)*x+(1/5*d+2*e)*x^10+(5/2*
d+45/4*e)*x^9-1/5*d+1/6*e*x^11)/x^5+(252*d+210*e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx$$
$$=\frac{10\,ex^{11}+12\,(d+10\,e)x^{10}+75\,(2\,d+9\,e)x^9+300\,(3\,d+8\,e)x^8+900\,(4\,d+7\,e)x^7+2520\,(5\,d+6\,e)x^6}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^6,x, algorithm="fricas")
```

output
```
1/60*(10*e*x^11 + 12*(d + 10*e)*x^10 + 75*(2*d + 9*e)*x^9 + 300*(3*d + 8*e
)*x^8 + 900*(4*d + 7*e)*x^7 + 2520*(5*d + 6*e)*x^6 + 2520*(6*d + 5*e)*x^5*
log(x) - 1800*(7*d + 4*e)*x^4 - 450*(8*d + 3*e)*x^3 - 100*(9*d + 2*e)*x^2
- 15*(10*d + e)*x - 12*d)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^6}\,dx = \frac{ex^6}{6} + x^5\left(\frac{d}{5}+2e\right) + x^4\cdot\left(\frac{5d}{2}+\frac{45e}{4}\right) + x^3\cdot(15d+40e) + x^2\cdot(60d+105e) + x(210d+252e) + 42\cdot(6d+5e)\log(x) + \frac{-12d + x^4(-12600d-7200e) + x^3(-3600d-1350e) + x^2(-900d-200e) + x(-150d-15e)}{60x^5}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**6,x)
```

output
```
e*x**6/6 + x**5*(d/5 + 2*e) + x**4*(5*d/2 + 45*e/4) + x**3*(15*d + 40*e) +
 x**2*(60*d + 105*e) + x*(210*d + 252*e) + 42*(6*d + 5*e)*log(x) + (-12*d
+ x**4*(-12600*d - 7200*e) + x**3*(-3600*d - 1350*e) + x**2*(-900*d - 200*
e) + x*(-150*d - 15*e))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^6}\,dx = \frac{1}{6}\,ex^6 + \frac{1}{5}\,(d+10\,e)x^5 + \frac{5}{4}\,(2\,d+9\,e)x^4 + 5\,(3\,d+8\,e)x^3 + 15\,(4\,d+7\,e)x^2 + 42\,(5\,d+6\,e)x + 42\,(6\,d+5\,e)\log(x) - \frac{1800\,(7\,d+4\,e)x^4 + 450\,(8\,d+3\,e)x^3 + 100\,(9\,d+2\,e)x^2 + 15\,(10\,d+e)x + 12\,d}{60\,x^5}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^6,x, algorithm="maxima")
```

output
```
1/6*e*x^6 + 1/5*(d + 10*e)*x^5 + 5/4*(2*d + 9*e)*x^4 + 5*(3*d + 8*e)*x^3 +
 15*(4*d + 7*e)*x^2 + 42*(5*d + 6*e)*x + 42*(6*d + 5*e)*log(x) - 1/60*(180
0*(7*d + 4*e)*x^4 + 450*(8*d + 3*e)*x^3 + 100*(9*d + 2*e)*x^2 + 15*(10*d +
 e)*x + 12*d)/x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx = \frac{1}{6}\,ex^6+\frac{1}{5}\,dx^5+2\,ex^5+\frac{5}{2}\,dx^4+\frac{45}{4}\,ex^4+15\,dx^3+40\,ex^3+60\,dx^2+105\,ex^2+210\,dx+252\,ex+42\,(6\,d+5\,e)\log\left(|x|\right)-\frac{1800\,(7\,d+4\,e)x^4+450\,(8\,d+3\,e)x^3+100\,(9\,d+2\,e)x^2+15\,(10\,d+e)x+12\,d}{60\,x^5}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^6,x, algorithm="giac")
```

output
```
1/6*e*x^6 + 1/5*d*x^5 + 2*e*x^5 + 5/2*d*x^4 + 45/4*e*x^4 + 15*d*x^3 + 40*e
*x^3 + 60*d*x^2 + 105*e*x^2 + 210*d*x + 252*e*x + 42*(6*d + 5*e)*log(abs(x
)) - 1/60*(1800*(7*d + 4*e)*x^4 + 450*(8*d + 3*e)*x^3 + 100*(9*d + 2*e)*x^
2 + 15*(10*d + e)*x + 12*d)/x^5
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx = x^5\left(\frac{d}{5}+2\,e\right)+x^3\,(15\,d+40\,e)+x^4\left(\frac{5\,d}{2}+\frac{45\,e}{4}\right)+x^2\,(60\,d+105\,e)+\ln\left(x\right)\,(252\,d+210\,e)-\frac{(210\,d+120\,e)\,x^4+\left(60\,d+\frac{45\,e}{2}\right)\,x^3+\left(15\,d+\frac{10\,e}{3}\right)\,x^2+\left(\frac{5\,d}{2}+\frac{e}{4}\right)\,x+\frac{d}{5}}{x^5}+\frac{e\,x^6}{6}+x\,(210\,d+252\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^6,x)
```

output
```
x^5*(d/5 + 2*e) + x^3*(15*d + 40*e) + x^4*((5*d)/2 + (45*e)/4) + x^2*(60*d
 + 105*e) + log(x)*(252*d + 210*e) - (d/5 + x^2*(15*d + (10*e)/3) + x^3*(6
0*d + (45*e)/2) + x^4*(210*d + 120*e) + x*((5*d)/2 + e/4))/x^5 + (e*x^6)/6
 + x*(210*d + 252*e)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^6}\,dx$$
$$= \frac{15120\log(x)\,d\,x^5+12600\log(x)\,e\,x^5+12d\,x^{10}+150d\,x^9+900d\,x^8+3600d\,x^7+12600d\,x^6-12600d\,x^4}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^6,x)
```

output
```
(15120*log(x)*d*x**5 + 12600*log(x)*e*x**5 + 12*d*x**10 + 150*d*x**9 + 900
*d*x**8 + 3600*d*x**7 + 12600*d*x**6 - 12600*d*x**4 - 3600*d*x**3 - 900*d*
x**2 - 150*d*x - 12*d + 10*e*x**11 + 120*e*x**10 + 675*e*x**9 + 2400*e*x**
8 + 6300*e*x**7 + 15120*e*x**6 - 7200*e*x**4 - 1350*e*x**3 - 200*e*x**2 -
15*e*x)/(60*x**5)
```

3.202 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx = -\frac{d}{6x^6}-\frac{10d+e}{5x^5}-\frac{5(9d+2e)}{4x^4}-\frac{5(8d+3e)}{x^3}-\frac{15(7d+4e)}{x^2}-\frac{42(6d+5e)}{x}+30(4d+7e)x+\frac{15}{2}(3d+8e)x^2+\frac{5}{3}(2d+9e)x^3+\frac{1}{4}(d+10e)x^4+\frac{ex^5}{5}+42(5d+6e)\log(x)$$

output
```
-1/6*d/x^6-1/5*(10*d+e)/x^5-5/4*(9*d+2*e)/x^4-5*(8*d+3*e)/x^3-15*(7*d+4*e)
/x^2-42*(6*d+5*e)/x+30*(4*d+7*e)*x+15/2*(3*d+8*e)*x^2+5/3*(2*d+9*e)*x^3+1/
4*(d+10*e)*x^4+1/5*e*x^5+42*(5*d+6*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7} dx = -\frac{d}{6x^6} + \frac{-10d-e}{5x^5} - \frac{5(9d+2e)}{4x^4} - \frac{5(8d+3e)}{x^3} - \frac{15(7d+4e)}{x^2} - \frac{42(6d+5e)}{x} + 30(4d+7e)x + \frac{15}{2}(3d+8e)x^2 + \frac{5}{3}(2d+9e)x^3 + \frac{1}{4}(d+10e)x^4 + \frac{ex^5}{5} + 42(5d+6e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^7,x]
```

output
```
-1/6*d/x^6 + (-10*d - e)/(5*x^5) - (5*(9*d + 2*e))/(4*x^4) - (5*(8*d + 3*e
))/x^3 - (15*(7*d + 4*e))/x^2 - (42*(6*d + 5*e))/x + 30*(4*d + 7*e)*x + (1
5*(3*d + 8*e)*x^2)/2 + (5*(2*d + 9*e)*x^3)/3 + ((d + 10*e)*x^4)/4 + (e*x^5
)/5 + 42*(5*d + 6*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^7} dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^7}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{10d+e}{x^6} + \frac{5(9d+2e)}{x^5} + \frac{15(8d+3e)}{x^4} + x^3(d+10e) + \frac{30(7d+4e)}{x^3} + 5x^2(2d+9e) + \frac{42(6d+5e)}{x^2} + 15x(3d\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{10d+e}{5x^5} + \frac{1}{4}x^4(d+10e) - \frac{5(9d+2e)}{4x^4} + \frac{5}{3}x^3(2d+9e) - \frac{5(8d+3e)}{x^3} + \frac{15}{2}x^2(3d+8e) - \frac{15(7d+4e)}{x^2} + 30x(4d+7e) - \frac{42(6d+5e)}{x} + 42(5d+6e)\log(x) - \frac{d}{6x^6} + \frac{ex^5}{5}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^7,x]
```

output
```
-1/6*d/x^6 - (10*d + e)/(5*x^5) - (5*(9*d + 2*e))/(4*x^4) - (5*(8*d + 3*e)
)/x^3 - (15*(7*d + 4*e))/x^2 - (42*(6*d + 5*e))/x + 30*(4*d + 7*e)*x + (15
*(3*d + 8*e)*x^2)/2 + (5*(2*d + 9*e)*x^3)/3 + ((d + 10*e)*x^4)/4 + (e*x^5)
/5 + 42*(5*d + 6*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

method	result
norman	$\frac{(-252d-210e)x^5+(-105d-60e)x^4+(-40d-15e)x^3+\left(-2d-\frac{e}{5}\right)x+(120d+210e)x^7+\left(-\frac{45d}{4}-\frac{5e}{2}\right)x^2+\left(\frac{d}{4}+\frac{5e}{2}\right)x^{10}+\left(\frac{10d}{3}+1\ldots\right.}{x^6}$
risch	$\frac{x^5 e}{5}+\frac{d\,x^4}{4}+\frac{5x^4 e}{2}+\frac{10d\,x^3}{3}+15x^3 e+\frac{45d\,x^2}{2}+60e\,x^2+120dx+210ex+\frac{(-252d-210e)x^5+(-105\ldots}{\ldots}$
default	$\frac{x^5 e}{5}+\frac{d\,x^4}{4}+\frac{5x^4 e}{2}+\frac{10d\,x^3}{3}+15x^3 e+\frac{45d\,x^2}{2}+60e\,x^2+120dx+210ex-\frac{10d+e}{5x^5}-\frac{d}{6x^6}-\frac{120d+\ldots}{3x^3}\ldots$
parallelrisch	$\frac{12e\,x^{11}+15d\,x^{10}+150e\,x^{10}+200d\,x^9+900e\,x^9+1350d\,x^8+3600e\,x^8+12600\ln(x)x^6 d+15120\ln(x)x^6 e+7200d\,x^7+12600e\,x^7\ldots}{60x^6}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
((-252*d-210*e)*x^5+(-105*d-60*e)*x^4+(-40*d-15*e)*x^3+(-2*d-1/5*e)*x+(120
*d+210*e)*x^7+(-45/4*d-5/2*e)*x^2+(1/4*d+5/2*e)*x^10+(10/3*d+15*e)*x^9+(45
/2*d+60*e)*x^8-1/6*d+1/5*e*x^11)/x^6+(210*d+252*e)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx$$
$$=\frac{12\,ex^{11}+15\,(d+10\,e)x^{10}+100\,(2\,d+9\,e)x^9+450\,(3\,d+8\,e)x^8+1800\,(4\,d+7\,e)x^7+2520\,(5\,d+6\,e)\ldots}{\ldots}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^7,x, algorithm="fricas")
```

output
```
1/60*(12*e*x^11 + 15*(d + 10*e)*x^10 + 100*(2*d + 9*e)*x^9 + 450*(3*d + 8*
e)*x^8 + 1800*(4*d + 7*e)*x^7 + 2520*(5*d + 6*e)*x^6*log(x) - 2520*(6*d +
5*e)*x^5 - 900*(7*d + 4*e)*x^4 - 300*(8*d + 3*e)*x^3 - 75*(9*d + 2*e)*x^2
- 12*(10*d + e)*x - 10*d)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{ex^5}{5}+x^4\left(\frac{d}{4}+\frac{5e}{2}\right)+x^3\cdot\left(\frac{10d}{3}+15e\right)+x^2\cdot\left(\frac{45d}{2}+60e\right)+x(120d+210e)+42\cdot(5d+6e)\log{(x)}+\frac{-10d+x^5(-15120d-12600e)+x^4(-6300d-3600e)+x^3(-2400d-900e)+x^2(-675d-150e)+}{60x^6}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**7,x)
```

output
```
e*x**5/5 + x**4*(d/4 + 5*e/2) + x**3*(10*d/3 + 15*e) + x**2*(45*d/2 + 60*e
) + x*(120*d + 210*e) + 42*(5*d + 6*e)*log(x) + (-10*d + x**5*(-15120*d -
12600*e) + x**4*(-6300*d - 3600*e) + x**3*(-2400*d - 900*e) + x**2*(-675*d
 - 150*e) + x*(-120*d - 12*e))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{1}{5}\,ex^5+\frac{1}{4}\,(d+10\,e)x^4+\frac{5}{3}\,(2\,d+9\,e)x^3+\frac{15}{2}\,(3\,d+8\,e)x^2+30\,(4\,d+7\,e)x+42\,(5\,d+6\,e)\log{(x)}-\frac{2520\,(6\,d+5\,e)x^5+900\,(7\,d+4\,e)x^4+300\,(8\,d+3\,e)x^3+75\,(9\,d+2\,e)x^2+12\,(10\,d+e)x+10\,d}{60\,x^6}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^7,x, algorithm="maxima")
```

output
```
1/5*e*x^5 + 1/4*(d + 10*e)*x^4 + 5/3*(2*d + 9*e)*x^3 + 15/2*(3*d + 8*e)*x^
2 + 30*(4*d + 7*e)*x + 42*(5*d + 6*e)*log(x) - 1/60*(2520*(6*d + 5*e)*x^5
+ 900*(7*d + 4*e)*x^4 + 300*(8*d + 3*e)*x^3 + 75*(9*d + 2*e)*x^2 + 12*(10*
d + e)*x + 10*d)/x^6
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{1}{5}\,ex^5 + \frac{1}{4}\,dx^4 + \frac{5}{2}\,ex^4 + \frac{10}{3}\,dx^3 + 15\,ex^3 + \frac{45}{2}\,dx^2 + 60\,ex^2 + 120\,dx + 210\,ex + 42\,(5\,d+6\,e)\log\left(|x|\right) - \frac{2520\,(6\,d+5\,e)x^5 + 900\,(7\,d+4\,e)x^4 + 300\,(8\,d+3\,e)x^3 + 75\,(9\,d+2\,e)x^2 + 12\,(10\,d+e)x + 10\,d}{60\,x^6}$$

input

```
integrate((e*x+d)*(x^2+2*x+1)^5/x^7,x, algorithm="giac")
```

output

```
1/5*e*x^5 + 1/4*d*x^4 + 5/2*e*x^4 + 10/3*d*x^3 + 15*e*x^3 + 45/2*d*x^2 + 6
0*e*x^2 + 120*d*x + 210*e*x + 42*(5*d + 6*e)*log(abs(x)) - 1/60*(2520*(6*d
 + 5*e)*x^5 + 900*(7*d + 4*e)*x^4 + 300*(8*d + 3*e)*x^3 + 75*(9*d + 2*e)*x
^2 + 12*(10*d + e)*x + 10*d)/x^6
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx = x^4\left(\frac{d}{4}+\frac{5\,e}{2}\right) + x^3\left(\frac{10\,d}{3}+15\,e\right) + x^2\left(\frac{45\,d}{2}+60\,e\right) + \ln\left(x\right)\,\left(210\,d+252\,e\right) - \frac{\left(252\,d+210\,e\right)\,x^5 + \left(105\,d+60\,e\right)\,x^4 + \left(40\,d+15\,e\right)\,x^3 + \left(\frac{45\,d}{4}+\frac{5\,e}{2}\right)\,x^2 + \left(2\,d+\frac{e}{5}\right)\,x + \frac{d}{6}}{x^6} + \frac{e\,x^5}{5} + x\,\left(120\,d+210\,e\right)$$

input

```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^7,x)
```

output

```
x^4*(d/4 + (5*e)/2) + x^3*((10*d)/3 + 15*e) + x^2*((45*d)/2 + 60*e) + log(
x)*(210*d + 252*e) - (d/6 + x^2*((45*d)/4 + (5*e)/2) + x^3*(40*d + 15*e) +
 x^4*(105*d + 60*e) + x^5*(252*d + 210*e) + x*(2*d + e/5))/x^6 + (e*x^5)/5
 + x*(120*d + 210*e)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^7}\,dx$$
$$= \frac{12600\log(x)\,d\,x^6+15120\log(x)\,e\,x^6+15d\,x^{10}+200d\,x^9+1350d\,x^8+7200d\,x^7-15120d\,x^5-6300d\,x}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^7,x)
```

output
```
(12600*log(x)*d*x**6 + 15120*log(x)*e*x**6 + 15*d*x**10 + 200*d*x**9 + 135
0*d*x**8 + 7200*d*x**7 - 15120*d*x**5 - 6300*d*x**4 - 2400*d*x**3 - 675*d*
x**2 - 120*d*x - 10*d + 12*e*x**11 + 150*e*x**10 + 900*e*x**9 + 3600*e*x**
8 + 12600*e*x**7 - 12600*e*x**5 - 3600*e*x**4 - 900*e*x**3 - 150*e*x**2 -
12*e*x)/(60*x**6)
```

3.203 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx = -\frac{d}{7x^7}-\frac{10d+e}{6x^6}-\frac{9d+2e}{x^5}-\frac{15(8d+3e)}{4x^4}-\frac{10(7d+4e)}{x^3} -\frac{21(6d+5e)}{x^2}-\frac{42(5d+6e)}{x}+15(3d+8e)x +\frac{5}{2}(2d+9e)x^2+\frac{1}{3}(d+10e)x^3+\frac{ex^4}{4}+30(4d+7e)\log(x)$$

output
```
-1/7*d/x^7-1/6*(10*d+e)/x^6-(9*d+2*e)/x^5-15/4*(8*d+3*e)/x^4-10*(7*d+4*e)/
x^3-21*(6*d+5*e)/x^2-42*(5*d+6*e)/x+15*(3*d+8*e)*x+5/2*(2*d+9*e)*x^2+1/3*(
d+10*e)*x^3+1/4*e*x^4+30*(4*d+7*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8} dx = -\frac{d}{7x^7} + \frac{-10d-e}{6x^6} + \frac{-9d-2e}{x^5} - \frac{15(8d+3e)}{4x^4} - \frac{10(7d+4e)}{x^3} - \frac{21(6d+5e)}{x^2} - \frac{42(5d+6e)}{x} + 15(3d+8e)x + \frac{5}{2}(2d+9e)x^2 + \frac{1}{3}(d+10e)x^3 + \frac{ex^4}{4} + 30(4d+7e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^8,x]
```

output
```
-1/7*d/x^7 + (-10*d - e)/(6*x^6) + (-9*d - 2*e)/x^5 - (15*(8*d + 3*e))/(4*
x^4) - (10*(7*d + 4*e))/x^3 - (21*(6*d + 5*e))/x^2 - (42*(5*d + 6*e))/x +
15*(3*d + 8*e)*x + (5*(2*d + 9*e)*x^2)/2 + ((d + 10*e)*x^3)/3 + (e*x^4)/4
+ 30*(4*d + 7*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^8} dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^8}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{10d+e}{x^7} + \frac{5(9d+2e)}{x^6} + \frac{15(8d+3e)}{x^5} + \frac{30(7d+4e)}{x^4} + \frac{42(6d+5e)}{x^3} + x^2(d+10e) + \frac{42(5d+6e)}{x^2} + 5x(2d+\right.$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{10d+e}{6x^6} - \frac{9d+2e}{x^5} - \frac{15(8d+3e)}{4x^4} + \frac{1}{3}x^3(d+10e) - \frac{10(7d+4e)}{x^3} + \frac{5}{2}x^2(2d+9e) - \frac{21(6d+5e)}{x^2} + 15x(3d+8e) - \frac{42(5d+6e)}{x} + 30(4d+7e)\log(x) - \frac{d}{7x^7} + \frac{ex^4}{4}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^8,x]
```

output
```
-1/7*d/x^7 - (10*d + e)/(6*x^6) - (9*d + 2*e)/x^5 - (15*(8*d + 3*e))/(4*x^
4) - (10*(7*d + 4*e))/x^3 - (21*(6*d + 5*e))/x^2 - (42*(5*d + 6*e))/x + 15
*(3*d + 8*e)*x + (5*(2*d + 9*e)*x^2)/2 + ((d + 10*e)*x^3)/3 + (e*x^4)/4 +
30*(4*d + 7*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x^4 e}{4}+\frac{d\,x^3}{3}+\frac{10x^3 e}{3}+5d\,x^2+\frac{45e\,x^2}{2}+45dx+120ex+\frac{(-210d-252e)x^6+(-126d-105e)x^5+(-70d-40e)}{}$
norman	$\frac{(-210d-252e)x^6+(-126d-105e)x^5+(-70d-40e)x^4+\left(-30d-\frac{45e}{4}\right)x^3+(-9d-2e)x^2+\left(5d+\frac{45e}{2}\right)x^9+(45d+120e)x^8+\left(-\frac{5d}{3}\right.}{x^7}$
default	$\frac{x^4 e}{4}+\frac{d\,x^3}{3}+\frac{10x^3 e}{3}+5d\,x^2+\frac{45e\,x^2}{2}+45dx+120ex-\frac{45d+10e}{5x^5}-\frac{10d+e}{6x^6}-\frac{210d+120e}{3x^3}-\frac{252d+210}{2x^2}$
parallelrisch	$\frac{21e\,x^{11}+28d\,x^{10}+280e\,x^{10}+420d\,x^9+1890e\,x^9+10080\ln(x)x^7 d+17640\ln(x)x^7 e+3780d\,x^8+10080e\,x^8-17640d\,x^6-21168e}{84x^7}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*e+1/3*d*x^3+10/3*x^3*e+5*d*x^2+45/2*e*x^2+45*d*x+120*e*x+((-210*d-
252*e)*x^6+(-126*d-105*e)*x^5+(-70*d-40*e)*x^4+(-30*d-45/4*e)*x^3+(-9*d-2*
e)*x^2+(-5/3*d-1/6*e)*x-1/7*d)/x^7+120*d*ln(x)+210*e*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$=\frac{21\,ex^{11}+28\,(d+10\,e)x^{10}+210\,(2\,d+9\,e)x^9+1260\,(3\,d+8\,e)x^8+2520\,(4\,d+7\,e)x^7\log{(x)}-3528\,(5}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^8,x, algorithm="fricas")
```

output
```
1/84*(21*e*x^11 + 28*(d + 10*e)*x^10 + 210*(2*d + 9*e)*x^9 + 1260*(3*d + 8
*e)*x^8 + 2520*(4*d + 7*e)*x^7*log(x) - 3528*(5*d + 6*e)*x^6 - 1764*(6*d +
 5*e)*x^5 - 840*(7*d + 4*e)*x^4 - 315*(8*d + 3*e)*x^3 - 84*(9*d + 2*e)*x^2
 - 14*(10*d + e)*x - 12*d)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$= \frac{ex^4}{4} + x^3\left(\frac{d}{3}+\frac{10e}{3}\right) + x^2\cdot\left(5d+\frac{45e}{2}\right) + x(45d+120e) + 30\cdot(4d+7e)\log(x)$$
$$+ \frac{-12d + x^6(-17640d-21168e) + x^5(-10584d-8820e) + x^4(-5880d-3360e) + x^3(-2520d-945e}{84x^7}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**8,x)
```

output
```
e*x**4/4 + x**3*(d/3 + 10*e/3) + x**2*(5*d + 45*e/2) + x*(45*d + 120*e) +
30*(4*d + 7*e)*log(x) + (-12*d + x**6*(-17640*d - 21168*e) + x**5*(-10584*
d - 8820*e) + x**4*(-5880*d - 3360*e) + x**3*(-2520*d - 945*e) + x**2*(-75
6*d - 168*e) + x*(-140*d - 14*e))/(84*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$= \frac{1}{4}ex^4 + \frac{1}{3}(d+10e)x^3 + \frac{5}{2}(2d+9e)x^2 + 15(3d+8e)x + 30(4d+7e)\log(x)$$
$$- \frac{3528(5d+6e)x^6 + 1764(6d+5e)x^5 + 840(7d+4e)x^4 + 315(8d+3e)x^3 + 84(9d+2e)x^2 + 14(}{84x^7}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^8,x, algorithm="maxima")
```

output
```
1/4*e*x^4 + 1/3*(d + 10*e)*x^3 + 5/2*(2*d + 9*e)*x^2 + 15*(3*d + 8*e)*x +
30*(4*d + 7*e)*log(x) - 1/84*(3528*(5*d + 6*e)*x^6 + 1764*(6*d + 5*e)*x^5
+ 840*(7*d + 4*e)*x^4 + 315*(8*d + 3*e)*x^3 + 84*(9*d + 2*e)*x^2 + 14*(10*
d + e)*x + 12*d)/x^7
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^8}\,dx$$
$$= \frac{1}{4}ex^4 + \frac{1}{3}dx^3 + \frac{10}{3}ex^3 + 5\,dx^2 + \frac{45}{2}ex^2 + 45\,dx + 120\,ex + 30\,(4\,d+7\,e)\log(|x|)$$
$$- \frac{3528\,(5\,d+6\,e)x^6 + 1764\,(6\,d+5\,e)x^5 + 840\,(7\,d+4\,e)x^4 + 315\,(8\,d+3\,e)x^3 + 84\,(9\,d+2\,e)x^2 + 14\,(}{84\,x^7}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^8,x, algorithm="giac")
```

output
```
1/4*e*x^4 + 1/3*d*x^3 + 10/3*e*x^3 + 5*d*x^2 + 45/2*e*x^2 + 45*d*x + 120*e
*x + 30*(4*d + 7*e)*log(abs(x)) - 1/84*(3528*(5*d + 6*e)*x^6 + 1764*(6*d +
 5*e)*x^5 + 840*(7*d + 4*e)*x^4 + 315*(8*d + 3*e)*x^3 + 84*(9*d + 2*e)*x^2
 + 14*(10*d + e)*x + 12*d)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^8}\,dx$$
$$= x^3\left(\frac{d}{3} + \frac{10\,e}{3}\right) + x^2\left(5\,d + \frac{45\,e}{2}\right) + \ln(x)\,(120\,d+210\,e) + \frac{e\,x^4}{4}$$
$$- \frac{(210\,d+252\,e)\,x^6 + (126\,d+105\,e)\,x^5 + (70\,d+40\,e)\,x^4 + \left(30\,d+\frac{45\,e}{4}\right)x^3 + (9\,d+2\,e)\,x^2 + \left(\frac{5\,d}{3}+\right.}{x^7}$$
$$+ x\,(45\,d+120\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^8,x)
```

output
```
x^3*(d/3 + (10*e)/3) + x^2*(5*d + (45*e)/2) + log(x)*(120*d + 210*e) + (e*
x^4)/4 - (d/7 + x^2*(9*d + 2*e) + x^3*(30*d + (45*e)/4) + x^4*(70*d + 40*e
) + x^5*(126*d + 105*e) + x^6*(210*d + 252*e) + x*((5*d)/3 + e/6))/x^7 + x
*(45*d + 120*e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$= \frac{10080\log(x)\,d\,x^7 + 17640\log(x)\,e\,x^7 + 28d\,x^{10} + 420d\,x^9 + 3780d\,x^8 - 17640d\,x^6 - 10584d\,x^5 - 5880d\,x}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^8,x)
```

output
```
(10080*log(x)*d*x**7 + 17640*log(x)*e*x**7 + 28*d*x**10 + 420*d*x**9 + 378
0*d*x**8 - 17640*d*x**6 - 10584*d*x**5 - 5880*d*x**4 - 2520*d*x**3 - 756*d
*x**2 - 140*d*x - 12*d + 21*e*x**11 + 280*e*x**10 + 1890*e*x**9 + 10080*e*
x**8 - 21168*e*x**6 - 8820*e*x**5 - 3360*e*x**4 - 945*e*x**3 - 168*e*x**2
- 14*e*x)/(84*x**7)
```

3.204 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^9} dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^9} dx = -\frac{d}{8x^8} - \frac{10d+e}{7x^7} - \frac{5(9d+2e)}{6x^6} - \frac{3(8d+3e)}{x^5} - \frac{15(7d+4e)}{2x^4} - \frac{14(6d+5e)}{x^3} - \frac{21(5d+6e)}{x^2} - \frac{30(4d+7e)}{x} + 5(2d+9e)x + \frac{1}{2}(d+10e)x^2 + \frac{ex^3}{3} + 15(3d+8e)\log(x)$$

output
```
-1/8*d/x^8-1/7*(10*d+e)/x^7-5/6*(9*d+2*e)/x^6-3*(8*d+3*e)/x^5-15/2*(7*d+4*
e)/x^4-14*(6*d+5*e)/x^3-21*(5*d+6*e)/x^2-30*(4*d+7*e)/x+5*(2*d+9*e)*x+1/2*
(d+10*e)*x^2+1/3*e*x^3+15*(3*d+8*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^9}\,dx = -\frac{d}{8x^8}+\frac{-10d-e}{7x^7}-\frac{5(9d+2e)}{6x^6} - \frac{3(8d+3e)}{x^5}-\frac{15(7d+4e)}{2x^4}-\frac{14(6d+5e)}{x^3} - \frac{21(5d+6e)}{x^2}-\frac{30(4d+7e)}{x}+5(2d+9e)x + \frac{1}{2}(d+10e)x^2+\frac{ex^3}{3}+15(3d+8e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^9,x]
```

output
```
-1/8*d/x^8 + (-10*d - e)/(7*x^7) - (5*(9*d + 2*e))/(6*x^6) - (3*(8*d + 3*e
))/x^5 - (15*(7*d + 4*e))/(2*x^4) - (14*(6*d + 5*e))/x^3 - (21*(5*d + 6*e)
)/x^2 - (30*(4*d + 7*e))/x + 5*(2*d + 9*e)*x + ((d + 10*e)*x^2)/2 + (e*x^3
)/3 + 15*(3*d + 8*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^9}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^9}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{10d+e}{x^8}+\frac{5(9d+2e)}{x^7}+\frac{15(8d+3e)}{x^6}+\frac{30(7d+4e)}{x^5}+\frac{42(6d+5e)}{x^4}+\frac{42(5d+6e)}{x^3}+\frac{30(4d+7e)}{x^2}+x(d+1$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10d+e}{7x^7}-\frac{5(9d+2e)}{6x^6}-\frac{3(8d+3e)}{x^5}-\frac{15(7d+4e)}{2x^4}-\frac{14(6d+5e)}{x^3}+\frac{1}{2}x^2(d+10e)-\frac{21(5d+6e)}{x^2}+5x(2d+9e)-\frac{30(4d+7e)}{x}+15(3d+8e)\log(x)-\frac{d}{8x^8}+\frac{ex^3}{3}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^9,x]
```

output
```
-1/8*d/x^8 - (10*d + e)/(7*x^7) - (5*(9*d + 2*e))/(6*x^6) - (3*(8*d + 3*e)
)/x^5 - (15*(7*d + 4*e))/(2*x^4) - (14*(6*d + 5*e))/x^3 - (21*(5*d + 6*e))
/x^2 - (30*(4*d + 7*e))/x + 5*(2*d + 9*e)*x + ((d + 10*e)*x^2)/2 + (e*x^3)
/3 + 15*(3*d + 8*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x^3 e}{3}+\frac{d\,x^2}{2}+5e\,x^2+10dx+45ex+\frac{(-120d-210e)x^7+(-105d-126e)x^6+(-84d-70e)x^5+\left(-\frac{105d}{2}-30e\right)x^4+}{x^8}$
norman	$\frac{(-120d-210e)x^7+(-105d-126e)x^6+(-84d-70e)x^5+(-24d-9e)x^3+(10d+45e)x^9+\left(-\frac{105d}{2}-30e\right)x^4+\left(-\frac{15d}{2}-\frac{5e}{3}\right)x^2+(}{x^8}$
default	$\frac{x^3 e}{3}+\frac{d\,x^2}{2}+5e\,x^2+10dx+45ex-\frac{120d+45e}{5x^5}-\frac{45d+10e}{6x^6}-\frac{252d+210e}{3x^3}-\frac{210d+252e}{2x^2}-\frac{d}{8x^8}-\frac{210d-}{4x}$
parallelrisch	$\frac{56e\,x^{11}+84d\,x^{10}+840e\,x^{10}+7560\ln(x)x^8 d+20160\ln(x)x^8 e+1680d\,x^9+7560e\,x^9-20160d\,x^7-35280e\,x^7-17640d\,x^6-21168}{168x^8}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*e+1/2*d*x^2+5*e*x^2+10*d*x+45*e*x+((-120*d-210*e)*x^7+(-105*d-126*
e)*x^6+(-84*d-70*e)*x^5+(-105/2*d-30*e)*x^4+(-24*d-9*e)*x^3+(-15/2*d-5/3*e
)*x^2+(-10/7*d-1/7*e)*x-1/8*d)/x^8+45*d*ln(x)+120*e*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx$$
$$=\frac{56\,ex^{11}+84\,(d+10\,e)x^{10}+840\,(2\,d+9\,e)x^9+2520\,(3\,d+8\,e)x^8\log(x)-5040\,(4\,d+7\,e)x^7-3528\,(5}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^9,x, algorithm="fricas")
```

output
```
1/168*(56*e*x^11 + 84*(d + 10*e)*x^10 + 840*(2*d + 9*e)*x^9 + 2520*(3*d +
8*e)*x^8*log(x) - 5040*(4*d + 7*e)*x^7 - 3528*(5*d + 6*e)*x^6 - 2352*(6*d
+ 5*e)*x^5 - 1260*(7*d + 4*e)*x^4 - 504*(8*d + 3*e)*x^3 - 140*(9*d + 2*e)*
x^2 - 24*(10*d + e)*x - 21*d)/x^8
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx = \frac{ex^3}{3}+x^2\left(\frac{d}{2}+5e\right)+x(10d+45e)+15\cdot(3d+8e)\log(x) + \frac{-21d+x^7(-20160d-35280e)+x^6(-17640d-21168e)+x^5(-14112d-11760e)+x^4(-8820d-5\cdots}{168x^8}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**9,x)
```

output
```
e*x**3/3 + x**2*(d/2 + 5*e) + x*(10*d + 45*e) + 15*(3*d + 8*e)*log(x) + (-
21*d + x**7*(-20160*d - 35280*e) + x**6*(-17640*d - 21168*e) + x**5*(-1411
2*d - 11760*e) + x**4*(-8820*d - 5040*e) + x**3*(-4032*d - 1512*e) + x**2*
(-1260*d - 280*e) + x*(-240*d - 24*e))/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx = \frac{1}{3}\,ex^3+\frac{1}{2}\,(d+10\,e)x^2+5\,(2\,d+9\,e)x+15\,(3\,d+8\,e)\log(x) - \frac{5040\,(4\,d+7\,e)x^7+3528\,(5\,d+6\,e)x^6+2352\,(6\,d+5\,e)x^5+1260\,(7\,d+4\,e)x^4+504\,(8\,d+3\,e)x^3+\cdots}{168\,x^8}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^9,x, algorithm="maxima")
```

output
```
1/3*e*x^3 + 1/2*(d + 10*e)*x^2 + 5*(2*d + 9*e)*x + 15*(3*d + 8*e)*log(x) -
 1/168*(5040*(4*d + 7*e)*x^7 + 3528*(5*d + 6*e)*x^6 + 2352*(6*d + 5*e)*x^5
 + 1260*(7*d + 4*e)*x^4 + 504*(8*d + 3*e)*x^3 + 140*(9*d + 2*e)*x^2 + 24*(
10*d + e)*x + 21*d)/x^8
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx$$
$$=\frac{1}{3}\,ex^3+\frac{1}{2}\,dx^2+5\,ex^2+10\,dx+45\,ex+15\,(3\,d+8\,e)\log\left(|x|\right)$$
$$-\frac{5040\,(4\,d+7\,e)x^7+3528\,(5\,d+6\,e)x^6+2352\,(6\,d+5\,e)x^5+1260\,(7\,d+4\,e)x^4+504\,(8\,d+3\,e)x^3+}{168\,x^8}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^9,x, algorithm="giac")
```

output
```
1/3*e*x^3 + 1/2*d*x^2 + 5*e*x^2 + 10*d*x + 45*e*x + 15*(3*d + 8*e)*log(abs
(x)) - 1/168*(5040*(4*d + 7*e)*x^7 + 3528*(5*d + 6*e)*x^6 + 2352*(6*d + 5*
e)*x^5 + 1260*(7*d + 4*e)*x^4 + 504*(8*d + 3*e)*x^3 + 140*(9*d + 2*e)*x^2
+ 24*(10*d + e)*x + 21*d)/x^8
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx=x^2\left(\frac{d}{2}+5\,e\right)+\ln\left(x\right)\,\left(45\,d+120\,e\right)+\frac{e\,x^3}{3}+x\,\left(10\,d+45\,e\right)$$
$$-\frac{\left(120\,d+210\,e\right)\,x^7+\left(105\,d+126\,e\right)\,x^6+\left(84\,d+70\,e\right)\,x^5+\left(\frac{105\,d}{2}+30\,e\right)\,x^4+\left(24\,d+9\,e\right)\,x^3+\left(\frac{15\,d}{2}\right.}{x^8}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^9,x)
```

output
```
x^2*(d/2 + 5*e) + log(x)*(45*d + 120*e) + (e*x^3)/3 + x*(10*d + 45*e) - (d
/8 + x^2*((15*d)/2 + (5*e)/3) + x^3*(24*d + 9*e) + x^4*((105*d)/2 + 30*e)
+ x^5*(84*d + 70*e) + x^6*(105*d + 126*e) + x^7*(120*d + 210*e) + x*((10*d
)/7 + e/7))/x^8
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^9}\,dx$$
$$=\frac{7560\log(x)\,d\,x^8+20160\log(x)\,e\,x^8+84d\,x^{10}+1680d\,x^9-20160d\,x^7-17640d\,x^6-14112d\,x^5-8820d$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^9,x)
```

output
```
(7560*log(x)*d*x**8 + 20160*log(x)*e*x**8 + 84*d*x**10 + 1680*d*x**9 - 201
60*d*x**7 - 17640*d*x**6 - 14112*d*x**5 - 8820*d*x**4 - 4032*d*x**3 - 1260
*d*x**2 - 240*d*x - 21*d + 56*e*x**11 + 840*e*x**10 + 7560*e*x**9 - 35280*
e*x**7 - 21168*e*x**6 - 11760*e*x**5 - 5040*e*x**4 - 1512*e*x**3 - 280*e*x
**2 - 24*e*x)/(168*x**8)
```

3.205 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}} \, dx = -\frac{d}{9x^9} - \frac{10d+e}{8x^8} - \frac{5(9d+2e)}{7x^7} - \frac{5(8d+3e)}{2x^6} - \frac{6(7d+4e)}{x^5} - \frac{21(6d+5e)}{2x^4} - \frac{14(5d+6e)}{x^3} - \frac{15(4d+7e)}{x^2} - \frac{15(3d+8e)}{x} + (d+10e)x + \frac{ex^2}{2} + 5(2d+9e)\log(x)$$

output

```
-1/9*d/x^9-1/8*(10*d+e)/x^8-5/7*(9*d+2*e)/x^7-5/2*(8*d+3*e)/x^6-6*(7*d+4*e
)/x^5-21/2*(6*d+5*e)/x^4-14*(5*d+6*e)/x^3-15*(4*d+7*e)/x^2-15*(3*d+8*e)/x+
(d+10*e)*x+1/2*e*x^2+5*(2*d+9*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}}\,dx = -\frac{d}{9x^9}+\frac{-10d-e}{8x^8}-\frac{5(9d+2e)}{7x^7}-\frac{5(8d+3e)}{2x^6}-\frac{6(7d+4e)}{x^5}-\frac{21(6d+5e)}{2x^4}-\frac{14(5d+6e)}{x^3}-\frac{15(4d+7e)}{x^2}-\frac{15(3d+8e)}{x}+(d+10e)x+\frac{ex^2}{2}+5(2d+9e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^10,x]
```

output
```
-1/9*d/x^9 + (-10*d - e)/(8*x^8) - (5*(9*d + 2*e))/(7*x^7) - (5*(8*d + 3*e
))/(2*x^6) - (6*(7*d + 4*e))/x^5 - (21*(6*d + 5*e))/(2*x^4) - (14*(5*d + 6
*e))/x^3 - (15*(4*d + 7*e))/x^2 - (15*(3*d + 8*e))/x + (d + 10*e)*x + (e*x
^2)/2 + 5*(2*d + 9*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5(d+ex)}{x^{10}}\,dx$$

$$\Big\downarrow \; 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{10}}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{10d+e}{x^9}+\frac{5(9d+2e)}{x^8}+\frac{15(8d+3e)}{x^7}+\frac{30(7d+4e)}{x^6}+\frac{42(6d+5e)}{x^5}+\frac{42(5d+6e)}{x^4}+\frac{30(4d+7e)}{x^3}+\frac{15(3d-}{x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10d+e}{8x^8}-\frac{5(9d+2e)}{7x^7}-\frac{5(8d+3e)}{2x^6}-\frac{6(7d+4e)}{x^5}-\frac{21(6d+5e)}{2x^4}-\frac{14(5d+6e)}{x^3}-\frac{15(4d+7e)}{x^2}+x(d+10e)-\frac{15(3d+8e)}{x}+5(2d+9e)\log(x)-\frac{d}{9x^9}+\frac{ex^2}{2}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^10,x]
```

output
```
-1/9*d/x^9 - (10*d + e)/(8*x^8) - (5*(9*d + 2*e))/(7*x^7) - (5*(8*d + 3*e)
)/(2*x^6) - (6*(7*d + 4*e))/x^5 - (21*(6*d + 5*e))/(2*x^4) - (14*(5*d + 6*
e))/x^3 - (15*(4*d + 7*e))/x^2 - (15*(3*d + 8*e))/x + (d + 10*e)*x + (e*x^
2)/2 + 5*(2*d + 9*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

method	result
risch	$\frac{e\,x^2}{2}+dx+10ex+\frac{(-45d-120e)x^8+(-60d-105e)x^7+(-70d-84e)x^6+\left(-63d-\frac{105e}{2}\right)x^5+(-42d-24e)x^4+(-20d-}{x^9}$
norman	$\frac{(-70d-84e)x^6+\left(-63d-\frac{105e}{2}\right)x^5+(-60d-105e)x^7+(-45d-120e)x^8+(-42d-24e)x^4+\left(-20d-\frac{15e}{2}\right)x^3+(d+10e)x^{10}+\left(-4\right.}{x^9}$
default	$\frac{e\,x^2}{2}+dx+10ex-\frac{210d+120e}{5x^5}-\frac{120d+45e}{6x^6}-\frac{210d+252e}{3x^3}-\frac{120d+210e}{2x^2}-\frac{10d+e}{8x^8}-\frac{252d+210e}{4x^4}+(10d$
parallelrisch	$\frac{252e\,x^{11}+5040\ln(x)x^9d+22680\ln(x)x^9e+504d\,x^{10}+5040e\,x^{10}-22680d\,x^8-60480e\,x^8-30240d\,x^7-52920e\,x^7-35280d\,x^6-}{504x^9}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2+d*x+10*e*x+((-45*d-120*e)*x^8+(-60*d-105*e)*x^7+(-70*d-84*e)*x^6
+(-63*d-105/2*e)*x^5+(-42*d-24*e)*x^4+(-20*d-15/2*e)*x^3+(-45/7*d-10/7*e)*
x^2+(-5/4*d-1/8*e)*x-1/9*d)/x^9+10*d*ln(x)+45*e*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}}\,dx$$
$$=\frac{252\,ex^{11}+504\,(d+10\,e)x^{10}+2520\,(2\,d+9\,e)x^9\log(x)-7560\,(3\,d+8\,e)x^8-7560\,(4\,d+7\,e)x^7-705}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^10,x, algorithm="fricas")
```

output
```
1/504*(252*e*x^11 + 504*(d + 10*e)*x^10 + 2520*(2*d + 9*e)*x^9*log(x) - 75
60*(3*d + 8*e)*x^8 - 7560*(4*d + 7*e)*x^7 - 7056*(5*d + 6*e)*x^6 - 5292*(6
*d + 5*e)*x^5 - 3024*(7*d + 4*e)*x^4 - 1260*(8*d + 3*e)*x^3 - 360*(9*d + 2
*e)*x^2 - 63*(10*d + e)*x - 56*d)/x^9
```

Sympy [A] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}}\,dx = \frac{ex^2}{2} + x(d+10e) + 5\cdot(2d+9e)\log(x) + \frac{-56d + x^8(-22680d - 60480e) + x^7(-30240d - 52920e) + x^6(-35280d - 42336e) + x^5(-31752d - }{504}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**10,x)
```

output
```
e*x**2/2 + x*(d + 10*e) + 5*(2*d + 9*e)*log(x) + (-56*d + x**8*(-22680*d -
 60480*e) + x**7*(-30240*d - 52920*e) + x**6*(-35280*d - 42336*e) + x**5*(
-31752*d - 26460*e) + x**4*(-21168*d - 12096*e) + x**3*(-10080*d - 3780*e)
 + x**2*(-3240*d - 720*e) + x*(-630*d - 63*e))/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{10}}\,dx = \frac{1}{2}\,ex^2 + (d+10\,e)x + 5\,(2\,d+9\,e)\log(x) - \frac{7560\,(3\,d+8\,e)x^8 + 7560\,(4\,d+7\,e)x^7 + 7056\,(5\,d+6\,e)x^6 + 5292\,(6\,d+5\,e)x^5 + 3024\,(7\,d+4\,e)x^4 -}{504\,x^9}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^10,x, algorithm="maxima")
```

output
```
1/2*e*x^2 + (d + 10*e)*x + 5*(2*d + 9*e)*log(x) - 1/504*(7560*(3*d + 8*e)*
x^8 + 7560*(4*d + 7*e)*x^7 + 7056*(5*d + 6*e)*x^6 + 5292*(6*d + 5*e)*x^5 +
 3024*(7*d + 4*e)*x^4 + 1260*(8*d + 3*e)*x^3 + 360*(9*d + 2*e)*x^2 + 63*(1
0*d + e)*x + 56*d)/x^9
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{10}}\,dx = \frac{1}{2}\,ex^2 + dx + 10\,ex + 5\,(2\,d+9\,e)\log(|x|) - \frac{7560\,(3\,d+8\,e)x^8 + 7560\,(4\,d+7\,e)x^7 + 7056\,(5\,d+6\,e)x^6 + 5292\,(6\,d+5\,e)x^5 + 3024\,(7\,d+4\,e)x^4 -}{504\,x^9}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^10,x, algorithm="giac")
```

output
```
1/2*e*x^2 + d*x + 10*e*x + 5*(2*d + 9*e)*log(abs(x)) - 1/504*(7560*(3*d +
8*e)*x^8 + 7560*(4*d + 7*e)*x^7 + 7056*(5*d + 6*e)*x^6 + 5292*(6*d + 5*e)*
x^5 + 3024*(7*d + 4*e)*x^4 + 1260*(8*d + 3*e)*x^3 + 360*(9*d + 2*e)*x^2 +
63*(10*d + e)*x + 56*d)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{10}}\,dx = \ln(x)\,(10\,d+45\,e) + x\,(d+10\,e) + \frac{e\,x^2}{2} - \frac{(45\,d+120\,e)\,x^8 + (60\,d+105\,e)\,x^7 + (70\,d+84\,e)\,x^6 + \left(63\,d+\frac{105\,e}{2}\right)\,x^5 + (42\,d+24\,e)\,x^4 + (20\,d}{x^9}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^10,x)
```

output
```
log(x)*(10*d + 45*e) + x*(d + 10*e) + (e*x^2)/2 - (d/9 + x^3*(20*d + (15*e
)/2) + x^4*(42*d + 24*e) + x^2*((45*d)/7 + (10*e)/7) + x^6*(70*d + 84*e) +
 x^7*(60*d + 105*e) + x^8*(45*d + 120*e) + x^5*(63*d + (105*e)/2) + x*((5*
d)/4 + e/8))/x^9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{10}}\,dx$$
$$= \frac{5040\log(x)\,d\,x^9 + 22680\log(x)\,e\,x^9 + 504d\,x^{10} - 22680d\,x^8 - 30240d\,x^7 - 35280d\,x^6 - 31752d\,x^5 - 211}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^10,x)
```

output
```
(5040*log(x)*d*x**9 + 22680*log(x)*e*x**9 + 504*d*x**10 - 22680*d*x**8 - 3
0240*d*x**7 - 35280*d*x**6 - 31752*d*x**5 - 21168*d*x**4 - 10080*d*x**3 -
3240*d*x**2 - 630*d*x - 56*d + 252*e*x**11 + 5040*e*x**10 - 60480*e*x**8 -
 52920*e*x**7 - 42336*e*x**6 - 26460*e*x**5 - 12096*e*x**4 - 3780*e*x**3 -
 720*e*x**2 - 63*e*x)/(504*x**9)
```

3.206 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}} dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\begin{aligned}\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}} dx = & -\frac{d}{10x^{10}} - \frac{10d+e}{9x^9} - \frac{5(9d+2e)}{8x^8} \\ & - \frac{15(8d+3e)}{7x^7} - \frac{5(7d+4e)}{x^6} - \frac{42(6d+5e)}{5x^5} \\ & - \frac{21(5d+6e)}{2x^4} - \frac{10(4d+7e)}{x^3} - \frac{15(3d+8e)}{2x^2} \\ & - \frac{5(2d+9e)}{x} + ex + (d+10e)\log(x)\end{aligned}$$

output
```
-1/10*d/x^10-1/9*(10*d+e)/x^9-5/8*(9*d+2*e)/x^8-15/7*(8*d+3*e)/x^7-5*(7*d+
4*e)/x^6-42/5*(6*d+5*e)/x^5-21/2*(5*d+6*e)/x^4-10*(4*d+7*e)/x^3-15/2*(3*d+
8*e)/x^2-5*(2*d+9*e)/x+e*x+(d+10*e)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}}\,dx = -\frac{d}{10x^{10}} + \frac{-10d-e}{9x^9} - \frac{5(9d+2e)}{8x^8} - \frac{15(8d+3e)}{7x^7} - \frac{5(7d+4e)}{x^6} - \frac{42(6d+5e)}{5x^5} - \frac{21(5d+6e)}{2x^4} - \frac{10(4d+7e)}{x^3} - \frac{15(3d+8e)}{2x^2} - \frac{5(2d+9e)}{x} + ex + (d+10e)\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^11,x]
```

output
```
-1/10*d/x^10 + (-10*d - e)/(9*x^9) - (5*(9*d + 2*e))/(8*x^8) - (15*(8*d +
3*e))/(7*x^7) - (5*(7*d + 4*e))/x^6 - (42*(6*d + 5*e))/(5*x^5) - (21*(5*d
+ 6*e))/(2*x^4) - (10*(4*d + 7*e))/x^3 - (15*(3*d + 8*e))/(2*x^2) - (5*(2*
d + 9*e))/x + e*x + (d + 10*e)*Log[x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^{11}}dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{11}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{10d+e}{x^{10}} + \frac{5(9d+2e)}{x^9} + \frac{15(8d+3e)}{x^8} + \frac{30(7d+4e)}{x^7} + \frac{42(6d+5e)}{x^6} + \frac{42(5d+6e)}{x^5} + \frac{30(4d+7e)}{x^4} + \frac{15(3d-}{x^3}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{10d+e}{9x^9} - \frac{5(9d+2e)}{8x^8} - \frac{15(8d+3e)}{7x^7} - \frac{5(7d+4e)}{x^6} - \frac{42(6d+5e)}{5x^5} - \frac{21(5d+6e)}{2x^4} - \frac{10(4d+7e)}{x^3} - \frac{15(3d+8e)}{2x^2} - \frac{5(2d+9e)}{x} + (d+10e)\log(x) - \frac{d}{10x^{10}} + ex$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^11,x]
```

output
```
-1/10*d/x^10 - (10*d + e)/(9*x^9) - (5*(9*d + 2*e))/(8*x^8) - (15*(8*d + 3
*e))/(7*x^7) - (5*(7*d + 4*e))/x^6 - (42*(6*d + 5*e))/(5*x^5) - (21*(5*d +
 6*e))/(2*x^4) - (10*(4*d + 7*e))/x^3 - (15*(3*d + 8*e))/(2*x^2) - (5*(2*d
 + 9*e))/x + e*x + (d + 10*e)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

method	result
risch	$ex+\frac{(-10d-45e)x^9+\left(-\frac{45d}{2}-60e\right)x^8+(-40d-70e)x^7+\left(-\frac{105d}{2}-63e\right)x^6+\left(-\frac{252d}{5}-42e\right)x^5+(-35d-20e)x^4+\left(-\frac{120d}{7}\right.}{x^{10}}$
norman	$\frac{e\,x^{11}+(-40d-70e)x^7+(-35d-20e)x^4+(-10d-45e)x^9+\left(-\frac{252d}{5}-42e\right)x^5+\left(-\frac{120d}{7}-\frac{45e}{7}\right)x^3+\left(-\frac{105d}{2}-63e\right)x^6+\left(-\frac{45d}{2}-\right.}{x^{10}}$
default	$ex-\frac{252d+210e}{5x^5}-\frac{210d+120e}{6x^6}-\frac{120d+210e}{3x^3}-\frac{45d+120e}{2x^2}-\frac{45d+10e}{8x^8}-\frac{210d+252e}{4x^4}+(d+10e)\ln(x)-$
parallelrisch	$2520\ln(x)x^{10}d+25200\ln(x)x^{10}e+2520e\,x^{11}-25200d\,x^9-113400e\,x^9-56700d\,x^8-151200e\,x^8-100800d\,x^7-176400e\,x^7-13$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^11,x,method=_RETURNVERBOSE)
```

output
```
e*x+((-10*d-45*e)*x^9+(-45/2*d-60*e)*x^8+(-40*d-70*e)*x^7+(-105/2*d-63*e)*
x^6+(-252/5*d-42*e)*x^5+(-35*d-20*e)*x^4+(-120/7*d-45/7*e)*x^3+(-45/8*d-5/
4*e)*x^2+(-10/9*d-1/9*e)*x-1/10*d)/x^10+d*ln(x)+10*e*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{11}}\,dx$$
$$=\frac{2520\,ex^{11}+2520\,(d+10\,e)x^{10}\log{(x)}-12600\,(2\,d+9\,e)x^9-18900\,(3\,d+8\,e)x^8-25200\,(4\,d+7\,e)x^7}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^11,x, algorithm="fricas")
```

output
```
1/2520*(2520*e*x^11 + 2520*(d + 10*e)*x^10*log(x) - 12600*(2*d + 9*e)*x^9
- 18900*(3*d + 8*e)*x^8 - 25200*(4*d + 7*e)*x^7 - 26460*(5*d + 6*e)*x^6 -
21168*(6*d + 5*e)*x^5 - 12600*(7*d + 4*e)*x^4 - 5400*(8*d + 3*e)*x^3 - 157
5*(9*d + 2*e)*x^2 - 280*(10*d + e)*x - 252*d)/x^10
```

Sympy [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}}\,dx = ex + (d+10e)\log(x) + \frac{-252d + x^9(-25200d - 113400e) + x^8(-56700d - 151200e) + x^7(-100800d - 176400e) + x^6(-132\ldots}{}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**11,x)
```

output
```
e*x + (d + 10*e)*log(x) + (-252*d + x**9*(-25200*d - 113400*e) + x**8*(-56
700*d - 151200*e) + x**7*(-100800*d - 176400*e) + x**6*(-132300*d - 158760
*e) + x**5*(-127008*d - 105840*e) + x**4*(-88200*d - 50400*e) + x**3*(-432
00*d - 16200*e) + x**2*(-14175*d - 3150*e) + x*(-2800*d - 280*e))/(2520*x*
*10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}}\,dx = ex + (d+10\,e)\log(x) - \frac{12600\,(2\,d+9\,e)x^9 + 18900\,(3\,d+8\,e)x^8 + 25200\,(4\,d+7\,e)x^7 + 26460\,(5\,d+6\,e)x^6 + 21168\,(6\,d+5\ldots}{2520\,x^{10}}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^11,x, algorithm="maxima")
```

output
```
e*x + (d + 10*e)*log(x) - 1/2520*(12600*(2*d + 9*e)*x^9 + 18900*(3*d + 8*e
)*x^8 + 25200*(4*d + 7*e)*x^7 + 26460*(5*d + 6*e)*x^6 + 21168*(6*d + 5*e)*
x^5 + 12600*(7*d + 4*e)*x^4 + 5400*(8*d + 3*e)*x^3 + 1575*(9*d + 2*e)*x^2
+ 280*(10*d + e)*x + 252*d)/x^10
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = ex + (d+10\,e)\log\left(|x|\right) - \frac{12600\,(2\,d+9\,e)x^9 + 18900\,(3\,d+8\,e)x^8 + 25200\,(4\,d+7\,e)x^7 + 26460\,(5\,d+6\,e)x^6 + 21168\,(6\,d+5}{2520\,x^{10}}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^11,x, algorithm="giac")
```

output
```
e*x + (d + 10*e)*log(abs(x)) - 1/2520*(12600*(2*d + 9*e)*x^9 + 18900*(3*d
+ 8*e)*x^8 + 25200*(4*d + 7*e)*x^7 + 26460*(5*d + 6*e)*x^6 + 21168*(6*d +
5*e)*x^5 + 12600*(7*d + 4*e)*x^4 + 5400*(8*d + 3*e)*x^3 + 1575*(9*d + 2*e)
*x^2 + 280*(10*d + e)*x + 252*d)/x^10
```

Mupad [B] (verification not implemented)

Time = 10.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = e\,x - \frac{(10\,d+45\,e)\ x^9 + \left(\frac{45\,d}{2}+60\,e\right)\ x^8 + (40\,d+70\,e)\ x^7 + \left(\frac{105\,d}{2}+63\,e\right)\ x^6 + \left(\frac{252\,d}{5}+42\,e\right)\ x^5 + (35\,d+}{x^{10}} + \ln\left(x\right)\ (d+10\,e)$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^11,x)
```

output
```
e*x - (d/10 + x^4*(35*d + 20*e) + x^2*((45*d)/8 + (5*e)/4) + x^9*(10*d + 4
5*e) + x^8*((45*d)/2 + 60*e) + x^7*(40*d + 70*e) + x^6*((105*d)/2 + 63*e)
+ x^3*((120*d)/7 + (45*e)/7) + x^5*((252*d)/5 + 42*e) + x*((10*d)/9 + e/9)
)/x^10 + log(x)*(d + 10*e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{11}}\,dx$$
$$= \frac{2520\log(x)\,d\,x^{10} + 25200\log(x)\,e\,x^{10} - 25200d\,x^9 - 56700d\,x^8 - 100800d\,x^7 - 132300d\,x^6 - 127008d\,x}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^11,x)
```

output
```
(2520*log(x)*d*x**10 + 25200*log(x)*e*x**10 - 25200*d*x**9 - 56700*d*x**8
- 100800*d*x**7 - 132300*d*x**6 - 127008*d*x**5 - 88200*d*x**4 - 43200*d*x
**3 - 14175*d*x**2 - 2800*d*x - 252*d + 2520*e*x**11 - 113400*e*x**9 - 151
200*e*x**8 - 176400*e*x**7 - 158760*e*x**6 - 105840*e*x**5 - 50400*e*x**4
- 16200*e*x**3 - 3150*e*x**2 - 280*e*x)/(2520*x**10)
```

3.207 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{12}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = -\frac{e}{10x^{10}}-\frac{10e}{9x^9}-\frac{45e}{8x^8}-\frac{120e}{7x^7}-\frac{35e}{x^6}-\frac{252e}{5x^5}-\frac{105e}{2x^4}-\frac{40e}{x^3}-\frac{45e}{2x^2}-\frac{10e}{x}-\frac{d(1+x)^{11}}{11x^{11}}+e\log(x)$$

output
```
-1/10*e/x^10-10/9*e/x^9-45/8*e/x^8-120/7*e/x^7-35*e/x^6-252/5*e/x^5-105/2*
e/x^4-40*e/x^3-45/2*e/x^2-10*e/x-1/11*d*(1+x)^11/x^11+e*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = -\frac{d}{11x^{11}}-\frac{10d+e}{10x^{10}}-\frac{5(9d+2e)}{9x^9}-\frac{15(8d+3e)}{8x^8}-\frac{30(7d+4e)}{7x^7}-\frac{7(6d+5e)}{x^6}-\frac{42(5d+6e)}{5x^5}-\frac{15(4d+7e)}{2x^4}-\frac{5(3d+8e)}{x^3}-\frac{5(2d+9e)}{2x^2}-\frac{d+10e}{x}+e\log(x)$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^12,x]
```

output
```
-1/11*d/x^11 - (10*d + e)/(10*x^10) - (5*(9*d + 2*e))/(9*x^9) - (15*(8*d +
 3*e))/(8*x^8) - (30*(7*d + 4*e))/(7*x^7) - (7*(6*d + 5*e))/x^6 - (42*(5*d
 + 6*e))/(5*x^5) - (15*(4*d + 7*e))/(2*x^4) - (5*(3*d + 8*e))/x^3 - (5*(2*
d + 9*e))/(2*x^2) - (d + 10*e)/x + e*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1184, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5 (d+ex)}{x^{12}} \, dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{12}}dx$$

$$\Big\downarrow \text{87}$$

$$e\int \frac{(x+1)^{10}}{x^{11}}dx-\frac{d(x+1)^{11}}{11x^{11}}$$

$$\Big\downarrow \text{49}$$

$$e\int \left(\frac{1}{x}+\frac{10}{x^2}+\frac{45}{x^3}+\frac{120}{x^4}+\frac{210}{x^5}+\frac{252}{x^6}+\frac{210}{x^7}+\frac{120}{x^8}+\frac{45}{x^9}+\frac{10}{x^{10}}+\frac{1}{x^{11}}\right)dx-\frac{d(x+1)^{11}}{11x^{11}}$$

$$\Big\downarrow \text{2009}$$

$$e\left(-\frac{1}{10x^{10}}-\frac{10}{9x^9}-\frac{45}{8x^8}-\frac{120}{7x^7}-\frac{35}{x^6}-\frac{252}{5x^5}-\frac{105}{2x^4}-\frac{40}{x^3}-\frac{45}{2x^2}-\frac{10}{x}+\log(x)\right)-\frac{d(x+1)^{11}}{11x^{11}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^12,x]
```

output
```
-1/11*(d*(1 + x)^11)/x^11 + e*(-1/10*1/x^10 - 10/(9*x^9) - 45/(8*x^8) - 12
0/(7*x^7) - 35/x^6 - 252/(5*x^5) - 105/(2*x^4) - 40/x^3 - 45/(2*x^2) - 10/
x + Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.33

method	result
norman	$\frac{(-42d-35e)x^5+(-42d-\frac{252e}{5})x^6+(-30d-\frac{120e}{7})x^4+(-30d-\frac{105e}{2})x^7+(-15d-40e)x^8+(-15d-\frac{45e}{8})x^3+(-5d-\frac{45e}{2})x^9+(}{x^{11}}$
risch	$\frac{(-42d-35e)x^5+(-42d-\frac{252e}{5})x^6+(-30d-\frac{120e}{7})x^4+(-30d-\frac{105e}{2})x^7+(-15d-40e)x^8+(-15d-\frac{45e}{8})x^3+(-5d-\frac{45e}{2})x^9+(}{x^{11}}$
default	$-\frac{210d+252e}{5x^5}-\frac{252d+210e}{6x^6}-\frac{45d+120e}{3x^3}-\frac{10d+45e}{2x^2}-\frac{120d+45e}{8x^8}-\frac{120d+210e}{4x^4}+e\ln(x)-\frac{45d+10e}{9x^9}-\frac{21}{}$
parallelrisch	$\frac{27720e\ln(x)x^{11}-27720d\,x^{10}-277200e\,x^{10}-138600d\,x^9-623700e\,x^9-415800d\,x^8-1108800e\,x^8-831600d\,x^7-1455300e\,x^7-}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^12,x,method=_RETURNVERBOSE)
```

output
```
((-42*d-35*e)*x^5+(-42*d-252/5*e)*x^6+(-30*d-120/7*e)*x^4+(-30*d-105/2*e)*
x^7+(-15*d-40*e)*x^8+(-15*d-45/8*e)*x^3+(-5*d-45/2*e)*x^9+(-5*d-10/9*e)*x^
2+(-d-10*e)*x^10+(-d-1/10*e)*x-1/11*d)/x^11+e*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = \frac{27720\,ex^{11}\log(x) - 27720\,(d+10\,e)x^{10} - 69300\,(2\,d+9\,e)x^9 - 138600\,(3\,d+8\,e)x^8 - 207900\,(4\,d+7\,e}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^12,x, algorithm="fricas")
```

output
```
1/27720*(27720*e*x^11*log(x) - 27720*(d + 10*e)*x^10 - 69300*(2*d + 9*e)*x
^9 - 138600*(3*d + 8*e)*x^8 - 207900*(4*d + 7*e)*x^7 - 232848*(5*d + 6*e)*
x^6 - 194040*(6*d + 5*e)*x^5 - 118800*(7*d + 4*e)*x^4 - 51975*(8*d + 3*e)*
x^3 - 15400*(9*d + 2*e)*x^2 - 2772*(10*d + e)*x - 2520*d)/x^11
```

Sympy [A] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = e\log(x) + \frac{-2520d + x^{10}(-27720d - 277200e) + x^9(-138600d - 623700e) + x^8(-415800d - 1108800e) + x^7(-}{}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**12,x)
```

output
```
e*log(x) + (-2520*d + x**10*(-27720*d - 277200*e) + x**9*(-138600*d - 6237
00*e) + x**8*(-415800*d - 1108800*e) + x**7*(-831600*d - 1455300*e) + x**6
*(-1164240*d - 1397088*e) + x**5*(-1164240*d - 970200*e) + x**4*(-831600*d
 - 475200*e) + x**3*(-415800*d - 155925*e) + x**2*(-138600*d - 30800*e) +
x*(-27720*d - 2772*e))/(27720*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{12}}\,dx = e\log(x) - \frac{27720\,(d+10\,e)x^{10} + 69300\,(2\,d+9\,e)x^9 + 138600\,(3\,d+8\,e)x^8 + 207900\,(4\,d+7\,e)x^7 + 232848\,(5\,d$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^12,x, algorithm="maxima")
```

output
```
e*log(x) - 1/27720*(27720*(d + 10*e)*x^10 + 69300*(2*d + 9*e)*x^9 + 138600
*(3*d + 8*e)*x^8 + 207900*(4*d + 7*e)*x^7 + 232848*(5*d + 6*e)*x^6 + 19404
0*(6*d + 5*e)*x^5 + 118800*(7*d + 4*e)*x^4 + 51975*(8*d + 3*e)*x^3 + 15400
*(9*d + 2*e)*x^2 + 2772*(10*d + e)*x + 2520*d)/x^11
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{12}}\,dx = e\log(|x|) - \frac{27720\,(d+10\,e)x^{10} + 69300\,(2\,d+9\,e)x^9 + 138600\,(3\,d+8\,e)x^8 + 207900\,(4\,d+7\,e)x^7 + 232848\,(5\,d$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^12,x, algorithm="giac")
```

output
```
e*log(abs(x)) - 1/27720*(27720*(d + 10*e)*x^10 + 69300*(2*d + 9*e)*x^9 + 1
38600*(3*d + 8*e)*x^8 + 207900*(4*d + 7*e)*x^7 + 232848*(5*d + 6*e)*x^6 +
194040*(6*d + 5*e)*x^5 + 118800*(7*d + 4*e)*x^4 + 51975*(8*d + 3*e)*x^3 +
15400*(9*d + 2*e)*x^2 + 2772*(10*d + e)*x + 2520*d)/x^11
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = e\,\ln(x) - \frac{(d+10\,e)\,x^{10}+\left(5\,d+\frac{45\,e}{2}\right)\,x^9+(15\,d+40\,e)\,x^8+\left(30\,d+\frac{105\,e}{2}\right)\,x^7+\left(42\,d+\frac{252\,e}{5}\right)\,x^6+(42\,d+35}{x^{11}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^12,x)
```

output
```
e*log(x) - (d/11 + x^2*(5*d + (10*e)/9) + x^9*(5*d + (45*e)/2) + x^8*(15*d
 + 40*e) + x^3*(15*d + (45*e)/8) + x^5*(42*d + 35*e) + x^7*(30*d + (105*e)
/2) + x^4*(30*d + (120*e)/7) + x^6*(42*d + (252*e)/5) + x*(d + e/10) + x^1
0*(d + 10*e))/x^11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = \frac{27720\log(x)\,e\,x^{11}-27720d\,x^{10}-138600d\,x^9-415800d\,x^8-831600d\,x^7-1164240d\,x^6-1164240d\,x^5}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^12,x)
```

output

```
(27720*log(x)*e*x**11 - 27720*d*x**10 - 138600*d*x**9 - 415800*d*x**8 - 83
1600*d*x**7 - 1164240*d*x**6 - 1164240*d*x**5 - 831600*d*x**4 - 415800*d*x
**3 - 138600*d*x**2 - 27720*d*x - 2520*d - 277200*e*x**10 - 623700*e*x**9
- 1108800*e*x**8 - 1455300*e*x**7 - 1397088*e*x**6 - 970200*e*x**5 - 47520
0*e*x**4 - 155925*e*x**3 - 30800*e*x**2 - 2772*e*x)/(27720*x**11)
```

3.208 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}} \, dx = -\frac{d(1+x)^{11}}{12x^{12}} + \frac{(d-12e)(1+x)^{11}}{132x^{11}}$$

output
```
-1/12*d*(1+x)^11/x^12+1/132*(d-12*e)*(1+x)^11/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}} \, dx = -\frac{12ex(1+11x+55x^2+165x^3+330x^4+462x^5+462x^6+330x^7+165x^8+55x^9+11x^{10})+d(11+ \cdots}{132x^{12}}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^13,x]
```

output
```
-1/132*(12*e*x*(1 + 11*x + 55*x^2 + 165*x^3 + 330*x^4 + 462*x^5 + 462*x^6
+ 330*x^7 + 165*x^8 + 55*x^9 + 11*x^10) + d*(11 + 120*x + 594*x^2 + 1760*x
^3 + 3465*x^4 + 4752*x^5 + 4620*x^6 + 3168*x^7 + 1485*x^8 + 440*x^9 + 66*x
^10))/x^12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^{13}} \, dx$$

$$\Big\downarrow \; 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{13}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{1}{12}(d-12e)\int \frac{(x+1)^{10}}{x^{12}}dx-\frac{d(x+1)^{11}}{12x^{12}}$$

$$\Big\downarrow \; 48$$

$$\frac{(x+1)^{11}(d-12e)}{132x^{11}}-\frac{d(x+1)^{11}}{12x^{12}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^13,x]
```

output
```
-1/12*(d*(1 + x)^11)/x^12 + ((d - 12*e)*(1 + x)^11)/(132*x^11)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(27) = 54$.

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.97

method	result
norman	$\frac{-e\,x^{11}+\left(-\frac{d}{2}-5e\right)x^{10}+\left(-\frac{10d}{3}-15e\right)x^{9}+\left(-\frac{45d}{4}-30e\right)x^{8}+(-24d-42e)x^{7}+(-35d-42e)x^{6}+(-36d-30e)x^{5}+\left(-\frac{105d}{4}-15e\right.}{x^{12}}$
risch	$\frac{-e\,x^{11}+\left(-\frac{d}{2}-5e\right)x^{10}+\left(-\frac{10d}{3}-15e\right)x^{9}+\left(-\frac{45d}{4}-30e\right)x^{8}+(-24d-42e)x^{7}+(-35d-42e)x^{6}+(-36d-30e)x^{5}+\left(-\frac{105d}{4}-15e\right.}{x^{12}}$
default	$-\frac{120d+210e}{5x^{5}}-\frac{210d+252e}{6x^{6}}-\frac{10d+45e}{3x^{3}}-\frac{d+10e}{2x^{2}}-\frac{210d+120e}{8x^{8}}-\frac{45d+120e}{4x^{4}}-\frac{120d+45e}{9x^{9}}-\frac{252d+210e}{7x^{7}}-\frac{e}{x}$
gosper	$-\frac{132e\,x^{11}+66d\,x^{10}+660e\,x^{10}+440d\,x^{9}+1980e\,x^{9}+1485d\,x^{8}+3960e\,x^{8}+3168d\,x^{7}+5544e\,x^{7}+4620d\,x^{6}+5544e\,x^{6}+4752d\,x}{132x^{12}}$
parallelrisch	$\frac{-132e\,x^{11}-66d\,x^{10}-660e\,x^{10}-440d\,x^{9}-1980e\,x^{9}-1485d\,x^{8}-3960e\,x^{8}-3168d\,x^{7}-5544e\,x^{7}-4620d\,x^{6}-5544e\,x^{6}-4752d\,x^{5}}{132x^{12}}$
orering	$-\frac{(132e\,x^{11}+66d\,x^{10}+660e\,x^{10}+440d\,x^{9}+1980e\,x^{9}+1485d\,x^{8}+3960e\,x^{8}+3168d\,x^{7}+5544e\,x^{7}+4620d\,x^{6}+5544e\,x^{6}+4752d}{132x^{12}(x+1)^{10}}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^13,x,method=_RETURNVERBOSE)
```

output
```
(-e*x^11+(-1/2*d-5*e)*x^10+(-10/3*d-15*e)*x^9+(-45/4*d-30*e)*x^8+(-24*d-42
*e)*x^7+(-35*d-42*e)*x^6+(-36*d-30*e)*x^5+(-105/4*d-15*e)*x^4+(-40/3*d-5*e
)*x^3+(-9/2*d-e)*x^2+(-10/11*d-1/11*e)*x-1/12*d)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(27) = 54$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.16

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = -\frac{132\,ex^{11} + 66\,(d+10\,e)x^{10} + 220\,(2\,d+9\,e)x^9 + 495\,(3\,d+8\,e)x^8 + 792\,(4\,d+7\,e)x^7 + 924\,(5\,d+6\,e}{13}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^13,x, algorithm="fricas")
```

output
```
-1/132*(132*e*x^11 + 66*(d + 10*e)*x^10 + 220*(2*d + 9*e)*x^9 + 495*(3*d +
 8*e)*x^8 + 792*(4*d + 7*e)*x^7 + 924*(5*d + 6*e)*x^6 + 792*(6*d + 5*e)*x^
5 + 495*(7*d + 4*e)*x^4 + 220*(8*d + 3*e)*x^3 + 66*(9*d + 2*e)*x^2 + 12*(1
0*d + e)*x + 11*d)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(26) = 52$.

Time = 6.91 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.23

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = \frac{-11d - 132ex^{11} + x^{10}(-66d-660e) + x^9(-440d-1980e) + x^8(-1485d-3960e) + x^7(-3168d-554}{}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**13,x)
```

output
```
(-11*d - 132*e*x**11 + x**10*(-66*d - 660*e) + x**9*(-440*d - 1980*e) + x*
*8*(-1485*d - 3960*e) + x**7*(-3168*d - 5544*e) + x**6*(-4620*d - 5544*e)
+ x**5*(-4752*d - 3960*e) + x**4*(-3465*d - 1980*e) + x**3*(-1760*d - 660*
e) + x**2*(-594*d - 132*e) + x*(-120*d - 12*e))/(132*x**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.16

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = -\frac{132\,ex^{11} + 66\,(d+10\,e)x^{10} + 220\,(2\,d+9\,e)x^9 + 495\,(3\,d+8\,e)x^8 + 792\,(4\,d+7\,e)x^7 + 924\,(5\,d+6\,e}{13}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^13,x, algorithm="maxima")
```

output
```
-1/132*(132*e*x^11 + 66*(d + 10*e)*x^10 + 220*(2*d + 9*e)*x^9 + 495*(3*d +
 8*e)*x^8 + 792*(4*d + 7*e)*x^7 + 924*(5*d + 6*e)*x^6 + 792*(6*d + 5*e)*x^
5 + 495*(7*d + 4*e)*x^4 + 220*(8*d + 3*e)*x^3 + 66*(9*d + 2*e)*x^2 + 12*(1
0*d + e)*x + 11*d)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.23

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = -\frac{132\,ex^{11} + 66\,dx^{10} + 660\,ex^{10} + 440\,dx^9 + 1980\,ex^9 + 1485\,dx^8 + 3960\,ex^8 + 3168\,dx^7 + 5544\,ex^7 + 4}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^13,x, algorithm="giac")
```

output
```
-1/132*(132*e*x^11 + 66*d*x^10 + 660*e*x^10 + 440*d*x^9 + 1980*e*x^9 + 148
5*d*x^8 + 3960*e*x^8 + 3168*d*x^7 + 5544*e*x^7 + 4620*d*x^6 + 5544*e*x^6 +
 4752*d*x^5 + 3960*e*x^5 + 3465*d*x^4 + 1980*e*x^4 + 1760*d*x^3 + 660*e*x^
3 + 594*d*x^2 + 132*e*x^2 + 120*d*x + 12*e*x + 11*d)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = -\frac{e\,x^{11}+\left(\frac{d}{2}+5\,e\right)\,x^{10}+\left(\frac{10\,d}{3}+15\,e\right)\,x^9+\left(\frac{45\,d}{4}+30\,e\right)\,x^8+(24\,d+42\,e)\,x^7+(35\,d+42\,e)\,x^6+(36}{x^{12}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^13,x)
```

output
```
-(d/12 + x^10*(d/2 + 5*e) + x^9*((10*d)/3 + 15*e) + x^3*((40*d)/3 + 5*e) +
 x^5*(36*d + 30*e) + x^7*(24*d + 42*e) + x^6*(35*d + 42*e) + x^8*((45*d)/4
 + 30*e) + x^4*((105*d)/4 + 15*e) + e*x^11 + x*((10*d)/11 + e/11) + x^2*((
9*d)/2 + e))/x^12
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.23

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{13}}\,dx = \frac{-132e\,x^{11}-66d\,x^{10}-660e\,x^{10}-440d\,x^9-1980e\,x^9-1485d\,x^8-3960e\,x^8-3168d\,x^7-5544e\,x^7-4}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^13,x)
```

output

```
( - 66*d*x**10 - 440*d*x**9 - 1485*d*x**8 - 3168*d*x**7 - 4620*d*x**6 - 47
52*d*x**5 - 3465*d*x**4 - 1760*d*x**3 - 594*d*x**2 - 120*d*x - 11*d - 132*
e*x**11 - 660*e*x**10 - 1980*e*x**9 - 3960*e*x**8 - 5544*e*x**7 - 5544*e*x
**6 - 3960*e*x**5 - 1980*e*x**4 - 660*e*x**3 - 132*e*x**2 - 12*e*x)/(132*x
**12)
```

3.209 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{14}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = -\frac{d(1+x)^{11}}{13x^{13}}+\frac{(2d-13e)(1+x)^{11}}{156x^{12}}-\frac{(2d-13e)(1+x)^{11}}{1716x^{11}}$$

output
```
-1/13*d*(1+x)^11/x^13+1/156*(2*d-13*e)*(1+x)^11/x^12-1/1716*(2*d-13*e)*(1+
x)^11/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(52) = 104.

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = -\frac{13ex(11+120x+594x^2+1760x^3+3465x^4+4752x^5+4620x^6+3168x^7+1485x^8+440x^9+66x^{10}}{}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^14,x]
```

output
```
-1/1716*(13*e*x*(11 + 120*x + 594*x^2 + 1760*x^3 + 3465*x^4 + 4752*x^5 + 4
620*x^6 + 3168*x^7 + 1485*x^8 + 440*x^9 + 66*x^10) + 2*d*(66 + 715*x + 351
0*x^2 + 10296*x^3 + 20020*x^4 + 27027*x^5 + 25740*x^6 + 17160*x^7 + 7722*x
^8 + 2145*x^9 + 286*x^10))/x^13
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1184, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^{14}}dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{14}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{1}{13}(2d-13e)\int \frac{(x+1)^{10}}{x^{13}}dx-\frac{d(x+1)^{11}}{13x^{13}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{1}{13}(2d-13e)\left(-\frac{1}{12}\int \frac{(x+1)^{10}}{x^{12}}dx-\frac{(x+1)^{11}}{12x^{12}}\right)-\frac{d(x+1)^{11}}{13x^{13}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{1}{13}\left(\frac{(x+1)^{11}}{132x^{11}}-\frac{(x+1)^{11}}{12x^{12}}\right)(2d-13e)-\frac{d(x+1)^{11}}{13x^{13}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^14,x]
```

output
```
-1/13*(d*(1 + x)^11)/x^13 - ((2*d - 13*e)*(-1/12*(1 + x)^11/x^12 + (1 + x)
^11/(132*x^11)))/13
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(46) = 92$.

Time = 0.86 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.37

method	result
norman	$\frac{-\frac{e x^{11}}{2}+\left(-\frac{d}{3}-\frac{10e}{3}\right)x^{10}+\left(-\frac{5d}{2}-\frac{45e}{4}\right)x^{9}+(-9d-24e)x^{8}+(-20d-35e)x^{7}+(-30d-36e)x^{6}+\left(-\frac{63d}{2}-\frac{105e}{4}\right)x^{5}+\left(-\frac{70d}{3}-\frac{40e}{3}\right.}{x^{13}}$
risch	$\frac{-\frac{e x^{11}}{2}+\left(-\frac{d}{3}-\frac{10e}{3}\right)x^{10}+\left(-\frac{5d}{2}-\frac{45e}{4}\right)x^{9}+(-9d-24e)x^{8}+(-20d-35e)x^{7}+(-30d-36e)x^{6}+\left(-\frac{63d}{2}-\frac{105e}{4}\right)x^{5}+\left(-\frac{70d}{3}-\frac{40e}{3}\right.}{x^{13}}$
default	$-\frac{45d+120e}{5x^{5}}-\frac{120d+210e}{6x^{6}}-\frac{d+10e}{3x^{3}}-\frac{e}{2x^{2}}-\frac{d}{13x^{13}}-\frac{252d+210e}{8x^{8}}-\frac{10d+45e}{4x^{4}}-\frac{210d+120e}{9x^{9}}-\frac{210d+252e}{7x^{7}}-$
gosper	$-\frac{858e x^{11}+572d x^{10}+5720e x^{10}+4290d x^{9}+19305e x^{9}+15444d x^{8}+41184e x^{8}+34320d x^{7}+60060e x^{7}+51480d x^{6}+61776e}{1716x^{1}}$
parallelrisch	$\frac{-858e x^{11}-572d x^{10}-5720e x^{10}-4290d x^{9}-19305e x^{9}-15444d x^{8}-41184e x^{8}-34320d x^{7}-60060e x^{7}-51480d x^{6}-61776e x}{1716x^{13}}$
orering	$-\frac{(858e x^{11}+572d x^{10}+5720e x^{10}+4290d x^{9}+19305e x^{9}+15444d x^{8}+41184e x^{8}+34320d x^{7}+60060e x^{7}+51480d x^{6}+61776e}{171}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^14,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*e*x^11+(-1/3*d-10/3*e)*x^10+(-5/2*d-45/4*e)*x^9+(-9*d-24*e)*x^8+(-20
*d-35*e)*x^7+(-30*d-36*e)*x^6+(-63/2*d-105/4*e)*x^5+(-70/3*d-40/3*e)*x^4+(
-12*d-9/2*e)*x^3+(-45/11*d-10/11*e)*x^2+(-5/6*d-1/12*e)*x-1/13*d)/x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(46) = 92$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{14}}\,dx =$$
$$-\frac{858\,ex^{11}+572\,(d+10\,e)x^{10}+2145\,(2\,d+9\,e)x^{9}+5148\,(3\,d+8\,e)x^{8}+8580\,(4\,d+7\,e)x^{7}+10296\,(5}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^14,x, algorithm="fricas")
```

output
```
-1/1716*(858*e*x^11 + 572*(d + 10*e)*x^10 + 2145*(2*d + 9*e)*x^9 + 5148*(3
*d + 8*e)*x^8 + 8580*(4*d + 7*e)*x^7 + 10296*(5*d + 6*e)*x^6 + 9009*(6*d +
 5*e)*x^5 + 5720*(7*d + 4*e)*x^4 + 2574*(8*d + 3*e)*x^3 + 780*(9*d + 2*e)*
x^2 + 143*(10*d + e)*x + 132*d)/x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(41) = 82$.

Time = 7.93 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{14}}\,dx = \frac{-132d - 858ex^{11} + x^{10}(-572d - 5720e) + x^9(-4290d - 19305e) + x^8(-15444d - 41184e) + x^7(-3432$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**14,x)
```

output
```
(-132*d - 858*e*x**11 + x**10*(-572*d - 5720*e) + x**9*(-4290*d - 19305*e)
 + x**8*(-15444*d - 41184*e) + x**7*(-34320*d - 60060*e) + x**6*(-51480*d
- 61776*e) + x**5*(-54054*d - 45045*e) + x**4*(-40040*d - 22880*e) + x**3*
(-20592*d - 7722*e) + x**2*(-7020*d - 1560*e) + x*(-1430*d - 143*e))/(1716
*x**13)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(46) = 92$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{14}}\,dx = -\frac{858\,ex^{11} + 572\,(d+10\,e)x^{10} + 2145\,(2\,d+9\,e)x^9 + 5148\,(3\,d+8\,e)x^8 + 8580\,(4\,d+7\,e)x^7 + 10296\,(5$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^14,x, algorithm="maxima")
```

output
```
-1/1716*(858*e*x^11 + 572*(d + 10*e)*x^10 + 2145*(2*d + 9*e)*x^9 + 5148*(3
*d + 8*e)*x^8 + 8580*(4*d + 7*e)*x^7 + 10296*(5*d + 6*e)*x^6 + 9009*(6*d +
 5*e)*x^5 + 5720*(7*d + 4*e)*x^4 + 2574*(8*d + 3*e)*x^3 + 780*(9*d + 2*e)*
x^2 + 143*(10*d + e)*x + 132*d)/x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(46) = 92$.

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{14}}\,dx = -\frac{858\,ex^{11}+572\,dx^{10}+5720\,ex^{10}+4290\,dx^9+19305\,ex^9+15444\,dx^8+41184\,ex^8+34320\,dx^7+6006$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^14,x, algorithm="giac")
```

output
```
-1/1716*(858*e*x^11 + 572*d*x^10 + 5720*e*x^10 + 4290*d*x^9 + 19305*e*x^9
+ 15444*d*x^8 + 41184*e*x^8 + 34320*d*x^7 + 60060*e*x^7 + 51480*d*x^6 + 61
776*e*x^6 + 54054*d*x^5 + 45045*e*x^5 + 40040*d*x^4 + 22880*e*x^4 + 20592*
d*x^3 + 7722*e*x^3 + 7020*d*x^2 + 1560*e*x^2 + 1430*d*x + 143*e*x + 132*d)
/x^13
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{14}}\,dx = -\frac{\frac{e\,x^{11}}{2}+\left(\frac{d}{3}+\frac{10\,e}{3}\right)\,x^{10}+\left(\frac{5\,d}{2}+\frac{45\,e}{4}\right)\,x^9+(9\,d+24\,e)\,x^8+(20\,d+35\,e)\,x^7+(30\,d+36\,e)\,x^6+\left(\frac{63\,d}{2}+\right.}{x^{13}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^14,x)
```

output
```
-(d/13 + x^3*(12*d + (9*e)/2) + x^10*(d/3 + (10*e)/3) + x^8*(9*d + 24*e) +
 x^7*(20*d + 35*e) + x^9*((5*d)/2 + (45*e)/4) + x^6*(30*d + 36*e) + x^2*((
45*d)/11 + (10*e)/11) + x^4*((70*d)/3 + (40*e)/3) + x^5*((63*d)/2 + (105*e
)/4) + (e*x^11)/2 + x*((5*d)/6 + e/12))/x^13
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{14}}\,dx$$
$$= \frac{-858e\,x^{11}-572d\,x^{10}-5720e\,x^{10}-4290d\,x^{9}-19305e\,x^{9}-15444d\,x^{8}-41184e\,x^{8}-34320d\,x^{7}-6006}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^14,x)
```

output
```
( - 572*d*x**10 - 4290*d*x**9 - 15444*d*x**8 - 34320*d*x**7 - 51480*d*x**6
 - 54054*d*x**5 - 40040*d*x**4 - 20592*d*x**3 - 7020*d*x**2 - 1430*d*x - 1
32*d - 858*e*x**11 - 5720*e*x**10 - 19305*e*x**9 - 41184*e*x**8 - 60060*e*
x**7 - 61776*e*x**6 - 45045*e*x**5 - 22880*e*x**4 - 7722*e*x**3 - 1560*e*x
**2 - 143*e*x)/(1716*x**13)
```

3.210 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx=-\frac{d(1+x)^{11}}{14x^{14}}+\frac{(3d-14e)(1+x)^{11}}{182x^{13}}-\frac{(3d-14e)(1+x)^{11}}{1092x^{12}}+\frac{(3d-14e)(1+x)^{11}}{12012x^{11}}$$

output
```
-1/14*d*(1+x)^11/x^14+1/182*(3*d-14*e)*(1+x)^11/x^13-1/1092*(3*d-14*e)*(1+
x)^11/x^12+1/12012*(3*d-14*e)*(1+x)^11/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. $2(71) = 142$.

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx=-\frac{d}{14x^{14}}-\frac{10d+e}{13x^{13}}-\frac{5(9d+2e)}{12x^{12}}-\frac{15(8d+3e)}{11x^{11}}-\frac{3(7d+4e)}{x^{10}}-\frac{14(6d+5e)}{3x^9}-\frac{21(5d+6e)}{4x^8}-\frac{30(4d+7e)}{7x^7}-\frac{5(3d+8e)}{2x^6}-\frac{2d+9e}{x^5}-\frac{d+10e}{4x^4}-\frac{e}{3x^3}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^15,x]
```

output
```
-1/14*d/x^14 - (10*d + e)/(13*x^13) - (5*(9*d + 2*e))/(12*x^12) - (15*(8*d
 + 3*e))/(11*x^11) - (3*(7*d + 4*e))/x^10 - (14*(6*d + 5*e))/(3*x^9) - (21
*(5*d + 6*e))/(4*x^8) - (30*(4*d + 7*e))/(7*x^7) - (5*(3*d + 8*e))/(2*x^6)
 - (2*d + 9*e)/x^5 - (d + 10*e)/(4*x^4) - e/(3*x^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1184, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5 (d+ex)}{x^{15}} dx$$

$$\Big\downarrow \ 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{15}}dx$$

$$\Big\downarrow \ 87$$

$$-\frac{1}{14}(3d-14e)\int \frac{(x+1)^{10}}{x^{14}}dx-\frac{d(x+1)^{11}}{14x^{14}}$$

$$\Big\downarrow \ 55$$

$$-\frac{1}{14}(3d-14e)\left(-\frac{2}{13}\int \frac{(x+1)^{10}}{x^{13}}dx-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{d(x+1)^{11}}{14x^{14}}$$

$$\Big\downarrow \ 55$$

$$-\frac{1}{14}(3d-14e)\left(-\frac{2}{13}\left(-\frac{1}{12}\int \frac{(x+1)^{10}}{x^{12}}dx-\frac{(x+1)^{11}}{12x^{12}}\right)-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{d(x+1)^{11}}{14x^{14}}$$

$$\Big\downarrow \ 48$$

$$-\frac{1}{14}\left(-\frac{(x+1)^{11}}{13x^{13}}-\frac{2}{13}\left(\frac{(x+1)^{11}}{132x^{11}}-\frac{(x+1)^{11}}{12x^{12}}\right)\right)(3d-14e)-\frac{d(x+1)^{11}}{14x^{14}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^15,x]
```

output
```
-1/14*(d*(1 + x)^11)/x^14 - ((3*d - 14*e)*(-1/13*(1 + x)^11/x^13 - (2*(-1/
12*(1 + x)^11/x^12 + (1 + x)^11/(132*x^11)))/13))/14
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.73

method	result
norman	$\frac{-\frac{d}{14}+\left(-\frac{10d}{13}-\frac{e}{13}\right)x+\left(-\frac{15d}{4}-\frac{5e}{6}\right)x^2+\left(-\frac{120d}{11}-\frac{45e}{11}\right)x^3+(-21d-12e)x^4+\left(-28d-\frac{70e}{3}\right)x^5+\left(-\frac{105d}{4}-\frac{63e}{2}\right)x^6+\left(-\frac{120d}{7}-3\right.}{x^{14}}$
risch	$\frac{-\frac{d}{14}+\left(-\frac{10d}{13}-\frac{e}{13}\right)x+\left(-\frac{15d}{4}-\frac{5e}{6}\right)x^2+\left(-\frac{120d}{11}-\frac{45e}{11}\right)x^3+(-21d-12e)x^4+\left(-28d-\frac{70e}{3}\right)x^5+\left(-\frac{105d}{4}-\frac{63e}{2}\right)x^6+\left(-\frac{120d}{7}-3\right.}{x^{14}}$
default	$-\frac{10d+45e}{5x^5}-\frac{45d+120e}{6x^6}-\frac{e}{3x^3}-\frac{10d+e}{13x^{13}}-\frac{210d+252e}{8x^8}-\frac{d+10e}{4x^4}-\frac{d}{14x^{14}}-\frac{252d+210e}{9x^9}-\frac{120d+210e}{7x^7}-\frac{210}{1}$
gosper	$-\frac{4004e\,x^{11}+3003d\,x^{10}+30030e\,x^{10}+24024d\,x^9+108108e\,x^9+90090d\,x^8+240240e\,x^8+205920d\,x^7+360360e\,x^7+315315d\,x}{}$
parallelrisch	$\frac{-4004e\,x^{11}-3003d\,x^{10}-30030e\,x^{10}-24024d\,x^9-108108e\,x^9-90090d\,x^8-240240e\,x^8-205920d\,x^7-360360e\,x^7-315315d\,x^6}{}$
orering	$-\frac{(4004e\,x^{11}+3003d\,x^{10}+30030e\,x^{10}+24024d\,x^9+108108e\,x^9+90090d\,x^8+240240e\,x^8+205920d\,x^7+360360e\,x^7+315315d}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^15,x,method=_RETURNVERBOSE)
```

output
```
(-1/14*d+(-10/13*d-1/13*e)*x+(-15/4*d-5/6*e)*x^2+(-120/11*d-45/11*e)*x^3+(
-21*d-12*e)*x^4+(-28*d-70/3*e)*x^5+(-105/4*d-63/2*e)*x^6+(-120/7*d-30*e)*x
^7+(-15/2*d-20*e)*x^8+(-2*d-9*e)*x^9+(-1/4*d-5/2*e)*x^10-1/3*e*x^11)/x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx =$$
$$-\frac{4004\,ex^{11}+3003\,(d+10\,e)x^{10}+12012\,(2\,d+9\,e)x^9+30030\,(3\,d+8\,e)x^8+51480\,(4\,d+7\,e)x^7+63}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^15,x, algorithm="fricas")
```

output
```
-1/12012*(4004*e*x^11 + 3003*(d + 10*e)*x^10 + 12012*(2*d + 9*e)*x^9 + 300
30*(3*d + 8*e)*x^8 + 51480*(4*d + 7*e)*x^7 + 63063*(5*d + 6*e)*x^6 + 56056
*(6*d + 5*e)*x^5 + 36036*(7*d + 4*e)*x^4 + 16380*(8*d + 3*e)*x^3 + 5005*(9
*d + 2*e)*x^2 + 924*(10*d + e)*x + 858*d)/x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(58) = 116$.

Time = 9.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = \frac{-858d - 4004ex^{11} + x^{10}(-3003d - 30030e) + x^9(-24024d - 108108e) + x^8(-90090d - 240240e) + x^7}{}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**15,x)
```

output
```
(-858*d - 4004*e*x**11 + x**10*(-3003*d - 30030*e) + x**9*(-24024*d - 1081
08*e) + x**8*(-90090*d - 240240*e) + x**7*(-205920*d - 360360*e) + x**6*(-
315315*d - 378378*e) + x**5*(-336336*d - 280280*e) + x**4*(-252252*d - 144
144*e) + x**3*(-131040*d - 49140*e) + x**2*(-45045*d - 10010*e) + x*(-9240
*d - 924*e))/(12012*x**14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{4004\,ex^{11} + 3003\,(d+10\,e)x^{10} + 12012\,(2\,d+9\,e)x^9 + 30030\,(3\,d+8\,e)x^8 + 51480\,(4\,d+7\,e)x^7 + 630}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^15,x, algorithm="maxima")
```

output
```
-1/12012*(4004*e*x^11 + 3003*(d + 10*e)*x^10 + 12012*(2*d + 9*e)*x^9 + 300
30*(3*d + 8*e)*x^8 + 51480*(4*d + 7*e)*x^7 + 63063*(5*d + 6*e)*x^6 + 56056
*(6*d + 5*e)*x^5 + 36036*(7*d + 4*e)*x^4 + 16380*(8*d + 3*e)*x^3 + 5005*(9
*d + 2*e)*x^2 + 924*(10*d + e)*x + 858*d)/x^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(63) = 126$.

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{4004\,ex^{11}+3003\,dx^{10}+30030\,ex^{10}+24024\,dx^9+108108\,ex^9+90090\,dx^8+240240\,ex^8+205920\,dx}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^15,x, algorithm="giac")
```

output
```
-1/12012*(4004*e*x^11 + 3003*d*x^10 + 30030*e*x^10 + 24024*d*x^9 + 108108*
e*x^9 + 90090*d*x^8 + 240240*e*x^8 + 205920*d*x^7 + 360360*e*x^7 + 315315*
d*x^6 + 378378*e*x^6 + 336336*d*x^5 + 280280*e*x^5 + 252252*d*x^4 + 144144
*e*x^4 + 131040*d*x^3 + 49140*e*x^3 + 45045*d*x^2 + 10010*e*x^2 + 9240*d*x
 + 924*e*x + 858*d)/x^14
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{\frac{e\,x^{11}}{3}+\left(\frac{d}{4}+\frac{5\,e}{2}\right)\,x^{10}+\left(2\,d+9\,e\right)\,x^9+\left(\frac{15\,d}{2}+20\,e\right)\,x^8+\left(\frac{120\,d}{7}+30\,e\right)\,x^7+\left(\frac{105\,d}{4}+\frac{63\,e}{2}\right)\,x^6+\left(28\,d\right.}{x^{14}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^15,x)
```

output
```
-(d/14 + x^9*(2*d + 9*e) + x^10*(d/4 + (5*e)/2) + x^2*((15*d)/4 + (5*e)/6)
 + x^4*(21*d + 12*e) + x^8*((15*d)/2 + 20*e) + x^5*(28*d + (70*e)/3) + x^7
*((120*d)/7 + 30*e) + x^6*((105*d)/4 + (63*e)/2) + x^3*((120*d)/11 + (45*e
)/11) + (e*x^11)/3 + x*((10*d)/13 + e/13))/x^14
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{15}}\,dx = \frac{-4004e\,x^{11}-3003d\,x^{10}-30030e\,x^{10}-24024d\,x^9-108108e\,x^9-90090d\,x^8-240240e\,x^8-205920d\,x^7}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^15,x)
```

output
```
( - 3003*d*x**10 - 24024*d*x**9 - 90090*d*x**8 - 205920*d*x**7 - 315315*d*
x**6 - 336336*d*x**5 - 252252*d*x**4 - 131040*d*x**3 - 45045*d*x**2 - 9240
*d*x - 858*d - 4004*e*x**11 - 30030*e*x**10 - 108108*e*x**9 - 240240*e*x**
8 - 360360*e*x**7 - 378378*e*x**6 - 280280*e*x**5 - 144144*e*x**4 - 49140*
e*x**3 - 10010*e*x**2 - 924*e*x)/(12012*x**14)
```

3.211 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{16}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{d(1+x)^{11}}{15x^{15}}+\frac{(4d-15e)(1+x)^{11}}{210x^{14}} -\frac{(4d-15e)(1+x)^{11}}{910x^{13}}+\frac{(4d-15e)(1+x)^{11}}{5460x^{12}} -\frac{(4d-15e)(1+x)^{11}}{60060x^{11}}$$

output
```
-1/15*d*(1+x)^11/x^15+1/210*(4*d-15*e)*(1+x)^11/x^14-1/910*(4*d-15*e)*(1+x
)^11/x^13+1/5460*(4*d-15*e)*(1+x)^11/x^12-1/60060*(4*d-15*e)*(1+x)^11/x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{d}{15x^{15}}-\frac{10d+e}{14x^{14}}-\frac{5(9d+2e)}{13x^{13}} -\frac{5(8d+3e)}{4x^{12}}-\frac{30(7d+4e)}{11x^{11}}-\frac{21(6d+5e)}{5x^{10}} -\frac{14(5d+6e)}{3x^9}-\frac{15(4d+7e)}{4x^8}-\frac{15(3d+8e)}{7x^7} -\frac{5(2d+9e)}{6x^6}-\frac{d+10e}{5x^5}-\frac{e}{4x^4}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^16,x]
```

output
```
-1/15*d/x^15 - (10*d + e)/(14*x^14) - (5*(9*d + 2*e))/(13*x^13) - (5*(8*d
+ 3*e))/(4*x^12) - (30*(7*d + 4*e))/(11*x^11) - (21*(6*d + 5*e))/(5*x^10)
- (14*(5*d + 6*e))/(3*x^9) - (15*(4*d + 7*e))/(4*x^8) - (15*(3*d + 8*e))/(
7*x^7) - (5*(2*d + 9*e))/(6*x^6) - (d + 10*e)/(5*x^5) - e/(4*x^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1184, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5 (d+ex)}{x^{16}} \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{16}}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{1}{15}(4d-15e)\int \frac{(x+1)^{10}}{x^{15}}dx-\frac{d(x+1)^{11}}{15x^{15}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{15}(4d-15e)\left(-\frac{3}{14}\int \frac{(x+1)^{10}}{x^{14}}dx-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{d(x+1)^{11}}{15x^{15}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{15}(4d-15e)\left(-\frac{3}{14}\left(-\frac{2}{13}\int \frac{(x+1)^{10}}{x^{13}}dx-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{d(x+1)^{11}}{15x^{15}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{15}(4d-15e)\left(-\frac{3}{14}\left(-\frac{2}{13}\left(-\frac{1}{12}\int\frac{(x+1)^{10}}{x^{12}}dx-\frac{(x+1)^{11}}{12x^{12}}\right)-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{d(x+1)^{11}}{15x^{15}}$$

$\downarrow$ 48

$$-\frac{1}{15}\left(-\frac{(x+1)^{11}}{14x^{14}}-\frac{3}{14}\left(-\frac{(x+1)^{11}}{13x^{13}}-\frac{2}{13}\left(\frac{(x+1)^{11}}{132x^{11}}-\frac{(x+1)^{11}}{12x^{12}}\right)\right)\right)(4d-15e)-\frac{d(x+1)^{11}}{15x^{15}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^16,x]
```

output
```
-1/15*(d*(1 + x)^11)/x^15 - ((4*d - 15*e)*(-1/14*(1 + x)^11/x^14 - (3*(-1/
13*(1 + x)^11/x^13 - (2*(-1/12*(1 + x)^11/x^12 + (1 + x)^11/(132*x^11)))/1
3))/14))/15
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

method	result
norman	$\frac{-\frac{d}{15}+\left(-\frac{5d}{7}-\frac{e}{14}\right)x+\left(-\frac{45d}{13}-\frac{10e}{13}\right)x^2+\left(-10d-\frac{15e}{4}\right)x^3+\left(-\frac{210d}{11}-\frac{120e}{11}\right)x^4+\left(-\frac{126d}{5}-21e\right)x^5+\left(-\frac{70d}{3}-28e\right)x^6+\left(-15d-\right.}{x^{15}}$
risch	$\frac{-\frac{d}{15}+\left(-\frac{5d}{7}-\frac{e}{14}\right)x+\left(-\frac{45d}{13}-\frac{10e}{13}\right)x^2+\left(-10d-\frac{15e}{4}\right)x^3+\left(-\frac{210d}{11}-\frac{120e}{11}\right)x^4+\left(-\frac{126d}{5}-21e\right)x^5+\left(-\frac{70d}{3}-28e\right)x^6+\left(-15d-\right.}{x^{15}}$
default	$-\frac{d+10e}{5x^5}-\frac{10d+45e}{6x^6}-\frac{45d+10e}{13x^{13}}-\frac{120d+210e}{8x^8}-\frac{e}{4x^4}-\frac{10d+e}{14x^{14}}-\frac{210d+252e}{9x^9}-\frac{45d+120e}{7x^7}-\frac{252d+210e}{10x^{10}}-$
gosper	$-\frac{15015e\,x^{11}+12012d\,x^{10}+120120e\,x^{10}+100100d\,x^9+450450e\,x^9+386100d\,x^8+1029600e\,x^8+900900d\,x^7+1576575e\,x^7+14}{}$
parallelrisch	$\frac{-15015e\,x^{11}-12012d\,x^{10}-120120e\,x^{10}-100100d\,x^9-450450e\,x^9-386100d\,x^8-1029600e\,x^8-900900d\,x^7-1576575e\,x^7-14}{}$
orering	$-\frac{(15015e\,x^{11}+12012d\,x^{10}+120120e\,x^{10}+100100d\,x^9+450450e\,x^9+386100d\,x^8+1029600e\,x^8+900900d\,x^7+1576575e\,x^7+1}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^16,x,method=_RETURNVERBOSE)
```

output
```
(-1/15*d+(-5/7*d-1/14*e)*x+(-45/13*d-10/13*e)*x^2+(-10*d-15/4*e)*x^3+(-210
/11*d-120/11*e)*x^4+(-126/5*d-21*e)*x^5+(-70/3*d-28*e)*x^6+(-15*d-105/4*e)
*x^7+(-45/7*d-120/7*e)*x^8+(-5/3*d-15/2*e)*x^9+(-1/5*d-2*e)*x^10-1/4*e*x^1
1)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx =$$
$$-\frac{15015\,ex^{11}+12012\,(d+10\,e)x^{10}+50050\,(2\,d+9\,e)x^9+128700\,(3\,d+8\,e)x^8+225225\,(4\,d+7\,e)x^7+}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^16,x, algorithm="fricas")
```

output
```
-1/60060*(15015*e*x^11 + 12012*(d + 10*e)*x^10 + 50050*(2*d + 9*e)*x^9 + 1
28700*(3*d + 8*e)*x^8 + 225225*(4*d + 7*e)*x^7 + 280280*(5*d + 6*e)*x^6 +
252252*(6*d + 5*e)*x^5 + 163800*(7*d + 4*e)*x^4 + 75075*(8*d + 3*e)*x^3 +
23100*(9*d + 2*e)*x^2 + 4290*(10*d + e)*x + 4004*d)/x^15
```

Sympy [A] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = \frac{-4004d-15015ex^{11}+x^{10}(-12012d-120120e)+x^9(-100100d-450450e)+x^8(-386100d-102960$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**16,x)
```

output
```
(-4004*d - 15015*e*x**11 + x**10*(-12012*d - 120120*e) + x**9*(-100100*d -
 450450*e) + x**8*(-386100*d - 1029600*e) + x**7*(-900900*d - 1576575*e) +
 x**6*(-1401400*d - 1681680*e) + x**5*(-1513512*d - 1261260*e) + x**4*(-11
46600*d - 655200*e) + x**3*(-600600*d - 225225*e) + x**2*(-207900*d - 4620
0*e) + x*(-42900*d - 4290*e))/(60060*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{15015\,ex^{11}+12012\,(d+10\,e)x^{10}+50050\,(2\,d+9\,e)x^9+128700\,(3\,d+8\,e)x^8+225225\,(4\,d+7\,e)x^7$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^16,x, algorithm="maxima")
```

output
```
-1/60060*(15015*e*x^11 + 12012*(d + 10*e)*x^10 + 50050*(2*d + 9*e)*x^9 + 1
28700*(3*d + 8*e)*x^8 + 225225*(4*d + 7*e)*x^7 + 280280*(5*d + 6*e)*x^6 +
252252*(6*d + 5*e)*x^5 + 163800*(7*d + 4*e)*x^4 + 75075*(8*d + 3*e)*x^3 +
23100*(9*d + 2*e)*x^2 + 4290*(10*d + e)*x + 4004*d)/x^15
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{16}}\,dx = -\frac{15015\,ex^{11}+12012\,dx^{10}+120120\,ex^{10}+100100\,dx^9+450450\,ex^9+386100\,dx^8+1029600\,ex^8+900}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^16,x, algorithm="giac")
```

output
```
-1/60060*(15015*e*x^11 + 12012*d*x^10 + 120120*e*x^10 + 100100*d*x^9 + 450
450*e*x^9 + 386100*d*x^8 + 1029600*e*x^8 + 900900*d*x^7 + 1576575*e*x^7 +
1401400*d*x^6 + 1681680*e*x^6 + 1513512*d*x^5 + 1261260*e*x^5 + 1146600*d*
x^4 + 655200*e*x^4 + 600600*d*x^3 + 225225*e*x^3 + 207900*d*x^2 + 46200*e*
x^2 + 42900*d*x + 4290*e*x + 4004*d)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{16}}\,dx = -\frac{\frac{e\,x^{11}}{4}+\left(\frac{d}{5}+2\,e\right)\,x^{10}+\left(\frac{5\,d}{3}+\frac{15\,e}{2}\right)\,x^9+\left(\frac{45\,d}{7}+\frac{120\,e}{7}\right)\,x^8+\left(15\,d+\frac{105\,e}{4}\right)\,x^7+\left(\frac{70\,d}{3}+28\,e\right)\,x^6+\left(\frac{126\,d}{5}\right.}{x^{15}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^16,x)
```

output
```
-(d/15 + x^10*(d/5 + 2*e) + x^3*(10*d + (15*e)/4) + x^9*((5*d)/3 + (15*e)/
2) + x^2*((45*d)/13 + (10*e)/13) + x^6*((70*d)/3 + 28*e) + x^7*(15*d + (10
5*e)/4) + x^5*((126*d)/5 + 21*e) + x^8*((45*d)/7 + (120*e)/7) + x^4*((210*
d)/11 + (120*e)/11) + (e*x^11)/4 + x*((5*d)/7 + e/14))/x^15
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{16}}\,dx$$
$$= \frac{-15015e\,x^{11}-12012d\,x^{10}-120120e\,x^{10}-100100d\,x^{9}-450450e\,x^{9}-386100d\,x^{8}-1029600e\,x^{8}-900}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^16,x)
```

output
```
( - 12012*d*x**10 - 100100*d*x**9 - 386100*d*x**8 - 900900*d*x**7 - 140140
0*d*x**6 - 1513512*d*x**5 - 1146600*d*x**4 - 600600*d*x**3 - 207900*d*x**2
 - 42900*d*x - 4004*d - 15015*e*x**11 - 120120*e*x**10 - 450450*e*x**9 - 1
029600*e*x**8 - 1576575*e*x**7 - 1681680*e*x**6 - 1261260*e*x**5 - 655200*
e*x**4 - 225225*e*x**3 - 46200*e*x**2 - 4290*e*x)/(60060*x**15)
```

3.212 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{17}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{17}}\,dx = -\frac{d(1+x)^{11}}{16x^{16}}+\frac{(5d-16e)(1+x)^{11}}{240x^{15}}-\frac{(5d-16e)(1+x)^{11}}{840x^{14}}+\frac{(5d-16e)(1+x)^{11}}{3640x^{13}}-\frac{(5d-16e)(1+x)^{11}}{21840x^{12}}+\frac{(5d-16e)(1+x)^{11}}{240240x^{11}}$$

output
```
-1/16*d*(1+x)^11/x^16+1/240*(5*d-16*e)*(1+x)^11/x^15-1/840*(5*d-16*e)*(1+x
)^11/x^14+1/3640*(5*d-16*e)*(1+x)^11/x^13-1/21840*(5*d-16*e)*(1+x)^11/x^12
+1/240240*(5*d-16*e)*(1+x)^11/x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{d}{16x^{16}}-\frac{10d+e}{15x^{15}}-\frac{5(9d+2e)}{14x^{14}}-\frac{15(8d+3e)}{13x^{13}}-\frac{5(7d+4e)}{2x^{12}}-\frac{42(6d+5e)}{11x^{11}}-\frac{21(5d+6e)}{5x^{10}}-\frac{10(4d+7e)}{3x^9}-\frac{15(3d+8e)}{8x^8}-\frac{5(2d+9e)}{7x^7}-\frac{d+10e}{6x^6}-\frac{e}{5x^5}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^17,x]
```

output
```
-1/16*d/x^16 - (10*d + e)/(15*x^15) - (5*(9*d + 2*e))/(14*x^14) - (15*(8*d
 + 3*e))/(13*x^13) - (5*(7*d + 4*e))/(2*x^12) - (42*(6*d + 5*e))/(11*x^11)
 - (21*(5*d + 6*e))/(5*x^10) - (10*(4*d + 7*e))/(3*x^9) - (15*(3*d + 8*e))
/(8*x^8) - (5*(2*d + 9*e))/(7*x^7) - (d + 10*e)/(6*x^6) - e/(5*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1184, 87, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5(d+ex)}{x^{17}}\,dx$$

$$\Big\downarrow \; 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{17}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{1}{16}(5d-16e)\int \frac{(x+1)^{10}}{x^{16}}dx-\frac{d(x+1)^{11}}{16x^{16}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{16}(5d-16e)\left(-\frac{4}{15}\int \frac{(x+1)^{10}}{x^{15}}dx-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{d(x+1)^{11}}{16x^{16}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{16}(5d-16e)\left(-\frac{4}{15}\left(-\frac{3}{14}\int \frac{(x+1)^{10}}{x^{14}}dx-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{d(x+1)^{11}}{16x^{16}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{16}(5d-16e)\left(-\frac{4}{15}\left(-\frac{3}{14}\left(-\frac{2}{13}\int \frac{(x+1)^{10}}{x^{13}}dx-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{d(x+1)^{11}}{16x^{16}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{16}(5d-16e)\left(-\frac{4}{15}\left(-\frac{3}{14}\left(-\frac{2}{13}\left(-\frac{1}{12}\int \frac{(x+1)^{10}}{x^{12}}dx-\frac{(x+1)^{11}}{12x^{12}}\right)-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{d(x+1)^{11}}{16x^{16}}$$

$$\Big\downarrow \; 48$$

$$-\frac{1}{16}\left(-\frac{(x+1)^{11}}{15x^{15}}-\frac{4}{15}\left(-\frac{(x+1)^{11}}{14x^{14}}-\frac{3}{14}\left(-\frac{(x+1)^{11}}{13x^{13}}-\frac{2}{13}\left(\frac{(x+1)^{11}}{132x^{11}}-\frac{(x+1)^{11}}{12x^{12}}\right)\right)\right)\right)(5d-16e)-\frac{d(x+1)^{11}}{16x^{16}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^17,x]
```

output
```
-1/16*(d*(1 + x)^11)/x^16 - ((5*d - 16*e)*(-1/15*(1 + x)^11/x^15 - (4*(-1/
14*(1 + x)^11/x^14 - (3*(-1/13*(1 + x)^11/x^13 - (2*(-1/12*(1 + x)^11/x^12
 + (1 + x)^11/(132*x^11)))/13))/14))/15))/16
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

method	result
norman	$\frac{-\frac{d}{16}+\left(-\frac{2d}{3}-\frac{e}{15}\right)x+\left(-\frac{45d}{14}-\frac{5e}{7}\right)x^2+\left(-\frac{120d}{13}-\frac{45e}{13}\right)x^3+\left(-\frac{35d}{2}-10e\right)x^4+\left(-\frac{252d}{11}-\frac{210e}{11}\right)x^5+\left(-21d-\frac{126e}{5}\right)x^6+\left(-\frac{40d}{3}-\right.}{x^{16}}$
risch	$\frac{-\frac{d}{16}+\left(-\frac{2d}{3}-\frac{e}{15}\right)x+\left(-\frac{45d}{14}-\frac{5e}{7}\right)x^2+\left(-\frac{120d}{13}-\frac{45e}{13}\right)x^3+\left(-\frac{35d}{2}-10e\right)x^4+\left(-\frac{252d}{11}-\frac{210e}{11}\right)x^5+\left(-21d-\frac{126e}{5}\right)x^6+\left(-\frac{40d}{3}-\right.}{x^{16}}$
default	$-\frac{e}{5x^5}-\frac{d+10e}{6x^6}-\frac{120d+45e}{13x^{13}}-\frac{45d+120e}{8x^8}-\frac{45d+10e}{14x^{14}}-\frac{120d+210e}{9x^9}-\frac{10d+45e}{7x^7}-\frac{210d+252e}{10x^{10}}-\frac{10d+e}{15x^{15}}-25$
gosper	$-\frac{48048e\,x^{11}+40040d\,x^{10}+400400e\,x^{10}+343200d\,x^9+1544400e\,x^9+1351350d\,x^8+3603600e\,x^8+3203200d\,x^7+5605600e\,x^7}{}$
parallelrisch	$\frac{-48048e\,x^{11}-40040d\,x^{10}-400400e\,x^{10}-343200d\,x^9-1544400e\,x^9-1351350d\,x^8-3603600e\,x^8-3203200d\,x^7-5605600e\,x^7-}{}$
orering	$-\frac{\left(48048e\,x^{11}+40040d\,x^{10}+400400e\,x^{10}+343200d\,x^9+1544400e\,x^9+1351350d\,x^8+3603600e\,x^8+3203200d\,x^7+5605600e\,x\right.}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^17,x,method=_RETURNVERBOSE)
```

output
```
(-1/16*d+(-2/3*d-1/15*e)*x+(-45/14*d-5/7*e)*x^2+(-120/13*d-45/13*e)*x^3+(-
35/2*d-10*e)*x^4+(-252/11*d-210/11*e)*x^5+(-21*d-126/5*e)*x^6+(-40/3*d-70/
3*e)*x^7+(-45/8*d-15*e)*x^8+(-10/7*d-45/7*e)*x^9+(-1/6*d-5/3*e)*x^10-1/5*e
*x^11)/x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{17}}\,dx =$$
$$-\frac{48048\,ex^{11}+40040\,(d+10\,e)x^{10}+171600\,(2\,d+9\,e)x^9+450450\,(3\,d+8\,e)x^8+800800\,(4\,d+7\,e)x^7}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^17,x, algorithm="fricas")
```

output
```
-1/240240*(48048*e*x^11 + 40040*(d + 10*e)*x^10 + 171600*(2*d + 9*e)*x^9 +
 450450*(3*d + 8*e)*x^8 + 800800*(4*d + 7*e)*x^7 + 1009008*(5*d + 6*e)*x^6
 + 917280*(6*d + 5*e)*x^5 + 600600*(7*d + 4*e)*x^4 + 277200*(8*d + 3*e)*x^
3 + 85800*(9*d + 2*e)*x^2 + 16016*(10*d + e)*x + 15015*d)/x^16
```

Sympy [A] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{17}}\,dx = \frac{-15015d-48048ex^{11}+x^{10}(-40040d-400400e)+x^9(-343200d-1544400e)+x^8(-1351350d-360$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**17,x)
```

output
```
(-15015*d - 48048*e*x**11 + x**10*(-40040*d - 400400*e) + x**9*(-343200*d
- 1544400*e) + x**8*(-1351350*d - 3603600*e) + x**7*(-3203200*d - 5605600*
e) + x**6*(-5045040*d - 6054048*e) + x**5*(-5503680*d - 4586400*e) + x**4*
(-4204200*d - 2402400*e) + x**3*(-2217600*d - 831600*e) + x**2*(-772200*d
- 171600*e) + x*(-160160*d - 16016*e))/(240240*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{17}}\,dx = -\frac{48048\,ex^{11}+40040\,(d+10\,e)x^{10}+171600\,(2\,d+9\,e)x^9+450450\,(3\,d+8\,e)x^8+800800\,(4\,d+7\,e)x^7$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^17,x, algorithm="maxima")
```

output
```
-1/240240*(48048*e*x^11 + 40040*(d + 10*e)*x^10 + 171600*(2*d + 9*e)*x^9 +
 450450*(3*d + 8*e)*x^8 + 800800*(4*d + 7*e)*x^7 + 1009008*(5*d + 6*e)*x^6
 + 917280*(6*d + 5*e)*x^5 + 600600*(7*d + 4*e)*x^4 + 277200*(8*d + 3*e)*x^
3 + 85800*(9*d + 2*e)*x^2 + 16016*(10*d + e)*x + 15015*d)/x^16
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{48048\,ex^{11}+40040\,dx^{10}+400400\,ex^{10}+343200\,dx^9+1544400\,ex^9+1351350\,dx^8+3603600\,ex^8+3}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^17,x, algorithm="giac")
```

output
```
-1/240240*(48048*e*x^11 + 40040*d*x^10 + 400400*e*x^10 + 343200*d*x^9 + 15
44400*e*x^9 + 1351350*d*x^8 + 3603600*e*x^8 + 3203200*d*x^7 + 5605600*e*x^
7 + 5045040*d*x^6 + 6054048*e*x^6 + 5503680*d*x^5 + 4586400*e*x^5 + 420420
0*d*x^4 + 2402400*e*x^4 + 2217600*d*x^3 + 831600*e*x^3 + 772200*d*x^2 + 17
1600*e*x^2 + 160160*d*x + 16016*e*x + 15015*d)/x^16
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{\frac{e\,x^{11}}{5}+\left(\frac{d}{6}+\frac{5\,e}{3}\right)\,x^{10}+\left(\frac{10\,d}{7}+\frac{45\,e}{7}\right)\,x^9+\left(\frac{45\,d}{8}+15\,e\right)\,x^8+\left(\frac{40\,d}{3}+\frac{70\,e}{3}\right)\,x^7+\left(21\,d+\frac{126\,e}{5}\right)\,x^6+\left(\frac{252\,d}{11}\right.}{x^{16}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^17,x)
```

output
```
-(d/16 + x^10*(d/6 + (5*e)/3) + x^4*((35*d)/2 + 10*e) + x^2*((45*d)/14 + (
5*e)/7) + x^8*((45*d)/8 + 15*e) + x^9*((10*d)/7 + (45*e)/7) + x^7*((40*d)/
3 + (70*e)/3) + x^6*(21*d + (126*e)/5) + x^3*((120*d)/13 + (45*e)/13) + x^
5*((252*d)/11 + (210*e)/11) + (e*x^11)/5 + x*((2*d)/3 + e/15))/x^16
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{17}}\,dx$$
$$= \frac{-48048e\,x^{11}-40040d\,x^{10}-400400e\,x^{10}-343200d\,x^9-1544400e\,x^9-1351350d\,x^8-3603600e\,x^8-3}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^17,x)
```

output
```
( - 40040*d*x**10 - 343200*d*x**9 - 1351350*d*x**8 - 3203200*d*x**7 - 5045
040*d*x**6 - 5503680*d*x**5 - 4204200*d*x**4 - 2217600*d*x**3 - 772200*d*x
**2 - 160160*d*x - 15015*d - 48048*e*x**11 - 400400*e*x**10 - 1544400*e*x*
*9 - 3603600*e*x**8 - 5605600*e*x**7 - 6054048*e*x**6 - 4586400*e*x**5 - 2
402400*e*x**4 - 831600*e*x**3 - 171600*e*x**2 - 16016*e*x)/(240240*x**16)
```

3.213 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{18}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 128

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{d(1+x)^{11}}{17x^{17}}+\frac{(6d-17e)(1+x)^{11}}{272x^{16}} -\frac{(6d-17e)(1+x)^{11}}{816x^{15}}+\frac{(6d-17e)(1+x)^{11}}{2856x^{14}} -\frac{(6d-17e)(1+x)^{11}}{12376x^{13}}+\frac{(6d-17e)(1+x)^{11}}{74256x^{12}} -\frac{(6d-17e)(1+x)^{11}}{816816x^{11}}$$

output
```
-1/17*d*(1+x)^11/x^17+1/272*(6*d-17*e)*(1+x)^11/x^16-1/816*(6*d-17*e)*(1+x
)^11/x^15+1/2856*(6*d-17*e)*(1+x)^11/x^14-1/12376*(6*d-17*e)*(1+x)^11/x^13
+1/74256*(6*d-17*e)*(1+x)^11/x^12-1/816816*(6*d-17*e)*(1+x)^11/x^11
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{d}{17x^{17}}-\frac{10d+e}{16x^{16}}-\frac{9d+2e}{3x^{15}}-\frac{15(8d+3e)}{14x^{14}}-\frac{30(7d+4e)}{13x^{13}}-\frac{7(6d+5e)}{2x^{12}}-\frac{42(5d+6e)}{11x^{11}}-\frac{3(4d+7e)}{x^{10}}-\frac{5(3d+8e)}{3x^9}-\frac{5(2d+9e)}{8x^8}-\frac{d+10e}{7x^7}-\frac{e}{6x^6}$$

input

```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^18,x]
```

output

```
-1/17*d/x^17 - (10*d + e)/(16*x^16) - (9*d + 2*e)/(3*x^15) - (15*(8*d + 3*
e))/(14*x^14) - (30*(7*d + 4*e))/(13*x^13) - (7*(6*d + 5*e))/(2*x^12) - (4
2*(5*d + 6*e))/(11*x^11) - (3*(4*d + 7*e))/x^10 - (5*(3*d + 8*e))/(3*x^9)
- (5*(2*d + 9*e))/(8*x^8) - (d + 10*e)/(7*x^7) - e/(6*x^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1184, 87, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5(d+ex)}{x^{18}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{18}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{1}{17}(6d-17e)\int \frac{(x+1)^{10}}{x^{17}}dx-\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{17}(6d-17e)\left(-\frac{5}{16}\int \frac{(x+1)^{10}}{x^{16}}dx-\frac{(x+1)^{11}}{16x^{16}}\right)-\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{17}(6d-17e)\left(-\frac{5}{16}\left(-\frac{4}{15}\int \frac{(x+1)^{10}}{x^{15}}dx-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{(x+1)^{11}}{16x^{16}}\right)-\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{17}(6d-17e)\left(-\frac{5}{16}\left(-\frac{4}{15}\left(-\frac{3}{14}\int \frac{(x+1)^{10}}{x^{14}}dx-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{(x+1)^{11}}{16x^{16}}\right)-\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{17}(6d-17e)\left(-\frac{5}{16}\left(-\frac{4}{15}\left(-\frac{3}{14}\left(-\frac{2}{13}\int \frac{(x+1)^{10}}{x^{13}}dx-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\frac{(x+1)^{11}}{16x^{16}}\right)-\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{17}(6d-17e)\left(-\frac{5}{16}\left(-\frac{4}{15}\left(-\frac{3}{14}\left(-\frac{2}{13}\left(-\frac{1}{12}\int \frac{(x+1)^{10}}{x^{12}}dx-\frac{(x+1)^{11}}{12x^{12}}\right)-\frac{(x+1)^{11}}{13x^{13}}\right)-\frac{(x+1)^{11}}{14x^{14}}\right)-\frac{(x+1)^{11}}{15x^{15}}\right)-\right.$$
$$\frac{d(x+1)^{11}}{17x^{17}}$$

$$\Big\downarrow \; 48$$

$$-\frac{1}{17}\left(-\frac{(x+1)^{11}}{16x^{16}}-\frac{5}{16}\left(-\frac{(x+1)^{11}}{15x^{15}}-\frac{4}{15}\left(-\frac{(x+1)^{11}}{14x^{14}}-\frac{3}{14}\left(-\frac{(x+1)^{11}}{13x^{13}}-\frac{2}{13}\left(\frac{(x+1)^{11}}{132x^{11}}-\frac{(x+1)^{11}}{12x^{12}}\right)\right)\right)\right)\right)$$
$$17e)-\frac{d(x+1)^{11}}{17x^{17}}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^18,x]
```

output
```
-1/17*(d*(1 + x)^11)/x^17 - ((6*d - 17*e)*(-1/16*(1 + x)^11/x^16 - (5*(-1/
15*(1 + x)^11/x^15 - (4*(-1/14*(1 + x)^11/x^14 - (3*(-1/13*(1 + x)^11/x^13
 - (2*(-1/12*(1 + x)^11/x^12 + (1 + x)^11/(132*x^11)))/13))/14))/15))/16))
/17
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

method	result
norman	$\frac{-\frac{d}{17}+\left(-\frac{5d}{8}-\frac{e}{16}\right)x+\left(-3d-\frac{2e}{3}\right)x^2+\left(-\frac{60d}{7}-\frac{45e}{14}\right)x^3+\left(-\frac{210d}{13}-\frac{120e}{13}\right)x^4+\left(-21d-\frac{35e}{2}\right)x^5+\left(-\frac{210d}{11}-\frac{252e}{11}\right)x^6+(-12d-21}{x^{17}}$
risch	$\frac{-\frac{d}{17}+\left(-\frac{5d}{8}-\frac{e}{16}\right)x+\left(-3d-\frac{2e}{3}\right)x^2+\left(-\frac{60d}{7}-\frac{45e}{14}\right)x^3+\left(-\frac{210d}{13}-\frac{120e}{13}\right)x^4+\left(-21d-\frac{35e}{2}\right)x^5+\left(-\frac{210d}{11}-\frac{252e}{11}\right)x^6+(-12d-21}{x^{17}}$
default	$-\frac{e}{6x^6}-\frac{210d+120e}{13x^{13}}-\frac{10d+45e}{8x^8}-\frac{120d+45e}{14x^{14}}-\frac{45d+120e}{9x^9}-\frac{d+10e}{7x^7}-\frac{120d+210e}{10x^{10}}-\frac{d}{17x^{17}}-\frac{45d+10e}{15x^{15}}-21$
gosper	$-\frac{136136e\,x^{11}+116688d\,x^{10}+1166880e\,x^{10}+1021020d\,x^9+4594590e\,x^9+4084080d\,x^8+10890880e\,x^8+9801792d\,x^7+171531}{}$
parallelrisch	$\frac{-136136e\,x^{11}-116688d\,x^{10}-1166880e\,x^{10}-1021020d\,x^9-4594590e\,x^9-4084080d\,x^8-10890880e\,x^8-9801792d\,x^7-171531}{}$
orering	$-\frac{(136136e\,x^{11}+116688d\,x^{10}+1166880e\,x^{10}+1021020d\,x^9+4594590e\,x^9+4084080d\,x^8+10890880e\,x^8+9801792d\,x^7+17153}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^18,x,method=_RETURNVERBOSE)
```

output
```
(-1/17*d+(-5/8*d-1/16*e)*x+(-3*d-2/3*e)*x^2+(-60/7*d-45/14*e)*x^3+(-210/13
*d-120/13*e)*x^4+(-21*d-35/2*e)*x^5+(-210/11*d-252/11*e)*x^6+(-12*d-21*e)*
x^7+(-5*d-40/3*e)*x^8+(-5/4*d-45/8*e)*x^9+(-1/7*d-10/7*e)*x^10-1/6*e*x^11)
/x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{136136\,ex^{11}+116688\,(d+10\,e)x^{10}+510510\,(2\,d+9\,e)x^9+1361360\,(3\,d+8\,e)x^8+2450448\,(4\,d+7}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^18,x, algorithm="fricas")
```

output
```
-1/816816*(136136*e*x^11 + 116688*(d + 10*e)*x^10 + 510510*(2*d + 9*e)*x^9
 + 1361360*(3*d + 8*e)*x^8 + 2450448*(4*d + 7*e)*x^7 + 3118752*(5*d + 6*e)
*x^6 + 2858856*(6*d + 5*e)*x^5 + 1884960*(7*d + 4*e)*x^4 + 875160*(8*d + 3
*e)*x^3 + 272272*(9*d + 2*e)*x^2 + 51051*(10*d + e)*x + 48048*d)/x^17
```

Sympy [A] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{18}}\,dx = \frac{-48048d - 136136ex^{11} + x^{10}(-116688d - 1166880e) + x^9(-1021020d - 4594590e) + x^8(-4084080d -$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**18,x)
```

output
```
(-48048*d - 136136*e*x**11 + x**10*(-116688*d - 1166880*e) + x**9*(-102102
0*d - 4594590*e) + x**8*(-4084080*d - 10890880*e) + x**7*(-9801792*d - 171
53136*e) + x**6*(-15593760*d - 18712512*e) + x**5*(-17153136*d - 14294280*
e) + x**4*(-13194720*d - 7539840*e) + x**3*(-7001280*d - 2625480*e) + x**2
*(-2450448*d - 544544*e) + x*(-510510*d - 51051*e))/(816816*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{18}}\,dx = -\frac{136136\,ex^{11} + 116688\,(d+10\,e)x^{10} + 510510\,(2\,d+9\,e)x^9 + 1361360\,(3\,d+8\,e)x^8 + 2450448\,(4\,d+7$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^18,x, algorithm="maxima")
```

output
```
-1/816816*(136136*e*x^11 + 116688*(d + 10*e)*x^10 + 510510*(2*d + 9*e)*x^9
 + 1361360*(3*d + 8*e)*x^8 + 2450448*(4*d + 7*e)*x^7 + 3118752*(5*d + 6*e)
*x^6 + 2858856*(6*d + 5*e)*x^5 + 1884960*(7*d + 4*e)*x^4 + 875160*(8*d + 3
*e)*x^3 + 272272*(9*d + 2*e)*x^2 + 51051*(10*d + e)*x + 48048*d)/x^17
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{18}}\,dx = -\frac{136136\,ex^{11}+116688\,dx^{10}+1166880\,ex^{10}+1021020\,dx^9+4594590\,ex^9+4084080\,dx^8+10890880\,e}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^18,x, algorithm="giac")
```

output
```
-1/816816*(136136*e*x^11 + 116688*d*x^10 + 1166880*e*x^10 + 1021020*d*x^9
+ 4594590*e*x^9 + 4084080*d*x^8 + 10890880*e*x^8 + 9801792*d*x^7 + 1715313
6*e*x^7 + 15593760*d*x^6 + 18712512*e*x^6 + 17153136*d*x^5 + 14294280*e*x^
5 + 13194720*d*x^4 + 7539840*e*x^4 + 7001280*d*x^3 + 2625480*e*x^3 + 24504
48*d*x^2 + 544544*e*x^2 + 510510*d*x + 51051*e*x + 48048*d)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{18}}\,dx = -\frac{\frac{e\,x^{11}}{6}+\left(\frac{d}{7}+\frac{10\,e}{7}\right)\,x^{10}+\left(\frac{5\,d}{4}+\frac{45\,e}{8}\right)\,x^9+\left(5\,d+\frac{40\,e}{3}\right)\,x^8+(12\,d+21\,e)\,x^7+\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)\,x^6+(21\,d}{x^{17}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^18,x)
```

output
```
-(d/17 + x^2*(3*d + (2*e)/3) + x^10*(d/7 + (10*e)/7) + x^7*(12*d + 21*e) +
 x^8*(5*d + (40*e)/3) + x^5*(21*d + (35*e)/2) + x^9*((5*d)/4 + (45*e)/8) +
 x^3*((60*d)/7 + (45*e)/14) + x^4*((210*d)/13 + (120*e)/13) + x^6*((210*d)
/11 + (252*e)/11) + (e*x^11)/6 + x*((5*d)/8 + e/16))/x^17
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{18}}\,dx$$
$$= \frac{-136136e\,x^{11} - 116688d\,x^{10} - 1166880e\,x^{10} - 1021020d\,x^9 - 4594590e\,x^9 - 4084080d\,x^8 - 10890880e\,\ldots}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^18,x)
```

output
```
( - 116688*d*x**10 - 1021020*d*x**9 - 4084080*d*x**8 - 9801792*d*x**7 - 15
593760*d*x**6 - 17153136*d*x**5 - 13194720*d*x**4 - 7001280*d*x**3 - 24504
48*d*x**2 - 510510*d*x - 48048*d - 136136*e*x**11 - 1166880*e*x**10 - 4594
590*e*x**9 - 10890880*e*x**8 - 17153136*e*x**7 - 18712512*e*x**6 - 1429428
0*e*x**5 - 7539840*e*x**4 - 2625480*e*x**3 - 544544*e*x**2 - 51051*e*x)/(8
16816*x**17)
```

3.214 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{19}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{d}{18x^{18}}-\frac{10d+e}{17x^{17}}-\frac{5(9d+2e)}{16x^{16}}-\frac{8d+3e}{x^{15}}-\frac{15(7d+4e)}{7x^{14}}-\frac{42(6d+5e)}{13x^{13}}-\frac{7(5d+6e)}{2x^{12}}-\frac{30(4d+7e)}{11x^{11}}-\frac{3(3d+8e)}{2x^{10}}-\frac{5(2d+9e)}{9x^9}-\frac{d+10e}{8x^8}-\frac{e}{7x^7}$$

output
```
-1/18*d/x^18-1/17*(10*d+e)/x^17-5/16*(9*d+2*e)/x^16-(8*d+3*e)/x^15-15/7*(7
*d+4*e)/x^14-42/13*(6*d+5*e)/x^13-7/2*(5*d+6*e)/x^12-30/11*(4*d+7*e)/x^11-
3/2*(3*d+8*e)/x^10-5/9*(2*d+9*e)/x^9-1/8*(d+10*e)/x^8-1/7*e/x^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{d}{18x^{18}}-\frac{10d+e}{17x^{17}}-\frac{5(9d+2e)}{16x^{16}}-\frac{8d+3e}{x^{15}}-\frac{15(7d+4e)}{7x^{14}}-\frac{42(6d+5e)}{13x^{13}}-\frac{7(5d+6e)}{2x^{12}}-\frac{30(4d+7e)}{11x^{11}}-\frac{3(3d+8e)}{2x^{10}}-\frac{5(2d+9e)}{9x^9}-\frac{d+10e}{8x^8}-\frac{e}{7x^7}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^19,x]
```

output
```
-1/18*d/x^18 - (10*d + e)/(17*x^17) - (5*(9*d + 2*e))/(16*x^16) - (8*d + 3
*e)/x^15 - (15*(7*d + 4*e))/(7*x^14) - (42*(6*d + 5*e))/(13*x^13) - (7*(5*
d + 6*e))/(2*x^12) - (30*(4*d + 7*e))/(11*x^11) - (3*(3*d + 8*e))/(2*x^10)
 - (5*(2*d + 9*e))/(9*x^9) - (d + 10*e)/(8*x^8) - e/(7*x^7)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^{19}}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{19}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{10d+e}{x^{18}}+\frac{5(9d+2e)}{x^{17}}+\frac{15(8d+3e)}{x^{16}}+\frac{30(7d+4e)}{x^{15}}+\frac{42(6d+5e)}{x^{14}}+\frac{42(5d+6e)}{x^{13}}+\frac{30(4d+7e)}{x^{12}}+\frac{15(3d-}{x^{1}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{10d+e}{17x^{17}}-\frac{5(9d+2e)}{16x^{16}}-\frac{8d+3e}{x^{15}}-\frac{15(7d+4e)}{7x^{14}}-\frac{42(6d+5e)}{13x^{13}}-\frac{7(5d+6e)}{2x^{12}}-\frac{30(4d+7e)}{11x^{11}}-\frac{3(3d+8e)}{2x^{10}}-\frac{5(2d+9e)}{9x^9}-\frac{d+10e}{8x^8}-\frac{d}{18x^{18}}-\frac{e}{7x^7}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^19,x]
```

output
```
-1/18*d/x^18 - (10*d + e)/(17*x^17) - (5*(9*d + 2*e))/(16*x^16) - (8*d + 3
*e)/x^15 - (15*(7*d + 4*e))/(7*x^14) - (42*(6*d + 5*e))/(13*x^13) - (7*(5*
d + 6*e))/(2*x^12) - (30*(4*d + 7*e))/(11*x^11) - (3*(3*d + 8*e))/(2*x^10)
 - (5*(2*d + 9*e))/(9*x^9) - (d + 10*e)/(8*x^8) - e/(7*x^7)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result
norman	$\frac{-\frac{d}{18}+\left(-\frac{10d}{17}-\frac{e}{17}\right)x+\left(-\frac{45d}{16}-\frac{5e}{8}\right)x^2+(-8d-3e)x^3+\left(-15d-\frac{60e}{7}\right)x^4+\left(-\frac{252d}{13}-\frac{210e}{13}\right)x^5+\left(-\frac{35d}{2}-21e\right)x^6+\left(-\frac{120d}{11}-\frac{210e}{11}\right.}{x^{18}}$
risch	$\frac{-\frac{d}{18}+\left(-\frac{10d}{17}-\frac{e}{17}\right)x+\left(-\frac{45d}{16}-\frac{5e}{8}\right)x^2+(-8d-3e)x^3+\left(-15d-\frac{60e}{7}\right)x^4+\left(-\frac{252d}{13}-\frac{210e}{13}\right)x^5+\left(-\frac{35d}{2}-21e\right)x^6+\left(-\frac{120d}{11}-\frac{210e}{11}\right.}{x^{18}}$
default	$-\frac{d}{18x^{18}}-\frac{252d+210e}{13x^{13}}-\frac{d+10e}{8x^8}-\frac{210d+120e}{14x^{14}}-\frac{10d+45e}{9x^9}-\frac{e}{7x^7}-\frac{45d+120e}{10x^{10}}-\frac{10d+e}{17x^{17}}-\frac{120d+45e}{15x^{15}}-\frac{120d}{11}$
gosper	$-\frac{350064e\,x^{11}+306306d\,x^{10}+3063060e\,x^{10}+2722720d\,x^9+12252240e\,x^9+11027016d\,x^8+29405376e\,x^8+26732160d\,x^7+46}{}$
parallelrisch	$\frac{-350064e\,x^{11}-306306d\,x^{10}-3063060e\,x^{10}-2722720d\,x^9-12252240e\,x^9-11027016d\,x^8-29405376e\,x^8-26732160d\,x^7-467}{}$
orering	$-\frac{(350064e\,x^{11}+306306d\,x^{10}+3063060e\,x^{10}+2722720d\,x^9+12252240e\,x^9+11027016d\,x^8+29405376e\,x^8+26732160d\,x^7+46}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^19,x,method=_RETURNVERBOSE)
```

output
```
(-1/18*d+(-10/17*d-1/17*e)*x+(-45/16*d-5/8*e)*x^2+(-8*d-3*e)*x^3+(-15*d-60
/7*e)*x^4+(-252/13*d-210/13*e)*x^5+(-35/2*d-21*e)*x^6+(-120/11*d-210/11*e)
*x^7+(-9/2*d-12*e)*x^8+(-10/9*d-5*e)*x^9+(-1/8*d-5/4*e)*x^10-1/7*e*x^11)/x
^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{350064\,ex^{11}+306306\,(d+10\,e)x^{10}+1361360\,(2\,d+9\,e)x^{9}+3675672\,(3\,d+8\,e)x^{8}+6683040\,(4\,d+$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^19,x, algorithm="fricas")
```

output
```
-1/2450448*(350064*e*x^11 + 306306*(d + 10*e)*x^10 + 1361360*(2*d + 9*e)*x
^9 + 3675672*(3*d + 8*e)*x^8 + 6683040*(4*d + 7*e)*x^7 + 8576568*(5*d + 6*
e)*x^6 + 7916832*(6*d + 5*e)*x^5 + 5250960*(7*d + 4*e)*x^4 + 2450448*(8*d
+ 3*e)*x^3 + 765765*(9*d + 2*e)*x^2 + 144144*(10*d + e)*x + 136136*d)/x^18
```

Sympy [A] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = \frac{-136136d-350064ex^{11}+x^{10}(-306306d-3063060e)+x^{9}(-2722720d-12252240e)+x^{8}(-11027016$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**19,x)
```

output
```
(-136136*d - 350064*e*x**11 + x**10*(-306306*d - 3063060*e) + x**9*(-27227
20*d - 12252240*e) + x**8*(-11027016*d - 29405376*e) + x**7*(-26732160*d -
 46781280*e) + x**6*(-42882840*d - 51459408*e) + x**5*(-47500992*d - 39584
160*e) + x**4*(-36756720*d - 21003840*e) + x**3*(-19603584*d - 7351344*e)
+ x**2*(-6891885*d - 1531530*e) + x*(-1441440*d - 144144*e))/(2450448*x**1
8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{19}}\,dx = -\frac{350064\,ex^{11} + 306306\,(d+10\,e)x^{10} + 1361360\,(2\,d+9\,e)x^9 + 3675672\,(3\,d+8\,e)x^8 + 6683040\,(4\,d+}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^19,x, algorithm="maxima")
```

output
```
-1/2450448*(350064*e*x^11 + 306306*(d + 10*e)*x^10 + 1361360*(2*d + 9*e)*x
^9 + 3675672*(3*d + 8*e)*x^8 + 6683040*(4*d + 7*e)*x^7 + 8576568*(5*d + 6*
e)*x^6 + 7916832*(6*d + 5*e)*x^5 + 5250960*(7*d + 4*e)*x^4 + 2450448*(8*d
+ 3*e)*x^3 + 765765*(9*d + 2*e)*x^2 + 144144*(10*d + e)*x + 136136*d)/x^18
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{19}}\,dx = -\frac{350064\,ex^{11} + 306306\,dx^{10} + 3063060\,ex^{10} + 2722720\,dx^9 + 12252240\,ex^9 + 11027016\,dx^8 + 2940537}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^19,x, algorithm="giac")
```

output
```
-1/2450448*(350064*e*x^11 + 306306*d*x^10 + 3063060*e*x^10 + 2722720*d*x^9
 + 12252240*e*x^9 + 11027016*d*x^8 + 29405376*e*x^8 + 26732160*d*x^7 + 467
81280*e*x^7 + 42882840*d*x^6 + 51459408*e*x^6 + 47500992*d*x^5 + 39584160*
e*x^5 + 36756720*d*x^4 + 21003840*e*x^4 + 19603584*d*x^3 + 7351344*e*x^3 +
 6891885*d*x^2 + 1531530*e*x^2 + 1441440*d*x + 144144*e*x + 136136*d)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{\frac{e\,x^{11}}{7}+\left(\frac{d}{8}+\frac{5\,e}{4}\right)\,x^{10}+\left(\frac{10\,d}{9}+5\,e\right)\,x^9+\left(\frac{9\,d}{2}+12\,e\right)\,x^8+\left(\frac{120\,d}{11}+\frac{210\,e}{11}\right)\,x^7+\left(\frac{35\,d}{2}+21\,e\right)\,x^6+\left(\frac{252\,d}{13}\right.}{x^{18}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^19,x)
```

output
```
-(d/18 + x^3*(8*d + 3*e) + x^10*(d/8 + (5*e)/4) + x^8*((9*d)/2 + 12*e) + x
^9*((10*d)/9 + 5*e) + x^6*((35*d)/2 + 21*e) + x^2*((45*d)/16 + (5*e)/8) +
x^4*(15*d + (60*e)/7) + x^7*((120*d)/11 + (210*e)/11) + x^5*((252*d)/13 +
(210*e)/13) + (e*x^11)/7 + x*((10*d)/17 + e/17))/x^18
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = \frac{-350064e\,x^{11}-306306d\,x^{10}-3063060e\,x^{10}-2722720d\,x^9-12252240e\,x^9-11027016d\,x^8-2940537}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^19,x)
```

output
```
( - 306306*d*x**10 - 2722720*d*x**9 - 11027016*d*x**8 - 26732160*d*x**7 -
42882840*d*x**6 - 47500992*d*x**5 - 36756720*d*x**4 - 19603584*d*x**3 - 68
91885*d*x**2 - 1441440*d*x - 136136*d - 350064*e*x**11 - 3063060*e*x**10 -
 12252240*e*x**9 - 29405376*e*x**8 - 46781280*e*x**7 - 51459408*e*x**6 - 3
9584160*e*x**5 - 21003840*e*x**4 - 7351344*e*x**3 - 1531530*e*x**2 - 14414
4*e*x)/(2450448*x**18)
```

3.215 $\int \frac{(d+ex)(1+2x+x^2)^5}{x^{20}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{20}}\,dx = -\frac{d}{19x^{19}}-\frac{10d+e}{18x^{18}}-\frac{5(9d+2e)}{17x^{17}}-\frac{15(8d+3e)}{16x^{16}}-\frac{2(7d+4e)}{x^{15}}-\frac{3(6d+5e)}{x^{14}}-\frac{42(5d+6e)}{13x^{13}}-\frac{5(4d+7e)}{2x^{12}}-\frac{15(3d+8e)}{11x^{11}}-\frac{2d+9e}{2x^{10}}-\frac{d+10e}{9x^9}-\frac{e}{8x^8}$$

output
```
-1/19*d/x^19-1/18*(10*d+e)/x^18-5/17*(9*d+2*e)/x^17-15/16*(8*d+3*e)/x^16-2
*(7*d+4*e)/x^15-3*(6*d+5*e)/x^14-42/13*(5*d+6*e)/x^13-5/2*(4*d+7*e)/x^12-1
5/11*(3*d+8*e)/x^11-1/2*(2*d+9*e)/x^10-1/9*(d+10*e)/x^9-1/8*e/x^8
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{20}}\,dx = -\frac{d}{19x^{19}}-\frac{10d+e}{18x^{18}}-\frac{5(9d+2e)}{17x^{17}}-\frac{15(8d+3e)}{16x^{16}}-\frac{2(7d+4e)}{x^{15}}-\frac{3(6d+5e)}{x^{14}}-\frac{42(5d+6e)}{13x^{13}}-\frac{5(4d+7e)}{2x^{12}}-\frac{15(3d+8e)}{11x^{11}}-\frac{2d+9e}{2x^{10}}-\frac{d+10e}{9x^9}-\frac{e}{8x^8}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^20,x]
```

output
```
-1/19*d/x^19 - (10*d + e)/(18*x^18) - (5*(9*d + 2*e))/(17*x^17) - (15*(8*d
 + 3*e))/(16*x^16) - (2*(7*d + 4*e))/x^15 - (3*(6*d + 5*e))/x^14 - (42*(5*
d + 6*e))/(13*x^13) - (5*(4*d + 7*e))/(2*x^12) - (15*(3*d + 8*e))/(11*x^11
) - (2*d + 9*e)/(2*x^10) - (d + 10*e)/(9*x^9) - e/(8*x^8)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2x+1)^5(d+ex)}{x^{20}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{20}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{10d+e}{x^{19}}+\frac{5(9d+2e)}{x^{18}}+\frac{15(8d+3e)}{x^{17}}+\frac{30(7d+4e)}{x^{16}}+\frac{42(6d+5e)}{x^{15}}+\frac{42(5d+6e)}{x^{14}}+\frac{30(4d+7e)}{x^{13}}+\frac{15(3d-}{x^{12}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10d+e}{18x^{18}}-\frac{5(9d+2e)}{17x^{17}}-\frac{15(8d+3e)}{16x^{16}}-\frac{2(7d+4e)}{x^{15}}-\frac{3(6d+5e)}{x^{14}}-\frac{42(5d+6e)}{13x^{13}}-\frac{5(4d+7e)}{2x^{12}}-\frac{15(3d+8e)}{11x^{11}}-\frac{2d+9e}{2x^{10}}-\frac{d+10e}{9x^9}-\frac{d}{19x^{19}}-\frac{e}{8x^8}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^20,x]
```

output
```
-1/19*d/x^19 - (10*d + e)/(18*x^18) - (5*(9*d + 2*e))/(17*x^17) - (15*(8*d
 + 3*e))/(16*x^16) - (2*(7*d + 4*e))/x^15 - (3*(6*d + 5*e))/x^14 - (42*(5*
d + 6*e))/(13*x^13) - (5*(4*d + 7*e))/(2*x^12) - (15*(3*d + 8*e))/(11*x^11
) - (2*d + 9*e)/(2*x^10) - (d + 10*e)/(9*x^9) - e/(8*x^8)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

method	result
norman	$\frac{-\frac{d}{19}+\left(-\frac{5d}{9}-\frac{e}{18}\right)x+\left(-\frac{45d}{17}-\frac{10e}{17}\right)x^2+\left(-\frac{15d}{2}-\frac{45e}{16}\right)x^3+(-14d-8e)x^4+(-18d-15e)x^5+\left(-\frac{210d}{13}-\frac{252e}{13}\right)x^6+\left(-10d-\frac{35e}{2}\right)}{x^{19}}$
risch	$\frac{-\frac{d}{19}+\left(-\frac{5d}{9}-\frac{e}{18}\right)x+\left(-\frac{45d}{17}-\frac{10e}{17}\right)x^2+\left(-\frac{15d}{2}-\frac{45e}{16}\right)x^3+(-14d-8e)x^4+(-18d-15e)x^5+\left(-\frac{210d}{13}-\frac{252e}{13}\right)x^6+\left(-10d-\frac{35e}{2}\right)}{x^{19}}$
default	$-\frac{10d+e}{18x^{18}}-\frac{210d+252e}{13x^{13}}-\frac{e}{8x^8}-\frac{252d+210e}{14x^{14}}-\frac{d+10e}{9x^9}-\frac{10d+45e}{10x^{10}}-\frac{45d+10e}{17x^{17}}-\frac{210d+120e}{15x^{15}}-\frac{45d+120e}{11x^{11}}-$
gosper	$-\frac{831402e\,x^{11}+739024d\,x^{10}+7390240e\,x^{10}+6651216d\,x^9+29930472e\,x^9+27209520d\,x^8+72558720e\,x^8+66512160d\,x^7+116}{}$
parallelrisch	$\frac{-831402e\,x^{11}-739024d\,x^{10}-7390240e\,x^{10}-6651216d\,x^9-29930472e\,x^9-27209520d\,x^8-72558720e\,x^8-66512160d\,x^7-116}{}$
orering	$-\frac{(831402e\,x^{11}+739024d\,x^{10}+7390240e\,x^{10}+6651216d\,x^9+29930472e\,x^9+27209520d\,x^8+72558720e\,x^8+66512160d\,x^7+11}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^20,x,method=_RETURNVERBOSE)
```

output
```
(-1/19*d+(-5/9*d-1/18*e)*x+(-45/17*d-10/17*e)*x^2+(-15/2*d-45/16*e)*x^3+(-
14*d-8*e)*x^4+(-18*d-15*e)*x^5+(-210/13*d-252/13*e)*x^6+(-10*d-35/2*e)*x^7
+(-45/11*d-120/11*e)*x^8+(-d-9/2*e)*x^9+(-1/9*d-10/9*e)*x^10-1/8*e*x^11)/x
^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{831402\,ex^{11}+739024\,(d+10\,e)x^{10}+3325608\,(2\,d+9\,e)x^9+9069840\,(3\,d+8\,e)x^8+16628040\,(4\,d-$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^20,x, algorithm="fricas")
```

output
```
-1/6651216*(831402*e*x^11 + 739024*(d + 10*e)*x^10 + 3325608*(2*d + 9*e)*x
^9 + 9069840*(3*d + 8*e)*x^8 + 16628040*(4*d + 7*e)*x^7 + 21488544*(5*d +
6*e)*x^6 + 19953648*(6*d + 5*e)*x^5 + 13302432*(7*d + 4*e)*x^4 + 6235515*(
8*d + 3*e)*x^3 + 1956240*(9*d + 2*e)*x^2 + 369512*(10*d + e)*x + 350064*d)
/x^19
```

Sympy [A] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = \frac{-350064d-831402ex^{11}+x^{10}(-739024d-7390240e)+x^9(-6651216d-29930472e)+x^8(-27209520$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**20,x)
```

output
```
(-350064*d - 831402*e*x***11 + x**10*(-739024*d - 7390240*e) + x**9*(-66512
16*d - 29930472*e) + x**8*(-27209520*d - 72558720*e) + x**7*(-66512160*d -
 116396280*e) + x**6*(-107442720*d - 128931264*e) + x**5*(-119721888*d - 9
9768240*e) + x**4*(-93117024*d - 53209728*e) + x**3*(-49884120*d - 1870654
5*e) + x**2*(-17606160*d - 3912480*e) + x*(-3695120*d - 369512*e))/(665121
6*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{831402\,ex^{11}+739024\,(d+10\,e)x^{10}+3325608\,(2\,d+9\,e)x^9+9069840\,(3\,d+8\,e)x^8+16628040\,(4\,d-$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^20,x, algorithm="maxima")
```

output
```
-1/6651216*(831402*e*x^11 + 739024*(d + 10*e)*x^10 + 3325608*(2*d + 9*e)*x
^9 + 9069840*(3*d + 8*e)*x^8 + 16628040*(4*d + 7*e)*x^7 + 21488544*(5*d +
6*e)*x^6 + 19953648*(6*d + 5*e)*x^5 + 13302432*(7*d + 4*e)*x^4 + 6235515*(
8*d + 3*e)*x^3 + 1956240*(9*d + 2*e)*x^2 + 369512*(10*d + e)*x + 350064*d)
/x^19
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{831402\,ex^{11}+739024\,dx^{10}+7390240\,ex^{10}+6651216\,dx^9+29930472\,ex^9+27209520\,dx^8+7255872$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^20,x, algorithm="giac")
```

output
```
-1/6651216*(831402*e*x^11 + 739024*d*x^10 + 7390240*e*x^10 + 6651216*d*x^9
 + 29930472*e*x^9 + 27209520*d*x^8 + 72558720*e*x^8 + 66512160*d*x^7 + 116
396280*e*x^7 + 107442720*d*x^6 + 128931264*e*x^6 + 119721888*d*x^5 + 99768
240*e*x^5 + 93117024*d*x^4 + 53209728*e*x^4 + 49884120*d*x^3 + 18706545*e*
x^3 + 17606160*d*x^2 + 3912480*e*x^2 + 3695120*d*x + 369512*e*x + 350064*d
)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{20}}\,dx = -\frac{\frac{e\,x^{11}}{8}+\left(\frac{d}{9}+\frac{10\,e}{9}\right)\,x^{10}+\left(d+\frac{9\,e}{2}\right)\,x^9+\left(\frac{45\,d}{11}+\frac{120\,e}{11}\right)\,x^8+\left(10\,d+\frac{35\,e}{2}\right)\,x^7+\left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)\,x^6+(18\,d-}{x^{19}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^20,x)
```

output
```
-(d/19 + x^4*(14*d + 8*e) + x^5*(18*d + 15*e) + x^10*(d/9 + (10*e)/9) + x^
7*(10*d + (35*e)/2) + x^3*((15*d)/2 + (45*e)/16) + x^2*((45*d)/17 + (10*e)
/17) + x^8*((45*d)/11 + (120*e)/11) + x^6*((210*d)/13 + (252*e)/13) + (e*x
^11)/8 + x*((5*d)/9 + e/18) + x^9*(d + (9*e)/2))/x^19
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{20}}\,dx = \frac{-831402e\,x^{11}-739024d\,x^{10}-7390240e\,x^{10}-6651216d\,x^9-29930472e\,x^9-27209520d\,x^8-7255872}{}$$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^20,x)
```

output
```
( - 739024*d*x**10 - 6651216*d*x**9 - 27209520*d*x**8 - 66512160*d*x**7 -
107442720*d*x**6 - 119721888*d*x**5 - 93117024*d*x**4 - 49884120*d*x**3 -
17606160*d*x**2 - 3695120*d*x - 350064*d - 831402*e*x**11 - 7390240*e*x**1
0 - 29930472*e*x**9 - 72558720*e*x**8 - 116396280*e*x**7 - 128931264*e*x**
6 - 99768240*e*x**5 - 53209728*e*x**4 - 18706545*e*x**3 - 3912480*e*x**2 -
 369512*e*x)/(6651216*x**19)
```

3.216 $\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{21}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{21}}\,dx = -\frac{d}{20x^{20}} - \frac{10d+e}{19x^{19}} - \frac{5(9d+2e)}{18x^{18}} - \frac{15(8d+3e)}{17x^{17}} - \frac{15(7d+4e)}{8x^{16}} - \frac{14(6d+5e)}{5x^{15}} - \frac{3(5d+6e)}{x^{14}} - \frac{30(4d+7e)}{13x^{13}} - \frac{5(3d+8e)}{4x^{12}} - \frac{5(2d+9e)}{11x^{11}} - \frac{d+10e}{10x^{10}} - \frac{e}{9x^9}$$

output
```
-1/20*d/x^20-1/19*(10*d+e)/x^19-5/18*(9*d+2*e)/x^18-15/17*(8*d+3*e)/x^17-1
5/8*(7*d+4*e)/x^16-14/5*(6*d+5*e)/x^15-3*(5*d+6*e)/x^14-30/13*(4*d+7*e)/x^
13-5/4*(3*d+8*e)/x^12-5/11*(2*d+9*e)/x^11-1/10*(d+10*e)/x^10-1/9*e/x^9
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = -\frac{d}{20x^{20}} - \frac{10d+e}{19x^{19}} - \frac{5(9d+2e)}{18x^{18}} - \frac{15(8d+3e)}{17x^{17}} - \frac{15(7d+4e)}{8x^{16}} - \frac{14(6d+5e)}{5x^{15}} - \frac{3(5d+6e)}{x^{14}} - \frac{30(4d+7e)}{13x^{13}} - \frac{5(3d+8e)}{4x^{12}} - \frac{5(2d+9e)}{11x^{11}} - \frac{d+10e}{10x^{10}} - \frac{e}{9x^9}$$

input
```
Integrate[((d + e*x)*(1 + 2*x + x^2)^5)/x^21,x]
```

output
```
-1/20*d/x^20 - (10*d + e)/(19*x^19) - (5*(9*d + 2*e))/(18*x^18) - (15*(8*d
 + 3*e))/(17*x^17) - (15*(7*d + 4*e))/(8*x^16) - (14*(6*d + 5*e))/(5*x^15)
 - (3*(5*d + 6*e))/x^14 - (30*(4*d + 7*e))/(13*x^13) - (5*(3*d + 8*e))/(4*
x^12) - (5*(2*d + 9*e))/(11*x^11) - (d + 10*e)/(10*x^10) - e/(9*x^9)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2+2x+1\right)^5 (d+ex)}{x^{21}}\,dx$$

$$\Big\downarrow \ 1184$$

$$\int \frac{(x+1)^{10}(d+ex)}{x^{21}}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{10d+e}{x^{20}} + \frac{5(9d+2e)}{x^{19}} + \frac{15(8d+3e)}{x^{18}} + \frac{30(7d+4e)}{x^{17}} + \frac{42(6d+5e)}{x^{16}} + \frac{42(5d+6e)}{x^{15}} + \frac{30(4d+7e)}{x^{14}} + \frac{15(3d-}{x^{13}} \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10d+e}{19x^{19}} - \frac{5(9d+2e)}{18x^{18}} - \frac{15(8d+3e)}{17x^{17}} - \frac{15(7d+4e)}{8x^{16}} - \frac{14(6d+5e)}{5x^{15}} - \frac{3(5d+6e)}{x^{14}} - \frac{30(4d+7e)}{13x^{13}} - \frac{5(3d+8e)}{4x^{12}} - \frac{5(2d+9e)}{11x^{11}} - \frac{d+10e}{10x^{10}} - \frac{d}{20x^{20}} - \frac{e}{9x^9}$$

input
```
Int[((d + e*x)*(1 + 2*x + x^2)^5)/x^21,x]
```

output
```
-1/20*d/x^20 - (10*d + e)/(19*x^19) - (5*(9*d + 2*e))/(18*x^18) - (15*(8*d
 + 3*e))/(17*x^17) - (15*(7*d + 4*e))/(8*x^16) - (14*(6*d + 5*e))/(5*x^15)
 - (3*(5*d + 6*e))/x^14 - (30*(4*d + 7*e))/(13*x^13) - (5*(3*d + 8*e))/(4*
x^12) - (5*(2*d + 9*e))/(11*x^11) - (d + 10*e)/(10*x^10) - e/(9*x^9)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result
norman	$\frac{-\frac{d}{20}+\left(-\frac{10d}{19}-\frac{e}{19}\right)x+\left(-\frac{5d}{2}-\frac{5e}{9}\right)x^{2}+\left(-\frac{120d}{17}-\frac{45e}{17}\right)x^{3}+\left(-\frac{105d}{8}-\frac{15e}{2}\right)x^{4}+\left(-\frac{84d}{5}-14e\right)x^{5}+(-15d-18e)x^{6}+\left(-\frac{120d}{13}-\right.}{x^{20}}$
risch	$\frac{-\frac{d}{20}+\left(-\frac{10d}{19}-\frac{e}{19}\right)x+\left(-\frac{5d}{2}-\frac{5e}{9}\right)x^{2}+\left(-\frac{120d}{17}-\frac{45e}{17}\right)x^{3}+\left(-\frac{105d}{8}-\frac{15e}{2}\right)x^{4}+\left(-\frac{84d}{5}-14e\right)x^{5}+(-15d-18e)x^{6}+\left(-\frac{120d}{13}-\right.}{x^{20}}$
default	$-\frac{45d+10e}{18x^{18}}-\frac{d}{20x^{20}}-\frac{120d+210e}{13x^{13}}-\frac{210d+252e}{14x^{14}}-\frac{e}{9x^{9}}-\frac{d+10e}{10x^{10}}-\frac{120d+45e}{17x^{17}}-\frac{252d+210e}{15x^{15}}-\frac{10d+45e}{11x^{11}}-$
gosper	$-\frac{1847560e\,x^{11}+1662804d\,x^{10}+16628040e\,x^{10}+15116400d\,x^{9}+68023800e\,x^{9}+62355150d\,x^{8}+166280400e\,x^{8}+153489600d}{}$
parallelrisch	$\frac{-1847560e\,x^{11}-1662804d\,x^{10}-16628040e\,x^{10}-15116400d\,x^{9}-68023800e\,x^{9}-62355150d\,x^{8}-166280400e\,x^{8}-153489600d\,x}{}$
orering	$-\frac{(1847560e\,x^{11}+1662804d\,x^{10}+16628040e\,x^{10}+15116400d\,x^{9}+68023800e\,x^{9}+62355150d\,x^{8}+166280400e\,x^{8}+153489600}{}$

input
```
int((e*x+d)*(x^2+2*x+1)^5/x^21,x,method=_RETURNVERBOSE)
```

output
```
(-1/20*d+(-10/19*d-1/19*e)*x+(-5/2*d-5/9*e)*x^2+(-120/17*d-45/17*e)*x^3+(-
105/8*d-15/2*e)*x^4+(-84/5*d-14*e)*x^5+(-15*d-18*e)*x^6+(-120/13*d-210/13*
e)*x^7+(-15/4*d-10*e)*x^8+(-10/11*d-45/11*e)*x^9+(-1/10*d-e)*x^10-1/9*e*x^
11)/x^20
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)\left(1+2x+x^{2}\right)^{5}}{x^{21}}\,dx = -\frac{1847560\,ex^{11}+1662804\,(d+10\,e)x^{10}+7558200\,(2\,d+9\,e)x^{9}+20785050\,(3\,d+8\,e)x^{8}+38372400\,(4}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^21,x, algorithm="fricas")
```

output
```
-1/16628040*(1847560*e*x^11 + 1662804*(d + 10*e)*x^10 + 7558200*(2*d + 9*e
)*x^9 + 20785050*(3*d + 8*e)*x^8 + 38372400*(4*d + 7*e)*x^7 + 49884120*(5*
d + 6*e)*x^6 + 46558512*(6*d + 5*e)*x^5 + 31177575*(7*d + 4*e)*x^4 + 14671
800*(8*d + 3*e)*x^3 + 4618900*(9*d + 2*e)*x^2 + 875160*(10*d + e)*x + 8314
02*d)/x^20
```

Sympy [A] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{21}}\,dx$$
$$= \frac{-831402d - 1847560ex^{11} + x^{10}(-1662804d - 16628040e) + x^9(-15116400d - 68023800e) + x^8(-6235\cdots}{}$$

input
```
integrate((e*x+d)*(x**2+2*x+1)**5/x**21,x)
```

output
```
(-831402*d - 1847560*e*x**11 + x**10*(-1662804*d - 16628040*e) + x**9*(-15
116400*d - 68023800*e) + x**8*(-62355150*d - 166280400*e) + x**7*(-1534896
00*d - 268606800*e) + x**6*(-249420600*d - 299304720*e) + x**5*(-279351072
*d - 232792560*e) + x**4*(-218243025*d - 124710300*e) + x**3*(-117374400*d
 - 44015400*e) + x**2*(-41570100*d - 9237800*e) + x*(-8751600*d - 875160*e
))/(16628040*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)\,(1+2x+x^2)^5}{x^{21}}\,dx =$$
$$-\frac{1847560\,ex^{11} + 1662804\,(d+10\,e)x^{10} + 7558200\,(2\,d+9\,e)x^9 + 20785050\,(3\,d+8\,e)x^8 + 38372400\,(4\cdots}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^21,x, algorithm="maxima")
```

output
```
-1/16628040*(1847560*e*x^11 + 1662804*(d + 10*e)*x^10 + 7558200*(2*d + 9*e
)*x^9 + 20785050*(3*d + 8*e)*x^8 + 38372400*(4*d + 7*e)*x^7 + 49884120*(5*
d + 6*e)*x^6 + 46558512*(6*d + 5*e)*x^5 + 31177575*(7*d + 4*e)*x^4 + 14671
800*(8*d + 3*e)*x^3 + 4618900*(9*d + 2*e)*x^2 + 875160*(10*d + e)*x + 8314
02*d)/x^20
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{21}}\,dx = -\frac{1847560\,ex^{11} + 1662804\,dx^{10} + 16628040\,ex^{10} + 15116400\,dx^9 + 68023800\,ex^9 + 62355150\,dx^8 + 166}{}$$

input
```
integrate((e*x+d)*(x^2+2*x+1)^5/x^21,x, algorithm="giac")
```

output
```
-1/16628040*(1847560*e*x^11 + 1662804*d*x^10 + 16628040*e*x^10 + 15116400*
d*x^9 + 68023800*e*x^9 + 62355150*d*x^8 + 166280400*e*x^8 + 153489600*d*x^
7 + 268606800*e*x^7 + 249420600*d*x^6 + 299304720*e*x^6 + 279351072*d*x^5
 + 232792560*e*x^5 + 218243025*d*x^4 + 124710300*e*x^4 + 117374400*d*x^3 +
44015400*e*x^3 + 41570100*d*x^2 + 9237800*e*x^2 + 8751600*d*x + 875160*e*x
 + 831402*d)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)(1+2x+x^2)^5}{x^{21}}\,dx = -\frac{\frac{e\,x^{11}}{9} + \left(\frac{d}{10}+e\right)x^{10} + \left(\frac{10\,d}{11}+\frac{45\,e}{11}\right)x^9 + \left(\frac{15\,d}{4}+10\,e\right)x^8 + \left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)x^7 + (15\,d+18\,e)\,x^6 + \left(\frac{84\,d}{5}\right.}{x^{20}}$$

input
```
int(((d + e*x)*(2*x + x^2 + 1)^5)/x^21,x)
```

output

```
-(d/20 + x^2*((5*d)/2 + (5*e)/9) + x^8*((15*d)/4 + 10*e) + x^6*(15*d + 18*
e) + x^9*((10*d)/11 + (45*e)/11) + x^5*((84*d)/5 + 14*e) + x^4*((105*d)/8
+ (15*e)/2) + x^3*((120*d)/17 + (45*e)/17) + x^7*((120*d)/13 + (210*e)/13)
 + (e*x^11)/9 + x*((10*d)/19 + e/19) + x^10*(d/10 + e))/x^20
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = \frac{-1847560e\,x^{11}-1662804d\,x^{10}-16628040e\,x^{10}-15116400d\,x^9-68023800e\,x^9-62355150d\,x^8-166}{}$$

input

```
int((e*x+d)*(x^2+2*x+1)^5/x^21,x)
```

output

```
( - 1662804*d*x**10 - 15116400*d*x**9 - 62355150*d*x**8 - 153489600*d*x**7
 - 249420600*d*x**6 - 279351072*d*x**5 - 218243025*d*x**4 - 117374400*d*x*
*3 - 41570100*d*x**2 - 8751600*d*x - 831402*d - 1847560*e*x**11 - 16628040
*e*x**10 - 68023800*e*x**9 - 166280400*e*x**8 - 268606800*e*x**7 - 2993047
20*e*x**6 - 232792560*e*x**5 - 124710300*e*x**4 - 44015400*e*x**3 - 923780
0*e*x**2 - 875160*e*x)/(16628040*x**20)
```

3.217 $\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{12}}{12}+\frac{11x^{13}}{13}+\frac{55x^{14}}{14}+11x^{15}+\frac{165x^{16}}{8}+\frac{462x^{17}}{17}+\frac{77x^{18}}{3}+\frac{330x^{19}}{19}+\frac{33x^{20}}{4}+\frac{55x^{21}}{21}+\frac{x^{22}}{2}+\frac{x^{23}}{23}$$

output
```
1/12*x^12+11/13*x^13+55/14*x^14+11*x^15+165/8*x^16+462/17*x^17+77/3*x^18+3
30/19*x^19+33/4*x^20+55/21*x^21+1/2*x^22+1/23*x^23
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{12}}{12}+\frac{11x^{13}}{13}+\frac{55x^{14}}{14}+11x^{15}+\frac{165x^{16}}{8}+\frac{462x^{17}}{17}+\frac{77x^{18}}{3}+\frac{330x^{19}}{19}+\frac{33x^{20}}{4}+\frac{55x^{21}}{21}+\frac{x^{22}}{2}+\frac{x^{23}}{23}$$

input
```
Integrate[x^11*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^12/12 + (11*x^13)/13 + (55*x^14)/14 + 11*x^15 + (165*x^16)/8 + (462*x^17
)/17 + (77*x^18)/3 + (330*x^19)/19 + (33*x^20)/4 + (55*x^21)/21 + x^22/2 +
 x^23/23
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}(x+1)\left(x^2+2x+1\right)^5\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^{11}(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^{22}+11x^{21}+55x^{20}+165x^{19}+330x^{18}+462x^{17}+462x^{16}+330x^{15}+165x^{14}+55x^{13}+11x^{12}+x^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{23}}{23}+\frac{x^{22}}{2}+\frac{55x^{21}}{21}+\frac{33x^{20}}{4}+\frac{330x^{19}}{19}+\frac{77x^{18}}{3}+\frac{462x^{17}}{17}+\frac{165x^{16}}{8}+11x^{15}+\frac{55x^{14}}{14}+\frac{11x^{13}}{13}+\frac{x^{12}}{12}$$

input
```
Int[x^11*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^12/12 + (11*x^13)/13 + (55*x^14)/14 + 11*x^15 + (165*x^16)/8 + (462*x^17
)/17 + (77*x^18)/3 + (330*x^19)/19 + (33*x^20)/4 + (55*x^21)/21 + x^22/2 +
 x^23/23
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{x^{12}(705432x^{11}+8112468x^{10}+42493880x^{9}+133855722x^{8}+281801520x^{7}+416440024x^{6}+440936496x^{5}+334639305x^{4}+1784}{16224936}$
default	$\frac{1}{12}x^{12}+\frac{11}{13}x^{13}+\frac{55}{14}x^{14}+11x^{15}+\frac{165}{8}x^{16}+\frac{462}{17}x^{17}+\frac{77}{3}x^{18}+\frac{330}{19}x^{19}+\frac{33}{4}x^{20}+\frac{55}{21}x^{21}+\frac{1}{2}x^{22}$
norman	$\frac{1}{12}x^{12}+\frac{11}{13}x^{13}+\frac{55}{14}x^{14}+11x^{15}+\frac{165}{8}x^{16}+\frac{462}{17}x^{17}+\frac{77}{3}x^{18}+\frac{330}{19}x^{19}+\frac{33}{4}x^{20}+\frac{55}{21}x^{21}+\frac{1}{2}x^{22}$
risch	$\frac{1}{12}x^{12}+\frac{11}{13}x^{13}+\frac{55}{14}x^{14}+11x^{15}+\frac{165}{8}x^{16}+\frac{462}{17}x^{17}+\frac{77}{3}x^{18}+\frac{330}{19}x^{19}+\frac{33}{4}x^{20}+\frac{55}{21}x^{21}+\frac{1}{2}x^{22}$
parallelrisch	$\frac{1}{12}x^{12}+\frac{11}{13}x^{13}+\frac{55}{14}x^{14}+11x^{15}+\frac{165}{8}x^{16}+\frac{462}{17}x^{17}+\frac{77}{3}x^{18}+\frac{330}{19}x^{19}+\frac{33}{4}x^{20}+\frac{55}{21}x^{21}+\frac{1}{2}x^{22}$
orering	$\frac{x^{12}(705432x^{11}+8112468x^{10}+42493880x^{9}+133855722x^{8}+281801520x^{7}+416440024x^{6}+440936496x^{5}+334639305x^{4}+1784}{16224936(x+1)^{10}}$

input
```
int(x^11*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16224936*x^12*(705432*x^11+8112468*x^10+42493880*x^9+133855722*x^8+28180
1520*x^7+416440024*x^6+440936496*x^5+334639305*x^4+178474296*x^3+63740820*
x^2+13728792*x+1352078)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{23}x^{23}+\frac{1}{2}x^{22}+\frac{55}{21}x^{21}+\frac{33}{4}x^{20}+\frac{330}{19}x^{19}+\frac{77}{3}x^{18}+\frac{462}{17}x^{17}+\frac{165}{8}x^{16}+11x^{15}+\frac{55}{14}x^{14}+\frac{11}{13}x^{13}+\frac{1}{12}x^{12}$$

input
```
integrate(x^11*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/23*x^23 + 1/2*x^22 + 55/21*x^21 + 33/4*x^20 + 330/19*x^19 + 77/3*x^18 +
462/17*x^17 + 165/8*x^16 + 11*x^15 + 55/14*x^14 + 11/13*x^13 + 1/12*x^12
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{23}}{23}+\frac{x^{22}}{2}+\frac{55x^{21}}{21}+\frac{33x^{20}}{4}+\frac{330x^{19}}{19}+\frac{77x^{18}}{3}+\frac{462x^{17}}{17}+\frac{165x^{16}}{8}+11x^{15}+\frac{55x^{14}}{14}+\frac{11x^{13}}{13}+\frac{x^{12}}{12}$$

input
```
integrate(x**11*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**23/23 + x**22/2 + 55*x**21/21 + 33*x**20/4 + 330*x**19/19 + 77*x**18/3
+ 462*x**17/17 + 165*x**16/8 + 11*x**15 + 55*x**14/14 + 11*x**13/13 + x**1
2/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{23}\,x^{23}+\frac{1}{2}\,x^{22}+\frac{55}{21}\,x^{21}+\frac{33}{4}\,x^{20}+\frac{330}{19}\,x^{19}+\frac{77}{3}\,x^{18}+\frac{462}{17}\,x^{17}+\frac{165}{8}\,x^{16}+11\,x^{15}+\frac{55}{14}\,x^{14}+\frac{11}{13}\,x^{13}+\frac{1}{12}\,x^{12}$$

input
```
integrate(x^11*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/23*x^23 + 1/2*x^22 + 55/21*x^21 + 33/4*x^20 + 330/19*x^19 + 77/3*x^18 +
462/17*x^17 + 165/8*x^16 + 11*x^15 + 55/14*x^14 + 11/13*x^13 + 1/12*x^12
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{23}\,x^{23}+\frac{1}{2}\,x^{22}+\frac{55}{21}\,x^{21}+\frac{33}{4}\,x^{20}+\frac{330}{19}\,x^{19}+\frac{77}{3}\,x^{18}+\frac{462}{17}\,x^{17}+\frac{165}{8}\,x^{16}+11\,x^{15}+\frac{55}{14}\,x^{14}+\frac{11}{13}\,x^{13}+\frac{1}{12}\,x^{12}$$

input
```
integrate(x^11*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/23*x^23 + 1/2*x^22 + 55/21*x^21 + 33/4*x^20 + 330/19*x^19 + 77/3*x^18 +
462/17*x^17 + 165/8*x^16 + 11*x^15 + 55/14*x^14 + 11/13*x^13 + 1/12*x^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{23}}{23}+\frac{x^{22}}{2}+\frac{55\,x^{21}}{21}+\frac{33\,x^{20}}{4}+\frac{330\,x^{19}}{19}+\frac{77\,x^{18}}{3}+\frac{462\,x^{17}}{17}+\frac{165\,x^{16}}{8}+11\,x^{15}+\frac{55\,x^{14}}{14}+\frac{11\,x^{13}}{13}+\frac{x^{12}}{12}$$

input
```
int(x^11*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^12/12 + (11*x^13)/13 + (55*x^14)/14 + 11*x^15 + (165*x^16)/8 + (462*x^17
)/17 + (77*x^18)/3 + (330*x^19)/19 + (33*x^20)/4 + (55*x^21)/21 + x^22/2 +
 x^23/23
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int x^{11}(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^{12}(705432x^{11}+8112468x^{10}+42493880x^9+133855722x^8+281801520x^7+416440024x^6+440936496}{16224936}$$

input
```
int(x^11*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**12*(705432*x**11 + 8112468*x**10 + 42493880*x**9 + 133855722*x**8 + 28
1801520*x**7 + 416440024*x**6 + 440936496*x**5 + 334639305*x**4 + 17847429
6*x**3 + 63740820*x**2 + 13728792*x + 1352078))/16224936
```

3.218 $\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{11}}{11}+\frac{11x^{12}}{12}+\frac{55x^{13}}{13}+\frac{165x^{14}}{14}+22x^{15}+\frac{231x^{16}}{8}+\frac{462x^{17}}{17}+\frac{55x^{18}}{3}+\frac{165x^{19}}{19}+\frac{11x^{20}}{4}+\frac{11x^{21}}{21}+\frac{x^{22}}{22}$$

output
```
1/11*x^11+11/12*x^12+55/13*x^13+165/14*x^14+22*x^15+231/8*x^16+462/17*x^17
+55/3*x^18+165/19*x^19+11/4*x^20+11/21*x^21+1/22*x^22
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{11}}{11}+\frac{11x^{12}}{12}+\frac{55x^{13}}{13}+\frac{165x^{14}}{14}+22x^{15}+\frac{231x^{16}}{8}+\frac{462x^{17}}{17}+\frac{55x^{18}}{3}+\frac{165x^{19}}{19}+\frac{11x^{20}}{4}+\frac{11x^{21}}{21}+\frac{x^{22}}{22}$$

input
```
Integrate[x^10*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^11/11 + (11*x^12)/12 + (55*x^13)/13 + (165*x^14)/14 + 22*x^15 + (231*x^1
6)/8 + (462*x^17)/17 + (55*x^18)/3 + (165*x^19)/19 + (11*x^20)/4 + (11*x^2
1)/21 + x^22/22
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}(x+1)\left(x^2+2x+1\right)^5\,dx$$
$$\Big\downarrow \text{1184}$$
$$\int x^{10}(x+1)^{11}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(x^{21}+11x^{20}+55x^{19}+165x^{18}+330x^{17}+462x^{16}+462x^{15}+330x^{14}+165x^{13}+55x^{12}+11x^{11}+x^{10}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^{22}}{22}+\frac{11x^{21}}{21}+\frac{11x^{20}}{4}+\frac{165x^{19}}{19}+\frac{55x^{18}}{3}+\frac{462x^{17}}{17}+\frac{231x^{16}}{8}+22x^{15}+\frac{165x^{14}}{14}+\frac{55x^{13}}{13}+\frac{11x^{12}}{12}+\frac{x^{11}}{11}$$

input
```
Int[x^10*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output

```
x^11/11 + (11*x^12)/12 + (55*x^13)/13 + (165*x^14)/14 + 22*x^15 + (231*x^1
6)/8 + (462*x^17)/17 + (55*x^18)/3 + (165*x^19)/19 + (11*x^20)/4 + (11*x^2
1)/21 + x^22/22
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{x^{11}(352716x^{11}+4064632x^{10}+21339318x^{9}+67387320x^{8}+142262120x^{7}+210882672x^{6}+224062839x^{5}+170714544x^{4}+91454}{7759752}$
default	$\frac{1}{11}x^{11}+\frac{11}{12}x^{12}+\frac{55}{13}x^{13}+\frac{165}{14}x^{14}+22x^{15}+\frac{231}{8}x^{16}+\frac{462}{17}x^{17}+\frac{55}{3}x^{18}+\frac{165}{19}x^{19}+\frac{11}{4}x^{20}+\frac{11}{21}x$
norman	$\frac{1}{11}x^{11}+\frac{11}{12}x^{12}+\frac{55}{13}x^{13}+\frac{165}{14}x^{14}+22x^{15}+\frac{231}{8}x^{16}+\frac{462}{17}x^{17}+\frac{55}{3}x^{18}+\frac{165}{19}x^{19}+\frac{11}{4}x^{20}+\frac{11}{21}x$
risch	$\frac{1}{11}x^{11}+\frac{11}{12}x^{12}+\frac{55}{13}x^{13}+\frac{165}{14}x^{14}+22x^{15}+\frac{231}{8}x^{16}+\frac{462}{17}x^{17}+\frac{55}{3}x^{18}+\frac{165}{19}x^{19}+\frac{11}{4}x^{20}+\frac{11}{21}x$
parallelrisch	$\frac{1}{11}x^{11}+\frac{11}{12}x^{12}+\frac{55}{13}x^{13}+\frac{165}{14}x^{14}+22x^{15}+\frac{231}{8}x^{16}+\frac{462}{17}x^{17}+\frac{55}{3}x^{18}+\frac{165}{19}x^{19}+\frac{11}{4}x^{20}+\frac{11}{21}x$
orering	$\frac{x^{11}(352716x^{11}+4064632x^{10}+21339318x^{9}+67387320x^{8}+142262120x^{7}+210882672x^{6}+224062839x^{5}+170714544x^{4}+91454}{7759752(x+1)^{10}}$

input
```
int(x^10*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7759752*x^11*(352716*x^11+4064632*x^10+21339318*x^9+67387320*x^8+1422621
20*x^7+210882672*x^6+224062839*x^5+170714544*x^4+91454220*x^3+32829720*x^2
+7113106*x+705432)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{22}\,x^{22}+\frac{11}{21}\,x^{21}+\frac{11}{4}\,x^{20}+\frac{165}{19}\,x^{19}+\frac{55}{3}\,x^{18}+\frac{462}{17}\,x^{17}+\frac{231}{8}\,x^{16}+22\,x^{15}+\frac{165}{14}\,x^{14}+\frac{55}{13}\,x^{13}+\frac{11}{12}\,x^{12}+\frac{1}{11}\,x^{11}$$

input
```
integrate(x^10*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/22*x^22 + 11/21*x^21 + 11/4*x^20 + 165/19*x^19 + 55/3*x^18 + 462/17*x^17
 + 231/8*x^16 + 22*x^15 + 165/14*x^14 + 55/13*x^13 + 11/12*x^12 + 1/11*x^1
1
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{22}}{22}+\frac{11x^{21}}{21}+\frac{11x^{20}}{4}+\frac{165x^{19}}{19}+\frac{55x^{18}}{3}+\frac{462x^{17}}{17}+\frac{231x^{16}}{8}+22x^{15}+\frac{165x^{14}}{14}+\frac{55x^{13}}{13}+\frac{11x^{12}}{12}+\frac{x^{11}}{11}$$

input
```
integrate(x**10*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**22/22 + 11*x**21/21 + 11*x**20/4 + 165*x**19/19 + 55*x**18/3 + 462*x**1
7/17 + 231*x**16/8 + 22*x**15 + 165*x**14/14 + 55*x**13/13 + 11*x**12/12 +
 x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{22}\,x^{22}+\frac{11}{21}\,x^{21}+\frac{11}{4}\,x^{20}+\frac{165}{19}\,x^{19}+\frac{55}{3}\,x^{18}+\frac{462}{17}\,x^{17}+\frac{231}{8}\,x^{16}+22\,x^{15}+\frac{165}{14}\,x^{14}+\frac{55}{13}\,x^{13}+\frac{11}{12}\,x^{12}+\frac{1}{11}\,x^{11}$$

input
```
integrate(x^10*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/22*x^22 + 11/21*x^21 + 11/4*x^20 + 165/19*x^19 + 55/3*x^18 + 462/17*x^17
 + 231/8*x^16 + 22*x^15 + 165/14*x^14 + 55/13*x^13 + 11/12*x^12 + 1/11*x^1
1
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{22}\,x^{22}+\frac{11}{21}\,x^{21}+\frac{11}{4}\,x^{20}+\frac{165}{19}\,x^{19}+\frac{55}{3}\,x^{18}+\frac{462}{17}\,x^{17}+\frac{231}{8}\,x^{16}+22\,x^{15}+\frac{165}{14}\,x^{14}+\frac{55}{13}\,x^{13}+\frac{11}{12}\,x^{12}+\frac{1}{11}\,x^{11}$$

input
```
integrate(x^10*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/22*x^22 + 11/21*x^21 + 11/4*x^20 + 165/19*x^19 + 55/3*x^18 + 462/17*x^17
 + 231/8*x^16 + 22*x^15 + 165/14*x^14 + 55/13*x^13 + 11/12*x^12 + 1/11*x^1
1
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{22}}{22}+\frac{11\,x^{21}}{21}+\frac{11\,x^{20}}{4}+\frac{165\,x^{19}}{19}+\frac{55\,x^{18}}{3}+\frac{462\,x^{17}}{17}+\frac{231\,x^{16}}{8}+22\,x^{15}+\frac{165\,x^{14}}{14}+\frac{55\,x^{13}}{13}+\frac{11\,x^{12}}{12}+\frac{x^{11}}{11}$$

input
```
int(x^10*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^11/11 + (11*x^12)/12 + (55*x^13)/13 + (165*x^14)/14 + 22*x^15 + (231*x^1
6)/8 + (462*x^17)/17 + (55*x^18)/3 + (165*x^19)/19 + (11*x^20)/4 + (11*x^2
1)/21 + x^22/22
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int x^{10}(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{11}(352716x^{11}+4064632x^{10}+21339318x^9+67387320x^8+142262120x^7+210882672x^6+224062839x}{7759752}$$

input
```
int(x^10*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**11*(352716*x**11 + 4064632*x**10 + 21339318*x**9 + 67387320*x**8 + 142
262120*x**7 + 210882672*x**6 + 224062839*x**5 + 170714544*x**4 + 91454220*
x**3 + 32829720*x**2 + 7113106*x + 705432))/7759752
```

3.219 $\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{12}(1+x)^{12}+\frac{9}{13}(1+x)^{13}-\frac{18}{7}(1+x)^{14}+\frac{28}{5}(1+x)^{15}-\frac{63}{8}(1+x)^{16}+\frac{126}{17}(1+x)^{17}-\frac{14}{3}(1+x)^{18}+\frac{36}{19}(1+x)^{19}-\frac{9}{20}(1+x)^{20}+\frac{1}{21}(1+x)^{21}$$

output
```
-1/12*(1+x)^12+9/13*(1+x)^13-18/7*(1+x)^14+28/5*(1+x)^15-63/8*(1+x)^16+126
/17*(1+x)^17-14/3*(1+x)^18+36/19*(1+x)^19-9/20*(1+x)^20+1/21*(1+x)^21
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{10}}{10}+x^{11}+\frac{55x^{12}}{12}+\frac{165x^{13}}{13}+\frac{165x^{14}}{7}+\frac{154x^{15}}{5}+\frac{231x^{16}}{8}+\frac{330x^{17}}{17}+\frac{55x^{18}}{6}+\frac{55x^{19}}{19}+\frac{11x^{20}}{20}+\frac{x^{21}}{21}$$

input
```
Integrate[x^9*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^10/10 + x^11 + (55*x^12)/12 + (165*x^13)/13 + (165*x^14)/7 + (154*x^15)/
5 + (231*x^16)/8 + (330*x^17)/17 + (55*x^18)/6 + (55*x^19)/19 + (11*x^20)/
20 + x^21/21
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9(x+1)\left(x^2+2x+1\right)^5\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int x^9(x+1)^{11}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left((x+1)^{20}-9(x+1)^{19}+36(x+1)^{18}-84(x+1)^{17}+126(x+1)^{16}-126(x+1)^{15}+84(x+1)^{14}-36(x+1\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{21}(x+1)^{21}-\frac{9}{20}(x+1)^{20}+\frac{36}{19}(x+1)^{19}-\frac{14}{3}(x+1)^{18}+\frac{126}{17}(x+1)^{17}-\frac{63}{8}(x+1)^{16}+\frac{28}{5}(x+1)^{15}-\frac{18}{7}(x+1)^{14}+\frac{9}{13}(x+1)^{13}-\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^9*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/12*(1 + x)^12 + (9*(1 + x)^13)/13 - (18*(1 + x)^14)/7 + (28*(1 + x)^15)
/5 - (63*(1 + x)^16)/8 + (126*(1 + x)^17)/17 - (14*(1 + x)^18)/3 + (36*(1
 + x)^19)/19 - (9*(1 + x)^20)/20 + (1 + x)^21/21
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

method	result
default	$\frac{1}{10}x^{10}+x^{11}+\frac{55}{12}x^{12}+\frac{165}{13}x^{13}+\frac{165}{7}x^{14}+\frac{154}{5}x^{15}+\frac{231}{8}x^{16}+\frac{330}{17}x^{17}+\frac{55}{6}x^{18}+\frac{55}{19}x^{19}+\frac{11}{20}x^{20}$
norman	$\frac{1}{10}x^{10}+x^{11}+\frac{55}{12}x^{12}+\frac{165}{13}x^{13}+\frac{165}{7}x^{14}+\frac{154}{5}x^{15}+\frac{231}{8}x^{16}+\frac{330}{17}x^{17}+\frac{55}{6}x^{18}+\frac{55}{19}x^{19}+\frac{11}{20}x^{20}$
risch	$\frac{1}{10}x^{10}+x^{11}+\frac{55}{12}x^{12}+\frac{165}{13}x^{13}+\frac{165}{7}x^{14}+\frac{154}{5}x^{15}+\frac{231}{8}x^{16}+\frac{330}{17}x^{17}+\frac{55}{6}x^{18}+\frac{55}{19}x^{19}+\frac{11}{20}x^{20}$
parallelrisch	$\frac{1}{10}x^{10}+x^{11}+\frac{55}{12}x^{12}+\frac{165}{13}x^{13}+\frac{165}{7}x^{14}+\frac{154}{5}x^{15}+\frac{231}{8}x^{16}+\frac{330}{17}x^{17}+\frac{55}{6}x^{18}+\frac{55}{19}x^{19}+\frac{11}{20}x^{20}$
gosper	$\frac{x^{10}(167960x^{11}+1939938x^{10}+10210200x^{9}+32332300x^{8}+68468400x^{7}+101846745x^{6}+108636528x^{5}+83140200x^{4}+4476780}{3527160}$
orering	$\frac{x^{10}(167960x^{11}+1939938x^{10}+10210200x^{9}+32332300x^{8}+68468400x^{7}+101846745x^{6}+108636528x^{5}+83140200x^{4}+4476780}{3527160(x+1)^{10}}$

input
```
int(x^9*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^10+x^11+55/12*x^12+165/13*x^13+165/7*x^14+154/5*x^15+231/8*x^16+330
/17*x^17+55/6*x^18+55/19*x^19+11/20*x^20+1/21*x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{21}\,x^{21}+\frac{11}{20}\,x^{20}+\frac{55}{19}\,x^{19}+\frac{55}{6}\,x^{18}+\frac{330}{17}\,x^{17}+\frac{231}{8}\,x^{16}+\frac{154}{5}\,x^{15}+\frac{165}{7}\,x^{14}+\frac{165}{13}\,x^{13}+\frac{55}{12}\,x^{12}+x^{11}+\frac{1}{10}\,x^{10}$$

input
```
integrate(x^9*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/21*x^21 + 11/20*x^20 + 55/19*x^19 + 55/6*x^18 + 330/17*x^17 + 231/8*x^16
 + 154/5*x^15 + 165/7*x^14 + 165/13*x^13 + 55/12*x^12 + x^11 + 1/10*x^10
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{21}}{21}+\frac{11x^{20}}{20}+\frac{55x^{19}}{19}+\frac{55x^{18}}{6}+\frac{330x^{17}}{17}+\frac{231x^{16}}{8}+\frac{154x^{15}}{5}+\frac{165x^{14}}{7}+\frac{165x^{13}}{13}+\frac{55x^{12}}{12}+x^{11}+\frac{x^{10}}{10}$$

input
```
integrate(x**9*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**21/21 + 11*x**20/20 + 55*x**19/19 + 55*x**18/6 + 330*x**17/17 + 231*x**
16/8 + 154*x**15/5 + 165*x**14/7 + 165*x**13/13 + 55*x**12/12 + x**11 + x*
*10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{21}x^{21}+\frac{11}{20}x^{20}+\frac{55}{19}x^{19}+\frac{55}{6}x^{18}+\frac{330}{17}x^{17}+\frac{231}{8}x^{16}+\frac{154}{5}x^{15}+\frac{165}{7}x^{14}+\frac{165}{13}x^{13}+\frac{55}{12}x^{12}+x^{11}+\frac{1}{10}x^{10}$$

input
```
integrate(x^9*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/21*x^21 + 11/20*x^20 + 55/19*x^19 + 55/6*x^18 + 330/17*x^17 + 231/8*x^16
 + 154/5*x^15 + 165/7*x^14 + 165/13*x^13 + 55/12*x^12 + x^11 + 1/10*x^10
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{21}x^{21}+\frac{11}{20}x^{20}+\frac{55}{19}x^{19}+\frac{55}{6}x^{18}+\frac{330}{17}x^{17}+\frac{231}{8}x^{16}+\frac{154}{5}x^{15}+\frac{165}{7}x^{14}+\frac{165}{13}x^{13}+\frac{55}{12}x^{12}+x^{11}+\frac{1}{10}x^{10}$$

input
```
integrate(x^9*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/21*x^21 + 11/20*x^20 + 55/19*x^19 + 55/6*x^18 + 330/17*x^17 + 231/8*x^16
 + 154/5*x^15 + 165/7*x^14 + 165/13*x^13 + 55/12*x^12 + x^11 + 1/10*x^10
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{21}}{21}+\frac{11\,x^{20}}{20}+\frac{55\,x^{19}}{19}+\frac{55\,x^{18}}{6}+\frac{330\,x^{17}}{17}+\frac{231\,x^{16}}{8}+\frac{154\,x^{15}}{5}+\frac{165\,x^{14}}{7}+\frac{165\,x^{13}}{13}+\frac{55\,x^{12}}{12}+x^{11}+\frac{x^{10}}{10}$$

input
```
int(x^9*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^10/10 + x^11 + (55*x^12)/12 + (165*x^13)/13 + (165*x^14)/7 + (154*x^15)/
5 + (231*x^16)/8 + (330*x^17)/17 + (55*x^18)/6 + (55*x^19)/19 + (11*x^20)/
20 + x^21/21
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int x^9(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^{10}(167960x^{11}+1939938x^{10}+10210200x^9+32332300x^8+68468400x^7+101846745x^6+108636528x^5}{3527160}$$

input
```
int(x^9*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**10*(167960*x**11 + 1939938*x**10 + 10210200*x**9 + 32332300*x**8 + 684
68400*x**7 + 101846745*x**6 + 108636528*x**5 + 83140200*x**4 + 44767800*x*
*3 + 16166150*x**2 + 3527160*x + 352716))/3527160
```

3.220 $\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{12}(1+x)^{12}-\frac{8}{13}(1+x)^{13}+2(1+x)^{14}-\frac{56}{15}(1+x)^{15}+\frac{35}{8}(1+x)^{16}-\frac{56}{17}(1+x)^{17}+\frac{14}{9}(1+x)^{18}-\frac{8}{19}(1+x)^{19}+\frac{1}{20}(1+x)^{20}$$

output
```
1/12*(1+x)^12-8/13*(1+x)^13+2*(1+x)^14-56/15*(1+x)^15+35/8*(1+x)^16-56/17*
(1+x)^17+14/9*(1+x)^18-8/19*(1+x)^19+1/20*(1+x)^20
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^9}{9}+\frac{11x^{10}}{10}+5x^{11}+\frac{55x^{12}}{4}+\frac{330x^{13}}{13}+33x^{14}+\frac{154x^{15}}{5}+\frac{165x^{16}}{8}+\frac{165x^{17}}{17}+\frac{55x^{18}}{18}+\frac{11x^{19}}{19}+\frac{x^{20}}{20}$$

input
```
Integrate[x^8*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^9/9 + (11*x^10)/10 + 5*x^11 + (55*x^12)/4 + (330*x^13)/13 + 33*x^14 + (1
54*x^15)/5 + (165*x^16)/8 + (165*x^17)/17 + (55*x^18)/18 + (11*x^19)/19 +
x^20/20
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(x+1)\left(x^2+2x+1\right)^5dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^8(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{19}-8(x+1)^{18}+28(x+1)^{17}-56(x+1)^{16}+70(x+1)^{15}-56(x+1)^{14}+28(x+1)^{13}-8(x+1)^{12}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{20}(x+1)^{20}-\frac{8}{19}(x+1)^{19}+\frac{14}{9}(x+1)^{18}-\frac{56}{17}(x+1)^{17}+\frac{35}{8}(x+1)^{16}-\frac{56}{15}(x+1)^{15}+2(x+1)^{14}-\frac{8}{13}(x+1)^{13}+\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^8*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12 - (8*(1 + x)^13)/13 + 2*(1 + x)^14 - (56*(1 + x)^15)/15 + (3
5*(1 + x)^16)/8 - (56*(1 + x)^17)/17 + (14*(1 + x)^18)/9 - (8*(1 + x)^19)/
19 + (1 + x)^20/20
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{x^{9}\left(75582x^{11}+875160x^{10}+4618900x^{9}+14671800x^{8}+31177575x^{7}+46558512x^{6}+49884120x^{5}+38372400x^{4}+20785050x^{3}+7\right.}{1511640}$
default	$\frac{1}{9}x^{9}+\frac{11}{10}x^{10}+5x^{11}+\frac{55}{4}x^{12}+\frac{330}{13}x^{13}+33x^{14}+\frac{154}{5}x^{15}+\frac{165}{8}x^{16}+\frac{165}{17}x^{17}+\frac{55}{18}x^{18}+\frac{11}{19}x^{19}$
norman	$\frac{1}{9}x^{9}+\frac{11}{10}x^{10}+5x^{11}+\frac{55}{4}x^{12}+\frac{330}{13}x^{13}+33x^{14}+\frac{154}{5}x^{15}+\frac{165}{8}x^{16}+\frac{165}{17}x^{17}+\frac{55}{18}x^{18}+\frac{11}{19}x^{19}$
risch	$\frac{1}{9}x^{9}+\frac{11}{10}x^{10}+5x^{11}+\frac{55}{4}x^{12}+\frac{330}{13}x^{13}+33x^{14}+\frac{154}{5}x^{15}+\frac{165}{8}x^{16}+\frac{165}{17}x^{17}+\frac{55}{18}x^{18}+\frac{11}{19}x^{19}$
parallelrisch	$\frac{1}{9}x^{9}+\frac{11}{10}x^{10}+5x^{11}+\frac{55}{4}x^{12}+\frac{330}{13}x^{13}+33x^{14}+\frac{154}{5}x^{15}+\frac{165}{8}x^{16}+\frac{165}{17}x^{17}+\frac{55}{18}x^{18}+\frac{11}{19}x^{19}$
orering	$\frac{x^{9}\left(75582x^{11}+875160x^{10}+4618900x^{9}+14671800x^{8}+31177575x^{7}+46558512x^{6}+49884120x^{5}+38372400x^{4}+20785050x^{3}+7\right.}{1511640(x+1)^{10}}$

input
```
int(x^8*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/1511640*x^9*(75582*x^11+875160*x^10+4618900*x^9+14671800*x^8+31177575*x^
7+46558512*x^6+49884120*x^5+38372400*x^4+20785050*x^3+7558200*x^2+1662804*
x+167960)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{20}\,x^{20}+\frac{11}{19}\,x^{19}+\frac{55}{18}\,x^{18}+\frac{165}{17}\,x^{17}+\frac{165}{8}\,x^{16}+\frac{154}{5}\,x^{15}+33\,x^{14}+\frac{330}{13}\,x^{13}+\frac{55}{4}\,x^{12}+5\,x^{11}+\frac{11}{10}\,x^{10}+\frac{1}{9}\,x^{9}$$

input
```
integrate(x^8*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/20*x^20 + 11/19*x^19 + 55/18*x^18 + 165/17*x^17 + 165/8*x^16 + 154/5*x^1
5 + 33*x^14 + 330/13*x^13 + 55/4*x^12 + 5*x^11 + 11/10*x^10 + 1/9*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{20}}{20}+\frac{11x^{19}}{19}+\frac{55x^{18}}{18}+\frac{165x^{17}}{17}+\frac{165x^{16}}{8}+\frac{154x^{15}}{5}+33x^{14}+\frac{330x^{13}}{13}+\frac{55x^{12}}{4}+5x^{11}+\frac{11x^{10}}{10}+\frac{x^{9}}{9}$$

input
```
integrate(x**8*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**20/20 + 11*x**19/19 + 55*x**18/18 + 165*x**17/17 + 165*x**16/8 + 154*x*
*15/5 + 33*x**14 + 330*x**13/13 + 55*x**12/4 + 5*x**11 + 11*x**10/10 + x**
9/9
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{20}x^{20}+\frac{11}{19}x^{19}+\frac{55}{18}x^{18}+\frac{165}{17}x^{17}+\frac{165}{8}x^{16}+\frac{154}{5}x^{15}+33x^{14}+\frac{330}{13}x^{13}+\frac{55}{4}x^{12}+5x^{11}+\frac{11}{10}x^{10}+\frac{1}{9}x^9$$

input
```
integrate(x^8*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/20*x^20 + 11/19*x^19 + 55/18*x^18 + 165/17*x^17 + 165/8*x^16 + 154/5*x^1
5 + 33*x^14 + 330/13*x^13 + 55/4*x^12 + 5*x^11 + 11/10*x^10 + 1/9*x^9
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{20}x^{20}+\frac{11}{19}x^{19}+\frac{55}{18}x^{18}+\frac{165}{17}x^{17}+\frac{165}{8}x^{16}+\frac{154}{5}x^{15}+33x^{14}+\frac{330}{13}x^{13}+\frac{55}{4}x^{12}+5x^{11}+\frac{11}{10}x^{10}+\frac{1}{9}x^9$$

input
```
integrate(x^8*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/20*x^20 + 11/19*x^19 + 55/18*x^18 + 165/17*x^17 + 165/8*x^16 + 154/5*x^1
5 + 33*x^14 + 330/13*x^13 + 55/4*x^12 + 5*x^11 + 11/10*x^10 + 1/9*x^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{20}}{20}+\frac{11\,x^{19}}{19}+\frac{55\,x^{18}}{18}+\frac{165\,x^{17}}{17}+\frac{165\,x^{16}}{8}+\frac{154\,x^{15}}{5}+33\,x^{14}+\frac{330\,x^{13}}{13}+\frac{55\,x^{12}}{4}+5\,x^{11}+\frac{11\,x^{10}}{10}+\frac{x^9}{9}$$

input
```
int(x^8*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^9/9 + (11*x^10)/10 + 5*x^11 + (55*x^12)/4 + (330*x^13)/13 + 33*x^14 + (1
54*x^15)/5 + (165*x^16)/8 + (165*x^17)/17 + (55*x^18)/18 + (11*x^19)/19 +
x^20/20
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^8(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^9(75582x^{11}+875160x^{10}+4618900x^9+14671800x^8+31177575x^7+46558512x^6+49884120x^5+383}{1511640}$$

input
```
int(x^8*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**9*(75582*x**11 + 875160*x**10 + 4618900*x**9 + 14671800*x**8 + 3117757
5*x**7 + 46558512*x**6 + 49884120*x**5 + 38372400*x**4 + 20785050*x**3 + 7
558200*x**2 + 1662804*x + 167960))/1511640
```

3.221 $\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{12}(1+x)^{12}+\frac{7}{13}(1+x)^{13}-\frac{3}{2}(1+x)^{14}+\frac{7}{3}(1+x)^{15}-\frac{35}{16}(1+x)^{16}+\frac{21}{17}(1+x)^{17}-\frac{7}{18}(1+x)^{18}+\frac{1}{19}(1+x)^{19}$$

output
```
-1/12*(1+x)^12+7/13*(1+x)^13-3/2*(1+x)^14+7/3*(1+x)^15-35/16*(1+x)^16+21/1
7*(1+x)^17-7/18*(1+x)^18+1/19*(1+x)^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^8}{8}+\frac{11x^9}{9}+\frac{11x^{10}}{2}+15x^{11}+\frac{55x^{12}}{2}+\frac{462x^{13}}{13}+33x^{14}+22x^{15}+\frac{165x^{16}}{16}+\frac{55x^{17}}{17}+\frac{11x^{18}}{18}+\frac{x^{19}}{19}$$

input
```
Integrate[x^7*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^8/8 + (11*x^9)/9 + (11*x^10)/2 + 15*x^11 + (55*x^12)/2 + (462*x^13)/13 +
 33*x^14 + 22*x^15 + (165*x^16)/16 + (55*x^17)/17 + (11*x^18)/18 + x^19/19
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7(x+1)\left(x^2+2x+1\right)^5 dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^7(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{18}-7(x+1)^{17}+21(x+1)^{16}-35(x+1)^{15}+35(x+1)^{14}-21(x+1)^{13}+7(x+1)^{12}-(x+1)^{11}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{19}(x+1)^{19}-\frac{7}{18}(x+1)^{18}+\frac{21}{17}(x+1)^{17}-\frac{35}{16}(x+1)^{16}+\frac{7}{3}(x+1)^{15}-\frac{3}{2}(x+1)^{14}+\frac{7}{13}(x+1)^{13}-\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^7*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/12*(1 + x)^12 + (7*(1 + x)^13)/13 - (3*(1 + x)^14)/2 + (7*(1 + x)^15)/3
 - (35*(1 + x)^16)/16 + (21*(1 + x)^17)/17 - (7*(1 + x)^18)/18 + (1 + x)^1
9/19
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{x^8(31824x^{11}+369512x^{10}+1956240x^9+6235515x^8+13302432x^7+19953648x^6+21488544x^5+16628040x^4+9069840x^3+332}{604656}$
default	$\frac{1}{8}x^8+\frac{11}{9}x^9+\frac{11}{2}x^{10}+15x^{11}+\frac{55}{2}x^{12}+\frac{462}{13}x^{13}+33x^{14}+22x^{15}+\frac{165}{16}x^{16}+\frac{55}{17}x^{17}+\frac{11}{18}x^{18}+$
norman	$\frac{1}{8}x^8+\frac{11}{9}x^9+\frac{11}{2}x^{10}+15x^{11}+\frac{55}{2}x^{12}+\frac{462}{13}x^{13}+33x^{14}+22x^{15}+\frac{165}{16}x^{16}+\frac{55}{17}x^{17}+\frac{11}{18}x^{18}+$
risch	$\frac{1}{8}x^8+\frac{11}{9}x^9+\frac{11}{2}x^{10}+15x^{11}+\frac{55}{2}x^{12}+\frac{462}{13}x^{13}+33x^{14}+22x^{15}+\frac{165}{16}x^{16}+\frac{55}{17}x^{17}+\frac{11}{18}x^{18}+$
parallelrisch	$\frac{1}{8}x^8+\frac{11}{9}x^9+\frac{11}{2}x^{10}+15x^{11}+\frac{55}{2}x^{12}+\frac{462}{13}x^{13}+33x^{14}+22x^{15}+\frac{165}{16}x^{16}+\frac{55}{17}x^{17}+\frac{11}{18}x^{18}+$
orering	$\frac{x^8(31824x^{11}+369512x^{10}+1956240x^9+6235515x^8+13302432x^7+19953648x^6+21488544x^5+16628040x^4+9069840x^3+332}{604656(x+1)^{10}}$

input
```
int(x^7*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/604656*x^8*(31824*x^11+369512*x^10+1956240*x^9+6235515*x^8+13302432*x^7+
19953648*x^6+21488544*x^5+16628040*x^4+9069840*x^3+3325608*x^2+739024*x+75
582)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{19}\,x^{19}+\frac{11}{18}\,x^{18}+\frac{55}{17}\,x^{17}+\frac{165}{16}\,x^{16}+22\,x^{15}+33\,x^{14}+\frac{462}{13}\,x^{13}+\frac{55}{2}\,x^{12}+15\,x^{11}+\frac{11}{2}\,x^{10}+\frac{11}{9}\,x^{9}+\frac{1}{8}\,x^{8}$$

input
```
integrate(x^7*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/19*x^19 + 11/18*x^18 + 55/17*x^17 + 165/16*x^16 + 22*x^15 + 33*x^14 + 46
2/13*x^13 + 55/2*x^12 + 15*x^11 + 11/2*x^10 + 11/9*x^9 + 1/8*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{19}}{19}+\frac{11x^{18}}{18}+\frac{55x^{17}}{17}+\frac{165x^{16}}{16}+22x^{15}+33x^{14}+\frac{462x^{13}}{13}+\frac{55x^{12}}{2}+15x^{11}+\frac{11x^{10}}{2}+\frac{11x^{9}}{9}+\frac{x^{8}}{8}$$

input
```
integrate(x**7*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**19/19 + 11*x**18/18 + 55*x**17/17 + 165*x**16/16 + 22*x**15 + 33*x**14
+ 462*x**13/13 + 55*x**12/2 + 15*x**11 + 11*x**10/2 + 11*x**9/9 + x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{19}\,x^{19}+\frac{11}{18}\,x^{18}+\frac{55}{17}\,x^{17}+\frac{165}{16}\,x^{16}+22\,x^{15}+33\,x^{14}+\frac{462}{13}\,x^{13}+\frac{55}{2}\,x^{12}+15\,x^{11}+\frac{11}{2}\,x^{10}+\frac{11}{9}\,x^{9}+\frac{1}{8}\,x^{8}$$

input
```
integrate(x^7*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/19*x^19 + 11/18*x^18 + 55/17*x^17 + 165/16*x^16 + 22*x^15 + 33*x^14 + 46
2/13*x^13 + 55/2*x^12 + 15*x^11 + 11/2*x^10 + 11/9*x^9 + 1/8*x^8
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{19}\,x^{19}+\frac{11}{18}\,x^{18}+\frac{55}{17}\,x^{17}+\frac{165}{16}\,x^{16}+22\,x^{15}+33\,x^{14}+\frac{462}{13}\,x^{13}+\frac{55}{2}\,x^{12}+15\,x^{11}+\frac{11}{2}\,x^{10}+\frac{11}{9}\,x^{9}+\frac{1}{8}\,x^{8}$$

input
```
integrate(x^7*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/19*x^19 + 11/18*x^18 + 55/17*x^17 + 165/16*x^16 + 22*x^15 + 33*x^14 + 46
2/13*x^13 + 55/2*x^12 + 15*x^11 + 11/2*x^10 + 11/9*x^9 + 1/8*x^8
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{19}}{19}+\frac{11\,x^{18}}{18}+\frac{55\,x^{17}}{17}+\frac{165\,x^{16}}{16}+22\,x^{15}+33\,x^{14}+\frac{462\,x^{13}}{13}+\frac{55\,x^{12}}{2}+15\,x^{11}+\frac{11\,x^{10}}{2}+\frac{11\,x^{9}}{9}+\frac{x^{8}}{8}$$

input
```
int(x^7*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^8/8 + (11*x^9)/9 + (11*x^10)/2 + 15*x^11 + (55*x^12)/2 + (462*x^13)/13 +
 33*x^14 + 22*x^15 + (165*x^16)/16 + (55*x^17)/17 + (11*x^18)/18 + x^19/19
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int x^7(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^8(31824x^{11}+369512x^{10}+1956240x^9+6235515x^8+13302432x^7+19953648x^6+21488544x^5+1662}{604656}$$

input
```
int(x^7*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**8*(31824*x**11 + 369512*x**10 + 1956240*x**9 + 6235515*x**8 + 13302432
*x**7 + 19953648*x**6 + 21488544*x**5 + 16628040*x**4 + 9069840*x**3 + 332
5608*x**2 + 739024*x + 75582))/604656
```

3.222 $\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{12}(1+x)^{12}-\frac{6}{13}(1+x)^{13}+\frac{15}{14}(1+x)^{14}-\frac{4}{3}(1+x)^{15}+\frac{15}{16}(1+x)^{16}-\frac{6}{17}(1+x)^{17}+\frac{1}{18}(1+x)^{18}$$

output
```
1/12*(1+x)^12-6/13*(1+x)^13+15/14*(1+x)^14-4/3*(1+x)^15+15/16*(1+x)^16-6/1
7*(1+x)^17+1/18*(1+x)^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^7}{7}+\frac{11x^8}{8}+\frac{55x^9}{9}+\frac{33x^{10}}{2}+30x^{11}+\frac{77x^{12}}{2}+\frac{462x^{13}}{13}+\frac{165x^{14}}{7}+11x^{15}+\frac{55x^{16}}{16}+\frac{11x^{17}}{17}+\frac{x^{18}}{18}$$

input
```
Integrate[x^6*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^7/7 + (11*x^8)/8 + (55*x^9)/9 + (33*x^10)/2 + 30*x^11 + (77*x^12)/2 + (4
62*x^13)/13 + (165*x^14)/7 + 11*x^15 + (55*x^16)/16 + (11*x^17)/17 + x^18/
18
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(x+1)\left(x^2+2x+1\right)^5\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^6(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{17}-6(x+1)^{16}+15(x+1)^{15}-20(x+1)^{14}+15(x+1)^{13}-6(x+1)^{12}+(x+1)^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{18}(x+1)^{18}-\frac{6}{17}(x+1)^{17}+\frac{15}{16}(x+1)^{16}-\frac{4}{3}(x+1)^{15}+\frac{15}{14}(x+1)^{14}-\frac{6}{13}(x+1)^{13}+\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^6*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12 - (6*(1 + x)^13)/13 + (15*(1 + x)^14)/14 - (4*(1 + x)^15)/3
+ (15*(1 + x)^16)/16 - (6*(1 + x)^17)/17 + (1 + x)^18/18
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
gosper	$\frac{x^7(12376x^{11}+144144x^{10}+765765x^9+2450448x^8+5250960x^7+7916832x^6+8576568x^5+6683040x^4+3675672x^3+1361360x}{222768}$
default	$\frac{1}{7}x^7+\frac{11}{8}x^8+\frac{55}{9}x^9+\frac{33}{2}x^{10}+30x^{11}+\frac{77}{2}x^{12}+\frac{462}{13}x^{13}+\frac{165}{7}x^{14}+11x^{15}+\frac{55}{16}x^{16}+\frac{11}{17}x^{17}+$
norman	$\frac{1}{7}x^7+\frac{11}{8}x^8+\frac{55}{9}x^9+\frac{33}{2}x^{10}+30x^{11}+\frac{77}{2}x^{12}+\frac{462}{13}x^{13}+\frac{165}{7}x^{14}+11x^{15}+\frac{55}{16}x^{16}+\frac{11}{17}x^{17}+$
risch	$\frac{1}{7}x^7+\frac{11}{8}x^8+\frac{55}{9}x^9+\frac{33}{2}x^{10}+30x^{11}+\frac{77}{2}x^{12}+\frac{462}{13}x^{13}+\frac{165}{7}x^{14}+11x^{15}+\frac{55}{16}x^{16}+\frac{11}{17}x^{17}+$
parallelrisch	$\frac{1}{7}x^7+\frac{11}{8}x^8+\frac{55}{9}x^9+\frac{33}{2}x^{10}+30x^{11}+\frac{77}{2}x^{12}+\frac{462}{13}x^{13}+\frac{165}{7}x^{14}+11x^{15}+\frac{55}{16}x^{16}+\frac{11}{17}x^{17}+$
orering	$\frac{x^7(12376x^{11}+144144x^{10}+765765x^9+2450448x^8+5250960x^7+7916832x^6+8576568x^5+6683040x^4+3675672x^3+1361360x}{222768(x+1)^{10}}$

input
```
int(x^6*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/222768*x^7*(12376*x^11+144144*x^10+765765*x^9+2450448*x^8+5250960*x^7+79
16832*x^6+8576568*x^5+6683040*x^4+3675672*x^3+1361360*x^2+306306*x+31824)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{18}\,x^{18}+\frac{11}{17}\,x^{17}+\frac{55}{16}\,x^{16}+11\,x^{15}+\frac{165}{7}\,x^{14}+\frac{462}{13}\,x^{13}+\frac{77}{2}\,x^{12}+30\,x^{11}+\frac{33}{2}\,x^{10}+\frac{55}{9}\,x^{9}+\frac{11}{8}\,x^{8}+\frac{1}{7}\,x^{7}$$

input
```
integrate(x^6*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/18*x^18 + 11/17*x^17 + 55/16*x^16 + 11*x^15 + 165/7*x^14 + 462/13*x^13 +
 77/2*x^12 + 30*x^11 + 33/2*x^10 + 55/9*x^9 + 11/8*x^8 + 1/7*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{18}}{18}+\frac{11x^{17}}{17}+\frac{55x^{16}}{16}+11x^{15}+\frac{165x^{14}}{7}+\frac{462x^{13}}{13}+\frac{77x^{12}}{2}+30x^{11}+\frac{33x^{10}}{2}+\frac{55x^{9}}{9}+\frac{11x^{8}}{8}+\frac{x^{7}}{7}$$

input
```
integrate(x**6*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**18/18 + 11*x**17/17 + 55*x**16/16 + 11*x**15 + 165*x**14/7 + 462*x**13/
13 + 77*x**12/2 + 30*x**11 + 33*x**10/2 + 55*x**9/9 + 11*x**8/8 + x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{18}x^{18}+\frac{11}{17}x^{17}+\frac{55}{16}x^{16}+11x^{15}+\frac{165}{7}x^{14}+\frac{462}{13}x^{13}+\frac{77}{2}x^{12}+30x^{11}+\frac{33}{2}x^{10}+\frac{55}{9}x^9+\frac{11}{8}x^8+\frac{1}{7}x^7$$

input
```
integrate(x^6*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/18*x^18 + 11/17*x^17 + 55/16*x^16 + 11*x^15 + 165/7*x^14 + 462/13*x^13 +
 77/2*x^12 + 30*x^11 + 33/2*x^10 + 55/9*x^9 + 11/8*x^8 + 1/7*x^7
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{18}x^{18}+\frac{11}{17}x^{17}+\frac{55}{16}x^{16}+11x^{15}+\frac{165}{7}x^{14}+\frac{462}{13}x^{13}+\frac{77}{2}x^{12}+30x^{11}+\frac{33}{2}x^{10}+\frac{55}{9}x^9+\frac{11}{8}x^8+\frac{1}{7}x^7$$

input
```
integrate(x^6*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/18*x^18 + 11/17*x^17 + 55/16*x^16 + 11*x^15 + 165/7*x^14 + 462/13*x^13 +
 77/2*x^12 + 30*x^11 + 33/2*x^10 + 55/9*x^9 + 11/8*x^8 + 1/7*x^7
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{18}}{18}+\frac{11\,x^{17}}{17}+\frac{55\,x^{16}}{16}+11\,x^{15}+\frac{165\,x^{14}}{7}+\frac{462\,x^{13}}{13}+\frac{77\,x^{12}}{2}+30\,x^{11}+\frac{33\,x^{10}}{2}+\frac{55\,x^9}{9}+\frac{11\,x^8}{8}+\frac{x^7}{7}$$

input
```
int(x^6*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^7/7 + (11*x^8)/8 + (55*x^9)/9 + (33*x^10)/2 + 30*x^11 + (77*x^12)/2 + (4
62*x^13)/13 + (165*x^14)/7 + 11*x^15 + (55*x^16)/16 + (11*x^17)/17 + x^18/
18
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int x^6(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$= \frac{x^7(12376x^{11}+144144x^{10}+765765x^9+2450448x^8+5250960x^7+7916832x^6+8576568x^5+6683040x}{222768}$$

input
```
int(x^6*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**7*(12376*x**11 + 144144*x**10 + 765765*x**9 + 2450448*x**8 + 5250960*x
**7 + 7916832*x**6 + 8576568*x**5 + 6683040*x**4 + 3675672*x**3 + 1361360*
x**2 + 306306*x + 31824))/222768
```

3.223 $\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{12}(1+x)^{12}+\frac{5}{13}(1+x)^{13}-\frac{5}{7}(1+x)^{14}+\frac{2}{3}(1+x)^{15}-\frac{5}{16}(1+x)^{16}+\frac{1}{17}(1+x)^{17}$$

output
```
-1/12*(1+x)^12+5/13*(1+x)^13-5/7*(1+x)^14+2/3*(1+x)^15-5/16*(1+x)^16+1/17*
(1+x)^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^6}{6}+\frac{11x^7}{7}+\frac{55x^8}{8}+\frac{55x^9}{3}+33x^{10}+42x^{11}+\frac{77x^{12}}{2}+\frac{330x^{13}}{13}+\frac{165x^{14}}{14}+\frac{11x^{15}}{3}+\frac{11x^{16}}{16}+\frac{x^{17}}{17}$$

input
```
Integrate[x^5*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^6/6 + (11*x^7)/7 + (55*x^8)/8 + (55*x^9)/3 + 33*x^10 + 42*x^11 + (77*x^1
2)/2 + (330*x^13)/13 + (165*x^14)/14 + (11*x^15)/3 + (11*x^16)/16 + x^17/1
7
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(x+1)\left(x^2+2x+1\right)^5\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^5(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{16}-5(x+1)^{15}+10(x+1)^{14}-10(x+1)^{13}+5(x+1)^{12}-(x+1)^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{17}(x+1)^{17}-\frac{5}{16}(x+1)^{16}+\frac{2}{3}(x+1)^{15}-\frac{5}{7}(x+1)^{14}+\frac{5}{13}(x+1)^{13}-\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^5*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/12*(1 + x)^12 + (5*(1 + x)^13)/13 - (5*(1 + x)^14)/7 + (2*(1 + x)^15)/3
 - (5*(1 + x)^16)/16 + (1 + x)^17/17
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

method	result
gosper	$\frac{x^{6}(4368x^{11}+51051x^{10}+272272x^{9}+875160x^{8}+1884960x^{7}+2858856x^{6}+3118752x^{5}+2450448x^{4}+1361360x^{3}+510510x^{2}+11}{74256}$
default	$\frac{1}{6}x^{6}+\frac{11}{7}x^{7}+\frac{55}{8}x^{8}+\frac{55}{3}x^{9}+33x^{10}+42x^{11}+\frac{77}{2}x^{12}+\frac{330}{13}x^{13}+\frac{165}{14}x^{14}+\frac{11}{3}x^{15}+\frac{11}{16}x^{16}+$
norman	$\frac{1}{6}x^{6}+\frac{11}{7}x^{7}+\frac{55}{8}x^{8}+\frac{55}{3}x^{9}+33x^{10}+42x^{11}+\frac{77}{2}x^{12}+\frac{330}{13}x^{13}+\frac{165}{14}x^{14}+\frac{11}{3}x^{15}+\frac{11}{16}x^{16}+$
risch	$\frac{1}{6}x^{6}+\frac{11}{7}x^{7}+\frac{55}{8}x^{8}+\frac{55}{3}x^{9}+33x^{10}+42x^{11}+\frac{77}{2}x^{12}+\frac{330}{13}x^{13}+\frac{165}{14}x^{14}+\frac{11}{3}x^{15}+\frac{11}{16}x^{16}+$
parallelrisch	$\frac{1}{6}x^{6}+\frac{11}{7}x^{7}+\frac{55}{8}x^{8}+\frac{55}{3}x^{9}+33x^{10}+42x^{11}+\frac{77}{2}x^{12}+\frac{330}{13}x^{13}+\frac{165}{14}x^{14}+\frac{11}{3}x^{15}+\frac{11}{16}x^{16}+$
orering	$\frac{x^{6}(4368x^{11}+51051x^{10}+272272x^{9}+875160x^{8}+1884960x^{7}+2858856x^{6}+3118752x^{5}+2450448x^{4}+1361360x^{3}+510510x^{2}+11}{74256(x+1)^{10}}$

input
```
int(x^5*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/74256*x^6*(4368*x^11+51051*x^10+272272*x^9+875160*x^8+1884960*x^7+285885
6*x^6+3118752*x^5+2450448*x^4+1361360*x^3+510510*x^2+116688*x+12376)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{17}\,x^{17}+\frac{11}{16}\,x^{16}+\frac{11}{3}\,x^{15}+\frac{165}{14}\,x^{14}+\frac{330}{13}\,x^{13}+\frac{77}{2}\,x^{12}+42\,x^{11}+33\,x^{10}+\frac{55}{3}\,x^9+\frac{55}{8}\,x^8+\frac{11}{7}\,x^7+\frac{1}{6}\,x^6$$

input
```
integrate(x^5*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/17*x^17 + 11/16*x^16 + 11/3*x^15 + 165/14*x^14 + 330/13*x^13 + 77/2*x^12
 + 42*x^11 + 33*x^10 + 55/3*x^9 + 55/8*x^8 + 11/7*x^7 + 1/6*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{17}}{17}+\frac{11x^{16}}{16}+\frac{11x^{15}}{3}+\frac{165x^{14}}{14}+\frac{330x^{13}}{13}+\frac{77x^{12}}{2}+42x^{11}+33x^{10}+\frac{55x^9}{3}+\frac{55x^8}{8}+\frac{11x^7}{7}+\frac{x^6}{6}$$

input
```
integrate(x**5*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**17/17 + 11*x**16/16 + 11*x**15/3 + 165*x**14/14 + 330*x**13/13 + 77*x**
12/2 + 42*x**11 + 33*x**10 + 55*x**9/3 + 55*x**8/8 + 11*x**7/7 + x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{17}x^{17}+\frac{11}{16}x^{16}+\frac{11}{3}x^{15}+\frac{165}{14}x^{14}+\frac{330}{13}x^{13}+\frac{77}{2}x^{12}+42x^{11}+33x^{10}+\frac{55}{3}x^9+\frac{55}{8}x^8+\frac{11}{7}x^7+\frac{1}{6}x^6$$

input
```
integrate(x^5*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/17*x^17 + 11/16*x^16 + 11/3*x^15 + 165/14*x^14 + 330/13*x^13 + 77/2*x^12
 + 42*x^11 + 33*x^10 + 55/3*x^9 + 55/8*x^8 + 11/7*x^7 + 1/6*x^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{17}x^{17}+\frac{11}{16}x^{16}+\frac{11}{3}x^{15}+\frac{165}{14}x^{14}+\frac{330}{13}x^{13}+\frac{77}{2}x^{12}+42x^{11}+33x^{10}+\frac{55}{3}x^9+\frac{55}{8}x^8+\frac{11}{7}x^7+\frac{1}{6}x^6$$

input
```
integrate(x^5*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/17*x^17 + 11/16*x^16 + 11/3*x^15 + 165/14*x^14 + 330/13*x^13 + 77/2*x^12
 + 42*x^11 + 33*x^10 + 55/3*x^9 + 55/8*x^8 + 11/7*x^7 + 1/6*x^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{17}}{17}+\frac{11\,x^{16}}{16}+\frac{11\,x^{15}}{3}+\frac{165\,x^{14}}{14}+\frac{330\,x^{13}}{13}+\frac{77\,x^{12}}{2}+42\,x^{11}+33\,x^{10}+\frac{55\,x^9}{3}+\frac{55\,x^8}{8}+\frac{11\,x^7}{7}+\frac{x^6}{6}$$

input
```
int(x^5*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^6/6 + (11*x^7)/7 + (55*x^8)/8 + (55*x^9)/3 + 33*x^10 + 42*x^11 + (77*x^1
2)/2 + (330*x^13)/13 + (165*x^14)/14 + (11*x^15)/3 + (11*x^16)/16 + x^17/1
7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int x^5(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^6(4368x^{11}+51051x^{10}+272272x^9+875160x^8+1884960x^7+2858856x^6+3118752x^5+2450448x^4+}{74256}$$

input
```
int(x^5*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**6*(4368*x**11 + 51051*x**10 + 272272*x**9 + 875160*x**8 + 1884960*x**7
 + 2858856*x**6 + 3118752*x**5 + 2450448*x**4 + 1361360*x**3 + 510510*x**2
 + 116688*x + 12376))/74256
```

3.224 $\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{12}(1+x)^{12}-\frac{4}{13}(1+x)^{13}+\frac{3}{7}(1+x)^{14}-\frac{4}{15}(1+x)^{15}+\frac{1}{16}(1+x)^{16}$$

output
```
1/12*(1+x)^12-4/13*(1+x)^13+3/7*(1+x)^14-4/15*(1+x)^15+1/16*(1+x)^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^5}{5}+\frac{11x^6}{6}+\frac{55x^7}{7}+\frac{165x^8}{8}+\frac{110x^9}{3}+\frac{231x^{10}}{5}+42x^{11}+\frac{55x^{12}}{2}+\frac{165x^{13}}{13}+\frac{55x^{14}}{14}+\frac{11x^{15}}{15}+\frac{x^{16}}{16}$$

input
```
Integrate[x^4*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^5/5 + (11*x^6)/6 + (55*x^7)/7 + (165*x^8)/8 + (110*x^9)/3 + (231*x^10)/5
 + 42*x^11 + (55*x^12)/2 + (165*x^13)/13 + (55*x^14)/14 + (11*x^15)/15 + x
^16/16
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(x+1)\left(x^2+2x+1\right)^5\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int x^4(x+1)^{11}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((x+1)^{15}-4(x+1)^{14}+6(x+1)^{13}-4(x+1)^{12}+(x+1)^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}(x+1)^{16}-\frac{4}{15}(x+1)^{15}+\frac{3}{7}(x+1)^{14}-\frac{4}{13}(x+1)^{13}+\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^4*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12 - (4*(1 + x)^13)/13 + (3*(1 + x)^14)/7 - (4*(1 + x)^15)/15 +
 (1 + x)^16/16
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

method	result
gosper	$\frac{x^5(1365x^{11}+16016x^{10}+85800x^9+277200x^8+600600x^7+917280x^6+1009008x^5+800800x^4+450450x^3+171600x^2+40040x}{21840}$
default	$\frac{1}{5}x^5+\frac{11}{6}x^6+\frac{55}{7}x^7+\frac{165}{8}x^8+\frac{110}{3}x^9+\frac{231}{5}x^{10}+42x^{11}+\frac{55}{2}x^{12}+\frac{165}{13}x^{13}+\frac{55}{14}x^{14}+\frac{11}{15}x^{15}+$
norman	$\frac{1}{5}x^5+\frac{11}{6}x^6+\frac{55}{7}x^7+\frac{165}{8}x^8+\frac{110}{3}x^9+\frac{231}{5}x^{10}+42x^{11}+\frac{55}{2}x^{12}+\frac{165}{13}x^{13}+\frac{55}{14}x^{14}+\frac{11}{15}x^{15}+$
risch	$\frac{1}{5}x^5+\frac{11}{6}x^6+\frac{55}{7}x^7+\frac{165}{8}x^8+\frac{110}{3}x^9+\frac{231}{5}x^{10}+42x^{11}+\frac{55}{2}x^{12}+\frac{165}{13}x^{13}+\frac{55}{14}x^{14}+\frac{11}{15}x^{15}+$
parallelrisch	$\frac{1}{5}x^5+\frac{11}{6}x^6+\frac{55}{7}x^7+\frac{165}{8}x^8+\frac{110}{3}x^9+\frac{231}{5}x^{10}+42x^{11}+\frac{55}{2}x^{12}+\frac{165}{13}x^{13}+\frac{55}{14}x^{14}+\frac{11}{15}x^{15}+$
orering	$\frac{x^5(1365x^{11}+16016x^{10}+85800x^9+277200x^8+600600x^7+917280x^6+1009008x^5+800800x^4+450450x^3+171600x^2+40040x}{21840(x+1)^{10}}$

input
```
int(x^4*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/21840*x^5*(1365*x^11+16016*x^10+85800*x^9+277200*x^8+600600*x^7+917280*x
^6+1009008*x^5+800800*x^4+450450*x^3+171600*x^2+40040*x+4368)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{16}\,x^{16}+\frac{11}{15}\,x^{15}+\frac{55}{14}\,x^{14}+\frac{165}{13}\,x^{13}+\frac{55}{2}\,x^{12}+42\,x^{11}+\frac{231}{5}\,x^{10}+\frac{110}{3}\,x^{9}+\frac{165}{8}\,x^{8}+\frac{55}{7}\,x^{7}+\frac{11}{6}\,x^{6}+\frac{1}{5}\,x^{5}$$

input
```
integrate(x^4*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/16*x^16 + 11/15*x^15 + 55/14*x^14 + 165/13*x^13 + 55/2*x^12 + 42*x^11 +
231/5*x^10 + 110/3*x^9 + 165/8*x^8 + 55/7*x^7 + 11/6*x^6 + 1/5*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{16}}{16}+\frac{11x^{15}}{15}+\frac{55x^{14}}{14}+\frac{165x^{13}}{13}+\frac{55x^{12}}{2}+42x^{11}+\frac{231x^{10}}{5}+\frac{110x^{9}}{3}+\frac{165x^{8}}{8}+\frac{55x^{7}}{7}+\frac{11x^{6}}{6}+\frac{x^{5}}{5}$$

input
```
integrate(x**4*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**16/16 + 11*x**15/15 + 55*x**14/14 + 165*x**13/13 + 55*x**12/2 + 42*x**1
1 + 231*x**10/5 + 110*x**9/3 + 165*x**8/8 + 55*x**7/7 + 11*x**6/6 + x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{16}\,x^{16} + \frac{11}{15}\,x^{15} + \frac{55}{14}\,x^{14} + \frac{165}{13}\,x^{13} + \frac{55}{2}\,x^{12} + 42\,x^{11} + \frac{231}{5}\,x^{10} + \frac{110}{3}\,x^9 + \frac{165}{8}\,x^8 + \frac{55}{7}\,x^7 + \frac{11}{6}\,x^6 + \frac{1}{5}\,x^5$$

input
```
integrate(x^4*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/16*x^16 + 11/15*x^15 + 55/14*x^14 + 165/13*x^13 + 55/2*x^12 + 42*x^11 +
231/5*x^10 + 110/3*x^9 + 165/8*x^8 + 55/7*x^7 + 11/6*x^6 + 1/5*x^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{16}\,x^{16} + \frac{11}{15}\,x^{15} + \frac{55}{14}\,x^{14} + \frac{165}{13}\,x^{13} + \frac{55}{2}\,x^{12} + 42\,x^{11} + \frac{231}{5}\,x^{10} + \frac{110}{3}\,x^9 + \frac{165}{8}\,x^8 + \frac{55}{7}\,x^7 + \frac{11}{6}\,x^6 + \frac{1}{5}\,x^5$$

input
```
integrate(x^4*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/16*x^16 + 11/15*x^15 + 55/14*x^14 + 165/13*x^13 + 55/2*x^12 + 42*x^11 +
231/5*x^10 + 110/3*x^9 + 165/8*x^8 + 55/7*x^7 + 11/6*x^6 + 1/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{16}}{16} + \frac{11\,x^{15}}{15} + \frac{55\,x^{14}}{14} + \frac{165\,x^{13}}{13} + \frac{55\,x^{12}}{2} + 42\,x^{11} + \frac{231\,x^{10}}{5} + \frac{110\,x^9}{3} + \frac{165\,x^8}{8} + \frac{55\,x^7}{7} + \frac{11\,x^6}{6} + \frac{x^5}{5}$$

input
```
int(x^4*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^5/5 + (11*x^6)/6 + (55*x^7)/7 + (165*x^8)/8 + (110*x^9)/3 + (231*x^10)/5
 + 42*x^11 + (55*x^12)/2 + (165*x^13)/13 + (55*x^14)/14 + (11*x^15)/15 + x
^16/16
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int x^4(1+x)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^5(1365x^{11}+16016x^{10}+85800x^9+277200x^8+600600x^7+917280x^6+1009008x^5+800800x^4+450}{21840}$$

input
```
int(x^4*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**5*(1365*x**11 + 16016*x**10 + 85800*x**9 + 277200*x**8 + 600600*x**7 +
 917280*x**6 + 1009008*x**5 + 800800*x**4 + 450450*x**3 + 171600*x**2 + 40
040*x + 4368))/21840
```

3.225 $\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{12}(1+x)^{12}+\frac{3}{13}(1+x)^{13}-\frac{3}{14}(1+x)^{14}+\frac{1}{15}(1+x)^{15}$$

output
```
-1/12*(1+x)^12+3/13*(1+x)^13-3/14*(1+x)^14+1/15*(1+x)^15
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.24

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^4}{4}+\frac{11x^5}{5}+\frac{55x^6}{6}+\frac{165x^7}{7}+\frac{165x^8}{4}+\frac{154x^9}{3}+\frac{231x^{10}}{5}+30x^{11}+\frac{55x^{12}}{4}+\frac{55x^{13}}{13}+\frac{11x^{14}}{14}+\frac{x^{15}}{15}$$

input
```
Integrate[x^3*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^4/4 + (11*x^5)/5 + (55*x^6)/6 + (165*x^7)/7 + (165*x^8)/4 + (154*x^9)/3
+ (231*x^10)/5 + 30*x^11 + (55*x^12)/4 + (55*x^13)/13 + (11*x^14)/14 + x^1
5/15
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)\left(x^2+2x+1\right)^5 dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int x^3(x+1)^{11}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left((x+1)^{14}-3(x+1)^{13}+3(x+1)^{12}-(x+1)^{11}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{15}(x+1)^{15}-\frac{3}{14}(x+1)^{14}+\frac{3}{13}(x+1)^{13}-\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^3*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/12*(1 + x)^12 + (3*(1 + x)^13)/13 - (3*(1 + x)^14)/14 + (1 + x)^15/15
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

method	result
gosper	$\frac{x^4(364x^{11}+4290x^{10}+23100x^9+75075x^8+163800x^7+252252x^6+280280x^5+225225x^4+128700x^3+50050x^2+12012x+1365}{5460}$
default	$\frac{1}{4}x^4+\frac{11}{5}x^5+\frac{55}{6}x^6+\frac{165}{7}x^7+\frac{165}{4}x^8+\frac{154}{3}x^9+\frac{231}{5}x^{10}+30x^{11}+\frac{55}{4}x^{12}+\frac{55}{13}x^{13}+\frac{11}{14}x^{14}+\frac{1}{15}$
norman	$\frac{1}{4}x^4+\frac{11}{5}x^5+\frac{55}{6}x^6+\frac{165}{7}x^7+\frac{165}{4}x^8+\frac{154}{3}x^9+\frac{231}{5}x^{10}+30x^{11}+\frac{55}{4}x^{12}+\frac{55}{13}x^{13}+\frac{11}{14}x^{14}+\frac{1}{15}$
risch	$\frac{1}{4}x^4+\frac{11}{5}x^5+\frac{55}{6}x^6+\frac{165}{7}x^7+\frac{165}{4}x^8+\frac{154}{3}x^9+\frac{231}{5}x^{10}+30x^{11}+\frac{55}{4}x^{12}+\frac{55}{13}x^{13}+\frac{11}{14}x^{14}+\frac{1}{15}$
parallelrisch	$\frac{1}{4}x^4+\frac{11}{5}x^5+\frac{55}{6}x^6+\frac{165}{7}x^7+\frac{165}{4}x^8+\frac{154}{3}x^9+\frac{231}{5}x^{10}+30x^{11}+\frac{55}{4}x^{12}+\frac{55}{13}x^{13}+\frac{11}{14}x^{14}+\frac{1}{15}$
orering	$\frac{x^4(364x^{11}+4290x^{10}+23100x^9+75075x^8+163800x^7+252252x^6+280280x^5+225225x^4+128700x^3+50050x^2+12012x+1365}{5460(x+1)^{10}}$

input
```
int(x^3*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5460*x^4*(364*x^11+4290*x^10+23100*x^9+75075*x^8+163800*x^7+252252*x^6+2
80280*x^5+225225*x^4+128700*x^3+50050*x^2+12012*x+1365)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{15}\,x^{15}+\frac{11}{14}\,x^{14}+\frac{55}{13}\,x^{13}+\frac{55}{4}\,x^{12}+30\,x^{11}+\frac{231}{5}\,x^{10}+\frac{154}{3}\,x^{9}+\frac{165}{4}\,x^{8}+\frac{165}{7}\,x^{7}+\frac{55}{6}\,x^{6}+\frac{11}{5}\,x^{5}+\frac{1}{4}\,x^{4}$$

input
```
integrate(x^3*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/15*x^15 + 11/14*x^14 + 55/13*x^13 + 55/4*x^12 + 30*x^11 + 231/5*x^10 + 1
54/3*x^9 + 165/4*x^8 + 165/7*x^7 + 55/6*x^6 + 11/5*x^5 + 1/4*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{15}}{15}+\frac{11x^{14}}{14}+\frac{55x^{13}}{13}+\frac{55x^{12}}{4}+30x^{11}+\frac{231x^{10}}{5}+\frac{154x^{9}}{3}+\frac{165x^{8}}{4}+\frac{165x^{7}}{7}+\frac{55x^{6}}{6}+\frac{11x^{5}}{5}+\frac{x^{4}}{4}$$

input
```
integrate(x**3*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**15/15 + 11*x**14/14 + 55*x**13/13 + 55*x**12/4 + 30*x**11 + 231*x**10/5
 + 154*x**9/3 + 165*x**8/4 + 165*x**7/7 + 55*x**6/6 + 11*x**5/5 + x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{15}x^{15} + \frac{11}{14}x^{14} + \frac{55}{13}x^{13} + \frac{55}{4}x^{12} + 30\,x^{11} + \frac{231}{5}x^{10} + \frac{154}{3}x^9 + \frac{165}{4}x^8 + \frac{165}{7}x^7 + \frac{55}{6}x^6 + \frac{11}{5}x^5 + \frac{1}{4}x^4$$

input
```
integrate(x^3*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/15*x^15 + 11/14*x^14 + 55/13*x^13 + 55/4*x^12 + 30*x^11 + 231/5*x^10 + 1
54/3*x^9 + 165/4*x^8 + 165/7*x^7 + 55/6*x^6 + 11/5*x^5 + 1/4*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{15}x^{15} + \frac{11}{14}x^{14} + \frac{55}{13}x^{13} + \frac{55}{4}x^{12} + 30\,x^{11} + \frac{231}{5}x^{10} + \frac{154}{3}x^9 + \frac{165}{4}x^8 + \frac{165}{7}x^7 + \frac{55}{6}x^6 + \frac{11}{5}x^5 + \frac{1}{4}x^4$$

input
```
integrate(x^3*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/15*x^15 + 11/14*x^14 + 55/13*x^13 + 55/4*x^12 + 30*x^11 + 231/5*x^10 + 1
54/3*x^9 + 165/4*x^8 + 165/7*x^7 + 55/6*x^6 + 11/5*x^5 + 1/4*x^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{15}}{15}+\frac{11\,x^{14}}{14}+\frac{55\,x^{13}}{13}+\frac{55\,x^{12}}{4}+30\,x^{11}+\frac{231\,x^{10}}{5}+\frac{154\,x^9}{3}+\frac{165\,x^8}{4}+\frac{165\,x^7}{7}+\frac{55\,x^6}{6}+\frac{11\,x^5}{5}+\frac{x^4}{4}$$

input
```
int(x^3*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^4/4 + (11*x^5)/5 + (55*x^6)/6 + (165*x^7)/7 + (165*x^8)/4 + (154*x^9)/3
+ (231*x^10)/5 + 30*x^11 + (55*x^12)/4 + (55*x^13)/13 + (11*x^14)/14 + x^1
5/15
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int x^3(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^4(364x^{11}+4290x^{10}+23100x^9+75075x^8+163800x^7+252252x^6+280280x^5+225225x^4+128700x^3}{5460}$$

input
```
int(x^3*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**4*(364*x**11 + 4290*x**10 + 23100*x**9 + 75075*x**8 + 163800*x**7 + 25
2252*x**6 + 280280*x**5 + 225225*x**4 + 128700*x**3 + 50050*x**2 + 12012*x
 + 1365))/5460
```

3.226 $\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{1}{12}(1+x)^{12}-\frac{2}{13}(1+x)^{13}+\frac{1}{14}(1+x)^{14}$$

output
```
1/12*(1+x)^12-2/13*(1+x)^13+1/14*(1+x)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(28) = 56.

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.82

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^3}{3}+\frac{11x^4}{4}+11x^5+\frac{55x^6}{2}+\frac{330x^7}{7}+\frac{231x^8}{4}+\frac{154x^9}{3}+33x^{10}+15x^{11}+\frac{55x^{12}}{12}+\frac{11x^{13}}{13}+\frac{x^{14}}{14}$$

input
```
Integrate[x^2*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^3/3 + (11*x^4)/4 + 11*x^5 + (55*x^6)/2 + (330*x^7)/7 + (231*x^8)/4 + (15
4*x^9)/3 + 33*x^10 + 15*x^11 + (55*x^12)/12 + (11*x^13)/13 + x^14/14
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(x+1)\left(x^2+2x+1\right)^5 dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int x^2(x+1)^{11}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left((x+1)^{13}-2(x+1)^{12}+(x+1)^{11}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{14}(x+1)^{14}-\frac{2}{13}(x+1)^{13}+\frac{1}{12}(x+1)^{12}$$

input
```
Int[x^2*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12 - (2*(1 + x)^13)/13 + (1 + x)^14/14
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(22) = 44$.

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

method	result
gosper	$\frac{x^3(78x^{11}+924x^{10}+5005x^9+16380x^8+36036x^7+56056x^6+63063x^5+51480x^4+30030x^3+12012x^2+3003x+364)}{1092}$
default	$\frac{1}{3}x^3+\frac{11}{4}x^4+11x^5+\frac{55}{2}x^6+\frac{330}{7}x^7+\frac{231}{4}x^8+\frac{154}{3}x^9+33x^{10}+15x^{11}+\frac{55}{12}x^{12}+\frac{11}{13}x^{13}+\frac{1}{14}x$
norman	$\frac{1}{3}x^3+\frac{11}{4}x^4+11x^5+\frac{55}{2}x^6+\frac{330}{7}x^7+\frac{231}{4}x^8+\frac{154}{3}x^9+33x^{10}+15x^{11}+\frac{55}{12}x^{12}+\frac{11}{13}x^{13}+\frac{1}{14}x$
risch	$\frac{1}{3}x^3+\frac{11}{4}x^4+11x^5+\frac{55}{2}x^6+\frac{330}{7}x^7+\frac{231}{4}x^8+\frac{154}{3}x^9+33x^{10}+15x^{11}+\frac{55}{12}x^{12}+\frac{11}{13}x^{13}+\frac{1}{14}x$
parallelrisch	$\frac{1}{3}x^3+\frac{11}{4}x^4+11x^5+\frac{55}{2}x^6+\frac{330}{7}x^7+\frac{231}{4}x^8+\frac{154}{3}x^9+33x^{10}+15x^{11}+\frac{55}{12}x^{12}+\frac{11}{13}x^{13}+\frac{1}{14}x$
orering	$\frac{x^3(78x^{11}+924x^{10}+5005x^9+16380x^8+36036x^7+56056x^6+63063x^5+51480x^4+30030x^3+12012x^2+3003x+364)(x^2+2x+1}{1092(x+1)^{10}}$

input
```
int(x^2*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/1092*x^3*(78*x^11+924*x^10+5005*x^9+16380*x^8+36036*x^7+56056*x^6+63063*
x^5+51480*x^4+30030*x^3+12012*x^2+3003*x+364)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{14}\,x^{14}+\frac{11}{13}\,x^{13}+\frac{55}{12}\,x^{12}+15\,x^{11}+33\,x^{10}+\frac{154}{3}\,x^{9}+\frac{231}{4}\,x^{8}+\frac{330}{7}\,x^{7}+\frac{55}{2}\,x^{6}+11\,x^{5}+\frac{11}{4}\,x^{4}+\frac{1}{3}\,x^{3}$$

input
```
integrate(x^2*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/14*x^14 + 11/13*x^13 + 55/12*x^12 + 15*x^11 + 33*x^10 + 154/3*x^9 + 231/
4*x^8 + 330/7*x^7 + 55/2*x^6 + 11*x^5 + 11/4*x^4 + 1/3*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{14}}{14}+\frac{11x^{13}}{13}+\frac{55x^{12}}{12}+15x^{11}+33x^{10}+\frac{154x^{9}}{3}+\frac{231x^{8}}{4}+\frac{330x^{7}}{7}+\frac{55x^{6}}{2}+11x^{5}+\frac{11x^{4}}{4}+\frac{x^{3}}{3}$$

input
```
integrate(x**2*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**14/14 + 11*x**13/13 + 55*x**12/12 + 15*x**11 + 33*x**10 + 154*x**9/3 +
231*x**8/4 + 330*x**7/7 + 55*x**6/2 + 11*x**5 + 11*x**4/4 + x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{14}\,x^{14} + \frac{11}{13}\,x^{13} + \frac{55}{12}\,x^{12} + 15\,x^{11} + 33\,x^{10} + \frac{154}{3}\,x^9 + \frac{231}{4}\,x^8 + \frac{330}{7}\,x^7 + \frac{55}{2}\,x^6 + 11\,x^5 + \frac{11}{4}\,x^4 + \frac{1}{3}\,x^3$$

input
```
integrate(x^2*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/14*x^14 + 11/13*x^13 + 55/12*x^12 + 15*x^11 + 33*x^10 + 154/3*x^9 + 231/
4*x^8 + 330/7*x^7 + 55/2*x^6 + 11*x^5 + 11/4*x^4 + 1/3*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{14}\,x^{14} + \frac{11}{13}\,x^{13} + \frac{55}{12}\,x^{12} + 15\,x^{11} + 33\,x^{10} + \frac{154}{3}\,x^9 + \frac{231}{4}\,x^8 + \frac{330}{7}\,x^7 + \frac{55}{2}\,x^6 + 11\,x^5 + \frac{11}{4}\,x^4 + \frac{1}{3}\,x^3$$

input
```
integrate(x^2*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/14*x^14 + 11/13*x^13 + 55/12*x^12 + 15*x^11 + 33*x^10 + 154/3*x^9 + 231/
4*x^8 + 330/7*x^7 + 55/2*x^6 + 11*x^5 + 11/4*x^4 + 1/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{14}}{14}+\frac{11\,x^{13}}{13}+\frac{55\,x^{12}}{12}+15\,x^{11}+33\,x^{10}+\frac{154\,x^9}{3}+\frac{231\,x^8}{4}+\frac{330\,x^7}{7}+\frac{55\,x^6}{2}+11\,x^5+\frac{11\,x^4}{4}+\frac{x^3}{3}$$

input
```
int(x^2*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^3/3 + (11*x^4)/4 + 11*x^5 + (55*x^6)/2 + (330*x^7)/7 + (231*x^8)/4 + (15
4*x^9)/3 + 33*x^10 + 15*x^11 + (55*x^12)/12 + (11*x^13)/13 + x^14/14
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int x^2(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^3(78x^{11}+924x^{10}+5005x^9+16380x^8+36036x^7+56056x^6+63063x^5+51480x^4+30030x^3+12012}{1092}$$

input
```
int(x^2*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**3*(78*x**11 + 924*x**10 + 5005*x**9 + 16380*x**8 + 36036*x**7 + 56056*
x**6 + 63063*x**5 + 51480*x**4 + 30030*x**3 + 12012*x**2 + 3003*x + 364))/
1092
```

3.227 $\int x(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx=-\frac{1}{12}(1+x)^{12}+\frac{1}{13}(1+x)^{13}$$

output
```
-1/12*(1+x)^12+1/13*(1+x)^13
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^2}{2}+\frac{11x^3}{3}+\frac{55x^4}{4}+33x^5+55x^6+66x^7+\frac{231x^8}{4}+\frac{110x^9}{3}+\frac{33x^{10}}{2}+5x^{11}+\frac{11x^{12}}{12}+\frac{x^{13}}{13}$$

input
```
Integrate[x*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^2/2 + (11*x^3)/3 + (55*x^4)/4 + 33*x^5 + 55*x^6 + 66*x^7 + (231*x^8)/4 +
 (110*x^9)/3 + (33*x^10)/2 + 5*x^11 + (11*x^12)/12 + x^13/13
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)\left(x^2+2x+1\right)^5 dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int x(x+1)^{11}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left((x+1)^{12}-(x+1)^{11}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{13}(x+1)^{13}-\frac{1}{12}(x+1)^{12}$$

input
```
Int[x*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
-1/12*(1 + x)^12 + (1 + x)^13/13
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(15) = 30$.

Time = 0.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

method	result
gosper	$\frac{x^{2}\left(12 x^{11}+143 x^{10}+780 x^{9}+2574 x^{8}+5720 x^{7}+9009 x^{6}+10296 x^{5}+8580 x^{4}+5148 x^{3}+2145 x^{2}+572 x +78\right)}{156}$
default	$\frac{1}{13}x^{13}+\frac{11}{12}x^{12}+5x^{11}+\frac{33}{2}x^{10}+\frac{110}{3}x^{9}+\frac{231}{4}x^{8}+66x^{7}+55x^{6}+33x^{5}+\frac{55}{4}x^{4}+\frac{11}{3}x^{3}+\frac{1}{2}x^{2}$
norman	$\frac{1}{13}x^{13}+\frac{11}{12}x^{12}+5x^{11}+\frac{33}{2}x^{10}+\frac{110}{3}x^{9}+\frac{231}{4}x^{8}+66x^{7}+55x^{6}+33x^{5}+\frac{55}{4}x^{4}+\frac{11}{3}x^{3}+\frac{1}{2}x^{2}$
risch	$\frac{1}{13}x^{13}+\frac{11}{12}x^{12}+5x^{11}+\frac{33}{2}x^{10}+\frac{110}{3}x^{9}+\frac{231}{4}x^{8}+66x^{7}+55x^{6}+33x^{5}+\frac{55}{4}x^{4}+\frac{11}{3}x^{3}+\frac{1}{2}x^{2}$
parallelrisch	$\frac{1}{13}x^{13}+\frac{11}{12}x^{12}+5x^{11}+\frac{33}{2}x^{10}+\frac{110}{3}x^{9}+\frac{231}{4}x^{8}+66x^{7}+55x^{6}+33x^{5}+\frac{55}{4}x^{4}+\frac{11}{3}x^{3}+\frac{1}{2}x^{2}$
orering	$\frac{x^{2}\left(12 x^{11}+143 x^{10}+780 x^{9}+2574 x^{8}+5720 x^{7}+9009 x^{6}+10296 x^{5}+8580 x^{4}+5148 x^{3}+2145 x^{2}+572 x +78\right)\left(x^{2}+2 x +1\right)^{5}}{156 \left(x +1\right)^{10}}$

input
```
int(x*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/156*x^2*(12*x^11+143*x^10+780*x^9+2574*x^8+5720*x^7+9009*x^6+10296*x^5+8
580*x^4+5148*x^3+2145*x^2+572*x+78)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{13}\,x^{13}+\frac{11}{12}\,x^{12}+5\,x^{11}+\frac{33}{2}\,x^{10}+\frac{110}{3}\,x^9+\frac{231}{4}\,x^8+66\,x^7+55\,x^6+33\,x^5+\frac{55}{4}\,x^4+\frac{11}{3}\,x^3+\frac{1}{2}\,x^2$$

input
```
integrate(x*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/13*x^13 + 11/12*x^12 + 5*x^11 + 33/2*x^10 + 110/3*x^9 + 231/4*x^8 + 66*x
^7 + 55*x^6 + 33*x^5 + 55/4*x^4 + 11/3*x^3 + 1/2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{13}}{13}+\frac{11x^{12}}{12}+5x^{11}+\frac{33x^{10}}{2}+\frac{110x^9}{3}+\frac{231x^8}{4}+66x^7+55x^6+33x^5+\frac{55x^4}{4}+\frac{11x^3}{3}+\frac{x^2}{2}$$

input
```
integrate(x*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**13/13 + 11*x**12/12 + 5*x**11 + 33*x**10/2 + 110*x**9/3 + 231*x**8/4 +
66*x**7 + 55*x**6 + 33*x**5 + 55*x**4/4 + 11*x**3/3 + x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{13}\,x^{13} + \frac{11}{12}\,x^{12} + 5\,x^{11} + \frac{33}{2}\,x^{10} + \frac{110}{3}\,x^{9} + \frac{231}{4}\,x^{8} + 66\,x^{7} + 55\,x^{6} + 33\,x^{5} + \frac{55}{4}\,x^{4} + \frac{11}{3}\,x^{3} + \frac{1}{2}\,x^{2}$$

input
```
integrate(x*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/13*x^13 + 11/12*x^12 + 5*x^11 + 33/2*x^10 + 110/3*x^9 + 231/4*x^8 + 66*x
^7 + 55*x^6 + 33*x^5 + 55/4*x^4 + 11/3*x^3 + 1/2*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{13}\,x^{13} + \frac{11}{12}\,x^{12} + 5\,x^{11} + \frac{33}{2}\,x^{10} + \frac{110}{3}\,x^{9} + \frac{231}{4}\,x^{8} + 66\,x^{7} + 55\,x^{6} + 33\,x^{5} + \frac{55}{4}\,x^{4} + \frac{11}{3}\,x^{3} + \frac{1}{2}\,x^{2}$$

input
```
integrate(x*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/13*x^13 + 11/12*x^12 + 5*x^11 + 33/2*x^10 + 110/3*x^9 + 231/4*x^8 + 66*x
^7 + 55*x^6 + 33*x^5 + 55/4*x^4 + 11/3*x^3 + 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{13}}{13}+\frac{11\,x^{12}}{12}+5\,x^{11}+\frac{33\,x^{10}}{2}+\frac{110\,x^9}{3}+\frac{231\,x^8}{4}+66\,x^7+55\,x^6+33\,x^5+\frac{55\,x^4}{4}+\frac{11\,x^3}{3}+\frac{x^2}{2}$$

input
```
int(x*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x^2/2 + (11*x^3)/3 + (55*x^4)/4 + 33*x^5 + 55*x^6 + 66*x^7 + (231*x^8)/4 +
 (110*x^9)/3 + (33*x^10)/2 + 5*x^11 + (11*x^12)/12 + x^13/13
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int x(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^2(12x^{11}+143x^{10}+780x^9+2574x^8+5720x^7+9009x^6+10296x^5+8580x^4+5148x^3+2145x^2+572}{156}$$

input
```
int(x*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**2*(12*x**11 + 143*x**10 + 780*x**9 + 2574*x**8 + 5720*x**7 + 9009*x**6
 + 10296*x**5 + 8580*x**4 + 5148*x**3 + 2145*x**2 + 572*x + 78))/156
```

3.228 $\int (1+x) \left(1+2x+x^2\right)^5 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 9

$$\int (1+x) \left(1+2x+x^2\right)^5 \, dx = \frac{1}{12}(1+x)^{12}$$

output
```
1/12*(1+x)^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (1+x) \left(1+2x+x^2\right)^5 \, dx = \frac{1}{12}(1+x)^{12}$$

input
```
Integrate[(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1098, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1) \left(x^2+2x+1\right)^5 \, dx$$

$$\Big\downarrow \ 1098$$

$$\int (x+1)^{11}dx$$

$$\Big\downarrow \ 17$$

$$\frac{1}{12}(x+1)^{12}$$

input
```
Int[(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
(1 + x)^12/12
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

method	result
default	$\frac{(x^2+2x+1)^6}{12}$
norman	$\frac{1}{12}x^{12}+x^{11}+\frac{11}{2}x^{10}+\frac{55}{3}x^9+\frac{165}{4}x^8+66x^7+77x^6+66x^5+\frac{165}{4}x^4+\frac{55}{3}x^3+\frac{11}{2}x^2+x$
parallelrisch	$\frac{1}{12}x^{12}+x^{11}+\frac{11}{2}x^{10}+\frac{55}{3}x^9+\frac{165}{4}x^8+66x^7+77x^6+66x^5+\frac{165}{4}x^4+\frac{55}{3}x^3+\frac{11}{2}x^2+x$
gosper	$\frac{x(x^{11}+12x^{10}+66x^9+220x^8+495x^7+792x^6+924x^5+792x^4+495x^3+220x^2+66x+12)}{12}$
risch	$\frac{1}{12}x^{12}+x^{11}+\frac{11}{2}x^{10}+\frac{55}{3}x^9+\frac{165}{4}x^8+66x^7+77x^6+66x^5+\frac{165}{4}x^4+\frac{55}{3}x^3+\frac{11}{2}x^2+x+\frac{1}{12}$
orering	$\frac{x(x^{11}+12x^{10}+66x^9+220x^8+495x^7+792x^6+924x^5+792x^4+495x^3+220x^2+66x+12)(x^2+2x+1)^5}{12(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(x^2+2*x+1)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 6.11

$$\int (1+x)\left(1+2x+x^2\right)^5 dx = \frac{1}{12}x^{12}+x^{11}+\frac{11}{2}x^{10}+\frac{55}{3}x^9+\frac{165}{4}x^8+66x^7+77x^6+66x^5+\frac{165}{4}x^4+\frac{55}{3}x^3+\frac{11}{2}x^2+x$$

input
```
integrate((1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
1/12*x^12 + x^11 + 11/2*x^10 + 55/3*x^9 + 165/4*x^8 + 66*x^7 + 77*x^6 + 66
*x^5 + 165/4*x^4 + 55/3*x^3 + 11/2*x^2 + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(5) = 10$.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 7.22

$$\int (1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{12}}{12} + x^{11} + \frac{11x^{10}}{2} + \frac{55x^9}{3} + \frac{165x^8}{4} + 66x^7 + 77x^6 + 66x^5 + \frac{165x^4}{4} + \frac{55x^3}{3} + \frac{11x^2}{2} + x$$

input
```
integrate((1+x)*(x**2+2*x+1)**5,x)
```

output
```
x**12/12 + x**11 + 11*x**10/2 + 55*x**9/3 + 165*x**8/4 + 66*x**7 + 77*x**6
 + 66*x**5 + 165*x**4/4 + 55*x**3/3 + 11*x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int (1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{12}\left(x^2 + 2\,x + 1\right)^6$$

input
```
integrate((1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
1/12*(x^2 + 2*x + 1)^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 6.89

$$\int (1+x)\left(1+2x+x^2\right)^5\,dx = \frac{1}{12}\left(x^2 + 2\,x\right)^6 + \frac{1}{2}\left(x^2 + 2\,x\right)^5 + \frac{5}{4}\left(x^2 + 2\,x\right)^4 + \frac{5}{3}\left(x^2 + 2\,x\right)^3 + \frac{5}{4}\left(x^2 + 2\,x\right)^2 + \frac{1}{2}\,x^2 + x$$

input
```
integrate((1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
1/12*(x^2 + 2*x)^6 + 1/2*(x^2 + 2*x)^5 + 5/4*(x^2 + 2*x)^4 + 5/3*(x^2 + 2*
x)^3 + 5/4*(x^2 + 2*x)^2 + 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 6.11

$$\int (1+x) \left(1+2x+x^2\right)^5 \, dx = \frac{x^{12}}{12} + x^{11} + \frac{11\,x^{10}}{2} + \frac{55\,x^9}{3} + \frac{165\,x^8}{4} + 66\,x^7 + 77\,x^6 + 66\,x^5 + \frac{165\,x^4}{4} + \frac{55\,x^3}{3} + \frac{11\,x^2}{2} + x$$

input
```
int((x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
x + (11*x^2)/2 + (55*x^3)/3 + (165*x^4)/4 + 66*x^5 + 77*x^6 + 66*x^7 + (16
5*x^8)/4 + (55*x^9)/3 + (11*x^10)/2 + x^11 + x^12/12
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 6.22

$$\int (1+x) \left(1+2x+x^2\right)^5 \, dx = \frac{x(x^{11} + 12x^{10} + 66x^9 + 220x^8 + 495x^7 + 792x^6 + 924x^5 + 792x^4 + 495x^3 + 220x^2 + 66x + 12)}{12}$$

input
```
int((1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x*(x**11 + 12*x**10 + 66*x**9 + 220*x**8 + 495*x**7 + 792*x**6 + 924*x**5
 + 792*x**4 + 495*x**3 + 220*x**2 + 66*x + 12))/12
```

3.229 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = 11x+\frac{55x^2}{2}+55x^3+\frac{165x^4}{2}+\frac{462x^5}{5}+77x^6+\frac{330x^7}{7}+\frac{165x^8}{8}+\frac{55x^9}{9}+\frac{11x^{10}}{10}+\frac{x^{11}}{11}+\log(x)$$

output
```
11*x+55/2*x^2+55*x^3+165/2*x^4+462/5*x^5+77*x^6+330/7*x^7+165/8*x^8+55/9*x
^9+11/10*x^10+1/11*x^11+ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = x+\frac{1}{2}(1+x)^2+\frac{1}{3}(1+x)^3+\frac{1}{4}(1+x)^4+\frac{1}{5}(1+x)^5+\frac{1}{6}(1+x)^6+\frac{1}{7}(1+x)^7+\frac{1}{8}(1+x)^8+\frac{1}{9}(1+x)^9+\frac{1}{10}(1+x)^{10}+\frac{1}{11}(1+x)^{11}+\log(-x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x,x]
```

output
```
x + (1 + x)^2/2 + (1 + x)^3/3 + (1 + x)^4/4 + (1 + x)^5/5 + (1 + x)^6/6 +
(1 + x)^7/7 + (1 + x)^8/8 + (1 + x)^9/9 + (1 + x)^10/10 + (1 + x)^11/11 +
Log[-x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x}\,dx$$
$$\Big\downarrow \text{1184}$$
$$\int \frac{(x+1)^{11}}{x}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(x^{10}+11x^9+55x^8+165x^7+330x^6+462x^5+462x^4+330x^3+165x^2+55x+\frac{1}{x}+11\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^{11}}{11}+\frac{11x^{10}}{10}+\frac{55x^9}{9}+\frac{165x^8}{8}+\frac{330x^7}{7}+77x^6+\frac{462x^5}{5}+\frac{165x^4}{2}+55x^3+\frac{55x^2}{2}+11x+\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x,x]
```

output
```
11*x + (55*x^2)/2 + 55*x^3 + (165*x^4)/2 + (462*x^5)/5 + 77*x^6 + (330*x^7
)/7 + (165*x^8)/8 + (55*x^9)/9 + (11*x^10)/10 + x^11/11 + Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	si
default	$11x+\frac{55x^{2}}{2}+55x^{3}+\frac{165x^{4}}{2}+\frac{462x^{5}}{5}+77x^{6}+\frac{330x^{7}}{7}+\frac{165x^{8}}{8}+\frac{55x^{9}}{9}+\frac{11x^{10}}{10}+\frac{x^{11}}{11}+\ln\left(x\right)$	57
norman	$11x+\frac{55x^{2}}{2}+55x^{3}+\frac{165x^{4}}{2}+\frac{462x^{5}}{5}+77x^{6}+\frac{330x^{7}}{7}+\frac{165x^{8}}{8}+\frac{55x^{9}}{9}+\frac{11x^{10}}{10}+\frac{x^{11}}{11}+\ln\left(x\right)$	57
risch	$11x+\frac{55x^{2}}{2}+55x^{3}+\frac{165x^{4}}{2}+\frac{462x^{5}}{5}+77x^{6}+\frac{330x^{7}}{7}+\frac{165x^{8}}{8}+\frac{55x^{9}}{9}+\frac{11x^{10}}{10}+\frac{x^{11}}{11}+\ln\left(x\right)$	57
parallelrisch	$11x+\frac{55x^{2}}{2}+55x^{3}+\frac{165x^{4}}{2}+\frac{462x^{5}}{5}+77x^{6}+\frac{330x^{7}}{7}+\frac{165x^{8}}{8}+\frac{55x^{9}}{9}+\frac{11x^{10}}{10}+\frac{x^{11}}{11}+\ln\left(x\right)$	57

input
```
int((x+1)*(x^2+2*x+1)^5/x,x,method=_RETURNVERBOSE)
```

output
```
11*x+55/2*x^2+55*x^3+165/2*x^4+462/5*x^5+77*x^6+330/7*x^7+165/8*x^8+55/9*x
^9+11/10*x^10+1/11*x^11+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x}\,dx = \frac{1}{11}\,x^{11} + \frac{11}{10}\,x^{10} + \frac{55}{9}\,x^9 + \frac{165}{8}\,x^8 + \frac{330}{7}\,x^7 + 77\,x^6 + \frac{462}{5}\,x^5 + \frac{165}{2}\,x^4 + 55\,x^3 + \frac{55}{2}\,x^2 + 11\,x + \log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x,x, algorithm="fricas")
```

output
```
1/11*x^11 + 11/10*x^10 + 55/9*x^9 + 165/8*x^8 + 330/7*x^7 + 77*x^6 + 462/5
*x^5 + 165/2*x^4 + 55*x^3 + 55/2*x^2 + 11*x + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x}\,dx = \frac{x^{11}}{11} + \frac{11x^{10}}{10} + \frac{55x^9}{9} + \frac{165x^8}{8} + \frac{330x^7}{7} + 77x^6 + \frac{462x^5}{5} + \frac{165x^4}{2} + 55x^3 + \frac{55x^2}{2} + 11x + \log\left(x\right)$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x,x)
```

output
```
x**11/11 + 11*x**10/10 + 55*x**9/9 + 165*x**8/8 + 330*x**7/7 + 77*x**6 + 4
62*x**5/5 + 165*x**4/2 + 55*x**3 + 55*x**2/2 + 11*x + log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = \frac{1}{11}\,x^{11} + \frac{11}{10}\,x^{10} + \frac{55}{9}\,x^9 + \frac{165}{8}\,x^8 + \frac{330}{7}\,x^7 + 77\,x^6 + \frac{462}{5}\,x^5 + \frac{165}{2}\,x^4 + 55\,x^3 + \frac{55}{2}\,x^2 + 11\,x + \log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x,x, algorithm="maxima")
```

output
```
1/11*x^11 + 11/10*x^10 + 55/9*x^9 + 165/8*x^8 + 330/7*x^7 + 77*x^6 + 462/5
*x^5 + 165/2*x^4 + 55*x^3 + 55/2*x^2 + 11*x + log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = \frac{1}{11}\,x^{11} + \frac{11}{10}\,x^{10} + \frac{55}{9}\,x^9 + \frac{165}{8}\,x^8 + \frac{330}{7}\,x^7 + 77\,x^6 + \frac{462}{5}\,x^5 + \frac{165}{2}\,x^4 + 55\,x^3 + \frac{55}{2}\,x^2 + 11\,x + \log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x,x, algorithm="giac")
```

output
```
1/11*x^11 + 11/10*x^10 + 55/9*x^9 + 165/8*x^8 + 330/7*x^7 + 77*x^6 + 462/5
*x^5 + 165/2*x^4 + 55*x^3 + 55/2*x^2 + 11*x + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = 11\,x+\ln\left(x\right)+\frac{55\,x^2}{2}+55\,x^3+\frac{165\,x^4}{2}+\frac{462\,x^5}{5}+77\,x^6+\frac{330\,x^7}{7}+\frac{165\,x^8}{8}+\frac{55\,x^9}{9}+\frac{11\,x^{10}}{10}+\frac{x^{11}}{11}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x,x)
```

output
```
11*x + log(x) + (55*x^2)/2 + 55*x^3 + (165*x^4)/2 + (462*x^5)/5 + 77*x^6 +
 (330*x^7)/7 + (165*x^8)/8 + (55*x^9)/9 + (11*x^10)/10 + x^11/11
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x}\,dx = \log(x)+\frac{x^{11}}{11}+\frac{11x^{10}}{10}+\frac{55x^9}{9}+\frac{165x^8}{8}+\frac{330x^7}{7}+77x^6+\frac{462x^5}{5}+\frac{165x^4}{2}+55x^3+\frac{55x^2}{2}+11x$$

input
```
int((1+x)*(x^2+2*x+1)^5/x,x)
```

output
```
(27720*log(x) + 2520*x**11 + 30492*x**10 + 169400*x**9 + 571725*x**8 + 130
6800*x**7 + 2134440*x**6 + 2561328*x**5 + 2286900*x**4 + 1524600*x**3 + 76
2300*x**2 + 304920*x)/27720
```

3.230 $\int \frac{(1+x)(1+2x+x^2)^5}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^2} \, dx = -\frac{1}{x} + 55x + \frac{165x^2}{2} + 110x^3 + \frac{231x^4}{2} + \frac{462x^5}{5} + 55x^6 + \frac{165x^7}{7} + \frac{55x^8}{8} + \frac{11x^9}{9} + \frac{x^{10}}{10} + 11\log(x)$$

output
```
-1/x+55*x+165/2*x^2+110*x^3+231/2*x^4+462/5*x^5+55*x^6+165/7*x^7+55/8*x^8+
11/9*x^9+1/10*x^10+11*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^2} \, dx = -\frac{1}{x} + 55x + \frac{165x^2}{2} + 110x^3 + \frac{231x^4}{2} + \frac{462x^5}{5} + 55x^6 + \frac{165x^7}{7} + \frac{55x^8}{8} + \frac{11x^9}{9} + \frac{x^{10}}{10} + 11\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^2,x]
```

output
```
-x^(-1) + 55*x + (165*x^2)/2 + 110*x^3 + (231*x^4)/2 + (462*x^5)/5 + 55*x^
6 + (165*x^7)/7 + (55*x^8)/8 + (11*x^9)/9 + x^10/10 + 11*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^2}\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{11}}{x^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(x^9+11x^8+55x^7+165x^6+330x^5+462x^4+462x^3+330x^2+\frac{1}{x^2}+165x+\frac{11}{x}+55\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{10}}{10}+\frac{11x^9}{9}+\frac{55x^8}{8}+\frac{165x^7}{7}+55x^6+\frac{462x^5}{5}+\frac{231x^4}{2}+110x^3+\frac{165x^2}{2}+55x-\frac{1}{x}+11\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^2,x]
```

output

```
-x^(-1) + 55*x + (165*x^2)/2 + 110*x^3 + (231*x^4)/2 + (462*x^5)/5 + 55*x^
6 + (165*x^7)/7 + (55*x^8)/8 + (11*x^9)/9 + x^10/10 + 11*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{x}+55x+\frac{165x^2}{2}+110x^3+\frac{231x^4}{2}+\frac{462x^5}{5}+55x^6+\frac{165x^7}{7}+\frac{55x^8}{8}+\frac{11x^9}{9}+\frac{x^{10}}{10}+11\ln\left(x\right)$
risch	$-\frac{1}{x}+55x+\frac{165x^2}{2}+110x^3+\frac{231x^4}{2}+\frac{462x^5}{5}+55x^6+\frac{165x^7}{7}+\frac{55x^8}{8}+\frac{11x^9}{9}+\frac{x^{10}}{10}+11\ln\left(x\right)$
norman	$\frac{-1+55x^2+\frac{165}{2}x^3+110x^4+\frac{231}{2}x^5+\frac{462}{5}x^6+55x^7+\frac{165}{7}x^8+\frac{55}{8}x^9+\frac{11}{9}x^{10}+\frac{1}{10}x^{11}}{x}+11\ln\left(x\right)$
parallelrisch	$\frac{252x^{11}+3080x^{10}+17325x^9+59400x^8+138600x^7+232848x^6+291060x^5+277200x^4+207900x^3+27720\ln(x)x+138600x^2-2}{2520x}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+55*x+165/2*x^2+110*x^3+231/2*x^4+462/5*x^5+55*x^6+165/7*x^7+55/8*x^8+
11/9*x^9+1/10*x^10+11*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{252\,x^{11} + 3080\,x^{10} + 17325\,x^9 + 59400\,x^8 + 138600\,x^7 + 232848\,x^6 + 291060\,x^5 + 277200\,x^4 + 207900\,x}{2520\,x}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^2,x, algorithm="fricas")
```

output
```
1/2520*(252*x^11 + 3080*x^10 + 17325*x^9 + 59400*x^8 + 138600*x^7 + 232848
*x^6 + 291060*x^5 + 277200*x^4 + 207900*x^3 + 138600*x^2 + 27720*x*log(x)
- 2520)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{x^{10}}{10} + \frac{11x^9}{9} + \frac{55x^8}{8} + \frac{165x^7}{7} + 55x^6 + \frac{462x^5}{5} + \frac{231x^4}{2} + 110x^3 + \frac{165x^2}{2} + 55x + 11\log(x) - \frac{1}{x}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**2,x)
```

output
```
x**10/10 + 11*x**9/9 + 55*x**8/8 + 165*x**7/7 + 55*x**6 + 462*x**5/5 + 231
*x**4/2 + 110*x**3 + 165*x**2/2 + 55*x + 11*log(x) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{1}{10}\,x^{10} + \frac{11}{9}\,x^9 + \frac{55}{8}\,x^8 + \frac{165}{7}\,x^7 + 55\,x^6 + \frac{462}{5}\,x^5 + \frac{231}{2}\,x^4 + 110\,x^3 + \frac{165}{2}\,x^2 + 55\,x - \frac{1}{x} + 11\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^2,x, algorithm="maxima")
```

output
```
1/10*x^10 + 11/9*x^9 + 55/8*x^8 + 165/7*x^7 + 55*x^6 + 462/5*x^5 + 231/2*x
^4 + 110*x^3 + 165/2*x^2 + 55*x - 1/x + 11*log(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{1}{10}\,x^{10} + \frac{11}{9}\,x^9 + \frac{55}{8}\,x^8 + \frac{165}{7}\,x^7 + 55\,x^6 + \frac{462}{5}\,x^5 + \frac{231}{2}\,x^4 + 110\,x^3 + \frac{165}{2}\,x^2 + 55\,x - \frac{1}{x} + 11\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^2,x, algorithm="giac")
```

output
```
1/10*x^10 + 11/9*x^9 + 55/8*x^8 + 165/7*x^7 + 55*x^6 + 462/5*x^5 + 231/2*x
^4 + 110*x^3 + 165/2*x^2 + 55*x - 1/x + 11*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = 55\,x+11\,\ln\left(x\right)-\frac{1}{x}+\frac{165\,x^2}{2}+110\,x^3+\frac{231\,x^4}{2}+\frac{462\,x^5}{5}+55\,x^6+\frac{165\,x^7}{7}+\frac{55\,x^8}{8}+\frac{11\,x^9}{9}+\frac{x^{10}}{10}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^2,x)
```

output
```
55*x + 11*log(x) - 1/x + (165*x^2)/2 + 110*x^3 + (231*x^4)/2 + (462*x^5)/5
 + 55*x^6 + (165*x^7)/7 + (55*x^8)/8 + (11*x^9)/9 + x^10/10
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^2}\,dx = \frac{27720\log(x)\,x+252x^{11}+3080x^{10}+17325x^9+59400x^8+138600x^7+232848x^6+291060x^5+277200}{2520x}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^2,x)
```

output
```
(27720*log(x)*x + 252*x**11 + 3080*x**10 + 17325*x**9 + 59400*x**8 + 13860
0*x**7 + 232848*x**6 + 291060*x**5 + 277200*x**4 + 207900*x**3 + 138600*x*
*2 - 2520)/(2520*x)
```

3.231 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3} \, dx = -\frac{1}{2x^2} - \frac{11}{x} + 165x + 165x^2 + 154x^3 + \frac{231x^4}{2} + 66x^5 + \frac{55x^6}{2} + \frac{55x^7}{7} + \frac{11x^8}{8} + \frac{x^9}{9} + 55\log(x)$$

output
```
-1/2/x^2-11/x+165*x+165*x^2+154*x^3+231/2*x^4+66*x^5+55/2*x^6+55/7*x^7+11/
8*x^8+1/9*x^9+55*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3} \, dx = -\frac{1}{2x^2} - \frac{11}{x} + 165x + 165x^2 + 154x^3 + \frac{231x^4}{2} + 66x^5 + \frac{55x^6}{2} + \frac{55x^7}{7} + \frac{11x^8}{8} + \frac{x^9}{9} + 55\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^3,x]
```

output
```
-1/2*1/x^2 - 11/x + 165*x + 165*x^2 + 154*x^3 + (231*x^4)/2 + 66*x^5 + (55
*x^6)/2 + (55*x^7)/7 + (11*x^8)/8 + x^9/9 + 55*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^3}dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^8+11x^7+55x^6+165x^5+330x^4+462x^3+\frac{1}{x^3}+462x^2+\frac{11}{x^2}+330x+\frac{55}{x}+165\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^9}{9}+\frac{11x^8}{8}+\frac{55x^7}{7}+\frac{55x^6}{2}+66x^5+\frac{231x^4}{2}+154x^3+165x^2-\frac{1}{2x^2}+165x-\frac{11}{x}+55\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^3,x]
```

output

```
-1/2*1/x^2 - 11/x + 165*x + 165*x^2 + 154*x^3 + (231*x^4)/2 + 66*x^5 + (55
*x^6)/2 + (55*x^7)/7 + (11*x^8)/8 + x^9/9 + 55*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^9}{9}+\frac{11x^8}{8}+\frac{55x^7}{7}+\frac{55x^6}{2}+66x^5+\frac{231x^4}{2}+154x^3+165x^2+165x+\frac{-11x-\frac{1}{2}}{x^2}+55\ln\left(x\right)$
default	$-\frac{1}{2x^2}-\frac{11}{x}+165x+165x^2+154x^3+\frac{231x^4}{2}+66x^5+\frac{55x^6}{2}+\frac{55x^7}{7}+\frac{11x^8}{8}+\frac{x^9}{9}+55\ln\left(x\right)$
norman	$\frac{-\frac{1}{2}-11x+165x^3+165x^4+154x^5+\frac{231}{2}x^6+66x^7+\frac{55}{2}x^8+\frac{55}{7}x^9+\frac{11}{8}x^{10}+\frac{1}{9}x^{11}}{x^2}+55\ln\left(x\right)$
parallelrisch	$\frac{56x^{11}+693x^{10}+3960x^9+13860x^8+33264x^7+58212x^6+77616x^5+83160x^4+27720\ln\left(x\right)x^2+83160x^3-252-5544x}{504x^2}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9+11/8*x^8+55/7*x^7+55/2*x^6+66*x^5+231/2*x^4+154*x^3+165*x^2+165*x+
(-11*x-1/2)/x^2+55*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^3}\,dx = \frac{56\,x^{11} + 693\,x^{10} + 3960\,x^9 + 13860\,x^8 + 33264\,x^7 + 58212\,x^6 + 77616\,x^5 + 83160\,x^4 + 83160\,x^3 + 27720}{504\,x^2}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^3,x, algorithm="fricas")
```

output
```
1/504*(56*x^11 + 693*x^10 + 3960*x^9 + 13860*x^8 + 33264*x^7 + 58212*x^6 +
 77616*x^5 + 83160*x^4 + 83160*x^3 + 27720*x^2*log(x) - 5544*x - 252)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^3}\,dx = \frac{x^9}{9} + \frac{11x^8}{8} + \frac{55x^7}{7} + \frac{55x^6}{2} + 66x^5 + \frac{231x^4}{2} + 154x^3 + 165x^2 + 165x + 55\log(x) + \frac{-22x-1}{2x^2}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**3,x)
```

output
```
x**9/9 + 11*x**8/8 + 55*x**7/7 + 55*x**6/2 + 66*x**5 + 231*x**4/2 + 154*x*
*3 + 165*x**2 + 165*x + 55*log(x) + (-22*x - 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{1}{9}\,x^9+\frac{11}{8}\,x^8+\frac{55}{7}\,x^7+\frac{55}{2}\,x^6+66\,x^5+\frac{231}{2}\,x^4+154\,x^3+165\,x^2+165\,x-\frac{22\,x+1}{2\,x^2}+55\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^3,x, algorithm="maxima")
```

output
```
1/9*x^9 + 11/8*x^8 + 55/7*x^7 + 55/2*x^6 + 66*x^5 + 231/2*x^4 + 154*x^3 +
165*x^2 + 165*x - 1/2*(22*x + 1)/x^2 + 55*log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{1}{9}\,x^9+\frac{11}{8}\,x^8+\frac{55}{7}\,x^7+\frac{55}{2}\,x^6+66\,x^5+\frac{231}{2}\,x^4+154\,x^3+165\,x^2+165\,x-\frac{22\,x+1}{2\,x^2}+55\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^3,x, algorithm="giac")
```

output
```
1/9*x^9 + 11/8*x^8 + 55/7*x^7 + 55/2*x^6 + 66*x^5 + 231/2*x^4 + 154*x^3 +
165*x^2 + 165*x - 1/2*(22*x + 1)/x^2 + 55*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3}\,dx = 165\,x+55\,\ln\left(x\right)-\frac{11\,x+\frac{1}{2}}{x^2}+165\,x^2+154\,x^3+\frac{231\,x^4}{2}+66\,x^5+\frac{55\,x^6}{2}+\frac{55\,x^7}{7}+\frac{11\,x^8}{8}+\frac{x^9}{9}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^3,x)
```

output
```
165*x + 55*log(x) - (11*x + 1/2)/x^2 + 165*x^2 + 154*x^3 + (231*x^4)/2 + 6
6*x^5 + (55*x^6)/2 + (55*x^7)/7 + (11*x^8)/8 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^3}\,dx = \frac{27720\log(x)\,x^2+56x^{11}+693x^{10}+3960x^9+13860x^8+33264x^7+58212x^6+77616x^5+83160x^4+83}{504x^2}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^3,x)
```

output
```
(27720*log(x)*x**2 + 56*x**11 + 693*x**10 + 3960*x**9 + 13860*x**8 + 33264
*x**7 + 58212*x**6 + 77616*x**5 + 83160*x**4 + 83160*x**3 - 5544*x - 252)/
(504*x**2)
```

3.232 $\int \frac{(1+x)(1+2x+x^2)^5}{x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^4} dx = -\frac{1}{3x^3} - \frac{11}{2x^2} - \frac{55}{x} + 330x + 231x^2 + 154x^3 + \frac{165x^4}{2} + 33x^5 + \frac{55x^6}{6} + \frac{11x^7}{7} + \frac{x^8}{8} + 165\log(x)$$

output
```
-1/3/x^3-11/2/x^2-55/x+330*x+231*x^2+154*x^3+165/2*x^4+33*x^5+55/6*x^6+11/
7*x^7+1/8*x^8+165*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^4} dx = -\frac{1}{3x^3} - \frac{11}{2x^2} - \frac{55}{x} + 330x + 231x^2 + 154x^3 + \frac{165x^4}{2} + 33x^5 + \frac{55x^6}{6} + \frac{11x^7}{7} + \frac{x^8}{8} + 165\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^4,x]
```

output
```
-1/3*1/x^3 - 11/(2*x^2) - 55/x + 330*x + 231*x^2 + 154*x^3 + (165*x^4)/2 +
 33*x^5 + (55*x^6)/6 + (11*x^7)/7 + x^8/8 + 165*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^4}dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^4}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^7+11x^6+55x^5+165x^4+\frac{1}{x^4}+330x^3+\frac{11}{x^3}+462x^2+\frac{55}{x^2}+462x+\frac{165}{x}+330\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8}{8}+\frac{11x^7}{7}+\frac{55x^6}{6}+33x^5+\frac{165x^4}{2}+154x^3-\frac{1}{3x^3}+231x^2-\frac{11}{2x^2}+330x-\frac{55}{x}+165\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^4,x]
```

output
```
-1/3*1/x^3 - 11/(2*x^2) - 55/x + 330*x + 231*x^2 + 154*x^3 + (165*x^4)/2 +
 33*x^5 + (55*x^6)/6 + (11*x^7)/7 + x^8/8 + 165*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^8}{8}+\frac{11x^7}{7}+\frac{55x^6}{6}+33x^5+\frac{165x^4}{2}+154x^3+231x^2+330x+\frac{-55x^2-\frac{11}{2}x-\frac{1}{3}}{x^3}+165\ln(x)$
default	$-\frac{1}{3x^3}-\frac{11}{2x^2}-\frac{55}{x}+330x+231x^2+154x^3+\frac{165x^4}{2}+33x^5+\frac{55x^6}{6}+\frac{11x^7}{7}+\frac{x^8}{8}+165\ln(x)$
norman	$\frac{-\frac{1}{3}-\frac{11}{2}x-55x^2+330x^4+231x^5+154x^6+\frac{165}{2}x^7+33x^8+\frac{55}{6}x^9+\frac{11}{7}x^{10}+\frac{1}{8}x^{11}}{x^3}+165\ln(x)$
parallelrisch	$\frac{21x^{11}+264x^{10}+1540x^9+5544x^8+13860x^7+25872x^6+38808x^5+27720\ln(x)x^3+55440x^4-56-9240x^2-924x}{168x^3}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8+11/7*x^7+55/6*x^6+33*x^5+165/2*x^4+154*x^3+231*x^2+330*x+(-55*x^2-
11/2*x-1/3)/x^3+165*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{21\,x^{11}+264\,x^{10}+1540\,x^9+5544\,x^8+13860\,x^7+25872\,x^6+38808\,x^5+55440\,x^4+27720\,x^3\log(x)-}{168\,x^3}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^4,x, algorithm="fricas")
```

output
```
1/168*(21*x^11 + 264*x^10 + 1540*x^9 + 5544*x^8 + 13860*x^7 + 25872*x^6 +
38808*x^5 + 55440*x^4 + 27720*x^3*log(x) - 9240*x^2 - 924*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{x^8}{8}+\frac{11x^7}{7}+\frac{55x^6}{6}+33x^5+\frac{165x^4}{2}+154x^3 + 231x^2+330x+165\log(x)+\frac{-330x^2-33x-2}{6x^3}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**4,x)
```

output
```
x**8/8 + 11*x**7/7 + 55*x**6/6 + 33*x**5 + 165*x**4/2 + 154*x**3 + 231*x**
2 + 330*x + 165*log(x) + (-330*x**2 - 33*x - 2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{1}{8}\,x^8+\frac{11}{7}\,x^7+\frac{55}{6}\,x^6+33\,x^5+\frac{165}{2}\,x^4+154\,x^3+231\,x^2+330\,x-\frac{330\,x^2+33\,x+2}{6\,x^3}+165\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^4,x, algorithm="maxima")
```

output
```
1/8*x^8 + 11/7*x^7 + 55/6*x^6 + 33*x^5 + 165/2*x^4 + 154*x^3 + 231*x^2 + 3
30*x - 1/6*(330*x^2 + 33*x + 2)/x^3 + 165*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{1}{8}\,x^8+\frac{11}{7}\,x^7+\frac{55}{6}\,x^6+33\,x^5+\frac{165}{2}\,x^4+154\,x^3+231\,x^2+330\,x-\frac{330\,x^2+33\,x+2}{6\,x^3}+165\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^4,x, algorithm="giac")
```

output
```
1/8*x^8 + 11/7*x^7 + 55/6*x^6 + 33*x^5 + 165/2*x^4 + 154*x^3 + 231*x^2 + 3
30*x - 1/6*(330*x^2 + 33*x + 2)/x^3 + 165*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = 330\,x+165\,\ln(x)-\frac{55\,x^2+\frac{11\,x}{2}+\frac{1}{3}}{x^3}+231\,x^2+154\,x^3+\frac{165\,x^4}{2}+33\,x^5+\frac{55\,x^6}{6}+\frac{11\,x^7}{7}+\frac{x^8}{8}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^4,x)
```

output
```
330*x + 165*log(x) - ((11*x)/2 + 55*x^2 + 1/3)/x^3 + 231*x^2 + 154*x^3 + (
165*x^4)/2 + 33*x^5 + (55*x^6)/6 + (11*x^7)/7 + x^8/8
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^4}\,dx = \frac{27720\log(x)\,x^3+21x^{11}+264x^{10}+1540x^9+5544x^8+13860x^7+25872x^6+38808x^5+55440x^4-924}{168x^3}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^4,x)
```

output
```
(27720*log(x)*x**3 + 21*x**11 + 264*x**10 + 1540*x**9 + 5544*x**8 + 13860*
x**7 + 25872*x**6 + 38808*x**5 + 55440*x**4 - 9240*x**2 - 924*x - 56)/(168
*x**3)
```

3.233 $\int \frac{(1+x)(1+2x+x^2)^5}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^5} dx = -\frac{1}{4x^4} - \frac{11}{3x^3} - \frac{55}{2x^2} - \frac{165}{x} + 462x + 231x^2 + 110x^3 + \frac{165x^4}{4} + 11x^5 + \frac{11x^6}{6} + \frac{x^7}{7} + 330\log(x)$$

output
```
-1/4/x^4-11/3/x^3-55/2/x^2-165/x+462*x+231*x^2+110*x^3+165/4*x^4+11*x^5+11
/6*x^6+1/7*x^7+330*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^5} dx = -\frac{1}{4x^4} - \frac{11}{3x^3} - \frac{55}{2x^2} - \frac{165}{x} + 462x + 231x^2 + 110x^3 + \frac{165x^4}{4} + 11x^5 + \frac{11x^6}{6} + \frac{x^7}{7} + 330\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^5,x]
```

output
```
-1/4*1/x^4 - 11/(3*x^3) - 55/(2*x^2) - 165/x + 462*x + 231*x^2 + 110*x^3 +
 (165*x^4)/4 + 11*x^5 + (11*x^6)/6 + x^7/7 + 330*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^5}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^5}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^6+11x^5+\frac{1}{x^5}+55x^4+\frac{11}{x^4}+165x^3+\frac{55}{x^3}+330x^2+\frac{165}{x^2}+462x+\frac{330}{x}+462\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^7}{7}+\frac{11x^6}{6}+11x^5+\frac{165x^4}{4}-\frac{1}{4x^4}+110x^3-\frac{11}{3x^3}+231x^2-\frac{55}{2x^2}+462x-\frac{165}{x}+330\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^5,x]
```

output

```
-1/4*1/x^4 - 11/(3*x^3) - 55/(2*x^2) - 165/x + 462*x + 231*x^2 + 110*x^3 +
 (165*x^4)/4 + 11*x^5 + (11*x^6)/6 + x^7/7 + 330*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^7}{7}+\frac{11x^6}{6}+11x^5+\frac{165x^4}{4}+110x^3+231x^2+462x+\frac{-165x^3-\frac{55}{2}x^2-\frac{11}{3}x-\frac{1}{4}}{x^4}+330\ln\left(x\right)$
default	$-\frac{1}{4x^4}-\frac{11}{3x^3}-\frac{55}{2x^2}-\frac{165}{x}+462x+231x^2+110x^3+\frac{165x^4}{4}+11x^5+\frac{11x^6}{6}+\frac{x^7}{7}+330\ln\left(x\right)$
norman	$\frac{-\frac{1}{4}-\frac{11}{3}x-\frac{55}{2}x^2-165x^3+462x^5+231x^6+110x^7+\frac{165}{4}x^8+11x^9+\frac{11}{6}x^{10}+\frac{1}{7}x^{11}}{x^4}+330\ln\left(x\right)$
parallelrisch	$\frac{12x^{11}+154x^{10}+924x^9+3465x^8+9240x^7+19404x^6+27720\ln\left(x\right)x^4+38808x^5-21-13860x^3-2310x^2-308x}{84x^4}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7+11/6*x^6+11*x^5+165/4*x^4+110*x^3+231*x^2+462*x+(-165*x^3-55/2*x^2
-11/3*x-1/4)/x^4+330*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = \frac{12\,x^{11}+154\,x^{10}+924\,x^{9}+3465\,x^{8}+9240\,x^{7}+19404\,x^{6}+38808\,x^{5}+27720\,x^{4}\log\left(x\right)-13860\,x^{3}-23}{84\,x^{4}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^5,x, algorithm="fricas")
```

output
```
1/84*(12*x^11 + 154*x^10 + 924*x^9 + 3465*x^8 + 9240*x^7 + 19404*x^6 + 388
08*x^5 + 27720*x^4*log(x) - 13860*x^3 - 2310*x^2 - 308*x - 21)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = \frac{x^7}{7}+\frac{11x^6}{6}+11x^5+\frac{165x^4}{4}+110x^3+231x^2 + 462x+330\log\left(x\right)+\frac{-1980x^3-330x^2-44x-3}{12x^4}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**5,x)
```

output
```
x**7/7 + 11*x**6/6 + 11*x**5 + 165*x**4/4 + 110*x**3 + 231*x**2 + 462*x +
330*log(x) + (-1980*x**3 - 330*x**2 - 44*x - 3)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = \frac{1}{7}\,x^7 + \frac{11}{6}\,x^6 + 11\,x^5 + \frac{165}{4}\,x^4 + 110\,x^3 + 231\,x^2 + 462\,x - \frac{1980\,x^3 + 330\,x^2 + 44\,x + 3}{12\,x^4} + 330\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^5,x, algorithm="maxima")
```

output
```
1/7*x^7 + 11/6*x^6 + 11*x^5 + 165/4*x^4 + 110*x^3 + 231*x^2 + 462*x - 1/12
*(1980*x^3 + 330*x^2 + 44*x + 3)/x^4 + 330*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = \frac{1}{7}\,x^7 + \frac{11}{6}\,x^6 + 11\,x^5 + \frac{165}{4}\,x^4 + 110\,x^3 + 231\,x^2 + 462\,x - \frac{1980\,x^3 + 330\,x^2 + 44\,x + 3}{12\,x^4} + 330\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^5,x, algorithm="giac")
```

output
```
1/7*x^7 + 11/6*x^6 + 11*x^5 + 165/4*x^4 + 110*x^3 + 231*x^2 + 462*x - 1/12
*(1980*x^3 + 330*x^2 + 44*x + 3)/x^4 + 330*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = 462\,x+330\,\ln\left(x\right)+231\,x^2+110\,x^3+\frac{165\,x^4}{4}+11\,x^5+\frac{11\,x^6}{6}+\frac{x^7}{7}-\frac{165\,x^3+\frac{55\,x^2}{2}+\frac{11\,x}{3}+\frac{1}{4}}{x^4}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^5,x)
```

output
```
462*x + 330*log(x) + 231*x^2 + 110*x^3 + (165*x^4)/4 + 11*x^5 + (11*x^6)/6
 + x^7/7 - ((11*x)/3 + (55*x^2)/2 + 165*x^3 + 1/4)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^5}\,dx = \frac{27720\log(x)\,x^4+12x^{11}+154x^{10}+924x^9+3465x^8+9240x^7+19404x^6+38808x^5-13860x^3-2310x}{84x^4}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^5,x)
```

output
```
(27720*log(x)*x**4 + 12*x**11 + 154*x**10 + 924*x**9 + 3465*x**8 + 9240*x*
*7 + 19404*x**6 + 38808*x**5 - 13860*x**3 - 2310*x**2 - 308*x - 21)/(84*x*
*4)
```

3.234 $\int \frac{(1+x)(1+2x+x^2)^5}{x^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^6} dx = -\frac{1}{5x^5} - \frac{11}{4x^4} - \frac{55}{3x^3} - \frac{165}{2x^2} - \frac{330}{x} + 462x + 165x^2 + 55x^3 + \frac{55x^4}{4} + \frac{11x^5}{5} + \frac{x^6}{6} + 462\log(x)$$

output
```
-1/5/x^5-11/4/x^4-55/3/x^3-165/2/x^2-330/x+462*x+165*x^2+55*x^3+55/4*x^4+1
1/5*x^5+1/6*x^6+462*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^6} dx = -\frac{1}{5x^5} - \frac{11}{4x^4} - \frac{55}{3x^3} - \frac{165}{2x^2} - \frac{330}{x} + 462x + 165x^2 + 55x^3 + \frac{55x^4}{4} + \frac{11x^5}{5} + \frac{x^6}{6} + 462\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^6,x]
```

output
```
-1/5*1/x^5 - 11/(4*x^4) - 55/(3*x^3) - 165/(2*x^2) - 330/x + 462*x + 165*x
^2 + 55*x^3 + (55*x^4)/4 + (11*x^5)/5 + x^6/6 + 462*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^6}\,dx$$
$$\Big\downarrow \text{ 1184}$$
$$\int \frac{(x+1)^{11}}{x^6}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{x^6}+x^5+\frac{11}{x^5}+11x^4+\frac{55}{x^4}+55x^3+\frac{165}{x^3}+165x^2+\frac{330}{x^2}+330x+\frac{462}{x}+462\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x^6}{6}+\frac{11x^5}{5}-\frac{1}{5x^5}+\frac{55x^4}{4}-\frac{11}{4x^4}+55x^3-\frac{55}{3x^3}+165x^2-\frac{165}{2x^2}+462x-\frac{330}{x}+462\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^6,x]
```

output
```
-1/5*1/x^5 - 11/(4*x^4) - 55/(3*x^3) - 165/(2*x^2) - 330/x + 462*x + 165*x
^2 + 55*x^3 + (55*x^4)/4 + (11*x^5)/5 + x^6/6 + 462*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	siz
risch	$\frac{x^6}{6}+\frac{11x^5}{5}+\frac{55x^4}{4}+55x^3+165x^2+462x+\frac{-330x^4-\frac{165}{2}x^3-\frac{55}{3}x^2-\frac{11}{4}x-\frac{1}{5}}{x^5}+462\ln\left(x\right)$	58
default	$-\frac{1}{5x^5}-\frac{11}{4x^4}-\frac{55}{3x^3}-\frac{165}{2x^2}-\frac{330}{x}+462x+165x^2+55x^3+\frac{55x^4}{4}+\frac{11x^5}{5}+\frac{x^6}{6}+462\ln\left(x\right)$	59
norman	$\frac{-\frac{1}{5}-\frac{11}{4}x-\frac{55}{3}x^2-\frac{165}{2}x^3-330x^4+462x^6+165x^7+55x^8+\frac{55}{4}x^9+\frac{11}{5}x^{10}+\frac{1}{6}x^{11}}{x^5}+462\ln\left(x\right)$	60
parallelrisch	$\frac{10x^{11}+132x^{10}+825x^9+3300x^8+9900x^7+27720\ln(x)x^5+27720x^6-12-19800x^4-4950x^3-1100x^2-165x}{60x^5}$	63

input
```
int((x+1)*(x^2+2*x+1)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6+11/5*x^5+55/4*x^4+55*x^3+165*x^2+462*x+(-330*x^4-165/2*x^3-55/3*x^
2-11/4*x-1/5)/x^5+462*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^6}\,dx = \frac{10\,x^{11}+132\,x^{10}+825\,x^{9}+3300\,x^{8}+9900\,x^{7}+27720\,x^{6}+27720\,x^{5}\log\left(x\right)-19800\,x^{4}-4950\,x^{3}-110}{60\,x^{5}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^6,x, algorithm="fricas")
```

output
```
1/60*(10*x^11 + 132*x^10 + 825*x^9 + 3300*x^8 + 9900*x^7 + 27720*x^6 + 277
20*x^5*log(x) - 19800*x^4 - 4950*x^3 - 1100*x^2 - 165*x - 12)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^6}\,dx = \frac{x^6}{6}+\frac{11x^5}{5}+\frac{55x^4}{4}+55x^3+165x^2+462x+462\log\left(x\right) + \frac{-19800x^4-4950x^3-1100x^2-165x-12}{60x^5}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**6,x)
```

output
```
x**6/6 + 11*x**5/5 + 55*x**4/4 + 55*x**3 + 165*x**2 + 462*x + 462*log(x) +
 (-19800*x**4 - 4950*x**3 - 1100*x**2 - 165*x - 12)/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^6}\,dx = \frac{1}{6}\,x^6 + \frac{11}{5}\,x^5 + \frac{55}{4}\,x^4 + 55\,x^3 + 165\,x^2 + 462\,x - \frac{19800\,x^4 + 4950\,x^3 + 1100\,x^2 + 165\,x + 12}{60\,x^5} + 462\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^6,x, algorithm="maxima")
```

output
```
1/6*x^6 + 11/5*x^5 + 55/4*x^4 + 55*x^3 + 165*x^2 + 462*x - 1/60*(19800*x^4
 + 4950*x^3 + 1100*x^2 + 165*x + 12)/x^5 + 462*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^6}\,dx = \frac{1}{6}\,x^6 + \frac{11}{5}\,x^5 + \frac{55}{4}\,x^4 + 55\,x^3 + 165\,x^2 + 462\,x - \frac{19800\,x^4 + 4950\,x^3 + 1100\,x^2 + 165\,x + 12}{60\,x^5} + 462\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^6,x, algorithm="giac")
```

output
```
1/6*x^6 + 11/5*x^5 + 55/4*x^4 + 55*x^3 + 165*x^2 + 462*x - 1/60*(19800*x^4
 + 4950*x^3 + 1100*x^2 + 165*x + 12)/x^5 + 462*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^6}\,dx = 462\,x+462\,\ln\left(x\right)-\frac{330\,x^4+\frac{165\,x^3}{2}+\frac{55\,x^2}{3}+\frac{11\,x}{4}+\frac{1}{5}}{x^5}+165\,x^2+55\,x^3+\frac{55\,x^4}{4}+\frac{11\,x^5}{5}+\frac{x^6}{6}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^6,x)
```

output
```
462*x + 462*log(x) - ((11*x)/4 + (55*x^2)/3 + (165*x^3)/2 + 330*x^4 + 1/5)
/x^5 + 165*x^2 + 55*x^3 + (55*x^4)/4 + (11*x^5)/5 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^6}\,dx = \frac{27720\log(x)\,x^5+10x^{11}+132x^{10}+825x^9+3300x^8+9900x^7+27720x^6-19800x^4-4950x^3-1100x^2}{60x^5}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^6,x)
```

output
```
(27720*log(x)*x**5 + 10*x**11 + 132*x**10 + 825*x**9 + 3300*x**8 + 9900*x*
*7 + 27720*x**6 - 19800*x**4 - 4950*x**3 - 1100*x**2 - 165*x - 12)/(60*x**
5)
```

3.235 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = -\frac{1}{6x^6}-\frac{11}{5x^5}-\frac{55}{4x^4}-\frac{55}{x^3}-\frac{165}{x^2}-\frac{462}{x}+330x+\frac{165x^2}{2}+\frac{55x^3}{3}+\frac{11x^4}{4}+\frac{x^5}{5}+462\log(x)$$

output
```
-1/6/x^6-11/5/x^5-55/4/x^4-55/x^3-165/x^2-462/x+330*x+165/2*x^2+55/3*x^3+1
1/4*x^4+1/5*x^5+462*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = -\frac{1}{6x^6}-\frac{11}{5x^5}-\frac{55}{4x^4}-\frac{55}{x^3}-\frac{165}{x^2}-\frac{462}{x}+330x+\frac{165x^2}{2}+\frac{55x^3}{3}+\frac{11x^4}{4}+\frac{x^5}{5}+462\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^7,x]
```

output
```
-1/6*1/x^6 - 11/(5*x^5) - 55/(4*x^4) - 55/x^3 - 165/x^2 - 462/x + 330*x +
(165*x^2)/2 + (55*x^3)/3 + (11*x^4)/4 + x^5/5 + 462*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^7}\,dx$$

$$\Big\downarrow \; 1184$$

$$\int \frac{(x+1)^{11}}{x^7}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{1}{x^7}+\frac{11}{x^6}+\frac{55}{x^5}+x^4+\frac{165}{x^4}+11x^3+\frac{330}{x^3}+55x^2+\frac{462}{x^2}+165x+\frac{462}{x}+330\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{6x^6}+\frac{x^5}{5}-\frac{11}{5x^5}+\frac{11x^4}{4}-\frac{55}{4x^4}+\frac{55x^3}{3}-\frac{55}{x^3}+\frac{165x^2}{2}-\frac{165}{x^2}+330x-\frac{462}{x}+462\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^7,x]
```

output
```
-1/6*1/x^6 - 11/(5*x^5) - 55/(4*x^4) - 55/x^3 - 165/x^2 - 462/x + 330*x +
(165*x^2)/2 + (55*x^3)/3 + (11*x^4)/4 + x^5/5 + 462*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x^5}{5}+\frac{11x^4}{4}+\frac{55x^3}{3}+\frac{165x^2}{2}+330x+\frac{-462x^5-165x^4-55x^3-\frac{55}{4}x^2-\frac{11}{5}x-\frac{1}{6}}{x^6}+462\ln\left(x\right)$	58
default	$-\frac{1}{6x^6}-\frac{11}{5x^5}-\frac{55}{4x^4}-\frac{55}{x^3}-\frac{165}{x^2}-\frac{462}{x}+330x+\frac{165x^2}{2}+\frac{55x^3}{3}+\frac{11x^4}{4}+\frac{x^5}{5}+462\ln\left(x\right)$	59
norman	$\frac{-\frac{1}{6}-\frac{11}{5}x-\frac{55}{4}x^2-55x^3-165x^4-462x^5+330x^7+\frac{165}{2}x^8+\frac{55}{3}x^9+\frac{11}{4}x^{10}+\frac{1}{5}x^{11}}{x^6}+462\ln\left(x\right)$	60
parallelrisch	$\frac{12x^{11}+165x^{10}+1100x^9+4950x^8+27720\ln\left(x\right)x^6+19800x^7-10-27720x^5-9900x^4-3300x^3-825x^2-132x}{60x^6}$	63

input
```
int((x+1)*(x^2+2*x+1)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5+11/4*x^4+55/3*x^3+165/2*x^2+330*x+(-462*x^5-165*x^4-55*x^3-55/4*x^
2-11/5*x-1/6)/x^6+462*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{12\,x^{11}+165\,x^{10}+1100\,x^9+4950\,x^8+19800\,x^7+27720\,x^6\log\left(x\right)-27720\,x^5-9900\,x^4-3300\,x^3-82}{60\,x^6}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^7,x, algorithm="fricas")
```

output
```
1/60*(12*x^11 + 165*x^10 + 1100*x^9 + 4950*x^8 + 19800*x^7 + 27720*x^6*log
(x) - 27720*x^5 - 9900*x^4 - 3300*x^3 - 825*x^2 - 132*x - 10)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{x^5}{5}+\frac{11x^4}{4}+\frac{55x^3}{3}+\frac{165x^2}{2}+330x+462\log\left(x\right) + \frac{-27720x^5-9900x^4-3300x^3-825x^2-132x-10}{60x^6}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**7,x)
```

output
```
x**5/5 + 11*x**4/4 + 55*x**3/3 + 165*x**2/2 + 330*x + 462*log(x) + (-27720
*x**5 - 9900*x**4 - 3300*x**3 - 825*x**2 - 132*x - 10)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{1}{5}\,x^5 + \frac{11}{4}\,x^4 + \frac{55}{3}\,x^3 + \frac{165}{2}\,x^2 + 330\,x - \frac{27720\,x^5 + 9900\,x^4 + 3300\,x^3 + 825\,x^2 + 132\,x + 10}{60\,x^6} + 462\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^7,x, algorithm="maxima")
```

output
```
1/5*x^5 + 11/4*x^4 + 55/3*x^3 + 165/2*x^2 + 330*x - 1/60*(27720*x^5 + 9900
*x^4 + 3300*x^3 + 825*x^2 + 132*x + 10)/x^6 + 462*log(x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^7}\,dx = \frac{1}{5}\,x^5 + \frac{11}{4}\,x^4 + \frac{55}{3}\,x^3 + \frac{165}{2}\,x^2 + 330\,x - \frac{27720\,x^5 + 9900\,x^4 + 3300\,x^3 + 825\,x^2 + 132\,x + 10}{60\,x^6} + 462\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^7,x, algorithm="giac")
```

output
```
1/5*x^5 + 11/4*x^4 + 55/3*x^3 + 165/2*x^2 + 330*x - 1/60*(27720*x^5 + 9900
*x^4 + 3300*x^3 + 825*x^2 + 132*x + 10)/x^6 + 462*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^7}\,dx = 330\,x+462\,\ln\left(x\right) - \frac{462\,x^5+165\,x^4+55\,x^3+\frac{55\,x^2}{4}+\frac{11\,x}{5}+\frac{1}{6}}{x^6} + \frac{165\,x^2}{2}+\frac{55\,x^3}{3}+\frac{11\,x^4}{4}+\frac{x^5}{5}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^7,x)
```

output
```
330*x + 462*log(x) - ((11*x)/5 + (55*x^2)/4 + 55*x^3 + 165*x^4 + 462*x^5 +
 1/6)/x^6 + (165*x^2)/2 + (55*x^3)/3 + (11*x^4)/4 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^7}\,dx = \frac{27720\log(x)\,x^6+12x^{11}+165x^{10}+1100x^9+4950x^8+19800x^7-27720x^5-9900x^4-3300x^3-825x^2}{60x^6}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^7,x)
```

output
```
(27720*log(x)*x**6 + 12*x**11 + 165*x**10 + 1100*x**9 + 4950*x**8 + 19800*
x**7 - 27720*x**5 - 9900*x**4 - 3300*x**3 - 825*x**2 - 132*x - 10)/(60*x**
6)
```

3.236 $\int \frac{(1+x)(1+2x+x^2)^5}{x^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^8} dx = -\frac{1}{7x^7} - \frac{11}{6x^6} - \frac{11}{x^5} - \frac{165}{4x^4} - \frac{110}{x^3} - \frac{231}{x^2} - \frac{462}{x} + 165x + \frac{55x^2}{2} + \frac{11x^3}{3} + \frac{x^4}{4} + 330\log(x)$$

output
```
-1/7/x^7-11/6/x^6-11/x^5-165/4/x^4-110/x^3-231/x^2-462/x+165*x+55/2*x^2+11
/3*x^3+1/4*x^4+330*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^8} dx = -\frac{1}{7x^7} - \frac{11}{6x^6} - \frac{11}{x^5} - \frac{165}{4x^4} - \frac{110}{x^3} - \frac{231}{x^2} - \frac{462}{x} + 165x + \frac{55x^2}{2} + \frac{11x^3}{3} + \frac{x^4}{4} + 330\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^8,x]
```

output
```
-1/7*1/x^7 - 11/(6*x^6) - 11/x^5 - 165/(4*x^4) - 110/x^3 - 231/x^2 - 462/x
 + 165*x + (55*x^2)/2 + (11*x^3)/3 + x^4/4 + 330*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^8}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{11}}{x^8}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{1}{x^8}+\frac{11}{x^7}+\frac{55}{x^6}+\frac{165}{x^5}+\frac{330}{x^4}+x^3+\frac{462}{x^3}+11x^2+\frac{462}{x^2}+55x+\frac{330}{x}+165\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{7x^7}-\frac{11}{6x^6}-\frac{11}{x^5}+\frac{x^4}{4}-\frac{165}{4x^4}+\frac{11x^3}{3}-\frac{110}{x^3}+\frac{55x^2}{2}-\frac{231}{x^2}+165x-\frac{462}{x}+330\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^8,x]
```

output

```
-1/7*1/x^7 - 11/(6*x^6) - 11/x^5 - 165/(4*x^4) - 110/x^3 - 231/x^2 - 462/x
 + 165*x + (55*x^2)/2 + (11*x^3)/3 + x^4/4 + 330*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x^{4}}{4}+\frac{11 x^{3}}{3}+\frac{55 x^{2}}{2}+165 x +\frac{-462 x^{6}-231 x^{5}-110 x^{4}-\frac{165}{4} x^{3}-11 x^{2}-\frac{11}{6} x -\frac{1}{7}}{x^{7}}+330 \ln \left(x \right)$	58
default	$-\frac{1}{7 x^{7}}-\frac{11}{6 x^{6}}-\frac{11}{x^{5}}-\frac{165}{4 x^{4}}-\frac{110}{x^{3}}-\frac{231}{x^{2}}-\frac{462}{x}+165 x +\frac{55 x^{2}}{2}+\frac{11 x^{3}}{3}+\frac{x^{4}}{4}+330 \ln \left(x \right)$	59
norman	$\frac{-\frac{1}{7}-\frac{11}{6} x -11 x^{2}-\frac{165}{4} x^{3}-110 x^{4}-231 x^{5}-462 x^{6}+165 x^{8}+\frac{55}{2} x^{9}+\frac{11}{3} x^{10}+\frac{1}{4} x^{11}}{x^{7}}+330 \ln \left(x \right)$	60
parallelrisch	$\frac{21 x^{11}+308 x^{10}+2310 x^{9}+27720 \ln \left(x \right) x^{7}+13860 x^{8}-12-38808 x^{6}-19404 x^{5}-9240 x^{4}-3465 x^{3}-924 x^{2}-154 x}{84 x^{7}}$	63

input
```
int((x+1)*(x^2+2*x+1)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+11/3*x^3+55/2*x^2+165*x+(-462*x^6-231*x^5-110*x^4-165/4*x^3-11*x^2
-11/6*x-1/7)/x^7+330*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^8}\,dx = \frac{21\,x^{11}+308\,x^{10}+2310\,x^9+13860\,x^8+27720\,x^7\log{(x)}-38808\,x^6-19404\,x^5-9240\,x^4-3465\,x^3-9}{84\,x^7}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^8,x, algorithm="fricas")
```

output
```
1/84*(21*x^11 + 308*x^10 + 2310*x^9 + 13860*x^8 + 27720*x^7*log(x) - 38808
*x^6 - 19404*x^5 - 9240*x^4 - 3465*x^3 - 924*x^2 - 154*x - 12)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^8}\,dx = \frac{x^4}{4}+\frac{11x^3}{3}+\frac{55x^2}{2}+165x+330\log{(x)} + \frac{-38808x^6-19404x^5-9240x^4-3465x^3-924x^2-154x-12}{84x^7}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**8,x)
```

output
```
x**4/4 + 11*x**3/3 + 55*x**2/2 + 165*x + 330*log(x) + (-38808*x**6 - 19404
*x**5 - 9240*x**4 - 3465*x**3 - 924*x**2 - 154*x - 12)/(84*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$=\frac{1}{4}\,x^4+\frac{11}{3}\,x^3+\frac{55}{2}\,x^2+165\,x-\frac{38808\,x^6+19404\,x^5+9240\,x^4+3465\,x^3+924\,x^2+154\,x+12}{84\,x^7}+330\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^8,x, algorithm="maxima")
```

output
```
1/4*x^4 + 11/3*x^3 + 55/2*x^2 + 165*x - 1/84*(38808*x^6 + 19404*x^5 + 9240
*x^4 + 3465*x^3 + 924*x^2 + 154*x + 12)/x^7 + 330*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^8}\,dx$$
$$=\frac{1}{4}\,x^4+\frac{11}{3}\,x^3+\frac{55}{2}\,x^2+165\,x-\frac{38808\,x^6+19404\,x^5+9240\,x^4+3465\,x^3+924\,x^2+154\,x+12}{84\,x^7}+330\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^8,x, algorithm="giac")
```

output
```
1/4*x^4 + 11/3*x^3 + 55/2*x^2 + 165*x - 1/84*(38808*x^6 + 19404*x^5 + 9240
*x^4 + 3465*x^3 + 924*x^2 + 154*x + 12)/x^7 + 330*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^8}\,dx = 165\,x+330\,\ln\left(x\right)-\frac{462\,x^6+231\,x^5+110\,x^4+\frac{165\,x^3}{4}+11\,x^2+\frac{11\,x}{6}+\frac{1}{7}}{x^7}+\frac{55\,x^2}{2}+\frac{11\,x^3}{3}+\frac{x^4}{4}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^8,x)
```

output
```
165*x + 330*log(x) - ((11*x)/6 + 11*x^2 + (165*x^3)/4 + 110*x^4 + 231*x^5
+ 462*x^6 + 1/7)/x^7 + (55*x^2)/2 + (11*x^3)/3 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^8}\,dx = \frac{27720\log(x)\,x^7+21x^{11}+308x^{10}+2310x^9+13860x^8-38808x^6-19404x^5-9240x^4-3465x^3-924x^2-154x-12}{84x^7}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^8,x)
```

output
```
(27720*log(x)*x**7 + 21*x**11 + 308*x**10 + 2310*x**9 + 13860*x**8 - 38808
*x**6 - 19404*x**5 - 9240*x**4 - 3465*x**3 - 924*x**2 - 154*x - 12)/(84*x*
*7)
```

3.237 $\int \frac{(1+x)(1+2x+x^2)^5}{x^9} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9} \, dx = -\frac{1}{8x^8} - \frac{11}{7x^7} - \frac{55}{6x^6} - \frac{33}{x^5} - \frac{165}{2x^4} - \frac{154}{x^3} - \frac{231}{x^2} - \frac{330}{x} + 55x + \frac{11x^2}{2} + \frac{x^3}{3} + 165\log(x)$$

output
```
-1/8/x^8-11/7/x^7-55/6/x^6-33/x^5-165/2/x^4-154/x^3-231/x^2-330/x+55*x+11/
2*x^2+1/3*x^3+165*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9} \, dx = -\frac{1}{8x^8} - \frac{11}{7x^7} - \frac{55}{6x^6} - \frac{33}{x^5} - \frac{165}{2x^4} - \frac{154}{x^3} - \frac{231}{x^2} - \frac{330}{x} + 55x + \frac{11x^2}{2} + \frac{x^3}{3} + 165\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^9,x]
```

output
```
-1/8*1/x^8 - 11/(7*x^7) - 55/(6*x^6) - 33/x^5 - 165/(2*x^4) - 154/x^3 - 23
1/x^2 - 330/x + 55*x + (11*x^2)/2 + x^3/3 + 165*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^9}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\int \frac{(x+1)^{11}}{x^9}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{1}{x^9}+\frac{11}{x^8}+\frac{55}{x^7}+\frac{165}{x^6}+\frac{330}{x^5}+\frac{462}{x^4}+\frac{462}{x^3}+x^2+\frac{330}{x^2}+11x+\frac{165}{x}+55\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8x^8}-\frac{11}{7x^7}-\frac{55}{6x^6}-\frac{33}{x^5}-\frac{165}{2x^4}+\frac{x^3}{3}-\frac{154}{x^3}+\frac{11x^2}{2}-\frac{231}{x^2}+55x-\frac{330}{x}+165\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^9,x]
```

output
```
-1/8*1/x^8 - 11/(7*x^7) - 55/(6*x^6) - 33/x^5 - 165/(2*x^4) - 154/x^3 - 23
1/x^2 - 330/x + 55*x + (11*x^2)/2 + x^3/3 + 165*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x^{3}}{3}+\frac{11 x^{2}}{2}+55 x +\frac{-330 x^{7}-231 x^{6}-154 x^{5}-\frac{165}{2} x^{4}-33 x^{3}-\frac{55}{6} x^{2}-\frac{11}{7} x -\frac{1}{8}}{x^{8}}+165 \ln \left(x \right)$	58
default	$-\frac{1}{8 x^{8}}-\frac{11}{7 x^{7}}-\frac{55}{6 x^{6}}-\frac{33}{x^{5}}-\frac{165}{2 x^{4}}-\frac{154}{x^{3}}-\frac{231}{x^{2}}-\frac{330}{x}+55 x +\frac{11 x^{2}}{2}+\frac{x^{3}}{3}+165 \ln \left(x \right)$	59
norman	$\frac{-\frac{1}{8}-\frac{11}{7} x -\frac{55}{6} x^{2}-33 x^{3}-\frac{165}{2} x^{4}-154 x^{5}-231 x^{6}-330 x^{7}+55 x^{9}+\frac{11}{2} x^{10}+\frac{1}{3} x^{11}}{x^{8}}+165 \ln \left(x \right)$	60
parallelrisch	$\frac{56 x^{11}+924 x^{10}+27720 \ln \left(x \right) x^{8}+9240 x^{9}-21-55440 x^{7}-38808 x^{6}-25872 x^{5}-13860 x^{4}-5544 x^{3}-1540 x^{2}-264 x}{168 x^{8}}$	63

input
```
int((x+1)*(x^2+2*x+1)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+11/2*x^2+55*x+(-330*x^7-231*x^6-154*x^5-165/2*x^4-33*x^3-55/6*x^2-
11/7*x-1/8)/x^8+165*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^9}\,dx$$
$$= \frac{56\,x^{11}+924\,x^{10}+9240\,x^9+27720\,x^8\log\left(x\right)-55440\,x^7-38808\,x^6-25872\,x^5-13860\,x^4-5544\,x^3-}{168\,x^8}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^9,x, algorithm="fricas")
```

output
```
1/168*(56*x^11 + 924*x^10 + 9240*x^9 + 27720*x^8*log(x) - 55440*x^7 - 3880
8*x^6 - 25872*x^5 - 13860*x^4 - 5544*x^3 - 1540*x^2 - 264*x - 21)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^9}\,dx$$
$$= \frac{x^3}{3}+\frac{11x^2}{2}+55x+165\log\left(x\right)$$
$$+\frac{-55440x^7-38808x^6-25872x^5-13860x^4-5544x^3-1540x^2-264x-21}{168x^8}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**9,x)
```

output
```
x**3/3 + 11*x**2/2 + 55*x + 165*log(x) + (-55440*x**7 - 38808*x**6 - 25872
*x**5 - 13860*x**4 - 5544*x**3 - 1540*x**2 - 264*x - 21)/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9}\,dx = \frac{1}{3}\,x^3 + \frac{11}{2}\,x^2 + 55\,x - \frac{55440\,x^7 + 38808\,x^6 + 25872\,x^5 + 13860\,x^4 + 5544\,x^3 + 1540\,x^2 + 264\,x + 21}{168\,x^8} + 165\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^9,x, algorithm="maxima")
```

output
```
1/3*x^3 + 11/2*x^2 + 55*x - 1/168*(55440*x^7 + 38808*x^6 + 25872*x^5 + 138
60*x^4 + 5544*x^3 + 1540*x^2 + 264*x + 21)/x^8 + 165*log(x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9}\,dx = \frac{1}{3}\,x^3 + \frac{11}{2}\,x^2 + 55\,x - \frac{55440\,x^7 + 38808\,x^6 + 25872\,x^5 + 13860\,x^4 + 5544\,x^3 + 1540\,x^2 + 264\,x + 21}{168\,x^8} + 165\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^9,x, algorithm="giac")
```

output
```
1/3*x^3 + 11/2*x^2 + 55*x - 1/168*(55440*x^7 + 38808*x^6 + 25872*x^5 + 138
60*x^4 + 5544*x^3 + 1540*x^2 + 264*x + 21)/x^8 + 165*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9}\,dx$$
$$= 55\,x + 165\,\ln(x) - \frac{330\,x^7 + 231\,x^6 + 154\,x^5 + \frac{165\,x^4}{2} + 33\,x^3 + \frac{55\,x^2}{6} + \frac{11\,x}{7} + \frac{1}{8}}{x^8} + \frac{11\,x^2}{2} + \frac{x^3}{3}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^9,x)
```

output
```
55*x + 165*log(x) - ((11*x)/7 + (55*x^2)/6 + 33*x^3 + (165*x^4)/2 + 154*x^
5 + 231*x^6 + 330*x^7 + 1/8)/x^8 + (11*x^2)/2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^9}\,dx$$
$$= \frac{27720\log(x)\,x^8 + 56x^{11} + 924x^{10} + 9240x^9 - 55440x^7 - 38808x^6 - 25872x^5 - 13860x^4 - 5544x^3 - 154}{168x^8}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^9,x)
```

output
```
(27720*log(x)*x**8 + 56*x**11 + 924*x**10 + 9240*x**9 - 55440*x**7 - 38808
*x**6 - 25872*x**5 - 13860*x**4 - 5544*x**3 - 1540*x**2 - 264*x - 21)/(168
*x**8)
```

3.238 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{10}} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{10}} dx = -\frac{1}{9x^9} - \frac{11}{8x^8} - \frac{55}{7x^7} - \frac{55}{2x^6} - \frac{66}{x^5} - \frac{231}{2x^4} - \frac{154}{x^3} - \frac{165}{x^2} - \frac{165}{x} + 11x + \frac{x^2}{2} + 55\log(x)$$

output
```
-1/9/x^9-11/8/x^8-55/7/x^7-55/2/x^6-66/x^5-231/2/x^4-154/x^3-165/x^2-165/x
+11*x+1/2*x^2+55*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{10}} dx = -\frac{1}{9x^9} - \frac{11}{8x^8} - \frac{55}{7x^7} - \frac{55}{2x^6} - \frac{66}{x^5} - \frac{231}{2x^4} - \frac{154}{x^3} - \frac{165}{x^2} - \frac{165}{x} + 11x + \frac{x^2}{2} + 55\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^10,x]
```

output
```
-1/9*1/x^9 - 11/(8*x^8) - 55/(7*x^7) - 55/(2*x^6) - 66/x^5 - 231/(2*x^4) -
 154/x^3 - 165/x^2 - 165/x + 11*x + x^2/2 + 55*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{10}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{10}}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{x^{10}}+\frac{11}{x^9}+\frac{55}{x^8}+\frac{165}{x^7}+\frac{330}{x^6}+\frac{462}{x^5}+\frac{462}{x^4}+\frac{330}{x^3}+\frac{165}{x^2}+x+\frac{55}{x}+11\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{9x^9}-\frac{11}{8x^8}-\frac{55}{7x^7}-\frac{55}{2x^6}-\frac{66}{x^5}-\frac{231}{2x^4}-\frac{154}{x^3}+\frac{x^2}{2}-\frac{165}{x^2}+11x-\frac{165}{x}+55\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^10,x]
```

output

```
-1/9*1/x^9 - 11/(8*x^8) - 55/(7*x^7) - 55/(2*x^6) - 66/x^5 - 231/(2*x^4) -
 154/x^3 - 165/x^2 - 165/x + 11*x + x^2/2 + 55*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x^2}{2}+11x+\frac{-165x^8-165x^7-154x^6-\frac{231}{2}x^5-66x^4-\frac{55}{2}x^3-\frac{55}{7}x^2-\frac{11}{8}x-\frac{1}{9}}{x^9}+55\ln\left(x\right)$	58
default	$-\frac{1}{9x^9}-\frac{11}{8x^8}-\frac{55}{7x^7}-\frac{55}{2x^6}-\frac{66}{x^5}-\frac{231}{2x^4}-\frac{154}{x^3}-\frac{165}{x^2}-\frac{165}{x}+11x+\frac{x^2}{2}+55\ln\left(x\right)$	59
norman	$\frac{-\frac{1}{9}-\frac{11}{8}x-\frac{55}{7}x^2-\frac{55}{2}x^3-66x^4-\frac{231}{2}x^5-154x^6-165x^7-165x^8+11x^{10}+\frac{1}{2}x^{11}}{x^9}+55\ln\left(x\right)$	60
parallelrisch	$\frac{252x^{11}+27720\ln\left(x\right)x^9+5544x^{10}-56-83160x^8-83160x^7-77616x^6-58212x^5-33264x^4-13860x^3-3960x^2-693x}{504x^9}$	63

input
```
int((x+1)*(x^2+2*x+1)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+11*x+(-165*x^8-165*x^7-154*x^6-231/2*x^5-66*x^4-55/2*x^3-55/7*x^2-
11/8*x-1/9)/x^9+55*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx$$
$$=\frac{252\,x^{11}+5544\,x^{10}+27720\,x^9\log\left(x\right)-83160\,x^8-83160\,x^7-77616\,x^6-58212\,x^5-33264\,x^4-13860}{504\,x^9}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^10,x, algorithm="fricas")
```

output
```
1/504*(252*x^11 + 5544*x^10 + 27720*x^9*log(x) - 83160*x^8 - 83160*x^7 - 7
7616*x^6 - 58212*x^5 - 33264*x^4 - 13860*x^3 - 3960*x^2 - 693*x - 56)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx=\frac{x^2}{2}+11x+55\log\left(x\right)$$
$$+\frac{-83160x^8-83160x^7-77616x^6-58212x^5-33264x^4-13860x^3-3960x^2-693x-56}{504x^9}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**10,x)
```

output
```
x**2/2 + 11*x + 55*log(x) + (-83160*x**8 - 83160*x**7 - 77616*x**6 - 58212
*x**5 - 33264*x**4 - 13860*x**3 - 3960*x**2 - 693*x - 56)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx = \frac{1}{2}\,x^2+11\,x - \frac{83160\,x^8+83160\,x^7+77616\,x^6+58212\,x^5+33264\,x^4+13860\,x^3+3960\,x^2+693\,x+56}{504\,x^9} + 55\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^10,x, algorithm="maxima")
```

output
```
1/2*x^2 + 11*x - 1/504*(83160*x^8 + 83160*x^7 + 77616*x^6 + 58212*x^5 + 33
264*x^4 + 13860*x^3 + 3960*x^2 + 693*x + 56)/x^9 + 55*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx = \frac{1}{2}\,x^2+11\,x - \frac{83160\,x^8+83160\,x^7+77616\,x^6+58212\,x^5+33264\,x^4+13860\,x^3+3960\,x^2+693\,x+56}{504\,x^9} + 55\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^10,x, algorithm="giac")
```

output
```
1/2*x^2 + 11*x - 1/504*(83160*x^8 + 83160*x^7 + 77616*x^6 + 58212*x^5 + 33
264*x^4 + 13860*x^3 + 3960*x^2 + 693*x + 56)/x^9 + 55*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx$$
$$= 11\,x + 55\,\ln(x) + \frac{x^2}{2} - \frac{165\,x^8 + 165\,x^7 + 154\,x^6 + \frac{231\,x^5}{2} + 66\,x^4 + \frac{55\,x^3}{2} + \frac{55\,x^2}{7} + \frac{11\,x}{8} + \frac{1}{9}}{x^9}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^10,x)
```

output
```
11*x + 55*log(x) + x^2/2 - ((11*x)/8 + (55*x^2)/7 + (55*x^3)/2 + 66*x^4 +
(231*x^5)/2 + 154*x^6 + 165*x^7 + 165*x^8 + 1/9)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{10}}\,dx$$
$$= \frac{27720\log(x)\,x^9 + 252x^{11} + 5544x^{10} - 83160x^8 - 83160x^7 - 77616x^6 - 58212x^5 - 33264x^4 - 13860x^3 -}{504x^9}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^10,x)
```

output
```
(27720*log(x)*x**9 + 252*x**11 + 5544*x**10 - 83160*x**8 - 83160*x**7 - 77
616*x**6 - 58212*x**5 - 33264*x**4 - 13860*x**3 - 3960*x**2 - 693*x - 56)/
(504*x**9)
```

3.239 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{11}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = -\frac{1}{10x^{10}}-\frac{11}{9x^9}-\frac{55}{8x^8}-\frac{165}{7x^7}-\frac{55}{x^6}-\frac{462}{5x^5}-\frac{231}{2x^4}-\frac{110}{x^3}-\frac{165}{2x^2}-\frac{55}{x}+x+11\log(x)$$

output
```
-1/10/x^10-11/9/x^9-55/8/x^8-165/7/x^7-55/x^6-462/5/x^5-231/2/x^4-110/x^3-
165/2/x^2-55/x+x+11*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = -\frac{1}{10x^{10}}-\frac{11}{9x^9}-\frac{55}{8x^8}-\frac{165}{7x^7}-\frac{55}{x^6}-\frac{462}{5x^5}-\frac{231}{2x^4}-\frac{110}{x^3}-\frac{165}{2x^2}-\frac{55}{x}+x+11\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^11,x]
```

output
```
-1/10*1/x^10 - 11/(9*x^9) - 55/(8*x^8) - 165/(7*x^7) - 55/x^6 - 462/(5*x^5
) - 231/(2*x^4) - 110/x^3 - 165/(2*x^2) - 55/x + x + 11*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{11}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{11}}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{x^{11}}+\frac{11}{x^{10}}+\frac{55}{x^9}+\frac{165}{x^8}+\frac{330}{x^7}+\frac{462}{x^6}+\frac{462}{x^5}+\frac{330}{x^4}+\frac{165}{x^3}+\frac{55}{x^2}+\frac{11}{x}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{10x^{10}}-\frac{11}{9x^9}-\frac{55}{8x^8}-\frac{165}{7x^7}-\frac{55}{x^6}-\frac{462}{5x^5}-\frac{231}{2x^4}-\frac{110}{x^3}-\frac{165}{2x^2}+x-\frac{55}{x}+11\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^11,x]
```

output
```
-1/10*1/x^10 - 11/(9*x^9) - 55/(8*x^8) - 165/(7*x^7) - 55/x^6 - 462/(5*x^5
) - 231/(2*x^4) - 110/x^3 - 165/(2*x^2) - 55/x + x + 11*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

method	result
risch	$x+\frac{-55x^{9}-\frac{165}{2}x^{8}-110x^{7}-\frac{231}{2}x^{6}-\frac{462}{5}x^{5}-55x^{4}-\frac{165}{7}x^{3}-\frac{55}{8}x^{2}-\frac{11}{9}x-\frac{1}{10}}{x^{10}}+11\ln\left(x\right)$
default	$-\frac{1}{10x^{10}}-\frac{11}{9x^{9}}-\frac{55}{8x^{8}}-\frac{165}{7x^{7}}-\frac{55}{x^{6}}-\frac{462}{5x^{5}}-\frac{231}{2x^{4}}-\frac{110}{x^{3}}-\frac{165}{2x^{2}}-\frac{55}{x}+x+11\ln\left(x\right)$
norman	$\frac{-\frac{1}{10}+x^{11}-\frac{11}{9}x-\frac{55}{8}x^{2}-\frac{165}{7}x^{3}-55x^{4}-\frac{462}{5}x^{5}-\frac{231}{2}x^{6}-110x^{7}-\frac{165}{2}x^{8}-55x^{9}}{x^{10}}+11\ln\left(x\right)$
parallelrisch	$\frac{27720\ln(x)x^{10}+2520x^{11}-252-138600x^{9}-207900x^{8}-277200x^{7}-291060x^{6}-232848x^{5}-138600x^{4}-59400x^{3}-17325x^{2}-308}{2520x^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^11,x,method=_RETURNVERBOSE)
```

output
```
x+(-55*x^9-165/2*x^8-110*x^7-231/2*x^6-462/5*x^5-55*x^4-165/7*x^3-55/8*x^2
-11/9*x-1/10)/x^10+11*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = \frac{2520\,x^{11} + 27720\,x^{10}\log\left(x\right) - 138600\,x^{9} - 207900\,x^{8} - 277200\,x^{7} - 291060\,x^{6} - 232848\,x^{5} - 138600\,x^{4}}{2520\,x^{10}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^11,x, algorithm="fricas")
```

output
```
1/2520*(2520*x^11 + 27720*x^10*log(x) - 138600*x^9 - 207900*x^8 - 277200*x
^7 - 291060*x^6 - 232848*x^5 - 138600*x^4 - 59400*x^3 - 17325*x^2 - 3080*x
 - 252)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}}\,dx = x + 11\log\left(x\right) + \frac{-138600x^{9} - 207900x^{8} - 277200x^{7} - 291060x^{6} - 232848x^{5} - 138600x^{4} - 59400x^{3} - 17325x^{2} - 308}{2520x^{10}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**11,x)
```

output
```
x + 11*log(x) + (-138600*x**9 - 207900*x**8 - 277200*x**7 - 291060*x**6 -
232848*x**5 - 138600*x**4 - 59400*x**3 - 17325*x**2 - 3080*x - 252)/(2520*
x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}} \, dx = x - \frac{138600\,x^9 + 207900\,x^8 + 277200\,x^7 + 291060\,x^6 + 232848\,x^5 + 138600\,x^4 + 59400\,x^3 + 17325\,x^2 + 30}{2520\,x^{10}} + 11\,\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^11,x, algorithm="maxima")
```

output
```
x - 1/2520*(138600*x^9 + 207900*x^8 + 277200*x^7 + 291060*x^6 + 232848*x^5
 + 138600*x^4 + 59400*x^3 + 17325*x^2 + 3080*x + 252)/x^10 + 11*log(x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{11}} \, dx = x - \frac{138600\,x^9 + 207900\,x^8 + 277200\,x^7 + 291060\,x^6 + 232848\,x^5 + 138600\,x^4 + 59400\,x^3 + 17325\,x^2 + 30}{2520\,x^{10}} + 11\,\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^11,x, algorithm="giac")
```

output
```
x - 1/2520*(138600*x^9 + 207900*x^8 + 277200*x^7 + 291060*x^6 + 232848*x^5
 + 138600*x^4 + 59400*x^3 + 17325*x^2 + 3080*x + 252)/x^10 + 11*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{11}}\,dx = -\frac{\frac{11\,x}{9}-11\,x^{10}\,\ln\left(x\right)+\frac{55\,x^2}{8}+\frac{165\,x^3}{7}+55\,x^4+\frac{462\,x^5}{5}+\frac{231\,x^6}{2}+110\,x^7+\frac{165\,x^8}{2}+55\,x^9-x^{11}+\frac{1}{10}}{x^{10}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^11,x)
```

output
```
-((11*x)/9 - 11*x^10*log(x) + (55*x^2)/8 + (165*x^3)/7 + 55*x^4 + (462*x^5
)/5 + (231*x^6)/2 + 110*x^7 + (165*x^8)/2 + 55*x^9 - x^11 + 1/10)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{11}}\,dx = \frac{27720\log(x)\,x^{10}+2520x^{11}-138600x^9-207900x^8-277200x^7-291060x^6-232848x^5-138600x^4-}{2520x^{10}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^11,x)
```

output
```
(27720*log(x)*x**10 + 2520*x**11 - 138600*x**9 - 207900*x**8 - 277200*x**7
 - 291060*x**6 - 232848*x**5 - 138600*x**4 - 59400*x**3 - 17325*x**2 - 308
0*x - 252)/(2520*x**10)
```

3.240 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = -\frac{1}{11x^{11}}-\frac{11}{10x^{10}}-\frac{55}{9x^9}-\frac{165}{8x^8}-\frac{330}{7x^7}-\frac{77}{x^6}-\frac{462}{5x^5}-\frac{165}{2x^4}-\frac{55}{x^3}-\frac{55}{2x^2}-\frac{11}{x}+\log(x)$$

output
```
-1/11/x^11-11/10/x^10-55/9/x^9-165/8/x^8-330/7/x^7-77/x^6-462/5/x^5-165/2/
x^4-55/x^3-55/2/x^2-11/x+ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = -\frac{1}{11x^{11}}-\frac{11}{10x^{10}}-\frac{55}{9x^9}-\frac{165}{8x^8}-\frac{330}{7x^7}-\frac{77}{x^6}-\frac{462}{5x^5}-\frac{165}{2x^4}-\frac{55}{x^3}-\frac{55}{2x^2}-\frac{11}{x}+\log(x)$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^12,x]
```

output
```
-1/11*1/x^11 - 11/(10*x^10) - 55/(9*x^9) - 165/(8*x^8) - 330/(7*x^7) - 77/
x^6 - 462/(5*x^5) - 165/(2*x^4) - 55/x^3 - 55/(2*x^2) - 11/x + Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{12}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{12}}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{x^{12}}+\frac{11}{x^{11}}+\frac{55}{x^{10}}+\frac{165}{x^9}+\frac{330}{x^8}+\frac{462}{x^7}+\frac{462}{x^6}+\frac{330}{x^5}+\frac{165}{x^4}+\frac{55}{x^3}+\frac{11}{x^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11x^{11}}-\frac{11}{10x^{10}}-\frac{55}{9x^9}-\frac{165}{8x^8}-\frac{330}{7x^7}-\frac{77}{x^6}-\frac{462}{5x^5}-\frac{165}{2x^4}-\frac{55}{x^3}-\frac{55}{2x^2}-\frac{11}{x}+\log(x)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^12,x]
```

output
```
-1/11*1/x^11 - 11/(10*x^10) - 55/(9*x^9) - 165/(8*x^8) - 330/(7*x^7) - 77/
x^6 - 462/(5*x^5) - 165/(2*x^4) - 55/x^3 - 55/(2*x^2) - 11/x + Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result
norman	$\frac{-\frac{1}{11}-\frac{11}{10}x-\frac{55}{9}x^{2}-\frac{165}{8}x^{3}-\frac{330}{7}x^{4}-77x^{5}-\frac{462}{5}x^{6}-\frac{165}{2}x^{7}-55x^{8}-\frac{55}{2}x^{9}-11x^{10}}{x^{11}}+\ln\left(x\right)$
risch	$\frac{-\frac{1}{11}-\frac{11}{10}x-\frac{55}{9}x^{2}-\frac{165}{8}x^{3}-\frac{330}{7}x^{4}-77x^{5}-\frac{462}{5}x^{6}-\frac{165}{2}x^{7}-55x^{8}-\frac{55}{2}x^{9}-11x^{10}}{x^{11}}+\ln\left(x\right)$
default	$-\frac{1}{11x^{11}}-\frac{11}{10x^{10}}-\frac{55}{9x^{9}}-\frac{165}{8x^{8}}-\frac{330}{7x^{7}}-\frac{77}{x^{6}}-\frac{462}{5x^{5}}-\frac{165}{2x^{4}}-\frac{55}{x^{3}}-\frac{55}{2x^{2}}-\frac{11}{x}+\ln\left(x\right)$
parallelrisch	$\frac{27720\ln(x)x^{11}-2520-304920x^{10}-762300x^{9}-1524600x^{8}-2286900x^{7}-2561328x^{6}-2134440x^{5}-1306800x^{4}-571725x^{3}-16}{27720x^{11}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^12,x,method=_RETURNVERBOSE)
```

output
```
(-1/11-11/10*x-55/9*x^2-165/8*x^3-330/7*x^4-77*x^5-462/5*x^6-165/2*x^7-55*
x^8-55/2*x^9-11*x^10)/x^11+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = \frac{27720\,x^{11}\log\left(x\right) - 304920\,x^{10} - 762300\,x^{9} - 1524600\,x^{8} - 2286900\,x^{7} - 2561328\,x^{6} - 2134440\,x^{5} - 13}{27720\,x^{11}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^12,x, algorithm="fricas")
```

output
```
1/27720*(27720*x^11*log(x) - 304920*x^10 - 762300*x^9 - 1524600*x^8 - 2286
900*x^7 - 2561328*x^6 - 2134440*x^5 - 1306800*x^4 - 571725*x^3 - 169400*x^
2 - 30492*x - 2520)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx = \log\left(x\right) + \frac{-304920x^{10} - 762300x^{9} - 1524600x^{8} - 2286900x^{7} - 2561328x^{6} - 2134440x^{5} - 1306800x^{4} - 571725}{27720x^{11}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**12,x)
```

output
```
log(x) + (-304920*x**10 - 762300*x**9 - 1524600*x**8 - 2286900*x**7 - 2561
328*x**6 - 2134440*x**5 - 1306800*x**4 - 571725*x**3 - 169400*x**2 - 30492
*x - 2520)/(27720*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx =$$
$$-\frac{304920\,x^{10}+762300\,x^9+1524600\,x^8+2286900\,x^7+2561328\,x^6+2134440\,x^5+1306800\,x^4+57172}{27720\,x^{11}}$$
$$+\log\left(x\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^12,x, algorithm="maxima")
```

output
```
-1/27720*(304920*x^10 + 762300*x^9 + 1524600*x^8 + 2286900*x^7 + 2561328*x
^6 + 2134440*x^5 + 1306800*x^4 + 571725*x^3 + 169400*x^2 + 30492*x + 2520)
/x^11 + log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{12}}\,dx =$$
$$-\frac{304920\,x^{10}+762300\,x^9+1524600\,x^8+2286900\,x^7+2561328\,x^6+2134440\,x^5+1306800\,x^4+57172}{27720\,x^{11}}$$
$$+\log\left(|x|\right)$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^12,x, algorithm="giac")
```

output
```
-1/27720*(304920*x^10 + 762300*x^9 + 1524600*x^8 + 2286900*x^7 + 2561328*x
^6 + 2134440*x^5 + 1306800*x^4 + 571725*x^3 + 169400*x^2 + 30492*x + 2520)
/x^11 + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{12}}\,dx$$
$$= \ln(x) - \frac{11\,x^{10}+\frac{55\,x^9}{2}+55\,x^8+\frac{165\,x^7}{2}+\frac{462\,x^6}{5}+77\,x^5+\frac{330\,x^4}{7}+\frac{165\,x^3}{8}+\frac{55\,x^2}{9}+\frac{11\,x}{10}+\frac{1}{11}}{x^{11}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^12,x)
```

output
```
log(x) - ((11*x)/10 + (55*x^2)/9 + (165*x^3)/8 + (330*x^4)/7 + 77*x^5 + (4
62*x^6)/5 + (165*x^7)/2 + 55*x^8 + (55*x^9)/2 + 11*x^10 + 1/11)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{12}}\,dx$$
$$= \frac{27720\log(x)\,x^{11}-304920x^{10}-762300x^9-1524600x^8-2286900x^7-2561328x^6-2134440x^5-1306}{27720x^{11}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^12,x)
```

output
```
(27720*log(x)*x**11 - 304920*x**10 - 762300*x**9 - 1524600*x**8 - 2286900*
x**7 - 2561328*x**6 - 2134440*x**5 - 1306800*x**4 - 571725*x**3 - 169400*x
**2 - 30492*x - 2520)/(27720*x**11)
```

3.241 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx = -\frac{(1+x)^{12}}{12x^{12}}$$

output
```
-1/12*(1+x)^12/x^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 75, normalized size of antiderivative = 6.25

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx = -\frac{1}{12x^{12}}-\frac{1}{x^{11}}-\frac{11}{2x^{10}}-\frac{55}{3x^9}-\frac{165}{4x^8}-\frac{66}{x^7}-\frac{77}{x^6}-\frac{66}{x^5}-\frac{165}{4x^4}-\frac{55}{3x^3}-\frac{11}{2x^2}-\frac{1}{x}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^13,x]
```

output
```
-1/12*1/x^12 - x^(-11) - 11/(2*x^10) - 55/(3*x^9) - 165/(4*x^8) - 66/x^7 -
 77/x^6 - 66/x^5 - 165/(4*x^4) - 55/(3*x^3) - 11/(2*x^2) - x^(-1)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1184, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{13}}dx$$

$$\Big\downarrow \ 1184$$

$$\int \frac{(x+1)^{11}}{x^{13}}dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(x+1)^{12}}{12x^{12}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^13,x]
```

output
```
-1/12*(1 + x)^12/x^12
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(10) = 20$.

Time = 0.74 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

method	result	size
norman	$\frac{-x^{11}-\frac{11}{2}x^{10}-\frac{55}{3}x^{9}-\frac{165}{4}x^{8}-66x^{7}-77x^{6}-66x^{5}-\frac{165}{4}x^{4}-\frac{55}{3}x^{3}-\frac{11}{2}x^{2}-x-\frac{1}{12}}{x^{12}}$	60
risch	$\frac{-x^{11}-\frac{11}{2}x^{10}-\frac{55}{3}x^{9}-\frac{165}{4}x^{8}-66x^{7}-77x^{6}-66x^{5}-\frac{165}{4}x^{4}-\frac{55}{3}x^{3}-\frac{11}{2}x^{2}-x-\frac{1}{12}}{x^{12}}$	60
gosper	$-\frac{12x^{11}+66x^{10}+220x^{9}+495x^{8}+792x^{7}+924x^{6}+792x^{5}+495x^{4}+220x^{3}+66x^{2}+12x+1}{12x^{12}}$	61
parallelrisch	$\frac{-12x^{11}-66x^{10}-220x^{9}-495x^{8}-792x^{7}-924x^{6}-792x^{5}-495x^{4}-220x^{3}-66x^{2}-12x-1}{12x^{12}}$	61
default	$-\frac{66}{x^{5}}-\frac{77}{x^{6}}-\frac{55}{3x^{3}}-\frac{11}{2x^{2}}-\frac{165}{4x^{8}}-\frac{165}{4x^{4}}-\frac{55}{3x^{9}}-\frac{66}{x^{7}}-\frac{1}{x}-\frac{11}{2x^{10}}-\frac{1}{x^{11}}-\frac{1}{12x^{12}}$	62
orering	$-\frac{(12x^{11}+66x^{10}+220x^{9}+495x^{8}+792x^{7}+924x^{6}+792x^{5}+495x^{4}+220x^{3}+66x^{2}+12x+1)(x^{2}+2x+1)^{5}}{12x^{12}(x+1)^{10}}$	76

input
```
int((x+1)*(x^2+2*x+1)^5/x^13,x,method=_RETURNVERBOSE)
```

output
```
(-x^11-11/2*x^10-55/3*x^9-165/4*x^8-66*x^7-77*x^6-66*x^5-165/4*x^4-55/3*x^
3-11/2*x^2-x-1/12)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx = -\frac{12\,x^{11}+66\,x^{10}+220\,x^{9}+495\,x^{8}+792\,x^{7}+924\,x^{6}+792\,x^{5}+495\,x^{4}+220\,x^{3}+66\,x^{2}+12\,x+1}{12\,x^{12}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^13,x, algorithm="fricas")
```

output
```
-1/12*(12*x^11 + 66*x^10 + 220*x^9 + 495*x^8 + 792*x^7 + 924*x^6 + 792*x^5
 + 495*x^4 + 220*x^3 + 66*x^2 + 12*x + 1)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.08

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{13}}\,dx = \frac{-12x^{11}-66x^{10}-220x^9-495x^8-792x^7-924x^6-792x^5-495x^4-220x^3-66x^2-12x-1}{12x^{12}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**13,x)
```

output
```
(-12*x**11 - 66*x**10 - 220*x**9 - 495*x**8 - 792*x**7 - 924*x**6 - 792*x*
*5 - 495*x**4 - 220*x**3 - 66*x**2 - 12*x - 1)/(12*x**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{13}}\,dx = -\frac{12\,x^{11}+66\,x^{10}+220\,x^9+495\,x^8+792\,x^7+924\,x^6+792\,x^5+495\,x^4+220\,x^3+66\,x^2+12\,x+1}{12\,x^{12}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^13,x, algorithm="maxima")
```

output
```
-1/12*(12*x^11 + 66*x^10 + 220*x^9 + 495*x^8 + 792*x^7 + 924*x^6 + 792*x^5
 + 495*x^4 + 220*x^3 + 66*x^2 + 12*x + 1)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx = -\frac{12\,x^{11}+66\,x^{10}+220\,x^{9}+495\,x^{8}+792\,x^{7}+924\,x^{6}+792\,x^{5}+495\,x^{4}+220\,x^{3}+66\,x^{2}+12\,x+1}{12\,x^{12}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^13,x, algorithm="giac")
```

output
```
-1/12*(12*x^11 + 66*x^10 + 220*x^9 + 495*x^8 + 792*x^7 + 924*x^6 + 792*x^5
 + 495*x^4 + 220*x^3 + 66*x^2 + 12*x + 1)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.67

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx = -\frac{x^{11}+\frac{11\,x^{10}}{2}+\frac{55\,x^9}{3}+\frac{165\,x^8}{4}+66\,x^7+77\,x^6+66\,x^5+\frac{165\,x^4}{4}+\frac{55\,x^3}{3}+\frac{11\,x^2}{2}+x+\frac{1}{12}}{x^{12}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^13,x)
```

output
```
-(x + (11*x^2)/2 + (55*x^3)/3 + (165*x^4)/4 + 66*x^5 + 77*x^6 + 66*x^7 + (
165*x^8)/4 + (55*x^9)/3 + (11*x^10)/2 + x^11 + 1/12)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{13}}\,dx$$
$$=\frac{-12x^{11}-66x^{10}-220x^9-495x^8-792x^7-924x^6-792x^5-495x^4-220x^3-66x^2-12x-1}{12x^{12}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^13,x)
```

output
```
( - 12*x**11 - 66*x**10 - 220*x**9 - 495*x**8 - 792*x**7 - 924*x**6 - 792*
x**5 - 495*x**4 - 220*x**3 - 66*x**2 - 12*x - 1)/(12*x**12)
```

3.242 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}} \, dx = -\frac{(1+x)^{12}}{13x^{13}}+\frac{(1+x)^{12}}{156x^{12}}$$

output
```
-1/13*(1+x)^12/x^13+1/156*(1+x)^12/x^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(25) = 50.

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}} \, dx = -\frac{1}{13x^{13}}-\frac{11}{12x^{12}}-\frac{5}{x^{11}}-\frac{33}{2x^{10}}-\frac{110}{3x^9}-\frac{231}{4x^8}-\frac{66}{x^7}-\frac{55}{x^6}-\frac{33}{x^5}-\frac{55}{4x^4}-\frac{11}{3x^3}-\frac{1}{2x^2}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^14,x]
```

output
```
-1/13*1/x^13 - 11/(12*x^12) - 5/x^11 - 33/(2*x^10) - 110/(3*x^9) - 231/(4*
x^8) - 66/x^7 - 55/x^6 - 33/x^5 - 55/(4*x^4) - 11/(3*x^3) - 1/(2*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{14}}dx$$
$$\Big\downarrow \text{1184}$$
$$\int \frac{(x+1)^{11}}{x^{14}}dx$$
$$\Big\downarrow \text{55}$$
$$-\frac{1}{13}\int \frac{(x+1)^{11}}{x^{13}}dx-\frac{(x+1)^{12}}{13x^{13}}$$
$$\Big\downarrow \text{48}$$
$$\frac{(x+1)^{12}}{156x^{12}}-\frac{(x+1)^{12}}{13x^{13}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^14,x]
```

output
```
-1/13*(1 + x)^12/x^13 + (1 + x)^12/(156*x^12)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(21) = 42$.

Time = 0.75 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

method	result	size
norman	$\frac{-\frac{1}{2}x^{11}-\frac{11}{3}x^{10}-\frac{55}{4}x^{9}-33x^{8}-55x^{7}-66x^{6}-\frac{231}{4}x^{5}-\frac{110}{3}x^{4}-\frac{33}{2}x^{3}-5x^{2}-\frac{11}{12}x-\frac{1}{13}}{x^{13}}$	60
risch	$\frac{-\frac{1}{2}x^{11}-\frac{11}{3}x^{10}-\frac{55}{4}x^{9}-33x^{8}-55x^{7}-66x^{6}-\frac{231}{4}x^{5}-\frac{110}{3}x^{4}-\frac{33}{2}x^{3}-5x^{2}-\frac{11}{12}x-\frac{1}{13}}{x^{13}}$	60
gosper	$-\frac{78x^{11}+572x^{10}+2145x^{9}+5148x^{8}+8580x^{7}+10296x^{6}+9009x^{5}+5720x^{4}+2574x^{3}+780x^{2}+143x+12}{156x^{13}}$	61
parallelrisch	$\frac{-78x^{11}-572x^{10}-2145x^{9}-5148x^{8}-8580x^{7}-10296x^{6}-9009x^{5}-5720x^{4}-2574x^{3}-780x^{2}-143x-12}{156x^{13}}$	61
default	$-\frac{33}{x^{5}}-\frac{55}{x^{6}}-\frac{11}{3x^{3}}-\frac{1}{2x^{2}}-\frac{1}{13x^{13}}-\frac{231}{4x^{8}}-\frac{55}{4x^{4}}-\frac{110}{3x^{9}}-\frac{66}{x^{7}}-\frac{33}{2x^{10}}-\frac{5}{x^{11}}-\frac{11}{12x^{12}}$	62
orering	$-\frac{(78x^{11}+572x^{10}+2145x^{9}+5148x^{8}+8580x^{7}+10296x^{6}+9009x^{5}+5720x^{4}+2574x^{3}+780x^{2}+143x+12)\left(x^{2}+2x+1\right)^{5}}{156x^{13}(x+1)^{10}}$	76

input
```
int((x+1)*(x^2+2*x+1)^5/x^14,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^11-11/3*x^10-55/4*x^9-33*x^8-55*x^7-66*x^6-231/4*x^5-110/3*x^4-33/
2*x^3-5*x^2-11/12*x-1/13)/x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = -\frac{78\,x^{11}+572\,x^{10}+2145\,x^{9}+5148\,x^{8}+8580\,x^{7}+10296\,x^{6}+9009\,x^{5}+5720\,x^{4}+2574\,x^{3}+780\,x^{2}+}{156\,x^{13}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^14,x, algorithm="fricas")
```

output
```
-1/156*(78*x^11 + 572*x^10 + 2145*x^9 + 5148*x^8 + 8580*x^7 + 10296*x^6 +
9009*x^5 + 5720*x^4 + 2574*x^3 + 780*x^2 + 143*x + 12)/x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = \frac{-78x^{11}-572x^{10}-2145x^{9}-5148x^{8}-8580x^{7}-10296x^{6}-9009x^{5}-5720x^{4}-2574x^{3}-780x^{2}-143}{156x^{13}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**14,x)
```

output
```
(-78*x**11 - 572*x**10 - 2145*x**9 - 5148*x**8 - 8580*x**7 - 10296*x**6 -
9009*x**5 - 5720*x**4 - 2574*x**3 - 780*x**2 - 143*x - 12)/(156*x**13)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = -\frac{78\,x^{11}+572\,x^{10}+2145\,x^{9}+5148\,x^{8}+8580\,x^{7}+10296\,x^{6}+9009\,x^{5}+5720\,x^{4}+2574\,x^{3}+780\,x^{2}+}{156\,x^{13}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^14,x, algorithm="maxima")
```

output
```
-1/156*(78*x^11 + 572*x^10 + 2145*x^9 + 5148*x^8 + 8580*x^7 + 10296*x^6 +
9009*x^5 + 5720*x^4 + 2574*x^3 + 780*x^2 + 143*x + 12)/x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx = -\frac{78\,x^{11}+572\,x^{10}+2145\,x^{9}+5148\,x^{8}+8580\,x^{7}+10296\,x^{6}+9009\,x^{5}+5720\,x^{4}+2574\,x^{3}+780\,x^{2}+}{156\,x^{13}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^14,x, algorithm="giac")
```

output
```
-1/156*(78*x^11 + 572*x^10 + 2145*x^9 + 5148*x^8 + 8580*x^7 + 10296*x^6 +
9009*x^5 + 5720*x^4 + 2574*x^3 + 780*x^2 + 143*x + 12)/x^13
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx$$
$$= -\frac{\frac{x^{11}}{2}+\frac{11\,x^{10}}{3}+\frac{55\,x^9}{4}+33\,x^8+55\,x^7+66\,x^6+\frac{231\,x^5}{4}+\frac{110\,x^4}{3}+\frac{33\,x^3}{2}+5\,x^2+\frac{11\,x}{12}+\frac{1}{13}}{x^{13}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^14,x)
```

output
```
-((11*x)/12 + 5*x^2 + (33*x^3)/2 + (110*x^4)/3 + (231*x^5)/4 + 66*x^6 + 55
*x^7 + 33*x^8 + (55*x^9)/4 + (11*x^10)/3 + x^11/2 + 1/13)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{14}}\,dx$$
$$= \frac{-78x^{11}-572x^{10}-2145x^9-5148x^8-8580x^7-10296x^6-9009x^5-5720x^4-2574x^3-780x^2-143x}{156x^{13}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^14,x)
```

output
```
( - 78*x**11 - 572*x**10 - 2145*x**9 - 5148*x**8 - 8580*x**7 - 10296*x**6
- 9009*x**5 - 5720*x**4 - 2574*x**3 - 780*x**2 - 143*x - 12)/(156*x**13)
```

3.243 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}} \, dx = -\frac{(1+x)^{12}}{14x^{14}}+\frac{(1+x)^{12}}{91x^{13}}-\frac{(1+x)^{12}}{1092x^{12}}$$

output
```
-1/14*(1+x)^12/x^14+1/91*(1+x)^12/x^13-1/1092*(1+x)^12/x^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.14

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}} \, dx = -\frac{1}{14x^{14}}-\frac{11}{13x^{13}}-\frac{55}{12x^{12}}-\frac{15}{x^{11}}-\frac{33}{x^{10}}-\frac{154}{3x^9}-\frac{231}{4x^8}-\frac{330}{7x^7}-\frac{55}{2x^6}-\frac{11}{x^5}-\frac{11}{4x^4}-\frac{1}{3x^3}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^15,x]
```

output
```
-1/14*1/x^14 - 11/(13*x^13) - 55/(12*x^12) - 15/x^11 - 33/x^10 - 154/(3*x^
9) - 231/(4*x^8) - 330/(7*x^7) - 55/(2*x^6) - 11/x^5 - 11/(4*x^4) - 1/(3*x
^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1184, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{15}}\,dx$$

$$\Big\downarrow \ 1184$$

$$\int \frac{(x+1)^{11}}{x^{15}}dx$$

$$\Big\downarrow \ 55$$

$$-\frac{1}{7}\int \frac{(x+1)^{11}}{x^{14}}dx-\frac{(x+1)^{12}}{14x^{14}}$$

$$\Big\downarrow \ 55$$

$$\frac{1}{7}\left(\frac{1}{13}\int \frac{(x+1)^{11}}{x^{13}}dx+\frac{(x+1)^{12}}{13x^{13}}\right)-\frac{(x+1)^{12}}{14x^{14}}$$

$$\Big\downarrow \ 48$$

$$\frac{1}{7}\left(\frac{(x+1)^{12}}{13x^{13}}-\frac{(x+1)^{12}}{156x^{12}}\right)-\frac{(x+1)^{12}}{14x^{14}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^15,x]
```

output
```
-1/14*(1 + x)^12/x^14 + ((1 + x)^12/(13*x^13) - (1 + x)^12/(156*x^12))/7
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

method	result
norman	$\frac{-\frac{1}{14}-\frac{11}{13}x-\frac{55}{12}x^2-15x^3-33x^4-\frac{154}{3}x^5-\frac{231}{4}x^6-\frac{330}{7}x^7-\frac{55}{2}x^8-11x^9-\frac{11}{4}x^{10}-\frac{1}{3}x^{11}}{x^{14}}$
risch	$\frac{-\frac{1}{14}-\frac{11}{13}x-\frac{55}{12}x^2-15x^3-33x^4-\frac{154}{3}x^5-\frac{231}{4}x^6-\frac{330}{7}x^7-\frac{55}{2}x^8-11x^9-\frac{11}{4}x^{10}-\frac{1}{3}x^{11}}{x^{14}}$
gosper	$-\frac{364x^{11}+3003x^{10}+12012x^9+30030x^8+51480x^7+63063x^6+56056x^5+36036x^4+16380x^3+5005x^2+924x+78}{1092x^{14}}$
parallelrisch	$\frac{-364x^{11}-3003x^{10}-12012x^9-30030x^8-51480x^7-63063x^6-56056x^5-36036x^4-16380x^3-5005x^2-924x-78}{1092x^{14}}$
default	$-\frac{11}{x^5}-\frac{55}{2x^6}-\frac{1}{3x^3}-\frac{11}{13x^{13}}-\frac{231}{4x^8}-\frac{11}{4x^4}-\frac{1}{14x^{14}}-\frac{154}{3x^9}-\frac{330}{7x^7}-\frac{33}{x^{10}}-\frac{15}{x^{11}}-\frac{55}{12x^{12}}$
orering	$-\frac{(364x^{11}+3003x^{10}+12012x^9+30030x^8+51480x^7+63063x^6+56056x^5+36036x^4+16380x^3+5005x^2+924x+78)(x^2+2x+1}{1092x^{14}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^15,x,method=_RETURNVERBOSE)
```

output
```
(-1/14-11/13*x-55/12*x^2-15*x^3-33*x^4-154/3*x^5-231/4*x^6-330/7*x^7-55/2*
x^8-11*x^9-11/4*x^10-1/3*x^11)/x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{364\,x^{11}+3003\,x^{10}+12012\,x^{9}+30030\,x^{8}+51480\,x^{7}+63063\,x^{6}+56056\,x^{5}+36036\,x^{4}+16380\,x^{3}+}{1092\,x^{14}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^15,x, algorithm="fricas")
```

output
```
-1/1092*(364*x^11 + 3003*x^10 + 12012*x^9 + 30030*x^8 + 51480*x^7 + 63063*
x^6 + 56056*x^5 + 36036*x^4 + 16380*x^3 + 5005*x^2 + 924*x + 78)/x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = \frac{-364x^{11}-3003x^{10}-12012x^{9}-30030x^{8}-51480x^{7}-63063x^{6}-56056x^{5}-36036x^{4}-16380x^{3}-500}{1092x^{14}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**15,x)
```

output
```
(-364*x**11 - 3003*x**10 - 12012*x**9 - 30030*x**8 - 51480*x**7 - 63063*x*
*6 - 56056*x**5 - 36036*x**4 - 16380*x**3 - 5005*x**2 - 924*x - 78)/(1092*
x**14)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{364\,x^{11}+3003\,x^{10}+12012\,x^9+30030\,x^8+51480\,x^7+63063\,x^6+56056\,x^5+36036\,x^4+16380\,x^3+}{1092\,x^{14}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^15,x, algorithm="maxima")
```

output
```
-1/1092*(364*x^11 + 3003*x^10 + 12012*x^9 + 30030*x^8 + 51480*x^7 + 63063*
x^6 + 56056*x^5 + 36036*x^4 + 16380*x^3 + 5005*x^2 + 924*x + 78)/x^14
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{364\,x^{11}+3003\,x^{10}+12012\,x^9+30030\,x^8+51480\,x^7+63063\,x^6+56056\,x^5+36036\,x^4+16380\,x^3+}{1092\,x^{14}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^15,x, algorithm="giac")
```

output
```
-1/1092*(364*x^11 + 3003*x^10 + 12012*x^9 + 30030*x^8 + 51480*x^7 + 63063*
x^6 + 56056*x^5 + 36036*x^4 + 16380*x^3 + 5005*x^2 + 924*x + 78)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = -\frac{\frac{x^{11}}{3}+\frac{11\,x^{10}}{4}+11\,x^9+\frac{55\,x^8}{2}+\frac{330\,x^7}{7}+\frac{231\,x^6}{4}+\frac{154\,x^5}{3}+33\,x^4+15\,x^3+\frac{55\,x^2}{12}+\frac{11\,x}{13}+\frac{1}{14}}{x^{14}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^15,x)
```

output
```
-((11*x)/13 + (55*x^2)/12 + 15*x^3 + 33*x^4 + (154*x^5)/3 + (231*x^6)/4 +
(330*x^7)/7 + (55*x^8)/2 + 11*x^9 + (11*x^10)/4 + x^11/3 + 1/14)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{15}}\,dx = \frac{-364x^{11}-3003x^{10}-12012x^9-30030x^8-51480x^7-63063x^6-56056x^5-36036x^4-16380x^3-500}{1092x^{14}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^15,x)
```

output
```
( - 364*x**11 - 3003*x**10 - 12012*x**9 - 30030*x**8 - 51480*x**7 - 63063*
x**6 - 56056*x**5 - 36036*x**4 - 16380*x**3 - 5005*x**2 - 924*x - 78)/(109
2*x**14)
```

3.244 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{(1+x)^{12}}{15x^{15}} + \frac{(1+x)^{12}}{70x^{14}} - \frac{(1+x)^{12}}{455x^{13}} + \frac{(1+x)^{12}}{5460x^{12}}$$

output
```
-1/15*(1+x)^12/x^15+1/70*(1+x)^12/x^14-1/455*(1+x)^12/x^13+1/5460*(1+x)^12
/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{1}{15x^{15}} - \frac{11}{14x^{14}} - \frac{55}{13x^{13}} - \frac{55}{4x^{12}} - \frac{30}{x^{11}} - \frac{231}{5x^{10}} - \frac{154}{3x^9} - \frac{165}{4x^8} - \frac{165}{7x^7} - \frac{55}{6x^6} - \frac{11}{5x^5} - \frac{1}{4x^4}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^16,x]
```

output
```
-1/15*1/x^15 - 11/(14*x^14) - 55/(13*x^13) - 55/(4*x^12) - 30/x^11 - 231/(
5*x^10) - 154/(3*x^9) - 165/(4*x^8) - 165/(7*x^7) - 55/(6*x^6) - 11/(5*x^5
) - 1/(4*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1184, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{16}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{16}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{1}{5}\int \frac{(x+1)^{11}}{x^{15}}dx-\frac{(x+1)^{12}}{15x^{15}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{5}\left(\frac{1}{7}\int \frac{(x+1)^{11}}{x^{14}}dx+\frac{(x+1)^{12}}{14x^{14}}\right)-\frac{(x+1)^{12}}{15x^{15}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{5}\left(\frac{1}{7}\left(-\frac{1}{13}\int \frac{(x+1)^{11}}{x^{13}}dx-\frac{(x+1)^{12}}{13x^{13}}\right)+\frac{(x+1)^{12}}{14x^{14}}\right)-\frac{(x+1)^{12}}{15x^{15}}$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{5}\left(\frac{(x+1)^{12}}{14x^{14}}+\frac{1}{7}\left(\frac{(x+1)^{12}}{156x^{12}}-\frac{(x+1)^{12}}{13x^{13}}\right)\right)-\frac{(x+1)^{12}}{15x^{15}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^16,x]
```

output
```
-1/15*(1 + x)^12/x^15 + ((1 + x)^12/(14*x^14) + (-1/13*(1 + x)^12/x^13 + (
1 + x)^12/(156*x^12))/7)/5
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result
norman	$\frac{-\frac{1}{15}-\frac{11}{14}x-\frac{55}{13}x^2-\frac{55}{4}x^3-30x^4-\frac{231}{5}x^5-\frac{154}{3}x^6-\frac{165}{4}x^7-\frac{165}{7}x^8-\frac{55}{6}x^9-\frac{11}{5}x^{10}-\frac{1}{4}x^{11}}{x^{15}}$
risch	$\frac{-\frac{1}{15}-\frac{11}{14}x-\frac{55}{13}x^2-\frac{55}{4}x^3-30x^4-\frac{231}{5}x^5-\frac{154}{3}x^6-\frac{165}{4}x^7-\frac{165}{7}x^8-\frac{55}{6}x^9-\frac{11}{5}x^{10}-\frac{1}{4}x^{11}}{x^{15}}$
gosper	$-\frac{1365x^{11}+12012x^{10}+50050x^9+128700x^8+225225x^7+280280x^6+252252x^5+163800x^4+75075x^3+23100x^2+4290x+364}{5460x^{15}}$
parallelrisch	$\frac{-1365x^{11}-12012x^{10}-50050x^9-128700x^8-225225x^7-280280x^6-252252x^5-163800x^4-75075x^3-23100x^2-4290x-364}{5460x^{15}}$
default	$-\frac{11}{5x^5}-\frac{55}{6x^6}-\frac{55}{13x^{13}}-\frac{165}{4x^8}-\frac{1}{4x^4}-\frac{11}{14x^{14}}-\frac{154}{3x^9}-\frac{165}{7x^7}-\frac{231}{5x^{10}}-\frac{1}{15x^{15}}-\frac{30}{x^{11}}-\frac{55}{4x^{12}}$
orering	$-\frac{(1365x^{11}+12012x^{10}+50050x^9+128700x^8+225225x^7+280280x^6+252252x^5+163800x^4+75075x^3+23100x^2+4290x+364}{5460x^{15}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^16,x,method=_RETURNVERBOSE)
```

output
```
(-1/15-11/14*x-55/13*x^2-55/4*x^3-30*x^4-231/5*x^5-154/3*x^6-165/4*x^7-165
/7*x^8-55/6*x^9-11/5*x^10-1/4*x^11)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{1365\,x^{11}+12012\,x^{10}+50050\,x^{9}+128700\,x^{8}+225225\,x^{7}+280280\,x^{6}+252252\,x^{5}+163800\,x^{4}+75}{5460\,x^{15}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^16,x, algorithm="fricas")
```

output
```
-1/5460*(1365*x^11 + 12012*x^10 + 50050*x^9 + 128700*x^8 + 225225*x^7 + 28
0280*x^6 + 252252*x^5 + 163800*x^4 + 75075*x^3 + 23100*x^2 + 4290*x + 364)
/x^15
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = \frac{-1365x^{11}-12012x^{10}-50050x^{9}-128700x^{8}-225225x^{7}-280280x^{6}-252252x^{5}-163800x^{4}-75075}{5460x^{15}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**16,x)
```

output
```
(-1365*x**11 - 12012*x**10 - 50050*x**9 - 128700*x**8 - 225225*x**7 - 2802
80*x**6 - 252252*x**5 - 163800*x**4 - 75075*x**3 - 23100*x**2 - 4290*x - 3
64)/(5460*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{1365\,x^{11}+12012\,x^{10}+50050\,x^9+128700\,x^8+225225\,x^7+280280\,x^6+252252\,x^5+163800\,x^4+75\ldots}{5460\,x^{15}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^16,x, algorithm="maxima")
```

output
```
-1/5460*(1365*x^11 + 12012*x^10 + 50050*x^9 + 128700*x^8 + 225225*x^7 + 28
0280*x^6 + 252252*x^5 + 163800*x^4 + 75075*x^3 + 23100*x^2 + 4290*x + 364)
/x^15
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{1365\,x^{11}+12012\,x^{10}+50050\,x^9+128700\,x^8+225225\,x^7+280280\,x^6+252252\,x^5+163800\,x^4+75\ldots}{5460\,x^{15}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^16,x, algorithm="giac")
```

output
```
-1/5460*(1365*x^11 + 12012*x^10 + 50050*x^9 + 128700*x^8 + 225225*x^7 + 28
0280*x^6 + 252252*x^5 + 163800*x^4 + 75075*x^3 + 23100*x^2 + 4290*x + 364)
/x^15
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = -\frac{\frac{x^{11}}{4}+\frac{11\,x^{10}}{5}+\frac{55\,x^9}{6}+\frac{165\,x^8}{7}+\frac{165\,x^7}{4}+\frac{154\,x^6}{3}+\frac{231\,x^5}{5}+30\,x^4+\frac{55\,x^3}{4}+\frac{55\,x^2}{13}+\frac{11\,x}{14}+\frac{1}{15}}{x^{15}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^16,x)
```

output
```
-((11*x)/14 + (55*x^2)/13 + (55*x^3)/4 + 30*x^4 + (231*x^5)/5 + (154*x^6)/
3 + (165*x^7)/4 + (165*x^8)/7 + (55*x^9)/6 + (11*x^10)/5 + x^11/4 + 1/15)/
x^15
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{16}}\,dx = \frac{-1365x^{11}-12012x^{10}-50050x^9-128700x^8-225225x^7-280280x^6-252252x^5-163800x^4-75075}{5460x^{15}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^16,x)
```

output
```
( - 1365*x**11 - 12012*x**10 - 50050*x**9 - 128700*x**8 - 225225*x**7 - 28
0280*x**6 - 252252*x**5 - 163800*x**4 - 75075*x**3 - 23100*x**2 - 4290*x -
 364)/(5460*x**15)
```

3.245 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}} \, dx = -\frac{(1+x)^{12}}{16x^{16}}+\frac{(1+x)^{12}}{60x^{15}}-\frac{(1+x)^{12}}{280x^{14}}+\frac{(1+x)^{12}}{1820x^{13}}-\frac{(1+x)^{12}}{21840x^{12}}$$

output
```
-1/16*(1+x)^12/x^16+1/60*(1+x)^12/x^15-1/280*(1+x)^12/x^14+1/1820*(1+x)^12
/x^13-1/21840*(1+x)^12/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}} \, dx = -\frac{1}{16x^{16}}-\frac{11}{15x^{15}}-\frac{55}{14x^{14}}-\frac{165}{13x^{13}}-\frac{55}{2x^{12}} -\frac{42}{x^{11}}-\frac{231}{5x^{10}}-\frac{110}{3x^9}-\frac{165}{8x^8}-\frac{55}{7x^7}-\frac{11}{6x^6}-\frac{1}{5x^5}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^17,x]
```

output
```
-1/16*1/x^16 - 11/(15*x^15) - 55/(14*x^14) - 165/(13*x^13) - 55/(2*x^12) -
 42/x^11 - 231/(5*x^10) - 110/(3*x^9) - 165/(8*x^8) - 55/(7*x^7) - 11/(6*x
^6) - 1/(5*x^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1184, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{17}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{17}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{1}{4}\int \frac{(x+1)^{11}}{x^{16}}dx-\frac{(x+1)^{12}}{16x^{16}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{4}\left(\frac{1}{5}\int \frac{(x+1)^{11}}{x^{15}}dx+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{7}\int \frac{(x+1)^{11}}{x^{14}}dx-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{4}\left(\frac{1}{5}\left(\frac{1}{7}\left(\frac{1}{13}\int \frac{(x+1)^{11}}{x^{13}}dx+\frac{(x+1)^{12}}{13x^{13}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{4}\left(\frac{(x+1)^{12}}{15x^{15}}+\frac{1}{5}\left(\frac{1}{7}\left(\frac{(x+1)^{12}}{13x^{13}}-\frac{(x+1)^{12}}{156x^{12}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)\right)-\frac{(x+1)^{12}}{16x^{16}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^17,x]
```

output
```
-1/16*(1 + x)^12/x^16 + ((1 + x)^12/(15*x^15) + (-1/14*(1 + x)^12/x^14 + (
(1 + x)^12/(13*x^13) - (1 + x)^12/(156*x^12))/7)/5)/4
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result
norman	$\frac{-\frac{1}{16}-\frac{11}{15}x-\frac{55}{14}x^2-\frac{165}{13}x^3-\frac{55}{2}x^4-42x^5-\frac{231}{5}x^6-\frac{110}{3}x^7-\frac{165}{8}x^8-\frac{55}{7}x^9-\frac{11}{6}x^{10}-\frac{1}{5}x^{11}}{x^{16}}$
risch	$\frac{-\frac{1}{16}-\frac{11}{15}x-\frac{55}{14}x^2-\frac{165}{13}x^3-\frac{55}{2}x^4-42x^5-\frac{231}{5}x^6-\frac{110}{3}x^7-\frac{165}{8}x^8-\frac{55}{7}x^9-\frac{11}{6}x^{10}-\frac{1}{5}x^{11}}{x^{16}}$
gosper	$-\frac{4368x^{11}+40040x^{10}+171600x^9+450450x^8+800800x^7+1009008x^6+917280x^5+600600x^4+277200x^3+85800x^2+16016x+}{21840x^{16}}$
parallelrisch	$\frac{-4368x^{11}-40040x^{10}-171600x^9-450450x^8-800800x^7-1009008x^6-917280x^5-600600x^4-277200x^3-85800x^2-16016x-}{21840x^{16}}$
default	$-\frac{1}{5x^5}-\frac{11}{6x^6}-\frac{165}{13x^{13}}-\frac{165}{8x^8}-\frac{55}{14x^{14}}-\frac{110}{3x^9}-\frac{55}{7x^7}-\frac{231}{5x^{10}}-\frac{11}{15x^{15}}-\frac{42}{x^{11}}-\frac{1}{16x^{16}}-\frac{55}{2x^{12}}$
orering	$-\frac{(4368x^{11}+40040x^{10}+171600x^9+450450x^8+800800x^7+1009008x^6+917280x^5+600600x^4+277200x^3+85800x^2+16016x}{21840x^{16}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^17,x,method=_RETURNVERBOSE)
```

output
```
(-1/16-11/15*x-55/14*x^2-165/13*x^3-55/2*x^4-42*x^5-231/5*x^6-110/3*x^7-16
5/8*x^8-55/7*x^9-11/6*x^10-1/5*x^11)/x^16
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{4368\,x^{11}+40040\,x^{10}+171600\,x^9+450450\,x^8+800800\,x^7+1009008\,x^6+917280\,x^5+600600\,x^4+}{21840\,x^{16}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^17,x, algorithm="fricas")
```

output
```
-1/21840*(4368*x^11 + 40040*x^10 + 171600*x^9 + 450450*x^8 + 800800*x^7 +
1009008*x^6 + 917280*x^5 + 600600*x^4 + 277200*x^3 + 85800*x^2 + 16016*x +
 1365)/x^16
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx$$
$$= \frac{-4368x^{11} - 40040x^{10} - 171600x^9 - 450450x^8 - 800800x^7 - 1009008x^6 - 917280x^5 - 600600x^4 - 277}{21840x^{16}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**17,x)
```

output
```
(-4368*x**11 - 40040*x**10 - 171600*x**9 - 450450*x**8 - 800800*x**7 - 100
9008*x**6 - 917280*x**5 - 600600*x**4 - 277200*x**3 - 85800*x**2 - 16016*x
 - 1365)/(21840*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx =$$
$$-\frac{4368\,x^{11} + 40040\,x^{10} + 171600\,x^9 + 450450\,x^8 + 800800\,x^7 + 1009008\,x^6 + 917280\,x^5 + 600600\,x^4 + }{21840\,x^{16}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^17,x, algorithm="maxima")
```

output
```
-1/21840*(4368*x^11 + 40040*x^10 + 171600*x^9 + 450450*x^8 + 800800*x^7 +
1009008*x^6 + 917280*x^5 + 600600*x^4 + 277200*x^3 + 85800*x^2 + 16016*x +
 1365)/x^16
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{4368\,x^{11}+40040\,x^{10}+171600\,x^9+450450\,x^8+800800\,x^7+1009008\,x^6+917280\,x^5+600600\,x^4+}{21840\,x^{16}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^17,x, algorithm="giac")
```

output
```
-1/21840*(4368*x^11 + 40040*x^10 + 171600*x^9 + 450450*x^8 + 800800*x^7 +
1009008*x^6 + 917280*x^5 + 600600*x^4 + 277200*x^3 + 85800*x^2 + 16016*x +
 1365)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx = -\frac{\frac{x^{11}}{5}+\frac{11\,x^{10}}{6}+\frac{55\,x^9}{7}+\frac{165\,x^8}{8}+\frac{110\,x^7}{3}+\frac{231\,x^6}{5}+42\,x^5+\frac{55\,x^4}{2}+\frac{165\,x^3}{13}+\frac{55\,x^2}{14}+\frac{11\,x}{15}+\frac{1}{16}}{x^{16}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^17,x)
```

output
```
-((11*x)/15 + (55*x^2)/14 + (165*x^3)/13 + (55*x^4)/2 + 42*x^5 + (231*x^6)
/5 + (110*x^7)/3 + (165*x^8)/8 + (55*x^9)/7 + (11*x^10)/6 + x^11/5 + 1/16)
/x^16
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{17}}\,dx$$
$$= \frac{-4368x^{11} - 40040x^{10} - 171600x^9 - 450450x^8 - 800800x^7 - 1009008x^6 - 917280x^5 - 600600x^4 - 277}{21840x^{16}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^17,x)
```

output
```
( - 4368*x**11 - 40040*x**10 - 171600*x**9 - 450450*x**8 - 800800*x**7 - 1
009008*x**6 - 917280*x**5 - 600600*x**4 - 277200*x**3 - 85800*x**2 - 16016
*x - 1365)/(21840*x**16)
```

3.246 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{18}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{(1+x)^{12}}{17x^{17}}+\frac{5(1+x)^{12}}{272x^{16}}-\frac{(1+x)^{12}}{204x^{15}}+\frac{(1+x)^{12}}{952x^{14}}-\frac{(1+x)^{12}}{6188x^{13}}+\frac{(1+x)^{12}}{74256x^{12}}$$

output
```
-1/17*(1+x)^12/x^17+5/272*(1+x)^12/x^16-1/204*(1+x)^12/x^15+1/952*(1+x)^12
/x^14-1/6188*(1+x)^12/x^13+1/74256*(1+x)^12/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{1}{17x^{17}}-\frac{11}{16x^{16}}-\frac{11}{3x^{15}}-\frac{165}{14x^{14}}-\frac{330}{13x^{13}}-\frac{77}{2x^{12}}-\frac{42}{x^{11}}-\frac{33}{x^{10}}-\frac{55}{3x^9}-\frac{55}{8x^8}-\frac{11}{7x^7}-\frac{1}{6x^6}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^18,x]
```

output
```
-1/17*1/x^17 - 11/(16*x^16) - 11/(3*x^15) - 165/(14*x^14) - 330/(13*x^13)
- 77/(2*x^12) - 42/x^11 - 33/x^10 - 55/(3*x^9) - 55/(8*x^8) - 11/(7*x^7) -
 1/(6*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1184, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{18}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{18}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{5}{17}\int \frac{(x+1)^{11}}{x^{17}}dx-\frac{(x+1)^{12}}{17x^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{5}{17}\left(-\frac{1}{4}\int \frac{(x+1)^{11}}{x^{16}}dx-\frac{(x+1)^{12}}{16x^{16}}\right)-\frac{(x+1)^{12}}{17x^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\int \frac{(x+1)^{11}}{x^{15}}dx+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)-\frac{(x+1)^{12}}{17x^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{7}\int \frac{(x+1)^{11}}{x^{14}}dx-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)-\frac{(x+1)^{12}}{17x^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(\frac{1}{7}\left(\frac{1}{13}\int\frac{(x+1)^{11}}{x^{13}}dx+\frac{(x+1)^{12}}{13x^{13}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)-\frac{(x+1)^{12}}{17x^{17}}$$

$$\Big\downarrow 48$$

$$-\frac{(x+1)^{12}}{17x^{17}}-\frac{5}{17}\left(\frac{1}{4}\left(\frac{(x+1)^{12}}{15x^{15}}+\frac{1}{5}\left(\frac{1}{7}\left(\frac{(x+1)^{12}}{13x^{13}}-\frac{(x+1)^{12}}{156x^{12}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)\right)-\frac{(x+1)^{12}}{16x^{16}}\right)$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^18,x]
```

output
```
-1/17*(1 + x)^12/x^17 - (5*(-1/16*(1 + x)^12/x^16 + ((1 + x)^12/(15*x^15)
+ (-1/14*(1 + x)^12/x^14 + ((1 + x)^12/(13*x^13) - (1 + x)^12/(156*x^12))/
7)/5)/4))/17
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result
norman	$\frac{-\frac{1}{17}-\frac{11}{16}x-\frac{11}{3}x^{2}-\frac{165}{14}x^{3}-\frac{330}{13}x^{4}-\frac{77}{2}x^{5}-42x^{6}-33x^{7}-\frac{55}{3}x^{8}-\frac{55}{8}x^{9}-\frac{11}{7}x^{10}-\frac{1}{6}x^{11}}{x^{17}}$
risch	$\frac{-\frac{1}{17}-\frac{11}{16}x-\frac{11}{3}x^{2}-\frac{165}{14}x^{3}-\frac{330}{13}x^{4}-\frac{77}{2}x^{5}-42x^{6}-33x^{7}-\frac{55}{3}x^{8}-\frac{55}{8}x^{9}-\frac{11}{7}x^{10}-\frac{1}{6}x^{11}}{x^{17}}$
gosper	$-\frac{12376x^{11}+116688x^{10}+510510x^{9}+1361360x^{8}+2450448x^{7}+3118752x^{6}+2858856x^{5}+1884960x^{4}+875160x^{3}+272272x^{2}+5}{74256x^{17}}$
parallelrisch	$\frac{-12376x^{11}-116688x^{10}-510510x^{9}-1361360x^{8}-2450448x^{7}-3118752x^{6}-2858856x^{5}-1884960x^{4}-875160x^{3}-272272x^{2}-5}{74256x^{17}}$
default	$-\frac{1}{6x^{6}}-\frac{330}{13x^{13}}-\frac{55}{8x^{8}}-\frac{165}{14x^{14}}-\frac{55}{3x^{9}}-\frac{11}{7x^{7}}-\frac{33}{x^{10}}-\frac{1}{17x^{17}}-\frac{11}{3x^{15}}-\frac{42}{x^{11}}-\frac{11}{16x^{16}}-\frac{77}{2x^{12}}$
orering	$-\frac{(12376x^{11}+116688x^{10}+510510x^{9}+1361360x^{8}+2450448x^{7}+3118752x^{6}+2858856x^{5}+1884960x^{4}+875160x^{3}+272272x^{2}+}{74256x^{17}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^18,x,method=_RETURNVERBOSE)
```

output
```
(-1/17-11/16*x-11/3*x^2-165/14*x^3-330/13*x^4-77/2*x^5-42*x^6-33*x^7-55/3*
x^8-55/8*x^9-11/7*x^10-1/6*x^11)/x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{18}}\,dx = -\frac{12376\,x^{11}+116688\,x^{10}+510510\,x^{9}+1361360\,x^{8}+2450448\,x^{7}+3118752\,x^{6}+2858856\,x^{5}+188496}{74256\,x^{17}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^18,x, algorithm="fricas")
```

output
```
-1/74256*(12376*x^11 + 116688*x^10 + 510510*x^9 + 1361360*x^8 + 2450448*x^
7 + 3118752*x^6 + 2858856*x^5 + 1884960*x^4 + 875160*x^3 + 272272*x^2 + 51
051*x + 4368)/x^17
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx$$
$$= \frac{-12376x^{11} - 116688x^{10} - 510510x^9 - 1361360x^8 - 2450448x^7 - 3118752x^6 - 2858856x^5 - 1884960x^4}{74256x^{17}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**18,x)
```

output
```
(-12376*x**11 - 116688*x**10 - 510510*x**9 - 1361360*x**8 - 2450448*x**7 -
 3118752*x**6 - 2858856*x**5 - 1884960*x**4 - 875160*x**3 - 272272*x**2 -
51051*x - 4368)/(74256*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx =$$
$$-\frac{12376\,x^{11} + 116688\,x^{10} + 510510\,x^9 + 1361360\,x^8 + 2450448\,x^7 + 3118752\,x^6 + 2858856\,x^5 + 188496}{74256\,x^{17}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^18,x, algorithm="maxima")
```

output
```
-1/74256*(12376*x^11 + 116688*x^10 + 510510*x^9 + 1361360*x^8 + 2450448*x^
7 + 3118752*x^6 + 2858856*x^5 + 1884960*x^4 + 875160*x^3 + 272272*x^2 + 51
051*x + 4368)/x^17
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{12376\,x^{11}+116688\,x^{10}+510510\,x^{9}+1361360\,x^{8}+2450448\,x^{7}+3118752\,x^{6}+2858856\,x^{5}+188496}{74256\,x^{17}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^18,x, algorithm="giac")
```

output
```
-1/74256*(12376*x^11 + 116688*x^10 + 510510*x^9 + 1361360*x^8 + 2450448*x^
7 + 3118752*x^6 + 2858856*x^5 + 1884960*x^4 + 875160*x^3 + 272272*x^2 + 51
051*x + 4368)/x^17
```

Mupad [B] (verification not implemented)

Time = 10.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx = -\frac{\frac{x^{11}}{6}+\frac{11\,x^{10}}{7}+\frac{55\,x^{9}}{8}+\frac{55\,x^{8}}{3}+33\,x^{7}+42\,x^{6}+\frac{77\,x^{5}}{2}+\frac{330\,x^{4}}{13}+\frac{165\,x^{3}}{14}+\frac{11\,x^{2}}{3}+\frac{11\,x}{16}+\frac{1}{17}}{x^{17}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^18,x)
```

output
```
-((11*x)/16 + (11*x^2)/3 + (165*x^3)/14 + (330*x^4)/13 + (77*x^5)/2 + 42*x
^6 + 33*x^7 + (55*x^8)/3 + (55*x^9)/8 + (11*x^10)/7 + x^11/6 + 1/17)/x^17
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{18}}\,dx$$
$$= \frac{-12376x^{11} - 116688x^{10} - 510510x^9 - 1361360x^8 - 2450448x^7 - 3118752x^6 - 2858856x^5 - 1884960x^4}{74256x^{17}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^18,x)
```

output
```
( - 12376*x**11 - 116688*x**10 - 510510*x**9 - 1361360*x**8 - 2450448*x**7
 - 3118752*x**6 - 2858856*x**5 - 1884960*x**4 - 875160*x**3 - 272272*x**2
- 51051*x - 4368)/(74256*x**17)
```

3.247 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{19}} dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{19}} dx = -\frac{(1+x)^{12}}{18x^{18}} + \frac{(1+x)^{12}}{51x^{17}} - \frac{5(1+x)^{12}}{816x^{16}} + \frac{(1+x)^{12}}{612x^{15}} - \frac{(1+x)^{12}}{2856x^{14}} + \frac{(1+x)^{12}}{18564x^{13}} - \frac{(1+x)^{12}}{222768x^{12}}$$

output
```
-1/18*(1+x)^12/x^18+1/51*(1+x)^12/x^17-5/816*(1+x)^12/x^16+1/612*(1+x)^12/
x^15-1/2856*(1+x)^12/x^14+1/18564*(1+x)^12/x^13-1/222768*(1+x)^12/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{19}} dx = -\frac{1}{18x^{18}} - \frac{11}{17x^{17}} - \frac{55}{16x^{16}} - \frac{11}{x^{15}} - \frac{165}{7x^{14}} - \frac{462}{13x^{13}} - \frac{77}{2x^{12}} - \frac{30}{x^{11}} - \frac{33}{2x^{10}} - \frac{55}{9x^9} - \frac{11}{8x^8} - \frac{1}{7x^7}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^19,x]
```

output
```
-1/18*1/x^18 - 11/(17*x^17) - 55/(16*x^16) - 11/x^15 - 165/(7*x^14) - 462/
(13*x^13) - 77/(2*x^12) - 30/x^11 - 33/(2*x^10) - 55/(9*x^9) - 11/(8*x^8)
- 1/(7*x^7)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1184, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{19}}dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{19}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{1}{3}\int \frac{(x+1)^{11}}{x^{18}}dx-\frac{(x+1)^{12}}{18x^{18}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{5}{17}\int \frac{(x+1)^{11}}{x^{17}}dx+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{5}{17}\left(-\frac{1}{4}\int \frac{(x+1)^{11}}{x^{16}}dx-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\int \frac{(x+1)^{11}}{x^{15}}dx+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{7}\int\frac{(x+1)^{11}}{x^{14}}dx-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}$$

$\Big\downarrow$ 55

$$\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(\frac{1}{7}\left(\frac{1}{13}\int\frac{(x+1)^{11}}{x^{13}}dx+\frac{(x+1)^{12}}{13x^{13}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}$$

$\Big\downarrow$ 48

$$\frac{1}{3}\left(\frac{(x+1)^{12}}{17x^{17}}+\frac{5}{17}\left(\frac{1}{4}\left(\frac{(x+1)^{12}}{15x^{15}}+\frac{1}{5}\left(\frac{1}{7}\left(\frac{(x+1)^{12}}{13x^{13}}-\frac{(x+1)^{12}}{156x^{12}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)\right)-\frac{(x+1)^{12}}{16x^{16}}\right)\right)-\frac{(x+1)^{12}}{18x^{18}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^19,x]
```

output
```
-1/18*(1 + x)^12/x^18 + ((1 + x)^12/(17*x^17) + (5*(-1/16*(1 + x)^12/x^16
+ ((1 + x)^12/(15*x^15) + (-1/14*(1 + x)^12/x^14 + ((1 + x)^12/(13*x^13) -
 (1 + x)^12/(156*x^12))/7)/5)/4))/17)/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

method	result
norman	$\frac{-\frac{1}{18}-\frac{11}{17}x-\frac{55}{16}x^2-11x^3-\frac{165}{7}x^4-\frac{462}{13}x^5-\frac{77}{2}x^6-30x^7-\frac{33}{2}x^8-\frac{55}{9}x^9-\frac{11}{8}x^{10}-\frac{1}{7}x^{11}}{x^{18}}$
risch	$\frac{-\frac{1}{18}-\frac{11}{17}x-\frac{55}{16}x^2-11x^3-\frac{165}{7}x^4-\frac{462}{13}x^5-\frac{77}{2}x^6-30x^7-\frac{33}{2}x^8-\frac{55}{9}x^9-\frac{11}{8}x^{10}-\frac{1}{7}x^{11}}{x^{18}}$
gosper	$-\frac{31824x^{11}+306306x^{10}+1361360x^9+3675672x^8+6683040x^7+8576568x^6+7916832x^5+5250960x^4+2450448x^3+765765x^2}{222768x^{18}}$
parallelrisch	$\frac{-31824x^{11}-306306x^{10}-1361360x^9-3675672x^8-6683040x^7-8576568x^6-7916832x^5-5250960x^4-2450448x^3-765765x^2-}{222768x^{18}}$
default	$-\frac{1}{18x^{18}}-\frac{462}{13x^{13}}-\frac{11}{8x^8}-\frac{165}{7x^{14}}-\frac{55}{9x^9}-\frac{1}{7x^7}-\frac{33}{2x^{10}}-\frac{11}{17x^{17}}-\frac{11}{x^{15}}-\frac{30}{x^{11}}-\frac{55}{16x^{16}}-\frac{77}{2x^{12}}$
orering	$-\frac{(31824x^{11}+306306x^{10}+1361360x^9+3675672x^8+6683040x^7+8576568x^6+7916832x^5+5250960x^4+2450448x^3+765765x}{222768x^{18}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^19,x,method=_RETURNVERBOSE)
```

output
```
(-1/18-11/17*x-55/16*x^2-11*x^3-165/7*x^4-462/13*x^5-77/2*x^6-30*x^7-33/2*
x^8-55/9*x^9-11/8*x^10-1/7*x^11)/x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{31824\,x^{11}+306306\,x^{10}+1361360\,x^9+3675672\,x^8+6683040\,x^7+8576568\,x^6+7916832\,x^5+52509}{222768\,x^{18}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^19,x, algorithm="fricas")
```

output
```
-1/222768*(31824*x^11 + 306306*x^10 + 1361360*x^9 + 3675672*x^8 + 6683040*
x^7 + 8576568*x^6 + 7916832*x^5 + 5250960*x^4 + 2450448*x^3 + 765765*x^2 +
 144144*x + 12376)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx$$
$$= \frac{-31824x^{11} - 306306x^{10} - 1361360x^9 - 3675672x^8 - 6683040x^7 - 8576568x^6 - 7916832x^5 - 5250960x}{222768x^{18}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**19,x)
```

output
```
(-31824*x**11 - 306306*x**10 - 1361360*x**9 - 3675672*x**8 - 6683040*x**7
- 8576568*x**6 - 7916832*x**5 - 5250960*x**4 - 2450448*x**3 - 765765*x**2
- 144144*x - 12376)/(222768*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx =$$
$$-\frac{31824\,x^{11} + 306306\,x^{10} + 1361360\,x^9 + 3675672\,x^8 + 6683040\,x^7 + 8576568\,x^6 + 7916832\,x^5 + 52509}{222768\,x^{18}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^19,x, algorithm="maxima")
```

output
```
-1/222768*(31824*x^11 + 306306*x^10 + 1361360*x^9 + 3675672*x^8 + 6683040*
x^7 + 8576568*x^6 + 7916832*x^5 + 5250960*x^4 + 2450448*x^3 + 765765*x^2 +
 144144*x + 12376)/x^18
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{31824\,x^{11}+306306\,x^{10}+1361360\,x^9+3675672\,x^8+6683040\,x^7+8576568\,x^6+7916832\,x^5+52509}{222768\,x^{18}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^19,x, algorithm="giac")
```

output
```
-1/222768*(31824*x^11 + 306306*x^10 + 1361360*x^9 + 3675672*x^8 + 6683040*
x^7 + 8576568*x^6 + 7916832*x^5 + 5250960*x^4 + 2450448*x^3 + 765765*x^2 +
 144144*x + 12376)/x^18
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx = -\frac{\frac{x^{11}}{7}+\frac{11\,x^{10}}{8}+\frac{55\,x^9}{9}+\frac{33\,x^8}{2}+30\,x^7+\frac{77\,x^6}{2}+\frac{462\,x^5}{13}+\frac{165\,x^4}{7}+11\,x^3+\frac{55\,x^2}{16}+\frac{11\,x}{17}+\frac{1}{18}}{x^{18}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^19,x)
```

output
```
-((11*x)/17 + (55*x^2)/16 + 11*x^3 + (165*x^4)/7 + (462*x^5)/13 + (77*x^6)
/2 + 30*x^7 + (33*x^8)/2 + (55*x^9)/9 + (11*x^10)/8 + x^11/7 + 1/18)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{19}}\,dx$$
$$= \frac{-31824x^{11} - 306306x^{10} - 1361360x^9 - 3675672x^8 - 6683040x^7 - 8576568x^6 - 7916832x^5 - 5250960x}{222768x^{18}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^19,x)
```

output
```
( - 31824*x**11 - 306306*x**10 - 1361360*x**9 - 3675672*x**8 - 6683040*x**
7 - 8576568*x**6 - 7916832*x**5 - 5250960*x**4 - 2450448*x**3 - 765765*x**
2 - 144144*x - 12376)/(222768*x**18)
```

3.248 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{20}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{20}} \, dx = -\frac{(1+x)^{12}}{19x^{19}} + \frac{7(1+x)^{12}}{342x^{18}} - \frac{7(1+x)^{12}}{969x^{17}} + \frac{35(1+x)^{12}}{15504x^{16}} - \frac{7(1+x)^{12}}{11628x^{15}} + \frac{(1+x)^{12}}{7752x^{14}} - \frac{(1+x)^{12}}{50388x^{13}} + \frac{(1+x)^{12}}{604656x^{12}}$$

output
```
-1/19*(1+x)^12/x^19+7/342*(1+x)^12/x^18-7/969*(1+x)^12/x^17+35/15504*(1+x)
^12/x^16-7/11628*(1+x)^12/x^15+1/7752*(1+x)^12/x^14-1/50388*(1+x)^12/x^13+
1/604656*(1+x)^12/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)(1+2x+x^2)^5}{x^{20}} \, dx = -\frac{1}{19x^{19}} - \frac{11}{18x^{18}} - \frac{55}{17x^{17}} - \frac{165}{16x^{16}} - \frac{22}{x^{15}} - \frac{33}{x^{14}} - \frac{462}{13x^{13}} - \frac{55}{2x^{12}} - \frac{15}{x^{11}} - \frac{11}{2x^{10}} - \frac{11}{9x^9} - \frac{1}{8x^8}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^20,x]
```

output
```
-1/19*1/x^19 - 11/(18*x^18) - 55/(17*x^17) - 165/(16*x^16) - 22/x^15 - 33/
x^14 - 462/(13*x^13) - 55/(2*x^12) - 15/x^11 - 11/(2*x^10) - 11/(9*x^9) -
1/(8*x^8)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1184, 55, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{20}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{20}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{7}{19}\int \frac{(x+1)^{11}}{x^{19}}dx-\frac{(x+1)^{12}}{19x^{19}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{7}{19}\left(-\frac{1}{3}\int \frac{(x+1)^{11}}{x^{18}}dx-\frac{(x+1)^{12}}{18x^{18}}\right)-\frac{(x+1)^{12}}{19x^{19}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{7}{19}\left(\frac{1}{3}\left(\frac{5}{17}\int \frac{(x+1)^{11}}{x^{17}}dx+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}\right)-\frac{(x+1)^{12}}{19x^{19}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{7}{19}\left(\frac{1}{3}\left(\frac{5}{17}\left(-\frac{1}{4}\int \frac{(x+1)^{11}}{x^{16}}dx-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}\right)-\frac{(x+1)^{12}}{19x^{19}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{7}{19}\left(\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\int\frac{(x+1)^{11}}{x^{15}}dx+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}\right)-\frac{(x+1)^{12}}{19x^{19}}$$

$\downarrow$ 55

$$-\frac{7}{19}\left(\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(-\frac{1}{7}\int\frac{(x+1)^{11}}{x^{14}}dx-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}\right)-\frac{(x+1)^{12}}{18x^{18}}\right)-\frac{(x+1)^{12}}{19x^{19}}$$

$\downarrow$ 55

$$-\frac{7}{19}\left(\frac{1}{3}\left(\frac{5}{17}\left(\frac{1}{4}\left(\frac{1}{5}\left(\frac{1}{7}\left(\frac{1}{13}\int\frac{(x+1)^{11}}{x^{13}}dx+\frac{(x+1)^{12}}{13x^{13}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)+\frac{(x+1)^{12}}{15x^{15}}\right)-\frac{(x+1)^{12}}{16x^{16}}\right)+\frac{(x+1)^{12}}{17x^{17}}-\frac{(x+1)^{12}}{19x^{19}}$$

$\downarrow$ 48

$$-\frac{(x+1)^{12}}{19x^{19}}-\frac{7}{19}\left(\frac{1}{3}\left(\frac{(x+1)^{12}}{17x^{17}}+\frac{5}{17}\left(\frac{1}{4}\left(\frac{(x+1)^{12}}{15x^{15}}+\frac{1}{5}\left(\frac{1}{7}\left(\frac{(x+1)^{12}}{13x^{13}}-\frac{(x+1)^{12}}{156x^{12}}\right)-\frac{(x+1)^{12}}{14x^{14}}\right)\right)-\frac{(x+1)^{12}}{16x^{16}}\right)\right)-\frac{(x+}{18x}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^20,x]
```

output
```
-1/19*(1 + x)^12/x^19 - (7*(-1/18*(1 + x)^12/x^18 + ((1 + x)^12/(17*x^17)
+ (5*(-1/16*(1 + x)^12/x^16 + ((1 + x)^12/(15*x^15) + (-1/14*(1 + x)^12/x^
14 + ((1 + x)^12/(13*x^13) - (1 + x)^12/(156*x^12))/7)/5)/4))/17)/3))/19
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result
norman	$\frac{-\frac{1}{19}-\frac{11}{18}x-\frac{55}{17}x^2-\frac{165}{16}x^3-22x^4-33x^5-\frac{462}{13}x^6-\frac{55}{2}x^7-15x^8-\frac{11}{2}x^9-\frac{11}{9}x^{10}-\frac{1}{8}x^{11}}{x^{19}}$
risch	$\frac{-\frac{1}{19}-\frac{11}{18}x-\frac{55}{17}x^2-\frac{165}{16}x^3-22x^4-33x^5-\frac{462}{13}x^6-\frac{55}{2}x^7-15x^8-\frac{11}{2}x^9-\frac{11}{9}x^{10}-\frac{1}{8}x^{11}}{x^{19}}$
gosper	$-\frac{75582x^{11}+739024x^{10}+3325608x^9+9069840x^8+16628040x^7+21488544x^6+19953648x^5+13302432x^4+6235515x^3+1956}{604656x^{19}}$
parallelrisch	$\frac{-75582x^{11}-739024x^{10}-3325608x^9-9069840x^8-16628040x^7-21488544x^6-19953648x^5-13302432x^4-6235515x^3-19562}{604656x^{19}}$
default	$-\frac{11}{18x^{18}}-\frac{462}{13x^{13}}-\frac{1}{8x^8}-\frac{33}{x^{14}}-\frac{11}{9x^9}-\frac{11}{2x^{10}}-\frac{55}{17x^{17}}-\frac{22}{x^{15}}-\frac{15}{x^{11}}-\frac{1}{19x^{19}}-\frac{165}{16x^{16}}-\frac{55}{2x^{12}}$
orering	$-\frac{(75582x^{11}+739024x^{10}+3325608x^9+9069840x^8+16628040x^7+21488544x^6+19953648x^5+13302432x^4+6235515x^3+195}{604656x^{19}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^20,x,method=_RETURNVERBOSE)
```

output
```
(-1/19-11/18*x-55/17*x^2-165/16*x^3-22*x^4-33*x^5-462/13*x^6-55/2*x^7-15*x
^8-11/2*x^9-11/9*x^10-1/8*x^11)/x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{75582\,x^{11}+739024\,x^{10}+3325608\,x^{9}+9069840\,x^{8}+16628040\,x^{7}+21488544\,x^{6}+19953648\,x^{5}+13}{604656\,x^{19}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^20,x, algorithm="fricas")
```

output
```
-1/604656*(75582*x^11 + 739024*x^10 + 3325608*x^9 + 9069840*x^8 + 16628040
*x^7 + 21488544*x^6 + 19953648*x^5 + 13302432*x^4 + 6235515*x^3 + 1956240*
x^2 + 369512*x + 31824)/x^19
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = \frac{-75582x^{11}-739024x^{10}-3325608x^{9}-9069840x^{8}-16628040x^{7}-21488544x^{6}-19953648x^{5}-13302}{604656x^{19}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**20,x)
```

output
```
(-75582*x**11 - 739024*x**10 - 3325608*x**9 - 9069840*x**8 - 16628040*x**7
 - 21488544*x**6 - 19953648*x**5 - 13302432*x**4 - 6235515*x**3 - 1956240*
x**2 - 369512*x - 31824)/(604656*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{75582\,x^{11}+739024\,x^{10}+3325608\,x^9+9069840\,x^8+16628040\,x^7+21488544\,x^6+19953648\,x^5+13}{604656\,x^{19}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^20,x, algorithm="maxima")
```

output
```
-1/604656*(75582*x^11 + 739024*x^10 + 3325608*x^9 + 9069840*x^8 + 16628040
*x^7 + 21488544*x^6 + 19953648*x^5 + 13302432*x^4 + 6235515*x^3 + 1956240*
x^2 + 369512*x + 31824)/x^19
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{75582\,x^{11}+739024\,x^{10}+3325608\,x^9+9069840\,x^8+16628040\,x^7+21488544\,x^6+19953648\,x^5+13}{604656\,x^{19}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^20,x, algorithm="giac")
```

output
```
-1/604656*(75582*x^11 + 739024*x^10 + 3325608*x^9 + 9069840*x^8 + 16628040
*x^7 + 21488544*x^6 + 19953648*x^5 + 13302432*x^4 + 6235515*x^3 + 1956240*
x^2 + 369512*x + 31824)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = -\frac{\frac{x^{11}}{8}+\frac{11\,x^{10}}{9}+\frac{11\,x^9}{2}+15\,x^8+\frac{55\,x^7}{2}+\frac{462\,x^6}{13}+33\,x^5+22\,x^4+\frac{165\,x^3}{16}+\frac{55\,x^2}{17}+\frac{11\,x}{18}+\frac{1}{19}}{x^{19}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^20,x)
```

output
```
-((11*x)/18 + (55*x^2)/17 + (165*x^3)/16 + 22*x^4 + 33*x^5 + (462*x^6)/13
+ (55*x^7)/2 + 15*x^8 + (11*x^9)/2 + (11*x^10)/9 + x^11/8 + 1/19)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{20}}\,dx = \frac{-75582x^{11}-739024x^{10}-3325608x^9-9069840x^8-16628040x^7-21488544x^6-19953648x^5-13302}{604656x^{19}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^20,x)
```

output
```
( - 75582*x**11 - 739024*x**10 - 3325608*x**9 - 9069840*x**8 - 16628040*x*
*7 - 21488544*x**6 - 19953648*x**5 - 13302432*x**4 - 6235515*x**3 - 195624
0*x**2 - 369512*x - 31824)/(604656*x**19)
```

3.249 $\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = -\frac{1}{20x^{20}}-\frac{11}{19x^{19}}-\frac{55}{18x^{18}}-\frac{165}{17x^{17}}-\frac{165}{8x^{16}}-\frac{154}{5x^{15}}-\frac{33}{x^{14}}-\frac{330}{13x^{13}}-\frac{55}{4x^{12}}-\frac{5}{x^{11}}-\frac{11}{10x^{10}}-\frac{1}{9x^9}$$

output
```
-1/20/x^20-11/19/x^19-55/18/x^18-165/17/x^17-165/8/x^16-154/5/x^15-33/x^14
-330/13/x^13-55/4/x^12-5/x^11-11/10/x^10-1/9/x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = -\frac{1}{20x^{20}}-\frac{11}{19x^{19}}-\frac{55}{18x^{18}}-\frac{165}{17x^{17}}-\frac{165}{8x^{16}}-\frac{154}{5x^{15}}-\frac{33}{x^{14}}-\frac{330}{13x^{13}}-\frac{55}{4x^{12}}-\frac{5}{x^{11}}-\frac{11}{10x^{10}}-\frac{1}{9x^9}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^21,x]
```

output
```
-1/20*1/x^20 - 11/(19*x^19) - 55/(18*x^18) - 165/(17*x^17) - 165/(8*x^16)
- 154/(5*x^15) - 33/x^14 - 330/(13*x^13) - 55/(4*x^12) - 5/x^11 - 11/(10*x
^10) - 1/(9*x^9)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{21}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{21}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{x^{21}}+\frac{11}{x^{20}}+\frac{55}{x^{19}}+\frac{165}{x^{18}}+\frac{330}{x^{17}}+\frac{462}{x^{16}}+\frac{462}{x^{15}}+\frac{330}{x^{14}}+\frac{165}{x^{13}}+\frac{55}{x^{12}}+\frac{11}{x^{11}}+\frac{1}{x^{10}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{20x^{20}}-\frac{11}{19x^{19}}-\frac{55}{18x^{18}}-\frac{165}{17x^{17}}-\frac{165}{8x^{16}}-\frac{154}{5x^{15}}-\frac{33}{x^{14}}-\frac{330}{13x^{13}}-\frac{55}{4x^{12}}-\frac{5}{x^{11}}-\frac{11}{10x^{10}}-\frac{1}{9x^9}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^21,x]
```

output

```
-1/20*1/x^20 - 11/(19*x^19) - 55/(18*x^18) - 165/(17*x^17) - 165/(8*x^16)
- 154/(5*x^15) - 33/x^14 - 330/(13*x^13) - 55/(4*x^12) - 5/x^11 - 11/(10*x
^10) - 1/(9*x^9)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

method	result
norman	$\frac{-\frac{1}{20}-\frac{11}{19}x-\frac{55}{18}x^2-\frac{165}{17}x^3-\frac{165}{8}x^4-\frac{154}{5}x^5-33x^6-\frac{330}{13}x^7-\frac{55}{4}x^8-5x^9-\frac{11}{10}x^{10}-\frac{1}{9}x^{11}}{x^{20}}$
risch	$\frac{-\frac{1}{20}-\frac{11}{19}x-\frac{55}{18}x^2-\frac{165}{17}x^3-\frac{165}{8}x^4-\frac{154}{5}x^5-33x^6-\frac{330}{13}x^7-\frac{55}{4}x^8-5x^9-\frac{11}{10}x^{10}-\frac{1}{9}x^{11}}{x^{20}}$
gosper	$-\frac{167960x^{11}+1662804x^{10}+7558200x^9+20785050x^8+38372400x^7+49884120x^6+46558512x^5+31177575x^4+14671800x^3+}{1511640x^{20}}$
parallelrisch	$\frac{-167960x^{11}-1662804x^{10}-7558200x^9-20785050x^8-38372400x^7-49884120x^6-46558512x^5-31177575x^4-14671800x^3-4}{1511640x^{20}}$
default	$-\frac{1}{20x^{20}}-\frac{11}{19x^{19}}-\frac{55}{18x^{18}}-\frac{165}{17x^{17}}-\frac{165}{8x^{16}}-\frac{154}{5x^{15}}-\frac{33}{x^{14}}-\frac{330}{13x^{13}}-\frac{55}{4x^{12}}-\frac{5}{x^{11}}-\frac{11}{10x^{10}}-\frac{1}{9x^9}$
orering	$-\frac{(167960x^{11}+1662804x^{10}+7558200x^9+20785050x^8+38372400x^7+49884120x^6+46558512x^5+31177575x^4+14671800x^3-}{1511640x^{20}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^21,x,method=_RETURNVERBOSE)
```

output
```
(-1/20-11/19*x-55/18*x^2-165/17*x^3-165/8*x^4-154/5*x^5-33*x^6-330/13*x^7-
55/4*x^8-5*x^9-11/10*x^10-1/9*x^11)/x^20
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = -\frac{167960\,x^{11}+1662804\,x^{10}+7558200\,x^{9}+20785050\,x^{8}+38372400\,x^{7}+49884120\,x^{6}+46558512\,x^{5}+}{1511640\,x^{20}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^21,x, algorithm="fricas")
```

output
```
-1/1511640*(167960*x^11 + 1662804*x^10 + 7558200*x^9 + 20785050*x^8 + 3837
2400*x^7 + 49884120*x^6 + 46558512*x^5 + 31177575*x^4 + 14671800*x^3 + 461
8900*x^2 + 875160*x + 75582)/x^20
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx = \frac{-167960x^{11}-1662804x^{10}-7558200x^{9}-20785050x^{8}-38372400x^{7}-49884120x^{6}-46558512x^{5}-31}{1511640x^{20}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**21,x)
```

output
```
(-167960*x**11 - 1662804*x**10 - 7558200*x**9 - 20785050*x**8 - 38372400*x
**7 - 49884120*x**6 - 46558512*x**5 - 31177575*x**4 - 14671800*x**3 - 4618
900*x**2 - 875160*x - 75582)/(1511640*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{21}}\,dx = -\frac{167960\,x^{11}+1662804\,x^{10}+7558200\,x^9+20785050\,x^8+38372400\,x^7+49884120\,x^6+46558512\,x^5+}{1511640\,x^{20}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^21,x, algorithm="maxima")
```

output
```
-1/1511640*(167960*x^11 + 1662804*x^10 + 7558200*x^9 + 20785050*x^8 + 3837
2400*x^7 + 49884120*x^6 + 46558512*x^5 + 31177575*x^4 + 14671800*x^3 + 461
8900*x^2 + 875160*x + 75582)/x^20
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{(1+x)\,(1+2x+x^2)^5}{x^{21}}\,dx = -\frac{167960\,x^{11}+1662804\,x^{10}+7558200\,x^9+20785050\,x^8+38372400\,x^7+49884120\,x^6+46558512\,x^5+}{1511640\,x^{20}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^21,x, algorithm="giac")
```

output
```
-1/1511640*(167960*x^11 + 1662804*x^10 + 7558200*x^9 + 20785050*x^8 + 3837
2400*x^7 + 49884120*x^6 + 46558512*x^5 + 31177575*x^4 + 14671800*x^3 + 461
8900*x^2 + 875160*x + 75582)/x^20
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx$$
$$= -\frac{\frac{x^{11}}{9} + \frac{11\,x^{10}}{10} + 5\,x^9 + \frac{55\,x^8}{4} + \frac{330\,x^7}{13} + 33\,x^6 + \frac{154\,x^5}{5} + \frac{165\,x^4}{8} + \frac{165\,x^3}{17} + \frac{55\,x^2}{18} + \frac{11\,x}{19} + \frac{1}{20}}{x^{20}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^21,x)
```

output
```
-((11*x)/19 + (55*x^2)/18 + (165*x^3)/17 + (165*x^4)/8 + (154*x^5)/5 + 33*
x^6 + (330*x^7)/13 + (55*x^8)/4 + 5*x^9 + (11*x^10)/10 + x^11/9 + 1/20)/x^
20
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{21}}\,dx$$
$$= \frac{-167960x^{11} - 1662804x^{10} - 7558200x^9 - 20785050x^8 - 38372400x^7 - 49884120x^6 - 46558512x^5 - 31}{1511640x^{20}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^21,x)
```

output
```
( - 167960*x**11 - 1662804*x**10 - 7558200*x**9 - 20785050*x**8 - 38372400
*x**7 - 49884120*x**6 - 46558512*x**5 - 31177575*x**4 - 14671800*x**3 - 46
18900*x**2 - 875160*x - 75582)/(1511640*x**20)
```

3.250 $\int \frac{(1+x)(1+2x+x^2)^5}{x^{22}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = -\frac{1}{21x^{21}}-\frac{11}{20x^{20}}-\frac{55}{19x^{19}}-\frac{55}{6x^{18}}-\frac{330}{17x^{17}}-\frac{231}{8x^{16}}-\frac{154}{5x^{15}}-\frac{165}{7x^{14}}-\frac{165}{13x^{13}}-\frac{55}{12x^{12}}-\frac{1}{x^{11}}-\frac{1}{10x^{10}}$$

output
```
-1/21/x^21-11/20/x^20-55/19/x^19-55/6/x^18-330/17/x^17-231/8/x^16-154/5/x^
15-165/7/x^14-165/13/x^13-55/12/x^12-1/x^11-1/10/x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = -\frac{1}{21x^{21}}-\frac{11}{20x^{20}}-\frac{55}{19x^{19}}-\frac{55}{6x^{18}}-\frac{330}{17x^{17}}-\frac{231}{8x^{16}}-\frac{154}{5x^{15}}-\frac{165}{7x^{14}}-\frac{165}{13x^{13}}-\frac{55}{12x^{12}}-\frac{1}{x^{11}}-\frac{1}{10x^{10}}$$

input
```
Integrate[((1 + x)*(1 + 2*x + x^2)^5)/x^22,x]
```

output
```
-1/21*1/x^21 - 11/(20*x^20) - 55/(19*x^19) - 55/(6*x^18) - 330/(17*x^17) -
 231/(8*x^16) - 154/(5*x^15) - 165/(7*x^14) - 165/(13*x^13) - 55/(12*x^12)
 - x^(-11) - 1/(10*x^10)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\left(x^2+2x+1\right)^5}{x^{22}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\int \frac{(x+1)^{11}}{x^{22}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{x^{22}}+\frac{11}{x^{21}}+\frac{55}{x^{20}}+\frac{165}{x^{19}}+\frac{330}{x^{18}}+\frac{462}{x^{17}}+\frac{462}{x^{16}}+\frac{330}{x^{15}}+\frac{165}{x^{14}}+\frac{55}{x^{13}}+\frac{11}{x^{12}}+\frac{1}{x^{11}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{21x^{21}}-\frac{11}{20x^{20}}-\frac{55}{19x^{19}}-\frac{55}{6x^{18}}-\frac{330}{17x^{17}}-\frac{231}{8x^{16}}-\frac{154}{5x^{15}}-\frac{165}{7x^{14}}-\frac{165}{13x^{13}}-\frac{55}{12x^{12}}-\frac{1}{x^{11}}-\frac{1}{10x^{10}}$$

input
```
Int[((1 + x)*(1 + 2*x + x^2)^5)/x^22,x]
```

output

```
-1/21*1/x^21 - 11/(20*x^20) - 55/(19*x^19) - 55/(6*x^18) - 330/(17*x^17) -
 231/(8*x^16) - 154/(5*x^15) - 165/(7*x^14) - 165/(13*x^13) - 55/(12*x^12)
 - x^(-11) - 1/(10*x^10)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result
norman	$\frac{-\frac{1}{21}-\frac{11}{20}x-\frac{55}{19}x^2-\frac{55}{6}x^3-\frac{330}{17}x^4-\frac{231}{8}x^5-\frac{154}{5}x^6-\frac{165}{7}x^7-\frac{165}{13}x^8-\frac{55}{12}x^9-x^{10}-\frac{1}{10}x^{11}}{x^{21}}$
risch	$\frac{-\frac{1}{21}-\frac{11}{20}x-\frac{55}{19}x^2-\frac{55}{6}x^3-\frac{330}{17}x^4-\frac{231}{8}x^5-\frac{154}{5}x^6-\frac{165}{7}x^7-\frac{165}{13}x^8-\frac{55}{12}x^9-x^{10}-\frac{1}{10}x^{11}}{x^{21}}$
gosper	$-\frac{352716x^{11}+3527160x^{10}+16166150x^9+44767800x^8+83140200x^7+108636528x^6+101846745x^5+68468400x^4+32332300x}{3527160x^{21}}$
parallelrisch	$\frac{-352716x^{11}-3527160x^{10}-16166150x^9-44767800x^8-83140200x^7-108636528x^6-101846745x^5-68468400x^4-32332300x}{3527160x^{21}}$
default	$-\frac{1}{21x^{21}}-\frac{11}{20x^{20}}-\frac{55}{19x^{19}}-\frac{55}{6x^{18}}-\frac{330}{17x^{17}}-\frac{231}{8x^{16}}-\frac{154}{5x^{15}}-\frac{165}{7x^{14}}-\frac{165}{13x^{13}}-\frac{55}{12x^{12}}-\frac{1}{x^{11}}-\frac{1}{10x^{10}}$
orering	$-\frac{(352716x^{11}+3527160x^{10}+16166150x^9+44767800x^8+83140200x^7+108636528x^6+101846745x^5+68468400x^4+32332300}{3527160x^{21}(x+1)^{10}}$

input
```
int((x+1)*(x^2+2*x+1)^5/x^22,x,method=_RETURNVERBOSE)
```

output
```
(-1/21-11/20*x-55/19*x^2-55/6*x^3-330/17*x^4-231/8*x^5-154/5*x^6-165/7*x^7
-165/13*x^8-55/12*x^9-x^10-1/10*x^11)/x^21
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx =$$
$$-\frac{352716\,x^{11}+3527160\,x^{10}+16166150\,x^{9}+44767800\,x^{8}+83140200\,x^{7}+108636528\,x^{6}+101846745\,}{3527160\,x^{21}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^22,x, algorithm="fricas")
```

output
```
-1/3527160*(352716*x^11 + 3527160*x^10 + 16166150*x^9 + 44767800*x^8 + 831
40200*x^7 + 108636528*x^6 + 101846745*x^5 + 68468400*x^4 + 32332300*x^3 +
10210200*x^2 + 1939938*x + 167960)/x^21
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx$$
$$= \frac{-352716x^{11}-3527160x^{10}-16166150x^{9}-44767800x^{8}-83140200x^{7}-108636528x^{6}-101846745x^{5}-}{3527160x^{21}}$$

input
```
integrate((1+x)*(x**2+2*x+1)**5/x**22,x)
```

output
```
(-352716*x**11 - 3527160*x**10 - 16166150*x**9 - 44767800*x**8 - 83140200*
x**7 - 108636528*x**6 - 101846745*x**5 - 68468400*x**4 - 32332300*x**3 - 1
0210200*x**2 - 1939938*x - 167960)/(3527160*x**21)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = -\frac{352716\,x^{11}+3527160\,x^{10}+16166150\,x^9+44767800\,x^8+83140200\,x^7+108636528\,x^6+101846745}{3527160\,x^{21}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^22,x, algorithm="maxima")
```

output
```
-1/3527160*(352716*x^11 + 3527160*x^10 + 16166150*x^9 + 44767800*x^8 + 831
40200*x^7 + 108636528*x^6 + 101846745*x^5 + 68468400*x^4 + 32332300*x^3 +
10210200*x^2 + 1939938*x + 167960)/x^21
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = -\frac{352716\,x^{11}+3527160\,x^{10}+16166150\,x^9+44767800\,x^8+83140200\,x^7+108636528\,x^6+101846745}{3527160\,x^{21}}$$

input
```
integrate((1+x)*(x^2+2*x+1)^5/x^22,x, algorithm="giac")
```

output
```
-1/3527160*(352716*x^11 + 3527160*x^10 + 16166150*x^9 + 44767800*x^8 + 831
40200*x^7 + 108636528*x^6 + 101846745*x^5 + 68468400*x^4 + 32332300*x^3 +
10210200*x^2 + 1939938*x + 167960)/x^21
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = -\frac{\frac{x^{11}}{10}+x^{10}+\frac{55\,x^9}{12}+\frac{165\,x^8}{13}+\frac{165\,x^7}{7}+\frac{154\,x^6}{5}+\frac{231\,x^5}{8}+\frac{330\,x^4}{17}+\frac{55\,x^3}{6}+\frac{55\,x^2}{19}+\frac{11\,x}{20}+\frac{1}{21}}{x^{21}}$$

input
```
int(((x + 1)*(2*x + x^2 + 1)^5)/x^22,x)
```

output
```
-((11*x)/20 + (55*x^2)/19 + (55*x^3)/6 + (330*x^4)/17 + (231*x^5)/8 + (154
*x^6)/5 + (165*x^7)/7 + (165*x^8)/13 + (55*x^9)/12 + x^10 + x^11/10 + 1/21
)/x^21
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(1+x)\left(1+2x+x^2\right)^5}{x^{22}}\,dx = \frac{-352716x^{11}-3527160x^{10}-16166150x^9-44767800x^8-83140200x^7-108636528x^6-101846745x^5-}{3527160x^{21}}$$

input
```
int((1+x)*(x^2+2*x+1)^5/x^22,x)
```

output
```
( - 352716*x**11 - 3527160*x**10 - 16166150*x**9 - 44767800*x**8 - 8314020
0*x**7 - 108636528*x**6 - 101846745*x**5 - 68468400*x**4 - 32332300*x**3 -
 10210200*x**2 - 1939938*x - 167960)/(3527160*x**21)
```

3.251 $\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{a^3(4Ab-5aB)x}{b^6}+\frac{a^2(3Ab-4aB)x^2}{2b^5} -\frac{a(2Ab-3aB)x^3}{3b^4}+\frac{(Ab-2aB)x^4}{4b^3}+\frac{Bx^5}{5b^2} +\frac{a^5(Ab-aB)}{b^7(a+bx)}+\frac{a^4(5Ab-6aB)\log(a+bx)}{b^7}$$

output
```
-a^3*(4*A*b-5*B*a)*x/b^6+1/2*a^2*(3*A*b-4*B*a)*x^2/b^5-1/3*a*(2*A*b-3*B*a)
*x^3/b^4+1/4*(A*b-2*B*a)*x^4/b^3+1/5*B*x^5/b^2+a^5*(A*b-B*a)/b^7/(b*x+a)+a
^4*(5*A*b-6*B*a)*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{60a^3b(-4Ab+5aB)x-30a^2b^2(-3Ab+4aB)x^2+20ab^3(-2Ab+3aB)x^3+15b^4(Ab-2aB)x^4+12b^5 B}{60b^7}$$

input
```
Integrate[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(60*a^3*b*(-4*A*b + 5*a*B)*x - 30*a^2*b^2*(-3*A*b + 4*a*B)*x^2 + 20*a*b^3*
(-2*A*b + 3*a*B)*x^3 + 15*b^4*(A*b - 2*a*B)*x^4 + 12*b^5*B*x^5 + (60*a^5*(
A*b - a*B))/(a + b*x) + 60*a^4*(5*A*b - 6*a*B)*Log[a + b*x])/(60*b^7)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^5(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^5(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^5(aB-Ab)}{b^6(a+bx)^2}-\frac{a^4(6aB-5Ab)}{b^6(a+bx)}+\frac{a^3(5aB-4Ab)}{b^6}-\frac{a^2x(4aB-3Ab)}{b^5}+\frac{ax^2(3aB-2Ab)}{b^4}+\frac{x^3(Ab-2aB)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5(Ab-aB)}{b^7(a+bx)}+\frac{a^4(5Ab-6aB)\log(a+bx)}{b^7}-\frac{a^3x(4Ab-5aB)}{b^6}+\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{x^4(Ab-2aB)}{4b^3}+\frac{Bx^5}{5b^2}$$

input
```
Int[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
-((a^3*(4*A*b - 5*a*B)*x)/b^6) + (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2
*A*b - 3*a*B)*x^3)/(3*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) + (B*x^5)/(5*b^2)
 + (a^5*(A*b - a*B))/(b^7*(a + b*x)) + (a^4*(5*A*b - 6*a*B)*Log[a + b*x])/
b^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\frac{a\left(5A\,a^{4}b-6B\,a^{5}\right)}{b^{7}}+\frac{B\,x^{6}}{5b}+\frac{(5Ab-6Ba)x^{5}}{20b^{2}}-\frac{a(5Ab-6Ba)x^{4}}{12b^{3}}+\frac{a^{2}(5Ab-6Ba)x^{3}}{6b^{4}}-\frac{a^{3}(5Ab-6Ba)x^{2}}{2b^{5}}}{bx+a}+\frac{a^{4}(5Ab-6Ba)\ln(bx+a)}{b^{7}}$
default	$-\frac{-\frac{1}{5}b^{4}B\,x^{5}-\frac{1}{4}A\,b^{4}x^{4}+\frac{1}{2}Ba\,b^{3}x^{4}+\frac{2}{3}Aa\,b^{3}x^{3}-B\,a^{2}b^{2}x^{3}-\frac{3}{2}A\,a^{2}b^{2}x^{2}+2B\,a^{3}b\,x^{2}+4A\,a^{3}bx-5a^{4}Bx}{b^{6}}+\frac{a^{5}(Ab-Ba)}{b^{7}(bx+a)}+$
risch	$\frac{B\,x^{5}}{5b^{2}}+\frac{A\,x^{4}}{4b^{2}}-\frac{Ba\,x^{4}}{2b^{3}}-\frac{2Aa\,x^{3}}{3b^{3}}+\frac{B\,a^{2}x^{3}}{b^{4}}+\frac{3A\,a^{2}x^{2}}{2b^{4}}-\frac{2B\,a^{3}x^{2}}{b^{5}}-\frac{4A\,a^{3}x}{b^{5}}+\frac{5a^{4}Bx}{b^{6}}+\frac{a^{5}A}{b^{6}(bx+a)}-$
parallelrisch	$\frac{12b^{6}B\,x^{6}+15A\,b^{6}x^{5}-18Ba\,b^{5}x^{5}-25Aa\,b^{5}x^{4}+30B\,a^{2}b^{4}x^{4}+50A\,a^{2}b^{4}x^{3}-60B\,a^{3}b^{3}x^{3}+300A\ln(bx+a)x\,a^{4}b^{2}-150A\,a^{3}b^{3}x^{2}}{60b^{7}(bx+a)}$

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
(a*(5*A*a^4*b-6*B*a^5)/b^7+1/5*B/b*x^6+1/20*(5*A*b-6*B*a)/b^2*x^5-1/12*a*(
5*A*b-6*B*a)/b^3*x^4+1/6*a^2*(5*A*b-6*B*a)/b^4*x^3-1/2*a^3*(5*A*b-6*B*a)/b
^5*x^2)/(b*x+a)+a^4*(5*A*b-6*B*a)*ln(b*x+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.40

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{12\,Bb^6x^6-60\,Ba^6+60\,Aa^5b-3\,(6\,Bab^5-5\,Ab^6)x^5+5\,(6\,Ba^2b^4-5\,Aab^5)x^4-10\,(6\,Ba^3b^3-5\,Aa^2b^4}{6}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/60*(12*B*b^6*x^6 - 60*B*a^6 + 60*A*a^5*b - 3*(6*B*a*b^5 - 5*A*b^6)*x^5 +
 5*(6*B*a^2*b^4 - 5*A*a*b^5)*x^4 - 10*(6*B*a^3*b^3 - 5*A*a^2*b^4)*x^3 + 30
*(6*B*a^4*b^2 - 5*A*a^3*b^3)*x^2 + 60*(5*B*a^5*b - 4*A*a^4*b^2)*x - 60*(6*
B*a^6 - 5*A*a^5*b + (6*B*a^5*b - 5*A*a^4*b^2)*x)*log(b*x + a))/(b^8*x + a*
b^7)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx^5}{5b^2} - \frac{a^4(-5Ab+6Ba)\log{(a+bx)}}{b^7} + x^4\left(\frac{A}{4b^2}-\frac{Ba}{2b^3}\right) + x^3\left(-\frac{2Aa}{3b^3}+\frac{Ba^2}{b^4}\right) + x^2\cdot\left(\frac{3Aa^2}{2b^4}-\frac{2Ba^3}{b^5}\right) + x\left(-\frac{4Aa^3}{b^5}+\frac{5Ba^4}{b^6}\right) + \frac{Aa^5b-Ba^6}{ab^7+b^8x}$$

input
```
integrate(x**5*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*x**5/(5*b**2) - a**4*(-5*A*b + 6*B*a)*log(a + b*x)/b**7 + x**4*(A/(4*b**
2) - B*a/(2*b**3)) + x**3*(-2*A*a/(3*b**3) + B*a**2/b**4) + x**2*(3*A*a**2
/(2*b**4) - 2*B*a**3/b**5) + x*(-4*A*a**3/b**5 + 5*B*a**4/b**6) + (A*a**5*
b - B*a**6)/(a*b**7 + b**8*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{Ba^6-Aa^5b}{b^8x+ab^7} + \frac{12\,Bb^4x^5-15\,(2\,Bab^3-Ab^4)x^4+20\,(3\,Ba^2b^2-2\,Aab^3)x^3-30\,(4\,Ba^3b-3\,Aa^2b^2)x^2+60\,(5\,Ba^4-}{60\,b^6} - \frac{(6\,Ba^5-5\,Aa^4b)\log(bx+a)}{b^7}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a^6 - A*a^5*b)/(b^8*x + a*b^7) + 1/60*(12*B*b^4*x^5 - 15*(2*B*a*b^3 -
A*b^4)*x^4 + 20*(3*B*a^2*b^2 - 2*A*a*b^3)*x^3 - 30*(4*B*a^3*b - 3*A*a^2*b^
2)*x^2 + 60*(5*B*a^4 - 4*A*a^3*b)*x)/b^6 - (6*B*a^5 - 5*A*a^4*b)*log(b*x +
 a)/b^7
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{(6\,Ba^5-5\,Aa^4b)\log(|bx+a|)}{b^7} - \frac{Ba^6-Aa^5b}{(bx+a)b^7} + \frac{12\,Bb^8x^5-30\,Bab^7x^4+15\,Ab^8x^4+60\,Ba^2b^6x^3-40\,Aab^7x^3-120\,Ba^3b^5x^2+90\,Aa^2b^6x^2+300\,Ba^4b}{60\,b^{10}}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(6*B*a^5 - 5*A*a^4*b)*log(abs(b*x + a))/b^7 - (B*a^6 - A*a^5*b)/((b*x + a
)*b^7) + 1/60*(12*B*b^8*x^5 - 30*B*a*b^7*x^4 + 15*A*b^8*x^4 + 60*B*a^2*b^6
*x^3 - 40*A*a*b^7*x^3 - 120*B*a^3*b^5*x^2 + 90*A*a^2*b^6*x^2 + 300*B*a^4*b
^4*x - 240*A*a^3*b^5*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.08

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = x^2\left(\frac{a\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{2\,b^2}\right)+x^4\,\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x^3\,\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{3\,b}+\frac{B\,a^2}{3\,b^4}\right)-x\left(\frac{2\,a\,\left(\frac{2\,a\,\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}\right)}{b}-\frac{a^2\,\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b^2}\right)-\frac{\ln\left(a+b\,x\right)\,\left(6\,B\,a^5-5\,A\,a^4\,b\right)}{b^7}+\frac{B\,x^5}{5\,b^2}-\frac{B\,a^6-A\,a^5\,b}{b\,\left(x\,b^7+a\,b^6\right)}$$

input
```
int((x^5*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^2*((a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (
2*B*a)/b^3))/(2*b^2)) + x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x^3*((2*a*(A/b^2
 - (2*B*a)/b^3))/(3*b) + (B*a^2)/(3*b^4)) - x*((2*a*((2*a*((2*a*(A/b^2 - (
2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (2*B*a)/b^3))/b^2))/b - (
a^2*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b^2) - (log(a + b*x)*(6
*B*a^5 - 5*A*a^4*b))/b^7 + (B*x^5)/(5*b^2) - (B*a^6 - A*a^5*b)/(b*(a*b^6 +
 b^7*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.47

$$\int \frac{x^5(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{-60\log(bx+a)\,a^5+60a^4bx-30a^3b^2x^2+20a^2b^3x^3-15a\,b^4x^4+12b^5x^5}{60b^6}$$

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 60*log(a + b*x)*a**5 + 60*a**4*b*x - 30*a**3*b**2*x**2 + 20*a**2*b**3*
x**3 - 15*a*b**4*x**4 + 12*b**5*x**5)/(60*b**6)
```

3.252 $\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2} \, dx = \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^2}{2b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^4}{4b^2} - \frac{a^4(Ab-aB)}{b^6(a+bx)} - \frac{a^3(4Ab-5aB)\log(a+bx)}{b^6}$$

output
```
a^2*(3*A*b-4*B*a)*x/b^5-1/2*a*(2*A*b-3*B*a)*x^2/b^4+1/3*(A*b-2*B*a)*x^3/b^
3+1/4*B*x^4/b^2-a^4*(A*b-B*a)/b^6/(b*x+a)-a^3*(4*A*b-5*B*a)*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2} \, dx = \frac{-12a^2b(-3Ab+4aB)x + 6ab^2(-2Ab+3aB)x^2 + 4b^3(Ab-2aB)x^3 + 3b^4Bx^4 + \frac{12a^4(-Ab+aB)}{a+bx} + 12a^3(-}{12b^6}$$

input
```
Integrate[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(-12*a^2*b*(-3*A*b + 4*a*B)*x + 6*a*b^2*(-2*A*b + 3*a*B)*x^2 + 4*b^3*(A*b
- 2*a*B)*x^3 + 3*b^4*B*x^4 + (12*a^4*(-(A*b) + a*B))/(a + b*x) + 12*a^3*(-
4*A*b + 5*a*B)*Log[a + b*x])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^2\int \frac{x^4(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^2}+\frac{a^3(5aB-4Ab)}{b^5(a+bx)}-\frac{a^2(4aB-3Ab)}{b^5}+\frac{ax(3aB-2Ab)}{b^4}+\frac{x^2(Ab-2aB)}{b^3}+\frac{Bx^3}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4(Ab-aB)}{b^6(a+bx)}-\frac{a^3(4Ab-5aB)\log(a+bx)}{b^6}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^4}{4b^2}$$

input
```
Int[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*
a*B)*x^3)/(3*b^3) + (B*x^4)/(4*b^2) - (a^4*(A*b - a*B))/(b^6*(a + b*x)) -
(a^3*(4*A*b - 5*a*B)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4} B\, b^{3} x^{4}+\frac{1}{3} A\, b^{3} x^{3}-\frac{2}{3} B a\, b^{2} x^{3}-A a\, b^{2} x^{2}+\frac{3}{2} B\, a^{2} b\, x^{2}+3 A\, a^{2} b x -4 B\, a^{3} x}{b^{5}}-\frac{a^{4} \left(A b -B a \right)}{b^{6} \left(b x +a \right)}-\frac{a^{3} \left(4 A b -5 B a \right) \ln \left(b x +a \right)}{b^{6}}$
norman	$\frac{\frac{B\, x^{5}}{4 b}-\frac{a \left(4 A\, a^{3} b -5 a^{4} B \right)}{b^{6}}+\frac{\left(4 A b -5 B a \right) x^{4}}{12 b^{2}}-\frac{a \left(4 A b -5 B a \right) x^{3}}{6 b^{3}}+\frac{a^{2} \left(4 A b -5 B a \right) x^{2}}{2 b^{4}}}{b x +a}-\frac{a^{3} \left(4 A b -5 B a \right) \ln \left(b x +a \right)}{b^{6}}$
risch	$\frac{B\, x^{4}}{4 b^{2}}+\frac{A\, x^{3}}{3 b^{2}}-\frac{2 B a\, x^{3}}{3 b^{3}}-\frac{A a\, x^{2}}{b^{3}}+\frac{3 B\, a^{2} x^{2}}{2 b^{4}}+\frac{3 A\, a^{2} x}{b^{4}}-\frac{4 B\, a^{3} x}{b^{5}}-\frac{a^{4} A}{b^{5} \left(b x +a \right)}+\frac{a^{5} B}{b^{6} \left(b x +a \right)}-\frac{4 a^{3} \ln \left(b x +a \right) A}{b^{5}}+$
parallelrisch	$-\frac{-3 B\, x^{5} b^{5}-4 A\, b^{5} x^{4}+5 B a\, b^{4} x^{4}+8 A a\, b^{4} x^{3}-10 B\, a^{2} b^{3} x^{3}+48 A \ln \left(b x +a \right) x\, a^{3} b^{2}-24 A\, a^{2} b^{3} x^{2}-60 B \ln \left(b x +a \right) x\, a^{4} b +30 B\, a^{3}}{12 b^{6} \left(b x +a \right)}$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/4*B*b^3*x^4+1/3*A*b^3*x^3-2/3*B*a*b^2*x^3-A*a*b^2*x^2+3/2*B*a^2*b
*x^2+3*A*a^2*b*x-4*B*a^3*x)-a^4*(A*b-B*a)/b^6/(b*x+a)-a^3*(4*A*b-5*B*a)*ln
(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{3\,Bb^5x^5+12\,Ba^5-12\,Aa^4b-(5\,Bab^4-4\,Ab^5)x^4+2\,(5\,Ba^2b^3-4\,Aab^4)x^3-6\,(5\,Ba^3b^2-4\,Aa^2b^3)x^2 \cdots}{12\,(b^7x+ab^6)}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/12*(3*B*b^5*x^5 + 12*B*a^5 - 12*A*a^4*b - (5*B*a*b^4 - 4*A*b^5)*x^4 + 2*
(5*B*a^2*b^3 - 4*A*a*b^4)*x^3 - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^2 - 12*(4*
B*a^4*b - 3*A*a^3*b^2)*x + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a^3*
b^2)*x)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx^4}{4b^2}+\frac{a^3(-4Ab+5Ba)\log{(a+bx)}}{b^6}+x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}\right) + x^2\left(-\frac{Aa}{b^3}+\frac{3Ba^2}{2b^4}\right)+x\left(\frac{3Aa^2}{b^4}-\frac{4Ba^3}{b^5}\right)+\frac{-Aa^4b+Ba^5}{ab^6+b^7x}$$

input
```
integrate(x**4*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*x**4/(4*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x)/b**6 + x**3*(A/(3*b**
2) - 2*B*a/(3*b**3)) + x**2*(-A*a/b**3 + 3*B*a**2/(2*b**4)) + x*(3*A*a**2/
b**4 - 4*B*a**3/b**5) + (-A*a**4*b + B*a**5)/(a*b**6 + b**7*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Ba^5-Aa^4b}{b^7x+ab^6} + \frac{3\,Bb^3x^4-4\,(2\,Bab^2-Ab^3)x^3+6\,(3\,Ba^2b-2\,Aab^2)x^2-12\,(4\,Ba^3-3\,Aa^2b)x}{12\,b^5} + \frac{(5\,Ba^4-4\,Aa^3b)\log{(bx+a)}}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
(B*a^5 - A*a^4*b)/(b^7*x + a*b^6) + 1/12*(3*B*b^3*x^4 - 4*(2*B*a*b^2 - A*b
^3)*x^3 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^2 - 12*(4*B*a^3 - 3*A*a^2*b)*x)/b^5
+ (5*B*a^4 - 4*A*a^3*b)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(5\,Ba^4-4\,Aa^3b)\log{(|bx+a|)}}{b^6} + \frac{Ba^5-Aa^4b}{(bx+a)b^6} + \frac{3\,Bb^6x^4-8\,Bab^5x^3+4\,Ab^6x^3+18\,Ba^2b^4x^2-12\,Aab^5x^2-48\,Ba^3b^3x+36\,Aa^2b^4x}{12\,b^8}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(5*B*a^4 - 4*A*a^3*b)*log(abs(b*x + a))/b^6 + (B*a^5 - A*a^4*b)/((b*x + a)
*b^6) + 1/12*(3*B*b^6*x^4 - 8*B*a*b^5*x^3 + 4*A*b^6*x^3 + 18*B*a^2*b^4*x^2
 - 12*A*a*b^5*x^2 - 48*B*a^3*b^3*x + 36*A*a^2*b^4*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.53

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = x\left(\frac{2\,a\,\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}\right) + x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)-x^2\left(\frac{a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{2\,b^4}\right) + \frac{\ln\left(a+b\,x\right)\,\left(5\,B\,a^4-4\,A\,a^3\,b\right)}{b^6}+\frac{B\,x^4}{4\,b^2}+\frac{B\,a^5-A\,a^4\,b}{b\,\left(x\,b^6+a\,b^5\right)}$$

input
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x*((2*a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (
2*B*a)/b^3))/b^2) + x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) - x^2*((a*(A/b^2 - (
2*B*a)/b^3))/b + (B*a^2)/(2*b^4)) + (log(a + b*x)*(5*B*a^4 - 4*A*a^3*b))/b
^6 + (B*x^4)/(4*b^2) + (B*a^5 - A*a^4*b)/(b*(a*b^5 + b^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.46

$$\int \frac{x^4(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4}{12b^5}$$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**3 + 3
*b**4*x**4)/(12*b**5)
```

3.253 $\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^3}{3b^2}+\frac{a^3(Ab-aB)}{b^5(a+bx)}+\frac{a^2(3Ab-4aB)\log(a+bx)}{b^5}$$

output
```
-a*(2*A*b-3*B*a)*x/b^4+1/2*(A*b-2*B*a)*x^2/b^3+1/3*B*x^3/b^2+a^3*(A*b-B*a)
/b^5/(b*x+a)+a^2*(3*A*b-4*B*a)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{6ab(-2Ab+3aB)x+3b^2(Ab-2aB)x^2+2b^3Bx^3+\frac{6a^3(Ab-aB)}{a+bx}+6a^2(3Ab-4aB)\log(a+bx)}{6b^5}$$

input
```
Integrate[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(6*a*b*(-2*A*b + 3*a*B)*x + 3*b^2*(A*b - 2*a*B)*x^2 + 2*b^3*B*x^3 + (6*a^3
*(A*b - a*B))/(a + b*x) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^3(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^2}-\frac{a^2(4aB-3Ab)}{b^4(a+bx)}+\frac{a(3aB-2Ab)}{b^4}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^2}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(Ab-aB)}{b^5(a+bx)}+\frac{a^2(3Ab-4aB)\log(a+bx)}{b^5}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^3}{3b^2}$$

input
```
Int[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
-((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^3)/(3*b^
2) + (a^3*(A*b - a*B))/(b^5*(a + b*x)) + (a^2*(3*A*b - 4*a*B)*Log[a + b*x]
)/b^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result
default	$-\frac{-\frac{1}{3}x^{3}B\,b^{2}-\frac{1}{2}x^{2}b^{2}A+Ba\,x^{2}b+2abAx-3a^{2}Bx}{b^{4}}+\frac{a^{3}\left(Ab-Ba\right)}{b^{5}\left(bx+a\right)}+\frac{a^{2}\left(3Ab-4Ba\right)\ln\left(bx+a\right)}{b^{5}}$
norman	$\frac{\frac{a\left(3A\,a^{2}b-4B\,a^{3}\right)}{b^{5}}+\frac{B\,x^{4}}{3b}+\frac{\left(3Ab-4Ba\right)x^{3}}{6b^{2}}-\frac{a\left(3Ab-4Ba\right)x^{2}}{2b^{3}}}{bx+a}+\frac{a^{2}\left(3Ab-4Ba\right)\ln\left(bx+a\right)}{b^{5}}$
risch	$\frac{B\,x^{3}}{3b^{2}}+\frac{x^{2}A}{2b^{2}}-\frac{Ba\,x^{2}}{b^{3}}-\frac{2aAx}{b^{3}}+\frac{3a^{2}Bx}{b^{4}}+\frac{a^{3}A}{b^{4}\left(bx+a\right)}-\frac{a^{4}B}{b^{5}\left(bx+a\right)}+\frac{3a^{2}\ln\left(bx+a\right)A}{b^{4}}-\frac{4a^{3}\ln\left(bx+a\right)B}{b^{5}}$
parallelrisch	$\frac{2b^{4}B\,x^{4}+3A\,b^{4}x^{3}-4Ba\,b^{3}x^{3}+18A\ln\left(bx+a\right)x\,a^{2}b^{2}-9Aa\,b^{3}x^{2}-24B\ln\left(bx+a\right)x\,a^{3}b+12B\,a^{2}b^{2}x^{2}+18A\ln\left(bx+a\right)a^{3}b-24B\ln}{6b^{5}\left(bx+a\right)}$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/3*x^3*B*b^2-1/2*x^2*b^2*A+B*a*x^2*b+2*a*b*A*x-3*a^2*B*x)+a^3*(A
*b-B*a)/b^5/(b*x+a)+a^2*(3*A*b-4*B*a)*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$
$$= \frac{2\,Bb^4x^4-6\,Ba^4+6\,Aa^3b-(4\,Bab^3-3\,Ab^4)x^3+3\,(4\,Ba^2b^2-3\,Aab^3)x^2+6\,(3\,Ba^3b-2\,Aa^2b^2)x-6\,(}{6\,(b^6x+ab^5)}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/6*(2*B*b^4*x^4 - 6*B*a^4 + 6*A*a^3*b - (4*B*a*b^3 - 3*A*b^4)*x^3 + 3*(4*
B*a^2*b^2 - 3*A*a*b^3)*x^2 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x - 6*(4*B*a^4 -
3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx^3}{3b^2} - \frac{a^2(-3Ab+4Ba)\log{(a+bx)}}{b^5} + x^2\left(\frac{A}{2b^2}-\frac{Ba}{b^3}\right) + x\left(-\frac{2Aa}{b^3}+\frac{3Ba^2}{b^4}\right) + \frac{Aa^3b-Ba^4}{ab^5+b^6x}$$

input
```
integrate(x**3*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*x**3/(3*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x)/b**5 + x**2*(A/(2*b**
2) - B*a/b**3) + x*(-2*A*a/b**3 + 3*B*a**2/b**4) + (A*a**3*b - B*a**4)/(a*
b**5 + b**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{Ba^4-Aa^3b}{b^6x+ab^5} + \frac{2\,Bb^2x^3-3\,(2\,Bab-Ab^2)x^2+6\,(3\,Ba^2-2\,Aab)x}{6\,b^4} - \frac{(4\,Ba^3-3\,Aa^2b)\log{(bx+a)}}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a^4 - A*a^3*b)/(b^6*x + a*b^5) + 1/6*(2*B*b^2*x^3 - 3*(2*B*a*b - A*b^2
)*x^2 + 6*(3*B*a^2 - 2*A*a*b)*x)/b^4 - (4*B*a^3 - 3*A*a^2*b)*log(b*x + a)/
b^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{(4\,Ba^3-3\,Aa^2b)\log{(|bx+a|)}}{b^5} + \frac{2\,Bb^4x^3-6\,Bab^3x^2+3\,Ab^4x^2+18\,Ba^2b^2x-12\,Aab^3x}{6\,b^6} - \frac{Ba^4-Aa^3b}{(bx+a)b^5}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(4*B*a^3 - 3*A*a^2*b)*log(abs(b*x + a))/b^5 + 1/6*(2*B*b^4*x^3 - 6*B*a*b^
3*x^2 + 3*A*b^4*x^2 + 18*B*a^2*b^2*x - 12*A*a*b^3*x)/b^6 - (B*a^4 - A*a^3*
b)/((b*x + a)*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)-\frac{\ln\left(a+b\,x\right)\,\left(4\,B\,a^3-3\,A\,a^2\,b\right)}{b^5}+\frac{B\,x^3}{3\,b^2}-\frac{B\,a^4-A\,a^3\,b}{b\,\left(x\,b^5+a\,b^4\right)}$$

input
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^2*(A/(2*b^2) - (B*a)/b^3) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b
^4) - (log(a + b*x)*(4*B*a^3 - 3*A*a^2*b))/b^5 + (B*x^3)/(3*b^2) - (B*a^4
- A*a^3*b)/(b*(a*b^4 + b^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.46

$$\int \frac{x^3(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3}{6b^4}$$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**3)/(6*b**
4)
```

3.254 $\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(Ab-2aB)x}{b^3}+\frac{Bx^2}{2b^2}-\frac{a^2(Ab-aB)}{b^4(a+bx)} - \frac{a(2Ab-3aB)\log(a+bx)}{b^4}$$

output
```
(A*b-2*B*a)*x/b^3+1/2*B*x^2/b^2-a^2*(A*b-B*a)/b^4/(b*x+a)-a*(2*A*b-3*B*a)*
ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{2b(Ab-2aB)x+b^2Bx^2+\frac{2a^2(-Ab+aB)}{a+bx}+2a(-2Ab+3aB)\log(a+bx)}{2b^4}$$

input
```
Integrate[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*b*(A*b - 2*a*B)*x + b^2*B*x^2 + (2*a^2*(-(A*b) + a*B))/(a + b*x) + 2*a*
(-2*A*b + 3*a*B)*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^2(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^2}+\frac{a(3aB-2Ab)}{b^3(a+bx)}+\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2(Ab-aB)}{b^4(a+bx)}-\frac{a(2Ab-3aB)\log(a+bx)}{b^4}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^2}{2b^2}$$

input
```
Int[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - 2*a*B)*x)/b^3 + (B*x^2)/(2*b^2) - (a^2*(A*b - a*B))/(b^4*(a + b*x)
) - (a*(2*A*b - 3*a*B)*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{2} B b\,x^{2}+A b x -2 B a x}{b^{3}}-\frac{a^{2} \left(A b -B a \right)}{b^{4} \left(b x +a \right)}-\frac{a \left(2 A b -3 B a \right) \ln \left(b x +a \right)}{b^{4}}$
norman	$\frac{\frac{B\,x^{3}}{2 b}-\frac{a \left(2 a b A -3 a^{2} B \right)}{b^{4}}+\frac{\left(2 A b -3 B a \right) x^{2}}{2 b^{2}}}{b x +a}-\frac{a \left(2 A b -3 B a \right) \ln \left(b x +a \right)}{b^{4}}$
risch	$\frac{B\,x^{2}}{2 b^{2}}+\frac{A x}{b^{2}}-\frac{2 B a x}{b^{3}}-\frac{a^{2} A}{b^{3} \left(b x +a \right)}+\frac{a^{3} B}{b^{4} \left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right) A}{b^{3}}+\frac{3 a^{2} \ln \left(b x +a \right) B}{b^{4}}$
parallelrisch	$-\frac{-x^{3} B\,b^{3}+4 A \ln \left(b x +a \right) x a\,b^{2}-2 A\,b^{3} x^{2}-6 B \ln \left(b x +a \right) x\,a^{2} b +3 B a\,b^{2} x^{2}+4 A \ln \left(b x +a \right) a^{2} b -6 B \ln \left(b x +a \right) a^{3}+4 A\,a^{2} b -6 B\,a^{3}}{2 b^{4} \left(b x +a \right)}$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*B*b*x^2+A*b*x-2*B*a*x)-a^2*(A*b-B*a)/b^4/(b*x+a)-a*(2*A*b-3*B*a
)*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bb^3x^3 + 2\,Ba^3 - 2\,Aa^2b - (3\,Bab^2 - 2\,Ab^3)x^2 - 2\,(2\,Ba^2b - Aab^2)x + 2\,(3\,Ba^3 - 2\,Aa^2b + (3\,Ba^2b - 2}{2\,(b^5x + ab^4)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
1/2*(B*b^3*x^3 + 2*B*a^3 - 2*A*a^2*b - (3*B*a*b^2 - 2*A*b^3)*x^2 - 2*(2*B*
a^2*b - A*a*b^2)*x + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x)*l
og(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx^2}{2b^2} + \frac{a(-2Ab+3Ba)\log{(a+bx)}}{b^4} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{-Aa^2b+Ba^3}{ab^4+b^5x}$$

input
```
integrate(x**2*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*x**2/(2*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x)/b**4 + x*(A/b**2 - 2*B*a
/b**3) + (-A*a**2*b + B*a**3)/(a*b**4 + b**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Ba^3-Aa^2b}{b^5x+ab^4}+\frac{Bbx^2-2\,(2\,Ba-Ab)x}{2\,b^3} + \frac{(3\,Ba^2-2\,Aab)\log(bx+a)}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
(B*a^3 - A*a^2*b)/(b^5*x + a*b^4) + 1/2*(B*b*x^2 - 2*(2*B*a - A*b)*x)/b^3
+ (3*B*a^2 - 2*A*a*b)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(3\,Ba^2-2\,Aab)\log(|bx+a|)}{b^4} + \frac{Bb^2x^2-4\,Babx+2\,Ab^2x}{2\,b^4}+\frac{Ba^3-Aa^2b}{(bx+a)b^4}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(3*B*a^2 - 2*A*a*b)*log(abs(b*x + a))/b^4 + 1/2*(B*b^2*x^2 - 4*B*a*b*x + 2
*A*b^2*x)/b^4 + (B*a^3 - A*a^2*b)/((b*x + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)+\frac{B\,x^2}{2\,b^2}+\frac{B\,a^3-A\,a^2\,b}{b\,\left(x\,b^4+a\,b^3\right)} + \frac{\ln\left(a+b\,x\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{b^4}$$

input
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) + (B*x^2)/(2*b^2) + (B*a^3 - A*a^2*b)/(b*(a*b^3 +
b^4*x)) + (log(a + b*x)*(3*B*a^2 - 2*A*a*b))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.42

$$\int \frac{x^2(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{2\log(bx+a)\,a^2-2abx+b^2x^2}{2b^3}$$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.255 $\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{Bx}{b^2}+\frac{a(Ab-aB)}{b^3(a+bx)}+\frac{(Ab-2aB)\log(a+bx)}{b^3}$$

output
```
B*x/b^2+a*(A*b-B*a)/b^3/(b*x+a)+(A*b-2*B*a)*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{bBx+\frac{a(Ab-aB)}{a+bx}+(Ab-2aB)\log(a+bx)}{b^3}$$

input
```
Integrate[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(b*B*x + (a*(A*b - a*B))/(a + b*x) + (A*b - 2*a*B)*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)}+\frac{a(aB-Ab)}{b^2(a+bx)^2}+\frac{B}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(Ab-aB)}{b^3(a+bx)}+\frac{(Ab-2aB)\log(a+bx)}{b^3}+\frac{Bx}{b^2}$$

input
```
Int[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(B*x)/b^2 + (a*(A*b - a*B))/(b^3*(a + b*x)) + ((A*b - 2*a*B)*Log[a + b*x])
/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{Bx}{b^{2}}+\frac{a(Ab-Ba)}{b^{3}(bx+a)}+\frac{(Ab-2Ba)\ln(bx+a)}{b^{3}}$	46
norman	$\frac{\frac{B\,x^{2}}{b}+\frac{a(Ab-2Ba)}{b^{3}}}{bx+a}+\frac{(Ab-2Ba)\ln(bx+a)}{b^{3}}$	50
risch	$\frac{Bx}{b^{2}}+\frac{aA}{b^{2}(bx+a)}-\frac{a^{2}B}{b^{3}(bx+a)}+\frac{\ln(bx+a)A}{b^{2}}-\frac{2\ln(bx+a)Ba}{b^{3}}$	61
parallelrisch	$\frac{A\ln(bx+a)x\,b^{2}-2B\ln(bx+a)xab+x^{2}B\,b^{2}+A\ln(bx+a)ab-2B\ln(bx+a)a^{2}+abA-2a^{2}B}{b^{3}(bx+a)}$	77

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+a*(A*b-B*a)/b^3/(b*x+a)+(A*b-2*B*a)*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bb^2x^2+Babx-Ba^2+Aab-(2\,Ba^2-Aab+(2\,Bab-Ab^2)x)\log(bx+a)}{b^4x+ab^3}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
(B*b^2*x^2 + B*a*b*x - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b - A*b^2
)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx}{b^2} + \frac{Aab-Ba^2}{ab^3+b^4x} - \frac{(-Ab+2Ba)\log(a+bx)}{b^3}$$

input
```
integrate(x*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*x/b**2 + (A*a*b - B*a**2)/(a*b**3 + b**4*x) - (-A*b + 2*B*a)*log(a + b*x
)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{Ba^2-Aab}{b^4x+ab^3} + \frac{Bx}{b^2} - \frac{(2\,Ba-Ab)\log(bx+a)}{b^3}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a^2 - A*a*b)/(b^4*x + a*b^3) + B*x/b^2 - (2*B*a - A*b)*log(b*x + a)/b^
3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{Bx}{b^2} - \frac{(2\,Ba-Ab)\log\left(|bx+a|\right)}{b^3} - \frac{Ba^2-Aab}{(bx+a)b^3}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
B*x/b^2 - (2*B*a - A*b)*log(abs(b*x + a))/b^3 - (B*a^2 - A*a*b)/((b*x + a)
*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{B\,x}{b^2} - \frac{B\,a^2-A\,a\,b}{b\,\left(x\,b^3+a\,b^2\right)} + \frac{\ln\left(a+b\,x\right)\,\left(A\,b-2\,B\,a\right)}{b^3}$$

input
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(B*x)/b^2 - (B*a^2 - A*a*b)/(b*(a*b^2 + b^3*x)) + (log(a + b*x)*(A*b - 2*B
*a))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.38

$$\int \frac{x(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{-\log(bx+a)\,a+bx}{b^2}$$

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.256 $\int \frac{A+Bx}{a^2+2abx+b^2x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2} \, dx = -\frac{Ab-aB}{b^2(a+bx)} + \frac{B\log(a+bx)}{b^2}$$

output
```
-(A*b-B*a)/b^2/(b*x+a)+B*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2} \, dx = \frac{-Ab+aB}{b^2(a+bx)} + \frac{B\log(a+bx)}{b^2}$$

input
```
Integrate[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(-(A*b) + a*B)/(b^2*(a + b*x)) + (B*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \ 1098$$

$$b^2\int \frac{A+Bx}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{A+Bx}{(a+bx)^2}dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{B\log(a+bx)}{b^2}-\frac{Ab-aB}{b^2(a+bx)}$$

input
```
Int[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
-((A*b - a*B)/(b^2*(a + b*x))) + (B*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{Ab-Ba}{b^{2}(bx+a)}+\frac{B\ln(bx+a)}{b^{2}}$	33
norman	$-\frac{Ab-Ba}{b^{2}(bx+a)}+\frac{B\ln(bx+a)}{b^{2}}$	33
risch	$-\frac{A}{b(bx+a)}+\frac{Ba}{b^{2}(bx+a)}+\frac{B\ln(bx+a)}{b^{2}}$	39
parallelrisch	$-\frac{-B\ln(bx+a)xb-B\ln(bx+a)a+Ab-Ba}{b^{2}(bx+a)}$	42

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-(A*b-B*a)/b^2/(b*x+a)+B*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx = \frac{Ba-Ab+(Bbx+Ba)\log(bx+a)}{b^3x+ab^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
(B*a - A*b + (B*b*x + B*a)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx = \frac{B\log(a+bx)}{b^2} + \frac{-Ab+Ba}{ab^2+b^3x}$$

input
```
integrate((B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
B*log(a + b*x)/b**2 + (-A*b + B*a)/(a*b**2 + b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx = \frac{Ba-Ab}{b^3x+ab^2} + \frac{B\log(bx+a)}{b^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
(B*a - A*b)/(b^3*x + a*b^2) + B*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx=\frac{B\log\left(|bx+a|\right)}{b^2}+\frac{Ba-Ab}{(bx+a)b^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/b^2 + (B*a - A*b)/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx=\frac{B\,\ln\left(a+b\,x\right)}{b^2}-\frac{A\,b-B\,a}{b^2\,\left(a+b\,x\right)}$$

input
```
int((A + B*x)/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(B*log(a + b*x))/b^2 - (A*b - B*a)/(b^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx}{a^2+2abx+b^2x^2}\,dx=\frac{\log(bx+a)}{b}$$

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
log(a + b*x)/b
```

3.257 $\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)}\,dx = \frac{Ab-aB}{ab(a+bx)} + \frac{A\log(x)}{a^2} - \frac{A\log(a+bx)}{a^2}$$

output
```
(A*b-B*a)/a/b/(b*x+a)+A*ln(x)/a^2-A*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)}\,dx = \frac{\frac{a(Ab-aB)}{b(a+bx)} + A\log(x) - A\log(a+bx)}{a^2}$$

input
```
Integrate[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x)) + A*Log[x] - A*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{Ab}{a^2(a+bx)}+\frac{A}{a^2x}+\frac{aB-Ab}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\log(a+bx)}{a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{ab(a+bx)}$$

input
```
Int[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(A*b - a*B)/(a*b*(a + b*x)) + (A*Log[x])/a^2 - (A*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{(Ab-Ba)x}{a^2(bx+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln(bx+a)}{a^2}$	42
default	$-\frac{-Ab+Ba}{ab(bx+a)}-\frac{A\ln(bx+a)}{a^2}+\frac{A\ln(x)}{a^2}$	44
risch	$\frac{A}{a(bx+a)}-\frac{B}{(bx+a)b}+\frac{A\ln(-x)}{a^2}-\frac{A\ln(bx+a)}{a^2}$	48
parallelrisch	$\frac{A\ln(x)xb-A\ln(bx+a)xb+aA\ln(x)-A\ln(bx+a)a-Abx+Bax}{a^2(bx+a)}$	54

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-(A*b-B*a)/a^2*x/(b*x+a)+A*ln(x)/a^2-A*ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{Ba^2-Aab+\left(Ab^2x+Aab\right)\log\left(bx+a\right)-\left(Ab^2x+Aab\right)\log\left(x\right)}{a^2b^2x+a^3b}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-(B*a^2 - A*a*b + (A*b^2*x + A*a*b)*log(b*x + a) - (A*b^2*x + A*a*b)*log(x
))/(a^2*b^2*x + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{A\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^2}+\frac{Ab-Ba}{a^2b+ab^2x}$$

input
```
integrate((B*x+A)/x/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
A*(log(x) - log(a/b + x))/a**2 + (A*b - B*a)/(a**2*b + a*b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{Ba-Ab}{ab^2x+a^2b}-\frac{A\log\left(bx+a\right)}{a^2}+\frac{A\log\left(x\right)}{a^2}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a - A*b)/(a*b^2*x + a^2*b) - A*log(b*x + a)/a^2 + A*log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{A\log\left(|bx+a|\right)}{a^2}+\frac{A\log\left(|x|\right)}{a^2}-\frac{Ba^2-Aab}{(bx+a)a^2b}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/a^2 + A*log(abs(x))/a^2 - (B*a^2 - A*a*b)/((b*x + a)*
a^2*b)
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{A\,b-B\,a}{a\,b\,\left(a+b\,x\right)}-\frac{2\,A\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

input
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
(A*b - B*a)/(a*b*(a + b*x)) - (2*A*atanh((2*b*x)/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.258 $\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{A}{a^2x}-\frac{Ab-aB}{a^2(a+bx)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log(a+bx)}{a^3}$$

output
```
-A/a^2/x-(A*b-B*a)/a^2/(b*x+a)-(2*A*b-B*a)*ln(x)/a^3+(2*A*b-B*a)*ln(b*x+a)
/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{-\frac{aA}{x}+\frac{a(-Ab+aB)}{a+bx}+(-2Ab+aB)\log(x)+(2Ab-aB)\log(a+bx)}{a^3}$$

input
```
Integrate[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(-((a*A)/x) + (a*(-(A*b) + a*B))/(a + b*x) + (-2*A*b + a*B)*Log[x] + (2*A*
b - a*B)*Log[a + b*x])/a^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^2\int \frac{A+Bx}{b^2x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{aB-2Ab}{a^3x}-\frac{b(aB-2Ab)}{a^3(a+bx)}-\frac{b(aB-Ab)}{a^2(a+bx)^2}+\frac{A}{a^2x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(x)(2Ab-aB)}{a^3}+\frac{(2Ab-aB)\log(a+bx)}{a^3}-\frac{Ab-aB}{a^2(a+bx)}-\frac{A}{a^2x}$$

input
```
Int[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-(A/(a^2*x)) - (A*b - a*B)/(a^2*(a + b*x)) - ((2*A*b - a*B)*Log[x])/a^3 +
((2*A*b - a*B)*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result
default	$\frac{(2Ab-Ba)\ln(bx+a)}{a^3}-\frac{Ab-Ba}{a^2(bx+a)}-\frac{A}{a^2x}+\frac{(-2Ab+Ba)\ln(x)}{a^3}$
norman	$\frac{-\frac{A}{a}-\frac{\left(2b^2A-abB\right)x}{a^2b}}{x(bx+a)}+\frac{(2Ab-Ba)\ln(bx+a)}{a^3}-\frac{(2Ab-Ba)\ln(x)}{a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x}{a^2}-\frac{A}{a}}{x(bx+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{B\ln(x)}{a^2}+\frac{2\ln(-bx-a)Ab}{a^3}-\frac{\ln(-bx-a)B}{a^2}$
parallelrisch	$-\frac{2A\ln(x)x^2b^3-2A\ln(bx+a)x^2b^3-B\ln(x)x^2a\,b^2+B\ln(bx+a)x^2a\,b^2+2A\ln(x)xa\,b^2-2A\ln(bx+a)xa\,b^2-B\ln(x)x\,a^2b+B\ln}{a^3bx(bx+a)}$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
(2*A*b-B*a)*ln(b*x+a)/a^3-(A*b-B*a)/a^2/(b*x+a)-A/a^2/x+(-2*A*b+B*a)/a^3*l
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)}\,dx = -\frac{Aa^2-(Ba^2-2\,Aab)x+((Bab-2\,Ab^2)x^2+(Ba^2-2\,Aab)x)\log(bx+a)-((Bab-2\,Ab^2)x^2+(B}{a^3bx^2+a^4x}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-(A*a^2 - (B*a^2 - 2*A*a*b)*x + ((B*a*b - 2*A*b^2)*x^2 + (B*a^2 - 2*A*a*b)
*x)*log(b*x + a) - ((B*a*b - 2*A*b^2)*x^2 + (B*a^2 - 2*A*a*b)*x)*log(x))/(
a^3*b*x^2 + a^4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)}\,dx = \frac{-Aa+x(-2Ab+Ba)}{a^3x+a^2bx^2} + \frac{(-2Ab+Ba)\log\left(x+\frac{-2Aab+Ba^2-a(-2Ab+Ba)}{-4Ab^2+2Bab}\right)}{a^3} - \frac{(-2Ab+Ba)\log\left(x+\frac{-2Aab+Ba^2+a(-2Ab+Ba)}{-4Ab^2+2Bab}\right)}{a^3}$$

input
```
integrate((B*x+A)/x**2/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
(-A*a + x*(-2*A*b + B*a))/(a**3*x + a**2*b*x**2) + (-2*A*b + B*a)*log(x +
(-2*A*a*b + B*a**2 - a*(-2*A*b + B*a))/(-4*A*b**2 + 2*B*a*b))/a**3 - (-2*A
*b + B*a)*log(x + (-2*A*a*b + B*a**2 + a*(-2*A*b + B*a))/(-4*A*b**2 + 2*B*
a*b))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{Aa-(Ba-2\,Ab)x}{a^2bx^2+a^3x} - \frac{(Ba-2\,Ab)\log\left(bx+a\right)}{a^3} + \frac{(Ba-2\,Ab)\log\left(x\right)}{a^3}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(A*a - (B*a - 2*A*b)*x)/(a^2*b*x^2 + a^3*x) - (B*a - 2*A*b)*log(b*x + a)/
a^3 + (B*a - 2*A*b)*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{(Ba-2\,Ab)\log\left(|x|\right)}{a^3} + \frac{Bax-2\,Abx-Aa}{(bx^2+ax)a^2} - \frac{(Bab-2\,Ab^2)\log\left(|bx+a|\right)}{a^3b}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(B*a - 2*A*b)*log(abs(x))/a^3 + (B*a*x - 2*A*b*x - A*a)/((b*x^2 + a*x)*a^2
) - (B*a*b - 2*A*b^2)*log(abs(b*x + a))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,\left(2\,A\,b-B\,a\right)}{a^3}-\frac{\frac{A}{a}+\frac{x\,\left(2\,A\,b-B\,a\right)}{a^2}}{b\,x^2+a\,x}$$

input
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(2*A*b - B*a))/a^3 - (A/a + (x*(2*A*b - B*a))/a^2)
/(a*x + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{\log(bx+a)\,bx-\log(x)\,bx-a}{a^2x}$$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.259 $\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)}\,dx = -\frac{A}{2a^2x^2}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)}{a^3(a+bx)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log(a+bx)}{a^4}$$

output
```
-1/2*A/a^2/x^2+(2*A*b-B*a)/a^3/x+b*(A*b-B*a)/a^3/(b*x+a)+b*(3*A*b-2*B*a)*l
n(x)/a^4-b*(3*A*b-2*B*a)*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)}\,dx = \frac{-\frac{a(-6Ab^2x^2+a^2(A+2Bx)+abx(-3A+4Bx))}{x^2(a+bx)}+2b(3Ab-2aB)\log(x)+2b(-3Ab+2aB)\log(a+bx)}{2a^4}$$

input
```
Integrate[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(-((a*(-6*A*b^2*x^2 + a^2*(A + 2*B*x) + a*b*x*(-3*A + 4*B*x)))/(x^2*(a + b
*x))) + 2*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(-3*A*b + 2*a*B)*Log[a + b*x])/(2
*a^4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^2\int \frac{A+Bx}{b^2x^3(a+bx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{x^3(a+bx)^2}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{b^2(2aB-3Ab)}{a^4(a+bx)}-\frac{b(2aB-3Ab)}{a^4x}+\frac{b^2(aB-Ab)}{a^3(a+bx)^2}+\frac{aB-2Ab}{a^3x^2}+\frac{A}{a^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{b(3Ab-2aB)\log(a+bx)}{a^4}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)}{a^3(a+bx)}-\frac{A}{2a^2x^2}$$

input
```
Int[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-1/2*A/(a^2*x^2) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B))/(a^3*(a + b*x))
 + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

method	result
default	$-\frac{b(3Ab-2Ba)\ln(bx+a)}{a^4}+\frac{b(Ab-Ba)}{a^3(bx+a)}-\frac{A}{2a^2x^2}-\frac{-2Ab+Ba}{x\,a^3}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}$
norman	$\frac{\frac{\left(3A\,b^3-2Ba\,b^2\right)x^2}{a^3b}-\frac{A}{2a}+\frac{(3Ab-2Ba)x}{2a^2}}{x^2(bx+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln(bx+a)}{a^4}$
risch	$\frac{\frac{b(3Ab-2Ba)x^2}{a^3}+\frac{(3Ab-2Ba)x}{2a^2}-\frac{A}{2a}}{x^2(bx+a)}-\frac{3b^2\ln(bx+a)A}{a^4}+\frac{2b\ln(bx+a)B}{a^3}+\frac{3b^2\ln(-x)A}{a^4}-\frac{2b\ln(-x)B}{a^3}$
parallelrisch	$\frac{6A\ln(x)x^3b^3-6A\ln(bx+a)x^3b^3-4B\ln(x)x^3a\,b^2+4B\ln(bx+a)x^3a\,b^2+6A\ln(x)x^2a\,b^2-6A\ln(bx+a)x^2a\,b^2-6A\,b^3x^3-4B\ln}{2a^4x^2(bx+a)}$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-b*(3*A*b-2*B*a)*ln(b*x+a)/a^4+b*(A*b-B*a)/a^3/(b*x+a)-1/2*A/a^2/x^2-(-2*A
*b+B*a)/x/a^3+b*(3*A*b-2*B*a)*ln(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{Aa^3+2\,(2\,Ba^2b-3\,Aab^2)x^2+(2\,Ba^3-3\,Aa^2b)x-2\,((2\,Bab^2-3\,Ab^3)x^3+(2\,Ba^2b-3\,Aab^2)x^2)\,\mathrm{lo}}{2\,(a^4bx^3+a^5x^2)}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-1/2*(A*a^3 + 2*(2*B*a^2*b - 3*A*a*b^2)*x^2 + (2*B*a^3 - 3*A*a^2*b)*x - 2*
((2*B*a*b^2 - 3*A*b^3)*x^3 + (2*B*a^2*b - 3*A*a*b^2)*x^2)*log(b*x + a) + 2
*((2*B*a*b^2 - 3*A*b^3)*x^3 + (2*B*a^2*b - 3*A*a*b^2)*x^2)*log(x))/(a^4*b*
x^3 + a^5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(80) = 160$.

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{-Aa^2+x^2\cdot(6Ab^2-4Bab)+x(3Aab-2Ba^2)}{2a^4x^2+2a^3bx^3} - \frac{b(-3Ab+2Ba)\log\left(x+\frac{-3Aab^2+2Ba^2b-ab(-3Ab+2Ba)}{-6Ab^3+4Bab^2}\right)}{a^4} + \frac{b(-3Ab+2Ba)\log\left(x+\frac{-3Aab^2+2Ba^2b+ab(-3Ab+2Ba)}{-6Ab^3+4Bab^2}\right)}{a^4}$$

input
```
integrate((B*x+A)/x**3/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
(-A*a**2 + x**2*(6*A*b**2 - 4*B*a*b) + x*(3*A*a*b - 2*B*a**2))/(2*a**4*x**
2 + 2*a**3*b*x**3) - b*(-3*A*b + 2*B*a)*log(x + (-3*A*a*b**2 + 2*B*a**2*b
- a*b*(-3*A*b + 2*B*a))/(-6*A*b**3 + 4*B*a*b**2))/a**4 + b*(-3*A*b + 2*B*a
)*log(x + (-3*A*a*b**2 + 2*B*a**2*b + a*b*(-3*A*b + 2*B*a))/(-6*A*b**3 + 4
*B*a*b**2))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^3\,(a^2+2abx+b^2x^2)}\,dx = -\frac{Aa^2+2\,(2\,Bab-3\,Ab^2)x^2+(2\,Ba^2-3\,Aab)x}{2\,(a^3bx^3+a^4x^2)} + \frac{(2\,Bab-3\,Ab^2)\log{(bx+a)}}{a^4} - \frac{(2\,Bab-3\,Ab^2)\log{(x)}}{a^4}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/2*(A*a^2 + 2*(2*B*a*b - 3*A*b^2)*x^2 + (2*B*a^2 - 3*A*a*b)*x)/(a^3*b*x^
3 + a^4*x^2) + (2*B*a*b - 3*A*b^2)*log(b*x + a)/a^4 - (2*B*a*b - 3*A*b^2)*
log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x^3\,(a^2+2abx+b^2x^2)}\,dx = -\frac{(2\,Bab-3\,Ab^2)\log{(|x|)}}{a^4} + \frac{(2\,Bab^2-3\,Ab^3)\log{(|bx+a|)}}{a^4b} - \frac{Aa^3+2\,(2\,Ba^2b-3\,Aab^2)x^2+(2\,Ba^3-3\,Aa^2b)x}{2\,(bx+a)a^4x^2}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + (2*B*a*b^2 - 3*A*b^3)*log(abs(b*x +
 a))/(a^4*b) - 1/2*(A*a^3 + 2*(2*B*a^2*b - 3*A*a*b^2)*x^2 + (2*B*a^3 - 3*A
*a^2*b)*x)/((b*x + a)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{x^3\,(a^2+2abx+b^2x^2)}\,dx = \frac{\frac{x\,(3\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{2\,a}+\frac{b\,x^2\,(3\,A\,b-2\,B\,a)}{a^3}}{b\,x^3+a\,x^2} - \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(3\,A\,b-2\,B\,a)\,(a+2\,b\,x)}{a\,(3\,A\,b^2-2\,B\,a\,b)}\right)\,(3\,A\,b-2\,B\,a)}{a^4}$$

input
```
int((A + B*x)/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
((x*(3*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (b*x^2*(3*A*b - 2*B*a))/a^3)/(a*x
^2 + b*x^3) - (2*b*atanh((b*(3*A*b - 2*B*a)*(a + 2*b*x))/(a*(3*A*b^2 - 2*B
*a*b)))*(3*A*b - 2*B*a))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx}{x^3\,(a^2+2abx+b^2x^2)}\,dx = \frac{-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx}{2a^3x^2}$$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.260 $\int \frac{A+Bx}{x^4(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{2a^3x^2}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)}{a^4(a+bx)} - \frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx)}{a^5}$$

output
```
-1/3*A/a^2/x^3+1/2*(2*A*b-B*a)/a^3/x^2-b*(3*A*b-2*B*a)/a^4/x-b^2*(A*b-B*a)
/a^4/(b*x+a)-b^2*(4*A*b-3*B*a)*ln(x)/a^5+b^2*(4*A*b-3*B*a)*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{-\frac{2a^3A}{x^3}-\frac{3a^2(-2Ab+aB)}{x^2}+\frac{6ab(-3Ab+2aB)}{x}+\frac{6ab^2(-Ab+aB)}{a+bx}+6b^2(-4Ab+3aB)\log(x)+6b^2(4Ab-3aB)\log(}{6a^5}$$

input
```
Integrate[(A + B*x)/(x^4*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
((-2*a^3*A)/x^3 - (3*a^2*(-2*A*b + a*B))/x^2 + (6*a*b*(-3*A*b + 2*a*B))/x
+ (6*a*b^2*(-(A*b) + a*B))/(a + b*x) + 6*b^2*(-4*A*b + 3*a*B)*Log[x] + 6*b
^2*(4*A*b - 3*a*B)*Log[a + b*x])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^4(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^4(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{b^3(3aB-4Ab)}{a^5(a+bx)}+\frac{b^2(3aB-4Ab)}{a^5x}-\frac{b^3(aB-Ab)}{a^4(a+bx)^2}-\frac{b(2aB-3Ab)}{a^4x^2}+\frac{aB-2Ab}{a^3x^3}+\frac{A}{a^2x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\log(x)(4Ab-3aB)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx)}{a^5}-\frac{b^2(Ab-aB)}{a^4(a+bx)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{3a^2x^3}$$

input
```
Int[(A + B*x)/(x^4*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-1/3*A/(a^2*x^3) + (2*A*b - a*B)/(2*a^3*x^2) - (b*(3*A*b - 2*a*B))/(a^4*x)
 - (b^2*(A*b - a*B))/(a^4*(a + b*x)) - (b^2*(4*A*b - 3*a*B)*Log[x])/a^5 +
(b^2*(4*A*b - 3*a*B)*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^{2}\left(4 A b -3 B a \right) \ln \left(b x +a \right)}{a^{5}}-\frac{b^{2}\left(A b -B a \right)}{a^{4}\left(b x +a \right)}-\frac{A}{3 a^{2} x^{3}}-\frac{-2 A b +B a}{2 x^{2} a^{3}}-\frac{b \left(3 A b -2 B a \right)}{a^{4} x}-\frac{b^{2}\left(4 A b -3 B a \right) \ln \left(x \right)}{a^{5}}$
norman	$\frac{\frac{b \left(4 A \,b^{3}-3 B a \,b^{2}\right) x^{4}}{a^{5}}-\frac{A}{3 a}+\frac{\left(4 A b -3 B a \right) x}{6 a^{2}}-\frac{b \left(4 A b -3 B a \right) x^{2}}{2 a^{3}}}{x^{3} \left(b x +a \right)}+\frac{b^{2}\left(4 A b -3 B a \right) \ln \left(b x +a \right)}{a^{5}}-\frac{b^{2}\left(4 A b -3 B a \right) \ln \left(x \right)}{a^{5}}$
risch	$\frac{-\frac{b^{2}\left(4 A b -3 B a \right) x^{3}}{a^{4}}-\frac{b \left(4 A b -3 B a \right) x^{2}}{2 a^{3}}+\frac{\left(4 A b -3 B a \right) x}{6 a^{2}}-\frac{A}{3 a}}{x^{3} \left(b x +a \right)}-\frac{4 b^{3} \ln \left(x \right) A}{a^{5}}+\frac{3 b^{2} \ln \left(x \right) B}{a^{4}}+\frac{4 b^{3} \ln \left(-b x -a \right) A}{a^{5}}-\frac{3 b^{2} \ln \left(-b x -a \right)}{a^{4}}$
parallelrisch	$-\frac{24 A \ln \left(x \right) x^{4} b^{4}-24 A \ln \left(b x +a \right) x^{4} b^{4}-18 B \ln \left(x \right) x^{4} a \,b^{3}+18 B \ln \left(b x +a \right) x^{4} a \,b^{3}+24 A \ln \left(x \right) x^{3} a \,b^{3}-24 A \ln \left(b x +a \right) x^{3} a \,b^{3}-24 A}{6 a^{5} \left(b x +a \right) x^{3}}$

input
```
int((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
b^2*(4*A*b-3*B*a)*ln(b*x+a)/a^5-b^2*(A*b-B*a)/a^4/(b*x+a)-1/3*A/a^2/x^3-1/
2*(-2*A*b+B*a)/x^2/a^3-b*(3*A*b-2*B*a)/a^4/x-b^2*(4*A*b-3*B*a)*ln(x)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{2\,Aa^4-6\,(3\,Ba^2b^2-4\,Aab^3)x^3-3\,(3\,Ba^3b-4\,Aa^2b^2)x^2+(3\,Ba^4-4\,Aa^3b)x+6\,((3\,Bab^3-4\,Ab^4)x}{6\,(a^5bx^4+}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-1/6*(2*A*a^4 - 6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 4*A*a^2*b
^2)*x^2 + (3*B*a^4 - 4*A*a^3*b)*x + 6*((3*B*a*b^3 - 4*A*b^4)*x^4 + (3*B*a^
2*b^2 - 4*A*a*b^3)*x^3)*log(b*x + a) - 6*((3*B*a*b^3 - 4*A*b^4)*x^4 + (3*B
*a^2*b^2 - 4*A*a*b^3)*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(104) = 208$.

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{-2Aa^3+x^3(-24Ab^3+18Bab^2)+x^2(-12Aab^2+9Ba^2b)+x(4Aa^2b-3Ba^3)}{6a^5x^3+6a^4bx^4} + \frac{b^2(-4Ab+3Ba)\log\left(x+\frac{-4Aab^3+3Ba^2b^2-ab^2(-4Ab+3Ba)}{-8Ab^4+6Bab^3}\right)}{a^5} - \frac{b^2(-4Ab+3Ba)\log\left(x+\frac{-4Aab^3+3Ba^2b^2+ab^2(-4Ab+3Ba)}{-8Ab^4+6Bab^3}\right)}{a^5}$$

input
```
integrate((B*x+A)/x**4/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
(-2*A*a**3 + x**3*(-24*A*b**3 + 18*B*a*b**2) + x**2*(-12*A*a*b**2 + 9*B*a*
*2*b) + x*(4*A*a**2*b - 3*B*a**3))/(6*a**5*x**3 + 6*a**4*b*x**4) + b**2*(-
4*A*b + 3*B*a)*log(x + (-4*A*a*b**3 + 3*B*a**2*b**2 - a*b**2*(-4*A*b + 3*B
*a))/(-8*A*b**4 + 6*B*a*b**3))/a**5 - b**2*(-4*A*b + 3*B*a)*log(x + (-4*A*
a*b**3 + 3*B*a**2*b**2 + a*b**2*(-4*A*b + 3*B*a))/(-8*A*b**4 + 6*B*a*b**3)
)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^4\,(a^2+2abx+b^2x^2)}\,dx = -\frac{2\,Aa^3-6\,(3\,Bab^2-4\,Ab^3)x^3-3\,(3\,Ba^2b-4\,Aab^2)x^2+(3\,Ba^3-4\,Aa^2b)x}{6\,(a^4bx^4+a^5x^3)} - \frac{(3\,Bab^2-4\,Ab^3)\log\left(bx+a\right)}{a^5} + \frac{(3\,Bab^2-4\,Ab^3)\log\left(x\right)}{a^5}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/6*(2*A*a^3 - 6*(3*B*a*b^2 - 4*A*b^3)*x^3 - 3*(3*B*a^2*b - 4*A*a*b^2)*x^
2 + (3*B*a^3 - 4*A*a^2*b)*x)/(a^4*b*x^4 + a^5*x^3) - (3*B*a*b^2 - 4*A*b^3)
*log(b*x + a)/a^5 + (3*B*a*b^2 - 4*A*b^3)*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^4\,(a^2+2abx+b^2x^2)}\,dx = \frac{(3\,Bab^2-4\,Ab^3)\log\left(|x|\right)}{a^5} - \frac{(3\,Bab^3-4\,Ab^4)\log\left(|bx+a|\right)}{a^5b} - \frac{2\,Aa^4-6\,(3\,Ba^2b^2-4\,Aab^3)x^3-3\,(3\,Ba^3b-4\,Aa^2b^2)x^2+(3\,Ba^4-4\,Aa^3b)x}{6\,(bx+a)a^5x^3}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(3*B*a*b^2 - 4*A*b^3)*log(abs(x))/a^5 - (3*B*a*b^3 - 4*A*b^4)*log(abs(b*x
+ a))/(a^5*b) - 1/6*(2*A*a^4 - 6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^3 - 3*(3*B*a^
3*b - 4*A*a^2*b^2)*x^2 + (3*B*a^4 - 4*A*a^3*b)*x)/((b*x + a)*a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{2\,b^2\,\mathrm{atanh}\left(\frac{b^2\,(4\,A\,b-3\,B\,a)\,(a+2\,b\,x)}{a\,(4\,A\,b^3-3\,B\,a\,b^2)}\right)\,(4\,A\,b-3\,B\,a)}{a^5}-\frac{\frac{A}{3\,a}-\frac{x\,(4\,A\,b-3\,B\,a)}{6\,a^2}+\frac{b^2\,x^3\,(4\,A\,b-3\,B\,a)}{a^4}+\frac{b\,x^2\,(4\,A\,b-3\,B\,a)}{2\,a^3}}{b\,x^4+a\,x^3}$$

input
```
int((A + B*x)/(x^4*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
(2*b^2*atanh((b^2*(4*A*b - 3*B*a)*(a + 2*b*x))/(a*(4*A*b^3 - 3*B*a*b^2)))*
(4*A*b - 3*B*a))/a^5 - (A/(3*a) - (x*(4*A*b - 3*B*a))/(6*a^2) + (b^2*x^3*(
4*A*b - 3*B*a))/a^4 + (b*x^2*(4*A*b - 3*B*a))/(2*a^3))/(a*x^3 + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2}{6a^4x^3}$$

input
```
int((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*b*x - 6*a
*b**2*x**2)/(6*a**4*x**3)
```

3.261 $\int \frac{A+Bx}{x^5(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\begin{aligned}\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx = &-\frac{A}{4a^2x^4}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{2a^4x^2}\\ &+\frac{b^2(4Ab-3aB)}{a^5x}+\frac{b^3(Ab-aB)}{a^5(a+bx)}\\ &+\frac{b^3(5Ab-4aB)\log(x)}{a^6}-\frac{b^3(5Ab-4aB)\log(a+bx)}{a^6}\end{aligned}$$

output
```
-1/4*A/a^2/x^4+1/3*(2*A*b-B*a)/a^3/x^3-1/2*b*(3*A*b-2*B*a)/a^4/x^2+b^2*(4*
A*b-3*B*a)/a^5/x+b^3*(A*b-B*a)/a^5/(b*x+a)+b^3*(5*A*b-4*B*a)*ln(x)/a^6-b^3
*(5*A*b-4*B*a)*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx$$
$$=\frac{\frac{a\left(60Ab^4x^4+6ab^3x^3(5A-8Bx)-a^4(3A+4Bx)+a^3bx(5A+8Bx)-2a^2b^2x^2(5A+12Bx)\right)}{x^4(a+bx)}+12b^3(5Ab-4aB)\log(x)+12b^3(-5A}{12a^6}$$

input
```
Integrate[(A + B*x)/(x^5*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
((a*(60*A*b^4*x^4 + 6*a*b^3*x^3*(5*A - 8*B*x) - a^4*(3*A + 4*B*x) + a^3*b*
x*(5*A + 8*B*x) - 2*a^2*b^2*x^2*(5*A + 12*B*x)))/(x^4*(a + b*x)) + 12*b^3*
(5*A*b - 4*a*B)*Log[x] + 12*b^3*(-5*A*b + 4*a*B)*Log[a + b*x])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5(a^2+2abx+b^2x^2)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^5(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^5(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^4(4aB-5Ab)}{a^6(a+bx)}-\frac{b^3(4aB-5Ab)}{a^6x}+\frac{b^4(aB-Ab)}{a^5(a+bx)^2}+\frac{b^2(3aB-4Ab)}{a^5x^2}-\frac{b(2aB-3Ab)}{a^4x^3}+\frac{aB-2Ab}{a^3x^4}+\frac{A}{a^2x^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\log(x)(5Ab-4aB)}{a^6}-\frac{b^3(5Ab-4aB)\log(a+bx)}{a^6}+\frac{b^3(Ab-aB)}{a^5(a+bx)}+\frac{b^2(4Ab-3aB)}{a^5x}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{4a^2x^4}$$

input
```
Int[(A + B*x)/(x^5*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-1/4*A/(a^2*x^4) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/(2*a^4*
x^2) + (b^2*(4*A*b - 3*a*B))/(a^5*x) + (b^3*(A*b - a*B))/(a^5*(a + b*x)) +
 (b^3*(5*A*b - 4*a*B)*Log[x])/a^6 - (b^3*(5*A*b - 4*a*B)*Log[a + b*x])/a^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

method	result
default	$-\frac{b^{3}(5Ab-4Ba)\ln(bx+a)}{a^{6}}+\frac{b^{3}(Ab-Ba)}{a^{5}(bx+a)}-\frac{A}{4a^{2}x^{4}}-\frac{-2Ab+Ba}{3x^{3}a^{3}}-\frac{b(3Ab-2Ba)}{2a^{4}x^{2}}+\frac{b^{3}(5Ab-4Ba)\ln(x)}{a^{6}}+\frac{b^{2}(4Ab-}{a^{5}}$
norman	$\frac{\frac{\left(5A\,b^{5}-4Ba\,b^{4}\right)x^{4}}{a^{5}b}-\frac{A}{4a}+\frac{(5Ab-4Ba)x}{12a^{2}}-\frac{b(5Ab-4Ba)x^{2}}{6a^{3}}+\frac{b^{2}(5Ab-4Ba)x^{3}}{2a^{4}}}{x^{4}(bx+a)}+\frac{b^{3}(5Ab-4Ba)\ln(x)}{a^{6}}-\frac{b^{3}(5Ab-4Ba)\ln(bx+a)}{a^{6}}$
risch	$\frac{\frac{b^{3}(5Ab-4Ba)x^{4}}{a^{5}}+\frac{b^{2}(5Ab-4Ba)x^{3}}{2a^{4}}-\frac{b(5Ab-4Ba)x^{2}}{6a^{3}}+\frac{(5Ab-4Ba)x}{12a^{2}}-\frac{A}{4a}}{x^{4}(bx+a)}-\frac{5b^{4}\ln(bx+a)A}{a^{6}}+\frac{4b^{3}\ln(bx+a)B}{a^{5}}+\frac{5b^{4}\ln(-x)A}{a^{6}}$
parallelrisch	$\frac{60A\ln(x)x^{5}b^{6}-60A\ln(bx+a)x^{5}b^{6}-48B\ln(x)x^{5}a\,b^{5}+48B\ln(bx+a)x^{5}a\,b^{5}+60A\ln(x)x^{4}a\,b^{5}-60A\ln(bx+a)x^{4}a\,b^{5}-48B\ln(}{}$

input
```
int((B*x+A)/x^5/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-b^3*(5*A*b-4*B*a)*ln(b*x+a)/a^6+b^3*(A*b-B*a)/a^5/(b*x+a)-1/4*A/a^2/x^4-1
/3*(-2*A*b+B*a)/x^3/a^3-1/2*b*(3*A*b-2*B*a)/a^4/x^2+b^3*(5*A*b-4*B*a)*ln(x
)/a^6+b^2*(4*A*b-3*B*a)/a^5/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{3\,Aa^5+12\,(4\,Ba^2b^3-5\,Aab^4)x^4+6\,(4\,Ba^3b^2-5\,Aa^2b^3)x^3-2\,(4\,Ba^4b-5\,Aa^3b^2)x^2+(4\,Ba^5-5\,A}{}$$

input
```
integrate((B*x+A)/x^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
-1/12*(3*A*a^5 + 12*(4*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 6*(4*B*a^3*b^2 - 5*A*a
^2*b^3)*x^3 - 2*(4*B*a^4*b - 5*A*a^3*b^2)*x^2 + (4*B*a^5 - 5*A*a^4*b)*x -
12*((4*B*a*b^4 - 5*A*b^5)*x^5 + (4*B*a^2*b^3 - 5*A*a*b^4)*x^4)*log(b*x + a
) + 12*((4*B*a*b^4 - 5*A*b^5)*x^5 + (4*B*a^2*b^3 - 5*A*a*b^4)*x^4)*log(x))
/(a^6*b*x^5 + a^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{-3Aa^4+x^4\cdot(60Ab^4-48Bab^3)+x^3\cdot(30Aab^3-24Ba^2b^2)+x^2(-10Aa^2b^2+8Ba^3b)+x(5Aa^3b-4B}{12a^6x^4+12a^5bx^5} - \frac{b^3(-5Ab+4Ba)\log\left(x+\frac{-5Aab^4+4Ba^2b^3-ab^3(-5Ab+4Ba)}{-10Ab^5+8Bab^4}\right)}{a^6} + \frac{b^3(-5Ab+4Ba)\log\left(x+\frac{-5Aab^4+4Ba^2b^3+ab^3(-5Ab+4Ba)}{-10Ab^5+8Bab^4}\right)}{a^6}$$

input
```
integrate((B*x+A)/x**5/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
(-3*A*a**4 + x**4*(60*A*b**4 - 48*B*a*b**3) + x**3*(30*A*a*b**3 - 24*B*a**
2*b**2) + x**2*(-10*A*a**2*b**2 + 8*B*a**3*b) + x*(5*A*a**3*b - 4*B*a**4))
/(12*a**6*x**4 + 12*a**5*b*x**5) - b**3*(-5*A*b + 4*B*a)*log(x + (-5*A*a*b
**4 + 4*B*a**2*b**3 - a*b**3*(-5*A*b + 4*B*a))/(-10*A*b**5 + 8*B*a*b**4))/
a**6 + b**3*(-5*A*b + 4*B*a)*log(x + (-5*A*a*b**4 + 4*B*a**2*b**3 + a*b**3
*(-5*A*b + 4*B*a))/(-10*A*b**5 + 8*B*a*b**4))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{3\,Aa^4+12\,(4\,Bab^3-5\,Ab^4)x^4+6\,(4\,Ba^2b^2-5\,Aab^3)x^3-2\,(4\,Ba^3b-5\,Aa^2b^2)x^2+(4\,Ba^4-5\,Aa^3b)}{12\,(a^5bx^5+a^6x^4)} + \frac{(4\,Bab^3-5\,Ab^4)\log(bx+a)}{a^6} - \frac{(4\,Bab^3-5\,Ab^4)\log(x)}{a^6}$$

input
```
integrate((B*x+A)/x^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/12*(3*A*a^4 + 12*(4*B*a*b^3 - 5*A*b^4)*x^4 + 6*(4*B*a^2*b^2 - 5*A*a*b^3
)*x^3 - 2*(4*B*a^3*b - 5*A*a^2*b^2)*x^2 + (4*B*a^4 - 5*A*a^3*b)*x)/(a^5*b*
x^5 + a^6*x^4) + (4*B*a*b^3 - 5*A*b^4)*log(b*x + a)/a^6 - (4*B*a*b^3 - 5*A
*b^4)*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^5\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{(4\,Bab^3-5\,Ab^4)\log(|x|)}{a^6} + \frac{(4\,Bab^4-5\,Ab^5)\log(|bx+a|)}{a^6b} - \frac{3\,Aa^5+12\,(4\,Ba^2b^3-5\,Aab^4)x^4+6\,(4\,Ba^3b^2-5\,Aa^2b^3)x^3-2\,(4\,Ba^4b-5\,Aa^3b^2)x^2+(4\,Ba^5-5\,Aa}{12\,(bx+a)a^6x^4}$$

input
```
integrate((B*x+A)/x^5/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(4*B*a*b^3 - 5*A*b^4)*log(abs(x))/a^6 + (4*B*a*b^4 - 5*A*b^5)*log(abs(b*x
 + a))/(a^6*b) - 1/12*(3*A*a^5 + 12*(4*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 6*(4*B
*a^3*b^2 - 5*A*a^2*b^3)*x^3 - 2*(4*B*a^4*b - 5*A*a^3*b^2)*x^2 + (4*B*a^5 -
 5*A*a^4*b)*x)/((b*x + a)*a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^5(a^2+2abx+b^2x^2)}\,dx = \frac{\frac{x\,(5\,A\,b-4\,B\,a)}{12\,a^2}-\frac{A}{4\,a}+\frac{b^2\,x^3\,(5\,A\,b-4\,B\,a)}{2\,a^4}+\frac{b^3\,x^4\,(5\,A\,b-4\,B\,a)}{a^5}-\frac{b\,x^2\,(5\,A\,b-4\,B\,a)}{6\,a^3}}{b\,x^5+a\,x^4} - \frac{2\,b^3\,\mathrm{atanh}\left(\frac{b^3\,(5\,A\,b-4\,B\,a)\,(a+2\,b\,x)}{a\,(5\,A\,b^4-4\,B\,a\,b^3)}\right)\,(5\,A\,b-4\,B\,a)}{a^6}$$

input
```
int((A + B*x)/(x^5*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
((x*(5*A*b - 4*B*a))/(12*a^2) - A/(4*a) + (b^2*x^3*(5*A*b - 4*B*a))/(2*a^4
) + (b^3*x^4*(5*A*b - 4*B*a))/a^5 - (b*x^2*(5*A*b - 4*B*a))/(6*a^3))/(a*x^
4 + b*x^5) - (2*b^3*atanh((b^3*(5*A*b - 4*B*a)*(a + 2*b*x))/(a*(5*A*b^4 -
4*B*a*b^3)))*(5*A*b - 4*B*a))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx}{x^5(a^2+2abx+b^2x^2)}\,dx = \frac{-12\log(bx+a)\,b^4x^4+12\log(x)\,b^4x^4-3a^4+4a^3bx-6a^2b^2x^2+12a\,b^3x^3}{12a^5x^4}$$

input
```
int((B*x+A)/x^5/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 12*log(a + b*x)*b**4*x**4 + 12*log(x)*b**4*x**4 - 3*a**4 + 4*a**3*b*x
- 6*a**2*b**2*x**2 + 12*a*b**3*x**3)/(12*a**5*x**4)
```

3.262 $\int \frac{x^5(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{x^5(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{2a(2Ab-5aB)x}{b^6}+\frac{(Ab-4aB)x^2}{2b^5} +\frac{Bx^3}{3b^4}+\frac{a^5(Ab-aB)}{3b^7(a+bx)^3}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx)^2} +\frac{5a^3(2Ab-3aB)}{b^7(a+bx)}+\frac{10a^2(Ab-2aB)\log(a+bx)}{b^7}$$

output
```
-2*a*(2*A*b-5*B*a)*x/b^6+1/2*(A*b-4*B*a)*x^2/b^5+1/3*B*x^3/b^4+1/3*a^5*(A*
b-B*a)/b^7/(b*x+a)^3-1/2*a^4*(5*A*b-6*B*a)/b^7/(b*x+a)^2+5*a^3*(2*A*b-3*B*
a)/b^7/(b*x+a)+10*a^2*(A*b-2*B*a)*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-74a^6B+a^5b(47A-102Bx)+b^6x^5(3A+2Bx)-3ab^5x^4(5A+2Bx)+3a^2b^4x^3(-21A+10Bx)+3a}{6b^7(a+bx)^3}$$

input
```
Integrate[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(-74*a^6*B + a^5*b*(47*A - 102*B*x) + b^6*x^5*(3*A + 2*B*x) - 3*a*b^5*x^4*
(5*A + 2*B*x) + 3*a^2*b^4*x^3*(-21*A + 10*B*x) + 3*a^4*b^2*x*(27*A + 26*B*
x) + a^3*b^3*x^2*(-9*A + 146*B*x) - 60*a^2*(-(A*b) + 2*a*B)*(a + b*x)^3*Lo
g[a + b*x])/(6*b^7*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{x^5(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^5(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^5(aB-Ab)}{b^6(a+bx)^4}-\frac{a^4(6aB-5Ab)}{b^6(a+bx)^3}+\frac{5a^3(3aB-2Ab)}{b^6(a+bx)^2}-\frac{10a^2(2aB-Ab)}{b^6(a+bx)}+\frac{2a(5aB-2Ab)}{b^6}+\frac{x(Ab-4aB)}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5(Ab-aB)}{3b^7(a+bx)^3}-\frac{a^4(5Ab-6aB)}{2b^7(a+bx)^2}+\frac{5a^3(2Ab-3aB)}{b^7(a+bx)}+\frac{10a^2(Ab-2aB)\log(a+bx)}{b^7}-\frac{2ax(2Ab-5aB)}{b^6}+\frac{x^2(Ab-4aB)}{2b^5}+\frac{Bx^3}{3b^4}$$

input
```
Int[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(-2*a*(2*A*b - 5*a*B)*x)/b^6 + ((A*b - 4*a*B)*x^2)/(2*b^5) + (B*x^3)/(3*b^
4) + (a^5*(A*b - a*B))/(3*b^7*(a + b*x)^3) - (a^4*(5*A*b - 6*a*B))/(2*b^7*
(a + b*x)^2) + (5*a^3*(2*A*b - 3*a*B))/(b^7*(a + b*x)) + (10*a^2*(A*b - 2*
a*B)*Log[a + b*x])/b^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

method	result
default	$-\frac{-\frac{1}{3}x^{3}B\,b^{2}-\frac{1}{2}x^{2}b^{2}A+2Ba\,x^{2}b+4abAx-10a^{2}Bx}{b^{6}}+\frac{a^{5}(Ab-Ba)}{3b^{7}(bx+a)^{3}}+\frac{5a^{3}(2Ab-3Ba)}{b^{7}(bx+a)}+\frac{10a^{2}(Ab-2Ba)\ln(bx+a)}{b^{7}}-$
norman	$\frac{\frac{B\,x^{6}}{3b}+\frac{a^{3}\left(55A\,a^{2}b-110B\,a^{3}\right)}{3b^{7}}+\frac{(Ab-2Ba)x^{5}}{2b^{2}}-\frac{5a(Ab-2Ba)x^{4}}{2b^{3}}+\frac{3a\left(10A\,a^{2}b-20B\,a^{3}\right)x^{2}}{b^{5}}+\frac{3a^{2}\left(15A\,a^{2}b-30B\,a^{3}\right)x}{b^{6}}}{(bx+a)^{3}}+\frac{10a^{2}(Ab-2}{}$
risch	$\frac{B\,x^{3}}{3b^{4}}+\frac{x^{2}A}{2b^{4}}-\frac{2Ba\,x^{2}}{b^{5}}-\frac{4aAx}{b^{5}}+\frac{10a^{2}Bx}{b^{6}}+\frac{(10a^{3}A\,b^{2}-15B\,a^{4}b)x^{2}+\frac{a^{4}(35Ab-54Ba)x}{2}+\frac{a^{5}(47Ab-74Ba)}{6b}}{b^{6}(bx+a)(b^{2}x^{2}+2abx+a^{2})}+\frac{10a^{2}\ln}{}$
parallelrisch	$2b^{6}B\,x^{6}+3A\,b^{6}x^{5}-6Ba\,b^{5}x^{5}+60A\ln(bx+a)x^{3}a^{2}b^{4}-15Aa\,b^{5}x^{4}-120B\ln(bx+a)x^{3}a^{3}b^{3}+30B\,a^{2}b^{4}x^{4}+180A\ln(bx+a)x^{2}a^{3}b$

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/3*x^3*B*b^2-1/2*x^2*b^2*A+2*B*a*x^2*b+4*a*b*A*x-10*a^2*B*x)+1/3
*a^5*(A*b-B*a)/b^7/(b*x+a)^3+5*a^3*(2*A*b-3*B*a)/b^7/(b*x+a)+10*a^2*(A*b-2
*B*a)*ln(b*x+a)/b^7-1/2*a^4*(5*A*b-6*B*a)/b^7/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.80

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$=\frac{2\,Bb^6x^6-74\,Ba^6+47\,Aa^5b-3\,(2\,Bab^5-Ab^6)x^5+15\,(2\,Ba^2b^4-Aab^5)x^4+(146\,Ba^3b^3-63\,Aa^2b^4)x^3}{}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^6*x^6 - 74*B*a^6 + 47*A*a^5*b - 3*(2*B*a*b^5 - A*b^6)*x^5 + 15*
(2*B*a^2*b^4 - A*a*b^5)*x^4 + (146*B*a^3*b^3 - 63*A*a^2*b^4)*x^3 + 3*(26*B
*a^4*b^2 - 3*A*a^3*b^3)*x^2 - 3*(34*B*a^5*b - 27*A*a^4*b^2)*x - 60*(2*B*a^
6 - A*a^5*b + (2*B*a^3*b^3 - A*a^2*b^4)*x^3 + 3*(2*B*a^4*b^2 - A*a^3*b^3)*
x^2 + 3*(2*B*a^5*b - A*a^4*b^2)*x)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 +
 3*a^2*b^8*x + a^3*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{x^5(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{Bx^3}{3b^4} - \frac{10a^2(-Ab+2Ba)\log{(a+bx)}}{b^7} + x^2\left(\frac{A}{2b^4} - \frac{2Ba}{b^5}\right) + x\left(-\frac{4Aa}{b^5} + \frac{10Ba^2}{b^6}\right) + \frac{47Aa^5b - 74Ba^6 + x^2\cdot(60Aa^3b^3 - 90Ba^4b^2) + x(105Aa^4b^2 - 162Ba^5b)}{6a^3b^7 + 18a^2b^8x + 18ab^9x^2 + 6b^{10}x^3}$$

input
```
integrate(x**5*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
B*x**3/(3*b**4) - 10*a**2*(-A*b + 2*B*a)*log(a + b*x)/b**7 + x**2*(A/(2*b*
*4) - 2*B*a/b**5) + x*(-4*A*a/b**5 + 10*B*a**2/b**6) + (47*A*a**5*b - 74*B
*a**6 + x**2*(60*A*a**3*b**3 - 90*B*a**4*b**2) + x*(105*A*a**4*b**2 - 162*
B*a**5*b))/(6*a**3*b**7 + 18*a**2*b**8*x + 18*a*b**9*x**2 + 6*b**10*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{x^5(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{74\,Ba^6 - 47\,Aa^5b + 30\,(3\,Ba^4b^2 - 2\,Aa^3b^3)x^2 + 3\,(54\,Ba^5b - 35\,Aa^4b^2)x}{6\,(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7)} + \frac{2\,Bb^2x^3 - 3\,(4\,Bab - Ab^2)x^2 + 12\,(5\,Ba^2 - 2\,Aab)x}{6\,b^6} - \frac{10\,(2\,Ba^3 - Aa^2b)\log{(bx+a)}}{b^7}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(74*B*a^6 - 47*A*a^5*b + 30*(3*B*a^4*b^2 - 2*A*a^3*b^3)*x^2 + 3*(54*B
*a^5*b - 35*A*a^4*b^2)*x)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
 + 1/6*(2*B*b^2*x^3 - 3*(4*B*a*b - A*b^2)*x^2 + 12*(5*B*a^2 - 2*A*a*b)*x)/
b^6 - 10*(2*B*a^3 - A*a^2*b)*log(b*x + a)/b^7
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= -\frac{10\,(2\,Ba^3-Aa^2b)\log\left(|bx+a|\right)}{b^7} - \frac{74\,Ba^6-47\,Aa^5b+30\,(3\,Ba^4b^2-2\,Aa^3b^3)x^2+3\,(54\,Ba^5b-35\,Aa^4b^2)x}{6\,(bx+a)^3b^7} + \frac{2\,Bb^8x^3-12\,Bab^7x^2+3\,Ab^8x^2+60\,Ba^2b^6x-24\,Aab^7x}{6\,b^{12}}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-10*(2*B*a^3 - A*a^2*b)*log(abs(b*x + a))/b^7 - 1/6*(74*B*a^6 - 47*A*a^5*b
 + 30*(3*B*a^4*b^2 - 2*A*a^3*b^3)*x^2 + 3*(54*B*a^5*b - 35*A*a^4*b^2)*x)/(
(b*x + a)^3*b^7) + 1/6*(2*B*b^8*x^3 - 12*B*a*b^7*x^2 + 3*A*b^8*x^2 + 60*B*
a^2*b^6*x - 24*A*a*b^7*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= x^2\left(\frac{A}{2\,b^4}-\frac{2\,B\,a}{b^5}\right) - \frac{x\left(27\,B\,a^5-\frac{35\,A\,a^4\,b}{2}\right)-x^2\,\left(10\,A\,a^3\,b^2-15\,B\,a^4\,b\right)+\frac{74\,B\,a^6-47\,A\,a^5\,b}{6\,b}}{a^3\,b^6+3\,a^2\,b^7\,x+3\,a\,b^8\,x^2+b^9\,x^3} - x\left(\frac{4\,a\,\left(\frac{A}{b^4}-\frac{4\,B\,a}{b^5}\right)}{b}+\frac{6\,B\,a^2}{b^6}\right) - \frac{\ln\left(a+b\,x\right)\,\left(20\,B\,a^3-10\,A\,a^2\,b\right)}{b^7} + \frac{B\,x^3}{3\,b^4}$$

input
```
int((x^5*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^2*(A/(2*b^4) - (2*B*a)/b^5) - (x*(27*B*a^5 - (35*A*a^4*b)/2) - x^2*(10*A
*a^3*b^2 - 15*B*a^4*b) + (74*B*a^6 - 47*A*a^5*b)/(6*b))/(a^3*b^6 + b^9*x^3
 + 3*a^2*b^7*x + 3*a*b^8*x^2) - x*((4*a*(A/b^4 - (4*B*a)/b^5))/b + (6*B*a^
2)/b^6) - (log(a + b*x)*(20*B*a^3 - 10*A*a^2*b))/b^7 + (B*x^3)/(3*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \frac{-60\log(bx+a)\,a^5-120\log(bx+a)\,a^4bx-60\log(bx+a)\,a^3b^2x^2-30a^5+60a^3b^2x^2+20a^2b^3x^3-5a\,b}{6b^6\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 60*log(a + b*x)*a**5 - 120*log(a + b*x)*a**4*b*x - 60*log(a + b*x)*a**
3*b**2*x**2 - 30*a**5 + 60*a**3*b**2*x**2 + 20*a**2*b**3*x**3 - 5*a*b**4*x
**4 + 2*b**5*x**5)/(6*b**6*(a**2 + 2*a*b*x + b**2*x**2))
```

3.263 $\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2} dx = \frac{(Ab-4aB)x}{b^5} + \frac{Bx^2}{2b^4} - \frac{a^4(Ab-aB)}{3b^6(a+bx)^3} + \frac{a^3(4Ab-5aB)}{2b^6(a+bx)^2} - \frac{2a^2(3Ab-5aB)}{b^6(a+bx)} - \frac{2a(2Ab-5aB)\log(a+bx)}{b^6}$$

output
```
(A*b-4*B*a)*x/b^5+1/2*B*x^2/b^4-1/3*a^4*(A*b-B*a)/b^6/(b*x+a)^3+1/2*a^3*(4
*A*b-5*B*a)/b^6/(b*x+a)^2-2*a^2*(3*A*b-5*B*a)/b^6/(b*x+a)-2*a*(2*A*b-5*B*a
)*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2} dx = \frac{(Ab-4aB)x}{b^5} + \frac{Bx^2}{2b^4} + \frac{-a^4Ab+a^5B}{3b^6(a+bx)^3} + \frac{4a^3Ab-5a^4B}{2b^6(a+bx)^2} + \frac{2(-3a^2Ab+5a^3B)}{b^6(a+bx)} + \frac{2(-2aAb+5a^2B)\log(a+bx)}{b^6}$$

input
```
Integrate[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - 4*a*B)*x)/b^5 + (B*x^2)/(2*b^4) + (-(a^4*A*b) + a^5*B)/(3*b^6*(a +
 b*x)^3) + (4*a^3*A*b - 5*a^4*B)/(2*b^6*(a + b*x)^2) + (2*(-3*a^2*A*b + 5*
a^3*B))/(b^6*(a + b*x)) + (2*(-2*a*A*b + 5*a^2*B)*Log[a + b*x])/b^6
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{x^4(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^4(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^4}+\frac{a^3(5aB-4Ab)}{b^5(a+bx)^3}-\frac{2a^2(5aB-3Ab)}{b^5(a+bx)^2}+\frac{2a(5aB-2Ab)}{b^5(a+bx)}+\frac{Ab-4aB}{b^5}+\frac{Bx}{b^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4(Ab-aB)}{3b^6(a+bx)^3}+\frac{a^3(4Ab-5aB)}{2b^6(a+bx)^2}-\frac{2a^2(3Ab-5aB)}{b^6(a+bx)}-\frac{2a(2Ab-5aB)\log(a+bx)}{b^6}+\frac{x(Ab-4aB)}{b^5}+\frac{Bx^2}{2b^4}$$

input
```
Int[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - 4*a*B)*x)/b^5 + (B*x^2)/(2*b^4) - (a^4*(A*b - a*B))/(3*b^6*(a + b*
x)^3) + (a^3*(4*A*b - 5*a*B))/(2*b^6*(a + b*x)^2) - (2*a^2*(3*A*b - 5*a*B)
)/(b^6*(a + b*x)) - (2*a*(2*A*b - 5*a*B)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{1}{2} B b\,x^{2}+A b x -4 B a x}{b^{5}}-\frac{a^{4} \left(A b -B a \right)}{3 b^{6} \left(b x +a \right)^{3}}-\frac{2 a^{2} \left(3 A b -5 B a \right)}{b^{6} \left(b x +a \right)}-\frac{2 a \left(2 A b -5 B a \right) \ln \left(b x +a \right)}{b^{6}}+\frac{a^{3} \left(4 A b -5 B a \right)}{2 b^{6} \left(b x +a \right)^{2}}$
norman	$\frac{\frac{B\,x^{5}}{2 b}-\frac{a^{3} \left(22 a b A -55 a^{2} B \right)}{3 b^{6}}+\frac{\left(2 A b -5 B a \right) x^{4}}{2 b^{2}}-\frac{3 a \left(4 a b A -10 a^{2} B \right) x^{2}}{b^{4}}-\frac{3 a^{2} \left(6 a b A -15 a^{2} B \right) x}{b^{5}}}{\left(b x +a \right)^{3}}-\frac{2 a \left(2 A b -5 B a \right) \ln \left(b x +a \right)}{b^{6}}$
risch	$\frac{B\,x^{2}}{2 b^{4}}+\frac{A x}{b^{4}}-\frac{4 B a x}{b^{5}}+\frac{\left(-6 a^{2} A\,b^{2}+10 B\,a^{3} b \right) x^{2}-\frac{5 a^{3} \left(4 A b -7 B a \right) x}{2}-\frac{a^{4} \left(26 A b -47 B a \right)}{6 b}}{b^{5} \left(b x +a \right) \left(b^{2} x^{2}+2 a b x +a^{2}\right)}-\frac{4 a \ln \left(b x +a \right) A}{b^{5}}+\frac{10 a^{2} \ln \left(b x +a \right)}{b^{6}}$
parallelrisch	$-\frac{-3 B\,x^{5} b^{5}+24 A \ln \left(b x +a \right) x^{3} a\,b^{4}-6 A\,b^{5} x^{4}-60 B \ln \left(b x +a \right) x^{3} a^{2} b^{3}+15 B a\,b^{4} x^{4}+72 A \ln \left(b x +a \right) x^{2} a^{2} b^{3}-180 B \ln \left(b x +a \right) x^{2} a^{3}}{}$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/2*B*b*x^2+A*b*x-4*B*a*x)-1/3*a^4*(A*b-B*a)/b^6/(b*x+a)^3-2*a^2*(3
*A*b-5*B*a)/b^6/(b*x+a)-2*a*(2*A*b-5*B*a)*ln(b*x+a)/b^6+1/2*a^3*(4*A*b-5*B
*a)/b^6/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.91

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{3\,Bb^5x^5+47\,Ba^5-26\,Aa^4b-3\,(5\,Bab^4-2\,Ab^5)x^4-9\,(7\,Ba^2b^3-2\,Aab^4)x^3-9\,(Ba^3b^2+2\,Aa^2b^3)x^2+}{6}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*B*b^5*x^5 + 47*B*a^5 - 26*A*a^4*b - 3*(5*B*a*b^4 - 2*A*b^5)*x^4 - 9
*(7*B*a^2*b^3 - 2*A*a*b^4)*x^3 - 9*(B*a^3*b^2 + 2*A*a^2*b^3)*x^2 + 27*(3*B
*a^4*b - 2*A*a^3*b^2)*x + 12*(5*B*a^5 - 2*A*a^4*b + (5*B*a^2*b^3 - 2*A*a*b
^4)*x^3 + 3*(5*B*a^3*b^2 - 2*A*a^2*b^3)*x^2 + 3*(5*B*a^4*b - 2*A*a^3*b^2)*
x)*log(b*x + a))/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx &= \frac{Bx^2}{2b^4}+\frac{2a(-2Ab+5Ba)\log{(a+bx)}}{b^6}+x\left(\frac{A}{b^4}-\frac{4Ba}{b^5}\right)\\ &\quad+\frac{-26Aa^4b+47Ba^5+x^2(-36Aa^2b^3+60Ba^3b^2)+x(-60Aa^3b^2+105Ba^4b)}{6a^3b^6+18a^2b^7x+18ab^8x^2+6b^9x^3}\end{aligned}$$

input
```
integrate(x**4*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
B*x**2/(2*b**4) + 2*a*(-2*A*b + 5*B*a)*log(a + b*x)/b**6 + x*(A/b**4 - 4*B
*a/b**5) + (-26*A*a**4*b + 47*B*a**5 + x**2*(-36*A*a**2*b**3 + 60*B*a**3*b
**2) + x*(-60*A*a**3*b**2 + 105*B*a**4*b))/(6*a**3*b**6 + 18*a**2*b**7*x +
 18*a*b**8*x**2 + 6*b**9*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{47\,Ba^5-26\,Aa^4b+12\,(5\,Ba^3b^2-3\,Aa^2b^3)x^2+15\,(7\,Ba^4b-4\,Aa^3b^2)x}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)} + \frac{Bbx^2-2\,(4\,Ba-Ab)x}{2\,b^5} + \frac{2\,(5\,Ba^2-2\,Aab)\log(bx+a)}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*(47*B*a^5 - 26*A*a^4*b + 12*(5*B*a^3*b^2 - 3*A*a^2*b^3)*x^2 + 15*(7*B*
a^4*b - 4*A*a^3*b^2)*x)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6) +
1/2*(B*b*x^2 - 2*(4*B*a - A*b)*x)/b^5 + 2*(5*B*a^2 - 2*A*a*b)*log(b*x + a)
/b^6
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{2\,(5\,Ba^2-2\,Aab)\log(|bx+a|)}{b^6} + \frac{Bb^4x^2-8\,Bab^3x+2\,Ab^4x}{2\,b^8} + \frac{47\,Ba^5-26\,Aa^4b+12\,(5\,Ba^3b^2-3\,Aa^2b^3)x^2+15\,(7\,Ba^4b-4\,Aa^3b^2)x}{6\,(bx+a)^3b^6}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2*(5*B*a^2 - 2*A*a*b)*log(abs(b*x + a))/b^6 + 1/2*(B*b^4*x^2 - 8*B*a*b^3*x
 + 2*A*b^4*x)/b^8 + 1/6*(47*B*a^5 - 26*A*a^4*b + 12*(5*B*a^3*b^2 - 3*A*a^2
*b^3)*x^2 + 15*(7*B*a^4*b - 4*A*a^3*b^2)*x)/((b*x + a)^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$=x\left(\frac{A}{b^4}-\frac{4\,B\,a}{b^5}\right)+\frac{x\left(\frac{35\,B\,a^4}{2}-10\,A\,a^3\,b\right)-x^2\,(6\,A\,a^2\,b^2-10\,B\,a^3\,b)+\frac{47\,B\,a^5-26\,A\,a^4\,b}{6\,b}}{a^3\,b^5+3\,a^2\,b^6\,x+3\,a\,b^7\,x^2+b^8\,x^3}$$
$$+\frac{B\,x^2}{2\,b^4}+\frac{\ln(a+b\,x)\,(10\,B\,a^2-4\,A\,a\,b)}{b^6}$$

input
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x*(A/b^4 - (4*B*a)/b^5) + (x*((35*B*a^4)/2 - 10*A*a^3*b) - x^2*(6*A*a^2*b^
2 - 10*B*a^3*b) + (47*B*a^5 - 26*A*a^4*b)/(6*b))/(a^3*b^5 + b^8*x^3 + 3*a^
2*b^6*x + 3*a*b^7*x^2) + (B*x^2)/(2*b^4) + (log(a + b*x)*(10*B*a^2 - 4*A*a
*b))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$=\frac{12\log(bx+a)\,a^4+24\log(bx+a)\,a^3bx+12\log(bx+a)\,a^2b^2x^2+6a^4-12a^2b^2x^2-4a\,b^3x^3+b^4x^4}{2b^5\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(12*log(a + b*x)*a**4 + 24*log(a + b*x)*a**3*b*x + 12*log(a + b*x)*a**2*b*
*2*x**2 + 6*a**4 - 12*a**2*b**2*x**2 - 4*a*b**3*x**3 + b**4*x**4)/(2*b**5*
(a**2 + 2*a*b*x + b**2*x**2))
```

3.264 $\int \frac{x^3(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{x^3(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{Bx}{b^4}+\frac{a^3(Ab-aB)}{3b^5(a+bx)^3}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx)^2}+\frac{3a(Ab-2aB)}{b^5(a+bx)}+\frac{(Ab-4aB)\log(a+bx)}{b^5}$$

output
```
B*x/b^4+1/3*a^3*(A*b-B*a)/b^5/(b*x+a)^3-1/2*a^2*(3*A*b-4*B*a)/b^5/(b*x+a)^
2+3*a*(A*b-2*B*a)/b^5/(b*x+a)+(A*b-4*B*a)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$$
$$=\frac{-26a^4B+6b^4Bx^4+a^3b(11A-54Bx)+9a^2b^2x(3A-2Bx)+18ab^3x^2(A+Bx)+6(Ab-4aB)(a+b}{6b^5(a+bx)^3}$$

input
```
Integrate[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(-26*a^4*B + 6*b^4*B*x^4 + a^3*b*(11*A - 54*B*x) + 9*a^2*b^2*x*(3*A - 2*B*
x) + 18*a*b^3*x^2*(A + B*x) + 6*(A*b - 4*a*B)*(a + b*x)^3*Log[a + b*x])/(6
*b^5*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{x^3(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^4}-\frac{a^2(4aB-3Ab)}{b^4(a+bx)^3}+\frac{3a(2aB-Ab)}{b^4(a+bx)^2}+\frac{Ab-4aB}{b^4(a+bx)}+\frac{B}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(Ab-aB)}{3b^5(a+bx)^3}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx)^2}+\frac{3a(Ab-2aB)}{b^5(a+bx)}+\frac{(Ab-4aB)\log(a+bx)}{b^5}+\frac{Bx}{b^4}$$

input
```
Int[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(B*x)/b^4 + (a^3*(A*b - a*B))/(3*b^5*(a + b*x)^3) - (a^2*(3*A*b - 4*a*B))/
(2*b^5*(a + b*x)^2) + (3*a*(A*b - 2*a*B))/(b^5*(a + b*x)) + ((A*b - 4*a*B)
*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{B\,x^4}{b}+\frac{a^3(11Ab-44Ba)}{6b^5}+\frac{3a(Ab-4Ba)x^2}{b^3}+\frac{3a^2(3Ab-12Ba)x}{2b^4}}{(bx+a)^3}+\frac{(Ab-4Ba)\ln(bx+a)}{b^5}$
default	$\frac{Bx}{b^4}+\frac{a^3(Ab-Ba)}{3b^5(bx+a)^3}-\frac{a^2(3Ab-4Ba)}{2b^5(bx+a)^2}+\frac{3a(Ab-2Ba)}{b^5(bx+a)}+\frac{(Ab-4Ba)\ln(bx+a)}{b^5}$
risch	$\frac{Bx}{b^4}+\frac{(3Aa\,b^2-6B\,a^2b)x^2+\frac{a^2(9Ab-20Ba)x}{2}+\frac{a^3(11Ab-26Ba)}{6b}}{b^4(bx+a)(b^2x^2+2abx+a^2)}+\frac{\ln(bx+a)A}{b^4}-\frac{4\ln(bx+a)Ba}{b^5}$
parallelrisch	$\frac{6A\ln(bx+a)x^3b^4-24B\ln(bx+a)x^3a\,b^3+6b^4B\,x^4+18A\ln(bx+a)x^2a\,b^3-72B\ln(bx+a)x^2a^2b^2+18A\ln(bx+a)x\,a^2b^2+18Aa\,b}{6b^5(b^2x^2+2abx+}$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(B/b*x^4+1/6*a^3*(11*A*b-44*B*a)/b^5+3*a*(A*b-4*B*a)/b^3*x^2+3/2*a^2*(3*A*
b-12*B*a)/b^4*x)/(b*x+a)^3+(A*b-4*B*a)*ln(b*x+a)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(96) = 192$.

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.99

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \frac{6\,Bb^4x^4 + 18\,Bab^3x^3 - 26\,Ba^4 + 11\,Aa^3b - 18\,(Ba^2b^2 - Aab^3)x^2 - 27\,(2\,Ba^3b - Aa^2b^2)x - 6\,(4\,Ba^4 -}{6\,(b^8x^3 + 3\,ab^7x^2 + 3\,a^2b^6x}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(6*B*b^4*x^4 + 18*B*a*b^3*x^3 - 26*B*a^4 + 11*A*a^3*b - 18*(B*a^2*b^2
- A*a*b^3)*x^2 - 27*(2*B*a^3*b - A*a^2*b^2)*x - 6*(4*B*a^4 - A*a^3*b + (4*
B*a*b^3 - A*b^4)*x^3 + 3*(4*B*a^2*b^2 - A*a*b^3)*x^2 + 3*(4*B*a^3*b - A*a^
2*b^2)*x)*log(b*x + a))/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \frac{Bx}{b^4} + \frac{11Aa^3b - 26Ba^4 + x^2\cdot(18Aab^3 - 36Ba^2b^2) + x(27Aa^2b^2 - 60Ba^3b)}{6a^3b^5 + 18a^2b^6x + 18ab^7x^2 + 6b^8x^3}$$
$$- \frac{(-Ab+4Ba)\log{(a+bx)}}{b^5}$$

input
```
integrate(x**3*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
B*x/b**4 + (11*A*a**3*b - 26*B*a**4 + x**2*(18*A*a*b**3 - 36*B*a**2*b**2)
+ x*(27*A*a**2*b**2 - 60*B*a**3*b))/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b
**7*x**2 + 6*b**8*x**3) - (-A*b + 4*B*a)*log(a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \frac{x^3(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{26\,Ba^4-11\,Aa^3b+18\,(2\,Ba^2b^2-Aab^3)x^2+3\,(20\,Ba^3b-9\,Aa^2b^2)x}{6\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)} + \frac{Bx}{b^4} - \frac{(4\,Ba-Ab)\log{(bx+a)}}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(26*B*a^4 - 11*A*a^3*b + 18*(2*B*a^2*b^2 - A*a*b^3)*x^2 + 3*(20*B*a^3
*b - 9*A*a^2*b^2)*x)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5) + B*x
/b^4 - (4*B*a - A*b)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{x^3(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{Bx}{b^4} - \frac{(4\,Ba-Ab)\log{(|bx+a|)}}{b^5} - \frac{26\,Ba^4-11\,Aa^3b+18\,(2\,Ba^2b^2-Aab^3)x^2+3\,(20\,Ba^3b-9\,Aa^2b^2)x}{6\,(bx+a)^3b^5}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
B*x/b^4 - (4*B*a - A*b)*log(abs(b*x + a))/b^5 - 1/6*(26*B*a^4 - 11*A*a^3*b
 + 18*(2*B*a^2*b^2 - A*a*b^3)*x^2 + 3*(20*B*a^3*b - 9*A*a^2*b^2)*x)/((b*x
+ a)^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{B\,x}{b^4} - \frac{x\left(10\,B\,a^3-\frac{9\,A\,a^2\,b}{2}\right)-x^2\,\left(3\,A\,a\,b^2-6\,B\,a^2\,b\right)+\frac{26\,B\,a^4-11\,A\,a^3\,b}{6\,b}}{a^3\,b^4+3\,a^2\,b^5\,x+3\,a\,b^6\,x^2+b^7\,x^3} + \frac{\ln\left(a+b\,x\right)\,\left(A\,b-4\,B\,a\right)}{b^5}$$

input
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(B*x)/b^4 - (x*(10*B*a^3 - (9*A*a^2*b)/2) - x^2*(3*A*a*b^2 - 6*B*a^2*b) +
(26*B*a^4 - 11*A*a^3*b)/(6*b))/(a^3*b^4 + b^7*x^3 + 3*a^2*b^5*x + 3*a*b^6*
x^2) + (log(a + b*x)*(A*b - 4*B*a))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{-6\log(bx+a)\,a^3-12\log(bx+a)\,a^2bx-6\log(bx+a)\,a\,b^2x^2-3a^3+6a\,b^2x^2+2b^3x^3}{2b^4\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 6*log(a + b*x)*a**3 - 12*log(a + b*x)*a**2*b*x - 6*log(a + b*x)*a*b**2
*x**2 - 3*a**3 + 6*a*b**2*x**2 + 2*b**3*x**3)/(2*b**4*(a**2 + 2*a*b*x + b*
*2*x**2))
```

3.265 $\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx = \frac{(Ab-aB)x^3}{3ab(a+bx)^3} - \frac{a^2B}{2b^4(a+bx)^2} + \frac{2aB}{b^4(a+bx)} + \frac{B\log(a+bx)}{b^4}$$

output
```
1/3*(A*b-B*a)*x^3/a/b/(b*x+a)^3-1/2*a^2*B/b^4/(b*x+a)^2+2*a*B/b^4/(b*x+a)+
B*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx = \frac{11a^3B-6Ab^3x^2-6ab^2x(A-3Bx)+a^2(-2Ab+27bBx)+6B(a+bx)^3\log(a+bx)}{6b^4(a+bx)^3}$$

input
```
Integrate[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(11*a^3*B - 6*A*b^3*x^2 - 6*a*b^2*x*(A - 3*B*x) + a^2*(-2*A*b + 27*b*B*x)
+ 6*B*(a + b*x)^3*Log[a + b*x])/(6*b^4*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1184, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{x^2(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{B\int \frac{x^2}{(a+bx)^3}dx}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{49}$$

$$\frac{B\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\left(-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\right)}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx)^3}$$

input
```
Int[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*x^3)/(3*a*b*(a + b*x)^3) + (B*(-1/2*a^2/(b^3*(a + b*x)^2) + (
2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

method	result
norman	$\frac{-\frac{a^2(2Ab-11Ba)}{6b^4}-\frac{(Ab-3Ba)x^2}{b^2}-\frac{a(2Ab-9Ba)x}{2b^3}}{(bx+a)^3}+\frac{B\ln(bx+a)}{b^4}$
default	$-\frac{a^2(Ab-Ba)}{3b^4(bx+a)^3}-\frac{Ab-3Ba}{b^4(bx+a)}+\frac{B\ln(bx+a)}{b^4}+\frac{a(2Ab-3Ba)}{2b^4(bx+a)^2}$
risch	$\frac{-\frac{a^2(2Ab-11Ba)}{6b^4}-\frac{(Ab-3Ba)x^2}{b^2}-\frac{a(2Ab-9Ba)x}{2b^3}}{(bx+a)(b^2x^2+2abx+a^2)}+\frac{B\ln(bx+a)}{b^4}$
parallelrisch	$-\frac{-6B\ln(bx+a)x^3b^3-18B\ln(bx+a)x^2a\,b^2+6A\,b^3x^2-18B\ln(bx+a)x\,a^2b-18Ba\,b^2x^2+6Aa\,b^2x-6B\ln(bx+a)a^3-27B\,a^2b}{6b^4(b^2x^2+2abx+a^2)(bx+a)}$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*a^2*(2*A*b-11*B*a)/b^4-(A*b-3*B*a)/b^2*x^2-1/2*a*(2*A*b-9*B*a)/b^3*x
)/(b*x+a)^3+B*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{11\,Ba^3-2\,Aa^2b+6\,(3\,Bab^2-Ab^3)x^2+3\,(9\,Ba^2b-2\,Aab^2)x+6\,(Bb^3x^3+3\,Bab^2x^2+3\,Ba^2bx+Ba^3}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(11*B*a^3 - 2*A*a^2*b + 6*(3*B*a*b^2 - A*b^3)*x^2 + 3*(9*B*a^2*b - 2*A
*a*b^2)*x + 6*(B*b^3*x^3 + 3*B*a*b^2*x^2 + 3*B*a^2*b*x + B*a^3)*log(b*x +
a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{B\log{(a+bx)}}{b^4} + \frac{-2Aa^2b+11Ba^3+x^2(-6Ab^3+18Bab^2)+x(-6Aab^2+27Ba^2b)}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}$$

input
```
integrate(x**2*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
B*log(a + b*x)/b**4 + (-2*A*a**2*b + 11*B*a**3 + x**2*(-6*A*b**3 + 18*B*a*
b**2) + x*(-6*A*a*b**2 + 27*B*a**2*b))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*
a*b**6*x**2 + 6*b**7*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{11\,Ba^3-2\,Aa^2b+6\,(3\,Bab^2-Ab^3)x^2+3\,(9\,Ba^2b-2\,Aab^2)x}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)} + \frac{B\log{(bx+a)}}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*(11*B*a^3 - 2*A*a^2*b + 6*(3*B*a*b^2 - A*b^3)*x^2 + 3*(9*B*a^2*b - 2*A
*a*b^2)*x)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + B*log(b*x + a
)/b^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{B\log(|bx+a|)}{b^4} + \frac{6\,(3\,Bab-Ab^2)x^2+3\,(9\,Ba^2-2\,Aab)x+\frac{11\,Ba^3-2\,Aa^2b}{b}}{6\,(bx+a)^3b^3}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/b^4 + 1/6*(6*(3*B*a*b - A*b^2)*x^2 + 3*(9*B*a^2 - 2*A*
a*b)*x + (11*B*a^3 - 2*A*a^2*b)/b)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\frac{11\,B\,a^3-2\,A\,a^2\,b}{6\,b^4}-\frac{x^2\,(A\,b-3\,B\,a)}{b^2}+\frac{x\,(9\,B\,a^2-2\,A\,a\,b)}{2\,b^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}+\frac{B\,\ln(a+b\,x)}{b^4}$$

input
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
((11*B*a^3 - 2*A*a^2*b)/(6*b^4) - (x^2*(A*b - 3*B*a))/b^2 + (x*(9*B*a^2 -
2*A*a*b))/(2*b^3))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) + (B*log(a +
b*x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{2\log(bx+a)\,a^2+4\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2+a^2-2b^2x^2}{2b^3\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(2*log(a + b*x)*a**2 + 4*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 + a
**2 - 2*b**2*x**2)/(2*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.266 $\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 59

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx = \frac{a(Ab-aB)}{3b^3(a+bx)^3} - \frac{Ab-2aB}{2b^3(a+bx)^2} - \frac{B}{b^3(a+bx)}$$

output
```
1/3*a*(A*b-B*a)/b^3/(b*x+a)^3-1/2*(A*b-2*B*a)/b^3/(b*x+a)^2-B/b^3/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2} \, dx = -\frac{2a^2B+3b^2x(A+2Bx)+ab(A+6Bx)}{6b^3(a+bx)^3}$$

input
```
Integrate[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/6*(2*a^2*B + 3*b^2*x*(A + 2*B*x) + a*b*(A + 6*B*x))/(b^3*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{x(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)^3}+\frac{a(aB-Ab)}{b^2(a+bx)^4}+\frac{B}{b^2(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab-2aB}{2b^3(a+bx)^2}+\frac{a(Ab-aB)}{3b^3(a+bx)^3}-\frac{B}{b^3(a+bx)}$$

input
```
Int[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a*(A*b - a*B))/(3*b^3*(a + b*x)^3) - (A*b - 2*a*B)/(2*b^3*(a + b*x)^2) -
B/(b^3*(a + b*x))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{B\,x^{2}}{b}-\frac{(Ab+2Ba)x}{2b^{2}}-\frac{a(Ab+2Ba)}{6b^{3}}}{(bx+a)^{3}}$	47
default	$\frac{a(Ab-Ba)}{3b^{3}(bx+a)^{3}}-\frac{Ab-2Ba}{2b^{3}(bx+a)^{2}}-\frac{B}{b^{3}(bx+a)}$	56
orering	$-\frac{(6x^{2}B\,b^{2}+3x\,b^{2}A+6xabB+abA+2a^{2}B)(bx+a)}{6b^{3}(b^{2}x^{2}+2abx+a^{2})^{2}}$	62
gosper	$-\frac{6x^{2}B\,b^{2}+3x\,b^{2}A+6xabB+abA+2a^{2}B}{6(bx+a)(b^{2}x^{2}+2abx+a^{2})b^{3}}$	64
parallelrisch	$-\frac{6x^{2}B\,b^{2}+3x\,b^{2}A+6xabB+abA+2a^{2}B}{6(bx+a)(b^{2}x^{2}+2abx+a^{2})b^{3}}$	64
risch	$\frac{-\frac{B\,x^{2}}{b}-\frac{(Ab+2Ba)x}{2b^{2}}-\frac{a(Ab+2Ba)}{6b^{3}}}{(bx+a)(b^{2}x^{2}+2abx+a^{2})}$	65

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-B*x^2/b-1/2*(A*b+2*B*a)/b^2*x-1/6*a*(A*b+2*B*a)/b^3)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{6\,Bb^2x^2+2\,Ba^2+Aab+3\,(2\,Bab+Ab^2)x}{6\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*B*b^2*x^2 + 2*B*a^2 + A*a*b + 3*(2*B*a*b + A*b^2)*x)/(b^6*x^3 + 3*
a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{-Aab-2Ba^2-6Bb^2x^2+x(-3Ab^2-6Bab)}{6a^3b^3+18a^2b^4x+18ab^5x^2+6b^6x^3}$$

input
```
integrate(x*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-A*a*b - 2*B*a**2 - 6*B*b**2*x**2 + x*(-3*A*b**2 - 6*B*a*b))/(6*a**3*b**3
 + 18*a**2*b**4*x + 18*a*b**5*x**2 + 6*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{6\,Bb^2x^2+2\,Ba^2+Aab+3\,(2\,Bab+Ab^2)x}{6\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(6*B*b^2*x^2 + 2*B*a^2 + A*a*b + 3*(2*B*a*b + A*b^2)*x)/(b^6*x^3 + 3*
a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{6\,Bb^2x^2+6\,Babx+3\,Ab^2x+2\,Ba^2+Aab}{6\,(bx+a)^3b^3}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/6*(6*B*b^2*x^2 + 6*B*a*b*x + 3*A*b^2*x + 2*B*a^2 + A*a*b)/((b*x + a)^3*
b^3)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{\frac{B\,x^2}{b}+\frac{a\,(A\,b+2\,B\,a)}{6\,b^3}+\frac{x\,(A\,b+2\,B\,a)}{2\,b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-((B*x^2)/b + (a*(A*b + 2*B*a))/(6*b^3) + (x*(A*b + 2*B*a))/(2*b^2))/(a^3
+ b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.44

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx=\frac{x^2}{2a\,(b^2x^2+2abx+a^2)}$$

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
x**2/(2*a*(a**2 + 2*a*b*x + b**2*x**2))
```

3.267 $\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx=\frac{-Ab+aB}{3b^2(a+bx)^3}-\frac{B}{2b^2(a+bx)^2}$$

output
```
1/3*(-A*b+B*a)/b^2/(b*x+a)^3-1/2*B/b^2/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx=-\frac{2Ab+B(a+3bx)}{6b^2(a+bx)^3}$$

input
```
Integrate[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/6*(2*A*b + B*(a + 3*b*x))/(b^2*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1098}$$

$$b^4\int \frac{A+Bx}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{(a+bx)^4}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^4}+\frac{B}{b(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab-aB}{3b^2(a+bx)^3}-\frac{B}{2b^2(a+bx)^2}$$

input
```
Int[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/3*(A*b - a*B)/(b^2*(a + b*x)^3) - B/(2*b^2*(a + b*x)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
norman	$\frac{-\frac{Bx}{2b}-\frac{2b^2A+abB}{6b^3}}{(bx+a)^3}$	33
default	$-\frac{Ab-Ba}{3b^2(bx+a)^3}-\frac{B}{2b^2(bx+a)^2}$	35
orering	$-\frac{(3Bbx+2Ab+Ba)(bx+a)}{6b^2(b^2x^2+2abx+a^2)^2}$	42
gosper	$-\frac{3Bbx+2Ab+Ba}{6b^2(bx+a)(b^2x^2+2abx+a^2)}$	44
risch	$\frac{-\frac{Bx}{2b}-\frac{2Ab+Ba}{6b^2}}{(bx+a)(b^2x^2+2abx+a^2)}$	48
parallelrisch	$-\frac{3xB\,b^2+2b^2A+abB}{6b^3(b^2x^2+2abx+a^2)(bx+a)}$	49

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*B*x/b-1/6*(2*A*b^2+B*a*b)/b^3)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,Bbx+Ba+2\,Ab}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*B*b*x + B*a + 2*A*b)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^
2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{-2Ab-Ba-3Bbx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

input
```
integrate((B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-2*A*b - B*a - 3*B*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 +
6*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,Bbx+Ba+2\,Ab}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*B*b*x + B*a + 2*A*b)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^
2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,Bbx+Ba+2\,Ab}{6\,(bx+a)^3b^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*B*b*x + B*a + 2*A*b)/((b*x + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{\frac{2\,A\,b+B\,a}{6\,b^2}+\frac{B\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int((A + B*x)/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
-((2*A*b + B*a)/(6*b^2) + (B*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^
2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{1}{2b\,(b^2x^2+2abx+a^2)}$$

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 1)/(2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.268 $\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2} dx = \frac{Ab-aB}{3ab(a+bx)^3}+\frac{A}{2a^2(a+bx)^2}+\frac{A}{a^3(a+bx)}+\frac{A\log(x)}{a^4}-\frac{A\log(a+bx)}{a^4}$$

output
```
1/3*(A*b-B*a)/a/b/(b*x+a)^3+1/2*A/a^2/(b*x+a)^2+A/a^3/(b*x+a)+A*ln(x)/a^4-
A*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2} dx = \frac{\frac{a(11a^2Ab-2a^3B+15aAb^2x+6Ab^3x^2)}{b(a+bx)^3}+6A\log(x)-6A\log(a+bx)}{6a^4}$$

input
```
Integrate[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((a*(11*a^2*A*b - 2*a^3*B + 15*a*A*b^2*x + 6*A*b^3*x^2))/(b*(a + b*x)^3) +
 6*A*Log[x] - 6*A*Log[a + b*x])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{A+Bx}{b^4x(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{x(a+bx)^4}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{Ab}{a^4(a+bx)}+\frac{A}{a^4x}-\frac{Ab}{a^3(a+bx)^2}-\frac{Ab}{a^2(a+bx)^3}+\frac{aB-Ab}{a(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{A\log(a+bx)}{a^4}+\frac{A\log(x)}{a^4}+\frac{A}{a^3(a+bx)}+\frac{A}{2a^2(a+bx)^2}+\frac{Ab-aB}{3ab(a+bx)^3}$$

input
```
Int[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*(a + b*x)^3) + A/(2*a^2*(a + b*x)^2) + A/(a^3*(a + b*x)
) + (A*Log[x])/a^4 - (A*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
default	$-\frac{-Ab+Ba}{3ab\left(bx+a\right)^{3}}-\frac{A\ln\left(bx+a\right)}{a^{4}}+\frac{A}{a^{3}\left(bx+a\right)}+\frac{A}{2a^{2}\left(bx+a\right)^{2}}+\frac{A\ln\left(x\right)}{a^{4}}$
norman	$\frac{-\frac{\left(3Ab-Ba\right)x}{a^{2}}-\frac{b\left(9Ab-2Ba\right)x^{2}}{2a^{3}}-\frac{b^{2}\left(11Ab-2Ba\right)x^{3}}{6a^{4}}}{\left(bx+a\right)^{3}}+\frac{A\ln\left(x\right)}{a^{4}}-\frac{A\ln\left(bx+a\right)}{a^{4}}$
risch	$\frac{\frac{b^{2}A\,x^{2}}{a^{3}}+\frac{5xAb}{2a^{2}}+\frac{11Ab-2Ba}{6ab}}{\left(bx+a\right)\left(b^{2}x^{2}+2abx+a^{2}\right)}+\frac{A\ln\left(-x\right)}{a^{4}}-\frac{A\ln\left(bx+a\right)}{a^{4}}$
parallelrisch	$\frac{6A\ln\left(x\right)x^{3}b^{3}-6A\ln\left(bx+a\right)x^{3}b^{3}+18A\ln\left(x\right)x^{2}a\,b^{2}-18A\ln\left(bx+a\right)x^{2}a\,b^{2}-11A\,b^{3}x^{3}+2Ba\,b^{2}x^{3}+18A\ln\left(x\right)x\,a^{2}b-18A\ln(bx+}{6a^{4}\left(b^{2}x^{2}+2abx+a^{2}\right)\left(bx+a\right)}$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-A*b+B*a)/a/b/(b*x+a)^3-A*ln(b*x+a)/a^4+A/a^3/(b*x+a)+1/2*A/a^2/(b*x
+a)^2+A*ln(x)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(68) = 136$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{6\,Aab^3x^2+15\,Aa^2b^2x-2\,Ba^4+11\,Aa^3b-6\left(Ab^4x^3+3\,Aab^3x^2+3\,Aa^2b^2x+Aa^3b\right)\log\left(bx+a\right)+6\,(A}{6\left(a^4b^4x^3+3\,a^5b^3x^2+3\,a^6b^2x+a^7b\right)}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(6*A*a*b^3*x^2 + 15*A*a^2*b^2*x - 2*B*a^4 + 11*A*a^3*b - 6*(A*b^4*x^3
+ 3*A*a*b^3*x^2 + 3*A*a^2*b^2*x + A*a^3*b)*log(b*x + a) + 6*(A*b^4*x^3 + 3
*A*a*b^3*x^2 + 3*A*a^2*b^2*x + A*a^3*b)*log(x))/(a^4*b^4*x^3 + 3*a^5*b^3*x
^2 + 3*a^6*b^2*x + a^7*b)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{A\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^4} + \frac{11Aa^2b+15Aab^2x+6Ab^3x^2-2Ba^3}{6a^6b+18a^5b^2x+18a^4b^3x^2+6a^3b^4x^3}$$

input
```
integrate((B*x+A)/x/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
A*(log(x) - log(a/b + x))/a**4 + (11*A*a**2*b + 15*A*a*b**2*x + 6*A*b**3*x
**2 - 2*B*a**3)/(6*a**6*b + 18*a**5*b**2*x + 18*a**4*b**3*x**2 + 6*a**3*b*
*4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^2}\,dx = \frac{6\,Ab^3x^2+15\,Aab^2x-2\,Ba^3+11\,Aa^2b}{6\,(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^6b)} - \frac{A\log(bx+a)}{a^4} + \frac{A\log(x)}{a^4}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/6*(6*A*b^3*x^2 + 15*A*a*b^2*x - 2*B*a^3 + 11*A*a^2*b)/(a^3*b^4*x^3 + 3*a
^4*b^3*x^2 + 3*a^5*b^2*x + a^6*b) - A*log(b*x + a)/a^4 + A*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^2}\,dx = -\frac{A\log(|bx+a|)}{a^4} + \frac{A\log(|x|)}{a^4} + \frac{6\,Aab^3x^2+15\,Aa^2b^2x-2\,Ba^4+11\,Aa^3b}{6\,(bx+a)^3a^4b}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/a^4 + A*log(abs(x))/a^4 + 1/6*(6*A*a*b^3*x^2 + 15*A*a
^2*b^2*x - 2*B*a^4 + 11*A*a^3*b)/((b*x + a)^3*a^4*b)
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{\frac{11\,A\,b-2\,B\,a}{6\,a\,b}+\frac{5\,A\,b\,x}{2\,a^2}+\frac{A\,b^2\,x^2}{a^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}-\frac{2\,A\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

input
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
((11*A*b - 2*B*a)/(6*a*b) + (5*A*b*x)/(2*a^2) + (A*b^2*x^2)/a^3)/(a^3 + b^
3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) - (2*A*atanh((2*b*x)/a + 1))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^2}\,dx$$
$$=\frac{-2\log(bx+a)\,a^2-4\log(bx+a)\,abx-2\log(bx+a)\,b^2x^2+2\log(x)\,a^2+4\log(x)\,abx+2\log(x)\,b^2x^2+}{2a^3\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 2*log(a + b*x)*a**2 - 4*log(a + b*x)*a*b*x - 2*log(a + b*x)*b**2*x**2
+ 2*log(x)*a**2 + 4*log(x)*a*b*x + 2*log(x)*b**2*x**2 + 2*a**2 - b**2*x**2
)/(2*a**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.269 $\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{A}{a^4x}-\frac{Ab-aB}{3a^2(a+bx)^3}-\frac{2Ab-aB}{2a^3(a+bx)^2}-\frac{3Ab-aB}{a^4(a+bx)} -\frac{(4Ab-aB)\log(x)}{a^5}+\frac{(4Ab-aB)\log(a+bx)}{a^5}$$

output
```
-A/a^4/x-1/3*(A*b-B*a)/a^2/(b*x+a)^3-1/2*(2*A*b-B*a)/a^3/(b*x+a)^2-(3*A*b-
B*a)/a^4/(b*x+a)-(4*A*b-B*a)*ln(x)/a^5+(4*A*b-B*a)*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-\frac{6aA}{x}+\frac{2a^3(-Ab+aB)}{(a+bx)^3}+\frac{3a^2(-2Ab+aB)}{(a+bx)^2}+\frac{6a(-3Ab+aB)}{a+bx}+6(-4Ab+aB)\log(x)+6(4Ab-aB)\log(a+bx)}{6a^5}$$

input
```
Integrate[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((-6*a*A)/x + (2*a^3*(-(A*b) + a*B))/(a + b*x)^3 + (3*a^2*(-2*A*b + a*B))/
(a + b*x)^2 + (6*a*(-3*A*b + a*B))/(a + b*x) + 6*(-4*A*b + a*B)*Log[x] + 6
*(4*A*b - a*B)*Log[a + b*x])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{A+Bx}{b^4x^2(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^2(a+bx)^4}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{aB-4Ab}{a^5x}-\frac{b(aB-4Ab)}{a^5(a+bx)}-\frac{b(aB-3Ab)}{a^4(a+bx)^2}+\frac{A}{a^4x^2}-\frac{b(aB-2Ab)}{a^3(a+bx)^3}-\frac{b(aB-Ab)}{a^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(x)(4Ab-aB)}{a^5}+\frac{(4Ab-aB)\log(a+bx)}{a^5}-\frac{3Ab-aB}{a^4(a+bx)}-\frac{A}{a^4x}-\frac{2Ab-aB}{2a^3(a+bx)^2}-\frac{Ab-aB}{3a^2(a+bx)^3}$$

input
```
Int[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
-(A/(a^4*x)) - (A*b - a*B)/(3*a^2*(a + b*x)^3) - (2*A*b - a*B)/(2*a^3*(a +
 b*x)^2) - (3*A*b - a*B)/(a^4*(a + b*x)) - ((4*A*b - a*B)*Log[x])/a^5 + ((
4*A*b - a*B)*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

method	result
default	$-\frac{2Ab-Ba}{2a^{3}\left(bx+a\right)^{2}}-\frac{3Ab-Ba}{a^{4}\left(bx+a\right)}+\frac{\left(4Ab-Ba\right)\ln\left(bx+a\right)}{a^{5}}-\frac{Ab-Ba}{3a^{2}\left(bx+a\right)^{3}}-\frac{A}{a^{4}x}+\frac{\left(-4Ab+Ba\right)\ln\left(x\right)}{a^{5}}$
norman	$\frac{-\frac{A}{a}+\frac{3b\left(4Ab-Ba\right)x^{2}}{a^{3}}+\frac{3b^{2}\left(12Ab-3Ba\right)x^{3}}{2a^{4}}+\frac{b^{3}\left(44Ab-11Ba\right)x^{4}}{6a^{5}}}{x\left(bx+a\right)^{3}}+\frac{\left(4Ab-Ba\right)\ln\left(bx+a\right)}{a^{5}}-\frac{\left(4Ab-Ba\right)\ln\left(x\right)}{a^{5}}$
risch	$\frac{-\frac{b^{2}\left(4Ab-Ba\right)x^{3}}{a^{4}}-\frac{5b\left(4Ab-Ba\right)x^{2}}{2a^{3}}-\frac{11\left(4Ab-Ba\right)x}{6a^{2}}-\frac{A}{a}}{x\left(b^{2}x^{2}+2abx+a^{2}\right)\left(bx+a\right)}+\frac{4\ln\left(-bx-a\right)Ab}{a^{5}}-\frac{\ln\left(-bx-a\right)B}{a^{4}}-\frac{4\ln\left(x\right)Ab}{a^{5}}+\frac{\ln\left(x\right)B}{a^{4}}$
parallelrisch	$-\frac{6B\ln\left(bx+a\right)x^{4}a\,b^{3}-24A\ln\left(bx+a\right)x^{4}b^{4}+11Ba\,b^{3}x^{4}-108Aa\,b^{3}x^{3}+18B\,a^{3}b\,x^{2}+6a^{4}A-6B\ln\left(x\right)x\,a^{4}+6B\ln\left(bx+a\right)x\,a^{4}-6}{}$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*A*b-B*a)/a^3/(b*x+a)^2-(3*A*b-B*a)/a^4/(b*x+a)+(4*A*b-B*a)*ln(b*x+
a)/a^5-1/3*(A*b-B*a)/a^2/(b*x+a)^3-A/a^4/x+(-4*A*b+B*a)/a^5*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(102) = 204.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{6\,Aa^4-6\,(Ba^2b^2-4\,Aab^3)x^3-15\,(Ba^3b-4\,Aa^2b^2)x^2-11\,(Ba^4-4\,Aa^3b)x+6\,((Bab^3-4\,Ab^4)x^4$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*A*a^4 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 - 15*(B*a^3*b - 4*A*a^2*b^2)
*x^2 - 11*(B*a^4 - 4*A*a^3*b)*x + 6*((B*a*b^3 - 4*A*b^4)*x^4 + 3*(B*a^2*b^
2 - 4*A*a*b^3)*x^3 + 3*(B*a^3*b - 4*A*a^2*b^2)*x^2 + (B*a^4 - 4*A*a^3*b)*x
)*log(b*x + a) - 6*((B*a*b^3 - 4*A*b^4)*x^4 + 3*(B*a^2*b^2 - 4*A*a*b^3)*x^
3 + 3*(B*a^3*b - 4*A*a^2*b^2)*x^2 + (B*a^4 - 4*A*a^3*b)*x)*log(x))/(a^5*b^
3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*b*x^2 + a^8*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(95) = 190.

Time = 0.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-6Aa^3+x^3(-24Ab^3+6Bab^2)+x^2(-60Aab^2+15Ba^2b)+x(-44Aa^2b+11Ba^3)}{6a^7x+18a^6bx^2+18a^5b^2x^3+6a^4b^3x^4} + \frac{(-4Ab+Ba)\log\left(x+\frac{-4Aab+Ba^2-a(-4Ab+Ba)}{-8Ab^2+2Bab}\right)}{a^5} - \frac{(-4Ab+Ba)\log\left(x+\frac{-4Aab+Ba^2+a(-4Ab+Ba)}{-8Ab^2+2Bab}\right)}{a^5}$$

input
```
integrate((B*x+A)/x**2/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-6*A*a**3 + x**3*(-24*A*b**3 + 6*B*a*b**2) + x**2*(-60*A*a*b**2 + 15*B*a*
*2*b) + x*(-44*A*a**2*b + 11*B*a**3))/(6*a**7*x + 18*a**6*b*x**2 + 18*a**5
*b**2*x**3 + 6*a**4*b**3*x**4) + (-4*A*b + B*a)*log(x + (-4*A*a*b + B*a**2
 - a*(-4*A*b + B*a))/(-8*A*b**2 + 2*B*a*b))/a**5 - (-4*A*b + B*a)*log(x +
(-4*A*a*b + B*a**2 + a*(-4*A*b + B*a))/(-8*A*b**2 + 2*B*a*b))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{6\,Aa^3-6\,(Bab^2-4\,Ab^3)x^3-15\,(Ba^2b-4\,Aab^2)x^2-11\,(Ba^3-4\,Aa^2b)x}{6\,(a^4b^3x^4+3\,a^5b^2x^3+3\,a^6bx^2+a^7x)} - \frac{(Ba-4\,Ab)\log(bx+a)}{a^5} + \frac{(Ba-4\,Ab)\log(x)}{a^5}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(6*A*a^3 - 6*(B*a*b^2 - 4*A*b^3)*x^3 - 15*(B*a^2*b - 4*A*a*b^2)*x^2 -
 11*(B*a^3 - 4*A*a^2*b)*x)/(a^4*b^3*x^4 + 3*a^5*b^2*x^3 + 3*a^6*b*x^2 + a^
7*x) - (B*a - 4*A*b)*log(b*x + a)/a^5 + (B*a - 4*A*b)*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{(Ba-4Ab)\log(|x|)}{a^5} - \frac{(Bab-4Ab^2)\log(|bx+a|)}{a^5b} - \frac{6Aa^4-6(Ba^2b^2-4Aab^3)x^3-15(Ba^3b-4Aa^2b^2)x^2-11(Ba^4-4Aa^3b)x}{6(bx+a)^3a^5x}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
(B*a - 4*A*b)*log(abs(x))/a^5 - (B*a*b - 4*A*b^2)*log(abs(b*x + a))/(a^5*b
) - 1/6*(6*A*a^4 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 - 15*(B*a^3*b - 4*A*a^2*b
^2)*x^2 - 11*(B*a^4 - 4*A*a^3*b)*x)/((b*x + a)^3*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 10.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,\left(4\,A\,b-B\,a\right)}{a^5} - \frac{\frac{A}{a}+\frac{11\,x\,\left(4\,A\,b-B\,a\right)}{6\,a^2}+\frac{b^2\,x^3\,\left(4\,A\,b-B\,a\right)}{a^4}+\frac{5\,b\,x^2\,\left(4\,A\,b-B\,a\right)}{2\,a^3}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

input
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(4*A*b - B*a))/a^5 - (A/a + (11*x*(4*A*b - B*a))/(
6*a^2) + (b^2*x^3*(4*A*b - B*a))/a^4 + (5*b*x^2*(4*A*b - B*a))/(2*a^3))/(a
^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^2}\,dx$$
$$= \frac{6\log(bx+a)\,a^2bx+12\log(bx+a)\,a\,b^2x^2+6\log(bx+a)\,b^3x^3-6\log(x)\,a^2bx-12\log(x)\,a\,b^2x^2-6\log(\cdots}{2a^4x\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(6*log(a + b*x)*a**2*b*x + 12*log(a + b*x)*a*b**2*x**2 + 6*log(a + b*x)*b*
*3*x**3 - 6*log(x)*a**2*b*x - 12*log(x)*a*b**2*x**2 - 6*log(x)*b**3*x**3 -
 2*a**3 - 6*a**2*b*x + 3*b**3*x**3)/(2*a**4*x*(a**2 + 2*a*b*x + b**2*x**2)
)
```

3.270 $\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{A}{2a^4x^2}+\frac{4Ab-aB}{a^5x}+\frac{b(Ab-aB)}{3a^3(a+bx)^3}+\frac{b(3Ab-2aB)}{2a^4(a+bx)^2}+\frac{3b(2Ab-aB)}{a^5(a+bx)}+\frac{2b(5Ab-2aB)\log(x)}{a^6}-\frac{2b(5Ab-2aB)\log(a+bx)}{a^6}$$

output
```
-1/2*A/a^4/x^2+(4*A*b-B*a)/a^5/x+1/3*b*(A*b-B*a)/a^3/(b*x+a)^3+1/2*b*(3*A*
b-2*B*a)/a^4/(b*x+a)^2+3*b*(2*A*b-B*a)/a^5/(b*x+a)+2*b*(5*A*b-2*B*a)*ln(x)
/a^6-2*b*(5*A*b-2*B*a)*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\frac{a\left(60Ab^4x^4+a^3bx(15A-44Bx)+10a^2b^2x^2(11A-6Bx)+6ab^3x^3(25A-4Bx)-3a^4(A+2Bx)\right)}{x^2(a+bx)^3}+12b(5Ab-2aB)\log(x)+12b(-5\ldots}{6a^6}$$

input
```
Integrate[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((a*(60*A*b^4*x^4 + a^3*b*x*(15*A - 44*B*x) + 10*a^2*b^2*x^2*(11*A - 6*B*x
) + 6*a*b^3*x^3*(25*A - 4*B*x) - 3*a^4*(A + 2*B*x)))/(x^2*(a + b*x)^3) + 1
2*b*(5*A*b - 2*a*B)*Log[x] + 12*b*(-5*A*b + 2*a*B)*Log[a + b*x])/(6*a^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3 \left(a^2+2abx+b^2x^2\right)^2} \, dx$$

$$\Big\downarrow \text{1184}$$

$$b^4 \int \frac{A+Bx}{b^4x^3(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^3(a+bx)^4}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2b^2(2aB-5Ab)}{a^6(a+bx)}-\frac{2b(2aB-5Ab)}{a^6x}+\frac{3b^2(aB-2Ab)}{a^5(a+bx)^2}+\frac{aB-4Ab}{a^5x^2}+\frac{b^2(2aB-3Ab)}{a^4(a+bx)^3}+\frac{A}{a^4x^3}+\frac{b^2(aB-Ab)}{a^3(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b\log(x)(5Ab-2aB)}{a^6}-\frac{2b(5Ab-2aB)\log(a+bx)}{a^6}+\frac{4Ab-aB}{a^5x}+\frac{3b(2Ab-aB)}{a^5(a+bx)}+\frac{b(3Ab-2aB)}{2a^4(a+bx)^2}-\frac{A}{2a^4x^2}+\frac{b(Ab-aB)}{3a^3(a+bx)^3}$$

input
```
Int[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
-1/2*A/(a^4*x^2) + (4*A*b - a*B)/(a^5*x) + (b*(A*b - a*B))/(3*a^3*(a + b*x
)^3) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x)^2) + (3*b*(2*A*b - a*B))/(a^5*
(a + b*x)) + (2*b*(5*A*b - 2*a*B)*Log[x])/a^6 - (2*b*(5*A*b - 2*a*B)*Log[a
 + b*x])/a^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

method	result
default	$\frac{3b(2Ab-Ba)}{a^5(bx+a)}+\frac{b(3Ab-2Ba)}{2a^4(bx+a)^2}-\frac{2b(5Ab-2Ba)\ln(bx+a)}{a^6}+\frac{b(Ab-Ba)}{3a^3(bx+a)^3}-\frac{A}{2a^4x^2}-\frac{-4Ab+Ba}{x\,a^5}+\frac{2b(5Ab-2Ba)\ln(}{a^6}$
norman	$\frac{-\frac{A}{2a}+\frac{(5Ab-2Ba)x}{2a^2}-\frac{3b\left(10b^2A-4abB\right)x^3}{a^4}-\frac{3b^2\left(15b^2A-6abB\right)x^4}{a^5}-\frac{b^3\left(55b^2A-22abB\right)x^5}{3a^6}}{(bx+a)^3x^2}+\frac{2b(5Ab-2Ba)\ln(x)}{a^6}-\frac{2b(5Ab-2}{}$
risch	$\frac{\frac{2b^3(5Ab-2Ba)x^4}{a^5}+\frac{5(5Ab-2Ba)b^2x^3}{a^4}+\frac{11b(5Ab-2Ba)x^2}{3a^3}+\frac{(5Ab-2Ba)x}{2a^2}-\frac{A}{2a}}{x^2(b^2x^2+2abx+a^2)(bx+a)}-\frac{10b^2\ln(bx+a)A}{a^6}+\frac{4b\ln(bx+a)B}{a^5}+\frac{10b^2\ln(}{a^6}$
parallelrisch	$\frac{-270Aa\,b^4x^4+15A\,a^4bx+44Ba\,b^4x^5-24B\ln(x)x^5a\,b^4+24B\ln(bx+a)x^5a\,b^4+180A\ln(x)x^4a\,b^4-180A\ln(bx+a)x^4a\,b^4-72}{}$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
3*b*(2*A*b-B*a)/a^5/(b*x+a)+1/2*b*(3*A*b-2*B*a)/a^4/(b*x+a)^2-2*b*(5*A*b-2
*B*a)*ln(b*x+a)/a^6+1/3*b*(A*b-B*a)/a^3/(b*x+a)^3-1/2*A/a^4/x^2-(-4*A*b+B*
a)/x/a^5+2*b*(5*A*b-2*B*a)*ln(x)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(128) = 256$.

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{3\,Aa^5+12\,(2\,Ba^2b^3-5\,Aab^4)x^4+30\,(2\,Ba^3b^2-5\,Aa^2b^3)x^3+22\,(2\,Ba^4b-5\,Aa^3b^2)x^2+3\,(2\,Ba^5-}{}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*A*a^5 + 12*(2*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 30*(2*B*a^3*b^2 - 5*A*a
^2*b^3)*x^3 + 22*(2*B*a^4*b - 5*A*a^3*b^2)*x^2 + 3*(2*B*a^5 - 5*A*a^4*b)*x
 - 12*((2*B*a*b^4 - 5*A*b^5)*x^5 + 3*(2*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 3*(2*
B*a^3*b^2 - 5*A*a^2*b^3)*x^3 + (2*B*a^4*b - 5*A*a^3*b^2)*x^2)*log(b*x + a)
 + 12*((2*B*a*b^4 - 5*A*b^5)*x^5 + 3*(2*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 3*(2*
B*a^3*b^2 - 5*A*a^2*b^3)*x^3 + (2*B*a^4*b - 5*A*a^3*b^2)*x^2)*log(x))/(a^6
*b^3*x^5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(131) = 262$.

Time = 0.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx}{x^3\,(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \frac{-3Aa^4 + x^4\cdot(60Ab^4 - 24Bab^3) + x^3\cdot(150Aab^3 - 60Ba^2b^2) + x^2\cdot(110Aa^2b^2 - 44Ba^3b) + x(15Aa^3b -}{6a^8x^2 + 18a^7bx^3 + 18a^6b^2x^4 + 6a^5b^3x^5}$$
$$- \frac{2b(-5Ab+2Ba)\log\left(x + \frac{-10Aab^2+4Ba^2b-2ab(-5Ab+2Ba)}{-20Ab^3+8Bab^2}\right)}{a^6}$$
$$+ \frac{2b(-5Ab+2Ba)\log\left(x + \frac{-10Aab^2+4Ba^2b+2ab(-5Ab+2Ba)}{-20Ab^3+8Bab^2}\right)}{a^6}$$

input
```
integrate((B*x+A)/x**3/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-3*A*a**4 + x**4*(60*A*b**4 - 24*B*a*b**3) + x**3*(150*A*a*b**3 - 60*B*a*
*2*b**2) + x**2*(110*A*a**2*b**2 - 44*B*a**3*b) + x*(15*A*a**3*b - 6*B*a**
4))/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5)
- 2*b*(-5*A*b + 2*B*a)*log(x + (-10*A*a*b**2 + 4*B*a**2*b - 2*a*b*(-5*A*b
+ 2*B*a))/(-20*A*b**3 + 8*B*a*b**2))/a**6 + 2*b*(-5*A*b + 2*B*a)*log(x + (
-10*A*a*b**2 + 4*B*a**2*b + 2*a*b*(-5*A*b + 2*B*a))/(-20*A*b**3 + 8*B*a*b*
*2))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{3\,Aa^4+12\,(2\,Bab^3-5\,Ab^4)x^4+30\,(2\,Ba^2b^2-5\,Aab^3)x^3+22\,(2\,Ba^3b-5\,Aa^2b^2)x^2+3\,(2\,Ba^4-5\,A}{6\,(a^5b^3x^5+3\,a^6b^2x^4+3\,a^7bx^3+a^8x^2)} + \frac{2\,(2\,Bab-5\,Ab^2)\log{(bx+a)}}{a^6} - \frac{2\,(2\,Bab-5\,Ab^2)\log{(x)}}{a^6}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*A*a^4 + 12*(2*B*a*b^3 - 5*A*b^4)*x^4 + 30*(2*B*a^2*b^2 - 5*A*a*b^3
)*x^3 + 22*(2*B*a^3*b - 5*A*a^2*b^2)*x^2 + 3*(2*B*a^4 - 5*A*a^3*b)*x)/(a^5
*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) + 2*(2*B*a*b - 5*A*b^2)*
log(b*x + a)/a^6 - 2*(2*B*a*b - 5*A*b^2)*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{2\,(2\,Bab-5\,Ab^2)\log{(|x|)}}{a^6} + \frac{2\,(2\,Bab^2-5\,Ab^3)\log{(|bx+a|)}}{a^6b} - \frac{3\,Aa^5+12\,(2\,Ba^2b^3-5\,Aab^4)x^4+30\,(2\,Ba^3b^2-5\,Aa^2b^3)x^3+22\,(2\,Ba^4b-5\,Aa^3b^2)x^2+3\,(2\,Ba^5-}{6\,(bx+a)^3a^6x^2}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-2*(2*B*a*b - 5*A*b^2)*log(abs(x))/a^6 + 2*(2*B*a*b^2 - 5*A*b^3)*log(abs(b
*x + a))/(a^6*b) - 1/6*(3*A*a^5 + 12*(2*B*a^2*b^3 - 5*A*a*b^4)*x^4 + 30*(2
*B*a^3*b^2 - 5*A*a^2*b^3)*x^3 + 22*(2*B*a^4*b - 5*A*a^3*b^2)*x^2 + 3*(2*B*
a^5 - 5*A*a^4*b)*x)/((b*x + a)^3*a^6*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\frac{x\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{2\,a}+\frac{5\,b^2\,x^3\,(5\,A\,b-2\,B\,a)}{a^4}+\frac{2\,b^3\,x^4\,(5\,A\,b-2\,B\,a)}{a^5}+\frac{11\,b\,x^2\,(5\,A\,b-2\,B\,a)}{3\,a^3}}{a^3\,x^2+3\,a^2\,b\,x^3+3\,a\,b^2\,x^4+b^3\,x^5} - \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,(5\,A\,b-2\,B\,a)\,(a+2\,b\,x)}{a\,(10\,A\,b^2-4\,B\,a\,b)}\right)\,(5\,A\,b-2\,B\,a)}{a^6}$$

input
```
int((A + B*x)/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
((x*(5*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (5*b^2*x^3*(5*A*b - 2*B*a))/a^4 +
 (2*b^3*x^4*(5*A*b - 2*B*a))/a^5 + (11*b*x^2*(5*A*b - 2*B*a))/(3*a^3))/(a^
3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (4*b*atanh((2*b*(5*A*b - 2*
B*a)*(a + 2*b*x))/(a*(10*A*b^2 - 4*B*a*b)))*(5*A*b - 2*B*a))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-12\log(bx+a)\,a^2b^2x^2-24\log(bx+a)\,a\,b^3x^3-12\log(bx+a)\,b^4x^4+12\log(x)\,a^2b^2x^2+24\log(x)\,a\,b^3x}{2a^5x^2\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 12*log(a + b*x)*a**2*b**2*x**2 - 24*log(a + b*x)*a*b**3*x**3 - 12*log(
a + b*x)*b**4*x**4 + 12*log(x)*a**2*b**2*x**2 + 24*log(x)*a*b**3*x**3 + 12
*log(x)*b**4*x**4 - a**4 + 4*a**3*b*x + 12*a**2*b**2*x**2 - 6*b**4*x**4)/(
2*a**5*x**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.271 $\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{A}{3a^4x^3}+\frac{4Ab-aB}{2a^5x^2}-\frac{2b(5Ab-2aB)}{a^6x} -\frac{b^2(Ab-aB)}{3a^4(a+bx)^3}-\frac{b^2(4Ab-3aB)}{2a^5(a+bx)^2}-\frac{2b^2(5Ab-3aB)}{a^6(a+bx)} -\frac{10b^2(2Ab-aB)\log(x)}{a^7}+\frac{10b^2(2Ab-aB)\log(a+bx)}{a^7}$$

output
```
-1/3*A/a^4/x^3+1/2*(4*A*b-B*a)/a^5/x^2-2*b*(5*A*b-2*B*a)/a^6/x-1/3*b^2*(A*
b-B*a)/a^4/(b*x+a)^3-1/2*b^2*(4*A*b-3*B*a)/a^5/(b*x+a)^2-2*b^2*(5*A*b-3*B*
a)/a^6/(b*x+a)-10*b^2*(2*A*b-B*a)*ln(x)/a^7+10*b^2*(2*A*b-B*a)*ln(b*x+a)/a
^7
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\frac{a\left(-120Ab^5x^5+60ab^4x^4(-5A+Bx)-a^5(2A+3Bx)+3a^4bx(2A+5Bx)+10a^3b^2x^2(-3A+11Bx)+10a^2b^3x^3(-22A+15Bx)\right)}{x^3(a+bx)^3}-60b^2(2Ab-}{6a^7}$$

input
```
Integrate[(A + B*x)/(x^4*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((a*(-120*A*b^5*x^5 + 60*a*b^4*x^4*(-5*A + B*x) - a^5*(2*A + 3*B*x) + 3*a^
4*b*x*(2*A + 5*B*x) + 10*a^3*b^2*x^2*(-3*A + 11*B*x) + 10*a^2*b^3*x^3*(-22
*A + 15*B*x)))/(x^3*(a + b*x)^3) - 60*b^2*(2*A*b - a*B)*Log[x] + 60*b^2*(2
*A*b - a*B)*Log[a + b*x])/(6*a^7)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx$$

$$\Big\downarrow \ 1184$$

$$b^4\int \frac{A+Bx}{b^4x^4(a+bx)^4}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{A+Bx}{x^4(a+bx)^4}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(-\frac{10b^3(aB-2Ab)}{a^7(a+bx)}+\frac{10b^2(aB-2Ab)}{a^7x}-\frac{2b^3(3aB-5Ab)}{a^6(a+bx)^2}-\frac{2b(2aB-5Ab)}{a^6x^2}-\frac{b^3(3aB-4Ab)}{a^5(a+bx)^3}+\frac{aB-4Ab}{a^5x^3}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{10b^2\log(x)(2Ab-aB)}{a^7}+\frac{10b^2(2Ab-aB)\log(a+bx)}{a^7}-\frac{2b^2(5Ab-3aB)}{a^6(a+bx)}-\frac{2b(5Ab-2aB)}{a^6x}-\frac{b^2(4Ab-3aB)}{2a^5(a+bx)^2}+\frac{4Ab-aB}{2a^5x^2}-\frac{b^2(Ab-aB)}{3a^4(a+bx)^3}-\frac{A}{3a^4x^3}$$

input
```
Int[(A + B*x)/(x^4*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
-1/3*A/(a^4*x^3) + (4*A*b - a*B)/(2*a^5*x^2) - (2*b*(5*A*b - 2*a*B))/(a^6*
x) - (b^2*(A*b - a*B))/(3*a^4*(a + b*x)^3) - (b^2*(4*A*b - 3*a*B))/(2*a^5*
(a + b*x)^2) - (2*b^2*(5*A*b - 3*a*B))/(a^6*(a + b*x)) - (10*b^2*(2*A*b -
a*B)*Log[x])/a^7 + (10*b^2*(2*A*b - a*B)*Log[a + b*x])/a^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

method	result
default	$-\frac{b^2(4Ab-3Ba)}{2a^5(bx+a)^2}-\frac{b^2(Ab-Ba)}{3a^4(bx+a)^3}+\frac{10b^2(2Ab-Ba)\ln(bx+a)}{a^7}-\frac{2b^2(5Ab-3Ba)}{a^6(bx+a)}-\frac{A}{3a^4x^3}-\frac{-4Ab+Ba}{2x^2a^5}-\frac{2b(5Ab-2}{a^6x}$
norman	$\frac{-\frac{A}{3a}+\frac{(2Ab-Ba)x}{2a^2}-\frac{5b(2Ab-Ba)x^2}{2a^3}+\frac{3b\left(20A\,b^3-10Ba\,b^2\right)x^4}{a^5}+\frac{3b^2\left(30A\,b^3-15Ba\,b^2\right)x^5}{a^6}+\frac{b^3\left(110A\,b^3-55Ba\,b^2\right)x^6}{3a^7}}{x^3(bx+a)^3}-\frac{10b^2(2Ab-}{a^7}$
risch	$\frac{-\frac{10b^4(2Ab-Ba)x^5}{a^6}-\frac{25b^3(2Ab-Ba)x^4}{a^5}-\frac{55b^2(2Ab-Ba)x^3}{3a^4}-\frac{5b(2Ab-Ba)x^2}{2a^3}+\frac{(2Ab-Ba)x}{2a^2}-\frac{A}{3a}}{x^3(b^2x^2+2abx+a^2)(bx+a)}+\frac{20b^3\ln(-bx-a)A}{a^7}-\frac{10b^2\ln(}{}$
parallelrisch	$-\frac{60B\ln(bx+a)x^6a\,b^5-360A\ln(bx+a)x^5a\,b^5+180B\ln(bx+a)x^5a^2b^4-360A\ln(bx+a)x^4a^2b^4+180B\ln(bx+a)x^4a^3b^3-120A}{}$

input
```
int((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*(4*A*b-3*B*a)/a^5/(b*x+a)^2-1/3*b^2*(A*b-B*a)/a^4/(b*x+a)^3+10*b^
2*(2*A*b-B*a)*ln(b*x+a)/a^7-2*b^2*(5*A*b-3*B*a)/a^6/(b*x+a)-1/3*A/a^4/x^3-
1/2*(-4*A*b+B*a)/x^2/a^5-2*b*(5*A*b-2*B*a)/a^6/x-10*b^2*(2*A*b-B*a)*ln(x)/
a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(155) = 310$.

Time = 0.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.01

$$\int \frac{A+Bx}{x^4(a^2+2abx+b^2x^2)^2}\,dx = -\frac{2\,Aa^6-60\,(Ba^2b^4-2\,Aab^5)x^5-150\,(Ba^3b^3-2\,Aa^2b^4)x^4-110\,(Ba^4b^2-2\,Aa^3b^3)x^3-15\,(Ba^5b-2}{}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*A*a^6 - 60*(B*a^2*b^4 - 2*A*a*b^5)*x^5 - 150*(B*a^3*b^3 - 2*A*a^2*
b^4)*x^4 - 110*(B*a^4*b^2 - 2*A*a^3*b^3)*x^3 - 15*(B*a^5*b - 2*A*a^4*b^2)*
x^2 + 3*(B*a^6 - 2*A*a^5*b)*x + 60*((B*a*b^5 - 2*A*b^6)*x^6 + 3*(B*a^2*b^4
 - 2*A*a*b^5)*x^5 + 3*(B*a^3*b^3 - 2*A*a^2*b^4)*x^4 + (B*a^4*b^2 - 2*A*a^3
*b^3)*x^3)*log(b*x + a) - 60*((B*a*b^5 - 2*A*b^6)*x^6 + 3*(B*a^2*b^4 - 2*A
*a*b^5)*x^5 + 3*(B*a^3*b^3 - 2*A*a^2*b^4)*x^4 + (B*a^4*b^2 - 2*A*a^3*b^3)*
x^3)*log(x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx}{x^4(a^2+2abx+b^2x^2)^2}\,dx = \frac{-2Aa^5+x^5(-120Ab^5+60Bab^4)+x^4(-300Aab^4+150Ba^2b^3)+x^3(-220Aa^2b^3+110Ba^3b^2)+x^2(-3}{6a^9x^3+18a^8bx^4+18a^7b^2x^5+6a^6b^3x^6} + \frac{10b^2(-2Ab+Ba)\log\left(x+\frac{-20Aab^3+10Ba^2b^2-10ab^2(-2Ab+Ba)}{-40Ab^4+20Bab^3}\right)}{a^7} - \frac{10b^2(-2Ab+Ba)\log\left(x+\frac{-20Aab^3+10Ba^2b^2+10ab^2(-2Ab+Ba)}{-40Ab^4+20Bab^3}\right)}{a^7}$$

input
```
integrate((B*x+A)/x**4/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
(-2*A*a**5 + x**5*(-120*A*b**5 + 60*B*a*b**4) + x**4*(-300*A*a*b**4 + 150*
B*a**2*b**3) + x**3*(-220*A*a**2*b**3 + 110*B*a**3*b**2) + x**2*(-30*A*a**
3*b**2 + 15*B*a**4*b) + x*(6*A*a**4*b - 3*B*a**5))/(6*a**9*x**3 + 18*a**8*
b*x**4 + 18*a**7*b**2*x**5 + 6*a**6*b**3*x**6) + 10*b**2*(-2*A*b + B*a)*lo
g(x + (-20*A*a*b**3 + 10*B*a**2*b**2 - 10*a*b**2*(-2*A*b + B*a))/(-40*A*b*
*4 + 20*B*a*b**3))/a**7 - 10*b**2*(-2*A*b + B*a)*log(x + (-20*A*a*b**3 + 1
0*B*a**2*b**2 + 10*a*b**2*(-2*A*b + B*a))/(-40*A*b**4 + 20*B*a*b**3))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{2\,Aa^5-60\,(Bab^4-2\,Ab^5)x^5-150\,(Ba^2b^3-2\,Aab^4)x^4-110\,(Ba^3b^2-2\,Aa^2b^3)x^3-15\,(Ba^4b-2\,A}{6\,(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3)} - \frac{10\,(Bab^2-2\,Ab^3)\log\left(bx+a\right)}{a^7} + \frac{10\,(Bab^2-2\,Ab^3)\log\left(x\right)}{a^7}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*A*a^5 - 60*(B*a*b^4 - 2*A*b^5)*x^5 - 150*(B*a^2*b^3 - 2*A*a*b^4)*x
^4 - 110*(B*a^3*b^2 - 2*A*a^2*b^3)*x^3 - 15*(B*a^4*b - 2*A*a^3*b^2)*x^2 +
3*(B*a^5 - 2*A*a^4*b)*x)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*
x^3) - 10*(B*a*b^2 - 2*A*b^3)*log(b*x + a)/a^7 + 10*(B*a*b^2 - 2*A*b^3)*lo
g(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x^4\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{10\,(Bab^2-2\,Ab^3)\log\left(|x|\right)}{a^7} - \frac{10\,(Bab^3-2\,Ab^4)\log\left(|bx+a|\right)}{a^7b} + \frac{60\,Bab^4x^5-120\,Ab^5x^5+150\,Ba^2b^3x^4-300\,Aab^4x^4+110\,Ba^3b^2x^3-220\,Aa^2b^3x^3+15\,Ba^4bx^2-30}{6\,(bx^2+ax)^3a^6}$$

input
```
integrate((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
10*(B*a*b^2 - 2*A*b^3)*log(abs(x))/a^7 - 10*(B*a*b^3 - 2*A*b^4)*log(abs(b*
x + a))/(a^7*b) + 1/6*(60*B*a*b^4*x^5 - 120*A*b^5*x^5 + 150*B*a^2*b^3*x^4
- 300*A*a*b^4*x^4 + 110*B*a^3*b^2*x^3 - 220*A*a^2*b^3*x^3 + 15*B*a^4*b*x^2
 - 30*A*a^3*b^2*x^2 - 3*B*a^5*x + 6*A*a^4*b*x - 2*A*a^5)/((b*x^2 + a*x)^3*
a^6)
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^4 (a^2+2abx+b^2x^2)^2}\,dx = \frac{20\,b^2\,\mathrm{atanh}\left(\frac{10\,b^2\,(2\,A\,b-B\,a)\,(a+2\,b\,x)}{a\,(20\,A\,b^3-10\,B\,a\,b^2)}\right)\,(2\,A\,b-B\,a)}{a^7} - \frac{\frac{A}{3\,a}-\frac{x\,(2\,A\,b-B\,a)}{2\,a^2}+\frac{55\,b^2\,x^3\,(2\,A\,b-B\,a)}{3\,a^4}+\frac{25\,b^3\,x^4\,(2\,A\,b-B\,a)}{a^5}+\frac{10\,b^4\,x^5\,(2\,A\,b-B\,a)}{a^6}+\frac{5\,b\,x^2\,(2\,A\,b-B\,a)}{2\,a^3}}{a^3\,x^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^5+b^3\,x^6}$$

input
```
int((A + B*x)/(x^4*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
(20*b^2*atanh((10*b^2*(2*A*b - B*a)*(a + 2*b*x))/(a*(20*A*b^3 - 10*B*a*b^2
)))*(2*A*b - B*a))/a^7 - (A/(3*a) - (x*(2*A*b - B*a))/(2*a^2) + (55*b^2*x^
3*(2*A*b - B*a))/(3*a^4) + (25*b^3*x^4*(2*A*b - B*a))/a^5 + (10*b^4*x^5*(2
*A*b - B*a))/a^6 + (5*b*x^2*(2*A*b - B*a))/(2*a^3))/(a^3*x^3 + b^3*x^6 + 3
*a^2*b*x^4 + 3*a*b^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^4 (a^2+2abx+b^2x^2)^2}\,dx = \frac{60\log(bx+a)\,a^2b^3x^3+120\log(bx+a)\,a\,b^4x^4+60\log(bx+a)\,b^5x^5-60\log(x)\,a^2b^3x^3-120\log(x)\,a\,b^4x^4 \cdots}{6a^6x^3\,(b^2x^2+2abx+a^2)}$$

input
```
int((B*x+A)/x^4/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(60*log(a + b*x)*a**2*b**3*x**3 + 120*log(a + b*x)*a*b**4*x**4 + 60*log(a
+ b*x)*b**5*x**5 - 60*log(x)*a**2*b**3*x**3 - 120*log(x)*a*b**4*x**4 - 60*
log(x)*b**5*x**5 - 2*a**5 + 5*a**4*b*x - 20*a**3*b**2*x**2 - 60*a**2*b**3*
x**3 + 30*b**5*x**5)/(6*a**6*x**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.272 $\int \frac{x^6(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int \frac{x^6(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{(Ab-6aB)x}{b^7}+\frac{Bx^2}{2b^6}-\frac{a^6(Ab-aB)}{5b^8(a+bx)^5}+\frac{a^5(6Ab-7aB)}{4b^8(a+bx)^4} -\frac{a^4(5Ab-7aB)}{b^8(a+bx)^3}+\frac{5a^3(4Ab-7aB)}{2b^8(a+bx)^2} -\frac{5a^2(3Ab-7aB)}{b^8(a+bx)}-\frac{3a(2Ab-7aB)\log(a+bx)}{b^8}$$

output
```
(A*b-6*B*a)*x/b^7+1/2*B*x^2/b^6-1/5*a^6*(A*b-B*a)/b^8/(b*x+a)^5+1/4*a^5*(6
*A*b-7*B*a)/b^8/(b*x+a)^4-a^4*(5*A*b-7*B*a)/b^8/(b*x+a)^3+5/2*a^3*(4*A*b-7
*B*a)/b^8/(b*x+a)^2-5*a^2*(3*A*b-7*B*a)/b^8/(b*x+a)-3*a*(2*A*b-7*B*a)*ln(b
*x+a)/b^8
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{20b(Ab-6aB)x+10b^2Bx^2+\frac{4a^6(-Ab+aB)}{(a+bx)^5}+\frac{5a^5(6Ab-7aB)}{(a+bx)^4}+\frac{20a^4(-5Ab+7aB)}{(a+bx)^3}-\frac{50a^3(-4Ab+7aB)}{(a+bx)^2}+\frac{100a^2(-3Ab+}{a+bx}}{20b^8}$$

input
```
Integrate[(x^6*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(20*b*(A*b - 6*a*B)*x + 10*b^2*B*x^2 + (4*a^6*(-(A*b) + a*B))/(a + b*x)^5
+ (5*a^5*(6*A*b - 7*a*B))/(a + b*x)^4 + (20*a^4*(-5*A*b + 7*a*B))/(a + b*x
)^3 - (50*a^3*(-4*A*b + 7*a*B))/(a + b*x)^2 + (100*a^2*(-3*A*b + 7*a*B))/(
a + b*x) + 60*a*(-2*A*b + 7*a*B)*Log[a + b*x])/(20*b^8)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$\Big\downarrow \; 1184$$
$$b^6\int \frac{x^6(A+Bx)}{b^6(a+bx)^6}dx$$
$$\Big\downarrow \; 27$$
$$\int \frac{x^6(A+Bx)}{(a+bx)^6}dx$$
$$\Big\downarrow \; 86$$

$$\int \left(-\frac{a^6(aB-Ab)}{b^7(a+bx)^6}+\frac{a^5(7aB-6Ab)}{b^7(a+bx)^5}-\frac{3a^4(7aB-5Ab)}{b^7(a+bx)^4}+\frac{5a^3(7aB-4Ab)}{b^7(a+bx)^3}-\frac{5a^2(7aB-3Ab)}{b^7(a+bx)^2}+\frac{3a(7aB-2}{b^7(a+bx}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6(Ab-aB)}{5b^8(a+bx)^5}+\frac{a^5(6Ab-7aB)}{4b^8(a+bx)^4}-\frac{a^4(5Ab-7aB)}{b^8(a+bx)^3}+\frac{5a^3(4Ab-7aB)}{2b^8(a+bx)^2}-\frac{5a^2(3Ab-7aB)}{b^8(a+bx)}-\frac{3a(2Ab-7aB)\log(a+bx)}{b^8}+\frac{x(Ab-6aB)}{b^7}+\frac{Bx^2}{2b^6}$$

input
```
Int[(x^6*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - 6*a*B)*x)/b^7 + (B*x^2)/(2*b^6) - (a^6*(A*b - a*B))/(5*b^8*(a + b*
x)^5) + (a^5*(6*A*b - 7*a*B))/(4*b^8*(a + b*x)^4) - (a^4*(5*A*b - 7*a*B))/
(b^8*(a + b*x)^3) + (5*a^3*(4*A*b - 7*a*B))/(2*b^8*(a + b*x)^2) - (5*a^2*(
3*A*b - 7*a*B))/(b^8*(a + b*x)) - (3*a*(2*A*b - 7*a*B)*Log[a + b*x])/b^8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{1}{2}Bb\,x^2+Abx-6Bax}{b^7}-\frac{a^6(Ab-Ba)}{5b^8(bx+a)^5}-\frac{a^4(5Ab-7Ba)}{b^8(bx+a)^3}-\frac{5a^2(3Ab-7Ba)}{b^8(bx+a)}-\frac{3a(2Ab-7Ba)\ln(bx+a)}{b^8}+\frac{5a^3(4Ab-7B}{2b^8(bx+a)^2}$
norman	$\frac{\frac{B\,x^7}{2b}-\frac{a^5\left(274abA-959a^2B\right)}{20b^8}+\frac{(2Ab-7Ba)x^6}{2b^2}-\frac{5a\left(6abA-21a^2B\right)x^4}{b^4}-\frac{5a^2\left(18abA-63a^2B\right)x^3}{b^5}-\frac{5a^3\left(22abA-77a^2B\right)x^2}{b^6}-\frac{5a^4\left(50abA-\right.}{4b}}{(bx+a)^5}$
risch	$\frac{B\,x^2}{2b^6}+\frac{Ax}{b^6}-\frac{6Bax}{b^7}+\frac{\left(-15A\,a^2b^4+35B\,a^3b^3\right)x^4-\frac{5a^3b^2(20Ab-49Ba)x^3}{2}+\left(-65A\,a^4b^2+\frac{329}{2}B\,a^5b\right)x^2+\left(-\frac{77}{2}A\,a^5b+\frac{399}{4}\right.}{b^7(bx+a)\left(b^2x^2+2abx+a^2\right)^2}$
parallelrisch	$-\frac{-7700B\,a^5b^2x^2+1250A\,a^5b^2x-4375B\,a^6bx+1200A\ln(bx+a)x^2a^4b^3-4200B\ln(bx+a)x^2a^5b^2+600A\ln(bx+a)x\,a^5b^2-21}{}$

input
```
int(x^6*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^7*(1/2*B*b*x^2+A*b*x-6*B*a*x)-1/5*a^6*(A*b-B*a)/b^8/(b*x+a)^5-a^4*(5*A
*b-7*B*a)/b^8/(b*x+a)^3-5*a^2*(3*A*b-7*B*a)/b^8/(b*x+a)-3*a*(2*A*b-7*B*a)*
ln(b*x+a)/b^8+5/2*a^3*(4*A*b-7*B*a)/b^8/(b*x+a)^2+1/4*a^5*(6*A*b-7*B*a)/b^
8/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(164) = 328.

Time = 0.08 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.04

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$=\frac{10\,Bb^7x^7+459\,Ba^7-174\,Aa^6b-10\,(7\,Bab^6-2\,Ab^7)x^6-100\,(5\,Ba^2b^5-Aab^6)x^5-100\,(4\,Ba^3b^4+A}{}$$

input
```
integrate(x^6*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/20*(10*B*b^7*x^7 + 459*B*a^7 - 174*A*a^6*b - 10*(7*B*a*b^6 - 2*A*b^7)*x^
6 - 100*(5*B*a^2*b^5 - A*a*b^6)*x^5 - 100*(4*B*a^3*b^4 + A*a^2*b^5)*x^4 +
100*(13*B*a^4*b^3 - 8*A*a^3*b^4)*x^3 + 300*(9*B*a^5*b^2 - 4*A*a^4*b^3)*x^2
 + 375*(5*B*a^6*b - 2*A*a^5*b^2)*x + 60*(7*B*a^7 - 2*A*a^6*b + (7*B*a^2*b^
5 - 2*A*a*b^6)*x^5 + 5*(7*B*a^3*b^4 - 2*A*a^2*b^5)*x^4 + 10*(7*B*a^4*b^3 -
 2*A*a^3*b^4)*x^3 + 10*(7*B*a^5*b^2 - 2*A*a^4*b^3)*x^2 + 5*(7*B*a^6*b - 2*
A*a^5*b^2)*x)*log(b*x + a))/(b^13*x^5 + 5*a*b^12*x^4 + 10*a^2*b^11*x^3 + 1
0*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.26

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{Bx^2}{2b^6} + \frac{3a(-2Ab+7Ba)\log{(a+bx)}}{b^8} + x\left(\frac{A}{b^6} - \frac{6Ba}{b^7}\right) + \frac{-174Aa^6b+459Ba^7+x^4(-300Aa^2b^5+700Ba^3b^4)+x^3(-1000Aa^3b^4+2450Ba^4b^3)+x^2(-1300Aa^4}{20a^5b^8+100a^4b^9x+200a^3b^{10}x^2+200a^2b^{11}x^3+100ab^{12}x^4+}$$

input
```
integrate(x**6*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
B*x**2/(2*b**6) + 3*a*(-2*A*b + 7*B*a)*log(a + b*x)/b**8 + x*(A/b**6 - 6*B
*a/b**7) + (-174*A*a**6*b + 459*B*a**7 + x**4*(-300*A*a**2*b**5 + 700*B*a*
*3*b**4) + x**3*(-1000*A*a**3*b**4 + 2450*B*a**4*b**3) + x**2*(-1300*A*a**
4*b**3 + 3290*B*a**5*b**2) + x*(-770*A*a**5*b**2 + 1995*B*a**6*b))/(20*a**
5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100
*a*b**12*x**4 + 20*b**13*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.25

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{459\,Ba^7-174\,Aa^6b+100\,(7\,Ba^3b^4-3\,Aa^2b^5)x^4+50\,(49\,Ba^4b^3-20\,Aa^3b^4)x^3+10\,(329\,Ba^5b^2-130\,A}{20\,(b^{13}x^5+5\,ab^{12}x^4+10\,a^2b^{11}x^3+10\,a^3b^{10}x^2+5\,a^4b^9x+a^5b^8)} + \frac{Bbx^2-2\,(6\,Ba-Ab)x}{2\,b^7} + \frac{3\,(7\,Ba^2-2\,Aab)\log{(bx+a)}}{b^8}$$

input
```
integrate(x^6*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/20*(459*B*a^7 - 174*A*a^6*b + 100*(7*B*a^3*b^4 - 3*A*a^2*b^5)*x^4 + 50*(
49*B*a^4*b^3 - 20*A*a^3*b^4)*x^3 + 10*(329*B*a^5*b^2 - 130*A*a^4*b^3)*x^2
+ 35*(57*B*a^6*b - 22*A*a^5*b^2)*x)/(b^13*x^5 + 5*a*b^12*x^4 + 10*a^2*b^11
*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*(B*b*x^2 - 2*(6*B*a
- A*b)*x)/b^7 + 3*(7*B*a^2 - 2*A*a*b)*log(b*x + a)/b^8
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{3\,(7\,Ba^2-2\,Aab)\log\left(|bx+a|\right)}{b^8}+\frac{Bb^6x^2-12\,Bab^5x+2\,Ab^6x}{2\,b^{12}}$$
$$+\frac{459\,Ba^7-174\,Aa^6b+100\,(7\,Ba^3b^4-3\,Aa^2b^5)x^4+50\,(49\,Ba^4b^3-20\,Aa^3b^4)x^3+10\,(329\,Ba^5b^2-13}{20\,(bx+a)^5b^8}$$

input
```
integrate(x^6*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
3*(7*B*a^2 - 2*A*a*b)*log(abs(b*x + a))/b^8 + 1/2*(B*b^6*x^2 - 12*B*a*b^5*
x + 2*A*b^6*x)/b^12 + 1/20*(459*B*a^7 - 174*A*a^6*b + 100*(7*B*a^3*b^4 - 3
*A*a^2*b^5)*x^4 + 50*(49*B*a^4*b^3 - 20*A*a^3*b^4)*x^3 + 10*(329*B*a^5*b^2
 - 130*A*a^4*b^3)*x^2 + 35*(57*B*a^6*b - 22*A*a^5*b^2)*x)/((b*x + a)^5*b^8
)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.23

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = x\left(\frac{A}{b^6}-\frac{6\,B\,a}{b^7}\right)$$
$$-\frac{x^2\left(65\,A\,a^4\,b^2-\frac{329\,B\,a^5\,b}{2}\right)-x\left(\frac{399\,B\,a^6}{4}-\frac{77\,A\,a^5\,b}{2}\right)-\frac{3\,(153\,B\,a^7-58\,A\,a^6\,b)}{20\,b}+x^4\,(15\,A\,a^2\,b^4-35\,B\,a^3\,b^3)+}{a^5\,b^7+5\,a^4\,b^8\,x+10\,a^3\,b^9\,x^2+10\,a^2\,b^{10}\,x^3+5\,a\,b^{11}\,x^4+b^{12}\,x^5}$$
$$+\frac{B\,x^2}{2\,b^6}+\frac{\ln(a+b\,x)\,(21\,B\,a^2-6\,A\,a\,b)}{b^8}$$

input
```
int((x^6*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x*(A/b^6 - (6*B*a)/b^7) - (x^2*(65*A*a^4*b^2 - (329*B*a^5*b)/2) - x*((399*
B*a^6)/4 - (77*A*a^5*b)/2) - (3*(153*B*a^7 - 58*A*a^6*b))/(20*b) + x^4*(15
*A*a^2*b^4 - 35*B*a^3*b^3) + x^3*(50*A*a^3*b^3 - (245*B*a^4*b^2)/2))/(a^5*
b^7 + b^12*x^5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10
*x^3) + (B*x^2)/(2*b^6) + (log(a + b*x)*(21*B*a^2 - 6*A*a*b))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$=\frac{60\log(bx+a)\,a^6+240\log(bx+a)\,a^5bx+360\log(bx+a)\,a^4b^2x^2+240\log(bx+a)\,a^3b^3x^3+60\log(bx+}{4b^7\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx}$$

input
```
int(x^6*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(60*log(a + b*x)*a**6 + 240*log(a + b*x)*a**5*b*x + 360*log(a + b*x)*a**4*
b**2*x**2 + 240*log(a + b*x)*a**3*b**3*x**3 + 60*log(a + b*x)*a**2*b**4*x*
*4 + 65*a**6 + 200*a**5*b*x + 180*a**4*b**2*x**2 - 60*a**2*b**4*x**4 - 12*
a*b**5*x**5 + 2*b**6*x**6)/(4*b**7*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4))
```

3.273 $\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{Bx}{b^6}+\frac{a^5(Ab-aB)}{5b^7(a+bx)^5}-\frac{a^4(5Ab-6aB)}{4b^7(a+bx)^4}+\frac{5a^3(2Ab-3aB)}{3b^7(a+bx)^3}-\frac{5a^2(Ab-2aB)}{b^7(a+bx)^2}+\frac{5a(Ab-3aB)}{b^7(a+bx)}+\frac{(Ab-6aB)\log(a+bx)}{b^7}$$

output
```
B*x/b^6+1/5*a^5*(A*b-B*a)/b^7/(b*x+a)^5-1/4*a^4*(5*A*b-6*B*a)/b^7/(b*x+a)^
4+5/3*a^3*(2*A*b-3*B*a)/b^7/(b*x+a)^3-5*a^2*(A*b-2*B*a)/b^7/(b*x+a)^2+5*a*
(A*b-3*B*a)/b^7/(b*x+a)+(A*b-6*B*a)*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{60bBx+\frac{12a^5(Ab-aB)}{(a+bx)^5}+\frac{15a^4(-5Ab+6aB)}{(a+bx)^4}+\frac{100a^3(2Ab-3aB)}{(a+bx)^3}+\frac{300a^2(-Ab+2aB)}{(a+bx)^2}+\frac{300a(Ab-3aB)}{a+bx}+60(Ab-6aB)\,\mathrm{l}}{60b^7}$$

input
```
Integrate[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(60*b*B*x + (12*a^5*(A*b - a*B))/(a + b*x)^5 + (15*a^4*(-5*A*b + 6*a*B))/(
a + b*x)^4 + (100*a^3*(2*A*b - 3*a*B))/(a + b*x)^3 + (300*a^2*(-(A*b) + 2*
a*B))/(a + b*x)^2 + (300*a*(A*b - 3*a*B))/(a + b*x) + 60*(A*b - 6*a*B)*Log
[a + b*x])/(60*b^7)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$

$$\Big\downarrow \; 1184$$

$$b^6\int \frac{x^5(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^5(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{a^5(aB-Ab)}{b^6(a+bx)^6}-\frac{a^4(6aB-5Ab)}{b^6(a+bx)^5}+\frac{5a^3(3aB-2Ab)}{b^6(a+bx)^4}-\frac{10a^2(2aB-Ab)}{b^6(a+bx)^3}+\frac{5a(3aB-Ab)}{b^6(a+bx)^2}+\frac{Ab-6aB}{b^6(a+bx)}+\frac{B}{b^6}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^5(Ab-aB)}{5b^7(a+bx)^5}-\frac{a^4(5Ab-6aB)}{4b^7(a+bx)^4}+\frac{5a^3(2Ab-3aB)}{3b^7(a+bx)^3}-\frac{5a^2(Ab-2aB)}{b^7(a+bx)^2}+\frac{5a(Ab-3aB)}{b^7(a+bx)}+\frac{(Ab-6aB)\log(a+bx)}{b^7}+\frac{Bx}{b^6}$$

input
```
Int[(x^5*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(B*x)/b^6 + (a^5*(A*b - a*B))/(5*b^7*(a + b*x)^5) - (a^4*(5*A*b - 6*a*B))/
(4*b^7*(a + b*x)^4) + (5*a^3*(2*A*b - 3*a*B))/(3*b^7*(a + b*x)^3) - (5*a^2
*(A*b - 2*a*B))/(b^7*(a + b*x)^2) + (5*a*(A*b - 3*a*B))/(b^7*(a + b*x)) +
((A*b - 6*a*B)*Log[a + b*x])/b^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

method	result
norman	$\frac{\frac{B\,x^6}{b}+\frac{a^5(137Ab-822Ba)}{60b^7}+\frac{5a(Ab-6Ba)x^4}{b^3}+\frac{5a^2(3Ab-18Ba)x^3}{b^4}+\frac{5a^3(11Ab-66Ba)x^2}{3b^5}+\frac{5a^4(25Ab-150Ba)x}{12b^6}}{(bx+a)^5}+\frac{(Ab-6Ba)\ln(bx+}{b^7}$
default	$\frac{Bx}{b^6}+\frac{a^5(Ab-Ba)}{5b^7(bx+a)^5}-\frac{a^4(5Ab-6Ba)}{4b^7(bx+a)^4}+\frac{5a^3(2Ab-3Ba)}{3b^7(bx+a)^3}-\frac{5a^2(Ab-2Ba)}{b^7(bx+a)^2}+\frac{5a(Ab-3Ba)}{b^7(bx+a)}+\frac{(Ab-6Ba)\ln(bx+a)}{b^7}$
risch	$\frac{Bx}{b^6}+\frac{(5Aa\,b^4-15B\,a^2b^3)x^4+5a^2b^2(3Ab-10Ba)x^3+(\frac{55}{3}a^3A\,b^2-65B\,a^4b)x^2+(\frac{125}{12}A\,a^4b-\frac{77}{2}B\,a^5)x+\frac{a^5(137Ab-522Ba)}{60b}}{b^6(bx+a)(b^2x^2+2abx+a^2)^2}$
parallelrisch	$600A\ln(bx+a)x^2a^3b^3+600A\ln(bx+a)x^3a^2b^4-3600B\ln(bx+a)x^3a^3b^3-3750B\,a^5bx+300Aa\,b^5x^4-6600B\,a^4b^2x^2+300A\ln($

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/b*x^6+1/60*a^5*(137*A*b-822*B*a)/b^7+5*a*(A*b-6*B*a)/b^3*x^4+5*a^2*(3*A
*b-18*B*a)/b^4*x^3+5/3*a^3*(11*A*b-66*B*a)/b^5*x^2+5/12*a^4*(25*A*b-150*B*
a)/b^6*x)/(b*x+a)^5+(A*b-6*B*a)*ln(b*x+a)/b^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(144) = 288.

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.13

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$=\frac{60\,Bb^6x^6+300\,Bab^5x^5-522\,Ba^6+137\,Aa^5b-300\,(Ba^2b^4-Aab^5)x^4-300\,(8\,Ba^3b^3-3\,Aa^2b^4)x^3-}{}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^6*x^6 + 300*B*a*b^5*x^5 - 522*B*a^6 + 137*A*a^5*b - 300*(B*a^
2*b^4 - A*a*b^5)*x^4 - 300*(8*B*a^3*b^3 - 3*A*a^2*b^4)*x^3 - 100*(36*B*a^4
*b^2 - 11*A*a^3*b^3)*x^2 - 125*(18*B*a^5*b - 5*A*a^4*b^2)*x - 60*(6*B*a^6
- A*a^5*b + (6*B*a*b^5 - A*b^6)*x^5 + 5*(6*B*a^2*b^4 - A*a*b^5)*x^4 + 10*(
6*B*a^3*b^3 - A*a^2*b^4)*x^3 + 10*(6*B*a^4*b^2 - A*a^3*b^3)*x^2 + 5*(6*B*a
^5*b - A*a^4*b^2)*x)*log(b*x + a))/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*
x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7)
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{Bx}{b^6} + \frac{137Aa^5b - 522Ba^6 + x^4\cdot(300Aab^5 - 900Ba^2b^4) + x^3\cdot(900Aa^2b^4 - 3000Ba^3b^3) + x^2\cdot(1100Aa^3b^3 - \ldots}{60a^5b^7 + 300a^4b^8x + 600a^3b^9x^2 + 600a^2b^{10}x^3 + 300ab^{11}x^4 + 60b\ldots} - \frac{(-Ab+6Ba)\log(a+bx)}{b^7}$$

input
```
integrate(x**5*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
B*x/b**6 + (137*A*a**5*b - 522*B*a**6 + x**4*(300*A*a*b**5 - 900*B*a**2*b*
*4) + x**3*(900*A*a**2*b**4 - 3000*B*a**3*b**3) + x**2*(1100*A*a**3*b**3 -
 3900*B*a**4*b**2) + x*(625*A*a**4*b**2 - 2310*B*a**5*b))/(60*a**5*b**7 +
300*a**4*b**8*x + 600*a**3*b**9*x**2 + 600*a**2*b**10*x**3 + 300*a*b**11*x
**4 + 60*b**12*x**5) - (-A*b + 6*B*a)*log(a + b*x)/b**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{522\,Ba^6 - 137\,Aa^5b + 300\,(3\,Ba^2b^4 - Aab^5)x^4 + 300\,(10\,Ba^3b^3 - 3\,Aa^2b^4)x^3 + 100\,(39\,Ba^4b^2 - 11\,A\ldots}{60\,(b^{12}x^5 + 5\,ab^{11}x^4 + 10\,a^2b^{10}x^3 + 10\,a^3b^9x^2 + 5\,a^4b^8x + a^5b^7)} + \frac{Bx}{b^6} - \frac{(6\,Ba - Ab)\log(bx+a)}{b^7}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(522*B*a^6 - 137*A*a^5*b + 300*(3*B*a^2*b^4 - A*a*b^5)*x^4 + 300*(10
*B*a^3*b^3 - 3*A*a^2*b^4)*x^3 + 100*(39*B*a^4*b^2 - 11*A*a^3*b^3)*x^2 + 5*
(462*B*a^5*b - 125*A*a^4*b^2)*x)/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^
3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x + a^5*b^7) + B*x/b^6 - (6*B*a - A*b)*log(
b*x + a)/b^7
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{Bx}{b^6} - \frac{(6\,Ba-Ab)\log(|bx+a|)}{b^7} - \frac{522\,Ba^6 - 137\,Aa^5b + 300\,(3\,Ba^2b^4 - Aab^5)x^4 + 300\,(10\,Ba^3b^3 - 3\,Aa^2b^4)x^3 + 100\,(39\,Ba^4b^2 - 11\,Aa}{60\,(bx+a)^5b^7}$$

input
```
integrate(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
B*x/b^6 - (6*B*a - A*b)*log(abs(b*x + a))/b^7 - 1/60*(522*B*a^6 - 137*A*a^
5*b + 300*(3*B*a^2*b^4 - A*a*b^5)*x^4 + 300*(10*B*a^3*b^3 - 3*A*a^2*b^4)*x
^3 + 100*(39*B*a^4*b^2 - 11*A*a^3*b^3)*x^2 + 5*(462*B*a^5*b - 125*A*a^4*b^
2)*x)/((b*x + a)^5*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{B\,x}{b^6} - \frac{x\left(\frac{77\,B\,a^5}{2} - \frac{125\,A\,a^4\,b}{12}\right) + x^4\left(15\,B\,a^2\,b^3 - 5\,A\,a\,b^4\right) - x^2\left(\frac{55\,A\,a^3\,b^2}{3} - 65\,B\,a^4\,b\right) + \frac{522\,B\,a^6-137\,A\,a^5\,b}{60\,b} - x^3\,(}{a^5\,b^6 + 5\,a^4\,b^7\,x + 10\,a^3\,b^8\,x^2 + 10\,a^2\,b^9\,x^3 + 5\,a\,b^{10}\,x^4 + b^{11}\,x^5} + \frac{\ln(a+b\,x)\,(A\,b-6\,B\,a)}{b^7}$$

input
```
int((x^5*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(B*x)/b^6 - (x*((77*B*a^5)/2 - (125*A*a^4*b)/12) + x^4*(15*B*a^2*b^3 - 5*A
*a*b^4) - x^2*((55*A*a^3*b^2)/3 - 65*B*a^4*b) + (522*B*a^6 - 137*A*a^5*b)/
(60*b) - x^3*(15*A*a^2*b^3 - 50*B*a^3*b^2))/(a^5*b^6 + b^11*x^5 + 5*a^4*b^
7*x + 5*a*b^10*x^4 + 10*a^3*b^8*x^2 + 10*a^2*b^9*x^3) + (log(a + b*x)*(A*b
 - 6*B*a))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{-60\log(bx+a)\,a^5 - 240\log(bx+a)\,a^4bx - 360\log(bx+a)\,a^3b^2x^2 - 240\log(bx+a)\,a^2b^3x^3 - 60\log(bx}{12b^6\,(b^4x^4 + 4a\,b^3x^3 + 6a^2b^2x^2 + 4a^3bx + a}$$

input
```
int(x^5*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 60*log(a + b*x)*a**5 - 240*log(a + b*x)*a**4*b*x - 360*log(a + b*x)*a*
*3*b**2*x**2 - 240*log(a + b*x)*a**2*b**3*x**3 - 60*log(a + b*x)*a*b**4*x*
*4 - 65*a**5 - 200*a**4*b*x - 180*a**3*b**2*x**2 + 60*a*b**4*x**4 + 12*b**
5*x**5)/(12*b**6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b
**4*x**4))
```

3.274 $\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 106

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{(Ab-aB)x^5}{5ab(a+bx)^5} - \frac{a^4B}{4b^6(a+bx)^4} + \frac{4a^3B}{3b^6(a+bx)^3} - \frac{3a^2B}{b^6(a+bx)^2} + \frac{4aB}{b^6(a+bx)} + \frac{B\log(a+bx)}{b^6}$$

output
```
1/5*(A*b-B*a)*x^5/a/b/(b*x+a)^5-1/4*a^4*B/b^6/(b*x+a)^4+4/3*a^3*B/b^6/(b*x
+a)^3-3*a^2*B/b^6/(b*x+a)^2+4*a*B/b^6/(b*x+a)+B*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{137a^5B - 60Ab^5x^4 + 60ab^4x^3(-2A+5Bx) + 60a^2b^3x^2(-2A+15Bx) + 20a^3b^2x(-3A+55Bx) + a^4(-}{60b^6(a+bx)^5}$$

input
```
Integrate[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(137*a^5*B - 60*A*b^5*x^4 + 60*a*b^4*x^3*(-2*A + 5*B*x) + 60*a^2*b^3*x^2*(
-2*A + 15*B*x) + 20*a^3*b^2*x*(-3*A + 55*B*x) + a^4*(-12*A*b + 625*b*B*x)
+ 60*B*(a + b*x)^5*Log[a + b*x])/(60*b^6*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1184, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{x^4(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{B\int \frac{x^4}{(a+bx)^5}dx}{b}+\frac{x^5(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{49}$$

$$\frac{B\int \left(\frac{a^4}{b^4(a+bx)^5}-\frac{4a^3}{b^4(a+bx)^4}+\frac{6a^2}{b^4(a+bx)^3}-\frac{4a}{b^4(a+bx)^2}+\frac{1}{b^4(a+bx)}\right)dx}{b}+\frac{x^5(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\left(-\frac{a^4}{4b^5(a+bx)^4}+\frac{4a^3}{3b^5(a+bx)^3}-\frac{3a^2}{b^5(a+bx)^2}+\frac{4a}{b^5(a+bx)}+\frac{\log(a+bx)}{b^5}\right)}{b}+\frac{x^5(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^5)/(5*a*b*(a + b*x)^5) + (B*(-1/4*a^4/(b^5*(a + b*x)^4) + (
4*a^3)/(3*b^5*(a + b*x)^3) - (3*a^2)/(b^5*(a + b*x)^2) + (4*a)/(b^5*(a + b
*x)) + Log[a + b*x]/b^5))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

method	result
norman	$\frac{-\frac{a^4(12Ab-137Ba)}{60b^6}-\frac{(Ab-5Ba)x^4}{b^2}-\frac{a(2Ab-15Ba)x^3}{b^3}-\frac{a^2(6Ab-55Ba)x^2}{3b^4}-\frac{a^3(12Ab-125Ba)x}{12b^5}}{(bx+a)^5}+\frac{B\ln(bx+a)}{b^6}$
default	$-\frac{a^4(Ab-Ba)}{5b^6(bx+a)^5}-\frac{2a^2(3Ab-5Ba)}{3b^6(bx+a)^3}-\frac{Ab-5Ba}{b^6(bx+a)}+\frac{B\ln(bx+a)}{b^6}+\frac{a(2Ab-5Ba)}{b^6(bx+a)^2}+\frac{a^3(4Ab-5Ba)}{4b^6(bx+a)^4}$
risch	$\frac{-\frac{a^4(12Ab-137Ba)}{60b^6}-\frac{(Ab-5Ba)x^4}{b^2}-\frac{a(2Ab-15Ba)x^3}{b^3}-\frac{a^2(6Ab-55Ba)x^2}{3b^4}-\frac{a^3(12Ab-125Ba)x}{12b^5}}{(bx+a)(b^2x^2+2abx+a^2)^2}+\frac{B\ln(bx+a)}{b^6}$
parallelrisch	$-\frac{-60B\ln(bx+a)x^5b^5-300B\ln(bx+a)x^4a\,b^4+60A\,b^5x^4-600B\ln(bx+a)x^3a^2b^3-300Ba\,b^4x^4+120Aa\,b^4x^3-600B\ln(bx+a}{60b^6(b}$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/60*a^4*(12*A*b-137*B*a)/b^6-(A*b-5*B*a)/b^2*x^4-a*(2*A*b-15*B*a)/b^3*x
^3-1/3*a^2*(6*A*b-55*B*a)/b^4*x^2-1/12*a^3*(12*A*b-125*B*a)/b^5*x)/(b*x+a)
^5+B*ln(b*x+a)/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(100) = 200.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.09

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$=\frac{137\,Ba^5-12\,Aa^4b+60\,(5\,Bab^4-Ab^5)x^4+60\,(15\,Ba^2b^3-2\,Aab^4)x^3+20\,(55\,Ba^3b^2-6\,Aa^2b^3)x^2+5}{60\,(b^{11}x^5+5\,ab^{10}x^4+10\,a^2b^9}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/60*(137*B*a^5 - 12*A*a^4*b + 60*(5*B*a*b^4 - A*b^5)*x^4 + 60*(15*B*a^2*b
^3 - 2*A*a*b^4)*x^3 + 20*(55*B*a^3*b^2 - 6*A*a^2*b^3)*x^2 + 5*(125*B*a^4*b
 - 12*A*a^3*b^2)*x + 60*(B*b^5*x^5 + 5*B*a*b^4*x^4 + 10*B*a^2*b^3*x^3 + 10
*B*a^3*b^2*x^2 + 5*B*a^4*b*x + B*a^5)*log(b*x + a))/(b^11*x^5 + 5*a*b^10*x
^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.62

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{B\log{(a+bx)}}{b^6} + \frac{-12Aa^4b+137Ba^5+x^4(-60Ab^5+300Bab^4)+x^3(-120Aab^4+900Ba^2b^3)+x^2(-120Aa^2b^3+1100}{60a^5b^6+300a^4b^7x+600a^3b^8x^2+600a^2b^9x^3+300ab^{10}x^4+60b^{11}x^5}$$

input
```
integrate(x**4*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
B*log(a + b*x)/b**6 + (-12*A*a**4*b + 137*B*a**5 + x**4*(-60*A*b**5 + 300*
B*a*b**4) + x**3*(-120*A*a*b**4 + 900*B*a**2*b**3) + x**2*(-120*A*a**2*b**
3 + 1100*B*a**3*b**2) + x*(-60*A*a**3*b**2 + 625*B*a**4*b))/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*
x**4 + 60*b**11*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{137\,Ba^5-12\,Aa^4b+60\,(5\,Bab^4-Ab^5)x^4+60\,(15\,Ba^2b^3-2\,Aab^4)x^3+20\,(55\,Ba^3b^2-6\,Aa^2b^3)x^2+5}{60\,(b^{11}x^5+5\,ab^{10}x^4+10\,a^2b^9x^3+10\,a^3b^8x^2+5\,a^4b^7x+a^5b^6)} + \frac{B\log{(bx+a)}}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/60*(137*B*a^5 - 12*A*a^4*b + 60*(5*B*a*b^4 - A*b^5)*x^4 + 60*(15*B*a^2*b
^3 - 2*A*a*b^4)*x^3 + 20*(55*B*a^3*b^2 - 6*A*a^2*b^3)*x^2 + 5*(125*B*a^4*b
 - 12*A*a^3*b^2)*x)/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8
*x^2 + 5*a^4*b^7*x + a^5*b^6) + B*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{B\log\left(|bx+a|\right)}{b^6} + \frac{60\,(5\,Bab^3-Ab^4)x^4+60\,(15\,Ba^2b^2-2\,Aab^3)x^3+20\,(55\,Ba^3b-6\,Aa^2b^2)x^2+5\,(125\,Ba^4-12\,Aa^3b)}{60\,(bx+a)^5b^5}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/b^6 + 1/60*(60*(5*B*a*b^3 - A*b^4)*x^4 + 60*(15*B*a^2*
b^2 - 2*A*a*b^3)*x^3 + 20*(55*B*a^3*b - 6*A*a^2*b^2)*x^2 + 5*(125*B*a^4 -
12*A*a^3*b)*x + (137*B*a^5 - 12*A*a^4*b)/b)/((b*x + a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.52

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{137\,B\,a^5-12\,A\,a^4\,b}{60\,b^6}+\frac{x^3\,\left(15\,B\,a^2-2\,A\,a\,b\right)}{b^3}+\frac{x\,\left(125\,B\,a^4-12\,A\,a^3\,b\right)}{12\,b^5}-\frac{x^4\,\left(A\,b-5\,B\,a\right)}{b^2}+\frac{x^2\,\left(55\,B\,a^3-6\,A\,a^2\,b\right)}{3\,b^4}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5} + \frac{B\,\ln\left(a+b\,x\right)}{b^6}$$

input
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
((137*B*a^5 - 12*A*a^4*b)/(60*b^6) + (x^3*(15*B*a^2 - 2*A*a*b))/b^3 + (x*(
125*B*a^4 - 12*A*a^3*b))/(12*b^5) - (x^4*(A*b - 5*B*a))/b^2 + (x^2*(55*B*a
^3 - 6*A*a^2*b))/(3*b^4))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 +
10*a^2*b^3*x^3 + 5*a^4*b*x) + (B*log(a + b*x))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{12\log(bx+a)\,a^4+48\log(bx+a)\,a^3bx+72\log(bx+a)\,a^2b^2x^2+48\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,}{12b^5\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)}$$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(12*log(a + b*x)*a**4 + 48*log(a + b*x)*a**3*b*x + 72*log(a + b*x)*a**2*b*
*2*x**2 + 48*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 + 13*a**
4 + 40*a**3*b*x + 36*a**2*b**2*x**2 - 12*b**4*x**4)/(12*b**5*(a**4 + 4*a**
3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.275 $\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 57

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3} dx = \frac{(Ab-aB)x^4}{5ab(a+bx)^5} + \frac{(Ab+4aB)x^4}{20a^2b(a+bx)^4}$$

output
```
1/5*(A*b-B*a)*x^4/a/b/(b*x+a)^5+1/20*(A*b+4*B*a)*x^4/a^2/b/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3} dx = -\frac{4a^4B+10b^4x^3(A+2Bx)+10ab^3x^2(A+4Bx)+5a^2b^2x(A+8Bx)+a^3b(A+20Bx)}{20b^5(a+bx)^5}$$

input
```
Integrate[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/20*(4*a^4*B + 10*b^4*x^3*(A + 2*B*x) + 10*a*b^3*x^2*(A + 4*B*x) + 5*a^2
*b^2*x*(A + 8*B*x) + a^3*b*(A + 20*B*x))/(b^5*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{x^3(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(4aB+Ab)\int \frac{x^3}{(a+bx)^5}dx}{5ab}+\frac{x^4(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{48}$$

$$\frac{x^4(4aB+Ab)}{20a^2b(a+bx)^4}+\frac{x^4(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^4)/(5*a*b*(a + b*x)^5) + ((A*b + 4*a*B)*x^4)/(20*a^2*b*(a +
 b*x)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

method	result	size
norman	$\frac{-\frac{B x^4}{b}-\frac{(Ab+4Ba)x^3}{2b^2}-\frac{a(Ab+4Ba)x^2}{2b^3}-\frac{a^2(Ab+4Ba)x}{4b^4}-\frac{a^3(Ab+4Ba)}{20b^5}}{(bx+a)^5}$	85
default	$\frac{a^3(Ab-Ba)}{5b^5(bx+a)^5}+\frac{a(Ab-2Ba)}{b^5(bx+a)^3}-\frac{B}{(bx+a)b^5}-\frac{Ab-4Ba}{2b^5(bx+a)^2}-\frac{a^2(3Ab-4Ba)}{4b^5(bx+a)^4}$	102
risch	$\frac{-\frac{B x^4}{b}-\frac{(Ab+4Ba)x^3}{2b^2}-\frac{a(Ab+4Ba)x^2}{2b^3}-\frac{a^2(Ab+4Ba)x}{4b^4}-\frac{a^3(Ab+4Ba)}{20b^5}}{(bx+a)(b^2x^2+2abx+a^2)^2}$	103
orering	$-\frac{(20b^4Bx^4+10Ab^4x^3+40Bab^3x^3+10Aab^3x^2+40Ba^2b^2x^2+5Aa^2b^2x+20Ba^3bx+Aa^3b+4a^4B)(bx+a)}{20b^5(b^2x^2+2abx+a^2)^3}$	110
gosper	$-\frac{20b^4Bx^4+10Ab^4x^3+40Bab^3x^3+10Aab^3x^2+40Ba^2b^2x^2+5Aa^2b^2x+20Ba^3bx+Aa^3b+4a^4B}{20(bx+a)(b^2x^2+2abx+a^2)^2b^5}$	112
parallelrisch	$-\frac{20b^4Bx^4+10Ab^4x^3+40Bab^3x^3+10Aab^3x^2+40Ba^2b^2x^2+5Aa^2b^2x+20Ba^3bx+Aa^3b+4a^4B}{20(bx+a)(b^2x^2+2abx+a^2)^2b^5}$	112

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-B/b*x^4-1/2*(A*b+4*B*a)/b^2*x^3-1/2*a*(A*b+4*B*a)/b^3*x^2-1/4*a^2*(A*b+4
*B*a)/b^4*x-1/20*a^3*(A*b+4*B*a)/b^5)/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(53) = 106.

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.44

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^4x^4+4\,Ba^4+Aa^3b+10\,(4\,Bab^3+Ab^4)x^3+10\,(4\,Ba^2b^2+Aab^3)x^2+5\,(4\,Ba^3b+Aa^2b^2)x}{20\,(b^{10}x^5+5\,ab^9x^4+10\,a^2b^8x^3+10\,a^3b^7x^2+5\,a^4b^6x+a^5b^5)}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(20*B*b^4*x^4 + 4*B*a^4 + A*a^3*b + 10*(4*B*a*b^3 + A*b^4)*x^3 + 10*
(4*B*a^2*b^2 + A*a*b^3)*x^2 + 5*(4*B*a^3*b + A*a^2*b^2)*x)/(b^10*x^5 + 5*a
*b^9*x^4 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b^6*x + a^5*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(48) = 96.

Time = 0.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-Aa^3b-4Ba^4-20Bb^4x^4+x^3(-10Ab^4-40Bab^3)+x^2(-10Aab^3-40Ba^2b^2)+x(-5Aa^2b^2-20Ba^3b}{20a^5b^5+100a^4b^6x+200a^3b^7x^2+200a^2b^8x^3+100ab^9x^4+20b^{10}x^5}$$

input
```
integrate(x**3*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-A*a**3*b - 4*B*a**4 - 20*B*b**4*x**4 + x**3*(-10*A*b**4 - 40*B*a*b**3) +
 x**2*(-10*A*a*b**3 - 40*B*a**2*b**2) + x*(-5*A*a**2*b**2 - 20*B*a**3*b))/
(20*a**5*b**5 + 100*a**4*b**6*x + 200*a**3*b**7*x**2 + 200*a**2*b**8*x**3
+ 100*a*b**9*x**4 + 20*b**10*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(53) = 106$.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.44

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^4x^4+4\,Ba^4+Aa^3b+10\,(4\,Bab^3+Ab^4)x^3+10\,(4\,Ba^2b^2+Aab^3)x^2+5\,(4\,Ba^3b+Aa^2b^2)x}{20\,(b^{10}x^5+5\,ab^9x^4+10\,a^2b^8x^3+10\,a^3b^7x^2+5\,a^4b^6x+a^5b^5)}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(20*B*b^4*x^4 + 4*B*a^4 + A*a^3*b + 10*(4*B*a*b^3 + A*b^4)*x^3 + 10*
(4*B*a^2*b^2 + A*a*b^3)*x^2 + 5*(4*B*a^3*b + A*a^2*b^2)*x)/(b^10*x^5 + 5*a
*b^9*x^4 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b^6*x + a^5*b^5)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^4x^4+40\,Bab^3x^3+10\,Ab^4x^3+40\,Ba^2b^2x^2+10\,Aab^3x^2+20\,Ba^3bx+5\,Aa^2b^2x+4\,Ba^4+Aa^3b}{20\,(bx+a)^5b^5}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/20*(20*B*b^4*x^4 + 40*B*a*b^3*x^3 + 10*A*b^4*x^3 + 40*B*a^2*b^2*x^2 + 1
0*A*a*b^3*x^2 + 20*B*a^3*b*x + 5*A*a^2*b^2*x + 4*B*a^4 + A*a^3*b)/((b*x +
a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.25

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{B\,x^4}{b}+\frac{a^3\,(A\,b+4\,B\,a)}{20\,b^5}+\frac{x^3\,(A\,b+4\,B\,a)}{2\,b^2}+\frac{a\,x^2\,(A\,b+4\,B\,a)}{2\,b^3}+\frac{a^2\,x\,(A\,b+4\,B\,a)}{4\,b^4}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-((B*x^4)/b + (a^3*(A*b + 4*B*a))/(20*b^5) + (x^3*(A*b + 4*B*a))/(2*b^2) +
 (a*x^2*(A*b + 4*B*a))/(2*b^3) + (a^2*x*(A*b + 4*B*a))/(4*b^4))/(a^5 + b^5
*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{x^4}{4a\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
x**4/(4*a*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**
4))
```

3.276 $\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{a^2(Ab-aB)}{5b^4(a+bx)^5}+\frac{a(2Ab-3aB)}{4b^4(a+bx)^4} - \frac{Ab-3aB}{3b^4(a+bx)^3}-\frac{B}{2b^4(a+bx)^2}$$

output
```
-1/5*a^2*(A*b-B*a)/b^4/(b*x+a)^5+1/4*a*(2*A*b-3*B*a)/b^4/(b*x+a)^4-1/3*(A*
b-3*B*a)/b^4/(b*x+a)^3-1/2*B/b^4/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{3a^3B+10ab^2x(A+3Bx)+10b^3x^2(2A+3Bx)+a^2b(2A+15Bx)}{60b^4(a+bx)^5}$$

input
```
Integrate[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/60*(3*a^3*B + 10*a*b^2*x*(A + 3*B*x) + 10*b^3*x^2*(2*A + 3*B*x) + a^2*b
*(2*A + 15*B*x))/(b^4*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{x^2(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^6}+\frac{a(3aB-2Ab)}{b^3(a+bx)^5}+\frac{Ab-3aB}{b^3(a+bx)^4}+\frac{B}{b^3(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2(Ab-aB)}{5b^4(a+bx)^5}+\frac{a(2Ab-3aB)}{4b^4(a+bx)^4}-\frac{Ab-3aB}{3b^4(a+bx)^3}-\frac{B}{2b^4(a+bx)^2}$$

input
```
Int[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/5*(a^2*(A*b - a*B))/(b^4*(a + b*x)^5) + (a*(2*A*b - 3*a*B))/(4*b^4*(a +
 b*x)^4) - (A*b - 3*a*B)/(3*b^4*(a + b*x)^3) - B/(2*b^4*(a + b*x)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result	size
norman	$\frac{-\frac{B\,x^{3}}{2b}-\frac{\left(2b^{2}A+3abB\right)x^{2}}{6b^{3}}-\frac{a\left(2b^{2}A+3abB\right)x}{12b^{4}}-\frac{a^{2}\left(2b^{2}A+3abB\right)}{60b^{5}}}{\left(bx+a\right)^{5}}$	78
default	$-\frac{a^{2}(Ab-Ba)}{5b^{4}(bx+a)^{5}}+\frac{a(2Ab-3Ba)}{4b^{4}(bx+a)^{4}}-\frac{Ab-3Ba}{3b^{4}(bx+a)^{3}}-\frac{B}{2b^{4}(bx+a)^{2}}$	80
risch	$\frac{-\frac{B\,x^{3}}{2b}-\frac{(2Ab+3Ba)x^{2}}{6b^{2}}-\frac{a(2Ab+3Ba)x}{12b^{3}}-\frac{a^{2}(2Ab+3Ba)}{60b^{4}}}{(bx+a)\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}}$	87
orering	$-\frac{\left(30x^{3}B\,b^{3}+20A\,b^{3}x^{2}+30Ba\,b^{2}x^{2}+10Aa\,b^{2}x+15B\,a^{2}bx+2A\,a^{2}b+3B\,a^{3}\right)(bx+a)}{60b^{4}\left(b^{2}x^{2}+2abx+a^{2}\right)^{3}}$	87
gosper	$-\frac{30x^{3}B\,b^{3}+20A\,b^{3}x^{2}+30Ba\,b^{2}x^{2}+10Aa\,b^{2}x+15B\,a^{2}bx+2A\,a^{2}b+3B\,a^{3}}{60b^{4}(bx+a)\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}}$	89
parallelrisch	$-\frac{30b^{4}B\,x^{3}+20A\,b^{4}x^{2}+30Ba\,b^{3}x^{2}+10aA\,b^{3}x+15B\,a^{2}b^{2}x+2a^{2}A\,b^{2}+3B\,a^{3}b}{60b^{5}\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}(bx+a)}$	94

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*B*x^3/b-1/6*(2*A*b^2+3*B*a*b)/b^3*x^2-1/12*a*(2*A*b^2+3*B*a*b)/b^4*x
-1/60*a^2*(2*A*b^2+3*B*a*b)/b^5)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{30\,Bb^3x^3+3\,Ba^3+2\,Aa^2b+10\,(3\,Bab^2+2\,Ab^3)x^2+5\,(3\,Ba^2b+2\,Aab^2)x}{60\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/60*(30*B*b^3*x^3 + 3*B*a^3 + 2*A*a^2*b + 10*(3*B*a*b^2 + 2*A*b^3)*x^2 +
 5*(3*B*a^2*b + 2*A*a*b^2)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10
*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-2Aa^2b-3Ba^3-30Bb^3x^3+x^2(-20Ab^3-30Bab^2)+x(-10Aab^2-15Ba^2b)}{60a^5b^4+300a^4b^5x+600a^3b^6x^2+600a^2b^7x^3+300ab^8x^4+60b^9x^5}$$

input
```
integrate(x**2*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-2*A*a**2*b - 3*B*a**3 - 30*B*b**3*x**3 + x**2*(-20*A*b**3 - 30*B*a*b**2)
 + x*(-10*A*a*b**2 - 15*B*a**2*b))/(60*a**5*b**4 + 300*a**4*b**5*x + 600*a
**3*b**6*x**2 + 600*a**2*b**7*x**3 + 300*a*b**8*x**4 + 60*b**9*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{30\,Bb^3x^3+3\,Ba^3+2\,Aa^2b+10\,(3\,Bab^2+2\,Ab^3)x^2+5\,(3\,Ba^2b+2\,Aab^2)x}{60\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(30*B*b^3*x^3 + 3*B*a^3 + 2*A*a^2*b + 10*(3*B*a*b^2 + 2*A*b^3)*x^2 +
 5*(3*B*a^2*b + 2*A*a*b^2)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10
*a^3*b^6*x^2 + 5*a^4*b^5*x + a^5*b^4)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{30\,Bb^3x^3+30\,Bab^2x^2+20\,Ab^3x^2+15\,Ba^2bx+10\,Aab^2x+3\,Ba^3+2\,Aa^2b}{60\,(bx+a)^5b^4}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/60*(30*B*b^3*x^3 + 30*B*a*b^2*x^2 + 20*A*b^3*x^2 + 15*B*a^2*b*x + 10*A*
a*b^2*x + 3*B*a^3 + 2*A*a^2*b)/((b*x + a)^5*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{B\,x^3}{2\,b}+\frac{a^2\,(2\,A\,b+3\,B\,a)}{60\,b^4}+\frac{x^2\,(2\,A\,b+3\,B\,a)}{6\,b^2}+\frac{a\,x\,(2\,A\,b+3\,B\,a)}{12\,b^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-((B*x^3)/(2*b) + (a^2*(2*A*b + 3*B*a))/(60*b^4) + (x^2*(2*A*b + 3*B*a))/(
6*b^2) + (a*x*(2*A*b + 3*B*a))/(12*b^3))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10
*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-6b^2x^2-4abx-a^2}{12b^3\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - a**2 - 4*a*b*x - 6*b**2*x**2)/(12*b**3*(a**4 + 4*a**3*b*x + 6*a**2*b**
2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.277 $\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3} \, dx = \frac{a(Ab-aB)}{5b^3(a+bx)^5} - \frac{Ab-2aB}{4b^3(a+bx)^4} - \frac{B}{3b^3(a+bx)^3}$$

output
```
1/5*a*(A*b-B*a)/b^3/(b*x+a)^5-1/4*(A*b-2*B*a)/b^3/(b*x+a)^4-1/3*B/b^3/(b*x
+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3} \, dx = -\frac{2a^2B+5b^2x(3A+4Bx)+ab(3A+10Bx)}{60b^3(a+bx)^5}$$

input
```
Integrate[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/60*(2*a^2*B + 5*b^2*x*(3*A + 4*B*x) + a*b*(3*A + 10*B*x))/(b^3*(a + b*x
)^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^6\int \frac{x(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)^5}+\frac{a(aB-Ab)}{b^2(a+bx)^6}+\frac{B}{b^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab-2aB}{4b^3(a+bx)^4}+\frac{a(Ab-aB)}{5b^3(a+bx)^5}-\frac{B}{3b^3(a+bx)^3}$$

input
```
Int[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a*(A*b - a*B))/(5*b^3*(a + b*x)^5) - (A*b - 2*a*B)/(4*b^3*(a + b*x)^4) -
B/(3*b^3*(a + b*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{a(Ab-Ba)}{5b^3(bx+a)^5}-\frac{Ab-2Ba}{4b^3(bx+a)^4}-\frac{B}{3b^3(bx+a)^3}$	56
norman	$\frac{-\frac{B\,x^2}{3b}-\frac{\left(3A\,b^3+2Ba\,b^2\right)x}{12b^4}-\frac{a\left(3A\,b^3+2Ba\,b^2\right)}{60b^5}}{(bx+a)^5}$	59
orering	$-\frac{\left(20x^2B\,b^2+15x\,b^2A+10xabB+3abA+2a^2B\right)(bx+a)}{60b^3(b^2x^2+2abx+a^2)^3}$	63
gosper	$-\frac{20x^2B\,b^2+15x\,b^2A+10xabB+3abA+2a^2B}{60b^3(bx+a)(b^2x^2+2abx+a^2)^2}$	65
risch	$\frac{-\frac{B\,x^2}{3b}-\frac{(3Ab+2Ba)x}{12b^2}-\frac{a(3Ab+2Ba)}{60b^3}}{(bx+a)(b^2x^2+2abx+a^2)^2}$	67
parallelrisch	$-\frac{20B\,b^4x^2+15A\,b^4x+10Ba\,b^3x+3Aa\,b^3+2B\,a^2b^2}{60b^5(b^2x^2+2abx+a^2)^2(bx+a)}$	72

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a*(A*b-B*a)/b^3/(b*x+a)^5-1/4*(A*b-2*B*a)/b^3/(b*x+a)^4-1/3*B/b^3/(b*x
+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^2x^2+2\,Ba^2+3\,Aab+5\,(2\,Bab+3\,Ab^2)x}{60\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/60*(20*B*b^2*x^2 + 2*B*a^2 + 3*A*a*b + 5*(2*B*a*b + 3*A*b^2)*x)/(b^8*x^
5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{-3Aab-2Ba^2-20Bb^2x^2+x(-15Ab^2-10Bab)}{60a^5b^3+300a^4b^4x+600a^3b^5x^2+600a^2b^6x^3+300ab^7x^4+60b^8x^5}$$

input
```
integrate(x*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-3*A*a*b - 2*B*a**2 - 20*B*b**2*x**2 + x*(-15*A*b**2 - 10*B*a*b))/(60*a**
5*b**3 + 300*a**4*b**4*x + 600*a**3*b**5*x**2 + 600*a**2*b**6*x**3 + 300*a
*b**7*x**4 + 60*b**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^2x^2+2\,Ba^2+3\,Aab+5\,(2\,Bab+3\,Ab^2)x}{60\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(20*B*b^2*x^2 + 2*B*a^2 + 3*A*a*b + 5*(2*B*a*b + 3*A*b^2)*x)/(b^8*x^
5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{20\,Bb^2x^2+10\,Babx+15\,Ab^2x+2\,Ba^2+3\,Aab}{60\,(bx+a)^5b^3}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/60*(20*B*b^2*x^2 + 10*B*a*b*x + 15*A*b^2*x + 2*B*a^2 + 3*A*a*b)/((b*x +
 a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{B\,x^2}{3\,b}+\frac{a\,(3\,A\,b+2\,B\,a)}{60\,b^3}+\frac{x\,(3\,A\,b+2\,B\,a)}{12\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-((B*x^2)/(3*b) + (a*(3*A*b + 2*B*a))/(60*b^3) + (x*(3*A*b + 2*B*a))/(12*b
^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{-4bx-a}{12b^2\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)}$$

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - a - 4*b*x)/(12*b**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x
**3 + b**4*x**4))
```

3.278 $\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx=\frac{-Ab+aB}{5b^2(a+bx)^5}-\frac{B}{4b^2(a+bx)^4}$$

output
```
1/5*(-A*b+B*a)/b^2/(b*x+a)^5-1/4*B/b^2/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx=-\frac{4Ab+B(a+5bx)}{20b^2(a+bx)^5}$$

input
```
Integrate[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/20*(4*A*b + B*(a + 5*b*x))/(b^2*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1098, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{1098}$$

$$b^6\int \frac{A+Bx}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{(a+bx)^6}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^6}+\frac{B}{b(a+bx)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab-aB}{5b^2(a+bx)^5}-\frac{B}{4b^2(a+bx)^4}$$

input
```
Int[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/5*(A*b - a*B)/(b^2*(a + b*x)^5) - B/(4*b^2*(a + b*x)^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{Ab-Ba}{5b^{2}\left(bx+a\right)^{5}}-\frac{B}{4b^{2}\left(bx+a\right)^{4}}$	35
norman	$\frac{-\frac{Bx}{4b}-\frac{4A\,b^{4}+Ba\,b^{3}}{20b^{5}}}{\left(bx+a\right)^{5}}$	35
orering	$-\frac{\left(5Bbx+4Ab+Ba\right)\left(bx+a\right)}{20b^{2}\left(b^{2}x^{2}+2abx+a^{2}\right)^{3}}$	42
gosper	$-\frac{5Bbx+4Ab+Ba}{20b^{2}\left(bx+a\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}}$	44
risch	$\frac{-\frac{Bx}{4b}-\frac{4Ab+Ba}{20b^{2}}}{\left(bx+a\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}}$	48
parallelrisch	$-\frac{5B\,b^{4}x+4A\,b^{4}+Ba\,b^{3}}{20b^{5}\left(b^{2}x^{2}+2abx+a^{2}\right)^{2}\left(bx+a\right)}$	51

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(A*b-B*a)/b^2/(b*x+a)^5-1/4*B/b^2/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(34) = 68.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{5\,Bbx+Ba+4\,Ab}{20\,\left(b^7x^5+5\,ab^6x^4+10\,a^2b^5x^3+10\,a^3b^4x^2+5\,a^4b^3x+a^5b^2\right)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(5*B*b*x + B*a + 4*A*b)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10
*a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{-4Ab-Ba-5Bbx}{20a^5b^2+100a^4b^3x+200a^3b^4x^2+200a^2b^5x^3+100ab^6x^4+20b^7x^5}$$

input
```
integrate((B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-4*A*b - B*a - 5*B*b*x)/(20*a**5*b**2 + 100*a**4*b**3*x + 200*a**3*b**4*x
**2 + 200*a**2*b**5*x**3 + 100*a*b**6*x**4 + 20*b**7*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{5\,Bbx+Ba+4\,Ab}{20\,(b^7x^5+5\,ab^6x^4+10\,a^2b^5x^3+10\,a^3b^4x^2+5\,a^4b^3x+a^5b^2)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(5*B*b*x + B*a + 4*A*b)/(b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10
*a^3*b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{5\,Bbx+Ba+4\,Ab}{20\,(bx+a)^5b^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-1/20*(5*B*b*x + B*a + 4*A*b)/((b*x + a)^5*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{\frac{4\,A\,b+B\,a}{20\,b^2}+\frac{B\,x}{4\,b}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((A + B*x)/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
-((4*A*b + B*a)/(20*b^2) + (B*x)/(4*b))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*
a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{1}{4b\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 1)/(4*b*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x
**4))
```

3.279 $\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{Ab-aB}{5ab(a+bx)^5}+\frac{A}{4a^2(a+bx)^4}+\frac{A}{3a^3(a+bx)^3}+\frac{A}{2a^4(a+bx)^2}+\frac{A}{a^5(a+bx)}+\frac{A\log(x)}{a^6}-\frac{A\log(a+bx)}{a^6}$$

output
```
1/5*(A*b-B*a)/a/b/(b*x+a)^5+1/4*A/a^2/(b*x+a)^4+1/3*A/a^3/(b*x+a)^3+1/2*A/
a^4/(b*x+a)^2+A/a^5/(b*x+a)+A*ln(x)/a^6-A*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{a\left(137a^4Ab-12a^5B+385a^3Ab^2x+470a^2Ab^3x^2+270aAb^4x^3+60Ab^5x^4\right)}{b(a+bx)^5}+60A\log(x)-60A\log(a+bx)}{60a^6}$$

input
```
Integrate[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((a*(137*a^4*A*b - 12*a^5*B + 385*a^3*A*b^2*x + 470*a^2*A*b^3*x^2 + 270*a*
A*b^4*x^3 + 60*A*b^5*x^4))/(b*(a + b*x)^5) + 60*A*Log[x] - 60*A*Log[a + b*
x])/(60*a^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{A+Bx}{b^6x(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x(a+bx)^6}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{Ab}{a^6(a+bx)}+\frac{A}{a^6x}-\frac{Ab}{a^5(a+bx)^2}-\frac{Ab}{a^4(a+bx)^3}-\frac{Ab}{a^3(a+bx)^4}-\frac{Ab}{a^2(a+bx)^5}+\frac{aB-Ab}{a(a+bx)^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\log(a+bx)}{a^6}+\frac{A\log(x)}{a^6}+\frac{A}{a^5(a+bx)}+\frac{A}{2a^4(a+bx)^2}+\frac{A}{3a^3(a+bx)^3}+\frac{A}{4a^2(a+bx)^4}+\frac{Ab-aB}{5ab(a+bx)^5}$$

input
```
Int[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output

```
(A*b - a*B)/(5*a*b*(a + b*x)^5) + A/(4*a^2*(a + b*x)^4) + A/(3*a^3*(a + b*
x)^3) + A/(2*a^4*(a + b*x)^2) + A/(a^5*(a + b*x)) + (A*Log[x])/a^6 - (A*Lo
g[a + b*x])/a^6
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result
default	$-\frac{-Ab+Ba}{5ab(bx+a)^{5}}-\frac{A\ln(bx+a)}{a^{6}}+\frac{A}{a^{5}(bx+a)}+\frac{A}{2a^{4}(bx+a)^{2}}+\frac{A}{3a^{3}(bx+a)^{3}}+\frac{A}{4a^{2}(bx+a)^{4}}+\frac{A\ln(x)}{a^{6}}$
risch	$\frac{\frac{b^{4}A\,x^{4}}{a^{5}}+\frac{9b^{3}A\,x^{3}}{2a^{4}}+\frac{47b^{2}A\,x^{2}}{6a^{3}}+\frac{77xAb}{12a^{2}}+\frac{137Ab-12Ba}{60ab}}{(bx+a)(b^{2}x^{2}+2abx+a^{2})^{2}}-\frac{A\ln(bx+a)}{a^{6}}+\frac{A\ln(-x)}{a^{6}}$
norman	$\frac{-\frac{(5Ab-Ba)x}{a^{2}}-\frac{b(15Ab-2Ba)x^{2}}{a^{3}}-\frac{b^{2}(55Ab-6Ba)x^{3}}{3a^{4}}-\frac{b^{3}(125Ab-12Ba)x^{4}}{12a^{5}}-\frac{b^{4}(137Ab-12Ba)x^{5}}{60a^{6}}}{(bx+a)^{5}}+\frac{A\ln(x)}{a^{6}}-\frac{A\ln(bx+a)}{a^{6}}$
parallelrisch	$-625Aa\,b^{4}x^{4}+120B\,a^{4}b\,x^{2}-300A\,a^{4}bx-300A\ln(bx+a)x\,a^{4}b+300A\ln(x)x\,a^{4}b+12Ba\,b^{4}x^{5}+300A\ln(x)x^{4}a\,b^{4}-300A\ln(bx$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-A*b+B*a)/a/b/(b*x+a)^5-A*ln(b*x+a)/a^6+A/a^5/(b*x+a)+1/2*A/a^4/(b*x
+a)^2+1/3*A/a^3/(b*x+a)^3+1/4*A/a^2/(b*x+a)^4+A*ln(x)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(94) = 188.

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{60\,Aab^5x^4+270\,Aa^2b^4x^3+470\,Aa^3b^3x^2+385\,Aa^4b^2x-12\,Ba^6+137\,Aa^5b-60\left(Ab^6x^5+5\,Aab^5x^4+\right.}{60\left(a^6b^6x^5+5\,a\right.}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
1/60*(60*A*a*b^5*x^4 + 270*A*a^2*b^4*x^3 + 470*A*a^3*b^3*x^2 + 385*A*a^4*b
^2*x - 12*B*a^6 + 137*A*a^5*b - 60*(A*b^6*x^5 + 5*A*a*b^5*x^4 + 10*A*a^2*b
^4*x^3 + 10*A*a^3*b^3*x^2 + 5*A*a^4*b^2*x + A*a^5*b)*log(b*x + a) + 60*(A*
b^6*x^5 + 5*A*a*b^5*x^4 + 10*A*a^2*b^4*x^3 + 10*A*a^3*b^3*x^2 + 5*A*a^4*b^
2*x + A*a^5*b)*log(x))/(a^6*b^6*x^5 + 5*a^7*b^5*x^4 + 10*a^8*b^4*x^3 + 10*
a^9*b^3*x^2 + 5*a^10*b^2*x + a^11*b)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{A\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6} + \frac{137Aa^4b+385Aa^3b^2x+470Aa^2b^3x^2+270Aab^4x^3+60Ab^5x^4-12Ba^5}{60a^{10}b+300a^9b^2x+600a^8b^3x^2+600a^7b^4x^3+300a^6b^5x^4+60a^5b^6x^5}$$

input
```
integrate((B*x+A)/x/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
A*(log(x) - log(a/b + x))/a**6 + (137*A*a**4*b + 385*A*a**3*b**2*x + 470*A
*a**2*b**3*x**2 + 270*A*a*b**4*x**3 + 60*A*b**5*x**4 - 12*B*a**5)/(60*a**1
0*b + 300*a**9*b**2*x + 600*a**8*b**3*x**2 + 600*a**7*b**4*x**3 + 300*a**6
*b**5*x**4 + 60*a**5*b**6*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{60\,Ab^5x^4+270\,Aab^4x^3+470\,Aa^2b^3x^2+385\,Aa^3b^2x-12\,Ba^5+137\,Aa^4b}{60\,\left(a^5b^6x^5+5\,a^6b^5x^4+10\,a^7b^4x^3+10\,a^8b^3x^2+5\,a^9b^2x+a^{10}b\right)} - \frac{A\log\left(bx+a\right)}{a^6}+\frac{A\log\left(x\right)}{a^6}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/60*(60*A*b^5*x^4 + 270*A*a*b^4*x^3 + 470*A*a^2*b^3*x^2 + 385*A*a^3*b^2*x
 - 12*B*a^5 + 137*A*a^4*b)/(a^5*b^6*x^5 + 5*a^6*b^5*x^4 + 10*a^7*b^4*x^3 +
 10*a^8*b^3*x^2 + 5*a^9*b^2*x + a^10*b) - A*log(b*x + a)/a^6 + A*log(x)/a^
6
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{A\log\left(|bx+a|\right)}{a^6}+\frac{A\log\left(|x|\right)}{a^6} + \frac{60\,Aab^5x^4+270\,Aa^2b^4x^3+470\,Aa^3b^3x^2+385\,Aa^4b^2x-12\,Ba^6+137\,Aa^5b}{60\,\left(bx+a\right)^5a^6b}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/a^6 + A*log(abs(x))/a^6 + 1/60*(60*A*a*b^5*x^4 + 270*
A*a^2*b^4*x^3 + 470*A*a^3*b^3*x^2 + 385*A*a^4*b^2*x - 12*B*a^6 + 137*A*a^5
*b)/((b*x + a)^5*a^6*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{137\,A\,b-12\,B\,a}{60\,a\,b}+\frac{77\,A\,b\,x}{12\,a^2}+\frac{47\,A\,b^2\,x^2}{6\,a^3}+\frac{9\,A\,b^3\,x^3}{2\,a^4}+\frac{A\,b^4\,x^4}{a^5}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5} - \frac{2\,A\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
((137*A*b - 12*B*a)/(60*a*b) + (77*A*b*x)/(12*a^2) + (47*A*b^2*x^2)/(6*a^3
) + (9*A*b^3*x^3)/(2*a^4) + (A*b^4*x^4)/a^5)/(a^5 + b^5*x^5 + 5*a*b^4*x^4
+ 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x) - (2*A*atanh((2*b*x)/a + 1)
)/a^6
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{-12\log(bx+a)\,a^4-48\log(bx+a)\,a^3bx-72\log(bx+a)\,a^2b^2x^2-48\log(bx+a)\,a\,b^3x^3-12\log(bx+a}{12a^5\,(b^4x^4}$$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 12*log(a + b*x)*a**4 - 48*log(a + b*x)*a**3*b*x - 72*log(a + b*x)*a**2
*b**2*x**2 - 48*log(a + b*x)*a*b**3*x**3 - 12*log(a + b*x)*b**4*x**4 + 12*
log(x)*a**4 + 48*log(x)*a**3*b*x + 72*log(x)*a**2*b**2*x**2 + 48*log(x)*a*
b**3*x**3 + 12*log(x)*b**4*x**4 + 22*a**4 + 40*a**3*b*x + 24*a**2*b**2*x**
2 - 3*b**4*x**4)/(12*a**5*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3
*x**3 + b**4*x**4))
```

3.280 $\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 157

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^3}\,dx = -\frac{A}{a^6x}-\frac{Ab-aB}{5a^2(a+bx)^5}-\frac{2Ab-aB}{4a^3(a+bx)^4}-\frac{3Ab-aB}{3a^4(a+bx)^3}-\frac{4Ab-aB}{2a^5(a+bx)^2}-\frac{5Ab-aB}{a^6(a+bx)}-\frac{(6Ab-aB)\log(x)}{a^7}+\frac{(6Ab-aB)\log(a+bx)}{a^7}$$

output
```
-A/a^6/x-1/5*(A*b-B*a)/a^2/(b*x+a)^5-1/4*(2*A*b-B*a)/a^3/(b*x+a)^4-1/3*(3*
A*b-B*a)/a^4/(b*x+a)^3-1/2*(4*A*b-B*a)/a^5/(b*x+a)^2-(5*A*b-B*a)/a^6/(b*x+
a)-(6*A*b-B*a)*ln(x)/a^7+(6*A*b-B*a)*ln(b*x+a)/a^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^3}\,dx = \frac{-\frac{60aA}{x}+\frac{12a^5(-Ab+aB)}{(a+bx)^5}+\frac{15a^4(-2Ab+aB)}{(a+bx)^4}+\frac{20a^3(-3Ab+aB)}{(a+bx)^3}+\frac{30a^2(-4Ab+aB)}{(a+bx)^2}+\frac{60a(-5Ab+aB)}{a+bx}+60(-6Ab+aB)}{60a^7}$$

input
```
Integrate[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((-60*a*A)/x + (12*a^5*(-(A*b) + a*B))/(a + b*x)^5 + (15*a^4*(-2*A*b + a*B
))/(a + b*x)^4 + (20*a^3*(-3*A*b + a*B))/(a + b*x)^3 + (30*a^2*(-4*A*b + a
*B))/(a + b*x)^2 + (60*a*(-5*A*b + a*B))/(a + b*x) + 60*(-6*A*b + a*B)*Log
[x] + 60*(6*A*b - a*B)*Log[a + b*x])/(60*a^7)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^3}dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{A+Bx}{b^6x^2(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^2(a+bx)^6}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{aB-6Ab}{a^7x}-\frac{b(aB-6Ab)}{a^7(a+bx)}-\frac{b(aB-5Ab)}{a^6(a+bx)^2}+\frac{A}{a^6x^2}-\frac{b(aB-4Ab)}{a^5(a+bx)^3}-\frac{b(aB-3Ab)}{a^4(a+bx)^4}-\frac{b(aB-2Ab)}{a^3(a+bx)^5}-\frac{b(aB}{a^2(a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(x)(6Ab-aB)}{a^7}+\frac{(6Ab-aB)\log(a+bx)}{a^7}-\frac{5Ab-aB}{a^6(a+bx)}-\frac{A}{a^6x}-\frac{4Ab-aB}{2a^5(a+bx)^2}-\frac{3Ab-aB}{3a^4(a+bx)^3}-\frac{2Ab-aB}{4a^3(a+bx)^4}-\frac{Ab-aB}{5a^2(a+bx)^5}$$

input
```
Int[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
-(A/(a^6*x)) - (A*b - a*B)/(5*a^2*(a + b*x)^5) - (2*A*b - a*B)/(4*a^3*(a +
 b*x)^4) - (3*A*b - a*B)/(3*a^4*(a + b*x)^3) - (4*A*b - a*B)/(2*a^5*(a + b
*x)^2) - (5*A*b - a*B)/(a^6*(a + b*x)) - ((6*A*b - a*B)*Log[x])/a^7 + ((6*
A*b - a*B)*Log[a + b*x])/a^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2Ab-Ba}{4a^3(bx+a)^4}-\frac{3Ab-Ba}{3a^4(bx+a)^3}-\frac{4Ab-Ba}{2a^5(bx+a)^2}-\frac{5Ab-Ba}{a^6(bx+a)}+\frac{(6Ab-Ba)\ln(bx+a)}{a^7}-\frac{Ab-Ba}{5a^2(bx+a)^5}-\frac{A}{a^6x}+\frac{(-6Ab}{}$
norman	$\frac{-\frac{A}{a}+\frac{5b(6Ab-Ba)x^2}{a^3}+\frac{5b^2(18Ab-3Ba)x^3}{a^4}+\frac{5b^3(66Ab-11Ba)x^4}{3a^5}+\frac{5b^4(150Ab-25Ba)x^5}{12a^6}+\frac{b^5(822Ab-137Ba)x^6}{60a^7}}{x(bx+a)^5}+\frac{(6Ab-Ba)\ln(bx}{a^7}$
risch	$\frac{-\frac{b^4(6Ab-Ba)x^5}{a^6}-\frac{9b^3(6Ab-Ba)x^4}{2a^5}-\frac{47b^2(6Ab-Ba)x^3}{6a^4}-\frac{77b(6Ab-Ba)x^2}{12a^3}-\frac{137(6Ab-Ba)x}{60a^2}-\frac{A}{a}}{x(b^2x^2+2abx+a^2)^2(bx+a)}+\frac{6\ln(-bx-a)Ab}{a^7}-\frac{\ln(-bx-a}{a^6}$
parallelrisch	$-\frac{60B\ln(bx+a)x^6ab^5-1800A\ln(bx+a)x^5ab^5+300B\ln(bx+a)x^5a^2b^4-3600A\ln(bx+a)x^4a^2b^4+600B\ln(bx+a)x^4a^3b^3-36}{}$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*A*b-B*a)/a^3/(b*x+a)^4-1/3*(3*A*b-B*a)/a^4/(b*x+a)^3-1/2*(4*A*b-B*
a)/a^5/(b*x+a)^2-(5*A*b-B*a)/a^6/(b*x+a)+(6*A*b-B*a)*ln(b*x+a)/a^7-1/5*(A*
b-B*a)/a^2/(b*x+a)^5-A/a^6/x+(-6*A*b+B*a)/a^7*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(142) = 284.

Time = 0.08 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.72

$$\int \frac{A+Bx}{x^2\,(a^2+2abx+b^2x^2)^3}\,dx = -\frac{60\,Aa^6-60\,(Ba^2b^4-6\,Aab^5)x^5-270\,(Ba^3b^3-6\,Aa^2b^4)x^4-470\,(Ba^4b^2-6\,Aa^3b^3)x^3-385\,(Ba^5b}{}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/60*(60*A*a^6 - 60*(B*a^2*b^4 - 6*A*a*b^5)*x^5 - 270*(B*a^3*b^3 - 6*A*a^
2*b^4)*x^4 - 470*(B*a^4*b^2 - 6*A*a^3*b^3)*x^3 - 385*(B*a^5*b - 6*A*a^4*b^
2)*x^2 - 137*(B*a^6 - 6*A*a^5*b)*x + 60*((B*a*b^5 - 6*A*b^6)*x^6 + 5*(B*a^
2*b^4 - 6*A*a*b^5)*x^5 + 10*(B*a^3*b^3 - 6*A*a^2*b^4)*x^4 + 10*(B*a^4*b^2
- 6*A*a^3*b^3)*x^3 + 5*(B*a^5*b - 6*A*a^4*b^2)*x^2 + (B*a^6 - 6*A*a^5*b)*x
)*log(b*x + a) - 60*((B*a*b^5 - 6*A*b^6)*x^6 + 5*(B*a^2*b^4 - 6*A*a*b^5)*x
^5 + 10*(B*a^3*b^3 - 6*A*a^2*b^4)*x^4 + 10*(B*a^4*b^2 - 6*A*a^3*b^3)*x^3 +
 5*(B*a^5*b - 6*A*a^4*b^2)*x^2 + (B*a^6 - 6*A*a^5*b)*x)*log(x))/(a^7*b^5*x
^6 + 5*a^8*b^4*x^5 + 10*a^9*b^3*x^4 + 10*a^10*b^2*x^3 + 5*a^11*b*x^2 + a^1
2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(136) = 272$.

Time = 0.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^3}\,dx = \frac{-60Aa^5+x^5(-360Ab^5+60Bab^4)+x^4(-1620Aab^4+270Ba^2b^3)+x^3(-2820Aa^2b^3+470Ba^3b^2)+x^2(\cdots}{60a^{11}x+300a^{10}bx^2+600a^9b^2x^3+600a^8b^3x^4+300a^7b^4x^5+\cdots} + \frac{(-6Ab+Ba)\log\left(x+\frac{-6Aab+Ba^2-a(-6Ab+Ba)}{-12Ab^2+2Bab}\right)}{a^7} - \frac{(-6Ab+Ba)\log\left(x+\frac{-6Aab+Ba^2+a(-6Ab+Ba)}{-12Ab^2+2Bab}\right)}{a^7}$$

input
```
integrate((B*x+A)/x**2/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-60*A*a**5 + x**5*(-360*A*b**5 + 60*B*a*b**4) + x**4*(-1620*A*a*b**4 + 27
0*B*a**2*b**3) + x**3*(-2820*A*a**2*b**3 + 470*B*a**3*b**2) + x**2*(-2310*
A*a**3*b**2 + 385*B*a**4*b) + x*(-822*A*a**4*b + 137*B*a**5))/(60*a**11*x
+ 300*a**10*b*x**2 + 600*a**9*b**2*x**3 + 600*a**8*b**3*x**4 + 300*a**7*b*
*4*x**5 + 60*a**6*b**5*x**6) + (-6*A*b + B*a)*log(x + (-6*A*a*b + B*a**2 -
 a*(-6*A*b + B*a))/(-12*A*b**2 + 2*B*a*b))/a**7 - (-6*A*b + B*a)*log(x + (
-6*A*a*b + B*a**2 + a*(-6*A*b + B*a))/(-12*A*b**2 + 2*B*a*b))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{60\,Aa^5-60\,(Bab^4-6\,Ab^5)x^5-270\,(Ba^2b^3-6\,Aab^4)x^4-470\,(Ba^3b^2-6\,Aa^2b^3)x^3-385\,(Ba^4b-6}{60\,(a^6b^5x^6+5\,a^7b^4x^5+10\,a^8b^3x^4+10\,a^9b^2x^3+5\,a^{10}bx^2+a^{11}x)} - \frac{(Ba-6\,Ab)\log(bx+a)}{a^7}+\frac{(Ba-6\,Ab)\log(x)}{a^7}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(60*A*a^5 - 60*(B*a*b^4 - 6*A*b^5)*x^5 - 270*(B*a^2*b^3 - 6*A*a*b^4)
*x^4 - 470*(B*a^3*b^2 - 6*A*a^2*b^3)*x^3 - 385*(B*a^4*b - 6*A*a^3*b^2)*x^2
 - 137*(B*a^5 - 6*A*a^4*b)*x)/(a^6*b^5*x^6 + 5*a^7*b^4*x^5 + 10*a^8*b^3*x^
4 + 10*a^9*b^2*x^3 + 5*a^10*b*x^2 + a^11*x) - (B*a - 6*A*b)*log(b*x + a)/a
^7 + (B*a - 6*A*b)*log(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{(Ba-6\,Ab)\log(|x|)}{a^7}-\frac{(Bab-6\,Ab^2)\log(|bx+a|)}{a^7b} - \frac{60\,Aa^6-60\,(Ba^2b^4-6\,Aab^5)x^5-270\,(Ba^3b^3-6\,Aa^2b^4)x^4-470\,(Ba^4b^2-6\,Aa^3b^3)x^3-385\,(Ba^5b}{60\,(bx+a)^5a^7x}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
(B*a - 6*A*b)*log(abs(x))/a^7 - (B*a*b - 6*A*b^2)*log(abs(b*x + a))/(a^7*b
) - 1/60*(60*A*a^6 - 60*(B*a^2*b^4 - 6*A*a*b^5)*x^5 - 270*(B*a^3*b^3 - 6*A
*a^2*b^4)*x^4 - 470*(B*a^4*b^2 - 6*A*a^3*b^3)*x^3 - 385*(B*a^5*b - 6*A*a^4
*b^2)*x^2 - 137*(B*a^6 - 6*A*a^5*b)*x)/((b*x + a)^5*a^7*x)
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,\left(6\,A\,b-B\,a\right)}{a^7} - \frac{\frac{A}{a}+\frac{137\,x\,\left(6\,A\,b-B\,a\right)}{60\,a^2}+\frac{47\,b^2\,x^3\,\left(6\,A\,b-B\,a\right)}{6\,a^4}+\frac{9\,b^3\,x^4\,\left(6\,A\,b-B\,a\right)}{2\,a^5}+\frac{b^4\,x^5\,\left(6\,A\,b-B\,a\right)}{a^6}+\frac{77\,b\,x^2\,\left(6\,A\,b-B\,a\right)}{12\,a^3}}{a^5\,x+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^3+10\,a^2\,b^3\,x^4+5\,a\,b^4\,x^5+b^5\,x^6}$$

input
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(6*A*b - B*a))/a^7 - (A/a + (137*x*(6*A*b - B*a))/
(60*a^2) + (47*b^2*x^3*(6*A*b - B*a))/(6*a^4) + (9*b^3*x^4*(6*A*b - B*a))/
(2*a^5) + (b^4*x^5*(6*A*b - B*a))/a^6 + (77*b*x^2*(6*A*b - B*a))/(12*a^3))
/(a^5*x + b^5*x^6 + 5*a^4*b*x^2 + 5*a*b^4*x^5 + 10*a^3*b^2*x^3 + 10*a^2*b^
3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$= \frac{60\log(bx+a)\,a^4bx+240\log(bx+a)\,a^3b^2x^2+360\log(bx+a)\,a^2b^3x^3+240\log(bx+a)\,a\,b^4x^4+60\log(}{}$$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(60*log(a + b*x)*a**4*b*x + 240*log(a + b*x)*a**3*b**2*x**2 + 360*log(a +
b*x)*a**2*b**3*x**3 + 240*log(a + b*x)*a*b**4*x**4 + 60*log(a + b*x)*b**5*
x**5 - 60*log(x)*a**4*b*x - 240*log(x)*a**3*b**2*x**2 - 360*log(x)*a**2*b*
*3*x**3 - 240*log(x)*a*b**4*x**4 - 60*log(x)*b**5*x**5 - 12*a**5 - 110*a**
4*b*x - 200*a**3*b**2*x**2 - 120*a**2*b**3*x**3 + 15*b**5*x**5)/(12*a**6*x
*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.281 $\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^3}\,dx = -\frac{A}{2a^6x^2}+\frac{6Ab-aB}{a^7x}+\frac{b(Ab-aB)}{5a^3(a+bx)^5}+\frac{b(3Ab-2aB)}{4a^4(a+bx)^4}+\frac{b(2Ab-aB)}{a^5(a+bx)^3}+\frac{b(5Ab-2aB)}{a^6(a+bx)^2}+\frac{5b(3Ab-aB)}{a^7(a+bx)}+\frac{3b(7Ab-2aB)\log(x)}{a^8}-\frac{3b(7Ab-2aB)\log(a+bx)}{a^8}$$

output
```
-1/2*A/a^6/x^2+(6*A*b-B*a)/a^7/x+1/5*b*(A*b-B*a)/a^3/(b*x+a)^5+1/4*b*(3*A*
b-2*B*a)/a^4/(b*x+a)^4+b*(2*A*b-B*a)/a^5/(b*x+a)^3+b*(5*A*b-2*B*a)/a^6/(b*
x+a)^2+5*b*(3*A*b-B*a)/a^7/(b*x+a)+3*b*(7*A*b-2*B*a)*ln(x)/a^8-3*b*(7*A*b-
2*B*a)*ln(b*x+a)/a^8
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{a(420Ab^6x^6+5a^3b^3x^3(539A-188Bx)+2a^5bx(35A-137Bx)+7a^4b^2x^2(137A-110Bx)+10a^2b^4x^4(329A-54Bx)+30ab^5x^5(63A-4Bx)-10a^6(A}{x^2(a+bx)^5}}{20a^8}$$

input
```
Integrate[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((a*(420*A*b^6*x^6 + 5*a^3*b^3*x^3*(539*A - 188*B*x) + 2*a^5*b*x*(35*A - 1
37*B*x) + 7*a^4*b^2*x^2*(137*A - 110*B*x) + 10*a^2*b^4*x^4*(329*A - 54*B*x
) + 30*a*b^5*x^5*(63*A - 4*B*x) - 10*a^6*(A + 2*B*x)))/(x^2*(a + b*x)^5) +
 60*b*(7*A*b - 2*a*B)*Log[x] + 60*b*(-7*A*b + 2*a*B)*Log[a + b*x])/(20*a^8
)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{A+Bx}{b^6x^3(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^3(a+bx)^6}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{3b^2(2aB-7Ab)}{a^8(a+bx)} - \frac{3b(2aB-7Ab)}{a^8x} + \frac{5b^2(aB-3Ab)}{a^7(a+bx)^2} + \frac{aB-6Ab}{a^7x^2} + \frac{2b^2(2aB-5Ab)}{a^6(a+bx)^3} + \frac{A}{a^6x^3} + \frac{3b^2(aB-}{a^5(a+b}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b\log(x)(7Ab-2aB)}{a^8} - \frac{3b(7Ab-2aB)\log(a+bx)}{a^8} + \frac{6Ab-aB}{a^7x} + \frac{5b(3Ab-aB)}{a^7(a+bx)} + \frac{b(5Ab-2aB)}{a^6(a+bx)^2} - \frac{A}{2a^6x^2} + \frac{b(2Ab-aB)}{a^5(a+bx)^3} + \frac{b(3Ab-2aB)}{4a^4(a+bx)^4} + \frac{b(Ab-aB)}{5a^3(a+bx)^5}$$

input
```
Int[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
-1/2*A/(a^6*x^2) + (6*A*b - a*B)/(a^7*x) + (b*(A*b - a*B))/(5*a^3*(a + b*x
)^5) + (b*(3*A*b - 2*a*B))/(4*a^4*(a + b*x)^4) + (b*(2*A*b - a*B))/(a^5*(a
 + b*x)^3) + (b*(5*A*b - 2*a*B))/(a^6*(a + b*x)^2) + (5*b*(3*A*b - a*B))/(
a^7*(a + b*x)) + (3*b*(7*A*b - 2*a*B)*Log[x])/a^8 - (3*b*(7*A*b - 2*a*B)*L
og[a + b*x])/a^8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

method	result
default	$\frac{b(2Ab-Ba)}{a^5(bx+a)^3}+\frac{b(3Ab-2Ba)}{4a^4(bx+a)^4}+\frac{5b(3Ab-Ba)}{a^7(bx+a)}+\frac{b(5Ab-2Ba)}{a^6(bx+a)^2}-\frac{3b(7Ab-2Ba)\ln(bx+a)}{a^8}+\frac{b(Ab-Ba)}{5a^3(bx+a)^5}-\frac{A}{2a^6x^2}-$
norman	$\frac{-\frac{A}{2a}+\frac{(7Ab-2Ba)x}{2a^2}-\frac{5b\left(21b^2A-6abB\right)x^3}{a^4}-\frac{5b^2\left(63b^2A-18abB\right)x^4}{a^5}-\frac{5b^3\left(77b^2A-22abB\right)x^5}{a^6}-\frac{5b^4\left(175b^2A-50abB\right)x^6}{4a^7}-\frac{b^5\left(959b^2A-\right.}{20a}}{x^2(bx+a)^5}$
risch	$\frac{\frac{3b^5(7Ab-2Ba)x^6}{a^7}+\frac{27(7Ab-2Ba)b^4x^5}{2a^6}+\frac{47b^3(7Ab-2Ba)x^4}{2a^5}+\frac{77b^2(7Ab-2Ba)x^3}{4a^4}+\frac{137b(7Ab-2Ba)x^2}{20a^3}+\frac{(7Ab-2Ba)x}{2a^2}-\frac{A}{2a}}{x^2(b^2x^2+2abx+a^2)^2(bx+a)}+\frac{21b^2\ln}{a}$
parallelrisch	$\frac{2100A\ln(x)x^3a^4b^3-2100A\ln(bx+a)x^3a^4b^3-600B\ln(x)x^3a^5b^2+600B\ln(bx+a)x^3a^5b^2+420A\ln(x)x^2a^5b^2+70Aa^6bx-120}{}$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b*(2*A*b-B*a)/a^5/(b*x+a)^3+1/4*b*(3*A*b-2*B*a)/a^4/(b*x+a)^4+5*b*(3*A*b-B
*a)/a^7/(b*x+a)+b*(5*A*b-2*B*a)/a^6/(b*x+a)^2-3*b*(7*A*b-2*B*a)*ln(b*x+a)/
a^8+1/5*b*(A*b-B*a)/a^3/(b*x+a)^5-1/2*A/a^6/x^2-(-6*A*b+B*a)/x/a^7+3*b*(7*
A*b-2*B*a)*ln(x)/a^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(171) = 342.

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx =$$
$$-\frac{10\,Aa^7+60\,(2\,Ba^2b^5-7\,Aab^6)x^6+270\,(2\,Ba^3b^4-7\,Aa^2b^5)x^5+470\,(2\,Ba^4b^3-7\,Aa^3b^4)x^4+385\,(2}{}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(10*A*a^7 + 60*(2*B*a^2*b^5 - 7*A*a*b^6)*x^6 + 270*(2*B*a^3*b^4 - 7*
A*a^2*b^5)*x^5 + 470*(2*B*a^4*b^3 - 7*A*a^3*b^4)*x^4 + 385*(2*B*a^5*b^2 -
7*A*a^4*b^3)*x^3 + 137*(2*B*a^6*b - 7*A*a^5*b^2)*x^2 + 10*(2*B*a^7 - 7*A*a
^6*b)*x - 60*((2*B*a*b^6 - 7*A*b^7)*x^7 + 5*(2*B*a^2*b^5 - 7*A*a*b^6)*x^6
+ 10*(2*B*a^3*b^4 - 7*A*a^2*b^5)*x^5 + 10*(2*B*a^4*b^3 - 7*A*a^3*b^4)*x^4
+ 5*(2*B*a^5*b^2 - 7*A*a^4*b^3)*x^3 + (2*B*a^6*b - 7*A*a^5*b^2)*x^2)*log(b
*x + a) + 60*((2*B*a*b^6 - 7*A*b^7)*x^7 + 5*(2*B*a^2*b^5 - 7*A*a*b^6)*x^6
+ 10*(2*B*a^3*b^4 - 7*A*a^2*b^5)*x^5 + 10*(2*B*a^4*b^3 - 7*A*a^3*b^4)*x^4
+ 5*(2*B*a^5*b^2 - 7*A*a^4*b^3)*x^3 + (2*B*a^6*b - 7*A*a^5*b^2)*x^2)*log(x
))/(a^8*b^5*x^7 + 5*a^9*b^4*x^6 + 10*a^10*b^3*x^5 + 10*a^11*b^2*x^4 + 5*a^
12*b*x^3 + a^13*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$= \frac{-10Aa^6+x^6\cdot(420Ab^6-120Bab^5)+x^5\cdot(1890Aab^5-540Ba^2b^4)+x^4\cdot(3290Aa^2b^4-940Ba^3b^3)+x^3}{20a^{12}x^2+100a^{11}bx^3+200a^{10}b^2x^4+200a^9b^3x^5}$$
$$-\frac{3b(-7Ab+2Ba)\log\left(x+\frac{-21Aab^2+6Ba^2b-3ab(-7Ab+2Ba)}{-42Ab^3+12Bab^2}\right)}{a^8}$$
$$+\frac{3b(-7Ab+2Ba)\log\left(x+\frac{-21Aab^2+6Ba^2b+3ab(-7Ab+2Ba)}{-42Ab^3+12Bab^2}\right)}{a^8}$$

input
```
integrate((B*x+A)/x**3/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
(-10*A*a**6 + x**6*(420*A*b**6 - 120*B*a*b**5) + x**5*(1890*A*a*b**5 - 540
*B*a**2*b**4) + x**4*(3290*A*a**2*b**4 - 940*B*a**3*b**3) + x**3*(2695*A*a
**3*b**3 - 770*B*a**4*b**2) + x**2*(959*A*a**4*b**2 - 274*B*a**5*b) + x*(7
0*A*a**5*b - 20*B*a**6))/(20*a**12*x**2 + 100*a**11*b*x**3 + 200*a**10*b**
2*x**4 + 200*a**9*b**3*x**5 + 100*a**8*b**4*x**6 + 20*a**7*b**5*x**7) - 3*
b*(-7*A*b + 2*B*a)*log(x + (-21*A*a*b**2 + 6*B*a**2*b - 3*a*b*(-7*A*b + 2*
B*a))/(-42*A*b**3 + 12*B*a*b**2))/a**8 + 3*b*(-7*A*b + 2*B*a)*log(x + (-21
*A*a*b**2 + 6*B*a**2*b + 3*a*b*(-7*A*b + 2*B*a))/(-42*A*b**3 + 12*B*a*b**2
))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^3}\,dx = -\frac{10\,Aa^6+60\,(2\,Bab^5-7\,Ab^6)x^6+270\,(2\,Ba^2b^4-7\,Aab^5)x^5+470\,(2\,Ba^3b^3-7\,Aa^2b^4)x^4+385\,(2\,Ba}{20\,(a^7b^5x^7+5\,a^8b^4x^6+10\,a^9b^3x^5+10\,a^{10}b^2x^4+}$$
$$+\frac{3\,(2\,Bab-7\,Ab^2)\log(bx+a)}{a^8}-\frac{3\,(2\,Bab-7\,Ab^2)\log(x)}{a^8}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(10*A*a^6 + 60*(2*B*a*b^5 - 7*A*b^6)*x^6 + 270*(2*B*a^2*b^4 - 7*A*a*
b^5)*x^5 + 470*(2*B*a^3*b^3 - 7*A*a^2*b^4)*x^4 + 385*(2*B*a^4*b^2 - 7*A*a^
3*b^3)*x^3 + 137*(2*B*a^5*b - 7*A*a^4*b^2)*x^2 + 10*(2*B*a^6 - 7*A*a^5*b)*
x)/(a^7*b^5*x^7 + 5*a^8*b^4*x^6 + 10*a^9*b^3*x^5 + 10*a^10*b^2*x^4 + 5*a^1
1*b*x^3 + a^12*x^2) + 3*(2*B*a*b - 7*A*b^2)*log(b*x + a)/a^8 - 3*(2*B*a*b
- 7*A*b^2)*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^3}\,dx = -\frac{3\,(2\,Bab-7\,Ab^2)\log(|x|)}{a^8}+\frac{3\,(2\,Bab^2-7\,Ab^3)\log(|bx+a|)}{a^8b}$$
$$-\frac{10\,Aa^7+60\,(2\,Ba^2b^5-7\,Aab^6)x^6+270\,(2\,Ba^3b^4-7\,Aa^2b^5)x^5+470\,(2\,Ba^4b^3-7\,Aa^3b^4)x^4+385\,(2}{20\,(bx+a)^5a^8x^2}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-3*(2*B*a*b - 7*A*b^2)*log(abs(x))/a^8 + 3*(2*B*a*b^2 - 7*A*b^3)*log(abs(b
*x + a))/(a^8*b) - 1/20*(10*A*a^7 + 60*(2*B*a^2*b^5 - 7*A*a*b^6)*x^6 + 270
*(2*B*a^3*b^4 - 7*A*a^2*b^5)*x^5 + 470*(2*B*a^4*b^3 - 7*A*a^3*b^4)*x^4 + 3
85*(2*B*a^5*b^2 - 7*A*a^4*b^3)*x^3 + 137*(2*B*a^6*b - 7*A*a^5*b^2)*x^2 + 1
0*(2*B*a^7 - 7*A*a^6*b)*x)/((b*x + a)^5*a^8*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$= \frac{\frac{x\,(7\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{2\,a}+\frac{77\,b^2\,x^3\,(7\,A\,b-2\,B\,a)}{4\,a^4}+\frac{47\,b^3\,x^4\,(7\,A\,b-2\,B\,a)}{2\,a^5}+\frac{27\,b^4\,x^5\,(7\,A\,b-2\,B\,a)}{2\,a^6}+\frac{3\,b^5\,x^6\,(7\,A\,b-2\,B\,a)}{a^7}+\frac{137\,b\,x^2\,(7}{20}}{a^5\,x^2+5\,a^4\,b\,x^3+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^5+5\,a\,b^4\,x^6+b^5\,x^7}$$
$$-\frac{6\,b\,\mathrm{atanh}\left(\frac{3\,b\,(7\,A\,b-2\,B\,a)\,(a+2\,b\,x)}{a\,(21\,A\,b^2-6\,B\,a\,b)}\right)\,(7\,A\,b-2\,B\,a)}{a^8}$$

input
```
int((A + B*x)/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
((x*(7*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (77*b^2*x^3*(7*A*b - 2*B*a))/(4*a
^4) + (47*b^3*x^4*(7*A*b - 2*B*a))/(2*a^5) + (27*b^4*x^5*(7*A*b - 2*B*a))/
(2*a^6) + (3*b^5*x^6*(7*A*b - 2*B*a))/a^7 + (137*b*x^2*(7*A*b - 2*B*a))/(2
0*a^3))/(a^5*x^2 + b^5*x^7 + 5*a^4*b*x^3 + 5*a*b^4*x^6 + 10*a^3*b^2*x^4 +
10*a^2*b^3*x^5) - (6*b*atanh((3*b*(7*A*b - 2*B*a)*(a + 2*b*x))/(a*(21*A*b^
2 - 6*B*a*b)))*(7*A*b - 2*B*a))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$= \frac{-60\log(bx+a)\,a^4b^2x^2-240\log(bx+a)\,a^3b^3x^3-360\log(bx+a)\,a^2b^4x^4-240\log(bx+a)\,a\,b^5x^5-60\,l}{}$$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 60*log(a + b*x)*a**4*b**2*x**2 - 240*log(a + b*x)*a**3*b**3*x**3 - 360
*log(a + b*x)*a**2*b**4*x**4 - 240*log(a + b*x)*a*b**5*x**5 - 60*log(a + b
*x)*b**6*x**6 + 60*log(x)*a**4*b**2*x**2 + 240*log(x)*a**3*b**3*x**3 + 360
*log(x)*a**2*b**4*x**4 + 240*log(x)*a*b**5*x**5 + 60*log(x)*b**6*x**6 - 2*
a**6 + 12*a**5*b*x + 110*a**4*b**2*x**2 + 200*a**3*b**3*x**3 + 120*a**2*b*
*4*x**4 - 15*b**6*x**6)/(4*a**7*x**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
 + 4*a*b**3*x**3 + b**4*x**4))
```

3.282 $\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{aAx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{(Ab+aB)x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)} + \frac{bBx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
a*A*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+(A*b+B*a)*x^6*((b*x+a)^2)^(1/2)/(6*b
*x+6*a)+b*B*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{x^5\sqrt{(a+bx)^2}(7a(6A+5Bx)+5bx(7A+6Bx))}{210(a+bx)}$$

input
```
Integrate[x^4*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^5*Sqrt[(a + b*x)^2]*(7*a*(6*A + 5*B*x) + 5*b*x*(7*A + 6*B*x)))/(210*(a
+ b*x))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^4(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^4(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^6+(Ab+aB)x^5+aAx^4\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{6}x^6(aB+Ab)+\frac{1}{5}aAx^5+\frac{1}{7}bBx^7\right)}{a+bx}$$

input
```
Int[x^4*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*A*x^5)/5 + ((A*b + a*B)*x^6)/6 + (b*B*x
^7)/7))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result
gosper	$\frac{x^{5}\left(30 B b\,x^{2}+35 A b x +35 B a x +42 A a \right) \sqrt{\left(b x +a \right)^{2}}}{210 b x +210 a}$
orering	$\frac{x^{5}\left(30 B b\,x^{2}+35 A b x +35 B a x +42 A a \right) \sqrt{\left(b x +a \right)^{2}}}{210 b x +210 a}$
risch	$\frac{a A\,x^{5} \sqrt{\left(b x +a \right)^{2}}}{5 b x +5 a}+\frac{\left(A b +B a \right) x^{6} \sqrt{\left(b x +a \right)^{2}}}{6 b x +6 a}+\frac{b B\,x^{7} \sqrt{\left(b x +a \right)^{2}}}{7 b x +7 a}$
default	$\frac{\operatorname{csgn}\left(b x +a \right) \left(b x +a \right)^{2} \left(30 B\,x^{5} b^{5}+35 A\,b^{5} x^{4}-25 B a\,b^{4} x^{4}-28 A a\,b^{4} x^{3}+20 B\,a^{2} b^{3} x^{3}+21 A\,a^{2} b^{3} x^{2}-15 B\,a^{3} b^{2} x^{2}-14 A\,a^{3} b^{2} x +10 B\,a^{4} b x}{210 b^{6}}$

input
```
int(x^4*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x^5*(30*B*b*x^2+35*A*b*x+35*B*a*x+42*A*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{5}\,Aax^5+\frac{1}{6}\,(Ba+Ab)x^6$$

input
```
integrate(x^4*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/7*B*b*x^7 + 1/5*A*a*x^5 + 1/6*(B*a + A*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.

Time = 2.57 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.67

$$\begin{aligned}
&\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx\\
&=A\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^5}{30b^5}-\frac{a^4x}{30b^4}+\frac{a^3x^2}{30b^3}-\frac{a^2x^3}{30b^2}+\frac{ax^4}{30b}+\frac{x^5}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^8(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{4a^6(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{6a^4(a^2+2abx)^{\frac{7}{2}}}{7}-\frac{4a^2(a^2+2abx)^{\frac{9}{2}}}{9}+\frac{(a^2+2abx)^{\frac{11}{2}}}{11}}{16a^5b^5} & \text{for } ab\neq 0\\ \frac{x^5\sqrt{a^2}}{5} & \text{otherwise}\end{cases}\right)\\
&+B\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^6}{42b^6}+\frac{a^5x}{42b^5}-\frac{a^4x^2}{42b^4}+\frac{a^3x^3}{42b^3}-\frac{a^2x^4}{42b^2}+\frac{ax^5}{42b}+\frac{x^6}{7}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^{10}(a^2+2abx)^{\frac{3}{2}}}{3}+a^8(a^2+2abx)^{\frac{5}{2}}-\frac{10a^6(a^2+2abx)^{\frac{7}{2}}}{7}+\frac{10a^4(a^2+2abx)^{\frac{9}{2}}}{9}-\frac{5a^2(a^2+2abx)^{\frac{11}{2}}}{11}+\frac{(a^2+2abx)^{\frac{13}{2}}}{13}}{32a^6b^6} & \text{for } ab\neq 0\\ \frac{x^6\sqrt{a^2}}{6} & \text{otherwise}\end{cases}\right)
\end{aligned}$$

input
```
integrate(x**4*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
A*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30
*b**4) + a**3*x**2/(30*b**3) - a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/
6), Ne(b**2, 0)), ((a**8*(a**2 + 2*a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*
x)**(5/2)/5 + 6*a**4*(a**2 + 2*a*b*x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**
(9/2)/9 + (a**2 + 2*a*b*x)**(11/2)/11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*
sqrt(a**2)/5, True)) + B*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**
6/(42*b**6) + a**5*x/(42*b**5) - a**4*x**2/(42*b**4) + a**3*x**3/(42*b**3)
 - a**2*x**4/(42*b**2) + a*x**5/(42*b) + x**6/7), Ne(b**2, 0)), ((-a**10*(
a**2 + 2*a*b*x)**(3/2)/3 + a**8*(a**2 + 2*a*b*x)**(5/2) - 10*a**6*(a**2 +
2*a*b*x)**(7/2)/7 + 10*a**4*(a**2 + 2*a*b*x)**(9/2)/9 - 5*a**2*(a**2 + 2*a
*b*x)**(11/2)/11 + (a**2 + 2*a*b*x)**(13/2)/13)/(32*a**6*b**6), Ne(a*b, 0)
), (x**6*sqrt(a**2)/6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.17

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bx^4}{7\,b^2} - \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bax^3}{42\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ax^3}{6\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^5x}{2\,b^5} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^4x}{2\,b^4} + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^2x^2}{14\,b^4} - \frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aax^2}{10\,b^3} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^6}{2\,b^6} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^5}{2\,b^5} - \frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^3x}{7\,b^5} + \frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aa^2x}{5\,b^4} + \frac{10\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^4}{21\,b^6} - \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aa^3}{15\,b^5}$$

input
```
integrate(x^4*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*x^4/b^2 - 11/42*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*B*a*x^3/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*x^3/b^2 -
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^5*x/b^5 + 1/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*A*a^4*x/b^4 + 5/14*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^2*x^2/b^4 -
 3/10*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a*x^2/b^3 - 1/2*sqrt(b^2*x^2 + 2*a
*b*x + a^2)*B*a^6/b^6 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^5/b^5 - 3/7*
(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^3*x/b^5 + 2/5*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*A*a^2*x/b^4 + 10/21*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^4/b^6 - 7/
15*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{7}\,Bbx^7\mathrm{sgn}(bx+a)+\frac{1}{6}\,Bax^6\mathrm{sgn}(bx+a)+\frac{1}{6}\,Abx^6\mathrm{sgn}(bx+a)+\frac{1}{5}\,Aax^5\mathrm{sgn}(bx+a)-\frac{(5\,Ba^7-7\,Aa^6b)\mathrm{sgn}(bx+a)}{210\,b^6}$$

input
```
integrate(x^4*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/7*B*b*x^7*sgn(b*x + a) + 1/6*B*a*x^6*sgn(b*x + a) + 1/6*A*b*x^6*sgn(b*x
+ a) + 1/5*A*a*x^5*sgn(b*x + a) - 1/210*(5*B*a^7 - 7*A*a^6*b)*sgn(b*x + a)
/b^6
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.78

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\frac{A\,x^3\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}{6\,b^2}+\frac{B\,x^4\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}{7\,b^2}$$
$$-\frac{11\,B\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^5+5\,b^3\,x^3\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-14\,a^3\,b^2\,x^2-13\,a^4\,b\,x-9\,a\,b^2\,x^2\,(a^2+\right.}{210\,b^6}$$
$$-\frac{B\,a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(4\,b^2\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-a^4+9\,a^2\,b^2\,x^2+8\,a^3\,b\,x-7\,a\,b\,x\,(a^2+2\,a\,b\right.}{35\,b^6}$$
$$-\frac{A\,a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^3-5\,a\,b^2\,x^2+3\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-4\,a^2\,b\,x\right)}{24\,b^5}$$
$$-\frac{3\,A\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(4\,b^2\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-a^4+9\,a^2\,b^2\,x^2+8\,a^3\,b\,x-7\,a\,b\,x\,(a^2+2\,a\,b\right.}{40\,b^5}$$

input
```
int(x^4*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
(A*x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(6*b^2) + (B*x^4*(a^2 + b^2*x^2 +
2*a*b*x)^(3/2))/(7*b^2) - (11*B*a*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^5 + 5
*b^3*x^3*(a^2 + b^2*x^2 + 2*a*b*x) - 14*a^3*b^2*x^2 - 13*a^4*b*x - 9*a*b^2
*x^2*(a^2 + b^2*x^2 + 2*a*b*x) + 12*a^2*b*x*(a^2 + b^2*x^2 + 2*a*b*x)))/(2
10*b^6) - (B*a^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(4*b^2*x^2*(a^2 + b^2*x^2
 + 2*a*b*x) - a^4 + 9*a^2*b^2*x^2 + 8*a^3*b*x - 7*a*b*x*(a^2 + b^2*x^2 + 2
*a*b*x)))/(35*b^6) - (A*a^2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*a*b^2
*x^2 + 3*b*x*(a^2 + b^2*x^2 + 2*a*b*x) - 4*a^2*b*x))/(24*b^5) - (3*A*a*(a^
2 + b^2*x^2 + 2*a*b*x)^(1/2)*(4*b^2*x^2*(a^2 + b^2*x^2 + 2*a*b*x) - a^4 +
9*a^2*b^2*x^2 + 8*a^3*b*x - 7*a*b*x*(a^2 + b^2*x^2 + 2*a*b*x)))/(40*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int x^4(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^5(15b^2x^2+35abx+21a^2)}{105}$$

input
```
int(x^4*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.283 $\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{aAx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} + \frac{(Ab+aB)x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{bBx^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}$$

output
```
a*A*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+(A*b+B*a)*x^5*((b*x+a)^2)^(1/2)/(5*b
*x+5*a)+b*B*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{x^4\sqrt{(a+bx)^2}(3a(5A+4Bx)+2bx(6A+5Bx))}{60(a+bx)}$$

input
```
Integrate[x^3*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^4*Sqrt[(a + b*x)^2]*(3*a*(5*A + 4*B*x) + 2*b*x*(6*A + 5*B*x)))/(60*(a +
 b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^3(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^5+(Ab+aB)x^4+aAx^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{5}x^5(aB+Ab)+\frac{1}{4}aAx^4+\frac{1}{6}bBx^6\right)}{a+bx}$$

input
```
Int[x^3*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*A*x^4)/4 + ((A*b + a*B)*x^5)/5 + (b*B*x
^6)/6))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result	size
gosper	$\frac{x^{4}\left(10Bb\,x^{2}+12Abx+12Bax+15Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60bx+60a}$	44
orering	$\frac{x^{4}\left(10Bb\,x^{2}+12Abx+12Bax+15Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60bx+60a}$	44
risch	$\frac{aA\,x^{4}\sqrt{\left(bx+a\right)^{2}}}{4bx+4a}+\frac{\left(Ab+Ba\right)x^{5}\sqrt{\left(bx+a\right)^{2}}}{5bx+5a}+\frac{bB\,x^{6}\sqrt{\left(bx+a\right)^{2}}}{6bx+6a}$	76
default	$-\frac{\operatorname{csgn}\left(bx+a\right)\left(bx+a\right)^{2}\left(-10b^{4}B\,x^{4}-12A\,b^{4}x^{3}+8Ba\,b^{3}x^{3}+9Aa\,b^{3}x^{2}-6B\,a^{2}b^{2}x^{2}-6A\,a^{2}b^{2}x+4B\,a^{3}bx+3A\,a^{3}b-2a^{4}B\right)}{60b^{5}}$	101

input
```
int(x^3*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*B*b*x^2+12*A*b*x+12*B*a*x+15*A*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{6}\,Bbx^6+\frac{1}{4}\,Aax^4+\frac{1}{5}\,(Ba+Ab)x^5$$

input
```
integrate(x^3*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*B*b*x^6 + 1/4*A*a*x^4 + 1/5*(B*a + A*b)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(78) = 156$.

Time = 1.90 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.13

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=A\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^4}{20b^4}+\frac{a^3x}{20b^3}-\frac{a^2x^2}{20b^2}+\frac{ax^3}{20b}+\frac{x^4}{5}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^6(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{3a^4(a^2+2abx)^{\frac{5}{2}}}{5}-\frac{3a^2(a^2+2abx)^{\frac{7}{2}}}{7}+\frac{(a^2+2abx)^{\frac{9}{2}}}{9}}{8a^4b^4} & \text{for } ab\neq 0\\ \frac{x^4\sqrt{a^2}}{4} & \text{otherwise}\end{cases}\right)$$
$$+B\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^5}{30b^5}-\frac{a^4x}{30b^4}+\frac{a^3x^2}{30b^3}-\frac{a^2x^3}{30b^2}+\frac{ax^4}{30b}+\frac{x^5}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^8(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{4a^6(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{6a^4(a^2+2abx)^{\frac{7}{2}}}{7}-\frac{4a^2(a^2+2abx)^{\frac{9}{2}}}{9}+\frac{(a^2+2abx)^{\frac{11}{2}}}{11}}{16a^5b^5} & \text{for } ab\neq 0\\ \frac{x^5\sqrt{a^2}}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
A*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*x/(2
0*b**3) - a**2*x**2/(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)), ((-
a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*a**2
*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4), Ne(
a*b, 0)), (x**4*sqrt(a**2)/4, True)) + B*Piecewise((sqrt(a**2 + 2*a*b*x +
b**2*x**2)*(a**5/(30*b**5) - a**4*x/(30*b**4) + a**3*x**2/(30*b**3) - a**2
*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6), Ne(b**2, 0)), ((a**8*(a**2 + 2*
a*b*x)**(3/2)/3 - 4*a**6*(a**2 + 2*a*b*x)**(5/2)/5 + 6*a**4*(a**2 + 2*a*b*
x)**(7/2)/7 - 4*a**2*(a**2 + 2*a*b*x)**(9/2)/9 + (a**2 + 2*a*b*x)**(11/2)/
11)/(16*a**5*b**5), Ne(a*b, 0)), (x**5*sqrt(a**2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(75) = 150$.

Time = 0.03 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.64

$$\begin{aligned}\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx =& \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bx^3}{6\,b^2} \\ &+ \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^4x}{2\,b^4} \\ &- \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^3x}{2\,b^3} \\ &- \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bax^2}{10\,b^3} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ax^2}{5\,b^2} \\ &+ \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^5}{2\,b^5} \\ &- \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^4}{2\,b^4} \\ &+ \frac{2\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^2x}{5\,b^4} \\ &- \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aax}{20\,b^3} \\ &- \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^3}{15\,b^5} \\ &+ \frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^2}{20\,b^4}\end{aligned}$$

input
```
integrate(x^3*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*x^3/b^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*B*a^4*x/b^4 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^3*x/b^3 - 3/10
*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a*x^2/b^3 + 1/5*(b^2*x^2 + 2*a*b*x + a^
2)^(3/2)*A*x^2/b^2 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^5/b^5 - 1/2*sqr
t(b^2*x^2 + 2*a*b*x + a^2)*A*a^4/b^4 + 2/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)
*B*a^2*x/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a*x/b^3 - 7/15*(b^2*
x^2 + 2*a*b*x + a^2)^(3/2)*B*a^3/b^5 + 9/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2
)*A*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{6}\,Bbx^6\mathrm{sgn}(bx+a)+\frac{1}{5}\,Bax^5\mathrm{sgn}(bx+a)+\frac{1}{5}\,Abx^5\mathrm{sgn}(bx+a)+\frac{1}{4}\,Aax^4\mathrm{sgn}(bx+a)+\frac{(2\,Ba^6-3\,Aa^5b)\mathrm{sgn}(bx+a)}{60\,b^5}$$

input
```
integrate(x^3*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*B*b*x^6*sgn(b*x + a) + 1/5*B*a*x^5*sgn(b*x + a) + 1/5*A*b*x^5*sgn(b*x
+ a) + 1/4*A*a*x^4*sgn(b*x + a) + 1/60*(2*B*a^6 - 3*A*a^5*b)*sgn(b*x + a)/
b^5
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.98

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$= \frac{A\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{5\,b^2}+\frac{B\,x^3\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{6\,b^2}$$
$$-\frac{7\,A\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^3-5\,a\,b^2\,x^2+3\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-4\,a^2\,b\,x\right)}{60\,b^4}$$
$$-\frac{B\,a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^3-5\,a\,b^2\,x^2+3\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-4\,a^2\,b\,x\right)}{24\,b^5}$$
$$-\frac{A\,a^2\,\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{60\,b^6}$$
$$-\frac{3\,B\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(4\,b^2\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-a^4+9\,a^2\,b^2\,x^2+8\,a^3\,b\,x-7\,a\,b\,x\,\left(a^2+2\,a\,b\right.}{40\,b^5}$$

input
```
int(x^3*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
(A*x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(5*b^2) + (B*x^3*(a^2 + b^2*x^2 +
2*a*b*x)^(3/2))/(6*b^2) - (7*A*a*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*
a*b^2*x^2 + 3*b*x*(a^2 + b^2*x^2 + 2*a*b*x) - 4*a^2*b*x))/(60*b^4) - (B*a^
2*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*a*b^2*x^2 + 3*b*x*(a^2 + b^2*x^
2 + 2*a*b*x) - 4*a^2*b*x))/(24*b^5) - (A*a^2*(8*b^2*(a^2 + b^2*x^2) - 12*a
^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(60*b^6) - (3*B*a*(a^
2 + b^2*x^2 + 2*a*b*x)^(1/2)*(4*b^2*x^2*(a^2 + b^2*x^2 + 2*a*b*x) - a^4 +
9*a^2*b^2*x^2 + 8*a^3*b*x - 7*a*b*x*(a^2 + b^2*x^2 + 2*a*b*x)))/(40*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int x^3(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int(x^3*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.284 $\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{aAx^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{(Ab+aB)x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{bBx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

output
```
a*A*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+(A*b+B*a)*x^4*((b*x+a)^2)^(1/2)/(4*b
*x+4*a)+b*B*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^3\sqrt{(a+bx)^2}(5a(4A+3Bx)+3bx(5A+4Bx))}{60(a+bx)}$$

input
```
Integrate[x^2*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^3*Sqrt[(a + b*x)^2]*(5*a*(4*A + 3*B*x) + 3*b*x*(5*A + 4*B*x)))/(60*(a +
b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^2(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^4+(Ab+aB)x^3+aAx^2\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{4}x^4(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{5}bBx^5\right)}{a+bx}$$

input
```
Int[x^2*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*A*x^3)/3 + ((A*b + a*B)*x^4)/4 + (b*B*x
^5)/5))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result	size
gosper	$\frac{x^{3}\left(12Bb\,x^{2}+15Abx+15Bax+20Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60bx+60a}$	44
orering	$\frac{x^{3}\left(12Bb\,x^{2}+15Abx+15Bax+20Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60bx+60a}$	44
risch	$\frac{aA\,x^{3}\sqrt{\left(bx+a\right)^{2}}}{3bx+3a}+\frac{\left(Ab+Ba\right)x^{4}\sqrt{\left(bx+a\right)^{2}}}{4bx+4a}+\frac{bB\,x^{5}\sqrt{\left(bx+a\right)^{2}}}{5bx+5a}$	76
default	$\frac{\operatorname{csgn}\left(bx+a\right)\left(bx+a\right)^{2}\left(12x^{3}B\,b^{3}+15A\,b^{3}x^{2}-9Ba\,b^{2}x^{2}-10Aa\,b^{2}x+6B\,a^{2}bx+5A\,a^{2}b-3B\,a^{3}\right)}{60b^{4}}$	77

input
```
int(x^2*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(12*B*b*x^2+15*A*b*x+15*B*a*x+20*A*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{5}\,Bbx^5+\frac{1}{3}\,Aax^3+\frac{1}{4}\,(Ba+Ab)x^4$$

input
```
integrate(x^2*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*B*b*x^5 + 1/3*A*a*x^3 + 1/4*(B*a + A*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.56

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$= A\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3}{12b^3}-\frac{a^2x}{12b^2}+\frac{ax^2}{12b}+\frac{x^3}{4}\right) & \text{for}\ b^2\neq 0\\ \frac{\frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for}\ ab\neq 0\\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise}\end{cases}\right)$$
$$+ B\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^4}{20b^4}+\frac{a^3x}{20b^3}-\frac{a^2x^2}{20b^2}+\frac{ax^3}{20b}+\frac{x^4}{5}\right) & \text{for}\ b^2\neq 0\\ \frac{-\frac{a^6(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{3a^4(a^2+2abx)^{\frac{5}{2}}}{5}-\frac{3a^2(a^2+2abx)^{\frac{7}{2}}}{7}+\frac{(a^2+2abx)^{\frac{9}{2}}}{9}}{8a^4b^4} & \text{for}\ ab\neq 0\\ \frac{x^4\sqrt{a^2}}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
A*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**2*x/(12
*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(
3/2)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*
a**3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True)) + B*Piecewise((sqrt(a*
*2 + 2*a*b*x + b**2*x**2)*(-a**4/(20*b**4) + a**3*x/(20*b**3) - a**2*x**2/
(20*b**2) + a*x**3/(20*b) + x**4/5), Ne(b**2, 0)), ((-a**6*(a**2 + 2*a*b*x
)**(3/2)/3 + 3*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 3*a**2*(a**2 + 2*a*b*x)**(
7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4), Ne(a*b, 0)), (x**4*sqrt
(a**2)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(75) = 150$.

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.11

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = -\frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^3x}{2\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^2x}{2\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bx^2}{5\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^4}{2\,b^4} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^3}{2\,b^3} - \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bax}{20\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ax}{4\,b^2} + \frac{9\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^2}{20\,b^4} - \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aa}{12\,b^3}$$

input
```
integrate(x^2*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^3*x/b^3 + 1/2*sqrt(b^2*x^2 + 2*a*b*
x + a^2)*A*a^2*x/b^2 + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*x^2/b^2 - 1/2
*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^4/b^4 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2)*A*a^3/b^3 - 7/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a*x/b^3 + 1/4*(b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*A*x/b^2 + 9/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B
*a^2/b^4 - 5/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{5}\,Bbx^5\text{sgn}(bx+a)+\frac{1}{4}\,Bax^4\text{sgn}(bx+a)+\frac{1}{4}\,Abx^4\text{sgn}(bx+a)+\frac{1}{3}\,Aax^3\text{sgn}(bx+a)-\frac{(3\,Ba^5-5\,Aa^4b)\text{sgn}(bx+a)}{60\,b^4}$$

input
```
integrate(x^2*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*B*b*x^5*sgn(b*x + a) + 1/4*B*a*x^4*sgn(b*x + a) + 1/4*A*b*x^4*sgn(b*x
+ a) + 1/3*A*a*x^3*sgn(b*x + a) - 1/60*(3*B*a^5 - 5*A*a^4*b)*sgn(b*x + a)/
b^4
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.38

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$= \frac{B\,x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{5\,b^2}+\frac{A\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{4\,b^2}$$
$$-\frac{7\,B\,a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(a^3-5\,a\,b^2\,x^2+3\,b\,x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)-4\,a^2\,b\,x\right)}{60\,b^4}$$
$$-\frac{5\,A\,a\,\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{96\,b^5}$$
$$-\frac{B\,a^2\,\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{60\,b^6}$$
$$-\frac{A\,a^2\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,b^2}$$

input
```
int(x^2*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
(B*x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(5*b^2) + (A*x*(a^2 + b^2*x^2 + 2*
a*b*x)^(3/2))/(4*b^2) - (7*B*a*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(a^3 - 5*a*
b^2*x^2 + 3*b*x*(a^2 + b^2*x^2 + 2*a*b*x) - 4*a^2*b*x))/(60*b^4) - (5*A*a*
(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)
^(1/2))/(96*b^5) - (B*a^2*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)
*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(60*b^6) - (A*a^2*(x/2 + a/(2*b))*(a^2 +
 b^2*x^2 + 2*a*b*x)^(1/2))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int x^2(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x^2*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.285 $\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{aAx^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{(Ab+aB)x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{bBx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}$$

output
```
a*A*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+(A*b+B*a)*x^3*((b*x+a)^2)^(1/2)/(3*b
*x+3*a)+b*B*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{x^2(bx(4A+3Bx)+a(6A+4Bx))\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{12\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)}$$

input
```
Integrate[x*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x^2*(b*x*(4*A + 3*B*x) + a*(6*A + 4*B*x))*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] -
 Sqrt[(a + b*x)^2])))/(12*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^3+(Ab+aB)x^2+aAx\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{3}x^3(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{4}bBx^4\right)}{a+bx}$$

input
```
Int[x*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a*A*x^2)/2 + ((A*b + a*B)*x^3)/3 + (b*B*x
^4)/4))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result	size
gosper	$\frac{x^{2}(3Bb\,x^{2}+4Abx+4Bax+6Aa)\sqrt{(bx+a)^{2}}}{12bx+12a}$	44
orering	$\frac{x^{2}(3Bb\,x^{2}+4Abx+4Bax+6Aa)\sqrt{(bx+a)^{2}}}{12bx+12a}$	44
default	$-\frac{\mathrm{csgn}(bx+a)(bx+a)^{2}(-3x^{2}B\,b^{2}-4x\,b^{2}A+2xabB+2abA-a^{2}B)}{12b^{3}}$	53
risch	$\frac{aA\,x^{2}\sqrt{(bx+a)^{2}}}{2bx+2a}+\frac{(Ab+Ba)x^{3}\sqrt{(bx+a)^{2}}}{3bx+3a}+\frac{bB\,x^{4}\sqrt{(bx+a)^{2}}}{4bx+4a}$	76

input
```
int(x*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*B*b*x^2+4*A*b*x+4*B*a*x+6*A*a)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{1}{4}\,Bbx^4+\frac{1}{2}\,Aax^2+\frac{1}{3}\,(Ba+Ab)x^3$$

input
```
integrate(x*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*B*b*x^4 + 1/2*A*a*x^2 + 1/3*(B*a + A*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.00

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$= A\left(\begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^2}{6b^2}+\frac{ax}{6b}+\frac{x^2}{3}\right) & \text{for } b^2\neq 0 \\ \frac{-\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{for } ab\neq 0 \\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise} \end{cases}\right)$$
$$+ B\left(\begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(\frac{a^3}{12b^3}-\frac{a^2x}{12b^2}+\frac{ax^2}{12b}+\frac{x^3}{4}\right) & \text{for } b^2\neq 0 \\ \frac{\frac{a^4(a^2+2abx)^{\frac{3}{2}}}{3}-\frac{2a^2(a^2+2abx)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab\neq 0 \\ \frac{x^3\sqrt{a^2}}{3} & \text{otherwise} \end{cases}\right)$$

input

```
integrate(x*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output

```
A*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b)
+ x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b
*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True)) + B*P
iecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(a**3/(12*b**3) - a**2*x/(12*b*
*2) + a*x**2/(12*b) + x**3/4), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x)**(3/2
)/3 - 2*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*a**
3*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(75) = 150$.

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^2x}{2\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Aax}{2\,b} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^3}{2\,b^3} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa^2}{2\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bx}{4\,b^2} - \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba}{12\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}A}{3\,b^2}$$

input
```
integrate(x*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^2*x/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*A*a*x/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^3/b^3 - 1/2*sqrt(b
^2*x^2 + 2*a*b*x + a^2)*A*a^2/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*
x/b^2 - 5/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a/b^3 + 1/3*(b^2*x^2 + 2*a*
b*x + a^2)^(3/2)*A/b^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{4}\,Bbx^4\mathrm{sgn}(bx+a) + \frac{1}{3}\,Bax^3\mathrm{sgn}(bx+a) + \frac{1}{3}\,Abx^3\mathrm{sgn}(bx+a) + \frac{1}{2}\,Aax^2\mathrm{sgn}(bx+a) + \frac{(Ba^4-2\,Aa^3b)\mathrm{sgn}(bx+a)}{12\,b^3}$$

input
```
integrate(x*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*B*b*x^4*sgn(b*x + a) + 1/3*B*a*x^3*sgn(b*x + a) + 1/3*A*b*x^3*sgn(b*x
+ a) + 1/2*A*a*x^2*sgn(b*x + a) + 1/12*(B*a^4 - 2*A*a^3*b)*sgn(b*x + a)/b^
3
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.54

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{A\,\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{24\,b^4} + \frac{B\,x\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}{4\,b^2} - \frac{5\,B\,a\,\left(8\,b^2\,\left(a^2+b^2\,x^2\right)-12\,a^2\,b^2+4\,a\,b^3\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{96\,b^5} - \frac{B\,a^2\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,b^2}$$

input
```
int(x*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
(A*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b
*x)^(1/2))/(24*b^4) + (B*x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/(4*b^2) - (5*B
*a*(8*b^2*(a^2 + b^2*x^2) - 12*a^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b
*x)^(1/2))/(96*b^5) - (B*a^2*(x/2 + a/(2*b))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/
2))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int x(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{x^2(3b^2x^2+8abx+6a^2)}{12}$$

input
```
int(x*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.286 $\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{(Ab-aB)(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2b^2}+\frac{B(a^2+2abx+b^2x^2)^{3/2}}{3b^2}$$

output
```
1/2*(A*b-B*a)*(b*x+a)*((b*x+a)^2)^(1/2)/b^2+1/3*B*(b^2*x^2+2*a*b*x+a^2)^(3
/2)/b^2
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x(3a(2A+Bx)+bx(3A+2Bx))\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}{-6a^2-6abx+6\sqrt{a^2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(x*(3*a*(2*A + B*x) + b*x*(3*A + 2*B*x))*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - S
qrt[(a + b*x)^2])))/(-6*a^2 - 6*a*b*x + 6*Sqrt[a^2]*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{(Ab-aB)\int \sqrt{a^2+2bxa+b^2x^2}dx}{b}+\frac{B\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \left(xb^2+ab\right)dx}{b^2(a+bx)}+\frac{B\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(Ab-aB)}{2b^2}+\frac{B\left(a^2+2abx+b^2x^2\right)^{3/2}}{3b^2}$$

input
```
Int[(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((A*b - a*B)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + (B*(a^2 +
2*a*b*x + b^2*x^2)^(3/2))/(3*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)(bx+a)^{2}(2Bbx+3Ab-Ba)}{6b^{2}}$	33
gosper	$\frac{x\left(2Bb\,x^{2}+3Abx+3Bax+6Aa\right)\sqrt{(bx+a)^{2}}}{6bx+6a}$	42
orering	$\frac{x\left(2Bb\,x^{2}+3Abx+3Bax+6Aa\right)\sqrt{(bx+a)^{2}}}{6bx+6a}$	42
risch	$\frac{\sqrt{(bx+a)^{2}}\,Bb\,x^{3}}{3bx+3a}+\frac{\sqrt{(bx+a)^{2}}\,(Ab+Ba)x^{2}}{2bx+2a}+\frac{\sqrt{(bx+a)^{2}}\,aAx}{bx+a}$	73

input
```
int((B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(b*x+a)*(b*x+a)^2*(2*B*b*x+3*A*b-B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{3}\,Bbx^3 + Aax + \frac{1}{2}\,(Ba+Ab)x^2$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*B*b*x^3 + A*a*x + 1/2*(B*a + A*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.26

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = A\left(\begin{cases}\left(\frac{a}{2b}+\frac{x}{2}\right)\sqrt{a^2+2abx+b^2x^2} & \text{for } b^2\neq 0\\ \frac{(a^2+2abx)^{\frac{3}{2}}}{3ab} & \text{for } ab\neq 0\\ x\sqrt{a^2} & \text{otherwise}\end{cases}\right) + B\left(\begin{cases}\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^2}{6b^2}+\frac{ax}{6b}+\frac{x^2}{3}\right) & \text{for } b^2\neq 0\\ \frac{-\frac{a^2(a^2+2abx)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
A*Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**2), Ne(b**2, 0)
), ((a**2 + 2*a*b*x)**(3/2)/(3*a*b), Ne(a*b, 0)), (x*sqrt(a**2), True)) +
B*Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(-a**2/(6*b**2) + a*x/(6*b)
+ x**2/3), Ne(b**2, 0)), ((-a**2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b
*x)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*sqrt(a**2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(52) = 104$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.81

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{2}\sqrt{b^2x^2+2\,abx+a^2}Ax - \frac{\sqrt{b^2x^2+2\,abx+a^2}Bax}{2\,b} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba^2}{2\,b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Aa}{2\,b} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}B}{3\,b^2}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*x - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
B*a*x/b - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^2/b^2 + 1/2*sqrt(b^2*x^2 +
 2*a*b*x + a^2)*A*a/b + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B/b^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{1}{3}\,Bbx^3\mathrm{sgn}(bx+a) + \frac{1}{2}\,Bax^2\mathrm{sgn}(bx+a) + \frac{1}{2}\,Abx^2\mathrm{sgn}(bx+a) + Aax\mathrm{sgn}(bx+a) - \frac{(Ba^3-3\,Aa^2b)\mathrm{sgn}(bx+a)}{6\,b^2}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*B*b*x^3*sgn(b*x + a) + 1/2*B*a*x^2*sgn(b*x + a) + 1/2*A*b*x^2*sgn(b*x
+ a) + A*a*x*sgn(b*x + a) - 1/6*(B*a^3 - 3*A*a^2*b)*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$$
$$=\frac{A\sqrt{(a+b\,x)^2}\,(a+b\,x)}{2\,b}$$
$$+\frac{B\,(8\,b^2\,(a^2+b^2\,x^2)-12\,a^2\,b^2+4\,a\,b^3\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{24\,b^4}$$

input
```
int(((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
(A*((a + b*x)^2)^(1/2)*(a + b*x))/(2*b) + (B*(8*b^2*(a^2 + b^2*x^2) - 12*a
^2*b^2 + 4*a*b^3*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(24*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int (A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{x(b^2x^2+3abx+3a^2)}{3}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.287 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 105

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x} \, dx = \frac{(Ab+aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{bBx^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{aA\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
(A*b+B*a)*x*((b*x+a)^2)^(1/2)/(b*x+a)+b*B*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a
)+a*A*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x} \, dx = \frac{\sqrt{(a+bx)^2}(x(2Ab+2aB+bBx)+2aA\log(x))}{2(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*(x*(2*A*b + 2*a*B + b*B*x) + 2*a*A*Log[x]))/(2*(a + b*x
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x}dx$$
$$\Big\downarrow \text{1187}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x}dx}{b(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x}dx}{a+bx}$$
$$\Big\downarrow \text{85}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bA+\frac{aA}{x}+aB+bBx\right)dx}{a+bx}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(x(aB+Ab)+aA\log(x)+\frac{1}{2}bBx^2\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((A*b + a*B)*x + (b*B*x^2)/2 + a*A*Log[x]))
/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(x^{2} B\, b^{2}+2 A a b \ln(-bx)+2 x\, b^{2} A+2 x a b B+2 a b A+a^{2} B\right)}{2 b}$	54
risch	$\frac{b B\, x^{2} \sqrt{(bx+a)^{2}}}{2 b x+2 a}+\frac{\sqrt{(bx+a)^{2}}\, A b x}{bx+a}+\frac{\sqrt{(bx+a)^{2}}\, B a x}{bx+a}+\frac{a A \sqrt{(bx+a)^{2}}\, \ln(x)}{bx+a}$	86

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x+a)*(x^2*B*b^2+2*A*a*b*ln(-b*x)+2*x*b^2*A+2*x*a*b*B+2*a*b*A+a^
2*B)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \frac{1}{2}\,Bbx^2 + Aa\log\left(x\right) + (Ba+Ab)x$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*B*b*x^2 + A*a*log(x) + (B*a + A*b)*x
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx &= (-1)^{2\,b^2x+2\,ab}\,Aa\log\left(2\,b^2x+2\,ab\right) \\ &\quad - (-1)^{2\,abx+2\,a^2}\,Aa\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) \\ &\quad + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Bx \\ &\quad + \sqrt{b^2x^2+2\,abx+a^2}A + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ba}{2\,b}\end{aligned}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*A*a*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*A
*a*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
B*x + sqrt(b^2*x^2 + 2*a*b*x + a^2)*A + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
B*a/b
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \frac{1}{2}Bbx^2\text{sgn}(bx+a) + Bax\text{sgn}(bx+a) + Abx\text{sgn}(bx+a) + Aa\log\left(|x|\right)\text{sgn}(bx+a)$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*B*b*x^2*sgn(b*x + a) + B*a*x*sgn(b*x + a) + A*b*x*sgn(b*x + a) + A*a*l
og(abs(x))*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = A\sqrt{a^2+2\,a\,b\,x+b^2\,x^2} - A\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)\sqrt{a^2} + \frac{B\,\sqrt{{(a+b\,x)}^2}\,(a+b\,x)}{2\,b} + \frac{A\,a\,b\,\ln\left(a\,b+\sqrt{{(a+b\,x)}^2}\,\sqrt{b^2}+b^2\,x\right)}{\sqrt{b^2}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x,x)
```

output
```
A*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2) - A*log(a*b + a^2/x + ((a^2)^(1/2)*(a^2
+ b^2*x^2 + 2*a*b*x)^(1/2))/x)*(a^2)^(1/2) + (B*((a + b*x)^2)^(1/2)*(a + b
*x))/(2*b) + (A*a*b*log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x))/(b
^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.19

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x}\,dx = \log(x)\,a^2+2abx+\frac{b^2x^2}{2}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x,x)
```

output
```
(2*log(x)*a**2 + 4*a*b*x + b**2*x**2)/2
```

3.288 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = -\frac{aA\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)}+\frac{bBx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{(Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-a*A*((b*x+a)^2)^(1/2)/x/(b*x+a)+b*B*x*((b*x+a)^2)^(1/2)/(b*x+a)+(A*b+B*a)
*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}(-aA+bBx^2+(Ab+aB)x\log(x))}{x(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-(a*A) + b*B*x^2 + (A*b + a*B)*x*Log[x]))/(x*(a + b*x)
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^2}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^2}+bB+\frac{Ab+aB}{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\log(x)(aB+Ab)-\frac{aA}{x}+bBx\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-((a*A)/x) + b*B*x + (A*b + a*B)*Log[x]))/
(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(A\ln(-bx)bx+B\ln(-bx)ax+Bb\,x^{2}+Bax-Aa\right)}{x}$	44
risch	$-\frac{aA\sqrt{(bx+a)^{2}}}{x(bx+a)}+\frac{bBx\sqrt{(bx+a)^{2}}}{bx+a}+\frac{(Ab+Ba)\sqrt{(bx+a)^{2}}\,\ln(x)}{bx+a}$	71

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
csgn(b*x+a)*(A*ln(-b*x)*b*x+B*ln(-b*x)*a*x+B*b*x^2+B*a*x-A*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.25

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \frac{Bbx^2+(Ba+Ab)x\log(x)-Aa}{x}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(B*b*x^2 + (B*a + A*b)*x*log(x) - A*a)/x
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x^2}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx &= (-1)^{2b^2x+2ab}Ba\log\left(2b^2x+2ab\right)\\ &\quad+(-1)^{2b^2x+2ab}Ab\log\left(2b^2x+2ab\right)\\ &\quad-(-1)^{2abx+2a^2}Ba\log\left(\frac{2abx}{|x|}+\frac{2a^2}{|x|}\right)\\ &\quad-(-1)^{2abx+2a^2}Ab\log\left(\frac{2abx}{|x|}+\frac{2a^2}{|x|}\right)\\ &\quad+\sqrt{b^2x^2+2abx+a^2}B-\frac{\sqrt{b^2x^2+2abx+a^2}A}{x}\end{aligned}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*a*log(2*b^2*x + 2*a*b) + (-1)^(2*b^2*x + 2*a*b)*A
*b*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*B*a*log(2*a*b*x/abs(x) +
2*a^2/abs(x)) - (-1)^(2*a*b*x + 2*a^2)*A*b*log(2*a*b*x/abs(x) + 2*a^2/abs(
x)) + sqrt(b^2*x^2 + 2*a*b*x + a^2)*B - sqrt(b^2*x^2 + 2*a*b*x + a^2)*A/x
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = Bbx\text{sgn}(bx+a) + (Ba\text{sgn}(bx+a)+Ab\text{sgn}(bx+a))\log(|x|) - \frac{Aa\text{sgn}(bx+a)}{x}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
B*b*x*sgn(b*x + a) + (B*a*sgn(b*x + a) + A*b*sgn(b*x + a))*log(abs(x)) - A
*a*sgn(b*x + a)/x
```

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.01

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = B\sqrt{a^2+2\,a\,b\,x+b^2\,x^2} - B\,\ln\left(a\,b+\frac{a^2}{x} + \frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)\sqrt{a^2} + A\,\ln\left(a\,b+\sqrt{(a+b\,x)^2}\,\sqrt{b^2}+b^2\,x\right)\sqrt{b^2} - \frac{A\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x} + \frac{B\,a\,b\,\ln\left(a\,b+\sqrt{(a+b\,x)^2}\,\sqrt{b^2}+b^2\,x\right)}{\sqrt{b^2}} - \frac{A\,a\,b\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^2,x)
```

output
```
B*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2) - B*log(a*b + a^2/x + ((a^2)^(1/2)*(a^2
+ b^2*x^2 + 2*a*b*x)^(1/2))/x)*(a^2)^(1/2) + A*log(a*b + ((a + b*x)^2)^(1/
2)*(b^2)^(1/2) + b^2*x)*(b^2)^(1/2) - (A*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/
x + (B*a*b*log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x))/(b^2)^(1/2)
 - (A*a*b*log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/
x))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx = \frac{2\log(x)\,abx-a^2+b^2x^2}{x}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^2,x)
```

output
```
(2*log(x)*a*b*x - a**2 + b**2*x**2)/x
```

3.289 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx = -\frac{aA\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{bB\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/2*a*A*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-(A*b+B*a)*((b*x+a)^2)^(1/2)/x/(b*x+
a)+b*B*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(108) = 216.

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx =$$

$$-\frac{-a\sqrt{a^2}A-2\sqrt{a^2}Abx-2a\sqrt{a^2}Bx+aA\sqrt{(a+bx)^2}+Abx\sqrt{(a+bx)^2}+2aBx\sqrt{(a+bx)^2}+4abBx^2}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^3,x]
```

output
```
-1/4*(-(a*Sqrt[a^2]*A) - 2*Sqrt[a^2]*A*b*x - 2*a*Sqrt[a^2]*B*x + a*A*Sqrt[
(a + b*x)^2] + A*b*x*Sqrt[(a + b*x)^2] + 2*a*B*x*Sqrt[(a + b*x)^2] + 4*a*b
*B*x^2*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] + 4*Sqrt[a^2]*b*B*x^
2*Log[x] - 2*Sqrt[a^2]*b*B*x^2*Log[a*(Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2])
] - 2*Sqrt[a^2]*b*B*x^2*Log[a*(Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2])])/(a*x
^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^3} \, dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^3}dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^3}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^3}+\frac{bB}{x}+\frac{Ab+aB}{x^2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{aB+Ab}{x}-\frac{aA}{2x^2}+bB\log(x)\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*(a*A)/x^2 - (A*b + a*B)/x + b*B*Log[x
]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\operatorname{csgn}(bx+a)\left(-2B\ln(-bx)b\,x^{2}+2Abx+2Bax+Aa\right)}{2x^{2}}$	38
risch	$\frac{\sqrt{(bx+a)^{2}}\left((-Ab-Ba)x-\frac{Aa}{2}\right)}{(bx+a)x^{2}}+\frac{bB\sqrt{(bx+a)^{2}}\,\ln(x)}{bx+a}$	59

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x+a)*(-2*B*ln(-b*x)*b*x^2+2*A*b*x+2*B*a*x+A*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx=\frac{2\,Bbx^2\log\left(x\right)-Aa-2\,(Ba+Ab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*b*x^2*log(x) - A*a - 2*(B*a + A*b)*x)/x^2
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx=\int \frac{(A+Bx)\,\sqrt{(a+bx)^2}}{x^3}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**3,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx = (-1)^{2\,b^2x+2\,ab}\,Bb\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,Bb\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^2}{2\,a^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}B}{x} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab}{2\,ax} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}A}{2\,a^2x^2}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*b*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*B
*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
A*b^2/a^2 - sqrt(b^2*x^2 + 2*a*b*x + a^2)*B/x + 1/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2)*A*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx = Bb\log\left(|x|\right)\mathrm{sgn}(bx+a) - \frac{Aa\mathrm{sgn}(bx+a)+2\left(Ba\mathrm{sgn}(bx+a)+Ab\mathrm{sgn}(bx+a)\right)x}{2\,x^2}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
B*b*log(abs(x))*sgn(b*x + a) - 1/2*(A*a*sgn(b*x + a) + 2*(B*a*sgn(b*x + a)
 + A*b*sgn(b*x + a))*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx = B\,\ln\left(a\,b+\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{b^2}+b^2\,x\right)\,\sqrt{b^2} - \frac{B\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x} - \frac{A\,\sqrt{{\left(a+b\,x\right)}^2}\,\left(a+2\,b\,x\right)}{2\,x^2\,\left(a+b\,x\right)} - \frac{B\,a\,b\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^3,x)
```

output
```
B*log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x)*(b^2)^(1/2) - (B*(a^2
 + b^2*x^2 + 2*a*b*x)^(1/2))/x - (A*((a + b*x)^2)^(1/2)*(a + 2*b*x))/(2*x^
2*(a + b*x)) - (B*a*b*log(a*b + a^2/x + ((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*
b*x)^(1/2))/x))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx = \frac{2\log(x)\,b^2x^2-a^2-4abx}{2x^2}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^3,x)
```

output
```
(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x)/(2*x**2)
```

3.290 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = \frac{(Ab-aB)(a+bx)\sqrt{a^2+2abx+b^2x^2}}{2a^2x^2} - \frac{A(a^2+2abx+b^2x^2)^{3/2}}{3a^2x^3}$$

output
```
1/2*(A*b-B*a)*(b*x+a)*((b*x+a)^2)^(1/2)/a^2/x^2-1/3*A*(b^2*x^2+2*a*b*x+a^2
)^(3/2)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = -\frac{\sqrt{(a+bx)^2}(3bx(A+2Bx)+a(2A+3Bx))}{6x^3(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^4,x]
```

output
```
-1/6*(Sqrt[(a + b*x)^2]*(3*b*x*(A + 2*B*x) + a*(2*A + 3*B*x)))/(x^3*(a + b
*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1186, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \text{ 1186}$$

$$-\frac{(Ab-aB)\int \frac{\sqrt{a^2+2bxa+b^2x^2}}{x^3}dx}{a}-\frac{A\left(a^2+2abx+b^2x^2\right)^{3/2}}{3a^2x^3}$$

$$\Big\downarrow \text{ 1102}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{b(a+bx)}{x^3}dx}{ab(a+bx)}-\frac{A\left(a^2+2abx+b^2x^2\right)^{3/2}}{3a^2x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{a+bx}{x^3}dx}{a(a+bx)}-\frac{A\left(a^2+2abx+b^2x^2\right)^{3/2}}{3a^2x^3}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(a+bx)\sqrt{a^2+2abx+b^2x^2}(Ab-aB)}{2a^2x^2}-\frac{A\left(a^2+2abx+b^2x^2\right)^{3/2}}{3a^2x^3}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^4,x]
```

output
```
((A*b - a*B)*(a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(2*a^2*x^2) - (A*(a^
2 + 2*a*b*x + b^2*x^2)^(3/2))/(3*a^2*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1186
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[-2*c*(e*f - d*g)*(d + e*x)^(m + 1)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e)^2)), x] + Simp[(2*c*f - b*g)/
(2*c*d - b*e)   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[
{a, b, c, d, e, f, g, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[m + 2*p + 3,
0] && NeQ[2*c*f - b*g, 0] && NeQ[2*c*d - b*e, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

method	result	size
default	$-\frac{\operatorname{csgn}\left(b x+a\right) \left(6 B b \,x^{2}+3 A b x+3 B a x+2 A a\right)}{6 x^{3}}$	34
gosper	$-\frac{\left(6 B b \,x^{2}+3 A b x+3 B a x+2 A a\right) \sqrt{\left(b x+a\right)^{2}}}{6 x^{3} \left(b x+a\right)}$	44
risch	$\frac{\left(-B b \,x^{2}+\left(-\frac{A b}{2}-\frac{B a}{2}\right) x-\frac{A a}{3}\right) \sqrt{\left(b x+a\right)^{2}}}{x^{3} \left(b x+a\right)}$	44
orering	$-\frac{\left(6 B b \,x^{2}+3 A b x+3 B a x+2 A a\right) \sqrt{\left(b x+a\right)^{2}}}{6 x^{3} \left(b x+a\right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x+a)*(6*B*b*x^2+3*A*b*x+3*B*a*x+2*A*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = -\frac{6\,Bbx^2+2\,Aa+3\,(Ba+Ab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(6*B*b*x^2 + 2*A*a + 3*(B*a + A*b)*x)/x^3
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x^4}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.60

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^2}{2\,a^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^3}{2\,a^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb}{2\,ax} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^2}{2\,a^2x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}B}{2\,a^2x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab}{2\,a^3x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}A}{3\,a^2x^3}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^2/a^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*A*b^3/a^3 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b/(a*x) - 1/2*sqrt(b
^2*x^2 + 2*a*b*x + a^2)*A*b^2/(a^2*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2
)*B/(a^2*x^2) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b/(a^3*x^2) - 1/3*(b
^2*x^2 + 2*a*b*x + a^2)^(3/2)*A/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = -\frac{(3\,Bab^2-Ab^3)\mathrm{sgn}(bx+a)}{6\,a^2} - \frac{6\,Bbx^2\mathrm{sgn}(bx+a)+3\,Bax\mathrm{sgn}(bx+a)+3\,Abx\mathrm{sgn}(bx+a)+2\,Aa\mathrm{sgn}(bx+a)}{6\,x^3}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/6*(3*B*a*b^2 - A*b^3)*sgn(b*x + a)/a^2 - 1/6*(6*B*b*x^2*sgn(b*x + a) +
3*B*a*x*sgn(b*x + a) + 3*A*b*x*sgn(b*x + a) + 2*A*a*sgn(b*x + a))/x^3
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = -\frac{\sqrt{(a+b\,x)^2}\,(2\,A\,a+3\,A\,b\,x+3\,B\,a\,x+6\,B\,b\,x^2)}{6\,x^3\,(a+b\,x)}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^4,x)
```

output
```
-(((a + b*x)^2)^(1/2)*(2*A*a + 3*A*b*x + 3*B*a*x + 6*B*b*x^2))/(6*x^3*(a +
 b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^4}\,dx = \frac{-3b^2x^2-3abx-a^2}{3x^3}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^4,x)
```

output
```
( - a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

3.291 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{aA\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{bB\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)}$$

output
```
-1/4*a*A*((b*x+a)^2)^(1/2)/x^4/(b*x+a)-1/3*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^3
/(b*x+a)-1/2*b*B*((b*x+a)^2)^(1/2)/x^2/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{\sqrt{(a+bx)^2}(3aA+4Abx+4aBx+6bBx^2)}{12x^4(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^5,x]
```

output
```
-1/12*(Sqrt[(a + b*x)^2]*(3*a*A + 4*A*b*x + 4*a*B*x + 6*b*B*x^2))/(x^4*(a
+ b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^5}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^5}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^5}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^5}+\frac{bB}{x^3}+\frac{Ab+aB}{x^4}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{aB+Ab}{3x^3}-\frac{aA}{4x^4}-\frac{bB}{2x^2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^5,x]
```

output
```
((-1/4*(a*A)/x^4 - (A*b + a*B)/(3*x^3) - (b*B)/(2*x^2))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

method	result	size
default	$-\frac{\operatorname{csgn}\left(bx+a\right)\left(6Bb\,x^{2}+4Abx+4Bax+3Aa\right)}{12x^{4}}$	34
gosper	$-\frac{\left(6Bb\,x^{2}+4Abx+4Bax+3Aa\right)\sqrt{\left(bx+a\right)^{2}}}{12x^{4}\left(bx+a\right)}$	44
risch	$\frac{\left(-\frac{Bb\,x^{2}}{2}+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x-\frac{Aa}{4}\right)\sqrt{\left(bx+a\right)^{2}}}{x^{4}\left(bx+a\right)}$	44
orering	$-\frac{\left(6Bb\,x^{2}+4Abx+4Bax+3Aa\right)\sqrt{\left(bx+a\right)^{2}}}{12x^{4}\left(bx+a\right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x+a)*(6*B*b*x^2+4*A*b*x+4*B*a*x+3*A*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{6\,Bbx^2+3\,Aa+4\,(Ba+Ab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*B*b*x^2 + 3*A*a + 4*(B*a + A*b)*x)/x^4
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = \int \frac{(A+Bx)\,\sqrt{(a+bx)^2}}{x^5}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**5,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^3}{2\,a^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^4}{2\,a^4} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^2}{2\,a^2x} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^3}{2\,a^3x} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb}{2\,a^3x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^2}{2\,a^4x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}B}{3\,a^2x^3} + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab}{12\,a^3x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}A}{4\,a^2x^4}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^3/a^3 + 1/2*sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*A*b^4/a^4 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^2/(a^2*x) + 1/2*s
qrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^3/(a^3*x) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)
^(3/2)*B*b/(a^3*x^2) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^2/(a^4*x^2)
 - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B/(a^2*x^3) + 5/12*(b^2*x^2 + 2*a*b
*x + a^2)^(3/2)*A*b/(a^3*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A/(a^2
*x^4)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = \frac{(2\,Bab^3-Ab^4)\mathrm{sgn}(bx+a)}{12\,a^3} - \frac{6\,Bbx^2\mathrm{sgn}(bx+a)+4\,Bax\mathrm{sgn}(bx+a)+4\,Abx\mathrm{sgn}(bx+a)+3\,Aa\mathrm{sgn}(bx+a)}{12\,x^4}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/12*(2*B*a*b^3 - A*b^4)*sgn(b*x + a)/a^3 - 1/12*(6*B*b*x^2*sgn(b*x + a) +
 4*B*a*x*sgn(b*x + a) + 4*A*b*x*sgn(b*x + a) + 3*A*a*sgn(b*x + a))/x^4
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = -\frac{\sqrt{(a+b\,x)^2}\,(3\,A\,a+4\,A\,b\,x+4\,B\,a\,x+6\,B\,b\,x^2)}{12\,x^4\,(a+b\,x)}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^5,x)
```

output
```
-(((a + b*x)^2)^(1/2)*(3*A*a + 4*A*b*x + 4*B*a*x + 6*B*b*x^2))/(12*x^4*(a
+ b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx = \frac{-6b^2x^2-8abx-3a^2}{12x^4}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^5,x)
```

output
```
( - 3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

3.292 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{aA\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{bB\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)}$$

output
```
-1/5*a*A*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^4
/(b*x+a)-1/3*b*B*((b*x+a)^2)^(1/2)/x^3/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{\sqrt{(a+bx)^2}(5bx(3A+4Bx)+3a(4A+5Bx))}{60x^5(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^6,x]
```

output
```
-1/60*(Sqrt[(a + b*x)^2]*(5*b*x*(3*A + 4*B*x) + 3*a*(4*A + 5*B*x)))/(x^5*(
a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^6}dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^6}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^6}+\frac{bB}{x^4}+\frac{Ab+aB}{x^5}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{aB+Ab}{4x^4}-\frac{aA}{5x^5}-\frac{bB}{3x^3}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^6,x]
```

output
```
((-1/5*(a*A)/x^5 - (A*b + a*B)/(4*x^4) - (b*B)/(3*x^3))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

method	result	size
default	$-\frac{\operatorname{csgn}\left(b x +a \right) \left(20 B b \,x^{2}+15 A b x +15 B a x +12 A a \right)}{60 x^{5}}$	34
gosper	$-\frac{\left(20 B b \,x^{2}+15 A b x +15 B a x +12 A a \right) \sqrt{\left(b x +a \right)^{2}}}{60 \left(b x +a \right) x^{5}}$	44
risch	$\frac{\left(-\frac{B b \,x^{2}}{3}+\left(-\frac{A b}{4}-\frac{B a}{4}\right) x -\frac{A a}{5}\right) \sqrt{\left(b x +a \right)^{2}}}{x^{5} \left(b x +a \right)}$	44
orering	$-\frac{\left(20 B b \,x^{2}+15 A b x +15 B a x +12 A a \right) \sqrt{\left(b x +a \right)^{2}}}{60 \left(b x +a \right) x^{5}}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60*csgn(b*x+a)*(20*B*b*x^2+15*A*b*x+15*B*a*x+12*A*a)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{20\,Bbx^2+12\,Aa+15\,(Ba+Ab)x}{60\,x^5}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(20*B*b*x^2 + 12*A*a + 15*(B*a + A*b)*x)/x^5
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x^6}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.76

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^4}{2\,a^4} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^5}{2\,a^5} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^3}{2\,a^3x} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^4}{2\,a^4x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^2}{2\,a^4x^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^3}{2\,a^5x^2} + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb}{12\,a^3x^3} - \frac{9\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^2}{20\,a^4x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}B}{4\,a^2x^4} + \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab}{20\,a^3x^4} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}A}{5\,a^2x^5}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^4/a^4 - 1/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*A*b^5/a^5 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^3/(a^3*x) - 1/2*sq
rt(b^2*x^2 + 2*a*b*x + a^2)*A*b^4/(a^4*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^
(3/2)*B*b^2/(a^4*x^2) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^3/(a^5*x^2
) + 5/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b/(a^3*x^3) - 9/20*(b^2*x^2 + 2
*a*b*x + a^2)^(3/2)*A*b^2/(a^4*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*
B/(a^2*x^4) + 7/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b/(a^3*x^4) - 1/5*(b^
2*x^2 + 2*a*b*x + a^2)^(3/2)*A/(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{(5\,Bab^4-3\,Ab^5)\mathrm{sgn}(bx+a)}{60\,a^4} - \frac{20\,Bbx^2\mathrm{sgn}(bx+a)+15\,Bax\mathrm{sgn}(bx+a)+15\,Abx\mathrm{sgn}(bx+a)+12\,Aa\mathrm{sgn}(bx+a)}{60\,x^5}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/60*(5*B*a*b^4 - 3*A*b^5)*sgn(b*x + a)/a^4 - 1/60*(20*B*b*x^2*sgn(b*x +
a) + 15*B*a*x*sgn(b*x + a) + 15*A*b*x*sgn(b*x + a) + 12*A*a*sgn(b*x + a))/
x^5
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = -\frac{\sqrt{(a+b\,x)^2}\,(12\,A\,a+15\,A\,b\,x+15\,B\,a\,x+20\,B\,b\,x^2)}{60\,x^5\,(a+b\,x)}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^6,x)
```

output
```
-(((a + b*x)^2)^(1/2)*(12*A*a + 15*A*b*x + 15*B*a*x + 20*B*b*x^2))/(60*x^5
*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx = \frac{-10b^2x^2-15abx-6a^2}{30x^5}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

3.293 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = -\frac{aA\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)} - \frac{(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{bB\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}$$

output
```
-1/6*a*A*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-1/5*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^5
/(b*x+a)-1/4*b*B*((b*x+a)^2)^(1/2)/x^4/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = -\frac{\sqrt{(a+bx)^2}(3bx(4A+5Bx)+2a(5A+6Bx))}{60x^6(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^7,x]
```

output
```
-1/60*(Sqrt[(a + b*x)^2]*(3*b*x*(4*A + 5*B*x) + 2*a*(5*A + 6*B*x)))/(x^6*(
a + b*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^7}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^7}dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^7}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^7}+\frac{bB}{x^5}+\frac{Ab+aB}{x^6}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{aB+Ab}{5x^5}-\frac{aA}{6x^6}-\frac{bB}{4x^4}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^7,x]
```

output
```
((-1/6*(a*A)/x^6 - (A*b + a*B)/(5*x^5) - (b*B)/(4*x^4))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.79 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

method	result	size
default	$-\frac{\operatorname{csgn}\left(bx+a\right)\left(15Bb\,x^{2}+12Abx+12Bax+10Aa\right)}{60x^{6}}$	34
gosper	$-\frac{\left(15Bb\,x^{2}+12Abx+12Bax+10Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60x^{6}\left(bx+a\right)}$	44
risch	$\frac{\left(-\frac{Bb\,x^{2}}{4}+\left(-\frac{Ab}{5}-\frac{Ba}{5}\right)x-\frac{Aa}{6}\right)\sqrt{\left(bx+a\right)^{2}}}{x^{6}\left(bx+a\right)}$	44
orering	$-\frac{\left(15Bb\,x^{2}+12Abx+12Bax+10Aa\right)\sqrt{\left(bx+a\right)^{2}}}{60x^{6}\left(bx+a\right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*csgn(b*x+a)*(15*B*b*x^2+12*A*b*x+12*B*a*x+10*A*a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx=-\frac{15\,Bbx^2+10\,Aa+12\,(Ba+Ab)x}{60\,x^6}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(15*B*b*x^2 + 10*A*a + 12*(B*a + A*b)*x)/x^6
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx=\int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x^7}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**7,x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.29

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = -\frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^5}{2\,a^5} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^6}{2\,a^6} - \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^4}{2\,a^4x} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^5}{2\,a^5x} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^3}{2\,a^5x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^4}{2\,a^6x^2} - \frac{9\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^2}{20\,a^4x^3} + \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^3}{15\,a^5x^3} + \frac{7\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb}{20\,a^3x^4} - \frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^2}{5\,a^4x^4} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}B}{5\,a^2x^5} + \frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab}{10\,a^3x^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}A}{6\,a^2x^6}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^5/a^5 + 1/2*sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*A*b^6/a^6 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^4/(a^4*x) + 1/2*s
qrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^5/(a^5*x) + 1/2*(b^2*x^2 + 2*a*b*x + a^2)
^(3/2)*B*b^3/(a^5*x^2) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^4/(a^6*x^
2) - 9/20*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^2/(a^4*x^3) + 7/15*(b^2*x^2
+ 2*a*b*x + a^2)^(3/2)*A*b^3/(a^5*x^3) + 7/20*(b^2*x^2 + 2*a*b*x + a^2)^(3
/2)*B*b/(a^3*x^4) - 2/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^2/(a^4*x^4) -
1/5*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B/(a^2*x^5) + 3/10*(b^2*x^2 + 2*a*b*x
+ a^2)^(3/2)*A*b/(a^3*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A/(a^2*x^
6)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = \frac{(3\,Bab^5-2\,Ab^6)\mathrm{sgn}(bx+a)}{60\,a^5} - \frac{15\,Bbx^2\mathrm{sgn}(bx+a)+12\,Bax\mathrm{sgn}(bx+a)+12\,Abx\mathrm{sgn}(bx+a)+10\,Aa\mathrm{sgn}(bx+a)}{60\,x^6}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/60*(3*B*a*b^5 - 2*A*b^6)*sgn(b*x + a)/a^5 - 1/60*(15*B*b*x^2*sgn(b*x + a
) + 12*B*a*x*sgn(b*x + a) + 12*A*b*x*sgn(b*x + a) + 10*A*a*sgn(b*x + a))/x
^6
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = -\frac{\sqrt{{(a+b\,x)}^2}\,\left(10\,A\,a+12\,A\,b\,x+12\,B\,a\,x+15\,B\,b\,x^2\right)}{60\,x^6\,\left(a+b\,x\right)}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^7,x)
```

output
```
-(((a + b*x)^2)^(1/2)*(10*A*a + 12*A*b*x + 12*B*a*x + 15*B*b*x^2))/(60*x^6
*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^7}\,dx = \frac{-15b^2x^2-24abx-10a^2}{60x^6}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^7,x)
```

output
```
( - 10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

3.294 $\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3Ax^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{a^2(3Ab+aB)x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{3ab(Ab+aB)x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{b^2(Ab+3aB)x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{b^3Bx^{10}\sqrt{a^2+2abx+b^2x^2}}{10(a+bx)}$$

output
```
a^3*A*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)+a^2*(3*A*b+B*a)*x^7*((b*x+a)^2)^(1
/2)/(7*b*x+7*a)+3*a*b*(A*b+B*a)*x^8*((b*x+a)^2)^(1/2)/(8*b*x+8*a)+b^2*(A*b
+3*B*a)*x^9*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+b^3*B*x^10*((b*x+a)^2)^(1/2)/(10
*b*x+10*a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^6\sqrt{(a+bx)^2}(60a^3(7A+6Bx)+135a^2bx(8A+7Bx)+105ab^2x^2(9A+8Bx)+28b^3x^3(1$$
$$2520(a+bx)$$

input
```
Integrate[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^6*Sqrt[(a + b*x)^2]*(60*a^3*(7*A + 6*B*x) + 135*a^2*b*x*(8*A + 7*B*x) +
 105*a*b^2*x^2*(9*A + 8*B*x) + 28*b^3*x^3*(10*A + 9*B*x)))/(2520*(a + b*x)
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^5(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^5(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^3Bx^9+b^2(Ab+3aB)x^8+3ab(Ab+aB)x^7+a^2(3Ab+aB)x^6+a^3Ax^5\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{6}a^3Ax^6+\frac{1}{7}a^2x^7(aB+3Ab)+\frac{1}{9}b^2x^9(3aB+Ab)+\frac{3}{8}abx^8(aB+Ab)+\frac{1}{10}b^3Bx^{10}\right)}{a+bx}$$

input

```
Int[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output

```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*A*x^6)/6 + (a^2*(3*A*b + a*B)*x^7)/7
+ (3*a*b*(A*b + a*B)*x^8)/8 + (b^2*(A*b + 3*a*B)*x^9)/9 + (b^3*B*x^10)/10)
)/(a + b*x)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{x^{6}\left(252B\,b^{3}x^{4}+280A\,b^{3}x^{3}+840Ba\,b^{2}x^{3}+945Aa\,b^{2}x^{2}+945B\,a^{2}b\,x^{2}+1080A\,a^{2}bx+360B\,a^{3}x+420a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
default	$\frac{x^{6}\left(252B\,b^{3}x^{4}+280A\,b^{3}x^{3}+840Ba\,b^{2}x^{3}+945Aa\,b^{2}x^{2}+945B\,a^{2}b\,x^{2}+1080A\,a^{2}bx+360B\,a^{3}x+420a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
orering	$\frac{x^{6}\left(252B\,b^{3}x^{4}+280A\,b^{3}x^{3}+840Ba\,b^{2}x^{3}+945Aa\,b^{2}x^{2}+945B\,a^{2}b\,x^{2}+1080A\,a^{2}bx+360B\,a^{3}x+420a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
risch	$\frac{b^{3}B\,x^{10}\sqrt{\left(bx+a\right)^{2}}}{10bx+10a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(A\,b^{3}+3Ba\,b^{2}\right)x^{9}}{9bx+9a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Aa\,b^{2}+3B\,a^{2}b\right)x^{8}}{8bx+8a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3A\,a^{2}b+B\,a^{3}\right)x^{7}}{7bx+7a}+\frac{a^{3}A}{}$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x^6*(252*B*b^3*x^4+280*A*b^3*x^3+840*B*a*b^2*x^3+945*A*a*b^2*x^2+94
5*B*a^2*b*x^2+1080*A*a^2*b*x+360*B*a^3*x+420*A*a^3)*((b*x+a)^2)^(3/2)/(b*x
+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{10}\,Bb^3x^{10}+\frac{1}{6}\,Aa^3x^6+\frac{1}{9}\left(3\,Bab^2+Ab^3\right)x^9+\frac{3}{8}\left(Ba^2b+Aab^2\right)x^8+\frac{1}{7}\left(Ba^3+3\,Aa^2b\right)x^7$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/10*B*b^3*x^10 + 1/6*A*a^3*x^6 + 1/9*(3*B*a*b^2 + A*b^3)*x^9 + 3/8*(B*a^2
*b + A*a*b^2)*x^8 + 1/7*(B*a^3 + 3*A*a^2*b)*x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8803 vs. $2(155) = 310$.

Time = 1.02 (sec) , antiderivative size = 8803, normalized size of antiderivative = 41.92

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**9/10 + x**8*(A*b**4
 + 21*B*a*b**3/10)/(9*b**2) + x**7*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*
(A*b**4 + 21*B*a*b**3/10)/(9*b))/(8*b**2) + x**6*(6*A*a**2*b**2 + 4*B*a**3
*b - 8*a**2*(A*b**4 + 21*B*a*b**3/10)/(9*b**2) - 15*a*(4*A*a*b**3 + 51*B*a
**2*b**2/10 - 17*a*(A*b**4 + 21*B*a*b**3/10)/(9*b))/(8*b))/(7*b**2) + x**5
*(4*A*a**3*b + B*a**4 - 7*a**2*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*(A*b
**4 + 21*B*a*b**3/10)/(9*b))/(8*b**2) - 13*a*(6*A*a**2*b**2 + 4*B*a**3*b -
 8*a**2*(A*b**4 + 21*B*a*b**3/10)/(9*b**2) - 15*a*(4*A*a*b**3 + 51*B*a**2*
b**2/10 - 17*a*(A*b**4 + 21*B*a*b**3/10)/(9*b))/(8*b))/(7*b))/(6*b**2) + x
**4*(A*a**4 - 6*a**2*(6*A*a**2*b**2 + 4*B*a**3*b - 8*a**2*(A*b**4 + 21*B*a
*b**3/10)/(9*b**2) - 15*a*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*(A*b**4 +
 21*B*a*b**3/10)/(9*b))/(8*b))/(7*b**2) - 11*a*(4*A*a**3*b + B*a**4 - 7*a*
*2*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*(A*b**4 + 21*B*a*b**3/10)/(9*b))
/(8*b**2) - 13*a*(6*A*a**2*b**2 + 4*B*a**3*b - 8*a**2*(A*b**4 + 21*B*a*b**
3/10)/(9*b**2) - 15*a*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*(A*b**4 + 21*
B*a*b**3/10)/(9*b))/(8*b))/(7*b))/(6*b))/(5*b**2) + x**3*(-5*a**2*(4*A*a**
3*b + B*a**4 - 7*a**2*(4*A*a*b**3 + 51*B*a**2*b**2/10 - 17*a*(A*b**4 + 21*
B*a*b**3/10)/(9*b))/(8*b**2) - 13*a*(6*A*a**2*b**2 + 4*B*a**3*b - 8*a**2*(
A*b**4 + 21*B*a*b**3/10)/(9*b**2) - 15*a*(4*A*a*b**3 + 51*B*a**2*b**2/10 -
 17*a*(A*b**4 + 21*B*a*b**3/10)/(9*b))/(8*b))/(7*b))/(6*b**2) - 9*a*(A*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.00

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bx^5}{10\,b^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bax^4}{6\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ax^4}{9\,b^2} + \frac{5\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^2x^3}{24\,b^4} - \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aax^3}{72\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^6x}{4\,b^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^5x}{4\,b^5} - \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^3x^2}{56\,b^5} + \frac{37\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^2x^2}{168\,b^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^7}{4\,b^7} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^6}{4\,b^6} + \frac{41\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^4x}{168\,b^6} - \frac{121\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^3x}{504\,b^5} - \frac{209\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^5}{840\,b^7} + \frac{125\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^4}{504\,b^6}$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/10*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*x^5/b^2 - 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*B*a*x^4/b^3 + 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*x^4/b^2 + 5
/24*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^2*x^3/b^4 - 13/72*(b^2*x^2 + 2*a*b
*x + a^2)^(5/2)*A*a*x^3/b^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^6*x/
b^6 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^5*x/b^5 - 13/56*(b^2*x^2 + 2
*a*b*x + a^2)^(5/2)*B*a^3*x^2/b^5 + 37/168*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*A*a^2*x^2/b^4 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^7/b^7 - 1/4*(b^2*
x^2 + 2*a*b*x + a^2)^(3/2)*A*a^6/b^6 + 41/168*(b^2*x^2 + 2*a*b*x + a^2)^(5
/2)*B*a^4*x/b^6 - 121/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^3*x/b^5 - 20
9/840*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^5/b^7 + 125/504*(b^2*x^2 + 2*a*b
*x + a^2)^(5/2)*A*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{10}Bb^3x^{10}\mathrm{sgn}(bx+a)+\frac{1}{3}Bab^2x^9\mathrm{sgn}(bx+a)+\frac{1}{9}Ab^3x^9\mathrm{sgn}(bx+a)+\frac{3}{8}Ba^2bx^8\mathrm{sgn}(bx+a)+\frac{3}{8}Aab^2x^8\mathrm{sgn}(bx+a)+\frac{1}{7}Ba^3x^7\mathrm{sgn}(bx+a)+\frac{3}{7}Aa^2bx^7\mathrm{sgn}(bx+a)+\frac{1}{6}Aa^3x^6\mathrm{sgn}(bx+a)+\frac{(3\,Ba^{10}-5\,Aa^9b)\mathrm{sgn}(bx+a)}{2520\,b^7}$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/10*B*b^3*x^10*sgn(b*x + a) + 1/3*B*a*b^2*x^9*sgn(b*x + a) + 1/9*A*b^3*x^
9*sgn(b*x + a) + 3/8*B*a^2*b*x^8*sgn(b*x + a) + 3/8*A*a*b^2*x^8*sgn(b*x +
a) + 1/7*B*a^3*x^7*sgn(b*x + a) + 3/7*A*a^2*b*x^7*sgn(b*x + a) + 1/6*A*a^3
*x^6*sgn(b*x + a) + 1/2520*(3*B*a^10 - 5*A*a^9*b)*sgn(b*x + a)/b^7
```

Mupad [F(-1)]

Timed out.

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^5\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^5*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^5*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^6(126b^4x^4+560a\,b^3x^3+945a^2b^2x^2+720a^3bx+210a^4)}{1260}$$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**6*(210*a**4 + 720*a**3*b*x + 945*a**2*b**2*x**2 + 560*a*b**3*x**3 + 12
6*b**4*x**4))/1260
```

3.295 $\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3Ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^2(3Ab+aB)x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{3ab(Ab+aB)x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{b^2(Ab+3aB)x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{b^3Bx^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}$$

output
```
a^3*A*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+a^2*(3*A*b+B*a)*x^6*((b*x+a)^2)^(1
/2)/(6*b*x+6*a)+3*a*b*(A*b+B*a)*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+b^2*(A*b
+3*B*a)*x^8*((b*x+a)^2)^(1/2)/(8*b*x+8*a)+b^3*B*x^9*((b*x+a)^2)^(1/2)/(9*b
*x+9*a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^5\sqrt{(a+bx)^2}(84a^3(6A+5Bx)+180a^2bx(7A+6Bx)+135ab^2x^2(8A+7Bx)+35b^3x^3(9}{2520(a+bx)}$$

input
```
Integrate[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^5*Sqrt[(a + b*x)^2]*(84*a^3*(6*A + 5*B*x) + 180*a^2*b*x*(7*A + 6*B*x) +
 135*a*b^2*x^2*(8*A + 7*B*x) + 35*b^3*x^3*(9*A + 8*B*x)))/(2520*(a + b*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^4(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^4(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^3Bx^8+b^2(Ab+3aB)x^7+3ab(Ab+aB)x^6+a^2(3Ab+aB)x^5+a^3Ax^4\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{5}a^3Ax^5+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{8}b^2x^8(3aB+Ab)+\frac{3}{7}abx^7(aB+Ab)+\frac{1}{9}b^3Bx^9\right)}{a+bx}$$

input
```
Int[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*A*x^5)/5 + (a^2*(3*A*b + a*B)*x^6)/6
+ (3*a*b*(A*b + a*B)*x^7)/7 + (b^2*(A*b + 3*a*B)*x^8)/8 + (b^3*B*x^9)/9))/
(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{x^{5}\left(280B\,b^{3}x^{4}+315A\,b^{3}x^{3}+945Ba\,b^{2}x^{3}+1080Aa\,b^{2}x^{2}+1080B\,a^{2}b\,x^{2}+1260A\,a^{2}bx+420B\,a^{3}x+504a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
default	$\frac{x^{5}\left(280B\,b^{3}x^{4}+315A\,b^{3}x^{3}+945Ba\,b^{2}x^{3}+1080Aa\,b^{2}x^{2}+1080B\,a^{2}b\,x^{2}+1260A\,a^{2}bx+420B\,a^{3}x+504a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
orering	$\frac{x^{5}\left(280B\,b^{3}x^{4}+315A\,b^{3}x^{3}+945Ba\,b^{2}x^{3}+1080Aa\,b^{2}x^{2}+1080B\,a^{2}b\,x^{2}+1260A\,a^{2}bx+420B\,a^{3}x+504a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{2520\left(bx+a\right)^{3}}$
risch	$\frac{b^{3}B\,x^{9}\sqrt{\left(bx+a\right)^{2}}}{9bx+9a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(A\,b^{3}+3Ba\,b^{2}\right)x^{8}}{8bx+8a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Aa\,b^{2}+3B\,a^{2}b\right)x^{7}}{7bx+7a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3A\,a^{2}b+B\,a^{3}\right)x^{6}}{6bx+6a}+\frac{a^{3}A}{}$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x^5*(280*B*b^3*x^4+315*A*b^3*x^3+945*B*a*b^2*x^3+1080*A*a*b^2*x^2+1
080*B*a^2*b*x^2+1260*A*a^2*b*x+420*B*a^3*x+504*A*a^3)*((b*x+a)^2)^(3/2)/(b
*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{9}\,Bb^3x^9+\frac{1}{5}\,Aa^3x^5+\frac{1}{8}\,(3\,Bab^2+Ab^3)x^8+\frac{3}{7}\,(Ba^2b+Aab^2)x^7+\frac{1}{6}\,(Ba^3+3\,Aa^2b)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/9*B*b^3*x^9 + 1/5*A*a^3*x^5 + 1/8*(3*B*a*b^2 + A*b^3)*x^8 + 3/7*(B*a^2*b
 + A*a*b^2)*x^7 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5448 vs. $2(155) = 310$.

Time = 0.94 (sec) , antiderivative size = 5448, normalized size of antiderivative = 25.94

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**8/9 + x**7*(A*b**4
+ 19*B*a*b**3/9)/(8*b**2) + x**6*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*
b**4 + 19*B*a*b**3/9)/(8*b))/(7*b**2) + x**5*(6*A*a**2*b**2 + 4*B*a**3*b -
 7*a**2*(A*b**4 + 19*B*a*b**3/9)/(8*b**2) - 13*a*(4*A*a*b**3 + 46*B*a**2*b
**2/9 - 15*a*(A*b**4 + 19*B*a*b**3/9)/(8*b))/(7*b))/(6*b**2) + x**4*(4*A*a
**3*b + B*a**4 - 6*a**2*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19
*B*a*b**3/9)/(8*b))/(7*b**2) - 11*a*(6*A*a**2*b**2 + 4*B*a**3*b - 7*a**2*(
A*b**4 + 19*B*a*b**3/9)/(8*b**2) - 13*a*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 1
5*a*(A*b**4 + 19*B*a*b**3/9)/(8*b))/(7*b))/(6*b))/(5*b**2) + x**3*(A*a**4
- 5*a**2*(6*A*a**2*b**2 + 4*B*a**3*b - 7*a**2*(A*b**4 + 19*B*a*b**3/9)/(8*
b**2) - 13*a*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19*B*a*b**3/9
)/(8*b))/(7*b))/(6*b**2) - 9*a*(4*A*a**3*b + B*a**4 - 6*a**2*(4*A*a*b**3 +
 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19*B*a*b**3/9)/(8*b))/(7*b**2) - 11*a*(
6*A*a**2*b**2 + 4*B*a**3*b - 7*a**2*(A*b**4 + 19*B*a*b**3/9)/(8*b**2) - 13
*a*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19*B*a*b**3/9)/(8*b))/(
7*b))/(6*b))/(5*b))/(4*b**2) + x**2*(-4*a**2*(4*A*a**3*b + B*a**4 - 6*a**2
*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19*B*a*b**3/9)/(8*b))/(7*
b**2) - 11*a*(6*A*a**2*b**2 + 4*B*a**3*b - 7*a**2*(A*b**4 + 19*B*a*b**3/9)
/(8*b**2) - 13*a*(4*A*a*b**3 + 46*B*a**2*b**2/9 - 15*a*(A*b**4 + 19*B*a*b*
*3/9)/(8*b))/(7*b))/(6*b))/(5*b**2) - 7*a*(A*a**4 - 5*a**2*(6*A*a**2*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.72

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bx^4}{9\,b^2} - \frac{13\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bax^3}{72\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ax^3}{8\,b^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^5x}{4\,b^5} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aa^4x}{4\,b^4} + \frac{37\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^2x^2}{168\,b^4} - \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aax^2}{56\,b^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ba^6}{4\,b^6} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Aa^5}{4\,b^5} - \frac{121\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^3x}{504\,b^5} + \frac{13\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^2x}{56\,b^4} + \frac{125\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^4}{504\,b^6} - \frac{69\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^3}{280\,b^5}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*x^4/b^2 - 13/72*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*B*a*x^3/b^3 + 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*x^3/b^2 -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^5*x/b^5 + 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*A*a^4*x/b^4 + 37/168*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^2*x^2
/b^4 - 11/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a*x^2/b^3 - 1/4*(b^2*x^2 +
2*a*b*x + a^2)^(3/2)*B*a^6/b^6 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^5
/b^5 - 121/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^3*x/b^5 + 13/56*(b^2*x^
2 + 2*a*b*x + a^2)^(5/2)*A*a^2*x/b^4 + 125/504*(b^2*x^2 + 2*a*b*x + a^2)^(
5/2)*B*a^4/b^6 - 69/280*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{9}\,Bb^3x^9\mathrm{sgn}(bx+a)+\frac{3}{8}\,Bab^2x^8\mathrm{sgn}(bx+a)+\frac{1}{8}\,Ab^3x^8\mathrm{sgn}(bx+a)+\frac{3}{7}\,Ba^2bx^7\mathrm{sgn}(bx+a)+\frac{3}{7}\,Aab^2x^7\mathrm{sgn}(bx+a)+\frac{1}{6}\,Ba^3x^6\mathrm{sgn}(bx+a)+\frac{1}{2}\,Aa^2bx^6\mathrm{sgn}(bx+a)+\frac{1}{5}\,Aa^3x^5\mathrm{sgn}(bx+a)-\frac{(5\,Ba^9-9\,Aa^8b)\mathrm{sgn}(bx+a)}{2520\,b^6}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/9*B*b^3*x^9*sgn(b*x + a) + 3/8*B*a*b^2*x^8*sgn(b*x + a) + 1/8*A*b^3*x^8*
sgn(b*x + a) + 3/7*B*a^2*b*x^7*sgn(b*x + a) + 3/7*A*a*b^2*x^7*sgn(b*x + a)
 + 1/6*B*a^3*x^6*sgn(b*x + a) + 1/2*A*a^2*b*x^6*sgn(b*x + a) + 1/5*A*a^3*x
^5*sgn(b*x + a) - 1/2520*(5*B*a^9 - 9*A*a^8*b)*sgn(b*x + a)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^4\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^5(70b^4x^4+315a\,b^3x^3+540a^2b^2x^2+420a^3bx+126a^4)}{630}$$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**5*(126*a**4 + 420*a**3*b*x + 540*a**2*b**2*x**2 + 315*a*b**3*x**3 + 70
*b**4*x**4))/630
```

3.296 $\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3Ax^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{a^2(3Ab+aB)x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{ab(Ab+aB)x^6\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^2(Ab+3aB)x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{b^3Bx^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}$$

output
```
a^3*A*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+a^2*(3*A*b+B*a)*x^5*((b*x+a)^2)^(1
/2)/(5*b*x+5*a)+a*b*(A*b+B*a)*x^6*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^2*(A*b+3
*B*a)*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+b^3*B*x^8*((b*x+a)^2)^(1/2)/(8*b*x
+8*a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^4\sqrt{(a+bx)^2}(14a^3(5A+4Bx)+28a^2bx(6A+5Bx)+20ab^2x^2(7A+6Bx)+5b^3x^3(8A+}{280(a+bx)}$$

input
```
Integrate[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^4*Sqrt[(a + b*x)^2]*(14*a^3*(5*A + 4*B*x) + 28*a^2*b*x*(6*A + 5*B*x) +
20*a*b^2*x^2*(7*A + 6*B*x) + 5*b^3*x^3*(8*A + 7*B*x)))/(280*(a + b*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^3(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^3Bx^7+b^2(Ab+3aB)x^6+3ab(Ab+aB)x^5+a^2(3Ab+aB)x^4+a^3Ax^3\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{4}a^3Ax^4+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{1}{7}b^2x^7(3aB+Ab)+\frac{1}{2}abx^6(aB+Ab)+\frac{1}{8}b^3Bx^8\right)}{a+bx}$$

input
```
Int[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^5)/5
+ (a*b*(A*b + a*B)*x^6)/2 + (b^2*(A*b + 3*a*B)*x^7)/7 + (b^3*B*x^8)/8))/(a
 + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{x^{4}\left(35 B\,b^{3} x^{4}+40 A\,b^{3} x^{3}+120 B a\,b^{2} x^{3}+140 A a\,b^{2} x^{2}+140 B\,a^{2} b\,x^{2}+168 A\,a^{2} b x+56 B\,a^{3} x+70 a^{3} A\right) \left(\left(b x+a\right)^{2}\right)^{\frac{3}{2}}}{280 \left(b x+a\right)^{3}}$
default	$\frac{x^{4}\left(35 B\,b^{3} x^{4}+40 A\,b^{3} x^{3}+120 B a\,b^{2} x^{3}+140 A a\,b^{2} x^{2}+140 B\,a^{2} b\,x^{2}+168 A\,a^{2} b x+56 B\,a^{3} x+70 a^{3} A\right) \left(\left(b x+a\right)^{2}\right)^{\frac{3}{2}}}{280 \left(b x+a\right)^{3}}$
orering	$\frac{x^{4}\left(35 B\,b^{3} x^{4}+40 A\,b^{3} x^{3}+120 B a\,b^{2} x^{3}+140 A a\,b^{2} x^{2}+140 B\,a^{2} b\,x^{2}+168 A\,a^{2} b x+56 B\,a^{3} x+70 a^{3} A\right) \left(b^{2} x^{2}+2 a b x+a^{2}\right)^{\frac{3}{2}}}{280 \left(b x+a\right)^{3}}$
risch	$\frac{b^{3} B\,x^{8} \sqrt{\left(b x+a\right)^{2}}}{8 b x+8 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(A\,b^{3}+3 B a\,b^{2}\right) x^{7}}{7 b x+7 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(3 A a\,b^{2}+3 B\,a^{2} b\right) x^{6}}{6 b x+6 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(3 A\,a^{2} b+B\,a^{3}\right) x^{5}}{5 b x+5 a}+\frac{a^{3} A}{}$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*x^4*(35*B*b^3*x^4+40*A*b^3*x^3+120*B*a*b^2*x^3+140*A*a*b^2*x^2+140*B
*a^2*b*x^2+168*A*a^2*b*x+56*B*a^3*x+70*A*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{8}\,Bb^3x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\left(3\,Bab^2+Ab^3\right)x^7+\frac{1}{2}\left(Ba^2b+Aab^2\right)x^6+\frac{1}{5}\left(Ba^3+3\,Aa^2b\right)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*B*b^3*x^8 + 1/4*A*a^3*x^4 + 1/7*(3*B*a*b^2 + A*b^3)*x^7 + 1/2*(B*a^2*b
 + A*a*b^2)*x^6 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3364 vs. $2(153) = 306$.

Time = 0.87 (sec) , antiderivative size = 3364, normalized size of antiderivative = 16.02

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**7/8 + x**6*(A*b**4
+ 17*B*a*b**3/8)/(7*b**2) + x**5*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*
b**4 + 17*B*a*b**3/8)/(7*b))/(6*b**2) + x**4*(6*A*a**2*b**2 + 4*B*a**3*b -
 6*a**2*(A*b**4 + 17*B*a*b**3/8)/(7*b**2) - 11*a*(4*A*a*b**3 + 41*B*a**2*b
**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)/(7*b))/(6*b))/(5*b**2) + x**3*(4*A*a
**3*b + B*a**4 - 5*a**2*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17
*B*a*b**3/8)/(7*b))/(6*b**2) - 9*a*(6*A*a**2*b**2 + 4*B*a**3*b - 6*a**2*(A
*b**4 + 17*B*a*b**3/8)/(7*b**2) - 11*a*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13
*a*(A*b**4 + 17*B*a*b**3/8)/(7*b))/(6*b))/(5*b))/(4*b**2) + x**2*(A*a**4 -
 4*a**2*(6*A*a**2*b**2 + 4*B*a**3*b - 6*a**2*(A*b**4 + 17*B*a*b**3/8)/(7*b
**2) - 11*a*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)
/(7*b))/(6*b))/(5*b**2) - 7*a*(4*A*a**3*b + B*a**4 - 5*a**2*(4*A*a*b**3 +
41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)/(7*b))/(6*b**2) - 9*a*(6*
A*a**2*b**2 + 4*B*a**3*b - 6*a**2*(A*b**4 + 17*B*a*b**3/8)/(7*b**2) - 11*a
*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)/(7*b))/(6*
b))/(5*b))/(4*b))/(3*b**2) + x*(-3*a**2*(4*A*a**3*b + B*a**4 - 5*a**2*(4*A
*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)/(7*b))/(6*b**2)
 - 9*a*(6*A*a**2*b**2 + 4*B*a**3*b - 6*a**2*(A*b**4 + 17*B*a*b**3/8)/(7*b*
*2) - 11*a*(4*A*a*b**3 + 41*B*a**2*b**2/8 - 13*a*(A*b**4 + 17*B*a*b**3/8)/
(7*b))/(6*b))/(5*b))/(4*b**2) - 5*a*(A*a**4 - 4*a**2*(6*A*a**2*b**2 + 4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.43

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bx^3}{8\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^4x}{4\,b^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^3x}{4\,b^3} - \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bax^2}{56\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ax^2}{7\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^5}{4\,b^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^4}{4\,b^4} + \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^2x}{56\,b^4} - \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aax}{14\,b^3} - \frac{69\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^3}{280\,b^5} + \frac{17\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^2}{70\,b^4}$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*x^3/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*B*a^4*x/b^4 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^3*x/b^3 -
11/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a*x^2/b^3 + 1/7*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*A*x^2/b^2 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^5/b^5 -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^4/b^4 + 13/56*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*B*a^2*x/b^4 - 3/14*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a*x/b^3 -
 69/280*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^3/b^5 + 17/70*(b^2*x^2 + 2*a*b
*x + a^2)^(5/2)*A*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{8}Bb^3x^8\mathrm{sgn}(bx+a)+\frac{3}{7}Bab^2x^7\mathrm{sgn}(bx+a)+\frac{1}{7}Ab^3x^7\mathrm{sgn}(bx+a)+\frac{1}{2}Ba^2bx^6\mathrm{sgn}(bx+a)+\frac{1}{2}Aab^2x^6\mathrm{sgn}(bx+a)+\frac{1}{5}Ba^3x^5\mathrm{sgn}(bx+a)+\frac{3}{5}Aa^2bx^5\mathrm{sgn}(bx+a)+\frac{1}{4}Aa^3x^4\mathrm{sgn}(bx+a)+\frac{(Ba^8-2\,Aa^7b)\mathrm{sgn}(bx+a)}{280\,b^5}$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*B*b^3*x^8*sgn(b*x + a) + 3/7*B*a*b^2*x^7*sgn(b*x + a) + 1/7*A*b^3*x^7*
sgn(b*x + a) + 1/2*B*a^2*b*x^6*sgn(b*x + a) + 1/2*A*a*b^2*x^6*sgn(b*x + a)
 + 1/5*B*a^3*x^5*sgn(b*x + a) + 3/5*A*a^2*b*x^5*sgn(b*x + a) + 1/4*A*a^3*x
^4*sgn(b*x + a) + 1/280*(B*a^8 - 2*A*a^7*b)*sgn(b*x + a)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^3\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^4(35b^4x^4+160a\,b^3x^3+280a^2b^2x^2+224a^3bx+70a^4)}{280}$$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**4*(70*a**4 + 224*a**3*b*x + 280*a**2*b**2*x**2 + 160*a*b**3*x**3 + 35*
b**4*x**4))/280
```

3.297 $\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{a^3Ax^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{a^2(3Ab+aB)x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{3ab(Ab+aB)x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{b^2(Ab+3aB)x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{b^3Bx^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
a^3*A*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+a^2*(3*A*b+B*a)*x^4*((b*x+a)^2)^(1
/2)/(4*b*x+4*a)+3*a*b*(A*b+B*a)*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+b^2*(A*b
+3*B*a)*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)+b^3*B*x^7*((b*x+a)^2)^(1/2)/(7*b
*x+7*a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^3\sqrt{(a+bx)^2}(35a^3(4A+3Bx)+63a^2bx(5A+4Bx)+42ab^2x^2(6A+5Bx)+10b^3x^3(7A}{420(a+bx)}$$

input
```
Integrate[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^3*Sqrt[(a + b*x)^2]*(35*a^3*(4*A + 3*B*x) + 63*a^2*b*x*(5*A + 4*B*x) +
42*a*b^2*x^2*(6*A + 5*B*x) + 10*b^3*x^3*(7*A + 6*B*x)))/(420*(a + b*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^2(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^6+b^2(Ab+3aB)x^5+3ab(Ab+aB)x^4+a^2(3Ab+aB)x^3+a^3Ax^2\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{6}b^2x^6(3aB+Ab)+\frac{3}{5}abx^5(aB+Ab)+\frac{1}{7}b^3Bx^7\right)}{a+bx}$$

input
```
Int[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^4)/4
+ (3*a*b*(A*b + a*B)*x^5)/5 + (b^2*(A*b + 3*a*B)*x^6)/6 + (b^3*B*x^7)/7))/
(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{x^3(60Bb^3x^4+70Ab^3x^3+210Bab^2x^3+252Aab^2x^2+252Ba^2bx^2+315Aa^2bx+105Ba^3x+140a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{420(bx+a)^3}$
default	$\frac{x^3(60Bb^3x^4+70Ab^3x^3+210Bab^2x^3+252Aab^2x^2+252Ba^2bx^2+315Aa^2bx+105Ba^3x+140a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{420(bx+a)^3}$
orering	$\frac{x^3(60Bb^3x^4+70Ab^3x^3+210Bab^2x^3+252Aab^2x^2+252Ba^2bx^2+315Aa^2bx+105Ba^3x+140a^3A)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{420(bx+a)^3}$
risch	$\frac{b^3Bx^7\sqrt{(bx+a)^2}}{7bx+7a}+\frac{\sqrt{(bx+a)^2}\,(Ab^3+3Bab^2)x^6}{6bx+6a}+\frac{\sqrt{(bx+a)^2}\,(3Aab^2+3Ba^2b)x^5}{5bx+5a}+\frac{\sqrt{(bx+a)^2}\,(3Aa^2b+Ba^3)x^4}{4bx+4a}+\frac{a^3A}{}$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/420*x^3*(60*B*b^3*x^4+70*A*b^3*x^3+210*B*a*b^2*x^3+252*A*a*b^2*x^2+252*B
*a^2*b*x^2+315*A*a^2*b*x+105*B*a^3*x+140*A*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^
3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{7}Bb^3x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}\left(3Bab^2+Ab^3\right)x^6+\frac{3}{5}\left(Ba^2b+Aab^2\right)x^5+\frac{1}{4}\left(Ba^3+3Aa^2b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*B*b^3*x^7 + 1/3*A*a^3*x^3 + 1/6*(3*B*a*b^2 + A*b^3)*x^6 + 3/5*(B*a^2*b
 + A*a*b^2)*x^5 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. $2(155) = 310$.

Time = 0.83 (sec) , antiderivative size = 2071, normalized size of antiderivative = 9.86

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**6/7 + x**5*(A*b**4
+ 15*B*a*b**3/7)/(6*b**2) + x**4*(4*A*a*b**3 + 36*B*a**2*b**2/7 - 11*a*(A*
b**4 + 15*B*a*b**3/7)/(6*b))/(5*b**2) + x**3*(6*A*a**2*b**2 + 4*B*a**3*b -
 5*a**2*(A*b**4 + 15*B*a*b**3/7)/(6*b**2) - 9*a*(4*A*a*b**3 + 36*B*a**2*b*
*2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*b))/(4*b**2) + x**2*(4*A*a*
*3*b + B*a**4 - 4*a**2*(4*A*a*b**3 + 36*B*a**2*b**2/7 - 11*a*(A*b**4 + 15*
B*a*b**3/7)/(6*b))/(5*b**2) - 7*a*(6*A*a**2*b**2 + 4*B*a**3*b - 5*a**2*(A*
b**4 + 15*B*a*b**3/7)/(6*b**2) - 9*a*(4*A*a*b**3 + 36*B*a**2*b**2/7 - 11*a
*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*b))/(4*b))/(3*b**2) + x*(A*a**4 - 3*a*
*2*(6*A*a**2*b**2 + 4*B*a**3*b - 5*a**2*(A*b**4 + 15*B*a*b**3/7)/(6*b**2)
- 9*a*(4*A*a*b**3 + 36*B*a**2*b**2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b)
)/(5*b))/(4*b**2) - 5*a*(4*A*a**3*b + B*a**4 - 4*a**2*(4*A*a*b**3 + 36*B*a
**2*b**2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*b**2) - 7*a*(6*A*a**2
*b**2 + 4*B*a**3*b - 5*a**2*(A*b**4 + 15*B*a*b**3/7)/(6*b**2) - 9*a*(4*A*a
*b**3 + 36*B*a**2*b**2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*b))/(4*
b))/(3*b))/(2*b**2) + (-2*a**2*(4*A*a**3*b + B*a**4 - 4*a**2*(4*A*a*b**3 +
 36*B*a**2*b**2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*b**2) - 7*a*(6
*A*a**2*b**2 + 4*B*a**3*b - 5*a**2*(A*b**4 + 15*B*a*b**3/7)/(6*b**2) - 9*a
*(4*A*a*b**3 + 36*B*a**2*b**2/7 - 11*a*(A*b**4 + 15*B*a*b**3/7)/(6*b))/(5*
b))/(4*b))/(3*b**2) - 3*a*(A*a**4 - 3*a**2*(6*A*a**2*b**2 + 4*B*a**3*b ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.15

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = -\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^3x}{4\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^2x}{4\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bx^2}{7\,b^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^4}{4\,b^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^3}{4\,b^3} - \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bax}{14\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ax}{6\,b^2} + \frac{17\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^2}{70\,b^4} - \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa}{30\,b^3}$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^3*x/b^3 + 1/4*(b^2*x^2 + 2*a*b*x
+ a^2)^(3/2)*A*a^2*x/b^2 + 1/7*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*x^2/b^2 -
 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^4/b^4 + 1/4*(b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*A*a^3/b^3 - 3/14*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a*x/b^3 + 1/
6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*x/b^2 + 17/70*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*B*a^2/b^4 - 7/30*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{1}{7}\,Bb^3x^7\mathrm{sgn}(bx+a) + \frac{1}{2}\,Bab^2x^6\mathrm{sgn}(bx+a) + \frac{1}{6}\,Ab^3x^6\mathrm{sgn}(bx+a) + \frac{3}{5}\,Ba^2bx^5\mathrm{sgn}(bx+a) + \frac{3}{5}\,Aab^2x^5\mathrm{sgn}(bx+a) + \frac{1}{4}\,Ba^3x^4\mathrm{sgn}(bx+a) + \frac{3}{4}\,Aa^2bx^4\mathrm{sgn}(bx+a) + \frac{1}{3}\,Aa^3x^3\mathrm{sgn}(bx+a) - \frac{(3\,Ba^7-7\,Aa^6b)\mathrm{sgn}(bx+a)}{420\,b^4}$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/7*B*b^3*x^7*sgn(b*x + a) + 1/2*B*a*b^2*x^6*sgn(b*x + a) + 1/6*A*b^3*x^6*
sgn(b*x + a) + 3/5*B*a^2*b*x^5*sgn(b*x + a) + 3/5*A*a*b^2*x^5*sgn(b*x + a)
 + 1/4*B*a^3*x^4*sgn(b*x + a) + 3/4*A*a^2*b*x^4*sgn(b*x + a) + 1/3*A*a^3*x
^3*sgn(b*x + a) - 1/420*(3*B*a^7 - 7*A*a^6*b)*sgn(b*x + a)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^2\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^3(15b^4x^4+70a\,b^3x^3+126a^2b^2x^2+105a^3bx+35a^4)}{105}$$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**3*(35*a**4 + 105*a**3*b*x + 126*a**2*b**2*x**2 + 70*a*b**3*x**3 + 15*b
**4*x**4))/105
```

3.298 $\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=-\frac{a(Ab-aB)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^3}+\frac{(Ab-2aB)(a+bx)^4\sqrt{a^2+2abx+b^2x^2}}{5b^3}+\frac{B(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^3}$$

output
```
-1/4*a*(A*b-B*a)*(b*x+a)^3*((b*x+a)^2)^(1/2)/b^3+1/5*(A*b-2*B*a)*(b*x+a)^4
*((b*x+a)^2)^(1/2)/b^3+1/6*B*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^2\sqrt{(a+bx)^2}(10a^3(3A+2Bx)+15a^2bx(4A+3Bx)+9ab^2x^2(5A+4Bx)+2b^3x^3(6A+}{60(a+bx)}$$

input
```
Integrate[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x^2*Sqrt[(a + b*x)^2]*(10*a^3*(3*A + 2*B*x) + 15*a^2*b*x*(4*A + 3*B*x) +
9*a*b^2*x^2*(5*A + 4*B*x) + 2*b^3*x^3*(6*A + 5*B*x)))/(60*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{B(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^4}{b^2}+\frac{a(aB-Ab)(a+bx)^3}{b^2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(a+bx)^5(Ab-2aB)}{5b^3}-\frac{a(a+bx)^4(Ab-aB)}{4b^3}+\frac{B(a+bx)^6}{6b^3}\right)}{a+bx}$$

input
```
Int[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*(a*(A*b - a*B)*(a + b*x)^4)/b^3 + ((A
*b - 2*a*B)*(a + b*x)^5)/(5*b^3) + (B*(a + b*x)^6)/(6*b^3)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{x^{2}\left(10B\,b^{3}x^{4}+12A\,b^{3}x^{3}+36Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+60A\,a^{2}bx+20B\,a^{3}x+30a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(bx+a\right)^{3}}$
default	$\frac{x^{2}\left(10B\,b^{3}x^{4}+12A\,b^{3}x^{3}+36Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+60A\,a^{2}bx+20B\,a^{3}x+30a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(bx+a\right)^{3}}$
orering	$\frac{x^{2}\left(10B\,b^{3}x^{4}+12A\,b^{3}x^{3}+36Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+60A\,a^{2}bx+20B\,a^{3}x+30a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{60\left(bx+a\right)^{3}}$
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,B\,b^{3}x^{6}}{6bx+6a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(A\,b^{3}+3Ba\,b^{2}\right)x^{5}}{5bx+5a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Aa\,b^{2}+3B\,a^{2}b\right)x^{4}}{4bx+4a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3A\,a^{2}b+B\,a^{3}\right)x^{3}}{3bx+3a}+\sqrt{(b$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^2*(10*B*b^3*x^4+12*A*b^3*x^3+36*B*a*b^2*x^3+45*A*a*b^2*x^2+45*B*a^2
*b*x^2+60*A*a^2*b*x+20*B*a^3*x+30*A*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{6}\,Bb^3x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\,\left(3\,Bab^2+Ab^3\right)x^5+\frac{3}{4}\,\left(Ba^2b+Aab^2\right)x^4+\frac{1}{3}\,\left(Ba^3+3\,Aa^2b\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*B*b^3*x^6 + 1/2*A*a^3*x^2 + 1/5*(3*B*a*b^2 + A*b^3)*x^5 + 3/4*(B*a^2*b
 + A*a*b^2)*x^4 + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. 2(85) = 170.

Time = 1.11 (sec) , antiderivative size = 1258, normalized size of antiderivative = 10.40

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**5/6 + x**4*(A*b**4
+ 13*B*a*b**3/6)/(5*b**2) + x**3*(4*A*a*b**3 + 31*B*a**2*b**2/6 - 9*a*(A*b
**4 + 13*B*a*b**3/6)/(5*b))/(4*b**2) + x**2*(6*A*a**2*b**2 + 4*B*a**3*b -
4*a**2*(A*b**4 + 13*B*a*b**3/6)/(5*b**2) - 7*a*(4*A*a*b**3 + 31*B*a**2*b**
2/6 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b))/(3*b**2) + x*(4*A*a**3*b
+ B*a**4 - 3*a**2*(4*A*a*b**3 + 31*B*a**2*b**2/6 - 9*a*(A*b**4 + 13*B*a*b*
*3/6)/(5*b))/(4*b**2) - 5*a*(6*A*a**2*b**2 + 4*B*a**3*b - 4*a**2*(A*b**4 +
 13*B*a*b**3/6)/(5*b**2) - 7*a*(4*A*a*b**3 + 31*B*a**2*b**2/6 - 9*a*(A*b**
4 + 13*B*a*b**3/6)/(5*b))/(4*b))/(3*b))/(2*b**2) + (A*a**4 - 2*a**2*(6*A*a
**2*b**2 + 4*B*a**3*b - 4*a**2*(A*b**4 + 13*B*a*b**3/6)/(5*b**2) - 7*a*(4*
A*a*b**3 + 31*B*a**2*b**2/6 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b))/(
3*b**2) - 3*a*(4*A*a**3*b + B*a**4 - 3*a**2*(4*A*a*b**3 + 31*B*a**2*b**2/6
 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b**2) - 5*a*(6*A*a**2*b**2 + 4*B
*a**3*b - 4*a**2*(A*b**4 + 13*B*a*b**3/6)/(5*b**2) - 7*a*(4*A*a*b**3 + 31*
B*a**2*b**2/6 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b))/(3*b))/(2*b))/b
**2) + (a/b + x)*(-a**2*(4*A*a**3*b + B*a**4 - 3*a**2*(4*A*a*b**3 + 31*B*a
**2*b**2/6 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b**2) - 5*a*(6*A*a**2*
b**2 + 4*B*a**3*b - 4*a**2*(A*b**4 + 13*B*a*b**3/6)/(5*b**2) - 7*a*(4*A*a*
b**3 + 31*B*a**2*b**2/6 - 9*a*(A*b**4 + 13*B*a*b**3/6)/(5*b))/(4*b))/(3*b)
)/(2*b**2) - a*(A*a**4 - 2*a**2*(6*A*a**2*b**2 + 4*B*a**3*b - 4*a**2*(A...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(83) = 166.

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.51

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^2x}{4\,b^2}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aax}{4\,b}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^3}{4\,b^3}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^2}{4\,b^2}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bx}{6\,b^2}-\frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba}{30\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{5\,b^2}$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^2*x/b^2 - 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*A*a*x/b + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^3/b^3 - 1/4*
(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^2/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^
(5/2)*B*x/b^2 - 7/30*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a/b^3 + 1/5*(b^2*x^
2 + 2*a*b*x + a^2)^(5/2)*A/b^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.22

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{6}\,Bb^3x^6\mathrm{sgn}(bx+a)+\frac{3}{5}\,Bab^2x^5\mathrm{sgn}(bx+a)+\frac{1}{5}\,Ab^3x^5\mathrm{sgn}(bx+a)+\frac{3}{4}\,Ba^2bx^4\mathrm{sgn}(bx+a)+\frac{3}{4}\,Aab^2x^4\mathrm{sgn}(bx+a)+\frac{1}{3}\,Ba^3x^3\mathrm{sgn}(bx+a)+Aa^2bx^3\mathrm{sgn}(bx+a)+\frac{1}{2}\,Aa^3x^2\mathrm{sgn}(bx+a)+\frac{(Ba^6-3\,Aa^5b)\mathrm{sgn}(bx+a)}{60\,b^3}$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/6*B*b^3*x^6*sgn(b*x + a) + 3/5*B*a*b^2*x^5*sgn(b*x + a) + 1/5*A*b^3*x^5*
sgn(b*x + a) + 3/4*B*a^2*b*x^4*sgn(b*x + a) + 3/4*A*a*b^2*x^4*sgn(b*x + a)
 + 1/3*B*a^3*x^3*sgn(b*x + a) + A*a^2*b*x^3*sgn(b*x + a) + 1/2*A*a^3*x^2*s
gn(b*x + a) + 1/60*(B*a^6 - 3*A*a^5*b)*sgn(b*x + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.38

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x^2(5b^4x^4+24a\,b^3x^3+45a^2b^2x^2+40a^3bx+15a^4)}{30}$$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x**2*(15*a**4 + 40*a**3*b*x + 45*a**2*b**2*x**2 + 24*a*b**3*x**3 + 5*b**4
*x**4))/30
```

3.299 $\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^{3/2} \, dx = \frac{(Ab-aB)(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b^2} + \frac{B(a^2+2abx+b^2x^2)^{5/2}}{5b^2}$$

output
```
1/4*(A*b-B*a)*(b*x+a)^3*((b*x+a)^2)^(1/2)/b^2+1/5*B*(b^2*x^2+2*a*b*x+a^2)^
(5/2)/b^2
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int (A+Bx) \left(a^2+2abx +b^2x^2\right)^{3/2} \, dx = \frac{x\sqrt{(a+bx)^2}(10a^3(2A+Bx)+10a^2bx(3A+2Bx)+5ab^2x^2(4A+3Bx)+b^3x^3(5A+4B}{20(a+bx)}$$

input
```
Integrate[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(x*Sqrt[(a + b*x)^2]*(10*a^3*(2*A + B*x) + 10*a^2*b*x*(3*A + 2*B*x) + 5*a*
b^2*x^2*(4*A + 3*B*x) + b^3*x^3*(5*A + 4*B*x)))/(20*(a + b*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{(Ab-aB)\int \left(a^2+2bxa+b^2x^2\right)^{3/2}dx}{b}+\frac{B\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \left(xb^2+ab\right)^3dx}{b^4(a+bx)}+\frac{B\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^3(Ab-aB)}{4b^2}+\frac{B\left(a^2+2abx+b^2x^2\right)^{5/2}}{5b^2}$$

input
```
Int[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((A*b - a*B)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*b^2) + (B*(a^2
+ 2*a*b*x + b^2*x^2)^(5/2))/(5*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

method	result
gosper	$\frac{x\left(4B\,b^{3}x^{4}+5A\,b^{3}x^{3}+15Ba\,b^{2}x^{3}+20Aa\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+30A\,a^{2}bx+10B\,a^{3}x+20a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{20\left(bx+a\right)^{3}}$
default	$\frac{x\left(4B\,b^{3}x^{4}+5A\,b^{3}x^{3}+15Ba\,b^{2}x^{3}+20Aa\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+30A\,a^{2}bx+10B\,a^{3}x+20a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{20\left(bx+a\right)^{3}}$
orering	$\frac{x\left(4B\,b^{3}x^{4}+5A\,b^{3}x^{3}+15Ba\,b^{2}x^{3}+20Aa\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+30A\,a^{2}bx+10B\,a^{3}x+20a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{20\left(bx+a\right)^{3}}$
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,B\,b^{3}x^{5}}{5bx+5a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(A\,b^{3}+3Ba\,b^{2}\right)x^{4}}{4bx+4a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Aa\,b^{2}+3B\,a^{2}b\right)x^{3}}{3bx+3a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3A\,a^{2}b+B\,a^{3}\right)x^{2}}{2bx+2a}+\sqrt{(b$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(4*B*b^3*x^4+5*A*b^3*x^3+15*B*a*b^2*x^3+20*A*a*b^2*x^2+20*B*a^2*b*x
^2+30*A*a^2*b*x+10*B*a^3*x+20*A*a^3)*((b*x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{1}{5}\,Bb^3x^5+Aa^3x+\frac{1}{4}\left(3\,Bab^2+Ab^3\right)x^4+\left(Ba^2b+Aab^2\right)x^3+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*B*b^3*x^5 + A*a^3*x + 1/4*(3*B*a*b^2 + A*b^3)*x^4 + (B*a^2*b + A*a*b^2
)*x^3 + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(56) = 112.

Time = 0.77 (sec) , antiderivative size = 745, normalized size of antiderivative = 10.49

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**2*x**4/5 + x**3*(A*b**4
+ 11*B*a*b**3/5)/(4*b**2) + x**2*(4*A*a*b**3 + 26*B*a**2*b**2/5 - 7*a*(A*b
**4 + 11*B*a*b**3/5)/(4*b))/(3*b**2) + x*(6*A*a**2*b**2 + 4*B*a**3*b - 3*a
**2*(A*b**4 + 11*B*a*b**3/5)/(4*b**2) - 5*a*(4*A*a*b**3 + 26*B*a**2*b**2/5
 - 7*a*(A*b**4 + 11*B*a*b**3/5)/(4*b))/(3*b))/(2*b**2) + (4*A*a**3*b + B*a
**4 - 2*a**2*(4*A*a*b**3 + 26*B*a**2*b**2/5 - 7*a*(A*b**4 + 11*B*a*b**3/5)
/(4*b))/(3*b**2) - 3*a*(6*A*a**2*b**2 + 4*B*a**3*b - 3*a**2*(A*b**4 + 11*B
*a*b**3/5)/(4*b**2) - 5*a*(4*A*a*b**3 + 26*B*a**2*b**2/5 - 7*a*(A*b**4 + 1
1*B*a*b**3/5)/(4*b))/(3*b))/(2*b))/b**2) + (a/b + x)*(A*a**4 - a**2*(6*A*a
**2*b**2 + 4*B*a**3*b - 3*a**2*(A*b**4 + 11*B*a*b**3/5)/(4*b**2) - 5*a*(4*
A*a*b**3 + 26*B*a**2*b**2/5 - 7*a*(A*b**4 + 11*B*a*b**3/5)/(4*b))/(3*b))/(
2*b**2) - a*(4*A*a**3*b + B*a**4 - 2*a**2*(4*A*a*b**3 + 26*B*a**2*b**2/5 -
 7*a*(A*b**4 + 11*B*a*b**3/5)/(4*b))/(3*b**2) - 3*a*(6*A*a**2*b**2 + 4*B*a
**3*b - 3*a**2*(A*b**4 + 11*B*a*b**3/5)/(4*b**2) - 5*a*(4*A*a*b**3 + 26*B*
a**2*b**2/5 - 7*a*(A*b**4 + 11*B*a*b**3/5)/(4*b))/(3*b))/(2*b))/b)*log(a/b
 + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((B*(a**2 + 2*a*b*x)**(7/2)/(
14*a*b) + (a**2 + 2*a*b*x)**(5/2)*(2*A*b - B*a)/(10*b))/(a*b), Ne(a*b, 0))
, ((A*x + B*x**2/2)*(a**2)**(3/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{1}{4}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ax - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bax}{4\,b} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^2}{4\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa}{4\,b} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{5\,b^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*x - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3
/2)*B*a*x/b - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^2/b^2 + 1/4*(b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*A*a/b + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.03

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{1}{5}\,Bb^3x^5\mathrm{sgn}(bx+a)+\frac{3}{4}\,Bab^2x^4\mathrm{sgn}(bx+a)+\frac{1}{4}\,Ab^3x^4\mathrm{sgn}(bx+a)+Ba^2bx^3\mathrm{sgn}(bx+a)+Aab^2x^3\mathrm{sgn}(bx+a)+\frac{1}{2}\,Ba^3x^2\mathrm{sgn}(bx+a)+\frac{3}{2}\,Aa^2bx^2\mathrm{sgn}(bx+a)+Aa^3x\mathrm{sgn}(bx+a)-\frac{(Ba^5-5\,Aa^4b)\mathrm{sgn}(bx+a)}{20\,b^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/5*B*b^3*x^5*sgn(b*x + a) + 3/4*B*a*b^2*x^4*sgn(b*x + a) + 1/4*A*b^3*x^4*
sgn(b*x + a) + B*a^2*b*x^3*sgn(b*x + a) + A*a*b^2*x^3*sgn(b*x + a) + 1/2*B
*a^3*x^2*sgn(b*x + a) + 3/2*A*a^2*b*x^2*sgn(b*x + a) + A*a^3*x*sgn(b*x + a
) - 1/20*(B*a^5 - 5*A*a^4*b)*sgn(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx = \frac{(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,(5\,A\,b-B\,a+4\,B\,b\,x)}{20\,b^2}$$

input
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
((a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)*(5*A*b - B*a + 4*B*b*x))/(20*b^
2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(x*(5*a**4 + 10*a**3*b*x + 10*a**2*b**2*x**2 + 5*a*b**3*x**3 + b**4*x**4))
/5
```

3.300 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 182

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\frac{3a^2Abx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{3aAb^2x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{Ab^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{B(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4b}+\frac{a^3A\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
3*a^2*A*b*x*((b*x+a)^2)^(1/2)/(b*x+a)+3*a*A*b^2*x^2*((b*x+a)^2)^(1/2)/(2*b
*x+2*a)+A*b^3*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+1/4*B*(b*x+a)^3*((b*x+a)^2
)^(1/2)/b+a^3*A*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\frac{\sqrt{(a+bx)^2}(x(12a^3B+18a^2b(2A+Bx)+6ab^2x(3A+2Bx)+b}{12(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*(x*(12*a^3*B + 18*a^2*b*(2*A + B*x) + 6*a*b^2*x*(3*A +
2*B*x) + b^3*x^2*(4*A + 3*B*x)) + 12*a^3*A*Log[x]))/(12*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.45, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x}dx}{a+bx}$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\int \frac{(a+bx)^3}{x}dx+\frac{B(a+bx)^4}{4b}\right)}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\int \left(\frac{a^3}{x}+3ba^2+3b^2xa+b^3x^2\right)dx+\frac{B(a+bx)^4}{4b}\right)}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\left(a^3\log(x)+3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}\right)+\frac{B(a+bx)^4}{4b}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((B*(a + b*x)^4)/(4*b) + A*(3*a^2*b*x + (3*
a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x])))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{3}{2}}\left(3B\,b^3x^4+4A\,b^3x^3+12Ba\,b^2x^3+18Aa\,b^2x^2+18B\,a^2b\,x^2+12a^3A\ln(x)+36A\,a^2bx+12B\,a^3x\right)}{12(bx+a)^3}$
risch	$\frac{\sqrt{(bx+a)^2}\,B\,b^3x^4}{4bx+4a}+\frac{A\,b^3x^3\sqrt{(bx+a)^2}}{3bx+3a}+\frac{\sqrt{(bx+a)^2}\,Ba\,b^2x^3}{bx+a}+\frac{3\sqrt{(bx+a)^2}\,Aa\,b^2x^2}{2(bx+a)}+\frac{3\sqrt{(bx+a)^2}\,B\,a^2b\,x^2}{2(bx+a)}+\frac{3a^2Abx\sqrt{(}}{bx+}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x+a)^2)^(3/2)*(3*B*b^3*x^4+4*A*b^3*x^3+12*B*a*b^2*x^3+18*A*a*b^2*
x^2+18*B*a^2*b*x^2+12*a^3*A*ln(x)+36*A*a^2*b*x+12*B*a^3*x)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{4}\,Bb^3x^4$$
$$+\,Aa^3\log(x)+\frac{1}{3}\left(3\,Bab^2+Ab^3\right)x^3+\frac{3}{2}\left(Ba^2b+Aab^2\right)x^2+\left(Ba^3+3\,Aa^2b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/4*B*b^3*x^4 + A*a^3*log(x) + 1/3*(3*B*a*b^2 + A*b^3)*x^3 + 3/2*(B*a^2*b
+ A*a*b^2)*x^2 + (B*a^3 + 3*A*a^2*b)*x
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=(-1)^{2\,b^2x+2\,ab}Aa^3\log\left(2\,b^2x+2\,ab\right)-(-1)^{2\,abx+2\,a^2}Aa^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+\frac{1}{2}\sqrt{b^2x^2+2\,abx+a^2}Aabx+\frac{3}{2}\sqrt{b^2x^2+2\,abx+a^2}Aa^2+\frac{1}{4}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bx+\frac{1}{3}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}A+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba}{4\,b}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*A*a^3*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)
*A*a^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*A*a*b*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^2 + 1/4*(b^2*x^2 + 2*a
*b*x + a^2)^(3/2)*B*x + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A + 1/4*(b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*B*a/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\frac{1}{4}\,Bb^3x^4\mathrm{sgn}(bx+a)+Bab^2x^3\mathrm{sgn}(bx+a)+\frac{1}{3}\,Ab^3x^3\mathrm{sgn}(bx+a)+\frac{3}{2}\,Ba^2bx^2\mathrm{sgn}(bx+a)+\frac{3}{2}\,Aab^2x^2\mathrm{sgn}(bx+a)+Ba^3x\mathrm{sgn}(bx+a)+3\,Aa^2bx\mathrm{sgn}(bx+a)+Aa^3\log\left(|x|\right)\mathrm{sgn}(bx+a)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/4*B*b^3*x^4*sgn(b*x + a) + B*a*b^2*x^3*sgn(b*x + a) + 1/3*A*b^3*x^3*sgn(
b*x + a) + 3/2*B*a^2*b*x^2*sgn(b*x + a) + 3/2*A*a*b^2*x^2*sgn(b*x + a) + B
*a^3*x*sgn(b*x + a) + 3*A*a^2*b*x*sgn(b*x + a) + A*a^3*log(abs(x))*sgn(b*x
 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}{x} \,d x$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x}\,dx=\log(x)\,a^4+4a^3bx+3a^2b^2x^2+\frac{4a\,b^3x^3}{3}+\frac{b^4x^4}{4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x,x)
```

output
```
(12*log(x)*a**4 + 48*a**3*b*x + 36*a**2*b**2*x**2 + 16*a*b**3*x**3 + 3*b**
4*x**4)/12
```

3.301 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 200

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{3ab(Ab+aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^2(Ab+3aB)x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{b^3Bx^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-a^3*A*((b*x+a)^2)^(1/2)/x/(b*x+a)+3*a*b*(A*b+B*a)*x*((b*x+a)^2)^(1/2)/(b*
x+a)+b^2*(A*b+3*B*a)*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^3*B*x^3*((b*x+a)^
2)^(1/2)/(3*b*x+3*a)+a^2*(3*A*b+B*a)*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}\left(-6a^3A+18a^2bBx^2+9ab^2x^2(2A+Bx)+b^3x^3(3A-\right.}{6x(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-6*a^3*A + 18*a^2*b*B*x^2 + 9*a*b^2*x^2*(2*A + B*x) +
b^3*x^3*(3*A + 2*B*x) + 6*a^2*(3*A*b + a*B)*x*Log[x]))/(6*x*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^2}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^2}+\frac{(3Ab+aB)a^2}{x}+3b(Ab+aB)a+b^3Bx^2+b^2(Ab+3aB)x\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{x}+a^2\log(x)(aB+3Ab)+\frac{1}{2}b^2x^2(3aB+Ab)+3abx(aB+Ab)+\frac{1}{3}b^3Bx^3\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-((a^3*A)/x) + 3*a*b*(A*b + a*B)*x + (b^2*
(A*b + 3*a*B)*x^2)/2 + (b^3*B*x^3)/3 + a^2*(3*A*b + a*B)*Log[x]))/(a + b*x
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}} \left(2B\,b^{3} x^{4}+3A\,b^{3} x^{3}+9Ba\,b^{2} x^{3}+18A\ln\left(x\right) x\,a^{2} b+18Aa\,b^{2} x^{2}+6B\ln\left(x\right) a^{3} x+18B\,a^{2} b\,x^{2}-6a^{3} A\right)}{6x\left(bx+a\right)^{3}}$	96
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\, b\left(\frac{1}{3} x^{3} B\,b^{2}+\frac{1}{2} x^{2} b^{2} A+\frac{3}{2} Ba\,x^{2} b+3abAx+3a^{2} Bx\right)}{bx+a}-\frac{a^{3} A\sqrt{\left(bx+a\right)^{2}}}{x\left(bx+a\right)}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3A\,a^{2} b+B\,a^{3}\right)\ln\left(x\right)}{bx+a}$	117

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(3/2)*(2*B*b^3*x^4+3*A*b^3*x^3+9*B*a*b^2*x^3+18*A*ln(x)*x*
a^2*b+18*A*a*b^2*x^2+6*B*ln(x)*a^3*x+18*B*a^2*b*x^2-6*a^3*A)/x/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = \frac{2\,Bb^3x^4-6\,Aa^3+3\,(3\,Bab^2+Ab^3)x^3+18\,(Ba^2b+Aab^2)x^2+6}{6\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^4 - 6*A*a^3 + 3*(3*B*a*b^2 + A*b^3)*x^3 + 18*(B*a^2*b + A*a
*b^2)*x^2 + 6*(B*a^3 + 3*A*a^2*b)*x*log(x))/x
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**2,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(141) = 282$.

Time = 0.03 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = (-1)^{2\,b^2x+2\,ab}Ba^3\log\left(2\,b^2x+2\,ab\right) + 3\,(-1)^{2\,b^2x+2\,ab}Aa^2b\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}Ba^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) - 3\,(-1)^{2\,abx+2\,a^2}Aa^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Babx + \frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ab^2x + \frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ba^2 + \frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Aab + \frac{1}{3}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}B - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}A}{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*a^3*log(2*b^2*x + 2*a*b) + 3*(-1)^(2*b^2*x + 2*a*
b)*A*a^2*b*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*B*a^3*log(2*a*b*x
/abs(x) + 2*a^2/abs(x)) - 3*(-1)^(2*a*b*x + 2*a^2)*A*a^2*b*log(2*a*b*x/abs
(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a*b*x + 3/2*sqrt
(b^2*x^2 + 2*a*b*x + a^2)*A*b^2*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^
2 + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a*b + 1/3*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*B - (b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A/x
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx = \frac{1}{3}\,Bb^3x^3\mathrm{sgn}(bx+a) + \frac{3}{2}\,Bab^2x^2\mathrm{sgn}(bx+a) + \frac{1}{2}\,Ab^3x^2\mathrm{sgn}(bx+a) + 3\,Ba^2bx\mathrm{sgn}(bx+a) + 3\,Aab^2x\mathrm{sgn}(bx+a) - \frac{Aa^3\mathrm{sgn}(bx+a)}{x} + \left(Ba^3\mathrm{sgn}(bx+a) + 3\,Aa^2b\mathrm{sgn}(bx+a)\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/3*B*b^3*x^3*sgn(b*x + a) + 3/2*B*a*b^2*x^2*sgn(b*x + a) + 1/2*A*b^3*x^2*
sgn(b*x + a) + 3*B*a^2*b*x*sgn(b*x + a) + 3*A*a*b^2*x*sgn(b*x + a) - A*a^3
*sgn(b*x + a)/x + (B*a^3*sgn(b*x + a) + 3*A*a^2*b*sgn(b*x + a))*log(abs(x)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^2,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^2}\,dx=\frac{12\log(x)\,a^3bx-3a^4+18a^2b^2x^2+6a\,b^3x^3+b^4x^4}{3x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^2,x)
```

output
```
(12*log(x)*a**3*b*x - 3*a**4 + 18*a**2*b**2*x**2 + 6*a*b**3*x**3 + b**4*x*
*4)/(3*x)
```

3.302 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 200

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3} \, dx = -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^2(Ab+3aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^3Bx^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{3ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/2*a^3*A*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-a^2*(3*A*b+B*a)*((b*x+a)^2)^(1/2)
/x/(b*x+a)+b^2*(A*b+3*B*a)*x*((b*x+a)^2)^(1/2)/(b*x+a)+b^3*B*x^2*((b*x+a)^
2)^(1/2)/(2*b*x+2*a)+3*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3} \, dx = \frac{\sqrt{(a+bx)^2}\left(-6a^2Abx+6ab^2Bx^3+b^3x^3(2A+Bx)-a^3(A+2B\right.}{2x^2(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-6*a^2*A*b*x + 6*a*b^2*B*x^3 + b^3*x^3*(2*A + B*x) - a
^3*(A + 2*B*x) + 6*a*b*(A*b + a*B)*x^2*Log[x]))/(2*x^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}(A+Bx)}{x^3}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^3}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^3}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^3}+\frac{(3Ab+aB)a^2}{x^2}+\frac{3b(Ab+aB)a}{x}+b^2(Ab+3aB)+b^3Bx\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{2x^2}-\frac{a^2(aB+3Ab)}{x}+b^2x(3aB+Ab)+3ab\log(x)(aB+Ab)+\frac{1}{2}b^3Bx^2\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*(a^3*A)/x^2 - (a^2*(3*A*b + a*B))/x +
 b^2*(A*b + 3*a*B)*x + (b^3*B*x^2)/2 + 3*a*b*(A*b + a*B)*Log[x]))/(a + b*x
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left((bx+a)^2\right)^{\frac{3}{2}} \left(B\,b^3x^4+6A\ln\left(x\right)x^2a\,b^2+2A\,b^3x^3+6B\ln\left(x\right)a^2b\,x^2+6Ba\,b^2x^3-6A\,a^2bx-2B\,a^3x-a^3A\right)}{2(bx+a)^3x^2}$	95
risch	$\frac{\sqrt{(bx+a)^2}\,b^2\left(\frac{1}{2}Bb\,x^2+Abx+3Bax\right)}{bx+a}+\frac{\sqrt{(bx+a)^2}\left(\left(-3A\,a^2b-B\,a^3\right)x-\frac{a^3A}{2}\right)}{(bx+a)x^2}+\frac{3\sqrt{(bx+a)^2}\,\left(Aa\,b^2+B\,a^2b\right)\ln\left(x\right)}{bx+a}$	115

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x+a)^2)^(3/2)*(B*b^3*x^4+6*A*ln(x)*x^2*a*b^2+2*A*b^3*x^3+6*B*ln(x)
*a^2*b*x^2+6*B*a*b^2*x^3-6*A*a^2*b*x-2*B*a^3*x-a^3*A)/(b*x+a)^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = \frac{Bb^3x^4 - Aa^3 + 2\,(3\,Bab^2 + Ab^3)x^3 + 6\,(Ba^2b + Aab^2)x^2\log{(x)} -}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(B*b^3*x^4 - A*a^3 + 2*(3*B*a*b^2 + A*b^3)*x^3 + 6*(B*a^2*b + A*a*b^2)
*x^2*log(x) - 2*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**3,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(141) = 282$.

Time = 0.04 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3}\,dx = 3\,(-1)^{2\,b^2x+2\,ab}\,Ba^2b\log\left(2\,b^2x+2\,ab\right) + 3\,(-1)^{2\,b^2x+2\,ab}\,Aab^2\log\left(2\,b^2x+2\,ab\right) - 3\,(-1)^{2\,abx+2\,a^2}\,Ba^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) - 3\,(-1)^{2\,abx+2\,a^2}\,Aab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Bb^2x + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Ab^3x}{2\,a} + \frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Bab + \frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ab^2 + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^2}{2\,a^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}B}{x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab}{2\,ax} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{2\,a^2x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3*(-1)^(2*b^2*x + 2*a*b)*B*a^2*b*log(2*b^2*x + 2*a*b) + 3*(-1)^(2*b^2*x +
2*a*b)*A*a*b^2*log(2*b^2*x + 2*a*b) - 3*(-1)^(2*a*b*x + 2*a^2)*B*a^2*b*log
(2*a*b*x/abs(x) + 2*a^2/abs(x)) - 3*(-1)^(2*a*b*x + 2*a^2)*A*a*b^2*log(2*a
*b*x/abs(x) + 2*a^2/abs(x)) + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^2*x +
3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^3*x/a + 9/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*B*a*b + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^2 + 1/2*(b^2*x^2 + 2*a
*b*x + a^2)^(3/2)*A*b^2/a^2 - (b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B/x - 1/2*(b
^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5
/2)*A/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\,Bb^3x^2\mathrm{sgn}(bx+a)+3\,Bab^2x\mathrm{sgn}(bx+a) + Ab^3x\mathrm{sgn}(bx+a)+3\,\left(Ba^2b\mathrm{sgn}(bx+a)+Aab^2\mathrm{sgn}(bx+a)\right)\log\left(|x|\right) - \frac{Aa^3\mathrm{sgn}(bx+a)+2\,(Ba^3\mathrm{sgn}(bx+a)+3\,Aa^2b\mathrm{sgn}(bx+a))x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/2*B*b^3*x^2*sgn(b*x + a) + 3*B*a*b^2*x*sgn(b*x + a) + A*b^3*x*sgn(b*x +
a) + 3*(B*a^2*b*sgn(b*x + a) + A*a*b^2*sgn(b*x + a))*log(abs(x)) - 1/2*(A*
a^3*sgn(b*x + a) + 2*(B*a^3*sgn(b*x + a) + 3*A*a^2*b*sgn(b*x + a))*x)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^3,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^3}\,dx=\frac{12\log(x)\,a^2b^2x^2-a^4-8a^3bx+8a\,b^3x^3+b^4x^4}{2x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^3,x)
```

output
```
(12*log(x)*a**2*b**2*x**2 - a**4 - 8*a**3*b*x + 8*a*b**3*x**3 + b**4*x**4)
/(2*x**2)
```

3.303 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 199

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^4}\,dx = -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{3ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^3Bx\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/3*a^3*A*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-1/2*a^2*(3*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^2/(b*x+a)-3*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x/(b*x+a)+b^3*B*x*((b*x
+a)^2)^(1/2)/(b*x+a)+b^2*(A*b+3*B*a)*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^4}\,dx = -\frac{\sqrt{(a+bx)^2}(18aAb^2x^2-6b^3Bx^4+9a^2bx(A+2Bx)+a^3(2A+3Bx)-6b^2(Ab+3aB)x^3\log(x))}{6x^3(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^4,x]
```

output
```
-1/6*(Sqrt[(a + b*x)^2]*(18*a*A*b^2*x^2 - 6*b^3*B*x^4 + 9*a^2*b*x*(A + 2*B
*x) + a^3*(2*A + 3*B*x) - 6*b^2*(A*b + 3*a*B)*x^3*Log[x]))/(x^3*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^4}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^4}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^4}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^4}+\frac{(3Ab+aB)a^2}{x^3}+\frac{3b(Ab+aB)a}{x^2}+b^3B+\frac{b^2(Ab+3aB)}{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{3x^3}-\frac{a^2(aB+3Ab)}{2x^2}+b^2\log(x)(3aB+Ab)-\frac{3ab(aB+Ab)}{x}+b^3Bx\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/3*(a^3*A)/x^3 - (a^2*(3*A*b + a*B))/(2*
x^2) - (3*a*b*(A*b + a*B))/x + b^3*B*x + b^2*(A*b + 3*a*B)*Log[x]))/(a + b
*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}} \left(6A\ln\left(x\right)x^{3}b^{3}+18B\ln\left(x\right)a\,b^{2}x^{3}+6B\,b^{3}x^{4}-18Aa\,b^{2}x^{2}-18B\,a^{2}b\,x^{2}-9A\,a^{2}bx-3B\,a^{3}x-2a^{3}A\right)}{6x^{3}\left(bx+a\right)^{3}}$	96
risch	$\frac{b^{3}Bx\sqrt{\left(bx+a\right)^{2}}}{bx+a}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(\left(-3Aa\,b^{2}-3B\,a^{2}b\right)x^{2}+\left(-\frac{3}{2}A\,a^{2}b-\frac{1}{2}B\,a^{3}\right)x-\frac{a^{3}A}{3}\right)}{\left(bx+a\right)x^{3}}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(A\,b^{3}+3Ba\,b^{2}\right)\ln\left(x\right)}{bx+a}$	118

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(3/2)*(6*A*ln(x)*x^3*b^3+18*B*ln(x)*a*b^2*x^3+6*B*b^3*x^4-
18*A*a*b^2*x^2-18*B*a^2*b*x^2-9*A*a^2*b*x-3*B*a^3*x-2*a^3*A)/x^3/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = \frac{6\,Bb^3x^4+6\,(3\,Bab^2+Ab^3)x^3\log\left(x\right)-2\,Aa^3-18\,(Ba^2b+Aab^2)}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*B*b^3*x^4 + 6*(3*B*a*b^2 + A*b^3)*x^3*log(x) - 2*A*a^3 - 18*(B*a^2*
b + A*a*b^2)*x^2 - 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**4,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(140) = 280$.

Time = 0.04 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = 3\,(-1)^{2\,b^2x+2\,ab}\,Bab^2\log\left(2\,b^2x+2\,ab\right) + (-1)^{2\,b^2x+2\,ab}\,Ab^3\log\left(2\,b^2x+2\,ab\right) - 3\,(-1)^{2\,abx+2\,a^2}\,Bab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) - (-1)^{2\,abx+2\,a^2}\,Ab^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Bb^3x}{2\,a} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ab^4x}{2\,a^2} + \frac{9}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Bb^2 + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Ab^3}{2\,a} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^2}{2\,a^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^3}{6\,a^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb}{2\,ax} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^2}{2\,a^2x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{2\,a^2x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{6\,a^3x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{3\,a^2x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
3*(-1)^(2*b^2*x + 2*a*b)*B*a*b^2*log(2*b^2*x + 2*a*b) + (-1)^(2*b^2*x + 2*
a*b)*A*b^3*log(2*b^2*x + 2*a*b) - 3*(-1)^(2*a*b*x + 2*a^2)*B*a*b^2*log(2*a
*b*x/abs(x) + 2*a^2/abs(x)) - (-1)^(2*a*b*x + 2*a^2)*A*b^3*log(2*a*b*x/abs
(x) + 2*a^2/abs(x)) + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^3*x/a + 1/2*sq
rt(b^2*x^2 + 2*a*b*x + a^2)*A*b^4*x/a^2 + 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2
)*B*b^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^3/a + 1/2*(b^2*x^2 + 2*a*b
*x + a^2)^(3/2)*B*b^2/a^2 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^3/a^3
- 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*A*b^2/(a^2*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/(a^2*x^2
) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b/(a^3*x^2) - 1/3*(b^2*x^2 + 2*a
*b*x + a^2)^(5/2)*A/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = Bb^3x\mathrm{sgn}(bx+a)$$
$$+\left(3\,Bab^2\mathrm{sgn}(bx+a)+Ab^3\mathrm{sgn}(bx+a)\right)\log\left(|x|\right)$$
$$-\frac{2\,Aa^3\mathrm{sgn}(bx+a)+18\,(Ba^2b\mathrm{sgn}(bx+a)+Aab^2\mathrm{sgn}(bx+a))x^2+3\,(Ba^3\mathrm{sgn}(bx+a)+3\,Aa^2b\mathrm{sgn}(bx+}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
B*b^3*x*sgn(b*x + a) + (3*B*a*b^2*sgn(b*x + a) + A*b^3*sgn(b*x + a))*log(a
bs(x)) - 1/6*(2*A*a^3*sgn(b*x + a) + 18*(B*a^2*b*sgn(b*x + a) + A*a*b^2*sg
n(b*x + a))*x^2 + 3*(B*a^3*sgn(b*x + a) + 3*A*a^2*b*sgn(b*x + a))*x)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = \int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^4,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^4}\,dx = \frac{12\log(x)\,a\,b^3x^3-a^4-6a^3bx-18a^2b^2x^2+3b^4x^4}{3x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^4,x)
```

output
```
(12*log(x)*a*b**3*x**3 - a**4 - 6*a**3*b*x - 18*a**2*b**2*x**2 + 3*b**4*x*
*4)/(3*x**3)
```

3.304 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 187

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = -\frac{a^3B\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{3a^2bB\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{3ab^2B\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} - \frac{A(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}{4ax^4} + \frac{b^3B\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/3*a^3*B*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-3/2*a^2*b*B*((b*x+a)^2)^(1/2)/x^2
/(b*x+a)-3*a*b^2*B*((b*x+a)^2)^(1/2)/x/(b*x+a)-1/4*A*(b*x+a)^3*((b*x+a)^2)
^(1/2)/a/x^4+b^3*B*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2}(12Ab^3x^3+18ab^2x^2(A+2Bx)+6a^2bx(2A+3Bx)+a^3(3A$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^5,x]
```

output
```
(Sqrt[a^2]*(12*A*b^3*x^3 + 18*a*b^2*x^2*(A + 2*B*x) + 6*a^2*b*x*(2*A + 3*B
*x) + a^3*(3*A + 4*B*x)) - Sqrt[(a + b*x)^2]*(3*A*b^3*x^3 + a^3*(3*A + 4*B
*x) + a^2*b*x*(9*A + 14*B*x) + a*b^2*x^2*(9*A + 22*B*x)) - 24*a*b^3*B*x^4*
ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 24*Sqrt[a^2]*b^3*B*x^4*Lo
g[x] + 12*Sqrt[a^2]*b^3*B*x^4*Log[a*(Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2])]
 + 12*Sqrt[a^2]*b^3*B*x^4*Log[a*(Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2])])/(2
4*a*x^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}(A+Bx)}{x^5}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^5}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^5}dx}{a+bx}$$

$$\Big\downarrow \text{87}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\int \frac{(a+bx)^3}{x^4}dx-\frac{A(a+bx)^4}{4ax^4}\right)}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^3}+\frac{3b^2a}{x^2}+\frac{b^3}{x}\right)dx-\frac{A(a+bx)^4}{4ax^4}\right)}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\left(-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\right)-\frac{A(a+bx)^4}{4ax^4}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*(A*(a + b*x)^4)/(a*x^4) + B*(-1/3*a^3
/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x])))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left((bx+a)^2\right)^{\frac{3}{2}}\left(-12B\,b^3\ln(x)x^4+12A\,b^3x^3+36Ba\,b^2x^3+18Aa\,b^2x^2+18B\,a^2b\,x^2+12A\,a^2bx+4B\,a^3x+3a^3A\right)}{12(bx+a)^3x^4}$	94
risch	$\frac{\sqrt{(bx+a)^2}\left((-A\,b^3-3Ba\,b^2)x^3+(-\frac{3}{2}Aa\,b^2-\frac{3}{2}B\,a^2b)x^2+(-A\,a^2b-\frac{1}{3}B\,a^3)x-\frac{a^3A}{4}\right)}{(bx+a)x^4}+\frac{b^3B\sqrt{(bx+a)^2}\,\ln(x)}{bx+a}$	105

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*((b*x+a)^2)^(3/2)*(-12*B*b^3*ln(x)*x^4+12*A*b^3*x^3+36*B*a*b^2*x^3+1
8*A*a*b^2*x^2+18*B*a^2*b*x^2+12*A*a^2*b*x+4*B*a^3*x+3*a^3*A)/(b*x+a)^3/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=\frac{12\,Bb^3x^4\log(x)-3\,Aa^3-12\,(3\,Bab^2+Ab^3)x^3-18\,(Ba^2b+Aa}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*B*b^3*x^4*log(x) - 3*A*a^3 - 12*(3*B*a*b^2 + A*b^3)*x^3 - 18*(B*a
^2*b + A*a*b^2)*x^2 - 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**5,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(126) = 252.

Time = 0.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = (-1)^{2\,b^2x+2\,ab}\,Bb^3\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,Bb^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^4x}{2\,a^2} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Bb^3}{2\,a} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^3}{6\,a^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^4}{4\,a^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^2}{2\,a^2x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^3}{4\,a^3x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{6\,a^3x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{4\,a^4x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{3\,a^2x^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{4\,a^3x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{4\,a^2x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*b^3*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)
*B*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*B*b^4*x/a^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^3/a - 1/6*(b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*B*b^3/a^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*
b^4/a^4 - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^2/(a^2*x) + 1/4*(b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*A*b^3/(a^3*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2
)*B*b/(a^3*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^2) - 1/
3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/(a^2*x^3) + 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*A*b/(a^3*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = Bb^3\log\left(|x|\right)\mathrm{sgn}(bx+a)$$
$$-\frac{3\,Aa^3\mathrm{sgn}(bx+a)+12\,(3\,Bab^2\mathrm{sgn}(bx+a)+Ab^3\mathrm{sgn}(bx+a))x^3+18\,(Ba^2b\mathrm{sgn}(bx+a)+Aab^2\mathrm{sgn}(bx+a)}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
B*b^3*log(abs(x))*sgn(b*x + a) - 1/12*(3*A*a^3*sgn(b*x + a) + 12*(3*B*a*b^
2*sgn(b*x + a) + A*b^3*sgn(b*x + a))*x^3 + 18*(B*a^2*b*sgn(b*x + a) + A*a*
b^2*sgn(b*x + a))*x^2 + 4*(B*a^3*sgn(b*x + a) + 3*A*a^2*b*sgn(b*x + a))*x)
/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^5}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^5,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx = \frac{12\log(x)\,b^4x^4-3a^4-16a^3bx-36a^2b^2x^2-48a\,b^3x^3}{12x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^5,x)
```

output
```
(12*log(x)*b**4*x**4 - 3*a**4 - 16*a**3*b*x - 36*a**2*b**2*x**2 - 48*a*b**
3*x**3)/(12*x**4)
```

3.305 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx=\frac{(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4a^2x^4}-\frac{A(a^2+2abx+b^2x^2)^{5/2}}{5a^2x^5}$$

output
```
1/4*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/a^2/x^4-1/5*A*(b^2*x^2+2
*a*b*x+a^2)^(5/2)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx=-\frac{(a+bx)^3\sqrt{(a+bx)^2}(4aA-Abx+5aBx)}{20a^2x^5}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^6,x]
```

output
```
-1/20*((a + b*x)^3*Sqrt[(a + b*x)^2]*(4*a*A - A*b*x + 5*a*B*x))/(a^2*x^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1186, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}(A+Bx)}{x^6}\,dx$$

$$\Big\downarrow \text{1186}$$

$$-\frac{(Ab-aB)\int \frac{(a^2+2bxa+b^2x^2)^{3/2}}{x^5}dx}{a}-\frac{A(a^2+2abx+b^2x^2)^{5/2}}{5a^2x^5}$$

$$\Big\downarrow \text{1102}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{b^3(a+bx)^3}{x^5}dx}{ab^3(a+bx)}-\frac{A(a^2+2abx+b^2x^2)^{5/2}}{5a^2x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{(a+bx)^3}{x^5}dx}{a(a+bx)}-\frac{A(a^2+2abx+b^2x^2)^{5/2}}{5a^2x^5}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^3\sqrt{a^2+2abx+b^2x^2}(Ab-aB)}{4a^2x^4}-\frac{A(a^2+2abx+b^2x^2)^{5/2}}{5a^2x^5}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^6,x]
```

output
```
((A*b - a*B)*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(4*a^2*x^4) - (A*(
a^2 + 2*a*b*x + b^2*x^2)^(5/2))/(5*a^2*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1186
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[-2*c*(e*f - d*g)*(d + e*x)^(m + 1)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e)^2)), x] + Simp[(2*c*f - b*g)/
(2*c*d - b*e)   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[
{a, b, c, d, e, f, g, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[m + 2*p + 3,
0] && NeQ[2*c*f - b*g, 0] && NeQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-B\,b^3x^4+(-\frac{1}{2}A\,b^3-\frac{3}{2}Ba\,b^2)x^3+(-Aa\,b^2-B\,a^2b)x^2+(-\frac{3}{4}A\,a^2b-\frac{1}{4}B\,a^3)x-\frac{a^3A}{5}\right)}{(bx+a)x^5}$	90
gosper	$-\frac{(20B\,b^3x^4+10A\,b^3x^3+30Ba\,b^2x^3+20Aa\,b^2x^2+20B\,a^2b\,x^2+15A\,a^2bx+5B\,a^3x+4a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{20x^5(bx+a)^3}$	92
default	$-\frac{(20B\,b^3x^4+10A\,b^3x^3+30Ba\,b^2x^3+20Aa\,b^2x^2+20B\,a^2b\,x^2+15A\,a^2bx+5B\,a^3x+4a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{20x^5(bx+a)^3}$	92
orering	$-\frac{(20B\,b^3x^4+10A\,b^3x^3+30Ba\,b^2x^3+20Aa\,b^2x^2+20B\,a^2b\,x^2+15A\,a^2bx+5B\,a^3x+4a^3A)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{20x^5(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-B*b^3*x^4+(-1/2*A*b^3-3/2*B*a*b^2)*x^3+(-A*a*b
^2-B*a^2*b)*x^2+(-3/4*A*a^2*b-1/4*B*a^3)*x-1/5*a^3*A)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx = -\frac{20\,Bb^3x^4+4\,Aa^3+10\,(3\,Bab^2+Ab^3)x^3+20\,(Ba^2b+Aab^2)x^2+5\,(Ba^3+3\,Aa^2b)x}{20\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(20*B*b^3*x^4 + 4*A*a^3 + 10*(3*B*a*b^2 + A*b^3)*x^3 + 20*(B*a^2*b +
 A*a*b^2)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Sympy [F]

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**6,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(67) = 134$.

Time = 0.05 (sec) , antiderivative size = 315, normalized size of antiderivative = 4.20

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^4}{4\,a^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^5}{4\,a^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^3}{4\,a^3x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^4}{4\,a^4x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^2}{4\,a^4x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^3}{4\,a^5x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{4\,a^3x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{4\,a^4x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{4\,a^2x^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{4\,a^3x^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{5\,a^2x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^4/a^4 - 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*A*b^5/a^5 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^3/(a^3*x) -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^4/(a^4*x) - 1/4*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*B*b^2/(a^4*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/
(a^5*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b/(a^3*x^3) - 1/4*(b^2*x
^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^
(5/2)*B/(a^2*x^4) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b/(a^3*x^4) - 1/
5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a^2*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx = -\frac{(5\,Bab^4 - Ab^5)\mathrm{sgn}(bx+a)}{20\,a^2} - \frac{20\,Bb^3x^4\mathrm{sgn}(bx+a) + 30\,Bab^2x^3\mathrm{sgn}(bx+a) + 10\,Ab^3x^3\mathrm{sgn}(bx+a) + 20\,Ba^2bx^2\mathrm{sgn}(bx+a) + 20\,Aab^2}{20\,x^5}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
-1/20*(5*B*a*b^4 - A*b^5)*sgn(b*x + a)/a^2 - 1/20*(20*B*b^3*x^4*sgn(b*x +
a) + 30*B*a*b^2*x^3*sgn(b*x + a) + 10*A*b^3*x^3*sgn(b*x + a) + 20*B*a^2*b*
x^2*sgn(b*x + a) + 20*A*a*b^2*x^2*sgn(b*x + a) + 5*B*a^3*x*sgn(b*x + a) +
15*A*a^2*b*x*sgn(b*x + a) + 4*A*a^3*sgn(b*x + a))/x^5
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.61

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^6}\,dx = -\frac{\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^4\,(a+b\,x)} - \frac{\left(\frac{A\,b^3}{2}+\frac{3\,B\,a\,b^2}{2}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^2\,(a+b\,x)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,(a+b\,x)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x\,(a+b\,x)} - \frac{a\,b\,(A\,b+B\,a)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^3\,(a+b\,x)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^6,x)
```

output
```
- (((B*a^3)/4 + (3*A*a^2*b)/4)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^4*(a +
b*x)) - (((A*b^3)/2 + (3*B*a*b^2)/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^2
*(a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) -
(B*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x*(a + b*x)) - (a*b*(A*b + B*a)*(
a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^3*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^6}\,dx=\frac{-5b^4x^4-10a\,b^3x^3-10a^2b^2x^2-5a^3bx-a^4}{5x^5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^6,x)
```

output
```
( - a**4 - 5*a**3*b*x - 10*a**2*b**2*x**2 - 10*a*b**3*x**3 - 5*b**4*x**4)/
(5*x**5)
```

3.306 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{b(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4a^3x^4} - \frac{A\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^2x^6} + \frac{(7Ab-6aB)\left(a^2+2abx+b^2x^2\right)^{5/2}}{30a^3x^5}$$

output
```
-1/4*b*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/a^3/x^4-1/6*A*(b^2*x^
2+2*a*b*x+a^2)^(5/2)/a^2/x^6+1/30*(7*A*b-6*B*a)*(b^2*x^2+2*a*b*x+a^2)^(5/2
)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{\sqrt{(a+bx)^2}\left(10b^3x^3(2A+3Bx)+15ab^2x^2(3A+4Bx)+9a^2bx(4A+5Bx)+2a^3(5A+6Bx)\right)}{60x^6(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^7,x]
```

output
```
-1/60*(Sqrt[(a + b*x)^2]*(10*b^3*x^3*(2*A + 3*B*x) + 15*a*b^2*x^2*(3*A + 4
*B*x) + 9*a^2*b*x*(4*A + 5*B*x) + 2*a^3*(5*A + 6*B*x)))/(x^6*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{3/2}(A+Bx)}{x^7}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^7}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^7}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^7}+\frac{(3Ab+aB)a^2}{x^6}+\frac{3b(Ab+aB)a}{x^5}+\frac{b^3B}{x^3}+\frac{b^2(Ab+3aB)}{x^4}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{6x^6}-\frac{a^2(aB+3Ab)}{5x^5}-\frac{b^2(3aB+Ab)}{3x^3}-\frac{3ab(aB+Ab)}{4x^4}-\frac{b^3B}{2x^2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^7,x]
```

output
```
((-1/6*(a^3*A)/x^6 - (a^2*(3*A*b + a*B))/(5*x^5) - (3*a*b*(A*b + a*B))/(4*
x^4) - (b^2*(A*b + 3*a*B))/(3*x^3) - (b^3*B)/(2*x^2))*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B\,b^3x^4}{2}+(-\frac{1}{3}A\,b^3-Ba\,b^2)x^3+(-\frac{3}{4}Aa\,b^2-\frac{3}{4}B\,a^2b)x^2+(-\frac{3}{5}A\,a^2b-\frac{1}{5}B\,a^3)x-\frac{a^3A}{6}\right)}{(bx+a)x^6}$	90
gosper	$-\frac{(30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45Aa\,b^2x^2+45B\,a^2b\,x^2+36A\,a^2bx+12B\,a^3x+10a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	92
default	$-\frac{(30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45Aa\,b^2x^2+45B\,a^2b\,x^2+36A\,a^2bx+12B\,a^3x+10a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	92
orering	$-\frac{(30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45Aa\,b^2x^2+45B\,a^2b\,x^2+36A\,a^2bx+12B\,a^3x+10a^3A)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{60x^6(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/2*B*b^3*x^4+(-1/3*A*b^3-B*a*b^2)*x^3+(-3/4*A
*a*b^2-3/4*B*a^2*b)*x^2+(-3/5*A*a^2*b-1/5*B*a^3)*x-1/6*a^3*A)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{30\,Bb^3x^4+10\,Aa^3+20\,(3\,Bab^2+Ab^3)x^3+45\,(Ba^2b+Aab^2)x^2+12\,(Ba^3+3\,Aa^2b)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(30*B*b^3*x^4 + 10*A*a^3 + 20*(3*B*a*b^2 + A*b^3)*x^3 + 45*(B*a^2*b
+ A*a*b^2)*x^2 + 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**7,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.26

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx=-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^5}{4\,a^5}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^6}{4\,a^6}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^4}{4\,a^4x}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^5}{4\,a^5x}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^3}{4\,a^5x^2}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^4}{4\,a^6x^2}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^2}{4\,a^4x^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^3}{4\,a^5x^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{4\,a^3x^4}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{4\,a^4x^4}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{5\,a^2x^5}+\frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{30\,a^3x^5}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{6\,a^2x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^5/a^5 + 1/4*(b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*A*b^6/a^6 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^4/(a^4*x) +
 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^5/(a^5*x) + 1/4*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*B*b^3/(a^5*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4
/(a^6*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^2/(a^4*x^3) + 1/4*(b^
2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/(a^5*x^3) + 1/4*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*B*b/(a^3*x^4) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^
4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/(a^2*x^5) + 7/30*(b^2*x^2 + 2*a
*b*x + a^2)^(5/2)*A*b/(a^3*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a
^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx = \frac{(3\,Bab^5-Ab^6)\mathrm{sgn}(bx+a)}{60\,a^3} - \frac{30\,Bb^3x^4\mathrm{sgn}(bx+a)+60\,Bab^2x^3\mathrm{sgn}(bx+a)+20\,Ab^3x^3\mathrm{sgn}(bx+a)+45\,Ba^2bx^2\mathrm{sgn}(bx+a)+45\,Aab}{60\,x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/60*(3*B*a*b^5 - A*b^6)*sgn(b*x + a)/a^3 - 1/60*(30*B*b^3*x^4*sgn(b*x + a
) + 60*B*a*b^2*x^3*sgn(b*x + a) + 20*A*b^3*x^3*sgn(b*x + a) + 45*B*a^2*b*x
^2*sgn(b*x + a) + 45*A*a*b^2*x^2*sgn(b*x + a) + 12*B*a^3*x*sgn(b*x + a) +
36*A*a^2*b*x*sgn(b*x + a) + 10*A*a^3*sgn(b*x + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^3}{3}+B\,a\,b^2\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^3\,\left(a+b\,x\right)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,\left(a+b\,x\right)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^2\,\left(a+b\,x\right)} - \frac{3\,a\,b\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^7,x)
```

output
```
- (((B*a^3)/5 + (3*A*a^2*b)/5)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a +
b*x)) - (((A*b^3)/3 + B*a*b^2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^3*(a +
b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (B*b^3
*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^2*(a + b*x)) - (3*a*b*(A*b + B*a)*(
a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx = \frac{-15b^4x^4-40a\,b^3x^3-45a^2b^2x^2-24a^3bx-5a^4}{30x^6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^7,x)
```

output
```
( - 5*a**4 - 24*a**3*b*x - 45*a**2*b**2*x**2 - 40*a*b**3*x**3 - 15*b**4*x*
*4)/(30*x**6)
```

3.307 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 157

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx = \frac{b^2(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{4a^4x^4} - \frac{A(a^2+2abx+b^2x^2)^{5/2}}{7a^2x^7} + \frac{(9Ab-7aB)\left(a^2+2abx+b^2x^2\right)^{5/2}}{42a^3x^6} - \frac{b(51Ab-49aB)\left(a^2+2abx+b^2x^2\right)^{5/2}}{210a^4x^5}$$

output
```
1/4*b^2*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/a^4/x^4-1/7*A*(b^2*x
^2+2*a*b*x+a^2)^(5/2)/a^2/x^7+1/42*(9*A*b-7*B*a)*(b^2*x^2+2*a*b*x+a^2)^(5/
2)/a^3/x^6-1/210*b*(51*A*b-49*B*a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^4/x^5
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx)^2}(35b^3x^3(3A+4Bx)+63ab^2x^2(4A+5Bx)+42a^2bx(5A+6Bx)+10a^3(6A+7Bx))}{420x^7(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^8,x]
```

output
```
-1/420*(Sqrt[(a + b*x)^2]*(35*b^3*x^3*(3*A + 4*B*x) + 63*a*b^2*x^2*(4*A +
5*B*x) + 42*a^2*b*x*(5*A + 6*B*x) + 10*a^3*(6*A + 7*B*x)))/(x^7*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^8}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^8}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^8}+\frac{(3Ab+aB)a^2}{x^7}+\frac{3b(Ab+aB)a}{x^6}+\frac{b^3B}{x^4}+\frac{b^2(Ab+3aB)}{x^5}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{7x^7}-\frac{a^2(aB+3Ab)}{6x^6}-\frac{b^2(3aB+Ab)}{4x^4}-\frac{3ab(aB+Ab)}{5x^5}-\frac{b^3B}{3x^3}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^8,x]
```

output
```
((-1/7*(a^3*A)/x^7 - (a^2*(3*A*b + a*B))/(6*x^6) - (3*a*b*(A*b + a*B))/(5*
x^5) - (b^2*(A*b + 3*a*B))/(4*x^4) - (b^3*B)/(3*x^3))*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B\,b^3x^4}{3}+(-\frac{1}{4}A\,b^3-\frac{3}{4}Ba\,b^2)x^3+(-\frac{3}{5}Aa\,b^2-\frac{3}{5}B\,a^2b)x^2+(-\frac{1}{2}A\,a^2b-\frac{1}{6}B\,a^3)x-\frac{a^3A}{7}\right)}{(bx+a)x^7}$	90
gosper	$-\frac{(140B\,b^3x^4+105A\,b^3x^3+315Ba\,b^2x^3+252Aa\,b^2x^2+252B\,a^2b\,x^2+210A\,a^2bx+70B\,a^3x+60a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{420x^7(bx+a)^3}$	92
default	$-\frac{(140B\,b^3x^4+105A\,b^3x^3+315Ba\,b^2x^3+252Aa\,b^2x^2+252B\,a^2b\,x^2+210A\,a^2bx+70B\,a^3x+60a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{420x^7(bx+a)^3}$	92
orering	$-\frac{(140B\,b^3x^4+105A\,b^3x^3+315Ba\,b^2x^3+252Aa\,b^2x^2+252B\,a^2b\,x^2+210A\,a^2bx+70B\,a^3x+60a^3A)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{420x^7(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/3*B*b^3*x^4+(-1/4*A*b^3-3/4*B*a*b^2)*x^3+(-3
/5*A*a*b^2-3/5*B*a^2*b)*x^2+(-1/2*A*a^2*b-1/6*B*a^3)*x-1/7*a^3*A)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx =$$
$$-\frac{140\,Bb^3x^4+60\,Aa^3+105\,(3\,Bab^2+Ab^3)x^3+252\,(Ba^2b+Aab^2)x^2+70\,(Ba^3+3\,Aa^2b)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-1/420*(140*B*b^3*x^4 + 60*A*a^3 + 105*(3*B*a*b^2 + A*b^3)*x^3 + 252*(B*a^
2*b + A*a*b^2)*x^2 + 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**8,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(141) = 282.

Time = 0.04 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx &= \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^6}{4\,a^6} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^7}{4\,a^7} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^5}{4\,a^5x} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^6}{4\,a^6x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^4}{4\,a^6x^2} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^5}{4\,a^7x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^3}{4\,a^5x^3} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^4}{4\,a^6x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^2}{4\,a^4x^4} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^3}{4\,a^5x^4} + \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{30\,a^3x^5} \\ &- \frac{17\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{70\,a^4x^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{6\,a^2x^6} \\ &+ \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{14\,a^3x^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{7\,a^2x^7}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^6/a^6 - 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*A*b^7/a^7 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^5/(a^5*x) -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^6/(a^6*x) - 1/4*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*B*b^4/(a^6*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/
(a^7*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^3/(a^5*x^3) - 1/4*(b^2
*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4/(a^6*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*B*b^2/(a^4*x^4) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/(a^5*x
^4) + 7/30*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b/(a^3*x^5) - 17/70*(b^2*x^2
+ 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/
2)*B/(a^2*x^6) + 3/14*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b/(a^3*x^6) - 1/7*
(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a^2*x^7)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{(7\,Bab^6-3\,Ab^7)\mathrm{sgn}(bx+a)}{420\,a^4} - \frac{140\,Bb^3x^4\mathrm{sgn}(bx+a)+315\,Bab^2x^3\mathrm{sgn}(bx+a)+105\,Ab^3x^3\mathrm{sgn}(bx+a)+252\,Ba^2bx^2\mathrm{sgn}(bx+a)+25}{420\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
-1/420*(7*B*a*b^6 - 3*A*b^7)*sgn(b*x + a)/a^4 - 1/420*(140*B*b^3*x^4*sgn(b
*x + a) + 315*B*a*b^2*x^3*sgn(b*x + a) + 105*A*b^3*x^3*sgn(b*x + a) + 252*
B*a^2*b*x^2*sgn(b*x + a) + 252*A*a*b^2*x^2*sgn(b*x + a) + 70*B*a^3*x*sgn(b
*x + a) + 210*A*a^2*b*x*sgn(b*x + a) + 60*A*a^3*sgn(b*x + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx=-\frac{\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^6\,\left(a+b\,x\right)}-\frac{\left(\frac{A\,b^3}{4}+\frac{3\,B\,a\,b^2}{4}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^4\,\left(a+b\,x\right)}-\frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)}-\frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,\left(a+b\,x\right)}-\frac{3\,a\,b\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^8,x)
```

output
```
- (((B*a^3)/6 + (A*a^2*b)/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^6*(a + b*
x)) - (((A*b^3)/4 + (3*B*a*b^2)/4)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^4*(
a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (B
*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a + b*x)) - (3*a*b*(A*b + B*
a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx=\frac{-35b^4x^4-105a\,b^3x^3-126a^2b^2x^2-70a^3bx-15a^4}{105x^7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^8,x)
```

output
```
( - 15*a**4 - 70*a**3*b*x - 126*a**2*b**2*x**2 - 105*a*b**3*x**3 - 35*b**4
*x**4)/(105*x**7)
```

3.308 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx &= -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} \\ &- \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{2x^6(a+bx)} \\ &- \frac{b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} - \frac{b^3B\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}\end{aligned}$$

output

```
-1/8*a^3*A*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-1/7*a^2*(3*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^7/(b*x+a)-1/2*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-1/5*b^2*(
A*b+3*B*a)*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*b^3*B*((b*x+a)^2)^(1/2)/x^4/(
b*x+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\sqrt{(a+bx)^2}\left(14b^3x^3(4A+5Bx)+28ab^2x^2(5A+6Bx)+20a^2bx(6A+7Bx)+5a^3(7A+8Bx)\right)}{280x^8(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^9,x]
```

output
```
-1/280*(Sqrt[(a + b*x)^2]*(14*b^3*x^3*(4*A + 5*B*x) + 28*a*b^2*x^2*(5*A +
6*B*x) + 20*a^2*b*x*(6*A + 7*B*x) + 5*a^3*(7*A + 8*B*x)))/(x^8*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^9}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^9}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^9}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^9}+\frac{(3Ab+aB)a^2}{x^8}+\frac{3b(Ab+aB)a}{x^7}+\frac{b^3B}{x^5}+\frac{b^2(Ab+3aB)}{x^6}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{8x^8}-\frac{a^2(aB+3Ab)}{7x^7}-\frac{b^2(3aB+Ab)}{5x^5}-\frac{ab(aB+Ab)}{2x^6}-\frac{b^3B}{4x^4}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^9,x]
```

output
```
((-1/8*(a^3*A)/x^8 - (a^2*(3*A*b + a*B))/(7*x^7) - (a*b*(A*b + a*B))/(2*x^
6) - (b^2*(A*b + 3*a*B))/(5*x^5) - (b^3*B)/(4*x^4))*Sqrt[a^2 + 2*a*b*x + b
^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B\,b^3x^4}{4}+\left(-\frac{1}{5}A\,b^3-\frac{3}{5}Ba\,b^2\right)x^3+\left(-\frac{1}{2}Aa\,b^2-\frac{1}{2}B\,a^2b\right)x^2+\left(-\frac{3}{7}A\,a^2b-\frac{1}{7}B\,a^3\right)x-\frac{a^3A}{8}\right)}{(bx+a)x^8}$	90
gosper	$-\frac{(70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140Aa\,b^2x^2+140B\,a^2b\,x^2+120A\,a^2bx+40B\,a^3x+35a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	92
default	$-\frac{(70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140Aa\,b^2x^2+140B\,a^2b\,x^2+120A\,a^2bx+40B\,a^3x+35a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	92
orering	$-\frac{(70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140Aa\,b^2x^2+140B\,a^2b\,x^2+120A\,a^2bx+40B\,a^3x+35a^3A)\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}}{280x^8(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/4*B*b^3*x^4+(-1/5*A*b^3-3/5*B*a*b^2)*x^3+(-1
/2*A*a*b^2-1/2*B*a^2*b)*x^2+(-3/7*A*a^2*b-1/7*B*a^3)*x-1/8*a^3*A)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{70\,Bb^3x^4+35\,Aa^3+56\,(3\,Bab^2+Ab^3)x^3+140\,(Ba^2b+Aab^2)x^2+40\,(Ba^3+3\,Aa^2b)x}{280\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/280*(70*B*b^3*x^4 + 35*A*a^3 + 56*(3*B*a*b^2 + A*b^3)*x^3 + 140*(B*a^2*
b + A*a*b^2)*x^2 + 40*(B*a^3 + 3*A*a^2*b)*x)/x^8
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**9,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = & -\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^7}{4\,a^7} \\ & + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^8}{4\,a^8} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^6}{4\,a^6x} \\ & + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^7}{4\,a^7x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^5}{4\,a^7x^2} \\ & - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^6}{4\,a^8x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^4}{4\,a^6x^3} \\ & + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^5}{4\,a^7x^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^3}{4\,a^5x^4} \\ & - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^4}{4\,a^6x^4} - \frac{17\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^2}{70\,a^4x^5} \\ & + \frac{69\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^3}{280\,a^5x^5} + \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{14\,a^3x^6} \\ & - \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{56\,a^4x^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{7\,a^2x^7} \\ & + \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{56\,a^3x^7} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{8\,a^2x^8}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^7/a^7 + 1/4*(b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*A*b^8/a^8 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^6/(a^6*x) +
 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^7/(a^7*x) + 1/4*(b^2*x^2 + 2*a*b*
x + a^2)^(5/2)*B*b^5/(a^7*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^6
/(a^8*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^4/(a^6*x^3) + 1/4*(b^
2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/(a^7*x^3) + 1/4*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*B*b^3/(a^5*x^4) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4/(a^6*
x^4) - 17/70*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^2/(a^4*x^5) + 69/280*(b^2
*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/(a^5*x^5) + 3/14*(b^2*x^2 + 2*a*b*x + a^
2)^(5/2)*B*b/(a^3*x^6) - 13/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*
x^6) - 1/7*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/(a^2*x^7) + 11/56*(b^2*x^2 +
2*a*b*x + a^2)^(5/2)*A*b/(a^3*x^7) - 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A
/(a^2*x^8)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = \frac{(2\,Bab^7-Ab^8)\mathrm{sgn}(bx+a)}{280\,a^5} - \frac{70\,Bb^3x^4\mathrm{sgn}(bx+a)+168\,Bab^2x^3\mathrm{sgn}(bx+a)+56\,Ab^3x^3\mathrm{sgn}(bx+a)+140\,Ba^2bx^2\mathrm{sgn}(bx+a)+140\,A}{280\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
1/280*(2*B*a*b^7 - A*b^8)*sgn(b*x + a)/a^5 - 1/280*(70*B*b^3*x^4*sgn(b*x +
 a) + 168*B*a*b^2*x^3*sgn(b*x + a) + 56*A*b^3*x^3*sgn(b*x + a) + 140*B*a^2
*b*x^2*sgn(b*x + a) + 140*A*a*b^2*x^2*sgn(b*x + a) + 40*B*a^3*x*sgn(b*x +
a) + 120*A*a^2*b*x*sgn(b*x + a) + 35*A*a^3*sgn(b*x + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\left(\frac{B\,a^3}{7}+\frac{3\,A\,b\,a^2}{7}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^7\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^3}{5}+\frac{3\,B\,a\,b^2}{5}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,\left(a+b\,x\right)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,\left(a+b\,x\right)} - \frac{a\,b\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^6\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^9,x)
```

output
```
- (((B*a^3)/7 + (3*A*a^2*b)/7)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^7*(a +
b*x)) - (((A*b^3)/5 + (3*B*a*b^2)/5)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5
*(a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) -
(B*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (a*b*(A*b + B*
a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^6*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx = \frac{-70b^4x^4-224a\,b^3x^3-280a^2b^2x^2-160a^3bx-35a^4}{280x^8}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^9,x)
```

output
```
( - 35*a**4 - 160*a**3*b*x - 280*a**2*b**2*x**2 - 224*a*b**3*x**3 - 70*b**
4*x**4)/(280*x**8)
```

3.309 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{3/2}}{x^{10}}\,dx = -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{3ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)} - \frac{b^3B\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)}$$

output
```
-1/9*a^3*A*((b*x+a)^2)^(1/2)/x^9/(b*x+a)-1/8*a^2*(3*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^8/(b*x+a)-3/7*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^7/(b*x+a)-1/6*b^2*(
A*b+3*B*a)*((b*x+a)^2)^(1/2)/x^6/(b*x+a)-1/5*b^3*B*((b*x+a)^2)^(1/2)/x^5/(
b*x+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = -\frac{\sqrt{(a+bx)^2}(84b^3x^3(5A+6Bx)+180ab^2x^2(6A+7Bx)+135a^2bx(7A+8Bx)+35a^3(8A+9Bx))}{2520x^9(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^10,x]
```

output
```
-1/2520*(Sqrt[(a + b*x)^2]*(84*b^3*x^3*(5*A + 6*B*x) + 180*a*b^2*x^2*(6*A
+ 7*B*x) + 135*a^2*b*x*(7*A + 8*B*x) + 35*a^3*(8*A + 9*B*x)))/(x^9*(a + b*
x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{10}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{10}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^3}{x^{10}}+\frac{(3Ab+aB)a^2}{x^9}+\frac{3b(Ab+aB)a}{x^8}+\frac{b^3B}{x^6}+\frac{b^2(Ab+3aB)}{x^7}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{9x^9}-\frac{a^2(aB+3Ab)}{8x^8}-\frac{b^2(3aB+Ab)}{6x^6}-\frac{3ab(aB+Ab)}{7x^7}-\frac{b^3B}{5x^5}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^10,x]
```

output
```
((-1/9*(a^3*A)/x^9 - (a^2*(3*A*b + a*B))/(8*x^8) - (3*a*b*(A*b + a*B))/(7*
x^7) - (b^2*(A*b + 3*a*B))/(6*x^6) - (b^3*B)/(5*x^5))*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

method	result	si
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^3 x^4}{5}+\left(-\frac{1}{6}A b^3-\frac{1}{2}B a b^2\right)x^3+\left(-\frac{3}{7}A a b^2-\frac{3}{7}B a^2 b\right)x^2+\left(-\frac{3}{8}A a^2 b-\frac{1}{8}B a^3\right)x-\frac{a^3 A}{9}\right)}{(bx+a)x^9}$	9
gosper	$-\frac{(504B b^3x^4+420A b^3x^3+1260Ba b^2x^3+1080Aa b^2x^2+1080B a^2b x^2+945A a^2bx+315B a^3x+280a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{2520x^9(bx+a)^3}$	9
default	$-\frac{(504B b^3x^4+420A b^3x^3+1260Ba b^2x^3+1080Aa b^2x^2+1080B a^2b x^2+945A a^2bx+315B a^3x+280a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{2520x^9(bx+a)^3}$	9
orering	$-\frac{(504B b^3x^4+420A b^3x^3+1260Ba b^2x^3+1080Aa b^2x^2+1080B a^2b x^2+945A a^2bx+315B a^3x+280a^3A)(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{2520x^9(bx+a)^3}$	1

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/5*B*b^3*x^4+(-1/6*A*b^3-1/2*B*a*b^2)*x^3+(-3
/7*A*a*b^2-3/7*B*a^2*b)*x^2+(-3/8*A*a^2*b-1/8*B*a^3)*x-1/9*a^3*A)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = -\frac{504\,Bb^3x^4+280\,Aa^3+420\,(3\,Bab^2+Ab^3)x^3+1080\,(Ba^2b+Aab^2)x^2+315\,(Ba^3+3\,Aa^2b)x}{2520\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-1/2520*(504*B*b^3*x^4 + 280*A*a^3 + 420*(3*B*a*b^2 + A*b^3)*x^3 + 1080*(B
*a^2*b + A*a*b^2)*x^2 + 315*(B*a^3 + 3*A*a^2*b)*x)/x^9
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**10,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(145) = 290.

Time = 0.05 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.64

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx &= \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^8}{4\,a^8} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^9}{4\,a^9} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^7}{4\,a^7x} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^8}{4\,a^8x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^6}{4\,a^8x^2} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^7}{4\,a^9x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^5}{4\,a^7x^3} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^6}{4\,a^8x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^4}{4\,a^6x^4} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^5}{4\,a^7x^4} + \frac{69\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^3}{280\,a^5x^5} \\ &- \frac{125\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^4}{504\,a^6x^5} - \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^2}{56\,a^4x^6} \\ &+ \frac{121\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^3}{504\,a^5x^6} + \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb}{56\,a^3x^7} \\ &- \frac{37\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{168\,a^4x^7} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{8\,a^2x^8} \\ &+ \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{72\,a^3x^8} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^8/a^8 - 1/4*(b^2*x^2 + 2*a*b*x + a
^2)^(3/2)*A*b^9/a^9 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^7/(a^7*x) -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^8/(a^8*x) - 1/4*(b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*B*b^6/(a^8*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^7/
(a^9*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^5/(a^7*x^3) - 1/4*(b^2
*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^6/(a^8*x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*B*b^4/(a^6*x^4) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/(a^7*x
^4) + 69/280*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^3/(a^5*x^5) - 125/504*(b^
2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4/(a^6*x^5) - 13/56*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*B*b^2/(a^4*x^6) + 121/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3
/(a^5*x^6) + 11/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b/(a^3*x^7) - 37/168*
(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^7) - 1/8*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*B/(a^2*x^8) + 13/72*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b/(a^3*x
^8) - 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = -\frac{(9\,Bab^8-5\,Ab^9)\mathrm{sgn}(bx+a)}{2520\,a^6} - \frac{504\,Bb^3x^4\mathrm{sgn}(bx+a)+1260\,Bab^2x^3\mathrm{sgn}(bx+a)+420\,Ab^3x^3\mathrm{sgn}(bx+a)+1080\,Ba^2bx^2\mathrm{sgn}(bx+a)+}{2520\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
-1/2520*(9*B*a*b^8 - 5*A*b^9)*sgn(b*x + a)/a^6 - 1/2520*(504*B*b^3*x^4*sgn
(b*x + a) + 1260*B*a*b^2*x^3*sgn(b*x + a) + 420*A*b^3*x^3*sgn(b*x + a) + 1
080*B*a^2*b*x^2*sgn(b*x + a) + 1080*A*a*b^2*x^2*sgn(b*x + a) + 315*B*a^3*x
*sgn(b*x + a) + 945*A*a^2*b*x*sgn(b*x + a) + 280*A*a^3*sgn(b*x + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = -\frac{\left(\frac{B\,a^3}{8}+\frac{3\,A\,b\,a^2}{8}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^8\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^3}{6}+\frac{B\,a\,b^2}{2}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^6\,\left(a+b\,x\right)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,\left(a+b\,x\right)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,\left(a+b\,x\right)} - \frac{3\,a\,b\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^10,x)
```

output
```
- (((B*a^3)/8 + (3*A*a^2*b)/8)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^8*(a +
b*x)) - (((A*b^3)/6 + (B*a*b^2)/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^6*(
a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x)) - (B
*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) - (3*a*b*(A*b + B*
a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{10}}\,dx = \frac{-126b^4x^4-420a\,b^3x^3-540a^2b^2x^2-315a^3bx-70a^4}{630x^9}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^10,x)
```

output
```
( - 70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b*
*4*x**4)/(630*x**9)
```

3.310 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx=-\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)}-\frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)}-\frac{3ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)}-\frac{b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}-\frac{b^3B\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}$$

output
```
-1/10*a^3*A*((b*x+a)^2)^(1/2)/x^10/(b*x+a)-1/9*a^2*(3*A*b+B*a)*((b*x+a)^2)
^(1/2)/x^9/(b*x+a)-3/8*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-1/7*b^2
*(A*b+3*B*a)*((b*x+a)^2)^(1/2)/x^7/(b*x+a)-1/6*b^3*B*((b*x+a)^2)^(1/2)/x^6
/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx)^2}\left(60b^3x^3(6A+7Bx)+135ab^2x^2(7A+8Bx)+105a^2bx(8A+9Bx)+28a^3(9A+10Bx)\right)}{2520x^{10}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^11,x]
```

output
```
-1/2520*(Sqrt[(a + b*x)^2]*(60*b^3*x^3*(6*A + 7*B*x) + 135*a*b^2*x^2*(7*A
+ 8*B*x) + 105*a^2*b*x*(8*A + 9*B*x) + 28*a^3*(9*A + 10*B*x)))/(x^10*(a +
b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{11}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{11}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{11}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^{11}}+\frac{(3Ab+aB)a^2}{x^{10}}+\frac{3b(Ab+aB)a}{x^9}+\frac{b^3B}{x^7}+\frac{b^2(Ab+3aB)}{x^8}\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{10x^{10}}-\frac{a^2(aB+3Ab)}{9x^9}-\frac{b^2(3aB+Ab)}{7x^7}-\frac{3ab(aB+Ab)}{8x^8}-\frac{b^3B}{6x^6}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^11,x]
```

output
```
((-1/10*(a^3*A)/x^10 - (a^2*(3*A*b + a*B))/(9*x^9) - (3*a*b*(A*b + a*B))/(
8*x^8) - (b^2*(A*b + 3*a*B))/(7*x^7) - (b^3*B)/(6*x^6))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^3 x^4}{6}+\left(-\frac{1}{7}A b^3-\frac{3}{7}B a b^2\right)x^3+\left(-\frac{3}{8}A a b^2-\frac{3}{8}B a^2 b\right)x^2+\left(-\frac{1}{3}A a^2 b-\frac{1}{9}B a^3\right)x-\frac{a^3 A}{10}\right)}{(bx+a)x^{10}}$	90
gosper	$-\frac{\left(420B b^3 x^4+360A b^3 x^3+1080B a b^2 x^3+945A a b^2 x^2+945B a^2 b x^2+840A a^2 b x+280B a^3 x+252a^3 A\right)\left((bx+a)^2\right)^{\frac{3}{2}}}{2520x^{10}(bx+a)^3}$	92
default	$-\frac{\left(420B b^3 x^4+360A b^3 x^3+1080B a b^2 x^3+945A a b^2 x^2+945B a^2 b x^2+840A a^2 b x+280B a^3 x+252a^3 A\right)\left((bx+a)^2\right)^{\frac{3}{2}}}{2520x^{10}(bx+a)^3}$	92
orering	$-\frac{\left(420B b^3 x^4+360A b^3 x^3+1080B a b^2 x^3+945A a b^2 x^2+945B a^2 b x^2+840A a^2 b x+280B a^3 x+252a^3 A\right)\left(b^2x^2+2abx+a^2\right)^{\frac{3}{2}}}{2520x^{10}(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/6*B*b^3*x^4+(-1/7*A*b^3-3/7*B*a*b^2)*x^3+(-3
/8*A*a*b^2-3/8*B*a^2*b)*x^2+(-1/3*A*a^2*b-1/9*B*a^3)*x-1/10*a^3*A)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx = -\frac{420\,Bb^3x^4+252\,Aa^3+360\,(3\,Bab^2+Ab^3)x^3+945\,(Ba^2b+Aab^2)x^2+280\,(Ba^3+3\,Aa^2b)x}{2520\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(420*B*b^3*x^4 + 252*A*a^3 + 360*(3*B*a*b^2 + A*b^3)*x^3 + 945*(B*
a^2*b + A*a*b^2)*x^2 + 280*(B*a^3 + 3*A*a^2*b)*x)/x^10
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**11,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^9/a^9 + 1/4*(b^2*x^2 + 2*a*b*x +
a^2)^(3/2)*A*b^10/a^10 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^8/(a^8*x)
 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^9/(a^9*x) + 1/4*(b^2*x^2 + 2*a*
b*x + a^2)^(5/2)*B*b^7/(a^9*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b
^8/(a^10*x^2) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^6/(a^8*x^3) + 1/4*
(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^7/(a^9*x^3) + 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*B*b^5/(a^7*x^4) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^6/(a
^8*x^4) - 125/504*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^4/(a^6*x^5) + 209/84
0*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/(a^7*x^5) + 121/504*(b^2*x^2 + 2*a
*b*x + a^2)^(5/2)*B*b^3/(a^5*x^6) - 41/168*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*A*b^4/(a^6*x^6) - 37/168*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^2/(a^4*x^7)
+ 13/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/(a^5*x^7) + 13/72*(b^2*x^2 +
 2*a*b*x + a^2)^(5/2)*B*b/(a^3*x^8) - 5/24*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*A*b^2/(a^4*x^8) - 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/(a^2*x^9) + 1/6*(
b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b/(a^3*x^9) - 1/10*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*A/(a^2*x^10)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx = \frac{(5\,Bab^9-3\,Ab^{10})\text{sgn}(bx+a)}{2520\,a^7} - \frac{420\,Bb^3x^4\text{sgn}(bx+a)+1080\,Bab^2x^3\text{sgn}(bx+a)+360\,Ab^3x^3\text{sgn}(bx+a)+945\,Ba^2bx^2\text{sgn}(bx+a)+9}{2520\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^11,x, algorithm="giac")
```

output
```
1/2520*(5*B*a*b^9 - 3*A*b^10)*sgn(b*x + a)/a^7 - 1/2520*(420*B*b^3*x^4*sgn
(b*x + a) + 1080*B*a*b^2*x^3*sgn(b*x + a) + 360*A*b^3*x^3*sgn(b*x + a) + 9
45*B*a^2*b*x^2*sgn(b*x + a) + 945*A*a*b^2*x^2*sgn(b*x + a) + 280*B*a^3*x*s
gn(b*x + a) + 840*A*a^2*b*x*sgn(b*x + a) + 252*A*a^3*sgn(b*x + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx = -\frac{\left(\frac{B\,a^3}{9}+\frac{A\,b\,a^2}{3}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^9\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^3}{7}+\frac{3\,B\,a\,b^2}{7}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^7\,\left(a+b\,x\right)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{10\,x^{10}\,\left(a+b\,x\right)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,\left(a+b\,x\right)} - \frac{3\,a\,b\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^11,x)
```

output
```
- (((B*a^3)/9 + (A*a^2*b)/3)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^9*(a + b*
x)) - (((A*b^3)/7 + (3*B*a*b^2)/7)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^7*(
a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(10*x^10*(a + b*x)) -
(B*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (3*a*b*(A*b +
B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{11}}\,dx = \frac{-210b^4x^4-720a\,b^3x^3-945a^2b^2x^2-560a^3bx-126a^4}{1260x^{10}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^11,x)
```

output
```
( - 126*a**4 - 560*a**3*b*x - 945*a**2*b**2*x**2 - 720*a*b**3*x**3 - 210*b
**4*x**4)/(1260*x**10)
```

3.311 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = -\frac{a^3A\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)} - \frac{a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{10x^{10}(a+bx)} - \frac{ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^9(a+bx)} - \frac{b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{b^3B\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}$$

output
```
-1/11*a^3*A*((b*x+a)^2)^(1/2)/x^11/(b*x+a)-1/10*a^2*(3*A*b+B*a)*((b*x+a)^2
)^(1/2)/x^10/(b*x+a)-1/3*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^9/(b*x+a)-1/8*b
^2*(A*b+3*B*a)*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-1/7*b^3*B*((b*x+a)^2)^(1/2)/x
^7/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx)^2}(165b^3x^3(7A+8Bx)+385ab^2x^2(8A+9Bx)+308a^2bx(9A+10Bx)+84a^3(10A+11Bx))}{9240x^{11}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^12,x]
```

output
```
-1/9240*(Sqrt[(a + b*x)^2]*(165*b^3*x^3*(7*A + 8*B*x) + 385*a*b^2*x^2*(8*A
 + 9*B*x) + 308*a^2*b*x*(9*A + 10*B*x) + 84*a^3*(10*A + 11*B*x)))/(x^11*(a
 + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{12}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{12}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{12}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^3}{x^{12}}+\frac{(3Ab+aB)a^2}{x^{11}}+\frac{3b(Ab+aB)a}{x^{10}}+\frac{b^3B}{x^8}+\frac{b^2(Ab+3aB)}{x^9}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3A}{11x^{11}}-\frac{a^2(aB+3Ab)}{10x^{10}}-\frac{b^2(3aB+Ab)}{8x^8}-\frac{ab(aB+Ab)}{3x^9}-\frac{b^3B}{7x^7}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^12,x]
```

output
```
((-1/11*(a^3*A)/x^11 - (a^2*(3*A*b + a*B))/(10*x^10) - (a*b*(A*b + a*B))/(
3*x^9) - (b^2*(A*b + 3*a*B))/(8*x^8) - (b^3*B)/(7*x^7))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B\,b^3x^4}{7}+(-\frac{1}{8}A\,b^3-\frac{3}{8}B a\,b^2)x^3+(-\frac{1}{3}A a\,b^2-\frac{1}{3}B\,a^2b)x^2+(-\frac{3}{10}A\,a^2b-\frac{1}{10}B\,a^3)x-\frac{a^3A}{11}\right)}{(bx+a)x^{11}}$
gosper	$-\frac{(1320B\,b^3x^4+1155A\,b^3x^3+3465Ba\,b^2x^3+3080Aa\,b^2x^2+3080B\,a^2b\,x^2+2772A\,a^2bx+924B\,a^3x+840a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{9240x^{11}(bx+a)^3}$
default	$-\frac{(1320B\,b^3x^4+1155A\,b^3x^3+3465Ba\,b^2x^3+3080Aa\,b^2x^2+3080B\,a^2b\,x^2+2772A\,a^2bx+924B\,a^3x+840a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{9240x^{11}(bx+a)^3}$
orering	$-\frac{(1320B\,b^3x^4+1155A\,b^3x^3+3465Ba\,b^2x^3+3080Aa\,b^2x^2+3080B\,a^2b\,x^2+2772A\,a^2bx+924B\,a^3x+840a^3A)\,(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{9240x^{11}(bx+a)^3}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/7*B*b^3*x^4+(-1/8*A*b^3-3/8*B*a*b^2)*x^3+(-1
/3*A*a*b^2-1/3*B*a^2*b)*x^2+(-3/10*A*a^2*b-1/10*B*a^3)*x-1/11*a^3*A)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = -\frac{1320\,Bb^3x^4+840\,Aa^3+1155\,(3\,Bab^2+Ab^3)x^3+3080\,(Ba^2b+Aab^2)x^2+924\,(Ba^3+3\,Aa^2b)x}{9240\,x^{11}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
-1/9240*(1320*B*b^3*x^4 + 840*A*a^3 + 1155*(3*B*a*b^2 + A*b^3)*x^3 + 3080*
(B*a^2*b + A*a*b^2)*x^2 + 924*(B*a^3 + 3*A*a^2*b)*x)/x^11
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**12,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**12, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(145) = 290.

Time = 0.05 (sec) , antiderivative size = 675, normalized size of antiderivative = 3.21

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^10/a^10 - 1/4*(b^2*x^2 + 2*a*b*x +
 a^2)^(3/2)*A*b^11/a^11 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^9/(a^9*x
) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^10/(a^10*x) - 1/4*(b^2*x^2 + 2
*a*b*x + a^2)^(5/2)*B*b^8/(a^10*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)
*A*b^9/(a^11*x^2) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^7/(a^9*x^3) -
1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^8/(a^10*x^3) - 1/4*(b^2*x^2 + 2*a*
b*x + a^2)^(5/2)*B*b^6/(a^8*x^4) + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b
^7/(a^9*x^4) + 209/840*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^5/(a^7*x^5) - 3
29/1320*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^6/(a^8*x^5) - 41/168*(b^2*x^2
+ 2*a*b*x + a^2)^(5/2)*B*b^4/(a^6*x^6) + 65/264*(b^2*x^2 + 2*a*b*x + a^2)^
(5/2)*A*b^5/(a^7*x^6) + 13/56*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^3/(a^5*x
^7) - 21/88*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4/(a^6*x^7) - 5/24*(b^2*x^
2 + 2*a*b*x + a^2)^(5/2)*B*b^2/(a^4*x^8) + 59/264*(b^2*x^2 + 2*a*b*x + a^2
)^(5/2)*A*b^3/(a^5*x^8) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b/(a^3*x^9
) - 13/66*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2/(a^4*x^9) - 1/10*(b^2*x^2
+ 2*a*b*x + a^2)^(5/2)*B/(a^2*x^10) + 17/110*(b^2*x^2 + 2*a*b*x + a^2)^(5/
2)*A*b/(a^3*x^10) - 1/11*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A/(a^2*x^11)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = -\frac{(11\,Bab^{10}-7\,Ab^{11})\mathrm{sgn}(bx+a)}{9240\,a^8} - \frac{1320\,Bb^3x^4\mathrm{sgn}(bx+a)+3465\,Bab^2x^3\mathrm{sgn}(bx+a)+1155\,Ab^3x^3\mathrm{sgn}(bx+a)+3080\,Ba^2bx^2\mathrm{sgn}(bx+a)}{9240\,x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^12,x, algorithm="giac")
```

output
```
-1/9240*(11*B*a*b^10 - 7*A*b^11)*sgn(b*x + a)/a^8 - 1/9240*(1320*B*b^3*x^4
*sgn(b*x + a) + 3465*B*a*b^2*x^3*sgn(b*x + a) + 1155*A*b^3*x^3*sgn(b*x + a
) + 3080*B*a^2*b*x^2*sgn(b*x + a) + 3080*A*a*b^2*x^2*sgn(b*x + a) + 924*B*
a^3*x*sgn(b*x + a) + 2772*A*a^2*b*x*sgn(b*x + a) + 840*A*a^3*sgn(b*x + a))
/x^11
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = -\frac{\left(\frac{B\,a^3}{10}+\frac{3\,A\,b\,a^2}{10}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^{10}\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^3}{8}+\frac{3\,B\,a\,b^2}{8}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^8\,\left(a+b\,x\right)} - \frac{A\,a^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{11\,x^{11}\,\left(a+b\,x\right)} - \frac{B\,b^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)} - \frac{a\,b\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^9\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^12,x)
```

output
```
- (((B*a^3)/10 + (3*A*a^2*b)/10)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^10*(a
 + b*x)) - (((A*b^3)/8 + (3*B*a*b^2)/8)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(
x^8*(a + b*x)) - (A*a^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(11*x^11*(a + b*x
)) - (B*b^3*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (a*b*(A*b
 + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^9*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{12}}\,dx = \frac{-330b^4x^4-1155a\,b^3x^3-1540a^2b^2x^2-924a^3bx-210a^4}{2310x^{11}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^12,x)
```

output
```
( - 210*a**4 - 924*a**3*b*x - 1540*a**2*b**2*x**2 - 1155*a*b**3*x**3 - 330
*b**4*x**4)/(2310*x**11)
```

3.312 $\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 303

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^5Ax^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{a^4(5Ab+aB)x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{5a^3b(2Ab+aB)x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{a^2b^2(Ab+aB)x^{10}\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{5ab^3(Ab+2aB)x^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{b^4(Ab+5aB)x^{12}\sqrt{a^2+2abx+b^2x^2}}{12(a+bx)}+\frac{b^5Bx^{13}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}$$

output

```
a^5*A*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+a^4*(5*A*b+B*a)*x^8*((b*x+a)^2)^(1
/2)/(8*b*x+8*a)+5*a^3*b*(2*A*b+B*a)*x^9*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+a^2*
b^2*(A*b+B*a)*x^10*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^3*(A*b+2*B*a)*x^11*((b*
x+a)^2)^(1/2)/(11*b*x+11*a)+b^4*(A*b+5*B*a)*x^12*((b*x+a)^2)^(1/2)/(12*b*x
+12*a)+b^5*B*x^13*((b*x+a)^2)^(1/2)/(13*b*x+13*a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^7\sqrt{(a+bx)^2}(1287a^5(8A+7Bx)+5005a^4bx(9A+8Bx)+8008a^3b^2x^2(10A+9Bx)+65}{72072(a+bx}$$

input
```
Integrate[x^6*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^7*Sqrt[(a + b*x)^2]*(1287*a^5*(8*A + 7*B*x) + 5005*a^4*b*x*(9*A + 8*B*x
) + 8008*a^3*b^2*x^2*(10*A + 9*B*x) + 6552*a^2*b^3*x^3*(11*A + 10*B*x) + 2
730*a*b^4*x^4*(12*A + 11*B*x) + 462*b^5*x^5*(13*A + 12*B*x)))/(72072*(a +
b*x))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^6(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^6(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^5Bx^{12}+b^4(Ab+5aB)x^{11}+5ab^3(Ab+2aB)x^{10}+10a^2b^2(Ab+aB)x^9+5a^3b(2Ab+aI\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{7}a^5Ax^7+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{12}b^4x^{12}(5aB+A\right.}{a+bx}$$

input
```
Int[x^6*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^8)/8
+ (5*a^3*b*(2*A*b + a*B)*x^9)/9 + a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b
 + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*B*x^13)/13))/(a +
b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.46

method	result
gosper	$\frac{x^7(5544B\,b^5x^6+6006A\,b^5x^5+30030Ba\,b^4x^5+32760Aa\,b^4x^4+65520B\,a^2b^3x^4+72072A\,a^2b^3x^3+72072B\,a^3b^2x^3+80080A\,a^3b^2x^2+}{72072(bx+a)^5}$
default	$\frac{x^7(5544B\,b^5x^6+6006A\,b^5x^5+30030Ba\,b^4x^5+32760Aa\,b^4x^4+65520B\,a^2b^3x^4+72072A\,a^2b^3x^3+72072B\,a^3b^2x^3+80080A\,a^3b^2x^2+}{72072(bx+a)^5}$
orering	$\frac{x^7(5544B\,b^5x^6+6006A\,b^5x^5+30030Ba\,b^4x^5+32760Aa\,b^4x^4+65520B\,a^2b^3x^4+72072A\,a^2b^3x^3+72072B\,a^3b^2x^3+80080A\,a^3b^2x^2+}{72072(bx+a)^5}$
risch	$\frac{b^5Bx^{13}\sqrt{(bx+a)^2}}{13bx+13a}+\frac{\sqrt{(bx+a)^2}\,(A\,b^5+5Ba\,b^4)x^{12}}{12bx+12a}+\frac{\sqrt{(bx+a)^2}\,(5Aa\,b^4+10B\,a^2b^3)x^{11}}{11bx+11a}+\frac{\sqrt{(bx+a)^2}\,(10A\,a^2b^3+10B\,a^3b^2)}{10bx+10a}$

input
```
int(x^6*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/72072*x^7*(5544*B*b^5*x^6+6006*A*b^5*x^5+30030*B*a*b^4*x^5+32760*A*a*b^4
*x^4+65520*B*a^2*b^3*x^4+72072*A*a^2*b^3*x^3+72072*B*a^3*b^2*x^3+80080*A*a
^3*b^2*x^2+40040*B*a^4*b*x^2+45045*A*a^4*b*x+9009*B*a^5*x+10296*A*a^5)*((b
*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{13}\,Bb^5x^{13}$$
$$+\frac{1}{7}\,Aa^5x^7+\frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12}+\frac{5}{11}\left(2\,Ba^2b^3+Aab^4\right)x^{11}$$
$$+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{8}\left(Ba^5+5\,Aa^4b\right)x^8$$

input
```
integrate(x^6*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/13*B*b^5*x^13 + 1/7*A*a^5*x^7 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/11*(2*
B*a^2*b^3 + A*a*b^4)*x^11 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/9*(B*a^4*b +
2*A*a^3*b^2)*x^9 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40368 vs. $2(228) = 456$.

Time = 1.25 (sec) , antiderivative size = 40368, normalized size of antiderivative = 133.23

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**12/13 + x**11*(A*b*
*6 + 53*B*a*b**5/13)/(12*b**2) + x**10*(6*A*a*b**5 + 183*B*a**2*b**4/13 -
23*a*(A*b**6 + 53*B*a*b**5/13)/(12*b))/(11*b**2) + x**9*(15*A*a**2*b**4 +
20*B*a**3*b**3 - 11*a**2*(A*b**6 + 53*B*a*b**5/13)/(12*b**2) - 21*a*(6*A*a
*b**5 + 183*B*a**2*b**4/13 - 23*a*(A*b**6 + 53*B*a*b**5/13)/(12*b))/(11*b)
)/(10*b**2) + x**8*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 10*a**2*(6*A*a*b**5
+ 183*B*a**2*b**4/13 - 23*a*(A*b**6 + 53*B*a*b**5/13)/(12*b))/(11*b**2) -
19*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 11*a**2*(A*b**6 + 53*B*a*b**5/13)/
(12*b**2) - 21*a*(6*A*a*b**5 + 183*B*a**2*b**4/13 - 23*a*(A*b**6 + 53*B*a*
b**5/13)/(12*b))/(11*b))/(10*b))/(9*b**2) + x**7*(15*A*a**4*b**2 + 6*B*a**
5*b - 9*a**2*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 11*a**2*(A*b**6 + 53*B*a*b
**5/13)/(12*b**2) - 21*a*(6*A*a*b**5 + 183*B*a**2*b**4/13 - 23*a*(A*b**6 +
 53*B*a*b**5/13)/(12*b))/(11*b))/(10*b**2) - 17*a*(20*A*a**3*b**3 + 15*B*a
**4*b**2 - 10*a**2*(6*A*a*b**5 + 183*B*a**2*b**4/13 - 23*a*(A*b**6 + 53*B*
a*b**5/13)/(12*b))/(11*b**2) - 19*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 11*
a**2*(A*b**6 + 53*B*a*b**5/13)/(12*b**2) - 21*a*(6*A*a*b**5 + 183*B*a**2*b
**4/13 - 23*a*(A*b**6 + 53*B*a*b**5/13)/(12*b))/(11*b))/(10*b))/(9*b))/(8*
b**2) + x**6*(6*A*a**5*b + B*a**6 - 8*a**2*(20*A*a**3*b**3 + 15*B*a**4*b**
2 - 10*a**2*(6*A*a*b**5 + 183*B*a**2*b**4/13 - 23*a*(A*b**6 + 53*B*a*b**5/
13)/(12*b))/(11*b**2) - 19*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 11*a**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(214) = 428.

Time = 0.05 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.59

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bx^6}{13\,b^2} - \frac{19\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bax^5}{156\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ax^5}{12\,b^2} + \frac{251\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^2x^4}{1716\,b^4} - \frac{17\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aax^4}{132\,b^3} - \frac{68\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^3x^3}{429\,b^5} + \frac{5\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^2x^3}{33\,b^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^7x}{6\,b^7} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^6x}{6\,b^6} + \frac{211\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^4x^2}{1287\,b^6} - \frac{16\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^3x^2}{99\,b^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^8}{6\,b^8} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^7}{6\,b^7} - \frac{1709\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^5x}{10296\,b^7} + \frac{131\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^4x}{792\,b^6} + \frac{1715\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^6}{10296\,b^8} - \frac{923\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^5}{5544\,b^7}$$

input
```
integrate(x^6*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/13*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*x^6/b^2 - 19/156*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*B*a*x^5/b^3 + 1/12*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*x^5/b^2
 + 251/1716*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^2*x^4/b^4 - 17/132*(b^2*x^
2 + 2*a*b*x + a^2)^(7/2)*A*a*x^4/b^3 - 68/429*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*B*a^3*x^3/b^5 + 5/33*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^2*x^3/b^4 - 1
/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^7*x/b^7 + 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*A*a^6*x/b^6 + 211/1287*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^4*x^
2/b^6 - 16/99*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^3*x^2/b^5 - 1/6*(b^2*x^2
 + 2*a*b*x + a^2)^(5/2)*B*a^8/b^8 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*
a^7/b^7 - 1709/10296*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^5*x/b^7 + 131/792
*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^4*x/b^6 + 1715/10296*(b^2*x^2 + 2*a*b
*x + a^2)^(7/2)*B*a^6/b^8 - 923/5544*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^5
/b^7
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.73

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{13}\,Bb^5x^{13}\mathrm{sgn}(bx+a)+\frac{5}{12}\,Bab^4x^{12}\mathrm{sgn}(bx+a)+\frac{1}{12}\,Ab^5x^{12}\mathrm{sgn}(bx+a)+\frac{10}{11}\,Ba^2b^3x^{11}\mathrm{sgn}(bx+a)+\frac{5}{11}\,Aab^4x^{11}\mathrm{sgn}(bx+a)+Ba^3b^2x^{10}\mathrm{sgn}(bx+a)+Aa^2b^3x^{10}\mathrm{sgn}(bx+a)+\frac{5}{9}\,Ba^4bx^9\mathrm{sgn}(bx+a)+\frac{10}{9}\,Aa^3b^2x^9\mathrm{sgn}(bx+a)+\frac{1}{8}\,Ba^5x^8\mathrm{sgn}(bx+a)+\frac{5}{8}\,Aa^4bx^8\mathrm{sgn}(bx+a)+\frac{1}{7}\,Aa^5x^7\mathrm{sgn}(bx+a)-\frac{(7\,Ba^{13}-13\,Aa^{12}b)\mathrm{sgn}(bx+a)}{72072\,b^8}$$

input
```
integrate(x^6*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/13*B*b^5*x^13*sgn(b*x + a) + 5/12*B*a*b^4*x^12*sgn(b*x + a) + 1/12*A*b^5
*x^12*sgn(b*x + a) + 10/11*B*a^2*b^3*x^11*sgn(b*x + a) + 5/11*A*a*b^4*x^11
*sgn(b*x + a) + B*a^3*b^2*x^10*sgn(b*x + a) + A*a^2*b^3*x^10*sgn(b*x + a)
+ 5/9*B*a^4*b*x^9*sgn(b*x + a) + 10/9*A*a^3*b^2*x^9*sgn(b*x + a) + 1/8*B*a
^5*x^8*sgn(b*x + a) + 5/8*A*a^4*b*x^8*sgn(b*x + a) + 1/7*A*a^5*x^7*sgn(b*x
 + a) - 1/72072*(7*B*a^13 - 13*A*a^12*b)*sgn(b*x + a)/b^8
```

Mupad [F(-1)]

Timed out.

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^6\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x^6*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^6*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x^6(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^7(924b^6x^6+6006a\,b^5x^5+16380a^2b^4x^4+24024a^3b^3x^3+20020a^4b^2x^2+9009a^5bx+1716}{12012}$$

input
```
int(x^6*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**7*(1716*a**6 + 9009*a**5*b*x + 20020*a**4*b**2*x**2 + 24024*a**3*b**3*
x**3 + 16380*a**2*b**4*x**4 + 6006*a*b**5*x**5 + 924*b**6*x**6))/12012
```

3.313 $\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 306

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^5Ax^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{a^4(5Ab+aB)x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{5a^3b(2Ab+aB)x^8\sqrt{a^2+2abx+b^2x^2}}{8(a+bx)}+\frac{10a^2b^2(Ab+aB)x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{ab^3(Ab+2aB)x^{10}\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)}+\frac{b^4(Ab+5aB)x^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{b^5Bx^{12}\sqrt{a^2+2abx+b^2x^2}}{12(a+bx)}$$

output
```
a^5*A*x^6*((b*x+a)^2)^(1/2)/(6*b*x+6*a)+a^4*(5*A*b+B*a)*x^7*((b*x+a)^2)^(1
/2)/(7*b*x+7*a)+5*a^3*b*(2*A*b+B*a)*x^8*((b*x+a)^2)^(1/2)/(8*b*x+8*a)+10*a
^2*b^2*(A*b+B*a)*x^9*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+a*b^3*(A*b+2*B*a)*x^10*
((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^4*(A*b+5*B*a)*x^11*((b*x+a)^2)^(1/2)/(11*b
*x+11*a)+b^5*B*x^12*((b*x+a)^2)^(1/2)/(12*b*x+12*a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^6\sqrt{(a+bx)^2}(132a^5(7A+6Bx)+495a^4bx(8A+7Bx)+770a^3b^2x^2(9A+8Bx)+616a^2b}{5544(a+bx)}$$

input
```
Integrate[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^6*Sqrt[(a + b*x)^2]*(132*a^5*(7*A + 6*B*x) + 495*a^4*b*x*(8*A + 7*B*x)
+ 770*a^3*b^2*x^2*(9*A + 8*B*x) + 616*a^2*b^3*x^3*(10*A + 9*B*x) + 252*a*b
^4*x^4*(11*A + 10*B*x) + 42*b^5*x^5*(12*A + 11*B*x)))/(5544*(a + b*x))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \ 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^5(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^5(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \ 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^5Bx^{11}+b^4(Ab+5aB)x^{10}+5ab^3(Ab+2aB)x^9+10a^2b^2(Ab+aB)x^8+5a^3b(2Ab+aB\right.}{a+bx}$$

$\big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{6}a^5Ax^6+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+\right.}{a+bx}$$

input
```
Int[x^5*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^5*A*x^6)/6 + (a^4*(5*A*b + a*B)*x^7)/7
+ (5*a^3*b*(2*A*b + a*B)*x^8)/8 + (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (a*b^3*
(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5*a*B)*x^11)/11 + (b^5*B*x^12)/12))/(a
 + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.46

method	result
gosper	$\frac{x^6(462B\,b^5x^6+504A\,b^5x^5+2520Ba\,b^4x^5+2772Aa\,b^4x^4+5544B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+6930A\,a^3b^2x^2+3465B\,a^4}{5544(bx+a)^5}$
default	$\frac{x^6(462B\,b^5x^6+504A\,b^5x^5+2520Ba\,b^4x^5+2772Aa\,b^4x^4+5544B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+6930A\,a^3b^2x^2+3465B\,a^4}{5544(bx+a)^5}$
orering	$\frac{x^6(462B\,b^5x^6+504A\,b^5x^5+2520Ba\,b^4x^5+2772Aa\,b^4x^4+5544B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+6930A\,a^3b^2x^2+3465B\,a^4}{5544(bx+a)^5}$
risch	$\frac{b^5B\,x^{12}\sqrt{(bx+a)^2}}{12bx+12a}+\frac{\sqrt{(bx+a)^2}\,(A\,b^5+5Ba\,b^4)x^{11}}{11bx+11a}+\frac{\sqrt{(bx+a)^2}\,(5Aa\,b^4+10B\,a^2b^3)x^{10}}{10bx+10a}+\frac{\sqrt{(bx+a)^2}\,(10A\,a^2b^3+10B\,a^3b^2)}{9bx+9a}$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5544*x^6*(462*B*b^5*x^6+504*A*b^5*x^5+2520*B*a*b^4*x^5+2772*A*a*b^4*x^4+
5544*B*a^2*b^3*x^4+6160*A*a^2*b^3*x^3+6160*B*a^3*b^2*x^3+6930*A*a^3*b^2*x^
2+3465*B*a^4*b*x^2+3960*A*a^4*b*x+792*B*a^5*x+924*A*a^5)*((b*x+a)^2)^(5/2)
/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{12}\,Bb^5x^{12}+\frac{1}{6}\,Aa^5x^6+\frac{1}{11}\,\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{1}{2}\,\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{9}\,\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{8}\,\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{7}\,\left(Ba^5+5\,Aa^4b\right)x^7$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*B*b^5*x^12 + 1/6*A*a^5*x^6 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 1/2*(2*B
*a^2*b^3 + A*a*b^4)*x^10 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/8*(B*a^4*b
 + 2*A*a^3*b^2)*x^8 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24941 vs. $2(231) = 462$.

Time = 1.27 (sec) , antiderivative size = 24941, normalized size of antiderivative = 81.51

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**11/12 + x**10*(A*b*
*6 + 49*B*a*b**5/12)/(11*b**2) + x**9*(6*A*a*b**5 + 169*B*a**2*b**4/12 - 2
1*a*(A*b**6 + 49*B*a*b**5/12)/(11*b))/(10*b**2) + x**8*(15*A*a**2*b**4 + 2
0*B*a**3*b**3 - 10*a**2*(A*b**6 + 49*B*a*b**5/12)/(11*b**2) - 19*a*(6*A*a*
b**5 + 169*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*B*a*b**5/12)/(11*b))/(10*b))
/(9*b**2) + x**7*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 9*a**2*(6*A*a*b**5 + 1
69*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*B*a*b**5/12)/(11*b))/(10*b**2) - 17*
a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 10*a**2*(A*b**6 + 49*B*a*b**5/12)/(11
*b**2) - 19*a*(6*A*a*b**5 + 169*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*B*a*b**
5/12)/(11*b))/(10*b))/(9*b))/(8*b**2) + x**6*(15*A*a**4*b**2 + 6*B*a**5*b
 - 8*a**2*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 10*a**2*(A*b**6 + 49*B*a*b**5/
12)/(11*b**2) - 19*a*(6*A*a*b**5 + 169*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*
B*a*b**5/12)/(11*b))/(10*b))/(9*b**2) - 15*a*(20*A*a**3*b**3 + 15*B*a**4*b
**2 - 9*a**2*(6*A*a*b**5 + 169*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*B*a*b**5
/12)/(11*b))/(10*b**2) - 17*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 10*a**2*(
A*b**6 + 49*B*a*b**5/12)/(11*b**2) - 19*a*(6*A*a*b**5 + 169*B*a**2*b**4/12
 - 21*a*(A*b**6 + 49*B*a*b**5/12)/(11*b))/(10*b))/(9*b))/(8*b))/(7*b**2) +
 x**5*(6*A*a**5*b + B*a**6 - 7*a**2*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 9*a
**2*(6*A*a*b**5 + 169*B*a**2*b**4/12 - 21*a*(A*b**6 + 49*B*a*b**5/12)/(11*
b))/(10*b**2) - 17*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 10*a**2*(A*b**6...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.38

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Bx^5}{12\,b^2}-\frac{17\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Bax^4}{132\,b^3}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ax^4}{11\,b^2}+\frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^2x^3}{33\,b^4}-\frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aax^3}{22\,b^3}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^6x}{6\,b^6}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^5x}{6\,b^5}-\frac{16\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^3x^2}{99\,b^5}+\frac{31\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aa^2x^2}{198\,b^4}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^7}{6\,b^7}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^6}{6\,b^6}+\frac{131\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^4x}{792\,b^6}-\frac{65\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aa^3x}{396\,b^5}-\frac{923\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^5}{5544\,b^7}+\frac{461\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aa^4}{2772\,b^6}$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*x^5/b^2 - 17/132*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*B*a*x^4/b^3 + 1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*x^4/b^2
 + 5/33*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^2*x^3/b^4 - 3/22*(b^2*x^2 + 2*
a*b*x + a^2)^(7/2)*A*a*x^3/b^3 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^6
*x/b^6 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^5*x/b^5 - 16/99*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*B*a^3*x^2/b^5 + 31/198*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*A*a^2*x^2/b^4 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^7/b^7 - 1/6*(b
^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^6/b^6 + 131/792*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*B*a^4*x/b^6 - 65/396*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^3*x/b^5 -
 923/5544*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^5/b^7 + 461/2772*(b^2*x^2 +
2*a*b*x + a^2)^(7/2)*A*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.72

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{12}Bb^5x^{12}\mathrm{sgn}(bx+a)+\frac{5}{11}Bab^4x^{11}\mathrm{sgn}(bx+a)+\frac{1}{11}Ab^5x^{11}\mathrm{sgn}(bx+a)+Ba^2b^3x^{10}\mathrm{sgn}(bx+a)+\frac{1}{2}Aab^4x^{10}\mathrm{sgn}(bx+a)+\frac{10}{9}Ba^3b^2x^9\mathrm{sgn}(bx+a)+\frac{10}{9}Aa^2b^3x^9\mathrm{sgn}(bx+a)+\frac{5}{8}Ba^4bx^8\mathrm{sgn}(bx+a)+\frac{5}{4}Aa^3b^2x^8\mathrm{sgn}(bx+a)+\frac{1}{7}Ba^5x^7\mathrm{sgn}(bx+a)+\frac{5}{7}Aa^4bx^7\mathrm{sgn}(bx+a)+\frac{1}{6}Aa^5x^6\mathrm{sgn}(bx+a)+\frac{(Ba^{12}-2\,Aa^{11}b)\mathrm{sgn}(bx+a)}{5544\,b^7}$$

input
```
integrate(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/12*B*b^5*x^12*sgn(b*x + a) + 5/11*B*a*b^4*x^11*sgn(b*x + a) + 1/11*A*b^5
*x^11*sgn(b*x + a) + B*a^2*b^3*x^10*sgn(b*x + a) + 1/2*A*a*b^4*x^10*sgn(b*
x + a) + 10/9*B*a^3*b^2*x^9*sgn(b*x + a) + 10/9*A*a^2*b^3*x^9*sgn(b*x + a)
 + 5/8*B*a^4*b*x^8*sgn(b*x + a) + 5/4*A*a^3*b^2*x^8*sgn(b*x + a) + 1/7*B*a
^5*x^7*sgn(b*x + a) + 5/7*A*a^4*b*x^7*sgn(b*x + a) + 1/6*A*a^5*x^6*sgn(b*x
 + a) + 1/5544*(B*a^12 - 2*A*a^11*b)*sgn(b*x + a)/b^7
```

Mupad [F(-1)]

Timed out.

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^5\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x^5*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^5*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x^5(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^6(462b^6x^6+3024a\,b^5x^5+8316a^2b^4x^4+12320a^3b^3x^3+10395a^4b^2x^2+4752a^5bx+924a^6)}{5544}$$

input
```
int(x^5*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**6*(924*a**6 + 4752*a**5*b*x + 10395*a**4*b**2*x**2 + 12320*a**3*b**3*x
**3 + 8316*a**2*b**4*x**4 + 3024*a*b**5*x**5 + 462*b**6*x**6))/5544
```

3.314 $\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 306

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^5Ax^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{a^4(5Ab+aB)x^6\sqrt{a^2+2abx+b^2x^2}}{6(a+bx)}+\frac{5a^3b(2Ab+aB)x^7\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{5a^2b^2(Ab+aB)x^8\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)}+\frac{5ab^3(Ab+2aB)x^9\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{b^4(Ab+5aB)x^{10}\sqrt{a^2+2abx+b^2x^2}}{10(a+bx)}+\frac{b^5Bx^{11}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}$$

output
```
a^5*A*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+a^4*(5*A*b+B*a)*x^6*((b*x+a)^2)^(1
/2)/(6*b*x+6*a)+5*a^3*b*(2*A*b+B*a)*x^7*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+5*a^
2*b^2*(A*b+B*a)*x^8*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+5*a*b^3*(A*b+2*B*a)*x^9*
((b*x+a)^2)^(1/2)/(9*b*x+9*a)+b^4*(A*b+5*B*a)*x^10*((b*x+a)^2)^(1/2)/(10*b
*x+10*a)+b^5*B*x^11*((b*x+a)^2)^(1/2)/(11*b*x+11*a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^5\sqrt{(a+bx)^2}(462a^5(6A+5Bx)+1650a^4bx(7A+6Bx)+2475a^3b^2x^2(8A+7Bx)+192}{13860(a+bx)}$$

input
```
Integrate[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^5*Sqrt[(a + b*x)^2]*(462*a^5*(6*A + 5*B*x) + 1650*a^4*b*x*(7*A + 6*B*x)
 + 2475*a^3*b^2*x^2*(8*A + 7*B*x) + 1925*a^2*b^3*x^3*(9*A + 8*B*x) + 770*a
*b^4*x^4*(10*A + 9*B*x) + 126*b^5*x^5*(11*A + 10*B*x)))/(13860*(a + b*x))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^4(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^4(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^5Bx^{10}+b^4(Ab+5aB)x^9+5ab^3(Ab+2aB)x^8+10a^2b^2(Ab+aB)x^7+5a^3b(2Ab+aB)\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{5}a^5Ax^5+\frac{1}{6}a^4x^6(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+A\right.}{a+bx}$$

input
```
Int[x^4*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^6)/6
+ (5*a^3*b*(2*A*b + a*B)*x^7)/7 + (5*a^2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3
*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^10)/10 + (b^5*B*x^11)/11))/(a
 + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.46

method	result
gosper	$\frac{x^{5}\left(1260 B\,b^{5} x^{6}+1386 A\,b^{5} x^{5}+6930 B a\,b^{4} x^{5}+7700 A a\,b^{4} x^{4}+15400 B\,a^{2} b^{3} x^{4}+17325 A\,a^{2} b^{3} x^{3}+17325 B\,a^{3} b^{2} x^{3}+19800 A\,a^{3} b^{2} x^{2}+99\right.}{13860 \left(b x+a\right)^{5}}$
default	$\frac{x^{5}\left(1260 B\,b^{5} x^{6}+1386 A\,b^{5} x^{5}+6930 B a\,b^{4} x^{5}+7700 A a\,b^{4} x^{4}+15400 B\,a^{2} b^{3} x^{4}+17325 A\,a^{2} b^{3} x^{3}+17325 B\,a^{3} b^{2} x^{3}+19800 A\,a^{3} b^{2} x^{2}+99\right.}{13860 \left(b x+a\right)^{5}}$
orering	$\frac{x^{5}\left(1260 B\,b^{5} x^{6}+1386 A\,b^{5} x^{5}+6930 B a\,b^{4} x^{5}+7700 A a\,b^{4} x^{4}+15400 B\,a^{2} b^{3} x^{4}+17325 A\,a^{2} b^{3} x^{3}+17325 B\,a^{3} b^{2} x^{3}+19800 A\,a^{3} b^{2} x^{2}+99\right.}{13860 \left(b x+a\right)^{5}}$
risch	$\frac{b^{5} B\,x^{11} \sqrt{\left(b x+a\right)^{2}}}{11 b x+11 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(A\,b^{5}+5 B a\,b^{4}\right) x^{10}}{10 b x+10 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(5 A a\,b^{4}+10 B\,a^{2} b^{3}\right) x^{9}}{9 b x+9 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(10 A\,a^{2} b^{3}+10 B\,a^{3} b^{2}\right)}{8 b x+8 a}$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/13860*x^5*(1260*B*b^5*x^6+1386*A*b^5*x^5+6930*B*a*b^4*x^5+7700*A*a*b^4*x
^4+15400*B*a^2*b^3*x^4+17325*A*a^2*b^3*x^3+17325*B*a^3*b^2*x^3+19800*A*a^3
*b^2*x^2+9900*B*a^4*b*x^2+11550*A*a^4*b*x+2310*B*a^5*x+2772*A*a^5)*((b*x+a
)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{11}\,Bb^5x^{11}+\frac{1}{5}\,Aa^5x^5+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\frac{5}{7}\left(Ba^4b+2\,Aa^3b^2\right)x^7+\frac{1}{6}\left(Ba^5+5\,Aa^4b\right)x^6$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/11*B*b^5*x^11 + 1/5*A*a^5*x^5 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/9*(2*B
*a^2*b^3 + A*a*b^4)*x^9 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 5/7*(B*a^4*b +
 2*A*a^3*b^2)*x^7 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15400 vs. $2(233) = 466$.

Time = 1.14 (sec) , antiderivative size = 15400, normalized size of antiderivative = 50.33

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**10/11 + x**9*(A*b**
6 + 45*B*a*b**5/11)/(10*b**2) + x**8*(6*A*a*b**5 + 155*B*a**2*b**4/11 - 19
*a*(A*b**6 + 45*B*a*b**5/11)/(10*b))/(9*b**2) + x**7*(15*A*a**2*b**4 + 20*
B*a**3*b**3 - 9*a**2*(A*b**6 + 45*B*a*b**5/11)/(10*b**2) - 17*a*(6*A*a*b**
5 + 155*B*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5/11)/(10*b))/(9*b))/(8*
b**2) + x**6*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 8*a**2*(6*A*a*b**5 + 155*B
*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5/11)/(10*b))/(9*b**2) - 15*a*(15
*A*a**2*b**4 + 20*B*a**3*b**3 - 9*a**2*(A*b**6 + 45*B*a*b**5/11)/(10*b**2)
 - 17*a*(6*A*a*b**5 + 155*B*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5/11)/
(10*b))/(9*b))/(8*b))/(7*b**2) + x**5*(15*A*a**4*b**2 + 6*B*a**5*b - 7*a**
2*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 9*a**2*(A*b**6 + 45*B*a*b**5/11)/(10*
b**2) - 17*a*(6*A*a*b**5 + 155*B*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5
/11)/(10*b))/(9*b))/(8*b**2) - 13*a*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 8*a
**2*(6*A*a*b**5 + 155*B*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5/11)/(10*
b))/(9*b**2) - 15*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 9*a**2*(A*b**6 + 45
*B*a*b**5/11)/(10*b**2) - 17*a*(6*A*a*b**5 + 155*B*a**2*b**4/11 - 19*a*(A*
b**6 + 45*B*a*b**5/11)/(10*b))/(9*b))/(8*b))/(7*b))/(6*b**2) + x**4*(6*A*a
**5*b + B*a**6 - 6*a**2*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 8*a**2*(6*A*a*b
**5 + 155*B*a**2*b**4/11 - 19*a*(A*b**6 + 45*B*a*b**5/11)/(10*b))/(9*b**2)
 - 15*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 9*a**2*(A*b**6 + 45*B*a*b**5...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.18

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bx^4}{11\,b^2} - \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bax^3}{22\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ax^3}{10\,b^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^5x}{6\,b^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^4x}{6\,b^4} + \frac{31\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^2x^2}{198\,b^4} - \frac{13\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aax^2}{90\,b^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^6}{6\,b^6} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^5}{6\,b^5} - \frac{65\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^3x}{396\,b^5} + \frac{29\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^2x}{180\,b^4} + \frac{461\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^4}{2772\,b^6} - \frac{209\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa^3}{1260\,b^5}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*x^4/b^2 - 3/22*(b^2*x^2 + 2*a*b*x +
 a^2)^(7/2)*B*a*x^3/b^3 + 1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*x^3/b^2 -
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^5*x/b^5 + 1/6*(b^2*x^2 + 2*a*b*x
+ a^2)^(5/2)*A*a^4*x/b^4 + 31/198*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^2*x^
2/b^4 - 13/90*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a*x^2/b^3 - 1/6*(b^2*x^2 +
 2*a*b*x + a^2)^(5/2)*B*a^6/b^6 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^
5/b^5 - 65/396*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^3*x/b^5 + 29/180*(b^2*x
^2 + 2*a*b*x + a^2)^(7/2)*A*a^2*x/b^4 + 461/2772*(b^2*x^2 + 2*a*b*x + a^2)
^(7/2)*B*a^4/b^6 - 209/1260*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.73

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{11}Bb^5x^{11}\mathrm{sgn}(bx+a)+\frac{1}{2}Bab^4x^{10}\mathrm{sgn}(bx+a)+\frac{1}{10}Ab^5x^{10}\mathrm{sgn}(bx+a)+\frac{10}{9}Ba^2b^3x^9\mathrm{sgn}(bx+a)+\frac{5}{9}Aab^4x^9\mathrm{sgn}(bx+a)+\frac{5}{4}Ba^3b^2x^8\mathrm{sgn}(bx+a)+\frac{5}{4}Aa^2b^3x^8\mathrm{sgn}(bx+a)+\frac{5}{7}Ba^4bx^7\mathrm{sgn}(bx+a)+\frac{10}{7}Aa^3b^2x^7\mathrm{sgn}(bx+a)+\frac{1}{6}Ba^5x^6\mathrm{sgn}(bx+a)+\frac{5}{6}Aa^4bx^6\mathrm{sgn}(bx+a)+\frac{1}{5}Aa^5x^5\mathrm{sgn}(bx+a)-\frac{(5\,Ba^{11}-11\,Aa^{10}b)\mathrm{sgn}(bx+a)}{13860\,b^6}$$

input
```
integrate(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/11*B*b^5*x^11*sgn(b*x + a) + 1/2*B*a*b^4*x^10*sgn(b*x + a) + 1/10*A*b^5*
x^10*sgn(b*x + a) + 10/9*B*a^2*b^3*x^9*sgn(b*x + a) + 5/9*A*a*b^4*x^9*sgn(
b*x + a) + 5/4*B*a^3*b^2*x^8*sgn(b*x + a) + 5/4*A*a^2*b^3*x^8*sgn(b*x + a)
 + 5/7*B*a^4*b*x^7*sgn(b*x + a) + 10/7*A*a^3*b^2*x^7*sgn(b*x + a) + 1/6*B*
a^5*x^6*sgn(b*x + a) + 5/6*A*a^4*b*x^6*sgn(b*x + a) + 1/5*A*a^5*x^5*sgn(b*
x + a) - 1/13860*(5*B*a^11 - 11*A*a^10*b)*sgn(b*x + a)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^4\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^4*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x^4(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^5(210b^6x^6+1386a\,b^5x^5+3850a^2b^4x^4+5775a^3b^3x^3+4950a^4b^2x^2+2310a^5bx+462a^6)}{2310}$$

input
```
int(x^4*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**5*(462*a**6 + 2310*a**5*b*x + 4950*a**4*b**2*x**2 + 5775*a**3*b**3*x**
3 + 3850*a**2*b**4*x**4 + 1386*a*b**5*x**5 + 210*b**6*x**6))/2310
```

3.315 $\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 212

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = -\frac{a^3(Ab-aB)(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^5} + \frac{a^2(3Ab-4aB)(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^5} - \frac{3a(Ab-2aB)(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^5} + \frac{(Ab-4aB)(a+bx)^8\sqrt{a^2+2abx+b^2x^2}}{9b^5} + \frac{B(a+bx)^9\sqrt{a^2+2abx+b^2x^2}}{10b^5}$$

output
```
-1/6*a^3*(A*b-B*a)*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^5+1/7*a^2*(3*A*b-4*B*a)*(
b*x+a)^6*((b*x+a)^2)^(1/2)/b^5-3/8*a*(A*b-2*B*a)*(b*x+a)^7*((b*x+a)^2)^(1/
2)/b^5+1/9*(A*b-4*B*a)*(b*x+a)^8*((b*x+a)^2)^(1/2)/b^5+1/10*B*(b*x+a)^9*((
b*x+a)^2)^(1/2)/b^5
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.59

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^4\sqrt{(a+bx)^2}(126a^5(5A+4Bx)+420a^4bx(6A+5Bx)+600a^3b^2x^2(7A+6Bx)+450a^2b}{2520(a+bx)}$$

input
```
Integrate[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^4*Sqrt[(a + b*x)^2]*(126*a^5*(5*A + 4*B*x) + 420*a^4*b*x*(6*A + 5*B*x)
+ 600*a^3*b^2*x^2*(7*A + 6*B*x) + 450*a^2*b^3*x^3*(8*A + 7*B*x) + 175*a*b^
4*x^4*(9*A + 8*B*x) + 28*b^5*x^5*(10*A + 9*B*x)))/(2520*(a + b*x))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^3(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^3(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{B(a+bx)^9}{b^4}+\frac{(Ab-4aB)(a+bx)^8}{b^4}+\frac{3a(2aB-Ab)(a+bx)^7}{b^4}-\frac{a^2(4aB-3Ab)(a+bx)^6}{b^4}+\frac{a^3(aB-Ab)(a+bx)^5}{b^4}\right)}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^3(a+bx)^6(Ab-aB)}{6b^5}+\frac{a^2(a+bx)^7(3Ab-4aB)}{7b^5}+\frac{(a+bx)^9(Ab-4aB)}{9b^5}-\frac{3a(a+bx)^8(Ab-2aB)}{8b^5}+\frac{B(a+bx)^{10}}{10b^5}\right)}{a+bx}$$

input
```
Int[x^3*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/6*(a^3*(A*b - a*B)*(a + b*x)^6)/b^5 + (
a^2*(3*A*b - 4*a*B)*(a + b*x)^7)/(7*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x)^8)
/(8*b^5) + ((A*b - 4*a*B)*(a + b*x)^9)/(9*b^5) + (B*(a + b*x)^10)/(10*b^5)
))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{x^{4}\left(252 B\,b^{5} x^{6}+280 A\,b^{5} x^{5}+1400 B a\,b^{4} x^{5}+1575 A a\,b^{4} x^{4}+3150 B\,a^{2} b^{3} x^{4}+3600 A\,a^{2} b^{3} x^{3}+3600 B\,a^{3} b^{2} x^{3}+4200 A\,a^{3} b^{2} x^{2}+2100 B\,a^{4}}{2520\left(b x+a\right)^{5}}$
default	$\frac{x^{4}\left(252 B\,b^{5} x^{6}+280 A\,b^{5} x^{5}+1400 B a\,b^{4} x^{5}+1575 A a\,b^{4} x^{4}+3150 B\,a^{2} b^{3} x^{4}+3600 A\,a^{2} b^{3} x^{3}+3600 B\,a^{3} b^{2} x^{3}+4200 A\,a^{3} b^{2} x^{2}+2100 B\,a^{4}}{2520\left(b x+a\right)^{5}}$
orering	$\frac{x^{4}\left(252 B\,b^{5} x^{6}+280 A\,b^{5} x^{5}+1400 B a\,b^{4} x^{5}+1575 A a\,b^{4} x^{4}+3150 B\,a^{2} b^{3} x^{4}+3600 A\,a^{2} b^{3} x^{3}+3600 B\,a^{3} b^{2} x^{3}+4200 A\,a^{3} b^{2} x^{2}+2100 B\,a^{4}}{2520\left(b x+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b x+a\right)^{2}}\, b^{5} B\,x^{10}}{10 b x+10 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(A\,b^{5}+5 B a\,b^{4}\right) x^{9}}{9 b x+9 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(5 A a\,b^{4}+10 B\,a^{2} b^{3}\right) x^{8}}{8 b x+8 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\,\left(10 A\,a^{2} b^{3}+10 B\,a^{3} b^{2}\right) x}{7 b x+7 a}$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x^4*(252*B*b^5*x^6+280*A*b^5*x^5+1400*B*a*b^4*x^5+1575*A*a*b^4*x^4+
3150*B*a^2*b^3*x^4+3600*A*a^2*b^3*x^3+3600*B*a^3*b^2*x^3+4200*A*a^3*b^2*x^
2+2100*B*a^4*b*x^2+2520*A*a^4*b*x+504*B*a^5*x+630*A*a^5)*((b*x+a)^2)^(5/2)
/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{10}Bb^5x^{10}+\frac{1}{4}Aa^5x^4+\frac{1}{9}\left(5\,Bab^4+Ab^5\right)x^9+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+\frac{5}{6}\left(Ba^4b+2\,Aa^3b^2\right)x^6+\frac{1}{5}\left(Ba^5+5\,Aa^4b\right)x^5$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/10*B*b^5*x^10 + 1/4*A*a^5*x^4 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/8*(2*B*a
^2*b^3 + A*a*b^4)*x^8 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9493 vs. $2(156) = 312$.

Time = 0.98 (sec) , antiderivative size = 9493, normalized size of antiderivative = 44.78

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**9/10 + x**8*(A*b**6
 + 41*B*a*b**5/10)/(9*b**2) + x**7*(6*A*a*b**5 + 141*B*a**2*b**4/10 - 17*a
*(A*b**6 + 41*B*a*b**5/10)/(9*b))/(8*b**2) + x**6*(15*A*a**2*b**4 + 20*B*a
**3*b**3 - 8*a**2*(A*b**6 + 41*B*a*b**5/10)/(9*b**2) - 15*a*(6*A*a*b**5 +
141*B*a**2*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b))/(8*b))/(7*b**2)
 + x**5*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 7*a**2*(6*A*a*b**5 + 141*B*a**2
*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b))/(8*b**2) - 13*a*(15*A*a**
2*b**4 + 20*B*a**3*b**3 - 8*a**2*(A*b**6 + 41*B*a*b**5/10)/(9*b**2) - 15*a
*(6*A*a*b**5 + 141*B*a**2*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b))/
(8*b))/(7*b))/(6*b**2) + x**4*(15*A*a**4*b**2 + 6*B*a**5*b - 6*a**2*(15*A*
a**2*b**4 + 20*B*a**3*b**3 - 8*a**2*(A*b**6 + 41*B*a*b**5/10)/(9*b**2) - 1
5*a*(6*A*a*b**5 + 141*B*a**2*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b
))/(8*b))/(7*b**2) - 11*a*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 7*a**2*(6*A*a
*b**5 + 141*B*a**2*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b))/(8*b**2
) - 13*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 8*a**2*(A*b**6 + 41*B*a*b**5/1
0)/(9*b**2) - 15*a*(6*A*a*b**5 + 141*B*a**2*b**4/10 - 17*a*(A*b**6 + 41*B*
a*b**5/10)/(9*b))/(8*b))/(7*b))/(6*b))/(5*b**2) + x**3*(6*A*a**5*b + B*a**
6 - 5*a**2*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 7*a**2*(6*A*a*b**5 + 141*B*a
**2*b**4/10 - 17*a*(A*b**6 + 41*B*a*b**5/10)/(9*b))/(8*b**2) - 13*a*(15*A*
a**2*b**4 + 20*B*a**3*b**3 - 8*a**2*(A*b**6 + 41*B*a*b**5/10)/(9*b**2) ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(149) = 298$.

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.42

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Bx^3}{10\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^4x}{6\,b^4} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^3x}{6\,b^3} - \frac{13\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Bax^2}{90\,b^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ax^2}{9\,b^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ba^5}{6\,b^5} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Aa^4}{6\,b^4} + \frac{29\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^2x}{180\,b^4} - \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aax}{72\,b^3} - \frac{209\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ba^3}{1260\,b^5} + \frac{83\,(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Aa^2}{504\,b^4}$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*x^3/b^2 + 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*B*a^4*x/b^4 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^3*x/b^3 -
 13/90*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a*x^2/b^3 + 1/9*(b^2*x^2 + 2*a*b*
x + a^2)^(7/2)*A*x^2/b^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^5/b^5 -
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^4/b^4 + 29/180*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*B*a^2*x/b^4 - 11/72*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a*x/b^
3 - 209/1260*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a^3/b^5 + 83/504*(b^2*x^2 +
 2*a*b*x + a^2)^(7/2)*A*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{10}Bb^5x^{10}\mathrm{sgn}(bx+a)+\frac{5}{9}Bab^4x^9\mathrm{sgn}(bx+a)+\frac{1}{9}Ab^5x^9\mathrm{sgn}(bx+a)+\frac{5}{4}Ba^2b^3x^8\mathrm{sgn}(bx+a)+\frac{5}{8}Aab^4x^8\mathrm{sgn}(bx+a)+\frac{10}{7}Ba^3b^2x^7\mathrm{sgn}(bx+a)+\frac{10}{7}Aa^2b^3x^7\mathrm{sgn}(bx+a)+\frac{5}{6}Ba^4bx^6\mathrm{sgn}(bx+a)+\frac{5}{3}Aa^3b^2x^6\mathrm{sgn}(bx+a)+\frac{1}{5}Ba^5x^5\mathrm{sgn}(bx+a)+Aa^4bx^5\mathrm{sgn}(bx+a)+\frac{1}{4}Aa^5x^4\mathrm{sgn}(bx+a)+\frac{(2\,Ba^{10}-5\,Aa^9b)\mathrm{sgn}(bx+a)}{2520\,b^5}$$

input
```
integrate(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/10*B*b^5*x^10*sgn(b*x + a) + 5/9*B*a*b^4*x^9*sgn(b*x + a) + 1/9*A*b^5*x^
9*sgn(b*x + a) + 5/4*B*a^2*b^3*x^8*sgn(b*x + a) + 5/8*A*a*b^4*x^8*sgn(b*x
+ a) + 10/7*B*a^3*b^2*x^7*sgn(b*x + a) + 10/7*A*a^2*b^3*x^7*sgn(b*x + a) +
 5/6*B*a^4*b*x^6*sgn(b*x + a) + 5/3*A*a^3*b^2*x^6*sgn(b*x + a) + 1/5*B*a^5
*x^5*sgn(b*x + a) + A*a^4*b*x^5*sgn(b*x + a) + 1/4*A*a^5*x^4*sgn(b*x + a)
+ 1/2520*(2*B*a^10 - 5*A*a^9*b)*sgn(b*x + a)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^3\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^3*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int x^3(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^4(84b^6x^6+560a\,b^5x^5+1575a^2b^4x^4+2400a^3b^3x^3+2100a^4b^2x^2+1008a^5bx+210a^6)}{840}$$

input
```
int(x^3*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**4*(210*a**6 + 1008*a**5*b*x + 2100*a**4*b**2*x**2 + 2400*a**3*b**3*x**
3 + 1575*a**2*b**4*x**4 + 560*a*b**5*x**5 + 84*b**6*x**6))/840
```

3.316 $\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{a^2(Ab-aB)(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^4}-\frac{a(2Ab-3aB)(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^4}+\frac{(Ab-3aB)(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^4}+\frac{B(a+bx)^8\sqrt{a^2+2abx+b^2x^2}}{9b^4}$$

output
```
1/6*a^2*(A*b-B*a)*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^4-1/7*a*(2*A*b-3*B*a)*(b*x
+a)^6*((b*x+a)^2)^(1/2)/b^4+1/8*(A*b-3*B*a)*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^
4+1/9*B*(b*x+a)^8*((b*x+a)^2)^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^3\sqrt{(a+bx)^2}(42a^5(4A+3Bx)+126a^4bx(5A+4Bx)+168a^3b^2x^2(6A+5Bx)+120a^2b^3}{504(a+bx)}$$

input
```
Integrate[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^3*Sqrt[(a + b*x)^2]*(42*a^5*(4*A + 3*B*x) + 126*a^4*b*x*(5*A + 4*B*x) +
 168*a^3*b^2*x^2*(6*A + 5*B*x) + 120*a^2*b^3*x^3*(7*A + 6*B*x) + 45*a*b^4*
x^4*(8*A + 7*B*x) + 7*b^5*x^5*(9*A + 8*B*x)))/(504*(a + b*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^2(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{B(a+bx)^8}{b^3}+\frac{(Ab-3aB)(a+bx)^7}{b^3}+\frac{a(3aB-2Ab)(a+bx)^6}{b^3}-\frac{a^2(aB-Ab)(a+bx)^5}{b^3}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{a^2(a+bx)^6(Ab-aB)}{6b^4}+\frac{(a+bx)^8(Ab-3aB)}{8b^4}-\frac{a(a+bx)^7(2Ab-3aB)}{7b^4}+\frac{B(a+bx)^9}{9b^4}\right)}{a+bx}$$

input
```
Int[x^2*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((a^2*(A*b - a*B)*(a + b*x)^6)/(6*b^4) - (a
*(2*A*b - 3*a*B)*(a + b*x)^7)/(7*b^4) + ((A*b - 3*a*B)*(a + b*x)^8)/(8*b^4
) + (B*(a + b*x)^9)/(9*b^4)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{x^{3}\left(56 B\,b^{5} x^{6}+63 A\,b^{5} x^{5}+315 B a\,b^{4} x^{5}+360 A a\,b^{4} x^{4}+720 B\,a^{2} b^{3} x^{4}+840 A\,a^{2} b^{3} x^{3}+840 B\,a^{3} b^{2} x^{3}+1008 A\,a^{3} b^{2} x^{2}+504 B\,a^{4} b\,x^{2}+630\right.}{504\left(b x+a\right)^{5}}$
default	$\frac{x^{3}\left(56 B\,b^{5} x^{6}+63 A\,b^{5} x^{5}+315 B a\,b^{4} x^{5}+360 A a\,b^{4} x^{4}+720 B\,a^{2} b^{3} x^{4}+840 A\,a^{2} b^{3} x^{3}+840 B\,a^{3} b^{2} x^{3}+1008 A\,a^{3} b^{2} x^{2}+504 B\,a^{4} b\,x^{2}+630\right.}{504\left(b x+a\right)^{5}}$
orering	$\frac{x^{3}\left(56 B\,b^{5} x^{6}+63 A\,b^{5} x^{5}+315 B a\,b^{4} x^{5}+360 A a\,b^{4} x^{4}+720 B\,a^{2} b^{3} x^{4}+840 A\,a^{2} b^{3} x^{3}+840 B\,a^{3} b^{2} x^{3}+1008 A\,a^{3} b^{2} x^{2}+504 B\,a^{4} b\,x^{2}+630\right.}{504\left(b x+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b x+a\right)^{2}}\, B\,b^{5} x^{9}}{9 b x+9 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(A\,b^{5}+5 B a\,b^{4}\right) x^{8}}{8 b x+8 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(5 A a\,b^{4}+10 B\,a^{2} b^{3}\right) x^{7}}{7 b x+7 a}+\frac{\sqrt{\left(b x+a\right)^{2}}\, \left(10 A\,a^{2} b^{3}+10 B\,a^{3} b^{2}\right) x^{6}}{6 b x+6 a}$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/504*x^3*(56*B*b^5*x^6+63*A*b^5*x^5+315*B*a*b^4*x^5+360*A*a*b^4*x^4+720*B
*a^2*b^3*x^4+840*A*a^2*b^3*x^3+840*B*a^3*b^2*x^3+1008*A*a^3*b^2*x^2+504*B*
a^4*b*x^2+630*A*a^4*b*x+126*B*a^5*x+168*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{9}\,Bb^5x^9 + \frac{1}{3}\,Aa^5x^3+\frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8+\frac{5}{7}\left(2\,Ba^2b^3+Aab^4\right)x^7 + \frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+\left(Ba^4b+2\,Aa^3b^2\right)x^5+\frac{1}{4}\left(Ba^5+5\,Aa^4b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/9*B*b^5*x^9 + 1/3*A*a^5*x^3 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/7*(2*B*a^2
*b^3 + A*a*b^4)*x^7 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + (B*a^4*b + 2*A*a^3
*b^2)*x^5 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5836 vs. $2(121) = 242$.

Time = 0.91 (sec) , antiderivative size = 5836, normalized size of antiderivative = 34.95

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**8/9 + x**7*(A*b**6
+ 37*B*a*b**5/9)/(8*b**2) + x**6*(6*A*a*b**5 + 127*B*a**2*b**4/9 - 15*a*(A
*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b**2) + x**5*(15*A*a**2*b**4 + 20*B*a**3*
b**3 - 7*a**2*(A*b**6 + 37*B*a*b**5/9)/(8*b**2) - 13*a*(6*A*a*b**5 + 127*B
*a**2*b**4/9 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b))/(6*b**2) + x**4
*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 6*a**2*(6*A*a*b**5 + 127*B*a**2*b**4/9
 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b**2) - 11*a*(15*A*a**2*b**4 +
20*B*a**3*b**3 - 7*a**2*(A*b**6 + 37*B*a*b**5/9)/(8*b**2) - 13*a*(6*A*a*b*
*5 + 127*B*a**2*b**4/9 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b))/(6*b)
)/(5*b**2) + x**3*(15*A*a**4*b**2 + 6*B*a**5*b - 5*a**2*(15*A*a**2*b**4 +
20*B*a**3*b**3 - 7*a**2*(A*b**6 + 37*B*a*b**5/9)/(8*b**2) - 13*a*(6*A*a*b*
*5 + 127*B*a**2*b**4/9 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b))/(6*b*
*2) - 9*a*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 6*a**2*(6*A*a*b**5 + 127*B*a*
*2*b**4/9 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*b**2) - 11*a*(15*A*a**
2*b**4 + 20*B*a**3*b**3 - 7*a**2*(A*b**6 + 37*B*a*b**5/9)/(8*b**2) - 13*a*
(6*A*a*b**5 + 127*B*a**2*b**4/9 - 15*a*(A*b**6 + 37*B*a*b**5/9)/(8*b))/(7*
b))/(6*b))/(5*b))/(4*b**2) + x**2*(6*A*a**5*b + B*a**6 - 4*a**2*(20*A*a**3
*b**3 + 15*B*a**4*b**2 - 6*a**2*(6*A*a*b**5 + 127*B*a**2*b**4/9 - 15*a*(A*
b**6 + 37*B*a*b**5/9)/(8*b))/(7*b**2) - 11*a*(15*A*a**2*b**4 + 20*B*a**3*b
**3 - 7*a**2*(A*b**6 + 37*B*a*b**5/9)/(8*b**2) - 13*a*(6*A*a*b**5 + 127...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(116) = 232.

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.44

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^3x}{6\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^2x}{6\,b^2}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bx^2}{9\,b^2}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^4}{6\,b^4}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^3}{6\,b^3}-\frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bax}{72\,b^3}+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ax}{8\,b^2}+\frac{83\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba^2}{504\,b^4}-\frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Aa}{56\,b^3}$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^3*x/b^3 + 1/6*(b^2*x^2 + 2*a*b*x
+ a^2)^(5/2)*A*a^2*x/b^2 + 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*x^2/b^2 -
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^4/b^4 + 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*A*a^3/b^3 - 11/72*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a*x/b^3 + 1
/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*x/b^2 + 83/504*(b^2*x^2 + 2*a*b*x + a
^2)^(7/2)*B*a^2/b^4 - 9/56*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.32

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{9}\,Bb^5x^9\mathrm{sgn}(bx+a)+\frac{5}{8}\,Bab^4x^8\mathrm{sgn}(bx+a)+\frac{1}{8}\,Ab^5x^8\mathrm{sgn}(bx+a)+\frac{10}{7}\,Ba^2b^3x^7\mathrm{sgn}(bx+a)+\frac{5}{7}\,Aab^4x^7\mathrm{sgn}(bx+a)+\frac{5}{3}\,Ba^3b^2x^6\mathrm{sgn}(bx+a)+\frac{5}{3}\,Aa^2b^3x^6\mathrm{sgn}(bx+a)+Ba^4bx^5\mathrm{sgn}(bx+a)+2\,Aa^3b^2x^5\mathrm{sgn}(bx+a)+\frac{1}{4}\,Ba^5x^4\mathrm{sgn}(bx+a)+\frac{5}{4}\,Aa^4bx^4\mathrm{sgn}(bx+a)+\frac{1}{3}\,Aa^5x^3\mathrm{sgn}(bx+a)-\frac{\left(Ba^9-3\,Aa^8b\right)\mathrm{sgn}(bx+a)}{504\,b^4}$$

input
```
integrate(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/9*B*b^5*x^9*sgn(b*x + a) + 5/8*B*a*b^4*x^8*sgn(b*x + a) + 1/8*A*b^5*x^8*
sgn(b*x + a) + 10/7*B*a^2*b^3*x^7*sgn(b*x + a) + 5/7*A*a*b^4*x^7*sgn(b*x +
 a) + 5/3*B*a^3*b^2*x^6*sgn(b*x + a) + 5/3*A*a^2*b^3*x^6*sgn(b*x + a) + B*
a^4*b*x^5*sgn(b*x + a) + 2*A*a^3*b^2*x^5*sgn(b*x + a) + 1/4*B*a^5*x^4*sgn(
b*x + a) + 5/4*A*a^4*b*x^4*sgn(b*x + a) + 1/3*A*a^5*x^3*sgn(b*x + a) - 1/5
04*(B*a^9 - 3*A*a^8*b)*sgn(b*x + a)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^2\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^2*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int x^2(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^3(28b^6x^6+189a\,b^5x^5+540a^2b^4x^4+840a^3b^3x^3+756a^4b^2x^2+378a^5bx+84a^6)}{252}$$

input
```
int(x^2*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**3*(84*a**6 + 378*a**5*b*x + 756*a**4*b**2*x**2 + 840*a**3*b**3*x**3 +
540*a**2*b**4*x**4 + 189*a*b**5*x**5 + 28*b**6*x**6))/252
```

3.317 $\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=-\frac{a(Ab-aB)(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^3}+\frac{(Ab-2aB)(a+bx)^6\sqrt{a^2+2abx+b^2x^2}}{7b^3}+\frac{B(a+bx)^7\sqrt{a^2+2abx+b^2x^2}}{8b^3}$$

output
```
-1/6*a*(A*b-B*a)*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^3+1/7*(A*b-2*B*a)*(b*x+a)^6
*((b*x+a)^2)^(1/2)/b^3+1/8*B*(b*x+a)^7*((b*x+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^2\sqrt{(a+bx)^2}(28a^5(3A+2Bx)+70a^4bx(4A+3Bx)+84a^3b^2x^2(5A+4Bx)+56a^2b^3x^3(}{168(a+bx)}$$

input
```
Integrate[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x^2*Sqrt[(a + b*x)^2]*(28*a^5*(3*A + 2*B*x) + 70*a^4*b*x*(4*A + 3*B*x) +
84*a^3*b^2*x^2*(5*A + 4*B*x) + 56*a^2*b^3*x^3*(6*A + 5*B*x) + 20*a*b^4*x^4
*(7*A + 6*B*x) + 3*b^5*x^5*(8*A + 7*B*x)))/(168*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{B(a+bx)^7}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{a(aB-Ab)(a+bx)^5}{b^2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(a+bx)^7(Ab-2aB)}{7b^3}-\frac{a(a+bx)^6(Ab-aB)}{6b^3}+\frac{B(a+bx)^8}{8b^3}\right)}{a+bx}$$

input
```
Int[x*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/6*(a*(A*b - a*B)*(a + b*x)^6)/b^3 + ((A
*b - 2*a*B)*(a + b*x)^7)/(7*b^3) + (B*(a + b*x)^8)/(8*b^3)))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

method	result
gosper	$\frac{x^2\left(21 B\,b^5 x^6+24 A\,b^5 x^5+120 B a\,b^4 x^5+140 A a\,b^4 x^4+280 B\,a^2 b^3 x^4+336 A\,a^2 b^3 x^3+336 B\,a^3 b^2 x^3+420 A\,a^3 b^2 x^2+210 B\,a^4 b\,x^2+280 A\right.}{168\left(b x+a\right)^5}$
default	$\frac{x^2\left(21 B\,b^5 x^6+24 A\,b^5 x^5+120 B a\,b^4 x^5+140 A a\,b^4 x^4+280 B\,a^2 b^3 x^4+336 A\,a^2 b^3 x^3+336 B\,a^3 b^2 x^3+420 A\,a^3 b^2 x^2+210 B\,a^4 b\,x^2+280 A\right.}{168\left(b x+a\right)^5}$
orering	$\frac{x^2\left(21 B\,b^5 x^6+24 A\,b^5 x^5+120 B a\,b^4 x^5+140 A a\,b^4 x^4+280 B\,a^2 b^3 x^4+336 A\,a^2 b^3 x^3+336 B\,a^3 b^2 x^3+420 A\,a^3 b^2 x^2+210 B\,a^4 b\,x^2+280 A\right.}{168\left(b x+a\right)^5}$
risch	$\frac{\sqrt{\left(b x+a\right)^2}\, B\,b^5 x^8}{8 b x+8 a}+\frac{\sqrt{\left(b x+a\right)^2}\,\left(A\,b^5+5 B a\,b^4\right) x^7}{7 b x+7 a}+\frac{\sqrt{\left(b x+a\right)^2}\,\left(5 A a\,b^4+10 B\,a^2 b^3\right) x^6}{6 b x+6 a}+\frac{\sqrt{\left(b x+a\right)^2}\,\left(10 A\,a^2 b^3+10 B\,a^3 b^2\right) x^5}{5 b x+5 a}$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/168*x^2*(21*B*b^5*x^6+24*A*b^5*x^5+120*B*a*b^4*x^5+140*A*a*b^4*x^4+280*B
*a^2*b^3*x^4+336*A*a^2*b^3*x^3+336*B*a^3*b^2*x^3+420*A*a^3*b^2*x^2+210*B*a
^4*b*x^2+280*A*a^4*b*x+56*B*a^5*x+84*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{8}\,Bb^5x^8+\frac{1}{2}\,Aa^5x^2+\frac{1}{7}\,\left(5\,Bab^4+Ab^5\right)x^7+\frac{5}{6}\,\left(2\,Ba^2b^3+Aab^4\right)x^6+2\,\left(Ba^3b^2+Aa^2b^3\right)x^5+\frac{5}{4}\,\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{3}\,\left(Ba^5+5\,Aa^4b\right)x^3$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/8*B*b^5*x^8 + 1/2*A*a^5*x^2 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/6*(2*B*a^2
*b^3 + A*a*b^4)*x^6 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/4*(B*a^4*b + 2*A*a
^3*b^2)*x^4 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3563 vs. 2(85) = 170.

Time = 1.12 (sec) , antiderivative size = 3563, normalized size of antiderivative = 29.45

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**7/8 + x**6*(A*b**6
+ 33*B*a*b**5/8)/(7*b**2) + x**5*(6*A*a*b**5 + 113*B*a**2*b**4/8 - 13*a*(A
*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b**2) + x**4*(15*A*a**2*b**4 + 20*B*a**3*
b**3 - 6*a**2*(A*b**6 + 33*B*a*b**5/8)/(7*b**2) - 11*a*(6*A*a*b**5 + 113*B
*a**2*b**4/8 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b))/(5*b**2) + x**3
*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 5*a**2*(6*A*a*b**5 + 113*B*a**2*b**4/8
 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b**2) - 9*a*(15*A*a**2*b**4 + 2
0*B*a**3*b**3 - 6*a**2*(A*b**6 + 33*B*a*b**5/8)/(7*b**2) - 11*a*(6*A*a*b**
5 + 113*B*a**2*b**4/8 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b))/(5*b))
/(4*b**2) + x**2*(15*A*a**4*b**2 + 6*B*a**5*b - 4*a**2*(15*A*a**2*b**4 + 2
0*B*a**3*b**3 - 6*a**2*(A*b**6 + 33*B*a*b**5/8)/(7*b**2) - 11*a*(6*A*a*b**
5 + 113*B*a**2*b**4/8 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b))/(5*b**
2) - 7*a*(20*A*a**3*b**3 + 15*B*a**4*b**2 - 5*a**2*(6*A*a*b**5 + 113*B*a**
2*b**4/8 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b**2) - 9*a*(15*A*a**2*
b**4 + 20*B*a**3*b**3 - 6*a**2*(A*b**6 + 33*B*a*b**5/8)/(7*b**2) - 11*a*(6
*A*a*b**5 + 113*B*a**2*b**4/8 - 13*a*(A*b**6 + 33*B*a*b**5/8)/(7*b))/(6*b)
)/(5*b))/(4*b))/(3*b**2) + x*(6*A*a**5*b + B*a**6 - 3*a**2*(20*A*a**3*b**3
 + 15*B*a**4*b**2 - 5*a**2*(6*A*a*b**5 + 113*B*a**2*b**4/8 - 13*a*(A*b**6
+ 33*B*a*b**5/8)/(7*b))/(6*b**2) - 9*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 -
6*a**2*(A*b**6 + 33*B*a*b**5/8)/(7*b**2) - 11*a*(6*A*a*b**5 + 113*B*a**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(83) = 166$.

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.51

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^2x}{6\,b^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aax}{6\,b} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^3}{6\,b^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa^2}{6\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bx}{8\,b^2} - \frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ba}{56\,b^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{7\,b^2}$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^2*x/b^2 - 1/6*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*A*a*x/b + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^3/b^3 - 1/6*
(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*a^2/b^2 + 1/8*(b^2*x^2 + 2*a*b*x + a^2)^
(7/2)*B*x/b^2 - 9/56*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*a/b^3 + 1/7*(b^2*x^
2 + 2*a*b*x + a^2)^(7/2)*A/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(83) = 166.

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.83

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{8}\,Bb^5x^8\mathrm{sgn}(bx+a)+\frac{5}{7}\,Bab^4x^7\mathrm{sgn}(bx+a)+\frac{1}{7}\,Ab^5x^7\mathrm{sgn}(bx+a)+\frac{5}{3}\,Ba^2b^3x^6\mathrm{sgn}(bx+a)+\frac{5}{6}\,Aab^4x^6\mathrm{sgn}(bx+a)+2\,Ba^3b^2x^5\mathrm{sgn}(bx+a)+2\,Aa^2b^3x^5\mathrm{sgn}(bx+a)+\frac{5}{4}\,Ba^4bx^4\mathrm{sgn}(bx+a)+\frac{5}{2}\,Aa^3b^2x^4\mathrm{sgn}(bx+a)+\frac{1}{3}\,Ba^5x^3\mathrm{sgn}(bx+a)+\frac{5}{3}\,Aa^4bx^3\mathrm{sgn}(bx+a)+\frac{1}{2}\,Aa^5x^2\mathrm{sgn}(bx+a)+\frac{(Ba^8-4\,Aa^7b)\mathrm{sgn}(bx+a)}{168\,b^3}$$

input
```
integrate(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/8*B*b^5*x^8*sgn(b*x + a) + 5/7*B*a*b^4*x^7*sgn(b*x + a) + 1/7*A*b^5*x^7*
sgn(b*x + a) + 5/3*B*a^2*b^3*x^6*sgn(b*x + a) + 5/6*A*a*b^4*x^6*sgn(b*x +
a) + 2*B*a^3*b^2*x^5*sgn(b*x + a) + 2*A*a^2*b^3*x^5*sgn(b*x + a) + 5/4*B*a
^4*b*x^4*sgn(b*x + a) + 5/2*A*a^3*b^2*x^4*sgn(b*x + a) + 1/3*B*a^5*x^3*sgn
(b*x + a) + 5/3*A*a^4*b*x^3*sgn(b*x + a) + 1/2*A*a^5*x^2*sgn(b*x + a) + 1/
168*(B*a^8 - 4*A*a^7*b)*sgn(b*x + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int x(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x^2(7b^6x^6+48a\,b^5x^5+140a^2b^4x^4+224a^3b^3x^3+210a^4b^2x^2+112a^5bx+28a^6)}{56}$$

input
```
int(x*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x**2*(28*a**6 + 112*a**5*b*x + 210*a**4*b**2*x**2 + 224*a**3*b**3*x**3 +
140*a**2*b**4*x**4 + 48*a*b**5*x**5 + 7*b**6*x**6))/56
```

3.318 $\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{(Ab-aB)(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b^2}+\frac{B(a^2+2abx+b^2x^2)^{7/2}}{7b^2}$$

output
```
1/6*(A*b-B*a)*(b*x+a)^5*((b*x+a)^2)^(1/2)/b^2+1/7*B*(b^2*x^2+2*a*b*x+a^2)^
(7/2)/b^2
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{x\sqrt{(a+bx)^2}(21a^5(2A+Bx)+35a^4bx(3A+2Bx)+35a^3b^2x^2(4A+3Bx)+21a^2b^3x^3(5A}{42(a+bx)}$$

input
```
Integrate[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(x*Sqrt[(a + b*x)^2]*(21*a^5*(2*A + B*x) + 35*a^4*b*x*(3*A + 2*B*x) + 35*a
^3*b^2*x^2*(4*A + 3*B*x) + 21*a^2*b^3*x^3*(5*A + 4*B*x) + 7*a*b^4*x^4*(6*A
 + 5*B*x) + b^5*x^5*(7*A + 6*B*x)))/(42*(a + b*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \ 1100$$

$$\frac{(Ab-aB)\int \left(a^2+2bxa+b^2x^2\right)^{5/2}dx}{b}+\frac{B\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

$$\Big\downarrow \ 1079$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \left(xb^2+ab\right)^5dx}{b^6(a+bx)}+\frac{B\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

$$\Big\downarrow \ 17$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a+bx)^5(Ab-aB)}{6b^2}+\frac{B\left(a^2+2abx+b^2x^2\right)^{7/2}}{7b^2}$$

input
```
Int[(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((A*b - a*B)*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*b^2) + (B*(a^2
+ 2*a*b*x + b^2*x^2)^(7/2))/(7*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(54) = 108$.

Time = 1.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

method	result
gosper	$\frac{x(6B\,b^5x^6+7A\,b^5x^5+35Ba\,b^4x^5+42Aa\,b^4x^4+84B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+140A\,a^3b^2x^2+70B\,a^4b\,x^2+105A\,a^4bx+2}{42(bx+a)^5}$
default	$\frac{x(6B\,b^5x^6+7A\,b^5x^5+35Ba\,b^4x^5+42Aa\,b^4x^4+84B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+140A\,a^3b^2x^2+70B\,a^4b\,x^2+105A\,a^4bx+2}{42(bx+a)^5}$
orering	$\frac{x(6B\,b^5x^6+7A\,b^5x^5+35Ba\,b^4x^5+42Aa\,b^4x^4+84B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+140A\,a^3b^2x^2+70B\,a^4b\,x^2+105A\,a^4bx+2}{42(bx+a)^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,b^5B\,x^7}{7bx+7a}+\frac{\sqrt{(bx+a)^2}\,(A\,b^5+5Ba\,b^4)x^6}{6bx+6a}+\frac{\sqrt{(bx+a)^2}\,(5Aa\,b^4+10B\,a^2b^3)x^5}{5bx+5a}+\frac{\sqrt{(bx+a)^2}\,(10A\,a^2b^3+10B\,a^3b^2)x^4}{4bx+4a}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*x*(6*B*b^5*x^6+7*A*b^5*x^5+35*B*a*b^4*x^5+42*A*a*b^4*x^4+84*B*a^2*b^3
*x^4+105*A*a^2*b^3*x^3+105*B*a^3*b^2*x^3+140*A*a^3*b^2*x^2+70*B*a^4*b*x^2+
105*A*a^4*b*x+21*B*a^5*x+42*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(54) = 108$.

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{1}{7}\,Bb^5x^7+Aa^5x+\frac{1}{6}\left(5\,Bab^4+Ab^5\right)x^6+\left(2\,Ba^2b^3+Aab^4\right)x^5+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/7*B*b^5*x^7 + A*a^5*x + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + (2*B*a^2*b^3 + A*a
*b^4)*x^5 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*
x^3 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(56) = 112$.

Time = 0.86 (sec) , antiderivative size = 2147, normalized size of antiderivative = 30.24

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*b**4*x**6/7 + x**5*(A*b**6
+ 29*B*a*b**5/7)/(6*b**2) + x**4*(6*A*a*b**5 + 99*B*a**2*b**4/7 - 11*a*(A*
b**6 + 29*B*a*b**5/7)/(6*b))/(5*b**2) + x**3*(15*A*a**2*b**4 + 20*B*a**3*b
**3 - 5*a**2*(A*b**6 + 29*B*a*b**5/7)/(6*b**2) - 9*a*(6*A*a*b**5 + 99*B*a*
*2*b**4/7 - 11*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b))/(4*b**2) + x**2*(2
0*A*a**3*b**3 + 15*B*a**4*b**2 - 4*a**2*(6*A*a*b**5 + 99*B*a**2*b**4/7 - 1
1*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b**2) - 7*a*(15*A*a**2*b**4 + 20*B*
a**3*b**3 - 5*a**2*(A*b**6 + 29*B*a*b**5/7)/(6*b**2) - 9*a*(6*A*a*b**5 + 9
9*B*a**2*b**4/7 - 11*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b))/(4*b))/(3*b*
*2) + x*(15*A*a**4*b**2 + 6*B*a**5*b - 3*a**2*(15*A*a**2*b**4 + 20*B*a**3*
b**3 - 5*a**2*(A*b**6 + 29*B*a*b**5/7)/(6*b**2) - 9*a*(6*A*a*b**5 + 99*B*a
**2*b**4/7 - 11*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b))/(4*b**2) - 5*a*(2
0*A*a**3*b**3 + 15*B*a**4*b**2 - 4*a**2*(6*A*a*b**5 + 99*B*a**2*b**4/7 - 1
1*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b**2) - 7*a*(15*A*a**2*b**4 + 20*B*
a**3*b**3 - 5*a**2*(A*b**6 + 29*B*a*b**5/7)/(6*b**2) - 9*a*(6*A*a*b**5 + 9
9*B*a**2*b**4/7 - 11*a*(A*b**6 + 29*B*a*b**5/7)/(6*b))/(5*b))/(4*b))/(3*b)
)/(2*b**2) + (6*A*a**5*b + B*a**6 - 2*a**2*(20*A*a**3*b**3 + 15*B*a**4*b**
2 - 4*a**2*(6*A*a*b**5 + 99*B*a**2*b**4/7 - 11*a*(A*b**6 + 29*B*a*b**5/7)/
(6*b))/(5*b**2) - 7*a*(15*A*a**2*b**4 + 20*B*a**3*b**3 - 5*a**2*(A*b**6 +
29*B*a*b**5/7)/(6*b**2) - 9*a*(6*A*a*b**5 + 99*B*a**2*b**4/7 - 11*a*(A*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{1}{6}\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ax - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bax}{6\,b} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba^2}{6\,b^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Aa}{6\,b} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}B}{7\,b^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*x - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5
/2)*B*a*x/b - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a^2/b^2 + 1/6*(b^2*x^2
 + 2*a*b*x + a^2)^(5/2)*A*a/b + 1/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.06

$$\int (A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{1}{7}Bb^5x^7\operatorname{sgn}(bx+a) + \frac{5}{6}Bab^4x^6\operatorname{sgn}(bx+a) + \frac{1}{6}Ab^5x^6\operatorname{sgn}(bx+a) + 2Ba^2b^3x^5\operatorname{sgn}(bx+a) + Aab^4x^5\operatorname{sgn}(bx+a) + \frac{5}{2}Ba^3b^2x^4\operatorname{sgn}(bx+a) + \frac{5}{2}Aa^2b^3x^4\operatorname{sgn}(bx+a) + \frac{5}{3}Ba^4bx^3\operatorname{sgn}(bx+a) + \frac{10}{3}Aa^3b^2x^3\operatorname{sgn}(bx+a) + \frac{1}{2}Ba^5x^2\operatorname{sgn}(bx+a) + \frac{5}{2}Aa^4bx^2\operatorname{sgn}(bx+a) + Aa^5x\operatorname{sgn}(bx+a) - \frac{(Ba^7-7Aa^6b)\operatorname{sgn}(bx+a)}{42b^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/7*B*b^5*x^7*sgn(b*x + a) + 5/6*B*a*b^4*x^6*sgn(b*x + a) + 1/6*A*b^5*x^6*
sgn(b*x + a) + 2*B*a^2*b^3*x^5*sgn(b*x + a) + A*a*b^4*x^5*sgn(b*x + a) + 5
/2*B*a^3*b^2*x^4*sgn(b*x + a) + 5/2*A*a^2*b^3*x^4*sgn(b*x + a) + 5/3*B*a^4
*b*x^3*sgn(b*x + a) + 10/3*A*a^3*b^2*x^3*sgn(b*x + a) + 1/2*B*a^5*x^2*sgn(
b*x + a) + 5/2*A*a^4*b*x^2*sgn(b*x + a) + A*a^5*x*sgn(b*x + a) - 1/42*(B*a
^7 - 7*A*a^6*b)*sgn(b*x + a)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx = \int (A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2} \,dx$$

input
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int (A+Bx) \left(a^2+2abx+b^2x^2\right)^{5/2} \, dx = \frac{x(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6)}{7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(x*(7*a**6 + 21*a**5*b*x + 35*a**4*b**2*x**2 + 35*a**3*b**3*x**3 + 21*a**2
*b**4*x**4 + 7*a*b**5*x**5 + b**6*x**6))/7
```

3.319 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 262

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx = \frac{5a^4Abx\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5a^3Ab^2x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{10a^2Ab^3x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{5aAb^4x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} + \frac{Ab^5x^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{B(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6b} + \frac{a^5A\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output

```
5*a^4*A*b*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*a^3*A*b^2*x^2*((b*x+a)^2)^(1/2)/(b
*x+a)+10*a^2*A*b^3*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+5*a*A*b^4*x^4*((b*x+a
)^2)^(1/2)/(4*b*x+4*a)+A*b^5*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+1/6*B*(b*x+
a)^5*((b*x+a)^2)^(1/2)/b+a^5*A*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx)^2}(x(60a^5B+150a^4b(2A+Bx)+100a^3b^2x(3A+2Bx}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*(x*(60*a^5*B + 150*a^4*b*(2*A + B*x) + 100*a^3*b^2*x*(3
*A + 2*B*x) + 50*a^2*b^3*x^2*(4*A + 3*B*x) + 15*a*b^4*x^3*(5*A + 4*B*x) +
2*b^5*x^4*(6*A + 5*B*x)) + 60*a^5*A*Log[x]))/(60*(a + b*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x}dx}{a+bx}$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\int \frac{(a+bx)^5}{x}dx+\frac{B(a+bx)^6}{6b}\right)}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\int\left(\frac{a^5}{x}+5ba^4+10b^2xa^3+10b^3x^2a^2+5b^4x^3a+b^5x^4\right)dx+\frac{B(a+bx)^6}{6b}\right)}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(A\left(a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}\right)+\frac{B(a+bx)^6}{6b}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((B*(a + b*x)^6)/(6*b) + A*(5*a^4*b*x + 5*a
^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5)/5 + a^5*Log[
x])))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.53

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(10B\,b^5x^6+12A\,b^5x^5+60Ba\,b^4x^5+75Aa\,b^4x^4+150B\,a^2b^3x^4+200A\,a^2b^3x^3+200B\,a^3b^2x^3+300A\,a^3b^2x^2+150B\,a^4b\,x\right.}{60(bx+a)^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,B\,b^5x^6}{6bx+6a}+\frac{A\,b^5x^5\sqrt{(bx+a)^2}}{5bx+5a}+\frac{\sqrt{(bx+a)^2}\,Ba\,b^4x^5}{bx+a}+\frac{5\sqrt{(bx+a)^2}\,Aa\,b^4x^4}{4(bx+a)}+\frac{5\sqrt{(bx+a)^2}\,B\,a^2b^3x^4}{2(bx+a)}+\frac{10\sqrt{(bx+}}{3(}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*((b*x+a)^2)^(5/2)*(10*B*b^5*x^6+12*A*b^5*x^5+60*B*a*b^4*x^5+75*A*a*b^
4*x^4+150*B*a^2*b^3*x^4+200*A*a^2*b^3*x^3+200*B*a^3*b^2*x^3+300*A*a^3*b^2*
x^2+150*B*a^4*b*x^2+60*A*ln(x)*a^5+300*A*a^4*b*x+60*B*a^5*x)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx=\frac{1}{6}\,Bb^5x^6$$
$$+Aa^5\log(x)+\frac{1}{5}\left(5\,Bab^4+Ab^5\right)x^5+\frac{5}{4}\left(2\,Ba^2b^3+Aab^4\right)x^4$$
$$+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^2+\left(Ba^5+5\,Aa^4b\right)x$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/6*B*b^5*x^6 + A*a^5*log(x) + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/4*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx&=(-1)^{2\,b^2x+2\,ab}\,Aa^5\log\left(2\,b^2x+2\,ab\right)\\&-(-1)^{2\,abx+2\,a^2}\,Aa^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Aa^3bx\\&+\frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Aa^4+\frac{1}{4}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aabx\\&+\frac{7}{12}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aa^2+\frac{1}{6}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bx\\&+\frac{1}{5}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ba}{6\,b}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*A*a^5*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)
*A*a^5*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*A*a^3*b*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^4 + 1/4*(b^2*x^2 + 2
*a*b*x + a^2)^(3/2)*A*a*b*x + 7/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*a^2 +
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*x + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(
5/2)*A + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*a/b
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx = \frac{1}{6}\,Bb^5x^6\mathrm{sgn}(bx+a) + Bab^4x^5\mathrm{sgn}(bx+a) + \frac{1}{5}\,Ab^5x^5\mathrm{sgn}(bx+a) + \frac{5}{2}\,Ba^2b^3x^4\mathrm{sgn}(bx+a) + \frac{5}{4}\,Aab^4x^4\mathrm{sgn}(bx+a) + \frac{10}{3}\,Ba^3b^2x^3\mathrm{sgn}(bx+a) + \frac{10}{3}\,Aa^2b^3x^3\mathrm{sgn}(bx+a) + \frac{5}{2}\,Ba^4bx^2\mathrm{sgn}(bx+a) + 5\,Aa^3b^2x^2\mathrm{sgn}(bx+a) + Ba^5x\mathrm{sgn}(bx+a) + 5\,Aa^4bx\mathrm{sgn}(bx+a) + Aa^5\log\left(|x|\right)\mathrm{sgn}(bx+a)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x, algorithm="giac")
```

output
```
1/6*B*b^5*x^6*sgn(b*x + a) + B*a*b^4*x^5*sgn(b*x + a) + 1/5*A*b^5*x^5*sgn(
b*x + a) + 5/2*B*a^2*b^3*x^4*sgn(b*x + a) + 5/4*A*a*b^4*x^4*sgn(b*x + a) +
 10/3*B*a^3*b^2*x^3*sgn(b*x + a) + 10/3*A*a^2*b^3*x^3*sgn(b*x + a) + 5/2*B
*a^4*b*x^2*sgn(b*x + a) + 5*A*a^3*b^2*x^2*sgn(b*x + a) + B*a^5*x*sgn(b*x +
 a) + 5*A*a^4*b*x*sgn(b*x + a) + A*a^5*log(abs(x))*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x}\,dx=\log(x)\,a^6+6a^5bx+\frac{15a^4b^2x^2}{2}+\frac{20a^3b^3x^3}{3}+\frac{15a^2b^4x^4}{4}+\frac{6a\,b^5x^5}{5}+\frac{b^6x^6}{6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x,x)
```

output
```
(60*log(x)*a**6 + 360*a**5*b*x + 450*a**4*b**2*x**2 + 400*a**3*b**3*x**3 +
 225*a**2*b**4*x**4 + 72*a*b**5*x**5 + 10*b**6*x**6)/60
```

3.320 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 294

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx &= -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} \\ &+ \frac{5a^3b(2Ab+aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5a^2b^2(Ab+aB)x^2\sqrt{a^2+2abx+b^2x^2}}{a+bx} \\ &+ \frac{5ab^3(Ab+2aB)x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{b^4(Ab+5aB)x^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} \\ &+ \frac{b^5Bx^5\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}\end{aligned}$$

output
```
-a^5*A*((b*x+a)^2)^(1/2)/x/(b*x+a)+5*a^3*b*(2*A*b+B*a)*x*((b*x+a)^2)^(1/2)
/(b*x+a)+5*a^2*b^2*(A*b+B*a)*x^2*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^3*(A*b+2*
B*a)*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+b^4*(A*b+5*B*a)*x^4*((b*x+a)^2)^(1/
2)/(4*b*x+4*a)+b^5*B*x^5*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+a^4*(5*A*b+B*a)*((b
*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx=\frac{\sqrt{(a+bx)^2}\left(-60a^5A+300a^4bBx^2+300a^3b^2x^2(2A+Bx)+100a\right.}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-60*a^5*A + 300*a^4*b*B*x^2 + 300*a^3*b^2*x^2*(2*A + B
*x) + 100*a^2*b^3*x^3*(3*A + 2*B*x) + 25*a*b^4*x^4*(4*A + 3*B*x) + 3*b^5*x
^5*(5*A + 4*B*x) + 60*a^4*(5*A*b + a*B)*x*Log[x]))/(60*x*(a + b*x))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^2}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^5}{x^2}+\frac{(5Ab+aB)a^4}{x}+5b(2Ab+aB)a^3+10b^2(Ab+aB)xa^2+5b^3(Ab+2aB)x^2a+b^5Bx^4\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{x}+a^4\log(x)(aB+5Ab)+5a^3bx(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-((a^5*A)/x) + 5*a^3*b*(2*A*b + a*B)*x + 5
*a^2*b^2*(A*b + a*B)*x^2 + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^5)/5 + a^4*(5*A*b + a*B)*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}(12B\,b^5x^6+15A\,b^5x^5+75Ba\,b^4x^5+100Aa\,b^4x^4+200B\,a^2b^3x^4+300A\,a^2b^3x^3+300B\,a^3b^2x^3+300A\ln(x)x\,a^4b+600A\,a}{60x(bx+a)^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,b(\frac{1}{5}b^4B\,x^5+\frac{1}{4}A\,b^4x^4+\frac{5}{4}Ba\,b^3x^4+\frac{5}{3}Aa\,b^3x^3+\frac{10}{3}B\,a^2b^2x^3+5A\,a^2b^2x^2+5B\,a^3b\,x^2+10A\,a^3bx+5a^4Bx)}{bx+a}-\frac{a^5A\sqrt{(bx+}}{x(bx+a)}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*((b*x+a)^2)^(5/2)*(12*B*b^5*x^6+15*A*b^5*x^5+75*B*a*b^4*x^5+100*A*a*b
^4*x^4+200*B*a^2*b^3*x^4+300*A*a^2*b^3*x^3+300*B*a^3*b^2*x^3+300*A*ln(x)*x
*a^4*b+600*A*a^3*b^2*x^2+60*B*ln(x)*a^5*x+300*B*a^4*b*x^2-60*A*a^5)/x/(b*x
+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx=\frac{12\,Bb^5x^6-60\,Aa^5+15\,(5\,Bab^4+Ab^5)x^5+100\,(2\,Ba^2b^3+Aab^4}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/60*(12*B*b^5*x^6 - 60*A*a^5 + 15*(5*B*a*b^4 + A*b^5)*x^5 + 100*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 300*(B*a^4*b + 2*A*
a^3*b^2)*x^2 + 60*(B*a^5 + 5*A*a^4*b)*x*log(x))/x
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**2,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx &= (-1)^{2\,b^2x+2\,ab}\,Ba^5\log\left(2\,b^2x+2\,ab\right)\\&+5\,(-1)^{2\,b^2x+2\,ab}\,Aa^4b\log\left(2\,b^2x+2\,ab\right)-(-1)^{2\,abx+2\,a^2}\,Ba^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)\\&-5\,(-1)^{2\,abx+2\,a^2}\,Aa^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)\\&+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ba^3bx+\frac{5}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Aa^2b^2x\\&+\frac{3}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ba^4+\frac{15}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Aa^3b\\&+\frac{1}{4}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Babx+\frac{5}{4}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^2x\\&+\frac{7}{12}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ba^2+\frac{35}{12}\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Aab\\&+\frac{1}{5}\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}A}{x}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*a^5*log(2*b^2*x + 2*a*b) + 5*(-1)^(2*b^2*x + 2*a*
b)*A*a^4*b*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)*B*a^5*log(2*a*b*x
/abs(x) + 2*a^2/abs(x)) - 5*(-1)^(2*a*b*x + 2*a^2)*A*a^4*b*log(2*a*b*x/abs
(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^3*b*x + 5/2*sq
rt(b^2*x^2 + 2*a*b*x + a^2)*A*a^2*b^2*x + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2
)*B*a^4 + 15/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^3*b + 1/4*(b^2*x^2 + 2*a*
b*x + a^2)^(3/2)*B*a*b*x + 5/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^2*x + 7
/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a^2 + 35/12*(b^2*x^2 + 2*a*b*x + a^2
)^(3/2)*A*a*b + 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B - (b^2*x^2 + 2*a*b*x
 + a^2)^(5/2)*A/x
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx = \frac{1}{5}\,Bb^5x^5\mathrm{sgn}(bx+a) + \frac{5}{4}\,Bab^4x^4\mathrm{sgn}(bx+a) + \frac{1}{4}\,Ab^5x^4\mathrm{sgn}(bx+a) + \frac{10}{3}\,Ba^2b^3x^3\mathrm{sgn}(bx+a) + \frac{5}{3}\,Aab^4x^3\mathrm{sgn}(bx+a) + 5\,Ba^3b^2x^2\mathrm{sgn}(bx+a) + 5\,Aa^2b^3x^2\mathrm{sgn}(bx+a) + 5\,Ba^4bx\mathrm{sgn}(bx+a) + 10\,Aa^3b^2x\mathrm{sgn}(bx+a) - \frac{Aa^5\mathrm{sgn}(bx+a)}{x} + \left(Ba^5\mathrm{sgn}(bx+a) + 5\,Aa^4b\mathrm{sgn}(bx+a)\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/5*B*b^5*x^5*sgn(b*x + a) + 5/4*B*a*b^4*x^4*sgn(b*x + a) + 1/4*A*b^5*x^4*
sgn(b*x + a) + 10/3*B*a^2*b^3*x^3*sgn(b*x + a) + 5/3*A*a*b^4*x^3*sgn(b*x +
 a) + 5*B*a^3*b^2*x^2*sgn(b*x + a) + 5*A*a^2*b^3*x^2*sgn(b*x + a) + 5*B*a^
4*b*x*sgn(b*x + a) + 10*A*a^3*b^2*x*sgn(b*x + a) - A*a^5*sgn(b*x + a)/x +
(B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^2}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^2,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^2}\,dx=\frac{60\log(x)\,a^5bx-10a^6+150a^4b^2x^2+100a^3b^3x^3+50a^2b^4x^4+15a}{10x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^2,x)
```

output
```
(60*log(x)*a**5*b*x - 10*a**6 + 150*a**4*b**2*x**2 + 100*a**3*b**3*x**3 +
50*a**2*b**4*x**4 + 15*a*b**5*x**5 + 2*b**6*x**6)/(10*x)
```

3.321 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 297

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3} dx = -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{10a^2b^2(Ab+aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{5ab^3(Ab+2aB)x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{b^4(Ab+5aB)x^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{b^5Bx^4\sqrt{a^2+2abx+b^2x^2}}{4(a+bx)} + \frac{5a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output

```
-1/2*a^5*A*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-a^4*(5*A*b+B*a)*((b*x+a)^2)^(1/2)
/x/(b*x+a)+10*a^2*b^2*(A*b+B*a)*x*((b*x+a)^2)^(1/2)/(b*x+a)+5*a*b^3*(A*b+2
*B*a)*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+b^4*(A*b+5*B*a)*x^3*((b*x+a)^2)^(1
/2)/(3*b*x+3*a)+b^5*B*x^4*((b*x+a)^2)^(1/2)/(4*b*x+4*a)+5*a^3*b*(2*A*b+B*a
)*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{(a+bx)^2}(-60a^4Abx+120a^3b^2Bx^3+60a^2b^3x^3(2A+Bx)-6a}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-60*a^4*A*b*x + 120*a^3*b^2*B*x^3 + 60*a^2*b^3*x^3*(2*
A + B*x) - 6*a^5*(A + 2*B*x) + 10*a*b^4*x^4*(3*A + 2*B*x) + b^5*x^5*(4*A +
 3*B*x) + 60*a^3*b*(2*A*b + a*B)*x^2*Log[x]))/(12*x^2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^3}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^3}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^3}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^5}{x^3}+\frac{(5Ab+aB)a^4}{x^2}+\frac{5b(2Ab+aB)a^3}{x}+10b^2(Ab+aB)a^2+5b^3(Ab+2aB)xa+b^5Bx^3+b^4(A}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{2x^2}-\frac{a^4(aB+5Ab)}{x}+5a^3b\log(x)(aB+2Ab)+10a^2b^2x(aB+Ab)+\frac{1}{3}b^4x^3(5aB+Ab)+\frac{5}{2}\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*(a^5*A)/x^2 - (a^4*(5*A*b + a*B))/x +
 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*
a*B)*x^3)/3 + (b^5*B*x^4)/4 + 5*a^3*b*(2*A*b + a*B)*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.48

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(3B\,b^5x^6+4A\,b^5x^5+20Ba\,b^4x^5+30Aa\,b^4x^4+60B\,a^2b^3x^4+120A\ln(x)x^2a^3b^2+120A\,a^2b^3x^3+60B\ln(x)a^4b\,x^2+120B\right.}{12x^2(bx+a)^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,b^2\left(\frac{1}{4}B\,b^3x^4+\frac{1}{3}A\,b^3x^3+\frac{5}{3}Ba\,b^2x^3+\frac{5}{2}Aa\,b^2x^2+5B\,a^2b\,x^2+10A\,a^2bx+10B\,a^3x\right)}{bx+a}+\frac{\sqrt{(bx+a)^2}\left((-5A\,a^4b-B\,a^5)x-\right.}{(bx+a)x^2}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x+a)^2)^(5/2)*(3*B*b^5*x^6+4*A*b^5*x^5+20*B*a*b^4*x^5+30*A*a*b^4*
x^4+60*B*a^2*b^3*x^4+120*A*ln(x)*x^2*a^3*b^2+120*A*a^2*b^3*x^3+60*B*ln(x)*
a^4*b*x^2+120*B*a^3*b^2*x^3-60*A*a^4*b*x-12*B*a^5*x-6*A*a^5)/x^2/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx = \frac{3\,Bb^5x^6-6\,Aa^5+4\,(5\,Bab^4+Ab^5)x^5+30\,(2\,Ba^2b^3+Aab^4)x^4+}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*B*b^5*x^6 - 6*A*a^5 + 4*(5*B*a*b^4 + A*b^5)*x^5 + 30*(2*B*a^2*b^3
+ A*a*b^4)*x^4 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 60*(B*a^4*b + 2*A*a^3*b
^2)*x^2*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x)/x^2
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx = \int \frac{(A+Bx)\,\left((a+bx)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**3,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(212) = 424.

Time = 0.05 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{5/2}}{x^3}\,dx &= 5\,(-1)^{2\,b^2x+2\,ab}\,Ba^4b\log\left(2\,b^2x\right.\\&\left.+2\,ab\right)+10\,(-1)^{2\,b^2x+2\,ab}\,Aa^3b^2\log\left(2\,b^2x+2\,ab\right)\\&-5\,(-1)^{2\,abx+2\,a^2}\,Ba^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)\\&-10\,(-1)^{2\,abx+2\,a^2}\,Aa^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)\\&+\frac{5}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ba^2b^2x+5\,\sqrt{b^2x^2+2\,abx+a^2}Aab^3x\\&+\frac{15}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ba^3b+15\,\sqrt{b^2x^2+2\,abx+a^2}Aa^2b^2\\&+\frac{5}{4}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^2x+\frac{5\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^3x}{2\,a}\\&+\frac{35}{12}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bab+\frac{35}{6}\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Ab^2\\&+\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^2}{2\,a^2}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}B}{x}\\&-\frac{3\,\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab}{2\,ax}-\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{2\,a^2x^2}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
5*(-1)^(2*b^2*x + 2*a*b)*B*a^4*b*log(2*b^2*x + 2*a*b) + 10*(-1)^(2*b^2*x +
 2*a*b)*A*a^3*b^2*log(2*b^2*x + 2*a*b) - 5*(-1)^(2*a*b*x + 2*a^2)*B*a^4*b*
log(2*a*b*x/abs(x) + 2*a^2/abs(x)) - 10*(-1)^(2*a*b*x + 2*a^2)*A*a^3*b^2*l
og(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^
2*b^2*x + 5*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a*b^3*x + 15/2*sqrt(b^2*x^2 +
2*a*b*x + a^2)*B*a^3*b + 15*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^2*b^2 + 5/4*
(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^2*x + 5/2*(b^2*x^2 + 2*a*b*x + a^2)^(3
/2)*A*b^3*x/a + 35/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*a*b + 35/6*(b^2*x^
2 + 2*a*b*x + a^2)^(3/2)*A*b^2 + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2
/a^2 - (b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B/x - 3/2*(b^2*x^2 + 2*a*b*x + a^2)
^(5/2)*A*b/(a*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx = \frac{1}{4}\,Bb^5x^4\mathrm{sgn}(bx+a) + \frac{5}{3}\,Bab^4x^3\mathrm{sgn}(bx+a) + \frac{1}{3}\,Ab^5x^3\mathrm{sgn}(bx+a) + 5\,Ba^2b^3x^2\mathrm{sgn}(bx+a) + \frac{5}{2}\,Aab^4x^2\mathrm{sgn}(bx+a) + 10\,Ba^3b^2x\mathrm{sgn}(bx+a) + 10\,Aa^2b^3x\mathrm{sgn}(bx+a) + 5\left(Ba^4b\mathrm{sgn}(bx+a) + 2\,Aa^3b^2\mathrm{sgn}(bx+a)\right)\log\left(|x|\right) - \frac{Aa^5\mathrm{sgn}(bx+a) + 2\left(Ba^5\mathrm{sgn}(bx+a) + 5\,Aa^4b\mathrm{sgn}(bx+a)\right)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/4*B*b^5*x^4*sgn(b*x + a) + 5/3*B*a*b^4*x^3*sgn(b*x + a) + 1/3*A*b^5*x^3*
sgn(b*x + a) + 5*B*a^2*b^3*x^2*sgn(b*x + a) + 5/2*A*a*b^4*x^2*sgn(b*x + a)
 + 10*B*a^3*b^2*x*sgn(b*x + a) + 10*A*a^2*b^3*x*sgn(b*x + a) + 5*(B*a^4*b*
sgn(b*x + a) + 2*A*a^3*b^2*sgn(b*x + a))*log(abs(x)) - 1/2*(A*a^5*sgn(b*x
+ a) + 2*(B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*x)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^3}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^3,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^3}\,dx=\frac{60\log(x)\,a^4b^2x^2-2a^6-24a^5bx+80a^3b^3x^3+30a^2b^4x^4+8a\,b^5x^5}{4x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^3,x)
```

output
```
(60*log(x)*a**4*b**2*x**2 - 2*a**6 - 24*a**5*b*x + 80*a**3*b**3*x**3 + 30*
a**2*b**4*x**4 + 8*a*b**5*x**5 + b**6*x**6)/(4*x**2)
```

3.322 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 29, antiderivative size = 297

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^4} dx = -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{5a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{5ab^3(Ab+2aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^4(Ab+5aB)x^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{b^5Bx^3\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{10a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/3*a^5*A*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-1/2*a^4*(5*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^2/(b*x+a)-5*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x/(b*x+a)+5*a*b^3*(
A*b+2*B*a)*x*((b*x+a)^2)^(1/2)/(b*x+a)+b^4*(A*b+5*B*a)*x^2*((b*x+a)^2)^(1/
2)/(2*b*x+2*a)+b^5*B*x^3*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+10*a^2*b^2*(A*b+B*a
)*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{(a+bx)^2}(-60a^3Ab^2x^2+60a^2b^3Bx^4+15ab^4x^4(2A+Bx)-15}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^4,x]
```

output
```
(Sqrt[(a + b*x)^2]*(-60*a^3*A*b^2*x^2 + 60*a^2*b^3*B*x^4 + 15*a*b^4*x^4*(2
*A + B*x) - 15*a^4*b*x*(A + 2*B*x) + b^5*x^5*(3*A + 2*B*x) - a^5*(2*A + 3*
B*x) + 60*a^2*b^2*(A*b + a*B)*x^3*Log[x]))/(6*x^3*(a + b*x))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^4}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^4}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^4}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^5}{x^4}+\frac{(5Ab+aB)a^4}{x^3}+\frac{5b(2Ab+aB)a^3}{x^2}+\frac{10b^2(Ab+aB)a^2}{x}+5b^3(Ab+2aB)a+b^5Bx^2+b^4(Ab+5\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{2x^2}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3x(\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/3*(a^5*A)/x^3 - (a^4*(5*A*b + a*B))/(2*
x^2) - (5*a^3*b*(2*A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5
*a*B)*x^2)/2 + (b^5*B*x^3)/3 + 10*a^2*b^2*(A*b + a*B)*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.48

method	result
default	$\frac{\left((bx+a)^{2}\right)^{\frac{5}{2}}\left(2B\,b^{5}x^{6}+3A\,b^{5}x^{5}+15Ba\,b^{4}x^{5}+60A\ln(x)x^{3}a^{2}b^{3}+30Aa\,b^{4}x^{4}+60B\ln(x)a^{3}b^{2}x^{3}+60B\,a^{2}b^{3}x^{4}-60A\,a^{3}b^{2}x^{2}-30B\,a^{4}b\cdots\right.}{6x^{3}(bx+a)^{5}}$
risch	$\frac{\sqrt{(bx+a)^{2}}\,b^{3}\left(\frac{1}{3}x^{3}B\,b^{2}+\frac{1}{2}x^{2}b^{2}A+\frac{5}{2}Ba\,x^{2}b+5abAx+10a^{2}Bx\right)}{bx+a}+\frac{\sqrt{(bx+a)^{2}}\left(\left(-10a^{3}A\,b^{2}-5B\,a^{4}b\right)x^{2}+\left(-\frac{5}{2}A\,a^{4}b-\frac{1}{2}B\,a^{5}\right)x-\cdots\right.}{(bx+a)x^{3}}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x+a)^2)^(5/2)*(2*B*b^5*x^6+3*A*b^5*x^5+15*B*a*b^4*x^5+60*A*ln(x)*x
^3*a^2*b^3+30*A*a*b^4*x^4+60*B*ln(x)*a^3*b^2*x^3+60*B*a^2*b^3*x^4-60*A*a^3
*b^2*x^2-30*B*a^4*b*x^2-15*A*a^4*b*x-3*B*a^5*x-2*A*a^5)/x^3/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^{2}+2abx+b^{2}x^{2}\right)^{5/2}}{x^{4}}\,dx=\frac{2\,Bb^{5}x^{6}-2\,Aa^{5}+3\,(5\,Bab^{4}+Ab^{5})x^{5}+30\,(2\,Ba^{2}b^{3}+Aab^{4})x^{4}+\cdots}{\cdots}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^5*x^6 - 2*A*a^5 + 3*(5*B*a*b^4 + A*b^5)*x^5 + 30*(2*B*a^2*b^3 +
 A*a*b^4)*x^4 + 60*(B*a^3*b^2 + A*a^2*b^3)*x^3*log(x) - 30*(B*a^4*b + 2*A*
a^3*b^2)*x^2 - 3*(B*a^5 + 5*A*a^4*b)*x)/x^3
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx=\int \frac{(A+Bx)\,\left((a+bx)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**4,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(212) = 424$.

Time = 0.04 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{5/2}}{x^4}\,dx &= 10\,(-1)^{2\,b^2x+2\,ab}\,Ba^3b^2\log\left(2\,b^2x\right.\\
&\left.+2\,ab\right)+10\,(-1)^{2\,b^2x+2\,ab}\,Aa^2b^3\log\left(2\,b^2x+2\,ab\right)\\
&-10\,(-1)^{2\,abx+2\,a^2}\,Ba^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)\\
&-10\,(-1)^{2\,abx+2\,a^2}\,Aa^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)+5\,\sqrt{b^2x^2+2\,abx+a^2}Bab^3x\\
&+5\,\sqrt{b^2x^2+2\,abx+a^2}Ab^4x+15\,\sqrt{b^2x^2+2\,abx+a^2}Ba^2b^2\\
&+15\,\sqrt{b^2x^2+2\,abx+a^2}Aab^3+\frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^3x}{2\,a}\\
&+\frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^4x}{2\,a^2}+\frac{35}{6}\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^2\\
&+\frac{35\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^3}{6\,a}+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bb^2}{2\,a^2}\\
&+\frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ab^3}{6\,a^3}-\frac{3\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bb}{2\,ax}\\
&-\frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ab^2}{6\,a^2x}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}B}{2\,a^2x^2}\\
&-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ab}{6\,a^3x^2}-\frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}A}{3\,a^2x^3}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
10*(-1)^(2*b^2*x + 2*a*b)*B*a^3*b^2*log(2*b^2*x + 2*a*b) + 10*(-1)^(2*b^2*
x + 2*a*b)*A*a^2*b^3*log(2*b^2*x + 2*a*b) - 10*(-1)^(2*a*b*x + 2*a^2)*B*a^
3*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) - 10*(-1)^(2*a*b*x + 2*a^2)*A*a^2
*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5*sqrt(b^2*x^2 + 2*a*b*x + a^2)*
B*a*b^3*x + 5*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^4*x + 15*sqrt(b^2*x^2 + 2*
a*b*x + a^2)*B*a^2*b^2 + 15*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a*b^3 + 5/2*(b
^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^3*x/a + 5/2*(b^2*x^2 + 2*a*b*x + a^2)^(3
/2)*A*b^4*x/a^2 + 35/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^2 + 35/6*(b^2*x
^2 + 2*a*b*x + a^2)^(3/2)*A*b^3/a + 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*
b^2/a^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^3/a^3 - 3/2*(b^2*x^2 + 2
*a*b*x + a^2)^(5/2)*B*b/(a*x) - 11/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^2
/(a^2*x) - 1/2*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^2) - 1/6*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^2) - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)
*A/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx = \frac{1}{3}\,Bb^5x^3\mathrm{sgn}(bx+a) + \frac{5}{2}\,Bab^4x^2\mathrm{sgn}(bx+a) + \frac{1}{2}\,Ab^5x^2\mathrm{sgn}(bx+a) + 10\,Ba^2b^3x\mathrm{sgn}(bx+a) + 5\,Aab^4x\mathrm{sgn}(bx+a) + 10\left(Ba^3b^2\mathrm{sgn}(bx+a) + Aa^2b^3\mathrm{sgn}(bx+a)\right)\log\left(|x|\right) - \frac{2\,Aa^5\mathrm{sgn}(bx+a) + 30\left(Ba^4b\mathrm{sgn}(bx+a) + 2\,Aa^3b^2\mathrm{sgn}(bx+a)\right)x^2 + 3\left(Ba^5\mathrm{sgn}(bx+a) + 5\,Aa^4b\mathrm{sgn}(bx\right.}{6\,x^3}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/3*B*b^5*x^3*sgn(b*x + a) + 5/2*B*a*b^4*x^2*sgn(b*x + a) + 1/2*A*b^5*x^2*
sgn(b*x + a) + 10*B*a^2*b^3*x*sgn(b*x + a) + 5*A*a*b^4*x*sgn(b*x + a) + 10
*(B*a^3*b^2*sgn(b*x + a) + A*a^2*b^3*sgn(b*x + a))*log(abs(x)) - 1/6*(2*A*
a^5*sgn(b*x + a) + 30*(B*a^4*b*sgn(b*x + a) + 2*A*a^3*b^2*sgn(b*x + a))*x^
2 + 3*(B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*x)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}{x^4}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^4,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^4}\,dx=\frac{60\log(x)\,a^3b^3x^3-a^6-9a^5bx-45a^4b^2x^2+45a^2b^4x^4+9a\,b^5x^5+}{3x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^4,x)
```

output
```
(60*log(x)*a**3*b**3*x**3 - a**6 - 9*a**5*b*x - 45*a**4*b**2*x**2 + 45*a**
2*b**4*x**4 + 9*a*b**5*x**5 + b**6*x**6)/(3*x**3)
```

3.323 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 296

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^5}\,dx = -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} - \frac{5a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{2x^2(a+bx)} - \frac{10a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} + \frac{b^4(Ab+5aB)x\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{b^5Bx^2\sqrt{a^2+2abx+b^2x^2}}{2(a+bx)} + \frac{5ab^3(Ab+2aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}$$

output
```
-1/4*a^5*A*((b*x+a)^2)^(1/2)/x^4/(b*x+a)-1/3*a^4*(5*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^3/(b*x+a)-5/2*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-10*a^
2*b^2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x/(b*x+a)+b^4*(A*b+5*B*a)*x*((b*x+a)^2)^
(1/2)/(b*x+a)+b^5*B*x^2*((b*x+a)^2)^(1/2)/(2*b*x+2*a)+5*a*b^3*(A*b+2*B*a)*
((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(120a^2Ab^3x^3-60ab^4Bx^5-6b^5x^5(2A+Bx)+60a^3b^2x^2(A+2Bx)+10a^4bx(2A+3Bx)+}{12x^4(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^5,x]
```

output
```
-1/12*(Sqrt[(a + b*x)^2]*(120*a^2*A*b^3*x^3 - 60*a*b^4*B*x^5 - 6*b^5*x^5*(
2*A + B*x) + 60*a^3*b^2*x^2*(A + 2*B*x) + 10*a^4*b*x*(2*A + 3*B*x) + a^5*(
3*A + 4*B*x) - 60*a*b^3*(A*b + 2*a*B)*x^4*Log[x]))/(x^4*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^5}dx$$
$$\Big\downarrow \text{1187}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^5}dx}{b^5(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^5}dx}{a+bx}$$
$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^5}+\frac{(5Ab+aB)a^4}{x^4}+\frac{5b(2Ab+aB)a^3}{x^3}+\frac{10b^2(Ab+aB)a^2}{x^2}+\frac{5b^3(Ab+2aB)a}{x}+b^4(Ab+5aB)+b^5Bx\right.}{a+bx}$$

$\big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{10a^2b^2(aB+Ab)}{x}+b^4x(5aB+Ab)+5ab^3\log(x)(2aB+\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(3*
x^3) - (5*a^3*b*(2*A*b + a*B))/(2*x^2) - (10*a^2*b^2*(A*b + a*B))/x + b^4*
(A*b + 5*a*B)*x + (b^5*B*x^2)/2 + 5*a*b^3*(A*b + 2*a*B)*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}(6B\,b^5x^6+60A\ln(x)x^4a\,b^4+12A\,b^5x^5+120B\ln(x)a^2b^3x^4+60Ba\,b^4x^5-120A\,a^2b^3x^3-120B\,a^3b^2x^3-60A\,a^3b^2x^2-30$
risch	$\frac{\sqrt{(bx+a)^2}\,b^4\left(\frac{1}{2}Bb\,x^2+Abx+5Bax\right)}{bx+a}+\frac{\sqrt{(bx+a)^2}\left(\left(-10A\,a^2b^3-10B\,a^3b^2\right)x^3+\left(-5a^3A\,b^2-\frac{5}{2}B\,a^4b\right)x^2+\left(-\frac{5}{3}A\,a^4b-\frac{1}{3}B\,a^5\right)x-\right.}{(bx+a)x^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x+a)^2)^(5/2)*(6*B*b^5*x^6+60*A*ln(x)*x^4*a*b^4+12*A*b^5*x^5+120*
B*ln(x)*a^2*b^3*x^4+60*B*a*b^4*x^5-120*A*a^2*b^3*x^3-120*B*a^3*b^2*x^3-60*
A*a^3*b^2*x^2-30*B*a^4*b*x^2-20*A*a^4*b*x-4*B*a^5*x-3*A*a^5)/x^4/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx = \frac{6\,Bb^5x^6-3\,Aa^5+12\,(5\,Bab^4+Ab^5)x^5+60\,(2\,Ba^2b^3+Aab^4)x^4}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/12*(6*B*b^5*x^6 - 3*A*a^5 + 12*(5*B*a*b^4 + A*b^5)*x^5 + 60*(2*B*a^2*b^3
 + A*a*b^4)*x^4*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 30*(B*a^4*b + 2
*A*a^3*b^2)*x^2 - 4*(B*a^5 + 5*A*a^4*b)*x)/x^4
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**5,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(211) = 422$.

Time = 0.05 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.08

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx = 10\,(-1)^{2\,b^2x+2\,ab}\,Ba^2b^3\log\left(2\,b^2x + 2\,ab\right) + 5\,(-1)^{2\,b^2x+2\,ab}\,Aab^4\log\left(2\,b^2x+2\,ab\right) - 10\,(-1)^{2\,abx+2\,a^2}\,Ba^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) - 5\,(-1)^{2\,abx+2\,a^2}\,Aab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + 5\,\sqrt{b^2x^2+2\,abx+a^2}Bb^4x + \frac{5\,\sqrt{b^2x^2+2\,abx+a^2}Ab^5x}{2\,a} + 15\,\sqrt{b^2x^2+2\,abx+a^2}Bab^3 + \frac{15}{2}\,\sqrt{b^2x^2+2\,abx+a^2}Ab^4 + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^4x}{2\,a^2} + \frac{5\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^5x}{4\,a^3} + \frac{35\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Bb^3}{6\,a} + \frac{35\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}Ab^4}{12\,a^2} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bb^3}{6\,a^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ab^4}{3\,a^4} - \frac{11\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Bb^2}{6\,a^2x} - \frac{2\,(b^2x^2+2\,abx+a^2)^{\frac{5}{2}}Ab^3}{3\,a^3x} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Bb}{6\,a^3x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ab^2}{3\,a^4x^2} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}B}{3\,a^2x^3} + \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}Ab}{12\,a^3x^3} - \frac{(b^2x^2+2\,abx+a^2)^{\frac{7}{2}}A}{4\,a^2x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
10*(-1)^(2*b^2*x + 2*a*b)*B*a^2*b^3*log(2*b^2*x + 2*a*b) + 5*(-1)^(2*b^2*x
 + 2*a*b)*A*a*b^4*log(2*b^2*x + 2*a*b) - 10*(-1)^(2*a*b*x + 2*a^2)*B*a^2*b
^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) - 5*(-1)^(2*a*b*x + 2*a^2)*A*a*b^4*l
og(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 5*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^4*
x + 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^5*x/a + 15*sqrt(b^2*x^2 + 2*a*b*
x + a^2)*B*a*b^3 + 15/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^4 + 5/2*(b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*B*b^4*x/a^2 + 5/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*
A*b^5*x/a^3 + 35/6*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^3/a + 35/12*(b^2*x^
2 + 2*a*b*x + a^2)^(3/2)*A*b^4/a^2 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B
*b^3/a^3 + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4/a^4 - 11/6*(b^2*x^2 +
 2*a*b*x + a^2)^(5/2)*B*b^2/(a^2*x) - 2/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*
A*b^3/(a^3*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^2) - 1/3*(b
^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^4*x^2) - 1/3*(b^2*x^2 + 2*a*b*x + a
^2)^(7/2)*B/(a^2*x^3) + 1/12*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^3)
 - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx = \frac{1}{2}\,Bb^5x^2\mathrm{sgn}(bx+a)+5\,Bab^4x\mathrm{sgn}(bx+a) + Ab^5x\mathrm{sgn}(bx+a)+5\left(2\,Ba^2b^3\mathrm{sgn}(bx+a)+Aab^4\mathrm{sgn}(bx+a)\right)\log\left(|x|\right) - \frac{3\,Aa^5\mathrm{sgn}(bx+a)+120\left(Ba^3b^2\mathrm{sgn}(bx+a)+Aa^2b^3\mathrm{sgn}(bx+a)\right)x^3+30\left(Ba^4b\mathrm{sgn}(bx+a)+2\,Aa^3b^2\mathrm{sgn}\right.}{12\,x^4}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b^5*x^2*sgn(b*x + a) + 5*B*a*b^4*x*sgn(b*x + a) + A*b^5*x*sgn(b*x +
a) + 5*(2*B*a^2*b^3*sgn(b*x + a) + A*a*b^4*sgn(b*x + a))*log(abs(x)) - 1/1
2*(3*A*a^5*sgn(b*x + a) + 120*(B*a^3*b^2*sgn(b*x + a) + A*a^2*b^3*sgn(b*x
+ a))*x^3 + 30*(B*a^4*b*sgn(b*x + a) + 2*A*a^3*b^2*sgn(b*x + a))*x^2 + 4*(
B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*x)/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}{x^5}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^5,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^5}\,dx=\frac{60\log(x)\,a^2b^4x^4-a^6-8a^5bx-30a^4b^2x^2-80a^3b^3x^3+24a\,b^5x^5+\cdots}{4x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^5,x)
```

output
```
(60*log(x)*a**2*b**4*x**4 - a**6 - 8*a**5*b*x - 30*a**4*b**2*x**2 - 80*a**
3*b**3*x**3 + 24*a*b**5*x**5 + 2*b**6*x**6)/(4*x**4)
```

3.324 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 293

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx = & -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} \\
& -\frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)}-\frac{5a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} \\
& -\frac{5a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)}-\frac{5ab^3(Ab+2aB)\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} \\
& +\frac{b^5Bx\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{b^4(Ab+5aB)\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}
\end{aligned}
$$

output

```
-1/5*a^5*A*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-1/4*a^4*(5*A*b+B*a)*((b*x+a)^2)^(
1/2)/x^4/(b*x+a)-5/3*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-5*a^2
*b^2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^2/(b*x+a)-5*a*b^3*(A*b+2*B*a)*((b*x+a)^
2)^(1/2)/x/(b*x+a)+b^5*B*x*((b*x+a)^2)^(1/2)/(b*x+a)+b^4*(A*b+5*B*a)*((b*x
+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(300aAb^4x^4-60b^5Bx^6+300a^2b^3x^3(A+2Bx)+100a^3b^2x^2(2A+3Bx)+25a^4bx(3A+4B}{60x^5(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^6,x]
```

output
```
-1/60*(Sqrt[(a + b*x)^2]*(300*a*A*b^4*x^4 - 60*b^5*B*x^6 + 300*a^2*b^3*x^3
*(A + 2*B*x) + 100*a^3*b^2*x^2*(2*A + 3*B*x) + 25*a^4*b*x*(3*A + 4*B*x) +
3*a^5*(4*A + 5*B*x) - 60*b^4*(A*b + 5*a*B)*x^5*Log[x]))/(x^5*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^6}\,dx$$

$$\Big\downarrow \ 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^6}dx}{b^5(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^6}dx}{a+bx}$$

$$\Big\downarrow \ 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^6}+\frac{(5Ab+aB)a^4}{x^5}+\frac{5b(2Ab+aB)a^3}{x^4}+\frac{10b^2(Ab+aB)a^2}{x^3}+\frac{5b^3(Ab+2aB)a}{x^2}+b^5B+\frac{b^4(Ab+5aB)}{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{5a^2b^2(aB+Ab)}{x^2}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(4*
x^4) - (5*a^3*b*(2*A*b + a*B))/(3*x^3) - (5*a^2*b^2*(A*b + a*B))/x^2 - (5*
a*b^3*(A*b + 2*a*B))/x + b^5*B*x + b^4*(A*b + 5*a*B)*Log[x]))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left((bx+a)^2\right)^{\frac{5}{2}}(60A\ln(x)x^5b^5+300B\ln(x)a\,b^4x^5+60B\,b^5x^6-300Aa\,b^4x^4-600B\,a^2b^3x^4-300A\,a^2b^3x^3-300B\,a^3b^2x^3-200A\,a^3b^2x^2}{60(bx+a)^5x^5}$
risch	$\frac{b^5Bx\sqrt{(bx+a)^2}}{bx+a}+\frac{\sqrt{(bx+a)^2}\left((-5Aa\,b^4-10B\,a^2b^3)x^4+(-5A\,a^2b^3-5B\,a^3b^2)x^3+(-\frac{10}{3}a^3A\,b^2-\frac{5}{3}B\,a^4b)x^2+(-\frac{5}{4}A\,a^4b-\frac{1}{4}B\,a\right.}{(bx+a)x^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/60*((b*x+a)^2)^(5/2)*(60*A*ln(x)*x^5*b^5+300*B*ln(x)*a*b^4*x^5+60*B*b^5*
x^6-300*A*a*b^4*x^4-600*B*a^2*b^3*x^4-300*A*a^2*b^3*x^3-300*B*a^3*b^2*x^3-
200*A*a^3*b^2*x^2-100*B*a^4*b*x^2-75*A*a^4*b*x-15*B*a^5*x-12*A*a^5)/(b*x+a
)^5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx = \frac{60\,Bb^5x^6+60\,(5\,Bab^4+Ab^5)x^5\log{(x)}-12\,Aa^5-300\,(2\,Ba^2b^3}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^5*x^6 + 60*(5*B*a*b^4 + A*b^5)*x^5*log(x) - 12*A*a^5 - 300*(2
*B*a^2*b^3 + A*a*b^4)*x^4 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 100*(B*a^4*b
 + 2*A*a^3*b^2)*x^2 - 15*(B*a^5 + 5*A*a^4*b)*x)/x^5
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**6,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(210) = 420$.

Time = 0.06 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
5*(-1)^(2*b^2*x + 2*a*b)*B*a*b^4*log(2*b^2*x + 2*a*b) + (-1)^(2*b^2*x + 2*
a*b)*A*b^5*log(2*b^2*x + 2*a*b) - 5*(-1)^(2*a*b*x + 2*a^2)*B*a*b^4*log(2*a
*b*x/abs(x) + 2*a^2/abs(x)) - (-1)^(2*a*b*x + 2*a^2)*A*b^5*log(2*a*b*x/abs
(x) + 2*a^2/abs(x)) + 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^5*x/a + 1/2*sq
rt(b^2*x^2 + 2*a*b*x + a^2)*A*b^6*x/a^2 + 15/2*sqrt(b^2*x^2 + 2*a*b*x + a^
2)*B*b^4 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^5/a + 5/4*(b^2*x^2 + 2*a*
b*x + a^2)^(3/2)*B*b^5*x/a^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*A*b^6*x
/a^4 + 35/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*B*b^4/a^2 + 7/12*(b^2*x^2 + 2
*a*b*x + a^2)^(3/2)*A*b^5/a^3 + 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^4/
a^4 - 2/15*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/a^5 - 2/3*(b^2*x^2 + 2*a*
b*x + a^2)^(5/2)*B*b^3/(a^3*x) - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^4
/(a^4*x) - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^4*x^2) + 2/15*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^2) + 1/12*(b^2*x^2 + 2*a*b*x + a^
2)^(7/2)*B*b/(a^3*x^3) - 11/60*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^4*
x^3) - 1/4*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^4) + 3/20*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)*A*b/(a^3*x^4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A/
(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx = Bb^5x\mathrm{sgn}(bx+a) + \left(5\,Bab^4\mathrm{sgn}(bx+a)+Ab^5\mathrm{sgn}(bx+a)\right)\log\left(|x|\right) - \frac{12\,Aa^5\mathrm{sgn}(bx+a)+300\left(2\,Ba^2b^3\mathrm{sgn}(bx+a)+Aab^4\mathrm{sgn}(bx+a)\right)x^4+300\left(Ba^3b^2\mathrm{sgn}(bx+a)+Aa^2b^3\mathrm{s}\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
B*b^5*x*sgn(b*x + a) + (5*B*a*b^4*sgn(b*x + a) + A*b^5*sgn(b*x + a))*log(a
bs(x)) - 1/60*(12*A*a^5*sgn(b*x + a) + 300*(2*B*a^2*b^3*sgn(b*x + a) + A*a
*b^4*sgn(b*x + a))*x^4 + 300*(B*a^3*b^2*sgn(b*x + a) + A*a^2*b^3*sgn(b*x +
 a))*x^3 + 100*(B*a^4*b*sgn(b*x + a) + 2*A*a^3*b^2*sgn(b*x + a))*x^2 + 15*
(B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*x)/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}{x^6}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^6,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^6}\,dx=\frac{60\log(x)\,a\,b^5x^5-2a^6-15a^5bx-50a^4b^2x^2-100a^3b^3x^3-150a^2b}{10x^5}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^6,x)
```

output
```
(60*log(x)*a*b**5*x**5 - 2*a**6 - 15*a**5*b*x - 50*a**4*b**2*x**2 - 100*a*
*3*b**3*x**3 - 150*a**2*b**4*x**4 + 10*b**6*x**6)/(10*x**5)
```

3.325 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 267

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx &= -\frac{a^5B\sqrt{a^2+2abx+b^2x^2}}{5x^5(a+bx)} \\ &- \frac{5a^4bB\sqrt{a^2+2abx+b^2x^2}}{4x^4(a+bx)} - \frac{10a^3b^2B\sqrt{a^2+2abx+b^2x^2}}{3x^3(a+bx)} \\ &- \frac{5a^2b^3B\sqrt{a^2+2abx+b^2x^2}}{x^2(a+bx)} - \frac{5ab^4B\sqrt{a^2+2abx+b^2x^2}}{x(a+bx)} \\ &- \frac{A(a+bx)^5\sqrt{a^2+2abx+b^2x^2}}{6ax^6} + \frac{b^5B\sqrt{a^2+2abx+b^2x^2}\log(x)}{a+bx}\end{aligned}$$

output
```
-1/5*a^5*B*((b*x+a)^2)^(1/2)/x^5/(b*x+a)-5/4*a^4*b*B*((b*x+a)^2)^(1/2)/x^4
/(b*x+a)-10/3*a^3*b^2*B*((b*x+a)^2)^(1/2)/x^3/(b*x+a)-5*a^2*b^3*B*((b*x+a)
^2)^(1/2)/x^2/(b*x+a)-5*a*b^4*B*((b*x+a)^2)^(1/2)/x/(b*x+a)-1/6*A*(b*x+a)^
5*((b*x+a)^2)^(1/2)/a/x^6+b^5*B*((b*x+a)^2)^(1/2)*ln(x)/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2}\left(60Ab^5x^5+150ab^4x^4(A+2Bx)+100a^2b^3x^3(2A+3Bx)+5\right.}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^7,x]
```

output
```
(Sqrt[a^2]*(60*A*b^5*x^5 + 150*a*b^4*x^4*(A + 2*B*x) + 100*a^2*b^3*x^3*(2*
A + 3*B*x) + 50*a^3*b^2*x^2*(3*A + 4*B*x) + 15*a^4*b*x*(4*A + 5*B*x) + 2*a
^5*(5*A + 6*B*x)) - Sqrt[(a + b*x)^2]*(10*A*b^5*x^5 + 2*a^5*(5*A + 6*B*x)
+ a^4*b*x*(50*A + 63*B*x) + a*b^4*x^4*(50*A + 137*B*x) + a^3*b^2*x^2*(100*
A + 137*B*x) + a^2*b^3*x^3*(100*A + 163*B*x)) - 120*a*b^5*B*x^6*ArcTanh[(b
*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])] - 120*Sqrt[a^2]*b^5*B*x^6*Log[x] + 60
*Sqrt[a^2]*b^5*B*x^6*Log[a*(Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2])] + 60*Sqr
t[a^2]*b^5*B*x^6*Log[a*(Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2])])/(120*a*x^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^7}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^7}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^7}dx}{a+bx}$$

$$\Big\downarrow \text{87}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\int\frac{(a+bx)^5}{x^6}dx-\frac{A(a+bx)^6}{6ax^6}\right)}{a+bx}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\int\left(\frac{a^5}{x^6}+\frac{5ba^4}{x^5}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^3}+\frac{5b^4a}{x^2}+\frac{b^5}{x}\right)dx-\frac{A(a+bx)^6}{6ax^6}\right)}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(B\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\right)-\frac{A(a+bx)^6}{6ax^6}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/6*(A*(a + b*x)^6)/(a*x^6) + B*(-1/5*a^5
/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 - (5*a*b
^4)/x + b^5*Log[x])))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.53

method	result
default	$-\frac{\left((bx+a)^2\right)^{\frac{5}{2}}\left(-60b^5B\ln(x)x^6+60A\,b^5x^5+300Ba\,b^4x^5+150Aa\,b^4x^4+300B\,a^2b^3x^4+200A\,a^2b^3x^3+200B\,a^3b^2x^3+150A\,a^3b^2x^2\right.}{60(bx+a)^5x^6}$
risch	$\frac{\sqrt{(bx+a)^2}\left(\left(-A\,b^5-5Ba\,b^4\right)x^5+\left(-\frac{5}{2}Aa\,b^4-5B\,a^2b^3\right)x^4+\left(-\frac{10}{3}A\,a^2b^3-\frac{10}{3}B\,a^3b^2\right)x^3+\left(-\frac{5}{2}a^3A\,b^2-\frac{5}{4}B\,a^4b\right)x^2+\left(-A\,a^4b-\frac{1}{5}B\,a\right.\right.}{(bx+a)x^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((b*x+a)^2)^(5/2)*(-60*b^5*B*ln(x)*x^6+60*A*b^5*x^5+300*B*a*b^4*x^5+
150*A*a*b^4*x^4+300*B*a^2*b^3*x^4+200*A*a^2*b^3*x^3+200*B*a^3*b^2*x^3+150*
A*a^3*b^2*x^2+75*B*a^4*b*x^2+60*A*a^4*b*x+12*B*a^5*x+10*A*a^5)/(b*x+a)^5/x
^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx=\frac{60\,Bb^5x^6\log\left(x\right)-10\,Aa^5-60\,(5\,Bab^4+Ab^5)x^5-150\,(2\,Ba^2b^3}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^5*x^6*log(x) - 10*A*a^5 - 60*(5*B*a*b^4 + A*b^5)*x^5 - 150*(2
*B*a^2*b^3 + A*a*b^4)*x^4 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 75*(B*a^4*b
+ 2*A*a^3*b^2)*x^2 - 12*(B*a^5 + 5*A*a^4*b)*x)/x^6
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**7,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(182) = 364$.

Time = 0.05 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.07

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx = (-1)^{2\,b^2x+2\,ab}\,Bb^5\log\left(2\,b^2x+2\,ab\right) - (-1)^{2\,abx+2\,a^2}\,Bb^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right) + \frac{\sqrt{b^2x^2+2\,abx+a^2}Bb^6x}{2\,a^2} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Bb^5}{2\,a} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^6x}{4\,a^4} + \frac{7\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}Bb^5}{12\,a^3} - \frac{2\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^5}{15\,a^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^6}{6\,a^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^4}{3\,a^4x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^5}{6\,a^5x} + \frac{2\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^3}{15\,a^5x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^4}{6\,a^6x^2} - \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^2}{60\,a^4x^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^3}{6\,a^5x^3} + \frac{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb}{20\,a^3x^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^2}{6\,a^4x^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}B}{5\,a^2x^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab}{6\,a^3x^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{6\,a^2x^6}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
(-1)^(2*b^2*x + 2*a*b)*B*b^5*log(2*b^2*x + 2*a*b) - (-1)^(2*a*b*x + 2*a^2)
*B*b^5*log(2*a*b*x/abs(x) + 2*a^2/abs(x)) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a
^2)*B*b^6*x/a^2 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b^5/a + 1/4*(b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*B*b^6*x/a^4 + 7/12*(b^2*x^2 + 2*a*b*x + a^2)^(3/2)
*B*b^5/a^3 - 2/15*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^5/a^5 + 1/6*(b^2*x^2
 + 2*a*b*x + a^2)^(5/2)*A*b^6/a^6 - 1/3*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*
b^4/(a^4*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^5/(a^5*x) + 2/15*(b^
2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^3/(a^5*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^
2)^(7/2)*A*b^4/(a^6*x^2) - 11/60*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^
4*x^3) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^3) + 3/20*(b^2*x
^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*A*b^2/(a^4*x^4) - 1/5*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^5) + 1/
6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^5) - 1/6*(b^2*x^2 + 2*a*b*x +
 a^2)^(7/2)*A/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx = Bb^5\log\left(|x|\right)\text{sgn}(bx+a) - \frac{10\,Aa^5\text{sgn}(bx+a)+60\left(5\,Bab^4\text{sgn}(bx+a)+Ab^5\text{sgn}(bx+a)\right)x^5+150\left(2\,Ba^2b^3\text{sgn}(bx+a)+Aab^4\text{sgn}\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
B*b^5*log(abs(x))*sgn(b*x + a) - 1/60*(10*A*a^5*sgn(b*x + a) + 60*(5*B*a*b
^4*sgn(b*x + a) + A*b^5*sgn(b*x + a))*x^5 + 150*(2*B*a^2*b^3*sgn(b*x + a)
+ A*a*b^4*sgn(b*x + a))*x^4 + 200*(B*a^3*b^2*sgn(b*x + a) + A*a^2*b^3*sgn(
b*x + a))*x^3 + 75*(B*a^4*b*sgn(b*x + a) + 2*A*a^3*b^2*sgn(b*x + a))*x^2 +
 12*(B*a^5*sgn(b*x + a) + 5*A*a^4*b*sgn(b*x + a))*x)/x^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}{x^7}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^7,x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.26

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx=\frac{60\log(x)\,b^6x^6-10a^6-72a^5bx-225a^4b^2x^2-400a^3b^3x^3-450a^2}{60x^6}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^7,x)
```

output
```
(60*log(x)*b**6*x**6 - 10*a**6 - 72*a**5*b*x - 225*a**4*b**2*x**2 - 400*a*
*3*b**3*x**3 - 450*a**2*b**4*x**4 - 360*a*b**5*x**5)/(60*x**6)
```

3.326 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = \frac{(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^2x^6} - \frac{A\left(a^2+2abx+b^2x^2\right)^{7/2}}{7a^2x^7}$$

output
```
1/6*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^2/x^6-1/7*A*(b^2*x^2+2
*a*b*x+a^2)^(7/2)/a^2/x^7
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx)^2}\left(21b^5x^5(A+2Bx)+35ab^4x^4(2A+3Bx)+35a^2b^3x^3(3A+4Bx)+21a^3b^2x^2(4A+5Bx)+\right.}{42x^7(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^8,x]
```

output
```
-1/42*(Sqrt[(a + b*x)^2]*(21*b^5*x^5*(A + 2*B*x) + 35*a*b^4*x^4*(2*A + 3*B
*x) + 35*a^2*b^3*x^3*(3*A + 4*B*x) + 21*a^3*b^2*x^2*(4*A + 5*B*x) + 7*a^4*
b*x*(5*A + 6*B*x) + a^5*(6*A + 7*B*x)))/(x^7*(a + b*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1186, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}(A+Bx)}{x^8}dx$$

$$\Big\downarrow \text{1186}$$

$$-\frac{(Ab-aB)\int \frac{(a^2+2bxa+b^2x^2)^{5/2}}{x^7}dx}{a}-\frac{A(a^2+2abx+b^2x^2)^{7/2}}{7a^2x^7}$$

$$\Big\downarrow \text{1102}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{b^5(a+bx)^5}{x^7}dx}{ab^5(a+bx)}-\frac{A(a^2+2abx+b^2x^2)^{7/2}}{7a^2x^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(Ab-aB)\int \frac{(a+bx)^5}{x^7}dx}{a(a+bx)}-\frac{A(a^2+2abx+b^2x^2)^{7/2}}{7a^2x^7}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^5\sqrt{a^2+2abx+b^2x^2}(Ab-aB)}{6a^2x^6}-\frac{A(a^2+2abx+b^2x^2)^{7/2}}{7a^2x^7}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^8,x]
```

output
```
((A*b - a*B)*(a + b*x)^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(6*a^2*x^6) - (A*(
a^2 + 2*a*b*x + b^2*x^2)^(7/2))/(7*a^2*x^7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1186
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[-2*c*(e*f - d*g)*(d + e*x)^(m + 1)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e)^2)), x] + Simp[(2*c*f - b*g)/
(2*c*d - b*e)   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[
{a, b, c, d, e, f, g, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[m + 2*p + 3,
0] && NeQ[2*c*f - b*g, 0] && NeQ[2*c*d - b*e, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.

Time = 1.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.81

method	result
risch	$\frac{\sqrt{(bx+a)^2}\,(-B\,b^5x^6+(-\frac{1}{2}A\,b^5-\frac{5}{2}Ba\,b^4)x^5+(-\frac{5}{3}Aa\,b^4-\frac{10}{3}B\,a^2b^3)x^4+(-\frac{5}{2}A\,a^2b^3-\frac{5}{2}B\,a^3b^2)x^3+(-2a^3A\,b^2-B\,a^4b)x^2+(-\frac{5}{6}A}{(bx+a)x^7}$
gosper	$-\frac{(42B\,b^5x^6+21A\,b^5x^5+105Ba\,b^4x^5+70Aa\,b^4x^4+140B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+84A\,a^3b^2x^2+42B\,a^4b\,x^2+35A\,a^4b}{42x^7(bx+a)^5}$
default	$-\frac{(42B\,b^5x^6+21A\,b^5x^5+105Ba\,b^4x^5+70Aa\,b^4x^4+140B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+84A\,a^3b^2x^2+42B\,a^4b\,x^2+35A\,a^4b}{42x^7(bx+a)^5}$
orering	$-\frac{(42B\,b^5x^6+21A\,b^5x^5+105Ba\,b^4x^5+70Aa\,b^4x^4+140B\,a^2b^3x^4+105A\,a^2b^3x^3+105B\,a^3b^2x^3+84A\,a^3b^2x^2+42B\,a^4b\,x^2+35A\,a^4b}{42x^7(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-B*b^5*x^6+(-1/2*A*b^5-5/2*B*a*b^4)*x^5+(-5/3*A
*a*b^4-10/3*B*a^2*b^3)*x^4+(-5/2*A*a^2*b^3-5/2*B*a^3*b^2)*x^3+(-2*A*a^3*b^
2-B*a^4*b)*x^2+(-5/6*A*a^4*b-1/6*B*a^5)*x-1/7*A*a^5)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{42\,Bb^5x^6+6\,Aa^5+21\,(5\,Bab^4+Ab^5)x^5+70\,(2\,Ba^2b^3+Aab^4)x^4+105\,(Ba^3b^2+Aa^2b^3)x^3+42\,(Ba^4b}{42\,x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
-1/42*(42*B*b^5*x^6 + 6*A*a^5 + 21*(5*B*a*b^4 + A*b^5)*x^5 + 70*(2*B*a^2*b
^3 + A*a*b^4)*x^4 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^2 + 7*(B*a^5 + 5*A*a^4*b)*x)/x^7
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**8,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(67) = 134$.

Time = 0.04 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^6}{6\,a^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^7}{6\,a^7} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^5}{6\,a^5x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^6}{6\,a^6x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^4}{6\,a^6x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^5}{6\,a^7x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^3}{6\,a^5x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^4}{6\,a^6x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^2}{6\,a^4x^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^3}{6\,a^5x^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb}{6\,a^3x^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^2}{6\,a^4x^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}B}{6\,a^2x^6} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab}{6\,a^3x^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{7\,a^2x^7}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^6/a^6 - 1/6*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*A*b^7/a^7 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^5/(a^5*x) -
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^6/(a^6*x) - 1/6*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*B*b^4/(a^6*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/
(a^7*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^3/(a^5*x^3) - 1/6*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*x^3) - 1/6*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*B*b^2/(a^4*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x
^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^5) - 1/6*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)*A*b^2/(a^4*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*
B/(a^2*x^6) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^6) - 1/7*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(67) = 134$.

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{(7\,Bab^6-Ab^7)\mathrm{sgn}(bx+a)}{42\,a^2}$$
$$-\frac{42\,Bb^5x^6\mathrm{sgn}(bx+a)+105\,Bab^4x^5\mathrm{sgn}(bx+a)+21\,Ab^5x^5\mathrm{sgn}(bx+a)+140\,Ba^2b^3x^4\mathrm{sgn}(bx+a)+70\,A}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
-1/42*(7*B*a*b^6 - A*b^7)*sgn(b*x + a)/a^2 - 1/42*(42*B*b^5*x^6*sgn(b*x +
a) + 105*B*a*b^4*x^5*sgn(b*x + a) + 21*A*b^5*x^5*sgn(b*x + a) + 140*B*a^2*
b^3*x^4*sgn(b*x + a) + 70*A*a*b^4*x^4*sgn(b*x + a) + 105*B*a^3*b^2*x^3*sgn
(b*x + a) + 105*A*a^2*b^3*x^3*sgn(b*x + a) + 42*B*a^4*b*x^2*sgn(b*x + a) +
 84*A*a^3*b^2*x^2*sgn(b*x + a) + 7*B*a^5*x*sgn(b*x + a) + 35*A*a^4*b*x*sgn
(b*x + a) + 6*A*a^5*sgn(b*x + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.79

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = -\frac{\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^6\,\left(a+b\,x\right)}$$
$$-\frac{\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^2\,\left(a+b\,x\right)}-\frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)}$$
$$-\frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x\,\left(a+b\,x\right)}-\frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,\left(a+b\,x\right)}$$
$$-\frac{a^3\,b\,\left(2\,A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)}$$
$$-\frac{5\,a^2\,b^2\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^4\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^8,x)
```

output
```
- (((B*a^5)/6 + (5*A*a^4*b)/6)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^6*(a +
b*x)) - (((A*b^5)/2 + (5*B*a*b^4)/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^2
*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) -
(B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x*(a + b*x)) - (5*a*b^3*(A*b + 2*
B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a + b*x)) - (a^3*b*(2*A*b +
B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a + b*x)) - (5*a^2*b^2*(A*b +
B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^4*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^8}\,dx = \frac{-7b^6x^6-21a\,b^5x^5-35a^2b^4x^4-35a^3b^3x^3-21a^4b^2x^2-7a^5bx-}{7x^7}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^8,x)
```

output
```
( - a**6 - 7*a**5*b*x - 21*a**4*b**2*x**2 - 35*a**3*b**3*x**3 - 35*a**2*b*
*4*x**4 - 21*a*b**5*x**5 - 7*b**6*x**6)/(7*x**7)
```

3.327 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{b(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^3x^6} - \frac{A\left(a^2+2abx+b^2x^2\right)^{7/2}}{8a^2x^8} + \frac{(9Ab-8aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{56a^3x^7}$$

output
```
-1/6*b*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^3/x^6-1/8*A*(b^2*x^
2+2*a*b*x+a^2)^(7/2)/a^2/x^8+1/56*(9*A*b-8*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2
)/a^3/x^7
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{\sqrt{(a+bx)^2}\left(28b^5x^5(2A+3Bx)+70ab^4x^4(3A+4Bx)+84a^2b^3x^3(4A+5Bx)+56a^3b^2x^2(5A+6Bx)+\right.}{168x^8(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^9,x]
```

output
```
-1/168*(Sqrt[(a + b*x)^2]*(28*b^5*x^5*(2*A + 3*B*x) + 70*a*b^4*x^4*(3*A +
4*B*x) + 84*a^2*b^3*x^3*(4*A + 5*B*x) + 56*a^3*b^2*x^2*(5*A + 6*B*x) + 20*
a^4*b*x*(6*A + 7*B*x) + 3*a^5*(7*A + 8*B*x)))/(x^8*(a + b*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^{5/2}(A+Bx)}{x^9}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^9}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^9}dx}{a+bx}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{(Ab-4aB)\int \frac{(a+bx)^5}{x^8}dx}{4a}-\frac{A(a+bx)^6}{8ax^8}\right)}{a+bx}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{(Ab-4aB)\left(-\frac{b\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{A(a+bx)^6}{8ax^8}\right)}{a+bx}$$

$$\downarrow 48$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{\left(\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}\right)(Ab-4aB)}{4a}-\frac{A(a+bx)^6}{8ax^8}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^9,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/8*(A*(a + b*x)^6)/(a*x^8) - ((A*b - 4*a
*B)*(-1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)))/(4*a)))/(a
+ b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B\,b^5x^6}{2}+\left(-\frac{1}{3}A\,b^5-\frac{5}{3}Ba\,b^4\right)x^5+\left(-\frac{5}{4}Aa\,b^4-\frac{5}{2}B\,a^2b^3\right)x^4+\left(-2A\,a^2b^3-2B\,a^3b^2\right)x^3+\left(-\frac{5}{3}a^3A\,b^2-\frac{5}{6}B\,a^4b\right)x^2+\left(-\frac{5}{7}A \cdots\right)\right.}{(bx+a)x^8}$
gosper	$-\frac{(84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210Aa\,b^4x^4+420B\,a^2b^3x^4+336A\,a^2b^3x^3+336B\,a^3b^2x^3+280A\,a^3b^2x^2+140B\,a^4b\,x^2+120A \cdots}{168x^8(bx+a)^5}$
default	$-\frac{(84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210Aa\,b^4x^4+420B\,a^2b^3x^4+336A\,a^2b^3x^3+336B\,a^3b^2x^3+280A\,a^3b^2x^2+140B\,a^4b\,x^2+120A \cdots}{168x^8(bx+a)^5}$
orering	$-\frac{(84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210Aa\,b^4x^4+420B\,a^2b^3x^4+336A\,a^2b^3x^3+336B\,a^3b^2x^3+280A\,a^3b^2x^2+140B\,a^4b\,x^2+120A \cdots}{168x^8(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/2*B*b^5*x^6+(-1/3*A*b^5-5/3*B*a*b^4)*x^5+(-5
/4*A*a*b^4-5/2*B*a^2*b^3)*x^4+(-2*A*a^2*b^3-2*B*a^3*b^2)*x^3+(-5/3*a^3*A*b
^2-5/6*B*a^4*b)*x^2+(-5/7*A*a^4*b-1/7*B*a^5)*x-1/8*A*a^5)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx =$$
$$-\frac{84\,Bb^5x^6+21\,Aa^5+56\,(5\,Bab^4+Ab^5)x^5+210\,(2\,Ba^2b^3+Aab^4)x^4+336\,(Ba^3b^2+Aa^2b^3)x^3+140\,(B}{168\,x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
-1/168*(84*B*b^5*x^6 + 21*A*a^5 + 56*(5*B*a*b^4 + A*b^5)*x^5 + 210*(2*B*a^
2*b^3 + A*a*b^4)*x^4 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^2 + 24*(B*a^5 + 5*A*a^4*b)*x)/x^8
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**9,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^7}{6\,a^7} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^8}{6\,a^8} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^6}{6\,a^6x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^7}{6\,a^7x} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^5}{6\,a^7x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^6}{6\,a^8x^2} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^4}{6\,a^6x^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^5}{6\,a^7x^3} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^3}{6\,a^5x^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^4}{6\,a^6x^4} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^2}{6\,a^4x^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^3}{6\,a^5x^5} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb}{6\,a^3x^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^2}{6\,a^4x^6} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}B}{7\,a^2x^7} + \frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab}{56\,a^3x^7} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{8\,a^2x^8}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^7/a^7 + 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*A*b^8/a^8 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^6/(a^6*x) +
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^7/(a^7*x) + 1/6*(b^2*x^2 + 2*a*b*
x + a^2)^(7/2)*B*b^5/(a^7*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^6
/(a^8*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^4/(a^6*x^3) + 1/6*(b^
2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/(a^7*x^3) + 1/6*(b^2*x^2 + 2*a*b*x + a^
2)^(7/2)*B*b^3/(a^5*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*
x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^4*x^5) + 1/6*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^5) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/
2)*B*b/(a^3*x^6) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^4*x^6) - 1
/7*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^7) + 9/56*(b^2*x^2 + 2*a*b*x +
 a^2)^(7/2)*A*b/(a^3*x^7) - 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^8
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(103) = 206$.

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = \frac{(4\,Bab^7-Ab^8)\mathrm{sgn}(bx+a)}{168\,a^3} - \frac{84\,Bb^5x^6\mathrm{sgn}(bx+a)+280\,Bab^4x^5\mathrm{sgn}(bx+a)+56\,Ab^5x^5\mathrm{sgn}(bx+a)+420\,Ba^2b^3x^4\mathrm{sgn}(bx+a)+210}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/168*(4*B*a*b^7 - A*b^8)*sgn(b*x + a)/a^3 - 1/168*(84*B*b^5*x^6*sgn(b*x +
 a) + 280*B*a*b^4*x^5*sgn(b*x + a) + 56*A*b^5*x^5*sgn(b*x + a) + 420*B*a^2
*b^3*x^4*sgn(b*x + a) + 210*A*a*b^4*x^4*sgn(b*x + a) + 336*B*a^3*b^2*x^3*s
gn(b*x + a) + 336*A*a^2*b^3*x^3*sgn(b*x + a) + 140*B*a^4*b*x^2*sgn(b*x + a
) + 280*A*a^3*b^2*x^2*sgn(b*x + a) + 24*B*a^5*x*sgn(b*x + a) + 120*A*a^4*b
*x*sgn(b*x + a) + 21*A*a^5*sgn(b*x + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.47

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^7\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^3\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^2\,\left(a+b\,x\right)} - \frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,\left(a+b\,x\right)} - \frac{5\,a^3\,b\,\left(2\,A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,\left(a+b\,x\right)} - \frac{2\,a^2\,b^2\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^9,x)
```

output
```
- (((B*a^5)/7 + (5*A*a^4*b)/7)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^7*(a +
b*x)) - (((A*b^5)/3 + (5*B*a*b^4)/3)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^3
*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) -
(B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^2*(a + b*x)) - (5*a*b^3*(A*b
+ 2*B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (5*a^3*b*(2*
A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (2*a^2*b^2
*(A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx = \frac{-28b^6x^6-112a\,b^5x^5-210a^2b^4x^4-224a^3b^3x^3-140a^4b^2x^2-48}{56x^8}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^9,x)
```

output
```
( - 7*a**6 - 48*a**5*b*x - 140*a**4*b**2*x**2 - 224*a**3*b**3*x**3 - 210*a
**2*b**4*x**4 - 112*a*b**5*x**5 - 28*b**6*x**6)/(56*x**8)
```

3.328 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 157

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx=\frac{b^2(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^4x^6}-\frac{A\left(a^2+2abx+b^2x^2\right)^{7/2}}{9a^2x^9}+\frac{(11Ab-9aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{72a^3x^8}-\frac{b(83Ab-81aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{504a^4x^7}$$

output
```
1/6*b^2*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^4/x^6-1/9*A*(b^2*x
^2+2*a*b*x+a^2)^(7/2)/a^2/x^9+1/72*(11*A*b-9*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7
/2)/a^3/x^8-1/504*b*(83*A*b-81*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^4/x^7
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx=-\frac{\sqrt{(a+bx)^2}(42b^5x^5(3A+4Bx)+126ab^4x^4(4A+5Bx)+168a^2b^3x^3(5A+6Bx)+120a^3b^2x^2(6A+7Bx}{504x^9(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^10,x]
```

output
```
-1/504*(Sqrt[(a + b*x)^2]*(42*b^5*x^5*(3*A + 4*B*x) + 126*a*b^4*x^4*(4*A +
 5*B*x) + 168*a^2*b^3*x^3*(5*A + 6*B*x) + 120*a^3*b^2*x^2*(6*A + 7*B*x) +
45*a^4*b*x*(7*A + 8*B*x) + 7*a^5*(8*A + 9*B*x)))/(x^9*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{10}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{10}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{10}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^5}{x^{10}}+\frac{(5Ab+aB)a^4}{x^9}+\frac{5b(2Ab+aB)a^3}{x^8}+\frac{10b^2(Ab+aB)a^2}{x^7}+\frac{5b^3(Ab+2aB)a}{x^6}+\frac{b^5B}{x^4}+\frac{b^4(Ab+5aB)}{x^5}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{3x^3}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^10,x]
```

output
```
((-1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))
/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (a*b^3*(A*b + 2*a*B))/x^5 - (
b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/(3*x^3))*Sqrt[a^2 + 2*a*b*x + b^2*x^2
])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^5 x^6}{3}+(-\frac{1}{4}A b^5-\frac{5}{4}B a b^4)x^5+(-A a b^4-2B a^2b^3)x^4+(-\frac{5}{3}A a^2b^3-\frac{5}{3}B a^3b^2)x^3+(-\frac{10}{7}a^3A b^2-\frac{5}{7}B a^4b)x^2+(-\frac{5}{8}A\right.}{(bx+a)x^9}$
gosper	$-\frac{(168B b^5x^6+126A b^5x^5+630Ba b^4x^5+504Aa b^4x^4+1008B a^2b^3x^4+840A a^2b^3x^3+840B a^3b^2x^3+720A a^3b^2x^2+360B a^4b x^2+3}{504x^9(bx+a)^5}$
default	$-\frac{(168B b^5x^6+126A b^5x^5+630Ba b^4x^5+504Aa b^4x^4+1008B a^2b^3x^4+840A a^2b^3x^3+840B a^3b^2x^3+720A a^3b^2x^2+360B a^4b x^2+3}{504x^9(bx+a)^5}$
orering	$-\frac{(168B b^5x^6+126A b^5x^5+630Ba b^4x^5+504Aa b^4x^4+1008B a^2b^3x^4+840A a^2b^3x^3+840B a^3b^2x^3+720A a^3b^2x^2+360B a^4b x^2+3}{504x^9(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/3*B*b^5*x^6+(-1/4*A*b^5-5/4*B*a*b^4)*x^5+(-A
*a*b^4-2*B*a^2*b^3)*x^4+(-5/3*A*a^2*b^3-5/3*B*a^3*b^2)*x^3+(-10/7*a^3*A*b^
2-5/7*B*a^4*b)*x^2+(-5/8*A*a^4*b-1/8*B*a^5)*x-1/9*A*a^5)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx =$$
$$-\frac{168\,Bb^5x^6+56\,Aa^5+126\,(5\,Bab^4+Ab^5)x^5+504\,(2\,Ba^2b^3+Aab^4)x^4+840\,(Ba^3b^2+Aa^2b^3)x^3+360\,(}{504\,x^9}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
-1/504*(168*B*b^5*x^6 + 56*A*a^5 + 126*(5*B*a*b^4 + A*b^5)*x^5 + 504*(2*B*
a^2*b^3 + A*a*b^4)*x^4 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 360*(B*a^4*b +
2*A*a^3*b^2)*x^2 + 63*(B*a^5 + 5*A*a^4*b)*x)/x^9
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = \int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**10,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(141) = 282.

Time = 0.05 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.54

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx &= \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^8}{6\,a^8} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^9}{6\,a^9} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Bb^7}{6\,a^7x} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{5}{2}}Ab^8}{6\,a^8x} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^6}{6\,a^8x^2} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^7}{6\,a^9x^2} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^5}{6\,a^7x^3} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^6}{6\,a^8x^3} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^4}{6\,a^6x^4} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^5}{6\,a^7x^4} + \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^3}{6\,a^5x^5} \\ &- \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^4}{6\,a^6x^5} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb^2}{6\,a^4x^6} \\ &+ \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^3}{6\,a^5x^6} + \frac{9\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Bb}{56\,a^3x^7} \\ &- \frac{83\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab^2}{504\,a^4x^7} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}B}{8\,a^2x^8} \\ &+ \frac{11\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}Ab}{72\,a^3x^8} - \frac{\left(b^2x^2+2\,abx+a^2\right)^{\frac{7}{2}}A}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^8/a^8 - 1/6*(b^2*x^2 + 2*a*b*x + a
^2)^(5/2)*A*b^9/a^9 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^7/(a^7*x) -
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^8/(a^8*x) - 1/6*(b^2*x^2 + 2*a*b*x
 + a^2)^(7/2)*B*b^6/(a^8*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^7/
(a^9*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^5/(a^7*x^3) - 1/6*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^6/(a^8*x^3) - 1/6*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*B*b^4/(a^6*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/(a^7*x
^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^3/(a^5*x^5) - 1/6*(b^2*x^2 +
 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*x^5) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2
)*B*b^2/(a^4*x^6) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^6) +
9/56*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^7) - 83/504*(b^2*x^2 + 2*a
*b*x + a^2)^(7/2)*A*b^2/(a^4*x^7) - 1/8*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/
(a^2*x^8) + 11/72*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^8) - 1/9*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{(3\,Bab^8-Ab^9)\mathrm{sgn}(bx+a)}{504\,a^4}$$
$$-\frac{168\,Bb^5x^6\mathrm{sgn}(bx+a)+630\,Bab^4x^5\mathrm{sgn}(bx+a)+126\,Ab^5x^5\mathrm{sgn}(bx+a)+1008\,Ba^2b^3x^4\mathrm{sgn}(bx+a)+}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
-1/504*(3*B*a*b^8 - A*b^9)*sgn(b*x + a)/a^4 - 1/504*(168*B*b^5*x^6*sgn(b*x
 + a) + 630*B*a*b^4*x^5*sgn(b*x + a) + 126*A*b^5*x^5*sgn(b*x + a) + 1008*B
*a^2*b^3*x^4*sgn(b*x + a) + 504*A*a*b^4*x^4*sgn(b*x + a) + 840*B*a^3*b^2*x
^3*sgn(b*x + a) + 840*A*a^2*b^3*x^3*sgn(b*x + a) + 360*B*a^4*b*x^2*sgn(b*x
 + a) + 720*A*a^3*b^2*x^2*sgn(b*x + a) + 63*B*a^5*x*sgn(b*x + a) + 315*A*a
^4*b*x*sgn(b*x + a) + 56*A*a^5*sgn(b*x + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^8\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^4\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^3\,\left(a+b\,x\right)} - \frac{a\,b^3\,\left(A\,b+2\,B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)} - \frac{5\,a^3\,b\,\left(2\,A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)} - \frac{5\,a^2\,b^2\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{3\,x^6\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^10,x)
```

output
```
- (((B*a^5)/8 + (5*A*a^4*b)/8)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^8*(a +
b*x)) - (((A*b^5)/4 + (5*B*a*b^4)/4)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^4
*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x)) -
(B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^3*(a + b*x)) - (a*b^3*(A*b +
2*B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a + b*x)) - (5*a^3*b*(2*A*b
+ B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (5*a^2*b^2*(A*
b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(3*x^6*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx = \frac{-84b^6x^6-378a\,b^5x^5-756a^2b^4x^4-840a^3b^3x^3-540a^4b^2x^2-18}{252x^9}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^10,x)
```

output
```
( - 28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756
*a**2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

3.329 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 199

$$\begin{aligned}\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx = &-\frac{b^3(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^5x^6}-\frac{A\left(a^2+2abx+b^2x^2\right)^{7/2}}{10a^2x^{10}}\\&+\frac{(13Ab-10aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{90a^3x^9}-\frac{b(58Ab-55aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{360a^4x^8}\\&+\frac{b^2(418Ab-415aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{2520a^5x^7}\end{aligned}$$

output
```
-1/6*b^3*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^5/x^6-1/10*A*(b^2
*x^2+2*a*b*x+a^2)^(7/2)/a^2/x^10+1/90*(13*A*b-10*B*a)*(b^2*x^2+2*a*b*x+a^2
)^(7/2)/a^3/x^9-1/360*b*(58*A*b-55*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^4/x^
8+1/2520*b^2*(418*A*b-415*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^5/x^7
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(126b^5x^5(4A+5Bx)+420ab^4x^4(5A+6Bx)+600a^2b^3x^3(6A+7Bx)+450a^3b^2x^2(7A+8}{2520x^{10}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^11,x]
```

output
```
-1/2520*(Sqrt[(a + b*x)^2]*(126*b^5*x^5*(4*A + 5*B*x) + 420*a*b^4*x^4*(5*A
 + 6*B*x) + 600*a^2*b^3*x^3*(6*A + 7*B*x) + 450*a^3*b^2*x^2*(7*A + 8*B*x)
+ 175*a^4*b*x*(8*A + 9*B*x) + 28*a^5*(9*A + 10*B*x)))/(x^10*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{11}}dx$$
$$\Big\downarrow \ 1187$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{11}}dx}{b^5(a+bx)}$$
$$\Big\downarrow \ 27$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{11}}dx}{a+bx}$$
$$\Big\downarrow \ 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{11}}+\frac{(5Ab+aB)a^4}{x^{10}}+\frac{5b(2Ab+aB)a^3}{x^9}+\frac{10b^2(Ab+aB)a^2}{x^8}+\frac{5b^3(Ab+2aB)a}{x^7}+\frac{b^5B}{x^5}+\frac{b^4(Ab+5aB)}{x^6}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{6x^6}-\frac{b^5B}{4x^4}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^11,x]
```

output
```
((-1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B
))/(8*x^8) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(6
*x^6) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(4*x^4))*Sqrt[a^2 + 2*a*b*x
+ b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^5 x^6}{4}+(-\frac{1}{5}A b^5-B a b^4)x^5+(-\frac{5}{6}A a b^4-\frac{5}{3}B a^2 b^3)x^4+(-\frac{10}{7}A a^2 b^3-\frac{10}{7}B a^3 b^2)x^3+(-\frac{5}{4}a^3 A b^2-\frac{5}{8}B a^4 b)x^2+(-\frac{5}{9}\right.}{(bx+a)x^{10}}$
gosper	$-\frac{(630B b^5 x^6+504A b^5 x^5+2520B a b^4 x^5+2100A a b^4 x^4+4200B a^2 b^3 x^4+3600A a^2 b^3 x^3+3600B a^3 b^2 x^3+3150A a^3 b^2 x^2+1575B a^4}{2520x^{10}(bx+a)^5}$
default	$-\frac{(630B b^5 x^6+504A b^5 x^5+2520B a b^4 x^5+2100A a b^4 x^4+4200B a^2 b^3 x^4+3600A a^2 b^3 x^3+3600B a^3 b^2 x^3+3150A a^3 b^2 x^2+1575B a^4}{2520x^{10}(bx+a)^5}$
orering	$-\frac{(630B b^5 x^6+504A b^5 x^5+2520B a b^4 x^5+2100A a b^4 x^4+4200B a^2 b^3 x^4+3600A a^2 b^3 x^3+3600B a^3 b^2 x^3+3150A a^3 b^2 x^2+1575B a^4}{2520x^{10}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/4*B*b^5*x^6+(-1/5*A*b^5-B*a*b^4)*x^5+(-5/6*A
*a*b^4-5/3*B*a^2*b^3)*x^4+(-10/7*A*a^2*b^3-10/7*B*a^3*b^2)*x^3+(-5/4*a^3*A
*b^2-5/8*B*a^4*b)*x^2+(-5/9*A*a^4*b-1/9*B*a^5)*x-1/10*A*a^5)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx =$$
$$-\frac{630\,Bb^5x^6+252\,Aa^5+504\,(5\,Bab^4+Ab^5)x^5+2100\,(2\,Ba^2b^3+Aab^4)x^4+3600\,(Ba^3b^2+Aa^2b^3)x^3+1}{2520\,x^{10}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(630*B*b^5*x^6 + 252*A*a^5 + 504*(5*B*a*b^4 + A*b^5)*x^5 + 2100*(2
*B*a^2*b^3 + A*a*b^4)*x^4 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^2 + 280*(B*a^5 + 5*A*a^4*b)*x)/x^10
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**11,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(179) = 358.

Time = 0.06 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.09

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^9/a^9 + 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(5/2)*A*b^10/a^10 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^8/(a^8*x)
 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^9/(a^9*x) + 1/6*(b^2*x^2 + 2*a*
b*x + a^2)^(7/2)*B*b^7/(a^9*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b
^8/(a^10*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^6/(a^8*x^3) + 1/6*
(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^7/(a^9*x^3) + 1/6*(b^2*x^2 + 2*a*b*x +
 a^2)^(7/2)*B*b^5/(a^7*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^6/(a
^8*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^4/(a^6*x^5) + 1/6*(b^2*x
^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/(a^7*x^5) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^
(7/2)*B*b^3/(a^5*x^6) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*x^6
) - 83/504*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^4*x^7) + 209/1260*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^7) + 11/72*(b^2*x^2 + 2*a*b*x + a
^2)^(7/2)*B*b/(a^3*x^8) - 29/180*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^
4*x^8) - 1/9*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^9) + 13/90*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^9) - 1/10*(b^2*x^2 + 2*a*b*x + a^2)^(7/2
)*A/(a^2*x^10)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx = \frac{(5\,Bab^9-2\,Ab^{10})\mathrm{sgn}(bx+a)}{2520\,a^5}$$
$$-\frac{630\,Bb^5x^6\mathrm{sgn}(bx+a)+2520\,Bab^4x^5\mathrm{sgn}(bx+a)+504\,Ab^5x^5\mathrm{sgn}(bx+a)+4200\,Ba^2b^3x^4\mathrm{sgn}(bx+a)+}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/2520*(5*B*a*b^9 - 2*A*b^10)*sgn(b*x + a)/a^5 - 1/2520*(630*B*b^5*x^6*sgn
(b*x + a) + 2520*B*a*b^4*x^5*sgn(b*x + a) + 504*A*b^5*x^5*sgn(b*x + a) + 4
200*B*a^2*b^3*x^4*sgn(b*x + a) + 2100*A*a*b^4*x^4*sgn(b*x + a) + 3600*B*a^
3*b^2*x^3*sgn(b*x + a) + 3600*A*a^2*b^3*x^3*sgn(b*x + a) + 1575*B*a^4*b*x^
2*sgn(b*x + a) + 3150*A*a^3*b^2*x^2*sgn(b*x + a) + 280*B*a^5*x*sgn(b*x + a
) + 1400*A*a^4*b*x*sgn(b*x + a) + 252*A*a^5*sgn(b*x + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx = -\frac{\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^9\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^5\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{10\,x^{10}\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^4\,\left(a+b\,x\right)} - \frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,\left(a+b\,x\right)} - \frac{5\,a^3\,b\,\left(2\,A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,\left(a+b\,x\right)} - \frac{10\,a^2\,b^2\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^11,x)
```

output
```
- (((B*a^5)/9 + (5*A*a^4*b)/9)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^9*(a +
b*x)) - (((A*b^5)/5 + B*a*b^4)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^5*(a +
b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(10*x^10*(a + b*x)) - (B*b
^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^4*(a + b*x)) - (5*a*b^3*(A*b + 2*
B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (5*a^3*b*(2*A*b
+ B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) - (10*a^2*b^2*(A
*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx = \frac{-210b^6x^6-1008a\,b^5x^5-2100a^2b^4x^4-2400a^3b^3x^3-1575a^4b^2x^2}{840x^{10}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^11,x)
```

output
```
( - 84*a**6 - 560*a**5*b*x - 1575*a**4*b**2*x**2 - 2400*a**3*b**3*x**3 - 2
100*a**2*b**4*x**4 - 1008*a*b**5*x**5 - 210*b**6*x**6)/(840*x**10)
```

3.330 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 241

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = \frac{b^4(Ab-aB)(a+bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{6a^6x^6} - \frac{A\left(a^2+2abx+b^2x^2\right)^{7/2}}{11a^2x^{11}} + \frac{(15Ab-11aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{110a^3x^{10}} - \frac{b(155Ab-143aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{990a^4x^9} + \frac{b^2(325Ab-319aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{1980a^5x^8} - \frac{b^3(2305Ab-2299aB)\left(a^2+2abx+b^2x^2\right)^{7/2}}{13860a^6x^7}$$

output

```
1/6*b^4*(A*b-B*a)*(b*x+a)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/a^6/x^6-1/11*A*(b^2*
x^2+2*a*b*x+a^2)^(7/2)/a^2/x^11+1/110*(15*A*b-11*B*a)*(b^2*x^2+2*a*b*x+a^2
)^(7/2)/a^3/x^10-1/990*b*(155*A*b-143*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^4
/x^9+1/1980*b^2*(325*A*b-319*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^5/x^8-1/13
860*b^3*(2305*A*b-2299*B*a)*(b^2*x^2+2*a*b*x+a^2)^(7/2)/a^6/x^7
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.52

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(462b^5x^5(5A+6Bx)+1650ab^4x^4(6A+7Bx)+2475a^2b^3x^3(7A+8Bx)+1925a^3b^2x^2(8A+}{13860x^{11}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^12,x]
```

output
```
-1/13860*(Sqrt[(a + b*x)^2]*(462*b^5*x^5*(5*A + 6*B*x) + 1650*a*b^4*x^4*(6
*A + 7*B*x) + 2475*a^2*b^3*x^3*(7*A + 8*B*x) + 1925*a^3*b^2*x^2*(8*A + 9*B
*x) + 770*a^4*b*x*(9*A + 10*B*x) + 126*a^5*(10*A + 11*B*x)))/(x^11*(a + b*
x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{12}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{12}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{12}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{12}}+\frac{(5Ab+aB)a^4}{x^{11}}+\frac{5b(2Ab+aB)a^3}{x^{10}}+\frac{10b^2(Ab+aB)a^2}{x^9}+\frac{5b^3(Ab+2aB)a}{x^8}+\frac{b^5B}{x^6}+\frac{b^4(Ab+5aB)}{x^7}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{6x^6}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^12,x]
```

output
```
((-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a
*B))/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (5*a*b^3*(A*b + 2*a*B))/(
7*x^7) - (b^4*(A*b + 5*a*B))/(6*x^6) - (b^5*B)/(5*x^5))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^5 x^6}{5}+(-\frac{1}{6}A b^5-\frac{5}{6}B a b^4)x^5+(-\frac{5}{7}A a b^4-\frac{10}{7}B a^2b^3)x^4+(-\frac{5}{4}A a^2b^3-\frac{5}{4}B a^3b^2)x^3+(-\frac{10}{9}a^3A b^2-\frac{5}{9}B a^4b)x^2+(-\right.}{(bx+a)x^{11}}$
gosper	$-\frac{(2772B b^5x^6+2310A b^5x^5+11550Ba b^4x^5+9900Aa b^4x^4+19800B a^2b^3x^4+17325A a^2b^3x^3+17325B a^3b^2x^3+15400A a^3b^2x^2+7}{13860x^{11}(bx+a)^5}$
default	$-\frac{(2772B b^5x^6+2310A b^5x^5+11550Ba b^4x^5+9900Aa b^4x^4+19800B a^2b^3x^4+17325A a^2b^3x^3+17325B a^3b^2x^3+15400A a^3b^2x^2+7}{13860x^{11}(bx+a)^5}$
orering	$-\frac{(2772B b^5x^6+2310A b^5x^5+11550Ba b^4x^5+9900Aa b^4x^4+19800B a^2b^3x^4+17325A a^2b^3x^3+17325B a^3b^2x^3+15400A a^3b^2x^2+7}{13860x^{11}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/5*B*b^5*x^6+(-1/6*A*b^5-5/6*B*a*b^4)*x^5+(-5
/7*A*a*b^4-10/7*B*a^2*b^3)*x^4+(-5/4*A*a^2*b^3-5/4*B*a^3*b^2)*x^3+(-10/9*a
^3*A*b^2-5/9*B*a^4*b)*x^2+(-1/2*A*a^4*b-1/10*B*a^5)*x-1/11*A*a^5)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.49

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = -\frac{2772\,Bb^5x^6+1260\,Aa^5+2310\,(5\,Bab^4+Ab^5)x^5+9900\,(2\,Ba^2b^3+Aab^4)x^4+17325\,(Ba^3b^2+Aa^2b^3)x^3}{13860\,x^{11}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-1/13860*(2772*B*b^5*x^6 + 1260*A*a^5 + 2310*(5*B*a*b^4 + A*b^5)*x^5 + 990
0*(2*B*a^2*b^3 + A*a*b^4)*x^4 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^2 + 1386*(B*a^5 + 5*A*a^4*b)*x)/x^11
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**12,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**12, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(217) = 434$.

Time = 0.06 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^10/a^10 - 1/6*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*A*b^11/a^11 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^9/(a^9*x
) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^10/(a^10*x) - 1/6*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)*B*b^8/(a^10*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)
*A*b^9/(a^11*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^7/(a^9*x^3) -
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^8/(a^10*x^3) - 1/6*(b^2*x^2 + 2*a*
b*x + a^2)^(7/2)*B*b^6/(a^8*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b
^7/(a^9*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^5/(a^7*x^5) - 1/6*(
b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^6/(a^8*x^5) - 1/6*(b^2*x^2 + 2*a*b*x +
a^2)^(7/2)*B*b^4/(a^6*x^6) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/(a^
7*x^6) + 209/1260*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^3/(a^5*x^7) - 461/27
72*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*x^7) - 29/180*(b^2*x^2 + 2*a
*b*x + a^2)^(7/2)*B*b^2/(a^4*x^8) + 65/396*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)
*A*b^3/(a^5*x^8) + 13/90*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^9) - 3
1/198*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^4*x^9) - 1/10*(b^2*x^2 + 2*
a*b*x + a^2)^(7/2)*B/(a^2*x^10) + 3/22*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b
/(a^3*x^10) - 1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^11)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = -\frac{(11\,Bab^{10}-5\,Ab^{11})\mathrm{sgn}(bx+a)}{13860\,a^6} - \frac{2772\,Bb^5x^6\mathrm{sgn}(bx+a)+11550\,Bab^4x^5\mathrm{sgn}(bx+a)+2310\,Ab^5x^5\mathrm{sgn}(bx+a)+19800\,Ba^2b^3x^4\mathrm{sgn}(bx+$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
-1/13860*(11*B*a*b^10 - 5*A*b^11)*sgn(b*x + a)/a^6 - 1/13860*(2772*B*b^5*x
^6*sgn(b*x + a) + 11550*B*a*b^4*x^5*sgn(b*x + a) + 2310*A*b^5*x^5*sgn(b*x
+ a) + 19800*B*a^2*b^3*x^4*sgn(b*x + a) + 9900*A*a*b^4*x^4*sgn(b*x + a) +
17325*B*a^3*b^2*x^3*sgn(b*x + a) + 17325*A*a^2*b^3*x^3*sgn(b*x + a) + 7700
*B*a^4*b*x^2*sgn(b*x + a) + 15400*A*a^3*b^2*x^2*sgn(b*x + a) + 1386*B*a^5*
x*sgn(b*x + a) + 6930*A*a^4*b*x*sgn(b*x + a) + 1260*A*a^5*sgn(b*x + a))/x^
11
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = -\frac{\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^{10}\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^6\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{11\,x^{11}\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{5\,x^5\,\left(a+b\,x\right)} - \frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)} - \frac{5\,a^3\,b\,\left(2\,A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,\left(a+b\,x\right)} - \frac{5\,a^2\,b^2\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{4\,x^8\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^12,x)
```

output
```
- (((B*a^5)/10 + (A*a^4*b)/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^10*(a +
b*x)) - (((A*b^5)/6 + (5*B*a*b^4)/6)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^6
*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(11*x^11*(a + b*x))
- (B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(5*x^5*(a + b*x)) - (5*a*b^3*(A*
b + 2*B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (5*a^3*b*(
2*A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x)) - (5*a^2*b
^2*(A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(4*x^8*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx = \frac{-462b^6x^6-2310a\,b^5x^5-4950a^2b^4x^4-5775a^3b^3x^3-3850a^4b^2x^2}{2310x^{11}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^12,x)
```

output
```
( - 210*a**6 - 1386*a**5*b*x - 3850*a**4*b**2*x**2 - 5775*a**3*b**3*x**3 -
 4950*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 462*b**6*x**6)/(2310*x**11)
```

3.331 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 306

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx = -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{12x^{12}(a+bx)} - \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)} - \frac{a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{2x^{10}(a+bx)} - \frac{10a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{5ab^3(Ab+2aB)\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{b^4(Ab+5aB)\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)} - \frac{b^5B\sqrt{a^2+2abx+b^2x^2}}{6x^6(a+bx)}$$

output
```
-1/12*a^5*A*((b*x+a)^2)^(1/2)/x^12/(b*x+a)-1/11*a^4*(5*A*b+B*a)*((b*x+a)^2
)^(1/2)/x^11/(b*x+a)-1/2*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^10/(b*x+a)-
10/9*a^2*b^2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^9/(b*x+a)-5/8*a*b^3*(A*b+2*B*a)
*((b*x+a)^2)^(1/2)/x^8/(b*x+a)-1/7*b^4*(A*b+5*B*a)*((b*x+a)^2)^(1/2)/x^7/(
b*x+a)-1/6*b^5*B*((b*x+a)^2)^(1/2)/x^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(132b^5x^5(6A+7Bx)+495ab^4x^4(7A+8Bx)+770a^2b^3x^3(8A+9Bx)+616a^3b^2x^2(9A+10}{5544x^{12}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^13,x]
```

output
```
-1/5544*(Sqrt[(a + b*x)^2]*(132*b^5*x^5*(6*A + 7*B*x) + 495*a*b^4*x^4*(7*A
 + 8*B*x) + 770*a^2*b^3*x^3*(8*A + 9*B*x) + 616*a^3*b^2*x^2*(9*A + 10*B*x)
 + 252*a^4*b*x*(10*A + 11*B*x) + 42*a^5*(11*A + 12*B*x)))/(x^12*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{13}}dx$$
$$\Big\downarrow \text{ 1187}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{13}}dx}{b^5(a+bx)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{13}}dx}{a+bx}$$
$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{13}}+\frac{(5Ab+aB)a^4}{x^{12}}+\frac{5b(2Ab+aB)a^3}{x^{11}}+\frac{10b^2(Ab+aB)a^2}{x^{10}}+\frac{5b^3(Ab+2aB)a}{x^9}+\frac{b^5B}{x^7}+\frac{b^4(Ab+5aB)}{x^8}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{12x^{12}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5B}{6x^6}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^13,x]
```

output
```
((-1/12*(a^5*A)/x^12 - (a^4*(5*A*b + a*B))/(11*x^11) - (a^3*b*(2*A*b + a*B
))/(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/(
8*x^8) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(6*x^6))*Sqrt[a^2 + 2*a*b*x
 + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\sqrt{(bx+a)^2}\,\left(-\frac{B\,b^5x^6}{6}+(-\frac{1}{7}A\,b^5-\frac{5}{7}Ba\,b^4)x^5+(-\frac{5}{8}Aa\,b^4-\frac{5}{4}B\,a^2b^3)x^4+(-\frac{10}{9}A\,a^2b^3-\frac{10}{9}B\,a^3b^2)x^3+(-a^3A\,b^2-\frac{1}{2}B\,a^4b)x^2+(-\right.}{(bx+a)x^{12}}$
gosper	$-\frac{(924B\,b^5x^6+792A\,b^5x^5+3960Ba\,b^4x^5+3465Aa\,b^4x^4+6930B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+5544A\,a^3b^2x^2+2772B\,a^4}{5544x^{12}(bx+a)^5}$
default	$-\frac{(924B\,b^5x^6+792A\,b^5x^5+3960Ba\,b^4x^5+3465Aa\,b^4x^4+6930B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+5544A\,a^3b^2x^2+2772B\,a^4}{5544x^{12}(bx+a)^5}$
orering	$-\frac{(924B\,b^5x^6+792A\,b^5x^5+3960Ba\,b^4x^5+3465Aa\,b^4x^4+6930B\,a^2b^3x^4+6160A\,a^2b^3x^3+6160B\,a^3b^2x^3+5544A\,a^3b^2x^2+2772B\,a^4}{5544x^{12}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/6*B*b^5*x^6+(-1/7*A*b^5-5/7*B*a*b^4)*x^5+(-5
/8*A*a*b^4-5/4*B*a^2*b^3)*x^4+(-10/9*A*a^2*b^3-10/9*B*a^3*b^2)*x^3+(-a^3*A
*b^2-1/2*B*a^4*b)*x^2+(-5/11*A*a^4*b-1/11*B*a^5)*x-1/12*A*a^5)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{5/2}}{x^{13}}\,dx = -\frac{924\,Bb^5x^6+462\,Aa^5+792\,(5\,Bab^4+Ab^5)x^5+3465\,(2\,Ba^2b^3+Aab^4)x^4+6160\,(Ba^3b^2+Aa^2b^3)x^3+2}{5544\,x^{12}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

output
```
-1/5544*(924*B*b^5*x^6 + 462*A*a^5 + 792*(5*B*a*b^4 + A*b^5)*x^5 + 3465*(2
*B*a^2*b^3 + A*a*b^4)*x^4 + 6160*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 2772*(B*a^4
*b + 2*A*a^3*b^2)*x^2 + 504*(B*a^5 + 5*A*a^4*b)*x)/x^12
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**13,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**13, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(215) = 430.

Time = 0.06 (sec) , antiderivative size = 735, normalized size of antiderivative = 2.40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

output
```
-1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^11/a^11 + 1/6*(b^2*x^2 + 2*a*b*x
+ a^2)^(5/2)*A*b^12/a^12 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^10/(a^1
0*x) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^11/(a^11*x) + 1/6*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*B*b^9/(a^11*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*A*b^10/(a^12*x^2) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^8/(a^10*x^
3) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^9/(a^11*x^3) + 1/6*(b^2*x^2 +
 2*a*b*x + a^2)^(7/2)*B*b^7/(a^9*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2
)*A*b^8/(a^10*x^4) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^6/(a^8*x^5) +
 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^7/(a^9*x^5) + 1/6*(b^2*x^2 + 2*a*
b*x + a^2)^(7/2)*B*b^5/(a^7*x^6) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b
^6/(a^8*x^6) - 461/2772*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^4/(a^6*x^7) +
923/5544*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^5/(a^7*x^7) + 65/396*(b^2*x^2
 + 2*a*b*x + a^2)^(7/2)*B*b^3/(a^5*x^8) - 131/792*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*A*b^4/(a^6*x^8) - 31/198*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^
4*x^9) + 16/99*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^9) + 3/22*(b^2
*x^2 + 2*a*b*x + a^2)^(7/2)*B*b/(a^3*x^10) - 5/33*(b^2*x^2 + 2*a*b*x + a^2
)^(7/2)*A*b^2/(a^4*x^10) - 1/11*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^1
1) + 17/132*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^11) - 1/12*(b^2*x^2
 + 2*a*b*x + a^2)^(7/2)*A/(a^2*x^12)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx = \frac{(2\,Bab^{11}-Ab^{12})\mathrm{sgn}(bx+a)}{5544\,a^7} - \frac{924\,Bb^5x^6\mathrm{sgn}(bx+a)+3960\,Bab^4x^5\mathrm{sgn}(bx+a)+792\,Ab^5x^5\mathrm{sgn}(bx+a)+6930\,Ba^2b^3x^4\mathrm{sgn}(bx+a)+\cdots}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^13,x, algorithm="giac")
```

output
```
1/5544*(2*B*a*b^11 - A*b^12)*sgn(b*x + a)/a^7 - 1/5544*(924*B*b^5*x^6*sgn(
b*x + a) + 3960*B*a*b^4*x^5*sgn(b*x + a) + 792*A*b^5*x^5*sgn(b*x + a) + 69
30*B*a^2*b^3*x^4*sgn(b*x + a) + 3465*A*a*b^4*x^4*sgn(b*x + a) + 6160*B*a^3
*b^2*x^3*sgn(b*x + a) + 6160*A*a^2*b^3*x^3*sgn(b*x + a) + 2772*B*a^4*b*x^2
*sgn(b*x + a) + 5544*A*a^3*b^2*x^2*sgn(b*x + a) + 504*B*a^5*x*sgn(b*x + a)
 + 2520*A*a^4*b*x*sgn(b*x + a) + 462*A*a^5*sgn(b*x + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx = -\frac{\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^{11}\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^7\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{12\,x^{12}\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{6\,x^6\,\left(a+b\,x\right)} - \frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{8\,x^8\,\left(a+b\,x\right)} - \frac{a^3\,b\,\left(2\,A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{2\,x^{10}\,\left(a+b\,x\right)} - \frac{10\,a^2\,b^2\,\left(A\,b+B\,a\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^13,x)
```

output
```
- (((B*a^5)/11 + (5*A*a^4*b)/11)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^11*(a
 + b*x)) - (((A*b^5)/7 + (5*B*a*b^4)/7)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(
x^7*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(12*x^12*(a + b*x
)) - (B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(6*x^6*(a + b*x)) - (5*a*b^3*
(A*b + 2*B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(8*x^8*(a + b*x)) - (a^3*b*
(2*A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(2*x^10*(a + b*x)) - (10*a^
2*b^2*(A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx = \frac{-924b^6x^6-4752a\,b^5x^5-10395a^2b^4x^4-12320a^3b^3x^3-8316a^4b^2}{5544x^{12}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^13,x)
```

output
```
( - 462*a**6 - 3024*a**5*b*x - 8316*a**4*b**2*x**2 - 12320*a**3*b**3*x**3
- 10395*a**2*b**4*x**4 - 4752*a*b**5*x**5 - 924*b**6*x**6)/(5544*x**12)
```

3.332 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 304

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx = -\frac{a^5A\sqrt{a^2+2abx+b^2x^2}}{13x^{13}(a+bx)} - \frac{a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{12x^{12}(a+bx)} - \frac{5a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{11x^{11}(a+bx)} - \frac{a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x^{10}(a+bx)} - \frac{5ab^3(Ab+2aB)\sqrt{a^2+2abx+b^2x^2}}{9x^9(a+bx)} - \frac{b^4(Ab+5aB)\sqrt{a^2+2abx+b^2x^2}}{8x^8(a+bx)} - \frac{b^5B\sqrt{a^2+2abx+b^2x^2}}{7x^7(a+bx)}$$

output
```
-1/13*a^5*A*((b*x+a)^2)^(1/2)/x^13/(b*x+a)-1/12*a^4*(5*A*b+B*a)*((b*x+a)^2
)^(1/2)/x^12/(b*x+a)-5/11*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^11/(b*x+a)
-a^2*b^2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^10/(b*x+a)-5/9*a*b^3*(A*b+2*B*a)*((
b*x+a)^2)^(1/2)/x^9/(b*x+a)-1/8*b^4*(A*b+5*B*a)*((b*x+a)^2)^(1/2)/x^8/(b*x
+a)-1/7*b^5*B*((b*x+a)^2)^(1/2)/x^7/(b*x+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx =$$
$$-\frac{\sqrt{(a+bx)^2}(1287b^5x^5(7A+8Bx)+5005ab^4x^4(8A+9Bx)+8008a^2b^3x^3(9A+10Bx)+6552a^3b^2x^2(10}{72072x^{13}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^14,x]
```

output
```
-1/72072*(Sqrt[(a + b*x)^2]*(1287*b^5*x^5*(7*A + 8*B*x) + 5005*a*b^4*x^4*(
8*A + 9*B*x) + 8008*a^2*b^3*x^3*(9*A + 10*B*x) + 6552*a^3*b^2*x^2*(10*A +
11*B*x) + 2730*a^4*b*x*(11*A + 12*B*x) + 462*a^5*(12*A + 13*B*x)))/(x^13*(
a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{14}}\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{14}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{14}}dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{14}}+\frac{(5Ab+aB)a^4}{x^{13}}+\frac{5b(2Ab+aB)a^3}{x^{12}}+\frac{10b^2(Ab+aB)a^2}{x^{11}}+\frac{5b^3(Ab+2aB)a}{x^{10}}+\frac{b^5B}{x^8}+\frac{b^4(Ab+5aB)}{x^9}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{7x^7}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^14,x]
```

output
```
((-1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a
*B))/(11*x^11) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(9*x
^9) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/(7*x^7))*Sqrt[a^2 + 2*a*b*x +
b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.45

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{B b^5 x^6}{7}+(-\frac{1}{8}A b^5-\frac{5}{8}B a b^4)x^5+(-\frac{5}{9}A a b^4-\frac{10}{9}B a^2b^3)x^4+(-A a^2b^3-B a^3b^2)x^3+(-\frac{10}{11}a^3 A b^2-\frac{5}{11}B a^4 b)x^2+(-\frac{5}{1}\right.}{(bx+a)x^{13}}$
gosper	$-\frac{(10296B b^5x^6+9009A b^5x^5+45045B a b^4x^5+40040A a b^4x^4+80080B a^2b^3x^4+72072A a^2b^3x^3+72072B a^3b^2x^3+65520A a^3b^2x^2}{72072x^{13}(bx+a)^5}$
default	$-\frac{(10296B b^5x^6+9009A b^5x^5+45045B a b^4x^5+40040A a b^4x^4+80080B a^2b^3x^4+72072A a^2b^3x^3+72072B a^3b^2x^3+65520A a^3b^2x^2}{72072x^{13}(bx+a)^5}$
orering	$-\frac{(10296B b^5x^6+9009A b^5x^5+45045B a b^4x^5+40040A a b^4x^4+80080B a^2b^3x^4+72072A a^2b^3x^3+72072B a^3b^2x^3+65520A a^3b^2x^2}{72072x^{13}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(-1/7*B*b^5*x^6+(-1/8*A*b^5-5/8*B*a*b^4)*x^5+(-5
/9*A*a*b^4-10/9*B*a^2*b^3)*x^4+(-A*a^2*b^3-B*a^3*b^2)*x^3+(-10/11*a^3*A*b^
2-5/11*B*a^4*b)*x^2+(-5/12*A*a^4*b-1/12*B*a^5)*x-1/13*A*a^5)/x^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx = -\frac{10296\,Bb^5x^6+5544\,Aa^5+9009\,(5\,Bab^4+Ab^5)x^5+40040\,(2\,Ba^2b^3+Aab^4)x^4+72072\,(Ba^3b^2+Aa^2b^3}{72072\,x^{13}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

output
```
-1/72072*(10296*B*b^5*x^6 + 5544*A*a^5 + 9009*(5*B*a*b^4 + A*b^5)*x^5 + 40
040*(2*B*a^2*b^3 + A*a*b^4)*x^4 + 72072*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 3276
0*(B*a^4*b + 2*A*a^3*b^2)*x^2 + 6006*(B*a^5 + 5*A*a^4*b)*x)/x^13
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**14,x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**14, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(215) = 430$.

Time = 0.06 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

output

```
1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^12/a^12 - 1/6*(b^2*x^2 + 2*a*b*x +
 a^2)^(5/2)*A*b^13/a^13 + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*B*b^11/(a^11
*x) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(5/2)*A*b^12/(a^12*x) - 1/6*(b^2*x^2 +
 2*a*b*x + a^2)^(7/2)*B*b^10/(a^12*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*A*b^11/(a^13*x^2) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^9/(a^11*x^
3) - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^10/(a^12*x^3) - 1/6*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*B*b^8/(a^10*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7
/2)*A*b^9/(a^11*x^4) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^7/(a^9*x^5)
 - 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^8/(a^10*x^5) - 1/6*(b^2*x^2 + 2
*a*b*x + a^2)^(7/2)*B*b^6/(a^8*x^6) + 1/6*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*
A*b^7/(a^9*x^6) + 923/5544*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^5/(a^7*x^7)
 - 1715/10296*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^6/(a^8*x^7) - 131/792*(b
^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^4/(a^6*x^8) + 1709/10296*(b^2*x^2 + 2*a*
b*x + a^2)^(7/2)*A*b^5/(a^7*x^8) + 16/99*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B
*b^3/(a^5*x^9) - 211/1287*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^4/(a^6*x^9)
- 5/33*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B*b^2/(a^4*x^10) + 68/429*(b^2*x^2
+ 2*a*b*x + a^2)^(7/2)*A*b^3/(a^5*x^10) + 17/132*(b^2*x^2 + 2*a*b*x + a^2)
^(7/2)*B*b/(a^3*x^11) - 251/1716*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*A*b^2/(a^
4*x^11) - 1/12*(b^2*x^2 + 2*a*b*x + a^2)^(7/2)*B/(a^2*x^12) + 19/156*(b^2*
x^2 + 2*a*b*x + a^2)^(7/2)*A*b/(a^3*x^12) - 1/13*(b^2*x^2 + 2*a*b*x + a...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx = -\frac{(13\,Bab^{12}-7\,Ab^{13})\mathrm{sgn}(bx+a)}{72072\,a^8}$$
$$-\frac{10296\,Bb^5x^6\mathrm{sgn}(bx+a)+45045\,Bab^4x^5\mathrm{sgn}(bx+a)+9009\,Ab^5x^5\mathrm{sgn}(bx+a)+80080\,Ba^2b^3x^4\mathrm{sgn}(bx\cdots}{}$$

input

```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^14,x, algorithm="giac")
```

output
```
-1/72072*(13*B*a*b^12 - 7*A*b^13)*sgn(b*x + a)/a^8 - 1/72072*(10296*B*b^5*
x^6*sgn(b*x + a) + 45045*B*a*b^4*x^5*sgn(b*x + a) + 9009*A*b^5*x^5*sgn(b*x
 + a) + 80080*B*a^2*b^3*x^4*sgn(b*x + a) + 40040*A*a*b^4*x^4*sgn(b*x + a)
+ 72072*B*a^3*b^2*x^3*sgn(b*x + a) + 72072*A*a^2*b^3*x^3*sgn(b*x + a) + 32
760*B*a^4*b*x^2*sgn(b*x + a) + 65520*A*a^3*b^2*x^2*sgn(b*x + a) + 6006*B*a
^5*x*sgn(b*x + a) + 30030*A*a^4*b*x*sgn(b*x + a) + 5544*A*a^5*sgn(b*x + a)
)/x^13
```

Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx = -\frac{\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^{12}\,\left(a+b\,x\right)} - \frac{\left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^8\,\left(a+b\,x\right)} - \frac{A\,a^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{13\,x^{13}\,\left(a+b\,x\right)} - \frac{B\,b^5\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{7\,x^7\,\left(a+b\,x\right)} - \frac{5\,a\,b^3\,\left(A\,b+2\,B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{9\,x^9\,\left(a+b\,x\right)} - \frac{5\,a^3\,b\,\left(2\,A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{11\,x^{11}\,\left(a+b\,x\right)} - \frac{a^2\,b^2\,\left(A\,b+B\,a\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x^{10}\,\left(a+b\,x\right)}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^14,x)
```

output
```
- (((B*a^5)/12 + (5*A*a^4*b)/12)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^12*(a
 + b*x)) - (((A*b^5)/8 + (5*B*a*b^4)/8)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(
x^8*(a + b*x)) - (A*a^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(13*x^13*(a + b*x
)) - (B*b^5*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(7*x^7*(a + b*x)) - (5*a*b^3*
(A*b + 2*B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(9*x^9*(a + b*x)) - (5*a^3*
b*(2*A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(11*x^11*(a + b*x)) - (a^
2*b^2*(A*b + B*a)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(x^10*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{14}}\,dx = \frac{-1716b^6x^6-9009a\,b^5x^5-20020a^2b^4x^4-24024a^3b^3x^3-16380a}{12012x^{13}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^14,x)
```

output
```
( - 924*a**6 - 6006*a**5*b*x - 16380*a**4*b**2*x**2 - 24024*a**3*b**3*x**3
 - 20020*a**2*b**4*x**4 - 9009*a*b**5*x**5 - 1716*b**6*x**6)/(12012*x**13)
```

3.333 $\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 258

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a^3(Ab-aB)x(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2(Ab-aB)x^2(a+bx)}{2b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{a(Ab-aB)x^3(a+bx)}{3b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-aB)x^4(a+bx)}{4b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx^5(a+bx)}{5b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^4(Ab-aB)(a+bx)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-a^3*(A*b-B*a)*x*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+1/2*a^2*(A*b-B*a)*x^2*(b*x+
a)/b^4/((b*x+a)^2)^(1/2)-1/3*a*(A*b-B*a)*x^3*(b*x+a)/b^3/((b*x+a)^2)^(1/2)
+1/4*(A*b-B*a)*x^4*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/5*B*x^5*(b*x+a)/b/((b*x
+a)^2)^(1/2)+a^4*(A*b-B*a)*(b*x+a)*ln(b*x+a)/b^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.45

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{(a+bx)\,(bx(60a^4B-30a^3b(2A+Bx)+10a^2b^2x(3A+2Bx)-5ab^3x^2(4A+3Bx)+3b^4x^3(5A+4Bx)}{60b^6\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^4*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(b*x*(60*a^4*B - 30*a^3*b*(2*A + B*x) + 10*a^2*b^2*x*(3*A + 2*B
*x) - 5*a*b^3*x^2*(4*A + 3*B*x) + 3*b^4*x^3*(5*A + 4*B*x)) - 60*a^4*(-(A*b
) + a*B)*Log[a + b*x]))/(60*b^6*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \; 1187$$

$$\frac{b(a+bx)\int \frac{x^4(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\int \frac{x^4(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 86$$

$$\frac{(a+bx)\int\left(-\frac{(aB-Ab)a^4}{b^5(a+bx)}+\frac{(aB-Ab)a^3}{b^5}-\frac{(aB-Ab)xa^2}{b^4}+\frac{(aB-Ab)x^2a}{b^3}+\frac{Bx^4}{b}+\frac{(Ab-aB)x^3}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)\left(\frac{a^4(Ab-aB)\log(a+bx)}{b^6}-\frac{a^3x(Ab-aB)}{b^5}+\frac{a^2x^2(Ab-aB)}{2b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^5}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^4*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(-((a^3*(A*b - a*B)*x)/b^5) + (a^2*(A*b - a*B)*x^2)/(2*b^4) - (
a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^5)/(5*b) + (
a^4*(A*b - a*B)*Log[a + b*x])/b^6))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

method	result
default	$\frac{(bx+a)(12B\,x^5b^5+15A\,b^5x^4-15Ba\,b^4x^4-20Aa\,b^4x^3+20B\,a^2b^3x^3+30A\,a^2b^3x^2-30B\,a^3b^2x^2+60A\ln(bx+a)a^4b-60A\,a^3b^2x-60B}{60\sqrt{(bx+a)^2}\,b^6}$
risch	$\frac{\sqrt{(bx+a)^2}\,(\frac{1}{5}b^4B\,x^5+\frac{1}{4}A\,b^4x^4-\frac{1}{4}Ba\,b^3x^4-\frac{1}{3}Aa\,b^3x^3+\frac{1}{3}B\,a^2b^2x^3+\frac{1}{2}A\,a^2b^2x^2-\frac{1}{2}B\,a^3b\,x^2-A\,a^3bx+a^4Bx)}{(bx+a)b^5}+\frac{\sqrt{(bx+a)^2}\,a^4(A}{(bx+}$

input
```
int(x^4*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(b*x+a)*(12*B*x^5*b^5+15*A*b^5*x^4-15*B*a*b^4*x^4-20*A*a*b^4*x^3+20*B
*a^2*b^3*x^3+30*A*a^2*b^3*x^2-30*B*a^3*b^2*x^2+60*A*ln(b*x+a)*a^4*b-60*A*a
^3*b^2*x-60*B*ln(b*x+a)*a^5+60*B*a^4*b*x)/((b*x+a)^2)^(1/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.45

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{12\,Bb^5x^5-15\,(Bab^4-Ab^5)x^4+20\,(Ba^2b^3-Aab^4)x^3-30\,(Ba^3b^2-Aa^2b^3)x^2+60\,(Ba^4b-Aa^3b^2)x-}{60\,b^6}$$

input
```
integrate(x^4*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/60*(12*B*b^5*x^5 - 15*(B*a*b^4 - A*b^5)*x^4 + 20*(B*a^2*b^3 - A*a*b^4)*x
^3 - 30*(B*a^3*b^2 - A*a^2*b^3)*x^2 + 60*(B*a^4*b - A*a^3*b^2)*x - 60*(B*a
^5 - A*a^4*b)*log(b*x + a))/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(187) = 374$.

Time = 1.59 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.81

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*x**4/(5*b**2) + x**3*(A - 9
*B*a/(5*b))/(4*b**2) + x**2*(-4*B*a**2/(5*b**2) - 7*a*(A - 9*B*a/(5*b))/(4
*b))/(3*b**2) + x*(-3*a**2*(A - 9*B*a/(5*b))/(4*b**2) - 5*a*(-4*B*a**2/(5*
b**2) - 7*a*(A - 9*B*a/(5*b))/(4*b))/(3*b))/(2*b**2) + (-2*a**2*(-4*B*a**2
/(5*b**2) - 7*a*(A - 9*B*a/(5*b))/(4*b))/(3*b**2) - 3*a*(-3*a**2*(A - 9*B*
a/(5*b))/(4*b**2) - 5*a*(-4*B*a**2/(5*b**2) - 7*a*(A - 9*B*a/(5*b))/(4*b))
/(3*b))/(2*b))/b**2) + (a/b + x)*(-a**2*(-3*a**2*(A - 9*B*a/(5*b))/(4*b**2
) - 5*a*(-4*B*a**2/(5*b**2) - 7*a*(A - 9*B*a/(5*b))/(4*b))/(3*b))/(2*b**2)
 - a*(-2*a**2*(-4*B*a**2/(5*b**2) - 7*a*(A - 9*B*a/(5*b))/(4*b))/(3*b**2)
- 3*a*(-3*a**2*(A - 9*B*a/(5*b))/(4*b**2) - 5*a*(-4*B*a**2/(5*b**2) - 7*a*
(A - 9*B*a/(5*b))/(4*b))/(3*b))/(2*b))/b)*log(a/b + x)/sqrt(b**2*(a/b + x)
**2), Ne(b**2, 0)), ((A*(a**8*sqrt(a**2 + 2*a*b*x) - 4*a**6*(a**2 + 2*a*b*
x)**(3/2)/3 + 6*a**4*(a**2 + 2*a*b*x)**(5/2)/5 - 4*a**2*(a**2 + 2*a*b*x)**
(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*a**4*b**4) + B*(-a**10*sqrt(a**2 +
 2*a*b*x) + 5*a**8*(a**2 + 2*a*b*x)**(3/2)/3 - 2*a**6*(a**2 + 2*a*b*x)**(5
/2) + 10*a**4*(a**2 + 2*a*b*x)**(7/2)/7 - 5*a**2*(a**2 + 2*a*b*x)**(9/2)/9
 + (a**2 + 2*a*b*x)**(11/2)/11)/(16*a**5*b**5))/(2*a*b), Ne(a*b, 0)), ((A*
x**5/5 + B*x**6/6)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}Bx^4}{5\,b^2} - \frac{9\sqrt{b^2x^2+2\,abx+a^2}Bax^3}{20\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ax^3}{4\,b^2} - \frac{77\,Ba^3x^2}{60\,b^4} + \frac{13\,Aa^2x^2}{12\,b^3} + \frac{47\sqrt{b^2x^2+2\,abx+a^2}Ba^2x^2}{60\,b^4} - \frac{7\sqrt{b^2x^2+2\,abx+a^2}Aax^2}{12\,b^3} + \frac{77\,Ba^4x}{30\,b^5} - \frac{13\,Aa^3x}{6\,b^4} - \frac{Ba^5\log\left(x+\frac{a}{b}\right)}{b^6} + \frac{Aa^4\log\left(x+\frac{a}{b}\right)}{b^5} - \frac{47\sqrt{b^2x^2+2\,abx+a^2}Ba^4}{30\,b^6} + \frac{7\sqrt{b^2x^2+2\,abx+a^2}Aa^3}{6\,b^5}$$

input
```
integrate(x^4*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*x^4/b^2 - 9/20*sqrt(b^2*x^2 + 2*a*b*x
+ a^2)*B*a*x^3/b^3 + 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*x^3/b^2 - 77/60*B
*a^3*x^2/b^4 + 13/12*A*a^2*x^2/b^3 + 47/60*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B
*a^2*x^2/b^4 - 7/12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a*x^2/b^3 + 77/30*B*a^
4*x/b^5 - 13/6*A*a^3*x/b^4 - B*a^5*log(x + a/b)/b^6 + A*a^4*log(x + a/b)/b
^5 - 47/30*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^4/b^6 + 7/6*sqrt(b^2*x^2 + 2*
a*b*x + a^2)*A*a^3/b^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.72

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{12\,Bb^4x^5\mathrm{sgn}(bx+a) - 15\,Bab^3x^4\mathrm{sgn}(bx+a) + 15\,Ab^4x^4\mathrm{sgn}(bx+a) + 20\,Ba^2b^2x^3\mathrm{sgn}(bx+a) - 20\,Aa}{} - \frac{(Ba^5\mathrm{sgn}(bx+a) - Aa^4b\mathrm{sgn}(bx+a))\log(|bx+a|)}{b^6}$$

input
```
integrate(x^4*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/60*(12*B*b^4*x^5*sgn(b*x + a) - 15*B*a*b^3*x^4*sgn(b*x + a) + 15*A*b^4*x
^4*sgn(b*x + a) + 20*B*a^2*b^2*x^3*sgn(b*x + a) - 20*A*a*b^3*x^3*sgn(b*x +
 a) - 30*B*a^3*b*x^2*sgn(b*x + a) + 30*A*a^2*b^2*x^2*sgn(b*x + a) + 60*B*a
^4*x*sgn(b*x + a) - 60*A*a^3*b*x*sgn(b*x + a))/b^5 - (B*a^5*sgn(b*x + a) -
 A*a^4*b*sgn(b*x + a))*log(abs(b*x + a))/b^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x^4\,(A+B\,x)}{\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((x^4*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{x^4(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{x^5}{5}$$

input
```
int(x^4*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
x**5/5
```

3.334 $\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 212

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{a^2(Ab-aB)x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{a(Ab-aB)x^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-aB)x^3(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx^4(a+bx)}{4b\sqrt{a^2+2abx+b^2x^2}} - \frac{a^3(Ab-aB)(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output

```
a^2*(A*b-B*a)*x*(b*x+a)/b^4/((b*x+a)^2)^(1/2)-1/2*a*(A*b-B*a)*x^2*(b*x+a)/
b^3/((b*x+a)^2)^(1/2)+1/3*(A*b-B*a)*x^3*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/4*
B*x^4*(b*x+a)/b/((b*x+a)^2)^(1/2)-a^3*(A*b-B*a)*(b*x+a)*ln(b*x+a)/b^5/((b*
x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.45

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$=\frac{(a+bx)\left(bx(-12a^3B+6a^2b(2A+Bx)-2ab^2x(3A+2Bx)+b^3x^2(4A+3Bx))+12a^3(-Ab+aB)\log\right.}{12b^5\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^3*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(b*x*(-12*a^3*B + 6*a^2*b*(2*A + B*x) - 2*a*b^2*x*(3*A + 2*B*x)
 + b^3*x^2*(4*A + 3*B*x)) + 12*a^3*(-(A*b) + a*B)*Log[a + b*x]))/(12*b^5*S
qrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$\Big\downarrow \; 1187$$
$$\frac{b(a+bx)\int \frac{x^3(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \; 27$$
$$\frac{(a+bx)\int \frac{x^3(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \; 86$$

$$\frac{(a+bx)\int \left(\frac{(aB-Ab)a^3}{b^4(a+bx)}-\frac{(aB-Ab)a^2}{b^4}+\frac{(aB-Ab)xa}{b^3}+\frac{Bx^3}{b}+\frac{(Ab-aB)x^2}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^3(Ab-aB)\log(a+bx)}{b^5}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^4}{4b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^3*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b
- a*B)*x^3)/(3*b^2) + (B*x^4)/(4*b) - (a^3*(A*b - a*B)*Log[a + b*x])/b^5))
/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.54

method	result
default	$-\frac{(bx+a)\left(-3b^4 B\,x^4-4A\,b^4 x^3+4Ba\,b^3 x^3+6Aa\,b^3 x^2-6B\,a^2 b^2 x^2+12A\ln(bx+a)a^3 b-12A\,a^2 b^2 x-12B\ln(bx+a)a^4+12B\,a^3 bx\right)}{12\sqrt{(bx+a)^2}\,b^5}$
risch	$\frac{\sqrt{(bx+a)^2}\,\left(\frac{1}{4}B\,b^3 x^4+\frac{1}{3}A\,b^3 x^3-\frac{1}{3}Ba\,b^2 x^3-\frac{1}{2}Aa\,b^2 x^2+\frac{1}{2}B\,a^2 b\,x^2+A\,a^2 bx-B\,a^3 x\right)}{(bx+a)b^4}-\frac{\sqrt{(bx+a)^2}\,a^3(Ab-Ba)\ln(bx+a)}{(bx+a)b^5}$

input
```
int(x^3*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)*(-3*b^4*B*x^4-4*A*b^4*x^3+4*B*a*b^3*x^3+6*A*a*b^3*x^2-6*B*a^
2*b^2*x^2+12*A*ln(b*x+a)*a^3*b-12*A*a^2*b^2*x-12*B*ln(b*x+a)*a^4+12*B*a^3*
b*x)/((b*x+a)^2)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.44

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{3\,Bb^4x^4-4\,(Bab^3-Ab^4)x^3+6\,(Ba^2b^2-Aab^3)x^2-12\,(Ba^3b-Aa^2b^2)x+12\,(Ba^4-Aa^3b)\log{(bx+}}{12\,b^5}$$

input
```
integrate(x^3*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*B*b^4*x^4 - 4*(B*a*b^3 - A*b^4)*x^3 + 6*(B*a^2*b^2 - A*a*b^3)*x^2
- 12*(B*a^3*b - A*a^2*b^2)*x + 12*(B*a^4 - A*a^3*b)*log(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(153) = 306$.

Time = 1.49 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.30

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$

$$= \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(\frac{Bx^3}{4b^2}+\frac{x^2\left(A-\frac{7Ba}{4b}\right)}{3b^2}+\frac{x\left(-\frac{3Ba^2}{4b^2}-\frac{5a\left(A-\frac{7Ba}{4b}\right)}{3b}\right)}{2b^2}+\frac{-\frac{2a^2\left(A-\frac{7Ba}{4b}\right)}{3b^2}-\frac{3a\left(-\frac{3Ba^2}{4b^2}-\frac{5a\left(A-\frac{7Ba}{4b}\right)}{3b}\right)}{2b}}{b^2}\right)+ \\ \frac{\frac{A\left(-a^6\sqrt{a^2+2abx}+a^4\left(a^2+2abx\right)^{\frac{3}{2}}-\frac{3a^2\left(a^2+2abx\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2abx\right)^{\frac{7}{2}}}{7}\right)}{4a^3b^3}+\frac{B\left(a^8\sqrt{a^2+2abx}-\frac{4a^6\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{6a^4\left(a^2+2abx\right)^{\frac{5}{2}}}{5}-\frac{4a^2\left(a^2+2abx\right)^{\frac{7}{2}}}{7}+\right.}{8a^4b^4}}{2ab} \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{\sqrt{a^2}} \end{cases}$$

input

```
integrate(x**3*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output

```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*x**3/(4*b**2) + x**2*(A - 7
*B*a/(4*b))/(3*b**2) + x*(-3*B*a**2/(4*b**2) - 5*a*(A - 7*B*a/(4*b))/(3*b)
)/(2*b**2) + (-2*a**2*(A - 7*B*a/(4*b))/(3*b**2) - 3*a*(-3*B*a**2/(4*b**2)
 - 5*a*(A - 7*B*a/(4*b))/(3*b))/(2*b))/b**2) + (a/b + x)*(-a**2*(-3*B*a**2
/(4*b**2) - 5*a*(A - 7*B*a/(4*b))/(3*b))/(2*b**2) - a*(-2*a**2*(A - 7*B*a/
(4*b))/(3*b**2) - 3*a*(-3*B*a**2/(4*b**2) - 5*a*(A - 7*B*a/(4*b))/(3*b))/(
2*b))/b)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((A*(-a**6*sq
rt(a**2 + 2*a*b*x) + a**4*(a**2 + 2*a*b*x)**(3/2) - 3*a**2*(a**2 + 2*a*b*x
)**(5/2)/5 + (a**2 + 2*a*b*x)**(7/2)/7)/(4*a**3*b**3) + B*(a**8*sqrt(a**2
+ 2*a*b*x) - 4*a**6*(a**2 + 2*a*b*x)**(3/2)/3 + 6*a**4*(a**2 + 2*a*b*x)**(
5/2)/5 - 4*a**2*(a**2 + 2*a*b*x)**(7/2)/7 + (a**2 + 2*a*b*x)**(9/2)/9)/(8*
a**4*b**4))/(2*a*b), Ne(a*b, 0)), ((A*x**4/4 + B*x**5/5)/sqrt(a**2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\sqrt{b^2x^2+2\,abx+a^2}Bx^3}{4\,b^2} + \frac{13\,Ba^2x^2}{12\,b^3} - \frac{5\,Aax^2}{6\,b^2} - \frac{7\,\sqrt{b^2x^2+2\,abx+a^2}Bax^2}{12\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Ax^2}{3\,b^2} - \frac{13\,Ba^3x}{6\,b^4} + \frac{5\,Aa^2x}{3\,b^3} + \frac{Ba^4\log\left(x+\frac{a}{b}\right)}{b^5} - \frac{Aa^3\log\left(x+\frac{a}{b}\right)}{b^4} + \frac{7\,\sqrt{b^2x^2+2\,abx+a^2}Ba^3}{6\,b^5} - \frac{2\,\sqrt{b^2x^2+2\,abx+a^2}Aa^2}{3\,b^4}$$

input
```
integrate(x^3*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*x^3/b^2 + 13/12*B*a^2*x^2/b^3 - 5/6*A*
a*x^2/b^2 - 7/12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a*x^2/b^3 + 1/3*sqrt(b^2*
x^2 + 2*a*b*x + a^2)*A*x^2/b^2 - 13/6*B*a^3*x/b^4 + 5/3*A*a^2*x/b^3 + B*a^
4*log(x + a/b)/b^5 - A*a^3*log(x + a/b)/b^4 + 7/6*sqrt(b^2*x^2 + 2*a*b*x +
 a^2)*B*a^3/b^5 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{3\,Bb^3x^4\mathrm{sgn}(bx+a) - 4\,Bab^2x^3\mathrm{sgn}(bx+a) + 4\,Ab^3x^3\mathrm{sgn}(bx+a) + 6\,Ba^2bx^2\mathrm{sgn}(bx+a) - 6\,Aab^2x^2\mathrm{sgn}}{12\,b^4} + \frac{(Ba^4\mathrm{sgn}(bx+a) - Aa^3b\mathrm{sgn}(bx+a))\log(|bx+a|)}{b^5}$$

input
```
integrate(x^3*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*B*b^3*x^4*sgn(b*x + a) - 4*B*a*b^2*x^3*sgn(b*x + a) + 4*A*b^3*x^3*
sgn(b*x + a) + 6*B*a^2*b*x^2*sgn(b*x + a) - 6*A*a*b^2*x^2*sgn(b*x + a) - 1
2*B*a^3*x*sgn(b*x + a) + 12*A*a^2*b*x*sgn(b*x + a))/b^4 + (B*a^4*sgn(b*x +
 a) - A*a^3*b*sgn(b*x + a))*log(abs(b*x + a))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x^3\,(A+B\,x)}{\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((x^3*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^3*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{x^3(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{x^4}{4}$$

input
```
int(x^3*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
x**4/4
```

3.335 $\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 166

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{a(Ab-aB)x(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-aB)x^2(a+bx)}{2b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{Bx^3(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}}+\frac{a^2(Ab-aB)(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-a*(A*b-B*a)*x*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+1/2*(A*b-B*a)*x^2*(b*x+a)/b^2
/((b*x+a)^2)^(1/2)+1/3*B*x^3*(b*x+a)/b/((b*x+a)^2)^(1/2)+a^2*(A*b-B*a)*(b*
x+a)*ln(b*x+a)/b^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.46

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(a+bx)\left(bx(6a^2B-3ab(2A+Bx)+b^2x(3A+2Bx))+6a^2(Ab-aB)\log(a+bx)\right)}{6b^4\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^2*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(b*x*(6*a^2*B - 3*a*b*(2*A + B*x) + b^2*x*(3*A + 2*B*x)) + 6*a^
2*(A*b - a*B)*Log[a + b*x]))/(6*b^4*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{x^2(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^2(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(-\frac{(aB-Ab)a^2}{b^3(a+bx)}+\frac{(aB-Ab)a}{b^3}+\frac{Bx^2}{b}+\frac{(Ab-aB)x}{b^2}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^2(Ab-aB)\log(a+bx)}{b^4}-\frac{ax(Ab-aB)}{b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^3}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^2*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(-((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^2)/(2*b^2) + (B*x^3)
/(3*b) + (a^2*(A*b - a*B)*Log[a + b*x])/b^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2
]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(bx+a)\left(2x^{3} B\,b^{3}+3A\,b^{3} x^{2}-3B a\,b^{2} x^{2}+6A\ln \left(bx+a\right) a^{2} b-6A a\,b^{2} x-6B\ln \left(bx+a\right) a^{3}+6B\,a^{2} b x\right)}{6\sqrt{\left(bx+a\right)^{2}}\, b^{4}}$	90
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(\frac{1}{3}x^{3} B\,b^{2}+\frac{1}{2}x^{2} b^{2} A-\frac{1}{2}B a\,x^{2} b-abAx+a^{2} B x\right)}{\left(bx+a\right) b^{3}}+\frac{\sqrt{\left(bx+a\right)^{2}}\, a^{2} \left(A b-B a\right) \ln \left(bx+a\right)}{\left(bx+a\right) b^{4}}$	98

input
```
int(x^2*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x+a)*(2*x^3*B*b^3+3*A*b^3*x^2-3*B*a*b^2*x^2+6*A*ln(b*x+a)*a^2*b-6*A
*a*b^2*x-6*B*ln(b*x+a)*a^3+6*B*a^2*b*x)/((b*x+a)^2)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.43

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,Bb^3x^3 - 3\,(Bab^2 - Ab^3)x^2 + 6\,(Ba^2b - Aab^2)x - 6\,(Ba^3 - Aa^2b)\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^2*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^3 - 3*(B*a*b^2 - A*b^3)*x^2 + 6*(B*a^2*b - A*a*b^2)*x - 6*(
B*a^3 - A*a^2*b)*log(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(119) = 238.

Time = 1.21 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.99

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} \sqrt{a^2+2abx+b^2x^2}\left(\frac{Bx^2}{3b^2} + \frac{x\left(A-\frac{5Ba}{3b}\right)}{2b^2} + \frac{-\frac{2Ba^2}{3b^2} - \frac{3a\left(A-\frac{5Ba}{3b}\right)}{2b}}{b^2}\right) + \frac{\left(\frac{a}{b}+x\right)\left(-\frac{a^2\left(A-\frac{5Ba}{3b}\right)}{2b^2} - \frac{a\left(-\frac{2Ba^2}{3b^2}-\frac{3a\left(A-\frac{5Ba}{3b}\right)}{2b}\right)}{b}\right)\text{lo}\cdots}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} \cdots \\ \frac{\frac{A\left(a^4\sqrt{a^2+2abx} - \frac{2a^2\left(a^2+2abx\right)^{\frac{3}{2}}}{3} + \frac{\left(a^2+2abx\right)^{\frac{5}{2}}}{5}\right)}{2a^2b^2} + \frac{B\left(-a^6\sqrt{a^2+2abx} + a^4\left(a^2+2abx\right)^{\frac{3}{2}} - \frac{3a^2\left(a^2+2abx\right)^{\frac{5}{2}}}{5} + \frac{\left(a^2+2abx\right)^{\frac{7}{2}}}{7}\right)}{4a^3b^3}}{2ab} \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{\sqrt{a^2}} \end{cases}$$

input
```
integrate(x**2*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(a**2 + 2*a*b*x + b**2*x**2)*(B*x**2/(3*b**2) + x*(A - 5*B*
a/(3*b))/(2*b**2) + (-2*B*a**2/(3*b**2) - 3*a*(A - 5*B*a/(3*b))/(2*b))/b**
2) + (a/b + x)*(-a**2*(A - 5*B*a/(3*b))/(2*b**2) - a*(-2*B*a**2/(3*b**2) -
 3*a*(A - 5*B*a/(3*b))/(2*b))/b)*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(
b**2, 0)), ((A*(a**4*sqrt(a**2 + 2*a*b*x) - 2*a**2*(a**2 + 2*a*b*x)**(3/2)
/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2) + B*(-a**6*sqrt(a**2 + 2*a*b
*x) + a**4*(a**2 + 2*a*b*x)**(3/2) - 3*a**2*(a**2 + 2*a*b*x)**(5/2)/5 + (a
**2 + 2*a*b*x)**(7/2)/7)/(4*a**3*b**3))/(2*a*b), Ne(a*b, 0)), ((A*x**3/3 +
 B*x**4/4)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{5\,Bax^2}{6\,b^2} + \frac{Ax^2}{2\,b} + \frac{\sqrt{b^2x^2+2\,abx+a^2}Bx^2}{3\,b^2} + \frac{5\,Ba^2x}{3\,b^3} - \frac{Aax}{b^2} - \frac{Ba^3\log\left(x+\frac{a}{b}\right)}{b^4} + \frac{Aa^2\log\left(x+\frac{a}{b}\right)}{b^3} - \frac{2\sqrt{b^2x^2+2\,abx+a^2}Ba^2}{3\,b^4}$$

input
```
integrate(x^2*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-5/6*B*a*x^2/b^2 + 1/2*A*x^2/b + 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*x^2/b
^2 + 5/3*B*a^2*x/b^3 - A*a*x/b^2 - B*a^3*log(x + a/b)/b^4 + A*a^2*log(x +
a/b)/b^3 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,Bb^2x^3\mathrm{sgn}(bx+a) - 3\,Babx^2\mathrm{sgn}(bx+a) + 3\,Ab^2x^2\mathrm{sgn}(bx+a) + 6\,Ba^2x\mathrm{sgn}(bx+a) - 6\,Aabx\mathrm{sgn}(bx}{6\,b^3} - \frac{(Ba^3\mathrm{sgn}(bx+a) - Aa^2b\mathrm{sgn}(bx+a))\log\left(|bx+a|\right)}{b^4}$$

input
```
integrate(x^2*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*B*b^2*x^3*sgn(b*x + a) - 3*B*a*b*x^2*sgn(b*x + a) + 3*A*b^2*x^2*sgn
(b*x + a) + 6*B*a^2*x*sgn(b*x + a) - 6*A*a*b*x*sgn(b*x + a))/b^3 - (B*a^3*
sgn(b*x + a) - A*a^2*b*sgn(b*x + a))*log(abs(b*x + a))/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x^2\,(A+B\,x)}{\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((x^2*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^2(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{x^3}{3}$$

input
```
int(x^2*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
x**3/3
```

3.336 $\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(Ab-aB)x(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx^2(a+bx)}{2b\sqrt{a^2+2abx+b^2x^2}} - \frac{a(Ab-aB)(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
(A*b-B*a)*x*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+1/2*B*x^2*(b*x+a)/b/((b*x+a)^2)^
(1/2)-a*(A*b-B*a)*(b*x+a)*ln(b*x+a)/b^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(a+bx)(bx(2Ab-2aB+bBx)+2a(-Ab+aB)\log(a+bx))}{2b^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(b*x*(2*A*b - 2*a*B + b*B*x) + 2*a*(-(A*b) + a*B)*Log[a + b*x])
)/(2*b^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{x(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(aB-Ab)}{b^2(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a(Ab-aB)\log(a+bx)}{b^3}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^2}{2b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*(((A*b - a*B)*x)/b^2 + (B*x^2)/(2*b) - (a*(A*b - a*B)*Log[a + b
*x])/b^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{(bx+a)\left(-x^{2} B\, b^{2}+2 A \ln \left(bx+a\right) a b-2 x\, b^{2} A-2 B \ln \left(bx+a\right) a^{2}+2 x a b B\right)}{2 \sqrt{\left(bx+a\right)^{2}}\, b^{3}}$	66
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\, \left(\frac{1}{2} B b\, x^{2}+A b x-B a x\right)}{\left(bx+a\right) b^{2}}-\frac{\sqrt{\left(bx+a\right)^{2}}\, a \left(A b-B a\right) \ln \left(bx+a\right)}{\left(bx+a\right) b^{3}}$	75

input
```
int(x*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)*(-x^2*B*b^2+2*A*ln(b*x+a)*a*b-2*x*b^2*A-2*B*ln(b*x+a)*a^2+2*x
*a*b*B)/((b*x+a)^2)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{Bb^2x^2-2\left(Bab-Ab^2\right)x+2\left(Ba^2-Aab\right)\log\left(bx+a\right)}{2\,b^3}$$

input
```
integrate(x*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(B*b^2*x^2 - 2*(B*a*b - A*b^2)*x + 2*(B*a^2 - A*a*b)*log(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(83) = 166.

Time = 1.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.82

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \begin{cases} \left(\frac{Bx}{2b^2}+\frac{A-\frac{3Ba}{2b}}{b^2}\right)\sqrt{a^2+2abx+b^2x^2}+\frac{\left(\frac{a}{b}+x\right)\left(-\frac{Ba^2}{2b^2}-\frac{a\left(A-\frac{3Ba}{2b}\right)}{b}\right)\log\left(\frac{a}{b}+x\right)}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} & \text{for } b^2\neq 0 \\ \frac{\frac{A\left(-a^2\sqrt{a^2+2abx}+\frac{\left(a^2+2abx\right)^{\frac{3}{2}}}{3}\right)}{ab}+\frac{B\left(a^4\sqrt{a^2+2abx}-\frac{2a^2\left(a^2+2abx\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx\right)^{\frac{5}{2}}}{5}\right)}{2a^2b^2}}{2ab} & \text{for } ab\neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((((B*x/(2*b**2) + (A - 3*B*a/(2*b))/b**2)*sqrt(a**2 + 2*a*b*x + b
**2*x**2) + (a/b + x)*(-B*a**2/(2*b**2) - a*(A - 3*B*a/(2*b))/b)*log(a/b +
 x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((A*(-a**2*sqrt(a**2 + 2*a*b*x)
 + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b) + B*(a**4*sqrt(a**2 + 2*a*b*x) - 2*a**
2*(a**2 + 2*a*b*x)**(3/2)/3 + (a**2 + 2*a*b*x)**(5/2)/5)/(2*a**2*b**2))/(2
*a*b), Ne(a*b, 0)), ((A*x**2/2 + B*x**3/3)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{Bx^2}{2\,b} - \frac{Bax}{b^2} + \frac{Ba^2\log\left(x+\frac{a}{b}\right)}{b^3} - \frac{Aa\log\left(x+\frac{a}{b}\right)}{b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2}A}{b^2}$$

input
```
integrate(x*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*B*x^2/b - B*a*x/b^2 + B*a^2*log(x + a/b)/b^3 - A*a*log(x + a/b)/b^2 +
sqrt(b^2*x^2 + 2*a*b*x + a^2)*A/b^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{Bbx^2\mathrm{sgn}(bx+a) - 2\,Bax\mathrm{sgn}(bx+a) + 2\,Abx\mathrm{sgn}(bx+a)}{2\,b^2} + \frac{(Ba^2\mathrm{sgn}(bx+a) - Aab\mathrm{sgn}(bx+a))\log\left(|bx+a|\right)}{b^3}$$

input
```
integrate(x*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(B*b*x^2*sgn(b*x + a) - 2*B*a*x*sgn(b*x + a) + 2*A*b*x*sgn(b*x + a))/b
^2 + (B*a^2*sgn(b*x + a) - A*a*b*sgn(b*x + a))*log(abs(b*x + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{x\,(A+B\,x)}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((x*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.04

$$\int \frac{x(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{x^2}{2}$$

input
```
int(x*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
x**2/2
```

3.337 $\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{B\sqrt{a^2+2abx+b^2x^2}}{b^2}+\frac{(Ab-aB)(a+bx)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

output
```
B*((b*x+a)^2)^(1/2)/b^2+(A*b-B*a)*(b*x+a)*ln(b*x+a)/b^2/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(69) = 138$.

Time = 0.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$=\frac{(2a+bx)\left(-bBx\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)-2(-Ab+aB)\left(-a^2-abx+\sqrt{a^2}\sqrt{(a+bx)^2}\right)a}{b^2\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}$$

input
```
Integrate[(A + B*x)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((2*a + b*x)*(-(b*B*x*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
 - 2*(-(A*b) + a*B)*(-a^2 - a*b*x + Sqrt[a^2]*Sqrt[(a + b*x)^2])*ArcTanh[(
b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])]))/(b^2*(Sqrt[a^2] - Sqrt[(a + b*x)^2
])*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a^2+2bxa+b^2x^2}}dx}{b}+\frac{B\sqrt{a^2+2abx+b^2x^2}}{b^2}$$

$$\Big\downarrow \text{1079}$$

$$\frac{(a+bx)(Ab-aB)\int \frac{1}{xb^2+ab}dx}{\sqrt{a^2+2abx+b^2x^2}}+\frac{B\sqrt{a^2+2abx+b^2x^2}}{b^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx)(Ab-aB)\log(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{B\sqrt{a^2+2abx+b^2x^2}}{b^2}$$

input
```
Int[(A + B*x)/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(B*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/b^2 + ((A*b - a*B)*(a + b*x)*Log[a + b*x
])/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{(bx+a)(A\ln(bx+a)b-B\ln(bx+a)a+Bbx)}{\sqrt{(bx+a)^2}\,b^2}$	43
risch	$\frac{\sqrt{(bx+a)^2}\,Bx}{(bx+a)b}+\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)\ln(bx+a)}{(bx+a)b^2}$	58

input
```
int((B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(A*ln(b*x+a)*b-B*ln(b*x+a)*a+B*b*x)/((b*x+a)^2)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{Bbx-(Ba-Ab)\log{(bx+a)}}{b^2}$$

input
```
integrate((B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(B*b*x - (B*a - A*b)*log(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.

Time = 0.82 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\begin{cases} \frac{B\sqrt{a^2+2abx+b^2x^2}}{b^2}+\frac{\left(A-\frac{Ba}{b}\right)\left(\frac{a}{b}+x\right)\log{\left(\frac{a}{b}+x\right)}}{\sqrt{b^2\left(\frac{a}{b}+x\right)^2}} & \text{for } b^2\neq 0 \\ \frac{2A\sqrt{a^2+2abx}+\frac{B\left(-a^2\sqrt{a^2+2abx}+\frac{\left(a^2+2abx\right)^{\frac{3}{2}}}{3}\right)}{ab}}{2ab} & \text{for } ab\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((B*sqrt(a**2 + 2*a*b*x + b**2*x**2)/b**2 + (A - B*a/b)*(a/b + x)
*log(a/b + x)/sqrt(b**2*(a/b + x)**2), Ne(b**2, 0)), ((2*A*sqrt(a**2 + 2*a
*b*x) + B*(-a**2*sqrt(a**2 + 2*a*b*x) + (a**2 + 2*a*b*x)**(3/2)/3)/(a*b))/
(2*a*b), Ne(a*b, 0)), ((A*x + B*x**2/2)/sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{Ba\log\left(x+\frac{a}{b}\right)}{b^2}+\frac{A\log\left(x+\frac{a}{b}\right)}{b}+\frac{\sqrt{b^2x^2+2\,abx+a^2}B}{b^2}$$

input
```
integrate((B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-B*a*log(x + a/b)/b^2 + A*log(x + a/b)/b + sqrt(b^2*x^2 + 2*a*b*x + a^2)*B
/b^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{Bx\mathrm{sgn}(bx+a)}{b} - \frac{(Ba\mathrm{sgn}(bx+a)-Ab\mathrm{sgn}(bx+a))\log\left(|bx+a|\right)}{b^2}$$

input
```
integrate((B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
B*x*sgn(b*x + a)/b - (B*a*sgn(b*x + a) - A*b*sgn(b*x + a))*log(abs(b*x + a
))/b^2
```

Mupad [B] (verification not implemented)

Time = 11.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{B\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^2}+\frac{A\,\ln\left(a+b\,x+\sqrt{{(a+b\,x)}^2}\right)}{b} - \frac{B\,a\,b\,\ln\left(a\,b+\sqrt{{(a+b\,x)}^2}\,\sqrt{b^2}+b^2\,x\right)}{{\left(b^2\right)}^{3/2}}$$

input
```
int((A + B*x)/((a + b*x)^2)^(1/2),x)
```

output
```
(B*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/b^2 + (A*log(a + b*x + ((a + b*x)^2)^(
1/2)))/b - (B*a*b*log(a*b + ((a + b*x)^2)^(1/2)*(b^2)^(1/2) + b^2*x))/(b^2
)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{A+Bx}{\sqrt{a^2+2abx+b^2x^2}}\,dx=x$$

input
```
int((B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
x
```

3.338 $\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{A(a+bx)\log(x)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{(Ab-aB)(a+bx)\log(a+bx)}{ab\sqrt{a^2+2abx+b^2x^2}}$$

output
```
A*(b*x+a)*ln(x)/a/((b*x+a)^2)^(1/2)-(A*b-B*a)*(b*x+a)*ln(b*x+a)/a/b/((b*x+
a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \left(\frac{A}{a}-\frac{2B}{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2}-\sqrt{(a+bx)^2}}{bx}\right) + \frac{A\left(-2\log(x)+\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)+\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)\right)}{2\sqrt{a^2}}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(A/a - (2*B)/b)*ArcTanh[(Sqrt[a^2] - Sqrt[(a + b*x)^2])/(b*x)] + (A*(-2*Lo
g[x] + Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + Log[Sqrt[a^2] + b*x - Sq
rt[(a + b*x)^2]]))/(2*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{A+Bx}{x(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 86$$

$$\frac{(a+bx)\int \left(\frac{A}{ax}+\frac{aB-Ab}{a(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{(a+bx)\left(\frac{A\log(x)}{a}-\frac{(Ab-aB)\log(a+bx)}{ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((A*Log[x])/a - ((A*b - a*B)*Log[a + b*x])/(a*b)))/Sqrt[a^2 + 2
*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{(bx+a)(A\ln(bx+a)b-Ab\ln(x)-B\ln(bx+a)a)}{\sqrt{(bx+a)^2}\,ab}$	49
risch	$\frac{\sqrt{(bx+a)^2}\,A\ln(-x)}{(bx+a)a}-\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)\ln(bx+a)}{(bx+a)ab}$	65

input
```
int((B*x+A)/x/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)*(A*ln(b*x+a)*b-A*b*ln(x)-B*ln(b*x+a)*a)/((b*x+a)^2)^(1/2)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{Ab\log(x)+(Ba-Ab)\log(bx+a)}{ab}$$

input
```
integrate((B*x+A)/x/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(A*b*log(x) + (B*a - A*b)*log(b*x + a))/(a*b)
```

Sympy [F]

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+Bx}{x\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(-1)^{2abx+2a^2}A\log\left(\frac{2abx}{|x|}+\frac{2a^2}{|x|}\right)}{a}+\frac{B\log\left(x+\frac{a}{b}\right)}{b}$$

input
```
integrate((B*x+A)/x/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*A*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a + B*log(x +
 a/b)/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{A\log(|x|)\operatorname{sgn}(bx+a)}{a} + \frac{(Ba\operatorname{sgn}(bx+a)-Ab\operatorname{sgn}(bx+a))\log(|bx+a|)}{ab}$$

input
```
integrate((B*x+A)/x/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
A*log(abs(x))*sgn(b*x + a)/a + (B*a*sgn(b*x + a) - A*b*sgn(b*x + a))*log(a
bs(b*x + a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{B\,\ln\left(a+b\,x+\sqrt{(a+b\,x)^2}\right)}{b} - \frac{A\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int((A + B*x)/(x*((a + b*x)^2)^(1/2)),x)
```

output
```
(B*log(a + b*x + ((a + b*x)^2)^(1/2)))/b - (A*log(a*b + a^2/x + ((a^2)^(1/
2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/x))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.02

$$\int \frac{A+Bx}{x\sqrt{a^2+2abx+b^2x^2}}\,dx=\log(x)$$

input
```
int((B*x+A)/x/((b*x+a)^2)^(1/2),x)
```

output
```
log(x)
```

3.339 $\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}+\frac{(Ab-aB)(a+bx)\log\left(\frac{b(a+bx)}{x}\right)}{a^2\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-A*((b*x+a)^2)^(1/2)/a^2/x+(A*b-B*a)*(b*x+a)*ln(b*(b*x+a)/x)/a^2/((b*x+a)^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{a^2A-\sqrt{a^2}A\sqrt{(a+bx)^2}+2a(Ab-aB)x\log(x)-\left(-a+\sqrt{a^2}\right)(-Ab+aB)x\log\left(\sqrt{a^2}-bx-\sqrt{(a+\right.}{2\left(a^2\right)^{3/2}x}$$

input
```
Integrate[(A + B*x)/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(a^2*A - Sqrt[a^2]*A*Sqrt[(a + b*x)^2] + 2*a*(A*b - a*B)*x*Log[x] - (-a +
Sqrt[a^2])*(-(A*b) + a*B)*x*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] + (a
+ Sqrt[a^2])*(-(A*b) + a*B)*x*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])/(2
*(a^2)^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1186, 1102, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1186}$$

$$-\frac{(Ab-aB)\int \frac{1}{x\sqrt{a^2+2bxa+b^2x^2}}dx}{a}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

$$\Big\downarrow \text{1102}$$

$$-\frac{b(a+bx)(Ab-aB)\int \frac{1}{bx(a+bx)}dx}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(a+bx)(Ab-aB)\int \frac{1}{x(a+bx)}dx}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

$$\Big\downarrow \text{47}$$

$$-\frac{(a+bx)(Ab-aB)\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

$$\Big\downarrow \text{14}$$

$$-\frac{(a+bx)(Ab-aB)\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(a+bx)(Ab-aB)\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{a\sqrt{a^2+2abx+b^2x^2}}-\frac{A\sqrt{a^2+2abx+b^2x^2}}{a^2x}$$

input
```
Int[(A + B*x)/(x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
-((A*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a^2*x)) - ((A*b - a*B)*(a + b*x)*(Log
[x]/a - Log[a + b*x]/a))/(a*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1186
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[-2*c*(e*f - d*g)*(d + e*x)^(m + 1)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e)^2)), x] + Simp[(2*c*f - b*g)/
(2*c*d - b*e)   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[
{a, b, c, d, e, f, g, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[m + 2*p + 3,
0] && NeQ[2*c*f - b*g, 0] && NeQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{(bx+a)(A\ln(bx+a)xb-A\ln(x)xb-B\ln(bx+a)ax+B\ln(x)ax-Aa)}{\sqrt{(bx+a)^2}\,a^2x}$	61
risch	$-\frac{\sqrt{(bx+a)^2}\,A}{(bx+a)ax}-\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)\ln(x)}{(bx+a)a^2}+\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)\ln(-bx-a)}{(bx+a)a^2}$	95

input
```
int((B*x+A)/x^2/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*(A*ln(b*x+a)*x*b-A*ln(x)*x*b-B*ln(b*x+a)*a*x+B*ln(x)*a*x-A*a)/((b*
x+a)^2)^(1/2)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{(Ba-Ab)x\log(bx+a)-(Ba-Ab)x\log(x)+Aa}{a^2x}$$

input
```
integrate((B*x+A)/x^2/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((B*a - A*b)*x*log(b*x + a) - (B*a - A*b)*x*log(x) + A*a)/(a^2*x)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+Bx}{x^2\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**2/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**2*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\,B\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a} + \frac{(-1)^{2\,abx+2\,a^2}\,Ab\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2}A}{a^2x}$$

input
```
integrate((B*x+A)/x^2/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*B*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a + (-1)^(2*a
*b*x + 2*a^2)*A*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^2 - sqrt(b^2*x^2 +
2*a*b*x + a^2)*A/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(Ba\mathrm{sgn}(bx+a)-Ab\mathrm{sgn}(bx+a))\log(|x|)}{a^2} - \frac{A\mathrm{sgn}(bx+a)}{ax} - \frac{(Bab\mathrm{sgn}(bx+a)-Ab^2\mathrm{sgn}(bx+a))\log(|bx+a|)}{a^2b}$$

input
```
integrate((B*x+A)/x^2/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
(B*a*sgn(b*x + a) - A*b*sgn(b*x + a))*log(abs(x))/a^2 - A*sgn(b*x + a)/(a*
x) - (B*a*b*sgn(b*x + a) - A*b^2*sgn(b*x + a))*log(abs(b*x + a))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 11.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{A\,a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}\right)}{{\left(a^2\right)}^{3/2}} - \frac{A\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{a^2\,x} - \frac{B\,\ln\left(a\,b+\frac{a^2}{x}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{x}\right)}{\sqrt{a^2}}$$

input
```
int((A + B*x)/(x^2*((a + b*x)^2)^(1/2)),x)
```

output
```
(A*a*b*atanh((a^2 + a*b*x)/((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))))
/(a^2)^(3/2) - (A*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(a^2*x) - (B*log(a*b +
a^2/x + ((a^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/x))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx}{x^2\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{1}{x}$$

input
```
int((B*x+A)/x^2/((b*x+a)^2)^(1/2),x)
```

output
```
( - 1)/x
```

3.340 $\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 162

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{A(a+bx)}{2ax^2\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-aB)(a+bx)}{a^2x\sqrt{a^2+2abx+b^2x^2}}+\frac{b(Ab-aB)(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b(Ab-aB)(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/2*A*(b*x+a)/a/x^2/((b*x+a)^2)^(1/2)+(A*b-B*a)*(b*x+a)/a^2/x/((b*x+a)^2)
^(1/2)+b*(A*b-B*a)*(b*x+a)*ln(x)/a^3/((b*x+a)^2)^(1/2)-b*(A*b-B*a)*(b*x+a)
*ln(b*x+a)/a^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{-\frac{a\sqrt{(a+bx)^2}(-3Abx+a(A+2Bx))}{x^2}+\frac{a^3(-2Abx+a(A+2Bx))}{\sqrt{a^2}x^2}+4\sqrt{a^2}b(-Ab+aB)\log(x)-2\left(-a+\sqrt{a^2}\right)b(-Ab+}{4a^4}$$

input
```
Integrate[(A + B*x)/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-((a*Sqrt[(a + b*x)^2]*(-3*A*b*x + a*(A + 2*B*x)))/x^2) + (a^3*(-2*A*b*x
+ a*(A + 2*B*x)))/(Sqrt[a^2]*x^2) + 4*Sqrt[a^2]*b*(-(A*b) + a*B)*Log[x] -
2*(-a + Sqrt[a^2])*b*(-(A*b) + a*B)*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2
]] - 2*(a + Sqrt[a^2])*b*(-(A*b) + a*B)*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*
x)^2]])/(4*a^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx^3(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^3(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{(aB-Ab)b^2}{a^3(a+bx)}-\frac{(aB-Ab)b}{a^3x}+\frac{aB-Ab}{a^2x^2}+\frac{A}{ax^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{b\log(x)(Ab-aB)}{a^3}-\frac{b(Ab-aB)\log(a+bx)}{a^3}+\frac{Ab-aB}{a^2x}-\frac{A}{2ax^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-1/2*A/(a*x^2) + (A*b - a*B)/(a^2*x) + (b*(A*b - a*B)*Log[x])/
a^3 - (b*(A*b - a*B)*Log[a + b*x])/a^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{(bx+a)\left(2A\ln(bx+a)b^2x^2-2A\ln(x)b^2x^2-2B\ln(bx+a)ab\,x^2+2B\ln(x)ab\,x^2-2abAx+2a^2Bx+a^2A\right)}{2\sqrt{(bx+a)^2}\,a^3x^2}$	92
risch	$\frac{\sqrt{(bx+a)^2}\,\left(\frac{(Ab-Ba)x}{a^2}-\frac{A}{2a}\right)}{(bx+a)x^2}+\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b\ln(-x)}{(bx+a)a^3}-\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b\ln(bx+a)}{(bx+a)a^3}$	111

input
```
int((B*x+A)/x^3/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)*(2*A*ln(b*x+a)*b^2*x^2-2*A*ln(x)*b^2*x^2-2*B*ln(b*x+a)*a*b*x^
2+2*B*ln(x)*a*b*x^2-2*a*b*A*x+2*a^2*B*x+a^2*A)/((b*x+a)^2)^(1/2)/a^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,(Bab-Ab^2)x^2\log(bx+a)-2\,(Bab-Ab^2)x^2\log(x)-Aa^2-2\,(Ba^2-Aab)x}{2\,a^3x^2}$$

input
```
integrate((B*x+A)/x^3/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*(B*a*b - A*b^2)*x^2*log(b*x + a) - 2*(B*a*b - A*b^2)*x^2*log(x) - A
*a^2 - 2*(B*a^2 - A*a*b)*x)/(a^3*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+Bx}{x^3\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**3/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**3*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{(-1)^{2\,abx+2\,a^2}Bb\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^2} - \frac{(-1)^{2\,abx+2\,a^2}Ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^3} - \frac{\sqrt{b^2x^2+2\,abx+a^2}B}{a^2x} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Ab}{2\,a^3x} - \frac{\sqrt{b^2x^2+2\,abx+a^2}A}{2\,a^2x^2}$$

input
```
integrate((B*x+A)/x^3/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x + 2*a^2)*B*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^2 - (-1)^(
2*a*b*x + 2*a^2)*A*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 - sqrt(b^2*x
^2 + 2*a*b*x + a^2)*B/(a^2*x) + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b/(a^3
*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(Bab\mathrm{sgn}(bx+a)-Ab^2\mathrm{sgn}(bx+a))\log(|x|)}{a^3} + \frac{(Bab^2\mathrm{sgn}(bx+a)-Ab^3\mathrm{sgn}(bx+a))\log(|bx+a|)}{a^3b} - \frac{Aa^2\mathrm{sgn}(bx+a)+2\,(Ba^2\mathrm{sgn}(bx+a)-Aab\mathrm{sgn}(bx+a))x}{2\,a^3x^2}$$

input
```
integrate((B*x+A)/x^3/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(B*a*b*sgn(b*x + a) - A*b^2*sgn(b*x + a))*log(abs(x))/a^3 + (B*a*b^2*sgn(
b*x + a) - A*b^3*sgn(b*x + a))*log(abs(b*x + a))/(a^3*b) - 1/2*(A*a^2*sgn(
b*x + a) + 2*(B*a^2*sgn(b*x + a) - A*a*b*sgn(b*x + a))*x)/(a^3*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^3\,\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((A + B*x)/(x^3*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^3*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx}{x^3\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{1}{2x^2}$$

input
```
int((B*x+A)/x^3/((b*x+a)^2)^(1/2),x)
```

output
```
( - 1)/(2*x**2)
```

3.341 $\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 211

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{A(a+bx)}{3ax^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-aB)(a+bx)}{2a^2x^2\sqrt{a^2+2abx+b^2x^2}} - \frac{b(Ab-aB)(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}} - \frac{b^2(Ab-aB)(a+bx)\log(x)}{a^4\sqrt{a^2+2abx+b^2x^2}} + \frac{b^2(Ab-aB)(a+bx)\log(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/3*A*(b*x+a)/a/x^3/((b*x+a)^2)^(1/2)+1/2*(A*b-B*a)*(b*x+a)/a^2/x^2/((b*x
+a)^2)^(1/2)-b*(A*b-B*a)*(b*x+a)/a^3/x/((b*x+a)^2)^(1/2)-b^2*(A*b-B*a)*(b*
x+a)*ln(x)/a^4/((b*x+a)^2)^(1/2)+b^2*(A*b-B*a)*(b*x+a)*ln(b*x+a)/a^4/((b*x
+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{1}{12}\left(\frac{6Ab^2x^2-3abx(A+2Bx)+a^2(2A+3Bx)}{(a^2)^{3/2}x^3} + \frac{\sqrt{(a+bx)^2}(-11Ab^2x^2-a^2(2A+3Bx)+abx(5A+9Bx))}{a^4x^3} - \frac{12\sqrt{a^2}b^2(-Ab+aB)\log(x)}{a^5} + \frac{6\left(-a+\sqrt{a^2}\right)b^2(-Ab+aB)\log\left(\sqrt{a^2}-bx-\sqrt{(a+bx)^2}\right)}{a^5} + \frac{6\left(a+\sqrt{a^2}\right)b^2(-Ab+aB)\log\left(\sqrt{a^2}+bx-\sqrt{(a+bx)^2}\right)}{a^5}\right)$$

input
```
Integrate[(A + B*x)/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((6*A*b^2*x^2 - 3*a*b*x*(A + 2*B*x) + a^2*(2*A + 3*B*x))/((a^2)^(3/2)*x^3)
 + (Sqrt[(a + b*x)^2]*(-11*A*b^2*x^2 - a^2*(2*A + 3*B*x) + a*b*x*(5*A + 9*
B*x)))/(a^4*x^3) - (12*Sqrt[a^2]*b^2*(-(A*b) + a*B)*Log[x])/a^5 + (6*(-a +
 Sqrt[a^2])*b^2*(-(A*b) + a*B)*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]])/a
^5 + (6*(a + Sqrt[a^2])*b^2*(-(A*b) + a*B)*Log[Sqrt[a^2] + b*x - Sqrt[(a +
 b*x)^2]])/a^5)/12
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$\Big\downarrow \text{ 1187}$$
$$\frac{b(a+bx)\int \frac{A+Bx}{bx^4(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{(a+bx)\int \frac{A+Bx}{x^4(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \text{ 86}$$
$$\frac{(a+bx)\int \left(-\frac{(aB-Ab)b^3}{a^4(a+bx)}+\frac{(aB-Ab)b^2}{a^4x}-\frac{(aB-Ab)b}{a^3x^2}+\frac{aB-Ab}{a^2x^3}+\frac{A}{ax^4}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{(a+bx)\left(-\frac{b^2\log(x)(Ab-aB)}{a^4}+\frac{b^2(Ab-aB)\log(a+bx)}{a^4}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{3ax^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^4*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-1/3*A/(a*x^3) + (A*b - a*B)/(2*a^2*x^2) - (b*(A*b - a*B))/(a^
3*x) - (b^2*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x])/a^4))
/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

method	result
default	$\frac{(bx+a)(6A\ln(bx+a)x^3b^3-6A\ln(x)x^3b^3-6B\ln(bx+a)a\,b^2x^3+6B\ln(x)a\,b^2x^3-6Aa\,b^2x^2+6B\,a^2b\,x^2+3A\,a^2bx-3B\,a^3x-2a^3A)}{6\sqrt{(bx+a)^2}\,a^4x^3}$
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{(Ab-Ba)b\,x^2}{a^3}+\frac{(Ab-Ba)x}{2a^2}-\frac{A}{3a}\right)}{(bx+a)x^3}+\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b^2\ln(-bx-a)}{(bx+a)a^4}-\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b^2\ln(x)}{(bx+a)a^4}$

input
```
int((B*x+A)/x^4/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x+a)*(6*A*ln(b*x+a)*x^3*b^3-6*A*ln(x)*x^3*b^3-6*B*ln(b*x+a)*a*b^2*x
^3+6*B*ln(x)*a*b^2*x^3-6*A*a*b^2*x^2+6*B*a^2*b*x^2+3*A*a^2*b*x-3*B*a^3*x-2
*a^3*A)/((b*x+a)^2)^(1/2)/a^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx =$$
$$-\frac{6\,(Bab^2-Ab^3)x^3\log\left(bx+a\right)-6\,(Bab^2-Ab^3)x^3\log\left(x\right)+2\,Aa^3-6\,(Ba^2b-Aab^2)x^2+3\,(Ba^3-A}{6\,a^4x^3}$$

input
```
integrate((B*x+A)/x^4/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(6*(B*a*b^2 - A*b^3)*x^3*log(b*x + a) - 6*(B*a*b^2 - A*b^3)*x^3*log(x
) + 2*A*a^3 - 6*(B*a^2*b - A*a*b^2)*x^2 + 3*(B*a^3 - A*a^2*b)*x)/(a^4*x^3)
```

Sympy [F]

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{A+Bx}{x^4\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**4/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**4*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = & -\frac{(-1)^{2\,abx+2\,a^2}\,Bb^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^3} \\ & +\frac{(-1)^{2\,abx+2\,a^2}\,Ab^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^4} \\ & +\frac{3\,\sqrt{b^2x^2+2\,abx+a^2}Bb}{2\,a^3x} \\ & -\frac{11\,\sqrt{b^2x^2+2\,abx+a^2}Ab^2}{6\,a^4x}-\frac{\sqrt{b^2x^2+2\,abx+a^2}B}{2\,a^2x^2} \\ & +\frac{5\,\sqrt{b^2x^2+2\,abx+a^2}Ab}{6\,a^3x^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2}A}{3\,a^2x^3}\end{aligned}$$

input
```
integrate((B*x+A)/x^4/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*B*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 + (-1
)^(2*a*b*x + 2*a^2)*A*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 + 3/2*sqr
t(b^2*x^2 + 2*a*b*x + a^2)*B*b/(a^3*x) - 11/6*sqrt(b^2*x^2 + 2*a*b*x + a^2
)*A*b^2/(a^4*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B/(a^2*x^2) + 5/6*sqrt
(b^2*x^2 + 2*a*b*x + a^2)*A*b/(a^3*x^2) - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2
)*A/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{\left(Bab^2\mathrm{sgn}(bx+a) - Ab^3\mathrm{sgn}(bx+a)\right)\log\left(|x|\right)}{a^4} - \frac{\left(Bab^3\mathrm{sgn}(bx+a) - Ab^4\mathrm{sgn}(bx+a)\right)\log\left(|bx+a|\right)}{a^4b} - \frac{2\,Aa^3\mathrm{sgn}(bx+a) - 6\left(Ba^2b\mathrm{sgn}(bx+a) - Aab^2\mathrm{sgn}(bx+a)\right)x^2 + 3\left(Ba^3\mathrm{sgn}(bx+a) - Aa^2b\mathrm{sgn}(bx+\right.}{6\,a^4x^3}$$

input
```
integrate((B*x+A)/x^4/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
(B*a*b^2*sgn(b*x + a) - A*b^3*sgn(b*x + a))*log(abs(x))/a^4 - (B*a*b^3*sgn
(b*x + a) - A*b^4*sgn(b*x + a))*log(abs(b*x + a))/(a^4*b) - 1/6*(2*A*a^3*s
gn(b*x + a) - 6*(B*a^2*b*sgn(b*x + a) - A*a*b^2*sgn(b*x + a))*x^2 + 3*(B*a
^3*sgn(b*x + a) - A*a^2*b*sgn(b*x + a))*x)/(a^4*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^4\,\sqrt{{\left(a+b\,x\right)}^2}}\,dx$$

input
```
int((A + B*x)/(x^4*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^4*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{A+Bx}{x^4\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{1}{3x^3}$$

input
```
int((B*x+A)/x^4/((b*x+a)^2)^(1/2),x)
```

output
```
( - 1)/(3*x**3)
```

3.342 $\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 256

$$\begin{aligned}\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx = &-\frac{A(a+bx)}{4ax^4\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-aB)(a+bx)}{3a^2x^3\sqrt{a^2+2abx+b^2x^2}}\\ &-\frac{b(Ab-aB)(a+bx)}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}}+\frac{b^2(Ab-aB)(a+bx)}{a^4x\sqrt{a^2+2abx+b^2x^2}}\\ &+\frac{b^3(Ab-aB)(a+bx)\log(x)}{a^5\sqrt{a^2+2abx+b^2x^2}}\\ &-\frac{b^3(Ab-aB)(a+bx)\log(a+bx)}{a^5\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/4*A*(b*x+a)/a/x^4/((b*x+a)^2)^(1/2)+1/3*(A*b-B*a)*(b*x+a)/a^2/x^3/((b*x
+a)^2)^(1/2)-1/2*b*(A*b-B*a)*(b*x+a)/a^3/x^2/((b*x+a)^2)^(1/2)+b^2*(A*b-B*
a)*(b*x+a)/a^4/x/((b*x+a)^2)^(1/2)+b^3*(A*b-B*a)*(b*x+a)*ln(x)/a^5/((b*x+a
)^2)^(1/2)-b^3*(A*b-B*a)*(b*x+a)*ln(b*x+a)/a^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{\frac{a^3(-12Ab^3x^3+6ab^2x^2(A+2Bx)-2a^2bx(2A+3Bx)+a^3(3A+4Bx))}{\sqrt{a^2}x^4}-\frac{a\sqrt{(a+bx)^2}(-25Ab^3x^3+a^3(3A+4Bx)-a^2bx(7A+10Bx)+ab^2x^2(13A}{x^4}}{}$$

input
```
Integrate[(A + B*x)/(x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a^3*(-12*A*b^3*x^3 + 6*a*b^2*x^2*(A + 2*B*x) - 2*a^2*b*x*(2*A + 3*B*x) +
 a^3*(3*A + 4*B*x)))/(Sqrt[a^2]*x^4) - (a*Sqrt[(a + b*x)^2]*(-25*A*b^3*x^3
 + a^3*(3*A + 4*B*x) - a^2*b*x*(7*A + 10*B*x) + a*b^2*x^2*(13*A + 22*B*x))
)/x^4 + 24*Sqrt[a^2]*b^3*(-(A*b) + a*B)*Log[x] - 12*(-a + Sqrt[a^2])*b^3*(
-(A*b) + a*B)*Log[Sqrt[a^2] - b*x - Sqrt[(a + b*x)^2]] - 12*(a + Sqrt[a^2]
)*b^3*(-(A*b) + a*B)*Log[Sqrt[a^2] + b*x - Sqrt[(a + b*x)^2]])/(24*a^6)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$\Big\downarrow \; 1187$$
$$\frac{b(a+bx)\int \frac{A+Bx}{bx^5(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$
$$\Big\downarrow \; 27$$
$$\frac{(a+bx)\int \frac{A+Bx}{x^5(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 86$$

$$\frac{(a+bx)\int \left(\frac{(aB-Ab)b^4}{a^5(a+bx)}-\frac{(aB-Ab)b^3}{a^5x}+\frac{(aB-Ab)b^2}{a^4x^2}-\frac{(aB-Ab)b}{a^3x^3}+\frac{aB-Ab}{a^2x^4}+\frac{A}{ax^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+bx)\left(\frac{b^3\log(x)(Ab-aB)}{a^5}-\frac{b^3(Ab-aB)\log(a+bx)}{a^5}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{4ax^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^5*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*(-1/4*A/(a*x^4) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(2*
a^3*x^2) + (b^2*(A*b - a*B))/(a^4*x) + (b^3*(A*b - a*B)*Log[x])/a^5 - (b^3
*(A*b - a*B)*Log[a + b*x])/a^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.56

method	result
default	$-\frac{(bx+a)(12A\ln(bx+a)x^4b^4-12A\ln(x)x^4b^4-12B\ln(bx+a)x^4ab^3+12B\ln(x)x^4ab^3-12Aab^3x^3+12Ba^2b^2x^3+6Aa^2b^2x^2-6Ba}{12\sqrt{(bx+a)^2}\,a^5x^4}$
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{(Ab-Ba)b^2x^3}{a^4}-\frac{(Ab-Ba)bx^2}{2a^3}+\frac{(Ab-Ba)x}{3a^2}-\frac{A}{4a}\right)}{(bx+a)x^4}-\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b^3\ln(bx+a)}{(bx+a)a^5}+\frac{\sqrt{(bx+a)^2}\,(Ab-Ba)b^3\ln(-x}{(bx+a)a^5}$

input
```
int((B*x+A)/x^5/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)*(12*A*ln(b*x+a)*x^4*b^4-12*A*ln(x)*x^4*b^4-12*B*ln(b*x+a)*x^
4*a*b^3+12*B*ln(x)*x^4*a*b^3-12*A*a*b^3*x^3+12*B*a^2*b^2*x^3+6*A*a^2*b^2*x
^2-6*B*a^3*b*x^2-4*A*a^3*b*x+4*a^4*B*x+3*a^4*A)/((b*x+a)^2)^(1/2)/a^5/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$=\frac{12\,(Bab^3-Ab^4)x^4\log(bx+a)-12\,(Bab^3-Ab^4)x^4\log(x)-3\,Aa^4-12\,(Ba^2b^2-Aab^3)x^3+6\,(Ba^3b-}{12\,a^5x^4}$$

input
```
integrate((B*x+A)/x^5/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(12*(B*a*b^3 - A*b^4)*x^4*log(b*x + a) - 12*(B*a*b^3 - A*b^4)*x^4*log
(x) - 3*A*a^4 - 12*(B*a^2*b^2 - A*a*b^3)*x^3 + 6*(B*a^3*b - A*a^2*b^2)*x^2
 - 4*(B*a^4 - A*a^3*b)*x)/(a^5*x^4)
```

Sympy [F]

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{A+Bx}{x^5\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**5/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**5*sqrt((a + b*x)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx=&\ \frac{(-1)^{2\,abx+2\,a^2}Bb^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^4}\\&-\frac{(-1)^{2\,abx+2\,a^2}Ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^5}\\&-\frac{11\sqrt{b^2x^2+2\,abx+a^2}Bb^2}{6\,a^4x}\\&+\frac{25\sqrt{b^2x^2+2\,abx+a^2}Ab^3}{12\,a^5x}+\frac{5\sqrt{b^2x^2+2\,abx+a^2}Bb}{6\,a^3x^2}\\&-\frac{13\sqrt{b^2x^2+2\,abx+a^2}Ab^2}{12\,a^4x^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2}B}{3\,a^2x^3}\\&+\frac{7\sqrt{b^2x^2+2\,abx+a^2}Ab}{12\,a^3x^3}-\frac{\sqrt{b^2x^2+2\,abx+a^2}A}{4\,a^2x^4}\end{aligned}$$

input
```
integrate((B*x+A)/x^5/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x + 2*a^2)*B*b^3*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 - (-1)
^(2*a*b*x + 2*a^2)*A*b^4*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 - 11/6*sqr
t(b^2*x^2 + 2*a*b*x + a^2)*B*b^2/(a^4*x) + 25/12*sqrt(b^2*x^2 + 2*a*b*x +
a^2)*A*b^3/(a^5*x) + 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2)*B*b/(a^3*x^2) - 13/
12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b^2/(a^4*x^2) - 1/3*sqrt(b^2*x^2 + 2*a*
b*x + a^2)*B/(a^2*x^3) + 7/12*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A*b/(a^3*x^3)
- 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2)*A/(a^2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{(Bab^3\mathrm{sgn}(bx+a)-Ab^4\mathrm{sgn}(bx+a))\log(|x|)}{a^5}$$
$$+\frac{(Bab^4\mathrm{sgn}(bx+a)-Ab^5\mathrm{sgn}(bx+a))\log(|bx+a|)}{a^5b}$$
$$-\frac{3\,Aa^4\mathrm{sgn}(bx+a)+12\,(Ba^2b^2\mathrm{sgn}(bx+a)-Aab^3\mathrm{sgn}(bx+a))x^3-6\,(Ba^3b\mathrm{sgn}(bx+a)-Aa^2b^2\mathrm{sgn}(b}{12\,a^5x^4}$$

input
```
integrate((B*x+A)/x^5/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(B*a*b^3*sgn(b*x + a) - A*b^4*sgn(b*x + a))*log(abs(x))/a^5 + (B*a*b^4*sg
n(b*x + a) - A*b^5*sgn(b*x + a))*log(abs(b*x + a))/(a^5*b) - 1/12*(3*A*a^4
*sgn(b*x + a) + 12*(B*a^2*b^2*sgn(b*x + a) - A*a*b^3*sgn(b*x + a))*x^3 - 6
*(B*a^3*b*sgn(b*x + a) - A*a^2*b^2*sgn(b*x + a))*x^2 + 4*(B*a^4*sgn(b*x +
a) - A*a^3*b*sgn(b*x + a))*x)/(a^5*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^5\,\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((A + B*x)/(x^5*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^5*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{A+Bx}{x^5\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{1}{4x^4}$$

input
```
int((B*x+A)/x^5/((b*x+a)^2)^(1/2),x)
```

output
```
( - 1)/(4*x**4)
```

3.343 $\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 249

$$\begin{aligned}\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}} \, dx &= \frac{a^3(4Ab-5aB)}{b^6\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{a^4(Ab-aB)}{2b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{3a(Ab-2aB)x(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{(Ab-3aB)x^2(a+bx)}{2b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx^3(a+bx)}{3b^3\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{2a^2(3Ab-5aB)(a+bx)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
a^3*(4*A*b-5*B*a)/b^6/((b*x+a)^2)^(1/2)-1/2*a^4*(A*b-B*a)/b^6/(b*x+a)/((b*
x+a)^2)^(1/2)-3*a*(A*b-2*B*a)*x*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+1/2*(A*b-3*B
*a)*x^2*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+1/3*B*x^3*(b*x+a)/b^3/((b*x+a)^2)^(1
/2)+2*a^2*(3*A*b-5*B*a)*(b*x+a)*ln(b*x+a)/b^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.56

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-27a^5B+b^5x^4(3A+2Bx)+3a^4b(7A+2Bx)-ab^4x^3(12A+5Bx)+a^2b^3\cdots}{6b^6(a\cdots}$$

input
```
Integrate[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(-27*a^5*B + b^5*x^4*(3*A + 2*B*x) + 3*a^4*b*(7*A + 2*B*x) - a*b^4*x^3*(12
*A + 5*B*x) + a^2*b^3*x^2*(-33*A + 20*B*x) + 3*a^3*b^2*x*(2*A + 21*B*x) -
12*a^2*(-3*A*b + 5*a*B)*(a + b*x)^2*Log[a + b*x])/(6*b^6*(a + b*x)*Sqrt[(a
 + b*x)^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^3(a+bx)\int \frac{x^4(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^4(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(-\frac{(aB-Ab)a^4}{b^5(a+bx)^3}+\frac{(5aB-4Ab)a^3}{b^5(a+bx)^2}-\frac{2(5aB-3Ab)a^2}{b^5(a+bx)}+\frac{3(2aB-Ab)a}{b^5}+\frac{Bx^2}{b^3}+\frac{(Ab-3aB)x}{b^4}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^4(Ab-aB)}{2b^6(a+bx)^2}+\frac{a^3(4Ab-5aB)}{b^6(a+bx)}+\frac{2a^2(3Ab-5aB)\log(a+bx)}{b^6}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^3}{3b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*((-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*
x^3)/(3*b^3) - (a^4*(A*b - a*B))/(2*b^6*(a + b*x)^2) + (a^3*(4*A*b - 5*a*B
))/(b^6*(a + b*x)) + (2*a^2*(3*A*b - 5*a*B)*Log[a + b*x])/b^6))/Sqrt[a^2 +
 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{1}{3}x^3Bb^2+\frac{1}{2}x^2b^2A-\frac{3}{2}Bax^2b-3abAx+6a^2Bx\right)}{(bx+a)b^5}+\frac{\sqrt{(bx+a)^2}\left((4Aa^3b-5a^4B)x+\frac{a^4(7Ab-9Ba)}{2b}\right)}{(bx+a)^3b^5}+\frac{2\sqrt{(bx+a)^2}\,a}{}$
default	$\frac{(2Bx^5b^5+3Ab^5x^4-5Bab^4x^4+36A\ln(bx+a)x^2a^2b^3-12Aab^4x^3-60B\ln(bx+a)x^2a^3b^2+20Ba^2b^3x^3+72A\ln(bx+a)xa^3b^2-33A}{6b^6\left((bx+\right.}$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*(1/3*x^3*B*b^2+1/2*x^2*b^2*A-3/2*B*a*x^2*b-3*a*b
*A*x+6*a^2*B*x)/b^5+((b*x+a)^2)^(1/2)/(b*x+a)^3*((4*A*a^3*b-5*B*a^4)*x+1/2
*a^4*(7*A*b-9*B*a)/b)/b^5+2*((b*x+a)^2)^(1/2)/(b*x+a)*a^2/b^6*(3*A*b-5*B*a
)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\,Bb^5x^5-27\,Ba^5+21\,Aa^4b-(5\,Bab^4-3\,Ab^5)x^4+4\,(5\,Ba^2b^3-3\,Aab^4)x^3}{}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*B*b^5*x^5 - 27*B*a^5 + 21*A*a^4*b - (5*B*a*b^4 - 3*A*b^5)*x^4 + 4*(
5*B*a^2*b^3 - 3*A*a*b^4)*x^3 + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^2 + 6*(B*
a^4*b + A*a^3*b^2)*x - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^2*b^
3)*x^2 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x)*log(b*x + a))/(b^8*x^2 + 2*a*b^7*x
 + a^2*b^6)
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\int \frac{x^4(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.22

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{Bx^4}{3\sqrt{b^2x^2+2\,abx+a^2}b^2}-\frac{7\,Bax^3}{6\sqrt{b^2x^2+2\,abx+a^2}b^3}+\frac{Ax^3}{2\sqrt{b^2x^2+2\,abx+a^2}b^2}+\frac{9\,Ba^2x^2}{2\sqrt{b^2x^2+2\,abx+a^2}b^4}-\frac{5\,Aax^2}{2\sqrt{b^2x^2+2\,abx+a^2}b^3}-\frac{10\,Ba^3\log\left(x+\frac{a}{b}\right)}{b^6}+\frac{6\,Aa^2\log\left(x+\frac{a}{b}\right)}{b^5}+\frac{9\,Ba^4}{\sqrt{b^2x^2+2\,abx+a^2}b^6}-\frac{5\,Aa^3}{\sqrt{b^2x^2+2\,abx+a^2}b^5}-\frac{20\,Ba^4x}{b^7\left(x+\frac{a}{b}\right)^2}+\frac{12\,Aa^3x}{b^6\left(x+\frac{a}{b}\right)^2}-\frac{39\,Ba^5}{2\,b^8\left(x+\frac{a}{b}\right)^2}+\frac{23\,Aa^4}{2\,b^7\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 7/6*B*a*x^3/(sqrt(b^2*x^2
+ 2*a*b*x + a^2)*b^3) + 1/2*A*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 9/
2*B*a^2*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) - 5/2*A*a*x^2/(sqrt(b^2*x^
2 + 2*a*b*x + a^2)*b^3) - 10*B*a^3*log(x + a/b)/b^6 + 6*A*a^2*log(x + a/b)
/b^5 + 9*B*a^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^6) - 5*A*a^3/(sqrt(b^2*x^2
 + 2*a*b*x + a^2)*b^5) - 20*B*a^4*x/(b^7*(x + a/b)^2) + 12*A*a^3*x/(b^6*(x
 + a/b)^2) - 39/2*B*a^5/(b^8*(x + a/b)^2) + 23/2*A*a^4/(b^7*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.60

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{2\,(5\,Ba^3-3\,Aa^2b)\log\left(|bx+a|\right)}{b^6\mathrm{sgn}\left(bx+a\right)} - \frac{9\,Ba^5-7\,Aa^4b+2\,(5\,Ba^4b-4\,Aa^3b^2)x}{2\,(bx+a)^2b^6\mathrm{sgn}\left(bx+a\right)} + \frac{2\,Bb^6x^3-9\,Bab^5x^2+3\,Ab^6x^2+36\,Ba^2b^4x-18\,Aab^5x}{6\,b^9\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x + a))/(b^6*sgn(b*x + a)) - 1/2*(9*B*a
^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x)/((b*x + a)^2*b^6*sgn(b*x +
 a)) + 1/6*(2*B*b^6*x^3 - 9*B*a*b^5*x^2 + 3*A*b^6*x^2 + 36*B*a^2*b^4*x - 1
8*A*a*b^5*x)/(b^9*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{x^4\,\left(A+B\,x\right)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}} \,d x$$

input
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.29

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-12\log(bx+a)\,a^4-12\log(bx+a)\,a^3bx+12a^3bx+6a^2b^2x^2-2a\,b^3x^3+b^4}{3b^5\,(bx+a)}$$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x + 6*a**2
*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4)/(3*b**5*(a + b*x))
```

3.344 $\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 202

$$\begin{aligned}\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = & -\frac{a^2(3Ab-4aB)}{b^5\sqrt{a^2+2abx+b^2x^2}} \\ & +\frac{a^3(Ab-aB)}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-3aB)x(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} \\ & +\frac{Bx^2(a+bx)}{2b^3\sqrt{a^2+2abx+b^2x^2}}-\frac{3a(Ab-2aB)(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-a^2*(3*A*b-4*B*a)/b^5/((b*x+a)^2)^(1/2)+1/2*a^3*(A*b-B*a)/b^5/(b*x+a)/((b
*x+a)^2)^(1/2)+(A*b-3*B*a)*x*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+1/2*B*x^2*(b*x+
a)/b^3/((b*x+a)^2)^(1/2)-3*a*(A*b-2*B*a)*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{7a^4B+4ab^3x^2(A-Bx)+b^4x^3(2A+Bx)-a^2b^2x(4A+11Bx)+a^3(-5A}{2b^5(a+bx)\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(7*a^4*B + 4*a*b^3*x^2*(A - B*x) + b^4*x^3*(2*A + B*x) - a^2*b^2*x*(4*A +
11*B*x) + a^3*(-5*A*b + 2*b*B*x) + 6*a*(-(A*b) + 2*a*B)*(a + b*x)^2*Log[a
+ b*x])/(2*b^5*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{x^3(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^3(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{(a+bx)\int \left(\frac{(aB-Ab)a^3}{b^4(a+bx)^3}-\frac{(4aB-3Ab)a^2}{b^4(a+bx)^2}+\frac{3(2aB-Ab)a}{b^4(a+bx)}+\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{a^3(Ab-aB)}{2b^5(a+bx)^2}-\frac{a^2(3Ab-4aB)}{b^5(a+bx)}-\frac{3a(Ab-2aB)\log(a+bx)}{b^5}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^2}{2b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(((A*b - 3*a*B)*x)/b^4 + (B*x^2)/(2*b^3) + (a^3*(A*b - a*B))/(2
*b^5*(a + b*x)^2) - (a^2*(3*A*b - 4*a*B))/(b^5*(a + b*x)) - (3*a*(A*b - 2*
a*B)*Log[a + b*x])/b^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\sqrt{(bx+a)^2}\,(\frac{1}{2}Bb\,x^2+Abx-3Bax)}{(bx+a)b^4}+\frac{\sqrt{(bx+a)^2}\left((-3A\,a^2b+4B\,a^3)x-\frac{a^3(5Ab-7Ba)}{2b}\right)}{(bx+a)^3b^4}-\frac{3\sqrt{(bx+a)^2}\,a(Ab-2Ba)\ln(bx+a)}{(bx+a)b^5}$
default	$-\frac{(-b^4B\,x^4+6A\ln(bx+a)a\,b^3x^2-2A\,b^4x^3-12B\ln(bx+a)a^2b^2x^2+4Ba\,b^3x^3+12A\ln(bx+a)a^2b^2x-4Aa\,b^3x^2-24B\ln(bx+a)a^3bx}{2b^5\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)/b^4*(1/2*B*b*x^2+A*b*x-3*B*a*x)+((b*x+a)^2)^(1/2
)/(b*x+a)^3*((-3*A*a^2*b+4*B*a^3)*x-1/2*a^3*(5*A*b-7*B*a)/b)/b^4-3*((b*x+a
)^2)^(1/2)/(b*x+a)*a/b^5*(A*b-2*B*a)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{Bb^4x^4+7\,Ba^4-5\,Aa^3b-2\,(2\,Bab^3-Ab^4)x^3-(11\,Ba^2b^2-4\,Aab^3)x^2+2}{2}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(B*b^4*x^4 + 7*B*a^4 - 5*A*a^3*b - 2*(2*B*a*b^3 - A*b^4)*x^3 - (11*B*a
^2*b^2 - 4*A*a*b^3)*x^2 + 2*(B*a^3*b - 2*A*a^2*b^2)*x + 6*(2*B*a^4 - A*a^3
*b + (2*B*a^2*b^2 - A*a*b^3)*x^2 + 2*(2*B*a^3*b - A*a^2*b^2)*x)*log(b*x +
a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^3(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx &= \frac{Bx^3}{2\sqrt{b^2x^2+2\,abx+a^2}b^2} \\ &\quad - \frac{5\,Bax^2}{2\sqrt{b^2x^2+2\,abx+a^2}b^3} + \frac{Ax^2}{\sqrt{b^2x^2+2\,abx+a^2}b^2} + \frac{6\,Ba^2\log\left(x+\frac{a}{b}\right)}{b^5} \\ &\quad - \frac{3\,Aa\log\left(x+\frac{a}{b}\right)}{b^4} - \frac{5\,Ba^3}{\sqrt{b^2x^2+2\,abx+a^2}b^5} + \frac{2\,Aa^2}{\sqrt{b^2x^2+2\,abx+a^2}b^4} \\ &\quad + \frac{12\,Ba^3x}{b^6\left(x+\frac{a}{b}\right)^2} - \frac{6\,Aa^2x}{b^5\left(x+\frac{a}{b}\right)^2} + \frac{23\,Ba^4}{2\,b^7\left(x+\frac{a}{b}\right)^2} - \frac{11\,Aa^3}{2\,b^6\left(x+\frac{a}{b}\right)^2}\end{aligned}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 5/2*B*a*x^2/(sqrt(b^2*x^2
+ 2*a*b*x + a^2)*b^3) + A*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 6*B*a^
2*log(x + a/b)/b^5 - 3*A*a*log(x + a/b)/b^4 - 5*B*a^3/(sqrt(b^2*x^2 + 2*a*
b*x + a^2)*b^5) + 2*A*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) + 12*B*a^3*x
/(b^6*(x + a/b)^2) - 6*A*a^2*x/(b^5*(x + a/b)^2) + 23/2*B*a^4/(b^7*(x + a/
b)^2) - 11/2*A*a^3/(b^6*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{3\,(2\,Ba^2-Aab)\log\left(|bx+a|\right)}{b^5\mathrm{sgn}\left(bx+a\right)} + \frac{Bb^3x^2\mathrm{sgn}(bx+a)-6\,Bab^2x\mathrm{sgn}(bx+a)+2\,Ab^3x\mathrm{sgn}(bx+a)}{2\,b^6} + \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x}{2\,(bx+a)^2b^5\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
3*(2*B*a^2 - A*a*b)*log(abs(b*x + a))/(b^5*sgn(b*x + a)) + 1/2*(B*b^3*x^2*
sgn(b*x + a) - 6*B*a*b^2*x*sgn(b*x + a) + 2*A*b^3*x*sgn(b*x + a))/b^6 + 1/
2*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x)/((b*x + a)^2*b^5*s
gn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2)}^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3}{2b^4\,(bx+a)}$$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a*b**2*x**
2 + b**3*x**3)/(2*b**4*(a + b*x))
```

3.345 $\int \frac{x^2(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 154

$$\int \frac{x^2(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{a(2Ab-3aB)}{b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{a^2(Ab-aB)}{2b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-3aB)(a+bx)\log(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}}$$

output
```
a*(2*A*b-3*B*a)/b^4/((b*x+a)^2)^(1/2)-1/2*a^2*(A*b-B*a)/b^4/(b*x+a)/((b*x+
a)^2)^(1/2)+B*x*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+(A*b-3*B*a)*(b*x+a)*ln(b*x+a
)/b^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\frac{b\left(ax\sqrt{(a+bx)^2}(6a^3B+Ab^3x^2-ab^2x(A+Bx)+a^2(-2Ab+3bBx))-\sqrt{a^2}x(6a^4B-Ab^4x^3+ab^3Bx^3+a^2b^2}{a^2(a+bx)\left(a^2+abx-\sqrt{a^2}\sqrt{(a+bx)^2}\right)}}{}$$

input
```
Integrate[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((b*(a*x*Sqrt[(a + b*x)^2]*(6*a^3*B + A*b^3*x^2 - a*b^2*x*(A + B*x) + a^2*
(-2*A*b + 3*b*B*x)) - Sqrt[a^2]*x*(6*a^4*B - A*b^4*x^3 + a*b^3*B*x^3 + a^2
*b^2*x*(-3*A + 2*B*x) + a^3*(-2*A*b + 9*b*B*x))))/(a^2*(a + b*x)*(a^2 + a*
b*x - Sqrt[a^2]*Sqrt[(a + b*x)^2])) - 4*A*b*ArcTanh[(b*x)/(Sqrt[a^2] - Sqr
t[(a + b*x)^2])] + 12*a*B*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])])/
(2*b^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b^3(a+bx)\int \frac{x^2(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{x^2(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int\left(-\frac{(aB-Ab)a^2}{b^3(a+bx)^3}+\frac{(3aB-2Ab)a}{b^3(a+bx)^2}+\frac{B}{b^3}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^2(Ab-aB)}{2b^4(a+bx)^2}+\frac{a(2Ab-3aB)}{b^4(a+bx)}+\frac{(Ab-3aB)\log(a+bx)}{b^4}+\frac{Bx}{b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*((B*x)/b^3 - (a^2*(A*b - a*B))/(2*b^4*(a + b*x)^2) + (a*(2*A*b
 - 3*a*B))/(b^4*(a + b*x)) + ((A*b - 3*a*B)*Log[a + b*x])/b^4))/Sqrt[a^2 +
2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\sqrt{(bx+a)^2}\, Bx}{(bx+a)b^3}+\frac{\sqrt{(bx+a)^2}\left((2abA-3a^2B)x+\frac{a^2(3Ab-5Ba)}{2b}\right)}{(bx+a)^3 b^3}+\frac{\sqrt{(bx+a)^2}\,(Ab-3Ba)\ln(bx+a)}{(bx+a)b^4}$
default	$\frac{(2A\ln(bx+a)b^3x^2-6B\ln(bx+a)a\,b^2x^2+2x^3B\,b^3+4A\ln(bx+a)xa\,b^2-12B\ln(bx+a)x\,a^2b+4Ba\,b^2x^2+2A\ln(bx+a)a^2b+4Aa\,b^2x-}{2b^4\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*B/b^3*x+((b*x+a)^2)^(1/2)/(b*x+a)^3*((2*A*a*b-3*
B*a^2)*x+1/2*a^2*(3*A*b-5*B*a)/b)/b^3+((b*x+a)^2)^(1/2)/(b*x+a)/b^4*(A*b-3
*B*a)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.87

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\,Bb^3x^3+4\,Bab^2x^2-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x-2\,(3\,Ba^3-Aa}{2\,(b^6x^2+2\,ab^5x+a^2}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*B*b^3*x^3 + 4*B*a*b^2*x^2 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*
b^2)*x - 2*(3*B*a^3 - A*a^2*b + (3*B*a*b^2 - A*b^3)*x^2 + 2*(3*B*a^2*b - A
*a*b^2)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx &= \frac{Bx^2}{\sqrt{b^2x^2+2\,abx+a^2}b^2} \\ &- \frac{3\,Ba\log\left(x+\frac{a}{b}\right)}{b^4} + \frac{A\log\left(x+\frac{a}{b}\right)}{b^3} + \frac{2\,Ba^2}{\sqrt{b^2x^2+2\,abx+a^2}b^4} \\ &- \frac{6\,Ba^2x}{b^5\left(x+\frac{a}{b}\right)^2} + \frac{2\,Aax}{b^4\left(x+\frac{a}{b}\right)^2} - \frac{11\,Ba^3}{2\,b^6\left(x+\frac{a}{b}\right)^2} + \frac{3\,Aa^2}{2\,b^5\left(x+\frac{a}{b}\right)^2}\end{aligned}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
B*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) - 3*B*a*log(x + a/b)/b^4 + A*log
(x + a/b)/b^3 + 2*B*a^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^4) - 6*B*a^2*x/(b
^5*(x + a/b)^2) + 2*A*a*x/(b^4*(x + a/b)^2) - 11/2*B*a^3/(b^6*(x + a/b)^2)
 + 3/2*A*a^2/(b^5*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{Bx}{b^3\mathrm{sgn}\,(bx+a)} - \frac{(3\,Ba-Ab)\log{(|bx+a|)}}{b^4\mathrm{sgn}\,(bx+a)} - \frac{5\,Ba^3-3\,Aa^2b+2\,(3\,Ba^2b-2\,Aab^2)x}{2\,(bx+a)^2b^4\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
B*x/(b^3*sgn(b*x + a)) - (3*B*a - A*b)*log(abs(b*x + a))/(b^4*sgn(b*x + a)
) - 1/2*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x)/((b*x + a)^2*b
^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^2\,(A+B\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2)}^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{-2\log(bx+a)\,a^2-2\log(bx+a)\,abx+2abx+b^2x^2}{b^3\,(bx+a)}$$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x**2)/(b**
3*(a + b*x))
```

3.346 $\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{Ab-2aB}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{a(Ab-aB)}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{B(a+bx)\log(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-(A*b-2*B*a)/b^3/((b*x+a)^2)^(1/2)+1/2*a*(A*b-B*a)/b^3/(b*x+a)/((b*x+a)^2)
^(1/2)+B*(b*x+a)*ln(b*x+a)/b^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.65

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\frac{bx\left(2a^5B-a^3Ab^2x+3a^4bBx-aAb^4x^3+a^2b^3Bx^3-\sqrt{a^2}\sqrt{(a+bx)^2}(2a^3B+a^2bBx+Ab^3x^2-ab^2x(A+Bx\right.}{a^3(a+bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}}{2b^3}$$

input
```
Integrate[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((b*x*(2*a^5*B - a^3*A*b^2*x + 3*a^4*b*B*x - a*A*b^4*x^3 + a^2*b^3*B*x^3 -
 Sqrt[a^2]*Sqrt[(a + b*x)^2]*(2*a^3*B + a^2*b*B*x + A*b^3*x^2 - a*b^2*x*(A
 + B*x))))/(a^3*(a + b*x)*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2
]))) - 4*B*ArcTanh[(b*x)/(Sqrt[a^2] - Sqrt[(a + b*x)^2])])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^3(a+bx)\int \frac{x(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{B}{b^2(a+bx)}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{a(aB-Ab)}{b^2(a+bx)^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{Ab-2aB}{b^3(a+bx)}+\frac{a(Ab-aB)}{2b^3(a+bx)^2}+\frac{B\log(a+bx)}{b^3}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*((a*(A*b - a*B))/(2*b^3*(a + b*x)^2) - (A*b - 2*a*B)/(b^3*(a +
b*x)) + (B*Log[a + b*x])/b^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{(Ab-2Ba)x}{b^2}-\frac{a(Ab-3Ba)}{2b^3}\right)}{(bx+a)^3}+\frac{\sqrt{(bx+a)^2}\,B\ln(bx+a)}{(bx+a)b^3}$	75
default	$-\frac{\left(-2B\ln(bx+a)b^2x^2-4B\ln(bx+a)abx+2x\,b^2A-2B\ln(bx+a)a^2-4xabB+abA-3a^2B\right)(bx+a)}{2b^3\left((bx+a)^2\right)^{\frac{3}{2}}}$	83

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(-(A*b-2*B*a)/b^2*x-1/2*a*(A*b-3*B*a)/b^3)+((b
*x+a)^2)^(1/2)/(b*x+a)*B/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{3\,Ba^2-Aab+2\,(2\,Bab-Ab^2)x+2\,(Bb^2x^2+2\,Babx+Ba^2)\log\left(bx+a\right)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x + 2*(B*b^2*x^2 + 2*B*a*b*x +
B*a^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F]

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{x(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{B\log\left(x+\frac{a}{b}\right)}{b^3} - \frac{A}{\sqrt{b^2x^2+2\,abx+a^2}b^2} + \frac{2\,Bax}{b^4\left(x+\frac{a}{b}\right)^2} + \frac{3\,Ba^2}{2\,b^5\left(x+\frac{a}{b}\right)^2} + \frac{Aa}{2\,b^4\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
B*log(x + a/b)/b^3 - A/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 2*B*a*x/(b^4*
(x + a/b)^2) + 3/2*B*a^2/(b^5*(x + a/b)^2) + 1/2*A*a/(b^4*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{B\log\left(|bx+a|\right)}{b^3\text{sgn}\left(bx+a\right)} + \frac{2\left(2\,Ba-Ab\right)x+\frac{3\,Ba^2-Aab}{b}}{2\left(bx+a\right)^2b^2\text{sgn}\left(bx+a\right)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/(b^3*sgn(b*x + a)) + 1/2*(2*(2*B*a - A*b)*x + (3*B*a^2
 - A*a*b)/b)/((b*x + a)^2*b^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x\left(A+B\,x\right)}{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\log(bx+a)\,a+\log(bx+a)\,bx-bx}{b^2\,(bx+a)}$$

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(b**2*(a + b*x))
```

3.347 $\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{B}{b^2\sqrt{a^2+2abx+b^2x^2}}-\frac{Ab-aB}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-B/b^2/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)/b^2/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(69) = 138.

Time = 0.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.55

$$\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{x\left(a^3Abx+a^2b^2Bx^3+a^4(2A+Bx)+abx\left(-Ab^2x^2+\sqrt{a^2}A\sqrt{(a+bx)^2}+\sqrt{a^2}Bx\sqrt{(a+bx)^2}\right)-\sqrt{a^2}\right.}{2a^4(a+bx)\left(\sqrt{a^2}bx+a\left(\sqrt{a^2}-\sqrt{(a+bx)^2}\right)\right)}$$

input
```
Integrate[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
-1/2*(x*(a^3*A*b*x + a^2*b^2*B*x^3 + a^4*(2*A + B*x) + a*b*x*(-(A*b^2*x^2)
 + Sqrt[a^2]*A*Sqrt[(a + b*x)^2] + Sqrt[a^2]*B*x*Sqrt[(a + b*x)^2]) - Sqrt
[a^2]*Sqrt[(a + b*x)^2]*(A*b^2*x^2 + a^2*(2*A + B*x))))/(a^4*(a + b*x)*(Sq
rt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{(Ab-aB)\int \frac{1}{(a^2+2bxa+b^2x^2)^{3/2}}dx}{b}-\frac{B}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{1078}$$

$$-\frac{Ab-aB}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{B}{b^2\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
-(B/(b^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2])) - (A*b - a*B)/(2*b^2*(a + b*x)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{(bx+a)(2Bbx+Ab+Ba)}{2b^{2}\left((bx+a)^{2}\right)^{\frac{3}{2}}}$	32
default	$-\frac{(bx+a)(2Bbx+Ab+Ba)}{2b^{2}\left((bx+a)^{2}\right)^{\frac{3}{2}}}$	32
risch	$\frac{\sqrt{(bx+a)^{2}}\,\left(-\frac{Bx}{b}-\frac{Ab+Ba}{2b^{2}}\right)}{(bx+a)^{3}}$	38
orering	$-\frac{(2Bbx+Ab+Ba)(bx+a)}{2b^{2}(b^{2}x^{2}+2abx+a^{2})^{\frac{3}{2}}}$	41

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)*(2*B*b*x+A*b+B*a)/b^2/((b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{2\,Bbx+Ba+Ab}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*B*b*x + B*a + A*b)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [F]

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{A+Bx}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{B}{\sqrt{b^2x^2+2\,abx+a^2}b^2} + \frac{Ba}{2\,b^4\left(x+\frac{a}{b}\right)^2} - \frac{A}{2\,b^3\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-B/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*b^2) + 1/2*B*a/(b^4*(x + a/b)^2) - 1/2*A
/(b^3*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{2\,Bbx+Ba+Ab}{2\,(bx+a)^2b^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*B*b*x + B*a + A*b)/((b*x + a)^2*b^2*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,(A\,b+B\,a+2\,B\,b\,x)}{2\,b^2\,(a+b\,x)^3}$$

input
```
int((A + B*x)/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(A*b + B*a + 2*B*b*x))/(2*b^2*(a + b*x)^
3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
x/(a*(a + b*x))
```

3.348 $\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^{3/2}}\, dx$

Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^{3/2}}\, dx = \frac{A}{a^2\sqrt{a^2+2abx+b^2x^2}} + \frac{Ab-aB}{2ab(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{A(a+bx)\log(x)}{a^3\sqrt{a^2+2abx+b^2x^2}} - \frac{A(a+bx)\log(a+bx)}{a^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
A/a^2/((b*x+a)^2)^(1/2)+1/2*(A*b-B*a)/a/b/(b*x+a)/((b*x+a)^2)^(1/2)+A*(b*x
+a)*ln(x)/a^3/((b*x+a)^2)^(1/2)-A*(b*x+a)*ln(b*x+a)/a^3/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^{3/2}}\, dx = \frac{a(3aAb-a^2B+2Ab^2x)+2Ab(a+bx)^2\log(x)-2Ab(a+bx)^2\log(a+bx}{2a^3b(a+bx)\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(a*(3*a*A*b - a^2*B + 2*A*b^2*x) + 2*A*b*(a + b*x)^2*Log[x] - 2*A*b*(a + b
*x)^2*Log[a + b*x])/(2*a^3*b*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{A}{a^3x}-\frac{bA}{a^3(a+bx)}-\frac{bA}{a^2(a+bx)^2}+\frac{aB-Ab}{a(a+bx)^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{A\log(a+bx)}{a^3}+\frac{A\log(x)}{a^3}+\frac{A}{a^2(a+bx)}+\frac{Ab-aB}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(2*a*b*(a + b*x)^2) + A/(a^2*(a + b*x)) + (A*Log[x
])/a^3 - (A*Log[a + b*x])/a^3))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{xAb}{a^2}+\frac{3Ab-Ba}{2ab}\right)}{(bx+a)^3}+\frac{\sqrt{(bx+a)^2}\,A\ln(-x)}{(bx+a)a^3}-\frac{\sqrt{(bx+a)^2}\,A\ln(bx+a)}{(bx+a)a^3}$
default	$-\frac{(2A\ln(bx+a)b^3x^2-2A\ln(x)x^2b^3+4A\ln(bx+a)xa\,b^2-4A\ln(x)xa\,b^2+2A\ln(bx+a)a^2b-2A\ln(x)a^2b-2Aa\,b^2x-3A\,a^2b+B\,a^3)(b}{2b\,a^3\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(1/a^2*x*A*b+1/2*(3*A*b-B*a)/a/b)+((b*x+a)^2)^
(1/2)/(b*x+a)*A/a^3*ln(-x)-((b*x+a)^2)^(1/2)/(b*x+a)*A/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{2\,Aab^2x - Ba^3 + 3\,Aa^2b - 2\left(Ab^3x^2 + 2\,Aab^2x + Aa^2b\right)\log\left(bx+a\right) + 2\,(A}{2\left(a^3b^3x^2 + 2\,a^4b^2x + a^5b\right)}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*A*a*b^2*x - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*
b)*log(b*x + a) + 2*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*b)*log(x))/(a^3*b^3*x
^2 + 2*a^4*b^2*x + a^5*b)
```

Sympy [F]

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x\left(\left(a+bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(x*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{\left(-1\right)^{2\,abx+2\,a^2} A\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{|x|}\right)}{a^3} + \frac{A}{\sqrt{b^2x^2+2\,abx+a^2}a^2} - \frac{B}{2\,b^3\left(x+\frac{a}{b}\right)^2} + \frac{A}{2\,ab^2\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*A*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 + A/(sqrt
(b^2*x^2 + 2*a*b*x + a^2)*a^2) - 1/2*B/(b^3*(x + a/b)^2) + 1/2*A/(a*b^2*(x
 + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{A\log\left(|bx+a|\right)}{a^3\mathrm{sgn}\left(bx+a\right)}+\frac{A\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx+a\right)}+\frac{2\,Aab^2x-Ba^3+3\,Aa^2b}{2\left(bx+a\right)^2a^3b\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/(a^3*sgn(b*x + a)) + A*log(abs(x))/(a^3*sgn(b*x + a))
 + 1/2*(2*A*a*b^2*x - B*a^3 + 3*A*a^2*b)/((b*x + a)^2*a^3*b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx}{a^2\,(bx+a)}$$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b*x)/(a**2
*(a + b*x))
```

3.349 $\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\begin{aligned}\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = &-\frac{2Ab-aB}{a^3\sqrt{a^2+2abx+b^2x^2}} \\ &-\frac{Ab-aB}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{A(a+bx)}{a^3x\sqrt{a^2+2abx+b^2x^2}} \\ &-\frac{(3Ab-aB)(a+bx)\log(x)}{a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{(3Ab-aB)(a+bx)\log(a+bx)}{a^4\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-(2*A*b-B*a)/a^3/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)/a^2/(b*x+a)/((b*x+a)^2)^(
1/2)-A*(b*x+a)/a^3/x/((b*x+a)^2)^(1/2)-(3*A*b-B*a)*(b*x+a)*ln(x)/a^4/((b*x
+a)^2)^(1/2)+(3*A*b-B*a)*(b*x+a)*ln(b*x+a)/a^4/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{a(-6Ab^2x^2+abx(-9A+2Bx)+a^2(-2A+3Bx))+2(-3Ab+aB)x(a}{2a^4x(a+bx)\sqrt{(a+bx}}$$

input
```
Integrate[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(a*(-6*A*b^2*x^2 + a*b*x*(-9*A + 2*B*x) + a^2*(-2*A + 3*B*x)) + 2*(-3*A*b
+ a*B)*x*(a + b*x)^2*Log[x] + 2*(3*A*b - a*B)*x*(a + b*x)^2*Log[a + b*x])/
(2*a^4*x*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2 (a^2+2abx+b^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x^2(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^2(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{(a+bx)\int \left(\frac{A}{a^3x^2}+\frac{aB-3Ab}{a^4x}-\frac{b(aB-3Ab)}{a^4(a+bx)}-\frac{b(aB-2Ab)}{a^3(a+bx)^2}-\frac{b(aB-Ab)}{a^2(a+bx)^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)\left(-\frac{\log(x)(3Ab-aB)}{a^4}+\frac{(3Ab-aB)\log(a+bx)}{a^4}-\frac{2Ab-aB}{a^3(a+bx)}-\frac{A}{a^3x}-\frac{Ab-aB}{2a^2(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(-(A/(a^3*x)) - (A*b - a*B)/(2*a^2*(a + b*x)^2) - (2*A*b - a*B)
/(a^3*(a + b*x)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x
])/a^4))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(-\frac{b\left(3Ab-Ba\right)x^{2}}{a^{3}}-\frac{3\left(3Ab-Ba\right)x}{2a^{2}}-\frac{A}{a}\right)}{\left(bx+a\right)^{3}x}+\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Ab-Ba\right)\ln\left(-bx-a\right)}{\left(bx+a\right)a^{4}}-\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(3Ab-Ba\right)\ln\left(x\right)}{\left(bx+a\right)a^{4}}$
default	$\frac{(6A\ln(bx+a)x^{3}b^{3}-6A\ln(x)x^{3}b^{3}-2B\ln(bx+a)a\,b^{2}x^{3}+2B\ln(x)a\,b^{2}x^{3}+12A\ln(bx+a)x^{2}a\,b^{2}-12A\ln(x)x^{2}a\,b^{2}-4B\ln(bx+a)a^{2}b\,x}{2x}$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(-b*(3*A*b-B*a)/a^3*x^2-3/2/a^2*(3*A*b-B*a)*x-
A/a)/x+((b*x+a)^2)^(1/2)/(b*x+a)*(3*A*b-B*a)/a^4*ln(-b*x-a)-((b*x+a)^2)^(1
/2)/(b*x+a)*(3*A*b-B*a)/a^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{2\,Aa^3-2\,(Ba^2b-3\,Aab^2)x^2-3\,(Ba^3-3\,Aa^2b)x+2\,((Bab^2-3\,Ab^3)x^3+2\,(Ba^2b-3\,Aab^2)x^2+(Ba^3}{2\,(a^4b^2x^3+2\,a^5}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*A*a^3 - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - 3*(B*a^3 - 3*A*a^2*b)*x + 2*
((B*a*b^2 - 3*A*b^3)*x^3 + 2*(B*a^2*b - 3*A*a*b^2)*x^2 + (B*a^3 - 3*A*a^2*
b)*x)*log(b*x + a) - 2*((B*a*b^2 - 3*A*b^3)*x^3 + 2*(B*a^2*b - 3*A*a*b^2)*
x^2 + (B*a^3 - 3*A*a^2*b)*x)*log(x))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x^2\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**2/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**2*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}\,B\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^3} + \frac{3\,(-1)^{2\,abx+2\,a^2}\,Ab\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^4} + \frac{B}{\sqrt{b^2x^2+2\,abx+a^2}a^2} - \frac{3\,Ab}{\sqrt{b^2x^2+2\,abx+a^2}a^3} - \frac{A}{\sqrt{b^2x^2+2\,abx+a^2}a^2x} + \frac{B}{2\,ab^2\left(x+\frac{a}{b}\right)^2} - \frac{A}{2\,a^2b\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*B*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^3 + 3*(-1)^
(2*a*b*x + 2*a^2)*A*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 + B/(sqrt(b^2
*x^2 + 2*a*b*x + a^2)*a^2) - 3*A*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3) - A
/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x) + 1/2*B/(a*b^2*(x + a/b)^2) - 1/2*A
/(a^2*b*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{(Ba-3\,Ab)\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx+a\right)} - \frac{(Bab-3\,Ab^2)\log\left(|bx+a|\right)}{a^4b\mathrm{sgn}\left(bx+a\right)} - \frac{2\,Aa^3-2\,(Ba^2b-3\,Aab^2)x^2-3\,(Ba^3-3\,Aa^2b)x}{2\,(bx+a)^2a^4x\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*log(abs(x))/(a^4*sgn(b*x + a)) - (B*a*b - 3*A*b^2)*log(abs(b
*x + a))/(a^4*b*sgn(b*x + a)) - 1/2*(2*A*a^3 - 2*(B*a^2*b - 3*A*a*b^2)*x^2
 - 3*(B*a^3 - 3*A*a^2*b)*x)/((b*x + a)^2*a^4*x*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^{3/2}}\,dx=\int \frac{A+B\,x}{x^2\,(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2}{a^3x\,(bx+a)}$$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.350 $\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 243

$$\begin{aligned}\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx &= \frac{b(3Ab-2aB)}{a^4\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{b(Ab-aB)}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{A(a+bx)}{2a^3x^2\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{(3Ab-aB)(a+bx)}{a^4x\sqrt{a^2+2abx+b^2x^2}} + \frac{3b(2Ab-aB)(a+bx)\log(x)}{a^5\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{3b(2Ab-aB)(a+bx)\log(a+bx)}{a^5\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
b*(3*A*b-2*B*a)/a^4/((b*x+a)^2)^(1/2)+1/2*b*(A*b-B*a)/a^3/(b*x+a)/((b*x+a)
^2)^(1/2)-1/2*A*(b*x+a)/a^3/x^2/((b*x+a)^2)^(1/2)+(3*A*b-B*a)*(b*x+a)/a^4/
x/((b*x+a)^2)^(1/2)+3*b*(2*A*b-B*a)*(b*x+a)*ln(x)/a^5/((b*x+a)^2)^(1/2)-3*
b*(2*A*b-B*a)*(b*x+a)*ln(b*x+a)/a^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{-a(-12Ab^3x^3+6ab^2x^2(-3A+Bx)+a^3(A+2Bx)+a^2bx(-4A+9Bx}{2a^5x^2(a+}$$

input
```
Integrate[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(-(a*(-12*A*b^3*x^3 + 6*a*b^2*x^2*(-3*A + B*x) + a^3*(A + 2*B*x) + a^2*b*x
*(-4*A + 9*B*x))) + 6*b*(2*A*b - a*B)*x^2*(a + b*x)^2*Log[x] + 6*b*(-2*A*b
 + a*B)*x^2*(a + b*x)^2*Log[a + b*x])/(2*a^5*x^2*(a + b*x)*Sqrt[(a + b*x)^
2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{(a+bx)\int \left(\frac{3(aB-2Ab)b^2}{a^5(a+bx)}+\frac{(2aB-3Ab)b^2}{a^4(a+bx)^2}+\frac{(aB-Ab)b^2}{a^3(a+bx)^3}-\frac{3(aB-2Ab)b}{a^5x}+\frac{aB-3Ab}{a^4x^2}+\frac{A}{a^3x^3}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{3b(2Ab-aB)\log(a+bx)}{a^5}+\frac{3Ab-aB}{a^4x}+\frac{b(3Ab-2aB)}{a^4(a+bx)}+\frac{b(Ab-aB)}{2a^3(a+bx)^2}-\frac{A}{2a^3x^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^3*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(-1/2*A/(a^3*x^2) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B))/(2*
a^3*(a + b*x)^2) + (b*(3*A*b - 2*a*B))/(a^4*(a + b*x)) + (3*b*(2*A*b - a*B
)*Log[x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x])/a^5))/Sqrt[a^2 + 2*a*b*x
+ b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{3b^2(2Ab-Ba)x^3}{a^4}+\frac{9b(2Ab-Ba)x^2}{2a^3}+\frac{(2Ab-Ba)x}{a^2}-\frac{A}{2a}\right)}{(bx+a)^3x^2}-\frac{3\sqrt{(bx+a)^2}\,b(2Ab-Ba)\ln(bx+a)}{(bx+a)a^5}+\frac{3\sqrt{(bx+a)^2}\,b(2Ab-Ba}{(bx+a)a^5}$
default	$-\frac{(12A\ln(bx+a)x^4b^4-12A\ln(x)x^4b^4-6B\ln(bx+a)x^4ab^3+6B\ln(x)x^4ab^3+24A\ln(bx+a)x^3ab^3-24A\ln(x)x^3ab^3-12B\ln(bx+a}{}$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^3*(3*b^2*(2*A*b-B*a)/a^4*x^3+9/2*b*(2*A*b-B*a)/a
^3*x^2+(2*A*b-B*a)/a^2*x-1/2*A/a)/x^2-3*((b*x+a)^2)^(1/2)/(b*x+a)*b*(2*A*b
-B*a)/a^5*ln(b*x+a)+3*((b*x+a)^2)^(1/2)/(b*x+a)*b*(2*A*b-B*a)/a^5*ln(-x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx =$$
$$-\frac{Aa^4+6\,(Ba^2b^2-2\,Aab^3)x^3+9\,(Ba^3b-2\,Aa^2b^2)x^2+2\,(Ba^4-2\,Aa^3b)x-6\,((Bab^3-2\,Ab^4)x^4+2\,(B}{2}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(A*a^4 + 6*(B*a^2*b^2 - 2*A*a*b^3)*x^3 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^
2 + 2*(B*a^4 - 2*A*a^3*b)*x - 6*((B*a*b^3 - 2*A*b^4)*x^4 + 2*(B*a^2*b^2 -
2*A*a*b^3)*x^3 + (B*a^3*b - 2*A*a^2*b^2)*x^2)*log(b*x + a) + 6*((B*a*b^3 -
 2*A*b^4)*x^4 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^3 + (B*a^3*b - 2*A*a^2*b^2)*x^
2)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{A+Bx}{x^3\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**3/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**3*((a + b*x)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^3(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{3\,(-1)^{2\,abx+2\,a^2}\,Bb\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^4} - \frac{6\,(-1)^{2\,abx+2\,a^2}\,Ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^5} - \frac{3\,Bb}{\sqrt{b^2x^2+2\,abx+a^2}a^3} + \frac{6\,Ab^2}{\sqrt{b^2x^2+2\,abx+a^2}a^4} - \frac{B}{\sqrt{b^2x^2+2\,abx+a^2}a^2x} + \frac{5\,Ab}{2\,\sqrt{b^2x^2+2\,abx+a^2}a^3x} - \frac{A}{2\,\sqrt{b^2x^2+2\,abx+a^2}a^2x^2} + \frac{A}{2\,a^3\left(x+\frac{a}{b}\right)^2} - \frac{B}{2\,a^2b\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima")
```

output
```
3*(-1)^(2*a*b*x + 2*a^2)*B*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^4 - 6*(-
1)^(2*a*b*x + 2*a^2)*A*b^2*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 - 3*B*b/
(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^3) + 6*A*b^2/(sqrt(b^2*x^2 + 2*a*b*x + a^
2)*a^4) - B/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x) + 5/2*A*b/(sqrt(b^2*x^2
+ 2*a*b*x + a^2)*a^3*x) - 1/2*A/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^2*x^2) +
1/2*A/(a^3*(x + a/b)^2) - 1/2*B/(a^2*b*(x + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = -\frac{3\,(Bab-2\,Ab^2)\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx+a\right)} + \frac{3\,(Bab^2-2\,Ab^3)\log\left(|bx+a|\right)}{a^5b\mathrm{sgn}\left(bx+a\right)} - \frac{6\,Bab^2x^3-12\,Ab^3x^3+9\,Ba^2bx^2-18\,Aab^2x^2+2\,Ba^3x-4\,Aa^2bx+Aa^3}{2\,(bx^2+ax)^2a^4\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-3*(B*a*b - 2*A*b^2)*log(abs(x))/(a^5*sgn(b*x + a)) + 3*(B*a*b^2 - 2*A*b^3
)*log(abs(b*x + a))/(a^5*b*sgn(b*x + a)) - 1/2*(6*B*a*b^2*x^3 - 12*A*b^3*x
^3 + 9*B*a^2*b*x^2 - 18*A*a*b^2*x^2 + 2*B*a^3*x - 4*A*a^2*b*x + A*a^3)/((b
*x^2 + a*x)^2*a^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{A+B\,x}{x^3\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^3*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx}{x^3\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{-6\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a\,b^2x^2+6\log(x)\,b^3x}{2a^4x^2\,(bx+a)}$$

input
```
int((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a*b**
2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3)/(2*a**4*x**
2*(a + b*x))
```

3.351 $\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 245

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}} \, dx = \frac{2a(2Ab-5aB)}{b^6\sqrt{a^2+2abx+b^2x^2}} - \frac{a^4(Ab-aB)}{4b^6(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{a^3(4Ab-5aB)}{3b^6(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{a^2(3Ab-5aB)}{b^6(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{Bx(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-5aB)(a+bx)\log(a+bx)}{b^6\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*a*(2*A*b-5*B*a)/b^6/((b*x+a)^2)^(1/2)-1/4*a^4*(A*b-B*a)/b^6/(b*x+a)^3/((
b*x+a)^2)^(1/2)+1/3*a^3*(4*A*b-5*B*a)/b^6/(b*x+a)^2/((b*x+a)^2)^(1/2)-a^2*
(3*A*b-5*B*a)/b^6/(b*x+a)/((b*x+a)^2)^(1/2)+B*x*(b*x+a)/b^5/((b*x+a)^2)^(1
/2)+(A*b-5*B*a)*(b*x+a)*ln(b*x+a)/b^6/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.52

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-77a^5B+12b^5Bx^5+a^4b(25A-248Bx)+4a^3b^2x(22A-63Bx)+12a^2b^3\cdots}{12b^6(a+bx)^3\cdots}$$

input
```
Integrate[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(-77*a^5*B + 12*b^5*B*x^5 + a^4*b*(25*A - 248*B*x) + 4*a^3*b^2*x*(22*A - 6
3*B*x) + 12*a^2*b^3*x^2*(9*A - 4*B*x) + 48*a*b^4*x^3*(A + B*x) + 12*(A*b -
 5*a*B)*(a + b*x)^4*Log[a + b*x])/(12*b^6*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{x^4(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^4(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(-\frac{(aB-Ab)a^4}{b^5(a+bx)^5}+\frac{(5aB-4Ab)a^3}{b^5(a+bx)^4}-\frac{2(5aB-3Ab)a^2}{b^5(a+bx)^3}+\frac{2(5aB-2Ab)a}{b^5(a+bx)^2}+\frac{B}{b^5}+\frac{Ab-5aB}{b^5(a+bx)}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{a^4(Ab-aB)}{4b^6(a+bx)^4}+\frac{a^3(4Ab-5aB)}{3b^6(a+bx)^3}-\frac{a^2(3Ab-5aB)}{b^6(a+bx)^2}+\frac{2a(2Ab-5aB)}{b^6(a+bx)}+\frac{(Ab-5aB)\log(a+bx)}{b^6}+\frac{Bx}{b^5}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^4*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*((B*x)/b^5 - (a^4*(A*b - a*B))/(4*b^6*(a + b*x)^4) + (a^3*(4*A*
b - 5*a*B))/(3*b^6*(a + b*x)^3) - (a^2*(3*A*b - 5*a*B))/(b^6*(a + b*x)^2)
+ (2*a*(2*A*b - 5*a*B))/(b^6*(a + b*x)) + ((A*b - 5*a*B)*Log[a + b*x])/b^6
))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\sqrt{(bx+a)^{2}}\, Bx}{(bx+a)b^{5}}+\frac{\sqrt{(bx+a)^{2}}\left(\left(4Aa\, b^{3}-10B\, a^{2}b^{2}\right)x^{3}+a^{2}b(9Ab-25Ba)x^{2}+\left(\frac{22}{3}A\, a^{3}b-\frac{65}{3}a^{4}B\right)x+\frac{a^{4}(25Ab-77Ba)}{12b}\right)}{(bx+a)^{5}b^{5}}+\frac{\sqrt{(bx+a}}{}$
default	$\frac{(12A\ln(bx+a)b^{5}x^{4}-60B\ln(bx+a)a\, b^{4}x^{4}+12B\, x^{5}b^{5}+48A\ln(bx+a)x^{3}a\, b^{4}-240B\ln(bx+a)x^{3}a^{2}b^{3}+48Ba\, b^{4}x^{4}+72A\ln(bx+a)x^{2}a}{}$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)*B/b^5*x+((b*x+a)^2)^(1/2)/(b*x+a)^5*((4*A*a*b^3-
10*B*a^2*b^2)*x^3+a^2*b*(9*A*b-25*B*a)*x^2+(22/3*A*a^3*b-65/3*a^4*B)*x+1/1
2*a^4*(25*A*b-77*B*a)/b)/b^5+((b*x+a)^2)^(1/2)/(b*x+a)/b^6*(A*b-5*B*a)*ln(
b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.03

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{12\,Bb^5x^5+48\,Bab^4x^4-77\,Ba^5+25\,Aa^4b-48\,(Ba^2b^3-Aab^4)x^3-36\,(7\,B}{}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(12*B*b^5*x^5 + 48*B*a*b^4*x^4 - 77*B*a^5 + 25*A*a^4*b - 48*(B*a^2*b^
3 - A*a*b^4)*x^3 - 36*(7*B*a^3*b^2 - 3*A*a^2*b^3)*x^2 - 8*(31*B*a^4*b - 11
*A*a^3*b^2)*x - 12*(5*B*a^5 - A*a^4*b + (5*B*a*b^4 - A*b^5)*x^4 + 4*(5*B*a
^2*b^3 - A*a*b^4)*x^3 + 6*(5*B*a^3*b^2 - A*a^2*b^3)*x^2 + 4*(5*B*a^4*b - A
*a^3*b^2)*x)*log(b*x + a))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3
*b^7*x + a^4*b^6)
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \int \frac{x^4(A+Bx)}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**4*(A + B*x)/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.86

$$\int \frac{x^4(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{1}{12}\,B\left(\frac{12\,b^5x^5+48\,ab^4x^4-48\,a^2b^3x^3-252\,a^3b^2x^2-248\,a^4bx-77\,a^5}{b^{10}x^4+4\,ab^9x^3+6\,a^2b^8x^2+4\,a^3b^7x+a^4b^6} - 60 \right. \\ + \frac{1}{12}\,A\left(\frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4}{b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5}+\frac{12\,\log\left(bx+a\right)}{b^5}\right)$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*B*((12*b^5*x^5 + 48*a*b^4*x^4 - 48*a^2*b^3*x^3 - 252*a^3*b^2*x^2 - 24
8*a^4*b*x - 77*a^5)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x
+ a^4*b^6) - 60*a*log(b*x + a)/b^6) + 1/12*A*((48*a*b^3*x^3 + 108*a^2*b^2*
x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*
b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.59

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{Bx}{b^5\mathrm{sgn}\,(bx+a)} - \frac{(5\,Ba-Ab)\log\left(|bx+a|\right)}{b^6\mathrm{sgn}\,(bx+a)} - \frac{77\,Ba^5-25\,Aa^4b+24\,(5\,Ba^2b^3-2\,Aab^4)x^3+12\,(25\,Ba^3b^2-9\,Aa^2b^3)x^2+4\,(65\,Ba^4b-22\,Aa^3b^2)x}{12\,(bx+a)^4b^6\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
B*x/(b^5*sgn(b*x + a)) - (5*B*a - A*b)*log(abs(b*x + a))/(b^6*sgn(b*x + a)
) - 1/12*(77*B*a^5 - 25*A*a^4*b + 24*(5*B*a^2*b^3 - 2*A*a*b^4)*x^3 + 12*(2
5*B*a^3*b^2 - 9*A*a^2*b^3)*x^2 + 4*(65*B*a^4*b - 22*A*a^3*b^2)*x)/((b*x +
a)^4*b^6*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^4\,(A+B\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^4*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.49

$$\int \frac{x^4(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-12\log(bx+a)\,a^4-36\log(bx+a)\,a^3bx-36\log(bx+a)\,a^2b^2x^2-12\log(b}{3b^5\left(b^3x^3+3a\,b^2x^2+3a^2bx+\right.}$$

input
```
int(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 12*log(a + b*x)*a**4 - 36*log(a + b*x)*a**3*b*x - 36*log(a + b*x)*a**2
*b**2*x**2 - 12*log(a + b*x)*a*b**3*x**3 - 10*a**4 - 18*a**3*b*x + 12*a*b*
*3*x**3 + 3*b**4*x**4)/(3*b**5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3))
```

3.352 $\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{3aB}{b^5\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-aB)x^4}{4ab(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{a^3B}{3b^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{3a^2B}{2b^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{B(a+bx)\log(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
3*a*B/b^5/((b*x+a)^2)^(1/2)+1/4*(A*b-B*a)*x^4/a/b/(b*x+a)^3/((b*x+a)^2)^(1
/2)+1/3*a^3*B/b^5/(b*x+a)^2/((b*x+a)^2)^(1/2)-3/2*a^2*B/b^5/(b*x+a)/((b*x+
a)^2)^(1/2)+B*(b*x+a)*ln(b*x+a)/b^5/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.55

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{25a^4B-12Ab^4x^3-12a^2b^2x(A-9Bx)+6ab^3x^2(-3A+8Bx)+a^3(-3Ab}{12b^5(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(25*a^4*B - 12*A*b^4*x^3 - 12*a^2*b^2*x*(A - 9*B*x) + 6*a*b^3*x^2*(-3*A +
8*B*x) + a^3*(-3*A*b + 88*b*B*x) + 12*B*(a + b*x)^4*Log[a + b*x])/(12*b^5*
(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1187, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^5(a+bx)\int \frac{x^3(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^3(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{B\int \frac{x^3}{(a+bx)^4}dx}{b}+\frac{x^4(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx)\left(\frac{B\int\left(-\frac{a^3}{b^3(a+bx)^4}+\frac{3a^2}{b^3(a+bx)^3}-\frac{3a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx}{b}+\frac{x^4(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(\frac{B\left(\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}\right)}{b}+\frac{x^4(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^3*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^4)/(4*a*b*(a + b*x)^4) + (B*(a^3/(3*b^4*(a + b*
x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x)) + Log[a + b*x]
/b^4))/b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{(Ab-4Ba)x^3}{b^2}-\frac{3a(Ab-6Ba)x^2}{2b^3}-\frac{a^2(3Ab-22Ba)x}{3b^4}-\frac{a^3(3Ab-25Ba)}{12b^5}\right)}{(bx+a)^5}+\frac{\sqrt{(bx+a)^2}\,B\ln(bx+a)}{(bx+a)b^5}$
default	$-\frac{(-12B\ln(bx+a)b^4x^4-48B\ln(bx+a)a\,b^3x^3+12A\,b^4x^3-72B\ln(bx+a)a^2b^2x^2-48Ba\,b^3x^3+18Aa\,b^3x^2-48B\ln(bx+a)a^3bx-108}{12b^5\left((bx+a)^2\right)^{\frac{5}{2}}}$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(-(A*b-4*B*a)/b^2*x^3-3/2*a*(A*b-6*B*a)/b^3*x^
2-1/3*a^2*(3*A*b-22*B*a)/b^4*x-1/12*a^3*(3*A*b-25*B*a)/b^5)+((b*x+a)^2)^(1
/2)/(b*x+a)*B/b^5*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{25\,Ba^4-3\,Aa^3b+12\,(4\,Bab^3-Ab^4)x^3+18\,(6\,Ba^2b^2-Aab^3)x^2+4\,(22\,B}{12\,(b^9x^4+4\,ab^8x^3}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(25*B*a^4 - 3*A*a^3*b + 12*(4*B*a*b^3 - A*b^4)*x^3 + 18*(6*B*a^2*b^2
- A*a*b^3)*x^2 + 4*(22*B*a^3*b - 3*A*a^2*b^2)*x + 12*(B*b^4*x^4 + 4*B*a*b^
3*x^3 + 6*B*a^2*b^2*x^2 + 4*B*a^3*b*x + B*a^4)*log(b*x + a))/(b^9*x^4 + 4*
a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5)
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\int \frac{x^3(A+Bx)}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**3*(A + B*x)/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{1}{12}\,B\left(\frac{48\,ab^3x^3+108\,a^2b^2x^2+88\,a^3bx+25\,a^4}{b^9x^4+4\,ab^8x^3+6\,a^2b^7x^2+4\,a^3b^6x+a^4b^5}+\frac{12\,\log\left(bx+a\right)}{b^5}\right)-\frac{1}{12}\,A\left(\frac{12\,x^2}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^2}+\frac{8\,a^2}{\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}b^4}+\frac{6\,a}{b^6\left(x+\frac{a}{b}\right)^2}-\frac{8\,a^2}{b^7\left(x+\frac{a}{b}\right)^3}-\frac{3\,a^3}{b^8\left(x+\frac{a}{b}\right)^4}\right)$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*B*((48*a*b^3*x^3 + 108*a^2*b^2*x^2 + 88*a^3*b*x + 25*a^4)/(b^9*x^4 +
4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5) + 12*log(b*x + a)/b^5
) - 1/12*A*(12*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 8*a^2/((b^2*x^2
 + 2*a*b*x + a^2)^(3/2)*b^4) + 6*a/(b^6*(x + a/b)^2) - 8*a^2/(b^7*(x + a/b
)^3) - 3*a^3/(b^8*(x + a/b)^4))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{B\log(|bx+a|)}{b^5\operatorname{sgn}(bx+a)} + \frac{12\,(4\,Bab^2-Ab^3)x^3+18\,(6\,Ba^2b-Aab^2)x^2+4\,(22\,Ba^3-3\,Aa^2b)x+\frac{25\,Ba^4-3\,Aa^3b}{b}}{12\,(bx+a)^4b^4\operatorname{sgn}(bx+a)}$$

input
```
integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/(b^5*sgn(b*x + a)) + 1/12*(12*(4*B*a*b^2 - A*b^3)*x^3
+ 18*(6*B*a^2*b - A*a*b^2)*x^2 + 4*(22*B*a^3 - 3*A*a^2*b)*x + (25*B*a^4 -
3*A*a^3*b)/b)/((b*x + a)^4*b^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2)}^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^3*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{6\log(bx+a)\,a^3+18\log(bx+a)\,a^2bx+18\log(bx+a)\,a\,b^2x^2+6\log(bx+a}{6b^4\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(6*log(a + b*x)*a**3 + 18*log(a + b*x)*a**2*b*x + 18*log(a + b*x)*a*b**2*x
**2 + 6*log(a + b*x)*b**3*x**3 + 5*a**3 + 9*a**2*b*x - 6*b**3*x**3)/(6*b**
4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.353 $\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=-\frac{(Ab-aB)x^4(a+bx)}{4a^2\left(a^2+2abx+b^2x^2\right)^{5/2}}+\frac{Ax^3}{3a^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

output
```
-1/4*(A*b-B*a)*x^4*(b*x+a)/a^2/(b^2*x^2+2*a*b*x+a^2)^(5/2)+1/3*A*x^3/a^2/(
b^2*x^2+2*a*b*x+a^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{-3a^3B-6b^3x^2(A+2Bx)-2ab^2x(2A+9Bx)-a^2b(A+12Bx)}{12b^4(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(-3*a^3*B - 6*b^3*x^2*(A + 2*B*x) - 2*a*b^2*x*(2*A + 9*B*x) - a^2*b*(A + 1
2*B*x))/(12*b^4*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1186, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1186}$$

$$\frac{Ax^3}{3a^2(a^2+2abx+b^2x^2)^{3/2}}-\frac{(Ab-aB)\int \frac{x^3}{(a^2+2bxa+b^2x^2)^{5/2}}dx}{a}$$

$$\Big\downarrow \text{1102}$$

$$\frac{Ax^3}{3a^2(a^2+2abx+b^2x^2)^{3/2}}-\frac{b^5(a+bx)(Ab-aB)\int \frac{x^3}{b^5(a+bx)^5}dx}{a\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{Ax^3}{3a^2(a^2+2abx+b^2x^2)^{3/2}}-\frac{(a+bx)(Ab-aB)\int \frac{x^3}{(a+bx)^5}dx}{a\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{Ax^3}{3a^2(a^2+2abx+b^2x^2)^{3/2}}-\frac{x^4(Ab-aB)}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^2*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(A*x^3)/(3*a^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) - ((A*b - a*B)*x^4)/(4*a^2
*(a + b*x)^3*Sqrt[a^2 + 2*a*b*x + b^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1186
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[-2*c*(e*f - d*g)*(d + e*x)^(m + 1)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e)^2)), x] + Simp[(2*c*f - b*g)/
(2*c*d - b*e)   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[
{a, b, c, d, e, f, g, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && EqQ[m + 2*p + 3,
0] && NeQ[2*c*f - b*g, 0] && NeQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\sqrt{(bx+a)^{2}}\,\left(-\frac{B\,x^{3}}{b}-\frac{(Ab+3Ba)x^{2}}{2b^{2}}-\frac{a(Ab+3Ba)x}{3b^{3}}-\frac{a^{2}(Ab+3Ba)}{12b^{4}}\right)}{(bx+a)^{5}}$	75
gosper	$-\frac{(bx+a)\left(12x^{3}B\,b^{3}+6A\,b^{3}x^{2}+18Ba\,b^{2}x^{2}+4Aa\,b^{2}x+12B\,a^{2}bx+A\,a^{2}b+3B\,a^{3}\right)}{12b^{4}\left((bx+a)^{2}\right)^{\frac{5}{2}}}$	77
default	$-\frac{(bx+a)\left(12x^{3}B\,b^{3}+6A\,b^{3}x^{2}+18Ba\,b^{2}x^{2}+4Aa\,b^{2}x+12B\,a^{2}bx+A\,a^{2}b+3B\,a^{3}\right)}{12b^{4}\left((bx+a)^{2}\right)^{\frac{5}{2}}}$	77
orering	$-\frac{\left(12x^{3}B\,b^{3}+6A\,b^{3}x^{2}+18Ba\,b^{2}x^{2}+4Aa\,b^{2}x+12B\,a^{2}bx+A\,a^{2}b+3B\,a^{3}\right)(bx+a)}{12b^{4}(b^{2}x^{2}+2abx+a^{2})^{\frac{5}{2}}}$	86

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(-B*x^3/b-1/2*(A*b+3*B*a)/b^2*x^2-1/3*a*(A*b+3
*B*a)/b^3*x-1/12*a^2*(A*b+3*B*a)/b^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{12\,Bb^3x^3+3\,Ba^3+Aa^2b+6\,(3\,Bab^2+Ab^3)x^2+4\,(3\,Ba^2b+Aab^2)x}{12\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(12*B*b^3*x^3 + 3*B*a^3 + A*a^2*b + 6*(3*B*a*b^2 + A*b^3)*x^2 + 4*(3
*B*a^2*b + A*a*b^2)*x)/(b^8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*
x + a^4*b^4)
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^2(A+Bx)}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x**2*(A + B*x)/((a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(67) = 134$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{Bx^2}{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2} - \frac{2\,Ba^2}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^4} - \frac{Ba}{2\,b^6\left(x+\frac{a}{b}\right)^2} - \frac{A}{2\,b^5\left(x+\frac{a}{b}\right)^2} + \frac{2\,Ba^2}{3\,b^7\left(x+\frac{a}{b}\right)^3} + \frac{2\,Aa}{3\,b^6\left(x+\frac{a}{b}\right)^3} + \frac{Ba^3}{4\,b^8\left(x+\frac{a}{b}\right)^4} - \frac{Aa^2}{4\,b^7\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-B*x^2/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 2/3*B*a^2/((b^2*x^2 + 2*a*b
*x + a^2)^(3/2)*b^4) - 1/2*B*a/(b^6*(x + a/b)^2) - 1/2*A/(b^5*(x + a/b)^2)
 + 2/3*B*a^2/(b^7*(x + a/b)^3) + 2/3*A*a/(b^6*(x + a/b)^3) + 1/4*B*a^3/(b^
8*(x + a/b)^4) - 1/4*A*a^2/(b^7*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{12\,Bb^3x^3+18\,Bab^2x^2+6\,Ab^3x^2+12\,Ba^2bx+4\,Aab^2x+3\,Ba^3+Aa^2b}{12\,(bx+a)^4b^4\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(12*B*b^3*x^3 + 18*B*a*b^2*x^2 + 6*A*b^3*x^2 + 12*B*a^2*b*x + 4*A*a*
b^2*x + 3*B*a^3 + A*a^2*b)/((b*x + a)^4*b^4*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.68

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\left(\frac{B\,a^2-A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{A\,b^2-B\,a\,b}{3\,b^4}-\frac{B\,a}{3\,b^3}\right)}{b}\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{{\left(a+b\,x\right)}^4} - \frac{\left(\frac{A\,b-2\,B\,a}{2\,b^4}-\frac{B\,a}{2\,b^4}\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{{\left(a+b\,x\right)}^3} - \frac{B\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^4\,{\left(a+b\,x\right)}^2} - \frac{a^2\,\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}}{b^2\,{\left(a+b\,x\right)}^5}$$

input
```
int((x^2*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
- (((B*a^2 - A*a*b)/(3*b^4) - (a*((A*b^2 - B*a*b)/(3*b^4) - (B*a)/(3*b^3))
)/b)*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(a + b*x)^4 - (((A*b - 2*B*a)/(2*b^4
) - (B*a)/(2*b^4))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(a + b*x)^3 - (B*(a^2
+ b^2*x^2 + 2*a*b*x)^(1/2))/(b^4*(a + b*x)^2) - (a^2*(A/(4*b) - (B*a)/(4*b
^2))*(a^2 + b^2*x^2 + 2*a*b*x)^(1/2))/(b^2*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{x^3}{3a\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
x**3/(3*a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.354 $\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{a(Ab-aB)}{4b^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{Ab-2aB}{3b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{B}{2b^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/4*a*(A*b-B*a)/b^3/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/3*(A*b-2*B*a)/b^3/(b*x+a
)^2/((b*x+a)^2)^(1/2)-1/2*B/b^3/(b*x+a)/((b*x+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(121) = 242.

Time = 1.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.64

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{x^2\left(-3a^2b^5Bx^6-3a^3\sqrt{a^2}b^2x^2\sqrt{(a+bx)^2}(A+Bx)+a^6bx(4A+Bx)+a^4\sqrt{a^2}bx\sqrt{(a+bx)^2}(2A+3Bx\right.}{}$$

input
```
Integrate[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/12*(x^2*(-3*a^2*b^5*B*x^6 - 3*a^3*Sqrt[a^2]*b^2*x^2*Sqrt[(a + b*x)^2]*(
A + B*x) + a^6*b*x*(4*A + B*x) + a^4*Sqrt[a^2]*b*x*Sqrt[(a + b*x)^2]*(2*A
+ 3*B*x) + a^7*(6*A + 4*B*x) - 3*a*b^4*x^4*(-(A*b^2*x^2) + Sqrt[a^2]*A*Sqr
t[(a + b*x)^2] + Sqrt[a^2]*B*x*Sqrt[(a + b*x)^2]) - a^5*(-(A*b^2*x^2) + 6*
Sqrt[a^2]*A*Sqrt[(a + b*x)^2] + 4*Sqrt[a^2]*B*x*Sqrt[(a + b*x)^2]) + 3*Sqr
t[a^2]*b^3*x^3*Sqrt[(a + b*x)^2]*(A*b^2*x^2 + a^2*(A + B*x))))/(a^7*(a + b
*x)^3*(Sqrt[a^2]*b*x + a*(Sqrt[a^2] - Sqrt[(a + b*x)^2])))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{x(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{B}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^4}+\frac{a(aB-Ab)}{b^2(a+bx)^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{Ab-2aB}{3b^3(a+bx)^3}+\frac{a(Ab-aB)}{4b^3(a+bx)^4}-\frac{B}{2b^3(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*((a*(A*b - a*B))/(4*b^3*(a + b*x)^4) - (A*b - 2*a*B)/(3*b^3*(a
+ b*x)^3) - B/(2*b^3*(a + b*x)^2)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{(bx+a)\left(6x^{2} B\,b^{2}+4x\,b^{2} A+4xabB+abA+a^{2} B\right)}{12 b^{3} \left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}$	52
default	$-\frac{(bx+a)\left(6x^{2} B\,b^{2}+4x\,b^{2} A+4xabB+abA+a^{2} B\right)}{12 b^{3} \left(\left(bx+a\right)^{2}\right)^{\frac{5}{2}}}$	52
risch	$\frac{\sqrt{\left(bx+a\right)^{2}}\,\left(-\frac{B\,x^{2}}{2b}-\frac{(Ab+Ba)x}{3b^{2}}-\frac{a(Ab+Ba)}{12b^{3}}\right)}{\left(bx+a\right)^{5}}$	54
orering	$-\frac{\left(6x^{2} B\,b^{2}+4x\,b^{2} A+4xabB+abA+a^{2} B\right)(bx+a)}{12 b^{3} \left(b^{2} x^{2}+2abx+a^{2}\right)^{\frac{5}{2}}}$	61

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)/b^3*(6*B*b^2*x^2+4*A*b^2*x+4*B*a*b*x+A*a*b+B*a^2)/((b*x+a)^2
)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{x(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=-\frac{6\,Bb^2x^2+Ba^2+Aab+4\,(Bab+Ab^2)x}{12\,(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*B*b^2*x^2 + B*a^2 + A*a*b + 4*(B*a*b + A*b^2)*x)/(b^7*x^4 + 4*a*b
^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x(A+Bx)}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(x*(A + B*x)/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{A}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2} - \frac{B}{2\,b^5\left(x+\frac{a}{b}\right)^2} + \frac{2\,Ba}{3\,b^6\left(x+\frac{a}{b}\right)^3} - \frac{Ba^2}{4\,b^7\left(x+\frac{a}{b}\right)^4} + \frac{Aa}{4\,b^6\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*A/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) - 1/2*B/(b^5*(x + a/b)^2) + 2
/3*B*a/(b^6*(x + a/b)^3) - 1/4*B*a^2/(b^7*(x + a/b)^4) + 1/4*A*a/(b^6*(x +
 a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{6\,Bb^2x^2+4\,Babx+4\,Ab^2x+Ba^2+Aab}{12\,(bx+a)^4b^3\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(6*B*b^2*x^2 + 4*B*a*b*x + 4*A*b^2*x + B*a^2 + A*a*b)/((b*x + a)^4*b
^3*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(B\,a^2+4\,B\,a\,b\,x+A\,a\,b+6\,B\,b^2\,x^2+4\,A\,b^2\,x\right)}{12\,b^3\,{\left(a+b\,x\right)}^5}$$

input
```
int((x*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(B*a^2 + 6*B*b^2*x^2 + A*a*b + 4*A*b^2*x
 + 4*B*a*b*x))/(12*b^3*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{x(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-3bx-a}{6b^2\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - a - 3*b*x)/(6*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.355 $\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}} \, dx = -\frac{B}{3b^2(a^2+2abx+b^2x^2)^{3/2}} - \frac{Ab-aB}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/3*B/b^2/(b^2*x^2+2*a*b*x+a^2)^(3/2)-1/4*(A*b-B*a)/b^2/(b*x+a)^3/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}} \, dx = \frac{-3Ab-B(a+4bx)}{12b^2(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(-3*A*b - B*(a + 4*b*x))/(12*b^2*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1100}$$

$$\frac{(Ab-aB)\int \frac{1}{\left(a^2+2bxa+b^2x^2\right)^{5/2}}dx}{b}-\frac{B}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1078}$$

$$-\frac{Ab-aB}{4b^2(a+bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}-\frac{B}{3b^2\left(a^2+2abx+b^2x^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
-1/3*B/(b^2*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)) - (A*b - a*B)/(4*b^2*(a + b*x
)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{(bx+a)(4Bbx+3Ab+Ba)}{12b^2\left((bx+a)^2\right)^{\frac{5}{2}}}$	33
default	$-\frac{(bx+a)(4Bbx+3Ab+Ba)}{12b^2\left((bx+a)^2\right)^{\frac{5}{2}}}$	33
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{Bx}{3b}-\frac{3Ab+Ba}{12b^2}\right)}{(bx+a)^5}$	39
orering	$-\frac{(4Bbx+3Ab+Ba)(bx+a)}{12b^2(b^2x^2+2abx+a^2)^{\frac{5}{2}}}$	42

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)/b^2*(4*B*b*x+3*A*b+B*a)/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,Bbx+Ba+3\,Ab}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*B*b*x + B*a + 3*A*b)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a
^3*b^3*x + a^4*b^2)
```

Sympy [F]

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{A+Bx}{\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{B}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}b^2} + \frac{Ba}{4\,b^6\left(x+\frac{a}{b}\right)^4} - \frac{A}{4\,b^5\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*b^2) + 1/4*B*a/(b^6*(x + a/b)^4) -
 1/4*A/(b^5*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{4\,Bbx+Ba+3\,Ab}{12\,(bx+a)^4b^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(4*B*b*x + B*a + 3*A*b)/((b*x + a)^4*b^2*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,(3\,A\,b+B\,a+4\,B\,b\,x)}{12\,b^2\,(a+b\,x)^5}$$

input
```
int((A + B*x)/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
-((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*(3*A*b + B*a + 4*B*b*x))/(12*b^2*(a + b*
x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{1}{3b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 1)/(3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.356 $\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{A}{a^4\sqrt{a^2+2abx+b^2x^2}} + \frac{Ab-aB}{4ab(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{A}{3a^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{A}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{A(a+bx)\log(x)}{a^5\sqrt{a^2+2abx+b^2x^2}} - \frac{A(a+bx)\log(a+bx)}{a^5\sqrt{a^2+2abx+b^2x^2}}$$

output
```
A/a^4/((b*x+a)^2)^(1/2)+1/4*(A*b-B*a)/a/b/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/3*
A/a^2/(b*x+a)^2/((b*x+a)^2)^(1/2)+1/2*A/a^3/(b*x+a)/((b*x+a)^2)^(1/2)+A*(b
*x+a)*ln(x)/a^5/((b*x+a)^2)^(1/2)-A*(b*x+a)*ln(b*x+a)/a^5/((b*x+a)^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{a(25a^3Ab-3a^4B+52a^2Ab^2x+42aAb^3x^2+12Ab^4x^3)+12Ab(a+bx)^4\,\mathrm{l}}{12a^5b(a+bx)^3\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(a*(25*a^3*A*b - 3*a^4*B + 52*a^2*A*b^2*x + 42*a*A*b^3*x^2 + 12*A*b^4*x^3)
 + 12*A*b*(a + b*x)^4*Log[x] - 12*A*b*(a + b*x)^4*Log[a + b*x])/(12*a^5*b*
(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5x(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{A+Bx}{x(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 86$$

$$\frac{(a+bx)\int \left(\frac{A}{a^5x}-\frac{bA}{a^5(a+bx)}-\frac{bA}{a^4(a+bx)^2}-\frac{bA}{a^3(a+bx)^3}-\frac{bA}{a^2(a+bx)^4}+\frac{aB-Ab}{a(a+bx)^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{A\log(a+bx)}{a^5}+\frac{A\log(x)}{a^5}+\frac{A}{a^4(a+bx)}+\frac{A}{2a^3(a+bx)^2}+\frac{A}{3a^2(a+bx)^3}+\frac{Ab-aB}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(4*a*b*(a + b*x)^4) + A/(3*a^2*(a + b*x)^3) + A/(2
*a^3*(a + b*x)^2) + A/(a^4*(a + b*x)) + (A*Log[x])/a^5 - (A*Log[a + b*x])/
a^5))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(\frac{b^3 A x^3}{a^4}+\frac{7b^2 A x^2}{2a^3}+\frac{13xAb}{3a^2}+\frac{25Ab-3Ba}{12ab}\right)}{(bx+a)^5}+\frac{\sqrt{(bx+a)^2}\, A\ln(-x)}{(bx+a)a^5}-\frac{\sqrt{(bx+a)^2}\, A\ln(bx+a)}{(bx+a)a^5}$
default	$-\frac{(12A\ln(bx+a)b^5x^4-12A\ln(x)x^4b^5+48A\ln(bx+a)x^3ab^4-48A\ln(x)x^3ab^4+72A\ln(bx+a)x^2a^2b^3-72A\ln(x)x^2a^2b^3-12Aab^4x}{12ba^5((bx}$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(b^3/a^4*A*x^3+7/2*b^2/a^3*A*x^2+13/3/a^2*x*A*
b+1/12*(25*A*b-3*B*a)/a/b)+((b*x+a)^2)^(1/2)/(b*x+a)*A/a^5*ln(-x)-((b*x+a)
^2)^(1/2)/(b*x+a)*A/a^5*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{12\,Aab^4x^3+42\,Aa^2b^3x^2+52\,Aa^3b^2x-3\,Ba^5+25\,Aa^4b-12\left(Ab^5x^4+4\,\right.}{12}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(12*A*a*b^4*x^3 + 42*A*a^2*b^3*x^2 + 52*A*a^3*b^2*x - 3*B*a^5 + 25*A*
a^4*b - 12*(A*b^5*x^4 + 4*A*a*b^4*x^3 + 6*A*a^2*b^3*x^2 + 4*A*a^3*b^2*x +
A*a^4*b)*log(b*x + a) + 12*(A*b^5*x^4 + 4*A*a*b^4*x^3 + 6*A*a^2*b^3*x^2 +
4*A*a^3*b^2*x + A*a^4*b)*log(x))/(a^5*b^5*x^4 + 4*a^6*b^4*x^3 + 6*a^7*b^3*
x^2 + 4*a^8*b^2*x + a^9*b)
```

Sympy [F]

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \int \frac{A+Bx}{x\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral((A + B*x)/(x*((a + b*x)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{(-1)^{2\,abx+2\,a^2}A\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^5} + \frac{A}{3\left(b^2x^2+2\,abx+a^2\right)^{\frac{3}{2}}a^2} + \frac{A}{\sqrt{b^2x^2+2\,abx+a^2}a^4} + \frac{A}{2\,a^3b^2\left(x+\frac{a}{b}\right)^2} - \frac{B}{4\,b^5\left(x+\frac{a}{b}\right)^4} + \frac{A}{4\,ab^4\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*A*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 + 1/3*A/(
(b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2) + A/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^
4) + 1/2*A/(a^3*b^2*(x + a/b)^2) - 1/4*B/(b^5*(x + a/b)^4) + 1/4*A/(a*b^4*
(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{A\log\left(|bx+a|\right)}{a^5\mathrm{sgn}\left(bx+a\right)}+\frac{A\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx+a\right)} + \frac{12\,Aab^4x^3+42\,Aa^2b^3x^2+52\,Aa^3b^2x-3\,Ba^5+25\,Aa^4b}{12\left(bx+a\right)^4a^5b\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/(a^5*sgn(b*x + a)) + A*log(abs(x))/(a^5*sgn(b*x + a))
 + 1/12*(12*A*a*b^4*x^3 + 42*A*a^2*b^3*x^2 + 52*A*a^3*b^2*x - 3*B*a^5 + 25
*A*a^4*b)/((b*x + a)^4*a^5*b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \int \frac{A+B\,x}{x\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}} \,d x$$

input
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{-6\log(bx+a)\,a^3-18\log(bx+a)\,a^2bx-18\log(bx+a)\,a\,b^2x^2-6\log(bx}{6a^4\,(b}$$

input
```
int((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 6*log(a + b*x)*a**3 - 18*log(a + b*x)*a**2*b*x - 18*log(a + b*x)*a*b**
2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a**3 + 18*log(x)*a**2*b*x + 1
8*log(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 + 9*a**3 + 9*a**2*b*x - 2*b**3*x
**3)/(6*a**4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.357 $\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 282

$$\begin{aligned}\int \frac{A+Bx}{x^2(a^2+2abx+b^2x^2)^{5/2}}\,dx &= -\frac{4Ab-aB}{a^5\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{Ab-aB}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{2Ab-aB}{3a^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{3Ab-aB}{2a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{A(a+bx)}{a^5x\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{(5Ab-aB)(a+bx)\log(x)}{a^6\sqrt{a^2+2abx+b^2x^2}}+\frac{(5Ab-aB)(a+bx)\log(a+bx)}{a^6\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-(4*A*b-B*a)/a^5/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)/a^2/(b*x+a)^3/((b*x+a)^2)
^(1/2)-1/3*(2*A*b-B*a)/a^3/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/2*(3*A*b-B*a)/a^4
/(b*x+a)/((b*x+a)^2)^(1/2)-A*(b*x+a)/a^5/x/((b*x+a)^2)^(1/2)-(5*A*b-B*a)*(
b*x+a)*ln(x)/a^6/((b*x+a)^2)^(1/2)+(5*A*b-B*a)*(b*x+a)*ln(b*x+a)/a^6/((b*x
+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx}{x^2 (a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{a(-60Ab^4x^4+6ab^3x^3(-35A+2Bx)+2a^2b^2x^2(-130A+21Bx)+a^4(-$$

input
```
Integrate[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(a*(-60*A*b^4*x^4 + 6*a*b^3*x^3*(-35*A + 2*B*x) + 2*a^2*b^2*x^2*(-130*A +
21*B*x) + a^4*(-12*A + 25*B*x) + a^3*b*x*(-125*A + 52*B*x)) + 12*(-5*A*b +
 a*B)*x*(a + b*x)^4*Log[x] + 12*(5*A*b - a*B)*x*(a + b*x)^4*Log[a + b*x])/
(12*a^6*x*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1187, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2 (a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5x^2(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^2(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx)\int \left(\frac{A}{a^5x^2}+\frac{aB-5Ab}{a^6x}-\frac{b(aB-5Ab)}{a^6(a+bx)}-\frac{b(aB-4Ab)}{a^5(a+bx)^2}-\frac{b(aB-3Ab)}{a^4(a+bx)^3}-\frac{b(aB-2Ab)}{a^3(a+bx)^4}-\frac{b(aB-Ab)}{a^2(a+bx)^5}\right)dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)\left(-\frac{\log(x)(5Ab-aB)}{a^6}+\frac{(5Ab-aB)\log(a+bx)}{a^6}-\frac{4Ab-aB}{a^5(a+bx)}-\frac{A}{a^5x}-\frac{3Ab-aB}{2a^4(a+bx)^2}-\frac{2Ab-aB}{3a^3(a+bx)^3}-\frac{Ab-aB}{4a^2(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^2*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*(-(A/(a^5*x)) - (A*b - a*B)/(4*a^2*(a + b*x)^4) - (2*A*b - a*B)
/(3*a^3*(a + b*x)^3) - (3*A*b - a*B)/(2*a^4*(a + b*x)^2) - (4*A*b - a*B)/(
a^5*(a + b*x)) - ((5*A*b - a*B)*Log[x])/a^6 + ((5*A*b - a*B)*Log[a + b*x])
/a^6))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\sqrt{(bx+a)^2}\left(-\frac{(5Ab-Ba)b^3x^4}{a^5}-\frac{7b^2(5Ab-Ba)x^3}{2a^4}-\frac{13(5Ab-Ba)b\,x^2}{3a^3}-\frac{25(5Ab-Ba)x}{12a^2}-\frac{A}{a}\right)}{(bx+a)^5x}+\frac{\sqrt{(bx+a)^2}\,(5Ab-Ba)\ln(-bx-a)}{(bx+a)a^6}-\sqrt{}$
default	$(-60Aa\,b^4x^4+72B\ln(x)a^3b^2x^3+12B\ln(x)a^5x+52B\,a^4b\,x^2-125A\,a^4bx+48B\ln(x)a^2b^3x^4-12B\ln(bx+a)a\,b^4x^5-48B\ln(bx+a)a$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)^2)^(1/2)/(b*x+a)^5*(-(5*A*b-B*a)/a^5*b^3*x^4-7/2/a^4*b^2*(5*A*b-B
*a)*x^3-13/3/a^3*(5*A*b-B*a)*b*x^2-25/12/a^2*(5*A*b-B*a)*x-A/a)/x+((b*x+a)
^2)^(1/2)/(b*x+a)*(5*A*b-B*a)/a^6*ln(-b*x-a)-((b*x+a)^2)^(1/2)/(b*x+a)*(5*
A*b-B*a)/a^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{12\,Aa^5-12\,(Ba^2b^3-5\,Aab^4)x^4-42\,(Ba^3b^2-5\,Aa^2b^3)x^3-52\,(Ba^4b-5\,Aa^3b^2)x^2-25\,(Ba^5-5\,Aa^4b}{}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(12*A*a^5 - 12*(B*a^2*b^3 - 5*A*a*b^4)*x^4 - 42*(B*a^3*b^2 - 5*A*a^2
*b^3)*x^3 - 52*(B*a^4*b - 5*A*a^3*b^2)*x^2 - 25*(B*a^5 - 5*A*a^4*b)*x + 12
*((B*a*b^4 - 5*A*b^5)*x^5 + 4*(B*a^2*b^3 - 5*A*a*b^4)*x^4 + 6*(B*a^3*b^2 -
 5*A*a^2*b^3)*x^3 + 4*(B*a^4*b - 5*A*a^3*b^2)*x^2 + (B*a^5 - 5*A*a^4*b)*x)
*log(b*x + a) - 12*((B*a*b^4 - 5*A*b^5)*x^5 + 4*(B*a^2*b^3 - 5*A*a*b^4)*x^
4 + 6*(B*a^3*b^2 - 5*A*a^2*b^3)*x^3 + 4*(B*a^4*b - 5*A*a^3*b^2)*x^2 + (B*a
^5 - 5*A*a^4*b)*x)*log(x))/(a^6*b^4*x^5 + 4*a^7*b^3*x^4 + 6*a^8*b^2*x^3 +
4*a^9*b*x^2 + a^10*x)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+Bx}{x^2\left((a+bx)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**2/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral((A + B*x)/(x**2*((a + b*x)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= -\frac{(-1)^{2\,abx+2\,a^2}\,B\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^5}\\&\quad+\frac{5\,(-1)^{2\,abx+2\,a^2}\,Ab\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{|x|}\right)}{a^6}+\frac{B}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^2}\\&\quad-\frac{5\,Ab}{3\,(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^3}+\frac{B}{\sqrt{b^2x^2+2\,abx+a^2}a^4}\\&\quad-\frac{5\,Ab}{\sqrt{b^2x^2+2\,abx+a^2}a^5}-\frac{A}{(b^2x^2+2\,abx+a^2)^{\frac{3}{2}}a^2x}\\&\quad+\frac{B}{2\,a^3b^2\left(x+\frac{a}{b}\right)^2}-\frac{5\,A}{2\,a^4b\left(x+\frac{a}{b}\right)^2}+\frac{B}{4\,ab^4\left(x+\frac{a}{b}\right)^4}-\frac{A}{4\,a^2b^3\left(x+\frac{a}{b}\right)^4}\end{aligned}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*b*x + 2*a^2)*B*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^5 + 5*(-1)^
(2*a*b*x + 2*a^2)*A*b*log(2*a*b*x/abs(x) + 2*a^2/abs(x))/a^6 + 1/3*B/((b^2
*x^2 + 2*a*b*x + a^2)^(3/2)*a^2) - 5/3*A*b/((b^2*x^2 + 2*a*b*x + a^2)^(3/2
)*a^3) + B/(sqrt(b^2*x^2 + 2*a*b*x + a^2)*a^4) - 5*A*b/(sqrt(b^2*x^2 + 2*a
*b*x + a^2)*a^5) - A/((b^2*x^2 + 2*a*b*x + a^2)^(3/2)*a^2*x) + 1/2*B/(a^3*
b^2*(x + a/b)^2) - 5/2*A/(a^4*b*(x + a/b)^2) + 1/4*B/(a*b^4*(x + a/b)^4) -
 1/4*A/(a^2*b^3*(x + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{(Ba-5\,Ab)\log\left(|x|\right)}{a^6\mathrm{sgn}\left(bx+a\right)}-\frac{(Bab-5\,Ab^2)\log\left(|bx+a|\right)}{a^6b\mathrm{sgn}\left(bx+a\right)}-\frac{12\,Aa^5-12\,(Ba^2b^3-5\,Aab^4)x^4-42\,(Ba^3b^2-5\,Aa^2b^3)x^3-52\,(Ba^4b-5\,Aa^3b^2)x^2-25\,(Ba^5-5\,Aa^4b}{12\,(bx+a)^4a^6x\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
(B*a - 5*A*b)*log(abs(x))/(a^6*sgn(b*x + a)) - (B*a*b - 5*A*b^2)*log(abs(b
*x + a))/(a^6*b*sgn(b*x + a)) - 1/12*(12*A*a^5 - 12*(B*a^2*b^3 - 5*A*a*b^4
)*x^4 - 42*(B*a^3*b^2 - 5*A*a^2*b^3)*x^3 - 52*(B*a^4*b - 5*A*a^3*b^2)*x^2
- 25*(B*a^5 - 5*A*a^4*b)*x)/((b*x + a)^4*a^6*x*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}} \,dx$$

input
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x^2*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^2\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{12\log(bx+a)\,a^3bx+36\log(bx+a)\,a^2b^2x^2+36\log(bx+a)\,a\,b^3x^3+12\,l}{}$$

input
```
int((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(12*log(a + b*x)*a**3*b*x + 36*log(a + b*x)*a**2*b**2*x**2 + 36*log(a + b*
x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a**3*b*x - 36*log(x
)*a**2*b**2*x**2 - 36*log(x)*a*b**3*x**3 - 12*log(x)*b**4*x**4 - 3*a**4 -
18*a**3*b*x - 18*a**2*b**2*x**2 + 4*b**4*x**4)/(3*a**5*x*(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3))
```

3.358 $\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 63

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{15}b^2Bx^{15/2}$$

output
```
2/9*a^2*A*x^(9/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+
2/15*b^2*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2x^{9/2}(65a^2(11A+9Bx)+90abx(13A+11Bx)+33b^2x^2(15A+13Bx))}{6435}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*x^(9/2)*(65*a^2*(11*A + 9*B*x) + 90*a*b*x*(13*A + 11*B*x) + 33*b^2*x^2*
(15*A + 13*B*x)))/6435
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^2x^{7/2}(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int x^{7/2}(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax^{7/2}+bx^{11/2}(2aB+Ab)+ax^{9/2}(aB+2Ab)+b^2Bx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)
*x^(13/2))/13 + (2*b^2*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{9}{2}}\left(429x^{3}B\,b^{2}+495x^{2}b^{2}A+990Ba\,x^{2}b+1170abAx+585a^{2}Bx+715a^{2}A\right)}{6435}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{2}A\,x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{2}A\,x^{\frac{9}{2}}}{9}$	52
trager	$\frac{2x^{\frac{9}{2}}\left(429x^{3}B\,b^{2}+495x^{2}b^{2}A+990Ba\,x^{2}b+1170abAx+585a^{2}Bx+715a^{2}A\right)}{6435}$	52
risch	$\frac{2x^{\frac{9}{2}}\left(429x^{3}B\,b^{2}+495x^{2}b^{2}A+990Ba\,x^{2}b+1170abAx+585a^{2}Bx+715a^{2}A\right)}{6435}$	52
orering	$\frac{2\left(429x^{3}B\,b^{2}+495x^{2}b^{2}A+990Ba\,x^{2}b+1170abAx+585a^{2}Bx+715a^{2}A\right)x^{\frac{9}{2}}\left(b^{2}x^{2}+2abx+a^{2}\right)}{6435\left(bx+a\right)^{2}}$	75

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/6435*x^(9/2)*(429*B*b^2*x^3+495*A*b^2*x^2+990*B*a*b*x^2+1170*A*a*b*x+585
*B*a^2*x+715*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{6435}\left(429\,Bb^2x^7+715\,Aa^2x^4+495\,\left(2\,Bab+Ab^2\right)x^6+585\,\left(Ba^2+2\,Aab\right)x^5\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
2/6435*(429*B*b^2*x^7 + 715*A*a^2*x^4 + 495*(2*B*a*b + A*b^2)*x^6 + 585*(B
*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(7/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(13/2)/13 + 2*B*a
**2*x**(11/2)/11 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\,(2\,Bab+Ab^2)x^{\frac{13}{2}}+\frac{2}{11}\,(Ba^2+2\,Aab)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
2/15*B*b^2*x^(15/2) + 2/9*A*a^2*x^(9/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2)
+ 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2/15*B*b^2*x^(15/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/11*B*a
^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{15/2}}{15}$$

input
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b
)/13) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2\sqrt{x}\,x^4(429b^3x^3+1485a\,b^2x^2+1755a^2bx+715a^3)}{6435}$$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*sqrt(x)*x**4*(715*a**3 + 1755*a**2*b*x + 1485*a*b**2*x**2 + 429*b**3*x*
*3))/6435
```

3.359 $\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 63

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{13}b^2Bx^{13/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/
13*b^2*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2x^{7/2}(143a^2(9A+7Bx)+182abx(11A+9Bx)+63b^2x^2(13A+11Bx))}{9009}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*x^(7/2)*(143*a^2*(9*A + 7*B*x) + 182*a*b*x*(11*A + 9*B*x) + 63*b^2*x^2*
(13*A + 11*B*x)))/9009
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \ 1184$$

$$\frac{\int b^2x^{5/2}(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int x^{5/2}(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \ 85$$

$$\int \left(a^2Ax^{5/2}+bx^{9/2}(2aB+Ab)+ax^{7/2}(aB+2Ab)+b^2Bx^{11/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x
^(11/2))/11 + (2*b^2*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(693x^{3}B\,b^{2}+819x^{2}b^{2}A+1638Ba\,x^{2}b+2002abAx+1001a^{2}Bx+1287a^{2}A\right)}{9009}$	52
derivativedivides	$\frac{2B\,b^{2}x^{\frac{13}{2}}}{13}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{11}{2}}}{11}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
default	$\frac{2B\,b^{2}x^{\frac{13}{2}}}{13}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{11}{2}}}{11}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
trager	$\frac{2x^{\frac{7}{2}}\left(693x^{3}B\,b^{2}+819x^{2}b^{2}A+1638Ba\,x^{2}b+2002abAx+1001a^{2}Bx+1287a^{2}A\right)}{9009}$	52
risch	$\frac{2x^{\frac{7}{2}}\left(693x^{3}B\,b^{2}+819x^{2}b^{2}A+1638Ba\,x^{2}b+2002abAx+1001a^{2}Bx+1287a^{2}A\right)}{9009}$	52
orering	$\frac{2\left(693x^{3}B\,b^{2}+819x^{2}b^{2}A+1638Ba\,x^{2}b+2002abAx+1001a^{2}Bx+1287a^{2}A\right)x^{\frac{7}{2}}\left(b^{2}x^{2}+2abx+a^{2}\right)}{9009\left(bx+a\right)^{2}}$	75

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*x^(7/2)*(693*B*b^2*x^3+819*A*b^2*x^2+1638*B*a*b*x^2+2002*A*a*b*x+10
01*B*a^2*x+1287*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{9009}\left(693\,Bb^2x^6+1287\,Aa^2x^3+819\,\left(2\,Bab+Ab^2\right)x^5+1001\,\left(Ba^2+2\,Aab\right)x^4\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
2/9009*(693*B*b^2*x^6 + 1287*A*a^2*x^3 + 819*(2*B*a*b + A*b^2)*x^5 + 1001*
(B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(5/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(11/2)/11 + 2*B*a**
2*x**(9/2)/9 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}+\frac{2}{11}\,\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{9}\,\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
2/13*B*b^2*x^(13/2) + 2/7*A*a^2*x^(7/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2)
+ 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{9}\,Ba^2x^{\frac{9}{2}}+\frac{4}{9}\,Aabx^{\frac{9}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2/13*B*b^2*x^(13/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/9*B*a^
2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

input
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/1
1) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2\sqrt{x}\,x^3(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3)}{3003}$$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*sqrt(x)*x**3*(429*a**3 + 1001*a**2*b*x + 819*a*b**2*x**2 + 231*b**3*x**
3))/3003
```

3.360 $\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 63

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{11}b^2Bx^{11/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/11
*b^2*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2x^{5/2}(99a^2(7A+5Bx)+110abx(9A+7Bx)+35b^2x^2(11A+9Bx))}{3465}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*x^(5/2)*(99*a^2*(7*A + 5*B*x) + 110*a*b*x*(9*A + 7*B*x) + 35*b^2*x^2*(1
1*A + 9*B*x)))/3465
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^2x^{3/2}(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int x^{3/2}(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax^{3/2}+bx^{7/2}(2aB+Ab)+ax^{5/2}(aB+2Ab)+b^2Bx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x
^(9/2))/9 + (2*b^2*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(315x^{3}B\,b^{2}+385x^{2}b^{2}A+770Ba\,x^{2}b+990abAx+495a^{2}Bx+693a^{2}A\right)}{3465}$	52
derivativedivides	$\frac{2B\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{2}A\,x^{\frac{5}{2}}}{5}$	52
default	$\frac{2B\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{2}A\,x^{\frac{5}{2}}}{5}$	52
trager	$\frac{2x^{\frac{5}{2}}\left(315x^{3}B\,b^{2}+385x^{2}b^{2}A+770Ba\,x^{2}b+990abAx+495a^{2}Bx+693a^{2}A\right)}{3465}$	52
risch	$\frac{2x^{\frac{5}{2}}\left(315x^{3}B\,b^{2}+385x^{2}b^{2}A+770Ba\,x^{2}b+990abAx+495a^{2}Bx+693a^{2}A\right)}{3465}$	52
orering	$\frac{2\left(315x^{3}B\,b^{2}+385x^{2}b^{2}A+770Ba\,x^{2}b+990abAx+495a^{2}Bx+693a^{2}A\right)x^{\frac{5}{2}}\left(b^{2}x^{2}+2abx+a^{2}\right)}{3465\left(bx+a\right)^{2}}$	75

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(5/2)*(315*B*b^2*x^3+385*A*b^2*x^2+770*B*a*b*x^2+990*A*a*b*x+495*
B*a^2*x+693*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{3465}\left(315\,Bb^2x^5+693\,Aa^2x^2+385\,\left(2\,Bab+Ab^2\right)x^4+495\,\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^2*x^5 + 693*A*a^2*x^2 + 385*(2*B*a*b + A*b^2)*x^4 + 495*(B
*a^2 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(3/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*
x**(7/2)/7 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}+\frac{2}{9}\,\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+\frac{2}{7}\,\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
2/11*B*b^2*x^(11/2) + 2/5*A*a^2*x^(5/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) +
2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2/11*B*b^2*x^(11/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/7*B*a^2*x^
(7/2) + 4/7*A*a*b*x^(7/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=x^{7/2}\,\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{9/2}\,\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{11/2}}{11}$$

input
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9)
+ (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2\sqrt{x}\,x^2(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3)}{1155}$$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*sqrt(x)*x**2*(231*a**3 + 495*a**2*b*x + 385*a*b**2*x**2 + 105*b**3*x**3
))/1155
```

3.361 $\int \sqrt{x}(A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 63

$$\int \sqrt{x}(A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = \frac{2}{3}a^2Ax^{3/2}+\frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{9}b^2Bx^{9/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/5*a*(2*A*b+B*a)*x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/9*
b^2*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \sqrt{x}(A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = \frac{2}{315}x^{3/2}\left(21a^2(5A+3Bx)+18abx(7A+5Bx) + 5b^2x^2(9A+7Bx)\right)$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*x^(3/2)*(21*a^2*(5*A + 3*B*x) + 18*a*b*x*(7*A + 5*B*x) + 5*b^2*x^2*(9*A
 + 7*B*x)))/315
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a^2+2abx+b^2x^2\right)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^2\sqrt{x}(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \sqrt{x}(a+bx)^2(A+Bx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2A\sqrt{x}+bx^{5/2}(2aB+Ab)+ax^{3/2}(aB+2Ab)+b^2Bx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x
^(7/2))/7 + (2*b^2*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35x^{3}B\,b^{2}+45x^{2}b^{2}A+90Ba\,x^{2}b+126abAx+63a^{2}Bx+105a^{2}A\right)}{315}$	52
derivativedivides	$\frac{2B\,b^{2}x^{\frac{9}{2}}}{9}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2B\,b^{2}x^{\frac{9}{2}}}{9}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
trager	$\frac{2x^{\frac{3}{2}}\left(35x^{3}B\,b^{2}+45x^{2}b^{2}A+90Ba\,x^{2}b+126abAx+63a^{2}Bx+105a^{2}A\right)}{315}$	52
risch	$\frac{2x^{\frac{3}{2}}\left(35x^{3}B\,b^{2}+45x^{2}b^{2}A+90Ba\,x^{2}b+126abAx+63a^{2}Bx+105a^{2}A\right)}{315}$	52
orering	$\frac{2x^{\frac{3}{2}}\left(35x^{3}B\,b^{2}+45x^{2}b^{2}A+90Ba\,x^{2}b+126abAx+63a^{2}Bx+105a^{2}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)}{315\left(bx+a\right)^{2}}$	75

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(3/2)*(35*B*b^2*x^3+45*A*b^2*x^2+90*B*a*b*x^2+126*A*a*b*x+63*B*a^2
*x+105*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{2}{315}\left(35\,Bb^2x^4+105\,Aa^2x+45\left(2\,Bab+Ab^2\right)x^3+63\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^2*x^4 + 105*A*a^2*x + 45*(2*B*a*b + A*b^2)*x^3 + 63*(B*a^2 +
 2*A*a*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{9}{2}}}{9}+\frac{2x^{\frac{7}{2}}(Ab^2+2Bab)}{7}+\frac{2x^{\frac{5}{2}}\cdot(2Aab+Ba^2)}{5}$$

input
```
integrate(x**(1/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(9/2)/9 + 2*x**(7/2)*(A*b**2 + 2*B*a*b)/
7 + 2*x**(5/2)*(2*A*a*b + B*a**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A + Bx) \left(a^2 + 2abx + b^2x^2\right) \, dx = \frac{2}{9} Bb^2x^{\frac{9}{2}} + \frac{2}{3} Aa^2x^{\frac{3}{2}} + \frac{2}{7} \left(2 Bab + Ab^2\right)x^{\frac{7}{2}} + \frac{2}{5} \left(Ba^2 + 2 Aab\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
2/9*B*b^2*x^(9/2) + 2/3*A*a^2*x^(3/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2/
5*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(A + Bx) \left(a^2 + 2abx + b^2x^2\right) \, dx = \frac{2}{9} Bb^2x^{\frac{9}{2}} + \frac{4}{7} Babx^{\frac{7}{2}} + \frac{2}{7} Ab^2x^{\frac{7}{2}} + \frac{2}{5} Ba^2x^{\frac{5}{2}} + \frac{4}{5} Aabx^{\frac{5}{2}} + \frac{2}{3} Aa^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2/9*B*b^2*x^(9/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/5*B*a^2*x^(5
/2) + 4/5*A*a*b*x^(5/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = x^{5/2} \left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{7/2} \left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right) + \frac{2\,A\,a^2\,x^{3/2}}{3} + \frac{2\,B\,b^2\,x^{9/2}}{9}$$

input
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7)
+ (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{2\sqrt{x}\,x(35b^3x^3+135a\,b^2x^2+189a^2bx+105a^3)}{315}$$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*sqrt(x)*x*(105*a**3 + 189*a**2*b*x + 135*a*b**2*x**2 + 35*b**3*x**3))/3
15
```

3.362 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 61

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{7}b^2Bx^{7/2}$$

output
```
2*a^2*A*x^(1/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/5*b*(A*b+2*B*a)*x^(5/2)+2/7*b^
2*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{\sqrt{x}}\,dx = \frac{2}{105}\sqrt{x}\left(35a^2(3A+Bx)+14abx(5A+3Bx)+3b^2x^2(7A+5Bx)\right)$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(35*a^2*(3*A + B*x) + 14*a*b*x*(5*A + 3*B*x) + 3*b^2*x^2*(7*A +
 5*B*x)))/105
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{\sqrt{x}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{\sqrt{x}}+bx^{3/2}(2aB+Ab)+a\sqrt{x}(aB+2Ab)+b^2Bx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2A\sqrt{x}+\frac{2}{5}bx^{5/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{7}b^2Bx^{7/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(5/
2))/5 + (2*b^2*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result	size
trager	$\left(\frac{2}{7}x^{3}B\,b^{2}+\frac{2}{5}x^{2}b^{2}A+\frac{4}{5}Ba\,x^{2}b+\frac{4}{3}abAx+\frac{2}{3}a^{2}Bx+2a^{2}A\right)\sqrt{x}$	51
gosper	$\frac{2\sqrt{x}\,(15x^{3}B\,b^{2}+21x^{2}b^{2}A+42Ba\,x^{2}b+70abAx+35a^{2}Bx+105a^{2}A)}{105}$	52
derivativedivides	$\frac{2B\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2(b^{2}A+2abB)x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^{2}B)x^{\frac{3}{2}}}{3}+2a^{2}A\sqrt{x}$	52
default	$\frac{2B\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2(b^{2}A+2abB)x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^{2}B)x^{\frac{3}{2}}}{3}+2a^{2}A\sqrt{x}$	52
risch	$\frac{2\sqrt{x}\,(15x^{3}B\,b^{2}+21x^{2}b^{2}A+42Ba\,x^{2}b+70abAx+35a^{2}Bx+105a^{2}A)}{105}$	52
orering	$\frac{2(15x^{3}B\,b^{2}+21x^{2}b^{2}A+42Ba\,x^{2}b+70abAx+35a^{2}Bx+105a^{2}A)\sqrt{x}\,(b^{2}x^{2}+2abx+a^{2})}{105(bx+a)^{2}}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*x^3*B*b^2+2/5*x^2*b^2*A+4/5*B*a*x^2*b+4/3*a*b*A*x+2/3*a^2*B*x+2*a^2*A
)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{\sqrt{x}}\,dx = \frac{2}{105}\left(15\,Bb^2x^3+105\,Aa^2+21\,\left(2\,Bab+Ab^2\right)x^2+35\,\left(Ba^2+2\,Aab\right)x\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^2*x^3 + 105*A*a^2 + 21*(2*B*a*b + A*b^2)*x^2 + 35*(B*a^2 + 2
*A*a*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{5}{2}}}{5}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*x**
(3/2)/3 + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + 2\,Aa^2\sqrt{x} + \frac{2}{5}\,\left(2\,Bab + Ab^2\right)x^{\frac{5}{2}} + \frac{2}{3}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/7*B*b^2*x^(7/2) + 2*A*a^2*sqrt(x) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) + 2/3*
(B*a^2 + 2*A*a*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{5}\,Babx^{\frac{5}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(1/2),x, algorithm="giac")
```

output
```
2/7*B*b^2*x^(7/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2/3*B*a^2*x^(3
/2) + 4/3*A*a*b*x^(3/2) + 2*A*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)+x^{5/2}\left(\frac{2\,A\,b^2}{5}+\frac{4\,B\,a\,b}{5}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{7/2}}{7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(5/2)*((2*A*b^2)/5 + (4*B*a*b)/5)
+ 2*A*a^2*x^(1/2) + (2*B*b^2*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,\left(5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3\right)}{35}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(35*a**3 + 35*a**2*b*x + 21*a*b**2*x**2 + 5*b**3*x**3))/35
```

3.363 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{3/2}} dx = -\frac{2a^2A}{\sqrt{x}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{3}b(Ab+2aB)x^{3/2} + \frac{2}{5}b^2Bx^{5/2}$$

output
```
-2*a^2*A/x^(1/2)+2*a*(2*A*b+B*a)*x^(1/2)+2/3*b*(A*b+2*B*a)*x^(3/2)+2/5*b^2
*B*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{3/2}} dx = \frac{-30a^2(A-Bx)+20abx(3A+Bx)+2b^2x^2(5A+3Bx)}{15\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(3/2),x]
```

output
```
(-30*a^2*(A - B*x) + 20*a*b*x*(3*A + B*x) + 2*b^2*x^2*(5*A + 3*B*x))/(15*S
qrt[x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^{3/2}}dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^2A}{x^{3/2}}+\frac{a(aB+2Ab)}{\sqrt{x}}+b\sqrt{x}(2aB+Ab)+b^2Bx^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}bx^{3/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{5}b^2Bx^{5/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(3/2
))/3 + (2*b^2*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{2\left(-3x^{3}B\,b^{2}-5x^{2}b^{2}A-10Ba\,x^{2}b-30abAx-15a^{2}Bx+15a^{2}A\right)}{15\sqrt{x}}$	52
trager	$-\frac{2\left(-3x^{3}B\,b^{2}-5x^{2}b^{2}A-10Ba\,x^{2}b-30abAx-15a^{2}Bx+15a^{2}A\right)}{15\sqrt{x}}$	52
risch	$-\frac{2\left(-3x^{3}B\,b^{2}-5x^{2}b^{2}A-10Ba\,x^{2}b-30abAx-15a^{2}Bx+15a^{2}A\right)}{15\sqrt{x}}$	52
derivativedivides	$\frac{2B\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}+4Aab\sqrt{x}+2B\,a^{2}\sqrt{x}-\frac{2a^{2}A}{\sqrt{x}}$	54
default	$\frac{2B\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}+4Aab\sqrt{x}+2B\,a^{2}\sqrt{x}-\frac{2a^{2}A}{\sqrt{x}}$	54
orering	$-\frac{2\left(-3x^{3}B\,b^{2}-5x^{2}b^{2}A-10Ba\,x^{2}b-30abAx-15a^{2}Bx+15a^{2}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)}{15\sqrt{x}\,\left(bx+a\right)^{2}}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^3-5*A*b^2*x^2-10*B*a*b*x^2-30*A*a*b*x-15*B*a^2*x+15*A*a^
2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{3/2}}\,dx=\frac{2\left(3\,Bb^2x^3-15\,Aa^2+5\,(2\,Bab+Ab^2)x^2+15\,(Ba^2+2\,Aab)x\right)}{15\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^2*x^3 - 15*A*a^2 + 5*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*A*a
*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{3/2}}\,dx=-\frac{2Aa^2}{\sqrt{x}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{3}{2}}}{3}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(3/2)/3 + 2*B*a**2*sqrt(
x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{3/2}}\,dx = \frac{2}{5}\,Bb^2x^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}} + \frac{2}{3}\left(2\,Bab+Ab^2\right)x^{\frac{3}{2}} + 2\left(Ba^2+2\,Aab\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/5*B*b^2*x^(5/2) - 2*A*a^2/sqrt(x) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) + 2*(B
*a^2 + 2*A*a*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{3/2}}\,dx = \frac{2}{5}\,Bb^2x^{\frac{5}{2}} + \frac{4}{3}\,Babx^{\frac{3}{2}} + \frac{2}{3}\,Ab^2x^{\frac{3}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(3/2),x, algorithm="giac")
```

output
```
2/5*B*b^2*x^(5/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) + 2*B*a^2*sqrt(x
) + 4*A*a*b*sqrt(x) - 2*A*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^{3/2}}\,dx=\sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{3/2}\,\left(\frac{2\,A\,b^2}{3}+\frac{4\,B\,a\,b}{3}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{5/2}}{5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(3/2)*((2*A*b^2)/3 + (4*B*a*b)/3) - (2*A*a
^2)/x^(1/2) + (2*B*b^2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^{3/2}}\,dx=\frac{\frac{2}{5}b^3x^3+2a\,b^2x^2+6a^2bx-2a^3}{\sqrt{x}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(3/2),x)
```

output
```
(2*( - 5*a**3 + 15*a**2*b*x + 5*a*b**2*x**2 + b**3*x**3))/(5*sqrt(x))
```

3.364 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{5/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{5/2}} \, dx = -\frac{2a^2A}{3x^{3/2}} - \frac{2a(2Ab+aB)}{\sqrt{x}} + 2b(Ab+2aB)\sqrt{x} + \frac{2}{3}b^2Bx^{3/2}$$

output
```
-2/3*a^2*A/x^(3/2)-2*a*(2*A*b+B*a)/x^(1/2)+2*b*(A*b+2*B*a)*x^(1/2)+2/3*b^2
*B*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{5/2}} \, dx = \frac{2(6abx(-A+Bx)+b^2x^2(3A+Bx)-a^2(A+3Bx))}{3x^{3/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(5/2),x]
```

output
```
(2*(6*a*b*x*(-A + B*x) + b^2*x^2*(3*A + B*x) - a^2*(A + 3*B*x)))/(3*x^(3/2
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^{5/2}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^{5/2}}+\frac{a(aB+2Ab)}{x^{3/2}}+\frac{b(2aB+Ab)}{\sqrt{x}}+b^2B\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{3x^{3/2}}-\frac{2a(aB+2Ab)}{\sqrt{x}}+2b\sqrt{x}(2aB+Ab)+\frac{2}{3}b^2Bx^{3/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + 2*b*(A*b + 2*a*B)*S
qrt[x] + (2*b^2*B*x^(3/2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2\left(-x^{3} B\,b^{2}-3 x^{2} b^{2} A-6 B a\,x^{2} b+6 a b A x+3 a^{2} B x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
derivativedivides	$\frac{2 B\,b^{2} x^{\frac{3}{2}}}{3}+2 A\,b^{2} \sqrt{x}+4 a b B \sqrt{x}-\frac{2 a^{2} A}{3 x^{\frac{3}{2}}}-\frac{2 a \left(2 A b+B a\right)}{\sqrt{x}}$	51
default	$\frac{2 B\,b^{2} x^{\frac{3}{2}}}{3}+2 A\,b^{2} \sqrt{x}+4 a b B \sqrt{x}-\frac{2 a^{2} A}{3 x^{\frac{3}{2}}}-\frac{2 a \left(2 A b+B a\right)}{\sqrt{x}}$	51
trager	$-\frac{2\left(-x^{3} B\,b^{2}-3 x^{2} b^{2} A-6 B a\,x^{2} b+6 a b A x+3 a^{2} B x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
risch	$-\frac{2\left(-x^{3} B\,b^{2}-3 x^{2} b^{2} A-6 B a\,x^{2} b+6 a b A x+3 a^{2} B x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
orering	$-\frac{2\left(-x^{3} B\,b^{2}-3 x^{2} b^{2} A-6 B a\,x^{2} b+6 a b A x+3 a^{2} B x+a^{2} A\right)\left(b^{2} x^{2}+2 a b x+a^{2}\right)}{3 x^{\frac{3}{2}} \left(b x+a\right)^{2}}$	74

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-B*b^2*x^3-3*A*b^2*x^2-6*B*a*b*x^2+6*A*a*b*x+3*B*a^2*x+A*a^2)/x^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{5/2}}\,dx = \frac{2\left(Bb^2x^3-Aa^2+3\left(2\,Bab+Ab^2\right)x^2-3\left(Ba^2+2\,Aab\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*b^2*x^3 - A*a^2 + 3*(2*B*a*b + A*b^2)*x^2 - 3*(B*a^2 + 2*A*a*b)*x)/
x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} - \frac{4Aab}{\sqrt{x}} + 2Ab^2\sqrt{x} - \frac{2Ba^2}{\sqrt{x}} + 4Bab\sqrt{x} + \frac{2Bb^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*sqrt(x) - 2*B*a**2/sqr
t(x) + 4*B*a*b*sqrt(x) + 2*B*b**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{5/2}}\,dx = \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + 2\left(2\,Bab+Ab^2\right)\sqrt{x} - \frac{2\left(Aa^2+3\left(Ba^2+2\,Aab\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*b^2*x^(3/2) + 2*(2*B*a*b + A*b^2)*sqrt(x) - 2/3*(A*a^2 + 3*(B*a^2 +
2*A*a*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{5/2}}\,dx = \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + 4\,Bab\sqrt{x} + 2\,Ab^2\sqrt{x} - \frac{2\left(3\,Ba^2x+6\,Aabx+Aa^2\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*B*b^2*x^(3/2) + 4*B*a*b*sqrt(x) + 2*A*b^2*sqrt(x) - 2/3*(3*B*a^2*x + 6
*A*a*b*x + A*a^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{5/2}}\,dx = -\frac{6\,B\,a^2\,x+2\,A\,a^2-12\,B\,a\,b\,x^2+12\,A\,a\,b\,x-2\,B\,b^2\,x^3-6\,A\,b^2\,x^2}{3\,x^{3/2}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^(5/2),x)
```

output
```
-(2*A*a^2 - 6*A*b^2*x^2 - 2*B*b^2*x^3 + 6*B*a^2*x - 12*B*a*b*x^2 + 12*A*a*
b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{5/2}}\,dx = \frac{\frac{2}{3}b^3x^3+6a\,b^2x^2-6a^2bx-\frac{2}{3}a^3}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(5/2),x)
```

output
```
(2*( - a**3 - 9*a**2*b*x + 9*a*b**2*x**2 + b**3*x**3))/(3*sqrt(x)*x)
```

3.365 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 59

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx=-\frac{2a^2A}{5x^{5/2}}-\frac{2a(2Ab+aB)}{3x^{3/2}}-\frac{2b(Ab+2aB)}{\sqrt{x}}+2b^2B\sqrt{x}$$

output
```
-2/5*a^2*A/x^(5/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2*b*(A*b+2*B*a)/x^(1/2)+2*b^2
*B*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = -\frac{2(15b^2x^2(A-Bx)+10abx(A+3Bx)+a^2(3A+5Bx))}{15x^{5/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(7/2),x]
```

output
```
(-2*(15*b^2*x^2*(A - B*x) + 10*a*b*x*(A + 3*B*x) + a^2*(3*A + 5*B*x)))/(15
*x^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^{7/2}}dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^2A}{x^{7/2}}+\frac{a(aB+2Ab)}{x^{5/2}}+\frac{b(2aB+Ab)}{x^{3/2}}+\frac{b^2B}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2A}{5x^{5/2}}-\frac{2a(aB+2Ab)}{3x^{3/2}}-\frac{2b(2aB+Ab)}{\sqrt{x}}+2b^2B\sqrt{x}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/(3*x^(3/2)) - (2*b*(A*b + 2*a
*B))/Sqrt[x] + 2*b^2*B*Sqrt[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2b(Ab+2Ba)}{\sqrt{x}}+2B\,b^2\sqrt{x}$	48
default	$-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2b(Ab+2Ba)}{\sqrt{x}}+2B\,b^2\sqrt{x}$	48
gosper	$-\frac{2(-15x^3B\,b^2+15x^2b^2A+30Ba\,x^2b+10abAx+5a^2Bx+3a^2A)}{15x^{\frac{5}{2}}}$	52
trager	$-\frac{2(-15x^3B\,b^2+15x^2b^2A+30Ba\,x^2b+10abAx+5a^2Bx+3a^2A)}{15x^{\frac{5}{2}}}$	52
risch	$-\frac{2(-15x^3B\,b^2+15x^2b^2A+30Ba\,x^2b+10abAx+5a^2Bx+3a^2A)}{15x^{\frac{5}{2}}}$	52
orering	$-\frac{2(-15x^3B\,b^2+15x^2b^2A+30Ba\,x^2b+10abAx+5a^2Bx+3a^2A)(b^2x^2+2abx+a^2)}{15x^{\frac{5}{2}}(bx+a)^2}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^2*A/x^(5/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2*b*(A*b+2*B*a)/x^(1/2)+2*B*b
^2*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = \frac{2\left(15\,Bb^2x^3-3\,Aa^2-15\,(2\,Bab+Ab^2)x^2-5\,(Ba^2+2\,Aab)x\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(15*B*b^2*x^3 - 3*A*a^2 - 15*(2*B*a*b + A*b^2)*x^2 - 5*(B*a^2 + 2*A*a
*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}}-\frac{4Aab}{3x^{\frac{3}{2}}}-\frac{2Ab^2}{\sqrt{x}}-\frac{2Ba^2}{3x^{\frac{3}{2}}}-\frac{4Bab}{\sqrt{x}}+2Bb^2\sqrt{x}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) - 4*A*a*b/(3*x**(3/2)) - 2*A*b**2/sqrt(x) - 2*B*a**
2/(3*x**(3/2)) - 4*B*a*b/sqrt(x) + 2*B*b**2*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = 2\,Bb^2\sqrt{x} - \frac{2\left(3\,Aa^2+15\,(2\,Bab+Ab^2)x^2+5\,(Ba^2+2\,Aab)x\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(7/2),x, algorithm="maxima")
```

output
```
2*B*b^2*sqrt(x) - 2/15*(3*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 5*(B*a^2 + 2*
A*a*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = 2\,Bb^2\sqrt{x} - \frac{2\left(30\,Babx^2+15\,Ab^2x^2+5\,Ba^2x+10\,Aabx+3\,Aa^2\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(7/2),x, algorithm="giac")
```

output
```
2*B*b^2*sqrt(x) - 2/15*(30*B*a*b*x^2 + 15*A*b^2*x^2 + 5*B*a^2*x + 10*A*a*b
*x + 3*A*a^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = 2\,B\,b^2\,\sqrt{x} - \frac{x^2\,\left(2\,A\,b^2+4\,B\,a\,b\right)+\frac{2\,A\,a^2}{5}+x\,\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)}{x^{5/2}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^(7/2),x)
```

output
```
2*B*b^2*x^(1/2) - (x^2*(2*A*b^2 + 4*B*a*b) + (2*A*a^2)/5 + x*((2*B*a^2)/3
+ (4*A*a*b)/3))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{7/2}}\,dx = \frac{2b^3x^3-6a\,b^2x^2-2a^2bx-\frac{2}{5}a^3}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(7/2),x)
```

output
```
(2*( - a**3 - 5*a**2*b*x - 15*a*b**2*x**2 + 5*b**3*x**3))/(5*sqrt(x)*x**2)
```

3.366 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 61

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{9/2}}\,dx = -\frac{2a^2A}{7x^{7/2}}-\frac{2a(2Ab+aB)}{5x^{5/2}}-\frac{2b(Ab+2aB)}{3x^{3/2}}-\frac{2b^2B}{\sqrt{x}}$$

output
```
-2/7*a^2*A/x^(7/2)-2/5*a*(2*A*b+B*a)/x^(5/2)-2/3*b*(A*b+2*B*a)/x^(3/2)-2*b
^2*B/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{9/2}}\,dx = -\frac{2(35b^2x^2(A+3Bx)+14abx(3A+5Bx)+3a^2(5A+7Bx))}{105x^{7/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(9/2),x]
```

output
```
(-2*(35*b^2*x^2*(A + 3*B*x) + 14*a*b*x*(3*A + 5*B*x) + 3*a^2*(5*A + 7*B*x)
))/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^2(a+bx)^2(A+Bx)}{x^{9/2}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^2(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^{9/2}}+\frac{a(aB+2Ab)}{x^{7/2}}+\frac{b(2aB+Ab)}{x^{5/2}}+\frac{b^2B}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{7x^{7/2}}-\frac{2a(aB+2Ab)}{5x^{5/2}}-\frac{2b(2aB+Ab)}{3x^{3/2}}-\frac{2b^2B}{\sqrt{x}}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2))/x^(9/2),x]
```

output
```
(-2*a^2*A)/(7*x^(7/2)) - (2*a*(2*A*b + a*B))/(5*x^(5/2)) - (2*b*(A*b + 2*a
*B))/(3*x^(3/2)) - (2*b^2*B)/Sqrt[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2a^2A}{7x^{\frac{7}{2}}}-\frac{2a(2Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{2b(Ab+2Ba)}{3x^{\frac{3}{2}}}-\frac{2Bb^2}{\sqrt{x}}$	48
default	$-\frac{2a^2A}{7x^{\frac{7}{2}}}-\frac{2a(2Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{2b(Ab+2Ba)}{3x^{\frac{3}{2}}}-\frac{2Bb^2}{\sqrt{x}}$	48
gosper	$-\frac{2(105x^3Bb^2+35x^2b^2A+70Bax^2b+42abAx+21a^2Bx+15a^2A)}{105x^{\frac{7}{2}}}$	52
trager	$-\frac{2(105x^3Bb^2+35x^2b^2A+70Bax^2b+42abAx+21a^2Bx+15a^2A)}{105x^{\frac{7}{2}}}$	52
risch	$-\frac{2(105x^3Bb^2+35x^2b^2A+70Bax^2b+42abAx+21a^2Bx+15a^2A)}{105x^{\frac{7}{2}}}$	52
orering	$-\frac{2(105x^3Bb^2+35x^2b^2A+70Bax^2b+42abAx+21a^2Bx+15a^2A)(b^2x^2+2abx+a^2)}{105x^{\frac{7}{2}}(bx+a)^2}$	75

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*a^2*A/x^(7/2)-2/5*a*(2*A*b+B*a)/x^(5/2)-2/3*b*(A*b+2*B*a)/x^(3/2)-2*B
*b^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{9/2}}\,dx = -\frac{2\,\left(105\,Bb^2x^3+15\,Aa^2+35\,(2\,Bab+Ab^2)x^2+21\,(Ba^2+2\,Aab)x\right)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(105*B*b^2*x^3 + 15*A*a^2 + 35*(2*B*a*b + A*b^2)*x^2 + 21*(B*a^2 +
2*A*a*b)*x)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)}{x^{9/2}}\,dx = -\frac{2Aa^2}{7x^{\frac{7}{2}}}-\frac{4Aab}{5x^{\frac{5}{2}}}-\frac{2Ab^2}{3x^{\frac{3}{2}}}-\frac{2Ba^2}{5x^{\frac{5}{2}}}-\frac{4Bab}{3x^{\frac{3}{2}}}-\frac{2Bb^2}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/x**(9/2),x)
```

output
```
-2*A*a**2/(7*x**(7/2)) - 4*A*a*b/(5*x**(5/2)) - 2*A*b**2/(3*x**(3/2)) - 2*
B*a**2/(5*x**(5/2)) - 4*B*a*b/(3*x**(3/2)) - 2*B*b**2/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^{9/2}}\,dx = -\frac{2\,(105\,Bb^2x^3+15\,Aa^2+35\,(2\,Bab+Ab^2)x^2+21\,(Ba^2+2\,Aab)x)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/105*(105*B*b^2*x^3 + 15*A*a^2 + 35*(2*B*a*b + A*b^2)*x^2 + 21*(B*a^2 +
2*A*a*b)*x)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)}{x^{9/2}}\,dx = -\frac{2\,(105\,Bb^2x^3+70\,Babx^2+35\,Ab^2x^2+21\,Ba^2x+42\,Aabx+15\,Aa^2)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(9/2),x, algorithm="giac")
```

output
```
-2/105*(105*B*b^2*x^3 + 70*B*a*b*x^2 + 35*A*b^2*x^2 + 21*B*a^2*x + 42*A*a*
b*x + 15*A*a^2)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{9/2}}\,dx = -\frac{x^2\left(\frac{2\,A\,b^2}{3}+\frac{4\,B\,a\,b}{3}\right)+\frac{2\,A\,a^2}{7}+x\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+2\,B\,b^2\,x^3}{x^{7/2}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x))/x^(9/2),x)
```

output
```
-(x^2*((2*A*b^2)/3 + (4*B*a*b)/3) + (2*A*a^2)/7 + x*((2*B*a^2)/5 + (4*A*a*
b)/5) + 2*B*b^2*x^3)/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)}{x^{9/2}}\,dx = \frac{-2b^3x^3-2a\,b^2x^2-\frac{6}{5}a^2bx-\frac{2}{7}a^3}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)/x^(9/2),x)
```

output
```
(2*( - 5*a**3 - 21*a**2*b*x - 35*a*b**2*x**2 - 35*b**3*x**3))/(35*sqrt(x)*
x**3)
```

3.367 $\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{9}a^4Ax^{9/2}+\frac{2}{11}a^3(4Ab+aB)x^{11/2}+\frac{4}{13}a^2b(3Ab+2aB)x^{13/2}+\frac{4}{15}ab^2(2Ab+3aB)x^{15/2}+\frac{2}{17}b^3(Ab+4aB)x^{17/2}+\frac{2}{19}b^4Bx^{19/2}$$

output
```
2/9*a^4*A*x^(9/2)+2/11*a^3*(4*A*b+B*a)*x^(11/2)+4/13*a^2*b*(3*A*b+2*B*a)*x
^(13/2)+4/15*a*b^2*(2*A*b+3*B*a)*x^(15/2)+2/17*b^3*(A*b+4*B*a)*x^(17/2)+2/
19*b^4*B*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2x^{9/2}(20995a^4(11A+9Bx)+58140a^3bx(13A+11Bx)+63954a^2b^2x^2(15A+13Bx)+326\ldots}{2078505}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*x^(9/2)*(20995*a^4*(11*A + 9*B*x) + 58140*a^3*b*x*(13*A + 11*B*x) + 639
54*a^2*b^2*x^2*(15*A + 13*B*x) + 32604*a*b^3*x^3*(17*A + 15*B*x) + 6435*b^
4*x^4*(19*A + 17*B*x)))/2078505
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = $\{1184, 27, 85, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2} \left(a^2+2abx+b^2x^2\right)^2 (A+Bx) \, dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^4 x^{7/2}(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^{7/2}(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^4Ax^{7/2}+a^3x^{9/2}(aB+4Ab)+2a^2bx^{11/2}(2aB+3Ab)+b^3x^{15/2}(4aB+Ab)+2ab^2x^{13/2}(3aB+2Ab)+b^4B\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^4Ax^{9/2}+\frac{2}{11}a^3x^{11/2}(aB+4Ab)+\frac{4}{13}a^2bx^{13/2}(2aB+3Ab)+\frac{2}{17}b^3x^{17/2}(4aB+Ab)+\frac{4}{15}ab^2x^{15/2}(3aB+2Ab)+\frac{2}{19}b^4Bx^{19/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*a^4*A*x^(9/2))/9 + (2*a^3*(4*A*b + a*B)*x^(11/2))/11 + (4*a^2*b*(3*A*b
+ 2*a*B)*x^(13/2))/13 + (4*a*b^2*(2*A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*(A*
b + 4*a*B)*x^(17/2))/17 + (2*b^4*B*x^(19/2))/19
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{2x^{\frac{9}{2}}(109395b^4Bx^5+122265Ab^4x^4+489060Bab^3x^4+554268Aab^3x^3+831402Ba^2b^2x^3+959310Aa^2b^2x^2+639540B}{2078505}$
derivativedivides	$\frac{2b^4Bx^{\frac{19}{2}}}{19}+\frac{2(Ab^4+4Bab^3)x^{\frac{17}{2}}}{17}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{15}{2}}}{15}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{13}{2}}}{13}+\frac{2(4Aa^3b+a^4B)x^{\frac{11}{2}}}{11}$
default	$\frac{2b^4Bx^{\frac{19}{2}}}{19}+\frac{2(Ab^4+4Bab^3)x^{\frac{17}{2}}}{17}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{15}{2}}}{15}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{13}{2}}}{13}+\frac{2(4Aa^3b+a^4B)x^{\frac{11}{2}}}{11}$
trager	$\frac{2x^{\frac{9}{2}}(109395b^4Bx^5+122265Ab^4x^4+489060Bab^3x^4+554268Aab^3x^3+831402Ba^2b^2x^3+959310Aa^2b^2x^2+639540B}{2078505}$
risch	$\frac{2x^{\frac{9}{2}}(109395b^4Bx^5+122265Ab^4x^4+489060Bab^3x^4+554268Aab^3x^3+831402Ba^2b^2x^3+959310Aa^2b^2x^2+639540B}{2078505}$
orering	$\frac{2(109395b^4Bx^5+122265Ab^4x^4+489060Bab^3x^4+554268Aab^3x^3+831402Ba^2b^2x^3+959310Aa^2b^2x^2+639540Ba^3b}{2078505(bx+a)^4}$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/2078505*x^(9/2)*(109395*B*b^4*x^5+122265*A*b^4*x^4+489060*B*a*b^3*x^4+55
4268*A*a*b^3*x^3+831402*B*a^2*b^2*x^3+959310*A*a^2*b^2*x^2+639540*B*a^3*b*
x^2+755820*A*a^3*b*x+188955*B*a^4*x+230945*A*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{2078505}\left(109395\,Bb^4x^9+230945\,Aa^4x^4+122265\left(4\,Bab^3+Ab^4\right)x^8+277134\left(3\,Ba^2b^2+2\right.\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
2/2078505*(109395*B*b^4*x^9 + 230945*A*a^4*x^4 + 122265*(4*B*a*b^3 + A*b^4
)*x^8 + 277134*(3*B*a^2*b^2 + 2*A*a*b^3)*x^7 + 319770*(2*B*a^3*b + 3*A*a^2
*b^2)*x^6 + 188955*(B*a^4 + 4*A*a^3*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2Aa^4x^{\frac{9}{2}}}{9}+\frac{8Aa^3bx^{\frac{11}{2}}}{11}+\frac{12Aa^2b^2x^{\frac{13}{2}}}{13}+\frac{8Aab^3x^{\frac{15}{2}}}{15}+\frac{2Ab^4x^{\frac{17}{2}}}{17}+\frac{2Ba^4x^{\frac{11}{2}}}{11}+\frac{8Ba^3bx^{\frac{13}{2}}}{13}+\frac{4Ba^2b^2x^{\frac{15}{2}}}{5}+\frac{8Bab^3x^{\frac{17}{2}}}{17}+\frac{2Bb^4x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(7/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
2*A*a**4*x**(9/2)/9 + 8*A*a**3*b*x**(11/2)/11 + 12*A*a**2*b**2*x**(13/2)/1
3 + 8*A*a*b**3*x**(15/2)/15 + 2*A*b**4*x**(17/2)/17 + 2*B*a**4*x**(11/2)/1
1 + 8*B*a**3*b*x**(13/2)/13 + 4*B*a**2*b**2*x**(15/2)/5 + 8*B*a*b**3*x**(1
7/2)/17 + 2*B*b**4*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{19}\,Bb^4x^{\frac{19}{2}}+\frac{2}{9}\,Aa^4x^{\frac{9}{2}}+\frac{2}{17}\left(4\,Bab^3+Ab^4\right)x^{\frac{17}{2}}+\frac{4}{15}\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{15}{2}}+\frac{4}{13}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^{\frac{13}{2}}+\frac{2}{11}\left(Ba^4+4\,Aa^3b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
2/19*B*b^4*x^(19/2) + 2/9*A*a^4*x^(9/2) + 2/17*(4*B*a*b^3 + A*b^4)*x^(17/2
) + 4/15*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(15/2) + 4/13*(2*B*a^3*b + 3*A*a^2*b^
2)*x^(13/2) + 2/11*(B*a^4 + 4*A*a^3*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{19}\,Bb^4x^{\frac{19}{2}}+\frac{8}{17}\,Bab^3x^{\frac{17}{2}}+\frac{2}{17}\,Ab^4x^{\frac{17}{2}}+\frac{4}{5}\,Ba^2b^2x^{\frac{15}{2}}+\frac{8}{15}\,Aab^3x^{\frac{15}{2}}+\frac{8}{13}\,Ba^3bx^{\frac{13}{2}}+\frac{12}{13}\,Aa^2b^2x^{\frac{13}{2}}+\frac{2}{11}\,Ba^4x^{\frac{11}{2}}+\frac{8}{11}\,Aa^3bx^{\frac{11}{2}}+\frac{2}{9}\,Aa^4x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2/19*B*b^4*x^(19/2) + 8/17*B*a*b^3*x^(17/2) + 2/17*A*b^4*x^(17/2) + 4/5*B*
a^2*b^2*x^(15/2) + 8/15*A*a*b^3*x^(15/2) + 8/13*B*a^3*b*x^(13/2) + 12/13*A
*a^2*b^2*x^(13/2) + 2/11*B*a^4*x^(11/2) + 8/11*A*a^3*b*x^(11/2) + 2/9*A*a^
4*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^{11/2}\left(\frac{2\,B\,a^4}{11}+\frac{8\,A\,b\,a^3}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^4}{17}+\frac{8\,B\,a\,b^3}{17}\right)+\frac{2\,A\,a^4\,x^{9/2}}{9}+\frac{2\,B\,b^4\,x^{19/2}}{19}+\frac{4\,a^2\,b\,x^{13/2}\,(3\,A\,b+2\,B\,a)}{13}+\frac{4\,a\,b^2\,x^{15/2}\,(2\,A\,b+3\,B}{15}$$

input
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^(11/2)*((2*B*a^4)/11 + (8*A*a^3*b)/11) + x^(17/2)*((2*A*b^4)/17 + (8*B*a
*b^3)/17) + (2*A*a^4*x^(9/2))/9 + (2*B*b^4*x^(19/2))/19 + (4*a^2*b*x^(13/2
)*(3*A*b + 2*B*a))/13 + (4*a*b^2*x^(15/2)*(2*A*b + 3*B*a))/15
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2\sqrt{x}\,x^4(21879b^5x^5+122265a\,b^4x^4+277134a^2b^3x^3+319770a^3b^2x^2+188955a^4bx+46189a^5}{415701}$$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(2*sqrt(x)*x**4*(46189*a**5 + 188955*a**4*b*x + 319770*a**3*b**2*x**2 + 27
7134*a**2*b**3*x**3 + 122265*a*b**4*x**4 + 21879*b**5*x**5))/415701
```

3.368 $\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{7}a^4Ax^{7/2}+\frac{2}{9}a^3(4Ab+aB)x^{9/2}+\frac{4}{11}a^2b(3Ab+2aB)x^{11/2}+\frac{4}{13}ab^2(2Ab+3aB)x^{13/2}+\frac{2}{15}b^3(Ab+4aB)x^{15/2}+\frac{2}{17}b^4Bx^{17/2}$$

output
```
2/7*a^4*A*x^(7/2)+2/9*a^3*(4*A*b+B*a)*x^(9/2)+4/11*a^2*b*(3*A*b+2*B*a)*x^(
11/2)+4/13*a*b^2*(2*A*b+3*B*a)*x^(13/2)+2/15*b^3*(A*b+4*B*a)*x^(15/2)+2/17
*b^4*B*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2x^{7/2}(12155a^4(9A+7Bx)+30940a^3bx(11A+9Bx)+32130a^2b^2x^2(13A+11Bx)+15708a\cdots}{765765}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*x^(7/2)*(12155*a^4*(9*A + 7*B*x) + 30940*a^3*b*x*(11*A + 9*B*x) + 32130
*a^2*b^2*x^2*(13*A + 11*B*x) + 15708*a*b^3*x^3*(15*A + 13*B*x) + 3003*b^4*
x^4*(17*A + 15*B*x)))/765765
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a^2+2abx+b^2x^2\right)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^4x^{5/2}(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^{5/2}(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^4Ax^{5/2}+a^3x^{7/2}(aB+4Ab)+2a^2bx^{9/2}(2aB+3Ab)+b^3x^{13/2}(4aB+Ab)+2ab^2x^{11/2}(3aB+2Ab)+b^4B\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^4Ax^{7/2}+\frac{2}{9}a^3x^{9/2}(aB+4Ab)+\frac{4}{11}a^2bx^{11/2}(2aB+3Ab)+\frac{2}{15}b^3x^{15/2}(4aB+Ab)+\frac{4}{13}ab^2x^{13/2}(3aB+2Ab)+\frac{2}{17}b^4Bx^{17/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*a^4*A*x^(7/2))/7 + (2*a^3*(4*A*b + a*B)*x^(9/2))/9 + (4*a^2*b*(3*A*b +
2*a*B)*x^(11/2))/11 + (4*a*b^2*(2*A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*(A*b
+ 4*a*B)*x^(15/2))/15 + (2*b^4*B*x^(17/2))/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{2x^{\frac{7}{2}}(45045b^4Bx^5+51051Ab^4x^4+204204Bab^3x^4+235620Aab^3x^3+353430Ba^2b^2x^3+417690Aa^2b^2x^2+278460Ba^3}{765765}$
derivativedivides	$\frac{2b^4Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^4+4Bab^3)x^{\frac{15}{2}}}{15}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{13}{2}}}{13}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{11}{2}}}{11}+\frac{2(4Aa^3b+a^4B)x^{\frac{9}{2}}}{9}$
default	$\frac{2b^4Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^4+4Bab^3)x^{\frac{15}{2}}}{15}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{13}{2}}}{13}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{11}{2}}}{11}+\frac{2(4Aa^3b+a^4B)x^{\frac{9}{2}}}{9}$
trager	$\frac{2x^{\frac{7}{2}}(45045b^4Bx^5+51051Ab^4x^4+204204Bab^3x^4+235620Aab^3x^3+353430Ba^2b^2x^3+417690Aa^2b^2x^2+278460Ba^3}{765765}$
risch	$\frac{2x^{\frac{7}{2}}(45045b^4Bx^5+51051Ab^4x^4+204204Bab^3x^4+235620Aab^3x^3+353430Ba^2b^2x^3+417690Aa^2b^2x^2+278460Ba^3}{765765}$
orering	$\frac{2(45045b^4Bx^5+51051Ab^4x^4+204204Bab^3x^4+235620Aab^3x^3+353430Ba^2b^2x^3+417690Aa^2b^2x^2+278460Ba^3bx^2}{765765(bx+a)^4}$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/765765*x^(7/2)*(45045*B*b^4*x^5+51051*A*b^4*x^4+204204*B*a*b^3*x^4+23562
0*A*a*b^3*x^3+353430*B*a^2*b^2*x^3+417690*A*a^2*b^2*x^2+278460*B*a^3*b*x^2
+340340*A*a^3*b*x+85085*B*a^4*x+109395*A*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{765765}\left(45045\,Bb^4x^8+109395\,Aa^4x^3+51051\left(4\,Bab^3+Ab^4\right)x^7+117810\left(3\,Ba^2b^2+2\,Aab\right.\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
2/765765*(45045*B*b^4*x^8 + 109395*A*a^4*x^3 + 51051*(4*B*a*b^3 + A*b^4)*x
^7 + 117810*(3*B*a^2*b^2 + 2*A*a*b^3)*x^6 + 139230*(2*B*a^3*b + 3*A*a^2*b^
2)*x^5 + 85085*(B*a^4 + 4*A*a^3*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2Aa^4x^{\frac{7}{2}}}{7}+\frac{8Aa^3bx^{\frac{9}{2}}}{9}+\frac{12Aa^2b^2x^{\frac{11}{2}}}{11}+\frac{8Aab^3x^{\frac{13}{2}}}{13}+\frac{2Ab^4x^{\frac{15}{2}}}{15}+\frac{2Ba^4x^{\frac{9}{2}}}{9}+\frac{8Ba^3bx^{\frac{11}{2}}}{11}+\frac{12Ba^2b^2x^{\frac{13}{2}}}{13}+\frac{8Bab^3x^{\frac{15}{2}}}{15}+\frac{2Bb^4x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(5/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
2*A*a**4*x**(7/2)/7 + 8*A*a**3*b*x**(9/2)/9 + 12*A*a**2*b**2*x**(11/2)/11
+ 8*A*a*b**3*x**(13/2)/13 + 2*A*b**4*x**(15/2)/15 + 2*B*a**4*x**(9/2)/9 +
8*B*a**3*b*x**(11/2)/11 + 12*B*a**2*b**2*x**(13/2)/13 + 8*B*a*b**3*x**(15/
2)/15 + 2*B*b**4*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{17}\,Bb^4x^{\frac{17}{2}}+\frac{2}{7}\,Aa^4x^{\frac{7}{2}}+\frac{2}{15}\,\left(4\,Bab^3+Ab^4\right)x^{\frac{15}{2}}+\frac{4}{13}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{13}{2}}+\frac{4}{11}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^{\frac{11}{2}}+\frac{2}{9}\,\left(Ba^4+4\,Aa^3b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
2/17*B*b^4*x^(17/2) + 2/7*A*a^4*x^(7/2) + 2/15*(4*B*a*b^3 + A*b^4)*x^(15/2
) + 4/13*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(13/2) + 4/11*(2*B*a^3*b + 3*A*a^2*b^
2)*x^(11/2) + 2/9*(B*a^4 + 4*A*a^3*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{17}Bb^4x^{\frac{17}{2}}+\frac{8}{15}Bab^3x^{\frac{15}{2}}+\frac{2}{15}Ab^4x^{\frac{15}{2}}+\frac{12}{13}Ba^2b^2x^{\frac{13}{2}}+\frac{8}{13}Aab^3x^{\frac{13}{2}}+\frac{8}{11}Ba^3bx^{\frac{11}{2}}+\frac{12}{11}Aa^2b^2x^{\frac{11}{2}}+\frac{2}{9}Ba^4x^{\frac{9}{2}}+\frac{8}{9}Aa^3bx^{\frac{9}{2}}+\frac{2}{7}Aa^4x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2/17*B*b^4*x^(17/2) + 8/15*B*a*b^3*x^(15/2) + 2/15*A*b^4*x^(15/2) + 12/13*
B*a^2*b^2*x^(13/2) + 8/13*A*a*b^3*x^(13/2) + 8/11*B*a^3*b*x^(11/2) + 12/11
*A*a^2*b^2*x^(11/2) + 2/9*B*a^4*x^(9/2) + 8/9*A*a^3*b*x^(9/2) + 2/7*A*a^4*
x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^{9/2}\left(\frac{2\,B\,a^4}{9}+\frac{8\,A\,b\,a^3}{9}\right)+x^{15/2}\left(\frac{2\,A\,b^4}{15}+\frac{8\,B\,a\,b^3}{15}\right)+\frac{2\,A\,a^4\,x^{7/2}}{7}+\frac{2\,B\,b^4\,x^{17/2}}{17}+\frac{4\,a^2\,b\,x^{11/2}\,(3\,A\,b+2\,B\,a)}{11}+\frac{4\,a\,b^2\,x^{13/2}\,(2\,A\,b+3\,B}{13}$$

input
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^(9/2)*((2*B*a^4)/9 + (8*A*a^3*b)/9) + x^(15/2)*((2*A*b^4)/15 + (8*B*a*b^
3)/15) + (2*A*a^4*x^(7/2))/7 + (2*B*b^4*x^(17/2))/17 + (4*a^2*b*x^(11/2)*(
3*A*b + 2*B*a))/11 + (4*a*b^2*x^(13/2)*(2*A*b + 3*B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2\sqrt{x}\,x^3(9009b^5x^5+51051a\,b^4x^4+117810a^2b^3x^3+139230a^3b^2x^2+85085a^4bx+21879a^5)}{153153}$$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(2*sqrt(x)*x**3*(21879*a**5 + 85085*a**4*b*x + 139230*a**3*b**2*x**2 + 117
810*a**2*b**3*x**3 + 51051*a*b**4*x**4 + 9009*b**5*x**5))/153153
```

3.369 $\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{5}a^4Ax^{5/2}+\frac{2}{7}a^3(4Ab+aB)x^{7/2}+\frac{4}{9}a^2b(3Ab+2aB)x^{9/2}+\frac{4}{11}ab^2(2Ab+3aB)x^{11/2}+\frac{2}{13}b^3(Ab+4aB)x^{13/2}+\frac{2}{15}b^4Bx^{15/2}$$

output
```
2/5*a^4*A*x^(5/2)+2/7*a^3*(4*A*b+B*a)*x^(7/2)+4/9*a^2*b*(3*A*b+2*B*a)*x^(9
/2)+4/11*a*b^2*(2*A*b+3*B*a)*x^(11/2)+2/13*b^3*(A*b+4*B*a)*x^(13/2)+2/15*b
^4*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2x^{5/2}(1287a^4(7A+5Bx)+2860a^3bx(9A+7Bx)+2730a^2b^2x^2(11A+9Bx)+1260ab^3x^3(1}{45045}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*x^(5/2)*(1287*a^4*(7*A + 5*B*x) + 2860*a^3*b*x*(9*A + 7*B*x) + 2730*a^2
*b^2*x^2*(11*A + 9*B*x) + 1260*a*b^3*x^3*(13*A + 11*B*x) + 231*b^4*x^4*(15
*A + 13*B*x)))/45045
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a^2+2abx+b^2x^2\right)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^4x^{3/2}(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^{3/2}(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^4Ax^{3/2}+a^3x^{5/2}(aB+4Ab)+2a^2bx^{7/2}(2aB+3Ab)+b^3x^{11/2}(4aB+Ab)+2ab^2x^{9/2}(3aB+2Ab)+b^4Bx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^4Ax^{5/2}+\frac{2}{7}a^3x^{7/2}(aB+4Ab)+\frac{4}{9}a^2bx^{9/2}(2aB+3Ab)+\frac{2}{13}b^3x^{13/2}(4aB+Ab)+\frac{4}{11}ab^2x^{11/2}(3aB+2Ab)+\frac{2}{15}b^4Bx^{15/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*a^4*A*x^(5/2))/5 + (2*a^3*(4*A*b + a*B)*x^(7/2))/7 + (4*a^2*b*(3*A*b +
2*a*B)*x^(9/2))/9 + (4*a*b^2*(2*A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*(A*b +
4*a*B)*x^(13/2))/13 + (2*b^4*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{2x^{\frac{5}{2}}\left(3003b^{4}B\,x^{5}+3465A\,b^{4}x^{4}+13860Ba\,b^{3}x^{4}+16380Aa\,b^{3}x^{3}+24570B\,a^{2}b^{2}x^{3}+30030A\,a^{2}b^{2}x^{2}+20020B\,a^{3}b\,x^{2}+25\right.}{45045}$
derivativedivides	$\frac{2b^{4}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(A\,b^{4}+4Ba\,b^{3}\right)x^{\frac{13}{2}}}{13}+\frac{2\left(4Aa\,b^{3}+6B\,a^{2}b^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2\left(6a^{2}A\,b^{2}+4B\,a^{3}b\right)x^{\frac{9}{2}}}{9}+\frac{2\left(4A\,a^{3}b+a^{4}B\right)x^{\frac{7}{2}}}{7}$
default	$\frac{2b^{4}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(A\,b^{4}+4Ba\,b^{3}\right)x^{\frac{13}{2}}}{13}+\frac{2\left(4Aa\,b^{3}+6B\,a^{2}b^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2\left(6a^{2}A\,b^{2}+4B\,a^{3}b\right)x^{\frac{9}{2}}}{9}+\frac{2\left(4A\,a^{3}b+a^{4}B\right)x^{\frac{7}{2}}}{7}$
trager	$\frac{2x^{\frac{5}{2}}\left(3003b^{4}B\,x^{5}+3465A\,b^{4}x^{4}+13860Ba\,b^{3}x^{4}+16380Aa\,b^{3}x^{3}+24570B\,a^{2}b^{2}x^{3}+30030A\,a^{2}b^{2}x^{2}+20020B\,a^{3}b\,x^{2}+25\right.}{45045}$
risch	$\frac{2x^{\frac{5}{2}}\left(3003b^{4}B\,x^{5}+3465A\,b^{4}x^{4}+13860Ba\,b^{3}x^{4}+16380Aa\,b^{3}x^{3}+24570B\,a^{2}b^{2}x^{3}+30030A\,a^{2}b^{2}x^{2}+20020B\,a^{3}b\,x^{2}+25\right.}{45045}$
orering	$\frac{2\left(3003b^{4}B\,x^{5}+3465A\,b^{4}x^{4}+13860Ba\,b^{3}x^{4}+16380Aa\,b^{3}x^{3}+24570B\,a^{2}b^{2}x^{3}+30030A\,a^{2}b^{2}x^{2}+20020B\,a^{3}b\,x^{2}+25740\right.}{45045(bx+a)^{4}}$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(5/2)*(3003*B*b^4*x^5+3465*A*b^4*x^4+13860*B*a*b^3*x^4+16380*A*a
*b^3*x^3+24570*B*a^2*b^2*x^3+30030*A*a^2*b^2*x^2+20020*B*a^3*b*x^2+25740*A
*a^3*b*x+6435*B*a^4*x+9009*A*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{45045}\left(3003\,Bb^4x^7+9009\,Aa^4x^2+3465\left(4\,Bab^3+Ab^4\right)x^6+8190\left(3\,Ba^2b^2+2\,Aab^3\right)x^5+\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*B*b^4*x^7 + 9009*A*a^4*x^2 + 3465*(4*B*a*b^3 + A*b^4)*x^6 +
8190*(3*B*a^2*b^2 + 2*A*a*b^3)*x^5 + 10010*(2*B*a^3*b + 3*A*a^2*b^2)*x^4 +
 6435*(B*a^4 + 4*A*a^3*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2Aa^4x^{\frac{5}{2}}}{5}+\frac{8Aa^3bx^{\frac{7}{2}}}{7}+\frac{4Aa^2b^2x^{\frac{9}{2}}}{3}+\frac{8Aab^3x^{\frac{11}{2}}}{11}+\frac{2Ab^4x^{\frac{13}{2}}}{13}+\frac{2Ba^4x^{\frac{7}{2}}}{7}+\frac{8Ba^3bx^{\frac{9}{2}}}{9}+\frac{12Ba^2b^2x^{\frac{11}{2}}}{11}+\frac{8Bab^3x^{\frac{13}{2}}}{13}+\frac{2Bb^4x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(3/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
2*A*a**4*x**(5/2)/5 + 8*A*a**3*b*x**(7/2)/7 + 4*A*a**2*b**2*x**(9/2)/3 + 8
*A*a*b**3*x**(11/2)/11 + 2*A*b**4*x**(13/2)/13 + 2*B*a**4*x**(7/2)/7 + 8*B
*a**3*b*x**(9/2)/9 + 12*B*a**2*b**2*x**(11/2)/11 + 8*B*a*b**3*x**(13/2)/13
 + 2*B*b**4*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{15}\,Bb^4x^{\frac{15}{2}}+\frac{2}{5}\,Aa^4x^{\frac{5}{2}}+\frac{2}{13}\left(4\,Bab^3+Ab^4\right)x^{\frac{13}{2}}+\frac{4}{11}\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{11}{2}}+\frac{4}{9}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^{\frac{9}{2}}+\frac{2}{7}\left(Ba^4+4\,Aa^3b\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
2/15*B*b^4*x^(15/2) + 2/5*A*a^4*x^(5/2) + 2/13*(4*B*a*b^3 + A*b^4)*x^(13/2
) + 4/11*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(11/2) + 4/9*(2*B*a^3*b + 3*A*a^2*b^2
)*x^(9/2) + 2/7*(B*a^4 + 4*A*a^3*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2}{15}Bb^4x^{\frac{15}{2}}+\frac{8}{13}Bab^3x^{\frac{13}{2}}+\frac{2}{13}Ab^4x^{\frac{13}{2}}+\frac{12}{11}Ba^2b^2x^{\frac{11}{2}}+\frac{8}{11}Aab^3x^{\frac{11}{2}}+\frac{8}{9}Ba^3bx^{\frac{9}{2}}+\frac{4}{3}Aa^2b^2x^{\frac{9}{2}}+\frac{2}{7}Ba^4x^{\frac{7}{2}}+\frac{8}{7}Aa^3bx^{\frac{7}{2}}+\frac{2}{5}Aa^4x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2/15*B*b^4*x^(15/2) + 8/13*B*a*b^3*x^(13/2) + 2/13*A*b^4*x^(13/2) + 12/11*
B*a^2*b^2*x^(11/2) + 8/11*A*a*b^3*x^(11/2) + 8/9*B*a^3*b*x^(9/2) + 4/3*A*a
^2*b^2*x^(9/2) + 2/7*B*a^4*x^(7/2) + 8/7*A*a^3*b*x^(7/2) + 2/5*A*a^4*x^(5/
2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=x^{7/2}\left(\frac{2\,B\,a^4}{7}+\frac{8\,A\,b\,a^3}{7}\right)+x^{13/2}\left(\frac{2\,A\,b^4}{13}+\frac{8\,B\,a\,b^3}{13}\right)+\frac{2\,A\,a^4\,x^{5/2}}{5}+\frac{2\,B\,b^4\,x^{15/2}}{15}+\frac{4\,a^2\,b\,x^{9/2}\,(3\,A\,b+2\,B\,a)}{9}+\frac{4\,a\,b^2\,x^{11/2}\,(2\,A\,b+3\,B$$

input
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^(7/2)*((2*B*a^4)/7 + (8*A*a^3*b)/7) + x^(13/2)*((2*A*b^4)/13 + (8*B*a*b^
3)/13) + (2*A*a^4*x^(5/2))/5 + (2*B*b^4*x^(15/2))/15 + (4*a^2*b*x^(9/2)*(3
*A*b + 2*B*a))/9 + (4*a*b^2*x^(11/2)*(2*A*b + 3*B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{2\sqrt{x}\,x^2(3003b^5x^5+17325a\,b^4x^4+40950a^2b^3x^3+50050a^3b^2x^2+32175a^4bx+9009a^5)}{45045}$$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(2*sqrt(x)*x**2*(9009*a**5 + 32175*a**4*b*x + 50050*a**3*b**2*x**2 + 40950
*a**2*b**3*x**3 + 17325*a*b**4*x**4 + 3003*b**5*x**5))/45045
```

3.370 $\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{2}{3}a^4Ax^{3/2}+\frac{2}{5}a^3(4Ab+aB)x^{5/2} +\frac{4}{7}a^2b(3Ab+2aB)x^{7/2}+\frac{4}{9}ab^2(2Ab+3aB)x^{9/2}+\frac{2}{11}b^3(Ab+4aB)x^{11/2}+\frac{2}{13}b^4Bx^{13/2}$$

output
```
2/3*a^4*A*x^(3/2)+2/5*a^3*(4*A*b+B*a)*x^(5/2)+4/7*a^2*b*(3*A*b+2*B*a)*x^(7
/2)+4/9*a*b^2*(2*A*b+3*B*a)*x^(9/2)+2/11*b^3*(A*b+4*B*a)*x^(11/2)+2/13*b^4
*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{2x^{3/2}(3003a^4(5A+3Bx)+5148a^3bx(7A+5Bx)+4290a^2b^2x^2(9A+7Bx)+1820ab^3x^3(11A+9Bx)+}{45045}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*x^(3/2)*(3003*a^4*(5*A + 3*B*x) + 5148*a^3*b*x*(7*A + 5*B*x) + 4290*a^2
*b^2*x^2*(9*A + 7*B*x) + 1820*a*b^3*x^3*(11*A + 9*B*x) + 315*b^4*x^4*(13*A
 + 11*B*x)))/45045
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a^2+2abx+b^2x^2\right)^2(A+Bx)\,dx$$

$$\Big\downarrow \ 1184$$

$$\frac{\int b^4\sqrt{x}(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \ 27$$

$$\int \sqrt{x}(a+bx)^4(A+Bx)dx$$

$$\Big\downarrow \ 85$$

$$\int \left(a^4A\sqrt{x}+a^3x^{3/2}(aB+4Ab)+2a^2bx^{5/2}(2aB+3Ab)+b^3x^{9/2}(4aB+Ab)+2ab^2x^{7/2}(3aB+2Ab)+b^4Bx^{11}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{3}a^4Ax^{3/2}+\frac{2}{5}a^3x^{5/2}(aB+4Ab)+\frac{4}{7}a^2bx^{7/2}(2aB+3Ab)+\frac{2}{11}b^3x^{11/2}(4aB+Ab)+\frac{4}{9}ab^2x^{9/2}(3aB+2Ab)+\frac{2}{13}b^4Bx^{13/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(2*a^4*A*x^(3/2))/3 + (2*a^3*(4*A*b + a*B)*x^(5/2))/5 + (4*a^2*b*(3*A*b +
2*a*B)*x^(7/2))/7 + (4*a*b^2*(2*A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*(A*b + 4*
a*B)*x^(11/2))/11 + (2*b^4*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{2x^{\frac{3}{2}}(3465b^4Bx^5+4095Ab^4x^4+16380Bab^3x^4+20020Aab^3x^3+30030Ba^2b^2x^3+38610Aa^2b^2x^2+25740Ba^3bx^2+360}{45045}$
derivativedivides	$\frac{2b^4Bx^{\frac{13}{2}}}{13}+\frac{2(Ab^4+4Bab^3)x^{\frac{11}{2}}}{11}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{9}{2}}}{9}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{7}{2}}}{7}+\frac{2(4Aa^3b+a^4B)x^{\frac{5}{2}}}{5}+$
default	$\frac{2b^4Bx^{\frac{13}{2}}}{13}+\frac{2(Ab^4+4Bab^3)x^{\frac{11}{2}}}{11}+\frac{2(4Aab^3+6Ba^2b^2)x^{\frac{9}{2}}}{9}+\frac{2(6a^2Ab^2+4Ba^3b)x^{\frac{7}{2}}}{7}+\frac{2(4Aa^3b+a^4B)x^{\frac{5}{2}}}{5}+$
trager	$\frac{2x^{\frac{3}{2}}(3465b^4Bx^5+4095Ab^4x^4+16380Bab^3x^4+20020Aab^3x^3+30030Ba^2b^2x^3+38610Aa^2b^2x^2+25740Ba^3bx^2+360}{45045}$
risch	$\frac{2x^{\frac{3}{2}}(3465b^4Bx^5+4095Ab^4x^4+16380Bab^3x^4+20020Aab^3x^3+30030Ba^2b^2x^3+38610Aa^2b^2x^2+25740Ba^3bx^2+360}{45045}$
orering	$\frac{2x^{\frac{3}{2}}(3465b^4Bx^5+4095Ab^4x^4+16380Bab^3x^4+20020Aab^3x^3+30030Ba^2b^2x^3+38610Aa^2b^2x^2+25740Ba^3bx^2+360}{45045(bx+a)^4}$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(3/2)*(3465*B*b^4*x^5+4095*A*b^4*x^4+16380*B*a*b^3*x^4+20020*A*a
*b^3*x^3+30030*B*a^2*b^2*x^3+38610*A*a^2*b^2*x^2+25740*B*a^3*b*x^2+36036*A
*a^3*b*x+9009*B*a^4*x+15015*A*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$=\frac{2}{45045}\left(3465\,Bb^4x^6+15015\,Aa^4x+4095\left(4\,Bab^3+Ab^4\right)x^5+10010\left(3\,Ba^2b^2+2\,Aab^3\right)x^4+12870\left(2\,B\right.\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*B*b^4*x^6 + 15015*A*a^4*x + 4095*(4*B*a*b^3 + A*b^4)*x^5 + 1
0010*(3*B*a^2*b^2 + 2*A*a*b^3)*x^4 + 12870*(2*B*a^3*b + 3*A*a^2*b^2)*x^3 +
 9009*(B*a^4 + 4*A*a^3*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{2Aa^4x^{\frac{3}{2}}}{3}+\frac{2Bb^4x^{\frac{13}{2}}}{13}+\frac{2x^{\frac{11}{2}}(Ab^4+4Bab^3)}{11}+\frac{2x^{\frac{9}{2}}\cdot(4Aab^3+6Ba^2b^2)}{9}+\frac{2x^{\frac{7}{2}}\cdot(6Aa^2b^2+4Ba^3b)}{7}+\frac{2x^{\frac{5}{2}}\cdot(4Aa^3b+Ba^4)}{5}$$

input
```
integrate(x**(1/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
2*A*a**4*x**(3/2)/3 + 2*B*b**4*x**(13/2)/13 + 2*x**(11/2)*(A*b**4 + 4*B*a*
b**3)/11 + 2*x**(9/2)*(4*A*a*b**3 + 6*B*a**2*b**2)/9 + 2*x**(7/2)*(6*A*a**
2*b**2 + 4*B*a**3*b)/7 + 2*x**(5/2)*(4*A*a**3*b + B*a**4)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{2}{13}\,Bb^4x^{\frac{13}{2}}+\frac{2}{3}\,Aa^4x^{\frac{3}{2}}+\frac{2}{11}\,\left(4\,Bab^3+Ab^4\right)x^{\frac{11}{2}}+\frac{4}{9}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{9}{2}}+\frac{4}{7}\,\left(2\,Ba^3b+3\,Aa^2b^2\right)x^{\frac{7}{2}}+\frac{2}{5}\,\left(Ba^4+4\,Aa^3b\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
2/13*B*b^4*x^(13/2) + 2/3*A*a^4*x^(3/2) + 2/11*(4*B*a*b^3 + A*b^4)*x^(11/2
) + 4/9*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(9/2) + 4/7*(2*B*a^3*b + 3*A*a^2*b^2)*
x^(7/2) + 2/5*(B*a^4 + 4*A*a^3*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \frac{2}{13}\,Bb^4x^{\frac{13}{2}} + \frac{8}{11}\,Bab^3x^{\frac{11}{2}} + \frac{2}{11}\,Ab^4x^{\frac{11}{2}} + \frac{4}{3}\,Ba^2b^2x^{\frac{9}{2}} + \frac{8}{9}\,Aab^3x^{\frac{9}{2}} + \frac{8}{7}\,Ba^3bx^{\frac{7}{2}} + \frac{12}{7}\,Aa^2b^2x^{\frac{7}{2}} + \frac{2}{5}\,Ba^4x^{\frac{5}{2}} + \frac{8}{5}\,Aa^3bx^{\frac{5}{2}} + \frac{2}{3}\,Aa^4x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2/13*B*b^4*x^(13/2) + 8/11*B*a*b^3*x^(11/2) + 2/11*A*b^4*x^(11/2) + 4/3*B*
a^2*b^2*x^(9/2) + 8/9*A*a*b^3*x^(9/2) + 8/7*B*a^3*b*x^(7/2) + 12/7*A*a^2*b
^2*x^(7/2) + 2/5*B*a^4*x^(5/2) + 8/5*A*a^3*b*x^(5/2) + 2/3*A*a^4*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = x^{5/2}\left(\frac{2\,B\,a^4}{5}+\frac{8\,A\,b\,a^3}{5}\right) + x^{11/2}\left(\frac{2\,A\,b^4}{11}+\frac{8\,B\,a\,b^3}{11}\right) + \frac{2\,A\,a^4\,x^{3/2}}{3} + \frac{2\,B\,b^4\,x^{13/2}}{13} + \frac{4\,a^2\,b\,x^{7/2}\,(3\,A\,b+2\,B\,a)}{7} + \frac{4\,a\,b^2\,x^{9/2}\,(2\,A\,b+3\,B\,a)}{9}$$

input
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^(5/2)*((2*B*a^4)/5 + (8*A*a^3*b)/5) + x^(11/2)*((2*A*b^4)/11 + (8*B*a*b^
3)/11) + (2*A*a^4*x^(3/2))/3 + (2*B*b^4*x^(13/2))/13 + (4*a^2*b*x^(7/2)*(3
*A*b + 2*B*a))/7 + (4*a*b^2*x^(9/2)*(2*A*b + 3*B*a))/9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$=\frac{2\sqrt{x}\,x(693b^5x^5+4095a\,b^4x^4+10010a^2b^3x^3+12870a^3b^2x^2+9009a^4bx+3003a^5)}{9009}$$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(2*sqrt(x)*x*(3003*a**5 + 9009*a**4*b*x + 12870*a**3*b**2*x**2 + 10010*a**
2*b**3*x**3 + 4095*a*b**4*x**4 + 693*b**5*x**5))/9009
```

3.371 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = 2a^4A\sqrt{x}+\frac{2}{3}a^3(4Ab+aB)x^{3/2} + \frac{4}{5}a^2b(3Ab+2aB)x^{5/2}+\frac{4}{7}ab^2(2Ab+3aB)x^{7/2} + \frac{2}{9}b^3(Ab+4aB)x^{9/2}+\frac{2}{11}b^4Bx^{11/2}$$

output
```
2*a^4*A*x^(1/2)+2/3*a^3*(4*A*b+B*a)*x^(3/2)+4/5*a^2*b*(3*A*b+2*B*a)*x^(5/2
)+4/7*a*b^2*(2*A*b+3*B*a)*x^(7/2)+2/9*b^3*(A*b+4*B*a)*x^(9/2)+2/11*b^4*B*x
^(11/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(1155a^4(3A+Bx)+924a^3bx(5A+3Bx)+594a^2b^2x^2(7A+5Bx)+220ab^3x^3(9A+7Bx)+35b^4x}{3465}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(1155*a^4*(3*A + B*x) + 924*a^3*b*x*(5*A + 3*B*x) + 594*a^2*b^2
*x^2*(7*A + 5*B*x) + 220*a*b^3*x^3*(9*A + 7*B*x) + 35*b^4*x^4*(11*A + 9*B*
x)))/3465
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2 (A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{\sqrt{x}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{\sqrt{x}}+a^3\sqrt{x}(aB+4Ab)+2a^2bx^{3/2}(2aB+3Ab)+b^3x^{7/2}(4aB+Ab)+2ab^2x^{5/2}(3aB+2Ab)+b^4Bx^{9/2}\right)$$

$$\Big\downarrow \text{2009}$$

$$2a^4A\sqrt{x}+\frac{2}{3}a^3x^{3/2}(aB+4Ab)+\frac{4}{5}a^2bx^{5/2}(2aB+3Ab)+\frac{2}{9}b^3x^{9/2}(4aB+Ab)+\frac{4}{7}ab^2x^{7/2}(3aB+2Ab)+\frac{2}{11}b^4Bx^{11/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/Sqrt[x],x]
```

output
```
2*a^4*A*Sqrt[x] + (2*a^3*(4*A*b + a*B)*x^(3/2))/3 + (4*a^2*b*(3*A*b + 2*a*
B)*x^(5/2))/5 + (4*a*b^2*(2*A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*(A*b + 4*a*B)
*x^(9/2))/9 + (2*b^4*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

method	result
trager	$\left(\frac{2}{11}b^4B\,x^5+\frac{2}{9}A\,b^4x^4+\frac{8}{9}Ba\,b^3x^4+\frac{8}{7}Aa\,b^3x^3+\frac{12}{7}B\,a^2b^2x^3+\frac{12}{5}A\,a^2b^2x^2+\frac{8}{5}B\,a^3b\,x^2\right.$
gosper	$\frac{2\sqrt{x}\,(315b^4B\,x^5+385A\,b^4x^4+1540Ba\,b^3x^4+1980Aa\,b^3x^3+2970B\,a^2b^2x^3+4158A\,a^2b^2x^2+2772B\,a^3b\,x^2+4620A\,a^3b$
derivativedivides	$\frac{2b^4B\,x^{\frac{11}{2}}}{11}+\frac{2(A\,b^4+4Ba\,b^3)x^{\frac{9}{2}}}{9}+\frac{2(4Aa\,b^3+6B\,a^2b^2)x^{\frac{7}{2}}}{7}+\frac{2(6a^2A\,b^2+4B\,a^3b)x^{\frac{5}{2}}}{5}+\frac{2(4A\,a^3b+a^4B)x^{\frac{3}{2}}}{3}+$
default	$\frac{2b^4B\,x^{\frac{11}{2}}}{11}+\frac{2(A\,b^4+4Ba\,b^3)x^{\frac{9}{2}}}{9}+\frac{2(4Aa\,b^3+6B\,a^2b^2)x^{\frac{7}{2}}}{7}+\frac{2(6a^2A\,b^2+4B\,a^3b)x^{\frac{5}{2}}}{5}+\frac{2(4A\,a^3b+a^4B)x^{\frac{3}{2}}}{3}+$
risch	$\frac{2\sqrt{x}\,(315b^4B\,x^5+385A\,b^4x^4+1540Ba\,b^3x^4+1980Aa\,b^3x^3+2970B\,a^2b^2x^3+4158A\,a^2b^2x^2+2772B\,a^3b\,x^2+4620A\,a^3b$
orering	$\frac{2(315b^4B\,x^5+385A\,b^4x^4+1540Ba\,b^3x^4+1980Aa\,b^3x^3+2970B\,a^2b^2x^3+4158A\,a^2b^2x^2+2772B\,a^3b\,x^2+4620A\,a^3bx+1}{3465(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/11*b^4*B*x^5+2/9*A*b^4*x^4+8/9*B*a*b^3*x^4+8/7*A*a*b^3*x^3+12/7*B*a^2*b
^2*x^3+12/5*A*a^2*b^2*x^2+8/5*B*a^3*b*x^2+8/3*A*a^3*b*x+2/3*a^4*B*x+2*a^4*
A)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{2}{3465}\left(315\,Bb^4x^5+3465\,Aa^4+385\left(4\,Bab^3+Ab^4\right)x^4+990\left(3\,Ba^2b^2+2\,Aab^3\right)x^3+1386\left(2\,Ba^3b+3\,A\right.\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^4*x^5 + 3465*A*a^4 + 385*(4*B*a*b^3 + A*b^4)*x^4 + 990*(3*
B*a^2*b^2 + 2*A*a*b^3)*x^3 + 1386*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 1155*(B*
a^4 + 4*A*a^3*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = 2Aa^4\sqrt{x} + \frac{8Aa^3bx^{\frac{3}{2}}}{3} + \frac{12Aa^2b^2x^{\frac{5}{2}}}{5} + \frac{8Aab^3x^{\frac{7}{2}}}{7} + \frac{2Ab^4x^{\frac{9}{2}}}{9} + \frac{2Ba^4x^{\frac{3}{2}}}{3} + \frac{8Ba^3bx^{\frac{5}{2}}}{5} + \frac{12Ba^2b^2x^{\frac{7}{2}}}{7} + \frac{8Bab^3x^{\frac{9}{2}}}{9} + \frac{2Bb^4x^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**(1/2),x)
```

output
```
2*A*a**4*sqrt(x) + 8*A*a**3*b*x**(3/2)/3 + 12*A*a**2*b**2*x**(5/2)/5 + 8*A
*a*b**3*x**(7/2)/7 + 2*A*b**4*x**(9/2)/9 + 2*B*a**4*x**(3/2)/3 + 8*B*a**3*
b*x**(5/2)/5 + 12*B*a**2*b**2*x**(7/2)/7 + 8*B*a*b**3*x**(9/2)/9 + 2*B*b**
4*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bb^4x^{\frac{11}{2}} + 2\,Aa^4\sqrt{x} + \frac{2}{9}\left(4\,Bab^3 + Ab^4\right)x^{\frac{9}{2}} + \frac{4}{7}\left(3\,Ba^2b^2 + 2\,Aab^3\right)x^{\frac{7}{2}} + \frac{4}{5}\left(2\,Ba^3b + 3\,Aa^2b^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Ba^4 + 4\,Aa^3b\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/11*B*b^4*x^(11/2) + 2*A*a^4*sqrt(x) + 2/9*(4*B*a*b^3 + A*b^4)*x^(9/2) +
4/7*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(7/2) + 4/5*(2*B*a^3*b + 3*A*a^2*b^2)*x^(5
/2) + 2/3*(B*a^4 + 4*A*a^3*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bb^4x^{\frac{11}{2}} + \frac{8}{9}\,Bab^3x^{\frac{9}{2}} + \frac{2}{9}\,Ab^4x^{\frac{9}{2}} + \frac{12}{7}\,Ba^2b^2x^{\frac{7}{2}} + \frac{8}{7}\,Aab^3x^{\frac{7}{2}} + \frac{8}{5}\,Ba^3bx^{\frac{5}{2}} + \frac{12}{5}\,Aa^2b^2x^{\frac{5}{2}} + \frac{2}{3}\,Ba^4x^{\frac{3}{2}} + \frac{8}{3}\,Aa^3bx^{\frac{3}{2}} + 2\,Aa^4\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/11*B*b^4*x^(11/2) + 8/9*B*a*b^3*x^(9/2) + 2/9*A*b^4*x^(9/2) + 12/7*B*a^2
*b^2*x^(7/2) + 8/7*A*a*b^3*x^(7/2) + 8/5*B*a^3*b*x^(5/2) + 12/5*A*a^2*b^2*
x^(5/2) + 2/3*B*a^4*x^(3/2) + 8/3*A*a^3*b*x^(3/2) + 2*A*a^4*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^4}{3}+\frac{8\,A\,b\,a^3}{3}\right) + x^{9/2}\left(\frac{2\,A\,b^4}{9}+\frac{8\,B\,a\,b^3}{9}\right) + 2\,A\,a^4\,\sqrt{x} + \frac{2\,B\,b^4\,x^{11/2}}{11} + \frac{4\,a^2\,b\,x^{5/2}\,(3\,A\,b+2\,B\,a)}{5} + \frac{4\,a\,b^2\,x^{7/2}\,(2\,A\,b+3\,B\,a)}{7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^4)/3 + (8*A*a^3*b)/3) + x^(9/2)*((2*A*b^4)/9 + (8*B*a*b^3)
/9) + 2*A*a^4*x^(1/2) + (2*B*b^4*x^(11/2))/11 + (4*a^2*b*x^(5/2)*(3*A*b +
2*B*a))/5 + (4*a*b^2*x^(7/2)*(2*A*b + 3*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(63b^5x^5+385a\,b^4x^4+990a^2b^3x^3+1386a^3b^2x^2+1155a^4bx+693a^5)}{693}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(693*a**5 + 1155*a**4*b*x + 1386*a**3*b**2*x**2 + 990*a**2*b**3
*x**3 + 385*a*b**4*x**4 + 63*b**5*x**5))/693
```

3.372 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx=-\frac{2a^4A}{\sqrt{x}}+2a^3(4Ab+aB)\sqrt{x}+\frac{4}{3}a^2b(3Ab+2aB)x^{3/2}+\frac{4}{5}ab^2(2Ab+3aB)x^{5/2}+\frac{2}{7}b^3(Ab+4aB)x^{7/2}+\frac{2}{9}b^4Bx^{9/2}$$

output
```
-2*a^4*A/x^(1/2)+2*a^3*(4*A*b+B*a)*x^(1/2)+4/3*a^2*b*(3*A*b+2*B*a)*x^(3/2)
+4/5*a*b^2*(2*A*b+3*B*a)*x^(5/2)+2/7*b^3*(A*b+4*B*a)*x^(7/2)+2/9*b^4*B*x^(
9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx=\frac{-630a^4(A-Bx)+840a^3bx(3A+Bx)+252a^2b^2x^2(5A+3Bx)+}{315\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(3/2),x]
```

output
```
(-630*a^4*(A - B*x) + 840*a^3*b*x*(3*A + B*x) + 252*a^2*b^2*x^2*(5*A + 3*B
*x) + 72*a*b^3*x^3*(7*A + 5*B*x) + 10*b^4*x^4*(9*A + 7*B*x))/(315*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{3/2}}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^4A}{x^{3/2}}+\frac{a^3(aB+4Ab)}{\sqrt{x}}+2a^2b\sqrt{x}(2aB+3Ab)+b^3x^{5/2}(4aB+Ab)+2ab^2x^{3/2}(3aB+2Ab)+b^4Bx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^4A}{\sqrt{x}}+2a^3\sqrt{x}(aB+4Ab)+\frac{4}{3}a^2bx^{3/2}(2aB+3Ab)+\frac{2}{7}b^3x^{7/2}(4aB+Ab)+\frac{4}{5}ab^2x^{5/2}(3aB+2Ab)+\frac{2}{9}b^4Bx^{9/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(3/2),x]
```

output
```
(-2*a^4*A)/Sqrt[x] + 2*a^3*(4*A*b + a*B)*Sqrt[x] + (4*a^2*b*(3*A*b + 2*a*B
)*x^(3/2))/3 + (4*a*b^2*(2*A*b + 3*a*B)*x^(5/2))/5 + (2*b^3*(A*b + 4*a*B)*
x^(7/2))/7 + (2*b^4*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{2(-35b^4B\,x^5-45A\,b^4x^4-180Ba\,b^3x^4-252Aa\,b^3x^3-378B\,a^2b^2x^3-630A\,a^2b^2x^2-420B\,a^3b\,x^2-1260A\,a^3bx-315a}{315\sqrt{x}}$
trager	$-\frac{2(-35b^4B\,x^5-45A\,b^4x^4-180Ba\,b^3x^4-252Aa\,b^3x^3-378B\,a^2b^2x^3-630A\,a^2b^2x^2-420B\,a^3b\,x^2-1260A\,a^3bx-315a}{315\sqrt{x}}$
risch	$-\frac{2(-35b^4B\,x^5-45A\,b^4x^4-180Ba\,b^3x^4-252Aa\,b^3x^3-378B\,a^2b^2x^3-630A\,a^2b^2x^2-420B\,a^3b\,x^2-1260A\,a^3bx-315a}{315\sqrt{x}}$
derivativedivides	$\frac{2b^4B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^4x^{\frac{7}{2}}}{7}+\frac{8Ba\,b^3x^{\frac{7}{2}}}{7}+\frac{8Aa\,b^3x^{\frac{5}{2}}}{5}+\frac{12B\,a^2b^2x^{\frac{5}{2}}}{5}+4A\,a^2b^2x^{\frac{3}{2}}+\frac{8B\,a^3b\,x^{\frac{3}{2}}}{3}+8A\,a^3b\sqrt{}$
default	$\frac{2b^4B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^4x^{\frac{7}{2}}}{7}+\frac{8Ba\,b^3x^{\frac{7}{2}}}{7}+\frac{8Aa\,b^3x^{\frac{5}{2}}}{5}+\frac{12B\,a^2b^2x^{\frac{5}{2}}}{5}+4A\,a^2b^2x^{\frac{3}{2}}+\frac{8B\,a^3b\,x^{\frac{3}{2}}}{3}+8A\,a^3b\sqrt{}$
orering	$-\frac{2(-35b^4B\,x^5-45A\,b^4x^4-180Ba\,b^3x^4-252Aa\,b^3x^3-378B\,a^2b^2x^3-630A\,a^2b^2x^2-420B\,a^3b\,x^2-1260A\,a^3bx-315a}{315\sqrt{x}\,(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*B*b^4*x^5-45*A*b^4*x^4-180*B*a*b^3*x^4-252*A*a*b^3*x^3-378*B*a
^2*b^2*x^3-630*A*a^2*b^2*x^2-420*B*a^3*b*x^2-1260*A*a^3*b*x-315*B*a^4*x+31
5*A*a^4)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^2}{x^{3/2}}\,dx = \frac{2\,(35\,Bb^4x^5-315\,Aa^4+45\,(4\,Bab^3+Ab^4)x^4+126\,(3\,Ba^2b^2+2\,A}{315\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^4*x^5 - 315*A*a^4 + 45*(4*B*a*b^3 + A*b^4)*x^4 + 126*(3*B*a^
2*b^2 + 2*A*a*b^3)*x^3 + 210*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 315*(B*a^4 +
4*A*a^3*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx = -\frac{2Aa^4}{\sqrt{x}}+8Aa^3b\sqrt{x}+4Aa^2b^2x^{\frac{3}{2}} + \frac{8Aab^3x^{\frac{5}{2}}}{5}+\frac{2Ab^4x^{\frac{7}{2}}}{7}+2Ba^4\sqrt{x}+\frac{8Ba^3bx^{\frac{3}{2}}}{3}+\frac{12Ba^2b^2x^{\frac{5}{2}}}{5}+\frac{8Bab^3x^{\frac{7}{2}}}{7}+\frac{2Bb^4x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**(3/2),x)
```

output
```
-2*A*a**4/sqrt(x) + 8*A*a**3*b*sqrt(x) + 4*A*a**2*b**2*x**(3/2) + 8*A*a*b*
*3*x**(5/2)/5 + 2*A*b**4*x**(7/2)/7 + 2*B*a**4*sqrt(x) + 8*B*a**3*b*x**(3/
2)/3 + 12*B*a**2*b**2*x**(5/2)/5 + 8*B*a*b**3*x**(7/2)/7 + 2*B*b**4*x**(9/
2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx = \frac{2}{9}\,Bb^4x^{\frac{9}{2}}-\frac{2\,Aa^4}{\sqrt{x}}+\frac{2}{7}\left(4\,Bab^3+Ab^4\right)x^{\frac{7}{2}} + \frac{4}{5}\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{5}{2}}+\frac{4}{3}\left(2\,Ba^3b+3\,Aa^2b^2\right)x^{\frac{3}{2}}+2\left(Ba^4+4\,Aa^3b\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/9*B*b^4*x^(9/2) - 2*A*a^4/sqrt(x) + 2/7*(4*B*a*b^3 + A*b^4)*x^(7/2) + 4/
5*(3*B*a^2*b^2 + 2*A*a*b^3)*x^(5/2) + 4/3*(2*B*a^3*b + 3*A*a^2*b^2)*x^(3/2
) + 2*(B*a^4 + 4*A*a^3*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx = \frac{2}{9}Bb^4x^{\frac{9}{2}} + \frac{8}{7}Bab^3x^{\frac{7}{2}} + \frac{2}{7}Ab^4x^{\frac{7}{2}} + \frac{12}{5}Ba^2b^2x^{\frac{5}{2}} + \frac{8}{5}Aab^3x^{\frac{5}{2}} + \frac{8}{3}Ba^3bx^{\frac{3}{2}} + 4\,Aa^2b^2x^{\frac{3}{2}} + 2\,Ba^4\sqrt{x} + 8\,Aa^3b\sqrt{x} - \frac{2\,Aa^4}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/9*B*b^4*x^(9/2) + 8/7*B*a*b^3*x^(7/2) + 2/7*A*b^4*x^(7/2) + 12/5*B*a^2*b
^2*x^(5/2) + 8/5*A*a*b^3*x^(5/2) + 8/3*B*a^3*b*x^(3/2) + 4*A*a^2*b^2*x^(3/
2) + 2*B*a^4*sqrt(x) + 8*A*a^3*b*sqrt(x) - 2*A*a^4/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx = \sqrt{x}\left(2\,B\,a^4+8\,A\,b\,a^3\right) + x^{7/2}\left(\frac{2\,A\,b^4}{7}+\frac{8\,B\,a\,b^3}{7}\right) - \frac{2\,A\,a^4}{\sqrt{x}} + \frac{2\,B\,b^4\,x^{9/2}}{9} + \frac{4\,a^2\,b\,x^{3/2}\,(3\,A\,b+2\,B\,a)}{3} + \frac{4\,a\,b^2\,x^{5/2}\,(2\,A\,b+3\,B\,a)}{5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^4 + 8*A*a^3*b) + x^(7/2)*((2*A*b^4)/7 + (8*B*a*b^3)/7) - (2
*A*a^4)/x^(1/2) + (2*B*b^4*x^(9/2))/9 + (4*a^2*b*x^(3/2)*(3*A*b + 2*B*a))/
3 + (4*a*b^2*x^(5/2)*(2*A*b + 3*B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{3/2}}\,dx = \frac{\frac{2}{9}b^5x^5+\frac{10}{7}a\,b^4x^4+4a^2b^3x^3+\frac{20}{3}a^3b^2x^2+10a^4bx-2a^5}{\sqrt{x}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(3/2),x)
```

output
```
(2*( - 63*a**5 + 315*a**4*b*x + 210*a**3*b**2*x**2 + 126*a**2*b**3*x**3 +
45*a*b**4*x**4 + 7*b**5*x**5))/(63*sqrt(x))
```

3.373 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx=-\frac{2a^4A}{3x^{3/2}}-\frac{2a^3(4Ab+aB)}{\sqrt{x}}+4a^2b(3Ab+2aB)\sqrt{x}+\frac{4}{3}ab^2(2Ab+3aB)x^{3/2}+\frac{2}{5}b^3(Ab+4aB)x^{5/2}+\frac{2}{7}b^4Bx^{7/2}$$

output
```
-2/3*a^4*A/x^(3/2)-2*a^3*(4*A*b+B*a)/x^(1/2)+4*a^2*b*(3*A*b+2*B*a)*x^(1/2)
+4/3*a*b^2*(2*A*b+3*B*a)*x^(3/2)+2/5*b^3*(A*b+4*B*a)*x^(5/2)+2/7*b^4*B*x^(
7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx=\frac{840a^3bx(-A+Bx)+420a^2b^2x^2(3A+Bx)-70a^4(A+3Bx)+56}{105x^{3/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(5/2),x]
```

output
```
(840*a^3*b*x*(-A + B*x) + 420*a^2*b^2*x^2*(3*A + B*x) - 70*a^4*(A + 3*B*x)
 + 56*a*b^3*x^3*(5*A + 3*B*x) + 6*b^4*x^4*(7*A + 5*B*x))/(105*x^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{5/2}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^{5/2}}+\frac{a^3(aB+4Ab)}{x^{3/2}}+\frac{2a^2b(2aB+3Ab)}{\sqrt{x}}+b^3x^{3/2}(4aB+Ab)+2ab^2\sqrt{x}(3aB+2Ab)+b^4Bx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4A}{3x^{3/2}}-\frac{2a^3(aB+4Ab)}{\sqrt{x}}+4a^2b\sqrt{x}(2aB+3Ab)+\frac{2}{5}b^3x^{5/2}(4aB+Ab)+\frac{4}{3}ab^2x^{3/2}(3aB+2Ab)+\frac{2}{7}b^4Bx^{7/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(5/2),x]
```

output
```
(-2*a^4*A)/(3*x^(3/2)) - (2*a^3*(4*A*b + a*B))/Sqrt[x] + 4*a^2*b*(3*A*b +
2*a*B)*Sqrt[x] + (4*a*b^2*(2*A*b + 3*a*B)*x^(3/2))/3 + (2*b^3*(A*b + 4*a*B
)*x^(5/2))/5 + (2*b^4*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^4 B x^{\frac{7}{2}}}{7}+\frac{2A b^4 x^{\frac{5}{2}}}{5}+\frac{8Ba b^3 x^{\frac{5}{2}}}{5}+\frac{8Aa b^3 x^{\frac{3}{2}}}{3}+4B a^2 b^2 x^{\frac{3}{2}}+12a^2 A b^2 \sqrt{x}+8B a^3 b \sqrt{x}-\frac{2a^3(}{}$
default	$\frac{2b^4 B x^{\frac{7}{2}}}{7}+\frac{2A b^4 x^{\frac{5}{2}}}{5}+\frac{8Ba b^3 x^{\frac{5}{2}}}{5}+\frac{8Aa b^3 x^{\frac{3}{2}}}{3}+4B a^2 b^2 x^{\frac{3}{2}}+12a^2 A b^2 \sqrt{x}+8B a^3 b \sqrt{x}-\frac{2a^3(}{}$
gosper	$-\frac{2(-15b^4 B x^5-21A b^4 x^4-84Ba b^3 x^4-140Aa b^3 x^3-210B a^2 b^2 x^3-630A a^2 b^2 x^2-420B a^3 b x^2+420A a^3 bx+105a^4 B}{105x^{\frac{3}{2}}}$
trager	$-\frac{2(-15b^4 B x^5-21A b^4 x^4-84Ba b^3 x^4-140Aa b^3 x^3-210B a^2 b^2 x^3-630A a^2 b^2 x^2-420B a^3 b x^2+420A a^3 bx+105a^4 B}{105x^{\frac{3}{2}}}$
risch	$-\frac{2(-15b^4 B x^5-21A b^4 x^4-84Ba b^3 x^4-140Aa b^3 x^3-210B a^2 b^2 x^3-630A a^2 b^2 x^2-420B a^3 b x^2+420A a^3 bx+105a^4 B}{105x^{\frac{3}{2}}}$
orering	$-\frac{2(-15b^4 B x^5-21A b^4 x^4-84Ba b^3 x^4-140Aa b^3 x^3-210B a^2 b^2 x^3-630A a^2 b^2 x^2-420B a^3 b x^2+420A a^3 bx+105a^4 B}{105x^{\frac{3}{2}}(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^4*B*x^(7/2)+2/5*A*b^4*x^(5/2)+8/5*B*a*b^3*x^(5/2)+8/3*A*a*b^3*x^(3/2
)+4*B*a^2*b^2*x^(3/2)+12*a^2*A*b^2*x^(1/2)+8*B*a^3*b*x^(1/2)-2*a^3*(4*A*b+
B*a)/x^(1/2)-2/3*a^4*A/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^{5/2}}\,dx=\frac{2\,(15\,Bb^4x^5-35\,Aa^4+21\,(4\,Bab^3+Ab^4)x^4+70\,(3\,Ba^2b^2+2\,Aab}{105\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^4*x^5 - 35*A*a^4 + 21*(4*B*a*b^3 + A*b^4)*x^4 + 70*(3*B*a^2*
b^2 + 2*A*a*b^3)*x^3 + 210*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 105*(B*a^4 + 4*
A*a^3*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx = -\frac{2Aa^4}{3x^{\frac{3}{2}}}-\frac{8Aa^3b}{\sqrt{x}}+12Aa^2b^2\sqrt{x}+\frac{8Aab^3x^{\frac{3}{2}}}{3}+\frac{2Ab^4x^{\frac{5}{2}}}{5}-\frac{2Ba^4}{\sqrt{x}}+8Ba^3b\sqrt{x}+4Ba^2b^2x^{\frac{3}{2}}+\frac{8Bab^3x^{\frac{5}{2}}}{5}+\frac{2Bb^4x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**(5/2),x)
```

output
```
-2*A*a**4/(3*x**(3/2)) - 8*A*a**3*b/sqrt(x) + 12*A*a**2*b**2*sqrt(x) + 8*A
*a*b**3*x**(3/2)/3 + 2*A*b**4*x**(5/2)/5 - 2*B*a**4/sqrt(x) + 8*B*a**3*b*s
qrt(x) + 4*B*a**2*b**2*x**(3/2) + 8*B*a*b**3*x**(5/2)/5 + 2*B*b**4*x**(7/2
)/7
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bb^4x^{\frac{7}{2}}+\frac{2}{5}\,\left(4\,Bab^3+Ab^4\right)x^{\frac{5}{2}}+\frac{4}{3}\,\left(3\,Ba^2b^2+2\,Aab^3\right)x^{\frac{3}{2}}+4\,\left(2\,Ba^3b+3\,Aa^2b^2\right)\sqrt{x}-\frac{2\,\left(Aa^4+3\,\left(Ba^4+4\,Aa^3b\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/7*B*b^4*x^(7/2) + 2/5*(4*B*a*b^3 + A*b^4)*x^(5/2) + 4/3*(3*B*a^2*b^2 + 2
*A*a*b^3)*x^(3/2) + 4*(2*B*a^3*b + 3*A*a^2*b^2)*sqrt(x) - 2/3*(A*a^4 + 3*(
B*a^4 + 4*A*a^3*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bb^4x^{\frac{7}{2}} + \frac{8}{5}\,Bab^3x^{\frac{5}{2}} + \frac{2}{5}\,Ab^4x^{\frac{5}{2}} + 4\,Ba^2b^2x^{\frac{3}{2}} + \frac{8}{3}\,Aab^3x^{\frac{3}{2}} + 8\,Ba^3b\sqrt{x} + 12\,Aa^2b^2\sqrt{x} - \frac{2\,(3\,Ba^4x + 12\,Aa^3bx + Aa^4)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/7*B*b^4*x^(7/2) + 8/5*B*a*b^3*x^(5/2) + 2/5*A*b^4*x^(5/2) + 4*B*a^2*b^2*
x^(3/2) + 8/3*A*a*b^3*x^(3/2) + 8*B*a^3*b*sqrt(x) + 12*A*a^2*b^2*sqrt(x) -
 2/3*(3*B*a^4*x + 12*A*a^3*b*x + A*a^4)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx = x^{5/2}\left(\frac{2\,A\,b^4}{5} + \frac{8\,B\,a\,b^3}{5}\right) - \frac{x\,(2\,B\,a^4 + 8\,A\,b\,a^3) + \frac{2\,A\,a^4}{3}}{x^{3/2}} + \frac{2\,B\,b^4\,x^{7/2}}{7} + 4\,a^2\,b\,\sqrt{x}\,(3\,A\,b + 2\,B\,a) + \frac{4\,a\,b^2\,x^{3/2}\,(2\,A\,b + 3\,B\,a)}{3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^(5/2),x)
```

output
```
x^(5/2)*((2*A*b^4)/5 + (8*B*a*b^3)/5) - (x*(2*B*a^4 + 8*A*a^3*b) + (2*A*a^
4)/3)/x^(3/2) + (2*B*b^4*x^(7/2))/7 + 4*a^2*b*x^(1/2)*(3*A*b + 2*B*a) + (4
*a*b^2*x^(3/2)*(2*A*b + 3*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{5/2}}\,dx = \frac{\frac{2}{7}b^5x^5+2a\,b^4x^4+\frac{20}{3}a^2b^3x^3+20a^3b^2x^2-10a^4bx-\frac{2}{3}a^5}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(5/2),x)
```

output
```
(2*( - 7*a**5 - 105*a**4*b*x + 210*a**3*b**2*x**2 + 70*a**2*b**3*x**3 + 21
*a*b**4*x**4 + 3*b**5*x**5))/(21*sqrt(x)*x)
```

3.374 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx=-\frac{2a^4A}{5x^{5/2}}-\frac{2a^3(4Ab+aB)}{3x^{3/2}}-\frac{4a^2b(3Ab+2aB)}{\sqrt{x}}+4ab^2(2Ab+3aB)\sqrt{x}+\frac{2}{3}b^3(Ab+4aB)x^{3/2}+\frac{2}{5}b^4Bx^{5/2}$$

output
```
-2/5*a^4*A/x^(5/2)-2/3*a^3*(4*A*b+B*a)/x^(3/2)-4*a^2*b*(3*A*b+2*B*a)/x^(1/
2)+4*a*b^2*(2*A*b+3*B*a)*x^(1/2)+2/3*b^3*(A*b+4*B*a)*x^(3/2)+2/5*b^4*B*x^(
5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx=\frac{2(90a^2b^2x^2(-A+Bx)+20ab^3x^3(3A+Bx)-20a^3bx(A+3Bx)+}{15x^{5/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(7/2),x]
```

output
```
(2*(90*a^2*b^2*x^2*(-A + B*x) + 20*a*b^3*x^3*(3*A + B*x) - 20*a^3*b*x*(A +
 3*B*x) + b^4*x^4*(5*A + 3*B*x) - a^4*(3*A + 5*B*x)))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^2(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{7/2}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^{7/2}}+\frac{a^3(aB+4Ab)}{x^{5/2}}+\frac{2a^2b(2aB+3Ab)}{x^{3/2}}+b^3\sqrt{x}(4aB+Ab)+\frac{2ab^2(3aB+2Ab)}{\sqrt{x}}+b^4Bx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4A}{5x^{5/2}}-\frac{2a^3(aB+4Ab)}{3x^{3/2}}-\frac{4a^2b(2aB+3Ab)}{\sqrt{x}}+\frac{2}{3}b^3x^{3/2}(4aB+Ab)+4ab^2\sqrt{x}(3aB+2Ab)+\frac{2}{5}b^4Bx^{5/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(7/2),x]
```

output
```
(-2*a^4*A)/(5*x^(5/2)) - (2*a^3*(4*A*b + a*B))/(3*x^(3/2)) - (4*a^2*b*(3*A
*b + 2*a*B))/Sqrt[x] + 4*a*b^2*(2*A*b + 3*a*B)*Sqrt[x] + (2*b^3*(A*b + 4*a
*B)*x^(3/2))/3 + (2*b^4*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^4Bx^{\frac{5}{2}}}{5}+\frac{2Ab^4x^{\frac{3}{2}}}{3}+\frac{8Bab^3x^{\frac{3}{2}}}{3}+8Aab^3\sqrt{x}+12Ba^2b^2\sqrt{x}-\frac{4a^2b(3Ab+2Ba)}{\sqrt{x}}-\frac{2a^4A}{5x^{\frac{5}{2}}}-\frac{2a^3(4}{3}$
default	$\frac{2b^4Bx^{\frac{5}{2}}}{5}+\frac{2Ab^4x^{\frac{3}{2}}}{3}+\frac{8Bab^3x^{\frac{3}{2}}}{3}+8Aab^3\sqrt{x}+12Ba^2b^2\sqrt{x}-\frac{4a^2b(3Ab+2Ba)}{\sqrt{x}}-\frac{2a^4A}{5x^{\frac{5}{2}}}-\frac{2a^3(4}{3}$
gosper	$-\frac{2(-3b^4Bx^5-5Ab^4x^4-20Bab^3x^4-60Aab^3x^3-90Ba^2b^2x^3+90Aa^2b^2x^2+60Ba^3bx^2+20Aa^3bx+5a^4Bx+3a^4A)}{15x^{\frac{5}{2}}}$
trager	$-\frac{2(-3b^4Bx^5-5Ab^4x^4-20Bab^3x^4-60Aab^3x^3-90Ba^2b^2x^3+90Aa^2b^2x^2+60Ba^3bx^2+20Aa^3bx+5a^4Bx+3a^4A)}{15x^{\frac{5}{2}}}$
risch	$-\frac{2(-3b^4Bx^5-5Ab^4x^4-20Bab^3x^4-60Aab^3x^3-90Ba^2b^2x^3+90Aa^2b^2x^2+60Ba^3bx^2+20Aa^3bx+5a^4Bx+3a^4A)}{15x^{\frac{5}{2}}}$
orering	$-\frac{2(-3b^4Bx^5-5Ab^4x^4-20Bab^3x^4-60Aab^3x^3-90Ba^2b^2x^3+90Aa^2b^2x^2+60Ba^3bx^2+20Aa^3bx+5a^4Bx+3a^4A)}{15x^{\frac{5}{2}}(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^4*B*x^(5/2)+2/3*A*b^4*x^(3/2)+8/3*B*a*b^3*x^(3/2)+8*A*a*b^3*x^(1/2)+
12*B*a^2*b^2*x^(1/2)-4*a^2*b*(3*A*b+2*B*a)/x^(1/2)-2/5*a^4*A/x^(5/2)-2/3*a
^3*(4*A*b+B*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx=\frac{2\left(3\,Bb^4x^5-3\,Aa^4+5\,(4\,Bab^3+Ab^4)x^4+30\,(3\,Ba^2b^2+2\,Aab^3)x^3}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^4*x^5 - 3*A*a^4 + 5*(4*B*a*b^3 + A*b^4)*x^4 + 30*(3*B*a^2*b^2
+ 2*A*a*b^3)*x^3 - 30*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 5*(B*a^4 + 4*A*a^3*b
)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx = -\frac{2Aa^4}{5x^{\frac{5}{2}}} - \frac{8Aa^3b}{3x^{\frac{3}{2}}} - \frac{12Aa^2b^2}{\sqrt{x}} + 8Aab^3\sqrt{x} + \frac{2Ab^4x^{\frac{3}{2}}}{3} - \frac{2Ba^4}{3x^{\frac{3}{2}}} - \frac{8Ba^3b}{\sqrt{x}} + 12Ba^2b^2\sqrt{x} + \frac{8Bab^3x^{\frac{3}{2}}}{3} + \frac{2Bb^4x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**(7/2),x)
```

output
```
-2*A*a**4/(5*x**(5/2)) - 8*A*a**3*b/(3*x**(3/2)) - 12*A*a**2*b**2/sqrt(x)
+ 8*A*a*b**3*sqrt(x) + 2*A*b**4*x**(3/2)/3 - 2*B*a**4/(3*x**(3/2)) - 8*B*a
**3*b/sqrt(x) + 12*B*a**2*b**2*sqrt(x) + 8*B*a*b**3*x**(3/2)/3 + 2*B*b**4*
x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bb^4x^{\frac{5}{2}} + \frac{2}{3}\left(4\,Bab^3+Ab^4\right)x^{\frac{3}{2}} + 4\left(3\,Ba^2b^2+2\,Aab^3\right)\sqrt{x} - \frac{2\left(3\,Aa^4+30\left(2\,Ba^3b+3\,Aa^2b^2\right)x^2+5\left(Ba^4+4\,Aa^3b\right)x\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/5*B*b^4*x^(5/2) + 2/3*(4*B*a*b^3 + A*b^4)*x^(3/2) + 4*(3*B*a^2*b^2 + 2*A
*a*b^3)*sqrt(x) - 2/15*(3*A*a^4 + 30*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 5*(B*
a^4 + 4*A*a^3*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bb^4x^{\frac{5}{2}} + \frac{8}{3}\,Bab^3x^{\frac{3}{2}} + \frac{2}{3}\,Ab^4x^{\frac{3}{2}} + 12\,Ba^2b^2\sqrt{x} + 8\,Aab^3\sqrt{x} - \frac{2\left(60\,Ba^3bx^2 + 90\,Aa^2b^2x^2 + 5\,Ba^4x + 20\,Aa^3bx + 3\,Aa^4\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/5*B*b^4*x^(5/2) + 8/3*B*a*b^3*x^(3/2) + 2/3*A*b^4*x^(3/2) + 12*B*a^2*b^2
*sqrt(x) + 8*A*a*b^3*sqrt(x) - 2/15*(60*B*a^3*b*x^2 + 90*A*a^2*b^2*x^2 + 5
*B*a^4*x + 20*A*a^3*b*x + 3*A*a^4)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx = x^{3/2}\left(\frac{2\,A\,b^4}{3}+\frac{8\,B\,a\,b^3}{3}\right) - \frac{x\left(\frac{2\,B\,a^4}{3}+\frac{8\,A\,b\,a^3}{3}\right)+\frac{2\,A\,a^4}{5}+x^2\left(8\,B\,a^3\,b+12\,A\,a^2\,b^2\right)}{x^{5/2}} + \frac{2\,B\,b^4\,x^{5/2}}{5} + 4\,a\,b^2\,\sqrt{x}\,\left(2\,A\,b+3\,B\,a\right)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^(7/2),x)
```

output
```
x^(3/2)*((2*A*b^4)/3 + (8*B*a*b^3)/3) - (x*((2*B*a^4)/3 + (8*A*a^3*b)/3) +
 (2*A*a^4)/5 + x^2*(12*A*a^2*b^2 + 8*B*a^3*b))/x^(5/2) + (2*B*b^4*x^(5/2))
/5 + 4*a*b^2*x^(1/2)*(2*A*b + 3*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{7/2}}\,dx = \frac{\frac{2}{5}b^5x^5+\frac{10}{3}a\,b^4x^4+20a^2b^3x^3-20a^3b^2x^2-\frac{10}{3}a^4bx-\frac{2}{5}a^5}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(7/2),x)
```

output
```
(2*( - 3*a**5 - 25*a**4*b*x - 150*a**3*b**2*x**2 + 150*a**2*b**3*x**3 + 25
*a*b**4*x**4 + 3*b**5*x**5))/(15*sqrt(x)*x**2)
```

3.375 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = -\frac{2a^4A}{7x^{7/2}} - \frac{2a^3(4Ab+aB)}{5x^{5/2}} - \frac{4a^2b(3Ab+2aB)}{3x^{3/2}} - \frac{4ab^2(2Ab+3aB)}{\sqrt{x}} + 2b^3(Ab+4aB)\sqrt{x} + \frac{2}{3}b^4Bx^{3/2}$$

output
```
-2/7*a^4*A/x^(7/2)-2/5*a^3*(4*A*b+B*a)/x^(5/2)-4/3*a^2*b*(3*A*b+2*B*a)/x^(
3/2)-4*a*b^2*(2*A*b+3*B*a)/x^(1/2)+2*b^3*(A*b+4*B*a)*x^(1/2)+2/3*b^4*B*x^(
3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = -\frac{2(420ab^3x^3(A-Bx)-35b^4x^4(3A+Bx)+210a^2b^2x^2(A+3Bx)+28a^3bx(3A+5Bx)+3a^4(5A+7Bx}{105x^{7/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(9/2),x]
```

output
```
(-2*(420*a*b^3*x^3*(A - B*x) - 35*b^4*x^4*(3*A + B*x) + 210*a^2*b^2*x^2*(A
 + 3*B*x) + 28*a^3*b*x*(3*A + 5*B*x) + 3*a^4*(5*A + 7*B*x)))/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^2 (A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^4(a+bx)^4(A+Bx)}{x^{9/2}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^4(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^4A}{x^{9/2}}+\frac{a^3(aB+4Ab)}{x^{7/2}}+\frac{2a^2b(2aB+3Ab)}{x^{5/2}}+\frac{b^3(4aB+Ab)}{\sqrt{x}}+\frac{2ab^2(3aB+2Ab)}{x^{3/2}}+b^4B\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4A}{7x^{7/2}}-\frac{2a^3(aB+4Ab)}{5x^{5/2}}-\frac{4a^2b(2aB+3Ab)}{3x^{3/2}}+2b^3\sqrt{x}(4aB+Ab)-\frac{4ab^2(3aB+2Ab)}{\sqrt{x}}+\frac{2}{3}b^4Bx^{3/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2)/x^(9/2),x]
```

output
```
(-2*a^4*A)/(7*x^(7/2)) - (2*a^3*(4*A*b + a*B))/(5*x^(5/2)) - (4*a^2*b*(3*A
*b + 2*a*B))/(3*x^(3/2)) - (4*a*b^2*(2*A*b + 3*a*B))/Sqrt[x] + 2*b^3*(A*b
+ 4*a*B)*Sqrt[x] + (2*b^4*B*x^(3/2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2b^4 B x^{\frac{3}{2}}}{3}+2A b^4\sqrt{x}+8Ba b^3\sqrt{x}-\frac{2a^4 A}{7x^{\frac{7}{2}}}-\frac{4a b^2(2Ab+3Ba)}{\sqrt{x}}-\frac{2a^3(4Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{4a^2b(3Ab+2Ba)}{3x^{\frac{3}{2}}}$
default	$\frac{2b^4 B x^{\frac{3}{2}}}{3}+2A b^4\sqrt{x}+8Ba b^3\sqrt{x}-\frac{2a^4 A}{7x^{\frac{7}{2}}}-\frac{4a b^2(2Ab+3Ba)}{\sqrt{x}}-\frac{2a^3(4Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{4a^2b(3Ab+2Ba)}{3x^{\frac{3}{2}}}$
gosper	$-\frac{2(-35b^4 B x^5-105A b^4x^4-420Ba b^3x^4+420Aa b^3x^3+630B a^2b^2x^3+210A a^2b^2x^2+140B a^3b x^2+84A a^3bx+21a^4 B}{105x^{\frac{7}{2}}}$
trager	$-\frac{2(-35b^4 B x^5-105A b^4x^4-420Ba b^3x^4+420Aa b^3x^3+630B a^2b^2x^3+210A a^2b^2x^2+140B a^3b x^2+84A a^3bx+21a^4 B}{105x^{\frac{7}{2}}}$
risch	$-\frac{2(-35b^4 B x^5-105A b^4x^4-420Ba b^3x^4+420Aa b^3x^3+630B a^2b^2x^3+210A a^2b^2x^2+140B a^3b x^2+84A a^3bx+21a^4 B}{105x^{\frac{7}{2}}}$
orering	$-\frac{2(-35b^4 B x^5-105A b^4x^4-420Ba b^3x^4+420Aa b^3x^3+630B a^2b^2x^3+210A a^2b^2x^2+140B a^3b x^2+84A a^3bx+21a^4 B}{105x^{\frac{7}{2}}(bx+a)^4}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^4*B*x^(3/2)+2*A*b^4*x^(1/2)+8*B*a*b^3*x^(1/2)-2/7*a^4*A/x^(7/2)-4*a*
b^2*(2*A*b+3*B*a)/x^(1/2)-2/5*a^3*(4*A*b+B*a)/x^(5/2)-4/3*a^2*b*(3*A*b+2*B
*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^2}{x^{9/2}}\,dx=\frac{2\,(35\,Bb^4x^5-15\,Aa^4+105\,(4\,Bab^3+Ab^4)x^4-210\,(3\,Ba^2b^2+2\,A}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(9/2),x, algorithm="fricas")
```

output
```
2/105*(35*B*b^4*x^5 - 15*A*a^4 + 105*(4*B*a*b^3 + A*b^4)*x^4 - 210*(3*B*a^
2*b^2 + 2*A*a*b^3)*x^3 - 70*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 - 21*(B*a^4 + 4*
A*a^3*b)*x)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = -\frac{2Aa^4}{7x^{\frac{7}{2}}} - \frac{8Aa^3b}{5x^{\frac{5}{2}}} - \frac{4Aa^2b^2}{x^{\frac{3}{2}}} - \frac{8Aab^3}{\sqrt{x}} + 2Ab^4\sqrt{x} - \frac{2Ba^4}{5x^{\frac{5}{2}}} - \frac{8Ba^3b}{3x^{\frac{3}{2}}} - \frac{12Ba^2b^2}{\sqrt{x}} + 8Bab^3\sqrt{x} + \frac{2Bb^4x^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/x**(9/2),x)
```

output
```
-2*A*a**4/(7*x**(7/2)) - 8*A*a**3*b/(5*x**(5/2)) - 4*A*a**2*b**2/x**(3/2)
- 8*A*a*b**3/sqrt(x) + 2*A*b**4*sqrt(x) - 2*B*a**4/(5*x**(5/2)) - 8*B*a**3
*b/(3*x**(3/2)) - 12*B*a**2*b**2/sqrt(x) + 8*B*a*b**3*sqrt(x) + 2*B*b**4*x
**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = \frac{2}{3}\,Bb^4x^{\frac{3}{2}} + 2\left(4\,Bab^3 + Ab^4\right)\sqrt{x} - \frac{2\,(15\,Aa^4 + 210\,(3\,Ba^2b^2 + 2\,Aab^3)x^3 + 70\,(2\,Ba^3b + 3\,Aa^2b^2)x^2 + 21\,(Ba^4 + 4\,Aa^3b)x)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(9/2),x, algorithm="maxima")
```

output
```
2/3*B*b^4*x^(3/2) + 2*(4*B*a*b^3 + A*b^4)*sqrt(x) - 2/105*(15*A*a^4 + 210*
(3*B*a^2*b^2 + 2*A*a*b^3)*x^3 + 70*(2*B*a^3*b + 3*A*a^2*b^2)*x^2 + 21*(B*a
^4 + 4*A*a^3*b)*x)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = \frac{2}{3}\,Bb^4x^{\frac{3}{2}} + 8\,Bab^3\sqrt{x} + 2\,Ab^4\sqrt{x} - \frac{2\,(630\,Ba^2b^2x^3 + 420\,Aab^3x^3 + 140\,Ba^3bx^2 + 210\,Aa^2b^2x^2 + 21\,Ba^4x + 84\,Aa^3bx + 15\,Aa^4)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(9/2),x, algorithm="giac")
```

output
```
2/3*B*b^4*x^(3/2) + 8*B*a*b^3*sqrt(x) + 2*A*b^4*sqrt(x) - 2/105*(630*B*a^2
*b^2*x^3 + 420*A*a*b^3*x^3 + 140*B*a^3*b*x^2 + 210*A*a^2*b^2*x^2 + 21*B*a^
4*x + 84*A*a^3*b*x + 15*A*a^4)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = \sqrt{x}\,\left(2\,A\,b^4+8\,B\,a\,b^3\right) - \frac{x\,\left(\frac{2\,B\,a^4}{5}+\frac{8\,A\,b\,a^3}{5}\right)+\frac{2\,A\,a^4}{7}+x^2\,\left(\frac{8\,B\,a^3\,b}{3}+4\,A\,a^2\,b^2\right)+x^3\,\left(12\,B\,a^2\,b^2+8\,A\,a\,b^3\right)}{x^{7/2}} + \frac{2\,B\,b^4\,x^{3/2}}{3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2)/x^(9/2),x)
```

output
```
x^(1/2)*(2*A*b^4 + 8*B*a*b^3) - (x*((2*B*a^4)/5 + (8*A*a^3*b)/5) + (2*A*a^
4)/7 + x^2*(4*A*a^2*b^2 + (8*B*a^3*b)/3) + x^3*(12*B*a^2*b^2 + 8*A*a*b^3))
/x^(7/2) + (2*B*b^4*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^2}{x^{9/2}}\,dx = \frac{\frac{2}{3}b^5x^5+10a\,b^4x^4-20a^2b^3x^3-\frac{20}{3}a^3b^2x^2-2a^4bx-\frac{2}{7}a^5}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2/x^(9/2),x)
```

output
```
(2*( - 3*a**5 - 21*a**4*b*x - 70*a**3*b**2*x**2 - 210*a**2*b**3*x**3 + 105
*a*b**4*x**4 + 7*b**5*x**5))/(21*sqrt(x)*x**3)
```

3.376 $\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{9}a^6Ax^{9/2}+\frac{2}{11}a^5(6Ab+aB)x^{11/2}$$
$$+\frac{6}{13}a^4b(5Ab+2aB)x^{13/2}+\frac{2}{3}a^3b^2(4Ab+3aB)x^{15/2}+\frac{10}{17}a^2b^3(3Ab+4aB)x^{17/2}+\frac{6}{19}ab^4(2Ab+5aB)x^{19/2}+\frac{2}{21}$$

output
```
2/9*a^6*A*x^(9/2)+2/11*a^5*(6*A*b+B*a)*x^(11/2)+6/13*a^4*b*(5*A*b+2*B*a)*x
^(13/2)+2/3*a^3*b^2*(4*A*b+3*B*a)*x^(15/2)+10/17*a^2*b^3*(3*A*b+4*B*a)*x^(
17/2)+6/19*a*b^4*(2*A*b+5*B*a)*x^(19/2)+2/21*b^5*(A*b+6*B*a)*x^(21/2)+2/23
*b^6*B*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{99}a^6x^{9/2}(11A+9Bx)$$
$$+\frac{12}{143}a^5bx^{11/2}(13A+11Bx)+\frac{2}{13}a^4b^2x^{13/2}(15A+13Bx)+\frac{8}{51}a^3b^3x^{15/2}(17A+15Bx)+\frac{30}{323}a^2b^4x^{17/2}(19A+1$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*x^(9/2)*(11*A + 9*B*x))/99 + (12*a^5*b*x^(11/2)*(13*A + 11*B*x))/14
3 + (2*a^4*b^2*x^(13/2)*(15*A + 13*B*x))/13 + (8*a^3*b^3*x^(15/2)*(17*A +
15*B*x))/51 + (30*a^2*b^4*x^(17/2)*(19*A + 17*B*x))/323 + (4*a*b^5*x^(19/2
)*(21*A + 19*B*x))/133 + (2*b^6*x^(21/2)*(23*A + 21*B*x))/483
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^{7/2}(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^{7/2}(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \Big(a^6Ax^{7/2}+a^5x^{9/2}(aB+6Ab)+3a^4bx^{11/2}(2aB+5Ab)+5a^3b^2x^{13/2}(3aB+4Ab)+5a^2b^3x^{15/2}(4aB+3Ab)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^6Ax^{9/2}+\frac{2}{11}a^5x^{11/2}(aB+6Ab)+\frac{6}{13}a^4bx^{13/2}(2aB+5Ab)+\frac{2}{3}a^3b^2x^{15/2}(3aB+4Ab)+\frac{10}{17}a^2b^3x^{17/2}(4aB+3Ab)+\frac{2}{21}b^5x^{21/2}(6aB+Ab)+\frac{6}{19}ab^4x^{19/2}(5aB+2Ab)+\frac{2}{23}b^6Bx^{23/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*A*x^(9/2))/9 + (2*a^5*(6*A*b + a*B)*x^(11/2))/11 + (6*a^4*b*(5*A*b
+ 2*a*B)*x^(13/2))/13 + (2*a^3*b^2*(4*A*b + 3*a*B)*x^(15/2))/3 + (10*a^2*b
^3*(3*A*b + 4*a*B)*x^(17/2))/17 + (6*a*b^4*(2*A*b + 5*a*B)*x^(19/2))/19 +
(2*b^5*(A*b + 6*a*B)*x^(21/2))/21 + (2*b^6*B*x^(23/2))/23
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{2x^{\frac{9}{2}}(2909907b^6Bx^7+3187041Ab^6x^6+19122246Bab^5x^6+21135114Aab^5x^5+52837785Ba^2b^4x^5+59053995Aa^2b^4x^4}{}$
derivativedivides	$\frac{2b^6Bx^{\frac{23}{2}}}{23}+\frac{2(Ab^6+6Bab^5)x^{\frac{21}{2}}}{21}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{19}{2}}}{19}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{17}{2}}}{17}+\frac{2(20Aa^3b^3+15}{15}$
default	$\frac{2b^6Bx^{\frac{23}{2}}}{23}+\frac{2(Ab^6+6Bab^5)x^{\frac{21}{2}}}{21}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{19}{2}}}{19}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{17}{2}}}{17}+\frac{2(20Aa^3b^3+15}{15}$
trager	$\frac{2x^{\frac{9}{2}}(2909907b^6Bx^7+3187041Ab^6x^6+19122246Bab^5x^6+21135114Aab^5x^5+52837785Ba^2b^4x^5+59053995Aa^2b^4x^4}{}$
risch	$\frac{2x^{\frac{9}{2}}(2909907b^6Bx^7+3187041Ab^6x^6+19122246Bab^5x^6+21135114Aab^5x^5+52837785Ba^2b^4x^5+59053995Aa^2b^4x^4}{}$
orering	$\frac{2(2909907b^6Bx^7+3187041Ab^6x^6+19122246Bab^5x^6+21135114Aab^5x^5+52837785Ba^2b^4x^5+59053995Aa^2b^4x^4+7}{}$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/66927861*x^(9/2)*(2909907*B*b^6*x^7+3187041*A*b^6*x^6+19122246*B*a*b^5*x
^6+21135114*A*a*b^5*x^5+52837785*B*a^2*b^4*x^5+59053995*A*a^2*b^4*x^4+7873
8660*B*a^3*b^3*x^4+89237148*A*a^3*b^3*x^3+66927861*B*a^4*b^2*x^3+77224455*
A*a^4*b^2*x^2+30889782*B*a^5*b*x^2+36506106*A*a^5*b*x+6084351*B*a^6*x+7436
429*A*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{66927861}\left(2909907\,Bb^6x^{11}+7436429\,Aa^6x^4+3187041\,\left(6\,Bab^5+Ab^6\right)x^{10}+10567557\,(5\,B\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
2/66927861*(2909907*B*b^6*x^11 + 7436429*A*a^6*x^4 + 3187041*(6*B*a*b^5 +
A*b^6)*x^10 + 10567557*(5*B*a^2*b^4 + 2*A*a*b^5)*x^9 + 19684665*(4*B*a^3*b
^3 + 3*A*a^2*b^4)*x^8 + 22309287*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^7 + 1544489
1*(2*B*a^5*b + 5*A*a^4*b^2)*x^6 + 6084351*(B*a^6 + 6*A*a^5*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2Aa^6x^{\frac{9}{2}}}{9}+\frac{12Aa^5bx^{\frac{11}{2}}}{11}+\frac{30Aa^4b^2x^{\frac{13}{2}}}{13}+\frac{8Aa^3b^3x^{\frac{15}{2}}}{3}+\frac{30Aa^2b^4x^{\frac{17}{2}}}{17}+\frac{12Aab^5x^{\frac{19}{2}}}{19}+\frac{2Ab^6x^{\frac{21}{2}}}{21}+\frac{2Ba^6x^{\frac{11}{2}}}{11}+\frac{12Ba^5bx^{\frac{13}{2}}}{13}+2Ba^4b^2x^{\frac{15}{2}}+\frac{40Ba^3b^3x^{\frac{17}{2}}}{17}+\frac{30Ba^2b^4x^{\frac{19}{2}}}{19}+\frac{4Bab^5x^{\frac{21}{2}}}{7}+\frac{2Bb^6x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(7/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
2*A*a**6*x**(9/2)/9 + 12*A*a**5*b*x**(11/2)/11 + 30*A*a**4*b**2*x**(13/2)/
13 + 8*A*a**3*b**3*x**(15/2)/3 + 30*A*a**2*b**4*x**(17/2)/17 + 12*A*a*b**5
*x**(19/2)/19 + 2*A*b**6*x**(21/2)/21 + 2*B*a**6*x**(11/2)/11 + 12*B*a**5*
b*x**(13/2)/13 + 2*B*a**4*b**2*x**(15/2) + 40*B*a**3*b**3*x**(17/2)/17 + 3
0*B*a**2*b**4*x**(19/2)/19 + 4*B*a*b**5*x**(21/2)/7 + 2*B*b**6*x**(23/2)/2
3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{23}\,Bb^6x^{\frac{23}{2}}+\frac{2}{9}\,Aa^6x^{\frac{9}{2}}+\frac{2}{21}\left(6\,Bab^5+Ab^6\right)x^{\frac{21}{2}}+\frac{6}{19}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{19}{2}}+\frac{10}{17}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{17}{2}}+\frac{2}{3}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{15}{2}}+\frac{6}{13}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{13}{2}}+\frac{2}{11}\left(Ba^6+6\,Aa^5b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
2/23*B*b^6*x^(23/2) + 2/9*A*a^6*x^(9/2) + 2/21*(6*B*a*b^5 + A*b^6)*x^(21/2
) + 6/19*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(19/2) + 10/17*(4*B*a^3*b^3 + 3*A*a^2
*b^4)*x^(17/2) + 2/3*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(15/2) + 6/13*(2*B*a^5*
b + 5*A*a^4*b^2)*x^(13/2) + 2/11*(B*a^6 + 6*A*a^5*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{23}\,Bb^6x^{\frac{23}{2}}+\frac{4}{7}\,Bab^5x^{\frac{21}{2}}+\frac{2}{21}\,Ab^6x^{\frac{21}{2}}+\frac{30}{19}\,Ba^2b^4x^{\frac{19}{2}}+\frac{12}{19}\,Aab^5x^{\frac{19}{2}}+\frac{40}{17}\,Ba^3b^3x^{\frac{17}{2}}+\frac{30}{17}\,Aa^2b^4x^{\frac{17}{2}}+2\,Ba^4b^2x^{\frac{15}{2}}+\frac{8}{3}\,Aa^3b^3x^{\frac{15}{2}}+\frac{12}{13}\,Ba^5bx^{\frac{13}{2}}+\frac{30}{13}\,Aa^4b^2x^{\frac{13}{2}}+\frac{2}{11}\,Ba^6x^{\frac{11}{2}}+\frac{12}{11}\,Aa^5bx^{\frac{11}{2}}+\frac{2}{9}\,Aa^6x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
2/23*B*b^6*x^(23/2) + 4/7*B*a*b^5*x^(21/2) + 2/21*A*b^6*x^(21/2) + 30/19*B
*a^2*b^4*x^(19/2) + 12/19*A*a*b^5*x^(19/2) + 40/17*B*a^3*b^3*x^(17/2) + 30
/17*A*a^2*b^4*x^(17/2) + 2*B*a^4*b^2*x^(15/2) + 8/3*A*a^3*b^3*x^(15/2) + 1
2/13*B*a^5*b*x^(13/2) + 30/13*A*a^4*b^2*x^(13/2) + 2/11*B*a^6*x^(11/2) + 1
2/11*A*a^5*b*x^(11/2) + 2/9*A*a^6*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=x^{11/2}\left(\frac{2\,B\,a^6}{11}+\frac{12\,A\,b\,a^5}{11}\right)+x^{21/2}\left(\frac{2\,A\,b^6}{21}+\frac{4\,B\,a\,b^5}{7}\right)+\frac{2\,A\,a^6\,x^{9/2}}{9}+\frac{2\,B\,b^6\,x^{23/2}}{23}+\frac{2\,a^3\,b^2\,x^{15/2}\,\left(4\,A\,b+3\,B\,a\right)}{3}+\frac{10\,a^2\,b^3\,x^{17/2}\,\left(3\,A\,b+\right.}{17}$$

input
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^(11/2)*((2*B*a^6)/11 + (12*A*a^5*b)/11) + x^(21/2)*((2*A*b^6)/21 + (4*B*
a*b^5)/7) + (2*A*a^6*x^(9/2))/9 + (2*B*b^6*x^(23/2))/23 + (2*a^3*b^2*x^(15
/2)*(4*A*b + 3*B*a))/3 + (10*a^2*b^3*x^(17/2)*(3*A*b + 4*B*a))/17 + (6*a^4
*b*x^(13/2)*(5*A*b + 2*B*a))/13 + (6*a*b^4*x^(19/2)*(2*A*b + 5*B*a))/19
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2\sqrt{x}\,x^4(415701b^7x^7+3187041a\,b^6x^6+10567557a^2b^5x^5+19684665a^3b^4x^4+22309287a^4b^3x}{9561123}$$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(2*sqrt(x)*x**4*(1062347*a**7 + 6084351*a**6*b*x + 15444891*a**5*b**2*x**2
 + 22309287*a**4*b**3*x**3 + 19684665*a**3*b**4*x**4 + 10567557*a**2*b**5*
x**5 + 3187041*a*b**6*x**6 + 415701*b**7*x**7))/9561123
```

3.377 $\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{7}a^6Ax^{7/2}+\frac{2}{9}a^5(6Ab+aB)x^{9/2}$$
$$+\frac{6}{11}a^4b(5Ab+2aB)x^{11/2}+\frac{10}{13}a^3b^2(4Ab+3aB)x^{13/2}+\frac{2}{3}a^2b^3(3Ab+4aB)x^{15/2}+\frac{6}{17}ab^4(2Ab+5aB)x^{17/2}+\frac{2}{19}$$

output
```
2/7*a^6*A*x^(7/2)+2/9*a^5*(6*A*b+B*a)*x^(9/2)+6/11*a^4*b*(5*A*b+2*B*a)*x^(
11/2)+10/13*a^3*b^2*(4*A*b+3*B*a)*x^(13/2)+2/3*a^2*b^3*(3*A*b+4*B*a)*x^(15
/2)+6/17*a*b^4*(2*A*b+5*B*a)*x^(17/2)+2/19*b^5*(A*b+6*B*a)*x^(19/2)+2/21*b
^6*B*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int x^{5/2}(A+Bx)\left(a^2+2abx\right.$$
$$\left.+b^2x^2\right)^3\,dx=\frac{2x^{7/2}(46189a^6(9A+7Bx)+176358a^5bx(11A+9Bx)+305235a^4b^2x^2(13A+11Bx)+298}{}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*x^(7/2)*(46189*a^6*(9*A + 7*B*x) + 176358*a^5*b*x*(11*A + 9*B*x) + 3052
35*a^4*b^2*x^2*(13*A + 11*B*x) + 298452*a^3*b^3*x^3*(15*A + 13*B*x) + 1711
71*a^2*b^4*x^4*(17*A + 15*B*x) + 54054*a*b^5*x^5*(19*A + 17*B*x) + 7293*b^
6*x^6*(21*A + 19*B*x)))/2909907
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \ 1184$$

$$\frac{\int b^6x^{5/2}(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \ 27$$

$$\int x^{5/2}(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \ 85$$

$$\int \left(a^6Ax^{5/2}+a^5x^{7/2}(aB+6Ab)+3a^4bx^{9/2}(2aB+5Ab)+5a^3b^2x^{11/2}(3aB+4Ab)+5a^2b^3x^{13/2}(4aB+3Ab)\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}a^6Ax^{7/2}+\frac{2}{9}a^5x^{9/2}(aB+6Ab)+\frac{6}{11}a^4bx^{11/2}(2aB+5Ab)+\frac{10}{13}a^3b^2x^{13/2}(3aB+4Ab)+\frac{2}{3}a^2b^3x^{15/2}(4aB+3Ab)+\frac{2}{19}b^5x^{19/2}(6aB+Ab)+\frac{6}{17}ab^4x^{17/2}(5aB+2Ab)+\frac{2}{21}b^6Bx^{21/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*A*x^(7/2))/7 + (2*a^5*(6*A*b + a*B)*x^(9/2))/9 + (6*a^4*b*(5*A*b +
2*a*B)*x^(11/2))/11 + (10*a^3*b^2*(4*A*b + 3*a*B)*x^(13/2))/13 + (2*a^2*b^
3*(3*A*b + 4*a*B)*x^(15/2))/3 + (6*a*b^4*(2*A*b + 5*a*B)*x^(17/2))/17 + (2
*b^5*(A*b + 6*a*B)*x^(19/2))/19 + (2*b^6*B*x^(21/2))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{2x^{\frac{7}{2}}(138567b^6Bx^7+153153Ab^6x^6+918918Bab^5x^6+1027026Aab^5x^5+2567565Ba^2b^4x^5+2909907Aa^2b^4x^4+38798}{}$
derivativedivides	$\frac{2b^6Bx^{\frac{21}{2}}}{21}+\frac{2(Ab^6+6Bab^5)x^{\frac{19}{2}}}{19}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{17}{2}}}{17}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{15}{2}}}{15}+\frac{2(20Aa^3b^3+15}{13}$
default	$\frac{2b^6Bx^{\frac{21}{2}}}{21}+\frac{2(Ab^6+6Bab^5)x^{\frac{19}{2}}}{19}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{17}{2}}}{17}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{15}{2}}}{15}+\frac{2(20Aa^3b^3+15}{13}$
trager	$\frac{2x^{\frac{7}{2}}(138567b^6Bx^7+153153Ab^6x^6+918918Bab^5x^6+1027026Aab^5x^5+2567565Ba^2b^4x^5+2909907Aa^2b^4x^4+38798}{}$
risch	$\frac{2x^{\frac{7}{2}}(138567b^6Bx^7+153153Ab^6x^6+918918Bab^5x^6+1027026Aab^5x^5+2567565Ba^2b^4x^5+2909907Aa^2b^4x^4+38798}{}$
orering	$\frac{2(138567b^6Bx^7+153153Ab^6x^6+918918Bab^5x^6+1027026Aab^5x^5+2567565Ba^2b^4x^5+2909907Aa^2b^4x^4+3879876B}{}$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/2909907*x^(7/2)*(138567*B*b^6*x^7+153153*A*b^6*x^6+918918*B*a*b^5*x^6+10
27026*A*a*b^5*x^5+2567565*B*a^2*b^4*x^5+2909907*A*a^2*b^4*x^4+3879876*B*a^
3*b^3*x^4+4476780*A*a^3*b^3*x^3+3357585*B*a^4*b^2*x^3+3968055*A*a^4*b^2*x^
2+1587222*B*a^5*b*x^2+1939938*A*a^5*b*x+323323*B*a^6*x+415701*A*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{2909907}\left(138567\,Bb^6x^{10}+415701\,Aa^6x^3+153153\left(6\,Bab^5+Ab^6\right)x^9+513513\left(5\,Ba^2b^4+2\right.\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
2/2909907*(138567*B*b^6*x^10 + 415701*A*a^6*x^3 + 153153*(6*B*a*b^5 + A*b^
6)*x^9 + 513513*(5*B*a^2*b^4 + 2*A*a*b^5)*x^8 + 969969*(4*B*a^3*b^3 + 3*A*
a^2*b^4)*x^7 + 1119195*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^6 + 793611*(2*B*a^5*b
 + 5*A*a^4*b^2)*x^5 + 323323*(B*a^6 + 6*A*a^5*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2Aa^6x^{\frac{7}{2}}}{7}+\frac{4Aa^5bx^{\frac{9}{2}}}{3}+\frac{30Aa^4b^2x^{\frac{11}{2}}}{11}+\frac{40Aa^3b^3x^{\frac{13}{2}}}{13}+2Aa^2b^4x^{\frac{15}{2}}+\frac{12Aab^5x^{\frac{17}{2}}}{17}+\frac{2Ab^6x^{\frac{19}{2}}}{19}+\frac{2Ba^6x^{\frac{9}{2}}}{9}+\frac{12Ba^5bx^{\frac{11}{2}}}{11}+\frac{30Ba^4b^2x^{\frac{13}{2}}}{13}+\frac{8Ba^3b^3x^{\frac{15}{2}}}{3}+\frac{30Ba^2b^4x^{\frac{17}{2}}}{17}+\frac{12Bab^5x^{\frac{19}{2}}}{19}+\frac{2Bb^6x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(5/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
2*A*a**6*x**(7/2)/7 + 4*A*a**5*b*x**(9/2)/3 + 30*A*a**4*b**2*x**(11/2)/11
+ 40*A*a**3*b**3*x**(13/2)/13 + 2*A*a**2*b**4*x**(15/2) + 12*A*a*b**5*x**(
17/2)/17 + 2*A*b**6*x**(19/2)/19 + 2*B*a**6*x**(9/2)/9 + 12*B*a**5*b*x**(1
1/2)/11 + 30*B*a**4*b**2*x**(13/2)/13 + 8*B*a**3*b**3*x**(15/2)/3 + 30*B*a
**2*b**4*x**(17/2)/17 + 12*B*a*b**5*x**(19/2)/19 + 2*B*b**6*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{21}\,Bb^6x^{\frac{21}{2}}+\frac{2}{7}\,Aa^6x^{\frac{7}{2}}+\frac{2}{19}\,\left(6\,Bab^5+Ab^6\right)x^{\frac{19}{2}}+\frac{6}{17}\,\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{17}{2}}+\frac{2}{3}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{15}{2}}+\frac{10}{13}\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{13}{2}}+\frac{6}{11}\,\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{11}{2}}+\frac{2}{9}\,\left(Ba^6+6\,Aa^5b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
2/21*B*b^6*x^(21/2) + 2/7*A*a^6*x^(7/2) + 2/19*(6*B*a*b^5 + A*b^6)*x^(19/2
) + 6/17*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(17/2) + 2/3*(4*B*a^3*b^3 + 3*A*a^2*b
^4)*x^(15/2) + 10/13*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(13/2) + 6/11*(2*B*a^5*
b + 5*A*a^4*b^2)*x^(11/2) + 2/9*(B*a^6 + 6*A*a^5*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{2}{21}\,Bb^6x^{\frac{21}{2}} + \frac{12}{19}\,Bab^5x^{\frac{19}{2}} + \frac{2}{19}\,Ab^6x^{\frac{19}{2}} + \frac{30}{17}\,Ba^2b^4x^{\frac{17}{2}} + \frac{12}{17}\,Aab^5x^{\frac{17}{2}} + \frac{8}{3}\,Ba^3b^3x^{\frac{15}{2}} + 2\,Aa^2b^4x^{\frac{15}{2}} + \frac{30}{13}\,Ba^4b^2x^{\frac{13}{2}} + \frac{40}{13}\,Aa^3b^3x^{\frac{13}{2}} + \frac{12}{11}\,Ba^5bx^{\frac{11}{2}} + \frac{30}{11}\,Aa^4b^2x^{\frac{11}{2}} + \frac{2}{9}\,Ba^6x^{\frac{9}{2}} + \frac{4}{3}\,Aa^5bx^{\frac{9}{2}} + \frac{2}{7}\,Aa^6x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
2/21*B*b^6*x^(21/2) + 12/19*B*a*b^5*x^(19/2) + 2/19*A*b^6*x^(19/2) + 30/17
*B*a^2*b^4*x^(17/2) + 12/17*A*a*b^5*x^(17/2) + 8/3*B*a^3*b^3*x^(15/2) + 2*
A*a^2*b^4*x^(15/2) + 30/13*B*a^4*b^2*x^(13/2) + 40/13*A*a^3*b^3*x^(13/2) +
 12/11*B*a^5*b*x^(11/2) + 30/11*A*a^4*b^2*x^(11/2) + 2/9*B*a^6*x^(9/2) + 4
/3*A*a^5*b*x^(9/2) + 2/7*A*a^6*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^{9/2}\left(\frac{2\,B\,a^6}{9}+\frac{4\,A\,b\,a^5}{3}\right) + x^{19/2}\left(\frac{2\,A\,b^6}{19}+\frac{12\,B\,a\,b^5}{19}\right)+\frac{2\,A\,a^6\,x^{7/2}}{7}+\frac{2\,B\,b^6\,x^{21/2}}{21}+\frac{10\,a^3\,b^2\,x^{13/2}\,(4\,A\,b+3\,B\,a)}{13}+\frac{2\,a^2\,b^3\,x^{15/2}\,(3\,A\,b+$$

input
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^(9/2)*((2*B*a^6)/9 + (4*A*a^5*b)/3) + x^(19/2)*((2*A*b^6)/19 + (12*B*a*b
^5)/19) + (2*A*a^6*x^(7/2))/7 + (2*B*b^6*x^(21/2))/21 + (10*a^3*b^2*x^(13/
2)*(4*A*b + 3*B*a))/13 + (2*a^2*b^3*x^(15/2)*(3*A*b + 4*B*a))/3 + (6*a^4*b
*x^(11/2)*(5*A*b + 2*B*a))/11 + (6*a*b^4*x^(17/2)*(2*A*b + 5*B*a))/17
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2\sqrt{x}\,x^3(138567b^7x^7+1072071a\,b^6x^6+3594591a^2b^5x^5+6789783a^3b^4x^4+7834365a^4b^3x^3+}{2909907}$$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(2*sqrt(x)*x**3*(415701*a**7 + 2263261*a**6*b*x + 5555277*a**5*b**2*x**2 +
 7834365*a**4*b**3*x**3 + 6789783*a**3*b**4*x**4 + 3594591*a**2*b**5*x**5
+ 1072071*a*b**6*x**6 + 138567*b**7*x**7))/2909907
```

3.378 $\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{5}a^6Ax^{5/2}+\frac{2}{7}a^5(6Ab+aB)x^{7/2}$$
$$+\frac{2}{3}a^4b(5Ab+2aB)x^{9/2}+\frac{10}{11}a^3b^2(4Ab+3aB)x^{11/2}+\frac{10}{13}a^2b^3(3Ab+4aB)x^{13/2}+\frac{2}{5}ab^4(2Ab+5aB)x^{15/2}+\frac{2}{17}b^5($$

output
```
2/5*a^6*A*x^(5/2)+2/7*a^5*(6*A*b+B*a)*x^(7/2)+2/3*a^4*b*(5*A*b+2*B*a)*x^(9
/2)+10/11*a^3*b^2*(4*A*b+3*B*a)*x^(11/2)+10/13*a^2*b^3*(3*A*b+4*B*a)*x^(13
/2)+2/5*a*b^4*(2*A*b+5*B*a)*x^(15/2)+2/17*b^5*(A*b+6*B*a)*x^(17/2)+2/19*b^
6*B*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{35}a^6x^{5/2}(7A+5Bx)$$
$$+\frac{4}{21}a^5bx^{7/2}(9A+7Bx)+\frac{10}{33}a^4b^2x^{9/2}(11A+9Bx)+\frac{40}{143}a^3b^3x^{11/2}(13A+11Bx)+\frac{2}{13}a^2b^4x^{13/2}(15A+13Bx)+$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*x^(5/2)*(7*A + 5*B*x))/35 + (4*a^5*b*x^(7/2)*(9*A + 7*B*x))/21 + (1
0*a^4*b^2*x^(9/2)*(11*A + 9*B*x))/33 + (40*a^3*b^3*x^(11/2)*(13*A + 11*B*x
))/143 + (2*a^2*b^4*x^(13/2)*(15*A + 13*B*x))/13 + (4*a*b^5*x^(15/2)*(17*A
 + 15*B*x))/85 + (2*b^6*x^(17/2)*(19*A + 17*B*x))/323
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int b^6x^{3/2}(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^{3/2}(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \Big(a^6Ax^{3/2}+a^5x^{5/2}(aB+6Ab)+3a^4bx^{7/2}(2aB+5Ab)+5a^3b^2x^{9/2}(3aB+4Ab)+5a^2b^3x^{11/2}(4aB+3Ab)+$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^6Ax^{5/2}+\frac{2}{7}a^5x^{7/2}(aB+6Ab)+\frac{2}{3}a^4bx^{9/2}(2aB+5Ab)+\frac{10}{11}a^3b^2x^{11/2}(3aB+4Ab)+\frac{10}{13}a^2b^3x^{13/2}(4aB+3Ab)+\frac{2}{17}b^5x^{17/2}(6aB+Ab)+\frac{2}{5}ab^4x^{15/2}(5aB+2Ab)+\frac{2}{19}b^6Bx^{19/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*A*x^(5/2))/5 + (2*a^5*(6*A*b + a*B)*x^(7/2))/7 + (2*a^4*b*(5*A*b +
2*a*B)*x^(9/2))/3 + (10*a^3*b^2*(4*A*b + 3*a*B)*x^(11/2))/11 + (10*a^2*b^3
*(3*A*b + 4*a*B)*x^(13/2))/13 + (2*a*b^4*(2*A*b + 5*a*B)*x^(15/2))/5 + (2*
b^5*(A*b + 6*a*B)*x^(17/2))/17 + (2*b^6*B*x^(19/2))/19
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{2x^{\frac{5}{2}}(255255b^6Bx^7+285285Ab^6x^6+1711710Bab^5x^6+1939938Aab^5x^5+4849845Ba^2b^4x^5+5595975Aa^2b^4x^4+74613}{}$
derivativedivides	$\frac{2b^6Bx^{\frac{19}{2}}}{19}+\frac{2(Ab^6+6Bab^5)x^{\frac{17}{2}}}{17}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{15}{2}}}{15}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{13}{2}}}{13}+\frac{2(20Aa^3b^3+15}{11}$
default	$\frac{2b^6Bx^{\frac{19}{2}}}{19}+\frac{2(Ab^6+6Bab^5)x^{\frac{17}{2}}}{17}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{15}{2}}}{15}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{13}{2}}}{13}+\frac{2(20Aa^3b^3+15}{11}$
trager	$\frac{2x^{\frac{5}{2}}(255255b^6Bx^7+285285Ab^6x^6+1711710Bab^5x^6+1939938Aab^5x^5+4849845Ba^2b^4x^5+5595975Aa^2b^4x^4+74613}{}$
risch	$\frac{2x^{\frac{5}{2}}(255255b^6Bx^7+285285Ab^6x^6+1711710Bab^5x^6+1939938Aab^5x^5+4849845Ba^2b^4x^5+5595975Aa^2b^4x^4+74613}{}$
orering	$\frac{2(255255b^6Bx^7+285285Ab^6x^6+1711710Bab^5x^6+1939938Aab^5x^5+4849845Ba^2b^4x^5+5595975Aa^2b^4x^4+7461300}{}$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/4849845*x^(5/2)*(255255*B*b^6*x^7+285285*A*b^6*x^6+1711710*B*a*b^5*x^6+1
939938*A*a*b^5*x^5+4849845*B*a^2*b^4*x^5+5595975*A*a^2*b^4*x^4+7461300*B*a
^3*b^3*x^4+8817900*A*a^3*b^3*x^3+6613425*B*a^4*b^2*x^3+8083075*A*a^4*b^2*x
^2+3233230*B*a^5*b*x^2+4157010*A*a^5*b*x+692835*B*a^6*x+969969*A*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{4849845}\left(255255\,Bb^6x^9+969969\,Aa^6x^2+285285\,\left(6\,Bab^5+Ab^6\right)x^8+969969\,\left(5\,Ba^2b^4+2\right.\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
2/4849845*(255255*B*b^6*x^9 + 969969*A*a^6*x^2 + 285285*(6*B*a*b^5 + A*b^6
)*x^8 + 969969*(5*B*a^2*b^4 + 2*A*a*b^5)*x^7 + 1865325*(4*B*a^3*b^3 + 3*A*
a^2*b^4)*x^6 + 2204475*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^5 + 1616615*(2*B*a^5*
b + 5*A*a^4*b^2)*x^4 + 692835*(B*a^6 + 6*A*a^5*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2Aa^6x^{\frac{5}{2}}}{5}+\frac{12Aa^5bx^{\frac{7}{2}}}{7}+\frac{10Aa^4b^2x^{\frac{9}{2}}}{3}+\frac{40Aa^3b^3x^{\frac{11}{2}}}{11}+\frac{30Aa^2b^4x^{\frac{13}{2}}}{13}+\frac{4Aab^5x^{\frac{15}{2}}}{5}+\frac{2Ab^6x^{\frac{17}{2}}}{17}+\frac{2Ba^6x^{\frac{7}{2}}}{7}+\frac{4Ba^5bx^{\frac{9}{2}}}{3}+\frac{30Ba^4b^2x^{\frac{11}{2}}}{11}+\frac{40Ba^3b^3x^{\frac{13}{2}}}{13}+2Ba^2b^4x^{\frac{15}{2}}+\frac{12Bab^5x^{\frac{17}{2}}}{17}+\frac{2Bb^6x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(3/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
2*A*a**6*x**(5/2)/5 + 12*A*a**5*b*x**(7/2)/7 + 10*A*a**4*b**2*x**(9/2)/3 +
 40*A*a**3*b**3*x**(11/2)/11 + 30*A*a**2*b**4*x**(13/2)/13 + 4*A*a*b**5*x**
*(15/2)/5 + 2*A*b**6*x**(17/2)/17 + 2*B*a**6*x**(7/2)/7 + 4*B*a**5*b*x**(9
/2)/3 + 30*B*a**4*b**2*x**(11/2)/11 + 40*B*a**3*b**3*x**(13/2)/13 + 2*B*a*
*2*b**4*x**(15/2) + 12*B*a*b**5*x**(17/2)/17 + 2*B*b**6*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{19}\,Bb^6x^{\frac{19}{2}}+\frac{2}{5}\,Aa^6x^{\frac{5}{2}}+\frac{2}{17}\,\left(6\,Bab^5+Ab^6\right)x^{\frac{17}{2}}+\frac{2}{5}\,\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{15}{2}}+\frac{10}{13}\,\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{13}{2}}+\frac{10}{11}\,\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{11}{2}}+\frac{2}{3}\,\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{9}{2}}+\frac{2}{7}\,\left(Ba^6+6\,Aa^5b\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
2/19*B*b^6*x^(19/2) + 2/5*A*a^6*x^(5/2) + 2/17*(6*B*a*b^5 + A*b^6)*x^(17/2
) + 2/5*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(15/2) + 10/13*(4*B*a^3*b^3 + 3*A*a^2*
b^4)*x^(13/2) + 10/11*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(11/2) + 2/3*(2*B*a^5*
b + 5*A*a^4*b^2)*x^(9/2) + 2/7*(B*a^6 + 6*A*a^5*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2}{19}\,Bb^6x^{\frac{19}{2}}+\frac{12}{17}\,Bab^5x^{\frac{17}{2}}+\frac{2}{17}\,Ab^6x^{\frac{17}{2}}+2\,Ba^2b^4x^{\frac{15}{2}}+\frac{4}{5}\,Aab^5x^{\frac{15}{2}}+\frac{40}{13}\,Ba^3b^3x^{\frac{13}{2}}+\frac{30}{13}\,Aa^2b^4x^{\frac{13}{2}}+\frac{30}{11}\,Ba^4b^2x^{\frac{11}{2}}+\frac{40}{11}\,Aa^3b^3x^{\frac{11}{2}}+\frac{4}{3}\,Ba^5bx^{\frac{9}{2}}+\frac{10}{3}\,Aa^4b^2x^{\frac{9}{2}}+\frac{2}{7}\,Ba^6x^{\frac{7}{2}}+\frac{12}{7}\,Aa^5bx^{\frac{7}{2}}+\frac{2}{5}\,Aa^6x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
2/19*B*b^6*x^(19/2) + 12/17*B*a*b^5*x^(17/2) + 2/17*A*b^6*x^(17/2) + 2*B*a
^2*b^4*x^(15/2) + 4/5*A*a*b^5*x^(15/2) + 40/13*B*a^3*b^3*x^(13/2) + 30/13*
A*a^2*b^4*x^(13/2) + 30/11*B*a^4*b^2*x^(11/2) + 40/11*A*a^3*b^3*x^(11/2) +
 4/3*B*a^5*b*x^(9/2) + 10/3*A*a^4*b^2*x^(9/2) + 2/7*B*a^6*x^(7/2) + 12/7*A
*a^5*b*x^(7/2) + 2/5*A*a^6*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=x^{7/2}\left(\frac{2\,B\,a^6}{7}+\frac{12\,A\,b\,a^5}{7}\right)+x^{17/2}\left(\frac{2\,A\,b^6}{17}+\frac{12\,B\,a\,b^5}{17}\right)+\frac{2\,A\,a^6\,x^{5/2}}{5}+\frac{2\,B\,b^6\,x^{19/2}}{19}+\frac{10\,a^3\,b^2\,x^{11/2}\,(4\,A\,b+3\,B\,a)}{11}+\frac{10\,a^2\,b^3\,x^{13/2}\,(3\,A\,b}{13}$$

input
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^(7/2)*((2*B*a^6)/7 + (12*A*a^5*b)/7) + x^(17/2)*((2*A*b^6)/17 + (12*B*a*
b^5)/17) + (2*A*a^6*x^(5/2))/5 + (2*B*b^6*x^(19/2))/19 + (10*a^3*b^2*x^(11
/2)*(4*A*b + 3*B*a))/11 + (10*a^2*b^3*x^(13/2)*(3*A*b + 4*B*a))/13 + (2*a^
4*b*x^(9/2)*(5*A*b + 2*B*a))/3 + (2*a*b^4*x^(15/2)*(2*A*b + 5*B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\frac{2\sqrt{x}\,x^2(36465b^7x^7+285285a\,b^6x^6+969969a^2b^5x^5+1865325a^3b^4x^4+2204475a^4b^3x^3+161}{692835}$$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(2*sqrt(x)*x**2*(138567*a**7 + 692835*a**6*b*x + 1616615*a**5*b**2*x**2 +
2204475*a**4*b**3*x**3 + 1865325*a**3*b**4*x**4 + 969969*a**2*b**5*x**5 +
285285*a*b**6*x**6 + 36465*b**7*x**7))/692835
```

3.379 $\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{2}{3}a^6Ax^{3/2}+\frac{2}{5}a^5(6Ab+aB)x^{5/2}+\frac{6}{7}a^4b(5Ab+2aB)x^{7/2}$$
$$+\frac{10}{9}a^3b^2(4Ab+3aB)x^{9/2}+\frac{10}{11}a^2b^3(3Ab+4aB)x^{11/2}+\frac{6}{13}ab^4(2Ab+5aB)x^{13/2}+\frac{2}{15}b^5(Ab+6aB)x^{15/2}+\frac{2}{17}$$

output
```
2/3*a^6*A*x^(3/2)+2/5*a^5*(6*A*b+B*a)*x^(5/2)+6/7*a^4*b*(5*A*b+2*B*a)*x^(7
/2)+10/9*a^3*b^2*(4*A*b+3*B*a)*x^(9/2)+10/11*a^2*b^3*(3*A*b+4*B*a)*x^(11/2
)+6/13*a*b^4*(2*A*b+5*B*a)*x^(13/2)+2/15*b^5*(A*b+6*B*a)*x^(15/2)+2/17*b^6
*B*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{2x^{3/2}(51051a^6(5A+3Bx)+131274a^5bx(7A+5Bx)+182325a^4b^2x^2(9A+7Bx)+154700a^3b^3x^3(11A}{765765}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*x^(3/2)*(51051*a^6*(5*A + 3*B*x) + 131274*a^5*b*x*(7*A + 5*B*x) + 18232
5*a^4*b^2*x^2*(9*A + 7*B*x) + 154700*a^3*b^3*x^3*(11*A + 9*B*x) + 80325*a^
2*b^4*x^4*(13*A + 11*B*x) + 23562*a*b^5*x^5*(15*A + 13*B*x) + 3003*b^6*x^6
*(17*A + 15*B*x)))/765765
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a^2+2abx+b^2x^2\right)^3(A+Bx)\,dx$$

$$\Big\downarrow \; 1184$$

$$\frac{\int b^6\sqrt{x}(a+bx)^6(A+Bx)dx}{b^6}$$

$$\Big\downarrow \; 27$$

$$\int \sqrt{x}(a+bx)^6(A+Bx)dx$$

$$\Big\downarrow \; 85$$

$$\int \left(a^6A\sqrt{x}+a^5x^{3/2}(aB+6Ab)+3a^4bx^{5/2}(2aB+5Ab)+5a^3b^2x^{7/2}(3aB+4Ab)+5a^2b^3x^{9/2}(4aB+3Ab)+b\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}a^6Ax^{3/2}+\frac{2}{5}a^5x^{5/2}(aB+6Ab)+\frac{6}{7}a^4bx^{7/2}(2aB+5Ab)+\frac{10}{9}a^3b^2x^{9/2}(3aB+4Ab)+\frac{10}{11}a^2b^3x^{11/2}(4aB+3Ab)+\frac{2}{15}b^5x^{15/2}(6aB+Ab)+\frac{6}{13}ab^4x^{13/2}(5aB+2Ab)+\frac{2}{17}b^6Bx^{17/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(2*a^6*A*x^(3/2))/3 + (2*a^5*(6*A*b + a*B)*x^(5/2))/5 + (6*a^4*b*(5*A*b +
2*a*B)*x^(7/2))/7 + (10*a^3*b^2*(4*A*b + 3*a*B)*x^(9/2))/9 + (10*a^2*b^3*(
3*A*b + 4*a*B)*x^(11/2))/11 + (6*a*b^4*(2*A*b + 5*a*B)*x^(13/2))/13 + (2*b
^5*(A*b + 6*a*B)*x^(15/2))/15 + (2*b^6*B*x^(17/2))/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{2x^{\frac{3}{2}}(45045b^6Bx^7+51051Ab^6x^6+306306Bab^5x^6+353430Aab^5x^5+883575Ba^2b^4x^5+1044225Aa^2b^4x^4+1392300B}{}$
derivativedivides	$\frac{2b^6Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^6+6Bab^5)x^{\frac{15}{2}}}{15}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{13}{2}}}{13}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{11}{2}}}{11}+\frac{2(20Aa^3b^3+15}{9}$
default	$\frac{2b^6Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^6+6Bab^5)x^{\frac{15}{2}}}{15}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{13}{2}}}{13}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{11}{2}}}{11}+\frac{2(20Aa^3b^3+15}{9}$
trager	$\frac{2x^{\frac{3}{2}}(45045b^6Bx^7+51051Ab^6x^6+306306Bab^5x^6+353430Aab^5x^5+883575Ba^2b^4x^5+1044225Aa^2b^4x^4+1392300B}{}$
risch	$\frac{2x^{\frac{3}{2}}(45045b^6Bx^7+51051Ab^6x^6+306306Bab^5x^6+353430Aab^5x^5+883575Ba^2b^4x^5+1044225Aa^2b^4x^4+1392300B}{}$
orering	$\frac{2x^{\frac{3}{2}}(45045b^6Bx^7+51051Ab^6x^6+306306Bab^5x^6+353430Aab^5x^5+883575Ba^2b^4x^5+1044225Aa^2b^4x^4+1392300B}{}$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/765765*x^(3/2)*(45045*B*b^6*x^7+51051*A*b^6*x^6+306306*B*a*b^5*x^6+35343
0*A*a*b^5*x^5+883575*B*a^2*b^4*x^5+1044225*A*a^2*b^4*x^4+1392300*B*a^3*b^3
*x^4+1701700*A*a^3*b^3*x^3+1276275*B*a^4*b^2*x^3+1640925*A*a^4*b^2*x^2+656
370*B*a^5*b*x^2+918918*A*a^5*b*x+153153*B*a^6*x+255255*A*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{2}{765765}\left(45045\,Bb^6x^8+255255\,Aa^6x+51051\left(6\,Bab^5+Ab^6\right)x^7+176715\left(5\,Ba^2b^4+2\,Aab^5\right)x^6+3480\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
2/765765*(45045*B*b^6*x^8 + 255255*A*a^6*x + 51051*(6*B*a*b^5 + A*b^6)*x^7
 + 176715*(5*B*a^2*b^4 + 2*A*a*b^5)*x^6 + 348075*(4*B*a^3*b^3 + 3*A*a^2*b^
4)*x^5 + 425425*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^4 + 328185*(2*B*a^5*b + 5*A*
a^4*b^2)*x^3 + 153153*(B*a^6 + 6*A*a^5*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{2Aa^6x^{\frac{3}{2}}}{3} + \frac{2Bb^6x^{\frac{17}{2}}}{17} + \frac{2x^{\frac{15}{2}}\left(Ab^6+6Bab^5\right)}{15} + \frac{2x^{\frac{13}{2}}\cdot\left(6Aab^5+15Ba^2b^4\right)}{13} + \frac{2x^{\frac{11}{2}}\cdot\left(15Aa^2b^4+20Ba^3b^3\right)}{11} + \frac{2x^{\frac{9}{2}}\cdot\left(20Aa^3b^3+15Ba^4b^2\right)}{9} + \frac{2x^{\frac{7}{2}}\cdot\left(15Aa^4b^2+6Ba^5b\right)}{7} + \frac{2x^{\frac{5}{2}}\cdot\left(6Aa^5b+Ba^6\right)}{5}$$

input
```
integrate(x**(1/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
2*A*a**6*x**(3/2)/3 + 2*B*b**6*x**(17/2)/17 + 2*x**(15/2)*(A*b**6 + 6*B*a*
b**5)/15 + 2*x**(13/2)*(6*A*a*b**5 + 15*B*a**2*b**4)/13 + 2*x**(11/2)*(15*
A*a**2*b**4 + 20*B*a**3*b**3)/11 + 2*x**(9/2)*(20*A*a**3*b**3 + 15*B*a**4*
b**2)/9 + 2*x**(7/2)*(15*A*a**4*b**2 + 6*B*a**5*b)/7 + 2*x**(5/2)*(6*A*a**
5*b + B*a**6)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{2}{17}\,Bb^6x^{\frac{17}{2}} + \frac{2}{3}\,Aa^6x^{\frac{3}{2}} + \frac{2}{15}\left(6\,Bab^5+Ab^6\right)x^{\frac{15}{2}} + \frac{6}{13}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{13}{2}} + \frac{10}{11}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{11}{2}} + \frac{10}{9}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{9}{2}} + \frac{6}{7}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{7}{2}} + \frac{2}{5}\left(Ba^6+6\,Aa^5b\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
2/17*B*b^6*x^(17/2) + 2/3*A*a^6*x^(3/2) + 2/15*(6*B*a*b^5 + A*b^6)*x^(15/2
) + 6/13*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(13/2) + 10/11*(4*B*a^3*b^3 + 3*A*a^2
*b^4)*x^(11/2) + 10/9*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(9/2) + 6/7*(2*B*a^5*b
 + 5*A*a^4*b^2)*x^(7/2) + 2/5*(B*a^6 + 6*A*a^5*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{2}{17}\,Bb^6x^{\frac{17}{2}} + \frac{4}{5}\,Bab^5x^{\frac{15}{2}} + \frac{2}{15}\,Ab^6x^{\frac{15}{2}} + \frac{30}{13}\,Ba^2b^4x^{\frac{13}{2}} + \frac{12}{13}\,Aab^5x^{\frac{13}{2}} + \frac{40}{11}\,Ba^3b^3x^{\frac{11}{2}} + \frac{30}{11}\,Aa^2b^4x^{\frac{11}{2}} + \frac{10}{3}\,Ba^4b^2x^{\frac{9}{2}} + \frac{40}{9}\,Aa^3b^3x^{\frac{9}{2}} + \frac{12}{7}\,Ba^5bx^{\frac{7}{2}} + \frac{30}{7}\,Aa^4b^2x^{\frac{7}{2}} + \frac{2}{5}\,Ba^6x^{\frac{5}{2}} + \frac{12}{5}\,Aa^5bx^{\frac{5}{2}} + \frac{2}{3}\,Aa^6x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
2/17*B*b^6*x^(17/2) + 4/5*B*a*b^5*x^(15/2) + 2/15*A*b^6*x^(15/2) + 30/13*B
*a^2*b^4*x^(13/2) + 12/13*A*a*b^5*x^(13/2) + 40/11*B*a^3*b^3*x^(11/2) + 30
/11*A*a^2*b^4*x^(11/2) + 10/3*B*a^4*b^2*x^(9/2) + 40/9*A*a^3*b^3*x^(9/2) +
 12/7*B*a^5*b*x^(7/2) + 30/7*A*a^4*b^2*x^(7/2) + 2/5*B*a^6*x^(5/2) + 12/5*
A*a^5*b*x^(5/2) + 2/3*A*a^6*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = x^{5/2}\left(\frac{2\,B\,a^6}{5}+\frac{12\,A\,b\,a^5}{5}\right) + x^{15/2}\left(\frac{2\,A\,b^6}{15}+\frac{4\,B\,a\,b^5}{5}\right)+\frac{2\,A\,a^6\,x^{3/2}}{3} + \frac{2\,B\,b^6\,x^{17/2}}{17}+\frac{10\,a^3\,b^2\,x^{9/2}\,(4\,A\,b+3\,B\,a)}{9} + \frac{10\,a^2\,b^3\,x^{11/2}\,(3\,A\,b+4\,B\,a)}{11} + \frac{6\,a^4\,b\,x^{7/2}\,(5\,A\,b+2\,B\,a)}{7} + \frac{6\,a\,b^4\,x^{13/2}\,(2\,A\,b+5\,B\,a)}{13}$$

input
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^(5/2)*((2*B*a^6)/5 + (12*A*a^5*b)/5) + x^(15/2)*((2*A*b^6)/15 + (4*B*a*b
^5)/5) + (2*A*a^6*x^(3/2))/3 + (2*B*b^6*x^(17/2))/17 + (10*a^3*b^2*x^(9/2)
*(4*A*b + 3*B*a))/9 + (10*a^2*b^3*x^(11/2)*(3*A*b + 4*B*a))/11 + (6*a^4*b*
x^(7/2)*(5*A*b + 2*B*a))/7 + (6*a*b^4*x^(13/2)*(2*A*b + 5*B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{2\sqrt{x}\,x(6435b^7x^7+51051a\,b^6x^6+176715a^2b^5x^5+348075a^3b^4x^4+425425a^4b^3x^3+328185a^5b^2x^2+1531}{109395}$$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(2*sqrt(x)*x*(36465*a**7 + 153153*a**6*b*x + 328185*a**5*b**2*x**2 + 42542
5*a**4*b**3*x**3 + 348075*a**3*b**4*x**4 + 176715*a**2*b**5*x**5 + 51051*a
*b**6*x**6 + 6435*b**7*x**7))/109395
```

3.380 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 157

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx$$
$$= 2a^6A\sqrt{x}+\frac{2}{3}a^5(6Ab+aB)x^{3/2}+\frac{6}{5}a^4b(5Ab+2aB)x^{5/2}+\frac{10}{7}a^3b^2(4Ab+3aB)x^{7/2}$$
$$+\frac{10}{9}a^2b^3(3Ab+4aB)x^{9/2}+\frac{6}{11}ab^4(2Ab+5aB)x^{11/2}+\frac{2}{13}b^5(Ab+6aB)x^{13/2}+\frac{2}{15}b^6Bx^{15/2}$$

output
```
2*a^6*A*x^(1/2)+2/3*a^5*(6*A*b+B*a)*x^(3/2)+6/5*a^4*b*(5*A*b+2*B*a)*x^(5/2
)+10/7*a^3*b^2*(4*A*b+3*B*a)*x^(7/2)+10/9*a^2*b^3*(3*A*b+4*B*a)*x^(9/2)+6/
11*a*b^4*(2*A*b+5*B*a)*x^(11/2)+2/13*b^5*(A*b+6*B*a)*x^(13/2)+2/15*b^6*B*x
^(15/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}(15015a^6(3A+Bx)+18018a^5bx(5A+3Bx)+19305a^4b^2x^2(7A+5Bx)+14300a^3b^3x^3(9A+7Bx}{45045}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(15015*a^6*(3*A + B*x) + 18018*a^5*b*x*(5*A + 3*B*x) + 19305*a^
4*b^2*x^2*(7*A + 5*B*x) + 14300*a^3*b^3*x^3*(9*A + 7*B*x) + 6825*a^2*b^4*x
^4*(11*A + 9*B*x) + 1890*a*b^5*x^5*(13*A + 11*B*x) + 231*b^6*x^6*(15*A + 1
3*B*x)))/45045
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 1184$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{\sqrt{x}}dx}{b^6}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a+bx)^6(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^6A}{\sqrt{x}} + a^5\sqrt{x}(aB+6Ab) + 3a^4bx^{3/2}(2aB+5Ab) + 5a^3b^2x^{5/2}(3aB+4Ab) + 5a^2b^3x^{7/2}(4aB+3Ab) + b^5x^1\right.$$

$$\Big\downarrow \text{2009}$$

$$2a^6A\sqrt{x} + \frac{2}{3}a^5x^{3/2}(aB+6Ab) + \frac{6}{5}a^4bx^{5/2}(2aB+5Ab) + \frac{10}{7}a^3b^2x^{7/2}(3aB+4Ab) + \frac{10}{9}a^2b^3x^{9/2}(4aB+3Ab) + \frac{2}{13}b^5x^{13/2}(6aB+Ab) + \frac{6}{11}ab^4x^{11/2}(5aB+2Ab) + \frac{2}{15}b^6Bx^{15/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/Sqrt[x],x]
```

output
```
2*a^6*A*Sqrt[x] + (2*a^5*(6*A*b + a*B)*x^(3/2))/3 + (6*a^4*b*(5*A*b + 2*a*
B)*x^(5/2))/5 + (10*a^3*b^2*(4*A*b + 3*a*B)*x^(7/2))/7 + (10*a^2*b^3*(3*A*
b + 4*a*B)*x^(9/2))/9 + (6*a*b^4*(2*A*b + 5*a*B)*x^(11/2))/11 + (2*b^5*(A*
b + 6*a*B)*x^(13/2))/13 + (2*b^6*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

method	result
trager	$\left(\frac{2}{15}b^6Bx^7+\frac{2}{13}Ab^6x^6+\frac{12}{13}Bab^5x^6+\frac{12}{11}Aab^5x^5+\frac{30}{11}Ba^2b^4x^5+\frac{10}{3}Aa^2b^4x^4+\frac{40}{9}Ba^3b\right.$
gosper	$\frac{2\sqrt{x}\,(3003b^6Bx^7+3465Ab^6x^6+20790Bab^5x^6+24570Aab^5x^5+61425Ba^2b^4x^5+75075Aa^2b^4x^4+100100Ba^3b^3x^4+}{45045}$
derivativedivides	$\frac{2b^6Bx^{\frac{15}{2}}}{15}+\frac{2(Ab^6+6Bab^5)x^{\frac{13}{2}}}{13}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{11}{2}}}{11}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{9}{2}}}{9}+\frac{2(20Aa^3b^3+15}{7}$
default	$\frac{2b^6Bx^{\frac{15}{2}}}{15}+\frac{2(Ab^6+6Bab^5)x^{\frac{13}{2}}}{13}+\frac{2(6Aab^5+15Ba^2b^4)x^{\frac{11}{2}}}{11}+\frac{2(15Aa^2b^4+20Ba^3b^3)x^{\frac{9}{2}}}{9}+\frac{2(20Aa^3b^3+15}{7}$
risch	$\frac{2\sqrt{x}\,(3003b^6Bx^7+3465Ab^6x^6+20790Bab^5x^6+24570Aab^5x^5+61425Ba^2b^4x^5+75075Aa^2b^4x^4+100100Ba^3b^3x^4+}{45045}$
orering	$\frac{2(3003b^6Bx^7+3465Ab^6x^6+20790Bab^5x^6+24570Aab^5x^5+61425Ba^2b^4x^5+75075Aa^2b^4x^4+100100Ba^3b^3x^4+1287}{45}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/15*b^6*B*x^7+2/13*A*b^6*x^6+12/13*B*a*b^5*x^6+12/11*A*a*b^5*x^5+30/11*B
*a^2*b^4*x^5+10/3*A*a^2*b^4*x^4+40/9*B*a^3*b^3*x^4+40/7*A*a^3*b^3*x^3+30/7
*B*a^4*b^2*x^3+6*A*a^4*b^2*x^2+12/5*B*a^5*b*x^2+4*A*a^5*b*x+2/3*B*a^6*x+2*
A*a^6)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{45045}\left(3003\,Bb^6x^7+45045\,Aa^6+3465\left(6\,Bab^5+Ab^6\right)x^6+12285\left(5\,Ba^2b^4+2\,Aab^5\right)x^5+25025\left(4\,Ba\right.\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*b^6*x^7 + 45045*A*a^6 + 3465*(6*B*a*b^5 + A*b^6)*x^6 + 122
85*(5*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 25025*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 +
 32175*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 27027*(2*B*a^5*b + 5*A*a^4*b^2)*x
^2 + 15015*(B*a^6 + 6*A*a^5*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx = 2Aa^6\sqrt{x}+4Aa^5bx^{\frac{3}{2}}+6Aa^4b^2x^{\frac{5}{2}}+\frac{40Aa^3b^3x^{\frac{7}{2}}}{7}+\frac{10Aa^2b^4x^{\frac{9}{2}}}{3}+\frac{12Aab^5x^{\frac{11}{2}}}{11}+\frac{2Ab^6x^{\frac{13}{2}}}{13}+\frac{2Ba^6x^{\frac{3}{2}}}{3}+\frac{12Ba^5bx^{\frac{5}{2}}}{5}+\frac{30Ba^4b^2x^{\frac{7}{2}}}{7}+\frac{40Ba^3b^3x^{\frac{9}{2}}}{9}+\frac{30Ba^2b^4x^{\frac{11}{2}}}{11}+\frac{12Bab^5x^{\frac{13}{2}}}{13}+\frac{2Bb^6x^{\frac{15}{2}}}{15}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(1/2),x)
```

output
```
2*A*a**6*sqrt(x) + 4*A*a**5*b*x**(3/2) + 6*A*a**4*b**2*x**(5/2) + 40*A*a**
3*b**3*x**(7/2)/7 + 10*A*a**2*b**4*x**(9/2)/3 + 12*A*a*b**5*x**(11/2)/11 +
 2*A*b**6*x**(13/2)/13 + 2*B*a**6*x**(3/2)/3 + 12*B*a**5*b*x**(5/2)/5 + 30
*B*a**4*b**2*x**(7/2)/7 + 40*B*a**3*b**3*x**(9/2)/9 + 30*B*a**2*b**4*x**(1
1/2)/11 + 12*B*a*b**5*x**(13/2)/13 + 2*B*b**6*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bb^6x^{\frac{15}{2}} + 2\,Aa^6\sqrt{x} + \frac{2}{13}\left(6\,Bab^5+Ab^6\right)x^{\frac{13}{2}} + \frac{6}{11}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{11}{2}} + \frac{10}{9}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{9}{2}} + \frac{10}{7}\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{7}{2}} + \frac{6}{5}\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Ba^6+6\,Aa^5b\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*b^6*x^(15/2) + 2*A*a^6*sqrt(x) + 2/13*(6*B*a*b^5 + A*b^6)*x^(13/2)
+ 6/11*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(11/2) + 10/9*(4*B*a^3*b^3 + 3*A*a^2*b^
4)*x^(9/2) + 10/7*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(7/2) + 6/5*(2*B*a^5*b + 5
*A*a^4*b^2)*x^(5/2) + 2/3*(B*a^6 + 6*A*a^5*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bb^6x^{\frac{15}{2}} + \frac{12}{13}\,Bab^5x^{\frac{13}{2}} + \frac{2}{13}\,Ab^6x^{\frac{13}{2}} + \frac{30}{11}\,Ba^2b^4x^{\frac{11}{2}} + \frac{12}{11}\,Aab^5x^{\frac{11}{2}} + \frac{40}{9}\,Ba^3b^3x^{\frac{9}{2}} + \frac{10}{3}\,Aa^2b^4x^{\frac{9}{2}} + \frac{30}{7}\,Ba^4b^2x^{\frac{7}{2}} + \frac{40}{7}\,Aa^3b^3x^{\frac{7}{2}} + \frac{12}{5}\,Ba^5bx^{\frac{5}{2}} + 6\,Aa^4b^2x^{\frac{5}{2}} + \frac{2}{3}\,Ba^6x^{\frac{3}{2}} + 4\,Aa^5bx^{\frac{3}{2}} + 2\,Aa^6\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*b^6*x^(15/2) + 12/13*B*a*b^5*x^(13/2) + 2/13*A*b^6*x^(13/2) + 30/11
*B*a^2*b^4*x^(11/2) + 12/11*A*a*b^5*x^(11/2) + 40/9*B*a^3*b^3*x^(9/2) + 10
/3*A*a^2*b^4*x^(9/2) + 30/7*B*a^4*b^2*x^(7/2) + 40/7*A*a^3*b^3*x^(7/2) + 1
2/5*B*a^5*b*x^(5/2) + 6*A*a^4*b^2*x^(5/2) + 2/3*B*a^6*x^(3/2) + 4*A*a^5*b*
x^(3/2) + 2*A*a^6*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^6}{3}+4\,A\,b\,a^5\right) + x^{13/2}\left(\frac{2\,A\,b^6}{13}+\frac{12\,B\,a\,b^5}{13}\right) + 2\,A\,a^6\,\sqrt{x} + \frac{2\,B\,b^6\,x^{15/2}}{15} + \frac{10\,a^3\,b^2\,x^{7/2}\,(4\,A\,b+3\,B\,a)}{7} + \frac{10\,a^2\,b^3\,x^{9/2}\,(3\,A\,b+4\,B\,a)}{9} + \frac{6\,a^4\,b\,x^{5/2}\,(5\,A\,b+2\,B\,a)}{5} + \frac{6\,a\,b^4\,x^{11/2}\,(2\,A\,b+5\,B\,a)}{11}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^6)/3 + 4*A*a^5*b) + x^(13/2)*((2*A*b^6)/13 + (12*B*a*b^5)/
13) + 2*A*a^6*x^(1/2) + (2*B*b^6*x^(15/2))/15 + (10*a^3*b^2*x^(7/2)*(4*A*b
 + 3*B*a))/7 + (10*a^2*b^3*x^(9/2)*(3*A*b + 4*B*a))/9 + (6*a^4*b*x^(5/2)*(
5*A*b + 2*B*a))/5 + (6*a*b^4*x^(11/2)*(2*A*b + 5*B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(429b^7x^7+3465a\,b^6x^6+12285a^2b^5x^5+25025a^3b^4x^4+32175a^4b^3x^3+27027a^5b^2x^2+15015a^6bx+}{6435}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(6435*a**7 + 15015*a**6*b*x + 27027*a**5*b**2*x**2 + 32175*a**4
*b**3*x**3 + 25025*a**3*b**4*x**4 + 12285*a**2*b**5*x**5 + 3465*a*b**6*x**
6 + 429*b**7*x**7))/6435
```

3.381 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = -\frac{2a^6A}{\sqrt{x}} + 2a^5(6Ab+aB)\sqrt{x} + 2a^4b(5Ab+2aB)x^{3/2} + 2a^3b^2(4Ab+3aB)x^{5/2} + \frac{10}{7}a^2b^3(3Ab+4aB)x^{7/2} + \frac{2}{3}ab^4(2Ab+5aB)x^{9/2} + \frac{2}{11}b^5(Ab+6aB)x^{11/2} + \frac{2}{13}b^6Bx^{13/2}$$

output
```
-2*a^6*A/x^(1/2)+2*a^5*(6*A*b+B*a)*x^(1/2)+2*a^4*b*(5*A*b+2*B*a)*x^(3/2)+2
*a^3*b^2*(4*A*b+3*B*a)*x^(5/2)+10/7*a^2*b^3*(3*A*b+4*B*a)*x^(7/2)+2/3*a*b^
4*(2*A*b+5*B*a)*x^(9/2)+2/11*b^5*(A*b+6*B*a)*x^(11/2)+2/13*b^6*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = \frac{2(-3003a^6(A-Bx)+6006a^5bx(3A+Bx)+3003a^4b^2x^2(5A+3B$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(3/2),x]
```

output
```
(2*(-3003*a^6*(A - B*x) + 6006*a^5*b*x*(3*A + B*x) + 3003*a^4*b^2*x^2*(5*A
 + 3*B*x) + 1716*a^3*b^3*x^3*(7*A + 5*B*x) + 715*a^2*b^4*x^4*(9*A + 7*B*x)
 + 182*a*b^5*x^5*(11*A + 9*B*x) + 21*b^6*x^6*(13*A + 11*B*x)))/(3003*Sqrt[
x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^{3/2}} dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{3/2}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{3/2}}+\frac{a^5(aB+6Ab)}{\sqrt{x}}+3a^4b\sqrt{x}(2aB+5Ab)+5a^3b^2x^{3/2}(3aB+4Ab)+5a^2b^3x^{5/2}(4aB+3Ab)+b^5x^{9/2}(6$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^6A}{\sqrt{x}}+2a^5\sqrt{x}(aB+6Ab)+2a^4bx^{3/2}(2aB+5Ab)+2a^3b^2x^{5/2}(3aB+4Ab)+ \frac{10}{7}a^2b^3x^{7/2}(4aB+3Ab)+\frac{2}{11}b^5x^{11/2}(6aB+Ab)+\frac{2}{3}ab^4x^{9/2}(5aB+2Ab)+\frac{2}{13}b^6Bx^{13/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(3/2),x]
```

output
```
(-2*a^6*A)/Sqrt[x] + 2*a^5*(6*A*b + a*B)*Sqrt[x] + 2*a^4*b*(5*A*b + 2*a*B)
*x^(3/2) + 2*a^3*b^2*(4*A*b + 3*a*B)*x^(5/2) + (10*a^2*b^3*(3*A*b + 4*a*B)
*x^(7/2))/7 + (2*a*b^4*(2*A*b + 5*a*B)*x^(9/2))/3 + (2*b^5*(A*b + 6*a*B)*x
^(11/2))/11 + (2*b^6*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

method	result
gosper	$-\frac{2(-231b^6Bx^7-273Ab^6x^6-1638Bab^5x^6-2002Aab^5x^5-5005Ba^2b^4x^5-6435Aa^2b^4x^4-8580Ba^3b^3x^4-12012Aa}{3003\sqrt{x}}$
trager	$-\frac{2(-231b^6Bx^7-273Ab^6x^6-1638Bab^5x^6-2002Aab^5x^5-5005Ba^2b^4x^5-6435Aa^2b^4x^4-8580Ba^3b^3x^4-12012Aa}{3003\sqrt{x}}$
risch	$-\frac{2(-231b^6Bx^7-273Ab^6x^6-1638Bab^5x^6-2002Aab^5x^5-5005Ba^2b^4x^5-6435Aa^2b^4x^4-8580Ba^3b^3x^4-12012Aa}{3003\sqrt{x}}$
derivativedivides	$\frac{2b^6Bx^{\frac{13}{2}}}{13}+\frac{2Ab^6x^{\frac{11}{2}}}{11}+\frac{12Bab^5x^{\frac{11}{2}}}{11}+\frac{4Aab^5x^{\frac{9}{2}}}{3}+\frac{10Ba^2b^4x^{\frac{9}{2}}}{3}+\frac{30Aa^2b^4x^{\frac{7}{2}}}{7}+\frac{40Ba^3b^3x^{\frac{7}{2}}}{7}+8Aa$
default	$\frac{2b^6Bx^{\frac{13}{2}}}{13}+\frac{2Ab^6x^{\frac{11}{2}}}{11}+\frac{12Bab^5x^{\frac{11}{2}}}{11}+\frac{4Aab^5x^{\frac{9}{2}}}{3}+\frac{10Ba^2b^4x^{\frac{9}{2}}}{3}+\frac{30Aa^2b^4x^{\frac{7}{2}}}{7}+\frac{40Ba^3b^3x^{\frac{7}{2}}}{7}+8Aa$
orering	$-\frac{2(-231b^6Bx^7-273Ab^6x^6-1638Bab^5x^6-2002Aab^5x^5-5005Ba^2b^4x^5-6435Aa^2b^4x^4-8580Ba^3b^3x^4-12012Aa}{3003\sqrt{x}\,(}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3003*(-231*B*b^6*x^7-273*A*b^6*x^6-1638*B*a*b^5*x^6-2002*A*a*b^5*x^5-50
05*B*a^2*b^4*x^5-6435*A*a^2*b^4*x^4-8580*B*a^3*b^3*x^4-12012*A*a^3*b^3*x^3
-9009*B*a^4*b^2*x^3-15015*A*a^4*b^2*x^2-6006*B*a^5*b*x^2-18018*A*a^5*b*x-3
003*B*a^6*x+3003*A*a^6)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^3}{x^{3/2}}\,dx = \frac{2\,(231\,Bb^6x^7-3003\,Aa^6+273\,(6\,Bab^5+Ab^6)x^6+1001\,(5\,Ba^2b^4-}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/3003*(231*B*b^6*x^7 - 3003*A*a^6 + 273*(6*B*a*b^5 + A*b^6)*x^6 + 1001*(5
*B*a^2*b^4 + 2*A*a*b^5)*x^5 + 2145*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 3003*
(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 3003*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 + 300
3*(B*a^6 + 6*A*a^5*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.35

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = -\frac{2Aa^6}{\sqrt{x}} + 12Aa^5b\sqrt{x} + 10Aa^4b^2x^{\frac{3}{2}} + 8Aa^3b^3x^{\frac{5}{2}} + \frac{30Aa^2b^4x^{\frac{7}{2}}}{7} + \frac{4Aab^5x^{\frac{9}{2}}}{3} + \frac{2Ab^6x^{\frac{11}{2}}}{11} + 2Ba^6\sqrt{x} + 4Ba^5bx^{\frac{3}{2}} + 6Ba^4b^2x^{\frac{5}{2}} + \frac{40Ba^3b^3x^{\frac{7}{2}}}{7} + \frac{10Ba^2b^4x^{\frac{9}{2}}}{3} + \frac{12Bab^5x^{\frac{11}{2}}}{11} + \frac{2Bb^6x^{\frac{13}{2}}}{13}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(3/2),x)
```

output
```
-2*A*a**6/sqrt(x) + 12*A*a**5*b*sqrt(x) + 10*A*a**4*b**2*x**(3/2) + 8*A*a*
*3*b**3*x**(5/2) + 30*A*a**2*b**4*x**(7/2)/7 + 4*A*a*b**5*x**(9/2)/3 + 2*A
*b**6*x**(11/2)/11 + 2*B*a**6*sqrt(x) + 4*B*a**5*b*x**(3/2) + 6*B*a**4*b**
2*x**(5/2) + 40*B*a**3*b**3*x**(7/2)/7 + 10*B*a**2*b**4*x**(9/2)/3 + 12*B*
a*b**5*x**(11/2)/11 + 2*B*b**6*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bb^6x^{\frac{13}{2}} - \frac{2\,Aa^6}{\sqrt{x}} + \frac{2}{11}\left(6\,Bab^5+Ab^6\right)x^{\frac{11}{2}} + \frac{2}{3}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{9}{2}} + \frac{10}{7}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{7}{2}} + 2\left(3\,Ba^4b^2+4\,Aa^3b^3\right)x^{\frac{5}{2}} + 2\left(2\,Ba^5b+5\,Aa^4b^2\right)x^{\frac{3}{2}} + 2\left(Ba^6+6\,Aa^5b\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/13*B*b^6*x^(13/2) - 2*A*a^6/sqrt(x) + 2/11*(6*B*a*b^5 + A*b^6)*x^(11/2)
+ 2/3*(5*B*a^2*b^4 + 2*A*a*b^5)*x^(9/2) + 10/7*(4*B*a^3*b^3 + 3*A*a^2*b^4)
*x^(7/2) + 2*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^(5/2) + 2*(2*B*a^5*b + 5*A*a^4*
b^2)*x^(3/2) + 2*(B*a^6 + 6*A*a^5*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bb^6x^{\frac{13}{2}} + \frac{12}{11}\,Bab^5x^{\frac{11}{2}} + \frac{2}{11}\,Ab^6x^{\frac{11}{2}} + \frac{10}{3}\,Ba^2b^4x^{\frac{9}{2}} + \frac{4}{3}\,Aab^5x^{\frac{9}{2}} + \frac{40}{7}\,Ba^3b^3x^{\frac{7}{2}} + \frac{30}{7}\,Aa^2b^4x^{\frac{7}{2}} + 6\,Ba^4b^2x^{\frac{5}{2}} + 8\,Aa^3b^3x^{\frac{5}{2}} + 4\,Ba^5bx^{\frac{3}{2}} + 10\,Aa^4b^2x^{\frac{3}{2}} + 2\,Ba^6\sqrt{x} + 12\,Aa^5b\sqrt{x} - \frac{2\,Aa^6}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/13*B*b^6*x^(13/2) + 12/11*B*a*b^5*x^(11/2) + 2/11*A*b^6*x^(11/2) + 10/3*
B*a^2*b^4*x^(9/2) + 4/3*A*a*b^5*x^(9/2) + 40/7*B*a^3*b^3*x^(7/2) + 30/7*A*
a^2*b^4*x^(7/2) + 6*B*a^4*b^2*x^(5/2) + 8*A*a^3*b^3*x^(5/2) + 4*B*a^5*b*x^
(3/2) + 10*A*a^4*b^2*x^(3/2) + 2*B*a^6*sqrt(x) + 12*A*a^5*b*sqrt(x) - 2*A*
a^6/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = \sqrt{x}\left(2\,B\,a^6+12\,A\,b\,a^5\right) + x^{11/2}\left(\frac{2\,A\,b^6}{11}+\frac{12\,B\,a\,b^5}{11}\right) - \frac{2\,A\,a^6}{\sqrt{x}} + \frac{2\,B\,b^6\,x^{13/2}}{13} + 2\,a^3\,b^2\,x^{5/2}\,(4\,A\,b+3\,B\,a) + \frac{10\,a^2\,b^3\,x^{7/2}\,(3\,A\,b+4\,B\,a)}{7} + 2\,a^4\,b\,x^{3/2}\,(5\,A\,b+2\,B\,a) + \frac{2\,a\,b^4\,x^{9/2}\,(2\,A\,b+5\,B}{3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^6 + 12*A*a^5*b) + x^(11/2)*((2*A*b^6)/11 + (12*B*a*b^5)/11)
 - (2*A*a^6)/x^(1/2) + (2*B*b^6*x^(13/2))/13 + 2*a^3*b^2*x^(5/2)*(4*A*b +
3*B*a) + (10*a^2*b^3*x^(7/2)*(3*A*b + 4*B*a))/7 + 2*a^4*b*x^(3/2)*(5*A*b +
 2*B*a) + (2*a*b^4*x^(9/2)*(2*A*b + 5*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{3/2}}\,dx = \frac{\frac{2}{13}b^7x^7+\frac{14}{11}a\,b^6x^6+\frac{14}{3}a^2b^5x^5+10a^3b^4x^4+14a^4b^3x^3+14a^5b^2x^2+}{\sqrt{x}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(3/2),x)
```

output
```
(2*( - 429*a**7 + 3003*a**6*b*x + 3003*a**5*b**2*x**2 + 3003*a**4*b**3*x**
3 + 2145*a**3*b**4*x**4 + 1001*a**2*b**5*x**5 + 273*a*b**6*x**6 + 33*b**7*
x**7))/(429*sqrt(x))
```

3.382 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 153

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = -\frac{2a^6A}{3x^{3/2}} - \frac{2a^5(6Ab+aB)}{\sqrt{x}} + 6a^4b(5Ab+2aB)\sqrt{x} + \frac{10}{3}a^3b^2(4Ab+3aB)x^{3/2} + 2a^2b^3(3Ab+4aB)x^{5/2} + \frac{6}{7}ab^4(2Ab+5aB)x^{7/2} + \frac{2}{9}b^5(Ab+6aB)x^{9/2} + \frac{2}{11}b^6Bx^{11/2}$$

output
```
-2/3*a^6*A/x^(3/2)-2*a^5*(6*A*b+B*a)/x^(1/2)+6*a^4*b*(5*A*b+2*B*a)*x^(1/2)
+10/3*a^3*b^2*(4*A*b+3*B*a)*x^(3/2)+2*a^2*b^3*(3*A*b+4*B*a)*x^(5/2)+6/7*a*
b^4*(2*A*b+5*B*a)*x^(7/2)+2/9*b^5*(A*b+6*B*a)*x^(9/2)+2/11*b^6*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = \frac{2(4158a^5bx(-A+Bx)+3465a^4b^2x^2(3A+Bx)-231a^6(A+3Bx$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(5/2),x]
```

output
```
(2*(4158*a^5*b*x*(-A + B*x) + 3465*a^4*b^2*x^2*(3*A + B*x) - 231*a^6*(A +
3*B*x) + 924*a^3*b^3*x^3*(5*A + 3*B*x) + 297*a^2*b^4*x^4*(7*A + 5*B*x) + 6
6*a*b^5*x^5*(9*A + 7*B*x) + 7*b^6*x^6*(11*A + 9*B*x)))/(693*x^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^3(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{5/2}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{5/2}}+\frac{a^5(aB+6Ab)}{x^{3/2}}+\frac{3a^4b(2aB+5Ab)}{\sqrt{x}}+5a^3b^2\sqrt{x}(3aB+4Ab)+5a^2b^3x^{3/2}(4aB+3Ab)+b^5x^{7/2}(6aB$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^6A}{3x^{3/2}}-\frac{2a^5(aB+6Ab)}{\sqrt{x}}+6a^4b\sqrt{x}(2aB+5Ab)+\frac{10}{3}a^3b^2x^{3/2}(3aB+4Ab)+2a^2b^3x^{5/2}(4aB+3Ab)+\frac{2}{9}b^5x^{9/2}(6aB+Ab)+\frac{6}{7}ab^4x^{7/2}(5aB+2Ab)+\frac{2}{11}b^6Bx^{11/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(5/2),x]
```

output
```
(-2*a^6*A)/(3*x^(3/2)) - (2*a^5*(6*A*b + a*B))/Sqrt[x] + 6*a^4*b*(5*A*b +
2*a*B)*Sqrt[x] + (10*a^3*b^2*(4*A*b + 3*a*B)*x^(3/2))/3 + 2*a^2*b^3*(3*A*b
 + 4*a*B)*x^(5/2) + (6*a*b^4*(2*A*b + 5*a*B)*x^(7/2))/7 + (2*b^5*(A*b + 6*
a*B)*x^(9/2))/9 + (2*b^6*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{2b^6 B x^{\frac{11}{2}}}{11}+\frac{2A b^6 x^{\frac{9}{2}}}{9}+\frac{4Ba b^5 x^{\frac{9}{2}}}{3}+\frac{12Aa b^5 x^{\frac{7}{2}}}{7}+\frac{30B a^2 b^4 x^{\frac{7}{2}}}{7}+6A a^2 b^4 x^{\frac{5}{2}}+8B a^3 b^3 x^{\frac{5}{2}}+$ 40A a
default	$\frac{2b^6 B x^{\frac{11}{2}}}{11}+\frac{2A b^6 x^{\frac{9}{2}}}{9}+\frac{4Ba b^5 x^{\frac{9}{2}}}{3}+\frac{12Aa b^5 x^{\frac{7}{2}}}{7}+\frac{30B a^2 b^4 x^{\frac{7}{2}}}{7}+6A a^2 b^4 x^{\frac{5}{2}}+8B a^3 b^3 x^{\frac{5}{2}}+$ 40A a
gosper	$-\frac{2(-63b^6 B x^7-77A b^6 x^6-462Ba b^5 x^6-594Aa b^5 x^5-1485B a^2 b^4 x^5-2079A a^2 b^4 x^4-2772B a^3 b^3 x^4-4620A a^3 b^3 x^3}{693x^{\frac{3}{2}}}$
trager	$-\frac{2(-63b^6 B x^7-77A b^6 x^6-462Ba b^5 x^6-594Aa b^5 x^5-1485B a^2 b^4 x^5-2079A a^2 b^4 x^4-2772B a^3 b^3 x^4-4620A a^3 b^3 x^3}{693x^{\frac{3}{2}}}$
risch	$-\frac{2(-63b^6 B x^7-77A b^6 x^6-462Ba b^5 x^6-594Aa b^5 x^5-1485B a^2 b^4 x^5-2079A a^2 b^4 x^4-2772B a^3 b^3 x^4-4620A a^3 b^3 x^3}{693x^{\frac{3}{2}}}$
orering	$-\frac{2(-63b^6 B x^7-77A b^6 x^6-462Ba b^5 x^6-594Aa b^5 x^5-1485B a^2 b^4 x^5-2079A a^2 b^4 x^4-2772B a^3 b^3 x^4-4620A a^3 b^3 x^3}{693x^{\frac{3}{2}}(bx+a)}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^6*B*x^(11/2)+2/9*A*b^6*x^(9/2)+4/3*B*a*b^5*x^(9/2)+12/7*A*a*b^5*x^(
7/2)+30/7*B*a^2*b^4*x^(7/2)+6*A*a^2*b^4*x^(5/2)+8*B*a^3*b^3*x^(5/2)+40/3*A
*a^3*b^3*x^(3/2)+10*B*a^4*b^2*x^(3/2)+30*A*a^4*b^2*x^(1/2)+12*B*a^5*b*x^(1
/2)-2*a^5*(6*A*b+B*a)/x^(1/2)-2/3*a^6*A/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = \frac{2\,(63\,Bb^6x^7-231\,Aa^6+77\,(6\,Bab^5+Ab^6)x^6+297\,(5\,Ba^2b^4+2\,A}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/693*(63*B*b^6*x^7 - 231*A*a^6 + 77*(6*B*a*b^5 + A*b^6)*x^6 + 297*(5*B*a^
2*b^4 + 2*A*a*b^5)*x^5 + 693*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 1155*(3*B*a
^4*b^2 + 4*A*a^3*b^3)*x^3 + 2079*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 693*(B*a^
6 + 6*A*a^5*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = -\frac{2Aa^6}{3x^{\frac{3}{2}}} - \frac{12Aa^5b}{\sqrt{x}} + 30Aa^4b^2\sqrt{x} + \frac{40Aa^3b^3x^{\frac{3}{2}}}{3} + 6Aa^2b^4x^{\frac{5}{2}} + \frac{12Aab^5x^{\frac{7}{2}}}{7} + \frac{2Ab^6x^{\frac{9}{2}}}{9} - \frac{2Ba^6}{\sqrt{x}} + 12Ba^5b\sqrt{x} + 10Ba^4b^2x^{\frac{3}{2}} + 8Ba^3b^3x^{\frac{5}{2}} + \frac{30Ba^2b^4x^{\frac{7}{2}}}{7} + \frac{4Bab^5x^{\frac{9}{2}}}{3} + \frac{2Bb^6x^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(5/2),x)
```

output
```
-2*A*a**6/(3*x**(3/2)) - 12*A*a**5*b/sqrt(x) + 30*A*a**4*b**2*sqrt(x) + 40
*A*a**3*b**3*x**(3/2)/3 + 6*A*a**2*b**4*x**(5/2) + 12*A*a*b**5*x**(7/2)/7
+ 2*A*b**6*x**(9/2)/9 - 2*B*a**6/sqrt(x) + 12*B*a**5*b*sqrt(x) + 10*B*a**4
*b**2*x**(3/2) + 8*B*a**3*b**3*x**(5/2) + 30*B*a**2*b**4*x**(7/2)/7 + 4*B*
a*b**5*x**(9/2)/3 + 2*B*b**6*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bb^6x^{\frac{11}{2}} + \frac{2}{9}\left(6\,Bab^5 + Ab^6\right)x^{\frac{9}{2}} + \frac{6}{7}\left(5\,Ba^2b^4 + 2\,Aab^5\right)x^{\frac{7}{2}} + 2\left(4\,Ba^3b^3 + 3\,Aa^2b^4\right)x^{\frac{5}{2}} + \frac{10}{3}\left(3\,Ba^4b^2 + 4\,Aa^3b^3\right)x^{\frac{3}{2}} + 6\left(2\,Ba^5b + 5\,Aa^4b^2\right)\sqrt{x} - \frac{2\left(Aa^6 + 3\left(Ba^6 + 6\,Aa^5b\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/11*B*b^6*x^(11/2) + 2/9*(6*B*a*b^5 + A*b^6)*x^(9/2) + 6/7*(5*B*a^2*b^4 +
 2*A*a*b^5)*x^(7/2) + 2*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^(5/2) + 10/3*(3*B*a^
4*b^2 + 4*A*a^3*b^3)*x^(3/2) + 6*(2*B*a^5*b + 5*A*a^4*b^2)*sqrt(x) - 2/3*(
A*a^6 + 3*(B*a^6 + 6*A*a^5*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bb^6x^{\frac{11}{2}} + \frac{4}{3}\,Bab^5x^{\frac{9}{2}} + \frac{2}{9}\,Ab^6x^{\frac{9}{2}} + \frac{30}{7}\,Ba^2b^4x^{\frac{7}{2}} + \frac{12}{7}\,Aab^5x^{\frac{7}{2}} + 8\,Ba^3b^3x^{\frac{5}{2}} + 6\,Aa^2b^4x^{\frac{5}{2}} + 10\,Ba^4b^2x^{\frac{3}{2}} + \frac{40}{3}\,Aa^3b^3x^{\frac{3}{2}} + 12\,Ba^5b\sqrt{x} + 30\,Aa^4b^2\sqrt{x} - \frac{2\left(3\,Ba^6x + 18\,Aa^5bx + Aa^6\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/11*B*b^6*x^(11/2) + 4/3*B*a*b^5*x^(9/2) + 2/9*A*b^6*x^(9/2) + 30/7*B*a^2
*b^4*x^(7/2) + 12/7*A*a*b^5*x^(7/2) + 8*B*a^3*b^3*x^(5/2) + 6*A*a^2*b^4*x^
(5/2) + 10*B*a^4*b^2*x^(3/2) + 40/3*A*a^3*b^3*x^(3/2) + 12*B*a^5*b*sqrt(x)
 + 30*A*a^4*b^2*sqrt(x) - 2/3*(3*B*a^6*x + 18*A*a^5*b*x + A*a^6)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = x^{9/2}\left(\frac{2\,A\,b^6}{9} + \frac{4\,B\,a\,b^5}{3}\right) - \frac{x\left(2\,B\,a^6 + 12\,A\,b\,a^5\right) + \frac{2\,A\,a^6}{3}}{x^{3/2}} + \frac{2\,B\,b^6\,x^{11/2}}{11} + \frac{10\,a^3\,b^2\,x^{3/2}\,(4\,A\,b + 3\,B\,a)}{3} + 2\,a^2\,b^3\,x^{5/2}\,(3\,A\,b + 4\,B\,a) + 6\,a^4\,b\,\sqrt{x}\,(5\,A\,b + 2\,B\,a) + \frac{6\,a\,b^4\,x^{7/2}\,(2\,A\,b + 5\,B\,a)}{7}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(5/2),x)
```

output
```
x^(9/2)*((2*A*b^6)/9 + (4*B*a*b^5)/3) - (x*(2*B*a^6 + 12*A*a^5*b) + (2*A*a
^6)/3)/x^(3/2) + (2*B*b^6*x^(11/2))/11 + (10*a^3*b^2*x^(3/2)*(4*A*b + 3*B*
a))/3 + 2*a^2*b^3*x^(5/2)*(3*A*b + 4*B*a) + 6*a^4*b*x^(1/2)*(5*A*b + 2*B*a
) + (6*a*b^4*x^(7/2)*(2*A*b + 5*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{5/2}}\,dx = \frac{\frac{2}{11}b^7x^7+\frac{14}{9}a\,b^6x^6+6a^2b^5x^5+14a^3b^4x^4+\frac{70}{3}a^4b^3x^3+42a^5b^2x^2-1}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(5/2),x)
```

output
```
(2*( - 33*a**7 - 693*a**6*b*x + 2079*a**5*b**2*x**2 + 1155*a**4*b**3*x**3
+ 693*a**3*b**4*x**4 + 297*a**2*b**5*x**5 + 77*a*b**6*x**6 + 9*b**7*x**7))
/(99*sqrt(x)*x)
```

3.383 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 155

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx=-\frac{2a^6A}{5x^{5/2}}-\frac{2a^5(6Ab+aB)}{3x^{3/2}}-\frac{6a^4b(5Ab+2aB)}{\sqrt{x}}+10a^3b^2(4Ab+3aB)\sqrt{x}+\frac{10}{3}a^2b^3(3Ab+4aB)x^{3/2}+\frac{6}{5}ab^4(2Ab+5aB)x^{5/2}+\frac{2}{7}b^5(Ab+6aB)x^{7/2}+\frac{2}{9}b^6Bx^{9/2}$$

output
```
-2/5*a^6*A/x^(5/2)-2/3*a^5*(6*A*b+B*a)/x^(3/2)-6*a^4*b*(5*A*b+2*B*a)/x^(1/
2)+10*a^3*b^2*(4*A*b+3*B*a)*x^(1/2)+10/3*a^2*b^3*(3*A*b+4*B*a)*x^(3/2)+6/5
*a*b^4*(2*A*b+5*B*a)*x^(5/2)+2/7*b^5*(A*b+6*B*a)*x^(7/2)+2/9*b^6*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx=\frac{2(4725a^4b^2x^2(-A+Bx)+2100a^3b^3x^3(3A+Bx)-630a^5bx(A+3\ldots}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(7/2),x]
```

output
```
(2*(4725*a^4*b^2*x^2*(-A + B*x) + 2100*a^3*b^3*x^3*(3*A + B*x) - 630*a^5*b
*x*(A + 3*B*x) + 315*a^2*b^4*x^4*(5*A + 3*B*x) - 21*a^6*(3*A + 5*B*x) + 54
*a*b^5*x^5*(7*A + 5*B*x) + 5*b^6*x^6*(9*A + 7*B*x)))/(315*x^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3(A+Bx)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{7/2}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^6A}{x^{7/2}}+\frac{a^5(aB+6Ab)}{x^{5/2}}+\frac{3a^4b(2aB+5Ab)}{x^{3/2}}+\frac{5a^3b^2(3aB+4Ab)}{\sqrt{x}}+5a^2b^3\sqrt{x}(4aB+3Ab)+b^5x^{5/2}(6aB+A$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^6A}{5x^{5/2}}-\frac{2a^5(aB+6Ab)}{3x^{3/2}}-\frac{6a^4b(2aB+5Ab)}{\sqrt{x}}+10a^3b^2\sqrt{x}(3aB+4Ab)+\frac{10}{3}a^2b^3x^{3/2}(4aB+3Ab)+\frac{2}{7}b^5x^{7/2}(6aB+Ab)+\frac{6}{5}ab^4x^{5/2}(5aB+2Ab)+\frac{2}{9}b^6Bx^{9/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(7/2),x]
```

output
```
(-2*a^6*A)/(5*x^(5/2)) - (2*a^5*(6*A*b + a*B))/(3*x^(3/2)) - (6*a^4*b*(5*A
*b + 2*a*B))/Sqrt[x] + 10*a^3*b^2*(4*A*b + 3*a*B)*Sqrt[x] + (10*a^2*b^3*(3
*A*b + 4*a*B)*x^(3/2))/3 + (6*a*b^4*(2*A*b + 5*a*B)*x^(5/2))/5 + (2*b^5*(A
*b + 6*a*B)*x^(7/2))/7 + (2*b^6*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^6Bx^{\frac{9}{2}}}{9}+\frac{2Ab^6x^{\frac{7}{2}}}{7}+\frac{12Bab^5x^{\frac{7}{2}}}{7}+\frac{12Aab^5x^{\frac{5}{2}}}{5}+6Ba^2b^4x^{\frac{5}{2}}+10Aa^2b^4x^{\frac{3}{2}}+\frac{40Ba^3b^3x^{\frac{3}{2}}}{3}+40A$
default	$\frac{2b^6Bx^{\frac{9}{2}}}{9}+\frac{2Ab^6x^{\frac{7}{2}}}{7}+\frac{12Bab^5x^{\frac{7}{2}}}{7}+\frac{12Aab^5x^{\frac{5}{2}}}{5}+6Ba^2b^4x^{\frac{5}{2}}+10Aa^2b^4x^{\frac{3}{2}}+\frac{40Ba^3b^3x^{\frac{3}{2}}}{3}+40A$
gosper	$-\frac{2(-35b^6Bx^7-45Ab^6x^6-270Bab^5x^6-378Aab^5x^5-945Ba^2b^4x^5-1575Aa^2b^4x^4-2100Ba^3b^3x^4-6300Aa^3b^3x^3-}{315x^{\frac{5}{2}}}$
trager	$-\frac{2(-35b^6Bx^7-45Ab^6x^6-270Bab^5x^6-378Aab^5x^5-945Ba^2b^4x^5-1575Aa^2b^4x^4-2100Ba^3b^3x^4-6300Aa^3b^3x^3-}{315x^{\frac{5}{2}}}$
risch	$-\frac{2(-35b^6Bx^7-45Ab^6x^6-270Bab^5x^6-378Aab^5x^5-945Ba^2b^4x^5-1575Aa^2b^4x^4-2100Ba^3b^3x^4-6300Aa^3b^3x^3-}{315x^{\frac{5}{2}}}$
orering	$-\frac{2(-35b^6Bx^7-45Ab^6x^6-270Bab^5x^6-378Aab^5x^5-945Ba^2b^4x^5-1575Aa^2b^4x^4-2100Ba^3b^3x^4-6300Aa^3b^3x^3-}{315x^{\frac{5}{2}}(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b^6*B*x^(9/2)+2/7*A*b^6*x^(7/2)+12/7*B*a*b^5*x^(7/2)+12/5*A*a*b^5*x^(5
/2)+6*B*a^2*b^4*x^(5/2)+10*A*a^2*b^4*x^(3/2)+40/3*B*a^3*b^3*x^(3/2)+40*A*a
^3*b^3*x^(1/2)+30*B*a^4*b^2*x^(1/2)-2/5*a^6*A/x^(5/2)-2/3*a^5*(6*A*b+B*a)/
x^(3/2)-6*a^4*b*(5*A*b+2*B*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx=\frac{2\left(35\,Bb^6x^7-63\,Aa^6+45\,(6\,Bab^5+Ab^6)x^6+189\,(5\,Ba^2b^4+2\,Aa\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^6*x^7 - 63*A*a^6 + 45*(6*B*a*b^5 + A*b^6)*x^6 + 189*(5*B*a^2
*b^4 + 2*A*a*b^5)*x^5 + 525*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 + 1575*(3*B*a^
4*b^2 + 4*A*a^3*b^3)*x^3 - 945*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 105*(B*a^6
+ 6*A*a^5*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx = -\frac{2Aa^6}{5x^{\frac{5}{2}}} - \frac{4Aa^5b}{x^{\frac{3}{2}}} - \frac{30Aa^4b^2}{\sqrt{x}} + 40Aa^3b^3\sqrt{x} + 10Aa^2b^4x^{\frac{3}{2}} + \frac{12Aab^5x^{\frac{5}{2}}}{5} + \frac{2Ab^6x^{\frac{7}{2}}}{7} - \frac{2Ba^6}{3x^{\frac{3}{2}}} - \frac{12Ba^5b}{\sqrt{x}} + 30Ba^4b^2\sqrt{x} + \frac{40Ba^3b^3x^{\frac{3}{2}}}{3} + 6Ba^2b^4x^{\frac{5}{2}} + \frac{12Bab^5x^{\frac{7}{2}}}{7} + \frac{2Bb^6x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(7/2),x)
```

output
```
-2*A*a**6/(5*x**(5/2)) - 4*A*a**5*b/x**(3/2) - 30*A*a**4*b**2/sqrt(x) + 40
*A*a**3*b**3*sqrt(x) + 10*A*a**2*b**4*x**(3/2) + 12*A*a*b**5*x**(5/2)/5 +
2*A*b**6*x**(7/2)/7 - 2*B*a**6/(3*x**(3/2)) - 12*B*a**5*b/sqrt(x) + 30*B*a
**4*b**2*sqrt(x) + 40*B*a**3*b**3*x**(3/2)/3 + 6*B*a**2*b**4*x**(5/2) + 12
*B*a*b**5*x**(7/2)/7 + 2*B*b**6*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx = \frac{2}{9}\,Bb^6x^{\frac{9}{2}} + \frac{2}{7}\left(6\,Bab^5+Ab^6\right)x^{\frac{7}{2}} + \frac{6}{5}\left(5\,Ba^2b^4+2\,Aab^5\right)x^{\frac{5}{2}} + \frac{10}{3}\left(4\,Ba^3b^3+3\,Aa^2b^4\right)x^{\frac{3}{2}} + 10\left(3\,Ba^4b^2+4\,Aa^3b^3\right)\sqrt{x} - \frac{2\left(3\,Aa^6+45\,(2\,Ba^5b+5\,Aa^4b^2)x^2+5\,(Ba^6+6\,Aa^5b)x\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/9*B*b^6*x^(9/2) + 2/7*(6*B*a*b^5 + A*b^6)*x^(7/2) + 6/5*(5*B*a^2*b^4 + 2
*A*a*b^5)*x^(5/2) + 10/3*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^(3/2) + 10*(3*B*a^4
*b^2 + 4*A*a^3*b^3)*sqrt(x) - 2/15*(3*A*a^6 + 45*(2*B*a^5*b + 5*A*a^4*b^2)
*x^2 + 5*(B*a^6 + 6*A*a^5*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx = \frac{2}{9}\,Bb^6x^{\frac{9}{2}} + \frac{12}{7}\,Bab^5x^{\frac{7}{2}} + \frac{2}{7}\,Ab^6x^{\frac{7}{2}} + 6\,Ba^2b^4x^{\frac{5}{2}} + \frac{12}{5}\,Aab^5x^{\frac{5}{2}} + \frac{40}{3}\,Ba^3b^3x^{\frac{3}{2}} + 10\,Aa^2b^4x^{\frac{3}{2}} + 30\,Ba^4b^2\sqrt{x} + 40\,Aa^3b^3\sqrt{x} - \frac{2\,(90\,Ba^5bx^2 + 225\,Aa^4b^2x^2 + 5\,Ba^6x + 30\,Aa^5bx + 3\,Aa^6)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/9*B*b^6*x^(9/2) + 12/7*B*a*b^5*x^(7/2) + 2/7*A*b^6*x^(7/2) + 6*B*a^2*b^4
*x^(5/2) + 12/5*A*a*b^5*x^(5/2) + 40/3*B*a^3*b^3*x^(3/2) + 10*A*a^2*b^4*x^
(3/2) + 30*B*a^4*b^2*sqrt(x) + 40*A*a^3*b^3*sqrt(x) - 2/15*(90*B*a^5*b*x^2
 + 225*A*a^4*b^2*x^2 + 5*B*a^6*x + 30*A*a^5*b*x + 3*A*a^6)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx = x^{7/2}\left(\frac{2\,A\,b^6}{7}+\frac{12\,B\,a\,b^5}{7}\right) - \frac{x\left(\frac{2\,B\,a^6}{3}+4\,A\,b\,a^5\right)+\frac{2\,A\,a^6}{5}+x^2\left(12\,B\,a^5\,b+30\,A\,a^4\,b^2\right)}{x^{5/2}} + \frac{2\,B\,b^6\,x^{9/2}}{9} + 10\,a^3\,b^2\,\sqrt{x}\,(4\,A\,b+3\,B\,a) + \frac{10\,a^2\,b^3\,x^{3/2}\,(3\,A\,b+4\,B\,a)}{3} + \frac{6\,a\,b^4\,x^{5/2}\,(2\,A\,b+5\,B\,a)}{5}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(7/2),x)
```

output
```
x^(7/2)*((2*A*b^6)/7 + (12*B*a*b^5)/7) - (x*((2*B*a^6)/3 + 4*A*a^5*b) + (2
*A*a^6)/5 + x^2*(30*A*a^4*b^2 + 12*B*a^5*b))/x^(5/2) + (2*B*b^6*x^(9/2))/9
 + 10*a^3*b^2*x^(1/2)*(4*A*b + 3*B*a) + (10*a^2*b^3*x^(3/2)*(3*A*b + 4*B*a
))/3 + (6*a*b^4*x^(5/2)*(2*A*b + 5*B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{7/2}}\,dx = \frac{\frac{2}{9}b^7x^7+2a\,b^6x^6+\frac{42}{5}a^2b^5x^5+\frac{70}{3}a^3b^4x^4+70a^4b^3x^3-42a^5b^2x^2-\frac{14}{3}}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(7/2),x)
```

output
```
(2*( - 9*a**7 - 105*a**6*b*x - 945*a**5*b**2*x**2 + 1575*a**4*b**3*x**3 +
525*a**3*b**4*x**4 + 189*a**2*b**5*x**5 + 45*a*b**6*x**6 + 5*b**7*x**7))/(
45*sqrt(x)*x**2)
```

3.384 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = -\frac{2a^6A}{7x^{7/2}} - \frac{2a^5(6Ab+aB)}{5x^{5/2}} - \frac{2a^4b(5Ab+2aB)}{x^{3/2}} - \frac{10a^3b^2(4Ab+3aB)}{\sqrt{x}} + 10a^2b^3(3Ab+4aB)\sqrt{x} + 2ab^4(2Ab+5aB)x^{3/2} + \frac{2}{5}b^5(Ab+6aB)x^{5/2} + \frac{2}{7}b^6Bx^{7/2}$$

output
```
-2/7*a^6*A/x^(7/2)-2/5*a^5*(6*A*b+B*a)/x^(5/2)-2*a^4*b*(5*A*b+2*B*a)/x^(3/
2)-10*a^3*b^2*(4*A*b+3*B*a)/x^(1/2)+10*a^2*b^3*(3*A*b+4*B*a)*x^(1/2)+2*a*b
^4*(2*A*b+5*B*a)*x^(3/2)+2/5*b^5*(A*b+6*B*a)*x^(5/2)+2/7*b^6*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = \frac{2(700a^3b^3x^3(-A+Bx)+175a^2b^4x^4(3A+Bx)-175a^4b^2x^2(A+3$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(9/2),x]
```

output
```
(2*(700*a^3*b^3*x^3*(-A + B*x) + 175*a^2*b^4*x^4*(3*A + B*x) - 175*a^4*b^2
*x^2*(A + 3*B*x) + 14*a*b^5*x^5*(5*A + 3*B*x) - 14*a^5*b*x*(3*A + 5*B*x) +
 b^6*x^6*(7*A + 5*B*x) - a^6*(5*A + 7*B*x)))/(35*x^(7/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3(A+Bx)}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{9/2}}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^6A}{x^{9/2}}+\frac{a^5(aB+6Ab)}{x^{7/2}}+\frac{3a^4b(2aB+5Ab)}{x^{5/2}}+\frac{5a^3b^2(3aB+4Ab)}{x^{3/2}}+\frac{5a^2b^3(4aB+3Ab)}{\sqrt{x}}+b^5x^{3/2}(6aB+Ab)\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^6A}{7x^{7/2}}-\frac{2a^5(aB+6Ab)}{5x^{5/2}}-\frac{2a^4b(2aB+5Ab)}{x^{3/2}}-\frac{10a^3b^2(3aB+4Ab)}{\sqrt{x}}+10a^2b^3\sqrt{x}(4aB+3Ab)+\frac{2}{5}b^5x^{5/2}(6aB+Ab)+2ab^4x^{3/2}(5aB+2Ab)+\frac{2}{7}b^6Bx^{7/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(9/2),x]
```

output
```
(-2*a^6*A)/(7*x^(7/2)) - (2*a^5*(6*A*b + a*B))/(5*x^(5/2)) - (2*a^4*b*(5*A
*b + 2*a*B))/x^(3/2) - (10*a^3*b^2*(4*A*b + 3*a*B))/Sqrt[x] + 10*a^2*b^3*(
3*A*b + 4*a*B)*Sqrt[x] + 2*a*b^4*(2*A*b + 5*a*B)*x^(3/2) + (2*b^5*(A*b + 6
*a*B)*x^(5/2))/5 + (2*b^6*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^6Bx^{\frac{7}{2}}}{7}+\frac{2Ab^6x^{\frac{5}{2}}}{5}+\frac{12Bab^5x^{\frac{5}{2}}}{5}+4Aab^5x^{\frac{3}{2}}+10Ba^2b^4x^{\frac{3}{2}}+30Aa^2b^4\sqrt{x}+40Ba^3b^3\sqrt{x}$
default	$\frac{2b^6Bx^{\frac{7}{2}}}{7}+\frac{2Ab^6x^{\frac{5}{2}}}{5}+\frac{12Bab^5x^{\frac{5}{2}}}{5}+4Aab^5x^{\frac{3}{2}}+10Ba^2b^4x^{\frac{3}{2}}+30Aa^2b^4\sqrt{x}+40Ba^3b^3\sqrt{x}$
gosper	$-\frac{2(-5b^6Bx^7-7Ab^6x^6-42Bab^5x^6-70Aab^5x^5-175Ba^2b^4x^5-525Aa^2b^4x^4-700Ba^3b^3x^4+700Aa^3b^3x^3+525Ba}{35x^{\frac{7}{2}}}$
trager	$-\frac{2(-5b^6Bx^7-7Ab^6x^6-42Bab^5x^6-70Aab^5x^5-175Ba^2b^4x^5-525Aa^2b^4x^4-700Ba^3b^3x^4+700Aa^3b^3x^3+525Ba}{35x^{\frac{7}{2}}}$
risch	$-\frac{2(-5b^6Bx^7-7Ab^6x^6-42Bab^5x^6-70Aab^5x^5-175Ba^2b^4x^5-525Aa^2b^4x^4-700Ba^3b^3x^4+700Aa^3b^3x^3+525Ba}{35x^{\frac{7}{2}}}$
orering	$-\frac{2(-5b^6Bx^7-7Ab^6x^6-42Bab^5x^6-70Aab^5x^5-175Ba^2b^4x^5-525Aa^2b^4x^4-700Ba^3b^3x^4+700Aa^3b^3x^3+525Ba}{35x^{\frac{7}{2}}(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^6*B*x^(7/2)+2/5*A*b^6*x^(5/2)+12/5*B*a*b^5*x^(5/2)+4*A*a*b^5*x^(3/2)
+10*B*a^2*b^4*x^(3/2)+30*A*a^2*b^4*x^(1/2)+40*B*a^3*b^3*x^(1/2)-2/5*a^5*(6
*A*b+B*a)/x^(5/2)-2*a^4*b*(5*A*b+2*B*a)/x^(3/2)-10*a^3*b^2*(4*A*b+3*B*a)/x
^(1/2)-2/7*a^6*A/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx=\frac{2\left(5\,Bb^6x^7-5\,Aa^6+7\left(6\,Bab^5+Ab^6\right)x^6+35\left(5\,Ba^2b^4+2\,Aab^5\right)x^5\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(9/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*b^6*x^7 - 5*A*a^6 + 7*(6*B*a*b^5 + A*b^6)*x^6 + 35*(5*B*a^2*b^4
+ 2*A*a*b^5)*x^5 + 175*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 - 175*(3*B*a^4*b^2
+ 4*A*a^3*b^3)*x^3 - 35*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 7*(B*a^6 + 6*A*a^5
*b)*x)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = -\frac{2Aa^6}{7x^{\frac{7}{2}}} - \frac{12Aa^5b}{5x^{\frac{5}{2}}} - \frac{10Aa^4b^2}{x^{\frac{3}{2}}} - \frac{40Aa^3b^3}{\sqrt{x}} + 30Aa^2b^4\sqrt{x} + 4Aab^5x^{\frac{3}{2}} + \frac{2Ab^6x^{\frac{5}{2}}}{5} - \frac{2Ba^6}{5x^{\frac{5}{2}}} - \frac{4Ba^5b}{x^{\frac{3}{2}}} - \frac{30Ba^4b^2}{\sqrt{x}} + 40Ba^3b^3\sqrt{x} + 10Ba^2b^4x^{\frac{3}{2}} + \frac{12Bab^5x^{\frac{5}{2}}}{5} + \frac{2Bb^6x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(9/2),x)
```

output
```
-2*A*a**6/(7*x**(7/2)) - 12*A*a**5*b/(5*x**(5/2)) - 10*A*a**4*b**2/x**(3/2
) - 40*A*a**3*b**3/sqrt(x) + 30*A*a**2*b**4*sqrt(x) + 4*A*a*b**5*x**(3/2)
+ 2*A*b**6*x**(5/2)/5 - 2*B*a**6/(5*x**(5/2)) - 4*B*a**5*b/x**(3/2) - 30*B
*a**4*b**2/sqrt(x) + 40*B*a**3*b**3*sqrt(x) + 10*B*a**2*b**4*x**(3/2) + 12
*B*a*b**5*x**(5/2)/5 + 2*B*b**6*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}\,Bb^6x^{\frac{7}{2}} + \frac{2}{5}\left(6\,Bab^5 + Ab^6\right)x^{\frac{5}{2}} + 2\left(5\,Ba^2b^4 + 2\,Aab^5\right)x^{\frac{3}{2}} + 10\left(4\,Ba^3b^3 + 3\,Aa^2b^4\right)\sqrt{x} - \frac{2\left(5\,Aa^6 + 175\,(3\,Ba^4b^2 + 4\,Aa^3b^3)x^3 + 35\,(2\,Ba^5b + 5\,Aa^4b^2)x^2 + 7\,(Ba^6 + 6\,Aa^5b)x\right)}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(9/2),x, algorithm="maxima")
```

output
```
2/7*B*b^6*x^(7/2) + 2/5*(6*B*a*b^5 + A*b^6)*x^(5/2) + 2*(5*B*a^2*b^4 + 2*A
*a*b^5)*x^(3/2) + 10*(4*B*a^3*b^3 + 3*A*a^2*b^4)*sqrt(x) - 2/35*(5*A*a^6 +
 175*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 35*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 +
7*(B*a^6 + 6*A*a^5*b)*x)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}Bb^6x^{\frac{7}{2}} + \frac{12}{5}Bab^5x^{\frac{5}{2}} + \frac{2}{5}Ab^6x^{\frac{5}{2}} + 10\,Ba^2b^4x^{\frac{3}{2}} + 4\,Aab^5x^{\frac{3}{2}} + 40\,Ba^3b^3\sqrt{x} + 30\,Aa^2b^4\sqrt{x} - \frac{2\left(525\,Ba^4b^2x^3 + 700\,Aa^3b^3x^3 + 70\,Ba^5bx^2 + 175\,Aa^4b^2x^2 + 7\,Ba^6x + 42\,Aa^5bx + 5\,Aa^6\right)}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(9/2),x, algorithm="giac")
```

output
```
2/7*B*b^6*x^(7/2) + 12/5*B*a*b^5*x^(5/2) + 2/5*A*b^6*x^(5/2) + 10*B*a^2*b^
4*x^(3/2) + 4*A*a*b^5*x^(3/2) + 40*B*a^3*b^3*sqrt(x) + 30*A*a^2*b^4*sqrt(x
) - 2/35*(525*B*a^4*b^2*x^3 + 700*A*a^3*b^3*x^3 + 70*B*a^5*b*x^2 + 175*A*a
^4*b^2*x^2 + 7*B*a^6*x + 42*A*a^5*b*x + 5*A*a^6)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = x^{5/2}\left(\frac{2\,A\,b^6}{5} + \frac{12\,B\,a\,b^5}{5}\right) - \frac{x\left(\frac{2\,B\,a^6}{5} + \frac{12\,A\,b\,a^5}{5}\right) + \frac{2\,A\,a^6}{7} + x^2\left(4\,B\,a^5\,b + 10\,A\,a^4\,b^2\right) + x^3\left(30\,B\,a^4\,b^2 + 40\,A\,a^3\,b^3\right)}{x^{7/2}} + \frac{2\,B\,b^6\,x^{7/2}}{7} + 10\,a^2\,b^3\,\sqrt{x}\,(3\,A\,b + 4\,B\,a) + 2\,a\,b^4\,x^{3/2}\,(2\,A\,b + 5\,B\,a)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(9/2),x)
```

output
```
x^(5/2)*((2*A*b^6)/5 + (12*B*a*b^5)/5) - (x*((2*B*a^6)/5 + (12*A*a^5*b)/5)
 + (2*A*a^6)/7 + x^2*(10*A*a^4*b^2 + 4*B*a^5*b) + x^3*(40*A*a^3*b^3 + 30*B
*a^4*b^2))/x^(7/2) + (2*B*b^6*x^(7/2))/7 + 10*a^2*b^3*x^(1/2)*(3*A*b + 4*B
*a) + 2*a*b^4*x^(3/2)*(2*A*b + 5*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{9/2}}\,dx = \frac{\frac{2}{7}b^7x^7+\frac{14}{5}a\,b^6x^6+14a^2b^5x^5+70a^3b^4x^4-70a^4b^3x^3-14a^5b^2x^2-}{\sqrt{x}\,x^3}$$

input

```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(9/2),x)
```

output

```
(2*( - 5*a**7 - 49*a**6*b*x - 245*a**5*b**2*x**2 - 1225*a**4*b**3*x**3 + 1
225*a**3*b**4*x**4 + 245*a**2*b**5*x**5 + 49*a*b**6*x**6 + 5*b**7*x**7))/(
35*sqrt(x)*x**3)
```

3.385 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 155

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = -\frac{2a^6A}{9x^{9/2}} - \frac{2a^5(6Ab+aB)}{7x^{7/2}} - \frac{6a^4b(5Ab+2aB)}{5x^{5/2}} - \frac{10a^3b^2(4Ab+3aB)}{3x^{3/2}} - \frac{10a^2b^3(3Ab+4aB)}{\sqrt{x}} + 6ab^4(2Ab+5aB)\sqrt{x} + \frac{2}{3}b^5(Ab+6aB)x^{3/2} + \frac{2}{5}b^6Bx^{5/2}$$

output
```
-2/9*a^6*A/x^(9/2)-2/7*a^5*(6*A*b+B*a)/x^(7/2)-6/5*a^4*b*(5*A*b+2*B*a)/x^(
5/2)-10/3*a^3*b^2*(4*A*b+3*B*a)/x^(3/2)-10*a^2*b^3*(3*A*b+4*B*a)/x^(1/2)+6
*a*b^4*(2*A*b+5*B*a)*x^(1/2)+2/3*b^5*(A*b+6*B*a)*x^(3/2)+2/5*b^6*B*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = -\frac{2(4725a^2b^4x^4(A-Bx) - 630ab^5x^5(3A+Bx) + 2100a^3b^3x^3(A+3Bx) - 21b^6x^6(5A+3Bx) + 315a^4b^2x}{315x^{9/2}}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(11/2),x]
```

output
```
(-2*(4725*a^2*b^4*x^4*(A - B*x) - 630*a*b^5*x^5*(3*A + B*x) + 2100*a^3*b^3
*x^3*(A + 3*B*x) - 21*b^6*x^6*(5*A + 3*B*x) + 315*a^4*b^2*x^2*(3*A + 5*B*x
) + 54*a^5*b*x*(5*A + 7*B*x) + 5*a^6*(7*A + 9*B*x)))/(315*x^(9/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2)^3 (A+Bx)}{x^{11/2}}dx$$

$$\Big\downarrow \textbf{1184}$$

$$\frac{\int \frac{b^6(a+bx)^6(A+Bx)}{x^{11/2}}dx}{b^6}$$

$$\Big\downarrow \textbf{27}$$

$$\int \frac{(a+bx)^6(A+Bx)}{x^{11/2}}dx$$

$$\Big\downarrow \textbf{85}$$

$$\int \left(\frac{a^6A}{x^{11/2}}+\frac{a^5(aB+6Ab)}{x^{9/2}}+\frac{3a^4b(2aB+5Ab)}{x^{7/2}}+\frac{5a^3b^2(3aB+4Ab)}{x^{5/2}}+\frac{5a^2b^3(4aB+3Ab)}{x^{3/2}}+b^5\sqrt{x}(6aB+Ab)\right.$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{2a^6A}{9x^{9/2}}-\frac{2a^5(aB+6Ab)}{7x^{7/2}}-\frac{6a^4b(2aB+5Ab)}{5x^{5/2}}-\frac{10a^3b^2(3aB+4Ab)}{3x^{3/2}}-\frac{10a^2b^3(4aB+3Ab)}{\sqrt{x}}+\frac{2}{3}b^5x^{3/2}(6aB+Ab)+6ab^4\sqrt{x}(5aB+2Ab)+\frac{2}{5}b^6Bx^{5/2}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3)/x^(11/2),x]
```

output
```
(-2*a^6*A)/(9*x^(9/2)) - (2*a^5*(6*A*b + a*B))/(7*x^(7/2)) - (6*a^4*b*(5*A
*b + 2*a*B))/(5*x^(5/2)) - (10*a^3*b^2*(4*A*b + 3*a*B))/(3*x^(3/2)) - (10*
a^2*b^3*(3*A*b + 4*a*B))/Sqrt[x] + 6*a*b^4*(2*A*b + 5*a*B)*Sqrt[x] + (2*b^
5*(A*b + 6*a*B)*x^(3/2))/3 + (2*b^6*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2b^6Bx^{\frac{5}{2}}}{5}+\frac{2Ab^6x^{\frac{3}{2}}}{3}+4Ba\,b^5x^{\frac{3}{2}}+12Aa\,b^5\sqrt{x}+30B\,a^2b^4\sqrt{x}-\frac{6a^4b(5Ab+2Ba)}{5x^{\frac{5}{2}}}-\frac{2a^5(6Ab+B}{7x^{\frac{7}{2}}}$
default	$\frac{2b^6Bx^{\frac{5}{2}}}{5}+\frac{2Ab^6x^{\frac{3}{2}}}{3}+4Ba\,b^5x^{\frac{3}{2}}+12Aa\,b^5\sqrt{x}+30B\,a^2b^4\sqrt{x}-\frac{6a^4b(5Ab+2Ba)}{5x^{\frac{5}{2}}}-\frac{2a^5(6Ab+B}{7x^{\frac{7}{2}}}$
gosper	$-\frac{2(-63b^6Bx^7-105Ab^6x^6-630Ba\,b^5x^6-1890Aa\,b^5x^5-4725B\,a^2b^4x^5+4725A\,a^2b^4x^4+6300B\,a^3b^3x^4+2100A\,a^3b^3}{315x^{\frac{9}{2}}}$
trager	$-\frac{2(-63b^6Bx^7-105Ab^6x^6-630Ba\,b^5x^6-1890Aa\,b^5x^5-4725B\,a^2b^4x^5+4725A\,a^2b^4x^4+6300B\,a^3b^3x^4+2100A\,a^3b^3}{315x^{\frac{9}{2}}}$
risch	$-\frac{2(-63b^6Bx^7-105Ab^6x^6-630Ba\,b^5x^6-1890Aa\,b^5x^5-4725B\,a^2b^4x^5+4725A\,a^2b^4x^4+6300B\,a^3b^3x^4+2100A\,a^3b^3}{315x^{\frac{9}{2}}}$
orering	$-\frac{2(-63b^6Bx^7-105Ab^6x^6-630Ba\,b^5x^6-1890Aa\,b^5x^5-4725B\,a^2b^4x^5+4725A\,a^2b^4x^4+6300B\,a^3b^3x^4+2100A\,a^3b^3}{315x^{\frac{9}{2}}(bx+a)^6}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^6*B*x^(5/2)+2/3*A*b^6*x^(3/2)+4*B*a*b^5*x^(3/2)+12*A*a*b^5*x^(1/2)+3
0*B*a^2*b^4*x^(1/2)-6/5*a^4*b*(5*A*b+2*B*a)/x^(5/2)-2/7*a^5*(6*A*b+B*a)/x^
(7/2)-10/3*a^3*b^2*(4*A*b+3*B*a)/x^(3/2)-10*a^2*b^3*(3*A*b+4*B*a)/x^(1/2)-
2/9*a^6*A/x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx=\frac{2\left(63\,Bb^6x^7-35\,Aa^6+105\,(6\,Bab^5+Ab^6)x^6+945\,(5\,Ba^2b^4+2\,A\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(11/2),x, algorithm="fricas")
```

output
```
2/315*(63*B*b^6*x^7 - 35*A*a^6 + 105*(6*B*a*b^5 + A*b^6)*x^6 + 945*(5*B*a^
2*b^4 + 2*A*a*b^5)*x^5 - 1575*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 - 525*(3*B*a
^4*b^2 + 4*A*a^3*b^3)*x^3 - 189*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 - 45*(B*a^6
+ 6*A*a^5*b)*x)/x^(9/2)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = -\frac{2Aa^6}{9x^{\frac{9}{2}}} - \frac{12Aa^5b}{7x^{\frac{7}{2}}} - \frac{6Aa^4b^2}{x^{\frac{5}{2}}} - \frac{40Aa^3b^3}{3x^{\frac{3}{2}}} - \frac{30Aa^2b^4}{\sqrt{x}} + 12Aab^5\sqrt{x} + \frac{2Ab^6x^{\frac{3}{2}}}{3} - \frac{2Ba^6}{7x^{\frac{7}{2}}} - \frac{12Ba^5b}{5x^{\frac{5}{2}}} - \frac{10Ba^4b^2}{x^{\frac{3}{2}}} - \frac{40Ba^3b^3}{\sqrt{x}} + 30Ba^2b^4\sqrt{x} + 4Bab^5x^{\frac{3}{2}} + \frac{2Bb^6x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3/x**(11/2),x)
```

output
```
-2*A*a**6/(9*x**(9/2)) - 12*A*a**5*b/(7*x**(7/2)) - 6*A*a**4*b**2/x**(5/2)
 - 40*A*a**3*b**3/(3*x**(3/2)) - 30*A*a**2*b**4/sqrt(x) + 12*A*a*b**5*sqrt
(x) + 2*A*b**6*x**(3/2)/3 - 2*B*a**6/(7*x**(7/2)) - 12*B*a**5*b/(5*x**(5/2
)) - 10*B*a**4*b**2/x**(3/2) - 40*B*a**3*b**3/sqrt(x) + 30*B*a**2*b**4*sqr
t(x) + 4*B*a*b**5*x**(3/2) + 2*B*b**6*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = \frac{2}{5}\,Bb^6x^{\frac{5}{2}} + \frac{2}{3}\left(6\,Bab^5 + Ab^6\right)x^{\frac{3}{2}} + 6\left(5\,Ba^2b^4 + 2\,Aab^5\right)\sqrt{x} - \frac{2\left(35\,Aa^6 + 1575\,(4\,Ba^3b^3 + 3\,Aa^2b^4)x^4 + 525\,(3\,Ba^4b^2 + 4\,Aa^3b^3)x^3 + 189\,(2\,Ba^5b + 5\,Aa^4b^2)x^2 + 45\,(\right.}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(11/2),x, algorithm="maxima")
```

output
```
2/5*B*b^6*x^(5/2) + 2/3*(6*B*a*b^5 + A*b^6)*x^(3/2) + 6*(5*B*a^2*b^4 + 2*A
*a*b^5)*sqrt(x) - 2/315*(35*A*a^6 + 1575*(4*B*a^3*b^3 + 3*A*a^2*b^4)*x^4 +
 525*(3*B*a^4*b^2 + 4*A*a^3*b^3)*x^3 + 189*(2*B*a^5*b + 5*A*a^4*b^2)*x^2 +
 45*(B*a^6 + 6*A*a^5*b)*x)/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = \frac{2}{5}Bb^6x^{\frac{5}{2}} + 4Bab^5x^{\frac{3}{2}} + \frac{2}{3}Ab^6x^{\frac{3}{2}} + 30Ba^2b^4\sqrt{x} + 12Aab^5\sqrt{x} - \frac{2\left(6300Ba^3b^3x^4 + 4725Aa^2b^4x^4 + 1575Ba^4b^2x^3 + 2100Aa^3b^3x^3 + 378Ba^5bx^2 + 945Aa^4b^2x^2 + 45Ba^6\right.}{315x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(11/2),x, algorithm="giac")
```

output
```
2/5*B*b^6*x^(5/2) + 4*B*a*b^5*x^(3/2) + 2/3*A*b^6*x^(3/2) + 30*B*a^2*b^4*s
qrt(x) + 12*A*a*b^5*sqrt(x) - 2/315*(6300*B*a^3*b^3*x^4 + 4725*A*a^2*b^4*x
^4 + 1575*B*a^4*b^2*x^3 + 2100*A*a^3*b^3*x^3 + 378*B*a^5*b*x^2 + 945*A*a^4
*b^2*x^2 + 45*B*a^6*x + 270*A*a^5*b*x + 35*A*a^6)/x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = x^{3/2}\left(\frac{2Ab^6}{3}+4Bab^5\right) - \frac{x\left(\frac{2Ba^6}{7}+\frac{12Aba^5}{7}\right)+\frac{2Aa^6}{9}+x^2\left(\frac{12Ba^5b}{5}+6Aa^4b^2\right)+x^3\left(10Ba^4b^2+\frac{40Aa^3b^3}{3}\right)+x^4\left(40Ba^3b^3+30\right.}{x^{9/2}} + \frac{2Bb^6x^{5/2}}{5} + 6ab^4\sqrt{x}\,(2Ab+5Ba)$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3)/x^(11/2),x)
```

output
```
x^(3/2)*((2*A*b^6)/3 + 4*B*a*b^5) - (x*((2*B*a^6)/7 + (12*A*a^5*b)/7) + (2
*A*a^6)/9 + x^2*(6*A*a^4*b^2 + (12*B*a^5*b)/5) + x^3*((40*A*a^3*b^3)/3 + 1
0*B*a^4*b^2) + x^4*(30*A*a^2*b^4 + 40*B*a^3*b^3))/x^(9/2) + (2*B*b^6*x^(5/
2))/5 + 6*a*b^4*x^(1/2)*(2*A*b + 5*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^3}{x^{11/2}}\,dx = \frac{\frac{2}{5}b^7x^7+\frac{14}{3}a\,b^6x^6+42a^2b^5x^5-70a^3b^4x^4-\frac{70}{3}a^4b^3x^3-\frac{42}{5}a^5b^2x^2-2}{\sqrt{x}\,x^4}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3/x^(11/2),x)
```

output
```
(2*( - 5*a**7 - 45*a**6*b*x - 189*a**5*b**2*x**2 - 525*a**4*b**3*x**3 - 15
75*a**3*b**4*x**4 + 945*a**2*b**5*x**5 + 105*a*b**6*x**6 + 9*b**7*x**7))/(
45*sqrt(x)*x**4)
```

3.386 $\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{a^2(7Ab-9aB)\sqrt{x}}{b^5}-\frac{a(7Ab-9aB)x^{3/2}}{3b^4}+\frac{(7Ab-9aB)x^{5/2}}{5b^3}+\frac{2Bx^{7/2}}{7b^2}-\frac{(Ab-aB)x^{7/2}}{b^2(a+bx)}-\frac{a^{5/2}(7Ab-9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

output
```
a^2*(7*A*b-9*B*a)*x^(1/2)/b^5-1/3*a*(7*A*b-9*B*a)*x^(3/2)/b^4+1/5*(7*A*b-9
*B*a)*x^(5/2)/b^3+2/7*B*x^(7/2)/b^2-(A*b-B*a)*x^(7/2)/b^2/(b*x+a)-a^(5/2)*
(7*A*b-9*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{\sqrt{x}(-945a^4B+105a^3b(7A-6Bx)+6b^4x^3(7A+5Bx)+14a^2b^2x(35A+9Bx)}{105b^5(a+bx)}+\frac{a^{5/2}(-7Ab+9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(Sqrt[x]*(-945*a^4*B + 105*a^3*b*(7*A - 6*B*x) + 6*b^4*x^3*(7*A + 5*B*x) +
 14*a^2*b^2*x*(35*A + 9*B*x) - 2*a*b^3*x^2*(49*A + 27*B*x)))/(105*b^5*(a +
 b*x)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(1
1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^{7/2}(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\int \frac{x^{7/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\int \frac{x^{5/2}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\int\frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}\right)}{2ab}$$

$\downarrow$ 60

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$\downarrow$ 60

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$\downarrow$ 73

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$\Big\downarrow$ 218

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*x^(9/2))/(a*b*(a + b*x)) - ((7*A*b - 9*a*B)*((2*x^(7/2))/(7*b
) - (a*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2(15x^3Bb^3+21Ab^3x^2-42Bab^2x^2-70Aab^2x+105Ba^2bx+315Aa^2b-420Ba^3)\sqrt{x}}{105b^5}-\frac{a^3\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-}{}\right.}{b^5}$
derivativedivides	$\frac{\frac{2Bx^{\frac{7}{2}}b^3}{7}+\frac{2Ab^3x^{\frac{5}{2}}}{5}-\frac{4Bab^2x^{\frac{5}{2}}}{5}-\frac{4Aab^2x^{\frac{3}{2}}}{3}+2Ba^2bx^{\frac{3}{2}}+6Aa^2b\sqrt{x}-8Ba^3\sqrt{x}}{b^5}-\frac{2a^3\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-9Ba)a}{2\sqrt{}}\right.}{b^5}$
default	$\frac{\frac{2Bx^{\frac{7}{2}}b^3}{7}+\frac{2Ab^3x^{\frac{5}{2}}}{5}-\frac{4Bab^2x^{\frac{5}{2}}}{5}-\frac{4Aab^2x^{\frac{3}{2}}}{3}+2Ba^2bx^{\frac{3}{2}}+6Aa^2b\sqrt{x}-8Ba^3\sqrt{x}}{b^5}-\frac{2a^3\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-9Ba)a}{2\sqrt{}}\right.}{b^5}$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*B*b^3*x^3+21*A*b^3*x^2-42*B*a*b^2*x^2-70*A*a*b^2*x+105*B*a^2*b*x
+315*A*a^2*b-420*B*a^3)*x^(1/2)/b^5-a^3/b^5*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/
(b*x+a)+(7*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.42

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \left[-\frac{105\,(9\,Ba^4-7\,Aa^3b+(9\,Ba^3b-7\,Aa^2b^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2}{}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/210*(105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(-a/b
)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(30*B*b^4*x^4 - 94
5*B*a^4 + 735*A*a^3*b - 6*(9*B*a*b^3 - 7*A*b^4)*x^3 + 14*(9*B*a^2*b^2 - 7*
A*a*b^3)*x^2 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(x))/(b^6*x + a*b^5), 1
/105*(105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) + (30*B*b^4*x^4 - 945*B*a^4 + 735*A*a^3*b - 6*
(9*B*a*b^3 - 7*A*b^4)*x^3 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^2 - 70*(9*B*a^3
*b - 7*A*a^2*b^2)*x)*sqrt(x))/(b^6*x + a*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(136) = 272$.

Time = 105.12 (sec) , antiderivative size = 986, normalized size of antiderivative = 6.99

$$\int \frac{x^{7/2}(A + Bx)}{a^2 + 2abx + b^2x^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B
*x**(7/2)/7)/b**2, Eq(a, 0)), (-735*A*a**4*b*log(sqrt(x) - sqrt(-a/b))/(21
0*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 735*A*a**4*b*log(sqrt(x) +
sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 1470*A*a**3*
b**2*sqrt(x)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) -
735*A*a**3*b**2*x*log(sqrt(x) - sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b
**7*x*sqrt(-a/b)) + 735*A*a**3*b**2*x*log(sqrt(x) + sqrt(-a/b))/(210*a*b**
6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 980*A*a**2*b**3*x**(3/2)*sqrt(-a/b
)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 196*A*a*b**4*x**(5/2)*
sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 84*A*b**5*x**
(7/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 945*B*a
**5*log(sqrt(x) - sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/
b)) - 945*B*a**5*log(sqrt(x) + sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b*
*7*x*sqrt(-a/b)) - 1890*B*a**4*b*sqrt(x)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b)
 + 210*b**7*x*sqrt(-a/b)) + 945*B*a**4*b*x*log(sqrt(x) - sqrt(-a/b))/(210*
a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 945*B*a**4*b*x*log(sqrt(x) +
sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 1260*B*a**3*
b**2*x**(3/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) +
 252*B*a**2*b**3*x**(5/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{(Ba^4-Aa^3b)\sqrt{x}}{b^6x+ab^5}+\frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}+\frac{2\left(15\,Bb^3x^{\frac{7}{2}}-21\,(2\,Bab^2-Ab^3)x^{\frac{5}{2}}+35\,(3\,Ba^2b-2\,Aab^2)x^{\frac{3}{2}}-105\,(4\,Ba^3-3\,Aa^2b)\sqrt{x}\right)}{105\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a^4 - A*a^3*b)*sqrt(x)/(b^6*x + a*b^5) + (9*B*a^4 - 7*A*a^3*b)*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 2/105*(15*B*b^3*x^(7/2) - 21*(2*B*a
*b^2 - A*b^3)*x^(5/2) + 35*(3*B*a^2*b - 2*A*a*b^2)*x^(3/2) - 105*(4*B*a^3
- 3*A*a^2*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}-\frac{Ba^4\sqrt{x}-Aa^3b\sqrt{x}}{(bx+a)b^5}+\frac{2\left(15\,Bb^{12}x^{\frac{7}{2}}-42\,Bab^{11}x^{\frac{5}{2}}+21\,Ab^{12}x^{\frac{5}{2}}+105\,Ba^2b^{10}x^{\frac{3}{2}}-70\,Aab^{11}x^{\frac{3}{2}}-420\,Ba^3b^9\sqrt{x}+315\,Aa^2b^{10}\sqrt{x}\right)}{105\,b^{14}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(9*B*a^4 - 7*A*a^3*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - (B*a^4
*sqrt(x) - A*a^3*b*sqrt(x))/((b*x + a)*b^5) + 2/105*(15*B*b^12*x^(7/2) - 4
2*B*a*b^11*x^(5/2) + 21*A*b^12*x^(5/2) + 105*B*a^2*b^10*x^(3/2) - 70*A*a*b
^11*x^(3/2) - 420*B*a^3*b^9*sqrt(x) + 315*A*a^2*b^10*sqrt(x))/b^14
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \sqrt{x}\left(\frac{2\,a\left(\frac{2\,a\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b}+\frac{2\,B\,a^2}{b^4}\right)}{b}-\frac{a^2\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b^2}\right)+x^{5/2}\left(\frac{2\,A}{5\,b^2}-\frac{4\,B\,a}{5\,b^3}\right)-x^{3/2}\left(\frac{2\,a\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{3\,b}+\frac{2\,B\,a^2}{3\,b^4}\right)+\frac{2\,B\,x^{7/2}}{7\,b^2}-\frac{\sqrt{x}\left(B\,a^4-A\,a^3\,b\right)}{x\,b^6+a\,b^5}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,\sqrt{x}\,(7\,A\,b-9\,B\,a)}{9\,B\,a^4-7\,A\,a^3\,b}\right)\,(7\,A\,b}{b^{11/2}}$$

input
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(1/2)*((2*a*((2*a*((2*A)/b^2 - (4*B*a)/b^3))/b + (2*B*a^2)/b^4))/b - (a^
2*((2*A)/b^2 - (4*B*a)/b^3))/b^2) + x^(5/2)*((2*A)/(5*b^2) - (4*B*a)/(5*b^
3)) - x^(3/2)*((2*a*((2*A)/b^2 - (4*B*a)/b^3))/(3*b) + (2*B*a^2)/(3*b^4))
+ (2*B*x^(7/2))/(7*b^2) - (x^(1/2)*(B*a^4 - A*a^3*b))/(a*b^5 + b^6*x) + (a
^(5/2)*atan((a^(5/2)*b^(1/2)*x^(1/2)*(7*A*b - 9*B*a))/(9*B*a^4 - 7*A*a^3*b
))*(7*A*b - 9*B*a))/b^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

$$\int \frac{x^{7/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3 - 2\sqrt{x}\,a^3b + \frac{2\sqrt{x}\,a^2b^2x}{3} - \frac{2\sqrt{x}\,a\,b^3x^2}{5} + \frac{2\sqrt{x}\,b^4x^3}{7}}{b^5}$$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqr
t(x)*a**3*b + 35*sqrt(x)*a**2*b**2*x - 21*sqrt(x)*a*b**3*x**2 + 15*sqrt(x)
*b**4*x**3))/(105*b**5)
```

3.387 $\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{a(5Ab-7aB)\sqrt{x}}{b^4}+\frac{(5Ab-7aB)x^{3/2}}{3b^3}+\frac{2Bx^{5/2}}{5b^2}-\frac{(Ab-aB)x^{5/2}}{b^2(a+bx)}+\frac{a^{3/2}(5Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
-a*(5*A*b-7*B*a)*x^(1/2)/b^4+1/3*(5*A*b-7*B*a)*x^(3/2)/b^3+2/5*B*x^(5/2)/b
^2-(A*b-B*a)*x^(5/2)/b^2/(b*x+a)+a^(3/2)*(5*A*b-7*B*a)*arctan(b^(1/2)*x^(1
/2)/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{\sqrt{x}(105a^3B+2b^3x^2(5A+3Bx)-2ab^2x(25A+7Bx)+a^2(-75Ab+70bBx))}{15b^4(a+bx)}-\frac{a^{3/2}(-5Ab+7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(Sqrt[x]*(105*a^3*B + 2*b^3*x^2*(5*A + 3*B*x) - 2*a*b^2*x*(25*A + 7*B*x) +
 a^2*(-75*A*b + 70*b*B*x)))/(15*b^4*(a + b*x)) - (a^(3/2)*(-5*A*b + 7*a*B)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1184, 27, 87, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^{5/2}(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\int \frac{x^{5/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{2ab}$$

$$\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$$\downarrow \text{73}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$$\downarrow \text{218}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*x^(7/2))/(a*b*(a + b*x)) - ((5*A*b - 7*a*B)*((2*x^(5/2))/(5*b
) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result	s
risch	$-\frac{2(-3x^2Bb^2-5xb^2A+10xabB+30abA-45a^2B)\sqrt{x}}{15b^4}+\frac{a^2\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^4}$	9
derivativedivides	$-\frac{2\left(-\frac{Bb^2x^{\frac{5}{2}}}{5}-\frac{Ab^2x^{\frac{3}{2}}}{3}+\frac{2Babx^{\frac{3}{2}}}{3}+2Aab\sqrt{x}-3Ba^2\sqrt{x}\right)}{b^4}+\frac{2a^2\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	1
default	$-\frac{2\left(-\frac{Bb^2x^{\frac{5}{2}}}{5}-\frac{Ab^2x^{\frac{3}{2}}}{3}+\frac{2Babx^{\frac{3}{2}}}{3}+2Aab\sqrt{x}-3Ba^2\sqrt{x}\right)}{b^4}+\frac{2a^2\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	1

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+10*B*a*b*x+30*A*a*b-45*B*a^2)*x^(1/2)/b^4+a^
2/b^4*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b*x+a)+(5*A*b-7*B*a)/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.48

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \left[-\frac{15\,(7\,Ba^3-5\,Aa^2b+(7\,Ba^2b-5\,Aab^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(6}{30\,(b}\right.$$

$$-\frac{15\,(7\,Ba^3-5\,Aa^2b+(7\,Ba^2b-5\,Aab^2)x)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(6\,Bb^3x^3+105\,Ba^3-75\,Aa^2b-2\,(7}{15\,(b^5x+ab^4)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/30*(15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x)*sqrt(-a/b)*lo
g((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(6*B*b^3*x^3 + 105*B*a
^3 - 75*A*a^2*b - 2*(7*B*a*b^2 - 5*A*b^3)*x^2 + 10*(7*B*a^2*b - 5*A*a*b^2)
*x)*sqrt(x))/(b^5*x + a*b^4), -1/15*(15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b
- 5*A*a*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - (6*B*b^3*x^3 + 1
05*B*a^3 - 75*A*a^2*b - 2*(7*B*a*b^2 - 5*A*b^3)*x^2 + 10*(7*B*a^2*b - 5*A*
a*b^2)*x)*sqrt(x))/(b^5*x + a*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(112) = 224$.

Time = 28.79 (sec) , antiderivative size = 877, normalized size of antiderivative = 7.50

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x
**(5/2)/5)/b**2, Eq(a, 0)), (75*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/(30*a*b
**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 75*A*a**3*b*log(sqrt(x) + sqrt(-a
/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 150*A*a**2*b**2*sqrt(
x)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 75*A*a**2*b*
*2*x*log(sqrt(x) - sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b
)) - 75*A*a**2*b**2*x*log(sqrt(x) + sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30
*b**6*x*sqrt(-a/b)) - 100*A*a*b**3*x**(3/2)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/
b) + 30*b**6*x*sqrt(-a/b)) + 20*A*b**4*x**(5/2)*sqrt(-a/b)/(30*a*b**5*sqrt
(-a/b) + 30*b**6*x*sqrt(-a/b)) - 105*B*a**4*log(sqrt(x) - sqrt(-a/b))/(30*
a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 105*B*a**4*log(sqrt(x) + sqrt(
-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 210*B*a**3*b*sqrt(x
)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 105*B*a**3*b*
x*log(sqrt(x) - sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b))
+ 105*B*a**3*b*x*log(sqrt(x) + sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6
*x*sqrt(-a/b)) + 140*B*a**2*b**2*x**(3/2)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b)
 + 30*b**6*x*sqrt(-a/b)) - 28*B*a*b**3*x**(5/2)*sqrt(-a/b)/(30*a*b**5*sqrt
(-a/b) + 30*b**6*x*sqrt(-a/b)) + 12*B*b**4*x**(7/2)*sqrt(-a/b)/(30*a*b**5*
sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(Ba^3-Aa^2b)\sqrt{x}}{b^5x+ab^4} - \frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(3\,Bb^2x^{\frac{5}{2}}-5\,(2\,Bab-Ab^2)x^{\frac{3}{2}}+15\,(3\,Ba^2-2\,Aab)\sqrt{x}\right)}{15\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
(B*a^3 - A*a^2*b)*sqrt(x)/(b^5*x + a*b^4) - (7*B*a^3 - 5*A*a^2*b)*arctan(b
*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/15*(3*B*b^2*x^(5/2) - 5*(2*B*a*b -
 A*b^2)*x^(3/2) + 15*(3*B*a^2 - 2*A*a*b)*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{Ba^3\sqrt{x}-Aa^2b\sqrt{x}}{(bx+a)b^4}+\frac{2\left(3\,Bb^8x^{\frac{5}{2}}-10\,Bab^7x^{\frac{3}{2}}+5\,Ab^8x^{\frac{3}{2}}+45\,Ba^2b^6\sqrt{x}-30\,Aab^7\sqrt{x}\right)}{15\,b^{10}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(7*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + (B*a^
3*sqrt(x) - A*a^2*b*sqrt(x))/((b*x + a)*b^4) + 2/15*(3*B*b^8*x^(5/2) - 10*
B*a*b^7*x^(3/2) + 5*A*b^8*x^(3/2) + 45*B*a^2*b^6*sqrt(x) - 30*A*a*b^7*sqrt
(x))/b^10
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b^2}-\frac{4\,B\,a}{3\,b^3}\right)-\sqrt{x}\left(\frac{2\,a\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b}+\frac{2\,B\,a^2}{b^4}\right)+\frac{2\,B\,x^{5/2}}{5\,b^2}+\frac{\sqrt{x}\,(B\,a^3-A\,a^2\,b)}{x\,b^5+a\,b^4}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,\sqrt{x}\,(5\,A\,b-7\,B\,a)}{7\,B\,a^3-5\,A\,a^2\,b}\right)\,(5\,A\,b-7\,B\,a)}{b^{9/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(3/2)*((2*A)/(3*b^2) - (4*B*a)/(3*b^3)) - x^(1/2)*((2*a*((2*A)/b^2 - (4*
B*a)/b^3))/b + (2*B*a^2)/b^4) + (2*B*x^(5/2))/(5*b^2) + (x^(1/2)*(B*a^3 -
A*a^2*b))/(a*b^4 + b^5*x) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x^(1/2)*(5*A*b
- 7*B*a))/(7*B*a^3 - 5*A*a^2*b))*(5*A*b - 7*B*a))/b^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{x^{5/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+2\sqrt{x}\,a^2b-\frac{2\sqrt{x}\,a\,b^2x}{3}+\frac{2\sqrt{x}\,b^3x^2}{5}}{b^4}$$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sq
rt(x)*a**2*b - 5*sqrt(x)*a*b**2*x + 3*sqrt(x)*b**3*x**2))/(15*b**4)
```

3.388 $\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(3Ab-5aB)\sqrt{x}}{b^3}+\frac{2Bx^{3/2}}{3b^2} - \frac{(Ab-aB)x^{3/2}}{b^2(a+bx)}-\frac{\sqrt{a}(3Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
(3*A*b-5*B*a)*x^(1/2)/b^3+2/3*B*x^(3/2)/b^2-(A*b-B*a)*x^(3/2)/b^2/(b*x+a)-
a^(1/2)*(3*A*b-5*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{\sqrt{x}(-15a^2B+ab(9A-10Bx)+2b^2x(3A+Bx))}{3b^3(a+bx)} + \frac{\sqrt{a}(-3Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(Sqrt[x]*(-15*a^2*B + a*b*(9*A - 10*B*x) + 2*b^2*x*(3*A + B*x)))/(3*b^3*(a
 + b*x)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^
(7/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1184, 27, 87, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{x^{3/2}(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\int \frac{x^{3/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2ab}$$

$$\downarrow \text{73}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2ab}$$

$$\downarrow \text{218}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*x^(5/2))/(a*b*(a + b*x)) - ((3*A*b - 5*a*B)*((2*x^(3/2))/(3*b
) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3
/2)))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{2(Bbx+3Ab-6Ba)\sqrt{x}}{3b^{3}}-\frac{a\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^{3}}$	77
derivativedivides	$\frac{\frac{2Bb\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	82
default	$\frac{\frac{2Bb\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	82

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-6*B*a)*x^(1/2)/b^3-a/b^3*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b
*x+a)+(3*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.43

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \left[-\frac{3\,(5\,Ba^2-3\,Aab+(5\,Bab-3\,Ab^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(2\,Bb^2}{6\,(b^4x+ab^3)}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x)*sqrt(-a/b)*log((b*x -
 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(2*B*b^2*x^2 - 15*B*a^2 + 9*A*
a*b - 2*(5*B*a*b - 3*A*b^2)*x)*sqrt(x))/(b^4*x + a*b^3), 1/3*(3*(5*B*a^2 -
 3*A*a*b + (5*B*a*b - 3*A*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (2*B*b^2*x^2 - 15*B*a^2 + 9*A*a*b - 2*(5*B*a*b - 3*A*b^2)*x)*sqrt(x))/(b
^4*x + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(88) = 176.

Time = 6.07 (sec) , antiderivative size = 762, normalized size of antiderivative = 8.02

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2
)/3)/b**2, Eq(a, 0)), (-9*A*a**2*b*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqr
t(-a/b) + 6*b**5*x*sqrt(-a/b)) + 9*A*a**2*b*log(sqrt(x) + sqrt(-a/b))/(6*a
*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 18*A*a*b**2*sqrt(x)*sqrt(-a/b)/(
6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 9*A*a*b**2*x*log(sqrt(x) - sq
rt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 9*A*a*b**2*x*log(s
qrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 12*A*b*
*3*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*B*
a**3*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b))
 - 15*B*a**3*log(sqrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqr
t(-a/b)) - 30*B*a**2*b*sqrt(x)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*
sqrt(-a/b)) + 15*B*a**2*b*x*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqrt(-a/b)
 + 6*b**5*x*sqrt(-a/b)) - 15*B*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(6*a*b**
4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 20*B*a*b**2*x**(3/2)*sqrt(-a/b)/(6*a
*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 4*B*b**3*x**(5/2)*sqrt(-a/b)/(6*
a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = -\frac{(Ba^2-Aab)\sqrt{x}}{b^4x+ab^3} + \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(2\,Ba-Ab)\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a^2 - A*a*b)*sqrt(x)/(b^4*x + a*b^3) + (5*B*a^2 - 3*A*a*b)*arctan(b*sq
rt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/3*(B*b*x^(3/2) - 3*(2*B*a - A*b)*sqrt
(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{Ba^2\sqrt{x}-Aab\sqrt{x}}{(bx+a)b^3} + \frac{2\left(Bb^4x^{\frac{3}{2}}-6\,Bab^3\sqrt{x}+3\,Ab^4\sqrt{x}\right)}{3\,b^6}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(5*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - (B*a^2*s
qrt(x) - A*a*b*sqrt(x))/((b*x + a)*b^3) + 2/3*(B*b^4*x^(3/2) - 6*B*a*b^3*s
qrt(x) + 3*A*b^4*sqrt(x))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \sqrt{x}\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right) - \frac{\sqrt{x}\,(B\,a^2-A\,a\,b)}{x\,b^4+a\,b^3} + \frac{2\,B\,x^{3/2}}{3\,b^2} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(3\,A\,b-5\,B\,a)}{5\,B\,a^2-3\,A\,a\,b}\right)(3\,A\,b-5\,B\,a)}{b^{7/2}}$$

input
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
x^(1/2)*((2*A)/b^2 - (4*B*a)/b^3) - (x^(1/2)*(B*a^2 - A*a*b))/(a*b^3 + b^4
*x) + (2*B*x^(3/2))/(3*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x^(1/2)*(3*A*
b - 5*B*a))/(5*B*a^2 - 3*A*a*b))*(3*A*b - 5*B*a))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{x^{3/2}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-2\sqrt{x}\,ab+\frac{2\sqrt{x}\,b^2x}{3}}{b^3}$$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(x)*a*
b + sqrt(x)*b**2*x))/(3*b**3)
```

3.389 $\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{2B\sqrt{x}}{b^2}-\frac{(Ab-aB)\sqrt{x}}{b^2(a+bx)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

output
```
2*B*x^(1/2)/b^2-(A*b-B*a)*x^(1/2)/b^2/(b*x+a)+(A*b-3*B*a)*arctan(b^(1/2)*x
^(1/2)/a^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{\sqrt{x}(-Ab+3aB+2bBx)}{b^2(a+bx)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(Sqrt[x]*(-(A*b) + 3*a*B + 2*b*B*x))/(b^2*(a + b*x)) + ((A*b - 3*a*B)*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1184, 27, 87, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{\sqrt{x}(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2ab}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2ab}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*x^(3/2))/(a*b*(a + b*x)) - ((A*b - 3*a*B)*((2*Sqrt[x])/b - (2
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	62
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	63
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	63

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^2+1/b^2*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b*x+a)+(A*b-3*B*a)/(a
*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$
$$=\left[\frac{(3\,Ba^2-Aab+(3\,Bab-Ab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(2\,Bab^2x+3\,Ba^2b-Aab^2)\sqrt{x}}{2\,(ab^4x+a^2b^3)},\ \frac{(3\,Ba\ldots}{\ldots}\right.$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[1/2*((3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*
sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(2*B*a*b^2*x + 3*B*a^2*b - A*a*b^2)*sqr
t(x))/(a*b^4*x + a^2*b^3), ((3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x)*sqrt(a
*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (2*B*a*b^2*x + 3*B*a^2*b - A*a*b^2)*sq
rt(x))/(a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(66) = 132$.

Time = 2.13 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.68

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+2B\sqrt{x}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{a^2} \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{b^2} \\ \frac{Aab\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{Aab\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{2Ab^2\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{Ab^2x\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{Ab^2x\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{3}{2} \end{cases}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2*A*x
**(3/2)/3 + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))
/b**2, Eq(a, 0)), (A*a*b*log(sqrt(x) - sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) +
2*b**4*x*sqrt(-a/b)) - A*a*b*log(sqrt(x) + sqrt(-a/b))/(2*a*b**3*sqrt(-a/b
) + 2*b**4*x*sqrt(-a/b)) - 2*A*b**2*sqrt(x)*sqrt(-a/b)/(2*a*b**3*sqrt(-a/b
) + 2*b**4*x*sqrt(-a/b)) + A*b**2*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**3*sq
rt(-a/b) + 2*b**4*x*sqrt(-a/b)) - A*b**2*x*log(sqrt(x) + sqrt(-a/b))/(2*a*
b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*B*a**2*log(sqrt(x) - sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*B*a**2*log(sqrt(x) + sqr
t(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*B*a*b*sqrt(x)*sqr
t(-a/b)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*B*a*b*x*log(sqrt(x
) - sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*B*a*b*x*lo
g(sqrt(x) + sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*B*
b**2*x**(3/2)*sqrt(-a/b)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{b^3x+ab^2} + \frac{2\,B\sqrt{x}}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
(B*a - A*b)*sqrt(x)/(b^3*x + a*b^2) + 2*B*sqrt(x)/b^2 - (3*B*a - A*b)*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{2\,B\sqrt{x}}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{(bx+a)b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^2 - (3*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2
) + (B*a*sqrt(x) - A*b*sqrt(x))/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{2\,B\,\sqrt{x}}{b^2}-\frac{\sqrt{x}\,(A\,b-B\,a)}{x\,b^3+a\,b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-3\,B\,a)}{\sqrt{a}\,b^{5/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(2*B*x^(1/2))/b^2 - (x^(1/2)*(A*b - B*a))/(a*b^2 + b^3*x) + (atan((b^(1/2)
*x^(1/2))/a^(1/2))*(A*b - 3*B*a))/(a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{x}(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)+2\sqrt{x}\,b}{b^2}$$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*( - sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + sqrt(x)*b))/b
**2
```

3.390 $\int \frac{A+Bx}{\sqrt{x}(a^2+2abx+b^2x^2)} dx$

Optimal result

Integrand size = 29, antiderivative size = 63

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{(Ab-aB)\sqrt{x}}{ab(a+bx)}+\frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

output
```
(A*b-B*a)*x^(1/2)/a/b/(b*x+a)+(A*b+B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^
(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{(-Ab+aB)\sqrt{x}}{ab(a+bx)}+\frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
-(((-(A*b) + a*B)*Sqrt[x])/(a*b*(a + b*x))) + ((A*b + a*B)*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1184, 27, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^2\int \frac{A+Bx}{b^2\sqrt{x}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{x}(a+bx)}dx}{2ab}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(aB+Ab)\int \frac{1}{a+bx}d\sqrt{x}}{ab}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(a*b*(a + b*x)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{ab(bx+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{ab\sqrt{ab}}$	57
default	$\frac{(Ab-Ba)\sqrt{x}}{ab(bx+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{ab\sqrt{ab}}$	57

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
(A*b-B*a)*x^(1/2)/a/b/(b*x+a)+(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(
a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = \left[-\frac{(Ba^2+Aab+(Bab+Ab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(Ba^2b-Aab^2)\sqrt{x}}{2\,(a^2b^3x+a^3b^2)}, -\frac{(Ba^2+Aab+(Bab+Ab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(Ba^2b-Aab^2)\sqrt{x}}{a^2b^3x+a^3b^2}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqr
t(-a*b)*sqrt(x))/(b*x + a)) + 2*(B*a^2*b - A*a*b^2)*sqrt(x))/(a^2*b^3*x +
a^3*b^2), -((B*a^2 + A*a*b + (B*a*b + A*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)
/(b*sqrt(x))) + (B*a^2*b - A*a*b^2)*sqrt(x))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(53) = 106$.

Time = 2.64 (sec) , antiderivative size = 615, normalized size of antiderivative = 9.76

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{a^2} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{b^2} \\ \frac{Aab\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}} - \frac{Aab\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}} + \frac{2Ab^2\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}} + \frac{Ab^2x\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}} - \frac{Ab^2x\log\left(\sqrt{x}+\sqrt{\right.}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x} \end{cases}$$

input
```
integrate((B*x+A)/x**(1/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*sqrt(x) + 2*B*x**(3/2)/3)/a**2, Eq(b, 0)), ((-2*A/(3*x**(3/2)) - 2*B/s
qrt(x))/b**2, Eq(a, 0)), (A*a*b*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqr
t(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - A*a*b*log(sqrt(x) + sqrt(-a/b))/(2*a**2
*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + 2*A*b**2*sqrt(x)*sqrt(-a/b)/(2
*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + A*b**2*x*log(sqrt(x) - sq
rt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - A*b**2*x*log(
sqrt(x) + sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + B
*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(
-a/b)) - B*a**2*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b*
*3*x*sqrt(-a/b)) - 2*B*a*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b**2*sqrt(-a/b) + 2*
a*b**3*x*sqrt(-a/b)) + B*a*b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqrt
(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - B*a*b*x*log(sqrt(x) + sqrt(-a/b))/(2*a**
2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{ab^2x+a^2b} + \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*sqrt(x)/(a*b^2*x + a^2*b) + (B*a + A*b)*arctan(b*sqrt(x)/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}ab} - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{(bx+a)ab}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) - (B*a*sqrt(x) - A
*b*sqrt(x))/((b*x + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b+B\,a)}{a^{3/2}\,b^{3/2}} + \frac{\sqrt{x}\,(A\,b-B\,a)}{a\,b\,(a+b\,x)}$$

input
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
(atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b + B*a))/(a^(3/2)*b^(3/2)) + (x^(1/2)
*(A*b - B*a))/(a*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.391 $\int \frac{A+Bx}{x^{3/2}(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{2A}{a^2\sqrt{x}}-\frac{(Ab-aB)\sqrt{x}}{a^2(a+bx)}-\frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

output
```
-2*A/a^2/x^(1/2)-(A*b-B*a)*x^(1/2)/a^2/(b*x+a)-(3*A*b-B*a)*arctan(b^(1/2)*
x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{-2aA-3Abx+aBx}{a^2\sqrt{x}(a+bx)}+\frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(-2*a*A - 3*A*b*x + a*B*x)/(a^2*Sqrt[x]*(a + b*x)) + ((-3*A*b + a*B)*ArcTa
n[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1184, 27, 87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a^2+2abx+b^2x^2)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(3Ab-aB)\int \frac{1}{x^{3/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(3Ab-aB)\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(3Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(A*b - a*B)/(a*b*Sqrt[x]*(a + b*x)) + ((3*A*b - a*B)*(-2/(a*Sqrt[x]) - (2*
Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2A}{a^2\sqrt{x}}$	64
default	$-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2A}{a^2\sqrt{x}}$	64
risch	$-\frac{2A}{a^2\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^2}$	64

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((1/2*A*b-1/2*B*a)*x^(1/2)/(b*x+a)+1/2*(3*A*b-B*a)/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))-2/a^2*A/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx=\left[\frac{((Bab-3Ab^2)x^2+(Ba^2-3Aab)x)\sqrt{-ab}\log\left(\frac{bx-a+2\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2(2A}{2(a^3b^2x^2+a^4bx)},\ -\frac{((Bab-3Ab^2)x^2+(Ba^2-3Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(2Aa^2b-(Ba^2b-3Aab^2)x)\sqrt{x}}{a^3b^2x^2+a^4bx}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[1/2*(((B*a*b - 3*A*b^2)*x^2 + (B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x -
a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2
)*x)*sqrt(x))/(a^3*b^2*x^2 + a^4*b*x), -(((B*a*b - 3*A*b^2)*x^2 + (B*a^2 -
 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (2*A*a^2*b - (B*a^2
*b - 3*A*a*b^2)*x)*sqrt(x))/(a^3*b^2*x^2 + a^4*b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(68) = 136$.

Time = 7.03 (sec) , antiderivative size = 794, normalized size of antiderivative = 10.59

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) - 2*
B/(3*x**(3/2)))/b**2, Eq(a, 0)), (-3*A*a*b*sqrt(x)*log(sqrt(x) - sqrt(-a/b
))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + 3*A*a
*b*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2
*b**2*x**(3/2)*sqrt(-a/b)) - 4*A*a*b*sqrt(-a/b)/(2*a**3*b*sqrt(x)*sqrt(-a/
b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) - 3*A*b**2*x**(3/2)*log(sqrt(x) - sq
rt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b))
+ 3*A*b**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b)
 + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) - 6*A*b**2*x*sqrt(-a/b)/(2*a**3*b*sqrt
(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + B*a**2*sqrt(x)*log(sqr
t(x) - sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqr
t(-a/b)) - B*a**2*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt
(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + B*a*b*x**(3/2)*log(sqrt(x) - s
qrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b))
 - B*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) +
 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + 2*B*a*b*x*sqrt(-a/b)/(2*a**3*b*sqrt(x)
*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{2\,Aa-(Ba-3\,Ab)x}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}+\frac{(Ba-3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-(2*A*a - (B*a - 3*A*b)*x)/(a^2*b*x^(3/2) + a^3*sqrt(x)) + (B*a - 3*A*b)*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{(Ba-3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{Bax-3\,Abx-2\,Aa}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + (B*a*x - 3*A*b
*x - 2*A*a)/((b*x^(3/2) + a*sqrt(x))*a^2)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{\frac{2\,A}{a}+\frac{x\,(3\,A\,b-B\,a)}{a^2}}{a\,\sqrt{x}+b\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(3\,A\,b-B\,a)}{a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
- ((2*A)/a + (x*(3*A*b - B*a))/a^2)/(a*x^(1/2) + b*x^(3/2)) - (atan((b^(1/
2)*x^(1/2))/a^(1/2))*(3*A*b - B*a))/(a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx}{x^{3/2}\,(a^2+2abx+b^2x^2)}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)-2a}{\sqrt{x}\,a^2}$$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + a))/(
sqrt(x)*a**2)
```

3.392 $\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)}\,dx = -\frac{2A}{3a^2x^{3/2}}+\frac{2(2Ab-aB)}{a^3\sqrt{x}}+\frac{b(Ab-aB)\sqrt{x}}{a^3(a+bx)}+\frac{\sqrt{b}(5Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-2/3*A/a^2/x^(3/2)+2*(2*A*b-B*a)/a^3/x^(1/2)+b*(A*b-B*a)*x^(1/2)/a^3/(b*x+
a)+b^(1/2)*(5*A*b-3*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)}\,dx = \frac{15Ab^2x^2+abx(10A-9Bx)-2a^2(A+3Bx)}{3a^3x^{3/2}(a+bx)}+\frac{\sqrt{b}(5Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(15*A*b^2*x^2 + a*b*x*(10*A - 9*B*x) - 2*a^2*(A + 3*B*x))/(3*a^3*x^(3/2)*(
a + b*x)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^
(7/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1184, 27, 87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^{5/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(5Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(A*b - a*B)/(a*b*x^(3/2)*(a + b*x)) + ((5*A*b - 3*a*B)*(-2/(3*a*x^(3/2)) -
 (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2
)))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{2(-6Abx+3Bax+Aa)}{3a^3x^{\frac{3}{2}}}+\frac{b\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	77
derivativedivides	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	81
default	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	81

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*A*b*x+3*B*a*x+A*a)/a^3/x^(3/2)+b/a^3*(2*(1/2*A*b-1/2*B*a)*x^(1/2)
/(b*x+a)+(5*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \left[-\frac{3\left((3\,Bab-5\,Ab^2)x^3+(3\,Ba^2-5\,Aab)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)}{6\left(a^3bx^3+a^4\right.}\right.$$

$$-\frac{3\left((3\,Bab-5\,Ab^2)x^3+(3\,Ba^2-5\,Aab)x^2\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+\left(2\,Aa^2+3\,(3\,Bab-5\,Ab^2)x^2+2\,(3\,B\right.}{3\left(a^3bx^3+a^4x^2\right)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/6*(3*((3*B*a*b - 5*A*b^2)*x^3 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(-b/a)*lo
g((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(2*A*a^2 + 3*(3*B*a*b
- 5*A*b^2)*x^2 + 2*(3*B*a^2 - 5*A*a*b)*x)*sqrt(x))/(a^3*b*x^3 + a^4*x^2),
-1/3*(3*((3*B*a*b - 5*A*b^2)*x^3 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(b/a)*arct
an(sqrt(x)*sqrt(b/a)) + (2*A*a^2 + 3*(3*B*a*b - 5*A*b^2)*x^2 + 2*(3*B*a^2
- 5*A*a*b)*x)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(90) = 180$.

Time = 27.55 (sec) , antiderivative size = 882, normalized size of antiderivative = 9.28

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \text{Too large to display}$$

input

```
integrate((B*x+A)/x**(5/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output

```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2))
 - 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), (-4*A*a**2*sqrt(-a/b)/(6*a**4*x**(3/
2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*A*a*b*x**(3/2)*log(sqrt
(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/
b)) - 15*A*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a
/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 20*A*a*b*x*sqrt(-a/b)/(6*a**4*x**(3/
2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*A*b**2*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a
/b)) - 15*A*b**2*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(
-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 30*A*b**2*x**2*sqrt(-a/b)/(6*a**4*
x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 9*B*a**2*x**(3/2)*lo
g(sqrt(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sq
rt(-a/b)) + 9*B*a**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*s
qrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 12*B*a**2*x*sqrt(-a/b)/(6*a**4
*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 9*B*a*b*x**(5/2)*lo
g(sqrt(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sq
rt(-a/b)) + 9*B*a*b*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sq
rt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 18*B*a*b*x**2*sqrt(-a/b)/(6*a**
4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{2\,Aa^2+3\,(3\,Bab-5\,Ab^2)x^2+2\,(3\,Ba^2-5\,Aab)x}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)} - \frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/3*(2*A*a^2 + 3*(3*B*a*b - 5*A*b^2)*x^2 + 2*(3*B*a^2 - 5*A*a*b)*x)/(a^3*
b*x^(5/2) + a^4*x^(3/2)) - (3*B*a*b - 5*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{Bab\sqrt{x}-Ab^2\sqrt{x}}{(bx+a)a^3} - \frac{2\,(3\,Bax-6\,Abx+Aa)}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(3*B*a*b - 5*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - (B*a*b*
sqrt(x) - A*b^2*sqrt(x))/((b*x + a)*a^3) - 2/3*(3*B*a*x - 6*A*b*x + A*a)/(
a^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)}\,dx = \frac{\frac{2\,x\,(5\,A\,b-3\,B\,a)}{3\,a^2}-\frac{2\,A}{3\,a}+\frac{b\,x^2\,(5\,A\,b-3\,B\,a)}{a^3}}{a\,x^{3/2}+b\,x^{5/2}} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(5\,A\,b-3\,B\,a)}{a^{7/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
((2*x*(5*A*b - 3*B*a))/(3*a^2) - (2*A)/(3*a) + (b*x^2*(5*A*b - 3*B*a))/a^3
)/(a*x^(3/2) + b*x^(5/2)) + (b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(5*A*
b - 3*B*a))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-\frac{2a^2}{3}+2abx}{\sqrt{x}\,a^3x}$$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*(3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - a*
*2 + 3*a*b*x))/(3*sqrt(x)*a**3*x)
```

3.393 $\int \frac{A+Bx}{x^{7/2}(a^2+2abx+b^2x^2)} dx$

Optimal result

Integrand size = 29, antiderivative size = 121

$$\int \frac{A+Bx}{x^{7/2}(a^2+2abx+b^2x^2)} dx = -\frac{2A}{5a^2x^{5/2}} + \frac{2(2Ab-aB)}{3a^3x^{3/2}} - \frac{2b(3Ab-2aB)}{a^4\sqrt{x}} - \frac{b^2(Ab-aB)\sqrt{x}}{a^4(a+bx)} - \frac{b^{3/2}(7Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-2/5*A/a^2/x^(5/2)+2/3*(2*A*b-B*a)/a^3/x^(3/2)-2*b*(3*A*b-2*B*a)/a^4/x^(1/
2)-b^2*(A*b-B*a)*x^(1/2)/a^4/(b*x+a)-b^(3/2)*(7*A*b-5*B*a)*arctan(b^(1/2)*
x^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{7/2}(a^2+2abx+b^2x^2)} dx = \frac{-105Ab^3x^3-2a^3(3A+5Bx)+5ab^2x^2(-14A+15Bx)+2a^2bx(7A+25}{15a^4x^{5/2}(a+bx)} + \frac{b^{3/2}(-7Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(-105*A*b^3*x^3 - 2*a^3*(3*A + 5*B*x) + 5*a*b^2*x^2*(-14*A + 15*B*x) + 2*a
^2*b*x*(7*A + 25*B*x))/(15*a^4*x^(5/2)*(a + b*x)) + (b^(3/2)*(-7*A*b + 5*a
*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1184, 27, 87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^{7/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(7Ab-5aB)\int \frac{1}{x^{7/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(A*b - a*B)/(a*b*x^(5/2)*(a + b*x)) + ((7*A*b - 5*a*B)*(-2/(5*a*x^(5/2)) -
 (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{3a^3x^{\frac{3}{2}}}-\frac{2b(3Ab-2Ba)}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$	101
default	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{3a^3x^{\frac{3}{2}}}-\frac{2b(3Ab-2Ba)}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$	101
risch	$-\frac{2(45x^2b^2A-30Ba\,x^2b-10abAx+5a^2Bx+3a^2A)}{15a^4x^{\frac{5}{2}}}-\frac{b^2\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^4}$	103

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^2/x^(5/2)-2/3*(-2*A*b+B*a)/a^3/x^(3/2)-2*b*(3*A*b-2*B*a)/a^4/x^(1
/2)-2*b^2/a^4*((1/2*A*b-1/2*B*a)*x^(1/2)/(b*x+a)+1/2*(7*A*b-5*B*a)/(a*b)^(
1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \left[-\frac{15\left(\left(5\,Bab^2-7\,Ab^3\right)x^4+\left(5\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}}{bx+a}\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/30*(15*((5*B*a*b^2 - 7*A*b^3)*x^4 + (5*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(
-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(6*A*a^3 - 15*
(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)*x^2 + 2*(5*B*a^3 -
7*A*a^2*b)*x)*sqrt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(15*((5*B*a*b^2 - 7*A*b
^3)*x^4 + (5*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a))
 - (6*A*a^3 - 15*(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)*x^
2 + 2*(5*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(117) = 234$.

Time = 113.34 (sec) , antiderivative size = 1017, normalized size of antiderivative = 8.40

$$\int \frac{A + Bx}{x^{7/2}\left(a^2 + 2abx + b^2x^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(7/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/a**2, Eq(b, 0)), ((-2*A/(9*x**(
9/2)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-12*A*a**3*sqrt(-a/b)/(30*a**5
*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 28*A*a**2*b*x*sqrt
(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) - 105
*A*a*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b)
+ 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 105*A*a*b**2*x**(5/2)*log(sqrt(x) + sqr
t(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) - 1
40*A*a*b**2*x**2*sqrt(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7
/2)*sqrt(-a/b)) - 105*A*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x
**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 105*A*b**3*x**(7/2)*
log(sqrt(x) + sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2
)*sqrt(-a/b)) - 210*A*b**3*x**3*sqrt(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) +
30*a**4*b*x**(7/2)*sqrt(-a/b)) - 20*B*a**3*x*sqrt(-a/b)/(30*a**5*x**(5/2)*
sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 75*B*a**2*b*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(
-a/b)) - 75*B*a**2*b*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(30*a**5*x**(5/2)*
sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 100*B*a**2*b*x**2*sqrt(-a/b)
/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 75*B*a*b*
*2*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30...
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{7/2}\,(a^2+2abx+b^2x^2)}\,dx = -\frac{6\,Aa^3-15\,(5\,Bab^2-7\,Ab^3)x^3-10\,(5\,Ba^2b-7\,Aab^2)x^2+2\,(5\,Ba^3-7\,Aa^2b)x}{15\left(a^4bx^{\frac{7}{2}}+a^5x^{\frac{5}{2}}\right)} + \frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/15*(6*A*a^3 - 15*(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)
*x^2 + 2*(5*B*a^3 - 7*A*a^2*b)*x)/(a^4*b*x^(7/2) + a^5*x^(5/2)) + (5*B*a*b
^2 - 7*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \frac{\left(5\,Bab^2-7\,Ab^3\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{Bab^2\sqrt{x}-Ab^3\sqrt{x}}{(bx+a)a^4} + \frac{2\left(30\,Babx^2-45\,Ab^2x^2-5\,Ba^2x+10\,Aabx-3\,Aa^2\right)}{15\,a^4x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
(5*B*a*b^2 - 7*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + (B*a*b
^2*sqrt(x) - A*b^3*sqrt(x))/((b*x + a)*a^4) + 2/15*(30*B*a*b*x^2 - 45*A*b^
2*x^2 - 5*B*a^2*x + 10*A*a*b*x - 3*A*a^2)/(a^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{\frac{2\,A}{5\,a}-\frac{2\,x\,(7\,A\,b-5\,B\,a)}{15\,a^2}+\frac{b^2\,x^3\,(7\,A\,b-5\,B\,a)}{a^4}+\frac{2\,b\,x^2\,(7\,A\,b-5\,B\,a)}{3\,a^3}}{a\,x^{5/2}+b\,x^{7/2}} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(7\,A\,b-5\,B\,a)}{a^{9/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
- ((2*A)/(5*a) - (2*x*(7*A*b - 5*B*a))/(15*a^2) + (b^2*x^3*(7*A*b - 5*B*a)
)/a^4 + (2*b*x^2*(7*A*b - 5*B*a))/(3*a^3))/(a*x^(5/2) + b*x^(7/2)) - (b^(3
/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(7*A*b - 5*B*a))/a^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx}{x^{7/2}\,(a^2+2abx+b^2x^2)}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-\frac{2a^3}{5}+\frac{2a^2bx}{3}-2a\,b^2x^2}{\sqrt{x}\,a^4x^2}$$

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2
*x**2 - 3*a**3 + 5*a**2*b*x - 15*a*b**2*x**2))/(15*sqrt(x)*a**4*x**2)
```

3.394 $\int \frac{A+Bx}{x^{9/2}(a^2+2abx+b^2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 143

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx=-\frac{2A}{7a^2x^{7/2}}+\frac{2(2Ab-aB)}{5a^3x^{5/2}}-\frac{2b(3Ab-2aB)}{3a^4x^{3/2}}+\frac{2b^2(4Ab-3aB)}{a^5\sqrt{x}}+\frac{b^3(Ab-aB)\sqrt{x}}{a^5(a+bx)}+\frac{b^{5/2}(9Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

output
```
-2/7*A/a^2/x^(7/2)+2/5*(2*A*b-B*a)/a^3/x^(5/2)-2/3*b*(3*A*b-2*B*a)/a^4/x^(
3/2)+2*b^2*(4*A*b-3*B*a)/a^5/x^(1/2)+b^3*(A*b-B*a)*x^(1/2)/a^5/(b*x+a)+b^(
5/2)*(9*A*b-7*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx=\frac{945Ab^4x^4+105ab^3x^3(6A-7Bx)-6a^4(5A+7Bx)-14a^2b^2x^2(9A+35I}{105a^5x^{7/2}(a+bx)}+\frac{b^{5/2}(9Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(945*A*b^4*x^4 + 105*a*b^3*x^3*(6*A - 7*B*x) - 6*a^4*(5*A + 7*B*x) - 14*a^
2*b^2*x^2*(9*A + 35*B*x) + 2*a^3*b*x*(27*A + 49*B*x))/(105*a^5*x^(7/2)*(a
+ b*x)) + (b^(5/2)*(9*A*b - 7*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(1
1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 61, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{A+Bx}{b^2x^{9/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(9Ab-7aB)\int \frac{1}{x^{9/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(9Ab-7aB)\left(-\frac{b\int \frac{1}{x^{7/2}(a+bx)}dx}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 61

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 61

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 73

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 218

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a^2 + 2*a*b*x + b^2*x^2)),x]
```

output
```
(A*b - a*B)/(a*b*x^(7/2)*(a + b*x)) + ((9*A*b - 7*a*B)*(-2/(7*a*x^(7/2)) -
 (b*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqr
t[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{2A}{7a^2x^{\frac{7}{2}}}-\frac{2(-2Ab+Ba)}{5a^3x^{\frac{5}{2}}}-\frac{2b(3Ab-2Ba)}{3a^4x^{\frac{3}{2}}}+\frac{2b^2(4Ab-3Ba)}{a^5\sqrt{x}}+\frac{2b^3\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^5}$
default	$-\frac{2A}{7a^2x^{\frac{7}{2}}}-\frac{2(-2Ab+Ba)}{5a^3x^{\frac{5}{2}}}-\frac{2b(3Ab-2Ba)}{3a^4x^{\frac{3}{2}}}+\frac{2b^2(4Ab-3Ba)}{a^5\sqrt{x}}+\frac{2b^3\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^5}$
risch	$-\frac{2(-420A\,b^3x^3+315Ba\,b^2x^3+105Aa\,b^2x^2-70B\,a^2b\,x^2-42A\,a^2bx+21B\,a^3x+15a^3A)}{105a^5x^{\frac{7}{2}}}+\frac{b^3\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9A\cdots}{\cdots}\right)}{a^5}$

input
```
int((B*x+A)/x^(9/2)/(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*A/a^2/x^(7/2)-2/5*(-2*A*b+B*a)/a^3/x^(5/2)-2/3*b*(3*A*b-2*B*a)/a^4/x^
(3/2)+2*b^2*(4*A*b-3*B*a)/a^5/x^(1/2)+2/a^5*b^3*((1/2*A*b-1/2*B*a)*x^(1/2)
/(b*x+a)+1/2*(9*A*b-7*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \left[-\frac{105\left((7\,Bab^3-9\,Ab^4)x^5+(7\,Ba^2b^2-9\,Aab^3)x^4\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+\cdots}{\cdots}\right.$$

$$-\frac{105\left((7\,Bab^3-9\,Ab^4)x^5+(7\,Ba^2b^2-9\,Aab^3)x^4\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+\left(30\,Aa^4+105\,(7\,Bab^3-9\,Ab^4)\cdots\right)}{105\left(a^5bx^5+a^6x^4\right)}$$

input
```
integrate((B*x+A)/x^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
[-1/210*(105*((7*B*a*b^3 - 9*A*b^4)*x^5 + (7*B*a^2*b^2 - 9*A*a*b^3)*x^4)*s
qrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(30*A*a^4
+ 105*(7*B*a*b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 14*(7
*B*a^3*b - 9*A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b*x
^5 + a^6*x^4), -1/105*(105*((7*B*a*b^3 - 9*A*b^4)*x^5 + (7*B*a^2*b^2 - 9*A
*a*b^3)*x^4)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (30*A*a^4 + 105*(7*B*a*
b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 14*(7*B*a^3*b - 9*
A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b*x^5 + a^6*x^4)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(9/2)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{30\,Aa^4+105\,(7\,Bab^3-9\,Ab^4)x^4+70\,(7\,Ba^2b^2-9\,Aab^3)x^3-14\,(7\,Ba^3b-9\,Aa^2b^2)x^2+6\,(7\,Ba^4-9\,A}{105\left(a^5bx^{\frac{9}{2}}+a^6x^{\frac{7}{2}}\right)}$$

$$-\frac{(7\,Bab^3-9\,Ab^4)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}$$

input
```
integrate((B*x+A)/x^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
-1/105*(30*A*a^4 + 105*(7*B*a*b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a
*b^3)*x^3 - 14*(7*B*a^3*b - 9*A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)/
(a^5*b*x^(9/2) + a^6*x^(7/2)) - (7*B*a*b^3 - 9*A*b^4)*arctan(b*sqrt(x)/sqr
t(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{9/2}\left(a^2+2abx+b^2x^2\right)}\,dx = -\frac{(7\,Bab^3-9\,Ab^4)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}-\frac{Bab^3\sqrt{x}-Ab^4\sqrt{x}}{(bx+a)a^5}$$
$$-\frac{2\,(315\,Bab^2x^3-420\,Ab^3x^3-70\,Ba^2bx^2+105\,Aab^2x^2+21\,Ba^3x-42\,Aa^2bx+15\,Aa^3)}{105\,a^5x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
-(7*B*a*b^3 - 9*A*b^4)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - (B*a*
b^3*sqrt(x) - A*b^4*sqrt(x))/((b*x + a)*a^5) - 2/105*(315*B*a*b^2*x^3 - 42
0*A*b^3*x^3 - 70*B*a^2*b*x^2 + 105*A*a*b^2*x^2 + 21*B*a^3*x - 42*A*a^2*b*x
 + 15*A*a^3)/(a^5*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 10.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{9/2}\,(a^2+2abx+b^2x^2)}\,dx = \frac{\frac{2\,x\,(9\,A\,b-7\,B\,a)}{35\,a^2}-\frac{2\,A}{7\,a}+\frac{2\,b^2\,x^3\,(9\,A\,b-7\,B\,a)}{3\,a^4}+\frac{b^3\,x^4\,(9\,A\,b-7\,B\,a)}{a^5}-\frac{2\,b\,x^2\,(9\,A\,b-7\,B\,a)}{15\,a^3}}{a\,x^{7/2}+b\,x^{9/2}} + \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(9\,A\,b-7\,B\,a)}{a^{11/2}}$$

input
```
int((A + B*x)/(x^(9/2)*(a^2 + b^2*x^2 + 2*a*b*x)),x)
```

output
```
((2*x*(9*A*b - 7*B*a))/(35*a^2) - (2*A)/(7*a) + (2*b^2*x^3*(9*A*b - 7*B*a)
)/(3*a^4) + (b^3*x^4*(9*A*b - 7*B*a))/a^5 - (2*b*x^2*(9*A*b - 7*B*a))/(15*
a^3))/(a*x^(7/2) + b*x^(9/2)) + (b^(5/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(
9*A*b - 7*B*a))/a^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx}{x^{9/2}\,(a^2+2abx+b^2x^2)}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^3x^3-\frac{2a^4}{7}+\frac{2a^3bx}{5}-\frac{2a^2b^2x^2}{3}+2a\,b^3x^3}{\sqrt{x}\,a^5x^3}$$

input
```
int((B*x+A)/x^(9/2)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(2*(105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x
**3 - 15*a**4 + 21*a**3*b*x - 35*a**2*b**2*x**2 + 105*a*b**3*x**3))/(105*s
qrt(x)*a**5*x**3)
```

3.395 $\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 156

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{35(Ab-3aB)\sqrt{x}}{8b^5}+\frac{2Bx^{3/2}}{3b^4}-\frac{(Ab-aB)x^{7/2}}{3b^2(a+bx)^3}-\frac{(7Ab-13aB)x^{5/2}}{12b^3(a+bx)^2}-\frac{(35Ab-89aB)x^{3/2}}{24b^4(a+bx)}-\frac{35\sqrt{a}(Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{11/2}}$$

output
```
35/8*(A*b-3*B*a)*x^(1/2)/b^5+2/3*B*x^(3/2)/b^4-1/3*(A*b-B*a)*x^(7/2)/b^2/(
b*x+a)^3-1/12*(7*A*b-13*B*a)*x^(5/2)/b^3/(b*x+a)^2-1/24*(35*A*b-89*B*a)*x^
(3/2)/b^4/(b*x+a)-35/8*a^(1/2)*(A*b-3*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))
/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{\sqrt{x}(-315a^4B+7a^2b^2x(40A-99Bx)+3ab^3x^2(77A-48Bx)+105a^3b(A-}{24b^5(a+bx)^3}+\frac{35\sqrt{a}(-Ab+3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(Sqrt[x]*(-315*a^4*B + 7*a^2*b^2*x*(40*A - 99*B*x) + 3*a*b^3*x^2*(77*A - 4
8*B*x) + 105*a^3*b*(A - 8*B*x) + 16*b^4*x^3*(3*A + B*x)))/(24*b^5*(a + b*x
)^3) + (35*Sqrt[a]*(-(A*b) + 3*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(8*
b^(11/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 51, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{x^{7/2}(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\int \frac{x^{7/2}}{(a+bx)^3}dx}{2ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\int \frac{x^{5/2}}{(a+bx)^2}dx}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\left(\frac{5\int \frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

$\Big\downarrow$ 60

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

$\Big\downarrow$ 60

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

$\Big\downarrow$ 73

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

$\downarrow$ 218

$$\frac{x^{9/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-3aB)\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2ab}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x)^3) - ((A*b - 3*a*B)*(-1/2*x^(7/2)/(
b*(a + b*x)^2) + (7*(-(x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/2))/(3*b) - (a
*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/
b))/(2*b)))/(4*b)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.76

method	result
risch	$\frac{2(Bbx+3Ab-12Ba)\sqrt{x}}{3b^5} - \frac{a\left(\frac{2\left(-\frac{29}{16}Ab^3+\frac{55}{16}Bab^2\right)x^{\frac{5}{2}}-\frac{ab(17Ab-35Ba)x^{\frac{3}{2}}}{3}+2\left(-\frac{19}{16}Aa^2b+\frac{41}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(Ab-3Ba)\,a}{8\sqrt{a}}\right.}{b^5}$
derivativedivides	$\frac{\frac{2Bbx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-8Ba\sqrt{x}}{b^5} - \frac{2a\left(\frac{\left(-\frac{29}{16}Ab^3+\frac{55}{16}Bab^2\right)x^{\frac{5}{2}}-\frac{ab(17Ab-35Ba)x^{\frac{3}{2}}}{6}+\left(-\frac{19}{16}Aa^2b+\frac{41}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(Ab-3Ba)}{16}\right.}{b^5}$
default	$\frac{\frac{2Bbx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-8Ba\sqrt{x}}{b^5} - \frac{2a\left(\frac{\left(-\frac{29}{16}Ab^3+\frac{55}{16}Bab^2\right)x^{\frac{5}{2}}-\frac{ab(17Ab-35Ba)x^{\frac{3}{2}}}{6}+\left(-\frac{19}{16}Aa^2b+\frac{41}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(Ab-3Ba)}{16}\right.}{b^5}$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-12*B*a)*x^(1/2)/b^5-a/b^5*(2*((-29/16*A*b^3+55/16*B*a*b^2
)*x^(5/2)-1/6*a*b*(17*A*b-35*B*a)*x^(3/2)+(-19/16*A*a^2*b+41/16*B*a^3)*x^(
1/2))/(b*x+a)^3+35/8*(A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.99

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \left[-\frac{105\,(3\,Ba^4-Aa^3b+(3\,Bab^3-Ab^4)x^3+3\,(3\,Ba^2b^2-Aab^3)x^2+3\,(3\,Ba^3}{}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/48*(105*(3*B*a^4 - A*a^3*b + (3*B*a*b^3 - A*b^4)*x^3 + 3*(3*B*a^2*b^2
- A*a*b^3)*x^2 + 3*(3*B*a^3*b - A*a^2*b^2)*x)*sqrt(-a/b)*log((b*x - 2*b*sq
rt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(16*B*b^4*x^4 - 315*B*a^4 + 105*A*a^3
*b - 48*(3*B*a*b^3 - A*b^4)*x^3 - 231*(3*B*a^2*b^2 - A*a*b^3)*x^2 - 280*(3
*B*a^3*b - A*a^2*b^2)*x)*sqrt(x))/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a
^3*b^5), 1/24*(105*(3*B*a^4 - A*a^3*b + (3*B*a*b^3 - A*b^4)*x^3 + 3*(3*B*a
^2*b^2 - A*a*b^3)*x^2 + 3*(3*B*a^3*b - A*a^2*b^2)*x)*sqrt(a/b)*arctan(b*sq
rt(x)*sqrt(a/b)/a) + (16*B*b^4*x^4 - 315*B*a^4 + 105*A*a^3*b - 48*(3*B*a*b
^3 - A*b^4)*x^3 - 231*(3*B*a^2*b^2 - A*a*b^3)*x^2 - 280*(3*B*a^3*b - A*a^2
*b^2)*x)*sqrt(x))/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,(55\,Ba^2b^2-29\,Aab^3)x^{\frac{5}{2}}+8\,(35\,Ba^3b-17\,Aa^2b^2)x^{\frac{3}{2}}+3\,(41\,Ba^4-19\,Aa^3b)\sqrt{x}}{24\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)} + \frac{35\,(3\,Ba^2-Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(4\,Ba-Ab)\sqrt{x}\right)}{3\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(3*(55*B*a^2*b^2 - 29*A*a*b^3)*x^(5/2) + 8*(35*B*a^3*b - 17*A*a^2*b^
2)*x^(3/2) + 3*(41*B*a^4 - 19*A*a^3*b)*sqrt(x))/(b^8*x^3 + 3*a*b^7*x^2 + 3
*a^2*b^6*x + a^3*b^5) + 35/8*(3*B*a^2 - A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*b^5) + 2/3*(B*b*x^(3/2) - 3*(4*B*a - A*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{35\,(3\,Ba^2-Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} - \frac{165\,Ba^2b^2x^{\frac{5}{2}}-87\,Aab^3x^{\frac{5}{2}}+280\,Ba^3bx^{\frac{3}{2}}-136\,Aa^2b^2x^{\frac{3}{2}}+123\,Ba^4\sqrt{x}-57\,Aa^3b\sqrt{x}}{24\,(bx+a)^3b^5} + \frac{2\left(Bb^8x^{\frac{3}{2}}-12\,Bab^7\sqrt{x}+3\,Ab^8\sqrt{x}\right)}{3\,b^{12}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
35/8*(3*B*a^2 - A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/24*
(165*B*a^2*b^2*x^(5/2) - 87*A*a*b^3*x^(5/2) + 280*B*a^3*b*x^(3/2) - 136*A*
a^2*b^2*x^(3/2) + 123*B*a^4*sqrt(x) - 57*A*a^3*b*sqrt(x))/((b*x + a)^3*b^5
) + 2/3*(B*b^8*x^(3/2) - 12*B*a*b^7*sqrt(x) + 3*A*b^8*sqrt(x))/b^12
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \sqrt{x}\left(\frac{2\,A}{b^4}-\frac{8\,B\,a}{b^5}\right) - \frac{x^{5/2}\left(\frac{55\,B\,a^2\,b^2}{8}-\frac{29\,A\,a\,b^3}{8}\right)-x^{3/2}\left(\frac{17\,A\,a^2\,b^2}{3}-\frac{35\,B\,a^3\,b}{3}\right)+\sqrt{x}\left(\frac{41\,B\,a^4}{8}-\frac{19\,A\,a^3\,b}{8}\right)}{a^3\,b^5+3\,a^2\,b^6\,x+3\,a\,b^7\,x^2+b^8\,x^3} + \frac{2\,B\,x^{3/2}}{3\,b^4} + \frac{35\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(A\,b-3\,B\,a)}{3\,B\,a^2-A\,a\,b}\right)\,(A\,b-3\,B\,a)}{8\,b^{11/2}}$$

input
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
x^(1/2)*((2*A)/b^4 - (8*B*a)/b^5) - (x^(5/2)*((55*B*a^2*b^2)/8 - (29*A*a*b
^3)/8) - x^(3/2)*((17*A*a^2*b^2)/3 - (35*B*a^3*b)/3) + x^(1/2)*((41*B*a^4)
/8 - (19*A*a^3*b)/8))/(a^3*b^5 + b^8*x^3 + 3*a^2*b^6*x + 3*a*b^7*x^2) + (2
*B*x^(3/2))/(3*b^4) + (35*a^(1/2)*atan((a^(1/2)*b^(1/2)*x^(1/2)*(A*b - 3*B
*a))/(3*B*a^2 - A*a*b))*(A*b - 3*B*a))/(8*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3+210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+105\sqrt{b}\,\sqrt{a}\,atan\Big(}{12b^5\,(b^2x^2+2ab}$$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 210*sqrt(b
)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 105*sqrt(b)*sqrt(
a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 - 105*sqrt(x)*a**3*b -
175*sqrt(x)*a**2*b**2*x - 56*sqrt(x)*a*b**3*x**2 + 8*sqrt(x)*b**4*x**3)/(1
2*b**5*(a**2 + 2*a*b*x + b**2*x**2))
```

3.396 $\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 134

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{2B\sqrt{x}}{b^4}-\frac{(Ab-aB)x^{5/2}}{3b^2(a+bx)^3}-\frac{(5Ab-11aB)x^{3/2}}{12b^3(a+bx)^2}-\frac{(5Ab-19aB)\sqrt{x}}{8b^4(a+bx)}+\frac{5(Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{a}b^{9/2}}$$

output
```
2*B*x^(1/2)/b^4-1/3*(A*b-B*a)*x^(5/2)/b^2/(b*x+a)^3-1/12*(5*A*b-11*B*a)*x^
(3/2)/b^3/(b*x+a)^2-1/8*(5*A*b-19*B*a)*x^(1/2)/b^4/(b*x+a)+5/8*(A*b-7*B*a)
*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{\sqrt{x}(105a^3B-5a^2b(3A-56Bx)+3b^3x^2(-11A+16Bx)+ab^2x(-40A+23}{24b^4(a+bx)^3}+\frac{5(Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{a}b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(Sqrt[x]*(105*a^3*B - 5*a^2*b*(3*A - 56*B*x) + 3*b^3*x^2*(-11*A + 16*B*x)
+ a*b^2*x*(-40*A + 231*B*x)))/(24*b^4*(a + b*x)^3) + (5*(A*b - 7*a*B)*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(8*Sqrt[a]*b^(9/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1184, 27, 87, 51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{x^{5/2}(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\int \frac{x^{5/2}}{(a+bx)^3}dx}{6ab}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\left(\frac{5\int \frac{x^{3/2}}{(a+bx)^2}dx}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6ab}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\left(\frac{5\left(\frac{3\int\frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6ab}$$

$\Big\downarrow$ 60

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6ab}$$

$\Big\downarrow$ 73

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6ab}$$

$\Big\downarrow$ 218

$$\frac{x^{7/2}(Ab-aB)}{3ab(a+bx)^3}-\frac{(Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x)^3) - ((A*b - 7*a*B)*(-1/2*x^(5/2)/(
b*(a + b*x)^2) + (5*(-(x^(3/2)/(b*(a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqr
t[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{b^4}+\frac{\frac{2\left(\left(\frac{29}{16}Ba\,b^2-\frac{11}{16}A\,b^3\right)x^{\frac{5}{2}}-\frac{ab(5Ab-17Ba)x^{\frac{3}{2}}}{6}+\left(\frac{19}{16}B\,a^3-\frac{5}{16}A\,a^2b\right)\sqrt{x}\right)}{(bx+a)^3}+\frac{5(Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^4}$	104
default	$\frac{2B\sqrt{x}}{b^4}+\frac{\frac{2\left(\left(\frac{29}{16}Ba\,b^2-\frac{11}{16}A\,b^3\right)x^{\frac{5}{2}}-\frac{ab(5Ab-17Ba)x^{\frac{3}{2}}}{6}+\left(\frac{19}{16}B\,a^3-\frac{5}{16}A\,a^2b\right)\sqrt{x}\right)}{(bx+a)^3}+\frac{5(Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^4}$	104
risch	$\frac{2B\sqrt{x}}{b^4}+\frac{\frac{2\left(\frac{29}{16}Ba\,b^2-\frac{11}{16}A\,b^3\right)x^{\frac{5}{2}}-\frac{ab(5Ab-17Ba)x^{\frac{3}{2}}}{3}+2\left(\frac{19}{16}B\,a^3-\frac{5}{16}A\,a^2b\right)\sqrt{x}}{(bx+a)^3}+\frac{5(Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^4}$	104

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^4+2/b^4*(((29/16*B*a*b^2-11/16*A*b^3)*x^(5/2)-1/6*a*b*(5*A*b
-17*B*a)*x^(3/2)+(19/16*B*a^3-5/16*A*a^2*b)*x^(1/2))/(b*x+a)^3+5/16*(A*b-7
*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.26

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \Bigg[\frac{15\,(7\,Ba^4 - Aa^3b + (7\,Bab^3 - Ab^4)x^3 + 3\,(7\,Ba^2b^2 - Aab^3)x^2 + 3\,(7\,Ba^3b -}{}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(15*(7*B*a^4 - A*a^3*b + (7*B*a*b^3 - A*b^4)*x^3 + 3*(7*B*a^2*b^2 -
A*a*b^3)*x^2 + 3*(7*B*a^3*b - A*a^2*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sq
rt(-a*b)*sqrt(x))/(b*x + a)) + 2*(48*B*a*b^4*x^3 + 105*B*a^4*b - 15*A*a^3*
b^2 + 33*(7*B*a^2*b^3 - A*a*b^4)*x^2 + 40*(7*B*a^3*b^2 - A*a^2*b^3)*x)*sqr
t(x))/(a*b^8*x^3 + 3*a^2*b^7*x^2 + 3*a^3*b^6*x + a^4*b^5), 1/24*(15*(7*B*a
^4 - A*a^3*b + (7*B*a*b^3 - A*b^4)*x^3 + 3*(7*B*a^2*b^2 - A*a*b^3)*x^2 + 3
*(7*B*a^3*b - A*a^2*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (48*
B*a*b^4*x^3 + 105*B*a^4*b - 15*A*a^3*b^2 + 33*(7*B*a^2*b^3 - A*a*b^4)*x^2
+ 40*(7*B*a^3*b^2 - A*a^2*b^3)*x)*sqrt(x))/(a*b^8*x^3 + 3*a^2*b^7*x^2 + 3*
a^3*b^6*x + a^4*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2271 vs. 2(133) = 266.

Time = 87.83 (sec) , antiderivative size = 2271, normalized size of antiderivative = 16.95

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2*A*x
**(7/2)/7 + 2*B*x**(9/2)/9)/a**4, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))
/b**4, Eq(a, 0)), (15*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/(48*a**3*b**5*sqr
t(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(-a/b) + 48*b**
8*x**3*sqrt(-a/b)) - 15*A*a**3*b*log(sqrt(x) + sqrt(-a/b))/(48*a**3*b**5*s
qrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(-a/b) + 48*b
**8*x**3*sqrt(-a/b)) - 30*A*a**2*b**2*sqrt(x)*sqrt(-a/b)/(48*a**3*b**5*sqr
t(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(-a/b) + 48*b**
8*x**3*sqrt(-a/b)) + 45*A*a**2*b**2*x*log(sqrt(x) - sqrt(-a/b))/(48*a**3*b
**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(-a/b) +
 48*b**8*x**3*sqrt(-a/b)) - 45*A*a**2*b**2*x*log(sqrt(x) + sqrt(-a/b))/(48
*a**3*b**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(
-a/b) + 48*b**8*x**3*sqrt(-a/b)) - 80*A*a*b**3*x**(3/2)*sqrt(-a/b)/(48*a**
3*b**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sqrt(-a/b
) + 48*b**8*x**3*sqrt(-a/b)) + 45*A*a*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/
(48*a**3*b**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**2*sq
rt(-a/b) + 48*b**8*x**3*sqrt(-a/b)) - 45*A*a*b**3*x**2*log(sqrt(x) + sqrt(
-a/b))/(48*a**3*b**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*
x**2*sqrt(-a/b) + 48*b**8*x**3*sqrt(-a/b)) - 66*A*b**4*x**(5/2)*sqrt(-a/b)
/(48*a**3*b**5*sqrt(-a/b) + 144*a**2*b**6*x*sqrt(-a/b) + 144*a*b**7*x**...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{3\,(29\,Bab^2-11\,Ab^3)x^{\frac{5}{2}}+8\,(17\,Ba^2b-5\,Aab^2)x^{\frac{3}{2}}+3\,(19\,Ba^3-5\,Aa^2b)\sqrt{x}}{24\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)} + \frac{2\,B\sqrt{x}}{b^4} - \frac{5\,(7\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/24*(3*(29*B*a*b^2 - 11*A*b^3)*x^(5/2) + 8*(17*B*a^2*b - 5*A*a*b^2)*x^(3/
2) + 3*(19*B*a^3 - 5*A*a^2*b)*sqrt(x))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*
x + a^3*b^4) + 2*B*sqrt(x)/b^4 - 5/8*(7*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a
*b))/(sqrt(a*b)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{2\,B\sqrt{x}}{b^4} - \frac{5\,(7\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} + \frac{87\,Bab^2x^{\frac{5}{2}} - 33\,Ab^3x^{\frac{5}{2}} + 136\,Ba^2bx^{\frac{3}{2}} - 40\,Aab^2x^{\frac{3}{2}} + 57\,Ba^3\sqrt{x} - 15\,Aa^2b\sqrt{x}}{24\,(bx+a)^3b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^4 - 5/8*(7*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^4) + 1/24*(87*B*a*b^2*x^(5/2) - 33*A*b^3*x^(5/2) + 136*B*a^2*b*x^(3/2)
- 40*A*a*b^2*x^(3/2) + 57*B*a^3*sqrt(x) - 15*A*a^2*b*sqrt(x))/((b*x + a)^3
*b^4)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{2\,B\,\sqrt{x}}{b^4} - \frac{x^{3/2}\left(\frac{5\,A\,a\,b^2}{3}-\frac{17\,B\,a^2\,b}{3}\right) - \sqrt{x}\left(\frac{19\,B\,a^3}{8}-\frac{5\,A\,a^2\,b}{8}\right) + x^{5/2}\left(\frac{11\,A\,b^3}{8}-\frac{29\,B\,a\,b^2}{8}\right)}{a^3\,b^4+3\,a^2\,b^5\,x+3\,a\,b^6\,x^2+b^7\,x^3} + \frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-7\,B\,a)}{8\,\sqrt{a}\,b^{9/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(2*B*x^(1/2))/b^4 - (x^(3/2)*((5*A*a*b^2)/3 - (17*B*a^2*b)/3) - x^(1/2)*((
19*B*a^3)/8 - (5*A*a^2*b)/8) + x^(5/2)*((11*A*b^3)/8 - (29*B*a*b^2)/8))/(a
^3*b^4 + b^7*x^3 + 3*a^2*b^5*x + 3*a*b^6*x^2) + (5*atan((b^(1/2)*x^(1/2))/
a^(1/2))*(A*b - 7*B*a))/(8*a^(1/2)*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{4b^4\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 - 30*sqrt(
b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 + 15*sqrt(x)*a**2*b + 25*sqr
t(x)*a*b**2*x + 8*sqrt(x)*b**3*x**2)/(4*b**4*(a**2 + 2*a*b*x + b**2*x**2))
```

3.397 $\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 124

$$\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{(Ab-aB)x^{3/2}}{3b^2(a+bx)^3}-\frac{(Ab-3aB)\sqrt{x}}{4b^3(a+bx)^2} +\frac{(Ab-11aB)\sqrt{x}}{8ab^3(a+bx)}+\frac{(Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{3/2}b^{7/2}}$$

output
```
-1/3*(A*b-B*a)*x^(3/2)/b^2/(b*x+a)^3-1/4*(A*b-3*B*a)*x^(1/2)/b^3/(b*x+a)^2
+1/8*(A*b-11*B*a)*x^(1/2)/a/b^3/(b*x+a)+1/8*(A*b+5*B*a)*arctan(b^(1/2)*x^(
1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{\sqrt{x}(15a^3B-3Ab^3x^2+ab^2x(8A+33Bx)+a^2b(3A+40Bx))}{24ab^3(a+bx)^3} +\frac{(Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{3/2}b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
-1/24*(Sqrt[x]*(15*a^3*B - 3*A*b^3*x^2 + a*b^2*x*(8*A + 33*B*x) + a^2*b*(3
*A + 40*B*x)))/(a*b^3*(a + b*x)^3) + ((A*b + 5*a*B)*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/(8*a^(3/2)*b^(7/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1184, 27, 87, 51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{x^{3/2}(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(5aB+Ab)\int \frac{x^{3/2}}{(a+bx)^3}dx}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{(5aB+Ab)\left(\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{(5aB+Ab)\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{(5aB+Ab)\left(\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{(5aB+Ab)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6ab}+\frac{x^{5/2}(Ab-aB)}{3ab(a+bx)^3}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x)^3) + ((A*b + 5*a*B)*(-1/2*x^(3/2)/(
b*(a + b*x)^2) + (3*(-(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]]/(Sqrt[a]*b^(3/2))))/(4*b)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{(Ab-11Ba)x^{\frac{5}{2}}}{8ab}-\frac{(Ab+5Ba)x^{\frac{3}{2}}}{3b^2}-\frac{(Ab+5Ba)a\sqrt{x}}{8b^3}}{(bx+a)^3}+\frac{(Ab+5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a\,b^3\sqrt{ab}}$	96
default	$\frac{\frac{(Ab-11Ba)x^{\frac{5}{2}}}{8ab}-\frac{(Ab+5Ba)x^{\frac{3}{2}}}{3b^2}-\frac{(Ab+5Ba)a\sqrt{x}}{8b^3}}{(bx+a)^3}+\frac{(Ab+5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a\,b^3\sqrt{ab}}$	96

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/16*(A*b-11*B*a)/a/b*x^(5/2)-1/6/b^2*(A*b+5*B*a)*x^(3/2)-1/16*(A*b+5*B
*a)*a/b^3*x^(1/2))/(b*x+a)^3+1/8*(A*b+5*B*a)/a/b^3/(a*b)^(1/2)*arctan(b*x^
(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.31

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \left[-\frac{3\,(5\,Ba^4+Aa^3b+(5\,Bab^3+Ab^4)x^3+3\,(5\,Ba^2b^2+Aab^3)x^2+3\,(5\,Ba^3b\right.$$

$$-\frac{3\,(5\,Ba^4+Aa^3b+(5\,Bab^3+Ab^4)x^3+3\,(5\,Ba^2b^2+Aab^3)x^2+3\,(5\,Ba^3b+Aa^2b^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{24\,(a^2b^7x^3+3\,a^3b^6x^2+3\,a^4b^5x+}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/48*(3*(5*B*a^4 + A*a^3*b + (5*B*a*b^3 + A*b^4)*x^3 + 3*(5*B*a^2*b^2 +
A*a*b^3)*x^2 + 3*(5*B*a^3*b + A*a^2*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sq
rt(-a*b)*sqrt(x))/(b*x + a)) + 2*(15*B*a^4*b + 3*A*a^3*b^2 + 3*(11*B*a^2*b
^3 - A*a*b^4)*x^2 + 8*(5*B*a^3*b^2 + A*a^2*b^3)*x)*sqrt(x))/(a^2*b^7*x^3 +
 3*a^3*b^6*x^2 + 3*a^4*b^5*x + a^5*b^4), -1/24*(3*(5*B*a^4 + A*a^3*b + (5*
B*a*b^3 + A*b^4)*x^3 + 3*(5*B*a^2*b^2 + A*a*b^3)*x^2 + 3*(5*B*a^3*b + A*a^
2*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (15*B*a^4*b + 3*A*a^3*
b^2 + 3*(11*B*a^2*b^3 - A*a*b^4)*x^2 + 8*(5*B*a^3*b^2 + A*a^2*b^3)*x)*sqrt
(x))/(a^2*b^7*x^3 + 3*a^3*b^6*x^2 + 3*a^4*b^5*x + a^5*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2261 vs. 2(117) = 234.

Time = 44.97 (sec) , antiderivative size = 2261, normalized size of antiderivative = 18.23

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(7/2)/7)/a**4, Eq(b, 0)), ((-2*A/(3*x**(3/2)) - 2*
B/sqrt(x))/b**4, Eq(a, 0)), (3*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/(48*a**4
*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/b) + 144*a**2*b**6*x**2*sqrt(-a
/b) + 48*a*b**7*x**3*sqrt(-a/b)) - 3*A*a**3*b*log(sqrt(x) + sqrt(-a/b))/(4
8*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/b) + 144*a**2*b**6*x**2*s
qrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b)) - 6*A*a**2*b**2*sqrt(x)*sqrt(-a/b)/
(48*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/b) + 144*a**2*b**6*x**2
*sqrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b)) + 9*A*a**2*b**2*x*log(sqrt(x) - s
qrt(-a/b))/(48*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/b) + 144*a**
2*b**6*x**2*sqrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b)) - 9*A*a**2*b**2*x*log(
sqrt(x) + sqrt(-a/b))/(48*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/b
) + 144*a**2*b**6*x**2*sqrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b)) - 16*A*a*b*
*3*x**(3/2)*sqrt(-a/b)/(48*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5*x*sqrt(-a/
b) + 144*a**2*b**6*x**2*sqrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b)) + 9*A*a*b*
*3*x**2*log(sqrt(x) - sqrt(-a/b))/(48*a**4*b**4*sqrt(-a/b) + 144*a**3*b**5
*x*sqrt(-a/b) + 144*a**2*b**6*x**2*sqrt(-a/b) + 48*a*b**7*x**3*sqrt(-a/b))
 - 9*A*a*b**3*x**2*log(sqrt(x) + sqrt(-a/b))/(48*a**4*b**4*sqrt(-a/b) + 14
4*a**3*b**5*x*sqrt(-a/b) + 144*a**2*b**6*x**2*sqrt(-a/b) + 48*a*b**7*x**3*
sqrt(-a/b)) + 6*A*b**4*x**(5/2)*sqrt(-a/b)/(48*a**4*b**4*sqrt(-a/b) + 1...
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{3\,(11\,Bab^2-Ab^3)x^{\frac{5}{2}}+8\,(5\,Ba^2b+Aab^2)x^{\frac{3}{2}}+3\,(5\,Ba^3+Aa^2b)\sqrt{x}}{24\,(ab^6x^3+3\,a^2b^5x^2+3\,a^3b^4x+a^4b^3)} + \frac{(5\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(3*(11*B*a*b^2 - A*b^3)*x^(5/2) + 8*(5*B*a^2*b + A*a*b^2)*x^(3/2) +
3*(5*B*a^3 + A*a^2*b)*sqrt(x))/(a*b^6*x^3 + 3*a^2*b^5*x^2 + 3*a^3*b^4*x +
a^4*b^3) + 1/8*(5*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{(5\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^3} - \frac{33\,Bab^2x^{\frac{5}{2}}-3\,Ab^3x^{\frac{5}{2}}+40\,Ba^2bx^{\frac{3}{2}}+8\,Aab^2x^{\frac{3}{2}}+15\,Ba^3\sqrt{x}+3\,Aa^2b\sqrt{x}}{24\,(bx+a)^3ab^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/8*(5*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^3) - 1/24*(33
*B*a*b^2*x^(5/2) - 3*A*b^3*x^(5/2) + 40*B*a^2*b*x^(3/2) + 8*A*a*b^2*x^(3/2
) + 15*B*a^3*sqrt(x) + 3*A*a^2*b*sqrt(x))/((b*x + a)^3*a*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b+5\,B\,a)}{8\,a^{3/2}\,b^{7/2}} - \frac{\frac{x^{3/2}\,(A\,b+5\,B\,a)}{3\,b^2}-\frac{x^{5/2}\,(A\,b-11\,B\,a)}{8\,a\,b}+\frac{a\,\sqrt{x}\,(A\,b+5\,B\,a)}{8\,b^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b + 5*B*a))/(8*a^(3/2)*b^(7/2)) - ((x^
(3/2)*(A*b + 5*B*a))/(3*b^2) - (x^(5/2)*(A*b - 11*B*a))/(8*a*b) + (a*x^(1/
2)*(A*b + 5*B*a))/(8*b^3))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b}{4a\,b^3\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*atan((
sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 3*sqrt(x)*a**2*b - 5*sqrt(x)*a*b
**2*x)/(4*a*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.398 $\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{(Ab-aB)\sqrt{x}}{3b^2(a+bx)^3}+\frac{(Ab-7aB)\sqrt{x}}{12ab^2(a+bx)^2}+\frac{(Ab+aB)\sqrt{x}}{8a^2b^2(a+bx)}+\frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

output
```
-1/3*(A*b-B*a)*x^(1/2)/b^2/(b*x+a)^3+1/12*(A*b-7*B*a)*x^(1/2)/a/b^2/(b*x+a
)^2+1/8*(A*b+B*a)*x^(1/2)/a^2/b^2/(b*x+a)+1/8*(A*b+B*a)*arctan(b^(1/2)*x^(
1/2)/a^(1/2))/a^(5/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{\sqrt{x}(-3a^3B+3Ab^3x^2+ab^2x(8A+3Bx)-a^2b(3A+8Bx))}{24a^2b^2(a+bx)^3}+\frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(Sqrt[x]*(-3*a^3*B + 3*A*b^3*x^2 + a*b^2*x*(8*A + 3*B*x) - a^2*b*(3*A + 8*
B*x)))/(24*a^2*b^2*(a + b*x)^3) + ((A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sq
rt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = $\{1184, 27, 87, 51, 52, 73, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4\int \frac{\sqrt{x}(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+Ab)\int \frac{\sqrt{x}}{(a+bx)^3}dx}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{(aB+Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{(aB+Ab)\left(\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{(aB+Ab)\left(\frac{\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{3ab(a+bx)^3}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x)^3) + ((A*b + a*B)*(-1/2*Sqrt[x]/(b*
(a + b*x)^2) + (Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/
(a^(3/2)*Sqrt[b]))/(4*b)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\frac{(Ab+Ba)x^{\frac{5}{2}}}{8a^2}+\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab}-\frac{(Ab+Ba)\sqrt{x}}{8b^2}}{(bx+a)^3}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$	92
default	$\frac{\frac{(Ab+Ba)x^{\frac{5}{2}}}{8a^2}+\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab}-\frac{(Ab+Ba)\sqrt{x}}{8b^2}}{(bx+a)^3}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$	92

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/16*(A*b+B*a)/a^2*x^(5/2)+1/6*(A*b-B*a)/a/b*x^(3/2)-1/16*(A*b+B*a)/b^2
*x^(1/2))/(b*x+a)^3+1/8*(A*b+B*a)/a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*
b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$$
$$= \Bigg[-\frac{3\,(Ba^4+Aa^3b+(Bab^3+Ab^4)x^3+3\,(Ba^2b^2+Aab^3)x^2+3\,(Ba^3b+Aa^2b^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-a}}{bx+a}\right.}{48\,(a^3b^6x^3+3\,a^4b^5x^2+3\,a^5b^4x}$$
$$-\frac{3\,(Ba^4+Aa^3b+(Bab^3+Ab^4)x^3+3\,(Ba^2b^2+Aab^3)x^2+3\,(Ba^3b+Aa^2b^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(3}{24\,(a^3b^6x^3+3\,a^4b^5x^2+3\,a^5b^4x+a^6b}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*a^4 + A*a^3*b + (B*a*b^3 + A*b^4)*x^3 + 3*(B*a^2*b^2 + A*a*b^
3)*x^2 + 3*(B*a^3*b + A*a^2*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)
*sqrt(x))/(b*x + a)) + 2*(3*B*a^4*b + 3*A*a^3*b^2 - 3*(B*a^2*b^3 + A*a*b^4
)*x^2 + 8*(B*a^3*b^2 - A*a^2*b^3)*x)*sqrt(x))/(a^3*b^6*x^3 + 3*a^4*b^5*x^2
 + 3*a^5*b^4*x + a^6*b^3), -1/24*(3*(B*a^4 + A*a^3*b + (B*a*b^3 + A*b^4)*x
^3 + 3*(B*a^2*b^2 + A*a*b^3)*x^2 + 3*(B*a^3*b + A*a^2*b^2)*x)*sqrt(a*b)*ar
ctan(sqrt(a*b)/(b*sqrt(x))) + (3*B*a^4*b + 3*A*a^3*b^2 - 3*(B*a^2*b^3 + A*
a*b^4)*x^2 + 8*(B*a^3*b^2 - A*a^2*b^3)*x)*sqrt(x))/(a^3*b^6*x^3 + 3*a^4*b^
5*x^2 + 3*a^5*b^4*x + a^6*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(117) = 234$.

Time = 22.53 (sec) , antiderivative size = 2302, normalized size of antiderivative = 18.42

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*x**(3/2)/3 + 2*B*x**(5/2)/5)/a**4, Eq(b, 0)), ((-2*A/(5*x**(5/2))
 - 2*B/(3*x**(3/2)))/b**4, Eq(a, 0)), (3*A*a**3*b*log(sqrt(x) - sqrt(-a/b)
)/(48*a**5*b**3*sqrt(-a/b) + 144*a**4*b**4*x*sqrt(-a/b) + 144*a**3*b**5*x*
*2*sqrt(-a/b) + 48*a**2*b**6*x**3*sqrt(-a/b)) - 3*A*a**3*b*log(sqrt(x) + s
qrt(-a/b))/(48*a**5*b**3*sqrt(-a/b) + 144*a**4*b**4*x*sqrt(-a/b) + 144*a**
3*b**5*x**2*sqrt(-a/b) + 48*a**2*b**6*x**3*sqrt(-a/b)) - 6*A*a**2*b**2*sqr
t(x)*sqrt(-a/b)/(48*a**5*b**3*sqrt(-a/b) + 144*a**4*b**4*x*sqrt(-a/b) + 14
4*a**3*b**5*x**2*sqrt(-a/b) + 48*a**2*b**6*x**3*sqrt(-a/b)) + 9*A*a**2*b**
2*x*log(sqrt(x) - sqrt(-a/b))/(48*a**5*b**3*sqrt(-a/b) + 144*a**4*b**4*x*s
qrt(-a/b) + 144*a**3*b**5*x**2*sqrt(-a/b) + 48*a**2*b**6*x**3*sqrt(-a/b))
- 9*A*a**2*b**2*x*log(sqrt(x) + sqrt(-a/b))/(48*a**5*b**3*sqrt(-a/b) + 144
*a**4*b**4*x*sqrt(-a/b) + 144*a**3*b**5*x**2*sqrt(-a/b) + 48*a**2*b**6*x**
3*sqrt(-a/b)) + 16*A*a*b**3*x**(3/2)*sqrt(-a/b)/(48*a**5*b**3*sqrt(-a/b) +
 144*a**4*b**4*x*sqrt(-a/b) + 144*a**3*b**5*x**2*sqrt(-a/b) + 48*a**2*b**6
*x**3*sqrt(-a/b)) + 9*A*a*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/(48*a**5*b**
3*sqrt(-a/b) + 144*a**4*b**4*x*sqrt(-a/b) + 144*a**3*b**5*x**2*sqrt(-a/b)
+ 48*a**2*b**6*x**3*sqrt(-a/b)) - 9*A*a*b**3*x**2*log(sqrt(x) + sqrt(-a/b)
)/(48*a**5*b**3*sqrt(-a/b) + 144*a**4*b**4*x*sqrt(-a/b) + 144*a**3*b**5*x*
*2*sqrt(-a/b) + 48*a**2*b**6*x**3*sqrt(-a/b)) + 6*A*b**4*x**(5/2)*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{3\left(Bab^2+Ab^3\right)x^{\frac{5}{2}}-8\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}}-3\left(Ba^3+Aa^2b\right)\sqrt{x}}{24\left(a^2b^5x^3+3\,a^3b^4x^2+3\,a^4b^3x+a^5b^2\right)} + \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/24*(3*(B*a*b^2 + A*b^3)*x^(5/2) - 8*(B*a^2*b - A*a*b^2)*x^(3/2) - 3*(B*a
^3 + A*a^2*b)*sqrt(x))/(a^2*b^5*x^3 + 3*a^3*b^4*x^2 + 3*a^4*b^3*x + a^5*b^
2) + 1/8*(B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2b^2} + \frac{3\,Bab^2x^{\frac{5}{2}}+3\,Ab^3x^{\frac{5}{2}}-8\,Ba^2bx^{\frac{3}{2}}+8\,Aab^2x^{\frac{3}{2}}-3\,Ba^3\sqrt{x}-3\,Aa^2b\sqrt{x}}{24\,(bx+a)^3a^2b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/8*(B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) + 1/24*(3*
B*a*b^2*x^(5/2) + 3*A*b^3*x^(5/2) - 8*B*a^2*b*x^(3/2) + 8*A*a*b^2*x^(3/2)
- 3*B*a^3*sqrt(x) - 3*A*a^2*b*sqrt(x))/((b*x + a)^3*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 11.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\frac{x^{5/2}\,(A\,b+B\,a)}{8\,a^2}-\frac{\sqrt{x}\,(A\,b+B\,a)}{8\,b^2}+\frac{x^{3/2}\,(A\,b-B\,a)}{3\,a\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b+B\,a)}{8\,a^{5/2}\,b^{5/2}}$$

input

```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output

```
((x^(5/2)*(A*b + B*a))/(8*a^2) - (x^(1/2)*(A*b + B*a))/(8*b^2) + (x^(3/2)*
(A*b - B*a))/(3*a*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) + (atan((b
^(1/2)*x^(1/2))/a^(1/2))*(A*b + B*a))/(8*a^(5/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-\sqrt{x}\,a^2b+\sqrt{x}\,a\,b^2x}{4a^2b^2\left(b^2x^2+2abx+a^2\right)}$$

input

```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output

```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 2*sqrt(b)*sqrt
(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + sqrt(b)*sqrt(a)*atan((sqrt
(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - sqrt(x)*a**2*b + sqrt(x)*a*b**2*x)/(
4*a**2*b**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.399 $\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{(Ab-aB)\sqrt{x}}{3ab(a+bx)^3}+\frac{(5Ab+aB)\sqrt{x}}{12a^2b(a+bx)^2}+\frac{(5Ab+aB)\sqrt{x}}{8a^3b(a+bx)}+\frac{(5Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{7/2}b^{3/2}}$$

output
```
1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x+a)^3+1/12*(5*A*b+B*a)*x^(1/2)/a^2/b/(b*x+a)
^2+1/8*(5*A*b+B*a)*x^(1/2)/a^3/b/(b*x+a)+1/8*(5*A*b+B*a)*arctan(b^(1/2)*x^
(1/2)/a^(1/2))/a^(7/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{\sqrt{x}(-3a^3B+15Ab^3x^2+ab^2x(40A+3Bx)+a^2b(33A+8Bx))}{24a^3b(a+bx)^3}+\frac{(5Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{7/2}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(Sqrt[x]*(-3*a^3*B + 15*A*b^3*x^2 + a*b^2*x*(40*A + 3*B*x) + a^2*b*(33*A +
 8*B*x)))/(24*a^3*b*(a + b*x)^3) + ((5*A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(8*a^(7/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1184, 27, 87, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{A+Bx}{b^4\sqrt{x}(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^4}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(aB+5Ab)\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(aB+5Ab)\left(\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(aB+5Ab)\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{(aB+5Ab)\left(\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab(a+bx)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+5Ab)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6ab}+\frac{\sqrt{x}(Ab-aB)}{3ab(a+bx)^3}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x)^3) + ((5*A*b + a*B)*(Sqrt[x]/(2*a*(
a + b*x)^2) + (3*(Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]
]/(a^(3/2)*Sqrt[b])))/(4*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{(5Ab+Ba)b\,x^{\frac{5}{2}}}{8a^{3}}+\frac{(5Ab+Ba)x^{\frac{3}{2}}}{3a^{2}}+\frac{(11Ab-Ba)\sqrt{x}}{8ab}}{(bx+a)^{3}}+\frac{(5Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a^{3}b\sqrt{ab}}$	97
default	$\frac{\frac{(5Ab+Ba)b\,x^{\frac{5}{2}}}{8a^{3}}+\frac{(5Ab+Ba)x^{\frac{3}{2}}}{3a^{2}}+\frac{(11Ab-Ba)\sqrt{x}}{8ab}}{(bx+a)^{3}}+\frac{(5Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8a^{3}b\sqrt{ab}}$	97

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/16*(5*A*b+B*a)/a^3*b*x^(5/2)+1/6/a^2*(5*A*b+B*a)*x^(3/2)+1/16*(11*A*b
-B*a)/a/b*x^(1/2))/(b*x+a)^3+1/8*(5*A*b+B*a)/a^3/b/(a*b)^(1/2)*arctan(b*x^
(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.15

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^2}\,dx$$

$$= \Bigg[-\frac{3\left(Ba^4+5\,Aa^3b+(Bab^3+5\,Ab^4)x^3+3\,(Ba^2b^2+5\,Aab^3)x^2+3\,(Ba^3b+5\,Aa^2b^2)x\right)\sqrt{-ab}\log\left(\frac{bx-a}{\cdots}\right.}{48\left(a^4b^5x^3+3\,a^5b^4x^2+3\,\cdots\right.}$$

$$-\frac{3\left(Ba^4+5\,Aa^3b+(Bab^3+5\,Ab^4)x^3+3\,(Ba^2b^2+5\,Aab^3)x^2+3\,(Ba^3b+5\,Aa^2b^2)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\cdots\right.}{24\left(a^4b^5x^3+3\,a^5b^4x^2+3\,a^6b^3x\cdots\right.}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*a^4 + 5*A*a^3*b + (B*a*b^3 + 5*A*b^4)*x^3 + 3*(B*a^2*b^2 + 5*
A*a*b^3)*x^2 + 3*(B*a^3*b + 5*A*a^2*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sq
rt(-a*b)*sqrt(x))/(b*x + a)) + 2*(3*B*a^4*b - 33*A*a^3*b^2 - 3*(B*a^2*b^3
+ 5*A*a*b^4)*x^2 - 8*(B*a^3*b^2 + 5*A*a^2*b^3)*x)*sqrt(x))/(a^4*b^5*x^3 +
3*a^5*b^4*x^2 + 3*a^6*b^3*x + a^7*b^2), -1/24*(3*(B*a^4 + 5*A*a^3*b + (B*a
*b^3 + 5*A*b^4)*x^3 + 3*(B*a^2*b^2 + 5*A*a*b^3)*x^2 + 3*(B*a^3*b + 5*A*a^2
*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (3*B*a^4*b - 33*A*a^3*b
^2 - 3*(B*a^2*b^3 + 5*A*a*b^4)*x^2 - 8*(B*a^3*b^2 + 5*A*a^2*b^3)*x)*sqrt(x
))/(a^4*b^5*x^3 + 3*a^5*b^4*x^2 + 3*a^6*b^3*x + a^7*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2300 vs. $2(119) = 238$.

Time = 32.98 (sec) , antiderivative size = 2300, normalized size of antiderivative = 17.69

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(1/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*sqrt(x) + 2*B*x**(3/2)/3)/a**4, Eq(b, 0)), ((-2*A/(7*x**(7/2)) -
2*B/(5*x**(5/2)))/b**4, Eq(a, 0)), (15*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/
(48*a**6*b**2*sqrt(-a/b) + 144*a**5*b**3*x*sqrt(-a/b) + 144*a**4*b**4*x**2
*sqrt(-a/b) + 48*a**3*b**5*x**3*sqrt(-a/b)) - 15*A*a**3*b*log(sqrt(x) + sq
rt(-a/b))/(48*a**6*b**2*sqrt(-a/b) + 144*a**5*b**3*x*sqrt(-a/b) + 144*a**4
*b**4*x**2*sqrt(-a/b) + 48*a**3*b**5*x**3*sqrt(-a/b)) + 66*A*a**2*b**2*sqr
t(x)*sqrt(-a/b)/(48*a**6*b**2*sqrt(-a/b) + 144*a**5*b**3*x*sqrt(-a/b) + 14
4*a**4*b**4*x**2*sqrt(-a/b) + 48*a**3*b**5*x**3*sqrt(-a/b)) + 45*A*a**2*b*
*2*x*log(sqrt(x) - sqrt(-a/b))/(48*a**6*b**2*sqrt(-a/b) + 144*a**5*b**3*x*
sqrt(-a/b) + 144*a**4*b**4*x**2*sqrt(-a/b) + 48*a**3*b**5*x**3*sqrt(-a/b))
 - 45*A*a**2*b**2*x*log(sqrt(x) + sqrt(-a/b))/(48*a**6*b**2*sqrt(-a/b) + 1
44*a**5*b**3*x*sqrt(-a/b) + 144*a**4*b**4*x**2*sqrt(-a/b) + 48*a**3*b**5*x
**3*sqrt(-a/b)) + 80*A*a*b**3*x**(3/2)*sqrt(-a/b)/(48*a**6*b**2*sqrt(-a/b)
 + 144*a**5*b**3*x*sqrt(-a/b) + 144*a**4*b**4*x**2*sqrt(-a/b) + 48*a**3*b*
*5*x**3*sqrt(-a/b)) + 45*A*a*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/(48*a**6*
b**2*sqrt(-a/b) + 144*a**5*b**3*x*sqrt(-a/b) + 144*a**4*b**4*x**2*sqrt(-a/
b) + 48*a**3*b**5*x**3*sqrt(-a/b)) - 45*A*a*b**3*x**2*log(sqrt(x) + sqrt(-
a/b))/(48*a**6*b**2*sqrt(-a/b) + 144*a**5*b**3*x*sqrt(-a/b) + 144*a**4*b**
4*x**2*sqrt(-a/b) + 48*a**3*b**5*x**3*sqrt(-a/b)) + 30*A*b**4*x**(5/2)*...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^2}\,dx = \frac{3\,(Bab^2+5\,Ab^3)x^{\frac{5}{2}}+8\,(Ba^2b+5\,Aab^2)x^{\frac{3}{2}}-3\,(Ba^3-11\,Aa^2b)\sqrt{x}}{24\,(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^6b)} + \frac{(Ba+5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
1/24*(3*(B*a*b^2 + 5*A*b^3)*x^(5/2) + 8*(B*a^2*b + 5*A*a*b^2)*x^(3/2) - 3*
(B*a^3 - 11*A*a^2*b)*sqrt(x))/(a^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*b^2*x +
 a^6*b) + 1/8*(B*a + 5*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^2}\,dx = \frac{(Ba+5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b} + \frac{3\,Bab^2x^{\frac{5}{2}}+15\,Ab^3x^{\frac{5}{2}}+8\,Ba^2bx^{\frac{3}{2}}+40\,Aab^2x^{\frac{3}{2}}-3\,Ba^3\sqrt{x}+33\,Aa^2b\sqrt{x}}{24\,(bx+a)^3a^3b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
1/8*(B*a + 5*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3*b) + 1/24*(3*
B*a*b^2*x^(5/2) + 15*A*b^3*x^(5/2) + 8*B*a^2*b*x^(3/2) + 40*A*a*b^2*x^(3/2
) - 3*B*a^3*sqrt(x) + 33*A*a^2*b*sqrt(x))/((b*x + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^2}\,dx = \frac{\frac{x^{3/2}\,(5\,A\,b+B\,a)}{3\,a^2}+\frac{\sqrt{x}\,(11\,A\,b-B\,a)}{8\,a\,b}+\frac{b\,x^{5/2}\,(5\,A\,b+B\,a)}{8\,a^3}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(5\,A\,b+B\,a)}{8\,a^{7/2}\,b^{3/2}}$$

input
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
((x^(3/2)*(5*A*b + B*a))/(3*a^2) + (x^(1/2)*(11*A*b - B*a))/(8*a*b) + (b*x
^(5/2)*(5*A*b + B*a))/(8*a^3))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) +
 (atan((b^(1/2)*x^(1/2))/a^(1/2))*(5*A*b + B*a))/(8*a^(7/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^2}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+5\sqrt{x}\,a^2b+3\sqrt{x}\,a\,b^2x}{4a^3b\,(b^2x^2+2abx+a^2)}$$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*atan((
sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 + 5*sqrt(x)*a**2*b + 3*sqrt(x)*a*b
**2*x)/(4*a**3*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.400 $\int \frac{A+Bx}{x^{3/2}(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 135

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx=-\frac{2A}{a^4\sqrt{x}}-\frac{(Ab-aB)\sqrt{x}}{3a^2(a+bx)^3}-\frac{(11Ab-5aB)\sqrt{x}}{12a^3(a+bx)^2}-\frac{(19Ab-5aB)\sqrt{x}}{8a^4(a+bx)}-\frac{5(7Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{b}}$$

output
```
-2*A/a^4/x^(1/2)-1/3*(A*b-B*a)*x^(1/2)/a^2/(b*x+a)^3-1/12*(11*A*b-5*B*a)*x
^(1/2)/a^3/(b*x+a)^2-1/8*(19*A*b-5*B*a)*x^(1/2)/a^4/(b*x+a)-5/8*(7*A*b-B*a
)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx=\frac{-105Ab^3x^3+5ab^2x^2(-56A+3Bx)+a^3(-48A+33Bx)+a^2bx(-231A}{24a^4\sqrt{x}(a+bx)^3}+\frac{5(-7Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(-105*A*b^3*x^3 + 5*a*b^2*x^2*(-56*A + 3*B*x) + a^3*(-48*A + 33*B*x) + a^2
*b*x*(-231*A + 40*B*x))/(24*a^4*Sqrt[x]*(a + b*x)^3) + (5*(-7*A*b + a*B)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(8*a^(9/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1184, 27, 87, 52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2} \left(a^2+2abx+b^2x^2\right)^2} \, dx$$

$$\Big\downarrow \text{1184}$$

$$b^4 \int \frac{A+Bx}{b^4x^{3/2}(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(7Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^3}dx}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{(7Ab-aB)\left(\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}dx}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{(7Ab-aB)\left(\frac{5\left(\frac{3\int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

$\Big\downarrow$ 61

$$\frac{(7Ab-aB)\left(\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

$\Big\downarrow$ 73

$$\frac{(7Ab-aB)\left(\frac{5\left(\frac{3\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

$\Big\downarrow$ 218

$$\frac{(7Ab-aB)\left(\frac{5\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6ab}+\frac{Ab-aB}{3ab\sqrt{x}(a+bx)^3}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x)^3) + ((7*A*b - a*B)*(1/(2*a*Sqrt[x]*(
a + b*x)^2) + (5*(1/(a*Sqrt[x]*(a + b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b
]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{19}{16}A\,b^{3}-\frac{5}{16}B a\,b^{2}\right)x^{\frac{5}{2}}+\frac{ab(17Ab-5Ba)x^{\frac{3}{2}}}{6}+\left(\frac{29}{16}A\,a^{2}b-\frac{11}{16}B\,a^{3}\right)\sqrt{x}}{(bx+a)^{3}}+\frac{5(7Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{4}}-\frac{2A}{a^{4}\sqrt{x}}$	105
default	$-\frac{2\left(\frac{\left(\frac{19}{16}A\,b^{3}-\frac{5}{16}B a\,b^{2}\right)x^{\frac{5}{2}}+\frac{ab(17Ab-5Ba)x^{\frac{3}{2}}}{6}+\left(\frac{29}{16}A\,a^{2}b-\frac{11}{16}B\,a^{3}\right)\sqrt{x}}{(bx+a)^{3}}+\frac{5(7Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{4}}-\frac{2A}{a^{4}\sqrt{x}}$	105
risch	$-\frac{2A}{a^{4}\sqrt{x}}-\frac{\frac{2\left(\frac{19}{16}A\,b^{3}-\frac{5}{16}B a\,b^{2}\right)x^{\frac{5}{2}}+\frac{ab(17Ab-5Ba)x^{\frac{3}{2}}}{3}+2\left(\frac{29}{16}A\,a^{2}b-\frac{11}{16}B\,a^{3}\right)\sqrt{x}}{(bx+a)^{3}}+\frac{5(7Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^{4}}$	106

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^4*(((19/16*A*b^3-5/16*B*a*b^2)*x^(5/2)+1/6*a*b*(17*A*b-5*B*a)*x^(3/2)
+(29/16*A*a^2*b-11/16*B*a^3)*x^(1/2))/(b*x+a)^3+5/16*(7*A*b-B*a)/(a*b)^(1/
2)*arctan(b*x^(1/2)/(a*b)^(1/2)))-2*A/a^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.30

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \left[\frac{15\left((Bab^3-7\,Ab^4)x^4+3\,(Ba^2b^2-7\,Aab^3)x^3+3\,(Ba^3b-7\,Aa^2b^2)x^2+\right.}{}\right.$$

$$-\frac{15\left((Bab^3-7\,Ab^4)x^4+3\,(Ba^2b^2-7\,Aab^3)x^3+3\,(Ba^3b-7\,Aa^2b^2)x^2+(Ba^4-7\,Aa^3b)x\right)\sqrt{ab}\arctan\left(\right.}{24\,(a^5b^4x^4+3\,a^6b^3x^3+}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(15*((B*a*b^3 - 7*A*b^4)*x^4 + 3*(B*a^2*b^2 - 7*A*a*b^3)*x^3 + 3*(B*
a^3*b - 7*A*a^2*b^2)*x^2 + (B*a^4 - 7*A*a^3*b)*x)*sqrt(-a*b)*log((b*x - a
+ 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) - 2*(48*A*a^4*b - 15*(B*a^2*b^3 - 7*A*a
*b^4)*x^3 - 40*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 - 33*(B*a^4*b - 7*A*a^3*b^2)*
x)*sqrt(x))/(a^5*b^4*x^4 + 3*a^6*b^3*x^3 + 3*a^7*b^2*x^2 + a^8*b*x), -1/24
*(15*((B*a*b^3 - 7*A*b^4)*x^4 + 3*(B*a^2*b^2 - 7*A*a*b^3)*x^3 + 3*(B*a^3*b
 - 7*A*a^2*b^2)*x^2 + (B*a^4 - 7*A*a^3*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b
*sqrt(x))) + (48*A*a^4*b - 15*(B*a^2*b^3 - 7*A*a*b^4)*x^3 - 40*(B*a^3*b^2
- 7*A*a^2*b^3)*x^2 - 33*(B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(x))/(a^5*b^4*x^4 +
 3*a^6*b^3*x^3 + 3*a^7*b^2*x^2 + a^8*b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. 2(134) = 268.

Time = 75.86 (sec) , antiderivative size = 2660, normalized size of antiderivative = 19.70

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/a**4, Eq(b, 0)), ((-2*A/(9*x**(9/2)) - 2*
B/(7*x**(7/2)))/b**4, Eq(a, 0)), (-105*A*a**3*b*sqrt(x)*log(sqrt(x) - sqrt
(-a/b))/(48*a**7*b*sqrt(x)*sqrt(-a/b) + 144*a**6*b**2*x**(3/2)*sqrt(-a/b)
+ 144*a**5*b**3*x**(5/2)*sqrt(-a/b) + 48*a**4*b**4*x**(7/2)*sqrt(-a/b)) +
105*A*a**3*b*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(48*a**7*b*sqrt(x)*sqrt(-a/
b) + 144*a**6*b**2*x**(3/2)*sqrt(-a/b) + 144*a**5*b**3*x**(5/2)*sqrt(-a/b)
 + 48*a**4*b**4*x**(7/2)*sqrt(-a/b)) - 96*A*a**3*b*sqrt(-a/b)/(48*a**7*b*s
qrt(x)*sqrt(-a/b) + 144*a**6*b**2*x**(3/2)*sqrt(-a/b) + 144*a**5*b**3*x**(
5/2)*sqrt(-a/b) + 48*a**4*b**4*x**(7/2)*sqrt(-a/b)) - 315*A*a**2*b**2*x**(
3/2)*log(sqrt(x) - sqrt(-a/b))/(48*a**7*b*sqrt(x)*sqrt(-a/b) + 144*a**6*b*
*2*x**(3/2)*sqrt(-a/b) + 144*a**5*b**3*x**(5/2)*sqrt(-a/b) + 48*a**4*b**4*
x**(7/2)*sqrt(-a/b)) + 315*A*a**2*b**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/
(48*a**7*b*sqrt(x)*sqrt(-a/b) + 144*a**6*b**2*x**(3/2)*sqrt(-a/b) + 144*a*
*5*b**3*x**(5/2)*sqrt(-a/b) + 48*a**4*b**4*x**(7/2)*sqrt(-a/b)) - 462*A*a*
*2*b**2*x*sqrt(-a/b)/(48*a**7*b*sqrt(x)*sqrt(-a/b) + 144*a**6*b**2*x**(3/2
)*sqrt(-a/b) + 144*a**5*b**3*x**(5/2)*sqrt(-a/b) + 48*a**4*b**4*x**(7/2)*s
qrt(-a/b)) - 315*A*a*b**3*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(48*a**7*b*sq
rt(x)*sqrt(-a/b) + 144*a**6*b**2*x**(3/2)*sqrt(-a/b) + 144*a**5*b**3*x**(5
/2)*sqrt(-a/b) + 48*a**4*b**4*x**(7/2)*sqrt(-a/b)) + 315*A*a*b**3*x**(5...
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{48\,Aa^3-15\,(Bab^2-7\,Ab^3)x^3-40\,(Ba^2b-7\,Aab^2)x^2-33\,(Ba^3-7\,Aa^2b)x}{24\left(a^4b^3x^{\frac{7}{2}}+3\,a^5b^2x^{\frac{5}{2}}+3\,a^6bx^{\frac{3}{2}}+a^7\sqrt{x}\right)} + \frac{5\,(Ba-7\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(48*A*a^3 - 15*(B*a*b^2 - 7*A*b^3)*x^3 - 40*(B*a^2*b - 7*A*a*b^2)*x^
2 - 33*(B*a^3 - 7*A*a^2*b)*x)/(a^4*b^3*x^(7/2) + 3*a^5*b^2*x^(5/2) + 3*a^6
*b*x^(3/2) + a^7*sqrt(x)) + 5/8*(B*a - 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/
(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{5\,(Ba-7\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}-\frac{2\,A}{a^4\sqrt{x}} + \frac{15\,Bab^2x^{\frac{5}{2}}-57\,Ab^3x^{\frac{5}{2}}+40\,Ba^2bx^{\frac{3}{2}}-136\,Aab^2x^{\frac{3}{2}}+33\,Ba^3\sqrt{x}-87\,Aa^2b\sqrt{x}}{24\,(bx+a)^3a^4}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
5/8*(B*a - 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 2*A/(a^4*s
qrt(x)) + 1/24*(15*B*a*b^2*x^(5/2) - 57*A*b^3*x^(5/2) + 40*B*a^2*b*x^(3/2)
 - 136*A*a*b^2*x^(3/2) + 33*B*a^3*sqrt(x) - 87*A*a^2*b*sqrt(x))/((b*x + a)
^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{\frac{2\,A}{a}+\frac{11\,x\,(7\,A\,b-B\,a)}{8\,a^2}+\frac{5\,b^2\,x^3\,(7\,A\,b-B\,a)}{8\,a^4}+\frac{5\,b\,x^2\,(7\,A\,b-B\,a)}{3\,a^3}}{a^3\,\sqrt{x}+b^3\,x^{7/2}+3\,a^2\,b\,x^{3/2}+3\,a\,b^2\,x^{5/2}} - \frac{5\,\mathrm{atan}\left(\frac{5\,\sqrt{b}\,\sqrt{x}\,(7\,A\,b-B\,a)}{\sqrt{a}\,(35\,A\,b-5\,B\,a)}\right)\,(7\,A\,b-B\,a)}{8\,a^{9/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
- ((2*A)/a + (11*x*(7*A*b - B*a))/(8*a^2) + (5*b^2*x^3*(7*A*b - B*a))/(8*a
^4) + (5*b*x^2*(7*A*b - B*a))/(3*a^3))/(a^3*x^(1/2) + b^3*x^(7/2) + 3*a^2*
b*x^(3/2) + 3*a*b^2*x^(5/2)) - (5*atan((5*b^(1/2)*x^(1/2)*(7*A*b - B*a))/(
a^(1/2)*(35*A*b - 5*B*a)))*(7*A*b - B*a))/(8*a^(9/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{-15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2 - 30\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx - 15\sqrt{x}}{4\sqrt{x}\,a^4\left(b^2x^2+2abx+\right.}$$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 -
30*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*
sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 8*
a**3 - 25*a**2*b*x - 15*a*b**2*x**2)/(4*sqrt(x)*a**4*(a**2 + 2*a*b*x + b**
2*x**2))
```

3.401 $\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{2A}{3a^4x^{3/2}}+\frac{2(4Ab-aB)}{a^5\sqrt{x}}+\frac{b(Ab-aB)\sqrt{x}}{3a^3(a+bx)^3}+\frac{b(17Ab-11aB)\sqrt{x}}{12a^4(a+bx)^2}+\frac{b(41Ab-19aB)\sqrt{x}}{8a^5(a+bx)}+\frac{35\sqrt{b}(3Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{11/2}}$$

output
```
-2/3*A/a^4/x^(3/2)+2*(4*A*b-B*a)/a^5/x^(1/2)+1/3*b*(A*b-B*a)*x^(1/2)/a^3/(
b*x+a)^3+1/12*b*(17*A*b-11*B*a)*x^(1/2)/a^4/(b*x+a)^2+1/8*b*(41*A*b-19*B*a
)*x^(1/2)/a^5/(b*x+a)+35/8*b^(1/2)*(3*A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1
/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{315Ab^4x^4+3a^3bx(48A-77Bx)+7a^2b^2x^2(99A-40Bx)-105ab^3x^3(-}{24a^5x^{3/2}(a+bx)^3} - \frac{35\sqrt{b}(-3Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(315*A*b^4*x^4 + 3*a^3*b*x*(48*A - 77*B*x) + 7*a^2*b^2*x^2*(99*A - 40*B*x)
 - 105*a*b^3*x^3*(-8*A + B*x) - 16*a^4*(A + 3*B*x))/(24*a^5*x^(3/2)*(a + b
*x)^3) - (35*Sqrt[b]*(-3*A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(8*
a^(11/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^4\int \frac{A+Bx}{b^4x^{5/2}(a+bx)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^4}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(3Ab-aB)\int \frac{1}{x^{5/2}(a+bx)^3}dx}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(3Ab-aB)\left(\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(3Ab-aB)\left(\frac{7\left(\frac{5\int\frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$\downarrow$ 61

$$\frac{(3Ab-aB)\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$\downarrow$ 61

$$\frac{(3Ab-aB)\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$\downarrow$ 73

$$\frac{(3Ab-aB)\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{(3Ab-aB)\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2ab}+\frac{Ab-aB}{3abx^{3/2}(a+bx)^3}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^2),x]
```

output
```
(A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x)^3) + ((3*A*b - a*B)*(1/(2*a*x^(3/2)*(
a + b*x)^2) + (7*(1/(a*x^(3/2)*(a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/
(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/
(2*a)))/(4*a)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{2(-12Abx+3Bax+Aa)}{3a^5x^{\frac{3}{2}}}+\frac{b\left(\frac{2\left(\frac{41}{16}Ab^3-\frac{19}{16}Bab^2\right)x^{\frac{5}{2}}+\frac{ab(35Ab-17Ba)x^{\frac{3}{2}}}{3}+2\left(\frac{55}{16}Aa^2b-\frac{29}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(3Ab-Ba)\,\mathrm{arc}\ldots}{8\sqrt{a\ldots}}\right.}{a^5}$ …
derivativedivides	$\frac{2b\left(\frac{\left(\frac{41}{16}Ab^3-\frac{19}{16}Bab^2\right)x^{\frac{5}{2}}+\frac{ab(35Ab-17Ba)x^{\frac{3}{2}}}{6}+\left(\frac{55}{16}Aa^2b-\frac{29}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^5}-\frac{2A}{3a^4x^{\frac{3}{2}}}-\frac{2(\ldots}{}$ …
default	$\frac{2b\left(\frac{\left(\frac{41}{16}Ab^3-\frac{19}{16}Bab^2\right)x^{\frac{5}{2}}+\frac{ab(35Ab-17Ba)x^{\frac{3}{2}}}{6}+\left(\frac{55}{16}Aa^2b-\frac{29}{16}Ba^3\right)\sqrt{x}}{(bx+a)^3}+\frac{35(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^5}-\frac{2A}{3a^4x^{\frac{3}{2}}}-\frac{2(\ldots}{}$ …

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-12*A*b*x+3*B*a*x+A*a)/a^5/x^(3/2)+1/a^5*b*(2*((41/16*A*b^3-19/16*B*
a*b^2)*x^(5/2)+1/6*a*b*(35*A*b-17*B*a)*x^(3/2)+(55/16*A*a^2*b-29/16*B*a^3)
*x^(1/2))/(b*x+a)^3+35/8*(3*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \left[-\frac{105\left((Bab^3-3\,Ab^4)x^5+3\,(Ba^2b^2-3\,Aab^3)x^4+3\,(Ba^3b-3\,Aa^2b^2)x\right.}{}\right.$$

$$-\frac{105\left((Bab^3-3\,Ab^4)x^5+3\,(Ba^2b^2-3\,Aab^3)x^4+3\,(Ba^3b-3\,Aa^2b^2)x^3+(Ba^4-3\,Aa^3b)x^2\right)\sqrt{\frac{b}{a}}\arctan}{24\,(a^5b^3x^5}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/48*(105*((B*a*b^3 - 3*A*b^4)*x^5 + 3*(B*a^2*b^2 - 3*A*a*b^3)*x^4 + 3*(
B*a^3*b - 3*A*a^2*b^2)*x^3 + (B*a^4 - 3*A*a^3*b)*x^2)*sqrt(-b/a)*log((b*x
+ 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(16*A*a^4 + 105*(B*a*b^3 - 3*
A*b^4)*x^4 + 280*(B*a^2*b^2 - 3*A*a*b^3)*x^3 + 231*(B*a^3*b - 3*A*a^2*b^2)
*x^2 + 48*(B*a^4 - 3*A*a^3*b)*x)*sqrt(x))/(a^5*b^3*x^5 + 3*a^6*b^2*x^4 + 3
*a^7*b*x^3 + a^8*x^2), -1/24*(105*((B*a*b^3 - 3*A*b^4)*x^5 + 3*(B*a^2*b^2
- 3*A*a*b^3)*x^4 + 3*(B*a^3*b - 3*A*a^2*b^2)*x^3 + (B*a^4 - 3*A*a^3*b)*x^2
)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (16*A*a^4 + 105*(B*a*b^3 - 3*A*b^4
)*x^4 + 280*(B*a^2*b^2 - 3*A*a*b^3)*x^3 + 231*(B*a^3*b - 3*A*a^2*b^2)*x^2
+ 48*(B*a^4 - 3*A*a^3*b)*x)*sqrt(x))/(a^5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*
b*x^3 + a^8*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(5/2)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{16\,Aa^4+105\,(Bab^3-3\,Ab^4)x^4+280\,(Ba^2b^2-3\,Aab^3)x^3+231\,(Ba^3b-3\,Aa^2b^2)x^2+48\,(Ba^4-3\,Aa^3b}{24\left(a^5b^3x^{\frac{9}{2}}+3\,a^6b^2x^{\frac{7}{2}}+3\,a^7bx^{\frac{5}{2}}+a^8x^{\frac{3}{2}}\right)} - \frac{35\,(Bab-3\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(16*A*a^4 + 105*(B*a*b^3 - 3*A*b^4)*x^4 + 280*(B*a^2*b^2 - 3*A*a*b^3
)*x^3 + 231*(B*a^3*b - 3*A*a^2*b^2)*x^2 + 48*(B*a^4 - 3*A*a^3*b)*x)/(a^5*b
^3*x^(9/2) + 3*a^6*b^2*x^(7/2) + 3*a^7*b*x^(5/2) + a^8*x^(3/2)) - 35/8*(B*
a*b - 3*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = -\frac{35\,(Bab-3\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5} - \frac{105\,Bab^3x^4-315\,Ab^4x^4+280\,Ba^2b^2x^3-840\,Aab^3x^3+231\,Ba^3bx^2-693\,Aa^2b^2x^2+48\,Ba^4x-144\,Aa}{24\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)^3a^5}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
-35/8*(B*a*b - 3*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/24
*(105*B*a*b^3*x^4 - 315*A*b^4*x^4 + 280*B*a^2*b^2*x^3 - 840*A*a*b^3*x^3 +
231*B*a^3*b*x^2 - 693*A*a^2*b^2*x^2 + 48*B*a^4*x - 144*A*a^3*b*x + 16*A*a^
4)/((b*x^(3/2) + a*sqrt(x))^3*a^5)
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{\frac{2\,x\,(3\,A\,b-B\,a)}{a^2}-\frac{2\,A}{3\,a}+\frac{35\,b^2\,x^3\,(3\,A\,b-B\,a)}{3\,a^4}+\frac{35\,b^3\,x^4\,(3\,A\,b-B\,a)}{8\,a^5}+\frac{77\,b\,x^2\,(3\,A\,b-B\,a)}{8\,a^3}}{a^3\,x^{3/2}+b^3\,x^{9/2}+3\,a^2\,b\,x^{5/2}+3\,a\,b^2\,x^{7/2}} + \frac{35\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(3\,A\,b-B\,a)}{8\,a^{11/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^2),x)
```

output
```
((2*x*(3*A*b - B*a))/a^2 - (2*A)/(3*a) + (35*b^2*x^3*(3*A*b - B*a))/(3*a^4
) + (35*b^3*x^4*(3*A*b - B*a))/(8*a^5) + (77*b*x^2*(3*A*b - B*a))/(8*a^3))
/(a^3*x^(3/2) + b^3*x^(9/2) + 3*a^2*b*x^(5/2) + 3*a*b^2*x^(7/2)) + (35*b^(
1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(3*A*b - B*a))/(8*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^2}\,dx = \frac{105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+210\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2+1}{12\sqrt{x}\,a^5x\,(b^2x^2}$$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x
+ 210*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x
**2 + 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3
*x**3 - 8*a**4 + 56*a**3*b*x + 175*a**2*b**2*x**2 + 105*a*b**3*x**3)/(12*s
qrt(x)*a**5*x*(a**2 + 2*a*b*x + b**2*x**2))
```

3.402 $\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 214

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{231(3Ab-13aB)\sqrt{x}}{128b^7}+\frac{2Bx^{3/2}}{3b^6}-\frac{(Ab-aB)x^{11/2}}{5b^2(a+bx)^5} -\frac{(11Ab-21aB)x^{9/2}}{40b^3(a+bx)^4}-\frac{(99Ab-269aB)x^{7/2}}{240b^4(a+bx)^3}-\frac{(693Ab-2363aB)x^{5/2}}{960b^5(a+bx)^2} -\frac{(693Ab-2747aB)x^{3/2}}{384b^6(a+bx)}-\frac{231\sqrt{a}(3Ab-13aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{15/2}}$$

output
```
231/128*(3*A*b-13*B*a)*x^(1/2)/b^7+2/3*B*x^(3/2)/b^6-1/5*(A*b-B*a)*x^(11/2
)/b^2/(b*x+a)^5-1/40*(11*A*b-21*B*a)*x^(9/2)/b^3/(b*x+a)^4-1/240*(99*A*b-2
69*B*a)*x^(7/2)/b^4/(b*x+a)^3-1/960*(693*A*b-2363*B*a)*x^(5/2)/b^5/(b*x+a)
^2-1/384*(693*A*b-2747*B*a)*x^(3/2)/b^6/(b*x+a)-231/128*a^(1/2)*(3*A*b-13*
B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(15/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\frac{\sqrt{b}\sqrt{x}(-45045a^6B+66a^3b^3x^2(1344A-5135Bx)+5ab^5x^4(6369A-3328Bx)+55a^2b^4x^3(1422A-2509Bx)+}{(a+bx)^5}}{}$$

input
```
Integrate[(x^(11/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((Sqrt[b]*Sqrt[x]*(-45045*a^6*B + 66*a^3*b^3*x^2*(1344*A - 5135*B*x) + 5*a
*b^5*x^4*(6369*A - 3328*B*x) + 55*a^2*b^4*x^3*(1422*A - 2509*B*x) + 462*a^
4*b^2*x*(105*A - 832*B*x) + 1155*a^5*b*(9*A - 182*B*x) + 1280*b^6*x^5*(3*A
 + B*x)))/(a + b*x)^5 + 3465*Sqrt[a]*(-3*A*b + 13*a*B)*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(1920*b^(15/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1184, 27, 87, 51, 51, 51, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{x^{11/2}(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{11/2}(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\int \frac{x^{11/2}}{(a+bx)^5}dx}{10ab}$$

$\Big\downarrow$ 51

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\left(\frac{11\int \frac{x^{9/2}}{(a+bx)^4}dx}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right)}{10ab}$$

$\Big\downarrow$ 51

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\left(\frac{11\left(\frac{3\int \frac{x^{7/2}}{(a+bx)^3}dx}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right)}{10ab}$$

$\Big\downarrow$ 51

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{x^{5/2}}{(a+bx)^2}dx}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right)}{10ab}$$

$\Big\downarrow$ 51

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-$$

$$(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right.$$

$\Big\downarrow$ 73

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5} -$$

$$(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}\,d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right.$$

↓ 218

$$\frac{x^{13/2}(Ab-aB)}{5ab(a+bx)^5}-(3Ab-13aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{11/2}}{4b(a+bx)^4}\right.$$

input
```
Int[(x^(11/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(13/2))/(5*a*b*(a + b*x)^5) - ((3*A*b - 13*a*B)*(-1/4*x^(11
/2)/(b*(a + b*x)^4) + (11*(-1/3*x^(9/2)/(b*(a + b*x)^3) + (3*(-1/2*x^(7/2)
/(b*(a + b*x)^2) + (7*(-(x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/2))/(3*b) -
(a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2))
)/b))/(2*b)))/(4*b)))/(2*b)))/(8*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77

method	result
risch	$\frac{2(Bbx+3Ab-18Ba)\sqrt{x}}{3b^7} - \frac{a\Bigl(\frac{2\left(-\frac{843}{256}Ab^5+\frac{2373}{256}Bab^4\right)x^{\frac{9}{2}}+2\left(-\frac{1327}{128}Aab^4+\frac{12131}{384}Ba^2b^3\right)x^{\frac{7}{2}}+2\left(-\frac{131}{10}Aa^2b^3+\frac{1253}{30}Ba^3\right.}{(bx+a)^5}}{}$
derivativedivides	$\frac{\frac{2Bbx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-12Ba\sqrt{x}}{b^7} - \frac{2a\Bigl(\frac{\left(-\frac{843}{256}Ab^5+\frac{2373}{256}Bab^4\right)x^{\frac{9}{2}}+\left(-\frac{1327}{128}Aab^4+\frac{12131}{384}Ba^2b^3\right)x^{\frac{7}{2}}+\left(-\frac{131}{10}Aa^2b^3+\frac{1253}{30}B\right.}{(bx+a)^5}}{}$
default	$\frac{\frac{2Bbx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-12Ba\sqrt{x}}{b^7} - \frac{2a\Bigl(\frac{\left(-\frac{843}{256}Ab^5+\frac{2373}{256}Bab^4\right)x^{\frac{9}{2}}+\left(-\frac{1327}{128}Aab^4+\frac{12131}{384}Ba^2b^3\right)x^{\frac{7}{2}}+\left(-\frac{131}{10}Aa^2b^3+\frac{1253}{30}B\right.}{(bx+a)^5}}{}$

input
```
int(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-18*B*a)*x^(1/2)/b^7-a/b^7*(2*((-843/256*A*b^5+2373/256*B*
a*b^4)*x^(9/2)+(-1327/128*A*a*b^4+12131/384*B*a^2*b^3)*x^(7/2)+(-131/10*A*
a^2*b^3+1253/30*B*a^3*b^2)*x^(5/2)-1/384*a^3*b*(2931*A*b-9629*B*a)*x^(3/2)
+(-437/256*A*a^4*b+1467/256*B*a^5)*x^(1/2))/(b*x+a)^5+231/128*(3*A*b-13*B*
a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 703, normalized size of antiderivative = 3.29

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(3465*(13*B*a^6 - 3*A*a^5*b + (13*B*a*b^5 - 3*A*b^6)*x^5 + 5*(13*
B*a^2*b^4 - 3*A*a*b^5)*x^4 + 10*(13*B*a^3*b^3 - 3*A*a^2*b^4)*x^3 + 10*(13*
B*a^4*b^2 - 3*A*a^3*b^3)*x^2 + 5*(13*B*a^5*b - 3*A*a^4*b^2)*x)*sqrt(-a/b)*
log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(1280*B*b^6*x^6 - 45
045*B*a^6 + 10395*A*a^5*b - 1280*(13*B*a*b^5 - 3*A*b^6)*x^5 - 10615*(13*B*
a^2*b^4 - 3*A*a*b^5)*x^4 - 26070*(13*B*a^3*b^3 - 3*A*a^2*b^4)*x^3 - 29568*
(13*B*a^4*b^2 - 3*A*a^3*b^3)*x^2 - 16170*(13*B*a^5*b - 3*A*a^4*b^2)*x)*sqr
t(x))/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*
b^8*x + a^5*b^7), 1/1920*(3465*(13*B*a^6 - 3*A*a^5*b + (13*B*a*b^5 - 3*A*b
^6)*x^5 + 5*(13*B*a^2*b^4 - 3*A*a*b^5)*x^4 + 10*(13*B*a^3*b^3 - 3*A*a^2*b^
4)*x^3 + 10*(13*B*a^4*b^2 - 3*A*a^3*b^3)*x^2 + 5*(13*B*a^5*b - 3*A*a^4*b^2
)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (1280*B*b^6*x^6 - 45045*B*a
^6 + 10395*A*a^5*b - 1280*(13*B*a*b^5 - 3*A*b^6)*x^5 - 10615*(13*B*a^2*b^4
 - 3*A*a*b^5)*x^4 - 26070*(13*B*a^3*b^3 - 3*A*a^2*b^4)*x^3 - 29568*(13*B*a
^4*b^2 - 3*A*a^3*b^3)*x^2 - 16170*(13*B*a^5*b - 3*A*a^4*b^2)*x)*sqrt(x))/(
b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a^4*b^8*x +
 a^5*b^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{45\,(791\,Ba^2b^4-281\,Aab^5)x^{\frac{9}{2}}+10\,(12131\,Ba^3b^3-3981\,Aa^2b^4)x^{\frac{7}{2}}+128\,(1253\,Ba^4b^2-393\,Aa^3b^3)x^{\frac{5}{2}}+}{1920\,(b^{12}x^5+5\,ab^{11}x^4+10\,a^2b^{10}x^3+10\,a^3b^9x^2+5\,a} + \frac{231\,(13\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^7} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(6\,Ba-Ab)\sqrt{x}\right)}{3\,b^7}$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1920*(45*(791*B*a^2*b^4 - 281*A*a*b^5)*x^(9/2) + 10*(12131*B*a^3*b^3 -
3981*A*a^2*b^4)*x^(7/2) + 128*(1253*B*a^4*b^2 - 393*A*a^3*b^3)*x^(5/2) + 1
0*(9629*B*a^5*b - 2931*A*a^4*b^2)*x^(3/2) + 15*(1467*B*a^6 - 437*A*a^5*b)*
sqrt(x))/(b^12*x^5 + 5*a*b^11*x^4 + 10*a^2*b^10*x^3 + 10*a^3*b^9*x^2 + 5*a
^4*b^8*x + a^5*b^7) + 231/128*(13*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a
*b))/(sqrt(a*b)*b^7) + 2/3*(B*b*x^(3/2) - 3*(6*B*a - A*b)*sqrt(x))/b^7
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{231\,(13\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^7} - \frac{35595\,Ba^2b^4x^{\frac{9}{2}}-12645\,Aab^5x^{\frac{9}{2}}+121310\,Ba^3b^3x^{\frac{7}{2}}-39810\,Aa^2b^4x^{\frac{7}{2}}+160384\,Ba^4b^2x^{\frac{5}{2}}-50304\,Aa^3b^3x}{1920\,(bx+a)^5b^7} + \frac{2\left(Bb^{12}x^{\frac{3}{2}}-18\,Bab^{11}\sqrt{x}+3\,Ab^{12}\sqrt{x}\right)}{3\,b^{18}}$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
231/128*(13*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^7) -
 1/1920*(35595*B*a^2*b^4*x^(9/2) - 12645*A*a*b^5*x^(9/2) + 121310*B*a^3*b^
3*x^(7/2) - 39810*A*a^2*b^4*x^(7/2) + 160384*B*a^4*b^2*x^(5/2) - 50304*A*a
^3*b^3*x^(5/2) + 96290*B*a^5*b*x^(3/2) - 29310*A*a^4*b^2*x^(3/2) + 22005*B
*a^6*sqrt(x) - 6555*A*a^5*b*sqrt(x))/((b*x + a)^5*b^7) + 2/3*(B*b^12*x^(3/
2) - 18*B*a*b^11*sqrt(x) + 3*A*b^12*sqrt(x))/b^18
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.15

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \sqrt{x}\left(\frac{2\,A}{b^6}-\frac{12\,B\,a}{b^7}\right) + \frac{x^{3/2}\left(\frac{977\,A\,a^4\,b^2}{64}-\frac{9629\,B\,a^5\,b}{192}\right)-x^{9/2}\left(\frac{2373\,B\,a^2\,b^4}{128}-\frac{843\,A\,a\,b^5}{128}\right)-\sqrt{x}\left(\frac{1467\,B\,a^6}{128}-\frac{437\,A\,a^5\,b}{128}\right)+x^{5/2}\left(\frac{131\,A\,a^3\,b^3}{5}-\right.}{a^5\,b^7+5\,a^4\,b^8\,x+10\,a^3\,b^9\,x^2+10\,a^2\,b^{10}\,x^3+5\,a\,b^{11}\,x^4+b^{12}\,x} + \frac{2\,B\,x^{3/2}}{3\,b^6} + \frac{231\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(3\,A\,b-13\,B\,a)}{13\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-13\,B\,a)}{128\,b^{15/2}}$$

input
```
int((x^(11/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
x^(1/2)*((2*A)/b^6 - (12*B*a)/b^7) + (x^(3/2)*((977*A*a^4*b^2)/64 - (9629*
B*a^5*b)/192) - x^(9/2)*((2373*B*a^2*b^4)/128 - (843*A*a*b^5)/128) - x^(1/
2)*((1467*B*a^6)/128 - (437*A*a^5*b)/128) + x^(5/2)*((131*A*a^3*b^3)/5 - (
1253*B*a^4*b^2)/15) + x^(7/2)*((1327*A*a^2*b^4)/64 - (12131*B*a^3*b^3)/192
))/(a^5*b^7 + b^12*x^5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*
a^2*b^10*x^3) + (2*B*x^(3/2))/(3*b^6) + (231*a^(1/2)*atan((a^(1/2)*b^(1/2)
*x^(1/2)*(3*A*b - 13*B*a))/(13*B*a^2 - 3*A*a*b))*(3*A*b - 13*B*a))/(128*b^
(15/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^5+13860\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4bx+20790\sqrt{b}\,\sqrt{a}\cdots}{}$$

input
```
int(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**5 + 13860*sqr
t(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4*b*x + 20790*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**2 + 13860*sqrt(b)
*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**3 + 3465*sqrt(b)
*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**4*x**4 - 3465*sqrt(x)*a*
*5*b - 12705*sqrt(x)*a**4*b**2*x - 16863*sqrt(x)*a**3*b**3*x**2 - 9207*sqr
t(x)*a**2*b**4*x**3 - 1408*sqrt(x)*a*b**5*x**4 + 128*sqrt(x)*b**6*x**5)/(1
92*b**7*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4)
)
```

3.403 $\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 190

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{2B\sqrt{x}}{b^6}-\frac{(Ab-aB)x^{9/2}}{5b^2(a+bx)^5}-\frac{(9Ab-19aB)x^{7/2}}{40b^3(a+bx)^4}-\frac{(21Ab-71aB)x^{5/2}}{80b^4(a+bx)^3}-\frac{(21Ab-103aB)x^{3/2}}{64b^5(a+bx)^2}-\frac{(63Ab-437aB)\sqrt{x}}{128b^6(a+bx)}+\frac{63(Ab-11aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128\sqrt{a}b^{13/2}}$$

output

```
2*B*x^(1/2)/b^6-1/5*(A*b-B*a)*x^(9/2)/b^2/(b*x+a)^5-1/40*(9*A*b-19*B*a)*x^
(7/2)/b^3/(b*x+a)^4-1/80*(21*A*b-71*B*a)*x^(5/2)/b^4/(b*x+a)^3-1/64*(21*A*
b-103*B*a)*x^(3/2)/b^5/(b*x+a)^2-1/128*(63*A*b-437*B*a)*x^(1/2)/b^6/(b*x+a
)+63/128*(A*b-11*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(13/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\frac{\sqrt{b}\sqrt{x}(3465a^5B-105a^4b(3A-154Bx)+5b^5x^4(-193A+256Bx)+42a^3b^2x(-35A+704Bx)+5ab^4x^3(-474A}{(a+bx)^5}}{640b^{13/2}}$$

input
```
Integrate[(x^(9/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((Sqrt[b]*Sqrt[x]*(3465*a^5*B - 105*a^4*b*(3*A - 154*B*x) + 5*b^5*x^4*(-19
3*A + 256*B*x) + 42*a^3*b^2*x*(-35*A + 704*B*x) + 5*a*b^4*x^3*(-474*A + 21
23*B*x) + 6*a^2*b^3*x^2*(-448*A + 4345*B*x)))/(a + b*x)^5 + (315*(A*b - 11
*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[a])/(640*b^(13/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1184, 27, 87, 51, 51, 51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{ 1184}$$

$$b^6\int \frac{x^{9/2}(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{9/2}(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\int \frac{x^{9/2}}{(a+bx)^5}dx}{10ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\int\frac{x^{7/2}}{(a+bx)^4}dx}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\left(\frac{7\int\frac{x^{5/2}}{(a+bx)^3}dx}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{x^{3/2}}{(a+bx)^2}dx}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}}{}$$

$$\downarrow 73$$

$$\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-\frac{(Ab-11aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

$\downarrow$ 218

$$\frac{\frac{x^{11/2}(Ab-aB)}{5ab(a+bx)^5}-(Ab-11aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{9/2}}{4b(a+bx)^4}\right)}{10ab}$$

input
```
Int[(x^(9/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(11/2))/(5*a*b*(a + b*x)^5) - ((A*b - 11*a*B)*(-1/4*x^(9/2)
/(b*(a + b*x)^4) + (9*(-1/3*x^(7/2)/(b*(a + b*x)^3) + (7*(-1/2*x^(5/2)/(b*
(a + b*x)^2) + (5*(-(x^(3/2)/(b*(a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqrt[
a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)))/(6*b)))/(8
*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^6}+\frac{\frac{2\left(\left(-\frac{193}{256}A\,b^5+\frac{843}{256}B a\,b^4\right)x^{\frac{9}{2}}+\left(\frac{1327}{128}B\,a^2b^3-\frac{237}{128}A a\,b^4\right)x^{\frac{7}{2}}+\left(\frac{131}{10}B\,a^3b^2-\frac{21}{10}A\,a^2b^3\right)x^{\frac{5}{2}}-\frac{a^3b(147Ab-977Ba)x^{\frac{3}{2}}}{128}\cdots\right.}{(bx+a)^5}}{b^6}$
default	$\frac{2B\sqrt{x}}{b^6}+\frac{\frac{2\left(\left(-\frac{193}{256}A\,b^5+\frac{843}{256}B a\,b^4\right)x^{\frac{9}{2}}+\left(\frac{1327}{128}B\,a^2b^3-\frac{237}{128}A a\,b^4\right)x^{\frac{7}{2}}+\left(\frac{131}{10}B\,a^3b^2-\frac{21}{10}A\,a^2b^3\right)x^{\frac{5}{2}}-\frac{a^3b(147Ab-977Ba)x^{\frac{3}{2}}}{128}\cdots\right.}{(bx+a)^5}}{b^6}$
risch	$\frac{2B\sqrt{x}}{b^6}+\frac{\frac{2\left(-\frac{193}{256}A\,b^5+\frac{843}{256}B a\,b^4\right)x^{\frac{9}{2}}+2\left(\frac{1327}{128}B\,a^2b^3-\frac{237}{128}A a\,b^4\right)x^{\frac{7}{2}}+2\left(\frac{131}{10}B\,a^3b^2-\frac{21}{10}A\,a^2b^3\right)x^{\frac{5}{2}}-\frac{a^3b(147Ab-977Ba)x^{\frac{3}{2}}}{64}\cdots}{(bx+a)^5}}{b^6}$

input
```
int(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^6+2/b^6*(((-193/256*A*b^5+843/256*B*a*b^4)*x^(9/2)+(1327/128
*B*a^2*b^3-237/128*A*a*b^4)*x^(7/2)+(131/10*B*a^3*b^2-21/10*A*a^2*b^3)*x^(
5/2)-1/128*a^3*b*(147*A*b-977*B*a)*x^(3/2)+(437/256*B*a^5-63/256*A*a^4*b)*
x^(1/2))/(b*x+a)^5+63/256*(A*b-11*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^
(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(157) = 314$.

Time = 0.10 (sec) , antiderivative size = 673, normalized size of antiderivative = 3.54

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \left[\frac{315\,(11\,Ba^6 - Aa^5b + (11\,Bab^5 - Ab^6)x^5 + 5\,(11\,Ba^2b^4 - Aab^5)x^4 + 10\,(11\right.$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[1/1280*(315*(11*B*a^6 - A*a^5*b + (11*B*a*b^5 - A*b^6)*x^5 + 5*(11*B*a^2*
b^4 - A*a*b^5)*x^4 + 10*(11*B*a^3*b^3 - A*a^2*b^4)*x^3 + 10*(11*B*a^4*b^2
- A*a^3*b^3)*x^2 + 5*(11*B*a^5*b - A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x - a -
 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(1280*B*a*b^6*x^5 + 3465*B*a^6*b - 3
15*A*a^5*b^2 + 965*(11*B*a^2*b^5 - A*a*b^6)*x^4 + 2370*(11*B*a^3*b^4 - A*a
^2*b^5)*x^3 + 2688*(11*B*a^4*b^3 - A*a^3*b^4)*x^2 + 1470*(11*B*a^5*b^2 - A
*a^4*b^3)*x)*sqrt(x))/(a*b^12*x^5 + 5*a^2*b^11*x^4 + 10*a^3*b^10*x^3 + 10*
a^4*b^9*x^2 + 5*a^5*b^8*x + a^6*b^7), 1/640*(315*(11*B*a^6 - A*a^5*b + (11
*B*a*b^5 - A*b^6)*x^5 + 5*(11*B*a^2*b^4 - A*a*b^5)*x^4 + 10*(11*B*a^3*b^3
- A*a^2*b^4)*x^3 + 10*(11*B*a^4*b^2 - A*a^3*b^3)*x^2 + 5*(11*B*a^5*b - A*a
^4*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (1280*B*a*b^6*x^5 + 3
465*B*a^6*b - 315*A*a^5*b^2 + 965*(11*B*a^2*b^5 - A*a*b^6)*x^4 + 2370*(11*
B*a^3*b^4 - A*a^2*b^5)*x^3 + 2688*(11*B*a^4*b^3 - A*a^3*b^4)*x^2 + 1470*(1
1*B*a^5*b^2 - A*a^4*b^3)*x)*sqrt(x))/(a*b^12*x^5 + 5*a^2*b^11*x^4 + 10*a^3
*b^10*x^3 + 10*a^4*b^9*x^2 + 5*a^5*b^8*x + a^6*b^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.08

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{5\,(843\,Bab^4-193\,Ab^5)x^{\frac{9}{2}}+10\,(1327\,Ba^2b^3-237\,Aab^4)x^{\frac{7}{2}}+128\,(131\,Ba^3b^2}{640\,(b^{11}x^5+5\,ab^{10}x^4+10\,a^2b^9x^3} + \frac{2\,B\sqrt{x}}{b^6} - \frac{63\,(11\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^6}$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/640*(5*(843*B*a*b^4 - 193*A*b^5)*x^(9/2) + 10*(1327*B*a^2*b^3 - 237*A*a*
b^4)*x^(7/2) + 128*(131*B*a^3*b^2 - 21*A*a^2*b^3)*x^(5/2) + 10*(977*B*a^4*
b - 147*A*a^3*b^2)*x^(3/2) + 5*(437*B*a^5 - 63*A*a^4*b)*sqrt(x))/(b^11*x^5
 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6)
 + 2*B*sqrt(x)/b^6 - 63/128*(11*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sq
rt(a*b)*b^6)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.84

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{2B\sqrt{x}}{b^6} - \frac{63\,(11\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^6} + \frac{4215\,Bab^4x^{\frac{9}{2}} - 965\,Ab^5x^{\frac{9}{2}} + 13270\,Ba^2b^3x^{\frac{7}{2}} - 2370\,Aab^4x^{\frac{7}{2}} + 16768\,Ba^3b^2x^{\frac{5}{2}} - 2688\,Aa^2b^3x^{\frac{5}{2}} + 9770\,B}{640\,(bx+a)^5b^6}$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^6 - 63/128*(11*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(
a*b)*b^6) + 1/640*(4215*B*a*b^4*x^(9/2) - 965*A*b^5*x^(9/2) + 13270*B*a^2*
b^3*x^(7/2) - 2370*A*a*b^4*x^(7/2) + 16768*B*a^3*b^2*x^(5/2) - 2688*A*a^2*
b^3*x^(5/2) + 9770*B*a^4*b*x^(3/2) - 1470*A*a^3*b^2*x^(3/2) + 2185*B*a^5*s
qrt(x) - 315*A*a^4*b*sqrt(x))/((b*x + a)^5*b^6)
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{2\,B\,\sqrt{x}}{b^6} - \frac{x^{3/2}\left(\frac{147\,A\,a^3\,b^2}{64} - \frac{977\,B\,a^4\,b}{64}\right) - x^{7/2}\left(\frac{1327\,B\,a^2\,b^3}{64} - \frac{237\,A\,a\,b^4}{64}\right) - \sqrt{x}\left(\frac{437\,B\,a^5}{128} - \frac{63\,A\,a^4\,b}{128}\right) + x^{9/2}\left(\frac{193\,A\,b^5}{128} - \frac{843}{}\right.}{a^5\,b^6 + 5\,a^4\,b^7\,x + 10\,a^3\,b^8\,x^2 + 10\,a^2\,b^9\,x^3 + 5\,a\,b^{10}\,x^4 + b^{11}\,x^5} + \frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b - 11\,B\,a)}{128\,\sqrt{a}\,b^{13/2}}$$

input
```
int((x^(9/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(2*B*x^(1/2))/b^6 - (x^(3/2)*((147*A*a^3*b^2)/64 - (977*B*a^4*b)/64) - x^(
7/2)*((1327*B*a^2*b^3)/64 - (237*A*a*b^4)/64) - x^(1/2)*((437*B*a^5)/128 -
 (63*A*a^4*b)/128) + x^(9/2)*((193*A*b^5)/128 - (843*B*a*b^4)/128) + x^(5/
2)*((21*A*a^2*b^3)/5 - (131*B*a^3*b^2)/5))/(a^5*b^6 + b^11*x^5 + 5*a^4*b^7
*x + 5*a*b^10*x^4 + 10*a^3*b^8*x^2 + 10*a^2*b^9*x^3) + (63*atan((b^(1/2)*x
^(1/2))/a^(1/2))*(A*b - 11*B*a))/(128*a^(1/2)*b^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.18

$$\int \frac{x^{9/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4-1260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx-1890\sqrt{b}\,\sqrt{a}\,at}{}$$

input
```
int(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 - 1260*sq
rt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x - 1890*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 - 1260*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 - 315*sqrt(b)*sqrt
(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 + 315*sqrt(x)*a**4*b + 1
155*sqrt(x)*a**3*b**2*x + 1533*sqrt(x)*a**2*b**3*x**2 + 837*sqrt(x)*a*b**4
*x**3 + 128*sqrt(x)*b**5*x**4)/(64*b**6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x
**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.404 $\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 182

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3} dx = -\frac{(Ab-aB)x^{7/2}}{5b^2(a+bx)^5} - \frac{(7Ab-17aB)x^{5/2}}{40b^3(a+bx)^4} - \frac{(7Ab-33aB)x^{3/2}}{48b^4(a+bx)^3} - \frac{(7Ab-65aB)\sqrt{x}}{64b^5(a+bx)^2} + \frac{(7Ab-193aB)\sqrt{x}}{128ab^5(a+bx)} + \frac{7(Ab+9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{3/2}b^{11/2}}$$

output
```
-1/5*(A*b-B*a)*x^(7/2)/b^2/(b*x+a)^5-1/40*(7*A*b-17*B*a)*x^(5/2)/b^3/(b*x+
a)^4-1/48*(7*A*b-33*B*a)*x^(3/2)/b^4/(b*x+a)^3-1/64*(7*A*b-65*B*a)*x^(1/2)
/b^5/(b*x+a)^2+1/128*(7*A*b-193*B*a)*x^(1/2)/a/b^5/(b*x+a)+7/128*(A*b+9*B*
a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.79

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx =$$
$$-\frac{\sqrt{x}(945a^5B-105Ab^5x^4+105a^4b(A+42Bx)+14a^3b^2x(35A+576Bx)+5ab^4x^3(158A+579Bx)+2a}{1920ab^5(a+bx)^5}$$
$$+\frac{7(Ab+9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{3/2}b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
-1/1920*(Sqrt[x]*(945*a^5*B - 105*A*b^5*x^4 + 105*a^4*b*(A + 42*B*x) + 14*
a^3*b^2*x*(35*A + 576*B*x) + 5*a*b^4*x^3*(158*A + 579*B*x) + 2*a^2*b^3*x^2
*(448*A + 3555*B*x)))/(a*b^5*(a + b*x)^5) + (7*(A*b + 9*a*B)*ArcTan[(Sqrt[
b]*Sqrt[x])/Sqrt[a]])/(128*a^(3/2)*b^(11/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1184, 27, 87, 51, 51, 51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$
$$\Big\downarrow \text{1184}$$
$$b^6\int \frac{x^{7/2}(A+Bx)}{b^6(a+bx)^6}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(9aB+Ab)\int \frac{x^{7/2}}{(a+bx)^5}dx}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(9aB+Ab)\left(\frac{7\int \frac{x^{5/2}}{(a+bx)^4}dx}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(9aB+Ab)\left(\frac{7\left(\frac{5\int \frac{x^{3/2}}{(a+bx)^3}dx}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(9aB+Ab)\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(9aB+Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \; 73$$

$$\frac{(9aB+Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \; 218$$

$$\frac{(9aB+Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{7/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{9/2}(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(9/2))/(5*a*b*(a + b*x)^5) + ((A*b + 9*a*B)*(-1/4*x^(7/2)/(
b*(a + b*x)^4) + (7*(-1/3*x^(5/2)/(b*(a + b*x)^3) + (5*(-1/2*x^(3/2)/(b*(a
 + b*x)^2) + (3*(-(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(Sqrt[a]*b^(3/2))))/(4*b)))/(6*b)))/(8*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{(7Ab-193Ba)x^{\frac{9}{2}}}{128ab}-\frac{79(Ab+9Ba)x^{\frac{7}{2}}}{192b^2}-\frac{7a(Ab+9Ba)x^{\frac{5}{2}}}{15b^3}-\frac{49a^2(Ab+9Ba)x^{\frac{3}{2}}}{192b^4}-\frac{7(Ab+9Ba)a^3\sqrt{x}}{128b^5}}{(bx+a)^5}+\frac{7(Ab+9Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a\,b^5\sqrt{ab}}$
default	$\frac{\frac{(7Ab-193Ba)x^{\frac{9}{2}}}{128ab}-\frac{79(Ab+9Ba)x^{\frac{7}{2}}}{192b^2}-\frac{7a(Ab+9Ba)x^{\frac{5}{2}}}{15b^3}-\frac{49a^2(Ab+9Ba)x^{\frac{3}{2}}}{192b^4}-\frac{7(Ab+9Ba)a^3\sqrt{x}}{128b^5}}{(bx+a)^5}+\frac{7(Ab+9Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a\,b^5\sqrt{ab}}$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/256*(7*A*b-193*B*a)/a/b*x^(9/2)-79/384*(A*b+9*B*a)/b^2*x^(7/2)-7/30*a
*(A*b+9*B*a)/b^3*x^(5/2)-49/384*a^2*(A*b+9*B*a)/b^4*x^(3/2)-7/256*(A*b+9*B
*a)*a^3/b^5*x^(1/2))/(b*x+a)^5+7/128*(A*b+9*B*a)/a/b^5/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(150) = 300$.

Time = 0.10 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.51

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \Bigg[-\frac{105\,(9\,Ba^6+Aa^5b+(9\,Bab^5+Ab^6)x^5+5\,(9\,Ba^2b^4+Aab^5)x^4+10\,(9\,Ba}{}$$

$$-\frac{105\,(9\,Ba^6+Aa^5b+(9\,Bab^5+Ab^6)x^5+5\,(9\,Ba^2b^4+Aab^5)x^4+10\,(9\,Ba^3b^3+Aa^2b^4)x^3+10\,(9\,Ba^4b^2}{}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(105*(9*B*a^6 + A*a^5*b + (9*B*a*b^5 + A*b^6)*x^5 + 5*(9*B*a^2*b^
4 + A*a*b^5)*x^4 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*x^3 + 10*(9*B*a^4*b^2 + A*
a^3*b^3)*x^2 + 5*(9*B*a^5*b + A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sq
rt(-a*b)*sqrt(x))/(b*x + a)) + 2*(945*B*a^6*b + 105*A*a^5*b^2 + 15*(193*B*
a^2*b^5 - 7*A*a*b^6)*x^4 + 790*(9*B*a^3*b^4 + A*a^2*b^5)*x^3 + 896*(9*B*a^
4*b^3 + A*a^3*b^4)*x^2 + 490*(9*B*a^5*b^2 + A*a^4*b^3)*x)*sqrt(x))/(a^2*b^
11*x^5 + 5*a^3*b^10*x^4 + 10*a^4*b^9*x^3 + 10*a^5*b^8*x^2 + 5*a^6*b^7*x +
a^7*b^6), -1/1920*(105*(9*B*a^6 + A*a^5*b + (9*B*a*b^5 + A*b^6)*x^5 + 5*(9
*B*a^2*b^4 + A*a*b^5)*x^4 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*x^3 + 10*(9*B*a^4
*b^2 + A*a^3*b^3)*x^2 + 5*(9*B*a^5*b + A*a^4*b^2)*x)*sqrt(a*b)*arctan(sqrt
(a*b)/(b*sqrt(x))) + (945*B*a^6*b + 105*A*a^5*b^2 + 15*(193*B*a^2*b^5 - 7*
A*a*b^6)*x^4 + 790*(9*B*a^3*b^4 + A*a^2*b^5)*x^3 + 896*(9*B*a^4*b^3 + A*a^
3*b^4)*x^2 + 490*(9*B*a^5*b^2 + A*a^4*b^3)*x)*sqrt(x))/(a^2*b^11*x^5 + 5*a
^3*b^10*x^4 + 10*a^4*b^9*x^3 + 10*a^5*b^8*x^2 + 5*a^6*b^7*x + a^7*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = -\frac{15\,(193\,Bab^4-7\,Ab^5)x^{\frac{9}{2}}+790\,(9\,Ba^2b^3+Aab^4)x^{\frac{7}{2}}+896\,(9\,Ba^3b^2+Aa^2b^3)x^{\frac{5}{2}}+490\,(9\,Ba^4b+Aa^3b^2)}{1920\,(ab^{10}x^5+5\,a^2b^9x^4+10\,a^3b^8x^3+10\,a^4b^7x^2+5\,a^5b^6x+a^6b^5)} + \frac{7\,(9\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}ab^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1920*(15*(193*B*a*b^4 - 7*A*b^5)*x^(9/2) + 790*(9*B*a^2*b^3 + A*a*b^4)*
x^(7/2) + 896*(9*B*a^3*b^2 + A*a^2*b^3)*x^(5/2) + 490*(9*B*a^4*b + A*a^3*b
^2)*x^(3/2) + 105*(9*B*a^5 + A*a^4*b)*sqrt(x))/(a*b^10*x^5 + 5*a^2*b^9*x^4
 + 10*a^3*b^8*x^3 + 10*a^4*b^7*x^2 + 5*a^5*b^6*x + a^6*b^5) + 7/128*(9*B*a
 + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^5)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{7\,(9\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}ab^5} - \frac{2895\,Bab^4x^{\frac{9}{2}}-105\,Ab^5x^{\frac{9}{2}}+7110\,Ba^2b^3x^{\frac{7}{2}}+790\,Aab^4x^{\frac{7}{2}}+8064\,Ba^3b^2x^{\frac{5}{2}}+896\,Aa^2b^3x^{\frac{5}{2}}+4410\,Ba^4bx}{1920\,(bx+a)^5ab^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
7/128*(9*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^5) - 1/1920
*(2895*B*a*b^4*x^(9/2) - 105*A*b^5*x^(9/2) + 7110*B*a^2*b^3*x^(7/2) + 790*
A*a*b^4*x^(7/2) + 8064*B*a^3*b^2*x^(5/2) + 896*A*a^2*b^3*x^(5/2) + 4410*B*
a^4*b*x^(3/2) + 490*A*a^3*b^2*x^(3/2) + 945*B*a^5*sqrt(x) + 105*A*a^4*b*sq
rt(x))/((b*x + a)^5*a*b^5)
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,\left(A\,b+9\,B\,a\right)}{128\,a^{3/2}\,b^{11/2}} - \frac{\frac{79\,x^{7/2}\,(A\,b+9\,B\,a)}{192\,b^2}+\frac{49\,a^2\,x^{3/2}\,(A\,b+9\,B\,a)}{192\,b^4}+\frac{7\,a^3\,\sqrt{x}\,(A\,b+9\,B\,a)}{128\,b^5}-\frac{x^{9/2}\,(7\,A\,b-193\,B\,a)}{128\,a\,b}+\frac{7\,a\,x^{5/2}\,(A\,b+9\,B\,a)}{15\,b^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(7*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b + 9*B*a))/(128*a^(3/2)*b^(11/2)) -
 ((79*x^(7/2)*(A*b + 9*B*a))/(192*b^2) + (49*a^2*x^(3/2)*(A*b + 9*B*a))/(1
92*b^4) + (7*a^3*x^(1/2)*(A*b + 9*B*a))/(128*b^5) - (x^(9/2)*(7*A*b - 193*
B*a))/(128*a*b) + (7*a*x^(5/2)*(A*b + 9*B*a))/(15*b^3))/(a^5 + b^5*x^5 + 5
*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+420\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+630\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 420*sqrt(b
)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 630*sqrt(b)*sqrt(
a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 420*sqrt(b)*sqrt(a
)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 + 105*sqrt(b)*sqrt(a)*at
an((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 105*sqrt(x)*a**4*b - 385*sqr
t(x)*a**3*b**2*x - 511*sqrt(x)*a**2*b**3*x**2 - 279*sqrt(x)*a*b**4*x**3)/(
192*a*b**5*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x*
*4))
```

3.405 $\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 185

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{(Ab-aB)x^{5/2}}{5b^2(a+bx)^5} - \frac{(Ab-3aB)x^{3/2}}{8b^3(a+bx)^4} - \frac{(3Ab-25aB)\sqrt{x}}{48b^4(a+bx)^3} + \frac{(3Ab-121aB)\sqrt{x}}{192ab^4(a+bx)^2} + \frac{(3Ab+7aB)\sqrt{x}}{128a^2b^4(a+bx)} + \frac{(3Ab+7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{5/2}b^{9/2}}$$

output

```
-1/5*(A*b-B*a)*x^(5/2)/b^2/(b*x+a)^5-1/8*(A*b-3*B*a)*x^(3/2)/b^3/(b*x+a)^4
-1/48*(3*A*b-25*B*a)*x^(1/2)/b^4/(b*x+a)^3+1/192*(3*A*b-121*B*a)*x^(1/2)/a
/b^4/(b*x+a)^2+1/128*(3*A*b+7*B*a)*x^(1/2)/a^2/b^4/(b*x+a)+1/128*(3*A*b+7*
B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\sqrt{x}(-105a^5B+45Ab^5x^4+105ab^4x^3(2A+Bx)-14a^3b^2x(15A+64Bx)-5}{1920a^2b^4(a+bx)^5}$$
$$+\frac{(3Ab+7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{5/2}b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(Sqrt[x]*(-105*a^5*B + 45*A*b^5*x^4 + 105*a*b^4*x^3*(2*A + B*x) - 14*a^3*b
^2*x*(15*A + 64*B*x) - 5*a^4*b*(9*A + 98*B*x) - 2*a^2*b^3*x^2*(192*A + 395
*B*x)))/(1920*a^2*b^4*(a + b*x)^5) + ((3*A*b + 7*a*B)*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(128*a^(5/2)*b^(9/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 51, 51, 51, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$
$$\Big\downarrow \ 1184$$
$$b^6\int \frac{x^{5/2}(A+Bx)}{b^6(a+bx)^6}dx$$
$$\Big\downarrow \ 27$$
$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^6}dx$$
$$\Big\downarrow \ 87$$

$$\frac{(7aB+3Ab)\int \frac{x^{5/2}}{(a+bx)^5}dx}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{(7aB+3Ab)\left(\frac{5\int \frac{x^{3/2}}{(a+bx)^4}dx}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{(7aB+3Ab)\left(\frac{5\left(\frac{\int \frac{\sqrt{x}}{(a+bx)^3}dx}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{(7aB+3Ab)\left(\frac{5\left(\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(7aB+3Ab)\left(\frac{5\left(\frac{\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{(7aB+3Ab)\left(\frac{5\left(\frac{\frac{\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{218}$$

$$\frac{(7aB+3Ab)\left(\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{5/2}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{7/2}(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(7/2))/(5*a*b*(a + b*x)^5) + ((3*A*b + 7*a*B)*(-1/4*x^(5/2)
/(b*(a + b*x)^4) + (5*(-1/3*x^(3/2)/(b*(a + b*x)^3) + (-1/2*Sqrt[x]/(b*(a
+ b*x)^2) + (Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^
(3/2)*Sqrt[b]))/(4*b))/(2*b)))/(8*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{(3Ab+7Ba)x^{\frac{9}{2}}}{128a^2}+\frac{(21Ab-79Ba)x^{\frac{7}{2}}}{192ab}-\frac{(3Ab+7Ba)x^{\frac{5}{2}}}{15b^2}-\frac{7a(3Ab+7Ba)x^{\frac{3}{2}}}{192b^3}-\frac{(3Ab+7Ba)a^2\sqrt{x}}{128b^4}}{(bx+a)^5}+\frac{(3Ab+7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^2b^4\sqrt{ab}}$
default	$\frac{\frac{(3Ab+7Ba)x^{\frac{9}{2}}}{128a^2}+\frac{(21Ab-79Ba)x^{\frac{7}{2}}}{192ab}-\frac{(3Ab+7Ba)x^{\frac{5}{2}}}{15b^2}-\frac{7a(3Ab+7Ba)x^{\frac{3}{2}}}{192b^3}-\frac{(3Ab+7Ba)a^2\sqrt{x}}{128b^4}}{(bx+a)^5}+\frac{(3Ab+7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^2b^4\sqrt{ab}}$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/256*(3*A*b+7*B*a)/a^2*x^(9/2)+1/384*(21*A*b-79*B*a)/a/b*x^(7/2)-1/30/
b^2*(3*A*b+7*B*a)*x^(5/2)-7/384*a/b^3*(3*A*b+7*B*a)*x^(3/2)-1/256*(3*A*b+7
*B*a)*a^2/b^4*x^(1/2))/(b*x+a)^5+1/128*(3*A*b+7*B*a)/a^2/b^4/(a*b)^(1/2)*a
rctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.55

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \left[-\frac{15\,(7\,Ba^6+3\,Aa^5b+(7\,Bab^5+3\,Ab^6)x^5+5\,(7\,Ba^2b^4+3\,Aab^5)x^4+10\,(7\cdots}{\cdots}\right.$$

$$-\frac{15\,(7\,Ba^6+3\,Aa^5b+(7\,Bab^5+3\,Ab^6)x^5+5\,(7\,Ba^2b^4+3\,Aab^5)x^4+10\,(7\,Ba^3b^3+3\,Aa^2b^4)x^3+10\,(7\,B\cdots}{\cdots}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(7*B*a^6 + 3*A*a^5*b + (7*B*a*b^5 + 3*A*b^6)*x^5 + 5*(7*B*a^2
*b^4 + 3*A*a*b^5)*x^4 + 10*(7*B*a^3*b^3 + 3*A*a^2*b^4)*x^3 + 10*(7*B*a^4*b
^2 + 3*A*a^3*b^3)*x^2 + 5*(7*B*a^5*b + 3*A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x
 - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(105*B*a^6*b + 45*A*a^5*b^2 -
15*(7*B*a^2*b^5 + 3*A*a*b^6)*x^4 + 10*(79*B*a^3*b^4 - 21*A*a^2*b^5)*x^3 +
128*(7*B*a^4*b^3 + 3*A*a^3*b^4)*x^2 + 70*(7*B*a^5*b^2 + 3*A*a^4*b^3)*x)*sq
rt(x))/(a^3*b^10*x^5 + 5*a^4*b^9*x^4 + 10*a^5*b^8*x^3 + 10*a^6*b^7*x^2 + 5
*a^7*b^6*x + a^8*b^5), -1/1920*(15*(7*B*a^6 + 3*A*a^5*b + (7*B*a*b^5 + 3*A
*b^6)*x^5 + 5*(7*B*a^2*b^4 + 3*A*a*b^5)*x^4 + 10*(7*B*a^3*b^3 + 3*A*a^2*b^
4)*x^3 + 10*(7*B*a^4*b^2 + 3*A*a^3*b^3)*x^2 + 5*(7*B*a^5*b + 3*A*a^4*b^2)*
x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (105*B*a^6*b + 45*A*a^5*b^2 -
 15*(7*B*a^2*b^5 + 3*A*a*b^6)*x^4 + 10*(79*B*a^3*b^4 - 21*A*a^2*b^5)*x^3 +
 128*(7*B*a^4*b^3 + 3*A*a^3*b^4)*x^2 + 70*(7*B*a^5*b^2 + 3*A*a^4*b^3)*x)*s
qrt(x))/(a^3*b^10*x^5 + 5*a^4*b^9*x^4 + 10*a^5*b^8*x^3 + 10*a^6*b^7*x^2 +
5*a^7*b^6*x + a^8*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{15\,(7\,Bab^4+3\,Ab^5)x^{\frac{9}{2}}-10\,(79\,Ba^2b^3-21\,Aab^4)x^{\frac{7}{2}}-128\,(7\,Ba^3b^2+3\,Aa^2b^{\cdots}}{1920\,(a^2b^9x^5+5\,a^3b^8x^4+10\,a^4b^7x^3+10\,a^{\cdots}} + \frac{(7\,Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^2b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/1920*(15*(7*B*a*b^4 + 3*A*b^5)*x^(9/2) - 10*(79*B*a^2*b^3 - 21*A*a*b^4)*
x^(7/2) - 128*(7*B*a^3*b^2 + 3*A*a^2*b^3)*x^(5/2) - 70*(7*B*a^4*b + 3*A*a^
3*b^2)*x^(3/2) - 15*(7*B*a^5 + 3*A*a^4*b)*sqrt(x))/(a^2*b^9*x^5 + 5*a^3*b^
8*x^4 + 10*a^4*b^7*x^3 + 10*a^5*b^6*x^2 + 5*a^6*b^5*x + a^7*b^4) + 1/128*(
7*B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b^4)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{(7\,Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^2b^4}$$
$$+\frac{105\,Bab^4x^{\frac{9}{2}}+45\,Ab^5x^{\frac{9}{2}}-790\,Ba^2b^3x^{\frac{7}{2}}+210\,Aab^4x^{\frac{7}{2}}-896\,Ba^3b^2x^{\frac{5}{2}}-384\,Aa^2b^3x^{\frac{5}{2}}-490\,Ba^4bx^{\frac{3}{2}}-21}{1920\,(bx+a)^5a^2b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/128*(7*B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/
1920*(105*B*a*b^4*x^(9/2) + 45*A*b^5*x^(9/2) - 790*B*a^2*b^3*x^(7/2) + 210
*A*a*b^4*x^(7/2) - 896*B*a^3*b^2*x^(5/2) - 384*A*a^2*b^3*x^(5/2) - 490*B*a
^4*b*x^(3/2) - 210*A*a^3*b^2*x^(3/2) - 105*B*a^5*sqrt(x) - 45*A*a^4*b*sqrt
(x))/((b*x + a)^5*a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(3\,A\,b+7\,B\,a)}{128\,a^{5/2}\,b^{9/2}}$$
$$-\frac{\frac{x^{5/2}\,(3\,A\,b+7\,B\,a)}{15\,b^2}-\frac{x^{9/2}\,(3\,A\,b+7\,B\,a)}{128\,a^2}+\frac{a^2\,\sqrt{x}\,(3\,A\,b+7\,B\,a)}{128\,b^4}-\frac{x^{7/2}\,(21\,A\,b-79\,B\,a)}{192\,a\,b}+\frac{7\,a\,x^{3/2}\,(3\,A\,b+7\,B\,a)}{192\,b^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(atan((b^(1/2)*x^(1/2))/a^(1/2))*(3*A*b + 7*B*a))/(128*a^(5/2)*b^(9/2)) -
((x^(5/2)*(3*A*b + 7*B*a))/(15*b^2) - (x^(9/2)*(3*A*b + 7*B*a))/(128*a^2)
+ (a^2*x^(1/2)*(3*A*b + 7*B*a))/(128*b^4) - (x^(7/2)*(21*A*b - 79*B*a))/(1
92*a*b) + (7*a*x^(3/2)*(3*A*b + 7*B*a))/(192*b^3))/(a^5 + b^5*x^5 + 5*a*b^
4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+90\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{}$$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 60*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 90*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 60*sqrt(b)*sqrt(a)*at
an((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 + 15*sqrt(b)*sqrt(a)*atan((s
qrt(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 15*sqrt(x)*a**4*b - 55*sqrt(x)*a*
*3*b**2*x - 73*sqrt(x)*a**2*b**3*x**2 + 15*sqrt(x)*a*b**4*x**3)/(192*a**2*
b**4*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.406 $\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 182

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx &= -\frac{(Ab-aB)x^{3/2}}{5b^2(a+bx)^5}\\ &\quad-\frac{(3Ab-13aB)\sqrt{x}}{40b^3(a+bx)^4}+\frac{(Ab-31aB)\sqrt{x}}{80ab^3(a+bx)^3}+\frac{(Ab+aB)\sqrt{x}}{64a^2b^3(a+bx)^2}\\ &\quad+\frac{3(Ab+aB)\sqrt{x}}{128a^3b^3(a+bx)}+\frac{3(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{7/2}b^{7/2}}\end{aligned}$$

output
```
-1/5*(A*b-B*a)*x^(3/2)/b^2/(b*x+a)^5-1/40*(3*A*b-13*B*a)*x^(1/2)/b^3/(b*x+
a)^4+1/80*(A*b-31*B*a)*x^(1/2)/a/b^3/(b*x+a)^3+1/64*(A*b+B*a)*x^(1/2)/a^2/
b^3/(b*x+a)^2+3/128*(A*b+B*a)*x^(1/2)/a^3/b^3/(b*x+a)+3/128*(A*b+B*a)*arct
an(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\sqrt{x}(-15a^5B+15Ab^5x^4+5ab^4x^3(14A+3Bx)-5a^4b(3A+14Bx)+2a^2b^3x}{640a^3b^3(a+bx)^5} + \frac{3(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{7/2}b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(Sqrt[x]*(-15*a^5*B + 15*A*b^5*x^4 + 5*a*b^4*x^3*(14*A + 3*B*x) - 5*a^4*b*
(3*A + 14*B*x) + 2*a^2*b^3*x^2*(64*A + 35*B*x) - 2*a^3*b^2*x*(35*A + 64*B*
x)))/(640*a^3*b^3*(a + b*x)^5) + (3*(A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(128*a^(7/2)*b^(7/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 51, 51, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}dx$$

$$\Big\downarrow \text{1184}$$

$$b^6\int \frac{x^{3/2}(A+Bx)}{b^6(a+bx)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+Ab)\int \frac{x^{3/2}}{(a+bx)^5}dx}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\downarrow$ 51

$$\frac{(aB+Ab)\left(\frac{3\int \frac{\sqrt{x}}{(a+bx)^4}dx}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\downarrow$ 51

$$\frac{(aB+Ab)\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\downarrow$ 52

$$\frac{(aB+Ab)\left(\frac{3\left(\frac{\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\downarrow$ 52

$$\frac{(aB+Ab)\left(\frac{3\left(\frac{\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\downarrow$ 73

$$\frac{(aB+Ab)\left(\frac{3\left(\frac{3\left(\frac{\int\frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\Big\downarrow$ 218

$$\frac{(aB+Ab)\left(\frac{3\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8b}-\frac{x^{3/2}}{4b(a+bx)^4}\right)}{2ab}+\frac{x^{5/2}(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(5/2))/(5*a*b*(a + b*x)^5) + ((A*b + a*B)*(-1/4*x^(3/2)/(b*
(a + b*x)^4) + (3*(-1/3*Sqrt[x]/(b*(a + b*x)^3) + (Sqrt[x]/(2*a*(a + b*x)^
2) + (3*(Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2
)*Sqrt[b])))/(4*a))/(6*b)))/(8*b)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{3(Ab+Ba)b\,x^{\frac{9}{2}}}{128a^{3}}+\frac{7(Ab+Ba)x^{\frac{7}{2}}}{64a^{2}}+\frac{(Ab-Ba)x^{\frac{5}{2}}}{5ab}-\frac{7(Ab+Ba)x^{\frac{3}{2}}}{64b^{2}}-\frac{3a(Ab+Ba)\sqrt{x}}{128b^{3}}}{(bx+a)^{5}}+\frac{3(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^{3}b^{3}\sqrt{ab}}$	124
default	$\frac{\frac{3(Ab+Ba)b\,x^{\frac{9}{2}}}{128a^{3}}+\frac{7(Ab+Ba)x^{\frac{7}{2}}}{64a^{2}}+\frac{(Ab-Ba)x^{\frac{5}{2}}}{5ab}-\frac{7(Ab+Ba)x^{\frac{3}{2}}}{64b^{2}}-\frac{3a(Ab+Ba)\sqrt{x}}{128b^{3}}}{(bx+a)^{5}}+\frac{3(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^{3}b^{3}\sqrt{ab}}$	124

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/256*(A*b+B*a)/a^3*b*x^(9/2)+7/128*(A*b+B*a)/a^2*x^(7/2)+1/10*(A*b-B*a
)/a/b*x^(5/2)-7/128*(A*b+B*a)/b^2*x^(3/2)-3/256*a/b^3*(A*b+B*a)*x^(1/2))/(
b*x+a)^5+3/128*(A*b+B*a)/a^3/b^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.40

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \left[-\frac{15\,(Ba^6+Aa^5b+(Bab^5+Ab^6)x^5+5\,(Ba^2b^4+Aab^5)x^4+10\,(Ba^3b^3+A\cdots}{\cdots}\right.$$

$$-\frac{15\,(Ba^6+Aa^5b+(Bab^5+Ab^6)x^5+5\,(Ba^2b^4+Aab^5)x^4+10\,(Ba^3b^3+Aa^2b^4)x^3+10\,(Ba^4b^2+Aa^3b^3)\cdots}{\cdots}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/1280*(15*(B*a^6 + A*a^5*b + (B*a*b^5 + A*b^6)*x^5 + 5*(B*a^2*b^4 + A*a
*b^5)*x^4 + 10*(B*a^3*b^3 + A*a^2*b^4)*x^3 + 10*(B*a^4*b^2 + A*a^3*b^3)*x^
2 + 5*(B*a^5*b + A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)) + 2*(15*B*a^6*b + 15*A*a^5*b^2 - 15*(B*a^2*b^5 + A*a*b^6)*
x^4 - 70*(B*a^3*b^4 + A*a^2*b^5)*x^3 + 128*(B*a^4*b^3 - A*a^3*b^4)*x^2 + 7
0*(B*a^5*b^2 + A*a^4*b^3)*x)*sqrt(x))/(a^4*b^9*x^5 + 5*a^5*b^8*x^4 + 10*a^
6*b^7*x^3 + 10*a^7*b^6*x^2 + 5*a^8*b^5*x + a^9*b^4), -1/640*(15*(B*a^6 + A
*a^5*b + (B*a*b^5 + A*b^6)*x^5 + 5*(B*a^2*b^4 + A*a*b^5)*x^4 + 10*(B*a^3*b
^3 + A*a^2*b^4)*x^3 + 10*(B*a^4*b^2 + A*a^3*b^3)*x^2 + 5*(B*a^5*b + A*a^4*
b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (15*B*a^6*b + 15*A*a^5*b
^2 - 15*(B*a^2*b^5 + A*a*b^6)*x^4 - 70*(B*a^3*b^4 + A*a^2*b^5)*x^3 + 128*(
B*a^4*b^3 - A*a^3*b^4)*x^2 + 70*(B*a^5*b^2 + A*a^4*b^3)*x)*sqrt(x))/(a^4*b
^9*x^5 + 5*a^5*b^8*x^4 + 10*a^6*b^7*x^3 + 10*a^7*b^6*x^2 + 5*a^8*b^5*x + a
^9*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{15\,(Bab^4+Ab^5)x^{\frac{9}{2}}+70\,(Ba^2b^3+Aab^4)x^{\frac{7}{2}}-128\,(Ba^3b^2-Aa^2b^3)x^{\frac{5}{2}}-70\,(B}{640\,(a^3b^8x^5+5\,a^4b^7x^4+10\,a^5b^6x^3+10\,a^6b^5x^2+5} + \frac{3\,(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^3b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/640*(15*(B*a*b^4 + A*b^5)*x^(9/2) + 70*(B*a^2*b^3 + A*a*b^4)*x^(7/2) - 1
28*(B*a^3*b^2 - A*a^2*b^3)*x^(5/2) - 70*(B*a^4*b + A*a^3*b^2)*x^(3/2) - 15
*(B*a^5 + A*a^4*b)*sqrt(x))/(a^3*b^8*x^5 + 5*a^4*b^7*x^4 + 10*a^5*b^6*x^3
+ 10*a^6*b^5*x^2 + 5*a^7*b^4*x + a^8*b^3) + 3/128*(B*a + A*b)*arctan(b*sqr
t(x)/sqrt(a*b))/(sqrt(a*b)*a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.85

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{3\,(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^3b^3} + \frac{15\,Bab^4x^{\frac{9}{2}}+15\,Ab^5x^{\frac{9}{2}}+70\,Ba^2b^3x^{\frac{7}{2}}+70\,Aab^4x^{\frac{7}{2}}-128\,Ba^3b^2x^{\frac{5}{2}}+128\,Aa^2b^3x^{\frac{5}{2}}-70\,Ba^4bx^{\frac{3}{2}}-70\,Aa^3}{640\,(bx+a)^5a^3b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
3/128*(B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3*b^3) + 1/640*
(15*B*a*b^4*x^(9/2) + 15*A*b^5*x^(9/2) + 70*B*a^2*b^3*x^(7/2) + 70*A*a*b^4
*x^(7/2) - 128*B*a^3*b^2*x^(5/2) + 128*A*a^2*b^3*x^(5/2) - 70*B*a^4*b*x^(3
/2) - 70*A*a^3*b^2*x^(3/2) - 15*B*a^5*sqrt(x) - 15*A*a^4*b*sqrt(x))/((b*x
+ a)^5*a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{\frac{7\,x^{7/2}\,(A\,b+B\,a)}{64\,a^2}-\frac{7\,x^{3/2}\,(A\,b+B\,a)}{64\,b^2}+\frac{x^{5/2}\,(A\,b-B\,a)}{5\,a\,b}-\frac{3\,a\,\sqrt{x}\,(A\,b+B\,a)}{128\,b^3}+\frac{3\,b\,x^{9/2}\,(A\,b+B\,a)}{128\,a^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b+B\,a)}{128\,a^{7/2}\,b^{7/2}}$$

input
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
((7*x^(7/2)*(A*b + B*a))/(64*a^2) - (7*x^(3/2)*(A*b + B*a))/(64*b^2) + (x^
(5/2)*(A*b - B*a))/(5*a*b) - (3*a*x^(1/2)*(A*b + B*a))/(128*b^3) + (3*b*x^
(9/2)*(A*b + B*a))/(128*a^3))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^
2 + 10*a^2*b^3*x^3 + 5*a^4*b*x) + (3*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b
+ B*a))/(128*a^(7/2)*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+18\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right.}{6}$$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 12*sqrt(b)*s
qrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 18*sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 12*sqrt(b)*sqrt(a)*ata
n((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 + 3*sqrt(b)*sqrt(a)*atan((sqr
t(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 3*sqrt(x)*a**4*b - 11*sqrt(x)*a**3*
b**2*x + 11*sqrt(x)*a**2*b**3*x**2 + 3*sqrt(x)*a*b**4*x**3)/(64*a**3*b**3*
(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.407 $\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3} dx = -\frac{(Ab-aB)\sqrt{x}}{5b^2(a+bx)^5}+\frac{(Ab-11aB)\sqrt{x}}{40ab^2(a+bx)^4}+\frac{(7Ab+3aB)\sqrt{x}}{240a^2b^2(a+bx)^3}+\frac{(7Ab+3aB)\sqrt{x}}{192a^3b^2(a+bx)^2}+\frac{(7Ab+3aB)\sqrt{x}}{128a^4b^2(a+bx)}+\frac{(7Ab+3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{9/2}b^{5/2}}$$

output
```
-1/5*(A*b-B*a)*x^(1/2)/b^2/(b*x+a)^5+1/40*(A*b-11*B*a)*x^(1/2)/a/b^2/(b*x+
a)^4+1/240*(7*A*b+3*B*a)*x^(1/2)/a^2/b^2/(b*x+a)^3+1/192*(7*A*b+3*B*a)*x^(
1/2)/a^3/b^2/(b*x+a)^2+1/128*(7*A*b+3*B*a)*x^(1/2)/a^4/b^2/(b*x+a)+1/128*(
7*A*b+3*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{\sqrt{x}(-45a^5B+105Ab^5x^4-105a^4b(A+2Bx)+5ab^4x^3(98A+9Bx)+14a^2b^3x^2(64A+15Bx)+2a^3b^2}{1920a^4b^2(a+bx)^5}$$
$$+\frac{(7Ab+3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{9/2}b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(Sqrt[x]*(-45*a^5*B + 105*A*b^5*x^4 - 105*a^4*b*(A + 2*B*x) + 5*a*b^4*x^3*
(98*A + 9*B*x) + 14*a^2*b^3*x^2*(64*A + 15*B*x) + 2*a^3*b^2*x*(395*A + 192
*B*x)))/(1920*a^4*b^2*(a + b*x)^5) + ((7*A*b + 3*a*B)*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(128*a^(9/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1184, 27, 87, 51, 52, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$\Big\downarrow \text{1184}$$
$$b^6\int \frac{\sqrt{x}(A+Bx)}{b^6(a+bx)^6}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(3aB+7Ab)\int \frac{\sqrt{x}}{(a+bx)^5}dx}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{(3aB+7Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)^4}dx}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(3aB+7Ab)\left(\frac{\frac{5\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(3aB+7Ab)\left(\frac{\frac{5\left(\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(3aB+7Ab)\left(\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{(3aB+7Ab)\left(\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

$\Big\downarrow$ 218

$$\frac{(3aB+7Ab)\left(\frac{\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}}{8b}-\frac{\sqrt{x}}{4b(a+bx)^4}\right)}{10ab}+\frac{x^{3/2}(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(5*a*b*(a + b*x)^5) + ((7*A*b + 3*a*B)*(-1/4*Sqrt[x]
/(b*(a + b*x)^4) + (Sqrt[x]/(3*a*(a + b*x)^3) + (5*(Sqrt[x]/(2*a*(a + b*x)
^2) + (3*(Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/
2)*Sqrt[b])))/(4*a)))/(6*a))/(8*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{(7Ab+3Ba)b^2x^{\frac{9}{2}}}{128a^4}+\frac{7b(7Ab+3Ba)x^{\frac{7}{2}}}{192a^3}+\frac{(7Ab+3Ba)x^{\frac{5}{2}}}{15a^2}+\frac{(79Ab-21Ba)x^{\frac{3}{2}}}{192ab}-\frac{(7Ab+3Ba)\sqrt{x}}{128b^2}}{(bx+a)^5}+\frac{(7Ab+3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^4b^2\sqrt{ab}}$
default	$\frac{\frac{(7Ab+3Ba)b^2x^{\frac{9}{2}}}{128a^4}+\frac{7b(7Ab+3Ba)x^{\frac{7}{2}}}{192a^3}+\frac{(7Ab+3Ba)x^{\frac{5}{2}}}{15a^2}+\frac{(79Ab-21Ba)x^{\frac{3}{2}}}{192ab}-\frac{(7Ab+3Ba)\sqrt{x}}{128b^2}}{(bx+a)^5}+\frac{(7Ab+3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^4b^2\sqrt{ab}}$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/256*(7*A*b+3*B*a)/a^4*b^2*x^(9/2)+7/384/a^3*b*(7*A*b+3*B*a)*x^(7/2)+1
/30/a^2*(7*A*b+3*B*a)*x^(5/2)+1/384*(79*A*b-21*B*a)/a/b*x^(3/2)-1/256*(7*A
*b+3*B*a)/b^2*x^(1/2))/(b*x+a)^5+1/128*(7*A*b+3*B*a)/a^4/b^2/(a*b)^(1/2)*a
rctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.44

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$

$$= \left[-\frac{15\,(3\,Ba^6+7\,Aa^5b+(3\,Bab^5+7\,Ab^6)x^5+5\,(3\,Ba^2b^4+7\,Aab^5)x^4+10\,(3\,Ba^3b^3+7\,Aa^2b^4)x^3+10}{}\right.$$

$$-\frac{15\,(3\,Ba^6+7\,Aa^5b+(3\,Bab^5+7\,Ab^6)x^5+5\,(3\,Ba^2b^4+7\,Aab^5)x^4+10\,(3\,Ba^3b^3+7\,Aa^2b^4)x^3+10\,(}{}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(3*B*a^6 + 7*A*a^5*b + (3*B*a*b^5 + 7*A*b^6)*x^5 + 5*(3*B*a^2
*b^4 + 7*A*a*b^5)*x^4 + 10*(3*B*a^3*b^3 + 7*A*a^2*b^4)*x^3 + 10*(3*B*a^4*b
^2 + 7*A*a^3*b^3)*x^2 + 5*(3*B*a^5*b + 7*A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x
 - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(45*B*a^6*b + 105*A*a^5*b^2 -
15*(3*B*a^2*b^5 + 7*A*a*b^6)*x^4 - 70*(3*B*a^3*b^4 + 7*A*a^2*b^5)*x^3 - 12
8*(3*B*a^4*b^3 + 7*A*a^3*b^4)*x^2 + 10*(21*B*a^5*b^2 - 79*A*a^4*b^3)*x)*sq
rt(x))/(a^5*b^8*x^5 + 5*a^6*b^7*x^4 + 10*a^7*b^6*x^3 + 10*a^8*b^5*x^2 + 5*
a^9*b^4*x + a^10*b^3), -1/1920*(15*(3*B*a^6 + 7*A*a^5*b + (3*B*a*b^5 + 7*A
*b^6)*x^5 + 5*(3*B*a^2*b^4 + 7*A*a*b^5)*x^4 + 10*(3*B*a^3*b^3 + 7*A*a^2*b^
4)*x^3 + 10*(3*B*a^4*b^2 + 7*A*a^3*b^3)*x^2 + 5*(3*B*a^5*b + 7*A*a^4*b^2)*
x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (45*B*a^6*b + 105*A*a^5*b^2 -
 15*(3*B*a^2*b^5 + 7*A*a*b^6)*x^4 - 70*(3*B*a^3*b^4 + 7*A*a^2*b^5)*x^3 - 1
28*(3*B*a^4*b^3 + 7*A*a^3*b^4)*x^2 + 10*(21*B*a^5*b^2 - 79*A*a^4*b^3)*x)*s
qrt(x))/(a^5*b^8*x^5 + 5*a^6*b^7*x^4 + 10*a^7*b^6*x^3 + 10*a^8*b^5*x^2 + 5
*a^9*b^4*x + a^10*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4933 vs. $2(185) = 370$.

Time = 131.44 (sec) , antiderivative size = 4933, normalized size of antiderivative = 25.83

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*x**(3/2)/3 + 2*B*x**(5/2)/5)/a**6, Eq(b, 0)), ((-2*A/(9*x**(9/2))
 - 2*B/(7*x**(7/2)))/b**6, Eq(a, 0)), (105*A*a**5*b*log(sqrt(x) - sqrt(-a/
b))/(3840*a**9*b**3*sqrt(-a/b) + 19200*a**8*b**4*x*sqrt(-a/b) + 38400*a**7
*b**5*x**2*sqrt(-a/b) + 38400*a**6*b**6*x**3*sqrt(-a/b) + 19200*a**5*b**7*
x**4*sqrt(-a/b) + 3840*a**4*b**8*x**5*sqrt(-a/b)) - 105*A*a**5*b*log(sqrt(
x) + sqrt(-a/b))/(3840*a**9*b**3*sqrt(-a/b) + 19200*a**8*b**4*x*sqrt(-a/b)
 + 38400*a**7*b**5*x**2*sqrt(-a/b) + 38400*a**6*b**6*x**3*sqrt(-a/b) + 192
00*a**5*b**7*x**4*sqrt(-a/b) + 3840*a**4*b**8*x**5*sqrt(-a/b)) - 210*A*a**
4*b**2*sqrt(x)*sqrt(-a/b)/(3840*a**9*b**3*sqrt(-a/b) + 19200*a**8*b**4*x*s
qrt(-a/b) + 38400*a**7*b**5*x**2*sqrt(-a/b) + 38400*a**6*b**6*x**3*sqrt(-a
/b) + 19200*a**5*b**7*x**4*sqrt(-a/b) + 3840*a**4*b**8*x**5*sqrt(-a/b)) +
525*A*a**4*b**2*x*log(sqrt(x) - sqrt(-a/b))/(3840*a**9*b**3*sqrt(-a/b) + 1
9200*a**8*b**4*x*sqrt(-a/b) + 38400*a**7*b**5*x**2*sqrt(-a/b) + 38400*a**6
*b**6*x**3*sqrt(-a/b) + 19200*a**5*b**7*x**4*sqrt(-a/b) + 3840*a**4*b**8*x
**5*sqrt(-a/b)) - 525*A*a**4*b**2*x*log(sqrt(x) + sqrt(-a/b))/(3840*a**9*b
**3*sqrt(-a/b) + 19200*a**8*b**4*x*sqrt(-a/b) + 38400*a**7*b**5*x**2*sqrt(
-a/b) + 38400*a**6*b**6*x**3*sqrt(-a/b) + 19200*a**5*b**7*x**4*sqrt(-a/b)
+ 3840*a**4*b**8*x**5*sqrt(-a/b)) + 1580*A*a**3*b**3*x**(3/2)*sqrt(-a/b)/(
3840*a**9*b**3*sqrt(-a/b) + 19200*a**8*b**4*x*sqrt(-a/b) + 38400*a**7*b...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{15\,(3\,Bab^4+7\,Ab^5)x^{\frac{9}{2}}+70\,(3\,Ba^2b^3+7\,Aab^4)x^{\frac{7}{2}}+128\,(3\,Ba^3b^2+7\,Aa^2b^3)x^{\frac{5}{2}}-10\,(21\,Ba^4b-79\,Aa^3b}{1920\,(a^4b^7x^5+5\,a^5b^6x^4+10\,a^6b^5x^3+10\,a^7b^4x^2+5\,a^8b^3x+a^9b^2)}$$
$$+\frac{(3\,Ba+7\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^4b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/1920*(15*(3*B*a*b^4 + 7*A*b^5)*x^(9/2) + 70*(3*B*a^2*b^3 + 7*A*a*b^4)*x^
(7/2) + 128*(3*B*a^3*b^2 + 7*A*a^2*b^3)*x^(5/2) - 10*(21*B*a^4*b - 79*A*a^
3*b^2)*x^(3/2) - 15*(3*B*a^5 + 7*A*a^4*b)*sqrt(x))/(a^4*b^7*x^5 + 5*a^5*b^
6*x^4 + 10*a^6*b^5*x^3 + 10*a^7*b^4*x^2 + 5*a^8*b^3*x + a^9*b^2) + 1/128*(
3*B*a + 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4*b^2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{(3\,Ba+7\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^4b^2} + \frac{45\,Bab^4x^{\frac{9}{2}}+105\,Ab^5x^{\frac{9}{2}}+210\,Ba^2b^3x^{\frac{7}{2}}+490\,Aab^4x^{\frac{7}{2}}+384\,Ba^3b^2x^{\frac{5}{2}}+896\,Aa^2b^3x^{\frac{5}{2}}-210\,Ba^4bx^{\frac{3}{2}}+}{1920\,(bx+a)^5a^4b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
1/128*(3*B*a + 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4*b^2) + 1/
1920*(45*B*a*b^4*x^(9/2) + 105*A*b^5*x^(9/2) + 210*B*a^2*b^3*x^(7/2) + 490
*A*a*b^4*x^(7/2) + 384*B*a^3*b^2*x^(5/2) + 896*A*a^2*b^3*x^(5/2) - 210*B*a
^4*b*x^(3/2) + 790*A*a^3*b^2*x^(3/2) - 45*B*a^5*sqrt(x) - 105*A*a^4*b*sqrt
(x))/((b*x + a)^5*a^4*b^2)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{x^{5/2}\,(7\,A\,b+3\,B\,a)}{15\,a^2}-\frac{\sqrt{x}\,(7\,A\,b+3\,B\,a)}{128\,b^2}+\frac{b^2\,x^{9/2}\,(7\,A\,b+3\,B\,a)}{128\,a^4}+\frac{x^{3/2}\,(79\,A\,b-21\,B\,a)}{192\,a\,b}+\frac{7\,b\,x^{7/2}\,(7\,A\,b+3\,B\,a)}{192\,a^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(7\,A\,b+3\,B\,a)}{128\,a^{9/2}\,b^{5/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
((x^(5/2)*(7*A*b + 3*B*a))/(15*a^2) - (x^(1/2)*(7*A*b + 3*B*a))/(128*b^2)
+ (b^2*x^(9/2)*(7*A*b + 3*B*a))/(128*a^4) + (x^(3/2)*(79*A*b - 21*B*a))/(1
92*a*b) + (7*b*x^(7/2)*(7*A*b + 3*B*a))/(192*a^3))/(a^5 + b^5*x^5 + 5*a*b^
4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x) + (atan((b^(1/2)*x^(1
/2))/a^(1/2))*(7*A*b + 3*B*a))/(128*a^(9/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^3}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+90\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2+60\sqrt{b}\,\sqrt{a}\,atan}{192a^4b^2\left(b^4x^4+4a\,b^3x^3+\right.}$$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 60*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 90*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 60*sqrt(b)*sqrt(a)*at
an((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 + 15*sqrt(b)*sqrt(a)*atan((s
qrt(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 15*sqrt(x)*a**4*b + 73*sqrt(x)*a*
*3*b**2*x + 55*sqrt(x)*a**2*b**3*x**2 + 15*sqrt(x)*a*b**4*x**3)/(192*a**4*
b**2*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.408 $\int \frac{A+Bx}{\sqrt{x}(a^2+2abx+b^2x^2)^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 190

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^3} \, dx = \frac{(Ab-aB)\sqrt{x}}{5ab(a+bx)^5}+\frac{(9Ab+aB)\sqrt{x}}{40a^2b(a+bx)^4} + \frac{7(9Ab+aB)\sqrt{x}}{240a^3b(a+bx)^3}+\frac{7(9Ab+aB)\sqrt{x}}{192a^4b(a+bx)^2} + \frac{7(9Ab+aB)\sqrt{x}}{128a^5b(a+bx)}+\frac{7(9Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{11/2}b^{3/2}}$$

output
```
1/5*(A*b-B*a)*x^(1/2)/a/b/(b*x+a)^5+1/40*(9*A*b+B*a)*x^(1/2)/a^2/b/(b*x+a)
^4+7/240*(9*A*b+B*a)*x^(1/2)/a^3/b/(b*x+a)^3+7/192*(9*A*b+B*a)*x^(1/2)/a^4
/b/(b*x+a)^2+7/128*(9*A*b+B*a)*x^(1/2)/a^5/b/(b*x+a)+7/128*(9*A*b+B*a)*arc
tan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$=\frac{\sqrt{x}(-105a^5B+945Ab^5x^4+105ab^4x^3(42A+Bx)+14a^2b^3x^2(576A+35Bx)+5a^4b(579A+158Bx)+}{1920a^5b(a+bx)^5}$$
$$+\frac{7(9Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{11/2}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
(Sqrt[x]*(-105*a^5*B + 945*A*b^5*x^4 + 105*a*b^4*x^3*(42*A + B*x) + 14*a^2
*b^3*x^2*(576*A + 35*B*x) + 5*a^4*b*(579*A + 158*B*x) + 2*a^3*b^2*x*(3555*
A + 448*B*x)))/(1920*a^5*b*(a + b*x)^5) + (7*(9*A*b + a*B)*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(128*a^(11/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1184, 27, 87, 52, 52, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^3}\,dx$$
$$\Big\downarrow \text{1184}$$
$$b^6\int \frac{A+Bx}{b^6\sqrt{x}(a+bx)^6}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^6}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+9Ab)\int \frac{1}{\sqrt{x}(a+bx)^5}dx}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(aB+9Ab)\left(\frac{7\int \frac{1}{\sqrt{x}(a+bx)^4}dx}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(aB+9Ab)\left(\frac{7\left(\frac{5\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(aB+9Ab)\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(aB+9Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$\Big\downarrow$ 73

$$\frac{(aB+9Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

$\Big\downarrow$ 218

$$\frac{(aB+9Ab)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8a}+\frac{\sqrt{x}}{4a(a+bx)^4}\right)}{10ab}+\frac{\sqrt{x}(Ab-aB)}{5ab(a+bx)^5}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(5*a*b*(a + b*x)^5) + ((9*A*b + a*B)*(Sqrt[x]/(4*a*(
a + b*x)^4) + (7*(Sqrt[x]/(3*a*(a + b*x)^3) + (5*(Sqrt[x]/(2*a*(a + b*x)^2
) + (3*(Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)
*Sqrt[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{7(9Ab+Ba)b^3x^{\frac{9}{2}}}{128a^5}+\frac{49b^2(9Ab+Ba)x^{\frac{7}{2}}}{192a^4}+\frac{7(9Ab+Ba)b\,x^{\frac{5}{2}}}{15a^3}+\frac{79(9Ab+Ba)x^{\frac{3}{2}}}{192a^2}+\frac{(193Ab-7Ba)\sqrt{x}}{128ab}}{(bx+a)^5}+\frac{7(9Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^5b\sqrt{ab}}$
default	$\frac{\frac{7(9Ab+Ba)b^3x^{\frac{9}{2}}}{128a^5}+\frac{49b^2(9Ab+Ba)x^{\frac{7}{2}}}{192a^4}+\frac{7(9Ab+Ba)b\,x^{\frac{5}{2}}}{15a^3}+\frac{79(9Ab+Ba)x^{\frac{3}{2}}}{192a^2}+\frac{(193Ab-7Ba)\sqrt{x}}{128ab}}{(bx+a)^5}+\frac{7(9Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128a^5b\sqrt{ab}}$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(7/256*(9*A*b+B*a)/a^5*b^3*x^(9/2)+49/384/a^4*b^2*(9*A*b+B*a)*x^(7/2)+7/
30/a^3*(9*A*b+B*a)*b*x^(5/2)+79/384/a^2*(9*A*b+B*a)*x^(3/2)+1/256*(193*A*b
-7*B*a)/a/b*x^(1/2))/(b*x+a)^5+7/128*(9*A*b+B*a)/a^5/b/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.35

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx$$

$$= \left[-\frac{105\,(Ba^6+9\,Aa^5b+(Bab^5+9\,Ab^6)x^5+5\,(Ba^2b^4+9\,Aab^5)x^4+10\,(Ba^3b^3+9\,Aa^2b^4)x^3+10\,(Ba^4 \cdots}{\cdots}\right.$$

$$-\frac{105\,(Ba^6+9\,Aa^5b+(Bab^5+9\,Ab^6)x^5+5\,(Ba^2b^4+9\,Aab^5)x^4+10\,(Ba^3b^3+9\,Aa^2b^4)x^3+10\,(Ba^4b \cdots}{\cdots}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(105*(B*a^6 + 9*A*a^5*b + (B*a*b^5 + 9*A*b^6)*x^5 + 5*(B*a^2*b^4
+ 9*A*a*b^5)*x^4 + 10*(B*a^3*b^3 + 9*A*a^2*b^4)*x^3 + 10*(B*a^4*b^2 + 9*A*
a^3*b^3)*x^2 + 5*(B*a^5*b + 9*A*a^4*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sq
rt(-a*b)*sqrt(x))/(b*x + a)) + 2*(105*B*a^6*b - 2895*A*a^5*b^2 - 105*(B*a^
2*b^5 + 9*A*a*b^6)*x^4 - 490*(B*a^3*b^4 + 9*A*a^2*b^5)*x^3 - 896*(B*a^4*b^
3 + 9*A*a^3*b^4)*x^2 - 790*(B*a^5*b^2 + 9*A*a^4*b^3)*x)*sqrt(x))/(a^6*b^7*
x^5 + 5*a^7*b^6*x^4 + 10*a^8*b^5*x^3 + 10*a^9*b^4*x^2 + 5*a^10*b^3*x + a^1
1*b^2), -1/1920*(105*(B*a^6 + 9*A*a^5*b + (B*a*b^5 + 9*A*b^6)*x^5 + 5*(B*a
^2*b^4 + 9*A*a*b^5)*x^4 + 10*(B*a^3*b^3 + 9*A*a^2*b^4)*x^3 + 10*(B*a^4*b^2
 + 9*A*a^3*b^3)*x^2 + 5*(B*a^5*b + 9*A*a^4*b^2)*x)*sqrt(a*b)*arctan(sqrt(a
*b)/(b*sqrt(x))) + (105*B*a^6*b - 2895*A*a^5*b^2 - 105*(B*a^2*b^5 + 9*A*a*
b^6)*x^4 - 490*(B*a^3*b^4 + 9*A*a^2*b^5)*x^3 - 896*(B*a^4*b^3 + 9*A*a^3*b^
4)*x^2 - 790*(B*a^5*b^2 + 9*A*a^4*b^3)*x)*sqrt(x))/(a^6*b^7*x^5 + 5*a^7*b^
6*x^4 + 10*a^8*b^5*x^3 + 10*a^9*b^4*x^2 + 5*a^10*b^3*x + a^11*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(1/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx = \frac{105\,(Bab^4+9\,Ab^5)x^{\frac{9}{2}}+490\,(Ba^2b^3+9\,Aab^4)x^{\frac{7}{2}}+896\,(Ba^3b^2+9\,Aa^2b^3)x^{\frac{5}{2}}+790\,(Ba^4b+9\,Aa^3b^2)x^{\frac{3}{2}}}{1920\,(a^5b^6x^5+5\,a^6b^5x^4+10\,a^7b^4x^3+10\,a^8b^3x^2+5\,a^9b^2x+a^{10}b)} + \frac{7\,(Ba+9\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^5b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
1/1920*(105*(B*a*b^4 + 9*A*b^5)*x^(9/2) + 490*(B*a^2*b^3 + 9*A*a*b^4)*x^(7
/2) + 896*(B*a^3*b^2 + 9*A*a^2*b^3)*x^(5/2) + 790*(B*a^4*b + 9*A*a^3*b^2)*
x^(3/2) - 15*(7*B*a^5 - 193*A*a^4*b)*sqrt(x))/(a^5*b^6*x^5 + 5*a^6*b^5*x^4
 + 10*a^7*b^4*x^3 + 10*a^8*b^3*x^2 + 5*a^9*b^2*x + a^10*b) + 7/128*(B*a +
9*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx = \frac{7\,(Ba+9\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^5b} + \frac{105\,Bab^4x^{\frac{9}{2}}+945\,Ab^5x^{\frac{9}{2}}+490\,Ba^2b^3x^{\frac{7}{2}}+4410\,Aab^4x^{\frac{7}{2}}+896\,Ba^3b^2x^{\frac{5}{2}}+8064\,Aa^2b^3x^{\frac{5}{2}}+790\,Ba^4bx}{1920\,(bx+a)^5a^5b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
7/128*(B*a + 9*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5*b) + 1/1920
*(105*B*a*b^4*x^(9/2) + 945*A*b^5*x^(9/2) + 490*B*a^2*b^3*x^(7/2) + 4410*A
*a*b^4*x^(7/2) + 896*B*a^3*b^2*x^(5/2) + 8064*A*a^2*b^3*x^(5/2) + 790*B*a^
4*b*x^(3/2) + 7110*A*a^3*b^2*x^(3/2) - 105*B*a^5*sqrt(x) + 2895*A*a^4*b*sq
rt(x))/((b*x + a)^5*a^5*b)
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{\frac{79\,x^{3/2}\,(9\,A\,b+B\,a)}{192\,a^2}+\frac{49\,b^2\,x^{7/2}\,(9\,A\,b+B\,a)}{192\,a^4}+\frac{7\,b^3\,x^{9/2}\,(9\,A\,b+B\,a)}{128\,a^5}+\frac{\sqrt{x}\,(193\,A\,b-7\,B\,a)}{128\,a\,b}+\frac{7\,b\,x^{5/2}\,(9\,A\,b+B\,a)}{15\,a^3}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$
$$+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(9\,A\,b+B\,a)}{128\,a^{11/2}\,b^{3/2}}$$

input
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
((79*x^(3/2)*(9*A*b + B*a))/(192*a^2) + (49*b^2*x^(7/2)*(9*A*b + B*a))/(19
2*a^4) + (7*b^3*x^(9/2)*(9*A*b + B*a))/(128*a^5) + (x^(1/2)*(193*A*b - 7*B
*a))/(128*a*b) + (7*b*x^(5/2)*(9*A*b + B*a))/(15*a^3))/(a^5 + b^5*x^5 + 5*
a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x) + (7*atan((b^(1/2
)*x^(1/2))/a^(1/2))*(9*A*b + B*a))/(128*a^(11/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^3}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+420\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+630\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2+420\sqrt{b}\,\sqrt{a}}{192a^5b\,(b^4x^4+4a\,b^3}$$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 420*sqrt(b
)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 630*sqrt(b)*sqrt(
a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 420*sqrt(b)*sqrt(a
)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 + 105*sqrt(b)*sqrt(a)*at
an((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**4*x**4 + 279*sqrt(x)*a**4*b + 511*sqr
t(x)*a**3*b**2*x + 385*sqrt(x)*a**2*b**3*x**2 + 105*sqrt(x)*a*b**4*x**3)/(
192*a**5*b*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x*
*4))
```

3.409 $\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{2A}{a^6\sqrt{x}}-\frac{(Ab-aB)\sqrt{x}}{5a^2(a+bx)^5}-\frac{(19Ab-9aB)\sqrt{x}}{40a^3(a+bx)^4}-\frac{(71Ab-21aB)\sqrt{x}}{80a^4(a+bx)^3}-\frac{(103Ab-21aB)\sqrt{x}}{64a^5(a+bx)^2}-\frac{(437Ab-63aB)\sqrt{x}}{128a^6(a+bx)}-\frac{63(11Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{b}}$$

output
```
-2*A/a^6/x^(1/2)-1/5*(A*b-B*a)*x^(1/2)/a^2/(b*x+a)^5-1/40*(19*A*b-9*B*a)*x
^(1/2)/a^3/(b*x+a)^4-1/80*(71*A*b-21*B*a)*x^(1/2)/a^4/(b*x+a)^3-1/64*(103*
A*b-21*B*a)*x^(1/2)/a^5/(b*x+a)^2-1/128*(437*A*b-63*B*a)*x^(1/2)/a^6/(b*x+
a)-63/128*(11*A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(13/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{\sqrt{a}(-3465Ab^5x^5+105ab^4x^4(-154A+3Bx)+42a^2b^3x^3(-704A+35Bx)+6a^3b^2x^2(-4345A+448Bx)}{\sqrt{x}(a+bx)^5}}{640a^{13/2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
((Sqrt[a]*(-3465*A*b^5*x^5 + 105*a*b^4*x^4*(-154*A + 3*B*x) + 42*a^2*b^3*x
^3*(-704*A + 35*B*x) + 6*a^3*b^2*x^2*(-4345*A + 448*B*x) + 5*a^4*b*x*(-212
3*A + 474*B*x) + a^5*(-1280*A + 965*B*x)))/(Sqrt[x]*(a + b*x)^5) + (315*(-
11*A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[b])/(640*a^(13/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1184, 27, 87, 52, 52, 52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx$$

$$\Big\downarrow \; 1184$$

$$b^6\int \frac{A+Bx}{b^6x^{3/2}(a+bx)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^6}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(11Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^5}dx}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(11Ab-aB)\left(\frac{9\int \frac{1}{x^{3/2}(a+bx)^4}dx}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\int \frac{1}{x^{3/2}(a+bx)^3}dx}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}dx}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$\big\downarrow$ 61

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$$\Big\downarrow \; 73$$

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

$\downarrow$ 218

$$\frac{(11Ab-aB)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8a}+\frac{1}{4a\sqrt{x}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5ab\sqrt{x}(a+bx)^5}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
(A*b - a*B)/(5*a*b*Sqrt[x]*(a + b*x)^5) + ((11*A*b - a*B)*(1/(4*a*Sqrt[x]*
(a + b*x)^4) + (9*(1/(3*a*Sqrt[x]*(a + b*x)^3) + (7*(1/(2*a*Sqrt[x]*(a + b
*x)^2) + (5*(1/(a*Sqrt[x]*(a + b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)))/(8*a)))/
(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{2\Big(\frac{(\frac{437}{256} A b^5-\frac{63}{256} B a b^4) x^{\frac{9}{2}}+\frac{a b^3 (977 A b-147 B a) x^{\frac{7}{2}}}{128}+(\frac{131}{10} A a^2 b^3-\frac{21}{10} B a^3 b^2) x^{\frac{5}{2}}+(\frac{1327}{128} a^3 A b^2-\frac{237}{128} B a^4 b) x^{\frac{3}{2}}+(\frac{843}{256} A}{(bx+a)^5}}{a^6}$
default	$-\frac{2\Big(\frac{(\frac{437}{256} A b^5-\frac{63}{256} B a b^4) x^{\frac{9}{2}}+\frac{a b^3 (977 A b-147 B a) x^{\frac{7}{2}}}{128}+(\frac{131}{10} A a^2 b^3-\frac{21}{10} B a^3 b^2) x^{\frac{5}{2}}+(\frac{1327}{128} a^3 A b^2-\frac{237}{128} B a^4 b) x^{\frac{3}{2}}+(\frac{843}{256} A}{(bx+a)^5}}{a^6}$
risch	$-\frac{2A}{a^6\sqrt{x}}-\frac{\frac{2(\frac{437}{256} A b^5-\frac{63}{256} B a b^4) x^{\frac{9}{2}}+\frac{a b^3 (977 A b-147 B a) x^{\frac{7}{2}}}{64}+2(\frac{131}{10} A a^2 b^3-\frac{21}{10} B a^3 b^2) x^{\frac{5}{2}}+2(\frac{1327}{128} a^3 A b^2-\frac{237}{128} B a^4 b) x}{(bx+a)^5}}{a^6}$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^6*(((437/256*A*b^5-63/256*B*a*b^4)*x^(9/2)+1/128*a*b^3*(977*A*b-147*B
*a)*x^(7/2)+(131/10*A*a^2*b^3-21/10*B*a^3*b^2)*x^(5/2)+(1327/128*a^3*A*b^2
-237/128*B*a^4*b)*x^(3/2)+(843/256*A*a^4*b-193/256*B*a^5)*x^(1/2))/(b*x+a)
^5+63/256*(11*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))-2*A/a^6/
x^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(156) = 312$.

Time = 0.10 (sec) , antiderivative size = 673, normalized size of antiderivative = 3.52

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \left[\frac{315\left((Bab^5-11\,Ab^6)x^6+5\,(Ba^2b^4-11\,Aab^5)x^5+10\,(Ba^3b^3-11\,Aa^2\right.}{}\right.$$

$$-\frac{315\left((Bab^5-11\,Ab^6)x^6+5\,(Ba^2b^4-11\,Aab^5)x^5+10\,(Ba^3b^3-11\,Aa^2b^4)x^4+10\,(Ba^4b^2-11\,Aa^3b^3)x^3\right.}{}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[1/1280*(315*((B*a*b^5 - 11*A*b^6)*x^6 + 5*(B*a^2*b^4 - 11*A*a*b^5)*x^5 +
10*(B*a^3*b^3 - 11*A*a^2*b^4)*x^4 + 10*(B*a^4*b^2 - 11*A*a^3*b^3)*x^3 + 5*
(B*a^5*b - 11*A*a^4*b^2)*x^2 + (B*a^6 - 11*A*a^5*b)*x)*sqrt(-a*b)*log((b*x
 - a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) - 2*(1280*A*a^6*b - 315*(B*a^2*b^5
 - 11*A*a*b^6)*x^5 - 1470*(B*a^3*b^4 - 11*A*a^2*b^5)*x^4 - 2688*(B*a^4*b^3
 - 11*A*a^3*b^4)*x^3 - 2370*(B*a^5*b^2 - 11*A*a^4*b^3)*x^2 - 965*(B*a^6*b
- 11*A*a^5*b^2)*x)*sqrt(x))/(a^7*b^6*x^6 + 5*a^8*b^5*x^5 + 10*a^9*b^4*x^4
+ 10*a^10*b^3*x^3 + 5*a^11*b^2*x^2 + a^12*b*x), -1/640*(315*((B*a*b^5 - 11
*A*b^6)*x^6 + 5*(B*a^2*b^4 - 11*A*a*b^5)*x^5 + 10*(B*a^3*b^3 - 11*A*a^2*b^
4)*x^4 + 10*(B*a^4*b^2 - 11*A*a^3*b^3)*x^3 + 5*(B*a^5*b - 11*A*a^4*b^2)*x^
2 + (B*a^6 - 11*A*a^5*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (128
0*A*a^6*b - 315*(B*a^2*b^5 - 11*A*a*b^6)*x^5 - 1470*(B*a^3*b^4 - 11*A*a^2*
b^5)*x^4 - 2688*(B*a^4*b^3 - 11*A*a^3*b^4)*x^3 - 2370*(B*a^5*b^2 - 11*A*a^
4*b^3)*x^2 - 965*(B*a^6*b - 11*A*a^5*b^2)*x)*sqrt(x))/(a^7*b^6*x^6 + 5*a^8
*b^5*x^5 + 10*a^9*b^4*x^4 + 10*a^10*b^3*x^3 + 5*a^11*b^2*x^2 + a^12*b*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(3/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx =$$
$$-\frac{1280\,Aa^5-315\,(Bab^4-11\,Ab^5)x^5-1470\,(Ba^2b^3-11\,Aab^4)x^4-2688\,(Ba^3b^2-11\,Aa^2b^3)x^3-2370\,(B}{640\left(a^6b^5x^{\frac{11}{2}}+5\,a^7b^4x^{\frac{9}{2}}+10\,a^8b^3x^{\frac{7}{2}}+10\,a^9b^2x^{\frac{5}{2}}+5\,a^{10}bx^{\frac{3}{2}}+a^{11}\right.}$$
$$+\frac{63\,(Ba-11\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^6}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/640*(1280*A*a^5 - 315*(B*a*b^4 - 11*A*b^5)*x^5 - 1470*(B*a^2*b^3 - 11*A
*a*b^4)*x^4 - 2688*(B*a^3*b^2 - 11*A*a^2*b^3)*x^3 - 2370*(B*a^4*b - 11*A*a
^3*b^2)*x^2 - 965*(B*a^5 - 11*A*a^4*b)*x)/(a^6*b^5*x^(11/2) + 5*a^7*b^4*x^
(9/2) + 10*a^8*b^3*x^(7/2) + 10*a^9*b^2*x^(5/2) + 5*a^10*b*x^(3/2) + a^11*
sqrt(x)) + 63/128*(B*a - 11*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^
6)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{63\,(Ba-11\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^6} - \frac{2\,A}{a^6\sqrt{x}} + \frac{315\,Bab^4x^{\frac{9}{2}} - 2185\,Ab^5x^{\frac{9}{2}} + 1470\,Ba^2b^3x^{\frac{7}{2}} - 9770\,Aab^4x^{\frac{7}{2}} + 2688\,Ba^3b^2x^{\frac{5}{2}} - 16768\,Aa^2b^3x^{\frac{5}{2}} + 2370\,Ba}{640\,(bx+a)^5a^6}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
63/128*(B*a - 11*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^6) - 2*A/(a
^6*sqrt(x)) + 1/640*(315*B*a*b^4*x^(9/2) - 2185*A*b^5*x^(9/2) + 1470*B*a^2
*b^3*x^(7/2) - 9770*A*a*b^4*x^(7/2) + 2688*B*a^3*b^2*x^(5/2) - 16768*A*a^2
*b^3*x^(5/2) + 2370*B*a^4*b*x^(3/2) - 13270*A*a^3*b^2*x^(3/2) + 965*B*a^5*
sqrt(x) - 4215*A*a^4*b*sqrt(x))/((b*x + a)^5*a^6)
```

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{\frac{2\,A}{a} + \frac{193\,x\,(11\,A\,b-B\,a)}{128\,a^2} + \frac{21\,b^2\,x^3\,(11\,A\,b-B\,a)}{5\,a^4} + \frac{147\,b^3\,x^4\,(11\,A\,b-B\,a)}{64\,a^5} + \frac{63\,b^4\,x^5\,(11\,A\,b-B\,a)}{128\,a^6} + \frac{237\,b\,x^2\,(11\,A\,b-B\,a)}{64\,a^3}}{a^5\,\sqrt{x} + b^5\,x^{11/2} + 5\,a^4\,b\,x^{3/2} + 5\,a\,b^4\,x^{9/2} + 10\,a^3\,b^2\,x^{5/2} + 10\,a^2\,b^3\,x^{7/2}} - \frac{63\,\mathrm{atan}\left(\frac{63\,\sqrt{b}\,\sqrt{x}\,(11\,A\,b-B\,a)}{\sqrt{a}\,(693\,A\,b-63\,B\,a)}\right)\,(11\,A\,b-B\,a)}{128\,a^{13/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
- ((2*A)/a + (193*x*(11*A*b - B*a))/(128*a^2) + (21*b^2*x^3*(11*A*b - B*a)
)/(5*a^4) + (147*b^3*x^4*(11*A*b - B*a))/(64*a^5) + (63*b^4*x^5*(11*A*b -
B*a))/(128*a^6) + (237*b*x^2*(11*A*b - B*a))/(64*a^3))/(a^5*x^(1/2) + b^5*
x^(11/2) + 5*a^4*b*x^(3/2) + 5*a*b^4*x^(9/2) + 10*a^3*b^2*x^(5/2) + 10*a^2
*b^3*x^(7/2)) - (63*atan((63*b^(1/2)*x^(1/2)*(11*A*b - B*a))/(a^(1/2)*(693
*A*b - 63*B*a)))*(11*A*b - B*a))/(128*a^(13/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{-315\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4-1260\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx-18}{}$$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
( - 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 -
 1260*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x
 - 1890*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b
**2*x**2 - 1260*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))
)*a*b**3*x**3 - 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt
(a)))*b**4*x**4 - 128*a**5 - 837*a**4*b*x - 1533*a**3*b**2*x**2 - 1155*a**
2*b**3*x**3 - 315*a*b**4*x**4)/(64*sqrt(x)*a**6*(a**4 + 4*a**3*b*x + 6*a**
2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.410 $\int \frac{A+Bx}{x^{5/2}(a^2+2abx+b^2x^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{2A}{3a^6x^{3/2}}+\frac{2(6Ab-aB)}{a^7\sqrt{x}}+\frac{b(Ab-aB)\sqrt{x}}{5a^3(a+bx)^5} + \frac{b(29Ab-19aB)\sqrt{x}}{40a^4(a+bx)^4}+\frac{b(443Ab-213aB)\sqrt{x}}{240a^5(a+bx)^3}+\frac{b(827Ab-309aB)\sqrt{x}}{192a^6(a+bx)^2} + \frac{b(1467Ab-437aB)\sqrt{x}}{128a^7(a+bx)}+\frac{231\sqrt{b}(13Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128a^{15/2}}$$

output

```
-2/3*A/a^6/x^(3/2)+2*(6*A*b-B*a)/a^7/x^(1/2)+1/5*b*(A*b-B*a)*x^(1/2)/a^3/(
b*x+a)^5+1/40*b*(29*A*b-19*B*a)*x^(1/2)/a^4/(b*x+a)^4+1/240*b*(443*A*b-213
*B*a)*x^(1/2)/a^5/(b*x+a)^3+1/192*b*(827*A*b-309*B*a)*x^(1/2)/a^6/(b*x+a)^
2+1/128*b*(1467*A*b-437*B*a)*x^(1/2)/a^7/(b*x+a)+231/128*b^(1/2)*(13*A*b-3
*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(15/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{\sqrt{a}(-45045Ab^6x^6+1280a^6(A+3Bx)+1155ab^5x^5(-182A+9Bx)+462a^2b^4x^4(-832A+105Bx)+6}{x^{3/2}(a+bx}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
(-((Sqrt[a]*(-45045*A*b^6*x^6 + 1280*a^6*(A + 3*B*x) + 1155*a*b^5*x^5*(-18
2*A + 9*B*x) + 462*a^2*b^4*x^4*(-832*A + 105*B*x) + 66*a^3*b^3*x^3*(-5135*
A + 1344*B*x) + 55*a^4*b^2*x^2*(-2509*A + 1422*B*x) + 5*a^5*b*x*(-3328*A +
 6369*B*x)))/(x^(3/2)*(a + b*x)^5)) + 3465*Sqrt[b]*(13*A*b - 3*a*B)*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(1920*a^(15/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1184, 27, 87, 52, 52, 52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}dx$$

$$\Big\downarrow \; 1184$$

$$b^6\int \frac{A+Bx}{b^6x^{5/2}(a+bx)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^6}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(13Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)^5}dx}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(13Ab-3aB)\left(\frac{11\int \frac{1}{x^{5/2}(a+bx)^4}dx}{8a}+\frac{1}{4ax^{3/2}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(13Ab-3aB)\left(\frac{11\left(\frac{3\int \frac{1}{x^{5/2}(a+bx)^3}dx}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$$\Big\downarrow \; 61$$

$$\frac{(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+bx)^4}\right)}{10ab}+\frac{Ab-aB}{5abx^{3/2}(a+bx)^5}$$

$\downarrow$ 61

$$(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+$$

↓ 73

$$(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}\,d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(a+bx)}\right.$$

↓ 218

$$(13Ab-3aB)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8a}+\frac{1}{4ax^{3/2}(}\right.$$

input
```
Int[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^3),x]
```

output
```
(A*b - a*B)/(5*a*b*x^(3/2)*(a + b*x)^5) + ((13*A*b - 3*a*B)*(1/(4*a*x^(3/2
)*(a + b*x)^4) + (11*(1/(3*a*x^(3/2)*(a + b*x)^3) + (3*(1/(2*a*x^(3/2)*(a
+ b*x)^2) + (7*(1/(a*x^(3/2)*(a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/(a
*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2
*a)))/(4*a)))/(2*a)))/(8*a)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-18Abx+3Bax+Aa)}{3a^7x^{\frac{3}{2}}}+\frac{b\left(\frac{2\left(\frac{1467}{256}Ab^5-\frac{437}{256}Bab^4\right)x^{\frac{9}{2}}+\frac{ab^3(9629Ab-2931Ba)x^{\frac{7}{2}}}{192}+2\left(\frac{1253}{30}Aa^2b^3-\frac{131}{10}Ba^3b^2\right)x^{\frac{5}{2}}+}{(bx+a)^5}\right.}{}$
derivativedivides	$\frac{2b\left(\frac{\left(\frac{1467}{256}Ab^5-\frac{437}{256}Bab^4\right)x^{\frac{9}{2}}+\frac{ab^3(9629Ab-2931Ba)x^{\frac{7}{2}}}{384}+\left(\frac{1253}{30}Aa^2b^3-\frac{131}{10}Ba^3b^2\right)x^{\frac{5}{2}}+\left(\frac{12131}{384}a^3Ab^2-\frac{1327}{128}Ba^4b\right)x^{\frac{3}{2}}+\left(\right.}{(bx+a)^5}\right.}{a^7}$
default	$\frac{2b\left(\frac{\left(\frac{1467}{256}Ab^5-\frac{437}{256}Bab^4\right)x^{\frac{9}{2}}+\frac{ab^3(9629Ab-2931Ba)x^{\frac{7}{2}}}{384}+\left(\frac{1253}{30}Aa^2b^3-\frac{131}{10}Ba^3b^2\right)x^{\frac{5}{2}}+\left(\frac{12131}{384}a^3Ab^2-\frac{1327}{128}Ba^4b\right)x^{\frac{3}{2}}+\left(\right.}{(bx+a)^5}\right.}{a^7}$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-18*A*b*x+3*B*a*x+A*a)/a^7/x^(3/2)+1/a^7*b*(2*((1467/256*A*b^5-437/2
56*B*a*b^4)*x^(9/2)+1/384*a*b^3*(9629*A*b-2931*B*a)*x^(7/2)+(1253/30*A*a^2
*b^3-131/10*B*a^3*b^2)*x^(5/2)+(12131/384*a^3*A*b^2-1327/128*B*a^4*b)*x^(3
/2)+(2373/256*A*a^4*b-843/256*B*a^5)*x^(1/2))/(b*x+a)^5+231/128*(13*A*b-3*
B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.36

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/3840*(3465*((3*B*a*b^5 - 13*A*b^6)*x^7 + 5*(3*B*a^2*b^4 - 13*A*a*b^5)*
x^6 + 10*(3*B*a^3*b^3 - 13*A*a^2*b^4)*x^5 + 10*(3*B*a^4*b^2 - 13*A*a^3*b^3
)*x^4 + 5*(3*B*a^5*b - 13*A*a^4*b^2)*x^3 + (3*B*a^6 - 13*A*a^5*b)*x^2)*sqr
t(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(1280*A*a^6
+ 3465*(3*B*a*b^5 - 13*A*b^6)*x^6 + 16170*(3*B*a^2*b^4 - 13*A*a*b^5)*x^5 +
 29568*(3*B*a^3*b^3 - 13*A*a^2*b^4)*x^4 + 26070*(3*B*a^4*b^2 - 13*A*a^3*b^
3)*x^3 + 10615*(3*B*a^5*b - 13*A*a^4*b^2)*x^2 + 1280*(3*B*a^6 - 13*A*a^5*b
)*x)*sqrt(x))/(a^7*b^5*x^7 + 5*a^8*b^4*x^6 + 10*a^9*b^3*x^5 + 10*a^10*b^2*
x^4 + 5*a^11*b*x^3 + a^12*x^2), -1/1920*(3465*((3*B*a*b^5 - 13*A*b^6)*x^7
+ 5*(3*B*a^2*b^4 - 13*A*a*b^5)*x^6 + 10*(3*B*a^3*b^3 - 13*A*a^2*b^4)*x^5 +
 10*(3*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + 5*(3*B*a^5*b - 13*A*a^4*b^2)*x^3 +
(3*B*a^6 - 13*A*a^5*b)*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (1280*A*
a^6 + 3465*(3*B*a*b^5 - 13*A*b^6)*x^6 + 16170*(3*B*a^2*b^4 - 13*A*a*b^5)*x
^5 + 29568*(3*B*a^3*b^3 - 13*A*a^2*b^4)*x^4 + 26070*(3*B*a^4*b^2 - 13*A*a^
3*b^3)*x^3 + 10615*(3*B*a^5*b - 13*A*a^4*b^2)*x^2 + 1280*(3*B*a^6 - 13*A*a
^5*b)*x)*sqrt(x))/(a^7*b^5*x^7 + 5*a^8*b^4*x^6 + 10*a^9*b^3*x^5 + 10*a^10*
b^2*x^4 + 5*a^11*b*x^3 + a^12*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(5/2)/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx=$$
$$-\frac{1280\,Aa^6+3465\,(3\,Bab^5-13\,Ab^6)x^6+16170\,(3\,Ba^2b^4-13\,Aab^5)x^5+29568\,(3\,Ba^3b^3-13\,Aa^2b^4)x^4+\cdots}{1920\left(a^7b^5x^{\frac{13}{2}}+5\,a^8b^4x^{\frac{11}{2}}+10\,a^9b^3x^{\frac{9}{2}}+10\cdots\right.}$$
$$-\frac{231\,(3\,Bab-13\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^7}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1920*(1280*A*a^6 + 3465*(3*B*a*b^5 - 13*A*b^6)*x^6 + 16170*(3*B*a^2*b^4
 - 13*A*a*b^5)*x^5 + 29568*(3*B*a^3*b^3 - 13*A*a^2*b^4)*x^4 + 26070*(3*B*a
^4*b^2 - 13*A*a^3*b^3)*x^3 + 10615*(3*B*a^5*b - 13*A*a^4*b^2)*x^2 + 1280*(
3*B*a^6 - 13*A*a^5*b)*x)/(a^7*b^5*x^(13/2) + 5*a^8*b^4*x^(11/2) + 10*a^9*b
^3*x^(9/2) + 10*a^10*b^2*x^(7/2) + 5*a^11*b*x^(5/2) + a^12*x^(3/2)) - 231/
128*(3*B*a*b - 13*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^7)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = -\frac{231\,(3\,Bab-13\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^7}-\frac{2\,(3\,Bax-18\,Abx+Aa)}{3\,a^7x^{\frac{3}{2}}} -\frac{6555\,Bab^5x^{\frac{9}{2}}-22005\,Ab^6x^{\frac{9}{2}}+29310\,Ba^2b^4x^{\frac{7}{2}}-96290\,Aab^5x^{\frac{7}{2}}+50304\,Ba^3b^3x^{\frac{5}{2}}-160384\,Aa^2b^4x^{\frac{5}{2}}+3}{1920\,(bx+a)^5a^7}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output
```
-231/128*(3*B*a*b - 13*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^7)
- 2/3*(3*B*a*x - 18*A*b*x + A*a)/(a^7*x^(3/2)) - 1/1920*(6555*B*a*b^5*x^(9
/2) - 22005*A*b^6*x^(9/2) + 29310*B*a^2*b^4*x^(7/2) - 96290*A*a*b^5*x^(7/2
) + 50304*B*a^3*b^3*x^(5/2) - 160384*A*a^2*b^4*x^(5/2) + 39810*B*a^4*b^2*x
^(3/2) - 121310*A*a^3*b^3*x^(3/2) + 12645*B*a^5*b*sqrt(x) - 35595*A*a^4*b^
2*sqrt(x))/((b*x + a)^5*a^7)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{\frac{2\,x\,(13\,A\,b-3\,B\,a)}{3\,a^2}-\frac{2\,A}{3\,a}+\frac{869\,b^2\,x^3\,(13\,A\,b-3\,B\,a)}{64\,a^4}+\frac{77\,b^3\,x^4\,(13\,A\,b-3\,B\,a)}{5\,a^5}+\frac{539\,b^4\,x^5\,(13}{64\,a}}{a^5\,x^{3/2}+b^5\,x^{13/2}+5\,a^4\,b\,x^{5/2}+5\,a\,b^4\,x^{11/2}+10} + \frac{231\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(13\,A\,b-3\,B\,a)}{128\,a^{15/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^3),x)
```

output
```
((2*x*(13*A*b - 3*B*a))/(3*a^2) - (2*A)/(3*a) + (869*b^2*x^3*(13*A*b - 3*B
*a))/(64*a^4) + (77*b^3*x^4*(13*A*b - 3*B*a))/(5*a^5) + (539*b^4*x^5*(13*A
*b - 3*B*a))/(64*a^6) + (231*b^5*x^6*(13*A*b - 3*B*a))/(128*a^7) + (2123*b
*x^2*(13*A*b - 3*B*a))/(384*a^3))/(a^5*x^(3/2) + b^5*x^(13/2) + 5*a^4*b*x^
(5/2) + 5*a*b^4*x^(11/2) + 10*a^3*b^2*x^(7/2) + 10*a^2*b^3*x^(9/2)) + (231
*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(13*A*b - 3*B*a))/(128*a^(15/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^3}\,dx = \frac{3465\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4bx+13860\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^2}{}$$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output
```
(3465*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4*b*x
 + 13860*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*
b**2*x**2 + 20790*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a
)))*a**2*b**3*x**3 + 13860*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(
b)*sqrt(a)))*a*b**4*x**4 + 3465*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(
sqrt(b)*sqrt(a)))*b**5*x**5 - 128*a**6 + 1408*a**5*b*x + 9207*a**4*b**2*x*
*2 + 16863*a**3*b**3*x**3 + 12705*a**2*b**4*x**4 + 3465*a*b**5*x**5)/(192*
sqrt(x)*a**7*x*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**
4*x**4))
```

3.411 $\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2aAx^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{2(Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{2bBx^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}$$

output
```
2*a*A*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+2*(A*b+B*a)*x^(11/2)*((b*x+a)^
2)^(1/2)/(11*b*x+11*a)+2*b*B*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13*a)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.41

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2x^{9/2}\sqrt{(a+bx)^2}(143aA+117Abx+117aBx+99bBx^2)}{1287(a+bx)}$$

input
```
Integrate[x^(7/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*x^(9/2)*Sqrt[(a + b*x)^2]*(143*a*A + 117*A*b*x + 117*a*B*x + 99*b*B*x^2
))/(1287*(a + b*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^{7/2}(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{7/2}(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^{11/2}+(Ab+aB)x^{9/2}+aAx^{7/2}\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{13}bBx^{13/2}\right)}{a+bx}$$

input
```
Int[x^(7/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(11/2
))/11 + (2*b*B*x^(13/2))/13))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{2\,\mathrm{csgn}(bx+a)x^{\frac{9}{2}}(99Bb\,x^{2}+117Abx+117Bax+143Aa)}{1287}$	34
gosper	$\frac{2x^{\frac{9}{2}}(99Bb\,x^{2}+117Abx+117Bax+143Aa)\sqrt{(bx+a)^{2}}}{1287(bx+a)}$	44
risch	$\frac{2x^{\frac{9}{2}}(99Bb\,x^{2}+117Abx+117Bax+143Aa)\sqrt{(bx+a)^{2}}}{1287(bx+a)}$	44
orering	$\frac{2x^{\frac{9}{2}}(99Bb\,x^{2}+117Abx+117Bax+143Aa)\sqrt{(bx+a)^{2}}}{1287(bx+a)}$	44

input
```
int(x^(7/2)*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/1287*csgn(b*x+a)*x^(9/2)*(99*B*b*x^2+117*A*b*x+117*B*a*x+143*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.27

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{1287}\left(99\,Bbx^6+143\,Aax^4+117\,(Ba+Ab)x^5\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/1287*(99*B*b*x^6 + 143*A*a*x^4 + 117*(B*a + A*b)*x^5)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{143}\,(11\,bx^2+13\,ax)\,Bx^{\frac{9}{2}} + \frac{2}{99}\,(9\,bx^2+11\,ax)\,Ax^{\frac{7}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/143*(11*b*x^2 + 13*a*x)*B*x^(9/2) + 2/99*(9*b*x^2 + 11*a*x)*A*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}}\operatorname{sgn}(bx+a) + \frac{2}{11}\,Bax^{\frac{11}{2}}\operatorname{sgn}(bx+a) + \frac{2}{11}\,Abx^{\frac{11}{2}}\operatorname{sgn}(bx+a) + \frac{2}{9}\,Aax^{\frac{9}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(7/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/13*B*b*x^(13/2)*sgn(b*x + a) + 2/11*B*a*x^(11/2)*sgn(b*x + a) + 2/11*A*b
*x^(11/2)*sgn(b*x + a) + 2/9*A*a*x^(9/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \int x^{7/2}\,\sqrt{{(a+b\,x)}^2}\,(A+B\,x)\,dx$$

input
```
int(x^(7/2)*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(7/2)*((a + b*x)^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int x^{7/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2\sqrt{x}\,x^4(99b^2x^2+234abx+143a^2)}{1287}$$

input
```
int(x^(7/2)*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**4*(143*a**2 + 234*a*b*x + 99*b**2*x**2))/1287
```

3.412 $\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2aAx^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2(Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{2bBx^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}$$

output
```
2*a*A*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*(A*b+B*a)*x^(9/2)*((b*x+a)^2
)^(1/2)/(9*b*x+9*a)+2*b*B*x^(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.41

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2x^{7/2}\sqrt{(a+bx)^2}(99aA+77Abx+77aBx+63bBx^2)}{693(a+bx)}$$

input
```
Integrate[x^(5/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*x^(7/2)*Sqrt[(a + b*x)^2]*(99*a*A + 77*A*b*x + 77*a*B*x + 63*b*B*x^2))/
(693*(a + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^{5/2}(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{5/2}(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^{9/2}+(Ab+aB)x^{7/2}+aAx^{5/2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{11}bBx^{11/2}\right)}{a+bx}$$

input
```
Int[x^(5/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(9/2)
)/9 + (2*b*B*x^(11/2))/11))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{2\,\mathrm{csgn}(bx+a)x^{\frac{7}{2}}\left(63Bb\,x^{2}+77Abx+77Bax+99Aa\right)}{693}$	34
gosper	$\frac{2x^{\frac{7}{2}}\left(63Bb\,x^{2}+77Abx+77Bax+99Aa\right)\sqrt{\left(bx+a\right)^{2}}}{693\left(bx+a\right)}$	44
risch	$\frac{2x^{\frac{7}{2}}\left(63Bb\,x^{2}+77Abx+77Bax+99Aa\right)\sqrt{\left(bx+a\right)^{2}}}{693\left(bx+a\right)}$	44
orering	$\frac{2x^{\frac{7}{2}}\left(63Bb\,x^{2}+77Abx+77Bax+99Aa\right)\sqrt{\left(bx+a\right)^{2}}}{693\left(bx+a\right)}$	44

input
```
int(x^(5/2)*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/693*csgn(b*x+a)*x^(7/2)*(63*B*b*x^2+77*A*b*x+77*B*a*x+99*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.27

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{693}\left(63\,Bbx^5+99\,Aax^3+77\,(Ba+Ab)x^4\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/693*(63*B*b*x^5 + 99*A*a*x^3 + 77*(B*a + A*b)*x^4)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{99}\left(9\,bx^2+11\,ax\right)Bx^{\frac{7}{2}}+\frac{2}{63}\left(7\,bx^2+9\,ax\right)Ax^{\frac{5}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/99*(9*b*x^2 + 11*a*x)*B*x^(7/2) + 2/63*(7*b*x^2 + 9*a*x)*A*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}}\mathrm{sgn}(bx+a) + \frac{2}{9}\,Bax^{\frac{9}{2}}\mathrm{sgn}(bx+a)+\frac{2}{9}\,Abx^{\frac{9}{2}}\mathrm{sgn}(bx+a)+\frac{2}{7}\,Aax^{\frac{7}{2}}\mathrm{sgn}(bx+a)$$

input
```
integrate(x^(5/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/11*B*b*x^(11/2)*sgn(b*x + a) + 2/9*B*a*x^(9/2)*sgn(b*x + a) + 2/9*A*b*x^
(9/2)*sgn(b*x + a) + 2/7*A*a*x^(7/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \int x^{5/2}\,\sqrt{{\left(a+b\,x\right)}^2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(5/2)*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(5/2)*((a + b*x)^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int x^{5/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2\sqrt{x}\,x^3(63b^2x^2+154abx+99a^2)}{693}$$

input
```
int(x^(5/2)*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**3*(99*a**2 + 154*a*b*x + 63*b**2*x**2))/693
```

3.413 $\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2aAx^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{2(Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2bBx^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}$$

output
```
2*a*A*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+2*(A*b+B*a)*x^(7/2)*((b*x+a)^2
)^(1/2)/(7*b*x+7*a)+2*b*B*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2x^{5/2}\sqrt{(a+bx)^2}(9a(7A+5Bx)+5bx(9A+7Bx))}{315(a+bx)}$$

input
```
Integrate[x^(3/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*x^(5/2)*Sqrt[(a + b*x)^2]*(9*a*(7*A + 5*B*x) + 5*b*x*(9*A + 7*B*x)))/(3
15*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int bx^{3/2}(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{3/2}(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^{7/2}+(Ab+aB)x^{5/2}+aAx^{3/2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{9}bBx^{9/2}\right)}{a+bx}$$

input
```
Int[x^(3/2)*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(7/2)
)/7 + (2*b*B*x^(9/2))/9))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{2\ \mathrm{csgn}(bx+a)x^{\frac{5}{2}}\left(35Bb\,x^{2}+45Abx+45Bax+63Aa\right)}{315}$	34
gosper	$\frac{2x^{\frac{5}{2}}\left(35Bb\,x^{2}+45Abx+45Bax+63Aa\right)\sqrt{\left(bx+a\right)^{2}}}{315\left(bx+a\right)}$	44
risch	$\frac{2x^{\frac{5}{2}}\left(35Bb\,x^{2}+45Abx+45Bax+63Aa\right)\sqrt{\left(bx+a\right)^{2}}}{315\left(bx+a\right)}$	44
orering	$\frac{2x^{\frac{5}{2}}\left(35Bb\,x^{2}+45Abx+45Bax+63Aa\right)\sqrt{\left(bx+a\right)^{2}}}{315\left(bx+a\right)}$	44

input
```
int(x^(3/2)*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*csgn(b*x+a)*x^(5/2)*(35*B*b*x^2+45*A*b*x+45*B*a*x+63*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.27

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\frac{2}{315}\left(35\,Bbx^4+63\,Aax^2+45\,(Ba+Ab)x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b*x^4 + 63*A*a*x^2 + 45*(B*a + A*b)*x^3)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{63}\left(7\,bx^2+9\,ax\right)Bx^{\frac{5}{2}}+\frac{2}{35}\left(5\,bx^2+7\,ax\right)Ax^{\frac{3}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/63*(7*b*x^2 + 9*a*x)*B*x^(5/2) + 2/35*(5*b*x^2 + 7*a*x)*A*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}}\mathrm{sgn}(bx+a) + \frac{2}{7}\,Bax^{\frac{7}{2}}\mathrm{sgn}(bx+a)+\frac{2}{7}\,Abx^{\frac{7}{2}}\mathrm{sgn}(bx+a)+\frac{2}{5}\,Aax^{\frac{5}{2}}\mathrm{sgn}(bx+a)$$

input
```
integrate(x^(3/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/9*B*b*x^(9/2)*sgn(b*x + a) + 2/7*B*a*x^(7/2)*sgn(b*x + a) + 2/7*A*b*x^(7
/2)*sgn(b*x + a) + 2/5*A*a*x^(5/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \int x^{3/2}\,\sqrt{{\left(a+b\,x\right)}^2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(3/2)*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(3/2)*((a + b*x)^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int x^{3/2}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2\sqrt{x}\,x^2(35b^2x^2+90abx+63a^2)}{315}$$

input
```
int(x^(3/2)*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**2*(63*a**2 + 90*a*b*x + 35*b**2*x**2))/315
```

3.414 $\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2aAx^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{2(Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{2bBx^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
2*a*A*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*(A*b+B*a)*x^(5/2)*((b*x+a)^2
)^(1/2)/(5*b*x+5*a)+2*b*B*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2x^{3/2}\sqrt{(a+bx)^2}(7a(5A+3Bx)+3bx(7A+5Bx))}{105(a+bx)}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*x^(3/2)*Sqrt[(a + b*x)^2]*(7*a*(5*A + 3*B*x) + 3*b*x*(7*A + 5*B*x)))/(1
05*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{a^2+2abx+b^2x^2}(A+Bx)\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b\sqrt{x}(a+bx)(A+Bx)dx}{b(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \sqrt{x}(a+bx)(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^{5/2}+(Ab+aB)x^{3/2}+aA\sqrt{x}\right)dx}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{5}x^{5/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{7}bBx^{7/2}\right)}{a+bx}$$

input
```
Int[Sqrt[x]*(A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(5/2)
)/5 + (2*b*B*x^(7/2))/7))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{2\,\operatorname{csgn}(bx+a)x^{\frac{3}{2}}\left(15Bb\,x^{2}+21Abx+21Bax+35Aa\right)}{105}$	34
gosper	$\frac{2x^{\frac{3}{2}}\left(15Bb\,x^{2}+21Abx+21Bax+35Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105\left(bx+a\right)}$	44
risch	$\frac{2x^{\frac{3}{2}}\left(15Bb\,x^{2}+21Abx+21Bax+35Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105\left(bx+a\right)}$	44
orering	$\frac{2x^{\frac{3}{2}}\left(15Bb\,x^{2}+21Abx+21Bax+35Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105\left(bx+a\right)}$	44

input
```
int(x^(1/2)*(B*x+A)*((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*csgn(b*x+a)*x^(3/2)*(15*B*b*x^2+21*A*b*x+21*B*a*x+35*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.25

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{105}\left(15\,Bbx^3+35\,Aax+21\,(Ba+Ab)x^2\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b*x^3 + 35*A*a*x + 21*(B*a + A*b)*x^2)*sqrt(x)
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \int \sqrt{x}(A+Bx)\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*((b*x+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)*sqrt((a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{35}\left(5\,bx^2+7\,ax\right)Bx^{\frac{3}{2}}+\frac{2}{15}\left(3\,bx^2+5\,ax\right)A\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b*x^2 + 7*a*x)*B*x^(3/2) + 2/15*(3*b*x^2 + 5*a*x)*A*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}}\mathrm{sgn}(bx+a)+\frac{2}{5}\,Bax^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{2}{5}\,Abx^{\frac{5}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Aax^{\frac{3}{2}}\mathrm{sgn}(bx+a)$$

input
```
integrate(x^(1/2)*(B*x+A)*((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/7*B*b*x^(7/2)*sgn(b*x + a) + 2/5*B*a*x^(5/2)*sgn(b*x + a) + 2/5*A*b*x^(5
/2)*sgn(b*x + a) + 2/3*A*a*x^(3/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \int \sqrt{x}\,\sqrt{{\left(a+b\,x\right)}^2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(1/2)*((a + b*x)^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(1/2)*((a + b*x)^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.20

$$\int \sqrt{x}(A+Bx)\sqrt{a^2+2abx+b^2x^2}\,dx = \frac{2\sqrt{x}\,x(15b^2x^2+42abx+35a^2)}{105}$$

input
```
int(x^(1/2)*(B*x+A)*((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x*(35*a**2 + 42*a*b*x + 15*b**2*x**2))/105
```

3.415 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2aA\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2(Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{2bBx^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

output
```
2*a*A*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*(A*b+B*a)*x^(3/2)*((b*x+a)^2)^(1
/2)/(3*b*x+3*a)+2*b*B*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{(a+bx)^2}(5a(3A+Bx)+bx(5A+3Bx))}{15(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[(a + b*x)^2]*(5*a*(3*A + B*x) + b*x*(5*A + 3*B*x)))/(15*(a
 + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{\sqrt{x}}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(bBx^{3/2}+(Ab+aB)\sqrt{x}+\frac{aA}{\sqrt{x}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{3}x^{3/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{5}bBx^{5/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/Sqrt[x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3
+ (2*b*B*x^(5/2))/5))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{2\,\mathrm{csgn}(bx+a)\sqrt{x}\,(3Bb\,x^{2}+5Abx+5Bax+15Aa)}{15}$	34
gosper	$\frac{2\sqrt{x}\,(3Bb\,x^{2}+5Abx+5Bax+15Aa)\sqrt{(bx+a)^{2}}}{15(bx+a)}$	44
risch	$\frac{2\sqrt{x}\,(3Bb\,x^{2}+5Abx+5Bax+15Aa)\sqrt{(bx+a)^{2}}}{15(bx+a)}$	44
orering	$\frac{2\sqrt{x}\,(3Bb\,x^{2}+5Abx+5Bax+15Aa)\sqrt{(bx+a)^{2}}}{15(bx+a)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*csgn(b*x+a)*x^(1/2)*(3*B*b*x^2+5*A*b*x+5*B*a*x+15*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2}{15}\left(3\,Bbx^2+15\,Aa+5\,(Ba+Ab)x\right)\sqrt{x}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b*x^2 + 15*A*a + 5*(B*a + A*b)*x)*sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/sqrt(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2}{15}\left(3\,bx^2+5\,ax\right)B\sqrt{x}+\frac{2\,(bx^2+3\,ax)A}{3\,\sqrt{x}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b*x^2 + 5*a*x)*B*sqrt(x) + 2/3*(b*x^2 + 3*a*x)*A/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2}{5}\,Bbx^{\frac{5}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Bax^{\frac{3}{2}}\mathrm{sgn}(bx+a) + \frac{2}{3}\,Abx^{\frac{3}{2}}\mathrm{sgn}(bx+a)+2\,Aa\sqrt{x}\mathrm{sgn}(bx+a)$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/5*B*b*x^(5/2)*sgn(b*x + a) + 2/3*B*a*x^(3/2)*sgn(b*x + a) + 2/3*A*b*x^(3
/2)*sgn(b*x + a) + 2*A*a*sqrt(x)*sgn(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{\sqrt{(a+b\,x)^2}\,\left(\frac{2\,B\,x^3}{5}+\frac{x^2\,(10\,A\,b+10\,B\,a)}{15\,b}+\frac{2\,A\,a\,x}{b}\right)}{x^{3/2}+\frac{a\,\sqrt{x}}{b}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^(1/2),x)
```

output
```
(((a + b*x)^2)^(1/2)*((2*B*x^3)/5 + (x^2*(10*A*b + 10*B*a))/(15*b) + (2*A*
a*x)/b))/(x^(3/2) + (a*x^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.19

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(15*a**2 + 10*a*b*x + 3*b**2*x**2))/15
```

3.416 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = -\frac{2aA\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2(Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2bBx^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}$$

output
```
-2*a*A*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)+2*(A*b+B*a)*x^(1/2)*((b*x+a)^2)^(
1/2)/(b*x+a)+2*b*B*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}(3aA-3Abx-3aBx-bBx^2)}{3\sqrt{x}(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(3/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(3*a*A - 3*A*b*x - 3*a*B*x - b*B*x^2))/(3*Sqrt[x]*(a
 + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^{3/2}}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^{3/2}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^{3/2}}+bB\sqrt{x}+\frac{Ab+aB}{\sqrt{x}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(2\sqrt{x}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(3/2),x]
```

output
```
(((-2*a*A)/Sqrt[x] + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(3/2))/3)*Sqrt[a^2 +
 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result	size
default	$-\frac{2\,\mathrm{csgn}(bx+a)\left(-Bb\,x^{2}-3Abx-3Bax+3Aa\right)}{3\sqrt{x}}$	34
gosper	$-\frac{2\left(-Bb\,x^{2}-3Abx-3Bax+3Aa\right)\sqrt{\left(bx+a\right)^{2}}}{3\sqrt{x}\,\left(bx+a\right)}$	44
risch	$-\frac{2\left(-Bb\,x^{2}-3Abx-3Bax+3Aa\right)\sqrt{\left(bx+a\right)^{2}}}{3\sqrt{x}\,\left(bx+a\right)}$	44
orering	$-\frac{2\left(-Bb\,x^{2}-3Abx-3Bax+3Aa\right)\sqrt{\left(bx+a\right)^{2}}}{3\sqrt{x}\,\left(bx+a\right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*csgn(b*x+a)*(-B*b*x^2-3*A*b*x-3*B*a*x+3*A*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \frac{2\left(Bbx^2-3\,Aa+3\,(Ba+Ab)x\right)}{3\,\sqrt{x}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*b*x^2 - 3*A*a + 3*(B*a + A*b)*x)/sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \int \frac{(A+Bx)\sqrt{(a+bx)^2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**(3/2),x)
```

output
```
Integral((A + B*x)*sqrt((a + b*x)**2)/x**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \frac{2\,(bx^2+3\,ax)B}{3\,\sqrt{x}} + \frac{2\,(bx^2-ax)A}{x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b*x^2 + 3*a*x)*B/sqrt(x) + 2*(b*x^2 - a*x)*A/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \frac{2}{3}\,Bbx^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 2\,Ba\sqrt{x}\mathrm{sgn}(bx+a) + 2\,Ab\sqrt{x}\mathrm{sgn}(bx+a) - \frac{2\,A a\mathrm{sgn}(bx+a)}{\sqrt{x}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
2/3*B*b*x^(3/2)*sgn(b*x + a) + 2*B*a*sqrt(x)*sgn(b*x + a) + 2*A*b*sqrt(x)*
sgn(b*x + a) - 2*A*a*sgn(b*x + a)/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \frac{\sqrt{(a+b\,x)^2}\,\left(\frac{2\,B\,x^2}{3} - \frac{2\,A\,a}{b} + \frac{x\,(6\,A\,b+6\,B\,a)}{3\,b}\right)}{x^{3/2} + \frac{a\,\sqrt{x}}{b}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^(3/2),x)
```

output
```
(((a + b*x)^2)^(1/2)*((2*B*x^2)/3 - (2*A*a)/b + (x*(6*A*b + 6*B*a))/(3*b))
)/(x^(3/2) + (a*x^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2+4abx-2a^2}{\sqrt{x}}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(3/2),x)
```

output
```
(2*( - 3*a**2 + 6*a*b*x + b**2*x**2))/(3*sqrt(x))
```

3.417 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = -\frac{2aA\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)} - \frac{2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2bB\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

output
```
-2/3*a*A*((b*x+a)^2)^(1/2)/x^(3/2)/(b*x+a)-2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x
^(1/2)/(b*x+a)+2*b*B*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}(3bx(A-Bx)+a(A+3Bx))}{3x^{3/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(5/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(3*b*x*(A - B*x) + a*(A + 3*B*x)))/(3*x^(3/2)*(a + b
*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^{5/2}}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^{5/2}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^{5/2}}+\frac{bB}{\sqrt{x}}+\frac{Ab+aB}{x^{3/2}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{3x^{3/2}}+2bB\sqrt{x}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(5/2),x]
```

output
```
(((-2*a*A)/(3*x^(3/2)) - (2*(A*b + a*B))/Sqrt[x] + 2*b*B*Sqrt[x])*Sqrt[a^2
 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.28

method	result	size
default	$-\frac{2\,\mathrm{csgn}\left(b x+a\right)\left(-3 B b\,x^{2}+3 A b x+3 B a x+A a\right)}{3 x^{\frac{3}{2}}}$	33
gosper	$-\frac{2\left(-3 B b\,x^{2}+3 A b x+3 B a x+A a\right)\sqrt{\left(b x+a\right)^{2}}}{3 x^{\frac{3}{2}}\left(b x+a\right)}$	43
risch	$-\frac{2\left(-3 B b\,x^{2}+3 A b x+3 B a x+A a\right)\sqrt{\left(b x+a\right)^{2}}}{3 x^{\frac{3}{2}}\left(b x+a\right)}$	43
orering	$-\frac{2\left(-3 B b\,x^{2}+3 A b x+3 B a x+A a\right)\sqrt{\left(b x+a\right)^{2}}}{3 x^{\frac{3}{2}}\left(b x+a\right)}$	43

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*csgn(b*x+a)*(-3*B*b*x^2+3*A*b*x+3*B*a*x+A*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = \frac{2\,(3\,Bbx^2-Aa-3\,(Ba+Ab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*B*b*x^2 - A*a - 3*(B*a + A*b)*x)/x^(3/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = \frac{2\,(bx^2-ax)B}{x^{\frac{3}{2}}} - \frac{2\,(3\,bx^2+ax)A}{3\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
2*(b*x^2 - a*x)*B/x^(3/2) - 2/3*(3*b*x^2 + a*x)*A/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = 2\,Bb\sqrt{x}\mathrm{sgn}(bx+a) - \frac{2\,(3\,Bax\mathrm{sgn}(bx+a)+3\,Abx\mathrm{sgn}(bx+a)+Aa\mathrm{sgn}(bx+a))}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
2*B*b*sqrt(x)*sgn(b*x + a) - 2/3*(3*B*a*x*sgn(b*x + a) + 3*A*b*x*sgn(b*x +
 a) + A*a*sgn(b*x + a))/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx = -\frac{\sqrt{(a+b\,x)^2}\,\left(\frac{2\,A\,a}{3\,b}-2\,B\,x^2+\frac{x\,(6\,A\,b+6\,B\,a)}{3\,b}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^(5/2),x)
```

output
```
-(((a + b*x)^2)^(1/2)*((2*A*a)/(3*b) - 2*B*x^2 + (x*(6*A*b + 6*B*a))/(3*b)
))/(x^(5/2) + (a*x^(3/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{5/2}}\,dx=\frac{2b^2x^2-4abx-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(5/2),x)
```

output
```
(2*( - a**2 - 6*a*b*x + 3*b**2*x**2))/(3*sqrt(x)*x)
```

3.418 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{2aA\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)} - \frac{2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)} - \frac{2bB\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)}$$

output
```
-2/5*a*A*((b*x+a)^2)^(1/2)/x^(5/2)/(b*x+a)-2/3*(A*b+B*a)*((b*x+a)^2)^(1/2)
/x^(3/2)/(b*x+a)-2*b*B*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}(3aA+5Abx+5aBx+15bBx^2)}{15x^{5/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(7/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(3*a*A + 5*A*b*x + 5*a*B*x + 15*b*B*x^2))/(15*x^(5/2
)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^{7/2}}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^{7/2}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^{7/2}}+\frac{bB}{x^{3/2}}+\frac{Ab+aB}{x^{5/2}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2(aB+Ab)}{3x^{3/2}}-\frac{2aA}{5x^{5/2}}-\frac{2bB}{\sqrt{x}}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(7/2),x]
```

output
```
(((-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/(3*x^(3/2)) - (2*b*B)/Sqrt[x])*Sq
rt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result	size
default	$-\frac{2\,\operatorname{csgn}\left(b x +a \right) \left(15 B b \,x^{2}+5 A b x +5 B a x +3 A a \right)}{15 x^{\frac{5}{2}}}$	34
gosper	$-\frac{2 \left(15 B b \,x^{2}+5 A b x +5 B a x +3 A a \right) \sqrt{\left(b x +a \right)^{2}}}{15 x^{\frac{5}{2}} \left(b x +a \right)}$	44
risch	$-\frac{2 \left(15 B b \,x^{2}+5 A b x +5 B a x +3 A a \right) \sqrt{\left(b x +a \right)^{2}}}{15 x^{\frac{5}{2}} \left(b x +a \right)}$	44
orering	$-\frac{2 \left(15 B b \,x^{2}+5 A b x +5 B a x +3 A a \right) \sqrt{\left(b x +a \right)^{2}}}{15 x^{\frac{5}{2}} \left(b x +a \right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*csgn(b*x+a)*(15*B*b*x^2+5*A*b*x+5*B*a*x+3*A*a)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{2\,(15\,Bbx^2+3\,Aa+5\,(Ba+Ab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(15*B*b*x^2 + 3*A*a + 5*(B*a + A*b)*x)/x^(5/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{2\,(3\,bx^2+ax)B}{3\,x^{\frac{5}{2}}} - \frac{2\,(5\,bx^2+3\,ax)A}{15\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/3*(3*b*x^2 + a*x)*B/x^(5/2) - 2/15*(5*b*x^2 + 3*a*x)*A/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{2\,(15\,Bbx^2\text{sgn}(bx+a)+5\,Bax\text{sgn}(bx+a)+5\,Abx\text{sgn}(bx+a)+3\,Aa\text{sgn}(bx+a))}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*B*b*x^2*sgn(b*x + a) + 5*B*a*x*sgn(b*x + a) + 5*A*b*x*sgn(b*x +
a) + 3*A*a*sgn(b*x + a))/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = -\frac{\sqrt{(a+b\,x)^2}\,\left(2\,B\,x^2+\frac{2\,A\,a}{5\,b}+\frac{x\,(10\,A\,b+10\,B\,a)}{15\,b}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^(7/2),x)
```

output
```
-(((a + b*x)^2)^(1/2)*(2*B*x^2 + (2*A*a)/(5*b) + (x*(10*A*b + 10*B*a))/(15
*b)))/(x^(7/2) + (a*x^(5/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{7/2}}\,dx = \frac{-2b^2x^2-\frac{4}{3}abx-\frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(7/2),x)
```

output
```
(2*( - 3*a**2 - 10*a*b*x - 15*b**2*x**2))/(15*sqrt(x)*x**2)
```

3.419 $\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 120

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = -\frac{2aA\sqrt{a^2+2abx+b^2x^2}}{7x^{7/2}(a+bx)} - \frac{2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)} - \frac{2bB\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)}$$

output
```
-2/7*a*A*((b*x+a)^2)^(1/2)/x^(7/2)/(b*x+a)-2/5*(A*b+B*a)*((b*x+a)^2)^(1/2)
/x^(5/2)/(b*x+a)-2/3*b*B*((b*x+a)^2)^(1/2)/x^(3/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}(15aA+21Abx+21aBx+35bBx^2)}{105x^{7/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(9/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(15*a*A + 21*A*b*x + 21*a*B*x + 35*b*B*x^2))/(105*x^
(7/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b(a+bx)(A+Bx)}{x^{9/2}}dx}{b(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)(A+Bx)}{x^{9/2}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{aA}{x^{9/2}}+\frac{bB}{x^{5/2}}+\frac{Ab+aB}{x^{7/2}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2(aB+Ab)}{5x^{5/2}}-\frac{2aA}{7x^{7/2}}-\frac{2bB}{3x^{3/2}}\right)}{a+bx}$$

input
```
Int[((A + B*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/x^(9/2),x]
```

output
```
(((-2*a*A)/(7*x^(7/2)) - (2*(A*b + a*B))/(5*x^(5/2)) - (2*b*B)/(3*x^(3/2))
)*Sqrt[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

method	result	size
default	$-\frac{2\,\mathrm{csgn}(bx+a)\left(35Bb\,x^{2}+21Abx+21Bax+15Aa\right)}{105x^{\frac{7}{2}}}$	34
gosper	$-\frac{2\left(35Bb\,x^{2}+21Abx+21Bax+15Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105x^{\frac{7}{2}}\left(bx+a\right)}$	44
risch	$-\frac{2\left(35Bb\,x^{2}+21Abx+21Bax+15Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105x^{\frac{7}{2}}\left(bx+a\right)}$	44
orering	$-\frac{2\left(35Bb\,x^{2}+21Abx+21Bax+15Aa\right)\sqrt{\left(bx+a\right)^{2}}}{105x^{\frac{7}{2}}\left(bx+a\right)}$	44

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*csgn(b*x+a)*(35*B*b*x^2+21*A*b*x+21*B*a*x+15*A*a)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = -\frac{2\,(35\,Bbx^2+15\,Aa+21\,(Ba+Ab)x)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(35*B*b*x^2 + 15*A*a + 21*(B*a + A*b)*x)/x^(7/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*((b*x+a)**2)**(1/2)/x**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx=-\frac{2\,(5\,bx^2+3\,ax)B}{15\,x^{\frac{7}{2}}}-\frac{2\,(7\,bx^2+5\,ax)A}{35\,x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/15*(5*b*x^2 + 3*a*x)*B/x^(7/2) - 2/35*(7*b*x^2 + 5*a*x)*A/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = -\frac{2\,(35\,Bbx^2\mathrm{sgn}(bx+a)+21\,Bax\mathrm{sgn}(bx+a)+21\,Abx\mathrm{sgn}(bx+a)+15\,Aa\mathrm{sgn}(bx+a))}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*((b*x+a)^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
-2/105*(35*B*b*x^2*sgn(b*x + a) + 21*B*a*x*sgn(b*x + a) + 21*A*b*x*sgn(b*x
 + a) + 15*A*a*sgn(b*x + a))/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx=-\frac{\sqrt{{\left(a+b\,x\right)}^2}\,\left(\frac{2\,B\,x^2}{3}+\frac{2\,A\,a}{7\,b}+\frac{x\,\left(42\,A\,b+42\,B\,a\right)}{105\,b}\right)}{x^{9/2}+\frac{a\,x^{7/2}}{b}}$$

input
```
int((((a + b*x)^2)^(1/2)*(A + B*x))/x^(9/2),x)
```

output
```
-(((a + b*x)^2)^(1/2)*((2*B*x^2)/3 + (2*A*a)/(7*b) + (x*(42*A*b + 42*B*a))
/(105*b)))/(x^(9/2) + (a*x^(7/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\sqrt{a^2+2abx+b^2x^2}}{x^{9/2}}\,dx = \frac{-\frac{2}{3}b^2x^2-\frac{4}{5}abx-\frac{2}{7}a^2}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*((b*x+a)^2)^(1/2)/x^(9/2),x)
```

output
```
(2*( - 15*a**2 - 42*a*b*x - 35*b**2*x**2))/(105*sqrt(x)*x**3)
```

3.420 $\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 220

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2a^3Ax^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{2a^2(3Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{6ab(Ab+aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{2b^2(Ab+3aB)x^{15/2}\sqrt{a^2+2abx+b^2x^2}}{15(a+bx)}+\frac{2b^3Bx^{17/2}\sqrt{a^2+2abx+b^2x^2}}{17(a+bx)}$$

output
```
2*a^3*A*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+2*a^2*(3*A*b+B*a)*x^(11/2)*(
(b*x+a)^2)^(1/2)/(11*b*x+11*a)+6*a*b*(A*b+B*a)*x^(13/2)*((b*x+a)^2)^(1/2)/
(13*b*x+13*a)+2*b^2*(A*b+3*B*a)*x^(15/2)*((b*x+a)^2)^(1/2)/(15*b*x+15*a)+2
*b^3*B*x^(17/2)*((b*x+a)^2)^(1/2)/(17*b*x+17*a)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2x^{9/2}\sqrt{(a+bx)^2}(1105a^3(11A+9Bx)+2295a^2bx(13A+11Bx)+1683ab^2x^2(15A+13B}{109395(a+bx)}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(2*x^(9/2)*Sqrt[(a + b*x)^2]*(1105*a^3*(11*A + 9*B*x) + 2295*a^2*b*x*(13*A
 + 11*B*x) + 1683*a*b^2*x^2*(15*A + 13*B*x) + 429*b^3*x^3*(17*A + 15*B*x))
)/(109395*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^{7/2}(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{7/2}(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^{15/2}+b^2(Ab+3aB)x^{13/2}+3ab(Ab+aB)x^{11/2}+a^2(3Ab+aB)x^{9/2}+a^3Ax^{7/2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{9}a^3Ax^{9/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{17}b^3Bx\right.}{a+bx}$$

input
```
Int[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)
*x^(11/2))/11 + (6*a*b*(A*b + a*B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(
15/2))/15 + (2*b^3*B*x^(17/2))/17))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^3x^4+7293A\,b^3x^3+21879Ba\,b^2x^3+25245Aa\,b^2x^2+25245B\,a^2b\,x^2+29835A\,a^2bx+9945B\,a^3x+12155a^3A\right)\left((bx+a)^2\right)}{109395(bx+a)^3}$
default	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^3x^4+7293A\,b^3x^3+21879Ba\,b^2x^3+25245Aa\,b^2x^2+25245B\,a^2b\,x^2+29835A\,a^2bx+9945B\,a^3x+12155a^3A\right)\left((bx+a)^2\right)}{109395(bx+a)^3}$
risch	$\frac{2\sqrt{(bx+a)^2}\,x^{\frac{9}{2}}\left(6435B\,b^3x^4+7293A\,b^3x^3+21879Ba\,b^2x^3+25245Aa\,b^2x^2+25245B\,a^2b\,x^2+29835A\,a^2bx+9945B\,a^3x+12155a^3A\right)}{109395(bx+a)}$
orering	$\frac{2\left(6435B\,b^3x^4+7293A\,b^3x^3+21879Ba\,b^2x^3+25245Aa\,b^2x^2+25245B\,a^2b\,x^2+29835A\,a^2bx+9945B\,a^3x+12155a^3A\right)x^{\frac{9}{2}}\left(b^2x^2+2ab\right.}{109395(bx+a)^3}$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/109395*x^(9/2)*(6435*B*b^3*x^4+7293*A*b^3*x^3+21879*B*a*b^2*x^3+25245*A*
a*b^2*x^2+25245*B*a^2*b*x^2+29835*A*a^2*b*x+9945*B*a^3*x+12155*A*a^3)*((b*
x+a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{109395}\left(6435\,Bb^3x^8+12155\,Aa^3x^4+7293\left(3\,Bab^2+Ab^3\right)x^7+25245\left(Ba^2b+Aab^2\right)x^6+\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
2/109395*(6435*B*b^3*x^8 + 12155*A*a^3*x^4 + 7293*(3*B*a*b^2 + A*b^3)*x^7
+ 25245*(B*a^2*b + A*a*b^2)*x^6 + 9945*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{6435}\left(33\left(13\,b^3x^2+15\,ab^2x\right)x^{\frac{11}{2}}+90\left(11\,ab^2x^2+13\,a^2bx\right)x^{\frac{9}{2}}+65\left(9\,a^2bx^2+11\,a^3x\right)x^{\frac{7}{2}}\right)A+\frac{2}{36465}\left(143\left(15\,b^3x^2+17\,ab^2x\right)x^{\frac{13}{2}}+374\left(13\,ab^2x^2+15\,a^2bx\right)x^{\frac{11}{2}}+255\left(11\,a^2bx^2+13\,a^3x\right)x^{\frac{9}{2}}\right)B$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
2/6435*(33*(13*b^3*x^2 + 15*a*b^2*x)*x^(11/2) + 90*(11*a*b^2*x^2 + 13*a^2*
b*x)*x^(9/2) + 65*(9*a^2*b*x^2 + 11*a^3*x)*x^(7/2))*A + 2/36465*(143*(15*b
^3*x^2 + 17*a*b^2*x)*x^(13/2) + 374*(13*a*b^2*x^2 + 15*a^2*b*x)*x^(11/2) +
 255*(11*a^2*b*x^2 + 13*a^3*x)*x^(9/2))*B
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{17}Bb^3x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{2}{5}Bab^2x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{2}{15}Ab^3x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{6}{13}Ba^2bx^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{6}{13}Aab^2x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{2}{11}Ba^3x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{6}{11}Aa^2bx^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{9}Aa^3x^{\frac{9}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/17*B*b^3*x^(17/2)*sgn(b*x + a) + 2/5*B*a*b^2*x^(15/2)*sgn(b*x + a) + 2/1
5*A*b^3*x^(15/2)*sgn(b*x + a) + 6/13*B*a^2*b*x^(13/2)*sgn(b*x + a) + 6/13*
A*a*b^2*x^(13/2)*sgn(b*x + a) + 2/11*B*a^3*x^(11/2)*sgn(b*x + a) + 6/11*A*
a^2*b*x^(11/2)*sgn(b*x + a) + 2/9*A*a^3*x^(9/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^{7/2}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.22

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{x}\,x^4(6435b^4x^4+29172a\,b^3x^3+50490a^2b^2x^2+39780a^3bx+12155a^4)}{109395}$$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*x**4*(12155*a**4 + 39780*a**3*b*x + 50490*a**2*b**2*x**2 + 2917
2*a*b**3*x**3 + 6435*b**4*x**4))/109395
```

3.421 $\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 220

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2a^3Ax^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2a^2(3Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{6ab(Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{2b^2(Ab+3aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{2b^3Bx^{15/2}\sqrt{a^2+2abx+b^2x^2}}{15(a+bx)}$$

output
```
2*a^3*A*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*a^2*(3*A*b+B*a)*x^(9/2)*((
b*x+a)^2)^(1/2)/(9*b*x+9*a)+6*a*b*(A*b+B*a)*x^(11/2)*((b*x+a)^2)^(1/2)/(11
*b*x+11*a)+2*b^2*(A*b+3*B*a)*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13*a)+2*b^
3*B*x^(15/2)*((b*x+a)^2)^(1/2)/(15*b*x+15*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2x^{7/2}\sqrt{(a+bx)^2}(715a^3(9A+7Bx)+1365a^2bx(11A+9Bx)+945ab^2x^2(13A+11Bx)+}{45045(a+bx)}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(2*x^(7/2)*Sqrt[(a + b*x)^2]*(715*a^3*(9*A + 7*B*x) + 1365*a^2*b*x*(11*A +
 9*B*x) + 945*a*b^2*x^2*(13*A + 11*B*x) + 231*b^3*x^3*(15*A + 13*B*x)))/(4
5045*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^{5/2}(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{5/2}(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^{13/2}+b^2(Ab+3aB)x^{11/2}+3ab(Ab+aB)x^{9/2}+a^2(3Ab+aB)x^{7/2}+a^3Ax^{5/2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{7}a^3Ax^{7/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15}\right.}{a+bx}$$

input
```
Int[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)
*x^(9/2))/9 + (6*a*b*(A*b + a*B)*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(13
/2))/13 + (2*b^3*B*x^(15/2))/15))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2x^{\frac{7}{2}}(3003B\,b^3x^4+3465A\,b^3x^3+10395Ba\,b^2x^3+12285Aa\,b^2x^2+12285B\,a^2b\,x^2+15015A\,a^2bx+5005B\,a^3x+6435a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{45045(bx+a)^3}$
default	$\frac{2x^{\frac{7}{2}}(3003B\,b^3x^4+3465A\,b^3x^3+10395Ba\,b^2x^3+12285Aa\,b^2x^2+12285B\,a^2b\,x^2+15015A\,a^2bx+5005B\,a^3x+6435a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{45045(bx+a)^3}$
risch	$\frac{2\sqrt{(bx+a)^2}\,x^{\frac{7}{2}}(3003B\,b^3x^4+3465A\,b^3x^3+10395Ba\,b^2x^3+12285Aa\,b^2x^2+12285B\,a^2b\,x^2+15015A\,a^2bx+5005B\,a^3x+6435a^3A)}{45045(bx+a)}$
orering	$\frac{2(3003B\,b^3x^4+3465A\,b^3x^3+10395Ba\,b^2x^3+12285Aa\,b^2x^2+12285B\,a^2b\,x^2+15015A\,a^2bx+5005B\,a^3x+6435a^3A)x^{\frac{7}{2}}(b^2x^2+2abx}{45045(bx+a)^3}$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(7/2)*(3003*B*b^3*x^4+3465*A*b^3*x^3+10395*B*a*b^2*x^3+12285*A*a
*b^2*x^2+12285*B*a^2*b*x^2+15015*A*a^2*b*x+5005*B*a^3*x+6435*A*a^3)*((b*x+
a)^2)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{45045}\left(3003\,Bb^3x^7+6435\,Aa^3x^3+3465\left(3\,Bab^2+Ab^3\right)x^6+12285\left(Ba^2b+Aab^2\right)x^5+50\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
2/45045*(3003*B*b^3*x^7 + 6435*A*a^3*x^3 + 3465*(3*B*a*b^2 + A*b^3)*x^6 +
12285*(B*a^2*b + A*a*b^2)*x^5 + 5005*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [F]

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^{\frac{5}{2}}(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x)*((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{9009}\left(63\left(11\,b^3x^2+13\,ab^2x\right)x^{\frac{9}{2}}+182\left(9\,ab^2x^2+11\,a^2bx\right)x^{\frac{7}{2}}+143\left(7\,a^2bx^2+9\,a^3x\right)x^{\frac{5}{2}}\right)A+\frac{2}{6435}\left(33\left(13\,b^3x^2+15\,ab^2x\right)x^{\frac{11}{2}}+90\left(11\,ab^2x^2+13\,a^2bx\right)x^{\frac{9}{2}}+65\left(9\,a^2bx^2+11\,a^3x\right)x^{\frac{7}{2}}\right)B$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
2/9009*(63*(11*b^3*x^2 + 13*a*b^2*x)*x^(9/2) + 182*(9*a*b^2*x^2 + 11*a^2*b
*x)*x^(7/2) + 143*(7*a^2*b*x^2 + 9*a^3*x)*x^(5/2))*A + 2/6435*(33*(13*b^3*
x^2 + 15*a*b^2*x)*x^(11/2) + 90*(11*a*b^2*x^2 + 13*a^2*b*x)*x^(9/2) + 65*(
9*a^2*b*x^2 + 11*a^3*x)*x^(7/2))*B
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{15}\,Bb^3x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{6}{13}\,Bab^2x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{6}{11}\,Ba^2bx^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{6}{11}\,Aab^2x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{9}\,Ba^3x^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{3}\,Aa^2bx^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/15*B*b^3*x^(15/2)*sgn(b*x + a) + 6/13*B*a*b^2*x^(13/2)*sgn(b*x + a) + 2/
13*A*b^3*x^(13/2)*sgn(b*x + a) + 6/11*B*a^2*b*x^(11/2)*sgn(b*x + a) + 6/11
*A*a*b^2*x^(11/2)*sgn(b*x + a) + 2/9*B*a^3*x^(9/2)*sgn(b*x + a) + 2/3*A*a^
2*b*x^(9/2)*sgn(b*x + a) + 2/7*A*a^3*x^(7/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^{5/2}\,\left(A+B\,x\right)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.22

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{x}\,x^3(3003b^4x^4+13860a\,b^3x^3+24570a^2b^2x^2+20020a^3bx+6435a^4)}{45045}$$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*x**3*(6435*a**4 + 20020*a**3*b*x + 24570*a**2*b**2*x**2 + 13860
*a*b**3*x**3 + 3003*b**4*x**4))/45045
```

3.422 $\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 220

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2a^3Ax^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{2a^2(3Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2ab(Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2b^2(Ab+3aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{2b^3Bx^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}$$

output
```
2*a^3*A*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+2*a^2*(3*A*b+B*a)*x^(7/2)*((
b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*a*b*(A*b+B*a)*x^(9/2)*((b*x+a)^2)^(1/2)/(3*b
*x+3*a)+2*b^2*(A*b+3*B*a)*x^(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a)+2*b^3*B
*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13*a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2x^{5/2}\sqrt{(a+bx)^2}(429a^3(7A+5Bx)+715a^2bx(9A+7Bx)+455ab^2x^2(11A+9Bx)+105}{15015(a+bx)}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(2*x^(5/2)*Sqrt[(a + b*x)^2]*(429*a^3*(7*A + 5*B*x) + 715*a^2*b*x*(9*A + 7
*B*x) + 455*a*b^2*x^2*(11*A + 9*B*x) + 105*b^3*x^3*(13*A + 11*B*x)))/(1501
5*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3x^{3/2}(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{3/2}(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^{11/2}+b^2(Ab+3aB)x^{9/2}+3ab(Ab+aB)x^{7/2}+a^2(3Ab+aB)x^{5/2}+a^3Ax^{3/2}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{5}a^3Ax^{5/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}\right)}{a+bx}$$

input
```
Int[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)
*x^(7/2))/7 + (2*a*b*(A*b + a*B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(11/2
))/11 + (2*b^3*B*x^(13/2))/13))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005Aa\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435A\,a^{2}bx+2145B\,a^{3}x+3003a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15015\left(bx+a\right)^{3}}$
default	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005Aa\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435A\,a^{2}bx+2145B\,a^{3}x+3003a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15015\left(bx+a\right)^{3}}$
risch	$\frac{2\sqrt{\left(bx+a\right)^{2}}\,x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005Aa\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435A\,a^{2}bx+2145B\,a^{3}x+3003a^{3}A\right)}{15015\left(bx+a\right)}$
orering	$\frac{2\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005Aa\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435A\,a^{2}bx+2145B\,a^{3}x+3003a^{3}A\right)x^{\frac{5}{2}}\left(b^{2}x^{2}+2abx+a^{2}\right)}{15015\left(bx+a\right)^{3}}$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*x^(5/2)*(1155*B*b^3*x^4+1365*A*b^3*x^3+4095*B*a*b^2*x^3+5005*A*a*b
^2*x^2+5005*B*a^2*b*x^2+6435*A*a^2*b*x+2145*B*a^3*x+3003*A*a^3)*((b*x+a)^2
)^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{15015}\left(1155\,Bb^3x^6+3003\,Aa^3x^2+1365\left(3\,Bab^2+Ab^3\right)x^5+5005\left(Ba^2b+Aab^2\right)x^4+21$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
2/15015*(1155*B*b^3*x^6 + 3003*A*a^3*x^2 + 1365*(3*B*a*b^2 + A*b^3)*x^5 +
5005*(B*a^2*b + A*a*b^2)*x^4 + 2145*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [F]

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^{\frac{3}{2}}(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)*((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{3465}\left(35\left(9\,b^3x^2+11\,ab^2x\right)x^{\frac{7}{2}}+110\left(7\,ab^2x^2+9\,a^2bx\right)x^{\frac{5}{2}}+99\left(5\,a^2bx^2+7\,a^3x\right)x^{\frac{3}{2}}\right)A+\frac{2}{9009}\left(63\left(11\,b^3x^2+13\,ab^2x\right)x^{\frac{9}{2}}+182\left(9\,ab^2x^2+11\,a^2bx\right)x^{\frac{7}{2}}+143\left(7\,a^2bx^2+9\,a^3x\right)x^{\frac{5}{2}}\right)B$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
2/3465*(35*(9*b^3*x^2 + 11*a*b^2*x)*x^(7/2) + 110*(7*a*b^2*x^2 + 9*a^2*b*x
)*x^(5/2) + 99*(5*a^2*b*x^2 + 7*a^3*x)*x^(3/2))*A + 2/9009*(63*(11*b^3*x^2
 + 13*a*b^2*x)*x^(9/2) + 182*(9*a*b^2*x^2 + 11*a^2*b*x)*x^(7/2) + 143*(7*a
^2*b*x^2 + 9*a^3*x)*x^(5/2))*B
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{13}Bb^3x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{6}{11}Bab^2x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{11}Ab^3x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{3}Ba^2bx^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{3}Aab^2x^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{7}Ba^3x^{\frac{7}{2}}\operatorname{sgn}(bx+a)+\frac{6}{7}Aa^2bx^{\frac{7}{2}}\operatorname{sgn}(bx+a)+\frac{2}{5}Aa^3x^{\frac{5}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/13*B*b^3*x^(13/2)*sgn(b*x + a) + 6/11*B*a*b^2*x^(11/2)*sgn(b*x + a) + 2/
11*A*b^3*x^(11/2)*sgn(b*x + a) + 2/3*B*a^2*b*x^(9/2)*sgn(b*x + a) + 2/3*A*
a*b^2*x^(9/2)*sgn(b*x + a) + 2/7*B*a^3*x^(7/2)*sgn(b*x + a) + 6/7*A*a^2*b*
x^(7/2)*sgn(b*x + a) + 2/5*A*a^3*x^(5/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int x^{3/2}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.22

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{x}\,x^2(1155b^4x^4+5460a\,b^3x^3+10010a^2b^2x^2+8580a^3bx+3003a^4)}{15015}$$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*x**2*(3003*a**4 + 8580*a**3*b*x + 10010*a**2*b**2*x**2 + 5460*a
*b**3*x**3 + 1155*b**4*x**4))/15015
```

3.423 $\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 220

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2a^3Ax^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2a^2(3Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{6ab(Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2b^2(Ab+3aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{2b^3Bx^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}$$

output

```
2*a^3*A*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*a^2*(3*A*b+B*a)*x^(5/2)*((
b*x+a)^2)^(1/2)/(5*b*x+5*a)+6*a*b*(A*b+B*a)*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b
*x+7*a)+2*b^2*(A*b+3*B*a)*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+2*b^3*B*x^
(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2x^{3/2}\sqrt{(a+bx)^2}(231a^3(5A+3Bx)+297a^2bx(7A+5Bx)+165ab^2x^2(9A+7Bx)+35b^3}{3465(a+bx)}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(2*x^(3/2)*Sqrt[(a + b*x)^2]*(231*a^3*(5*A + 3*B*x) + 297*a^2*b*x*(7*A + 5
*B*x) + 165*a*b^2*x^2*(9*A + 7*B*x) + 35*b^3*x^3*(11*A + 9*B*x)))/(3465*(a
 + b*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^3\sqrt{x}(a+bx)^3(A+Bx)dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \sqrt{x}(a+bx)^3(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^{9/2}+b^2(Ab+3aB)x^{7/2}+3ab(Ab+aB)x^{5/2}+a^2(3Ab+aB)x^{3/2}+a^3A\sqrt{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{3}a^3Ax^{3/2}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}\right)}{a+bx}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)
*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(9/2)
)/9 + (2*b^3*B*x^(11/2))/11))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485Aa\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079A\,a^{2}bx+693B\,a^{3}x+1155a^{3}A\right)\left((bx+a)^{2}\right)^{\frac{3}{2}}}{3465(bx+a)^{3}}$
default	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485Aa\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079A\,a^{2}bx+693B\,a^{3}x+1155a^{3}A\right)\left((bx+a)^{2}\right)^{\frac{3}{2}}}{3465(bx+a)^{3}}$
risch	$\frac{2\sqrt{(bx+a)^{2}}\,x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485Aa\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079A\,a^{2}bx+693B\,a^{3}x+1155a^{3}A\right)}{3465(bx+a)}$
orering	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485Aa\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079A\,a^{2}bx+693B\,a^{3}x+1155a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{3465(bx+a)^{3}}$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(3/2)*(315*B*b^3*x^4+385*A*b^3*x^3+1155*B*a*b^2*x^3+1485*A*a*b^2*
x^2+1485*B*a^2*b*x^2+2079*A*a^2*b*x+693*B*a^3*x+1155*A*a^3)*((b*x+a)^2)^(3
/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.35

$$\int \sqrt{x}(A+Bx)\left(a^{2}+2abx+b^{2}x^{2}\right)^{3/2}\,dx=\frac{2}{3465}\left(315\,Bb^{3}x^{5}+1155\,Aa^{3}x+385\left(3\,Bab^{2}+Ab^{3}\right)x^{4}+1485\left(Ba^{2}b+Aab^{2}\right)x^{3}+693\left(B\right.\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
2/3465*(315*B*b^3*x^5 + 1155*A*a^3*x + 385*(3*B*a*b^2 + A*b^3)*x^4 + 1485*
(B*a^2*b + A*a*b^2)*x^3 + 693*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int \sqrt{x}(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)*((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{315}\left(5\left(7\,b^3x^2+9\,ab^2x\right)x^{\frac{5}{2}}+18\left(5\,ab^2x^2+7\,a^2bx\right)x^{\frac{3}{2}}+21\left(3\,a^2bx^2+5\,a^3x\right)\sqrt{x}\right)A+\frac{2}{3465}\left(35\left(9\,b^3x^2+11\,ab^2x\right)x^{\frac{7}{2}}+110\left(7\,ab^2x^2+9\,a^2bx\right)x^{\frac{5}{2}}+99\left(5\,a^2bx^2+7\,a^3x\right)x^{\frac{3}{2}}\right)B$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
2/315*(5*(7*b^3*x^2 + 9*a*b^2*x)*x^(5/2) + 18*(5*a*b^2*x^2 + 7*a^2*b*x)*x^
(3/2) + 21*(3*a^2*b*x^2 + 5*a^3*x)*sqrt(x))*A + 2/3465*(35*(9*b^3*x^2 + 11
*a*b^2*x)*x^(7/2) + 110*(7*a*b^2*x^2 + 9*a^2*b*x)*x^(5/2) + 99*(5*a^2*b*x^
2 + 7*a^3*x)*x^(3/2))*B
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2}{11}\,Bb^3x^{\frac{11}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Bab^2x^{\frac{9}{2}}\mathrm{sgn}(bx+a)+\frac{2}{9}\,Ab^3x^{\frac{9}{2}}\mathrm{sgn}(bx+a)+\frac{6}{7}\,Ba^2bx^{\frac{7}{2}}\mathrm{sgn}(bx+a)+\frac{6}{7}\,Aab^2x^{\frac{7}{2}}\mathrm{sgn}(bx+a)+\frac{2}{5}\,Ba^3x^{\frac{5}{2}}\mathrm{sgn}(bx+a)+\frac{6}{5}\,Aa^2bx^{\frac{5}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a)$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/11*B*b^3*x^(11/2)*sgn(b*x + a) + 2/3*B*a*b^2*x^(9/2)*sgn(b*x + a) + 2/9*
A*b^3*x^(9/2)*sgn(b*x + a) + 6/7*B*a^2*b*x^(7/2)*sgn(b*x + a) + 6/7*A*a*b^
2*x^(7/2)*sgn(b*x + a) + 2/5*B*a^3*x^(5/2)*sgn(b*x + a) + 6/5*A*a^2*b*x^(5
/2)*sgn(b*x + a) + 2/3*A*a^3*x^(3/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\int \sqrt{x}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.21

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{x}\,x(315b^4x^4+1540a\,b^3x^3+2970a^2b^2x^2+2772a^3bx+1155a^4)}{3465}$$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*x*(1155*a**4 + 2772*a**3*b*x + 2970*a**2*b**2*x**2 + 1540*a*b**
3*x**3 + 315*b**4*x**4))/3465
```

3.424 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 218

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2a^3A\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2a^2(3Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{6ab(Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{2b^2(Ab+3aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)} + \frac{2b^3Bx^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}$$

output
```
2*a^3*A*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*a^2*(3*A*b+B*a)*x^(3/2)*((b*x+
a)^2)^(1/2)/(3*b*x+3*a)+6*a*b*(A*b+B*a)*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5
*a)+2*b^2*(A*b+3*B*a)*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*b^3*B*x^(9/2
)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{(a+bx)^2}(105a^3(3A+Bx)+63a^2bx(5A+3Bx)+27ab^2x}{315(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[(a + b*x)^2]*(105*a^3*(3*A + B*x) + 63*a^2*b*x*(5*A + 3*B*
x) + 27*a*b^2*x^2*(7*A + 5*B*x) + 5*b^3*x^3*(9*A + 7*B*x)))/(315*(a + b*x)
)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{\sqrt{x}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^3Bx^{7/2}+b^2(Ab+3aB)x^{5/2}+3ab(Ab+aB)x^{3/2}+a^2(3Ab+aB)\sqrt{x}+\frac{a^3A}{\sqrt{x}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(2a^3A\sqrt{x}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{9}b^3Bx^{9/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(
3/2))/3 + (6*a*b*(A*b + a*B)*x^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7
+ (2*b^3*B*x^(9/2))/9))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{2\sqrt{x}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189Aa\,b^2x^2+189B\,a^2b\,x^2+315A\,a^2bx+105B\,a^3x+315a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{315(bx+a)^3}$	92
default	$\frac{2\sqrt{x}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189Aa\,b^2x^2+189B\,a^2b\,x^2+315A\,a^2bx+105B\,a^3x+315a^3A)\left((bx+a)^2\right)^{\frac{3}{2}}}{315(bx+a)^3}$	92
risch	$\frac{2\sqrt{(bx+a)^2}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189Aa\,b^2x^2+189B\,a^2b\,x^2+315A\,a^2bx+105B\,a^3x+315a^3A)\sqrt{x}}{315(bx+a)}$	92
orering	$\frac{2(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189Aa\,b^2x^2+189B\,a^2b\,x^2+315A\,a^2bx+105B\,a^3x+315a^3A)\sqrt{x}\,(b^2x^2+2abx+a^2)^{\frac{3}{2}}}{315(bx+a)^3}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(1/2)*(35*B*b^3*x^4+45*A*b^3*x^3+135*B*a*b^2*x^3+189*A*a*b^2*x^2+1
89*B*a^2*b*x^2+315*A*a^2*b*x+105*B*a^3*x+315*A*a^3)*((b*x+a)^2)^(3/2)/(b*x
+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

$$\int \frac{(A+Bx)\,(a^2+2abx+b^2x^2)^{3/2}}{\sqrt{x}}\,dx = \frac{2}{315}\left(35\,Bb^3x^4+315\,Aa^3+45\left(3\,Bab^2+Ab^3\right)x^3+189\left(Ba^2b+\right.\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(1/2),x, algorithm="fricas
")
```

output
```
2/315*(35*B*b^3*x^4 + 315*A*a^3 + 45*(3*B*a*b^2 + A*b^3)*x^3 + 189*(B*a^2*
b + A*a*b^2)*x^2 + 105*(B*a^3 + 3*A*a^2*b)*x)*sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**(1/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/sqrt(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx=\frac{2}{105}\left(3\left(5\,b^3x^2+7\,ab^2x\right)x^{\frac{3}{2}}+14\left(3\,ab^2x^2+5\,a^2bx\right)\sqrt{x}+\frac{35\,(a^2bx}{}\right.$$
$$+\frac{2}{315}\left(5\left(7\,b^3x^2+9\,ab^2x\right)x^{\frac{5}{2}}+18\left(5\,ab^2x^2+7\,a^2bx\right)x^{\frac{3}{2}}+21\left(3\,a^2bx^2+5\,a^3x\right)\sqrt{x}\right)B$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(1/2),x, algorithm="maxima
")
```

output
```
2/105*(3*(5*b^3*x^2 + 7*a*b^2*x)*x^(3/2) + 14*(3*a*b^2*x^2 + 5*a^2*b*x)*sq
rt(x) + 35*(a^2*b*x^2 + 3*a^3*x)/sqrt(x))*A + 2/315*(5*(7*b^3*x^2 + 9*a*b^
2*x)*x^(5/2) + 18*(5*a*b^2*x^2 + 7*a^2*b*x)*x^(3/2) + 21*(3*a^2*b*x^2 + 5*
a^3*x)*sqrt(x))*B
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2}{9}\,Bb^3x^{\frac{9}{2}}\mathrm{sgn}(bx+a) + \frac{6}{7}\,Bab^2x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + \frac{2}{7}\,Ab^3x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + \frac{6}{5}\,Ba^2bx^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{6}{5}\,Aab^2x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{2}{3}\,Ba^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 2\,Aa^2bx^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 2\,Aa^3\sqrt{x}\mathrm{sgn}(bx+a)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*B*b^3*x^(9/2)*sgn(b*x + a) + 6/7*B*a*b^2*x^(7/2)*sgn(b*x + a) + 2/7*A*
b^3*x^(7/2)*sgn(b*x + a) + 6/5*B*a^2*b*x^(5/2)*sgn(b*x + a) + 6/5*A*a*b^2*
x^(5/2)*sgn(b*x + a) + 2/3*B*a^3*x^(3/2)*sgn(b*x + a) + 2*A*a^2*b*x^(3/2)*
sgn(b*x + a) + 2*A*a^3*sqrt(x)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(1/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{\sqrt{x}}\,dx=\frac{2\sqrt{x}\,(35b^4x^4+180a\,b^3x^3+378a^2b^2x^2+420a^3bx+315a^4)}{315}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(315*a**4 + 420*a**3*b*x + 378*a**2*b**2*x**2 + 180*a*b**3*x**3
 + 35*b**4*x**4))/315
```

3.425 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 214

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx = -\frac{2a^3A\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2a^2(3Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2ab(Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2b^2(Ab+3aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)} + \frac{2b^3Bx^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
-2*a^3*A*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)+2*a^2*(3*A*b+B*a)*x^(1/2)*((b*x
+a)^2)^(1/2)/(b*x+a)+2*a*b*(A*b+B*a)*x^(3/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b
^2*(A*b+3*B*a)*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+2*b^3*B*x^(7/2)*((b*x
+a)^2)^(1/2)/(7*b*x+7*a)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{(a+bx)^2}(-35a^3(A-Bx)+35a^2bx(3A+Bx)+7ab^2x^2(5A}{35\sqrt{x}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(3/2),x]
```

output
```
(2*Sqrt[(a + b*x)^2]*(-35*a^3*(A - B*x) + 35*a^2*b*x*(3*A + B*x) + 7*a*b^2
*x^2*(5*A + 3*B*x) + b^3*x^3*(7*A + 5*B*x)))/(35*Sqrt[x]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{3/2}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}dx}{a+bx}$$

$$\Big\downarrow \ 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^{3/2}}+\frac{(3Ab+aB)a^2}{\sqrt{x}}+3b(Ab+aB)\sqrt{x}a+b^3Bx^{5/2}+b^2(Ab+3aB)x^{3/2}\right)dx}{a+bx}$$

$\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^3A}{\sqrt{x}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{5}b^2x^{5/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{7}b^3Bx^{7/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^3*A)/Sqrt[x] + 2*a^2*(3*A*b + a*B)*S
qrt[x] + 2*a*b*(A*b + a*B)*x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(5/2))/5 + (2*
b^3*B*x^(7/2))/7))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_^2))^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2\left(-5B\,b^{3}x^{4}-7A\,b^{3}x^{3}-21Ba\,b^{2}x^{3}-35Aa\,b^{2}x^{2}-35B\,a^{2}b\,x^{2}-105A\,a^{2}bx-35B\,a^{3}x+35a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{35\sqrt{x}\,\left(bx+a\right)^{3}}$	92
default	$-\frac{2\left(-5B\,b^{3}x^{4}-7A\,b^{3}x^{3}-21Ba\,b^{2}x^{3}-35Aa\,b^{2}x^{2}-35B\,a^{2}b\,x^{2}-105A\,a^{2}bx-35B\,a^{3}x+35a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{35\sqrt{x}\,\left(bx+a\right)^{3}}$	92
risch	$-\frac{2\sqrt{\left(bx+a\right)^{2}}\,\left(-5B\,b^{3}x^{4}-7A\,b^{3}x^{3}-21Ba\,b^{2}x^{3}-35Aa\,b^{2}x^{2}-35B\,a^{2}b\,x^{2}-105A\,a^{2}bx-35B\,a^{3}x+35a^{3}A\right)}{35\left(bx+a\right)\sqrt{x}}$	92
orering	$-\frac{2\left(-5B\,b^{3}x^{4}-7A\,b^{3}x^{3}-21Ba\,b^{2}x^{3}-35Aa\,b^{2}x^{2}-35B\,a^{2}b\,x^{2}-105A\,a^{2}bx-35B\,a^{3}x+35a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{35\sqrt{x}\,\left(bx+a\right)^{3}}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5*B*b^3*x^4-7*A*b^3*x^3-21*B*a*b^2*x^3-35*A*a*b^2*x^2-35*B*a^2*b*x
^2-105*A*a^2*b*x-35*B*a^3*x+35*A*a^3)*((b*x+a)^2)^(3/2)/x^(1/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{2\left(5\,Bb^3x^4-35\,Aa^3+7\,(3\,Bab^2+Ab^3)x^3+35\,(Ba^2b+Aab^2)x^2\right.}{35\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(3/2),x, algorithm="fricas
")
```

output
```
2/35*(5*B*b^3*x^4 - 35*A*a^3 + 7*(3*B*a*b^2 + A*b^3)*x^3 + 35*(B*a^2*b + A
*a*b^2)*x^2 + 35*(B*a^3 + 3*A*a^2*b)*x)/sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**(3/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{2}{15}\left(\left(3\,b^3x^2+5\,ab^2x\right)\sqrt{x}+\frac{10\left(ab^2x^2+3\,a^2bx\right)}{\sqrt{x}}+\frac{15\left(a^2bx^2-a^3\right.}{x^{\frac{3}{2}}}\right.$$
$$+\frac{2}{105}\left(3\left(5\,b^3x^2+7\,ab^2x\right)x^{\frac{3}{2}}+14\left(3\,ab^2x^2+5\,a^2bx\right)\sqrt{x}+\frac{35\left(a^2bx^2+3\,a^3x\right)}{\sqrt{x}}\right)B$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(3/2),x, algorithm="maxima
")
```

output
```
2/15*((3*b^3*x^2 + 5*a*b^2*x)*sqrt(x) + 10*(a*b^2*x^2 + 3*a^2*b*x)/sqrt(x)
 + 15*(a^2*b*x^2 - a^3*x)/x^(3/2))*A + 2/105*(3*(5*b^3*x^2 + 7*a*b^2*x)*x^
(3/2) + 14*(3*a*b^2*x^2 + 5*a^2*b*x)*sqrt(x) + 35*(a^2*b*x^2 + 3*a^3*x)/sq
rt(x))*B
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{2}{7}\,Bb^3x^{\frac{7}{2}}\operatorname{sgn}(bx+a) + \frac{6}{5}\,Bab^2x^{\frac{5}{2}}\operatorname{sgn}(bx+a) + \frac{2}{5}\,Ab^3x^{\frac{5}{2}}\operatorname{sgn}(bx+a) + 2\,Ba^2bx^{\frac{3}{2}}\operatorname{sgn}(bx+a) + 2\,Aab^2x^{\frac{3}{2}}\operatorname{sgn}(bx+a) + 2\,Ba^3\sqrt{x}\operatorname{sgn}(bx+a) + 6\,Aa^2b\sqrt{x}\operatorname{sgn}(bx+a) - \frac{2\,Aa^3\operatorname{sgn}(bx+a)}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*B*b^3*x^(7/2)*sgn(b*x + a) + 6/5*B*a*b^2*x^(5/2)*sgn(b*x + a) + 2/5*A*
b^3*x^(5/2)*sgn(b*x + a) + 2*B*a^2*b*x^(3/2)*sgn(b*x + a) + 2*A*a*b^2*x^(3
/2)*sgn(b*x + a) + 2*B*a^3*sqrt(x)*sgn(b*x + a) + 6*A*a^2*b*sqrt(x)*sgn(b*
x + a) - 2*A*a^3*sgn(b*x + a)/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(\frac{x\,(70\,B\,a^3+210\,A\,b\,a^2)}{35\,b}-\frac{2\,A\,a^3}{b}+\frac{2\,B\,b^2\,x^4}{7}+\frac{x^3\,(14}{}\right.}{x^{3/2}+\frac{a\,\sqrt{x}}{b}}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(3/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((x*(70*B*a^3 + 210*A*a^2*b))/(35*b) - (2
*A*a^3)/b + (2*B*b^2*x^4)/7 + (x^3*(14*A*b^3 + 42*B*a*b^2))/(35*b) + 2*a*x
^2*(A*b + B*a)))/(x^(3/2) + (a*x^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{\frac{2}{7}b^4x^4+\frac{8}{5}a\,b^3x^3+4a^2b^2x^2+8a^3bx-2a^4}{\sqrt{x}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(3/2),x)
```

output
```
(2*( - 35*a**4 + 140*a**3*b*x + 70*a**2*b**2*x**2 + 28*a*b**3*x**3 + 5*b**
4*x**4))/(35*sqrt(x))
```

3.426 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 216

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx = -\frac{2a^3A\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)} - \frac{2a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{6ab(Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2b^2(Ab+3aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{2b^3Bx^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

output
```
-2/3*a^3*A*((b*x+a)^2)^(1/2)/x^(3/2)/(b*x+a)-2*a^2*(3*A*b+B*a)*((b*x+a)^2)
^(1/2)/x^(1/2)/(b*x+a)+6*a*b*(A*b+B*a)*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2
*b^2*(A*b+3*B*a)*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*b^3*B*x^(5/2)*((b
*x+a)^2)^(1/2)/(5*b*x+5*a)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}\left(45a^2bx(A-Bx)-15ab^2x^2(3A+Bx)+5a^3(A+3Bx)-b^3x^3(5A+3Bx)\right)}{15x^{3/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(5/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(45*a^2*b*x*(A - B*x) - 15*a*b^2*x^2*(3*A + B*x) + 5
*a^3*(A + 3*B*x) - b^3*x^3*(5*A + 3*B*x)))/(15*x^(3/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{5/2}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^{5/2}}+\frac{(3Ab+aB)a^2}{x^{3/2}}+\frac{3b(Ab+aB)a}{\sqrt{x}}+b^3Bx^{3/2}+b^2(Ab+3aB)\sqrt{x}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^3A}{3x^{3/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}(3aB+Ab)+6ab\sqrt{x}(aB+Ab)+\frac{2}{5}b^3Bx^{5/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^3*A)/(3*x^(3/2)) - (2*a^2*(3*A*b + a
*B))/Sqrt[x] + 6*a*b*(A*b + a*B)*Sqrt[x] + (2*b^2*(A*b + 3*a*B)*x^(3/2))/3
 + (2*b^3*B*x^(5/2))/5))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_^2))^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2\left(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45Aa\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45A\,a^{2}bx+15B\,a^{3}x+5a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15x^{\frac{3}{2}}\left(bx+a\right)^{3}}$	92
default	$-\frac{2\left(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45Aa\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45A\,a^{2}bx+15B\,a^{3}x+5a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15x^{\frac{3}{2}}\left(bx+a\right)^{3}}$	92
risch	$-\frac{2\sqrt{\left(bx+a\right)^{2}}\,\left(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45Aa\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45A\,a^{2}bx+15B\,a^{3}x+5a^{3}A\right)}{15\left(bx+a\right)x^{\frac{3}{2}}}$	92
orering	$-\frac{2\left(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45Aa\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45A\,a^{2}bx+15B\,a^{3}x+5a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{15x^{\frac{3}{2}}\left(bx+a\right)^{3}}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^3*x^4-5*A*b^3*x^3-15*B*a*b^2*x^3-45*A*a*b^2*x^2-45*B*a^2*b*x
^2+45*A*a^2*b*x+15*B*a^3*x+5*A*a^3)*((b*x+a)^2)^(3/2)/x^(3/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{2\left(3\,Bb^3x^4-5\,Aa^3+5\,(3\,Bab^2+Ab^3)x^3+45\,(Ba^2b+Aab^2)x^2-\right.}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(5/2),x, algorithm="fricas
")
```

output
```
2/15*(3*B*b^3*x^4 - 5*A*a^3 + 5*(3*B*a*b^2 + A*b^3)*x^3 + 45*(B*a^2*b + A*
a*b^2)*x^2 - 15*(B*a^3 + 3*A*a^2*b)*x)/x^(3/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**(5/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx=\frac{2}{15}\left(\left(3\,b^3x^2+5\,ab^2x\right)\sqrt{x}+\frac{10\left(ab^2x^2+3\,a^2bx\right)}{\sqrt{x}}+\frac{15\left(a^2bx^2-a^3\right.}{x^{\frac{3}{2}}}\right.$$
$$+\frac{2}{3}\,A\left(\frac{b^3x^2+3\,ab^2x}{\sqrt{x}}+\frac{6\left(ab^2x^2-a^2bx\right)}{x^{\frac{3}{2}}}-\frac{3\,a^2bx^2+a^3x}{x^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(5/2),x, algorithm="maxima
")
```

output
```
2/15*((3*b^3*x^2 + 5*a*b^2*x)*sqrt(x) + 10*(a*b^2*x^2 + 3*a^2*b*x)/sqrt(x)
 + 15*(a^2*b*x^2 - a^3*x)/x^(3/2))*B + 2/3*A*((b^3*x^2 + 3*a*b^2*x)/sqrt(x
) + 6*(a*b^2*x^2 - a^2*b*x)/x^(3/2) - (3*a^2*b*x^2 + a^3*x)/x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx=\frac{2}{5}\,Bb^3x^{\frac{5}{2}}\mathrm{sgn}(bx+a)$$
$$+2\,Bab^2x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Ab^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a)$$
$$+6\,Ba^2b\sqrt{x}\mathrm{sgn}(bx+a)+6\,Aab^2\sqrt{x}\mathrm{sgn}(bx+a)$$
$$-\frac{2\left(3\,Ba^3x\mathrm{sgn}(bx+a)+9\,Aa^2bx\mathrm{sgn}(bx+a)+Aa^3\mathrm{sgn}(bx+a)\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*B*b^3*x^(5/2)*sgn(b*x + a) + 2*B*a*b^2*x^(3/2)*sgn(b*x + a) + 2/3*A*b^
3*x^(3/2)*sgn(b*x + a) + 6*B*a^2*b*sqrt(x)*sgn(b*x + a) + 6*A*a*b^2*sqrt(x
)*sgn(b*x + a) - 2/3*(3*B*a^3*x*sgn(b*x + a) + 9*A*a^2*b*x*sgn(b*x + a) +
A*a^3*sgn(b*x + a))/x^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{5/2}}\,dx=\frac{\frac{2}{5}b^4x^4+\frac{8}{3}a\,b^3x^3+12a^2b^2x^2-8a^3bx-\frac{2}{3}a^4}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(5/2),x)
```

output
```
(2*( - 5*a**4 - 60*a**3*b*x + 90*a**2*b**2*x**2 + 20*a*b**3*x**3 + 3*b**4*
x**4))/(15*sqrt(x)*x)
```

3.427 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 216

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx = -\frac{2a^3A\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)} - \frac{2a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)} - \frac{6ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2b^2(Ab+3aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{2b^3Bx^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}$$

output
```
-2/5*a^3*A*((b*x+a)^2)^(1/2)/x^(5/2)/(b*x+a)-2/3*a^2*(3*A*b+B*a)*((b*x+a)^
2)^(1/2)/x^(3/2)/(b*x+a)-6*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)
+2*b^2*(A*b+3*B*a)*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b^3*B*x^(3/2)*((b*x
+a)^2)^(1/2)/(3*b*x+3*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}(45ab^2x^2(A-Bx)-5b^3x^3(3A+Bx)+15a^2bx(A+3Bx)+a^3(3A+5Bx))}{15x^{5/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(7/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(45*a*b^2*x^2*(A - B*x) - 5*b^3*x^3*(3*A + B*x) + 15
*a^2*b*x*(A + 3*B*x) + a^3*(3*A + 5*B*x)))/(15*x^(5/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{7/2}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^{7/2}}+\frac{(3Ab+aB)a^2}{x^{5/2}}+\frac{3b(Ab+aB)a}{x^{3/2}}+b^3B\sqrt{x}+\frac{b^2(Ab+3aB)}{\sqrt{x}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{3x^{3/2}}+2b^2\sqrt{x}(3aB+Ab)-\frac{6ab(aB+Ab)}{\sqrt{x}}+\frac{2}{3}b^3Bx^{3/2}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(7/2),x]
```

output
```
(((-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/(3*x^(3/2)) - (6*a*b*(A*b
 + a*B))/Sqrt[x] + 2*b^2*(A*b + 3*a*B)*Sqrt[x] + (2*b^3*B*x^(3/2))/3)*Sqrt
[a^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2\left(-5B\,b^{3}x^{4}-15A\,b^{3}x^{3}-45Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+15A\,a^{2}bx+5B\,a^{3}x+3a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15x^{\frac{5}{2}}\left(bx+a\right)^{3}}$	92
default	$-\frac{2\left(-5B\,b^{3}x^{4}-15A\,b^{3}x^{3}-45Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+15A\,a^{2}bx+5B\,a^{3}x+3a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{15x^{\frac{5}{2}}\left(bx+a\right)^{3}}$	92
risch	$-\frac{2\sqrt{\left(bx+a\right)^{2}}\,\left(-5B\,b^{3}x^{4}-15A\,b^{3}x^{3}-45Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+15A\,a^{2}bx+5B\,a^{3}x+3a^{3}A\right)}{15\left(bx+a\right)x^{\frac{5}{2}}}$	92
orering	$-\frac{2\left(-5B\,b^{3}x^{4}-15A\,b^{3}x^{3}-45Ba\,b^{2}x^{3}+45Aa\,b^{2}x^{2}+45B\,a^{2}b\,x^{2}+15A\,a^{2}bx+5B\,a^{3}x+3a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{15x^{\frac{5}{2}}\left(bx+a\right)^{3}}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-5*B*b^3*x^4-15*A*b^3*x^3-45*B*a*b^2*x^3+45*A*a*b^2*x^2+45*B*a^2*b*
x^2+15*A*a^2*b*x+5*B*a^3*x+3*A*a^3)*((b*x+a)^2)^(3/2)/x^(5/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx = \frac{2\left(5\,Bb^3x^4-3\,Aa^3+15\,(3\,Bab^2+Ab^3)x^3-45\,(Ba^2b+Aab^2)x^2\right.}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(7/2),x, algorithm="fricas
")
```

output
```
2/15*(5*B*b^3*x^4 - 3*A*a^3 + 15*(3*B*a*b^2 + A*b^3)*x^3 - 45*(B*a^2*b + A
*a*b^2)*x^2 - 5*(B*a^3 + 3*A*a^2*b)*x)/x^(5/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**(7/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx=\frac{2}{3}\,B\left(\frac{b^3x^2+3\,ab^2x}{\sqrt{x}}+\frac{6\,(ab^2x^2-a^2bx)}{x^{\frac{3}{2}}}-\frac{3\,a^2bx^2+a^3x}{x^{\frac{5}{2}}}\right)+\frac{2}{15}\,A\left(\frac{15\,(b^3x^2-ab^2x)}{x^{\frac{3}{2}}}-\frac{10\,(3\,ab^2x^2+a^2bx)}{x^{\frac{5}{2}}}-\frac{5\,a^2bx^2+3\,a^3x}{x^{\frac{7}{2}}}\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(7/2),x, algorithm="maxima
")
```

output
```
2/3*B*((b^3*x^2 + 3*a*b^2*x)/sqrt(x) + 6*(a*b^2*x^2 - a^2*b*x)/x^(3/2) - (
3*a^2*b*x^2 + a^3*x)/x^(5/2)) + 2/15*A*(15*(b^3*x^2 - a*b^2*x)/x^(3/2) - 1
0*(3*a*b^2*x^2 + a^2*b*x)/x^(5/2) - (5*a^2*b*x^2 + 3*a^3*x)/x^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx=\frac{2}{3}\,Bb^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+6\,Bab^2\sqrt{x}\mathrm{sgn}(bx+a)+2\,Ab^3\sqrt{x}\mathrm{sgn}(bx+a)-\frac{2\,(45\,Ba^2bx^2\mathrm{sgn}(bx+a)+45\,Aab^2x^2\mathrm{sgn}(bx+a)+5\,Ba^3x\mathrm{sgn}(bx+a)+15\,Aa^2bx\mathrm{sgn}(bx+a)+3\,Aa^3\mathrm{s}}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*B*b^3*x^(3/2)*sgn(b*x + a) + 6*B*a*b^2*sqrt(x)*sgn(b*x + a) + 2*A*b^3*
sqrt(x)*sgn(b*x + a) - 2/15*(45*B*a^2*b*x^2*sgn(b*x + a) + 45*A*a*b^2*x^2*
sgn(b*x + a) + 5*B*a^3*x*sgn(b*x + a) + 15*A*a^2*b*x*sgn(b*x + a) + 3*A*a^
3*sgn(b*x + a))/x^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(7/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{7/2}}\,dx=\frac{\frac{2}{3}b^4x^4+8a\,b^3x^3-12a^2b^2x^2-\frac{8}{3}a^3bx-\frac{2}{5}a^4}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(7/2),x)
```

output
```
(2*( - 3*a**4 - 20*a**3*b*x - 90*a**2*b**2*x**2 + 60*a*b**3*x**3 + 5*b**4*
x**4))/(15*sqrt(x)*x**2)
```

3.428 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 214

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2a^3A\sqrt{a^2+2abx+b^2x^2}}{7x^{7/2}(a+bx)} - \frac{2a^2(3Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)} - \frac{2ab(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{x^{3/2}(a+bx)} - \frac{2b^2(Ab+3aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2b^3B\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx}$$

output
```
-2/7*a^3*A*((b*x+a)^2)^(1/2)/x^(7/2)/(b*x+a)-2/5*a^2*(3*A*b+B*a)*((b*x+a)^
2)^(1/2)/x^(5/2)/(b*x+a)-2*a*b*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^(3/2)/(b*x+a)
-2*b^2*(A*b+3*B*a)*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)+2*b^3*B*x^(1/2)*((b*x
+a)^2)^(1/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2\sqrt{(a+bx)^2}\left(35b^3x^3(A-Bx)+35ab^2x^2(A+3Bx)+7a^2bx(3A+5Bx)+a^3(5A+7Bx)\right)}{35x^{7/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(9/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(35*b^3*x^3*(A - B*x) + 35*a*b^2*x^2*(A + 3*B*x) + 7
*a^2*b*x*(3*A + 5*B*x) + a^3*(5*A + 7*B*x)))/(35*x^(7/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}(A+Bx)}{x^{9/2}}\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^3(a+bx)^3(A+Bx)}{x^{9/2}}dx}{b^3(a+bx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^3(A+Bx)}{x^{9/2}}dx}{a+bx}$$

$$\Big\downarrow \; 85$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^3}{x^{9/2}}+\frac{(3Ab+aB)a^2}{x^{7/2}}+\frac{3b(Ab+aB)a}{x^{5/2}}+\frac{b^3B}{\sqrt{x}}+\frac{b^2(Ab+3aB)}{x^{3/2}}\right)dx}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^3A}{7x^{7/2}}-\frac{2a^2(aB+3Ab)}{5x^{5/2}}-\frac{2b^2(3aB+Ab)}{\sqrt{x}}-\frac{2ab(aB+Ab)}{x^{3/2}}+2b^3B\sqrt{x}\right)}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2))/x^(9/2),x]
```

output
```
(((-2*a^3*A)/(7*x^(7/2)) - (2*a^2*(3*A*b + a*B))/(5*x^(5/2)) - (2*a*b*(A*b
 + a*B))/x^(3/2) - (2*b^2*(A*b + 3*a*B))/Sqrt[x] + 2*b^3*B*Sqrt[x])*Sqrt[a
^2 + 2*a*b*x + b^2*x^2])/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2\left(-35B\,b^{3}x^{4}+35A\,b^{3}x^{3}+105Ba\,b^{2}x^{3}+35Aa\,b^{2}x^{2}+35B\,a^{2}b\,x^{2}+21A\,a^{2}bx+7B\,a^{3}x+5a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{35x^{\frac{7}{2}}\left(bx+a\right)^{3}}$	92
default	$-\frac{2\left(-35B\,b^{3}x^{4}+35A\,b^{3}x^{3}+105Ba\,b^{2}x^{3}+35Aa\,b^{2}x^{2}+35B\,a^{2}b\,x^{2}+21A\,a^{2}bx+7B\,a^{3}x+5a^{3}A\right)\left(\left(bx+a\right)^{2}\right)^{\frac{3}{2}}}{35x^{\frac{7}{2}}\left(bx+a\right)^{3}}$	92
risch	$-\frac{2\sqrt{\left(bx+a\right)^{2}}\,\left(-35B\,b^{3}x^{4}+35A\,b^{3}x^{3}+105Ba\,b^{2}x^{3}+35Aa\,b^{2}x^{2}+35B\,a^{2}b\,x^{2}+21A\,a^{2}bx+7B\,a^{3}x+5a^{3}A\right)}{35\left(bx+a\right)x^{\frac{7}{2}}}$	92
orering	$-\frac{2\left(-35B\,b^{3}x^{4}+35A\,b^{3}x^{3}+105Ba\,b^{2}x^{3}+35Aa\,b^{2}x^{2}+35B\,a^{2}b\,x^{2}+21A\,a^{2}bx+7B\,a^{3}x+5a^{3}A\right)\left(b^{2}x^{2}+2abx+a^{2}\right)^{\frac{3}{2}}}{35x^{\frac{7}{2}}\left(bx+a\right)^{3}}$	101

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-35*B*b^3*x^4+35*A*b^3*x^3+105*B*a*b^2*x^3+35*A*a*b^2*x^2+35*B*a^2*
b*x^2+21*A*a^2*b*x+7*B*a^3*x+5*A*a^3)*((b*x+a)^2)^(3/2)/x^(7/2)/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.34

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{2\left(35\,Bb^3x^4-5\,Aa^3-35\,(3\,Bab^2+Ab^3)x^3-35\,(Ba^2b+Aab^2)x^2\right.}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(9/2),x, algorithm="fricas
")
```

output
```
2/35*(35*B*b^3*x^4 - 5*A*a^3 - 35*(3*B*a*b^2 + A*b^3)*x^3 - 35*(B*a^2*b +
A*a*b^2)*x^2 - 7*(B*a^3 + 3*A*a^2*b)*x)/x^(7/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(3/2)/x**(9/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(3/2)/x**(9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx=\frac{2}{15}\,B\left(\frac{15\,(b^3x^2-ab^2x)}{x^{\frac{3}{2}}}-\frac{10\,(3\,ab^2x^2+a^2bx)}{x^{\frac{5}{2}}}-\frac{5\,a^2bx^2+3\,a^3x}{x^{\frac{7}{2}}}\right)$$
$$-\frac{2}{105}\,A\left(\frac{35\,(3\,b^3x^2+ab^2x)}{x^{\frac{5}{2}}}+\frac{14\,(5\,ab^2x^2+3\,a^2bx)}{x^{\frac{7}{2}}}+\frac{3\,(7\,a^2bx^2+5\,a^3x)}{x^{\frac{9}{2}}}\right)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(9/2),x, algorithm="maxima
")
```

output
```
2/15*B*(15*(b^3*x^2 - a*b^2*x)/x^(3/2) - 10*(3*a*b^2*x^2 + a^2*b*x)/x^(5/2
) - (5*a^2*b*x^2 + 3*a^3*x)/x^(7/2)) - 2/105*A*(35*(3*b^3*x^2 + a*b^2*x)/x
^(5/2) + 14*(5*a*b^2*x^2 + 3*a^2*b*x)/x^(7/2) + 3*(7*a^2*b*x^2 + 5*a^3*x)/
x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = 2\,Bb^3\sqrt{x}\mathrm{sgn}(bx+a)$$
$$-\frac{2\,(105\,Bab^2x^3\mathrm{sgn}(bx+a)+35\,Ab^3x^3\mathrm{sgn}(bx+a)+35\,Ba^2bx^2\mathrm{sgn}(bx+a)+35\,Aab^2x^2\mathrm{sgn}(bx+a)+7\,I}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
2*B*b^3*sqrt(x)*sgn(b*x + a) - 2/35*(105*B*a*b^2*x^3*sgn(b*x + a) + 35*A*b
^3*x^3*sgn(b*x + a) + 35*B*a^2*b*x^2*sgn(b*x + a) + 35*A*a*b^2*x^2*sgn(b*x
 + a) + 7*B*a^3*x*sgn(b*x + a) + 21*A*a^2*b*x*sgn(b*x + a) + 5*A*a^3*sgn(b
*x + a))/x^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(9/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{2b^4x^4-8a\,b^3x^3-4a^2b^2x^2-\frac{8}{5}a^3bx-\frac{2}{7}a^4}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(3/2)/x^(9/2),x)
```

output
```
(2*( - 5*a**4 - 28*a**3*b*x - 70*a**2*b**2*x**2 - 140*a*b**3*x**3 + 35*b**
4*x**4))/(35*sqrt(x)*x**3)
```

3.429 $\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 320

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2a^5Ax^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{2a^4(5Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{10a^3b(2Ab+aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{4a^2b^2(Ab+aB)x^{15/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{10ab^3(Ab+2aB)x^{17/2}\sqrt{a^2+2abx+b^2x^2}}{17(a+bx)}+\frac{2b^4(Ab+5aB)x^{19/2}\sqrt{a^2+2abx+b^2x^2}}{19(a+bx)}+\frac{2b^5Bx^{21/2}\sqrt{a^2+2abx+b^2x^2}}{21(a+bx)}$$

output
```
2*a^5*A*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+2*a^4*(5*A*b+B*a)*x^(11/2)*(
(b*x+a)^2)^(1/2)/(11*b*x+11*a)+10*a^3*b*(2*A*b+B*a)*x^(13/2)*((b*x+a)^2)^(
1/2)/(13*b*x+13*a)+4*a^2*b^2*(A*b+B*a)*x^(15/2)*((b*x+a)^2)^(1/2)/(3*b*x+3
*a)+10*a*b^3*(A*b+2*B*a)*x^(17/2)*((b*x+a)^2)^(1/2)/(17*b*x+17*a)+2*b^4*(A
*b+5*B*a)*x^(19/2)*((b*x+a)^2)^(1/2)/(19*b*x+19*a)+2*b^5*B*x^(21/2)*((b*x+
a)^2)^(1/2)/(21*b*x+21*a)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.40

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2x^{9/2}\sqrt{(a+bx)^2}(29393a^5(11A+9Bx)+101745a^4bx(13A+11Bx)+149226a^3b^2x^2(15A$$

$$2909$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(2*x^(9/2)*Sqrt[(a + b*x)^2]*(29393*a^5*(11*A + 9*B*x) + 101745*a^4*b*x*(1
3*A + 11*B*x) + 149226*a^3*b^2*x^2*(15*A + 13*B*x) + 114114*a^2*b^3*x^3*(1
7*A + 15*B*x) + 45045*a*b^4*x^4*(19*A + 17*B*x) + 7293*b^5*x^5*(21*A + 19*
B*x)))/(2909907*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^{7/2}(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{7/2}(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^5Bx^{19/2}+b^4(Ab+5aB)x^{17/2}+5ab^3(Ab+2aB)x^{15/2}+10a^2b^2(Ab+aB)x^{13/2}+5a^3b(2\right.}{a+bx}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{9}a^5Ax^{9/2}+\frac{2}{11}a^4x^{11/2}(aB+5Ab)+\frac{10}{13}a^3bx^{13/2}(aB+2Ab)+\frac{4}{3}a^2b^2x^{15/2}(aB+Ab)+\frac{2}{19}b^4x\right.}{a+bx}$$

input
```
Int[x^(7/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^5*A*x^(9/2))/9 + (2*a^4*(5*A*b + a*B)
*x^(11/2))/11 + (10*a^3*b*(2*A*b + a*B)*x^(13/2))/13 + (4*a^2*b^2*(A*b + a
*B)*x^(15/2))/3 + (10*a*b^3*(A*b + 2*a*B)*x^(17/2))/17 + (2*b^4*(A*b + 5*a
*B)*x^(19/2))/19 + (2*b^5*B*x^(21/2))/21))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{2x^{\frac{9}{2}}(138567B\,b^5x^6+153153A\,b^5x^5+765765Ba\,b^4x^5+855855Aa\,b^4x^4+1711710B\,a^2b^3x^4+1939938A\,a^2b^3x^3+1939938B\,a^3b^2x^3+2}{2909907(bx+a)^5}$
default	$\frac{2x^{\frac{9}{2}}(138567B\,b^5x^6+153153A\,b^5x^5+765765Ba\,b^4x^5+855855Aa\,b^4x^4+1711710B\,a^2b^3x^4+1939938A\,a^2b^3x^3+1939938B\,a^3b^2x^3+2}{2909907(bx+a)^5}$
risch	$\frac{2\sqrt{(bx+a)^2}\,x^{\frac{9}{2}}(138567B\,b^5x^6+153153A\,b^5x^5+765765Ba\,b^4x^5+855855Aa\,b^4x^4+1711710B\,a^2b^3x^4+1939938A\,a^2b^3x^3+1939938B}{2909907(bx+a)}$
orering	$\frac{2(138567B\,b^5x^6+153153A\,b^5x^5+765765Ba\,b^4x^5+855855Aa\,b^4x^4+1711710B\,a^2b^3x^4+1939938A\,a^2b^3x^3+1939938B\,a^3b^2x^3+2238}{2909907(bx+a)^5}$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/2909907*x^(9/2)*(138567*B*b^5*x^6+153153*A*b^5*x^5+765765*B*a*b^4*x^5+85
5855*A*a*b^4*x^4+1711710*B*a^2*b^3*x^4+1939938*A*a^2*b^3*x^3+1939938*B*a^3
*b^2*x^3+2238390*A*a^3*b^2*x^2+1119195*B*a^4*b*x^2+1322685*A*a^4*b*x+26453
7*B*a^5*x+323323*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{2909907}\left(138567\,Bb^5x^{10}+323323\,Aa^5x^4+153153\left(5\,Bab^4+Ab^5\right)x^9+855855\left(2\,Ba^2b^3+\right.\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
2/2909907*(138567*B*b^5*x^10 + 323323*A*a^5*x^4 + 153153*(5*B*a*b^4 + A*b^
5)*x^9 + 855855*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b
^3)*x^7 + 1119195*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 264537*(B*a^5 + 5*A*a^4*b)
*x^5)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{2078505}\left(6435\left(17\,b^5x^2+19\,ab^4x\right)x^{\frac{15}{2}}+32604\left(15\,ab^4x^2+17\,a^2b^3x\right)x^{\frac{13}{2}}+63954\left(13\,a^2b^3\right.\right. \\ +\frac{2}{4849845}\left(12155\left(19\,b^5x^2+21\,ab^4x\right)x^{\frac{17}{2}}+60060\left(17\,ab^4x^2+19\,a^2b^3x\right)x^{\frac{15}{2}}+114114\left(15\,a^2b^3x^2+17\,a^3b^2\right.\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
2/2078505*(6435*(17*b^5*x^2 + 19*a*b^4*x)*x^(15/2) + 32604*(15*a*b^4*x^2 +
 17*a^2*b^3*x)*x^(13/2) + 63954*(13*a^2*b^3*x^2 + 15*a^3*b^2*x)*x^(11/2) +
 58140*(11*a^3*b^2*x^2 + 13*a^4*b*x)*x^(9/2) + 20995*(9*a^4*b*x^2 + 11*a^5
*x)*x^(7/2))*A + 2/4849845*(12155*(19*b^5*x^2 + 21*a*b^4*x)*x^(17/2) + 600
60*(17*a*b^4*x^2 + 19*a^2*b^3*x)*x^(15/2) + 114114*(15*a^2*b^3*x^2 + 17*a^
3*b^2*x)*x^(13/2) + 99484*(13*a^3*b^2*x^2 + 15*a^4*b*x)*x^(11/2) + 33915*(
11*a^4*b*x^2 + 13*a^5*x)*x^(9/2))*B
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{21}\,Bb^5x^{\frac{21}{2}}\operatorname{sgn}(bx+a)+\frac{10}{19}\,Bab^4x^{\frac{19}{2}}\operatorname{sgn}(bx+a)+\frac{2}{19}\,Ab^5x^{\frac{19}{2}}\operatorname{sgn}(bx+a)+\frac{20}{17}\,Ba^2b^3x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{10}{17}\,Aab^4x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{4}{3}\,Ba^3b^2x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{4}{3}\,Aa^2b^3x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{10}{13}\,Ba^4bx^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{20}{13}\,Aa^3b^2x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{2}{11}\,Ba^5x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{10}{11}\,Aa^4bx^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{9}\,Aa^5x^{\frac{9}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/21*B*b^5*x^(21/2)*sgn(b*x + a) + 10/19*B*a*b^4*x^(19/2)*sgn(b*x + a) + 2
/19*A*b^5*x^(19/2)*sgn(b*x + a) + 20/17*B*a^2*b^3*x^(17/2)*sgn(b*x + a) +
10/17*A*a*b^4*x^(17/2)*sgn(b*x + a) + 4/3*B*a^3*b^2*x^(15/2)*sgn(b*x + a)
+ 4/3*A*a^2*b^3*x^(15/2)*sgn(b*x + a) + 10/13*B*a^4*b*x^(13/2)*sgn(b*x + a
) + 20/13*A*a^3*b^2*x^(13/2)*sgn(b*x + a) + 2/11*B*a^5*x^(11/2)*sgn(b*x +
a) + 10/11*A*a^4*b*x^(11/2)*sgn(b*x + a) + 2/9*A*a^5*x^(9/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^{7/2}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^(7/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int x^{7/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2\sqrt{x}\,x^4(138567b^6x^6+918918a\,b^5x^5+2567565a^2b^4x^4+3879876a^3b^3x^3+3357585a^4b^2x^2+}{2909907}$$

input
```
int(x^(7/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*x**4*(323323*a**6 + 1587222*a**5*b*x + 3357585*a**4*b**2*x**2 +
 3879876*a**3*b**3*x**3 + 2567565*a**2*b**4*x**4 + 918918*a*b**5*x**5 + 13
8567*b**6*x**6))/2909907
```

3.430 $\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 320

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2a^5Ax^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2a^4(5Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{10a^3b(2Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{20a^2b^2(Ab+aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{2ab^3(Ab+2aB)x^{15/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2b^4(Ab+5aB)x^{17/2}\sqrt{a^2+2abx+b^2x^2}}{17(a+bx)}+\frac{2b^5Bx^{19/2}\sqrt{a^2+2abx+b^2x^2}}{19(a+bx)}$$

output
```
2*a^5*A*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*a^4*(5*A*b+B*a)*x^(9/2)*((
b*x+a)^2)^(1/2)/(9*b*x+9*a)+10*a^3*b*(2*A*b+B*a)*x^(11/2)*((b*x+a)^2)^(1/2
)/(11*b*x+11*a)+20*a^2*b^2*(A*b+B*a)*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13
*a)+2*a*b^3*(A*b+2*B*a)*x^(15/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*b^4*(A*b+
5*B*a)*x^(17/2)*((b*x+a)^2)^(1/2)/(17*b*x+17*a)+2*b^5*B*x^(19/2)*((b*x+a)^
2)^(1/2)/(19*b*x+19*a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.40

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{2x^{7/2}\sqrt{(a+bx)^2}(46189a^5(9A+7Bx)+146965a^4bx(11A+9Bx)+203490a^3b^2x^2(13A+$$

2909907

input
```
Integrate[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(2*x^(7/2)*Sqrt[(a + b*x)^2]*(46189*a^5*(9*A + 7*B*x) + 146965*a^4*b*x*(11
*A + 9*B*x) + 203490*a^3*b^2*x^2*(13*A + 11*B*x) + 149226*a^2*b^3*x^3*(15*
A + 13*B*x) + 57057*a*b^4*x^4*(17*A + 15*B*x) + 9009*b^5*x^5*(19*A + 17*B*
x)))/(2909907*(a + b*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^{5/2}(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{5/2}(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(b^5Bx^{17/2}+b^4(Ab+5aB)x^{15/2}+5ab^3(Ab+2aB)x^{13/2}+10a^2b^2(Ab+aB)x^{11/2}+5a^3b(2\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{7}a^5Ax^{7/2}+\frac{2}{9}a^4x^{9/2}(aB+5Ab)+\frac{10}{11}a^3bx^{11/2}(aB+2Ab)+\frac{20}{13}a^2b^2x^{13/2}(aB+Ab)+\frac{2}{17}b^4x\right.}{a+bx}$$

input
```
Int[x^(5/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^5*A*x^(7/2))/7 + (2*a^4*(5*A*b + a*B)
*x^(9/2))/9 + (10*a^3*b*(2*A*b + a*B)*x^(11/2))/11 + (20*a^2*b^2*(A*b + a*
B)*x^(13/2))/13 + (2*a*b^3*(A*b + 2*a*B)*x^(15/2))/3 + (2*b^4*(A*b + 5*a*B
)*x^(17/2))/17 + (2*b^5*B*x^(19/2))/19))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{2x^{\frac{7}{2}}(153153Bb^5x^6+171171Ab^5x^5+855855Bab^4x^5+969969Aab^4x^4+1939938Ba^2b^3x^4+2238390Aa^2b^3x^3+2238390Ba^3b^2x^3+2}{2909907(bx+a)^5}$
default	$\frac{2x^{\frac{7}{2}}(153153Bb^5x^6+171171Ab^5x^5+855855Bab^4x^5+969969Aab^4x^4+1939938Ba^2b^3x^4+2238390Aa^2b^3x^3+2238390Ba^3b^2x^3+2}{2909907(bx+a)^5}$
risch	$\frac{2\sqrt{(bx+a)^2}\,x^{\frac{7}{2}}(153153Bb^5x^6+171171Ab^5x^5+855855Bab^4x^5+969969Aab^4x^4+1939938Ba^2b^3x^4+2238390Aa^2b^3x^3+2238390B}{2909907(bx+a)}$
orering	$\frac{2(153153Bb^5x^6+171171Ab^5x^5+855855Bab^4x^5+969969Aab^4x^4+1939938Ba^2b^3x^4+2238390Aa^2b^3x^3+2238390Ba^3b^2x^3+2645}{2909907(bx+a)^5}$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/2909907*x^(7/2)*(153153*B*b^5*x^6+171171*A*b^5*x^5+855855*B*a*b^4*x^5+96
9969*A*a*b^4*x^4+1939938*B*a^2*b^3*x^4+2238390*A*a^2*b^3*x^3+2238390*B*a^3
*b^2*x^3+2645370*A*a^3*b^2*x^2+1322685*B*a^4*b*x^2+1616615*A*a^4*b*x+32332
3*B*a^5*x+415701*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}dx=\frac{2}{2909907}\left(153153\,Bb^5x^9+415701\,Aa^5x^3+171171\left(5\,Bab^4+Ab^5\right)x^8+969969\left(2\,Ba^2b^3+\right.\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
2/2909907*(153153*B*b^5*x^9 + 415701*A*a^5*x^3 + 171171*(5*B*a*b^4 + A*b^5
)*x^8 + 969969*(2*B*a^2*b^3 + A*a*b^4)*x^7 + 2238390*(B*a^3*b^2 + A*a^2*b^
3)*x^6 + 1322685*(B*a^4*b + 2*A*a^3*b^2)*x^5 + 323323*(B*a^5 + 5*A*a^4*b)*
x^4)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{765765}\left(3003\left(15\,b^5x^2+17\,ab^4x\right)x^{\frac{13}{2}}+15708\left(13\,ab^4x^2+15\,a^2b^3x\right)x^{\frac{11}{2}}+32130\left(11\,a^2b^3x^2\right.\right.$$
$$+\frac{2}{2078505}\left(6435\left(17\,b^5x^2+19\,ab^4x\right)x^{\frac{15}{2}}+32604\left(15\,ab^4x^2+17\,a^2b^3x\right)x^{\frac{13}{2}}+63954\left(13\,a^2b^3x^2+15\,a^3b^2x\right)\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
2/765765*(3003*(15*b^5*x^2 + 17*a*b^4*x)*x^(13/2) + 15708*(13*a*b^4*x^2 +
15*a^2*b^3*x)*x^(11/2) + 32130*(11*a^2*b^3*x^2 + 13*a^3*b^2*x)*x^(9/2) + 3
0940*(9*a^3*b^2*x^2 + 11*a^4*b*x)*x^(7/2) + 12155*(7*a^4*b*x^2 + 9*a^5*x)*
x^(5/2))*A + 2/2078505*(6435*(17*b^5*x^2 + 19*a*b^4*x)*x^(15/2) + 32604*(1
5*a*b^4*x^2 + 17*a^2*b^3*x)*x^(13/2) + 63954*(13*a^2*b^3*x^2 + 15*a^3*b^2*
x)*x^(11/2) + 58140*(11*a^3*b^2*x^2 + 13*a^4*b*x)*x^(9/2) + 20995*(9*a^4*b
*x^2 + 11*a^5*x)*x^(7/2))*B
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{19}Bb^5x^{\frac{19}{2}}\operatorname{sgn}(bx+a)+\frac{10}{17}Bab^4x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{2}{17}Ab^5x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{4}{3}Ba^2b^3x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{2}{3}Aab^4x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{20}{13}Ba^3b^2x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{20}{13}Aa^2b^3x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{10}{11}Ba^4bx^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{20}{11}Aa^3b^2x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{2}{9}Ba^5x^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{10}{9}Aa^4bx^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{7}Aa^5x^{\frac{7}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/19*B*b^5*x^(19/2)*sgn(b*x + a) + 10/17*B*a*b^4*x^(17/2)*sgn(b*x + a) + 2
/17*A*b^5*x^(17/2)*sgn(b*x + a) + 4/3*B*a^2*b^3*x^(15/2)*sgn(b*x + a) + 2/
3*A*a*b^4*x^(15/2)*sgn(b*x + a) + 20/13*B*a^3*b^2*x^(13/2)*sgn(b*x + a) +
20/13*A*a^2*b^3*x^(13/2)*sgn(b*x + a) + 10/11*B*a^4*b*x^(11/2)*sgn(b*x + a
) + 20/11*A*a^3*b^2*x^(11/2)*sgn(b*x + a) + 2/9*B*a^5*x^(9/2)*sgn(b*x + a)
 + 10/9*A*a^4*b*x^(9/2)*sgn(b*x + a) + 2/7*A*a^5*x^(7/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^{5/2}\,(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}\,dx$$

input
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^(5/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int x^{5/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2\sqrt{x}\,x^3(51051b^6x^6+342342a\,b^5x^5+969969a^2b^4x^4+1492260a^3b^3x^3+1322685a^4b^2x^2+6}{969969}$$

input
```
int(x^(5/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*x**3*(138567*a**6 + 646646*a**5*b*x + 1322685*a**4*b**2*x**2 +
1492260*a**3*b**3*x**3 + 969969*a**2*b**4*x**4 + 342342*a*b**5*x**5 + 5105
1*b**6*x**6))/969969
```

3.431 $\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 320

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2a^5Ax^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{2a^4(5Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{10a^3b(2Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{20a^2b^2(Ab+aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{10ab^3(Ab+2aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{2b^4(Ab+5aB)x^{15/2}\sqrt{a^2+2abx+b^2x^2}}{15(a+bx)}+\frac{2b^5Bx^{17/2}\sqrt{a^2+2abx+b^2x^2}}{17(a+bx)}$$

output

```
2*a^5*A*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)+2*a^4*(5*A*b+B*a)*x^(7/2)*((
b*x+a)^2)^(1/2)/(7*b*x+7*a)+10*a^3*b*(2*A*b+B*a)*x^(9/2)*((b*x+a)^2)^(1/2)
/(9*b*x+9*a)+20*a^2*b^2*(A*b+B*a)*x^(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a)
+10*a*b^3*(A*b+2*B*a)*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13*a)+2*b^4*(A*b+
5*B*a)*x^(15/2)*((b*x+a)^2)^(1/2)/(15*b*x+15*a)+2*b^5*B*x^(17/2)*((b*x+a)^
2)^(1/2)/(17*b*x+17*a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.40

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2x^{5/2}\sqrt{(a+bx)^2}(21879a^5(7A+5Bx)+60775a^4bx(9A+7Bx)+77350a^3b^2x^2(11A+9Bx$$

$$765765(a$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(2*x^(5/2)*Sqrt[(a + b*x)^2]*(21879*a^5*(7*A + 5*B*x) + 60775*a^4*b*x*(9*A
 + 7*B*x) + 77350*a^3*b^2*x^2*(11*A + 9*B*x) + 53550*a^2*b^3*x^3*(13*A + 1
1*B*x) + 19635*a*b^4*x^4*(15*A + 13*B*x) + 3003*b^5*x^5*(17*A + 15*B*x)))/
(765765*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5x^{3/2}(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^{3/2}(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^5Bx^{15/2}+b^4(Ab+5aB)x^{13/2}+5ab^3(Ab+2aB)x^{11/2}+10a^2b^2(Ab+aB)x^{9/2}+5a^3b(2A\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{5}a^5Ax^{5/2}+\frac{2}{7}a^4x^{7/2}(aB+5Ab)+\frac{10}{9}a^3bx^{9/2}(aB+2Ab)+\frac{20}{11}a^2b^2x^{11/2}(aB+Ab)+\frac{2}{15}b^4x^{1}\right.}{a+bx}$$

input
```
Int[x^(3/2)*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^5*A*x^(5/2))/5 + (2*a^4*(5*A*b + a*B)
*x^(7/2))/7 + (10*a^3*b*(2*A*b + a*B)*x^(9/2))/9 + (20*a^2*b^2*(A*b + a*B)
*x^(11/2))/11 + (10*a*b^3*(A*b + 2*a*B)*x^(13/2))/13 + (2*b^4*(A*b + 5*a*B
)*x^(15/2))/15 + (2*b^5*B*x^(17/2))/17))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{2x^{\frac{5}{2}}\left(45045B\,b^{5}x^{6}+51051A\,b^{5}x^{5}+255255Ba\,b^{4}x^{5}+294525Aa\,b^{4}x^{4}+589050B\,a^{2}b^{3}x^{4}+696150A\,a^{2}b^{3}x^{3}+696150B\,a^{3}b^{2}x^{3}+850850\right.}{765765(bx+a)^{5}}$
default	$\frac{2x^{\frac{5}{2}}\left(45045B\,b^{5}x^{6}+51051A\,b^{5}x^{5}+255255Ba\,b^{4}x^{5}+294525Aa\,b^{4}x^{4}+589050B\,a^{2}b^{3}x^{4}+696150A\,a^{2}b^{3}x^{3}+696150B\,a^{3}b^{2}x^{3}+850850\right.}{765765(bx+a)^{5}}$
risch	$\frac{2\sqrt{(bx+a)^{2}}\,x^{\frac{5}{2}}\left(45045B\,b^{5}x^{6}+51051A\,b^{5}x^{5}+255255Ba\,b^{4}x^{5}+294525Aa\,b^{4}x^{4}+589050B\,a^{2}b^{3}x^{4}+696150A\,a^{2}b^{3}x^{3}+696150B\,a^{3}b^{2}\right.}{765765(bx+a)}$
orering	$\frac{2\left(45045B\,b^{5}x^{6}+51051A\,b^{5}x^{5}+255255Ba\,b^{4}x^{5}+294525Aa\,b^{4}x^{4}+589050B\,a^{2}b^{3}x^{4}+696150A\,a^{2}b^{3}x^{3}+696150B\,a^{3}b^{2}x^{3}+850850A\,a\right.}{765765(bx+a)^{5}}$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/765765*x^(5/2)*(45045*B*b^5*x^6+51051*A*b^5*x^5+255255*B*a*b^4*x^5+29452
5*A*a*b^4*x^4+589050*B*a^2*b^3*x^4+696150*A*a^2*b^3*x^3+696150*B*a^3*b^2*x
^3+850850*A*a^3*b^2*x^2+425425*B*a^4*b*x^2+546975*A*a^4*b*x+109395*B*a^5*x
+153153*A*a^5)*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{765765}\left(45045\,Bb^5x^8+153153\,Aa^5x^2+51051\left(5\,Bab^4+Ab^5\right)x^7+294525\left(2\,Ba^2b^3+Aa\right.\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
2/765765*(45045*B*b^5*x^8 + 153153*A*a^5*x^2 + 51051*(5*B*a*b^4 + A*b^5)*x
^7 + 294525*(2*B*a^2*b^3 + A*a*b^4)*x^6 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x
^5 + 425425*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 109395*(B*a^5 + 5*A*a^4*b)*x^3)*
sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{45045}\left(231\left(13\,b^5x^2+15\,ab^4x\right)x^{\frac{11}{2}}+1260\left(11\,ab^4x^2+13\,a^2b^3x\right)x^{\frac{9}{2}}+2730\left(9\,a^2b^3x^2+11\,a\right.\right.$$
$$+\frac{2}{765765}\left(3003\left(15\,b^5x^2+17\,ab^4x\right)x^{\frac{13}{2}}+15708\left(13\,ab^4x^2+15\,a^2b^3x\right)x^{\frac{11}{2}}+32130\left(11\,a^2b^3x^2+13\,a^3b^2x\right)x\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
2/45045*(231*(13*b^5*x^2 + 15*a*b^4*x)*x^(11/2) + 1260*(11*a*b^4*x^2 + 13*
a^2*b^3*x)*x^(9/2) + 2730*(9*a^2*b^3*x^2 + 11*a^3*b^2*x)*x^(7/2) + 2860*(7
*a^3*b^2*x^2 + 9*a^4*b*x)*x^(5/2) + 1287*(5*a^4*b*x^2 + 7*a^5*x)*x^(3/2))*
A + 2/765765*(3003*(15*b^5*x^2 + 17*a*b^4*x)*x^(13/2) + 15708*(13*a*b^4*x^
2 + 15*a^2*b^3*x)*x^(11/2) + 32130*(11*a^2*b^3*x^2 + 13*a^3*b^2*x)*x^(9/2)
 + 30940*(9*a^3*b^2*x^2 + 11*a^4*b*x)*x^(7/2) + 12155*(7*a^4*b*x^2 + 9*a^5
*x)*x^(5/2))*B
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{17}Bb^5x^{\frac{17}{2}}\operatorname{sgn}(bx+a)+\frac{2}{3}Bab^4x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{2}{15}Ab^5x^{\frac{15}{2}}\operatorname{sgn}(bx+a)+\frac{20}{13}Ba^2b^3x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{10}{13}Aab^4x^{\frac{13}{2}}\operatorname{sgn}(bx+a)+\frac{20}{11}Ba^3b^2x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{20}{11}Aa^2b^3x^{\frac{11}{2}}\operatorname{sgn}(bx+a)+\frac{10}{9}Ba^4bx^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{20}{9}Aa^3b^2x^{\frac{9}{2}}\operatorname{sgn}(bx+a)+\frac{2}{7}Ba^5x^{\frac{7}{2}}\operatorname{sgn}(bx+a)+\frac{10}{7}Aa^4bx^{\frac{7}{2}}\operatorname{sgn}(bx+a)+\frac{2}{5}Aa^5x^{\frac{5}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/17*B*b^5*x^(17/2)*sgn(b*x + a) + 2/3*B*a*b^4*x^(15/2)*sgn(b*x + a) + 2/1
5*A*b^5*x^(15/2)*sgn(b*x + a) + 20/13*B*a^2*b^3*x^(13/2)*sgn(b*x + a) + 10
/13*A*a*b^4*x^(13/2)*sgn(b*x + a) + 20/11*B*a^3*b^2*x^(11/2)*sgn(b*x + a)
+ 20/11*A*a^2*b^3*x^(11/2)*sgn(b*x + a) + 10/9*B*a^4*b*x^(9/2)*sgn(b*x + a
) + 20/9*A*a^3*b^2*x^(9/2)*sgn(b*x + a) + 2/7*B*a^5*x^(7/2)*sgn(b*x + a) +
 10/7*A*a^4*b*x^(7/2)*sgn(b*x + a) + 2/5*A*a^5*x^(5/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int x^{3/2}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}\,dx$$

input
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^(3/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int x^{3/2}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{2\sqrt{x}\,x^2(15015b^6x^6+102102a\,b^5x^5+294525a^2b^4x^4+464100a^3b^3x^3+425425a^4b^2x^2+218}{255255}$$

input
```
int(x^(3/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*x**2*(51051*a**6 + 218790*a**5*b*x + 425425*a**4*b**2*x**2 + 46
4100*a**3*b**3*x**3 + 294525*a**2*b**4*x**4 + 102102*a*b**5*x**5 + 15015*b
**6*x**6))/255255
```

3.432 $\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 320

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2a^5Ax^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2a^4(5Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{10a^3b(2Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{20a^2b^2(Ab+aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}+\frac{10ab^3(Ab+2aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{2b^4(Ab+5aB)x^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}+\frac{2b^5Bx^{15/2}\sqrt{a^2+2abx+b^2x^2}}{15(a+bx)}$$

output
```
2*a^5*A*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*a^4*(5*A*b+B*a)*x^(5/2)*((
b*x+a)^2)^(1/2)/(5*b*x+5*a)+10*a^3*b*(2*A*b+B*a)*x^(7/2)*((b*x+a)^2)^(1/2)
/(7*b*x+7*a)+20*a^2*b^2*(A*b+B*a)*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+10
*a*b^3*(A*b+2*B*a)*x^(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a)+2*b^4*(A*b+5*B
*a)*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*x+13*a)+2*b^5*B*x^(15/2)*((b*x+a)^2)^
(1/2)/(15*b*x+15*a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.40

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2x^{3/2}\sqrt{(a+bx)^2}(3003a^5(5A+3Bx)+6435a^4bx(7A+5Bx)+7150a^3b^2x^2(9A+7Bx)+}{45045(a+bx}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*Sqrt[(a + b*x)^2]*(3003*a^5*(5*A + 3*B*x) + 6435*a^4*b*x*(7*A +
 5*B*x) + 7150*a^3*b^2*x^2*(9*A + 7*B*x) + 4550*a^2*b^3*x^3*(11*A + 9*B*x)
 + 1575*a*b^4*x^4*(13*A + 11*B*x) + 231*b^5*x^5*(15*A + 13*B*x)))/(45045*(
a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int b^5\sqrt{x}(a+bx)^5(A+Bx)dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \sqrt{x}(a+bx)^5(A+Bx)dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^5Bx^{13/2}+b^4(Ab+5aB)x^{11/2}+5ab^3(Ab+2aB)x^{9/2}+10a^2b^2(Ab+aB)x^{7/2}+5a^3b(2A\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2}{3}a^5Ax^{3/2}+\frac{2}{5}a^4x^{5/2}(aB+5Ab)+\frac{10}{7}a^3bx^{7/2}(aB+2Ab)+\frac{20}{9}a^2b^2x^{9/2}(aB+Ab)+\frac{2}{13}b^4x^{13}\right.}{a+bx}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((2*a^5*A*x^(3/2))/3 + (2*a^4*(5*A*b + a*B)
*x^(5/2))/5 + (10*a^3*b*(2*A*b + a*B)*x^(7/2))/7 + (20*a^2*b^2*(A*b + a*B)
*x^(9/2))/9 + (10*a*b^3*(A*b + 2*a*B)*x^(11/2))/11 + (2*b^4*(A*b + 5*a*B)*
x^(13/2))/13 + (2*b^5*B*x^(15/2))/15))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(3003B\,b^{5}x^{6}+3465A\,b^{5}x^{5}+17325Ba\,b^{4}x^{5}+20475Aa\,b^{4}x^{4}+40950B\,a^{2}b^{3}x^{4}+50050A\,a^{2}b^{3}x^{3}+50050B\,a^{3}b^{2}x^{3}+64350A\,a^{3}b^{2}x^{2}}{45045(bx+a)^{5}}$
default	$\frac{2x^{\frac{3}{2}}\left(3003B\,b^{5}x^{6}+3465A\,b^{5}x^{5}+17325Ba\,b^{4}x^{5}+20475Aa\,b^{4}x^{4}+40950B\,a^{2}b^{3}x^{4}+50050A\,a^{2}b^{3}x^{3}+50050B\,a^{3}b^{2}x^{3}+64350A\,a^{3}b^{2}x^{2}}{45045(bx+a)^{5}}$
risch	$\frac{2\sqrt{(bx+a)^{2}}\,x^{\frac{3}{2}}\left(3003B\,b^{5}x^{6}+3465A\,b^{5}x^{5}+17325Ba\,b^{4}x^{5}+20475Aa\,b^{4}x^{4}+40950B\,a^{2}b^{3}x^{4}+50050A\,a^{2}b^{3}x^{3}+50050B\,a^{3}b^{2}x^{3}+6435}{45045(bx+a)}$
orering	$\frac{2x^{\frac{3}{2}}\left(3003B\,b^{5}x^{6}+3465A\,b^{5}x^{5}+17325Ba\,b^{4}x^{5}+20475Aa\,b^{4}x^{4}+40950B\,a^{2}b^{3}x^{4}+50050A\,a^{2}b^{3}x^{3}+50050B\,a^{3}b^{2}x^{3}+64350A\,a^{3}b^{2}x^{2}}{45045(bx+a)^{5}}$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(3/2)*(3003*B*b^5*x^6+3465*A*b^5*x^5+17325*B*a*b^4*x^5+20475*A*a
*b^4*x^4+40950*B*a^2*b^3*x^4+50050*A*a^2*b^3*x^3+50050*B*a^3*b^2*x^3+64350
*A*a^3*b^2*x^2+32175*B*a^4*b*x^2+45045*A*a^4*b*x+9009*B*a^5*x+15015*A*a^5)
*((b*x+a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.38

$$\int \sqrt{x}(A+Bx)\left(a^{2}+2abx+b^{2}x^{2}\right)^{5/2}\,dx=\frac{2}{45045}\left(3003\,Bb^{5}x^{7}+15015\,Aa^{5}x+3465\left(5\,Bab^{4}+Ab^{5}\right)x^{6}+20475\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{5}+\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
2/45045*(3003*B*b^5*x^7 + 15015*A*a^5*x + 3465*(5*B*a*b^4 + A*b^5)*x^6 + 2
0475*(2*B*a^2*b^3 + A*a*b^4)*x^5 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 321
75*(B*a^4*b + 2*A*a^3*b^2)*x^3 + 9009*(B*a^5 + 5*A*a^4*b)*x^2)*sqrt(x)
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\int \sqrt{x}(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)*((a + b*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx=\frac{2}{45045}\left(315\left(11\,b^5x^2+13\,ab^4x\right)x^{\frac{9}{2}}+1820\left(9\,ab^4x^2+11\,a^2b^3x\right)x^{\frac{7}{2}}+4290\left(7\,a^2b^3x^2+9\,a^3b^2x\right)\dots\right)+\frac{2}{45045}\left(231\left(13\,b^5x^2+15\,ab^4x\right)x^{\frac{11}{2}}+1260\left(11\,ab^4x^2+13\,a^2b^3x\right)x^{\frac{9}{2}}+2730\left(9\,a^2b^3x^2+11\,a^3b^2x\right)x^{\frac{7}{2}}+28\dots\right)$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
2/45045*(315*(11*b^5*x^2 + 13*a*b^4*x)*x^(9/2) + 1820*(9*a*b^4*x^2 + 11*a^
2*b^3*x)*x^(7/2) + 4290*(7*a^2*b^3*x^2 + 9*a^3*b^2*x)*x^(5/2) + 5148*(5*a^
3*b^2*x^2 + 7*a^4*b*x)*x^(3/2) + 3003*(3*a^4*b*x^2 + 5*a^5*x)*sqrt(x))*A +
 2/45045*(231*(13*b^5*x^2 + 15*a*b^4*x)*x^(11/2) + 1260*(11*a*b^4*x^2 + 13
*a^2*b^3*x)*x^(9/2) + 2730*(9*a^2*b^3*x^2 + 11*a^3*b^2*x)*x^(7/2) + 2860*(
7*a^3*b^2*x^2 + 9*a^4*b*x)*x^(5/2) + 1287*(5*a^4*b*x^2 + 7*a^5*x)*x^(3/2))
*B
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \frac{2}{15}\,Bb^5x^{\frac{15}{2}}\operatorname{sgn}(bx+a) + \frac{10}{13}\,Bab^4x^{\frac{13}{2}}\operatorname{sgn}(bx+a) + \frac{2}{13}\,Ab^5x^{\frac{13}{2}}\operatorname{sgn}(bx+a) + \frac{20}{11}\,Ba^2b^3x^{\frac{11}{2}}\operatorname{sgn}(bx+a) + \frac{10}{11}\,Aab^4x^{\frac{11}{2}}\operatorname{sgn}(bx+a) + \frac{20}{9}\,Ba^3b^2x^{\frac{9}{2}}\operatorname{sgn}(bx+a) + \frac{20}{9}\,Aa^2b^3x^{\frac{9}{2}}\operatorname{sgn}(bx+a) + \frac{10}{7}\,Ba^4bx^{\frac{7}{2}}\operatorname{sgn}(bx+a) + \frac{20}{7}\,Aa^3b^2x^{\frac{7}{2}}\operatorname{sgn}(bx+a) + \frac{2}{5}\,Ba^5x^{\frac{5}{2}}\operatorname{sgn}(bx+a) + 2\,Aa^4bx^{\frac{5}{2}}\operatorname{sgn}(bx+a) + \frac{2}{3}\,Aa^5x^{\frac{3}{2}}\operatorname{sgn}(bx+a)$$

input
```
integrate(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/15*B*b^5*x^(15/2)*sgn(b*x + a) + 10/13*B*a*b^4*x^(13/2)*sgn(b*x + a) + 2
/13*A*b^5*x^(13/2)*sgn(b*x + a) + 20/11*B*a^2*b^3*x^(11/2)*sgn(b*x + a) +
10/11*A*a*b^4*x^(11/2)*sgn(b*x + a) + 20/9*B*a^3*b^2*x^(9/2)*sgn(b*x + a)
+ 20/9*A*a^2*b^3*x^(9/2)*sgn(b*x + a) + 10/7*B*a^4*b*x^(7/2)*sgn(b*x + a)
+ 20/7*A*a^3*b^2*x^(7/2)*sgn(b*x + a) + 2/5*B*a^5*x^(5/2)*sgn(b*x + a) + 2
*A*a^4*b*x^(5/2)*sgn(b*x + a) + 2/3*A*a^5*x^(3/2)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx = \int \sqrt{x}\,\left(A+B\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2} \,d x$$

input
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int(x^(1/2)*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.21

$$\int \sqrt{x}(A+Bx)\left(a^2+2abx +b^2x^2\right)^{5/2}\,dx = \frac{2\sqrt{x}\,x(3003b^6x^6+20790a\,b^5x^5+61425a^2b^4x^4+100100a^3b^3x^3+96525a^4b^2x^2+54054a^5bx}{45045}$$

input
```
int(x^(1/2)*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*x*(15015*a**6 + 54054*a**5*b*x + 96525*a**4*b**2*x**2 + 100100*
a**3*b**3*x**3 + 61425*a**2*b**4*x**4 + 20790*a*b**5*x**5 + 3003*b**6*x**6
))/45045
```

3.433 $\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 316

$$\begin{aligned}\int \frac{(A+Bx)(a^2+2abx+b^2x^2)^{5/2}}{\sqrt{x}}\,dx &= \frac{2a^5A\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx}\\ &+\frac{2a^4(5Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}\\ &+\frac{2a^3b(2Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{a+bx}\\ &+\frac{20a^2b^2(Ab+aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}\\ &+\frac{10ab^3(Ab+2aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}\\ &+\frac{2b^4(Ab+5aB)x^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}+\frac{2b^5Bx^{13/2}\sqrt{a^2+2abx+b^2x^2}}{13(a+bx)}\end{aligned}$$

output
```
2*a^5*A*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*a^4*(5*A*b+B*a)*x^(3/2)*((b*x+
a)^2)^(1/2)/(3*b*x+3*a)+2*a^3*b*(2*A*b+B*a)*x^(5/2)*((b*x+a)^2)^(1/2)/(b*x
+a)+20*a^2*b^2*(A*b+B*a)*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+10*a*b^3*(A
*b+2*B*a)*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)+2*b^4*(A*b+5*B*a)*x^(11/2)
*((b*x+a)^2)^(1/2)/(11*b*x+11*a)+2*b^5*B*x^(13/2)*((b*x+a)^2)^(1/2)/(13*b*
x+13*a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.40

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{(a+bx)^2}(3003a^5(3A+Bx)+3003a^4bx(5A+3Bx)+257}{}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[(a + b*x)^2]*(3003*a^5*(3*A + B*x) + 3003*a^4*b*x*(5*A + 3
*B*x) + 2574*a^3*b^2*x^2*(7*A + 5*B*x) + 1430*a^2*b^3*x^3*(9*A + 7*B*x) +
455*a*b^4*x^4*(11*A + 9*B*x) + 63*b^5*x^5*(13*A + 11*B*x)))/(9009*(a + b*x
))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{\sqrt{x}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{\sqrt{x}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(b^5Bx^{11/2}+b^4(Ab+5aB)x^{9/2}+5ab^3(Ab+2aB)x^{7/2}+10a^2b^2(Ab+aB)x^{5/2}+5a^3b(2A\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(2a^5A\sqrt{x}+\frac{2}{3}a^4x^{3/2}(aB+5Ab)+2a^3bx^{5/2}(aB+2Ab)+\frac{20}{7}a^2b^2x^{7/2}(aB+Ab)+\frac{2}{11}b^4x^{11/2}(\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/Sqrt[x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(2*a^5*A*Sqrt[x] + (2*a^4*(5*A*b + a*B)*x^(
3/2))/3 + 2*a^3*b*(2*A*b + a*B)*x^(5/2) + (20*a^2*b^2*(A*b + a*B)*x^(7/2))
/7 + (10*a*b^3*(A*b + 2*a*B)*x^(9/2))/9 + (2*b^4*(A*b + 5*a*B)*x^(11/2))/1
1 + (2*b^5*B*x^(13/2))/13))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{2\sqrt{x}\,(693B\,b^5x^6+819A\,b^5x^5+4095Ba\,b^4x^5+5005Aa\,b^4x^4+10010B\,a^2b^3x^4+12870A\,a^2b^3x^3+12870B\,a^3b^2x^3+18018A\,a^3b^2x^2+90}{9009(bx+a)^5}$
default	$\frac{2\sqrt{x}\,(693B\,b^5x^6+819A\,b^5x^5+4095Ba\,b^4x^5+5005Aa\,b^4x^4+10010B\,a^2b^3x^4+12870A\,a^2b^3x^3+12870B\,a^3b^2x^3+18018A\,a^3b^2x^2+90}{9009(bx+a)^5}$
risch	$\frac{2\sqrt{(bx+a)^2}\,(693B\,b^5x^6+819A\,b^5x^5+4095Ba\,b^4x^5+5005Aa\,b^4x^4+10010B\,a^2b^3x^4+12870A\,a^2b^3x^3+12870B\,a^3b^2x^3+18018A\,a^3b}{9009(bx+a)}$
orering	$\frac{2(693B\,b^5x^6+819A\,b^5x^5+4095Ba\,b^4x^5+5005Aa\,b^4x^4+10010B\,a^2b^3x^4+12870A\,a^2b^3x^3+12870B\,a^3b^2x^3+18018A\,a^3b^2x^2+9009B}{9009(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*x^(1/2)*(693*B*b^5*x^6+819*A*b^5*x^5+4095*B*a*b^4*x^5+5005*A*a*b^4*
x^4+10010*B*a^2*b^3*x^4+12870*A*a^2*b^3*x^3+12870*B*a^3*b^2*x^3+18018*A*a^
3*b^2*x^2+9009*B*a^4*b*x^2+15015*A*a^4*b*x+3003*B*a^5*x+9009*A*a^5)*((b*x+
a)^2)^(5/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2}{9009}\left(693\,Bb^5x^6+9009\,Aa^5+819\left(5\,Bab^4+Ab^5\right)x^5+5005\left(2\,B\right.\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(1/2),x, algorithm="fricas
")
```

output
```
2/9009*(693*B*b^5*x^6 + 9009*A*a^5 + 819*(5*B*a*b^4 + A*b^5)*x^5 + 5005*(2
*B*a^2*b^3 + A*a*b^4)*x^4 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 9009*(B*a^
4*b + 2*A*a^3*b^2)*x^2 + 3003*(B*a^5 + 5*A*a^4*b)*x)*sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**(1/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/sqrt(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx=\frac{2}{3465}\left(35\left(9\,b^5x^2+11\,ab^4x\right)x^{\frac{7}{2}}+220\left(7\,ab^4x^2+9\,a^2b^3x\right)x^{\frac{5}{2}}+59\right.$$
$$+\frac{2}{45045}\left(315\left(11\,b^5x^2+13\,ab^4x\right)x^{\frac{9}{2}}+1820\left(9\,ab^4x^2+11\,a^2b^3x\right)x^{\frac{7}{2}}+4290\left(7\,a^2b^3x^2+9\,a^3b^2x\right)x^{\frac{5}{2}}+5148\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(1/2),x, algorithm="maxima
")
```

output
```
2/3465*(35*(9*b^5*x^2 + 11*a*b^4*x)*x^(7/2) + 220*(7*a*b^4*x^2 + 9*a^2*b^3
*x)*x^(5/2) + 594*(5*a^2*b^3*x^2 + 7*a^3*b^2*x)*x^(3/2) + 924*(3*a^3*b^2*x
^2 + 5*a^4*b*x)*sqrt(x) + 1155*(a^4*b*x^2 + 3*a^5*x)/sqrt(x))*A + 2/45045*
(315*(11*b^5*x^2 + 13*a*b^4*x)*x^(9/2) + 1820*(9*a*b^4*x^2 + 11*a^2*b^3*x)
*x^(7/2) + 4290*(7*a^2*b^3*x^2 + 9*a^3*b^2*x)*x^(5/2) + 5148*(5*a^3*b^2*x^
2 + 7*a^4*b*x)*x^(3/2) + 3003*(3*a^4*b*x^2 + 5*a^5*x)*sqrt(x))*B
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx=\frac{2}{13}\,Bb^5x^{\frac{13}{2}}\mathrm{sgn}(bx+a)$$
$$+\frac{10}{11}\,Bab^4x^{\frac{11}{2}}\mathrm{sgn}(bx+a)+\frac{2}{11}\,Ab^5x^{\frac{11}{2}}\mathrm{sgn}(bx+a)+\frac{20}{9}\,Ba^2b^3x^{\frac{9}{2}}\mathrm{sgn}(bx+a)$$
$$+\frac{10}{9}\,Aab^4x^{\frac{9}{2}}\mathrm{sgn}(bx+a)+\frac{20}{7}\,Ba^3b^2x^{\frac{7}{2}}\mathrm{sgn}(bx+a)$$
$$+\frac{20}{7}\,Aa^2b^3x^{\frac{7}{2}}\mathrm{sgn}(bx+a)+2\,Ba^4bx^{\frac{5}{2}}\mathrm{sgn}(bx+a)+4\,Aa^3b^2x^{\frac{5}{2}}\mathrm{sgn}(bx+a)$$
$$+\frac{2}{3}\,Ba^5x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+\frac{10}{3}\,Aa^4bx^{\frac{3}{2}}\mathrm{sgn}(bx+a)+2\,Aa^5\sqrt{x}\mathrm{sgn}(bx+a)$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*B*b^5*x^(13/2)*sgn(b*x + a) + 10/11*B*a*b^4*x^(11/2)*sgn(b*x + a) + 2
/11*A*b^5*x^(11/2)*sgn(b*x + a) + 20/9*B*a^2*b^3*x^(9/2)*sgn(b*x + a) + 10
/9*A*a*b^4*x^(9/2)*sgn(b*x + a) + 20/7*B*a^3*b^2*x^(7/2)*sgn(b*x + a) + 20
/7*A*a^2*b^3*x^(7/2)*sgn(b*x + a) + 2*B*a^4*b*x^(5/2)*sgn(b*x + a) + 4*A*a
^3*b^2*x^(5/2)*sgn(b*x + a) + 2/3*B*a^5*x^(3/2)*sgn(b*x + a) + 10/3*A*a^4*
b*x^(3/2)*sgn(b*x + a) + 2*A*a^5*sqrt(x)*sgn(b*x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx=\int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(1/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(231b^6x^6+1638a\,b^5x^5+5005a^2b^4x^4+8580a^3b^3x^3+9009a^4}{3003}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(3003*a**6 + 6006*a**5*b*x + 9009*a**4*b**2*x**2 + 8580*a**3*b*
*3*x**3 + 5005*a**2*b**4*x**4 + 1638*a*b**5*x**5 + 231*b**6*x**6))/3003
```

3.434 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 314

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx = -\frac{2a^5A\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)} + \frac{2a^4(5Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{10a^3b(2Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} + \frac{4a^2b^2(Ab+aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{a+bx} + \frac{10ab^3(Ab+2aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)} + \frac{2b^4(Ab+5aB)x^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)} + \frac{2b^5Bx^{11/2}\sqrt{a^2+2abx+b^2x^2}}{11(a+bx)}$$

output
```
-2*a^5*A*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)+2*a^4*(5*A*b+B*a)*x^(1/2)*((b*x
+a)^2)^(1/2)/(b*x+a)+10*a^3*b*(2*A*b+B*a)*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x
+3*a)+4*a^2*b^2*(A*b+B*a)*x^(5/2)*((b*x+a)^2)^(1/2)/(b*x+a)+10*a*b^3*(A*b+
2*B*a)*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)+2*b^4*(A*b+5*B*a)*x^(9/2)*((b
*x+a)^2)^(1/2)/(9*b*x+9*a)+2*b^5*B*x^(11/2)*((b*x+a)^2)^(1/2)/(11*b*x+11*a
)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx=\frac{2\sqrt{(a+bx)^2}(-693a^5(A-Bx)+1155a^4bx(3A+Bx)+462a^3b^2$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(3/2),x]
```

output
```
(2*Sqrt[(a + b*x)^2]*(-693*a^5*(A - B*x) + 1155*a^4*b*x*(3*A + B*x) + 462*
a^3*b^2*x^2*(5*A + 3*B*x) + 198*a^2*b^3*x^3*(7*A + 5*B*x) + 55*a*b^4*x^4*(
9*A + 7*B*x) + 7*b^5*x^5*(11*A + 9*B*x)))/(693*Sqrt[x]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{3/2}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{3/2}}dx}{a+bx}$$

$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \left(\frac{Aa^5}{x^{3/2}}+\frac{(5Ab+aB)a^4}{\sqrt{x}}+5b(2Ab+aB)\sqrt{x}a^3+10b^2(Ab+aB)x^{3/2}a^2+5b^3(Ab+2aB)x^{5/2}a\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^5A}{\sqrt{x}}+2a^4\sqrt{x}(aB+5Ab)+\frac{10}{3}a^3bx^{3/2}(aB+2Ab)+4a^2b^2x^{5/2}(aB+Ab)+\frac{2}{9}b^4x^{9/2}(5aB\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^5*A)/Sqrt[x] + 2*a^4*(5*A*b + a*B)*S
qrt[x] + (10*a^3*b*(2*A*b + a*B)*x^(3/2))/3 + 4*a^2*b^2*(A*b + a*B)*x^(5/2
) + (10*a*b^3*(A*b + 2*a*B)*x^(7/2))/7 + (2*b^4*(A*b + 5*a*B)*x^(9/2))/9 +
 (2*b^5*B*x^(11/2))/11))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.45

method	result
gosper	$-\frac{2(-63B\,b^5x^6-77A\,b^5x^5-385Ba\,b^4x^5-495Aa\,b^4x^4-990B\,a^2b^3x^4-1386A\,a^2b^3x^3-1386B\,a^3b^2x^3-2310A\,a^3b^2x^2-1155B\,a^4b\,x}{693\sqrt{x}\,(bx+a)^5}$
default	$-\frac{2(-63B\,b^5x^6-77A\,b^5x^5-385Ba\,b^4x^5-495Aa\,b^4x^4-990B\,a^2b^3x^4-1386A\,a^2b^3x^3-1386B\,a^3b^2x^3-2310A\,a^3b^2x^2-1155B\,a^4b\,x}{693\sqrt{x}\,(bx+a)^5}$
risch	$-\frac{2\sqrt{(bx+a)^2}\,(-63B\,b^5x^6-77A\,b^5x^5-385Ba\,b^4x^5-495Aa\,b^4x^4-990B\,a^2b^3x^4-1386A\,a^2b^3x^3-1386B\,a^3b^2x^3-2310A\,a^3b^2x^2-1}{693(bx+a)\sqrt{x}}$
orering	$-\frac{2(-63B\,b^5x^6-77A\,b^5x^5-385Ba\,b^4x^5-495Aa\,b^4x^4-990B\,a^2b^3x^4-1386A\,a^2b^3x^3-1386B\,a^3b^2x^3-2310A\,a^3b^2x^2-1155B\,a^4b\,x}{693\sqrt{x}\,(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*(-63*B*b^5*x^6-77*A*b^5*x^5-385*B*a*b^4*x^5-495*A*a*b^4*x^4-990*B*a
^2*b^3*x^4-1386*A*a^2*b^3*x^3-1386*B*a^3*b^2*x^3-2310*A*a^3*b^2*x^2-1155*B
*a^4*b*x^2-3465*A*a^4*b*x-693*B*a^5*x+693*A*a^5)*((b*x+a)^2)^(5/2)/x^(1/2)
/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{2\,(63\,Bb^5x^6-693\,Aa^5+77\,(5\,Bab^4+Ab^5)x^5+495\,(2\,Ba^2b^3+A}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(3/2),x, algorithm="fricas
")
```

output
```
2/693*(63*B*b^5*x^6 - 693*A*a^5 + 77*(5*B*a*b^4 + A*b^5)*x^5 + 495*(2*B*a^
2*b^3 + A*a*b^4)*x^4 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 1155*(B*a^4*b +
2*A*a^3*b^2)*x^2 + 693*(B*a^5 + 5*A*a^4*b)*x)/sqrt(x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**(3/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx=\frac{2}{315}\left(5\left(7\,b^5x^2+9\,ab^4x\right)x^{\frac{5}{2}}+36\left(5\,ab^4x^2+7\,a^2b^3x\right)x^{\frac{3}{2}}+126\,(3\,a\right.$$
$$\left.+\frac{2}{3465}\left(35\left(9\,b^5x^2+11\,ab^4x\right)x^{\frac{7}{2}}+220\left(7\,ab^4x^2+9\,a^2b^3x\right)x^{\frac{5}{2}}+594\left(5\,a^2b^3x^2+7\,a^3b^2x\right)x^{\frac{3}{2}}+924\,(3\,a^3b^2x\right.\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(3/2),x, algorithm="maxima
")
```

output
```
2/315*(5*(7*b^5*x^2 + 9*a*b^4*x)*x^(5/2) + 36*(5*a*b^4*x^2 + 7*a^2*b^3*x)*
x^(3/2) + 126*(3*a^2*b^3*x^2 + 5*a^3*b^2*x)*sqrt(x) + 420*(a^3*b^2*x^2 + 3
*a^4*b*x)/sqrt(x) + 315*(a^4*b*x^2 - a^5*x)/x^(3/2))*A + 2/3465*(35*(9*b^5
*x^2 + 11*a*b^4*x)*x^(7/2) + 220*(7*a*b^4*x^2 + 9*a^2*b^3*x)*x^(5/2) + 594
*(5*a^2*b^3*x^2 + 7*a^3*b^2*x)*x^(3/2) + 924*(3*a^3*b^2*x^2 + 5*a^4*b*x)*s
qrt(x) + 1155*(a^4*b*x^2 + 3*a^5*x)/sqrt(x))*B
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{2}{11}\,Bb^5x^{\frac{11}{2}}\mathrm{sgn}(bx+a) + \frac{10}{9}\,Bab^4x^{\frac{9}{2}}\mathrm{sgn}(bx+a) + \frac{2}{9}\,Ab^5x^{\frac{9}{2}}\mathrm{sgn}(bx+a) + \frac{20}{7}\,Ba^2b^3x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + \frac{10}{7}\,Aab^4x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + 4\,Ba^3b^2x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + 4\,Aa^2b^3x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{10}{3}\,Ba^4bx^{\frac{3}{2}}\mathrm{sgn}(bx+a) + \frac{20}{3}\,Aa^3b^2x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 2\,Ba^5\sqrt{x}\mathrm{sgn}(bx+a) + 10\,Aa^4b\sqrt{x}\mathrm{sgn}(bx+a) - \frac{2\,Aa^5\mathrm{sgn}(bx+a)}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*B*b^5*x^(11/2)*sgn(b*x + a) + 10/9*B*a*b^4*x^(9/2)*sgn(b*x + a) + 2/9
*A*b^5*x^(9/2)*sgn(b*x + a) + 20/7*B*a^2*b^3*x^(7/2)*sgn(b*x + a) + 10/7*A
*a*b^4*x^(7/2)*sgn(b*x + a) + 4*B*a^3*b^2*x^(5/2)*sgn(b*x + a) + 4*A*a^2*b
^3*x^(5/2)*sgn(b*x + a) + 10/3*B*a^4*b*x^(3/2)*sgn(b*x + a) + 20/3*A*a^3*b
^2*x^(3/2)*sgn(b*x + a) + 2*B*a^5*sqrt(x)*sgn(b*x + a) + 10*A*a^4*b*sqrt(x
)*sgn(b*x + a) - 2*A*a^5*sgn(b*x + a)/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2}\,\left(\frac{2\,B\,b^4\,x^6}{11}-\frac{2\,A\,a^5}{b}+\frac{10\,a^3\,x^2\,(2\,A\,b+B\,a)}{3}+\frac{x^5\,(154\,A}{}\right.}{x^3}$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(3/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x)^(1/2)*((2*B*b^4*x^6)/11 - (2*A*a^5)/b + (10*a^3
*x^2*(2*A*b + B*a))/3 + (x^5*(154*A*b^5 + 770*B*a*b^4))/(693*b) + 4*a^2*b*
x^3*(A*b + B*a) + (10*a*b^2*x^4*(A*b + 2*B*a))/7 + (2*a^4*x*(5*A*b + B*a))
/b))/(x^(3/2) + (a*x^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{\frac{2}{11}b^6x^6+\frac{4}{3}a\,b^5x^5+\frac{30}{7}a^2b^4x^4+8a^3b^3x^3+10a^4b^2x^2+12a^5bx-2a}{\sqrt{x}}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(3/2),x)
```

output
```
(2*( - 231*a**6 + 1386*a**5*b*x + 1155*a**4*b**2*x**2 + 924*a**3*b**3*x**3
 + 495*a**2*b**4*x**4 + 154*a*b**5*x**5 + 21*b**6*x**6))/(231*sqrt(x))
```

3.435 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 314

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx=-\frac{2a^5A\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)} -\frac{2a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)}+\frac{10a^3b(2Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx} +\frac{20a^2b^2(Ab+aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)} +\frac{2ab^3(Ab+2aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{a+bx} +\frac{2b^4(Ab+5aB)x^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}+\frac{2b^5Bx^{9/2}\sqrt{a^2+2abx+b^2x^2}}{9(a+bx)}$$

output
```
-2/3*a^5*A*((b*x+a)^2)^(1/2)/x^(3/2)/(b*x+a)-2*a^4*(5*A*b+B*a)*((b*x+a)^2)
^(1/2)/x^(1/2)/(b*x+a)+10*a^3*b*(2*A*b+B*a)*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x
+a)+20*a^2*b^2*(A*b+B*a)*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*a*b^3*(A*
b+2*B*a)*x^(5/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b^4*(A*b+5*B*a)*x^(7/2)*((b*x
+a)^2)^(1/2)/(7*b*x+7*a)+2*b^5*B*x^(9/2)*((b*x+a)^2)^(1/2)/(9*b*x+9*a)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx =$$
$$-\frac{2\sqrt{(a+bx)^2}(315a^4bx(A-Bx)-210a^3b^2x^2(3A+Bx)+21a^5(A+3Bx)-42a^2b^3x^3(5A+3Bx)-9ab}{63x^{3/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(5/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(315*a^4*b*x*(A - B*x) - 210*a^3*b^2*x^2*(3*A + B*x)
 + 21*a^5*(A + 3*B*x) - 42*a^2*b^3*x^3*(5*A + 3*B*x) - 9*a*b^4*x^4*(7*A +
5*B*x) - b^5*x^5*(9*A + 7*B*x)))/(63*x^(3/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{5/2}}dx$$
$$\Big\downarrow \text{1187}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{5/2}}dx}{b^5(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{5/2}}dx}{a+bx}$$
$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{5/2}}+\frac{(5Ab+aB)a^4}{x^{3/2}}+\frac{5b(2Ab+aB)a^3}{\sqrt{x}}+10b^2(Ab+aB)\sqrt{x}a^2+5b^3(Ab+2aB)x^{3/2}a+b^5Bx^{7}\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^5A}{3x^{3/2}}-\frac{2a^4(aB+5Ab)}{\sqrt{x}}+10a^3b\sqrt{x}(aB+2Ab)+\frac{20}{3}a^2b^2x^{3/2}(aB+Ab)+\frac{2}{7}b^4x^{7/2}(5aB+Ab)\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^5*A)/(3*x^(3/2)) - (2*a^4*(5*A*b + a
*B))/Sqrt[x] + 10*a^3*b*(2*A*b + a*B)*Sqrt[x] + (20*a^2*b^2*(A*b + a*B)*x^
(3/2))/3 + 2*a*b^3*(A*b + 2*a*B)*x^(5/2) + (2*b^4*(A*b + 5*a*B)*x^(7/2))/7
 + (2*b^5*B*x^(9/2))/9))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.45

method	result
gosper	$-\frac{2(-7B\,b^5x^6-9A\,b^5x^5-45Ba\,b^4x^5-63Aa\,b^4x^4-126B\,a^2b^3x^4-210A\,a^2b^3x^3-210B\,a^3b^2x^3-630A\,a^3b^2x^2-315B\,a^4b\,x^2+315A\,a}{63x^{\frac{3}{2}}(bx+a)^5}$
default	$-\frac{2(-7B\,b^5x^6-9A\,b^5x^5-45Ba\,b^4x^5-63Aa\,b^4x^4-126B\,a^2b^3x^4-210A\,a^2b^3x^3-210B\,a^3b^2x^3-630A\,a^3b^2x^2-315B\,a^4b\,x^2+315A\,a}{63x^{\frac{3}{2}}(bx+a)^5}$
risch	$-\frac{2\sqrt{(bx+a)^2}\,(-7B\,b^5x^6-9A\,b^5x^5-45Ba\,b^4x^5-63Aa\,b^4x^4-126B\,a^2b^3x^4-210A\,a^2b^3x^3-210B\,a^3b^2x^3-630A\,a^3b^2x^2-315B\,a^4b}{63(bx+a)x^{\frac{3}{2}}}$
orering	$-\frac{2(-7B\,b^5x^6-9A\,b^5x^5-45Ba\,b^4x^5-63Aa\,b^4x^4-126B\,a^2b^3x^4-210A\,a^2b^3x^3-210B\,a^3b^2x^3-630A\,a^3b^2x^2-315B\,a^4b\,x^2+315A\,a}{63x^{\frac{3}{2}}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(-7*B*b^5*x^6-9*A*b^5*x^5-45*B*a*b^4*x^5-63*A*a*b^4*x^4-126*B*a^2*b^
3*x^4-210*A*a^2*b^3*x^3-210*B*a^3*b^2*x^3-630*A*a^3*b^2*x^2-315*B*a^4*b*x^
2+315*A*a^4*b*x+63*B*a^5*x+21*A*a^5)*((b*x+a)^2)^(5/2)/x^(3/2)/(b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx = \frac{2\left(7\,Bb^5x^6-21\,Aa^5+9\,(5\,Bab^4+Ab^5)x^5+63\,(2\,Ba^2b^3+Aab^4)\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(5/2),x, algorithm="fricas
")
```

output
```
2/63*(7*B*b^5*x^6 - 21*A*a^5 + 9*(5*B*a*b^4 + A*b^5)*x^5 + 63*(2*B*a^2*b^3
 + A*a*b^4)*x^4 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 315*(B*a^4*b + 2*A*a^3
*b^2)*x^2 - 63*(B*a^5 + 5*A*a^4*b)*x)/x^(3/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**(5/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx=\frac{2}{105}\left(3\left(5\,b^5x^2+7\,ab^4x\right)x^{\frac{3}{2}}+28\left(3\,ab^4x^2+5\,a^2b^3x\right)\sqrt{x}+\frac{210\,(a^2\cdots}{\cdots}\right.$$
$$+\frac{2}{315}\left(5\left(7\,b^5x^2+9\,ab^4x\right)x^{\frac{5}{2}}+36\left(5\,ab^4x^2+7\,a^2b^3x\right)x^{\frac{3}{2}}+126\left(3\,a^2b^3x^2+5\,a^3b^2x\right)\sqrt{x}+\frac{420\left(a^3b^2x^2+3\cdots\right.}{\sqrt{x}}\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(5/2),x, algorithm="maxima
")
```

output
```
2/105*(3*(5*b^5*x^2 + 7*a*b^4*x)*x^(3/2) + 28*(3*a*b^4*x^2 + 5*a^2*b^3*x)*
sqrt(x) + 210*(a^2*b^3*x^2 + 3*a^3*b^2*x)/sqrt(x) + 420*(a^3*b^2*x^2 - a^4
*b*x)/x^(3/2) - 35*(3*a^4*b*x^2 + a^5*x)/x^(5/2))*A + 2/315*(5*(7*b^5*x^2
+ 9*a*b^4*x)*x^(5/2) + 36*(5*a*b^4*x^2 + 7*a^2*b^3*x)*x^(3/2) + 126*(3*a^2
*b^3*x^2 + 5*a^3*b^2*x)*sqrt(x) + 420*(a^3*b^2*x^2 + 3*a^4*b*x)/sqrt(x) +
315*(a^4*b*x^2 - a^5*x)/x^(3/2))*B
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx = \frac{2}{9}\,Bb^5x^{\frac{9}{2}}\mathrm{sgn}(bx+a) + \frac{10}{7}\,Bab^4x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + \frac{2}{7}\,Ab^5x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + 4\,Ba^2b^3x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + 2\,Aab^4x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{20}{3}\,Ba^3b^2x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + \frac{20}{3}\,Aa^2b^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 10\,Ba^4b\sqrt{x}\mathrm{sgn}(bx+a) + 20\,Aa^3b^2\sqrt{x}\mathrm{sgn}(bx+a) - \frac{2\,(3\,Ba^5x\mathrm{sgn}(bx+a) + 15\,Aa^4bx\mathrm{sgn}(bx+a) + Aa^5\mathrm{sgn}(bx+a))}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*B*b^5*x^(9/2)*sgn(b*x + a) + 10/7*B*a*b^4*x^(7/2)*sgn(b*x + a) + 2/7*A
*b^5*x^(7/2)*sgn(b*x + a) + 4*B*a^2*b^3*x^(5/2)*sgn(b*x + a) + 2*A*a*b^4*x
^(5/2)*sgn(b*x + a) + 20/3*B*a^3*b^2*x^(3/2)*sgn(b*x + a) + 20/3*A*a^2*b^3
*x^(3/2)*sgn(b*x + a) + 10*B*a^4*b*sqrt(x)*sgn(b*x + a) + 20*A*a^3*b^2*sqr
t(x)*sgn(b*x + a) - 2/3*(3*B*a^5*x*sgn(b*x + a) + 15*A*a^4*b*x*sgn(b*x + a
) + A*a^5*sgn(b*x + a))/x^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{5/2}}\,dx = \frac{\frac{2}{9}b^6x^6+\frac{12}{7}a\,b^5x^5+6a^2b^4x^4+\frac{40}{3}a^3b^3x^3+30a^4b^2x^2-12a^5bx-\frac{2}{3}a}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(5/2),x)
```

output
```
(2*( - 21*a**6 - 378*a**5*b*x + 945*a**4*b**2*x**2 + 420*a**3*b**3*x**3 +
189*a**2*b**4*x**4 + 54*a*b**5*x**5 + 7*b**6*x**6))/(63*sqrt(x)*x)
```

3.436 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 316

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx=-\frac{2a^5A\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)}-\frac{2a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)}-\frac{10a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)}+\frac{20a^2b^2(Ab+aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{10ab^3(Ab+2aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2b^4(Ab+5aB)x^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}+\frac{2b^5Bx^{7/2}\sqrt{a^2+2abx+b^2x^2}}{7(a+bx)}$$

output
```
-2/5*a^5*A*((b*x+a)^2)^(1/2)/x^(5/2)/(b*x+a)-2/3*a^4*(5*A*b+B*a)*((b*x+a)^
2)^(1/2)/x^(3/2)/(b*x+a)-10*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^(1/2)/(b
*x+a)+20*a^2*b^2*(A*b+B*a)*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+10*a*b^3*(A*b
+2*B*a)*x^(3/2)*((b*x+a)^2)^(1/2)/(3*b*x+3*a)+2*b^4*(A*b+5*B*a)*x^(5/2)*((
b*x+a)^2)^(1/2)/(5*b*x+5*a)+2*b^5*B*x^(7/2)*((b*x+a)^2)^(1/2)/(7*b*x+7*a)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx =$$
$$-\frac{2\sqrt{(a+bx)^2}(1050a^3b^2x^2(A-Bx)-350a^2b^3x^3(3A+Bx)+175a^4bx(A+3Bx)-35ab^4x^4(5A+3Bx)}{105x^{5/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(7/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(1050*a^3*b^2*x^2*(A - B*x) - 350*a^2*b^3*x^3*(3*A +
 B*x) + 175*a^4*b*x*(A + 3*B*x) - 35*a*b^4*x^4*(5*A + 3*B*x) + 7*a^5*(3*A
+ 5*B*x) - 3*b^5*x^5*(7*A + 5*B*x)))/(105*x^(5/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{7/2}}dx$$
$$\Big\downarrow \text{1187}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{7/2}}dx}{b^5(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{7/2}}dx}{a+bx}$$
$$\Big\downarrow \text{85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{7/2}}+\frac{(5Ab+aB)a^4}{x^{5/2}}+\frac{5b(2Ab+aB)a^3}{x^{3/2}}+\frac{10b^2(Ab+aB)a^2}{\sqrt{x}}+5b^3(Ab+2aB)\sqrt{x}a+b^5Bx^{5/2}+b^4(A\right.}{a+bx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^5A}{5x^{5/2}}-\frac{2a^4(aB+5Ab)}{3x^{3/2}}-\frac{10a^3b(aB+2Ab)}{\sqrt{x}}+20a^2b^2\sqrt{x}(aB+Ab)+\frac{2}{5}b^4x^{5/2}(5aB+Ab)+\frac{10}{3}ab^3\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(7/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^5*A)/(5*x^(5/2)) - (2*a^4*(5*A*b + a
*B))/(3*x^(3/2)) - (10*a^3*b*(2*A*b + a*B))/Sqrt[x] + 20*a^2*b^2*(A*b + a*
B)*Sqrt[x] + (10*a*b^3*(A*b + 2*a*B)*x^(3/2))/3 + (2*b^4*(A*b + 5*a*B)*x^(
5/2))/5 + (2*b^5*B*x^(7/2))/7))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$-\frac{2(-15B\,b^5x^6-21A\,b^5x^5-105Ba\,b^4x^5-175Aa\,b^4x^4-350B\,a^2b^3x^4-1050A\,a^2b^3x^3-1050B\,a^3b^2x^3+1050A\,a^3b^2x^2+525B\,a^4b\,x^2}{105x^{\frac{5}{2}}(bx+a)^5}$
default	$-\frac{2(-15B\,b^5x^6-21A\,b^5x^5-105Ba\,b^4x^5-175Aa\,b^4x^4-350B\,a^2b^3x^4-1050A\,a^2b^3x^3-1050B\,a^3b^2x^3+1050A\,a^3b^2x^2+525B\,a^4b\,x^2}{105x^{\frac{5}{2}}(bx+a)^5}$
risch	$-\frac{2\sqrt{(bx+a)^2}\,(-15B\,b^5x^6-21A\,b^5x^5-105Ba\,b^4x^5-175Aa\,b^4x^4-350B\,a^2b^3x^4-1050A\,a^2b^3x^3-1050B\,a^3b^2x^3+1050A\,a^3b^2x^2+5}{105(bx+a)x^{\frac{5}{2}}}$
orering	$-\frac{2(-15B\,b^5x^6-21A\,b^5x^5-105Ba\,b^4x^5-175Aa\,b^4x^4-350B\,a^2b^3x^4-1050A\,a^2b^3x^3-1050B\,a^3b^2x^3+1050A\,a^3b^2x^2+525B\,a^4b\,x^2}{105x^{\frac{5}{2}}(bx+a)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-15*B*b^5*x^6-21*A*b^5*x^5-105*B*a*b^4*x^5-175*A*a*b^4*x^4-350*B*a
^2*b^3*x^4-1050*A*a^2*b^3*x^3-1050*B*a^3*b^2*x^3+1050*A*a^3*b^2*x^2+525*B*
a^4*b*x^2+175*A*a^4*b*x+35*B*a^5*x+21*A*a^5)*((b*x+a)^2)^(5/2)/x^(5/2)/(b*
x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{2\left(15\,Bb^5x^6-21\,Aa^5+21\,(5\,Bab^4+Ab^5)x^5+175\,(2\,Ba^2b^3+Aa\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(7/2),x, algorithm="fricas
")
```

output
```
2/105*(15*B*b^5*x^6 - 21*A*a^5 + 21*(5*B*a*b^4 + A*b^5)*x^5 + 175*(2*B*a^2
*b^3 + A*a*b^4)*x^4 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 525*(B*a^4*b + 2*
A*a^3*b^2)*x^2 - 35*(B*a^5 + 5*A*a^4*b)*x)/x^(5/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**(7/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx=\frac{2}{15}\left(\left(3\,b^5x^2+5\,ab^4x\right)\sqrt{x}+\frac{20\left(ab^4x^2+3\,a^2b^3x\right)}{\sqrt{x}}+\frac{90\left(a^2b^3x^2-\right.}{x^{\frac{3}{2}}}\right.$$
$$+\frac{2}{105}\left(3\left(5\,b^5x^2+7\,ab^4x\right)x^{\frac{3}{2}}+28\left(3\,ab^4x^2+5\,a^2b^3x\right)\sqrt{x}+\frac{210\left(a^2b^3x^2+3\,a^3b^2x\right)}{\sqrt{x}}+\frac{420\left(a^3b^2x^2-a^4bx\right)}{x^{\frac{3}{2}}}\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(7/2),x, algorithm="maxima
")
```

output
```
2/15*((3*b^5*x^2 + 5*a*b^4*x)*sqrt(x) + 20*(a*b^4*x^2 + 3*a^2*b^3*x)/sqrt(
x) + 90*(a^2*b^3*x^2 - a^3*b^2*x)/x^(3/2) - 20*(3*a^3*b^2*x^2 + a^4*b*x)/x
^(5/2) - (5*a^4*b*x^2 + 3*a^5*x)/x^(7/2))*A + 2/105*(3*(5*b^5*x^2 + 7*a*b^
4*x)*x^(3/2) + 28*(3*a*b^4*x^2 + 5*a^2*b^3*x)*sqrt(x) + 210*(a^2*b^3*x^2 +
 3*a^3*b^2*x)/sqrt(x) + 420*(a^3*b^2*x^2 - a^4*b*x)/x^(3/2) - 35*(3*a^4*b*
x^2 + a^5*x)/x^(5/2))*B
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{2}{7}\,Bb^5x^{\frac{7}{2}}\mathrm{sgn}(bx+a) + 2\,Bab^4x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{2}{5}\,Ab^5x^{\frac{5}{2}}\mathrm{sgn}(bx+a) + \frac{20}{3}\,Ba^2b^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + \frac{10}{3}\,Aab^4x^{\frac{3}{2}}\mathrm{sgn}(bx+a) + 20\,Ba^3b^2\sqrt{x}\mathrm{sgn}(bx+a) + 20\,Aa^2b^3\sqrt{x}\mathrm{sgn}(bx+a) - \frac{2\,(75\,Ba^4bx^2\mathrm{sgn}(bx+a) + 150\,Aa^3b^2x^2\mathrm{sgn}(bx+a) + 5\,Ba^5x\mathrm{sgn}(bx+a) + 25\,Aa^4bx\mathrm{sgn}(bx+a) + 3\,Aa}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*B*b^5*x^(7/2)*sgn(b*x + a) + 2*B*a*b^4*x^(5/2)*sgn(b*x + a) + 2/5*A*b^
5*x^(5/2)*sgn(b*x + a) + 20/3*B*a^2*b^3*x^(3/2)*sgn(b*x + a) + 10/3*A*a*b^
4*x^(3/2)*sgn(b*x + a) + 20*B*a^3*b^2*sqrt(x)*sgn(b*x + a) + 20*A*a^2*b^3*
sqrt(x)*sgn(b*x + a) - 2/15*(75*B*a^4*b*x^2*sgn(b*x + a) + 150*A*a^3*b^2*x
^2*sgn(b*x + a) + 5*B*a^5*x*sgn(b*x + a) + 25*A*a^4*b*x*sgn(b*x + a) + 3*A
*a^5*sgn(b*x + a))/x^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(7/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{\frac{2}{7}b^6x^6+\frac{12}{5}a\,b^5x^5+10a^2b^4x^4+40a^3b^3x^3-30a^4b^2x^2-4a^5bx-\frac{2}{5}a^6}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(7/2),x)
```

output
```
(2*( - 7*a**6 - 70*a**5*b*x - 525*a**4*b**2*x**2 + 700*a**3*b**3*x**3 + 17
5*a**2*b**4*x**4 + 42*a*b**5*x**5 + 5*b**6*x**6))/(35*sqrt(x)*x**2)
```

3.437 $\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 316

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx = -\frac{2a^5A\sqrt{a^2+2abx+b^2x^2}}{7x^{7/2}(a+bx)}-\frac{2a^4(5Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{5x^{5/2}(a+bx)}-\frac{10a^3b(2Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{3x^{3/2}(a+bx)}-\frac{20a^2b^2(Ab+aB)\sqrt{a^2+2abx+b^2x^2}}{\sqrt{x}(a+bx)}+\frac{10ab^3(Ab+2aB)\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}{a+bx}+\frac{2b^4(Ab+5aB)x^{3/2}\sqrt{a^2+2abx+b^2x^2}}{3(a+bx)}+\frac{2b^5Bx^{5/2}\sqrt{a^2+2abx+b^2x^2}}{5(a+bx)}$$

output
```
-2/7*a^5*A*((b*x+a)^2)^(1/2)/x^(7/2)/(b*x+a)-2/5*a^4*(5*A*b+B*a)*((b*x+a)^
2)^(1/2)/x^(5/2)/(b*x+a)-10/3*a^3*b*(2*A*b+B*a)*((b*x+a)^2)^(1/2)/x^(3/2)/
(b*x+a)-20*a^2*b^2*(A*b+B*a)*((b*x+a)^2)^(1/2)/x^(1/2)/(b*x+a)+10*a*b^3*(A
*b+2*B*a)*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b^4*(A*b+5*B*a)*x^(3/2)*((b*
x+a)^2)^(1/2)/(3*b*x+3*a)+2*b^5*B*x^(5/2)*((b*x+a)^2)^(1/2)/(5*b*x+5*a)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.39

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx =$$
$$-\frac{2\sqrt{(a+bx)^2}(1050a^2b^3x^3(A-Bx)-175ab^4x^4(3A+Bx)+350a^3b^2x^2(A+3Bx)-7b^5x^5(5A+3Bx)+}{105x^{7/2}(a+bx)}$$

input
```
Integrate[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(9/2),x]
```

output
```
(-2*Sqrt[(a + b*x)^2]*(1050*a^2*b^3*x^3*(A - B*x) - 175*a*b^4*x^4*(3*A + B
*x) + 350*a^3*b^2*x^2*(A + 3*B*x) - 7*b^5*x^5*(5*A + 3*B*x) + 35*a^4*b*x*(
3*A + 5*B*x) + 3*a^5*(5*A + 7*B*x)))/(105*x^(7/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1187, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{b^5(a+bx)^5(A+Bx)}{x^{9/2}}dx}{b^5(a+bx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)^5(A+Bx)}{x^{9/2}}dx}{a+bx}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int\left(\frac{Aa^5}{x^{9/2}}+\frac{(5Ab+aB)a^4}{x^{7/2}}+\frac{5b(2Ab+aB)a^3}{x^{5/2}}+\frac{10b^2(Ab+aB)a^2}{x^{3/2}}+\frac{5b^3(Ab+2aB)a}{\sqrt{x}}+b^5Bx^{3/2}+b^4(Ab+5a\right.}{a+bx}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{2a^5A}{7x^{7/2}}-\frac{2a^4(aB+5Ab)}{5x^{5/2}}-\frac{10a^3b(aB+2Ab)}{3x^{3/2}}-\frac{20a^2b^2(aB+Ab)}{\sqrt{x}}+\frac{2}{3}b^4x^{3/2}(5aB+Ab)+10ab^3\sqrt{x}(2a\right.}{a+bx}$$

input
```
Int[((A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2))/x^(9/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((-2*a^5*A)/(7*x^(7/2)) - (2*a^4*(5*A*b + a
*B))/(5*x^(5/2)) - (10*a^3*b*(2*A*b + a*B))/(3*x^(3/2)) - (20*a^2*b^2*(A*b
 + a*B))/Sqrt[x] + 10*a*b^3*(A*b + 2*a*B)*Sqrt[x] + (2*b^4*(A*b + 5*a*B)*x
^(3/2))/3 + (2*b^5*B*x^(5/2))/5))/(a + b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

method	result
gosper	$-\frac{2\left(-21B\,b^5x^6-35A\,b^5x^5-175Ba\,b^4x^5-525Aa\,b^4x^4-1050B\,a^2b^3x^4+1050A\,a^2b^3x^3+1050B\,a^3b^2x^3+350A\,a^3b^2x^2+175B\,a^4b\,x^2\right.}{105x^{\frac{7}{2}}\left(bx+a\right)^5}$
default	$-\frac{2\left(-21B\,b^5x^6-35A\,b^5x^5-175Ba\,b^4x^5-525Aa\,b^4x^4-1050B\,a^2b^3x^4+1050A\,a^2b^3x^3+1050B\,a^3b^2x^3+350A\,a^3b^2x^2+175B\,a^4b\,x^2\right.}{105x^{\frac{7}{2}}\left(bx+a\right)^5}$
risch	$-\frac{2\sqrt{\left(bx+a\right)^2}\,\left(-21B\,b^5x^6-35A\,b^5x^5-175Ba\,b^4x^5-525Aa\,b^4x^4-1050B\,a^2b^3x^4+1050A\,a^2b^3x^3+1050B\,a^3b^2x^3+350A\,a^3b^2x^2+1\right.}{105(bx+a)x^{\frac{7}{2}}}$
orering	$-\frac{2\left(-21B\,b^5x^6-35A\,b^5x^5-175Ba\,b^4x^5-525Aa\,b^4x^4-1050B\,a^2b^3x^4+1050A\,a^2b^3x^3+1050B\,a^3b^2x^3+350A\,a^3b^2x^2+175B\,a^4b\,x^2\right.}{105x^{\frac{7}{2}}\left(bx+a\right)^5}$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-21*B*b^5*x^6-35*A*b^5*x^5-175*B*a*b^4*x^5-525*A*a*b^4*x^4-1050*B*
a^2*b^3*x^4+1050*A*a^2*b^3*x^3+1050*B*a^3*b^2*x^3+350*A*a^3*b^2*x^2+175*B*
a^4*b*x^2+105*A*a^4*b*x+21*B*a^5*x+15*A*a^5)*((b*x+a)^2)^(5/2)/x^(7/2)/(b*
x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.38

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx = \frac{2\left(21\,Bb^5x^6-15\,Aa^5+35\,(5\,Bab^4+Ab^5)x^5+525\,(2\,Ba^2b^3+Aa\right.}{}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(9/2),x, algorithm="fricas
")
```

output
```
2/105*(21*B*b^5*x^6 - 15*A*a^5 + 35*(5*B*a*b^4 + A*b^5)*x^5 + 525*(2*B*a^2
*b^3 + A*a*b^4)*x^4 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 175*(B*a^4*b + 2*
A*a^3*b^2)*x^2 - 21*(B*a^5 + 5*A*a^4*b)*x)/x^(7/2)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx=\int \frac{(A+Bx)\left((a+bx)^2\right)^{\frac{5}{2}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**(5/2)/x**(9/2),x)
```

output
```
Integral((A + B*x)*((a + b*x)**2)**(5/2)/x**(9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx=\frac{2}{15}\left(\left(3\,b^5x^2+5\,ab^4x\right)\sqrt{x}+\frac{20\left(ab^4x^2+3\,a^2b^3x\right)}{\sqrt{x}}+\frac{90\left(a^2b^3x^2-\right.}{x^{\frac{3}{2}}}\right.$$
$$+\frac{2}{105}\,A\left(\frac{35\left(b^5x^2+3\,ab^4x\right)}{\sqrt{x}}+\frac{420\left(ab^4x^2-a^2b^3x\right)}{x^{\frac{3}{2}}}-\frac{210\left(3\,a^2b^3x^2+a^3b^2x\right)}{x^{\frac{5}{2}}}-\frac{28\left(5\,a^3b^2x^2+3\,a^4bx\right)}{x^{\frac{7}{2}}}-\frac{3}{\ }\right.$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(9/2),x, algorithm="maxima
")
```

output
```
2/15*((3*b^5*x^2 + 5*a*b^4*x)*sqrt(x) + 20*(a*b^4*x^2 + 3*a^2*b^3*x)/sqrt(
x) + 90*(a^2*b^3*x^2 - a^3*b^2*x)/x^(3/2) - 20*(3*a^3*b^2*x^2 + a^4*b*x)/x
^(5/2) - (5*a^4*b*x^2 + 3*a^5*x)/x^(7/2))*B + 2/105*A*(35*(b^5*x^2 + 3*a*b
^4*x)/sqrt(x) + 420*(a*b^4*x^2 - a^2*b^3*x)/x^(3/2) - 210*(3*a^2*b^3*x^2 +
 a^3*b^2*x)/x^(5/2) - 28*(5*a^3*b^2*x^2 + 3*a^4*b*x)/x^(7/2) - 3*(7*a^4*b*
x^2 + 5*a^5*x)/x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx = \frac{2}{5}\,Bb^5x^{\frac{5}{2}}\mathrm{sgn}(bx+a)+\frac{10}{3}\,Bab^4x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+\frac{2}{3}\,Ab^5x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+20\,Ba^2b^3\sqrt{x}\mathrm{sgn}(bx+a)+10\,Aab^4\sqrt{x}\mathrm{sgn}(bx+a)-\frac{2\,(1050\,Ba^3b^2x^3\mathrm{sgn}(bx+a)+1050\,Aa^2b^3x^3\mathrm{sgn}(bx+a)+175\,Ba^4bx^2\mathrm{sgn}(bx+a)+350\,Aa^3b^2x^2\mathrm{sgn}(bx}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(9/2),x, algorithm="giac")
```

output
```
2/5*B*b^5*x^(5/2)*sgn(b*x + a) + 10/3*B*a*b^4*x^(3/2)*sgn(b*x + a) + 2/3*A
*b^5*x^(3/2)*sgn(b*x + a) + 20*B*a^2*b^3*sqrt(x)*sgn(b*x + a) + 10*A*a*b^4
*sqrt(x)*sgn(b*x + a) - 2/105*(1050*B*a^3*b^2*x^3*sgn(b*x + a) + 1050*A*a^
2*b^3*x^3*sgn(b*x + a) + 175*B*a^4*b*x^2*sgn(b*x + a) + 350*A*a^3*b^2*x^2*
sgn(b*x + a) + 21*B*a^5*x*sgn(b*x + a) + 105*A*a^4*b*x*sgn(b*x + a) + 15*A
*a^5*sgn(b*x + a))/x^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx = \int \frac{(A+B\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(9/2),x)
```

output
```
int(((A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx)\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{9/2}}\,dx = \frac{\frac{2}{5}b^6x^6+4a\,b^5x^5+30a^2b^4x^4-40a^3b^3x^3-10a^4b^2x^2-\frac{12}{5}a^5bx-\frac{2}{7}}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(b^2*x^2+2*a*b*x+a^2)^(5/2)/x^(9/2),x)
```

output
```
(2*( - 5*a**6 - 42*a**5*b*x - 175*a**4*b**2*x**2 - 700*a**3*b**3*x**3 + 52
5*a**2*b**4*x**4 + 70*a*b**5*x**5 + 7*b**6*x**6))/(35*sqrt(x)*x**3)
```

3.438 $\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 286

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2a^3(Ab-aB)\sqrt{x}(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}}+\frac{2a^2(Ab-aB)x^{3/2}(a+bx)}{3b^4\sqrt{a^2+2abx+b^2x^2}}-\frac{2a(Ab-aB)x^{5/2}(a+bx)}{5b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(Ab-aB)x^{7/2}(a+bx)}{7b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2Bx^{9/2}(a+bx)}{9b\sqrt{a^2+2abx+b^2x^2}}+\frac{2a^{7/2}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2*a^3*(A*b-B*a)*x^(1/2)*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+2/3*a^2*(A*b-B*a)*x
^(3/2)*(b*x+a)/b^4/((b*x+a)^2)^(1/2)-2/5*a*(A*b-B*a)*x^(5/2)*(b*x+a)/b^3/(
(b*x+a)^2)^(1/2)+2/7*(A*b-B*a)*x^(7/2)*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+2/9*B
*x^(9/2)*(b*x+a)/b/((b*x+a)^2)^(1/2)+2*a^(7/2)*(A*b-B*a)*(b*x+a)*arctan(b^
(1/2)*x^(1/2)/a^(1/2))/b^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.49

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2(a+bx)\left(\sqrt{b}\sqrt{x}(315a^4B-105a^3b(3A+Bx)+21a^2b^2x(5A+3Bx)-9ab^3x\right.}{315b^{11/2}\sqrt{(}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*(a + b*x)*(Sqrt[b]*Sqrt[x]*(315*a^4*B - 105*a^3*b*(3*A + B*x) + 21*a^2*
b^2*x*(5*A + 3*B*x) - 9*a*b^3*x^2*(7*A + 5*B*x) + 5*b^4*x^3*(9*A + 7*B*x))
 - 315*a^(7/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(315*b^(
11/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {1187, 27, 90, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b(a+bx)\int \frac{x^{7/2}(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{x^{7/2}(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 90$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\int \frac{x^{7/2}}{a+bx}dx}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\int \frac{x^{5/2}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(7/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((2*B*x^(9/2))/(9*b) + ((A*b - a*B)*((2*x^(7/2))/(7*b) - (a*((2
*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/b))/b))/Sqrt[a^2 + 2*a
*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{2(-35b^4Bx^4-45Ab^4x^3+45Bab^3x^3+63Aab^3x^2-63Ba^2b^2x^2-105Aa^2b^2x+105Ba^3bx+315Aa^3b-315a^4B)\sqrt{x}\sqrt{(bx+a)^2}}{315b^5(bx+a)}+$
default	$\frac{2(bx+a)\left(35B\sqrt{ab}\,x^{\frac{9}{2}}b^4+45A\sqrt{ab}\,x^{\frac{7}{2}}b^4-45B\sqrt{ab}\,x^{\frac{7}{2}}ab^3-63A\sqrt{ab}\,x^{\frac{5}{2}}ab^3+63B\sqrt{ab}\,x^{\frac{5}{2}}a^2b^2+105A\sqrt{ab}\,x^{\frac{3}{2}}a^2b^2-105B\sqrt{ab}\,x^{\frac{3}{2}}a^3\right.}{315\sqrt{(bx+a)^2}\,b^5\sqrt{ab}}$

input
```
int(x^(7/2)*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*B*b^4*x^4-45*A*b^4*x^3+45*B*a*b^3*x^3+63*A*a*b^3*x^2-63*B*a^2*
b^2*x^2-105*A*a^2*b^2*x+105*B*a^3*b*x+315*A*a^3*b-315*B*a^4)*x^(1/2)/b^5*(
(b*x+a)^2)^(1/2)/(b*x+a)+2*a^4*(A*b-B*a)/b^5/(a*b)^(1/2)*arctan(b*x^(1/2)/
(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.97

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[-\frac{315\,(Ba^4-Aa^3b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(35\,Bb^4x^4+315\,Ba^4-3}{}\right.$$

$$-\frac{2\left(315\,(Ba^4-Aa^3b)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(35\,Bb^4x^4+315\,Ba^4-315\,Aa^3b-45\,(Bab^3-Ab^4)x^3+6\right.}{315\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/315*(315*(B*a^4 - A*a^3*b)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b
) - a)/(b*x + a)) - 2*(35*B*b^4*x^4 + 315*B*a^4 - 315*A*a^3*b - 45*(B*a*b^
3 - A*b^4)*x^3 + 63*(B*a^2*b^2 - A*a*b^3)*x^2 - 105*(B*a^3*b - A*a^2*b^2)*
x)*sqrt(x))/b^5, -2/315*(315*(B*a^4 - A*a^3*b)*sqrt(a/b)*arctan(b*sqrt(x)*
sqrt(a/b)/a) - (35*B*b^4*x^4 + 315*B*a^4 - 315*A*a^3*b - 45*(B*a*b^3 - A*b
^4)*x^3 + 63*(B*a^2*b^2 - A*a*b^3)*x^2 - 105*(B*a^3*b - A*a^2*b^2)*x)*sqrt
(x))/b^5]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.90

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{10\,(7\,Bb^4x^2+9\,Bab^3x)x^{\frac{7}{2}}-2\,(5\,(11\,Bab^3-9\,Ab^4)x^2+9\,(9\,Ba^2b^2-7\,Aab^3)x\cdots}{\cdots}-\frac{2\,(Ba^5-Aa^4b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}-\frac{(11\,Ba^3b-9\,Aa^2b^2)x^{\frac{3}{2}}-6\,(Ba^4-Aa^3b)\sqrt{x}}{3\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/315*(10*(7*B*b^4*x^2 + 9*B*a*b^3*x)*x^(7/2) - 2*(5*(11*B*a*b^3 - 9*A*b^4
)*x^2 + 9*(9*B*a^2*b^2 - 7*A*a*b^3)*x)*x^(5/2) + 6*(3*(11*B*a^2*b^2 - 9*A*
a*b^3)*x^2 + 7*(9*B*a^3*b - 7*A*a^2*b^2)*x)*x^(3/2) + 21*(3*(11*B*a^3*b -
9*A*a^2*b^2)*x^2 + 5*(9*B*a^4 - 7*A*a^3*b)*x)*sqrt(x))/(b^5*x + a*b^4) - 2
*(B*a^5 - A*a^4*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/3*((11*
B*a^3*b - 9*A*a^2*b^2)*x^(3/2) - 6*(B*a^4 - A*a^3*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2\left(Ba^5\mathrm{sgn}(bx+a)-Aa^4b\mathrm{sgn}(bx+a)\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}$$
$$+\frac{2\left(35\,Bb^8x^{\frac{9}{2}}\mathrm{sgn}(bx+a)-45\,Bab^7x^{\frac{7}{2}}\mathrm{sgn}(bx+a)+45\,Ab^8x^{\frac{7}{2}}\mathrm{sgn}(bx+a)+63\,Ba^2b^6x^{\frac{5}{2}}\mathrm{sgn}(bx+a)-63\right.}{}$$

input
```
integrate(x^(7/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*a^5*sgn(b*x + a) - A*a^4*b*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*b^5) + 2/315*(35*B*b^8*x^(9/2)*sgn(b*x + a) - 45*B*a*b^7*x^(7/
2)*sgn(b*x + a) + 45*A*b^8*x^(7/2)*sgn(b*x + a) + 63*B*a^2*b^6*x^(5/2)*sgn
(b*x + a) - 63*A*a*b^7*x^(5/2)*sgn(b*x + a) - 105*B*a^3*b^5*x^(3/2)*sgn(b*
x + a) + 105*A*a^2*b^6*x^(3/2)*sgn(b*x + a) + 315*B*a^4*b^4*sqrt(x)*sgn(b*
x + a) - 315*A*a^3*b^5*sqrt(x)*sgn(b*x + a))/b^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{x^{7/2}\,(A+B\,x)}{\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.02

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\sqrt{x}\,x^4}{9}$$

input
```
int(x^(7/2)*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**4)/9
```

3.439 $\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2a^2(Ab-aB)\sqrt{x}(a+bx)}{b^4\sqrt{a^2+2abx+b^2x^2}} - \frac{2a(Ab-aB)x^{3/2}(a+bx)}{3b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{2(Ab-aB)x^{5/2}(a+bx)}{5b^2\sqrt{a^2+2abx+b^2x^2}} + \frac{2Bx^{7/2}(a+bx)}{7b\sqrt{a^2+2abx+b^2x^2}} - \frac{2a^{5/2}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
2*a^2*(A*b-B*a)*x^(1/2)*(b*x+a)/b^4/((b*x+a)^2)^(1/2)-2/3*a*(A*b-B*a)*x^(3
/2)*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+2/5*(A*b-B*a)*x^(5/2)*(b*x+a)/b^2/((b*x+
a)^2)^(1/2)+2/7*B*x^(7/2)*(b*x+a)/b/((b*x+a)^2)^(1/2)-2*a^(5/2)*(A*b-B*a)*
(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.50

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2(a+bx)\left(\sqrt{b}\sqrt{x}(-105a^3B+35a^2b(3A+Bx)-7ab^2x(5A+3Bx)+3b^3x^2(\right.}{105b^{9/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*(a + b*x)*(Sqrt[b]*Sqrt[x]*(-105*a^3*B + 35*a^2*b*(3*A + B*x) - 7*a*b^2
*x*(5*A + 3*B*x) + 3*b^3*x^2*(7*A + 5*B*x)) + 105*a^(5/2)*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(105*b^(9/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 90, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b(a+bx)\int \frac{x^{5/2}(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^{5/2}(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\int \frac{x^{5/2}}{a+bx}dx}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(5/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((2*B*x^(7/2))/(7*b) + ((A*b - a*B)*((2*x^(5/2))/(5*b) - (a*((2
*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/b^(3/2)))/b))/b))/b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(15x^3Bb^3+21Ab^3x^2-21Bab^2x^2-35Aab^2x+35Ba^2bx+105Aa^2b-105Ba^3)\sqrt{x}\sqrt{(bx+a)^2}}{105b^4(bx+a)}-\frac{2a^3(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{(b}}{b^4\sqrt{ab}\,(bx+a)}$
default	$\frac{2(bx+a)\left(15B\sqrt{ab}\,x^{\frac{7}{2}}b^3+21A\sqrt{ab}\,x^{\frac{5}{2}}b^3-21B\sqrt{ab}\,x^{\frac{5}{2}}ab^2-35A\sqrt{ab}\,x^{\frac{3}{2}}ab^2+35B\sqrt{ab}\,x^{\frac{3}{2}}a^2b+105A\sqrt{ab}\sqrt{x}\,a^2b-105A\arctan\left(\frac{b\sqrt{}}{\sqrt{a}}\right.\right.}{105\sqrt{(bx+a)^2}\,b^4\sqrt{ab}}$

input
```
int(x^(5/2)*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*B*b^3*x^3+21*A*b^3*x^2-21*B*a*b^2*x^2-35*A*a*b^2*x+35*B*a^2*b*x+
105*A*a^2*b-105*B*a^3)*x^(1/2)/b^4*((b*x+a)^2)^(1/2)/(b*x+a)-2*a^3*(A*b-B*
a)/b^4/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.96

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[-\frac{105\,(Ba^3-Aa^2b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(15\,Bb^3x^3-105\,Ba^3+1}{105\,b^4}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b
) - a)/(b*x + a)) - 2*(15*B*b^3*x^3 - 105*B*a^3 + 105*A*a^2*b - 21*(B*a*b^
2 - A*b^3)*x^2 + 35*(B*a^2*b - A*a*b^2)*x)*sqrt(x))/b^4, 2/105*(105*(B*a^3
 - A*a^2*b)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*B*b^3*x^3 - 105*
B*a^3 + 105*A*a^2*b - 21*(B*a*b^2 - A*b^3)*x^2 + 35*(B*a^2*b - A*a*b^2)*x)
*sqrt(x))/b^4]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{6\,(5\,Bb^3x^2+7\,Bab^2x)x^{\frac{5}{2}}-2\,(3\,(9\,Bab^2-7\,Ab^3)x^2+7\,(7\,Ba^2b-5\,Aab^2)x)x^{\frac{3}{2}}}{105\,(b^4x+ab^3)} + \frac{2\,(Ba^4-Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{(9\,Ba^2b-7\,Aab^2)x^{\frac{3}{2}}-6\,(Ba^3-Aa^2b)\sqrt{x}}{3\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/105*(6*(5*B*b^3*x^2 + 7*B*a*b^2*x)*x^(5/2) - 2*(3*(9*B*a*b^2 - 7*A*b^3)*
x^2 + 7*(7*B*a^2*b - 5*A*a*b^2)*x)*x^(3/2) - 7*(3*(9*B*a^2*b - 7*A*a*b^2)*
x^2 + 5*(7*B*a^3 - 5*A*a^2*b)*x)*sqrt(x))/(b^4*x + a*b^3) + 2*(B*a^4 - A*a
^3*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/3*((9*B*a^2*b - 7*A*
a*b^2)*x^(3/2) - 6*(B*a^3 - A*a^2*b)*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,(Ba^4\mathrm{sgn}(bx+a)-Aa^3b\mathrm{sgn}(bx+a))\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(15\,Bb^6x^{\frac{7}{2}}\mathrm{sgn}(bx+a)-21\,Bab^5x^{\frac{5}{2}}\mathrm{sgn}(bx+a)+21\,Ab^6x^{\frac{5}{2}}\mathrm{sgn}(bx+a)+35\,Ba^2b^4x^{\frac{3}{2}}\mathrm{sgn}(bx+a)-35}{105\,b^7}$$

input
```
integrate(x^(5/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*(B*a^4*sgn(b*x + a) - A*a^3*b*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))/
(sqrt(a*b)*b^4) + 2/105*(15*B*b^6*x^(7/2)*sgn(b*x + a) - 21*B*a*b^5*x^(5/2
)*sgn(b*x + a) + 21*A*b^6*x^(5/2)*sgn(b*x + a) + 35*B*a^2*b^4*x^(3/2)*sgn(
b*x + a) - 35*A*a*b^5*x^(3/2)*sgn(b*x + a) - 105*B*a^3*b^3*sqrt(x)*sgn(b*x
 + a) + 105*A*a^2*b^4*sqrt(x)*sgn(b*x + a))/b^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.03

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{2\sqrt{x}\,x^3}{7}$$

input
```
int(x^(5/2)*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**3)/7
```

3.440 $\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2a(Ab-aB)\sqrt{x}(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}}+\frac{2(Ab-aB)x^{3/2}(a+bx)}{3b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2Bx^{5/2}(a+bx)}{5b\sqrt{a^2+2abx+b^2x^2}}+\frac{2a^{3/2}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2*a*(A*b-B*a)*x^(1/2)*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+2/3*(A*b-B*a)*x^(3/2)
*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+2/5*B*x^(5/2)*(b*x+a)/b/((b*x+a)^2)^(1/2)+2
*a^(3/2)*(A*b-B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{2(a+bx)\left(\sqrt{b}\sqrt{x}(15a^2B-5ab(3A+Bx)+b^2x(5A+3Bx))-15a^{3/2}(-Ab-\right.}{15b^{7/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*(a + b*x)*(Sqrt[b]*Sqrt[x]*(15*a^2*B - 5*a*b*(3*A + B*x) + b^2*x*(5*A +
 3*B*x)) - 15*a^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(
15*b^(7/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1187, 27, 90, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{x^{3/2}(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^{3/2}(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{90}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\int \frac{x^{3/2}}{a+bx}dx}{b}+\frac{2Bx^{5/2}}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(3/2)*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((2*B*x^(5/2))/(5*b) + ((A*b - a*B)*((2*x^(3/2))/(3*b) - (a*((2
*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/
b))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{2\left(-3x^{2}B\,b^{2}-5x\,b^{2}A+5xabB+15abA-15a^{2}B\right)\sqrt{x}\,\sqrt{\left(bx+a\right)^{2}}}{15b^{3}\left(bx+a\right)}+\frac{2a^{2}\left(Ab-Ba\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{\left(bx+a\right)^{2}}}{b^{3}\sqrt{ab}\,\left(bx+a\right)}$
default	$\frac{2(bx+a)\left(3B\,x^{\frac{5}{2}}\sqrt{ab}\,b^{2}+5A\,x^{\frac{3}{2}}\sqrt{ab}\,b^{2}-5B\,x^{\frac{3}{2}}\sqrt{ab}\,ab-15A\sqrt{x}\,\sqrt{ab}\,ab+15A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^{2}b+15B\sqrt{x}\,\sqrt{ab}\,a^{2}-15B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right.}{15\sqrt{\left(bx+a\right)^{2}}\,b^{3}\sqrt{ab}}$

input
```
int(x^(3/2)*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+5*B*a*b*x+15*A*a*b-15*B*a^2)*x^(1/2)/b^3*((b
*x+a)^2)^(1/2)/(b*x+a)+2*a^2*(A*b-B*a)/b^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a
*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[-\frac{15\,(Ba^2-Aab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(3\,Bb^2x^2+15\,Ba^2-15\,Aa}{15\,b^3}\right.$$

$$\left.-\frac{2\left(15\,(Ba^2-Aab)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,Bb^2x^2+15\,Ba^2-15\,Aab-5\,(Bab-Ab^2)x)\sqrt{x}\right)}{15\,b^3}\right]$$

input
```
integrate(x^(3/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*a^2 - A*a*b)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) -
a)/(b*x + a)) - 2*(3*B*b^2*x^2 + 15*B*a^2 - 15*A*a*b - 5*(B*a*b - A*b^2)*x
)*sqrt(x))/b^3, -2/15*(15*(B*a^2 - A*a*b)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(
a/b)/a) - (3*B*b^2*x^2 + 15*B*a^2 - 15*A*a*b - 5*(B*a*b - A*b^2)*x)*sqrt(x
))/b^3]
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx)}{\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)/sqrt((a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,(3\,Bb^2x^2+5\,Babx)x^{\frac{3}{2}}+(3\,(7\,Bab-5\,Ab^2)x^2+5\,(5\,Ba^2-3\,Aab)x)\sqrt{x}}{15\,(b^3x+ab^2)} - \frac{2\,(Ba^3-Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{(7\,Bab-5\,Ab^2)x^{\frac{3}{2}}-6\,(Ba^2-Aab)\sqrt{x}}{3\,b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(2*(3*B*b^2*x^2 + 5*B*a*b*x)*x^(3/2) + (3*(7*B*a*b - 5*A*b^2)*x^2 + 5
*(5*B*a^2 - 3*A*a*b)*x)*sqrt(x))/(b^3*x + a*b^2) - 2*(B*a^3 - A*a^2*b)*arc
tan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/3*((7*B*a*b - 5*A*b^2)*x^(3/2
) - 6*(B*a^2 - A*a*b)*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2\,(Ba^3\mathrm{sgn}(bx+a)-Aa^2b\mathrm{sgn}(bx+a))\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,Bb^4x^{\frac{5}{2}}\mathrm{sgn}(bx+a)-5\,Bab^3x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+5\,Ab^4x^{\frac{3}{2}}\mathrm{sgn}(bx+a)+15\,Ba^2b^2\sqrt{x}\mathrm{sgn}(bx+a)-15\,Aa\right.}{15\,b^5}$$

input
```
integrate(x^(3/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*a^3*sgn(b*x + a) - A*a^2*b*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*b^3) + 2/15*(3*B*b^4*x^(5/2)*sgn(b*x + a) - 5*B*a*b^3*x^(3/2)*
sgn(b*x + a) + 5*A*b^4*x^(3/2)*sgn(b*x + a) + 15*B*a^2*b^2*sqrt(x)*sgn(b*x
 + a) - 15*A*a*b^3*sqrt(x)*sgn(b*x + a))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{x^{3/2}\,(A+B\,x)}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.04

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{2\sqrt{x}\,x^2}{5}$$

input
```
int(x^(3/2)*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x**2)/5
```

3.441 $\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 144

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2(Ab-aB)\sqrt{x}(a+bx)}{b^2\sqrt{a^2+2abx+b^2x^2}}+\frac{2Bx^{3/2}(a+bx)}{3b\sqrt{a^2+2abx+b^2x^2}} - \frac{2\sqrt{a}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
2*(A*b-B*a)*x^(1/2)*(b*x+a)/b^2/((b*x+a)^2)^(1/2)+2/3*B*x^(3/2)*(b*x+a)/b/
((b*x+a)^2)^(1/2)-2*a^(1/2)*(A*b-B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/
2))/b^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2(a+bx)\left(\sqrt{b}\sqrt{x}(3Ab-3aB+bBx)+3\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(2*(a + b*x)*(Sqrt[b]*Sqrt[x]*(3*A*b - 3*a*B + b*B*x) + 3*Sqrt[a]*(-(A*b)
+ a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1187, 27, 90, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{\sqrt{x}(A+Bx)}{b(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{\sqrt{x}(A+Bx)}{a+bx}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{90}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\int \frac{\sqrt{x}}{a+bx}dx}{b}+\frac{2Bx^{3/2}}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}+\frac{2Bx^{3/2}}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{3/2}}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}+\frac{2Bx^{3/2}}{3b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/Sqrt[a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((a + b*x)*((2*B*x^(3/2))/(3*b) + ((A*b - a*B)*((2*Sqrt[x])/b - (2*Sqrt[a]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/Sqrt[a^2 + 2*a*b*x + b^2
*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{2(Bbx+3Ab-3Ba)\sqrt{x}\,\sqrt{(bx+a)^2}}{3b^2(bx+a)}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{(bx+a)^2}}{b^2\sqrt{ab}\,(bx+a)}$	85
default	$\frac{2(bx+a)\left(B\sqrt{ab}\,x^{\frac{3}{2}}b+3A\sqrt{ab}\,\sqrt{x}\,b-3A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)ab-3B\sqrt{ab}\,\sqrt{x}\,a+3B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^2\right)}{3\sqrt{(bx+a)^2}\,b^2\sqrt{ab}}$	94

input
```
int(x^(1/2)*(B*x+A)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-3*B*a)*x^(1/2)/b^2*((b*x+a)^2)^(1/2)/(b*x+a)-2*a*(A*b-B*a
)/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx$$
$$= \left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(Bbx-3\,Ba+3\,Ab)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,(Ba-Ab)\sqrt{\frac{a}{b}}\arctan\left(\right.\right.}{}\right.$$

input
```
integrate(x^(1/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*
x + a)) - 2*(B*b*x - 3*B*a + 3*A*b)*sqrt(x))/b^2, 2/3*(3*(B*a - A*b)*sqrt(
a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (B*b*x - 3*B*a + 3*A*b)*sqrt(x))/b^2]
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{\sqrt{x}(A+Bx)}{\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)/((b*x+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)/sqrt((a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{((Bab-Ab^2)x^2+(Ba^2-Aab)x)\sqrt{x}}{ab^2x+a^2b} + \frac{2\,(Ba^2-Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{(5\,Bab-3\,Ab^2)x^{\frac{3}{2}}-6\,(Ba^2-Aab)\sqrt{x}}{3\,ab^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-((B*a*b - A*b^2)*x^2 + (B*a^2 - A*a*b)*x)*sqrt(x)/(a*b^2*x + a^2*b) + 2*(
B*a^2 - A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*((5*B*a*b
 - 3*A*b^2)*x^(3/2) - 6*(B*a^2 - A*a*b)*sqrt(x))/(a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,(Ba^2\mathrm{sgn}(bx+a)-Aab\mathrm{sgn}(bx+a))\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(Bb^2x^{\frac{3}{2}}\mathrm{sgn}(bx+a)-3\,Bab\sqrt{x}\mathrm{sgn}(bx+a)+3\,Ab^2\sqrt{x}\mathrm{sgn}(bx+a)\right)}{3\,b^3}$$

input
```
integrate(x^(1/2)*(B*x+A)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*(B*a^2*sgn(b*x + a) - A*a*b*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))/(s
qrt(a*b)*b^2) + 2/3*(B*b^2*x^(3/2)*sgn(b*x + a) - 3*B*a*b*sqrt(x)*sgn(b*x
+ a) + 3*A*b^2*sqrt(x)*sgn(b*x + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{\sqrt{x}\,(A+B\,x)}{\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/((a + b*x)^2)^(1/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/((a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\sqrt{x}\,x}{3}$$

input
```
int(x^(1/2)*(B*x+A)/((b*x+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*x)/3
```

3.442 $\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 99

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2B\sqrt{x}(a+bx)}{b\sqrt{a^2+2abx+b^2x^2}} + \frac{2(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
2*B*x^(1/2)*(b*x+a)/b/((b*x+a)^2)^(1/2)+2*(A*b-B*a)*(b*x+a)*arctan(b^(1/2)
*x^(1/2)/a^(1/2))/a^(1/2)/b^(3/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} dx = \frac{2(a+bx)\left(\sqrt{a}\sqrt{b}B\sqrt{x}+(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{\sqrt{a}b^{3/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(2*(a + b*x)*(Sqrt[a]*Sqrt[b]*B*Sqrt[x] + (A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1187, 27, 90, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{b\sqrt{x}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{\sqrt{x}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{90}$$

$$\frac{(a+bx)\left(\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}+\frac{2B\sqrt{x}}{b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{2(Ab-aB)\int \frac{1}{a+bx}d\sqrt{x}}{b}+\frac{2B\sqrt{x}}{b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{2B\sqrt{x}}{b}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((2*B*Sqrt[x])/b + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/(Sqrt[a]*b^(3/2))))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2(bx+a)\left(A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b+B\sqrt{x}\,\sqrt{ab}-B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\right)}{\sqrt{(bx+a)^{2}}\,b\sqrt{ab}}$	65
risch	$\frac{2B\sqrt{x}\,\sqrt{(bx+a)^{2}}}{b(bx+a)}+\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{(bx+a)^{2}}}{b\sqrt{ab}\,(bx+a)}$	72

input
```
int((B*x+A)/x^(1/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*(A*arctan(b*x^(1/2)/(a*b)^(1/2))*b+B*x^(1/2)*(a*b)^(1/2)-B*arcta
n(b*x^(1/2)/(a*b)^(1/2))*a)/((b*x+a)^2)^(1/2)/b/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[\frac{2\,Bab\sqrt{x}+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab^2}, \frac{2\left(Bab\sqrt{x}+(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)\right)}{ab^2}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[(2*B*a*b*sqrt(x) + (B*a - A*b)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^2), 2*(B*a*b*sqrt(x) + (B*a - A*b)*sqrt(a*b)*arctan
(sqrt(a*b)/(b*sqrt(x))))/(a*b^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+Bx}{\sqrt{x}\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**(1/2)/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(x)*sqrt((a + b*x)**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(65) = 130.

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx &= -\frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} \\ &\quad+ \frac{((3\,Bab-Ab^2)x^2+3\,(Ba^2+Aab)x)\sqrt{x}+\frac{2\,(Aabx^2+3\,Aa^2x)}{\sqrt{x}}}{3\,(a^2bx+a^3)} \\ &\quad- \frac{(3\,Bab-Ab^2)x^{\frac{3}{2}}-6\,(Ba^2-Aab)\sqrt{x}}{3\,a^2b}\end{aligned}$$

input
```
integrate((B*x+A)/x^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 1/3*(((3*B*a*b
- A*b^2)*x^2 + 3*(B*a^2 + A*a*b)*x)*sqrt(x) + 2*(A*a*b*x^2 + 3*A*a^2*x)/sq
rt(x))/(a^2*b*x + a^3) - 1/3*((3*B*a*b - A*b^2)*x^(3/2) - 6*(B*a^2 - A*a*b
)*sqrt(x))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\,B\sqrt{x}\mathrm{sgn}(bx+a)}{b} - \frac{2\,(Ba\mathrm{sgn}(bx+a)-Ab\mathrm{sgn}(bx+a))\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x+A)/x^(1/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)*sgn(b*x + a)/b - 2*(B*a*sgn(b*x + a) - A*b*sgn(b*x + a))*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{\sqrt{x}\,\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.04

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}\,dx = 2\sqrt{x}$$

input
```
int((B*x+A)/x^(1/2)/((b*x+a)^2)^(1/2),x)
```

output
```
2*sqrt(x)
```

3.443 $\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 99

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2A(a+bx)}{a\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} - \frac{2(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2*A*(b*x+a)/a/x^(1/2)/((b*x+a)^2)^(1/2)-2*(A*b-B*a)*(b*x+a)*arctan(b^(1/2
)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2A(a+bx)}{a\sqrt{x}\sqrt{(a+bx)^2}} + \frac{2(-Ab+aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*A*(a + b*x))/(a*Sqrt[x]*Sqrt[(a + b*x)^2]) + (2*(-(A*b) + a*B)*(a + b*
x)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1187, 27, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx^{3/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{3/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2A}{a\sqrt{x}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(-\frac{2(Ab-aB)\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2A}{a\sqrt{x}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{2A}{a\sqrt{x}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((-2*A)/(a*Sqrt[x]) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(a^(3/2)*Sqrt[b])))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2(bx+a)\left(A\sqrt{x}\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b-B\sqrt{x}\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a+A\sqrt{ab}\right)}{\sqrt{(bx+a)^{2}}\,a\sqrt{x}\,\sqrt{ab}}$	71
risch	$-\frac{2A\sqrt{(bx+a)^{2}}}{a\sqrt{x}\,(bx+a)}-\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{(bx+a)^{2}}}{a\sqrt{ab}\,(bx+a)}$	72

input
```
int((B*x+A)/x^(3/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)*(A*x^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))*b-B*x^(1/2)*arctan(b*x
^(1/2)/(a*b)^(1/2))*a+A*(a*b)^(1/2))/((b*x+a)^2)^(1/2)/a/x^(1/2)/(a*b)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=\left[-\frac{2\,Aab\sqrt{x}-(Ba-Ab)\sqrt{-ab}x\log\left(\frac{bx-a+2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{a^2bx},\ -\frac{2\left(Aab\sqrt{x}+(Ba-Ab)\sqrt{ab}x\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)\right)}{a^2bx}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-(2*A*a*b*sqrt(x) - (B*a - A*b)*sqrt(-a*b)*x*log((b*x - a + 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)))/(a^2*b*x), -2*(A*a*b*sqrt(x) + (B*a - A*b)*sqrt(a*b)*
x*arctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=\int \frac{A+Bx}{x^{\frac{3}{2}}\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**(3/2)/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**(3/2)*sqrt((a + b*x)**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=\frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{((Bab^2+Ab^3)x^2-3\,(Ba^2b-3\,Aab^2)x)\sqrt{x}-\frac{2\,((Ba^2b+Aab^2)x^2+3\,(Ba^3-3\,Aa^2b)x)}{\sqrt{x}}-\frac{6\,(Aa^2bx^2-Aa^3x)}{x^{\frac{3}{2}}}}{3\,(a^3bx+a^4)}+\frac{(Bab+Ab^2)x^{\frac{3}{2}}-6\,(Ba^2-Aab)\sqrt{x}}{3\,a^3}$$

input
```
integrate((B*x+A)/x^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*(((B*a*b^2 +
 A*b^3)*x^2 - 3*(B*a^2*b - 3*A*a*b^2)*x)*sqrt(x) - 2*((B*a^2*b + A*a*b^2)*
x^2 + 3*(B*a^3 - 3*A*a^2*b)*x)/sqrt(x) - 6*(A*a^2*b*x^2 - A*a^3*x)/x^(3/2)
)/(a^3*b*x + a^4) + 1/3*((B*a*b + A*b^2)*x^(3/2) - 6*(B*a^2 - A*a*b)*sqrt(
x))/a^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\left(Ba\mathrm{sgn}(bx+a)-Ab\mathrm{sgn}(bx+a)\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2\,A\mathrm{sgn}(bx+a)}{a\sqrt{x}}$$

input
```
integrate((B*x+A)/x^(3/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*(B*a*sgn(b*x + a) - A*b*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(
a*b)*a) - 2*A*sgn(b*x + a)/(a*sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^{3/2}\,\sqrt{{\left(a+b\,x\right)}^2}}\,dx$$

input
```
int((A + B*x)/(x^(3/2)*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2}{\sqrt{x}}$$

input
```
int((B*x+A)/x^(3/2)/((b*x+a)^2)^(1/2),x)
```

output
```
( - 2)/sqrt(x)
```

3.444 $\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 144

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2A(a+bx)}{3ax^{3/2}\sqrt{a^2+2abx+b^2x^2}}+\frac{2(Ab-aB)(a+bx)}{a^2\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}+\frac{2\sqrt{b}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2/3*A*(b*x+a)/a/x^(3/2)/((b*x+a)^2)^(1/2)+2*(A*b-B*a)*(b*x+a)/a^2/x^(1/2)
/((b*x+a)^2)^(1/2)+2*b^(1/2)*(A*b-B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1
/2))/a^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2(a+bx)\left(\sqrt{a}(-3Abx+a(A+3Bx))-3\sqrt{b}(Ab-aB)x^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{3a^{5/2}x^{3/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*(a + b*x)*(Sqrt[a]*(-3*A*b*x + a*(A + 3*B*x)) - 3*Sqrt[b]*(A*b - a*B)*
x^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*a^(5/2)*x^(3/2)*Sqrt[(a + b
*x)^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1187, 27, 87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx^{5/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{5/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2A}{3ax^{3/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 218$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((-2*A)/(3*a*x^(3/2)) - ((A*b - a*B)*(-2/(a*Sqrt[x]) - (2*Sqrt[
b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/Sqrt[a^2 + 2*a*b*x + b
^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

method	result	size
risch	$-\frac{2(-3Abx+3Bax+Aa)\sqrt{(bx+a)^{2}}}{3a^{2}x^{\frac{3}{2}}(bx+a)}+\frac{2b(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{(bx+a)^{2}}}{a^{2}\sqrt{ab}\,(bx+a)}$	86
default	$\frac{2(bx+a)\left(3A\,x^{\frac{3}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^{2}-3B\,x^{\frac{3}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)ab+3Ax\sqrt{ab}\,b-3Bx\sqrt{ab}\,a-Aa\sqrt{ab}\right)}{3\sqrt{(bx+a)^{2}}\,a^{2}x^{\frac{3}{2}}\sqrt{ab}}$	97

input
```
int((B*x+A)/x^(5/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*A*b*x+3*B*a*x+A*a)/a^2/x^(3/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b*(A*b
-B*a)/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x
+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[-\frac{3\,(Ba-Ab)x^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(Aa+3\,(Ba-Ab)x)\sqrt{x}}{3\,a^2x^2}, -\frac{2\left(3\,(Ba-Ab)x^2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(Aa+3\,(Ba-Ab)x)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

input
```
integrate((B*x+A)/x^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*a - A*b)*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)
/(b*x + a)) + 2*(A*a + 3*(B*a - A*b)*x)*sqrt(x))/(a^2*x^2), -2/3*(3*(B*a -
 A*b)*x^2*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (A*a + 3*(B*a - A*b)*x)*sq
rt(x))/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+Bx}{x^{\frac{5}{2}}\sqrt{(a+bx)^2}}\,dx$$

input
```
integrate((B*x+A)/x**(5/2)/((b*x+a)**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**(5/2)*sqrt((a + b*x)**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(95) = 190$.

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{((Bab^3-3\,Ab^4)x^2+3\,(3\,Ba^2b^2-5\,Aab^3)x)\sqrt{x}-\frac{2\,((Ba^2b^2-3\,Aab^3)x^2-3\,(3\,Ba^3b-5\,Aa^2b^2)x)}{\sqrt{x}}-\frac{2\,(3\,(Ba^3b-3\,Aa^2b^2)}{x}}{3\,(a^4bx+a^5)} - \frac{2\,(Bab-Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{(Bab^2-3\,Ab^3)x^{\frac{3}{2}}+6\,(Ba^2b-Aab^2)\sqrt{x}}{3\,a^4}$$

input
```
integrate((B*x+A)/x^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(((B*a*b^3 - 3*A*b^4)*x^2 + 3*(3*B*a^2*b^2 - 5*A*a*b^3)*x)*sqrt(x) -
2*((B*a^2*b^2 - 3*A*a*b^3)*x^2 - 3*(3*B*a^3*b - 5*A*a^2*b^2)*x)/sqrt(x) -
2*(3*(B*a^3*b - 3*A*a^2*b^2)*x^2 - (3*B*a^4 - 5*A*a^3*b)*x)/x^(3/2) + 2*(3
*A*a^3*b*x^2 + A*a^4*x)/x^(5/2))/(a^4*b*x + a^5) - 2*(B*a*b - A*b^2)*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*((B*a*b^2 - 3*A*b^3)*x^(3/2)
+ 6*(B*a^2*b - A*a*b^2)*sqrt(x))/a^4
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2\,(Bab\mathrm{sgn}(bx+a)-Ab^2\mathrm{sgn}(bx+a))\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{2\,(3\,Bax\mathrm{sgn}(bx+a)-3\,Abx\mathrm{sgn}(bx+a)+Aa\mathrm{sgn}(bx+a))}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*a*b*sgn(b*x + a) - A*b^2*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))/(
sqrt(a*b)*a^2) - 2/3*(3*B*a*x*sgn(b*x + a) - 3*A*b*x*sgn(b*x + a) + A*a*sg
n(b*x + a))/(a^2*x^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^{5/2}\,\sqrt{(a+b\,x)^2}}\,dx$$

input
```
int((A + B*x)/(x^(5/2)*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(5/2)*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2}{3\sqrt{x}\,x}$$

input
```
int((B*x+A)/x^(5/2)/((b*x+a)^2)^(1/2),x)
```

output
```
( - 2)/(3*sqrt(x)*x)
```

3.445 $\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2A(a+bx)}{5ax^{5/2}\sqrt{a^2+2abx+b^2x^2}} + \frac{2(Ab-aB)(a+bx)}{3a^2x^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{2b(Ab-aB)(a+bx)}{a^3\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} - \frac{2b^{3/2}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2/5*A*(b*x+a)/a/x^(5/2)/((b*x+a)^2)^(1/2)+2/3*(A*b-B*a)*(b*x+a)/a^2/x^(3/
2)/((b*x+a)^2)^(1/2)-2*b*(A*b-B*a)*(b*x+a)/a^3/x^(1/2)/((b*x+a)^2)^(1/2)-2
*b^(3/2)*(A*b-B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2(a+bx)\left(\sqrt{a}(15Ab^2x^2-5abx(A+3Bx)+a^2(3A+5Bx))+15b^{3/2}(Ab-aB)x^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{15a^{7/2}x^{5/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*(a + b*x)*(Sqrt[a]*(15*A*b^2*x^2 - 5*a*b*x*(A + 3*B*x) + a^2*(3*A + 5*
B*x)) + 15*b^(3/2)*(A*b - a*B)*x^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))
/(15*a^(7/2)*x^(5/2)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1187, 27, 87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx^{7/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{7/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2A}{5ax^{5/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((-2*A)/(5*a*x^(5/2)) - ((A*b - a*B)*(-2/(3*a*x^(3/2)) - (b*(-2
/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))
/a))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2\left(15x^{2}b^{2}A-15Ba\,x^{2}b-5abAx+5a^{2}Bx+3a^{2}A\right)\sqrt{\left(bx+a\right)^{2}}}{15a^{3}x^{\frac{5}{2}}\left(bx+a\right)}-\frac{2b^{2}\left(Ab-Ba\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{\left(bx+a\right)^{2}}}{a^{3}\sqrt{ab}\,\left(bx+a\right)}$
default	$-\frac{2\left(bx+a\right)\left(15A\,x^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^{3}-15B\,x^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{2}+15A\,x^{2}\sqrt{ab}\,b^{2}-15B\,x^{2}\sqrt{ab}\,ab-5Ax\sqrt{ab}\,ab+5Bx\sqrt{ab}\,a^{2}+3A\,a^{2}\sqrt{\ }}{15\sqrt{\left(bx+a\right)^{2}}\,a^{3}x^{\frac{5}{2}}\sqrt{ab}}$

input
```
int((B*x+A)/x^(7/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*A*b^2*x^2-15*B*a*b*x^2-5*A*a*b*x+5*B*a^2*x+3*A*a^2)/a^3/x^(5/2)*
((b*x+a)^2)^(1/2)/(b*x+a)-2*b^2*(A*b-B*a)/a^3/(a*b)^(1/2)*arctan(b*x^(1/2)
/(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int\frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=\left[-\frac{15\,(Bab-Ab^2)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,Aa^2-15\,(Bab-}{15\,a^3x^3}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*a*b - A*b^2)*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a
) - a)/(b*x + a)) + 2*(3*A*a^2 - 15*(B*a*b - A*b^2)*x^2 + 5*(B*a^2 - A*a*b
)*x)*sqrt(x))/(a^3*x^3), 2/15*(15*(B*a*b - A*b^2)*x^3*sqrt(b/a)*arctan(sqr
t(x)*sqrt(b/a)) - (3*A*a^2 - 15*(B*a*b - A*b^2)*x^2 + 5*(B*a^2 - A*a*b)*x)
*sqrt(x))/(a^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(7/2)/((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(126) = 252.

Time = 0.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{5\left((3\,Bab^4-5\,Ab^5)x^2+3\,(5\,Ba^2b^3-7\,Aab^4)x\right)\sqrt{x}-\frac{10\left((3\,Ba^2b^3-5\,Aab^4)x^2-3\right.}{\sqrt{x}}}{}$$
$$+\frac{2\,(Bab^2-Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}-\frac{(3\,Bab^3-5\,Ab^4)x^{\frac{3}{2}}+6\,(Ba^2b^2-Aab^3)\sqrt{x}}{3\,a^5}$$

input
```
integrate((B*x+A)/x^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(5*((3*B*a*b^4 - 5*A*b^5)*x^2 + 3*(5*B*a^2*b^3 - 7*A*a*b^4)*x)*sqrt(x
) - 10*((3*B*a^2*b^3 - 5*A*a*b^4)*x^2 - 3*(5*B*a^3*b^2 - 7*A*a^2*b^3)*x)/s
qrt(x) - 10*(3*(3*B*a^3*b^2 - 5*A*a^2*b^3)*x^2 - (5*B*a^4*b - 7*A*a^3*b^2)
*x)/x^(3/2) - 2*(5*(3*B*a^4*b - 5*A*a^3*b^2)*x^2 + (5*B*a^5 - 7*A*a^4*b)*x
)/x^(5/2) - 2*(5*A*a^4*b*x^2 + 3*A*a^5*x)/x^(7/2))/(a^5*b*x + a^6) + 2*(B*
a*b^2 - A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/3*((3*B*a*b
^3 - 5*A*b^4)*x^(3/2) + 6*(B*a^2*b^2 - A*a*b^3)*sqrt(x))/a^5
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \frac{2\left(Bab^2\text{sgn}(bx+a)-Ab^3\text{sgn}(bx+a)\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{2\left(15\,Babx^2\text{sgn}(bx+a)-15\,Ab^2x^2\text{sgn}(bx+a)-5\,Ba^2x\text{sgn}(bx+a)+5\,Aabx\text{sgn}(bx+a)-3\,Aa^2\text{sgn}(b\right.}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*(B*a*b^2*sgn(b*x + a) - A*b^3*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))/
(sqrt(a*b)*a^3) + 2/15*(15*B*a*b*x^2*sgn(b*x + a) - 15*A*b^2*x^2*sgn(b*x +
 a) - 5*B*a^2*x*sgn(b*x + a) + 5*A*a*b*x*sgn(b*x + a) - 3*A*a^2*sgn(b*x +
a))/(a^3*x^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^{7/2}\,\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((A + B*x)/(x^(7/2)*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(7/2)*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.05

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2}{5\sqrt{x}\,x^2}$$

input
```
int((B*x+A)/x^(7/2)/((b*x+a)^2)^(1/2),x)
```

output
```
( - 2)/(5*sqrt(x)*x**2)
```

3.446 $\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2A(a+bx)}{7ax^{7/2}\sqrt{a^2+2abx+b^2x^2}}+\frac{2(Ab-aB)(a+bx)}{5a^2x^{5/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{2b(Ab-aB)(a+bx)}{3a^3x^{3/2}\sqrt{a^2+2abx+b^2x^2}}+\frac{2b^2(Ab-aB)(a+bx)}{a^4\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}+\frac{2b^{5/2}(Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-2/7*A*(b*x+a)/a/x^(7/2)/((b*x+a)^2)^(1/2)+2/5*(A*b-B*a)*(b*x+a)/a^2/x^(5/
2)/((b*x+a)^2)^(1/2)-2/3*b*(A*b-B*a)*(b*x+a)/a^3/x^(3/2)/((b*x+a)^2)^(1/2)
+2*b^2*(A*b-B*a)*(b*x+a)/a^4/x^(1/2)/((b*x+a)^2)^(1/2)+2*b^(5/2)*(A*b-B*a)
*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx =$$
$$-\frac{2(a+bx)\left(\sqrt{a}(-105Ab^3x^3+35ab^2x^2(A+3Bx)-7a^2bx(3A+5Bx)+3a^3(5A+7Bx))-105b^{5/2}(Ab-\right.}{105a^{9/2}x^{7/2}\sqrt{(a+bx)^2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
(-2*(a + b*x)*(Sqrt[a]*(-105*A*b^3*x^3 + 35*a*b^2*x^2*(A + 3*B*x) - 7*a^2*
b*x*(3*A + 5*B*x) + 3*a^3*(5*A + 7*B*x)) - 105*b^(5/2)*(A*b - a*B)*x^(7/2)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(105*a^(9/2)*x^(7/2)*Sqrt[(a + b*x)^2
])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b(a+bx)\int \frac{A+Bx}{bx^{9/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{9/2}(a+bx)}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\int \frac{1}{x^{7/2}(a+bx)}dx}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx)\left(-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(9/2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]),x]
```

output
```
((a + b*x)*((-2*A)/(7*a*x^(7/2)) - ((A*b - a*B)*(-2/(5*a*x^(5/2)) - (b*(-2
/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/a^(3/2)))/a))/a))/a))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{2(-105A\,b^3x^3+105Ba\,b^2x^3+35Aa\,b^2x^2-35B\,a^2b\,x^2-21A\,a^2bx+21B\,a^3x+15a^3A)\sqrt{(bx+a)^2}}{105a^4x^{\frac{7}{2}}(bx+a)}+\frac{2b^3(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)\sqrt{}}{a^4\sqrt{ab}\,(bx+a)}$
default	$\frac{2(bx+a)\left(105A\,x^{\frac{7}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^4-105B\,x^{\frac{7}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^3+105A\,x^3\sqrt{ab}\,b^3-105B\,x^3\sqrt{ab}\,a\,b^2-35A\,x^2\sqrt{ab}\,a\,b^2+35B\,x^2\sqrt{ab}\right.}{105\sqrt{(bx+a)^2}\,a^4x^{\frac{7}{2}}\sqrt{ab}}$

input
```
int((B*x+A)/x^(9/2)/((b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-105*A*b^3*x^3+105*B*a*b^2*x^3+35*A*a*b^2*x^2-35*B*a^2*b*x^2-21*A*
a^2*b*x+21*B*a^3*x+15*A*a^3)/a^4/x^(7/2)*((b*x+a)^2)^(1/2)/(b*x+a)+2*b^3*(
A*b-B*a)/a^4/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))*((b*x+a)^2)^(1/2)/(
b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \left[-\frac{105\,(Bab^2-Ab^3)x^4\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,Aa^3+105\,(Ba}{105\,a^4x^4}\right.$$

$$-\frac{2\left(105\,(Bab^2-Ab^3)x^4\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(15\,Aa^3+105\,(Bab^2-Ab^3)x^3-35\,(Ba^2b-Aab^2)x^2+2\right.}{105\,a^4x^4}$$

input
```
integrate((B*x+A)/x^(9/2)/((b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*a*b^2 - A*b^3)*x^4*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(
-b/a) - a)/(b*x + a)) + 2*(15*A*a^3 + 105*(B*a*b^2 - A*b^3)*x^3 - 35*(B*a^
2*b - A*a*b^2)*x^2 + 21*(B*a^3 - A*a^2*b)*x)*sqrt(x))/(a^4*x^4), -2/105*(1
05*(B*a*b^2 - A*b^3)*x^4*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (15*A*a^3 +
 105*(B*a*b^2 - A*b^3)*x^3 - 35*(B*a^2*b - A*a*b^2)*x^2 + 21*(B*a^3 - A*a^
2*b)*x)*sqrt(x))/(a^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(9/2)/((b*x+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(159) = 318$.

Time = 0.17 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{35\left(\left(5\,Bab^5-7\,Ab^6\right)x^2+3\left(7\,Ba^2b^4-9\,Aab^5\right)x\right)\sqrt{x}-\frac{70\left(\left(5\,Ba^2b^4-7\,Aab^5\right)x^2-3\left(7\,Ba^3b^3-9\,Aa^2b^4\right)x\right)}{\sqrt{x}}-\frac{70\left(3\left(5\,Ba^3\right.\right.}{}}{} - \frac{2\left(Bab^3-Ab^4\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{\left(5\,Bab^4-7\,Ab^5\right)x^{\frac{3}{2}}+6\left(Ba^2b^3-Aab^4\right)\sqrt{x}}{3\,a^6}$$

input
```
integrate((B*x+A)/x^(9/2)/((b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/105*(35*((5*B*a*b^5 - 7*A*b^6)*x^2 + 3*(7*B*a^2*b^4 - 9*A*a*b^5)*x)*sqr
t(x) - 70*((5*B*a^2*b^4 - 7*A*a*b^5)*x^2 - 3*(7*B*a^3*b^3 - 9*A*a^2*b^4)*x
)/sqrt(x) - 70*(3*(5*B*a^3*b^3 - 7*A*a^2*b^4)*x^2 - (7*B*a^4*b^2 - 9*A*a^3
*b^3)*x)/x^(3/2) - 14*(5*(5*B*a^4*b^2 - 7*A*a^3*b^3)*x^2 + (7*B*a^5*b - 9*
A*a^4*b^2)*x)/x^(5/2) + 2*(7*(5*B*a^5*b - 7*A*a^4*b^2)*x^2 + 3*(7*B*a^6 -
9*A*a^5*b)*x)/x^(7/2) + 6*(7*A*a^5*b*x^2 + 5*A*a^6*x)/x^(9/2))/(a^6*b*x +
a^7) - 2*(B*a*b^3 - A*b^4)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 1
/3*((5*B*a*b^4 - 7*A*b^5)*x^(3/2) + 6*(B*a^2*b^3 - A*a*b^4)*sqrt(x))/a^6
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = -\frac{2\left(Bab^3\mathrm{sgn}(bx+a)-Ab^4\mathrm{sgn}(bx+a)\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{2\left(105\,Bab^2x^3\mathrm{sgn}(bx+a)-105\,Ab^3x^3\mathrm{sgn}(bx+a)-35\,Ba^2bx^2\mathrm{sgn}(bx+a)+35\,Aab^2x^2\mathrm{sgn}(bx+a)+2\right.}{105\,a^4x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/((b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*a*b^3*sgn(b*x + a) - A*b^4*sgn(b*x + a))*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*a^4) - 2/105*(105*B*a*b^2*x^3*sgn(b*x + a) - 105*A*b^3*x^3*sgn
(b*x + a) - 35*B*a^2*b*x^2*sgn(b*x + a) + 35*A*a*b^2*x^2*sgn(b*x + a) + 21
*B*a^3*x*sgn(b*x + a) - 21*A*a^2*b*x*sgn(b*x + a) + 15*A*a^3*sgn(b*x + a))
/(a^4*x^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx = \int \frac{A+B\,x}{x^{9/2}\,\sqrt{{(a+b\,x)}^2}}\,dx$$

input
```
int((A + B*x)/(x^(9/2)*((a + b*x)^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(9/2)*((a + b*x)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a^2+2abx+b^2x^2}}\,dx=-\frac{2}{7\sqrt{x}\,x^3}$$

input
```
int((B*x+A)/x^(9/2)/((b*x+a)^2)^(1/2),x)
```

output
```
( - 2)/(7*sqrt(x)*x**3)
```

3.447 $\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 285

$$\begin{aligned}\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= -\frac{(7Ab-11aB)x^{5/2}}{4b^3\sqrt{a^2+2abx+b^2x^2}}\\ &-\frac{(Ab-aB)x^{7/2}}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{7a(5Ab-9aB)\sqrt{x}(a+bx)}{4b^5\sqrt{a^2+2abx+b^2x^2}}\\ &+\frac{7(5Ab-9aB)x^{3/2}(a+bx)}{12b^4\sqrt{a^2+2abx+b^2x^2}}+\frac{2Bx^{5/2}(a+bx)}{5b^3\sqrt{a^2+2abx+b^2x^2}}\\ &+\frac{7a^{3/2}(5Ab-9aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{11/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/4*(7*A*b-11*B*a)*x^(5/2)/b^3/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)*x^(7/2)/b^
2/(b*x+a)/((b*x+a)^2)^(1/2)-7/4*a*(5*A*b-9*B*a)*x^(1/2)*(b*x+a)/b^5/((b*x+
a)^2)^(1/2)+7/12*(5*A*b-9*B*a)*x^(3/2)*(b*x+a)/b^4/((b*x+a)^2)^(1/2)+2/5*B
*x^(5/2)*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+7/4*a^(3/2)*(5*A*b-9*B*a)*(b*x+a)*a
rctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.52

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\sqrt{b}\sqrt{x}(945a^4B-525a^3b(A-3Bx)+8b^4x^3(5A+3Bx)-8ab^3x^2(35A+9}{60b^{11/2}(a-}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*(945*a^4*B - 525*a^3*b*(A - 3*B*x) + 8*b^4*x^3*(5*A + 3*B
*x) - 8*a*b^3*x^2*(35*A + 9*B*x) + 7*a^2*b^2*x*(-125*A + 72*B*x)) - 105*a^
(3/2)*(-5*A*b + 9*a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(60*
b^(11/2)*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {1187, 27, 87, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{x^{7/2}(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\int \frac{x^{7/2}}{(a+bx)^2}dx}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\int \frac{x^{5/2}}{a+bx}dx}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input

```
Int[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x)^2) - ((5*A*b - 9*a*B)*(
-(x^(7/2)/(b*(a + b*x))) + (7*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) -
 (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
))/b))/b))/(2*b)))/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2\left(-3x^{2}B\,b^{2}-5x\,b^{2}A+15xabB+45abA-90a^{2}B\right)\sqrt{x}\,\sqrt{\left(bx+a\right)^{2}}}{15b^{5}\left(bx+a\right)}+\frac{a^{2}\left(\frac{2\left(-\frac{13}{8}b^{2}A+\frac{17}{8}abB\right)x^{\frac{3}{2}}-\frac{a\left(11Ab-15Ba\right)\sqrt{x}}{4}}{\left(bx+a\right)^{2}}+\frac{7\left(5Ab-9Ba\right)}{4\sqrt{}}\right.}{b^{5}\left(bx+a\right)}$
default	$\frac{\left(24B\sqrt{ab}\,x^{\frac{9}{2}}b^{4}+40A\sqrt{ab}\,x^{\frac{7}{2}}b^{4}-72B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^{3}-280A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^{3}+504B\sqrt{ab}\,x^{\frac{5}{2}}a^{2}b^{2}-875A\sqrt{ab}\,x^{\frac{3}{2}}a^{2}b^{2}+525A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^{2}\right.}{}$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+15*B*a*b*x+45*A*a*b-90*B*a^2)*x^(1/2)/b^5*((
b*x+a)^2)^(1/2)/(b*x+a)+a^2/b^5*(2*((-13/8*b^2*A+17/8*a*b*B)*x^(3/2)-1/8*a
*(11*A*b-15*B*a)*x^(1/2))/(b*x+a)^2+7/4*(5*A*b-9*B*a)/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.43

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \left[-\frac{105\,(9\,Ba^4-5\,Aa^3b+(9\,Ba^2b^2-5\,Aab^3)x^2+2\,(9\,Ba^3b-5\,Aa^2b^2)x)\sqrt{}}{}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/120*(105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 2*(9*B
*a^3*b - 5*A*a^2*b^2)*x)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)
/(b*x + a)) - 2*(24*B*b^4*x^4 + 945*B*a^4 - 525*A*a^3*b - 8*(9*B*a*b^3 - 5
*A*b^4)*x^3 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 175*(9*B*a^3*b - 5*A*a^2*
b^2)*x)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/60*(105*(9*B*a^4 - 5*
A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x)*s
qrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - (24*B*b^4*x^4 + 945*B*a^4 - 525*A
*a^3*b - 8*(9*B*a*b^3 - 5*A*b^4)*x^3 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^2 +
175*(9*B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.96

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{16\,(3\,Bb^4x^2+5\,Bab^3x)x^{\frac{7}{2}}+(89\,(11\,Bab^3-5\,Ab^4)x^2+285\,(3\,Ba^2b^2-Aab}{}$$
$$-\frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^5}-\frac{7\left((11\,Bab-5\,Ab^2)x^{\frac{3}{2}}-2\,(9\,Ba^2-5\,Aab)\sqrt{x}\right)}{8\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/120*(16*(3*B*b^4*x^2 + 5*B*a*b^3*x)*x^(7/2) + (89*(11*B*a*b^3 - 5*A*b^4)
*x^2 + 285*(3*B*a^2*b^2 - A*a*b^3)*x)*x^(5/2) + 12*(12*(11*B*a^2*b^2 - 5*A
*a*b^3)*x^2 + 35*(3*B*a^3*b - A*a^2*b^2)*x)*x^(3/2) + 7*(9*(11*B*a^3*b - 5
*A*a^2*b^2)*x^2 + 25*(3*B*a^4 - A*a^3*b)*x)*sqrt(x))/(b^7*x^3 + 3*a*b^6*x^
2 + 3*a^2*b^5*x + a^3*b^4) - 7/4*(9*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sq
rt(a*b))/(sqrt(a*b)*b^5) - 7/8*((11*B*a*b - 5*A*b^2)*x^(3/2) - 2*(9*B*a^2
- 5*A*a*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.60

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^5\mathrm{sgn}\,(bx+a)}$$
$$+\frac{17\,Ba^3bx^{\frac{3}{2}}-13\,Aa^2b^2x^{\frac{3}{2}}+15\,Ba^4\sqrt{x}-11\,Aa^3b\sqrt{x}}{4\,(bx+a)^2b^5\mathrm{sgn}\,(bx+a)}$$
$$+\frac{2\left(3\,Bb^{12}x^{\frac{5}{2}}-15\,Bab^{11}x^{\frac{3}{2}}+5\,Ab^{12}x^{\frac{3}{2}}+90\,Ba^2b^{10}\sqrt{x}-45\,Aab^{11}\sqrt{x}\right)}{15\,b^{15}\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-7/4*(9*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5*sgn(
b*x + a)) + 1/4*(17*B*a^3*b*x^(3/2) - 13*A*a^2*b^2*x^(3/2) + 15*B*a^4*sqrt
(x) - 11*A*a^3*b*sqrt(x))/((b*x + a)^2*b^5*sgn(b*x + a)) + 2/15*(3*B*b^12*
x^(5/2) - 15*B*a*b^11*x^(3/2) + 5*A*b^12*x^(3/2) + 90*B*a^2*b^10*sqrt(x) -
 45*A*a*b^11*sqrt(x))/(b^15*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\int \frac{x^{7/2}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.35

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+105\sqrt{x}\,a^3b+}{15b^5\,(bx+a)}$$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqr
t(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 105*sqrt(x)*a*
*3*b + 70*sqrt(x)*a**2*b**2*x - 14*sqrt(x)*a*b**3*x**2 + 6*sqrt(x)*b**4*x*
*3)/(15*b**5*(a + b*x))
```

3.448 $\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{(5Ab-9aB)x^{3/2}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(Ab-aB)x^{5/2}}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{5(3Ab-7aB)\sqrt{x}(a+bx)}{4b^4\sqrt{a^2+2abx+b^2x^2}} + \frac{2Bx^{3/2}(a+bx)}{3b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{5\sqrt{a}(3Ab-7aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/4*(5*A*b-9*B*a)*x^(3/2)/b^3/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)*x^(5/2)/b^2
/(b*x+a)/((b*x+a)^2)^(1/2)+5/4*(3*A*b-7*B*a)*x^(1/2)*(b*x+a)/b^4/((b*x+a)^
2)^(1/2)+2/3*B*x^(3/2)*(b*x+a)/b^3/((b*x+a)^2)^(1/2)-5/4*a^(1/2)*(3*A*b-7*
B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.53

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{(a+bx)\left(\sqrt{b}\sqrt{x}(-105a^3B+ab^2x(75A-56Bx)+5a^2b(9A-35Bx)+8b^3\right.}{12b^{9/2}\left((a+bx)\right.}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(Sqrt[b]*Sqrt[x]*(-105*a^3*B + a*b^2*x*(75*A - 56*B*x) + 5*a^2*
b*(9*A - 35*B*x) + 8*b^3*x^2*(3*A + B*x)) + 15*Sqrt[a]*(-3*A*b + 7*a*B)*(a
 + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(12*b^(9/2)*((a + b*x)^2)^(3
/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{x^{5/2}(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\int\frac{x^{5/2}}{(a+bx)^2}dx}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{51}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\int\frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*x)^2) - ((3*A*b - 7*a*B)*(
-(x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*
Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/(2*b)))/(4*a*b)))
/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(Bbx+3Ab-9Ba)\sqrt{x}\,\sqrt{(bx+a)^{2}}}{3b^{4}(bx+a)}-\frac{a\left(\frac{2\left(-\frac{9}{8}b^{2}A+\frac{13}{8}abB\right)x^{\frac{3}{2}}-\frac{a(7Ab-11Ba)\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{5(3Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)\sqrt{(bx+a)^{2}}}{b^{4}(bx+a)}$
default	$\frac{\left(8B\sqrt{ab}\,x^{\frac{7}{2}}b^{3}-56B\sqrt{ab}\,x^{\frac{5}{2}}a\,b^{2}+75A\sqrt{ab}\,x^{\frac{3}{2}}a\,b^{2}+24A\sqrt{ab}\,x^{\frac{5}{2}}b^{3}-45A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{3}x^{2}-175B\sqrt{ab}\,x^{\frac{3}{2}}a^{2}b+105B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right.\right.}{}$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-9*B*a)*x^(1/2)/b^4*((b*x+a)^2)^(1/2)/(b*x+a)-a/b^4*(2*((-
9/8*b^2*A+13/8*a*b*B)*x^(3/2)-1/8*a*(7*A*b-11*B*a)*x^(1/2))/(b*x+a)^2+5/4*
(3*A*b-7*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)
/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\left[-\frac{15\left(7\,Ba^3-3\,Aa^2b+\left(7\,Bab^2-3\,Ab^3\right)x^2+2\left(7\,Ba^2b-3\,Aab^2\right)x\right)\sqrt{-\frac{a}{b}}}{}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/24*(15*(7*B*a^3 - 3*A*a^2*b + (7*B*a*b^2 - 3*A*b^3)*x^2 + 2*(7*B*a^2*b
 - 3*A*a*b^2)*x)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) - 2*(8*B*b^3*x^3 - 105*B*a^3 + 45*A*a^2*b - 8*(7*B*a*b^2 - 3*A*b^3)*x^
2 - 25*(7*B*a^2*b - 3*A*a*b^2)*x)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
, 1/12*(15*(7*B*a^3 - 3*A*a^2*b + (7*B*a*b^2 - 3*A*b^3)*x^2 + 2*(7*B*a^2*b
 - 3*A*a*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*B*b^3*x^3 -
105*B*a^3 + 45*A*a^2*b - 8*(7*B*a*b^2 - 3*A*b^3)*x^2 - 25*(7*B*a^2*b - 3*A
*a*b^2)*x)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx =$$
$$-\frac{((89\,Bab^3-35\,Ab^4)x^2+3\,(19\,Ba^2b^2-5\,Aab^3)x)x^{\frac{5}{2}}+12\,(4\,(3\,Ba^2b^2-Aab^3)x^2+(7\,Ba^3b-Aa^2b^2)x)x}{24\,(ab^6x^3+3\,a^2b^5x^2+3\,a^3b^4x+a^4b^3)}$$
$$+\frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4}+\frac{5\left(7\,(3\,Bab-Ab^2)x^{\frac{3}{2}}-6\,(7\,Ba^2-3\,Aab)\sqrt{x}\right)}{24\,ab^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/24*(((89*B*a*b^3 - 35*A*b^4)*x^2 + 3*(19*B*a^2*b^2 - 5*A*a*b^3)*x)*x^(5
/2) + 12*(4*(3*B*a^2*b^2 - A*a*b^3)*x^2 + (7*B*a^3*b - A*a^2*b^2)*x)*x^(3/
2) + (21*(3*B*a^3*b - A*a^2*b^2)*x^2 + 5*(7*B*a^4 - A*a^3*b)*x)*sqrt(x))/(
a*b^6*x^3 + 3*a^2*b^5*x^2 + 3*a^3*b^4*x + a^4*b^3) + 5/4*(7*B*a^2 - 3*A*a*
b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 5/24*(7*(3*B*a*b - A*b^2)
*x^(3/2) - 6*(7*B*a^2 - 3*A*a*b)*sqrt(x))/(a*b^4)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4\mathrm{sgn}\,(bx+a)} - \frac{13\,Ba^2bx^{\frac{3}{2}}-9\,Aab^2x^{\frac{3}{2}}+11\,Ba^3\sqrt{x}-7\,Aa^2b\sqrt{x}}{4\,(bx+a)^2b^4\mathrm{sgn}\,(bx+a)} + \frac{2\left(Bb^6x^{\frac{3}{2}}-9\,Bab^5\sqrt{x}+3\,Ab^6\sqrt{x}\right)}{3\,b^9\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
5/4*(7*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4*sgn(b*x
 + a)) - 1/4*(13*B*a^2*b*x^(3/2) - 9*A*a*b^2*x^(3/2) + 11*B*a^3*sqrt(x) -
7*A*a^2*b*sqrt(x))/((b*x + a)^2*b^4*sgn(b*x + a)) + 2/3*(B*b^6*x^(3/2) - 9
*B*a*b^5*sqrt(x) + 3*A*b^6*sqrt(x))/(b^9*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2)}^{3/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.35

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx-15\sqrt{x}\,a^2b-10\sqrt{x}\,a}{3b^4\,(bx+a)}$$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(x)*a**2*b - 10
*sqrt(x)*a*b**2*x + 2*sqrt(x)*b**3*x**2)/(3*b**4*(a + b*x))
```

3.449 $\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 189

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = -\frac{(3Ab-7aB)\sqrt{x}}{4b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(Ab-aB)x^{3/2}}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{2B\sqrt{x}(a+bx)}{b^3\sqrt{a^2+2abx+b^2x^2}} + \frac{3(Ab-5aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
-1/4*(3*A*b-7*B*a)*x^(1/2)/b^3/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)*x^(3/2)/b^2
/(b*x+a)/((b*x+a)^2)^(1/2)+2*B*x^(1/2)*(b*x+a)/b^3/((b*x+a)^2)^(1/2)+3/4*(
A*b-5*B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(7/2)/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{(a+bx)\left(\sqrt{b}\sqrt{x}(15a^2B+b^2x(-5A+8Bx)+a(-3Ab+25bBx))+\frac{3(Ab-5\cdots}{\cdots}\right.}{4b^{7/2}\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(Sqrt[b]*Sqrt[x]*(15*a^2*B + b^2*x*(-5*A + 8*B*x) + a*(-3*A*b +
 25*b*B*x)) + (3*(A*b - 5*a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
]])/Sqrt[a]))/(4*b^(7/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1187, 27, 87, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b^3(a+bx)\int \frac{x^{3/2}(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 87$$

$$\frac{(a+bx)\left(\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\int\frac{x^{3/2}}{(a+bx)^2}dx}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(a+bx)\left(\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\int\frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a+bx)\left(\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(a+bx)\left(\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx)\left(\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x)^2) - ((A*b - 5*a*B)*(-(
x^(3/2)/(b*(a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

method	result
risch	$\frac{2B\sqrt{x}\,\sqrt{(bx+a)^2}}{b^3(bx+a)}+\frac{\left(\frac{2\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{3}{2}}-\frac{a(3Ab-7Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{3(Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)\sqrt{(bx+a)^2}}{b^3(bx+a)}$
default	$-\frac{\left(5A\,x^{\frac{3}{2}}\sqrt{ab}\,b^2-3A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^3x^2-25B\,x^{\frac{3}{2}}\sqrt{ab}\,ab-8B\,x^{\frac{5}{2}}\sqrt{ab}\,b^2+15B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^2x^2-6A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^2x+30B\right.}{4\sqrt{ab}\,b^3\left((bx+a)^2\right)^{\frac{3}{2}}}$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*B/b^3*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/b^3*(2*((-5/8*b^2*A+9/8*a*b*B)
*x^(3/2)-1/8*a*(3*A*b-7*B*a)*x^(1/2))/(b*x+a)^2+3/4*(A*b-5*B*a)/(a*b)^(1/2
)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.69

$$\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \left[\frac{3\left(5\,Ba^3-Aa^2b+(5\,Bab^2-Ab^3)x^2+2\,(5\,Ba^2b-Aab^2)x\right)\sqrt{-ab}\log\left(\frac{bx-}{}\right.}{8\left(ab^6x^2+2\right.}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[1/8*(3*(5*B*a^3 - A*a^2*b + (5*B*a*b^2 - A*b^3)*x^2 + 2*(5*B*a^2*b - A*a*
b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(8*
B*a*b^3*x^2 + 15*B*a^3*b - 3*A*a^2*b^2 + 5*(5*B*a^2*b^2 - A*a*b^3)*x)*sqrt
(x))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/4*(3*(5*B*a^3 - A*a^2*b + (5*B
*a*b^2 - A*b^3)*x^2 + 2*(5*B*a^2*b - A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (8*B*a*b^3*x^2 + 15*B*a^3*b - 3*A*a^2*b^2 + 5*(5*B*a^2*b^
2 - A*a*b^3)*x)*sqrt(x))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{(5\,(7\,Bab^3-Ab^4)x^2+3\,(5\,Ba^2b^2+Aab^3)x)x^{\frac{5}{2}}+12\,(4\,Ba^2b^2x^2+(Ba^3b+}{24\,(a^2b^5x^3+3\,a^3b^4x^2+3\,a}$$
$$-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}-\frac{5\,(7\,Bab-Ab^2)x^{\frac{3}{2}}-18\,(5\,Ba^2-Aab)\sqrt{x}}{24\,a^2b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/24*((5*(7*B*a*b^3 - A*b^4)*x^2 + 3*(5*B*a^2*b^2 + A*a*b^3)*x)*x^(5/2) +
12*(4*B*a^2*b^2*x^2 + (B*a^3*b + A*a^2*b^2)*x)*x^(3/2) + (3*(7*B*a^3*b - A
*a^2*b^2)*x^2 + (5*B*a^4 + A*a^3*b)*x)*sqrt(x))/(a^2*b^5*x^3 + 3*a^3*b^4*x
^2 + 3*a^4*b^3*x + a^5*b^2) - 3/4*(5*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b)
)/(sqrt(a*b)*b^3) - 1/24*(5*(7*B*a*b - A*b^2)*x^(3/2) - 18*(5*B*a^2 - A*a*
b)*sqrt(x))/(a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\,B\sqrt{x}}{b^3\mathrm{sgn}\,(bx+a)}-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3\mathrm{sgn}\,(bx+a)}$$
$$+\frac{9\,Babx^{\frac{3}{2}}-5\,Ab^2x^{\frac{3}{2}}+7\,Ba^2\sqrt{x}-3\,Aab\sqrt{x}}{4\,(bx+a)^2b^3\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/(b^3*sgn(b*x + a)) - 3/4*(5*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a
*b))/(sqrt(a*b)*b^3*sgn(b*x + a)) + 1/4*(9*B*a*b*x^(3/2) - 5*A*b^2*x^(3/2)
 + 7*B*a^2*sqrt(x) - 3*A*a*b*sqrt(x))/((b*x + a)^2*b^3*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\int \frac{x^{3/2}\,(A+B\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{3/2}} \,dx$$

input
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.35

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx=\frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx+3\sqrt{x}\,ab+2\sqrt{x}\,b^2x}{b^3\,(bx+a)}$$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + 3*sqrt(x)*a*b + 2*sqrt(x)*
b**2*x)/(b**3*(a + b*x))
```

3.450 $\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 155

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{(Ab-5aB)\sqrt{x}}{4ab^2\sqrt{a^2+2abx+b^2x^2}}-\frac{(Ab-aB)\sqrt{x}}{2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab+3aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{5/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/4*(A*b-5*B*a)*x^(1/2)/a/b^2/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)*x^(1/2)/b^2/
(b*x+a)/((b*x+a)^2)^(1/2)+1/4*(A*b+3*B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a
^(1/2))/a^(3/2)/b^(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{(a+bx)\left(\sqrt{a}\sqrt{b}\sqrt{x}\left(-3a^2B+Ab^2x-ab(A+5Bx)\right)+(Ab+3aB)(a+bx\right.}{4a^{3/2}b^{5/2}\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(Sqrt[a]*Sqrt[b]*Sqrt[x]*(-3*a^2*B + A*b^2*x - a*b*(A + 5*B*x))
 + (A*b + 3*a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(4*a^(3/2
)*b^(5/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1187, 27, 87, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^3(a+bx)\int \frac{\sqrt{x}(A+Bx)}{b^3(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(\frac{(3aB+Ab)\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{51}$$

$$\frac{(a+bx)\left(\frac{(3aB+Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{(3aB+Ab)\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx)\left(\frac{(3aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x)^2) + ((A*b + 3*a*B)*(-(
Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2
))))/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(A\,x^{\frac{3}{2}}\sqrt{a b}\,b^{2}+A\arctan\left(\frac{b\sqrt{x}}{\sqrt{a b}}\right)b^{3}x^{2}-5B\,x^{\frac{3}{2}}\sqrt{a b}\,a b+3B\arctan\left(\frac{b\sqrt{x}}{\sqrt{a b}}\right)a\,b^{2}x^{2}+2A\arctan\left(\frac{b\sqrt{x}}{\sqrt{a b}}\right)a\,b^{2}x+6B\arctan\left(\frac{b\sqrt{x}}{\sqrt{a b}}\right)a^{2}b x-A\sqrt{}}{4\sqrt{a b}\,b^{2}a\left(\left(b x+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(A*x^(3/2)*(a*b)^(1/2)*b^2+A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^3*x^2-5*B
*x^(3/2)*(a*b)^(1/2)*a*b+3*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^2*x^2+2*A*a
rctan(b*x^(1/2)/(a*b)^(1/2))*a*b^2*x+6*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^2
*b*x-A*x^(1/2)*(a*b)^(1/2)*a*b+A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^2*b-3*B*x
^(1/2)*(a*b)^(1/2)*a^2+3*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3)*(b*x+a)/(a*b
)^(1/2)/b^2/a/((b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \left[-\frac{(3\,Ba^3+Aa^2b+(3\,Bab^2+Ab^3)x^2+2\,(3\,Ba^2b+Aab^2)x)\sqrt{-ab}\log\left(\frac{bx-}{}\right.}{8\,(a^2b^5x^2+2\,a^3b^4x}\right.$$

$$-\frac{(3\,Ba^3+Aa^2b+(3\,Bab^2+Ab^3)x^2+2\,(3\,Ba^2b+Aab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(3\,Ba^3b+Aa^2b^2+(5\,Ba}{4\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/8*((3*B*a^3 + A*a^2*b + (3*B*a*b^2 + A*b^3)*x^2 + 2*(3*B*a^2*b + A*a*b
^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(3*B
*a^3*b + A*a^2*b^2 + (5*B*a^2*b^2 - A*a*b^3)*x)*sqrt(x))/(a^2*b^5*x^2 + 2*
a^3*b^4*x + a^4*b^3), -1/4*((3*B*a^3 + A*a^2*b + (3*B*a*b^2 + A*b^3)*x^2 +
 2*(3*B*a^2*b + A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (3*B
*a^3*b + A*a^2*b^2 + (5*B*a^2*b^2 - A*a*b^3)*x)*sqrt(x))/(a^2*b^5*x^2 + 2*
a^3*b^4*x + a^4*b^3)]
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{x}(A+Bx)}{\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)/((a + b*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(103) = 206$.

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{12\,(Ba^3b+Aa^2b^2)x^{\frac{5}{2}} - ((5\,Bab^3+Ab^4)x^2 - 3\,(Ba^2b^2+Aab^3)x)x^{\frac{5}{2}} - (3\,(B}{24\,(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^{}} + \frac{(3\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab^2} + \frac{(5\,Bab+Ab^2)x^{\frac{3}{2}} - 6\,(3\,Ba^2+Aab)\sqrt{x}}{24\,a^3b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/24*(12*(B*a^3*b + A*a^2*b^2)*x^(5/2) - ((5*B*a*b^3 + A*b^4)*x^2 - 3*(B*a
^2*b^2 + A*a*b^3)*x)*x^(5/2) - (3*(B*a^3*b - 3*A*a^2*b^2)*x^2 - (B*a^4 + 1
7*A*a^3*b)*x)*sqrt(x))/(a^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*b^2*x + a^6*b)
 + 1/4*(3*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/24*
((5*B*a*b + A*b^2)*x^(3/2) - 6*(3*B*a^2 + A*a*b)*sqrt(x))/(a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab^2\mathrm{sgn}\left(bx+a\right)} - \frac{5\,Babx^{\frac{3}{2}} - Ab^2x^{\frac{3}{2}} + 3\,Ba^2\sqrt{x} + Aab\sqrt{x}}{4\,(bx+a)^2ab^2\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(3*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2*sgn(b*x + a
)) - 1/4*(5*B*a*b*x^(3/2) - A*b^2*x^(3/2) + 3*B*a^2*sqrt(x) + A*a*b*sqrt(x
))/((b*x + a)^2*a*b^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{x}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{3/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-\sqrt{x}\,ab}{a\,b^2\,(bx+a)}$$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - sqrt(x)*a*b)/(a*b**2*(a + b*x))
```

3.451 $\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 158

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{(3Ab+aB)\sqrt{x}}{4a^2b\sqrt{a^2+2abx+b^2x^2}}+\frac{(Ab-aB)\sqrt{x}}{2ab(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{(3Ab+aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/4*(3*A*b+B*a)*x^(1/2)/a^2/b/((b*x+a)^2)^(1/2)+1/2*(A*b-B*a)*x^(1/2)/a/b/
(b*x+a)/((b*x+a)^2)^(1/2)+1/4*(3*A*b+B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a
^(1/2))/a^(5/2)/b^(3/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{(a+bx)\left(\sqrt{a}\sqrt{b}\sqrt{x}(-a^2B+3Ab^2x+ab(5A+Bx))+(3Ab+aB)(a+\right.}{4a^{5/2}b^{3/2}\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(Sqrt[a]*Sqrt[b]*Sqrt[x]*(-(a^2*B) + 3*A*b^2*x + a*b*(5*A + B*x
)) + (3*A*b + a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(4*a^(5
/2)*b^(3/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1187, 27, 87, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3\sqrt{x}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{(aB+3Ab)\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(a+bx)\left(\frac{(aB+3Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(a+bx)\left(\frac{(aB+3Ab)\left(\frac{\int\frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x)^2) + ((3*A*b + a*B)*(Sq
rt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]))
)/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(3A\,x^{\frac{3}{2}}\sqrt{ab}\,b^{2}+3A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^{3}x^{2}+B\,x^{\frac{3}{2}}\sqrt{ab}\,ab+B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{2}x^{2}+6A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{2}x+2B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^{2}bx+5A\sqrt{\cdots}\right.}{4\sqrt{ab}\,b\,a^{2}\left((bx+a)^{2}\right)^{\frac{3}{2}}}$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*A*x^(3/2)*(a*b)^(1/2)*b^2+3*A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^3*x^2
+B*x^(3/2)*(a*b)^(1/2)*a*b+B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^2*x^2+6*A*a
rctan(b*x^(1/2)/(a*b)^(1/2))*a*b^2*x+2*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^2
*b*x+5*A*x^(1/2)*(a*b)^(1/2)*a*b+3*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^2*b-B
*x^(1/2)*(a*b)^(1/2)*a^2+B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3)*(b*x+a)/(a*b
)^(1/2)/b/a^2/((b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{3/2}}\,dx = \left[-\frac{(Ba^3+3\,Aa^2b+(Bab^2+3\,Ab^3)x^2+2\,(Ba^2b+3\,Aab^2)x)\sqrt{-ab}\log\ldots}{8\,(a^3b^4x^2+2\,a^4\ldots}\right.$$

$$-\frac{(Ba^3+3\,Aa^2b+(Bab^2+3\,Ab^3)x^2+2\,(Ba^2b+3\,Aab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(Ba^3b-5\,Aa^2b^2-(Ba^2\ldots}{4\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/8*((B*a^3 + 3*A*a^2*b + (B*a*b^2 + 3*A*b^3)*x^2 + 2*(B*a^2*b + 3*A*a*b
^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(B*a
^3*b - 5*A*a^2*b^2 - (B*a^2*b^2 + 3*A*a*b^3)*x)*sqrt(x))/(a^3*b^4*x^2 + 2*
a^4*b^3*x + a^5*b^2), -1/4*((B*a^3 + 3*A*a^2*b + (B*a*b^2 + 3*A*b^3)*x^2 +
 2*(B*a^2*b + 3*A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (B*a
^3*b - 5*A*a^2*b^2 - (B*a^2*b^2 + 3*A*a*b^3)*x)*sqrt(x))/(a^3*b^4*x^2 + 2*
a^4*b^3*x + a^5*b^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{A+Bx}{\sqrt{x}\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(sqrt(x)*((a + b*x)**2)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(105) = 210$.

Time = 0.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{12\,(Ba^3b+5\,Aa^2b^2)x^{\frac{5}{2}}-((Bab^3+Ab^4)x^2-3\,(Ba^2b^2+5\,Aab^3)x)x^{\frac{5}{2}}+}{24\,(a^4b^3x^3+3\,a^5b^2x^2}$$
$$+\frac{(Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2b}+\frac{(Bab+Ab^2)x^{\frac{3}{2}}-6\,(Ba^2+3\,Aab)\sqrt{x}}{24\,a^4b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/24*(12*(B*a^3*b + 5*A*a^2*b^2)*x^(5/2) - ((B*a*b^3 + A*b^4)*x^2 - 3*(B*a
^2*b^2 + 5*A*a*b^3)*x)*x^(5/2) + (9*(B*a^3*b + A*a^2*b^2)*x^2 + 17*(B*a^4
+ 5*A*a^3*b)*x)*sqrt(x) + 16*(A*a^3*b*x^2 + 3*A*a^4*x)/sqrt(x))/(a^4*b^3*x
^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7) + 1/4*(B*a + 3*A*b)*arctan(b*sqrt(x)
/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/24*((B*a*b + A*b^2)*x^(3/2) - 6*(B*a^2 +
 3*A*a*b)*sqrt(x))/(a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2b\mathrm{sgn}\,(bx+a)}$$
$$+\frac{Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}-Ba^2\sqrt{x}+5\,Aab\sqrt{x}}{4\,(bx+a)^2a^2b\mathrm{sgn}\,(bx+a)}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b*sgn(b*x + a
)) + 1/4*(B*a*b*x^(3/2) + 3*A*b^2*x^(3/2) - B*a^2*sqrt(x) + 5*A*a*b*sqrt(x
))/((b*x + a)^2*a^2*b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{\sqrt{x}\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx+\sqrt{x}\,ab}{a^2b\,(bx+a)}$$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + sqrt(x)*a*b)/(a**2*b*(a + b*x))
```

3.452 $\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{(7Ab-3aB)\sqrt{x}}{4a^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(Ab-aB)\sqrt{x}}{2a^2(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2A(a+bx)}{a^3\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}-\frac{3(5Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
-1/4*(7*A*b-3*B*a)*x^(1/2)/a^3/((b*x+a)^2)^(1/2)-1/2*(A*b-B*a)*x^(1/2)/a^2
/(b*x+a)/((b*x+a)^2)^(1/2)-2*A*(b*x+a)/a^3/x^(1/2)/((b*x+a)^2)^(1/2)-3/4*(
5*A*b-B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)/((b*x+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{(a+bx)\left(\frac{\sqrt{a}(-15Ab^2x^2+abx(-25A+3Bx)+a^2(-8A+5Bx))}{\sqrt{x}}+\frac{3(-5Ab+aB)(a+bx)^2\,\mathrm{arc}}{\sqrt{b}}\right.}{4a^{7/2}\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*((Sqrt[a]*(-15*A*b^2*x^2 + a*b*x*(-25*A + 3*B*x) + a^2*(-8*A +
5*B*x)))/Sqrt[x] + (3*(-5*A*b + a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/Sqrt[b]))/(4*a^(7/2)*((a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1187, 27, 87, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x^{3/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{3/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 87$$

$$\frac{(a+bx)\left(\frac{(5Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 52

$$\frac{(a+bx)\left(\frac{(5Ab-aB)\left(\frac{3\int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 61

$$\frac{(a+bx)\left(\frac{(5Ab-aB)\left(\frac{3\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 73

$$\frac{(a+bx)\left(\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx)\left(\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x)^2) + ((5*A*b - a*B)*(1/(a
*Sqrt[x]*(a + b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{2 A \sqrt{\left(b x +a \right)^{2}}}{a^{3} \sqrt{x}\, \left(b x +a \right)}-\frac{\left(\frac{2 \left(\frac{7}{8} b^{2} A -\frac{3}{8} a b B \right) x^{\frac{3}{2}}+\frac{a \left(9 A b -5 B a \right) \sqrt{x}}{4}}{\left(b x +a \right)^{2}}+\frac{3 \left(5 A b -B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4 \sqrt{a b}}\right) \sqrt{\left(b x +a \right)^{2}}}{a^{3} \left(b x +a \right)}$
default	$-\frac{\left(15 A \,x^{2} \sqrt{a b}\, b^{2}+15 A \,x^{\frac{5}{2}} \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right) b^{3}-3 B \,x^{2} \sqrt{a b}\, a b -3 B \,x^{\frac{5}{2}} \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right) a \,b^{2}+30 A \,x^{\frac{3}{2}} \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right) a \,b^{2}-6 B \,x^{\frac{3}{2}} \arctan \left(\frac{b}{\sqrt{}}\right.}{4 \sqrt{x}\, \sqrt{a b}\, a^{3} \left(\left(b x +a \right)^{2}\right.}$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/a^3*(2*((7/8*b^2*A-3/8*a*b*B)
*x^(3/2)+1/8*a*(9*A*b-5*B*a)*x^(1/2))/(b*x+a)^2+3/4*(5*A*b-B*a)/(a*b)^(1/2
)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \left[\frac{3\left((Bab^2-5\,Ab^3)x^3+2\,(Ba^2b-5\,Aab^2)x^2+(Ba^3-5\,Aa^2b)x\right)\sqrt{-ab}}{8\,(a^4b^3}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[1/8*(3*((B*a*b^2 - 5*A*b^3)*x^3 + 2*(B*a^2*b - 5*A*a*b^2)*x^2 + (B*a^3 -
5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x - a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) -
 2*(8*A*a^3*b - 3*(B*a^2*b^2 - 5*A*a*b^3)*x^2 - 5*(B*a^3*b - 5*A*a^2*b^2)*
x)*sqrt(x))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x), -1/4*(3*((B*a*b^2 - 5
*A*b^3)*x^3 + 2*(B*a^2*b - 5*A*a*b^2)*x^2 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*
b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (8*A*a^3*b - 3*(B*a^2*b^2 - 5*A*a*b^3)*
x^2 - 5*(B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 +
 a^6*b*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x^{\frac{3}{2}}\left((a+bx)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**(3/2)*((a + b*x)**2)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(123) = 246$.

Time = 0.18 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{60\,(Ba^3b^2-7\,Aa^2b^3)x^{\frac{5}{2}}-((Bab^4+5\,Ab^5)x^2-15\,(Ba^2b^3-7\,Aab^4)x)x}{}$$
$$+\frac{3\,(Ba-5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}+\frac{(Bab+5\,Ab^2)x^{\frac{3}{2}}-18\,(Ba^2-5\,Aab)\sqrt{x}}{24\,a^5}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/24*(60*(B*a^3*b^2 - 7*A*a^2*b^3)*x^(5/2) - ((B*a*b^4 + 5*A*b^5)*x^2 - 15
*(B*a^2*b^3 - 7*A*a*b^4)*x)*x^(5/2) + (9*(B*a^3*b^2 + 5*A*a^2*b^3)*x^2 + 8
5*(B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(x) + 16*((B*a^4*b + 5*A*a^3*b^2)*x^2 + 3
*(B*a^5 - 7*A*a^4*b)*x)/sqrt(x) + 48*(A*a^4*b*x^2 - A*a^5*x)/x^(3/2))/(a^5
*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8) + 3/4*(B*a - 5*A*b)*arctan(b*s
qrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/24*((B*a*b + 5*A*b^2)*x^(3/2) - 18*(
B*a^2 - 5*A*a*b)*sqrt(x))/a^5
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{3\,(Ba-5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3\mathrm{sgn}\left(bx+a\right)}$$
$$-\frac{2\,A}{a^3\sqrt{x}\mathrm{sgn}\left(bx+a\right)}+\frac{3\,Babx^{\frac{3}{2}}-7\,Ab^2x^{\frac{3}{2}}+5\,Ba^2\sqrt{x}-9\,Aab\sqrt{x}}{4\,(bx+a)^2a^3\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
3/4*(B*a - 5*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x + a))
 - 2*A/(a^3*sqrt(x)*sgn(b*x + a)) + 1/4*(3*B*a*b*x^(3/2) - 7*A*b^2*x^(3/2)
 + 5*B*a^2*sqrt(x) - 9*A*a*b*sqrt(x))/((b*x + a)^2*a^3*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^{3/2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-2a^2-3ab}{\sqrt{x}\,a^3\,(bx+a)}$$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sq
rt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - 2*a**2 - 3
*a*b*x)/(sqrt(x)*a**3*(a + b*x))
```

3.453 $\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 238

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{b(11Ab-7aB)\sqrt{x}}{4a^4\sqrt{a^2+2abx+b^2x^2}}+\frac{b(Ab-aB)\sqrt{x}}{2a^3(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2A(a+bx)}{3a^3x^{3/2}\sqrt{a^2+2abx+b^2x^2}}+\frac{2(3Ab-aB)(a+bx)}{a^4\sqrt{x}\sqrt{a^2+2abx+b^2x^2}}+\frac{5\sqrt{b}(7Ab-3aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output

```
1/4*b*(11*A*b-7*B*a)*x^(1/2)/a^4/((b*x+a)^2)^(1/2)+1/2*b*(A*b-B*a)*x^(1/2)
/a^3/(b*x+a)/((b*x+a)^2)^(1/2)-2/3*A*(b*x+a)/a^3/x^(3/2)/((b*x+a)^2)^(1/2)
+2*(3*A*b-B*a)*(b*x+a)/a^4/x^(1/2)/((b*x+a)^2)^(1/2)+5/4*b^(1/2)*(7*A*b-3*
B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{(a+bx)\left(-\frac{\sqrt{a}(-105Ab^3x^3+8a^3(A+3Bx)+5ab^2x^2(-35A+9Bx)+a^2bx(-56A+75Bx))}{x^{3/2}}+\right.}{12a^{9/2}\left((a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*(-((Sqrt[a]*(-105*A*b^3*x^3 + 8*a^3*(A + 3*B*x) + 5*a*b^2*x^2*(
-35*A + 9*B*x) + a^2*b*x*(-56*A + 75*B*x)))/x^(3/2)) + 15*Sqrt[b]*(7*A*b -
 3*a*B)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(12*a^(9/2)*((a +
b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x^{5/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{5/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\left(\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx)\left(\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x)^2) + ((7*A*b - 3*a*B)*(1/
(a*x^(3/2)*(a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqr
t[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2*a)))/(4*a*b)))/Sq
rt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{2(-9Abx+3Bax+Aa)\sqrt{(bx+a)^2}}{3a^4x^{\frac{3}{2}}(bx+a)}+\frac{b\left(\frac{2\left(\frac{11}{8}b^2A-\frac{7}{8}abB\right)x^{\frac{3}{2}}+\frac{a(13Ab-9Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{5(7Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)\sqrt{(bx+a)^2}}{a^4(bx+a)}$
default	$-\frac{\left(-105A\,x^{\frac{7}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^4+45B\,x^{\frac{7}{2}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^3-210A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{5}{2}}a\,b^3+90B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{5}{2}}a^2b^2-105A\arctan\left(\right.\right.}{}$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-9*A*b*x+3*B*a*x+A*a)/a^4/x^(3/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/a^4*b*
(2*((11/8*b^2*A-7/8*a*b*B)*x^(3/2)+1/8*a*(13*A*b-9*B*a)*x^(1/2))/(b*x+a)^2
+5/4*(7*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^
(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\left[-\frac{15\left((3\,Bab^2-7\,Ab^3)x^4+2\,(3\,Ba^2b-7\,Aab^2)x^3+(3\,Ba^3-7\,Aa^2b)\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/24*(15*((3*B*a*b^2 - 7*A*b^3)*x^4 + 2*(3*B*a^2*b - 7*A*a*b^2)*x^3 + (3
*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)
/(b*x + a)) + 2*(8*A*a^3 + 15*(3*B*a*b^2 - 7*A*b^3)*x^3 + 25*(3*B*a^2*b -
7*A*a*b^2)*x^2 + 8*(3*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*
b*x^3 + a^6*x^2), -1/12*(15*((3*B*a*b^2 - 7*A*b^3)*x^4 + 2*(3*B*a^2*b - 7*
A*a*b^2)*x^3 + (3*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/
a)) + (8*A*a^3 + 15*(3*B*a*b^2 - 7*A*b^3)*x^3 + 25*(3*B*a^2*b - 7*A*a*b^2)
*x^2 + 8*(3*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^
6*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+Bx}{x^{\frac{5}{2}}\left(\left(a+bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**(5/2)*((a + b*x)**2)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(156) = 312$.

Time = 0.19 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=$$

$$-\frac{420\left(Ba^3b^3-3\,Aa^2b^4\right)x^{\frac{5}{2}}+5\left(\left(Bab^5-7\,Ab^6\right)x^2+21\left(Ba^2b^4-3\,Aab^5\right)x\right)x^{\frac{5}{2}}-5\left(9\left(Ba^3b^3-7\,Aa^2b^4\right)x^2\right.}{24\left(a^6b^3\right.}$$

$$-\frac{5\left(3\,Bab-7\,Ab^2\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^4}+\frac{5\left(\left(Bab^2-7\,Ab^3\right)x^{\frac{3}{2}}+6\left(3\,Ba^2b-7\,Aab^2\right)\sqrt{x}\right)}{24\,a^6}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/24*(420*(B*a^3*b^3 - 3*A*a^2*b^4)*x^(5/2) + 5*((B*a*b^5 - 7*A*b^6)*x^2
+ 21*(B*a^2*b^4 - 3*A*a*b^5)*x)*x^(5/2) - 5*(9*(B*a^3*b^3 - 7*A*a^2*b^4)*x
^2 - 119*(B*a^4*b^2 - 3*A*a^3*b^3)*x)*sqrt(x) - 16*(5*(B*a^4*b^2 - 7*A*a^3
*b^3)*x^2 - 21*(B*a^5*b - 3*A*a^4*b^2)*x)/sqrt(x) - 48*((B*a^5*b - 7*A*a^4
*b^2)*x^2 - (B*a^6 - 3*A*a^5*b)*x)/x^(3/2) + 16*(3*A*a^5*b*x^2 + A*a^6*x)/
x^(5/2))/(a^6*b^3*x^3 + 3*a^7*b^2*x^2 + 3*a^8*b*x + a^9) - 5/4*(3*B*a*b -
7*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 5/24*((B*a*b^2 - 7*
A*b^3)*x^(3/2) + 6*(3*B*a^2*b - 7*A*a*b^2)*sqrt(x))/a^6
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=-\frac{5\left(3\,Bab-7\,Ab^2\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4\mathrm{sgn}\left(bx+a\right)}-\frac{2\left(3\,Bax-9\,Abx+Aa\right)}{3\,a^4x^{\frac{3}{2}}\mathrm{sgn}\left(bx+a\right)}-\frac{7\,Bab^2x^{\frac{3}{2}}-11\,Ab^3x^{\frac{3}{2}}+9\,Ba^2b\sqrt{x}-13\,Aab^2\sqrt{x}}{4\left(bx+a\right)^2a^4\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
-5/4*(3*B*a*b - 7*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4*sgn(b*
x + a)) - 2/3*(3*B*a*x - 9*A*b*x + A*a)/(a^4*x^(3/2)*sgn(b*x + a)) - 1/4*(
7*B*a*b^2*x^(3/2) - 11*A*b^3*x^(3/2) + 9*B*a^2*b*sqrt(x) - 13*A*a*b^2*sqrt
(x))/((b*x + a)^2*a^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^{5/2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-2a^3 -}{3\sqrt{x}\,a^4x\,(bx+a)}$$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
(15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 15
*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 2
*a**3 + 10*a**2*b*x + 15*a*b**2*x**2)/(3*sqrt(x)*a**4*x*(a + b*x))
```

3.454 $\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 289

$$\begin{aligned}\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= -\frac{b^2(15Ab-11aB)\sqrt{x}}{4a^5\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{b^2(Ab-aB)\sqrt{x}}{2a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{2A(a+bx)}{5a^3x^{5/2}\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{2(3Ab-aB)(a+bx)}{3a^4x^{3/2}\sqrt{a^2+2abx+b^2x^2}} - \frac{6b(2Ab-aB)(a+bx)}{a^5\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{7b^{3/2}(9Ab-5aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{11/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output

```
-1/4*b^2*(15*A*b-11*B*a)*x^(1/2)/a^5/((b*x+a)^2)^(1/2)-1/2*b^2*(A*b-B*a)*x
^(1/2)/a^4/(b*x+a)/((b*x+a)^2)^(1/2)-2/5*A*(b*x+a)/a^3/x^(5/2)/((b*x+a)^2)
^(1/2)+2/3*(3*A*b-B*a)*(b*x+a)/a^4/x^(3/2)/((b*x+a)^2)^(1/2)-6*b*(2*A*b-B*
a)*(b*x+a)/a^5/x^(1/2)/((b*x+a)^2)^(1/2)-7/4*b^(3/2)*(9*A*b-5*B*a)*(b*x+a)
*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{a}(-945Ab^4x^4+525ab^3x^3(-3A+Bx)-8a^4(3A+5Bx)+8a^3bx(9A}{60a^{11}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*(-945*A*b^4*x^4 + 525*a*b^3*x^3*(-3*A + B*x) - 8*a^4*(3*A + 5*B*x
) + 8*a^3*b*x*(9*A + 35*B*x) + 7*a^2*b^2*x^2*(-72*A + 125*B*x)) + 105*b^(3
/2)*(-9*A*b + 5*a*B)*x^(5/2)*(a + b*x)^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/(60*a^(11/2)*x^(5/2)*(a + b*x)*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.62, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {1187, 27, 87, 52, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^3(a+bx)\int \frac{A+Bx}{b^3x^{7/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{7/2}(a+bx)^3}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\int \frac{1}{x^{7/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\int \frac{1}{x^{7/2}(a+bx)}dx}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx)\left(\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(3/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x)^2) + ((9*A*b - 5*a*B)*(1/
(a*x^(5/2)*(a + b*x)) + (7*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(
-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a
))/a))/(2*a)))/(4*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{2(90x^2b^2A-45Bax^2b-15abAx+5a^2Bx+3a^2A)\sqrt{(bx+a)^2}}{15a^5x^{\frac{5}{2}}(bx+a)}-\frac{b^2\left(\frac{2\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^{\frac{3}{2}}+\frac{a(17Ab-13Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{7(9Ab-5Ba)\arctan}{4\sqrt{ab}}\right.}{a^5(bx+a)}$
default	$-\frac{\left(945A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{9}{2}}b^5-525B\sqrt{ab}\,x^4ab^3+1575A\sqrt{ab}\,x^3ab^3+1890A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{7}{2}}ab^4-1050B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{7}{2}}a^2b^3-52\right.}{}$

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(90*A*b^2*x^2-45*B*a*b*x^2-15*A*a*b*x+5*B*a^2*x+3*A*a^2)/a^5/x^(5/2)
*((b*x+a)^2)^(1/2)/(b*x+a)-1/a^5*b^2*(2*((15/8*b^2*A-11/8*a*b*B)*x^(3/2)+1
/8*a*(17*A*b-13*B*a)*x^(1/2))/(b*x+a)^2+7/4*(9*A*b-5*B*a)/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \left[-\frac{105\left((5\,Bab^3-9\,Ab^4)x^5+2\,(5\,Ba^2b^2-9\,Aab^3)x^4+(5\,Ba^3b-9\,Aa\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/120*(105*((5*B*a*b^3 - 9*A*b^4)*x^5 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^4
+ (5*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b
/a) - a)/(b*x + a)) + 2*(24*A*a^4 - 105*(5*B*a*b^3 - 9*A*b^4)*x^4 - 175*(5
*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^2 + 8*(5*B*a^
4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3), 1/60*(10
5*((5*B*a*b^3 - 9*A*b^4)*x^5 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^4 + (5*B*a^3*
b - 9*A*a^2*b^2)*x^3)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) - (24*A*a^4 - 10
5*(5*B*a*b^3 - 9*A*b^4)*x^4 - 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 56*(5*B*
a^3*b - 9*A*a^2*b^2)*x^2 + 8*(5*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b^2*x^
5 + 2*a^6*b*x^4 + a^7*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(191) = 382$.

Time = 0.18 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{1260\left(5\,Ba^3b^4-11\,Aa^2b^5\right)x^{\frac{5}{2}}+35\left(5\left(Bab^6-3\,Ab^7\right)x^2+9\left(5\,Ba^2b^5-\right.\right.}{}$$
$$+\frac{7\left(5\,Bab^2-9\,Ab^3\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^5}-\frac{7\left(5\left(Bab^3-3\,Ab^4\right)x^{\frac{3}{2}}+6\left(5\,Ba^2b^2-9\,Aab^3\right)\sqrt{x}\right)}{24\,a^7}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/120*(1260*(5*B*a^3*b^4 - 11*A*a^2*b^5)*x^(5/2) + 35*(5*(B*a*b^6 - 3*A*b^
7)*x^2 + 9*(5*B*a^2*b^5 - 11*A*a*b^6)*x)*x^(5/2) - 105*(15*(B*a^3*b^4 - 3*
A*a^2*b^5)*x^2 - 17*(5*B*a^4*b^3 - 11*A*a^3*b^4)*x)*sqrt(x) - 112*(25*(B*a
^4*b^3 - 3*A*a^3*b^4)*x^2 - 9*(5*B*a^5*b^2 - 11*A*a^4*b^3)*x)/sqrt(x) - 48
*(35*(B*a^5*b^2 - 3*A*a^4*b^3)*x^2 - 3*(5*B*a^6*b - 11*A*a^5*b^2)*x)/x^(3/
2) - 16*(15*(B*a^6*b - 3*A*a^5*b^2)*x^2 + (5*B*a^7 - 11*A*a^6*b)*x)/x^(5/2
) - 16*(5*A*a^6*b*x^2 + 3*A*a^7*x)/x^(7/2))/(a^7*b^3*x^3 + 3*a^8*b^2*x^2 +
 3*a^9*b*x + a^10) + 7/4*(5*B*a*b^2 - 9*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*a^5) - 7/24*(5*(B*a*b^3 - 3*A*b^4)*x^(3/2) + 6*(5*B*a^2*b^2 -
9*A*a*b^3)*sqrt(x))/a^7
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{7\left(5\,Bab^2-9\,Ab^3\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^5\operatorname{sgn}(bx+a)}$$
$$+\frac{11\,Bab^3x^{\frac{3}{2}}-15\,Ab^4x^{\frac{3}{2}}+13\,Ba^2b^2\sqrt{x}-17\,Aab^3\sqrt{x}}{4\left(bx+a\right)^2a^5\operatorname{sgn}(bx+a)}$$
$$+\frac{2\left(45\,Babx^2-90\,Ab^2x^2-5\,Ba^2x+15\,Aabx-3\,Aa^2\right)}{15\,a^5x^{\frac{5}{2}}\operatorname{sgn}(bx+a)}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

output
```
7/4*(5*B*a*b^2 - 9*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5*sgn(b
*x + a)) + 1/4*(11*B*a*b^3*x^(3/2) - 15*A*b^4*x^(3/2) + 13*B*a^2*b^2*sqrt(
x) - 17*A*a*b^3*sqrt(x))/((b*x + a)^2*a^5*sgn(b*x + a)) + 2/15*(45*B*a*b*x
^2 - 90*A*b^2*x^2 - 5*B*a^2*x + 15*A*a*b*x - 3*A*a^2)/(a^5*x^(5/2)*sgn(b*x
 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^{7/2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\frac{-105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2-105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^3x^3}{15\sqrt{x}\,a^5x^2\left(bx+a\right)}$$

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2
*x**2 - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*
*3*x**3 - 6*a**4 + 14*a**3*b*x - 70*a**2*b**2*x**2 - 105*a*b**3*x**3)/(15*
sqrt(x)*a**5*x**2*(a + b*x))
```

3.455 $\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 381

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx &= -\frac{(231Ab-575aB)x^{5/2}}{64b^5\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{(Ab-aB)x^{11/2}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(11Ab-19aB)x^{9/2}}{24b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{(33Ab-73aB)x^{7/2}}{32b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{231a(5Ab-13aB)\sqrt{x}(a+bx)}{64b^7\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{77(5Ab-13aB)x^{3/2}(a+bx)}{64b^6\sqrt{a^2+2abx+b^2x^2}} + \frac{2Bx^{5/2}(a+bx)}{5b^5\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{231a^{3/2}(5Ab-13aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{15/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/64*(231*A*b-575*B*a)*x^(5/2)/b^5/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(11/
2)/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(11*A*b-19*B*a)*x^(9/2)/b^3/(b*x+a
)^2/((b*x+a)^2)^(1/2)-1/32*(33*A*b-73*B*a)*x^(7/2)/b^4/(b*x+a)/((b*x+a)^2)
^(1/2)-231/64*a*(5*A*b-13*B*a)*x^(1/2)*(b*x+a)/b^7/((b*x+a)^2)^(1/2)+77/64
*(5*A*b-13*B*a)*x^(3/2)*(b*x+a)/b^6/((b*x+a)^2)^(1/2)+2/5*B*x^(5/2)*(b*x+a
)/b^5/((b*x+a)^2)^(1/2)+231/64*a^(3/2)*(5*A*b-13*B*a)*(b*x+a)*arctan(b^(1/
2)*x^(1/2)/a^(1/2))/b^(15/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.49

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\sqrt{b}\sqrt{x}(45045a^6B-1155a^5b(15A-143Bx)+128b^6x^5(5A+3Bx)-128ab}{}$$

input
```
Integrate[(x^(11/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*(45045*a^6*B - 1155*a^5*b*(15*A - 143*B*x) + 128*b^6*x^5*
(5*A + 3*B*x) - 128*a*b^5*x^4*(55*A + 13*B*x) + 231*a^4*b^2*x*(-275*A + 94
9*B*x) + 11*a^2*b^4*x^3*(-4185*A + 1664*B*x) + 33*a^3*b^3*x^2*(-2555*A + 3
627*B*x)) - 3465*a^(3/2)*(-5*A*b + 13*a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(960*b^(15/2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.61, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {1187, 27, 87, 51, 51, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^5(a+bx)\int \frac{x^{11/2}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^{11/2}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\int \frac{x^{11/2}}{(a+bx)^4}dx}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\downarrow$ 51

$$\frac{(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\left(\frac{11\int \frac{x^{9/2}}{(a+bx)^3}dx}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\downarrow$ 51

$$\frac{(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\left(\frac{11\left(\frac{9\int \frac{x^{7/2}}{(a+bx)^2}dx}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\downarrow$ 51

$$\frac{(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{x^{5/2}}{a+bx}dx}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\downarrow$ 60

$$\frac{(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\int\frac{x^{3/2}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow 60$$

$$(a+bx)\left(\frac{x^{13/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}\,dx}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)}{8ab}\right.$$

↓ 60

$$\left((5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)}\right.\right.$$

↓ 73

$$(5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11/2}}{3b(a+bx)^3}\right)$$

$$x^{13/2}(Ab-aB)$$

↓ 218

$$\left((5Ab-13aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4b}-\frac{x^{9/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{11}}{3b(a+}\right.\right.$$

input
```
Int[(x^(11/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(13/2))/(4*a*b*(a + b*x)^4) - ((5*A*b - 13*a*B)
*(-1/3*x^(11/2)/(b*(a + b*x)^3) + (11*(-1/2*x^(9/2)/(b*(a + b*x)^2) + (9*(
-(x^(7/2)/(b*(a + b*x))) + (7*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) -
 (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
))/b))/b))/(2*b)))/(4*b)))/(6*b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{2\left(-3x^2 B\,b^2-5x\,b^2 A+25xabB+75abA-225a^2 B\right)\sqrt{x}\,\sqrt{\left(bx+a\right)^2}}{15b^7\left(bx+a\right)}+\frac{a^2\left(\frac{2\left(-\frac{765}{128}A\,b^4+\frac{1477}{128}Ba\,b^3\right)x^{\frac{7}{2}}+2\left(-\frac{5855}{384}Aa\,b^3+\frac{11767}{384}B\,a^2 \cdots\right.}{\left(\cdots\right.}\right.}{\cdots}$
default	$\frac{\left(384B\,x^{\frac{13}{2}}\sqrt{ab}\,b^6+640A\,x^{\frac{11}{2}}\sqrt{ab}\,b^6-1664B\,x^{\frac{11}{2}}\sqrt{ab}\,a\,b^5-7040A\,x^{\frac{9}{2}}\sqrt{ab}\,a\,b^5+18304B\,x^{\frac{9}{2}}\sqrt{ab}\,a^2b^4-46035A\,x^{\frac{7}{2}}\sqrt{ab}\,a^2b^4+11969\cdots\right.}{\cdots}$

input
```
int(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+25*B*a*b*x+75*A*a*b-225*B*a^2)*x^(1/2)/b^7*(
(b*x+a)^2)^(1/2)/(b*x+a)+a^2/b^7*(2*((-765/128*A*b^4+1477/128*B*a*b^3)*x^(
7/2)+(-5855/384*A*a*b^3+11767/384*B*a^2*b^2)*x^(5/2)-1/384*a^2*b*(5153*A*b
-10633*B*a)*x^(3/2)+(-515/128*A*a^3*b+1083/128*a^4*B)*x^(1/2))/(b*x+a)^4+2
31/64*(5*A*b-13*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2
)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.69

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{3465\,(13\,Ba^6-5\,Aa^5b+(13\,Ba^2b^4-5\,Aab^5)x^4+4\,(13\,Ba^3b^3-5\,Aa^2b^4}{}\right.$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="frica
s")
```

output
```
[-1/1920*(3465*(13*B*a^6 - 5*A*a^5*b + (13*B*a^2*b^4 - 5*A*a*b^5)*x^4 + 4*
(13*B*a^3*b^3 - 5*A*a^2*b^4)*x^3 + 6*(13*B*a^4*b^2 - 5*A*a^3*b^3)*x^2 + 4*
(13*B*a^5*b - 5*A*a^4*b^2)*x)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b)
 - a)/(b*x + a)) - 2*(384*B*b^6*x^6 + 45045*B*a^6 - 17325*A*a^5*b - 128*(1
3*B*a*b^5 - 5*A*b^6)*x^5 + 1408*(13*B*a^2*b^4 - 5*A*a*b^5)*x^4 + 9207*(13*
B*a^3*b^3 - 5*A*a^2*b^4)*x^3 + 16863*(13*B*a^4*b^2 - 5*A*a^3*b^3)*x^2 + 12
705*(13*B*a^5*b - 5*A*a^4*b^2)*x)*sqrt(x))/(b^11*x^4 + 4*a*b^10*x^3 + 6*a^
2*b^9*x^2 + 4*a^3*b^8*x + a^4*b^7), -1/960*(3465*(13*B*a^6 - 5*A*a^5*b + (
13*B*a^2*b^4 - 5*A*a*b^5)*x^4 + 4*(13*B*a^3*b^3 - 5*A*a^2*b^4)*x^3 + 6*(13
*B*a^4*b^2 - 5*A*a^3*b^3)*x^2 + 4*(13*B*a^5*b - 5*A*a^4*b^2)*x)*sqrt(a/b)*
arctan(b*sqrt(x)*sqrt(a/b)/a) - (384*B*b^6*x^6 + 45045*B*a^6 - 17325*A*a^5
*b - 128*(13*B*a*b^5 - 5*A*b^6)*x^5 + 1408*(13*B*a^2*b^4 - 5*A*a*b^5)*x^4
+ 9207*(13*B*a^3*b^3 - 5*A*a^2*b^4)*x^3 + 16863*(13*B*a^4*b^2 - 5*A*a^3*b^
3)*x^2 + 12705*(13*B*a^5*b - 5*A*a^4*b^2)*x)*sqrt(x))/(b^11*x^4 + 4*a*b^10
*x^3 + 6*a^2*b^9*x^2 + 4*a^3*b^8*x + a^4*b^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.05

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{256\,(3\,Bb^6x^2+5\,Bab^5x)x^{\frac{11}{2}}+5\,(2747\,(3\,Bab^5-Ab^6)x^2+437\,(13\,Ba^2b^4-$$
$$-\frac{231\,(13\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^7}$$
$$-\frac{77\left(13\,(3\,Bab-Ab^2)x^{\frac{3}{2}}-6\,(13\,Ba^2-5\,Aab)\sqrt{x}\right)}{128\,b^7}$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxim
a")
```

output
```
1/1920*(256*(3*B*b^6*x^2 + 5*B*a*b^5*x)*x^(11/2) + 5*(2747*(3*B*a*b^5 - A*
b^6)*x^2 + 437*(13*B*a^2*b^4 - 3*A*a*b^5)*x)*x^(9/2) + 10*(4667*(3*B*a^2*b
^4 - A*a*b^5)*x^2 + 671*(13*B*a^3*b^3 - 3*A*a^2*b^4)*x)*x^(7/2) + 2860*(22
*(3*B*a^3*b^3 - A*a^2*b^4)*x^2 + 3*(13*B*a^4*b^2 - 3*A*a^3*b^3)*x)*x^(5/2)
 + 66*(585*(3*B*a^4*b^2 - A*a^3*b^3)*x^2 + 77*(13*B*a^5*b - 3*A*a^4*b^2)*x
)*x^(3/2) + 231*(39*(3*B*a^5*b - A*a^4*b^2)*x^2 + 5*(13*B*a^6 - 3*A*a^5*b)
*x)*sqrt(x))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^8*x^2 +
5*a^4*b^7*x + a^5*b^6) - 231/64*(13*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sq
rt(a*b))/(sqrt(a*b)*b^7) - 77/128*(13*(3*B*a*b - A*b^2)*x^(3/2) - 6*(13*B*
a^2 - 5*A*a*b)*sqrt(x))/b^7
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.57

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{231\,(13\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^7\mathrm{sgn}\,(bx+a)}$$
$$+\frac{4431\,Ba^3b^3x^{\frac{7}{2}}-2295\,Aa^2b^4x^{\frac{7}{2}}+11767\,Ba^4b^2x^{\frac{5}{2}}-5855\,Aa^3b^3x^{\frac{5}{2}}+10633\,Ba^5bx^{\frac{3}{2}}-5153\,Aa^4b^2x^{\frac{3}{2}}+324}{192\,(bx+a)^4b^7\mathrm{sgn}\,(bx+a)}$$
$$+\frac{2\left(3\,Bb^{20}x^{\frac{5}{2}}-25\,Bab^{19}x^{\frac{3}{2}}+5\,Ab^{20}x^{\frac{3}{2}}+225\,Ba^2b^{18}\sqrt{x}-75\,Aab^{19}\sqrt{x}\right)}{15\,b^{25}\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac"
)
```

output
```
-231/64*(13*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^7*
sgn(b*x + a)) + 1/192*(4431*B*a^3*b^3*x^(7/2) - 2295*A*a^2*b^4*x^(7/2) + 1
1767*B*a^4*b^2*x^(5/2) - 5855*A*a^3*b^3*x^(5/2) + 10633*B*a^5*b*x^(3/2) -
5153*A*a^4*b^2*x^(3/2) + 3249*B*a^6*sqrt(x) - 1545*A*a^5*b*sqrt(x))/((b*x
+ a)^4*b^7*sgn(b*x + a)) + 2/15*(3*B*b^20*x^(5/2) - 25*B*a*b^19*x^(3/2) +
5*A*b^20*x^(3/2) + 225*B*a^2*b^18*sqrt(x) - 75*A*a*b^19*sqrt(x))/(b^25*sgn
(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^{11/2}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int((x^(11/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(11/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.53

$$\int \frac{x^{11/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^5-10395\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4bx-10395\sqrt{b}}{}$$

input
```
int(x^(11/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**5 - 10395*
sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4*b*x - 10395*sqrt(
b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**2 - 3465*sqrt(
b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**3 + 3465*sqrt(
x)*a**5*b + 9240*sqrt(x)*a**4*b**2*x + 7623*sqrt(x)*a**3*b**3*x**2 + 1584*
sqrt(x)*a**2*b**4*x**3 - 176*sqrt(x)*a*b**5*x**4 + 48*sqrt(x)*b**6*x**5)/(
120*b**7*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.456 $\int \frac{x^{9/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 334

$$\begin{aligned}\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx &= -\frac{(315Ab-1027aB)x^{3/2}}{192b^5\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{(Ab-aB)x^{9/2}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(9Ab-17aB)x^{7/2}}{24b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{(63Ab-167aB)x^{5/2}}{96b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{105(3Ab-11aB)\sqrt{x}(a+bx)}{64b^6\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{2Bx^{3/2}(a+bx)}{3b^5\sqrt{a^2+2abx+b^2x^2}} - \frac{105\sqrt{a}(3Ab-11aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{13/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/192*(315*A*b-1027*B*a)*x^(3/2)/b^5/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(9
/2)/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(9*A*b-17*B*a)*x^(7/2)/b^3/(b*x+a
)^2/((b*x+a)^2)^(1/2)-1/96*(63*A*b-167*B*a)*x^(5/2)/b^4/(b*x+a)/((b*x+a)^2
)^(1/2)+105/64*(3*A*b-11*B*a)*x^(1/2)*(b*x+a)/b^6/((b*x+a)^2)^(1/2)+2/3*B*
x^(3/2)*(b*x+a)/b^5/((b*x+a)^2)^(1/2)-105/64*a^(1/2)*(3*A*b-11*B*a)*(b*x+a
)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(13/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.50

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\sqrt{b}\sqrt{x}(-3465a^5B+ab^4x^3(2511A-1408Bx)+9a^2b^3x^2(511A-1023Bx)\ldots}{}$$

input
```
Integrate[(x^(9/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*(-3465*a^5*B + a*b^4*x^3*(2511*A - 1408*B*x) + 9*a^2*b^3*
x^2*(511*A - 1023*B*x) + 105*a^4*b*(9*A - 121*B*x) + 231*a^3*b^2*x*(15*A -
 73*B*x) + 128*b^5*x^4*(3*A + B*x)) + 315*Sqrt[a]*(-3*A*b + 11*a*B)*(a + b
*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*b^(13/2)*(a + b*x)^3*Sqrt[(a
 + b*x)^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1187, 27, 87, 51, 51, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{x^{9/2}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{x^{9/2}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\int\frac{x^{9/2}}{(a+bx)^4}dx}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 51$$

$$\frac{(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\int\frac{x^{7/2}}{(a+bx)^3}dx}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 51$$

$$\frac{(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\int\frac{x^{5/2}}{(a+bx)^2}dx}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 51$$

$$\frac{(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 60$$

$$\frac{(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\downarrow$ 60

$$(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right.$$

$\downarrow$ 73

$$(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}\,d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right.$$

$\Big\downarrow$ 218

$$(a+bx)\left(\frac{x^{11/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(3Ab-11aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}\right)}{2b}-\frac{x^{9/2}}{3b(a+bx)^3}\right)}{8ab}\right.$$

input
```
Int[(x^(9/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(11/2))/(4*a*b*(a + b*x)^4) - ((3*A*b - 11*a*B)
*(-1/3*x^(9/2)/(b*(a + b*x)^3) + (3*(-1/2*x^(7/2)/(b*(a + b*x)^2) + (7*(-(
x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sq
rt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/(2*b)))/(4*b)))/(2*
b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.52

method	result
risch	$\frac{2(Bbx+3Ab-15Ba)\sqrt{x}\sqrt{(bx+a)^2}}{3b^6(bx+a)} - \frac{a\Big(\frac{2\left(-\frac{325}{128}Ab^4+\frac{765}{128}Bab^3\right)x^{\frac{7}{2}}+2\left(-\frac{765}{128}Aab^3+\frac{5855}{384}Ba^2b^2\right)x^{\frac{5}{2}}-\frac{a^2b(1929Ab-5153Ba)x^{\frac{3}{2}}}{192}+2(-}{(bx+a)^4}}{b^6(bx+a)}$
default	$\frac{\Big(3465A\sqrt{ab}\,x^{\frac{3}{2}}a^3b^2-5670A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^3b^3x^2-12705B\sqrt{ab}\,x^{\frac{3}{2}}a^4b-9207B\sqrt{ab}\,x^{\frac{7}{2}}a^2b^3+20790B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a^4b^2x^2-945A}{}$

input
```
int(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-15*B*a)*x^(1/2)/b^6*((b*x+a)^2)^(1/2)/(b*x+a)-a/b^6*(2*((
-325/128*A*b^4+765/128*B*a*b^3)*x^(7/2)+(-765/128*A*a*b^3+5855/384*B*a^2*b
^2)*x^(5/2)-1/384*a^2*b*(1929*A*b-5153*B*a)*x^(3/2)+(-187/128*A*a^3*b+515/
128*a^4*B)*x^(1/2))/(b*x+a)^4+105/64*(3*A*b-11*B*a)/(a*b)^(1/2)*arctan(b*x
^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.75

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{315\,(11\,Ba^5-3\,Aa^4b+(11\,Bab^4-3\,Ab^5)x^4+4\,(11\,Ba^2b^3-3\,Aab^4)x^3}{}\right.$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(315*(11*B*a^5 - 3*A*a^4*b + (11*B*a*b^4 - 3*A*b^5)*x^4 + 4*(11*B*
a^2*b^3 - 3*A*a*b^4)*x^3 + 6*(11*B*a^3*b^2 - 3*A*a^2*b^3)*x^2 + 4*(11*B*a^
4*b - 3*A*a^3*b^2)*x)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) - 2*(128*B*b^5*x^5 - 3465*B*a^5 + 945*A*a^4*b - 128*(11*B*a*b^4 -
 3*A*b^5)*x^4 - 837*(11*B*a^2*b^3 - 3*A*a*b^4)*x^3 - 1533*(11*B*a^3*b^2 -
3*A*a^2*b^3)*x^2 - 1155*(11*B*a^4*b - 3*A*a^3*b^2)*x)*sqrt(x))/(b^10*x^4 +
 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6), 1/192*(315*(11*B*a^
5 - 3*A*a^4*b + (11*B*a*b^4 - 3*A*b^5)*x^4 + 4*(11*B*a^2*b^3 - 3*A*a*b^4)*
x^3 + 6*(11*B*a^3*b^2 - 3*A*a^2*b^3)*x^2 + 4*(11*B*a^4*b - 3*A*a^3*b^2)*x)
*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (128*B*b^5*x^5 - 3465*B*a^5 + 9
45*A*a^4*b - 128*(11*B*a*b^4 - 3*A*b^5)*x^4 - 837*(11*B*a^2*b^3 - 3*A*a*b^
4)*x^3 - 1533*(11*B*a^3*b^2 - 3*A*a^2*b^3)*x^2 - 1155*(11*B*a^4*b - 3*A*a^
3*b^2)*x)*sqrt(x))/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 + 4*a^3*b^7*x +
 a^4*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.14

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{5\left((2747\,Bab^5-693\,Ab^6)x^2+3\,(437\,Ba^2b^4-63\,Aab^5)x\right)x^{\frac{9}{2}}+10\,(359\,(13\,Ba^2b^4-3\,Aab^5)x^2+183\,(11\,B}{}$$

$$+\frac{105\,(11\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^6}+\frac{7\left(11\,(13\,Bab-3\,Ab^2)x^{\frac{3}{2}}-30\,(11\,Ba^2-3\,Aab)\sqrt{x}\right)}{128\,ab^6}$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(5*((2747*B*a*b^5 - 693*A*b^6)*x^2 + 3*(437*B*a^2*b^4 - 63*A*a*b^5
)*x)*x^(9/2) + 10*(359*(13*B*a^2*b^4 - 3*A*a*b^5)*x^2 + 183*(11*B*a^3*b^3
- A*a^2*b^4)*x)*x^(7/2) + 20*(242*(13*B*a^3*b^3 - 3*A*a^2*b^4)*x^2 + 117*(
11*B*a^4*b^2 - A*a^3*b^3)*x)*x^(5/2) + 198*(15*(13*B*a^4*b^2 - 3*A*a^3*b^3
)*x^2 + 7*(11*B*a^5*b - A*a^4*b^2)*x)*x^(3/2) + 63*(11*(13*B*a^5*b - 3*A*a
^4*b^2)*x^2 + 5*(11*B*a^6 - A*a^5*b)*x)*sqrt(x))/(a*b^10*x^5 + 5*a^2*b^9*x
^4 + 10*a^3*b^8*x^3 + 10*a^4*b^7*x^2 + 5*a^5*b^6*x + a^6*b^5) + 105/64*(11
*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^6) + 7/128*(11*
(13*B*a*b - 3*A*b^2)*x^(3/2) - 30*(11*B*a^2 - 3*A*a*b)*sqrt(x))/(a*b^6)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{105\,(11\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^6\mathrm{sgn}\,(bx+a)}$$

$$-\frac{2295\,Ba^2b^3x^{\frac{7}{2}}-975\,Aab^4x^{\frac{7}{2}}+5855\,Ba^3b^2x^{\frac{5}{2}}-2295\,Aa^2b^3x^{\frac{5}{2}}+5153\,Ba^4bx^{\frac{3}{2}}-1929\,Aa^3b^2x^{\frac{3}{2}}+1545\,B}{192\,(bx+a)^4b^6\mathrm{sgn}\,(bx+a)}$$

$$+\frac{2\left(Bb^{10}x^{\frac{3}{2}}-15\,Bab^9\sqrt{x}+3\,Ab^{10}\sqrt{x}\right)}{3\,b^{15}\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
105/64*(11*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^6*sgn
(b*x + a)) - 1/192*(2295*B*a^2*b^3*x^(7/2) - 975*A*a*b^4*x^(7/2) + 5855*B*
a^3*b^2*x^(5/2) - 2295*A*a^2*b^3*x^(5/2) + 5153*B*a^4*b*x^(3/2) - 1929*A*a
^3*b^2*x^(3/2) + 1545*B*a^5*sqrt(x) - 561*A*a^4*b*sqrt(x))/((b*x + a)^4*b^
6*sgn(b*x + a)) + 2/3*(B*b^10*x^(3/2) - 15*B*a*b^9*sqrt(x) + 3*A*b^10*sqrt
(x))/(b^15*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\int \frac{x^{9/2}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int((x^(9/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(9/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.56

$$\int \frac{x^{9/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^4+945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+945\sqrt{b}\,\sqrt{a}\,atan}{}$$

input
```
int(x^(9/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 945*sqrt(b
)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x + 945*sqrt(b)*sqrt(
a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 315*sqrt(b)*sqrt(a
)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**3*x**3 - 315*sqrt(x)*a**4*b - 8
40*sqrt(x)*a**3*b**2*x - 693*sqrt(x)*a**2*b**3*x**2 - 144*sqrt(x)*a*b**4*x
**3 + 16*sqrt(x)*b**5*x**4)/(24*b**6*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3))
```

3.457 $\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 285

$$\begin{aligned}\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= -\frac{(35Ab-187aB)\sqrt{x}}{64b^5\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{(Ab-aB)x^{7/2}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(7Ab-15aB)x^{5/2}}{24b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &\quad -\frac{(35Ab-123aB)x^{3/2}}{96b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}+\frac{2B\sqrt{x}(a+bx)}{b^5\sqrt{a^2+2abx+b^2x^2}} \\ &\quad +\frac{35(Ab-9aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64\sqrt{a}b^{11/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/64*(35*A*b-187*B*a)*x^(1/2)/b^5/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(7/2)
/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(7*A*b-15*B*a)*x^(5/2)/b^3/(b*x+a)^2
/((b*x+a)^2)^(1/2)-1/96*(35*A*b-123*B*a)*x^(3/2)/b^4/(b*x+a)/((b*x+a)^2)^(
1/2)+2*B*x^(1/2)*(b*x+a)/b^5/((b*x+a)^2)^(1/2)+35/64*(A*b-9*B*a)*(b*x+a)*a
rctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(11/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.51

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\sqrt{b}\sqrt{x}(945a^4B-105a^3b(A-33Bx)+3b^4x^3(-93A+128Bx)+7a^2b^2x(-\cdots}{192b^{11/2}(a+b\cdots}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*(945*a^4*B - 105*a^3*b*(A - 33*B*x) + 3*b^4*x^3*(-93*A +
128*B*x) + 7*a^2*b^2*x*(-55*A + 657*B*x) + a*b^3*x^2*(-511*A + 2511*B*x))
+ (105*(A*b - 9*a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[a
])/(192*b^(11/2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {1187, 27, 87, 51, 51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \ 1187$$

$$\frac{b^5(a+bx)\int \frac{x^{7/2}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)\int \frac{x^{7/2}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 87$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\int \frac{x^{7/2}}{(a+bx)^4}dx}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\int \frac{x^{5/2}}{(a+bx)^3}dx}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\left(\frac{5\int \frac{x^{3/2}}{(a+bx)^2}dx}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow 73$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx)\left(\frac{x^{9/2}(Ab-aB)}{4ab(a+bx)^4}-\frac{(Ab-9aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{7/2}}{3b(a+bx)^3}\right)}{8ab}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(9/2))/(4*a*b*(a + b*x)^4) - ((A*b - 9*a*B)*(-1
/3*x^(7/2)/(b*(a + b*x)^3) + (7*(-1/2*x^(5/2)/(b*(a + b*x)^2) + (5*(-(x^(3
/2)/(b*(a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)))/(6*b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*
x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

method	result
risch	$\frac{2B\sqrt{x}\,\sqrt{(bx+a)^2}}{b^5(bx+a)}+\frac{\left(\frac{2\left(-\frac{93}{128}A\,b^4+\frac{325}{128}Ba\,b^3\right)x^{\frac{7}{2}}+2\left(\frac{765}{128}B\,a^2b^2-\frac{511}{384}Aa\,b^3\right)x^{\frac{5}{2}}-\frac{a^2b(385Ab-1929Ba)x^{\frac{3}{2}}}{192}+2\left(\frac{187}{128}a^4B-\frac{35}{128}A\,a^3b\right)\sqrt{x}}{(bx+a)^4}+\right.}{b^5(bx+a)}$
default	$-\frac{\left(279A\sqrt{ab}\,x^{\frac{7}{2}}b^4-2511B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^3+511A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^3-105A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^5x^4-4599B\sqrt{ab}\,x^{\frac{5}{2}}a^2b^2-384B\sqrt{ab}\,x^{\frac{9}{2}}b^4+945B\right.}{}$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*B/b^5*x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/b^5*(2*((-93/128*A*b^4+325/128
*B*a*b^3)*x^(7/2)+(765/128*B*a^2*b^2-511/384*A*a*b^3)*x^(5/2)-1/384*a^2*b*
(385*A*b-1929*B*a)*x^(3/2)+(187/128*a^4*B-35/128*A*a^3*b)*x^(1/2))/(b*x+a)
^4+35/64*(A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2
)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.95

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[\frac{105\,(9\,Ba^5-Aa^4b+(9\,Bab^4-Ab^5)x^4+4\,(9\,Ba^2b^3-Aab^4)x^3+6\,(9\,Ba^3\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[1/384*(105*(9*B*a^5 - A*a^4*b + (9*B*a*b^4 - A*b^5)*x^4 + 4*(9*B*a^2*b^3
- A*a*b^4)*x^3 + 6*(9*B*a^3*b^2 - A*a^2*b^3)*x^2 + 4*(9*B*a^4*b - A*a^3*b^
2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(384*
B*a*b^5*x^4 + 945*B*a^5*b - 105*A*a^4*b^2 + 279*(9*B*a^2*b^4 - A*a*b^5)*x^
3 + 511*(9*B*a^3*b^3 - A*a^2*b^4)*x^2 + 385*(9*B*a^4*b^2 - A*a^3*b^3)*x)*s
qrt(x))/(a*b^10*x^4 + 4*a^2*b^9*x^3 + 6*a^3*b^8*x^2 + 4*a^4*b^7*x + a^5*b^
6), 1/192*(105*(9*B*a^5 - A*a^4*b + (9*B*a*b^4 - A*b^5)*x^4 + 4*(9*B*a^2*b
^3 - A*a*b^4)*x^3 + 6*(9*B*a^3*b^2 - A*a^2*b^3)*x^2 + 4*(9*B*a^4*b - A*a^3
*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (384*B*a*b^5*x^4 + 945*
B*a^5*b - 105*A*a^4*b^2 + 279*(9*B*a^2*b^4 - A*a*b^5)*x^3 + 511*(9*B*a^3*b
^3 - A*a^2*b^4)*x^2 + 385*(9*B*a^4*b^2 - A*a^3*b^3)*x)*sqrt(x))/(a*b^10*x^
4 + 4*a^2*b^9*x^3 + 6*a^3*b^8*x^2 + 4*a^4*b^7*x + a^5*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.31

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{105\,(3\,(11\,Bab^5-Ab^6)x^2+(9\,Ba^2b^4+Aab^5)x)x^{\frac{9}{2}}+30\,((359\,Ba^2b^4-21\,A}{}$$
$$-\frac{35\,(9\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^5}-\frac{7\left(3\,(11\,Bab-Ab^2)x^{\frac{3}{2}}-10\,(9\,Ba^2-Aab)\sqrt{x}\right)}{128\,a^2b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
1/1920*(105*(3*(11*B*a*b^5 - A*b^6)*x^2 + (9*B*a^2*b^4 + A*a*b^5)*x)*x^(9/
2) + 30*((359*B*a^2*b^4 - 21*A*a*b^5)*x^2 + (61*B*a^3*b^3 + 21*A*a^2*b^4)*
x)*x^(7/2) + 20*(66*(11*B*a^3*b^3 - A*a^2*b^4)*x^2 + 13*(9*B*a^4*b^2 + A*a
^3*b^3)*x)*x^(5/2) + 2*(405*(11*B*a^4*b^2 - A*a^3*b^3)*x^2 + 77*(9*B*a^5*b
 + A*a^4*b^2)*x)*x^(3/2) + 7*(27*(11*B*a^5*b - A*a^4*b^2)*x^2 + 5*(9*B*a^6
 + A*a^5*b)*x)*sqrt(x))/(a^2*b^9*x^5 + 5*a^3*b^8*x^4 + 10*a^4*b^7*x^3 + 10
*a^5*b^6*x^2 + 5*a^6*b^5*x + a^7*b^4) - 35/64*(9*B*a - A*b)*arctan(b*sqrt(
x)/sqrt(a*b))/(sqrt(a*b)*b^5) - 7/128*(3*(11*B*a*b - A*b^2)*x^(3/2) - 10*(
9*B*a^2 - A*a*b)*sqrt(x))/(a^2*b^5)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

$$\int \frac{x^{7/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{2\,B\sqrt{x}}{b^5\mathrm{sgn}\,(bx+a)}-\frac{35\,(9\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}b^5\mathrm{sgn}\,(bx+a)}$$
$$+\frac{975\,Bab^3x^{\frac{7}{2}}-279\,Ab^4x^{\frac{7}{2}}+2295\,Ba^2b^2x^{\frac{5}{2}}-511\,Aab^3x^{\frac{5}{2}}+1929\,Ba^3bx^{\frac{3}{2}}-385\,Aa^2b^2x^{\frac{3}{2}}+561\,Ba^4\sqrt{x}-}{192\,(bx+a)^4b^5\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/(b^5*sgn(b*x + a)) - 35/64*(9*B*a - A*b)*arctan(b*sqrt(x)/sqrt
(a*b))/(sqrt(a*b)*b^5*sgn(b*x + a)) + 1/192*(975*B*a*b^3*x^(7/2) - 279*A*b
^4*x^(7/2) + 2295*B*a^2*b^2*x^(5/2) - 511*A*a*b^3*x^(5/2) + 1929*B*a^3*b*x
^(3/2) - 385*A*a^2*b^2*x^(3/2) + 561*B*a^4*sqrt(x) - 105*A*a^3*b*sqrt(x))/
((b*x + a)^4*b^5*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{x^{7/2}\,(A+B\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.60

$$\int \frac{x^{7/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx-315\sqrt{b}\,\sqrt{a}\,ata}{24}$$

input
```
int(x^(7/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 - 315*sqr
t(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x - 315*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 - 105*sqrt(b)*sqrt(a
)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x**3 + 105*sqrt(x)*a**3*b + 280
*sqrt(x)*a**2*b**2*x + 231*sqrt(x)*a*b**3*x**2 + 48*sqrt(x)*b**4*x**3)/(24
*b**5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.458 $\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 252

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{(5Ab-93aB)\sqrt{x}}{64ab^4\sqrt{a^2+2abx+b^2x^2}} - \frac{(Ab-aB)x^{5/2}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(5Ab-13aB)x^{3/2}}{24b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} - \frac{(5Ab-29aB)\sqrt{x}}{32b^4(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{5(Ab+7aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{3/2}b^{9/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/64*(5*A*b-93*B*a)*x^(1/2)/a/b^4/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(5/2)/
b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(5*A*b-13*B*a)*x^(3/2)/b^3/(b*x+a)^2/
((b*x+a)^2)^(1/2)-1/32*(5*A*b-29*B*a)*x^(1/2)/b^4/(b*x+a)/((b*x+a)^2)^(1/2
)+5/64*(A*b+7*B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(9/2)
/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{-\sqrt{a}\sqrt{b}\sqrt{x}(105a^4B-15Ab^4x^3+5a^3b(3A+77Bx)+ab^3x^2(73A+279Bx}{192a^{3/2}b^{9/2}(a+bx)^3}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(-(Sqrt[a]*Sqrt[b]*Sqrt[x]*(105*a^4*B - 15*A*b^4*x^3 + 5*a^3*b*(3*A + 77*B
*x) + a*b^3*x^2*(73*A + 279*B*x) + a^2*b^2*x*(55*A + 511*B*x))) + 15*(A*b
+ 7*a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*a^(3/2)*b^(9/
2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 51, 51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1187$$

$$\frac{b^5(a+bx)\int \frac{x^{5/2}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\int \frac{x^{5/2}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 87$$

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\int \frac{x^{5/2}}{(a+bx)^4}dx}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 51$$

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\left(\frac{5\int \frac{x^{3/2}}{(a+bx)^3}dx}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 51$$

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\left(\frac{5\left(\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 51$$

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$\big\downarrow$ 218

$$\frac{(a+bx)\left(\frac{(7aB+Ab)\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}\right)}{6b}-\frac{x^{5/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x)^4) + ((A*b + 7*a*B)*(-1
/3*x^(5/2)/(b*(a + b*x)^3) + (5*(-1/2*x^(3/2)/(b*(a + b*x)^2) + (3*(-(Sqrt
[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2))))
/(4*b)))/(6*b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(169) = 338.

Time = 1.24 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(15A\sqrt{ab}\,x^{\frac{7}{2}}b^{4}-279B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^{3}-73A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^{3}+15A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^{5}x^{4}-511B\sqrt{ab}\,x^{\frac{5}{2}}a^{2}b^{2}+105B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{4}x^{4}+60A\right.}{}$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(15*A*(a*b)^(1/2)*x^(7/2)*b^4-279*B*(a*b)^(1/2)*x^(7/2)*a*b^3-73*A*(
a*b)^(1/2)*x^(5/2)*a*b^3+15*A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^5*x^4-511*B*
(a*b)^(1/2)*x^(5/2)*a^2*b^2+105*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^4+
60*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^3+420*B*arctan(b*x^(1/2)/(a*b)^
(1/2))*a^2*b^3*x^3-55*A*(a*b)^(1/2)*x^(3/2)*a^2*b^2+90*A*arctan(b*x^(1/2)/
(a*b)^(1/2))*a^2*b^3*x^2-385*B*(a*b)^(1/2)*x^(3/2)*a^3*b+630*B*arctan(b*x^
(1/2)/(a*b)^(1/2))*a^3*b^2*x^2+60*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3*b^2*
x+420*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b*x-15*A*(a*b)^(1/2)*x^(1/2)*a^3
*b+15*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b-105*B*(a*b)^(1/2)*x^(1/2)*a^4+
105*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^5)*(b*x+a)/(a*b)^(1/2)/b^4/a/((b*x+a
)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.08

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{15\,(7\,Ba^5+Aa^4b+(7\,Bab^4+Ab^5)x^4+4\,(7\,Ba^2b^3+Aab^4)x^3+6\,(7\,Ba}{}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(15*(7*B*a^5 + A*a^4*b + (7*B*a*b^4 + A*b^5)*x^4 + 4*(7*B*a^2*b^3
+ A*a*b^4)*x^3 + 6*(7*B*a^3*b^2 + A*a^2*b^3)*x^2 + 4*(7*B*a^4*b + A*a^3*b^
2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(105*
B*a^5*b + 15*A*a^4*b^2 + 3*(93*B*a^2*b^4 - 5*A*a*b^5)*x^3 + 73*(7*B*a^3*b^
3 + A*a^2*b^4)*x^2 + 55*(7*B*a^4*b^2 + A*a^3*b^3)*x)*sqrt(x))/(a^2*b^9*x^4
 + 4*a^3*b^8*x^3 + 6*a^4*b^7*x^2 + 4*a^5*b^6*x + a^6*b^5), -1/192*(15*(7*B
*a^5 + A*a^4*b + (7*B*a*b^4 + A*b^5)*x^4 + 4*(7*B*a^2*b^3 + A*a*b^4)*x^3 +
 6*(7*B*a^3*b^2 + A*a^2*b^3)*x^2 + 4*(7*B*a^4*b + A*a^3*b^2)*x)*sqrt(a*b)*
arctan(sqrt(a*b)/(b*sqrt(x))) + (105*B*a^5*b + 15*A*a^4*b^2 + 3*(93*B*a^2*
b^4 - 5*A*a*b^5)*x^3 + 73*(7*B*a^3*b^3 + A*a^2*b^4)*x^2 + 55*(7*B*a^4*b^2
+ A*a^3*b^3)*x)*sqrt(x))/(a^2*b^9*x^4 + 4*a^3*b^8*x^3 + 6*a^4*b^7*x^2 + 4*
a^5*b^6*x + a^6*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(169) = 338$.

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.49

$$\int \frac{x^{5/2}(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{5\,(7\,(9\,Bab^5+Ab^6)x^2-3\,(7\,Ba^2b^4+3\,Aab^5)x)x^{\frac{9}{2}}+10\,(7\,(9\,Ba^2b^4+Aab^5)x^2-9\,(7\,Ba^3b^3+3\,Aa^2b^4)x)}{}$$

$$+\frac{5\,(7\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}ab^4}+\frac{7\,(9\,Bab+Ab^2)x^{\frac{3}{2}}-30\,(7\,Ba^2+Aab)\sqrt{x}}{384\,a^3b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(5*(7*(9*B*a*b^5 + A*b^6)*x^2 - 3*(7*B*a^2*b^4 + 3*A*a*b^5)*x)*x^(
9/2) + 10*(7*(9*B*a^2*b^4 + A*a*b^5)*x^2 - 9*(7*B*a^3*b^3 + 3*A*a^2*b^4)*x
)*x^(7/2) + 20*(2*(33*B*a^3*b^3 - 7*A*a^2*b^4)*x^2 - (13*B*a^4*b^2 + 33*A*
a^3*b^3)*x)*x^(5/2) + 2*(45*(9*B*a^4*b^2 + A*a^3*b^3)*x^2 - 11*(7*B*a^5*b
+ 3*A*a^4*b^2)*x)*x^(3/2) + (21*(9*B*a^5*b + A*a^4*b^2)*x^2 - 5*(7*B*a^6 +
 3*A*a^5*b)*x)*sqrt(x))/(a^3*b^8*x^5 + 5*a^4*b^7*x^4 + 10*a^5*b^6*x^3 + 10
*a^6*b^5*x^2 + 5*a^7*b^4*x + a^8*b^3) + 5/64*(7*B*a + A*b)*arctan(b*sqrt(x
)/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/384*(7*(9*B*a*b + A*b^2)*x^(3/2) - 30*(
7*B*a^2 + A*a*b)*sqrt(x))/(a^3*b^4)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{5\,(7\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}ab^4\mathrm{sgn}\,(bx+a)} - \frac{279\,Bab^3x^{\frac{7}{2}} - 15\,Ab^4x^{\frac{7}{2}} + 511\,Ba^2b^2x^{\frac{5}{2}} + 73\,Aab^3x^{\frac{5}{2}} + 385\,Ba^3bx^{\frac{3}{2}} + 55\,Aa^2b^2x^{\frac{3}{2}} + 105\,Ba^4\sqrt{x} + 15\,Aa}{192\,(bx+a)^4ab^4\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/64*(7*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^4*sgn(b*x +
a)) - 1/192*(279*B*a*b^3*x^(7/2) - 15*A*b^4*x^(7/2) + 511*B*a^2*b^2*x^(5/2
) + 73*A*a*b^3*x^(5/2) + 385*B*a^3*b*x^(3/2) + 55*A*a^2*b^2*x^(3/2) + 105*
B*a^4*sqrt(x) + 15*A*a^3*b*sqrt(x))/((b*x + a)^4*a*b^4*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\int \frac{x^{5/2}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.65

$$\int \frac{x^{5/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx=\frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+45\sqrt{b}\,\sqrt{a}\,atan\left(\cdots\right.}{24a\,b^4\left(b^3x^3+\cdots\right.}$$

input
```
int(x^(5/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 45*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 45*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 15*sqrt(b)*sqrt(a)*atan(
(sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x**3 - 15*sqrt(x)*a**3*b - 40*sqrt(x)*
a**2*b**2*x - 33*sqrt(x)*a*b**3*x**2)/(24*a*b**4*(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3))
```

3.459 $\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 256

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{(3Ab+5aB)\sqrt{x}}{64a^2b^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(Ab-aB)x^{3/2}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} - \frac{(3Ab-11aB)\sqrt{x}}{24b^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{(3Ab-59aB)\sqrt{x}}{96ab^3(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(3Ab+5aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{5/2}b^{7/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
1/64*(3*A*b+5*B*a)*x^(1/2)/a^2/b^3/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(3/2)
/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(3*A*b-11*B*a)*x^(1/2)/b^3/(b*x+a)^2
/((b*x+a)^2)^(1/2)+1/96*(3*A*b-59*B*a)*x^(1/2)/a/b^3/(b*x+a)/((b*x+a)^2)^(
1/2)+1/64*(3*A*b+5*B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^
(7/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{x}(-15a^4B+9Ab^4x^3+3ab^3x^2(11A+5Bx)-a^3b(9A+55Bx)-a}{192a^{5/2}b^{7/2}(a+bx)^3\sqrt{}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*Sqrt[x]*(-15*a^4*B + 9*A*b^4*x^3 + 3*a*b^3*x^2*(11*A + 5*
B*x) - a^3*b*(9*A + 55*B*x) - a^2*b^2*x*(33*A + 73*B*x)) + 3*(3*A*b + 5*a*
B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*a^(5/2)*b^(7/2)*(a
+ b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 51, 51, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^5(a+bx)\int \frac{x^{3/2}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{x^{3/2}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\int \frac{x^{3/2}}{(a+bx)^4}dx}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{51}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\left(\frac{\int \frac{\sqrt{x}}{(a+bx)^3}dx}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{51}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\left(\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}}{2b}-\frac{x^{3/2}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x)^4) + ((3*A*b + 5*a*B)*(
-1/3*x^(3/2)/(b*(a + b*x)^3) + (-1/2*Sqrt[x]/(b*(a + b*x)^2) + (Sqrt[x]/(a
*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]))/(4*b))/
(2*b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.

Time = 1.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.39

method	result
default	$\frac{\left(9A\sqrt{ab}\,x^{\frac{7}{2}}b^4+15B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^3+33A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^3+9A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^5x^4-73B\sqrt{ab}\,x^{\frac{5}{2}}a^2b^2+15B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^4x^4+36A\arctan\right.}{}$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(9*A*(a*b)^(1/2)*x^(7/2)*b^4+15*B*(a*b)^(1/2)*x^(7/2)*a*b^3+33*A*(a*
b)^(1/2)*x^(5/2)*a*b^3+9*A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^5*x^4-73*B*(a*b
)^(1/2)*x^(5/2)*a^2*b^2+15*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^4+36*A*
arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^3+60*B*arctan(b*x^(1/2)/(a*b)^(1/2))
*a^2*b^3*x^3-33*A*(a*b)^(1/2)*x^(3/2)*a^2*b^2+54*A*arctan(b*x^(1/2)/(a*b)^
(1/2))*a^2*b^3*x^2-55*B*(a*b)^(1/2)*x^(3/2)*a^3*b+90*B*arctan(b*x^(1/2)/(a
*b)^(1/2))*a^3*b^2*x^2+36*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3*b^2*x+60*B*a
rctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b*x-9*A*(a*b)^(1/2)*x^(1/2)*a^3*b+9*A*arc
tan(b*x^(1/2)/(a*b)^(1/2))*a^4*b-15*B*(a*b)^(1/2)*x^(1/2)*a^4+15*B*arctan(
b*x^(1/2)/(a*b)^(1/2))*a^5)*(b*x+a)/(a*b)^(1/2)/b^3/a^2/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.10

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{3\,(5\,Ba^5+3\,Aa^4b+(5\,Bab^4+3\,Ab^5)x^4+4\,(5\,Ba^2b^3+3\,Aab^4)x^3+6\,(5\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(3*(5*B*a^5 + 3*A*a^4*b + (5*B*a*b^4 + 3*A*b^5)*x^4 + 4*(5*B*a^2*b
^3 + 3*A*a*b^4)*x^3 + 6*(5*B*a^3*b^2 + 3*A*a^2*b^3)*x^2 + 4*(5*B*a^4*b + 3
*A*a^3*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))
+ 2*(15*B*a^5*b + 9*A*a^4*b^2 - 3*(5*B*a^2*b^4 + 3*A*a*b^5)*x^3 + (73*B*a^
3*b^3 - 33*A*a^2*b^4)*x^2 + 11*(5*B*a^4*b^2 + 3*A*a^3*b^3)*x)*sqrt(x))/(a^
3*b^8*x^4 + 4*a^4*b^7*x^3 + 6*a^5*b^6*x^2 + 4*a^6*b^5*x + a^7*b^4), -1/192
*(3*(5*B*a^5 + 3*A*a^4*b + (5*B*a*b^4 + 3*A*b^5)*x^4 + 4*(5*B*a^2*b^3 + 3*
A*a*b^4)*x^3 + 6*(5*B*a^3*b^2 + 3*A*a^2*b^3)*x^2 + 4*(5*B*a^4*b + 3*A*a^3*
b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (15*B*a^5*b + 9*A*a^4*b^
2 - 3*(5*B*a^2*b^4 + 3*A*a*b^5)*x^3 + (73*B*a^3*b^3 - 33*A*a^2*b^4)*x^2 +
11*(5*B*a^4*b^2 + 3*A*a^3*b^3)*x)*sqrt(x))/(a^3*b^8*x^4 + 4*a^4*b^7*x^3 +
6*a^5*b^6*x^2 + 4*a^6*b^5*x + a^7*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(173) = 346.

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.45

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = -\frac{5\left((7\,Bab^5+3\,Ab^6)x^2-9\,(Ba^2b^4+Aab^5)x\right)x^{\frac{9}{2}}+10\left((7\,Ba^2b^4+3\,Aab^5)x^2-27\,(Ba^3b^3+Aa^2b^4)x\right)x^{\frac{7}{2}}-\cdots}{1\cdots} + \frac{(5\,Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^2b^3} + \frac{(7\,Bab+3\,Ab^2)x^{\frac{3}{2}}-6\,(5\,Ba^2+3\,Aab)\sqrt{x}}{384\,a^4b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(5*((7*B*a*b^5 + 3*A*b^6)*x^2 - 9*(B*a^2*b^4 + A*a*b^5)*x)*x^(9/2)
 + 10*((7*B*a^2*b^4 + 3*A*a*b^5)*x^2 - 27*(B*a^3*b^3 + A*a^2*b^4)*x)*x^(7/
2) - 20*(2*(7*B*a^3*b^3 + 3*A*a^2*b^4)*x^2 + 33*(B*a^4*b^2 + A*a^3*b^3)*x)
*x^(5/2) + 6*(5*(3*B*a^4*b^2 - 17*A*a^3*b^3)*x^2 - (11*B*a^5*b + 139*A*a^4
*b^2)*x)*x^(3/2) + 3*((7*B*a^5*b + 3*A*a^4*b^2)*x^2 - 5*(B*a^6 + A*a^5*b)*
x)*sqrt(x))/(a^4*b^7*x^5 + 5*a^5*b^6*x^4 + 10*a^6*b^5*x^3 + 10*a^7*b^4*x^2
 + 5*a^8*b^3*x + a^9*b^2) + 1/64*(5*B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b
))/(sqrt(a*b)*a^2*b^3) + 1/384*((7*B*a*b + 3*A*b^2)*x^(3/2) - 6*(5*B*a^2 +
 3*A*a*b)*sqrt(x))/(a^4*b^3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{(5\,Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^2b^3\mathrm{sgn}\,(bx+a)} + \frac{15\,Bab^3x^{\frac{7}{2}}+9\,Ab^4x^{\frac{7}{2}}-73\,Ba^2b^2x^{\frac{5}{2}}+33\,Aab^3x^{\frac{5}{2}}-55\,Ba^3bx^{\frac{3}{2}}-33\,Aa^2b^2x^{\frac{3}{2}}-15\,Ba^4\sqrt{x}-9\,Aa^3b\sqrt{x}}{192\,(bx+a)^4a^2b^3\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*(5*B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b^3*sgn(b*
x + a)) + 1/192*(15*B*a*b^3*x^(7/2) + 9*A*b^4*x^(7/2) - 73*B*a^2*b^2*x^(5/
2) + 33*A*a*b^3*x^(5/2) - 55*B*a^3*b*x^(3/2) - 33*A*a^2*b^2*x^(3/2) - 15*B
*a^4*sqrt(x) - 9*A*a^3*b*sqrt(x))/((b*x + a)^4*a^2*b^3*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{x^{3/2}\,(A+B\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2)^{5/2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\int \frac{x^{3/2}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{24a^2b^3\left(b^3x^3+3a\,b\right.}$$

input
```
int(x^(3/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 9*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 9*sqrt(b)*sqrt(a)*ata
n((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 3*sqrt(b)*sqrt(a)*atan((sqr
t(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x**3 - 3*sqrt(x)*a**3*b - 8*sqrt(x)*a**2*b
**2*x + 3*sqrt(x)*a*b**3*x**2)/(24*a**2*b**3*(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3))
```

3.460 $\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\begin{aligned}\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx &= \frac{(5Ab+3aB)\sqrt{x}}{64a^3b^2\sqrt{a^2+2abx+b^2x^2}} \\ &- \frac{(Ab-aB)\sqrt{x}}{4b^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-9aB)\sqrt{x}}{24ab^2(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{(5Ab+3aB)\sqrt{x}}{96a^2b^2(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{(5Ab+3aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{7/2}b^{5/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
1/64*(5*A*b+3*B*a)*x^(1/2)/a^3/b^2/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(1/2)
/b^2/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/24*(A*b-9*B*a)*x^(1/2)/a/b^2/(b*x+a)^2/
((b*x+a)^2)^(1/2)+1/96*(5*A*b+3*B*a)*x^(1/2)/a^2/b^2/(b*x+a)/((b*x+a)^2)^(
1/2)+1/64*(5*A*b+3*B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)/b^
(5/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{x}(-9a^4B+15Ab^4x^3+ab^3x^2(55A+9Bx)-3a^3b(5A+11Bx)+a}{192a^{7/2}b^{5/2}(a+bx)^3\sqrt{}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*Sqrt[x]*(-9*a^4*B + 15*A*b^4*x^3 + a*b^3*x^2*(55*A + 9*B*
x) - 3*a^3*b*(5*A + 11*B*x) + a^2*b^2*x*(73*A + 33*B*x)) + 3*(5*A*b + 3*a*
B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*a^(7/2)*b^(5/2)*(a
+ b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 51, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}dx$$

$$\Big\downarrow \; 1187$$

$$\frac{b^5(a+bx)\int \frac{\sqrt{x}(A+Bx)}{b^5(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 87$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\int \frac{\sqrt{x}}{(a+bx)^4}dx}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 51$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 52$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\left(\frac{\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 52$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\left(\frac{\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\left(\frac{\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\downarrow \text{218}$$

$$\frac{(a+bx)\left(\frac{(3aB+5Ab)\left(\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}}{6b}-\frac{\sqrt{x}}{3b(a+bx)^3}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^(5/2),x]
```

output
```
((a + b*x)*(((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x)^4) + ((5*A*b + 3*a*B)*(
-1/3*Sqrt[x]/(b*(a + b*x)^3) + (Sqrt[x]/(2*a*(a + b*x)^2) + (3*(Sqrt[x]/(a
*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])))/(4*a))
/(6*b)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(175) = 350$.

Time = 1.17 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.38

method	result
default	$\frac{\left(15A\sqrt{ab}\,x^{\frac{7}{2}}b^{4}+9B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^{3}+55A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^{3}+15A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^{5}x^{4}+33B\sqrt{ab}\,x^{\frac{5}{2}}a^{2}b^{2}+9B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^{4}x^{4}+60A\arctan\right.}{}$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(15*A*(a*b)^(1/2)*x^(7/2)*b^4+9*B*(a*b)^(1/2)*x^(7/2)*a*b^3+55*A*(a*
b)^(1/2)*x^(5/2)*a*b^3+15*A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^5*x^4+33*B*(a*
b)^(1/2)*x^(5/2)*a^2*b^2+9*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^4+60*A*
arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^3+36*B*arctan(b*x^(1/2)/(a*b)^(1/2))
*a^2*b^3*x^3+73*A*(a*b)^(1/2)*x^(3/2)*a^2*b^2+90*A*arctan(b*x^(1/2)/(a*b)^
(1/2))*a^2*b^3*x^2-33*B*(a*b)^(1/2)*x^(3/2)*a^3*b+54*B*arctan(b*x^(1/2)/(a
*b)^(1/2))*a^3*b^2*x^2+60*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3*b^2*x+36*B*a
rctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b*x-15*A*(a*b)^(1/2)*x^(1/2)*a^3*b+15*A*a
rctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b-9*B*(a*b)^(1/2)*x^(1/2)*a^4+9*B*arctan(
b*x^(1/2)/(a*b)^(1/2))*a^5)*(b*x+a)/(a*b)^(1/2)/b^2/a^3/((b*x+a)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{3\,(3\,Ba^5+5\,Aa^4b+(3\,Bab^4+5\,Ab^5)x^4+4\,(3\,Ba^2b^3+5\,Aab^4)x^3+6\,(3\cdots}{\cdots}\right.$$

$$-\frac{3\,(3\,Ba^5+5\,Aa^4b+(3\,Bab^4+5\,Ab^5)x^4+4\,(3\,Ba^2b^3+5\,Aab^4)x^3+6\,(3\,Ba^3b^2+5\,Aa^2b^3)x^2+4\,(3\,Ba^4b\cdots}{192\,(a^4b\cdots}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(3*(3*B*a^5 + 5*A*a^4*b + (3*B*a*b^4 + 5*A*b^5)*x^4 + 4*(3*B*a^2*b
^3 + 5*A*a*b^4)*x^3 + 6*(3*B*a^3*b^2 + 5*A*a^2*b^3)*x^2 + 4*(3*B*a^4*b + 5
*A*a^3*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))
+ 2*(9*B*a^5*b + 15*A*a^4*b^2 - 3*(3*B*a^2*b^4 + 5*A*a*b^5)*x^3 - 11*(3*B*
a^3*b^3 + 5*A*a^2*b^4)*x^2 + (33*B*a^4*b^2 - 73*A*a^3*b^3)*x)*sqrt(x))/(a^
4*b^7*x^4 + 4*a^5*b^6*x^3 + 6*a^6*b^5*x^2 + 4*a^7*b^4*x + a^8*b^3), -1/192
*(3*(3*B*a^5 + 5*A*a^4*b + (3*B*a*b^4 + 5*A*b^5)*x^4 + 4*(3*B*a^2*b^3 + 5*
A*a*b^4)*x^3 + 6*(3*B*a^3*b^2 + 5*A*a^2*b^3)*x^2 + 4*(3*B*a^4*b + 5*A*a^3*
b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (9*B*a^5*b + 15*A*a^4*b^
2 - 3*(3*B*a^2*b^4 + 5*A*a*b^5)*x^3 - 11*(3*B*a^3*b^3 + 5*A*a^2*b^4)*x^2 +
 (33*B*a^4*b^2 - 73*A*a^3*b^3)*x)*sqrt(x))/(a^4*b^7*x^4 + 4*a^5*b^6*x^3 +
6*a^6*b^5*x^2 + 4*a^7*b^4*x + a^8*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(176) = 352.

Time = 0.19 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx =$$

$$-\frac{15\left((Bab^5+Ab^6)x^2-(3\,Ba^2b^4+7\,Aab^5)x\right)x^{\frac{9}{2}}+30\left((Ba^2b^4+Aab^5)x^2-3\,(3\,Ba^3b^3+7\,Aa^2b^4)x\right)x^{\frac{7}{2}}-2}{}$$

$$+\frac{(3\,Ba+5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^3b^2}+\frac{(Bab+Ab^2)x^{\frac{3}{2}}-2\,(3\,Ba^2+5\,Aab)\sqrt{x}}{128\,a^5b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(15*((B*a*b^5 + A*b^6)*x^2 - (3*B*a^2*b^4 + 7*A*a*b^5)*x)*x^(9/2)
+ 30*((B*a^2*b^4 + A*a*b^5)*x^2 - 3*(3*B*a^3*b^3 + 7*A*a^2*b^4)*x)*x^(7/2)
 - 20*(6*(B*a^3*b^3 + A*a^2*b^4)*x^2 + 11*(3*B*a^4*b^2 + 7*A*a^3*b^3)*x)*x
^(5/2) - 2*(255*(B*a^4*b^2 + A*a^3*b^3)*x^2 + 139*(3*B*a^5*b + 7*A*a^4*b^2
)*x)*x^(3/2) + (3*(3*B*a^5*b - 253*A*a^4*b^2)*x^2 - 5*(3*B*a^6 + 263*A*a^5
*b)*x)*sqrt(x))/(a^5*b^6*x^5 + 5*a^6*b^5*x^4 + 10*a^7*b^4*x^3 + 10*a^8*b^3
*x^2 + 5*a^9*b^2*x + a^10*b) + 1/64*(3*B*a + 5*A*b)*arctan(b*sqrt(x)/sqrt(
a*b))/(sqrt(a*b)*a^3*b^2) + 1/128*((B*a*b + A*b^2)*x^(3/2) - 2*(3*B*a^2 +
5*A*a*b)*sqrt(x))/(a^5*b^2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{(3\,Ba+5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^3b^2\mathrm{sgn}\,(bx+a)} + \frac{9\,Bab^3x^{\frac{7}{2}}+15\,Ab^4x^{\frac{7}{2}}+33\,Ba^2b^2x^{\frac{5}{2}}+55\,Aab^3x^{\frac{5}{2}}-33\,Ba^3bx^{\frac{3}{2}}+73\,Aa^2b^2x^{\frac{3}{2}}-9\,Ba^4\sqrt{x}-15\,Aa^3b\sqrt{x}}{192\,(bx+a)^4a^3b^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*(3*B*a + 5*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3*b^2*sgn(b*
x + a)) + 1/192*(9*B*a*b^3*x^(7/2) + 15*A*b^4*x^(7/2) + 33*B*a^2*b^2*x^(5/
2) + 55*A*a*b^3*x^(5/2) - 33*B*a^3*b*x^(3/2) + 73*A*a^2*b^2*x^(3/2) - 9*B*
a^4*sqrt(x) - 15*A*a^3*b*sqrt(x))/((b*x + a)^4*a^3*b^2*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \int \frac{\sqrt{x}\,(A+B\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2)}^{5/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{x}(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right.}{24a^3b^2\,(b^3x^3+3a\,b}$$

input
```
int(x^(1/2)*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 9*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 9*sqrt(b)*sqrt(a)*ata
n((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 3*sqrt(b)*sqrt(a)*atan((sqr
t(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x**3 - 3*sqrt(x)*a**3*b + 8*sqrt(x)*a**2*b
**2*x + 3*sqrt(x)*a*b**3*x**2)/(24*a**3*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3))
```

3.461 $\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{5(7Ab+aB)\sqrt{x}}{64a^4b\sqrt{a^2+2abx+b^2x^2}} + \frac{(Ab-aB)\sqrt{x}}{4ab(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{(7Ab+aB)\sqrt{x}}{24a^2b(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} + \frac{5(7Ab+aB)\sqrt{x}}{96a^3b(a+bx)\sqrt{a^2+2abx+b^2x^2}} + \frac{5(7Ab+aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{9/2}b^{3/2}\sqrt{a^2+2abx+b^2x^2}}$$

output
```
5/64*(7*A*b+B*a)*x^(1/2)/a^4/b/((b*x+a)^2)^(1/2)+1/4*(A*b-B*a)*x^(1/2)/a/b
/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/24*(7*A*b+B*a)*x^(1/2)/a^2/b/(b*x+a)^2/((b*
x+a)^2)^(1/2)+5/96*(7*A*b+B*a)*x^(1/2)/a^3/b/(b*x+a)/((b*x+a)^2)^(1/2)+5/6
4*(7*A*b+B*a)*(b*x+a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)/b^(3/2)/((b*
x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{x}(-15a^4B+105Ab^4x^3+5ab^3x^2(77A+3Bx)+a^2b^2x(511A+\cdots}{192a^{9/2}b^{3/2}(a+\cdots}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[a]*Sqrt[b]*Sqrt[x]*(-15*a^4*B + 105*A*b^4*x^3 + 5*a*b^3*x^2*(77*A + 
3*B*x) + a^2*b^2*x*(511*A + 55*B*x) + a^3*b*(279*A + 73*B*x)) + 15*(7*A*b 
+ a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*a^(9/2)*b^(3/2)
*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1187, 27, 87, 52, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5\sqrt{x}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{\sqrt{x}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\int \frac{1}{\sqrt{x}(a+bx)^4}dx}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\left(\frac{5\int \frac{1}{\sqrt{x}(a+bx)^3}dx}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\left(\frac{5\left(\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(\frac{(aB+7Ab)\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4a}+\frac{\sqrt{x}}{2a(a+bx)^2}\right)}{6a}+\frac{\sqrt{x}}{3a(a+bx)^3}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input

```
Int[(A + B*x)/(Sqrt[x]*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output

```
((a + b*x)*(((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x)^4) + ((7*A*b + a*B)*(Sq
rt[x]/(3*a*(a + b*x)^3) + (5*(Sqrt[x]/(2*a*(a + b*x)^2) + (3*(Sqrt[x]/(a*(
a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])))/(4*a)))/
(6*a)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(175) = 350.

Time = 1.06 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.38

method	result
default	$\frac{\left(105A\sqrt{ab}\,x^{\frac{7}{2}}b^4+15B\sqrt{ab}\,x^{\frac{7}{2}}a\,b^3+385A\sqrt{ab}\,x^{\frac{5}{2}}a\,b^3+105A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)b^5x^4+55B\sqrt{ab}\,x^{\frac{5}{2}}a^2b^2+15B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)a\,b^4x^4+420A\right.}{}$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(105*A*(a*b)^(1/2)*x^(7/2)*b^4+15*B*(a*b)^(1/2)*x^(7/2)*a*b^3+385*A*
(a*b)^(1/2)*x^(5/2)*a*b^3+105*A*arctan(b*x^(1/2)/(a*b)^(1/2))*b^5*x^4+55*B
*(a*b)^(1/2)*x^(5/2)*a^2*b^2+15*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^4+
420*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a*b^4*x^3+60*B*arctan(b*x^(1/2)/(a*b)^
(1/2))*a^2*b^3*x^3+511*A*(a*b)^(1/2)*x^(3/2)*a^2*b^2+630*A*arctan(b*x^(1/2
)/(a*b)^(1/2))*a^2*b^3*x^2+73*B*(a*b)^(1/2)*x^(3/2)*a^3*b+90*B*arctan(b*x^
(1/2)/(a*b)^(1/2))*a^3*b^2*x^2+420*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^3*b^2
*x+60*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b*x+279*A*(a*b)^(1/2)*x^(1/2)*a^
3*b+105*A*arctan(b*x^(1/2)/(a*b)^(1/2))*a^4*b-15*B*(a*b)^(1/2)*x^(1/2)*a^4
+15*B*arctan(b*x^(1/2)/(a*b)^(1/2))*a^5)*(b*x+a)/(a*b)^(1/2)/b/a^4/((b*x+a
)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{5/2}}\,dx = \left[-\frac{15\,(Ba^5+7\,Aa^4b+(Bab^4+7\,Ab^5)x^4+4\,(Ba^2b^3+7\,Aab^4)x^3+6\,(B\right.}{}$$

$$-\frac{15\,(Ba^5+7\,Aa^4b+(Bab^4+7\,Ab^5)x^4+4\,(Ba^2b^3+7\,Aab^4)x^3+6\,(Ba^3b^2+7\,Aa^2b^3)x^2+4\,(Ba^4b+7\,Aa}{192\,(a^5b^6x^4}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(15*(B*a^5 + 7*A*a^4*b + (B*a*b^4 + 7*A*b^5)*x^4 + 4*(B*a^2*b^3 +
7*A*a*b^4)*x^3 + 6*(B*a^3*b^2 + 7*A*a^2*b^3)*x^2 + 4*(B*a^4*b + 7*A*a^3*b^
2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(15*B
*a^5*b - 279*A*a^4*b^2 - 15*(B*a^2*b^4 + 7*A*a*b^5)*x^3 - 55*(B*a^3*b^3 +
7*A*a^2*b^4)*x^2 - 73*(B*a^4*b^2 + 7*A*a^3*b^3)*x)*sqrt(x))/(a^5*b^6*x^4 +
 4*a^6*b^5*x^3 + 6*a^7*b^4*x^2 + 4*a^8*b^3*x + a^9*b^2), -1/192*(15*(B*a^5
 + 7*A*a^4*b + (B*a*b^4 + 7*A*b^5)*x^4 + 4*(B*a^2*b^3 + 7*A*a*b^4)*x^3 + 6
*(B*a^3*b^2 + 7*A*a^2*b^3)*x^2 + 4*(B*a^4*b + 7*A*a^3*b^2)*x)*sqrt(a*b)*ar
ctan(sqrt(a*b)/(b*sqrt(x))) + (15*B*a^5*b - 279*A*a^4*b^2 - 15*(B*a^2*b^4
+ 7*A*a*b^5)*x^3 - 55*(B*a^3*b^3 + 7*A*a^2*b^4)*x^2 - 73*(B*a^4*b^2 + 7*A*
a^3*b^3)*x)*sqrt(x))/(a^5*b^6*x^4 + 4*a^6*b^5*x^3 + 6*a^7*b^4*x^2 + 4*a^8*
b^3*x + a^9*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(1/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(175) = 350$.

Time = 0.19 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=$$

$$-\frac{5\left(\left(3\,Bab^5+7\,Ab^6\right)x^2-21\left(Ba^2b^4+9\,Aab^5\right)x\right)x^{\frac{9}{2}}+10\left(\left(3\,Ba^2b^4+7\,Aab^5\right)x^2-63\left(Ba^3b^3+9\,Aa^2b^4\right)x\right)\ldots}{\ldots}$$

$$+\frac{5\left(Ba+7\,Ab\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^4b}+\frac{\left(3\,Bab+7\,Ab^2\right)x^{\frac{3}{2}}-30\left(Ba^2+7\,Aab\right)\sqrt{x}}{384\,a^6b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(5*((3*B*a*b^5 + 7*A*b^6)*x^2 - 21*(B*a^2*b^4 + 9*A*a*b^5)*x)*x^(9
/2) + 10*((3*B*a^2*b^4 + 7*A*a*b^5)*x^2 - 63*(B*a^3*b^3 + 9*A*a^2*b^4)*x)*
x^(7/2) - 20*(2*(3*B*a^3*b^3 + 7*A*a^2*b^4)*x^2 + 77*(B*a^4*b^2 + 9*A*a^3*
b^3)*x)*x^(5/2) - 2*(85*(3*B*a^4*b^2 + 7*A*a^3*b^3)*x^2 + 973*(B*a^5*b + 9
*A*a^4*b^2)*x)*x^(3/2) - (253*(3*B*a^5*b + 7*A*a^4*b^2)*x^2 + 1315*(B*a^6
+ 9*A*a^5*b)*x)*sqrt(x) - 1280*(A*a^5*b*x^2 + 3*A*a^6*x)/sqrt(x))/(a^6*b^5
*x^5 + 5*a^7*b^4*x^4 + 10*a^8*b^3*x^3 + 10*a^9*b^2*x^2 + 5*a^10*b*x + a^11
) + 5/64*(B*a + 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3
84*((3*B*a*b + 7*A*b^2)*x^(3/2) - 30*(B*a^2 + 7*A*a*b)*sqrt(x))/(a^6*b)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx}{\sqrt{x}\,(a^2+2abx+b^2x^2)^{5/2}}\,dx = \frac{5\,(Ba+7\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^4b\text{sgn}\,(bx+a)} + \frac{15\,Bab^3x^{\frac{7}{2}}+105\,Ab^4x^{\frac{7}{2}}+55\,Ba^2b^2x^{\frac{5}{2}}+385\,Aab^3x^{\frac{5}{2}}+73\,Ba^3bx^{\frac{3}{2}}+511\,Aa^2b^2x^{\frac{3}{2}}-15\,Ba^4\sqrt{x}+279\,Aa}{192\,(bx+a)^4a^4b\text{sgn}\,(bx+a)}$$

input
```
integrate((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/64*(B*a + 7*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4*b*sgn(b*x +
a)) + 1/192*(15*B*a*b^3*x^(7/2) + 105*A*b^4*x^(7/2) + 55*B*a^2*b^2*x^(5/2)
 + 385*A*a*b^3*x^(5/2) + 73*B*a^3*b*x^(3/2) + 511*A*a^2*b^2*x^(3/2) - 15*B
*a^4*sqrt(x) + 279*A*a^3*b*sqrt(x))/((b*x + a)^4*a^4*b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{\sqrt{x}\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{\sqrt{x}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+45\sqrt{b}\,\sqrt{a}\,atan}{24a^4b\,(b^3x^3}$$

input
```
int((B*x+A)/x^(1/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 45*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 45*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 15*sqrt(b)*sqrt(a)*atan(
(sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x**3 + 33*sqrt(x)*a**3*b + 40*sqrt(x)*
a**2*b**2*x + 15*sqrt(x)*a*b**3*x**2)/(24*a**4*b*(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3))
```

3.462 $\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 286

$$\begin{aligned}\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = & -\frac{(187Ab-35aB)\sqrt{x}}{64a^5\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{(Ab-aB)\sqrt{x}}{4a^2(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{(15Ab-7aB)\sqrt{x}}{24a^3(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{(123Ab-35aB)\sqrt{x}}{96a^4(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2A(a+bx)}{a^5\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{35(9Ab-aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{11/2}\sqrt{b}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/64*(187*A*b-35*B*a)*x^(1/2)/a^5/((b*x+a)^2)^(1/2)-1/4*(A*b-B*a)*x^(1/2)
/a^2/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*(15*A*b-7*B*a)*x^(1/2)/a^3/(b*x+a)^2
/((b*x+a)^2)^(1/2)-1/96*(123*A*b-35*B*a)*x^(1/2)/a^4/(b*x+a)/((b*x+a)^2)^(
1/2)-2*A*(b*x+a)/a^5/x^(1/2)/((b*x+a)^2)^(1/2)-35/64*(9*A*b-B*a)*(b*x+a)*a
rctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)/b^(1/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\frac{\sqrt{a}\left(-945Ab^4x^4+105ab^3x^3(-33A+Bx)+7a^2b^2x^2(-657A+55Bx)+a^4(-384A+279Bx)+a^3bx(-\right.}{\sqrt{x}}}{192a^{11/2}(a+bx)^3\sqrt{(a+bx}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((Sqrt[a]*(-945*A*b^4*x^4 + 105*a*b^3*x^3*(-33*A + B*x) + 7*a^2*b^2*x^2*(-
657*A + 55*B*x) + a^4*(-384*A + 279*B*x) + a^3*b*x*(-2511*A + 511*B*x)))/S
qrt[x] + (105*(-9*A*b + a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[b])/(192*a^(11/2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {1187, 27, 87, 52, 52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1187}$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5x^{3/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{3/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^4}dx}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\int \frac{1}{x^{3/2}(a+bx)^3}dx}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\left(\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}dx}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow\ 73$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx)\left(\frac{(9Ab-aB)\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}\right)}{6a}+\frac{1}{3a\sqrt{x}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x)^4) + ((9*A*b - a*B)*(1/(3
*a*Sqrt[x]*(a + b*x)^3) + (7*(1/(2*a*Sqrt[x]*(a + b*x)^2) + (5*(1/(a*Sqrt[
x]*(a + b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x +
b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{2A\sqrt{(bx+a)^2}}{a^5\sqrt{x}\,(bx+a)}-\frac{\left(\frac{2\left(\frac{187}{128}A\,b^4-\frac{35}{128}Ba\,b^3\right)x^{\frac{7}{2}}+\frac{a\,b^2(1929Ab-385Ba)x^{\frac{5}{2}}}{192}+2\left(\frac{765}{128}a^2A\,b^2-\frac{511}{384}B\,a^3b\right)x^{\frac{3}{2}}+2\left(\frac{325}{128}A\,a^3b-\frac{93}{128}a^4B\right)\sqrt{x}}{(bx+a)^4}+\cdots\right.}{a^5(bx+a)}$
default	$-\frac{\left(945A\sqrt{ab}\,x^4b^4-105B\sqrt{ab}\,x^4a\,b^3+3465A\sqrt{ab}\,x^3a\,b^3+945A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{9}{2}}b^5-385B\sqrt{ab}\,x^3a^2b^2-105B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{9}{2}}a\,b^4\cdots\right.}{}$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^5/x^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/a^5*(2*((187/128*A*b^4-35/128
*B*a*b^3)*x^(7/2)+1/384*a*b^2*(1929*A*b-385*B*a)*x^(5/2)+(765/128*a^2*A*b^
2-511/384*B*a^3*b)*x^(3/2)+(325/128*A*a^3*b-93/128*a^4*B)*x^(1/2))/(b*x+a)
^4+35/64*(9*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2
)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \left[\frac{105\left((Bab^4-9\,Ab^5)x^5+4\left(Ba^2b^3-9\,Aab^4\right)x^4+6\left(Ba^3b^2-9\,Aa^2b^3\right)\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[1/384*(105*((B*a*b^4 - 9*A*b^5)*x^5 + 4*(B*a^2*b^3 - 9*A*a*b^4)*x^4 + 6*(
B*a^3*b^2 - 9*A*a^2*b^3)*x^3 + 4*(B*a^4*b - 9*A*a^3*b^2)*x^2 + (B*a^5 - 9*
A*a^4*b)*x)*sqrt(-a*b)*log((b*x - a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) - 2
*(384*A*a^5*b - 105*(B*a^2*b^4 - 9*A*a*b^5)*x^4 - 385*(B*a^3*b^3 - 9*A*a^2
*b^4)*x^3 - 511*(B*a^4*b^2 - 9*A*a^3*b^3)*x^2 - 279*(B*a^5*b - 9*A*a^4*b^2
)*x)*sqrt(x))/(a^6*b^5*x^5 + 4*a^7*b^4*x^4 + 6*a^8*b^3*x^3 + 4*a^9*b^2*x^2
 + a^10*b*x), -1/192*(105*((B*a*b^4 - 9*A*b^5)*x^5 + 4*(B*a^2*b^3 - 9*A*a*
b^4)*x^4 + 6*(B*a^3*b^2 - 9*A*a^2*b^3)*x^3 + 4*(B*a^4*b - 9*A*a^3*b^2)*x^2
 + (B*a^5 - 9*A*a^4*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (384*A
*a^5*b - 105*(B*a^2*b^4 - 9*A*a*b^5)*x^4 - 385*(B*a^3*b^3 - 9*A*a^2*b^4)*x
^3 - 511*(B*a^4*b^2 - 9*A*a^3*b^3)*x^2 - 279*(B*a^5*b - 9*A*a^4*b^2)*x)*sq
rt(x))/(a^6*b^5*x^5 + 4*a^7*b^4*x^4 + 6*a^8*b^3*x^3 + 4*a^9*b^2*x^2 + a^10
*b*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(3/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(189) = 378$.

Time = 0.19 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{35\left((Bab^6+9\,Ab^7)x^2-27\,(Ba^2b^5-11\,Aab^6)x\right)x^{\frac{9}{2}}+70\left((Ba^2b^5+9\,Aab^6)x^2-81\,(Ba^3b^4-11\,Aa^2b^5)x\right)}{}$$

$$+\frac{35\,(Ba-9\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^5}+\frac{7\left((Bab+9\,Ab^2)x^{\frac{3}{2}}-30\,(Ba^2-9\,Aab)\sqrt{x}\right)}{384\,a^7}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(35*((B*a*b^6 + 9*A*b^7)*x^2 - 27*(B*a^2*b^5 - 11*A*a*b^6)*x)*x^(9
/2) + 70*((B*a^2*b^5 + 9*A*a*b^6)*x^2 - 81*(B*a^3*b^4 - 11*A*a^2*b^5)*x)*x
^(7/2) - 140*(2*(B*a^3*b^4 + 9*A*a^2*b^5)*x^2 + 99*(B*a^4*b^3 - 11*A*a^3*b
^4)*x)*x^(5/2) - 14*(85*(B*a^4*b^3 + 9*A*a^3*b^4)*x^2 + 1251*(B*a^5*b^2 -
11*A*a^4*b^3)*x)*x^(3/2) - (1771*(B*a^5*b^2 + 9*A*a^4*b^3)*x^2 + 11835*(B*
a^6*b - 11*A*a^5*b^2)*x)*sqrt(x) - 1280*((B*a^6*b + 9*A*a^5*b^2)*x^2 + 3*(
B*a^7 - 11*A*a^6*b)*x)/sqrt(x) - 3840*(A*a^6*b*x^2 - A*a^7*x)/x^(3/2))/(a^
7*b^5*x^5 + 5*a^8*b^4*x^4 + 10*a^9*b^3*x^3 + 10*a^10*b^2*x^2 + 5*a^11*b*x
+ a^12) + 35/64*(B*a - 9*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5)
+ 7/384*((B*a*b + 9*A*b^2)*x^(3/2) - 30*(B*a^2 - 9*A*a*b)*sqrt(x))/a^7
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{35\,(Ba-9\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^5\mathrm{sgn}\left(bx+a\right)}-\frac{2\,A}{a^5\sqrt{x}\mathrm{sgn}\left(bx+a\right)}$$

$$+\frac{105\,Bab^3x^{\frac{7}{2}}-561\,Ab^4x^{\frac{7}{2}}+385\,Ba^2b^2x^{\frac{5}{2}}-1929\,Aab^3x^{\frac{5}{2}}+511\,Ba^3bx^{\frac{3}{2}}-2295\,Aa^2b^2x^{\frac{3}{2}}+279\,Ba^4\sqrt{x}-}{192\,(bx+a)^4a^5\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
35/64*(B*a - 9*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5*sgn(b*x + a
)) - 2*A/(a^5*sqrt(x)*sgn(b*x + a)) + 1/192*(105*B*a*b^3*x^(7/2) - 561*A*b
^4*x^(7/2) + 385*B*a^2*b^2*x^(5/2) - 1929*A*a*b^3*x^(5/2) + 511*B*a^3*b*x^
(3/2) - 2295*A*a^2*b^2*x^(3/2) + 279*B*a^4*sqrt(x) - 975*A*a^3*b*sqrt(x))/
((b*x + a)^4*a^5*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^{3/2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}} \,d x$$

input
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x^{3/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{-105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3-315\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx-3$$

input
```
int((B*x+A)/x^(3/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 -
 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x
- 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x
**2 - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3
*x**3 - 48*a**4 - 231*a**3*b*x - 280*a**2*b**2*x**2 - 105*a*b**3*x**3)/(24
*sqrt(x)*a**5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.463 $\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 336

$$\begin{aligned}\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx &= \frac{b(515Ab-187aB)\sqrt{x}}{64a^6\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{b(Ab-aB)\sqrt{x}}{4a^3(a+bx)^3\sqrt{a^2+2abx+b^2x^2}} + \frac{b(23Ab-15aB)\sqrt{x}}{24a^4(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{b(259Ab-123aB)\sqrt{x}}{96a^5(a+bx)\sqrt{a^2+2abx+b^2x^2}} - \frac{2A(a+bx)}{3a^5x^{3/2}\sqrt{a^2+2abx+b^2x^2}} \\ &+ \frac{2(5Ab-aB)(a+bx)}{a^6\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} + \frac{105\sqrt{b}(11Ab-3aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{13/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
1/64*b*(515*A*b-187*B*a)*x^(1/2)/a^6/((b*x+a)^2)^(1/2)+1/4*b*(A*b-B*a)*x^(
1/2)/a^3/(b*x+a)^3/((b*x+a)^2)^(1/2)+1/24*b*(23*A*b-15*B*a)*x^(1/2)/a^4/(b
*x+a)^2/((b*x+a)^2)^(1/2)+1/96*b*(259*A*b-123*B*a)*x^(1/2)/a^5/(b*x+a)/((b
*x+a)^2)^(1/2)-2/3*A*(b*x+a)/a^5/x^(3/2)/((b*x+a)^2)^(1/2)+2*(5*A*b-B*a)*(
b*x+a)/a^6/x^(1/2)/((b*x+a)^2)^(1/2)+105/64*b^(1/2)*(11*A*b-3*B*a)*(b*x+a)
*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(13/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = -\frac{\frac{\sqrt{a}(-3465Ab^5x^5+128a^5(A+3Bx)+105ab^4x^4(-121A+9Bx)+231a^2b^3x^3(-73A+15Bx)+9a^3b}{x^{3/2}}}{192a^{13/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
(-((Sqrt[a]*(-3465*A*b^5*x^5 + 128*a^5*(A + 3*B*x) + 105*a*b^4*x^4*(-121*A
 + 9*B*x) + 231*a^2*b^3*x^3*(-73*A + 15*B*x) + 9*a^3*b^2*x^2*(-1023*A + 51
1*B*x) + a^4*b*x*(-1408*A + 2511*B*x)))/x^(3/2)) + 315*Sqrt[b]*(11*A*b - 3
*a*B)*(a + b*x)^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(192*a^(13/2)*(a + b*
x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.63, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1187, 27, 87, 52, 52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1187}$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5x^{5/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{5/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{(a+bx)\left(\frac{(11Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)^4}dx}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\int \frac{1}{x^{5/2}(a+bx)^3}dx}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 61$$

$$(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-}{4abx^{3/2}}\right.$$

↓ 73

$$(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-a}{4abx^{3/2}(a}\right.$$

$\Big\downarrow$ 218

$$(a+bx)\left(\frac{(11Ab-3aB)\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}\right)}{2a}+\frac{1}{3ax^{3/2}(a+bx)^3}\right)}{8ab}+\frac{}{4abx\ldots}\right.$$

input
```
Int[(A + B*x)/(x^(5/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(4*a*b*x^(3/2)*(a + b*x)^4) + ((11*A*b - 3*a*B)*(1
/(3*a*x^(3/2)*(a + b*x)^3) + (3*(1/(2*a*x^(3/2)*(a + b*x)^2) + (7*(1/(a*x^
(3/2)*(a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2*a)))/(4*a)))/(2*a)))/(
8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{2(-15Abx+3Bax+Aa)\sqrt{(bx+a)^2}}{3a^6x^{\frac{3}{2}}(bx+a)}+\frac{b\left(\frac{2\left(\frac{515}{128}Ab^4-\frac{187}{128}Bab^3\right)x^{\frac{7}{2}}+\frac{ab^2(5153Ab-1929Ba)x^{\frac{5}{2}}}{192}+2\left(\frac{5855}{384}a^2Ab^2-\frac{765}{128}Ba^3b\right)x^{\frac{3}{2}}+2\left(\right.}{(bx+a)^4}\right.}{a^6(bx+a)}$
default	$-\frac{\left(-3465A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{11}{2}}b^6+945B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{11}{2}}ab^5-13860A\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{9}{2}}ab^5+3780B\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)x^{\frac{9}{2}}a^2b^4-20790\right.}{}$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-15*A*b*x+3*B*a*x+A*a)/a^6/x^(3/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/a^6*b
*(2*((515/128*A*b^4-187/128*B*a*b^3)*x^(7/2)+1/384*a*b^2*(5153*A*b-1929*B*
a)*x^(5/2)+(5855/384*a^2*A*b^2-765/128*B*a^3*b)*x^(3/2)+(765/128*A*a^3*b-3
25/128*a^4*B)*x^(1/2))/(b*x+a)^4+105/64*(11*A*b-3*B*a)/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \left[-\frac{315\left((3\,Bab^4-11\,Ab^5)x^6+4\,(3\,Ba^2b^3-11\,Aab^4)x^5+6\,(3\,Ba^3b^2-\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/384*(315*((3*B*a*b^4 - 11*A*b^5)*x^6 + 4*(3*B*a^2*b^3 - 11*A*a*b^4)*x^
5 + 6*(3*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 4*(3*B*a^4*b - 11*A*a^3*b^2)*x^3
+ (3*B*a^5 - 11*A*a^4*b)*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a)
 - a)/(b*x + a)) + 2*(128*A*a^5 + 315*(3*B*a*b^4 - 11*A*b^5)*x^5 + 1155*(3
*B*a^2*b^3 - 11*A*a*b^4)*x^4 + 1533*(3*B*a^3*b^2 - 11*A*a^2*b^3)*x^3 + 837
*(3*B*a^4*b - 11*A*a^3*b^2)*x^2 + 128*(3*B*a^5 - 11*A*a^4*b)*x)*sqrt(x))/(
a^6*b^4*x^6 + 4*a^7*b^3*x^5 + 6*a^8*b^2*x^4 + 4*a^9*b*x^3 + a^10*x^2), -1/
192*(315*((3*B*a*b^4 - 11*A*b^5)*x^6 + 4*(3*B*a^2*b^3 - 11*A*a*b^4)*x^5 +
6*(3*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 4*(3*B*a^4*b - 11*A*a^3*b^2)*x^3 + (3
*B*a^5 - 11*A*a^4*b)*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (128*A*a^5
 + 315*(3*B*a*b^4 - 11*A*b^5)*x^5 + 1155*(3*B*a^2*b^3 - 11*A*a*b^4)*x^4 +
1533*(3*B*a^3*b^2 - 11*A*a^2*b^3)*x^3 + 837*(3*B*a^4*b - 11*A*a^3*b^2)*x^2
 + 128*(3*B*a^5 - 11*A*a^4*b)*x)*sqrt(x))/(a^6*b^4*x^6 + 4*a^7*b^3*x^5 + 6
*a^8*b^2*x^4 + 4*a^9*b*x^3 + a^10*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(5/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(224) = 448$.

Time = 0.21 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{315\left(\left(Bab^7-11\,Ab^8\right)x^2+11\left(3\,Ba^2b^6-13\,Aab^7\right)x\right)x^{\frac{9}{2}}+630\left(\left(Ba^2b^6-11\,Aab^7\right)x^2+33\left(3\,Ba^3b^5-13\,A\right.\right.}{}$$
$$-\frac{105\left(3\,Bab-11\,Ab^2\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^6}$$
$$+\frac{21\left(\left(Bab^2-11\,Ab^3\right)x^{\frac{3}{2}}+10\left(3\,Ba^2b-11\,Aab^2\right)\sqrt{x}\right)}{128\,a^8}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/1920*(315*((B*a*b^7 - 11*A*b^8)*x^2 + 11*(3*B*a^2*b^6 - 13*A*a*b^7)*x)*
x^(9/2) + 630*((B*a^2*b^6 - 11*A*a*b^7)*x^2 + 33*(3*B*a^3*b^5 - 13*A*a^2*b
^6)*x)*x^(7/2) - 420*(6*(B*a^3*b^5 - 11*A*a^2*b^6)*x^2 - 121*(3*B*a^4*b^4
- 13*A*a^3*b^5)*x)*x^(5/2) - 42*(255*(B*a^4*b^4 - 11*A*a^3*b^5)*x^2 - 1529
*(3*B*a^5*b^3 - 13*A*a^4*b^4)*x)*x^(3/2) - 33*(483*(B*a^5*b^3 - 11*A*a^4*b
^4)*x^2 - 1315*(3*B*a^6*b^2 - 13*A*a^5*b^3)*x)*sqrt(x) - 1280*(9*(B*a^6*b^
2 - 11*A*a^5*b^3)*x^2 - 11*(3*B*a^7*b - 13*A*a^6*b^2)*x)/sqrt(x) - 1280*(3
*(B*a^7*b - 11*A*a^6*b^2)*x^2 - (3*B*a^8 - 13*A*a^7*b)*x)/x^(3/2) + 1280*(
3*A*a^7*b*x^2 + A*a^8*x)/x^(5/2))/(a^8*b^5*x^5 + 5*a^9*b^4*x^4 + 10*a^10*b
^3*x^3 + 10*a^11*b^2*x^2 + 5*a^12*b*x + a^13) - 105/64*(3*B*a*b - 11*A*b^2
)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^6) + 21/128*((B*a*b^2 - 11*A*b^
3)*x^(3/2) + 10*(3*B*a^2*b - 11*A*a*b^2)*sqrt(x))/a^8
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{105\left(3\,Bab-11\,Ab^2\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^6\mathrm{sgn}\left(bx+a\right)}-\frac{2\left(3\,Bax-15\,Abx+Aa\right)}{3\,a^6x^{\frac{3}{2}}\mathrm{sgn}\left(bx+a\right)}$$
$$-\frac{561\,Bab^4x^{\frac{7}{2}}-1545\,Ab^5x^{\frac{7}{2}}+1929\,Ba^2b^3x^{\frac{5}{2}}-5153\,Aab^4x^{\frac{5}{2}}+2295\,Ba^3b^2x^{\frac{3}{2}}-5855\,Aa^2b^3x^{\frac{3}{2}}+975\,Ba^4b}{192\left(bx+a\right)^4a^6\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
-105/64*(3*B*a*b - 11*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^6*sg
n(b*x + a)) - 2/3*(3*B*a*x - 15*A*b*x + A*a)/(a^6*x^(3/2)*sgn(b*x + a)) -
1/192*(561*B*a*b^4*x^(7/2) - 1545*A*b^5*x^(7/2) + 1929*B*a^2*b^3*x^(5/2) -
 5153*A*a*b^4*x^(5/2) + 2295*B*a^3*b^2*x^(3/2) - 5855*A*a^2*b^3*x^(3/2) +
975*B*a^4*b*sqrt(x) - 2295*A*a^3*b^2*sqrt(x))/((b*x + a)^4*a^6*sgn(b*x + a
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^{5/2}\,{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}^{5/2}} \,d x$$

input
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(5/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{x^{5/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{315\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3bx+945\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2+}{}$$

input
```
int((B*x+A)/x^(5/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
(315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*b*x
+ 945*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**
2*x**2 + 945*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a
*b**3*x**3 + 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)
))*b**4*x**4 - 16*a**5 + 144*a**4*b*x + 693*a**3*b**2*x**2 + 840*a**2*b**3
*x**3 + 315*a*b**4*x**4)/(24*sqrt(x)*a**6*x*(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3))
```

3.464 $\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 391

$$\begin{aligned}\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = & -\frac{b^2(1083Ab-515aB)\sqrt{x}}{64a^7\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{b^2(Ab-aB)\sqrt{x}}{4a^4(a+bx)^3\sqrt{a^2+2abx+b^2x^2}}-\frac{b^2(31Ab-23aB)\sqrt{x}}{24a^5(a+bx)^2\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{b^2(443Ab-259aB)\sqrt{x}}{96a^6(a+bx)\sqrt{a^2+2abx+b^2x^2}}-\frac{2A(a+bx)}{5a^5x^{5/2}\sqrt{a^2+2abx+b^2x^2}} \\ & +\frac{2(5Ab-aB)(a+bx)}{3a^6x^{3/2}\sqrt{a^2+2abx+b^2x^2}}-\frac{10b(3Ab-aB)(a+bx)}{a^7\sqrt{x}\sqrt{a^2+2abx+b^2x^2}} \\ & -\frac{231b^{3/2}(13Ab-5aB)(a+bx)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{15/2}\sqrt{a^2+2abx+b^2x^2}}\end{aligned}$$

output
```
-1/64*b^2*(1083*A*b-515*B*a)*x^(1/2)/a^7/((b*x+a)^2)^(1/2)-1/4*b^2*(A*b-B*
a)*x^(1/2)/a^4/(b*x+a)^3/((b*x+a)^2)^(1/2)-1/24*b^2*(31*A*b-23*B*a)*x^(1/2
)/a^5/(b*x+a)^2/((b*x+a)^2)^(1/2)-1/96*b^2*(443*A*b-259*B*a)*x^(1/2)/a^6/(
b*x+a)/((b*x+a)^2)^(1/2)-2/5*A*(b*x+a)/a^5/x^(5/2)/((b*x+a)^2)^(1/2)+2/3*(
5*A*b-B*a)*(b*x+a)/a^6/x^(3/2)/((b*x+a)^2)^(1/2)-10*b*(3*A*b-B*a)*(b*x+a)/
a^7/x^(1/2)/((b*x+a)^2)^(1/2)-231/64*b^(3/2)*(13*A*b-5*B*a)*(b*x+a)*arctan
(b^(1/2)*x^(1/2)/a^(1/2))/a^(15/2)/((b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{\frac{\sqrt{a}(-45045Ab^6x^6-128a^6(3A+5Bx)+1155ab^5x^5(-143A+15Bx)+128a^5bx(13A+55Bx)+231a^2\cdots}{x^{5/2}}}{\cdots}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((Sqrt[a]*(-45045*A*b^6*x^6 - 128*a^6*(3*A + 5*B*x) + 1155*a*b^5*x^5*(-143
*A + 15*B*x) + 128*a^5*b*x*(13*A + 55*B*x) + 231*a^2*b^4*x^4*(-949*A + 275
*B*x) + 33*a^3*b^3*x^3*(-3627*A + 2555*B*x) + 11*a^4*b^2*x^2*(-1664*A + 41
85*B*x)))/x^(5/2) + 3465*b^(3/2)*(-13*A*b + 5*a*B)*(a + b*x)^4*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/(960*a^(15/2)*(a + b*x)^3*Sqrt[(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {1187, 27, 87, 52, 52, 52, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1187$$

$$\frac{b^5(a+bx)\int \frac{A+Bx}{b^5x^{7/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx)\int \frac{A+Bx}{x^{7/2}(a+bx)^5}dx}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \; 87$$

$$\frac{(a+bx)\left(\frac{(13Ab-5aB)\int \frac{1}{x^{7/2}(a+bx)^4}dx}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(a+bx)\left(\frac{(13Ab-5aB)\left(\frac{11\int \frac{1}{x^{7/2}(a+bx)^3}dx}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(a+bx)\left(\frac{(13Ab-5aB)\left(\frac{11\left(\frac{9\int \frac{1}{x^{7/2}(a+bx)^2}dx}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{(a+bx)\left(\frac{(13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^{7/2}(a+bx)}dx}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{(a+bx)\left(\frac{(13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\int\frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx)^4}\right)}{\sqrt{a^2+2abx+b^2x^2}}$$

$$\Big\downarrow \ 61$$

$$(a+bx)\left(\frac{(13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right)}{8ab}+\frac{}{4a\cdots}$$

↓ 61

$$\left((13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)}\right.\right.$$

↓ 73

$$\left((13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}\,d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a+bx)^3}\right.\right.$$

$(a+bx)$

↓ 218

$$\left((13Ab-5aB)\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx)^2}\right)}{6a}+\frac{1}{3ax^{5/2}(a\cdots}\right.\right.$$

input
```
Int[(A + B*x)/(x^(7/2)*(a^2 + 2*a*b*x + b^2*x^2)^(5/2)),x]
```

output
```
((a + b*x)*((A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x)^4) + ((13*A*b - 5*a*B)*(1
/(3*a*x^(5/2)*(a + b*x)^3) + (11*(1/(2*a*x^(5/2)*(a + b*x)^2) + (9*(1/(a*x
^(5/2)*(a + b*x)) + (7*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(
a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a
))/(2*a)))/(4*a)))/(6*a)))/(8*a*b)))/Sqrt[a^2 + 2*a*b*x + b^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1187
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^
IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(d + e*x)^m*(f + g*x)^n*(b/2
+ c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b^2
 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{2\left(225x^2b^2A-75Ba\,x^2b-25abAx+5a^2Bx+3a^2A\right)\sqrt{(bx+a)^2}}{15a^7x^{\frac{5}{2}}(bx+a)}-\frac{b^2\left(\frac{2\left(\frac{1083}{128}A\,b^4-\frac{515}{128}Ba\,b^3\right)x^{\frac{7}{2}}+\frac{a\,b^2(10633Ab-5153Ba)x^{\frac{5}{2}}}{192}+2\left(\frac{11}{3}\right.}{(bx+a)}\right.}{}$
default	$-\frac{\left(-7040B\,x^2\sqrt{ab}\,a^5b-1664Ax\sqrt{ab}\,a^5b+219219A\,x^4\sqrt{ab}\,a^2b^4-84315B\,x^4\sqrt{ab}\,a^3b^3+119691A\,x^3\sqrt{ab}\,a^3b^3+45045A\,x^{\frac{5}{2}}\arctan\left(\right.\right.}{}$

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(225*A*b^2*x^2-75*B*a*b*x^2-25*A*a*b*x+5*B*a^2*x+3*A*a^2)/a^7/x^(5/2
)*((b*x+a)^2)^(1/2)/(b*x+a)-1/a^7*b^2*(2*((1083/128*A*b^4-515/128*B*a*b^3)
*x^(7/2)+1/384*a*b^2*(10633*A*b-5153*B*a)*x^(5/2)+(11767/384*a^2*A*b^2-585
5/384*B*a^3*b)*x^(3/2)+(1477/128*A*a^3*b-765/128*a^4*B)*x^(1/2))/(b*x+a)^4
+231/64*(13*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))*((b*x+a)
^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/1920*(3465*((5*B*a*b^5 - 13*A*b^6)*x^7 + 4*(5*B*a^2*b^4 - 13*A*a*b^5)*
x^6 + 6*(5*B*a^3*b^3 - 13*A*a^2*b^4)*x^5 + 4*(5*B*a^4*b^2 - 13*A*a^3*b^3)*
x^4 + (5*B*a^5*b - 13*A*a^4*b^2)*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) + 2*(384*A*a^6 - 3465*(5*B*a*b^5 - 13*A*b^6)*x^6
- 12705*(5*B*a^2*b^4 - 13*A*a*b^5)*x^5 - 16863*(5*B*a^3*b^3 - 13*A*a^2*b^4
)*x^4 - 9207*(5*B*a^4*b^2 - 13*A*a^3*b^3)*x^3 - 1408*(5*B*a^5*b - 13*A*a^4
*b^2)*x^2 + 128*(5*B*a^6 - 13*A*a^5*b)*x)*sqrt(x))/(a^7*b^4*x^7 + 4*a^8*b^
3*x^6 + 6*a^9*b^2*x^5 + 4*a^10*b*x^4 + a^11*x^3), 1/960*(3465*((5*B*a*b^5
- 13*A*b^6)*x^7 + 4*(5*B*a^2*b^4 - 13*A*a*b^5)*x^6 + 6*(5*B*a^3*b^3 - 13*A
*a^2*b^4)*x^5 + 4*(5*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + (5*B*a^5*b - 13*A*a^4
*b^2)*x^3)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) - (384*A*a^6 - 3465*(5*B*a*
b^5 - 13*A*b^6)*x^6 - 12705*(5*B*a^2*b^4 - 13*A*a*b^5)*x^5 - 16863*(5*B*a^
3*b^3 - 13*A*a^2*b^4)*x^4 - 9207*(5*B*a^4*b^2 - 13*A*a^3*b^3)*x^3 - 1408*(
5*B*a^5*b - 13*A*a^4*b^2)*x^2 + 128*(5*B*a^6 - 13*A*a^5*b)*x)*sqrt(x))/(a^
7*b^4*x^7 + 4*a^8*b^3*x^6 + 6*a^9*b^2*x^5 + 4*a^10*b*x^4 + a^11*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(7/2)/(b**2*x**2+2*a*b*x+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(263) = 526.

Time = 0.21 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{1155\left((3\,Bab^8-13\,Ab^9)x^2+39\,(Ba^2b^7-3\,Aab^8)x\right)x^{\frac{9}{2}}+2310\left((3\,Ba^2b\right.}{}$$
$$+\frac{231\,(5\,Bab^2-13\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^7}$$
$$-\frac{77\left((3\,Bab^3-13\,Ab^4)x^{\frac{3}{2}}+6\,(5\,Ba^2b^2-13\,Aab^3)\sqrt{x}\right)}{128\,a^9}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
1/1920*(1155*((3*B*a*b^8 - 13*A*b^9)*x^2 + 39*(B*a^2*b^7 - 3*A*a*b^8)*x)*x
^(9/2) + 2310*((3*B*a^2*b^7 - 13*A*a*b^8)*x^2 + 117*(B*a^3*b^6 - 3*A*a^2*b
^7)*x)*x^(7/2) - 4620*(2*(3*B*a^3*b^6 - 13*A*a^2*b^7)*x^2 - 143*(B*a^4*b^5
 - 3*A*a^3*b^6)*x)*x^(5/2) - 462*(85*(3*B*a^4*b^5 - 13*A*a^3*b^6)*x^2 - 18
07*(B*a^5*b^4 - 3*A*a^4*b^5)*x)*x^(3/2) - 33*(1771*(3*B*a^5*b^4 - 13*A*a^4
*b^5)*x^2 - 17095*(B*a^6*b^3 - 3*A*a^5*b^4)*x)*sqrt(x) - 14080*(3*(3*B*a^6
*b^3 - 13*A*a^5*b^4)*x^2 - 13*(B*a^7*b^2 - 3*A*a^6*b^3)*x)/sqrt(x) - 1280*
(11*(3*B*a^7*b^2 - 13*A*a^6*b^3)*x^2 - 13*(B*a^8*b - 3*A*a^7*b^2)*x)/x^(3/
2) - 1280*((3*B*a^8*b - 13*A*a^7*b^2)*x^2 + (B*a^9 - 3*A*a^8*b)*x)/x^(5/2)
 - 256*(5*A*a^8*b*x^2 + 3*A*a^9*x)/x^(7/2))/(a^9*b^5*x^5 + 5*a^10*b^4*x^4
+ 10*a^11*b^3*x^3 + 10*a^12*b^2*x^2 + 5*a^13*b*x + a^14) + 231/64*(5*B*a*b
^2 - 13*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^7) - 77/128*((3*B*
a*b^3 - 13*A*b^4)*x^(3/2) + 6*(5*B*a^2*b^2 - 13*A*a*b^3)*sqrt(x))/a^9
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx = \frac{231\left(5\,Bab^2-13\,Ab^3\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{64\,\sqrt{ab}a^7\mathrm{sgn}\left(bx+a\right)} + \frac{2\left(75\,Babx^2-225\,Ab^2x^2-5\,Ba^2x+25\,Aabx-3\,Aa^2\right)}{15\,a^7x^{\frac{5}{2}}\mathrm{sgn}\left(bx+a\right)} + \frac{1545\,Bab^5x^{\frac{7}{2}}-3249\,Ab^6x^{\frac{7}{2}}+5153\,Ba^2b^4x^{\frac{5}{2}}-10633\,Aab^5x^{\frac{5}{2}}+5855\,Ba^3b^3x^{\frac{3}{2}}-11767\,Aa^2b^4x^{\frac{3}{2}}+2295\,B}{192\left(bx+a\right)^4a^7\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

output
```
231/64*(5*B*a*b^2 - 13*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^7*s
gn(b*x + a)) + 2/15*(75*B*a*b*x^2 - 225*A*b^2*x^2 - 5*B*a^2*x + 25*A*a*b*x
 - 3*A*a^2)/(a^7*x^(5/2)*sgn(b*x + a)) + 1/192*(1545*B*a*b^5*x^(7/2) - 324
9*A*b^6*x^(7/2) + 5153*B*a^2*b^4*x^(5/2) - 10633*A*a*b^5*x^(5/2) + 5855*B*
a^3*b^3*x^(3/2) - 11767*A*a^2*b^4*x^(3/2) + 2295*B*a^4*b^2*sqrt(x) - 4431*
A*a^3*b^3*sqrt(x))/((b*x + a)^4*a^7*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^{7/2}\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(7/2)*(a^2 + b^2*x^2 + 2*a*b*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{x^{7/2}\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx=\frac{-3465\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^2-10395\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a$$

input
```
int((B*x+A)/x^(7/2)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x)
```

output
```
( - 3465*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3*
b**2*x**2 - 10395*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a
)))*a**2*b**3*x**3 - 10395*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(
b)*sqrt(a)))*a*b**4*x**4 - 3465*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(
sqrt(b)*sqrt(a)))*b**5*x**5 - 48*a**6 + 176*a**5*b*x - 1584*a**4*b**2*x**2
 - 7623*a**3*b**3*x**3 - 9240*a**2*b**4*x**4 - 3465*a*b**5*x**5)/(120*sqrt
(x)*a**7*x**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.465 $\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 219

$$\begin{aligned}\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx &= \frac{a^6A(gx)^{1+m}}{g(1+m)}+\frac{a^5(6Ab+aB)(gx)^{2+m}}{g^2(2+m)}\\&\quad+\frac{3a^4b(5Ab+2aB)(gx)^{3+m}}{g^3(3+m)}\\&\quad+\frac{5a^3b^2(4Ab+3aB)(gx)^{4+m}}{g^4(4+m)}\\&\quad+\frac{5a^2b^3(3Ab+4aB)(gx)^{5+m}}{g^5(5+m)}\\&\quad+\frac{3ab^4(2Ab+5aB)(gx)^{6+m}}{g^6(6+m)}\\&\quad+\frac{b^5(Ab+6aB)(gx)^{7+m}}{g^7(7+m)}+\frac{b^6B(gx)^{8+m}}{g^8(8+m)}\end{aligned}$$

output
```
a^6*A*(g*x)^(1+m)/g/(1+m)+a^5*(6*A*b+B*a)*(g*x)^(2+m)/g^2/(2+m)+3*a^4*b*(5
*A*b+2*B*a)*(g*x)^(3+m)/g^3/(3+m)+5*a^3*b^2*(4*A*b+3*B*a)*(g*x)^(4+m)/g^4/
(4+m)+5*a^2*b^3*(3*A*b+4*B*a)*(g*x)^(5+m)/g^5/(5+m)+3*a*b^4*(2*A*b+5*B*a)*
(g*x)^(6+m)/g^6/(6+m)+b^5*(A*b+6*B*a)*(g*x)^(7+m)/g^7/(7+m)+b^6*B*(g*x)^(8
+m)/g^8/(8+m)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.63

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx$$
$$=\frac{(gx)^m\left(Bx(a+bx)^7+(-aB(1+m)+Ab(8+m))x\left(\frac{a^6}{1+m}+\frac{6a^5bx}{2+m}+\frac{15a^4b^2x^2}{3+m}+\frac{20a^3b^3x^3}{4+m}+\frac{15a^2b^4x^4}{5+m}+\frac{6ab^5}{6+m}\right.\right.}{b(8+m)}$$

input
```
Integrate[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
((g*x)^m*(B*x*(a + b*x)^7 + (-(a*B*(1 + m)) + A*b*(8 + m))*x*(a^6/(1 + m)
+ (6*a^5*b*x)/(2 + m) + (15*a^4*b^2*x^2)/(3 + m) + (20*a^3*b^3*x^3)/(4 + m
) + (15*a^2*b^4*x^4)/(5 + m) + (6*a*b^5*x^5)/(6 + m) + (b^6*x^6)/(7 + m)))
)/(b*(8 + m))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^3(A+Bx)(gx)^m\,dx$$
$$\Big\downarrow \; 1184$$
$$\frac{\int b^6(gx)^m(a+bx)^6(A+Bx)dx}{b^6}$$
$$\Big\downarrow \; 27$$
$$\int (a+bx)^6(A+Bx)(gx)^mdx$$
$$\Big\downarrow \; 85$$

$$\int \left(a^6A(gx)^m + \frac{a^5(gx)^{m+1}(aB+6Ab)}{g} + \frac{3a^4b(gx)^{m+2}(2aB+5Ab)}{g^2} + \frac{5a^3b^2(gx)^{m+3}(3aB+4Ab)}{g^3} + \frac{5a^2b^3(gx}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6A(gx)^{m+1}}{g(m+1)} + \frac{a^5(gx)^{m+2}(aB+6Ab)}{g^2(m+2)} + \frac{3a^4b(gx)^{m+3}(2aB+5Ab)}{g^3(m+3)} + \frac{5a^3b^2(gx)^{m+4}(3aB+4Ab)}{g^4(m+4)} + \frac{5a^2b^3(gx)^{m+5}(4aB+3Ab)}{g^5(m+5)} + \frac{b^5(gx)^{m+7}(6aB+Ab)}{g^7(m+7)} + \frac{3ab^4(gx)^{m+6}(5aB+2Ab)}{g^6(m+6)} + \frac{b^6B(gx)^{m+8}}{g^8(m+8)}$$

input
```
Int[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^3,x]
```

output
```
(a^6*A*(g*x)^(1 + m))/(g*(1 + m)) + (a^5*(6*A*b + a*B)*(g*x)^(2 + m))/(g^2
*(2 + m)) + (3*a^4*b*(5*A*b + 2*a*B)*(g*x)^(3 + m))/(g^3*(3 + m)) + (5*a^3
*b^2*(4*A*b + 3*a*B)*(g*x)^(4 + m))/(g^4*(4 + m)) + (5*a^2*b^3*(3*A*b + 4*
a*B)*(g*x)^(5 + m))/(g^5*(5 + m)) + (3*a*b^4*(2*A*b + 5*a*B)*(g*x)^(6 + m)
)/(g^6*(6 + m)) + (b^5*(A*b + 6*a*B)*(g*x)^(7 + m))/(g^7*(7 + m)) + (b^6*B
*(g*x)^(8 + m))/(g^8*(8 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

method	result
norman	$\frac{A\,a^{6} x\,\mathrm{e}^{m \ln(g x)}}{1+m}+\frac{B\,b^{6} x^{8} \mathrm{e}^{m \ln(g x)}}{8+m}+\frac{a^{5} (6 A b+B a) x^{2} \mathrm{e}^{m \ln(g x)}}{2+m}+\frac{b^{5} (A b+6 B a) x^{7} \mathrm{e}^{m \ln(g x)}}{7+m}+\frac{3 a\,b^{4} (2 A b+5 B a) x^{6} \mathrm{e}^{m \ln(}}{6+m}$
gosper	Expression too large to display
risch	Expression too large to display
orering	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
A*a^6/(1+m)*x*exp(m*ln(g*x))+B*b^6/(8+m)*x^8*exp(m*ln(g*x))+a^5*(6*A*b+B*a
)/(2+m)*x^2*exp(m*ln(g*x))+b^5*(A*b+6*B*a)/(7+m)*x^7*exp(m*ln(g*x))+3*a*b^
4*(2*A*b+5*B*a)/(6+m)*x^6*exp(m*ln(g*x))+5*a^2*b^3*(3*A*b+4*B*a)/(5+m)*x^5
*exp(m*ln(g*x))+5*a^3*b^2*(4*A*b+3*B*a)/(4+m)*x^4*exp(m*ln(g*x))+3*a^4*b*(
5*A*b+2*B*a)/(3+m)*x^3*exp(m*ln(g*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(219) = 438.

Time = 0.09 (sec) , antiderivative size = 1193, normalized size of antiderivative = 5.45

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="fricas")
```

output
```
((B*b^6*m^7 + 28*B*b^6*m^6 + 322*B*b^6*m^5 + 1960*B*b^6*m^4 + 6769*B*b^6*m
^3 + 13132*B*b^6*m^2 + 13068*B*b^6*m + 5040*B*b^6)*x^8 + ((6*B*a*b^5 + A*b
^6)*m^7 + 34560*B*a*b^5 + 5760*A*b^6 + 29*(6*B*a*b^5 + A*b^6)*m^6 + 343*(6
*B*a*b^5 + A*b^6)*m^5 + 2135*(6*B*a*b^5 + A*b^6)*m^4 + 7504*(6*B*a*b^5 + A
*b^6)*m^3 + 14756*(6*B*a*b^5 + A*b^6)*m^2 + 14832*(6*B*a*b^5 + A*b^6)*m)*x
^7 + 3*((5*B*a^2*b^4 + 2*A*a*b^5)*m^7 + 33600*B*a^2*b^4 + 13440*A*a*b^5 +
30*(5*B*a^2*b^4 + 2*A*a*b^5)*m^6 + 366*(5*B*a^2*b^4 + 2*A*a*b^5)*m^5 + 234
0*(5*B*a^2*b^4 + 2*A*a*b^5)*m^4 + 8409*(5*B*a^2*b^4 + 2*A*a*b^5)*m^3 + 168
30*(5*B*a^2*b^4 + 2*A*a*b^5)*m^2 + 17144*(5*B*a^2*b^4 + 2*A*a*b^5)*m)*x^6
+ 5*((4*B*a^3*b^3 + 3*A*a^2*b^4)*m^7 + 32256*B*a^3*b^3 + 24192*A*a^2*b^4 +
 31*(4*B*a^3*b^3 + 3*A*a^2*b^4)*m^6 + 391*(4*B*a^3*b^3 + 3*A*a^2*b^4)*m^5
+ 2581*(4*B*a^3*b^3 + 3*A*a^2*b^4)*m^4 + 9544*(4*B*a^3*b^3 + 3*A*a^2*b^4)*
m^3 + 19564*(4*B*a^3*b^3 + 3*A*a^2*b^4)*m^2 + 20304*(4*B*a^3*b^3 + 3*A*a^2
*b^4)*m)*x^5 + 5*((3*B*a^4*b^2 + 4*A*a^3*b^3)*m^7 + 30240*B*a^4*b^2 + 4032
0*A*a^3*b^3 + 32*(3*B*a^4*b^2 + 4*A*a^3*b^3)*m^6 + 418*(3*B*a^4*b^2 + 4*A*
a^3*b^3)*m^5 + 2864*(3*B*a^4*b^2 + 4*A*a^3*b^3)*m^4 + 10993*(3*B*a^4*b^2 +
 4*A*a^3*b^3)*m^3 + 23312*(3*B*a^4*b^2 + 4*A*a^3*b^3)*m^2 + 24876*(3*B*a^4
*b^2 + 4*A*a^3*b^3)*m)*x^4 + 3*((2*B*a^5*b + 5*A*a^4*b^2)*m^7 + 26880*B*a^
5*b + 67200*A*a^4*b^2 + 33*(2*B*a^5*b + 5*A*a^4*b^2)*m^6 + 447*(2*B*a^5*b
+ 5*A*a^4*b^2)*m^5 + 3195*(2*B*a^5*b + 5*A*a^4*b^2)*m^4 + 12864*(2*B*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7961 vs. $2(211) = 422$.

Time = 0.87 (sec) , antiderivative size = 7961, normalized size of antiderivative = 36.35

$$\int (gx)^m(A + Bx) \left(a^2 + 2abx + b^2x^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((g*x)**m*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**3,x)
```

output
```
Piecewise(((-A*a**6/(7*x**7) - A*a**5*b/x**6 - 3*A*a**4*b**2/x**5 - 5*A*a*
*3*b**3/x**4 - 5*A*a**2*b**4/x**3 - 3*A*a*b**5/x**2 - A*b**6/x - B*a**6/(6
*x**6) - 6*B*a**5*b/(5*x**5) - 15*B*a**4*b**2/(4*x**4) - 20*B*a**3*b**3/(3
*x**3) - 15*B*a**2*b**4/(2*x**2) - 6*B*a*b**5/x + B*b**6*log(x))/g**8, Eq(
m, -8)), ((-A*a**6/(6*x**6) - 6*A*a**5*b/(5*x**5) - 15*A*a**4*b**2/(4*x**4
) - 20*A*a**3*b**3/(3*x**3) - 15*A*a**2*b**4/(2*x**2) - 6*A*a*b**5/x + A*b
**6*log(x) - B*a**6/(5*x**5) - 3*B*a**5*b/(2*x**4) - 5*B*a**4*b**2/x**3 -
10*B*a**3*b**3/x**2 - 15*B*a**2*b**4/x + 6*B*a*b**5*log(x) + B*b**6*x)/g**
7, Eq(m, -7)), ((-A*a**6/(5*x**5) - 3*A*a**5*b/(2*x**4) - 5*A*a**4*b**2/x*
*3 - 10*A*a**3*b**3/x**2 - 15*A*a**2*b**4/x + 6*A*a*b**5*log(x) + A*b**6*x
 - B*a**6/(4*x**4) - 2*B*a**5*b/x**3 - 15*B*a**4*b**2/(2*x**2) - 20*B*a**3
*b**3/x + 15*B*a**2*b**4*log(x) + 6*B*a*b**5*x + B*b**6*x**2/2)/g**6, Eq(m
, -6)), ((-A*a**6/(4*x**4) - 2*A*a**5*b/x**3 - 15*A*a**4*b**2/(2*x**2) - 2
0*A*a**3*b**3/x + 15*A*a**2*b**4*log(x) + 6*A*a*b**5*x + A*b**6*x**2/2 - B
*a**6/(3*x**3) - 3*B*a**5*b/x**2 - 15*B*a**4*b**2/x + 20*B*a**3*b**3*log(x
) + 15*B*a**2*b**4*x + 3*B*a*b**5*x**2 + B*b**6*x**3/3)/g**5, Eq(m, -5)),
((-A*a**6/(3*x**3) - 3*A*a**5*b/x**2 - 15*A*a**4*b**2/x + 20*A*a**3*b**3*l
og(x) + 15*A*a**2*b**4*x + 3*A*a*b**5*x**2 + A*b**6*x**3/3 - B*a**6/(2*x**
2) - 6*B*a**5*b/x + 15*B*a**4*b**2*log(x) + 20*B*a**3*b**3*x + 15*B*a**2*b
**4*x**2/2 + 2*B*a*b**5*x**3 + B*b**6*x**4/4)/g**4, Eq(m, -4)), ((-A*a*...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.37

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{Bb^6g^mx^8x^m}{m+8} + \frac{6\,Bab^5g^mx^7x^m}{m+7} + \frac{Ab^6g^mx^7x^m}{m+7} + \frac{15\,Ba^2b^4g^mx^6x^m}{m+6} + \frac{6\,Aab^5g^mx^6x^m}{m+6} + \frac{20\,Ba^3b^3g^mx^5x^m}{m+5} + \frac{15\,Aa^2b^4g^mx^5x^m}{m+5} + \frac{15\,Ba^4b^2g^mx^4x^m}{m+4} + \frac{20\,Aa^3b^3g^mx^4x^m}{m+4} + \frac{6\,Ba^5bg^mx^3x^m}{m+3} + \frac{15\,Aa^4b^2g^mx^3x^m}{m+3} + \frac{Ba^6g^mx^2x^m}{m+2} + \frac{6\,Aa^5bg^mx^2x^m}{m+2} + \frac{(gx)^{m+1}Aa^6}{g(m+1)}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="maxima")
```

output
```
B*b^6*g^m*x^8*x^m/(m + 8) + 6*B*a*b^5*g^m*x^7*x^m/(m + 7) + A*b^6*g^m*x^7*
x^m/(m + 7) + 15*B*a^2*b^4*g^m*x^6*x^m/(m + 6) + 6*A*a*b^5*g^m*x^6*x^m/(m
+ 6) + 20*B*a^3*b^3*g^m*x^5*x^m/(m + 5) + 15*A*a^2*b^4*g^m*x^5*x^m/(m + 5)
 + 15*B*a^4*b^2*g^m*x^4*x^m/(m + 4) + 20*A*a^3*b^3*g^m*x^4*x^m/(m + 4) + 6
*B*a^5*b*g^m*x^3*x^m/(m + 3) + 15*A*a^4*b^2*g^m*x^3*x^m/(m + 3) + B*a^6*g^
m*x^2*x^m/(m + 2) + 6*A*a^5*b*g^m*x^2*x^m/(m + 2) + (g*x)^(m + 1)*A*a^6/(g
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. $2(219) = 438$.

Time = 0.28 (sec) , antiderivative size = 2032, normalized size of antiderivative = 9.28

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x, algorithm="giac")
```

output

```
((g*x)^m*B*b^6*m^7*x^8 + 6*(g*x)^m*B*a*b^5*m^7*x^7 + (g*x)^m*A*b^6*m^7*x^7
 + 28*(g*x)^m*B*b^6*m^6*x^8 + 15*(g*x)^m*B*a^2*b^4*m^7*x^6 + 6*(g*x)^m*A*a
*b^5*m^7*x^6 + 174*(g*x)^m*B*a*b^5*m^6*x^7 + 29*(g*x)^m*A*b^6*m^6*x^7 + 32
2*(g*x)^m*B*b^6*m^5*x^8 + 20*(g*x)^m*B*a^3*b^3*m^7*x^5 + 15*(g*x)^m*A*a^2*
b^4*m^7*x^5 + 450*(g*x)^m*B*a^2*b^4*m^6*x^6 + 180*(g*x)^m*A*a*b^5*m^6*x^6
+ 2058*(g*x)^m*B*a*b^5*m^5*x^7 + 343*(g*x)^m*A*b^6*m^5*x^7 + 1960*(g*x)^m*
B*b^6*m^4*x^8 + 15*(g*x)^m*B*a^4*b^2*m^7*x^4 + 20*(g*x)^m*A*a^3*b^3*m^7*x^
4 + 620*(g*x)^m*B*a^3*b^3*m^6*x^5 + 465*(g*x)^m*A*a^2*b^4*m^6*x^5 + 5490*(
g*x)^m*B*a^2*b^4*m^5*x^6 + 2196*(g*x)^m*A*a*b^5*m^5*x^6 + 12810*(g*x)^m*B*
a*b^5*m^4*x^7 + 2135*(g*x)^m*A*b^6*m^4*x^7 + 6769*(g*x)^m*B*b^6*m^3*x^8 +
6*(g*x)^m*B*a^5*b*m^7*x^3 + 15*(g*x)^m*A*a^4*b^2*m^7*x^3 + 480*(g*x)^m*B*a
^4*b^2*m^6*x^4 + 640*(g*x)^m*A*a^3*b^3*m^6*x^4 + 7820*(g*x)^m*B*a^3*b^3*m^
5*x^5 + 5865*(g*x)^m*A*a^2*b^4*m^5*x^5 + 35100*(g*x)^m*B*a^2*b^4*m^4*x^6 +
 14040*(g*x)^m*A*a*b^5*m^4*x^6 + 45024*(g*x)^m*B*a*b^5*m^3*x^7 + 7504*(g*x
)^m*A*b^6*m^3*x^7 + 13132*(g*x)^m*B*b^6*m^2*x^8 + (g*x)^m*B*a^6*m^7*x^2 +
6*(g*x)^m*A*a^5*b*m^7*x^2 + 198*(g*x)^m*B*a^5*b*m^6*x^3 + 495*(g*x)^m*A*a^
4*b^2*m^6*x^3 + 6270*(g*x)^m*B*a^4*b^2*m^5*x^4 + 8360*(g*x)^m*A*a^3*b^3*m^
5*x^4 + 51620*(g*x)^m*B*a^3*b^3*m^4*x^5 + 38715*(g*x)^m*A*a^2*b^4*m^4*x^5
+ 126135*(g*x)^m*B*a^2*b^4*m^3*x^6 + 50454*(g*x)^m*A*a*b^5*m^3*x^6 + 88536
*(g*x)^m*B*a*b^5*m^2*x^7 + 14756*(g*x)^m*A*b^6*m^2*x^7 + 13068*(g*x)^m*...
```

Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.40

$$
\begin{aligned}
&\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx \\
&= \frac{A\,a^6\,x\,(g\,x)^m\,(m^7+35\,m^6+511\,m^5+4025\,m^4+18424\,m^3+48860\,m^2+69264\,m+40320)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{B\,b^6\,x^8\,(g\,x)^m\,(m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{a^5\,x^2\,(g\,x)^m\,(6\,A\,b+B\,a)\,\left(m^7+34\,m^6+478\,m^5+3580\,m^4+15289\,m^3+36706\,m^2+44712\,m+201\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{b^5\,x^7\,(g\,x)^m\,(A\,b+6\,B\,a)\,\left(m^7+29\,m^6+343\,m^5+2135\,m^4+7504\,m^3+14756\,m^2+14832\,m+5760\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{5\,a^2\,b^3\,x^5\,(g\,x)^m\,(3\,A\,b+4\,B\,a)\,\left(m^7+31\,m^6+391\,m^5+2581\,m^4+9544\,m^3+19564\,m^2+20304\,m+\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{5\,a^3\,b^2\,x^4\,(g\,x)^m\,(4\,A\,b+3\,B\,a)\,\left(m^7+32\,m^6+418\,m^5+2864\,m^4+10993\,m^3+23312\,m^2+24876\,m\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{3\,a\,b^4\,x^6\,(g\,x)^m\,(2\,A\,b+5\,B\,a)\,\left(m^7+30\,m^6+366\,m^5+2340\,m^4+8409\,m^3+16830\,m^2+17144\,m+\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320} \\
&\quad+\frac{3\,a^4\,b\,x^3\,(g\,x)^m\,(5\,A\,b+2\,B\,a)\,\left(m^7+33\,m^6+447\,m^5+3195\,m^4+12864\,m^3+28692\,m^2+32048\,m+\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}
\end{aligned}
$$

input
```
int((g*x)^m*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^3,x)
```

output
```
(A*a^6*x*(g*x)^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4 + 511*m^5 + 3
5*m^6 + m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 453
6*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (B*b^6*x^8*(g*x)^m*(13068*m + 13
132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m
+ 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 +
 40320) + (a^5*x^2*(g*x)^m*(6*A*b + B*a)*(44712*m + 36706*m^2 + 15289*m^3
+ 3580*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(109584*m + 118124*m^2 + 672
84*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (b^5*x^7
*(g*x)^m*(A*b + 6*B*a)*(14832*m + 14756*m^2 + 7504*m^3 + 2135*m^4 + 343*m^
5 + 29*m^6 + m^7 + 5760))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 +
 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (5*a^2*b^3*x^5*(g*x)^m*(3*A*
b + 4*B*a)*(20304*m + 19564*m^2 + 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 +
 m^7 + 8064))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (5*a^3*b^2*x^4*(g*x)^m*(4*A*b + 3*B*a)*(
24876*m + 23312*m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 + 32*m^6 + m^7 + 1008
0))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 +
36*m^7 + m^8 + 40320) + (3*a*b^4*x^6*(g*x)^m*(2*A*b + 5*B*a)*(17144*m + 16
830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m^6 + m^7 + 6720))/(109584*m
+ 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 +
 40320) + (3*a^4*b*x^3*(g*x)^m*(5*A*b + 2*B*a)*(32048*m + 28692*m^2 + 1...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.58

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^3\,dx = \frac{x^m g^m x(b^7m^7x^7+7a\,b^6m^7x^6+28b^7m^6x^7+21a^2b^5m^7x^5+203a\,b^6m^6x^6+322b^7m^5x^7+35a^3b^4m^7x^4+6}{}$$

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^3,x)
```

output

```
(x**m*g**m*x*(a**7*m**7 + 35*a**7*m**6 + 511*a**7*m**5 + 4025*a**7*m**4 +
18424*a**7*m**3 + 48860*a**7*m**2 + 69264*a**7*m + 40320*a**7 + 7*a**6*b*m
**7*x + 238*a**6*b*m**6*x + 3346*a**6*b*m**5*x + 25060*a**6*b*m**4*x + 107
023*a**6*b*m**3*x + 256942*a**6*b*m**2*x + 312984*a**6*b*m*x + 141120*a**6
*b*x + 21*a**5*b**2*m**7*x**2 + 693*a**5*b**2*m**6*x**2 + 9387*a**5*b**2*m
**5*x**2 + 67095*a**5*b**2*m**4*x**2 + 270144*a**5*b**2*m**3*x**2 + 602532
*a**5*b**2*m**2*x**2 + 673008*a**5*b**2*m*x**2 + 282240*a**5*b**2*x**2 + 3
5*a**4*b**3*m**7*x**3 + 1120*a**4*b**3*m**6*x**3 + 14630*a**4*b**3*m**5*x*
*3 + 100240*a**4*b**3*m**4*x**3 + 384755*a**4*b**3*m**3*x**3 + 815920*a**4
*b**3*m**2*x**3 + 870660*a**4*b**3*m*x**3 + 352800*a**4*b**3*x**3 + 35*a**
3*b**4*m**7*x**4 + 1085*a**3*b**4*m**6*x**4 + 13685*a**3*b**4*m**5*x**4 +
90335*a**3*b**4*m**4*x**4 + 334040*a**3*b**4*m**3*x**4 + 684740*a**3*b**4*
m**2*x**4 + 710640*a**3*b**4*m*x**4 + 282240*a**3*b**4*x**4 + 21*a**2*b**5
*m**7*x**5 + 630*a**2*b**5*m**6*x**5 + 7686*a**2*b**5*m**5*x**5 + 49140*a*
*2*b**5*m**4*x**5 + 176589*a**2*b**5*m**3*x**5 + 353430*a**2*b**5*m**2*x**
5 + 360024*a**2*b**5*m*x**5 + 141120*a**2*b**5*x**5 + 7*a*b**6*m**7*x**6 +
 203*a*b**6*m**6*x**6 + 2401*a*b**6*m**5*x**6 + 14945*a*b**6*m**4*x**6 + 5
2528*a*b**6*m**3*x**6 + 103292*a*b**6*m**2*x**6 + 103824*a*b**6*m*x**6 + 4
0320*a*b**6*x**6 + b**7*m**7*x**7 + 28*b**7*m**6*x**7 + 322*b**7*m**5*x**7
 + 1960*b**7*m**4*x**7 + 6769*b**7*m**3*x**7 + 13132*b**7*m**2*x**7 + 1...
```

3.466 $\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 155

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{a^4A(gx)^{1+m}}{g(1+m)}+\frac{a^3(4Ab+aB)(gx)^{2+m}}{g^2(2+m)}+\frac{2a^2b(3Ab+2aB)(gx)^{3+m}}{g^3(3+m)}+\frac{2ab^2(2Ab+3aB)(gx)^{4+m}}{g^4(4+m)}+\frac{b^3(Ab+4aB)(gx)^{5+m}}{g^5(5+m)}+\frac{b^4B(gx)^{6+m}}{g^6(6+m)}$$

output
```
a^4*A*(g*x)^(1+m)/g/(1+m)+a^3*(4*A*b+B*a)*(g*x)^(2+m)/g^2/(2+m)+2*a^2*b*(3
*A*b+2*B*a)*(g*x)^(3+m)/g^3/(3+m)+2*a*b^2*(2*A*b+3*B*a)*(g*x)^(4+m)/g^4/(4
+m)+b^3*(A*b+4*B*a)*(g*x)^(5+m)/g^5/(5+m)+b^4*B*(g*x)^(6+m)/g^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int (gx)^m (A+Bx) \left(a^2+2abx+b^2x^2\right)^2 \, dx = \frac{(gx)^m \left(Bx(a+bx)^5+(-aB(1+m)+Ab(6+m))x\left(\frac{a^4}{1+m}+\frac{4a^3bx}{2+m}+\frac{6a^2b^2x^2}{3+m}+\frac{4ab^3x^3}{4+m}+\frac{b^4x^4}{5+m}\right)\right)}{b(6+m)}$$

input
```
Integrate[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((g*x)^m*(B*x*(a + b*x)^5 + (-(a*B*(1 + m)) + A*b*(6 + m))*x*(a^4/(1 + m)
+ (4*a^3*b*x)/(2 + m) + (6*a^2*b^2*x^2)/(3 + m) + (4*a*b^3*x^3)/(4 + m) +
(b^4*x^4)/(5 + m))))/(b*(6 + m))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)^2 (A+Bx)(gx)^m \, dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^4(gx)^m(a+bx)^4(A+Bx)dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int (a+bx)^4(A+Bx)(gx)^m dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^4A(gx)^m + \frac{a^3(gx)^{m+1}(aB+4Ab)}{g} + \frac{2a^2b(gx)^{m+2}(2aB+3Ab)}{g^2} + \frac{b^3(gx)^{m+4}(4aB+Ab)}{g^4} + \frac{2ab^2(gx)^{m+3}(}{g^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^4A(gx)^{m+1}}{g(m+1)} + \frac{a^3(gx)^{m+2}(aB+4Ab)}{g^2(m+2)} + \frac{2a^2b(gx)^{m+3}(2aB+3Ab)}{g^3(m+3)} + \frac{b^3(gx)^{m+5}(4aB+Ab)}{g^5(m+5)} + \frac{2ab^2(gx)^{m+4}(3aB+2Ab)}{g^4(m+4)} + \frac{b^4B(gx)^{m+6}}{g^6(m+6)}$$

input
```
Int[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(a^4*A*(g*x)^(1 + m))/(g*(1 + m)) + (a^3*(4*A*b + a*B)*(g*x)^(2 + m))/(g^2
*(2 + m)) + (2*a^2*b*(3*A*b + 2*a*B)*(g*x)^(3 + m))/(g^3*(3 + m)) + (2*a*b
^2*(2*A*b + 3*a*B)*(g*x)^(4 + m))/(g^4*(4 + m)) + (b^3*(A*b + 4*a*B)*(g*x)
^(5 + m))/(g^5*(5 + m)) + (b^4*B*(g*x)^(6 + m))/(g^6*(6 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
norman	$\frac{B\,b^{4} x^{6} {\mathrm e}^{m \ln \left(g x \right)}}{6+m}+\frac{a^{3} \left(4 A b +B a \right) x^{2} {\mathrm e}^{m \ln \left(g x \right)}}{2+m}+\frac{a^{4} A x \,{\mathrm e}^{m \ln \left(g x \right)}}{1+m}+\frac{b^{3} \left(A b +4 B a \right) x^{5} {\mathrm e}^{m \ln \left(g x \right)}}{5+m}+\frac{2 a \,b^{2} \left(2 A b +3 B a \right) x^{4} {\mathrm e}^{m \ln \left(g x \right)}}{4+m}$
gosper	$\frac{\left(g x \right)^{m} \left(B\,b^{4} m^{5} x^{5}+A\,b^{4} m^{5} x^{4}+4 B a \,b^{3} m^{5} x^{4}+15 B\,b^{4} m^{4} x^{5}+4 A a \,b^{3} m^{5} x^{3}+16 A\,b^{4} m^{4} x^{4}+6 B\,a^{2} b^{2} m^{5} x^{3}+64 B a \,b^{3} m^{4} x^{4}+85 B\right.}{}$
risch	$\frac{\left(g x \right)^{m} \left(B\,b^{4} m^{5} x^{5}+A\,b^{4} m^{5} x^{4}+4 B a \,b^{3} m^{5} x^{4}+15 B\,b^{4} m^{4} x^{5}+4 A a \,b^{3} m^{5} x^{3}+16 A\,b^{4} m^{4} x^{4}+6 B\,a^{2} b^{2} m^{5} x^{3}+64 B a \,b^{3} m^{4} x^{4}+85 B\right.}{}$
orering	$\frac{\left(B\,b^{4} m^{5} x^{5}+A\,b^{4} m^{5} x^{4}+4 B a \,b^{3} m^{5} x^{4}+15 B\,b^{4} m^{4} x^{5}+4 A a \,b^{3} m^{5} x^{3}+16 A\,b^{4} m^{4} x^{4}+6 B\,a^{2} b^{2} m^{5} x^{3}+64 B a \,b^{3} m^{4} x^{4}+85 B\,b^{4} m^{3}\right.}{}$
parallelrisch	Expression too large to display

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
B*b^4/(6+m)*x^6*exp(m*ln(g*x))+a^3*(4*A*b+B*a)/(2+m)*x^2*exp(m*ln(g*x))+a^
4*A/(1+m)*x*exp(m*ln(g*x))+b^3*(A*b+4*B*a)/(5+m)*x^5*exp(m*ln(g*x))+2*a*b^
2*(2*A*b+3*B*a)/(4+m)*x^4*exp(m*ln(g*x))+2*a^2*b*(3*A*b+2*B*a)/(3+m)*x^3*e
xp(m*ln(g*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(155) = 310.

Time = 0.08 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.93

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$=\frac{((Bb^4m^5+15\,Bb^4m^4+85\,Bb^4m^3+225\,Bb^4m^2+274\,Bb^4m+120\,Bb^4)x^6+((4\,Bab^3+Ab^4)m^5+576}{}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
((B*b^4*m^5 + 15*B*b^4*m^4 + 85*B*b^4*m^3 + 225*B*b^4*m^2 + 274*B*b^4*m +
120*B*b^4)*x^6 + ((4*B*a*b^3 + A*b^4)*m^5 + 576*B*a*b^3 + 144*A*b^4 + 16*(
4*B*a*b^3 + A*b^4)*m^4 + 95*(4*B*a*b^3 + A*b^4)*m^3 + 260*(4*B*a*b^3 + A*b
^4)*m^2 + 324*(4*B*a*b^3 + A*b^4)*m)*x^5 + 2*((3*B*a^2*b^2 + 2*A*a*b^3)*m^
5 + 540*B*a^2*b^2 + 360*A*a*b^3 + 17*(3*B*a^2*b^2 + 2*A*a*b^3)*m^4 + 107*(
3*B*a^2*b^2 + 2*A*a*b^3)*m^3 + 307*(3*B*a^2*b^2 + 2*A*a*b^3)*m^2 + 396*(3*
B*a^2*b^2 + 2*A*a*b^3)*m)*x^4 + 2*((2*B*a^3*b + 3*A*a^2*b^2)*m^5 + 480*B*a
^3*b + 720*A*a^2*b^2 + 18*(2*B*a^3*b + 3*A*a^2*b^2)*m^4 + 121*(2*B*a^3*b +
 3*A*a^2*b^2)*m^3 + 372*(2*B*a^3*b + 3*A*a^2*b^2)*m^2 + 508*(2*B*a^3*b + 3
*A*a^2*b^2)*m)*x^3 + ((B*a^4 + 4*A*a^3*b)*m^5 + 360*B*a^4 + 1440*A*a^3*b +
 19*(B*a^4 + 4*A*a^3*b)*m^4 + 137*(B*a^4 + 4*A*a^3*b)*m^3 + 461*(B*a^4 + 4
*A*a^3*b)*m^2 + 702*(B*a^4 + 4*A*a^3*b)*m)*x^2 + (A*a^4*m^5 + 20*A*a^4*m^4
 + 155*A*a^4*m^3 + 580*A*a^4*m^2 + 1044*A*a^4*m + 720*A*a^4)*x)*(g*x)^m/(m
^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3538 vs. $2(146) = 292$.

Time = 0.54 (sec) , antiderivative size = 3538, normalized size of antiderivative = 22.83

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((g*x)**m*(B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Piecewise(((-A*a**4/(5*x**5) - A*a**3*b/x**4 - 2*A*a**2*b**2/x**3 - 2*A*a*
b**3/x**2 - A*b**4/x - B*a**4/(4*x**4) - 4*B*a**3*b/(3*x**3) - 3*B*a**2*b*
*2/x**2 - 4*B*a*b**3/x + B*b**4*log(x))/g**6, Eq(m, -6)), ((-A*a**4/(4*x**
4) - 4*A*a**3*b/(3*x**3) - 3*A*a**2*b**2/x**2 - 4*A*a*b**3/x + A*b**4*log(
x) - B*a**4/(3*x**3) - 2*B*a**3*b/x**2 - 6*B*a**2*b**2/x + 4*B*a*b**3*log(
x) + B*b**4*x)/g**5, Eq(m, -5)), ((-A*a**4/(3*x**3) - 2*A*a**3*b/x**2 - 6*
A*a**2*b**2/x + 4*A*a*b**3*log(x) + A*b**4*x - B*a**4/(2*x**2) - 4*B*a**3*
b/x + 6*B*a**2*b**2*log(x) + 4*B*a*b**3*x + B*b**4*x**2/2)/g**4, Eq(m, -4)
), ((-A*a**4/(2*x**2) - 4*A*a**3*b/x + 6*A*a**2*b**2*log(x) + 4*A*a*b**3*x
 + A*b**4*x**2/2 - B*a**4/x + 4*B*a**3*b*log(x) + 6*B*a**2*b**2*x + 2*B*a*
b**3*x**2 + B*b**4*x**3/3)/g**3, Eq(m, -3)), ((-A*a**4/x + 4*A*a**3*b*log(
x) + 6*A*a**2*b**2*x + 2*A*a*b**3*x**2 + A*b**4*x**3/3 + B*a**4*log(x) + 4
*B*a**3*b*x + 3*B*a**2*b**2*x**2 + 4*B*a*b**3*x**3/3 + B*b**4*x**4/4)/g**2
, Eq(m, -2)), ((A*a**4*log(x) + 4*A*a**3*b*x + 3*A*a**2*b**2*x**2 + 4*A*a*
b**3*x**3/3 + A*b**4*x**4/4 + B*a**4*x + 2*B*a**3*b*x**2 + 2*B*a**2*b**2*x
**3 + B*a*b**3*x**4 + B*b**4*x**5/5)/g, Eq(m, -1)), (A*a**4*m**5*x*(g*x)**
m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 20*A
*a**4*m**4*x*(g*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 +
1764*m + 720) + 155*A*a**4*m**3*x*(g*x)**m/(m**6 + 21*m**5 + 175*m**4 + 73
5*m**3 + 1624*m**2 + 1764*m + 720) + 580*A*a**4*m**2*x*(g*x)**m/(m**6 +...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.34

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx=\frac{Bb^4g^mx^6x^m}{m+6}+\frac{4\,Bab^3g^mx^5x^m}{m+5}+\frac{Ab^4g^mx^5x^m}{m+5}+\frac{6\,Ba^2b^2g^mx^4x^m}{m+4}+\frac{4\,Aab^3g^mx^4x^m}{m+4}+\frac{4\,Ba^3bg^mx^3x^m}{m+3}+\frac{6\,Aa^2b^2g^mx^3x^m}{m+3}+\frac{Ba^4g^mx^2x^m}{m+2}+\frac{4\,Aa^3bg^mx^2x^m}{m+2}+\frac{(gx)^{m+1}Aa^4}{g(m+1)}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
B*b^4*g^m*x^6*x^m/(m + 6) + 4*B*a*b^3*g^m*x^5*x^m/(m + 5) + A*b^4*g^m*x^5*
x^m/(m + 5) + 6*B*a^2*b^2*g^m*x^4*x^m/(m + 4) + 4*A*a*b^3*g^m*x^4*x^m/(m +
 4) + 4*B*a^3*b*g^m*x^3*x^m/(m + 3) + 6*A*a^2*b^2*g^m*x^3*x^m/(m + 3) + B*
a^4*g^m*x^2*x^m/(m + 2) + 4*A*a^3*b*g^m*x^2*x^m/(m + 2) + (g*x)^(m + 1)*A*
a^4/(g*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(155) = 310$.

Time = 0.28 (sec) , antiderivative size = 1046, normalized size of antiderivative = 6.75

$$\int (gx)^m(A + Bx)\left(a^2 + 2abx + b^2x^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
((g*x)^m*B*b^4*m^5*x^6 + 4*(g*x)^m*B*a*b^3*m^5*x^5 + (g*x)^m*A*b^4*m^5*x^5
 + 15*(g*x)^m*B*b^4*m^4*x^6 + 6*(g*x)^m*B*a^2*b^2*m^5*x^4 + 4*(g*x)^m*A*a*
b^3*m^5*x^4 + 64*(g*x)^m*B*a*b^3*m^4*x^5 + 16*(g*x)^m*A*b^4*m^4*x^5 + 85*(
g*x)^m*B*b^4*m^3*x^6 + 4*(g*x)^m*B*a^3*b*m^5*x^3 + 6*(g*x)^m*A*a^2*b^2*m^5
*x^3 + 102*(g*x)^m*B*a^2*b^2*m^4*x^4 + 68*(g*x)^m*A*a*b^3*m^4*x^4 + 380*(g
*x)^m*B*a*b^3*m^3*x^5 + 95*(g*x)^m*A*b^4*m^3*x^5 + 225*(g*x)^m*B*b^4*m^2*x
^6 + (g*x)^m*B*a^4*m^5*x^2 + 4*(g*x)^m*A*a^3*b*m^5*x^2 + 72*(g*x)^m*B*a^3*
b*m^4*x^3 + 108*(g*x)^m*A*a^2*b^2*m^4*x^3 + 642*(g*x)^m*B*a^2*b^2*m^3*x^4
+ 428*(g*x)^m*A*a*b^3*m^3*x^4 + 1040*(g*x)^m*B*a*b^3*m^2*x^5 + 260*(g*x)^m
*A*b^4*m^2*x^5 + 274*(g*x)^m*B*b^4*m*x^6 + (g*x)^m*A*a^4*m^5*x + 19*(g*x)^
m*B*a^4*m^4*x^2 + 76*(g*x)^m*A*a^3*b*m^4*x^2 + 484*(g*x)^m*B*a^3*b*m^3*x^3
 + 726*(g*x)^m*A*a^2*b^2*m^3*x^3 + 1842*(g*x)^m*B*a^2*b^2*m^2*x^4 + 1228*(
g*x)^m*A*a*b^3*m^2*x^4 + 1296*(g*x)^m*B*a*b^3*m*x^5 + 324*(g*x)^m*A*b^4*m*
x^5 + 120*(g*x)^m*B*b^4*x^6 + 20*(g*x)^m*A*a^4*m^4*x + 137*(g*x)^m*B*a^4*m
^3*x^2 + 548*(g*x)^m*A*a^3*b*m^3*x^2 + 1488*(g*x)^m*B*a^3*b*m^2*x^3 + 2232
*(g*x)^m*A*a^2*b^2*m^2*x^3 + 2376*(g*x)^m*B*a^2*b^2*m*x^4 + 1584*(g*x)^m*A
*a*b^3*m*x^4 + 576*(g*x)^m*B*a*b^3*x^5 + 144*(g*x)^m*A*b^4*x^5 + 155*(g*x)
^m*A*a^4*m^3*x + 461*(g*x)^m*B*a^4*m^2*x^2 + 1844*(g*x)^m*A*a^3*b*m^2*x^2
+ 2032*(g*x)^m*B*a^3*b*m*x^3 + 3048*(g*x)^m*A*a^2*b^2*m*x^3 + 1080*(g*x)^m
*B*a^2*b^2*x^4 + 720*(g*x)^m*A*a*b^3*x^4 + 580*(g*x)^m*A*a^4*m^2*x + 70...
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.61

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$= (g\,x)^m\left(\frac{A\,a^4\,x\,(m^5+20\,m^4+155\,m^3+580\,m^2+1044\,m+720)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{B\,b^4\,x^6\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{a^3\,x^2\,(4\,A\,b+B\,a)\,(m^5+19\,m^4+137\,m^3+461\,m^2+702\,m+360)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{b^3\,x^5\,(A\,b+4\,B\,a)\,(m^5+16\,m^4+95\,m^3+260\,m^2+324\,m+144)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{2\,a\,b^2\,x^4\,(2\,A\,b+3\,B\,a)\,(m^5+17\,m^4+107\,m^3+307\,m^2+396\,m+180)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{2\,a^2\,b\,x^3\,(3\,A\,b+2\,B\,a)\,(m^5+18\,m^4+121\,m^3+372\,m^2+508\,m+240)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}\right)$$

input
```
int((g*x)^m*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
(g*x)^m*((A*a^4*x*(1044*m + 580*m^2 + 155*m^3 + 20*m^4 + m^5 + 720))/(1764
*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (B*b^4*x^6*(274*
m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 +
 175*m^4 + 21*m^5 + m^6 + 720) + (a^3*x^2*(4*A*b + B*a)*(702*m + 461*m^2 +
 137*m^3 + 19*m^4 + m^5 + 360))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 2
1*m^5 + m^6 + 720) + (b^3*x^5*(A*b + 4*B*a)*(324*m + 260*m^2 + 95*m^3 + 16
*m^4 + m^5 + 144))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 +
 720) + (2*a*b^2*x^4*(2*A*b + 3*B*a)*(396*m + 307*m^2 + 107*m^3 + 17*m^4 +
 m^5 + 180))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)
+ (2*a^2*b*x^3*(3*A*b + 2*B*a)*(508*m + 372*m^2 + 121*m^3 + 18*m^4 + m^5 +
 240))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.72

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)^2\,dx$$
$$= \frac{x^m g^m x(b^5m^5x^5+5a\,b^4m^5x^4+15b^5m^4x^5+10a^2b^3m^5x^3+80a\,b^4m^4x^4+85b^5m^3x^5+10a^3b^2m^5x^2+170}{}$$

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(x**m*g**m*x*(a**5*m**5 + 20*a**5*m**4 + 155*a**5*m**3 + 580*a**5*m**2 + 1
044*a**5*m + 720*a**5 + 5*a**4*b*m**5*x + 95*a**4*b*m**4*x + 685*a**4*b*m*
*3*x + 2305*a**4*b*m**2*x + 3510*a**4*b*m*x + 1800*a**4*b*x + 10*a**3*b**2
*m**5*x**2 + 180*a**3*b**2*m**4*x**2 + 1210*a**3*b**2*m**3*x**2 + 3720*a**
3*b**2*m**2*x**2 + 5080*a**3*b**2*m*x**2 + 2400*a**3*b**2*x**2 + 10*a**2*b
**3*m**5*x**3 + 170*a**2*b**3*m**4*x**3 + 1070*a**2*b**3*m**3*x**3 + 3070*
a**2*b**3*m**2*x**3 + 3960*a**2*b**3*m*x**3 + 1800*a**2*b**3*x**3 + 5*a*b*
*4*m**5*x**4 + 80*a*b**4*m**4*x**4 + 475*a*b**4*m**3*x**4 + 1300*a*b**4*m*
*2*x**4 + 1620*a*b**4*m*x**4 + 720*a*b**4*x**4 + b**5*m**5*x**5 + 15*b**5*
m**4*x**5 + 85*b**5*m**3*x**5 + 225*b**5*m**2*x**5 + 274*b**5*m*x**5 + 120
*b**5*x**5))/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m +
720)
```

3.467 $\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{a^2A(gx)^{1+m}}{g(1+m)}+\frac{a(2Ab+aB)(gx)^{2+m}}{g^2(2+m)}+\frac{b(Ab+2aB)(gx)^{3+m}}{g^3(3+m)}+\frac{b^2B(gx)^{4+m}}{g^4(4+m)}$$

output
```
a^2*A*(g*x)^(1+m)/g/(1+m)+a*(2*A*b+B*a)*(g*x)^(2+m)/g^2/(2+m)+b*(A*b+2*B*a
)*(g*x)^(3+m)/g^3/(3+m)+b^2*B*(g*x)^(4+m)/g^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx=\frac{(gx)^m\left(Bx(a+bx)^3+(-aB(1+m)+Ab(4+m))x\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)\right)}{b(4+m)}$$

input
```
Integrate[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((g*x)^m*(B*x*(a + b*x)^3 + (-(a*B*(1 + m)) + A*b*(4 + m))*x*(a^2/(1 + m)
+ (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))))/(b*(4 + m))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1184, 27, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2\right)(A+Bx)(gx)^m\,dx$$

$$\Big\downarrow \text{ 1184}$$

$$\frac{\int b^2(gx)^m(a+bx)^2(A+Bx)dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int (a+bx)^2(A+Bx)(gx)^m dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2A(gx)^m+\frac{b(gx)^{m+2}(2aB+Ab)}{g^2}+\frac{a(gx)^{m+1}(aB+2Ab)}{g}+\frac{b^2B(gx)^{m+3}}{g^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2A(gx)^{m+1}}{g(m+1)}+\frac{b(gx)^{m+3}(2aB+Ab)}{g^3(m+3)}+\frac{a(gx)^{m+2}(aB+2Ab)}{g^2(m+2)}+\frac{b^2B(gx)^{m+4}}{g^4(m+4)}$$

input
```
Int[(g*x)^m*(A + B*x)*(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(a^2*A*(g*x)^(1 + m))/(g*(1 + m)) + (a*(2*A*b + a*B)*(g*x)^(2 + m))/(g^2*(
2 + m)) + (b*(A*b + 2*a*B)*(g*x)^(3 + m))/(g^3*(3 + m)) + (b^2*B*(g*x)^(4
+ m))/(g^4*(4 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result
norman	$\frac{B\,b^{2}x^{4}{\mathrm e}^{m\ln(gx)}}{4+m}+\frac{a(2Ab+Ba)x^{2}{\mathrm e}^{m\ln(gx)}}{2+m}+\frac{a^{2}Ax\,{\mathrm e}^{m\ln(gx)}}{1+m}+\frac{b(Ab+2Ba)x^{3}{\mathrm e}^{m\ln(gx)}}{3+m}$
gosper	$\frac{(gx)^{m}(B\,b^{2}m^{3}x^{3}+A\,b^{2}m^{3}x^{2}+2Bab\,m^{3}x^{2}+6B\,b^{2}m^{2}x^{3}+2Aab\,m^{3}x+7A\,b^{2}m^{2}x^{2}+B\,a^{2}m^{3}x+14Bab\,m^{2}x^{2}+11B\,b^{2}m\,x^{3}+A}{}$
risch	$\frac{(gx)^{m}(B\,b^{2}m^{3}x^{3}+A\,b^{2}m^{3}x^{2}+2Bab\,m^{3}x^{2}+6B\,b^{2}m^{2}x^{3}+2Aab\,m^{3}x+7A\,b^{2}m^{2}x^{2}+B\,a^{2}m^{3}x+14Bab\,m^{2}x^{2}+11B\,b^{2}m\,x^{3}+A}{}$
orering	$\frac{(B\,b^{2}m^{3}x^{3}+A\,b^{2}m^{3}x^{2}+2Bab\,m^{3}x^{2}+6B\,b^{2}m^{2}x^{3}+2Aab\,m^{3}x+7A\,b^{2}m^{2}x^{2}+B\,a^{2}m^{3}x+14Bab\,m^{2}x^{2}+11B\,b^{2}m\,x^{3}+A\,a^{2}m^{3}}{}$
parallelrisch	$\frac{26Ax(gx)^{m}a^{2}m+7A\,x^{3}(gx)^{m}b^{2}m^{2}+11B\,x^{4}(gx)^{m}b^{2}m+B\,x^{2}(gx)^{m}a^{2}m^{3}+14A\,x^{3}(gx)^{m}b^{2}m+Ax(gx)^{m}a^{2}m^{3}+9Ax(gx)^{m}a}{}$

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x,method=_RETURNVERBOSE)
```

output
```
B*b^2/(4+m)*x^4*exp(m*ln(g*x))+a*(2*A*b+B*a)/(2+m)*x^2*exp(m*ln(g*x))+a^2*
A/(1+m)*x*exp(m*ln(g*x))+b*(A*b+2*B*a)/(3+m)*x^3*exp(m*ln(g*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.38

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{((Bb^2m^3+6\,Bb^2m^2+11\,Bb^2m+6\,Bb^2)x^4+((2\,Bab+Ab^2)m^3+16\,Bab+8\,Ab^2+7\,(2\,Bab+Ab^2)m^2}{}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 6*B*b^2*m^2 + 11*B*b^2*m + 6*B*b^2)*x^4 + ((2*B*a*b + A*b^2)
*m^3 + 16*B*a*b + 8*A*b^2 + 7*(2*B*a*b + A*b^2)*m^2 + 14*(2*B*a*b + A*b^2)
*m)*x^3 + ((B*a^2 + 2*A*a*b)*m^3 + 12*B*a^2 + 24*A*a*b + 8*(B*a^2 + 2*A*a*
b)*m^2 + 19*(B*a^2 + 2*A*a*b)*m)*x^2 + (A*a^2*m^3 + 9*A*a^2*m^2 + 26*A*a^2
*m + 24*A*a^2)*x)*(g*x)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(82) = 164$.

Time = 0.33 (sec) , antiderivative size = 1073, normalized size of antiderivative = 11.79

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((g*x)**m*(B*x+A)*(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Piecewise(((-A*a**2/(3*x**3) - A*a*b/x**2 - A*b**2/x - B*a**2/(2*x**2) - 2
*B*a*b/x + B*b**2*log(x))/g**4, Eq(m, -4)), ((-A*a**2/(2*x**2) - 2*A*a*b/x
 + A*b**2*log(x) - B*a**2/x + 2*B*a*b*log(x) + B*b**2*x)/g**3, Eq(m, -3)),
 ((-A*a**2/x + 2*A*a*b*log(x) + A*b**2*x + B*a**2*log(x) + 2*B*a*b*x + B*b
**2*x**2/2)/g**2, Eq(m, -2)), ((A*a**2*log(x) + 2*A*a*b*x + A*b**2*x**2/2
+ B*a**2*x + B*a*b*x**2 + B*b**2*x**3/3)/g, Eq(m, -1)), (A*a**2*m**3*x*(g*
x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*A*a**2*m**2*x*(g*x)**m/(m
**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*A*a**2*m*x*(g*x)**m/(m**4 + 10*m
**3 + 35*m**2 + 50*m + 24) + 24*A*a**2*x*(g*x)**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) + 2*A*a*b*m**3*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50
*m + 24) + 16*A*a*b*m**2*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m +
24) + 38*A*a*b*m*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24
*A*a*b*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + A*b**2*m**3*
x**3*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 7*A*b**2*m**2*x**3*
(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 14*A*b**2*m*x**3*(g*x)**
m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 8*A*b**2*x**3*(g*x)**m/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + B*a**2*m**3*x**2*(g*x)**m/(m**4 + 10*m**3
 + 35*m**2 + 50*m + 24) + 8*B*a**2*m**2*x**2*(g*x)**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 19*B*a**2*m*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 +
 50*m + 24) + 12*B*a**2*x**2*(g*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx = \frac{Bb^2g^mx^4x^m}{m+4}+\frac{2\,Babg^mx^3x^m}{m+3}+\frac{Ab^2g^mx^3x^m}{m+3}+\frac{Ba^2g^mx^2x^m}{m+2}+\frac{2\,Aabg^mx^2x^m}{m+2}+\frac{(gx)^{m+1}\,Aa^2}{g(m+1)}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
B*b^2*g^m*x^4*x^m/(m + 4) + 2*B*a*b*g^m*x^3*x^m/(m + 3) + A*b^2*g^m*x^3*x^
m/(m + 3) + B*a^2*g^m*x^2*x^m/(m + 2) + 2*A*a*b*g^m*x^2*x^m/(m + 2) + (g*x
)^(m + 1)*A*a^2/(g*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(91) = 182$.

Time = 0.26 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.18

$$\int (gx)^m (A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = \frac{(gx)^m Bb^2m^3x^4 + 2\,(gx)^m Babm^3x^3 + (gx)^m Ab^2m^3x^3 + 6\,(gx)^m Bb^2m^2x^4 + (gx)^m Ba^2m^3x^2 + 2\,(gx)^m}{}$$

input
```
integrate((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
((g*x)^m*B*b^2*m^3*x^4 + 2*(g*x)^m*B*a*b*m^3*x^3 + (g*x)^m*A*b^2*m^3*x^3 +
 6*(g*x)^m*B*b^2*m^2*x^4 + (g*x)^m*B*a^2*m^3*x^2 + 2*(g*x)^m*A*a*b*m^3*x^2
 + 14*(g*x)^m*B*a*b*m^2*x^3 + 7*(g*x)^m*A*b^2*m^2*x^3 + 11*(g*x)^m*B*b^2*m
*x^4 + (g*x)^m*A*a^2*m^3*x + 8*(g*x)^m*B*a^2*m^2*x^2 + 16*(g*x)^m*A*a*b*m^
2*x^2 + 28*(g*x)^m*B*a*b*m*x^3 + 14*(g*x)^m*A*b^2*m*x^3 + 6*(g*x)^m*B*b^2*
x^4 + 9*(g*x)^m*A*a^2*m^2*x + 19*(g*x)^m*B*a^2*m*x^2 + 38*(g*x)^m*A*a*b*m*
x^2 + 16*(g*x)^m*B*a*b*x^3 + 8*(g*x)^m*A*b^2*x^3 + 26*(g*x)^m*A*a^2*m*x +
12*(g*x)^m*B*a^2*x^2 + 24*(g*x)^m*A*a*b*x^2 + 24*(g*x)^m*A*a^2*x)/(m^4 + 1
0*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int (gx)^m (A+Bx) \left(a^2+2abx+b^2x^2\right) \, dx = (g\,x)^m \left(\frac{B\,b^2\,x^4\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{A\,a^2\,x\,(m^3+9\,m^2+26\,m+24)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{a\,x^2\,(2\,A\,b+B\,a)\,(m^3+8\,m^2+19\,m+12)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{b\,x^3\,(A\,b+2\,B\,a)\,(m^3+7\,m^2+14\,m+8)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

input
```
int((g*x)^m*(A + B*x)*(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
(g*x)^m*((B*b^2*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (A*a^2*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*x^2*(2*A*b + B*a)*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 +
 10*m^3 + m^4 + 24) + (b*x^3*(A*b + 2*B*a)*(14*m + 7*m^2 + m^3 + 8))/(50*m
 + 35*m^2 + 10*m^3 + m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int (gx)^m(A+Bx)\left(a^2+2abx+b^2x^2\right)\,dx$$
$$= \frac{x^m g^m x(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11b^3m\,x^3+a^3m^3+24a^2b\,m^2x+4}{m^4+10m^3+35m^2+50m+}$$

input
```
int((g*x)^m*(B*x+A)*(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(x**m*g**m*x*(a**3*m**3 + 9*a**3*m**2 + 26*a**3*m + 24*a**3 + 3*a**2*b*m**
3*x + 24*a**2*b*m**2*x + 57*a**2*b*m*x + 36*a**2*b*x + 3*a*b**2*m**3*x**2
+ 21*a*b**2*m**2*x**2 + 42*a*b**2*m*x**2 + 24*a*b**2*x**2 + b**3*m**3*x**3
 + 6*b**3*m**2*x**3 + 11*b**3*m*x**3 + 6*b**3*x**3))/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24)
```

3.468 $\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{B(gx)^{1+m}}{bgm(a+bx)} + \frac{(Abm-aB(1+m))(gx)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{a}\right)}{a^2bgm(1+m)}$$

output
```
B*(g*x)^(1+m)/b/g/m/(b*x+a)+(A*b*m-a*B*(1+m))*(g*x)^(1+m)*hypergeom([2, 1+
m],[2+m],-b*x/a)/a^2/b/g/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{x(gx)^m\left(\frac{a(Ab-aB)}{a+bx}+\frac{(-Abm+aB(1+m))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{a^2b}$$

input
```
Integrate[((g*x)^m*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
(x*(g*x)^m*((a*(A*b - a*B))/(a + b*x) + ((-(A*b*m) + a*B*(1 + m))*Hypergeo
metric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(1 + m)))/(a^2*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(gx)^m}{a^2+2abx+b^2x^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^2\int \frac{(gx)^m(A+Bx)}{b^2(a+bx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(A+Bx)(gx)^m}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(gx)^{m+1}(Ab-aB)}{abg(a+bx)}-\frac{(Abm-aB(m+1))\int \frac{(gx)^m}{a+bx}dx}{ab}$$

$$\Big\downarrow \text{74}$$

$$\frac{(gx)^{m+1}(Ab-aB)}{abg(a+bx)}-\frac{(gx)^{m+1}(Abm-aB(m+1))\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{a^2bg(m+1)}$$

input
```
Int[((g*x)^m*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2),x]
```

output
```
((A*b - a*B)*(g*x)^(1 + m))/(a*b*g*(a + b*x)) - ((A*b*m - a*B*(1 + m))*(g*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a^2*b*g*(1 + m
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(gx)^m (Bx + A)}{b^2x^2 + 2abx + a^2} dx$$

input
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

Fricas [F]

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\int \frac{(Bx+A)(gx)^m}{b^2x^2+2\,abx+a^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(g*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\int \frac{(gx)^m\,(A+Bx)}{(a+bx)^2}\,dx$$

input
```
integrate((g*x)**m*(B*x+A)/(b**2*x**2+2*a*b*x+a**2),x)
```

output
```
Integral((g*x)**m*(A + B*x)/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx=\int \frac{(Bx+A)(gx)^m}{b^2x^2+2\,abx+a^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(g*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Giac [F]

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \int \frac{(Bx+A)(gx)^m}{b^2x^2+2\,abx+a^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(g*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \int \frac{(g\,x)^m\,(A+B\,x)}{a^2+2\,a\,b\,x+b^2\,x^2}\,dx$$

input
```
int(((g*x)^m*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x),x)
```

output
```
int(((g*x)^m*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x), x)
```

Reduce [F]

$$\int \frac{(gx)^m(A+Bx)}{a^2+2abx+b^2x^2}\,dx = \frac{g^m\left(x^m-\left(\int \frac{x^m}{b\,x^2+ax}dx\right)am\right)}{bm}$$

input
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2),x)
```

output
```
(g**m*(x**m - int(x**m/(a*x + b*x**2),x)*a*m))/(b*m)
```

3.469 $\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 81

$$\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = -\frac{B(gx)^{1+m}}{bg(2-m)(a+bx)^3} + \frac{\left(\frac{A}{1+m}+\frac{aB}{2b-bm}\right)(gx)^{1+m}\,\text{Hypergeometric2F1}\left(4,1+m,2+m,-\frac{bx}{a}\right)}{a^4g}$$

output
```
-B*(g*x)^(1+m)/b/g/(2-m)/(b*x+a)^3+(A/(1+m)+a*B/(-b*m+2*b))*(g*x)^(1+m)*hy
pergeom([4, 1+m],[2+m],-b*x/a)/a^4/g
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx = \frac{x(gx)^m\left(\frac{a^3(Ab-aB)}{(a+bx)^3}-\frac{(Ab(-2+m)-aB(1+m))\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{3a^4b}$$

input
```
Integrate[((g*x)^m*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
(x*(g*x)^m*((a^3*(A*b - a*B))/(a + b*x)^3 - ((A*b*(-2 + m) - a*B*(1 + m))*
Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(1 + m)))/(3*a^4*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1184, 27, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(gx)^m}{(a^2+2abx+b^2x^2)^2}dx$$

$$\Big\downarrow \text{1184}$$

$$b^4 \int \frac{(gx)^m(A+Bx)}{b^4(a+bx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(A+Bx)(gx)^m}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB(m+1)+Ab(2-m))\int \frac{(gx)^m}{(a+bx)^3}dx}{3ab}+\frac{(gx)^{m+1}(Ab-aB)}{3abg(a+bx)^3}$$

$$\Big\downarrow \text{74}$$

$$\frac{(gx)^{m+1}(aB(m+1)+Ab(2-m))\,\text{Hypergeometric2F1}\left(3,m+1,m+2,-\frac{bx}{a}\right)}{3a^4bg(m+1)}+\frac{(gx)^{m+1}(Ab-aB)}{3abg(a+bx)^3}$$

input
```
Int[((g*x)^m*(A + B*x))/(a^2 + 2*a*b*x + b^2*x^2)^2,x]
```

output
```
((A*b - a*B)*(g*x)^(1 + m))/(3*a*b*g*(a + b*x)^3) + ((A*b*(2 - m) + a*B*(1
 + m))*(g*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(3*a^
4*b*g*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(gx)^m (Bx + A)}{(b^2x^2 + 2abx + a^2)^2} dx$$

input
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

Fricas [F]

$$\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx=\int \frac{(Bx+A)(gx)^m}{(b^2x^2+2\,abx+a^2)^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(g*x)^m/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*
b*x + a^4), x)
```

Sympy [F]

$$\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx=\int \frac{(gx)^m\,(A+Bx)}{(a+bx)^4}\,dx$$

input
```
integrate((g*x)**m*(B*x+A)/(b**2*x**2+2*a*b*x+a**2)**2,x)
```

output
```
Integral((g*x)**m*(A + B*x)/(a + b*x)**4, x)
```

Maxima [F]

$$\int \frac{(gx)^m(A+Bx)}{(a^2+2abx+b^2x^2)^2}\,dx=\int \frac{(Bx+A)(gx)^m}{(b^2x^2+2\,abx+a^2)^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(g*x)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)
```

Giac [F]

$$\int \frac{(gx)^m(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\int \frac{(Bx+A)(gx)^m}{\left(b^2x^2+2\,abx+a^2\right)^2}\,dx$$

input
```
integrate((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(g*x)^m/(b^2*x^2 + 2*a*b*x + a^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx=\int \frac{(g\,x)^m\,(A+B\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2\right)^2}\,dx$$

input
```
int(((g*x)^m*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2,x)
```

output
```
int(((g*x)^m*(A + B*x))/(a^2 + b^2*x^2 + 2*a*b*x)^2, x)
```

Reduce [F]

$$\int \frac{(gx)^m(A+Bx)}{\left(a^2+2abx+b^2x^2\right)^2}\,dx$$
$$=\frac{g^m\left(x^m-\left(\int \frac{x^m}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a^2bm\,x^2-6a\,b^2x^3+a^3mx-6a^2b\,x^2-2a^3x}dx\right)a^3m^2+2\left(\int \frac{}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a}\right.\right.}{}$$

input
```
int((g*x)^m*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^2,x)
```

output
```
(g**m*(x**m - int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x
**2 + 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3
*m**2 + 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2
+ 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3*m -
 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m**2*x +
 4*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m*x - in
t(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**2*m
*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m**2*x**2 + 2
*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**
2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m*x**2))/(
b*(a**2*m - 2*a**2 + 2*a*b*m*x - 4*a*b*x + b**2*m*x**2 - 2*b**2*x**2))
```

3.470 $\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 143

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx=\frac{x^{1+m}}{1+m}+\frac{11x^{2+m}}{2+m}+\frac{55x^{3+m}}{3+m}+\frac{165x^{4+m}}{4+m}+\frac{330x^{5+m}}{5+m}+\frac{462x^{6+m}}{6+m}+\frac{462x^{7+m}}{7+m}+\frac{330x^{8+m}}{8+m}+\frac{165x^{9+m}}{9+m}+\frac{55x^{10+m}}{10+m}+\frac{11x^{11+m}}{11+m}+\frac{x^{12+m}}{12+m}$$

output

```
x^(1+m)/(1+m)+11*x^(2+m)/(2+m)+55*x^(3+m)/(3+m)+165*x^(4+m)/(4+m)+330*x^(5
+m)/(5+m)+462*x^(6+m)/(6+m)+462*x^(7+m)/(7+m)+330*x^(8+m)/(8+m)+165*x^(9+m
)/(9+m)+55*x^(10+m)/(10+m)+11*x^(11+m)/(11+m)+x^(12+m)/(12+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx=x^{1+m}\left(\frac{1}{1+m}+\frac{11x}{2+m}+\frac{55x^2}{3+m}+\frac{165x^3}{4+m}+\frac{330x^4}{5+m}+\frac{462x^5}{6+m}+\frac{462x^6}{7+m}+\frac{330x^7}{8+m}+\frac{165x^8}{9+m}+\frac{55x^9}{10+m}+\frac{11x^{10}}{11+m}+\frac{x^{11}}{12+m}\right)$$

input
```
Integrate[x^m*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^(1 + m)*((1 + m)^(-1) + (11*x)/(2 + m) + (55*x^2)/(3 + m) + (165*x^3)/(4
 + m) + (330*x^4)/(5 + m) + (462*x^5)/(6 + m) + (462*x^6)/(7 + m) + (330*x
^7)/(8 + m) + (165*x^8)/(9 + m) + (55*x^9)/(10 + m) + (11*x^10)/(11 + m) +
 x^11/(12 + m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1184, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1) \left(x^2+2x+1\right)^5 x^m \, dx$$

$$\Big\downarrow \text{1184}$$

$$\int (x+1)^{11} x^m dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(11x^{m+1}+55x^{m+2}+165x^{m+3}+330x^{m+4}+462x^{m+5}+462x^{m+6}+330x^{m+7}+165x^{m+8}+55x^{m+9}+11x^{m\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}}{m+1}+\frac{11x^{m+2}}{m+2}+\frac{55x^{m+3}}{m+3}+\frac{165x^{m+4}}{m+4}+\frac{330x^{m+5}}{m+5}+\frac{462x^{m+6}}{m+6}+\frac{462x^{m+7}}{m+7}+\frac{330x^{m+8}}{m+8}+\frac{165x^{m+9}}{m+9}+\frac{55x^{m+10}}{m+10}+\frac{11x^{m+11}}{m+11}+\frac{x^{m+12}}{m+12}$$

input
```
Int[x^m*(1 + x)*(1 + 2*x + x^2)^5,x]
```

output
```
x^(1 + m)/(1 + m) + (11*x^(2 + m))/(2 + m) + (55*x^(3 + m))/(3 + m) + (165
*x^(4 + m))/(4 + m) + (330*x^(5 + m))/(5 + m) + (462*x^(6 + m))/(6 + m) +
(462*x^(7 + m))/(7 + m) + (330*x^(8 + m))/(8 + m) + (165*x^(9 + m))/(9 + m
) + (55*x^(10 + m))/(10 + m) + (11*x^(11 + m))/(11 + m) + x^(12 + m)/(12 +
 m)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(143) = 286$.

Time = 0.92 (sec) , antiderivative size = 1095, normalized size of antiderivative = 7.66

method	result	size
risch	Expression too large to display	1095
gosper	Expression too large to display	1096
orering	Expression too large to display	1110
parallelrisch	Expression too large to display	1561

input
```
int(x^m*(x+1)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
x^m*(m^11*x^11+11*m^11*x^10+66*m^10*x^11+55*m^11*x^9+737*m^10*x^10+1925*m^
9*x^11+165*m^11*x^8+3740*m^10*x^9+21780*m^9*x^10+32670*m^8*x^11+330*m^11*x
^7+11385*m^10*x^8+112035*m^9*x^9+373890*m^8*x^10+357423*m^7*x^11+462*m^11*
x^6+23100*m^10*x^7+345840*m^9*x^8+1947000*m^8*x^9+4131303*m^7*x^10+2637558
*m^6*x^11+462*m^11*x^5+32802*m^10*x^6+711810*m^9*x^7+6089490*m^8*x^8+21750
465*m^7*x^9+30748641*m^6*x^10+13339535*m^5*x^11+330*m^11*x^4+33264*m^10*x^
5+1025640*m^9*x^6+12709620*m^8*x^7+68855985*m^7*x^8+163460220*m^6*x^9+1566
57490*m^5*x^10+45995730*m^4*x^11+165*m^11*x^3+24090*m^10*x^4+1055670*m^9*x
^5+18586260*m^8*x^6+145645830*m^7*x^7+523190745*m^6*x^8+839860505*m^5*x^9+
543539260*m^4*x^10+105258076*m^3*x^11+55*m^11*x^2+12210*m^10*x^3+776160*m^
9*x^4+19431720*m^8*x^5+216148086*m^7*x^6+1120622580*m^6*x^7+2714671410*m^5
*x^8+2935253200*m^4*x^9+1250343336*m^3*x^10+150917976*m^2*x^11+11*m^11*x+4
125*m^10*x^2+399465*m^9*x^3+14523300*m^8*x^4+229661586*m^7*x^5+1687068306*
m^6*x^6+5881795590*m^5*x^7+9569532060*m^4*x^8+6793843980*m^3*x^9+180038707
2*m^2*x^10+120543840*m*x^11+m^11+836*m^10*x+137060*m^9*x^2+7604190*m^8*x^3
+174706290*m^7*x^4+1822135392*m^6*x^5+8976008580*m^5*x^6+20948784780*m^4*x
^7+22313339400*m^3*x^8+9832379040*m^2*x^9+1442897280*m*x^10+39916800*x^11+
77*m^10+28215*m^9*x+2656170*m^8*x^2+93244635*m^7*x^3+1412257770*m^6*x^4+98
52674370*m^5*x^5+32372349240*m^4*x^6+49287977640*m^3*x^7+32492401920*m^2*x
^8+7911984960*m*x^9+479001600*x^10+2640*m^9+557040*m^8*x+33251955*m^7*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(143) = 286$.

Time = 0.09 (sec) , antiderivative size = 757, normalized size of antiderivative = 5.29

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
integrate(x^m*(1+x)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
((m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339
535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 399
16800)*x^12 + 11*(m^11 + 67*m^10 + 1980*m^9 + 33990*m^8 + 375573*m^7 + 279
5331*m^6 + 14241590*m^5 + 49412660*m^4 + 113667576*m^3 + 163671552*m^2 + 1
31172480*m + 43545600)*x^11 + 55*(m^11 + 68*m^10 + 2037*m^9 + 35400*m^8 +
395463*m^7 + 2972004*m^6 + 15270191*m^5 + 53368240*m^4 + 123524436*m^3 + 1
78770528*m^2 + 143854272*m + 47900160)*x^10 + 165*(m^11 + 69*m^10 + 2096*m
^9 + 36906*m^8 + 417309*m^7 + 3170853*m^6 + 16452554*m^5 + 57997164*m^4 +
135232360*m^3 + 196923648*m^2 + 159246720*m + 53222400)*x^9 + 330*(m^11 +
70*m^10 + 2157*m^9 + 38514*m^8 + 441351*m^7 + 3395826*m^6 + 17823623*m^5 +
 63481166*m^4 + 149357508*m^3 + 219154824*m^2 + 178320960*m + 59875200)*x^
8 + 462*(m^11 + 71*m^10 + 2220*m^9 + 40230*m^8 + 467853*m^7 + 3651663*m^6
+ 19428590*m^5 + 70070020*m^4 + 166716696*m^3 + 246998016*m^2 + 202573440*
m + 68428800)*x^7 + 462*(m^11 + 72*m^10 + 2285*m^9 + 42060*m^8 + 497103*m^
7 + 3944016*m^6 + 21326135*m^5 + 78113340*m^4 + 188526796*m^3 + 282854112*
m^2 + 234434880*m + 79833600)*x^6 + 330*(m^11 + 73*m^10 + 2352*m^9 + 44010
*m^8 + 529413*m^7 + 4279569*m^6 + 23592386*m^5 + 88108220*m^4 + 216665736*
m^3 + 330686208*m^2 + 278128512*m + 95800320)*x^5 + 165*(m^11 + 74*m^10 +
2421*m^9 + 46086*m^8 + 565119*m^7 + 4666158*m^6 + 26325599*m^5 + 100767754
*m^4 + 254135820*m^3 + 397471608*m^2 + 341673120*m + 119750400)*x^4 + 5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11008 vs. $2(117) = 234$.

Time = 1.04 (sec) , antiderivative size = 11008, normalized size of antiderivative = 76.98

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(1+x)*(x**2+2*x+1)**5,x)
```

output
```
Piecewise((log(x) - 11/x - 55/(2*x**2) - 55/x**3 - 165/(2*x**4) - 462/(5*x
**5) - 77/x**6 - 330/(7*x**7) - 165/(8*x**8) - 55/(9*x**9) - 11/(10*x**10)
 - 1/(11*x**11), Eq(m, -12)), (x + 11*log(x) - 55/x - 165/(2*x**2) - 110/x
**3 - 231/(2*x**4) - 462/(5*x**5) - 55/x**6 - 165/(7*x**7) - 55/(8*x**8) -
 11/(9*x**9) - 1/(10*x**10), Eq(m, -11)), (x**2/2 + 11*x + 55*log(x) - 165
/x - 165/x**2 - 154/x**3 - 231/(2*x**4) - 66/x**5 - 55/(2*x**6) - 55/(7*x*
*7) - 11/(8*x**8) - 1/(9*x**9), Eq(m, -10)), (x**3/3 + 11*x**2/2 + 55*x +
165*log(x) - 330/x - 231/x**2 - 154/x**3 - 165/(2*x**4) - 33/x**5 - 55/(6*
x**6) - 11/(7*x**7) - 1/(8*x**8), Eq(m, -9)), (x**4/4 + 11*x**3/3 + 55*x**
2/2 + 165*x + 330*log(x) - 462/x - 231/x**2 - 110/x**3 - 165/(4*x**4) - 11
/x**5 - 11/(6*x**6) - 1/(7*x**7), Eq(m, -8)), (x**5/5 + 11*x**4/4 + 55*x**
3/3 + 165*x**2/2 + 330*x + 462*log(x) - 462/x - 165/x**2 - 55/x**3 - 55/(4
*x**4) - 11/(5*x**5) - 1/(6*x**6), Eq(m, -7)), (x**6/6 + 11*x**5/5 + 55*x*
*4/4 + 55*x**3 + 165*x**2 + 462*x + 462*log(x) - 330/x - 165/(2*x**2) - 55
/(3*x**3) - 11/(4*x**4) - 1/(5*x**5), Eq(m, -6)), (x**7/7 + 11*x**6/6 + 11
*x**5 + 165*x**4/4 + 110*x**3 + 231*x**2 + 462*x + 330*log(x) - 165/x - 55
/(2*x**2) - 11/(3*x**3) - 1/(4*x**4), Eq(m, -5)), (x**8/8 + 11*x**7/7 + 55
*x**6/6 + 33*x**5 + 165*x**4/2 + 154*x**3 + 231*x**2 + 330*x + 165*log(x)
- 55/x - 11/(2*x**2) - 1/(3*x**3), Eq(m, -4)), (x**9/9 + 11*x**8/8 + 55*x*
*7/7 + 55*x**6/2 + 66*x**5 + 231*x**4/2 + 154*x**3 + 165*x**2 + 165*x +...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx = \frac{x^{m+12}}{m+12}+\frac{11\,x^{m+11}}{m+11}+\frac{55\,x^{m+10}}{m+10}+\frac{165\,x^{m+9}}{m+9}+\frac{330\,x^{m+8}}{m+8}+\frac{462\,x^{m+7}}{m+7}+\frac{462\,x^{m+6}}{m+6}+\frac{330\,x^{m+5}}{m+5}+\frac{165\,x^{m+4}}{m+4}+\frac{55\,x^{m+3}}{m+3}+\frac{11\,x^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

input
```
integrate(x^m*(1+x)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
x^(m + 12)/(m + 12) + 11*x^(m + 11)/(m + 11) + 55*x^(m + 10)/(m + 10) + 16
5*x^(m + 9)/(m + 9) + 330*x^(m + 8)/(m + 8) + 462*x^(m + 7)/(m + 7) + 462*
x^(m + 6)/(m + 6) + 330*x^(m + 5)/(m + 5) + 165*x^(m + 4)/(m + 4) + 55*x^(
m + 3)/(m + 3) + 11*x^(m + 2)/(m + 2) + x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1560 vs. $2(143) = 286$.

Time = 0.27 (sec) , antiderivative size = 1560, normalized size of antiderivative = 10.91

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
integrate(x^m*(1+x)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
(m^11*x^12*x^m + 11*m^11*x^11*x^m + 66*m^10*x^12*x^m + 55*m^11*x^10*x^m +
737*m^10*x^11*x^m + 1925*m^9*x^12*x^m + 165*m^11*x^9*x^m + 3740*m^10*x^10*
x^m + 21780*m^9*x^11*x^m + 32670*m^8*x^12*x^m + 330*m^11*x^8*x^m + 11385*m
^10*x^9*x^m + 112035*m^9*x^10*x^m + 373890*m^8*x^11*x^m + 357423*m^7*x^12*
x^m + 462*m^11*x^7*x^m + 23100*m^10*x^8*x^m + 345840*m^9*x^9*x^m + 1947000
*m^8*x^10*x^m + 4131303*m^7*x^11*x^m + 2637558*m^6*x^12*x^m + 462*m^11*x^6
*x^m + 32802*m^10*x^7*x^m + 711810*m^9*x^8*x^m + 6089490*m^8*x^9*x^m + 217
50465*m^7*x^10*x^m + 30748641*m^6*x^11*x^m + 13339535*m^5*x^12*x^m + 330*m
^11*x^5*x^m + 33264*m^10*x^6*x^m + 1025640*m^9*x^7*x^m + 12709620*m^8*x^8*
x^m + 68855985*m^7*x^9*x^m + 163460220*m^6*x^10*x^m + 156657490*m^5*x^11*x
^m + 45995730*m^4*x^12*x^m + 165*m^11*x^4*x^m + 24090*m^10*x^5*x^m + 10556
70*m^9*x^6*x^m + 18586260*m^8*x^7*x^m + 145645830*m^7*x^8*x^m + 523190745*
m^6*x^9*x^m + 839860505*m^5*x^10*x^m + 543539260*m^4*x^11*x^m + 105258076*
m^3*x^12*x^m + 55*m^11*x^3*x^m + 12210*m^10*x^4*x^m + 776160*m^9*x^5*x^m +
 19431720*m^8*x^6*x^m + 216148086*m^7*x^7*x^m + 1120622580*m^6*x^8*x^m + 2
714671410*m^5*x^9*x^m + 2935253200*m^4*x^10*x^m + 1250343336*m^3*x^11*x^m
+ 150917976*m^2*x^12*x^m + 11*m^11*x^2*x^m + 4125*m^10*x^3*x^m + 399465*m^
9*x^4*x^m + 14523300*m^8*x^5*x^m + 229661586*m^7*x^6*x^m + 1687068306*m^6*
x^7*x^m + 5881795590*m^5*x^8*x^m + 9569532060*m^4*x^9*x^m + 6793843980*m^3
*x^10*x^m + 1800387072*m^2*x^11*x^m + 120543840*m*x^12*x^m + m^11*x*x^m...
```

Mupad [B] (verification not implemented)

Time = 11.70 (sec) , antiderivative size = 1459, normalized size of antiderivative = 10.20

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx=\text{Too large to display}$$

input
```
int(x^m*(x + 1)*(2*x + x^2 + 1)^5,x)
```

output
```
(x^m*x^8*(58845916800*m + 72321091920*m^2 + 49287977640*m^3 + 20948784780*
m^4 + 5881795590*m^5 + 1120622580*m^6 + 145645830*m^7 + 12709620*m^8 + 711
810*m^9 + 23100*m^10 + 330*m^11 + 19758816000))/(1486442880*m + 1931559552
*m^2 + 1414014888*m^3 + 657206836*m^4 + 206070150*m^5 + 44990231*m^6 + 692
6634*m^7 + 749463*m^8 + 55770*m^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600
) + (x^m*x^10*(7911984960*m + 9832379040*m^2 + 6793843980*m^3 + 2935253200
*m^4 + 839860505*m^5 + 163460220*m^6 + 21750465*m^7 + 1947000*m^8 + 112035
*m^9 + 3740*m^10 + 55*m^11 + 2634508800))/(1486442880*m + 1931559552*m^2 +
 1414014888*m^3 + 657206836*m^4 + 206070150*m^5 + 44990231*m^6 + 6926634*m
^7 + 749463*m^8 + 55770*m^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600) + (x
^m*x^2*(6858181440*m + 7194486816*m^2 + 4179838476*m^3 + 1524718360*m^4 +
371026645*m^5 + 61932948*m^6 + 7130013*m^7 + 557040*m^8 + 28215*m^9 + 836*
m^10 + 11*m^11 + 2634508800))/(1486442880*m + 1931559552*m^2 + 1414014888*
m^3 + 657206836*m^4 + 206070150*m^5 + 44990231*m^6 + 6926634*m^7 + 749463*
m^8 + 55770*m^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600) + (x^m*x^11*(144
2897280*m + 1800387072*m^2 + 1250343336*m^3 + 543539260*m^4 + 156657490*m^
5 + 30748641*m^6 + 4131303*m^7 + 373890*m^8 + 21780*m^9 + 737*m^10 + 11*m^
11 + 479001600))/(1486442880*m + 1931559552*m^2 + 1414014888*m^3 + 6572068
36*m^4 + 206070150*m^5 + 44990231*m^6 + 6926634*m^7 + 749463*m^8 + 55770*m
^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600) + (x^m*x^6*(108308914560*m...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1094, normalized size of antiderivative = 7.65

$$\int x^m(1+x)\left(1+2x+x^2\right)^5\,dx=\text{Too large to display}$$

input
```
int(x^m*(1+x)*(x^2+2*x+1)^5,x)
```

output
```
(x**m*x*(m**11*x**11 + 11*m**11*x**10 + 55*m**11*x**9 + 165*m**11*x**8 + 3
30*m**11*x**7 + 462*m**11*x**6 + 462*m**11*x**5 + 330*m**11*x**4 + 165*m**
11*x**3 + 55*m**11*x**2 + 11*m**11*x + m**11 + 66*m**10*x**11 + 737*m**10*
x**10 + 3740*m**10*x**9 + 11385*m**10*x**8 + 23100*m**10*x**7 + 32802*m**1
0*x**6 + 33264*m**10*x**5 + 24090*m**10*x**4 + 12210*m**10*x**3 + 4125*m**
10*x**2 + 836*m**10*x + 77*m**10 + 1925*m**9*x**11 + 21780*m**9*x**10 + 11
2035*m**9*x**9 + 345840*m**9*x**8 + 711810*m**9*x**7 + 1025640*m**9*x**6 +
 1055670*m**9*x**5 + 776160*m**9*x**4 + 399465*m**9*x**3 + 137060*m**9*x**
2 + 28215*m**9*x + 2640*m**9 + 32670*m**8*x**11 + 373890*m**8*x**10 + 1947
000*m**8*x**9 + 6089490*m**8*x**8 + 12709620*m**8*x**7 + 18586260*m**8*x**
6 + 19431720*m**8*x**5 + 14523300*m**8*x**4 + 7604190*m**8*x**3 + 2656170*
m**8*x**2 + 557040*m**8*x + 53130*m**8 + 357423*m**7*x**11 + 4131303*m**7*
x**10 + 21750465*m**7*x**9 + 68855985*m**7*x**8 + 145645830*m**7*x**7 + 21
6148086*m**7*x**6 + 229661586*m**7*x**5 + 174706290*m**7*x**4 + 93244635*m
**7*x**3 + 33251955*m**7*x**2 + 7130013*m**7*x + 696333*m**7 + 2637558*m**
6*x**11 + 30748641*m**6*x**10 + 163460220*m**6*x**9 + 523190745*m**6*x**8
+ 1120622580*m**6*x**7 + 1687068306*m**6*x**6 + 1822135392*m**6*x**5 + 141
2257770*m**6*x**4 + 769916070*m**6*x**3 + 281209005*m**6*x**2 + 61932948*m
**6*x + 6230301*m**6 + 13339535*m**5*x**11 + 156657490*m**5*x**10 + 839860
505*m**5*x**9 + 2714671410*m**5*x**8 + 5881795590*m**5*x**7 + 897600858...
```

3.471 $\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{dx^{1+m}}{1+m}+\frac{(10d+e)x^{2+m}}{2+m}+\frac{5(9d+2e)x^{3+m}}{3+m}+\frac{15(8d+3e)x^{4+m}}{4+m}+\frac{30(7d+4e)x^{5+m}}{5+m}+\frac{42(6d+5e)x^{6+m}}{6+m}+\frac{42(5d+6e)x^{7+m}}{7+m}+\frac{30(4d+7e)x^{8+m}}{8+m}+\frac{15(3d+8e)x^{9+m}}{9+m}+\frac{5(2d+9e)x^{10+m}}{10+m}+\frac{(d+10e)x^{11+m}}{11+m}+\frac{ex^{12+m}}{12+m}$$

output
```
d*x^(1+m)/(1+m)+(10*d+e)*x^(2+m)/(2+m)+5*(9*d+2*e)*x^(3+m)/(3+m)+15*(8*d+3
*e)*x^(4+m)/(4+m)+30*(7*d+4*e)*x^(5+m)/(5+m)+42*(6*d+5*e)*x^(6+m)/(6+m)+42
*(5*d+6*e)*x^(7+m)/(7+m)+30*(4*d+7*e)*x^(8+m)/(8+m)+15*(3*d+8*e)*x^(9+m)/(
9+m)+5*(2*d+9*e)*x^(10+m)/(10+m)+(d+10*e)*x^(11+m)/(11+m)+e*x^(12+m)/(12+m
)
```

Mathematica [A] (verified)

Time = 0.55 (sec), antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx$$
$$=\frac{x^{1+m}\left(e(1+x)^{11}+(-e(1+m)+d(12+m))\left(\frac{1}{1+m}+\frac{10x}{2+m}+\frac{45x^2}{3+m}+\frac{120x^3}{4+m}+\frac{210x^4}{5+m}+\frac{252x^5}{6+m}+\frac{210x^6}{7+m}+\frac{120x}{8+m}\right.\right.}{12+m}$$

input
```
Integrate[x^m*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(x^(1 + m)*(e*(1 + x)^11 + (-(e*(1 + m)) + d*(12 + m))*((1 + m)^(-1) + (10
*x)/(2 + m) + (45*x^2)/(3 + m) + (120*x^3)/(4 + m) + (210*x^4)/(5 + m) + (
252*x^5)/(6 + m) + (210*x^6)/(7 + m) + (120*x^7)/(8 + m) + (45*x^8)/(9 + m
) + (10*x^9)/(10 + m) + x^10/(11 + m))))/(12 + m)
```

Rubi [A] (verified)

Time = 0.64 (sec), antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1184, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+2x+1\right)^5 x^m(d+ex)\,dx$$
$$\Big\downarrow \text{1184}$$
$$\int (x+1)^{10}x^m(d+ex)dx$$
$$\Big\downarrow \text{85}$$
$$\int \left((10d+e)x^{m+1}+5(9d+2e)x^{m+2}+15(8d+3e)x^{m+3}+30(7d+4e)x^{m+4}+42(6d+5e)x^{m+5}+42(5d+6e)\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(10d+e)x^{m+2}}{m+2}+\frac{5(9d+2e)x^{m+3}}{m+3}+\frac{15(8d+3e)x^{m+4}}{m+4}+\frac{30(7d+4e)x^{m+5}}{m+5}+\frac{42(6d+5e)x^{m+6}}{m+6}+\frac{42(5d+6e)x^{m+7}}{m+7}+\frac{30(4d+7e)x^{m+8}}{m+8}+\frac{15(3d+8e)x^{m+9}}{m+9}+\frac{5(2d+9e)x^{m+10}}{m+10}+\frac{(d+10e)x^{m+11}}{m+11}+\frac{dx^{m+1}}{m+1}+\frac{ex^{m+12}}{m+12}$$

input
```
Int[x^m*(d + e*x)*(1 + 2*x + x^2)^5,x]
```

output
```
(d*x^(1 + m))/(1 + m) + ((10*d + e)*x^(2 + m))/(2 + m) + (5*(9*d + 2*e)*x^
(3 + m))/(3 + m) + (15*(8*d + 3*e)*x^(4 + m))/(4 + m) + (30*(7*d + 4*e)*x^
(5 + m))/(5 + m) + (42*(6*d + 5*e)*x^(6 + m))/(6 + m) + (42*(5*d + 6*e)*x^
(7 + m))/(7 + m) + (30*(4*d + 7*e)*x^(8 + m))/(8 + m) + (15*(3*d + 8*e)*x^
(9 + m))/(9 + m) + (5*(2*d + 9*e)*x^(10 + m))/(10 + m) + ((d + 10*e)*x^(11
 + m))/(11 + m) + (e*x^(12 + m))/(12 + m)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(209) = 418$.

Time = 0.88 (sec) , antiderivative size = 2245, normalized size of antiderivative = 10.74

method	result	size
risch	Expression too large to display	2245
gosper	Expression too large to display	2246
orering	Expression too large to display	2260
parallelrisch	Expression too large to display	3093

input
```
int(x^m*(e*x+d)*(x^2+2*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
x^m*(e*m^11*x^11+d*m^11*x^10+10*e*m^11*x^10+66*e*m^10*x^11+10*d*m^11*x^9+6
7*d*m^10*x^10+45*e*m^11*x^9+670*e*m^10*x^10+1925*e*m^9*x^11+45*d*m^11*x^8+
680*d*m^10*x^9+1980*d*m^9*x^10+120*e*m^11*x^8+3060*e*m^10*x^9+19800*e*m^9*
x^10+32670*e*m^8*x^11+120*d*m^11*x^7+3105*d*m^10*x^8+20370*d*m^9*x^9+33990
*d*m^8*x^10+210*e*m^11*x^7+8280*e*m^10*x^8+91665*e*m^9*x^9+339900*e*m^8*x^
10+357423*e*m^7*x^11+210*d*m^11*x^6+8400*d*m^10*x^7+94320*d*m^9*x^8+354000
*d*m^8*x^9+375573*d*m^7*x^10+252*e*m^11*x^6+14700*e*m^10*x^7+251520*e*m^9*
x^8+1593000*e*m^8*x^9+3755730*e*m^7*x^10+2637558*e*m^6*x^11+252*d*m^11*x^5
+14910*d*m^10*x^6+258840*d*m^9*x^7+1660770*d*m^8*x^8+3954630*d*m^7*x^9+279
5331*d*m^6*x^10+210*e*m^11*x^5+17892*e*m^10*x^6+452970*e*m^9*x^7+4428720*e
*m^8*x^8+17795835*e*m^7*x^9+27953310*e*m^6*x^10+13339535*e*m^5*x^11+210*d*
m^11*x^4+18144*d*m^10*x^5+466200*d*m^9*x^6+4621680*d*m^8*x^7+18778905*d*m^
7*x^8+29720040*d*m^6*x^9+14241590*d*m^5*x^10+120*e*m^11*x^4+15120*e*m^10*x
^5+559440*e*m^9*x^6+8087940*e*m^8*x^7+50077080*e*m^7*x^8+133740180*e*m^6*x
^9+142415900*e*m^5*x^10+45995730*e*m^4*x^11+120*d*m^11*x^3+15330*d*m^10*x^
4+575820*d*m^9*x^5+8448300*d*m^8*x^6+52962120*d*m^7*x^7+142688385*d*m^6*x^
8+152701910*d*m^5*x^9+49412660*d*m^4*x^10+45*e*m^11*x^3+8760*e*m^10*x^4+47
9850*e*m^9*x^5+10137960*e*m^8*x^6+92683710*e*m^7*x^7+380502360*e*m^6*x^8+6
87158595*e*m^5*x^9+494126600*e*m^4*x^10+105258076*e*m^3*x^11+45*d*m^11*x^2
+8880*d*m^10*x^3+493920*d*m^9*x^4+10599120*d*m^8*x^5+98249130*d*m^7*x^6...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(209) = 418$.

Time = 0.09 (sec) , antiderivative size = 1569, normalized size of antiderivative = 7.51

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
integrate(x^m*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="fricas")
```

output
```
((e*m^11 + 66*e*m^10 + 1925*e*m^9 + 32670*e*m^8 + 357423*e*m^7 + 2637558*e
*m^6 + 13339535*e*m^5 + 45995730*e*m^4 + 105258076*e*m^3 + 150917976*e*m^2
 + 120543840*e*m + 39916800*e)*x^12 + ((d + 10*e)*m^11 + 67*(d + 10*e)*m^1
0 + 1980*(d + 10*e)*m^9 + 33990*(d + 10*e)*m^8 + 375573*(d + 10*e)*m^7 + 2
795331*(d + 10*e)*m^6 + 14241590*(d + 10*e)*m^5 + 49412660*(d + 10*e)*m^4
+ 113667576*(d + 10*e)*m^3 + 163671552*(d + 10*e)*m^2 + 131172480*(d + 10*
e)*m + 43545600*d + 435456000*e)*x^11 + 5*((2*d + 9*e)*m^11 + 68*(2*d + 9*
e)*m^10 + 2037*(2*d + 9*e)*m^9 + 35400*(2*d + 9*e)*m^8 + 395463*(2*d + 9*e
)*m^7 + 2972004*(2*d + 9*e)*m^6 + 15270191*(2*d + 9*e)*m^5 + 53368240*(2*d
 + 9*e)*m^4 + 123524436*(2*d + 9*e)*m^3 + 178770528*(2*d + 9*e)*m^2 + 1438
54272*(2*d + 9*e)*m + 95800320*d + 431101440*e)*x^10 + 15*((3*d + 8*e)*m^1
1 + 69*(3*d + 8*e)*m^10 + 2096*(3*d + 8*e)*m^9 + 36906*(3*d + 8*e)*m^8 + 4
17309*(3*d + 8*e)*m^7 + 3170853*(3*d + 8*e)*m^6 + 16452554*(3*d + 8*e)*m^5
 + 57997164*(3*d + 8*e)*m^4 + 135232360*(3*d + 8*e)*m^3 + 196923648*(3*d +
 8*e)*m^2 + 159246720*(3*d + 8*e)*m + 159667200*d + 425779200*e)*x^9 + 30*
((4*d + 7*e)*m^11 + 70*(4*d + 7*e)*m^10 + 2157*(4*d + 7*e)*m^9 + 38514*(4*
d + 7*e)*m^8 + 441351*(4*d + 7*e)*m^7 + 3395826*(4*d + 7*e)*m^6 + 17823623
*(4*d + 7*e)*m^5 + 63481166*(4*d + 7*e)*m^4 + 149357508*(4*d + 7*e)*m^3 +
219154824*(4*d + 7*e)*m^2 + 178320960*(4*d + 7*e)*m + 239500800*d + 419126
400*e)*x^8 + 42*((5*d + 6*e)*m^11 + 71*(5*d + 6*e)*m^10 + 2220*(5*d + 6...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20971 vs. $2(168) = 336$.

Time = 1.59 (sec) , antiderivative size = 20971, normalized size of antiderivative = 100.34

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(e*x+d)*(x**2+2*x+1)**5,x)
```

output
```
Piecewise((-d/x - 5*d/x**2 - 15*d/x**3 - 30*d/x**4 - 42*d/x**5 - 42*d/x**6
 - 30*d/x**7 - 15*d/x**8 - 5*d/x**9 - d/x**10 - d/(11*x**11) + e*log(x) -
10*e/x - 45*e/(2*x**2) - 40*e/x**3 - 105*e/(2*x**4) - 252*e/(5*x**5) - 35*
e/x**6 - 120*e/(7*x**7) - 45*e/(8*x**8) - 10*e/(9*x**9) - e/(10*x**10), Eq
(m, -12)), (d*log(x) - 10*d/x - 45*d/(2*x**2) - 40*d/x**3 - 105*d/(2*x**4)
 - 252*d/(5*x**5) - 35*d/x**6 - 120*d/(7*x**7) - 45*d/(8*x**8) - 10*d/(9*x
**9) - d/(10*x**10) + e*x + 10*e*log(x) - 45*e/x - 60*e/x**2 - 70*e/x**3 -
 63*e/x**4 - 42*e/x**5 - 20*e/x**6 - 45*e/(7*x**7) - 5*e/(4*x**8) - e/(9*x
**9), Eq(m, -11)), (d*x + 10*d*log(x) - 45*d/x - 60*d/x**2 - 70*d/x**3 - 6
3*d/x**4 - 42*d/x**5 - 20*d/x**6 - 45*d/(7*x**7) - 5*d/(4*x**8) - d/(9*x**
9) + e*x**2/2 + 10*e*x + 45*e*log(x) - 120*e/x - 105*e/x**2 - 84*e/x**3 -
105*e/(2*x**4) - 24*e/x**5 - 15*e/(2*x**6) - 10*e/(7*x**7) - e/(8*x**8), E
q(m, -10)), (d*x**2/2 + 10*d*x + 45*d*log(x) - 120*d/x - 105*d/x**2 - 84*d
/x**3 - 105*d/(2*x**4) - 24*d/x**5 - 15*d/(2*x**6) - 10*d/(7*x**7) - d/(8*
x**8) + e*x**3/3 + 5*e*x**2 + 45*e*x + 120*e*log(x) - 210*e/x - 126*e/x**2
 - 70*e/x**3 - 30*e/x**4 - 9*e/x**5 - 5*e/(3*x**6) - e/(7*x**7), Eq(m, -9)
), (d*x**3/3 + 5*d*x**2 + 45*d*x + 120*d*log(x) - 210*d/x - 126*d/x**2 - 7
0*d/x**3 - 30*d/x**4 - 9*d/x**5 - 5*d/(3*x**6) - d/(7*x**7) + e*x**4/4 + 1
0*e*x**3/3 + 45*e*x**2/2 + 120*e*x + 210*e*log(x) - 252*e/x - 105*e/x**2 -
 40*e/x**3 - 45*e/(4*x**4) - 2*e/x**5 - e/(6*x**6), Eq(m, -8)), (d*x**4...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.35

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx=\frac{ex^{m+12}}{m+12}+\frac{dx^{m+11}}{m+11}+\frac{10\,ex^{m+11}}{m+11}+\frac{10\,dx^{m+10}}{m+10}+\frac{45\,ex^{m+10}}{m+10}+\frac{45\,dx^{m+9}}{m+9}+\frac{120\,ex^{m+9}}{m+9}+\frac{120\,dx^{m+8}}{m+8}+\frac{210\,ex^{m+8}}{m+8}+\frac{210\,dx^{m+7}}{m+7}+\frac{252\,ex^{m+7}}{m+7}+\frac{252\,dx^{m+6}}{m+6}+\frac{210\,ex^{m+6}}{m+6}+\frac{210\,dx^{m+5}}{m+5}+\frac{120\,ex^{m+5}}{m+5}+\frac{120\,dx^{m+4}}{m+4}+\frac{45\,ex^{m+4}}{m+4}+\frac{45\,dx^{m+3}}{m+3}+\frac{10\,ex^{m+3}}{m+3}+\frac{10\,dx^{m+2}}{m+2}+\frac{ex^{m+2}}{m+2}+\frac{dx^{m+1}}{m+1}$$

input
```
integrate(x^m*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="maxima")
```

output
```
e*x^(m + 12)/(m + 12) + d*x^(m + 11)/(m + 11) + 10*e*x^(m + 11)/(m + 11) +
 10*d*x^(m + 10)/(m + 10) + 45*e*x^(m + 10)/(m + 10) + 45*d*x^(m + 9)/(m +
 9) + 120*e*x^(m + 9)/(m + 9) + 120*d*x^(m + 8)/(m + 8) + 210*e*x^(m + 8)/
(m + 8) + 210*d*x^(m + 7)/(m + 7) + 252*e*x^(m + 7)/(m + 7) + 252*d*x^(m +
 6)/(m + 6) + 210*e*x^(m + 6)/(m + 6) + 210*d*x^(m + 5)/(m + 5) + 120*e*x^
(m + 5)/(m + 5) + 120*d*x^(m + 4)/(m + 4) + 45*e*x^(m + 4)/(m + 4) + 45*d*
x^(m + 3)/(m + 3) + 10*e*x^(m + 3)/(m + 3) + 10*d*x^(m + 2)/(m + 2) + e*x^
(m + 2)/(m + 2) + d*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3092 vs. 2(209) = 418.

Time = 0.27 (sec) , antiderivative size = 3092, normalized size of antiderivative = 14.79

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx=\text{Too large to display}$$

input
```
integrate(x^m*(e*x+d)*(x^2+2*x+1)^5,x, algorithm="giac")
```

output
```
(e*m^11*x^12*x^m + d*m^11*x^11*x^m + 10*e*m^11*x^11*x^m + 66*e*m^10*x^12*x
^m + 10*d*m^11*x^10*x^m + 45*e*m^11*x^10*x^m + 67*d*m^10*x^11*x^m + 670*e*
m^10*x^11*x^m + 1925*e*m^9*x^12*x^m + 45*d*m^11*x^9*x^m + 120*e*m^11*x^9*x
^m + 680*d*m^10*x^10*x^m + 3060*e*m^10*x^10*x^m + 1980*d*m^9*x^11*x^m + 19
800*e*m^9*x^11*x^m + 32670*e*m^8*x^12*x^m + 120*d*m^11*x^8*x^m + 210*e*m^1
1*x^8*x^m + 3105*d*m^10*x^9*x^m + 8280*e*m^10*x^9*x^m + 20370*d*m^9*x^10*x
^m + 91665*e*m^9*x^10*x^m + 33990*d*m^8*x^11*x^m + 339900*e*m^8*x^11*x^m +
 357423*e*m^7*x^12*x^m + 210*d*m^11*x^7*x^m + 252*e*m^11*x^7*x^m + 8400*d*
m^10*x^8*x^m + 14700*e*m^10*x^8*x^m + 94320*d*m^9*x^9*x^m + 251520*e*m^9*x
^9*x^m + 354000*d*m^8*x^10*x^m + 1593000*e*m^8*x^10*x^m + 375573*d*m^7*x^1
1*x^m + 3755730*e*m^7*x^11*x^m + 2637558*e*m^6*x^12*x^m + 252*d*m^11*x^6*x
^m + 210*e*m^11*x^6*x^m + 14910*d*m^10*x^7*x^m + 17892*e*m^10*x^7*x^m + 25
8840*d*m^9*x^8*x^m + 452970*e*m^9*x^8*x^m + 1660770*d*m^8*x^9*x^m + 442872
0*e*m^8*x^9*x^m + 3954630*d*m^7*x^10*x^m + 17795835*e*m^7*x^10*x^m + 27953
31*d*m^6*x^11*x^m + 27953310*e*m^6*x^11*x^m + 13339535*e*m^5*x^12*x^m + 21
0*d*m^11*x^5*x^m + 120*e*m^11*x^5*x^m + 18144*d*m^10*x^6*x^m + 15120*e*m^1
0*x^6*x^m + 466200*d*m^9*x^7*x^m + 559440*e*m^9*x^7*x^m + 4621680*d*m^8*x^
8*x^m + 8087940*e*m^8*x^8*x^m + 18778905*d*m^7*x^9*x^m + 50077080*e*m^7*x^
9*x^m + 29720040*d*m^6*x^10*x^m + 133740180*e*m^6*x^10*x^m + 14241590*d*m^
5*x^11*x^m + 142415900*e*m^5*x^11*x^m + 45995730*e*m^4*x^12*x^m + 120*d...
```

Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 1515, normalized size of antiderivative = 7.25

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
int(x^m*(d + e*x)*(2*x + x^2 + 1)^5,x)
```

output
```
(e*x^m*x^12*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 +
13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 +
 m^11 + 39916800))/(1486442880*m + 1931559552*m^2 + 1414014888*m^3 + 65720
6836*m^4 + 206070150*m^5 + 44990231*m^6 + 6926634*m^7 + 749463*m^8 + 55770
*m^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600) + (x^m*x^11*(d + 10*e)*(131
172480*m + 163671552*m^2 + 113667576*m^3 + 49412660*m^4 + 14241590*m^5 + 2
795331*m^6 + 375573*m^7 + 33990*m^8 + 1980*m^9 + 67*m^10 + m^11 + 43545600
))/(1486442880*m + 1931559552*m^2 + 1414014888*m^3 + 657206836*m^4 + 20607
0150*m^5 + 44990231*m^6 + 6926634*m^7 + 749463*m^8 + 55770*m^9 + 2717*m^10
 + 78*m^11 + m^12 + 479001600) + (d*x*x^m*(1007441280*m + 924118272*m^2 +
489896616*m^3 + 167310220*m^4 + 38759930*m^5 + 6230301*m^6 + 696333*m^7 +
53130*m^8 + 2640*m^9 + 77*m^10 + m^11 + 479001600))/(1486442880*m + 193155
9552*m^2 + 1414014888*m^3 + 657206836*m^4 + 206070150*m^5 + 44990231*m^6 +
 6926634*m^7 + 749463*m^8 + 55770*m^9 + 2717*m^10 + 78*m^11 + m^12 + 47900
1600) + (x^m*x^2*(10*d + e)*(623471040*m + 654044256*m^2 + 379985316*m^3 +
 138610760*m^4 + 33729695*m^5 + 5630268*m^6 + 648183*m^7 + 50640*m^8 + 256
5*m^9 + 76*m^10 + m^11 + 239500800))/(1486442880*m + 1931559552*m^2 + 1414
014888*m^3 + 657206836*m^4 + 206070150*m^5 + 44990231*m^6 + 6926634*m^7 +
749463*m^8 + 55770*m^9 + 2717*m^10 + 78*m^11 + m^12 + 479001600) + (5*x^m*
x^10*(2*d + 9*e)*(143854272*m + 178770528*m^2 + 123524436*m^3 + 5336824...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2244, normalized size of antiderivative = 10.74

$$\int x^m(d+ex)\left(1+2x+x^2\right)^5\,dx = \text{Too large to display}$$

input
```
int(x^m*(e*x+d)*(x^2+2*x+1)^5,x)
```

output

```
(x**m*x*(d*m**11*x**10 + 10*d*m**11*x**9 + 45*d*m**11*x**8 + 120*d*m**11*x
**7 + 210*d*m**11*x**6 + 252*d*m**11*x**5 + 210*d*m**11*x**4 + 120*d*m**11
*x**3 + 45*d*m**11*x**2 + 10*d*m**11*x + d*m**11 + 67*d*m**10*x**10 + 680*
d*m**10*x**9 + 3105*d*m**10*x**8 + 8400*d*m**10*x**7 + 14910*d*m**10*x**6
+ 18144*d*m**10*x**5 + 15330*d*m**10*x**4 + 8880*d*m**10*x**3 + 3375*d*m**
10*x**2 + 760*d*m**10*x + 77*d*m**10 + 1980*d*m**9*x**10 + 20370*d*m**9*x*
*9 + 94320*d*m**9*x**8 + 258840*d*m**9*x**7 + 466200*d*m**9*x**6 + 575820*
d*m**9*x**5 + 493920*d*m**9*x**4 + 290520*d*m**9*x**3 + 112140*d*m**9*x**2
 + 25650*d*m**9*x + 2640*d*m**9 + 33990*d*m**8*x**10 + 354000*d*m**8*x**9
+ 1660770*d*m**8*x**8 + 4621680*d*m**8*x**7 + 8448300*d*m**8*x**6 + 105991
20*d*m**8*x**5 + 9242100*d*m**8*x**4 + 5530320*d*m**8*x**3 + 2173230*d*m**
8*x**2 + 506400*d*m**8*x + 53130*d*m**8 + 375573*d*m**7*x**10 + 3954630*d*
m**7*x**9 + 18778905*d*m**7*x**8 + 52962120*d*m**7*x**7 + 98249130*d*m**7*
x**6 + 125269956*d*m**7*x**5 + 111176730*d*m**7*x**4 + 67814280*d*m**7*x**
3 + 27206145*d*m**7*x**2 + 6481830*d*m**7*x + 696333*d*m**7 + 2795331*d*m*
*6*x**10 + 29720040*d*m**6*x**9 + 142688385*d*m**6*x**8 + 407499120*d*m**6
*x**7 + 766849230*d*m**6*x**6 + 993892032*d*m**6*x**5 + 898709490*d*m**6*x
**4 + 559938960*d*m**6*x**3 + 230080095*d*m**6*x**2 + 56302680*d*m**6*x +
6230301*d*m**6 + 14241590*d*m**5*x**10 + 152701910*d*m**5*x**9 + 740364930
*d*m**5*x**8 + 2138834760*d*m**5*x**7 + 4080003900*d*m**5*x**6 + 537418...
```

3.472 $\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
6/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)-4/
55*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1
)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2
*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.30

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$
$$= \frac{2\left(x\sqrt{1+x}(3-3x+3x^2+5x^3-5x^4+5x^5)+\sqrt{-\frac{6i}{3i+\sqrt{3}}}(3i+\sqrt{3})\,(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{}}{(3i+}}\right.}{55\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/
(3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqr
t[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3]
)*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqr
t[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x
^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x^3\sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\int \frac{x^3}{\sqrt{x^3+1}}dx+\frac{2}{11}\sqrt{x^3+1}x^4\right)}{\sqrt{x^3+1}}$$

$$\downarrow 843$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int\frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{11}\sqrt{x^3+1}x^4\right)}{\sqrt{x^3+1}}$$

$$\downarrow 759$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{11}\sqrt{x^3+1}x\right.}{\sqrt{x^3+1}}$$

input
```
Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^4*Sqrt[1 + x^3])/11 + (3*((2*x*Sqrt[1
 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt
[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/11))/S
qrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x^4\sqrt{x^3+1}}{11}+\frac{6x\sqrt{x^3+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$\frac{2x(5x^3+3)\sqrt{x+1}\sqrt{x^2-x+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(}}{55\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\left(5x^7+3i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x}{i\sqrt{3}+3}}\right.}{55(x^3+1)}$

input
```
int(x^3*(x+1)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/11*x^4*(x^3+1)^(1/2
)+6/55*x*(x^3+1)^(1/2)-12/55*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)
))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*
3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{55}\left(5\,x^4+3\,x\right)\sqrt{x^2-x+1}\sqrt{x+1} - \frac{12}{55}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 + 3*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 12/55*weierstrassPInver
se(0, -4, x)
```

Sympy [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4}{11}+\frac{6\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{55}-\frac{6\left(\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx\right)}{55}$$

input
```
int(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

output
```
(2*(5*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**4 + 3*sqrt(x + 1)*sqrt(x**2 - x +
1)*x - 3*int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**3 + 1),x)))/55
```

3.473 $\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

output
```
2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

input
```
Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \; 1208$$

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

input
```
Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{9}$	18
orering	$\frac{2(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{9}$	18
default	$\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(x^{3}+1\right)}{9}$	23
risch	$\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(x^{3}+1\right)}{9}$	23
elliptic	$\frac{\sqrt{(x+1)(x^{2}-x+1)}\left(\frac{2x^{3}\sqrt{x^{3}+1}}{9}+\frac{2\sqrt{x^{3}+1}}{9}\right)}{\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	53

input
```
int(x^2*(x+1)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(x+1)^(3/2)*(x^2-x+1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\begin{aligned}&\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx\\&=\frac{2}{315}\left(\left(5\left(7x-23\right)(x+1)+258\right)(x+1)-213\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}\\&\quad+\frac{2}{105}\left(3\left(5x-12\right)(x+1)+71\right)\sqrt{(x+1)^2-3x}\sqrt{x+1}\end{aligned}$$

input
```
integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*s
qrt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(
x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\,(x^3+1)\,\sqrt{x+1}\,\sqrt{x^2-x+1}}{9}$$

input
```
int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}{9}$$

input
```
int(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*(x**3 + 1))/9
```

3.474 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 294

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}+\frac{2\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+6*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(7+7*3^(
1/2)+7*x)-3/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)+2/7*2^(1/2)*3^(3/4)*(
1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+
x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3
+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.64 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$$
$$= \frac{\sqrt{1+x}\left(4x^2\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}(1-x+x^2)-3\sqrt{2}(-3i+\sqrt{3})\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right.\right.\right.}{14\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1}}$$

input
```
Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3
*Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sq
rt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*
Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] +
3*Sqrt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqr
t[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(
14*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1210, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$
$$\Big\downarrow \text{1210}$$
$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x\sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\int \frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{7}\sqrt{x\cdots}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2\cdots}{(x+\cdots}}\cdots}{\sqrt{\cdots}}\cdots\right)\cdots\right)}{\sqrt{x^3+1}}$$

input

```
Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output

```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^2*Sqrt[1 + x^3])/7 + (3*((2*Sqrt[1 +
x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
 - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt
[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/7))/
Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.73

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\cdots\right.}{7\sqrt{x^3+1}}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$\frac{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}\cdots\right.\right.}{7\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)+2x^5-18\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x\cdots}{i\sqrt{3}+3}}\cdots\right.}{\cdots}$

input
```
int(x*(x+1)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/7*x^2*(x^3+1)^(1/2)
+6/7*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2
+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{7}\sqrt{x^2-x+1}\sqrt{x+1}x^2 - \frac{6}{7}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 - 6/7*weierstrassZeta(0, -4, weierst
rassPInverse(0, -4, x))
```

Sympy [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Giac [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

input
```
integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int x\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2}{7} + \frac{3\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{x^3+1}dx\right)}{7}$$

input
```
int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**2 + 3*int((sqrt(x + 1)*sqrt(x**2 - x
+ 1)*x)/(x**3 + 1),x))/7
```

3.475 $\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/5*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1
+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+
1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2x\sqrt{1+x}(1-x+x^2)+\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{5\sqrt{1-x+x^2}}$$

input
```
Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3
])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSin
h[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3
])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(5*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1151, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

$$\Big\downarrow \text{1151}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \sqrt{x^3+1}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{5}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)}{\sqrt{x^3+1}}$$

input
```
Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x*Sqrt[1 + x^3])/5 + (2*3^(3/4)*Sqrt[2
+ Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(
1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1)}}{5\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-9\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{5(x^3+1)}$

input
```
int((x+1)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)+6
/5*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.17

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x+\frac{6}{5}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x + 6/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Maxima [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Giac [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \int \sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

input
```
int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

output
```
int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{5} + \frac{3\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx\right)}{5}$$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*x + 3*int((sqrt(x + 1)*sqrt(x**2 - x + 1
))/(x**3 + 1),x))/5
```

3.476 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2} - \frac{2\sqrt{1+x}\sqrt{1-x+x^2}\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*arctanh((x
^3+1)^(1/2))/(x^3+1)^(1/2)
```

Mathematica [A] (verified)

Time = 17.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} \, dx = \frac{2}{3}\left(\sqrt{1+x}\sqrt{1-x+x^2} - \text{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

output
```
(2*(Sqrt[1 + x]*Sqrt[1 - x + x^2] - ArcTanh[Sqrt[1 + x]*Sqrt[1 - x + x^2]]
))/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 798, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x} \, dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^3}dx^3}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\sqrt{x^3+1}-2\text{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2*Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1 + x^3]]
))/(3*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(-\sqrt{x^{3}+1}+\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\right)}{3\sqrt{x^{3}+1}}$	43
elliptic	$\frac{\sqrt{(x+1)(x^{2}-x+1)}\,\left(\frac{2\sqrt{x^{3}+1}}{3}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	51

input
```
int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(
x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \frac{2}{3}\,\sqrt{x^{2}-x+1}\sqrt{x+1}-\frac{1}{3}\,\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\,\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1)
+ 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}dx$$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x)
```

output
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x)
```

3.477 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2} dx &= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} \\ &- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)} \\ &+ \frac{\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}\end{aligned}$$

output
```
-(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))
-3/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x
+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)
+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)+2^(1/2)*3^(3/4)*(1+x)^(3/2)*(x
^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+\frac{2i(1+x)}{-3i+\sqrt{3}}}\sqrt{1-\frac{2i(1+x)}{3i+\sqrt{3}}}\left(-\frac{\left(-3i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1+x}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}} + \frac{\left(-i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{\cdots}}{\cdots}\right.}{2\sqrt{2}\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{3-3(1+x)+(1+x)^2}}$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

output
```
-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
 Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*
Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-
I)*(1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]
*Sqrt[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3
])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]))/(2*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]
)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}\,dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 809$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 832$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2 \cdots}{(x+\cdots}}\cdots}{\sqrt{\cdots}}\cdots\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-(Sqrt[1 + x^3]/x) + (3*((2*Sqrt[1 + x^3])
/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)
/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2
*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/2))/Sqrt[
1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+(}{\sqrt{x^3+1}}}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}-\right.\right.}{\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(3i\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+9\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{2}$

input
```
int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+3*
(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*
I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = -\frac{3\,x\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(3*x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x
 + 1)*sqrt(x + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}+3\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^5+x^2}dx\right)x}{x}$$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1) + 3*int((sqrt(x + 1)*sqrt(x**2 - x + 1))
/(x**5 + x**2),x)*x)/x
```

3.478 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

output
```
-1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+1/2*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))
*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((
1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x
^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{\sqrt{1+x}\left(-\frac{2(1-x+x^2)}{x^2}-\frac{3i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{4\sqrt{1-x+x^2}}$$

input
```
Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

output
```
(Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] -
(2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I +
Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[
1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\sqrt{x^3+1}}{x^3}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3}{4}\int \frac{1}{\sqrt{x^3+1}}dx - \frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x^3+1}}$$

input
```
Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-1/2*Sqrt[1 + x^3]/x^2 + (3^(3/4)*Sqrt[2 +
 Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1)}}{2\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^2-9\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}}{4(x^3+1)x^2}$

input
```
int((x+1)^(1/2)*(x^2-x+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)+3/2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{3\,x^2\text{weierstrassPInverse}(0,-4,x)-\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(3*x^2*weierstrassPInverse(0, -4, x) - sqrt(x^2 - x + 1)*sqrt(x + 1))/
x^2
```

Sympy [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

input
```
integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

input
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3,x)
```

output
```
int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{-2\sqrt{x+1}\,\sqrt{x^2-x+1}-3\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^6+x^3}dx\right)x^2}{x^2}$$

input
```
int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(x + 1)*sqrt(x**2 - x + 1) - 3*int((sqrt(x + 1)*sqrt(x**2 - x +
1))/(x**6 + x**3),x)*x**2)/x**2
```

3.479 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{935\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output

```
54/935*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/187*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2
)+2/17*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(x^3+1)-36/935*3^(3/4)*(1/2*6^(1/2)
+1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2
)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.03 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.17

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}dx=\frac{2\left(x\sqrt{1+x}(27-27x+27x^2+100x^3-100x^4+100x^5+55x^6-55x^7+55x^8)-\frac{9i\sqrt{6}(1+x)\sqrt{}}{}\right.}{935\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x
^6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I
 + Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + S
qrt[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I
 + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*
I + Sqrt[3])]))/(935*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 811, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x^3\left(x^3+1\right)^{3/2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\int x^3\sqrt{x^3+1}dx+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\int \frac{x^3}{\sqrt{x^3+1}}dx+\frac{2}{11}\sqrt{x^3+1}x^4\right)+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{11}\sqrt{x^3+1}x^4\right)+\frac{2}{17}\left(x^3+1\right)^{3/2}x^4\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{17}\left(\frac{3}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{11}\sqrt{x^3\ldots}\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^4*(1 + x^3)^(3/2))/17 + (9*((2*x^4*Sq
rt[1 + x^3])/11 + (3*((2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*Sqrt[1 + x^3])))/11))/17))/Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2x(55x^6+100x^3+27)\sqrt{x+1}\,\sqrt{x^2-x+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots\right.}{935\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\sqrt{(x+1)(x^2-x+1)}\left(\frac{40x^4\sqrt{x^3+1}}{187}+\frac{54x\sqrt{x^3+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{935\sqrt{x^3+1}}\cdots\right.}{x^3+1}$
default	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\left(55x^{10}+155x^7+27i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-81\sqrt{-\frac{2(x+\cdots}{i\sqrt{3}-\cdots}}\cdots\right.}{935(x^3+1)}$

input
```
int(x^3*(x+1)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*x^6+100*x^3+27)*(x+1)^(1/2)*(x^2-x+1)^(1/2)-108/935*(3/2-1/2*I
*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-
x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{935}\left(55\,x^7+100\,x^4+27\,x\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{108}{935}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/935*(55*x^7 + 100*x^4 + 27*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 108/935*we
ierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^3\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

input
```
int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

Reduce [F]

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^7}{17}+\frac{40\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4}{187}+\frac{54\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{935}-\frac{54\left(\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx\right)}{935}$$

input
```
int(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

output
```
(2*(55*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**7 + 100*sqrt(x + 1)*sqrt(x**2 - x
 + 1)*x**4 + 27*sqrt(x + 1)*sqrt(x**2 - x + 1)*x - 27*int((sqrt(x + 1)*sqr
t(x**2 - x + 1))/(x**3 + 1),x)))/935
```

3.480 $\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

output
```
2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

input
```
Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1208}$$

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

input
```
Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(x+1)^{\frac{5}{2}}(x^{2}-x+1)^{\frac{5}{2}}}{15}$	18
orering	$\frac{2(x+1)^{\frac{5}{2}}(x^{2}-x+1)^{\frac{5}{2}}}{15}$	18
default	$\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,(x^{6}+2x^{3}+1)}{15}$	28
risch	$\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,(x^{6}+2x^{3}+1)}{15}$	28
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\sqrt{(x+1)(x^{2}-x+1)}\left(\frac{2x^{6}\sqrt{x^{3}+1}}{15}+\frac{4x^{3}\sqrt{x^{3}+1}}{15}+\frac{2\sqrt{x^{3}+1}}{15}\right)}{x^{3}+1}$	72

input
```
int(x^2*(x+1)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x+1)^(5/2)*(x^2-x+1)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x^2(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 7.52

$$\begin{aligned}&\int x^2(1+x)^{3/2}\left(1-x\right.\\&\left.+x^2\right)^{3/2}\,dx=\frac{2}{45045}\left(\left(\left(7\left(3\left(11\left(13\,x-80\right)(x+1)+3165\right)(x+1)-16442\right)(x+1)+121227\right)(x+1)-80\right.\\&+\frac{2}{45045}\left(\left(5\left(7\left(9\left(11\,x-57\right)(x+1)+1601\right)(x+1)-15837\right)(x+1)+65172\right)(x+1)-34077\right)\sqrt{(x+1)^2}\\&+\frac{2}{315}\left(\left(5\left(7\,x-23\right)(x+1)+258\right)(x+1)-213\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\\&+\frac{2}{105}\left(3\left(5\,x-12\right)(x+1)+71\right)\sqrt{(x+1)^2-3\,x}\sqrt{x+1}\end{aligned}$$

input
```
integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) +
 121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x +
1) + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1
) + 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*
(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
 + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2\,\sqrt{x+1}\,{\left(x^2-x+1\right)}^{5/2}\,\left(x^2+2\,x+1\right)}{15}$$

input
```
int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
(2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^6+2x^3+1)}{15}$$

input
```
int(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*(x**6 + 2*x**3 + 1))/15
```

3.481 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 325

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}+\frac{18\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
18/91*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+54*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(91+9
1*3^(1/2)+91*x)+2/13*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(x^3+1)-27/91*3^(1/4)
*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)
/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)+18/91*2^(1/2)*3^(3/4)*(1+x)^(3/2)*(x^2-x+1
)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\sqrt{1+x}\left(4x^2(1-x+x^2)\left(16+7x^3\right)-\frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right.}{\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2i}}}\right.}{182\sqrt{1-x+x^2}}$$

input
```
Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sq
rt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I +
 Sqrt[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1210, 811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int x\left(x^3+1\right)^{3/2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 811$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\int x\sqrt{x^3+1}dx+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\int \frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)+\frac{2}{13}\left(x^3+1\right)^{3/2}x^2\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\right.\right.}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{13}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)}{}\right.\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x^2*(1 + x^3)^(3/2))/13 + (9*((2*x^2*Sq
rt[1 + x^3])/7 + (3*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2
 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1
+ x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt
[3] + x)^2]*Sqrt[1 + x^3])))/7))/13))/Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2x^2(7x^3+16)\sqrt{x+1}\,\sqrt{x^2-x+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\cdots}{-\frac{3}{2}-\cdots}}\right.\right.}{91\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-\cdots}}$
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\sqrt{(x+1)(x^2-x+1)}\left(\frac{32x^2\sqrt{x^3+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x\cdots}{\frac{3}{2}-\cdots}}\right.\right.}{91\sqrt{x^3+1}\cdots}\right.}{x^3+1}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(14x^8+27i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)+46x^5+81\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\cdots\right.}{\cdots}$

input
```
int(x*(x+1)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*x^3+16)*(x+1)^(1/2)*(x^2-x+1)^(1/2)+54/91*(3/2-1/2*I*3^(1/2))*
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF
(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.12

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{91}\left(7x^5+16x^2\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{54}{91}\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/91*(7*x^5 + 16*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 54/91*weierstrassZet
a(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Giac [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\int x\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

input
```
int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

Reduce [F]

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^5}{13}+\frac{32\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2}{91}+\frac{27\left(\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{x^3+1}dx\right)}{91}$$

input
```
int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

output
```
(14*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**5 + 32*sqrt(x + 1)*sqrt(x**2 - x + 1
)*x**2 + 27*int((sqrt(x + 1)*sqrt(x**2 - x + 1)*x)/(x**3 + 1),x))/91
```

3.482 $\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \frac{18}{55} x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11} x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output
```
18/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(x^
3+1)+18/55*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*(
(x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3
^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.83 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2x\sqrt{1+x}(1-x+x^2)\left(14+5x^3\right)+\frac{9i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\ldots}}{\ldots}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{55\sqrt{1-x+x^2}}$$

input
```
Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I
)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*E
llipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[
3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1151, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^{3/2} \left(x^2-x+1\right)^{3/2} \, dx$$

$$\Big\downarrow \; 1151$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1} \int \left(x^3+1\right)^{3/2} dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \; 748$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\int\sqrt{x^3+1}dx+\frac{2}{11}x\left(x^3+1\right)^{3/2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\left(\frac{3}{5}\int\frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)+\frac{2}{11}x\left(x^3+1\right)^{3/2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{11}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)+\frac{2}{11}x\left(x^3\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*((2*x*(1 + x^3)^(3/2))/11 + (9*((2*x*Sqrt[1
 + x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/11))/S
qrt[1 + x^3]
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

method	result
risch	$\frac{2x(5x^3+14)\sqrt{x+1}\,\sqrt{x^2-x+1}}{55}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)}}{55\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\sqrt{(x+1)(x^2-x+1)}\left(\frac{28x\sqrt{x^3+1}}{55}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+i}{-\frac{3}{2}-i}}\right.}{55\sqrt{x^3+1}}\right.}{x^3+1}$
default	$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(-10x^7+27i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-81\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{i}\right.}{55(x^3+1)}$

input
```
int((x+1)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*x^3+14)*(x+1)^(1/2)*(x^2-x+1)^(1/2)+54/55*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*
EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.19

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \frac{2}{55} \left(5 \, x^4 + 14 \, x\right) \sqrt{x^2 - x + 1} \sqrt{x + 1} + \frac{54}{55} \, \text{weierstrassPInverse}(0, -4, x)$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 + 14*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 54/55*weierstrassPInve
rse(0, -4, x)
```

Sympy [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int \left(x + 1\right)^{\frac{3}{2}} \left(x^{2} - x + 1\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Maxima [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx = \int { {\left(x^{2} - x + 1\right)}^{\frac{3}{2}} {\left(x + 1\right)}^{\frac{3}{2}} \,d x}$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Giac [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int \left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}} dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \int (x+1)^{3/2} \left(x^2-x+1\right)^{3/2} dx$$

input
```
int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

output
```
int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

Reduce [F]

$$\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4}{11} + \frac{28\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{55} + \frac{27\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx\right)}{55}$$

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

output
```
(10*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**4 + 28*sqrt(x + 1)*sqrt(x**2 - x + 1
)*x + 27*int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**3 + 1),x))/55
```

3.483 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(x^3+1)-2/
3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*arctanh((x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\frac{2}{9}\left(\sqrt{1+x}\sqrt{1-x+x^2}\left(4+x^3\right)-3\text{arctanh}\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\right)$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

output
```
(2*(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(4 + x^3) - 3*ArcTanh[Sqrt[1 + x]*Sqrt[1
 - x + x^2]]))/9
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 798, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x^3}dx^3}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{\sqrt{x^3+1}}{x^3}dx^3+\frac{2}{3}\left(x^3+1\right)^{3/2}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{3}\left(x^3+1\right)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{3}\left(x^3+1\right)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(-2\text{arctanh}\left(\sqrt{x^3+1}\right)+\frac{2}{3}(x^3+1)^{3/2}+2\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2*Sqrt[1 + x^3] + (2*(1 + x^3)^(3/2))/3 -
2*ArcTanh[Sqrt[1 + x^3]]))/(3*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(-x^{3}\sqrt{x^{3}+1}+3\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)-4\sqrt{x^{3}+1}\right)}{9\sqrt{x^{3}+1}}$	57
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\sqrt{(x+1)(x^{2}-x+1)}\,\left(\frac{2x^{3}\sqrt{x^{3}+1}}{9}+\frac{8\sqrt{x^{3}+1}}{9}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{x^{3}+1}$	70

input
```
int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/
2))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \frac{2}{9}\left(x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1} - \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sq
rt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx=\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

output
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

3.484 $\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 318

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7(1+\sqrt{3}+x)} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}(1+x^3)} + \frac{9\sqrt{2}3^{3/4}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}(1+x^3)}$$

output
```
-(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+27*(1+x
)^(1/2)*(x^2-x+1)^(1/2)/(7+7*3^(1/2)+7*x)-27/14*3^(1/4)*(1/2*6^(1/2)-1/2*2
^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Elli
pticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)/(x^3+1)+9/7*2^(1/2)*3^(3/4)*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.77

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{1+x}\left(\frac{4(1-x+x^2)(-7+2x^3)}{x} - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\cdots}\right.\right.\right.}{\cdots}\right.}{28\sqrt{1-x+x^2}\cdots}$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sq
rt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I +
 Sqrt[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 809, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{\left(x^3+1\right)^{3/2}}{x^2}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\int x\sqrt{x^3+1}dx-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{811}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\int \frac{x}{\sqrt{x^3+1}}dx+\frac{2}{7}\sqrt{x^3+1}x^2\right)-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)+\frac{2}{7}\sqrt{x^3+1}x^2\right)-\frac{\left(x^3+1\right)^{3/2}}{x}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)+\frac{2}{7}\right.\right.}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{2}\left(\frac{3}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{}}{}\right.\right.\right.}{\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-((1 + x^3)^(3/2)/x) + (9*((2*x^2*Sqrt[1 +
 x^3])/7 + (3*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
- Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqr
t[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])))/7))/2))/Sqrt[1 + x^3]
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,(2x^{3}-7)}{7x}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots\right.\right.}{7\sqrt{x^{3}+1}\,\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\sqrt{(x+1)(x^{2}-x+1)}\left(-\frac{\sqrt{x^{3}+1}}{x}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\cdots}}\cdots\right.\right.}{7\sqrt{x^{3}+1}}\cdots\right.}{x^{3}+1}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\left(27i\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+4x^{6}+81\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-\cdots}{i\sqrt{3}\cdots}}\cdots\right.}{\cdots}$

input
```
int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(2*x^3-7)/x+27/7*(3/2-1/2*I*3^(1/2))*((x+1
)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-
3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.12

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^2} \, dx = \frac{(2\,x^3-7)\sqrt{x^2-x+1}\sqrt{x+1}-27\,x\text{weierstrassZeta}(0,-4,\text{weierstrass}$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/7*((2*x^3 - 7)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 27*x*weierstrassZeta(0, -
4, weierstrassPInverse(0, -4, x)))/x
```

Sympy [F]

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^2} \, dx = \int \frac{(x+1)^{\frac{3}{2}} \left(x^2-x+1\right)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}{x^2} \, dx = \int \frac{\left(x^2-x+1\right)^{\frac{3}{2}} (x+1)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^2}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx=\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^3+20\sqrt{x+1}\,\sqrt{x^2-x+1}+27\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^5+x^2}\right.}{7x}$$

input
```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**3 + 20*sqrt(x + 1)*sqrt(x**2 - x + 1)
 + 27*int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**5 + x**2),x)*x)/(7*x)
```

3.485 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx=-\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2}+\frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{10\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

output

```
-1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+9/1
0*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(3/2)*(x^2-x+1)^(1/2)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*
I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^3}\,dx = \frac{\sqrt{1+x}\left(\frac{2(1-x+x^2)(-5+4x^3)}{x^2} - \frac{27i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\right.\right.}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right.}{20\sqrt{1-x+x^2}}$$

input
```
Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
 + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I
 + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3
])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]))/(20*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 809, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}(x^2-x+1)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\int \frac{(x^3+1)^{3/2}}{x^3}dx}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{809}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\int \sqrt{x^3+1}dx-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\left(\frac{3}{5}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2}{5}\sqrt{x^3+1}x\right)-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(\frac{9}{4}\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{5}\sqrt{x^3+1}x\right)-\frac{\left(x^3+1\right)^{3/2}}{2x^2}\right)}{\sqrt{x^3+1}}$$

input
```
Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(-1/2*(1 + x^3)^(3/2)/x^2 + (9*((2*x*Sqrt[1
 + x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(5*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/4))/Sq
rt[1 + x^3]
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

method	result
risch	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,(4x^3-5)}{10x^2}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2}}{10\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
elliptic	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{}}{}}{-\frac{3}{2}-\frac{i\sqrt{}}{}}}\right.}{10\sqrt{x^3+1}}\right.}{x^3+1}$
default	$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(27i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^2-81\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x}{i\sqrt{3}+}}\right.}{20x^2(x^3+1)}$

input
```
int((x+1)^(3/2)*(x^2-x+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(4*x^3-5)/x^2+27/10*(3/2-1/2*I*3^(1/2))*(
(x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)
*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{27\,x^2\text{weierstrassPInverse}(0,-4,x)+(4\,x^3-5)\sqrt{x^2-x+1}\sqrt{x+1}}{10\,x^2}$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/10*(27*x^2*weierstrassPInverse(0, -4, x) + (4*x^3 - 5)*sqrt(x^2 - x + 1)
*sqrt(x + 1))/x^2
```

Sympy [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \int \frac{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}{x^3}\,dx$$

input
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3,x)
```

output
```
int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^3-16\sqrt{x+1}\,\sqrt{x^2-x+1}-27\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^6+x^3}\right.}{5x^2}$$

input

```
int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x)
```

output

```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1)*x**3 - 16*sqrt(x + 1)*sqrt(x**2 - x + 1)
 - 27*int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**6 + x**3),x)*x**2)/(5*x**2)
```

3.486 $\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/5*x*(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-4/15*(1/2*6^(1/2)+1/2*2^(1/2))*(
1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+
3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.75 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{6x\sqrt{1+x}(1-x+x^2) - \frac{2i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{15\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Ar
cSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sq
rt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x^3}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int\frac{1}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*((2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}\right)}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{x+1}\,\sqrt{x^2-x+1}}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1)}}{5\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{i}\right.}{5(x^3+1)}$

input
```
int(x^3/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/5*x*(x^3+1)^(1/2)-4
/5*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.18

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{5}\sqrt{x^2-x+1}\sqrt{x+1}x - \frac{4}{5}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 4/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{5} - \frac{2\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx\right)}{5}$$

input
```
int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
(2*(sqrt(x + 1)*sqrt(x**2 - x + 1)*x - int((sqrt(x + 1)*sqrt(x**2 - x + 1)
)/(x**3 + 1),x)))/5
```

3.487 $\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

output
```
2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

input
```
Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}dx$$
$$\Big\downarrow \text{ 1208}$$
$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

input
```
Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$	18
default	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$	18
risch	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$	18
orering	$\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$	18
elliptic	$\frac{2\sqrt{(x+1)(x^2-x+1)}\,\sqrt{x^3+1}}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$	39

input
```
int(x^2/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\,\sqrt{x^2-x+1}\sqrt{x+1}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\,(x^3+1)}{3\,\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{(x+1)^2-3\,x}\sqrt{x+1}$$

input
```
integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{x^3+1}}{3}$$

input
```
int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
(2*(x^3 + 1)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}}{3}$$

input
```
int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
(2*sqrt(x + 1)*sqrt(x**2 - x + 1))/3
```

3.488 $\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 253

$$\begin{aligned}
&\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx \\
&= \frac{2(1+x^3)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&\quad + \frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

output
```
2*(x^3+1)/(1+x)^(1/2)/(1+x+3^(1/2))/(x^2-x+1)^(1/2)-3^(1/4)*(1/2*6^(1/2)-1
/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+
1)^(1/2)+2/3*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Ellipti
cF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))
^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$$
$$= \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}(3i}{}\right.}{6\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 +
(3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqr
t[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
 Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] -
 (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3
*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x
]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sq
rt[3])]*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1210, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sq
rt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticF}\left(\sqrt{\frac{x+\cdots}{\frac{3}{2}-\cdots}}\right.\right.}{\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(i\sqrt{3}-3\right)\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\left(i\operatorname{EllipticE}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}-i\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+\cdots}{i\sqrt{3}-\cdots}}\right.\right.}{2x^3+2}$

input
```
int(x/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*
3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/
2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))*((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^
2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{x^3+1}dx$$

input
```
int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1)*x)/(x**3 + 1),x)
```

3.489 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2
)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3
*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]
*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1151, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{ 1151}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

input
```
Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\left(-i\sqrt{3}+3\right)\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)}{x^{3}+1}$	137
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^{2}-x+1)}}{\sqrt{x^{3}+1}\,\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	145

input
```
int(1/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-I*3^(1/2)+3)*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(-2*(x+1)/(I*3^(1/2)-3))^(1/2)*
((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^
(1/2)*EllipticF((-2*(x+1)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+
3))^(1/2))/(x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = 2\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3+1}dx$$

input
```
int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**3 + 1),x)
```

3.490 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2\sqrt{1+x^3}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
-2/3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 7.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{2}{3}\operatorname{arctanh}\left(\sqrt{1+x}\sqrt{3-3(1+x)+(1+x)^2}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 + x]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{x^3+1}\int \frac{1}{x^6-1}d\sqrt{x^3+1}}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{2\sqrt{x^3+1}\text{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3\sqrt{x^{3}+1}}$	33
elliptic	$-\frac{2\sqrt{\left(x+1\right)\left(x^{2}-x+1\right)}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	40

input
```
int(1/x/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*arctanh((x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sq
rt(x + 1) - 1)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

input
```
integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x\sqrt{x+1}\,\sqrt{x^2-x+1}}dx$$

input
```
int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

3.491 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&\quad + \frac{\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

output

```
-(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+(x^3+1)/(1+x)^(1/2)/(1+x+3^(1/2))/(
x^2-x+1)^(1/2)-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1
)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2
*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)+1/3*2^(1/2)*(1+x)^(1/2)*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*
3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\cdots}{\cdots}\right)}{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I
+ Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*
I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)
/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sq
rt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
])/Sqrt[1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1210, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 847$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 832$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{arcs}\ldots\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3\ldots}}\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x)
- (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])
/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt
[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2))/(Sqrt[1 + x]*Sqrt[1 - x +
 x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\cdots\right)\right)}{\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\cdots\right)\right)}{\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(i\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x-6\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\cdots}\right)}{2x\cdots}$

input
```
int(1/x^2/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-1/x*(x^3+1)^(1/2)+(3
/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1
/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\sqrt{x+1}}{x}$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-(x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x +
 1)*sqrt(x + 1))/x
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^2\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^5+x^2}dx$$

input
```
int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**5 + x**2),x)
```

3.492 $\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{-1-x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output

```
1/2*(-x^3-1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-1/6*(1/2*6^(1/2)+1/2*2^(1/2))
*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+
x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{-\frac{6\sqrt{1+x}(1-x+x^2)}{x^2}-\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{12\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + S
qrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*A
rcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - S
qrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\sqrt{x^3+1}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x^3+1}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

output
```
(Sqrt[1 + x^3]*(-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1)}}{2\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^2-3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\right.}{4x^2(x^3+1)}$

input
```
int(1/x^3/(x+1)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-1/2/x^2*(x^3+1)^(1/2
)-1/2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = -\frac{x^2\text{weierstrassPInverse}(0,-4,x)+\sqrt{x^2-x+1}\sqrt{x+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^2*weierstrassPInverse(0, -4, x) + sqrt(x^2 - x + 1)*sqrt(x + 1))/x
^2
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

input
```
int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^6+x^3}dx$$

input
```
int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**6 + x**3),x)
```

3.493 $\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
-2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+4/9*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/
2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2))
,I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.67 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{-\frac{6x}{\sqrt{1+x}} + \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{9\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 +
 x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[
(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sq
rt[(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1210, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1} \int \frac{x^3}{\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{\sqrt{x^3+1} \left(\frac{2}{3} \int \frac{1}{\sqrt{x^3+1}}dx-\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{x^3+1} \left(\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}} \text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((-2*x)/(3*Sqrt[1 + x^3]) + (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{2x}{3\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
risch	$-\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1}}{3\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{3(x^3+1)}$

input
```
int(x^3/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/3*x/(x^3+1)^(1/2)+
4/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x-2\left(x^3+1\right)\mathrm{weierstrassPInverse}(0,-4,x)\right)}{3\left(x^3+1\right)}$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 2*(x^3 + 1)*weierstrassPInverse(0,
 -4, x))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^3}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{-2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+2\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^6+2x^3+1}dx\right)x^3+2\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+}}{x^6+2x^3+1}\right.}{x^3+1}$$

input
```
int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
(2*( - sqrt(x + 1)*sqrt(x**2 - x + 1)*x + int((sqrt(x + 1)*sqrt(x**2 - x +
 1))/(x**6 + 2*x**3 + 1),x)*x**3 + int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x
**6 + 2*x**3 + 1),x)))/(x**3 + 1)
```

3.494 $\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
-2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=-\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

input
```
Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1208}$$

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	18
risch	$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	18
orering	$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	18
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}}{3(x^{3}+1)}$	25
elliptic	$-\frac{2\sqrt{(x+1)(x^{2}-x+1)}}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\sqrt{x^{3}+1}}$	39

input
```
int(x^2/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x+1)^(1/2)/(x^2-x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{3/2}\,(1-x+x^2)^{3/2}}\,dx = -\frac{2\,\sqrt{x^2-x+1}\sqrt{x+1}}{3\,(x^3+1)}$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^2}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [F]

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}\, dx = -\frac{2}{3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

input
```
int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
-2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(1+x)^{3/2} \left(1-x+x^2\right)^{3/2}}\, dx = -\frac{2\sqrt{x^2-x+1}}{3\sqrt{x+1}\,\left(x^2-x+1\right)}$$

input
```
int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
( - 2*sqrt(x**2 - x + 1))/(3*sqrt(x + 1)*(x**2 - x + 1))
```

3.495 $\int \frac{x}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 282

$$\begin{aligned}\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\ &-\frac{2(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}\\ &+\frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\\ &-\frac{2\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
2/3*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)/(1+x)^(1/2)/(1+x+3^(1/2))/
(x^2-x+1)^(1/2)+1/3*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+
2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)-2/9*2^(1/2)*(1+x)^(1/2)
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I
*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.43

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i \cdots\right.}{\cdots}\right)}{18\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/
(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqr
t[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] +
(6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sq
rt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*S
qrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3]
)]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqr
t[3])])/Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1210, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{(x^3+1)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{1}{3}\int \frac{x}{\sqrt{x^3+1}}dx\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{x^3+1}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+}}\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x^2)/(3*Sqrt[1 + x^3]) + ((-2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[
3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*(1 - Sqrt[
3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/3))/(Sqrt[1 + x]*Sqrt
[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+(}{3\sqrt{x^3+1}}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$
risch	$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\right.\right.}{3\sqrt{x^3+1}\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)+3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{3(}$

input
```
int(x/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/3/(x^3+1)^(1/2)*x^2
-2/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2
+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x^2+(x^3+1)\text{weierstrassZeta}(0,-4,\text{weierstrassP}\right.}{3\left(x^3+1\right)}$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 + (x^3 + 1)*weierstrassZeta(0, -4,
weierstrassPInverse(0, -4, x)))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{x}{(x+1)^{3/2} (x^2-x+1)^{3/2}} \, dx$$

input
```
int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{x^6+2x^3+1} dx$$

input
```
int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1)*x)/(x**6 + 2*x**3 + 1),x)
```

3.496 $\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/2
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\sqrt{3-3(1+x)+(1+x)^2}\left(-\frac{2}{9\sqrt{1+x}}+\frac{2(1+x)^{3/2}}{9\left(3-3(1+x)+(1+x)^2\right)}\right)+$$

input
```
Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9
*(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 -
 I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*Ar
cSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3
])])/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1151, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{3/2} \left(x^2-x+1\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 1151$$

$$\frac{\sqrt{x^3+1} \int \frac{1}{\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 749$$

$$\frac{\sqrt{x^3+1} \left(\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 759$$

$$\frac{\sqrt{x^3+1} \left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2x}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x)/(3*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}\right)}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^2-x+1)}}{3\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)-3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\cdots}\right.}{3(x^3+1)}$

input
```
int(1/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/3*x/(x^3+1)^(1/2)+2
/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}x+(x^3+1)\mathrm{weierstrassPInverse}(0,-4,x)\right)}{3\left(x^3+1\right)}$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x + (x^3 + 1)*weierstrassPInverse(0, -4
, x))/(x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{1}{(x+1)^{\frac{3}{2}} (x^2-x+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}} (x+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^{3/2} (1-x+x^2)^{3/2}} \, dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}} (x+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^6+2x^3+1}dx$$

input
```
int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**6 + 2*x**3 + 1),x)
```

3.497 $\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}} dx = \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2\sqrt{1+x^3}\operatorname{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))/(
1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 12.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}} dx = \frac{2\left(\sqrt{1+x} - (1+x)^2\sqrt{\frac{1-x+x^2}{(1+x)^2}}\operatorname{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)\right)}{3(1+x)\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(2*(Sqrt[1 + x] - (1 + x)^2*Sqrt[(1 - x + x^2)/(1 + x)^2]*ArcTanh[1/((1 +
x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])]))/(3*(1 + x)*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 798, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{\sqrt{x^3+1}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{x^3+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{\sqrt{x^3+1}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{\sqrt{x^3+1}}-2\text{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1 + x^3]]))/(3*Sqrt[1 + x
]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\sqrt{x^{3}+1}-1\right)}{3(x^{3}+1)}$	43
elliptic	$\frac{\sqrt{(x+1)(x^{2}-x+1)}\,\left(\frac{2}{3\sqrt{x^{3}+1}}-\frac{2\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	51
risch	$\frac{2}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}-\frac{2\sqrt{(x+1)(x^{2}-x+1)}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$	58

input
```
int(1/x/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x+1)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/
(x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}+1\right)-\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}-1\right)-2\sqrt{x^{2}-x+1}\sqrt{x+}}{3\left(x^{3}+1\right)}$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqr
t(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 +
1)
```

Sympy [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(x+1\right)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(x+1\right)^{3/2}\,\left(x^2-x+1\right)^{3/2}}\,dx$$

input
```
int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(x+1\right)^{\frac{3}{2}}\,\left(x^2-x+1\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

3.498 $\int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} \\ &- \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\ &- \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\ &+ \frac{5\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
2/3/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-5/3*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2
)+5/3*(x^3+1)/(1+x)^(1/2)/(1+x+3^(1/2))/(x^2-x+1)^(1/2)-5/6*3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Elliptic
E((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
/(x^2-x+1)^(1/2)+5/9*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)
*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+
3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{3+5x^3}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(3\cdots}{\cdots}\right.}{36\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
-1/3*(3 + 5*x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12
*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*S
qrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I +
 Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I
)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1
+ x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*
I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Elli
pticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\left(x^3+1\right)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 819$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\int \frac{1}{x^2\sqrt{x^3+1}}dx+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 847$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 832$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{3}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E(\ldots}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\ldots}\ldots}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*x*Sqrt[1 + x^3]) + (5*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[
1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 -
 x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x
^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2
))/3))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{2x^2}{3\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{x}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}}{-\frac{3}{2}}}\right.\right.}{3\sqrt{x^3+1}}\right.}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
risch	$-\frac{5x^3+3}{3x\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\right.\right.}{3\sqrt{x^3+1}\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\left(5i\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x+15\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\right.}{}$

input
```
int(1/x^2/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/3/(x^3+1)^(1/2)*x^
2-1/x*(x^3+1)^(1/2)+5/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/
2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*Ellipt
icE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(
1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(5\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+5\,(x^4+x)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{3\,(x^4+x)}$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((5*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^4 + x)*weierstrassZ
eta(0, -4, weierstrassPInverse(0, -4, x)))/(x^4 + x)
```

Sympy [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

input
```
int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^8+2x^5+x^2}dx$$

input
```
int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**8 + 2*x**5 + x**2),x)
```

3.499 $\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx = \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
2/3/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-7/6*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^
(1/2)-7/18*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+
x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{-\frac{6(3+7x^3)}{x^2\sqrt{1+x}} - \frac{7i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{36\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
 + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF
[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I
 - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3(x^3+1)^{3/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\int \frac{1}{x^3\sqrt{x^3+1}}dx+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\left(-\frac{1}{4}\int\frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{3}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*x^2*Sqrt[1 + x^3]) + (7*(-1/2*Sqrt[1 + x^3]/x^2 - (Sq
rt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/3))/(Sqrt[1 + x]*Sqrt[1
- x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(x+1)(x^{2}-x+1)}\left(-\frac{2x}{3\sqrt{x^{3}+1}}-\frac{\sqrt{x^{3}+1}}{2x^{2}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{x^{3}+1}}\right)}{\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$
risch	$-\frac{7x^{3}+3}{6x^{2}\sqrt{x+1}\,\sqrt{x^{2}-x+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(x+1)(x^{2}-x+ \cdots}}{6\sqrt{x^{3}+1}\,\sqrt{x+1}\,\sqrt{x^{2}-x+1}}$
default	$\frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}\left(7i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^{2}-21\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \cdots\right.}{12(x^{3}+1)x^{2}}$

input
```
int(1/x^3/(x+1)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/3*x/(x^3+1)^(1/2)-
1/2/x^2*(x^3+1)^(1/2)-7/6*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(
1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*
I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{(7\,x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+7\,(x^5+x^2)\text{weierstrassPInverse}(0,-4,x)}{6\,(x^5+x^2)}$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*((7*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^5 + x^2)*weierstras
sPInverse(0, -4, x))/(x^5 + x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

output
```
Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

input
```
int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^9+2x^6+x^3}\,dx$$

input
```
int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**9 + 2*x**6 + x**3),x)
```

3.500 $\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
4/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/9*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)/(x^3+
1)+4/81*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^
(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/
(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{6x\left(-1+2x^3\right)}{(1+x)^{3/2}(1-x+x^2)} + \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

input
```
Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1
+ (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 +
x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x
^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1210, 817, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{x^3}{\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\int \frac{1}{(x^3+1)^{3/2}}dx-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 749$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\left(\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{2x}{3\sqrt{x^3+1}}\right)-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{x^3+1}\left(\frac{2}{9}\left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2x}{3\sqrt{x^3+1}}\right)-\frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*((-2*x)/(9*(1 + x^3)^(3/2)) + (2*((2*x)/(3*Sqrt[1 + x^3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/9))/(Sqrt[1 + x]*S
qrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}+\frac{4x}{27\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right.}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$-\frac{2\left(i\sqrt{3}\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}-3\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-\right.}{}$

input
```
int(x^3/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/9*x/(x^3+1)^(3/2)+
4/27*x/(x^3+1)^(1/2)+4/27*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(
1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*
I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left(\left(2\,x^4-x\right)\sqrt{x^2-x+1}\sqrt{x+1}+2\left(x^6+2\,x^3+1\right)\text{weierstrassPInvers}\right.}{27\left(x^6+2\,x^3+1\right)}$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((2*x^4 - x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 2*(x^6 + 2*x^3 + 1)*weie
rstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x^3}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{-2\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+2\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^9+3x^6+3x^3+1}dx\right)x^6+4\left(\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^9+3x^6+3x^3+1}\right.}{7x^6+14x^3+7}$$

input
```
int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
(2*( - sqrt(x + 1)*sqrt(x**2 - x + 1)*x + int((sqrt(x + 1)*sqrt(x**2 - x +
 1))/(x**9 + 3*x**6 + 3*x**3 + 1),x)*x**6 + 2*int((sqrt(x + 1)*sqrt(x**2 -
 x + 1))/(x**9 + 3*x**6 + 3*x**3 + 1),x)*x**3 + int((sqrt(x + 1)*sqrt(x**2
 - x + 1))/(x**9 + 3*x**6 + 3*x**3 + 1),x)))/(7*(x**6 + 2*x**3 + 1))
```

3.501 $\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

output
```
-2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Mathematica [A] (verified)

Time = 10.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

input
```
Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x+1)^{5/2} (x^2-x+1)^{5/2}} \, dx$$

$$\Big\downarrow \text{1208}$$

$$-\frac{2}{9(x+1)^{3/2} (x^2-x+1)^{3/2}}$$

input
```
Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Defintions of rubi rules used

rule 1208
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^2*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g^2*(d + e*x)^(m + 1)*((a + b*x + c
*x^2)^(p + 1)/(c*e*(m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}
, x] && EqQ[b*e*g*(m + p + 2) + 2*c*(d*g*(p + 1) - e*f*(m + 2*p + 3)), 0] &
& EqQ[e*(c*f^2 - b*f*g + a*g^2)*(m + 1) + (2*c*f - b*g)*(e*f - d*g)*(p + 1)
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2}{9(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}$	18
orering	$-\frac{2}{9(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}$	18
default	$-\frac{2}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}$	25
elliptic	$-\frac{2\sqrt{(x+1)(x^2-x+1)}}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)^{\frac{3}{2}}}$	39

input
```
int(x^2/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(x+1)^(3/2)/(x^2-x+1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = -\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{9\,(x^6+2\,x^3+1)}$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x^2}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = -\frac{2}{9(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
-2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [F]

$$\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{x^2}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.57

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{18\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}-18\,x\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}}{(x+1)\,\left(81\,x\,\left(x^2-x+1\right)^4-162\,\left(x^2-x+1\right)^4+81\,\left(x^2-x+1\right)^5\right)}$$

input
```
int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
(18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(
5/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x +
 1)^5))
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2\sqrt{x^2-x+1}}{9\sqrt{x+1}\,\left(x^5-x^4+x^3+x^2-x+1\right)}$$

input
```
int(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
( - 2*sqrt(x**2 - x + 1))/(9*sqrt(x + 1)*(x**5 - x**4 + x**3 + x**2 - x +
1))
```

3.502 $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 318

$$\begin{aligned}\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx &= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} \\ &+ \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{10(1+x^3)}{27\sqrt{1+x}\,(1+\sqrt{3}+x)\,\sqrt{1-x+x^2}} \\ &+ \frac{5\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}} \\ &- \frac{10\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

output
```
10/27*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)/
(x^3+1)-10/27*(x^3+1)/(1+x)^(1/2)/(1+x+3^(1/2))/(x^2-x+1)^(1/2)+5/27*3^(1/
4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)
*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))
^2)^(1/2)/(x^2-x+1)^(1/2)-10/81*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2
))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.13 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2x^2(8+5x^3)}{27(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} - \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}(\cdots}{\cdots}\right)}{162\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3
/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*
(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt
[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])
/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 +
 x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3
])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*S
qrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1210, 819, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1210$$

$$\frac{\sqrt{x^3+1}\int \frac{x}{\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 819$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\int \frac{x}{\left(x^3+1\right)^{3/2}}dx+\frac{2x^2}{9\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 819$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{1}{3}\int \frac{x}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{9\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 832$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)+\frac{2x^2}{9\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 759$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx\right)+\frac{2x^2}{3\sqrt{x^3+1}}\right)+\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \ 2416$$

$$\frac{\sqrt{x^3+1}\left(\frac{5}{9}\left(\frac{1}{3}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\mathrm{a}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{}}\right.\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x^2)/(9*(1 + x^3)^(3/2)) + (5*((2*x^2)/(3*Sqrt[1 + x^3]
) + ((-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1
+ x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3]) + (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt
[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x^2}{9(x^3+1)^{\frac{3}{2}}}+\frac{10x^2}{27\sqrt{x^3+1}}-\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\cdots}\right.\right.}{27\sqrt{x^3+1}}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$-\frac{5i\sqrt{3}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}+15\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-\cdots}}{\cdots}$

input
```
int(x/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x^2/(x^3+1)^(3/2)
+10/27/(x^3+1)^(1/2)*x^2-10/27*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))
*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1
/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.19

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left((5\,x^5+8\,x^2)\sqrt{x^2-x+1}\sqrt{x+1}+5\,(x^6+2\,x^3+1)\text{weierstrassZeta}(\right.}{27\,(x^6+2\,x^3+1)}$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((5*x^5 + 8*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^6 + 2*x^3 + 1)*
weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{x}{(x^2-x+1)^{\frac{5}{2}}\,(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{x}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{x}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,x}{x^9+3x^6+3x^3+1}dx$$

input
```
int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1)*x)/(x**9 + 3*x**6 + 3*x**3 + 1),x)
```

3.503 $\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
14/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)/(x^3
+1)+14/81*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.94 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{6x(10+7x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\text{EllipticF}\left(i\text{arcsinh}\left(\right.\right.}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

input
```
Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1
+ (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 +
x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x
^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1151, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1151}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\int \frac{1}{(x^3+1)^{3/2}}dx + \frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \textbf{749}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\left(\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx + \frac{2x}{3\sqrt{x^3+1}}\right) + \frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \textbf{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{7}{9}\left(\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2x}{3\sqrt{x^3+1}}\right) + \frac{2x}{9(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*((2*x)/(9*(1 + x^3)^(3/2)) + (7*((2*x)/(3*Sqrt[1 + x^3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/9))/(Sqrt[1 + x]*Sq
rt[1 - x + x^2])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1151
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p])   Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; F
reeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
 IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2x}{9(x^3+1)^{\frac{3}{2}}}+\frac{14x}{27\sqrt{x^3+1}}+\frac{14\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{27\sqrt{x^3+1}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$-\frac{7i\sqrt{3}\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}-21\operatorname{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-}}{}$

input
```
int(1/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/9*x/(x^3+1)^(3/2)+1
4/27*x/(x^3+1)^(1/2)+14/27*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left(\left(7\,x^4+10\,x\right)\sqrt{x^2-x+1}\sqrt{x+1}+7\left(x^6+2\,x^3+1\right)\text{weierstrassPInve}\right.}{27\left(x^6+2\,x^3+1\right)}$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
2/27*((7*x^4 + 10*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^6 + 2*x^3 + 1)*w
eierstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{1}{(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^9+3x^6+3x^3+1}dx$$

input
```
int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**9 + 3*x**6 + 3*x**3 + 1),x)
```

3.504 $\int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{2\sqrt{1+x^3}\,\text{arctanh}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

output
```
2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/(1+x)^(1/2)/(x^2-x+1)^(1/2)/(x^3+1)-2/
3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\frac{2(4+3x^3)}{3(1+x)^{3/2}(1-x+x^2)} - 2(1+x)\sqrt{\frac{1-x+x^2}{(1+x)^2}}\,\text{arctanh}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)}{3\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((2*(4 + 3*x^3))/(3*(1 + x)^(3/2)*(1 - x + x^2)) - 2*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + x)^2]*ArcTanh[1/((1 + x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])])
/(3*Sqrt[1 - x + x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1210, 798, 61, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{5/2}}dx^3}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\left(x^3+1\right)^{3/2}}dx^3+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{x^3+1}\left(\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x^3+1}\left(2\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{x^3+1}\left(-2\text{arctanh}\left(\sqrt{x^3+1}\right)+\frac{2}{\sqrt{x^3+1}}+\frac{2}{3(x^3+1)^{3/2}}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(3*(1 + x^3)^(3/2)) + 2/Sqrt[1 + x^3] - 2*ArcTanh[Sqrt[1
 + x^3]]))/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(\frac{2}{9(x^3+1)^{\frac{3}{2}}}+\frac{2}{3\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$	60
default	$-\frac{2\left(3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}\,x^3-3x^3+3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-4\right)}{9(x^3+1)\sqrt{x^2-x+1}\,\sqrt{x+1}}$	69

input
```
int(1/x/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(2/9/(x^3+1)^(3/2)+2/3
/(x^3+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{2\left(3\,x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1}-3\left(x^6+2\,x^3+1\right)\log\left(\sqrt{x^2-x+1}\sqrt{\right.}}{9\left(x^6+2\,x^3\right.}$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log
(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x
 + 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

Sympy [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{1}{x\,(x+1)^{5/2}\,\left(x^2-x+1\right)^{5/2}}\,dx$$

input
```
int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{1}{x\,(x+1)^{\frac{5}{2}}\,\left(x^2-x+1\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

3.505 $\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 349

$$\begin{aligned}
\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx &= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} \\
&+ \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{55(1+x^3)}{27x\sqrt{1+x}\sqrt{1-x+x^2}} \\
&+ \frac{55(1+x^3)}{27\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} \\
&- \frac{55\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}} \\
&+ \frac{55\sqrt{2}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}
\end{aligned}$$

output
```
22/27/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)/(x^3
+1)-55/27*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/27*(x^3+1)/(1+x)^(1/2)/
(1+x+3^(1/2))/(x^2-x+1)^(1/2)-55/54*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x
)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)+55/81*2
^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2
))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^2
-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.98 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{27+88x^3+55x^6}{27x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} + \frac{55(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}(\cdots}{\cdots}\right)}{324\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

input
```
Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
-1/27*(27 + 88*x^3 + 55*x^6)/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(
1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3
*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[
3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*A
rcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - S
qrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (
6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I
 + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]]
, (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sq
rt[3])]*Sqrt[1 - x + x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1210, 819, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^2\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\int \frac{1}{x^2\left(x^3+1\right)^{3/2}}dx+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\int \frac{1}{x^2\sqrt{x^3+1}}dx+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)+\frac{2}{3x\sqrt{x^3+1}}\right)+\frac{2}{9x\left(x^3+1\right)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}\right)\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\downarrow \text{2416}$$

$$\frac{\sqrt{x^3+1}\left(\frac{11}{9}\left(\frac{5}{3}\left(\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+\cdots}{\left(x+\sqrt{3}+\cdots\right.}}}{\sqrt{\frac{x-\cdots}{\left(x+\sqrt{\cdots}\right.}}}\right.\right.\right.\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(9*x*(1 + x^3)^(3/2)) + (11*(2/(3*x*Sqrt[1 + x^3]) + (5*
(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[Ar
cSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(
1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3]))/2))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.69

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{2x^2}{9(x^3+1)^{\frac{3}{2}}}-\frac{28x^2}{27\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{x}+\frac{55\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\right.}\right.}{27\sqrt{x^3}}\right.}{\sqrt{x+1}\sqrt{x^2-x+1}}$
default	$\frac{55i\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^4\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}+165\,\text{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^4\sqrt{-}}{}$

input
```
int(1/x^2/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/9*x^2/(x^3+1)^(3/2
)-28/27/(x^3+1)^(1/2)*x^2-1/x*(x^3+1)^(1/2)+55/27*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((
-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx =$$
$$-\frac{(55\,x^6+88\,x^3+27)\sqrt{x^2-x+1}\sqrt{x+1}+55\,(x^7+2\,x^4+x)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}}{27\,(x^7+2\,x^4+x)}$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*((55*x^6 + 88*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 55*(x^7 + 2*
x^4 + x)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^7 + 2*x
^4 + x)
```

Sympy [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x**2*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{1}{x^2\,{\left(x+1\right)}^{5/2}\,{\left(x^2-x+1\right)}^{5/2}} \,d x$$

input
```
int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{x+1}\,\sqrt{x^{2}-x+1}}{x^{11}+3 x^{8}+3 x^{5}+x^{2}}d x$$

input
```
int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**11 + 3*x**8 + 3*x**5 + x**2),x)
```

3.506 $\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)}-\frac{91(1+x^3)}{54x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{91\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

output
```
26/27/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)/
(x^3+1)-91/54*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-91/162*(1/2*6^(1/2)+
1/2*2^(1/2))*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-
3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2
)/(x^2-x+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=\frac{-\frac{6\left(27+130x^3+91x^6\right)}{x^2(1+x)^{3/2}}-\frac{91i(1+x)\left(1-x+x^2\right)\sqrt{6+\frac{36i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6i}{\left(3i+\sqrt{3}\right)(1+x)}}\,\text{Ellipt}}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{324\left(1-x+x^2\right)^{3/2}}$$

input
```
Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
 + x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I +
 Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[
1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(32
4*(1 - x + x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1210, 819, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1210}$$

$$\frac{\sqrt{x^3+1}\int \frac{1}{x^3\left(x^3+1\right)^{5/2}}dx}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\int \frac{1}{x^3(x^3+1)^{3/2}}dx+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\int \frac{1}{x^3\sqrt{x^3+1}}dx+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\left(-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{x^3+1}\left(\frac{13}{9}\left(\frac{7}{3}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)+\frac{2}{3x^2\sqrt{x^3+1}}\right)+\frac{2}{9x^2(x^3+1)^{3/2}}\right)}{\sqrt{x+1}\sqrt{x^2-x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + x^3]*(2/(9*x^2*(1 + x^3)^(3/2)) + (13*(2/(3*x^2*Sqrt[1 + x^3]) +
 (7*(-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2
)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
 x^3])))/3))/9))/(Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1210
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^FracPart[p]*((a + b*x +
c*x^2)^FracPart[p]/(a*d + c*e*x^3)^FracPart[p])   Int[(f + g*x)^n*(a*d + c*
e*x^3)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[b*d + a*e,
 0] && EqQ[c*d + b*e, 0] && EqQ[m, p]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\sqrt{(x+1)(x^2-x+1)}\left(-\frac{2x}{9(x^3+1)^{\frac{3}{2}}}-\frac{32x}{27\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}-\frac{91\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{-\ldots}\right.}{54\sqrt{x^3+1}}\right.}{\sqrt{x+1}\,\sqrt{x^2-x+1}}$
default	$\frac{91i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^5\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}-273\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^5\sqrt{-\ldots}}{\ldots}$

input
```
int(1/x^3/(x+1)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)*(x^2-x+1))^(1/2)/(x+1)^(1/2)/(x^2-x+1)^(1/2)*(-2/9*x/(x^3+1)^(3/2)-
32/27*x/(x^3+1)^(1/2)-1/2/x^2*(x^3+1)^(1/2)-91/54*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*El
lipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{(91\,x^6+130\,x^3+27)\sqrt{x^2-x+1}\sqrt{x+1}+91\,(x^8+2\,x^5+x^2)\text{weierstrassPInverse}(0,-4,x)}{54\,(x^8+2\,x^5+x^2)}$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/54*((91*x^6 + 130*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 91*(x^8 + 2
*x^5 + x^2)*weierstrassPInverse(0, -4, x))/(x^8 + 2*x^5 + x^2)
```

Sympy [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3\left(x+1\right)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

output
```
Integral(1/(x**3*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx = \int \frac{1}{x^3\,(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

input
```
int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

output
```
int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^{12}+3x^9+3x^6+x^3}dx$$

input
```
int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x**2 - x + 1))/(x**12 + 3*x**9 + 3*x**6 + x**3),x)
```

3.507 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 89

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = -\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
-2*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticPi(1/2*(-c*x+1)^(1
/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))/(1-1/c^2/x^2)^(1/2)/x/(e*x+d)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx =$$

$$-\frac{2i\sqrt{\frac{e(-1+cx)}{c(d+ex)}}(d+ex)\sqrt{\frac{e+cex}{cd+cex}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right),\frac{cd-e}{cd+e}\right)-\text{EllipticPi}\left(\frac{cd}{cd+e},i\text{arcsinh}\left(\frac{\sqrt{-}}{\sqrt{}}\right.\right.\right.}{d\sqrt{-\frac{cd+e}{c}}\sqrt{1-\frac{1}{c^2x^2}}x}$$

input
```
Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

output
```
((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
 + c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d
- e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c
)]/Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 -
 1/(c^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1898, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1898}$$

$$\frac{\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 633}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 632}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{ 186}$$

$$-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

input
```
Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

output
```
(-2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e/c - (e*(1 - c*x))/c])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{2(cd-e)\,\mathrm{EllipticPi}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\frac{cd-e}{dc},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{-\frac{e(cx+1)}{cd-e}}\,\sqrt{-\frac{e(cx-1)}{cd+e}}\,\sqrt{\frac{c(ex+d)}{cd-e}}}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x\sqrt{ex+d}\,cd}$	148

input
```
int(1/(1-1/c^2/x^2)^(1/2)/x^2/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi((c*(e*x+d)/(c*d-e))^(1
/2),(c*d-e)/d/c,((c*d-e)/(c*d+e))^(1/2))*(-e*(c*x+1)/(c*d-e))^(1/2)*(-e*(c
*x-1)/(c*d+e))^(1/2)*(c*(e*x+d)/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

Fricas [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/(1-1/c^2/x^2)^(1/2)/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*c^2*sqrt((c^2*x^2 - 1)/(c^2*x^2))/(c^2*e*x^3 + c^2*
d*x^2 - e*x - d), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(1-1/c**2/x**2)**(1/2)/x**2/(e*x+d)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/(1-1/c^2/x^2)^(1/2)/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

input
```
integrate(1/(1-1/c^2/x^2)^(1/2)/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \int \frac{1}{x^2\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+e\,x}}\,dx$$

input
```
int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx = \left(\int \frac{\sqrt{ex+d}\,\sqrt{c^2x^2-1}}{c^2e\,x^4+c^2d\,x^3-e\,x^2-dx}dx\right)c$$

input
```
int(1/(1-1/c^2/x^2)^(1/2)/x^2/(e*x+d)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(c**2*x**2 - 1))/(c**2*d*x**3 + c**2*e*x**4 - d*x -
 e*x**2),x)*c
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file