Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.4/105-1.2.1.4-b

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**180**]. This is test number [105].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	96.11 (173)	3.89 (7)
Rubi	93.89 (169)	6.11 (11)
Maple	90.56 (163)	9.44 (17)
Fricas	85.56 (154)	14.44 (26)
Giac	75.00 (135)	25.00 (45)
Reduce	67.22 (121)	32.78 (59)
Maxima	58.89 (106)	41.11 (74)
Sympy	56.67 (102)	43.33 (78)
Mupad	51.11 (92)	48.89 (88)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

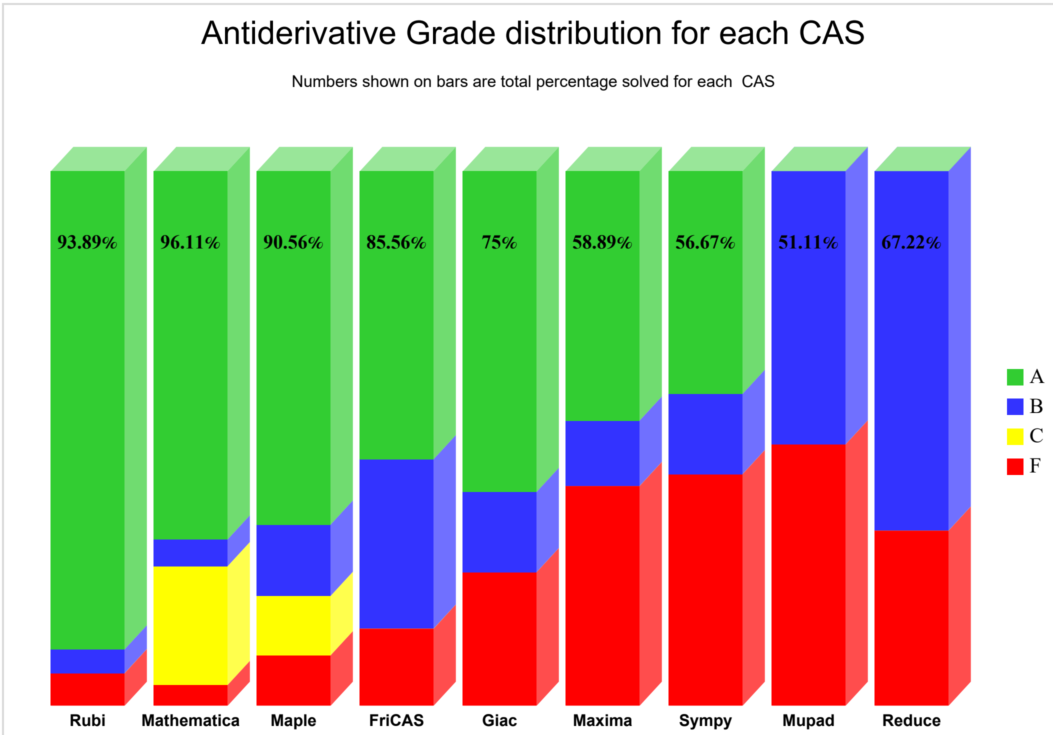
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

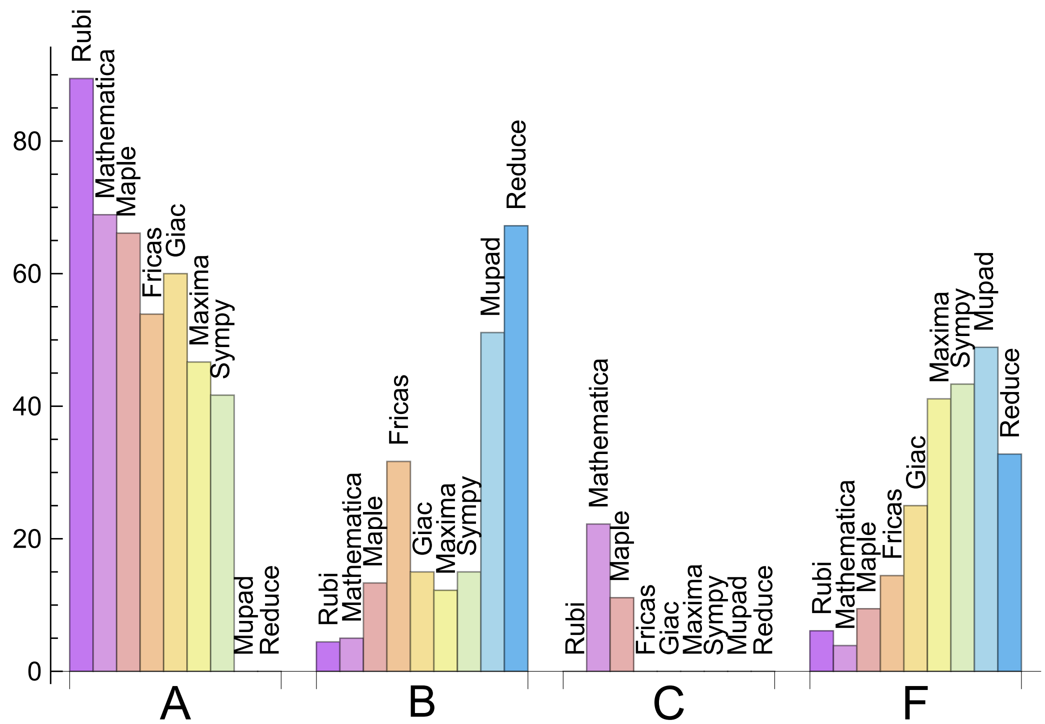
System	% A grade	% B grade	% C grade	% F grade
Rubi	89.444	4.444	0.000	6.111
Mathematica	68.889	5.000	22.222	3.889
Maple	66.111	13.333	11.111	9.444
Giac	60.000	15.000	0.000	25.000
Fricas	53.889	31.667	0.000	14.444
Maxima	46.667	12.222	0.000	41.111
Sympy	41.667	15.000	0.000	43.333
Mupad	0.000	51.111	0.000	48.889
Reduce	0.000	67.222	0.000	32.778

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	7	100.00	0.00	0.00
Rubi	11	100.00	0.00	0.00
Maple	17	100.00	0.00	0.00
Fricas	26	73.08	26.92	0.00
Giac	45	42.22	13.33	44.44
Reduce	59	100.00	0.00	0.00
Maxima	74	71.62	0.00	28.38
Sympy	78	75.64	24.36	0.00
Mupad	88	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.18
Fricas	0.25
Reduce	0.43
Rubi	0.53
Mathematica	1.37
Maple	2.58
Sympy	2.66
Mupad	8.96

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	100.24	1.40	64.00	0.89
Mupad	144.49	1.72	62.50	1.00
Rubi	181.20	1.17	116.00	1.05
Sympy	204.12	1.94	79.00	1.00
Mathematica	280.55	1.45	90.00	1.00
Maple	432.86	1.71	65.00	0.94
Fricas	433.82	2.50	108.00	1.54
Giac	436.70	2.57	72.00	0.84
Reduce	605.13	3.64	122.00	1.74

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

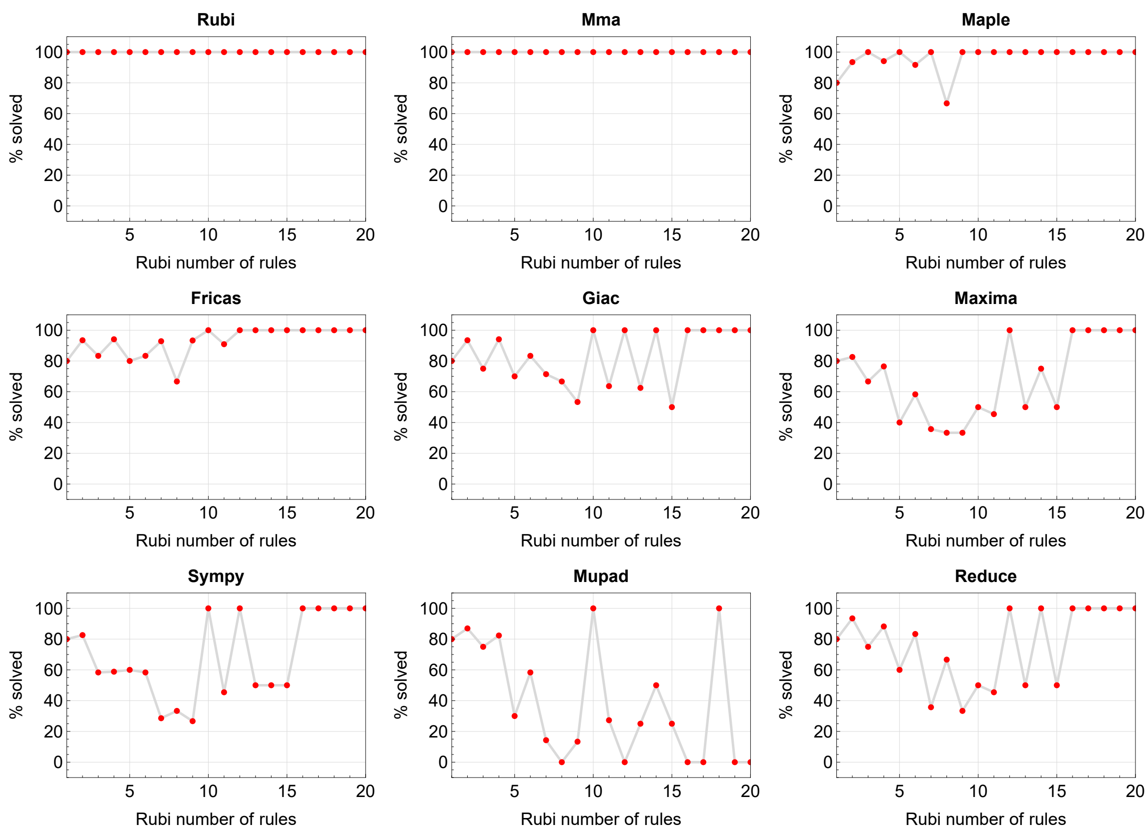


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

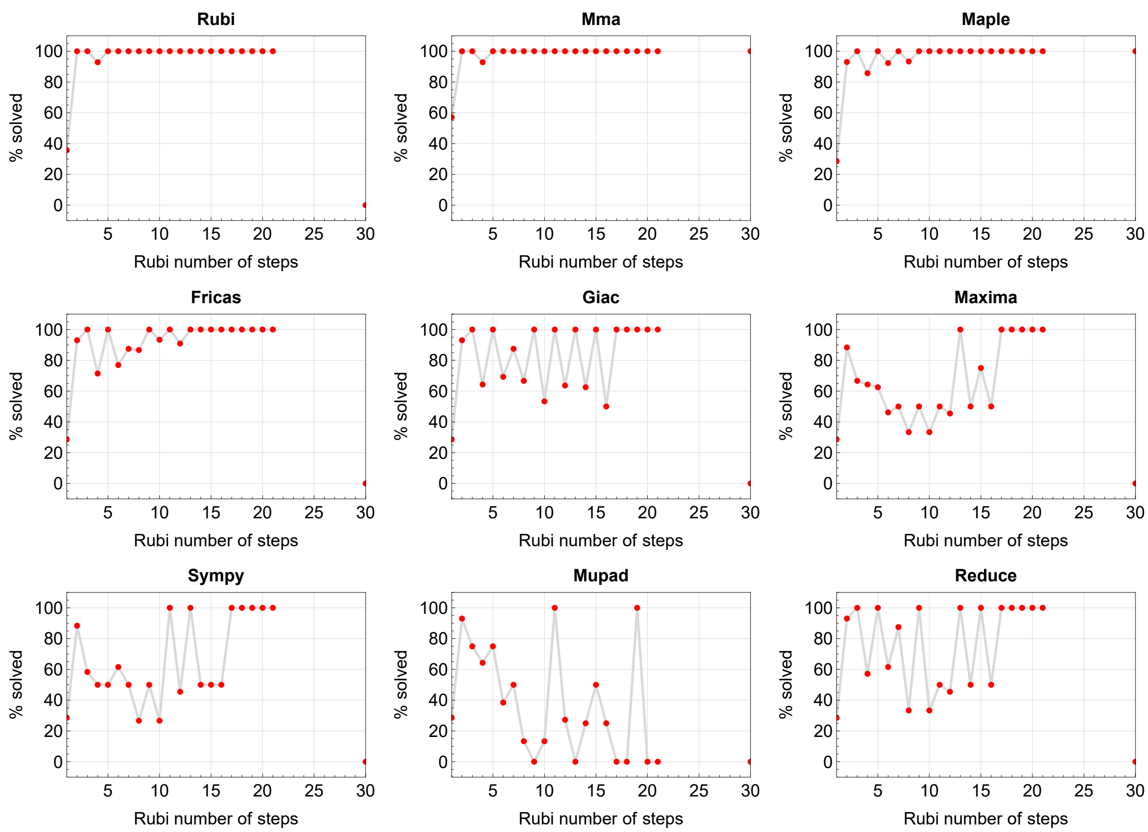


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

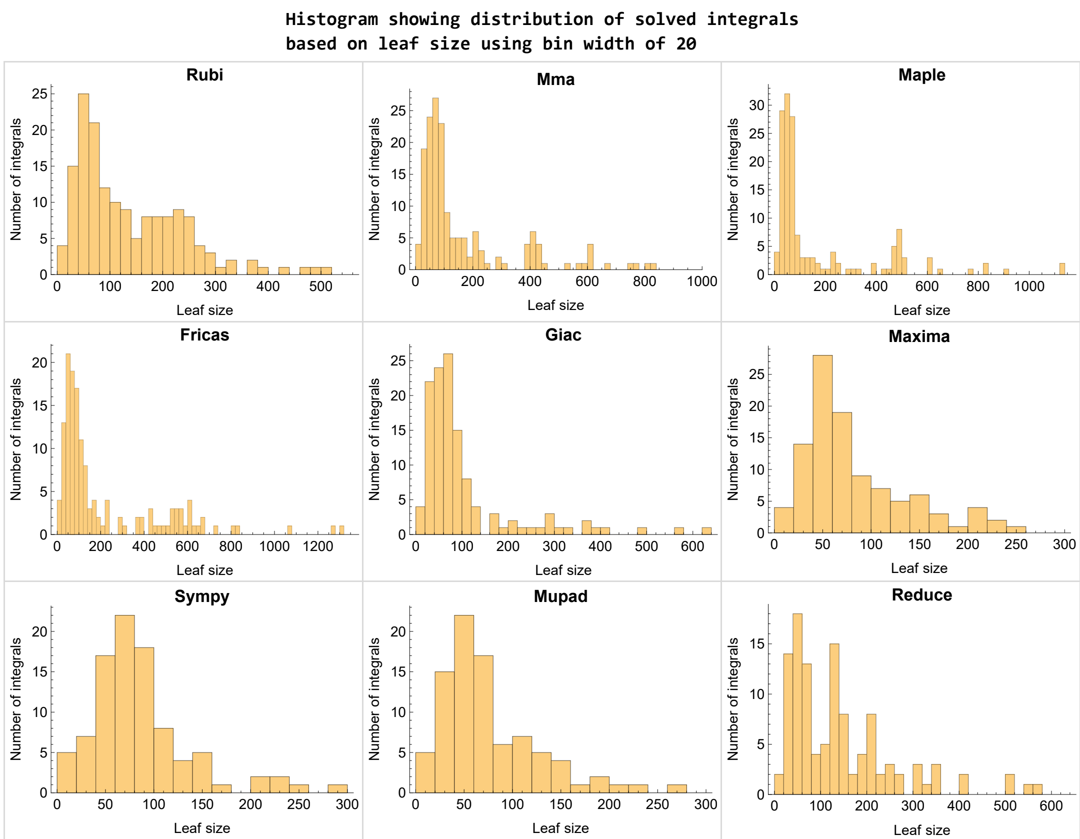


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

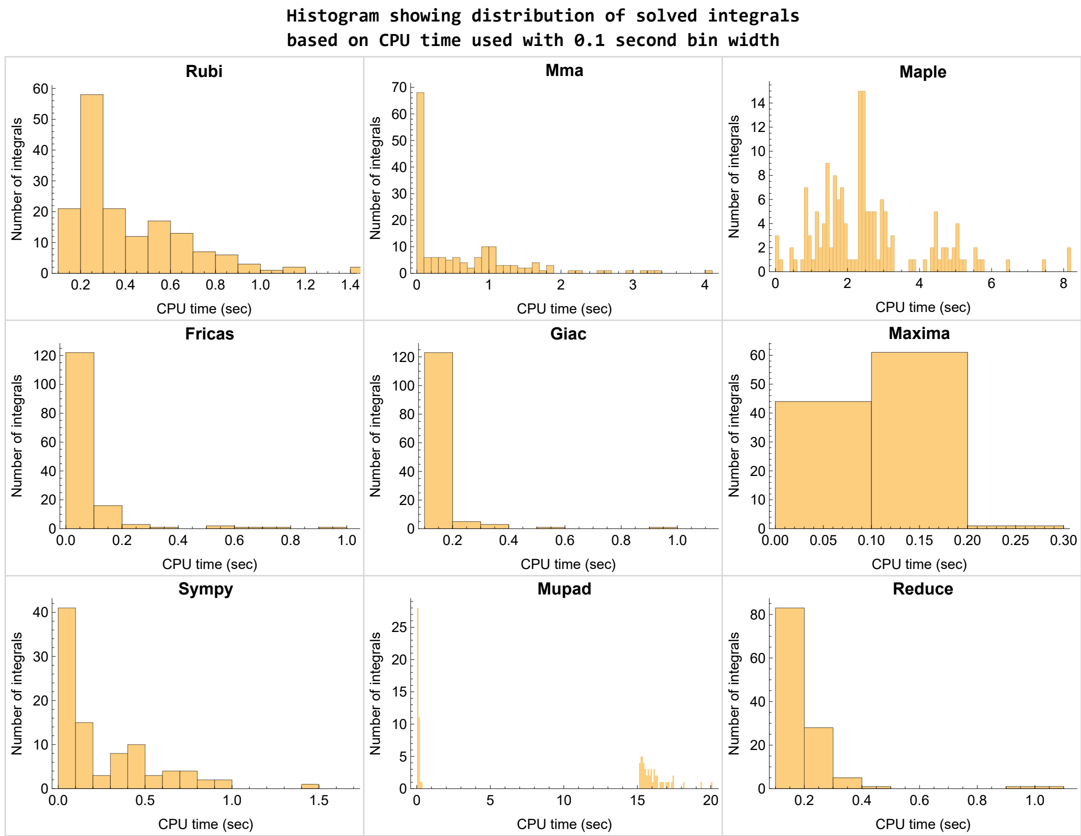


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

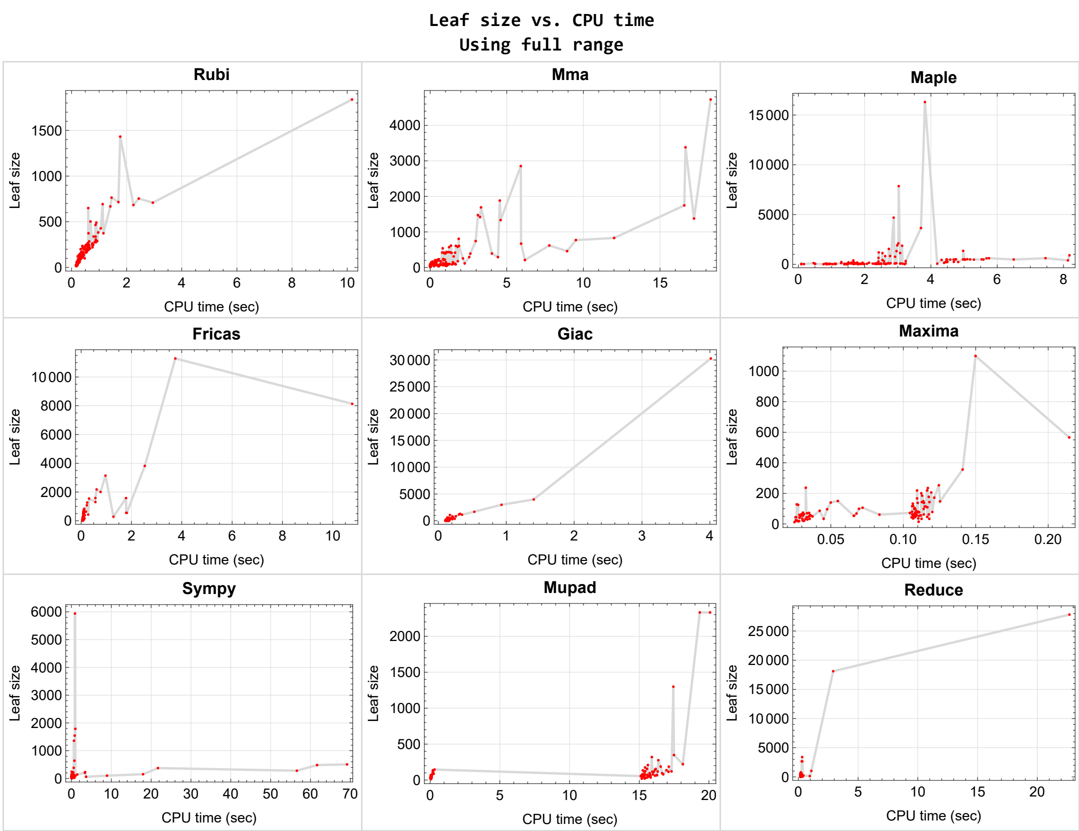


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{142, 143, 144, 146, 151, 164, 166, 167, 168\}$

Mathematica $\{43, 144, 151, 164, 165, 166, 167, 168, 174, 179, 180\}$

Maple {96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 126, 131, 132, 138, 143, 144, 161, 164}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

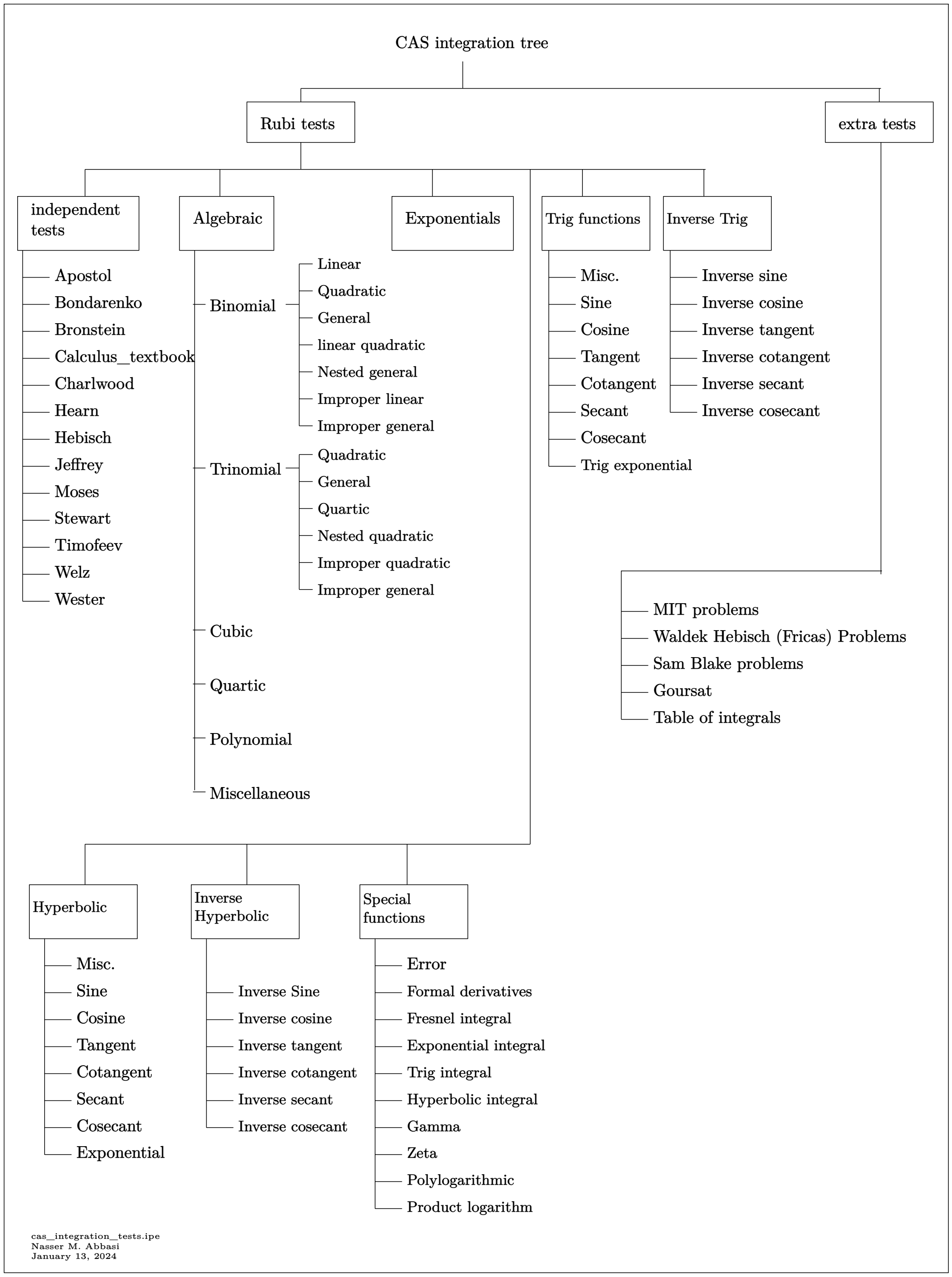
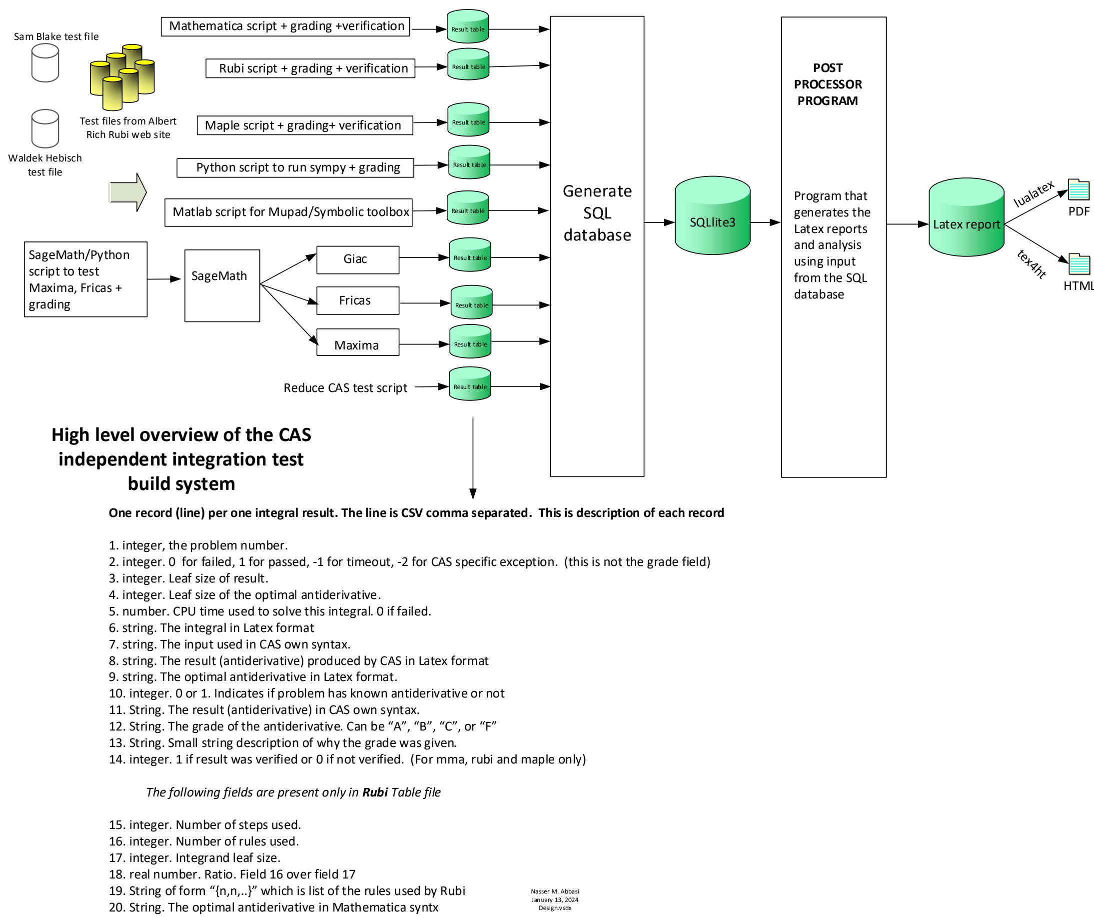


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 180 }

B grade { 5, 6, 15, 16, 21, 26, 27, 164 }

C grade { }

F normal fail { 156, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 123, 127, 128, 129, 130, 134, 135, 136, 140, 141, 146, 147, 148, 149, 151, 153, 154, 155, 158, 159, 160, 163, 164, 165, 169, 174, 177, 179 }

B grade { 5, 6, 15, 16, 26, 38, 39, 144, 180 }

C grade { 2, 3, 4, 35, 36, 37, 40, 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 125, 126, 131, 132, 133, 137, 138, 139, 142, 143, 145, 150, 152, 156, 157, 161, 162, 166, 167, 168 }

F normal fail { 170, 171, 172, 173, 175, 176, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 123, 127, 128, 129, 130, 134, 135, 136, 139, 141, 145, 147, 148, 149, 152, 153, 154, 157, 158, 160, 162, 164, 165 }

B grade { 5, 6, 15, 16, 26, 27, 35, 36, 37, 38, 39, 40, 137, 138, 140, 142, 143, 144, 150, 151, 155, 156, 159, 163 }

C grade { 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 125, 126, 131, 132, 133, 146, 161 }

F normal fail { 42, 43, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 17, 18, 19, 28, 29, 30, 31, 34, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 127, 128, 130, 134, 135, 136, 141, 146, 147, 148, 149, 158, 159 }

B grade { 5, 6, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33, 35, 36, 37, 38, 39, 40, 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 123, 124, 125, 126, 129, 131, 132, 133, 139, 140, 145, 150, 152, 153, 154, 155, 157, 160, 161, 162, 163 }

C grade { }

F normal fail { 42, 43, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-1) timedout fail { 137, 138, 142, 143, 144, 151, 156 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 13, 17, 18, 19, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 123, 130, 134 }

B grade { 5, 6, 11, 12, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33, 127, 128, 129, 158, 161 }

C grade { }

F normal fail { 37, 38, 39, 40, 41, 42, 43, 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 125, 126, 131, 132, 133, 138, 139, 143, 144, 145, 146, 151, 152, 157, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { 34, 35, 36, 89, 90, 135, 136, 137, 140, 141, 142, 147, 148, 149, 150, 153, 154, 155, 156, 159, 160 }

Giac

A grade { 1, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 123, 127, 128, 129, 130, 134, 135, 136, 141, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162 }

B grade { 2, 3, 4, 5, 6, 13, 14, 15, 16, 23, 24, 25, 26, 27, 35, 36, 37, 38, 39, 40, 41, 139, 140, 145, 146, 152, 163 }

C grade { }

F normal fail { 42, 43, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-1) timedout fail { 138, 143, 144, 150, 151, 156 }

F(-2) exception fail { 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 125, 126, 131, 132, 133, 137, 142 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 123, 130, 134, 135, 136, 155, 160 }

C grade { }

F normal fail { }

F(-1) timedout fail { 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-2) exception fail { }

Sympy

A grade { 1, 7, 8, 9, 10, 17, 18, 19, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 134 }

B grade { 2, 3, 4, 5, 6, 11, 12, 13, 15, 16, 20, 21, 22, 23, 24, 26, 27, 32, 33, 34, 135, 136, 140, 141, 147, 148, 149 }

C grade { }

F normal fail { 35, 36, 37, 40, 41, 43, 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 137, 138, 139, 142, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 168, 169, 174, 176, 177, 179 }

F(-1) timedout fail { 14, 25, 38, 39, 42, 89, 90, 143, 144, 159, 166, 167, 170, 171, 172, 173, 175, 178, 180 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 122, 123, 127, 128, 129, 130, 134, 136, 141, 146, 149, 155, 158, 159, 160 }

C grade { }

F normal fail { 25, 42, 43, 96, 97, 98, 103, 104, 105, 110, 111, 112, 117, 118, 119, 124, 125, 126, 131, 132, 133, 135, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 150, 151, 152, 153, 154, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	20	14	15	14	17	10	14	25	14
N.S.	1	1.43	1.00	1.07	1.00	1.21	0.71	1.00	1.79	1.00
time (sec)	N/A	0.169	0.012	3.210	0.111	0.058	0.058	0.104	0.203	0.044

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	125	37	53	61	230	108	64	48
N.S.	1	1.14	3.57	1.06	1.51	1.74	6.57	3.09	1.83	1.37
time (sec)	N/A	0.202	0.398	0.923	0.066	0.070	3.438	0.111	0.211	15.666

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	41	118	36	54	64	207	130	68	76
N.S.	1	1.17	3.37	1.03	1.54	1.83	5.91	3.71	1.94	2.17
time (sec)	N/A	0.205	0.381	0.871	0.066	0.072	3.379	0.116	0.203	15.368

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	67	209	55	61	93	374	269	70	123
N.S.	1	1.24	3.87	1.02	1.13	1.72	6.93	4.98	1.30	2.28
time (sec)	N/A	0.231	6.184	0.965	0.084	0.074	21.712	0.175	0.194	15.428

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	55	73	67	72	72	80	72	67	72
N.S.	1	2.50	3.32	3.05	3.27	3.27	3.64	3.27	3.05	3.27
time (sec)	N/A	0.263	0.007	0.812	0.035	0.057	0.022	0.105	0.200	15.178

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	50	47	43	48	48	54	48	43	48
N.S.	1	2.27	2.14	1.95	2.18	2.18	2.45	2.18	1.95	2.18
time (sec)	N/A	0.255	0.005	0.510	0.035	0.059	0.019	0.104	0.196	15.250

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	24	27	24	20	20
N.S.	1	1.00	1.00	0.88	1.00	1.00	1.12	1.00	0.83	0.83
time (sec)	N/A	0.166	0.000	0.086	0.026	0.059	0.017	0.106	0.188	15.383

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	24	19	18	18	19	18	18	18
N.S.	1	1.00	1.33	1.06	1.00	1.00	1.06	1.00	1.00	1.00
time (sec)	N/A	0.198	0.012	0.941	0.028	0.061	0.064	0.106	0.298	0.050

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	21	21	15	21	20	19
N.S.	1	1.00	1.00	1.05	1.11	1.11	0.79	1.11	1.05	1.00
time (sec)	N/A	0.175	0.019	0.865	0.032	0.061	0.184	0.114	0.199	15.181

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	32	32	31	21	33	35
N.S.	1	1.00	1.05	0.95	1.45	1.45	1.41	0.95	1.50	1.59
time (sec)	N/A	0.177	0.018	1.032	0.033	0.061	0.302	0.119	0.187	15.196

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	43	43	42	21	44	33
N.S.	1	1.00	1.05	0.95	1.95	1.95	1.91	0.95	2.00	1.50
time (sec)	N/A	0.177	0.019	0.862	0.029	0.063	0.418	0.114	0.190	15.469

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	54	54	54	21	55	33
N.S.	1	1.00	1.05	0.95	2.45	2.45	2.45	0.95	2.50	1.50
time (sec)	N/A	0.178	0.018	0.845	0.036	0.060	0.543	0.128	0.194	0.071

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	45	33	38	68	82	280	182	76	79
N.S.	1	1.32	0.97	1.12	2.00	2.41	8.24	5.35	2.24	2.32
time (sec)	N/A	0.235	0.580	1.466	0.068	0.093	56.590	0.149	0.196	15.979

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	171	118	133	356	300	0	1690	275	276
N.S.	1	1.44	0.99	1.12	2.99	2.52	0.00	14.20	2.31	2.32
time (sec)	N/A	0.362	0.841	1.743	0.141	0.077	0.000	0.531	0.179	16.371

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	116	75	120	125	125	143	134	119	115
N.S.	1	4.64	3.00	4.80	5.00	5.00	5.72	5.36	4.76	4.60
time (sec)	N/A	0.409	0.028	1.290	0.027	0.056	0.033	0.131	0.191	16.035

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	69	51	65	70	70	78	73	64	69
N.S.	1	2.76	2.04	2.60	2.80	2.80	3.12	2.92	2.56	2.76
time (sec)	N/A	0.289	0.018	0.494	0.034	0.056	0.030	0.106	0.202	0.044

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	22	22	24	22	20	24
N.S.	1	1.00	1.05	0.95	1.00	1.00	1.09	1.00	0.91	1.09
time (sec)	N/A	0.166	0.000	0.092	0.030	0.055	0.048	0.109	0.186	0.029

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	20	20	20	20	20
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.202	0.013	1.489	0.027	0.061	0.127	0.106	0.175	0.062

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	17	17	26	17
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.00	1.00	1.53	1.00
time (sec)	N/A	0.163	0.015	1.392	0.028	0.060	0.286	0.109	0.175	0.057

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	49	49	51	24	51	48
N.S.	1	1.00	0.96	0.96	2.13	2.13	2.22	1.04	2.22	2.09
time (sec)	N/A	0.190	0.023	1.477	0.037	0.063	0.701	0.140	0.172	0.101

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	50	22	22	86	86	92	24	93	87
N.S.	1	2.17	0.96	0.96	3.74	3.74	4.00	1.04	4.04	3.78
time (sec)	N/A	0.210	0.024	1.393	0.042	0.063	0.986	0.114	0.176	0.163

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	140	140	143	24	149	137
N.S.	1	1.00	1.00	0.96	5.60	5.60	5.72	0.96	5.96	5.48
time (sec)	N/A	0.194	0.026	1.464	0.050	0.066	1.452	0.138	0.173	16.951

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	59	45	57	106	126	507	321	136	115
N.S.	1	1.28	0.98	1.24	2.30	2.74	11.02	6.98	2.96	2.50
time (sec)	N/A	0.286	1.081	1.496	0.072	0.082	69.176	0.134	0.180	17.096

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	56	44	50	99	122	483	297	127	121
N.S.	1	1.27	1.00	1.14	2.25	2.77	10.98	6.75	2.89	2.75
time (sec)	N/A	0.287	1.027	1.458	0.069	0.075	61.659	0.129	0.169	17.317

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	212	146	174	566	518	0	3983	881	348
N.S.	1	1.43	0.99	1.18	3.82	3.50	0.00	26.91	5.95	2.35
time (sec)	N/A	0.404	1.028	1.776	0.215	0.087	0.000	1.405	0.261	17.488

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	205	110	222	237	237	245	259	255	209
N.S.	1	5.54	2.97	6.00	6.41	6.41	6.62	7.00	6.89	5.65
time (sec)	N/A	0.537	0.060	1.299	0.033	0.059	0.047	0.113	0.203	15.620

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	108	72	121	128	128	131	138	135	112
N.S.	1	2.92	1.95	3.27	3.46	3.46	3.54	3.73	3.65	3.03
time (sec)	N/A	0.371	0.039	0.460	0.026	0.059	0.030	0.121	0.957	15.379

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	45	41	43	42	41	42	44
N.S.	1	1.00	1.12	1.10	1.00	1.05	1.02	1.00	1.02	1.07
time (sec)	N/A	0.194	0.000	0.093	0.028	0.085	0.028	0.130	0.178	0.051

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	31	33	33	27	40	39	40
N.S.	1	1.00	0.94	0.94	1.00	1.00	0.82	1.21	1.18	1.21
time (sec)	N/A	0.225	0.026	1.476	0.035	0.060	0.256	0.115	0.199	0.091

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	55	26	27	29	29	24	29	36	27
N.S.	1	1.90	0.90	0.93	1.00	1.00	0.83	1.00	1.24	0.93
time (sec)	N/A	0.226	0.028	1.382	0.034	0.065	0.736	0.113	0.194	0.068

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	58	58	61	33	61	57
N.S.	1	1.00	0.91	0.91	1.66	1.66	1.74	0.94	1.74	1.63
time (sec)	N/A	0.224	0.044	1.389	0.036	0.067	3.683	0.108	0.183	15.263

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	33	96	96	102	33	103	96
N.S.	1	1.00	0.97	0.92	2.67	2.67	2.83	0.92	2.86	2.67
time (sec)	N/A	0.217	0.040	1.412	0.047	0.075	8.931	0.111	0.192	15.747

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	33	150	150	153	33	158	146
N.S.	1	1.00	0.97	0.92	4.17	4.17	4.25	0.92	4.39	4.06
time (sec)	N/A	0.223	0.045	1.427	0.055	0.072	17.971	0.158	0.194	0.312

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	103	87	79	0	205	214	81	173	0
N.S.	1	1.01	0.85	0.77	0.00	2.01	2.10	0.79	1.70	0.00
time (sec)	N/A	0.282	0.961	1.608	0.000	0.086	0.467	0.139	0.201	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	204	491	0	1079	0	851	316	0
N.S.	1	1.00	2.49	5.99	0.00	13.16	0.00	10.38	3.85	0.00
time (sec)	N/A	0.251	0.471	4.470	0.000	0.227	0.000	0.258	0.195	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	189	307	0	813	0	703	225	0
N.S.	1	1.00	2.86	4.65	0.00	12.32	0.00	10.65	3.41	0.00
time (sec)	N/A	0.230	0.454	2.582	0.000	0.119	0.000	0.217	0.215	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	1333	827	0	1544	0	1170	2661	0
N.S.	1	1.00	10.33	6.41	0.00	11.97	0.00	9.07	20.63	0.00
time (sec)	N/A	0.310	4.598	2.621	0.000	0.315	0.000	0.304	0.277	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	243	1746	1868	0	3818	0	2986	18107	0
N.S.	1	1.08	7.79	8.34	0.00	17.04	0.00	13.33	80.83	0.00
time (sec)	N/A	0.601	16.586	3.131	0.000	2.529	0.000	0.930	2.915	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	374	3382	3649	0	8134	0	30280	27811	0
N.S.	1	1.14	10.31	11.12	0.00	24.80	0.00	92.32	84.79	0.00
time (sec)	N/A	1.157	16.662	3.701	0.000	10.771	0.000	4.014	22.694	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	1420	1352	0	2005	0	1305	3406	0
N.S.	1	1.00	8.77	8.35	0.00	12.38	0.00	8.06	21.02	0.00
time (sec)	N/A	0.388	3.254	4.974	0.000	0.772	0.000	0.321	0.321	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	27	0	38	0	52	42	0
N.S.	1	1.00	1.39	0.96	0.00	1.36	0.00	1.86	1.50	0.00
time (sec)	N/A	0.167	0.241	2.638	0.000	0.066	0.000	0.116	0.190	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	136	142	0	0	0	0	0	803	0
N.S.	1	1.26	1.31	0.00	0.00	0.00	0.00	0.00	7.44	0.00
time (sec)	N/A	0.267	0.238	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	200	172	0	0	0	0	0	925	0
N.S.	1	1.52	1.30	0.00	0.00	0.00	0.00	0.00	7.01	0.00
time (sec)	N/A	0.329	0.678	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	54	54	54	65	54	53	54
N.S.	1	1.00	1.00	0.79	0.79	0.79	0.96	0.79	0.78	0.79
time (sec)	N/A	0.258	0.005	1.102	0.031	0.056	0.038	0.124	0.190	0.086

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	44	44	44	53	44	43	44
N.S.	1	1.00	1.00	0.79	0.79	0.79	0.95	0.79	0.77	0.79
time (sec)	N/A	0.236	0.002	1.133	0.026	0.056	0.031	0.137	0.194	0.035

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	34	34	34	41	34	33	34
N.S.	1	1.00	1.00	0.77	0.77	0.77	0.93	0.77	0.75	0.77
time (sec)	N/A	0.217	0.002	1.109	0.030	0.058	0.034	0.106	0.189	0.026

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	23	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.77	0.80
time (sec)	N/A	0.193	0.001	0.168	0.026	0.058	0.027	0.111	0.185	0.023

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	49	33	32	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.76	0.83
time (sec)	N/A	0.208	0.021	1.825	0.118	0.063	0.076	0.108	0.194	15.821

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	69	35
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	1.60	0.81
time (sec)	N/A	0.192	0.017	1.817	0.110	0.063	0.078	0.148	0.188	0.045

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	53	47	56	75	63	46	123	55
N.S.	1	1.08	0.83	0.73	0.88	1.17	0.98	0.72	1.92	0.86
time (sec)	N/A	0.217	0.029	1.827	0.109	0.062	0.097	0.121	0.190	15.806

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	64	64	64	76	64	63	64
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.95	0.80	0.79	0.80
time (sec)	N/A	0.272	0.005	1.687	0.030	0.060	0.068	0.131	0.189	0.106

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	54	54	54	63	54	53	54
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.95	0.82	0.80	0.82
time (sec)	N/A	0.259	0.003	1.651	0.036	0.056	0.038	0.128	0.197	0.077

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	44	44	44	51	44	43	44
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.94	0.81	0.80	0.81
time (sec)	N/A	0.233	0.003	1.676	0.033	0.056	0.028	0.118	0.193	0.035

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	34	34	34	41	34	33	34
N.S.	1	1.00	1.00	0.74	0.74	0.74	0.89	0.74	0.72	0.74
time (sec)	N/A	0.216	0.001	1.101	0.045	0.055	0.023	0.159	0.188	0.027

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	43	43	63	43	42	45
N.S.	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.75	0.80
time (sec)	N/A	0.222	0.024	2.360	0.108	0.064	0.078	0.112	0.191	15.236

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	66	59	50	52	78	65	52	117	51
N.S.	1	1.05	0.94	0.79	0.83	1.24	1.03	0.83	1.86	0.81
time (sec)	N/A	0.271	0.035	2.378	0.108	0.073	0.093	0.108	0.189	0.052

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	53	47	56	75	63	46	123	55
N.S.	1	1.08	0.83	0.73	0.88	1.17	0.98	0.72	1.92	0.86
time (sec)	N/A	0.250	0.027	2.068	0.112	0.064	0.099	0.115	0.196	15.105

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	63	57	76	105	83	56	179	75
N.S.	1	1.12	0.74	0.67	0.89	1.24	0.98	0.66	2.11	0.88
time (sec)	N/A	0.279	0.048	2.312	0.111	0.064	0.107	0.121	0.193	15.423

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	74	74	74	92	74	73	74
N.S.	1	1.00	1.00	0.77	0.77	0.77	0.96	0.77	0.76	0.77
time (sec)	N/A	0.301	0.004	1.708	0.031	0.058	0.049	0.137	0.191	0.143

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	64	64	64	78	64	63	64
N.S.	1	1.00	1.00	0.78	0.78	0.78	0.95	0.78	0.77	0.78
time (sec)	N/A	0.261	0.003	1.723	0.031	0.056	0.032	0.110	0.190	0.108

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	54	54	54	65	54	53	54
N.S.	1	1.00	1.00	0.79	0.79	0.79	0.96	0.79	0.78	0.79
time (sec)	N/A	0.264	0.003	1.707	0.029	0.060	0.032	0.119	0.189	0.077

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	44	44	44	53	44	43	44
N.S.	1	1.00	1.00	0.79	0.79	0.79	0.95	0.79	0.77	0.79
time (sec)	N/A	0.236	0.002	1.125	0.026	0.057	0.028	0.139	0.186	0.038

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	76	53	52	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.74	0.79
time (sec)	N/A	0.243	0.025	2.467	0.111	0.064	0.080	0.152	0.189	0.045

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	77	60	62	88	78	62	127	61
N.S.	1	1.01	1.00	0.78	0.81	1.14	1.01	0.81	1.65	0.79
time (sec)	N/A	0.297	0.031	2.415	0.114	0.069	0.090	0.106	0.197	15.336

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	78	63	72	118	85	62	203	71
N.S.	1	1.10	0.93	0.75	0.86	1.40	1.01	0.74	2.42	0.85
time (sec)	N/A	0.329	0.043	2.441	0.109	0.077	0.115	0.118	0.189	16.270

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	63	63	87	63	62	65
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.74	0.77
time (sec)	N/A	0.243	0.032	4.191	0.108	0.067	0.079	0.113	0.175	16.266

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	73	53	52	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.74	0.79
time (sec)	N/A	0.234	0.026	2.359	0.107	0.069	0.099	0.133	0.194	0.047

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	43	43	60	43	42	45
N.S.	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.75	0.80
time (sec)	N/A	0.220	0.024	2.339	0.106	0.067	0.069	0.110	0.192	0.043

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	46	33	32	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.76	0.83
time (sec)	N/A	0.199	0.013	1.830	0.113	0.073	0.085	0.126	0.198	0.041

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	73	60	59	59	83	59	57	79
N.S.	1	1.08	1.00	0.82	0.81	0.81	1.14	0.81	0.78	1.08
time (sec)	N/A	0.251	0.036	3.071	0.107	0.076	0.121	0.109	0.178	16.722

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	105	94	76	78	117	102	78	204	95
N.S.	1	1.12	1.00	0.81	0.83	1.24	1.09	0.83	2.17	1.01
time (sec)	N/A	0.342	0.085	2.905	0.111	0.067	0.154	0.110	0.198	16.640

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	131	104	89	98	177	119	88	358	115
N.S.	1	1.14	0.90	0.77	0.85	1.54	1.03	0.77	3.11	1.00
time (sec)	N/A	0.440	0.161	2.920	0.108	0.070	0.195	0.138	0.199	0.191

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	86	91	70	72	98	90	72	137	72
N.S.	1	0.95	1.00	0.77	0.79	1.08	0.99	0.79	1.51	0.79
time (sec)	N/A	0.300	0.059	2.246	0.109	0.060	0.095	0.127	0.182	15.776

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	77	60	62	88	75	62	127	61
N.S.	1	0.99	1.00	0.78	0.81	1.14	0.97	0.81	1.65	0.79
time (sec)	N/A	0.288	0.035	2.345	0.106	0.060	0.105	0.110	0.197	16.126

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	63	50	52	78	61	52	117	52
N.S.	1	1.02	1.00	0.79	0.83	1.24	0.97	0.83	1.86	0.83
time (sec)	N/A	0.263	0.038	2.365	0.109	0.063	0.084	0.118	0.196	0.050

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	69	36
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	1.60	0.84
time (sec)	N/A	0.191	0.017	1.781	0.107	0.062	0.081	0.103	0.193	15.558

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	105	94	76	78	117	102	78	204	96
N.S.	1	1.12	1.00	0.81	0.83	1.24	1.09	0.83	2.17	1.02
time (sec)	N/A	0.345	0.064	2.742	0.120	0.071	0.165	0.113	0.175	0.190

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	143	106	94	96	167	122	96	358	115
N.S.	1	1.13	0.83	0.74	0.76	1.31	0.96	0.76	2.82	0.91
time (sec)	N/A	0.443	0.061	3.079	0.108	0.070	0.179	0.111	0.196	0.200

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	169	136	106	118	237	143	110	514	135
N.S.	1	1.14	0.92	0.72	0.80	1.60	0.97	0.74	3.47	0.91
time (sec)	N/A	0.553	0.068	2.958	0.107	0.073	0.192	0.114	0.189	15.257

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	98	73	82	128	95	72	213	81
N.S.	1	1.08	1.00	0.74	0.84	1.31	0.97	0.73	2.17	0.83
time (sec)	N/A	0.379	0.040	2.343	0.109	0.063	0.123	0.109	0.184	15.271

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	94	84	63	72	118	82	62	203	72
N.S.	1	1.12	1.00	0.75	0.86	1.40	0.98	0.74	2.42	0.86
time (sec)	N/A	0.339	0.040	2.368	0.105	0.066	0.112	0.111	0.193	15.582

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	51	47	56	75	63	46	123	56
N.S.	1	1.08	0.80	0.73	0.88	1.17	0.98	0.72	1.92	0.88
time (sec)	N/A	0.253	0.033	2.399	0.109	0.064	0.095	0.118	0.191	0.055

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	51	47	56	75	61	46	123	55
N.S.	1	1.08	0.80	0.73	0.88	1.17	0.95	0.72	1.92	0.86
time (sec)	N/A	0.217	0.031	1.661	0.109	0.066	0.090	0.109	0.199	15.333

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	131	99	89	98	177	122	88	358	116
N.S.	1	1.14	0.86	0.77	0.85	1.54	1.06	0.77	3.11	1.01
time (sec)	N/A	0.434	0.166	3.237	0.114	0.070	0.193	0.113	0.169	15.939

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	181	136	106	116	227	143	110	514	136
N.S.	1	1.13	0.85	0.66	0.72	1.42	0.89	0.69	3.21	0.85
time (sec)	N/A	0.565	0.114	2.783	0.106	0.071	0.200	0.129	0.194	0.192

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	207	151	118	138	297	163	116	670	155
N.S.	1	1.14	0.83	0.65	0.76	1.64	0.90	0.64	3.70	0.86
time (sec)	N/A	0.670	0.087	2.715	0.110	0.070	0.218	0.109	0.195	15.558

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	120	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	1.74	0.87
time (sec)	N/A	0.212	0.059	0.873	0.107	0.071	0.377	0.109	0.184	0.111

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	120	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	1.74	0.87
time (sec)	N/A	0.230	0.016	0.853	0.118	0.068	0.386	0.137	0.175	15.711

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	164	158	0	553	0	172	419	2331
N.S.	1	1.00	1.03	0.99	0.00	3.48	0.00	1.08	2.64	14.66
time (sec)	N/A	0.416	0.160	1.937	0.000	1.815	0.000	0.110	0.172	20.080

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	164	152	158	0	553	0	172	419	2331
N.S.	1	1.03	0.96	0.99	0.00	3.48	0.00	1.08	2.64	14.66
time (sec)	N/A	0.380	0.195	2.129	0.000	1.791	0.000	0.127	0.170	19.342

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86	0.86
time (sec)	N/A	0.176	0.006	0.757	0.025	0.060	0.059	0.106	0.166	0.058

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	248	95	75	177	98	97	93	186	221
N.S.	1	1.19	0.46	0.36	0.85	0.47	0.47	0.45	0.89	1.06
time (sec)	N/A	0.757	1.226	2.418	0.118	0.070	0.467	0.118	0.236	18.125

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	196	85	65	143	88	83	83	154	187
N.S.	1	1.18	0.51	0.39	0.86	0.53	0.50	0.50	0.93	1.13
time (sec)	N/A	0.554	0.993	2.409	0.118	0.069	0.409	0.120	0.200	17.121

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	144	75	55	109	78	70	73	122	153
N.S.	1	1.16	0.60	0.44	0.88	0.63	0.56	0.59	0.98	1.23
time (sec)	N/A	0.379	0.654	2.381	0.117	0.069	0.387	0.143	0.175	16.178

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	92	65	45	75	68	56	63	90	119
N.S.	1	1.12	0.79	0.55	0.91	0.83	0.68	0.77	1.10	1.45
time (sec)	N/A	0.241	0.372	1.958	0.108	0.073	0.351	0.116	0.198	16.328

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	174	180	206	483	0	492	0	0	99	0
N.S.	1	1.03	1.18	2.78	0.00	2.83	0.00	0.00	0.57	0.00
time (sec)	N/A	0.520	0.329	4.688	0.000	0.091	0.000	0.000	0.411	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	194	419	473	0	526	0	0	346	0
N.S.	1	1.03	2.23	2.52	0.00	2.80	0.00	0.00	1.84	0.00
time (sec)	N/A	0.528	0.781	5.298	0.000	0.087	0.000	0.000	0.216	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	234	392	483	0	586	0	0	698	0
N.S.	1	1.05	1.76	2.17	0.00	2.63	0.00	0.00	3.13	0.00
time (sec)	N/A	0.640	0.975	5.586	0.000	0.099	0.000	0.000	0.248	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	276	105	85	206	108	110	103	218	0
N.S.	1	1.19	0.45	0.37	0.89	0.47	0.48	0.45	0.94	0.00
time (sec)	N/A	0.842	1.568	2.427	0.119	0.073	0.596	0.124	0.314	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	224	95	75	172	98	95	93	186	0
N.S.	1	1.19	0.50	0.40	0.91	0.52	0.50	0.49	0.98	0.00
time (sec)	N/A	0.586	1.273	2.431	0.121	0.071	0.601	0.125	0.270	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	172	85	65	138	88	83	83	154	0
N.S.	1	1.17	0.58	0.44	0.94	0.60	0.56	0.56	1.05	0.00
time (sec)	N/A	0.408	0.913	3.146	0.114	0.081	0.456	0.119	0.233	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	75	55	104	78	70	73	122	0
N.S.	1	1.14	0.71	0.52	0.99	0.74	0.67	0.70	1.16	0.00
time (sec)	N/A	0.274	0.600	1.876	0.108	0.068	0.400	0.113	0.195	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	197	208	228	501	0	542	0	0	229	0
N.S.	1	1.06	1.16	2.54	0.00	2.75	0.00	0.00	1.16	0.00
time (sec)	N/A	0.653	0.549	5.009	0.000	0.096	0.000	0.000	0.509	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	248	416	604	0	574	0	0	425	0
N.S.	1	1.01	1.69	2.46	0.00	2.33	0.00	0.00	1.73	0.00
time (sec)	N/A	0.788	0.900	5.679	0.000	0.101	0.000	0.000	0.238	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	234	572	483	0	618	0	0	698	0
N.S.	1	1.05	2.57	2.17	0.00	2.77	0.00	0.00	3.13	0.00
time (sec)	N/A	0.594	1.236	6.499	0.000	0.092	0.000	0.000	0.279	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	304	115	95	235	118	124	113	250	0
N.S.	1	1.20	0.45	0.37	0.93	0.46	0.49	0.44	0.98	0.00
time (sec)	N/A	0.881	2.253	2.365	0.117	0.076	0.726	0.136	0.425	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	252	105	85	201	108	110	103	218	0
N.S.	1	1.19	0.50	0.40	0.95	0.51	0.52	0.49	1.03	0.00
time (sec)	N/A	0.624	1.617	2.490	0.112	0.081	0.652	0.131	0.324	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	200	95	75	167	98	95	93	186	0
N.S.	1	1.18	0.56	0.44	0.98	0.58	0.56	0.55	1.09	0.00
time (sec)	N/A	0.448	1.138	2.438	0.110	0.070	0.604	0.123	0.256	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	148	85	65	133	88	83	83	154	0
N.S.	1	1.16	0.66	0.51	1.04	0.69	0.65	0.65	1.20	0.00
time (sec)	N/A	0.300	1.052	1.912	0.107	0.068	0.481	0.126	0.217	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	236	240	238	508	0	565	0	0	261	0
N.S.	1	1.02	1.01	2.15	0.00	2.39	0.00	0.00	1.11	0.00
time (sec)	N/A	0.765	0.849	4.756	0.000	0.122	0.000	0.000	0.560	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	288	276	433	613	0	616	0	0	457	0
N.S.	1	0.96	1.50	2.13	0.00	2.14	0.00	0.00	1.59	0.00
time (sec)	N/A	0.884	1.069	5.754	0.000	0.106	0.000	0.000	0.293	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	288	616	614	0	644	0	0	833	0
N.S.	1	0.98	2.09	2.08	0.00	2.18	0.00	0.00	2.82	0.00
time (sec)	N/A	0.931	1.359	7.458	0.000	0.152	0.000	0.000	0.331	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	220	85	65	148	88	83	83	154	0
N.S.	1	1.19	0.46	0.35	0.80	0.48	0.45	0.45	0.83	0.00
time (sec)	N/A	0.760	0.920	2.394	0.114	0.069	0.479	0.117	0.221	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	168	75	55	114	78	70	73	122	0
N.S.	1	1.17	0.52	0.38	0.80	0.55	0.49	0.51	0.85	0.00
time (sec)	N/A	0.514	0.670	2.474	0.115	0.068	0.421	0.154	0.204	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	65	45	80	68	56	63	90	0
N.S.	1	1.15	0.64	0.45	0.79	0.67	0.55	0.62	0.89	0.00
time (sec)	N/A	0.348	0.479	2.448	0.106	0.068	0.387	0.120	0.200	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	55	35	46	58	42	53	58	0
N.S.	1	1.08	0.93	0.59	0.78	0.98	0.71	0.90	0.98	0.00
time (sec)	N/A	0.218	0.274	1.892	0.108	0.078	0.384	0.114	0.201	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	154	135	443	0	454	0	0	34	0
N.S.	1	1.04	0.91	2.99	0.00	3.07	0.00	0.00	0.23	0.00
time (sec)	N/A	0.421	0.396	4.323	0.000	0.081	0.000	0.000	0.298	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	194	230	473	0	526	0	0	46	0
N.S.	1	1.03	1.22	2.52	0.00	2.80	0.00	0.00	0.24	0.00
time (sec)	N/A	0.533	0.571	4.724	0.000	0.086	0.000	0.000	0.215	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	234	396	483	0	618	0	0	56	0
N.S.	1	1.05	1.78	2.17	0.00	2.77	0.00	0.00	0.25	0.00
time (sec)	N/A	0.635	0.941	5.017	0.000	0.092	0.000	0.000	0.230	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	192	85	65	148	112	0	82	247	0
N.S.	1	1.16	0.51	0.39	0.89	0.67	0.00	0.49	1.49	0.00
time (sec)	N/A	0.617	1.435	2.313	0.125	0.074	0.000	0.121	0.209	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	137	75	55	114	102	0	72	215	0
N.S.	1	1.10	0.60	0.44	0.92	0.82	0.00	0.58	1.73	0.00
time (sec)	N/A	0.433	0.906	2.667	0.115	0.072	0.000	0.117	0.206	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	65	45	80	92	0	62	183	0
N.S.	1	1.10	0.79	0.55	0.98	1.12	0.00	0.76	2.23	0.00
time (sec)	N/A	0.286	0.811	2.620	0.110	0.073	0.000	0.138	0.208	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	55	35	46	82	0	53	151	87
N.S.	1	1.00	1.22	0.78	1.02	1.82	0.00	1.18	3.36	1.93
time (sec)	N/A	0.195	0.477	1.983	0.116	0.110	0.000	0.120	0.199	0.257

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	176	182	199	461	0	527	0	0	44	0
N.S.	1	1.03	1.13	2.62	0.00	2.99	0.00	0.00	0.25	0.00
time (sec)	N/A	0.516	0.524	5.027	0.000	0.086	0.000	0.000	0.284	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	222	414	490	0	577	0	0	56	0
N.S.	1	1.05	1.96	2.32	0.00	2.73	0.00	0.00	0.27	0.00
time (sec)	N/A	0.628	0.980	5.183	0.000	0.088	0.000	0.000	0.225	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	262	607	493	0	678	0	0	66	0
N.S.	1	1.07	2.47	2.00	0.00	2.76	0.00	0.00	0.27	0.00
time (sec)	N/A	0.755	1.680	5.533	0.000	0.102	0.000	0.000	0.231	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	163	85	65	253	132	0	81	337	0
N.S.	1	1.11	0.58	0.44	1.72	0.90	0.00	0.55	2.29	0.00
time (sec)	N/A	0.527	1.667	2.305	0.125	0.074	0.000	0.127	0.227	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	116	75	55	219	122	0	72	305	0
N.S.	1	1.10	0.71	0.52	2.09	1.16	0.00	0.69	2.90	0.00
time (sec)	N/A	0.371	1.093	2.480	0.116	0.088	0.000	0.115	0.214	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	65	45	185	112	0	61	273	0
N.S.	1	1.04	0.96	0.66	2.72	1.65	0.00	0.90	4.01	0.00
time (sec)	N/A	0.264	0.902	2.502	0.113	0.072	0.000	0.121	0.202	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	29	114	29
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	2.43	0.62
time (sec)	N/A	0.188	0.531	1.803	0.028	0.072	0.000	0.135	0.185	0.101

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	212	209	471	0	605	0	0	56	0
N.S.	1	1.07	1.05	2.37	0.00	3.04	0.00	0.00	0.28	0.00
time (sec)	N/A	0.620	0.877	4.582	0.000	0.087	0.000	0.000	0.314	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	234	250	416	493	0	668	0	0	66	0
N.S.	1	1.07	1.78	2.11	0.00	2.85	0.00	0.00	0.28	0.00
time (sec)	N/A	0.751	1.623	4.968	0.000	0.098	0.000	0.000	0.231	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	290	605	511	0	738	0	0	76	0
N.S.	1	1.08	2.25	1.90	0.00	2.74	0.00	0.00	0.28	0.00
time (sec)	N/A	0.875	1.879	5.096	0.000	0.102	0.000	0.000	0.230	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	196	90	65	143	88	82	83	154	187
N.S.	1	1.18	0.54	0.39	0.86	0.53	0.49	0.50	0.93	1.13
time (sec)	N/A	0.572	1.024	2.583	0.113	0.071	0.420	0.132	0.231	16.537

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	383	459	645	0	1269	1544	627	27	1299
N.S.	1	0.88	1.05	1.48	0.00	2.91	3.54	1.44	0.06	2.98
time (sec)	N/A	0.967	8.938	2.418	0.000	0.242	0.819	0.133	200.031	17.449

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	174	197	0	465	384	204	557	320
N.S.	1	0.95	0.99	1.13	0.00	2.66	2.19	1.17	3.18	1.83
time (sec)	N/A	0.353	1.805	1.513	0.000	0.093	0.562	0.123	0.281	15.905

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	491	539	1547	0	0	0	0	27	0
N.S.	1	1.14	1.25	3.59	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.900	0.819	2.731	0.000	0.000	0.000	0.000	200.027	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	488	503	1691	4691	0	0	0	0	27	0
N.S.	1	1.03	3.47	9.61	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.687	3.319	2.876	0.000	0.000	0.000	0.000	200.033	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	247	431	231	0	380	0	378	25	0
N.S.	1	1.13	1.97	1.05	0.00	1.74	0.00	1.73	0.11	0.00
time (sec)	N/A	0.561	1.194	4.497	0.000	0.097	0.000	0.241	200.030	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	428	829	1200	0	2179	5942	1132	27	0
N.S.	1	0.76	1.47	2.13	0.00	3.86	10.54	2.01	0.05	0.00
time (sec)	N/A	1.060	12.005	2.426	0.000	0.615	0.885	0.349	200.025	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	210	293	398	0	839	1360	403	1037	0
N.S.	1	0.89	1.24	1.69	0.00	3.56	5.76	1.71	4.39	0.00
time (sec)	N/A	0.395	4.423	1.660	0.000	0.123	0.637	0.196	1.086	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	678	684	1472	1326	0	0	0	0	27	0
N.S.	1	1.01	2.17	1.96	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.246	3.117	2.963	0.000	0.000	0.000	0.000	200.030	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	730	716	2854	7856	0	0	0	0	27	0
N.S.	1	0.98	3.91	10.76	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.700	5.916	3.028	0.000	0.000	0.000	0.000	200.025	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	669	693	4727	16309	0	0	0	0	27	0
N.S.	1	1.04	7.07	24.38	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.128	18.304	3.820	0.000	0.000	0.000	0.000	200.030	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	285	582	241	0	434	0	480	25	0
N.S.	1	1.07	2.19	0.91	0.00	1.63	0.00	1.80	0.09	0.00
time (sec)	N/A	0.639	1.792	4.444	0.000	0.118	0.000	0.185	200.027	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	75	325	0	161	0	171	95	0
N.S.	1	1.01	0.77	3.32	0.00	1.64	0.00	1.74	0.97	0.00
time (sec)	N/A	0.477	0.252	3.240	0.000	0.137	0.000	0.123	0.383	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	754	618	837	0	1583	1787	818	27	0
N.S.	1	1.05	0.86	1.17	0.00	2.21	2.49	1.14	0.04	0.00
time (sec)	N/A	2.439	7.778	2.767	0.000	0.567	0.985	0.208	200.017	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	337	254	286	0	637	641	296	27	0
N.S.	1	1.07	0.80	0.91	0.00	2.02	2.03	0.94	0.09	0.00
time (sec)	N/A	0.873	2.138	2.415	0.000	0.192	0.748	0.147	200.025	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	98	86	0	227	226	93	231	0
N.S.	1	1.04	0.84	0.74	0.00	1.96	1.95	0.80	1.99	0.00
time (sec)	N/A	0.295	0.680	1.547	0.000	0.098	0.359	0.152	0.155	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	218	761	0	11287	0	0	27	0
N.S.	1	1.00	0.58	2.03	0.00	30.18	0.00	0.00	0.07	0.00
time (sec)	N/A	0.615	0.478	2.918	0.000	3.743	0.000	0.000	200.024	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	764	1377	2108	0	0	0	0	27	0
N.S.	1	0.97	1.75	2.68	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.452	17.219	3.017	0.000	0.000	0.000	0.000	200.026	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	247	431	231	0	376	0	378	25	0
N.S.	1	1.06	1.85	0.99	0.00	1.61	0.00	1.62	0.11	0.00
time (sec)	N/A	0.522	1.080	4.457	0.000	0.116	0.000	0.173	200.023	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	709	771	1136	0	3143	0	1093	27	0
N.S.	1	1.09	1.19	1.75	0.00	4.84	0.00	1.68	0.04	0.00
time (sec)	N/A	2.950	9.510	3.072	0.000	0.968	0.000	0.171	200.018	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	339	291	435	0	1305	0	399	27	0
N.S.	1	1.10	0.94	1.41	0.00	4.22	0.00	1.29	0.09	0.00
time (sec)	N/A	0.800	2.549	2.539	0.000	0.566	0.000	0.158	200.022	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	111	106	201	0	429	0	118	574	143
N.S.	1	1.03	0.98	1.86	0.00	3.97	0.00	1.09	5.31	1.32
time (sec)	N/A	0.277	1.011	1.671	0.000	0.281	0.000	0.128	0.161	16.132

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	0	806	1906	0	0	0	0	154	0
N.S.	1	0.00	1.21	2.86	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	1.861	2.982	0.000	0.000	0.000	0.000	164.289	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	237	416	231	0	366	0	295	25	0
N.S.	1	1.06	1.87	1.04	0.00	1.64	0.00	1.32	0.11	0.00
time (sec)	N/A	0.504	1.044	4.823	0.000	0.109	0.000	0.169	200.025	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	80	55	219	122	0	72	305	0
N.S.	1	1.04	0.76	0.52	2.09	1.16	0.00	0.69	2.90	0.00
time (sec)	N/A	0.377	1.175	2.641	0.110	0.080	0.000	0.159	0.167	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	466	392	1129	0	1581	0	579	2663	0
N.S.	1	1.06	0.89	2.57	0.00	3.59	0.00	1.32	6.05	0.00
time (sec)	N/A	0.870	4.030	2.538	0.000	1.785	0.000	0.152	0.334	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	131	147	176	0	286	0	234	793	175
N.S.	1	0.99	1.11	1.33	0.00	2.17	0.00	1.77	6.01	1.33
time (sec)	N/A	0.277	1.485	1.608	0.000	1.285	0.000	0.149	0.203	15.417

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	227	219	471	1099	380	0	122	25	0
N.S.	1	1.10	1.06	2.29	5.33	1.84	0.00	0.59	0.12	0.00
time (sec)	N/A	0.480	0.969	4.488	0.150	0.143	0.000	0.142	200.026	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	265	426	241	0	436	0	304	25	0
N.S.	1	1.08	1.73	0.98	0.00	1.77	0.00	1.24	0.10	0.00
time (sec)	N/A	0.586	1.315	4.692	0.000	0.109	0.000	0.206	200.023	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	144	0	154	0	205	48	0
N.S.	1	1.00	1.04	2.82	0.00	3.02	0.00	4.02	0.94	0.00
time (sec)	N/A	0.221	0.248	4.398	0.000	0.091	0.000	0.143	0.146	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	700	1432	670	905	0	0	0	0	27	0
N.S.	1	2.05	0.96	1.29	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.766	5.933	8.181	0.000	0.000	0.000	0.000	200.027	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	650	390	395	0	0	0	0	27	0
N.S.	1	1.00	0.60	0.61	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.606	2.624	8.135	0.000	0.000	0.000	0.000	200.025	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1520	1838	1884	0	0	0	0	0	0	0
N.S.	1	1.21	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	10.175	4.544	0.000	0.000	0.000	0.000	0.000	0.523	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	578	668	742	0	0	0	0	0	0	0
N.S.	1	1.16	1.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.408	2.970	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	236	409	0	0	0	0	0	0	0
N.S.	1	1.18	2.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	1.399	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	122	126	0	0	0	0	0	243	0
N.S.	1	1.44	1.48	0.00	0.00	0.00	0.00	0.00	2.86	0.00
time (sec)	N/A	0.225	0.113	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	527	0	0	0	0	0	0	0	27	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1127	0	0	0	0	0	0	0	54	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	525	0	0	0	0	0	0	0	27	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	0	0	0	0	0	0	0	24	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	311	0	0	0	0	0	26	0
N.S.	1	0.00	0.92	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	1.532	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	0	0	0	0	0	0	0	26	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0	23	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	0	84	0	0	0	0	0	25	0
N.S.	1	0.00	1.58	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.000	0.294	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	0	0	0	0	0	0	0	24	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	170	0	0	0	0	0	23	0
N.S.	1	0.00	0.96	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.705	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	21	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.182	0.311	0.000	0.000	0.000	0.000	0.000	0.148	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [106] had the largest ratio of [.7407409999999999983]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.43	21	0.190
2	A	3	3	1.14	29	0.103
3	A	3	3	1.17	34	0.088
4	A	3	3	1.24	41	0.073
5	B	3	3	2.50	31	0.097
6	B	2	2	2.27	29	0.069
7	A	1	1	1.00	21	0.048
8	A	2	2	1.00	31	0.065
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	31	0.065
11	A	2	2	1.00	31	0.065
12	A	2	2	1.00	31	0.065
13	A	3	3	1.32	45	0.067
14	A	4	4	1.44	52	0.077
15	B	2	2	4.64	35	0.057
16	B	2	2	2.76	33	0.061
17	A	1	1	1.00	22	0.045
18	A	2	2	1.00	33	0.061
19	A	1	1	1.00	28	0.036
20	A	2	2	1.00	35	0.057
21	B	2	2	2.17	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	35	0.057
23	A	4	4	1.28	69	0.058
24	A	3	3	1.27	67	0.045
25	A	4	4	1.43	65	0.062
26	B	2	2	5.54	54	0.037
27	B	2	2	2.92	52	0.038
28	A	1	1	1.00	41	0.024
29	A	2	2	1.00	50	0.040
30	A	2	2	1.90	39	0.051
31	A	2	2	1.00	54	0.037
32	A	2	2	1.00	54	0.037
33	A	2	2	1.00	54	0.037
34	A	6	5	1.01	29	0.172
35	A	3	2	1.00	31	0.065
36	A	3	2	1.00	27	0.074
37	A	5	4	1.00	27	0.148
38	A	7	6	1.08	27	0.222
39	A	9	8	1.14	27	0.296
40	A	5	4	1.00	31	0.129
41	A	3	2	1.00	23	0.087
42	A	2	2	1.26	31	0.065
43	A	2	2	1.52	34	0.059
44	A	2	2	1.00	23	0.087
45	A	2	2	1.00	23	0.087
46	A	2	2	1.00	23	0.087
47	A	2	2	1.00	21	0.095
48	A	2	2	1.00	23	0.087
49	A	5	4	1.00	23	0.174
50	A	6	5	1.08	23	0.217
51	A	2	2	1.00	25	0.080
52	A	2	2	1.00	25	0.080
53	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	23	0.087
55	A	2	2	1.00	25	0.080
56	A	4	4	1.05	25	0.160
57	A	7	6	1.08	25	0.240
58	A	8	7	1.12	25	0.280
59	A	2	2	1.00	25	0.080
60	A	2	2	1.00	25	0.080
61	A	2	2	1.00	25	0.080
62	A	2	2	1.00	23	0.087
63	A	2	2	1.00	25	0.080
64	A	4	4	1.01	25	0.160
65	A	6	6	1.10	25	0.240
66	A	2	2	1.00	25	0.080
67	A	2	2	1.00	25	0.080
68	A	2	2	1.00	25	0.080
69	A	2	2	1.00	23	0.087
70	A	8	7	1.08	25	0.280
71	A	10	9	1.12	25	0.360
72	A	12	11	1.14	25	0.440
73	A	4	4	0.95	25	0.160
74	A	4	4	0.99	25	0.160
75	A	4	4	1.02	25	0.160
76	A	5	4	1.00	23	0.174
77	A	10	9	1.12	25	0.360
78	A	12	11	1.13	25	0.440
79	A	14	13	1.14	25	0.520
80	A	6	6	1.08	25	0.240
81	A	6	6	1.12	25	0.240
82	A	7	6	1.08	25	0.240
83	A	6	5	1.08	23	0.217
84	A	12	11	1.14	25	0.440
85	A	14	13	1.13	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	16	15	1.14	25	0.600
87	A	4	4	1.00	20	0.200
88	A	5	5	1.00	19	0.263
89	A	2	2	1.00	25	0.080
90	A	2	2	1.03	25	0.080
91	A	3	3	1.00	15	0.200
92	A	19	18	1.19	27	0.667
93	A	15	14	1.18	27	0.519
94	A	11	10	1.16	27	0.370
95	A	7	6	1.12	25	0.240
96	A	10	9	1.03	27	0.333
97	A	8	7	1.03	27	0.259
98	A	10	9	1.05	27	0.333
99	A	20	19	1.19	27	0.704
100	A	16	15	1.19	27	0.556
101	A	12	11	1.17	27	0.407
102	A	8	7	1.14	25	0.280
103	A	12	11	1.06	27	0.407
104	A	14	13	1.01	27	0.481
105	A	10	9	1.05	27	0.333
106	A	21	20	1.20	27	0.741
107	A	17	16	1.19	27	0.593
108	A	13	12	1.18	27	0.444
109	A	9	8	1.16	25	0.320
110	A	14	13	1.02	27	0.481
111	A	16	15	0.96	27	0.556
112	A	16	15	0.98	27	0.556
113	A	18	17	1.19	27	0.630
114	A	14	13	1.17	27	0.481
115	A	10	9	1.15	27	0.333
116	A	6	5	1.08	25	0.200
117	A	6	5	1.04	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	7	1.03	27	0.259
119	A	10	9	1.05	27	0.333
120	A	15	14	1.16	27	0.519
121	A	12	11	1.10	27	0.407
122	A	8	7	1.10	27	0.259
123	A	5	4	1.00	25	0.160
124	A	8	7	1.03	27	0.259
125	A	10	9	1.05	27	0.333
126	A	12	11	1.07	27	0.407
127	A	14	13	1.11	27	0.481
128	A	10	9	1.10	27	0.333
129	A	7	6	1.04	27	0.222
130	A	3	3	1.00	25	0.120
131	A	10	9	1.07	27	0.333
132	A	12	11	1.07	27	0.407
133	A	14	13	1.08	27	0.481
134	A	15	14	1.18	25	0.560
135	A	11	10	0.88	27	0.370
136	A	7	6	0.95	25	0.240
137	A	7	6	1.14	27	0.222
138	A	6	5	1.03	27	0.185
139	A	8	7	1.13	25	0.280
140	A	12	11	0.76	27	0.407
141	A	8	7	0.89	25	0.280
142	A	10	9	1.01	27	0.333
143	A	12	11	0.98	27	0.407
144	A	8	7	1.04	27	0.259
145	A	10	9	1.07	25	0.360
146	A	15	14	1.01	27	0.519
147	A	14	13	1.05	27	0.481
148	A	10	9	1.07	27	0.333
149	A	6	5	1.04	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	1.00	27	0.111
151	A	6	5	0.97	27	0.185
152	A	8	7	1.06	25	0.280
153	A	12	11	1.09	27	0.407
154	A	8	7	1.10	27	0.259
155	A	5	4	1.03	25	0.160
156	F	0	0	N/A	0.000	N/A
157	A	8	7	1.06	25	0.280
158	A	10	9	1.04	25	0.360
159	A	7	6	1.06	27	0.222
160	A	3	3	0.99	25	0.120
161	A	8	7	1.10	25	0.280
162	A	10	9	1.08	25	0.360
163	A	6	5	1.00	27	0.185
164	B	4	3	2.05	29	0.103
165	A	4	3	1.00	29	0.103
166	A	8	8	1.21	25	0.320
167	A	6	6	1.16	25	0.240
168	A	4	4	1.18	23	0.174
169	A	1	1	1.44	12	0.083
170	F	0	0	N/A	0.000	N/A
171	F	0	0	N/A	0.000	N/A
172	F	0	0	N/A	0.000	N/A
173	F	0	0	N/A	0.000	N/A
174	F	0	0	N/A	0.000	N/A
175	F	0	0	N/A	0.000	N/A
176	F	0	0	N/A	0.000	N/A
177	F	0	0	N/A	0.000	N/A
178	F	0	0	N/A	0.000	N/A
179	F	0	0	N/A	0.000	N/A
180	A	2	2	1.00	19	0.105

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \frac{1}{2-x} + \arctan(2-x)$$

output
```
1/(2-x)-arctan(-2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{-2+x} + \arctan(2-x)$$

input
```
Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
-(-2 + x)^(-1) + ArcTan[2 - x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1294, 1117, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-4x+4)(x^2-4x+5)}dx$$

$$\Big\downarrow \text{ 1294}$$

$$\int \frac{1}{(2-x)^2(x^2-4x+5)}dx$$

$$\Big\downarrow \text{ 1117}$$

$$\frac{1}{2-x}-\int \frac{1}{x^2-4x+5}dx$$

$$\Big\downarrow \text{ 1083}$$

$$2\int \frac{1}{-(2x-4)^2-4}d(2x-4)+\frac{1}{2-x}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2-x}-\arctan\left(\frac{1}{2}(2x-4)\right)$$

input
```
Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
(2 - x)^(-1) - ArcTan[(-4 + 2*x)/2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1117
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Simp[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[m, -1] &
& (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || IntegerQ[(m + 2*p + 3)
/2])
```

rule 1294
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(b/2 + c*x)^(2*p)*(d + e*x + f*x
^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && EqQ[b^2 - 4*a*c, 0] &&
IntegerQ[p]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\arctan\left(x-2\right)-\frac{1}{x-2}$	15
risch	$-\arctan\left(x-2\right)-\frac{1}{x-2}$	15
parallelrisch	$\frac{i\ln(x-2-i)x-i\ln(x-2+i)x-2i\ln(x-2-i)+2i\ln(x-2+i)-x}{2x-4}$	50

input
```
int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x-2)-1/(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{(x-2)\arctan(x-2)+1}{x-2}$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

output
```
-((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\operatorname{atan}(x-2) - \frac{1}{x-2}$$

input
```
integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

output
```
-atan(x - 2) - 1/(x - 2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\mathrm{atan}(x-2) - \frac{1}{x-2}$$

input
```
int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

output
```
- atan(x - 2) - 1/(x - 2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \frac{-2atan(x-2)\,x+4atan(x-2)-x}{2x-4}$$

input
```
int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
```

output
```
( - 2*atan(x - 2)*x + 4*atan(x - 2) - x)/(2*(x - 2))
```

3.2 $\int (a+cx^2)^p (ad+aex+cd(3+2p)x^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 35

$$\int (a+cx^2)^p (ad+aex+cd(3+2p)x^2)\,dx=\frac{(ae+2cd(1+p)x)(a+cx^2)^{1+p}}{2c(1+p)}$$

output
```
1/2*(a*e+2*c*d*(p+1)*x)*(c*x^2+a)^(p+1)/c/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.57

$$\int (a+cx^2)^p (ad+aex+cd(3+2p)x^2)\,dx$$
$$=\frac{(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(3ae(a+cx^2)\left(1+\frac{cx^2}{a}\right)^p+6acd(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{cx^2}{a}\right)\right.}{6c(1+p)}$$

input
```
Integrate[(a + c*x^2)^p*(a*d + a*e*x + c*d*(3 + 2*p)*x^2),x]
```

output
```
((a + c*x^2)^p*(3*a*e*(a + c*x^2)*(1 + (c*x^2)/a)^p + 6*a*c*d*(1 + p)*x*Hy
pergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)] + 2*c^2*d*(3 + 5*p + 2*p^2)*x^
3*Hypergeometric2F1[3/2, -p, 5/2, -((c*x^2)/a)]))/(6*c*(1 + p)*(1 + (c*x^2
)/a)^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2346, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(2p+3)x^2\right) \, dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int ace(2p+3)x\left(cx^2+a\right)^p dx}{c(2p+3)}+dx\left(a+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{ 27}$$

$$ae\int x\left(cx^2+a\right)^p dx+dx\left(a+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{ 241}$$

$$dx\left(a+cx^2\right)^{p+1}+\frac{ae\left(a+cx^2\right)^{p+1}}{2c(p+1)}$$

input
```
Int[(a + c*x^2)^p*(a*d + a*e*x + c*d*(3 + 2*p)*x^2),x]
```

output
```
(a*e*(a + c*x^2)^(1 + p))/(2*c*(1 + p)) + d*x*(a + c*x^2)^(1 + p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(c\,x^2+a)^{p+1}(2cdxp+2cdx+ae)}{2c(p+1)}$	37
risch	$\frac{(2c^2dp\,x^3+2c^2d\,x^3+2acdpx+ace\,x^2+2adxc+a^2e)(c\,x^2+a)^p}{2c(p+1)}$	65
norman	$adx\,e^{p\ln(c\,x^2+a)}+cd\,x^3e^{p\ln(c\,x^2+a)}+\frac{ae\,x^2e^{p\ln(c\,x^2+a)}}{2+2p}+\frac{a^2e\,e^{p\ln(c\,x^2+a)}}{2c(p+1)}$	82
orering	$\frac{(2cdxp+2cdx+ae)(c\,x^2+a)(c\,x^2+a)^p(ad+aex+cd(3+2p)x^2)}{2c(p+1)(2cd\,x^2p+3cd\,x^2+aex+ad)}$	86
parallelrisch	$\frac{2x^3(c\,x^2+a)^pc^2dp+2x^3(c\,x^2+a)^pc^2d+x^2(c\,x^2+a)^pace+2x(c\,x^2+a)^pacdp+2x(c\,x^2+a)^pacd+(c\,x^2+a)^pa^2e}{2c(p+1)}$	110

input
```
int((c*x^2+a)^p*(a*d+a*e*x+c*d*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/(p+1)*(c*x^2+a)^(p+1)*(2*c*d*p*x+2*c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right)\,dx = \frac{(acex^2+2\,(c^2dp+c^2d)x^3+a^2e+2\,(acdp+acd)x)(cx^2+a)^p}{2\,(cp+c)}$$

input
```
integrate((c*x^2+a)^p*(a*d+a*e*x+c*d*(3+2*p)*x^2),x, algorithm="fricas")
```

output
```
1/2*(a*c*e*x^2 + 2*(c^2*d*p + c^2*d)*x^3 + a^2*e + 2*(a*c*d*p + a*c*d)*x)*
(c*x^2 + a)^p/(c*p + c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(27) = 54.

Time = 3.44 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.57

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right)\,dx = \begin{cases} \frac{adx+\frac{aex^2}{2}}{a} & \text{for } c=0 \wedge \\ a^p\left(adx+\frac{aex^2}{2}\right) & \text{for } c=0 \\ \frac{ae\log\left(x-\sqrt{-\frac{a}{c}}\right)}{2c}+\frac{ae\log\left(x+\sqrt{-\frac{a}{c}}\right)}{2c}+dx & \text{for } p=-1 \\ \frac{a^2e(a+cx^2)^p}{2cp+2c}+\frac{2acdpx(a+cx^2)^p}{2cp+2c}+\frac{2acdx(a+cx^2)^p}{2cp+2c}+\frac{acex^2(a+cx^2)^p}{2cp+2c}+\frac{2c^2dpx^3(a+cx^2)^p}{2cp+2c}+\frac{2c^2dx^3(a+cx^2)^p}{2cp+2c} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)**p*(a*d+a*e*x+c*d*(3+2*p)*x**2),x)
```

output
```
Piecewise(((a*d*x + a*e*x**2/2)/a, Eq(c, 0) & Eq(p, -1)), (a**p*(a*d*x + a
*e*x**2/2), Eq(c, 0)), (a*e*log(x - sqrt(-a/c))/(2*c) + a*e*log(x + sqrt(-
a/c))/(2*c) + d*x, Eq(p, -1)), (a**2*e*(a + c*x**2)**p/(2*c*p + 2*c) + 2*a
*c*d*p*x*(a + c*x**2)**p/(2*c*p + 2*c) + 2*a*c*d*x*(a + c*x**2)**p/(2*c*p
+ 2*c) + a*c*e*x**2*(a + c*x**2)**p/(2*c*p + 2*c) + 2*c**2*d*p*x**3*(a + c
*x**2)**p/(2*c*p + 2*c) + 2*c**2*d*x**3*(a + c*x**2)**p/(2*c*p + 2*c), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right) \, dx = \frac{(2\,c^2d(p+1)x^3+2\,acd(p+1)x+acex^2+a^2e)(cx^2+a)^p}{2\,c(p+1)}$$

input
```
integrate((c*x^2+a)^p*(a*d+a*e*x+c*d*(3+2*p)*x^2),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*d*(p + 1)*x^3 + 2*a*c*d*(p + 1)*x + a*c*e*x^2 + a^2*e)*(c*x^2 +
 a)^p/(c*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(33) = 66.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.09

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right) \, dx = \frac{2\left(cx^2+a\right)^p c^2dpx^3+2\left(cx^2+a\right)^p c^2dx^3+2\left(cx^2+a\right)^p acdpx+\left(cx^2+a\right)^p acex^2+2\left(cx^2+a\right)^p acdx+(ca}{2\left(cp+c\right)}$$

input
```
integrate((c*x^2+a)^p*(a*d+a*e*x+c*d*(3+2*p)*x^2),x, algorithm="giac")
```

output
```
1/2*(2*(c*x^2 + a)^p*c^2*d*p*x^3 + 2*(c*x^2 + a)^p*c^2*d*x^3 + 2*(c*x^2 +
a)^p*a*c*d*p*x + (c*x^2 + a)^p*a*c*e*x^2 + 2*(c*x^2 + a)^p*a*c*d*x + (c*x^
2 + a)^p*a^2*e)/(c*p + c)
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right)\,dx = \left(c\,x^2+a\right)^p\,\left(a\,d\,x+c\,d\,x^3+\frac{a\,e\,x^2}{2\,\left(p+1\right)}+\frac{a^2\,e}{2\,c\,\left(p+1\right)}\right)$$

input
```
int((a + c*x^2)^p*(a*d + a*e*x + c*d*x^2*(2*p + 3)),x)
```

output
```
(a + c*x^2)^p*(a*d*x + c*d*x^3 + (a*e*x^2)/(2*(p + 1)) + (a^2*e)/(2*c*(p +
 1)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \left(a+cx^2\right)^p \left(ad+aex+cd(3+2p)x^2\right)\,dx = \frac{\left(c\,x^2+a\right)^p \left(2c^2dp\,x^3+2c^2d\,x^3+2acdpx+ace\,x^2+2acdx+a^2e\right)}{2c\left(p+1\right)}$$

input
```
int((c*x^2+a)^p*(a*d+a*e*x+c*d*(3+2*p)*x^2),x)
```

output
```
((a + c*x**2)**p*(a**2*e + 2*a*c*d*p*x + 2*a*c*d*x + a*c*e*x**2 + 2*c**2*d
*p*x**3 + 2*c**2*d*x**3))/(2*c*(p + 1))
```

3.3 $\int (a+cx^2)^p (ad+bc(3+2p)x+cd(3+2p)x^2)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 35

$$\int (a+cx^2)^p (ad+bc(3+2p)x+cd(3+2p)x^2)\,dx = \frac{(b(3+2p)+2d(1+p)x)(a+cx^2)^{1+p}}{2(1+p)}$$

output
```
(b*(3+2*p)+2*d*(p+1)*x)*(c*x^2+a)^(p+1)/(2*p+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.37

$$\int (a+cx^2)^p (ad+bc(3+2p)x+cd(3+2p)x^2)\,dx = \frac{(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(6ad(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{cx^2}{a}\right)+(3+2p)\left(3b(a+cx^2)\left(1+\right.\right.\right.}{6(1+p)}$$

input
```
Integrate[(a + c*x^2)^p*(a*d + b*c*(3 + 2*p)*x + c*d*(3 + 2*p)*x^2),x]
```

output
```
((a + c*x^2)^p*(6*a*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/
a)] + (3 + 2*p)*(3*b*(a + c*x^2)*(1 + (c*x^2)/a)^p + 2*c*d*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((c*x^2)/a)])))/(6*(1 + p)*(1 + (c*x^2)/a)^p
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2346, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^p \left(ad+bc(2p+3)x+cd(2p+3)x^2\right) dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int bc^2(2p+3)^2x\left(cx^2+a\right)^p dx}{c(2p+3)}+dx\left(a+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{27}$$

$$bc(2p+3)\int x\left(cx^2+a\right)^p dx+dx\left(a+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{241}$$

$$\frac{b(2p+3)\left(a+cx^2\right)^{p+1}}{2(p+1)}+dx\left(a+cx^2\right)^{p+1}$$

input
```
Int[(a + c*x^2)^p*(a*d + b*c*(3 + 2*p)*x + c*d*(3 + 2*p)*x^2),x]
```

output
```
(b*(3 + 2*p)*(a + c*x^2)^(1 + p))/(2*(1 + p)) + d*x*(a + c*x^2)^(1 + p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{\left(c\,x^{2}+a\right)^{p+1} \left(2dpx+2bp+2dx+3b\right)}{2+2p}$
risch	$\frac{\left(2cdp\,x^{3}+2bcp\,x^{2}+2cd\,x^{3}+2adpx+3bc\,x^{2}+2abp+2adx+3ab\right) \left(c\,x^{2}+a\right)^{p}}{2+2p}$
norman	$adx\,{\mathrm e}^{p\ln\left(c\,x^{2}+a\right)}+cd\,x^{3}{\mathrm e}^{p\ln\left(c\,x^{2}+a\right)}+\frac{ab(3+2p){\mathrm e}^{p\ln\left(c\,x^{2}+a\right)}}{2+2p}+\frac{bc(3+2p)x^{2}{\mathrm e}^{p\ln\left(c\,x^{2}+a\right)}}{2+2p}$
orering	$\frac{\left(2dpx+2bp+2dx+3b\right) \left(c\,x^{2}+a\right) \left(c\,x^{2}+a\right)^{p} \left(ad+bc(3+2p)x+cd(3+2p)x^{2}\right)}{2(p+1)\left(2cd\,x^{2}p+2cbxp+3cd\,x^{2}+3cbx+ad\right)}$
parallelrisch	$\frac{2x^{3}\left(c\,x^{2}+a\right)^{p}c^{2}dp+2x^{3}\left(c\,x^{2}+a\right)^{p}c^{2}d+2x^{2}\left(c\,x^{2}+a\right)^{p}b\,c^{2}p+3x^{2}\left(c\,x^{2}+a\right)^{p}b\,c^{2}+2x\left(c\,x^{2}+a\right)^{p}acdp+2x\left(c\,x^{2}+a\right)^{p}acd+2(c\,x^{2}\cdots}{2c(p+1)}$

input
```
int((c*x^2+a)^p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x^2),x,method=_RETURNVERBOS
E)
```

output
```
1/2/(p+1)*(c*x^2+a)^(p+1)*(2*d*p*x+2*b*p+2*d*x+3*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a+cx^2)^p (ad+bc(3+2p)x+cd(3+2p)x^2) \, dx = \frac{(2\,(cdp+cd)x^3+2\,abp+(2\,bcp+3\,bc)x^2+3\,ab+2\,(adp+ad)x)(cx^2+a)^p}{2\,(p+1)}$$

input
```
integrate((c*x^2+a)^p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x^2),x, algorithm="fr
icas")
```

output
```
1/2*(2*(c*d*p + c*d)*x^3 + 2*a*b*p + (2*b*c*p + 3*b*c)*x^2 + 3*a*b + 2*(a*
d*p + a*d)*x)*(c*x^2 + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(29) = 58.

Time = 3.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.91

$$\int (a+cx^2)^p (ad+bc(3+2p)x+cd(3+2p)x^2) \, dx = \begin{cases} \frac{2abp(a+cx^2)^p}{2p+2} + \frac{3ab(a+cx^2)^p}{2p+2} + \frac{2adpx(a+cx^2)^p}{2p+2} + \frac{2adx(a+cx^2)^p}{2p+2} + \frac{2bcpx^2(a+cx^2)^p}{2p+2} + \frac{3bcx^2(a+cx^2)^p}{2p+2} + \frac{2cdpx^3(a+cx^2)^p}{2p+2} + \\ \frac{b\log\left(x-\sqrt{-\frac{a}{c}}\right)}{2} + \frac{b\log\left(x+\sqrt{-\frac{a}{c}}\right)}{2} + dx \end{cases}$$

input
```
integrate((c*x**2+a)**p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x**2),x)
```

output
```
Piecewise((2*a*b*p*(a + c*x**2)**p/(2*p + 2) + 3*a*b*(a + c*x**2)**p/(2*p
+ 2) + 2*a*d*p*x*(a + c*x**2)**p/(2*p + 2) + 2*a*d*x*(a + c*x**2)**p/(2*p
+ 2) + 2*b*c*p*x**2*(a + c*x**2)**p/(2*p + 2) + 3*b*c*x**2*(a + c*x**2)**p
/(2*p + 2) + 2*c*d*p*x**3*(a + c*x**2)**p/(2*p + 2) + 2*c*d*x**3*(a + c*x*
*2)**p/(2*p + 2), Ne(p, -1)), (b*log(x - sqrt(-a/c))/2 + b*log(x + sqrt(-a
/c))/2 + d*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+cx^2)^p\left(ad+bc(3+2p)x+cd(3+2p)x^2\right)\,dx = \frac{(2\,cd(p+1)x^3+bc(2\,p+3)x^2+2\,ad(p+1)x+ab(2\,p+3))(cx^2+a)^p}{2\,(p+1)}$$

input
```
integrate((c*x^2+a)^p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x^2),x, algorithm="ma
xima")
```

output
```
1/2*(2*c*d*(p + 1)*x^3 + b*c*(2*p + 3)*x^2 + 2*a*d*(p + 1)*x + a*b*(2*p +
3))*(c*x^2 + a)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(33) = 66$.

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.71

$$\int (a+cx^2)^p\left(ad+bc(3+2p)x+cd(3+2p)x^2\right)\,dx = \frac{2\,(cx^2+a)^p cdpx^3+2\,(cx^2+a)^p bcpx^2+2\,(cx^2+a)^p cdx^3+2\,(cx^2+a)^p adpx+3\,(cx^2+a)^p bcx^2+2\,(cx}{2\,(p+1)}$$

input
```
integrate((c*x^2+a)^p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x^2),x, algorithm="gi
ac")
```

output
```
1/2*(2*(c*x^2 + a)^p*c*d*p*x^3 + 2*(c*x^2 + a)^p*b*c*p*x^2 + 2*(c*x^2 + a)
^p*c*d*x^3 + 2*(c*x^2 + a)^p*a*d*p*x + 3*(c*x^2 + a)^p*b*c*x^2 + 2*(c*x^2
+ a)^p*a*b*p + 2*(c*x^2 + a)^p*a*d*x + 3*(c*x^2 + a)^p*a*b)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int (a+cx^2)^p \left(ad+bc(3+2p)x+cd(3+2p)x^2\right) dx = (c\,x^2+a)^p\left(\frac{a\,b\,(2\,p+3)}{2\,p+2}+\frac{2\,c\,d\,x^3\,(p+1)}{2\,p+2}+\frac{b\,c\,x^2\,(2\,p+3)}{2\,p+2}+\frac{2\,a\,d\,x\,(p+1)}{2\,p+2}\right)$$

input
```
int((a + c*x^2)^p*(a*d + c*d*x^2*(2*p + 3) + b*c*x*(2*p + 3)),x)
```

output
```
(a + c*x^2)^p*((a*b*(2*p + 3))/(2*p + 2) + (2*c*d*x^3*(p + 1))/(2*p + 2) +
 (b*c*x^2*(2*p + 3))/(2*p + 2) + (2*a*d*x*(p + 1))/(2*p + 2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int (a+cx^2)^p \left(ad+bc(3+2p)x+cd(3+2p)x^2\right) dx = \frac{(c\,x^2+a)^p\,(2cdp\,x^3+2bcp\,x^2+2cd\,x^3+2adpx+3bc\,x^2+2abp+2adx+3ab)}{2p+2}$$

input
```
int((c*x^2+a)^p*(a*d+b*c*(3+2*p)*x+c*d*(3+2*p)*x^2),x)
```

output
```
((a + c*x**2)**p*(2*a*b*p + 3*a*b + 2*a*d*p*x + 2*a*d*x + 2*b*c*p*x**2 + 3
*b*c*x**2 + 2*c*d*p*x**3 + 2*c*d*x**3))/(2*(p + 1))
```

3.4 $\int (a+cx^2)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 54

$$\int (a+cx^2)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx=-\frac{d(ae-(cd-af)x)\left(a+cx^2\right)^{\frac{1}{2}\left(-1+\frac{af}{cd}\right)}}{a(cd-af)}$$

output
```
-d*(a*e-(-a*f+c*d)*x)*(c*x^2+a)^(-1/2+1/2*a*f/c/d)/a/(-a*f+c*d)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.87

$$\int (a+cx^2)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{(a+cx^2)^{\frac{1}{2}\left(-1+\frac{af}{cd}\right)}\left(1+\frac{cx^2}{a}\right)^{-\frac{af}{2cd}}\left(3ade\left(1+\frac{cx^2}{a}\right)^{\frac{af}{2cd}}+3d(-cd+af)x\sqrt{1+\frac{cx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{3a(-cd+}$$

input
```
Integrate[(a + c*x^2)^((-6*c^2*d + 2*a*c*f)/(4*c^2*d))*(d + e*x + f*x^2),x
]
```

output
```
((a + c*x^2)^((-1 + (a*f)/(c*d))/2)*(3*a*d*e*(1 + (c*x^2)/a)^((a*f)/(2*c*d
)) + 3*d*(-(c*d) + a*f)*x*Sqrt[1 + (c*x^2)/a]*Hypergeometric2F1[1/2, (3 -
(a*f)/(c*d))/2, 3/2, -((c*x^2)/a)] + f*(-(c*d) + a*f)*x^3*Sqrt[1 + (c*x^2)
/a]*Hypergeometric2F1[3/2, (3 - (a*f)/(c*d))/2, 5/2, -((c*x^2)/a)]))/(3*a*
(-(c*d) + a*f)*(1 + (c*x^2)/a)^((a*f)/(2*c*d)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {2346, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex+fx^2)\left(a+cx^2\right)^{\frac{2acf-6c^2d}{4c^2d}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{d\int \frac{aefx(cx^2+a)^{\frac{1}{2}\left(\frac{af}{cd}-3\right)}}{d}dx}{af}+\frac{dx\left(a+cx^2\right)^{\frac{1}{2}\left(\frac{af}{cd}-1\right)}}{a}$$

$$\Big\downarrow \text{27}$$

$$e\int x\left(cx^2+a\right)^{\frac{1}{2}\left(\frac{af}{cd}-3\right)}dx+\frac{dx\left(a+cx^2\right)^{\frac{1}{2}\left(\frac{af}{cd}-1\right)}}{a}$$

$$\Big\downarrow \text{241}$$

$$\frac{dx\left(a+cx^2\right)^{\frac{1}{2}\left(\frac{af}{cd}-1\right)}}{a}-\frac{de\left(a+cx^2\right)^{\frac{1}{2}\left(\frac{af}{cd}-1\right)}}{cd-af}$$

input
```
Int[(a + c*x^2)^((-6*c^2*d + 2*a*c*f)/(4*c^2*d))*(d + e*x + f*x^2),x]
```

output
```
-((d*e*(a + c*x^2)^((-1 + (a*f)/(c*d))/2))/(c*d - a*f)) + (d*x*(a + c*x^2)
^((-1 + (a*f)/(c*d))/2))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

method	result
gosper	$\frac{d\left(c\,x^{2}+a\right)^{1+\frac{af-3cd}{2cd}}\left(afx-cdx+ae\right)}{a\left(af-cd\right)}$
orering	$\frac{\left(afx-cdx+ae\right)d\left(c\,x^{2}+a\right)\left(c\,x^{2}+a\right)^{\frac{2acf-6c^{2}d}{4c^{2}d}}}{a\left(af-cd\right)}$
risch	$\frac{\left(acf\,x^{3}-c^{2}d\,x^{3}+ace\,x^{2}+f\,a^{2}x-adxc+a^{2}e\right)d\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}}{a\left(af-cd\right)}$
norman	$dx\,\mathrm{e}^{\frac{\left(2acf-6c^{2}d\right)\ln\left(c\,x^{2}+a\right)}{4c^{2}d}}+\frac{ade\,\mathrm{e}^{\frac{\left(2acf-6c^{2}d\right)\ln\left(c\,x^{2}+a\right)}{4c^{2}d}}}{af-cd}+\frac{cd\,x^{3}\mathrm{e}^{\frac{\left(2acf-6c^{2}d\right)\ln\left(c\,x^{2}+a\right)}{4c^{2}d}}}{a}+\frac{dec\,x^{2}\mathrm{e}^{\frac{\left(2acf-6c^{2}d\right)\ln}{4c^{2}d}}}{af-cd}$
parallelrisch	$\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}acdf-x^{3}\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}c^{2}d^{2}+acde\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}x^{2}+x\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}a^{2}df-x\left(c\,x^{2}+a\right)^{\frac{af-3cd}{2cd}}ac\,d}{a\left(af-cd\right)}$

input
```
int((c*x^2+a)^(1/4*(2*a*c*f-6*c^2*d)/c^2/d)*(f*x^2+e*x+d),x,method=_RETURN
VERBOSE)
```

output
```
d/a*(c*x^2+a)^(1+1/2/c*(a*f-3*c*d)/d)/(a*f-c*d)*(a*f*x-c*d*x+a*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx = -\frac{acdex^2+a^2de-(c^2d^2-acdf)x^3-(acd^2-a^2df)x}{(acd-a^2f)(cx^2+a)^{\frac{3\,cd-af}{2\,cd}}}$$

input
```
integrate((c*x^2+a)^(1/4*(2*a*c*f-6*c^2*d)/c^2/d)*(f*x^2+e*x+d),x, algorit
hm="fricas")
```

output
```
-(a*c*d*e*x^2 + a^2*d*e - (c^2*d^2 - a*c*d*f)*x^3 - (a*c*d^2 - a^2*d*f)*x)
/((a*c*d - a^2*f)*(c*x^2 + a)^(1/2*(3*c*d - a*f)/(c*d)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(41) = 82.

Time = 21.71 (sec) , antiderivative size = 374, normalized size of antiderivative = 6.93

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx = \begin{cases} \tilde{\infty}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \\ e\left(\begin{cases}\tilde{\infty}x^2 & \text{for } c=0 \\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise}\end{cases}\right)+\frac{fx^3\log(x)}{(cx^2)^{\frac{3}{2}}} \\ -\frac{dx}{2(cx^2)^{\frac{3}{2}}}+e\left(\begin{cases}\tilde{\infty}x^2 & \text{for } c=0 \\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise}\end{cases}\right)+\frac{fx^3\log(x)}{(cx^2)^{\frac{3}{2}}} \\ \frac{e\log\left(x-\sqrt{-\frac{d}{f}}\right)}{2c}+\frac{e\log\left(x+\sqrt{-\frac{d}{f}}\right)}{2c}+\frac{fx}{c} \\ \frac{a^2de(a+cx^2)^{\frac{af}{2cd}-\frac{3}{2}}}{a^2f-acd}+\frac{a^2dfx(a+cx^2)^{\frac{af}{2cd}-\frac{3}{2}}}{a^2f-acd}-\frac{acd^2x(a+cx^2)^{\frac{af}{2cd}-\frac{3}{2}}}{a^2f-acd}+\frac{acdex^2(a+cx^2)^{\frac{af}{2cd}-\frac{3}{2}}}{a^2f-acd}+\frac{acdfx^3(a+cx^2)^{\frac{af}{2cd}-\frac{3}{2}}}{a^2f-acd}-\frac{c^2d^2x^3}{} \end{cases}$$

input
```
integrate((c*x**2+a)**(1/4*(2*a*c*f-6*c**2*d)/c**2/d)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((zoo*(d*x + e*x**2/2 + f*x**3/3), Eq(a, 0) & Eq(c, 0)), (e*Piece
wise((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True)) + f*x**3*log(x)/(c
*x**2)**(3/2), Eq(a, 0) & Eq(d, 0)), (-d*x/(2*(c*x**2)**(3/2)) + e*Piecewi
se((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True)) + f*x**3*log(x)/(c*x
**2)**(3/2), Eq(a, 0)), (e*log(x - sqrt(-d/f))/(2*c) + e*log(x + sqrt(-d/f
))/(2*c) + f*x/c, Eq(a, c*d/f)), (a**2*d*e*(a + c*x**2)**(a*f/(2*c*d) - 3/
2)/(a**2*f - a*c*d) + a**2*d*f*x*(a + c*x**2)**(a*f/(2*c*d) - 3/2)/(a**2*f
 - a*c*d) - a*c*d**2*x*(a + c*x**2)**(a*f/(2*c*d) - 3/2)/(a**2*f - a*c*d)
+ a*c*d*e*x**2*(a + c*x**2)**(a*f/(2*c*d) - 3/2)/(a**2*f - a*c*d) + a*c*d*
f*x**3*(a + c*x**2)**(a*f/(2*c*d) - 3/2)/(a**2*f - a*c*d) - c**2*d**2*x**3
*(a + c*x**2)**(a*f/(2*c*d) - 3/2)/(a**2*f - a*c*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}} \left(d+ex+fx^2\right) \, dx = -\frac{(ade-(cd^2-adf)x)(cx^2+a)^{\frac{af}{2\,cd}}}{(acd-a^2f)\sqrt{cx^2+a}}$$

input
```
integrate((c*x^2+a)^(1/4*(2*a*c*f-6*c^2*d)/c^2/d)*(f*x^2+e*x+d),x, algorit
hm="maxima")
```

output
```
-(a*d*e - (c*d^2 - a*d*f)*x)*(c*x^2 + a)^(1/2*a*f/(c*d))/((a*c*d - a^2*f)*
sqrt(c*x^2 + a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(51) = 102.

Time = 0.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.98

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}} \left(d+ex+fx^2\right) \, dx$$

$$= \frac{c^2d^2x^3e^{\left(-\frac{3\,cd\log\left(cx^2+a\right)-af\log\left(cx^2+a\right)}{2\,cd}\right)} - acdfx^3e^{\left(-\frac{3\,cd\log\left(cx^2+a\right)-af\log\left(cx^2+a\right)}{2\,cd}\right)} - acdex^2e^{\left(-\frac{3\,cd\log\left(cx^2+a\right)-af\log\left(cx^2\right.}{2\,cd}\right.}}{a}$$

input
```
integrate((c*x^2+a)^(1/4*(2*a*c*f-6*c^2*d)/c^2/d)*(f*x^2+e*x+d),x, algorit
hm="giac")
```

output
```
(c^2*d^2*x^3*e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)) -
a*c*d*f*x^3*e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)) - a
*c*d*e*x^2*e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)) + a*
c*d^2*x*e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)) - a^2*d
*f*x*e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)) - a^2*d*e*
e^(-1/2*(3*c*d*log(c*x^2 + a) - a*f*log(c*x^2 + a))/(c*d)))/(a*c*d - a^2*f
)
```

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.28

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx=\frac{\frac{a^2\,d\,e}{a^2\,f-a\,c\,d}+\frac{a\,d\,x\,\left(a\,f-c\,d\right)}{a^2\,f-a\,c\,d}+\frac{c\,d\,x^3\,\left(a\,f-c\,d\right)}{a^2\,f-a\,c\,d}+\frac{a\,c\,d\,e\,x^2}{a^2\,f-a\,c\,d}}{{\left(c\,x^2+a\right)}^{\frac{\frac{3\,c^2\,d}{2}-\frac{a\,c\,f}{2}}{c^2\,d}}}$$

input
```
int((d + e*x + f*x^2)/(a + c*x^2)^(((3*c^2*d)/2 - (a*c*f)/2)/(c^2*d)),x)
```

output
```
((a^2*d*e)/(a^2*f - a*c*d) + (a*d*x*(a*f - c*d))/(a^2*f - a*c*d) + (c*d*x^
3*(a*f - c*d))/(a^2*f - a*c*d) + (a*c*d*e*x^2)/(a^2*f - a*c*d))/(a + c*x^2
)^(((3*c^2*d)/2 - (a*c*f)/2)/(c^2*d))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \left(a+cx^2\right)^{\frac{-6c^2d+2acf}{4c^2d}}\left(d+ex+fx^2\right)\,dx=\frac{\left(c\,x^2+a\right)^{\frac{af+cd}{2cd}}d(afx-cdx+ae)}{a\left(acf\,x^2-c^2d\,x^2+a^2f-acd\right)}$$

input
```
int((c*x^2+a)^(1/4*(2*a*c*f-6*c^2*d)/c^2/d)*(f*x^2+e*x+d),x)
```

output
```
((a + c*x**2)**((a*f + c*d)/(2*c*d))*d*(a*e + a*f*x - c*d*x))/(a*(a**2*f -
 a*c*d + a*c*f*x**2 - c**2*d*x**2))
```

3.5 $\int (a+cx^2)^2 (-2acd-2acex-14c^2dx^2)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 22

$$\int (a+cx^2)^2 (-2acd-2acex-14c^2dx^2)\,dx = -\frac{1}{3}(ae+6cdx)(a+cx^2)^3$$

output
```
-1/3*(6*c*d*x+a*e)*(c*x^2+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int (a+cx^2)^2 (-2acd-2acex-14c^2dx^2)\,dx = -2c\left(a^3dx+\frac{1}{2}a^3ex^2+3a^2cdx^3+\frac{1}{2}a^2cex^4+3ac^2dx^5+\frac{1}{6}ac^2ex^6+c^3dx^7\right)$$

input
```
Integrate[(a + c*x^2)^2*(-2*a*c*d - 2*a*c*e*x - 14*c^2*d*x^2),x]
```

output
```
-2*c*(a^3*d*x + (a^3*e*x^2)/2 + 3*a^2*c*d*x^3 + (a^2*c*e*x^4)/2 + 3*a*c^2*
d*x^5 + (a*c^2*e*x^6)/6 + c^3*d*x^7)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 55 vs. $2(22) = 44$.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2017, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 \left(-2acd-2acex-14c^2dx^2\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(cx^2+a\right)^2 \left(-14c^2dx^2-2acd\right)dx-\frac{1}{3}ae\left(a+cx^2\right)^3$$

$$\Big\downarrow \text{290}$$

$$\int \left(-14c^4dx^6-30ac^3dx^4-18a^2c^2dx^2-2a^3cd\right)dx-\frac{1}{3}ae\left(a+cx^2\right)^3$$

$$\Big\downarrow \text{2009}$$

$$-2a^3cdx-6a^2c^2dx^3-6ac^3dx^5-\frac{1}{3}ae\left(a+cx^2\right)^3-2c^4dx^7$$

input
```
Int[(a + c*x^2)^2*(-2*a*c*d - 2*a*c*e*x - 14*c^2*d*x^2),x]
```

output
```
-2*a^3*c*d*x - 6*a^2*c^2*d*x^3 - 6*a*c^3*d*x^5 - 2*c^4*d*x^7 - (a*e*(a + c
*x^2)^3)/3
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(20) = 40$.

Time = 0.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

method	result	size
gosper	$-\frac{cx\left(6c^{3}d\,x^{6}+a\,c^{2}e\,x^{5}+18a\,c^{2}d\,x^{4}+3a^{2}ce\,x^{3}+18a^{2}cd\,x^{2}+3a^{3}ex+6a^{3}d\right)}{3}$	67
default	$-2c^{4}d\,x^{7}-\frac{1}{3}c^{3}ae\,x^{6}-6a\,c^{3}d\,x^{5}-a^{2}c^{2}e\,x^{4}-6a^{2}c^{2}d\,x^{3}-ce\,a^{3}x^{2}-2a^{3}cdx$	73
norman	$-2c^{4}d\,x^{7}-\frac{1}{3}c^{3}ae\,x^{6}-6a\,c^{3}d\,x^{5}-a^{2}c^{2}e\,x^{4}-6a^{2}c^{2}d\,x^{3}-ce\,a^{3}x^{2}-2a^{3}cdx$	73
risch	$-2c^{4}d\,x^{7}-\frac{1}{3}c^{3}ae\,x^{6}-6a\,c^{3}d\,x^{5}-a^{2}c^{2}e\,x^{4}-6a^{2}c^{2}d\,x^{3}-ce\,a^{3}x^{2}-2a^{3}cdx$	73
parallelrisch	$-2c^{4}d\,x^{7}-\frac{1}{3}c^{3}ae\,x^{6}-6a\,c^{3}d\,x^{5}-a^{2}c^{2}e\,x^{4}-6a^{2}c^{2}d\,x^{3}-ce\,a^{3}x^{2}-2a^{3}cdx$	73
orering	$\frac{x\left(6c^{3}d\,x^{6}+a\,c^{2}e\,x^{5}+18a\,c^{2}d\,x^{4}+3a^{2}ce\,x^{3}+18a^{2}cd\,x^{2}+3a^{3}ex+6a^{3}d\right)\left(-14c^{2}d\,x^{2}-2acex-2acd\right)}{42cd\,x^{2}+6aex+6ad}$	104

input
```
int((c*x^2+a)^2*(-14*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*x*(6*c^3*d*x^6+a*c^2*e*x^5+18*a*c^2*d*x^4+3*a^2*c*e*x^3+18*a^2*c*d*
x^2+3*a^3*e*x+6*a^3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(20) = 40.

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int \left(a+cx^2\right)^2 \left(-2acd-2acex-14c^2dx^2\right)\,dx$$
$$= -2\,c^4dx^7 - \frac{1}{3}\,ac^3ex^6 - 6\,ac^3dx^5 - a^2c^2ex^4 - 6\,a^2c^2dx^3 - a^3cex^2 - 2\,a^3cdx$$

input
```
integrate((c*x^2+a)^2*(-14*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="fric
as")
```

output
```
-2*c^4*d*x^7 - 1/3*a*c^3*e*x^6 - 6*a*c^3*d*x^5 - a^2*c^2*e*x^4 - 6*a^2*c^2
*d*x^3 - a^3*c*e*x^2 - 2*a^3*c*d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \left(a+cx^2\right)^2 \left(-2acd-2acex-14c^2dx^2\right)\,dx$$
$$= -2a^3cdx - a^3cex^2 - 6a^2c^2dx^3 - a^2c^2ex^4 - 6ac^3dx^5 - \frac{ac^3ex^6}{3} - 2c^4dx^7$$

input
```
integrate((c*x**2+a)**2*(-14*c**2*d*x**2-2*a*c*e*x-2*a*c*d),x)
```

output
```
-2*a**3*c*d*x - a**3*c*e*x**2 - 6*a**2*c**2*d*x**3 - a**2*c**2*e*x**4 - 6*
a*c**3*d*x**5 - a*c**3*e*x**6/3 - 2*c**4*d*x**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int (a+cx^2)^2 (-2acd-2acex-14c^2dx^2)\,dx$$
$$= -2\,c^4dx^7 - \frac{1}{3}\,ac^3ex^6 - 6\,ac^3dx^5 - a^2c^2ex^4 - 6\,a^2c^2dx^3 - a^3cex^2 - 2\,a^3cdx$$

input
```
integrate((c*x^2+a)^2*(-14*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="maxi
ma")
```

output
```
-2*c^4*d*x^7 - 1/3*a*c^3*e*x^6 - 6*a*c^3*d*x^5 - a^2*c^2*e*x^4 - 6*a^2*c^2
*d*x^3 - a^3*c*e*x^2 - 2*a^3*c*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(20) = 40$.

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int (a+cx^2)^2 (-2acd-2acex-14c^2dx^2)\,dx$$
$$= -2\,c^4dx^7 - \frac{1}{3}\,ac^3ex^6 - 6\,ac^3dx^5 - a^2c^2ex^4 - 6\,a^2c^2dx^3 - a^3cex^2 - 2\,a^3cdx$$

input
```
integrate((c*x^2+a)^2*(-14*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="giac
")
```

output
```
-2*c^4*d*x^7 - 1/3*a*c^3*e*x^6 - 6*a*c^3*d*x^5 - a^2*c^2*e*x^4 - 6*a^2*c^2
*d*x^3 - a^3*c*e*x^2 - 2*a^3*c*d*x
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int (a+cx^2)^2 \left(-2acd-2acex-14c^2dx^2\right) dx$$
$$= -e\,a^3\,c\,x^2-2\,d\,a^3\,c\,x-e\,a^2\,c^2\,x^4-6\,d\,a^2\,c^2\,x^3-\frac{e\,a\,c^3\,x^6}{3}-6\,d\,a\,c^3\,x^5-2\,d\,c^4\,x^7$$

input
```
int(-(a + c*x^2)^2*(14*c^2*d*x^2 + 2*a*c*d + 2*a*c*e*x),x)
```

output
```
- 2*c^4*d*x^7 - 6*a^2*c^2*d*x^3 - a^2*c^2*e*x^4 - 2*a^3*c*d*x - 6*a*c^3*d*
x^5 - a^3*c*e*x^2 - (a*c^3*e*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int (a+cx^2)^2 \left(-2acd-2acex-14c^2dx^2\right) dx$$
$$= \frac{cx(-6c^3d\,x^6-a\,c^2e\,x^5-18a\,c^2d\,x^4-3a^2ce\,x^3-18a^2cd\,x^2-3a^3ex-6a^3d)}{3}$$

input
```
int((c*x^2+a)^2*(-14*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x)
```

output
```
(c*x*( - 6*a**3*d - 3*a**3*e*x - 18*a**2*c*d*x**2 - 3*a**2*c*e*x**3 - 18*a
*c**2*d*x**4 - a*c**2*e*x**5 - 6*c**3*d*x**6))/3
```

3.6 $\int (a+cx^2)(-2acd-2acex-10c^2dx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 22

$$\int (a+cx^2)\left(-2acd-2acex-10c^2dx^2\right)\,dx=-\frac{1}{2}(ae+4cdx)\left(a+cx^2\right)^2$$

output
```
-1/2*(4*c*d*x+a*e)*(c*x^2+a)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int (a+cx^2)\left(-2acd-2acex-10c^2dx^2\right)\,dx$$
$$=-2c\left(a^2dx+\frac{1}{2}a^2ex^2+2acdx^3+\frac{1}{4}acex^4+c^2dx^5\right)$$

input
```
Integrate[(a + c*x^2)*(-2*a*c*d - 2*a*c*e*x - 10*c^2*d*x^2),x]
```

output
```
-2*c*(a^2*d*x + (a^2*e*x^2)/2 + 2*a*c*d*x^3 + (a*c*e*x^4)/4 + c^2*d*x^5)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(-2acd-2acex-10c^2dx^2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-2a^2cd-2a^2cex-12ac^2dx^2-2ac^2ex^3-10c^3dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2cdx-a^2cex^2-4ac^2dx^3-\frac{1}{2}ac^2ex^4-2c^3dx^5$$

input
```
Int[(a + c*x^2)*(-2*a*c*d - 2*a*c*e*x - 10*c^2*d*x^2),x]
```

output
```
-2*a^2*c*d*x - a^2*c*e*x^2 - 4*a*c^2*d*x^3 - (a*c^2*e*x^4)/2 - 2*c^3*d*x^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

method	result	size
gosper	$-\frac{cx\left(4c^{2}d\,x^{4}+ace\,x^{3}+8ad\,x^{2}c+2a^{2}ex+4a^{2}d\right)}{2}$	43
default	$-2c^{3}d\,x^{5}-\frac{1}{2}c^{2}ae\,x^{4}-4a\,c^{2}d\,x^{3}-a^{2}ce\,x^{2}-2a^{2}cdx$	49
norman	$-2c^{3}d\,x^{5}-\frac{1}{2}c^{2}ae\,x^{4}-4a\,c^{2}d\,x^{3}-a^{2}ce\,x^{2}-2a^{2}cdx$	49
risch	$-2c^{3}d\,x^{5}-\frac{1}{2}c^{2}ae\,x^{4}-4a\,c^{2}d\,x^{3}-a^{2}ce\,x^{2}-2a^{2}cdx$	49
parallelrisch	$-2c^{3}d\,x^{5}-\frac{1}{2}c^{2}ae\,x^{4}-4a\,c^{2}d\,x^{3}-a^{2}ce\,x^{2}-2a^{2}cdx$	49
orering	$\frac{x\left(4c^{2}d\,x^{4}+ace\,x^{3}+8ad\,x^{2}c+2a^{2}ex+4a^{2}d\right)\left(-10c^{2}d\,x^{2}-2acex-2acd\right)}{20cd\,x^{2}+4aex+4ad}$	80

input
```
int((c*x^2+a)*(-10*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*x*(4*c^2*d*x^4+a*c*e*x^3+8*a*c*d*x^2+2*a^2*e*x+4*a^2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \left(a+cx^{2}\right)\left(-2acd-2acex-10c^{2}dx^{2}\right)\,dx$$
$$=-2\,c^{3}dx^{5}-\frac{1}{2}\,ac^{2}ex^{4}-4\,ac^{2}dx^{3}-a^{2}cex^{2}-2\,a^{2}cdx$$

input
```
integrate((c*x^2+a)*(-10*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="fricas
")
```

output
```
-2*c^3*d*x^5 - 1/2*a*c^2*e*x^4 - 4*a*c^2*d*x^3 - a^2*c*e*x^2 - 2*a^2*c*d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int (a + cx^2) \left(-2acd - 2acex - 10c^2dx^2\right) dx$$
$$= -2a^2cdx - a^2cex^2 - 4ac^2dx^3 - \frac{ac^2ex^4}{2} - 2c^3dx^5$$

input
```
integrate((c*x**2+a)*(-10*c**2*d*x**2-2*a*c*e*x-2*a*c*d),x)
```

output
```
-2*a**2*c*d*x - a**2*c*e*x**2 - 4*a*c**2*d*x**3 - a*c**2*e*x**4/2 - 2*c**3
*d*x**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (a + cx^2) \left(-2acd - 2acex - 10c^2dx^2\right) dx$$
$$= -2\,c^3dx^5 - \frac{1}{2}\,ac^2ex^4 - 4\,ac^2dx^3 - a^2cex^2 - 2\,a^2cdx$$

input
```
integrate((c*x^2+a)*(-10*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="maxima
")
```

output
```
-2*c^3*d*x^5 - 1/2*a*c^2*e*x^4 - 4*a*c^2*d*x^3 - a^2*c*e*x^2 - 2*a^2*c*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (a+cx^2)(-2acd-2acex-10c^2dx^2)\,dx$$
$$= -2\,c^3dx^5 - \frac{1}{2}\,ac^2ex^4 - 4\,ac^2dx^3 - a^2cex^2 - 2\,a^2cdx$$

input
```
integrate((c*x^2+a)*(-10*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x, algorithm="giac")
```

output
```
-2*c^3*d*x^5 - 1/2*a*c^2*e*x^4 - 4*a*c^2*d*x^3 - a^2*c*e*x^2 - 2*a^2*c*d*x
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (a+cx^2)(-2acd-2acex-10c^2dx^2)\,dx$$
$$= -e\,a^2\,c\,x^2 - 2\,d\,a^2\,c\,x - \frac{e\,a\,c^2\,x^4}{2} - 4\,d\,a\,c^2\,x^3 - 2\,d\,c^3\,x^5$$

input
```
int(-(a + c*x^2)*(10*c^2*d*x^2 + 2*a*c*d + 2*a*c*e*x),x)
```

output
```
- 2*c^3*d*x^5 - 2*a^2*c*d*x - 4*a*c^2*d*x^3 - a^2*c*e*x^2 - (a*c^2*e*x^4)/
2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \left(a+cx^2\right) \left(-2acd-2acex-10c^2dx^2\right) \, dx = \frac{cx(-4c^2d\,x^4-ace\,x^3-8acd\,x^2-2a^2ex-4a^2d)}{2}$$

input
```
int((c*x^2+a)*(-10*c^2*d*x^2-2*a*c*e*x-2*a*c*d),x)
```

output
```
(c*x*( - 4*a**2*d - 2*a**2*e*x - 8*a*c*d*x**2 - a*c*e*x**3 - 4*c**2*d*x**4
))/2
```

3.7 $\int \left(-2acd-2acex-6c^2dx^2\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \left(-2acd-2acex-6c^2dx^2\right) \, dx=-2acdx-acex^2-2c^2dx^3$$

output
```
-2*c^2*d*x^3-a*c*e*x^2-2*a*c*d*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(-2acd-2acex-6c^2dx^2\right) \, dx=-2acdx-acex^2-2c^2dx^3$$

input
```
Integrate[-2*a*c*d - 2*a*c*e*x - 6*c^2*d*x^2,x]
```

output
```
-2*a*c*d*x - a*c*e*x^2 - 2*c^2*d*x^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2acd-2acex-6c^2dx^2\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$-2acdx-acex^2-2c^2dx^3$$

input
```
Int[-2*a*c*d - 2*a*c*e*x - 6*c^2*d*x^2,x]
```

output
```
-2*a*c*d*x - a*c*e*x^2 - 2*c^2*d*x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
gosper	$-cx(2cd\,x^2+aex+2ad)$	21
default	$-2c\left(cd\,x^3+\frac{1}{2}ae\,x^2+adx\right)$	22
norman	$-2c^2d\,x^3-ace\,x^2-2adxc$	25
risch	$-2c^2d\,x^3-ace\,x^2-2adxc$	25
parallelrisch	$-2c^2d\,x^3-ace\,x^2-2adxc$	25
parts	$-2c^2d\,x^3-ace\,x^2-2adxc$	25
orering	$\frac{x(2cd\,x^2+aex+2ad)\left(-6c^2d\,x^2-2acex-2acd\right)}{6cd\,x^2+2aex+2ad}$	58

input
```
int(-6*c^2*d*x^2-2*a*c*e*x-2*a*c*d,x,method=_RETURNVERBOSE)
```

output
```
-c*x*(2*c*d*x^2+a*e*x+2*a*d)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=-2\,c^2dx^3-acex^2-2\,acdx$$

input
```
integrate(-6*c^2*d*x^2-2*a*c*e*x-2*a*c*d,x, algorithm="fricas")
```

output
```
-2*c^2*d*x^3 - a*c*e*x^2 - 2*a*c*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=-2acdx-acex^2-2c^2dx^3$$

input
```
integrate(-6*c**2*d*x**2-2*a*c*e*x-2*a*c*d,x)
```

output
```
-2*a*c*d*x - a*c*e*x**2 - 2*c**2*d*x**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=-2\,c^2dx^3-acex^2-2\,acdx$$

input
```
integrate(-6*c^2*d*x^2-2*a*c*e*x-2*a*c*d,x, algorithm="maxima")
```

output
```
-2*c^2*d*x^3 - a*c*e*x^2 - 2*a*c*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=-2\,c^2dx^3-acex^2-2\,acdx$$

input
```
integrate(-6*c^2*d*x^2-2*a*c*e*x-2*a*c*d,x, algorithm="giac")
```

output
```
-2*c^2*d*x^3 - a*c*e*x^2 - 2*a*c*d*x
```

Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=-c\,x\,\left(2\,c\,d\,x^2+a\,e\,x+2\,a\,d\right)$$

input
```
int(- 6*c^2*d*x^2 - 2*a*c*d - 2*a*c*e*x,x)
```

output
```
-c*x*(2*a*d + a*e*x + 2*c*d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(-2acd-2acex-6c^2dx^2\right)\,dx=cx\left(-2cd\,x^2-aex-2ad\right)$$

input
```
int(-6*c^2*d*x^2-2*a*c*e*x-2*a*c*d,x)
```

output
```
c*x*( - 2*a*d - a*e*x - 2*c*d*x**2)
```

3.8 $\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 18

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx=-2cdx-ae\log\left(a+cx^2\right)$$

output
```
-2*c*d*x-a*e*ln(c*x^2+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx=-2c\left(dx+\frac{ae\log\left(a+cx^2\right)}{2c}\right)$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x - 2*c^2*d*x^2)/(a + c*x^2),x]
```

output
```
-2*c*(d*x + (a*e*Log[a + c*x^2])/(2*c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(-\frac{2acex}{a+cx^2}-2cd\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ae\log\left(a+cx^2\right)-2cdx$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x - 2*c^2*d*x^2)/(a + c*x^2),x]
```

output
```
-2*c*d*x - a*e*Log[a + c*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
norman	$-2cdx - ae\ln\left(c\,x^{2}+a\right)$	19
risch	$-2cdx - ae\ln\left(c\,x^{2}+a\right)$	19
parallelrisch	$-2cdx - ae\ln\left(c\,x^{2}+a\right)$	19
default	$2c\left(-dx - \frac{ae\ln\left(c\,x^{2}+a\right)}{2c}\right)$	24

input
```
int((-2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*c*d*x-a*e*ln(c*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx = -2\,cdx - ae\log\left(cx^2+a\right)$$

input
```
integrate((-2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a),x, algorithm="fricas"
)
```

output
```
-2*c*d*x - a*e*log(c*x^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx = -ae\log\left(a+cx^2\right) - 2cdx$$

input
```
integrate((-2*c**2*d*x**2-2*a*c*e*x-2*a*c*d)/(c*x**2+a),x)
```

output
```
-a*e*log(a + c*x**2) - 2*c*d*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx=-2\,cdx-ae\log\left(cx^2+a\right)$$

input
```
integrate((-2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a),x, algorithm="maxima"
)
```

output
```
-2*c*d*x - a*e*log(c*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx=-2\,cdx-ae\log\left(cx^2+a\right)$$

input
```
integrate((-2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a),x, algorithm="giac")
```

output
```
-2*c*d*x - a*e*log(c*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex-2c^2dx^2}{a+cx^2}\,dx=-2\,c\,d\,x-a\,e\,\ln\left(c\,x^2+a\right)$$

input
```
int(-(2*c^2*d*x^2 + 2*a*c*d + 2*a*c*e*x)/(a + c*x^2),x)
```

output
```
- 2*c*d*x - a*e*log(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2acd - 2acex - 2c^2dx^2}{a + cx^2} \, dx = -\log\left(c\,x^2 + a\right) ae - 2cdx$$

input
```
int((-2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a),x)
```

output
```
 - log(a + c*x**2)*a*e - 2*c*d*x
```

3.9 $\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 19

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=\frac{ae-2cdx}{a+cx^2}$$

output
```
(-2*c*d*x+a*e)/(c*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=\frac{ae-2cdx}{a+cx^2}$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x + 2*c^2*d*x^2)/(a + c*x^2)^2,x]
```

output
```
(a*e - 2*c*d*x)/(a + c*x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2345, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2} dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{ae-2cdx}{a+cx^2} - \frac{\int 0dx}{2a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{ae-2cdx}{a+cx^2}$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x + 2*c^2*d*x^2)/(a + c*x^2)^2,x]
```

output
```
(a*e - 2*c*d*x)/(a + c*x^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{-2cdx+ae}{c\,x^{2}+a}$	20
risch	$\frac{-2cdx+ae}{c\,x^{2}+a}$	20
default	$-\frac{2c\left(dx-\frac{ae}{2c}\right)}{c\,x^{2}+a}$	24
norman	$\frac{-ce\,x^{2}-2cdx}{c\,x^{2}+a}$	24
parallelrisch	$\frac{-2c^{2}dx+ace}{c\left(c\,x^{2}+a\right)}$	26
orering	$-\frac{(-2cdx+ae)\left(2c^{2}d\,x^{2}-2acex-2acd\right)}{2c\left(-cd\,x^{2}+aex+ad\right)\left(c\,x^{2}+a\right)}$	62

input
```
int((2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*c*d*x+a*e)/(c*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=-\frac{2\,cdx-ae}{cx^2+a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^2,x, algorithm="fricas
")
```

output
```
-(2*c*d*x - a*e)/(c*x^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=\frac{ae-2cdx}{a+cx^2}$$

input
```
integrate((2*c**2*d*x**2-2*a*c*e*x-2*a*c*d)/(c*x**2+a)**2,x)
```

output
```
(a*e - 2*c*d*x)/(a + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=-\frac{2\,cdx-ae}{cx^2+a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-(2*c*d*x - a*e)/(c*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=-\frac{2\,cdx-ae}{cx^2+a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-(2*c*d*x - a*e)/(c*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=\frac{a\,e-2\,c\,d\,x}{c\,x^2+a}$$

input
```
int(-(2*a*c*d - 2*c^2*d*x^2 + 2*a*c*e*x)/(a + c*x^2)^2,x)
```

output
```
(a*e - 2*c*d*x)/(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{-2acd-2acex+2c^2dx^2}{(a+cx^2)^2}\,dx=\frac{cx(-ex-2d)}{c\,x^2+a}$$

input
```
int((2*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^2,x)
```

output
```
(c*x*( - 2*d - e*x))/(a + c*x**2)
```

3.10 $\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 22

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx = \frac{ae-4cdx}{2(a+cx^2)^2}$$

output
```
1/2*(-4*c*d*x+a*e)/(c*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx = -\frac{-ae+4cdx}{2(a+cx^2)^2}$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x + 6*c^2*d*x^2)/(a + c*x^2)^3,x]
```

output
```
-1/2*(-(a*e) + 4*c*d*x)/(a + c*x^2)^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2345, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{ae-4cdx}{2(a+cx^2)^2}-\frac{\int 0dx}{4a}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-4cdx}{2(a+cx^2)^2}$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x + 6*c^2*d*x^2)/(a + c*x^2)^3,x]
```

output
```
(a*e - 4*c*d*x)/(2*(a + c*x^2)^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{-4cdx+ae}{2(c\,x^{2}+a)^{2}}$	21
norman	$\frac{\frac{ae}{2}-2cdx}{(c\,x^{2}+a)^{2}}$	21
risch	$\frac{\frac{ae}{2}-2cdx}{(c\,x^{2}+a)^{2}}$	21
default	$-\frac{2c\left(dx-\frac{ae}{4c}\right)}{(c\,x^{2}+a)^{2}}$	24
parallelrisch	$\frac{-4c^{3}dx+c^{2}ae}{2c^{2}(c\,x^{2}+a)^{2}}$	29
orering	$-\frac{(-4cdx+ae)(6c^{2}d\,x^{2}-2acex-2acd)}{4c(-3cd\,x^{2}+aex+ad)(c\,x^{2}+a)^{2}}$	62

input
```
int((6*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-4*c*d*x+a*e)/(c*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx=-\frac{4\,cdx-ae}{2\,(c^2x^4+2\,acx^2+a^2)}$$

input
```
integrate((6*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^3,x, algorithm="fricas
")
```

output
```
-1/2*(4*c*d*x - a*e)/(c^2*x^4 + 2*a*c*x^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{-2acd - 2acex + 6c^2dx^2}{(a + cx^2)^3} \, dx = \frac{ae - 4cdx}{2a^2 + 4acx^2 + 2c^2x^4}$$

input
```
integrate((6*c**2*d*x**2-2*a*c*e*x-2*a*c*d)/(c*x**2+a)**3,x)
```

output
```
(a*e - 4*c*d*x)/(2*a**2 + 4*a*c*x**2 + 2*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{-2acd - 2acex + 6c^2dx^2}{(a + cx^2)^3} \, dx = -\frac{4\,cdx - ae}{2\,(c^2x^4 + 2\,acx^2 + a^2)}$$

input
```
integrate((6*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^3,x, algorithm="maxima
")
```

output
```
-1/2*(4*c*d*x - a*e)/(c^2*x^4 + 2*a*c*x^2 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{-2acd - 2acex + 6c^2dx^2}{(a + cx^2)^3} \, dx = -\frac{4\,cdx - ae}{2\,(cx^2 + a)^2}$$

input
```
integrate((6*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(4*c*d*x - a*e)/(c*x^2 + a)^2
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx = \frac{e}{2\,(c\,x^2+a)} - \frac{c\,x\,(4\,d+e\,x)}{2\,(c\,x^2+a)^2}$$

input
```
int(-(2*a*c*d - 6*c^2*d*x^2 + 2*a*c*e*x)/(a + c*x^2)^3,x)
```

output
```
e/(2*(a + c*x^2)) - (c*x*(4*d + e*x))/(2*(a + c*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{-2acd-2acex+6c^2dx^2}{(a+cx^2)^3}\,dx = \frac{-4cdx+ae}{2c^2x^4+4ac\,x^2+2a^2}$$

input
```
int((6*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^3,x)
```

output
```
(a*e - 4*c*d*x)/(2*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.11 $\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 22

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx=\frac{ae-6cdx}{3(a+cx^2)^3}$$

output
```
1/3*(-6*c*d*x+a*e)/(c*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx=-\frac{-ae+6cdx}{3(a+cx^2)^3}$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x + 10*c^2*d*x^2)/(a + c*x^2)^4,x]
```

output
```
-1/3*(-(a*e) + 6*c*d*x)/(a + c*x^2)^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2345, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{ae-6cdx}{3(a+cx^2)^3}-\frac{\int 0dx}{6a}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-6cdx}{3(a+cx^2)^3}$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x + 10*c^2*d*x^2)/(a + c*x^2)^4,x]
```

output
```
(a*e - 6*c*d*x)/(3*(a + c*x^2)^3)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{-6cdx+ae}{3(cx^2+a)^3}$	21
norman	$\frac{\frac{ae}{3}-2cdx}{(cx^2+a)^3}$	21
risch	$\frac{\frac{ae}{3}-2cdx}{(cx^2+a)^3}$	21
default	$-\frac{2c\left(dx-\frac{ae}{6c}\right)}{(cx^2+a)^3}$	24
parallelrisch	$\frac{-6c^4dx+c^3ae}{3c^3(cx^2+a)^3}$	29
orering	$-\frac{(-6cdx+ae)(10c^2dx^2-2acex-2acd)}{6c(-5cdx^2+aex+ad)(cx^2+a)^3}$	62

input
```
int((10*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-6*c*d*x+a*e)/(c*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx = -\frac{6\,cdx-ae}{3\,(c^3x^6+3\,ac^2x^4+3\,a^2cx^2+a^3)}$$

input
```
integrate((10*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^4,x, algorithm="frica
s")
```

output
```
-1/3*(6*c*d*x - a*e)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{-2acd - 2acex + 10c^2dx^2}{(a + cx^2)^4}\,dx = \frac{ae - 6cdx}{3a^3 + 9a^2cx^2 + 9ac^2x^4 + 3c^3x^6}$$

input
```
integrate((10*c**2*d*x**2-2*a*c*e*x-2*a*c*d)/(c*x**2+a)**4,x)
```

output
```
(a*e - 6*c*d*x)/(3*a**3 + 9*a**2*c*x**2 + 9*a*c**2*x**4 + 3*c**3*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{-2acd - 2acex + 10c^2dx^2}{(a + cx^2)^4}\,dx = -\frac{6\,cdx - ae}{3\,(c^3x^6 + 3\,ac^2x^4 + 3\,a^2cx^2 + a^3)}$$

input
```
integrate((10*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^4,x, algorithm="maxim
a")
```

output
```
-1/3*(6*c*d*x - a*e)/(c^3*x^6 + 3*a*c^2*x^4 + 3*a^2*c*x^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{-2acd - 2acex + 10c^2dx^2}{(a + cx^2)^4}\,dx = -\frac{6\,cdx - ae}{3\,(cx^2 + a)^3}$$

input
```
integrate((10*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^4,x, algorithm="giac"
)
```

output
```
-1/3*(6*c*d*x - a*e)/(c*x^2 + a)^3
```

Mupad [B] (verification not implemented)

Time = 15.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx = \frac{e}{3\,(c\,x^2+a)^2} - \frac{c\,x\,(6\,d+e\,x)}{3\,(c\,x^2+a)^3}$$

input
```
int(-(2*a*c*d - 10*c^2*d*x^2 + 2*a*c*e*x)/(a + c*x^2)^4,x)
```

output
```
e/(3*(a + c*x^2)^2) - (c*x*(6*d + e*x))/(3*(a + c*x^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{-2acd-2acex+10c^2dx^2}{(a+cx^2)^4}\,dx = \frac{-6cdx+ae}{3c^3x^6+9a\,c^2x^4+9a^2c\,x^2+3a^3}$$

input
```
int((10*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^4,x)
```

output
```
(a*e - 6*c*d*x)/(3*(a**3 + 3*a**2*c*x**2 + 3*a*c**2*x**4 + c**3*x**6))
```

3.12 $\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5} dx$

Optimal result

Integrand size = 31, antiderivative size = 22

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5} dx = \frac{ae-8cdx}{4(a+cx^2)^4}$$

output
```
1/4*(-8*c*d*x+a*e)/(c*x^2+a)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5} dx = -\frac{-ae+8cdx}{4(a+cx^2)^4}$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x + 14*c^2*d*x^2)/(a + c*x^2)^5,x]
```

output
```
-1/4*(-(a*e) + 8*c*d*x)/(a + c*x^2)^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2345, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{ae-8cdx}{4(a+cx^2)^4}-\frac{\int 0dx}{8a}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-8cdx}{4(a+cx^2)^4}$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x + 14*c^2*d*x^2)/(a + c*x^2)^5,x]
```

output
```
(a*e - 8*c*d*x)/(4*(a + c*x^2)^4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{-8cdx+ae}{4(c\,x^2+a)^4}$	21
norman	$\frac{\frac{ae}{4}-2cdx}{(c\,x^2+a)^4}$	21
risch	$\frac{\frac{ae}{4}-2cdx}{(c\,x^2+a)^4}$	21
default	$-\frac{2c\left(dx-\frac{ae}{8c}\right)}{(c\,x^2+a)^4}$	24
parallelrisch	$\frac{-8c^5dx+a\,c^4e}{4c^4(c\,x^2+a)^4}$	29
orering	$-\frac{(-8cdx+ae)(14c^2d\,x^2-2acex-2acd)}{8c(c\,x^2+a)^4(-7cd\,x^2+aex+ad)}$	62

input
```
int((14*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-8*c*d*x+a*e)/(c*x^2+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5}\,dx=-\frac{8\,cdx-ae}{4\,(c^4x^8+4\,ac^3x^6+6\,a^2c^2x^4+4\,a^3cx^2+a^4)}$$

input
```
integrate((14*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^5,x, algorithm="frica
s")
```

output
```
-1/4*(8*c*d*x - a*e)/(c^4*x^8 + 4*a*c^3*x^6 + 6*a^2*c^2*x^4 + 4*a^3*c*x^2
+ a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{-2acd - 2acex + 14c^2dx^2}{(a + cx^2)^5}\,dx = \frac{ae - 8cdx}{4a^4 + 16a^3cx^2 + 24a^2c^2x^4 + 16ac^3x^6 + 4c^4x^8}$$

input
```
integrate((14*c**2*d*x**2-2*a*c*e*x-2*a*c*d)/(c*x**2+a)**5,x)
```

output
```
(a*e - 8*c*d*x)/(4*a**4 + 16*a**3*c*x**2 + 24*a**2*c**2*x**4 + 16*a*c**3*x
**6 + 4*c**4*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{-2acd - 2acex + 14c^2dx^2}{(a + cx^2)^5}\,dx = -\frac{8\,cdx - ae}{4\,(c^4x^8 + 4\,ac^3x^6 + 6\,a^2c^2x^4 + 4\,a^3cx^2 + a^4)}$$

input
```
integrate((14*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^5,x, algorithm="maxim
a")
```

output
```
-1/4*(8*c*d*x - a*e)/(c^4*x^8 + 4*a*c^3*x^6 + 6*a^2*c^2*x^4 + 4*a^3*c*x^2
+ a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5}\,dx=-\frac{8\,cdx-ae}{4\,(cx^2+a)^4}$$

input
```
integrate((14*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^5,x, algorithm="giac"
)
```

output
```
-1/4*(8*c*d*x - a*e)/(c*x^2 + a)^4
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5}\,dx=\frac{e}{4\,(c\,x^2+a)^3}-\frac{c\,x\,(8\,d+e\,x)}{4\,(c\,x^2+a)^4}$$

input
```
int(-(2*a*c*d - 14*c^2*d*x^2 + 2*a*c*e*x)/(a + c*x^2)^5,x)
```

output
```
e/(4*(a + c*x^2)^3) - (c*x*(8*d + e*x))/(4*(a + c*x^2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{-2acd-2acex+14c^2dx^2}{(a+cx^2)^5}\,dx=\frac{-8cdx+ae}{4c^4x^8+16a\,c^3x^6+24a^2c^2x^4+16a^3c\,x^2+4a^4}$$

input
```
int((14*c^2*d*x^2-2*a*c*e*x-2*a*c*d)/(c*x^2+a)^5,x)
```

output
```
(a*e - 8*c*d*x)/(4*(a**4 + 4*a**3*c*x**2 + 6*a**2*c**2*x**4 + 4*a*c**3*x**
6 + c**4*x**8))
```

3.13 $\int (a+bx+cx^2)^p (d(2b^2-2ac+b^2p)-2c^2d(3+2p)$

Optimal result

Integrand size = 45, antiderivative size = 34

$$\int (a+bx+cx^2)^p \left(d\left(2b^2-2ac+b^2p\right)-2c^2d(3+2p)x^2\right)\,dx = \frac{d(b(2+p)-2c(1+p)x)\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

output
```
d*(b*(2+p)-2*c*(p+1)*x)*(c*x^2+b*x+a)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a+bx+cx^2)^p \left(d\left(2b^2-2ac+b^2p\right)-2c^2d(3+2p)x^2\right)\,dx = \frac{d(b(2+p)-2c(1+p)x)(a+x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(a + b*x + c*x^2)^p*(d*(2*b^2 - 2*a*c + b^2*p) - 2*c^2*d*(3 + 2*
p)*x^2),x]
```

output
```
(d*(b*(2 + p) - 2*c*(1 + p)*x)*(a + x*(b + c*x))^(1 + p))/(1 + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2192, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^p \left(d\left(-2ac+b^2p+2b^2\right)-2c^2d(2p+3)x^2\right) dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int bcd(p+2)(2p+3)(b+2cx)\left(cx^2+bx+a\right)^p dx}{c(2p+3)}-2cdx\left(a+bx+cx^2\right)^{p+1}$$

$$\Big\downarrow \; 27$$

$$bd(p+2)\int (b+2cx)\left(cx^2+bx+a\right)^p dx-2cdx\left(a+bx+cx^2\right)^{p+1}$$

$$\Big\downarrow \; 1104$$

$$\frac{bd(p+2)\left(a+bx+cx^2\right)^{p+1}}{p+1}-2cdx\left(a+bx+cx^2\right)^{p+1}$$

input
```
Int[(a + b*x + c*x^2)^p*(d*(2*b^2 - 2*a*c + b^2*p) - 2*c^2*d*(3 + 2*p)*x^2
),x]
```

output
```
(b*d*(2 + p)*(a + b*x + c*x^2)^(1 + p))/(1 + p) - 2*c*d*x*(a + b*x + c*x^2
)^(1 + p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{d\left(c\,x^{2}+bx+a\right)^{p+1}\left(-2cpx+bp-2cx+2b\right)}{p+1}$
risch	$\frac{d\left(-2p\,c^{2}x^{3}-bcp\,x^{2}-2x^{3}c^{2}-2acpx+b^{2}px+abp-2acx+2b^{2}x+2ab\right)\left(c\,x^{2}+bx+a\right)^{p}}{p+1}$
orering	$-\frac{\left(-2cpx+bp-2cx+2b\right)\left(c\,x^{2}+bx+a\right)\left(c\,x^{2}+bx+a\right)^{p}\left(d\left(b^{2}p-2ac+2b^{2}\right)-2c^{2}d(3+2p)x^{2}\right)}{(p+1)(4c^{2}x^{2}p+6c^{2}x^{2}-b^{2}p+2ac-2b^{2})}$
norman	$\frac{dab(2+p)\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}}{p+1}-2c^{2}d\,x^{3}\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}-\frac{d\left(2acp-b^{2}p+2ac-2b^{2}\right)x\,\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}}{p+1}-\frac{bcdp\,x^{2}\mathrm{e}^{p\ln(c}}{p+1}$
parallelrisch	$-\frac{2x^{3}\left(c\,x^{2}+bx+a\right)^{p}a\,c^{2}dp+2x^{3}\left(c\,x^{2}+bx+a\right)^{p}a\,c^{2}d+x^{2}\left(c\,x^{2}+bx+a\right)^{p}abcdp+2x\left(c\,x^{2}+bx+a\right)^{p}a^{2}cdp-x\left(c\,x^{2}+bx+a\right)^{p}a\,b^{2}d}{(p+1)a}$

input
```
int((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)-2*c^2*d*(3+2*p)*x^2),x,method=_
RETURNVERBOSE)
```

output
```
d/(p+1)*(c*x^2+b*x+a)^(p+1)*(-2*c*p*x+b*p-2*c*x+2*b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right)\,dx = -\frac{(bcdpx^2-abdp+2\,(c^2dp+c^2d)x^3-2\,abd-((b^2-2\,ac)dp+2\,(b^2-ac)d)x)(cx^2+bx+a)^p}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)-2*c^2*d*(3+2*p)*x^2),x, a
lgorithm="fricas")
```

output
```
-(b*c*d*p*x^2 - a*b*d*p + 2*(c^2*d*p + c^2*d)*x^3 - 2*a*b*d - ((b^2 - 2*a*
c)*d*p + 2*(b^2 - a*c)*d)*x)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(31) = 62.

Time = 56.59 (sec) , antiderivative size = 280, normalized size of antiderivative = 8.24

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right)\,dx$$

$$= \begin{cases} \frac{abdp(a+bx+cx^2)^p}{p+1} + \frac{2abd(a+bx+cx^2)^p}{p+1} - \frac{2acdpx(a+bx+cx^2)^p}{p+1} - \frac{2acdx(a+bx+cx^2)^p}{p+1} + \frac{b^2dpx(a+bx+cx^2)^p}{p+1} + \frac{2b^2dx(a+bx+cx^2}{p+1} \\ bd\log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right)+bd\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)-2cdx \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**p*(d*(b**2*p-2*a*c+2*b**2)-2*c**2*d*(3+2*p)*x**2
),x)
```

output
```
Piecewise((a*b*d*p*(a + b*x + c*x**2)**p/(p + 1) + 2*a*b*d*(a + b*x + c*x*
*2)**p/(p + 1) - 2*a*c*d*p*x*(a + b*x + c*x**2)**p/(p + 1) - 2*a*c*d*x*(a
+ b*x + c*x**2)**p/(p + 1) + b**2*d*p*x*(a + b*x + c*x**2)**p/(p + 1) + 2*
b**2*d*x*(a + b*x + c*x**2)**p/(p + 1) - b*c*d*p*x**2*(a + b*x + c*x**2)**
p/(p + 1) - 2*c**2*d*p*x**3*(a + b*x + c*x**2)**p/(p + 1) - 2*c**2*d*x**3*
(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (b*d*log(b/(2*c) + x - sqrt(-4*
a*c + b**2)/(2*c)) + b*d*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)) - 2*
c*d*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right) dx = -\frac{(2\,c^2d(p+1)x^3+bcdpx^2-abd(p+2)-(b^2d(p+2)-2\,acd(p+1))x)(cx^2+bx+a)^p}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)-2*c^2*d*(3+2*p)*x^2),x, a
lgorithm="maxima")
```

output
```
-(2*c^2*d*(p + 1)*x^3 + b*c*d*p*x^2 - a*b*d*(p + 2) - (b^2*d*(p + 2) - 2*a
*c*d*(p + 1))*x)*(c*x^2 + b*x + a)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.35

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right) dx = -\frac{2\,(cx^2+bx+a)^pc^2dpx^3+(cx^2+bx+a)^pbcdpx^2+2\,(cx^2+bx+a)^pc^2dx^3-(cx^2+bx+a)^pb^2dpx+}{}$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)-2*c^2*d*(3+2*p)*x^2),x, a
lgorithm="giac")
```

output
```
-(2*(c*x^2 + b*x + a)^p*c^2*d*p*x^3 + (c*x^2 + b*x + a)^p*b*c*d*p*x^2 + 2*
(c*x^2 + b*x + a)^p*c^2*d*x^3 - (c*x^2 + b*x + a)^p*b^2*d*p*x + 2*(c*x^2 +
 b*x + a)^p*a*c*d*p*x - (c*x^2 + b*x + a)^p*a*b*d*p - 2*(c*x^2 + b*x + a)^
p*b^2*d*x + 2*(c*x^2 + b*x + a)^p*a*c*d*x - 2*(c*x^2 + b*x + a)^p*a*b*d)/(
p + 1)
```

Mupad [B] (verification not implemented)

Time = 15.98 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right) dx = -(c\,x^2+b\,x+a)^p \left(2\,c^2\,d\,x^3+\frac{d\,x\,(2\,a\,c-b^2\,p-2\,b^2+2\,a\,c\,p)}{p+1}-\frac{a\,b\,d\,(p+2)}{p+1}+\frac{b\,c\,d\,p\,x^2}{p+1}\right)$$

input
```
int((d*(b^2*p - 2*a*c + 2*b^2) - 2*c^2*d*x^2*(2*p + 3))*(a + b*x + c*x^2)^
p,x)
```

output
```
-(a + b*x + c*x^2)^p*(2*c^2*d*x^3 + (d*x*(2*a*c - b^2*p - 2*b^2 + 2*a*c*p)
)/(p + 1) - (a*b*d*(p + 2))/(p + 1) + (b*c*d*p*x^2)/(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)-2c^2d(3+2p)x^2\right) dx = \frac{(c\,x^2+bx+a)^p\,d(-2c^2p\,x^3-bcp\,x^2-2c^2x^3-2acpx+b^2px+abp-2acx+2b^2x+2ab)}{p+1}$$

input
```
int((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)-2*c^2*d*(3+2*p)*x^2),x)
```

output
```
((a + b*x + c*x**2)**p*d*(a*b*p + 2*a*b - 2*a*c*p*x - 2*a*c*x + b**2*p*x +
 2*b**2*x - b*c*p*x**2 - 2*c**2*p*x**3 - 2*c**2*x**3))/(p + 1)
```

3.14 $\int (a+bx+cx^2)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}} (d+fx^2)\,dx$

Optimal result

Integrand size = 52, antiderivative size = 119

$$\int (a+bx+cx^2)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}} (d+fx^2)\,dx = -\frac{(4c^2d+b^2f)\,(b(cd+af)+(2c^2d+b^2f-2acf)\,x)\,(a+bx+cx^2)^{-\frac{2c^2d+b^2f-2acf}{4c^2d+b^2f}}}{c\,(b^2-4ac)\,(2c^2d+b^2f-2acf)}$$

output
```
-(b^2*f+4*c^2*d)*(b*(a*f+c*d)+(-2*a*c*f+b^2*f+2*c^2*d)*x)/c/(-4*a*c+b^2)/(
-2*a*c*f+b^2*f+2*c^2*d)/((c*x^2+b*x+a)^((-2*a*c*f+b^2*f+2*c^2*d)/(b^2*f+4*
c^2*d)))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int (a+bx+cx^2)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}} (d+fx^2)\,dx = \frac{(4c^2d+b^2f)\,(b(cd+af)+b^2fx+2c(cd-af)x)\,(a+x(b+cx))^{-\frac{2c^2d+b^2f-2acf}{4c^2d+b^2f}}}{c\,(-b^2+4ac)\,(2c^2d+b^2f-2acf)}$$

input
```
Integrate[(a + b*x + c*x^2)^((-6*c^2*d - 2*b^2*f + 2*a*c*f)/(4*c^2*d + b^2
*f))*(d + f*x^2),x]
```

output
```
((4*c^2*d + b^2*f)*(b*(c*d + a*f) + b^2*f*x + 2*c*(c*d - a*f)*x))/(c*(-b^2
 + 4*a*c)*(2*c^2*d + b^2*f - 2*a*c*f)*(a + x*(b + c*x))^((2*c^2*d + b^2*f
- 2*a*c*f)/(4*c^2*d + b^2*f)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2192, 25, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+fx^2\right)\left(a+bx+cx^2\right)^{\frac{2acf-2b^2f-6c^2d}{b^2f+4c^2d}}dx$$

$$\Big\downarrow \text{2192}$$

$$-\frac{\left(b^2f+4c^2d\right)\int -\frac{bf(cd+af)(b+2cx)\left(cx^2+bx+a\right)^{-\frac{2\left(fb^2+3c^2d-acf\right)}{fb^2+4c^2d}}}{fb^2+4c^2d}dx}{cf\left(b^2-4ac\right)}-\frac{x\left(b^2f+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f+2c^2d}{b^2f+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(b^2f+4c^2d\right)\int \frac{bf(cd+af)(b+2cx)\left(cx^2+bx+a\right)^{-\frac{2\left(fb^2+3c^2d-acf\right)}{fb^2+4c^2d}}}{fb^2+4c^2d}dx}{cf\left(b^2-4ac\right)}-\frac{x\left(b^2f+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f+2c^2d}{b^2f+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(af+cd)\int (b+2cx)\left(cx^2+bx+a\right)^{-\frac{2\left(fb^2+3c^2d-acf\right)}{fb^2+4c^2d}}dx}{c\left(b^2-4ac\right)}-\frac{x\left(b^2f+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f+2c^2d}{b^2f+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{1104}$$

$$-\frac{x\left(b^2f+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f+2c^2d}{b^2f+4c^2d}}}{c\left(b^2-4ac\right)}-\frac{b(af+cd)\left(b^2f+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f+2c^2d}{b^2f+4c^2d}}}{c\left(b^2-4ac\right)\left(-2acf+b^2f+2c^2d\right)}$$

input
```
Int[(a + b*x + c*x^2)^((-6*c^2*d - 2*b^2*f + 2*a*c*f)/(4*c^2*d + b^2*f))*(
d + f*x^2),x]
```

output
```
-((b*(c*d + a*f)*(4*c^2*d + b^2*f))/(c*(b^2 - 4*a*c)*(2*c^2*d + b^2*f - 2*
a*c*f)*(a + b*x + c*x^2)^((2*c^2*d + b^2*f - 2*a*c*f)/(4*c^2*d + b^2*f))))
 - ((4*c^2*d + b^2*f)*x)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^((2*c^2*d + b^
2*f - 2*a*c*f)/(4*c^2*d + b^2*f)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

method	result
gosper	$-\frac{\left(b^{2} f+4 c^{2} d\right) \left(c \,x^{2}+b x +a \right)^{\frac{2 a c f -2 b^{2} f -6 c^{2} d}{b^{2} f +4 c^{2} d}+1} \left(-2 a c f x +b^{2} f x +2 c^{2} d x +a b f +d b c \right)}{c \left(8 a^{2} c^{2} f -6 a \,b^{2} c f -8 a \,c^{3} d +b^{4} f +2 b^{2} c^{2} d \right)}$
orering	$-\frac{\left(-2 a \,b^{2} c \,f^{2} x -8 a \,c^{3} d f x +b^{4} f^{2} x +6 b^{2} c^{2} d f x +8 c^{4} d^{2} x +a \,b^{3} f^{2}+4 a b \,c^{2} d f +b^{3} c d f +4 b \,c^{3} d^{2}\right) \left(c \,x^{2}+b x +a \right) \left(c \,x^{2}+b x +a \right)^{\frac{2 a c f -2 b}{b^{2} f +}}}{c \left(8 a^{2} c^{2} f -6 a \,b^{2} c f -8 a \,c^{3} d +b^{4} f +2 b^{2} c^{2} d \right)}$
risch	$-\frac{\left(b^{2} f+4 c^{2} d\right) \left(-2 a \,c^{2} f \,x^{3}+b^{2} c f \,x^{3}+2 c^{3} d \,x^{3}-a b c f \,x^{2}+b^{3} f \,x^{2}+3 b \,c^{2} d \,x^{2}-2 a^{2} c f x +2 a \,b^{2} f x +2 a d x \,c^{2}+b^{2} c x d +f \,a^{2} b +a b c d \right)}{c \left(4 a c -b^{2}\right) \left(2 a c f -b^{2} f -2 c^{2} d \right)}$
norman	$\frac{\left(b^{2} f +4 c^{2} d \right) x^{3} {\mathrm e}^{\frac{\left(2 a c f -2 b^{2} f -6 c^{2} d \right) \ln \left(c \,x^{2}+b x +a \right)}{b^{2} f +4 c^{2} d}}}{4 a c -b^{2}}+\frac{\left(2 a^{2} b^{2} c \,f^{2}+8 a^{2} c^{3} d f -2 a \,b^{4} f^{2}-10 a \,b^{2} c^{2} d f -8 a \,c^{4} d^{2}-b^{4} c d f -4 c^{3} b^{2} d^{2}\right) x}{c \left(8 a^{2} c^{2} f -6 a \,b^{2} c f -8 a \,c^{3} d +b^{4} f +2 b^{2} c}$
parallelrisch	Expression too large to display

input
```
int((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f-6*c^2*d)/(b^2*f+4*c^2*d))*(f*x^2+d),x,
method=_RETURNVERBOSE)
```

output
```
-1/c*(b^2*f+4*c^2*d)*(c*x^2+b*x+a)^(1+2*(a*c*f-b^2*f-3*c^2*d)/(b^2*f+4*c^2
*d))/(8*a^2*c^2*f-6*a*b^2*c*f-8*a*c^3*d+b^4*f+2*b^2*c^2*d)*(-2*a*c*f*x+b^2
*f*x+2*c^2*d*x+a*b*f+b*c*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(120) = 240$.

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.52

$$\int (a + bx + cx^2)^{\frac{-6c^2d - 2b^2f + 2acf}{4c^2d + b^2f}} (d + fx^2)\, dx =$$
$$-\frac{4\,abc^3d^2 + a^2b^3f^2 + (8\,c^5d^2 + 2\,(3\,b^2c^3 - 4\,ac^4)df + (b^4c - 2\,ab^2c^2)f^2)x^3 + (ab^3c + 4\,a^2bc^2)df + (12\,bc}{(2\,(b^2c^3 - 4\,ac^4)d + (b^4c - 6\,a}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f-6*c^2*d)/(b^2*f+4*c^2*d))*(f*x^2
+d),x, algorithm="fricas")
```

output
```
-(4*a*b*c^3*d^2 + a^2*b^3*f^2 + (8*c^5*d^2 + 2*(3*b^2*c^3 - 4*a*c^4)*d*f +
 (b^4*c - 2*a*b^2*c^2)*f^2)*x^3 + (a*b^3*c + 4*a^2*b*c^2)*d*f + (12*b*c^4*
d^2 + (7*b^3*c^2 - 4*a*b*c^3)*d*f + (b^5 - a*b^3*c)*f^2)*x^2 + (4*(b^2*c^3
 + 2*a*c^4)*d^2 + (b^4*c + 10*a*b^2*c^2 - 8*a^2*c^3)*d*f + 2*(a*b^4 - a^2*
b^2*c)*f^2)*x)/((2*(b^2*c^3 - 4*a*c^4)*d + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^
3)*f)*(c*x^2 + b*x + a)^(2*(3*c^2*d + (b^2 - a*c)*f)/(4*c^2*d + b^2*f)))
```

Sympy [F(-1)]

Timed out.

$$\int (a + bx + cx^2)^{\frac{-6c^2d - 2b^2f + 2acf}{4c^2d + b^2f}} (d + fx^2)\, dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**((2*a*c*f-2*b**2*f-6*c**2*d)/(b**2*f+4*c**2*d))*
(f*x**2+d),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(120) = 240$.

Time = 0.14 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.99

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}}\left(d+fx^2\right)\,dx =$$

$$-\frac{((8\,c^5d^2+6\,b^2c^3df+b^4cf^2-2\,(4\,c^4df+b^2c^2f^2)a)x^3+(4\,bc^2df+b^3f^2)a^2+(12\,bc^4d^2+7\,b^3c^2df+b^5}{}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f-6*c^2*d)/(b^2*f+4*c^2*d))*(f*x^2
+d),x, algorithm="maxima")
```

output
```
-((8*c^5*d^2 + 6*b^2*c^3*d*f + b^4*c*f^2 - 2*(4*c^4*d*f + b^2*c^2*f^2)*a)*
x^3 + (4*b*c^2*d*f + b^3*f^2)*a^2 + (12*b*c^4*d^2 + 7*b^3*c^2*d*f + b^5*f^
2 - (4*b*c^3*d*f + b^3*c*f^2)*a)*x^2 + (4*b*c^3*d^2 + b^3*c*d*f)*a + (4*b^
2*c^3*d^2 + b^4*c*d*f - 2*(4*c^3*d*f + b^2*c*f^2)*a^2 + 2*(4*c^4*d^2 + 5*b
^2*c^2*d*f + b^4*f^2)*a)*x)*e^(-6*c^2*d*log(c*x^2 + b*x + a)/(4*c^2*d + b^
2*f) - 2*b^2*f*log(c*x^2 + b*x + a)/(4*c^2*d + b^2*f) + 2*a*c*f*log(c*x^2
+ b*x + a)/(4*c^2*d + b^2*f))/(2*b^2*c^3*d + b^4*c*f + 8*a^2*c^3*f - 2*(4*
c^4*d + 3*b^2*c^2*f)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(120) = 240$.

Time = 0.53 (sec) , antiderivative size = 1690, normalized size of antiderivative = 14.20

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}}\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f-6*c^2*d)/(b^2*f+4*c^2*d))*(f*x^2
+d),x, algorithm="giac")
```

output

```
-(8*c^5*d^2*x^3*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*
x + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) + 6*b^2*c^3*d*f*x^
3*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f
*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) - 8*a*c^4*d*f*x^3*e^(-2*(3*c^2*d
*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*x
 + a))/(4*c^2*d + b^2*f)) + b^4*c*f^2*x^3*e^(-2*(3*c^2*d*log(c*x^2 + b*x +
 a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d +
b^2*f)) - 2*a*b^2*c^2*f^2*x^3*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*
log(c*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) + 12
*b*c^4*d^2*x^2*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x
 + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) + 7*b^3*c^2*d*f*x^2
*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f*
log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) - 4*a*b*c^3*d*f*x^2*e^(-2*(3*c^2*
d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*
x + a))/(4*c^2*d + b^2*f)) + b^5*f^2*x^2*e^(-2*(3*c^2*d*log(c*x^2 + b*x +
a) + b^2*f*log(c*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b
^2*f)) - a*b^3*c*f^2*x^2*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c
*x^2 + b*x + a) - a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) + 4*b^2*c
^3*d^2*x*e^(-2*(3*c^2*d*log(c*x^2 + b*x + a) + b^2*f*log(c*x^2 + b*x + a)
- a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d + b^2*f)) + 8*a*c^4*d^2*x*e^(-2*...
```

Mupad [B] (verification not implemented)

Time = 16.37 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.32

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}}\left(d+fx^2\right)\,dx$$
$$=\frac{\frac{x^3\,\left(f\,b^2+4\,d\,c^2\right)}{4\,a\,c-b^2}+\frac{x\,\left(f\,b^2+4\,d\,c^2\right)\,\left(-2\,f\,a^2\,c+2\,f\,a\,b^2+2\,d\,a\,c^2+d\,b^2\,c\right)}{c\,\left(4\,a\,c-b^2\right)\,\left(f\,b^2+2\,d\,c^2-2\,a\,f\,c\right)}+\frac{b\,x^2\,\left(f\,b^2+4\,d\,c^2\right)\,\left(f\,b^2+3\,d\,c^2-a\,f\,c\right)}{c\,\left(4\,a\,c-b^2\right)\,\left(f\,b^2+2\,d\,c^2-2\,a\,f\,c\right)}+\frac{a\,b\,\left(f\,b^2+4\,d\,c^2\right)\,\left(a\,f\right.}{c\,\left(4\,a\,c-b^2\right)\,\left(f\,b^2+2\,d\,c^2\right.}}{{\left(c\,x^2+b\,x+a\right)}^{\frac{2\,f\,b^2+6\,d\,c^2-2\,a\,f\,c}{f\,b^2+4\,d\,c^2}}}$$

input

```
int((d + f*x^2)/(a + b*x + c*x^2)^((6*c^2*d + 2*b^2*f - 2*a*c*f)/(4*c^2*d
+ b^2*f)),x)
```

output
```
((x^3*(4*c^2*d + b^2*f))/(4*a*c - b^2) + (x*(4*c^2*d + b^2*f)*(2*a*c^2*d +
 2*a*b^2*f + b^2*c*d - 2*a^2*c*f))/(c*(4*a*c - b^2)*(2*c^2*d + b^2*f - 2*a
*c*f)) + (b*x^2*(4*c^2*d + b^2*f)*(3*c^2*d + b^2*f - a*c*f))/(c*(4*a*c - b
^2)*(2*c^2*d + b^2*f - 2*a*c*f)) + (a*b*(4*c^2*d + b^2*f)*(a*f + c*d))/(c*
(4*a*c - b^2)*(2*c^2*d + b^2*f - 2*a*c*f)))/(a + b*x + c*x^2)^((6*c^2*d +
2*b^2*f - 2*a*c*f)/(4*c^2*d + b^2*f))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.31

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d-2b^2f+2acf}{4c^2d+b^2f}} \left(d+fx^2\right) \, dx$$

$$= \frac{(c\,x^2+bx+a)^{\frac{2acf+2c^2d}{b^2f+4c^2d}} \left(2a\,b^2c\,f^2x+8a\,c^3dfx-b^4f^2x-6b^2c^2dfx-8c^4d^2x-a\right.}{c\,(8a^2c^3f\,x^2-6a\,b^2c^2f\,x^2-8a\,c^4d\,x^2+b^4cf\,x^2+2b^2c^3d\,x^2+8a^2b\,c^2fx-6a\,b^3cfx-8ab\,c^3dx+b^5fx+}$$

input
```
int((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f-6*c^2*d)/(b^2*f+4*c^2*d))*(f*x^2+d),x)
```

output
```
((a + b*x + c*x**2)**((2*a*c*f + 2*c**2*d)/(b**2*f + 4*c**2*d))*( - a*b**3
*f**2 + 2*a*b**2*c*f**2*x - 4*a*b*c**2*d*f + 8*a*c**3*d*f*x - b**4*f**2*x
- b**3*c*d*f - 6*b**2*c**2*d*f*x - 4*b*c**3*d**2 - 8*c**4*d**2*x))/(c*(8*a
**3*c**2*f - 6*a**2*b**2*c*f + 8*a**2*b*c**2*f*x - 8*a**2*c**3*d + 8*a**2*
c**3*f*x**2 + a*b**4*f - 6*a*b**3*c*f*x + 2*a*b**2*c**2*d - 6*a*b**2*c**2*
f*x**2 - 8*a*b*c**3*d*x - 8*a*c**4*d*x**2 + b**5*f*x + b**4*c*f*x**2 + 2*b
**3*c**2*d*x + 2*b**2*c**3*d*x**2))
```

3.15 $\int (a+bx+cx^2)^2 \left((4b^2-2ac)d-14c^2dx^2\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 25

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)d-14c^2dx^2\right) dx = \frac{2}{3}d(2b-3cx)\left(a+bx+cx^2\right)^3$$

output
```
2/3*d*(-3*c*x+2*b)*(c*x^2+b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(25) = 50.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\begin{aligned}\int (a+bx+cx^2)^2 \left((4b^2-2ac)d-14c^2dx^2\right) dx \\ = -2d\bigg(a^3cx+ax^2(b+cx)^2(-2b+3cx)+\frac{1}{3}x^3(b+cx)^3(-2b+3cx) \\ +a^2x\left(-2b^2+bcx+3c^2x^2\right)\bigg)\end{aligned}$$

input
```
Integrate[(a + b*x + c*x^2)^2*((4*b^2 - 2*a*c)*d - 14*c^2*d*x^2),x]
```

output
```
-2*d*(a^3*c*x + a*x^2*(b + c*x)^2*(-2*b + 3*c*x) + (x^3*(b + c*x)^3*(-2*b
+ 3*c*x))/3 + a^2*x*(-2*b^2 + b*c*x + 3*c^2*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. $2(25) = 50$.

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)^2 \left(d(4b^2-2ac)-14c^2dx^2\right) dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-2a^2d(ac-2b^2)-10c^2dx^4(3ac+b^2)+8bcdx^3(b^2-4ac)+2dx^2(2b^2-3ac)(3ac+b^2)-4abdx(ac-2b^2)\right.$$

$$\Big\downarrow \text{2009}$$

$$2a^2dx(2b^2-ac)-2c^2dx^5(3ac+b^2)+2bcdx^4(b^2-4ac)+\frac{2}{3}dx^3(2b^2-3ac)(3ac+b^2)+ 2abdx^2(2b^2-ac)-\frac{14}{3}bc^3dx^6-2c^4dx^7$$

input
```
Int[(a + b*x + c*x^2)^2*((4*b^2 - 2*a*c)*d - 14*c^2*d*x^2),x]
```

output
```
2*a^2*(2*b^2 - a*c)*d*x + 2*a*b*(2*b^2 - a*c)*d*x^2 + (2*(2*b^2 - 3*a*c)*(
b^2 + 3*a*c)*d*x^3)/3 + 2*b*c*(b^2 - 4*a*c)*d*x^4 - 2*c^2*(b^2 + 3*a*c)*d*
x^5 - (14*b*c^3*d*x^6)/3 - 2*c^4*d*x^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(23) = 46.

Time = 1.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.80

method	result
gosper	$-\frac{2dx(3c^4x^6+7bc^3x^5+9ac^3x^4+3b^2c^2x^4+12abc^2x^3-3b^3cx^3+9a^2c^2x^2-3ab^2cx^2-2b^4x^2+3a^2bcx-6ab^3x+3ca^3-6a^2b^2)}{3}$
norman	$(-6a^2c^2d+2cdab^2+\frac{4}{3}b^4d)x^3+(-6ac^3d-2b^2c^2d)x^5+(-8abc^2d+2b^3cd)x^4+(-2a^2b$
risch	$-2c^4dx^7-\frac{14}{3}bc^3dx^6-6ac^3dx^5-2b^2c^2dx^5-8abc^2dx^4+2b^3cdx^4-6a^2c^2dx^3+2ab^2c$
parallelrisch	$-2c^4dx^7-\frac{14}{3}bc^3dx^6-6ac^3dx^5-2b^2c^2dx^5-8abc^2dx^4+2b^3cdx^4-6a^2c^2dx^3+2ab^2c$
default	$-2c^4dx^7-\frac{14bc^3dx^6}{3}+\frac{(-14(2ac+b^2)c^2d+c^2(-2ac+4b^2)d)x^5}{5}+\frac{(-28abc^2d+2bc(-2ac+4b^2)d)x^4}{4}+\frac{(-14a^2c^2}{}$
orering	$\frac{x(3c^4x^6+7bc^3x^5+9ac^3x^4+3b^2c^2x^4+12abc^2x^3-3b^3cx^3+9a^2c^2x^2-3ab^2cx^2-2b^4x^2+3a^2bcx-6ab^3x+3ca^3-6a^2b^2)((-2c}{21c^2x^2+3ac-6b^2}$

input
```
int((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d-14*c^2*d*x^2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*d*x*(3*c^4*x^6+7*b*c^3*x^5+9*a*c^3*x^4+3*b^2*c^2*x^4+12*a*b*c^2*x^3-3
*b^3*c*x^3+9*a^2*c^2*x^2-3*a*b^2*c*x^2-2*b^4*x^2+3*a^2*b*c*x-6*a*b^3*x+3*a
^3*c-6*a^2*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(23) = 46$.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.00

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)d-14c^2dx^2\right) dx$$
$$= -2\,c^4dx^7 - \frac{14}{3}\,bc^3dx^6 - 2\left(b^2c^2+3\,ac^3\right)dx^5 + 2\left(b^3c-4\,abc^2\right)dx^4 + \frac{2}{3}\left(2\,b^4+3\,ab^2c-9\,a^2c^2\right)dx^3 + 2\left(2\,ab^3-a^2bc\right)dx^2 + 2\left(2\,a^2b^2-a^3c\right)dx$$

input
```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d-14*c^2*d*x^2),x, algorithm="fr
icas")
```

output
```
-2*c^4*d*x^7 - 14/3*b*c^3*d*x^6 - 2*(b^2*c^2 + 3*a*c^3)*d*x^5 + 2*(b^3*c -
 4*a*b*c^2)*d*x^4 + 2/3*(2*b^4 + 3*a*b^2*c - 9*a^2*c^2)*d*x^3 + 2*(2*a*b^3
 - a^2*b*c)*d*x^2 + 2*(2*a^2*b^2 - a^3*c)*d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.72

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)d-14c^2dx^2\right) dx$$
$$= -\frac{14bc^3dx^6}{3} - 2c^4dx^7 + x^5\left(-6ac^3d-2b^2c^2d\right) + x^4\left(-8abc^2d+2b^3cd\right) + x^3\left(-6a^2c^2d+2ab^2cd+\frac{4b^4d}{3}\right) + x^2\left(-2a^2bcd+4ab^3d\right) + x\left(-2a^3cd+4a^2b^2d\right)$$

input
```
integrate((c*x**2+b*x+a)**2*((-2*a*c+4*b**2)*d-14*c**2*d*x**2),x)
```

output
```
-14*b*c**3*d*x**6/3 - 2*c**4*d*x**7 + x**5*(-6*a*c**3*d - 2*b**2*c**2*d) +
 x**4*(-8*a*b*c**2*d + 2*b**3*c*d) + x**3*(-6*a**2*c**2*d + 2*a*b**2*c*d +
 4*b**4*d/3) + x**2*(-2*a**2*b*c*d + 4*a*b**3*d) + x*(-2*a**3*c*d + 4*a**2
*b**2*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.00

$$\int \left(a+bx+cx^2\right)^2 \left(\left(4b^2-2ac\right)d-14c^2dx^2\right) dx = -2\,c^4dx^7 - \frac{14}{3}\,bc^3dx^6 - 2\left(b^2c^2+3\,ac^3\right)dx^5 + 2\left(b^3c-4\,abc^2\right)dx^4 + \frac{2}{3}\left(2\,b^4+3\,ab^2c-9\,a^2c^2\right)dx^3 + 2\left(2\,ab^3-a^2bc\right)dx^2 + 2\left(2\,a^2b^2-a^3c\right)dx$$

input
```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d-14*c^2*d*x^2),x, algorithm="ma
xima")
```

output
```
-2*c^4*d*x^7 - 14/3*b*c^3*d*x^6 - 2*(b^2*c^2 + 3*a*c^3)*d*x^5 + 2*(b^3*c -
 4*a*b*c^2)*d*x^4 + 2/3*(2*b^4 + 3*a*b^2*c - 9*a^2*c^2)*d*x^3 + 2*(2*a*b^3
 - a^2*b*c)*d*x^2 + 2*(2*a^2*b^2 - a^3*c)*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.36

$$\int \left(a+bx+cx^2\right)^2 \left(\left(4b^2-2ac\right)d-14c^2dx^2\right) dx = -2\,c^4dx^7 - \frac{14}{3}\,bc^3dx^6 - 2\,b^2c^2dx^5 - 6\,ac^3dx^5 + 2\,b^3cdx^4 - 8\,abc^2dx^4 + \frac{4}{3}\,b^4dx^3 + 2\,ab^2cdx^3 - 6\,a^2c^2dx^3 + 4\,ab^3dx^2 - 2\,a^2bcdx^2 + 4\,a^2b^2dx - 2\,a^3cdx$$

input
```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d-14*c^2*d*x^2),x, algorithm="gi
ac")
```

output
```
-2*c^4*d*x^7 - 14/3*b*c^3*d*x^6 - 2*b^2*c^2*d*x^5 - 6*a*c^3*d*x^5 + 2*b^3*
c*d*x^4 - 8*a*b*c^2*d*x^4 + 4/3*b^4*d*x^3 + 2*a*b^2*c*d*x^3 - 6*a^2*c^2*d*
x^3 + 4*a*b^3*d*x^2 - 2*a^2*b*c*d*x^2 + 4*a^2*b^2*d*x - 2*a^3*c*d*x
```

Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.60

$$\int \left(a+bx+cx^2\right)^2 \left(\left(4b^2-2ac\right)d-14c^2dx^2\right) \, dx \\ = \frac{2\,d\,x^3\,\left(-9\,a^2\,c^2+3\,a\,b^2\,c+2\,b^4\right)}{3}-2\,c^4\,d\,x^7-2\,a^2\,d\,x\,\left(a\,c-2\,b^2\right) \\ -2\,c^2\,d\,x^5\,\left(b^2+3\,a\,c\right)-\frac{14\,b\,c^3\,d\,x^6}{3}-2\,a\,b\,d\,x^2\,\left(a\,c-2\,b^2\right)-2\,b\,c\,d\,x^4\,\left(4\,a\,c-b^2\right)$$

input
```
int(-(d*(2*a*c - 4*b^2) + 14*c^2*d*x^2)*(a + b*x + c*x^2)^2,x)
```

output
```
(2*d*x^3*(2*b^4 - 9*a^2*c^2 + 3*a*b^2*c))/3 - 2*c^4*d*x^7 - 2*a^2*d*x*(a*c
 - 2*b^2) - 2*c^2*d*x^5*(3*a*c + b^2) - (14*b*c^3*d*x^6)/3 - 2*a*b*d*x^2*(
a*c - 2*b^2) - 2*b*c*d*x^4*(4*a*c - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.76

$$\int \left(a+bx+cx^2\right)^2 \left(\left(4b^2-2ac\right)d-14c^2dx^2\right) \, dx \\ = \frac{2dx(-3c^4x^6-7b\,c^3x^5-9a\,c^3x^4-3b^2c^2x^4-12ab\,c^2x^3+3b^3c\,x^3-9a^2c^2x^2+3a\,b^2c\,x^2+2b^4x^2-3a^2bcx}{3}$$

input
```
int((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d-14*c^2*d*x^2),x)
```

output
```
(2*d*x*( - 3*a**3*c + 6*a**2*b**2 - 3*a**2*b*c*x - 9*a**2*c**2*x**2 + 6*a*
b**3*x + 3*a*b**2*c*x**2 - 12*a*b*c**2*x**3 - 9*a*c**3*x**4 + 2*b**4*x**2
+ 3*b**3*c*x**3 - 3*b**2*c**2*x**4 - 7*b*c**3*x**5 - 3*c**4*x**6))/3
```

3.16 $\int (a+bx+cx^2) ((3b^2-2ac) d-10c^2dx^2) dx$

Optimal result

Integrand size = 33, antiderivative size = 25

$$\int (a+bx+cx^2) ((3b^2-2ac) d-10c^2dx^2) dx = \frac{1}{2}d(3b-4cx) (a+bx+cx^2)^2$$

output
```
1/2*d*(-4*c*x+3*b)*(c*x^2+b*x+a)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int (a+bx+cx^2) ((3b^2-2ac) d-10c^2dx^2) dx = -\frac{1}{2}dx\left(4a^2c+x(b+cx)^2(-3b+4cx)+a\left(-6b^2+2bcx+8c^2x^2\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)*((3*b^2 - 2*a*c)*d - 10*c^2*d*x^2),x]
```

output
```
-1/2*(d*x*(4*a^2*c + x*(b + c*x)^2*(-3*b + 4*c*x) + a*(-6*b^2 + 2*b*c*x +
8*c^2*x^2)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)\left(d(3b^2-2ac)-10c^2dx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-3cdx^2(4ac-b^2)+bdx(3b^2-2ac)-ad(2ac-3b^2)-10bc^2dx^3-10c^3dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cdx^3(b^2-4ac)+\frac{1}{2}bdx^2(3b^2-2ac)+adx(3b^2-2ac)-\frac{5}{2}bc^2dx^4-2c^3dx^5$$

input
```
Int[(a + b*x + c*x^2)*((3*b^2 - 2*a*c)*d - 10*c^2*d*x^2),x]
```

output
```
a*(3*b^2 - 2*a*c)*d*x + (b*(3*b^2 - 2*a*c)*d*x^2)/2 + c*(b^2 - 4*a*c)*d*x^
3 - (5*b*c^2*d*x^4)/2 - 2*c^3*d*x^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(23) = 46.

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.60

method	result	size
gosper	$-\frac{dx\left(4c^{3}x^{4}+5b\,c^{2}x^{3}+8a\,c^{2}x^{2}-2b^{2}c\,x^{2}+2abcx-3b^{3}x+4a^{2}c-6a\,b^{2}\right)}{2}$	65
norman	$\left(-abcd+\frac{3}{2}b^{3}d\right)x^{2}+\left(-4a\,c^{2}d+cd\,b^{2}\right)x^{3}+\left(-2a^{2}cd+3da\,b^{2}\right)x-2c^{3}d\,x^{5}-\frac{5b\,c^{2}d\,x^{4}}{2}$	73
risch	$-2c^{3}d\,x^{5}-\frac{5}{2}b\,c^{2}d\,x^{4}-4a\,c^{2}d\,x^{3}+b^{2}cd\,x^{3}-abcd\,x^{2}+\frac{3}{2}b^{3}d\,x^{2}-2a^{2}cdx+3da\,b^{2}x$	74
parallelrisch	$-2c^{3}d\,x^{5}-\frac{5}{2}b\,c^{2}d\,x^{4}-4a\,c^{2}d\,x^{3}+b^{2}cd\,x^{3}-abcd\,x^{2}+\frac{3}{2}b^{3}d\,x^{2}-2a^{2}cdx+3da\,b^{2}x$	74
default	$-2c^{3}d\,x^{5}-\frac{5b\,c^{2}d\,x^{4}}{2}+\frac{\left(-10a\,c^{2}d+c\left(-2ac+3b^{2}\right)d\right)x^{3}}{3}+\frac{b\left(-2ac+3b^{2}\right)d\,x^{2}}{2}+a\left(-2ac+3b^{2}\right)dx$	78
orering	$\frac{x\left(4c^{3}x^{4}+5b\,c^{2}x^{3}+8a\,c^{2}x^{2}-2b^{2}c\,x^{2}+2abcx-3b^{3}x+4a^{2}c-6a\,b^{2}\right)\left(\left(-2ac+3b^{2}\right)d-10c^{2}d\,x^{2}\right)}{20c^{2}x^{2}+4ac-6b^{2}}$	106

input
```
int((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d-10*c^2*d*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*x*(4*c^3*x^4+5*b*c^2*x^3+8*a*c^2*x^2-2*b^2*c*x^2+2*a*b*c*x-3*b^3*x+
4*a^2*c-6*a*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(23) = 46.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int (a+bx+cx^2)\left((3b^2-2ac)\,d-10c^2dx^2\right)\,dx$$
$$=-2\,c^3dx^5-\frac{5}{2}\,bc^2dx^4+\left(b^2c-4\,ac^2\right)dx^3+\frac{1}{2}\left(3\,b^3-2\,abc\right)dx^2+\left(3\,ab^2-2\,a^2c\right)dx$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d-10*c^2*d*x^2),x, algorithm="fric
as")
```

output
```
-2*c^3*d*x^5 - 5/2*b*c^2*d*x^4 + (b^2*c - 4*a*c^2)*d*x^3 + 1/2*(3*b^3 - 2*
a*b*c)*d*x^2 + (3*a*b^2 - 2*a^2*c)*d*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int (a+bx+cx^2)\left((3b^2-2ac)\,d-10c^2dx^2\right)\,dx$$
$$= -\frac{5bc^2dx^4}{2} - 2c^3dx^5 + x^3(-4ac^2d + b^2cd) + x^2\left(-abcd + \frac{3b^3d}{2}\right) + x(-2a^2cd + 3ab^2d)$$

input
```
integrate((c*x**2+b*x+a)*((-2*a*c+3*b**2)*d-10*c**2*d*x**2),x)
```

output
```
-5*b*c**2*d*x**4/2 - 2*c**3*d*x**5 + x**3*(-4*a*c**2*d + b**2*c*d) + x**2*
(-a*b*c*d + 3*b**3*d/2) + x*(-2*a**2*c*d + 3*a*b**2*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int (a+bx+cx^2)\left((3b^2-2ac)\,d-10c^2dx^2\right)\,dx$$
$$= -2\,c^3dx^5 - \frac{5}{2}\,bc^2dx^4 + (b^2c - 4\,ac^2)dx^3 + \frac{1}{2}\,(3\,b^3 - 2\,abc)dx^2 + (3\,ab^2 - 2\,a^2c)dx$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d-10*c^2*d*x^2),x, algorithm="maxi
ma")
```

output
```
-2*c^3*d*x^5 - 5/2*b*c^2*d*x^4 + (b^2*c - 4*a*c^2)*d*x^3 + 1/2*(3*b^3 - 2*
a*b*c)*d*x^2 + (3*a*b^2 - 2*a^2*c)*d*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.92

$$\int (a+bx+cx^2)\left((3b^2-2ac)d-10c^2dx^2\right)\,dx$$
$$= -2\,c^3dx^5 - \frac{5}{2}\,bc^2dx^4 + b^2cdx^3 - 4\,ac^2dx^3 + \frac{3}{2}\,b^3dx^2 - abcdx^2 + 3\,ab^2dx - 2\,a^2cdx$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d-10*c^2*d*x^2),x, algorithm="giac
")
```

output
```
-2*c^3*d*x^5 - 5/2*b*c^2*d*x^4 + b^2*c*d*x^3 - 4*a*c^2*d*x^3 + 3/2*b^3*d*x
^2 - a*b*c*d*x^2 + 3*a*b^2*d*x - 2*a^2*c*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int (a+bx+cx^2)\left((3b^2-2ac)d-10c^2dx^2\right)\,dx$$
$$= -2\,c^3\,d\,x^5 - a\,d\,x\,\left(2\,a\,c-3\,b^2\right) - \frac{b\,d\,x^2\,\left(2\,a\,c-3\,b^2\right)}{2} - c\,d\,x^3\,\left(4\,a\,c-b^2\right) - \frac{5\,b\,c^2\,d\,x^4}{2}$$

input
```
int(-(d*(2*a*c - 3*b^2) + 10*c^2*d*x^2)*(a + b*x + c*x^2),x)
```

output
```
- 2*c^3*d*x^5 - a*d*x*(2*a*c - 3*b^2) - (b*d*x^2*(2*a*c - 3*b^2))/2 - c*d*
x^3*(4*a*c - b^2) - (5*b*c^2*d*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int (a+bx+cx^2) \left((3b^2-2ac) d-10c^2dx^2\right) dx$$
$$= \frac{dx(-4c^3x^4-5b\,c^2x^3-8a\,c^2x^2+2b^2c\,x^2-2abcx+3b^3x-4a^2c+6a\,b^2)}{2}$$

input
```
int((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d-10*c^2*d*x^2),x)
```

output
```
(d*x*( - 4*a**2*c + 6*a*b**2 - 2*a*b*c*x - 8*a*c**2*x**2 + 3*b**3*x + 2*b*
*2*c*x**2 - 5*b*c**2*x**3 - 4*c**3*x**4))/2
```

3.17 $\int \left(\left(2 b^2-2 a c\right) d-6 c^2 d x^2\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \left(\left(2 b^2-2 a c\right) d-6 c^2 d x^2\right) \, dx=2 \left(b^2-a c\right) d x-2 c^2 d x^3$$

output
```
2*(-a*c+b^2)*d*x-2*c^2*d*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \left(\left(2 b^2-2 a c\right) d-6 c^2 d x^2\right) \, dx=2 b^2 d x-2 a c d x-2 c^2 d x^3$$

input
```
Integrate[(2*b^2 - 2*a*c)*d - 6*c^2*d*x^2,x]
```

output
```
2*b^2*d*x - 2*a*c*d*x - 2*c^2*d*x^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d\left(2b^2-2ac\right)-6c^2dx^2\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$2dx\left(b^2-ac\right)-2c^2dx^3$$

input
```
Int[(2*b^2 - 2*a*c)*d - 6*c^2*d*x^2,x]
```

output
```
2*(b^2 - a*c)*d*x - 2*c^2*d*x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$-2dx(c^2x^2+ac-b^2)$	21
default	$-2d(x^3c^2+acx-b^2x)$	22
risch	$-2c^2d\,x^3-2adxc+2b^2dx$	24
parallelrisch	$(-2ac+2b^2)\,dx-2c^2d\,x^3$	24
parts	$-2c^2d\,x^3-2adxc+2b^2dx$	24
norman	$(-2acd+2b^2d)\,x-2c^2d\,x^3$	25
orering	$\frac{x(c^2x^2+ac-b^2)((-2ac+2b^2)d-6c^2d\,x^2)}{3c^2x^2+ac-b^2}$	60

input
```
int((-2*a*c+2*b^2)*d-6*c^2*d*x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d*x*(c^2*x^2+a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left((2b^2-2ac)\,d-6c^2dx^2\right)\,dx=-2\,c^2dx^3+2\,(b^2-ac)dx$$

input
```
integrate((-2*a*c+2*b^2)*d-6*c^2*d*x^2,x, algorithm="fricas")
```

output
```
-2*c^2*d*x^3 + 2*(b^2 - a*c)*d*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(\left(2b^2-2ac\right)d-6c^2dx^2\right)\,dx=-2c^2dx^3+x\left(-2acd+2b^2d\right)$$

input
```
integrate((-2*a*c+2*b**2)*d-6*c**2*d*x**2,x)
```

output
```
-2*c**2*d*x**3 + x*(-2*a*c*d + 2*b**2*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(\left(2b^2-2ac\right)d-6c^2dx^2\right)\,dx=-2\,c^2dx^3+2\left(b^2-ac\right)dx$$

input
```
integrate((-2*a*c+2*b^2)*d-6*c^2*d*x^2,x, algorithm="maxima")
```

output
```
-2*c^2*d*x^3 + 2*(b^2 - a*c)*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(\left(2b^2-2ac\right)d-6c^2dx^2\right)\,dx=-2\,c^2dx^3+2\left(b^2-ac\right)dx$$

input
```
integrate((-2*a*c+2*b^2)*d-6*c^2*d*x^2,x, algorithm="giac")
```

output
```
-2*c^2*d*x^3 + 2*(b^2 - a*c)*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(\left(2b^2-2ac\right)d-6c^2dx^2\right)\,dx=-2\,c^2\,d\,x^3-d\,x\,\left(2\,a\,c-2\,b^2\right)$$

input
```
int(- d*(2*a*c - 2*b^2) - 6*c^2*d*x^2,x)
```

output
```
- 2*c^2*d*x^3 - d*x*(2*a*c - 2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(\left(2b^2-2ac\right)d-6c^2dx^2\right)\,dx=2dx\left(-c^2x^2-ac+b^2\right)$$

input
```
int((-2*a*c+2*b^2)*d-6*c^2*d*x^2,x)
```

output
```
2*d*x*( - a*c + b**2 - c**2*x**2)
```

3.18 $\int \frac{(b^2-2ac)d-2c^2dx^2}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = -2cdx + bd\log\left(a+bx+cx^2\right)$$

output
```
-2*c*d*x+b*d*ln(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = d\big(-2cx + b\log\left(a+bx+cx^2\right)\big)$$

input
```
Integrate[((b^2 - 2*a*c)*d - 2*c^2*d*x^2)/(a + b*x + c*x^2),x]
```

output
```
d*(-2*c*x + b*Log[a + b*x + c*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(b^2-2ac)-2c^2dx^2}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{b^2d+2bcdx}{a+bx+cx^2}-2cd\right)dx$$

$$\Big\downarrow \text{2009}$$

$$bd\log\left(a+bx+cx^2\right)-2cdx$$

input
```
Int[((b^2 - 2*a*c)*d - 2*c^2*d*x^2)/(a + b*x + c*x^2),x]
```

output
```
-2*c*d*x + b*d*Log[a + b*x + c*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
default	$d(-2cx+b\ln(c\,x^2+bx+a))$	21
norman	$-2cdx+bd\ln(c\,x^2+bx+a)$	21
risch	$-2cdx+bd\ln(c\,x^2+bx+a)$	21
parallelrisch	$-2cdx+bd\ln(c\,x^2+bx+a)$	21

input
```
int(((-2*a*c+b^2)*d-2*c^2*d*x^2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
d*(-2*c*x+b*ln(c*x^2+b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = -2\,cdx + bd\log\left(cx^2+bx+a\right)$$

input
```
integrate(((-2*a*c+b^2)*d-2*c^2*d*x^2)/(c*x^2+b*x+a),x, algorithm="fricas"
)
```

output
```
-2*c*d*x + b*d*log(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = bd\log\left(a+bx+cx^2\right) - 2cdx$$

input
```
integrate(((-2*a*c+b**2)*d-2*c**2*d*x**2)/(c*x**2+b*x+a),x)
```

output
```
b*d*log(a + b*x + c*x**2) - 2*c*d*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = -2\,cdx + bd\log\left(cx^2+bx+a\right)$$

input
```
integrate(((-2*a*c+b^2)*d-2*c^2*d*x^2)/(c*x^2+b*x+a),x, algorithm="maxima"
)
```

output
```
-2*c*d*x + b*d*log(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = -2\,cdx + bd\log\left(cx^2+bx+a\right)$$

input
```
integrate(((-2*a*c+b^2)*d-2*c^2*d*x^2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-2*c*d*x + b*d*log(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = b\,d\,\ln\left(c\,x^2+b\,x+a\right)-2\,c\,d\,x$$

input
```
int(-(d*(2*a*c - b^2) + 2*c^2*d*x^2)/(a + b*x + c*x^2),x)
```

output
```
b*d*log(a + b*x + c*x^2) - 2*c*d*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d-2c^2dx^2}{a+bx+cx^2}\,dx = d\big(\log\big(c\,x^2+bx+a\big)\,b-2cx\big)$$

input
```
int(((-2*a*c+b^2)*d-2*c^2*d*x^2)/(c*x^2+b*x+a),x)
```

output
```
d*(log(a + b*x + c*x**2)*b - 2*c*x)
```

3.19 $\int \frac{-2acd+2c^2dx^2}{(a+bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{-2acd+2c^2dx^2}{(a+bx+cx^2)^2} \, dx = -\frac{2cdx}{a+bx+cx^2}$$

output
```
-2*c*d*x/(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd+2c^2dx^2}{(a+bx+cx^2)^2} \, dx = -\frac{2cdx}{a+bx+cx^2}$$

input
```
Integrate[(-2*a*c*d + 2*c^2*d*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(-2*c*d*x)/(a + b*x + c*x^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2c^2dx^2-2acd}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{2cdx}{a+bx+cx^2}$$

input
```
Int[(-2*a*c*d + 2*c^2*d*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(-2*c*d*x)/(a + b*x + c*x^2)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
gosper	$-\frac{2cdx}{c\,x^2+bx+a}$	18
default	$-\frac{2cdx}{c\,x^2+bx+a}$	18
risch	$-\frac{2cdx}{c\,x^2+bx+a}$	18
parallelrisch	$-\frac{2cdx}{c\,x^2+bx+a}$	18
norman	$\frac{\frac{2adc}{b}+\frac{2c^2d\,x^2}{b}}{c\,x^2+bx+a}$	35
orering	$-\frac{(c\,x^2+a)\,(2c^2d\,x^2-2acd)}{b(-c\,x^2+a)(c\,x^2+bx+a)}$	50

input
```
int((2*c^2*d*x^2-2*a*c*d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*c*d*x/(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd+2c^2dx^2}{(a+bx+cx^2)^2}\,dx=-\frac{2\,cdx}{cx^2+bx+a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*c*d*x/(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd+2c^2dx^2}{(a+bx+cx^2)^2}\,dx=-\frac{2cdx}{a+bx+cx^2}$$

input
```
integrate((2*c**2*d*x**2-2*a*c*d)/(c*x**2+b*x+a)**2,x)
```

output
```
-2*c*d*x/(a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd + 2c^2dx^2}{(a + bx + cx^2)^2}\, dx = -\frac{2\, cdx}{cx^2 + bx + a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*c*d*x/(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd + 2c^2dx^2}{(a + bx + cx^2)^2}\, dx = -\frac{2\, cdx}{cx^2 + bx + a}$$

input
```
integrate((2*c^2*d*x^2-2*a*c*d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-2*c*d*x/(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2acd + 2c^2dx^2}{(a + bx + cx^2)^2}\, dx = -\frac{2\, c\, d\, x}{c\, x^2 + b\, x + a}$$

input
```
int((2*c^2*d*x^2 - 2*a*c*d)/(a + b*x + c*x^2)^2,x)
```

output
```
-(2*c*d*x)/(a + b*x + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{-2acd + 2c^2dx^2}{(a + bx + cx^2)^2} \, dx = \frac{2cd(c\,x^2 + a)}{b\,(c\,x^2 + bx + a)}$$

input
```
int((2*c^2*d*x^2-2*a*c*d)/(c*x^2+b*x+a)^2,x)
```

output
```
(2*c*d*(a + c*x**2))/(b*(a + b*x + c*x**2))
```

3.20 $\int \frac{(-b^2-2ac)d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 23

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx=\frac{d(b-4cx)}{2\,(a+bx+cx^2)^2}$$

output
```
1/2*d*(-4*c*x+b)/(c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx=\frac{d(b-4cx)}{2(a+x(b+cx))^2}$$

input
```
Integrate[((-b^2 - 2*a*c)*d + 6*c^2*d*x^2)/(a + b*x + c*x^2)^3,x]
```

output
```
(d*(b - 4*c*x))/(2*(a + x*(b + c*x))^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-b^2)+6c^2dx^2}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{d(b-4cx)}{2(a+bx+cx^2)^2}-\frac{\int 0dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{d(b-4cx)}{2(a+bx+cx^2)^2}$$

input
```
Int[((-b^2 - 2*a*c)*d + 6*c^2*d*x^2)/(a + b*x + c*x^2)^3,x]
```

output
```
(d*(b - 4*c*x))/(2*(a + b*x + c*x^2)^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{d(-4cx+b)}{2(c\,x^2+bx+a)^2}$	22
default	$-\frac{d\left(2cx-\frac{b}{2}\right)}{(c\,x^2+bx+a)^2}$	24
norman	$\frac{\frac{1}{2}bd-2cdx}{(c\,x^2+bx+a)^2}$	24
risch	$\frac{\frac{1}{2}bd-2cdx}{(c\,x^2+bx+a)^2}$	24
parallelrisch	$\frac{-4c^3dx+b\,c^2d}{2c^2(c\,x^2+bx+a)^2}$	32
orering	$-\frac{(-4cx+b)\left((-2ac-b^2)d+6c^2d\,x^2\right)}{2(c\,x^2+bx+a)^2(-6c^2x^2+2ac+b^2)}$	61

input
```
int(((-2*a*c-b^2)*d+6*c^2*d*x^2)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(-4*c*x+b)/(c*x^2+b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx = -\frac{4\,cdx-bd}{2\,(c^2x^4+2\,bcx^3+2\,abx+(b^2+2\,ac)x^2+a^2)}$$

input
```
integrate(((-2*a*c-b^2)*d+6*c^2*d*x^2)/(c*x^2+b*x+a)^3,x, algorithm="frica
s")
```

output
```
-1/2*(4*c*d*x - b*d)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 +
a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(20) = 40$.

Time = 0.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{(-b^2 - 2ac)\,d + 6c^2dx^2}{(a + bx + cx^2)^3}\,dx = \frac{bd - 4cdx}{2a^2 + 4abx + 4bcx^3 + 2c^2x^4 + x^2 \cdot (4ac + 2b^2)}$$

input
```
integrate(((-2*a*c-b**2)*d+6*c**2*d*x**2)/(c*x**2+b*x+a)**3,x)
```

output
```
(b*d - 4*c*d*x)/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2*x**4 + x**2*(4*a*c
 + 2*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \frac{(-b^2 - 2ac)\,d + 6c^2dx^2}{(a + bx + cx^2)^3}\,dx = -\frac{4\,cdx - bd}{2\,(c^2x^4 + 2\,bcx^3 + 2\,abx + (b^2 + 2\,ac)x^2 + a^2)}$$

input
```
integrate(((-2*a*c-b^2)*d+6*c^2*d*x^2)/(c*x^2+b*x+a)^3,x, algorithm="maxim
a")
```

output
```
-1/2*(4*c*d*x - b*d)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 +
a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx = -\frac{4\,cdx-bd}{2\,(cx^2+bx+a)^2}$$

input
```
integrate(((-2*a*c-b^2)*d+6*c^2*d*x^2)/(c*x^2+b*x+a)^3,x, algorithm="giac"
)
```

output
```
-1/2*(4*c*d*x - b*d)/(c*x^2 + b*x + a)^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx = \frac{\frac{b\,d}{2}-2\,c\,d\,x}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

input
```
int(-(d*(2*a*c + b^2) - 6*c^2*d*x^2)/(a + b*x + c*x^2)^3,x)
```

output
```
((b*d)/2 - 2*c*d*x)/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x
^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{(-b^2-2ac)\,d+6c^2dx^2}{(a+bx+cx^2)^3}\,dx = \frac{d(-4cx+b)}{2c^2x^4+4bc\,x^3+4ac\,x^2+2b^2x^2+4abx+2a^2}$$

input
```
int(((-2*a*c-b^2)*d+6*c^2*d*x^2)/(c*x^2+b*x+a)^3,x)
```

output
```
(d*(b - 4*c*x))/(2*(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 +
 c**2*x**4))
```

3.21 $\int \frac{(-2b^2-2ac)d+10c^2dx^2}{(a+bx+cx^2)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 23

$$\int \frac{(-2b^2-2ac)\,d+10c^2dx^2}{(a+bx+cx^2)^4}\,dx=\frac{2d(b-3cx)}{3\,(a+bx+cx^2)^3}$$

output
```
2/3*d*(-3*c*x+b)/(c*x^2+b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(-2b^2-2ac)\,d+10c^2dx^2}{(a+bx+cx^2)^4}\,dx=\frac{2d(b-3cx)}{3(a+x(b+cx))^3}$$

input
```
Integrate[((-2*b^2 - 2*a*c)*d + 10*c^2*d*x^2)/(a + b*x + c*x^2)^4,x]
```

output
```
(2*d*(b - 3*c*x))/(3*(a + x*(b + c*x))^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-2b^2)+10c^2dx^2}{(a+bx+cx^2)^4}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2d(b(b^2-4ac)-3cx(b^2-4ac))}{3(b^2-4ac)(a+bx+cx^2)^3}-\frac{\int 0dx}{3(b^2-4ac)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2d(b(b^2-4ac)-3cx(b^2-4ac))}{3(b^2-4ac)(a+bx+cx^2)^3}$$

input
```
Int[((-2*b^2 - 2*a*c)*d + 10*c^2*d*x^2)/(a + b*x + c*x^2)^4,x]
```

output
```
(2*d*(b*(b^2 - 4*a*c) - 3*c*(b^2 - 4*a*c)*x))/(3*(b^2 - 4*a*c)*(a + b*x +
c*x^2)^3)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{2d(-3cx+b)}{3(c\,x^{2}+bx+a)^{3}}$	22
default	$-\frac{2d\left(cx-\frac{b}{3}\right)}{(c\,x^{2}+bx+a)^{3}}$	23
norman	$\frac{-2cdx+\frac{2}{3}bd}{(c\,x^{2}+bx+a)^{3}}$	24
risch	$\frac{-2cdx+\frac{2}{3}bd}{(c\,x^{2}+bx+a)^{3}}$	24
parallelrisch	$\frac{-6c^{4}dx+2b\,c^{3}d}{3c^{3}(c\,x^{2}+bx+a)^{3}}$	33
orering	$-\frac{(-3cx+b)\left((-2ac-2b^{2})d+10c^{2}d\,x^{2}\right)}{3(c\,x^{2}+bx+a)^{3}(-5c^{2}x^{2}+ac+b^{2})}$	60

input
```
int(((-2*a*c-2*b^2)*d+10*c^2*d*x^2)/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOS
E)
```

output
```
2/3*d*(-3*c*x+b)/(c*x^2+b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \frac{(-2b^2 - 2ac)\,d + 10c^2dx^2}{(a + bx + cx^2)^4}\,dx$$
$$= -\frac{2\,(3\,cdx - bd)}{3\,(c^3x^6 + 3\,bc^2x^5 + 3\,(b^2c + ac^2)x^4 + 3\,a^2bx + (b^3 + 6\,abc)x^3 + a^3 + 3\,(ab^2 + a^2c)x^2)}$$

input
```
integrate(((-2*a*c-2*b^2)*d+10*c^2*d*x^2)/(c*x^2+b*x+a)^4,x, algorithm="fr
icas")
```

output
```
-2/3*(3*c*d*x - b*d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^
2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(22) = 44$.

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \frac{(-2b^2 - 2ac)\,d + 10c^2dx^2}{(a + bx + cx^2)^4}\,dx$$
$$= \frac{2bd - 6cdx}{3a^3 + 9a^2bx + 9bc^2x^5 + 3c^3x^6 + x^4 \cdot (9ac^2 + 9b^2c) + x^3 \cdot (18abc + 3b^3) + x^2 \cdot (9a^2c + 9ab^2)}$$

input
```
integrate(((-2*a*c-2*b**2)*d+10*c**2*d*x**2)/(c*x**2+b*x+a)**4,x)
```

output
```
(2*b*d - 6*c*d*x)/(3*a**3 + 9*a**2*b*x + 9*b*c**2*x**5 + 3*c**3*x**6 + x**
4*(9*a*c**2 + 9*b**2*c) + x**3*(18*a*b*c + 3*b**3) + x**2*(9*a**2*c + 9*a*
b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \frac{(-2b^2 - 2ac)\,d + 10c^2dx^2}{(a + bx + cx^2)^4}\,dx$$
$$= -\frac{2\,(3\,cdx - bd)}{3\,(c^3x^6 + 3\,bc^2x^5 + 3\,(b^2c + ac^2)x^4 + 3\,a^2bx + (b^3 + 6\,abc)x^3 + a^3 + 3\,(ab^2 + a^2c)x^2)}$$

input
```
integrate(((-2*a*c-2*b^2)*d+10*c^2*d*x^2)/(c*x^2+b*x+a)^4,x, algorithm="ma
xima")
```

output
```
-2/3*(3*c*d*x - b*d)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^
2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(-2b^2 - 2ac)\,d + 10c^2dx^2}{(a + bx + cx^2)^4}\,dx = -\frac{2\,(3\,cdx - bd)}{3\,(cx^2 + bx + a)^3}$$

input
```
integrate(((-2*a*c-2*b^2)*d+10*c^2*d*x^2)/(c*x^2+b*x+a)^4,x, algorithm="gi
ac")
```

output
```
-2/3*(3*c*d*x - b*d)/(c*x^2 + b*x + a)^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int \frac{(-2b^2-2ac)\,d+10c^2dx^2}{\left(a+bx+cx^2\right)^4}\,dx$$
$$=\frac{\frac{2\,b\,d}{3}-2\,c\,d\,x}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a^3+x^3\,\left(b^3+6\,a\,c\,b\right)+c^3\,x^6+3\,b\,c^2\,x^5+3\,a^2\,b\,x}$$

input
```
int(-(d*(2*a*c + 2*b^2) - 10*c^2*d*x^2)/(a + b*x + c*x^2)^4,x)
```

output
```
((2*b*d)/3 - 2*c*d*x)/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) +
 a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.04

$$\int \frac{(-2b^2-2ac)\,d+10c^2dx^2}{\left(a+bx+cx^2\right)^4}\,dx$$
$$=\frac{2d(-3cx+b)}{3c^3x^6+9b\,c^2x^5+9a\,c^2x^4+9b^2c\,x^4+18abc\,x^3+3b^3x^3+9a^2c\,x^2+9a\,b^2x^2+9a^2bx+3a^3}$$

input
```
int(((-2*a*c-2*b^2)*d+10*c^2*d*x^2)/(c*x^2+b*x+a)^4,x)
```

output
```
(2*d*(b - 3*c*x))/(3*(a**3 + 3*a**2*b*x + 3*a**2*c*x**2 + 3*a*b**2*x**2 +
6*a*b*c*x**3 + 3*a*c**2*x**4 + b**3*x**3 + 3*b**2*c*x**4 + 3*b*c**2*x**5 +
 c**3*x**6))
```

3.22 $\int \frac{(-3b^2-2ac)d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 25

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx=\frac{d(3b-8cx)}{4\left(a+bx+cx^2\right)^4}$$

output
```
1/4*d*(-8*c*x+3*b)/(c*x^2+b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx=-\frac{d(-3b+8cx)}{4\left(a+bx+cx^2\right)^4}$$

input
```
Integrate[((-3*b^2 - 2*a*c)*d + 14*c^2*d*x^2)/(a + b*x + c*x^2)^5,x]
```

output
```
-1/4*(d*(-3*b + 8*c*x))/(a + b*x + c*x^2)^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-3b^2)+14c^2dx^2}{(a+bx+cx^2)^5}dx$$

$$\Big\downarrow \ 2191$$

$$\frac{d(3b-8cx)}{4(a+bx+cx^2)^4}-\frac{\int 0dx}{4(b^2-4ac)}$$

$$\Big\downarrow \ 24$$

$$\frac{d(3b-8cx)}{4(a+bx+cx^2)^4}$$

input
```
Int[((-3*b^2 - 2*a*c)*d + 14*c^2*d*x^2)/(a + b*x + c*x^2)^5,x]
```

output
```
(d*(3*b - 8*c*x))/(4*(a + b*x + c*x^2)^4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{d(-8cx+3b)}{4(c\,x^2+bx+a)^4}$	24
default	$-\frac{d\left(2cx-\frac{3b}{4}\right)}{(c\,x^2+bx+a)^4}$	24
norman	$\frac{\frac{3}{4}bd-2cdx}{(c\,x^2+bx+a)^4}$	24
risch	$\frac{\frac{3}{4}bd-2cdx}{(c\,x^2+bx+a)^4}$	24
parallelrisch	$\frac{-8c^5dx+3b\,c^4d}{4c^4(c\,x^2+bx+a)^4}$	33
orering	$-\frac{(-8cx+3b)\left((-2ac-3b^2)d+14c^2d\,x^2\right)}{4(c\,x^2+bx+a)^4(-14c^2x^2+2ac+3b^2)}$	65

input
```
int(((-2*a*c-3*b^2)*d+14*c^2*d*x^2)/(c*x^2+b*x+a)^5,x,method=_RETURNVERBOS
E)
```

output
```
1/4*d*(-8*c*x+3*b)/(c*x^2+b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.60

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx =$$

$$-\frac{8\,cdx-3\,bd}{4\,(c^4x^8+4\,bc^3x^7+2\,(3\,b^2c^2+2\,ac^3)x^6+4\,(b^3c+3\,abc^2)x^5+4\,a^3bx+(b^4+12\,ab^2c+6\,a^2c^2)x^4+a^4}$$

input
```
integrate(((-2*a*c-3*b^2)*d+14*c^2*d*x^2)/(c*x^2+b*x+a)^5,x, algorithm="fr
icas")
```

output
```
-1/4*(8*c*d*x - 3*b*d)/(c^4*x^8 + 4*b*c^3*x^7 + 2*(3*b^2*c^2 + 2*a*c^3)*x^
6 + 4*(b^3*c + 3*a*b*c^2)*x^5 + 4*a^3*b*x + (b^4 + 12*a*b^2*c + 6*a^2*c^2)
*x^4 + a^4 + 4*(a*b^3 + 3*a^2*b*c)*x^3 + 2*(3*a^2*b^2 + 2*a^3*c)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(22) = 44$.

Time = 1.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.72

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{3bd-8cdx}{4a^4+16a^3bx+16bc^3x^7+4c^4x^8+x^6\cdot(16ac^3+24b^2c^2)+x^5\cdot(48abc^2+16b^3c)+x^4\cdot(24a^2c^2+48ab^2c$$

input
```
integrate(((-2*a*c-3*b**2)*d+14*c**2*d*x**2)/(c*x**2+b*x+a)**5,x)
```

output
```
(3*b*d - 8*c*d*x)/(4*a**4 + 16*a**3*b*x + 16*b*c**3*x**7 + 4*c**4*x**8 + x
**6*(16*a*c**3 + 24*b**2*c**2) + x**5*(48*a*b*c**2 + 16*b**3*c) + x**4*(24
*a**2*c**2 + 48*a*b**2*c + 4*b**4) + x**3*(48*a**2*b*c + 16*a*b**3) + x**2
*(16*a**3*c + 24*a**2*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(23) = 46$.

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.60

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx =$$
$$-\frac{8\,cdx-3\,bd}{4\,(c^4x^8+4\,bc^3x^7+2\,(3\,b^2c^2+2\,ac^3)x^6+4\,(b^3c+3\,abc^2)x^5+4\,a^3bx+(b^4+12\,ab^2c+6\,a^2c^2)x^4+a^4$$

input
```
integrate(((-2*a*c-3*b^2)*d+14*c^2*d*x^2)/(c*x^2+b*x+a)^5,x, algorithm="ma
xima")
```

output
```
-1/4*(8*c*d*x - 3*b*d)/(c^4*x^8 + 4*b*c^3*x^7 + 2*(3*b^2*c^2 + 2*a*c^3)*x^
6 + 4*(b^3*c + 3*a*b*c^2)*x^5 + 4*a^3*b*x + (b^4 + 12*a*b^2*c + 6*a^2*c^2)
*x^4 + a^4 + 4*(a*b^3 + 3*a^2*b*c)*x^3 + 2*(3*a^2*b^2 + 2*a^3*c)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx = -\frac{8\,cdx-3\,bd}{4\,(cx^2+bx+a)^4}$$

input
```
integrate(((-2*a*c-3*b^2)*d+14*c^2*d*x^2)/(c*x^2+b*x+a)^5,x, algorithm="gi
ac")
```

output
```
-1/4*(8*c*d*x - 3*b*d)/(c*x^2 + b*x + a)^4
```

Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{\frac{3\,b\,d}{4}-2\,c\,d\,x}{x^4\,\left(6\,a^2\,c^2+12\,a\,b^2\,c+b^4\right)+a^4+c^4\,x^8+x^2\,\left(4\,c\,a^3+6\,a^2\,b^2\right)+x^6\,\left(6\,b^2\,c^2+4\,a\,c^3\right)+x^3\,\left(12\,c\,a^2\,b+4\,a\right.}$$

input
```
int(-(d*(2*a*c + 3*b^2) - 14*c^2*d*x^2)/(a + b*x + c*x^2)^5,x)
```

output
```
((3*b*d)/4 - 2*c*d*x)/(x^4*(b^4 + 6*a^2*c^2 + 12*a*b^2*c) + a^4 + c^4*x^8
+ x^2*(4*a^3*c + 6*a^2*b^2) + x^6*(4*a*c^3 + 6*b^2*c^2) + x^3*(4*a*b^3 + 1
2*a^2*b*c) + x^5*(4*b^3*c + 12*a*b*c^2) + 4*b*c^3*x^7 + 4*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.96

$$\int \frac{(-3b^2-2ac)\,d+14c^2dx^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{d(-8cx+3b)}{4c^4x^8+16b\,c^3x^7+16a\,c^3x^6+24b^2c^2x^6+48ab\,c^2x^5+16b^3c\,x^5+24a^2c^2x^4+48a\,b^2c\,x^4+4b^4x^4+48a^2b}$$

input
```
int(((-2*a*c-3*b^2)*d+14*c^2*d*x^2)/(c*x^2+b*x+a)^5,x)
```

output
```
(d*(3*b - 8*c*x))/(4*(a**4 + 4*a**3*b*x + 4*a**3*c*x**2 + 6*a**2*b**2*x**2
 + 12*a**2*b*c*x**3 + 6*a**2*c**2*x**4 + 4*a*b**3*x**3 + 12*a*b**2*c*x**4
+ 12*a*b*c**2*x**5 + 4*a*c**3*x**6 + b**4*x**4 + 4*b**3*c*x**5 + 6*b**2*c*
*2*x**6 + 4*b*c**3*x**7 + c**4*x**8))
```

3.23 $\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac\right.$

Optimal result

Integrand size = 69, antiderivative size = 46

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx = -\frac{(ae-bd(2+p)+(2cd-be)(1+p)x)\,(a+bx+cx^2)^{1+p}}{1+p}$$

output
```
-(a*e-b*d*(2+p)+(-b*e+2*c*d)*(p+1)*x)*(c*x^2+b*x+a)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx = \frac{(-ae+bd(2+p)-2cd(1+p)x+be(1+p)x)(a+x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(a + b*x + c*x^2)^p*(d*(2*b^2 - 2*a*c + b^2*p) + e*(2*b^2 - 2*a*
c + b^2*p)*x - c*(2*c*d - b*e)*(3 + 2*p)*x^2),x]
```

output
```
((-(a*e) + b*d*(2 + p) - 2*c*d*(1 + p)*x + b*e*(1 + p)*x)*(a + x*(b + c*x)
)^(1 + p))/(1 + p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {2192, 25, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^p \left(d\left(-2ac+b^2p+2b^2\right)+ex\left(-2ac+b^2p+2b^2\right)-c(2p+3)x^2(2cd-be)\right) dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int -c(2p+3)(b(ae-bd(p+2))+2cx(ae-bd(p+2)))\left(cx^2+bx+a\right)^p dx}{c(2p+3)}-x(2cd-be)\left(a+bx+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int c(2p+3)(ae-bd(p+2))(b+2cx)\left(cx^2+bx+a\right)^p dx}{c(2p+3)}-\left(x(2cd-be)\left(a+bx+cx^2\right)^{p+1}\right)$$

$$\Big\downarrow \text{27}$$

$$-(ae-bd(p+2))\int (b+2cx)\left(cx^2+bx+a\right)^p dx-\left(x(2cd-be)\left(a+bx+cx^2\right)^{p+1}\right)$$

$$\Big\downarrow \text{1104}$$

$$-\frac{\left(a+bx+cx^2\right)^{p+1}(ae-bd(p+2))}{p+1}-x(2cd-be)\left(a+bx+cx^2\right)^{p+1}$$

input
```
Int[(a + b*x + c*x^2)^p*(d*(2*b^2 - 2*a*c + b^2*p) + e*(2*b^2 - 2*a*c + b^
2*p)*x - c*(2*c*d - b*e)*(3 + 2*p)*x^2),x]
```

output
```
-(((a*e - b*d*(2 + p))*(a + b*x + c*x^2)^(1 + p))/(1 + p)) - (2*c*d - b*e)
*x*(a + b*x + c*x^2)^(1 + p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

method	result
gosper	$-\frac{(c\,x^2+bx+a)^{p+1}(-bepx+2cdxp-bdp-bex+2cdx+ae-2bd)}{p+1}$
risch	$-\frac{(-bcep\,x^3+2c^2dp\,x^3-b^2ep\,x^2+bcdp\,x^2-bce\,x^3+2c^2d\,x^3-abepx+2acdpx+ace\,x^2-b^2dpx-b^2e\,x^2-abdp+2adxc-2b^2dx+a}{p+1}$
norman	$(be-2cd)\,c\,x^3\mathrm{e}^{p\ln(c\,x^2+bx+a)}+\frac{(abep-2acdp+b^2dp-2acd+2b^2d)x\,\mathrm{e}^{p\ln\left(c\,x^2+bx+a\right)}}{p+1}-\frac{a(-bdp+ae-2bd)\mathrm{e}^{p\ln\left(c\right.}}{p+1}$
orering	$\frac{(-bepx+2cdxp-bdp-bex+2cdx+ae-2bd)(c\,x^2+bx+a)(c\,x^2+bx+a)^p\left(d\left(b^2p-2ac+2b^2\right)+e\left(b^2p-2ac+2b^2\right)x-c(-be+2cd)\right(}{(p+1)(-2bcep\,x^2+4c^2d\,x^2p-b^2epx-3bce\,x^2+6c^2d\,x^2+2acex-b^2dp-2b^2ex+2acd-2b^2d)}$
parallelrisch	$\frac{x^3(c\,x^2+bx+a)^p abcep-2x^3(c\,x^2+bx+a)^p a\,c^2dp+x^3(c\,x^2+bx+a)^p abce-2x^3(c\,x^2+bx+a)^p a\,c^2d+x^2(c\,x^2+bx+a)^p a\,b^2ep-}{}$

input
```
int((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)+e*(b^2*p-2*a*c+2*b^2)*x-c*(-b*e
+2*c*d)*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(p+1)*(c*x^2+b*x+a)^(p+1)*(-b*e*p*x+2*c*d*p*x-b*d*p-b*e*x+2*c*d*x+a*e-2
*b*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(46) = 92$.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx$$
$$= \frac{(abdp-(2\,c^2d-bce+(2\,c^2d-bce)p)x^3+2\,abd-a^2e+((b^2-ac)e-(bcd-b^2e)p)x^2+(2\,(b^2-ac)d}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)+e*(b^2*p-2*a*c+2*b^2)*x-c
*(-b*e+2*c*d)*(3+2*p)*x^2),x, algorithm="fricas")
```

output
```
(a*b*d*p - (2*c^2*d - b*c*e + (2*c^2*d - b*c*e)*p)*x^3 + 2*a*b*d - a^2*e +
 ((b^2 - a*c)*e - (b*c*d - b^2*e)*p)*x^2 + (2*(b^2 - a*c)*d + (a*b*e + (b^
2 - 2*a*c)*d)*p)*x)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(39) = 78$.

Time = 69.18 (sec) , antiderivative size = 507, normalized size of antiderivative = 11.02

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx$$
$$= \begin{cases} -\frac{a^2e(a+bx+cx^2)^p}{p+1} + \frac{abdp(a+bx+cx^2)^p}{p+1} + \frac{2abd(a+bx+cx^2)^p}{p+1} + \frac{abepx(a+bx+cx^2)^p}{p+1} - \frac{2acdpx(a+bx+cx^2)^p}{p+1} - \frac{2acdx(a+bx+cx^2)}{p+1} \\ -ae\log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right) - ae\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right) + bd\log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right) + bd\log\left(\frac{b}{2c}\right. \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**p*(d*(b**2*p-2*a*c+2*b**2)+e*(b**2*p-2*a*c+2*b**
2)*x-c*(-b*e+2*c*d)*(3+2*p)*x**2),x)
```

output
```
Piecewise((-a**2*e*(a + b*x + c*x**2)**p/(p + 1) + a*b*d*p*(a + b*x + c*x*
*2)**p/(p + 1) + 2*a*b*d*(a + b*x + c*x**2)**p/(p + 1) + a*b*e*p*x*(a + b*
x + c*x**2)**p/(p + 1) - 2*a*c*d*p*x*(a + b*x + c*x**2)**p/(p + 1) - 2*a*c
*d*x*(a + b*x + c*x**2)**p/(p + 1) - a*c*e*x**2*(a + b*x + c*x**2)**p/(p +
 1) + b**2*d*p*x*(a + b*x + c*x**2)**p/(p + 1) + 2*b**2*d*x*(a + b*x + c*x
**2)**p/(p + 1) + b**2*e*p*x**2*(a + b*x + c*x**2)**p/(p + 1) + b**2*e*x**
2*(a + b*x + c*x**2)**p/(p + 1) - b*c*d*p*x**2*(a + b*x + c*x**2)**p/(p +
1) + b*c*e*p*x**3*(a + b*x + c*x**2)**p/(p + 1) + b*c*e*x**3*(a + b*x + c*
x**2)**p/(p + 1) - 2*c**2*d*p*x**3*(a + b*x + c*x**2)**p/(p + 1) - 2*c**2*
d*x**3*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (-a*e*log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) - a*e*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*
c)) + b*d*log(b/(2*c) + x - sqrt(-4*a*c + b**2)/(2*c)) + b*d*log(b/(2*c) +
 x + sqrt(-4*a*c + b**2)/(2*c)) + b*e*x - 2*c*d*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx$$
$$= \frac{(abd(p+2)-(2\,c^2d(p+1)-bce(p+1))x^3-a^2e+(b^2e(p+1)-bcdp-ace)x^2+(b^2d(p+2)-(2\,cd(}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)+e*(b^2*p-2*a*c+2*b^2)*x-c
*(-b*e+2*c*d)*(3+2*p)*x^2),x, algorithm="maxima")
```

output
```
(a*b*d*(p + 2) - (2*c^2*d*(p + 1) - b*c*e*(p + 1))*x^3 - a^2*e + (b^2*e*(p
 + 1) - b*c*d*p - a*c*e)*x^2 + (b^2*d*(p + 2) - (2*c*d*(p + 1) - b*e*p)*a)
*x)*(c*x^2 + b*x + a)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(46) = 92$.

Time = 0.13 (sec) , antiderivative size = 321, normalized size of antiderivative = 6.98

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx = -\frac{2\,(cx^2+bx+a)^p c^2 dpx^3 - (cx^2+bx+a)^p bcepx^3 + (cx^2+bx+a)^p bcdpx^2 - (cx^2+bx+a)^p b^2 epx^2 +$$

input
```
integrate((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)+e*(b^2*p-2*a*c+2*b^2)*x-c
*(-b*e+2*c*d)*(3+2*p)*x^2),x, algorithm="giac")
```

output
```
-(2*(c*x^2 + b*x + a)^p*c^2*d*p*x^3 - (c*x^2 + b*x + a)^p*b*c*e*p*x^3 + (c
*x^2 + b*x + a)^p*b*c*d*p*x^2 - (c*x^2 + b*x + a)^p*b^2*e*p*x^2 + 2*(c*x^2
 + b*x + a)^p*c^2*d*x^3 - (c*x^2 + b*x + a)^p*b*c*e*x^3 - (c*x^2 + b*x + a
)^p*b^2*d*p*x + 2*(c*x^2 + b*x + a)^p*a*c*d*p*x - (c*x^2 + b*x + a)^p*a*b*
e*p*x - (c*x^2 + b*x + a)^p*b^2*e*x^2 + (c*x^2 + b*x + a)^p*a*c*e*x^2 - (c
*x^2 + b*x + a)^p*a*b*d*p - 2*(c*x^2 + b*x + a)^p*b^2*d*x + 2*(c*x^2 + b*x
 + a)^p*a*c*d*x - 2*(c*x^2 + b*x + a)^p*a*b*d + (c*x^2 + b*x + a)^p*a^2*e)
/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x - c(2cd-be)(3+2p)x^2\right)\,dx = (c\,x^2+b\,x+a)^p \left(\frac{a\,(2\,b\,d-a\,e+b\,d\,p)}{p+1} + c\,x^3\,(b\,e-2\,c\,d) + \frac{x\,(2\,b^2\,d-2\,a\,c\,d+b^2\,d\,p+a\,b\,e\,p-2\,a\,c\,d\,p)}{p+1} + \frac{x^2\,(b^2\,e-a\,c\,e+b^2\,e\,p-b\,c\,d\,p)}{p+1}\right)$$

input
```
int((d*(b^2*p - 2*a*c + 2*b^2) + e*x*(b^2*p - 2*a*c + 2*b^2) + c*x^2*(2*p
+ 3)*(b*e - 2*c*d))*(a + b*x + c*x^2)^p,x)
```

output
```
(a + b*x + c*x^2)^p*((a*(2*b*d - a*e + b*d*p))/(p + 1) + c*x^3*(b*e - 2*c*
d) + (x*(2*b^2*d - 2*a*c*d + b^2*d*p + a*b*e*p - 2*a*c*d*p))/(p + 1) + (x^
2*(b^2*e - a*c*e + b^2*e*p - b*c*d*p))/(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int (a+bx+cx^2)^p \left(d(2b^2-2ac+b^2p)+e(2b^2-2ac+b^2p)\,x-c(2cd-be)(3+2p)x^2\right)\,dx$$
$$=\frac{(c\,x^2+bx+a)^p\,(bcep\,x^3-2c^2dp\,x^3+b^2ep\,x^2-bcdp\,x^2+bce\,x^3-2c^2d\,x^3+abepx-2acdpx-ace\,x^2+}{p+1}$$

input
```
int((c*x^2+b*x+a)^p*(d*(b^2*p-2*a*c+2*b^2)+e*(b^2*p-2*a*c+2*b^2)*x-c*(-b*e
+2*c*d)*(3+2*p)*x^2),x)
```

output
```
((a + b*x + c*x**2)**p*( - a**2*e + a*b*d*p + 2*a*b*d + a*b*e*p*x - 2*a*c*
d*p*x - 2*a*c*d*x - a*c*e*x**2 + b**2*d*p*x + 2*b**2*d*x + b**2*e*p*x**2 +
 b**2*e*x**2 - b*c*d*p*x**2 + b*c*e*p*x**3 + b*c*e*x**3 - 2*c**2*d*p*x**3
- 2*c**2*d*x**3))/(p + 1)
```

3.24 $\int (a+bx+cx^2)^p (-bcf(3+2p)+d(2b^2-2ac+b^2p$

Optimal result

Integrand size = 67, antiderivative size = 44

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d\left(2b^2-2ac+b^2p\right)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right)\,dx=\frac{(bd(2+p)-cf(3+2p)-2cd(1+p)x)\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

output
```
(b*d*(2+p)-c*f*(3+2*p)-2*c*d*(p+1)*x)*(c*x^2+b*x+a)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d\left(2b^2-2ac+b^2p\right)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right)\,dx=\frac{(a+x(b+cx))^{1+p}(bd(2+p)-c(f(3+2p)+2d(1+p)x))}{1+p}$$

input
```
Integrate[(a + b*x + c*x^2)^p*(-(b*c*f*(3 + 2*p)) + d*(2*b^2 - 2*a*c + b^2
*p) - 2*c^2*f*(3 + 2*p)*x - 2*c^2*d*(3 + 2*p)*x^2),x]
```

output
```
((a + x*(b + c*x))^(1 + p)*(b*d*(2 + p) - c*(f*(3 + 2*p) + 2*d*(1 + p)*x))
)/(1 + p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2192, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^p \left(d\left(-2ac+b^2p+2b^2\right)-bcf(2p+3)-2c^2d(2p+3)x^2-2c^2f(2p+3)x\right) \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int c(2p+3)(bd(p+2)-cf(2p+3))(b+2cx)\left(cx^2+bx+a\right)^p dx}{c(2p+3)}-2cdx\left(a+bx+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{27}$$

$$(bd(p+2)-cf(2p+3))\int (b+2cx)\left(cx^2+bx+a\right)^p dx-2cdx\left(a+bx+cx^2\right)^{p+1}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(a+bx+cx^2\right)^{p+1}(bd(p+2)-cf(2p+3))}{p+1}-2cdx\left(a+bx+cx^2\right)^{p+1}$$

input
```
Int[(a + b*x + c*x^2)^p*(-(b*c*f*(3 + 2*p)) + d*(2*b^2 - 2*a*c + b^2*p) -
2*c^2*f*(3 + 2*p)*x - 2*c^2*d*(3 + 2*p)*x^2),x]
```

output
```
((b*d*(2 + p) - c*f*(3 + 2*p))*(a + b*x + c*x^2)^(1 + p))/(1 + p) - 2*c*d*
x*(a + b*x + c*x^2)^(1 + p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
gosper	$\frac{(c x^2+bx+a)^{p+1}(-2cdxp+bdp-2cdx-2cfp+2bd-3cf)}{p+1}$
risch	$\frac{(-2c^2dp x^3-bcdp x^2-2c^2d x^3-2c^2fp x^2-2acdpx+b^2dpx-2bcfpx-3c^2f x^2+abdp-2adxc-2acfp+2b^2dx-3bcfx+2dab-3a}{p+1}$
norman	$\frac{a(bdp-2cfp+2bd-3cf)e^{p\ln(c x^2+bx+a)}}{p+1}-2c^2d x^3 e^{p\ln(c x^2+bx+a)}-\frac{(2acdp-b^2dp+2bcfp+2acd-2b^2d+3bcf)x e^{p \ln}}{p+1}$
orering	$-\frac{(-2cdxp+bdp-2cdx-2cfp+2bd-3cf)(c x^2+bx+a)(c x^2+bx+a)^p(-bcf(3+2p)+d(b^2p-2ac+2b^2)-2c^2f(3+2p)x-2c^2d(}{(p+1)(4c^2d x^2p+6c^2d x^2+4c^2fxp-b^2dp+2bcfp+6c^2fx+2acd-2b^2d+3bcf)}$
parallelrisch	$-\frac{2x^3(c x^2+bx+a)^p a c^2dp+2x^3(c x^2+bx+a)^p a c^2d+x^2(c x^2+bx+a)^p abcdp+2x^2(c x^2+bx+a)^p a c^2fp+3x^2(c x^2+bx+a)^p a}{}$

input
```
int((c*x^2+b*x+a)^p*(-b*c*f*(3+2*p)+d*(b^2*p-2*a*c+2*b^2)-2*c^2*f*(3+2*p)*
x-2*c^2*d*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(p+1)*(c*x^2+b*x+a)^(p+1)*(-2*c*d*p*x+b*d*p-2*c*d*x-2*c*f*p+2*b*d-3*c*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.77

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d(2b^2-2ac+b^2p)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right) dx$$
$$= -\frac{(2\,(c^2dp+c^2d)x^3-2\,abd+3\,acf+(3\,c^2f+(bcd+2\,c^2f)p)x^2-(abd-2\,acf)p+(3\,bcf-2\,(b^2-ac}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(-b*c*f*(3+2*p)+d*(b^2*p-2*a*c+2*b^2)-2*c^2*f*(3
+2*p)*x-2*c^2*d*(3+2*p)*x^2),x, algorithm="fricas")
```

output
```
-(2*(c^2*d*p + c^2*d)*x^3 - 2*a*b*d + 3*a*c*f + (3*c^2*f + (b*c*d + 2*c^2*
f)*p)*x^2 - (a*b*d - 2*a*c*f)*p + (3*b*c*f - 2*(b^2 - a*c)*d + (2*b*c*f -
(b^2 - 2*a*c)*d)*p)*x)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(41) = 82$.

Time = 61.66 (sec) , antiderivative size = 483, normalized size of antiderivative = 10.98

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d(2b^2-2ac+b^2p)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right) dx$$
$$= \begin{cases} \frac{abdp(a+bx+cx^2)^p}{p+1} + \frac{2abd(a+bx+cx^2)^p}{p+1} - \frac{2acdpx(a+bx+cx^2)^p}{p+1} - \frac{2acdx(a+bx+cx^2)^p}{p+1} - \frac{2acfp(a+bx+cx^2)^p}{p+1} - \frac{3acf(a+bx+cx^2)}{p+1} \\ bd\log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right) + bd\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right) - 2cdx - cf\log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right) - cf\,\mathrm{lo} \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**p*(-b*c*f*(3+2*p)+d*(b**2*p-2*a*c+2*b**2)-2*c**2
*f*(3+2*p)*x-2*c**2*d*(3+2*p)*x**2),x)
```

output
```
Piecewise((a*b*d*p*(a + b*x + c*x**2)**p/(p + 1) + 2*a*b*d*(a + b*x + c*x*
*2)**p/(p + 1) - 2*a*c*d*p*x*(a + b*x + c*x**2)**p/(p + 1) - 2*a*c*d*x*(a
+ b*x + c*x**2)**p/(p + 1) - 2*a*c*f*p*(a + b*x + c*x**2)**p/(p + 1) - 3*a
*c*f*(a + b*x + c*x**2)**p/(p + 1) + b**2*d*p*x*(a + b*x + c*x**2)**p/(p +
 1) + 2*b**2*d*x*(a + b*x + c*x**2)**p/(p + 1) - b*c*d*p*x**2*(a + b*x + c
*x**2)**p/(p + 1) - 2*b*c*f*p*x*(a + b*x + c*x**2)**p/(p + 1) - 3*b*c*f*x*
(a + b*x + c*x**2)**p/(p + 1) - 2*c**2*d*p*x**3*(a + b*x + c*x**2)**p/(p +
 1) - 2*c**2*d*x**3*(a + b*x + c*x**2)**p/(p + 1) - 2*c**2*f*p*x**2*(a + b
*x + c*x**2)**p/(p + 1) - 3*c**2*f*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(
p, -1)), (b*d*log(b/(2*c) + x - sqrt(-4*a*c + b**2)/(2*c)) + b*d*log(b/(2*
c) + x + sqrt(-4*a*c + b**2)/(2*c)) - 2*c*d*x - c*f*log(b/(2*c) + x - sqrt
(-4*a*c + b**2)/(2*c)) - c*f*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(45) = 90$.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d(2b^2-2ac+b^2p)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right) dx$$
$$= -\frac{(2\,c^2d(p+1)x^3+(c^2f(2\,p+3)+bcdp)x^2+(cf(2\,p+3)-bd(p+2))a+(bcf(2\,p+3)-b^2d(p+2)+}{p+1}$$

input
```
integrate((c*x^2+b*x+a)^p*(-b*c*f*(3+2*p)+d*(b^2*p-2*a*c+2*b^2)-2*c^2*f*(3
+2*p)*x-2*c^2*d*(3+2*p)*x^2),x, algorithm="maxima")
```

output
```
-(2*c^2*d*(p + 1)*x^3 + (c^2*f*(2*p + 3) + b*c*d*p)*x^2 + (c*f*(2*p + 3) -
 b*d*(p + 2))*a + (b*c*f*(2*p + 3) - b^2*d*(p + 2) + 2*a*c*d*(p + 1))*x)*(
c*x^2 + b*x + a)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 6.75

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d(2b^2-2ac+b^2p)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right) dx$$
$$= -\frac{2\,(cx^2+bx+a)^p c^2dpx^3 + (cx^2+bx+a)^p bcdpx^2 + 2\,(cx^2+bx+a)^p c^2fpx^2 + 2\,(cx^2+bx+a)^p c^2dx^3}{}$$

input
```
integrate((c*x^2+b*x+a)^p*(-b*c*f*(3+2*p)+d*(b^2*p-2*a*c+2*b^2)-2*c^2*f*(3
+2*p)*x-2*c^2*d*(3+2*p)*x^2),x, algorithm="giac")
```

output
```
-(2*(c*x^2 + b*x + a)^p*c^2*d*p*x^3 + (c*x^2 + b*x + a)^p*b*c*d*p*x^2 + 2*
(c*x^2 + b*x + a)^p*c^2*f*p*x^2 + 2*(c*x^2 + b*x + a)^p*c^2*d*x^3 - (c*x^2
 + b*x + a)^p*b^2*d*p*x + 2*(c*x^2 + b*x + a)^p*a*c*d*p*x + 2*(c*x^2 + b*x
 + a)^p*b*c*f*p*x + 3*(c*x^2 + b*x + a)^p*c^2*f*x^2 - (c*x^2 + b*x + a)^p*
a*b*d*p + 2*(c*x^2 + b*x + a)^p*a*c*f*p - 2*(c*x^2 + b*x + a)^p*b^2*d*x +
2*(c*x^2 + b*x + a)^p*a*c*d*x + 3*(c*x^2 + b*x + a)^p*b*c*f*x - 2*(c*x^2 +
 b*x + a)^p*a*b*d + 3*(c*x^2 + b*x + a)^p*a*c*f)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d(2b^2-2ac+b^2p)-2c^2f(3+2p)x - 2c^2d(3+2p)x^2\right) dx = -(c\,x^2+b\,x+a)^p \left(\frac{x^2\,(3\,c^2\,f+2\,c^2\,f\,p+b\,c\,d\,p)}{p+1} + 2\,c^2\,d\,x^3 - \frac{a\,(2\,b\,d-3\,c\,f+b\,d\,p-2\,c\,f\,p)}{p+1} + \frac{x\,(2\,a\,c\,d-2\,b^2\,d+3\,b\,c\,f-b^2\,d\,p+2\,a\,c\,d\,p+2\,b\,c\,f\,p)}{p+1}\right)$$

input
```
int(-(a + b*x + c*x^2)^p*(2*c^2*f*x*(2*p + 3) - d*(b^2*p - 2*a*c + 2*b^2)
+ 2*c^2*d*x^2*(2*p + 3) + b*c*f*(2*p + 3)),x)
```

output
```
-(a + b*x + c*x^2)^p*((x^2*(3*c^2*f + 2*c^2*f*p + b*c*d*p))/(p + 1) + 2*c^
2*d*x^3 - (a*(2*b*d - 3*c*f + b*d*p - 2*c*f*p))/(p + 1) + (x*(2*a*c*d - 2*
b^2*d + 3*b*c*f - b^2*d*p + 2*a*c*d*p + 2*b*c*f*p))/(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.89

$$\int (a+bx+cx^2)^p \left(-bcf(3+2p)+d\left(2b^2-2ac+b^2p\right)-2c^2f(3+2p)x-2c^2d(3+2p)x^2\right) dx$$
$$= \frac{(c\,x^2+bx+a)^p\left(-2c^2dp\,x^3-bcdp\,x^2-2c^2d\,x^3-2c^2fp\,x^2-2acdpx+b^2dpx-2bcfpx-3c^2f\,x^2+abc\right.}{p+1}$$

input
```
int((c*x^2+b*x+a)^p*(-b*c*f*(3+2*p)+d*(b^2*p-2*a*c+2*b^2)-2*c^2*f*(3+2*p)*
x-2*c^2*d*(3+2*p)*x^2),x)
```

output
```
((a + b*x + c*x**2)**p*(a*b*d*p + 2*a*b*d - 2*a*c*d*p*x - 2*a*c*d*x - 2*a*
c*f*p - 3*a*c*f + b**2*d*p*x + 2*b**2*d*x - b*c*d*p*x**2 - 2*b*c*f*p*x - 3
*b*c*f*x - 2*c**2*d*p*x**3 - 2*c**2*d*x**3 - 2*c**2*f*p*x**2 - 3*c**2*f*x*
*2))/(p + 1)
```

3.25 $\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 65, antiderivative size = 148

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right) dx = -\frac{\left(4c^2d-2bce+b^2f\right)\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x\right)\left(a+bx+cx^2\right)^{-\frac{2c^2d-bce+b^2f-2acf}{4c^2d-2bce+b^2f}}}{c\left(b^2-4ac\right)\left(2c^2d-bce+b^2f-2acf\right)}$$

output
```
-(b^2*f-2*b*c*e+4*c^2*d)*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*
d)*x)/c/(-4*a*c+b^2)/(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)/((c*x^2+b*x+a)^((-2*a*
c*f+b^2*f-b*c*e+2*c^2*d)/(b^2*f-2*b*c*e+4*c^2*d)))
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right) dx = \frac{\left(4c^2d-2bce+b^2f\right)\left(a+x(b+cx)\right)^{\frac{-2c^2d+bce-b^2f+2acf}{4c^2d-2bce+b^2f}}\left(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx)\right)}{c\left(-b^2+4ac\right)\left(2c^2d+b^2f-c(be+2af)\right)}$$

input
```
Integrate[(a + b*x + c*x^2)^((-6*c^2*d + 3*b*c*e - 2*b^2*f + 2*a*c*f)/(4*c
^2*d - 2*b*c*e + b^2*f))*(d + e*x + f*x^2),x]
```

output
```
((4*c^2*d - 2*b*c*e + b^2*f)*(a + x*(b + c*x))^((-2*c^2*d + b*c*e - b^2*f
+ 2*a*c*f)/(4*c^2*d - 2*b*c*e + b^2*f))*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c
*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b^2 + 4*a*c)*(2*c^2*d + b^2*f - c*(b*e
 + 2*a*f)))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2192, 25, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)\left(a+bx+cx^2\right)^{\frac{2acf-2b^2f+3bce-6c^2d}{b^2f-2bce+4c^2d}}dx$$

$$\Big\downarrow \text{2192}$$

$$-\frac{\left(b^2f-2bce+4c^2d\right)\int -\frac{f(bcd-2ace+abf)(b+2cx)\left(cx^2+bx+a\right)^{-\frac{2fb^2-3ceb+6c^2d-2acf}{fb^2-2ceb+4c^2d}}}{fb^2-2ceb+4c^2d}dx}{cf\left(b^2-4ac\right)}-\frac{x\left(b^2f-2bce+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f-bce+2c^2d}{b^2f-2bce+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(b^2f-2bce+4c^2d\right)\int \frac{f(bcd-2ace+abf)(b+2cx)\left(cx^2+bx+a\right)^{-\frac{2fb^2-3ceb+6c^2d-2acf}{fb^2-2ceb+4c^2d}}}{fb^2-2ceb+4c^2d}dx}{cf\left(b^2-4ac\right)}-\frac{x\left(b^2f-2bce+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f-bce+2c^2d}{b^2f-2bce+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(abf-2ace+bcd)\int(b+2cx)\left(cx^2+bx+a\right)^{-\frac{2fb^2-3ceb+6c^2d-2acf}{fb^2-2ceb+4c^2d}}dx}{c\left(b^2-4ac\right)}-\frac{x\left(b^2f-2bce+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f-bce+2c^2d}{b^2f-2bce+4c^2d}}}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1104}$$

$$-\frac{x\left(b^2f-2bce+4c^2d\right)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f-bce+2c^2d}{b^2f-2bce+4c^2d}}}{c\left(b^2-4ac\right)}-\frac{\left(b^2f-2bce+4c^2d\right)(abf-2ace+bcd)\left(a+bx+cx^2\right)^{-\frac{-2acf+b^2f-bce+2c^2d}{b^2f-2bce+4c^2d}}}{c\left(b^2-4ac\right)\left(-2acf+b^2f-bce+2c^2d\right)}$$

input
```
Int[(a + b*x + c*x^2)^((-6*c^2*d + 3*b*c*e - 2*b^2*f + 2*a*c*f)/(4*c^2*d -
 2*b*c*e + b^2*f))*(d + e*x + f*x^2),x]
```

output
```
-(((b*c*d - 2*a*c*e + a*b*f)*(4*c^2*d - 2*b*c*e + b^2*f))/(c*(b^2 - 4*a*c)
*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(a + b*x + c*x^2)^((2*c^2*d - b*c*e +
 b^2*f - 2*a*c*f)/(4*c^2*d - 2*b*c*e + b^2*f)))) - ((4*c^2*d - 2*b*c*e + b
^2*f)*x)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^((2*c^2*d - b*c*e + b^2*f - 2*
a*c*f)/(4*c^2*d - 2*b*c*e + b^2*f)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

method	result
gosper	$-\frac{\left(c\,x^{2}+bx+a\right)^{1+\frac{2acf-2b^{2}f+3bce-6c^{2}d}{b^{2}f-2bce+4c^{2}d}}\left(b^{2}f-2bce+4c^{2}d\right)\left(-2acfx+b^{2}fx-bcex+2c^{2}dx+abf-2ace+dbc\right)}{c\left(8a^{2}c^{2}f-6a\,b^{2}cf+4ab\,c^{2}e-8a\,c^{3}d+b^{4}f-b^{3}ce+2b^{2}c^{2}d\right)}$
risch	$-\frac{\left(b^{2}f-2bce+4c^{2}d\right)\left(-2a\,c^{2}f\,x^{3}+b^{2}cf\,x^{3}-b\,c^{2}x^{3}e+2c^{3}d\,x^{3}-abcf\,x^{2}-2a\,c^{2}e\,x^{2}+b^{3}f\,x^{2}-b^{2}ce\,x^{2}+3b\,c^{2}d\,x^{2}-2a^{2}cfx+2a\,b}{c\left(4ac-b^{2}\right)\left(2acf-b^{2}f+bce-2c^{2}d\right)}$
orering	$-\frac{\left(-2a\,b^{2}c\,f^{2}x+4ab\,c^{2}efx-8a\,c^{3}dfx+b^{4}f^{2}x-3b^{3}cefx+6b^{2}c^{2}dfx+2b^{2}c^{2}e^{2}x-8b\,c^{3}dex+8c^{4}d^{2}x+a\,b^{3}f^{2}-4a\,b^{2}cef+4ab\,c^{2}a}{c\left(8a^{2}c^{2}f-6a\,b^{2}cf+4ab\,c^{2}e-8a\,c^{3}d+b^{4}f}$
norman	$\frac{\left(b^{2}f-2bce+4c^{2}d\right)x^{3}\mathrm{e}^{\frac{\left(2acf-2b^{2}f+3bce-6c^{2}d\right)\ln\left(c\,x^{2}+bx+a\right)}{b^{2}f-2bce+4c^{2}d}}}{4ac-b^{2}}+\frac{\left(a\,b^{3}c\,f^{2}+4ab\,c^{3}df-4ab\,c^{3}e^{2}+8a\,c^{4}de-b^{5}f^{2}+3b^{4}cef-7b^{3}}{c\left(8a^{2}c^{2}f-6a\,b^{2}cf}$
parallelrisch	Expression too large to display

input
```
int((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+4*c^2*
d))*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c*x^2+b*x+a)^(1+(2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+4*c
^2*d))*(b^2*f-2*b*c*e+4*c^2*d)/(8*a^2*c^2*f-6*a*b^2*c*f+4*a*b*c^2*e-8*a*c^
3*d+b^4*f-b^3*c*e+2*b^2*c^2*d)*(-2*a*c*f*x+b^2*f*x-b*c*e*x+2*c^2*d*x+a*b*f
-2*a*c*e+b*c*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(149) = 298$.

Time = 0.09 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.50

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right)\,dx = -\frac{4\,abc^3d^2+4\,a^2bc^2e^2+a^2b^3f^2+(8\,c^5d^2-8\,bc^4de+2\,b^2c^3e^2+(b^4c-2\,ab^2c^2)f^2+(2\,(3\,b^2c^3-4\,ac^4)d}{}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+
4*c^2*d))*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-(4*a*b*c^3*d^2 + 4*a^2*b*c^2*e^2 + a^2*b^3*f^2 + (8*c^5*d^2 - 8*b*c^4*d*e
 + 2*b^2*c^3*e^2 + (b^4*c - 2*a*b^2*c^2)*f^2 + (2*(3*b^2*c^3 - 4*a*c^4)*d
- (3*b^3*c^2 - 4*a*b*c^3)*e)*f)*x^3 - 2*(a*b^2*c^2 + 4*a^2*c^3)*d*e + (12*
b*c^4*d^2 - 2*(5*b^2*c^3 + 4*a*c^4)*d*e + 2*(b^3*c^2 + 2*a*b*c^3)*e^2 + (b
^5 - a*b^3*c)*f^2 - (3*b^4*c*e - (7*b^3*c^2 - 4*a*b*c^3)*d)*f)*x^2 - (4*a^
2*b^2*c*e - (a*b^3*c + 4*a^2*b*c^2)*d)*f + (6*a*b^2*c^2*e^2 + 4*(b^2*c^3 +
 2*a*c^4)*d^2 - 2*(b^3*c^2 + 8*a*b*c^3)*d*e + 2*(a*b^4 - a^2*b^2*c)*f^2 +
((b^4*c + 10*a*b^2*c^2 - 8*a^2*c^3)*d - (7*a*b^3*c - 4*a^2*b*c^2)*e)*f)*x)
/((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b^4*c - 6*a*b^2*c^
2 + 8*a^2*c^3)*f)*(c*x^2 + b*x + a)^((6*c^2*d - 3*b*c*e + 2*(b^2 - a*c)*f)
/(4*c^2*d - 2*b*c*e + b^2*f)))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right)\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**((2*a*c*f-2*b**2*f+3*b*c*e-6*c**2*d)/(b**2*f-2*b
*c*e+4*c**2*d))*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(149) = 298$.

Time = 0.21 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.82

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}}\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{((8\,c^5d^2-8\,bc^4de-3\,b^3c^2ef+b^4cf^2+2\,(e^2+3\,df)b^2c^3-2\,(4\,c^4df-2\,bc^3ef+b^2c^2f^2)a)x^3-(8\,c^3de}{}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+
4*c^2*d))*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-((8*c^5*d^2 - 8*b*c^4*d*e - 3*b^3*c^2*e*f + b^4*c*f^2 + 2*(e^2 + 3*d*f)*b
^2*c^3 - 2*(4*c^4*d*f - 2*b*c^3*e*f + b^2*c^2*f^2)*a)*x^3 - (8*c^3*d*e + 4
*b^2*c*e*f - b^3*f^2 - 4*(e^2 + d*f)*b*c^2)*a^2 + (12*b*c^4*d^2 - 10*b^2*c
^3*d*e - 3*b^4*c*e*f + b^5*f^2 + (2*e^2 + 7*d*f)*b^3*c^2 - (8*c^4*d*e + b^
3*c*f^2 - 4*(e^2 - d*f)*b*c^3)*a)*x^2 + (4*b*c^3*d^2 - 2*b^2*c^2*d*e + b^3
*c*d*f)*a + (4*b^2*c^3*d^2 - 2*b^3*c^2*d*e + b^4*c*d*f - 2*(4*c^3*d*f - 2*
b*c^2*e*f + b^2*c*f^2)*a^2 + (8*c^4*d^2 - 16*b*c^3*d*e - 7*b^3*c*e*f + 2*b
^4*f^2 + 2*(3*e^2 + 5*d*f)*b^2*c^2)*a)*x)*e^(-6*c^2*d*log(c*x^2 + b*x + a)
/(4*c^2*d - 2*b*c*e + b^2*f) + 3*b*c*e*log(c*x^2 + b*x + a)/(4*c^2*d - 2*b
*c*e + b^2*f) - 2*b^2*f*log(c*x^2 + b*x + a)/(4*c^2*d - 2*b*c*e + b^2*f) +
 2*a*c*f*log(c*x^2 + b*x + a)/(4*c^2*d - 2*b*c*e + b^2*f))/(2*b^2*c^3*d -
b^3*c^2*e + b^4*c*f + 8*a^2*c^3*f - 2*(4*c^4*d - 2*b*c^3*e + 3*b^2*c^2*f)*
a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3983 vs. $2(149) = 298$.

Time = 1.41 (sec) , antiderivative size = 3983, normalized size of antiderivative = 26.91

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+
4*c^2*d))*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-(8*c^5*d^2*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x + a) - 3*b*c*e*log(c*x^2 + b*
x + a) + 2*b^2*f*log(c*x^2 + b*x + a) - 2*a*c*f*log(c*x^2 + b*x + a))/(4*c
^2*d - 2*b*c*e + b^2*f)) - 8*b*c^4*d*e*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x +
a) - 3*b*c*e*log(c*x^2 + b*x + a) + 2*b^2*f*log(c*x^2 + b*x + a) - 2*a*c*f
*log(c*x^2 + b*x + a))/(4*c^2*d - 2*b*c*e + b^2*f)) + 2*b^2*c^3*e^2*x^3*e^
(-(6*c^2*d*log(c*x^2 + b*x + a) - 3*b*c*e*log(c*x^2 + b*x + a) + 2*b^2*f*l
og(c*x^2 + b*x + a) - 2*a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d - 2*b*c*e + b
^2*f)) + 6*b^2*c^3*d*f*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x + a) - 3*b*c*e*log
(c*x^2 + b*x + a) + 2*b^2*f*log(c*x^2 + b*x + a) - 2*a*c*f*log(c*x^2 + b*x
 + a))/(4*c^2*d - 2*b*c*e + b^2*f)) - 8*a*c^4*d*f*x^3*e^(-(6*c^2*d*log(c*x
^2 + b*x + a) - 3*b*c*e*log(c*x^2 + b*x + a) + 2*b^2*f*log(c*x^2 + b*x + a
) - 2*a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d - 2*b*c*e + b^2*f)) - 3*b^3*c^2
*e*f*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x + a) - 3*b*c*e*log(c*x^2 + b*x + a)
+ 2*b^2*f*log(c*x^2 + b*x + a) - 2*a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d -
2*b*c*e + b^2*f)) + 4*a*b*c^3*e*f*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x + a) -
3*b*c*e*log(c*x^2 + b*x + a) + 2*b^2*f*log(c*x^2 + b*x + a) - 2*a*c*f*log(
c*x^2 + b*x + a))/(4*c^2*d - 2*b*c*e + b^2*f)) + b^4*c*f^2*x^3*e^(-(6*c^2*
d*log(c*x^2 + b*x + a) - 3*b*c*e*log(c*x^2 + b*x + a) + 2*b^2*f*log(c*x^2
+ b*x + a) - 2*a*c*f*log(c*x^2 + b*x + a))/(4*c^2*d - 2*b*c*e + b^2*f)) -
2*a*b^2*c^2*f^2*x^3*e^(-(6*c^2*d*log(c*x^2 + b*x + a) - 3*b*c*e*log(c*x...
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.35

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right)\,dx = \frac{\frac{x^3\,\left(f\,b^2-2\,e\,b\,c+4\,d\,c^2\right)}{4\,a\,c-b^2}+\frac{a\,\left(f\,b^2-2\,e\,b\,c+4\,d\,c^2\right)\,\left(a\,b\,f-2\,a\,c\,e+b\,c\,d\right)}{c\,\left(4\,a\,c-b^2\right)\,\left(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c\right)}+\frac{x\,\left(f\,b^2-2\,e\,b\,c+4\,d\,c^2\right)\,\left(-2\,f\,a^2\,c+2\,f\,a\,b^2-3\,e\,a\,b\,c+2\,d\,a\,c^2+d\,b^2\,c\right)}{c\,\left(4\,a\,c-b^2\right)\,\left(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c\right)}}{{\left(c\,x^2+b\,x+a\right)}^{\frac{2\,f\,b^2-3\,e\,b\,c+6\,d\,c^2-2\,a\,f\,c}{f\,b^2-2\,e\,b\,c+4\,d\,c^2}}}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^((6*c^2*d + 2*b^2*f - 2*a*c*f - 3*
b*c*e)/(4*c^2*d + b^2*f - 2*b*c*e)),x)
```

output
```
((x^3*(4*c^2*d + b^2*f - 2*b*c*e))/(4*a*c - b^2) + (a*(4*c^2*d + b^2*f - 2
*b*c*e)*(a*b*f - 2*a*c*e + b*c*d))/(c*(4*a*c - b^2)*(2*c^2*d + b^2*f - 2*a
*c*f - b*c*e)) + (x*(4*c^2*d + b^2*f - 2*b*c*e)*(2*a*c^2*d + 2*a*b^2*f + b
^2*c*d - 2*a^2*c*f - 3*a*b*c*e))/(c*(4*a*c - b^2)*(2*c^2*d + b^2*f - 2*a*c
*f - b*c*e)) - (x^2*(4*c^2*d + b^2*f - 2*b*c*e)*(2*a*c^2*e - b^3*f - 3*b*c
^2*d + b^2*c*e + a*b*c*f))/(c*(4*a*c - b^2)*(2*c^2*d + b^2*f - 2*a*c*f - b
*c*e)))/(a + b*x + c*x^2)^((6*c^2*d + 2*b^2*f - 2*a*c*f - 3*b*c*e)/(4*c^2*
d + b^2*f - 2*b*c*e))
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^{\frac{-6c^2d+3bce-2b^2f+2acf}{4c^2d-2bce+b^2f}} \left(d+ex+fx^2\right) dx$$

$$= \left(\left(\int \frac{\left(cx^2+bx\ \cdots\right.}{\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} a^2+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} abx+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} acx^2\ \cdots}\right.\right.$$

$$+\left(\left(\int \frac{\left(cx^2+bx\ \cdots\right.}{\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} a^2+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} abx+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} ac\ \cdots}\right.\right.$$

$$+\left(\left(\int \frac{\left(cx^2+bx\ \cdots\right.}{\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} a^2+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} abx+2\left(cx^2+bx+a\right)^{\frac{bce}{b^2f-2bce+4c^2d}} ac\ \cdots}\right.\right.$$

input
```
int((c*x^2+b*x+a)^((2*a*c*f-2*b^2*f+3*b*c*e-6*c^2*d)/(b^2*f-2*b*c*e+4*c^2*
d))*(f*x^2+e*x+d),x)
```

output

```
int((a + b*x + c*x**2)**((2*a*c*f + 2*c**2*d)/(b**2*f - 2*b*c*e + 4*c**2*d
))/((a + b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a**2 + 2*(
a + b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*b*x + 2*(a +
b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*c*x**2 + (a + b*x
 + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b**2*x**2 + 2*(a + b*x
 + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b*c*x**3 + (a + b*x +
c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*c**2*x**4),x)*d + int(((a
 + b*x + c*x**2)**((2*a*c*f + 2*c**2*d)/(b**2*f - 2*b*c*e + 4*c**2*d))*x**
2)/((a + b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a**2 + 2*(
a + b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*b*x + 2*(a +
b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*c*x**2 + (a + b*x
 + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b**2*x**2 + 2*(a + b*x
 + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b*c*x**3 + (a + b*x +
c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*c**2*x**4),x)*f + int(((a
 + b*x + c*x**2)**((2*a*c*f + 2*c**2*d)/(b**2*f - 2*b*c*e + 4*c**2*d))*x)/
((a + b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a**2 + 2*(a +
 b*x + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*b*x + 2*(a + b*x
 + c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*a*c*x**2 + (a + b*x +
c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b**2*x**2 + 2*(a + b*x +
c*x**2)**((b*c*e)/(b**2*f - 2*b*c*e + 4*c**2*d))*b*c*x**3 + (a + b*x + ...
```

3.26 $\int (a+bx+cx^2)^2 ((4b^2-2ac) d+(4b^2-2ac) ex-7$

Optimal result

Integrand size = 54, antiderivative size = 37

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac) d+(4b^2-2ac) ex-7c(2cd-be)x^2\right) dx = \frac{1}{3}(4bd-ae-3(2cd-be)x) (a+bx+cx^2)^3$$

output
```
1/3*(4*b*d-a*e-3*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 110 vs. 2(37) = 74.

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.97

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac) d+(4b^2-2ac) ex-7c(2cd-be)x^2\right) dx = \frac{1}{3}x\left(-3a^3c(2d+ex)+x^2(b+cx)^3(4bd-6cdx+3bex)+3a^2(b+cx)(2b(2d+ex)-cx(6d+ex))+ax(b+cx)^2(-cx(18d+ex)+4b(3d+2ex))\right)$$

input
```
Integrate[(a + b*x + c*x^2)^2*((4*b^2 - 2*a*c)*d + (4*b^2 - 2*a*c)*e*x - 7
*c*(2*c*d - b*e)*x^2),x]
```

output
```
(x*(-3*a^3*c*(2*d + e*x) + x^2*(b + c*x)^3*(4*b*d - 6*c*d*x + 3*b*e*x) + 3
*a^2*(b + c*x)*(2*b*(2*d + e*x) - c*x*(6*d + e*x)) + a*x*(b + c*x)^2*(-(c*
x*(18*d + e*x)) + 4*b*(3*d + 2*e*x))))/3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. $2(37) = 74$.

Time = 0.54 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.54, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^2 \left(d\left(4b^2-2ac\right)+ex\left(4b^2-2ac\right)-7cx^2(2cd-be)\right) \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(-2a^2d\left(ac-2b^2\right)+4x^3\left(-a^2c^2e+5ab^2ce-8abc^2d+b^4e+2b^3cd\right)+x^2\left(3a^2bce-18a^2c^2d+8ab^3e+6ab^2cd\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2a^2dx\left(2b^2-ac\right)+x^4\left(-a^2c^2e+5ab^2ce-8abc^2d+b^4e+2b^3cd\right)+\frac{1}{3}x^3\left(3a^2bce-18a^2c^2d+8ab^3e+6ab^2cd+4b^4d\right)-\frac{1}{3}c^2x^6\left(ace-9b^2e+14bcd\right)+ax^2\left(2b^2-ac\right)(ae+2bd)-cx^5\left(-2abce+6ac^2d-3b^3e+2b^2cd\right)-c^3x^7(2cd-be)$$

input
```
Int[(a + b*x + c*x^2)^2*((4*b^2 - 2*a*c)*d + (4*b^2 - 2*a*c)*e*x - 7*c*(2*
c*d - b*e)*x^2),x]
```

output
```
2*a^2*(2*b^2 - a*c)*d*x + a*(2*b^2 - a*c)*(2*b*d + a*e)*x^2 + ((4*b^4*d +
6*a*b^2*c*d - 18*a^2*c^2*d + 8*a*b^3*e + 3*a^2*b*c*e)*x^3)/3 + (2*b^3*c*d
- 8*a*b*c^2*d + b^4*e + 5*a*b^2*c*e - a^2*c^2*e)*x^4 - c*(2*b^2*c*d + 6*a*
c^2*d - 3*b^3*e - 2*a*b*c*e)*x^5 - (c^2*(14*b*c*d - 9*b^2*e + a*c*e)*x^6)/
3 - c^3*(2*c*d - b*e)*x^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(35) = 70$.

Time = 1.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 6.00

method	result
norman	$\left(-\frac{1}{3}c^3ae+3b^2c^2e-\frac{14}{3}b\,c^3d\right)x^6+\left(a^2bce-6a^2c^2d+\frac{8}{3}a\,b^3e+2cda\,b^2+\frac{4}{3}b^4d\right)x^3+\left(-2a^3c\right.$
gosper	$-\frac{x\left(-3b\,c^3e\,x^6+6c^4d\,x^6+x^5c^3ae-9x^5b^2c^2e+14b\,c^3d\,x^5-6ab\,c^2e\,x^4+18a\,c^3d\,x^4-9b^3ce\,x^4+6b^2c^2d\,x^4+3a^2c^2e\,x^3-15a\,b^2ce\right.}{}$
risch	$-8ab\,c^2d\,x^4+4a^2b^2dx+2b^3cd\,x^4+2a\,b^2cd\,x^3+5a\,b^2ce\,x^4-6a\,c^3d\,x^5-2b^2c^2d\,x^5-2x^2a$
parallelrisch	$-8ab\,c^2d\,x^4+4a^2b^2dx+2b^3cd\,x^4+2a\,b^2cd\,x^3+5a\,b^2ce\,x^4-6a\,c^3d\,x^5-2b^2c^2d\,x^5-2x^2a$
default	$-c^3(-be+2cd)\,x^7+\frac{\left(-14b\,c^2(-be+2cd)+c^2\left(-2ac+4b^2\right)e\right)x^6}{6}+\frac{\left(-7\left(2ac+b^2\right)c(-be+2cd)+2bc\left(-2ac+4b^2\right)e+c^2\right.}{5}$
orering	$\frac{x\left(-3b\,c^3e\,x^6+6c^4d\,x^6+x^5c^3ae-9x^5b^2c^2e+14b\,c^3d\,x^5-6ab\,c^2e\,x^4+18a\,c^3d\,x^4-9b^3ce\,x^4+6b^2c^2d\,x^4+3a^2c^2e\,x^3-15a\,b^2ce\,x^3\right.}{}$

input
```
int((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d+(-2*a*c+4*b^2)*e*x-7*c*(-b*e+2*c*d)*
x^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c^3*a*e+3*b^2*c^2*e-14/3*b*c^3*d)*x^6+(a^2*b*c*e-6*a^2*c^2*d+8/3*a*b
^3*e+2*c*d*a*b^2+4/3*b^4*d)*x^3+(-2*a^3*c*d+4*a^2*b^2*d)*x+(b*c^3*e-2*c^4*
d)*x^7+(2*a*b*c^2*e-6*a*c^3*d+3*b^3*c*e-2*b^2*c^2*d)*x^5+(-a^3*c*e+2*a^2*b
^2*e-2*a^2*b*c*d+4*a*b^3*d)*x^2+(-a^2*c^2*e+5*a*b^2*c*e-8*a*b*c^2*d+b^4*e+
2*b^3*c*d)*x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 6.41

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)\,d+(4b^2-2ac)\,ex-7c(2cd-be)x^2\right)\,dx$$
$$= -(2\,c^4d-bc^3e)x^7 - \frac{1}{3}\left(14\,bc^3d-\left(9\,b^2c^2-ac^3\right)e\right)x^6 - \left(2\left(b^2c^2+3\,ac^3\right)d-\left(3\,b^3c+2\,abc^2\right)e\right)x^5 + \left(2\left(b^3c-4\,abc^2\right)d+\left(b^4+5\,ab^2c-a^2c^2\right)e\right)x^4 + \frac{1}{3}\left(2\left(2\,b^4+3\,ab^2c-9\,a^2c^2\right)d+\left(8\,ab^3+3\,a^2bc\right)e\right)x^3 + 2\left(2\,a^2b^2-a^3c\right)dx + \left(2\left(2\,ab^3-a^2bc\right)d+\left(2\,a^2b^2-a^3c\right)e\right)x^2$$

input

```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d+(-2*a*c+4*b^2)*e*x-7*c*(-b*e+2
*c*d)*x^2),x, algorithm="fricas")
```

output

```
-(2*c^4*d - b*c^3*e)*x^7 - 1/3*(14*b*c^3*d - (9*b^2*c^2 - a*c^3)*e)*x^6 -
(2*(b^2*c^2 + 3*a*c^3)*d - (3*b^3*c + 2*a*b*c^2)*e)*x^5 + (2*(b^3*c - 4*a*
b*c^2)*d + (b^4 + 5*a*b^2*c - a^2*c^2)*e)*x^4 + 1/3*(2*(2*b^4 + 3*a*b^2*c
- 9*a^2*c^2)*d + (8*a*b^3 + 3*a^2*b*c)*e)*x^3 + 2*(2*a^2*b^2 - a^3*c)*d*x
+ (2*(2*a*b^3 - a^2*b*c)*d + (2*a^2*b^2 - a^3*c)*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(36) = 72$.

Time = 0.05 (sec) , antiderivative size = 245, normalized size of antiderivative = 6.62

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)\,d+(4b^2-2ac)\,ex-7c(2cd-be)x^2\right)\,dx$$
$$= x^7\left(bc^3e-2c^4d\right) + x^6\left(-\frac{ac^3e}{3}+3b^2c^2e-\frac{14bc^3d}{3}\right) + x^5 \cdot \left(2abc^2e-6ac^3d+3b^3ce-2b^2c^2d\right) + x^4\left(-a^2c^2e+5ab^2ce-8abc^2d+b^4e+2b^3cd\right) + x^3\left(a^2bce-6a^2c^2d+\frac{8ab^3e}{3}+2ab^2cd+\frac{4b^4d}{3}\right) + x^2\left(-a^3ce+2a^2b^2e-2a^2bcd+4ab^3d\right) + x\left(-2a^3cd+4a^2b^2d\right)$$

input
```
integrate((c*x**2+b*x+a)**2*((-2*a*c+4*b**2)*d+(-2*a*c+4*b**2)*e*x-7*c*(-b
*e+2*c*d)*x**2),x)
```

output
```
x**7*(b*c**3*e - 2*c**4*d) + x**6*(-a*c**3*e/3 + 3*b**2*c**2*e - 14*b*c**3
*d/3) + x**5*(2*a*b*c**2*e - 6*a*c**3*d + 3*b**3*c*e - 2*b**2*c**2*d) + x*
*4*(-a**2*c**2*e + 5*a*b**2*c*e - 8*a*b*c**2*d + b**4*e + 2*b**3*c*d) + x*
*3*(a**2*b*c*e - 6*a**2*c**2*d + 8*a*b**3*e/3 + 2*a*b**2*c*d + 4*b**4*d/3)
 + x**2*(-a**3*c*e + 2*a**2*b**2*e - 2*a**2*b*c*d + 4*a*b**3*d) + x*(-2*a*
*3*c*d + 4*a**2*b**2*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 237, normalized size of antiderivative = 6.41

$$\begin{aligned}
&\int (a+bx+cx^2)^2 \left((4b^2-2ac)\,d+(4b^2-2ac)\,ex-7c(2cd-be)x^2\right)\,dx \\
&= -\left(2\,c^4d-bc^3e\right)x^7-\frac{1}{3}\left(14\,bc^3d-\left(9\,b^2c^2-ac^3\right)e\right)x^6 \\
&\quad -\left(2\left(b^2c^2+3\,ac^3\right)d-\left(3\,b^3c+2\,abc^2\right)e\right)x^5 \\
&\quad +\left(2\left(b^3c-4\,abc^2\right)d+\left(b^4+5\,ab^2c-a^2c^2\right)e\right)x^4 \\
&\quad +\frac{1}{3}\left(2\left(2\,b^4+3\,ab^2c-9\,a^2c^2\right)d+\left(8\,ab^3+3\,a^2bc\right)e\right)x^3 \\
&\quad +2\left(2\,a^2b^2-a^3c\right)dx+\left(2\left(2\,ab^3-a^2bc\right)d+\left(2\,a^2b^2-a^3c\right)e\right)x^2
\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d+(-2*a*c+4*b^2)*e*x-7*c*(-b*e+2
*c*d)*x^2),x, algorithm="maxima")
```

output
```
-(2*c^4*d - b*c^3*e)*x^7 - 1/3*(14*b*c^3*d - (9*b^2*c^2 - a*c^3)*e)*x^6 -
(2*(b^2*c^2 + 3*a*c^3)*d - (3*b^3*c + 2*a*b*c^2)*e)*x^5 + (2*(b^3*c - 4*a*
b*c^2)*d + (b^4 + 5*a*b^2*c - a^2*c^2)*e)*x^4 + 1/3*(2*(2*b^4 + 3*a*b^2*c
- 9*a^2*c^2)*d + (8*a*b^3 + 3*a^2*b*c)*e)*x^3 + 2*(2*a^2*b^2 - a^3*c)*d*x
+ (2*(2*a*b^3 - a^2*b*c)*d + (2*a^2*b^2 - a^3*c)*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.00

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)d+(4b^2-2ac)ex-7c(2cd-be)x^2\right) dx$$
$$= -2c^4dx^7 + bc^3ex^7 - \frac{14}{3}bc^3dx^6 + 3b^2c^2ex^6 - \frac{1}{3}ac^3ex^6 - 2b^2c^2dx^5 - 6ac^3dx^5 + 3b^3cex^5 + 2abc^2ex^5 + 2b^3cdx^4 - 8abc^2dx^4 + b^4ex^4 + 5ab^2cex^4 - a^2c^2ex^4 + \frac{4}{3}b^4dx^3 + 2ab^2cdx^3 - 6a^2c^2dx^3 + \frac{8}{3}ab^3ex^3 + a^2bcex^3 + 4ab^3dx^2 - 2a^2bcdx^2 + 2a^2b^2ex^2 - a^3cex^2 + 4a^2b^2dx - 2a^3cdx$$

input
```
integrate((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d+(-2*a*c+4*b^2)*e*x-7*c*(-b*e+2
*c*d)*x^2),x, algorithm="giac")
```

output
```
-2*c^4*d*x^7 + b*c^3*e*x^7 - 14/3*b*c^3*d*x^6 + 3*b^2*c^2*e*x^6 - 1/3*a*c^
3*e*x^6 - 2*b^2*c^2*d*x^5 - 6*a*c^3*d*x^5 + 3*b^3*c*e*x^5 + 2*a*b*c^2*e*x^
5 + 2*b^3*c*d*x^4 - 8*a*b*c^2*d*x^4 + b^4*e*x^4 + 5*a*b^2*c*e*x^4 - a^2*c^
2*e*x^4 + 4/3*b^4*d*x^3 + 2*a*b^2*c*d*x^3 - 6*a^2*c^2*d*x^3 + 8/3*a*b^3*e*
x^3 + a^2*b*c*e*x^3 + 4*a*b^3*d*x^2 - 2*a^2*b*c*d*x^2 + 2*a^2*b^2*e*x^2 -
a^3*c*e*x^2 + 4*a^2*b^2*d*x - 2*a^3*c*d*x
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.65

$$\int (a+bx+cx^2)^2 \left((4b^2-2ac)d+(4b^2-2ac)ex-7c(2cd-be)x^2\right) dx$$
$$= x^3\left(e\,a^2\,b\,c-6\,d\,a^2\,c^2+\frac{8\,e\,a\,b^3}{3}+2\,d\,a\,b^2\,c+\frac{4\,d\,b^4}{3}\right) - x^6\left(-3\,e\,b^2\,c^2+\frac{14\,d\,b\,c^3}{3}+\frac{a\,e\,c^3}{3}\right) - x^7\left(2\,c^4\,d-b\,c^3\,e\right) + x^4\left(-e\,a^2\,c^2+5\,e\,a\,b^2\,c-8\,d\,a\,b\,c^2+e\,b^4+2\,d\,b^3\,c\right) - x^5\left(-3\,e\,b^3\,c+2\,d\,b^2\,c^2-2\,a\,e\,b\,c^2+6\,a\,d\,c^3\right) - 2\,a^2\,d\,x\left(a\,c-2\,b^2\right) - a\,x^2\left(a\,c-2\,b^2\right)\left(a\,e+2\,b\,d\right)$$

input
```
int(-(a + b*x + c*x^2)^2*(d*(2*a*c - 4*b^2) - 7*c*x^2*(b*e - 2*c*d) + e*x*
(2*a*c - 4*b^2)),x)
```

output
```
x^3*((4*b^4*d)/3 - 6*a^2*c^2*d + (8*a*b^3*e)/3 + 2*a*b^2*c*d + a^2*b*c*e)
- x^6*((a*c^3*e)/3 - 3*b^2*c^2*e + (14*b*c^3*d)/3) - x^7*(2*c^4*d - b*c^3*
e) + x^4*(b^4*e - a^2*c^2*e + 2*b^3*c*d - 8*a*b*c^2*d + 5*a*b^2*c*e) - x^5
*(2*b^2*c^2*d + 6*a*c^3*d - 3*b^3*c*e - 2*a*b*c^2*e) - 2*a^2*d*x*(a*c - 2*
b^2) - a*x^2*(a*c - 2*b^2)*(a*e + 2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 6.89

$$\int (a+bx+cx^2)^2 \left(\left(4b^2-2ac\right)d+\left(4b^2-2ac\right)ex-7c(2cd-be)x^2\right) dx$$
$$= \frac{x(3bc^3ex^6-6c^4dx^6-ac^3ex^5+9b^2c^2ex^5-14bc^3dx^5+6abc^2ex^4-18ac^3dx^4+9b^3cex^4-6b^2c^2dx^4}{}$$

input
```
int((c*x^2+b*x+a)^2*((-2*a*c+4*b^2)*d+(-2*a*c+4*b^2)*e*x-7*c*(-b*e+2*c*d)*
x^2),x)
```

output
```
(x*( - 6*a**3*c*d - 3*a**3*c*e*x + 12*a**2*b**2*d + 6*a**2*b**2*e*x - 6*a*
*2*b*c*d*x + 3*a**2*b*c*e*x**2 - 18*a**2*c**2*d*x**2 - 3*a**2*c**2*e*x**3
+ 12*a*b**3*d*x + 8*a*b**3*e*x**2 + 6*a*b**2*c*d*x**2 + 15*a*b**2*c*e*x**3
 - 24*a*b*c**2*d*x**3 + 6*a*b*c**2*e*x**4 - 18*a*c**3*d*x**4 - a*c**3*e*x*
*5 + 4*b**4*d*x**2 + 3*b**4*e*x**3 + 6*b**3*c*d*x**3 + 9*b**3*c*e*x**4 - 6
*b**2*c**2*d*x**4 + 9*b**2*c**2*e*x**5 - 14*b*c**3*d*x**5 + 3*b*c**3*e*x**
6 - 6*c**4*d*x**6))/3
```

3.27 $\int (a+bx+cx^2) \left(\left(3b^2-2ac\right) d+\left(3b^2-2ac\right) ex-5c$

Optimal result

Integrand size = 52, antiderivative size = 37

$$\int (a+bx+cx^2) \left(\left(3b^2-2ac\right) d+\left(3b^2-2ac\right) ex-5c(2cd-be)x^2\right) \, dx = \frac{1}{2}(3bd-ae-2(2cd-be)x) \left(a+bx+cx^2\right)^2$$

output
```
1/2*(3*b*d-a*e-2*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

$$\int (a+bx+cx^2) \left(\left(3b^2-2ac\right) d+\left(3b^2-2ac\right) ex-5c(2cd-be)x^2\right) \, dx = \frac{1}{2}x\left(-2a^2c(2d+ex)+x(b+cx)^2(3bd-4cdx+2bex) + a(b+cx)(3b(2d+ex)-cx(8d+ex))\right)$$

input
```
Integrate[(a + b*x + c*x^2)*((3*b^2 - 2*a*c)*d + (3*b^2 - 2*a*c)*e*x - 5*c
*(2*c*d - b*e)*x^2),x]
```

output
```
(x*(-2*a^2*c*(2*d + e*x) + x*(b + c*x)^2*(3*b*d - 4*c*d*x + 2*b*e*x) + a*(
b + c*x)*(3*b*(2*d + e*x) - c*x*(8*d + e*x))))/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(37) = 74$.

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)\left(d\left(3b^2-2ac\right)+ex\left(3b^2-2ac\right)-5cx^2(2cd-be)\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-2cx^3\left(ace-4b^2e+5bcd\right)+x\left(3b^2-2ac\right)(ae+bd)-ad\left(2ac-3b^2\right)+3x^2\left(abce-4ac^2d+b^3e+b^2cd\right)-5c \text{[truncated]}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}cx^4\left(ace-4b^2e+5bcd\right)+\frac{1}{2}x^2\left(3b^2-2ac\right)(ae+bd)+adx\left(3b^2-2ac\right)+x^3\left(abce-4ac^2d+b^3e+b^2cd\right)-c^2x^5(2cd-be)$$

input
```
Int[(a + b*x + c*x^2)*((3*b^2 - 2*a*c)*d + (3*b^2 - 2*a*c)*e*x - 5*c*(2*c*
d - b*e)*x^2),x]
```

output
```
a*(3*b^2 - 2*a*c)*d*x + ((3*b^2 - 2*a*c)*(b*d + a*e)*x^2)/2 + (b^2*c*d - 4
*a*c^2*d + b^3*e + a*b*c*e)*x^3 - (c*(5*b*c*d - 4*b^2*e + a*c*e)*x^4)/2 -
c^2*(2*c*d - b*e)*x^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(35) = 70$.

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.27

method	result
norman	$\left(-\frac{1}{2}c^2ae+2ce\,b^2-\frac{5}{2}b\,c^2d\right)x^4+\left(-a^2ce+\frac{3}{2}ea\,b^2-abcd+\frac{3}{2}b^3d\right)x^2+\left(-2a^2cd+3da\,b^2\right)x$
gosper	$-\frac{x\left(-2b\,c^2e\,x^4+4c^3d\,x^4+a\,c^2e\,x^3-4b^2ce\,x^3+5b\,c^2d\,x^3-2abce\,x^2+8a\,c^2d\,x^2-2b^3e\,x^2-2b^2c\,x^2d+2a^2cex-3a\,b^2ex+2abcdx-\right.}{2}$
risch	$b\,c^2e\,x^5-2c^3d\,x^5-\frac{1}{2}c^2ae\,x^4+2x^4ce\,b^2-\frac{5}{2}b\,c^2d\,x^4+abce\,x^3-4a\,c^2d\,x^3+b^3e\,x^3+b^2cd\,x$
parallelrisch	$b\,c^2e\,x^5-2c^3d\,x^5-\frac{1}{2}c^2ae\,x^4+2x^4ce\,b^2-\frac{5}{2}b\,c^2d\,x^4+abce\,x^3-4a\,c^2d\,x^3+b^3e\,x^3+b^2cd\,x$
default	$-c^2(-be+2cd)\,x^5+\frac{\left(-5bc(-be+2cd)+c\left(-2ac+3b^2\right)e\right)x^4}{4}+\frac{\left(-5ac(-be+2cd)+b\left(-2ac+3b^2\right)e+c\left(-2ac+3b^2\right)d\right)}{3}$
orering	$\frac{x\left(-2b\,c^2e\,x^4+4c^3d\,x^4+a\,c^2e\,x^3-4b^2ce\,x^3+5b\,c^2d\,x^3-2abce\,x^2+8a\,c^2d\,x^2-2b^3e\,x^2-2b^2c\,x^2d+2a^2cex-3a\,b^2ex+2abcdx-3\right.}{-10bce\,x^2+20c^2d\,x^2+4acex-6b^2ex+4acd-6b^2d}$

input
```
int((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d+(-2*a*c+3*b^2)*e*x-5*c*(-b*e+2*c*d)*x^
2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^2*a*e+2*c*e*b^2-5/2*b*c^2*d)*x^4+(-a^2*c*e+3/2*e*a*b^2-a*b*c*d+3/2
*b^3*d)*x^2+(-2*a^2*c*d+3*a*b^2*d)*x+(b*c^2*e-2*c^3*d)*x^5+(a*b*c*e-4*a*c^
2*d+b^3*e+b^2*c*d)*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(35) = 70$.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.46

$$\int (a+bx+cx^2)\left((3b^2-2ac)d+(3b^2-2ac)ex-5c(2cd-be)x^2\right)dx$$
$$=-(2c^3d-bc^2e)x^5-\frac{1}{2}(5bc^2d-(4b^2c-ac^2)e)x^4+((b^2c-4ac^2)d+(b^3+abc)e)x^3+(3ab^2-2a^2c)dx+\frac{1}{2}((3b^3-2abc)d+(3ab^2-2a^2c)e)x^2$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d+(-2*a*c+3*b^2)*e*x-5*c*(-b*e+2*c
*d)*x^2),x, algorithm="fricas")
```

output
```
-(2*c^3*d - b*c^2*e)*x^5 - 1/2*(5*b*c^2*d - (4*b^2*c - a*c^2)*e)*x^4 + ((b
^2*c - 4*a*c^2)*d + (b^3 + a*b*c)*e)*x^3 + (3*a*b^2 - 2*a^2*c)*d*x + 1/2*(
(3*b^3 - 2*a*b*c)*d + (3*a*b^2 - 2*a^2*c)*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int (a+bx+cx^2)\left((3b^2-2ac)d+(3b^2-2ac)ex-5c(2cd-be)x^2\right)dx$$
$$=x^5(bc^2e-2c^3d)+x^4\left(-\frac{ac^2e}{2}+2b^2ce-\frac{5bc^2d}{2}\right)+x^3(abce-4ac^2d+b^3e+b^2cd)+x^2\left(-a^2ce+\frac{3ab^2e}{2}-abcd+\frac{3b^3d}{2}\right)+x(-2a^2cd+3ab^2d)$$

input
```
integrate((c*x**2+b*x+a)*((-2*a*c+3*b**2)*d+(-2*a*c+3*b**2)*e*x-5*c*(-b*e+
2*c*d)*x**2),x)
```

output
```
x**5*(b*c**2*e - 2*c**3*d) + x**4*(-a*c**2*e/2 + 2*b**2*c*e - 5*b*c**2*d/2
) + x**3*(a*b*c*e - 4*a*c**2*d + b**3*e + b**2*c*d) + x**2*(-a**2*c*e + 3*
a*b**2*e/2 - a*b*c*d + 3*b**3*d/2) + x*(-2*a**2*c*d + 3*a*b**2*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.46

$$\int (a+bx+cx^2)\left((3b^2-2ac)\,d+(3b^2-2ac)\,ex-5c(2cd-be)x^2\right)\,dx$$
$$=-(2\,c^3d-bc^2e)x^5-\frac{1}{2}\left(5\,bc^2d-\left(4\,b^2c-ac^2\right)e\right)x^4+\left(\left(b^2c-4\,ac^2\right)d+\left(b^3+abc\right)e\right)x^3+\left(3\,ab^2-2\,a^2c\right)dx+\frac{1}{2}\left(\left(3\,b^3-2\,abc\right)d+\left(3\,ab^2-2\,a^2c\right)e\right)x^2$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d+(-2*a*c+3*b^2)*e*x-5*c*(-b*e+2*c
*d)*x^2),x, algorithm="maxima")
```

output
```
-(2*c^3*d - b*c^2*e)*x^5 - 1/2*(5*b*c^2*d - (4*b^2*c - a*c^2)*e)*x^4 + ((b
^2*c - 4*a*c^2)*d + (b^3 + a*b*c)*e)*x^3 + (3*a*b^2 - 2*a^2*c)*d*x + 1/2*(
(3*b^3 - 2*a*b*c)*d + (3*a*b^2 - 2*a^2*c)*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.73

$$\int (a+bx+cx^2)\left((3b^2-2ac)\,d+(3b^2-2ac)\,ex-5c(2cd-be)x^2\right)\,dx$$
$$=-2\,c^3dx^5+bc^2ex^5-\frac{5}{2}\,bc^2dx^4+2\,b^2cex^4-\frac{1}{2}\,ac^2ex^4+b^2cdx^3-4\,ac^2dx^3+b^3ex^3+abcex^3+\frac{3}{2}\,b^3dx^2-abcdx^2+\frac{3}{2}\,ab^2ex^2-a^2cex^2+3\,ab^2dx-2\,a^2cdx$$

input
```
integrate((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d+(-2*a*c+3*b^2)*e*x-5*c*(-b*e+2*c
*d)*x^2),x, algorithm="giac")
```

output
```
-2*c^3*d*x^5 + b*c^2*e*x^5 - 5/2*b*c^2*d*x^4 + 2*b^2*c*e*x^4 - 1/2*a*c^2*e
*x^4 + b^2*c*d*x^3 - 4*a*c^2*d*x^3 + b^3*e*x^3 + a*b*c*e*x^3 + 3/2*b^3*d*x
^2 - a*b*c*d*x^2 + 3/2*a*b^2*e*x^2 - a^2*c*e*x^2 + 3*a*b^2*d*x - 2*a^2*c*d
*x
```

Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.03

$$\int (a+bx+cx^2) \left((3b^2-2ac)\,d+(3b^2-2ac)\,ex-5c(2cd-be)x^2\right)\,dx$$
$$=\left(b\,c^2\,e-2\,c^3\,d\right)\,x^5+\left(2\,e\,b^2\,c-\frac{5\,d\,b\,c^2}{2}-\frac{a\,e\,c^2}{2}\right)\,x^4+\left(e\,b^3+d\,b^2\,c+a\,e\,b\,c-4\,a\,d\,c^2\right)\,x^3-\frac{\left(2\,a\,c-3\,b^2\right)\,\left(a\,e+b\,d\right)\,x^2}{2}-a\,d\,\left(2\,a\,c-3\,b^2\right)\,x$$

input
```
int(-(a + b*x + c*x^2)*(d*(2*a*c - 3*b^2) - 5*c*x^2*(b*e - 2*c*d) + e*x*(2
*a*c - 3*b^2)),x)
```

output
```
x^3*(b^3*e - 4*a*c^2*d + b^2*c*d + a*b*c*e) - x^4*((a*c^2*e)/2 + (5*b*c^2*
d)/2 - 2*b^2*c*e) - x^5*(2*c^3*d - b*c^2*e) - (x^2*(2*a*c - 3*b^2)*(a*e +
b*d))/2 - a*d*x*(2*a*c - 3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.65

$$\int (a+bx+cx^2) \left((3b^2-2ac)\,d+(3b^2-2ac)\,ex-5c(2cd-be)x^2\right)\,dx$$
$$=\frac{x(2b\,c^2e\,x^4-4c^3d\,x^4-a\,c^2e\,x^3+4b^2ce\,x^3-5b\,c^2d\,x^3+2abce\,x^2-8a\,c^2d\,x^2+2b^3e\,x^2+2b^2cd\,x^2-2a^2c}{2}$$

input
```
int((c*x^2+b*x+a)*((-2*a*c+3*b^2)*d+(-2*a*c+3*b^2)*e*x-5*c*(-b*e+2*c*d)*x^
2),x)
```

output
```
(x*( - 4*a**2*c*d - 2*a**2*c*e*x + 6*a*b**2*d + 3*a*b**2*e*x - 2*a*b*c*d*x
 + 2*a*b*c*e*x**2 - 8*a*c**2*d*x**2 - a*c**2*e*x**3 + 3*b**3*d*x + 2*b**3*
e*x**2 + 2*b**2*c*d*x**2 + 4*b**2*c*e*x**3 - 5*b*c**2*d*x**3 + 2*b*c**2*e*
x**4 - 4*c**3*d*x**4))/2
```

3.28 $\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)$

Optimal result

Integrand size = 41, antiderivative size = 41

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx = 2\left(b^2-ac\right)dx+\left(b^2-ac\right)ex^2-c(2cd-be)x^3$$

output
```
2*(-a*c+b^2)*d*x+(-a*c+b^2)*e*x^2-c*(-b*e+2*c*d)*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx = 2b^2dx-2acdx+b^2ex^2-acex^2-2c^2dx^3+bcex^3$$

input
```
Integrate[(2*b^2 - 2*a*c)*d + (2*b^2 - 2*a*c)*e*x - 3*c*(2*c*d - b*e)*x^2,
x]
```

output
```
2*b^2*d*x - 2*a*c*d*x + b^2*e*x^2 - a*c*e*x^2 - 2*c^2*d*x^3 + b*c*e*x^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d\left(2b^2-2ac\right)+ex\left(2b^2-2ac\right)-3cx^2(2cd-be)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$2dx\left(b^2-ac\right)+ex^2\left(b^2-ac\right)-cx^3(2cd-be)$$

input
```
Int[(2*b^2 - 2*a*c)*d + (2*b^2 - 2*a*c)*e*x - 3*c*(2*c*d - b*e)*x^2,x]
```

output
```
2*(b^2 - a*c)*d*x + (b^2 - a*c)*e*x^2 - c*(2*c*d - b*e)*x^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

method	result	size
gosper	$-x(-bce\,x^2+2c^2d\,x^2+acex-b^2ex+2acd-2b^2d)$	45
norman	$(-2acd+2b^2d)\,x+(-ace+e\,b^2)\,x^2+(bce-2c^2d)\,x^3$	46
default	$bce\,x^3-2c^2d\,x^3-ace\,x^2+b^2e\,x^2-2adxc+2b^2dx$	47
risch	$bce\,x^3-2c^2d\,x^3-ace\,x^2+b^2e\,x^2-2adxc+2b^2dx$	47
parallelrisch	$bce\,x^3-2c^2d\,x^3-ace\,x^2+b^2e\,x^2+(-2ac+2b^2)\,dx$	47
parts	$bce\,x^3-2c^2d\,x^3-ace\,x^2+b^2e\,x^2-2adxc+2b^2dx$	47
orering	$\frac{x(-bce\,x^2+2c^2d\,x^2+acex-b^2ex+2acd-2b^2d)((-2ac+2b^2)d+(-2ac+2b^2)ex-3c(-be+2cd)x^2)}{-3bce\,x^2+6c^2d\,x^2+2acex-2b^2ex+2acd-2b^2d}$	129

input
```
int((-2*a*c+2*b^2)*d+(-2*a*c+2*b^2)*e*x-3*c*(-b*e+2*c*d)*x^2,x,method=_RET
URNVERBOSE)
```

output
```
-x*(-b*c*e*x^2+2*c^2*d*x^2+a*c*e*x-b^2*e*x+2*a*c*d-2*b^2*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \left((2b^2-2ac)\,d+(2b^2-2ac)\,ex-3c(2cd-be)x^2\right)\,dx$$
$$=(b^2-ac)ex^2-(2\,c^2d-bce)x^3+2\,(b^2-ac)dx$$

input
```
integrate((-2*a*c+2*b^2)*d+(-2*a*c+2*b^2)*e*x-3*c*(-b*e+2*c*d)*x^2,x, algo
rithm="fricas")
```

output
```
(b^2 - a*c)*e*x^2 - (2*c^2*d - b*c*e)*x^3 + 2*(b^2 - a*c)*d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx$$
$$=x^3\left(bce-2c^2d\right)+x^2\left(-ace+b^2e\right)+x\left(-2acd+2b^2d\right)$$

input
```
integrate((-2*a*c+2*b**2)*d+(-2*a*c+2*b**2)*e*x-3*c*(-b*e+2*c*d)*x**2,x)
```

output
```
x**3*(b*c*e - 2*c**2*d) + x**2*(-a*c*e + b**2*e) + x*(-2*a*c*d + 2*b**2*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx$$
$$=-(2\,cd-be)cx^3+\left(b^2-ac\right)ex^2+2\left(b^2-ac\right)dx$$

input
```
integrate((-2*a*c+2*b^2)*d+(-2*a*c+2*b^2)*e*x-3*c*(-b*e+2*c*d)*x^2,x, algo
rithm="maxima")
```

output
```
-(2*c*d - b*e)*c*x^3 + (b^2 - a*c)*e*x^2 + 2*(b^2 - a*c)*d*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx$$
$$=-(2\,cd-be)cx^3+\left(b^2-ac\right)ex^2+2\left(b^2-ac\right)dx$$

input
```
integrate((-2*a*c+2*b^2)*d+(-2*a*c+2*b^2)*e*x-3*c*(-b*e+2*c*d)*x^2,x, algo
rithm="giac")
```

output
```
-(2*c*d - b*e)*c*x^3 + (b^2 - a*c)*e*x^2 + 2*(b^2 - a*c)*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx$$
$$=c\,(b\,e-2\,c\,d)\,x^3-\frac{e\,(2\,a\,c-2\,b^2)\,x^2}{2}-d\,\left(2\,a\,c-2\,b^2\right)\,x$$

input
```
int(3*c*x^2*(b*e - 2*c*d) - d*(2*a*c - 2*b^2) - e*x*(2*a*c - 2*b^2),x)
```

output
```
c*x^3*(b*e - 2*c*d) - (e*x^2*(2*a*c - 2*b^2))/2 - d*x*(2*a*c - 2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \left(\left(2b^2-2ac\right)d+\left(2b^2-2ac\right)ex-3c(2cd-be)x^2\right)\,dx$$
$$=x\left(bce\,x^2-2c^2d\,x^2-acex+b^2ex-2acd+2b^2d\right)$$

input
```
int((-2*a*c+2*b^2)*d+(-2*a*c+2*b^2)*e*x-3*c*(-b*e+2*c*d)*x^2,x)
```

output
```
x*( - 2*a*c*d - a*c*e*x + 2*b**2*d + b**2*e*x + b*c*e*x**2 - 2*c**2*d*x**2
)
```

3.29 $\int \frac{(b^2-2ac)d+(b^2-2ac)ex-c(2cd-be)x^2}{a+bx+cx^2} dx$

Optimal result

Integrand size = 50, antiderivative size = 33

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx = -((2cd-be)x)+(bd-ae)\log\left(a+bx+cx^2\right)$$

output
```
-(-b*e+2*c*d)*x+(-a*e+b*d)*ln(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx = (-2cd+be)x+(bd-ae)\log\left(a+bx+cx^2\right)$$

input
```
Integrate[((b^2 - 2*a*c)*d + (b^2 - 2*a*c)*e*x - c*(2*c*d - b*e)*x^2)/(a +
 b*x + c*x^2),x]
```

output
```
(-2*c*d + b*e)*x + (b*d - a*e)*Log[a + b*x + c*x^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(b^2-2ac)+ex(b^2-2ac)-cx^2(2cd-be)}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{2cx(bd-ae)+b(bd-ae)}{a+bx+cx^2}+be-2cd\right)dx$$

$$\Big\downarrow \text{2009}$$

$$(bd-ae)\log\left(a+bx+cx^2\right)-x(2cd-be)$$

input
```
Int[((b^2 - 2*a*c)*d + (b^2 - 2*a*c)*e*x - c*(2*c*d - b*e)*x^2)/(a + b*x +
 c*x^2),x]
```

output
```
-((2*c*d - b*e)*x) + (b*d - a*e)*Log[a + b*x + c*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
default	$bex - 2cdx + (-ae + bd) \ln (c x^2 + bx + a)$	31
norman	$(be - 2cd) x + (-ae + bd) \ln (c x^2 + bx + a)$	32
risch	$bex - 2cdx - \ln (c x^2 + bx + a) ae + bd \ln (c x^2 + bx + a)$	40
parallelrisch	$bex - 2cdx - \ln (c x^2 + bx + a) ae + bd \ln (c x^2 + bx + a)$	40

input
```
int(((-2*a*c+b^2)*d+(-2*a*c+b^2)*e*x-c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a),x,m
ethod=_RETURNVERBOSE)
```

output
```
b*e*x-2*c*d*x+(-a*e+b*d)*ln(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(b^2 - 2ac)\, d + (b^2 - 2ac)\, ex - c(2cd - be)x^2}{a + bx + cx^2}\, dx \\ = -(2\, cd - be)x + (bd - ae) \log \left(cx^2 + bx + a\right)$$

input
```
integrate(((-2*a*c+b^2)*d+(-2*a*c+b^2)*e*x-c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a),x, algorithm="fricas")
```

output
```
-(2*c*d - b*e)*x + (b*d - a*e)*log(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx$$
$$= x(be-2cd)+(-ae+bd)\log\left(a+bx+cx^2\right)$$

input
```
integrate(((-2*a*c+b**2)*d+(-2*a*c+b**2)*e*x-c*(-b*e+2*c*d)*x**2)/(c*x**2+
b*x+a),x)
```

output
```
x*(b*e - 2*c*d) + (-a*e + b*d)*log(a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx$$
$$= -(2\,cd-be)x+(bd-ae)\log\left(cx^2+bx+a\right)$$

input
```
integrate(((-2*a*c+b^2)*d+(-2*a*c+b^2)*e*x-c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a),x, algorithm="maxima")
```

output
```
-(2*c*d - b*e)*x + (b*d - a*e)*log(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx$$
$$= (bd-ae)\log\left(cx^2+bx+a\right)-\frac{2\,c^2dx-bcex}{c}$$

input
```
integrate(((-2*a*c+b^2)*d+(-2*a*c+b^2)*e*x-c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a),x, algorithm="giac")
```

output
```
(b*d - a*e)*log(c*x^2 + b*x + a) - (2*c^2*d*x - b*c*e*x)/c
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx$$
$$=-\ln\left(c\,x^2+b\,x+a\right)\,\left(a\,e-b\,d\right)-\frac{x\,\left(2\,c^2\,d-b\,c\,e\right)}{c}$$

input
```
int(-(d*(2*a*c - b^2) - c*x^2*(b*e - 2*c*d) + e*x*(2*a*c - b^2))/(a + b*x
+ c*x^2),x)
```

output
```
- log(a + b*x + c*x^2)*(a*e - b*d) - (x*(2*c^2*d - b*c*e))/c
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{(b^2-2ac)\,d+(b^2-2ac)\,ex-c(2cd-be)x^2}{a+bx+cx^2}\,dx$$
$$=-\log\left(c\,x^2+bx+a\right)ae+\log\left(c\,x^2+bx+a\right)bd+bex-2cdx$$

input
```
int(((-2*a*c+b^2)*d+(-2*a*c+b^2)*e*x-c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a),x)
```

output
```
- log(a + b*x + c*x**2)*a*e + log(a + b*x + c*x**2)*b*d + b*e*x - 2*c*d*x
```

3.30 $\int \frac{-2acd-2acex+c(2cd-be)x^2}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 29

$$\int \frac{-2acd-2acex+c(2cd-be)x^2}{(a+bx+cx^2)^2}\,dx=\frac{ae-(2cd-be)x}{a+bx+cx^2}$$

output
```
(a*e-(-b*e+2*c*d)*x)/(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{-2acd-2acex+c(2cd-be)x^2}{(a+bx+cx^2)^2}\,dx=\frac{ae-2cdx+bex}{a+bx+cx^2}$$

input
```
Integrate[(-2*a*c*d - 2*a*c*e*x + c*(2*c*d - b*e)*x^2)/(a + b*x + c*x^2)^2
,x]
```

output
```
(a*e - 2*c*d*x + b*e*x)/(a + b*x + c*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2acd-2acex+cx^2(2cd-be)}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{ae(b^2-4ac)-x(b^2-4ac)(2cd-be)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int 0dx}{b^2-4ac}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae(b^2-4ac)-x(b^2-4ac)(2cd-be)}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(-2*a*c*d - 2*a*c*e*x + c*(2*c*d - b*e)*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(a*(b^2 - 4*a*c)*e - (b^2 - 4*a*c)*(2*c*d - b*e)*x)/((b^2 - 4*a*c)*(a + b*
x + c*x^2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{bex-2cdx+ae}{c\,x^{2}+bx+a}$	27
risch	$\frac{(be-2cd)x+ae}{c\,x^{2}+bx+a}$	28
parallelrisch	$-\frac{ace\,x^{2}+2adxc}{a(c\,x^{2}+bx+a)}$	32
default	$-\frac{c\left(-\frac{(be-2cd)x}{c}-\frac{ae}{c}\right)}{c\,x^{2}+bx+a}$	38
norman	$\frac{\frac{2adc}{b}-\frac{c(be-2cd)x^{2}}{b}}{c\,x^{2}+bx+a}$	40
orering	$-\frac{(bex-2cdx+ae)\left(-2acd-2acex+c(-be+2cd)x^{2}\right)}{(c\,x^{2}+bx+a)c(be\,x^{2}-2cd\,x^{2}+2aex+2ad)}$	82

input
```
int((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^2,x,method=_RETU
RNVERBOSE)
```

output
```
(b*e*x-2*c*d*x+a*e)/(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2}\,dx = \frac{ae - (2\,cd - be)x}{cx^2 + bx + a}$$

input
```
integrate((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^2,x, algor
ithm="fricas")
```

output
```
(a*e - (2*c*d - b*e)*x)/(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2}\,dx = -\frac{-ae + x(-be + 2cd)}{a + bx + cx^2}$$

input
```
integrate((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x**2)/(c*x**2+b*x+a)**2,x)
```

output
```
-(-a*e + x*(-b*e + 2*c*d))/(a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2}\,dx = \frac{ae - (2\,cd - be)x}{cx^2 + bx + a}$$

input
```
integrate((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^2,x, algor
ithm="maxima")
```

output
```
(a*e - (2*c*d - b*e)*x)/(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2} \, dx = -\frac{2\, cdx - bex - ae}{cx^2 + bx + a}$$

input
```
integrate((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^2,x, algor
ithm="giac")
```

output
```
-(2*c*d*x - b*e*x - a*e)/(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2} \, dx = \frac{a\, e + x\, (b\, e - 2\, c\, d)}{c\, x^2 + b\, x + a}$$

input
```
int(-(c*x^2*(b*e - 2*c*d) + 2*a*c*d + 2*a*c*e*x)/(a + b*x + c*x^2)^2,x)
```

output
```
(a*e + x*(b*e - 2*c*d))/(a + b*x + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{-2acd - 2acex + c(2cd - be)x^2}{(a + bx + cx^2)^2} \, dx = \frac{c(-be\, x^2 + 2cd\, x^2 + 2ad)}{b\, (c\, x^2 + bx + a)}$$

input
```
int((-2*a*c*d-2*a*c*e*x+c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^2,x)
```

output
```
(c*(2*a*d - b*e*x**2 + 2*c*d*x**2))/(b*(a + b*x + c*x**2))
```

3.31 $\int \frac{(-b^2-2ac)d+(-b^2-2ac)ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 35

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx=\frac{bd+ae-2(2cd-be)x}{2\,(a+bx+cx^2)^2}$$

output
```
1/2*(b*d+a*e-2*(-b*e+2*c*d)*x)/(c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx=\frac{ae-4cdx+b(d+2ex)}{2(a+x(b+cx))^2}$$

input
```
Integrate[((-b^2 - 2*a*c)*d + (-b^2 - 2*a*c)*e*x + 3*c*(2*c*d - b*e)*x^2)/
(a + b*x + c*x^2)^3,x]
```

output
```
(a*e - 4*c*d*x + b*(d + 2*e*x))/(2*(a + x*(b + c*x))^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-b^2)+ex(-2ac-b^2)+3cx^2(2cd-be)}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{ae-2x(2cd-be)+bd}{2\left(a+bx+cx^2\right)^2}-\frac{\int 0dx}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-2x(2cd-be)+bd}{2\left(a+bx+cx^2\right)^2}$$

input
```
Int[((-b^2 - 2*a*c)*d + (-b^2 - 2*a*c)*e*x + 3*c*(2*c*d - b*e)*x^2)/(a + b
*x + c*x^2)^3,x]
```

output
```
(b*d + a*e - 2*(2*c*d - b*e)*x)/(2*(a + b*x + c*x^2)^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{2bex-4cdx+ae+bd}{2(c\,x^2+bx+a)^2}$	32
risch	$\frac{(be-2cd)x+\frac{ae}{2}+\frac{bd}{2}}{(c\,x^2+bx+a)^2}$	33
default	$-\frac{(-be+2cd)x-\frac{ae}{2}-\frac{bd}{2}}{(c\,x^2+bx+a)^2}$	35
parallelrisch	$\frac{2b\,c^2ex-4c^3dx+c^2ae+b\,c^2d}{2c^2(c\,x^2+bx+a)^2}$	46
norman	$\frac{\frac{(b\,c^2e-2d\,c^3)x}{c^2}+\frac{c^2ae+b\,c^2d}{2c^2}}{(c\,x^2+bx+a)^2}$	51
orering	$-\frac{(2bex-4cdx+ae+bd)\left((-2ac-b^2)d+(-2ac-b^2)ex+3c(-be+2cd)x^2\right)}{2(c\,x^2+bx+a)^2(3bce\,x^2-6c^2d\,x^2+2acex+b^2ex+2acd+b^2d)}$	115

input
```
int(((-2*a*c-b^2)*d+(-2*a*c-b^2)*e*x+3*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^3
,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b*e*x-4*c*d*x+a*e+b*d)/(c*x^2+b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx = \frac{bd+ae-2\,(2\,cd-be)x}{2\,(c^2x^4+2\,bcx^3+2\,abx+(b^2+2\,ac)x^2+a^2)}$$

input
```
integrate(((-2*a*c-b^2)*d+(-2*a*c-b^2)*e*x+3*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*
x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b*d + a*e - 2*(2*c*d - b*e)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2
+ 2*a*c)*x^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx = -\frac{-ae-bd+x(-2be+4cd)}{2a^2+4abx+4bcx^3+2c^2x^4+x^2\cdot(4ac+2b^2)}$$

input
```
integrate(((-2*a*c-b**2)*d+(-2*a*c-b**2)*e*x+3*c*(-b*e+2*c*d)*x**2)/(c*x**
2+b*x+a)**3,x)
```

output
```
-(-a*e - b*d + x*(-2*b*e + 4*c*d))/(2*a**2 + 4*a*b*x + 4*b*c*x**3 + 2*c**2
*x**4 + x**2*(4*a*c + 2*b**2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx = \frac{bd+ae-2\,(2\,cd-be)x}{2\,(c^2x^4+2\,bcx^3+2\,abx+(b^2+2\,ac)x^2+a^2)}$$

input
```
integrate(((-2*a*c-b^2)*d+(-2*a*c-b^2)*e*x+3*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*
x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(b*d + a*e - 2*(2*c*d - b*e)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2
+ 2*a*c)*x^2 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx = -\frac{4\,cdx-2\,bex-bd-ae}{2\,(cx^2+bx+a)^2}$$

input
```
integrate(((-2*a*c-b^2)*d+(-2*a*c-b^2)*e*x+3*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*
x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(4*c*d*x - 2*b*e*x - b*d - a*e)/(c*x^2 + b*x + a)^2
```

Mupad [B] (verification not implemented)

Time = 15.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{(-b^2-2ac)\,d+(-b^2-2ac)\,ex+3c(2cd-be)x^2}{(a+bx+cx^2)^3}\,dx = \frac{b\left(\frac{d}{2}+e\,x\right)+\frac{a\,e}{2}-2\,c\,d\,x}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

input
```
int(-(d*(2*a*c + b^2) + 3*c*x^2*(b*e - 2*c*d) + e*x*(2*a*c + b^2))/(a + b*
x + c*x^2)^3,x)
```

output
```
(b*(d/2 + e*x) + (a*e)/2 - 2*c*d*x)/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2
*a*b*x + 2*b*c*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{(-b^2 - 2ac)\, d + (-b^2 - 2ac)\, ex + 3c(2cd - be)x^2}{(a + bx + cx^2)^3}\, dx = \frac{2bex - 4cdx + ae + bd}{2c^2x^4 + 4bc\, x^3 + 4ac\, x^2 + 2b^2x^2 + 4abx + 2a^2}$$

input
```
int(((-2*a*c-b^2)*d+(-2*a*c-b^2)*e*x+3*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+a)^3
,x)
```

output
```
(a*e + b*d + 2*b*e*x - 4*c*d*x)/(2*(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**
2 + 2*b*c*x**3 + c**2*x**4))
```

3.32 $\int \frac{(-2b^2-2ac)d+(-2b^2-2ac)ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4} dx$

Optimal result

Integrand size = 54, antiderivative size = 36

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx=\frac{2bd+ae-3(2cd-be)x}{3\,(a+bx+cx^2)^3}$$

output
```
1/3*(2*b*d+a*e-3*(-b*e+2*c*d)*x)/(c*x^2+b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx=-\frac{-2bd-ae+6cdx-3bex}{3\,(a+bx+cx^2)^3}$$

input
```
Integrate[((-2*b^2 - 2*a*c)*d + (-2*b^2 - 2*a*c)*e*x + 5*c*(2*c*d - b*e)*x
^2)/(a + b*x + c*x^2)^4,x]
```

output
```
-1/3*(-2*b*d - a*e + 6*c*d*x - 3*b*e*x)/(a + b*x + c*x^2)^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-2b^2)+ex(-2ac-2b^2)+5cx^2(2cd-be)}{(a+bx+cx^2)^4}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{ae-3x(2cd-be)+2bd}{3(a+bx+cx^2)^3}-\frac{\int 0dx}{3(b^2-4ac)}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-3x(2cd-be)+2bd}{3(a+bx+cx^2)^3}$$

input
```
Int[((-2*b^2 - 2*a*c)*d + (-2*b^2 - 2*a*c)*e*x + 5*c*(2*c*d - b*e)*x^2)/(a
 + b*x + c*x^2)^4,x]
```

output
```
(2*b*d + a*e - 3*(2*c*d - b*e)*x)/(3*(a + b*x + c*x^2)^3)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{3bex-6cdx+ae+2bd}{3(c\,x^{2}+bx+a)^{3}}$	33
risch	$\frac{(be-2cd)x+\frac{ae}{3}+\frac{2bd}{3}}{(c\,x^{2}+bx+a)^{3}}$	33
default	$-\frac{(-be+2cd)x-\frac{ae}{3}-\frac{2bd}{3}}{(c\,x^{2}+bx+a)^{3}}$	35
parallelrisch	$\frac{3b\,c^{3}ex-6c^{4}dx+c^{3}ae+2b\,c^{3}d}{3c^{3}(c\,x^{2}+bx+a)^{3}}$	47
norman	$\frac{\frac{\left(be\,c^{3}-2c^{4}d\right)x}{c^{3}}+\frac{c^{3}ae+2b\,c^{3}d}{3c^{3}}}{(c\,x^{2}+bx+a)^{3}}$	52
orering	$-\frac{(3bex-6cdx+ae+2bd)\left(\left(-2ac-2b^{2}\right)d+\left(-2ac-2b^{2}\right)ex+5c(-be+2cd)x^{2}\right)}{3(c\,x^{2}+bx+a)^{3}(5bce\,x^{2}-10c^{2}d\,x^{2}+2acex+2b^{2}ex+2acd+2b^{2}d)}$	118

input
```
int(((-2*a*c-2*b^2)*d+(-2*a*c-2*b^2)*e*x+5*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*b*e*x-6*c*d*x+a*e+2*b*d)/(c*x^2+b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx$$
$$= \frac{2\,bd+ae-3\,(2\,cd-be)x}{3\,(c^3x^6+3\,bc^2x^5+3\,(b^2c+ac^2)x^4+3\,a^2bx+(b^3+6\,abc)x^3+a^3+3\,(ab^2+a^2c)x^2)}$$

input
```
integrate(((-2*a*c-2*b^2)*d+(-2*a*c-2*b^2)*e*x+5*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(2*b*d + a*e - 3*(2*c*d - b*e)*x)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c +
a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time = 8.93 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx =$$
$$-\frac{-ae-2bd+x(-3be+6cd)}{3a^3+9a^2bx+9bc^2x^5+3c^3x^6+x^4\cdot(9ac^2+9b^2c)+x^3\cdot(18abc+3b^3)+x^2\cdot(9a^2c+9ab^2)}$$

input
```
integrate(((-2*a*c-2*b**2)*d+(-2*a*c-2*b**2)*e*x+5*c*(-b*e+2*c*d)*x**2)/(c
*x**2+b*x+a)**4,x)
```

output
```
-(-a*e - 2*b*d + x*(-3*b*e + 6*c*d))/(3*a**3 + 9*a**2*b*x + 9*b*c**2*x**5
+ 3*c**3*x**6 + x**4*(9*a*c**2 + 9*b**2*c) + x**3*(18*a*b*c + 3*b**3) + x*
*2*(9*a**2*c + 9*a*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{(-2b^2 - 2ac)\,d + (-2b^2 - 2ac)\,ex + 5c(2cd - be)x^2}{(a + bx + cx^2)^4}\,dx$$
$$= \frac{2\,bd + ae - 3\,(2\,cd - be)x}{3\,(c^3x^6 + 3\,bc^2x^5 + 3\,(b^2c + ac^2)x^4 + 3\,a^2bx + (b^3 + 6\,abc)x^3 + a^3 + 3\,(ab^2 + a^2c)x^2)}$$

input
```
integrate(((-2*a*c-2*b^2)*d+(-2*a*c-2*b^2)*e*x+5*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(2*b*d + a*e - 3*(2*c*d - b*e)*x)/(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c +
a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{(-2b^2 - 2ac)\,d + (-2b^2 - 2ac)\,ex + 5c(2cd - be)x^2}{(a + bx + cx^2)^4}\,dx = -\frac{6\,cdx - 3\,bex - 2\,bd - ae}{3\,(cx^2 + bx + a)^3}$$

input
```
integrate(((-2*a*c-2*b^2)*d+(-2*a*c-2*b^2)*e*x+5*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^4,x, algorithm="giac")
```

output
```
-1/3*(6*c*d*x - 3*b*e*x - 2*b*d - a*e)/(c*x^2 + b*x + a)^3
```

Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx$$
$$=\frac{b\left(\frac{2\,d}{3}+e\,x\right)+\frac{a\,e}{3}-2\,c\,d\,x}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a^3+x^3\,\left(b^3+6\,a\,c\,b\right)+c^3\,x^6+3\,b\,c^2\,x^5+3\,a^2\,b\,x}$$

input
```
int(-(d*(2*a*c + 2*b^2) + 5*c*x^2*(b*e - 2*c*d) + e*x*(2*a*c + 2*b^2))/(a
+ b*x + c*x^2)^4,x)
```

output
```
(b*((2*d)/3 + e*x) + (a*e)/3 - 2*c*d*x)/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*
a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a
^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \frac{(-2b^2-2ac)\,d+(-2b^2-2ac)\,ex+5c(2cd-be)x^2}{(a+bx+cx^2)^4}\,dx$$
$$=\frac{3bex-6cdx+ae+2bd}{3c^3x^6+9b\,c^2x^5+9a\,c^2x^4+9b^2c\,x^4+18abc\,x^3+3b^3x^3+9a^2c\,x^2+9a\,b^2x^2+9a^2bx+3a^3}$$

input
```
int(((-2*a*c-2*b^2)*d+(-2*a*c-2*b^2)*e*x+5*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a)^4,x)
```

output
```
(a*e + 2*b*d + 3*b*e*x - 6*c*d*x)/(3*(a**3 + 3*a**2*b*x + 3*a**2*c*x**2 +
3*a*b**2*x**2 + 6*a*b*c*x**3 + 3*a*c**2*x**4 + b**3*x**3 + 3*b**2*c*x**4 +
 3*b*c**2*x**5 + c**3*x**6))
```

3.33 $\int \frac{(-3b^2-2ac)d+(-3b^2-2ac)ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 36

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx=\frac{3bd+ae-4(2cd-be)x}{4\,(a+bx+cx^2)^4}$$

output
```
1/4*(3*b*d+a*e-4*(-b*e+2*c*d)*x)/(c*x^2+b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx=-\frac{-3bd-ae+8cdx-4bex}{4\,(a+bx+cx^2)^4}$$

input
```
Integrate[((-3*b^2 - 2*a*c)*d + (-3*b^2 - 2*a*c)*e*x + 7*c*(2*c*d - b*e)*x
^2)/(a + b*x + c*x^2)^5,x]
```

output
```
-1/4*(-3*b*d - a*e + 8*c*d*x - 4*b*e*x)/(a + b*x + c*x^2)^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d(-2ac-3b^2)+ex(-2ac-3b^2)+7cx^2(2cd-be)}{(a+bx+cx^2)^5}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{ae-4x(2cd-be)+3bd}{4(a+bx+cx^2)^4}-\frac{\int 0dx}{4(b^2-4ac)}$$

$$\Big\downarrow \text{24}$$

$$\frac{ae-4x(2cd-be)+3bd}{4(a+bx+cx^2)^4}$$

input
```
Int[((-3*b^2 - 2*a*c)*d + (-3*b^2 - 2*a*c)*e*x + 7*c*(2*c*d - b*e)*x^2)/(a
 + b*x + c*x^2)^5,x]
```

output
```
(3*b*d + a*e - 4*(2*c*d - b*e)*x)/(4*(a + b*x + c*x^2)^4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{4bex-8cdx+ae+3bd}{4(c\,x^{2}+bx+a)^{4}}$	33
risch	$\frac{(be-2cd)x+\frac{ae}{4}+\frac{3bd}{4}}{(c\,x^{2}+bx+a)^{4}}$	33
default	$-\frac{(-be+2cd)x-\frac{ae}{4}-\frac{3bd}{4}}{(c\,x^{2}+bx+a)^{4}}$	35
parallelrisch	$\frac{4b\,c^{4}ex-8c^{5}dx+a\,c^{4}e+3b\,c^{4}d}{4c^{4}(c\,x^{2}+bx+a)^{4}}$	47
norman	$\frac{\frac{\left(b\,c^{4}e-2c^{5}d\right)x}{c^{4}}+\frac{a\,c^{4}e+3b\,c^{4}d}{4c^{4}}}{(c\,x^{2}+bx+a)^{4}}$	52
orering	$-\frac{(4bex-8cdx+ae+3bd)\left(\left(-2ac-3b^{2}\right)d+\left(-2ac-3b^{2}\right)ex+7c(-be+2cd)x^{2}\right)}{4(c\,x^{2}+bx+a)^{4}(7bce\,x^{2}-14c^{2}d\,x^{2}+2acex+3b^{2}ex+2acd+3b^{2}d)}$	118

input
```
int(((-2*a*c-3*b^2)*d+(-2*a*c-3*b^2)*e*x+7*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*b*e*x-8*c*d*x+a*e+3*b*d)/(c*x^2+b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(34) = 68.

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.17

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{3\,bd+ae-4\,(2\,cd-be)x}{4\,(c^4x^8+4\,bc^3x^7+2\,(3\,b^2c^2+2\,ac^3)x^6+4\,(b^3c+3\,abc^2)x^5+4\,a^3bx+(b^4+12\,ab^2c+6\,a^2c^2)x^4+a^4+}$$

input
```
integrate(((-2*a*c-3*b^2)*d+(-2*a*c-3*b^2)*e*x+7*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(3*b*d + a*e - 4*(2*c*d - b*e)*x)/(c^4*x^8 + 4*b*c^3*x^7 + 2*(3*b^2*c^
2 + 2*a*c^3)*x^6 + 4*(b^3*c + 3*a*b*c^2)*x^5 + 4*a^3*b*x + (b^4 + 12*a*b^2
*c + 6*a^2*c^2)*x^4 + a^4 + 4*(a*b^3 + 3*a^2*b*c)*x^3 + 2*(3*a^2*b^2 + 2*a
^3*c)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(36) = 72.

Time = 17.97 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.25

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx =$$
$$-\frac{-ae-3bd+x(-4be+8cd)}{4a^4+16a^3bx+16bc^3x^7+4c^4x^8+x^6\cdot(16ac^3+24b^2c^2)+x^5\cdot(48abc^2+16b^3c)+x^4\cdot(24a^2c^2+48ab}$$

input
```
integrate(((-2*a*c-3*b**2)*d+(-2*a*c-3*b**2)*e*x+7*c*(-b*e+2*c*d)*x**2)/(c
*x**2+b*x+a)**5,x)
```

output
```
-(-a*e - 3*b*d + x*(-4*b*e + 8*c*d))/(4*a**4 + 16*a**3*b*x + 16*b*c**3*x**
7 + 4*c**4*x**8 + x**6*(16*a*c**3 + 24*b**2*c**2) + x**5*(48*a*b*c**2 + 16
*b**3*c) + x**4*(24*a**2*c**2 + 48*a*b**2*c + 4*b**4) + x**3*(48*a**2*b*c
+ 16*a*b**3) + x**2*(16*a**3*c + 24*a**2*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(34) = 68$.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.17

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{3\,bd+ae-4\,(2\,cd-be)x}{4\,(c^4x^8+4\,bc^3x^7+2\,(3\,b^2c^2+2\,ac^3)x^6+4\,(b^3c+3\,abc^2)x^5+4\,a^3bx+(b^4+12\,ab^2c+6\,a^2c^2)x^4+a^4+}$$

input
```
integrate(((-2*a*c-3*b^2)*d+(-2*a*c-3*b^2)*e*x+7*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^5,x, algorithm="maxima")
```

output
```
1/4*(3*b*d + a*e - 4*(2*c*d - b*e)*x)/(c^4*x^8 + 4*b*c^3*x^7 + 2*(3*b^2*c^
2 + 2*a*c^3)*x^6 + 4*(b^3*c + 3*a*b*c^2)*x^5 + 4*a^3*b*x + (b^4 + 12*a*b^2
*c + 6*a^2*c^2)*x^4 + a^4 + 4*(a*b^3 + 3*a^2*b*c)*x^3 + 2*(3*a^2*b^2 + 2*a
^3*c)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx = -\frac{8\,cdx-4\,bex-3\,bd-ae}{4\,(cx^2+bx+a)^4}$$

input
```
integrate(((-2*a*c-3*b^2)*d+(-2*a*c-3*b^2)*e*x+7*c*(-b*e+2*c*d)*x^2)/(c*x^
2+b*x+a)^5,x, algorithm="giac")
```

output
```
-1/4*(8*c*d*x - 4*b*e*x - 3*b*d - a*e)/(c*x^2 + b*x + a)^4
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.06

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{b\left(\frac{3\,d}{4}+e\,x\right)+\frac{a\,e}{4}-2\,c\,d\,x}{x^4\,\left(6\,a^2\,c^2+12\,a\,b^2\,c+b^4\right)+a^4+c^4\,x^8+x^2\,\left(4\,c\,a^3+6\,a^2\,b^2\right)+x^6\,\left(6\,b^2\,c^2+4\,a\,c^3\right)+x^3\,\left(12\,c\,a^2\,b+4\,a\right.}$$

input
```
int(-(d*(2*a*c + 3*b^2) + 7*c*x^2*(b*e - 2*c*d) + e*x*(2*a*c + 3*b^2))/(a
+ b*x + c*x^2)^5,x)
```

output
```
(b*((3*d)/4 + e*x) + (a*e)/4 - 2*c*d*x)/(x^4*(b^4 + 6*a^2*c^2 + 12*a*b^2*c
) + a^4 + c^4*x^8 + x^2*(4*a^3*c + 6*a^2*b^2) + x^6*(4*a*c^3 + 6*b^2*c^2)
+ x^3*(4*a*b^3 + 12*a^2*b*c) + x^5*(4*b^3*c + 12*a*b*c^2) + 4*b*c^3*x^7 +
4*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.39

$$\int \frac{(-3b^2-2ac)\,d+(-3b^2-2ac)\,ex+7c(2cd-be)x^2}{(a+bx+cx^2)^5}\,dx$$
$$= \frac{4bex-8cdx+ae+3bd}{4c^4x^8+16b\,c^3x^7+16a\,c^3x^6+24b^2c^2x^6+48ab\,c^2x^5+16b^3c\,x^5+24a^2c^2x^4+48a\,b^2c\,x^4+4b^4x^4+48a^2b}$$

input
```
int(((-2*a*c-3*b^2)*d+(-2*a*c-3*b^2)*e*x+7*c*(-b*e+2*c*d)*x^2)/(c*x^2+b*x+
a)^5,x)
```

output
```
(a*e + 3*b*d + 4*b*e*x - 8*c*d*x)/(4*(a**4 + 4*a**3*b*x + 4*a**3*c*x**2 +
6*a**2*b**2*x**2 + 12*a**2*b*c*x**3 + 6*a**2*c**2*x**4 + 4*a*b**3*x**3 + 1
2*a*b**2*c*x**4 + 12*a*b*c**2*x**5 + 4*a*c**3*x**6 + b**4*x**4 + 4*b**3*c*
x**5 + 6*b**2*c**2*x**6 + 4*b*c**3*x**7 + c**4*x**8))
```

3.34 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \frac{b\sqrt{d+ex+fx^2}}{4f} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{\left(8af-b\left(e+\frac{4df}{e}\right)\right)\operatorname{arctanh}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}$$

output

```
1/4*b*(f*x^2+e*x+d)^(1/2)/f+1/2*b*x*(f*x^2+e*x+d)^(1/2)/e+1/8*(8*a*f-b*(e+
4*d*f/e))*arctanh(1/2*(2*f*x+e)/f^(1/2)/(f*x^2+e*x+d)^(1/2))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \frac{b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)}+(-8aef+b(e^2+4df))\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{d}-\sqrt{d+x(e+fx)}}\right)}{4ef^{3/2}}$$

input
```
Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

output
```
(b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] + (-8*a*e*f + b*(e^2 + 4*d*f)
)*ArcTanh[(Sqrt[f]*x)/(Sqrt[d] - Sqrt[d + x*(e + f*x)])])/(4*e*f^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{bfx^2}{e}+bx}{\sqrt{d+ex+fx^2}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{f\left(2\left(2a-\frac{bd}{e}\right)+bx\right)}{2\sqrt{fx^2+ex+d}}dx}{2f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{2(2a-\frac{bd}{e})+bx}{\sqrt{fx^2+ex+d}}dx+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\int \frac{1}{\sqrt{fx^2+ex+d}}dx}{2f}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\int \frac{1}{4f-\frac{(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}}{f}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\operatorname{arctanh}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{2f^{3/2}}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

input
```
Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

output
```
(b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((b*Sqrt[d + e*x + f*x^2])/f - ((b*e -
 8*a*f + (4*b*d*f)/e)*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2]
)])/(2*f^(3/2)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
risch	$\frac{b(2fx+e)\sqrt{f\,x^2+ex+d}}{4fe}+\frac{(8aef-4bdf-b\,e^2)\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{8f^{\frac{3}{2}}e}$
default	$\frac{\frac{ae\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{\sqrt{f}}+be\left(\frac{\sqrt{f\,x^2+ex+d}}{f}-\frac{e\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{2f^{\frac{3}{2}}}\right)+fb\left(\frac{x\sqrt{f\,x^2+ex+d}}{2f}-\frac{3e\left(\frac{\sqrt{f\,x^2+ex+d}}{f}-\frac{e\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\cdots\right)}{2\cdots}\right)}{4f}\cdots\right.}{e}$

input
```
int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/f/e+1/8*(8*a*e*f-4*b*d*f-b*e^2)/f^(3/2
)*ln((f*x+1/2*e)/f^(1/2)+(f*x^2+e*x+d)^(1/2))/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.01

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$$
$$=\left[-\frac{(be^2+4\,(bd-2\,ae)f)\sqrt{f}\log\left(-8\,f^2x^2-8\,efx-e^2-4\,\sqrt{fx^2+ex+d}(2\,fx+e)\sqrt{f}-4\,df\right)-4\,(2\,\cdots}{16\,ef^2}\right.$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(f)*log(-8*f^2*x^2 - 8*e*f*x - e^2
 - 4*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(f) - 4*d*f) - 4*(2*b*f^2*x + b
*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqr
t(-f)*arctan(1/2*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + e*f
*x + d*f)) + 2*(2*b*f^2*x + b*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(85) = 170$.

Time = 0.47 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.10

$$\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}}\, dx = \begin{cases} \left(\frac{b}{4f} + \frac{bx}{2e}\right)\sqrt{d + ex + fx^2} + \left(a - \frac{bd}{2e} - \frac{be}{8f}\right)\left(\begin{cases} \frac{\log\left(e + 2\sqrt{f}\sqrt{d+ex+fx^2} + 2fx\right)}{\sqrt{f}} & \text{for}\ d - \frac{e^2}{4f} \neq 0 \\ \frac{\left(\frac{e}{2f}+x\right)\log\left(\frac{e}{2f}+x\right)}{\sqrt{f\left(\frac{e}{2f}+x\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for}\ f \neq 0 \\ \frac{2a\sqrt{d+ex} + \frac{2b\left(-d\sqrt{d+ex} + \frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e} + \frac{2bf\left(d^2\sqrt{d+ex} - \frac{2d(d+ex)^{\frac{3}{2}}}{3} + \frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^3}}{e} & \text{for}\ e \neq 0 \\ \frac{ax + \tilde{\infty}bfx^3 + \frac{bx^2}{2}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Piecewise(((b/(4*f) + b*x/(2*e))*sqrt(d + e*x + f*x**2) + (a - b*d/(2*e) -
 b*e/(8*f))*Piecewise((log(e + 2*sqrt(f)*sqrt(d + e*x + f*x**2) + 2*f*x)/s
qrt(f), Ne(d - e**2/(4*f), 0)), ((e/(2*f) + x)*log(e/(2*f) + x)/sqrt(f*(e/
(2*f) + x)**2), True)), Ne(f, 0)), ((2*a*sqrt(d + e*x) + 2*b*(-d*sqrt(d +
e*x) + (d + e*x)**(3/2)/3)/e + 2*b*f*(d**2*sqrt(d + e*x) - 2*d*(d + e*x)**
(3/2)/3 + (d + e*x)**(5/2)/5)/e**3)/e, Ne(e, 0)), ((a*x + zoo*b*f*x**3 + b
*x**2/2)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}}\,dx = \frac{1}{4}\sqrt{fx^2 + ex + d}\left(\frac{2\,bx}{e} + \frac{b}{f}\right) + \frac{(be^2 + 4\,bdf - 8\,aef)\log\left(\left|2\left(\sqrt{f}x - \sqrt{fx^2 + ex + d}\right)\sqrt{f} + e\right|\right)}{8\,ef^{\frac{3}{2}}}$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(f*x^2 + e*x + d)*(2*b*x/e + b/f) + 1/8*(b*e^2 + 4*b*d*f - 8*a*e*f
)*log(abs(2*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*sqrt(f) + e))/(e*f^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx=\int \frac{a+b\,x+\frac{b\,f\,x^2}{e}}{\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)
```

output
```
int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \frac{2\sqrt{f\,x^2+ex+d}\,bef+4\sqrt{f\,x^2+ex+d}\,b\,f^2x+8\sqrt{f}\log\left(\frac{2\sqrt{f}\,\sqrt{f\,x^2+ex+d}+e+2fx}{\sqrt{4df-e^2}}\right)aef-4\sqrt{f}\log\left(\frac{2\sqrt{f}\,\sqrt{f}}{}\right.}{8e\,f^2}$$

input
```
int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x + f*x**2)*b*e*f + 4*sqrt(d + e*x + f*x**2)*b*f**2*x + 8*sq
rt(f)*log((2*sqrt(f)*sqrt(d + e*x + f*x**2) + e + 2*f*x)/sqrt(4*d*f - e**2
))*a*e*f - 4*sqrt(f)*log((2*sqrt(f)*sqrt(d + e*x + f*x**2) + e + 2*f*x)/sq
rt(4*d*f - e**2))*b*d*f - sqrt(f)*log((2*sqrt(f)*sqrt(d + e*x + f*x**2) +
e + 2*f*x)/sqrt(4*d*f - e**2))*b*e**2)/(8*e*f**2)
```

3.35 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 82

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx=-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

output
```
-2*e^(1/2)*arctanh((-a*e+b*d)^(1/2)*(2*f*x+e)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(
f*x^2+e*x+d)^(1/2))/(-a*e+b*d)^(1/2)/(-4*a*f+b*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$$
$$= e\text{RootSum}\Big[aef^2-2b\sqrt{d}ef\#1+be^2\#1^2+4bdf\#1^2-2aef\#1^2-2b\sqrt{d}e\#1^3$$
$$+ae\#1^4\&,\frac{-f\log(x)+f\log\left(-\sqrt{d}+\sqrt{d+ex+fx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{d}+\sqrt{d+ex+\right.}{-b\sqrt{d}ef+be^2\#1+4bdf\#1-2aef\#1-3b\sqrt{d}e\#1^2+2ae\#1^3}$$

input
```
Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

output
```
e*RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e
*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-(f*Log[x]) + f*Log[-Sqrt[d]
+ Sqrt[d + e*x + f*x^2] - x*#1] + Log[x]*#1^2 - Log[-Sqrt[d] + Sqrt[d + e*
x + f*x^2] - x*#1]*#1^2)/(-(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e
*f*#1 - 3*b*Sqrt[d]*e*#1^2 + 2*a*e*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+\frac{bfx^2}{e}+bx\right)}dx$$
$$\Big\downarrow \text{ 1313}$$

$$-2e\int \frac{1}{e(be-4af)-\frac{(bd-ae)(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\sqrt{e}\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

input
```
Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

output
```
(-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*
f]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(68) = 136$.

Time = 4.47 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.99

method	result
default	$e\left(-\frac{\ln\left(\frac{-\frac{2(ae-bd)}{b}+\frac{\sqrt{-be(4af-be)}\left(x-\frac{-be+\sqrt{-be(4af-be)}}{2fb}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x-\frac{-be+\sqrt{-be(4af-be)}}{2fb}\right)^2 f+\frac{\sqrt{-be(4af-be)}\left(x-\frac{-be}{}\right.}{b}}}{x-\frac{-be+\sqrt{-be(4af-be)}}{2fb}}\right)}{\sqrt{-be(4af-be)}\sqrt{-\frac{ae-bd}{b}}}\right.$

input
```
int(1/(f*x^2+e*x+d)^(1/2)/(a+b*x+b*f*x^2/e),x,method=_RETURNVERBOSE)
```

output
```
e*(-1/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b
*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)+2*(-(a
*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f+(-b*e*(4
*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b
)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b))+1/(-b*e*(4*a*f-b*e))
^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*
(4*a*f-b*e))^(1/2))/f/b)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(68) = 136$.

Time = 0.23 (sec) , antiderivative size = 1079, normalized size of antiderivative = 13.16

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(a+b*x+b*f*x^2/e),x, algorithm="fricas")
```

output
```
[1/2*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*log((8*b^2*d^2*e^4
- 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2
- 8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b
^2*e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 -
 4*a*b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*
(3*b^2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b
*e^5)*f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*
e^6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^
4 + 2*a^2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16
*(a*b^2*d^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2
+ 3*a^2*b*e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2
- a^3*d*e^3)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4
*(b^3*d^2*e^2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)
*f)*x^2 - 4*(3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a
*b^2*e^6 + 32*(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*
d*e^3 + a^3*e^4)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)
*x)*sqrt(f*x^2 + e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)
))/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b
*e*f)*x^2)), -sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1
/2*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate(1/(f*x**2+e*x+d)**(1/2)/(a+b*x+b*f*x**2/e),x)
```

output
```
e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) +
b*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(a+b*x+b*f*x^2/e),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(4*a*f-b*e)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 851, normalized size of antiderivative = 10.38

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(a+b*x+b*f*x^2/e),x, algorithm="giac")
```

output
```
-sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*log(abs(-(sqrt(f)*x
 - sqrt(f*x^2 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d)
)^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x
 - sqrt(f*x^2 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x
+ d))*b*d*e*f^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2)
- 3*b*d*e^2*f + 2*a*e^3*f + 4*b*d^2*f^2 + 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a
*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) + 4*
sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*
x^2 + e*x + d))*e*f + sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f
)*e^2*sqrt(f)))/(b^2*d*e - a*b*e^2 - 4*a*b*d*f + 4*a^2*e*f) + sqrt(b^2*d*e
^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2
 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*d*f^2 +
 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2
 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f
^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f
 + 2*a*e^3*f + 4*b*d^2*f^2 - 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*
a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) - 4*sqrt(b^2*d*e^
2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d
))*e*f - sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)
))/(b^2*d*e - a*b*e^2 - 4*a*b*d*f + 4*a^2*e*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx=\int \frac{1}{\sqrt{f\,x^2+e\,x+d}\,\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

input
```
int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)
```

output
```
int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.85

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$$

$$= \frac{\sqrt{e}\,\sqrt{4a^2ef-4abdf-ab\,e^2+b^2de}\left(\log\left(-\sqrt{4\sqrt{f}\,\sqrt{e}\,\sqrt{4a^2ef-4abdf-ab\,e^2+b^2de}-8aef+4bdf+\right.\right.}{}$$

input
```
int(1/(f*x^2+e*x+d)^(1/2)/(a+b*x+b*f*x^2/e),x)
```

output
```
(sqrt(e)*sqrt(4*a**2*e*f - 4*a*b*d*f - a*b*e**2 + b**2*d*e)*(log( - sqrt(4
*sqrt(f)*sqrt(e)*sqrt(4*a**2*e*f - 4*a*b*d*f - a*b*e**2 + b**2*d*e) - 8*a*
e*f + 4*b*d*f + b*e**2) + 2*sqrt(f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(
b)*e + 2*sqrt(b)*f*x) + log(sqrt(4*sqrt(f)*sqrt(e)*sqrt(4*a**2*e*f - 4*a*b
*d*f - a*b*e**2 + b**2*d*e) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqrt(f)*sqrt
(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x) - log(4*sqrt(f)*sq
rt(e)*sqrt(4*a**2*e*f - 4*a*b*d*f - a*b*e**2 + b**2*d*e) + 4*sqrt(f)*sqrt(
d + e*x + f*x**2)*b*e + 8*sqrt(f)*sqrt(d + e*x + f*x**2)*b*f*x + 8*a*e*f +
 8*b*e*f*x + 8*b*f**2*x**2)))/(4*a**2*e*f - 4*a*b*d*f - a*b*e**2 + b**2*d*
e)
```

3.36 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

output
```
-2*arctanh((a-d)^(1/2)*(2*c*x+b)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a
-d)^(1/2)/(b^2-4*c*d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx$$
$$=\text{RootSum}\Big[c^2d-2\sqrt{a}bc\#1+b^2\#1^2+4ac\#1^2-2cd\#1^2-2\sqrt{a}b\#1^3$$
$$+d\#1^4\&,\frac{-c\log(x)+c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}\right.}{-\sqrt{a}bc+b^2\#1+4ac\#1-2cd\#1-3\sqrt{a}b\#1^2+2d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

output
```
RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2 + 4*a*c*#1^2 - 2*c*d*#1^2 - 2*
Sqrt[a]*b*#1^3 + d*#1^4 & , (-(c*Log[x]) + c*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1] + Log[x]*#1^2 - Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1]*#1^2)/(-(Sqrt[a]*b*c) + b^2*#1 + 4*a*c*#1 - 2*c*d*#1 - 3*Sqrt[a]*b*#1^2
 + 2*d*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)}dx$$

$$\Big\downarrow \text{1313}$$

$$-2b\int \frac{1}{b\,(b^2-4cd)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

output
```
(-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(56) = 112$.

Time = 2.58 (sec) , antiderivative size = 307, normalized size of antiderivative = 4.65

method	result
default	$-\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{c\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+a-d}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}+\frac{\ln\left(\frac{2a-2d-\sqrt{b^2}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+
(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c))+1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*
c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x+1/2*(b+(b^2
-4*c*d)^(1/2))/c)^2-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)
^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 813, normalized size of antiderivative = 12.32

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d),x, algorithm="fricas")
```

output
```
[1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 +
128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a
^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3
)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*
(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2
- (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(
a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^
2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b
^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^
2 + 2*c*d)*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b
^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 -
8*c^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2
- 4*c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2
+ 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b
*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2
*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)
)/(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(c*x**2+b*x+d),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*d-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(56) = 112.

Time = 0.22 (sec) , antiderivative size = 703, normalized size of antiderivative = 10.65

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx =$$

$$-\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2b^2c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^2c\right.\right.}{}$$

$$+\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2b^2c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^2d\right.\right.}{}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d),x, algorithm="giac")
```

output
```
-log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*
d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2
 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sq
rt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4
*a*c*d + 4*c*d^2) + log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c -
 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d +
4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2
*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqr
t(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.41

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx$$
$$= \frac{\sqrt{a-d}\,\sqrt{b^2-4cd}\,\Big(\log\Big(-\sqrt{4\sqrt{c}\,\sqrt{a-d}\,\sqrt{b^2-4cd}+4ac+b^2-8cd}+2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx\Big)}{}$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d),x)
```

output
```
(sqrt(a - d)*sqrt(b**2 - 4*c*d)*(log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b*
*2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
 + 2*c*x) + log(sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b*
*2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x) - log(4*sqrt(c
)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*
sqrt(c)*sqrt(a + b*x + c*x**2)*c*x + 8*b*c*x + 8*c**2*x**2 + 8*c*d)))/(a*b
**2 - 4*a*c*d - b**2*d + 4*c*d**2)
```

3.37 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 129

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b^2+4c(a-2d)\right)\operatorname{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

output
```
-(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)+(b^2+4*c*(a
-2*d))*arctanh((a-d)^(1/2)*(2*c*x+b)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/(a-d)^(3/2)/(b^2-4*c*d)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.60 (sec) , antiderivative size = 1333, normalized size of antiderivative = 10.33

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

output
```
-(RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2 + 4*a*c*#1^2 - 2*c*d*#1^2 -
2*Sqrt[a]*b*#1^3 + d*#1^4 & , (-4*a*b^2*Log[x] + b^2*d*Log[x] + 4*a*c*d*Lo
g[x] + c*d^2*Log[x] + 4*a*b^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
 - b^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a*c*d*Log[-Sqrt[
a] + Sqrt[a + b*x + c*x^2] - x*#1] - c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] - 2*Sqrt[a]*b*d*Log[x]*#1 + 2*Sqrt[a]*b*d*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1]*#1 - d^2*Log[x]*#1^2 + d^2*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*c) + b^2*#1 + 4*a*c*#1 - 2*c*d
*#1 - 3*Sqrt[a]*b*#1^2 + 2*d*#1^3) & ]/d^3) + ((2*(b + 2*c*x)*Sqrt[a + x*(
b + c*x)])/(d + x*(b + c*x)) - RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2
 + 4*a*c*#1^2 - 2*c*d*#1^2 - 2*Sqrt[a]*b*#1^3 + d*#1^4 & , (-8*a^2*b^4*Log
[x] + 10*a*b^4*d*Log[x] + 40*a^2*b^2*c*d*Log[x] - 2*b^4*d^2*Log[x] - 46*a*
b^2*c*d^2*Log[x] - 32*a^2*c^2*d^2*Log[x] + 7*b^2*c*d^3*Log[x] + 28*a*c^2*d
^3*Log[x] + 8*a^2*b^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 10*a*
b^4*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 40*a^2*b^2*c*d*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*b^4*d^2*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1] + 46*a*b^2*c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1] + 32*a^2*c^2*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] -
7*b^2*c*d^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 28*a*c^2*d^3*Lo
g[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^(3/2)*b^3*d*Log[x]*#1 ...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1305, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)^2}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{cx^2+bx+a}\,(cx^2+bx+d)}dx}{c^2(a-d)^2(b^2-4cd)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(4c(a-2d)+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)}dx}{2(a-d)\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

$$\Big\downarrow \text{ 1313}$$

$$\frac{b\left(4c(a-2d)+b^2\right)\int \frac{1}{b(b^2-4cd)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{(a-d)\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(4c(a-2d)+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

output
```
-(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^
2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. 2(117) = 234.

Time = 2.62 (sec) , antiderivative size = 827, normalized size of antiderivative = 6.41

method	result
default	$\frac{-\frac{\sqrt{c\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+a-d}}{(a-d)\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}+\frac{\sqrt{b^2-4cd}\,\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{c\left(x-\frac{-b+}{}\right.}}{x-\frac{-b+\sqrt{b^2-}}{2c}}\right.}{2(a-d)^{\frac{3}{2}}}}{b^2-4cd}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-4*c*d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*(c*(x-1/2*(-b+(b^
2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-
d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(
x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(
x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*
a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d
)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+1/(b^2-4*c*d)*(-
1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)*(c*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
^2-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4
*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d
)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2-(b^2-4*c*d)
^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(
1/2))/c)))-2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)
*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x+1/2*(b+(b^2-4*c*d)^(1
/2))/c)^2-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x
+1/2*(b+(b^2-4*c*d)^(1/2))/c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(117) = 234$.

Time = 0.32 (sec) , antiderivative size = 1544, normalized size of antiderivative = 11.97

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2
- 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2
*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 +
4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 -
 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (
b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b
^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*
c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*
c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2
 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*
(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c
 + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
+ d^2)) - 4*(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^
2 - (b^2*c + 4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5
 - 8*(b^2*c + 4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^
4 + 4*a^2*b^2*c)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3
 + (b^4*c + 16*a*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d
)*x^2 + (a^2*b^5 + 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*
b^3*c + 16*a^2*b*c^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b
^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx=\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)^2}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(c*x**2+b*x+d)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^2}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 1170, normalized size of antiderivative = 9.07

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^2,x, algorithm="giac")
```

output
```
1/2*((b^2 + 4*a*c - 8*c*d)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^2*c - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt
(c) - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b*c^(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a
*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2
+ 2*b^2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c
)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(ab
s(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d - (sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d
 - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*
d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d
+ 4*c*d^2))/(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*b^2*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(
3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2)*d + (sqrt(c)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx=\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^2}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2661, normalized size of antiderivative = 20.63

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^2,x)
```

output
```
( - 4*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqr
t(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
 + b + 2*c*x)*a*b*c*x - 4*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqr
t(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)*a*c**2*x**2 - 4*sqrt(a - d)*sqrt(b**2 -
4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2
 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c*d - sqrt(a -
 d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d
) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*
b**3*x - sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*
sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + b + 2*c*x)*b**2*c*x**2 - sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(
4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**2*d + 8*sqrt(a - d)*sqrt(b**2 -
4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2
 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b*c*d*x + 8*sqrt
(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4
*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*c**2*d*x**2 + 8*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*s
qrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(...
```

3.38 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)} \\ &\quad -\frac{\left(3b^4+8b^2c(a-4d)+16c^2\left(3a^2-8ad+8d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}\end{aligned}$$

output

```
-1/2*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^2+3/4*(
b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2/(b^2-4*c*d)^2/(c*x^
2+b*x+d)-1/4*(3*b^4+8*b^2*c*(a-4*d)+16*c^2*(3*a^2-8*a*d+8*d^2))*arctanh((a
-d)^(1/2)*(2*c*x+b)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(5/2)/(b^
2-4*c*d)^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1746 vs. $2(224) = 448$.

Time = 16.59 (sec) , antiderivative size = 1746, normalized size of antiderivative = 7.79

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

output
```
(-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*
c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d
)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) +
 (2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*
c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d
)*(b^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) +
 (6*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] -
 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[
a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*Sqrt[a + b*x + c*
x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)
/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(2*(a - d)^(3/2)*(b^2 - 4*c*d)^
(3/2)*Sqrt[a + x*(b + c*x)]) + (6*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c
 - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*S
qrt[a + b*x + c*x^2])]])/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c
*x)]) + (3*c*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*
d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(
2*(a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)*Sqrt[a + x*(b + c*x)]) + (4*c^3*Sqrt[a
 + b*x + c*x^2]*(((-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2
 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*
d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) +...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1305, 27, 2135, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)^3}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{\int -\frac{8(a-d)x^2c^4+8b(a-d)xc^3+(a-d)\left(3b^2+12ac-16cd\right)c^2}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{2c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{8(a-d)x^2c^4+8b(a-d)xc^3+(a-d)\left(3b^2+12ac-16cd\right)c^2}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \ 2135$$

$$-\frac{-\frac{\int \frac{c^4(a-d)^2\left(3b^4+8c(a-4d)b^2+16c^2\left(3a^2-8da+8d^2\right)\right)}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{c^2(a-d)^2\left(b^2-4cd\right)}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{c^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{2\left(b^2-4cd\right)}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \ 1313$$

$$-\frac{\frac{bc^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\int \frac{1}{b\left(b^2-4cd\right)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{b^2-4cd}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{c^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\left(b^2-4cd\right)^{3/2}}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

output
```
-1/2*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
 c*x^2)^2) - ((-3*c^2*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x
^2])/((b^2 - 4*c*d)*(d + b*x + c*x^2)) + (c^2*(3*b^4 + 8*b^2*c*(a - 4*d) +
 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b
^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(3/2)))/(4
*c^2*(a - d)^2*(b^2 - 4*c*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(204) = 408$.

Time = 3.13 (sec) , antiderivative size = 1868, normalized size of antiderivative = 8.34

method	result	size
default	Expression too large to display	1868

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-4*c*d)^(3/2)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*(c*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2
)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(
a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+
2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
)-3/(b^2-4*c*d)^2*c*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*(c*(x-1/2*(
-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(
1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1
/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))-6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1/2
)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(204) = 408$.

Time = 2.53 (sec) , antiderivative size = 3818, normalized size of antiderivative = 17.04

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(c*x**2+b*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2986 vs. $2(204) = 408$.

Time $= 0.93$ (sec) , antiderivative size $= 2986$, normalized size of antiderivative $= 13.33$

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^3,x, algorithm="giac")
```

output
```
-1/8*((3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d + 128*c^2
*d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*b*c^(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
 + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*
d - 4*a*c*d + 4*c*d^2) - (3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 12
8*a*c^2*d + 128*c^2*d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^
2*c - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c
) - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 +
2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c))
)/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2*a*b^4*d - 8*a^2*b^
2*c*d + b^4*d^2 + 16*a*b^2*c*d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^3}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 18107, normalized size of antiderivative = 80.83

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^3,x)
```

output

```
(192*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt
(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
+ b + 2*c*x)*a**3*b**2*c**3*x**2 + 384*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log(
 - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) +
 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**3*b*c**4*x**3 + 384*sqrt
(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4
*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a**3*b*c**3*d*x + 192*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqr
t(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)*a**3*c**5*x**4 + 384*sqrt(a - d)*sqrt(b*
*2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c +
 b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**3*c**4*d
*x**2 + 192*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a -
d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)*a**3*c**3*d**2 + 80*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log
( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d)
+ 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**2*b**4*c**2*x**2 + 160*
sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2
 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
 2*c*x)*a**2*b**3*c**3*x**3 + 160*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( -...
```

3.39 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3}+\frac{5(b^2+4c(a-2d))\,(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2\,(b^2-4cd)^2\,(d+bx+cx^2)^2}\\ &\quad-\frac{(15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))\,(b+2cx)\sqrt{a+bx+cx^2}}{24(a-d)^3\,(b^2-4cd)^3\,(d+bx+cx^2)}\\ &\quad+\frac{(b^2+4c(a-2d))\,(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+8d^2))\,\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{8(a-d)^{7/2}\,(b^2-4cd)^{7/2}}\end{aligned}$$

output
```
-1/3*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^3+5/12*
(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2/(b^2-4*c*d)^2/(c*x
^2+b*x+d)^2-1/24*(15*b^4+8*b^2*c*(7*a-22*d)+16*c^2*(15*a^2-44*a*d+44*d^2))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^3/(b^2-4*c*d)^3/(c*x^2+b*x+d)+1/8*(b^
2+4*c*(a-2*d))*(5*b^4-8*b^2*c*(a+4*d)+16*c^2*(5*a^2-8*a*d+8*d^2))*arctanh(
(a-d)^(1/2)*(2*c*x+b)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(7/2)/(
b^2-4*c*d)^(7/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3382 vs. $2(328) = 656$.

Time = 16.66 (sec) , antiderivative size = 3382, normalized size of antiderivative = 10.31

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

output
```
(-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*
d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*
(b^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)
]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4
*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a -
d)*(b^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]
) - (8*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 -
 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a
 - d)*(b^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)
]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c
*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(
Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*
c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/((a - d)^(3/2)*(b^2 - 4*
c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[
(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a
- d]*Sqrt[a + b*x + c*x^2]])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*
(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^
2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x
^2])]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)^4}\,dx$$

$$\Big\downarrow \; 1305$$

$$\frac{\int -\frac{16(a-d)x^2c^4+16b(a-d)xc^3+(a-d)\left(5b^2+20ac-24cd\right)c^2}{2\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}dx}{3c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{16(a-d)x^2c^4+16b(a-d)xc^3+(a-d)\left(5b^2+20ac-24cd\right)c^2}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}dx}{6c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$$\Big\downarrow \; 2135$$

$$-\frac{-\frac{\int \frac{40\left(b^2+4c(a-2d)\right)(a-d)^2x^2c^6+40b\left(b^2+4c(a-2d)\right)(a-d)^2xc^5+(a-d)^2\left(15b^4+8c(7a-17d)b^2+16c^2\left(15a^2-34da+24d^2\right)\right)c^4}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{2c^2(a-d)^2(b^2-4cd)}-\frac{5c^2(b+2cx)(4c(a-\cdots}{2(b^2-4cd)(\cdots}}{6c^2(a-d)^2\left(b^2-4cd\right)}$$
$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{40\left(b^2+4c(a-2d)\right)(a-d)^2x^2c^6+40b\left(b^2+4c(a-2d)\right)(a-d)^2xc^5+(a-d)^2\left(15b^4+8c(7a-17d)b^2+16c^2\left(15a^2-34da+24d^2\right)\right)c^4}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{4c^2(a-d)^2(b^2-4cd)}-\frac{5c^2(b+2cx)(4c(a-\cdots}{2(b^2-4cd)(\cdots}}{6c^2(a-d)^2\left(b^2-4cd\right)}$$
$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$$\Big\downarrow \; 2135$$

$$-\frac{-\frac{-\frac{\int \frac{3c^6\left(b^2+4c(a-2d)\right)(a-d)^3\left(5b^4-8c(a+4d)b^2+16c^2\left(5a^2-8da+8d^2\right)\right)}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{c^2(a-d)^2\left(b^2-4cd\right)}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+c\cdots}}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)}}{4c^2(a-d)^2(b^2-4cd)}\cdots}{6c^2(a-d)^2\left(b^2-4cd\right)}\cdots$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{3c^4(a-d)\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{2\left(b^2-4cd\right)}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7\cdots\right.}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)\cdots}}{4c^2(a-d)^2(b^2-4cd)}\cdots}{6c^2(a-d)^2\left(b^2-4cd\right)}\cdots$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$\Big\downarrow$ 1313

$$-\frac{-\frac{\frac{3bc^4(a-d)\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\int \frac{1}{b\left(b^2-4cd\right)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{b^2-4cd}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad\cdots\right.\right.}{\left(b^2-4c\cdots\right.}}{4c^2(a-d)^2(b^2-4cd)}\cdots}{6c^2(a-d)^2\left(b^2-4cd\right)}\cdots$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\frac{3c^4\sqrt{a-d}\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\left(b^2-4cd\right)^{3/2}}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b\cdots\right.}{\left(b^2-4cd\right)\left(bx+cx^2\cdots\right.}}{4c^2(a-d)^2(b^2-4cd)}\cdots}{6c^2(a-d)^2\left(b^2-4cd\right)}\cdots$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

output
```
-1/3*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
 c*x^2)^3) - ((-5*c^2*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x
^2])/(2*(b^2 - 4*c*d)*(d + b*x + c*x^2)^2) - (-((c^4*(a - d)*(15*b^4 + 8*b
^2*c*(7*a - 22*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a
+ b*x + c*x^2])/((b^2 - 4*c*d)*(d + b*x + c*x^2))) + (3*c^4*(b^2 + 4*c*(a
- 2*d))*Sqrt[a - d]*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8
*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x +
 c*x^2])])/(b^2 - 4*c*d)^(3/2))/(4*c^2*(a - d)^2*(b^2 - 4*c*d)))/(6*c^2*(a
 - d)^2*(b^2 - 4*c*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. $2(304) = 608$.

Time = 3.70 (sec) , antiderivative size = 3649, normalized size of antiderivative = 11.12

method	result	size
default	Expression too large to display	3649

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-4*c*d)^2*(-1/3/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^3*(c*(x-1/2*(
-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)+a-d)^(1/2)-5/6*(b^2-4*c*d)^(1/2)/(a-d)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)^2*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/
(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-
4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c
*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)))+1/2*c/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2
+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)))-2/3*c/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)*(c*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(
b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-
2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(c*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))))-4/(b^2-4*c*d)^(5
/2)*c*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*(c*(x-1/2*(-b+(b^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3962 vs. $2(304) = 608$.

Time = 10.77 (sec) , antiderivative size = 8134, normalized size of antiderivative = 24.80

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(c*x**2+b*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^4}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^4,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30280 vs. $2(304) = 608$.

Time $= 4.01$ (sec) , antiderivative size $= 30280$, normalized size of antiderivative $= 92.32$

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^4,x, algorithm="giac")
```

output
```
-1/16*((5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*c^3 - 72*b^4*c*d - 1
92*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 1536*a*c^3*d^2 - 1024*
c^3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d + 4*sqr
t(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^
2*d - 4*a*c*d + 4*c*d^2) - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*
c^3 - 72*b^4*c*d - 192*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 15
36*a*c^3*d^2 - 1024*c^3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqr
t(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*
a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2
 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^4}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4), x)
```

Reduce [B] (verification not implemented)

Time = 22.69 (sec) , antiderivative size = 27811, normalized size of antiderivative = 84.79

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(c*x^2+b*x+d)^4,x)
```

output
```
( - 960*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*s
qrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**
2) + b + 2*c*x)*a**3*b**3*c**3*x**3 - 2880*sqrt(a - d)*sqrt(b**2 - 4*c*d)*
log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*
d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**3*b**2*c**4*x**4 - 2
880*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(
b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)*a**3*b**2*c**3*d*x**2 - 2880*sqrt(a - d)*sqrt(b**2 - 4*c*d)*lo
g( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d)
 + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**3*b*c**5*x**5 - 5760*s
qrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2
- 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*a**3*b*c**4*d*x**3 - 2880*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqr
t(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqr
t(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**3*b*c**3*d**2*x - 960*sqrt(a -
 d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)*sqrt(a - d)*sqrt(b**2 - 4*c*d
) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*
a**3*c**6*x**6 - 2880*sqrt(a - d)*sqrt(b**2 - 4*c*d)*log( - sqrt(4*sqrt(c)
*sqrt(a - d)*sqrt(b**2 - 4*c*d) + 4*a*c + b**2 - 8*c*d) + 2*sqrt(c)*sqrt(a
 + b*x + c*x**2) + b + 2*c*x)*a**3*c**5*d*x**4 - 2880*sqrt(a - d)*sqrt(...
```

3.40 $\int \frac{1}{\sqrt{d+ex+fx^2}(ae+bex+bfx^2)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 162

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} - \frac{(8aef-b(e^2+4df))\,\text{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\,\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}$$

output
```
-b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+
a*e)-(8*a*e*f-b*(4*d*f+e^2))*arctanh((-a*e+b*d)^(1/2)*(2*f*x+e)/e^(1/2)/(-
4*a*f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/(-4*a*f+b*e
)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.25 (sec) , antiderivative size = 1420, normalized size of antiderivative = 8.77

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
((-2*RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*
a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-4*b^2*d*e*Log[x] + a*b*e^
2*Log[x] + 4*a*b*d*f*Log[x] + a^2*e*f*Log[x] + 4*b^2*d*e*Log[-Sqrt[d] + Sq
rt[d + e*x + f*x^2] - x*#1] - a*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2]
 - x*#1] - 4*a*b*d*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - a^2*e*
f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 2*a*b*Sqrt[d]*e*Log[x]*#1
 + 2*a*b*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - a^2*e
*Log[x]*#1^2 + a^2*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1^2)/(-
(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e*f*#1 - 3*b*Sqrt[d]*e*#1^2
+ 2*a*e*#1^3) & ])/a^3 + (b*((-2*e*(e + 2*f*x)*Sqrt[d + x*(e + f*x)])/(a*e
 + b*x*(e + f*x)) + RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*
b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-8*b^3*d^2*
e^2*Log[x] + 10*a*b^2*d*e^3*Log[x] - 2*a^2*b*e^4*Log[x] + 40*a*b^2*d^2*e*f
*Log[x] - 46*a^2*b*d*e^2*f*Log[x] + 7*a^3*e^3*f*Log[x] - 32*a^2*b*d^2*f^2*
Log[x] + 28*a^3*d*e*f^2*Log[x] + 8*b^3*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x
 + f*x^2] - x*#1] - 10*a*b^2*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] -
x*#1] + 2*a^2*b*e^4*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 40*a*b^
2*d^2*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] + 46*a^2*b*d*e^2*f*
Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 7*a^3*e^3*f*Log[-Sqrt[d] +
Sqrt[d + e*x + f*x^2] - x*#1] + 32*a^2*b*d^2*f^2*Log[-Sqrt[d] + Sqrt[d ...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1305, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}dx$$

$$\Big\downarrow \; 1305$$

$$\frac{\int \frac{b(bd-ae)f^2\left(8aef-b\left(e^2+4df\right)\right)}{2\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{bef^2(bd-ae)^2(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2e(bd-ae)(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 1313$$

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{e^2(be-4af)-\frac{e(bd-ae)(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}}{(bd-ae)(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

input
```
Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
-((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e
+ b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*
e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/
2)*(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. 2(146) = 292.

Time = 4.97 (sec) , antiderivative size = 1352, normalized size of antiderivative = 8.35

method	result	size
default	Expression too large to display	1352

input
```
int(1/(f*x^2+e*x+d)^(1/2)/(b*f*x^2+b*e*x+a*e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e/(4*a*f-b*e)/b*(1/(a*e-b*d)*b/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f
/b)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f+(-b*e*(4*a*f-b*e))^(1
/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b)^(1/2)-1/2*(
-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+
(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)+2*(
-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d
)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)))-1/e/(4*a*f-b*e)/
b*(1/(a*e-b*d)*b/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)*((x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b)^(1/2)+1/2*(-b*e*(4*a*f-b*e))^(1
/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(
1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)+2*(-(a*e-b*d)/b)^(1/2)*(
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/f/b)))-2/e/(4*a*f-b*e)*f/(-b*e*(4*a*f-b*e))^(1/
2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1
/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b
*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-
b*e+(-b*e*(4*a*f-b*e))^(1/2))/f/b)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(146) = 292$.

Time = 0.77 (sec) , antiderivative size = 2005, normalized size of antiderivative = 12.38

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(b*f*x^2+b*e*x+a*e)^2,x, algorithm="fricas
")
```

output
```
[-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2
*e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e
^3 + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*
e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*
e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d
^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3
 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*
a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqr
t(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*
d*e^2*f + 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e
- 2*a*e^2)*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)
*sqrt(f*x^2 + e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^
5 - 3*a*b*e^6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 -
5*a*b*d*e^4 + 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x
+ a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*
b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 -
 a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6
+ a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(
a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*
a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx$$

input
```
integrate(1/(f*x**2+e*x+d)**(1/2)/(b*f*x**2+b*e*x+a*e)**2,x)
```

output
```
Integral(1/(sqrt(d + e*x + f*x**2)*(a*e + b*e*x + b*f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{1}{(bfx^2+bex+ae)^2\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(b*f*x^2+b*e*x+a*e)^2,x, algorithm="maxima
")
```

output
```
integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(146) = 292$.

Time = 0.32 (sec) , antiderivative size = 1305, normalized size of antiderivative = 8.06

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x^2+e*x+d)^(1/2)/(b*f*x^2+b*e*x+a*e)^2,x, algorithm="giac")
```

output
```
1/2*((b*e^2 + 4*b*d*f - 8*a*e*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2 + e*x +
d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*d*f^2 + 8*(sqrt(
f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2 + e*x +
d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f^(3/2) +
8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f + 2*a*e^
3*f + 4*b*d^2*f^2 + 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f
)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) + 4*sqrt(b^2*d*e^2 - a*b*e
^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*e*f +
sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)))/sqrt(b
^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f) - (b*e^2 + 4*b*d*f - 8*a*e
*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x
- sqrt(f*x^2 + e*x + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))
^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f
)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f + 2*a*e^3*f + 4*b*d^2*f^2 - 4*sqrt(b^
2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))^2*f^(3/2) - 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*
f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*e*f - sqrt(b^2*d*e^2 - a*b*e^3 - 4*
a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)))/sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*
e*f + 4*a^2*e^2*f))/(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f) +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{1}{(b\,f\,x^2+b\,e\,x+a\,e)^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 3406, normalized size of antiderivative = 21.02

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(f*x^2+e*x+d)^(1/2)/(b*f*x^2+b*e*x+a*e)^2,x)
```

output
```
(8*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)*sqrt(e)
*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqrt(
f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*a**2*e**2*f
 - 4*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)*sqrt(
e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqr
t(f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*a*b*d*e*f
 - sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)*sqrt(e)
*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqrt(
f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*a*b*e**3 +
8*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)*sqrt(e)*
sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqrt(f
)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*a*b*e**2*f*x
 + 8*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)*sqrt(
e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) + 2*sqr
t(f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*a*b*e*f**
2*x**2 - 4*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt(f)
*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e**2) +
 2*sqrt(f)*sqrt(b)*sqrt(d + e*x + f*x**2) + sqrt(b)*e + 2*sqrt(b)*f*x)*b**
2*d*e*f*x - 4*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d)*log( - sqrt(4*sqrt
(f)*sqrt(e)*sqrt(4*a*f - b*e)*sqrt(a*e - b*d) - 8*a*e*f + 4*b*d*f + b*e...
```

3.41 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx=\frac{\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx=-\frac{\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
-(ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]/Sqrt[3])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1313, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{1313}$$

$$-4\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+5}-24}d\frac{2(x+1)}{\sqrt{x^2+2x+5}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

input
```
Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2+2x)}{6\sqrt{x^{2}+2x+5}}\right)}{3}$	27
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+3\right)x^{2}+3\sqrt{x^{2}+2x+5}\,x-2\operatorname{RootOf}\left(\_Z^{2}+3\right)x+3\sqrt{x^{2}+2x+5}-7\operatorname{RootOf}\left(\_Z^{2}+3\right)}{x^{2}+2x+4}\right)}{6}$	75

input
```
int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2+2*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}(x+1) - \frac{1}{3}\sqrt{3}(x^2+2x+4)\right)$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(
x^2 + 2*x + 4))
```

Sympy [F]

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

input
```
integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

output
```
Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Maxima [F]

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time $= 0.12$ (sec) , antiderivative size $= 52$, normalized size of antiderivative $= 1.86$

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right)$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt
(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{(x^2+2\,x+4)\,\sqrt{x^2+2\,x+5}}\,dx$$

input
```
int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{\sqrt{3}\left(-atan\left(\frac{\sqrt{x^2+2x+5}+x+2}{\sqrt{3}}\right)+atan\left(\frac{\sqrt{x^2+2x+5}+x}{\sqrt{3}}\right)\right)}{3}$$

input
```
int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

output
```
(sqrt(3)*( - atan((sqrt(x**2 + 2*x + 5) + x + 2)/sqrt(3)) + atan((sqrt(x**
2 + 2*x + 5) + x)/sqrt(3))))/3
```

3.42 $\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx$

Optimal result

Integrand size = 31, antiderivative size = 108

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \frac{2^{-2-4p-3q}(e+4cx)\left(2a+ex+2cx^2\right)^{p+q}\left(\frac{c(2a+ex+2cx^2)}{16ac-e^2}\right)^{-p-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-p-q,\frac{3}{2},-\frac{(e+4cx)^2}{16ac-e^2}\right.}{c}$$

output

```
2^(-2-4*p-3*q)*(4*c*x+e)*(2*c*x^2+e*x+2*a)^(p+q)*(c*(2*c*x^2+e*x+2*a)/(16*
a*c-e^2))^(-p-q)*hypergeom([1/2, -p-q],[3/2],-(4*c*x+e)^2/(16*a*c-e^2))/c
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.31

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \frac{2^{-2+q}\left(e-\sqrt{-16ac+e^2}+4cx\right)\left(\frac{e+\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-p-q}(2a+x(e+2cx))^{p+q}\text{Hypergeometric2F1}\left(-p-\right.}{c(1+p+q)}$$

input

```
Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

output
```
(2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] +
4*c*x)/Sqrt[-16*a*c + e^2])^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hyperge
ometric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x
)/(2*Sqrt[-16*a*c + e^2])])/(c*(1 + p + q))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1295, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2+\frac{ex}{2}\right)^p \left(2a+2cx^2+ex\right)^q \, dx$$

$$\Big\downarrow \; 1295$$

$$2^{-p}\int \left(2cx^2+ex+2a\right)^{p+q}dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\text{Hypergeometric2F1}\left(-p-q,p+q+1,p+q+2,\frac{e+\cdots}{\cdots}\right.}{(p+q+1)\sqrt{e^2-16ac}}$$

input
```
Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

output
```
-((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-
1 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 +
 p + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2
])])/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1295
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_
)^2)^(q_.), x_Symbol] :> Simp[(c/f)^p   Int[(d + e*x + f*x^2)^(p + q), x],
x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d -
a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !IntegerQ[q] || LeafCount[d +
e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Maple [F]

$$\int \left(a+\frac{1}{2}ex+c\,x^{2}\right)^{p} \left(2c\,x^{2}+ex+2a\right)^{q} dx$$

input
```
int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

output
```
int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \,dx=\int \left(2\,cx^{2}+ex+2\,a\right)^{q} \left(cx^{2}+\frac{1}{2}\,ex+a\right)^{p} \,dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \int \left(2\,cx^2+ex+2\,a\right)^q \left(cx^2+\frac{1}{2}\,ex+a\right)^p \, dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

Giac [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \int \left(2\,cx^2+ex+2\,a\right)^q \left(cx^2+\frac{1}{2}\,ex+a\right)^p \, dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \int \left(c\,x^2+\frac{e\,x}{2}+a\right)^p \left(2\,c\,x^2+e\,x+2\,a\right)^q \,dx$$

input
```
int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x)
```

output
```
int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx$$
$$= \frac{4(2c\,x^2+ex+2a)^{p+q}\,a+(2c\,x^2+ex+2a)^{p+q}\,ex-32\left(\int \frac{(2c\,x^2+ex+2a)^{p+q}x}{4cp\,x^2+4cq\,x^2+2c\,x^2+2epx+2eqx+4ap+4aq+ex+2a}dx\right) a$$

input
```
int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

output
```
(4*(2*a + 2*c*x**2 + e*x)**(p + q)*a + (2*a + 2*c*x**2 + e*x)**(p + q)*e*x
 - 32*int(((2*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p
*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*a*c*p**2 - 64*
int(((2*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2
+ 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*a*c*p*q - 16*int(((2
*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q
*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*a*c*p - 32*int(((2*a + 2*c*
x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2
*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*a*c*q**2 - 16*int(((2*a + 2*c*x**2 +
 e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2*c*x**
2 + 2*e*p*x + 2*e*q*x + e*x),x)*a*c*q + 2*int(((2*a + 2*c*x**2 + e*x)**(p
+ q)*x)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*
x + 2*e*q*x + e*x),x)*e**2*p**2 + 4*int(((2*a + 2*c*x**2 + e*x)**(p + q)*x
)/(4*a*p + 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*
e*q*x + e*x),x)*e**2*p*q + int(((2*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p
+ 4*a*q + 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e
*x),x)*e**2*p + 2*int(((2*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q +
 2*a + 4*c*p*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*e*
*2*q**2 + int(((2*a + 2*c*x**2 + e*x)**(p + q)*x)/(4*a*p + 4*a*q + 2*a + 4
*c*p*x**2 + 4*c*q*x**2 + 2*c*x**2 + 2*e*p*x + 2*e*q*x + e*x),x)*e**2*q)...
```

3.43 $\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx = \frac{2^{-1-2p-2q}(e+2fx)\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \left(-\frac{cf\left(\frac{af}{c}+ex+fx^2\right)}{ce^2-4af^2}\right)^{-p-q} \text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{f}$$

output
```
2^(-1-2*p-2*q)*(2*f*x+e)*(a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q*(-c*f*(a*
f/c+e*x+f*x^2)/(-4*a*f^2+c*e^2))^(-p-q)*hypergeom([1/2, -p-q],[3/2],c*(2*f
*x+e)^2/(-4*a*f^2+c*e^2))/f
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx = \frac{2^{-1+p+q}\left(\frac{af}{c}+x(e+fx)\right)^q \left(a+\frac{cx(e+fx)}{f}\right)^p \left(-\sqrt{ce^2-4af^2}+\sqrt{c}(e+2fx)\right)\left(1+\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2-4af^2}}\right)^{-p-q} \text{Hyper}}{\sqrt{c}f(1+p+q)}$$

input
```
Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

output
```
(2^(-1 + p + q)*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt
[c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c
*e^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
1/2 - (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])])/(Sqrt[c]*f*(1 + p
+ q))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1296, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$

$$\Big\downarrow \text{1296}$$

$$\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{-p} \int \left(fx^2+ex+\frac{af}{c}\right)^{p+q}dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{\sqrt{c}2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{q+1} \left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1} \text{Hypergeometric2F1}\Bigg(-p}{(p+q+1)\sqrt{ce^2-4af^2}}$$

input
```
Int[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

output
```
-((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] +
2*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a
*f)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p +
q, (Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4
*a*f^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1296
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x + c*x^2)^FracPart[p]/(d
^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]))   Int[(d + e*x + f*x^2)^(p + q)
, x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[
b*d - a*e, 0] &&  !IntegerQ[p] &&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

Maple [F]

$$\int \left(a+\frac{cex}{f}+c\,x^{2}\right)^{p} \left(\frac{af}{c}+ex+f\,x^{2}\right)^{q}dx$$

input
```
int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

output
```
int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx = \int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
```

output
```
integral(((c*f*x^2 + c*e*x + a*f)/c)^q*((c*f*x^2 + c*e*x + a*f)/f)^p, x)
```

Sympy [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$
$$= \int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
```

output
```
Integral((a + c*e*x/f + c*x**2)**p*(a*f/c + e*x + f*x**2)**q, x)
```

Maxima [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx = \int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

Giac [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx=\int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")
```

output
```
integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$
$$=\int \left(e\,x+f\,x^2+\frac{a\,f}{c}\right)^q \left(a+c\,x^2+\frac{c\,e\,x}{f}\right)^p dx$$

input
```
int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p,x)
```

output
```
int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p, x)
```

Reduce [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx=\text{Too large to display}$$

input
```
int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

output

```
(2*(a*f + c*e*x + c*f*x**2)**(p + q)*a*f + (a*f + c*e*x + c*f*x**2)**(p +
q)*c*e*x - 8*int(((a*f + c*e*x + c*f*x**2)**(p + q)*x)/(2*a*f*p + 2*a*f*q
+ a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*c*f*q*x**2 + c*f*
x**2),x)*a*c*f**2*p**2 - 16*int(((a*f + c*e*x + c*f*x**2)**(p + q)*x)/(2*a
*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*c*
f*q*x**2 + c*f*x**2),x)*a*c*f**2*p*q - 4*int(((a*f + c*e*x + c*f*x**2)**(p
 + q)*x)/(2*a*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*
p*x**2 + 2*c*f*q*x**2 + c*f*x**2),x)*a*c*f**2*p - 8*int(((a*f + c*e*x + c*
f*x**2)**(p + q)*x)/(2*a*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e
*x + 2*c*f*p*x**2 + 2*c*f*q*x**2 + c*f*x**2),x)*a*c*f**2*q**2 - 4*int(((a*
f + c*e*x + c*f*x**2)**(p + q)*x)/(2*a*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2
*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*c*f*q*x**2 + c*f*x**2),x)*a*c*f**2*q +
 2*int(((a*f + c*e*x + c*f*x**2)**(p + q)*x)/(2*a*f*p + 2*a*f*q + a*f + 2*
c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*c*f*q*x**2 + c*f*x**2),x)*c
**2*e**2*p**2 + 4*int(((a*f + c*e*x + c*f*x**2)**(p + q)*x)/(2*a*f*p + 2*a
*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*c*f*q*x**2 +
 c*f*x**2),x)*c**2*e**2*p*q + int(((a*f + c*e*x + c*f*x**2)**(p + q)*x)/(2
*a*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*c*f*p*x**2 + 2*
c*f*q*x**2 + c*f*x**2),x)*c**2*e**2*p + 2*int(((a*f + c*e*x + c*f*x**2)**(
p + q)*x)/(2*a*f*p + 2*a*f*q + a*f + 2*c*e*p*x + 2*c*e*q*x + c*e*x + 2*...
```

3.44 $\int (3-x+2x^2) (2+3x+5x^2)^4 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (3-x+2x^2) (2+3x+5x^2)^4 \, dx = 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

output
```
48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+
5075/9*x^9+475/2*x^10+1250/11*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2) (2+3x+5x^2)^4 \, dx = 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)\left(5x^2+3x+2\right)^4 dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(1250x^{10}+2375x^9+5075x^8+6830x^7+8232x^6+7131x^5+5099x^4+2624x^3+1064x^2+272x+48\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

output

```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

method	result
orering	$\frac{x(225000x^{10}+470250x^9+1116500x^8+1690425x^7+2328480x^6+2353230x^5+2019204x^4+1298880x^3+702240x^2+269280x+9}{1980}$
gosper	$48x + 136x^2 + \frac{1064}{3}x^3 + 656x^4 + \frac{5099}{5}x^5 + \frac{2377}{2}x^6 + 1176x^7 + \frac{3415}{4}x^8 + \frac{5075}{9}x^9 + \frac{475}{2}x^{10} + 1$
default	$48x + 136x^2 + \frac{1064}{3}x^3 + 656x^4 + \frac{5099}{5}x^5 + \frac{2377}{2}x^6 + 1176x^7 + \frac{3415}{4}x^8 + \frac{5075}{9}x^9 + \frac{475}{2}x^{10} + 1$
norman	$48x + 136x^2 + \frac{1064}{3}x^3 + 656x^4 + \frac{5099}{5}x^5 + \frac{2377}{2}x^6 + 1176x^7 + \frac{3415}{4}x^8 + \frac{5075}{9}x^9 + \frac{475}{2}x^{10} + 1$
risch	$48x + 136x^2 + \frac{1064}{3}x^3 + 656x^4 + \frac{5099}{5}x^5 + \frac{2377}{2}x^6 + 1176x^7 + \frac{3415}{4}x^8 + \frac{5075}{9}x^9 + \frac{475}{2}x^{10} + 1$
parallelrisch	$48x + 136x^2 + \frac{1064}{3}x^3 + 656x^4 + \frac{5099}{5}x^5 + \frac{2377}{2}x^6 + 1176x^7 + \frac{3415}{4}x^8 + \frac{5075}{9}x^9 + \frac{475}{2}x^{10} + 1$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/1980*x*(225000*x^10+470250*x^9+1116500*x^8+1690425*x^7+2328480*x^6+23532
30*x^5+2019204*x^4+1298880*x^3+702240*x^2+269280*x+95040)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

output
```
1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377
*x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176x^7+\frac{2377}{2}x^6+\frac{5099}{5}x^5+656x^4+\frac{1064}{3}x^3+136x^2+48x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176x^7+\frac{2377}{2}x^6+\frac{5099}{5}x^5+656x^4+\frac{1064}{3}x^3+136x^2+48x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx$$
$$=\frac{x(225000x^{10}+470250x^9+1116500x^8+1690425x^7+2328480x^6+2353230x^5+2019204x^4+1298880x}{1980}$$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x)
```

output
```
(x*(225000*x**10 + 470250*x**9 + 1116500*x**8 + 1690425*x**7 + 2328480*x**
6 + 2353230*x**5 + 2019204*x**4 + 1298880*x**3 + 702240*x**2 + 269280*x +
95040))/1980
```

3.45 $\int (3-x+2x^2)(2+3x+5x^2)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^3\,dx = 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

output
```
24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/
9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^3\,dx = 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2 x^2-x+3\right) \left(5 x^2+3 x+2\right)^3 \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(250 x^8+325 x^7+720 x^6+804 x^5+876 x^4+579 x^3+322 x^2+100 x+24\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{250 x^9}{9}+\frac{325 x^8}{8}+\frac{720 x^7}{7}+134 x^6+\frac{876 x^5}{5}+\frac{579 x^4}{4}+\frac{322 x^3}{3}+50 x^2+24 x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
orering	$\frac{x(70000x^8+102375x^7+259200x^6+337680x^5+441504x^4+364770x^3+270480x^2+126000x+60480)}{2520}$	44
gosper	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
default	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
norman	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
risch	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
parallelrisch	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2520*x*(70000*x^8+102375*x^7+259200*x^6+337680*x^5+441504*x^4+364770*x^3
+270480*x^2+126000*x+60480)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx = \frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134\,x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50\,x^2+24\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^3\,dx = \frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

output
```
250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4
+ 322*x**3/3 + 50*x**2 + 24*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^3\,dx = \frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\,(2+3x+5x^2)^3\,dx = \frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\,(2+3x+5x^2)^3\,dx = \frac{250\,x^9}{9}+\frac{325\,x^8}{8}+\frac{720\,x^7}{7}+134\,x^6+\frac{876\,x^5}{5}+\frac{579\,x^4}{4}+\frac{322\,x^3}{3}+50\,x^2+24\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx$$
$$=\frac{x(70000x^8+102375x^7+259200x^6+337680x^5+441504x^4+364770x^3+270480x^2+126000x+6048}{2520}$$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x)
```

output
```
(x*(70000*x**8 + 102375*x**7 + 259200*x**6 + 337680*x**5 + 441504*x**4 + 3
64770*x**3 + 270480*x**2 + 126000*x + 60480))/2520
```

3.46 $\int (3-x+2x^2)(2+3x+5x^2)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx = 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}$$

output
```
12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx = 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2 - x + 3)(5x^2 + 3x + 2)^2 \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int (50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

method	result	size
orering	$\frac{x(3000x^6+2450x^5+8652x^4+8925x^3+11620x^2+6720x+5040)}{420}$	34
gosper	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
default	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
norman	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
risch	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
parallelrisch	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/420*x*(3000*x^6+2450*x^5+8652*x^4+8925*x^3+11620*x^2+6720*x+5040)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx=\frac{50}{7}x^7+\frac{35}{6}x^6+\frac{103}{5}x^5+\frac{85}{4}x^4+\frac{83}{3}x^3+16x^2+12x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx=\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

output
```
50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*
x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx=\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx=\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2) (2+3x+5x^2)^2 \, dx = \frac{50\,x^7}{7}+\frac{35\,x^6}{6}+\frac{103\,x^5}{5}+\frac{85\,x^4}{4}+\frac{83\,x^3}{3}+16\,x^2+12\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int (3-x+2x^2) (2+3x+5x^2)^2 \, dx = \frac{x(3000x^6+2450x^5+8652x^4+8925x^3+11620x^2+6720x+5040)}{420}$$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x)
```

output
```
(x*(3000*x**6 + 2450*x**5 + 8652*x**4 + 8925*x**3 + 11620*x**2 + 6720*x +
5040))/420
```

3.47 $\int (3-x+2x^2)(2+3x+5x^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int (3-x+2x^2)(2+3x+5x^2)\,dx = 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

output
```
6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)(2+3x+5x^2)\,dx = 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2 - x + 3)(5x^2 + 3x + 2) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int (10x^4 + x^3 + 16x^2 + 7x + 6) \, dx$$

$$\Big\downarrow \text{2009}$$

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
orering	$\frac{x(24x^4+3x^3+64x^2+42x+72)}{12}$	24
gosper	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
default	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
norman	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
risch	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
parallelrisch	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4$	25

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(24*x^4+3*x^3+64*x^2+42*x+72)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)(2+3x+5x^2)\,dx = 2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx = 2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

output
```
2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx = 2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx = 2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)\,(2+3x+5x^2)\,dx=2\,x^5+\frac{x^4}{4}+\frac{16\,x^3}{3}+\frac{7\,x^2}{2}+6\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2),x)
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)\,(2+3x+5x^2)\,dx=\frac{x(24x^4+3x^3+64x^2+42x+72)}{12}$$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2),x)
```

output
```
(x*(24*x**4 + 3*x**3 + 64*x**2 + 42*x + 72))/12
```

3.48 $\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx = \frac{2x}{5}+\frac{143 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)$$

output
```
2/5*x+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-11/50*ln(5*x^2+3*x+2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx = \frac{2x}{5}+\frac{143 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

output
```
(2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*
x + 5*x^2])/50
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{5x^2+3x+2}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{11(1-x)}{5\left(5x^2+3x+2\right)}+\frac{2}{5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{143\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

output
```
(2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*
x + 5*x^2])/50
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2x}{5}+\frac{143\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{775}-\frac{11\ln\left(5x^2+3x+2\right)}{50}$	34
risch	$\frac{2x}{5}-\frac{11\ln\left(100x^2+60x+40\right)}{50}+\frac{143\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{775}$	34

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x+143/775*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)-11/50*ln(5*x^2+3*x+2
)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{143}{775}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

output
```
2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/775
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx = \frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10 x+3)\right) + \frac{2}{5} x - \frac{11}{50} \log\left(5 x^2+3 x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx = \frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10 x+3)\right) + \frac{2}{5} x - \frac{11}{50} \log\left(5 x^2+3 x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 15.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{2\,x}{5}-\frac{11\,\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{775}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2),x)
```

output
```
(2*x)/5 - (11*log(3*x + 5*x^2 + 2))/50 + (143*31^(1/2)*atan((10*31^(1/2)*x
)/31 + (3*31^(1/2))/31))/775
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{143\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)}{775}-\frac{11\log(5x^2+3x+2)}{50}+\frac{2x}{5}$$

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2),x)
```

output
```
(286*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 341*log(5*x**2 + 3*x + 2) + 620*
x)/1550
```

3.49 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx=\frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

output
```
11*(7+13*x)/(775*x^2+465*x+310)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31^(
1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx=\frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])
/(31*Sqrt[31])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{31}\int \frac{41}{5x^2+3x+2}dx+\frac{11(13x+7)}{155\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{41}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{11(13x+7)}{155\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{11(13x+7)}{155\,(5x^2+3x+2)}-\frac{82}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)$$

$$\Big\downarrow \text{217}$$

$$\frac{82\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{11(13x+7)}{155\,(5x^2+3x+2)}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])
/(31*Sqrt[31])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34
risch	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*arctan(1/31*(10*x+3)*31^(1/2))*3
1^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{410\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+4433x+2387}{4805\,(5x^2+3x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) +
4433*x + 2387)/(5*x^2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{143x+77}{775x^2+465x+310} + \frac{82\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

output
```
(143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/961
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{11\,(13x+7)}{155\,(5x^2+3x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^
2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{11\,(13x+7)}{155\,(5x^2+3x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^
2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{\frac{143\,x}{775}+\frac{77}{775}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{82\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^(1/2)*atan((10*31^(1
/2)*x)/31 + (3*31^(1/2))/31))/961
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{1230\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+738\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x+492\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)-4433x^2-341}{14415x^2+8649x+5766}$$

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

output
```
(1230*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 738*sqrt(31)*atan((10*x +
3)/sqrt(31))*x + 492*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 4433*x**2 - 341)
/(2883*(5*x**2 + 3*x + 2))
```

3.50 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} \, dx = \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2} + \frac{553(3+10x)}{9610\left(2+3x+5x^2\right)} + \frac{1106\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

output
```
11/310*(7+13*x)/(5*x^2+3*x+2)^2+553*(3+10*x)/(48050*x^2+28830*x+19220)+110
6/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} \, dx = \frac{\frac{31(1141+4094x+4977x^2+5530x^3)}{(2+3x+5x^2)^2} + 2212\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{59582}$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqr
t[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{62}\int \frac{553}{5(5x^2+3x+2)^2}dx+\frac{11(13x+7)}{310(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{553}{310}\int \frac{1}{(5x^2+3x+2)^2}dx+\frac{11(13x+7)}{310(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{553}{310}\left(\frac{10}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(13x+7)}{310(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{553}{310}\left(\frac{10x+3}{31(5x^2+3x+2)}-\frac{20}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)+\frac{11(13x+7)}{310(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{553}{310}\left(\frac{20\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(13x+7)}{310(5x^2+3x+2)^2}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*((3 + 10*x)/(31*(2 + 3*x
+ 5*x^2)) + (20*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])))/310
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+1
106/29791*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx$$
$$=\frac{171430\,x^3+2212\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+154287\,x^2+12691}{59582\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*
arctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{5530x^3+4977x^2+4094x+1141}{48050x^4+57660x^3+55738x^2+23064x+7688} + \frac{1106\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

output
```
(5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x
**2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/3
1)/29791
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 +
4977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\,(5\,x^2+3\,x+2)^2}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 +
4977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

Mupad [B] (verification not implemented)

Time = 15.81 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791} + \frac{\frac{553\,x^3}{4805}+\frac{4977\,x^2}{48050}+\frac{2047\,x}{24025}+\frac{1141}{48050}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(1106*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((2047*
x)/24025 + (4977*x^2)/48050 + (553*x^3)/4805 + 1141/48050)/((12*x)/25 + (2
9*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx$$
$$= \frac{165900\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4+199080\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3+192444\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+79632\sqrt{31}\,at}{4468650x^4+5362380x^3+5183634x^2+}$$

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

output
```
(165900*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 199080*sqrt(31)*atan((10
*x + 3)/sqrt(31))*x**3 + 192444*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 +
79632*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 26544*sqrt(31)*atan((10*x + 3
)/sqrt(31)) - 428575*x**4 - 34286*x**2 + 175026*x + 37541)/(178746*(25*x**
4 + 30*x**3 + 29*x**2 + 12*x + 4))
```

3.51 $\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x^{12}}{3}+\frac{2500x^{13}}{13}$$

output
```
144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315
*x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x^{12}}{3}+\frac{2500x^{13}}{13}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^4 \, dx$$

$$\Big\downarrow \ 2188$$

$$\int \left(2500x^{12}+3500x^{11}+11525x^{10}+15710x^9+24859x^8+26520x^7+27763x^6+21542x^5+14801x^4+7352x^3$$

$$\Big\downarrow \ 2009$$

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

method	result
orering	$\frac{x(8662500x^{12}+13138125x^{11}+47194875x^{10}+70765695x^{9}+124419295x^{8}+149324175x^{7}+178654905x^{6}+161726565x^{5}+1333}{45045}$
gosper	$144x+384x^{2}+\frac{3016}{3}x^{3}+1838x^{4}+\frac{14801}{5}x^{5}+\frac{10771}{3}x^{6}+\frac{27763}{7}x^{7}+3315x^{8}+\frac{24859}{9}x^{9}+1571$
default	$144x+384x^{2}+\frac{3016}{3}x^{3}+1838x^{4}+\frac{14801}{5}x^{5}+\frac{10771}{3}x^{6}+\frac{27763}{7}x^{7}+3315x^{8}+\frac{24859}{9}x^{9}+1571$
norman	$144x+384x^{2}+\frac{3016}{3}x^{3}+1838x^{4}+\frac{14801}{5}x^{5}+\frac{10771}{3}x^{6}+\frac{27763}{7}x^{7}+3315x^{8}+\frac{24859}{9}x^{9}+1571$
risch	$144x+384x^{2}+\frac{3016}{3}x^{3}+1838x^{4}+\frac{14801}{5}x^{5}+\frac{10771}{3}x^{6}+\frac{27763}{7}x^{7}+3315x^{8}+\frac{24859}{9}x^{9}+1571$
parallelrisch	$144x+384x^{2}+\frac{3016}{3}x^{3}+1838x^{4}+\frac{14801}{5}x^{5}+\frac{10771}{3}x^{6}+\frac{27763}{7}x^{7}+3315x^{8}+\frac{24859}{9}x^{9}+1571$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/45045*x*(8662500*x^12+13138125*x^11+47194875*x^10+70765695*x^9+124419295
*x^8+149324175*x^7+178654905*x^6+161726565*x^5+133342209*x^4+82792710*x^3+
45285240*x^2+17297280*x+6486480)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^9+3315\,x^8+\frac{27763}{7}\,x^7+\frac{10771}{3}\,x^6+\frac{14801}{5}\,x^5+1838\,x^4+\frac{3016}{3}\,x^3+384\,x^2+144\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

output
```
2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
 3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016
*x**3/3 + 384*x**2 + 144*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \frac{2500}{13}\,x^{13} + \frac{875}{3}\,x^{12} + \frac{11525}{11}\,x^{11} + 1571\,x^{10} + \frac{24859}{9}\,x^9 + 3315\,x^8 + \frac{27763}{7}\,x^7 + \frac{10771}{3}\,x^6 + \frac{14801}{5}\,x^5 + 1838\,x^4 + \frac{3016}{3}\,x^3 + 384\,x^2 + 144\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \frac{2500}{13}\,x^{13} + \frac{875}{3}\,x^{12} + \frac{11525}{11}\,x^{11} + 1571\,x^{10} + \frac{24859}{9}\,x^9 + 3315\,x^8 + \frac{27763}{7}\,x^7 + \frac{10771}{3}\,x^6 + \frac{14801}{5}\,x^5 + 1838\,x^4 + \frac{3016}{3}\,x^3 + 384\,x^2 + 144\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500\,x^{13}}{13}+\frac{875\,x^{12}}{3}+\frac{11525\,x^{11}}{11}+1571\,x^{10}+\frac{24859\,x^9}{9}+3315\,x^8+\frac{27763\,x^7}{7}+\frac{10771\,x^6}{3}+\frac{14801\,x^5}{5}+1838\,x^4+\frac{3016\,x^3}{3}+384\,x^2+144\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^4,x)
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{x(8662500x^{12}+13138125x^{11}+47194875x^{10}+70765695x^9+124419295x^8+149324175x^7+17865490}{45045}$$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x)
```

output
```
(x*(8662500*x**12 + 13138125*x**11 + 47194875*x**10 + 70765695*x**9 + 1244
19295*x**8 + 149324175*x**7 + 178654905*x**6 + 161726565*x**5 + 133342209*
x**4 + 82792710*x**3 + 45285240*x**2 + 17297280*x + 6486480))/45045
```

3.52 $\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

output
```
72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+18
65/9*x^9+40*x^10+500/11*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

output
```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^3 dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(500x^{10}+400x^9+1865x^8+1863x^7+3108x^6+2694x^5+2693x^4+1615x^3+914x^2+276x+72\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

output

```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

method	result
orering	$\frac{x\left(180000x^{10}+158400x^{9}+820600x^{8}+922185x^{7}+1758240x^{6}+1778040x^{5}+2132856x^{4}+1598850x^{3}+1206480x^{2}+546480x+28\right.}{3960}$
gosper	$72x+138x^{2}+\frac{914}{3}x^{3}+\frac{1615}{4}x^{4}+\frac{2693}{5}x^{5}+449x^{6}+444x^{7}+\frac{1863}{8}x^{8}+\frac{1865}{9}x^{9}+40x^{10}+\frac{500}{11}x$
default	$72x+138x^{2}+\frac{914}{3}x^{3}+\frac{1615}{4}x^{4}+\frac{2693}{5}x^{5}+449x^{6}+444x^{7}+\frac{1863}{8}x^{8}+\frac{1865}{9}x^{9}+40x^{10}+\frac{500}{11}x$
norman	$72x+138x^{2}+\frac{914}{3}x^{3}+\frac{1615}{4}x^{4}+\frac{2693}{5}x^{5}+449x^{6}+444x^{7}+\frac{1863}{8}x^{8}+\frac{1865}{9}x^{9}+40x^{10}+\frac{500}{11}x$
risch	$72x+138x^{2}+\frac{914}{3}x^{3}+\frac{1615}{4}x^{4}+\frac{2693}{5}x^{5}+449x^{6}+444x^{7}+\frac{1863}{8}x^{8}+\frac{1865}{9}x^{9}+40x^{10}+\frac{500}{11}x$
parallelrisch	$72x+138x^{2}+\frac{914}{3}x^{3}+\frac{1615}{4}x^{4}+\frac{2693}{5}x^{5}+449x^{6}+444x^{7}+\frac{1863}{8}x^{8}+\frac{1865}{9}x^{9}+40x^{10}+\frac{500}{11}x$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3960*x*(180000*x^10+158400*x^9+820600*x^8+922185*x^7+1758240*x^6+1778040
*x^5+2132856*x^4+1598850*x^3+1206480*x^2+546480*x+285120)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^{2}\right)^{2}\left(2+3x+5x^{2}\right)^{3}\,dx=\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^{9}+\frac{1863}{8}\,x^{8}+444\,x^{7}+449\,x^{6}+\frac{2693}{5}\,x^{5}+\frac{1615}{4}\,x^{4}+\frac{914}{3}\,x^{3}+138\,x^{2}+72\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

output
```
500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6
+ 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^9+\frac{1863}{8}\,x^8+444\,x^7 + 449\,x^6 + \frac{2693}{5}\,x^5 + \frac{1615}{4}\,x^4 + \frac{914}{3}\,x^3 + 138\,x^2 + 72\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^9+\frac{1863}{8}\,x^8+444\,x^7+449\,x^6+\frac{2693}{5}\,x^5+\frac{1615}{4}\,x^4+\frac{914}{3}\,x^3+138\,x^2+72\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500\,x^{11}}{11}+40\,x^{10}+\frac{1865\,x^9}{9}+\frac{1863\,x^8}{8}+444\,x^7+449\,x^6+\frac{2693\,x^5}{5}+\frac{1615\,x^4}{4}+\frac{914\,x^3}{3}+138\,x^2+72\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)
```

output
```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 dx$$
$$= \frac{x(180000x^{10} + 158400x^9 + 820600x^8 + 922185x^7 + 1758240x^6 + 1778040x^5 + 2132856x^4 + 1598850x^3}{3960}$$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x)
```

output
```
(x*(180000*x**10 + 158400*x**9 + 820600*x**8 + 922185*x**7 + 1758240*x**6
+ 1778040*x**5 + 2132856*x**4 + 1598850*x**3 + 1206480*x**2 + 546480*x + 2
85120))/3960
```

3.53 $\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5 +\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

output
```
36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5 +\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^2 \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(100x^8+20x^7+321x^6+172x^5+390x^4+236x^3+241x^2+84x+36\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result	size
orering	$\frac{x(1400x^8+315x^7+5778x^6+3612x^5+9828x^4+7434x^3+10122x^2+5292x+4536)}{126}$	44
gosper	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
default	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
norman	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
risch	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
parallelrisch	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/126*x*(1400*x^8+315*x^7+5778*x^6+3612*x^5+9828*x^4+7434*x^3+10122*x^2+52
92*x+4536)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = \frac{100}{9}x^9+\frac{5}{2}x^8+\frac{321}{7}x^7+\frac{86}{3}x^6+78x^5+59x^4+\frac{241}{3}x^3+42x^2+36x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \,dx = \frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

output
```
100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x
**3/3 + 42*x**2 + 36*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \,dx = \frac{100}{9}\,x^9 + \frac{5}{2}\,x^8 + \frac{321}{7}\,x^7 + \frac{86}{3}\,x^6 + 78\,x^5 + 59\,x^4 + \frac{241}{3}\,x^3 + 42\,x^2 + 36\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = \frac{100}{9}\,x^9 + \frac{5}{2}\,x^8 + \frac{321}{7}\,x^7 + \frac{86}{3}\,x^6 + 78\,x^5 + 59\,x^4 + \frac{241}{3}\,x^3 + 42\,x^2 + 36\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = \frac{100\,x^9}{9} + \frac{5\,x^8}{2} + \frac{321\,x^7}{7} + \frac{86\,x^6}{3} + 78\,x^5 + 59\,x^4 + \frac{241\,x^3}{3} + 42\,x^2 + 36\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx$$
$$= \frac{x(1400x^8+315x^7+5778x^6+3612x^5+9828x^4+7434x^3+10122x^2+5292x+4536)}{126}$$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x)
```

output
```
(x*(1400*x**8 + 315*x**7 + 5778*x**6 + 3612*x**5 + 9828*x**4 + 7434*x**3 +
 10122*x**2 + 5292*x + 4536))/126
```

3.54 $\int (3-x+2x^2)^2 (2+3x+5x^2)\, dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int (3-x+2x^2)^2 (2+3x+5x^2)\, dx = 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}$$

output
```
18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)\, dx = 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(20x^6-8x^5+61x^4+x^3+53x^2+15x+18\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
orering	$\frac{x(1200x^6-560x^5+5124x^4+105x^3+7420x^2+3150x+7560)}{420}$	34
gosper	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
default	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
norman	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
risch	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
parallelrisch	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/420*x*(1200*x^6-560*x^5+5124*x^4+105*x^3+7420*x^2+3150*x+7560)
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) dx = \frac{20}{7}\,x^7-\frac{4}{3}\,x^6+\frac{61}{5}\,x^5+\frac{1}{4}\,x^4+\frac{53}{3}\,x^3+\frac{15}{2}\,x^2+18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)\,dx=\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

output
```
20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)\,dx=\frac{20}{7}\,x^7-\frac{4}{3}\,x^6+\frac{61}{5}\,x^5+\frac{1}{4}\,x^4+\frac{53}{3}\,x^3+\frac{15}{2}\,x^2+18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)\,dx=\frac{20}{7}\,x^7-\frac{4}{3}\,x^6+\frac{61}{5}\,x^5+\frac{1}{4}\,x^4+\frac{53}{3}\,x^3+\frac{15}{2}\,x^2+18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)\,dx=\frac{20\,x^7}{7}-\frac{4\,x^6}{3}+\frac{61\,x^5}{5}+\frac{x^4}{4}+\frac{53\,x^3}{3}+\frac{15\,x^2}{2}+18\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2),x)
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)\,dx$$
$$=\frac{x(1200x^6-560x^5+5124x^4+105x^3+7420x^2+3150x+7560)}{420}$$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2),x)
```

output
```
(x*(1200*x**6 - 560*x**5 + 5124*x**4 + 105*x**3 + 7420*x**2 + 3150*x + 756
0))/420
```

3.55 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx = \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{8349 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}} - \frac{1573 \log(2+3x+5x^2)}{1250}$$

output
```
381/125*x-16/25*x^2+4/15*x^3+8349/19375*arctan(1/31*(3+10*x)*31^(1/2))*31^
(1/2)-1573/1250*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx = \frac{8349 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}} + \frac{10x(1143-240x+100x^2) - 4719 \log(2+3x+5x^2)}{3750}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

output
```
(8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x +
100*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{5x^2+3x+2} \, dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(\frac{4x^2}{5}+\frac{121(3-13x)}{125\,(5x^2+3x+2)}-\frac{32x}{25}+\frac{381}{125}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{8349\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}+\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

output
```
(381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]]
)/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{19375}-\frac{1573\ln\left(5x^2+3x+2\right)}{1250}$	44
risch	$\frac{4x^3}{15}-\frac{16x^2}{25}+\frac{381x}{125}-\frac{1573\ln\left(100x^2+60x+40\right)}{1250}+\frac{8349\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{19375}$	44

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
381/125*x-16/25*x^2+4/15*x^3+8349/19375*arctan(1/31*(10*x+3)*31^(1/2))*31^
(1/2)-1573/1250*ln(5*x^2+3*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{16}{25}\,x^2+\frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x-\frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

output
```
4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 8
349*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{381\,x}{125} - \frac{1573\,\ln\left(5\,x^2+3\,x+2\right)}{1250} + \frac{8349\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{19375} - \frac{16\,x^2}{25} + \frac{4\,x^3}{15}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2),x)
```

output
```
(381*x)/125 - (1573*log(3*x + 5*x^2 + 2))/1250 + (8349*31^(1/2)*atan((10*3
1^(1/2)*x)/31 + (3*31^(1/2))/31))/19375 - (16*x^2)/25 + (4*x^3)/15
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{8349\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)}{19375} - \frac{1573\log(5x^2+3x+2)}{1250} + \frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125}$$

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

output
```
(50094*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 146289*log(5*x**2 + 3*x + 2) +
 31000*x**3 - 74400*x**2 + 354330*x)/116250
```

3.56 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)}+\frac{41932\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)$$

output
```
4/25*x+121*(61+69*x)/(19375*x^2+11625*x+7750)+41932/120125*arctan(1/31*(3+
10*x)*31^(1/2))*31^(1/2)-22/125*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{19220x+\frac{3751(61+69x)}{2+3x+5x^2}+41932\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)-21142\log\left(2+3x+5x^2\right)}{120125}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3
 + 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{31}\int \frac{4(155x^2-248x+1008)}{25(5x^2+3x+2)}dx+\frac{121(69x+61)}{3875(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{775}\int \frac{155x^2-248x+1008}{5x^2+3x+2}dx+\frac{121(69x+61)}{3875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{4}{775}\int \left(\frac{11(86-31x)}{5x^2+3x+2}+31\right)dx+\frac{121(69x+61)}{3875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{775}\left(\frac{10483\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}-\frac{341}{10}\log\left(5x^2+3x+2\right)+31x\right)+\frac{121(69x+61)}{3875(5x^2+3x+2)}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (4*(31*x + (10483*ArcTan[(3 +
 10*x)/Sqrt[31]])/(5*Sqrt[31]) - (341*Log[2 + 3*x + 5*x^2])/10))/775
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{4x}{25}+\frac{\frac{8349x}{19375}+\frac{7381}{19375}}{x^{2}+\frac{3}{5}x+\frac{2}{5}}-\frac{22\ln\left(100x^{2}+60x+40\right)}{125}+\frac{41932\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	50
default	$\frac{4x}{25}-\frac{11\left(-\frac{759x}{775}-\frac{671}{775}\right)}{25\left(x^{2}+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{22\ln\left(5x^{2}+3x+2\right)}{125}+\frac{41932\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	51

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
4/25*x+(8349/19375*x+7381/19375)/(x^2+3/5*x+2/5)-22/125*ln(100*x^2+60*x+40
)+41932/120125*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{96100\,x^3+41932\,\sqrt{31}(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+57660\,x^2-21142\,(5\,x^2+3\,x+2)\,\mathrm{log}}{120125\,(5\,x^2+3\,x+2)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31
)*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) +
 297259*x + 228811)/(5*x^2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4x}{25}+\frac{8349x+7381}{19375x^2+11625x+7750}-\frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125}$$
$$+\frac{41932\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

output
```
4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x
/5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/1201
25
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{41932}{120125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x + \frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875
*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{41932}{120125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x + \frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875
*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4\,x}{25} - \frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125} + \frac{\frac{8349\,x}{19375}+\frac{7381}{19375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{41932\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{120125}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)
```

output
```
(4*x)/25 - (22*log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/(
(3*x)/5 + x^2 + 2/5) + (41932*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/
2))/31))/120125
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{628980\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+377388\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x+251592\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)-317130\log(5x^2 -}{1801875x^2+10}$$

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

output
```
(628980*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 377388*sqrt(31)*atan((10
*x + 3)/sqrt(31))*x + 251592*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 317130*l
og(5*x**2 + 3*x + 2)*x**2 - 190278*log(5*x**2 + 3*x + 2)*x - 126852*log(5*
x**2 + 3*x + 2) + 288300*x**3 - 1313315*x**2 + 91915)/(360375*(5*x**2 + 3*
x + 2))
```

3.57 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}+\frac{4330\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

output
```
121/7750*(61+69*x)/(5*x^2+3*x+2)^2+11*(17557+45710*x)/(1201250*x^2+720750*
x+480500)+4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{11(11183+33524x+44983x^2+45710x^3)}{48050\,(2+3x+5x^2)^2}+\frac{4330\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
 + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{62}\int \frac{6200x^2-9920x+48669}{125\left(5x^2+3x+2\right)^2}dx+\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6200x^2-9920x+48669}{\left(5x^2+3x+2\right)^2}dx}{7750}+\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{31}\int \frac{541250}{5x^2+3x+2}dx+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{541250}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{11(45710x+17557)}{31(5x^2+3x+2)}-\frac{1082500}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)}{7750}+\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 217$$

$$\frac{\frac{1082500\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750\,(5x^2+3x+2)^2}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + ((11*(17557 + 45710*x))/(31
*(2 + 3*x + 5*x^2)) + (1082500*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31]))
/7750
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5
*x^2+3*x+2)^2+4330/29791*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{15587110\,x^3+216500\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+15339203\,x^2 +}{1489550\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
 + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 38134
03)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{502810x^3+494813x^2+368764x+123013}{1201250x^4+1441500x^3+1393450x^2+576600x+192200} + \frac{4330\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

output
```
(502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x*
*3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/
31 + 3*sqrt(31)/31)/29791
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{11\,(45710\,x^3+44983\,x^2+33524\,x+11183)}{48050\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
 + 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{11\,(45710\,x^3+44983\,x^2+33524\,x+11183)}{48050\,(5\,x^2+3\,x+2)^2}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
 + 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281\,x^3}{120125}+\frac{494813\,x^2}{1201250}+\frac{184382\,x}{600625}+\frac{123013}{1201250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(4330*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((18438
2*x)/600625 + (494813*x^2)/1201250 + (50281*x^3)/120125 + 123013/1201250)/
((12*x)/25 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$$
$$= \frac{3247500\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4+3897000\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3+3767100\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+1558800\sqrt{}}{22343250x^4+26811900x^3+2591817}$$

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

output
```
(3247500*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 3897000*sqrt(31)*atan((
10*x + 3)/sqrt(31))*x**3 + 3767100*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2
 + 1558800*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 519600*sqrt(31)*atan((10
*x + 3)/sqrt(31)) - 7793555*x**4 + 162998*x**2 + 3118104*x + 1041073)/(893
730*(25*x**4 + 30*x**3 + 29*x**2 + 12*x + 4))
```

3.58 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{66752\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

output
```
121/11625*(61+69*x)/(5*x^2+3*x+2)^3+11/120125*(4579+12060*x)/(5*x^2+3*x+2)
^2+16688*(3+10*x)/(744775*x^2+446865*x+297910)+66752/923521*arctan(1/31*(3
+10*x)*31^(1/2))*31^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{1259239+5674908x+12780597x^2+21491796x^3+18774000x^4+12516000x^5}{446865(2+3x+5x^2)^3}+\frac{66752\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

output
```
(1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 125160
00*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])
/(29791*Sqrt[31])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2191, 27, 2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{(5x^2+3x+2)^4}dx$$
$$\Big\downarrow \text{ 2191}$$
$$\frac{1}{93}\int \frac{6(1550x^2-2480x+12863)}{125\,(5x^2+3x+2)^3}dx+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{2\int \frac{1550x^2-2480x+12863}{(5x^2+3x+2)^3}dx}{3875}+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$
$$\Big\downarrow \text{ 2191}$$
$$\frac{2\left(\frac{1}{62}\int \frac{417200}{(5x^2+3x+2)^2}dx+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{2\left(\frac{208600}{31}\int \frac{1}{(5x^2+3x+2)^2}dx+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$
$$\Big\downarrow \text{ 1086}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{10}{31}\int\frac{1}{5x^2+3x+2}dx+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{10x+3}{31(5x^2+3x+2)}-\frac{20}{31}\int\frac{1}{-(10x+3)^2-31}d(10x+3)\right)+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{20\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

output
```
(121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (2*((11*(4579 + 12060*x))/
(62*(2 + 3*x + 5*x^2)^2) + (208600*((3 + 10*x)/(31*(2 + 3*x + 5*x^2)) + (2
0*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])))/31))/3875
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57
risch	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/186193
75*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*a
rctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{387996000\,x^5+581994000\,x^4+666245676\,x^3+1001280\,\sqrt{31}(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,}{13852815\,(125\,x^6+225\,x^5+285\,x^4+207\,}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(3
1)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/3
1*sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6
+ 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{55858125x^6+100544625x^5+127356525x^4+92501055x^3+50942610x^2+16087140x+3574920} + \frac{66752\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

output
```
(12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
 + 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x*
*3 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/923521
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$$
$$= \frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)$$
$$+ \frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(1251600
0*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/
(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$$
$$= \frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)$$
$$+ \frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\,(5\,x^2+3\,x+2)^3}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(1251600
0*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/
(5*x^2 + 3*x + 2)^3
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{66752\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521} + \frac{\frac{33376\,x^5}{148955}+\frac{50064\,x^4}{148955}+\frac{7163932\,x^3}{18619375}+\frac{4260199\,x^2}{18619375}+\frac{1891636\,x}{18619375}+\frac{1259239}{55858125}}{x^6+\frac{9\,x^5}{5}+\frac{57\,x^4}{25}+\frac{207\,x^3}{125}+\frac{114\,x^2}{125}+\frac{36\,x}{125}+\frac{8}{125}}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4,x)
```

output
```
(66752*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/923521 + ((189
1636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (5006
4*x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*
x^2)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.11

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{375480000\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^6+675864000\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^5+856094400\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4+62}{}$$

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x)
```

output
```
(375480000*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**6 + 675864000*sqrt(31)*at
an((10*x + 3)/sqrt(31))*x**5 + 856094400*sqrt(31)*atan((10*x + 3)/sqrt(31)
)*x**4 + 621794880*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**3 + 342437760*sqr
t(31)*atan((10*x + 3)/sqrt(31))*x**2 + 108138240*sqrt(31)*atan((10*x + 3)/
sqrt(31))*x + 24030720*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 646660000*x**6
 + 271597200*x**4 + 927868068*x**3 + 598841601*x**2 + 341528364*x + 757229
87)/(41558445*(125*x**6 + 225*x**5 + 285*x**4 + 207*x**3 + 114*x**2 + 36*x
 + 8))
```

3.59 $\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = 432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8}+\frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11}+\frac{30395x^{12}}{12}+\frac{27050x^{13}}{13}+\frac{2250x^{14}}{7}+\frac{1000x^{15}}{3}$$

output
```
432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881
/8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^1
3+2250/7*x^14+1000/3*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = 432x+1080x^2+2856x^3+5144x^4 + \frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8} + \frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11} + \frac{30395x^{12}}{12}+\frac{27050x^{13}}{13}+\frac{2250x^{14}}{7}+\frac{1000x^{15}}{3}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^3 (5x^2+3x+2)^4 \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int (5000x^{14}+4500x^{13}+27050x^{12}+30395x^{11}+68583x^{10}+75311x^9+103583x^8+94881x^7+91349x^6+6452$$

$$\Big\downarrow \text{2009}$$

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+\frac{43083x^5}{5}+5144x^4+2856x^3+1080x^2+432x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result
orering	$\frac{x(120120000x^{14}+115830000x^{13}+749826000x^{12}+912761850x^{11}+2246779080x^{10}+2713907196x^9+4147463320x^8+4273914}{360360}$
gosper	$432x+1080x^2+2856x^3+5144x^4+\frac{43083}{5}x^5+\frac{64529}{6}x^6+\frac{91349}{7}x^7+\frac{94881}{8}x^8+\frac{103583}{9}x^9+75$
default	$432x+1080x^2+2856x^3+5144x^4+\frac{43083}{5}x^5+\frac{64529}{6}x^6+\frac{91349}{7}x^7+\frac{94881}{8}x^8+\frac{103583}{9}x^9+75$
norman	$432x+1080x^2+2856x^3+5144x^4+\frac{43083}{5}x^5+\frac{64529}{6}x^6+\frac{91349}{7}x^7+\frac{94881}{8}x^8+\frac{103583}{9}x^9+75$
risch	$432x+1080x^2+2856x^3+5144x^4+\frac{43083}{5}x^5+\frac{64529}{6}x^6+\frac{91349}{7}x^7+\frac{94881}{8}x^8+\frac{103583}{9}x^9+75$
parallelrisch	$432x+1080x^2+2856x^3+5144x^4+\frac{43083}{5}x^5+\frac{64529}{6}x^6+\frac{91349}{7}x^7+\frac{94881}{8}x^8+\frac{103583}{9}x^9+75$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/360360*x*(120120000*x^14+115830000*x^13+749826000*x^12+912761850*x^11+22
46779080*x^10+2713907196*x^9+4147463320*x^8+4273914645*x^7+4702646520*x^6+
3875611740*x^5+3105077976*x^4+1853691840*x^3+1029188160*x^2+389188800*x+15
5675520)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \frac{1000}{3}\,x^{15} + \frac{2250}{7}\,x^{14} + \frac{27050}{13}\,x^{13} + \frac{30395}{12}\,x^{12} + \frac{68583}{11}\,x^{11} + \frac{75311}{10}\,x^{10} + \frac{103583}{9}\,x^{9} + \frac{94881}{8}\,x^{8} + \frac{91349}{7}\,x^{7} + \frac{64529}{6}\,x^{6} + \frac{43083}{5}\,x^{5} + 5144\,x^{4} + 2856\,x^{3} + 1080\,x^{2} + 432\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^{9}}{9} + \frac{94881x^{8}}{8} + \frac{91349x^{7}}{7} + \frac{64529x^{6}}{6} + \frac{43083x^{5}}{5} + 5144x^{4} + 2856x^{3} + 1080x^{2} + 432x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

output
```
1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**1
1/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 6452
9*x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^{9} + \frac{94881}{8}x^{8} + \frac{91349}{7}x^{7} + \frac{64529}{6}x^{6} + \frac{43083}{5}x^{5} + 5144\,x^4 + 2856\,x^3 + 1080\,x^2 + 432\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^{9} + \frac{94881}{8}x^{8} + \frac{91349}{7}x^{7} + \frac{64529}{6}x^{6} + \frac{43083}{5}x^{5} + 5144\,x^4 + 2856\,x^3 + 1080\,x^2 + 432\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{1000\,x^{15}}{3} + \frac{2250\,x^{14}}{7} + \frac{27050\,x^{13}}{13} + \frac{30395\,x^{12}}{12} + \frac{68583\,x^{11}}{11} + \frac{75311\,x^{10}}{10} + \frac{103583\,x^9}{9} + \frac{94881\,x^8}{8} + \frac{91349\,x^7}{7} + \frac{64529\,x^6}{6} + \frac{43083\,x^5}{5} + 5144\,x^4 + 2856\,x^3 + 1080\,x^2 + 432\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^4,x)
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{x(120120000x^{14} + 115830000x^{13} + 749826000x^{12} + 912761850x^{11} + 2246779080x^{10} + 2713907196x^9 +}{}$$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x)
```

output
```
(x*(120120000*x**14 + 115830000*x**13 + 749826000*x**12 + 912761850*x**11
+ 2246779080*x**10 + 2713907196*x**9 + 4147463320*x**8 + 4273914645*x**7 +
 4702646520*x**6 + 3875611740*x**5 + 3105077976*x**4 + 1853691840*x**3 + 1
029188160*x**2 + 389188800*x + 155675520))/360360
```

3.60 $\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+\frac{1000x^{13}}{13}$$

output
```
216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*
x^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+\frac{1000x^{13}}{13}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right)^3 dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(1000x^{12}+300x^{11}+4830x^{10}+3061x^9+9948x^8+7869x^7+12016x^6+8619x^5+8292x^4+4483x^3+2610x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

method	result
orering	$\frac{x(9240000x^{12}+3003000x^{11}+52743600x^{10}+36768732x^9+132772640x^8+118153035x^7+206194560x^6+172552380x^5+19920}{120120}$
gosper	$216x + 378x^2 + 870x^3 + \frac{4483}{4}x^4 + \frac{8292}{5}x^5 + \frac{2873}{2}x^6 + \frac{12016}{7}x^7 + \frac{7869}{8}x^8 + \frac{3316}{3}x^9 + \frac{3061}{10}x^{10} +$
default	$216x + 378x^2 + 870x^3 + \frac{4483}{4}x^4 + \frac{8292}{5}x^5 + \frac{2873}{2}x^6 + \frac{12016}{7}x^7 + \frac{7869}{8}x^8 + \frac{3316}{3}x^9 + \frac{3061}{10}x^{10} +$
norman	$216x + 378x^2 + 870x^3 + \frac{4483}{4}x^4 + \frac{8292}{5}x^5 + \frac{2873}{2}x^6 + \frac{12016}{7}x^7 + \frac{7869}{8}x^8 + \frac{3316}{3}x^9 + \frac{3061}{10}x^{10} +$
risch	$216x + 378x^2 + 870x^3 + \frac{4483}{4}x^4 + \frac{8292}{5}x^5 + \frac{2873}{2}x^6 + \frac{12016}{7}x^7 + \frac{7869}{8}x^8 + \frac{3316}{3}x^9 + \frac{3061}{10}x^{10} +$
parallelrisch	$216x + 378x^2 + 870x^3 + \frac{4483}{4}x^4 + \frac{8292}{5}x^5 + \frac{2873}{2}x^6 + \frac{12016}{7}x^7 + \frac{7869}{8}x^8 + \frac{3316}{3}x^9 + \frac{3061}{10}x^{10} +$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/120120*x*(9240000*x^12+3003000*x^11+52743600*x^10+36768732*x^9+132772640
*x^8+118153035*x^7+206194560*x^6+172552380*x^5+199207008*x^4+134624490*x^3
+104504400*x^2+45405360*x+25945920)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3\,dx = \frac{1000}{13}\,x^{13} + 25\,x^{12} + \frac{4830}{11}\,x^{11} + \frac{3061}{10}\,x^{10} + \frac{3316}{3}\,x^9 + \frac{7869}{8}\,x^8 + \frac{12016}{7}\,x^7 + \frac{2873}{2}\,x^6 + \frac{8292}{5}\,x^5 + \frac{4483}{4}\,x^4 + 870\,x^3 + 378\,x^2 + 216\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3\,dx = \frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

output
```
1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 7
869*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*
x**3 + 378*x**2 + 216*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000}{13}\, x^{13} + 25\, x^{12} + \frac{4830}{11}\, x^{11} + \frac{3061}{10}\, x^{10} + \frac{3316}{3}\, x^9 + \frac{7869}{8}\, x^8 + \frac{12016}{7}\, x^7 + \frac{2873}{2}\, x^6 + \frac{8292}{5}\, x^5 + \frac{4483}{4}\, x^4 + 870\, x^3 + 378\, x^2 + 216\, x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000}{13}\, x^{13} + 25\, x^{12} + \frac{4830}{11}\, x^{11} + \frac{3061}{10}\, x^{10} + \frac{3316}{3}\, x^9 + \frac{7869}{8}\, x^8 + \frac{12016}{7}\, x^7 + \frac{2873}{2}\, x^6 + \frac{8292}{5}\, x^5 + \frac{4483}{4}\, x^4 + 870\, x^3 + 378\, x^2 + 216\, x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx = \frac{1000\,x^{13}}{13}+25\,x^{12}+\frac{4830\,x^{11}}{11}+\frac{3061\,x^{10}}{10}+\frac{3316\,x^9}{3}+\frac{7869\,x^8}{8}+\frac{12016\,x^7}{7}+\frac{2873\,x^6}{2}+\frac{8292\,x^5}{5}+\frac{4483\,x^4}{4}+870\,x^3+378\,x^2+216\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3,x)
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx = \frac{x(9240000x^{12}+3003000x^{11}+52743600x^{10}+36768732x^9+132772640x^8+118153035x^7+206194560}{120120}$$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x)
```

output
```
(x*(9240000*x**12 + 3003000*x**11 + 52743600*x**10 + 36768732*x**9 + 13277
2640*x**8 + 118153035*x**7 + 206194560*x**6 + 172552380*x**5 + 199207008*x
**4 + 134624490*x**3 + 104504400*x**2 + 45405360*x + 25945920))/120120
```

3.61 $\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

output
```
108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+9
22/9*x^9-6*x^10+200/11*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

output
```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right)^2 \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(200x^{10}-60x^9+922x^8+83x^7+1571x^6+598x^5+1416x^4+635x^3+711x^2+216x+108\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

output

```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

method	result
orering	$\frac{x(504000x^{10}-166320x^{9}+2839760x^{8}+287595x^{7}+6221160x^{6}+2762760x^{5}+7850304x^{4}+4400550x^{3}+6569640x^{2}+2993760x+}{27720}$
gosper	$108x+108x^{2}+237x^{3}+\frac{635}{4}x^{4}+\frac{1416}{5}x^{5}+\frac{299}{3}x^{6}+\frac{1571}{7}x^{7}+\frac{83}{8}x^{8}+\frac{922}{9}x^{9}-6x^{10}+\frac{200}{11}x^{11}$
default	$108x+108x^{2}+237x^{3}+\frac{635}{4}x^{4}+\frac{1416}{5}x^{5}+\frac{299}{3}x^{6}+\frac{1571}{7}x^{7}+\frac{83}{8}x^{8}+\frac{922}{9}x^{9}-6x^{10}+\frac{200}{11}x^{11}$
norman	$108x+108x^{2}+237x^{3}+\frac{635}{4}x^{4}+\frac{1416}{5}x^{5}+\frac{299}{3}x^{6}+\frac{1571}{7}x^{7}+\frac{83}{8}x^{8}+\frac{922}{9}x^{9}-6x^{10}+\frac{200}{11}x^{11}$
risch	$108x+108x^{2}+237x^{3}+\frac{635}{4}x^{4}+\frac{1416}{5}x^{5}+\frac{299}{3}x^{6}+\frac{1571}{7}x^{7}+\frac{83}{8}x^{8}+\frac{922}{9}x^{9}-6x^{10}+\frac{200}{11}x^{11}$
parallelrisch	$108x+108x^{2}+237x^{3}+\frac{635}{4}x^{4}+\frac{1416}{5}x^{5}+\frac{299}{3}x^{6}+\frac{1571}{7}x^{7}+\frac{83}{8}x^{8}+\frac{922}{9}x^{9}-6x^{10}+\frac{200}{11}x^{11}$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/27720*x*(504000*x^10-166320*x^9+2839760*x^8+287595*x^7+6221160*x^6+27627
60*x^5+7850304*x^4+4400550*x^3+6569640*x^2+2993760*x+2993760)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2\,dx=\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

output
```
200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
 + 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^9 + \frac{83}{8}\,x^8 + \frac{1571}{7}\,x^7 + \frac{299}{3}\,x^6 + \frac{1416}{5}\,x^5 + \frac{635}{4}\,x^4 + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^9 + \frac{83}{8}\,x^8 + \frac{1571}{7}\,x^7 + \frac{299}{3}\,x^6 + \frac{1416}{5}\,x^5 + \frac{635}{4}\,x^4 + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200\,x^{11}}{11} - 6\,x^{10} + \frac{922\,x^9}{9} + \frac{83\,x^8}{8} + \frac{1571\,x^7}{7} + \frac{299\,x^6}{3} + \frac{1416\,x^5}{5} + \frac{635\,x^4}{4} + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)
```

output
```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx$$
$$= \frac{x(504000x^{10} - 166320x^9 + 2839760x^8 + 287595x^7 + 6221160x^6 + 2762760x^5 + 7850304x^4 + 4400550x}{27720}$$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x)
```

output
```
(x*(504000*x**10 - 166320*x**9 + 2839760*x**8 + 287595*x**7 + 6221160*x**6
 + 2762760*x**5 + 7850304*x**4 + 4400550*x**3 + 6569640*x**2 + 2993760*x +
 2993760))/27720
```

3.62 $\int (3-x+2x^2)^3 (2+3x+5x^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\,dx = 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}$$

output
```
54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\,dx = 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

output
```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(40x^8-36x^7+190x^6-83x^5+288x^4-20x^3+180x^2+27x+54\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

output
```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
orering	$\frac{x(2800x^8-2835x^7+17100x^6-8715x^5+36288x^4-3150x^3+37800x^2+8505x+34020)}{630}$	44
gosper	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
default	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
norman	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
risch	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
parallelrisch	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/630*x*(2800*x^8-2835*x^7+17100*x^6-8715*x^5+36288*x^4-3150*x^3+37800*x^2
+8505*x+34020)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx = \frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx = \frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

output
```
40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x
**3 + 27*x**2/2 + 54*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx = \frac{40}{9}\,x^9 - \frac{9}{2}\,x^8 + \frac{190}{7}\,x^7 - \frac{83}{6}\,x^6 + \frac{288}{5}\,x^5 - 5\,x^4 + 60\,x^3 + \frac{27}{2}\,x^2 + 54\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\ dx = \frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\ dx = \frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2),x)
```

output
```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \left(3 - x + 2x^2\right)^3 \left(2 + 3x + 5x^2\right)\,dx$$
$$= \frac{x(2800x^8 - 2835x^7 + 17100x^6 - 8715x^5 + 36288x^4 - 3150x^3 + 37800x^2 + 8505x + 34020)}{630}$$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2),x)
```

output
```
(x*(2800*x**8 - 2835*x**7 + 17100*x**6 - 8715*x**5 + 36288*x**4 - 3150*x**
3 + 37800*x**2 + 8505*x + 34020))/630
```

3.63 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} \, dx = \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} + \frac{328757 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}} - \frac{158389 \log(2+3x+5x^2)}{31250}$$

output
```
49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5+328757/484375*a
rctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-158389/31250*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} \, dx = \frac{1972542\sqrt{31} \arctan\left(\frac{3+10x}{\sqrt{31}}\right) + 31(5x(297048 - 111765x + 61100x^2 - 15750x^3 + 6000x^4) - 475167 \log}{2906250}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

output
```
(1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
 + 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/29062
50
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^3}{5x^2+3x+2} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{8x^4}{5}-\frac{84x^3}{25}+\frac{1222x^2}{125}-\frac{1331(119x+11)}{3125\,(5x^2+3x+2)}-\frac{7451x}{625}+\frac{49508}{3125}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{328757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}+\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

output
```
(49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/
25 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2
 + 3*x + 5*x^2])/31250
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{484375}-\frac{158389\ln(5x^2+3x+2)}{31250}$	54
risch	$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}-\frac{158389\ln(100x^2+60x+40)}{31250}+\frac{328757\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{484375}$	54

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5+328757/484375*a
rctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)-158389/31250*ln(5*x^2+3*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

output
```
8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 1
58389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/3
1 + 3*sqrt(31)/31)/484375
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{49508\,x}{3125} - \frac{158389\,\ln\left(5\,x^2+3\,x+2\right)}{31250} + \frac{328757\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{484375} - \frac{7451\,x^2}{1250} + \frac{1222\,x^3}{375} - \frac{21\,x^4}{25} + \frac{8\,x^5}{25}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)
```

output
```
(49508*x)/3125 - (158389*log(3*x + 5*x^2 + 2))/31250 + (328757*31^(1/2)*at
an((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/484375 - (7451*x^2)/1250 + (1222
*x^3)/375 - (21*x^4)/25 + (8*x^5)/25
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{328757\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)}{484375} - \frac{158389\log(5x^2+3x+2)}{31250} + \frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125}$$

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

output
```
(1972542*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 14730177*log(5*x**2 + 3*x +
2) + 930000*x**5 - 2441250*x**4 + 9470500*x**3 - 17323575*x**2 + 46042440*
x)/2906250
```

3.64 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx = \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{3819607\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}} - \frac{10769\log(2+3x+5x^2)}{6250}$$

output
```
1466/625*x-54/125*x^2+8/75*x^3+1331*(443+247*x)/(484375*x^2+290625*x+19375
0)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-10769/6250*ln(5
*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx = \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{3819607\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}} - \frac{10769\log(2+3x+5x^2)}{6250}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2
+ 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) -
 (10769*Log[2 + 3*x + 5*x^2])/6250
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^3}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{31}\int \frac{31000x^4-65100x^3+189410x^2-230981x+372701}{625(5x^2+3x+2)}dx+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{31000x^4-65100x^3+189410x^2-230981x+372701}{5x^2+3x+2}dx}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\int \left(6200x^2-16740x+\frac{121(2329-2759x)}{5x^2+3x+2}+45446\right)dx}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3819607\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}+\frac{6200x^3}{3}-8370x^2-\frac{333839}{10}\log(5x^2+3x+2)+45446x}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(1331*(443 + 247*x))/(96875*(2 + 3*x + 5*x^2)) + (45446*x - 8370*x^2 + (62
00*x^3)/3 + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(5*Sqrt[31]) - (333839*L
og[2 + 3*x + 5*x^2])/10)/19375
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{\frac{328757x}{484375}+\frac{589633}{484375}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{10769\ln(100x^2+60x+40)}{6250}+\frac{3819607\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	60
default	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}-\frac{121\left(-\frac{2717x}{775}-\frac{4873}{775}\right)}{625\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{10769\ln(5x^2+3x+2)}{6250}+\frac{3819607\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	61

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
8/75*x^3-54/125*x^2+1466/625*x+(328757/484375*x+589633/484375)/(x^2+3/5*x+
2/5)-10769/6250*ln(100*x^2+60*x+40)+3819607/3003125*arctan(1/31*(10*x+3)*3
1^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{9610000\,x^5-33154500\,x^4+191815600\,x^3+22917642\,\sqrt{31}(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+}{18018750\,(5\,x^2+3}$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)
*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 3104
7027*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*
x^2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x+589633}{484375x^2+290625x+193750} - \frac{10769\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{3003125}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

output
```
8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 +
290625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)
*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8}{75}\,x^3 - \frac{54}{125}\,x^2 + \frac{3819607}{3003125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{1466}{625}\,x + \frac{1331\,(247\,x+443)}{96875\,(5\,x^2+3\,x+2)} - \frac{10769}{6250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*
x + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/
6250*log(5*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8}{75}\,x^3 - \frac{54}{125}\,x^2 + \frac{3819607}{3003125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{1466}{625}\,x + \frac{1331\,(247\,x+443)}{96875\,(5\,x^2+3\,x+2)} - \frac{10769}{6250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*
x + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/
6250*log(5*x^2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{1466\,x}{625} - \frac{10769\,\ln\left(5\,x^2+3\,x+2\right)}{6250} + \frac{\frac{328757\,x}{484375}+\frac{589633}{484375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{3819607\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{3003125} - \frac{54\,x^2}{125} + \frac{8\,x^3}{75}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)
```

output
```
(1466*x)/625 - (10769*log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 58
9633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^(1/2)*atan((10*31^(1/2)*x
)/31 + (3*31^(1/2))/31))/3003125 - (54*x^2)/125 + (8*x^3)/75
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{114588210\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+68752926\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x+45835284\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)-1552351}{}$$

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

output
```
(114588210*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 68752926*sqrt(31)*ata
n((10*x + 3)/sqrt(31))*x + 45835284*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 1
55235135*log(5*x**2 + 3*x + 2)*x**2 - 93141081*log(5*x**2 + 3*x + 2)*x - 6
2094054*log(5*x**2 + 3*x + 2) + 9610000*x**5 - 33154500*x**4 + 191815600*x
**3 - 131571130*x**2 + 12552830)/(18018750*(5*x**2 + 3*x + 2))
```

3.65 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx=\frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{11341176\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}}-\frac{66}{625}\log\left(2+3x+5x^2\right)$$

output
```
8/125*x+1331/193750*(443+247*x)/(5*x^2+3*x+2)^2+121*(188381+342840*x)/(300
31250*x^2+18018750*x+12012500)+11341176/18619375*arctan(1/31*(3+10*x)*31^(
1/2))*31^(1/2)-66/625*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$$
$$=\frac{11916400x+\frac{1279091(443+247x)}{(2+3x+5x^2)^2}+\frac{3751(188381+342840x)}{2+3x+5x^2}+113411760\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)-19662060\log(2+}{186193750}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381
+ 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt
[31]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^3}{(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{62}\int \frac{310000x^4-651000x^3+1894100x^2-2309810x+4055767}{3125(5x^2+3x+2)^2}dx+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{310000x^4-651000x^3+1894100x^2-2309810x+4055767}{(5x^2+3x+2)^2}dx}{193750}+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{31}\int \frac{100(19220x^2-51894x+555719)}{5x^2+3x+2}dx+\frac{121(342840x+188381)}{31(5x^2+3x+2)}}{193750}+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{100}{31}\int \frac{19220x^2-51894x+555719}{5x^2+3x+2}dx+\frac{121(342840x+188381)}{31(5x^2+3x+2)}}{193750}+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{100}{31}\int \left(\frac{33(16607-1922x)}{5x^2+3x+2}+3844\right)dx+\frac{121(342840x+188381)}{31(5x^2+3x+2)}}{193750}+\frac{1331(247x+443)}{193750\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{100}{31}\left(\frac{5670588\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}-\frac{31713}{5}\log\left(5x^2+3x+2\right)+3844x\right)+\frac{121(342840x+188381)}{31(5x^2+3x+2)}}{193750}+\frac{1331(247x+443)}{193750\,(5x^2+3x+2)^2}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + ((121*(188381 + 342840
*x))/(31*(2 + 3*x + 5*x^2)) + (100*(3844*x + (5670588*ArcTan[(3 + 10*x)/Sq
rt[31]])/(5*Sqrt[31]) - (31713*Log[2 + 3*x + 5*x^2])/5))/31)/193750
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{8x}{125}-\frac{11\left(-\frac{377124}{24025}x^{3}-\frac{866987}{48050}x^{2}-\frac{293711}{24025}x-\frac{232243}{48050}\right)}{5\left(5x^{2}+3x+2\right)^{2}}-\frac{66\ln\left(5x^{2}+3x+2\right)}{625}+\frac{11341176\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63
risch	$\frac{8x}{125}+\frac{\frac{4148364}{120125}x^{3}+\frac{9536857}{240250}x^{2}+\frac{3230821}{120125}x+\frac{2554673}{240250}}{\left(5x^{2}+3x+2\right)^{2}}-\frac{66\ln\left(100x^{2}+60x+40\right)}{625}+\frac{11341176\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/480
50)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(
10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{\left(3-x+2x^{2}\right)^{3}}{\left(2+3x+5x^{2}\right)^{3}}\,dx$$
$$=\frac{59582000\,x^{5}+71498400\,x^{4}+1355107960\,x^{3}+22682352\sqrt{31}(25\,x^{4}+30\,x^{3}+29\,x^{2}+12\,x+4)\arctan\left(\right.}{3723875}$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(3
1)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3))
+ 1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2
 + 3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x +
 4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{8x}{125} + \frac{8296728x^3+9536857x^2+6461642x+2554673}{6006250x^4+7207500x^3+6967250x^2+2883000x+961000} - \frac{66\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{18619375}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

output
```
8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**
4 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/
5 + 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18
619375
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121\,(68568\,x^3+78817\,x^2+53402\,x+21113)}{240250\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)} - \frac{66}{625}\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 12
1/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x
^2 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{8}{125}x + \frac{121\,(68568\,x^3+78817\,x^2+53402\,x+21113)}{240250\,(5\,x^2+3\,x+2)^2} - \frac{66}{625}\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 12
1/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 6
6/625*log(5*x^2 + 3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{8\,x}{125}-\frac{66\,\ln\left(5\,x^2+3\,x+2\right)}{625} + \frac{11341176\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{18619375} + \frac{\frac{4148364\,x^3}{3003125}+\frac{9536857\,x^2}{6006250}+\frac{3230821\,x}{3003125}+\frac{2554673}{6006250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(8*x)/125 - (66*log(3*x + 5*x^2 + 2))/625 + (11341176*31^(1/2)*atan((10*31
^(1/2)*x)/31 + (3*31^(1/2))/31))/18619375 + ((3230821*x)/3003125 + (953685
7*x^2)/6006250 + (4148364*x^3)/3003125 + 2554673/6006250)/((12*x)/25 + (29
*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.42

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$$
$$= \frac{1701176400\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4 + 2041411680\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3 + 1973364624\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2 -}{}$$

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

output
```
(1701176400*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 2041411680*sqrt(31)*
atan((10*x + 3)/sqrt(31))*x**3 + 1973364624*sqrt(31)*atan((10*x + 3)/sqrt(
31))*x**2 + 816564672*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 272188224*sqr
t(31)*atan((10*x + 3)/sqrt(31)) - 294930900*log(5*x**2 + 3*x + 2)*x**4 - 3
53917080*log(5*x**2 + 3*x + 2)*x**3 - 342119844*log(5*x**2 + 3*x + 2)*x**2
 - 141566832*log(5*x**2 + 3*x + 2)*x - 47188944*log(5*x**2 + 3*x + 2) + 17
8746000*x**5 - 3173274700*x**4 + 590623501*x**2 + 1407133338*x + 645879761
)/(111716250*(25*x**4 + 30*x**3 + 29*x**2 + 12*x + 4))
```

3.66 $\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx = \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}} + \frac{307461}{512} \log\left(3-x+2x^2\right)$$

output
```
122691/128*x-28747/128*x^2-21229/96*x^3+6245/64*x^4+1855/8*x^5+3625/24*x^6
+625/14*x^7+1156639/5888*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+307461/512
*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx = \frac{x(2576511-603687x-594412x^2+262290x^3+623280x^4+406000x^5+120000x^6)}{2688} - \frac{1156639 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{256\sqrt{23}} + \frac{307461}{512} \log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

output
```
(x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
 + 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23]
) + (307461*Log[3 - x + 2*x^2])/512
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{2x^2-x+3}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(\frac{625x^6}{2}+\frac{3625x^5}{4}+\frac{9275x^4}{8}+\frac{6245x^3}{16}-\frac{21229x^2}{32}-\frac{14641(25-21x)}{128(2x^2-x+3)}-\frac{28747x}{64}+\frac{122691}{128}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1156639\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

output
```
(122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*
x^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]
])/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

method	result
default	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(2x^2-x+3)}{512}-\frac{1156639\sqrt{23}\ \mathrm{arcta}}{5888}$
risch	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(16x^2-8x+24)}{512}-\frac{1156639\sqrt{23}\ \mathrm{a}}{5}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+1
22691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(4*
x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}\,x^7+\frac{3625}{24}\,x^6+\frac{1855}{8}\,x^5+\frac{6245}{64}\,x^4-\frac{21229}{96}\,x^3 - \frac{28747}{128}\,x^2-\frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{122691}{128}\,x+\frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{512} - \frac{1156639\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{5888}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

output
```
625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 -
28747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639
*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512}\log\left(2x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}\,x^7 + \frac{3625}{24}\,x^6 + \frac{1855}{8}\,x^5 + \frac{6245}{64}\,x^4 - \frac{21229}{96}\,x^3 - \frac{28747}{128}\,x^2 - \frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{122691}{128}\,x + \frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{122691\,x}{128} + \frac{307461\,\ln\left(2\,x^2-x+3\right)}{512} - \frac{1156639\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888} - \frac{28747\,x^2}{128} - \frac{21229\,x^3}{96} + \frac{6245\,x^4}{64} + \frac{1855\,x^5}{8} + \frac{3625\,x^6}{24} + \frac{625\,x^7}{14}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3),x)
```

output
```
(122691*x)/128 + (307461*log(2*x^2 - x + 3))/512 - (1156639*23^(1/2)*atan(
(4*23^(1/2)*x)/23 - 23^(1/2)/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96
+ (6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = -\frac{1156639\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{5888} + \frac{307461\log(2x^2-x+3)}{512} + \frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3),x)
```

output
```
( - 48578838*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 148503663*log(2*x**2 - x
+ 3) + 11040000*x**7 + 37352000*x**6 + 57341760*x**5 + 24130680*x**4 - 546
85904*x**3 - 55539204*x**2 + 237039012*x)/247296
```

3.67 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2} - \frac{59895\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log\left(3-x+2x^2\right)$$

output
```
-4795/32*x-829/32*x^2+965/24*x^3+575/16*x^4+25/2*x^5-59895/1472*arctan(1/2
3*(1-4*x)*23^(1/2))*23^(1/2)+1331/128*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{59895\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{64\sqrt{23}} + \frac{1}{384}\left(4x\left(-14385-2487x+3860x^2+3450x^3+1200x^4\right) + 3993\log\left(3-x+2x^2\right)\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

output
```
(59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x
+ 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{2x^2-x+3}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(\frac{125x^4}{2}+\frac{575x^3}{4}+\frac{965x^2}{8}+\frac{1331(x+11)}{32(2x^2-x+3)}-\frac{829x}{16}-\frac{4795}{32}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{59895\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

output
```
(-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (
59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2]
)/128
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln\left(2x^2-x+3\right)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472}$	54
risch	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln\left(16x^2-8x+24\right)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472}$	54

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)
+59895/1472*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}\,x^5+\frac{575}{16}\,x^4+\frac{965}{24}\,x^3-\frac{829}{32}\,x^2 + \frac{59895}{1472}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

output
```
25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log
(x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/1472
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}\,x^5+\frac{575}{16}\,x^4+\frac{965}{24}\,x^3-\frac{829}{32}\,x^2 + \frac{59895}{1472}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{1331\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{4795\,x}{32} + \frac{59895\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{1472} - \frac{829\,x^2}{32}+\frac{965\,x^3}{24}+\frac{575\,x^4}{16}+\frac{25\,x^5}{2}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3),x)
```

output
```
(1331*log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^(1/2)*atan((4*23^(
1/2)*x)/23 - 23^(1/2)/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/
16 + (25*x^5)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{59895\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{1472} + \frac{1331\log(2x^2-x+3)}{128} + \frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3),x)
```

output
```
(359370*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 91839*log(2*x**2 - x + 3) + 11
0400*x**5 + 317400*x**4 + 355120*x**3 - 228804*x**2 - 1323420*x)/8832
```

3.68 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx = \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} + \frac{847 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}} - \frac{363}{32} \log\left(3-x+2x^2\right)$$

output
```
51/8*x+85/8*x^2+25/6*x^3+847/368*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)-36
3/32*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx = \frac{1}{24} x\left(153 + 255x + 100x^2\right) - \frac{847 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{16\sqrt{23}} - \frac{363}{32} \log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

output
```
(x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqr
t[23]) - (363*Log[3 - x + 2*x^2])/32
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^2}{2x^2-x+3}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(\frac{25x^2}{2}-\frac{121(3x+1)}{8\left(2x^2-x+3\right)}+\frac{85x}{4}+\frac{51}{8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{847\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}+\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

output
```
(51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*
Sqrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(2x^2-x+3)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}$	44
risch	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(16x^2-8x+24)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}$	44

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23
*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32} - \frac{847\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{368}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

output
```
25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(2
3)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3 + \frac{85}{8}\,x^2 - \frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x - \frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3 + \frac{85}{8}\,x^2 - \frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x - \frac{363}{32}\,\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{51\,x}{8} - \frac{363\,\ln\left(2\,x^2-x+3\right)}{32} - \frac{847\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{368} + \frac{85\,x^2}{8} + \frac{25\,x^3}{6}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3),x)
```

output
```
(51*x)/8 - (363*log(2*x^2 - x + 3))/32 - (847*23^(1/2)*atan((4*23^(1/2)*x)
/23 - 23^(1/2)/23))/368 + (85*x^2)/8 + (25*x^3)/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = -\frac{847\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{368} - \frac{363\log(2x^2-x+3)}{32} + \frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3),x)
```

output
```
( - 5082*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 25047*log(2*x**2 - x + 3) + 9
200*x**3 + 23460*x**2 + 14076*x)/2208
```

3.69 $\int \frac{2+3x+5x^2}{3-x+2x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{2+3x+5x^2}{3-x+2x^2} \, dx = \frac{5x}{2} + \frac{33 \arctan \left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}} + \frac{11}{8} \log \left(3-x+2x^2\right)$$

output
```
5/2*x+33/92*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+11/8*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{2+3x+5x^2}{3-x+2x^2} \, dx = \frac{5x}{2} - \frac{33 \arctan \left(\frac{-1+4x}{\sqrt{23}}\right)}{4\sqrt{23}} + \frac{11}{8} \log \left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

output
```
(5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 
2*x^2])/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{2x^2-x+3}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(2x^2-x+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{33\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

output
```
(5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{5x}{2}+\frac{11\ln(2x^2-x+3)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}$	34
risch	$\frac{5x}{2}+\frac{11\ln(16x^2-8x+24)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}$	34

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = \frac{5x}{2}+\frac{11\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

output
```
5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/92
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx=\frac{5\,x}{2}+\frac{11\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{33\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3),x)
```

output
```
(5*x)/2 + (11*log(2*x^2 - x + 3))/8 - (33*23^(1/2)*atan((4*23^(1/2)*x)/23
- 23^(1/2)/23))/92
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx=-\frac{33\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{92}+\frac{11\log(2x^2-x+3)}{8}+\frac{5x}{2}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3),x)
```

output
```
( - 66*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 253*log(2*x**2 - x + 3) + 460*x
)/184
```

3.70 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{3\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right)$$

output
```
3/506*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+13/682*arctan(1/31*(3+10*x)*3
1^(1/2))*31^(1/2)-1/44*ln(2*x^2-x+3)+1/44*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = -\frac{3\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqr
t[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1311, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{1311}$$

$$\frac{1}{242}\int -\frac{11(2x+1)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(5x+8)}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{22}\int \frac{5x+8}{5x^2+3x+2}dx-\frac{1}{22}\int \frac{2x+1}{2x^2-x+3}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{22}\left(-\frac{3}{2}\int \frac{1}{2x^2-x+3}dx-\frac{1}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{13}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{3}{2}\int \frac{1}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{13}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx+3\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{1}{22}\left(\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx-13\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{3\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{1}{22}\left(\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx+\frac{13\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{22}\left(-\frac{3\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{1}{2}\log\left(2x^2-x+3\right)\right)+\frac{1}{22}\left(\frac{13\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{1}{2}\log\left(5x^2+3x+2\right)\right)$$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

output
```
((-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - Log[3 - x + 2*x^2]/2)/22 + ((
13*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + Log[2 + 3*x + 5*x^2]/2)/22
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1311
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Simp[1/q   Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^
2*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*e^2 - c*d*f -
 b*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f,
 0]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\ln\left(2x^{2}-x+3\right)}{44}-\frac{3\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}+\frac{\ln\left(5x^{2}+3x+2\right)}{44}+\frac{13\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{682}$	60
risch	$-\frac{3\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}-\frac{\ln\left(16x^{2}-8x+24\right)}{44}+\frac{\ln\left(100x^{2}+60x+40\right)}{44}+\frac{13\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{682}$	60

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/44*ln(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+1/44*ln(5
*x^2+3*x+2)+13/682*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{1}{44}\log\left(5\,x^2+3\,x+2\right) - \frac{1}{44}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44} + \frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44} - \frac{3\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{506} + \frac{13\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

output
```
-log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4
*sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*
sqrt(31)/31)/682
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{1}{44}\log\left(5x^2+3x+2\right) - \frac{1}{44}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{1}{44}\log\left(5x^2+3x+2\right) - \frac{1}{44}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

Mupad [B] (verification not implemented)

Time = 16.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 - 1/44) - log(x + (23^(
1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 + 1/44) - log(x - (31^(1/2)*1i)/10 +
 3/10)*((31^(1/2)*13i)/1364 - 1/44) + log(x + (31^(1/2)*1i)/10 + 3/10)*((3
1^(1/2)*13i)/1364 + 1/44)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)}{682} - \frac{3\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{506} + \frac{\log(5x^2+3x+2)}{44} - \frac{\log(2x^2-x+3)}{44}$$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x)
```

output
```
(598*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 186*sqrt(23)*atan((4*x - 1)/sqrt
(23)) + 713*log(5*x**2 + 3*x + 2) - 713*log(2*x**2 - x + 3))/31372
```

3.71 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(2+3x+5x^2)} + \frac{7\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right)$$

output
```
(4+65*x)/(3410*x^2+2046*x+1364)+7/11132*arctan(1/23*(1-4*x)*23^(1/2))*23^(
1/2)+2891/465124*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+3/968*ln(2*x^2-x+
3)-3/968*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(2+3x+5x^2)} - \frac{7\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*
Sqrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3
 - x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1305, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{682(5x^2+3x+2)} - \frac{\int -\frac{11(130x^2-127x+164)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{682}\int \frac{130x^2-127x+164}{(2x^2-x+3)(5x^2+3x+2)}dx + \frac{65x+4}{682(5x^2+3x+2)}$$

$$\Big\downarrow \text{2141}$$

$$\frac{1}{682}\left(\frac{1}{242}\int -\frac{341(5-6x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(1306-465x)}{5x^2+3x+2}dx\right)+\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{682}\left(\frac{1}{22}\int \frac{1306-465x}{5x^2+3x+2}dx-\frac{31}{22}\int \frac{5-6x}{2x^2-x+3}dx\right)+\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{93}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{7}{2}\int \frac{1}{2x^2-x+3}dx-\frac{3}{2}\int -\frac{1-4x}{2x^2-x+3}d\right.\right.$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{93}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{7}{2}\int \frac{1}{2x^2-x+3}dx+\frac{3}{2}\int \frac{1-4x}{2x^2-x+3}dx\right)\right.$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(-\frac{93}{2}\int \frac{10x+3}{5x^2+3x+2}dx-2891\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{3}{2}\int \frac{1-4x}{2x^2-x+3}dx-7\int -\right.\right.$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{3}{2}\int \frac{1-4x}{2x^2-x+3}dx+\frac{7\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)\right)+$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{7\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{3}{2}\log\left(2x^2-x+3\right)\right)\right)+$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + ((-31*((7*ArcTan[(-1 + 4*x)/Sqrt[23]]
)/Sqrt[23] - (3*Log[3 - x + 2*x^2])/2))/22 + ((2891*ArcTan[(3 + 10*x)/Sqrt
[31]])/Sqrt[31] - (93*Log[2 + 3*x + 5*x^2])/2)/22)/682
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\frac{13x}{682}+\frac{2}{1705}}{x^{2}+\frac{3}{5}x+\frac{2}{5}}+\frac{3\ln\left(16x^{2}-8x+24\right)}{968}-\frac{7\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132}-\frac{3\ln\left(100x^{2}+60x+40\right)}{968}+\frac{2891\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{465124}$
default	$\frac{3\ln\left(2x^{2}-x+3\right)}{968}-\frac{7\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132}-\frac{-\frac{286x}{31}-\frac{88}{155}}{484\left(x^{2}+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{3\ln\left(5x^{2}+3x+2\right)}{968}+\frac{2891\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{465124}$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(13/682*x+2/1705)/(x^2+3/5*x+2/5)+3/968*ln(16*x^2-8*x+24)-7/11132*23^(1/2)
*arctan(1/23*(4*x-1)*23^(1/2))-3/968*ln(100*x^2+60*x+40)+2891/465124*arcta
n(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$$
$$= \frac{132986\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - 13454\sqrt{23}(5x^2+3x+2)\arctan\left(\frac{1}{23}\sqrt{23}(4x\right.}{213957}$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
 3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) -
66309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log
(2*x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{65x+4}{3410x^2+2046x+1364} + \frac{3\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968} - \frac{3\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968} - \frac{7\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{11132} + \frac{2891\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{465124}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

output
```
(65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*l
og(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{65\,x+4}{682\,(5\,x^2+3\,x+2)} - \frac{3}{968}\log\left(5\,x^2+3\,x+2\right) + \frac{3}{968}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/96
8*log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{65\,x+4}{682\,(5\,x^2+3\,x+2)} - \frac{3}{968}\log\left(5\,x^2+3\,x+2\right) + \frac{3}{968}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/96
8*log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{\frac{13\,x}{682}+\frac{2}{1705}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
((13*x)/682 + 2/1705)/((3*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)*1i)/4 - 1/
4)*((23^(1/2)*7i)/22264 + 3/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/
2)*7i)/22264 - 3/968) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)
/930248 + 3/968) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/9302
48 - 3/968)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.17

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$$
$$= \frac{1994790\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+1196874\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x+797916\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)-201810\sqrt{23}\,at}{}$$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

output
```
(1994790*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 1196874*sqrt(31)*atan((
10*x + 3)/sqrt(31))*x + 797916*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 201810
*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 121086*sqrt(23)*atan((4*x - 1)/s
qrt(23))*x - 80724*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 994635*log(5*x**2 +
 3*x + 2)*x**2 - 596781*log(5*x**2 + 3*x + 2)*x - 397854*log(5*x**2 + 3*x
+ 2) + 994635*log(2*x**2 - x + 3)*x**2 + 596781*log(2*x**2 - x + 3)*x + 39
7854*log(2*x**2 - x + 3) - 10195900*x**2 - 3701896)/(64187112*(5*x**2 + 3*
x + 2))
```

3.72 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx = \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10232728\sqrt{31}} - \frac{\log(3-x+2x^2)}{21296} + \frac{\log(2+3x+5x^2)}{21296}$$

output
```
1/1364*(4+65*x)/(5*x^2+3*x+2)^2+(7923+21605*x)/(2325620*x^2+1395372*x+9302
48)-45/244904*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+847793/317214568*arct
an(1/31*(3+10*x)*31^(1/2))*31^(1/2)-1/21296*ln(2*x^2-x+3)+1/21296*ln(5*x^2
+3*x+2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{45\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{1695586\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)+31\left(\frac{44(17210+89144x+104430x^2+108025x^3)}{(2+3x+5x^2)^2}-961\log(3-x+2x^2)+961\log(\ldots\right.}{634429136}$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcT
an[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x
^3))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^
2]))/634429136
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{65x+4}{1364(5x^2+3x+2)^2}-\frac{\int -\frac{11(390x^2-319x+523)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{15004}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{390x^2-319x+523}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{1364}+\frac{65x+4}{1364(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{22\left(43210x^2-15839x+60010\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx}{7502}+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{341}\int \frac{43210x^2-15839x+60010}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2141}$$

$$\frac{\frac{1}{341}\left(\frac{1}{242}\int \frac{10571(23-2x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(4805x+425338)}{5x^2+3x+2}dx\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\int \frac{23-2x}{2x^2-x+3}dx+\frac{1}{22}\int \frac{4805x+425338}{5x^2+3x+2}dx\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45}{2}\int \frac{1}{2x^2-x+3}dx-\frac{1}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{847793}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{961}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45}{2}\int \frac{1}{2x^2-x+3}dx+\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{847793}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{961}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx-45\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{1}{22}\left(\frac{961}{2}\int \frac{10x+3}{5x^2+3x+2}dx-847793\int \frac{1}{-(10x+3)^2-31}d(10x+\right.\right.}{1364}$$
$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx+\frac{45\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{1}{22}\left(\frac{961}{2}\int \frac{10x+3}{5x^2+3x+2}dx+\frac{847793\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$\big\downarrow$ 1103

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{1}{2}\log\left(2x^2-x+3\right)\right)+\frac{1}{22}\left(\frac{847793\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{961}{2}\log\left(5x^2+3x+2\right)\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + ((7923 + 21605*x)/(341*(2 + 3*x +
5*x^2)) + ((961*((45*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - Log[3 - x + 2
*x^2]/2))/22 + ((847793*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + (961*Log[2
 + 3*x + 5*x^2])/2)/22)/341)/1364
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\ln(2x^2-x+3)}{21296}+\frac{45\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{244904}+\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{\ln(5x^2+3x+2)}{21296}+\frac{847793\arctan}{317}$
risch	$\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{\ln(100x^2+60x+40)}{21296}+\frac{847793\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{317214568}-\frac{\ln(16x^2-8x+24)}{21296}+\frac{45\sqrt{}}{}$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+25
/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+3
*x+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*arctan(1/31*(10*x+3)*31^(
1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{3388960300\,x^3+38998478\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+2681190}{}$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^
2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960
*x^2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) -
 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625
568*x + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{108025x^3+104430x^2+89144x+17210}{11628100x^4+13953720x^3+13488596x^2+5581488x+1860496} - \frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296} + \frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296} + \frac{45\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{244904} + \frac{847793\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{317214568}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

output
```
(108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x*
*3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 +
log(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23
)/23)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3172
14568
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{108025x^3+104430x^2+89144x+17210}{465124(25x^4+30x^3+29x^2+12x+4)} + \frac{1}{21296}\log\left(5x^2+3x+2\right) - \frac{1}{21296}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2
+ 89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x
^2 + 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{108025\,x^3+104430\,x^2+89144\,x+17210}{465124\,(5\,x^2+3\,x+2)^2} + \frac{1}{21296}\log\left(5\,x^2+3\,x+2\right) - \frac{1}{21296}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2
+ 89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/
21296*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{\frac{4321\,x^3}{465124}+\frac{10443\,x^2}{1162810}+\frac{2026\,x}{264275}+\frac{1721}{1162810}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}} + \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 - 1/21296) - log(x -
 (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 + 1/21296) - log(x - (31^(1
/2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 - 1/21296) + log(x + (31^
(1/2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 + 1/21296) + ((2026*x)/
264275 + (10443*x^2)/1162810 + (4321*x^3)/465124 + 1721/1162810)/((12*x)/2
5 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.11

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{2924885850\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4+3509863020\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3+3392867586\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2 -}{}$$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

output
```
(2924885850*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 3509863020*sqrt(31)*
atan((10*x + 3)/sqrt(31))*x**3 + 3392867586*sqrt(31)*atan((10*x + 3)/sqrt(
31))*x**2 + 1403945208*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 467981736*sq
rt(31)*atan((10*x + 3)/sqrt(31)) + 201089250*sqrt(23)*atan((4*x - 1)/sqrt(
23))*x**4 + 241307100*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**3 + 233263530*s
qrt(23)*atan((4*x - 1)/sqrt(23))*x**2 + 96522840*sqrt(23)*atan((4*x - 1)/s
qrt(23))*x + 32174280*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 51389475*log(5*x
**2 + 3*x + 2)*x**4 + 61667370*log(5*x**2 + 3*x + 2)*x**3 + 59611791*log(5
*x**2 + 3*x + 2)*x**2 + 24666948*log(5*x**2 + 3*x + 2)*x + 8222316*log(5*x
**2 + 3*x + 2) - 51389475*log(2*x**2 - x + 3)*x**4 - 61667370*log(2*x**2 -
 x + 3)*x**3 - 59611791*log(2*x**2 - x + 3)*x**2 - 24666948*log(2*x**2 - x
 + 3)*x - 8222316*log(2*x**2 - x + 3) - 8472400750*x**4 + 549010*x**2 + 43
23124344*x + 264152240)/(43775610384*(25*x**4 + 30*x**3 + 29*x**2 + 12*x +
 4))
```

3.73 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} \, dx = -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{13292697 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}} - \frac{30613}{128}\log\left(3-x+2x^2\right)$$

output
```
-89359/64*x-1185/8*x^2+9775/48*x^3+2125/16*x^4+125/4*x^5-14641*(101+79*x)/
(5888*x^2-2944*x+8832)-13292697/33856*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/
2)-30613/128*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\,(3-x+2x^2)}+\frac{13292697\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{1472\sqrt{23}}-\frac{30613}{128}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

output
```
(-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
 - (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*
x)/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^2}\,dx$$

$$\Big\downarrow \; 2191$$

$$\frac{1}{23}\int \frac{460000x^6+1334000x^5+1706600x^4+574540x^3-976534x^2-661181x+832627}{64\,(2x^2-x+3)}dx-\frac{14641(79x+101)}{2944\,(2x^2-x+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{460000x^6+1334000x^5+1706600x^4+574540x^3-976534x^2-661181x+832627}{2x^2-x+3}dx}{1472} - \frac{14641(79x+101)}{2944\,(2x^2-x+3)}$$

$$\Big\downarrow \; 2188$$

$$\frac{\int \left(230000x^4+782000x^3+899300x^2-436080x+\frac{2662(2629-529x)}{2x^2-x+3}-2055257\right)dx}{1472} - \frac{14641(79x+101)}{2944\,(2x^2-x+3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{13292697\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+46000x^5+195500x^4+\frac{899300x^3}{3}-218040x^2-\frac{704099}{2}\log\left(2x^2-x+3\right)-2055257x}{1472} - \frac{14641(79x+101)}{2944\,(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

output
```
(-14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (-2055257*x - 218040*x^2 +
(899300*x^3)/3 + 195500*x^4 + 46000*x^5 - (13292697*ArcTan[(1 - 4*x)/Sqrt[
23]])/Sqrt[23] - (704099*Log[3 - x + 2*x^2])/2)/1472
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
risch	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-\frac{1156639x}{5888}-\frac{1478741}{5888}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{30613\ln(16x^2-8x+24)}{128}+\frac{13292697\sqrt{23}\,\mathrm{arct}}{338}$
default	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}-\frac{1331\left(\frac{869x}{92}+\frac{1111}{92}\right)}{64\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{30613\ln(2x^2-x+3)}{128}+\frac{13292697\sqrt{23}\,\arctan\left(\right.}{33856}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x+(-1156639/5888*x-1
478741/5888)/(x^2-1/2*x+3/2)-30613/128*ln(16*x^2-8*x+24)+13292697/33856*23
^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx$$
$$=\frac{12696000\,x^7+47610000\,x^6+74800600\,x^5-20609840\,x^4-413058012\,x^3+79756182\,\sqrt{23}(2\,x^2-x+3}{2031}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 4130
58012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x -
1)) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684
489*x - 102033129)/(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{-1156639x - 1478741}{5888x^2 - 2944x + 8832} - \frac{30613\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{33856}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

output
```
125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 + (-11
56639*x - 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/
2)/128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125}{4}\,x^5 + \frac{2125}{16}\,x^4 + \frac{9775}{48}\,x^3 - \frac{1185}{8}\,x^2 + \frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{89359}{64}\,x - \frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)} - \frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(2
3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/
(2*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125}{4}\,x^5 + \frac{2125}{16}\,x^4 + \frac{9775}{48}\,x^3 - \frac{1185}{8}\,x^2 + \frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{89359}{64}\,x - \frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)} - \frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

output

```
125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(2
3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/
(2*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```


Mupad [B] (verification not implemented)

Time = 15.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{13292697\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856} - \frac{30613\,\ln\left(2\,x^2-x+3\right)}{128} - \frac{\frac{1156639\,x}{5888}+\frac{1478741}{5888}}{x^2-\frac{x}{2}+\frac{3}{2}} - \frac{89359\,x}{64} - \frac{1185\,x^2}{8} + \frac{9775\,x^3}{48} + \frac{2125\,x^4}{16} + \frac{125\,x^5}{4}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)
```

output
```
(13292697*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/33856 - (30613*l
og(2*x^2 - x + 3))/128 - ((1156639*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/
2) - (89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^
5)/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{159512364\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2-79756182\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x+239268546\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)-97165662}{}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x)
```

output
```
(159512364*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 79756182*sqrt(23)*atan
((4*x - 1)/sqrt(23))*x + 239268546*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 971
65662*log(2*x**2 - x + 3)*x**2 + 48582831*log(2*x**2 - x + 3)*x - 14574849
3*log(2*x**2 - x + 3) + 12696000*x**7 + 47610000*x**6 + 74800600*x**5 - 20
609840*x**4 - 413058012*x**3 - 1668012072*x**2 - 2894086596)/(203136*(2*x*
*2 - x + 3))
```

3.74 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\,(3-x+2x^2)}+\frac{223971\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

output
```
915/16*x+175/4*x^2+125/12*x^3-1331*(17-45*x)/(1472*x^2-736*x+2208)+223971/
8464*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)-2057/32*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}+\frac{1331(-17+45x)}{736\,(3-x+2x^2)}-\frac{223971\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

output
```
(915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x
+ 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Lo
g[3 - x + 2*x^2])/32
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^2} dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{23}\int -\frac{-23000x^4-52900x^3-44390x^2+19067x+25195}{16\left(2x^2-x+3\right)}dx-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{368}\int \frac{-23000x^4-52900x^3-44390x^2+19067x+25195}{2x^2-x+3}dx-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 2188}$$

$$-\frac{1}{368}\int \left(-11500x^2-32200x+\frac{242(391x+365)}{2x^2-x+3}-21045\right)dx-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{368}\left(\frac{223971\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{11500x^3}{3}+16100x^2-\frac{47311}{2}\log\left(2x^2-x+3\right)+21045x\right)-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

output
```
(-1331*(17 - 45*x))/(736*(3 - x + 2*x^2)) + (21045*x + 16100*x^2 + (11500*
x^3)/3 + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/Sqrt[23] - (47311*Log[3 - x +
 2*x^2])/2)/368
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{\frac{59895x}{1472}-\frac{22627}{1472}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{2057\ln(16x^2-8x+24)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}$	60
default	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}-\frac{121\left(-\frac{495x}{92}+\frac{187}{92}\right)}{16\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{2057\ln(2x^2-x+3)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}$	61

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
125/12*x^3+175/4*x^2+915/16*x+(59895/1472*x-22627/1472)/(x^2-1/2*x+3/2)-20
57/32*ln(16*x^2-8*x+24)-223971/8464*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{1058000\,x^5+3914600\,x^4+5173620\,x^3-1343826\,\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+376119}{50784\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 -
 x + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

output
```
125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
 + 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)
*x/23 - sqrt(23)/23)/8464
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125}{12}\,x^3 + \frac{175}{4}\,x^2 - \frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{915}{16}\,x + \frac{1331\,(45\,x-17)}{736\,(2\,x^2-x+3)} - \frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x -
1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2
- x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125}{12}\,x^3 + \frac{175}{4}\,x^2 - \frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{915}{16}\,x + \frac{1331\,(45\,x-17)}{736\,(2\,x^2-x+3)} - \frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x -
1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2
- x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{915\,x}{16} - \frac{2057\,\ln\left(2\,x^2-x+3\right)}{32} + \frac{\frac{59895\,x}{1472}-\frac{22627}{1472}}{x^2-\frac{x}{2}+\frac{3}{2}} - \frac{223971\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464} + \frac{175\,x^2}{4} + \frac{125\,x^3}{12}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)
```

output
```
(915*x)/16 - (2057*log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/
(x^2 - x/2 + 3/2) - (223971*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23)
)/8464 + (175*x^2)/4 + (125*x^3)/12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$$
$$= \frac{-2687652\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2+1343826\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x-4031478\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)-6528918\log(2}{}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)
```

output
```
( - 2687652*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 + 1343826*sqrt(23)*atan
((4*x - 1)/sqrt(23))*x - 4031478*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 65289
18*log(2*x**2 - x + 3)*x**2 + 3264459*log(2*x**2 - x + 3)*x - 9793377*log(
2*x**2 - x + 3) + 1058000*x**5 + 3914600*x**4 + 5173620*x**3 + 29451960*x*
*2 + 36974892)/(50784*(2*x**2 - x + 3))
```

3.75 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4}+\frac{121(19-7x)}{184\,(3-x+2x^2)}+\frac{1859\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

output
```
25/4*x+121*(19-7*x)/(368*x^2-184*x+552)+1859/2116*arctan(1/23*(1-4*x)*23^(
1/2))*23^(1/2)+55/8*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4}-\frac{121(-19+7x)}{184\,(3-x+2x^2)}-\frac{1859\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

output
```
(25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*
x)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{23}\int \frac{1150x^2+1955x+163}{4(2x^2-x+3)}dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{92}\int \frac{1150x^2+1955x+163}{2x^2-x+3}dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{92}\int \left(575-\frac{22(71-115x)}{2x^2-x+3}\right)dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{92}\left(\frac{1859\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{1265}{2}\log\left(2x^2-x+3\right)+575x\right)+\frac{121(19-7x)}{184(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

output
```
(121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (575*x + (1859*ArcTan[(1 - 4*x)/S
qrt[23]])/Sqrt[23] + (1265*Log[3 - x + 2*x^2])/2)/92
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln\left(16x^{2}-8x+24\right)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}$	50
default	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln\left(2x^{2}-x+3\right)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}$	51

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
25/4*x+(-847/368*x+2299/368)/(x^2-1/2*x+3/2)+55/8*ln(16*x^2-8*x+24)-1859/2
116*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx$$
$$= \frac{52900\,x^3 - 3718\,\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - 26450\,x^2 + 29095\,(2\,x^2-x+3)\log\left(2\,x^2\right.}{4232\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*
x - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x +
 52877)/(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4} + \frac{2299-847x}{368x^2-184x+552} + \frac{55\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8} - \frac{1859\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

output
```
25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2
)/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = -\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x - \frac{121\,(7x-19)}{184\,(2x^2-x+3)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
-1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*
x - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = -\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x - \frac{121\,(7x-19)}{184\,(2x^2-x+3)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
-1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*
x - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25\,x}{4} + \frac{55\,\ln\left(2\,x^2-x+3\right)}{8} - \frac{\frac{847\,x}{368}-\frac{2299}{368}}{x^2-\frac{x}{2}+\frac{3}{2}} - \frac{1859\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^2,x)
```

output
```
(25*x)/4 + (55*log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2
 + 3/2) - (1859*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/2116
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{-7436\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2+3718\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x-11154\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)+58190\log(2x^2-x+3)}{8464x^2-4232x+1}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x)
```

output
```
( - 7436*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 + 3718*sqrt(23)*atan((4*x
- 1)/sqrt(23))*x - 11154*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 58190*log(2*x
**2 - x + 3)*x**2 - 29095*log(2*x**2 - x + 3)*x + 87285*log(2*x**2 - x + 3
) + 52900*x**3 + 93288*x**2 + 232484)/(4232*(2*x**2 - x + 3))
```

3.76 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx=-\frac{11(5+3x)}{46\,(3-x+2x^2)}-\frac{82\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

output
```
(-55-33*x)/(92*x^2-46*x+138)-82/529*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx=-\frac{11(5+3x)}{46\,(3-x+2x^2)}+\frac{82\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

output
```
(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(2
3*Sqrt[23])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{23}\int \frac{41}{2x^2-x+3}dx-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{41}{23}\int \frac{1}{2x^2-x+3}dx-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{82}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{82\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{11(3x+5)}{46(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

output
```
(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(2
3*Sqrt[23])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529}$	34
risch	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529}$	34

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(4*x-1)*23^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{164\,\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-759\,x-1265}{1058\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759
*x - 1265)/(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{-33x-55}{92x^2-46x+138}+\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

output
```
(-33*x - 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - s
qrt(23)/23)/529
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx=\frac{82}{529}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-\frac{11\,(3\,x+5)}{46\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
 x + 3)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx=\frac{82}{529}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-\frac{11\,(3\,x+5)}{46\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
 x + 3)
```

Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx=\frac{82\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}-\frac{\frac{33\,x}{92}+\frac{55}{92}}{x^2-\frac{x}{2}+\frac{3}{2}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)
```

output
```
(82*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/529 - ((33*x)/92 + 55/
92)/(x^2 - x/2 + 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx$$
$$= \frac{164\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2-82\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x+246\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)-759x^2-1771}{1058x^2-529x+1587}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)
```

output
```
(164*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 82*sqrt(23)*atan((4*x - 1)/s
qrt(23))*x + 246*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 759*x**2 - 1771)/(529
*(2*x**2 - x + 3))
```

3.77 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{506(3-x+2x^2)} + \frac{241\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

output
```
(13-6*x)/(1012*x^2-506*x+1518)+241/256036*arctan(1/23*(1-4*x)*23^(1/2))*23
^(1/2)+69/15004*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-13/968*ln(2*x^2-x+
3)+13/968*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{506(3-x+2x^2)} - \frac{241\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(1113
2*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3
- x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1305, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{\int -\frac{11(-30x^2+97x+172)}{(2x^2-x+3)(5x^2+3x+2)}dx}{5566}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\int \frac{-30x^2+97x+172}{(2x^2-x+3)(5x^2+3x+2)}dx + \frac{13-6x}{506(2x^2-x+3)}$$

$$\Big\downarrow \text{ 2141}$$

$$\frac{1}{506}\left(\frac{1}{242}\int \frac{11(29-598x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(65x+54)}{5x^2+3x+2}dx\right)+\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\left(\frac{1}{22}\int \frac{29-598x}{2x^2-x+3}dx+\frac{23}{22}\int \frac{65x+54}{5x^2+3x+2}dx\right)+\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(-\frac{241}{2}\int \frac{1}{2x^2-x+3}dx-\frac{299}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{69}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{13}{2}\int \frac{10x+3}{5x^2+3x+}\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{241}{2}\int \frac{1}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{69}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{13}{2}\int \frac{10x+3}{5x^2+3x+2}d\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int \frac{1-4x}{2x^2-x+3}dx+241\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{23}{22}\left(\frac{13}{2}\int \frac{10x+3}{5x^2+3x+2}dx-69\int \frac{}{-(1}\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{241\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{23}{22}\left(\frac{13}{2}\int \frac{10x+3}{5x^2+3x+2}dx+\frac{69\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)\right)$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(-\frac{241\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{299}{2}\log\left(2x^2-x+3\right)\right)+\frac{23}{22}\left(\frac{69\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{13}{2}\log\left(5x^2+3x+2\right)\right)\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)) + (((-241*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sq
rt[23] - (299*Log[3 - x + 2*x^2])/2)/22 + (23*((69*ArcTan[(3 + 10*x)/Sqrt[
31]])/Sqrt[31] + (13*Log[2 + 3*x + 5*x^2])/2))/22)/506
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

method	result
risch	$\frac{-\frac{3x}{506}+\frac{13}{1012}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}+\frac{13\ln\left(100x^{2}+60x+40\right)}{968}+\frac{69\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{15004}-\frac{13\ln\left(16x^{2}-8x+24\right)}{968}-\frac{241\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{256036}$
default	$-\frac{\frac{66x}{23}-\frac{143}{23}}{484\left(x^{2}-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{13\ln\left(2x^{2}-x+3\right)}{968}-\frac{241\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{256036}+\frac{13\ln\left(5x^{2}+3x+2\right)}{968}+\frac{69\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{15004}$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-3/506*x+13/1012)/(x^2-1/2*x+3/2)+13/968*ln(100*x^2+60*x+40)+69/15004*arc
tan(1/31*(10*x+3)*31^(1/2))*31^(1/2)-13/968*ln(16*x^2-8*x+24)-241/256036*2
3^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{73002\sqrt{31}(2x^2-x+3)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-14942\sqrt{23}(2x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)}{15874232}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3)
) - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 21318
7*(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2
- x + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{1012x^2-506x+1518} - \frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968} + \frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968} - \frac{241\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{256036} + \frac{69\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{15004}$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

output
```
(13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*
log(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)
/23)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6x-13}{506(2x^2-x+3)} + \frac{13}{968}\log\left(5x^2+3x+2\right) - \frac{13}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968
*log(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6x-13}{506(2x^2-x+3)} + \frac{13}{968}\log\left(5x^2+3x+2\right) - \frac{13}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968
*log(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = -\frac{\frac{3x}{506}-\frac{13}{1012}}{x^2-\frac{x}{2}+\frac{3}{2}} - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right)$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 + 13/968) - log(x -
 (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 - 13/968) - ((3*x)/506 - 1
3/1012)/(x^2 - x/2 + 3/2) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i
)/512072 - 13/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/51207
2 + 13/968)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.17

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx$$
$$= \frac{146004\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2 - 73002\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x + 219006\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right) - 29884\sqrt{23}\,atan\left(\right.}{}$$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

output
```
(146004*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 - 73002*sqrt(31)*atan((10*
x + 3)/sqrt(31))*x + 219006*sqrt(31)*atan((10*x + 3)/sqrt(31)) - 29884*sqr
t(23)*atan((4*x - 1)/sqrt(23))*x**2 + 14942*sqrt(23)*atan((4*x - 1)/sqrt(2
3))*x - 44826*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 426374*log(5*x**2 + 3*x
+ 2)*x**2 - 213187*log(5*x**2 + 3*x + 2)*x + 639561*log(5*x**2 + 3*x + 2)
- 426374*log(2*x**2 - x + 3)*x**2 + 213187*log(2*x**2 - x + 3)*x - 639561*
log(2*x**2 - x + 3) - 376464*x**2 - 156860)/(15874232*(2*x**2 - x + 3))
```

3.78 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = &-\frac{25(117-137x)}{172546(2+3x+5x^2)} \\ &+\frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} \\ &+\frac{2769\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}+\frac{12643\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{165044\sqrt{31}} \\ &+\frac{19\log(3-x+2x^2)}{10648}-\frac{19\log(2+3x+5x^2)}{10648}\end{aligned}$$

output
```
(-2925+3425*x)/(862730*x^2+517638*x+345092)+1/506*(13-6*x)/(2*x^2-x+3)/(5*
x^2+3*x+2)+2769/2816396*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+12643/51163
64*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+19/10648*ln(2*x^2-x+3)-19/10648
*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$$
$$= \frac{\frac{31372(-4342+11154x-9275x^2+6850x^3)}{6+7x+16x^2+x^3+10x^4} - 5322018\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right) + 13376294\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right) + 96590}{5413113112}$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

output
```
((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
 10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31
]*ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2
 + 3*x + 5*x^2])/5413113112
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2(5x^2+3x+2)^2}\,dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} - \frac{\int -\frac{11(-90x^2+209x+211)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{5566}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\int \frac{-90x^2+209x+211}{(2x^2-x+3)(5x^2+3x+2)^2}dx + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{1}{506}\left(\frac{\int \frac{22(6850x^2-19235x+12538)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 27

$$\frac{1}{506}\left(\frac{1}{341}\int \frac{6850x^2-19235x+12538}{(2x^2-x+3)(5x^2+3x+2)}dx-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 2141

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{1}{242}\int -\frac{341(1603-874x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(5438-2945x)}{5x^2+3x+2}dx\right)-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 27

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\int \frac{5438-2945x}{5x^2+3x+2}dx-\frac{31}{22}\int \frac{1603-874x}{2x^2-x+3}dx\right)-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 1142

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2769}{2}\int \frac{1}{2x^2-x+3}dx-\frac{437}{2}\int -\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 25

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2769}{2}\int \frac{1}{2x^2-x+3}dx+\frac{437}{2}\int \frac{}{2:}\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 1083

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx-12643\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{437}{2}\int \frac{1-4x}{2x^2-x+3}d\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$\big\downarrow$ 217

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{589}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{437}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{2769\arctan}{\sqrt{23}}\right.\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{589}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{2769\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{437}{2}\log\left(2x^2-\right.\right.\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + ((-25*(117 - 137*x))/
(341*(2 + 3*x + 5*x^2)) + ((-31*((2769*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[2
3] - (437*Log[3 - x + 2*x^2])/2))/22 + (23*((12643*ArcTan[(3 + 10*x)/Sqrt[
31]])/Sqrt[31] - (589*Log[2 + 3*x + 5*x^2])/2))/22)/341)/506
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q
, 0]
```

rule 2135

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141

```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

method	result
default	$\frac{-\frac{77x}{23}-\frac{341}{46}}{5324x^2-2662x+7986}+\frac{19\ln(2x^2-x+3)}{10648}-\frac{2769\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}-\frac{-\frac{759x}{31}+\frac{1078}{155}}{5324\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{19\ln(5x^2+3x+2)}{10648}+$
risch	$\frac{\frac{3425}{86273}x^3-\frac{9275}{172546}x^2+\frac{507}{7843}x-\frac{2171}{86273}}{(2x^2-x+3)(5x^2+3x+2)}+\frac{19\ln(16x^2-8x+24)}{10648}-\frac{2769\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}-\frac{19\ln(100x^2+60x+40)}{10648}+$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5324*(-77/23*x-341/46)/(x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/28163
96*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))-1/5324*(-759/31*x+1078/155)/(x^2
+3/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+12643/5116364*arctan(1/31*(10*x+3)*31
^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int \frac{1}{(3-x+2x^2)^2\,(2+3x+5x^2)^2}\,dx$$
$$= \frac{214898200\,x^3+13376294\,\sqrt{31}(10\,x^4+x^3+16\,x^2+7\,x+6)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)-5322018\,\sqrt{23}(}{}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7
*x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3
+ 16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659
011*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x
^4 + x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)
/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$
\begin{aligned}
&\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx \\
&= \frac{6850x^3-9275x^2+11154x-4342}{1725460x^4+172546x^3+2760736x^2+1207822x+1035276} \\
&\quad+\frac{19\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{10648}-\frac{19\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{10648} \\
&\quad-\frac{2769\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2816396}+\frac{12643\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{5116364}
\end{aligned}
$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

output
```
(6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 276
0736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log
(x**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)
/23)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/51163
64
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$
\begin{aligned}
\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx &= \frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) \\
&\quad-\frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) \\
&\quad+\frac{6850x^3-9275x^2+11154x-4342}{172546\left(10x^4+x^3+16x^2+7x+6\right)} \\
&\quad-\frac{19}{10648}\log\left(5x^2+3x+2\right) \\
&\quad+\frac{19}{10648}\log\left(2x^2-x+3\right)
\end{aligned}
$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11
154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x
 + 2) + 19/10648*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{6850x^3-9275x^2+11154x-4342}{172546(10x^4+x^3+16x^2+7x+6)} - \frac{19}{10648}\log\left(5x^2+3x+2\right) + \frac{19}{10648}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output

```
12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11
154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x
 + 2) + 19/10648*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right) + \frac{\frac{685\,x^3}{172546}-\frac{1855\,x^2}{345092}+\frac{507\,x}{78430}-\frac{2171}{862730}}{x^4+\frac{x^3}{10}+\frac{8\,x^2}{5}+\frac{7\,x}{10}+\frac{3}{5}}$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2),x)
```

output
```
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 + 19/10648) - log
(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 - 19/10648) - log(x
- (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 + 19/10648) + log(x
 + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 - 19/10648) + ((50
7*x)/78430 - (1855*x^2)/345092 + (685*x^3)/172546 - 2171/862730)/((7*x)/10
 + (8*x^2)/5 + x^3/10 + x^4 + 3/5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.82

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{133762940\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4+13376294\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3+214020704\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2+936}{}$$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

output
```
(133762940*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 13376294*sqrt(31)*ata
n((10*x + 3)/sqrt(31))*x**3 + 214020704*sqrt(31)*atan((10*x + 3)/sqrt(31))
*x**2 + 93634058*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 80257764*sqrt(31)*
atan((10*x + 3)/sqrt(31)) - 53220180*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**
4 - 5322018*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**3 - 85152288*sqrt(23)*ata
n((4*x - 1)/sqrt(23))*x**2 - 37254126*sqrt(23)*atan((4*x - 1)/sqrt(23))*x
- 31932108*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 96590110*log(5*x**2 + 3*x +
 2)*x**4 - 9659011*log(5*x**2 + 3*x + 2)*x**3 - 154544176*log(5*x**2 + 3*x
 + 2)*x**2 - 67613077*log(5*x**2 + 3*x + 2)*x - 57954066*log(5*x**2 + 3*x
+ 2) + 96590110*log(2*x**2 - x + 3)*x**4 + 9659011*log(2*x**2 - x + 3)*x**
3 + 154544176*log(2*x**2 - x + 3)*x**2 + 67613077*log(2*x**2 - x + 3)*x +
57954066*log(2*x**2 - x + 3) - 2148982000*x**4 - 3729346500*x**2 - 1154364
112*x - 1425606424)/(5413113112*(10*x**4 + x**3 + 16*x**2 + 7*x + 6))
```

3.79 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 148

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx &= \frac{-9446+5765x}{690184(2+3x+5x^2)^2} \\ &+ \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} \\ &+ \frac{1765599+3996965x}{235352744(2+3x+5x^2)} \\ &- \frac{25557\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{5387888\sqrt{23}} \\ &+ \frac{4464079\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{225120016\sqrt{31}} \\ &+ \frac{97\log(3-x+2x^2)}{468512} - \frac{97\log(2+3x+5x^2)}{468512}\end{aligned}$$

output
```
1/690184*(-9446+5765*x)/(5*x^2+3*x+2)^2+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+
3*x+2)^2+(1765599+3996965*x)/(1176763720*x^2+706058232*x+470705488)-25557/
123921424*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+4464079/6978720496*arctan
(1/31*(3+10*x)*31^(1/2))*31^(1/2)+97/468512*ln(2*x^2-x+3)-97/468512*ln(5*x
^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92

$$\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^3} \, dx = \frac{-11+90x}{244904 (3-x+2x^2)} + \frac{-98+345x}{30008 (2+3x+5x^2)^2} + \frac{67573+164380x}{10232728 (2+3x+5x^2)} + \frac{25557 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{5387888\sqrt{23}} + \frac{4464079 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{225120016\sqrt{31}} + \frac{97 \log(3-x+2x^2)}{468512} - \frac{97 \log(2+3x+5x^2)}{468512}$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*
x^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[
(-1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt
[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 +
 3*x + 5*x^2])/468512
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1305, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2 (5x^2+3x+2)^3} \, dx$$

$$\Big\downarrow \ 1305$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}-\frac{\int -\frac{11\left(-150x^2+321x+250\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^3}dx}{5566}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\int \frac{-150x^2+321x+250}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^3}dx+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{1}{506}\left(\frac{\int \frac{11\left(34590x^2-106699x+68191\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}dx}{15004}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\left(\frac{\int \frac{34590x^2-106699x+68191}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}dx}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{1}{506}\left(\frac{\frac{\int \frac{22\left(7993930x^2-1730927x+7580866\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx}{7502}+\frac{3996965x+1765599}{341\left(5x^2+3x+2\right)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\int \frac{7993930x^2-1730927x+7580866}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx+\frac{3996965x+1765599}{341\left(5x^2+3x+2\right)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 2141}$$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{1}{242}\int \frac{10571(4462x+11663)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(2092214-466085x)}{5x^2+3x+2}dx\right)+\frac{3996965x+1765599}{341\left(5x^2+3x+2\right)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\int\frac{4462x+11663}{2x^2-x+3}dx+\frac{23}{22}\int\frac{2092214-466085x}{5x^2+3x+2}dx\right)+\frac{3996965x+1765599}{341(5x^2+3x+2)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557}{2}\int\frac{1}{2x^2-x+3}dx+\frac{2231}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+}{1364}\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557}{2}\int\frac{1}{2x^2-x+3}dx-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+39}{1364}\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx-25557\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{23}{22}\left(-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx-4464079\int\frac{}{-(}\right.\right.}{1364}\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+39}{1364}\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{2231}{2}\log\left(2x^2-x+3\right)\right)+\frac{23}{22}\left(\frac{4464079\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93217}{2}\log\left(5x^2+3x+2\right.\right.\right.}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (-1/1364*(9446 - 57
65*x)/(2 + 3*x + 5*x^2)^2 + ((1765599 + 3996965*x)/(341*(2 + 3*x + 5*x^2))
 + ((961*((25557*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] + (2231*Log[3 - x +
 2*x^2])/2))/22 + (23*((4464079*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] - (9
3217*Log[2 + 3*x + 5*x^2])/2))/22)/341)/1364)/506
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

method	result
default	$\frac{\frac{990x}{23}-\frac{121}{23}}{234256x^2-117128x+351384}+\frac{97\ln(2x^2-x+3)}{468512}+\frac{25557\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424}-\frac{25\left(-\frac{723272}{961}x^3-\frac{3656422}{4805}x^2-\frac{14280728}{24025}x-\right.}{234256(5x^2+3x+2)^2}$
risch	$\frac{\frac{19984825}{117676372}x^5+\frac{21652955}{235352744}x^4+\frac{69648769}{235352744}x^3+\frac{23910151}{117676372}x^2+\frac{5333615}{29419093}x+\frac{158567}{5348926}}{(2x^2-x+3)(5x^2+3x+2)^2}+\frac{97\ln(16x^2-8x+24)}{468512}+\frac{25557\sqrt{23}\,\arctan\left(\frac{(4x-}{2}\right.}{123921424}$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/468512*ln(2*x^2-x+3)+25557/1
23921424*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))-25/234256*(-723272/961*x^3
-3656422/4805*x^2-14280728/24025*x-2238016/24025)/(5*x^2+3*x+2)^2-97/46851
2*ln(5*x^2+3*x+2)+4464079/6978720496*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.60

$$\int \frac{1}{(3-x+2x^2)^2\,(2+3x+5x^2)^3}\,dx$$
$$=\frac{1253927859800\,x^5+679296504260\,x^4+2185021181068\,x^3+4722995582\,\sqrt{31}(50\,x^6+35\,x^5+103\,x^4+}{}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3
+ 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*
x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 +
83*x^2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 1
03*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x
 + 218880812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{11767637200x^6+8237346040x^5+24241332632x^4+20004983240x^3+19534277752x^2+7531287808x} + \frac{97\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{468512} - \frac{97\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{468512} + \frac{25557\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{123921424} + \frac{4464079\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{6978720496}$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

output
```
(39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*
x + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 2000
4983240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2
 - x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*
atan(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*s
qrt(31)*x/31 + 3*sqrt(31)/31)/6978720496
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)} - \frac{97}{468512}\log\left(5x^2+3x+2\right) + \frac{97}{468512}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/12392
1424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5
+ 21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x
^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2
 + 3*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$$
$$= \frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744(5x^2+3x+2)^2(2x^2-x+3)} - \frac{97}{468512}\log(5x^2+3x+2) + \frac{97}{468512}\log(2x^2-x+3)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/12392
1424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5
+ 21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x
^2 + 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468
512*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 15.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx = \frac{\frac{799393\,x^5}{235352744}+\frac{4330591\,x^4}{2353527440}+\frac{69648769\,x^3}{11767637200}+\frac{23910151\,x^2}{5883818600}+\frac{1066723\,x}{294190930}+\frac{158567}{267446300}}{x^6+\frac{7\,x^5}{10}+\frac{103\,x^4}{50}+\frac{17\,x^3}{10}+\frac{83\,x^2}{50}+\frac{16\,x}{25}+\frac{6}{25}} + \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 + 97/468512) -
 log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 - 97/468512)
+ ((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176
7637200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/26744
6300)/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 +
 x^6 + 6/25) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957
440992 + 97/468512) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i
)/13957440992 - 97/468512)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.47

$$\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

output
```
(236149779100*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**6 + 165304845370*sqrt(
31)*atan((10*x + 3)/sqrt(31))*x**5 + 486468544946*sqrt(31)*atan((10*x + 3)
/sqrt(31))*x**4 + 401454624470*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**3 + 3
92008633306*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 151135858624*sqrt(31
)*atan((10*x + 3)/sqrt(31))*x + 56675946984*sqrt(31)*atan((10*x + 3)/sqrt(
31)) + 76136858700*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**6 + 53295801090*sq
rt(23)*atan((4*x - 1)/sqrt(23))*x**5 + 156841928922*sqrt(23)*atan((4*x - 1
)/sqrt(23))*x**4 + 129432659790*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**3 + 1
26387185442*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 + 48727589568*sqrt(23)*
atan((4*x - 1)/sqrt(23))*x + 18272846088*sqrt(23)*atan((4*x - 1)/sqrt(23))
 - 76433279150*log(5*x**2 + 3*x + 2)*x**6 - 53503295405*log(5*x**2 + 3*x +
 2)*x**5 - 157452555049*log(5*x**2 + 3*x + 2)*x**4 - 129936574555*log(5*x*
*2 + 3*x + 2)*x**3 - 126879243389*log(5*x**2 + 3*x + 2)*x**2 - 48917298656
*log(5*x**2 + 3*x + 2)*x - 18343986996*log(5*x**2 + 3*x + 2) + 76433279150
*log(2*x**2 - x + 3)*x**6 + 53503295405*log(2*x**2 - x + 3)*x**5 + 1574525
55049*log(2*x**2 - x + 3)*x**4 + 129936574555*log(2*x**2 - x + 3)*x**3 + 1
26879243389*log(2*x**2 - x + 3)*x**2 + 48917298656*log(2*x**2 - x + 3)*x +
 18343986996*log(2*x**2 - x + 3) - 1791325514000*x**6 - 3010834054580*x**4
 - 860232192732*x**3 - 1473381838896*x**2 + 192161029280*x - 211037310704)
/(7383486284768*(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*...
```

3.80 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} \, dx = \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} + \frac{63799791 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}} - \frac{13915}{64} \log\left(3-x+2x^2\right)$$

output

```
2725/8*x+4875/32*x^2+625/24*x^3-14641/5888*(101+79*x)/(2*x^2-x+3)^2+1331*(
5229+76420*x)/(270848*x^2-135424*x+406272)+63799791/389344*arctan(1/23*(1-
4*x)*23^(1/2))*23^(1/2)-13915/64*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)}-\frac{63799791\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{16928\sqrt{23}} - \frac{13915}{64}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

output
```
(2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3
- x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (6379
9791*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*
x^2])/64
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^3}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{46}\int \frac{1840000x^6+5336000x^5+6826400x^4+2298160x^3-3906136x^2-2644724x+2173869}{128\,(2x^2-x+3)^2}dx - \frac{14641(79x+101)}{5888\,(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1840000x^6+5336000x^5+6826400x^4+2298160x^3-3906136x^2-2644724x+2173869}{(2x^2-x+3)^2}dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{23}\int -\frac{16(-1322500x^4-4496500x^3-5170975x^2+2507460x+5460539)}{2x^2-x+3}dx+\frac{1331(76420x+5229)}{23(2x^2-x+3)}}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\int \frac{-1322500x^4-4496500x^3-5170975x^2+2507460x+5460539}{2x^2-x+3}dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\int \left(-661250x^2-2578875x+\frac{121(60835x+116609)}{2x^2-x+3}-2883050\right)dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\left(-\frac{63799791\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{23}}-\frac{661250x^3}{3}-\frac{2578875x^2}{2}+\frac{7361035}{4}\log\left(2x^2-x+3\right)-2883050x\right)}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

output
```
(-14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + ((1331*(5229 + 76420*x))/
(23*(3 - x + 2*x^2)) - (16*(-2883050*x - (2578875*x^2)/2 - (661250*x^3)/3
- (63799791*ArcTan[(1 - 4*x)/Sqrt[23]])/(2*Sqrt[23]) + (7361035*Log[3 - x
+ 2*x^2])/4))/23)/5888
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

method	result
default	$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}-\frac{121\left(-\frac{210155}{4232}x^3+\frac{362791}{16928}x^2-\frac{561121}{8464}x+\frac{54263}{16928}\right)}{4\left(2x^2-x+3\right)^2}-\frac{13915\ln\left(2x^2-x+3\right)}{64}-\frac{63799791\sqrt{23}\ \arctan(}{389344}$
risch	$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{\frac{25428755}{16928}x^3-\frac{43897711}{67712}x^2+\frac{67895641}{33856}x-\frac{6565823}{67712}}{\left(2x^2-x+3\right)^2}-\frac{13915\ln\left(16x^2-8x+24\right)}{64}-\frac{63799791\sqrt{23}\ \mathrm{arc}}{389}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-5
61121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/38
9344*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.31

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx$$
$$= \frac{486680000\,x^7 + 2360398000\,x^6 + 5100406400\,x^5 + 2157209100\,x^4 + 24531516180\,x^3 - 765597492\,\sqrt{23}(\cdots}{\cdots}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^
4 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4
*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(
4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} + \frac{101715020x^3 - 43897711x^2 + 135791282x - 6565823}{270848x^4 - 270848x^3 + 880256x^2 - 406272x + 609408} - \frac{13915\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{64} - \frac{63799791\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{389344}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

output
```
625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 +
135791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x
 + 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqr
t(23)*x/23 - sqrt(23)/23)/389344
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} \, dx = \frac{625}{24} x^3 + \frac{4875}{32} x^2 - \frac{63799791}{389344} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{2725}{8} x + \frac{1331 (76420 x^3 - 32981 x^2 + 102022 x - 4933)}{67712 (4 x^4 - 4 x^3 + 13 x^2 - 6 x + 9)} - \frac{13915}{64} \log(2 x^2 - x + 3)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(
4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933
)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} \, dx = \frac{625}{24} x^3 + \frac{4875}{32} x^2 - \frac{63799791}{389344} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{2725}{8} x + \frac{1331 (76420 x^3 - 32981 x^2 + 102022 x - 4933)}{67712 (2 x^2 - x + 3)^2} - \frac{13915}{64} \log(2 x^2 - x + 3)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(
4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933
)/(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{2725\,x}{8} - \frac{13915\,\ln\left(2\,x^2-x+3\right)}{64} - \frac{63799791\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{389344} + \frac{4875\,x^2}{32} + \frac{625\,x^3}{24} + \frac{\frac{25428755\,x^3}{67712}-\frac{43897711\,x^2}{270848}+\frac{67895641\,x}{135424}-\frac{6565823}{270848}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)
```

output
```
(2725*x)/8 - (13915*log(2*x^2 - x + 3))/64 - (63799791*23^(1/2)*atan((4*23
^(1/2)*x)/23 - 23^(1/2)/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((678
95641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/2
70848)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{-1531194984\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^4 + 1531194984\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^3 - 4976383698\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2 + \cdots}{}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x)
```

output

```
( - 1531194984*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**4 + 1531194984*sqrt(23
)*atan((4*x - 1)/sqrt(23))*x**3 - 4976383698*sqrt(23)*atan((4*x - 1)/sqrt(
23))*x**2 + 2296792476*sqrt(23)*atan((4*x - 1)/sqrt(23))*x - 3445188714*sq
rt(23)*atan((4*x - 1)/sqrt(23)) - 2031645660*log(2*x**2 - x + 3)*x**4 + 20
31645660*log(2*x**2 - x + 3)*x**3 - 6602848395*log(2*x**2 - x + 3)*x**2 +
3047468490*log(2*x**2 - x + 3)*x - 4571202735*log(2*x**2 - x + 3) + 243340
000*x**7 + 1180199000*x**6 + 2550203200*x**5 + 13344362640*x**4 + 36777874
713*x**2 - 6552341706*x + 27371434809)/(2336064*(4*x**4 - 4*x**3 + 13*x**2
 - 6*x + 9))
```

3.81 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} \, dx = \frac{125x}{8} - \frac{1331(17-45x)}{1472\,(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856\,(3-x+2x^2)} + \frac{165099 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log\left(3-x+2x^2\right)$$

output
```
125/8*x-1331/1472*(17-45*x)/(2*x^2-x+3)^2+121*(21193-12828*x)/(67712*x^2-3
3856*x+101568)+165099/194672*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+825/32
*ln(2*x^2-x+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} \, dx = \frac{125x}{8} + \frac{1331(-17+45x)}{1472\,(3-x+2x^2)^2} - \frac{121(-21193+12828x)}{33856\,(3-x+2x^2)} - \frac{165099 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

output
```
(125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 +
12828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(
8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{46}\int -\frac{-92000x^4-211600x^3-177560x^2+76268x+40885}{32(2x^2-x+3)^2}dx-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-92000x^4-211600x^3-177560x^2+76268x+40885}{(2x^2-x+3)^2}dx}{1472}-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{121(21193-12828x)}{23(2x^2-x+3)}-\frac{1}{23}\int -\frac{16(66125x^2+185150x+23997)}{2x^2-x+3}dx}{1472}-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{23}\int \frac{66125x^2+185150x+23997}{2x^2-x+3}dx+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{16}{23}\int \left(\frac{66125}{2}-\frac{33(4557-13225x)}{2(2x^2-x+3)}\right)dx+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{16}{23}\left(\frac{165099\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{436425}{8}\log\left(2x^2-x+3\right)+\frac{66125x}{2}\right)+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

output
```
(-1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + ((121*(21193 - 12828*x))/(2
3*(3 - x + 2*x^2)) + (16*((66125*x)/2 + (165099*ArcTan[(1 - 4*x)/Sqrt[23]]
)/(4*Sqrt[23]) + (436425*Log[3 - x + 2*x^2])/8))/23)/1472
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^{3}+\frac{3340447}{16928}x^{2}-\frac{1460833}{8464}x+\frac{3586319}{16928}}{\left(2x^{2}-x+3\right)^{2}}+\frac{825\ln\left(2x^{2}-x+3\right)}{32}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}$	63
risch	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^{3}+\frac{3340447}{16928}x^{2}-\frac{1460833}{8464}x+\frac{3586319}{16928}}{\left(2x^{2}-x+3\right)^{2}}+\frac{825\ln\left(16x^{2}-8x+24\right)}{32}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}$	63

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(
2*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(4*x-
1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx$$
$$= \frac{24334000\,x^{5}-24334000\,x^{4}+43385176\,x^{3}-330198\,\sqrt{23}(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9)\arctan\left(\frac{1}{23}\sqrt{23}(\right.}{389344\,(4\,x^{4}-}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*
x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281
*x^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12
446818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = \frac{125x}{8} + \frac{-1552188x^3+3340447x^2-2921666x+3586319}{67712x^4-67712x^3+220064x^2-101568x+152352} + \frac{825\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32} - \frac{165099\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

output
```
125*x/8 + (-1552188*x**3 + 3340447*x**2 - 2921666*x + 3586319)/(67712*x**4
 - 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/
2)/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = -\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121\,(12828\,x^3-27607\,x^2+24146\,x-29639)}{16928\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)} + \frac{825}{32}\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
-165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16
928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9) + 825/32*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = -\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x - \frac{121\,(12828\,x^3-27607\,x^2+24146\,x-29639)}{16928\,(2\,x^2-x+3)^2} + \frac{825}{32}\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
-165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16
928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*l
og(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = \frac{125\,x}{8}+\frac{825\,\ln\left(2\,x^2-x+3\right)}{32} - \frac{165099\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{194672} - \frac{\frac{388047\,x^3}{16928}-\frac{3340447\,x^2}{67712}+\frac{1460833\,x}{33856}-\frac{3586319}{67712}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^3,x)
```

output
```
(125*x)/8 + (825*log(2*x^2 - x + 3))/32 - (165099*23^(1/2)*atan((4*23^(1/2
)*x)/23 - 23^(1/2)/23))/194672 - ((1460833*x)/33856 - (3340447*x^2)/67712
+ (388047*x^3)/16928 - 3586319/67712)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 +
9/4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.42

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx$$
$$= \frac{-1320792\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^4+1320792\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^3-4292574\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2+1981188\sqrt{}}{}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x)
```

output
```
( - 1320792*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**4 + 1320792*sqrt(23)*atan
((4*x - 1)/sqrt(23))*x**3 - 4292574*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2
 + 1981188*sqrt(23)*atan((4*x - 1)/sqrt(23))*x - 2971782*sqrt(23)*atan((4*
x - 1)/sqrt(23)) + 40151100*log(2*x**2 - x + 3)*x**4 - 40151100*log(2*x**2
 - x + 3)*x**3 + 130491075*log(2*x**2 - x + 3)*x**2 - 60226650*log(2*x**2
- x + 3)*x + 90339975*log(2*x**2 - x + 3) + 24334000*x**5 + 19051176*x**4
+ 181331103*x**2 - 77524582*x + 180101983)/(389344*(4*x**4 - 4*x**3 + 13*x
**2 - 6*x + 9))
```

3.82 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx = \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} - \frac{4330\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

output
```
121/368*(19-7*x)/(2*x^2-x+3)^2-55*(975+332*x)/(16928*x^2-8464*x+25392)-433
0/12167*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx = -\frac{11(4909+938x+4045x^2+1660x^3)}{4232(-3+x-2x^2)^2} + \frac{4330\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

output
```
(-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (43
30*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^3}dx$$

$$\Big\downarrow \ 2191$$

$$\frac{1}{46}\int -\frac{5(-920x^2-1564x+39)}{8(2x^2-x+3)^2}dx+\frac{121(19-7x)}{368(2x^2-x+3)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\int \frac{-920x^2-1564x+39}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \ 2191$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{1}{23}\int -\frac{6928}{2x^2-x+3}dx+\frac{11(332x+975)}{23(2x^2-x+3)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{11(332x+975)}{23(2x^2-x+3)}-\frac{6928}{23}\int \frac{1}{2x^2-x+3}dx\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{13856}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)+\frac{11(332x+975)}{23(2x^2-x+3)}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{11(332x+975)}{23(2x^2-x+3)}-\frac{13856\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}\right)$$

input

```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

output
```
(121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (5*((11*(975 + 332*x))/(23*(3 -
 x + 2*x^2)) - (13856*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23*Sqrt[23])))/368
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{4565}{1058}x^{3}-\frac{44495}{4232}x^{2}-\frac{5159}{2116}x-\frac{53999}{4232}}{\left(2x^{2}-x+3\right)^{2}}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{-\frac{4565}{1058}x^{3}-\frac{44495}{4232}x^{2}-\frac{5159}{2116}x-\frac{53999}{4232}}{\left(2x^{2}-x+3\right)^{2}}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+4
330/12167*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = -\frac{419980\,x^3 - 34640\,\sqrt{23}(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + 1023385\,x^2 + 23731}{97336\,(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
-1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4
- 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{-18260x^3 - 44495x^2 - 10318x - 53999}{16928x^4 - 16928x^3 + 55016x^2 - 25392x + 38088} + \frac{4330\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

output
```
(-18260*x**3 - 44495*x**2 - 10318*x - 53999)/(16928*x**4 - 16928*x**3 + 55
016*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23
)/23)/12167
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 +
4045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(2\,x^2-x+3)^2}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 +
4045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx=\frac{4330\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}-\frac{\frac{4565\,x^3}{4232}+\frac{44495\,x^2}{16928}+\frac{5159\,x}{8464}+\frac{53999}{16928}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^3,x)
```

output
```
(4330*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 - ((5159*x)/84
64 + (44495*x^2)/16928 + (4565*x^3)/4232 + 53999/16928)/((13*x^2)/4 - (3*x
)/2 - x^3 + x^4 + 9/4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx$$
$$=\frac{34640\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^4-34640\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^3+112580\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2-51960\sqrt{23}\,atan\left(\right.}{97336x^4-97336x^3+316342x^2-1460}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x)
```

output
```
(34640*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**4 - 34640*sqrt(23)*atan((4*x -
 1)/sqrt(23))*x**3 + 112580*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 51960
*sqrt(23)*atan((4*x - 1)/sqrt(23))*x + 77940*sqrt(23)*atan((4*x - 1)/sqrt(
23)) - 104995*x**4 - 597080*x**2 + 98164*x - 546733)/(24334*(4*x**4 - 4*x*
*3 + 13*x**2 - 6*x + 9))
```

3.83 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3} \, dx = -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

output
```
1/92*(-55-33*x)/(2*x^2-x+3)^2-131*(1-4*x)/(4232*x^2-2116*x+6348)-262/12167
*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3} \, dx = \frac{\frac{46\left(-829+472x-393x^2+524x^3\right)}{\left(-3+x-2x^2\right)^2}+1048\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{48668}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

output
```
((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23
]*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^3}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{46}\int \frac{131}{2(2x^2-x+3)^2}dx-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{131}{92}\int \frac{1}{(2x^2-x+3)^2}dx-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{131}{92}\left(\frac{4}{23}\int \frac{1}{2x^2-x+3}dx-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{131}{92}\left(-\frac{8}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{131}{92}\left(\frac{8\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

output
```
(-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) + (131*(-1/23*(1 - 4*x)/(3 - x + 2*
x^2) + (8*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23*Sqrt[23])))/92
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23
^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx$$
$$=\frac{12052\,x^3+524\,\sqrt{23}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-9039\,x^2+10856\,x-1906}{24334\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 +
13*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{524x^3-393x^2+472x-829}{4232x^4-4232x^3+13754x^2-6348x+9522} + \frac{262\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

output
```
(524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 -
6348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{524x^3-393x^2+472x-829}{1058(4x^4-4x^3+13x^2-6x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393
*x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{524x^3-393x^2+472x-829}{1058(2x^2-x+3)^2}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393
*x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167} + \frac{\frac{131x^3}{1058}-\frac{393x^2}{4232}+\frac{59x}{529}-\frac{829}{4232}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^3,x)
```

output
```
(262*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 + ((59*x)/529 -
 (393*x^2)/4232 + (131*x^3)/1058 - 829/4232)/((13*x^2)/4 - (3*x)/2 - x^3 +
 x^4 + 9/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx$$
$$= \frac{1048\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^4-1048\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^3+3406\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)x^2-1572\sqrt{23}\,atan\left(\frac{4x-1}{\sqrt{23}}\right)}{48668x^4-48668x^3+158171x^2-73002x+1}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x)
```

output
```
(1048*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**4 - 1048*sqrt(23)*atan((4*x - 1
)/sqrt(23))*x**3 + 3406*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 1572*sqrt
(23)*atan((4*x - 1)/sqrt(23))*x + 2358*sqrt(23)*atan((4*x - 1)/sqrt(23)) +
 6026*x**4 + 15065*x**2 - 3611*x + 4025)/(12167*(4*x**4 - 4*x**3 + 13*x**2
 - 6*x + 9))
```

3.84 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{13-6x}{1012(3-x+2x^2)^2}+\frac{3625-746x}{256036(3-x+2x^2)} - \frac{53403\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+\frac{247\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10648\sqrt{31}} - \frac{119\log(3-x+2x^2)}{21296}+\frac{119\log(2+3x+5x^2)}{21296}$$

output
```
1/1012*(13-6*x)/(2*x^2-x+3)^2+(3625-746*x)/(512072*x^2-256036*x+768108)-53
403/129554216*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+247/330088*arctan(1/3
1*(3+10*x)*31^(1/2))*31^(1/2)-119/21296*ln(2*x^2-x+3)+119/21296*ln(5*x^2+3
*x+2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$$
$$= \frac{3310986\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)+6010498\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)+713\left(-\frac{44(-14164+7381x-7996x^2+1492x^3)}{(-3+x-2x^2)^2}-629\right.}{8032361392}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

output
```
(3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3
 + 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-
3 + x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))
/8032361392
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3(5x^2+3x+2)}dx$$
$$\Big\downarrow \text{ 1305}$$
$$\frac{13-6x}{1012(2x^2-x+3)^2}-\frac{\int -\frac{22(-45x^2+88x+166)}{(2x^2-x+3)^2(5x^2+3x+2)}dx}{11132}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{506}\int \frac{-45x^2+88x+166}{(2x^2-x+3)^2(5x^2+3x+2)}dx+\frac{13-6x}{1012(2x^2-x+3)^2}$$
$$\Big\downarrow \text{ 2135}$$

$$\frac{1}{506}\left(\frac{\int \frac{11\left(-3730x^2+32147x+27074\right)}{(2x^2-x+3)(5x^2+3x+2)}dx}{5566}+\frac{3625-746x}{506\left(2x^2-x+3\right)}\right)+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\left(\frac{1}{506}\int \frac{-3730x^2+32147x+27074}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx+\frac{3625-746x}{506\left(2x^2-x+3\right)}\right)+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 2141$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{1}{242}\int \frac{77(8311-17986x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{5819(595x+302)}{5x^2+3x+2}dx\right)+\frac{3625-746x}{506\left(2x^2-x+3\right)}\right)+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\int \frac{8311-17986x}{2x^2-x+3}dx+\frac{529}{22}\int \frac{595x+302}{5x^2+3x+2}dx\right)+\frac{3625-746x}{506\left(2x^2-x+3\right)}\right)+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629}{2}\int \frac{1}{2x^2-x+3}dx-\frac{8993}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{247}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{119}{2}\int \frac{5}{}\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629}{2}\int \frac{1}{2x^2-x+3}dx+\frac{8993}{2}\int \frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{247}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{119}{2}\int \frac{}{5x}\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{8993}{2}\int \frac{1-4x}{2x^2-x+3}dx-7629\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{529}{22}\left(\frac{119}{2}\int \frac{10x+3}{5x^2+3x+2}dx-\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{8993}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{7629\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{529}{22}\left(\frac{119}{2}\int\frac{10x+3}{5x^2+3x+2}dx+\frac{247\arctan}{\sqrt{31}}\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{8993}{2}\log\left(2x^2-x+3\right)\right)+\frac{529}{22}\left(\frac{247\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{119}{2}\log\left(5x^2+\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + ((3625 - 746*x)/(506*(3 - x + 2*x^2)
) + ((7*((7629*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - (8993*Log[3 - x + 2
*x^2])/2))/22 + (529*((247*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + (119*Lo
g[2 + 3*x + 5*x^2])/2))/22)/506)/506
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\frac{8206}{529}x^3-\frac{43978}{529}x^2+\frac{81191}{1058}x-\frac{77902}{529}}{2662(2x^2-x+3)^2}-\frac{119\ln(2x^2-x+3)}{21296}+\frac{53403\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216}+\frac{119\ln(5x^2+3x+2)}{21296}+$ 247 arc
risch	$\frac{-\frac{373}{64009}x^3+\frac{1999}{64009}x^2-\frac{61}{2116}x+\frac{3541}{64009}}{(2x^2-x+3)^2}+\frac{119\ln(100x^2+60x+40)}{21296}+\frac{247\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{330088}-\frac{119\ln(16x^2-8x+24)}{21296}+$ 5

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-
119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(4*x-1)*23^(1
/2))+119/21296*ln(5*x^2+3*x+2)+247/330088*arctan(1/31*(10*x+3)*31^(1/2))*3
1^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)}\,dx = -\frac{46807024\,x^3-6010498\,\sqrt{31}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)-3310986\,\sqrt{23}}{}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6
*x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448
84063*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(
4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 44435
3008)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{-1492x^3+7996x^2-7381x+14164}{1024144x^4-1024144x^3+3328468x^2-1536216x+2304324} - \frac{119\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296} + \frac{119\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296} + \frac{53403\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{129554216} + \frac{247\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{330088}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

output
```
(-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 3
328468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119
*log(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqr
t(23)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/
330088
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{1492\,x^3-7996\,x^2+7381\,x-14164}{256036\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)} + \frac{119}{21296}\log\left(5\,x^2+3\,x+2\right) - \frac{119}{21296}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 73
81*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3
*x + 2) - 119/21296*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = {} & \frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) \\ & + \frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) \\ & - \frac{1492x^3-7996x^2+7381x-14164}{256036(2x^2-x+3)^2} \\ & + \frac{119}{21296}\log\left(5x^2+3x+2\right) \\ & - \frac{119}{21296}\log\left(2x^2-x+3\right)\end{aligned}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 73
81*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/212
96*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)} \, dx = -\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right) - \frac{\frac{373\,x^3}{256036}-\frac{1999\,x^2}{256036}+\frac{61\,x}{8464}-\frac{3541}{256036}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 + 119/21296) - lo
g(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 - 119/21296) - log(
x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 + 119/21296) + log
(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 - 119/21296) - ((
61*x)/8464 - (1999*x^2)/256036 + (373*x^3)/256036 - 3541/256036)/((13*x^2)
/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.11

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)} \, dx = \frac{24041992\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^4 - 24041992\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^3 + 78136474\sqrt{31}\,atan\left(\frac{10x+3}{\sqrt{31}}\right)x^2 - 36062}{}$$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

output

```
(24041992*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 - 24041992*sqrt(31)*atan
((10*x + 3)/sqrt(31))*x**3 + 78136474*sqrt(31)*atan((10*x + 3)/sqrt(31))*x
**2 - 36062988*sqrt(31)*atan((10*x + 3)/sqrt(31))*x + 54094482*sqrt(31)*at
an((10*x + 3)/sqrt(31)) + 13243944*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**4
- 13243944*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**3 + 43042818*sqrt(23)*atan
((4*x - 1)/sqrt(23))*x**2 - 19865916*sqrt(23)*atan((4*x - 1)/sqrt(23))*x +
 29798874*sqrt(23)*atan((4*x - 1)/sqrt(23)) + 179536252*log(5*x**2 + 3*x +
 2)*x**4 - 179536252*log(5*x**2 + 3*x + 2)*x**3 + 583492819*log(5*x**2 + 3
*x + 2)*x**2 - 269304378*log(5*x**2 + 3*x + 2)*x + 403956567*log(5*x**2 +
3*x + 2) - 179536252*log(2*x**2 - x + 3)*x**4 + 179536252*log(2*x**2 - x +
 3)*x**3 - 583492819*log(2*x**2 - x + 3)*x**2 + 269304378*log(2*x**2 - x +
 3)*x - 403956567*log(2*x**2 - x + 3) - 46807024*x**4 + 98727684*x**2 - 16
1346196*x + 339037204)/(8032361392*(4*x**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.85 $\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^2} \, dx = \frac{-2328909-252815x}{174616552 (2+3x+5x^2)} + \frac{13-6x}{1012 (3-x+2x^2)^2 (2+3x+5x^2)} + \frac{9665-1446x}{512072 (3-x+2x^2) (2+3x+5x^2)} + \frac{2038497 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{7261936\sqrt{31}} + \frac{181 \log(3-x+2x^2)}{468512} - \frac{181 \log(2+3x+5x^2)}{468512}$$

output
```
(-2328909-252815*x)/(873082760*x^2+523849656*x+349233104)+1/1012*(13-6*x)/
(2*x^2-x+3)^2/(5*x^2+3*x+2)+1/512072*(9665-1446*x)/(2*x^2-x+3)/(5*x^2+3*x+
2)+2038497/2850192752*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+246757/225120
016*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+181/468512*ln(2*x^2-x+3)-181/4
68512*ln(5*x^2+3*x+2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{-31-14x}{22264(3-x+2x^2)^2}+\frac{-1782-2923x}{1408198(3-x+2x^2)} + \frac{-1474+1235x}{330088(2+3x+5x^2)} - \frac{2038497\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{7261936\sqrt{31}} + \frac{181\log(3-x+2x^2)}{468512} - \frac{181\log(2+3x+5x^2)}{468512}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x
+ 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[
(-1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqr
t[31]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2
+ 3*x + 5*x^2])/468512
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1305, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3(5x^2+3x+2)^2}\,dx$$

$$\Big\downarrow \; 1305$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}-\frac{\int -\frac{11\left(-150x^2+288x+371\right)}{\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}dx}{11132}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-150x^2+288x+371}{\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}dx}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 2135$$

$$\frac{\frac{\int \frac{11\left(-21690x^2+181009x+88019\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}dx}{5566}+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{506}\int \frac{-21690x^2+181009x+88019}{(2x^2-x+3)(5x^2+3x+2)^2}dx+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 2135$$

$$\frac{\frac{1}{506}\left(\frac{\int \frac{22\left(-505630x^2-8759443x+5285594\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx}{7502}-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\int \frac{-505630x^2-8759443x+5285594}{(2x^2-x+3)(5x^2+3x+2)}dx-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 2141$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{1}{242}\int -\frac{341(1067123-191498x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{5819(114962-28055x)}{5x^2+3x+2}dx\right)-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\int\frac{114962-28055x}{5x^2+3x+2}dx-\frac{31}{22}\int\frac{1067123-191498x}{2x^2-x+3}dx\right)-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2038497}{2}\int\frac{1}{2x^2-x+3}dx-\frac{95749}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)\right)-\right.}{1012}\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2038497}{2}\int\frac{1}{2x^2-x+3}dx+\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)\right)-\frac{25}{3}\right.}{1012}\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx-246757\int\frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx-2038497\int\frac{}{-(}\right.\right.\right.}{1012}\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{2038497\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)\right)-\frac{25}{3}\right.}{1012}\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{5611}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{2038497\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{95749}{2}\log\left(2x^2-x+3\right.\right.\right.\right.}{1012}\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + ((9665 - 1446*x)/(
506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (-1/341*(2328909 + 252815*x)/(2 +
 3*x + 5*x^2) + ((-31*((2038497*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - (9
5749*Log[3 - x + 2*x^2])/2))/22 + (529*((246757*ArcTan[(3 + 10*x)/Sqrt[31]
])/Sqrt[31] - (5611*Log[2 + 3*x + 5*x^2])/2))/22)/341)/506)/1012
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

method	result
default	$\frac{-\frac{128612}{529}x^3-\frac{14102}{529}x^2-\frac{173195}{529}x-\frac{321497}{1058}}{58564(2x^2-x+3)^2}+\frac{181\ln(2x^2-x+3)}{468512}-\frac{2038497\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2850192752}-\frac{-\frac{5434x}{31}+\frac{32428}{155}}{234256\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-$
risch	$\frac{-\frac{252815}{43654138}x^5-\frac{1038047}{21827069}x^4+\frac{5042869}{174616552}x^3-\frac{21674311}{174616552}x^2+\frac{1471955}{43654138}x-\frac{200677}{3968558}}{(2x^2-x+3)^2(5x^2+3x+2)}-\frac{181\ln(100x^2+60x+40)}{468512}+\frac{246757\arctan\left(\frac{(10x+}{3}\right.}{22512001}$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/58564*(-128612/529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+
3)^2+181/468512*ln(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(4*x
-1)*23^(1/2))-1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*l
n(5*x^2+3*x+2)+246757/225120016*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.42

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^2}\,dx =$$
$$-\frac{31725248720\,x^5+260524883872\,x^4-158204886268\,x^3-6004584838\,\sqrt{31}(20\,x^6-8\,x^5+61\,x^4+x^3}{}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 -
6004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*a
rctan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61
*x^4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 6799664
84692*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18
)*log(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^
2 + 15*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6
 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{-1011260x^5-8304376x^4+5042869x^3-21674311x^2+5887820x-8829788}{3492331040x^6-1396932416x^5+10651609672x^4+174616552x^3+9254677256x^2+2619248280x+31} + \frac{181\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{468512} - \frac{181\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{468512} - \frac{2038497\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2850192752} + \frac{246757\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{225120016}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

output
```
(-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
 8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 17461655
2*x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2
 + 3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*ata
n(4*sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt
(31)*x/31 + 3*sqrt(31)/31)/225120016
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.72

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\,(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18)} - \frac{181}{468512}\log\left(5\,x^2+3\,x+2\right) + \frac{181}{468512}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/28501
92752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5
+ 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6
- 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x
+ 2) + 181/468512*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\,(5\,x^2+3\,x+2)(2\,x^2-x+3)^2} - \frac{181}{468512}\log\left(5\,x^2+3\,x+2\right) + \frac{181}{468512}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/28501
92752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5
+ 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2
+ 3*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/4685
12*log(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^2} dx$$
$$= -\frac{\frac{50563\,x^5}{174616552}+\frac{1038047\,x^4}{436541380}-\frac{5042869\,x^3}{3492331040}+\frac{21674311\,x^2}{3492331040}-\frac{294391\,x}{174616552}+\frac{200677}{79371160}}{x^6-\frac{2\,x^5}{5}+\frac{61\,x^4}{20}+\frac{x^3}{20}+\frac{53\,x^2}{20}+\frac{3\,x}{4}+\frac{9}{10}}$$
$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right)$$
$$+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right)$$
$$+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)$$
$$-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 - 181/46851
2) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 + 181/
468512) - ((21674311*x^2)/3492331040 - (294391*x)/174616552 - (5042869*x^3
)/3492331040 + (1038047*x^4)/436541380 + (50563*x^5)/174616552 + 200677/79
371160)/((3*x)/4 + (53*x^2)/20 + x^3/20 + (61*x^4)/20 - (2*x^5)/5 + x^6 +
9/10) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 + 1
81/468512) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/570038550
4 - 181/468512)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.21

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

output
```
(120091696760*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**6 - 48036678704*sqrt(3
1)*atan((10*x + 3)/sqrt(31))*x**5 + 366279675118*sqrt(31)*atan((10*x + 3)/
sqrt(31))*x**4 + 6004584838*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**3 + 3182
42996414*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**2 + 90068772570*sqrt(31)*at
an((10*x + 3)/sqrt(31))*x + 108082527084*sqrt(31)*atan((10*x + 3)/sqrt(31)
) - 78359824680*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**6 + 31343929872*sqrt(
23)*atan((4*x - 1)/sqrt(23))*x**5 - 238997465274*sqrt(23)*atan((4*x - 1)/s
qrt(23))*x**4 - 3917991234*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**3 - 207653
535402*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2 - 58769868510*sqrt(23)*atan(
(4*x - 1)/sqrt(23))*x - 70523842212*sqrt(23)*atan((4*x - 1)/sqrt(23)) - 42
326802940*log(5*x**2 + 3*x + 2)*x**6 + 16930721176*log(5*x**2 + 3*x + 2)*x
**5 - 129096748967*log(5*x**2 + 3*x + 2)*x**4 - 2116340147*log(5*x**2 + 3*
x + 2)*x**3 - 112166027791*log(5*x**2 + 3*x + 2)*x**2 - 31745102205*log(5*
x**2 + 3*x + 2)*x - 38094122646*log(5*x**2 + 3*x + 2) + 42326802940*log(2*
x**2 - x + 3)*x**6 - 16930721176*log(2*x**2 - x + 3)*x**5 + 129096748967*l
og(2*x**2 - x + 3)*x**4 + 2116340147*log(2*x**2 - x + 3)*x**3 + 1121660277
91*log(2*x**2 - x + 3)*x**2 + 31745102205*log(2*x**2 - x + 3)*x + 38094122
646*log(2*x**2 - x + 3) - 79313121800*x**6 - 502429905362*x**4 + 154239230
178*x**3 - 890146257462*x**2 + 125227847690*x - 348389918756)/(54780704693
44*(20*x**6 - 8*x**5 + 61*x**4 + x**3 + 53*x**2 + 15*x + 18))
```

3.86 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx = &-\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} \\ &+\frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} \\ &+\frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} \\ &+\frac{15(2618306+7140435x)}{14886061058(2+3x+5x^2)} \\ &-\frac{880575\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{340783916\sqrt{23}} \\ &+\frac{2768835\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{619080044\sqrt{31}} \\ &+\frac{405\log(3-x+2x^2)}{1288408} \\ &-\frac{405\log(2+3x+5x^2)}{1288408}\end{aligned}$$

output
```
1/87308276*(-1118535-385100*x)/(5*x^2+3*x+2)^2+1/1012*(13-6*x)/(2*x^2-x+3)
^2/(5*x^2+3*x+2)^2+5/64009*(302-35*x)/(2*x^2-x+3)/(5*x^2+3*x+2)^2+15*(2618
306+7140435*x)/(74430305290*x^2+44658183174*x+29772122116)-880575/78380300
68*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+2768835/19191481364*arctan(1/31*
(3+10*x)*31^(1/2))*31^(1/2)+405/1288408*ln(2*x^2-x+3)-405/1288408*ln(5*x^2
+3*x+2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^3}\,dx\\&=\frac{-4342+11154x-9275x^2+6850x^3}{345092\,(6+7x+16x^2+x^3+10x^4)^2}\\&\quad+\frac{5(14085977+51156233x-5711469x^2+42842610x^3)}{14886061058\,(6+7x+16x^2+x^3+10x^4)}+\frac{880575\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{340783916\sqrt{23}}\\&\quad+\frac{2768835\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{619080044\sqrt{31}}+\frac{405\log(3-x+2x^2)}{1288408}-\frac{405\log(2+3x+5x^2)}{1288408}\end{aligned}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 +
10*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(148
86061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sq
rt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619
080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5
*x^2])/1288408
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1305, 27, 2135, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}-\frac{\int -\frac{110(-21x^2+40x+41)}{(2x^2-x+3)^2(5x^2+3x+2)^3}dx}{11132}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{506}\int \frac{-21x^2+40x+41}{(2x^2-x+3)^2(5x^2+3x+2)^3}dx+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{5}{506}\left(\frac{\int \frac{22(-1750x^2+17315x+6819)}{(2x^2-x+3)(5x^2+3x+2)^3}dx}{5566}+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{506}\left(\frac{1}{253}\int \frac{-1750x^2+17315x+6819}{(2x^2-x+3)(5x^2+3x+2)^3}dx+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{\int \frac{264(-38510x^2-114479x+45248)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{15004}-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 27$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\int\frac{-38510x^2-114479x+45248}{(2x^2-x+3)(5x^2+3x+2)^2}dx-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)}\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 2135$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{\int\frac{11(14280870x^2-5235733x+5790640)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{}{253(\ldots}\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 27$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\int\frac{14280870x^2-5235733x+5790640}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 2141$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{1}{242}\int\frac{10571(28566x+22211)}{2x^2-x+3}dx+\frac{1}{242}\int\frac{5819(53374-129735x)}{5x^2+3x+2}dx\right)+\frac{7140435x+2\ldots}{682(5x^2+3\ldots}\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 27$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\int\frac{28566x+22211}{2x^2-x+3}dx+\frac{529}{22}\int\frac{53374-129735x}{5x^2+3x+2}dx\right)+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{\ldots}{68\ldots}\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow 1142$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705}{2}\int\frac{1}{2x^2-x+3}dx+\frac{14283}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589}{2}\int\frac{1}{5x^2+3x\ldots}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705}{2}\int\frac{1}{2x^2-x+3}dx-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589}{2}\int\frac{1}{5x^2+3x+\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx-58705\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{529}{22}\left(-\frac{25947}{2}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589\arctan\left(\frac{10x+3}{\sqrt{31}}\right.}{\sqrt{31}}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{14283}{2}\log\left(2x^2-x+3\right)\right)+\frac{529}{22}\left(\frac{184589\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*((2*(302 - 35
*x))/(253*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (-1/682*(223707 + 77020*x
)/(2 + 3*x + 5*x^2)^2 + (6*((2618306 + 7140435*x)/(682*(2 + 3*x + 5*x^2))
+ ((961*((58705*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] + (14283*Log[3 - x +
 2*x^2])/2))/22 + (529*((184589*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] - (2
5947*Log[2 + 3*x + 5*x^2])/2))/22)/682))/341)/253))/506
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.65

method	result
default	$\frac{\frac{302907}{529}x^3-\frac{368291}{529}x^2+\frac{2501587}{2116}x-\frac{665819}{1058}}{644204(2x^2-x+3)^2}+\frac{405\ln(2x^2-x+3)}{1288408}+\frac{880575\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{7838030068}-\frac{25(-\frac{3013197}{961}x^3-\frac{14516062}{4805}}{2576816(5x}$
risch	$\frac{\frac{1071065250}{7443030529}x^7-\frac{35680200}{7443030529}x^6+\frac{5956663105}{14886061058}x^5+\frac{2002653845}{14886061058}x^4+\frac{5543790435}{14886061058}x^3+\frac{4691822415}{29772122116}x^2+\frac{1254420353}{7443030529}x+\frac{235280627}{14886061058}}{(2x^2-x+3)^2(5x^2+3x+2)^2}-\frac{405\,\mathrm{l}}{}$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/644204*(302907/529*x^3-368291/529*x^2+2501587/2116*x-665819/1058)/(2*x^2
-x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7838030068*23^(1/2)*arctan(1/23*(
4*x-1)*23^(1/2))-25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/2
4025*x-5423968/24025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/
19191481364*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.64

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$$
$$= \frac{67202918046000\,x^7 - 2238718468800\,x^6 + 186872434930060\,x^5 + 62827256425340\,x^4 + 1739197935268}{}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 18687243493006
0*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(1
00*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x +
 36)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20
*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan
(1/23*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 2
0*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5
*x^2 + 3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*
x + 7381223830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x
^3 + 241*x^2 + 84*x + 36)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$$
$$= \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 110875808\ldots}{2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7\ldots}$$
$$+ \frac{405\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{1288408} - \frac{405\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1288408}$$
$$+ \frac{880575\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{7838030068} + \frac{2768835\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19191481364}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

output
```
(4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 1
1087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(29772122116
00*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11
611127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 250085825774
4*x + 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 +
3*x/5 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)
/7838030068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/1919
1481364
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3}\,dx = \frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{4284261000\,x^7 - 142720800\,x^6 + 11913326210\,x^5 + 4005307690\,x^4 + 11087580870\,x^3 + 4691822415\,x}{29772122116\,(100\,x^8 + 20\,x^7 + 321\,x^6 + 172\,x^5 + 390\,x^4 + 236\,x^3 + 241\,x^2 + 8} - \frac{405}{1288408}\log\left(5\,x^2+3\,x+2\right) + \frac{405}{1288408}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/783
8030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(42842610
00*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^
3 + 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6
 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*
x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^3}\,dx = \frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)$$
$$+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)$$
$$+\frac{4284261000\,x^7-142720800\,x^6+11913326210\,x^5+4005307690\,x^4+11087580870\,x^3+4691822415\,x}{29772122116\,(10\,x^4+x^3+16\,x^2+7\,x+6)^2}$$
$$-\frac{405}{1288408}\log\left(5\,x^2+3\,x+2\right)+\frac{405}{1288408}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/783
8030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(42842610
00*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^
3 + 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x +
3)
```

Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$$
$$= \frac{\frac{21421305\,x^7}{14886061058}-\frac{356802\,x^6}{7443030529}+\frac{1191332621\,x^5}{297721221160}+\frac{400530769\,x^4}{297721221160}+\frac{1108758087\,x^3}{297721221160}+\frac{938364483\,x^2}{595442442320}+\frac{1254420353\,x}{744303052900}+\frac{235280627}{1488606105800}}{x^8+\frac{x^7}{5}+\frac{321\,x^6}{100}+\frac{43\,x^5}{25}+\frac{39\,x^4}{10}+\frac{59\,x^3}{25}+\frac{241\,x^2}{100}+\frac{21\,x}{25}+\frac{9}{25}}$$
$$+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$
$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)$$
$$+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)$$
$$-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/12884
08) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405
/1288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962
728 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)
/38382962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^
2)/595442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/29772122
1160 + (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305
*x^7)/14886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 +
(59*x^3)/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/
25)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.70

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

output
```
(6737683089000*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**8 + 1347536617800*sqr
t(31)*atan((10*x + 3)/sqrt(31))*x**7 + 21627962715690*sqrt(31)*atan((10*x
+ 3)/sqrt(31))*x**6 + 11588814913080*sqrt(31)*atan((10*x + 3)/sqrt(31))*x*
*5 + 26276964047100*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**4 + 159009320900
40*sqrt(31)*atan((10*x + 3)/sqrt(31))*x**3 + 16237816244490*sqrt(31)*atan(
(10*x + 3)/sqrt(31))*x**2 + 5659653794760*sqrt(31)*atan((10*x + 3)/sqrt(31
))*x + 2425565912040*sqrt(31)*atan((10*x + 3)/sqrt(31)) + 5246641965000*sq
rt(23)*atan((4*x - 1)/sqrt(23))*x**8 + 1049328393000*sqrt(23)*atan((4*x -
1)/sqrt(23))*x**7 + 16841720707650*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**6
+ 9024224179800*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**5 + 20461903663500*sq
rt(23)*atan((4*x - 1)/sqrt(23))*x**4 + 12382075037400*sqrt(23)*atan((4*x -
 1)/sqrt(23))*x**3 + 12644407135650*sqrt(23)*atan((4*x - 1)/sqrt(23))*x**2
 + 4407179250600*sqrt(23)*atan((4*x - 1)/sqrt(23))*x + 1888791107400*sqrt(
23)*atan((4*x - 1)/sqrt(23)) - 14679917428500*log(5*x**2 + 3*x + 2)*x**8 -
 2935983485700*log(5*x**2 + 3*x + 2)*x**7 - 47122534945485*log(5*x**2 + 3*
x + 2)*x**6 - 25249457977020*log(5*x**2 + 3*x + 2)*x**5 - 57251677971150*l
og(5*x**2 + 3*x + 2)*x**4 - 34644605131260*log(5*x**2 + 3*x + 2)*x**3 - 35
378601002685*log(5*x**2 + 3*x + 2)*x**2 - 12331130639940*log(5*x**2 + 3*x
+ 2)*x - 5284770274260*log(5*x**2 + 3*x + 2) + 14679917428500*log(2*x**2 -
 x + 3)*x**8 + 2935983485700*log(2*x**2 - x + 3)*x**7 + 47122534945485*...
```

3.87 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = -\frac{aB-(Ab-aC)x}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
-1/2*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(b^(1/2)*x/a^(1/2
))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = \frac{-aB+Abx-aCx}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^2,x]
```

output
```
(-(a*B) + A*b*x - a*C*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx}{2ab}-\frac{aB-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{aB-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	65
risch	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a}-\frac{\ln\left(bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}$	130

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*B/b)/(b*x^2+a)+1/2/a*(A*b+C*a)/b/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = \left[-\frac{2\,Ba^2b+(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Ca^2b-Aab^2)x}{4\,(a^2b^3x^2+a^3b^2)}, -\frac{Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Ca^2b-Aab^2)x}{2\,(a^2b^3x^2+a^3b^2)}\right]$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^2*b - A*a*b^2)*x)/(a^2*b^
3*x^2 + a^3*b^2), -1/2*(B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (C*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*
b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{-Ba+x(Ab-Ca)}{2a^2b+2ab^2x^2}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ (-B*a + x*(A*b - C*a))/(2*a**2*b + 2*a*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = -\frac{Ba+(Ca-Ab)x}{2\,(ab^2x^2+a^2b)}+\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a + (C*a - A*b)*x)/(a*b^2*x^2 + a^2*b) + 1/2*(C*a + A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{Cax-Abx+Ba}{2\,(bx^2+a)ab}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a*x - A*b*x
 + B*a)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b+C\,a\right)}{2\,a^{3/2}\,b^{3/2}}-\frac{\frac{B}{2\,b}-\frac{x\,\left(A\,b-C\,a\right)}{2\,a\,b}}{b\,x^2+a}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + C*a))/(2*a^(3/2)*b^(3/2)) - (B/(2*b) - (
x*(A*b - C*a))/(2*a*b))/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^2}\,dx$$
$$=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\left(b\,x^2+a\right)}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + a*
b**2*x - a*b*c*x + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

3.88 $\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx = -\frac{aB-(Ab-aC)x}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
-1/2*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(b^(1/2)*x/a^(1/2
))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx = \frac{-aB+Abx-aCx}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + x*(B + C*x))/(a + b*x^2)^2,x]
```

output
```
(-(a*B) + A*b*x - a*C*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2083, 2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2083}$$

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx}{2ab}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + x*(B + C*x))/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2083
```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && QuadraticQ[{u, v}, x] &&  !QuadraticMat
chQ[{u, v}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	65
risch	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a}-\frac{\ln\left(bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}$	130

input
```
int((A+x*(C*x+B))/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*B/b)/(b*x^2+a)+1/2/a*(A*b+C*a)/b/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx = \left[-\frac{2\,Ba^2b+(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Ca^2b-Aab^2)x}{4\,(a^2b^3x^2+a^3b^2)}, -\frac{Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Ca^2b-Aab^2)x}{2\,(a^2b^3x^2+a^3b^2)}\right]$$

input
```
integrate((A+x*(C*x+B))/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^2*b - A*a*b^2)*x)/(a^2*b^
3*x^2 + a^3*b^2), -1/2*(B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (C*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*
b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{-Ba+x(Ab-Ca)}{2a^2b+2ab^2x^2}$$

input
```
integrate((A+x*(C*x+B))/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ (-B*a + x*(A*b - C*a))/(2*a**2*b + 2*a*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx=-\frac{Ba+(Ca-Ab)x}{2\,(ab^2x^2+a^2b)}+\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((A+x*(C*x+B))/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a + (C*a - A*b)*x)/(a*b^2*x^2 + a^2*b) + 1/2*(C*a + A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx=\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{Cax-Abx+Ba}{2\,(bx^2+a)ab}$$

input
```
integrate((A+x*(C*x+B))/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a*x - A*b*x
 + B*a)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 15.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+C\,a)}{2\,a^{3/2}\,b^{3/2}} - \frac{\frac{B}{2\,b}-\frac{x\,(A\,b-C\,a)}{2\,a\,b}}{b\,x^2+a}$$

input
```
int((A + x*(B + C*x))/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + C*a))/(2*a^(3/2)*b^(3/2)) - (B/(2*b) - (
x*(A*b - C*a))/(2*a*b))/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{A+x(B+Cx)}{(a+bx^2)^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\,(b\,x^2+a)}$$

input
```
int((A+x*(C*x+B))/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + a*
b**2*x - a*b*c*x + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

3.89 $\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)} \, dx = \frac{(3b^2-6ac-13bc-20c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(9a+6b+4c)(a-5b+25c)\sqrt{b^2-4ac}} - \frac{9\log(2-3x)}{17(9a+6b+4c)} + \frac{\log(5+x)}{17(a-5b+25c)} - \frac{(3b-13c)\log\left(a+bx+cx^2\right)}{2(9a+6b+4c)(a-5b+25c)}$$

output
```
(-6*a*c+3*b^2-13*b*c-20*c^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(9*a+6*
b+4*c)/(a-5*b+25*c)/(-4*a*c+b^2)^(1/2)-9*ln(2-3*x)/(153*a+102*b+68*c)+ln(5
+x)/(17*a-85*b+425*c)-1/2*(3*b-13*c)*ln(c*x^2+b*x+a)/(9*a+6*b+4*c)/(a-5*b+
25*c)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx = -\frac{\left(3b^2-13bc-2c(3a+10c)\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(9a+6b+4c)(a-5b+25c)\sqrt{-b^2+4ac}} - \frac{9\log(2-3x)}{17(9a+6b+4c)} + \frac{\log(5+x)}{17(a-5b+25c)} - \frac{(3b-13c)\log(a+x(b+cx))}{2(9a+6b+4c)(a-5b+25c)}$$

input
```
Integrate[1/((2 - 3*x)*(5 + x)*(a + b*x + c*x^2)),x]
```

output
```
-(((3*b^2 - 13*b*c - 2*c*(3*a + 10*c))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/((9*a + 6*b + 4*c)*(a - 5*b + 25*c)*Sqrt[-b^2 + 4*a*c])) - (9*Log[2 -
 3*x])/(17*(9*a + 6*b + 4*c)) + Log[5 + x]/(17*(a - 5*b + 25*c)) - ((3*b -
 13*c)*Log[a + x*(b + c*x)])/(2*(9*a + 6*b + 4*c)*(a - 5*b + 25*c))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x)(x+5)\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \ 1200$$

$$\int \left(\frac{c(3a+10c)-3b^2-cx(3b-13c)+13bc}{(9a+6b+4c)(a-5b+25c)\left(a+bx+cx^2\right)}+\frac{1}{17(x+5)(a-5b+25c)}-\frac{27}{17(3x-2)(9a+6b+4c)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(-6ac+3b^2-13bc-20c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(9a+6b+4c)(a-5b+25c)\sqrt{b^2-4ac}}-\frac{(3b-13c)\log\left(a+bx+cx^2\right)}{2(9a+6b+4c)(a-5b+25c)}-\frac{9\log(2-3x)}{17(9a+6b+4c)}+\frac{\log(x+5)}{17(a-5b+25c)}$$

input
```
Int[1/((2 - 3*x)*(5 + x)*(a + b*x + c*x^2)),x]
```

output
```
((3*b^2 - 6*a*c - 13*b*c - 20*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((9*a + 6*b + 4*c)*(a - 5*b + 25*c)*Sqrt[b^2 - 4*a*c]) - (9*Log[2 - 3*x])
/(17*(9*a + 6*b + 4*c)) + Log[5 + x]/(17*(a - 5*b + 25*c)) - ((3*b - 13*c)
*Log[a + b*x + c*x^2])/(2*(9*a + 6*b + 4*c)*(a - 5*b + 25*c))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(x+5)}{17a-85b+425c}-\frac{9\ln(-2+3x)}{153a+102b+68c}+\frac{\frac{(-3bc+13c^2)\ln(cx^2+bx+a)}{2c}+\frac{2\left(3ac-3b^2+13bc+10c^2-\frac{(-3bc+13c^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a-5b+25c)(9a+6b+4c)}$
risch	Expression too large to display

input
```
int(1/(2-3*x)/(x+5)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x+5)/(17*a-85*b+425*c)-9/(153*a+102*b+68*c)*ln(-2+3*x)+1/(a-5*b+25*c)/(
9*a+6*b+4*c)*(1/2*(-3*b*c+13*c^2)/c*ln(c*x^2+b*x+a)+2*(3*a*c-3*b^2+13*b*c+
10*c^2-1/2*(-3*b*c+13*c^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 553, normalized size of antiderivative = 3.48

$$\int \frac{1}{(2-3x)(5+x)\,(a+bx+cx^2)}\,dx$$
$$= \left[-\frac{17\,(3\,b^2-(6\,a+13\,b)c-20\,c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+17\,(3\,b^3+52\,ac^2}{34\,(9\,a^2b^2-39\,ab^3-30}\right.$$

input
```
integrate(1/(2-3*x)/(5+x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/34*(17*(3*b^2 - (6*a + 13*b)*c - 20*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 17*(3*b^3 + 52*a*c^2 - (12*a*b + 13*b^2)*c)*log(c*x^2 + b*x + a) +
 18*(a*b^2 - 5*b^3 - 100*a*c^2 - (4*a^2 - 20*a*b - 25*b^2)*c)*log(3*x - 2)
 - 2*(9*a*b^2 + 6*b^3 - 16*a*c^2 - 4*(9*a^2 + 6*a*b - b^2)*c)*log(x + 5))/
(9*a^2*b^2 - 39*a*b^3 - 30*b^4 - 400*a*c^3 - 4*(229*a^2 + 130*a*b - 25*b^2
)*c^2 - (36*a^3 - 156*a^2*b - 349*a*b^2 - 130*b^3)*c), 1/34*(34*(3*b^2 - (
6*a + 13*b)*c - 20*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c
*x + b)/(b^2 - 4*a*c)) - 17*(3*b^3 + 52*a*c^2 - (12*a*b + 13*b^2)*c)*log(c
*x^2 + b*x + a) - 18*(a*b^2 - 5*b^3 - 100*a*c^2 - (4*a^2 - 20*a*b - 25*b^2
)*c)*log(3*x - 2) + 2*(9*a*b^2 + 6*b^3 - 16*a*c^2 - 4*(9*a^2 + 6*a*b - b^2
)*c)*log(x + 5))/(9*a^2*b^2 - 39*a*b^3 - 30*b^4 - 400*a*c^3 - 4*(229*a^2 +
 130*a*b - 25*b^2)*c^2 - (36*a^3 - 156*a^2*b - 349*a*b^2 - 130*b^3)*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(2-3*x)/(5+x)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(2-3*x)/(5+x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

$$\begin{aligned}
&\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{(3\,b-13\,c)\log\left(cx^2+bx+a\right)}{2\left(9\,a^2-39\,ab-30\,b^2+229\,ac+130\,bc+100\,c^2\right)} \\
&\quad -\frac{\left(3\,b^2-6\,ac-13\,bc-20\,c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(9\,a^2-39\,ab-30\,b^2+229\,ac+130\,bc+100\,c^2\right)\sqrt{-b^2+4\,ac}} \\
&\quad -\frac{9\,\log\left(|3\,x-2|\right)}{17\left(9\,a+6\,b+4\,c\right)}+\frac{\log\left(|x+5|\right)}{17\left(a-5\,b+25\,c\right)}
\end{aligned}$$

input
```
integrate(1/(2-3*x)/(5+x)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(3*b - 13*c)*log(c*x^2 + b*x + a)/(9*a^2 - 39*a*b - 30*b^2 + 229*a*c
+ 130*b*c + 100*c^2) - (3*b^2 - 6*a*c - 13*b*c - 20*c^2)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((9*a^2 - 39*a*b - 30*b^2 + 229*a*c + 130*b*c + 100*
c^2)*sqrt(-b^2 + 4*a*c)) - 9/17*log(abs(3*x - 2))/(9*a + 6*b + 4*c) + 1/17
*log(abs(x + 5))/(a - 5*b + 25*c)
```

Mupad [B] (verification not implemented)

Time = 20.08 (sec) , antiderivative size = 2331, normalized size of antiderivative = 14.66

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int(-1/((3*x - 2)*(x + 5)*(a + b*x + c*x^2)),x)
```

output
```
log(x + 5)/(17*a - 85*b + 425*c) - (9*log(x - 2/3))/(17*(9*a + 6*b + 4*c))
 - (log(3757000*a*c^5 - 16416*a*b^5 + 289000*b*c^5 + 2700*b^5*c - 16416*b^
6*x + 578000*c^6*x - 289000*c^5*(b^2 - 4*a*c)^(1/2) - 1053*a^2*b^4 + 243*a
^3*b^3 - 1461798*a^2*c^4 + 552474*a^3*c^3 + 8424*a^4*c^2 - 375700*b^2*c^4
+ 77700*b^3*c^3 - 11700*b^4*c^2 - 16416*a*b^4*(b^2 - 4*a*c)^(1/2) + 312277
0*a*c^4*(b^2 - 4*a*c)^(1/2) - 972*a^4*c*(b^2 - 4*a*c)^(1/2) + 375700*b*c^4
*(b^2 - 4*a*c)^(1/2) - 2700*b^4*c*(b^2 - 4*a*c)^(1/2) - 16416*b^5*x*(b^2 -
 4*a*c)^(1/2) + 1878500*c^5*x*(b^2 - 4*a*c)^(1/2) + 391586*a*b^2*c^3 - 808
218*a*b^3*c^2 + 2828133*a^2*b*c^3 + 95769*a^2*b^3*c - 128223*a^3*b*c^2 + 2
106*a^3*b^2*c + 243*a^2*b^4*x - 1708782*a^2*c^4*x + 71550*a^3*c^3*x + 1944
*a^4*c^2*x + 2163950*b^2*c^4*x + 107666*b^3*c^3*x - 756198*b^4*c^2*x - 105
3*a^2*b^3*(b^2 - 4*a*c)^(1/2) + 243*a^3*b^2*(b^2 - 4*a*c)^(1/2) + 854391*a
^2*c^3*(b^2 - 4*a*c)^(1/2) - 35775*a^3*c^2*(b^2 - 4*a*c)^(1/2) - 77700*b^2
*c^3*(b^2 - 4*a*c)^(1/2) + 11700*b^3*c^2*(b^2 - 4*a*c)^(1/2) - 955071*a^2*
b^2*c^2 + 894330*a*b*c^4 + 216918*a*b^4*c - 972*a^4*b*c - 1053*a*b^5*x - 6
245540*a*c^5*x + 1127100*b*c^5*x + 209898*b^5*c*x - 196101*a^2*b^2*c^2*x -
 471991*a*b*c^4*x + 109242*a*b^4*c*x - 129454*a*b*c^3*(b^2 - 4*a*c)^(1/2)
+ 202878*a*b^3*c*(b^2 - 4*a*c)^(1/2) + 4212*a^3*b*c*(b^2 - 4*a*c)^(1/2) -
1053*a*b^4*x*(b^2 - 4*a*c)^(1/2) - 730899*a*c^4*x*(b^2 - 4*a*c)^(1/2) + 20
08550*b*c^4*x*(b^2 - 4*a*c)^(1/2) + 209898*b^4*c*x*(b^2 - 4*a*c)^(1/2) ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.64

$$\int \frac{1}{(2-3x)(5+x)\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{204\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac-102\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2+442\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc+}{}$$

input
```
int(1/(2-3*x)/(5+x)/(c*x^2+b*x+a),x)
```

output

```
(204*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c - 102*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2 + 442*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c + 680*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2 - 72*log(3*x - 2)*a**2*c + 18*l
og(3*x - 2)*a*b**2 + 360*log(3*x - 2)*a*b*c - 1800*log(3*x - 2)*a*c**2 - 9
0*log(3*x - 2)*b**3 + 450*log(3*x - 2)*b**2*c - 204*log(a + b*x + c*x**2)*
a*b*c + 884*log(a + b*x + c*x**2)*a*c**2 + 51*log(a + b*x + c*x**2)*b**3 -
 221*log(a + b*x + c*x**2)*b**2*c + 72*log(x + 5)*a**2*c - 18*log(x + 5)*a
*b**2 + 48*log(x + 5)*a*b*c + 32*log(x + 5)*a*c**2 - 12*log(x + 5)*b**3 -
8*log(x + 5)*b**2*c)/(34*(36*a**3*c - 9*a**2*b**2 - 156*a**2*b*c + 916*a**
2*c**2 + 39*a*b**3 - 349*a*b**2*c + 520*a*b*c**2 + 400*a*c**3 + 30*b**4 -
130*b**3*c - 100*b**2*c**2))
```

3.90 $\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx = \frac{(3b^2-6ac-13bc-20c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(9a+6b+4c)(a-5b+25c)\sqrt{b^2-4ac}} - \frac{9\log(2-3x)}{17(9a+6b+4c)} + \frac{\log(5+x)}{17(a-5b+25c)} - \frac{(3b-13c)\log(a+bx+cx^2)}{2(9a+6b+4c)(a-5b+25c)}$$

output

```
(-6*a*c+3*b^2-13*b*c-20*c^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(9*a+6*
b+4*c)/(a-5*b+25*c)/(-4*a*c+b^2)^(1/2)-9*ln(2-3*x)/(153*a+102*b+68*c)+ln(5
+x)/(17*a-85*b+425*c)-1/2*(3*b-13*c)*ln(c*x^2+b*x+a)/(9*a+6*b+4*c)/(a-5*b+
25*c)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx =$$
$$-\frac{34(3b^2-13bc-2c(3a+10c))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}(18(a-5b+25c)\log(2-3x)-2(9a}{34(9a+6b+4c)(a-5b+25c)\sqrt{-b^2+4ac}}$$

input
```
Integrate[1/((10 - 13*x - 3*x^2)*(a + b*x + c*x^2)),x]
```

output
```
-1/34*(34*(3*b^2 - 13*b*c - 2*c*(3*a + 10*c))*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(18*(a - 5*b + 25*c)*Log[2 - 3*x] - 2*(9*a
 + 6*b + 4*c)*Log[5 + x] + 17*(3*b - 13*c)*Log[a + x*(b + c*x)]))/((9*a +
6*b + 4*c)*(a - 5*b + 25*c)*Sqrt[-b^2 + 4*a*c])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1299, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-13x+10)(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1299}$$

$$-3\int \left(\frac{3b^2-13cb-10c^2-3ac+(3b-13c)cx}{3(9a+6b+4c)(a-5b+25c)(cx^2+bx+a)}-\frac{9}{17(9a+6b+4c)(2-3x)}-\frac{1}{51(a-5b+25c)(x+5)}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-3\left(-\frac{(-6ac+3b^2-13bc-20c^2)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3(9a+6b+4c)(a-5b+25c)\sqrt{b^2-4ac}}+\frac{(3b-13c)\log(a+bx+cx^2)}{6(9a+6b+4c)(a-5b+25c)}+\frac{3\log(2-3x)}{17(9a+6b+4c)}-\frac{}{5}\right.$$

input
```
Int[1/((10 - 13*x - 3*x^2)*(a + b*x + c*x^2)),x]
```

output
```
-3*(-1/3*((3*b^2 - 6*a*c - 13*b*c - 20*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/((9*a + 6*b + 4*c)*(a - 5*b + 25*c)*Sqrt[b^2 - 4*a*c]) + (3*Log[
2 - 3*x])/(17*(9*a + 6*b + 4*c)) - Log[5 + x]/(51*(a - 5*b + 25*c)) + ((3*
b - 13*c)*Log[a + b*x + c*x^2])/(6*(9*a + 6*b + 4*c)*(a - 5*b + 25*c)))
```

Defintions of rubi rules used

rule 1299
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[Ex
pandIntegrand[(b/2 - r/2 + c*x)^p*(b/2 + r/2 + c*x)^p*(d + e*x + f*x^2)^q,
x], x], x] /; EqQ[p, -1] ||  !FractionalPowerFactorQ[r]] /; FreeQ[{a, b, c,
 d, e, f}, x] && ILtQ[p, 0] && IntegerQ[q] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(x+5)}{17a-85b+425c}-\frac{9\ln(-2+3x)}{153a+102b+68c}+\frac{\frac{\left(-3bc+13c^{2}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(3ac-3b^{2}+13bc+10c^{2}-\frac{\left(-3bc+13c^{2}\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{(a-5b+25c)(9a+6b+4c)}$
risch	Expression too large to display

input
```
int(1/(-3*x^2-13*x+10)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x+5)/(17*a-85*b+425*c)-9/(153*a+102*b+68*c)*ln(-2+3*x)+1/(a-5*b+25*c)/(
9*a+6*b+4*c)*(1/2*(-3*b*c+13*c^2)/c*ln(c*x^2+b*x+a)+2*(3*a*c-3*b^2+13*b*c+
10*c^2-1/2*(-3*b*c+13*c^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 553, normalized size of antiderivative = 3.48

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx$$
$$=\left[-\frac{17\,(3\,b^2-(6\,a+13\,b)c-20\,c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+17\,(3\,b^3+52\,ac^2}{34\,(9\,a^2b^2-39\,ab^3-30}\right.$$

input
```
integrate(1/(-3*x^2-13*x+10)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/34*(17*(3*b^2 - (6*a + 13*b)*c - 20*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 17*(3*b^3 + 52*a*c^2 - (12*a*b + 13*b^2)*c)*log(c*x^2 + b*x + a) +
 18*(a*b^2 - 5*b^3 - 100*a*c^2 - (4*a^2 - 20*a*b - 25*b^2)*c)*log(3*x - 2)
 - 2*(9*a*b^2 + 6*b^3 - 16*a*c^2 - 4*(9*a^2 + 6*a*b - b^2)*c)*log(x + 5))/
(9*a^2*b^2 - 39*a*b^3 - 30*b^4 - 400*a*c^3 - 4*(229*a^2 + 130*a*b - 25*b^2
)*c^2 - (36*a^3 - 156*a^2*b - 349*a*b^2 - 130*b^3)*c), 1/34*(34*(3*b^2 - (
6*a + 13*b)*c - 20*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c
*x + b)/(b^2 - 4*a*c)) - 17*(3*b^3 + 52*a*c^2 - (12*a*b + 13*b^2)*c)*log(c
*x^2 + b*x + a) - 18*(a*b^2 - 5*b^3 - 100*a*c^2 - (4*a^2 - 20*a*b - 25*b^2
)*c)*log(3*x - 2) + 2*(9*a*b^2 + 6*b^3 - 16*a*c^2 - 4*(9*a^2 + 6*a*b - b^2
)*c)*log(x + 5))/(9*a^2*b^2 - 39*a*b^3 - 30*b^4 - 400*a*c^3 - 4*(229*a^2 +
 130*a*b - 25*b^2)*c^2 - (36*a^3 - 156*a^2*b - 349*a*b^2 - 130*b^3)*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-3*x**2-13*x+10)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(10-13x-3x^2)\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-3*x^2-13*x+10)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{(10-13x-3x^2)\,(a+bx+cx^2)}\,dx &= -\frac{(3\,b-13\,c)\log\left(cx^2+bx+a\right)}{2\,(9\,a^2-39\,ab-30\,b^2+229\,ac+130\,bc+100\,c^2)} \\ &\quad - \frac{(3\,b^2-6\,ac-13\,bc-20\,c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(9\,a^2-39\,ab-30\,b^2+229\,ac+130\,bc+100\,c^2)\sqrt{-b^2+4\,ac}} \\ &\quad - \frac{9\,\log\left(|3\,x-2|\right)}{17\,(9\,a+6\,b+4\,c)} + \frac{\log\left(|x+5|\right)}{17\,(a-5\,b+25\,c)}\end{aligned}$$

input
```
integrate(1/(-3*x^2-13*x+10)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(3*b - 13*c)*log(c*x^2 + b*x + a)/(9*a^2 - 39*a*b - 30*b^2 + 229*a*c
+ 130*b*c + 100*c^2) - (3*b^2 - 6*a*c - 13*b*c - 20*c^2)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((9*a^2 - 39*a*b - 30*b^2 + 229*a*c + 130*b*c + 100*
c^2)*sqrt(-b^2 + 4*a*c)) - 9/17*log(abs(3*x - 2))/(9*a + 6*b + 4*c) + 1/17
*log(abs(x + 5))/(a - 5*b + 25*c)
```

Mupad [B] (verification not implemented)

Time = 19.34 (sec) , antiderivative size = 2331, normalized size of antiderivative = 14.66

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(-1/((13*x + 3*x^2 - 10)*(a + b*x + c*x^2)),x)
```

output
```
log(x + 5)/(17*a - 85*b + 425*c) - (9*log(x - 2/3))/(17*(9*a + 6*b + 4*c))
 - (log(3757000*a*c^5 - 16416*a*b^5 + 289000*b*c^5 + 2700*b^5*c - 16416*b^
6*x + 578000*c^6*x - 289000*c^5*(b^2 - 4*a*c)^(1/2) - 1053*a^2*b^4 + 243*a
^3*b^3 - 1461798*a^2*c^4 + 552474*a^3*c^3 + 8424*a^4*c^2 - 375700*b^2*c^4
+ 77700*b^3*c^3 - 11700*b^4*c^2 - 16416*a*b^4*(b^2 - 4*a*c)^(1/2) + 312277
0*a*c^4*(b^2 - 4*a*c)^(1/2) - 972*a^4*c*(b^2 - 4*a*c)^(1/2) + 375700*b*c^4
*(b^2 - 4*a*c)^(1/2) - 2700*b^4*c*(b^2 - 4*a*c)^(1/2) - 16416*b^5*x*(b^2 -
 4*a*c)^(1/2) + 1878500*c^5*x*(b^2 - 4*a*c)^(1/2) + 391586*a*b^2*c^3 - 808
218*a*b^3*c^2 + 2828133*a^2*b*c^3 + 95769*a^2*b^3*c - 128223*a^3*b*c^2 + 2
106*a^3*b^2*c + 243*a^2*b^4*x - 1708782*a^2*c^4*x + 71550*a^3*c^3*x + 1944
*a^4*c^2*x + 2163950*b^2*c^4*x + 107666*b^3*c^3*x - 756198*b^4*c^2*x - 105
3*a^2*b^3*(b^2 - 4*a*c)^(1/2) + 243*a^3*b^2*(b^2 - 4*a*c)^(1/2) + 854391*a
^2*c^3*(b^2 - 4*a*c)^(1/2) - 35775*a^3*c^2*(b^2 - 4*a*c)^(1/2) - 77700*b^2
*c^3*(b^2 - 4*a*c)^(1/2) + 11700*b^3*c^2*(b^2 - 4*a*c)^(1/2) - 955071*a^2*
b^2*c^2 + 894330*a*b*c^4 + 216918*a*b^4*c - 972*a^4*b*c - 1053*a*b^5*x - 6
245540*a*c^5*x + 1127100*b*c^5*x + 209898*b^5*c*x - 196101*a^2*b^2*c^2*x -
 471991*a*b*c^4*x + 109242*a*b^4*c*x - 129454*a*b*c^3*(b^2 - 4*a*c)^(1/2)
+ 202878*a*b^3*c*(b^2 - 4*a*c)^(1/2) + 4212*a^3*b*c*(b^2 - 4*a*c)^(1/2) -
1053*a*b^4*x*(b^2 - 4*a*c)^(1/2) - 730899*a*c^4*x*(b^2 - 4*a*c)^(1/2) + 20
08550*b*c^4*x*(b^2 - 4*a*c)^(1/2) + 209898*b^4*c*x*(b^2 - 4*a*c)^(1/2) ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.64

$$\int \frac{1}{(10-13x-3x^2)(a+bx+cx^2)}\,dx$$

$$= \frac{204\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac - 102\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2 + 442\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc +}{}$$

input
```
int(1/(-3*x^2-13*x+10)/(c*x^2+b*x+a),x)
```

output
```
(204*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c - 102*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2 + 442*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c + 680*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2 - 72*log(3*x - 2)*a**2*c + 18*l
og(3*x - 2)*a*b**2 + 360*log(3*x - 2)*a*b*c - 1800*log(3*x - 2)*a*c**2 - 9
0*log(3*x - 2)*b**3 + 450*log(3*x - 2)*b**2*c - 204*log(a + b*x + c*x**2)*
a*b*c + 884*log(a + b*x + c*x**2)*a*c**2 + 51*log(a + b*x + c*x**2)*b**3 -
 221*log(a + b*x + c*x**2)*b**2*c + 72*log(x + 5)*a**2*c - 18*log(x + 5)*a
*b**2 + 48*log(x + 5)*a*b*c + 32*log(x + 5)*a*c**2 - 12*log(x + 5)*b**3 -
8*log(x + 5)*b**2*c)/(34*(36*a**3*c - 9*a**2*b**2 - 156*a**2*b*c + 916*a**
2*c**2 + 39*a*b**3 - 349*a*b**2*c + 520*a*b*c**2 + 400*a*c**3 + 30*b**4 -
130*b**3*c - 100*b**2*c**2))
```

3.91 $\int \frac{1+x^2}{-x+x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{1+x^2}{-x+x^2} \, dx = x+2\log(1-x)-\log(x)$$

output
```
x+2*ln(1-x)-ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2} \, dx = x+2\log(1-x)-\log(x)$$

input
```
Integrate[(1 + x^2)/(-x + x^2),x]
```

output
```
x + 2*Log[1 - x] - Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2026, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^2-x}\,dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^2+1}{(x-1)x}dx$$
$$\Big\downarrow \text{ 522}$$
$$\int \left(-\frac{1}{x}+\frac{2}{x-1}+1\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$x+2\log(1-x)-\log(x)$$

input
```
Int[(1 + x^2)/(-x + x^2),x]
```

output
```
x + 2*Log[1 - x] - Log[x]
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x-\ln \left(x \right)+2 \ln \left(x -1\right)$	13
norman	$x-\ln \left(x \right)+2 \ln \left(x -1\right)$	13
risch	$x-\ln \left(x \right)+2 \ln \left(x -1\right)$	13
parallelrisch	$x-\ln \left(x \right)+2 \ln \left(x -1\right)$	13
meijerg	$2 \ln \left(1-x \right)-\ln \left(x \right)-i \pi +x$	19

input
```
int((x^2+1)/(x^2-x),x,method=_RETURNVERBOSE)
```

output
```
x-ln(x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx=x+2\,\log\left(x-1\right)-\log\left(x\right)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
```

output
```
x + 2*log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{-x+x^2}\,dx = x - \log(x) + 2\log(x-1)$$

input
```
integrate((x**2+1)/(x**2-x),x)
```

output
```
x - log(x) + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx = x + 2\log(x-1) - \log(x)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
```

output
```
x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2}\,dx = x + 2\log(|x-1|) - \log(|x|)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="giac")
```

output
```
x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx = x + 2\,\ln(x-1) - \ln(x)$$

input
```
int(-(x^2 + 1)/(x - x^2),x)
```

output
```
x + 2*log(x - 1) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx = 2\log(x-1) - \log(x) + x$$

input
```
int((x^2+1)/(x^2-x),x)
```

output
```
2*log(x - 1) - log(x) + x
```

3.92 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$

Optimal result

Integrand size = 27, antiderivative size = 208

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx &= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} \\ &+ \frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920} + \frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160} \\ &- \frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760} + \frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192} \\ &+ \frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008} + \frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536} \\ &+ \frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2} + \frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2} - \frac{8267844569\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}\end{aligned}$$

output
```
-359471503/67108864*(1-4*x)*(2*x^2-x+3)^(1/2)+27185733541/440401920*(2*x^2
-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(3/2)-83948353/2293760*x^2*(2
*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3)^(3/2)+4796405/43008*x^4*(2
*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3/2)+14125/144*x^6*(2*x^2-x+3
)^(3/2)+125/4*x^7*(2*x^2-x+3)^(3/2)-8267844569/268435456*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.46

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$= \frac{4\sqrt{3-x+2x^2}(3801512106459+537752185764x-174418077792x^2+2211683657856x^3+535474199}{}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2
+ 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 772596203520
0*x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043
71039235*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/84557168640
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^4\,dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{20}\int \frac{5}{2}\sqrt{2x^2-x+3}\left(14125x^7+13550x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(14125x^7+13550x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(233225x^6+55140x^5+169056x^4+89856x^3+36096x^2+9216x+1536\right)dx+\frac{14125}{18}\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\int \sqrt{2x^2-x+3}\left(233225x^6+55140x^5+169056x^4+89856x^3+36096x^2+9216x+1536\right)dx+\frac{14125}{18}\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{16}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(4796405x^5-1586958x^4+2875392x^3+1155072x^2+294912x+49152\right)dx+\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\int \sqrt{2x^2-x+3}\left(4796405x^5-1586958x^4+2875392x^3+1155072x^2+294912x+49152\right)dx+\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(8325631x^4-34602744x^3+32342016x^2+8257536x+1376256\right)dx+\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\int \sqrt{2x^2-x+3}\left(8325631x^4-34602744x^3+32342016x^2+8257536x+1376256\right)dx+\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{9}{2}\sqrt{2x^2-x+3}\left(-83948353x^3+69594114x^2+22020096x+3670016\right)dx+\frac{8325631}{12}\left(2x\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\int \sqrt{2x^2-x+3}\left(-83948353x^3+69594114x^2+22020096x+3670016\right)dx+\frac{8325631}{12}\left(2x^2\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{10}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(804243809x^2+1447782156x+73400320\right)dx-\frac{83948353}{10}x^2\left(2x^2-x\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\int \sqrt{2x^2-x+3}\left(804243809x^2+1447782156x+73400320\right)dx-\frac{83948353}{10}x^2\left(2x^2-x\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int -\frac{1}{2}(3651057734-27185733541x)\sqrt{2x^2-x+3}dx+\frac{804243809}{8}x\left(2x^2-x+3\right)^{3/}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{804243809}{8}x\left(2x^2-x+3\right)^{3/2}-\frac{1}{16}\int (3651057734-27185733541x)\sqrt{2x^2-x+3}dx\right)\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\int\sqrt{2x^2-x+3}dx+\frac{27185733541}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{804243809}{8}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$\Big\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{27185733541}{6}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$\Big\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$\Big\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{27185733541}{6}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(125*x^7*(3 - x + 2*x^2)^(3/2))/4 + ((14125*x^6*(3 - x + 2*x^2)^(3/2))/18
+ ((233225*x^5*(3 - x + 2*x^2)^(3/2))/16 + ((4796405*x^4*(3 - x + 2*x^2)^(
3/2))/14 + ((8325631*x^3*(3 - x + 2*x^2)^(3/2))/12 + (3*((-83948353*x^2*(3
 - x + 2*x^2)^(3/2))/10 + ((804243809*x*(3 - x + 2*x^2)^(3/2))/8 + ((27185
733541*(3 - x + 2*x^2)^(3/2))/6 + (12581502605*(-1/8*((1 - 4*x)*Sqrt[3 - x
 + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/4)/16)/20))/
8)/28)/32)/12)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

method	result
risch	$\frac{(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+5354741991424x^4+221168365}{21139292160}$
trager	$\left(\frac{125}{2}x^9+\frac{11875}{72}x^8+\frac{689675}{2304}x^7+\frac{7859255}{21504}x^6+\frac{185859571}{516096}x^5+\frac{373517159}{1474560}x^4+\frac{5759592859}{55050240}x^3-\frac{259550711}{31457280}x^2\right.$
default	$\frac{359471503(4x-1)\sqrt{2x^2-x+3}}{67108864}+\frac{8267844569\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{268435456}+\frac{27185733541\left(2x^2-x+3\right)^{\frac{3}{2}}}{440401920}+\frac{804243809x\left(2x^2-x\right.}{36700160}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/21139292160*(1321205760000*x^9+3486515200000*x^8+6327795712000*x^7+77259
62035200*x^6+7612808028160*x^5+5354741991424*x^4+2211683657856*x^3-1744180
77792*x^2+537752185764*x+3801512106459)*(2*x^2-x+3)^(1/2)+8267844569/26843
5456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.47

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$=\frac{1}{21139292160}\left(1321205760000\,x^9+3486515200000\,x^8+6327795712000\,x^7+7725962035200\,x^6+7612\right.$$
$$+\frac{8267844569}{536870912}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
 7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856
*x^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x +
 3) + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.47

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{125x^9}{2}+\frac{11875x^8}{72}+\frac{689675x^7}{2304}+\frac{7859255x^6}{21504}+\frac{185859571x^5}{516096}+\frac{373517159x^4}{1474560}+\frac{5759592859x^3}{55050240}-\frac{259550711x^2}{31457280}+\frac{44812682147x}{1761607680}+\frac{422390234051}{2348810240}\right)+\frac{8267844569\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{268435456}$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**2+3*x+2)**4,x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**9/2 + 11875*x**8/72 + 689675*x**7/2304 + 7859
255*x**6/21504 + 185859571*x**5/516096 + 373517159*x**4/1474560 + 57595928
59*x**3/55050240 - 259550711*x**2/31457280 + 44812682147*x/1761607680 + 42
2390234051/2348810240) + 8267844569*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)
/268435456
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx=\frac{125}{4}\left(2x^2-x+3\right)^{\frac{3}{2}}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{\frac{3}{2}}x^6+\frac{233225}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{4796405}{43008}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{8325631}{1032192}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3-\frac{83948353}{2293760}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{804243809}{36700160}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{27185733541}{440401920}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{359471503}{16777216}\sqrt{2x^2-x+3}\,x+\frac{8267844569}{268435456}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{359471503}{67108864}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 23
3225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*
x^4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2
- x + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733
541/440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x +
3)*x + 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359
471503/67108864*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.45

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$=\frac{1}{21139292160}\left(4\left(8\left(4\left(16\left(20\left(40\left(140\left(160\left(36\,x+95\right)x+27587\right)x+4715553\right)x+185859571\right)x+26146\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{8267844569}{268435456}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 47
15553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x +
 134438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/2684354
56*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$=\frac{8325631\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{1032192}-\frac{83948353\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{2293760}$$
$$+\frac{4796405\,x^4\,\left(2\,x^2-x+3\right)^{3/2}}{43008}+\frac{233225\,x^5\,\left(2\,x^2-x+3\right)^{3/2}}{1536}$$
$$+\frac{14125\,x^6\,\left(2\,x^2-x+3\right)^{3/2}}{144}+\frac{125\,x^7\,\left(2\,x^2-x+3\right)^{3/2}}{4}$$
$$-\frac{41987163941\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{1174405120}$$
$$-\frac{1825528867\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{36700160}$$
$$+\frac{27185733541\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{7046430720}+\frac{804243809\,x\,\left(2\,x^2-x+3\right)^{3/2}}{36700160}$$
$$+\frac{625271871443\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{9395240960}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
(8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3
)^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5
*(2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (12
5*x^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^
(1/2) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (1825528867*(x/2 - 1/8)*(2*
x^2 - x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2
- 4*x + 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (
625271871443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))
/9395240960
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \frac{125\sqrt{2x^2-x+3}\,x^9}{2}+\frac{11875\sqrt{2x^2-x+3}\,x^8}{72} + \frac{689675\sqrt{2x^2-x+3}\,x^7}{2304} + \frac{7859255\sqrt{2x^2-x+3}\,x^6}{21504} + \frac{185859571\sqrt{2x^2-x+3}\,x^5}{516096} + \frac{373517159\sqrt{2x^2-x+3}\,x^4}{1474560} + \frac{5759592859\sqrt{2x^2-x+3}\,x^3}{55050240} - \frac{259550711\sqrt{2x^2-x+3}\,x^2}{31457280} + \frac{44812682147\sqrt{2x^2-x+3}\,x}{1761607680} + \frac{422390234051\sqrt{2x^2-x+3}}{2348810240} + \frac{8267844569\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{268435456}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^4,x)
```

output
```
(5284823040000*sqrt(2*x**2 - x + 3)*x**9 + 13946060800000*sqrt(2*x**2 - x
+ 3)*x**8 + 25311182848000*sqrt(2*x**2 - x + 3)*x**7 + 30903848140800*sqrt
(2*x**2 - x + 3)*x**6 + 30451232112640*sqrt(2*x**2 - x + 3)*x**5 + 2141896
7965696*sqrt(2*x**2 - x + 3)*x**4 + 8846734631424*sqrt(2*x**2 - x + 3)*x**
3 - 697672311168*sqrt(2*x**2 - x + 3)*x**2 + 2151008743056*sqrt(2*x**2 - x
 + 3)*x + 15206048425836*sqrt(2*x**2 - x + 3) + 2604371039235*sqrt(2)*log(
(2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/84557168640
```

3.93 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx &= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} \\ &\quad -\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880} \\ &\quad +\frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752} \\ &\quad +\frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}-\frac{155620231\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}\end{aligned}$$

output
```
-6766097/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)-22548119/4587520*(2*x^2-x+3)^(3
/2)-9627393/1146880*x*(2*x^2-x+3)^(3/2)+531681/71680*x^2*(2*x^2-x+3)^(3/2)
+247435/10752*x^3*(2*x^2-x+3)^(3/2)+8825/448*x^4*(2*x^2-x+3)^(3/2)+125/16*
x^5*(2*x^2-x+3)^(3/2)-155620231/8388608*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3 dx$$
$$= \frac{4\sqrt{3-x+2x^2}(-3957369321-1621307916x+4583812128x^2+9872163456x^3+11212171264x^4+10\ldots}{8808038\ldots}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 9872
163456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 34406400
00*x^7) - 16340124255*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8808
03840
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^3 dx$$
$$\Big\downarrow \text{2192}$$
$$\frac{1}{16}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(8825x^5+5370x^4+6624x^3+3648x^2+1152x+256\right)dx+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{32}\int \sqrt{2x^2-x+3}\left(8825x^5+5370x^4+6624x^3+3648x^2+1152x+256\right)dx+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(247435x^4-26328x^3+102144x^2+32256x+7168\right)dx+\frac{8825}{14}\left(2x^2-x+3\right)^{3/2}x^4\right)
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\int \sqrt{2x^2-x+3}\left(247435x^4-26328x^3+102144x^2+32256x+7168\right)dx+\frac{8825}{14}\left(2x^2-x+3\right)^{3/2}x^4\right)+
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(531681x^3-667458x^2+258048x+57344\right)dx+\frac{247435}{12}\left(2x^2-x+3\right)^{3/2}x^3\right)+
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\int \sqrt{2x^2-x+3}\left(531681x^3-667458x^2+258048x+57344\right)dx+\frac{247435}{12}\left(2x^2-x+3\right)^{3/2}x^3\right)+\frac{88}{1}
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{10}\int \frac{1}{2}\left(-9627393x^2-1219212x+1146880\right)\sqrt{2x^2-x+3}dx+\frac{531681}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{24}{}
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\int \left(-9627393x^2-1219212x+1146880\right)\sqrt{2x^2-x+3}dx+\frac{531681}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{247}{1}
\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int \frac{1}{2}(76114438-67644357x)\sqrt{2x^2-x+3}dx-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{531681}{10}\left(2x^2\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\int(76114438-67644357x)\sqrt{2x^2-x+3}dx-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{531681}{10}\left(2x^2\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$\big\downarrow$ 1160

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\int\sqrt{2x^2-x+3}dx-\frac{22548119}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3}\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$\big\downarrow$ 1087

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{22548119}{2}\left(2x^2-x+3\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$\big\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{225}{}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$\big\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{22548119}{2}\left(2x^2-x+3\right)^{3/}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(125*x^5*(3 - x + 2*x^2)^(3/2))/16 + ((8825*x^4*(3 - x + 2*x^2)^(3/2))/14
+ ((247435*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((531681*x^2*(3 - x + 2*x^2)^(3
/2))/10 + ((-9627393*x*(3 - x + 2*x^2)^(3/2))/8 + ((-22548119*(3 - x + 2*x
^2)^(3/2))/2 + (236813395*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcS
inh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/4)/16)/20)/8)/28)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321)\sqrt{}}{220200960}$
trager	$\left(\frac{125}{8}x^7+\frac{7075}{224}x^6+\frac{267535}{5376}x^5+\frac{782099}{15360}x^4+\frac{25708759}{573440}x^3+\frac{6821149}{327680}x^2-\frac{135108993}{18350080}x-\frac{1319123107}{73400320}\right)\sqrt{2x^2-}$
default	$\frac{6766097(4x-1)\sqrt{2x^2-x+3}}{2097152}+\frac{155620231\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}-\frac{22548119(2x^2-x+3)^{\frac{3}{2}}}{4587520}-\frac{9627393x(2x^2-x+3)^{\frac{3}{2}}}{1146880}+$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/220200960*(3440640000*x^7+6955008000*x^6+10958233600*x^5+11212171264*x^4
+9872163456*x^3+4583812128*x^2-1621307916*x-3957369321)*(2*x^2-x+3)^(1/2)+
155620231/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$$
$$=\frac{1}{220200960}\left(3440640000\,x^7+6955008000\,x^6+10958233600\,x^5+11212171264\,x^4+9872163456\,x^3+45\right.$$
$$+\frac{155620231}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 112121712
64*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt
(2*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{125x^7}{8}+\frac{7075x^6}{224}+\frac{267535x^5}{5376}+\frac{782099x^4}{15360}+\frac{25708759x^3}{573440}+\frac{6821149x^2}{327680}-\frac{135108993x}{18350080}-\frac{1319123107}{73400320}\right)+\frac{155620231\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**2+3*x+2)**3,x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**7/8 + 7075*x**6/224 + 267535*x**5/5376 + 7820
99*x**4/15360 + 25708759*x**3/573440 + 6821149*x**2/327680 - 135108993*x/1
8350080 - 1319123107/73400320) + 155620231*sqrt(2)*asinh(4*sqrt(23)*(x - 1
/4)/23)/8388608
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx = \frac{125}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4 +\frac{247435}{10752}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3 +\frac{531681}{71680}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2 -\frac{9627393}{1146880}\left(2x^2-x+3\right)^{\frac{3}{2}}x -\frac{22548119}{4587520}\left(2x^2-x+3\right)^{\frac{3}{2}} +\frac{6766097}{524288}\sqrt{2x^2-x+3}x +\frac{155620231}{8388608}\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) -\frac{6766097}{2097152}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 24
7435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*
x^2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 -
x + 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x +
3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$$

$$=\frac{1}{220200960}\left(4\left(8\left(4\left(16\left(100\left(120\left(140x+283\right)x+53507\right)x+5474693\right)x+77126277\right)x+143244129\right)x -\right.\right. -\frac{155620231}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
 + 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x
+ 3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$$
$$= \frac{531681\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{71680}+\frac{247435\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{10752}+\frac{8825\,x^4\,\left(2\,x^2-x+3\right)^{3/2}}{448}+\frac{125\,x^5\,\left(2\,x^2-x+3\right)^{3/2}}{16}+\frac{875316037\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{36700160}+\frac{38057219\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{1146880}-\frac{22548119\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{73400320}-\frac{9627393\,x\,\left(2\,x^2-x+3\right)^{3/2}}{1146880}-\frac{1555820211\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{293601280}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
(531681*x^2*(2*x^2 - x + 3)^(3/2))/71680 + (247435*x^3*(2*x^2 - x + 3)^(3/
2))/10752 + (8825*x^4*(2*x^2 - x + 3)^(3/2))/448 + (125*x^5*(2*x^2 - x + 3
)^(3/2))/16 + (875316037*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x
 - 1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/11468
80 - (22548119*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/73400320 - (9627
393*x*(2*x^2 - x + 3)^(3/2))/1146880 - (1555820211*2^(1/2)*log(2*(2*x^2 -
x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/293601280
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx = \frac{125\sqrt{2x^2-x+3}\,x^7}{8}+\frac{7075\sqrt{2x^2-x+3}\,x^6}{224}+\frac{267535\sqrt{2x^2-x+3}\,x^5}{5376}+\frac{782099\sqrt{2x^2-x+3}\,x^4}{15360}+\frac{25708759\sqrt{2x^2-x+3}\,x^3}{573440}+\frac{6821149\sqrt{2x^2-x+3}\,x^2}{327680}-\frac{135108993\sqrt{2x^2-x+3}\,x}{18350080}-\frac{1319123107\sqrt{2x^2-x+3}}{73400320}+\frac{155620231\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{8388608}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^3,x)
```

output
```
(13762560000*sqrt(2*x**2 - x + 3)*x**7 + 27820032000*sqrt(2*x**2 - x + 3)*
x**6 + 43832934400*sqrt(2*x**2 - x + 3)*x**5 + 44848685056*sqrt(2*x**2 - x
 + 3)*x**4 + 39488653824*sqrt(2*x**2 - x + 3)*x**3 + 18335248512*sqrt(2*x*
*2 - x + 3)*x**2 - 6485231664*sqrt(2*x**2 - x + 3)*x - 15829477284*sqrt(2*
x**2 - x + 3) + 16340124255*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) +
4*x - 1)/sqrt(23)))/880803840
```

3.94 $\int \sqrt{3-x+2x^2}(2+3x+5x^2)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx &= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}\\ &\quad-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}\\ &\quad+\frac{769}{256}x\left(3-x+2x^2\right)^{3/2}+\frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}\\ &\quad+\frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{284533\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}\end{aligned}$$

output
```
12371/16384*(1-4*x)*(2*x^2-x+3)^(1/2)-2107/3072*(2*x^2-x+3)^(3/2)+769/256*
x*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x+3)^(3/2)+25/12*x^3*(2*x^2-x+3)^(3/2
)+284533/65536*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$$
$$=\frac{4\sqrt{3-x+2x^2}(-64023+328204x+365536x^2+408960x^3+284672x^4+204800x^5)+853599\sqrt{2}\log\left(\right.}{196608}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 2846
72*x^4 + 204800*x^5) + 853599*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/196608
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(315x^3+82x^2+96x+32\right)dx+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(315x^3+82x^2+96x+32\right)dx+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{10}\int \frac{5}{2}\sqrt{2x^2-x+3}(769x^2-372x+128)\,dx+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{4}\int \sqrt{2x^2-x+3}(769x^2-372x+128)\,dx+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\int -\frac{1}{2}(2107x+2566)\sqrt{2x^2-x+3}dx+\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}-\frac{1}{16}\int(2107x+2566)\sqrt{2x^2-x+3}dx\right)+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 1160

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\int \sqrt{2x^2-x+3}dx-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{63}{2}\left(2x^2-x+3\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x(2x^2\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$\big\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right.\right.\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x\left(2x^2-\right.\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(25*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((63*x^2*(3 - x + 2*x^2)^(3/2))/2 + ((
769*x*(3 - x + 2*x^2)^(3/2))/8 + ((-2107*(3 - x + 2*x^2)^(3/2))/6 - (12371
*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(16*Sqrt[2])))/4)/16)/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023)\sqrt{2x^2-x+3}}{49152} - \frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{25}{6}x^5 + \frac{139}{24}x^4 + \frac{1065}{128}x^3 + \frac{11423}{1536}x^2 + \frac{82051}{12288}x - \frac{21341}{16384}\right)\sqrt{2x^2-x+3} - \frac{284533\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{Roo}\right.}{}$
default	$-\frac{12371(4x-1)\sqrt{2x^2-x+3}}{16384} - \frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536} - \frac{2107(2x^2-x+3)^{\frac{3}{2}}}{3072} + \frac{769x(2x^2-x+3)^{\frac{3}{2}}}{256} + \frac{63x^2(2x^2-}{16}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/49152*(204800*x^5+284672*x^4+408960*x^3+365536*x^2+328204*x-64023)*(2*x^
2-x+3)^(1/2)-284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{1}{49152}\left(204800\,x^5+284672\,x^4+408960\,x^3+365536\,x^2+328204\,x-64023\right)\sqrt{2\,x^2-x+3} + \frac{284533}{131072}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 64
023)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^5}{6}+\frac{139x^4}{24}+\frac{1065x^3}{128}+\frac{11423x^2}{1536}+\frac{82051x}{12288}-\frac{21341}{16384}\right) - \frac{284533\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**2+3*x+2)**2,x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**5/6 + 139*x**4/24 + 1065*x**3/128 + 11423*x**2
/1536 + 82051*x/12288 - 21341/16384) - 284533*sqrt(2)*asinh(4*sqrt(23)*(x
- 1/4)/23)/65536
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3 + \frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2 + \frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{12371}{4096}\sqrt{2x^2-x+3}x - \frac{284533}{65536}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{12371}{16384}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/25
6*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*s
qrt(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1
)) + 12371/16384*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{1}{49152}\left(4\left(8\left(4\left(16\left(100x+139\right)x+3195\right)x+11423\right)x+82051\right)x-64023\right)\sqrt{2x^2-x+3} + \frac{284533}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{63\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{16}+\frac{25\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{12} - \frac{29509\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{8192} - \frac{1283\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{256} - \frac{2107\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{49152} + \frac{769\,x\,\left(2\,x^2-x+3\right)^{3/2}}{256} - \frac{48461\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{65536}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
(63*x^2*(2*x^2 - x + 3)^(3/2))/16 + (25*x^3*(2*x^2 - x + 3)^(3/2))/12 - (2
9509*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/8192 -
(1283*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/256 - (2107*(2*x^2 - x + 3)^(1/2)
*(32*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^(3/2))/256 - (48461*2
^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/65536
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{25\sqrt{2x^2-x+3}\,x^5}{6}+\frac{139\sqrt{2x^2-x+3}\,x^4}{24} + \frac{1065\sqrt{2x^2-x+3}\,x^3}{128}+\frac{11423\sqrt{2x^2-x+3}\,x^2}{1536} + \frac{82051\sqrt{2x^2-x+3}\,x}{12288}-\frac{21341\sqrt{2x^2-x+3}}{16384} - \frac{284533\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{65536}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2)^2,x)
```

output
```
(819200*sqrt(2*x**2 - x + 3)*x**5 + 1138688*sqrt(2*x**2 - x + 3)*x**4 + 16
35840*sqrt(2*x**2 - x + 3)*x**3 + 1462144*sqrt(2*x**2 - x + 3)*x**2 + 1312
816*sqrt(2*x**2 - x + 3)*x - 256092*sqrt(2*x**2 - x + 3) - 853599*sqrt(2)*
log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/196608
```

3.95 $\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2} + \frac{5}{8}x\left(3-x+2x^2\right)^{3/2}-\frac{1863\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

output
```
-81/512*(1-4*x)*(2*x^2-x+3)^(1/2)+73/96*(2*x^2-x+3)^(3/2)+5/8*x*(2*x^2-x+3
)^(3/2)-1863/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = \frac{4\sqrt{3-x+2x^2}(3261+2684x+1376x^2+1920x^3)-5589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{6144}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6144
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{1}{2}(73x+2)\sqrt{2x^2-x+3}dx+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int (73x+2)\sqrt{2x^2-x+3}dx+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{16}\left(\frac{81}{4}\int \sqrt{2x^2-x+3}dx+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(3/2))/8 + ((73*(3 - x + 2*x^2)^(3/2))/6 + (81*(-1/8*
((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sq
rt[2])))/4)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(1920x^3+1376x^2+2684x+3261)\sqrt{2x^2-x+3}}{1536}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}$
default	$\frac{81(4x-1)\sqrt{2x^2-x+3}}{512}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}+\frac{73\left(2x^2-x+3\right)^{\frac{3}{2}}}{96}+\frac{5x\left(2x^2-x+3\right)^{\frac{3}{2}}}{8}$
trager	$\left(\frac{5}{4}x^3+\frac{43}{48}x^2+\frac{671}{384}x+\frac{1087}{512}\right)\sqrt{2x^2-x+3}+\frac{1863\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\mathrm{R}\right.}{2048}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(1920*x^3+1376*x^2+2684*x+3261)*(2*x^2-x+3)^(1/2)+1863/2048*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx$$
$$=\frac{1}{1536}\left(1920\,x^3+1376\,x^2+2684\,x+3261\right)\sqrt{2\,x^2-x+3}$$
$$+\frac{1863}{4096}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/40
96*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
25)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^3}{4}+\frac{43x^2}{48}+\frac{671x}{384}+\frac{1087}{512}\right) + \frac{1863\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**2+3*x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**3/4 + 43*x**2/48 + 671*x/384 + 1087/512) + 1863
*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/2048
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \frac{5}{8}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{81}{128}\sqrt{2\,x^2-x+3}x + \frac{1863}{2048}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{81}{512}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*
x^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/5
12*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx$$
$$=\frac{1}{1536}\left(4\left(8\left(60\,x+43\right)x+671\right)x+3261\right)\sqrt{2\,x^2-x+3}-\frac{1863}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/204
8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx=\frac{23\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{256}+\frac{\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2\,x^2-x+3}}{8}+\frac{73\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{1536}+\frac{5\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{8}+\frac{1679\sqrt{2}\,\ln\left(2\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{2048}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2),x)
```

output
```
(23*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/256 + ((
x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/8 + (73*(2*x^2 - x + 3)^(1/2)*(32*x^2 -
4*x + 45))/1536 + (5*x*(2*x^2 - x + 3)^(3/2))/8 + (1679*2^(1/2)*log(2*(2*x
^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/2048
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \sqrt{3-x+2x^2}\,(2+3x+5x^2)\;dx = \frac{5\sqrt{2x^2-x+3}\,x^3}{4}+\frac{43\sqrt{2x^2-x+3}\,x^2}{48}+\frac{671\sqrt{2x^2-x+3}\,x}{384}+\frac{1087\sqrt{2x^2-x+3}}{512}+\frac{1863\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{2048}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^2+3*x+2),x)
```

output
```
(7680*sqrt(2*x**2 - x + 3)*x**3 + 5504*sqrt(2*x**2 - x + 3)*x**2 + 10736*s
qrt(2*x**2 - x + 3)*x + 13044*sqrt(2*x**2 - x + 3) + 5589*sqrt(2)*log((2*s
qrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/6144
```

3.96 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 174

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx \\
&= -\frac{1}{5}\sqrt{2}\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right) \\
&\quad + \frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) \\
&\quad - \frac{1}{5}\sqrt{\frac{11}{31}\left(-13+10\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-13+10\sqrt{2}\right)}}\left(6-7\sqrt{2}+\left(20-13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)
\end{aligned}
$$

output

```
-1/5*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/155*(4433+3410*2^(1/2))^(1/2
)*arctan(11^(1/2)/(806+620*2^(1/2))^(1/2)*(6+7*2^(1/2)+(20+13*2^(1/2))*x)/
(2*x^2-x+3)^(1/2))-1/155*(-4433+3410*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-806
+620*2^(1/2))^(1/2)*(6-7*2^(1/2)+(20-13*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \frac{1}{5}\Bigg(-\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right) + 11\text{RootSum}\Bigg[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3 -5\#1^4\&, \frac{-2\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+\log\left(\cdot\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

output
```
(-(Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 11*RootSum[-56 - 26*S
qrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1 + Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[
2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1320, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{5x^2+3x+2}dx$$

$$\Big\downarrow \; 1320$$

$$\frac{2}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{5}\int -\frac{11(1-x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx$$

$$\Big\downarrow \; 1090$$

$$\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{1}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)$$

$$\Big\downarrow \; 222$$

$$\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 1368$$

$$\frac{11}{5}\left(\frac{\int -\frac{11\left(\sqrt{2}x-\sqrt{2}+2\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\sqrt{2}x+\sqrt{2}+2\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{11}{5}\left(\frac{\int \frac{-\sqrt{2}x+\sqrt{2}+2}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}x-\sqrt{2}+2}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 1362$$

$$\frac{11}{5}\left(\sqrt{2}\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)^2}{2x^2-x+3}-62\left(13-10\sqrt{2}\right)}d\frac{\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6}{\sqrt{2x^2-x+3}}-\sqrt{2}\left(13+10\right.\right.$$
$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{11}{5}\left(\sqrt{2}\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)^2}{2x^2-x+3}-62\left(13-10\sqrt{2}\right)}d\frac{\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6}{\sqrt{2x^2-x+3}}+\sqrt{\frac{1}{341}\left(13+\right.}\right.$$
$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+$$
$$\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(13-10\sqrt{2}\right)\text{arctanh}\left(\sqrt{\frac{}{62(10}}\right.}{\sqrt{341\,(10}}\right.$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

output
```
(Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/5 + (11*(Sqrt[(13 + 10*Sqrt[2])/341
]*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2
])*x))/Sqrt[3 - x + 2*x^2]] + ((13 - 10*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-13
 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/Sqrt[341*(-13 + 10*Sqrt[2])]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.69 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.78

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{5}-\frac{\operatorname{RootOf}\left(\_Z^2+24025\operatorname{RootOf}\left(24025\_Z\right.\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/5*RootOf(_Z^2-2)*ln(4*RootOf(_Z^2-2)*x+4*(2*x^2-x+3)^(1/2)-RootOf(_Z^2-2
))-1/155*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*ln(-(5
194205*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*RootOf(2
4025*_Z^4+4433*_Z^2+242)^4*x+446710*RootOf(24025*_Z^4+4433*_Z^2+242)^2*Roo
tOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*x+66745294*(2*x^2-
x+3)^(1/2)*RootOf(24025*_Z^4+4433*_Z^2+242)^2-641080*RootOf(24025*_Z^4+443
3*_Z^2+242)^2*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)-3
8115*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*x+6024106*
(2*x^2-x+3)^(1/2)+33880*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)
^2+4433))/(775*x*RootOf(24025*_Z^4+4433*_Z^2+242)^2+55*x-22))-RootOf(24025
*_Z^4+4433*_Z^2+242)*ln((-649275625*x*RootOf(24025*_Z^4+4433*_Z^2+242)^5-1
83764900*RootOf(24025*_Z^4+4433*_Z^2+242)^3*x+53826850*(2*x^2-x+3)^(1/2)*R
ootOf(24025*_Z^4+4433*_Z^2+242)^2-80135000*RootOf(24025*_Z^4+4433*_Z^2+242
)^3-7037844*x*RootOf(24025*_Z^4+4433*_Z^2+242)+5073772*(2*x^2-x+3)^(1/2)-1
9021200*RootOf(24025*_Z^4+4433*_Z^2+242))/(775*x*RootOf(24025*_Z^4+4433*_Z
^2+242)^2+88*x+22))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(124) = 248$.

Time = 0.09 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-1/10*sqrt(110/31*sqrt(2) + 143/31)*arctan(-1/11*(88*(25*x^3 - 61*x^2 - sq
rt(2)*(22*x^3 - 47*x^2 - 8*x + 24) - 32*x + 24)*sqrt(2*x^2 - x + 3) + (171
*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) -
3936*x)*sqrt(110/31*sqrt(2) - 143/31))*sqrt(110/31*sqrt(2) + 143/31)/(343*
x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + 1/10*sqrt(110/31*sqrt(2) + 143/
31)*arctan(1/11*(88*(25*x^3 - 61*x^2 - sqrt(2)*(22*x^3 - 47*x^2 - 8*x + 24
) - 32*x + 24)*sqrt(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*
sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(110/31*sqrt(2) - 143
/31))*sqrt(110/31*sqrt(2) + 143/31)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x
- 576)) + 1/10*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25) - 1/20*sqrt(110/31*sqrt(2) - 143/31)*log((49*x^2 + 2*sqrt(
2*x^2 - x + 3)*(sqrt(2)*(22*x - 63) + 41*x - 85)*sqrt(110/31*sqrt(2) - 143
/31) + 44*sqrt(2)*(2*x^2 - x + 3) - 151*x + 200)/x^2) + 1/20*sqrt(110/31*s
qrt(2) - 143/31)*log((49*x^2 - 2*sqrt(2*x^2 - x + 3)*(sqrt(2)*(22*x - 63)
+ 41*x - 85)*sqrt(110/31*sqrt(2) - 143/31) + 44*sqrt(2)*(2*x^2 - x + 3) -
151*x + 200)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \frac{\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\,\sqrt{2}-4x+1\right)}{5} + \frac{11\left(\int \frac{\sqrt{2x^2-x+3}}{10x^4+x^3+16x^2+7x+6}dx\right)}{5} - \frac{11\left(\int \frac{\sqrt{2x^2-x+3}\,x}{10x^4+x^3+16x^2+7x+6}dx\right)}{5}$$

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

output
```
(sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 11*int(sqrt(2*
x**2 - x + 3)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x) - 11*int((sqrt(2*x**
2 - x + 3)*x)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x))/5
```

3.97 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx=\frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\sqrt{2}+\left(973+696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)-\frac{1}{62}\sqrt{\frac{1}{682}\left(-70517+49942\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-70517+49942\sqrt{2}\right)}}\left(419-277\sqrt{2}+\left(973-696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
(3+10*x)*(2*x^2-x+3)^(1/2)/(155*x^2+93*x+62)+1/42284*(48092594+34060444*2^
(1/2))^(1/2)*arctan(11^(1/2)/(2186027+1548202*2^(1/2))^(1/2)*(419+277*2^(1
/2)+(973+696*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/42284*(-48092594+34060444*2^
(1/2))^(1/2)*arctanh(11^(1/2)/(-2186027+1548202*2^(1/2))^(1/2)*(419-277*2^
(1/2)+(973-696*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{\frac{50(3+10x)\sqrt{3-x+2x^2}}{2+3x+5x^2} - 6151\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2}\right.}{}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((50*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 6151*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ] + 124*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^
4 & , (49*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ] - 10*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[
2]*#1^3 - 5*#1^4 & , (191*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1 + 55*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(13*Sqrt[2
] - 17*#1 - 9*Sqrt[2]*#1^2 + 10*#1^3) & ])/1550
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1302, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^2}\,dx$$
$$\Big\downarrow \; 1302$$

$$\frac{(10x+3)\sqrt{2x^2-x+3}}{31\left(5x^2+3x+2\right)}-\frac{1}{31}\int -\frac{63-22x}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{62}\int \frac{63-22x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1368$$

$$\frac{1}{62}\left(\frac{\int -\frac{11\left(-\left(\left(41-22\sqrt{2}\right)x\right)-63\sqrt{2}+85\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(41+22\sqrt{2}\right)x\right)+63\sqrt{2}+85\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{62}\left(\frac{\int \frac{-\left(\left(41+22\sqrt{2}\right)x\right)+63\sqrt{2}+85}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(41-22\sqrt{2}\right)x\right)-63\sqrt{2}+85}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1362$$

$$\frac{1}{62}\left(\frac{\left(70517-49942\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)^2}{2x^2-x+3}-31\left(70517-49942\sqrt{2}\right)}d\frac{\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{(70517+4\cdots}{\cdots}\right.$$
$$+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{62}\left(\frac{\left(70517-49942\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)^2}{2x^2-x+3}-31\left(70517-49942\sqrt{2}\right)}d\frac{\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}}\left(705\cdots\right.\right.$$
$$+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{62}\left(\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)+\frac{(70517-}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 4
9942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 27
7*Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((70517 - 49942
*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419 - 277*Sqrt[
2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-70517 + 4994
2*Sqrt[2])])/62
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.30 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.52

method	result
trager	$\frac{(10x+3)\sqrt{2x^2-x+3}}{155x^2+93x+62} - \frac{2\,\mathrm{RootOf}\left(59535872\_\_Z^4+384740752\_\_Z^2+623550841\right)\ln\left(-\frac{150744827904x\,\mathrm{RootOf}\left(59535872\_\_Z^4+3847\right.}{}\right.}{}$
risch	$\frac{(10x+3)\sqrt{2x^2-x+3}}{155x^2+93x+62} + \frac{\sqrt{\frac{8(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}\,(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(26569\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\arctan\left(\frac{\sqrt{-775687+5493}}{}\right.\right.}{}$
default	Expression too large to display

input

```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/31*(10*x+3)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)-2/31*RootOf(59535872*_Z^4+38
4740752*_Z^2+623550841)*ln(-(150744827904*x*RootOf(59535872*_Z^4+384740752
*_Z^2+623550841)^5+232524550016*RootOf(59535872*_Z^4+384740752*_Z^2+623550
841)^3*x+2424286162144*(2*x^2-x+3)^(1/2)*RootOf(59535872*_Z^4+384740752*_Z
^2+623550841)^2-85650470400*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)
^3-988525310334*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)*x+781092138
3613*(2*x^2-x+3)^(1/2)+163005849200*RootOf(59535872*_Z^4+384740752*_Z^2+62
3550841))/(10912*x*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)^2+34189*
x-1426))+1/42284*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+6
23550841)^2+48092594)*ln((18843103488*RootOf(_Z^2+7441984*RootOf(59535872*
_Z^4+384740752*_Z^2+623550841)^2+48092594)*RootOf(59535872*_Z^4+384740752*
_Z^2+623550841)^4*x+214475327264*RootOf(59535872*_Z^4+384740752*_Z^2+62355
0841)^2*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)
^2+48092594)*x+826681581291104*(2*x^2-x+3)^(1/2)*RootOf(59535872*_Z^4+3847
40752*_Z^2+623550841)^2+10706308800*RootOf(59535872*_Z^4+384740752*_Z^2+62
3550841)^2*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+6235508
41)^2+48092594)+475524326173*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+3847
40752*_Z^2+623550841)^2+48092594)*x+2678769161185106*(2*x^2-x+3)^(1/2)+895
63485700*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+623550841
)^2+48092594))/(5456*x*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(139) = 278$.

Time = 0.09 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/248*(2*(5*x^2 + 3*x + 2)*sqrt(24971/341*sqrt(2) + 70517/682)*arctan(-22/
713*(4*(2904*x^3 - 6538*x^2 - sqrt(2)*(1993*x^3 - 4569*x^2 - 1664*x + 2040
) - 2064*x + 3024)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^3 - 1640*x^2 -
176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(24971/341*sqrt(2
) - 70517/682))*sqrt(24971/341*sqrt(2) + 70517/682)/(343*x^4 - 400*x^3 + 1
136*x^2 + 384*x - 576)) - 2*(5*x^2 + 3*x + 2)*sqrt(24971/341*sqrt(2) + 705
17/682)*arctan(22/713*(4*(2904*x^3 - 6538*x^2 - sqrt(2)*(1993*x^3 - 4569*x
^2 - 1664*x + 2040) - 2064*x + 3024)*sqrt(2*x^2 - x + 3) - (171*x^4 + 1212
*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqr
t(24971/341*sqrt(2) - 70517/682))*sqrt(24971/341*sqrt(2) + 70517/682)/(343
*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + (5*x^2 + 3*x + 2)*sqrt(24971/3
41*sqrt(2) - 70517/682)*log((34937*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
2649*x - 6257) + 3608*x - 8906)*sqrt(24971/341*sqrt(2) - 70517/682) + 3137
2*sqrt(2)*(2*x^2 - x + 3) - 107663*x + 142600)/x^2) - (5*x^2 + 3*x + 2)*sq
rt(24971/341*sqrt(2) - 70517/682)*log((34937*x^2 - 22*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(2649*x - 6257) + 3608*x - 8906)*sqrt(24971/341*sqrt(2) - 70517/6
82) + 31372*sqrt(2)*(2*x^2 - x + 3) - 107663*x + 142600)/x^2) + 8*sqrt(2*x
^2 - x + 3)*(10*x + 3))/(5*x^2 + 3*x + 2)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx=\int \frac{\sqrt{2\,x^2-x+3}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \frac{40\sqrt{2x^2-x+3}\,x+34\sqrt{2x^2-x+3}+14795\left(\int \frac{\sqrt{2x^2-x+3}}{50x^6+35x^5+103x^4+85x^3+83x^2+32x+12}dx\right)x^2+8877\left(\int \frac{}{50x^6+}\right.}{}$$

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

output
```
(40*sqrt(2*x**2 - x + 3)*x + 34*sqrt(2*x**2 - x + 3) + 14795*int(sqrt(2*x*
*2 - x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12)
,x)*x**2 + 8877*int(sqrt(2*x**2 - x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 8
5*x**3 + 83*x**2 + 32*x + 12),x)*x + 5918*int(sqrt(2*x**2 - x + 3)/(50*x**
6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x) + 9955*int((sqr
t(2*x**2 - x + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2
+ 32*x + 12),x)*x**2 + 5973*int((sqrt(2*x**2 - x + 3)*x**2)/(50*x**6 + 35*
x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x)*x + 3982*int((sqrt(2*x
**2 - x + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*
x + 12),x))/(953*(5*x**2 + 3*x + 2))
```

3.98 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)}$$
$$+ \frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(509587+362788\sqrt{2}+\left(1235163+8\right.\right.}{\sqrt{3-x+2x^2}}\right.}{169136}$$
$$- \frac{\sqrt{\frac{1}{682}\left(-112285869463+79399380740\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-112285869463+79399380740\sqrt{2}\right)}}\left(509587-362788\sqrt{2}+\left(12351\right.\right.}{\sqrt{3-x+2x^2}}\right.}{169136}$$

output
```
1/62*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+(3464+13665*x)*(2*x^2-x+3)
^(1/2)/(422840*x^2+253704*x+169136)+1/115350752*(76578962973766+5415037766
4680*2^(1/2))^(1/2)*arctan(11^(1/2)/(3480861953353+2461380802940*2^(1/2))^
(1/2)*(509587+362788*2^(1/2)+(1235163+872375*2^(1/2))*x)/(2*x^2-x+3)^(1/2)
)-1/115350752*(-76578962973766+54150377664680*2^(1/2))^(1/2)*arctanh(11^(1
/2)/(-3480861953353+2461380802940*2^(1/2))^(1/2)*(509587-362788*2^(1/2)+(1
235163-872375*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$$
$$= \frac{\frac{661250\sqrt{3-x+2x^2}(11020+51362x+58315x^2+68325x^3)}{(2+3x+5x^2)^2} + \text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\right.}{}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((661250*Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3))/(2
 + 3*x + 5*x^2)^2 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
 - 5*#1^4 & , (-537295920831*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
+ 120146195680*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 4
5923442075*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2]
 + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 248*RootSum[-56 - 26*Sqrt[2]*#1
 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2139373897*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 277937160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 - 228643025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1
]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/55920590000
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1302, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^3}\,dx$$
$$\Big\downarrow \; 1302$$

$$\frac{(10x+3)\sqrt{2x^2-x+3}}{62\left(5x^2+3x+2\right)^2}-\frac{1}{62}\int -\frac{80x^2-62x+183}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}dx$$

$$\downarrow \text{27}$$

$$\frac{1}{124}\int \frac{80x^2-62x+183}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}dx+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\downarrow \text{2135}$$

$$\frac{1}{124}\left(\frac{\int \frac{11(77456-32605x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{7502}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\downarrow \text{27}$$

$$\frac{1}{124}\left(\frac{\int \frac{77456-32605x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{1364}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\downarrow \text{1368}$$

$$\frac{1}{124}\left(\frac{\frac{\int -\frac{11\left(-\left(\left(44851-32605\sqrt{2}\right)x\right)-77456\sqrt{2}+110061\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(44851+32605\sqrt{2}\right)x\right)+77456\sqrt{2}+110061\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(13665}{682\left(5x^2+3x\right.}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\downarrow \text{27}$$

$$\frac{1}{124}\left(\frac{\frac{\int \frac{-\left(\left(44851+32605\sqrt{2}\right)x\right)+77456\sqrt{2}+110061}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(44851-32605\sqrt{2}\right)x\right)-77456\sqrt{2}+110061}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\downarrow \text{1362}$$

$$\frac{1}{124}\left(\frac{\frac{\left(112285869463-79399380740\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)^2}{2x^2-x+3}-31\left(112285869463-79399380740\sqrt{2}\right)}d\frac{\left(1235163-872375\right.}{\sqrt{}}}{\sqrt{2}}}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 217

$$\frac{1}{124}\left(\frac{\frac{\left(112285869463-79399380740\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)^2}{2x^2-x+3}-31\left(112285869463-79399380740\sqrt{2}\right)}d\frac{\left(1235163-872375\right.}{\sqrt{}}}{\sqrt{2}}}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$\frac{1}{124}\left(\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(\left(1235163+872375\sqrt{2}\right)x+362788\right.}{\sqrt{2x^2-x+3}}\right.}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + (((3464 + 1366
5*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 +
79399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*
Sqrt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]] + ((112285869463 - 79399380740*Sqrt[2])*ArcTanh[(Sqrt[11/(
31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*Sqrt[2] + (123
5163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-112285869463 +
 79399380740*Sqrt[2])])/1364)/124
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.59 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{(68325x^3+58315x^2+51362x+11020)\sqrt{2x^2-x+3}}{84568(5x^2+3x+2)^2}+\frac{\sqrt{\frac{8(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}\,(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(33504619\sqrt{2}\,\sqrt{-8866+\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/84568*(68325*x^3+58315*x^2+51362*x+11020)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(1
/2)-1/115350752*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+612631703790128
*_Z^2+39401635386846768422 5)^2+76578962973766)*ln(-(8901118918912*RootOf(_
Z^2+29767936*RootOf(238143488*_Z^4+612631703790128*_Z^2+394016353868467684
225)^2+76578962973766)*RootOf(238143488*_Z^4+612631703790128*_Z^2+39401635
3868467684225)^4*x+21996649948054194864*RootOf(238143488*_Z^4+612631703790
128*_Z^2+394016353868467684225)^2*RootOf(_Z^2+29767936*RootOf(238143488*_Z
^4+612631703790128*_Z^2+394016353868467684225)^2+76578962973766)*x-2031490
73871924400*RootOf(238143488*_Z^4+612631703790128*_Z^2+3940163538684676842
25)^2*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+612631703790128*_Z^2+3940
16353868467684225)^2+76578962973766)+1055850314121696708832 5712*RootOf(238
143488*_Z^4+612631703790128*_Z^2+394016353868467684225)^2*(2*x^2-x+3)^(1/2
)+1355838783404196779258 3352*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+61
2631703790128*_Z^2+394016353868467684225)^2+76578962973766)*x-238972679575
677054341025*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+612631703790128*_Z
^2+394016353868467684225)^2+76578962973766)+135771321202551192561528744000
47*(2*x^2-x+3)^(1/2))/(21824*x*RootOf(238143488*_Z^4+612631703790128*_Z^2+
394016353868467684225)^2+27985547479*x-114559849))+1/21142*RootOf(23814348
8*_Z^4+612631703790128*_Z^2+394016353868467684225)*ln((113934322162 0736*x*
RootOf(238143488*_Z^4+612631703790128*_Z^2+394016353868467684225)^5+304...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(170) = 340$.

Time = 0.10 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{3 - x + 2x^2}}{(2 + 3x + 5x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
-1/676544*(2*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(39699690370/341*sq
rt(2) + 112285869463/682)*arctan(-22/114559849*(4*(3603694*x^3 - 8193804*x
^2 - sqrt(2)*(2555895*x^3 - 5800781*x^2 - 1956912*x + 2641464) - 2804336*x
 + 3717888)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqr
t(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(39699690370/341*sqrt(2)
 - 112285869463/682))*sqrt(39699690370/341*sqrt(2) + 112285869463/682)/(34
3*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 2*(25*x^4 + 30*x^3 + 29*x^2 +
 12*x + 4)*sqrt(39699690370/341*sqrt(2) + 112285869463/682)*arctan(22/1145
59849*(4*(3603694*x^3 - 8193804*x^2 - sqrt(2)*(2555895*x^3 - 5800781*x^2 -
 1956912*x + 2641464) - 2804336*x + 3717888)*sqrt(2*x^2 - x + 3) - (171*x^
4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 393
6*x)*sqrt(39699690370/341*sqrt(2) - 112285869463/682))*sqrt(39699690370/34
1*sqrt(2) + 112285869463/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)
) + (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(39699690370/341*sqrt(2) - 1
12285869463/682)*log((5613432601*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(32
73511*x - 7920565) + 4647054*x - 11194076)*sqrt(39699690370/341*sqrt(2) -
112285869463/682) + 5040633356*sqrt(2)*(2*x^2 - x + 3) - 17298537199*x + 2
2911969800)/x^2) - (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(39699690370/
341*sqrt(2) - 112285869463/682)*log((5613432601*x^2 - 22*sqrt(2*x^2 - x +
3)*(sqrt(2)*(3273511*x - 7920565) + 4647054*x - 11194076)*sqrt(39699690...
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx=\int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$$
$$= \frac{-6400\sqrt{2x^2-x+3}\,x^3 - 9280\sqrt{2x^2-x+3}\,x^2 - 5144\sqrt{2x^2-x+3}\,x + 3462\sqrt{2x^2-x+3} + 1951042}{}$$

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)
```

output
```
( - 6400*sqrt(2*x**2 - x + 3)*x**3 - 9280*sqrt(2*x**2 - x + 3)*x**2 - 5144
*sqrt(2*x**2 - x + 3)*x + 3462*sqrt(2*x**2 - x + 3) + 19510425*int(sqrt(2*
x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*
x**3 + 322*x**2 + 100*x + 24),x)*x**4 + 23412510*int(sqrt(2*x**2 - x + 3)/
(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**
2 + 100*x + 24),x)*x**3 + 22632093*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 32
5*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24
),x)*x**2 + 9365004*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x*
*6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x + 312166
8*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 87
6*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x) + 7555625*int((sqrt(2*x**2 -
 x + 3)*x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*
x**3 + 322*x**2 + 100*x + 24),x)*x**4 + 9066750*int((sqrt(2*x**2 - x + 3)*
x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 3
22*x**2 + 100*x + 24),x)*x**3 + 8764525*int((sqrt(2*x**2 - x + 3)*x**2)/(2
50*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2
+ 100*x + 24),x)*x**2 + 3626700*int((sqrt(2*x**2 - x + 3)*x**2)/(250*x**8
+ 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x
+ 24),x)*x + 1208900*int((sqrt(2*x**2 - x + 3)*x**2)/(250*x**8 + 325*x**7
+ 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)...
```

3.99 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 231

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x)(3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283(3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x(3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2(3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3(3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4(3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768}x^5(3-x+2x^2)^{5/2} + \frac{7625}{96}x^6(3-x+2x^2)^{5/2} + \frac{625}{24}x^7(3-x+2x^2)^{5/2} - \frac{606427533063\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4294967296\sqrt{2}}$$

output
```
-26366414481/2147483648*(1-4*x)*(2*x^2-x+3)^(1/2)-382121949/134217728*(1-4
*x)*(2*x^2-x+3)^(3/2)+2124689283/146800640*(2*x^2-x+3)^(5/2)+48669967/2202
0096*x*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2*x^2-x+3)^(5/2)+10444117/2
94912*x^3*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x^2-x+3)^(5/2)+95165/768*x^
5*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+625/24*x^7*(2*x^2-x+3)^(
5/2)-606427533063/8589934592*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.45

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{4\sqrt{3-x+2x^2}(74032009514181+12971175524316x+65151998063712x^2+2390211842231$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712
*x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 7
65087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651
174400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845
*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/2705829396480
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.19, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2} \left(5x^2+3x+2\right)^4 \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{24} \int \frac{1}{2} \left(2x^2-x+3\right)^{3/2} \left(83875x^7+86550x^6+112320x^5+84528x^4+44928x^3+18048x^2+4608x+768\right) dx + \frac{625}{24} \left(2x^2-x+3\right)^{5/2} x^7$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{48}\int \left(2x^2-x+3\right)^{3/2}\left(83875x^7+86550x^6+112320x^5+84528x^4+44928x^3+18048x^2+4608x+768\right)dx+ \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{22}\int \frac{11}{2}\left(2x^2-x+3\right)^{3/2}\left(475825x^6+174780x^5+338112x^4+179712x^3+72192x^2+18432x+3072\right)dx\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\int \left(2x^2-x+3\right)^{3/2}\left(475825x^6+174780x^5+338112x^4+179712x^3+72192x^2+18432x+3072\right)dx+\frac{76}{2}\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{1}{20}\int \frac{15}{2}\left(2x^2-x+3\right)^{3/2}\left(941905x^5-50018x^4+479232x^3+192512x^2+49152x+8192\right)dx+\frac{95165}{4}\right.\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\int \left(2x^2-x+3\right)^{3/2}\left(941905x^5-50018x^4+479232x^3+192512x^2+49152x+8192\right)dx+\frac{95165}{4}\left(2x^2\right.\right.\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(10444117x^4-5353368x^3+6930432x^2+1769472x+294912\right)dx+\frac{941905}{18}\right.\right.\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{3/2}\left(10444117x^4-5353368x^3+6930432x^2+1769472x+294912\right)dx+\frac{941905}{18}\right.\right.\right. \frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(-56422489x^3+33779718x^2+56623104x+9437184\right)dx+\frac{10444117}{16}\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\int \left(2x^2-x+3\right)^{3/2}\left(-56422489x^3+33779718x^2+56623104x+9437184\right)dx+\frac{10444117}{16}(2\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{3}{2}\left(2x^2-x+3\right)^{3/2}\left(146009901x^2+754172260x+88080384\right)dx-\frac{56422489}{14}x^2\left(2x^2-\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\int \left(2x^2-x+3\right)^{3/2}\left(146009901x^2+754172260x+88080384\right)dx-\frac{56422489}{14}x^2\left(2x^2-\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{12}\int \frac{27}{2}(708229761x+45847030)\left(2x^2-x+3\right)^{3/2}dx+\frac{48669967}{4}x\left(2x^2-x+3\right)^{5/2}\right)\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\int (708229761x+45847030)\left(2x^2-x+3\right)^{3/2}dx+\frac{48669967}{4}x\left(2x^2-x+3\right)^{5/2}\right)-\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\int \left(2x^2-x+3\right)^{3/2}dx+\frac{708229761}{10}\left(2x^2-x+3\right)^{5/2}\right)+\frac{48669967}{4}x(2x$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$\big\downarrow$ 1087

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{708229761}{10}($$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$\big\downarrow$ 1087

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$\big\downarrow$ 1090

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$\big\downarrow$ 222

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(625*x^7*(3 - x + 2*x^2)^(5/2))/24 + ((7625*x^6*(3 - x + 2*x^2)^(5/2))/2 +
 ((95165*x^5*(3 - x + 2*x^2)^(5/2))/4 + (3*((941905*x^4*(3 - x + 2*x^2)^(5
/2))/18 + ((10444117*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((-56422489*x^2*(3 -
x + 2*x^2)^(5/2))/14 + (3*((48669967*x*(3 - x + 2*x^2)^(5/2))/4 + (9*((708
229761*(3 - x + 2*x^2)^(5/2))/10 + (891617881*(-1/16*((1 - 4*x)*(3 - x + 2
*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4))/8))/28)/32)/36))/8)/4)/48
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

method	result
risch	$\frac{(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^{9}+534038708224000x^{8}+745133229998080x^{7}+765087080448000x}{67}$
trager	$\left(\frac{625}{6}x^{11}+\frac{5125}{24}x^{10}+\frac{33055}{64}x^{9}+\frac{1818925}{2304}x^{8}+\frac{81213077}{73728}x^{7}+\frac{778286825}{688128}x^{6}+\frac{16491197869}{16515072}x^{5}+\frac{31499817401}{47185920}x^{4}\right.$
default	$\frac{382121949(4x-1)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{134217728}+\frac{26366414481(4x-1)\sqrt{2x^{2}-x+3}}{2147483648}+\frac{606427533063\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8589934592}+\frac{212468928}{146}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/676457349120*(70464307200000*x^11+144451829760000*x^10+349379651174400*x
^9+534038708224000*x^8+745133229998080*x^7+765087080448000*x^6+67547946471
4240*x^5+451581382260736*x^4+239021184223104*x^3+65151998063712*x^2+129711
75524316*x+74032009514181)*(2*x^2-x+3)^(1/2)+606427533063/8589934592*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int \left(3-x+2x^{2}\right)^{3/2}\left(2+3x+5x^{2}\right)^{4}\,dx=\frac{1}{676457349120}\left(70464307200000\,x^{11}+144451829760000\,x^{10}+349379651174400\,x^{9}+534038\right.$$
$$+\frac{606427533063}{17179869184}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^{2}-x+3}(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 3493796511744
00*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 +
 675479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199
8063712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 606
427533063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1)
 - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.48

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{625x^{11}}{6} + \frac{5125x^{10}}{24} + \frac{33055x^9}{64} + \frac{1818925x^8}{2304} + \frac{81213077x^7}{73728} + \frac{778286825x^6}{688128} + \frac{16491197869x^5}{16515072} + \frac{31499817401x^4}{47185920} + \frac{622451000581x^3}{1761607680} + \frac{32317459357x^2}{335544320} + \frac{360310431231x}{18790481920} + \frac{8225778834909}{75161927680}\right) + \frac{606427533063\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8589934592}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

output
```
sqrt(2*x**2 - x + 3)*(625*x**11/6 + 5125*x**10/24 + 33055*x**9/64 + 181892
5*x**8/2304 + 81213077*x**7/73728 + 778286825*x**6/688128 + 16491197869*x*
*5/16515072 + 31499817401*x**4/47185920 + 622451000581*x**3/1761607680 + 3
2317459357*x**2/335544320 + 360310431231*x/18790481920 + 8225778834909/751
61927680) + 606427533063*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/8589934592
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = \frac{625}{24} (2x^2-x+3)^{\frac{5}{2}} x^7 + \frac{7625}{96} (2x^2-x+3)^{\frac{5}{2}} x^6 + \frac{95165}{768} (2x^2-x+3)^{\frac{5}{2}} x^5 + \frac{941905}{9216} (2x^2-x+3)^{\frac{5}{2}} x^4 + \frac{10444117}{294912} (2x^2-x+3)^{\frac{5}{2}} x^3 - \frac{56422489}{8257536} (2x^2-x+3)^{\frac{5}{2}} x^2 + \frac{48669967}{22020096} (2x^2-x+3)^{\frac{5}{2}} x + \frac{2124689283}{146800640} (2x^2-x+3)^{\frac{5}{2}} + \frac{382121949}{33554432} (2x^2-x+3)^{\frac{3}{2}} x - \frac{382121949}{134217728} (2x^2-x+3)^{\frac{3}{2}} + \frac{26366414481}{536870912} \sqrt{2x^2-x+3} x + \frac{606427533063}{8589934592} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{26366414481}{2147483648} \sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 951
65/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 +
 10444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x +
 3)^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/146
800640*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x
- 382121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2
*x^2 - x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4
*x - 1)) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.45

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = \frac{1}{676457349120} (4 (8 (4 (16 (20 (8 (28 (160 (12 (200 (20 x + 41)x + 19833)x + 363785)x + 8121 - \frac{606427533063}{8589934592} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) + 1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833
)*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 220498721
807)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009
514181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(
2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx=\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^4 \,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx &= \frac{625\sqrt{2x^2-x+3}\,x^{11}}{6} \\ &+ \frac{5125\sqrt{2x^2-x+3}\,x^{10}}{24} + \frac{33055\sqrt{2x^2-x+3}\,x^{9}}{64} \\ &+ \frac{1818925\sqrt{2x^2-x+3}\,x^{8}}{2304} + \frac{81213077\sqrt{2x^2-x+3}\,x^{7}}{73728} \\ &+ \frac{778286825\sqrt{2x^2-x+3}\,x^{6}}{688128} + \frac{16491197869\sqrt{2x^2-x+3}\,x^{5}}{16515072} \\ &+ \frac{31499817401\sqrt{2x^2-x+3}\,x^{4}}{47185920} + \frac{622451000581\sqrt{2x^2-x+3}\,x^{3}}{1761607680} \\ &+ \frac{32317459357\sqrt{2x^2-x+3}\,x^{2}}{335544320} + \frac{360310431231\sqrt{2x^2-x+3}\,x}{18790481920} \\ &+ \frac{8225778834909\sqrt{2x^2-x+3}}{75161927680} + \frac{606427533063\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{8589934592}\end{aligned}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x)
```

output
```
(281857228800000*sqrt(2*x**2 - x + 3)*x**11 + 577807319040000*sqrt(2*x**2
- x + 3)*x**10 + 1397518604697600*sqrt(2*x**2 - x + 3)*x**9 + 213615483289
6000*sqrt(2*x**2 - x + 3)*x**8 + 2980532919992320*sqrt(2*x**2 - x + 3)*x**
7 + 3060348321792000*sqrt(2*x**2 - x + 3)*x**6 + 2701917858856960*sqrt(2*x
**2 - x + 3)*x**5 + 1806325529042944*sqrt(2*x**2 - x + 3)*x**4 + 956084736
892416*sqrt(2*x**2 - x + 3)*x**3 + 260607992254848*sqrt(2*x**2 - x + 3)*x*
*2 + 51884702097264*sqrt(2*x**2 - x + 3)*x + 296128038056724*sqrt(2*x**2 -
 x + 3) + 191024672914845*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*
x - 1)/sqrt(23)))/2705829396480
```

3.100 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 189

$$\begin{aligned}\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 \, dx = & -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} \\ & - \frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} \\ & + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} \\ & + \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} - \frac{1059790665\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{67108864\sqrt{2}}\end{aligned}$$

output
```
-46077855/33554432*(1-4*x)*(2*x^2-x+3)^(1/2)-667795/2097152*(1-4*x)*(2*x^2
-x+3)^(3/2)-4625907/2293760*(2*x^2-x+3)^(5/2)-81685/114688*x*(2*x^2-x+3)^(
5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)+27785/1536*x^3*(2*x^2-x+3)^(5/2)+7
25/48*x^4*(2*x^2-x+3)^(5/2)+25/4*x^5*(2*x^2-x+3)^(5/2)-1059790665/13421772
8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 \, dx = \frac{4\sqrt{3-x+2x^2}(-72152399943+53985432012x+199615064544x^2+389257196928x^3+487\cdots}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 +
389257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6
+ 328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sq
rt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/14092861440
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{3/2} (5x^2+3x+2)^3 \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{20} \int \frac{5}{2} (2x^2-x+3)^{3/2} (2175x^5+1530x^4+1656x^3+912x^2+288x+64) \, dx + \frac{25}{4} (2x^2-x+3)^{5/2} x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8} \int (2x^2-x+3)^{3/2} (2175x^5+1530x^4+1656x^3+912x^2+288x+64) \, dx + \frac{25}{4} (2x^2-x+3)^{5/2} x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{3}{2}\left(2x^2-x+3\right)^{3/2}\left(27785x^4+2472x^3+10944x^2+3456x+768\right)dx+\frac{725}{6}\left(2x^2-x+3\right)^{5/2}x^4\right)+\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\int \left(2x^2-x+3\right)^{3/2}\left(27785x^4+2472x^3+10944x^2+3456x+768\right)dx+\frac{725}{6}\left(2x^2-x+3\right)^{5/2}x^4\right)+\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{16}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(384739x^3-149922x^2+110592x+24576\right)dx+\frac{27785}{16}\left(2x^2-x+3\right)^{5/2}x^3\right)+\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\int \left(2x^2-x+3\right)^{3/2}\left(384739x^3-149922x^2+110592x+24576\right)dx+\frac{27785}{16}\left(2x^2-x+3\right)^{5/2}x^3\right)+\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{3}{2}\left(-245055x^2-506764x+229376\right)\left(2x^2-x+3\right)^{3/2}dx+\frac{384739}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{27}{1}\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\int \left(-245055x^2-506764x+229376\right)\left(2x^2-x+3\right)^{3/2}dx+\frac{384739}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{2778}{16}\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{12}\int \frac{3}{2}(2325118-4625907x)\left(2x^2-x+3\right)^{3/2}dx-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{384739}{14}x^2(2x\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\int (2325118-4625907x)\left(2x^2-x+3\right)^{3/2}dx-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{384739}{14}x^2\left(2x^2\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 1160

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{4625907}{10}\left(2x^2-x+3\right)^{5/2}\right)-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{4625907}{10}\left(2x^2-x+3\right)\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(25*x^5*(3 - x + 2*x^2)^(5/2))/4 + ((725*x^4*(3 - x + 2*x^2)^(5/2))/6 + ((
27785*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((384739*x^2*(3 - x + 2*x^2)^(5/2))/
14 + (3*((-81685*x*(3 - x + 2*x^2)^(5/2))/4 + ((-4625907*(3 - x + 2*x^2)^(
5/2))/10 + (4674565*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*(
(1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqr
t[2])))/32))/4)/8))/28)/32)/12)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+1}{3523215360}$
trager	$\left(25x^9+\frac{425}{12}x^8+\frac{35785}{384}x^7+\frac{438253}{3584}x^6+\frac{13947713}{86016}x^5+\frac{34032637}{245760}x^4+\frac{1013690617}{9175040}x^3+\frac{297046227}{5242880}x^2+\frac{44987}{2936}\right.$
default	$\frac{667795(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{2097152}+\frac{46077855(4x-1)\sqrt{2x^2-x+3}}{33554432}+\frac{1059790665\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{134217728}-\frac{4625907(2x^2-x+3)}{2293760}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3523215360*(88080384000*x^9+124780544000*x^8+328328806400*x^7+4308202291
20*x^6+571298324480*x^5+487891884032*x^4+389257196928*x^3+199615064544*x^2
+53985432012*x-72152399943)*(2*x^2-x+3)^(1/2)+1059790665/134217728*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{1}{3523215360} \left(88080384000\,x^9 + 124780544000\,x^8 + 328328806400\,x^7 + 430820229120\,x^6 + 5\right. + \frac{1059790665}{268435456}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 4308
20229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 19
9615064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979
0665/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 dx = \sqrt{2x^2-x+3} \cdot \left(25x^9 + \frac{425x^8}{12} + \frac{35785x^7}{384} + \frac{438253x^6}{3584} + \frac{13947713x^5}{86016} + \frac{34032637x^4}{245760} + \frac{1013690617x^3}{9175040} + \frac{297046227x^2}{5242880} + \frac{4498786001x}{293601280} - \frac{24050799981}{1174405120}\right) + \frac{1059790665\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{134217728}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**9 + 425*x**8/12 + 35785*x**7/384 + 438253*x**6
/3584 + 13947713*x**5/86016 + 34032637*x**4/245760 + 1013690617*x**3/91750
40 + 297046227*x**2/5242880 + 4498786001*x/293601280 - 24050799981/1174405
120) + 1059790665*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/134217728
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx = \frac{25}{4}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^5 + \frac{725}{48}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4 + \frac{27785}{1536}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3 + \frac{384739}{43008}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2 - \frac{81685}{114688}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x - \frac{4625907}{2293760}\left(2\,x^2-x+3\right)^{\frac{5}{2}} + \frac{667795}{524288}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x - \frac{667795}{2097152}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{46077855}{8388608}\sqrt{2\,x^2-x+3}\,x + \frac{1059790665}{134217728}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{46077855}{33554432}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/
1536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 -
81685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/
2) + 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3
)^(3/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x
+ 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \,dx = \frac{1}{3523215360}(4(8(4(16(20(8(140(160(12x+17)x+7157)x+1314759)x+13947713)x+ - \frac{1059790665}{134217728}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 13147
59)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 13496
358003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2
)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \,dx = \int \left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^3 \,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx = 25\sqrt{2x^2-x+3}\,x^9 + \frac{425\sqrt{2x^2-x+3}\,x^8}{12} + \frac{35785\sqrt{2x^2-x+3}\,x^7}{384} + \frac{438253\sqrt{2x^2-x+3}\,x^6}{3584} + \frac{13947713\sqrt{2x^2-x+3}\,x^5}{86016} + \frac{34032637\sqrt{2x^2-x+3}\,x^4}{245760} + \frac{1013690617\sqrt{2x^2-x+3}\,x^3}{9175040} + \frac{297046227\sqrt{2x^2-x+3}\,x^2}{5242880} + \frac{4498786001\sqrt{2x^2-x+3}\,x}{293601280} - \frac{24050799981\sqrt{2x^2-x+3}}{1174405120} + \frac{1059790665\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{134217728}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x)
```

output
```
(352321536000*sqrt(2*x**2 - x + 3)*x**9 + 499122176000*sqrt(2*x**2 - x + 3
)*x**8 + 1313315225600*sqrt(2*x**2 - x + 3)*x**7 + 1723280916480*sqrt(2*x*
*2 - x + 3)*x**6 + 2285193297920*sqrt(2*x**2 - x + 3)*x**5 + 1951567536128
*sqrt(2*x**2 - x + 3)*x**4 + 1557028787712*sqrt(2*x**2 - x + 3)*x**3 + 798
460258176*sqrt(2*x**2 - x + 3)*x**2 + 215941728048*sqrt(2*x**2 - x + 3)*x
- 288609599772*sqrt(2*x**2 - x + 3) + 111278019825*sqrt(2)*log((2*sqrt(2*x
**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/14092861440
```

3.101 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx = \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{12850997\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

output
```
558739/1048576*(1-4*x)*(2*x^2-x+3)^(1/2)+24293/196608*(1-4*x)*(2*x^2-x+3)^
(3/2)+73861/215040*(2*x^2-x+3)^(5/2)+24499/10752*x*(2*x^2-x+3)^(5/2)+1235/
448*x^2*(2*x^2-x+3)^(5/2)+25/16*x^3*(2*x^2-x+3)^(5/2)+12850997/4194304*arc
sinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx = \frac{4\sqrt{3-x+2x^2}(439831323+1619403428x+1799647136x^2+2728413312x^3+2061273088x}{44}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 272841
3312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7
) + 1349354685*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/440401920
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{3/2} (5x^2+3x+2)^2 \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\int \frac{1}{2}(2x^2-x+3)^{3/2} (1235x^3+478x^2+384x+128) \, dx + \frac{25}{16}(2x^2-x+3)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int (2x^2-x+3)^{3/2} (1235x^3+478x^2+384x+128) \, dx + \frac{25}{16}(2x^2-x+3)^{5/2} x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(24499x^2-4068x+3584\right)dx+\frac{1235}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\int \left(2x^2-x+3\right)^{3/2}\left(24499x^2-4068x+3584\right)dx+\frac{1235}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int -\frac{1}{2}(60978-73861x)\left(2x^2-x+3\right)^{3/2}dx+\frac{24499}{12}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{1235}{14}x^2\left(2x^2-x+3\right)^{5/}\right.$$
$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{24499}{12}x\left(2x^2-x+3\right)^{5/2}-\frac{1}{24}\int(60978-73861x)\left(2x^2-x+3\right)^{3/2}dx\right)+\frac{1235}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+$$
$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\int\left(2x^2-x+3\right)^{3/2}dx\right)+\frac{24499}{12}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{1235}{14}x^2\right.$$
$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\int\sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)\right)+\frac{24}{}\right.\right.$$
$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right.\right.\right.\right.$$
$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x\right.\right.\right.\right.\right.$$

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-\right.\right.\right.\right.$$

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(25*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((1235*x^2*(3 - x + 2*x^2)^(5/2))/14 +
 ((24499*x*(3 - x + 2*x^2)^(5/2))/12 + ((73861*(3 - x + 2*x^2)^(5/2))/10 -
 (170051*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*S
qrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32)
)/4)/24)/28)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)\sqrt{2x^2-x}}{110100480}$
trager	$\left(\frac{25}{4}x^7+\frac{535}{112}x^6+\frac{49459}{2688}x^5+\frac{143783}{7680}x^4+\frac{7105243}{286720}x^3+\frac{8034139}{491520}x^2+\frac{404850857}{27525120}x+\frac{146610441}{36700160}\right)\sqrt{2x^2-x+}$
default	$-\frac{24293(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{196608}-\frac{558739(4x-1)\sqrt{2x^2-x+3}}{1048576}-\frac{12850997\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{73861(2x^2-x+3)^{\frac{5}{2}}}{215040}+$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/110100480*(688128000*x^7+525926400*x^6+2025840640*x^5+2061273088*x^4+272
8413312*x^3+1799647136*x^2+1619403428*x+439831323)*(2*x^2-x+3)^(1/2)-12850
997/4194304*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{1}{110100480}\left(688128000\,x^7+525926400\,x^6+2025840640\,x^5+2061273088\,x^4+2728413312\,x\right.$$
$$+\frac{12850997}{8388608}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x
^4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^
2 - x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4
*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^7}{4}+\frac{535x^6}{112}+\frac{49459x^5}{2688}+\frac{143783x^4}{7680}+\frac{7105243x^3}{286720}+\frac{8034139x^2}{491520}+\frac{404850857x}{27525120}+\frac{146610441}{36700160}\right)-\frac{12850997\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**7/4 + 535*x**6/112 + 49459*x**5/2688 + 143783*
x**4/7680 + 7105243*x**3/286720 + 8034139*x**2/491520 + 404850857*x/275251
20 + 146610441/36700160) - 12850997*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)
/4194304
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{16}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3 + \frac{1235}{448}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2 + \frac{24499}{10752}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x + \frac{73861}{215040}\left(2\,x^2-x+3\right)^{\frac{5}{2}} - \frac{24293}{49152}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{24293}{196608}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{558739}{262144}\sqrt{2\,x^2-x+3}x - \frac{12850997}{4194304}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{558739}{1048576}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 244
99/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 24
293/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5
58739/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23
*sqrt(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{1}{110100480}\left(4\left(8\left(4\left(16\left(20\left(120\left(140\,x+107\right)x+49459\right)x+1006481\right)x+21315729\right)x+5623\right.\right.\right.\right. + \frac{12850997}{4194304}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x
+ 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3
) + 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx = \int \left(2\,x^2-x+3\right)^{3/2} \left(5\,x^2+3\,x+2\right)^2 dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx &= \frac{25\sqrt{2x^2-x+3}\,x^7}{4} \\ &+ \frac{535\sqrt{2x^2-x+3}\,x^6}{112} + \frac{49459\sqrt{2x^2-x+3}\,x^5}{2688} \\ &+ \frac{143783\sqrt{2x^2-x+3}\,x^4}{7680} + \frac{7105243\sqrt{2x^2-x+3}\,x^3}{286720} \\ &+ \frac{8034139\sqrt{2x^2-x+3}\,x^2}{491520} + \frac{404850857\sqrt{2x^2-x+3}\,x}{27525120} \\ &+ \frac{146610441\sqrt{2x^2-x+3}}{36700160} - \frac{12850997\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{4194304}\end{aligned}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)
```

output
```
(2752512000*sqrt(2*x**2 - x + 3)*x**7 + 2103705600*sqrt(2*x**2 - x + 3)*x**
*6 + 8103362560*sqrt(2*x**2 - x + 3)*x**5 + 8245092352*sqrt(2*x**2 - x + 3
)*x**4 + 10913653248*sqrt(2*x**2 - x + 3)*x**3 + 7198588544*sqrt(2*x**2 -
x + 3)*x**2 + 6477613712*sqrt(2*x**2 - x + 3)*x + 1759325292*sqrt(2*x**2 -
 x + 3) - 1349354685*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1
)/sqrt(23)))/440401920
```

3.102 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx = -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240}(3-x+2x^2)^{5/2} + \frac{5}{12}x(3-x+2x^2)^{5/2} - \frac{94691\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

output
```
-4117/8192*(1-4*x)*(2*x^2-x+3)^(1/2)-179/1536*(1-4*x)*(2*x^2-x+3)^(3/2)+10
7/240*(2*x^2-x+3)^(5/2)+5/12*x*(2*x^2-x+3)^(5/2)-94691/32768*arcsinh(1/23*
(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx = \frac{4\sqrt{3-x+2x^2}(388341+565276x+319072x^2+561024x^3+14336x^4+204800x^5) - 142036}{491520}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 1433
6*x^4 + 204800*x^5) - 1420365*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/491520
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2} \left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12} \int \frac{1}{2} (107x+18) \left(2x^2-x+3\right)^{3/2}dx+\frac{5}{12} x \left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24} \int (107x+18) \left(2x^2-x+3\right)^{3/2}dx+\frac{5}{12} x \left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{24} \left(\frac{179}{4} \int \left(2x^2-x+3\right)^{3/2}dx+\frac{107}{10} \left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{12} x \left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{24} \left(\frac{179}{4} \left(\frac{69}{32} \int \sqrt{2x^2-x+3}dx-\frac{1}{16} (1-4x) \left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10} \left(2x^2-x+3\right)^{5/2}\right)+ \frac{5}{12} x \left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10}\left(2x^2\right.\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$\Big\downarrow$ 1090

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right.\right.\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$\Big\downarrow$ 222

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10}\left(2x^2-x\right.\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(5/2))/12 + ((107*(3 - x + 2*x^2)^(5/2))/10 + (179*(-
1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x +
 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4)/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341)\sqrt{2x^2-x+3}}{122880}+\frac{94691\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{32768}$
trager	$\left(\frac{5}{3}x^5+\frac{7}{60}x^4+\frac{1461}{320}x^3+\frac{9971}{3840}x^2+\frac{141319}{30720}x+\frac{129447}{40960}\right)\sqrt{2x^2-x+3}-\frac{94691\ \text{RootOf}\left(\_Z^2-2\right)\ln\left(-4\ \text{Root}\right.}{}$
default	$\frac{179(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{1536}+\frac{4117(4x-1)\sqrt{2x^2-x+3}}{8192}+\frac{94691\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{32768}+\frac{107(2x^2-x+3)^{\frac{5}{2}}}{240}+\frac{5x(2x^2-x+}{12}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/122880*(204800*x^5+14336*x^4+561024*x^3+319072*x^2+565276*x+388341)*(2*x
^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\frac{1}{122880}\left(204800\,x^5+14336\,x^4+561024\,x^3+319072\,x^2+565276\,x+388341\right)\sqrt{2\,x^2-x+3}+\frac{94691}{65536}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 38
8341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^5}{3}+\frac{7x^4}{60}+\frac{1461x^3}{320}+\frac{9971x^2}{3840}+\frac{141319x}{30720}+\frac{129447}{40960}\right)+\frac{94691\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**5/3 + 7*x**4/60 + 1461*x**3/320 + 9971*x**2/384
0 + 141319*x/30720 + 129447/40960) + 94691*sqrt(2)*asinh(4*sqrt(23)*(x - 1
/4)/23)/32768
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)\,dx = \frac{5}{12}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{107}{240}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{179}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{179}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4117}{2048}\sqrt{2x^2-x+3}x + \frac{94691}{32768}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4117}{8192}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*
x^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x
^2 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 411
7/8192*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)\,dx = \frac{1}{122880}(4(8(4(16(100x+7)x+4383)x+9971)x+141319)x+388341)\sqrt{2x^2-x+3} - \frac{94691}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 3883
41)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)\,dx=\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)\,dx=\frac{5\sqrt{2x^2-x+3}\,x^5}{3}+\frac{7\sqrt{2x^2-x+3}\,x^4}{60}+\frac{1461\sqrt{2x^2-x+3}\,x^3}{320}+\frac{9971\sqrt{2x^2-x+3}\,x^2}{3840}+\frac{141319\sqrt{2x^2-x+3}\,x}{30720}+\frac{129447\sqrt{2x^2-x+3}}{40960}+\frac{94691\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{32768}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x)
```

output
```
(819200*sqrt(2*x**2 - x + 3)*x**5 + 57344*sqrt(2*x**2 - x + 3)*x**4 + 2244
096*sqrt(2*x**2 - x + 3)*x**3 + 1276288*sqrt(2*x**2 - x + 3)*x**2 + 226110
4*sqrt(2*x**2 - x + 3)*x + 1553364*sqrt(2*x**2 - x + 3) + 1420365*sqrt(2)*
log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/491520
```

3.103 $\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} \, dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(8+61\sqrt{2}+\left(130+69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{11}{125}\sqrt{\frac{11}{31}\left(-247+500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-247+500\sqrt{2}\right)}}\left(8-61\sqrt{2}+\left(130-69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
-1/100*(49-20*x)*(2*x^2-x+3)^(1/2)-2203/2000*arcsinh(1/23*(1-4*x)*23^(1/2)
)*2^(1/2)+11/3875*(84227+170500*2^(1/2))^(1/2)*arctan(11^(1/2)/(15314+3100
0*2^(1/2))^(1/2)*(8+61*2^(1/2)+(130+69*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-11/3
875*(-84227+170500*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-15314+31000*2^(1/2))^
(1/2)*(8-61*2^(1/2)+(130-69*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \frac{20(-49+20x)\sqrt{3-x+2x^2}-2203\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)+1936\text{R}}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

output
```
(20*(-49 + 20*x)*Sqrt[3 - x + 2*x^2] - 2203*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6
 - 2*x + 4*x^2]] + 1936*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*
#1^3 - 5*#1^4 & , (-36*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 6*Sq
rt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 13*Log[-(Sqrt[2]*x
) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
- 10*#1^3) & ])/2000
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1308, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{1308}$$

$$-\frac{1}{50}\int -\frac{2203x^2-1195x+1462}{4\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{200}\int \frac{2203x^2-1195x+1462}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{200}\left(\frac{2203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{968(3-13x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{200}\left(\frac{2203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{200}\left(\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{2203\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{200}\left(\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\frac{\int -\frac{11\left(\left(10+13\sqrt{2}\right)x-3\sqrt{2}+16\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(10-13\sqrt{2}\right)x+3\sqrt{2}+16\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\frac{\int \frac{\left(10-13\sqrt{2}\right)x+3\sqrt{2}+16}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(10+13\sqrt{2}\right)x-3\sqrt{2}+16}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{1362}$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\sqrt{2}\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)^2}{2x^2-x+3}-62\left(247-500\sqrt{2}\right)}d\frac{\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8}{\sqrt{2x^2-x+3}}\right.\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\sqrt{2}\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)^2}{2x^2-x+3}-62\left(247-500\sqrt{2}\right)}d\frac{\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8}{\sqrt{2x^2-x+3}}\right.\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{200}\left(\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}+\frac{968}{5}\left(\sqrt{\frac{1}{341}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+\right.}{\sqrt{2x^2-x+3}}\right.\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

output
```
-1/100*((49 - 20*x)*Sqrt[3 - x + 2*x^2]) + ((2203*ArcSinh[(-1 + 4*x)/Sqrt[
23]])/(5*Sqrt[2]) + (968*(Sqrt[(247 + 500*Sqrt[2])/341]*ArcTan[(Sqrt[11/(6
2*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]] + ((247 - 500*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-247 + 500*Sqrt[2
]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[3
41*(-247 + 500*Sqrt[2])]))/5)/200
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.01 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.54

method	result
trager	$\left(-\frac{49}{100}+\frac{x}{5}\right)\sqrt{2x^2-x+3}+\frac{2203\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\mathrm{RootOf}\left(\_Z^2-2\right)\right)}{2000}-$
risch	$\frac{(-49+20x)\sqrt{2x^2-x+3}}{100}+\frac{2203\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2000}+\frac{11\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1535\sqrt{2}\sqrt{\right.}{}$
default	Expression too large to display

input

```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output

```
(-49/100+1/5*x)*(2*x^2-x+3)^(1/2)+2203/2000*RootOf(_Z^2-2)*ln(4*RootOf(_Z^
2-2)*x+4*(2*x^2-x+3)^(1/2)-RootOf(_Z^2-2))-1/100*RootOf(24025*_Z^4+1630634
72*_Z^2+2267598080000)*ln((-5429049375*x*RootOf(24025*_Z^4+163063472*_Z^2+
2267598080000)^5-40888264630400*RootOf(24025*_Z^4+163063472*_Z^2+226759808
0000)^3*x+13640929511440000*(2*x^2-x+3)^(1/2)*RootOf(24025*_Z^4+163063472*
_Z^2+2267598080000)^2-154372254960000*RootOf(24025*_Z^4+163063472*_Z^2+226
7598080000)^3+592661349855657984*RootOf(24025*_Z^4+163063472*_Z^2+22675980
80000)*x+52586627694873395200*(2*x^2-x+3)^(1/2)-2295791036224716800*RootOf
(24025*_Z^4+163063472*_Z^2+2267598080000))/(775*x*RootOf(24025*_Z^4+163063
472*_Z^2+2267598080000)^2+6431392*x+5068448))-1/15500*RootOf(_Z^2+24025*Ro
otOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2+163063472)*ln(-(8686479*Ro
otOf(_Z^2+24025*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2+16306347
2)*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^4*x+52493234464*RootOf(
24025*_Z^4+163063472*_Z^2+2267598080000)^2*RootOf(_Z^2+24025*RootOf(24025*
_Z^4+163063472*_Z^2+2267598080000)^2+163063472)*x-246995607936*RootOf(2402
5*_Z^4+163063472*_Z^2+2267598080000)^2*RootOf(_Z^2+24025*RootOf(24025*_Z^4
+163063472*_Z^2+2267598080000)^2+163063472)+3382950518837120*(2*x^2-x+3)^(
1/2)*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2-992130849952000*Roo
tOf(_Z^2+24025*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2+163063472
)*x+1996846869248000*RootOf(_Z^2+24025*RootOf(24025*_Z^4+163063472*_Z^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(143) = 286$.

Time = 0.10 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.75

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-11/250*sqrt(11/31)*sqrt(500*sqrt(2) + 247)*arctan(-1/1309*sqrt(11/31)*(sq
rt(11/31)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*
x^2 + 12*x) - 3936*x)*sqrt(500*sqrt(2) - 247) + 88*(105*x^3 - 323*x^2 - sq
rt(2)*(161*x^3 - 306*x^2 + 56*x + 192) - 336*x + 72)*sqrt(2*x^2 - x + 3))*
sqrt(500*sqrt(2) + 247)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + 11
/250*sqrt(11/31)*sqrt(500*sqrt(2) + 247)*arctan(-1/1309*sqrt(11/31)*(sqrt(
11/31)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2
 + 12*x) - 3936*x)*sqrt(500*sqrt(2) - 247) - 88*(105*x^3 - 323*x^2 - sqrt(
2)*(161*x^3 - 306*x^2 + 56*x + 192) - 336*x + 72)*sqrt(2*x^2 - x + 3))*sqr
t(500*sqrt(2) + 247)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 11/50
0*sqrt(11/31)*sqrt(500*sqrt(2) - 247)*log(11*(2*sqrt(11/31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(81*x - 394) + 313*x - 475)*sqrt(500*sqrt(2) - 247) + 5831*
x^2 + 5236*sqrt(2)*(2*x^2 - x + 3) - 17969*x + 23800)/x^2) + 11/500*sqrt(1
1/31)*sqrt(500*sqrt(2) - 247)*log(-11*(2*sqrt(11/31)*sqrt(2*x^2 - x + 3)*(
sqrt(2)*(81*x - 394) + 313*x - 475)*sqrt(500*sqrt(2) - 247) - 5831*x^2 - 5
236*sqrt(2)*(2*x^2 - x + 3) + 17969*x - 23800)/x^2) + 1/100*sqrt(2*x^2 - x
 + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx=\int \frac{\left(2\,x^2-x+3\right)^{3/2}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx &= \frac{\sqrt{2x^2-x+3}\,x}{5}-\frac{80961\sqrt{2x^2-x+3}}{62500}\\ &+\frac{143163\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\,\sqrt{2}-4x+1\right)}{50000}\\ &+\frac{11011\sqrt{2}\log\left(2\sqrt{2x^2-x+3}\,\sqrt{2}-4x+1\right)}{6250}\\ &+\frac{32791\left(\int \frac{\sqrt{2x^2-x+3}}{10x^4+x^3+16x^2+7x+6}dx\right)}{15625}+\frac{25168\left(\int \frac{\sqrt{2x^2-x+3}\,x^3}{10x^4+x^3+16x^2+7x+6}dx\right)}{3125}\\ &+\frac{44044\left(\int \frac{\sqrt{2x^2-x+3}\,x^2}{10x^4+x^3+16x^2+7x+6}dx\right)}{15625}-\frac{33033\left(\int \frac{\sqrt{2x^2-x+3}\,x}{10x^4+x^3+16x^2+7x+6}dx\right)}{3125}\end{aligned}$$

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

output
```
(50000*sqrt(2*x**2 - x + 3)*x - 323844*sqrt(2*x**2 - x + 3) + 715815*sqrt(
2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 440440*sqrt(2)*log(2
*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 524656*int(sqrt(2*x**2 - x + 3)
/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x) + 2013440*int((sqrt(2*x**2 - x +
3)*x**3)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x) + 704704*int((sqrt(2*x**2
 - x + 3)*x**2)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x) - 2642640*int((sqr
t(2*x**2 - x + 3)*x)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x))/250000
```

3.104 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx = \frac{16}{155}\sqrt{3-x+2x^2} - \frac{4}{31}x\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2}\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})}\arctan\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}\left(3514+2963\sqrt{2}+\left(9440+6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1550} - \frac{\sqrt{\frac{11}{31}(-3169333+2265350\sqrt{2})}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62(-3169333+2265350\sqrt{2})}}\left(3514-2963\sqrt{2}+\left(9440-6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1550}$$

output
```
16/155*(2*x^2-x+3)^(1/2)-4/31*x*(2*x^2-x+3)^(1/2)+(3+10*x)*(2*x^2-x+3)^(3/
2)/(155*x^2+93*x+62)-2/25*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/48050*(
1080742553+772484350*2^(1/2))^(1/2)*arctan(11^(1/2)/(196498646+140451700*2
^(1/2))^(1/2)*(3514+2963*2^(1/2)+(9440+6477*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
-1/48050*(-1080742553+772484350*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-19649864
6+140451700*2^(1/2))^(1/2)*(3514-2963*2^(1/2)+(9440-6477*2^(1/2))*x)/(2*x^
2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.90 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.69

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \frac{50\left(\frac{55(7+13x)\sqrt{3-x+2x^2}}{2+3x+5x^2} - 62\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\right) + 682\text{RootSum}\Big[\cdots}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(50*((55*(7 + 13*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 62*Sqrt[2]*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 682*RootSum[-56 - 26*Sqrt[2]*#1 +
17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (999*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
 2*x^2] - #1] + 310*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1 + 100*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 11*RootSum[-56 - 26*Sqrt[2]*#1 + 1
7*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-72888*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1] + 8230*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1
]*#1 + 2025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2
] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/38750
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1302, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} - \frac{1}{31}\int -\frac{(-80x^2-26x+69)\sqrt{2x^2-x+3}}{2(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{62}\int \frac{\left(-80x^2-26x+69\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2138}$$

$$\frac{1}{62}\left(\frac{8}{5}(4-5x)\sqrt{2x^2-x+3}-\frac{1}{100}\int -\frac{20\left(248x^2-575x+1307\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{62}\left(\frac{1}{5}\int \frac{248x^2-575x+1307}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{248}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{11(549-329x)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{248}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{124}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{8}{5}\sqrt{2x^2-x+3}(\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\frac{\int -\frac{11\left(-\left(\left(220-329\sqrt{2}\right)x\right)-549\sqrt{2}+878\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(220+329\sqrt{2}\right)x\right)+549\sqrt{2}+878\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{124}{5}\sqrt{2}\text{arcsinh}\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\frac{\int \frac{-\left(\left(220+329\sqrt{2}\right)x\right)+549\sqrt{2}+878}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(220-329\sqrt{2}\right)x\right)-549\sqrt{2}+878}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1362}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\sqrt{2}\left(3169333-2265350\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)^2}{2x^2-x+3}-62\left(3169333-2265350\sqrt{2}\right)}d\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\sqrt{2}\left(3169333-2265350\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)^2}{2x^2-x+3}-62\left(3169333-2265350\sqrt{2}\right)}d\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(3169333+2265350\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}((944}{\sqrt{2x}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(31*(2 + 3*x + 5*x^2)) + ((8*(4 - 5*x)*
Sqrt[3 - x + 2*x^2])/5 + ((124*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/5 + (
11*(Sqrt[(3169333 + 2265350*Sqrt[2])/341]*ArcTan[(Sqrt[11/(62*(3169333 + 2
265350*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]] + ((3169333 - 2265350*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-316933
3 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sqrt[2])*x))/Sq
rt[3 - x + 2*x^2]])/Sqrt[341*(-3169333 + 2265350*Sqrt[2])]))/5)/5)/62
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2138

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143

```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.68 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.46

method	result
trager	Expression too large to display
risch	$\frac{11(7+13x)\sqrt{2x^2-x+3}}{155(5x^2+3x+2)}+\frac{2\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{25}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(126130\sqrt{2}\sqrt{-88}\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
11/155*(7+13*x)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)+1/48050*RootOf(_Z^2+384400
*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+1080742553)*ln((6571
0627584*RootOf(_Z^2+384400*RootOf(3075200*_Z^4+8645940424*_Z^2+62094908532
25)^2+1080742553)*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^4*x+1
01773581037216*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2*RootOf
(_Z^2+384400*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+10807425
53)*x-42599057069184*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2*
RootOf(_Z^2+384400*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+10
80742553)-4066278786433881248*(2*x^2-x+3)^(1/2)*RootOf(3075200*_Z^4+864594
0424*_Z^2+6209490853225)^2-11025935123814325*RootOf(_Z^2+384400*RootOf(307
5200*_Z^4+8645940424*_Z^2+6209490853225)^2+1080742553)*x+4330816090653800*
RootOf(_Z^2+384400*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+10
80742553)-5673402550436157361550*(2*x^2-x+3)^(1/2))/(12400*x*RootOf(307520
0*_Z^4+8645940424*_Z^2+6209490853225)^2+16044655*x-1848902))-2/155*RootOf(
3075200*_Z^4+8645940424*_Z^2+6209490853225)*ln(-(41069142240000*x*RootOf(3
075200*_Z^4+8645940424*_Z^2+6209490853225)^5+167323715982737600*RootOf(307
5200*_Z^4+8645940424*_Z^2+6209490853225)^3*x+26624410668240000*RootOf(3075
200*_Z^4+8645940424*_Z^2+6209490853225)^3-409907135729221900*(2*x^2-x+3)^
(1/2)*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+138906735546068
157756*x*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)+77561425919...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(184) = 368$.

Time = 0.10 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.33

$$\int \frac{(3 - x + 2x^2)^{3/2}}{(2 + 3x + 5x^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/6200*(2*(5*x^2 + 3*x + 2)*sqrt(24918850/31*sqrt(2) + 34862663/31)*arcta
n(-1/924451*(88*(13065*x^3 - 30409*x^2 - sqrt(2)*(9988*x^3 - 22173*x^2 - 6
152*x + 10536) - 12288*x + 13176)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^
3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(2
4918850/31*sqrt(2) - 34862663/31))*sqrt(24918850/31*sqrt(2) + 34862663/31)
/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 2*(5*x^2 + 3*x + 2)*sqrt(
24918850/31*sqrt(2) + 34862663/31)*arctan(1/924451*(88*(13065*x^3 - 30409*
x^2 - sqrt(2)*(9988*x^3 - 22173*x^2 - 6152*x + 10536) - 12288*x + 13176)*s
qrt(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 +
 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(24918850/31*sqrt(2) - 34862663/31))*
sqrt(24918850/31*sqrt(2) + 34862663/31)/(343*x^4 - 400*x^3 + 1136*x^2 + 38
4*x - 576)) - 248*sqrt(2)*(5*x^2 + 3*x + 2)*log(-4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + (5*x^2 + 3*x + 2)*sqrt(24918850/31*
sqrt(2) - 34862663/31)*log((4118009*x^2 + 2*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
11748*x - 30077) + 18329*x - 41825)*sqrt(24918850/31*sqrt(2) - 34862663/31
) + 3697804*sqrt(2)*(2*x^2 - x + 3) - 12690191*x + 16808200)/x^2) - (5*x^2
 + 3*x + 2)*sqrt(24918850/31*sqrt(2) - 34862663/31)*log((4118009*x^2 - 2*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(11748*x - 30077) + 18329*x - 41825)*sqrt(2491
8850/31*sqrt(2) - 34862663/31) + 3697804*sqrt(2)*(2*x^2 - x + 3) - 1269019
1*x + 16808200)/x^2) - 440*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x...
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}}{(5x^2+3x+2)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{174900625,[8]%%%}+%%%{%%{[-419761500,0]:[1,0,-2]%%},[7]%%%
}+%%%{-68
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

Reduce [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \frac{-13640\sqrt{2x^2-x+3}\,x+9372\sqrt{2x^2-x+3}+9530\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\,\sqrt{2}\right.}{}$$

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

output
```
( - 13640*sqrt(2*x**2 - x + 3)*x + 9372*sqrt(2*x**2 - x + 3) + 9530*sqrt(2
)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 5718*sqrt(2)*log
( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 3812*sqrt(2)*log( - 2*sq
rt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 1873685*int(sqrt(2*x**2 - x + 3)/(
50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x)*x**2 + 11
24211*int(sqrt(2*x**2 - x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 8
3*x**2 + 32*x + 12),x)*x + 749474*int(sqrt(2*x**2 - x + 3)/(50*x**6 + 35*x
**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x) + 64735*int((sqrt(2*x**
2 - x + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x
+ 12),x)*x**2 + 38841*int((sqrt(2*x**2 - x + 3)*x**2)/(50*x**6 + 35*x**5 +
 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x)*x + 25894*int((sqrt(2*x**2 -
 x + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 1
2),x))/(23825*(5*x**2 + 3*x + 2))
```

3.105 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2}+\frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)}$$
$$+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(29367+20575\sqrt{2}+\left(70517+49942\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{7688}$$
$$-\frac{3\sqrt{\frac{1}{682}\left(-366990269+259509026\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-366990269+259509026\sqrt{2}\right)}}\left(29367-20575\sqrt{2}+\left(70517-49942\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{7688}$$

output

```
1/62*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+3*(277+696*x)*(2*x^2-x+3)^
(1/2)/(19220*x^2+11532*x+7688)+3/5243216*(250287363458+176985155732*2^(1/2
))^(1/2)*arctan(11^(1/2)/(11376698339+8044779806*2^(1/2))^(1/2)*(29367+205
75*2^(1/2)+(70517+49942*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-3/5243216*(-2502873
63458+176985155732*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-11376698339+804477980
6*2^(1/2))^(1/2)*(29367-20575*2^(1/2)+(70517-49942*2^(1/2))*x)/(2*x^2-x+3)
^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.24 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.57

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{\frac{3306250\sqrt{3-x+2x^2}(2220+8343x+10171x^2+11680x^3)}{(2+3x+5x^2)^2} - 42578694225\text{RootSum}\Big[-56-26\sqrt{2}\cdots}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3306250*Sqrt[3 - x + 2*x^2]*(2220 + 8343*x + 10171*x^2 + 11680*x^3))/(2
+ 3*x + 5*x^2)^2 - 42578694225*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*S
qrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 406695200*RootSum[-56 -
 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (93*Log[-(Sqrt[2]*x
) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 1
4*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492
6449381*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 26609914
65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 186*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (155209944*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1 - 248390285*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^
2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/12709
225000
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1302, 27, 1346, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2x^2-x+3\right)^{3/2}}{\left(5x^2+3x+2\right)^3}dx$$

$$\Big\downarrow \text{ 1302}$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}-\frac{1}{62}\int -\frac{3(63-22x)\sqrt{2x^2-x+3}}{2\left(5x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{124}\int \frac{(63-22x)\sqrt{2x^2-x+3}}{\left(5x^2+3x+2\right)^2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 1346}$$

$$\frac{3}{124}\left(\frac{(696x+277)\sqrt{2x^2-x+3}}{31\left(5x^2+3x+2\right)}-\frac{1}{31}\int -\frac{4453-1804x}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{124}\left(\frac{1}{62}\int \frac{4453-1804x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{\sqrt{2x^2-x+3}(696x+277)}{31\left(5x^2+3x+2\right)}\right)+$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 1368}$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\int -\frac{11\left(-\left(\left(2649-1804\sqrt{2}\right)x\right)-4453\sqrt{2}+6257\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(2649+1804\sqrt{2}\right)x\right)+4453\sqrt{2}+6257\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)+\frac{\sqrt{2x^2-x}}{31\,(5}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\int \frac{-\left(\left(2649+1804\sqrt{2}\right)x\right)+4453\sqrt{2}+6257}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(2649-1804\sqrt{2}\right)x\right)-4453\sqrt{2}+6257}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(696x}{31\left(5x^2+3x+2\right.}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 1362$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\left(366990269-259509026\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)^2}{2x^2-x+3}-31\left(366990269-259509026\sqrt{2}\right)}d\frac{\left(70517-49\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\left(366990269-259509026\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)^2}{2x^2-x+3}-31\left(366990269-259509026\sqrt{2}\right)}d\frac{\left(70517-49\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(\left(70517+49942\sqrt{2}\right)x+20\right.}{\sqrt{2x^2-x+3}}\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(((277 +
696*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))
]*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]
] + ((366990269 - 259509026*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-366990269 + 25
9509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*Sqrt[2])*x))/Sq
rt[3 - x + 2*x^2]])/Sqrt[682*(-366990269 + 259509026*Sqrt[2])])/62))/124
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1346
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
 (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*
(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.50 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{(11680x^3+10171x^2+8343x+2220)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)^2}+\frac{3\sqrt{\frac{8(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}\,(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1915561\sqrt{2}\,\sqrt{-8866+68\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3844*(11680*x^3+10171*x^2+8343*x+2220)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2)
-3/5243216*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+1
6836233643867169)^2+250287363458)*ln(-(36607893262336*RootOf(_Z^2+29767936
*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2+25028736345
8)*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^4*x+3262645
23201744512*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2*
RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+168362336438
67169)^2+250287363458)*x+3707589189779200*RootOf(952573952*_Z^4+8009195630
656*_Z^2+16836233643867169)^2*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8
009195630656*_Z^2+16836233643867169)^2+250287363458)-122510784267145793066
24*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2*(2*x^2-x+
3)^(1/2)+723862202733749385201*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+
8009195630656*_Z^2+16836233643867169)^2+250287363458)*x+175986874983483557
00*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+168362336
43867169)^2+250287363458)-5152337237574050505771805 4*(2*x^2-x+3)^(1/2))/(2
1824*x*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2+92128
844*x+508369))+3/961*RootOf(952573952*_Z^4+8009195630656*_Z^2+168362336438
67169)*ln((585726292197376*x*RootOf(952573952*_Z^4+8009195630656*_Z^2+1683
6233643867169)^5+4629284194657700864*RootOf(952573952*_Z^4+8009195630656*_
Z^2+16836233643867169)^3*x-5932142703646720 0*RootOf(952573952*_Z^4+8009...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(170) = 340$.

Time = 0.09 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.77

$$\int \frac{(3 - x + 2x^2)^{3/2}}{(2 + 3x + 5x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/30752*(6*sqrt(1/682)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(25950902
6*sqrt(2) + 366990269)*arctan(-22/508369*sqrt(1/682)*(sqrt(1/682)*(171*x^4
 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936
*x)*sqrt(259509026*sqrt(2) - 366990269) + 4*(206756*x^3 - 469094*x^2 - sqr
t(2)*(145601*x^3 - 331165*x^2 - 113632*x + 150168) - 157840*x + 213744)*sq
rt(2*x^2 - x + 3))*sqrt(259509026*sqrt(2) + 366990269)/(343*x^4 - 400*x^3
+ 1136*x^2 + 384*x - 576)) - 6*sqrt(1/682)*(25*x^4 + 30*x^3 + 29*x^2 + 12*
x + 4)*sqrt(259509026*sqrt(2) + 366990269)*arctan(-22/508369*sqrt(1/682)*(
sqrt(1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 +
5*x^2 + 12*x) - 3936*x)*sqrt(259509026*sqrt(2) - 366990269) - 4*(206756*x^
3 - 469094*x^2 - sqrt(2)*(145601*x^3 - 331165*x^2 - 113632*x + 150168) - 1
57840*x + 213744)*sqrt(2*x^2 - x + 3))*sqrt(259509026*sqrt(2) + 366990269)
/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + 3*sqrt(1/682)*(25*x^4 + 3
0*x^3 + 29*x^2 + 12*x + 4)*sqrt(259509026*sqrt(2) - 366990269)*log(3*(22*s
qrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(187985*x - 452469) + 264484*x - 6
40454)*sqrt(259509026*sqrt(2) - 366990269) + 24910081*x^2 + 22368236*sqrt(
2)*(2*x^2 - x + 3) - 76763719*x + 101673800)/x^2) - 3*sqrt(1/682)*(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(259509026*sqrt(2) - 366990269)*log(-3*(
22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(187985*x - 452469) + 264484*x
 - 640454)*sqrt(259509026*sqrt(2) - 366990269) - 24910081*x^2 - 2236823...
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```

Reduce [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{-186400\sqrt{2x^2-x+3}\,x^3-270280\sqrt{2x^2-x+3}\,x^2-377744\sqrt{2x^2-x+3}\,x+}{}$$

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

output
```
( - 186400*sqrt(2*x**2 - x + 3)*x**3 - 270280*sqrt(2*x**2 - x + 3)*x**2 -
377744*sqrt(2*x**2 - x + 3)*x + 157812*sqrt(2*x**2 - x + 3) + 388037925*in
t(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x*
*4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**4 + 465645510*int(sqrt(2*x**2
 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3
 + 322*x**2 + 100*x + 24),x)*x**3 + 450123993*int(sqrt(2*x**2 - x + 3)/(25
0*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 +
 100*x + 24),x)*x**2 + 186258204*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*
x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),
x)*x + 62086068*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 +
 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x) + 39854375*int
((sqrt(2*x**2 - x + 3)*x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 +
876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**4 + 47825250*int((sqrt(
2*x**2 - x + 3)*x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**
4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**3 + 46231075*int((sqrt(2*x**2
- x + 3)*x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579
*x**3 + 322*x**2 + 100*x + 24),x)*x**2 + 19130100*int((sqrt(2*x**2 - x + 3
)*x**2)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 +
 322*x**2 + 100*x + 24),x)*x + 6376700*int((sqrt(2*x**2 - x + 3)*x**2)/(25
0*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**...
```

3.106 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx = -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648} - \frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x(3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2(3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3(3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4(3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5(3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2} + \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2} - \frac{14641852251147\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{68719476736\sqrt{2}}$$

output

```
-636602271789/34359738368*(1-4*x)*(2*x^2-x+3)^(1/2)-9226119881/2147483648*
(1-4*x)*(2*x^2-x+3)^(3/2)-401135647/335544320*(1-4*x)*(2*x^2-x+3)^(5/2)+25
250178739/5725224960*(2*x^2-x+3)^(7/2)+112244125/122683392*x*(2*x^2-x+3)^(
7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+23460839/532480*x^3*(2*x^2-x
+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+1046225/9984*x^5*(2*x^2-x+3)
^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2-x+3)^(7/2)-146418
52251147/137438953472*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.45

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)^4\,dx = \frac{4\sqrt{3-x+2x^2}(10820567498568669+12071614275862524x+50064174038215008x^2+1424\ldots}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 50064174
038215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 40546838
2284161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645
58846345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 13723
3466130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 59
958384968446965*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/5628125144
67840
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.20, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{5/2}(5x^2+3x+2)^4\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{28}\int \frac{7}{2}(2x^2-x+3)^{5/2}(13875x^7+15050x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128)\,dx+\frac{625}{28}(2x^2-x+3)^{7/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\int \left(2x^2-x+3\right)^{5/2}\left(13875x^7+15050x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+ \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{26}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(1046225x^6+473940x^5+732576x^4+389376x^3+156416x^2+39936x+6656\right)dx\right. \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{52}\int \left(2x^2-x+3\right)^{5/2}\left(1046225x^6+473940x^5+732576x^4+389376x^3+156416x^2+39936x+6656\right)dx+\right. \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{24}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(40534945x^5+3776898x^4+18690048x^3+7507968x^2+1916928x+319488\right)\right.\right. \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\int \left(2x^2-x+3\right)^{5/2}\left(40534945x^5+3776898x^4+18690048x^3+7507968x^2+1916928x+319488\right)dx\right.\right. \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{1}{22}\int \frac{33}{2}\left(2x^2-x+3\right)^{5/2}\left(23460839x^4-4559896x^3+10010624x^2+2555904x+425984\right)dx+\frac{368}{}\right.\right.\right. \frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\int \left(2x^2-x+3\right)^{5/2}\left(23460839x^4-4559896x^3+10010624x^2+2555904x+425984\right)dx+\frac{368499}{2}\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{20}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(122595067x^3-21870142x^2+102236160x+17039360\right)dx+\frac{2346083}{20}\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\int \left(2x^2-x+3\right)^{5/2}\left(122595067x^3-21870142x^2+102236160x+17039360\right)dx+\frac{23460839}{20}\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(561220625x^2+2209360956x+613416960\right)dx+\frac{122595067}{18}x^2(2x\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(561220625x^2+2209360956x+613416960\right)dx+\frac{122595067}{18}x^2(2x^2\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{3}{2}(25250178739x+5420672990)\left(2x^2-x+3\right)^{5/2}dx+\frac{561220625}{16}x(2x^2-x+3\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\int (25250178739x+5420672990)\left(2x^2-x+3\right)^{5/2}dx+\frac{561220625}{16}x(2x^2-x+3)^{7}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\int \left(2x^2-x+3\right)^{5/2}dx+\frac{25250178739}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{5612206}{16}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{2525}{}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(625*x^7*(3 - x + 2*x^2)^(7/2))/28 + ((13875*x^6*(3 - x + 2*x^2)^(7/2))/26
 + ((1046225*x^5*(3 - x + 2*x^2)^(7/2))/24 + ((3684995*x^4*(3 - x + 2*x^2)
^(7/2))/2 + (3*((23460839*x^3*(3 - x + 2*x^2)^(7/2))/20 + ((122595067*x^2*
(3 - x + 2*x^2)^(7/2))/18 + ((561220625*x*(3 - x + 2*x^2)^(7/2))/16 + (3*(
(25250178739*(3 - x + 2*x^2)^(7/2))/14 + (46932870699*(-1/24*((1 - 4*x)*(3
 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69
*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(16*Sqrt[2])))/32))/48))/4))/32)/36)/40))/4)/48)/52)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.37

method	result
risch	$\frac{(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600x^{10}+363646430503501824x^{9}+}{}$
trager	$\left(\frac{1250}{7}x^{13}+\frac{48375}{182}x^{12}+\frac{1217225}{1248}x^{11}+\frac{50895515}{34944}x^{10}+\frac{172023939}{66560}x^{9}+\frac{52340574127}{16773120}x^{8}+\frac{2023708176167}{536739840}x^{7}+2$
default	$\frac{401135647(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{335544320}+\frac{9226119881(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{2147483648}+\frac{636602271789(4x-1)\sqrt{2x^2-x+3}}{34359738368}+\frac{14641852251147\sqrt{}}{137}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/140703128616960*(25125558681600000*x^13+37398427729920000*x^12+137233466
130432000*x^11+204932411660697600*x^10+363646430503501824*x^9+439064558846
345216*x^8+530502956133122048*x^7+485091164642279424*x^6+40546838228416102
4*x^5+257786732552566784*x^4+142490931553577856*x^3+50064174038215008*x^2+
12071614275862524*x+10820567498568669)*(2*x^2-x+3)^(1/2)+14641852251147/13
7438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.46

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{1}{140703128616960} \left(25125558681600000\,x^{13} + 37398427729920000\,x^{12} + 13723346613043200\ldots\right. + \frac{14641852251147}{274877906944}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 13723
3466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 43
9064558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4
05468382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 +
50064174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^
2 - x + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 dx = \sqrt{2x^2-x+3} \cdot \left(\frac{1250x^{13}}{7} + \frac{48375x^{12}}{182} + \frac{1217225x^{11}}{1248} + \frac{50895515x^{10}}{34944} + \frac{172023939x^9}{66560} + \frac{52340574127x^8}{16773120} + \frac{2023708176167x^7}{536739840} + \frac{2467301252453x^6}{715653120} + \frac{49495652134297x^5}{17175674880} + \frac{17981775429169x^4}{9814671360} + \frac{371070134254109x^3}{366414397440} + \frac{24833419661813x^2}{69793218560} + \frac{335322618773959x}{3908420239360} + \frac{1202285277618741}{15633680957440}\right) + \frac{14641852251147\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{137438953472}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

output
```
sqrt(2*x**2 - x + 3)*(1250*x**13/7 + 48375*x**12/182 + 1217225*x**11/1248
+ 50895515*x**10/34944 + 172023939*x**9/66560 + 52340574127*x**8/16773120
+ 2023708176167*x**7/536739840 + 2467301252453*x**6/715653120 + 4949565213
4297*x**5/17175674880 + 17981775429169*x**4/9814671360 + 371070134254109*x
**3/366414397440 + 24833419661813*x**2/69793218560 + 335322618773959*x/390
8420239360 + 1202285277618741/15633680957440) + 14641852251147*sqrt(2)*asi
nh(4*sqrt(23)*(x - 1/4)/23)/137438953472
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)^4\,dx = \frac{625}{28}\left(2x^2-x+3\right)^{\frac{7}{2}}x^7 + \frac{13875}{208}\left(2x^2-x+3\right)^{\frac{7}{2}}x^6 + \frac{1046225}{9984}\left(2x^2-x+3\right)^{\frac{7}{2}}x^5 + \frac{3684995}{39936}\left(2x^2-x+3\right)^{\frac{7}{2}}x^4 + \frac{23460839}{532480}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3 + \frac{122595067}{19169280}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2 + \frac{112244125}{122683392}\left(2x^2-x+3\right)^{\frac{7}{2}}x + \frac{25250178739}{5725224960}\left(2x^2-x+3\right)^{\frac{7}{2}} + \frac{401135647}{83886080}\left(2x^2-x+3\right)^{\frac{5}{2}}x - \frac{401135647}{335544320}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{9226119881}{536870912}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{9226119881}{2147483648}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{636602271789}{8589934592}\sqrt{2x^2-x+3}\,x + \frac{14641852251147}{137438953472}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{636602271789}{34359738368}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 1
046225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2
)*x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*
x^2 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 252
50178739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x
+ 3)^(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/5368
70912*(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2
) + 636602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/1374389
53472*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*
sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.44

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{1}{140703128616960} (4 (8 (4 (16 (4 (8 (4 (32 (12 (200 (20 (240 (260 x + 387)x + 340823)x + 1017 - \frac{14641852251147}{137438953472} \sqrt{2} \log \left(-2 \sqrt{2}\left(\sqrt{2}x - \sqrt{2 x^2 - x + 3}\right) + 1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387
)*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 202370817
6167)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113
210402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 108205674985
68669)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \int \left(2\,x^2-x+3\right)^{5/2} \left(5\,x^2+3\,x+2\right)^4 \, dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx &= \frac{1250\sqrt{2x^2-x+3}\,x^{13}}{7} \\ &+ \frac{48375\sqrt{2x^2-x+3}\,x^{12}}{182} + \frac{1217225\sqrt{2x^2-x+3}\,x^{11}}{1248} \\ &+ \frac{50895515\sqrt{2x^2-x+3}\,x^{10}}{34944} + \frac{172023939\sqrt{2x^2-x+3}\,x^{9}}{66560} \\ &+ \frac{52340574127\sqrt{2x^2-x+3}\,x^{8}}{16773120} + \frac{2023708176167\sqrt{2x^2-x+3}\,x^{7}}{536739840} \\ &+ \frac{2467301252453\sqrt{2x^2-x+3}\,x^{6}}{715653120} + \frac{49495652134297\sqrt{2x^2-x+3}\,x^{5}}{17175674880} \\ &+ \frac{17981775429169\sqrt{2x^2-x+3}\,x^{4}}{9814671360} \\ &+ \frac{371070134254109\sqrt{2x^2-x+3}\,x^{3}}{366414397440} + \frac{24833419661813\sqrt{2x^2-x+3}\,x^{2}}{69793218560} \\ &+ \frac{335322618773959\sqrt{2x^2-x+3}\,x}{3908420239360} + \frac{1202285277618741\sqrt{2x^2-x+3}}{15633680957440} \\ &+ \frac{14641852251147\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{137438953472}\end{aligned}$$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)
```

output
```
(100502234726400000*sqrt(2*x**2 - x + 3)*x**13 + 149593710919680000*sqrt(2
*x**2 - x + 3)*x**12 + 548933864521728000*sqrt(2*x**2 - x + 3)*x**11 + 819
729646642790400*sqrt(2*x**2 - x + 3)*x**10 + 1454585722014007296*sqrt(2*x*
*2 - x + 3)*x**9 + 1756258235385380864*sqrt(2*x**2 - x + 3)*x**8 + 2122011
824532488192*sqrt(2*x**2 - x + 3)*x**7 + 1940364658569117696*sqrt(2*x**2 -
 x + 3)*x**6 + 1621873529136644096*sqrt(2*x**2 - x + 3)*x**5 + 10311469302
10267136*sqrt(2*x**2 - x + 3)*x**4 + 569963726214311424*sqrt(2*x**2 - x +
3)*x**3 + 200256696152860032*sqrt(2*x**2 - x + 3)*x**2 + 48286457103450096
*sqrt(2*x**2 - x + 3)*x + 43282269994274676*sqrt(2*x**2 - x + 3) + 5995838
4968446965*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)
))/562812514467840
```

3.107 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 212

$$\begin{aligned}\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx = &-\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} \\ &-\frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864}-\frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} \\ &-\frac{1696165(3-x+2x^2)^{7/2}}{2752512}+\frac{509257x(3-x+2x^2)^{7/2}}{294912}+\frac{80483x^2(3-x+2x^2)^{7/2}}{9216} \\ &+\frac{3823}{256}x^3(3-x+2x^2)^{7/2}+\frac{1175}{96}x^4(3-x+2x^2)^{7/2}+\frac{125}{24}x^5(3-x+2x^2)^{7/2}-\frac{10569777075\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2147483648\sqrt{2}}\end{aligned}$$

output
```
-459555525/1073741824*(1-4*x)*(2*x^2-x+3)^(1/2)-6660225/67108864*(1-4*x)*(
2*x^2-x+3)^(3/2)-57915/2097152*(1-4*x)*(2*x^2-x+3)^(5/2)-1696165/2752512*(
2*x^2-x+3)^(7/2)+509257/294912*x*(2*x^2-x+3)^(7/2)+80483/9216*x^2*(2*x^2-x
+3)^(7/2)+3823/256*x^3*(2*x^2-x+3)^(7/2)+1175/96*x^4*(2*x^2-x+3)^(7/2)+125
/24*x^5*(2*x^2-x+3)^(7/2)-10569777075/4294967296*arcsinh(1/23*(1-4*x)*23^(
1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{4\sqrt{3-x+2x^2}(-1191399152715+4560943728924x+10060731582048x^2+2038482468441}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*
x^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 28347
538538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x
^9 + 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/270582939648
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2} \left(5x^2+3x+2\right)^3 \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{24}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2} \left(12925x^5+9930x^4+9936x^3+5472x^2+1728x+384\right)dx+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{48}\int \left(2x^2-x+3\right)^{5/2}\left(12925x^5+9930x^4+9936x^3+5472x^2+1728x+384\right)dx+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 2192

$$\frac{1}{48}\left(\frac{1}{22}\int \frac{33}{2}\left(2x^2-x+3\right)^{5/2}\left(19115x^4+3848x^3+7296x^2+2304x+512\right)dx+\frac{1175}{2}\left(2x^2-x+3\right)^{7/2}x^4\right)+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{48}\left(\frac{3}{4}\int \left(2x^2-x+3\right)^{5/2}\left(19115x^4+3848x^3+7296x^2+2304x+512\right)dx+\frac{1175}{2}\left(2x^2-x+3\right)^{7/2}x^4\right)+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 2192

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{20}\int \frac{5}{2}\left(2x^2-x+3\right)^{5/2}\left(80483x^3-10446x^2+18432x+4096\right)dx+\frac{3823}{4}x^3\left(2x^2-x+3\right)^{7/2}\right)+\frac{1175}{2}\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\int \left(2x^2-x+3\right)^{5/2}\left(80483x^3-10446x^2+18432x+4096\right)dx+\frac{3823}{4}x^3\left(2x^2-x+3\right)^{7/2}\right)+\frac{1175}{2}(2x\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 2192

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(509257x^2-302244x+147456\right)dx+\frac{80483}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{3823}{4}x^3\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(509257x^2-302244x+147456\right)dx+\frac{80483}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{3823}{4}x^3(\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{15}{2}(110870-339233x)\left(2x^2-x+3\right)^{5/2}dx+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{80483}{18}x^2\left(2x^2\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 27$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\int (110870-339233x)\left(2x^2-x+3\right)^{5/2}dx+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{80483}{18}x^2\left(2x^2-\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\int \left(2x^2-x+3\right)^{5/2}dx-\frac{339233}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)-\frac{339233}{14}\left(2x^2-x+\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-4x)\left(2x^2-\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$\downarrow$ 222

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(125*x^5*(3 - x + 2*x^2)^(7/2))/24 + ((1175*x^4*(3 - x + 2*x^2)^(7/2))/2 +
 (3*((3823*x^3*(3 - x + 2*x^2)^(7/2))/4 + ((80483*x^2*(3 - x + 2*x^2)^(7/2
))/18 + ((509257*x*(3 - x + 2*x^2)^(7/2))/16 + (15*((-339233*(3 - x + 2*x^
2)^(7/2))/14 + (104247*(-1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1
/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x +
2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4))/3
2)/36)/8))/4)/48
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^{9}+14341894045696x^{8}+27835561148416x^{7}+28347538538496x^{6}+3437}{6764573}$
trager	$\left(\frac{125}{3}x^{11}+\frac{425}{12}x^{10}+\frac{6123}{32}x^{9}+\frac{244241}{1152}x^{8}+\frac{15169177}{36864}x^{7}+\frac{144183037}{344064}x^{6}+\frac{4196608145}{8257536}x^{5}+\frac{1826627177}{4718592}x^{4}+5\right.$
default	$\frac{57915(4x-1)\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2097152}+\frac{6660225(4x-1)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{67108864}+\frac{459555525(4x-1)\sqrt{2x^{2}-x+3}}{1073741824}+\frac{10569777075\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{2}}{}\right.}{4294967296}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/67645734912*(2818572288000*x^11+2395786444800*x^10+12943588589568*x^9+14
341894045696*x^8+27835561148416*x^7+28347538538496*x^6+34378613923840*x^5+
26186527209472*x^4+20384824684416*x^3+10060731582048*x^2+4560943728924*x-1
191399152715)*(2*x^2-x+3)^(1/2)+10569777075/4294967296*2^(1/2)*arcsinh(4/2
3*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 dx = \frac{1}{67645734912} \left(2818572288000\, x^{11} + 2395786444800\, x^{10} + 12943588589568\, x^9 + 14341894045\right. + \frac{10569777075}{8589934592} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^
9 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 3437861
3923840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2
 + 4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/8589
934592*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 dx = \sqrt{2x^2-x+3} \cdot \left(\frac{125x^{11}}{3} + \frac{425x^{10}}{12} + \frac{6123x^9}{32} + \frac{244241x^8}{1152} + \frac{15169177x^7}{36864} + \frac{144183037x^6}{344064} + \frac{4196608145x^5}{8257536} + \frac{1826627177x^4}{4718592} + \frac{53085480949x^3}{176160768} + \frac{4990442253x^2}{33554432} + \frac{126692881359x}{1879048192} - \frac{132377683635}{7516192768}\right) + \frac{10569777075\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4294967296}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**11/3 + 425*x**10/12 + 6123*x**9/32 + 244241*x
**8/1152 + 15169177*x**7/36864 + 144183037*x**6/344064 + 4196608145*x**5/8
257536 + 1826627177*x**4/4718592 + 53085480949*x**3/176160768 + 4990442253
*x**2/33554432 + 126692881359*x/1879048192 - 132377683635/7516192768) + 10
569777075*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4294967296
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx = \frac{125}{24}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^5 + \frac{1175}{96}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^4 + \frac{3823}{256}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^3 + \frac{80483}{9216}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^2 + \frac{509257}{294912}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x - \frac{1696165}{2752512}\left(2\,x^2-x+3\right)^{\frac{7}{2}} + \frac{57915}{524288}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x - \frac{57915}{2097152}\left(2\,x^2-x+3\right)^{\frac{5}{2}} + \frac{6660225}{16777216}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x - \frac{6660225}{67108864}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{459555525}{268435456}\sqrt{2\,x^2-x+3}x + \frac{10569777075}{4294967296}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{459555525}{1073741824}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 382
3/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 5
09257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/
2) + 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^
(5/2) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2
 - x + 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/
4294967296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824
*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

$$\int \left(3-x+2x^2\right)^{5/2} (2+3x +5x^2)^3 \, dx = \frac{1}{67645734912} (4 (8 (4 (16 (4 (8 (28 (32 (12 (200 (20 x + 17)x + 18369)x + 244241)x + 1516917 - \frac{10569777075}{4294967296} \sqrt{2} \log \left(-2 \sqrt{2}\left(\sqrt{2}x - \sqrt{2 x^2 - x + 3}\right) + 1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
 + 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x
 + 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*s
qrt(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx = \int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3 \,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03

$$
\begin{aligned}
&\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{125\sqrt{2x^2-x+3}\,x^{11}}{3} \\
&+ \frac{425\sqrt{2x^2-x+3}\,x^{10}}{12} + \frac{6123\sqrt{2x^2-x+3}\,x^9}{32} \\
&+ \frac{244241\sqrt{2x^2-x+3}\,x^8}{1152} + \frac{15169177\sqrt{2x^2-x+3}\,x^7}{36864} \\
&+ \frac{144183037\sqrt{2x^2-x+3}\,x^6}{344064} + \frac{4196608145\sqrt{2x^2-x+3}\,x^5}{8257536} \\
&+ \frac{1826627177\sqrt{2x^2-x+3}\,x^4}{4718592} + \frac{53085480949\sqrt{2x^2-x+3}\,x^3}{176160768} \\
&+ \frac{4990442253\sqrt{2x^2-x+3}\,x^2}{33554432} + \frac{126692881359\sqrt{2x^2-x+3}\,x}{1879048192} \\
&- \frac{132377683635\sqrt{2x^2-x+3}}{7516192768} + \frac{10569777075\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{4294967296}
\end{aligned}
$$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x)
```

output
```
(11274289152000*sqrt(2*x**2 - x + 3)*x**11 + 9583145779200*sqrt(2*x**2 - x
 + 3)*x**10 + 51774354358272*sqrt(2*x**2 - x + 3)*x**9 + 57367576182784*sq
rt(2*x**2 - x + 3)*x**8 + 111342244593664*sqrt(2*x**2 - x + 3)*x**7 + 1133
90154153984*sqrt(2*x**2 - x + 3)*x**6 + 137514455695360*sqrt(2*x**2 - x +
3)*x**5 + 104746108837888*sqrt(2*x**2 - x + 3)*x**4 + 81539298737664*sqrt(
2*x**2 - x + 3)*x**3 + 40242926328192*sqrt(2*x**2 - x + 3)*x**2 + 18243774
915696*sqrt(2*x**2 - x + 3)*x - 4765596610860*sqrt(2*x**2 - x + 3) + 66589
5955725*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/
270582939648
```

3.108 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 170

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx = -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} - \frac{94111745\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

output
```
-4091815/16777216*(1-4*x)*(2*x^2-x+3)^(1/2)-177905/3145728*(1-4*x)*(2*x^2-
x+3)^(3/2)-1547/98304*(1-4*x)*(2*x^2-x+3)^(5/2)+23225/43008*(2*x^2-x+3)^(7
/2)+8467/4608*x*(2*x^2-x+3)^(7/2)+305/144*x^2*(2*x^2-x+3)^(7/2)+5/4*x^3*(2
*x^2-x+3)^(7/2)-94111745/67108864*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx = \frac{4\sqrt{3-x+2x^2}(14824182519+39533249652x+42992644128x^2+77872272000x^3+571474\ldots}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 77
872272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 4416
3137536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/4227858432
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{5/2} (5x^2+3x+2)^2 \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{20} \int \frac{5}{2} (2x^2-x+3)^{5/2} (305x^3+142x^2+96x+32) \, dx + \frac{5}{4} x^3 (2x^2-x+3)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8} \int (2x^2-x+3)^{5/2} (305x^3+142x^2+96x+32) \, dx + \frac{5}{4} x^3 (2x^2-x+3)^{7/2}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(8467x^2-204x+1152\right)dx+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(8467x^2-204x+1152\right)dx+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int -\frac{3}{2}(4646-23225x)\left(2x^2-x+3\right)^{5/2}dx+\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\int (4646-23225x)\left(2x^2-x+3\right)^{5/2}dx\right)+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\int \left(2x^2-x+3\right)^{5/2}dx-\frac{23225}{14}\left(2x^2-x+3\right)^{7/2}\right)\right)+\frac{305}{18}x^2(2x^2\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)-\frac{2}{}\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\right.\right.\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right.\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(5*x^3*(3 - x + 2*x^2)^(7/2))/4 + ((305*x^2*(3 - x + 2*x^2)^(7/2))/18 + ((
8467*x*(3 - x + 2*x^2)^(7/2))/16 - (3*((-23225*(3 - x + 2*x^2)^(7/2))/14 -
 (4641*(-1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(
3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4))/32)/36)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644}{1056964608}$
trager	$\left(10x^9+\frac{35}{18}x^8+\frac{24067}{576}x^7+\frac{134287}{5376}x^6+\frac{9202859}{129024}x^5+\frac{3986291}{73728}x^4+\frac{202792375}{2752512}x^3+\frac{63977149}{1572864}x^2+\frac{3294437471}{88080384}\right.$
default	$\frac{1547(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{98304}+\frac{177905(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{3145728}+\frac{4091815(4x-1)\sqrt{2x^2-x+3}}{16777216}+\frac{94111745\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right.}{67108864}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1056964608*(10569646080*x^9+2055208960*x^8+44163137536*x^7+26401898496*x
^6+75389820928*x^5+57147467776*x^4+77872272000*x^3+42992644128*x^2+3953324
9652*x+14824182519)*(2*x^2-x+3)^(1/2)+94111745/67108864*2^(1/2)*arcsinh(4/
23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x +5x^2\right)^2 \, dx = \frac{1}{1056964608} \left(10569646080\,x^9 + 2055208960\,x^8 + 44163137536\,x^7 + 26401898496\,x^6 + 75389\right. + \frac{94111745}{134217728}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 2640189
8496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 429926441
28*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421
7728*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x
- 25)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \sqrt{2x^2-x+3} \cdot \left(10x^9 + \frac{35x^8}{18} + \frac{24067x^7}{576} + \frac{134287x^6}{5376} + \frac{9202859x^5}{129024} + \frac{3986291x^4}{73728} + \frac{202792375x^3}{2752512} + \frac{63977149x^2}{1572864} + \frac{3294437471x}{88080384} + \frac{1647131391}{117440512}\right) + \frac{94111745\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{67108864}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

output
```
sqrt(2*x**2 - x + 3)*(10*x**9 + 35*x**8/18 + 24067*x**7/576 + 134287*x**6/
5376 + 9202859*x**5/129024 + 3986291*x**4/73728 + 202792375*x**3/2752512 +
 63977149*x**2/1572864 + 3294437471*x/88080384 + 1647131391/117440512) + 9
4111745*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/67108864
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \frac{5}{4} \left(2\,x^2-x+3\right)^{\frac{7}{2}} x^3 + \frac{305}{144} \left(2\,x^2-x+3\right)^{\frac{7}{2}} x^2 + \frac{8467}{4608} \left(2\,x^2-x+3\right)^{\frac{7}{2}} x + \frac{23225}{43008} \left(2\,x^2-x+3\right)^{\frac{7}{2}} + \frac{1547}{24576} \left(2\,x^2-x+3\right)^{\frac{5}{2}} x - \frac{1547}{98304} \left(2\,x^2-x+3\right)^{\frac{5}{2}} + \frac{177905}{786432} \left(2\,x^2-x+3\right)^{\frac{3}{2}} x - \frac{177905}{3145728} \left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{4091815}{4194304} \sqrt{2\,x^2-x+3}x + \frac{94111745}{67108864} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{4091815}{16777216} \sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/4
608*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/245
76*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786
432*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 40918
15/4194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*
sqrt(23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)^2\,dx = \frac{1}{1056964608}(4(8(4(16(4(8(28(160(36x+7)x+24067)x+402861)x+9202859)x+2790$$
$$-\frac{94111745}{67108864}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)
*x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)
*x + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)^2\,dx = \int (2x^2-x+3)^{5/2}(5x^2+3x+2)^2\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx = 10\sqrt{2x^2-x+3}\,x^9 + \frac{35\sqrt{2x^2-x+3}\,x^8}{18} + \frac{24067\sqrt{2x^2-x+3}\,x^7}{576} + \frac{134287\sqrt{2x^2-x+3}\,x^6}{5376} + \frac{9202859\sqrt{2x^2-x+3}\,x^5}{129024} + \frac{3986291\sqrt{2x^2-x+3}\,x^4}{73728} + \frac{202792375\sqrt{2x^2-x+3}\,x^3}{2752512} + \frac{63977149\sqrt{2x^2-x+3}\,x^2}{1572864} + \frac{3294437471\sqrt{2x^2-x+3}\,x}{88080384} + \frac{1647131391\sqrt{2x^2-x+3}}{117440512} + \frac{94111745\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{67108864}$$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x)
```

output
```
(42278584320*sqrt(2*x**2 - x + 3)*x**9 + 8220835840*sqrt(2*x**2 - x + 3)*x
**8 + 176652550144*sqrt(2*x**2 - x + 3)*x**7 + 105607593984*sqrt(2*x**2 -
x + 3)*x**6 + 301559283712*sqrt(2*x**2 - x + 3)*x**5 + 228589871104*sqrt(2
*x**2 - x + 3)*x**4 + 311489088000*sqrt(2*x**2 - x + 3)*x**3 + 17197057651
2*sqrt(2*x**2 - x + 3)*x**2 + 158132998608*sqrt(2*x**2 - x + 3)*x + 592967
30076*sqrt(2*x**2 - x + 3) + 5929039935*sqrt(2)*log((2*sqrt(2*x**2 - x + 3
)*sqrt(2) + 4*x - 1)/sqrt(23)))/4227858432
```

3.109 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) \, dx = -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304} - \frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072} + \frac{141}{448}\left(3-x+2x^2\right)^{7/2} + \frac{5}{16}x\left(3-x+2x^2\right)^{7/2} - \frac{16851295\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

output
```
-732665/524288*(1-4*x)*(2*x^2-x+3)^(1/2)-31855/98304*(1-4*x)*(2*x^2-x+3)^(
3/2)-277/3072*(1-4*x)*(2*x^2-x+3)^(5/2)+141/448*(2*x^2-x+3)^(7/2)+5/16*x*(
2*x^2-x+3)^(7/2)-16851295/2097152*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) \, dx = \frac{4\sqrt{3-x+2x^2}(58536675+148957444x+67272352x^2+172684416x^3-1619968x^4+118808}{44040192}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*
x^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 3538771
95*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/44040192
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2} \left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16} \int \frac{1}{2}(141x+34) \left(2x^2-x+3\right)^{5/2}dx+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \int (141x+34) \left(2x^2-x+3\right)^{5/2}dx+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{32}\left(\frac{277}{4}\int \left(2x^2-x+3\right)^{5/2}dx+\frac{141}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{141}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int\sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{141}{14}\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}\right.\right.\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-\right.\right.\right.\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-\right.\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(7/2))/16 + ((141*(3 - x + 2*x^2)^(7/2))/14 + (277*(-
1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(3 - x + 2
*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

method	result
risch	$\frac{(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+58536675)\sqrt{2x^2-x+3}}{11010048}+\frac{1685}{}$
trager	$\left(\frac{5}{2}x^7-\frac{69}{56}x^6+\frac{14503}{1344}x^5-\frac{113}{768}x^4+\frac{449699}{28672}x^3+\frac{300323}{49152}x^2+\frac{37239361}{2752512}x+\frac{19512225}{3670016}\right)\sqrt{2x^2-x+3}-\frac{168}{}$
default	$\frac{277(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{3072}+\frac{31855(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{98304}+\frac{732665(4x-1)\sqrt{2x^2-x+3}}{524288}+\frac{16851295\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{2097152}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/11010048*(27525120*x^7-13565952*x^6+118808576*x^5-1619968*x^4+172684416*
x^3+67272352*x^2+148957444*x+58536675)*(2*x^2-x+3)^(1/2)+16851295/2097152*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)\,dx = \frac{1}{11010048}\left(27525120\,x^7-13565952\,x^6+118808576\,x^5-1619968\,x^4+172684416\,x^3+67272\right.$$
$$+\frac{16851295}{4194304}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 17
2684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) +
 16851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 3
2*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^7}{2}-\frac{69x^6}{56}+\frac{14503x^5}{1344}-\frac{113x^4}{768}+\frac{449699x^3}{28672}+\frac{300323x^2}{49152}+\frac{37239361x}{2752512}+\frac{19512225}{3670016}\right)+\frac{16851295\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2097152}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**7/2 - 69*x**6/56 + 14503*x**5/1344 - 113*x**4/7
68 + 449699*x**3/28672 + 300323*x**2/49152 + 37239361*x/2752512 + 19512225
/3670016) + 16851295*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/2097152
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)\,dx=\frac{5}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{277}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{277}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{31855}{24576}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{31855}{98304}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{732665}{131072}\sqrt{2x^2-x+3}\,x+\frac{16851295}{2097152}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{732665}{524288}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*
x^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2
 - x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt
(2*x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
1)) - 732665/524288*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)\, dx = \frac{1}{11010048} (4 (8 (4 (16 (4 (24 (140 x-69)x+14503)x-791)x+1349097)x+2102261)x+372 - \frac{16851295}{2097152} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2 x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 134909
7)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295
/2097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)\, dx = \int \left(2 x^2-x+3\right)^{5/2} \left(5 x^2+3 x+2\right) dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)\,dx = \frac{5\sqrt{2x^2-x+3}\,x^7}{2} - \frac{69\sqrt{2x^2-x+3}\,x^6}{56} + \frac{14503\sqrt{2x^2-x+3}\,x^5}{1344} - \frac{113\sqrt{2x^2-x+3}\,x^4}{768} + \frac{449699\sqrt{2x^2-x+3}\,x^3}{28672} + \frac{300323\sqrt{2x^2-x+3}\,x^2}{49152} + \frac{37239361\sqrt{2x^2-x+3}\,x}{2752512} + \frac{19512225\sqrt{2x^2-x+3}}{3670016} + \frac{16851295\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{2097152}$$

input

```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)
```

output

```
(110100480*sqrt(2*x**2 - x + 3)*x**7 - 54263808*sqrt(2*x**2 - x + 3)*x**6
+ 475234304*sqrt(2*x**2 - x + 3)*x**5 - 6479872*sqrt(2*x**2 - x + 3)*x**4
+ 690737664*sqrt(2*x**2 - x + 3)*x**3 + 269089408*sqrt(2*x**2 - x + 3)*x**
2 + 595829776*sqrt(2*x**2 - x + 3)*x + 234146700*sqrt(2*x**2 - x + 3) + 35
3877195*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/
44040192
```

3.110 $\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 236

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} \, dx = -\frac{226249\sqrt{3-x+2x^2}}{80000} + \frac{4981x\sqrt{3-x+2x^2}}{4000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}} - \frac{121\sqrt{\frac{11}{31}\left(-15457+25000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(-15457+25000\sqrt{2}\right)}}\left(196-443\sqrt{2}-\left(690+247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{3125} + \frac{121\sqrt{\frac{11}{31}\left(15457+25000\sqrt{2}\right)}\text{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(15457+25000\sqrt{2}\right)}}\left(196+443\sqrt{2}-\left(690-247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{3125}$$

output

```
-226249/80000*(2*x^2-x+3)^(1/2)+4981/4000*x*(2*x^2-x+3)^(1/2)-1/600*(103-6
0*x)*(2*x^2-x+3)^(3/2)-7216203/1600000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)-121/96875*(-5270837+8525000*2^(1/2))^(1/2)*arctan(11^(1/2)/(-958334+15
50000*2^(1/2))^(1/2)*(196-443*2^(1/2)-(690+247*2^(1/2))*x)/(2*x^2-x+3)^(1/
2))+121/96875*(5270837+8525000*2^(1/2))^(1/2)*arctanh(11^(1/2)/(958334+155
0000*2^(1/2))^(1/2)*(196+443*2^(1/2)-(690-247*2^(1/2))*x)/(2*x^2-x+3)^(1/2
))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \frac{20\sqrt{3-x+2x^2}(-802347+412060x-106400x^2+48000x^3)-21648609\sqrt{2}\log}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

output
```
(20*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) - 21
648609*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] - 2044416*RootSum[-5
6 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (368*Log[-(Sqrt[
2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 22*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 -
 x + 2*x^2] - #1]*#1 - 119*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1
^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/4800000
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1308, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{5/2}}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{ 1308}$$

$$-\frac{1}{300}\int -\frac{3\sqrt{2x^2-x+3}(4981x^2-2045x+3154)}{4(5x^2+3x+2)}dx-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{400}\int \frac{\sqrt{2x^2-x+3}(4981x^2-2045x+3154)}{5x^2+3x+2}dx-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{2138}$$

$$\frac{1}{400}\left(-\frac{1}{100}\int -\frac{7216203x^2-3779795x+2136862}{4\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{200}\sqrt{2x^2-x+3}(226249-99620x)\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{27}$$

$$\frac{1}{400}\left(\frac{1}{400}\int \frac{7216203x^2-3779795x+2136862}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{2143}$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int -\frac{340736(119x+11)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{27}$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{1090}$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{222}$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\downarrow \text{1368}$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\frac{\int \frac{11\left(\left(130+119\sqrt{2}\right)x+11\sqrt{2}+108\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int \frac{11\left(\left(130-119\sqrt{2}\right)x-11\sqrt{2}+108\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\frac{\int \frac{\left(130+119\sqrt{2}\right)x+11\sqrt{2}+108}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(130-119\sqrt{2}\right)x-11\sqrt{2}+108}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)\right)\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 1362

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\sqrt{2}\left(15457+25000\sqrt{2}\right)\int \frac{1}{62\left(15457+25000\sqrt{2}\right)-\frac{11\left(-\left(\left(690-\right.\right.\right.}{}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 217

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\sqrt{2}\left(15457+25000\sqrt{2}\right)\int \frac{1}{62\left(15457+25000\sqrt{2}\right)-\frac{11\left(-\left(\left(690-\right.\right.\right.}{}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 219

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(-\frac{\left(15457-25000\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{11}{62\left(25000\sqrt{2}-15457\right)}}\left(-\left(\left(690+247\sqrt{2}\right)\right.\right.}{\sqrt{2x^2-x+3}}\right.}{\sqrt{341\left(25000\sqrt{2}-15457\right)}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

output
```
-1/600*((103 - 60*x)*(3 - x + 2*x^2)^(3/2)) + (-1/200*((226249 - 99620*x)*
Sqrt[3 - x + 2*x^2]) + ((7216203*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(5*Sqrt[2])
 - (340736*(-(((15457 - 25000*Sqrt[2])*ArcTan[(Sqrt[11/(62*(-15457 + 25000
*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2*x^
2]])/Sqrt[341*(-15457 + 25000*Sqrt[2])]) - Sqrt[(15457 + 25000*Sqrt[2])/34
1]*ArcTanh[(Sqrt[11/(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (69
0 - 247*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]))/5)/400)/400
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.76 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.15

method	result
trager	Expression too large to display
risch	$\frac{(48000x^3-106400x^2+412060x-802347)\sqrt{2x^2-x+3}}{240000}+\frac{7216203\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1600000}+\frac{121\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\right.}{\left(\sqrt{2}+1\right.}}}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^3-133/300*x^2+20603/12000*x-267449/80000)*(2*x^2-x+3)^(1/2)-7216203
/1600000*RootOf(_Z^2-2)*ln(-4*RootOf(_Z^2-2)*x+RootOf(_Z^2-2)+4*(2*x^2-x+3
)^(1/2))+1/80000*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031555002938163
200000000)*ln((27172875625*x*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031
555002938163200000000)^5-236919309332543242240O*RootOf(24025*_Z^4-12643585
96886528*_Z^2+87031555002938163200000000)^3*x+459442941042663424000O*RootO
f(24025*_Z^4-1264358596886528*_Z^2+87031555002938163200000000)^3+713801017
384745500672000000*RootOf(24025*_Z^4-1264358596886528*_Z^2+870315550029381
63200000000)^2*(2*x^2-x+3)^(1/2)-72306759653812432951569243 17696*RootOf(24
025*_Z^4-1264358596886528*_Z^2+87031555002938163200000000)*x+1356280148925
085081474611281920O*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031555002938
163200000000)-168239350222065201258083534661222400O*(2*x^2-x+3)^(1/2))/(7
75*x*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031555002938163200000000)^2
+2211196731392*x+30138769768448))+1/12400000*RootOf(_Z^2+24025*RootOf(2402
5*_Z^4-1264358596886528*_Z^2+87031555002938163200000000)^2-126435859688652
8)*ln((43476601*RootOf(_Z^2+24025*RootOf(24025*_Z^4-1264358596886528*_Z^2+
87031555002938163200000000)^2-1264358596886528)*RootOf(24025*_Z^4-12643585
96886528*_Z^2+87031555002938163200000000)^4*x-7853588332187811184*RootOf(24
025*_Z^4-1264358596886528*_Z^2+87031555002938163200000000)^2*RootOf(_Z^2+2
4025*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031555002938163200000000...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(172) = 344$.

Time = 0.12 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.39

$$\int \frac{(3 - x + 2x^2)^{5/2}}{2 + 3x + 5x^2} \, dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-121/6250*sqrt(11/31)*sqrt(25000*sqrt(2) - 15457)*arctan(-1/177041*(248*sq
rt(11/31)*(115*x^3 - 1149*x^2 - sqrt(2)*(993*x^3 - 1628*x^2 + 1128*x + 129
6) - 2768*x - 264)*sqrt(2*x^2 - x + 3)*sqrt(25000*sqrt(2) - 15457) + (171*
x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3
936*x)*sqrt(25000*sqrt(2) + 15457)*sqrt(25000*sqrt(2) - 15457))/(343*x^4 -
 400*x^3 + 1136*x^2 + 384*x - 576)) + 121/6250*sqrt(11/31)*sqrt(25000*sqrt
(2) - 15457)*arctan(1/177041*(248*sqrt(11/31)*(115*x^3 - 1149*x^2 - sqrt(2
)*(993*x^3 - 1628*x^2 + 1128*x + 1296) - 2768*x - 264)*sqrt(2*x^2 - x + 3)
*sqrt(25000*sqrt(2) - 15457) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2
)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(25000*sqrt(2) + 15457)*sqr
t(25000*sqrt(2) - 15457))/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) +
121/12500*sqrt(11/31)*sqrt(25000*sqrt(2) + 15457)*log(121*(2*sqrt(11/31)*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(47*x + 1972) - 2019*x + 1925)*sqrt(25000*sqrt
(2) + 15457) + 279839*x^2 + 251284*sqrt(2)*(2*x^2 - x + 3) - 862361*x + 11
42200)/x^2) - 121/12500*sqrt(11/31)*sqrt(25000*sqrt(2) + 15457)*log(-121*(
2*sqrt(11/31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(47*x + 1972) - 2019*x + 1925)*
sqrt(25000*sqrt(2) + 15457) - 279839*x^2 - 251284*sqrt(2)*(2*x^2 - x + 3)
+ 862361*x - 1142200)/x^2) + 1/240000*(48000*x^3 - 106400*x^2 + 412060*x -
 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*sqrt(2)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{5}{2}}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{5}{2}}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx=\int \frac{(2\,x^2-x+3)^{5/2}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx&=\frac{\sqrt{2x^2-x+3}\,x^3}{5}-\frac{133\sqrt{2x^2-x+3}\,x^2}{300}\\&+\frac{20603\sqrt{2x^2-x+3}\,x}{12000}-\frac{329345961\sqrt{2x^2-x+3}}{50000000}\\&+\frac{464238163\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\,\sqrt{2}-4x+1\right)}{40000000}\\&+\frac{1108723\sqrt{2}\log\left(2\sqrt{2x^2-x+3}\,\sqrt{2}-4x+1\right)}{156250}\\&-\frac{3097237\left(\int \frac{\sqrt{2x^2-x+3}}{10x^4+x^3+16x^2+7x+6}dx\right)}{390625}+\frac{2534224\left(\int \frac{\sqrt{2x^2-x+3}\,x^3}{10x^4+x^3+16x^2+7x+6}dx\right)}{78125}\\&+\frac{4434892\left(\int \frac{\sqrt{2x^2-x+3}\,x^2}{10x^4+x^3+16x^2+7x+6}dx\right)}{390625}-\frac{3326169\left(\int \frac{\sqrt{2x^2-x+3}\,x}{10x^4+x^3+16x^2+7x+6}dx\right)}{78125}\end{aligned}$$

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

output

```
(120000000*sqrt(2*x**2 - x + 3)*x**3 - 266000000*sqrt(2*x**2 - x + 3)*x**2
 + 1030150000*sqrt(2*x**2 - x + 3)*x - 3952151532*sqrt(2*x**2 - x + 3) + 6
963572445*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 42574
96320*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 4757356032*i
nt(sqrt(2*x**2 - x + 3)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x) + 19462840
320*int((sqrt(2*x**2 - x + 3)*x**3)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x
) + 6811994112*int((sqrt(2*x**2 - x + 3)*x**2)/(10*x**4 + x**3 + 16*x**2 +
 7*x + 6),x) - 25544977920*int((sqrt(2*x**2 - x + 3)*x)/(10*x**4 + x**3 +
16*x**2 + 7*x + 6),x))/600000000
```

3.111 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 288

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \frac{1123\sqrt{3-x+2x^2}}{7750}-\frac{604}{775}x\sqrt{3-x+2x^2}+\frac{52}{155}x^2\sqrt{3-x+2x^2} -\frac{8}{31}x^3\sqrt{3-x+2x^2}+\frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)}-\frac{4799\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2500\sqrt{2}} + \frac{11\sqrt{\frac{11}{31}\left(224510383+194487500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(224510383+194487500\sqrt{2}\right)}}\left(21136+33287\sqrt{2}+\left(87710+54423\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{38750} - \frac{11\sqrt{\frac{11}{31}\left(-224510383+194487500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-224510383+194487500\sqrt{2}\right)}}\left(21136-33287\sqrt{2}+\left(87710-54423\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{38750}$$

output
```
1123/7750*(2*x^2-x+3)^(1/2)-604/775*x*(2*x^2-x+3)^(1/2)+52/155*x^2*(2*x^2-
x+3)^(1/2)-8/31*x^3*(2*x^2-x+3)^(1/2)+(3+10*x)*(2*x^2-x+3)^(5/2)/(155*x^2+
93*x+62)-4799/5000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+11/1201250*(7655
8040603+66320237500*2^(1/2))^(1/2)*arctan(11^(1/2)/(13919643746+1205822500
0*2^(1/2))^(1/2)*(21136+33287*2^(1/2)+(87710+54423*2^(1/2))*x)/(2*x^2-x+3)
^(1/2))-11/1201250*(-76558040603+66320237500*2^(1/2))^(1/2)*arctanh(11^(1/
2)/(-13919643746+12058225000*2^(1/2))^(1/2)*(21136-33287*2^(1/2)+(87710-54
423*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.07 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.50

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \frac{\frac{500\sqrt{3-x+2x^2}(8996+9289x-12555x^2+3100x^3)}{2+3x+5x^2} - 3719225\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((500*Sqrt[3 - x + 2*x^2]*(8996 + 9289*x - 12555*x^2 + 3100*x^3))/(2 + 3*x
 + 5*x^2) - 3719225*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] + 30008
*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (5237
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2880*Sqrt[2]*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 2225*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] -
242*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^
4 & , (639994*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 22980
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1175*Sqrt[2]*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*
#1^2 - 10*#1^3) & ])/3875000
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1302, 27, 2138, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{5/2}}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \ 1302$$

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}-\frac{1}{31}\int -\frac{5(-32x^2-6x+15)(2x^2-x+3)^{3/2}}{2(5x^2+3x+2)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{5}{62}\int \frac{(-32x^2-6x+15)(2x^2-x+3)^{3/2}}{5x^2+3x+2}dx+\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}$$

$$\Big\downarrow \ 2138$$

$$\frac{5}{62}\left(\frac{8}{25}(4-5x)(2x^2-x+3)^{3/2}-\frac{1}{600}\int -\frac{24(-896x^2-905x+1461)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx\right)+\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{5}{62}\left(\frac{1}{25}\int \frac{(-896x^2-905x+1461)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{8}{25}(4-5x)(2x^2-x+3)^{3/2}\right)+\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}$$

$$\Big\downarrow \ 2138$$

$$\frac{5}{62}\left(\frac{1}{25}\left(-\frac{1}{100}\int -\frac{2(148769x^2-175535x+243476)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx-\frac{1}{25}\sqrt{2x^2-x+3}(2240x+1277)\right)+\frac{8}{25}(4-5x)(2x\right.$$
$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\int \frac{148769x^2-175535x+243476}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{25}(2240x+1277)\sqrt{2x^2-x+3}\right)+\frac{8}{25}(4-5x)\left(2x^2-x+\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{2143}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{242(3801-5471x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{148769\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}\right)-\frac{1}{25}(2240x+1277\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{148769\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{1368}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\frac{\int -\frac{11\left(\left(1670+5471\sqrt{2}\right)x-3801\sqrt{2}+9272\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(1670-5471\sqrt{2}\right)x+3801\sqrt{2}+9272\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{14876\cdots}{}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\frac{\int \frac{\left(1670-5471\sqrt{2}\right)x+3801\sqrt{2}+9272}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(1670+5471\sqrt{2}\right)x-3801\sqrt{2}+9272}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{148769\text{arcsinh}\left(\frac{4x}{\sqrt{\cdots}}\right.}{5\sqrt{2}}\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1362$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\sqrt{2}\left(224510383-194487500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)^2}{2x^2-x+3}-62\left(22451038\cdots\right.}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\sqrt{2}\left(224510383-194487500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)^2}{2x^2-x+3}-62\left(22451038\cdots\right.}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}+\frac{242}{5}\left(\sqrt{\frac{1}{341}\left(224510383+194487500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(224510383+19448\right.}}}{}\right.\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) + (5*((8*(4 - 5*
x)*(3 - x + 2*x^2)^(3/2))/25 + (-1/25*((1277 + 2240*x)*Sqrt[3 - x + 2*x^2]
) + ((148769*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(5*Sqrt[2]) + (242*(Sqrt[(22451
0383 + 194487500*Sqrt[2])/341]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*
Sqrt[2]))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]] + ((224510383 - 194487500*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-22451
0383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710 - 54423*Sqrt[2
])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[341*(-224510383 + 194487500*Sqrt[2])]))/
5)/50)/25))/62
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.75 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.13

method	result
trager	Expression too large to display
risch	$\frac{(3100x^3-12555x^2+9289x+8996)\sqrt{2x^2-x+3}}{38750x^2+23250x+15500}+\frac{4799\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{5000}+\frac{11\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8}}{}$
default	Expression too large to display

input

```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/7750*(3100*x^3-12555*x^2+9289*x+8996)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)-1/
1201250*RootOf(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+33504990790
8469531250)^2+9263522912963)*ln(-(50635674288*RootOf(_Z^2+96100*RootOf(961
00*_Z^4+9263522912963*_Z^2+335049907908469531250)^2+9263522912963)*RootOf(
96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^4*x+24839430743324904
08*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2*RootOf(_Z
^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2+926
3522912963)*x-5394974354905351392*RootOf(96100*_Z^4+9263522912963*_Z^2+335
049907908469531250)^2*RootOf(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z
^2+335049907908469531250)^2+9263522912963)+19722622249345627868333000*Root
Of(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2*(2*x^2-x+3)^(1/2
)-176231528680762367884053125*RootOf(_Z^2+96100*RootOf(96100*_Z^4+92635229
12963*_Z^2+335049907908469531250)^2+9263522912963)*x+212361413450122530324
775000*RootOf(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908
469531250)^2+9263522912963)+916476229734634762910552969993750*(2*x^2-x+3)^
(1/2))/(3100*x*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)
^2+92436758755*x-75966534842))+1/3875*RootOf(96100*_Z^4+9263522912963*_Z^2
+335049907908469531250)*ln((7911824107500*x*RootOf(96100*_Z^4+926352291296
3*_Z^2+335049907908469531250)^5+113719843996646972200*RootOf(96100*_Z^4+9
263522912963*_Z^2+335049907908469531250)^3*x+842964742953961155000*Root...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(218) = 436$.

Time = 0.11 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.14

$$\int \frac{(3 - x + 2x^2)^{5/2}}{(2 + 3x + 5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/310000*(44*sqrt(11/31)*(5*x^2 + 3*x + 2)*sqrt(194487500*sqrt(2) + 22451
0383)*arctan(-1/313911301*sqrt(11/31)*(sqrt(11/31)*(171*x^4 + 1212*x^3 - 1
640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(194487
500*sqrt(2) - 224510383) + 88*(100035*x^3 - 255241*x^2 - sqrt(2)*(99487*x^
3 - 206202*x^2 - 17048*x + 111264) - 161712*x + 91224)*sqrt(2*x^2 - x + 3)
)*sqrt(194487500*sqrt(2) + 224510383)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*
x - 576)) - 44*sqrt(11/31)*(5*x^2 + 3*x + 2)*sqrt(194487500*sqrt(2) + 2245
10383)*arctan(-1/313911301*sqrt(11/31)*(sqrt(11/31)*(171*x^4 + 1212*x^3 -
1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(19448
7500*sqrt(2) - 224510383) - 88*(100035*x^3 - 255241*x^2 - sqrt(2)*(99487*x
^3 - 206202*x^2 - 17048*x + 111264) - 161712*x + 91224)*sqrt(2*x^2 - x + 3
))*sqrt(194487500*sqrt(2) + 224510383)/(343*x^4 - 400*x^3 + 1136*x^2 + 384
*x - 576)) + 22*sqrt(11/31)*(5*x^2 + 3*x + 2)*sqrt(194487500*sqrt(2) - 224
510383)*log(11*(2*sqrt(11/31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(86127*x - 2736
98) + 187571*x - 359825)*sqrt(194487500*sqrt(2) - 224510383) + 1398332159*
x^2 + 1255645204*sqrt(2)*(2*x^2 - x + 3) - 4309146041*x + 5707478200)/x^2)
 - 22*sqrt(11/31)*(5*x^2 + 3*x + 2)*sqrt(194487500*sqrt(2) - 224510383)*lo
g(-11*(2*sqrt(11/31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(86127*x - 273698) + 187
571*x - 359825)*sqrt(194487500*sqrt(2) - 224510383) - 1398332159*x^2 - 125
5645204*sqrt(2)*(2*x^2 - x + 3) + 4309146041*x - 5707478200)/x^2) - 148...
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{5}{2}}}{(5x^2+3x+2)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{5}{2}}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{15625,[8]%%%}+%%%{%%{[-37500,0]:[1,0,-2]%%},[7]%%%}+%%%{-6
1250,[6]%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx=\int \frac{(2\,x^2-x+3)^{5/2}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)
```

Reduce [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx=\frac{1906000\sqrt{2x^2-x+3}\,x^3-7719300\sqrt{2x^2-x+3}\,x^2-36431580\sqrt{2x^2-x+3}\,x}{}$$

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

output
```
(1906000*sqrt(2*x**2 - x + 3)*x**3 - 7719300*sqrt(2*x**2 - x + 3)*x**2 - 3
6431580*sqrt(2*x**2 - x + 3)*x + 10411464*sqrt(2*x**2 - x + 3) + 22867235*
sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 13720341*s
qrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 9146894*sqrt(2
)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 2073857720*int(sqrt(2
*x**2 - x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x +
12),x)*x**2 + 1244314632*int(sqrt(2*x**2 - x + 3)/(50*x**6 + 35*x**5 + 103
*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x)*x + 829543088*int(sqrt(2*x**2 -
x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x) -
 1781250680*int((sqrt(2*x**2 - x + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4
+ 85*x**3 + 83*x**2 + 32*x + 12),x)*x**2 - 1068750408*int((sqrt(2*x**2 - x
 + 3)*x**2)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2 + 32*x + 12)
,x)*x - 712500272*int((sqrt(2*x**2 - x + 3)*x**2)/(50*x**6 + 35*x**5 + 103
*x**4 + 85*x**3 + 83*x**2 + 32*x + 12),x))/(4765000*(5*x**2 + 3*x + 2))
```

3.112 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 295

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \frac{38}{775}\sqrt{3-x+2x^2}$$

$$-\frac{4}{155}x\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2}$$

$$+\frac{11(8517+14843x)\sqrt{3-x+2x^2}}{96100\,(2+3x+5x^2)} - \frac{4}{125}\sqrt{2}\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)$$

$$+\frac{\sqrt{11\,(1+4\sqrt{2})}\,(2937349+1978861\sqrt{2})\arctan\Bigg(\frac{\sqrt{\frac{11}{62\left(3531015707557+2498852071250\sqrt{2}\right)}}\,(3957722+2937349\sqrt{2}+(9832420+}{\sqrt{3-x+2x^2}}}{29791000}$$

$$-\frac{(2937349-1978861\sqrt{2})\,\sqrt{11\,(-1+4\sqrt{2})}\operatorname{arctanh}\Bigg(\frac{\sqrt{\frac{11}{62\left(-3531015707557+2498852071250\sqrt{2}\right)}}\,(3957722-2937349\sqrt{2}+(983}{\sqrt{3-x+2x^2}}}{29791000}$$

output
```
38/775*(2*x^2-x+3)^(1/2)-4/155*x*(2*x^2-x+3)^(1/2)+1/62*(3+10*x)*(2*x^2-x+
3)^(5/2)/(5*x^2+3*x+2)^2+11*(8517+14843*x)*(2*x^2-x+3)^(1/2)/(480500*x^2+2
88300*x+192200)-4/125*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/29791000*(1
1+44*2^(1/2))^(1/2)*(2937349+1978861*2^(1/2))*arctan(11^(1/2)/(21892297386
8534+154928828417500*2^(1/2))^(1/2)*(3957722+2937349*2^(1/2)+(9832420+6895
071*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/29791000*(2937349-1978861*2^(1/2))*(-
11+44*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-218922973868534+154928828417500*2^
(1/2))^(1/2)*(3957722-2937349*2^(1/2)+(9832420-6895071*2^(1/2))*x)/(2*x^2-
x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.36 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.09

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \frac{\frac{15812500\sqrt{3-x+2x^2}(22552+69621x+93872x^2+97155x^3)}{(2+3x+5x^2)^2} - 4420600000\sqrt{2}\log\left(1-4x+2\sqrt{6}\right.}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((15812500*Sqrt[3 - x + 2*x^2]*(22552 + 69621*x + 93872*x^2 + 97155*x^3))/
(2 + 3*x + 5*x^2)^2 - 4420600000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]] + 972532000*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
 5*#1^4 & , (3781*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 630*Sqrt[
2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 150*Log[-(Sqrt[2]*x)
+ Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 -
10*#1^3) & ] + 682*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[
2]*#1^3 - 5*#1^4 & , (4978708507*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2
*x^2] - #1] - 165870920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 +
1110955025*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 11*Sqrt[2]*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492740319684*Sqrt
[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 128644699540*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 55365920925*Sqrt[2]*Log[-(Sqrt[2]*
x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
 - 10*#1^3) & ])/138143750000
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1302, 27, 2132, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2x^2-x+3\right)^{5/2}}{\left(5x^2+3x+2\right)^3}dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}-\frac{1}{62}\int -\frac{5\left(-16x^2-14x+39\right)\left(2x^2-x+3\right)^{3/2}}{2\left(5x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{124}\int \frac{\left(-16x^2-14x+39\right)\left(2x^2-x+3\right)^{3/2}}{\left(5x^2+3x+2\right)^2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2132}$$

$$\frac{5}{124}\left(\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{155\left(5x^2+3x+2\right)}-\frac{1}{155}\int -\frac{\left(-20672x^2-5900x+13347\right)\sqrt{2x^2-x+3}}{2\left(5x^2+3x+2\right)}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{124}\left(\frac{1}{310}\int \frac{\left(-20672x^2-5900x+13347\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{155\left(5x^2+3x+2\right)}\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2138}$$

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{4}{25}(11359-12920x)\sqrt{2x^2-x+3}-\frac{1}{100}\int -\frac{4\left(61504x^2-579685x+1356541\right)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)+\frac{(2336x+76}{155\,(5}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\int \frac{61504x^2-579685x+1356541}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{4}{25}\sqrt{2x^2-x+3}(11359-12920x)\right)+\frac{(2336x+769)\,(2x}{155\,(5x^2+3}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 2143

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{61504}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{11(605427-280267x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)+\frac{4}{25}\sqrt{2x^2-x+3}(11359-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{61504}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)+\frac{4}{25}\sqrt{2x^2-x+3}(11359\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1090

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{30752}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{4}{25}\sqrt{2}\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 222

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{30752}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{4}{25}\sqrt{2x^2-x+3}(1135$$

$$\frac{(10x+3)\,(2x^2-x+3)^{5/2}}{62\,(5x^2+3x+2)^2}$$

$\downarrow$ 1368

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\frac{\int -\frac{11\left(-\left(\left(325160-280267\sqrt{2}\right)x\right)-605427\sqrt{2}+885694\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(325160+280267\sqrt{2}\right)x\right)+605427\sqrt{2}\right.}{\sqrt{2x^2-x+3}(5x^2+3x+2)}}{22\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\,(2x^2-x+3)^{5/2}}{62\,(5x^2+3x+2)^2}$$

$\downarrow$ 27

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\frac{\int \frac{-\left(\left(325160+280267\sqrt{2}\right)x\right)+605427\sqrt{2}+885694}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(325160-280267\sqrt{2}\right)x\right)-605427\sqrt{2}+885694}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\,(2x^2-x+3)^{5/2}}{62\,(5x^2+3x+2)^2}$$

$\downarrow$ 1362

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\sqrt{2}\left(3531015707557-2498852071250\sqrt{2}\right)\int \frac{}{-\frac{11\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)}{2x^2-x+3}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\,(2x^2-x+3)^{5/2}}{62\,(5x^2+3x+2)^2}$$

$\downarrow$ 217

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\sqrt{2}\left(3531015707557-2498852071250\sqrt{2}\right)\int \frac{}{-\frac{11\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)}{2x^2-x+3}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\,(2x^2-x+3)^{5/2}}{62\,(5x^2+3x+2)^2}$$

$\downarrow$ 219

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{30752}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(3531015707557+2498852071250\sqrt{2}\right)}\arctan\left(\sqrt{\frac{}{62}}\right.\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + (5*(((769 +
2336*x)*(3 - x + 2*x^2)^(3/2))/(155*(2 + 3*x + 5*x^2)) + ((4*(11359 - 1292
0*x)*Sqrt[3 - x + 2*x^2])/25 + ((30752*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/5 + (11*(Sqrt[(3531015707557 + 2498852071250*Sqrt[2])/341]*ArcTan[(Sqrt
[11/(62*(3531015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[
2] + (9832420 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((353101570755
7 - 2498852071250*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-3531015707557 + 24988520
71250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*Sqrt[2])*
x))/Sqrt[3 - x + 2*x^2]])/Sqrt[341*(-3531015707557 + 2498852071250*Sqrt[2]
)]))/5)/25)/310))/124
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.46 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.08

method	result
trager	Expression too large to display
risch	$\frac{11(97155x^3+93872x^2+69621x+22552)\sqrt{2x^2-x+3}}{96100(5x^2+3x+2)^2}+\frac{4\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{125}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
11/96100*(97155*x^3+93872*x^2+69621*x+22552)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(
1/2)-1/24025*RootOf(49203200*_Z^4+38530443400861984*_Z^2+75555566255284022
42640625)*ln((-36815059399680000*x*RootOf(49203200*_Z^4+38530443400861984*
_Z^2+7555556625528402242640625)^5-3509377981550580814808320 0*RootOf(492032
00*_Z^4+38530443400861984*_Z^2+7555556625528402242640625)^3*x+240332528860
32327378002870000 0*RootOf(49203200*_Z^4+38530443400861984*_Z^2+75555566255
28402242640625)^2*(2*x^2-x+3)^(1/2)-18235579580711357356800 00*RootOf(49203
200*_Z^4+38530443400861984*_Z^2+7555556625528402242640625)^3-7966757013299
67949736262207059959 2*RootOf(49203200*_Z^4+38530443400861984*_Z^2+75555566
25528402242640625)*x+94293965883068184162712639837646059625*(2*x^2-x+3)^(1
/2)-10583927361598319517687004630216 00*RootOf(49203200*_Z^4+38530443400861
984*_Z^2+7555556625528402242640625))/(24800*x*RootOf(49203200*_Z^4+3853044
3400861984*_Z^2+7555556625528402242640625)^2+9922195093316*x+282535863379)
)-1/29791000*RootOf(_Z^2+1537600*RootOf(49203200*_Z^4+38530443400861984*_Z
^2+7555556625528402242640625)^2+1204076356276937)*ln(-(29452047519744*Root
Of(_Z^2+1537600*RootOf(49203200*_Z^4+38530443400861984*_Z^2+75555566255284
02242640625)^2+1204076356276937)*RootOf(49203200*_Z^4+38530443400861984*_Z
^2+7555556625528402242640625)^4*x+180520756045003421090 56*RootOf(49203200*
_Z^4+38530443400861984*_Z^2+7555556625528402242640625)^2*RootOf(_Z^2+15376
00*RootOf(49203200*_Z^4+38530443400861984*_Z^2+755555662552840224264062...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(229) = 458$.

Time = 0.15 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.18

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
-1/3844000*(2*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(27487372783750/31
*sqrt(2) + 38841172783127/31)*arctan(-1/282535863379*(88*(14160195*x^3 - 3
2378807*x^2 - sqrt(2)*(10230374*x^3 - 23089929*x^2 - 7444696*x + 10628328)
 - 11569824*x + 14530248)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^3 - 1640
*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(274873727
83750/31*sqrt(2) - 38841172783127/31))*sqrt(27487372783750/31*sqrt(2) + 38
841172783127/31)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 2*(25*x^4
 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(27487372783750/31*sqrt(2) + 3884117278
3127/31)*arctan(1/282535863379*(88*(14160195*x^3 - 32378807*x^2 - sqrt(2)*
(10230374*x^3 - 23089929*x^2 - 7444696*x + 10628328) - 11569824*x + 145302
48)*sqrt(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*
x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(27487372783750/31*sqrt(2) - 388
41172783127/31))*sqrt(27487372783750/31*sqrt(2) + 38841172783127/31)/(343*
x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 61504*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25) + (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(27487372783750/
31*sqrt(2) - 38841172783127/31)*log((54720384607*x^2 + 2*sqrt(2*x^2 - x +
3)*(sqrt(2)*(557898*x - 1368527) + 810629*x - 1926425)*sqrt(27487372783750
/31*sqrt(2) - 38841172783127/31) + 49136671892*sqrt(2)*(2*x^2 - x + 3) - 1
68628123993*x + 223348508600)/x^2) - (25*x^4 + 30*x^3 + 29*x^2 + 12*x +...
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{5}{2}}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

Maxima [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{5}{2}}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{5/2}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)
```

Reduce [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \text{Too large to display}$$

input

```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

output

```
( - 518696200*sqrt(2*x**2 - x + 3)*x**3 + 376119260*sqrt(2*x**2 - x + 3)*x
**2 - 163050602*sqrt(2*x**2 - x + 3)*x + 368803446*sqrt(2*x**2 - x + 3) +
341887500*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 +
410265000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 +
396589500*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 +
164106000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 547
02000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) + 482636605
275*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 +
876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**4 + 579163926330*int(sq
rt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 +
 579*x**3 + 322*x**2 + 100*x + 24),x)*x**3 + 559858462119*int(sqrt(2*x**2
- x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3
+ 322*x**2 + 100*x + 24),x)*x**2 + 231665570532*int(sqrt(2*x**2 - x + 3)/(
250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2
 + 100*x + 24),x)*x + 77221856844*int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325
*x**7 + 720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24)
,x) + 624830463125*int((sqrt(2*x**2 - x + 3)*x**2)/(250*x**8 + 325*x**7 +
720*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**4
 + 749796555750*int((sqrt(2*x**2 - x + 3)*x**2)/(250*x**8 + 325*x**7 + 720
*x**6 + 804*x**5 + 876*x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)*x**3...
```

3.113 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 185

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx &= \frac{16493087661\sqrt{3-x+2x^2}}{29360128}+\frac{1572007407x\sqrt{3-x+2x^2}}{7340032}\\&-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{19750457x^3\sqrt{3-x+2x^2}}{229376}\\&+\frac{686531x^4\sqrt{3-x+2x^2}}{6144}+\frac{2116475x^5\sqrt{3-x+2x^2}}{10752}\\&+\frac{57375}{448}x^6\sqrt{3-x+2x^2}+\frac{625}{16}x^7\sqrt{3-x+2x^2}\\&+\frac{2899366573\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8388608\sqrt{2}}\end{aligned}$$

output
```
16493087661/29360128*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1
/2)-15428243/131072*x^2*(2*x^2-x+3)^(1/2)-19750457/229376*x^3*(2*x^2-x+3)^
(1/2)+686531/6144*x^4*(2*x^2-x+3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2
)+57375/448*x^6*(2*x^2-x+3)^(1/2)+625/16*x^7*(2*x^2-x+3)^(1/2)+2899366573/
16777216*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(49479262983+18864088884x-10367779296x^2-7584175488x^3+9842108416x^4+17}{352321}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 75
84175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 344064
0000*x^7) + 60886698033*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/35
2321536
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\int \frac{57375x^7+48950x^6+74880x^5+56352x^4+29952x^3+12032x^2+3072x+512}{2\sqrt{2x^2-x+3}}dx+\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int \frac{57375x^7+48950x^6+74880x^5+56352x^4+29952x^3+12032x^2+3072x+512}{\sqrt{2x^2-x+3}}dx+\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{2116475x^6+31140x^5+1577856x^4+838656x^3+336896x^2+86016x+14336}{2\sqrt{2x^2-x+3}}dx+\frac{57375}{14}\sqrt{2x^2-x\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\int \frac{2116475x^6+31140x^5+1577856x^4+838656x^3+336896x^2+86016x+14336}{\sqrt{2x^2-x+3}}dx+\frac{57375}{14}\sqrt{2x^2-x\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{24028585x^5-25625706x^4+20127744x^3+8085504x^2+2064384x+344064}{2\sqrt{2x^2-x+3}}dx+\frac{2116475}{12}\sqrt{2x\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\int \frac{24028585x^5-25625706x^4+20127744x^3+8085504x^2+2064384x+344064}{\sqrt{2x^2-x+3}}dx+\frac{2116475}{12}\sqrt{2x\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{1}{10}\int \frac{15(-19750457x^4-11608744x^3+10780672x^2+2752512x+458752)}{2\sqrt{2x^2-x+3}}dx+\frac{4805717}{2}\sqrt{2x^2-\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\int \frac{-19750457x^4-11608744x^3+10780672x^2+2752512x+458752}{\sqrt{2x^2-x+3}}dx+\frac{4805717}{2}\sqrt{2x^2-x+3\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{-323993103x^3+527998978x^2+44040192x+7340032}{2\sqrt{2x^2-x+3}}dx-\frac{19750457}{8}x^3\sqrt{2x^2-x+3}\right)\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\int \frac{-323993103x^3+527998978x^2+44040192x+7340032}{\sqrt{2x^2-x+3}}dx-\frac{19750457}{8}x^3\sqrt{2x^2-x+3}\right)\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{6}\int \frac{3(1572007407x^2+1472133180x+29360128)}{2\sqrt{2x^2-x+3}}dx-\frac{107997701}{2}x^2\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{1572007407x^2+1472133180x+29360128}{\sqrt{2x^2-x+3}}dx-\frac{107997701}{2}x^2\sqrt{2x^2-x+3}\right)-\frac{1975}{8}\right.\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\int -\frac{9197163418-16493087661x}{2\sqrt{2x^2-x+3}}dx+\frac{1572007407}{4}\sqrt{2x^2-x+3}x\right)-\frac{107997701}{2}x^2\right.\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1572007407}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{9197163418-16493087661x}{\sqrt{2x^2-x+3}}dx\right)-\frac{107997701}{2}x^2\sqrt{}\right.\right.\right.\right.\right.}{\frac{625}{16}\sqrt{2x^2-x+3}x^7}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1572007407}{4}\sqrt{\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}\right)+\frac{157200}{4}\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}\right)+\frac{1572007407}{4}\sqrt{2x^2}\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

input
```
Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

output
```
(625*x^7*Sqrt[3 - x + 2*x^2])/16 + ((57375*x^6*Sqrt[3 - x + 2*x^2])/14 + (
(2116475*x^5*Sqrt[3 - x + 2*x^2])/12 + ((4805717*x^4*Sqrt[3 - x + 2*x^2])/
2 + (3*((-19750457*x^3*Sqrt[3 - x + 2*x^2])/8 + ((-107997701*x^2*Sqrt[3 -
x + 2*x^2])/2 + ((1572007407*x*Sqrt[3 - x + 2*x^2])/4 + ((16493087661*Sqrt
[3 - x + 2*x^2])/2 - (20295566011*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]
))/8)/4)/16))/4)/24)/28)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result
risch	$\frac{(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983}{88080384}$
trager	$\left(\frac{625}{16}x^7+\frac{57375}{448}x^6+\frac{2116475}{10752}x^5+\frac{686531}{6144}x^4-\frac{19750457}{229376}x^3-\frac{15428243}{131072}x^2+\frac{1572007407}{7340032}x+\frac{16493087661}{29360128}\right)\sqrt{2}$
default	$-\frac{2899366573\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{1572007407x\sqrt{2x^2-x+3}}{7340032}-\frac{15428243x^2\sqrt{2x^2-x+3}}{131072}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/88080384*(3440640000*x^7+11280384000*x^6+17338163200*x^5+9842108416*x^4-
7584175488*x^3-10367779296*x^2+18864088884*x+49479262983)*(2*x^2-x+3)^(1/2
)-2899366573/16777216*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{88080384}\left(3440640000\,x^7 + 11280384000\,x^6 + 17338163200\,x^5 + 9842108416\,x^4 - 7584175488\,x^3 - 103\right.$$
$$+ \frac{2899366573}{33554432}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 984210841
6*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sq
rt(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{625x^7}{16} + \frac{57375x^6}{448} + \frac{2116475x^5}{10752} + \frac{686531x^4}{6144} - \frac{19750457x^3}{229376} - \frac{15428243x^2}{131072} + \frac{1572007407x}{7340032} + \frac{16493087661}{29360128}\right) - \frac{2899366573\sqrt{2}\,\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(625*x**7/16 + 57375*x**6/448 + 2116475*x**5/10752 +
686531*x**4/6144 - 19750457*x**3/229376 - 15428243*x**2/131072 + 157200740
7*x/7340032 + 16493087661/29360128) - 2899366573*sqrt(2)*asinh(4*sqrt(23)*
(x - 1/4)/23)/16777216
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}\sqrt{2\,x^2-x+3}x^7 + \frac{57375}{448}\sqrt{2\,x^2-x+3}x^6 + \frac{2116475}{10752}\sqrt{2\,x^2-x+3}x^5 + \frac{686531}{6144}\sqrt{2\,x^2-x+3}x^4 - \frac{19750457}{229376}\sqrt{2\,x^2-x+3}x^3 - \frac{15428243}{131072}\sqrt{2\,x^2-x+3}x^2 + \frac{1572007407}{7340032}\sqrt{2\,x^2-x+3}x - \frac{2899366573}{16777216}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{16493087661}{29360128}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 21164
75/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 1
9750457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x +
3)*x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 -
 x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{88080384}(4\,(8\,(4\,(16\,(100\,(120\,(140\,x+459)x+84659)x+4805717)x-59251371)x-323993103)x+ + \frac{2899366573}{16777216}\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x
- 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x
 + 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^4}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx &= \frac{625\sqrt{2x^2-x+3}\,x^7}{16} + \frac{57375\sqrt{2x^2-x+3}\,x^6}{448} \\ &\quad + \frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752} + \frac{686531\sqrt{2x^2-x+3}\,x^4}{6144} \\ &\quad - \frac{19750457\sqrt{2x^2-x+3}\,x^3}{229376} - \frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072} \\ &\quad + \frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032} + \frac{16493087661\sqrt{2x^2-x+3}}{29360128} \\ &\quad - \frac{2899366573\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{16777216}\end{aligned}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x)
```

output

```
(13762560000*sqrt(2*x**2 - x + 3)*x**7 + 45121536000*sqrt(2*x**2 - x + 3)*
x**6 + 69352652800*sqrt(2*x**2 - x + 3)*x**5 + 39368433664*sqrt(2*x**2 - x
 + 3)*x**4 - 30336701952*sqrt(2*x**2 - x + 3)*x**3 - 41471117184*sqrt(2*x*
*2 - x + 3)*x**2 + 75456355536*sqrt(2*x**2 - x + 3)*x + 197917051932*sqrt(
2*x**2 - x + 3) - 60886698033*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 4*x - 1)/sqrt(23)))/352321536
```

3.114 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = -\frac{203373\sqrt{3-x+2x^2}}{32768}-\frac{372783x\sqrt{3-x+2x^2}}{8192} -\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2} +\frac{1355}{48}x^4\sqrt{3-x+2x^2}+\frac{125}{12}x^5\sqrt{3-x+2x^2} -\frac{9267707\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

output

```
-203373/32768*(2*x^2-x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-3387/1024*
x^2*(2*x^2-x+3)^(1/2)+8185/256*x^3*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+
3)^(1/2)+125/12*x^5*(2*x^2-x+3)^(1/2)-9267707/131072*arcsinh(1/23*(1-4*x)*
23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-610119-4473396x-325152x^2+3143040x^3+2775040x^4+1024000x^5)-27803121}{393216}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 2
775040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x
+ 4*x^2]])/393216
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{6775x^5+3090x^4+4968x^3+2736x^2+864x+192}{2\sqrt{2x^2-x+3}}dx+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{6775x^5+3090x^4+4968x^3+2736x^2+864x+192}{\sqrt{2x^2-x+3}}dx+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{24}\left(\frac{1}{10}\int \frac{15\left(8185x^4-4216x^3+3648x^2+1152x+256\right)}{2\sqrt{2x^2-x+3}}dx+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\int \frac{8185x^4-4216x^3+3648x^2+1152x+256}{\sqrt{2x^2-x+3}}dx+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{-10161x^3-88962x^2+18432x+4096}{2\sqrt{2x^2-x+3}}dx+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\int \frac{-10161x^3-88962x^2+18432x+4096}{\sqrt{2x^2-x+3}}dx+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{6}\int \frac{3\left(-372783x^2+114372x+16384\right)}{2\sqrt{2x^2-x+3}}dx-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\ldots\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{-372783x^2+114372x+16384}{\sqrt{2x^2-x+3}}dx-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\ldots\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{2367770-203373x}{2\sqrt{2x^2-x+3}}dx-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2}\ldots\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\int \frac{2367770-203373x}{\sqrt{2x^2-x+3}}dx-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

input
```
Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

output
```
(125*x^5*Sqrt[3 - x + 2*x^2])/12 + ((1355*x^4*Sqrt[3 - x + 2*x^2])/2 + (3*
((8185*x^3*Sqrt[3 - x + 2*x^2])/8 + ((-3387*x^2*Sqrt[3 - x + 2*x^2])/2 + (
(-372783*x*Sqrt[3 - x + 2*x^2])/4 + ((-203373*Sqrt[3 - x + 2*x^2])/2 + (92
67707*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8)/4)/16))/4)/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119)\sqrt{2x^2-x+3}}{98304}+\frac{9267707\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$
trager	$\left(\frac{125}{12}x^5+\frac{1355}{48}x^4+\frac{8185}{256}x^3-\frac{3387}{1024}x^2-\frac{372783}{8192}x-\frac{203373}{32768}\right)\sqrt{2x^2-x+3}-\frac{9267707\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\right.}{}$
default	$\frac{9267707\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}-\frac{203373\sqrt{2x^2-x+3}}{32768}-\frac{372783x\sqrt{2x^2-x+3}}{8192}-\frac{3387x^2\sqrt{2x^2-x+3}}{1024}+\frac{8185x^3\sqrt{2x^2-x+}}{256}$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/98304*(1024000*x^5+2775040*x^4+3143040*x^3-325152*x^2-4473396*x-610119)*
(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3} + \frac{9267707}{262144}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x
- 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{125x^5}{12}+\frac{1355x^4}{48}+\frac{8185x^3}{256}-\frac{3387x^2}{1024}-\frac{372783x}{8192}-\frac{203373}{32768}\right) + \frac{9267707\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**5/12 + 1355*x**4/48 + 8185*x**3/256 - 3387*x*
*2/1024 - 372783*x/8192 - 203373/32768) + 9267707*sqrt(2)*asinh(4*sqrt(23)
*(x - 1/4)/23)/131072
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}x^4 + \frac{8185}{256}\sqrt{2x^2-x+3}x^3 - \frac{3387}{1024}\sqrt{2x^2-x+3}x^2 - \frac{372783}{8192}\sqrt{2x^2-x+3}x + \frac{9267707}{131072}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{203373}{32768}\sqrt{2x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/25
6*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/819
2*sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*
x - 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{98304}(4(8(20(16(100x+271)x+4911)x-10161)x-1118349)x-610119)\sqrt{2x^2-x+3} - \frac{9267707}{131072}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x -
610119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^3}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125\sqrt{2x^2-x+3}\,x^5}{12} + \frac{1355\sqrt{2x^2-x+3}\,x^4}{48} + \frac{8185\sqrt{2x^2-x+3}\,x^3}{256} - \frac{3387\sqrt{2x^2-x+3}\,x^2}{1024} - \frac{372783\sqrt{2x^2-x+3}\,x}{8192} - \frac{203373\sqrt{2x^2-x+3}}{32768} + \frac{9267707\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{131072}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

output
```
(4096000*sqrt(2*x**2 - x + 3)*x**5 + 11100160*sqrt(2*x**2 - x + 3)*x**4 +
12572160*sqrt(2*x**2 - x + 3)*x**3 - 1300608*sqrt(2*x**2 - x + 3)*x**2 - 1
7893584*sqrt(2*x**2 - x + 3)*x - 2440476*sqrt(2*x**2 - x + 3) + 27803121*s
qrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/393216
```

3.115 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = -\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{30725\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

output
```
-11373/1024*(2*x^2-x+3)^(1/2)+3443/768*x*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x
^2-x+3)^(1/2)+25/8*x^3*(2*x^2-x+3)^(1/2)+30725/4096*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-34119+13772x+20960x^2+9600x^3)+92175\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*S
qrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{655x^3+14x^2+192x+64}{2\sqrt{2x^2-x+3}}dx+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{655x^3+14x^2+192x+64}{\sqrt{2x^2-x+3}}dx+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{6}\int \frac{3443x^2-5556x+768}{2\sqrt{2x^2-x+3}}dx+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\int \frac{3443x^2-5556x+768}{\sqrt{2x^2-x+3}}dx+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{1}{4}\int -\frac{3(11373x+4838)}{2\sqrt{2x^2-x+3}}dx+\frac{3443}{4}\sqrt{2x^2-x+3}x\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\int \frac{11373x+4838}{\sqrt{2x^2-x+3}}dx\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-x+3}x\right.$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-\right.$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)-$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

input
```
Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

output
```
(25*x^3*Sqrt[3 - x + 2*x^2])/8 + ((655*x^2*Sqrt[3 - x + 2*x^2])/6 + ((3443
*x*Sqrt[3 - x + 2*x^2])/4 - (3*((11373*Sqrt[3 - x + 2*x^2])/2 + (30725*Arc
Sinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/12)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$\frac{(9600x^3+20960x^2+13772x-34119)\sqrt{2x^2-x+3}}{3072}-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$
trager	$\left(\frac{25}{8}x^3+\frac{655}{96}x^2+\frac{3443}{768}x-\frac{11373}{1024}\right)\sqrt{2x^2-x+3}-\frac{30725\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x-}\right.}{4096}$
default	$-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{3443x\sqrt{2x^2-x+3}}{768}+\frac{655x^2\sqrt{2x^2-x+3}}{96}+\frac{25x^3\sqrt{2x^2-x+3}}{8}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(9600*x^3+20960*x^2+13772*x-34119)*(2*x^2-x+3)^(1/2)-30725/4096*2^(
1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3} + \frac{30725}{8192}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x - 25\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 3072
5/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x
 - 25)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^3}{8}+\frac{655x^2}{96}+\frac{3443x}{768}-\frac{11373}{1024}\right) - \frac{30725\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**3/8 + 655*x**2/96 + 3443*x/768 - 11373/1024) -
 30725*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4096
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}\sqrt{2x^2-x+3}x^3+\frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{30725}{4096}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{11373}{1024}\sqrt{2x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*s
qrt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
 - 11373/1024*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(4\left(40\left(60\,x+131\right)x+3443\right)x-34119\right)\sqrt{2\,x^2-x+3} + \frac{30725}{4096}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 3072
5/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \int \frac{\left(5\,x^2+3\,x+2\right)^2}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25\sqrt{2x^2-x+3}\,x^3}{8} + \frac{655\sqrt{2x^2-x+3}\,x^2}{96} + \frac{3443\sqrt{2x^2-x+3}\,x}{768} - \frac{11373\sqrt{2x^2-x+3}}{1024} - \frac{30725\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{4096}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

output
```
(38400*sqrt(2*x**2 - x + 3)*x**3 + 83840*sqrt(2*x**2 - x + 3)*x**2 + 55088
*sqrt(2*x**2 - x + 3)*x - 136476*sqrt(2*x**2 - x + 3) - 92175*sqrt(2)*log(
(2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/12288
```

3.116 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 59

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx=\frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{17\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

output
```
39/16*(2*x^2-x+3)^(1/2)+5/4*x*(2*x^2-x+3)^(1/2)+17/64*arcsinh(1/23*(1-4*x)
*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx=\frac{1}{64}\Big(4(39+20x)\sqrt{3-x+2x^2}+17\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\Big)$$

input
```
Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2
*x + 4*x^2]])/64
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{4}\int -\frac{14-39x}{2\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{14-39x}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

input
```
Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(5*x*Sqrt[3 - x + 2*x^2])/4 + ((39*Sqrt[3 - x + 2*x^2])/2 - (17*ArcSinh[(-
1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(20x+39)\sqrt{2x^2-x+3}}{16}-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}$	35
default	$-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{39\sqrt{2x^2-x+3}}{16}+\frac{5x\sqrt{2x^2-x+3}}{4}$	45
trager	$\left(\frac{5x}{4}+\frac{39}{16}\right)\sqrt{2x^2-x+3}-\frac{17\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{64}$	61

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(20*x+39)*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/
4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}\,\sqrt{2\,x^2-x+3}(20\,x+39) + \frac{17}{128}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \left(\frac{5x}{4}+\frac{39}{16}\right)\sqrt{2x^2-x+3} - \frac{17\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
(5*x/4 + 39/16)*sqrt(2*x**2 - x + 3) - 17*sqrt(2)*asinh(4*sqrt(23)*(x - 1/
4)/23)/64
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{4}\,\sqrt{2\,x^2-x+3}x - \frac{17}{64}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{39}{16}\,\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
 + 39/16*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}\,\sqrt{2\,x^2-x+3}(20\,x+39) + \frac{17}{64}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^2+3\,x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{5\sqrt{2x^2-x+3}\,x}{4} + \frac{39\sqrt{2x^2-x+3}}{16} - \frac{17\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{64}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

output
```
(80*sqrt(2*x**2 - x + 3)*x + 156*sqrt(2*x**2 - x + 3) - 17*sqrt(2)*log((2*
sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/64
```

3.117 $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \sqrt{\frac{1}{682}\left(-13+10\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-13+10\sqrt{2}\right)}}\left(7-3\sqrt{2}+\left(13-10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output

```
1/682*(8866+6820*2^(1/2))^(1/2)*arctan(11^(1/2)/(403+310*2^(1/2))^(1/2)*(7
+3*2^(1/2)+(13+10*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/682*(-8866+6820*2^(1/2)
)^(1/2)*arctanh(11^(1/2)/(-403+310*2^(1/2))^(1/2)*(7-3*2^(1/2)+(13-10*2^(1
/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3 -5\#1^4\&, \frac{\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\Big]$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

output
```
RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (Log[-
(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1317, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}\,dx$$

$$\Big\downarrow \text{1317}$$

$$\frac{\int \frac{11\left(-x+\sqrt{2}+1\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int \frac{11\left(-x-\sqrt{2}+1\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-x+\sqrt{2}+1}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-x-\sqrt{2}+1}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}$$

$\Big\downarrow$ 1362

$$\frac{\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13-10\sqrt{2}\right)}d\frac{\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(13+10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13+10\sqrt{2}\right)}d\frac{\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}$$

$\Big\downarrow$ 217

$$\frac{\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13-10\sqrt{2}\right)}d\frac{\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(13-10\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)}{\sqrt{682\left(10\sqrt{2}-13\right)}}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

output
```
Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3
*Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((13 - 10*Sqrt[2])
*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 10*Sqrt[
2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-13 + 10*Sqrt[2])]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.32 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.99

method	result
trager	$-\operatorname{RootOf}\left(232562\_Z^4+4433\_Z^2+25\right)\ln\left(-\frac{-74884964\operatorname{RootOf}\left(232562\_Z^4+4433\_Z^2+25\right)^5 x-3976060}{}\right.$
default	$\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(369\sqrt{2}\sqrt{-8866+6820\sqrt{2}}\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\sqrt{-23\left(8+3\sqrt{2}\right)\left(-\frac{23(-}{(\sqrt{}}\right.}}{}\right.\right.}{}$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-RootOf(232562*_Z^4+4433*_Z^2+25)*ln(-(-74884964*RootOf(232562*_Z^4+4433*_
Z^2+25)^5*x-3976060*RootOf(232562*_Z^4+4433*_Z^2+25)^3*x+391468*RootOf(232
562*_Z^4+4433*_Z^2+25)^2*(2*x^2-x+3)^(1/2)+954800*RootOf(232562*_Z^4+4433*
_Z^2+25)^3-41625*RootOf(232562*_Z^4+4433*_Z^2+25)*x+3650*(2*x^2-x+3)^(1/2)
+37000*RootOf(232562*_Z^4+4433*_Z^2+25))/(682*RootOf(232562*_Z^4+4433*_Z^2
+25)^2*x+5*x-2))+1/682*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)
^2+8866)*ln(-(18721241*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)
^2+8866)*RootOf(232562*_Z^4+4433*_Z^2+25)^4*x-280302*RootOf(232562*_Z^4+44
33*_Z^2+25)^2*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+8866)*
x+238700*RootOf(232562*_Z^4+4433*_Z^2+25)^2*RootOf(_Z^2+465124*RootOf(2325
62*_Z^4+4433*_Z^2+25)^2+8866)-66745294*RootOf(232562*_Z^4+4433*_Z^2+25)^2*
(2*x^2-x+3)^(1/2)-1739*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)
^2+8866)*x-4700*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+8866
)-649946*(2*x^2-x+3)^(1/2))/(341*RootOf(232562*_Z^4+4433*_Z^2+25)^2*x+4*x+
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(108) = 216$.

Time = 0.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.07

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{5}{341}\sqrt{2}+\frac{13}{682}}\arctan\left(-\frac{22\left(4\left(44\,x^3-94\,x^2-\sqrt{2}(25\,x^3-61\,x^2-32\,x+24)-16\,x+48\right)\sqrt{2\,x^2}\right.}{}\right.$$

$$-\frac{1}{2}\sqrt{\frac{5}{341}\sqrt{2}+\frac{13}{682}}\arctan\left(\frac{22\left(4\left(44\,x^3-94\,x^2-\sqrt{2}(25\,x^3-61\,x^2-32\,x+24)-16\,x+48\right)\sqrt{2\,x^2}\right.}{}\right.$$

$$+\frac{1}{4}\sqrt{\frac{5}{341}\sqrt{2}-\frac{13}{682}}\log\left(\frac{49\,x^2+22\sqrt{2\,x^2-x+3}\left(\sqrt{2}(41\,x-85)+44\,x-126\right)\sqrt{\frac{5}{341}\sqrt{2}-\frac{13}{682}}+44}{x^2}\right.$$

$$-\frac{1}{4}\sqrt{\frac{5}{341}\sqrt{2}-\frac{13}{682}}\log\left(\frac{49\,x^2-22\sqrt{2\,x^2-x+3}\left(\sqrt{2}(41\,x-85)+44\,x-126\right)\sqrt{\frac{5}{341}\sqrt{2}-\frac{13}{682}}+44}{x^2}\right.$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/2*sqrt(5/341*sqrt(2) + 13/682)*arctan(-22*(4*(44*x^3 - 94*x^2 - sqrt(2)*
(25*x^3 - 61*x^2 - 32*x + 24) - 16*x + 48)*sqrt(2*x^2 - x + 3) + (171*x^4
+ 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*
x)*sqrt(5/341*sqrt(2) - 13/682))*sqrt(5/341*sqrt(2) + 13/682)/(343*x^4 - 4
00*x^3 + 1136*x^2 + 384*x - 576)) - 1/2*sqrt(5/341*sqrt(2) + 13/682)*arcta
n(22*(4*(44*x^3 - 94*x^2 - sqrt(2)*(25*x^3 - 61*x^2 - 32*x + 24) - 16*x +
48)*sqrt(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*
x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(5/341*sqrt(2) - 13/682))*sqrt(5
/341*sqrt(2) + 13/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + 1/4
*sqrt(5/341*sqrt(2) - 13/682)*log((49*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2
)*(41*x - 85) + 44*x - 126)*sqrt(5/341*sqrt(2) - 13/682) + 44*sqrt(2)*(2*x
^2 - x + 3) - 151*x + 200)/x^2) - 1/4*sqrt(5/341*sqrt(2) - 13/682)*log((49
*x^2 - 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(41*x - 85) + 44*x - 126)*sqrt(5/34
1*sqrt(2) - 13/682) + 44*sqrt(2)*(2*x^2 - x + 3) - 151*x + 200)/x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{\sqrt{2x^2-x+3}}{10x^4+x^3+16x^2+7x+6}dx$$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(10*x**4 + x**3 + 16*x**2 + 7*x + 6),x)
```

3.118 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(2119+1816\sqrt{2}+\left(5751+3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1364} - \frac{\sqrt{\frac{1}{682}\left(-2343727+1678700\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-2343727+1678700\sqrt{2}\right)}}\left(2119-1816\sqrt{2}+\left(5751-3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1364}$$

output

```
(4+65*x)*(2*x^2-x+3)^(1/2)/(3410*x^2+2046*x+1364)+1/930248*(1598421814+114
4873400*2^(1/2))^(1/2)*arctan(11^(1/2)/(72655537+52039700*2^(1/2))^(1/2)*(
2119+1816*2^(1/2)+(5751+3935*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/930248*(-159
8421814+1144873400*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-72655537+52039700*2^(
1/2))^(1/2)*(2119-1816*2^(1/2)+(5751-3935*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} + \frac{\text{RootSum}\Big[-10580-2024\sqrt{2}\#1+68\#1^2+44\sqrt{2}\#1^3-5\#1^4\&,\ \frac{-9430\sqrt{2}\log\left(\sqrt{2}(-1+4x)-4\sqrt{3-x+2x^2}+\#1\right)}{}}{682\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + RootSum[-10580
- 2024*Sqrt[2]*#1 + 68*#1^2 + 44*Sqrt[2]*#1^3 - 5*#1^4 & , (-9430*Sqrt[2]*
Log[Sqrt[2]*(-1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1] + 4492*Log[Sqrt[2]*(-
1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1 + 205*Sqrt[2]*Log[Sqrt[2]*(-1 +
4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1^2)/(-506*Sqrt[2] + 34*#1 + 33*Sqrt[2
]*#1^2 - 5*#1^3) & ]/(682*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1305, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{(65x+4)\sqrt{2x^2-x+3}}{682\,(5x^2+3x+2)} - \frac{\int -\frac{11(332-205x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{332-205x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{1364}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \ 1368$$

$$\frac{\frac{\int -\frac{11\left(-\left(\left(127-205\sqrt{2}\right)x\right)-332\sqrt{2}+537\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(127+205\sqrt{2}\right)x\right)+332\sqrt{2}+537\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-\left(\left(127+205\sqrt{2}\right)x\right)+332\sqrt{2}+537}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(127-205\sqrt{2}\right)x\right)-332\sqrt{2}+537}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \ 1362$$

$$\frac{\frac{\left(2343727-1678700\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)^2}{2x^2-x+3}-31\left(2343727-1678700\sqrt{2}\right)}d\frac{\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(2343727+167870}{}}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\left(2343727-1678700\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)^2}{2x^2-x+3}-31\left(2343727-1678700\sqrt{2}\right)}d\frac{\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}}\left(234372\right.}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(2343727-1678700\sqrt{\right.}}{}}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727
+ 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*
(2119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((
2343727 - 1678700*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-2343727 + 1678700*Sqrt[2
]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
)/Sqrt[682*(-2343727 + 1678700*Sqrt[2])])/1364
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.72 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.52

method	result
trager	$\frac{(4+65x)\sqrt{2x^2-x+3}}{3410x^2+2046x+1364} - \frac{\mathrm{RootOf}\left(\_Z^2+66977856\,\mathrm{RootOf}\left(1205601408\_Z^4+28771592652\_Z^2+176127105625\right)^2+15984218\right.}{}$
risch	$\frac{(4+65x)\sqrt{2x^2-x+3}}{3410x^2+2046x+1364} + \frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(153463\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+54}}{}\right.\right.}{}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/682*(4+65*x)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)-1/930248*RootOf(_Z^2+669778
56*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2+1598421814)*ln(
-(2055399700464*x*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^4*
RootOf(_Z^2+66977856*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)
^2+1598421814)+26334199149588*RootOf(1205601408*_Z^4+28771592652*_Z^2+1761
27105625)^2*RootOf(_Z^2+66977856*RootOf(1205601408*_Z^4+28771592652*_Z^2+1
76127105625)^2+1598421814)*x+1369841493940634484*RootOf(1205601408*_Z^4+28
771592652*_Z^2+176127105625)^2*(2*x^2-x+3)^(1/2)-12361058255700*RootOf(120
5601408*_Z^4+28771592652*_Z^2+176127105625)^2*RootOf(_Z^2+66977856*RootOf(
1205601408*_Z^4+28771592652*_Z^2+176127105625)^2+1598421814)-4059260333683
2*RootOf(_Z^2+66977856*RootOf(1205601408*_Z^4+28771592652*_Z^2+17612710562
5)^2+1598421814)*x+16211894834030320271*(2*x^2-x+3)^(1/2)+17188361642025*R
ootOf(_Z^2+66977856*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^
2+1598421814))/(49104*x*RootOf(1205601408*_Z^4+28771592652*_Z^2+1761271056
25)^2+534991*x-67921))-3/341*RootOf(1205601408*_Z^4+28771592652*_Z^2+17612
7105625)*ln((-394636742489088*x*RootOf(1205601408*_Z^4+28771592652*_Z^2+17
6127105625)^5-13779789867949248*RootOf(1205601408*_Z^4+28771592652*_Z^2+17
6127105625)^3*x+32137043846114592*RootOf(1205601408*_Z^4+28771592652*_Z^2+
176127105625)^2*(2*x^2-x+3)^(1/2)-2373323185094400*RootOf(1205601408*_Z^4+
28771592652*_Z^2+176127105625)^3-96300551793960525*x*RootOf(1205601408*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(139) = 278$.

Time = 0.09 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.80

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/5456*(2*(5*x^2 + 3*x + 2)*sqrt(839350/341*sqrt(2) + 2343727/682)*arctan
(-22/67921*(4*(15838*x^3 - 36948*x^2 - sqrt(2)*(12195*x^3 - 27017*x^2 - 73
44*x + 12888) - 15152*x + 15936)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^3
 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(83
9350/341*sqrt(2) - 2343727/682))*sqrt(839350/341*sqrt(2) + 2343727/682)/(3
43*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 2*(5*x^2 + 3*x + 2)*sqrt(839
350/341*sqrt(2) + 2343727/682)*arctan(22/67921*(4*(15838*x^3 - 36948*x^2 -
 sqrt(2)*(12195*x^3 - 27017*x^2 - 7344*x + 12888) - 15152*x + 15936)*sqrt(
2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x
^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(839350/341*sqrt(2) - 2343727/682))*sqrt(
839350/341*sqrt(2) + 2343727/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x -
576)) + (5*x^2 + 3*x + 2)*sqrt(839350/341*sqrt(2) - 2343727/682)*log((3328
129*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(14227*x - 36625) + 22398*x - 50
852)*sqrt(839350/341*sqrt(2) - 2343727/682) + 2988524*sqrt(2)*(2*x^2 - x +
 3) - 10256071*x + 13584200)/x^2) - (5*x^2 + 3*x + 2)*sqrt(839350/341*sqrt
(2) - 2343727/682)*log((3328129*x^2 - 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(142
27*x - 36625) + 22398*x - 50852)*sqrt(839350/341*sqrt(2) - 2343727/682) +
2988524*sqrt(2)*(2*x^2 - x + 3) - 10256071*x + 13584200)/x^2) - 8*sqrt(2*x
^2 - x + 3)*(65*x + 4))/(5*x^2 + 3*x + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^2\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{50x^6+35x^5+103x^4+85x^3+83x^2+32x+12}dx$$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(50*x**6 + 35*x**5 + 103*x**4 + 85*x**3 + 83*x**2
 + 32*x + 12),x)
```

3.119 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$$
$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2}+\frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)}$$
$$+ \frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(123161+85754\sqrt{2}+\left(294669+208915\right.\right.}{\sqrt{3-x+2x^2}}\right.}{3720992}$$
$$- \frac{25\sqrt{\frac{1}{682}\left(-6414867847+4536374600\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-6414867847+4536374600\sqrt{2}\right)}}\left(123161-85754\sqrt{2}+\left(294669-20\right.\right.}{\sqrt{3-x+2x^2}}\right.}{3720992}$$

output
```
1/1364*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+(26794+86265*x)*(2*x^2-x
+3)^(1/2)/(9302480*x^2+5581488*x+3720992)+25/2537716544*(4374939871654+309
3807477200*2^(1/2))^(1/2)*arctan(11^(1/2)/(198860903257+140627612600*2^(1/
2))^(1/2)*(123161+85754*2^(1/2)+(294669+208915*2^(1/2))*x)/(2*x^2-x+3)^(1/
2))-25/2537716544*(-4374939871654+3093807477200*2^(1/2))^(1/2)*arctanh(11^
(1/2)/(-198860903257+140627612600*2^(1/2))^(1/2)*(123161-85754*2^(1/2)+(29
4669-208915*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.94 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$$
$$= \frac{\sqrt{3-x+2x^2}(59044+341572x+392765x^2+431325x^3)}{1860496\left(2+3x+5x^2\right)^2}$$
$$+ \frac{3\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-42330420383\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+1162930\cdots}{\cdots}\right.}{49210119200}$$
$$- \frac{16\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-720397\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+129160\sqrt{2}\,\text{lo}\cdots}{-13\sqrt{\cdots}}\right.}{4509725}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(Sqrt[3 - x + 2*x^2]*(59044 + 341572*x + 392765*x^2 + 431325*x^3))/(186049
6*(2 + 3*x + 5*x^2)^2) + (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt
[2]*#1^3 - 5*#1^4 & , (-42330420383*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
 - #1] + 11629301740*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*
#1 - 2992879225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sq
rt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/49210119200 - (16*RootSum[-
56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-720397*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 129160*Sqrt[2]*Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]*#1 - 65525*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^
2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/45097
25
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^3}dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{(65x+4)\sqrt{2x^2-x+3}}{1364\left(5x^2+3x+2\right)^2}-\frac{\int -\frac{11\left(520x^2-589x+1050\right)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}dx}{15004}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{520x^2-589x+1050}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}dx}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{\frac{\int \frac{275(18658-7445x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{7502}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{25\int \frac{18658-7445x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 1368}$$

$$\frac{\frac{25\left(\frac{\int -\frac{11\left(-\left(\left(11213-7445\sqrt{2}\right)x\right)-18658\sqrt{2}+26103\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(11213+7445\sqrt{2}\right)x\right)+18658\sqrt{2}+26103\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{25\left(\frac{\int -\frac{\left(\left(11213+7445\sqrt{2}\right)x\right)+18658\sqrt{2}+26103}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int -\frac{\left(\left(11213-7445\sqrt{2}\right)x\right)-18658\sqrt{2}+26103}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$\big\downarrow$ 1362

$$\frac{25\left(\frac{\left(6414867847-4536374600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)^2}{2x^2-x+3}-31\left(6414867847-4536374600\sqrt{2}\right)}d\frac{\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+12}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.}{}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$\big\downarrow$ 217

$$\frac{25\left(\frac{\left(6414867847-4536374600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)^2}{2x^2-x+3}-31\left(6414867847-4536374600\sqrt{2}\right)}d\frac{\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+12}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$\big\downarrow$ 219

$$\frac{\frac{25\left(\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(\left(294669+208915\sqrt{2}\right)x+85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(6414867847-45363\right.}{}\right.}{1364}}{2728}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + (((26794 + 8
6265*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (25*(Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600
*Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3
 - x + 2*x^2]] + ((6414867847 - 4536374600*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(
-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqrt[2] + (294669 - 20
8915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-6414867847 + 4536374600
*Sqrt[2])]))/1364)/2728
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.02 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{(431325x^3+392765x^2+341572x+59044)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)^2}+\frac{25\sqrt{\frac{8(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}(-1+\sqrt{2}+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\sqrt{2}\left(11325170\sqrt{-8866\right.}{}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1860496*(431325*x^3+392765*x^2+341572*x+59044)/(5*x^2+3*x+2)^2*(2*x^2-x+
3)^(1/2)+25/2537716544*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+787489
17689772*_Z^2+321542101742580625)^2+4374939871654)*ln((741143465568O*x*Roo
tOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)^4*RootOf(_Z^2+
267911424*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)^2
+4374939871654)+133779516386184108*RootOf(4822405632*_Z^4+78748917689772*_
Z^2+321542101742580625)^2*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+787
48917689772*_Z^2+321542101742580625)^2+4374939871654)*x+352484900285398187
57496*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)^2*(2*
x^2-x+3)^(1/2)+2440416054631500*RootOf(4822405632*_Z^4+78748917689772*_Z^2
+321542101742580625)^2*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+787489
17689772*_Z^2+321542101742580625)^2+4374939871654)+59650904312154126l413*R
ootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101 7
42580625)^2+4374939871654)*x+2879887428875757890l6260666*(2*x^2-x+3)^(1/2)
+244281337180512680 25*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+7874891
7689772*_Z^2+321542101742580625)^2+4374939871654))/(196416*x*RootOf(482240
5632*_Z^4+78748917689772*_Z^2+321542101742580625)^2+1614873451*x+14875319)
)-75/465124*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)
*ln(-(569198181556224O*x*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542
101742580625)^5+83155176470593979136*RootOf(4822405632*_Z^4+78748917689...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(170) = 340$.

Time = 0.09 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{3 - x + 2x^2}\,(2 + 3x + 5x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/14883968*(50*sqrt(1/682)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(4536
374600*sqrt(2) + 6414867847)*arctan(-22/14875319*sqrt(1/682)*(sqrt(1/682)*
(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x
) - 3936*x)*sqrt(4536374600*sqrt(2) - 6414867847) + 4*(865622*x^3 - 196231
2*x^2 - sqrt(2)*(607905*x^3 - 1383823*x^2 - 477936*x + 626472) - 655888*x
+ 895584)*sqrt(2*x^2 - x + 3))*sqrt(4536374600*sqrt(2) + 6414867847)/(343*
x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 50*sqrt(1/682)*(25*x^4 + 30*x^3
 + 29*x^2 + 12*x + 4)*sqrt(4536374600*sqrt(2) + 6414867847)*arctan(-22/148
75319*sqrt(1/682)*(sqrt(1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2
)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(4536374600*sqrt(2) - 64148
67847) - 4*(865622*x^3 - 1962312*x^2 - sqrt(2)*(607905*x^3 - 1383823*x^2 -
 477936*x + 626472) - 655888*x + 895584)*sqrt(2*x^2 - x + 3))*sqrt(4536374
600*sqrt(2) + 6414867847)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) +
25*sqrt(1/682)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(4536374600*sqrt(
2) - 6414867847)*log(25*(22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(3423
1*x - 82225) + 47994*x - 116456)*sqrt(4536374600*sqrt(2) - 6414867847) + 3
1690897*x^2 + 28457132*sqrt(2)*(2*x^2 - x + 3) - 97659703*x + 129350600)/x
^2) - 25*sqrt(1/682)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(4536374600
*sqrt(2) - 6414867847)*log(-25*(22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2
)*(34231*x - 82225) + 47994*x - 116456)*sqrt(4536374600*sqrt(2) - 64148...
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx=\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx$$
$$=\int \frac{\sqrt{2x^2-x+3}}{250x^8+325x^7+720x^6+804x^5+876x^4+579x^3+322x^2+100x+24}dx$$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(250*x**8 + 325*x**7 + 720*x**6 + 804*x**5 + 876*
x**4 + 579*x**3 + 322*x**2 + 100*x + 24),x)
```

3.120 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = & -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}}-\frac{31009685\sqrt{3-x+2x^2}}{65536} \\ & -\frac{8992487x\sqrt{3-x+2x^2}}{16384}-\frac{111315x^2\sqrt{3-x+2x^2}}{2048}+\frac{79425}{512}x^3\sqrt{3-x+2x^2} \\ & +\frac{10075}{96}x^4\sqrt{3-x+2x^2}+\frac{625}{24}x^5\sqrt{3-x+2x^2}-\frac{310445587\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}\end{aligned}$$

output

```
1/1472*(-1478741-1156639*x)/(2*x^2-x+3)^(1/2)-31009685/65536*(2*x^2-x+3)^(
1/2)-8992487/16384*x*(2*x^2-x+3)^(1/2)-111315/2048*x^2*(2*x^2-x+3)^(1/2)+7
9425/512*x^3*(2*x^2-x+3)^(1/2)+10075/96*x^4*(2*x^2-x+3)^(1/2)+625/24*x^5*(
2*x^2-x+3)^(1/2)-310445587/262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{4(-10961697147-8859305979x-2534760678x^2-2613624504x^3+230669760x^4+1281670400x^5+831385600x^6-}{\sqrt{3-x+2x^2}}}{18087936}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

output
```
((4*(-10961697147 - 8859305979*x - 2534760678*x^2 - 2613624504*x^3 + 23066
9760*x^4 + 1281670400*x^5 + 831385600*x^6 + 235520000*x^7))/Sqrt[3 - x + 2
*x^2] - 21420745503*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/180879
36
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{23}\int \frac{23(40000x^6+116000x^5+148400x^4+49960x^3-84916x^2-57494x+122691)}{256\sqrt{2x^2-x+3}}dx - \frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{128}\int \frac{40000x^6+116000x^5+148400x^4+49960x^3-84916x^2-57494x+122691}{\sqrt{2x^2-x+3}}dx - \frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{128}\left(\frac{1}{12}\int \frac{4\left(403000x^5+295200x^4+149880x^3-254748x^2-172482x+368073\right)}{\sqrt{2x^2-x+3}}dx+\frac{10000}{3}\sqrt{2x^2-x+3}x^5\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{128}\left(\frac{1}{3}\int \frac{403000x^5+295200x^4+149880x^3-254748x^2-172482x+368073}{\sqrt{2x^2-x+3}}dx+\frac{10000}{3}\sqrt{2x^2-x+3}x^5\right)-$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{128}\left(\frac{1}{3}\left(\frac{1}{10}\int \frac{30\left(158850x^4-111240x^3-84916x^2-57494x+122691\right)}{\sqrt{2x^2-x+3}}dx+40300\sqrt{2x^2-x+3}x^4\right)+\frac{10000}{3}\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{128}\left(\frac{1}{3}\left(3\int \frac{158850x^4-111240x^3-84916x^2-57494x+122691}{\sqrt{2x^2-x+3}}dx+40300\sqrt{2x^2-x+3}x^4\right)+\frac{10000}{3}\sqrt{2x^2}\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\int \frac{-333945x^3-2108978x^2-459952x+981528}{\sqrt{2x^2-x+3}}dx+\frac{79425}{4}\sqrt{2x^2-x+3}x^3\right)+40300\sqrt{2x^2-x}\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{6}\int \frac{3\left(-8992487x^2-504028x+3926112\right)}{2\sqrt{2x^2-x+3}}dx-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{79425}{4}\sqrt{2x^2-x+}\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\int\frac{-8992487x^2-504028x+3926112}{\sqrt{2x^2-x+3}}dx-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{79425}{4}\sqrt{2x^2-x+3}x^3\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\downarrow$ 2192

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{4}\int\frac{85363818-31009685x}{2\sqrt{2x^2-x+3}}dx-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\downarrow$ 27

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\int\frac{85363818-31009685x}{\sqrt{2x^2-x+3}}dx-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\downarrow$ 1160

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)\right.\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\downarrow$ 1090

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-\right.\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\downarrow$ 222

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) + ((10000*x^5*Sqrt[3 - x
+ 2*x^2])/3 + (40300*x^4*Sqrt[3 - x + 2*x^2] + 3*((79425*x^3*Sqrt[3 - x +
2*x^2])/4 + ((-111315*x^2*Sqrt[3 - x + 2*x^2])/2 + ((-8992487*x*Sqrt[3 - x
 + 2*x^2])/4 + ((-31009685*Sqrt[3 - x + 2*x^2])/2 + (310445587*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8)/4)/8))/3)/128
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}+\frac{3104\cdots}{\cdots}$
trager	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}-\frac{3104\cdots}{\cdots}$
default	$\frac{\frac{1234044515x}{3014656}-\frac{1234044515}{12058624}}{\sqrt{2x^2-x+3}}-\frac{1217267299}{524288\sqrt{2x^2-x+3}}-\frac{310445587x}{131072\sqrt{2x^2-x+3}}+\frac{310445587\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{262144}-\frac{1836783\cdots}{32768\sqrt{2x\cdots}}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4521984*(235520000*x^7+831385600*x^6+1281670400*x^5+230669760*x^4-261362
4504*x^3-2534760678*x^2-8859305979*x-10961697147)/(2*x^2-x+3)^(1/2)+310445
587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{21420745503\,\sqrt{2}(2\,x^2-x+3)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x\cdots\right.}{\cdots}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^
6 + 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885
9305979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx &= \frac{625\,x^7}{12\,\sqrt{2\,x^2-x+3}} + \frac{8825\,x^6}{48\,\sqrt{2\,x^2-x+3}} \\ &+ \frac{217675\,x^5}{768\,\sqrt{2\,x^2-x+3}} + \frac{52235\,x^4}{1024\,\sqrt{2\,x^2-x+3}} - \frac{4734827\,x^3}{8192\,\sqrt{2\,x^2-x+3}} \\ &- \frac{18367831\,x^2}{32768\,\sqrt{2\,x^2-x+3}} + \frac{310445587}{262144}\,\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) \\ &- \frac{2953101993\,x}{1507328\,\sqrt{2\,x^2-x+3}} - \frac{3653899049}{1507328\,\sqrt{2\,x^2-x+3}}\end{aligned}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/
768*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827
/8192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3
10445587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507
328*x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = -\frac{310445587}{262144}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{(46(4(40(20(16(100x+353)x+8707)x+31341)x-14204481)x-55103493)x-8859305979)x-10}{4521984\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x
 - 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x
 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.49

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{7536640000\sqrt{2x^2-x+3}\,x^7+26604339200\sqrt{2x^2-x+3}\,x^6+41013452800\sqrt{2x}}{}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x)
```

output
```
(7536640000*sqrt(2*x**2 - x + 3)*x**7 + 26604339200*sqrt(2*x**2 - x + 3)*x
**6 + 41013452800*sqrt(2*x**2 - x + 3)*x**5 + 7381432320*sqrt(2*x**2 - x +
 3)*x**4 - 83635984128*sqrt(2*x**2 - x + 3)*x**3 - 81112341696*sqrt(2*x**2
 - x + 3)*x**2 - 283497791328*sqrt(2*x**2 - x + 3)*x - 350774308704*sqrt(2
*x**2 - x + 3) + 342731928048*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 4*x - 1)/sqrt(23))*x**2 - 171365964024*sqrt(2)*log((2*sqrt(2*x**2 - x +
3)*sqrt(2) + 4*x - 1)/sqrt(23))*x + 514097892072*sqrt(2)*log((2*sqrt(2*x**
2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) - 188762163462*sqrt(2)*x**2 + 9438
1081731*sqrt(2)*x - 283143245193*sqrt(2))/(144703488*(2*x**2 - x + 3))
```

3.121 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}-\frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2}+\frac{1168881\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

output
```
1/368*(-22627+59895*x)/(2*x^2-x+3)^(1/2)-181561/2048*(2*x^2-x+3)^(1/2)+155
65/512*x*(2*x^2-x+3)^(1/2)+1825/64*x^2*(2*x^2-x+3)^(1/2)+125/16*x^3*(2*x^2
-x+3)^(1/2)+1168881/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{4(-15423965+16138403x-5754186x^2+2624760x^3+2318400x^4+736000x^5)}{\sqrt{3-x+2x^2}}+26884263\sqrt{2}\log\left(1-\right.}{188416}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

output
```
((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 73
6000*x^5))/Sqrt[3 - x + 2*x^2] + 26884263*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
 2*x + 4*x^2]])/188416
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23(-2000x^4-4600x^3-3860x^2+1658x+4795)}{64\sqrt{2x^2-x+3}}dx-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{32}\int \frac{-2000x^4-4600x^3-3860x^2+1658x+4795}{\sqrt{2x^2-x+3}}dx-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(250x^3\sqrt{2x^2-x+3}-\frac{1}{8}\int\frac{8\left(-5475x^3-1610x^2+1658x+4795\right)}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(250x^3\sqrt{2x^2-x+3}-\int\frac{-5475x^3-1610x^2+1658x+4795}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{32}\left(-\frac{1}{6}\int\frac{3\left(-15565x^2+28532x+19180\right)}{2\sqrt{2x^2-x+3}}dx+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(-\frac{1}{4}\int\frac{-15565x^2+28532x+19180}{\sqrt{2x^2-x+3}}dx+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{15565}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int\frac{181561x+246830}{2\sqrt{2x^2-x+3}}dx\right)+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{15565}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int\frac{181561x+246830}{\sqrt{2x^2-x+3}}dx\right)+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 1160

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{2x^2-\right.\right.$$
$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{}\right.$$

$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{2x^2-x+}\right.$$

$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) + ((1825*x^2*Sqrt[3 - x + 2*
x^2])/2 + 250*x^3*Sqrt[3 - x + 2*x^2] + ((15565*x*Sqrt[3 - x + 2*x^2])/4 +
 ((-181561*Sqrt[3 - x + 2*x^2])/2 - (1168881*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(4*Sqrt[2]))/8)/4)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x\right.}{8192}$
default	$\frac{\frac{5392543x}{94208}-\frac{5392543}{376832}}{\sqrt{2x^2-x+3}}-\frac{5130399}{16384\sqrt{2x^2-x+3}}+\frac{1168881x}{4096\sqrt{2x^2-x+3}}-\frac{1168881\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}-\frac{125091x^2}{1024\sqrt{2x^2-x+3}}+$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/47104*(736000*x^5+2318400*x^4+2624760*x^3-5754186*x^2+16138403*x-1542396
5)/(2*x^2-x+3)^(1/2)-1168881/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{26884263\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)}{3}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760
*x^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 -
x + 3)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{125\,x^5}{8\,\sqrt{2\,x^2-x+3}}+\frac{1575\,x^4}{32\,\sqrt{2\,x^2-x+3}}+\frac{14265\,x^3}{256\,\sqrt{2\,x^2-x+3}} - \frac{125091\,x^2}{1024\,\sqrt{2\,x^2-x+3}}-\frac{1168881}{8192}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{16138403\,x}{47104\,\sqrt{2\,x^2-x+3}}-\frac{15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/25
6*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/
8192*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^
2 - x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{1168881}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(46\,(20\,(40\,(20\,x+63)x+2853)x-125091)x+16138403)x-15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
 + 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x
- 15423965)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^3}{(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.73

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{2944000\sqrt{2x^2-x+3}\,x^5+9273600\sqrt{2x^2-x+3}\,x^4+10499040\sqrt{2x^2-x+3}\,x}{}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x)
```

output
```
(2944000*sqrt(2*x**2 - x + 3)*x**5 + 9273600*sqrt(2*x**2 - x + 3)*x**4 + 1
0499040*sqrt(2*x**2 - x + 3)*x**3 - 23016744*sqrt(2*x**2 - x + 3)*x**2 + 6
4553612*sqrt(2*x**2 - x + 3)*x - 61695860*sqrt(2*x**2 - x + 3) - 53768526*
sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 + 26
884263*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x
- 80652789*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)
) + 36151970*sqrt(2)*x**2 - 18075985*sqrt(2)*x + 54227955*sqrt(2))/(188416
*(2*x**2 - x + 3))
```

3.122 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} - \frac{223\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

output
```
121/92*(19-7*x)/(2*x^2-x+3)^(1/2)+415/32*(2*x^2-x+3)^(1/2)+25/8*x*(2*x^2-x
+3)^(1/2)-223/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{47027-9421x+16790x^2+4600x^3}{736\sqrt{3-x+2x^2}} - \frac{223\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (223*L
og[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{3/2}}dx$$

$$\big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23(100x^2+170x+51)}{16\sqrt{2x^2-x+3}}dx+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{100x^2+170x+51}{\sqrt{2x^2-x+3}}dx+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{4}\int -\frac{2(48-415x)}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{8}\left(25x\sqrt{2x^2-x+3}-\frac{1}{2}\int \frac{48-415x}{\sqrt{2x^2-x+3}}dx\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (25*x*Sqrt[3 - x + 2*x^2] + ((
415*Sqrt[3 - x + 2*x^2])/2 + (223*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]
))/2)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$
trager	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}-\frac{223\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{128}$
default	$-\frac{13713(4x-1)}{5888\sqrt{2x^2-x+3}}+\frac{15761}{256\sqrt{2x^2-x+3}}-\frac{223x}{64\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{365x^2}{16\sqrt{2x^2-x+3}}+\frac{25x^3}{4\sqrt{2x^2-x+3}}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/736*(4600*x^3+16790*x^2-9421*x+47027)/(2*x^2-x+3)^(1/2)+223/128*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{5129\sqrt{2}(2\,x^2-x+3)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+}{5888\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4
*x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*
sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{25\,x^3}{4\sqrt{2\,x^2-x+3}} + \frac{365\,x^2}{16\sqrt{2\,x^2-x+3}} + \frac{223}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{9421\,x}{736\sqrt{2\,x^2-x+3}} + \frac{47027}{736\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) +
47027/736/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(230\,(20\,x+73)x-9421)x+47027}{736\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^2}{(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.23

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{18400\sqrt{2x^2-x+3}\,x^3+67160\sqrt{2x^2-x+3}\,x^2-37684\sqrt{2x^2-x+3}\,x+1881\ldots}{\ldots}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x)
```

output
```
(18400*sqrt(2*x**2 - x + 3)*x**3 + 67160*sqrt(2*x**2 - x + 3)*x**2 - 37684
*sqrt(2*x**2 - x + 3)*x + 188108*sqrt(2*x**2 - x + 3) + 10258*sqrt(2)*log(
(2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 - 5129*sqrt(2)*l
og((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x + 15387*sqrt(2)*
log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) - 27104*sqrt(2)*x
**2 + 13552*sqrt(2)*x - 40656*sqrt(2))/(2944*(2*x**2 - x + 3))
```

3.123 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

output
```
1/23*(-55-33*x)/(2*x^2-x+3)^(1/2)-5/4*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{2\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x
 + 4*x^2]])/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{115}{4\sqrt{2x^2-x+3}}dx-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{2}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]]
)/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4}$	35
trager	$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{4}$	60
default	$\frac{\frac{49x}{46}-\frac{49}{184}}{\sqrt{2x^2-x+3}}-\frac{17}{8\sqrt{2x^2-x+3}}-\frac{5x}{2\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4}$	64

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-11/23*(3*x+5)/(2*x^2-x+3)^(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{115\sqrt{2}(2x^2-x+3)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-}{184\,(2x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x
 + 3)
```

Sympy [F]

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{5}{4}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{33\,x}{23\sqrt{2x^2-x+3}} - \frac{55}{23\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
 - 55/23/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{11\,(3x+5)}{23\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23
*(3*x + 5)/sqrt(2*x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{5\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{4} + \frac{3\,\left(2\,x-12\right)}{23\,\sqrt{2\,x^2-x+3}} - \frac{10\,\left(\frac{11\,x}{2}+\frac{3}{2}\right)}{23\,\sqrt{2\,x^2-x+3}} + \frac{16\,x-4}{23\,\sqrt{2\,x^2-x+3}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

output
```
(5*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2
*x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 -
x + 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{-132\sqrt{2x^2-x+3}\,x-220\sqrt{2x^2-x+3}+230\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)x^2 -}{}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x)
```

output
```
( - 132*sqrt(2*x**2 - x + 3)*x - 220*sqrt(2*x**2 - x + 3) + 230*sqrt(2)*lo
g((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 - 115*sqrt(2)*
log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x + 345*sqrt(2)*l
og((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) - 132*sqrt(2)*x**2
 + 66*sqrt(2)*x - 198*sqrt(2))/(92*(2*x**2 - x + 3))
```

3.124 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 176

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(61+4\sqrt{2}+\left(69+65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{1}{22}\sqrt{\frac{1}{682}\left(-247+500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-247+500\sqrt{2}\right)}}\left(61-4\sqrt{2}+\left(69-65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
1/253*(13-6*x)/(2*x^2-x+3)^(1/2)+1/15004*(168454+341000*2^(1/2))^(1/2)*arc
tan(11^(1/2)/(7657+15500*2^(1/2))^(1/2)*(61+4*2^(1/2)+(69+65*2^(1/2))*x)/(
2*x^2-x+3)^(1/2))-1/15004*(-168454+341000*2^(1/2))^(1/2)*arctanh(11^(1/2)/
(-7657+15500*2^(1/2))^(1/2)*(61-4*2^(1/2)+(69-65*2^(1/2))*x)/(2*x^2-x+3)^(
1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.13

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}}$$
$$+\frac{1}{22}\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3$$
$$-5\#1^4\&,\frac{23\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+16\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-5\log\left(-\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (23*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1] + 16*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 5*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9
*Sqrt[2]*#1^2 - 10*#1^3) & ]/22
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1305, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}\,dx$$
$$\Big\downarrow \text{1305}$$
$$\frac{13-6x}{253\sqrt{2x^2-x+3}}-\frac{\int -\frac{253(5x+8)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{22}\int \frac{5x+8}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 1368$$

$$\frac{1}{22}\left(\frac{\int -\frac{11\left(-\left(\left(13+5\sqrt{2}\right)x\right)-8\sqrt{2}+3\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(13-5\sqrt{2}\right)x\right)+8\sqrt{2}+3\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{22}\left(\frac{\int \frac{-\left(\left(13-5\sqrt{2}\right)x\right)+8\sqrt{2}+3}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(13+5\sqrt{2}\right)x\right)-8\sqrt{2}+3}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 1362$$

$$\frac{1}{22}\left(\frac{\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)^2}{2x^2-x+3}-31\left(247-500\sqrt{2}\right)}d\frac{\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(247+500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69+65\sqrt{\right.}\right.}{2x}}}{}\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{22}\left(\frac{\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)^2}{2x^2-x+3}-31\left(247-500\sqrt{2}\right)}d\frac{\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\text{arcta}\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{22}\left(\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(247-500\sqrt{2}\right)\operatorname{arctanh}}{\sqrt{6}}\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

input
```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcT
an[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*
x))/Sqrt[3 - x + 2*x^2]] + ((247 - 500*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-247
 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[3 - x + 2*x
^2]])/Sqrt[682*(-247 + 500*Sqrt[2])])/22
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.03 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.62

method	result
trager	$-\frac{-13+6x}{253\sqrt{2x^2-x+3}}+\frac{9\,\mathrm{RootOf}\left(6103357128\_Z^4+6822387\_Z^2+15625\right)\ln\left(-\frac{373964897946816x\,\mathrm{RootOf}\left(6103357128\_Z^4+682238\right.}{}\right.}{}$
risch	$-\frac{-13+6x}{253\sqrt{2x^2-x+3}}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(2197\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+549362\sqrt{}}}{}\right.\right.}{}$
default	$-\frac{3(4x-1)}{506\sqrt{2x^2-x+3}}+\frac{1}{22\sqrt{2x^2-x+3}}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(2197\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\right.\right.}{}$

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/253*(-13+6*x)/(2*x^2-x+3)^(1/2)+9/11*RootOf(6103357128*_Z^4+6822387*_Z^
2+15625)*ln(-(373964897946816*x*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)
^5+2007171224784*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^3*x+7546720104
0*(2*x^2-x+3)^(1/2)*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2-175125978
7200*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^3+2645619075*RootOf(610335
7128*_Z^4+6822387*_Z^2+15625)*x+36443750*(2*x^2-x+3)^(1/2)-5324797800*Root
Of(6103357128*_Z^4+6822387*_Z^2+15625))/(220968*x*RootOf(6103357128*_Z^4+6
822387*_Z^2+15625)^2-55*x-238))+1/15004*RootOf(_Z^2+150700176*RootOf(61033
57128*_Z^4+6822387*_Z^2+15625)^2+168454)*ln((-649244614491*RootOf(_Z^2+150
700176*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+168454)*RootOf(6103357
128*_Z^4+6822387*_Z^2+15625)^4*x+2033209431*RootOf(6103357128*_Z^4+6822387
*_Z^2+15625)^2*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2+1
5625)^2+168454)*x+1608394722165*(2*x^2-x+3)^(1/2)*RootOf(6103357128*_Z^4+6
822387*_Z^2+15625)^2-3040381575*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)
^2*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+1684
54)-1509120*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2+1562
5)^2+168454)*x+1021170535*(2*x^2-x+3)^(1/2)+5845875*RootOf(_Z^2+150700176*
RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+168454))/(110484*x*RootOf(610
3357128*_Z^4+6822387*_Z^2+15625)^2+151*x+119))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(127) = 254$.

Time = 0.09 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.99

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/2024*(46*(2*x^2 - x + 3)*sqrt(250/341*sqrt(2) + 247/682)*arctan(-22/119*
(4*(322*x^3 - 612*x^2 - sqrt(2)*(105*x^3 - 323*x^2 - 336*x + 72) + 112*x +
 384)*sqrt(2*x^2 - x + 3) + (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(
6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(250/341*sqrt(2) - 247/682))*s
qrt(250/341*sqrt(2) + 247/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576
)) - 46*(2*x^2 - x + 3)*sqrt(250/341*sqrt(2) + 247/682)*arctan(22/119*(4*(
322*x^3 - 612*x^2 - sqrt(2)*(105*x^3 - 323*x^2 - 336*x + 72) + 112*x + 384
)*sqrt(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^
4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(250/341*sqrt(2) - 247/682))*sqrt(
250/341*sqrt(2) + 247/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) +
 23*(2*x^2 - x + 3)*sqrt(250/341*sqrt(2) - 247/682)*log((5831*x^2 + 22*sqr
t(2*x^2 - x + 3)*(sqrt(2)*(313*x - 475) + 162*x - 788)*sqrt(250/341*sqrt(2
) - 247/682) + 5236*sqrt(2)*(2*x^2 - x + 3) - 17969*x + 23800)/x^2) - 23*(
2*x^2 - x + 3)*sqrt(250/341*sqrt(2) - 247/682)*log((5831*x^2 - 22*sqrt(2*x
^2 - x + 3)*(sqrt(2)*(313*x - 475) + 162*x - 788)*sqrt(250/341*sqrt(2) - 2
47/682) + 5236*sqrt(2)*(2*x^2 - x + 3) - 17969*x + 23800)/x^2) - 8*sqrt(2*
x^2 - x + 3)*(6*x - 13))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)(2\,x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{\sqrt{2x^2-x+3}}{20x^6-8x^5+61x^4+x^3+53x^2+15x+18}dx$$

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(20*x**6 - 8*x**5 + 61*x**4 + x**3 + 53*x**2 + 15
*x + 18),x)
```

3.125 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 211

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)}$$
$$+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(12611+16454\sqrt{2}+\left(45519+29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{30008}$$
$$-\frac{\sqrt{\frac{1}{682}\left(-129694447+103775000\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-129694447+103775000\sqrt{2}\right)}}\left(12611-16454\sqrt{2}+\left(45519-29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{30008}$$

output

```
-1/345092*(6315-2306*x)/(2*x^2-x+3)^(1/2)+1/682*(4+65*x)/(2*x^2-x+3)^(1/2)
/(5*x^2+3*x+2)+1/20465456*(88451612854+70774550000*2^(1/2))^(1/2)*arctan(1
1^(1/2)/(4020527857+3217025000*2^(1/2))^(1/2)*(12611+16454*2^(1/2)+(45519+
29065*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/20465456*(-88451612854+70774550000*
2^(1/2))^(1/2)*arctanh(11^(1/2)/(-4020527857+3217025000*2^(1/2))^(1/2)*(12
611-16454*2^(1/2)+(45519-29065*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.96

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = \frac{\sqrt{3-x+2x^2}(-10606+18557x-24657x^2+11530x^3)}{345092(6+7x+16x^2+x^3+10x^4)} - \frac{1}{484}\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3 -5\#1^4\&, \frac{225\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+8\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-15\log\left(\cdot\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3} + \frac{\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{8623\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+9624\log\left(-\sqrt{2}x+\sqrt{3\right.}{-13\sqrt{2}+17\#1}}{30008\sqrt{2}}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(Sqrt[3 - x + 2*x^2]*(-10606 + 18557*x - 24657*x^2 + 11530*x^3))/(345092*(
6 + 7*x + 16*x^2 + x^3 + 10*x^4)) - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2
+ 6*Sqrt[2]*#1^3 - 5*#1^4 & , (225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1] + 8*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 15*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*S
qrt[2]*#1^2 - 10*#1^3) & ]/484 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6
*Sqrt[2]*#1^3 - 5*#1^4 & , (8623*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2
*x^2] - #1] + 9624*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1565*
Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(30008*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}-\frac{\int -\frac{11(520x^2-303x+324)}{2(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{520x^2-303x+324}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{253(2158-2495x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{22}\int \frac{2158-2495x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$

$$\Big\downarrow \text{1368}$$

$$\frac{\frac{1}{22}\left(\frac{\int -\frac{11\left(\left(337+2495\sqrt{2}\right)x-2158\sqrt{2}+4653\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(337-2495\sqrt{2}\right)x+2158\sqrt{2}+4653\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{22}\left(\frac{\int \frac{\left(337-2495\sqrt{2}\right)x+2158\sqrt{2}+4653}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(337+2495\sqrt{2}\right)x-2158\sqrt{2}+4653}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$

$$\Big\downarrow \text{1362}$$

$$\frac{\frac{1}{22}\left(\frac{\left(129694447-103775000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)^2}{2x^2-x+3}-31\left(129694447-103775000\sqrt{2}\right)}d\frac{\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.}{}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

$\downarrow$ 217

$$\frac{\frac{1}{22}\left(\frac{\left(129694447-103775000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)^2}{2x^2-x+3}-31\left(129694447-103775000\sqrt{2}\right)}d\frac{\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.}{}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

$\downarrow$ 219

$$\frac{\frac{1}{22}\left(\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(\left(45519+29065\sqrt{2}\right)x+16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)+\right.}{1364}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

input

```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

output

```
(4 + 65*x)/(682*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (-1/253*(6315 - 2
306*x)/Sqrt[3 - x + 2*x^2] + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
 + (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((129694447 - 103775
000*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*Sqrt[2]))]*(1261
1 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt
[682*(-129694447 + 103775000*Sqrt[2])])/22)/1364
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.18 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.32

method	result
trager	Expression too large to display
risch	$\frac{11530x^3-24657x^2+18557x-10606}{345092(5x^2+3x+2)\sqrt{2x^2-x+3}}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1173047\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\arctan\right.}{}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/345092*(11530*x^3-24657*x^2+18557*x-10606)/(10*x^4+x^3+16*x^2+7*x+6)*(2*
x^2-x+3)^(1/2)+9/7502*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541
015625)*ln((1593373465716747264*x*RootOf(24413428512*_Z^4+3582290320587*_Z
^2+168269541015625)^5+356051747055336070464*RootOf(24413428512*_Z^4+358229
0320587*_Z^2+168269541015625)^3*x+193384498382205292800*RootOf(24413428512
*_Z^4+3582290320587*_Z^2+168269541015625)^3+32475459835859538960000*RootOf(
24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2*(2*x^2-x+3)^(1/2)+8
600066701843343049675*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541
015625)*x+3788537843234711874780 0*RootOf(24413428512*_Z^4+3582290320587*_Z
^2+168269541015625)+24465971384883001859375 0*(2*x^2-x+3)^(1/2))/(883872*x*
RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2+83351945*x+2
4672962))+1/20465456*RootOf(_Z^2+602800704*RootOf(24413428512*_Z^4+3582290
320587*_Z^2+168269541015625)^2+88451612854)*ln((691568344495116*x*RootOf(2
4413428512*_Z^4+3582290320587*_Z^2+168269541015625)^4*RootOf(_Z^2+60280070
4*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2+8845161285
4)+48417413991953391*RootOf(24413428512*_Z^4+3582290320587*_Z^2+1682695410
15625)^2*RootOf(_Z^2+602800704*RootOf(24413428512*_Z^4+3582290320587*_Z^2+
168269541015625)^2+88451612854)*x-3460666188758782120 4250*RootOf(244134285
12*_Z^4+3582290320587*_Z^2+168269541015625)^2*(2*x^2-x+3)^(1/2)-8393424408
9498825*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2*R...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.73

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/2760736*(46*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(51887500/341*sqrt(2)
 + 129694447/682)*arctan(-22/12336481*(4*(109922*x^3 - 272712*x^2 - sqrt(2
)*(101355*x^3 - 213973*x^2 - 29136*x + 111672) - 154288*x + 103584)*sqrt(2
*x^2 - x + 3) + (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^
3 + 5*x^2 + 12*x) - 3936*x)*sqrt(51887500/341*sqrt(2) - 129694447/682))*sq
rt(51887500/341*sqrt(2) + 129694447/682)/(343*x^4 - 400*x^3 + 1136*x^2 + 3
84*x - 576)) - 46*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(51887500/341*sqrt
(2) + 129694447/682)*arctan(22/12336481*(4*(109922*x^3 - 272712*x^2 - sqrt
(2)*(101355*x^3 - 213973*x^2 - 29136*x + 111672) - 154288*x + 103584)*sqrt
(2*x^2 - x + 3) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*
x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(51887500/341*sqrt(2) - 129694447/682))*
sqrt(51887500/341*sqrt(2) + 129694447/682)/(343*x^4 - 400*x^3 + 1136*x^2 +
 384*x - 576)) + 23*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(51887500/341*sq
rt(2) - 129694447/682)*log((604487569*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2
)*(95963*x - 285725) + 189762*x - 381688)*sqrt(51887500/341*sqrt(2) - 1296
94447/682) + 542805164*sqrt(2)*(2*x^2 - x + 3) - 1862808631*x + 2467296200
)/x^2) - 23*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(51887500/341*sqrt(2) -
129694447/682)*log((604487569*x^2 - 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(95963
*x - 285725) + 189762*x - 381688)*sqrt(51887500/341*sqrt(2) - 129694447/68
2) + 542805164*sqrt(2)*(2*x^2 - x + 3) - 1862808631*x + 2467296200)/x^2...
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\,(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^2(2\,x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx=\int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx=\int \frac{\sqrt{2x^2-x+3}}{100x^8+20x^7+321x^6+172x^5+390x^4+236x^3+241x^2+8}$$

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(100*x**8 + 20*x**7 + 321*x**6 + 172*x**5 + 390*x
**4 + 236*x**3 + 241*x**2 + 84*x + 36),x)
```

3.126 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}$$
$$+ \frac{3\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393+4123702\sqrt{2}+\left(\right.\right.}{\sqrt{3-x+2x^2}}\right.}{81861824}$$
$$- \frac{3\sqrt{\frac{1}{682}\left(-13874275807943+9819738650000\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393-4123702\right.}{\sqrt{3-x+2x^2}}\right.}{81861824}$$

output
```
-1/941410976*(4353943-6508666*x)/(2*x^2-x+3)^(1/2)+1/1364*(4+65*x)/(2*x^2-
x+3)^(1/2)/(5*x^2+3*x+2)^2+5/1860496*(7318+17315*x)/(2*x^2-x+3)^(1/2)/(5*x
^2+3*x+2)+3/55829763968*(9462256101017126+6697061759300000*2^(1/2))^(1/2)*
arctan(11^(1/2)/(430102550046233+304411898150000*2^(1/2))^(1/2)*(5538393+4
123702*2^(1/2)+(13785797+9662095*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-3/55829763
968*(-9462256101017126+6697061759300000*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-
430102550046233+304411898150000*2^(1/2))^(1/2)*(5538393-4123702*2^(1/2)+(1
3785797-9662095*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.68 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.47

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \frac{\frac{4(22374044+161806828x+175833195x^2+277167774x^3+86411405x^4+162716650x^5)}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} -}{}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
((4*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*x^4
 + 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) - 176824*Sqrt
[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-
491*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 208*Log[-(Sqrt[
2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 5*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
 & ] + 124*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
- 5*#1^4 & , (7194481*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
 - 3798456*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 575915*Sqrt[2
]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
+ 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 7*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 +
17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (143178771*Sqrt[2]*Log[-(Sqrt[2]*x)
+ Sqrt[3 - x + 2*x^2] - #1] - 105962920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*
x^2] - #1]*#1 + 6180225*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #
1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/3765643904
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^3}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}-\frac{\int -\frac{11\left(1040x^2-687x+1042\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}dx}{15004}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1040x^2-687x+1042}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}dx}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\int \frac{11\left(692600x^2-28425x+483914\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{7502}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{692600x^2-28425x+483914}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\frac{\int \frac{759(847654-395185x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783}-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3}{22}\int \frac{847654-395185x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 1368$$

$$\frac{\frac{\frac{3}{22}\left(\frac{\int -\frac{11\left(-\left(\left(452469-395185\sqrt{2}\right)x\right)-847654\sqrt{2}+1242839\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(452469+395185\sqrt{2}\right)x\right)+847654\sqrt{2}+1242839\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{\cdots}{682\cdots}}{2728}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{22}\left(\frac{\int \frac{-\left(\left(452469+395185\sqrt{2}\right)x\right)+847654\sqrt{2}+1242839}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(452469-395185\sqrt{2}\right)x\right)-847654\sqrt{2}+1242839}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x\cdots}{682\sqrt{2x^2-x+3}\cdots}}{2728}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1362$$

$$\frac{\frac{3}{22}\left(\frac{\left(13874275807943-9819738650000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)^2}{2x^2-x+3}-31\left(13874275807943-9819738650000\sqrt{2}\right)}d\frac{\left(13785797-96\cdots\right.}{\cdots}}{\sqrt{2}}\cdots\right.}{\cdots}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{3}{22}\left(\frac{\left(13874275807943-9819738650000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)^2}{2x^2-x+3}-31\left(13874275807943-9819738650000\sqrt{2}\right)}d\frac{\left(13785797-96\cdots\right.}{\cdots}}{\sqrt{2}}\cdots\right.}{\cdots}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3}{22}\left(\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(\left(13785797+9662095\sqrt{2}\right)x+4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right.\right.}{}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(4 + 65*x)/(1364*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + ((5*(7318 + 17
315*x))/(682*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (-1/253*(4353943 - 6
508666*x)/Sqrt[3 - x + 2*x^2] + (3*(Sqrt[(13874275807943 + 9819738650000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2]))
]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]] + ((13874275807943 - 9819738650000*Sqrt[2])*ArcTanh[(Sqrt[11/(3
1*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538393 - 4123702*Sqrt[2] +
 (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-1387427
5807943 + 9819738650000*Sqrt[2])]))/22)/1364)/2728
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.53 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.00

method	result
trager	Expression too large to display
risch	$\frac{162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044}{941410976(5x^2+3x+2)^2\sqrt{2x^2-x+3}} + \frac{3\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3}}{}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/941410976*(162716650*x^5+86411405*x^4+277167774*x^3+175833195*x^2+161806
828*x+22374044)/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2)-3/55829763968*RootOf(_Z^
2+2411202816*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+3766690123214
99306640625)^2+9462256101017126)*ln(-(239326023754418832*RootOf(_Z^2+24112
02816*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+37666901232149930664
0625)^2+9462256101017126)*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+
376669012321499306640625)^4*x+725400753296830666567677*RootOf(97653714048*
_Z^4+383221372091193603*_Z^2+376669012321499306640625)^2*RootOf(_Z^2+24112
02816*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+37666901232149930664
0625)^2+9462256101017126)*x+2856571331932648444940522338950O*(2*x^2-x+3)^(
1/2)*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+376669012321499306640
625)^2-633449775355269709227 5*RootOf(97653714048*_Z^4+383221372091193603*
_Z^2+376669012321499306640625)^2*RootOf(_Z^2+2411202816*RootOf(97653714048
*_Z^4+383221372091193603*_Z^2+376669012321499306640625)^2+9462256101017126
)+478183388985603888097387392375*RootOf(_Z^2+2411202816*RootOf(97653714048
*_Z^4+383221372091193603*_Z^2+376669012321499306640625)^2+9462256101017126
)*x+559273555720414611567695913992325845OO*(2*x^2-x+3)^(1/2)-6137834418289
6989866287615625*RootOf(_Z^2+2411202816*RootOf(97653714048*_Z^4+3832213720
91193603*_Z^2+376669012321499306640625)^2+9462256101017126))/(1767744*x*Ro
otOf(97653714048*_Z^4+383221372091193603*_Z^2+376669012321499306640625)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(189) = 378$.

Time = 0.10 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.76

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
-1/7531287808*(138*sqrt(1/682)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^
2 + 32*x + 12)*sqrt(9819738650000*sqrt(2) + 13874275807943)*arctan(-22/107
614172489*sqrt(1/682)*(sqrt(1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sq
rt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(9819738650000*sqrt(2)
- 13874275807943) + 4*(39667886*x^3 - 90744656*x^2 - sqrt(2)*(28699865*x^3
 - 64747999*x^2 - 20801968*x + 29828136) - 32531344*x + 40687392)*sqrt(2*x
^2 - x + 3))*sqrt(9819738650000*sqrt(2) + 13874275807943)/(343*x^4 - 400*x
^3 + 1136*x^2 + 384*x - 576)) - 138*sqrt(1/682)*(50*x^6 + 35*x^5 + 103*x^4
 + 85*x^3 + 83*x^2 + 32*x + 12)*sqrt(9819738650000*sqrt(2) + 1387427580794
3)*arctan(-22/107614172489*sqrt(1/682)*(sqrt(1/682)*(171*x^4 + 1212*x^3 -
1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(98197
38650000*sqrt(2) - 13874275807943) - 4*(39667886*x^3 - 90744656*x^2 - sqrt
(2)*(28699865*x^3 - 64747999*x^2 - 20801968*x + 29828136) - 32531344*x + 4
0687392)*sqrt(2*x^2 - x + 3))*sqrt(9819738650000*sqrt(2) + 13874275807943)
/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) + 69*sqrt(1/682)*(50*x^6 +
35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*sqrt(9819738650000*sqrt(2)
 - 13874275807943)*log(3*(22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(359
39369*x - 88253175) + 52313806*x - 124192544)*sqrt(9819738650000*sqrt(2) -
 13874275807943) + 5273094451961*x^2 + 4735023589516*sqrt(2)*(2*x^2 - x +
3) - 16249740045839*x + 21522834497800)/x^2) - 69*sqrt(1/682)*(50*x^6 +...
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\,(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3(2\,x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \int \frac{\sqrt{2x^2-x+3}}{500x^{10}+400x^9+1865x^8+1863x^7+3108x^6+2694x^5+269}$$

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(500*x**10 + 400*x**9 + 1865*x**8 + 1863*x**7 + 3
108*x**6 + 2694*x**5 + 2693*x**4 + 1615*x**3 + 914*x**2 + 276*x + 72),x)
```

3.127 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096}+\frac{526075x\sqrt{3-x+2x^2}}{3072} + \frac{38375}{384}x^2\sqrt{3-x+2x^2}+\frac{625}{32}x^3\sqrt{3-x+2x^2}+\frac{16955197\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

output
```
1/4416*(-1478741-1156639*x)/(2*x^2-x+3)^(3/2)+1331/101568*(7409+116368*x)/
(2*x^2-x+3)^(1/2)-1308645/4096*(2*x^2-x+3)^(1/2)+526075/3072*x*(2*x^2-x+3)
^(1/2)+38375/384*x^2*(2*x^2-x+3)^(1/2)+625/32*x^3*(2*x^2-x+3)^(1/2)+169551
97/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{-18974698519+49883864262x-36481630395x^2+39848900984x^3-50767812}{6500352\,(3-x+2x^2)^{3/2}}$$
$$+\frac{16955197\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 507678
1260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 -
x + 2*x^2)^(3/2)) + (16955197*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(819
2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {2191, 27, 2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{69}\int \frac{2760000x^6+8004000x^5+10239600x^4+3447240x^3-5859204x^2-3967086x+3839123}{256\,(2x^2-x+3)^{3/2}}dx - \frac{14641(79x+101)}{4416\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2760000x^6+8004000x^5+10239600x^4+3447240x^3-5859204x^2-3967086x+3839123}{(2x^2-x+3)^{3/2}}dx}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{\frac{2}{23}\int -\frac{1587(-10000x^4-34000x^3-39100x^2+18960x+89359)}{\sqrt{2x^2-x+3}}dx+\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\int \frac{-10000x^4-34000x^3-39100x^2+18960x+89359}{\sqrt{2x^2-x+3}}dx}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{8}\int \frac{8(-38375x^3-27850x^2+18960x+89359)}{\sqrt{2x^2-x+3}}dx-1250x^3\sqrt{2x^2-x+3}\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\int \frac{-38375x^3-27850x^2+18960x+89359}{\sqrt{2x^2-x+3}}dx-1250x^3\sqrt{2x^2-x+3}\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{6}\int \frac{-526075x^2+688020x+1072308}{2\sqrt{2x^2-x+3}}dx-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x^2-x+3}x^3\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\int \frac{-526075x^2+688020x+1072308}{\sqrt{2x^2-x+3}}dx-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x^2-x+3}x^3\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{1}{4}\int\frac{3(1308645x+3911638)}{2\sqrt{2x^2-x+3}}dx-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{\cdots}\right.}{8832}$$
$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\int\frac{1308645x+3911638}{\sqrt{2x^2-x+3}}dx-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x\cdots}\right.}{8832}$$
$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\cdots\right.}{8832}$$
$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1090

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)\cdots\right.}{8832}$$
$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 222

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{383\cdots}{6}\cdots\right.}{8832}$$
$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + ((2662*(7409 + 116368
*x))/(23*Sqrt[3 - x + 2*x^2]) - 138*((-38375*x^2*Sqrt[3 - x + 2*x^2])/6 -
1250*x^3*Sqrt[3 - x + 2*x^2] + ((-526075*x*Sqrt[3 - x + 2*x^2])/4 + (3*((1
308645*Sqrt[3 - x + 2*x^2])/2 + (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4
*Sqrt[2])))/8)/12))/8832
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} -$
trager	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} +$
default	$\frac{16955197}{32768\sqrt{2x^2-x+3}} - \frac{2149616639}{524288(2x^2-x+3)^{\frac{3}{2}}} + \frac{138025x^5}{256(2x^2-x+3)^{\frac{3}{2}}} + \frac{\frac{5141612725x}{9043968}-\frac{5141612725}{36175872}}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{67488035x^2}{16384(2x^2-x+3)^{\frac{3}{2}}} - \frac{169551}{}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6500352*(507840000*x^7+2090608000*x^6+3504730800*x^5-5076781260*x^4+3984
8900984*x^3-36481630395*x^2+49883864262*x-18974698519)/(2*x^2-x+3)^(3/2)-1
6955197/16384*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{26907897639\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)\right.}{}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*s
qrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*
x^7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 -
 36481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(118) = 236.

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx &= \frac{625\,x^7}{8\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{30875\,x^6}{96\,(2\,x^2-x+3)^{\frac{3}{2}}}\\
&+ \frac{138025\,x^5}{256\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{799745\,x^4}{1024\,(2\,x^2-x+3)^{\frac{3}{2}}}\\
&- \frac{16955197}{13000704}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)\\
&- \frac{16955197}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{1203818987}{6500352}\sqrt{2\,x^2-x+3}\\
&+ \frac{3536205583\,x}{3250176\sqrt{2\,x^2-x+3}} - \frac{2638851\,x^2}{512\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{257773037}{1083392\sqrt{2\,x^2-x+3}}\\
&+ \frac{29484067\,x}{23552\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{374445479}{70656\,(2\,x^2-x+3)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 138
025/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2)
- 16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3
)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x
^2 - x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1
)) + 1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*
x^2 - x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/s
qrt(2*x^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70
656/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{16955197}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{((4(2645(20(40(60x+247)x+16563)x-479847)x+9962225246)x-36481630395)x+4988386426}{6500352\,(2x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) +
1) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x +
9962225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x
+ 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^4}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.29

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{16250880000\sqrt{2x^2-x+3}\,x^7+66899456000\sqrt{2x^2-x+3}\,x^6+112151385600\sqrt{}}{}$$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)
```

output
```
(16250880000*sqrt(2*x**2 - x + 3)*x**7 + 66899456000*sqrt(2*x**2 - x + 3)*
x**6 + 112151385600*sqrt(2*x**2 - x + 3)*x**5 - 162457000320*sqrt(2*x**2 -
 x + 3)*x**4 + 1275164831488*sqrt(2*x**2 - x + 3)*x**3 - 1167412172640*sqr
t(2*x**2 - x + 3)*x**2 + 1596283656384*sqrt(2*x**2 - x + 3)*x - 6071903526
08*sqrt(2*x**2 - x + 3) - 861052724448*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)
*sqrt(2) + 4*x - 1)/sqrt(23))*x**4 + 861052724448*sqrt(2)*log((2*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**3 - 2798421354456*sqrt(2)*log(
(2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 + 1291579086672*
sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x - 19373
68630008*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))
- 203665714820*sqrt(2)*x**4 + 203665714820*sqrt(2)*x**3 - 661913573165*sqr
t(2)*x**2 + 305498572230*sqrt(2)*x - 458247858345*sqrt(2))/(208011264*(4*x
**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.128 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\,(3-x+2x^2)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64}\sqrt{3-x+2x^2} + \frac{125}{16}x\sqrt{3-x+2x^2} - \frac{7495\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

output
```
1/1104*(-22627+59895*x)/(2*x^2-x+3)^(3/2)+121/8464*(10679-6744*x)/(2*x^2-x
+3)^(1/2)+3175/64*(2*x^2-x+3)^(1/2)+125/16*x*(2*x^2-x+3)^(1/2)-7495/256*ar
csinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{89784565 - 62463282x + 101546529x^2 - 29423976x^3 + 16980900x^4 + 3174000x}{101568\,(3-x+2x^2)^{3/2}} - \frac{7495\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 317
4000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) - (7495*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/(128*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int -\frac{3(-46000x^4-105800x^3-88780x^2+38134x+30425)}{64(2x^2-x+3)^{3/2}}dx-\frac{1331(17-45x)}{1104(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{736}\int \frac{-46000x^4-105800x^3-88780x^2+38134x+30425}{(2x^2-x+3)^{3/2}}dx-\frac{1331(17-45x)}{1104(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{736}\left(\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}-\frac{2}{23}\int -\frac{2645(100x^2+280x+183)}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{1104(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{736}\left(230\int \frac{100x^2+280x+183}{\sqrt{2x^2-x+3}}dx+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{736}\left(230\left(\frac{1}{4}\int \frac{2(635x+216)}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\int \frac{635x+216}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + ((242*(10679 - 6744*x))
/(23*Sqrt[3 - x + 2*x^2]) + 230*(25*x*Sqrt[3 - x + 2*x^2] + ((635*Sqrt[3 -
 x + 2*x^2])/2 + (1499*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/2))/736
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)\right.}{256}$
default	$-\frac{14081711(4x-1)}{565248(2x^2-x+3)^{\frac{3}{2}}}-\frac{3391139(4x-1)}{203136\sqrt{2x^2-x+3}}+\frac{20961031}{24576(2x^2-x+3)^{\frac{3}{2}}}-\frac{281177x}{2048(2x^2-x+3)^{\frac{3}{2}}}+\frac{222809x^2}{256(2x^2-x+3)^{\frac{3}{2}}}-\frac{7495}{192(2x^2-}$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/101568*(3174000*x^5+16980900*x^4-29423976*x^3+101546529*x^2-62463282*x+8
9784565)/(2*x^2-x+3)^(3/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{11894565\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-\right.}{}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(
2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 +
16980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(
2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(84) = 168.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx &= \frac{125\,x^5}{4\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{2675\,x^4}{16\,(2\,x^2-x+3)^{\frac{3}{2}}} \\ &+ \frac{7495}{203136}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) \\ &+ \frac{7495}{256}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{532145}{101568}\,\sqrt{2\,x^2-x+3} - \frac{4515389\,x}{50784\,\sqrt{2\,x^2-x+3}} \\ &+ \frac{7197\,x^2}{8\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{396211}{50784\,\sqrt{2\,x^2-x+3}} - \frac{269783\,x}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{1002137}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495
/203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71
/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^
(3/2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568
*sqrt(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2
*x^2 - x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*
x^2 - x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = -\frac{7495}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{3\,((4\,(13225\,(20\,x+107)x-2451998)x+33848843)x-20821094)x+89784565}{101568\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
1/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 2082109
4)*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^3}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.90

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{25392000\sqrt{2x^2-x+3}\,x^5+135847200\sqrt{2x^2-x+3}\,x^4-235391808\sqrt{2x^2-x}}{}$$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x)
```

output

```
(25392000*sqrt(2*x**2 - x + 3)*x**5 + 135847200*sqrt(2*x**2 - x + 3)*x**4
- 235391808*sqrt(2*x**2 - x + 3)*x**3 + 812372232*sqrt(2*x**2 - x + 3)*x**
2 - 499706256*sqrt(2*x**2 - x + 3)*x + 718276520*sqrt(2*x**2 - x + 3) + 95
156520*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x*
*4 - 95156520*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(
23))*x**3 + 309258690*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x -
1)/sqrt(23))*x**2 - 142734780*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 4*x - 1)/sqrt(23))*x + 214102170*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqr
t(2) + 4*x - 1)/sqrt(23)) + 5253716*sqrt(2)*x**4 - 5253716*sqrt(2)*x**3 +
17074577*sqrt(2)*x**2 - 7880574*sqrt(2)*x + 11820861*sqrt(2))/(812544*(4*x
**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.129 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 68

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx=\frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}-\frac{25\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

output
```
121/276*(19-7*x)/(2*x^2-x+3)^(3/2)-11/6348*(7351+2336*x)/(2*x^2-x+3)^(1/2)
-25/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx=-\frac{11(8623+714x+6183x^2+2336x^3)}{3174\,(3-x+2x^2)^{3/2}}-\frac{25\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) -
(25*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{6900x^2+11730x+131}{16(2x^2-x+3)^{3/2}}dx+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{552}\int \frac{6900x^2+11730x+131}{(2x^2-x+3)^{3/2}}dx+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{552}\left(\frac{2}{23}\int \frac{39675}{\sqrt{2x^2-x+3}}dx-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{552}\left(3450\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{552}\left(75\sqrt{46}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{552}\left(1725\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) + ((-22*(7351 + 2336*x))/(23*
Sqrt[3 - x + 2*x^2]) + 1725*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/552
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{11(2336x^3+6183x^2+714x+8623)}{3174(2x^2-x+3)^{\frac{3}{2}}}+\frac{25\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}$
trager	$-\frac{11(2336x^3+6183x^2+714x+8623)}{3174(2x^2-x+3)^{\frac{3}{2}}}+\frac{25\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{8}$
default	$\frac{\frac{8493x}{1472}-\frac{8493}{5888}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{2267x}{529}-\frac{2267}{2116}}{\sqrt{2x^2-x+3}}-\frac{15775}{768(2x^2-x+3)^{\frac{3}{2}}}-\frac{319x}{64(2x^2-x+3)^{\frac{3}{2}}}-\frac{145x^2}{8(2x^2-x+3)^{\frac{3}{2}}}-\frac{25x^3}{6(2x^2-x+3)^{\frac{3}{2}}}-\frac{25x}{4\sqrt{2x^2-x+3}}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-11/3174*(2336*x^3+6183*x^2+714*x+8623)/(2*x^2-x+3)^(3/2)+25/8*2^(1/2)*arc
sinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{39675\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32\right.}{25392\,(4x^4-4x^3+13}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x
^2 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx=\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx=\frac{25}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}}-\right) +\frac{25}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{1775}{3174}\sqrt{2\,x^2-x+3}+\frac{1017\,x}{529\sqrt{2\,x^2-x+3}} -\frac{15\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{6413}{3174\sqrt{2\,x^2-x+3}}-\frac{x}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{2593}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71
/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^
(3/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*
x^2 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/
2) - 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/
138/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = -\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{11\left(((2336x+6183)x+714)x+8623\right)}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/3
174*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.01

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{-102784\sqrt{2x^2-x+3}\,x^3-272052\sqrt{2x^2-x+3}\,x^2-31416\sqrt{2x^2-x+3}\,x-3}{}$$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x)
```

output
```
( - 102784*sqrt(2*x**2 - x + 3)*x**3 - 272052*sqrt(2*x**2 - x + 3)*x**2 -
31416*sqrt(2*x**2 - x + 3)*x - 379412*sqrt(2*x**2 - x + 3) + 158700*sqrt(2
)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**4 - 158700*s
qrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**3 + 515
775*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2
- 238050*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*
x + 357075*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)
) + 70400*sqrt(2)*x**4 - 70400*sqrt(2)*x**3 + 228800*sqrt(2)*x**2 - 105600
*sqrt(2)*x + 158400*sqrt(2))/(12696*(4*x**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.130 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} \, dx = -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}}$$

output
```
1/69*(-55-33*x)/(2*x^2-x+3)^(3/2)-71/529*(1-4*x)/(2*x^2-x+3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} \, dx = \frac{2(-952+1005x-639x^2+852x^3)}{1587(3-x+2x^2)^{3/2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2191, 27, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{69}\int \frac{213}{4(2x^2-x+3)^{3/2}}dx-\frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{71}{46}\int \frac{1}{(2x^2-x+3)^{3/2}}dx-\frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 -
x + 2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
orering	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
default	$\frac{\frac{71x}{92}-\frac{71}{368}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{284x}{529}-\frac{71}{529}}{\sqrt{2x^2-x+3}}-\frac{29}{48(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x}{4(2x^2-x+3)^{\frac{3}{2}}}$	69

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(852\,x^3-639\,x^2+1005\,x-952)\sqrt{2\,x^2-x+3}}{1587\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x
^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{284\,x}{529\,\sqrt{2\,x^2-x+3}} - \frac{71}{529\,\sqrt{2\,x^2-x+3}} - \frac{11\,x}{23\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{55}{69\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^
2 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(3\,(71\,(4\,x-3)x+335)x-952)}{1587\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(852\,x^3-639\,x^2+1005\,x-952)}{1587\,(2\,x^2-x+3)^{3/2}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

output
```
(2*(1005*x - 639*x^2 + 852*x^3 - 952))/(1587*(2*x^2 - x + 3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.43

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{1704\sqrt{2x^2-x+3}\,x^3-1278\sqrt{2x^2-x+3}\,x^2+2010\sqrt{2x^2-x+3}\,x-1904\sqrt{2x}}{6348x^4-6348x^3+20631x^2 -}$$

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
(2*(852*sqrt(2*x**2 - x + 3)*x**3 - 639*sqrt(2*x**2 - x + 3)*x**2 + 1005*s
qrt(2*x**2 - x + 3)*x - 952*sqrt(2*x**2 - x + 3) + 68*sqrt(2)*x**4 - 68*sq
rt(2)*x**3 + 221*sqrt(2)*x**2 - 102*sqrt(2)*x + 153*sqrt(2)))/(1587*(4*x**
4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.131 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{759(3-x+2x^2)^{3/2}}+\frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+25000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(-15457+25000\sqrt{2}\right)}}\left(443-98\sqrt{2}+\left(247+345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{1}{484}\sqrt{\frac{1}{682}\left(15457+25000\sqrt{2}\right)}\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(15457+25000\sqrt{2}\right)}}\left(443+98\sqrt{2}+\left(247-345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
1/759*(13-6*x)/(2*x^2-x+3)^(3/2)+1/128018*(3603-658*x)/(2*x^2-x+3)^(1/2)+1
/330088*(-10541674+17050000*2^(1/2))^(1/2)*arctan(11^(1/2)/(-479167+775000
*2^(1/2))^(1/2)*(443-98*2^(1/2)+(247+345*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-1/
330088*(10541674+17050000*2^(1/2))^(1/2)*arctanh(11^(1/2)/(479167+775000*2
^(1/2))^(1/2)*(443+98*2^(1/2)+(247-345*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{39005-19767x+23592x^2-3948x^3}{384054(3-x+2x^2)^{3/2}} + \frac{1}{484}\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{249\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+108\right.$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) +
RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (249*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 108*Sqrt[2]*Log[-(Sqrt[2]*x)
 + Sqrt[3 - x + 2*x^2] - #1]*#1 - 65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/484
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}(5x^2+3x+2)}\,dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{13-6x}{759(2x^2-x+3)^{3/2}} - \frac{\int -\frac{33(-40x^2+91x+168)}{2(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{8349}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{506}\int \frac{-40x^2+91x+168}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx+\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 2135

$$\frac{1}{506}\left(\frac{\int \frac{5819(65x+54)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{506}\left(\frac{23}{22}\int \frac{65x+54}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 1368

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\int \frac{11\left(\left(119+65\sqrt{2}\right)x+54\sqrt{2}+11\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int \frac{11\left(\left(119-65\sqrt{2}\right)x-54\sqrt{2}+11\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\int \frac{\left(119+65\sqrt{2}\right)x+54\sqrt{2}+11}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(119-65\sqrt{2}\right)x-54\sqrt{2}+11}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 1362

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\left(15457+25000\sqrt{2}\right)\int \frac{1}{31\left(15457+25000\sqrt{2}\right)-\frac{11\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)^2}{2x^2-x+3}}d\left(-\frac{\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443}{\sqrt{2x^2-x+3}}\right)}{\sqrt{2}}-\frac{(15\cdots}{\cdots}\right.\right.$$

$$\frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$\downarrow$ 217

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\left(15457+25000\sqrt{2}\right)\int \frac{1}{31\left(15457+25000\sqrt{2}\right)-\frac{11\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)^2}{2x^2-x+3}}d\left(-\frac{\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443}{\sqrt{2x^2-x+3}}\right)}{\sqrt{2}} - \right.\right.$$

$$\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{506}\left(\frac{23}{22}\left(-\frac{\left(15457-25000\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\sqrt{2}\right)x-98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)}{\sqrt{682\left(25000\sqrt{2}-15457\right)}} - \sqrt{\frac{1}{682}\left(15457+25000\right.}\right.\right.$$

$$\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + ((3603 - 658*x)/(253*Sqrt[3 - x +
 2*x^2]) + (23*(-(((15457 - 25000*Sqrt[2])*ArcTan[(Sqrt[11/(31*(-15457 + 2
5000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/Sqrt[682*(-15457 + 25000*Sqrt[2])]) - Sqrt[(15457 + 25000*Sqrt[2])
/682]*ArcTanh[(Sqrt[11/(31*(15457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (
247 - 345*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]))/22)/506
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.58 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.37

method	result
trager	$-\frac{3948x^3-23592x^2+19767x-39005}{384054(2x^2-x+3)^{\frac{3}{2}}}+\frac{27\,\mathrm{RootOf}\left(494371927368\_Z^4-3842440173\_Z^2+39062500\right)\ln\left(-\frac{100943233115249606}{}\right.}{}$
default	$-\frac{329(4x-1)}{256036\sqrt{2x^2-x+3}}+\frac{13}{484\sqrt{2x^2-x+3}}-\frac{4x-1}{506(2x^2-x+3)^{\frac{3}{2}}}+\frac{1}{66(2x^2-x+3)^{\frac{3}{2}}}+\frac{\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-}}{}$

input

```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output

```
-1/384054*(3948*x^3-23592*x^2+19767*x-39005)/(2*x^2-x+3)^(3/2)+27/242*Root
Of(494371927368*_Z^4-3842440173*_Z^2+39062500)*ln(-(1009432331152496064*x*
RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^5+7463993424020496*Root
Of(494371927368*_Z^4-3842440173*_Z^2+39062500)^3*x-25206911474500800*RootO
f(494371927368*_Z^4-3842440173*_Z^2+39062500)^3+1629806162292000*RootOf(49
4371927368*_Z^4-3842440173*_Z^2+39062500)^2*(2*x^2-x+3)^(1/2)+103675333731
75*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)*x-46727026216200*Roo
tOf(494371927368*_Z^4-3842440173*_Z^2+39062500)-6994190312500*(2*x^2-x+3)^
(1/2))/(1988712*x*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2-162
95*x-11422))-1/330088*RootOf(_Z^2+1356301584*RootOf(494371927368*_Z^4-3842
440173*_Z^2+39062500)^2-10541674)*ln((584162228676213*x*RootOf(49437192736
8*_Z^4-3842440173*_Z^2+39062500)^4*RootOf(_Z^2+1356301584*RootOf(494371927
368*_Z^4-3842440173*_Z^2+39062500)^2-10541674)-13400087377743*RootOf(49437
1927368*_Z^4-3842440173*_Z^2+39062500)^2*RootOf(_Z^2+1356301584*RootOf(494
371927368*_Z^4-3842440173*_Z^2+39062500)^2-10541674)*x+14587333029225*Root
Of(494371927368*_Z^4-3842440173*_Z^2+39062500)^2*RootOf(_Z^2+1356301584*Ro
otOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2-10541674)+3473524383384
8250*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2*(2*x^2-x+3)^(1/2
)+74861069500*RootOf(_Z^2+1356301584*RootOf(494371927368*_Z^4-3842440173*_
Z^2+39062500)^2-10541674)*x-140419212500*RootOf(_Z^2+1356301584*RootOf(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(146) = 292$.

Time = 0.09 (sec) , antiderivative size = 605, normalized size of antiderivative = 3.04

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/3072432*(3174*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(12500/341*sqrt(2)
 - 15457/682)*arctan(-22/5711*(4*(1986*x^3 - 3256*x^2 - sqrt(2)*(115*x^3 -
1149*x^2 - 2768*x - 264) + 2256*x + 2592)*sqrt(2*x^2 - x + 3)*sqrt(12500/3
41*sqrt(2) - 15457/682) + (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*
x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(12500/341*sqrt(2) + 15457/682)*
sqrt(12500/341*sqrt(2) - 15457/682))/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x
 - 576)) - 3174*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(12500/341*sqrt(2)
 - 15457/682)*arctan(22/5711*(4*(1986*x^3 - 3256*x^2 - sqrt(2)*(115*x^3 - 1
149*x^2 - 2768*x - 264) + 2256*x + 2592)*sqrt(2*x^2 - x + 3)*sqrt(12500/34
1*sqrt(2) - 15457/682) - (171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x
^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(12500/341*sqrt(2) + 15457/682)*s
qrt(12500/341*sqrt(2) - 15457/682))/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x
 - 576)) + 1587*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(12500/341*sqrt(2) +
 15457/682)*log((279839*x^2 + 22*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2019*x - 19
25) - 94*x - 3944)*sqrt(12500/341*sqrt(2) + 15457/682) + 251284*sqrt(2)*(2
*x^2 - x + 3) - 862361*x + 1142200)/x^2) - 1587*(4*x^4 - 4*x^3 + 13*x^2 -
6*x + 9)*sqrt(12500/341*sqrt(2) + 15457/682)*log((279839*x^2 - 22*sqrt(2*x
^2 - x + 3)*(sqrt(2)*(2019*x - 1925) - 94*x - 3944)*sqrt(12500/341*sqrt(2)
 + 15457/682) + 251284*sqrt(2)*(2*x^2 - x + 3) - 862361*x + 1142200)/x^2)
 - 8*(3948*x^3 - 23592*x^2 + 19767*x - 39005)*sqrt(2*x^2 - x + 3))/(4*x^...
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)(2\,x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{\sqrt{2x^2-x+3}}{40x^8-36x^7+190x^6-83x^5+288x^4-20x^3+180x^2+27x+}$$

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(40*x**8 - 36*x**7 + 190*x**6 - 83*x**5 + 288*x**
4 - 20*x**3 + 180*x**2 + 27*x + 54),x)
```

3.132 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = &-\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} \\ &-\frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}}+\frac{4+65x}{682(3-x+2x^2)^{3/2}(2+3x+5x^2)} \\ &+\frac{625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(203+242\sqrt{2}+\left(687+445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{660176} \\ &-\frac{625\sqrt{\frac{1}{682}\left(-30463+23600\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-30463+23600\sqrt{2}\right)}}\left(203-242\sqrt{2}+\left(687-445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{660176}\end{aligned}$$

output
```
-1/1035276*(15101-8654*x)/(2*x^2-x+3)^(3/2)-1/523849656*(3133427+1352542*x
)/(2*x^2-x+3)^(1/2)+1/682*(4+65*x)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)+625/450
240032*(20775766+16095200*2^(1/2))^(1/2)*arctan(11^(1/2)/(944353+731600*2^
(1/2))^(1/2)*(203+242*2^(1/2)+(687+445*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-625/
450240032*(-20775766+16095200*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-944353+731
600*2^(1/2))^(1/2)*(203-242*2^(1/2)+(687-445*2^(1/2))*x)/(2*x^2-x+3)^(1/2)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.62 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.78

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \frac{-31010342+5712309x-84671384x^2-2879479x^3-32686812}{523849656\,(3-x+2x^2)^{3/2}\,(2+3x+5x^2)}$$
$$+\frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{-1376\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+106\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3}\right.}{-13\sqrt{2}+17\#1+9}\right.}{5324}$$
$$+\frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{126249\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+58712\log\left(-\sqrt{2}x+\right.}{-13\sqrt{2}+17\#}\right.}{660176\sqrt{2}}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(-31010342 + 5712309*x - 84671384*x^2 - 2879479*x^3 - 32686812*x^4 - 13525
420*x^5)/(523849656*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + RootSum[-56
 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-1376*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 106*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1 + 95*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/5324 + RootSum[-
56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (126249*Sqrt[2]
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 58712*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1]*#1 + 10095*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
+ 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/
(660176*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)} - \frac{\int -\frac{11(1040x^2-401x+316)}{2(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1040x^2-401x+316}{(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{1364} + \frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\int \frac{11\left(173080x^2-284277x+155482\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{8349} - \frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364} + \frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{173080x^2-284277x+155482}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{1518} - \frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364} + \frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\frac{\int \frac{10910625(34-35x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783} - \frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518} - \frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364} + \frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{43125}{22}\int \frac{34-35x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx - \frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518} - \frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364} + \frac{65x+4}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}$$

$$\Big\downarrow \ 1368$$

$$\frac{\frac{\frac{43125}{22}\left(\frac{\int -\frac{11\left(\left(1+35\sqrt{2}\right)x-34\sqrt{2}+69\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(1-35\sqrt{2}\right)x+34\sqrt{2}+69\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{43125}{22}\left(\frac{\int \frac{\left(1-35\sqrt{2}\right)x+34\sqrt{2}+69}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(1+35\sqrt{2}\right)x-34\sqrt{2}+69}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 1362

$$\frac{\frac{\frac{43125}{22}\left(\frac{\left(30463-23600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)^2}{2x^2-x+3}-31\left(30463-23600\sqrt{2}\right)}d\frac{\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(30463+23600\sqrt{2}\right)\int \frac{}{-\frac{11\left(\left(687+445\cdots\right.\right.}{\cdots}}\cdots}{\cdots}\right.\cdots}{1518}\cdots}{1364}\cdots$$

$$\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 217

$$\frac{\frac{\frac{43125}{22}\left(\frac{\left(30463-23600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)^2}{2x^2-x+3}-31\left(30463-23600\sqrt{2}\right)}d\frac{\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\cdots\right.}{1518}\cdots}{1364}\cdots$$

$$\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\frac{43125}{22}\left(\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(\left(687+445\sqrt{2}\right)x+242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(30463-23600\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(23600\sqrt{2}-3046\right.}}}{\sqrt{682\left(23600\sqrt{2}-304\right.}}\right.}{}\right.}{1518}}{1364}$$

$$\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (-1/759*(15101
- 8654*x)/(3 - x + 2*x^2)^(3/2) + (-1/253*(3133427 + 1352542*x)/Sqrt[3 - x
 + 2*x^2] + (43125*(Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*
(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]] + ((30463 - 23600*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-30463
+ 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]])/Sqrt[682*(-30463 + 23600*Sqrt[2])]))/22)/1518)/1364
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.97 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.11

method	result
trager	$-\frac{13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342}{523849656(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)} - \frac{16875\,\mathrm{RootOf}\left(1977487709472\_Z^4+75727667\ldots\right.}{}$
risch	$-\frac{13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342}{523849656(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)} + \frac{625\sqrt{\frac{8\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(-1+\sqrt{2}+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{\ldots}}}{}$
default	Expression too large to display

input

```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/523849656*(13525420*x^5+32686812*x^4+2879479*x^3+84671384*x^2-5712309*x
+31010342)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)-16875/165044*RootOf(19774877094
72*_Z^4+7572766707*_Z^2+8702500)*ln(-(5226452556429468672*x*RootOf(1977487
709472*_Z^4+7572766707*_Z^2+8702500)^5+30671894532413376*RootOf(1977487709
472*_Z^4+7572766707*_Z^2+8702500)^3*x+13991557820486400*RootOf(19774877094
72*_Z^4+7572766707*_Z^2+8702500)^3-1319515725838464*RootOf(1977487709472*_
Z^4+7572766707*_Z^2+8702500)^2*(2*x^2-x+3)^(1/2)+23459476566825*RootOf(197
7487709472*_Z^4+7572766707*_Z^2+8702500)*x+69460188136200*RootOf(197748770
9472*_Z^4+7572766707*_Z^2+8702500)-2583860307500*(2*x^2-x+3)^(1/2))/(79548
48*x*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+18905*x+4898))+6
25/450240032*RootOf(_Z^2+5425206336*RootOf(1977487709472*_Z^4+7572766707*_
Z^2+8702500)^2+20775766)*ln((756141862909356*RootOf(_Z^2+5425206336*RootOf
(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+20775766)*RootOf(1977487709
472*_Z^4+7572766707*_Z^2+8702500)^4*x+1353788597799*RootOf(1977487709472*_
Z^4+7572766707*_Z^2+8702500)^2*RootOf(_Z^2+5425206336*RootOf(1977487709472
*_Z^4+7572766707*_Z^2+8702500)^2+20775766)*x-2024241582825*RootOf(19774877
09472*_Z^4+7572766707*_Z^2+8702500)^2*RootOf(_Z^2+5425206336*RootOf(197748
7709472*_Z^4+7572766707*_Z^2+8702500)^2+20775766)-14061089453466132*RootOf
(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2*(2*x^2-x+3)^(1/2)-251046817
2*RootOf(_Z^2+5425206336*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(177) = 354$.

Time = 0.10 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.85

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/4190797248*(1983750*sqrt(1/682)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2
 + 15*x + 18)*sqrt(23600*sqrt(2) + 30463)*arctan(-22/2449*sqrt(1/682)*(sqr
t(1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x
^2 + 12*x) - 3936*x)*sqrt(23600*sqrt(2) - 30463) + 4*(1706*x^3 - 4176*x^2
- sqrt(2)*(1515*x^3 - 3229*x^2 - 528*x + 1656) - 2224*x + 1632)*sqrt(2*x^2
 - x + 3))*sqrt(23600*sqrt(2) + 30463)/(343*x^4 - 400*x^3 + 1136*x^2 + 384
*x - 576)) - 1983750*sqrt(1/682)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 +
 15*x + 18)*sqrt(23600*sqrt(2) + 30463)*arctan(-22/2449*sqrt(1/682)*(sqrt(
1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2
 + 12*x) - 3936*x)*sqrt(23600*sqrt(2) - 30463) - 4*(1706*x^3 - 4176*x^2 -
sqrt(2)*(1515*x^3 - 3229*x^2 - 528*x + 1656) - 2224*x + 1632)*sqrt(2*x^2 -
 x + 3))*sqrt(23600*sqrt(2) + 30463)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x
 - 576)) + 991875*sqrt(1/682)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15
*x + 18)*sqrt(23600*sqrt(2) - 30463)*log(625*(22*sqrt(1/682)*sqrt(2*x^2 -
x + 3)*(sqrt(2)*(1499*x - 4325) + 2826*x - 5824)*sqrt(23600*sqrt(2) - 3046
3) + 120001*x^2 + 107756*sqrt(2)*(2*x^2 - x + 3) - 369799*x + 489800)/x^2)
 - 991875*sqrt(1/682)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
*sqrt(23600*sqrt(2) - 30463)*log(-625*(22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(1499*x - 4325) + 2826*x - 5824)*sqrt(23600*sqrt(2) - 30463) - 12
0001*x^2 - 107756*sqrt(2)*(2*x^2 - x + 3) + 369799*x - 489800)/x^2) + 8...
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5x^2+3x+2)^2(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{200x^{10}-60x^9+922x^8+83x^7+1571x^6+598x^5+1416x^4+$$

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(200*x**10 - 60*x**9 + 922*x**8 + 83*x**7 + 1571*
x**6 + 598*x**5 + 1416*x**4 + 635*x**3 + 711*x**2 + 216*x + 108),x)
```

3.133 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 269

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3} dx = -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{4+65x}{1364(3-x+2x^2)^{3/2}(2+3x+5x^2)^2} + \frac{46386+86885x}{1860496(3-x+2x^2)^{3/2}(2+3x+5x^2)} + \frac{35\sqrt{\frac{1}{682}(2243059557247+2011748500000\sqrt{2})}\arctan\left(\frac{\sqrt{\frac{11}{31(2243059557247+2011748500000\sqrt{2})}}\left(1432939+2428746\sqrt{2}+(6\right.}{\sqrt{3-x+2x^2}}\right.}{1800960128} - \frac{35\sqrt{\frac{1}{682}(-2243059557247+2011748500000\sqrt{2})}\text{arctanh}\left(\frac{\sqrt{\frac{11}{31(-2243059557247+2011748500000\sqrt{2})}}\left(1432939-2428746\sqrt{\right.}{\sqrt{3-x+2x^2}}\right.}{1800960128}$$

output
```
-1/2824232928*(12280939-19536786*x)/(2*x^2-x+3)^(3/2)-1/476353953856*(1134
826571-1504660754*x)/(2*x^2-x+3)^(1/2)+1/1364*(4+65*x)/(2*x^2-x+3)^(3/2)/(
5*x^2+3*x+2)^2+1/1860496*(46386+86885*x)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)+3
5/1228254807296*(1529766618042454+1372012477000000*2^(1/2))^(1/2)*arctan(1
1^(1/2)/(69534846274657+62364203500000*2^(1/2))^(1/2)*(1432939+2428746*2^(
1/2)+(6290431+3861685*2^(1/2))*x)/(2*x^2-x+3)^(1/2))-35/1228254807296*(-15
29766618042454+1372012477000000*2^(1/2))^(1/2)*arctanh(11^(1/2)/(-69534846
274657+62364203500000*2^(1/2))^(1/2)*(1432939-2428746*2^(1/2)+(6290431-386
1685*2^(1/2))*x)/(2*x^2-x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.88 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.25

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)^3}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}\,(9739335532+218659985088x+178650961091x^2+519223213785x^3+174241}{(6+7x+16x^2+x^3+10x^4)}}{}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(9739335532 + 218659985088*x + 178650961091*x^2 +
519223213785*x^3 + 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 +
 225699113100*x^7))/(6 + 7*x + 16*x^2 + x^3 + 10*x^4)^2 - 2976*RootSum[-56
 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-26154346*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 37230166*Sqrt[2]*Log[-(Sqrt[2]*x)
 + Sqrt[3 - x + 2*x^2] - #1]*#1 - 1193705*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] -
24401712*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 &
 , (-3647*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 3172*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 485*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ] + 15*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
- 5*#1^4 & , (-9138129081*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1] + 16445754136*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 10044
12885*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt
[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5716247446272
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {1305, 27, 2135, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{1364(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}-\frac{\int -\frac{11(1560x^2-785x+1034)}{2(2x^2-x+3)^{5/2}(5x^2+3x+2)^2}dx}{15004}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1560x^2-785x+1034}{(2x^2-x+3)^{5/2}(5x^2+3x+2)^2}dx}{2728}+\frac{65x+4}{1364(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{11(1390160x^2+284771x+462194)}{2(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{7502}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{1390160x^2+284771x+462194}{(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\frac{\int \frac{33(130245240x^2+4179719x+60094966)}{2(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{8349}-\frac{12280939-19536786x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{506}\int \frac{130245240x^2+4179719x+60094966}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 2135

$$\frac{\frac{\frac{1}{506}\left(\frac{\int \frac{203665(263242-409755x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783}-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\int \frac{263242-409755x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 1368

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{\int -\frac{11\left(\left(146513+409755\sqrt{2}\right)x-263242\sqrt{2}+672997\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(146513-409755\sqrt{2}\right)x+263242\sqrt{2}+672997\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-1}{1364}}{2728}$$

$$\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{\int \frac{(146513-409755\sqrt{2})x+263242\sqrt{2}+672997}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{(146513+409755\sqrt{2})x-263242\sqrt{2}+672997}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx}{2\sqrt{2}}\right)-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-195\cdots}{759(2x^2-x+\cdots}}{1364}}{2728}$$

$$\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \; 1362$$

$$\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{(2243059557247-2011748500000\sqrt{2})\int \frac{1}{-\frac{11((6290431-3861685\sqrt{2})x-2428746\sqrt{2}+1432939)^2}{2x^2-x+3}-31(2243059557247-2011748500000\sqrt{2})}\,d\frac{(6290431\cdots}{\cdots}}{\sqrt{2}}\cdots\right)\cdots\right)\cdots}{\cdots}$$

$$\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{(2243059557247-2011748500000\sqrt{2})\int \frac{1}{-\frac{11((6290431-3861685\sqrt{2})x-2428746\sqrt{2}+1432939)^2}{2x^2-x+3}-31(2243059557247-2011748500000\sqrt{2})}\,d\frac{(6290431\cdots}{\cdots}}{\sqrt{2}}\cdots\right)\cdots\right)\cdots}{\cdots}$$

$$\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{506}\left(\frac{805}{22}\left(\sqrt{\frac{1}{682}(2243059557247+2011748500000\sqrt{2})}\arctan\left(\frac{\sqrt{\frac{11}{31(2243059557247+2011748500000\sqrt{2})}}\,((6290431+3861685\sqrt{2})x+2428746\sqrt{2}+14329\cdots}{\sqrt{2x^2-x+3}}\cdots\right)\cdots\right)\cdots\right)\cdots}{\cdots}$$

$$\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}\,(5x^2+3x+2)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(4 + 65*x)/(1364*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + ((46386 + 86
885*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (-1/759*(12280939 -
 19536786*x)/(3 - x + 2*x^2)^(3/2) + (-1/253*(1134826571 - 1504660754*x)/S
qrt[3 - x + 2*x^2] + (805*(Sqrt[(2243059557247 + 2011748500000*Sqrt[2])/68
2]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*(1432939
+ 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] +
 ((2243059557247 - 2011748500000*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-224305955
7247 + 2011748500000*Sqrt[2]))]*(1432939 - 2428746*Sqrt[2] + (6290431 - 38
61685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-2243059557247 + 201174
8500000*Sqrt[2])]))/22)/506)/1364)/2728
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.10 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.90

method	result
trager	Expression too large to display
risch	$\frac{225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+973}{1429061861568(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)^2}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1429061861568*(225699113100*x^7-12234606480*x^6+592923725931*x^5+1742416
14961*x^4+519223213785*x^3+178650961091*x^2+218659985088*x+9739335532)/(10
*x^4+x^3+16*x^2+7*x+6)^2*(2*x^2-x+3)^(1/2)-35/1228254807296*RootOf(_Z^2+21
700825344*RootOf(7909950837888*_Z^4+557599932276474483*_Z^2+15809109481454
101562500)^2+1529766618042454)*ln(-(3609214479402775056*RootOf(_Z^2+217008
25344*RootOf(7909950837888*_Z^4+557599932276474483*_Z^2+158091094814541015
62500)^2+1529766618042454)*RootOf(7909950837888*_Z^4+557599932276474483*_Z
^2+15809109481454101562500)^4*x+13326758501298020522162 1*RootOf(7909950837
888*_Z^4+557599932276474483*_Z^2+15809109481454101562500)^2*RootOf(_Z^2+21
700825344*RootOf(7909950837888*_Z^4+557599932276474483*_Z^2+15809109481454
101562500)^2+1529766618042454)*x+1398582662884576755415558690 5000*(2*x^2-x
+3)^(1/2)*RootOf(7909950837888*_Z^4+557599932276474483*_Z^2+15809109481454
101562500)^2-310035860689884712026075*RootOf(7909950837888*_Z^4+5575999322
76474483*_Z^2+15809109481454101562500)^2*RootOf(_Z^2+21700825344*RootOf(79
09950837888*_Z^4+557599932276474483*_Z^2+15809109481454101562500)^2+152976
6618042454)-751077958109653657562967 2500*RootOf(_Z^2+21700825344*RootOf(79
09950837888*_Z^4+557599932276474483*_Z^2+15809109481454101562500)^2+152976
6618042454)*x+473458193348490384034058466836980000*(2*x^2-x+3)^(1/2)+95336
58115508780780226687500*RootOf(_Z^2+21700825344*RootOf(7909950837888*_Z^4+
557599932276474483*_Z^2+15809109481454101562500)^2+1529766618042454))/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(208) = 416$.

Time = 0.10 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.74

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
-1/11432494892544*(111090*sqrt(1/682)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2011748500000*sqrt(2) +
2243059557247)*arctan(-22/314332260881*sqrt(1/682)*(sqrt(1/682)*(171*x^4 +
 1212*x^3 - 1640*x^2 - 176*sqrt(2)*(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x
)*sqrt(2011748500000*sqrt(2) - 2243059557247) + 4*(14041178*x^3 - 36217888
*x^2 - sqrt(2)*(14366395*x^3 - 29579677*x^2 - 1867664*x + 16151928) - 2388
0112*x + 12635616)*sqrt(2*x^2 - x + 3))*sqrt(2011748500000*sqrt(2) + 22430
59557247)/(343*x^4 - 400*x^3 + 1136*x^2 + 384*x - 576)) - 111090*sqrt(1/68
2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 8
4*x + 36)*sqrt(2011748500000*sqrt(2) + 2243059557247)*arctan(-22/314332260
881*sqrt(1/682)*(sqrt(1/682)*(171*x^4 + 1212*x^3 - 1640*x^2 - 176*sqrt(2)*
(6*x^4 + 5*x^3 + 5*x^2 + 12*x) - 3936*x)*sqrt(2011748500000*sqrt(2) - 2243
059557247) - 4*(14041178*x^3 - 36217888*x^2 - sqrt(2)*(14366395*x^3 - 2957
9677*x^2 - 1867664*x + 16151928) - 23880112*x + 12635616)*sqrt(2*x^2 - x +
 3))*sqrt(2011748500000*sqrt(2) + 2243059557247)/(343*x^4 - 400*x^3 + 1136
*x^2 + 384*x - 576)) + 55545*sqrt(1/682)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2011748500000*sqrt(2)
 - 2243059557247)*log(35*(22*sqrt(1/682)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(120
22187*x - 39175525) + 27153338*x - 51197712)*sqrt(2011748500000*sqrt(2) -
2243059557247) + 15402280783169*x^2 + 13830619478764*sqrt(2)*(2*x^2 - x...
```

Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}\,(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3(2\,x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{\sqrt{2x^2 -}}{1000x^{12}+300x^{11}+4830x^{10}+3061x^9+9948x^8+7869x^7+1}$$

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

output
```
int(sqrt(2*x**2 - x + 3)/(1000*x**12 + 300*x**11 + 4830*x**10 + 3061*x**9
+ 9948*x**8 + 7869*x**7 + 12016*x**6 + 8619*x**5 + 8292*x**4 + 4483*x**3 +
 2610*x**2 + 756*x + 216),x)
```

3.134 $\int (4+x-2x^2)^3 \sqrt{2+3x+5x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\begin{aligned}\int (4+x-2x^2)^3 \sqrt{2+3x+5x^2}\,dx &= \frac{364734979(3+10x)\sqrt{2+3x+5x^2}}{160000000} \\ &+ \frac{461470657(2+3x+5x^2)^{3/2}}{84000000} - \frac{44194603x(2+3x+5x^2)^{3/2}}{14000000} \\ &- \frac{938857x^2(2+3x+5x^2)^{3/2}}{350000} + \frac{25553x^3(2+3x+5x^2)^{3/2}}{21000} \\ &+ \frac{159}{350}x^4(2+3x+5x^2)^{3/2} - \frac{1}{5}x^5(2+3x+5x^2)^{3/2} + \frac{11306784349\,\text{arcsinh}\left(\frac{3+10x}{\sqrt{31}}\right)}{320000000\sqrt{5}}\end{aligned}$$

output
```
364734979/160000000*(3+10*x)*(5*x^2+3*x+2)^(1/2)+461470657/84000000*(5*x^2
+3*x+2)^(3/2)-44194603/14000000*x*(5*x^2+3*x+2)^(3/2)-938857/350000*x^2*(5
*x^2+3*x+2)^(3/2)+25553/21000*x^3*(5*x^2+3*x+2)^(3/2)+159/350*x^4*(5*x^2+3
*x+2)^(3/2)-1/5*x^5*(5*x^2+3*x+2)^(3/2)+11306784349/1600000000*arcsinh(1/3
1*(3+10*x)*31^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \left(4+x-2x^2\right)^3 \sqrt{2+3x+5x^2}\, dx$$
$$= \frac{\sqrt{2+3x+5x^2}(59895956237+110757414990x+42447962840x^2-71895645200x^3-29746896000x^4+}{3360000000}$$
$$- \frac{11306784349 \log \left(-3-10x+2\sqrt{5}\sqrt{2+3x+5x^2}\right)}{320000000\sqrt{5}}$$

input
```
Integrate[(4 + x - 2*x^2)^3*Sqrt[2 + 3*x + 5*x^2],x]
```

output
```
(Sqrt[2 + 3*x + 5*x^2]*(59895956237 + 110757414990*x + 42447962840*x^2 - 7
1895645200*x^3 - 29746896000*x^4 + 23677600000*x^5 + 5616000000*x^6 - 3360
000000*x^7))/3360000000 - (11306784349*Log[-3 - 10*x + 2*Sqrt[5]*Sqrt[2 +
3*x + 5*x^2]])/(320000000*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2+x+4\right)^3 \sqrt{5x^2+3x+2}\, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{40} \int 4\sqrt{5x^2+3x+2}\left(159x^5+440x^4-470x^3-840x^2+480x+640\right) dx - \frac{1}{5}x^5 \left(5x^2+3x+2\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}\int \sqrt{5x^2+3x+2}\left(159x^5+440x^4-470x^3-840x^2+480x+640\right)dx-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 2192

$$\frac{1}{10}\left(\frac{1}{35}\int \frac{1}{2}\sqrt{5x^2+3x+2}\left(25553x^4-35444x^3-58800x^2+33600x+44800\right)dx+\frac{159}{35}\left(5x^2+3x+2\right)^{3/2}x^4\right)-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{70}\int \sqrt{5x^2+3x+2}\left(25553x^4-35444x^3-58800x^2+33600x+44800\right)dx+\frac{159}{35}\left(5x^2+3x+2\right)^{3/2}x^4\right)-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 2192

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{30}\int \frac{3}{2}\sqrt{5x^2+3x+2}\left(-938857x^3-1278212x^2+672000x+896000\right)dx+\frac{25553}{30}\left(5x^2+3x+2\right)^{3/2}x\right.\right.$$
$$-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\int \sqrt{5x^2+3x+2}\left(-938857x^3-1278212x^2+672000x+896000\right)dx+\frac{25553}{30}\left(5x^2+3x+2\right)^{3/2}x^3\right.\right.$$
$$-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 2192

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{25}\int \frac{1}{2}\left(-44194603x^2+41110856x+44800000\right)\sqrt{5x^2+3x+2}dx-\frac{938857}{25}x^2\left(5x^2+3x+2\right)^{3/2}\right.\right.\right.$$
$$-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\int \left(-44194603x^2+41110856x+44800000\right)\sqrt{5x^2+3x+2}dx-\frac{938857}{25}x^2\left(5x^2+3x+2\right)^{3/2}\right)\right.\right.$$
$$-\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{2192}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{20}\int \frac{1}{2}(2307353285x+1968778412)\sqrt{5x^2+3x+2}dx-\frac{44194603}{20}x\left(5x^2+3x+2\right)^{3/2}\right)-\frac{93\ldots}{}\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{40}\int (2307353285x+1968778412)\sqrt{5x^2+3x+2}dx-\frac{44194603}{20}x\left(5x^2+3x+2\right)^{3/2}\right)-\frac{938\ldots}{2\ldots}\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{1160}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{40}\left(\frac{2553144853}{2}\int \sqrt{5x^2+3x+2}dx+\frac{461470657}{3}\left(5x^2+3x+2\right)^{3/2}\right)-\frac{44194603}{20}x\left(5x^2+3\ldots\right.\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{1087}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{40}\left(\frac{2553144853}{2}\left(\frac{31}{40}\int \frac{1}{\sqrt{5x^2+3x+2}}dx+\frac{1}{20}\sqrt{5x^2+3x+2}(10x+3)\right)+\frac{461470657}{3}\left(5x^2\ldots\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{1090}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{40}\left(\frac{2553144853}{2}\left(\frac{1}{40}\sqrt{\frac{31}{5}}\int \frac{1}{\sqrt{\frac{1}{31}(10x+3)^2+1}}d(10x+3)+\frac{1}{20}\sqrt{5x^2+3x+2}(10x+3)\right)\ldots\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

$$\big\downarrow \text{222}$$

$$\frac{1}{10}\left(\frac{1}{70}\left(\frac{1}{20}\left(\frac{1}{50}\left(\frac{1}{40}\left(\frac{2553144853}{2}\left(\frac{31\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)}{40\sqrt{5}}+\frac{1}{20}\sqrt{5x^2+3x+2}(10x+3)\right)+\frac{461470657}{3}\left(5x^2+\ldots\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{5}x^5\left(5x^2+3x+2\right)^{3/2}$$

input
```
Int[(4 + x - 2*x^2)^3*Sqrt[2 + 3*x + 5*x^2],x]
```

output
```
-1/5*(x^5*(2 + 3*x + 5*x^2)^(3/2)) + ((159*x^4*(2 + 3*x + 5*x^2)^(3/2))/35
 + ((25553*x^3*(2 + 3*x + 5*x^2)^(3/2))/30 + ((-938857*x^2*(2 + 3*x + 5*x^
2)^(3/2))/25 + ((-44194603*x*(2 + 3*x + 5*x^2)^(3/2))/20 + ((461470657*(2
+ 3*x + 5*x^2)^(3/2))/3 + (2553144853*(((3 + 10*x)*Sqrt[2 + 3*x + 5*x^2])/
20 + (31*ArcSinh[(3 + 10*x)/Sqrt[31]])/(40*Sqrt[5])))/2)/40)/50)/20)/70)/1
0
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{(3360000000x^7-5616000000x^6-23677600000x^5+29746896000x^4+71895645200x^3-42447962840x^2-110757414990x-5989595}{3360000000}$
trager	$\left(-x^7+\frac{117}{70}x^6+\frac{29597}{4200}x^5-\frac{619727}{70000}x^4-\frac{179739113}{8400000}x^3+\frac{1061199071}{84000000}x^2+\frac{3691913833}{112000000}x+\frac{59895956237}{3360000000}\right)\sqrt{5x^2}$
default	$\frac{364734979(10x+3)\sqrt{5x^2+3x+2}}{160000000}+\frac{11306784349\sqrt{5}\,\operatorname{arcsinh}\left(\frac{10\sqrt{31}\left(x+\frac{3}{10}\right)}{31}\right)}{1600000000}+\frac{461470657(5x^2+3x+2)^{\frac{3}{2}}}{84000000}-\frac{44194603x(5x^2}{140000}$

input
```
int((-2*x^2+x+4)^3*(5*x^2+3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3360000000*(3360000000*x^7-5616000000*x^6-23677600000*x^5+29746896000*x
^4+71895645200*x^3-42447962840*x^2-110757414990*x-59895956237)*(5*x^2+3*x+
2)^(1/2)+11306784349/1600000000*5^(1/2)*arcsinh(10/31*31^(1/2)*(x+3/10))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int \left(4+x-2x^2\right)^3\sqrt{2+3x+5x^2}\,dx =$$
$$-\frac{1}{3360000000}\left(3360000000\,x^7-5616000000\,x^6-23677600000\,x^5+29746896000\,x^4+71895645200\,x^3\right.$$
$$+\frac{11306784349}{3200000000}\sqrt{5}\log\left(-4\sqrt{5}\sqrt{5\,x^2+3\,x+2}(10\,x+3)-200\,x^2-120\,x-49\right)$$

input
```
integrate((-2*x^2+x+4)^3*(5*x^2+3*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3360000000*(3360000000*x^7 - 5616000000*x^6 - 23677600000*x^5 + 2974689
6000*x^4 + 71895645200*x^3 - 42447962840*x^2 - 110757414990*x - 5989595623
7)*sqrt(5*x^2 + 3*x + 2) + 11306784349/3200000000*sqrt(5)*log(-4*sqrt(5)*s
qrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int (4+x-2x^2)^3\sqrt{2+3x+5x^2}\,dx = \sqrt{5x^2+3x+2}\left(-x^7+\frac{117x^6}{70}+\frac{29597x^5}{4200}-\frac{619727x^4}{70000}-\frac{179739113x^3}{8400000}+\frac{1061199071x^2}{84000000}+\frac{3691913833x}{112000000}+\frac{59895956237}{3360000000}\right)+\frac{11306784349\sqrt{5}\operatorname{asinh}\left(\frac{10\sqrt{31}\left(x+\frac{3}{10}\right)}{31}\right)}{1600000000}$$

input
```
integrate((-2*x**2+x+4)**3*(5*x**2+3*x+2)**(1/2),x)
```

output
```
sqrt(5*x**2 + 3*x + 2)*(-x**7 + 117*x**6/70 + 29597*x**5/4200 - 619727*x**
4/70000 - 179739113*x**3/8400000 + 1061199071*x**2/84000000 + 3691913833*x
/112000000 + 59895956237/3360000000) + 11306784349*sqrt(5)*asinh(10*sqrt(3
1)*(x + 3/10)/31)/1600000000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int (4+x-2x^2)^3\sqrt{2+3x+5x^2}\,dx = -\frac{1}{5}\left(5x^2+3x+2\right)^{\frac{3}{2}}x^5+\frac{159}{350}\left(5x^2+3x+2\right)^{\frac{3}{2}}x^4 + \frac{25553}{21000}\left(5x^2+3x+2\right)^{\frac{3}{2}}x^3 - \frac{938857}{350000}\left(5x^2+3x+2\right)^{\frac{3}{2}}x^2 - \frac{44194603}{14000000}\left(5x^2+3x+2\right)^{\frac{3}{2}}x + \frac{461470657}{84000000}\left(5x^2+3x+2\right)^{\frac{3}{2}} + \frac{364734979}{16000000}\sqrt{5x^2+3x+2}x + \frac{11306784349}{1600000000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1094204937}{160000000}\sqrt{5x^2+3x+2}$$

input
```
integrate((-2*x^2+x+4)^3*(5*x^2+3*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*(5*x^2 + 3*x + 2)^(3/2)*x^5 + 159/350*(5*x^2 + 3*x + 2)^(3/2)*x^4 + 2
5553/21000*(5*x^2 + 3*x + 2)^(3/2)*x^3 - 938857/350000*(5*x^2 + 3*x + 2)^(
3/2)*x^2 - 44194603/14000000*(5*x^2 + 3*x + 2)^(3/2)*x + 461470657/8400000
0*(5*x^2 + 3*x + 2)^(3/2) + 364734979/16000000*sqrt(5*x^2 + 3*x + 2)*x + 1
1306784349/1600000000*sqrt(5)*arcsinh(1/31*sqrt(31)*(10*x + 3)) + 10942049
37/160000000*sqrt(5*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50

$$\int (4+x-2x^2)^3\sqrt{2+3x+5x^2}\,dx = -\frac{1}{3360000000}(10(4(10(40(50(60(70x-117)x-29597)x+1859181)x+179739113)x-106119907 - \frac{11306784349}{1600000000}\sqrt{5}\log\left(-2\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)-3\right)$$

input
```
integrate((-2*x^2+x+4)^3*(5*x^2+3*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/3360000000*(10*(4*(10*(40*(50*(60*(70*x - 117)*x - 29597)*x + 1859181)*
x + 179739113)*x - 1061199071)*x - 11075741499)*x - 59895956237)*sqrt(5*x^
2 + 3*x + 2) - 11306784349/1600000000*sqrt(5)*log(-2*sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 3*x + 2)) - 3)
```

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int (4+x-2x^2)^3\sqrt{2+3x+5x^2}\,dx = \frac{25553\,x^3\,{\left(5\,x^2+3\,x+2\right)}^{3/2}}{21000}-\frac{938857\,x^2\,{\left(5\,x^2+3\,x+2\right)}^{3/2}}{350000}+\frac{159\,x^4\,{\left(5\,x^2+3\,x+2\right)}^{3/2}}{350}-\frac{x^5\,{\left(5\,x^2+3\,x+2\right)}^{3/2}}{5}+\frac{15258032693\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+3\,x+2}+\frac{\sqrt{5}\,\left(5\,x+\frac{3}{2}\right)}{5}\right)}{1400000000}+\frac{492194603\,\left(\frac{x}{2}+\frac{3}{20}\right)\,\sqrt{5\,x^2+3\,x+2}}{7000000}+\frac{461470657\,\sqrt{5\,x^2+3\,x+2}\,\left(200\,x^2+30\,x+53\right)}{3360000000}-\frac{44194603\,x\,{\left(5\,x^2+3\,x+2\right)}^{3/2}}{14000000}-\frac{42916771101\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+3\,x+2}+\frac{\sqrt{5}\,\left(10\,x+3\right)}{5}\right)}{11200000000}$$

input
```
int((x - 2*x^2 + 4)^3*(3*x + 5*x^2 + 2)^(1/2),x)
```

output
```
(25553*x^3*(3*x + 5*x^2 + 2)^(3/2))/21000 - (938857*x^2*(3*x + 5*x^2 + 2)^
(3/2))/350000 + (159*x^4*(3*x + 5*x^2 + 2)^(3/2))/350 - (x^5*(3*x + 5*x^2
+ 2)^(3/2))/5 + (15258032693*5^(1/2)*log((3*x + 5*x^2 + 2)^(1/2) + (5^(1/2
)*(5*x + 3/2))/5))/1400000000 + (492194603*(x/2 + 3/20)*(3*x + 5*x^2 + 2)^
(1/2))/7000000 + (461470657*(3*x + 5*x^2 + 2)^(1/2)*(30*x + 200*x^2 + 53))
/3360000000 - (44194603*x*(3*x + 5*x^2 + 2)^(3/2))/14000000 - (42916771101
*5^(1/2)*log(2*(3*x + 5*x^2 + 2)^(1/2) + (5^(1/2)*(10*x + 3))/5))/11200000
000
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \left(4+x-2x^2\right)^3 \sqrt{2+3x+5x^2}\,dx = -\sqrt{5x^2+3x+2}\,x^7 + \frac{117\sqrt{5x^2+3x+2}\,x^6}{70} + \frac{29597\sqrt{5x^2+3x+2}\,x^5}{4200} - \frac{619727\sqrt{5x^2+3x+2}\,x^4}{70000} - \frac{179739113\sqrt{5x^2+3x+2}\,x^3}{8400000} + \frac{1061199071\sqrt{5x^2+3x+2}\,x^2}{84000000} + \frac{3691913833\sqrt{5x^2+3x+2}\,x}{112000000} + \frac{59895956237\sqrt{5x^2+3x+2}}{3360000000} + \frac{11306784349\sqrt{5}\log\left(\frac{2\sqrt{5x^2+3x+2}\,\sqrt{5}+10x+3}{\sqrt{31}}\right)}{1600000000}$$

input

```
int((-2*x^2+x+4)^3*(5*x^2+3*x+2)^(1/2),x)
```

output

```
( - 33600000000*sqrt(5*x**2 + 3*x + 2)*x**7 + 56160000000*sqrt(5*x**2 + 3*
x + 2)*x**6 + 236776000000*sqrt(5*x**2 + 3*x + 2)*x**5 - 297468960000*sqrt
(5*x**2 + 3*x + 2)*x**4 - 718956452000*sqrt(5*x**2 + 3*x + 2)*x**3 + 42447
9628400*sqrt(5*x**2 + 3*x + 2)*x**2 + 1107574149900*sqrt(5*x**2 + 3*x + 2)
*x + 598959562370*sqrt(5*x**2 + 3*x + 2) + 237442471329*sqrt(5)*log((2*sqr
t(5*x**2 + 3*x + 2)*sqrt(5) + 10*x + 3)/sqrt(31)))/33600000000
```

3.135 $\int \sqrt{a+bx+cx^2}(d+ex+fx^2)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 436

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$
$$= \frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+5b^2(e^2+2df}{512c^5}$$
$$+\frac{(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2(32aef+25b(e^2+2df)))\left(a+bx+cx^2\right)^{3/2}}{960c^4}$$
$$+\frac{(21b^2f^2-4cf(14be+5af)+40c^2(e^2+2df))\,x\left(a+bx+cx^2\right)^{3/2}}{160c^3}$$
$$+\frac{f(8ce-3bf)x^2\left(a+bx+cx^2\right)^{3/2}}{20c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$
$$-\frac{(b^2-4ac)\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+\right.}{1024c^{11/2}}$$

output
```
1/512*(128*c^4*d^2+21*b^4*f^2-56*b^2*c*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*
f+e^2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b
*x+a)^(1/2)/c^5+1/960*(640*c^3*d*e-105*b^3*f^2+28*b*c*f*(7*a*f+10*b*e)-8*c
^2*(32*a*e*f+25*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(3/2)/c^4+1/160*(21*b^2*f^2-
4*c*f*(5*a*f+14*b*e)+40*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(3/2)/c^3+1/20*f*
(-3*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^(3/2)/c^2+1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)
/c-1/1024*(-4*a*c+b^2)*(128*c^4*d^2+21*b^4*f^2-56*b^2*c*f*(a*f+b*e)-32*c^3
*(4*b*d*e+a*(2*d*f+e^2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b^2*(2*d*f+e^2)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 8.94 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.05

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$

$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(315b^5f^2-210b^4cf(4e+fx)-16b^2c^2(-2af(115e+28fx)+c(120de+25e^2x+50}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f^2 - 210*b^4*c*f*(4*e + f*x) - 16
*b^2*c^2*(-2*a*f*(115*e + 28*f*x) + c*(120*d*e + 25*e^2*x + 50*d*f*x + 28*
e*f*x^2 + 9*f^2*x^3)) + 8*b^3*c*(-210*a*f^2 + c*(75*e^2 + 70*e*f*x + 3*f*(
50*d + 7*f*x^2))) + 16*b*c^2*(113*a^2*f^2 - 2*a*c*(65*e^2 + 58*e*f*x + f*(
130*d + 17*f*x^2)) + 4*c^2*(30*d^2 + 10*d*x*(2*e + f*x) + x^2*(5*e^2 + 6*e
*f*x + 2*f^2*x^2))) - 32*c^3*(a^2*f*(64*e + 15*f*x) - 2*a*c*(80*d*e + 15*e
^2*x + 30*d*f*x + 16*e*f*x^2 + 5*f^2*x^3) - 4*c^2*x*(30*d^2 + 10*d*x*(4*e
+ 3*f*x) + x^2*(15*e^2 + 24*e*f*x + 10*f^2*x^2)))) - 15*(b^2 - 4*a*c)*(128
*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2
+ 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(11/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{3}{2}\sqrt{cx^2+bx+a}\left(f(8ce-3bf)x^3-2\left(af^2-2c\left(e^2+2df\right)\right)x^2+8cdex+4cd^2\right)dx}{6c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{cx^2+bx+a}\left(f(8ce-3bf)x^3-2\left(af^2-2c\left(e^2+2df\right)\right)x^2+8cdex+4cd^2\right)dx}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{1}{2}\sqrt{cx^2+bx+a}\left(40c^2d^2+\left(40\left(e^2+2df\right)c^2-4f(14be+5af)c+21b^2f^2\right)x^2+4\left(20dec^2-8aefc+3abf^2\right)x\right)dx}{5c}+\frac{fx^2\left(a+bx+cx^2\right)^{3/2}(8ce-3bf)}{5c}}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{cx^2+bx+a}\left(40c^2d^2+\left(40\left(e^2+2df\right)c^2-4f(14be+5af)c+21b^2f^2\right)x^2+4\left(20dec^2-8aefc+3abf^2\right)x\right)dx}{10c}+\frac{fx^2\left(a+bx+cx^2\right)^{3/2}(8ce-3bf)}{5c}}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{1}{2}\left(320d^2c^3-80a\left(e^2+2df\right)c^2+8af(14be+5af)c-42ab^2f^2+\left(-105f^2b^3+28cf(10be+7af)b+640c^3de-8c^2\left(32aef+25b\left(e^2+2df\right)\right)\right)x\right)\sqrt{cx^2+bx+a}dx}{4c}+\frac{x\left(a+bx\right.}{}}{10c}}{4c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 27

$$\frac{\frac{\int \left(2\left(160d^2c^3-40a\left(e^2+2df\right)c^2+4af(14be+5af)c-21ab^2f^2\right)+\left(-105f^2b^3+28cf(10be+7af)b+640c^3de-8c^2\left(32aef+25b\left(e^2+2df\right)\right)\right)x\right)\sqrt{cx^2+bx+a}dx}{8c}+\frac{x\left(a+b\right.}{}}{10c}}{4c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}\left(-8c^2\left(32aef+25b\left(2\right.\right.\right.}{}}{8c}}{10c}}{4c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 1087

$$\frac{\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+}{8c}}{10c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 1092

$$\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{}{\sqrt{}}}{4c}\right.}{2c}}{8c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 219

$$\frac{\frac{\frac{5\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2\cdots\right)}{2c}}{8c}}{}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

output
```
(f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) + ((f*(8*c*e - 3*b*f)*x^2*(a + b*x
 + c*x^2)^(3/2))/(5*c) + (((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(
e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((640*c^3*d*e - 105*b^3*
f^2 + 28*b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(
a + b*x + c*x^2)^(3/2))/(3*c) + (5*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*
(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a
^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c)
- ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2))))/(2*c))/(8*c))/(10*c))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.48

method	result
risch	$\underline{(1280 f^2 c^5 x^5+128 b c^4 f^2 x^4+3072 c^5 e f x^4+320 a c^4 f^2 x^3-144 b^2 c^3 f^2 x^3+384 b c^4 e f x^3+3840 c^5 d f x^3+1920 c^5 e^2 x^3-544 a b c^3 f^2 x^2+1}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7680*(1280*c^5*f^2*x^5+128*b*c^4*f^2*x^4+3072*c^5*e*f*x^4+320*a*c^4*f^2*
x^3-144*b^2*c^3*f^2*x^3+384*b*c^4*e*f*x^3+3840*c^5*d*f*x^3+1920*c^5*e^2*x^
3-544*a*b*c^3*f^2*x^2+1024*a*c^4*e*f*x^2+168*b^3*c^2*f^2*x^2-448*b^2*c^3*e
*f*x^2+640*b*c^4*d*f*x^2+320*b*c^4*e^2*x^2+5120*c^5*d*e*x^2-480*a^2*c^3*f^
2*x+896*a*b^2*c^2*f^2*x-1856*a*b*c^3*e*f*x+1920*a*c^4*d*f*x+960*a*c^4*e^2*
x-210*b^4*c*f^2*x+560*b^3*c^2*e*f*x-800*b^2*c^3*d*f*x-400*b^2*c^3*e^2*x+12
80*b*c^4*d*e*x+3840*c^5*d^2*x+1808*a^2*b*c^2*f^2-2048*a^2*c^3*e*f-1680*a*b
^3*c*f^2+3680*a*b^2*c^2*e*f-4160*a*b*c^3*d*f-2080*a*b*c^3*e^2+5120*a*c^4*d
*e+315*b^5*f^2-840*b^4*c*e*f+1200*b^3*c^2*d*f+600*b^3*c^2*e^2-1920*b^2*c^3
*d*e+1920*b*c^4*d^2)/c^5*(c*x^2+b*x+a)^(1/2)+1/1024*(64*a^3*c^3*f^2-240*a^
2*b^2*c^2*f^2+384*a^2*b*c^3*e*f-256*a^2*c^4*d*f-128*a^2*c^4*e^2+140*a*b^4*
c*f^2-320*a*b^3*c^2*e*f+384*a*b^2*c^3*d*f+192*a*b^2*c^3*e^2-512*a*b*c^4*d*
e+512*a*c^5*d^2-21*b^6*f^2+56*b^5*c*e*f-80*b^4*c^2*d*f-40*b^4*c^2*e^2+128*
b^3*c^3*d*e-128*b^2*c^4*d^2)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.91

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e
 + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c +
 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a
^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(c)*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) - 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*
x^4 + 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*
e)*f)*x^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e
^2 + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e +
40*b*c^5*e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4
- 16*a*c^5)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 -
 460*a*b^2*c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*
(5*b^2*c^4 - 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f
^2 - 8*(10*(5*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*
sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128
*(b^3*c^3 - 4*a*b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2
 + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4
*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(444) = 888$.

Time = 0.82 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.54

$$\int \sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**2*x**5/6 + x**4*(b*f**2/12 + 2*c*e*f
)/(5*c) + x**3*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*
c*d*f + c*e**2)/(4*c) + x**2*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) +
2*b*d*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10
*c) + 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(3*c) + x*(2*a*d*f + a*e**2 - 3*a
*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2
)/(4*c) + 2*b*d*e - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d
*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) +
 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(2*c) + (2*a*d*e - 2*a
*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d*f + b*e**2 - 7*b*(a*f*
*2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2)/(8*c
) + 2*c*d*e)/(3*c) + b*d**2 - 3*b*(2*a*d*f + a*e**2 - 3*a*(a*f**2/6 + 2*b*
e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2)/(4*c) + 2*b*d*e
 - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d*f + b*e**2 - 7*b
*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2
)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(4*c))/c) + (a*d**2 - a*(2*a*d*f + a*e*
*2 - 3*a*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f
+ c*e**2)/(4*c) + 2*b*d*e - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c)
 + 2*b*d*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/
(10*c) + 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(2*c) - b*(...
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.44

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 \, dx$$
$$= \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10f^2x+\frac{24c^5ef+bc^4f^2}{c^5}\right)x+\frac{120c^5e^2+240c^5df+24bc^4ef-9b^2\right.\right.\right.\right.\right.$$
$$+\frac{(128b^2c^4d^2-512ac^5d^2-128b^3c^3de+512abc^4de+40b^4c^2e^2-192ab^2c^3e^2+128a^2c^4e^2+80b^4c^2df}{}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (24*c^5*e*f + b*c^4*f
^2)/c^5)*x + (120*c^5*e^2 + 240*c^5*d*f + 24*b*c^4*e*f - 9*b^2*c^3*f^2 + 2
0*a*c^4*f^2)/c^5)*x + (640*c^5*d*e + 40*b*c^4*e^2 + 80*b*c^4*d*f - 56*b^2*
c^3*e*f + 128*a*c^4*e*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2)/c^5)*x + (1920*
c^5*d^2 + 640*b*c^4*d*e - 200*b^2*c^3*e^2 + 480*a*c^4*e^2 - 400*b^2*c^3*d*
f + 960*a*c^4*d*f + 280*b^3*c^2*e*f - 928*a*b*c^3*e*f - 105*b^4*c*f^2 + 44
8*a*b^2*c^2*f^2 - 240*a^2*c^3*f^2)/c^5)*x + (1920*b*c^4*d^2 - 1920*b^2*c^3
*d*e + 5120*a*c^4*d*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2 + 1200*b^3*c^2*
d*f - 4160*a*b*c^3*d*f - 840*b^4*c*e*f + 3680*a*b^2*c^2*e*f - 2048*a^2*c^3
*e*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2)/c^5) + 1/1024*
(128*b^2*c^4*d^2 - 512*a*c^5*d^2 - 128*b^3*c^3*d*e + 512*a*b*c^4*d*e + 40*
b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2 + 80*b^4*c^2*d*f - 384*a
*b^2*c^3*d*f + 256*a^2*c^4*d*f - 56*b^5*c*e*f + 320*a*b^3*c^2*e*f - 384*a^
2*b*c^3*e*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*
c^3*f^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11
/2)
```

Mupad [B] (verification not implemented)

Time = 17.45 (sec) , antiderivative size = 1299, normalized size of antiderivative = 2.98

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

output
```
d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/
2))/(4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)
*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(
4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c)
- (3*b*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/
2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2
) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) -
 (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f
^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x +
 c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/
2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a +
 c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - ...
```

Reduce [F]

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx=\int \sqrt{c\,x^2+bx+a}\,\left(f\,x^2+ex+d\right)^2dx$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x)
```

output
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x)
```

3.136 $\int \sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx &= \frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} \\ &\quad + \frac{(8ce-5bf)\,(a+bx+cx^2)^{3/2}}{24c^2} + \frac{fx(a+bx+cx^2)^{3/2}}{4c} \\ &\quad - \frac{(b^2-4ac)\,(16c^2d+5b^2f-4c(2be+af))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

output
```
1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
+1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c
-1/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+
b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+\ldots}{192c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*
(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e
+ a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(7
/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\int \frac{1}{2}(8cd-2af+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{4c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (2(4cd-af)+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(-4acf+5b^2f-8bce+16c^2d\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

input

```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output

```
(f*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((8*c*e - 5*b*f)*(a + b*x + c*x^2)^
(3/2))/(3*c) + ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(2*c))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(-48 f \,c^{3} x^{3}-8 b \,c^{2} f \,x^{2}-64 c^{3} e \,x^{2}-24 a \,c^{2} f x +10 b^{2} c f x -16 b \,c^{2} e x -96 c^{3} d x +52 a b c f -64 c^{2} a e -15 b^{3} f +24 c e \,b^{2}-48 b \,c^{2} d \right) \sqrt{c \,x^{2}+b x}}{192 c^{3}}$
default	$d \left(\frac{\left(2 c x +b \right) \sqrt{c \,x^{2}+b x +a}}{4 c}+\frac{\left(4 a c -b^{2}\right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b x +a}\right)}{8 c^{\frac{3}{2}}}\right)+e \left(\frac{\left(c \,x^{2}+b x +a \right)^{\frac{3}{2}}}{3 c}-\frac{b \left(\frac{\left(2 c x +b \right) \sqrt{c \,x^{2}+b x +a}}{4 c}+\frac{\left(4 a c -b\right.}{}\right.}{2 c}\right.$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*c^3*f*x^3-8*b*c^2*f*x^2-64*c^3*e*x^2-24*a*c^2*f*x+10*b^2*c*f*x
-16*b*c^2*e*x-96*c^3*d*x+52*a*b*c*f-64*a*c^2*e-15*b^3*f+24*b^2*c*e-48*b*c^
2*d)/c^3*(c*x^2+b*x+a)^(1/2)-1/128*(16*a^2*c^2*f-24*a*b^2*c*f+32*a*b*c^2*e
-64*a*c^3*d+5*b^4*f-8*b^3*c*e+16*b^2*c^2*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.66

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 2
4*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^
3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c -
52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqr
t(c*x^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4
*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c
^4*f*x^3 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3
)*e + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 1
2*a*c^3)*f)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time $= 0.56$ (sec) , antiderivative size $= 384$, normalized size of antiderivative $= 2.19$

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fx^3}{4}+\frac{x^2\left(\frac{bf}{8}+ce\right)}{3c}+\frac{x\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{2c}+\frac{ae-\frac{2a\left(\frac{bf}{8}+ce\right)}{3c}+bd-\frac{3b\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{4c}}{c}\right)+ \\ \frac{2\left(\frac{f(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{5}{2}}(-2af+be)}{5b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(a^2f-abe+b^2d\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*x**3/4 + x**2*(b*f/8 + c*e)/(3*c) + x
*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) + (a*e - 2*a*(b*f/8 +
 c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(4*c
))/c) + (a*d - a*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) - b*(
a*e - 2*a*(b*f/8 + c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)
/(6*c) + c*d)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*(a + b*x)**(7/2)/
(7*b**2) + (a + b*x)**(5/2)*(-2*a*f + b*e)/(5*b**2) + (a + b*x)**(3/2)*(a*
*2*f - a*b*e + b**2*d)/(3*b**2))/b, Ne(b, 0)), (sqrt(a)*(d*x + e*x**2/2 +
f*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{8\,c^3e+bc^2f}{c^3}\right)x+\frac{48\,c^3d+8\,bc^2e-5\,b^2cf+12\,ac^2f}{c^3}\right)x+\frac{48\,bc^2d-\right.$$
$$+\frac{(16\,b^2c^2d-64\,ac^3d-8\,b^3ce+32\,abc^2e+5\,b^4f-24\,ab^2cf+16\,a^2c^2f)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\right.\right.}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (8*c^3*e + b*c^2*f)/c^3)*x + (4
8*c^3*d + 8*b*c^2*e - 5*b^2*c*f + 12*a*c^2*f)/c^3)*x + (48*b*c^2*d - 24*b^
2*c*e + 64*a*c^2*e + 15*b^3*f - 52*a*b*c*f)/c^3) + 1/128*(16*b^2*c^2*d - 6
4*a*c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2
*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.83

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a} - \frac{a\,f\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} + \frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} + \frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} - \frac{5\,b\,f\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} + \frac{e\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2} + \frac{f\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b +
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
+ ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2) + (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.18

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{-104\sqrt{c\,x^2+bx+a}\,ab\,c^2 f+128\sqrt{c\,x^2+bx+a}\,a\,c^3 e+48\sqrt{c\,x^2+bx+a}\,a\,c^3 fx+30\sqrt{c\,x^2+bx+a}\,b^3}{}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
( - 104*sqrt(a + b*x + c*x**2)*a*b*c**2*f + 128*sqrt(a + b*x + c*x**2)*a*c
**3*e + 48*sqrt(a + b*x + c*x**2)*a*c**3*f*x + 30*sqrt(a + b*x + c*x**2)*b
**3*c*f - 48*sqrt(a + b*x + c*x**2)*b**2*c**2*e - 20*sqrt(a + b*x + c*x**2
)*b**2*c**2*f*x + 96*sqrt(a + b*x + c*x**2)*b*c**3*d + 32*sqrt(a + b*x + c
*x**2)*b*c**3*e*x + 16*sqrt(a + b*x + c*x**2)*b*c**3*f*x**2 + 192*sqrt(a +
 b*x + c*x**2)*c**4*d*x + 128*sqrt(a + b*x + c*x**2)*c**4*e*x**2 + 96*sqrt
(a + b*x + c*x**2)*c**4*f*x**3 - 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*f + 72*sqrt(c)*log((2*s
qrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*f
- 96*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c
 - b**2))*a*b*c**2*e + 192*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**3*d - 15*sqrt(c)*log((2*sqrt(c)*sqrt(
a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*f + 24*sqrt(c)*log
((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c
*e - 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*
a*c - b**2))*b**2*c**2*d)/(384*c**4)
```

3.137 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 431

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2\cdots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}} + \frac{\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2\cdots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

output
```
c^(1/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f-1/2*(c*(e^2-2
*d*f-e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2))))^(1/2)*arcta
nh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))*x)
*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
/(c*x^2+b*x+a)^(1/2))*2^(1/2)/f/(-4*d*f+e^2)^(1/2)+1/2*(c*(e^2-2*d*f+e*(-4
*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)*arctanh(1/4*(4*
a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*
x+a)^(1/2))*2^(1/2)/f/(-4*d*f+e^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)+\operatorname{RootSum}\left[c^2d-bce+b^2f+2\sqrt{a}ce\#1-4\sqrt{a}bf\#1-2cd\#1^2+be\#1\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + RootS
um[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^
2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d*Log[x])
 + b*c*e*Log[x] - b^2*f*Log[x] + a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1] - b*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*
#1] + b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*c*f*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*Log[x]*#1 + 2*Sqrt[a]*
b*f*Log[x]*#1 + 2*Sqrt[a]*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
*#1 - 2*Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*
Log[x]*#1^2 - a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1]*#1^2 + a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sq
rt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1
^2 - 2*d*#1^3) & ])/f
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1320, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}dx$$

$$\Big\downarrow \; 1320$$

$$\frac{c\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 1092$$

$$\frac{2c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 1365$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-$$
$$\frac{\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{c\ldots}}}{\sqrt{e^2-4df}}}{f}$$

$$\Big\downarrow \; 1154$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-$$
$$\frac{2\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af\ldots}{\ldots}}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-$$
$$\frac{\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(c\ldots\right.}{\sqrt{2}\sqrt{e\ldots}}}{f}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (-(((
2*f*(c*d - a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((2*f*(c*d - a*f) -
(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(376) = 752$.

Time = 2.73 (sec) , antiderivative size = 1547, normalized size of antiderivative = 3.59

method	result	size
default	Expression too large to display	1547

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*f+e^2)^(1/2)*(1/2*(4*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+4*(c*(-
4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(f*b*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/
2)+1/2*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+f*b-
c*e)/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c^(1/2)+(c*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))+1/2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*d*f*c+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2*2^(1/2)/((f*b*(-4*d*f+e^2)^(
1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*ln(((f
*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/
f^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2
*2^(1/2)*((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d
*f*c+c*e^2)/f^2)^(1/2)*(4*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+4*(c*(-4*d
*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(f*b*(-4*d*f+
e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2))
/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))-1/(-4*d*f+e^2)^(1/2)*(1/2*(4*c*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2+4/f*(-c*(-4*d*f+e^2)^(1/2)+f*b-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+2*(-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+f*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

3.138 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 488

$$\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)} - \frac{\left(f(be-4af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\,(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\left(f(be-4af)-(ce-bf)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\,(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-(2*f*x+e)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)-1/2*(f*(-4*a*f+b
*e)-(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))*arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2
)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/
(-4*d*f+e^2)^(3/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1
/2))^(1/2)+1/2*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e+(-4*d*f+e^2)^(1/2)))*arctanh(
1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c
*x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(3/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.32 (sec) , antiderivative size = 1691, normalized size of antiderivative = 3.47

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

output
```
-1/2*(2*d^3*e*Sqrt[a + x*(b + c*x)] + 4*d^3*f*x*Sqrt[a + x*(b + c*x)] - 2*
(e^2 - 4*d*f)*(d + x*(e + f*x))*RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*
c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]
*e*#1^3 + d*#1^4 & , (-(b^2*d^2*Log[x]) - 3*a*c*d^2*Log[x] + 5*a*b*d*e*Log
[x] - 4*a^2*e^2*Log[x] + 4*a^2*d*f*Log[x] + b^2*d^2*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1] + 3*a*c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
x*#1] - 5*a*b*d*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 4*a^2*e^2
*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^2*d*f*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] + 2*Sqrt[a]*b*d^2*Log[x]*#1 - 2*a^(3/2)*d*e*L
og[x]*#1 - 2*Sqrt[a]*b*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1
 + 2*a^(3/2)*d*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - a*d^2*L
og[x]*#1^2 + a*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(S
qrt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#
1^2 - 2*d*#1^3) & ] + (d + x*(e + f*x))*RootSum[c^2*d - b*c*e + b^2*f + 2*
Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2
*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(b*c*d^3*e*Log[x]) + 2*b^2*d^2*e^2*Log[x] +
 6*a*c*d^2*e^2*Log[x] - 10*a*b*d*e^3*Log[x] + 8*a^2*e^4*Log[x] - 6*b^2*d^3
*f*Log[x] - 28*a*c*d^3*f*Log[x] + 40*a*b*d^2*e*f*Log[x] - 40*a^2*d*e^2*f*L
og[x] + 32*a^2*d^2*f^2*Log[x] + b*c*d^3*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*
x^2] - x*#1] - 2*b^2*d^2*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#...
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1302, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx$$

$$\Big\downarrow \; 1302$$

$$\frac{\int \frac{be-4af+2(ce-bf)x}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{e^2-4df}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{be-4af+2(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2(e^2-4df)}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{2\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{2\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{\ldots}}}{\sqrt{e^2-4df}}}{2(e^2-4df)}$$
$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{4\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af\ldots}{\ldots}}{\sqrt{e^2-4df}}}{\ldots}$$

$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sqrt{2}\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\left(f(be-4af)-\left(e-\sqrt{e^2-4\right.\right.}{}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

output
```
-(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) +
 (-((Sqrt[2]*(f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTa
nh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*(f*(b
*e - 4*a*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sq
rt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4690 vs. $2(439) = 878$.

Time = 2.88 (sec) , antiderivative size = 4691, normalized size of antiderivative = 9.61

method	result	size
default	Expression too large to display	4691

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(4*d*f-e^2)*(-2/(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*d*f*c+c*e^2)*f^2/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(c*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))+1/2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*d*f*c+c*e^2)/f^2)^(3/2)+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)*f/(f*b*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)*(1/2*
(4*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+4*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/
f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1/
2)+f*b-c*e)/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f+c*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2))))/c^(1/2)+(c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+(c*(-4*d*
f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(f*b*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)
)/c^(1/2)-1/2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*d*f*c+c*e^2)/f^2*2^(1/2)/((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*ln(((f*b*(-4*d*f+e^2)^(1/2)-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+
f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((f*b*(-4*d*f+e^2
)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*(4*
c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+4*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f\,x^2+e\,x+d\right)^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}}{\left(f\,x^2+ex+d\right)^2}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

output
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

3.139 $\int \frac{\sqrt{2+3x+5x^2}}{\left(4+x-2x^2\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \frac{\sqrt{2+3x+5x^2}}{\left(4+x-2x^2\right)^3}\,dx = -\frac{(1-4x)\sqrt{2+3x+5x^2}}{66\left(4+x-2x^2\right)^2}-\frac{(835-10358x)\sqrt{2+3x+5x^2}}{676632\left(4+x-2x^2\right)} - \frac{\sqrt{\frac{38080240681-6621689063\sqrt{33}}{2563}}\,\mathrm{arctanh}\left(\frac{19-3\sqrt{33}+2\left(11-5\sqrt{33}\right)x}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{902176} + \frac{\sqrt{\frac{38080240681+6621689063\sqrt{33}}{2563}}\,\mathrm{arctanh}\left(\frac{19+3\sqrt{33}+2\left(11+5\sqrt{33}\right)x}{2\sqrt{2\left(107+11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{902176}$$

output

```
-1/66*(1-4*x)*(5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^2-(835-10358*x)*(5*x^2+3*x+
2)^(1/2)/(-1353264*x^2+676632*x+2706528)-1/2312277088*(97599656865403-1697
1389068469*33^(1/2))^(1/2)*arctanh(1/2*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)
/(214-22*33^(1/2))^(1/2)/(5*x^2+3*x+2)^(1/2))+1/2312277088*(97599656865403
+16971389068469*33^(1/2))^(1/2)*arctanh(1/2*(19+3*33^(1/2)+2*(11+5*33^(1/2
))*x)/(214+22*33^(1/2))^(1/2)/(5*x^2+3*x+2)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx = \frac{-\frac{1922\sqrt{2+3x+5x^2}(13592-81605x-12028x^2+20716x^3)}{(4+x-2x^2)^2} - 118995825\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3\right.}{}$$

input
```
Integrate[Sqrt[2 + 3*x + 5*x^2]/(4 + x - 2*x^2)^3,x]
```

output
```
((-1922*Sqrt[2 + 3*x + 5*x^2]*(13592 - 81605*x - 12028*x^2 + 20716*x^3))/(
4 + x - 2*x^2)^2 - 118995825*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqr
t[5]*#1^3 + 2*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]/(22*
Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ] + 220*RootSum[-22 + 44*Sqrt
[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (2349842713*Sqrt[5]*Log[-(S
qrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 1544419745*Log[-(Sqrt[5]*x) +
 Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 +
4*#1^3) & ] - 2*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2
*#1^4 & , (258395294519*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] -
 #1]*#1 + 169921975927*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2
)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ])/1300486704
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1302, 27, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{5x^2+3x+2}}{(-2x^2+x+4)^3}\,dx$$

$\Big\downarrow$ 1302

$$\frac{1}{66}\int \frac{80x^2+70x+51}{2\left(-2x^2+x+4\right)^2\sqrt{5x^2+3x+2}}dx-\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{132}\int \frac{80x^2+70x+51}{\left(-2x^2+x+4\right)^2\sqrt{5x^2+3x+2}}dx-\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$\Big\downarrow$ 2135

$$\frac{1}{132}\left(-\frac{\int -\frac{9(24838x+30317)}{2(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx}{15378}-\frac{\sqrt{5x^2+3x+2}(835-10358x)}{5126\left(-2x^2+x+4\right)}\right)-\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{132}\left(\frac{3\int \frac{24838x+30317}{(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx}{10252}-\frac{(835-10358x)\sqrt{5x^2+3x+2}}{5126\left(-2x^2+x+4\right)}\right)-\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$\Big\downarrow$ 1365

$$\frac{1}{132}\left(\frac{3\left(\frac{2}{11}\left(136609-24351\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx+\frac{2}{11}\left(136609+24351\sqrt{33}\right)\int \frac{1}{\left(-4x+\sqrt{33}+1\right)\sqrt{5x^2+3x}}\right.}{10252}\right.$$
$$\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$\Big\downarrow$ 1154

$$\frac{1}{132}\left(\frac{3\left(-\frac{4}{11}\left(136609-24351\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)-\frac{4}{11}\left(1366\right.\right.}{10252}\right.$$
$$\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow 219$$

$$\frac{1}{132}\left(\frac{3\left(\frac{1}{11}\left(136609-24351\sqrt{33}\right)\sqrt{\frac{2}{107-11\sqrt{33}}}\text{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)+\frac{1}{11}\sqrt{\frac{2}{107+11\sqrt{33}}}\left(136609+\right.\right.}{10252}\right.$$

$$\frac{(1-4x)\sqrt{5x^2+3x+2}}{66\left(-2x^2+x+4\right)^2}$$

input
```
Int[Sqrt[2 + 3*x + 5*x^2]/(4 + x - 2*x^2)^3,x]
```

output
```
-1/66*((1 - 4*x)*Sqrt[2 + 3*x + 5*x^2])/(4 + x - 2*x^2)^2 + (-1/5126*((835
 - 10358*x)*Sqrt[2 + 3*x + 5*x^2])/(4 + x - 2*x^2) + (3*(((136609 - 24351*
Sqrt[33])*Sqrt[2/(107 - 11*Sqrt[33])]*ArcTanh[(19 - 3*Sqrt[33] + 2*(11 - 5
*Sqrt[33])*x)/(2*Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])])/11 +
 (Sqrt[2/(107 + 11*Sqrt[33])]*(136609 + 24351*Sqrt[33])*ArcTanh[(19 + 3*Sq
rt[33] + 2*(11 + 5*Sqrt[33])*x)/(2*Sqrt[2*(107 + 11*Sqrt[33])]*Sqrt[2 + 3*
x + 5*x^2])])/11))/10252)/132
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{(20716x^3-12028x^2-81605x+13592)\sqrt{5x^2+3x+2}}{676632(2x^2-x-4)^2}+\frac{\left(-73053+12419\sqrt{33}\right)\sqrt{33}\ \operatorname{arctanh}\left(\frac{214-22\sqrt{33}+8\left(\frac{11}{2}-\right.}{\sqrt{214-22\sqrt{33}}\sqrt{80\left(x-\frac{1}{4}+\frac{\sqrt{33}}{4}\right)^2+16}}\right.}{7442952\sqrt{214-22\sqrt{33}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/676632*(20716*x^3-12028*x^2-81605*x+13592)/(2*x^2-x-4)^2*(5*x^2+3*x+2)^
(1/2)+1/7442952*(-73053+12419*33^(1/2))*33^(1/2)/(214-22*33^(1/2))^(1/2)*a
rctanh(8*(107/4-11/4*33^(1/2)+(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2)))/(2
14-22*33^(1/2))^(1/2)/(80*(x-1/4+1/4*33^(1/2))^2+16*(11/2-5/2*33^(1/2))*(x
-1/4+1/4*33^(1/2))+214-22*33^(1/2))^(1/2))+1/7442952*(73053+12419*33^(1/2)
)*33^(1/2)/(214+22*33^(1/2))^(1/2)*arctanh(8*(107/4+11/4*33^(1/2)+(11/2+5/
2*33^(1/2))*(x-1/4*33^(1/2)-1/4))/(214+22*33^(1/2))^(1/2)/(80*(x-1/4*33^(1
/2)-1/4)^2+16*(11/2+5/2*33^(1/2))*(x-1/4*33^(1/2)-1/4)+214+22*33^(1/2))^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx =$$

$$-\frac{3\left(4x^4-4x^3-15x^2+8x+16\right)\sqrt{\frac{6621689063}{233}\sqrt{\frac{3}{11}}+\frac{38080240681}{2563}}\log\left(\frac{11\sqrt{5x^2+3x+2}\sqrt{\frac{6621689063}{233}\sqrt{\frac{3}{11}}+\frac{3808024\ldots}{2563}}}{\ldots}\right.}{\ldots}$$

input
```
integrate((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x, algorithm="fricas")
```

output
```
-1/5413056*(3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(6621689063/233*sqrt
(3/11) + 38080240681/2563)*log((11*sqrt(5*x^2 + 3*x + 2)*sqrt(6621689063/2
33*sqrt(3/11) + 38080240681/2563)*(215019*sqrt(3/11) - 119807) + 113253844
*sqrt(3/11)*(3*x + 4) + 792776908*x + 205916080)/x) - 3*(4*x^4 - 4*x^3 - 1
5*x^2 + 8*x + 16)*sqrt(6621689063/233*sqrt(3/11) + 38080240681/2563)*log(-
(11*sqrt(5*x^2 + 3*x + 2)*sqrt(6621689063/233*sqrt(3/11) + 38080240681/256
3)*(215019*sqrt(3/11) - 119807) - 113253844*sqrt(3/11)*(3*x + 4) - 7927769
08*x - 205916080)/x) + 3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(-6621689
063/233*sqrt(3/11) + 38080240681/2563)*log(-(11*sqrt(5*x^2 + 3*x + 2)*(215
019*sqrt(3/11) + 119807)*sqrt(-6621689063/233*sqrt(3/11) + 38080240681/256
3) + 113253844*sqrt(3/11)*(3*x + 4) - 792776908*x - 205916080)/x) - 3*(4*x
^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(-6621689063/233*sqrt(3/11) + 38080240
681/2563)*log((11*sqrt(5*x^2 + 3*x + 2)*(215019*sqrt(3/11) + 119807)*sqrt(
-6621689063/233*sqrt(3/11) + 38080240681/2563) - 113253844*sqrt(3/11)*(3*x
 + 4) + 792776908*x + 205916080)/x) + 8*(20716*x^3 - 12028*x^2 - 81605*x +
 13592)*sqrt(5*x^2 + 3*x + 2))/(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx = -\int \frac{\sqrt{5x^2+3x+2}}{8x^6-12x^5-42x^4+47x^3+84x^2-48x-64}\,dx$$

input
```
integrate((5*x**2+3*x+2)**(1/2)/(-2*x**2+x+4)**3,x)
```

output
```
-Integral(sqrt(5*x**2 + 3*x + 2)/(8*x**6 - 12*x**5 - 42*x**4 + 47*x**3 + 8
4*x**2 - 48*x - 64), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx = \int -\frac{\sqrt{5\,x^2+3\,x+2}}{(2\,x^2-x-4)^3}\,dx$$

input
```
integrate((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(5*x^2 + 3*x + 2)/(2*x^2 - x - 4)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(167) = 334.

Time = 0.24 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx$$
$$= \frac{149028\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^7 - 41640\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^6 - 12183040\left(\sqrt{5}x-\sqrt{5x^2+3\right.}{676632\left(2\left(\sqrt{5}x-\sqrt{5x^2+\right.\right.}$$
$$+ 0.00604061027347214\log\left(-\sqrt{5}x+\sqrt{5x^2+3x+2}+8.38267526007000\right)$$
$$- 0.000141101376098677\log\left(-\sqrt{5}x+\sqrt{5x^2+3x+2}-0.312157316296000\right)$$
$$- 0.00604061027347214\log\left(-\sqrt{5}x+\sqrt{5x^2+3x+2}-0.842024981991000\right)$$
$$+ 0.000141101376098677\log\left(-\sqrt{5}x+\sqrt{5x^2+3x+2}-4.99242498429000\right)$$

input
```
integrate((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x, algorithm="giac")
```

output
```
1/676632*(149028*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 41640*sqrt(5)*(sq
rt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 - 12183040*(sqrt(5)*x - sqrt(5*x^2 + 3*
x + 2))^5 - 46222852*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 18395
0431*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 50816348*sqrt(5)*(sqrt(5)*x -
 sqrt(5*x^2 + 3*x + 2))^2 - 19592650*sqrt(5)*x + 544984*sqrt(5) + 19592650
*sqrt(5*x^2 + 3*x + 2))/(2*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(
5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 91*(sqrt(5)*x - sqrt(5*x^2 + 3*
x + 2))^2 - 44*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) - 22)^2 + 0.006
04061027347214*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) + 8.38267526007000)
- 0.000141101376098677*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.31215731
6296000) - 0.00604061027347214*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.
842024981991000) + 0.000141101376098677*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x
+ 2) - 4.99242498429000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx = \int \frac{\sqrt{5\,x^2+3\,x+2}}{(-2\,x^2+x+4)^3}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^(1/2)/(x - 2*x^2 + 4)^3,x)
```

output
```
int((3*x + 5*x^2 + 2)^(1/2)/(x - 2*x^2 + 4)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{2+3x+5x^2}}{(4+x-2x^2)^3}\,dx = \int \frac{\sqrt{5x^2+3x+2}}{(-2x^2+x+4)^3}dx$$

input
```
int((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x)
```

output
```
int((5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^3,x)
```

3.140 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 564

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx =$$

$$-\frac{(b^2-4ac)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2\right.}{16384c^6}$$

$$+\frac{\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2+14b^2(e^2\right.}{6144c^5}$$

$$+\frac{(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2(48aef+49b(e^2+2df)))\left(a+bx+cx^2\right)^{5/2}}{13440c^4}$$

$$+\frac{(99b^2f^2-12cf(24be+7af)+224c^2(e^2+2df))\,x\left(a+bx+cx^2\right)^{5/2}}{1344c^3}$$

$$+\frac{f(32ce-11bf)x^2\left(a+bx+cx^2\right)^{5/2}}{112c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$+\frac{(b^2-4ac)^2\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2\right.}{32768c^{13/2}}$$

output
```
-1/16384*(-4*a*c+b^2)*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-128
*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+e^
2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6+1/6144*(768*c^4*d^2+99*b^4*f^2-72*b
^2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*
a^2*f^2+14*b^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^5+1/13440*(53
76*c^3*d*e-693*b^3*f^2+36*b*c*f*(31*a*f+56*b*e)-32*c^2*(48*a*e*f+49*b*(2*d
*f+e^2)))*(c*x^2+b*x+a)^(5/2)/c^4+1/1344*(99*b^2*f^2-12*c*f*(7*a*f+24*b*e)
+224*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(5/2)/c^3+1/112*f*(-11*b*f+32*c*e)*x
^2*(c*x^2+b*x+a)^(5/2)/c^2+1/8*f^2*x^3*(c*x^2+b*x+a)^(5/2)/c+1/32768*(-4*a
*c+b^2)^2*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*
e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+e^2)))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 12.01 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.47

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 dx = \frac{430080d^2(b+2cx)(a+x(b+cx))^{3/2}+1376256de(a+x(b+cx))^{5/2}+573440(e^2+2df)\,x(a}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

output
```
(430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b +
c*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*
x^2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80
640*(b^2 - 4*a*c)*d^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2
 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2)
 - (26880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 -
 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-2
56*c^(5/2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3
*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4
*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(e^2 +
2*d*f)*(1792*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(
3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b +
 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a +
 x*(b + c*x))^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280
*a*c^2*x)*(a + x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[
c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x...
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{1}{2}\left(cx^2+bx+a\right)^{3/2} \left(f(32ce-11bf)x^3-2\left(3af^2-8c\left(e^2+2df\right)\right)x^2+32cdex+16cd^2\right)dx}{8c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(cx^2+bx+a\right)^{3/2}\left(f(32ce-11bf)x^3-2\left(3af^2-8c\left(e^2+2df\right)\right)x^2+32cdex+16cd^2\right)dx}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\int \frac{1}{2}\left(cx^2+bx+a\right)^{3/2}\left(224c^2d^2+\left(224\left(e^2+2df\right)c^2-12f(24be+7af)c+99b^2f^2\right)x^2+4\left(112dec^2-32aefc+11abf^2\right)x\right)dx}{7c}+\frac{fx^2\left(a+bx+cx^2\right)^{5/2}(32ce\cdots}{7c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \left(cx^2+bx+a\right)^{3/2}\left(224c^2d^2+\left(224\left(e^2+2df\right)c^2-12f(24be+7af)c+99b^2f^2\right)x^2+4\left(112dec^2-32aefc+11abf^2\right)x\right)dx}{14c}+\frac{fx^2\left(a+bx+cx^2\right)^{5/2}(32ce-\cdots}{7c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{1}{2}\left(2688d^2c^3-448a\left(e^2+2df\right)c^2+24af(24be+7af)c-198ab^2f^2+\left(-693f^2b^3+36cf(56be+31af)b+5376c^3de-32c^2\left(48aef+49b\left(e^2+2df\right)\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx\cdots}{6c}}{14c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \left(2\left(1344d^2c^3-224a\left(e^2+2df\right)c^2+12af(24be+7af)c-99ab^2f^2\right)+\left(-693f^2b^3+36cf(56be+31af)b+5376c^3de-32c^2\left(48aef+49b\left(e^2+2df\right)\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx\cdots}{12c}}{14c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}\left(-32c^2\left(48\cdots\right.\right.}{\cdots}}{12c}}{14c}}{\cdots}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^2+bx\cdots}}{16c}\right.}{2c}}{12c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 1087

$$\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{2c}}{12c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 1092

$$\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{2c}}{12c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\big\downarrow$ 219

$$\frac{\frac{7\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2\cdots\right.}{2c}}{12c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

output
```
(f^2*x^3*(a + b*x + c*x^2)^(5/2))/(8*c) + ((f*(32*c*e - 11*b*f)*x^2*(a + b
*x + c*x^2)^(5/2))/(7*c) + (((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 224*c
^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((5376*c^3*d*e - 693
*b^3*f^2 + 36*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d
*f)))*(a + b*x + c*x^2)^(5/2))/(5*c) + (7*(768*c^4*d^2 + 99*b^4*f^2 - 72*b
^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*
a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(((b + 2*c*x)*(a + b*x + c*x^
2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4
*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c))/(14*c))/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(530) = 1060$.

Time = 2.43 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.13

method	result	size
risch	Expression too large to display	1200
default	Expression too large to display	1654

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1720320/c^6*(215040*c^7*f^2*x^7+261120*b*c^6*f^2*x^6+491520*c^7*e*f*x^6+
322560*a*c^6*f^2*x^5+3840*b^2*c^5*f^2*x^5+614400*b*c^6*e*f*x^5+573440*c^7*
d*f*x^5+286720*c^7*e^2*x^5+19968*a*b*c^5*f^2*x^4+786432*a*c^6*e*f*x^4-4224
*b^3*c^4*f^2*x^4+12288*b^2*c^5*e*f*x^4+745472*b*c^6*d*f*x^4+372736*b*c^6*e
^2*x^4+688128*c^7*d*e*x^4+26880*a^2*c^5*f^2*x^3-27264*a*b^2*c^4*f^2*x^3+67
584*a*b*c^5*e*f*x^3+1003520*a*c^6*d*f*x^3+501760*a*c^6*e^2*x^3+4752*b^4*c^
3*f^2*x^3-13824*b^3*c^4*e*f*x^3+21504*b^2*c^5*d*f*x^3+10752*b^2*c^5*e^2*x^
3+946176*b*c^6*d*e*x^3+430080*c^7*d^2*x^3-57984*a^2*b*c^4*f^2*x^2+98304*a^
2*c^5*e*f*x^2+37440*a*b^3*c^3*f^2*x^2-95232*a*b^2*c^4*e*f*x^2+129024*a*b*c
^5*d*f*x^2+64512*a*b*c^5*e^2*x^2+1376256*a*c^6*d*e*x^2-5544*b^5*c^2*f^2*x^
2+16128*b^4*c^3*e*f*x^2-25088*b^3*c^4*d*f*x^2-12544*b^3*c^4*e^2*x^2+43008*
b^2*c^5*d*e*x^2+645120*b*c^6*d^2*x^2-40320*a^3*c^4*f^2*x+113376*a^2*b^2*c^
3*f^2*x-224256*a^2*b*c^4*e*f*x+215040*a^2*c^5*d*f*x+107520*a^2*c^5*e^2*x-5
3928*a*b^4*c^2*f^2*x+139776*a*b^3*c^3*e*f*x-193536*a*b^2*c^4*d*f*x-96768*a
*b^2*c^4*e^2*x+301056*a*b*c^5*d*e*x+1075200*a*c^6*d^2*x+6930*b^6*c*f^2*x-2
0160*b^5*c^2*e*f*x+31360*b^4*c^3*d*f*x+15680*b^4*c^3*e^2*x-53760*b^3*c^4*d
*e*x+53760*b^2*c^5*d^2*x+176448*a^3*b*c^3*f^2-196608*a^3*c^4*e*f-244944*a^
2*b^3*c^2*f^2+526848*a^2*b^2*c^3*e*f-580608*a^2*b*c^4*d*f-290304*a^2*b*c^4
*e^2+688128*a^2*c^5*d*e+91980*a*b^5*c*f^2-241920*a*b^4*c^2*e*f+340480*a*b^
3*c^3*d*f+170240*a*b^3*c^3*e^2-537600*a*b^2*c^4*d*e+537600*a*b*c^5*d^2-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(530) = 1060.

Time = 0.62 (sec) , antiderivative size = 2179, normalized size of antiderivative = 3.86

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*
a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2
 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 +
 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*
c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15
360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*
a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*
b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6
 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*
c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 +
32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 -
26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 12
8*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 -
3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2
 + 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 3
6*a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 3
2*(14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2
*c^6)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5942 vs. $2(580) = 1160$.

Time = 0.89 (sec) , antiderivative size = 5942, normalized size of antiderivative = 10.54

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f**2*x**7/8 + x**6*(17*b*c*f**2/16 +
2*c**2*e*f)/(7*c) + x**5*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*
b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + x**4*(2
*a*b*f**2 + 4*a*c*e*f - 6*a*(17*b*c*f**2/16 + 2*c**2*e*f)/(7*c) + 2*b**2*e
*f + 4*b*c*d*f + 2*b*c*e**2 - 11*b*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f -
 13*b*(17*b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(12*c
) + 2*c**2*d*e)/(5*c) + x**3*(a**2*f**2 + 4*a*b*e*f + 4*a*c*d*f + 2*a*c*e*
*2 - 5*a*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 + 2*
c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + 2*b**2*d*f + b**2*e**2
+ 4*b*c*d*e - 9*b*(2*a*b*f**2 + 4*a*c*e*f - 6*a*(17*b*c*f**2/16 + 2*c**2*e
*f)/(7*c) + 2*b**2*e*f + 4*b*c*d*f + 2*b*c*e**2 - 11*b*(9*a*c*f**2/8 + b**
2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*
f + c**2*e**2)/(12*c) + 2*c**2*d*e)/(10*c) + c**2*d**2)/(4*c) + x**2*(2*a*
*2*e*f + 4*a*b*d*f + 2*a*b*e**2 + 4*a*c*d*e - 4*a*(2*a*b*f**2 + 4*a*c*e*f
- 6*a*(17*b*c*f**2/16 + 2*c**2*e*f)/(7*c) + 2*b**2*e*f + 4*b*c*d*f + 2*b*c
*e**2 - 11*b*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16
+ 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(12*c) + 2*c**2*d*e)/(5*c)
+ 2*b**2*d*e + 2*b*c*d**2 - 7*b*(a**2*f**2 + 4*a*b*e*f + 4*a*c*d*f + 2*a*c
*e**2 - 5*a*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 +
 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + 2*b**2*d*f + b**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(530) = 1060$.

Time = 0.35 (sec) , antiderivative size = 1132, normalized size of antiderivative = 2.01

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (32*c^8*
e*f + 17*b*c^7*f^2)/c^7)*x + (224*c^8*e^2 + 448*c^8*d*f + 480*b*c^7*e*f +
3*b^2*c^6*f^2 + 252*a*c^7*f^2)/c^7)*x + (5376*c^8*d*e + 2912*b*c^7*e^2 + 5
824*b*c^7*d*f + 96*b^2*c^6*e*f + 6144*a*c^7*e*f - 33*b^3*c^5*f^2 + 156*a*b
*c^6*f^2)/c^7)*x + (26880*c^8*d^2 + 59136*b*c^7*d*e + 672*b^2*c^6*e^2 + 31
360*a*c^7*e^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f - 864*b^3*c^5*e*f + 422
4*a*b*c^6*e*f + 297*b^4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2)/c
^7)*x + (80640*b*c^7*d^2 + 5376*b^2*c^6*d*e + 172032*a*c^7*d*e - 1568*b^3*
c^5*e^2 + 8064*a*b*c^6*e^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f + 2016*b
^4*c^4*e*f - 11904*a*b^2*c^5*e*f + 12288*a^2*c^6*e*f - 693*b^5*c^3*f^2 + 4
680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2)/c^7)*x + (26880*b^2*c^6*d^2 + 5376
00*a*c^7*d^2 - 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e + 7840*b^4*c^4*e^2 -
 48384*a*b^2*c^5*e^2 + 53760*a^2*c^6*e^2 + 15680*b^4*c^4*d*f - 96768*a*b^2
*c^5*d*f + 107520*a^2*c^6*d*f - 10080*b^5*c^3*e*f + 69888*a*b^3*c^4*e*f -
112128*a^2*b*c^5*e*f + 3465*b^6*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*
b^2*c^4*f^2 - 20160*a^3*c^5*f^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*
c^6*d^2 - 80640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e +
23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2 + 47040*b^
5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f - 30240*b^6*c^2*e*
f + 241920*a*b^4*c^3*e*f - 526848*a^2*b^2*c^4*e*f + 196608*a^3*c^5*e*f ...
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx=\int \left(c\,x^2+b\,x+a\right)^{3/2}\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx=\int \left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(f\,x^2+ex+d\right)^2dx$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x)
```

output
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x)
```

3.141 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = & -\frac{(b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\ & +\frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} \\ & +\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} \\ & +\frac{(b^2-4ac)^2\left(24c^2d+7b^2f-4c(3be+af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

output

```
-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*
x+a)^(1/2)/c^4+1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2
+b*x+a)^(3/2)/c^3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*
x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e)
)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 4.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.24

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48\ldots}{\ldots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*b
^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(
7*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*
e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*
e + 35*f*x)))) + 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f
))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{1}{2}(12cd-2af+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{6c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (2(6cd-af)+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-4acf+7b^2f-12bce+24c^2d\right)}{2c}+\frac{(a+bx\cdots}{\cdots}}{12c}$$

$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((12*c*e - 7*b*f)*(a + b*x + c*x^2)
^(5/2))/(5*c) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*
x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{(-1280c^5 f x^5-1664b c^4 f x^4-1536c^5 e x^4-2240a c^4 f x^3-48b^2c^3 f x^3-2112b c^4 e x^3-1920c^5 d x^3-288ab c^3 f x^2-3072a c^4 e x^2+5}{}$
default	$d\left(\frac{(2cx+b)(c x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+e\left(\frac{(c x^2+bx+a)^{\frac{5}{2}}}{5c}\right.$

input

```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output

```
-1/7680/c^4*(-1280*c^5*f*x^5-1664*b*c^4*f*x^4-1536*c^5*e*x^4-2240*a*c^4*f*
x^3-48*b^2*c^3*f*x^3-2112*b*c^4*e*x^3-1920*c^5*d*x^3-288*a*b*c^3*f*x^2-307
2*a*c^4*e*x^2+56*b^3*c^2*f*x^2-96*b^2*c^3*e*x^2-2880*b*c^4*d*x^2-480*a^2*c
^3*f*x+432*a*b^2*c^2*f*x-672*a*b*c^3*e*x-4800*a*c^4*d*x-70*b^4*c*f*x+120*b
^3*c^2*e*x-240*b^2*c^3*d*x+1296*a^2*b*c^2*f-1536*a^2*c^3*e-760*a*b^3*c*f+1
200*a*b^2*c^2*e-2400*a*b*c^3*d+105*b^5*f-180*b^4*c*e+360*b^3*c^2*d)*(c*x^2
+b*x+a)^(1/2)-1/1024*(64*a^3*c^3*f-144*a^2*b^2*c^2*f+192*a^2*b*c^3*e-384*a
^2*c^4*d+60*a*b^4*c*f-96*a*b^3*c^2*e+192*a*b^2*c^3*d-7*b^6*f+12*b^5*c*e-24
*b^4*c^2*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.56

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^
3*c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c
^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 +
 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*
x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 12
8*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^
2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*
b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*
d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^
2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 2
0*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*
c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12
*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(230) = 460.

Time = 0.64 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.76

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*x**5/6 + x**4*(13*b*c*f/12 + c**2*e
)/(5*c) + x**3*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/
(10*c) + c**2*d)/(4*c) + x**2*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2
*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13
*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(3*c) + x*(a**2*f + 2*a*b*e +
2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(
10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12
 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e -
9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) + (a**2*e
+ 2*a*b*d - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b*
*2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c*
*2*e)/(10*c) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*
a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2
*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(
5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/c) + (a**2*d - a*(a**
2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e -
4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**
2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int \left(a+bx+cx^2\right)^{3/2}(d+ex+fx^2)\,dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{12\,c^6e+13\,bc^5f}{c^5}\right)x+\frac{120\,c^6d+132\,bc^5e+3\,b^2c^4}{c^5}\right.\right.\right.\right.$$
$$-\frac{(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (12*c^6*e + 13*b*c^5*
f)/c^5)*x + (120*c^6*d + 132*b*c^5*e + 3*b^2*c^4*f + 140*a*c^5*f)/c^5)*x +
 (360*b*c^5*d + 12*b^2*c^4*e + 384*a*c^5*e - 7*b^3*c^3*f + 36*a*b*c^4*f)/c
^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d - 60*b^3*c^3*e + 336*a*b*c^4*e + 35*
b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f)/c^5)*x - (360*b^3*c^3*d - 240
0*a*b*c^4*d - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e + 105*b^5*
c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f)/c^5) - 1/1024*(24*b^4*c^2*d - 19
2*a*b^2*c^3*d + 384*a^2*c^4*d - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^
3*e + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2}(d+ex+fx^2)\,dx = \int \left(c\,x^2+b\,x+a\right)^{3/2}\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1037, normalized size of antiderivative = 4.39

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

output
```
( - 2592*sqrt(a + b*x + c*x**2)*a**2*b*c**3*f + 3072*sqrt(a + b*x + c*x**2
)*a**2*c**4*e + 960*sqrt(a + b*x + c*x**2)*a**2*c**4*f*x + 1520*sqrt(a + b
*x + c*x**2)*a*b**3*c**2*f - 2400*sqrt(a + b*x + c*x**2)*a*b**2*c**3*e - 8
64*sqrt(a + b*x + c*x**2)*a*b**2*c**3*f*x + 4800*sqrt(a + b*x + c*x**2)*a*
b*c**4*d + 1344*sqrt(a + b*x + c*x**2)*a*b*c**4*e*x + 576*sqrt(a + b*x + c
*x**2)*a*b*c**4*f*x**2 + 9600*sqrt(a + b*x + c*x**2)*a*c**5*d*x + 6144*sqr
t(a + b*x + c*x**2)*a*c**5*e*x**2 + 4480*sqrt(a + b*x + c*x**2)*a*c**5*f*x
**3 - 210*sqrt(a + b*x + c*x**2)*b**5*c*f + 360*sqrt(a + b*x + c*x**2)*b**
4*c**2*e + 140*sqrt(a + b*x + c*x**2)*b**4*c**2*f*x - 720*sqrt(a + b*x + c
*x**2)*b**3*c**3*d - 240*sqrt(a + b*x + c*x**2)*b**3*c**3*e*x - 112*sqrt(a
 + b*x + c*x**2)*b**3*c**3*f*x**2 + 480*sqrt(a + b*x + c*x**2)*b**2*c**4*d
*x + 192*sqrt(a + b*x + c*x**2)*b**2*c**4*e*x**2 + 96*sqrt(a + b*x + c*x**
2)*b**2*c**4*f*x**3 + 5760*sqrt(a + b*x + c*x**2)*b*c**5*d*x**2 + 4224*sqr
t(a + b*x + c*x**2)*b*c**5*e*x**3 + 3328*sqrt(a + b*x + c*x**2)*b*c**5*f*x
**4 + 3840*sqrt(a + b*x + c*x**2)*c**6*d*x**3 + 3072*sqrt(a + b*x + c*x**2
)*c**6*e*x**4 + 2560*sqrt(a + b*x + c*x**2)*c**6*f*x**5 - 960*sqrt(c)*log(
(2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c*
*3*f + 2160*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqr
t(4*a*c - b**2))*a**2*b**2*c**2*f - 2880*sqrt(c)*log((2*sqrt(c)*sqrt(a + b
*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*e + 5760*sqrt...
```

3.142 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 678

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2}$$
$$-\frac{\left(c^2\left(8d-\frac{8e^2}{f}\right)-3b^2f+12c(be-af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}$$
$$+\frac{\left((ce-bf)\left(e-\sqrt{e^2-4df}\right)(f(be-2af)-c(e^2-2df))-2f(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-b}}$$
$$+\frac{\left(4cdf^2(be-af)-2f^3(b^2d-a^2f)-2c^2df(e^2-df)-(ce-bf)\left(e+\sqrt{e^2-4df}\right)(f(be-2af)-c(e^2-\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-b}}$$

output
```
-1/4*(-2*c*f*x-5*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/f^2-1/8*(c^2*(8*d-8*e^2/f)
-3*b^2*f+12*c*(-a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(1/2)/f^2+1/2*((-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2))*(f*(-2*a*f+b*e)-c*(-
2*d*f+e^2))-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*
arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2)
))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^
(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/f^3/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*(4*c*d*f^2*(-a*f+b*e
)-2*f^3*(-a^2*f+b^2*d)-2*c^2*d*f*(-d*f+e^2)-(-b*f+c*e)*(e+(-4*d*f+e^2)^(1/
2))*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2)))*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^
(1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/f^
3/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^
(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.12 (sec) , antiderivative size = 1472, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

output
```
(f*(-4*c*e + 5*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + ((3*b^2*f^2 + 12*c*f
*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[
a + x*(b + c*x)])])/Sqrt[c] - 4*RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*
c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]
*e*#1^3 + d*#1^4 & , (c^3*d*e^2*Log[x] - b*c^2*e^3*Log[x] - c^3*d^2*f*Log[
x] + 2*b^2*c*e^2*f*Log[x] - b^2*c*d*f^2*Log[x] + 2*a*c^2*d*f^2*Log[x] - b^
3*e*f^2*Log[x] - 2*a*b*c*e*f^2*Log[x] + 2*a*b^2*f^3*Log[x] - a^2*c*f^3*Log
[x] - c^3*d*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b*c^2*e^3*L
og[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + c^3*d^2*f*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1] - 2*b^2*c*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] + b^2*c*d*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] -
 2*a*c^2*d*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^3*e*f^2*Lo
g[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*a*b*c*e*f^2*Log[-Sqrt[a] +
Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*b^2*f^3*Log[-Sqrt[a] + Sqrt[a + b*x +
c*x^2] - x*#1] + a^2*c*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] +
2*Sqrt[a]*c^2*e^3*Log[x]*#1 - 4*Sqrt[a]*c^2*d*e*f*Log[x]*#1 - 4*Sqrt[a]*b*
c*e^2*f*Log[x]*#1 + 4*Sqrt[a]*b*c*d*f^2*Log[x]*#1 + 2*Sqrt[a]*b^2*e*f^2*Lo
g[x]*#1 + 4*a^(3/2)*c*e*f^2*Log[x]*#1 - 4*a^(3/2)*b*f^3*Log[x]*#1 - 2*Sqrt
[a]*c^2*e^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*c^
2*d*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*b*c...
```

Rubi [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1308, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \; 1308$$

$$-\frac{\int -\frac{-5dfb^2+4cdeb+\left(8\left(e^2-df\right)c^2-12f(be-af)c+3b^2f^2\right)x^2-4af(cd-2af)+\left(8dec^2+4\left(be^2-afe-4bdf\right)c-bf(5be-16af)\right)x}{4\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{2f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-5dfb^2+4cdeb+(8(e^2-df)c^2-12f(be-af)c+3b^2f^2)x^2-4af(cd-2af)+(8dec^2-4aefc+4b(e^2-4df)c-bf(5be-16af))x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 2143$$

$$\frac{\frac{\int \frac{8\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)+(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{(-12cf(be-af)+3b^2f^2+8c^2(e^2-df))\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{(-12cf(be-af)+3b^2f^2+8c^2(e^2-df))\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{2(-12cf(be-af)+3b^2f^2+8c^2(e^2-df))\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-12cf(be-af)+3b^2f^2+8c^2(e^2-df))}{\sqrt{c}f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 1365$$

$$8\left(\frac{\left(-2f^3\left(b^2d-a^2f\right)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\int\frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(-2f^3\left(b^2\right.\right.}{}\right.$$

$$f$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$\Big\downarrow$ 1154

$$8\left(\frac{2\left(-2f^3\left(b^2d-a^2f\right)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\int\frac{}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right.}}{\sqrt{e^2-4df}}\right.$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$\Big\downarrow$ 219

$$8\left(\frac{\left(-2f^3\left(b^2d-a^2f\right)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

output
```
-1/4*((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/f^2 + (((3*b^2*f^2
- 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*f) + (8*(-(((4*c*d*f^2*(b*e - a*f) - 2*f^
3*(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*
d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*
(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*
f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 -
 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])))/f)/(8*f^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. $2(618) = 1236$.

Time = 2.96 (sec) , antiderivative size = 1326, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1326
default	Expression too large to display	2860

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*x+5*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/f^2+1/8/f^2*(1/f*(12*a*c*f^2
+3*b^2*f^2-12*b*c*e*f-8*c^2*d*f+8*c^2*e^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))/c^(1/2)-4/f^2*(2*a*b*f^3*(-4*d*f+e^2)^(1/2)-2*a*c*e*f^2*(-4*d
*f+e^2)^(1/2)-b^2*e*f^2*(-4*d*f+e^2)^(1/2)-2*b*c*d*f^2*(-4*d*f+e^2)^(1/2)+
2*b*c*e^2*f*(-4*d*f+e^2)^(1/2)+2*c^2*d*e*f*(-4*d*f+e^2)^(1/2)-c^2*e^3*(-4*
d*f+e^2)^(1/2)-2*a^2*f^4+2*a*b*f^3*e+4*a*c*d*f^3-2*a*c*e^2*f^2+2*b^2*d*f^3
-b^2*e^2*f^2-6*b*c*d*e*f^2+2*b*c*e^3*f-2*f^2*c^2*d^2+4*c^2*d*e^2*f-c^2*e^4
)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*ln(((-f*b*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2
)^(1/2)+f*b-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-f*b*(-4*d
*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/
2)*(4*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2+4/f*(-c*(-4*d*f+e^2)^(1/2)+f*b-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))-4/f^2*(2*a*b*f^3*(-4*d*f+e^2)^(1/2)-2*a*c*e*f^2*(-4*d*f+e^2
)^(1/2)-b^2*e*f^2*(-4*d*f+e^2)^(1/2)-2*b*c*d*f^2*(-4*d*f+e^2)^(1/2)+2*b*c*
e^2*f*(-4*d*f+e^2)^(1/2)+2*c^2*d*e*f*(-4*d*f+e^2)^(1/2)-c^2*e^3*(-4*d*f+e^
2)^(1/2)+2*a^2*f^4-2*a*b*f^3*e-4*a*c*d*f^3+2*a*c*e^2*f^2-2*b^2*d*f^3+b^2*e
^2*f^2+6*b*c*d*e*f^2-2*b*c*e^3*f+2*f^2*c^2*d^2-4*c^2*d*e^2*f+c^2*e^4)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

output
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

3.143 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 730

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \frac{\left(2b-\frac{ce}{f}\right)\sqrt{a+bx+cx^2}}{e^2-4df} + \frac{2cx\sqrt{a+bx+cx^2}}{e^2-4df}$$
$$-\frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{c^{3/2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2}$$
$$-\frac{\left((ce-bf)\,(f(be-2af)+2c(e^2-5df))\,\left(e-\sqrt{e^2-4df}\right)+2f(cdf(be-4af)-f^2(2b^2d-3abe+4a^2f)}{2\sqrt{2}f^2\,(e^2-4df)^{3/2}\,\sqrt{ce^2-2cdf-bef+2af^2-}$$
$$+\frac{\left((ce-bf)\,(f(be-2af)+2c(e^2-5df))\,\left(e+\sqrt{e^2-4df}\right)+2f(cdf(be-4af)-f^2(2b^2d-3abe+4a^2f)}{2\sqrt{2}f^2\,(e^2-4df)^{3/2}\,\sqrt{ce^2-2cdf-bef+2af^2+}$$

output
```
(2*b-c*e/f)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)+2*c*x*(c*x^2+b*x+a)^(1/2)/(-4
*d*f+e^2)-(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)+c^(3/2)
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2-1/4*((-b*f+c*e)*(f
*(-2*a*f+b*e)+2*c*(-5*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2))+2*f*(c*d*f*(-4*a*f+
b*e)-f^2*(4*a^2*f-3*a*b*e+2*b^2*d)-2*c^2*d*(-4*d*f+e^2)))*arctanh(1/4*(4*a
*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c
*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x
+a)^(1/2))*2^(1/2)/f^2/(-4*d*f+e^2)^(3/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b
*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/4*((-b*f+c*e)*(f*(-2*a*f+b*e)+2*c*(-5*
d*f+e^2))*(e+(-4*d*f+e^2)^(1/2))+2*f*(c*d*f*(-4*a*f+b*e)-f^2*(4*a^2*f-3*a*
b*e+2*b^2*d)-2*c^2*d*(-4*d*f+e^2)))*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(
1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/f^2
/(-4*d*f+e^2)^(3/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(
1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.92 (sec) , antiderivative size = 2854, normalized size of antiderivative = 3.91

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

output
```
((-2*f*Sqrt[a + x*(b + c*x)]*(c*e^2*x - b*f*(2*d + e*x) + c*d*(e - 2*f*x)
+ a*f*(e + 2*f*x)))/((e^2 - 4*d*f)*(d + x*(e + f*x))) + 4*c^(3/2)*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + (2*RootSum[c^2*d - b*c*e
 + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4
*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^3*d^4*Log[x]) + b*c^2*d^3*e
*Log[x] + 4*a*c^2*d^3*f*Log[x] - 6*a*b^2*d^2*f^2*Log[x] - 7*a^2*c*d^2*f^2*
Log[x] + 9*a^2*b*d*e*f^2*Log[x] - 4*a^3*e^2*f^2*Log[x] + 4*a^3*d*f^3*Log[x
] + c^3*d^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - b*c^2*d^3*e*Log
[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a*c^2*d^3*f*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] + 6*a*b^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x
 + c*x^2] - x*#1] + 7*a^2*c*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1] - 9*a^2*b*d*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 4*
a^3*e^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^3*d*f^3*Log
[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c^2*d^3*e*Log[x]*#1
+ 4*a^(3/2)*b*d^2*f^2*Log[x]*#1 - 2*a^(5/2)*d*e*f^2*Log[x]*#1 + 2*Sqrt[a]*
c^2*d^3*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 4*a^(3/2)*b*d^
2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*a^(5/2)*d*e*f^2*
Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c^2*d^4*Log[x]*#1^2 - a^
2*d^2*f^2*Log[x]*#1^2 - c^2*d^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1]*#1^2 + a^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2...
```

Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 716, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1302, 27, 2138, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}(8cfx^2+2(3ce+bf)x+3be-4af)}{2(fx^2+ex+d)}dx}{e^2-4df}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}\left(8cfx^2+2(3ce+bf)x+3be-4af\right)}{fx^2+ex+d}dx}{2\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\big\downarrow$ 2138

$$\frac{-\frac{\int \frac{2\left(-2f\left(e^2-4df\right)x^2c^3+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)c-f\left(2dec^2+2aefc+b\left(e^2-10df\right)c+bf(be-2af)\right)xc\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{2cf^2}-\frac{2\sqrt{a+bx+cx^2}(-2bf+ce-2cfx)}{f}}{2\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\big\downarrow$ 27

$$\frac{-\frac{\int \frac{-2f\left(e^2-4df\right)x^2c^3+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)c-f\left(2dec^2+2aefc+b\left(e^2-10df\right)c+bf(be-2af)\right)xc}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{cf^2}-\frac{2\sqrt{a+bx+cx^2}(-2bf+ce-2cfx)}{f}-}{2\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\big\downarrow$ 2143

$$\frac{-\frac{\frac{\int \frac{cf\left(2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-2c^3\left(e^2-4df\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{cf^2}-2\sqrt{a+bx+cx^2}}{2\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\big\downarrow$ 27

$$\frac{-\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-2c^3\left(e^2-4df\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{cf^2}-\frac{2\sqrt{a+bx+cx^2}(-}{f}}{2\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\big\downarrow$ 1092

$$-\frac{-\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-4c^3\left(e^2-4df\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{cf^2}-\frac{2\sqrt{c}\cdots}{\cdots}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 219

$$-\frac{-\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-2c^{5/2}\left(e^2-4df\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{cf^2}-\frac{2\sqrt{\cdots}}{\cdots}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 1365

$$-\frac{c\left(\frac{\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx\cdots}{\sqrt{e^2-4df}}\cdots\right.}{\cdots}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 1154

$$-\frac{c\left(\frac{2\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{\cdots}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\cdots\right)\right)\cdots}}{\sqrt{e^2-4df}}\cdots\right.}{\cdots}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 219

$$-\frac{c\left(\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{\right.\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

input

```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

output

```
-(((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)))
 + ((-2*(c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/f - (-2*c^(5/2)*(e^
2 - 4*d*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])] + c*((((
c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) -
 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d
+ 3*a*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2
- 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4
*d*f]]) - (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^
2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f
) - b*e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt
[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)
*Sqrt[e^2 - 4*d*f]])))/(c*f^2))/(2*(e^2 - 4*d*f))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7855 vs. $2(664) = 1328$.

Time = 3.03 (sec) , antiderivative size = 7856, normalized size of antiderivative = 10.76

method	result	size
default	Expression too large to display	7856

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2+ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f\,x^2+e\,x+d)^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{(f\,x^2+ex+d)^2}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

output
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

3.144 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 669

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$
$$+\frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+4af^2)\,x)\sqrt{a+bx+cx^2}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$
$$+\frac{3\left(4bef(cd+3af)-b^2f(e^2+4df)-4af(ce^2+4af^2)-2(2cd-be+2af)(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\text{ ar}}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{}}}$$
$$-\frac{3\left(4bef(cd+3af)-b^2f(e^2+4df)-4af(ce^2+4af^2)-2(2cd-be+2af)(ce-bf)\left(e+\sqrt{e^2-4df}\right)\right)\text{ a}}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{}}}$$

output
```
-1/2*(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)^2+3/4*(4*c*d
*e+4*a*e*f-b*(4*d*f+e^2)+2*(4*a*f^2-2*b*e*f+c*e^2)*x)*(c*x^2+b*x+a)^(1/2)/
(-4*d*f+e^2)^2/(f*x^2+e*x+d)+3/8*(4*b*e*f*(3*a*f+c*d)-b^2*f*(4*d*f+e^2)-4*
a*f*(4*a*f^2+c*e^2)-2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))
*arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2
)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))
^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(5/2)/(c*e^2-2*c*d*f-b*e*
f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-3/8*(4*b*e*f*(3*a*f+c*d)-b^
2*f*(4*d*f+e^2)-4*a*f*(4*a*f^2+c*e^2)-2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e+(-
4*d*f+e^2)^(1/2)))*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e
+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(
-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(5/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4727 vs. $2(669) = 1338$.

Time = 18.30 (sec) , antiderivative size = 4727, normalized size of antiderivative = 7.07

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

output
```
(-2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/2)*(e - Sqrt[e^2 - 4*d*
f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^2*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x)) + (2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^
(3/2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))
/((e^2 - 4*d*f)^2*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (9*f^2*(a + x*(b + c*
x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c^2*f
*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f])))*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
- 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^2 - 4*d*f]) - 4*a*c*f*(e
 - Sqrt[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + 4*c*(
-e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + S
qrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2
- 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^
2*(a + b*x + c*x^2)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((-4*c*f*(4*a
*f - b*(e - Sqrt[e^2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(...
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1302, 27, 1346, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{\int \frac{3(be-4af+2(ce-bf)x)\sqrt{cx^2+bx+a}}{2(fx^2+ex+d)^2}dx}{2(e^2-4df)}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{(be-4af+2(ce-bf)x)\sqrt{cx^2+bx+a}}{(fx^2+ex+d)^2}dx}{4\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\Big\downarrow$ 1346

$$\frac{3\left(\frac{\int -\frac{-\left(\left(e^2+4df\right)b^2\right)+4e(cd+3af)b-4a\left(ce^2+4af^2\right)+4(2cd-be+2af)(ce-bf)x}{2\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{e^2-4df}+\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} \cdots\right.}{4\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\int \frac{-\left(\left(e^2+4df\right)b^2\right)+4e(cd+3af)b-4a\left(ce^2+4af^2\right)+4(2cd-be+2af)(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{2\left(e^2-4df\right)}\right) \cdots}{4\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\Big\downarrow$ 1365

$$3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\frac{2\left(2\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2 \cdots\right.\right.\right.}{\cdots}}{\sqrt{e^2-4df} \cdots} \cdots\right.$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\Big\downarrow$ 1154

$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{4\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(\right.\right.\right.\right.}{}\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\downarrow$ 219

$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\frac{\sqrt{2}\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b\right.\right.\right.}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e}}}}{}\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

output
```
-1/2*((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2
)^2) + (3*(((4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*
a*f^2)*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2)) - ((Sqr
t[2]*(2*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - f*(4*b
*e*(c*d + 3*a*f) - b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*
a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(
2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (Sqrt[2]*(2*(2*c*d -
b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3*a*f)
- b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f))))/(4*(e^2 - 4*d*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1346
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
 (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*
(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 16308 vs. 2(609) = 1218.

Time = 3.82 (sec) , antiderivative size = 16309, normalized size of antiderivative = 24.38

method	result	size
default	Expression too large to display	16309

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2+ex+d)^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f\,x^2+e\,x+d)^3}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{(f\,x^2+ex+d)^3}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)
```

output
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)
```

3.145 $\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 266

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \frac{(983+1150x)\sqrt{2+3x+5x^2}}{13068\,(4+x-2x^2)^2} - \frac{23(40605+1414x)\sqrt{2+3x+5x^2}}{133973136\,(4+x-2x^2)} - \frac{(1-4x)\,(2+3x+5x^2)^{3/2}}{99\,(4+x-2x^2)^3} - \frac{\left(687839+4799663\sqrt{33}\right)\operatorname{arctanh}\left(\frac{19-3\sqrt{33}+2\left(11-5\sqrt{33}\right)x}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{133973136\sqrt{66\,\left(107-11\sqrt{33}\right)}} + \frac{\left(687839-4799663\sqrt{33}\right)\operatorname{arctanh}\left(\frac{19+3\sqrt{33}+2\left(11+5\sqrt{33}\right)x}{2\sqrt{2\left(107+11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{133973136\sqrt{66\,\left(107+11\sqrt{33}\right)}}$$

output
```
1/13068*(983+1150*x)*(5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^2-23*(40605+1414*x)*
(5*x^2+3*x+2)^(1/2)/(-267946272*x^2+133973136*x+535892544)-1/99*(1-4*x)*(5
*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^3-1/133973136*(687839+4799663*33^(1/2))*arc
tanh(1/2*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)/(214-22*33^(1/2))^(1/2)/(5*x^
2+3*x+2)^(1/2))/(7062-726*33^(1/2))^(1/2)+1/133973136*(687839-4799663*33^(
1/2))*arctanh(1/2*(19+3*33^(1/2)+2*(11+5*33^(1/2))*x)/(214+22*33^(1/2))^(1
/2)/(5*x^2+3*x+2)^(1/2))/(7062+726*33^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.79 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.19

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \frac{-\frac{1721443144\sqrt{2+3x+5x^2}(-22661696-56011564x-14855765x^2-7709170x^3+3605572x^4+130088x^5)}{(4+x-2x^2)^3} - 2}{}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^(3/2)/(4 + x - 2*x^2)^4,x]
```

output
```
((-1721443144*Sqrt[2 + 3*x + 5*x^2]*(-22661696 - 56011564*x - 14855765*x^2
 - 7709170*x^3 + 3605572*x^4 + 130088*x^5))/(4 + x - 2*x^2)^3 - 2360951095
1410660*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 &
, Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]/(22*Sqrt[5] - 91*#1 + 3*S
qrt[5]*#1^2 + 4*#1^3) & ] - 97536502800*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#
1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (7727219261*Sqrt[5]*Log[-(Sqrt[5]*x) + S
qrt[2 + 3*x + 5*x^2] - #1]*#1 + 5054548389*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x
 + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ] -
 99*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (4
521207380523978672487*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #
1]*#1 + 2982791612362287724911*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] -
#1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ] + RootSum[-22
 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (4483532213381719
76800157*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 29578
93808593098836158611*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(
22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ])/230627136447379584
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1302, 27, 2132, 25, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^{3/2}}{(-2x^2+x+4)^4}dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{1}{99}\int \frac{\sqrt{5x^2+3x+2}(80x^2+114x+89)}{2(-2x^2+x+4)^3}dx-\frac{(1-4x)(5x^2+3x+2)^{3/2}}{99(-2x^2+x+4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{198}\int \frac{\sqrt{5x^2+3x+2}(80x^2+114x+89)}{(-2x^2+x+4)^3}dx-\frac{(1-4x)(5x^2+3x+2)^{3/2}}{99(-2x^2+x+4)^3}$$

$$\Big\downarrow \text{2132}$$

$$\frac{1}{198}\left(\frac{(1150x+983)\sqrt{5x^2+3x+2}}{66(-2x^2+x+4)^2}-\frac{1}{132}\int -\frac{-3400x^2-8420x+291}{(-2x^2+x+4)^2\sqrt{5x^2+3x+2}}dx\right)-\frac{(1-4x)(5x^2+3x+2)^{3/2}}{99(-2x^2+x+4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{198}\left(\frac{1}{132}\int \frac{-3400x^2-8420x+291}{(-2x^2+x+4)^2\sqrt{5x^2+3x+2}}dx+\frac{\sqrt{5x^2+3x+2}(1150x+983)}{66(-2x^2+x+4)^2}\right)-\frac{(1-4x)(5x^2+3x+2)^{3/2}}{99(-2x^2+x+4)^3}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{198}\left(\frac{1}{132}\left(-\frac{\int -\frac{3(2743751-9599326x)}{2(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx}{15378}-\frac{23\sqrt{5x^2+3x+2}(1414x+40605)}{5126(-2x^2+x+4)}\right)+\frac{\sqrt{5x^2+3x+2}(1150x+98}{66(-2x^2+x+4)^2}\right.$$
$$\frac{(1-4x)(5x^2+3x+2)^{3/2}}{99(-2x^2+x+4)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{198}\left(\frac{1}{132}\left(\frac{\int \frac{2743751-9599326x}{(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx}{10252}-\frac{23(1414x+40605)\sqrt{5x^2+3x+2}}{5126\left(-2x^2+x+4\right)}\right)+\frac{\sqrt{5x^2+3x+2}(1150x+983)}{66\left(-2x^2+x+4\right)^2}\right)$$
$$\frac{(1-4x)\left(5x^2+3x+2\right)^{3/2}}{99\left(-2x^2+x+4\right)^3}$$

$$\Big\downarrow \ 1365$$

$$\frac{1}{198}\left(\frac{1}{132}\left(\frac{-\frac{2}{33}\left(158388879+687839\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx-\frac{2}{33}\left(158388879-687839\sqrt{33}\right)\int \frac{1}{\left(-4x+\right.}}{10252}\right.\right.$$
$$\frac{(1-4x)\left(5x^2+3x+2\right)^{3/2}}{99\left(-2x^2+x+4\right)^3}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{198}\left(\frac{1}{132}\left(\frac{\frac{4}{33}\left(158388879+687839\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)+\frac{4}{33}}{10252}\right.\right.$$
$$\frac{(1-4x)\left(5x^2+3x+2\right)^{3/2}}{99\left(-2x^2+x+4\right)^3}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{198}\left(\frac{1}{132}\left(\frac{-\frac{1}{33}\sqrt{\frac{2}{107-11\sqrt{33}}}\left(158388879+687839\sqrt{33}\right)\operatorname{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)-\frac{1}{33}\left(158388879-\right.}{10252}\right.\right.$$
$$\frac{(1-4x)\left(5x^2+3x+2\right)^{3/2}}{99\left(-2x^2+x+4\right)^3}$$

input
```
Int[(2 + 3*x + 5*x^2)^(3/2)/(4 + x - 2*x^2)^4,x]
```

output
```
-1/99*((1 - 4*x)*(2 + 3*x + 5*x^2)^(3/2))/(4 + x - 2*x^2)^3 + (((983 + 115
0*x)*Sqrt[2 + 3*x + 5*x^2])/(66*(4 + x - 2*x^2)^2) + ((-23*(40605 + 1414*x
)*Sqrt[2 + 3*x + 5*x^2])/(5126*(4 + x - 2*x^2)) + (-1/33*(Sqrt[2/(107 - 11
*Sqrt[33])]*(158388879 + 687839*Sqrt[33])*ArcTanh[(19 - 3*Sqrt[33] + 2*(11
 - 5*Sqrt[33])*x)/(2*Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])])
- ((158388879 - 687839*Sqrt[33])*Sqrt[2/(107 + 11*Sqrt[33])]*ArcTanh[(19 +
 3*Sqrt[33] + 2*(11 + 5*Sqrt[33])*x)/(2*Sqrt[2*(107 + 11*Sqrt[33])]*Sqrt[2
 + 3*x + 5*x^2])])/33)/10252)/132)/198
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\left(130088x^5+3605572x^4-7709170x^3-14855765x^2-56011564x-22661696\right)\sqrt{5x^2+3x+2}}{133973136\left(2x^2-x-4\right)^3}-\frac{\left(-687839+4799663\sqrt{33}\right)\sqrt{33}\ \arctan}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x,method=_RETURNVERBOSE)
```

output
```
1/133973136*(130088*x^5+3605572*x^4-7709170*x^3-14855765*x^2-56011564*x-22
661696)/(2*x^2-x-4)^3*(5*x^2+3*x+2)^(1/2)-1/4421113488*(-687839+4799663*33
^(1/2))*33^(1/2)/(214+22*33^(1/2))^(1/2)*arctanh(8*(107/4+11/4*33^(1/2)+(1
1/2+5/2*33^(1/2))*(x-1/4*33^(1/2)-1/4))/(214+22*33^(1/2))^(1/2)/(80*(x-1/4
*33^(1/2)-1/4)^2+16*(11/2+5/2*33^(1/2))*(x-1/4*33^(1/2)-1/4)+214+22*33^(1/
2))^(1/2))-1/4421113488*(687839+4799663*33^(1/2))*33^(1/2)/(214-22*33^(1/2
))^(1/2)*arctanh(8*(107/4-11/4*33^(1/2)+(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^
(1/2)))/(214-22*33^(1/2))^(1/2)/(80*(x-1/4+1/4*33^(1/2))^2+16*(11/2-5/2*33
^(1/2))*(x-1/4+1/4*33^(1/2))+214-22*33^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(212) = 424$.

Time = 0.12 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.63

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x, algorithm="fricas")
```

output
```
-1/1071785088*((8*x^6 - 12*x^5 - 42*x^4 + 47*x^3 + 84*x^2 - 48*x - 64)*sqr
t(68742792623043/233*sqrt(33) + 20947563518842067/7689)*log(-(sqrt(5*x^2 +
 3*x + 2)*sqrt(68742792623043/233*sqrt(33) + 20947563518842067/7689)*(4806
69349*sqrt(33) - 1335207093) + 94967514957232*sqrt(33)*(3*x + 4) - 7312498
651706864*x - 1899350299144640)/x) - (8*x^6 - 12*x^5 - 42*x^4 + 47*x^3 + 8
4*x^2 - 48*x - 64)*sqrt(68742792623043/233*sqrt(33) + 20947563518842067/76
89)*log((sqrt(5*x^2 + 3*x + 2)*sqrt(68742792623043/233*sqrt(33) + 20947563
518842067/7689)*(480669349*sqrt(33) - 1335207093) - 94967514957232*sqrt(33
)*(3*x + 4) + 7312498651706864*x + 1899350299144640)/x) + (8*x^6 - 12*x^5
- 42*x^4 + 47*x^3 + 84*x^2 - 48*x - 64)*sqrt(-68742792623043/233*sqrt(33)
+ 20947563518842067/7689)*log((sqrt(5*x^2 + 3*x + 2)*(480669349*sqrt(33) +
 1335207093)*sqrt(-68742792623043/233*sqrt(33) + 20947563518842067/7689) +
 94967514957232*sqrt(33)*(3*x + 4) + 7312498651706864*x + 1899350299144640
)/x) - (8*x^6 - 12*x^5 - 42*x^4 + 47*x^3 + 84*x^2 - 48*x - 64)*sqrt(-68742
792623043/233*sqrt(33) + 20947563518842067/7689)*log(-(sqrt(5*x^2 + 3*x +
2)*(480669349*sqrt(33) + 1335207093)*sqrt(-68742792623043/233*sqrt(33) + 2
0947563518842067/7689) - 94967514957232*sqrt(33)*(3*x + 4) - 7312498651706
864*x - 1899350299144640)/x) - 8*(130088*x^5 + 3605572*x^4 - 7709170*x^3 -
 14855765*x^2 - 56011564*x - 22661696)*sqrt(5*x^2 + 3*x + 2))/(8*x^6 - 12*
x^5 - 42*x^4 + 47*x^3 + 84*x^2 - 48*x - 64)
```

Sympy [F]

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \int \frac{(5x^2+3x+2)^{\frac{3}{2}}}{(2x^2-x-4)^4}\,dx$$

input
```
integrate((5*x**2+3*x+2)**(3/2)/(-2*x**2+x+4)**4,x)
```

output
```
Integral((5*x**2 + 3*x + 2)**(3/2)/(2*x**2 - x - 4)**4, x)
```

Maxima [F]

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \int \frac{(5\,x^2+3\,x+2)^{\frac{3}{2}}}{(2\,x^2-x-4)^4}\,dx$$

input
```
integrate((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 3*x + 2)^(3/2)/(2*x^2 - x - 4)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(212) = 424$.

Time = 0.18 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.80

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx =$$

$$-\frac{38397304\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^{11}-106968264\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^{10}-4215429092\left(\sqrt{5}x-\right.}{}$$

$$-0.00189338515268464\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}+8.38267526007000\right)$$
$$-0.00392277827816914\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.312157316296000\right)$$
$$+0.00189338515268464\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.842024981991000\right)$$
$$+0.00392277827816914\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-4.99242498429000\right)$$

input
```
integrate((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x, algorithm="giac")
```

output
```
-1/133973136*(38397304*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^11 - 106968264*
sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^10 - 4215429092*(sqrt(5)*x - s
qrt(5*x^2 + 3*x + 2))^9 - 27627624304*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*
x + 2))^8 - 293612574754*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 304441105
682*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 - 962237337353*(sqrt(5)*
x - sqrt(5*x^2 + 3*x + 2))^5 - 368536654772*sqrt(5)*(sqrt(5)*x - sqrt(5*x^
2 + 3*x + 2))^4 - 353763969768*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 421
6436664*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 48130637060*sqrt(5
)*x + 5546394128*sqrt(5) - 48130637060*sqrt(5*x^2 + 3*x + 2))/(2*(sqrt(5)*
x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2
))^3 - 91*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 44*sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 3*x + 2)) - 22)^3 - 0.00189338515268464*log(-sqrt(5)*x + sqrt
(5*x^2 + 3*x + 2) + 8.38267526007000) - 0.00392277827816914*log(-sqrt(5)*x
 + sqrt(5*x^2 + 3*x + 2) - 0.312157316296000) + 0.00189338515268464*log(-s
qrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.842024981991000) + 0.0039227782781691
4*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 4.99242498429000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \int \frac{(5\,x^2+3\,x+2)^{3/2}}{(-2\,x^2+x+4)^4}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^(3/2)/(x - 2*x^2 + 4)^4,x)
```

output
```
int((3*x + 5*x^2 + 2)^(3/2)/(x - 2*x^2 + 4)^4, x)
```

Reduce [F]

$$\int \frac{(2+3x+5x^2)^{3/2}}{(4+x-2x^2)^4}\,dx = \int \frac{(5x^2+3x+2)^{\frac{3}{2}}}{(-2x^2+x+4)^4}dx$$

input
```
int((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x)
```

output
```
int((5*x^2+3*x+2)^(3/2)/(-2*x^2+x+4)^4,x)
```

3.146 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 98

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx = -\frac{1}{2}\arcsin(2+x) - \frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output

```
-1/2*arcsin(2+x)-1/2*arctan(1/2*(1-(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1
/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/2*arcta
nh(x/(-x^2-4*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx = \frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}} + \arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

output
```
ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2] + ArcTan[Sqrt[-3
- 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {1320, 1090, 223, 1361, 27, 1317, 27, 1359, 27, 1360, 219, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-4x-3}}{2x^2+4x+3}dx$$

$$\Big\downarrow \text{1320}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{-x^2-4x-3}}dx-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-4)^2}}d(-2x-4)-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1361}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1317

$$\frac{1}{2}\left(3\left(\frac{1}{6}\int -\frac{4x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{6}\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)-2\int \frac{2}{\sqrt{-x^2-4x}}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{2}{3}\int \frac{x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)-2\int \frac{2x+}{\sqrt{-x^2-4x-3}}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1359

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{16}{3}\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-2\int\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1360

$$\frac{1}{2}\left(3\left(\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-6\int \frac{}{3-}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(3\left(\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{}{\sqrt{}}\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \; 1475$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{}{\frac{(x}{9(-x}}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-}}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)+$$
$$\frac{1}{2}\left(3\left(\frac{2}{3}\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)$$

input
```
Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

output
```
ArcSin[(-4 - 2*x)/2]/2 + (3*((2*Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[-3
- 4*x - x^2])])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3) - 2*ArcTanh[x/Sqrt[
-3 - 4*x - x^2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

rule 1361
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(2*h*d - g*e)/e   Int[1/((a +
b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/e   Int[(2*d + e*x)/((
a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e,
0] && NeQ[2*h*d - g*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.24 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.32

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)x+2\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-4x-3}\right)}{2}+\operatorname{RootOf}\left(16\_Z^2+8\_Z+3\right)\ln$
default	$-\frac{\arcsin(2+x)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}-\frac{\sqrt{3}\,\sqrt{}}{}$

input

```
int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x,method=_RETURNVERBOSE)
```

output

```
1/2*RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*x+2*RootOf(_Z^2+1)+(-x^2-4*x-3)^(1/2)
)+RootOf(16*_Z^2+8*_Z+3)*ln(-(-16*RootOf(16*_Z^2+8*_Z+3)^2*x-24*RootOf(16*
_Z^2+8*_Z+3)*x+6*(-x^2-4*x-3)^(1/2)-24*RootOf(16*_Z^2+8*_Z+3)-5*x-6)/(4*Ro
otOf(16*_Z^2+8*_Z+3)*x-x-3))-1/2*ln(-(-16*RootOf(16*_Z^2+8*_Z+3)^2*x+8*Roo
tOf(16*_Z^2+8*_Z+3)*x+6*(-x^2-4*x-3)^(1/2)+24*RootOf(16*_Z^2+8*_Z+3)+3*x+6
)/(4*RootOf(16*_Z^2+8*_Z+3)*x+3*x+3))-ln(-(-16*RootOf(16*_Z^2+8*_Z+3)^2*x+
8*RootOf(16*_Z^2+8*_Z+3)*x+6*(-x^2-4*x-3)^(1/2)+24*RootOf(16*_Z^2+8*_Z+3)+
3*x+6)/(4*RootOf(16*_Z^2+8*_Z+3)*x+3*x+3))*RootOf(16*_Z^2+8*_Z+3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4\,x-3}(x+2)}{x^2+4\,x+3}\right) + \frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) - \frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x
+ 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3
))/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) +
 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2
 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = \int \frac{\sqrt{-(x+1)(x+3)}}{2x^2+4x+3}\,dx$$

input
```
integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

output
```
Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = \int \frac{\sqrt{-x^2-4x-3}}{2x^2+4x+3}\,dx$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx &= -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) \\ &\quad -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) \\ &\quad -\frac{1}{2}\arcsin\left(x+2\right)-\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}\right. \\ &\qquad \left.+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{\left(x+2\right)^2}+1\right) \\ &\quad +\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{\left(x+2\right)^2}\right. \\ &\qquad \left.+3\right)\end{aligned}$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*
(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx=\int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\,dx$$

input
```
int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

output
```
int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx=-\frac{asin(x+2)}{2}+\frac{\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin(x+2)}{2}\right)-1}{\sqrt{2}}\right)}{2}+\frac{\sqrt{2}\,atan\left(\frac{3\tan\left(\frac{asin(x+2)}{2}\right)-1}{\sqrt{2}}\right)}{2}+\frac{\log\left(\tan\left(\frac{asin(x+2)}{2}\right)^2-2\tan\left(\frac{asin(x+2)}{2}\right)+3\right)}{4}-\frac{\log\left(3\tan\left(\frac{asin(x+2)}{2}\right)^2-2\tan\left(\frac{asin(x+2)}{2}\right)+1\right)}{4}$$

input
```
int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x)
```

output
```
( - 2*asin(x + 2) + 2*sqrt(2)*atan((tan(asin(x + 2)/2) - 1)/sqrt(2)) + 2*s
qrt(2)*atan((3*tan(asin(x + 2)/2) - 1)/sqrt(2)) + log(tan(asin(x + 2)/2)**
2 - 2*tan(asin(x + 2)/2) + 3) - log(3*tan(asin(x + 2)/2)**2 - 2*tan(asin(x
 + 2)/2) + 1))/4
```

3.147 $\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 717

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{(23040c^5d^2e - 3465b^5f^3 + 420b^3cf^2(27be+34af) - 504bc^2f(70abef+22a^2f^2+25b^2(e^2+df)) - 640c \cdots}{76 \cdots}$$

$$+ \frac{(1155b^4f^3 - 252b^2cf^2(15be+14af) + 5760c^4d(e^2+df) + 24c^2f(322abef+50a^2f^2+175b^2(e^2+df) \cdots}{3840c^5}$$

$$- \frac{(231b^3f^3 - 36bcf^2(21be+13af) - 320c^3(e^3+6def) + 24c^2f(32aef+35b(e^2+df)))\,x^2\sqrt{a+bx+c \cdots}}{960c^4}$$

$$+ \frac{f(99b^2f^2 - 4cf(81be+25af) + 360c^2(e^2+df))\,x^3\sqrt{a+bx+cx^2}}{480c^3}$$

$$+ \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$+ \frac{(1024c^6d^3 + 231b^6f^3 - 252b^4cf^2(3be+5af) - 1536c^5d(bde+a(e^2+df)) + 840b^2c^2f(4abef+2a^2f^2 \cdots}{\cdots}$$

output
```
1/7680*(23040*c^5*d^2*e-3465*b^5*f^3+420*b^3*c*f^2*(34*a*f+27*b*e)-504*b*c
^2*f*(70*a*b*e*f+22*a^2*f^2+25*b^2*(d*f+e^2))-640*c^4*(27*b*d*(d*f+e^2)+8*
a*e*(6*d*f+e^2))+96*c^3*(128*a^2*e*f^2+275*a*b*f*(d*f+e^2)+50*b^2*(6*d*e*f
+e^3)))*(c*x^2+b*x+a)^(1/2)/c^6+1/3840*(1155*b^4*f^3-252*b^2*c*f^2*(14*a*f
+15*b*e)+5760*c^4*d*(d*f+e^2)+24*c^2*f*(322*a*b*e*f+50*a^2*f^2+175*b^2*(d*
f+e^2))-160*c^3*(27*a*f*(d*f+e^2)+10*b*(6*d*e*f+e^3)))*x*(c*x^2+b*x+a)^(1/
2)/c^5-1/960*(231*b^3*f^3-36*b*c*f^2*(13*a*f+21*b*e)-320*c^3*(6*d*e*f+e^3)
+24*c^2*f*(32*a*e*f+35*b*(d*f+e^2)))*x^2*(c*x^2+b*x+a)^(1/2)/c^4+1/480*f*(
99*b^2*f^2-4*c*f*(25*a*f+81*b*e)+360*c^2*(d*f+e^2))*x^3*(c*x^2+b*x+a)^(1/2
)/c^3+1/60*f^2*(-11*b*f+36*c*e)*x^4*(c*x^2+b*x+a)^(1/2)/c^2+1/6*f^3*x^5*(c
*x^2+b*x+a)^(1/2)/c+1/1024*(1024*c^6*d^3+231*b^6*f^3-252*b^4*c*f^2*(5*a*f+
3*b*e)-1536*c^5*d*(b*d*e+a*(d*f+e^2))+840*b^2*c^2*f*(4*a*b*e*f+2*a^2*f^2+b
^2*(d*f+e^2))+384*c^4*(3*b^2*d*(d*f+e^2)+3*a^2*f*(d*f+e^2)+2*a*b*e*(6*d*f+
e^2))-320*c^3*(9*a^2*b*e*f^2+a^3*f^3+9*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 7.78 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^5f^3+210b^3cf^2(54be+68af+11bfx)-168bc^2f(66a^2f^2+42abf(5e+fx)}{}$$

input
```
Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68
*a*f + 11*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(7
5*e^2 + 75*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 1
5*d*x*(6*e^2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*
x^2 + 10*f^3*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 1
75*e^2*f*x + 6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*
f*(275*e^2 + 161*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*
e^2*f*x + 96*e*f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f
+ 15*d*(18*e^2 + 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^
2*x^2 + 22*f^3*x^3)))) + 15*(1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3
*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*
e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*
f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3
+ 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt
[a] + Sqrt[a + x*(b + c*x)])]])/(7680*c^(13/2))
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\int \frac{f^2(36ce-11bf)x^5-2f(5af^2-18c(e^2+df))x^4+12ce(e^2+6df)x^3+36cd(e^2+df)x^2+36cd^2ex+12cd^3}{2\sqrt{cx^2+bx+a}}dx}{6c}+\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{f^2(36ce-11bf)x^5-2f(5af^2-18c(e^2+df))x^4+12ce(e^2+6df)x^3+36cd(e^2+df)x^2+36cd^2ex+12cd^3}{\sqrt{cx^2+bx+a}}dx}{12c}+\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\frac{\int \frac{f\left(360\left(e^2+df\right)c^2-4f(81be+25af)c+99b^2f^2\right)x^4-8\left(-11abf^3+36acef^2-15c^2\left(e^3+6dfe\right)\right)x^3+360c^2d\left(e^2+df\right)x^2+360c^2d^2ex+120c^2d^3}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{f^2x^4\sqrt{a+bx+}}{}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{f\left(360\left(e^2+df\right)c^2-4f(81be+25af)c+99b^2f^2\right)x^4-8\left(-11abf^3+36acef^2-15c^2\left(e^3+6dfe\right)\right)x^3+360c^2d\left(e^2+df\right)x^2+360c^2d^2ex+120c^2d^3}{\sqrt{cx^2+bx+a}}dx}{10c}+\frac{f^2x^4\sqrt{a+bx+}}{}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\frac{\frac{\int \frac{3\left(320d^3c^3+960d^2exc^3-\left(-320\left(e^3+6dfe\right)c^3+24f\left(32aef+35b\left(e^2+df\right)\right)c^2-36bf^2(21be+13af)c+231b^3f^3\right)x^3-2\left(-480d\left(e^2+df\right)c^3+360af\left(e^2+df\right)c^2-4af^2(\right.\right.}{2\sqrt{cx^2+bx+a}}}{4c}}{10c}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int \frac{320d^3c^3+960d^2exc^3-\left(-320\left(e^3+6dfe\right)c^3+24f\left(32aef+35b\left(e^2+df\right)\right)c^2-36bf^2(21be+13af)c+231b^3f^3\right)x^3-2\left(-480d\left(e^2+df\right)c^3+360af\left(e^2+df\right)c^2-4af^2(8\right.}{\sqrt{cx^2+bx+a}}}{8c}}{10c}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 2192}$$

$$3\left(\frac{\int \frac{1920d^3c^4+\left(1155f^3b^4-252cf^2(15be+14af)b^2+5760c^4d\left(e^2+df\right)+24c^2f\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)-160c^3\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)\right)x^2+4}{2\sqrt{cx^2+bx+a}}}{3c}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \text{ 27}$$

$$3\left(\frac{\int \frac{1920d^3c^4+\left(1155f^3b^4-252cf^2(15be+14af)b^2+5760c^4d\left(e^2+df\right)+24c^2f\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)-160c^3\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)\right)x^2+4}{\sqrt{cx^2+bx+a}}}{6c}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 2192

$$3\left(\int \frac{7680d^3c^5-11520ad\left(e^2+df\right)c^4+320a\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)c^3-48af\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)c^2+504ab^2f^2(15be+14af)c-2310ab^4f^3}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 27

$$3\left(\int \frac{2\left(3840d^3c^5-5760ad\left(e^2+df\right)c^4+160a\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)c^3-24af\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)c^2+252ab^2f^2(15be+14af)c-1155ab^4f\right.}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 1160

$$3\left(\frac{15\left(384c^4\left(3a^2f\left(df+e^2\right)+2abe\left(6df+e^2\right)+3b^2d\left(df+e^2\right)\right)+840b^2c^2f\left(2a^2f^2+4abef+b^2\left(df+e^2\right)\right)-320c^3\left(a^3f^3+9a^2bef^2+9ab^2f\left(df+e^2\right)+b^3\left(6def+e^3\right)\right)-\right.}{2c}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 1092

$$3\left(\frac{15\left(384c^4\left(3a^2f\left(df+e^2\right)+2abe\left(6df+e^2\right)+3b^2d\left(df+e^2\right)\right)+840b^2c^2f\left(2a^2f^2+4abef+b^2\left(df+e^2\right)\right)-320c^3\left(a^3f^3+9a^2bef^2+9ab^2f\left(df+e^2\right)+b^3\left(6def+e^3\right)\right)-\cdots}{c}\cdots\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\downarrow$ 219

$$3\left(\frac{x\sqrt{a+bx+cx^2}\left(24c^2f\left(50a^2f^2+322abef+175b^2\left(df+e^2\right)\right)-252b^2cf^2(14af+15be)-160c^3\left(27af\left(df+e^2\right)+10b\left(6def+e^3\right)\right)+1155b^4f^3+5760c^4d\left(df+e^2\right)\right)}{2c}+\cdots\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

input

```
Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a
 + b*x + c*x^2])/(5*c) + ((f*(99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c
^2*(e^2 + d*f))*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + (3*(-1/3*((231*b^3*f^3
- 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f*(32*a*
e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/c + (((1155*b^4*f^3 -
252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*f*(322*a
*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 + d*f) +
 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((23040*c^5*d^2*
e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*c^2*f*(70*a*b*e
*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2 + d*f) + 8*a*
e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d*f) + 50*b^2*
(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/c + (15*(1024*c^6*d^3 + 231*b^6*f
^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d
*(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a
^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c))/(6*
c)))/(8*c))/(10*c))/(12*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{(-1280f^3c^5x^5+1408b\,c^4f^3x^4-4608c^5e\,f^2x^4+1600a\,c^4f^3x^3-1584b^2c^3f^3x^3+5184b\,c^4e\,f^2x^3-5760c^5d\,f^2x^3-5760c^5e^2f\,x^3-37}{}$
default	Expression too large to display

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-1280*c^5*f^3*x^5+1408*b*c^4*f^3*x^4-4608*c^5*e*f^2*x^4+1600*a*c^
4*f^3*x^3-1584*b^2*c^3*f^3*x^3+5184*b*c^4*e*f^2*x^3-5760*c^5*d*f^2*x^3-576
0*c^5*e^2*f*x^3-3744*a*b*c^3*f^3*x^2+6144*a*c^4*e*f^2*x^2+1848*b^3*c^2*f^3
*x^2-6048*b^2*c^3*e*f^2*x^2+6720*b*c^4*d*f^2*x^2+6720*b*c^4*e^2*f*x^2-1536
0*c^5*d*e*f*x^2-2560*c^5*e^3*x^2-2400*a^2*c^3*f^3*x+7056*a*b^2*c^2*f^3*x-1
5456*a*b*c^3*e*f^2*x+8640*a*c^4*d*f^2*x+8640*a*c^4*e^2*f*x-2310*b^4*c*f^3*
x+7560*b^3*c^2*e*f^2*x-8400*b^2*c^3*d*f^2*x-8400*b^2*c^3*e^2*f*x+19200*b*c
^4*d*e*f*x+3200*b*c^4*e^3*x-11520*c^5*d^2*f*x-11520*c^5*d*e^2*x+11088*a^2*
b*c^2*f^3-12288*a^2*c^3*e*f^2-14280*a*b^3*c*f^3+35280*a*b^2*c^2*e*f^2-2640
0*a*b*c^3*d*f^2-26400*a*b*c^3*e^2*f+30720*a*c^4*d*e*f+5120*a*c^4*e^3+3465*
b^5*f^3-11340*b^4*c*e*f^2+12600*b^3*c^2*d*f^2+12600*b^3*c^2*e^2*f-28800*b^
2*c^3*d*e*f-4800*b^2*c^3*e^3+17280*b*c^4*d^2*f+17280*b*c^4*d*e^2-23040*c^5
*d^2*e)/c^6*(c*x^2+b*x+a)^(1/2)-1/1024*(320*a^3*c^3*f^3-1680*a^2*b^2*c^2*f
^3+2880*a^2*b*c^3*e*f^2-1152*a^2*c^4*d*f^2-1152*a^2*c^4*e^2*f+1260*a*b^4*c
*f^3-3360*a*b^3*c^2*e*f^2+2880*a*b^2*c^3*d*f^2+2880*a*b^2*c^3*e^2*f-4608*a
*b*c^4*d*e*f-768*a*b*c^4*e^3+1536*a*c^5*d^2*f+1536*a*c^5*d*e^2-231*b^6*f^3
+756*b^5*c*e*f^2-840*b^4*c^2*d*f^2-840*b^4*c^2*e^2*f+1920*b^3*c^3*d*e*f+32
0*b^3*c^3*e^3-1152*b^2*c^4*d^2*f-1152*b^2*c^4*d*e^2+1536*b*c^5*d^2*e-1024*
c^6*d^3)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 1583, normalized size of antiderivative = 2.21

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)
*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*
a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^
2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b
^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 12
0*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 +
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^
5 + 23040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3
)*x^4 + 320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 +
528*a^2*b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36
*(10*c^6*d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (9
45*b^4*c^2 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(7
7*b^3*c^3 - 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)
*f^2 + 120*(16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15
*b^2*c^4 - 16*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^
6*d*e^2 - 1600*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*
f^3 + 12*(10*(35*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^
2 + 120*(48*c^6*d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sq
rt(c*x^2 + b*x + a))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e +
384*(3*b^2*c^4 - 4*a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (23...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(750) = 1500$.

Time = 0.98 (sec) , antiderivative size = 1787, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**3*x**5/(6*c) + x**4*(-11*b*f**3/(12*
c) + 3*e*f**2)/(5*c) + x**3*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*
e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) + x**2*(-4*a*(-11*b*f**3/(12*c
) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*
f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e**3)/(3*c) + x*(-3*
a*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2
+ 3*e**2*f)/(4*c) - 5*b*(-4*a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(
-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3
*e**2*f)/(8*c) + 6*d*e*f + e**3)/(6*c) + 3*d**2*f + 3*d*e**2)/(2*c) + (-2*
a*(-4*a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*
(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e
*f + e**3)/(3*c) - 3*b*(-3*a*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3
*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) - 5*b*(-4*a*(-11*b*f**3/(12*c
) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*
f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e**3)/(6*c) + 3*d**2
*f + 3*d*e**2)/(4*c) + 3*d**2*e)/c) + (-a*(-3*a*(-5*a*f**3/(6*c) - 9*b*(-1
1*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) - 5*b*(-4*
a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b
*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e
**3)/(6*c) + 3*d**2*f + 3*d*e**2)/(2*c) - b*(-2*a*(-4*a*(-11*b*f**3/(12...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c + (36*c^5*e*f^2 - 11*
b*c^4*f^3)/c^6)*x + (360*c^5*e^2*f + 360*c^5*d*f^2 - 324*b*c^4*e*f^2 + 99*
b^2*c^3*f^3 - 100*a*c^4*f^3)/c^6)*x + (320*c^5*e^3 + 1920*c^5*d*e*f - 840*
b*c^4*e^2*f - 840*b*c^4*d*f^2 + 756*b^2*c^3*e*f^2 - 768*a*c^4*e*f^2 - 231*
b^3*c^2*f^3 + 468*a*b*c^3*f^3)/c^6)*x + (5760*c^5*d*e^2 - 1600*b*c^4*e^3 +
 5760*c^5*d^2*f - 9600*b*c^4*d*e*f + 4200*b^2*c^3*e^2*f - 4320*a*c^4*e^2*f
 + 4200*b^2*c^3*d*f^2 - 4320*a*c^4*d*f^2 - 3780*b^3*c^2*e*f^2 + 7728*a*b*c
^3*e*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3)/c^6)*x
+ (23040*c^5*d^2*e - 17280*b*c^4*d*e^2 + 4800*b^2*c^3*e^3 - 5120*a*c^4*e^3
 - 17280*b*c^4*d^2*f + 28800*b^2*c^3*d*e*f - 30720*a*c^4*d*e*f - 12600*b^3
*c^2*e^2*f + 26400*a*b*c^3*e^2*f - 12600*b^3*c^2*d*f^2 + 26400*a*b*c^3*d*f
^2 + 11340*b^4*c*e*f^2 - 35280*a*b^2*c^2*e*f^2 + 12288*a^2*c^3*e*f^2 - 346
5*b^5*f^3 + 14280*a*b^3*c*f^3 - 11088*a^2*b*c^2*f^3)/c^6) - 1/1024*(1024*c
^6*d^3 - 1536*b*c^5*d^2*e + 1152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 - 320*b^
3*c^3*e^3 + 768*a*b*c^4*e^3 + 1152*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f - 1920
*b^3*c^3*d*e*f + 4608*a*b*c^4*d*e*f + 840*b^4*c^2*e^2*f - 2880*a*b^2*c^3*e
^2*f + 1152*a^2*c^4*e^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^2 + 115
2*a^2*c^4*d*f^2 - 756*b^5*c*e*f^2 + 3360*a*b^3*c^2*e*f^2 - 2880*a^2*b*c^3*
e*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*f^3 - 320*a^3*c^
3*f^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f\,x^2+ex+d)^3}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

3.148 $\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 316

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2(16aef+9b(e^2+2df)))\sqrt{a+bx+cx^2}}{192c^4} + \frac{(35b^2f^2-4cf(20be+9af)+48c^2(e^2+2df))\,x\sqrt{a+bx+cx^2}}{96c^3} + \frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{(128c^4d^2+35b^4f^2-40b^2cf(2be+3af)-64c^3(2bde+a(e^2+2df))+48c^2(4abef+a^2f^2+b^2(e^2+2}{128c^{9/2}}$$

output

```
1/192*(384*c^3*d*e-105*b^3*f^2+20*b*c*f*(11*a*f+12*b*e)-16*c^2*(16*a*e*f+9
*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(1/2)/c^4+1/96*(35*b^2*f^2-4*c*f*(9*a*f+20*
b*e)+48*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(1/2)/c^3+1/24*f*(-7*b*f+16*c*e)*
x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*f^2*x^3*(c*x^2+b*x+a)^(1/2)/c+1/128*(128*c
^4*d^2+35*b^4*f^2-40*b^2*c*f*(3*a*f+2*b*e)-64*c^3*(2*b*d*e+a*(2*d*f+e^2))+
48*c^2*(4*a*b*e*f+a^2*f^2+b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3f^2+10bcf(24be+22af+7bfx)+16c^3(12d(2e+fx)+x(6e^2+8efx+3f\cdots}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f +
 7*b*f*x) + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) -
8*c^2*(a*f*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))) +
 3*(128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*
e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*A
rcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(9/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{f(16ce-7bf)x^3-2(3af^2-4c(e^2+2df))x^2+16cdex+8cd^2}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{f(16ce-7bf)x^3-2(3af^2-4c(e^2+2df))x^2+16cdex+8cd^2}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{48c^2d^2+\left(48\left(e^2+2df\right)c^2-4f(20be+9af)c+35b^2f^2\right)x^2+4\left(24dec^2-16aefc+7abf^2\right)x}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{fx^2\sqrt{a+bx+cx^2}(16ce-7bf)}{3c}}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{48c^2d^2+\left(48\left(e^2+2df\right)c^2-4f(20be+9af)c+35b^2f^2\right)x^2+4\left(24dec^2-16aefc+7abf^2\right)x}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{fx^2\sqrt{a+bx+cx^2}(16ce-7bf)}{3c}}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\frac{\int \frac{192d^2c^3-96a\left(e^2+2df\right)c^2+8af(20be+9af)c-70ab^2f^2+\left(-105f^2b^3+20cf(12be+11af)b+384c^3de-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{x\sqrt{a+bx+cx^2}\left(-4cf(9a\cdots\right.}{\cdots}}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{2\left(96d^2c^3-48a\left(e^2+2df\right)c^2+4af(20be+9af)c-35ab^2f^2\right)+\left(-105f^2b^3+20cf(12be+11af)b+384c^3de-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{x\sqrt{a+bx+cx^2}\left(-4cf(\cdots\right.}{\cdots}}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\frac{\frac{3\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2\left(16aef+9b\left(2df+\cdots\right.\right.\right.}{\cdots}}{4c}\cdots}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\frac{\frac{3\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}}{4c}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2\left(1\ldots\right.\right.}{\ldots}}{6c}}{8\ldots}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2\left(1\ldots\right.\right.}{\ldots}}{4c}}{6c}}{8\ldots}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

input
```
Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f^2*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + ((f*(16*c*e - 7*b*f)*x^2*Sqrt[a +
b*x + c*x^2])/(3*c) + (((35*b^2*f^2 - 4*c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2
 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((384*c^3*d*e - 105*b^3*f^2 +
 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f + 9*b*(e^2 + 2*d*f)))*Sqrt[
a + b*x + c*x^2])/c + (3*(128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3
*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 +
 b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(2*c^(3/2)))/(4*c))/(6*c))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(48c^3f^2x^3-56b\,c^2f^2x^2+128c^3ef\,x^2-72a\,c^2f^2x+70b^2c\,f^2x-160b\,c^2efx+192c^3dfx+96c^3e^2x+220abc\,f^2-256a\,c^2ef-105b^3f^2+2\cdots}{192c^4}$
default	$\frac{d^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+f^2\left(\frac{x^3\sqrt{c\,x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\cdots\right)}{2\cdots}\right)}{4c}\cdots\right)}{6c}\cdots\right)}{\cdots}\cdots\right)\cdots$

input

```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/192*(48*c^3*f^2*x^3-56*b*c^2*f^2*x^2+128*c^3*e*f*x^2-72*a*c^2*f^2*x+70*b
^2*c*f^2*x-160*b*c^2*e*f*x+192*c^3*d*f*x+96*c^3*e^2*x+220*a*b*c*f^2-256*a*
c^2*e*f-105*b^3*f^2+240*b^2*c*e*f-288*b*c^2*d*f-144*b*c^2*e^2+384*c^3*d*e)
/c^4*(c*x^2+b*x+a)^(1/2)+1/128*(48*a^2*c^2*f^2-120*a*b^2*c*f^2+192*a*b*c^2
*e*f-128*a*c^3*d*f-64*a*c^3*e^2+35*b^4*f^2-80*b^3*c*e*f+96*b^2*c^2*d*f+48*
b^2*c^2*e^2-128*b*c^3*d*e+128*c^4*d^2)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 637, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\,(128\,c^4d^2 - 128\,bc^3de + 16\,(3\,b^2c^2 - 4\,ac^3)e^2 + (35\,b^4 - 120\,ab^2c + 48\,a^2c^2)f^2 + 16\,(2\,(3\,b^2c^2 - 4\,ac^3)\right.$$
$$-\frac{3\,(128\,c^4d^2 - 128\,bc^3de + 16\,(3\,b^2c^2 - 4\,ac^3)e^2 + (35\,b^4 - 120\,ab^2c + 48\,a^2c^2)f^2 + 16\,(2\,(3\,b^2c^2 - 4\,ac^3}{}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (3
5*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5
*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c
^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7
*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^
4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(
c*x^2 + b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*
c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^
2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^
4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 +
8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3
)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c
^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(318) = 636$.

Time = 0.75 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.03

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}}\, dx$$

$$= \begin{cases} \sqrt{a + bx + cx^2}\left(\frac{f^2x^3}{4c} + \frac{x^2\left(-\frac{7bf^2}{8c} + 2ef\right)}{3c} + \frac{x\left(-\frac{3af^2}{4c} - \frac{5b\left(-\frac{7bf^2}{8c}+2ef\right)}{6c} + 2df + e^2\right)}{2c} + \frac{-\frac{2a\left(-\frac{7bf^2}{8c}+2ef\right)}{3c} - \frac{3b\left(-\frac{3af^2}{4c} - \frac{5b\left(-\frac{7}{}\right.}{}\right.}{}}{c}\right. \\ \frac{2\left(\frac{f^2(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{(a+bx)^{\frac{7}{2}}\left(-4af^2+2bef\right)}{7b^4} + \frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2f^2-6abef+2b^2df+b^2e^2\right)}{5b^4} + \frac{(a+bx)^{\frac{3}{2}}\left(-4a^3f^2+6a^2bef-4ab^2df-2ab^2e^2+2b^3de\right)}{3b^4} + \frac{\sqrt{a+bx}\left(a^4\right.}{}\right.}{b} \\ \frac{d^2x+dex^2+\frac{efx^4}{2}+\frac{f^2x^5}{5}+\frac{x^3\cdot\left(2df+e^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**2*x**3/(4*c) + x**2*(-7*b*f**2/(8*c)
 + 2*e*f)/(3*c) + x*(-3*a*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c)
 + 2*d*f + e**2)/(2*c) + (-2*a*(-7*b*f**2/(8*c) + 2*e*f)/(3*c) - 3*b*(-3*a
*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2*d*f + e**2)/(4*c) +
2*d*e)/c) + (-a*(-3*a*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2
*d*f + e**2)/(2*c) - b*(-2*a*(-7*b*f**2/(8*c) + 2*e*f)/(3*c) - 3*b*(-3*a*f
**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2*d*f + e**2)/(4*c) + 2*
d*e)/(2*c) + d**2)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqr
t(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f**2*(a + b*x)**(9/2)/(9*b**
4) + (a + b*x)**(7/2)*(-4*a*f**2 + 2*b*e*f)/(7*b**4) + (a + b*x)**(5/2)*(6
*a**2*f**2 - 6*a*b*e*f + 2*b**2*d*f + b**2*e**2)/(5*b**4) + (a + b*x)**(3/
2)*(-4*a**3*f**2 + 6*a**2*b*e*f - 4*a*b**2*d*f - 2*a*b**2*e**2 + 2*b**3*d*
e)/(3*b**4) + sqrt(a + b*x)*(a**4*f**2 - 2*a**3*b*e*f + 2*a**2*b**2*d*f +
a**2*b**2*e**2 - 2*a*b**3*d*e + b**4*d**2)/b**4)/b, Ne(b, 0)), ((d**2*x +
d*e*x**2 + e*f*x**4/2 + f**2*x**5/5 + x**3*(2*d*f + e**2)/3)/sqrt(a), True
))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,f^2x}{c}+\frac{16\,c^3ef-7\,bc^2f^2}{c^4}\right)x+\frac{48\,c^3e^2+96\,c^3df-80\,bc^2ef+35\,b^2cf^2-36\right.\right.$$
$$-\frac{(128\,c^4d^2-128\,bc^3de+48\,b^2c^2e^2-64\,ac^3e^2+96\,b^2c^2df-128\,ac^3df-80\,b^3cef+192\,abc^2ef+35\,b^4f}{128\,c^{\frac{9}{2}}}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c + (16*c^3*e*f - 7*b*c^2*f^2)/
c^4)*x + (48*c^3*e^2 + 96*c^3*d*f - 80*b*c^2*e*f + 35*b^2*c*f^2 - 36*a*c^2
*f^2)/c^4)*x + (384*c^3*d*e - 144*b*c^2*e^2 - 288*b*c^2*d*f + 240*b^2*c*e*
f - 256*a*c^2*e*f - 105*b^3*f^2 + 220*a*b*c*f^2)/c^4) - 1/128*(128*c^4*d^2
 - 128*b*c^3*d*e + 48*b^2*c^2*e^2 - 64*a*c^3*e^2 + 96*b^2*c^2*d*f - 128*a*
c^3*d*f - 80*b^3*c*e*f + 192*a*b*c^2*e*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 +
48*a^2*c^2*f^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b)
)/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{{\left(f\,x^2+e\,x+d\right)}^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}}\, dx = \int \frac{(f\,x^2 + ex + d)^2}{\sqrt{c\,x^2 + bx + a}} dx$$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

3.149 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{(8c^2d+3b^2f-4c(be+af))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x^2+b*x+a)^(1/2)/c+1
/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{c}(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4c(be+af))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^
2*f - 4*c*(b*e + a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x
)])])/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{4cd-2af+(4ce-3bf)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2(2cd-af)+(4ce-3bf)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2
])/c + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-2cfx+3fb-4ce)\sqrt{c\,x^2+bx+a}}{4c^2}-\frac{(4acf-3b^2f+4bce-8c^2d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+e\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+c}}{c}\right.}{}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*c*f*x+3*b*f-4*c*e)/c^2*(c*x^2+b*x+a)^(1/2)-1/8*(4*a*c*f-3*b^2*f+4
*b*c*e-8*c^2*d)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)}{16\,c^3}\right.$$
$$-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx+4\,c^2e-3\,bcf)\sqrt{cx^2+}}{8\,c^3}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2
*c^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d -
 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b
*c*f)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \left(\frac{fx}{2c}+\frac{-\frac{3bf}{4c}+e}{c}\right)\sqrt{a+bx+cx^2}+\left(-\frac{af}{2c}-\frac{b\left(-\frac{3bf}{4c}+e\right)}{2c}+d\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((f*x/(2*c) + (-3*b*f/(4*c) + e)/c)*sqrt(a + b*x + c*x**2) + (-a
*f/(2*c) - b*(-3*b*f/(4*c) + e)/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) +
 x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*d*sq
rt(a + b*x) + 2*e*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sq
rt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b, Ne(b,
0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}+\frac{4\,ce-3\,bf}{c^2}\right) - \frac{(8\,c^2d-4\,bce+3\,b^2f-4\,acf)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c + (4*c*e - 3*b*f)/c^2) - 1/8*(8*c^2*d -
 4*b*c*e + 3*b^2*f - 4*a*c*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{-6\sqrt{c\,x^2+bx+a}\,bcf+8\sqrt{c\,x^2+bx+a}\,c^2e+4\sqrt{c\,x^2+bx+a}\,c^2fx-4\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)a}{}$$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x + c*x**2)*b*c*f + 8*sqrt(a + b*x + c*x**2)*c**2*e + 4*s
qrt(a + b*x + c*x**2)*c**2*f*x - 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*f + 3*sqrt(c)*log((2*sqrt(c)*s
qrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*f - 4*sqrt(c)*
log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b*c
*e + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a
*c - b**2))*c**2*d)/(8*c**3)
```

3.150 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output

```
-2^(1/2)*f*arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f
+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*
e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+2^(1/2)*f*arctanh(1/4*(4*
a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*
x+a)^(1/2))/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\text{RootSum}\Big[c^2d-bce+b^2f+2\sqrt{a}ce\#1 - 4\sqrt{a}bf\#1-2cd\#1^2+be\#1^2+4af\#1^2-2\sqrt{a}e\#1^3 + d\#1^4\&, \frac{c\log(x)-c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)-\log(x)\#1^2+\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-\right.}{\sqrt{a}ce-2\sqrt{a}bf-2cd\#1+be\#1+4af\#1-3\sqrt{a}e\#1^2+2d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c
*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c*Log[x]
- c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - Log[x]*#1^2 + Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2
*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1314, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1314}$$

$$\frac{2f\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

input

```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output

```
-((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e -
Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d
*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]
])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1314
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[1/(
(b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[2*(c/q)   Int[1/((b +
 q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ
[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(330) = 660$.

Time = 2.92 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{2}\,\ln\left(\frac{fb\sqrt{-4df+e^2}-\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2dfc+c\,e^2}{f^2}+\frac{\left(c\sqrt{-4df+e^2}+fb-ce\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\sqrt{2}\sqrt{\frac{fb\sqrt{-4df+e^2}-\sqrt{-4d}}{}}\right.}{\sqrt{-4df+e^2}\sqrt{\frac{f}{}}}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*ln(((f*b*(-4*d*f+e^2)^(1/2)-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2
)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((f*b*(-4*d*f+e
^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*(
4*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+4*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2))))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*ln(((-f*b*(-4*d*f+
e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2+1/f*(-c
*(-4*d*f+e^2)^(1/2)+f*b-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*
((-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e
^2)/f^2)^(1/2)*(4*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2+4/f*(-c*(-4*d*f+e^2
)^(1/2)+f*b-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-f*b*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11287 vs. $2(328) = 656$.

Time = 3.74 (sec) , antiderivative size = 11287, normalized size of antiderivative = 30.18

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+bx+a}\,(f\,x^2+ex+d)}dx$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

output
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

3.151 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 787

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx$$
$$=\frac{(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2df))\,x)\sqrt{a+bx+cx^2}}{(e^2-4df)\,((cd-af)^2-(bd-ae)(ce-bf))\,(d+ex+fx^2)}$$
$$+\frac{\left(f(2cd-be+2af)(ce-bf)\left(e-\sqrt{e^2-4df}\right)-2f(3abef^2-4a^2f^3+b^2f(e^2-6df)+2c^2d(e^2-4df\right.}{2\sqrt{2}\,(e^2-4df)^{3/2}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{}}$$
$$-\frac{\left(f(2cd-be+2af)(ce-bf)\left(e+\sqrt{e^2-4df}\right)-2f(3abef^2-4a^2f^3+b^2f(e^2-6df)+2c^2d(e^2-4df\right.}{2\sqrt{2}\,(e^2-4df)^{3/2}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{}}$$

output
```
(f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)+f*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2
))*x)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e)
)/(f*x^2+e*x+d)+1/4*(f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2))
-2*f*(3*a*b*e*f^2-4*a^2*f^3+b^2*f*(-6*d*f+e^2)+2*c^2*d*(-4*d*f+e^2)-4*a*c*
f*(-3*d*f+e^2)-b*c*(-5*d*e*f+e^3)))*arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(
1/2))+2*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/(-4
*d*f+e^2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/4*(f*(2*a*f-b*e+2*c*d)*(-b*
f+c*e)*(e+(-4*d*f+e^2)^(1/2))-2*f*(3*a*b*e*f^2-4*a^2*f^3+b^2*f*(-6*d*f+e^2
)+2*c^2*d*(-4*d*f+e^2)-4*a*c*f*(-3*d*f+e^2)-b*c*(-5*d*e*f+e^3)))*arctanh(1
/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(
1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*
x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*
f+c*e))/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 17.22 (sec) , antiderivative size = 1377, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

output
```
(-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sq
rt[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 -
 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^
2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x +
 c*x^2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*
d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]
*(2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*
a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*
f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)*(4*a*f
^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*
f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f
- b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sq...
```

Rubi [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 764, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1305, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{\int \frac{-4a^2f^3+3abef^2+b^2(e^2-6df)f-4ac(e^2-3df)f+(2cd-be+2af)(ce-bf)xf+2c^2d(e^2-4df)-bc(e^3-5def)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c(e^2-2df)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{(e^2-4df)\,(d+ex+fx^2)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-4a^2f^3+3abef^2+b^2(e^2-6df)f-4ac(e^2-3df)f+(2cd-be+2af)(ce-bf)xf+2c^2d(e^2-4df)-bc(e^3-5def)}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{f\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{\cdots}{\left(e+2fx+\sqrt{e^2-\cdots}\right.}}{\sqrt{e^2-4df}}\cdots}{2\left(e^2-\cdots\right.}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2f\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{\cdots}{4\left(4af^2-2b\left(e\cdots\right.\right.}}{\sqrt{e^2-4df}}\cdots}{\cdots}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{f\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)+2c^2d\left(e^2-4df\right)\right)\right)\operatorname{arctanh}\left(\frac{\cdots}{2\sqrt{2}\cdots}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\cdots}{\cdots}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

output
```
((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c
*(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*((2*c*d - b*e + 2*a*f)*(
c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e
^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5
*d*e*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 -
4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]) - (f*((2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*
(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4
*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f)*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2107 vs. $2(733) = 1466$.

Time = 3.02 (sec) , antiderivative size = 2108, normalized size of antiderivative = 2.68

method	result	size
default	Expression too large to display	2108

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(4*d*f-e^2)*(-2/(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*d*f*c+c*e^2)*f^2/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(c*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))+1/2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)*f/(f*b*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)*2^(1/
2)/((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c
*e^2)/f^2)^(1/2)*ln(((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*d*f*c+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*(4*c*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))^2+4*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^
(1/2)))+2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d
*f*c+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))-1/(4*d*f-e^2)*
(-2/(-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+
c*e^2)*f^2/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*(c*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+f*b-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*(-f*b*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f
*c+c*e^2)/f^2)^(1/2)+f*(-c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/(-f*b*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)*2^(1/2)/((-f*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)^2}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx=\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)^2}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx=\int \frac{1}{\sqrt{c\,x^2+bx+a}\,\left(f\,x^2+ex+d\right)^2}dx$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

output
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

3.152 $\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 233

$$\begin{aligned}\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx &= -\frac{(235-214x)\sqrt{2+3x+5x^2}}{30756\,(4+x-2x^2)^2}-\frac{(1763725-1644506x)\sqrt{2+3x+5x^2}}{315310512\,(4+x-2x^2)} \\ &\quad -\frac{\left(22434211+746867\sqrt{33}\right)\operatorname{arctanh}\left(\frac{19-3\sqrt{33}+2\left(11-5\sqrt{33}\right)x}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{105103504\sqrt{66\left(107-11\sqrt{33}\right)}} \\ &\quad +\frac{\left(22434211-746867\sqrt{33}\right)\operatorname{arctanh}\left(\frac{19+3\sqrt{33}+2\left(11+5\sqrt{33}\right)x}{2\sqrt{2\left(107+11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{105103504\sqrt{66\left(107+11\sqrt{33}\right)}}\end{aligned}$$

output
```
-1/30756*(235-214*x)*(5*x^2+3*x+2)^(1/2)/(-2*x^2+x+4)^2-(1763725-1644506*x
)*(5*x^2+3*x+2)^(1/2)/(-630621024*x^2+315310512*x+1261242048)-1/105103504*
(22434211+746867*33^(1/2))*arctanh(1/2*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)
/(214-22*33^(1/2))^(1/2)/(5*x^2+3*x+2)^(1/2))/(7062-726*33^(1/2))^(1/2)+1/
105103504*(22434211-746867*33^(1/2))*arctanh(1/2*(19+3*33^(1/2)+2*(11+5*33
^(1/2))*x)/(214+22*33^(1/2))^(1/2)/(5*x^2+3*x+2)^(1/2))/(7062+726*33^(1/2)
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.85

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx$$

$$= \frac{-\frac{1922\sqrt{2+3x+5x^2}(9464120-7008227x-5171956x^2+3289012x^3)}{(4+x-2x^2)^2} - 108859442055\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2\right.}{}$$

input
```
Integrate[1/((4 + x - 2*x^2)^3*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
((-1922*Sqrt[2 + 3*x + 5*x^2]*(9464120 - 7008227*x - 5171956*x^2 + 3289012
*x^3))/(4 + x - 2*x^2)^2 - 108859442055*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#
1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2]
 - #1]/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ] + 8201600*RootSu
m[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (1147243*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 757075*Log[-(Sqrt
[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]
*#1^2 + 4*#1^3) & ] - 2*RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*
#1^3 + 2*#1^4 & , (4671198570463*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x +
 5*x^2] - #1]*#1 + 3102459942439*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2]
- #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ])/6060268040
64
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1305, 27, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^2+x+4)^3\sqrt{5x^2+3x+2}}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{\int \frac{4280x^2-1700x+14757}{2(-2x^2+x+4)^2\sqrt{5x^2+3x+2}}dx}{30756}-\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4280x^2-1700x+14757}{(-2x^2+x+4)^2\sqrt{5x^2+3x+2}}dx}{61512}-\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{-\frac{\int -\frac{9(11590539-1493734x)}{2\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx}{15378}-\frac{\sqrt{5x^2+3x+2}(1763725-1644506x)}{5126(-2x^2+x+4)}}{61512}-\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{11590539-1493734x}{\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx}{10252}-\frac{(1763725-1644506x)\sqrt{5x^2+3x+2}}{5126(-2x^2+x+4)}}{61512}-\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{1365}$$

$$\frac{\frac{3\left(-\frac{2}{33}\left(24646611+22434211\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx-\frac{2}{33}\left(24646611-22434211\sqrt{33}\right)\int \frac{1}{\left(-4x+\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx\right)}{10252}-(176372}{61512}-\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{3\left(\frac{4}{33}\left(24646611+22434211\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)+\frac{4}{33}\left(24646611-22434211\sqrt{33}\right)\int\right.}{10252}$$

$$61512$$

$$\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(-\frac{1}{33}\sqrt{\frac{2}{107-11\sqrt{33}}}\left(24646611+22434211\sqrt{33}\right)\text{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)-\frac{1}{33}\left(24646611-22434211\sqrt{33}\right)\sqrt{\frac{2}{107+11\sqrt{33}}}\text{arcta}\right.}{10252}$$

$$61512$$

$$\frac{(235-214x)\sqrt{5x^2+3x+2}}{30756\left(-2x^2+x+4\right)^2}$$

input
```
Int[1/((4 + x - 2*x^2)^3*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
-1/30756*((235 - 214*x)*Sqrt[2 + 3*x + 5*x^2])/(4 + x - 2*x^2)^2 + (-1/512
6*((1763725 - 1644506*x)*Sqrt[2 + 3*x + 5*x^2])/(4 + x - 2*x^2) + (3*(-1/3
3*(Sqrt[2/(107 - 11*Sqrt[33])]*(24646611 + 22434211*Sqrt[33])*ArcTanh[(19
- 3*Sqrt[33] + 2*(11 - 5*Sqrt[33])*x)/(2*Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[
2 + 3*x + 5*x^2])]) - ((24646611 - 22434211*Sqrt[33])*Sqrt[2/(107 + 11*Sqr
t[33])]*ArcTanh[(19 + 3*Sqrt[33] + 2*(11 + 5*Sqrt[33])*x)/(2*Sqrt[2*(107 +
 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])])/33))/10252)/61512
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{(3289012x^3-5171956x^2-7008227x+9464120)\sqrt{5x^2+3x+2}}{315310512(2x^2-x-4)^2}-\frac{\left(-22434211+746867\sqrt{33}\right)\sqrt{33}\ \operatorname{arctanh}\left(\frac{214\cdots}{\sqrt{214+22\sqrt{33}}\sqrt{80\left(x-\cdots\right.}}\right.}{3468415632\sqrt{214\cdots}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315310512*(3289012*x^3-5171956*x^2-7008227*x+9464120)/(2*x^2-x-4)^2*(5*
x^2+3*x+2)^(1/2)-1/3468415632*(-22434211+746867*33^(1/2))*33^(1/2)/(214+22
*33^(1/2))^(1/2)*arctanh(8*(107/4+11/4*33^(1/2)+(11/2+5/2*33^(1/2))*(x-1/4
*33^(1/2)-1/4))/(214+22*33^(1/2))^(1/2)/(80*(x-1/4*33^(1/2)-1/4)^2+16*(11/
2+5/2*33^(1/2))*(x-1/4*33^(1/2)-1/4)+214+22*33^(1/2))^(1/2))-1/3468415632*
(22434211+746867*33^(1/2))*33^(1/2)/(214-22*33^(1/2))^(1/2)*arctanh(8*(107
/4-11/4*33^(1/2)+(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2)))/(214-22*33^(1/2
))^(1/2)/(80*(x-1/4+1/4*33^(1/2))^2+16*(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(
1/2))+214-22*33^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.61

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx$$
$$= \frac{3\left(4\,x^4-4\,x^3-15\,x^2+8\,x+16\right)\sqrt{\frac{211917426800849}{699}\,\sqrt{33}+\frac{16996616820423467}{7689}}\log\left(-\frac{\sqrt{5\,x^2+3\,x+2}\sqrt{\frac{211917426800849}{699}\,\sqrt{\cdots}}}{\cdots}\right.}{\cdots}$$

input
```
integrate(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2522484096*(3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(211917426800849/6
99*sqrt(33) + 16996616820423467/7689)*log(-(sqrt(5*x^2 + 3*x + 2)*sqrt(211
917426800849/699*sqrt(33) + 16996616820423467/7689)*(253413521*sqrt(33) -
2418159777) + 60610760346848*sqrt(33)*(3*x + 4) - 4667028546707296*x - 121
2215206936960)/x) - 3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(21191742680
0849/699*sqrt(33) + 16996616820423467/7689)*log((sqrt(5*x^2 + 3*x + 2)*sqr
t(211917426800849/699*sqrt(33) + 16996616820423467/7689)*(253413521*sqrt(3
3) - 2418159777) - 60610760346848*sqrt(33)*(3*x + 4) + 4667028546707296*x
+ 1212215206936960)/x) + 3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt(-21191
7426800849/699*sqrt(33) + 16996616820423467/7689)*log((sqrt(5*x^2 + 3*x +
2)*(253413521*sqrt(33) + 2418159777)*sqrt(-211917426800849/699*sqrt(33) +
16996616820423467/7689) + 60610760346848*sqrt(33)*(3*x + 4) + 466702854670
7296*x + 1212215206936960)/x) - 3*(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 16)*sqrt
(-211917426800849/699*sqrt(33) + 16996616820423467/7689)*log(-(sqrt(5*x^2
+ 3*x + 2)*(253413521*sqrt(33) + 2418159777)*sqrt(-211917426800849/699*sqr
t(33) + 16996616820423467/7689) - 60610760346848*sqrt(33)*(3*x + 4) - 4667
028546707296*x - 1212215206936960)/x) - 8*(3289012*x^3 - 5171956*x^2 - 700
8227*x + 9464120)*sqrt(5*x^2 + 3*x + 2))/(4*x^4 - 4*x^3 - 15*x^2 + 8*x + 1
6)
```

Sympy [F]

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx = -\int \frac{1}{8x^6\sqrt{5x^2+3x+2}-12x^5\sqrt{5x^2+3x+2}-42x^4\sqrt{5x^2+3x+2}+47x^3\sqrt{5x^2+3x+2}+84x^2\sqrt{5x}$$

input
```
integrate(1/(-2*x**2+x+4)**3/(5*x**2+3*x+2)**(1/2),x)
```

output
```
-Integral(1/(8*x**6*sqrt(5*x**2 + 3*x + 2) - 12*x**5*sqrt(5*x**2 + 3*x + 2
) - 42*x**4*sqrt(5*x**2 + 3*x + 2) + 47*x**3*sqrt(5*x**2 + 3*x + 2) + 84*x
**2*sqrt(5*x**2 + 3*x + 2) - 48*x*sqrt(5*x**2 + 3*x + 2) - 64*sqrt(5*x**2
+ 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx=\int -\frac{1}{\sqrt{5\,x^2+3\,x+2}(2\,x^2-x-4)^3}\,dx$$

input
```
integrate(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(5*x^2 + 3*x + 2)*(2*x^2 - x - 4)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(181) = 362.

Time = 0.17 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.62

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx=$$

$$-\frac{8962404\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^7-82986840\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^6-276608240\left(\sqrt{5}x-\sqrt{\right.}\right.}{315310512\left(2\left(\sqrt{5}\right.\right.}$$

$$+0.00162881405851131\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}+8.38267526007000\right)$$

$$-0.00472864743789132\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.312157316296000\right)$$

$$-0.00162881405850180\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.842024981991000\right)$$

$$+0.00472864743789132\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-4.99242498429000\right)$$

input
```
integrate(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/315310512*(8962404*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 82986840*sqr
t(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 - 276608240*(sqrt(5)*x - sqrt(5
*x^2 + 3*x + 2))^5 + 4781277004*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)
)^4 + 20669978041*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 + 5000096948*sqrt(
5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 700654966*sqrt(5)*x - 188637064
*sqrt(5) - 700654966*sqrt(5*x^2 + 3*x + 2))/(2*(sqrt(5)*x - sqrt(5*x^2 + 3
*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 91*(sqrt(5)
*x - sqrt(5*x^2 + 3*x + 2))^2 - 44*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x +
 2)) - 22)^2 + 0.00162881405851131*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2)
+ 8.38267526007000) - 0.00472864743789132*log(-sqrt(5)*x + sqrt(5*x^2 + 3*
x + 2) - 0.312157316296000) - 0.00162881405850180*log(-sqrt(5)*x + sqrt(5*
x^2 + 3*x + 2) - 0.842024981991000) + 0.00472864743789132*log(-sqrt(5)*x +
 sqrt(5*x^2 + 3*x + 2) - 4.99242498429000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{(-2\,x^2+x+4)^3\,\sqrt{5\,x^2+3\,x+2}}\,dx$$

input
```
int(1/((x - 2*x^2 + 4)^3*(3*x + 5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((x - 2*x^2 + 4)^3*(3*x + 5*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(4+x-2x^2)^3\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{(-2x^2+x+4)^3\,\sqrt{5x^2+3x+2}}dx$$

input
```
int(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x)
```

output
```
int(1/(-2*x^2+x+4)^3/(5*x^2+3*x+2)^(1/2),x)
```

3.153 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 649

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(3ab^4cef^2-ab^5f^3+ab^3cf(5af^2-3c(e^2+df))-bc^2(c^3d^3+5a^3f^3+3ac^2d(e^2$$

$$-\frac{(187b^3f^3-4bcf^2(114be+73af)-64c^3(e^3+6def)+16c^2f(20aef+21b(e^2+df)))\sqrt{a+bx+cx^2}}{64c^5}$$

$$+\frac{f(41b^2f^2-4cf(22be+7af)+48c^2(e^2+df))x\sqrt{a+bx+cx^2}}{32c^4}$$

$$+\frac{f^2(8ce-5bf)x^2\sqrt{a+bx+cx^2}}{8c^3}+\frac{f^3x^3\sqrt{a+bx+cx^2}}{4c^2}$$

$$+\frac{3(105b^4f^3-280b^2cf^2(be+af)+128c^4d(e^2+df)+80c^2f(6abef+a^2f^2+3b^2(e^2+df))-64c^3(3af(e^2}{128c^{11/2}}$$

output
```
2*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*
d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f
^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*
f+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*
f-3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^
2)*x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)-1/64*(187*b^3*f^3-4*b*c*f^2*(73
*a*f+114*b*e)-64*c^3*(6*d*e*f+e^3)+16*c^2*f*(20*a*e*f+21*b*(d*f+e^2)))*(c*
x^2+b*x+a)^(1/2)/c^5+1/32*f*(41*b^2*f^2-4*c*f*(7*a*f+22*b*e)+48*c^2*(d*f+e
^2))*x*(c*x^2+b*x+a)^(1/2)/c^4+1/8*f^2*(-5*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^(1
/2)/c^3+1/4*f^3*x^3*(c*x^2+b*x+a)^(1/2)/c^2+3/128*(105*b^4*f^3-280*b^2*c*f
^2*(a*f+b*e)+128*c^4*d*(d*f+e^2)+80*c^2*f*(6*a*b*e*f+a^2*f^2+3*b^2*(d*f+e^
2))-64*c^3*(3*a*f*(d*f+e^2)+b*(6*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 9.51 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{c}(315b^6f^3x+105b^5f^2(3af+cx(-8e+fx))-2b^4cf(105af(4e+9fx)+cx(\cdots$$

input
```
Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*(315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c
*f*(105*a*f*(4*e + 9*f*x) + c*x*(-360*e^2 + 140*e*f*x + 3*f*(-120*d + 7*f*
x^2))) + 8*b^3*c*(-210*a^2*f^3 + a*c*f*(90*e^2 + 530*e*f*x + f*(90*d - 77*
f*x^2)) + c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2*e*f*(-7
2*d + 7*f*x^2))) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3
+ 186*e^2*f*x + 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*
x*(-24*d^2*f + 6*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4
*e*f^2*x^2 + f^3*x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) +
a^2*c*(16*e^3 + 36*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2
)) + 2*a*c^2*(-12*d^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2
*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8
*c^3*d^2*(d - 3*e*x) + a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^
2)) + 2*a*c^2*(12*d^2*f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3
+ 30*e^2*f*x + 14*e*f^2*x^2 + 3*f^3*x^3)))) - 3*(b^2 - 4*a*c)*(105*b^4*f^3
 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e*f
 + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6*d
*e*f)))*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(
b + c*x)])])/(64*c^(11/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\ldots\right.\right.}{\ldots}$$

$$\frac{2\int -\frac{\frac{(b^2-4ac)f^3x^4}{c}+\frac{(b^2-4ac)f^2(3ce-bf)x^3}{c^2}+\frac{(b^2-4ac)f\left(3(e^2+df)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}-\frac{(b^2-4ac)\left(-\left((e^3+6dfe)c^3\right)+3f\left(aef+b(e^2+df)\right)c^2-\ldots\right.}{c^4}}{2\sqrt{cx^2+bx\ldots}}}{b^2-4ac\ldots}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{(b^2-4ac)f^3x^4}{c}+\frac{(b^2-4ac)f^2(3ce-bf)x^3}{c^2}+\frac{(b^2-4ac)f\left(3(e^2+df)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}-\frac{(b^2-4ac)\left(-\left((e^3+6dfe)c^3\right)+3f\left(aef+b(e^2+df)\right)c^2-bf^2(3\cdots}{c^4}}{\sqrt{cx^2+bx+a}}}{b^2-4ac}}{2\left(-x(-2acf+b^2f-bce+2c^2d)\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\cdots\right.}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{\frac{3(b^2-4ac)f^2(8ce-5bf)x^3}{c}+\frac{2(b^2-4ac)f\left(12(e^2+df)c^2-f(12be+7af)c+4b^2f^2\right)x^2}{c^2}-\frac{8(b^2-4ac)\left(-\left((e^3+6dfe)c^3\right)+3f\left(aef+b(e^2+df)\right)c^2-bf^2(3be+2af)c\cdots}{c^3}}{2\sqrt{cx^2+bx+a}}}{4c}}{b\cdots}}{2\left(-x(-2acf+b^2f-bce+2c^2d)\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\cdots\right.}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\frac{3(b^2-4ac)f^2(8ce-5bf)x^3}{c}+\frac{2(b^2-4ac)f\left(12(e^2+df)c^2-f(12be+7af)c+4b^2f^2\right)x^2}{c^2}-\frac{8(b^2-4ac)\left(-\left((e^3+6dfe)c^3\right)+3f\left(aef+b(e^2+df)\right)c^2-bf^2(3be+2af)c\cdots}{c^3}}{\sqrt{cx^2+bx+a}}}{8c}}{b\cdots}}{2\left(-x(-2acf+b^2f-bce+2c^2d)\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\cdots\right.}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{3\left(\frac{(b^2-4ac)f\left(48(e^2+df)c^2-4f(22be+7af)c+41b^2f^2\right)x^2}{c}-\frac{4(b^2-4ac)\left(-4(e^3+6dfe)c^3+4f\left(5aef+3b(e^2+df)\right)c^2-bf^2(12be+13af)c+4b^3f^3\right)x}{c^2}+\frac{16(b^2-4\cdots}{\cdots}\right.}{2\sqrt{cx^2+bx+a}}}{3c}}{8c\cdots}}{2\left(-x(-2acf+b^2f-bce+2c^2d)\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\cdots\right.}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\frac{(b^2-4ac)f\left(48(e^2+df)c^2-4f(22be+7af)c+41b^2f^2\right)x^2}{c}-\frac{4(b^2-4ac)\left(-4(e^3+6dfe)c^3+4f\left(5aef+3b(e^2+df)\right)c^2-bf^2(12be+13af)c+4b^3f^3\right)x}{c^2}+\frac{16(b^2-4ac)\cdots}{\cdots}}{\sqrt{cx^2+bx+a}}}{2c}}{8c\cdots}}{2\left(-x(-2acf+b^2f-bce+2c^2d)\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\cdots\right.}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{(b^2-4ac)\left(2\left(32f^3b^4-cf^2(96be+137af)b^2+96c^4d\left(e^2+df\right)+4c^2f\left(24\left(e^2+df\right)b^2+70aefb+15a^2f^2\right)-16c^3\left(9af\left(e^2+df\right)+2b\left(e^3+6dfe\right)\right)\right)-c\left(-64\left(e^3+6dfe\right)c^3\right.}{2c^2\sqrt{cx^2+bx+a}}}{2c}}{2c}}{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\right.\right.}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{(b^2-4ac)\int \frac{2\left(32f^3b^4-cf^2(96be+137af)b^2+96c^4d\left(e^2+df\right)+4c^2f\left(24\left(e^2+df\right)b^2+70aefb+15a^2f^2\right)-16c^3\left(9af\left(e^2+df\right)+2b\left(e^3+6dfe\right)\right)\right)-c\left(-64\left(e^3+6dfe\right)c^3\right.}{\sqrt{cx^2+bx+a}}}{4c^3}}{2c}}{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\right.\right.}$$

$$\downarrow 1160$$

$$\frac{\frac{\frac{(b^2-4ac)\left(\frac{3}{2}\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx-\sqrt{a+b}\right.}{4c^3}}{2}}{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\right.\right.}$$

$$\downarrow 1092$$

$$\frac{\frac{\frac{(b^2-4ac)\left(3\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2}{\sqrt{cx^2-}}\right.}{4c^3}}{}}{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\right.\right.}$$

$$\downarrow 219$$

$$\frac{\frac{\frac{(b^2-4ac)\left(\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+\right.\right.}{2\sqrt{c}}\right.}{4c^3}}{}}{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+\right.\right.}$$

input
```
Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) -
b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)
) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^
2*f^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2
- b*c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^
3*c*e*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*
(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a*c)*f^3*x^3*Sqrt[a + b*
x + c*x^2])/(4*c^2) + (((b^2 - 4*a*c)*f^2*(8*c*e - 5*b*f)*x^2*Sqrt[a + b*x
 + c*x^2])/c^2 + (((b^2 - 4*a*c)*f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) +
48*c^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((b^2 - 4*a*c)*(-((
187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c
^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[a + b*x + c*x^2]) + (3*(105*b^4*f
^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e
*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6
*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[
c]))/(4*c^3))/(2*c))/(8*c))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 1136, normalized size of antiderivative = 1.75

method	result	size
risch	Expression too large to display	1136
default	Expression too large to display	2266

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*c^3*f^3*x^3-40*b*c^2*f^3*x^2+64*c^3*e*f^2*x^2-56*a*c^2*f^3*x+82*b
^2*c*f^3*x-176*b*c^2*e*f^2*x+96*c^3*d*f^2*x+96*c^3*e^2*f*x+292*a*b*c*f^3-3
20*a*c^2*e*f^2-187*b^3*f^3+456*b^2*c*e*f^2-336*b*c^2*d*f^2-336*b*c^2*e^2*f
+384*c^3*d*e*f+64*c^3*e^3)/c^5*(c*x^2+b*x+a)^(1/2)+1/128/c^5*(3*c*(80*a^2*
c^2*f^3-280*a*b^2*c*f^3+480*a*b*c^2*e*f^2-192*a*c^3*d*f^2-192*a*c^3*e^2*f+
105*b^4*f^3-280*b^3*c*e*f^2+240*b^2*c^2*d*f^2+240*b^2*c^2*e^2*f-384*b*c^3*
d*e*f-64*b*c^3*e^3+128*c^4*d^2*f+128*c^4*d*e^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-
1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-144*a^2*b*
c^2*f^3+384*a^2*c^3*e*f^2-328*a*b^3*c*f^3+288*a*b^2*c^2*e*f^2+192*a*b*c^3*
d*f^2+192*a*b*c^3*e^2*f-768*a*c^4*d*e*f-128*a*c^4*e^3+187*b^5*f^3-456*b^4*
c*e*f^2+336*b^3*c^2*d*f^2+336*b^3*c^2*e^2*f-384*b^2*c^3*d*e*f-64*b^2*c^3*e
^3+384*c^5*d^2*e)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2))+256*c^5*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+374
*a*b^4*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+224*a^3*c^2*f^3*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-128*a*b*c^3*e^3*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)-912*a^2*b^2*c*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)-384*a^2*c^3*d*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-384*a^2
*c^3*e^2*f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+672*a*b^2*c^2*d*f^2*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+672*a*b^2*c^2*e^2*f*(2*c*x+b)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1570 vs. $2(621) = 1242$.

Time = 0.97 (sec) , antiderivative size = 3143, normalized size of antiderivative = 4.84

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) + (8*b^2
*c^4*e*f^2 - 32*a*c^5*e*f^2 - 3*b^3*c^3*f^3 + 12*a*b*c^4*f^3)/(b^2*c^5 - 4
*a*c^6))*x + (48*b^2*c^4*e^2*f - 192*a*c^5*e^2*f + 48*b^2*c^4*d*f^2 - 192*
a*c^5*d*f^2 - 56*b^3*c^3*e*f^2 + 224*a*b*c^4*e*f^2 + 21*b^4*c^2*f^3 - 104*
a*b^2*c^3*f^3 + 80*a^2*c^4*f^3)/(b^2*c^5 - 4*a*c^6))*x + (64*b^2*c^4*e^3 -
 256*a*c^5*e^3 + 384*b^2*c^4*d*e*f - 1536*a*c^5*d*e*f - 240*b^3*c^3*e^2*f
+ 960*a*b*c^4*e^2*f - 240*b^3*c^3*d*f^2 + 960*a*b*c^4*d*f^2 + 280*b^4*c^2*
e*f^2 - 1376*a*b^2*c^3*e*f^2 + 1024*a^2*c^4*e*f^2 - 105*b^5*c*f^3 + 616*a*
b^3*c^2*f^3 - 784*a^2*b*c^3*f^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 - 3
84*b*c^5*d^2*e + 384*b^2*c^4*d*e^2 - 768*a*c^5*d*e^2 - 192*b^3*c^3*e^3 + 6
40*a*b*c^4*e^3 + 384*b^2*c^4*d^2*f - 768*a*c^5*d^2*f - 1152*b^3*c^3*d*e*f
+ 3840*a*b*c^4*d*e*f + 720*b^4*c^2*e^2*f - 2976*a*b^2*c^3*e^2*f + 1152*a^2
*c^4*e^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152*a^2*c^4*d*f^2
 - 840*b^5*c*e*f^2 + 4240*a*b^3*c^2*e*f^2 - 3904*a^2*b*c^3*e*f^2 + 315*b^6
*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 480*a^3*c^3*f^3)/(b^2*c^5
 - 4*a*c^6))*x - (128*b*c^5*d^3 - 768*a*c^5*d^2*e + 384*a*b*c^4*d*e^2 - 19
2*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3 + 384*a*b*c^4*d^2*f - 1152*a*b^2*c^3*d*e
*f + 3072*a^2*c^4*d*e*f + 720*a*b^3*c^2*e^2*f - 2496*a^2*b*c^3*e^2*f + 720
*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3*d*f^2 - 840*a*b^4*c*e*f^2 + 3680*a^2*b^2
*c^2*e*f^2 - 2048*a^3*c^3*e*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^3}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{3/2}}\, dx = \int \frac{(f\,x^2 + ex + d)^3}{(c\,x^2 + bx + a)^{\frac{3}{2}}} dx$$

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

3.154 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 309

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd-af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2+2df)) - (2c^4d}{c^3(b^2 -}$$
$$+ \frac{f(8ce-7bf)\sqrt{a+bx+cx^2}}{4c^3} + \frac{f^2x\sqrt{a+bx+cx^2}}{2c^2}$$
$$+ \frac{(15b^2f^2 - 12cf(2be+af) + 8c^2(e^2+2df))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

output

```
2*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*c
*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d
*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c
*x^2+b*x+a)^(1/2)+1/4*f*(-7*b*f+8*c*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/2*f^2*x*(
c*x^2+b*x+a)^(1/2)/c^2+1/8*(15*b^2*f^2-12*c*f*(a*f+2*b*e)+8*c^2*(2*d*f+e^2
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-\frac{\sqrt{c}(15b^4f^2x+b^3f(15af+cx(-24e+5fx))+4bc(-13a^2f^2+2c^2d(d-2ex)+ac(2e^2+4df+20efx-5f^2x^2))-2\ldots}{(b^2\ldots}}{}$$

input
```
Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-((Sqrt[c]*(15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(
-13*a^2*f^2 + 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*
x^2)) - 2*b^2*c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2
*x^2)) + 8*c^2*(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) +
x*(-2*e^2 + 4*e*f*x + f^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]))
+ (15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(Sqrt[
c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4*c^(7/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2191, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3\ldots}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}\ldots$$

$$\frac{2\int -\frac{\frac{(b^2-4ac)f^2x^2}{c}+\frac{(b^2-4ac)f(2ce-bf)x}{c^2}+\frac{(b^2-4ac)\left(\left(e^2+2df\right)c^2-f(2be+af)c+b^2f^2\right)}{c^3}}{2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{(b^2-4ac)f^2x^2}{c}+\frac{(b^2-4ac)f(2ce-bf)x}{c^2}+\frac{(b^2-4ac)((e^2+2df)c^2-f(2be+af)c+b^2f^2)}{c^3}}{\sqrt{cx^2+bx+a}}dx}{b^2-4ac}+$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\int \frac{(b^2-4ac)(2(2(e^2+2df)c^2-f(4be+3af)c+2b^2f^2)+cf(8ce-7bf)x)}{2c^2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{f^2x(b^2-4ac)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{(b^2-4ac)\int \frac{2(2(e^2+2df)c^2-f(4be+3af)c+2b^2f^2)+cf(8ce-7bf)x}{\sqrt{cx^2+bx+a}}dx}{4c^3}+\frac{f^2x(b^2-4ac)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{(b^2-4ac)\left(\frac{1}{2}(-12cf(af+2be)+15b^2f^2+8c^2(2df+e^2))\int \frac{1}{\sqrt{cx^2+bx+a}}dx+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x(b^2-4ac)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{(b^2-4ac)\left((-12cf(af+2be)+15b^2f^2+8c^2(2df+e^2))\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x(b^2-4ac)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \quad \frac{\left(b^2-4ac\right)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)}{2\sqrt{c}}+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}$$

input
```
Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a
^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*
f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e
^2 + 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a
*c)*f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((b^2 - 4*a*c)*(f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2] + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^
2 + 2*d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqr
t[c])))/(4*c^3))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{f(-2cfx+7fb-8ce)\sqrt{c\,x^2+bx+a}}{4c^3}-\frac{c(12ac\,f^2-15b^2f^2+24bcef-16c^2df-8c^2e^2)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{}{c\left(4ac-\right.}\right.}{2c}\right.}{}$
default	$\frac{2d^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f^2\left(\frac{x^3}{2c\sqrt{c\,x^2+bx+a}}-\frac{5b\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+}}\right.}{2c}\right.}{2c}\right.}{}\right.$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*f*(-2*c*f*x+7*b*f-8*c*e)/c^3*(c*x^2+b*x+a)^(1/2)-1/8/c^3*(c*(12*a*c*f
^2-15*b^2*f^2+24*b*c*e*f-16*c^2*d*f-8*c^2*e^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1
/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-4*a*b*c*f^2
+16*a*c^2*e*f-7*b^3*f^2+8*b^2*c*e*f-16*c^3*d*e)*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-16*c^3*d^2*(2*c*x+b)/(4*a*c
-b^2)/(c*x^2+b*x+a)^(1/2)-14*a*b^2*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)+8*a^2*c*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*a*b*c*e*f*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(289) = 578$.

Time = 0.57 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.22

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^
3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a
^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x
^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*
(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*
(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(
-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) -
 4*a*c) + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a
*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e
*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*
a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (
15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (
3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2
*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2
*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(
b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(
2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*
c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*
c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a
*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c...
```

Sympy [F]

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(b^2c^2f^2-4\,ac^3f^2)x}{b^2c^3-4\,ac^4}+\frac{8\,b^2c^2ef-32\,ac^3ef-5\,b^3cf^2+20\,abc^2f^2}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4d^2-16\,bc^3de+8\,b^2c^2e^2-16}{}\right.}{} - \frac{(8\,c^2e^2+16\,c^2df-24\,bcef+15\,b^2f^2-12\,acf^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2*e*
f - 32*a*c^3*e*f - 5*b^3*c*f^2 + 20*a*b*c^2*f^2)/(b^2*c^3 - 4*a*c^4))*x -
(16*c^4*d^2 - 16*b*c^3*d*e + 8*b^2*c^2*e^2 - 16*a*c^3*e^2 + 16*b^2*c^2*d*f
 - 32*a*c^3*d*f - 24*b^3*c*e*f + 80*a*b*c^2*e*f + 15*b^4*f^2 - 62*a*b^2*c*
f^2 + 24*a^2*c^2*f^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 - 32*a*c^3*d*e
 + 8*a*b*c^2*e^2 + 16*a*b*c^2*d*f - 24*a*b^2*c*e*f + 64*a^2*c^2*e*f + 15*a
*b^3*f^2 - 52*a^2*b*c*f^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/
8*(8*c^2*e^2 + 16*c^2*d*f - 24*b*c*e*f + 15*b^2*f^2 - 12*a*c*f^2)*log(abs(
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^2}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+ex+d)^2}{(c\,x^2+bx+a)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

3.155 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c
*x^2+b*x+a)^(1/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
3/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{c\left(-b^2+4ac\right)\sqrt{a+x(b+cx)}} + \frac{2f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b
^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (2*f*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
Sqrt[a + x*(b + c*x)])])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2f\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(98) = 196$.

Time = 1.67 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.86

method	result
default	$\frac{2d(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+e\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)+f\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{}{c}\right.}{2}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+e*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+f*(-x/c/(c*x^2+b*x+a)^(1/2)
-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(98) = 196$.

Time = 0.28 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.97

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-\right.}{2\,(ab^2c}\right.$$

$$-\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(bc^2d-2\,ac^2}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f
)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^
2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^
2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b
*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x
)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
+ (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} \, dx = \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2\,c^2d-bce+b^2f-2\,acf)x}{b^2c-4\,ac^2}+\frac{bcd-2\,ace+abf}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{f\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x/(b^2*c - 4*a*c^2) + (b*c*d - 2*a
*c*e + a*b*f)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.32

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} - \frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}} + \frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}} + \frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c -
 b^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c
 - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.31

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a\,c^2e+2\sqrt{c\,x^2+bx+a}\,b\,c^2d+4\sqrt{c\,x^2+bx+a}\,c^3dx-4\sqrt{c}\,c}{}$$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*a*b*c*f - 4*sqrt(a + b*x + c*x**2)*a*c**2*e - 4*
sqrt(a + b*x + c*x**2)*a*c**2*f*x + 2*sqrt(a + b*x + c*x**2)*b**2*c*f*x +
2*sqrt(a + b*x + c*x**2)*b*c**2*d - 2*sqrt(a + b*x + c*x**2)*b*c**2*e*x +
4*sqrt(a + b*x + c*x**2)*c**3*d*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c*f - sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*f + 4*sq
rt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b*c*f*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)/sqrt(4*a*c - b**2))*a*c**2*f*x**2 - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*f*x - sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*f*x**2 -
 4*sqrt(c)*a**2*c*f + 2*sqrt(c)*a*b**2*f - 2*sqrt(c)*a*b*c*e - 4*sqrt(c)*a
*b*c*f*x + 4*sqrt(c)*a*c**2*d - 4*sqrt(c)*a*c**2*f*x**2 + 2*sqrt(c)*b**3*f
*x - 2*sqrt(c)*b**2*c*e*x + 2*sqrt(c)*b**2*c*f*x**2 + 4*sqrt(c)*b*c**2*d*x
 - 2*sqrt(c)*b*c**2*e*x**2 + 4*sqrt(c)*c**3*d*x**2)/(c**2*(4*a**2*c - a*b*
*2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.156 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 666

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2ac}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

$$-\frac{f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e}\right.\right.}}$$

$$+\frac{f\left(c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2}\right.\right.}}$$

output
```
2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*
d)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2
)-1/2*f*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/
2))))*arctanh(1/4*(4*a*f-b*(e-(-4*d*f+e^2)^(1/2))+2*(b*f-c*(e-(-4*d*f+e^2)
^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(
1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/((-a*f+c*d)^2-
(-a*e+b*d)*(-b*f+c*e))/(c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e-(
-4*d*f+e^2)^(1/2))))^(1/2)+1/2*f*(c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))+f*(2*
a*f-b*(e-(-4*d*f+e^2)^(1/2))))*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))
+2*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))*x)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+
(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)/(c*x^2+b*x+a)^(1/2))*2^(1/2)/(-4*d*f+
e^2)^(1/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.86 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-2(b^3f+b^2c(-e+fx)+bc(-3af+c(d-ex))+2c^2(cdx+a(}{}$$

input
```
Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x
+ a*(e - f*x))) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b*c*
e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 +
4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*e^2*Log[x]) + c^2*d*f*Lo
g[x] + 2*b*c*e*f*Log[x] - b^2*f^2*Log[x] - a*c*f^2*Log[x] + c^2*e^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b
*x + c*x^2] - x*#1] - 2*b*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1] + b^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*f*Log[x]*#1 + 2*Sqr
t[a]*b*f^2*Log[x]*#1 + 2*Sqrt[a]*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1]*#1 - 2*Sqrt[a]*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
]*#1 + c*e^2*Log[x]*#1^2 - c*d*f*Log[x]*#1^2 - b*e*f*Log[x]*#1^2 + a*f^2*L
og[x]*#1^2 - c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + c*d
*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + b*e*f*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*f^2*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2*c*d*#1 + b*e*#1 + 4
*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*
d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)]
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{2\int -\frac{(b^2-4ac)(f(be-af)-c(e^2-df)-f(ce-bf)x)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))}+\frac{2(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)\sqrt{a+bx+cx^2}((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$\downarrow$ 25

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$\downarrow$ 25

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$\downarrow$ 25

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$\downarrow$ 25

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$\downarrow$ 25

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$\downarrow$ 25

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

input
```
Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(609) = 1218$.

Time = 2.98 (sec) , antiderivative size = 1906, normalized size of antiderivative = 2.86

method	result	size
default	Expression too large to display	1906

input
```
int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*f+e^2)^(1/2)*(2/(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*d*f*c+c*e^2)*f^2/(c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+(c*(-
4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(f*b*(-4
*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(
1/2)-2*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)*f/(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f)/(2*c*(f*b*(-4*d*f+e^2)^(1/2)-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+
f*b-c*e)^2/f^2)/(c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2+(c*(-4*d*f+e^2)^(1/
2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(f*b*(-4*d*f+e^2)^(1/2
)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)-2/(f*b*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)*f^2*2
^(1/2)/((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*d*f
*c+c*e^2)/f^2)^(1/2)*ln(((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*d*f*c+c*e^2)/f^2)^(1/2)*(4*c*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))^2+4*(c*(-4*d*f+e^2)^(1/2)+f*b-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))+2*(f*b*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*d*f*c+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))-1/(-4*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx=\int \frac{1}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx=\int \frac{}{c^2f\,x^6+2bcf\,x^5+c^2e\,x^5+2acf\,x^4+b^2f\,x^4+2bce\,x^4+c^2d\,x}$$

input
```
int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

output
```
int(sqrt(a + b*x + c*x**2)/(a**2*d + a**2*e*x + a**2*f*x**2 + 2*a*b*d*x +
2*a*b*e*x**2 + 2*a*b*f*x**3 + 2*a*c*d*x**2 + 2*a*c*e*x**3 + 2*a*c*f*x**4 +
 b**2*d*x**2 + b**2*e*x**3 + b**2*f*x**4 + 2*b*c*d*x**3 + 2*b*c*e*x**4 + 2
*b*c*f*x**5 + c**2*d*x**4 + c**2*e*x**5 + c**2*f*x**6),x)
```

3.157 $\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}} \, dx = \frac{4752569+5254415x}{222150588\sqrt{2+3x+5x^2}} - \frac{235-214x}{15378 (4+x-2x^2) \sqrt{2+3x+5x^2}} - \frac{\left(1668967+204919\sqrt{33}\right) \operatorname{arctanh}\left(\frac{19-3\sqrt{33}+2\left(11-5\sqrt{33}\right)x}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{7166148\sqrt{66\left(107-11\sqrt{33}\right)}} + \frac{\left(1668967-204919\sqrt{33}\right) \operatorname{arctanh}\left(\frac{19+3\sqrt{33}+2\left(11+5\sqrt{33}\right)x}{2\sqrt{2\left(107+11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{7166148\sqrt{66\left(107+11\sqrt{33}\right)}}$$

output

```
1/222150588*(4752569+5254415*x)/(5*x^2+3*x+2)^(1/2)-1/15378*(235-214*x)/(-
2*x^2+x+4)/(5*x^2+3*x+2)^(1/2)-1/7166148*(1668967+204919*33^(1/2))*arctanh
(1/2*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)/(214-22*33^(1/2))^(1/2)/(5*x^2+3*
x+2)^(1/2))/(7062-726*33^(1/2))^(1/2)+1/7166148*(1668967-204919*33^(1/2))*
arctanh(1/2*(19+3*33^(1/2)+2*(11+5*33^(1/2))*x)/(214+22*33^(1/2))^(1/2)/(5
*x^2+3*x+2)^(1/2))/(7062+726*33^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.04 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.87

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}} dx = \frac{-15615466 - 28861673x + 4250723x^2 + 10508830x^3}{222150588 (-4-x+2x^2) \sqrt{2+3x+5x^2}} + \frac{\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3+2\#1^4\&, \frac{-134287\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{2+3x+5x^2}-\#1\right)+75530\log\left(-\sqrt{5}\right.}{22\sqrt{5}-91}\right.}{217156\sqrt{5}} + \frac{\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3+2\#1^4\&, \frac{5232437\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{2+3x+5x^2}-\#1\right)+4384450\log\left(-\right.}{22\sqrt{5}-}\right.}{14332296\sqrt{5}}$$

input
```
Integrate[1/((4 + x - 2*x^2)^2*(2 + 3*x + 5*x^2)^(3/2)),x]
```

output
```
(-15615466 - 28861673*x + 4250723*x^2 + 10508830*x^3)/(222150588*(-4 - x +
 2*x^2)*Sqrt[2 + 3*x + 5*x^2]) + RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2
*Sqrt[5]*#1^3 + 2*#1^4 & , (-134287*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*
x + 5*x^2] - #1] + 75530*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1
 + 4554*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*S
qrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ]/(217156*Sqrt[5]) + RootSum[-
22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (5232437*Sqrt[5
]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1] + 4384450*Log[-(Sqrt[5]*x
) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 109274*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqr
t[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1
^3) & ]/(14332296*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1305, 27, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^2+x+4)^2 (5x^2+3x+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1305$$

$$\frac{\int \frac{4280x^2-4054x+5893}{2(-2x^2+x+4)(5x^2+3x+2)^{3/2}}dx}{15378} - \frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4280x^2-4054x+5893}{(-2x^2+x+4)(5x^2+3x+2)^{3/2}}dx}{30756} - \frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 2135$$

$$\frac{\frac{\int \frac{31(936943-409838x)}{2\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx}{7223} + \frac{5254415x+4752569}{7223\sqrt{5x^2+3x+2}}}{30756} - \frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{466}\int \frac{936943-409838x}{(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx + \frac{5254415x+4752569}{7223\sqrt{5x^2+3x+2}}}{30756} - \frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{1}{466}\left(-\frac{2}{33}\left(6762327+1668967\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx - \frac{2}{33}\left(6762327-1668967\sqrt{33}\right)\int \frac{1}{\left(-4x+\sqrt{33}+1\right)\sqrt{5}}\right.}{30756}$$
$$\frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{466}\left(\frac{4}{33}\left(6762327+1668967\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)+\frac{4}{33}\left(676232\right.\right.}{30756}$$
$$\frac{235-214x}{15378(-2x^2+x+4)\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{466}\left(-\frac{1}{33}\sqrt{\frac{2}{107-11\sqrt{33}}}\left(6762327+1668967\sqrt{33}\right)\operatorname{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)-\frac{1}{33}\left(6762327-1668967\right.\right.}{30756}$$

$$\frac{235-214x}{15378\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}$$

input
```
Int[1/((4 + x - 2*x^2)^2*(2 + 3*x + 5*x^2)^(3/2)),x]
```

output
```
-1/15378*(235 - 214*x)/((4 + x - 2*x^2)*Sqrt[2 + 3*x + 5*x^2]) + ((4752569
 + 5254415*x)/(7223*Sqrt[2 + 3*x + 5*x^2]) + (-1/33*(Sqrt[2/(107 - 11*Sqrt
[33])]*(6762327 + 1668967*Sqrt[33])*ArcTanh[(19 - 3*Sqrt[33] + 2*(11 - 5*S
qrt[33])*x)/(2*Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])]) - ((67
62327 - 1668967*Sqrt[33])*Sqrt[2/(107 + 11*Sqrt[33])]*ArcTanh[(19 + 3*Sqrt
[33] + 2*(11 + 5*Sqrt[33])*x)/(2*Sqrt[2*(107 + 11*Sqrt[33])]*Sqrt[2 + 3*x
+ 5*x^2])])/33)/466)/30756
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.04

method	result
risch	$\frac{10508830x^3+4250723x^2-28861673x-15615466}{222150588(2x^2-x-4)\sqrt{5x^2+3x+2}} - \frac{\left(-1668967+204919\sqrt{33}\right)\sqrt{33}\,\operatorname{arctanh}\left(\frac{214+22\sqrt{33}+8\left(\frac{11}{2}+\right.}{\sqrt{214+22\sqrt{33}}\sqrt{80\left(x-\frac{\sqrt{33}}{4}-\frac{1}{4}\right)^2+16\left(\right.}}\right.}{236482884\sqrt{214+22\sqrt{33}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/222150588*(10508830*x^3+4250723*x^2-28861673*x-15615466)/(2*x^2-x-4)/(5*
x^2+3*x+2)^(1/2)-1/236482884*(-1668967+204919*33^(1/2))*33^(1/2)/(214+22*3
3^(1/2))^(1/2)*arctanh(8*(107/4+11/4*33^(1/2)+(11/2+5/2*33^(1/2))*(x-1/4*3
3^(1/2)-1/4))/(214+22*33^(1/2))^(1/2)/(80*(x-1/4*33^(1/2)-1/4)^2+16*(11/2+
5/2*33^(1/2))*(x-1/4*33^(1/2)-1/4)+214+22*33^(1/2))^(1/2))-1/236482884*(16
68967+204919*33^(1/2))*33^(1/2)/(214-22*33^(1/2))^(1/2)*arctanh(8*(107/4-1
1/4*33^(1/2)+(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2)))/(214-22*33^(1/2))^(
1/2)/(80*(x-1/4+1/4*33^(1/2))^2+16*(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2)
)+214-22*33^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(169) = 338$.

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.64

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}}\,dx = \frac{31\,(10\,x^4+x^3-19\,x^2-14\,x-8)\sqrt{\frac{9020578324 67}{233}\sqrt{33}+\frac{17365262190}{7689}}}{}$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1777204704*(31*(10*x^4 + x^3 - 19*x^2 - 14*x - 8)*sqrt(902057832467/233*
sqrt(33) + 173652621908003/7689)*log(-(sqrt(5*x^2 + 3*x + 2)*sqrt(90205783
2467/233*sqrt(33) + 173652621908003/7689)*(33638717*sqrt(33) - 219326349)
+ 174965195072*sqrt(33)*(3*x + 4) - 13472320020544*x - 3499303901440)/x) -
 31*(10*x^4 + x^3 - 19*x^2 - 14*x - 8)*sqrt(902057832467/233*sqrt(33) + 17
3652621908003/7689)*log((sqrt(5*x^2 + 3*x + 2)*sqrt(902057832467/233*sqrt(
33) + 173652621908003/7689)*(33638717*sqrt(33) - 219326349) - 174965195072
*sqrt(33)*(3*x + 4) + 13472320020544*x + 3499303901440)/x) + 31*(10*x^4 +
x^3 - 19*x^2 - 14*x - 8)*sqrt(-902057832467/233*sqrt(33) + 173652621908003
/7689)*log((sqrt(5*x^2 + 3*x + 2)*(33638717*sqrt(33) + 219326349)*sqrt(-90
2057832467/233*sqrt(33) + 173652621908003/7689) + 174965195072*sqrt(33)*(3
*x + 4) + 13472320020544*x + 3499303901440)/x) - 31*(10*x^4 + x^3 - 19*x^2
 - 14*x - 8)*sqrt(-902057832467/233*sqrt(33) + 173652621908003/7689)*log(-
(sqrt(5*x^2 + 3*x + 2)*(33638717*sqrt(33) + 219326349)*sqrt(-902057832467/
233*sqrt(33) + 173652621908003/7689) - 174965195072*sqrt(33)*(3*x + 4) - 1
3472320020544*x - 3499303901440)/x) + 8*(10508830*x^3 + 4250723*x^2 - 2886
1673*x - 15615466)*sqrt(5*x^2 + 3*x + 2))/(10*x^4 + x^3 - 19*x^2 - 14*x -
8)
```

Sympy [F]

$$\int \frac{1}{(4+x-2x^2)^2\,(2+3x+5x^2)^{3/2}}\,dx = \int \frac{1}{(2x^2-x-4)^2\,(5x^2+3x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-2*x**2+x+4)**2/(5*x**2+3*x+2)**(3/2),x)
```

output
```
Integral(1/((2*x**2 - x - 4)**2*(5*x**2 + 3*x + 2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^{\frac{3}{2}}(2\,x^2-x-4)^2}\,dx$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^(3/2)*(2*x^2 - x - 4)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.32

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}}\,dx = \frac{97745\,x+64617}{3365918\,\sqrt{5\,x^2+3\,x+2}}$$
$$-\frac{109274\,\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3-493082\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2-930819\,\sqrt{5}x+315}{7166148\,\left(2\,\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^4-2\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3-91\,\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2\right.}$$
$$+0.000647531417292805\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}+8.38267526007000\right)$$
$$-0.00738605225631678\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.312157316296000\right)$$
$$-0.000647531417292805\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.842024981991000\right)$$
$$+0.00738605225631678\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-4.99242498429000\right)$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x, algorithm="giac")
```

output
```
1/3365918*(97745*x + 64617)/sqrt(5*x^2 + 3*x + 2) - 1/7166148*(109274*(sqr
t(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 493082*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2
 + 3*x + 2))^2 - 930819*sqrt(5)*x + 31592*sqrt(5) + 930819*sqrt(5*x^2 + 3*
x + 2))/(2*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 3*x + 2))^3 - 91*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 44*s
qrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) - 22) + 0.000647531417292805*lo
g(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) + 8.38267526007000) - 0.0073860522563
1678*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.312157316296000) - 0.00064
7531417292805*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.842024981991000)
+ 0.00738605225631678*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 4.992424984
29000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}}\,dx = \int \frac{1}{(-2\,x^2+x+4)^2\,(5\,x^2+3\,x+2)^{3/2}}\,dx$$

input
```
int(1/((x - 2*x^2 + 4)^2*(3*x + 5*x^2 + 2)^(3/2)),x)
```

output
```
int(1/((x - 2*x^2 + 4)^2*(3*x + 5*x^2 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{3/2}}\,dx = \int \frac{1}{(-2x^2+x+4)^2 (5x^2+3x+2)^{\frac{3}{2}}}dx$$

input
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x)
```

output
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(3/2),x)
```

3.158 $\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = -\frac{2(573358-1635093x)}{290625\,(2+3x+5x^2)^{3/2}}+\frac{2(10505303+15267525x)}{3003125\sqrt{2+3x+5x^2}}+\frac{126}{625}\sqrt{2+3x+5x^2}-\frac{4}{125}x\sqrt{2+3x+5x^2}+\frac{97\operatorname{arcsinh}\left(\frac{3+10x}{\sqrt{31}}\right)}{125\sqrt{5}}$$

output
```
1/290625*(-1146716+3270186*x)/(5*x^2+3*x+2)^(3/2)+2/3003125*(10505303+1526
7525*x)/(5*x^2+3*x+2)^(1/2)+126/625*(5*x^2+3*x+2)^(1/2)-4/125*x*(5*x^2+3*x
+2)^(1/2)+97/625*arcsinh(1/31*(3+10*x)*31^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = -\frac{2(-1955612-9886476x-12783747x^2-10083075x^3-735165x^4+144150x^5)}{360375\,(2+3x+5x^2)^{3/2}}-\frac{97\log\left(-3-10x+2\sqrt{5}\sqrt{2+3x+5x^2}\right)}{125\sqrt{5}}$$

input
```
Integrate[(4 + x - 2*x^2)^3/(2 + 3*x + 5*x^2)^(5/2),x]
```

output
```
(-2*(-1955612 - 9886476*x - 12783747*x^2 - 10083075*x^3 - 735165*x^4 + 144
150*x^5))/(360375*(2 + 3*x + 5*x^2)^(3/2)) - (97*Log[-3 - 10*x + 2*Sqrt[5]
*Sqrt[2 + 3*x + 5*x^2]])/(125*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+x+4)^3}{(5x^2+3x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{93}\int \frac{3(-155000x^4+325500x^3+680450x^2-1449095x+1149901)}{6250(5x^2+3x+2)^{3/2}}dx-\frac{2(573358-1635093x)}{290625(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-155000x^4+325500x^3+680450x^2-1449095x+1149901}{(5x^2+3x+2)^{3/2}}dx}{96875}-\frac{2(573358-1635093x)}{290625(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{\frac{2}{31}\int \frac{4805(-100x^2+270x+317)}{\sqrt{5x^2+3x+2}}dx+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{310\int \frac{-100x^2+270x+317}{\sqrt{5x^2+3x+2}}dx+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \; 2192$$

$$\frac{310\left(\frac{1}{10}\int \frac{10(315x+337)}{\sqrt{5x^2+3x+2}}dx-10x\sqrt{5x^2+3x+2}\right)+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{310\left(\int \frac{315x+337}{\sqrt{5x^2+3x+2}}dx-10x\sqrt{5x^2+3x+2}\right)+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \; 1160$$

$$\frac{310\left(\frac{485}{2}\int \frac{1}{\sqrt{5x^2+3x+2}}dx-10\sqrt{5x^2+3x+2}x+63\sqrt{5x^2+3x+2}\right)+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \; 1090$$

$$\frac{310\left(\frac{97}{2}\sqrt{\frac{5}{31}}\int \frac{1}{\sqrt{\frac{1}{31}(10x+3)^2+1}}d(10x+3)-10\sqrt{5x^2+3x+2}x+63\sqrt{5x^2+3x+2}\right)+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{310\left(\frac{97}{2}\sqrt{5}\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)-10\sqrt{5x^2+3x+2}x+63\sqrt{5x^2+3x+2}\right)+\frac{2(15267525x+10505303)}{31\sqrt{5x^2+3x+2}}}{96875}-\frac{2(573358-1635093x)}{290625\left(5x^2+3x+2\right)^{3/2}}$$

input

```
Int[(4 + x - 2*x^2)^3/(2 + 3*x + 5*x^2)^(5/2),x]
```

output

```
(-2*(573358 - 1635093*x))/(290625*(2 + 3*x + 5*x^2)^(3/2)) + ((2*(10505303
 + 15267525*x))/(31*Sqrt[2 + 3*x + 5*x^2]) + 310*(63*Sqrt[2 + 3*x + 5*x^2]
 - 10*x*Sqrt[2 + 3*x + 5*x^2] + (97*Sqrt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]])/
2))/96875
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{2(144150x^5-735165x^4-10083075x^3-12783747x^2-9886476x-1955612)}{360375(5x^2+3x+2)^{\frac{3}{2}}}+\frac{97\sqrt{5}\,\operatorname{arcsinh}\left(\frac{10\sqrt{31}\left(x+\frac{3}{10}\right)}{31}\right)}{625}$
trager	$-\frac{2(144150x^5-735165x^4-10083075x^3-12783747x^2-9886476x-1955612)}{360375(5x^2+3x+2)^{\frac{3}{2}}}+\frac{97\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(10\,\mathrm{RootOf}\left(\_Z^2-5\right)x\right.}{625}$
default	$\frac{\frac{4324909x}{465000}+\frac{4324909}{1550000}}{(5x^2+3x+2)^{\frac{3}{2}}}+\frac{\frac{4406098x}{360375}+\frac{2203049}{600625}}{\sqrt{5x^2+3x+2}}+\frac{39223}{150000(5x^2+3x+2)^{\frac{3}{2}}}+\frac{54819x}{5000(5x^2+3x+2)^{\frac{3}{2}}}+\frac{4273x^2}{250(5x^2+3x+2)^{\frac{3}{2}}}-\frac{97x}{75(5x^2+3}$

input
```
int((-2*x^2+x+4)^3/(5*x^2+3*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/360375*(144150*x^5-735165*x^4-10083075*x^3-12783747*x^2-9886476*x-19556
12)/(5*x^2+3*x+2)^(3/2)+97/625*5^(1/2)*arcsinh(10/31*31^(1/2)*(x+3/10))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = \frac{279651\sqrt{5}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\log\left(-4\sqrt{5}\sqrt{5\,x^2+3\,x+2}(10\,x+\right.}{}$$

input
```
integrate((-2*x^2+x+4)^3/(5*x^2+3*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/3603750*(279651*sqrt(5)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(-4*sqr
t(5)*sqrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - 20*(144150
*x^5 - 735165*x^4 - 10083075*x^3 - 12783747*x^2 - 9886476*x - 1955612)*sqr
t(5*x^2 + 3*x + 2))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx =$$

$$-\int \left(-\frac{48x}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2-}}\right.$$

$$-\int \frac{84x^2}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2+3x}}$$

$$-\int \frac{47x^3}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2+3x}}$$

$$-\int \left(-\frac{42x^4}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2-}}\right.$$

$$-\int \left(-\frac{12x^5}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2-}}\right.$$

$$-\int \frac{8x^6}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2+3x}}$$

$$-\int \left(-\frac{64}{25x^4\sqrt{5x^2+3x+2}+30x^3\sqrt{5x^2+3x+2}+29x^2\sqrt{5x^2+3x+2}+12x\sqrt{5x^2+3x+2}+4\sqrt{5x^2-}}\right.$$

input
```
integrate((-2*x**2+x+4)**3/(5*x**2+3*x+2)**(5/2),x)
```

output
```
-Integral(-48*x/(25*x**4*sqrt(5*x**2 + 3*x + 2) + 30*x**3*sqrt(5*x**2 + 3*
x + 2) + 29*x**2*sqrt(5*x**2 + 3*x + 2) + 12*x*sqrt(5*x**2 + 3*x + 2) + 4*
sqrt(5*x**2 + 3*x + 2)), x) - Integral(84*x**2/(25*x**4*sqrt(5*x**2 + 3*x
+ 2) + 30*x**3*sqrt(5*x**2 + 3*x + 2) + 29*x**2*sqrt(5*x**2 + 3*x + 2) + 1
2*x*sqrt(5*x**2 + 3*x + 2) + 4*sqrt(5*x**2 + 3*x + 2)), x) - Integral(47*x
**3/(25*x**4*sqrt(5*x**2 + 3*x + 2) + 30*x**3*sqrt(5*x**2 + 3*x + 2) + 29*
x**2*sqrt(5*x**2 + 3*x + 2) + 12*x*sqrt(5*x**2 + 3*x + 2) + 4*sqrt(5*x**2
+ 3*x + 2)), x) - Integral(-42*x**4/(25*x**4*sqrt(5*x**2 + 3*x + 2) + 30*x
**3*sqrt(5*x**2 + 3*x + 2) + 29*x**2*sqrt(5*x**2 + 3*x + 2) + 12*x*sqrt(5*
x**2 + 3*x + 2) + 4*sqrt(5*x**2 + 3*x + 2)), x) - Integral(-12*x**5/(25*x*
*4*sqrt(5*x**2 + 3*x + 2) + 30*x**3*sqrt(5*x**2 + 3*x + 2) + 29*x**2*sqrt(
5*x**2 + 3*x + 2) + 12*x*sqrt(5*x**2 + 3*x + 2) + 4*sqrt(5*x**2 + 3*x + 2)
), x) - Integral(8*x**6/(25*x**4*sqrt(5*x**2 + 3*x + 2) + 30*x**3*sqrt(5*x
**2 + 3*x + 2) + 29*x**2*sqrt(5*x**2 + 3*x + 2) + 12*x*sqrt(5*x**2 + 3*x +
 2) + 4*sqrt(5*x**2 + 3*x + 2)), x) - Integral(-64/(25*x**4*sqrt(5*x**2 +
3*x + 2) + 30*x**3*sqrt(5*x**2 + 3*x + 2) + 29*x**2*sqrt(5*x**2 + 3*x + 2)
 + 12*x*sqrt(5*x**2 + 3*x + 2) + 4*sqrt(5*x**2 + 3*x + 2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(84) = 168$.

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.09

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = -\frac{4\,x^5}{5\,(5\,x^2+3\,x+2)^{\frac{3}{2}}} + \frac{102\,x^4}{25\,(5\,x^2+3\,x+2)^{\frac{3}{2}}} - \frac{97}{360375}\,x\left(\frac{3330\,x}{\sqrt{5\,x^2+3\,x+2}} + \frac{14415\,x^2}{(5\,x^2+3\,x+2)^{\frac{3}{2}}} + \frac{999}{\sqrt{5\,x^2+3\,x+2}} + \frac{4743\,x}{(5\,x^2+3\,x+2)^{\frac{3}{2}}} + \frac{4402}{(5\,x^2+3\,x+\right. + \frac{97}{625}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{21534}{120125}\sqrt{5\,x^2+3\,x+2} + \frac{1405326\,x}{120125\,\sqrt{5\,x^2+3\,x+2}} + \frac{1991\,x^2}{125\,(5\,x^2+3\,x+2)^{\frac{3}{2}}} + \frac{2234283}{600625\,\sqrt{5\,x^2+3\,x+2}} + \frac{1155623\,x}{58125\,(5\,x^2+3\,x+2)^{\frac{3}{2}}} + \frac{156722}{58125\,(5\,x^2+3\,x+2)^{\frac{3}{2}}}$$

input
```
integrate((-2*x^2+x+4)^3/(5*x^2+3*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-4/5*x^5/(5*x^2 + 3*x + 2)^(3/2) + 102/25*x^4/(5*x^2 + 3*x + 2)^(3/2) - 97
/360375*x*(3330*x/sqrt(5*x^2 + 3*x + 2) + 14415*x^2/(5*x^2 + 3*x + 2)^(3/2
) + 999/sqrt(5*x^2 + 3*x + 2) + 4743*x/(5*x^2 + 3*x + 2)^(3/2) + 4402/(5*x
^2 + 3*x + 2)^(3/2)) + 97/625*sqrt(5)*arcsinh(1/31*sqrt(31)*(10*x + 3)) +
21534/120125*sqrt(5*x^2 + 3*x + 2) + 1405326/120125*x/sqrt(5*x^2 + 3*x + 2
) + 1991/125*x^2/(5*x^2 + 3*x + 2)^(3/2) + 2234283/600625/sqrt(5*x^2 + 3*x
 + 2) + 1155623/58125*x/(5*x^2 + 3*x + 2)^(3/2) + 156722/58125/(5*x^2 + 3*
x + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = -\frac{97}{625}\sqrt{5}\log\left(-2\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)-3\right) - \frac{2\,(3\,((5\,(961\,(10\,x-51)x-672205)x-4261249)x-3295492)x-1955612)}{360375\,(5\,x^2+3\,x+2)^{\frac{3}{2}}}$$

input
```
integrate((-2*x^2+x+4)^3/(5*x^2+3*x+2)^(5/2),x, algorithm="giac")
```

output
```
-97/625*sqrt(5)*log(-2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) - 3) -
2/360375*(3*((5*(961*(10*x - 51)*x - 672205)*x - 4261249)*x - 3295492)*x -
 1955612)/(5*x^2 + 3*x + 2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = \int \frac{(-2\,x^2+x+4)^3}{(5\,x^2+3\,x+2)^{5/2}}\,dx$$

input
```
int((x - 2*x^2 + 4)^3/(3*x + 5*x^2 + 2)^(5/2),x)
```

output
```
int((x - 2*x^2 + 4)^3/(3*x + 5*x^2 + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.90

$$\int \frac{(4+x-2x^2)^3}{(2+3x+5x^2)^{5/2}}\,dx = \frac{-14415000\sqrt{5x^2+3x+2}\,x^5+73516500\sqrt{5x^2+3x+2}\,x^4+1008307500\sqrt{5x^2}}{}$$

input
```
int((-2*x^2+x+4)^3/(5*x^2+3*x+2)^(5/2),x)
```

output
```
( - 14415000*sqrt(5*x**2 + 3*x + 2)*x**5 + 73516500*sqrt(5*x**2 + 3*x + 2)
*x**4 + 1008307500*sqrt(5*x**2 + 3*x + 2)*x**3 + 1278374700*sqrt(5*x**2 +
3*x + 2)*x**2 + 988647600*sqrt(5*x**2 + 3*x + 2)*x + 195561200*sqrt(5*x**2
 + 3*x + 2) + 69912750*sqrt(5)*log((2*sqrt(5*x**2 + 3*x + 2)*sqrt(5) + 10*
x + 3)/sqrt(31))*x**4 + 83895300*sqrt(5)*log((2*sqrt(5*x**2 + 3*x + 2)*sqr
t(5) + 10*x + 3)/sqrt(31))*x**3 + 81098790*sqrt(5)*log((2*sqrt(5*x**2 + 3*
x + 2)*sqrt(5) + 10*x + 3)/sqrt(31))*x**2 + 33558120*sqrt(5)*log((2*sqrt(5
*x**2 + 3*x + 2)*sqrt(5) + 10*x + 3)/sqrt(31))*x + 11186040*sqrt(5)*log((2
*sqrt(5*x**2 + 3*x + 2)*sqrt(5) + 10*x + 3)/sqrt(31)) - 1176655325*sqrt(5)
*x**4 - 1411986390*sqrt(5)*x**3 - 1364920177*sqrt(5)*x**2 - 564794556*sqrt
(5)*x - 188264852*sqrt(5))/(18018750*(25*x**4 + 30*x**3 + 29*x**2 + 12*x +
 4))
```

3.159 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 440

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd-af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2+2df)) - (2c^4d}{3c^3\,(b^2 -}$$
$$-\frac{2\left(2b^4ef + 48a^2c^2ef - \frac{b^5f^2}{c} + 4b^2ce(2cd-3af) + b^3(10af^2 - c(e^2+2df)) - 4bc(2c^2d^2 + 8a^2f^2 + ac(e^2\right.}{3c^2\,(b^2 -}$$
$$+\frac{f^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a
*c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2
*d*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/
(c*x^2+b*x+a)^(3/2)-2/3*(2*b^4*e*f+48*a^2*c^2*e*f-b^5*f^2/c+4*b^2*c*e*(-3*
a*f+2*c*d)+b^3*(10*a*f^2-c*(2*d*f+e^2))-4*b*c*(2*c^2*d^2+8*a^2*f^2+a*c*(2*
d*f+e^2))-2*(8*c^4*d^2-2*b^4*f^2+b^2*c*f*(14*a*f+b*e)-c^3*(8*b*d*e-4*a*(2*
d*f+e^2))-c^2*(12*a*b*e*f+16*a^2*f^2-b^2*(2*d*f+e^2)))*x)/c^2/(-4*a*c+b^2)
^2/(c*x^2+b*x+a)^(1/2)+f^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/
2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-3b^5f^2x^2 - 2b^4f^2x(3a+2cx^2) + 4bc(5a^3f^2 + 2c^3dx^2(3d-2ex) + 2a^2c(e^2 +}{} \\ + \frac{2f^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^
3*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x
)*(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6
*d*x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*
e + 3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*
(3*e + 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(
-6*e + f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2)))))
/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (2*f^2*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2191, 27, 2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2)))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{2\int \frac{3\left(4a-\frac{b^2}{c}\right)f^2x^2-\frac{3\left(b^2-4ac\right)f(2ce-bf)x}{c^2}+\frac{f^2b^4-cf(2be+af)b^2+8c^4d^2-c^3\left(8bde-4a\left(e^2+2df\right)\right)-c^2\left(4a^2f^2-b^2\left(e^2+2df\right)\right)}{c^3}}{2(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$\downarrow$ 27

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2)))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\int \frac{\frac{f^2b^4}{c^3}-\frac{f(2be+af)b^2}{c^2}-8deb+8cd^2+3\left(4a-\frac{b^2}{c}\right)f^2x^2+4a(e^2+2df)-\frac{4a^2f^2-b^2\left(e^2+2df\right)}{c}-\frac{3\left(b^2-4ac\right)f(2ce-bf)x}{c^2}}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$\downarrow$ 2191

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2)))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2))))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+4}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\downarrow$ 27

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2)))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2))))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+4}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\downarrow$ 1092

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2)))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2))))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+4}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\downarrow$ 219

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\frac{\frac{2\left(-2cx\left(-c^2\left(16a^2f^2+12abef-\left(b^2\left(2df+e^2\right)\right)\right)+b^2cf(14af+be)-c^3\left(8bde-4a\left(2df+e^2\right)\right)-2b^4f^2+8c^4d^2\right)-4bc^2\left(8a^2f^2+ac\left(2df+e^2\right)+2c^2d^2\right)+4\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}}{3\left(b^2-4ac\right)}$$

input
```
Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a
^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*
f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e
^2 + 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - ((2*(2*b
^4*c*e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*
(10*a*f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 +
 2*d*f)) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*
d*e - 4*a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f
)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*c)*f^2*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2))/(3*(b^2 -
4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(424) = 848$.

Time = 2.54 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.57

method	result	size
default	Expression too large to display	1129

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^2*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2))+f^2*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(
-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(
-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(
c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+
b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^
(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(2*d*f+e^2)*(-1/2*x/c/(c*x^2+b*x+a)^(3/
2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/
(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1
/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2
*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*d*e*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c
*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*
x+b)/(c*x^2+b*x+a)^(1/2)))+2*e*f*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2
*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(
c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(...
```

Fricas [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 1581, normalized size of antiderivative = 3.59

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a
^4*c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b
*c^5*d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)
*f^2 + (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*
c^3 - 12*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*
a^3*b*c^2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^
2 - (b^5*c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3
+ 4*a^2*c^4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3
*e^2 + (b^2*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*
a^2*b^2*c^2 + 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(
c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^
4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^
4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*f^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^
4 - 8*a^3*b^2*c + 16*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{f^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}} + \frac{2\left(\left(\left(\frac{2\,(8\,c^5d^2-8\,bc^4de+b^2c^3e^2+4\,ac^4e^2+2\,b^2c^3df+8\,ac^4df+b^3c^2ef-12\,abc^3ef-2\,b^4cf^2+14\,ab^2c^2f^2-16\,a^2c^3f^2)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4} + \frac{3\,(8\,bc^4d^2-8\,b^2c^3de-}{}\right.\right.\right.}{}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-f^2*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 - 8*b*c^4*d*e + b^2*c^3*e^2 + 4*a*c^4*e^2 + 2*b^2*c^3
*d*f + 8*a*c^4*d*f + b^3*c^2*e*f - 12*a*b*c^3*e*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b
*c^4*d^2 - 8*b^2*c^3*d*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2 + 2*b^3*c^2*d*f + 8
*a*b*c^3*d*f - 4*a*b^2*c^2*e*f - 16*a^2*c^3*e*f - b^5*f^2 + 6*a*b^3*c*f^2)
/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 -
b^3*c^2*d*e - 4*a*b*c^3*d*e + 2*a*b^2*c^2*e^2 + 4*a*b^2*c^2*d*f - 8*a^2*b*
c^2*e*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2)/(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 + 4*a*b^2*c^2*d*e + 1
6*a^2*c^3*d*e - 8*a^2*b*c^2*e^2 - 16*a^2*b*c^2*d*f + 32*a^3*c^2*e*f + 3*a^
2*b^3*f^2 - 20*a^3*b*c*f^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 +
 b*x + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^2}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 2663, normalized size of antiderivative = 6.05

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(40*sqrt(a + b*x + c*x**2)*a**3*b*c**2*f**2 - 64*sqrt(a + b*x + c*x**2)*a*
*3*c**3*e*f - 48*sqrt(a + b*x + c*x**2)*a**3*c**3*f**2*x - 6*sqrt(a + b*x
+ c*x**2)*a**2*b**3*c*f**2 + 84*sqrt(a + b*x + c*x**2)*a**2*b**2*c**2*f**2
*x + 32*sqrt(a + b*x + c*x**2)*a**2*b*c**3*d*f + 16*sqrt(a + b*x + c*x**2)
*a**2*b*c**3*e**2 - 96*sqrt(a + b*x + c*x**2)*a**2*b*c**3*e*f*x - 32*sqrt(
a + b*x + c*x**2)*a**2*c**4*d*e - 96*sqrt(a + b*x + c*x**2)*a**2*c**4*e*f*
x**2 - 64*sqrt(a + b*x + c*x**2)*a**2*c**4*f**2*x**3 - 12*sqrt(a + b*x + c
*x**2)*a*b**4*c*f**2*x + 36*sqrt(a + b*x + c*x**2)*a*b**3*c**2*f**2*x**2 -
 8*sqrt(a + b*x + c*x**2)*a*b**2*c**3*d*e + 48*sqrt(a + b*x + c*x**2)*a*b*
*2*c**3*d*f*x + 24*sqrt(a + b*x + c*x**2)*a*b**2*c**3*e**2*x - 24*sqrt(a +
 b*x + c*x**2)*a*b**2*c**3*e*f*x**2 + 56*sqrt(a + b*x + c*x**2)*a*b**2*c**
3*f**2*x**3 + 24*sqrt(a + b*x + c*x**2)*a*b*c**4*d**2 - 48*sqrt(a + b*x +
c*x**2)*a*b*c**4*d*e*x + 48*sqrt(a + b*x + c*x**2)*a*b*c**4*d*f*x**2 + 24*
sqrt(a + b*x + c*x**2)*a*b*c**4*e**2*x**2 - 48*sqrt(a + b*x + c*x**2)*a*b*
c**4*e*f*x**3 + 48*sqrt(a + b*x + c*x**2)*a*c**5*d**2*x + 32*sqrt(a + b*x
+ c*x**2)*a*c**5*d*f*x**3 + 16*sqrt(a + b*x + c*x**2)*a*c**5*e**2*x**3 - 6
*sqrt(a + b*x + c*x**2)*b**5*c*f**2*x**2 - 8*sqrt(a + b*x + c*x**2)*b**4*c
**2*f**2*x**3 - 2*sqrt(a + b*x + c*x**2)*b**3*c**3*d**2 - 12*sqrt(a + b*x
+ c*x**2)*b**3*c**3*d*e*x + 12*sqrt(a + b*x + c*x**2)*b**3*c**3*d*f*x**2 +
 6*sqrt(a + b*x + c*x**2)*b**3*c**3*e**2*x**2 + 4*sqrt(a + b*x + c*x**2...
```

3.160 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8c^2d-4bce+b^2f+4acf\right)(b+2cx)}{3c\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

output
```
2/3*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/
(c*x^2+b*x+a)^(3/2)+2/3*(4*a*c*f+b^2*f-4*b*c*e+8*c^2*d)*(2*c*x+b)/c/(-4*a*
c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{-2b^3(d+3x(e-fx))+16c(-a^2e+2c^2dx^3+acx(3d+fx^2))-4b^2(a(e-6fx}{3\left(b^2-4ac\right)^2(a+}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d +
f*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f
 - 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(
a + x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2191, 27, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int \frac{\frac{fb^2}{c}-4eb+8cd+4af}{2(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(4af+\frac{b^2f}{c}-4be+8cd\right)\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f +
(b^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.33

method	result
trager	$\frac{\frac{16}{3}a\,c^2f\,x^3+\frac{4}{3}b^2cf\,x^3-\frac{16}{3}b\,c^2x^3e+\frac{32}{3}c^3d\,x^3+8abcf\,x^2+2b^3f\,x^2-8b^2ce\,x^2+16b\,c^2d\,x^2+8a\,b^2fx-8abcex+16adx\,c^2-2b^3ex+4b^2cx}{(4ac-b^2)^2(c\,x^2+bx+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{16}{3}a\,c^2f\,x^3+\frac{4}{3}b^2cf\,x^3-\frac{16}{3}b\,c^2x^3e+\frac{32}{3}c^3d\,x^3+8abcf\,x^2+2b^3f\,x^2-8b^2ce\,x^2+16b\,c^2d\,x^2+8a\,b^2fx-8abcex+16adx\,c^2-2b^3ex+4b^2cx}{(c\,x^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ca\,b^2+b^4)}$
orering	$\frac{\frac{16}{3}a\,c^2f\,x^3+\frac{4}{3}b^2cf\,x^3-\frac{16}{3}b\,c^2x^3e+\frac{32}{3}c^3d\,x^3+8abcf\,x^2+2b^3f\,x^2-8b^2ce\,x^2+16b\,c^2d\,x^2+8a\,b^2fx-8abcex+16adx\,c^2-2b^3ex+4b^2cx}{(c\,x^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ca\,b^2+b^4)}$
default	$d\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{c\,x^2+bx+a}}\right)+e\left(-\frac{1}{3c(c\,x^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{1}{3\left(4ac-b\right.}\right.}{2c}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d*x^3+12*a*b*c*f*x^2
+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-12*a*b*c*e*x+24*a*
c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2*e+12*a*b*c*d-b^3
*d)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(124) = 248$.

Time = 1.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.17

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,a^2bf+2\,(8\,c^3d-4\,bc^2e+(b^2c+4\,ac^2)f)x^3+3\,(8\,bc^2d-4\,b^2ce+(b^3+4\,ab}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*
b*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b
^2 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e
)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x)
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.77

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{2\,(8\,c^3d-4\,bc^2e+b^2cf+4\,ac^2f)\,x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(8\,bc^2d-4\,b^2ce+b^3f+4\,abcf)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{3\,(2\,b^2cd+8\,ac^2d-b^3e-4}{b^4-8\,ab^2c+16\,a}\right.\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(((2*(8*c^3*d - 4*b*c^2*e + b^2*c*f + 4*a*c^2*f)*x/(b^4 - 8*a*b^2*c +
16*a^2*c^2) + 3*(8*b*c^2*d - 4*b^2*c*e + b^3*f + 4*a*b*c*f)/(b^4 - 8*a*b^2
*c + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d - b^3*e - 4*a*b*c*e + 4*a*b
^2*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d + 2*a*b^2*e
+ 8*a^2*c*e - 8*a^2*b*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.33

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,f\,a^2\,b-8\,e\,a^2\,c+12\,f\,a\,b^2\,x-2\,e\,a\,b^2+12\,f\,a\,b\,c\,x^2-12\,e\,a\,b\,c\,x+12\,d\,a\,b\,c}{}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

output
```
(2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e
 - 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2
- 12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*
c*d - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)
^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 793, normalized size of antiderivative = 6.01

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(2*(8*sqrt(a + b*x + c*x**2)*a**2*b*c*f - 8*sqrt(a + b*x + c*x**2)*a**2*c*
*2*e - 2*sqrt(a + b*x + c*x**2)*a*b**2*c*e + 12*sqrt(a + b*x + c*x**2)*a*b
**2*c*f*x + 12*sqrt(a + b*x + c*x**2)*a*b*c**2*d - 12*sqrt(a + b*x + c*x**
2)*a*b*c**2*e*x + 12*sqrt(a + b*x + c*x**2)*a*b*c**2*f*x**2 + 24*sqrt(a +
b*x + c*x**2)*a*c**3*d*x + 8*sqrt(a + b*x + c*x**2)*a*c**3*f*x**3 - sqrt(a
 + b*x + c*x**2)*b**3*c*d - 3*sqrt(a + b*x + c*x**2)*b**3*c*e*x + 3*sqrt(a
 + b*x + c*x**2)*b**3*c*f*x**2 + 6*sqrt(a + b*x + c*x**2)*b**2*c**2*d*x -
12*sqrt(a + b*x + c*x**2)*b**2*c**2*e*x**2 + 2*sqrt(a + b*x + c*x**2)*b**2
*c**2*f*x**3 + 24*sqrt(a + b*x + c*x**2)*b*c**3*d*x**2 - 8*sqrt(a + b*x +
c*x**2)*b*c**3*e*x**3 + 16*sqrt(a + b*x + c*x**2)*c**4*d*x**3 + 8*sqrt(c)*
a**3*c*f - 6*sqrt(c)*a**2*b**2*f + 8*sqrt(c)*a**2*b*c*e + 16*sqrt(c)*a**2*
b*c*f*x - 16*sqrt(c)*a**2*c**2*d + 16*sqrt(c)*a**2*c**2*f*x**2 - 12*sqrt(c
)*a*b**3*f*x + 16*sqrt(c)*a*b**2*c*e*x - 4*sqrt(c)*a*b**2*c*f*x**2 - 32*sq
rt(c)*a*b*c**2*d*x + 16*sqrt(c)*a*b*c**2*e*x**2 + 16*sqrt(c)*a*b*c**2*f*x*
*3 - 32*sqrt(c)*a*c**3*d*x**2 + 8*sqrt(c)*a*c**3*f*x**4 - 6*sqrt(c)*b**4*f
*x**2 + 8*sqrt(c)*b**3*c*e*x**2 - 12*sqrt(c)*b**3*c*f*x**3 - 16*sqrt(c)*b*
*2*c**2*d*x**2 + 16*sqrt(c)*b**2*c**2*e*x**3 - 6*sqrt(c)*b**2*c**2*f*x**4
- 32*sqrt(c)*b*c**3*d*x**3 + 8*sqrt(c)*b*c**3*e*x**4 - 16*sqrt(c)*c**4*d*x
**4))/(3*c*(16*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*b*c**2*x + 32*a**3*c**3
*x**2 + a**2*b**4 - 16*a**2*b**3*c*x + 32*a**2*b*c**3*x**3 + 16*a**2*c*...
```

3.161 $\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\begin{aligned}\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx &= \frac{481+1035x}{21669(2+3x+5x^2)^{3/2}} \\ &+ \frac{107(28891+78685x)}{104343458\sqrt{2+3x+5x^2}} \\ &- \frac{\sqrt{\frac{4466971907+775844179\sqrt{33}}{7689}}\operatorname{arctanh}\left(\frac{19-3\sqrt{33}+2(11-5\sqrt{33})x}{2\sqrt{2(107-11\sqrt{33})}\sqrt{2+3x+5x^2}}\right)}{434312} \\ &+ \frac{(7721-1177\sqrt{33})\operatorname{arctanh}\left(\frac{19+3\sqrt{33}+2(11+5\sqrt{33})x}{2\sqrt{2(107+11\sqrt{33})}\sqrt{2+3x+5x^2}}\right)}{108578\sqrt{66(107+11\sqrt{33})}}\end{aligned}$$

output
```
1/21669*(481+1035*x)/(5*x^2+3*x+2)^(3/2)+107/104343458*(28891+78685*x)/(5*
x^2+3*x+2)^(1/2)-1/3339424968*(34346546992923+5965465892331*33^(1/2))^(1/2
)*arctanh(1/2*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)/(214-22*33^(1/2))^(1/2)/
(5*x^2+3*x+2)^(1/2))+1/108578*(7721-1177*33^(1/2))*arctanh(1/2*(19+3*33^(1
/2)+2*(11+5*33^(1/2))*x)/(214+22*33^(1/2))^(1/2)/(5*x^2+3*x+2)^(1/2))/(706
2+726*33^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.97 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06

$$\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx = \frac{25496548+93289413x+122143710x^2+126289425x^3}{313030374(2+3x+5x^2)^{3/2}} + \frac{\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3+2\#1^4\&,\frac{-18055\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{2+3x+5x^2}-\#1\right)+44490\log\left(-\sqrt{5}x\right.}{22\sqrt{5}-91\#}\right.}{217156\sqrt{5}}$$

input
```
Integrate[1/((4 + x - 2*x^2)*(2 + 3*x + 5*x^2)^(5/2)),x]
```

output
```
(25496548 + 93289413*x + 122143710*x^2 + 126289425*x^3)/(313030374*(2 + 3*
x + 5*x^2)^(3/2)) + RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3
 + 2*#1^4 & , (-18055*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #
1] + 44490*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 2354*Sqrt[5
]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1
 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ]/(217156*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1305, 27, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^2+x+4)(5x^2+3x+2)^{5/2}}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{1035x+481}{21669(5x^2+3x+2)^{3/2}} - \frac{\int -\frac{3(-2760x^2+698x+7349)}{2(-2x^2+x+4)(5x^2+3x+2)^{3/2}}dx}{21669}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-2760x^2+698x+7349}{(-2x^2+x+4)(5x^2+3x+2)^{3/2}}dx}{14446}+\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\int \frac{961(4449-2354x)}{2\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx}{7223}+\frac{107(78685x+28891)}{7223\sqrt{5x^2+3x+2}}}{14446}+\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{31}{466}\int \frac{4449-2354x}{(-2x^2+x+4)\sqrt{5x^2+3x+2}}dx+\frac{107(78685x+28891)}{7223\sqrt{5x^2+3x+2}}}{14446}+\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \ 1365$$

$$\frac{\frac{31}{466}\left(-\frac{2}{33}\left(38841+7721\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx-\frac{2}{33}\left(38841-7721\sqrt{33}\right)\int \frac{1}{\left(-4x+\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx\right)}{14446}$$
$$\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{31}{466}\left(\frac{4}{33}\left(38841+7721\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)+\frac{4}{33}\left(38841-772\right.\right.}{14446}$$
$$\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{31}{466}\left(-\frac{1}{33}\sqrt{\frac{2}{107-11\sqrt{33}}}\left(38841+7721\sqrt{33}\right)\text{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)-\frac{1}{33}\left(38841-7721\sqrt{33}\right)\sqrt{\frac{}{107+}}\right.}{14446}$$
$$\frac{1035x+481}{21669\,(5x^2+3x+2)^{3/2}}$$

input
```
Int[1/((4 + x - 2*x^2)*(2 + 3*x + 5*x^2)^(5/2)),x]
```

output
```
(481 + 1035*x)/(21669*(2 + 3*x + 5*x^2)^(3/2)) + ((107*(28891 + 78685*x))/
(7223*Sqrt[2 + 3*x + 5*x^2]) + (31*(-1/33*(Sqrt[2/(107 - 11*Sqrt[33])]*(38
841 + 7721*Sqrt[33])*ArcTanh[(19 - 3*Sqrt[33] + 2*(11 - 5*Sqrt[33])*x)/(2*
Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])]) - ((38841 - 7721*Sqrt
[33])*Sqrt[2/(107 + 11*Sqrt[33])]*ArcTanh[(19 + 3*Sqrt[33] + 2*(11 + 5*Sqr
t[33])*x)/(2*Sqrt[2*(107 + 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])])/33))/466
)/14446
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.49 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.29

method	result	size
trager	Expression too large to display	471
default	Expression too large to display	868

input
```
int(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/313030374*(126289425*x^3+122143710*x^2+93289413*x+25496548)/(5*x^2+3*x+2
)^(3/2)+1/3339424968*RootOf(_Z^2+60539618304*RootOf(129913344*_Z^4-1474100
72931*_Z^2+188626913344)^2-68693093985846)*ln(-(-15507309802822 0416*x*Root
Of(129913344*_Z^4-147410072931*_Z^2+188626913344)^4*RootOf(_Z^2+6053961830
4*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2-68693093985846)+
18570003256984386 2400*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344
)^2*RootOf(_Z^2+60539618304*RootOf(129913344*_Z^4-147410072931*_Z^2+188626
913344)^2-68693093985846)*x+10473263815690235 5239720960*(5*x^2+3*x+2)^(1/2
)*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2-7886774808737240
064*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2*RootOf(_Z^2+60
539618304*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2-68693093
985846)-27828862679762090275069*RootOf(_Z^2+60539618304*RootOf(129913344*_
Z^4-147410072931*_Z^2+188626913344)^2-68693093985846)*x-120846116382610297
241654369124*(5*x^2+3*x+2)^(1/2)+13850181226853609 36344*RootOf(_Z^2+605396
18304*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2-686930939858
46))/(3936768*x*RootOf(129913344*_Z^4-147410072931*_Z^2+188626913344)^2-26
21408043*x-3103376716))+4/54289*RootOf(129913344*_Z^4-147410072931*_Z^2+18
8626913344)*ln((18066015920287678464*x*RootOf(129913344*_Z^4-147410072931*
_Z^2+188626913344)^5-193642400280278261 26272*RootOf(129913344*_Z^4-1474100
72931*_Z^2+188626913344)^3*x+49589317309139372746080*(5*x^2+3*x+2)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(150) = 300$.

Time = 0.14 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.84

$$\int \frac{1}{(4+x-2x^2)\,(2+3x+5x^2)^{5/2}}\,dx = \frac{2883\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\sqrt{\frac{70531289}{233}\,\sqrt{\frac{11}{3}}+\frac{4466971907}{7689}}}{}$$

input
```
integrate(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/2504242992*(2883*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(70531289/233
*sqrt(11/3) + 4466971907/7689)*log(-(3*sqrt(5*x^2 + 3*x + 2)*sqrt(70531289
/233*sqrt(11/3) + 4466971907/7689)*(178291*sqrt(11/3) - 358369) + 5211744*
sqrt(11/3)*(3*x + 4) - 133768096*x - 34744960)/x) - 2883*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4)*sqrt(70531289/233*sqrt(11/3) + 4466971907/7689)*log((
3*sqrt(5*x^2 + 3*x + 2)*sqrt(70531289/233*sqrt(11/3) + 4466971907/7689)*(1
78291*sqrt(11/3) - 358369) - 5211744*sqrt(11/3)*(3*x + 4) + 133768096*x +
34744960)/x) + 2883*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-70531289/2
33*sqrt(11/3) + 4466971907/7689)*log((3*sqrt(5*x^2 + 3*x + 2)*(178291*sqrt
(11/3) + 358369)*sqrt(-70531289/233*sqrt(11/3) + 4466971907/7689) + 521174
4*sqrt(11/3)*(3*x + 4) + 133768096*x + 34744960)/x) - 2883*(25*x^4 + 30*x^
3 + 29*x^2 + 12*x + 4)*sqrt(-70531289/233*sqrt(11/3) + 4466971907/7689)*lo
g(-(3*sqrt(5*x^2 + 3*x + 2)*(178291*sqrt(11/3) + 358369)*sqrt(-70531289/23
3*sqrt(11/3) + 4466971907/7689) - 5211744*sqrt(11/3)*(3*x + 4) - 133768096
*x - 34744960)/x) + 8*(126289425*x^3 + 122143710*x^2 + 93289413*x + 254965
48)*sqrt(5*x^2 + 3*x + 2))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1}{(4+x-2x^2)\left(2+3x+5x^2\right)^{5/2}}\,dx =$$
$$-\int \frac{1}{50x^6\sqrt{5x^2+3x+2}+35x^5\sqrt{5x^2+3x+2}-72x^4\sqrt{5x^2+3x+2}-125x^3\sqrt{5x^2+3x+2}-120x^2\sqrt{5}$$

input
```
integrate(1/(-2*x**2+x+4)/(5*x**2+3*x+2)**(5/2),x)
```

output
```
-Integral(1/(50*x**6*sqrt(5*x**2 + 3*x + 2) + 35*x**5*sqrt(5*x**2 + 3*x +
2) - 72*x**4*sqrt(5*x**2 + 3*x + 2) - 125*x**3*sqrt(5*x**2 + 3*x + 2) - 12
0*x**2*sqrt(5*x**2 + 3*x + 2) - 52*x*sqrt(5*x**2 + 3*x + 2) - 16*sqrt(5*x*
*2 + 3*x + 2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 1099, normalized size of antiderivative = 5.33

$$\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x, algorithm="maxima")
```

output
```
1/95139*sqrt(33)*(6200*sqrt(33)*x/(11*sqrt(33)*(5*x^2 + 3*x + 2)^(3/2) + 1
07*(5*x^2 + 3*x + 2)^(3/2)) - 6200*sqrt(33)*x/(11*sqrt(33)*(5*x^2 + 3*x +
2)^(3/2) - 107*(5*x^2 + 3*x + 2)^(3/2)) + 74400*sqrt(33)*x/(1177*sqrt(33)*
sqrt(5*x^2 + 3*x + 2) + 7721*sqrt(5*x^2 + 3*x + 2)) - 74400*sqrt(33)*x/(11
77*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 7721*sqrt(5*x^2 + 3*x + 2)) + 8000*sqr
t(33)*x/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) + 107*sqrt(5*x^2 + 3*x + 2)) -
8000*sqrt(33)*x/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 107*sqrt(5*x^2 + 3*x
+ 2)) + 13640*x/(11*sqrt(33)*(5*x^2 + 3*x + 2)^(3/2) + 107*(5*x^2 + 3*x +
2)^(3/2)) + 13640*x/(11*sqrt(33)*(5*x^2 + 3*x + 2)^(3/2) - 107*(5*x^2 + 3*
x + 2)^(3/2)) + 163680*x/(1177*sqrt(33)*sqrt(5*x^2 + 3*x + 2) + 7721*sqrt(
5*x^2 + 3*x + 2)) + 163680*x/(1177*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 7721*s
qrt(5*x^2 + 3*x + 2)) + 17600*x/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) + 107*s
qrt(5*x^2 + 3*x + 2)) + 17600*x/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 107*s
qrt(5*x^2 + 3*x + 2)) + 1860*sqrt(33)/(11*sqrt(33)*(5*x^2 + 3*x + 2)^(3/2)
 + 107*(5*x^2 + 3*x + 2)^(3/2)) - 1860*sqrt(33)/(11*sqrt(33)*(5*x^2 + 3*x
+ 2)^(3/2) - 107*(5*x^2 + 3*x + 2)^(3/2)) + 22320*sqrt(33)/(1177*sqrt(33)*
sqrt(5*x^2 + 3*x + 2) + 7721*sqrt(5*x^2 + 3*x + 2)) - 22320*sqrt(33)/(1177
*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 7721*sqrt(5*x^2 + 3*x + 2)) + 2400*sqrt(
33)/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) + 107*sqrt(5*x^2 + 3*x + 2)) - 2400
*sqrt(33)/(11*sqrt(33)*sqrt(5*x^2 + 3*x + 2) - 107*sqrt(5*x^2 + 3*x + 2...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.59

$$\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx = \frac{3\,(535\,(78685\,x+76102)x+31096471)x+25496548}{313030374\,(5\,x^2+3\,x+2)^{\frac{3}{2}}} + 0.0000833934028287498\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}+8.38267526007000\right) - 0.00248050182749728\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.312157316296000\right) - 0.0000833934028286578\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.842024981991000\right) + 0.00248050182749728\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-4.99242498429000\right)$$

input
```
integrate(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/313030374*(3*(535*(78685*x + 76102)*x + 31096471)*x + 25496548)/(5*x^2 +
 3*x + 2)^(3/2) + 0.0000833934028287498*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x
+ 2) + 8.38267526007000) - 0.00248050182749728*log(-sqrt(5)*x + sqrt(5*x^2
 + 3*x + 2) - 0.312157316296000) - 0.0000833934028286578*log(-sqrt(5)*x +
sqrt(5*x^2 + 3*x + 2) - 0.842024981991000) + 0.00248050182749728*log(-sqrt
(5)*x + sqrt(5*x^2 + 3*x + 2) - 4.99242498429000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx = \int \frac{1}{(-2\,x^2+x+4)\,(5\,x^2+3\,x+2)^{5/2}}\,dx$$

input
```
int(1/((x - 2*x^2 + 4)*(3*x + 5*x^2 + 2)^(5/2)),x)
```

output
```
int(1/((x - 2*x^2 + 4)*(3*x + 5*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(4+x-2x^2)(2+3x+5x^2)^{5/2}}\,dx = \int \frac{1}{(-2x^2+x+4)(5x^2+3x+2)^{\frac{5}{2}}}dx$$

input
```
int(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x)
```

output
```
int(1/(-2*x^2+x+4)/(5*x^2+3*x+2)^(5/2),x)
```

3.162 $\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\begin{aligned}\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}} \, dx &= \frac{1909421+953635x}{222150588 (2+3x+5x^2)^{3/2}} \\ &- \frac{235-214x}{15378 (4+x-2x^2) (2+3x+5x^2)^{3/2}} + \frac{34495558451+65901205285x}{3209187394248\sqrt{2+3x+5x^2}} \\ &- \frac{\left(191614963+30645571\sqrt{33}\right) \operatorname{arctanh}\left(\frac{19-3\sqrt{33}+2\left(11-5\sqrt{33}\right)x}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{3339424968\sqrt{66\left(107-11\sqrt{33}\right)}} \\ &+ \frac{\left(191614963-30645571\sqrt{33}\right) \operatorname{arctanh}\left(\frac{19+3\sqrt{33}+2\left(11+5\sqrt{33}\right)x}{2\sqrt{2\left(107+11\sqrt{33}\right)}\sqrt{2+3x+5x^2}}\right)}{3339424968\sqrt{66\left(107+11\sqrt{33}\right)}}\end{aligned}$$

output
```
1/222150588*(1909421+953635*x)/(5*x^2+3*x+2)^(3/2)-1/15378*(235-214*x)/(-2
*x^2+x+4)/(5*x^2+3*x+2)^(3/2)+1/3209187394248*(34495558451+65901205285*x)/
(5*x^2+3*x+2)^(1/2)-1/3339424968*(191614963+30645571*33^(1/2))*arctanh(1/2
*(19-3*33^(1/2)+2*(11-5*33^(1/2))*x)/(214-22*33^(1/2))^(1/2)/(5*x^2+3*x+2)
^(1/2))/(7062-726*33^(1/2))^(1/2)+1/3339424968*(191614963-30645571*33^(1/2
))*arctanh(1/2*(19+3*33^(1/2)+2*(11+5*33^(1/2))*x)/(214+22*33^(1/2))^(1/2)
/(5*x^2+3*x+2)^(1/2))/(7062+726*33^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.73

$$\int \frac{1}{(4+x-2x^2)^2(2+3x+5x^2)^{5/2}}\,dx = \frac{-337257025412-1137494801224x-1536641704237x^2-1190\cdots}{3209187394248(-4-x+2x^2)\cdots} + \frac{\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3+2\#1^4\&, \frac{-8023187\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{2+3x+5x^2}-\#1\right)+5896170\log\left(\cdots\right.}{22\sqrt{5}-\cdots}\cdots\right.}{50597348\sqrt{5}} + \frac{\text{RootSum}\left[-22+44\sqrt{5}\#1-91\#1^2+2\sqrt{5}\#1^3+2\#1^4\&, \frac{603087599\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{2+3x+5x^2}-\#1\right)+333008230\log\cdots}{22\sqrt{\cdots}}\cdots\right.}{6678849936\sqrt{5}}$$

input
```
Integrate[1/((4 + x - 2*x^2)^2*(2 + 3*x + 5*x^2)^(5/2)),x]
```

output
```
(-337257025412 - 1137494801224*x - 1536641704237*x^2 - 1190074919544*x^3 +
 410856789795*x^4 + 659012052850*x^5)/(3209187394248*(-4 - x + 2*x^2)*(2 +
 3*x + 5*x^2)^(3/2)) + RootSum[-22 + 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#
1^3 + 2*#1^4 & , (-8023187*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2
] - #1] + 5896170*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 3847
58*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5
] - 91*#1 + 3*Sqrt[5]*#1^2 + 4*#1^3) & ]/(50597348*Sqrt[5]) + RootSum[-22
+ 44*Sqrt[5]*#1 - 91*#1^2 + 2*Sqrt[5]*#1^3 + 2*#1^4 & , (603087599*Sqrt[5]
*Log[-(Sqrt[5]*x) + Sqrt[2 + 3*x + 5*x^2] - #1] + 333008230*Log[-(Sqrt[5]*
x) + Sqrt[2 + 3*x + 5*x^2] - #1]*#1 + 10503086*Sqrt[5]*Log[-(Sqrt[5]*x) +
Sqrt[2 + 3*x + 5*x^2] - #1]*#1^2)/(22*Sqrt[5] - 91*#1 + 3*Sqrt[5]*#1^2 + 4
*#1^3) & ]/(6678849936*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2x^2+x+4)^2 (5x^2+3x+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{\int \frac{8560x^2-7470x+4483}{2(-2x^2+x+4)(5x^2+3x+2)^{5/2}}dx}{15378} - \frac{235-214x}{15378(-2x^2+x+4)(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{8560x^2-7470x+4483}{(-2x^2+x+4)(5x^2+3x+2)^{5/2}}dx}{30756} - \frac{235-214x}{15378(-2x^2+x+4)(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{\frac{\int \frac{3\left(-7629080x^2-15665426x+59121207\right)}{2\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}dx}{21669} + \frac{953635x+1909421}{7223(5x^2+3x+2)^{3/2}}}{30756} - \frac{235-214x}{15378(-2x^2+x+4)(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-7629080x^2-15665426x+59121207}{\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}dx}{14446} + \frac{953635x+1909421}{7223(5x^2+3x+2)^{3/2}}}{30756} - \frac{235-214x}{15378(-2x^2+x+4)(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{\frac{\frac{\int \frac{961(111130267-61291142x)}{2\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx}{7223} + \frac{65901205285x+34495558451}{7223\sqrt{5x^2+3x+2}}}{14446} + \frac{953635x+1909421}{7223(5x^2+3x+2)^{3/2}}}{30756} - \frac{235-214x}{15378(-2x^2+x+4)(5x^2+3x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{31}{466}\int \frac{111130267-61291142x}{\left(-2x^2+x+4\right)\sqrt{5x^2+3x+2}}dx+\frac{65901205285x+34495558451}{7223\sqrt{5x^2+3x+2}}}{14446}+\frac{953635x+1909421}{7223(5x^2+3x+2)^{3/2}}}{30756}-\frac{235-214x}{15378\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1365$$

$$\frac{\frac{\frac{31}{466}\left(-\frac{2}{33}\left(1011303843+191614963\sqrt{33}\right)\int \frac{1}{\left(-4x-\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx-\frac{2}{33}\left(1011303843-191614963\sqrt{33}\right)\int \frac{1}{\left(-4x+\sqrt{33}+1\right)\sqrt{5x^2+3x+2}}dx\right)+6}{14446}}{30756}-\frac{235-214x}{15378\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{\frac{31}{466}\left(\frac{4}{33}\left(1011303843+191614963\sqrt{33}\right)\int \frac{1}{8\left(107-11\sqrt{33}\right)-\frac{\left(2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19\right)^2}{5x^2+3x+2}}d\left(-\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{\sqrt{5x^2+3x+2}}\right)+\frac{4}{33}\left(1011303843-19161496\right.\right.}{14446}}{30756}-\frac{235-214x}{15378\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{31}{466}\left(-\frac{1}{33}\sqrt{\frac{2}{107-11\sqrt{33}}}\left(1011303843+191614963\sqrt{33}\right)\text{arctanh}\left(\frac{2\left(11-5\sqrt{33}\right)x-3\sqrt{33}+19}{2\sqrt{2\left(107-11\sqrt{33}\right)}\sqrt{5x^2+3x+2}}\right)-\frac{1}{33}\left(1011303843-191614963\sqrt{33}\right)\sqrt{\frac{2}{107+11\sqrt{\ }}}\right.}{14446}}{30756}-\frac{235-214x}{15378\left(-2x^2+x+4\right)\left(5x^2+3x+2\right)^{3/2}}$$

input

```
Int[1/((4 + x - 2*x^2)^2*(2 + 3*x + 5*x^2)^(5/2)),x]
```

output
```
-1/15378*(235 - 214*x)/((4 + x - 2*x^2)*(2 + 3*x + 5*x^2)^(3/2)) + ((19094
21 + 953635*x)/(7223*(2 + 3*x + 5*x^2)^(3/2)) + ((34495558451 + 6590120528
5*x)/(7223*Sqrt[2 + 3*x + 5*x^2]) + (31*(-1/33*(Sqrt[2/(107 - 11*Sqrt[33])
]*(1011303843 + 191614963*Sqrt[33])*ArcTanh[(19 - 3*Sqrt[33] + 2*(11 - 5*S
qrt[33])*x)/(2*Sqrt[2*(107 - 11*Sqrt[33])]*Sqrt[2 + 3*x + 5*x^2])]) - ((10
11303843 - 191614963*Sqrt[33])*Sqrt[2/(107 + 11*Sqrt[33])]*ArcTanh[(19 + 3
*Sqrt[33] + 2*(11 + 5*Sqrt[33])*x)/(2*Sqrt[2*(107 + 11*Sqrt[33])]*Sqrt[2 +
 3*x + 5*x^2])])/33))/466)/14446)/30756
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

method	result
risch	$\frac{659012052850x^5+410856789795x^4-1190074919544x^3-1536641704237x^2-1137494801224x-337257025412}{3209187394248(5x^2+3x+2)^{\frac{3}{2}}(2x^2-x-4)} -$ $\left(-191614963+3\right.$
trager	Expression too large to display
default	Expression too large to display

input
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3209187394248*(659012052850*x^5+410856789795*x^4-1190074919544*x^3-15366
41704237*x^2-1137494801224*x-337257025412)/(5*x^2+3*x+2)^(3/2)/(2*x^2-x-4)
-1/110201023944*(-191614963+30645571*33^(1/2))*33^(1/2)/(214+22*33^(1/2))^
(1/2)*arctanh(8*(107/4+11/4*33^(1/2)+(11/2+5/2*33^(1/2))*(x-1/4*33^(1/2)-1
/4))/(214+22*33^(1/2))^(1/2)/(80*(x-1/4*33^(1/2)-1/4)^2+16*(11/2+5/2*33^(1
/2))*(x-1/4*33^(1/2)-1/4)+214+22*33^(1/2))^(1/2))-1/110201023944*(19161496
3+30645571*33^(1/2))*33^(1/2)/(214-22*33^(1/2))^(1/2)*arctanh(8*(107/4-11/
4*33^(1/2)+(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2)))/(214-22*33^(1/2))^(1/
2)/(80*(x-1/4+1/4*33^(1/2))^2+16*(11/2-5/2*33^(1/2))*(x-1/4+1/4*33^(1/2))+
214-22*33^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(188) = 376$.

Time = 0.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.77

$$\int \frac{1}{(4+x-2x^2)^2(2+3x+5x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/25673499153984*(961*(50*x^6 + 35*x^5 - 72*x^4 - 125*x^3 - 120*x^2 - 52*x
 - 16)*sqrt(15162357162557627/233*sqrt(33) + 287699164683339 4763/7689)*log
(-(sqrt(5*x^2 + 3*x + 2)*sqrt(15162357162557627/233*sqrt(33) + 28769916468
33394763/7689)*(4566831233*sqrt(33) - 27060312081) + 715538790282752*sqrt(
33)*(3*x + 4) - 55096486851771904*x - 14310775805655040)/x) - 961*(50*x^6
+ 35*x^5 - 72*x^4 - 125*x^3 - 120*x^2 - 52*x - 16)*sqrt(15162357162557627/
233*sqrt(33) + 2876991646833394763/7689)*log((sqrt(5*x^2 + 3*x + 2)*sqrt(1
5162357162557627/233*sqrt(33) + 2876991646833394763/7689)*(4566831233*sqrt
(33) - 27060312081) - 715538790282752*sqrt(33)*(3*x + 4) + 550964868517719
04*x + 14310775805655040)/x) + 961*(50*x^6 + 35*x^5 - 72*x^4 - 125*x^3 - 1
20*x^2 - 52*x - 16)*sqrt(-15162357162557627/233*sqrt(33) + 287699164683339
4763/7689)*log((sqrt(5*x^2 + 3*x + 2)*(4566831233*sqrt(33) + 27060312081)*
sqrt(-15162357162557627/233*sqrt(33) + 2876991646833394763/7689) + 7155387
90282752*sqrt(33)*(3*x + 4) + 55096486851771904*x + 14310775805655040)/x)
- 961*(50*x^6 + 35*x^5 - 72*x^4 - 125*x^3 - 120*x^2 - 52*x - 16)*sqrt(-151
62357162557627/233*sqrt(33) + 2876991646833394763/7689)*log(-(sqrt(5*x^2 +
 3*x + 2)*(4566831233*sqrt(33) + 27060312081)*sqrt(-15162357162557627/233*
sqrt(33) + 2876991646833394763/7689) - 715538790282752*sqrt(33)*(3*x + 4)
- 55096486851771904*x - 14310775805655040)/x) + 8*(659012052850*x^5 + 4108
56789795*x^4 - 1190074919544*x^3 - 1536641704237*x^2 - 1137494801224*x ...
```

Sympy [F]

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}}\, dx = \int \frac{1}{(2x^2-x-4)^2 (5x^2+3x+2)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/(-2*x**2+x+4)**2/(5*x**2+3*x+2)**(5/2),x)
```

output
```
Integral(1/((2*x**2 - x - 4)**2*(5*x**2 + 3*x + 2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^{\frac{5}{2}}(2\,x^2-x-4)^2}\,dx$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^(5/2)*(2*x^2 - x - 4)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.24

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}}\,dx =$$
$$-\frac{10503086\,(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2})^3-38552366\,\sqrt{5}(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2})^2-70815609\,\sqrt{5}x+}{3339424968\,\left(2\,(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2})^4-2\,\sqrt{5}(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2})^3-91\,(\sqrt{5}x-\sqrt{5\,x^2+3\,x+}\right.}$$
$$+\frac{(5\,(1566193045\,x+1686888069)x+6079914951)x+1961073075}{72936077142\,(5\,x^2+3\,x+2)^{\frac{3}{2}}}$$
$$+0.0000439911622979157\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}+8.38267526007000\right)$$
$$-0.00204746962514176\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.312157316296000\right)$$
$$-0.0000439911622979157\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-0.842024981991000\right)$$
$$+0.00204746962514176\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+3\,x+2}-4.99242498429000\right)$$

input
```
integrate(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x, algorithm="giac")
```

output
```
-1/3339424968*(10503086*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 38552366*s
qrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 70815609*sqrt(5)*x + 354437
6*sqrt(5) + 70815609*sqrt(5*x^2 + 3*x + 2))/(2*(sqrt(5)*x - sqrt(5*x^2 + 3
*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 91*(sqrt(5)
*x - sqrt(5*x^2 + 3*x + 2))^2 - 44*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x +
 2)) - 22) + 1/72936077142*((5*(1566193045*x + 1686888069)*x + 6079914951)
*x + 1961073075)/(5*x^2 + 3*x + 2)^(3/2) + 0.0000439911622979157*log(-sqrt
(5)*x + sqrt(5*x^2 + 3*x + 2) + 8.38267526007000) - 0.00204746962514176*lo
g(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.312157316296000) - 0.000043991162
2979157*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 0.842024981991000) + 0.00
204746962514176*log(-sqrt(5)*x + sqrt(5*x^2 + 3*x + 2) - 4.99242498429000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}}\,dx = \int \frac{1}{(-2\,x^2+x+4)^2\,(5\,x^2+3\,x+2)^{5/2}}\,dx$$

input
```
int(1/((x - 2*x^2 + 4)^2*(3*x + 5*x^2 + 2)^(5/2)),x)
```

output
```
int(1/((x - 2*x^2 + 4)^2*(3*x + 5*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(4+x-2x^2)^2 (2+3x+5x^2)^{5/2}}\,dx = \int \frac{1}{(-2x^2+x+4)^2\,(5x^2+3x+2)^{\frac{5}{2}}}dx$$

input
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x)
```

output
```
int(1/(-2*x^2+x+4)^2/(5*x^2+3*x+2)^(5/2),x)
```

3.163 $\int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 51

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}\,dx = \frac{1}{10}\arctan\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right) + \frac{1}{5}\text{arctanh}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)$$

output
```
1/10*arctan(5/2*(2+x)/(5*x^2+2*x-7)^(1/2))+1/5*arctanh(5*(1+x)/(5*x^2+2*x-
7)^(1/2))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}\,dx = \frac{1}{10}\arctan\left(\frac{5+\frac{5x}{2}}{\sqrt{-7+2x+5x^2}}\right) + \frac{1}{5}\text{arctanh}\left(\frac{5+5x}{\sqrt{-7+2x+5x^2}}\right)$$

input
```
Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

output
```
ArcTan[(5 + (5*x)/2)/Sqrt[-7 + 2*x + 5*x^2]]/10 + ArcTanh[(5 + 5*x)/Sqrt[-
7 + 2*x + 5*x^2]]/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1317, 27, 1362, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \ 1317$$

$$\frac{1}{50}\int -\frac{50(x+1)}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx-\frac{1}{50}\int -\frac{50(x+2)}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x+2}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx-\int \frac{x+1}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \ 1362$$

$$-32\int \frac{1}{\frac{6400(x+1)^2}{5x^2+2x-7}-256}d\frac{8(x+1)}{\sqrt{5x^2+2x-7}}-8\int \frac{1}{\frac{400(x+2)^2}{5x^2+2x-7}+64}d\left(-\frac{2(x+2)}{\sqrt{5x^2+2x-7}}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{10}\arctan\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)-32\int \frac{1}{\frac{6400(x+1)^2}{5x^2+2x-7}-256}d\frac{8(x+1)}{\sqrt{5x^2+2x-7}}$$

$$\Big\downarrow \ 220$$

$$\frac{1}{10}\arctan\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\text{arctanh}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

input
```
Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

output
```
ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sq
rt[-7 + 2*x + 5*x^2]]/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(41) = 82$.

Time = 4.40 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\sqrt{-\frac{4(2+x)^{2}}{(-x-1)^{2}}+9}\left(2\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{4(2+x)^{2}}{(-x-1)^{2}}+9}}{5}\right)+\arctan\left(\frac{5\sqrt{-\frac{4(2+x)^{2}}{(-x-1)^{2}}+9}\,(2+x)}{2\left(\frac{4(2+x)^{2}}{(-x-1)^{2}}-9\right)(-x-1)}\right)\right)}{10\sqrt{-\frac{\frac{4(2+x)^{2}}{(-x-1)^{2}}-9}{\left(1+\frac{2+x}{-x-1}\right)^{2}}}\left(1+\frac{2+x}{-x-1}\right)}$
trager	$-\frac{\ln\left(-\frac{129600\,\mathrm{RootOf}\left(80\_Z^{2}+16\_Z+1\right)^{2}x+8750\sqrt{5x^{2}+2x-7}\,\mathrm{RootOf}\left(80\_Z^{2}+16\_Z+1\right)+33210\,\mathrm{RootOf}\left(80\_Z^{2}+16\_Z+1\right)x+3}{20\,\mathrm{RootOf}\left(80\_Z^{2}+16\_Z+1\right)x-x-4}\right.}{5}$

input
```
int(1/(5*x^2+2*x-7)^(1/2)/(5*x^2+12*x+8),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-4*(2+x)^2/(-x-1)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(2+x)^2/(-x-1)^2+9)
^(1/2))+arctan(5/2*(-4*(2+x)^2/(-x-1)^2+9)^(1/2)/(4*(2+x)^2/(-x-1)^2-9)*(2
+x)/(-x-1)))/(-(4*(2+x)^2/(-x-1)^2-9)/(1+(2+x)/(-x-1))^2)^(1/2)/(1+(2+x)/(
-x-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(41) = 82$.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.02

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \frac{1}{20}\arctan\left(\frac{27\,x^2+20\,\sqrt{5\,x^2+2\,x-7}(x+2)+36\,x}{31\,x^2+16\,x-56}\right) + \frac{1}{20}\arctan\left(-\frac{27\,x^2-20\,\sqrt{5\,x^2+2\,x-7}(x+2)+36\,x}{31\,x^2+16\,x-56}\right) + \frac{1}{20}\log\left(\frac{15\,x^2+5\,\sqrt{5\,x^2+2\,x-7}(x+1)+26\,x+9}{x^2}\right) - \frac{1}{20}\log\left(\frac{15\,x^2-5\,\sqrt{5\,x^2+2\,x-7}(x+1)+26\,x+9}{x^2}\right)$$

input
```
integrate(1/(5*x^2+2*x-7)^(1/2)/(5*x^2+12*x+8),x, algorithm="fricas")
```

output
```
1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 1
6*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*
x)/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x +
 1) + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
+ 26*x + 9)/x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{\sqrt{(x-1)\,(5x+7)}\,(5x^2+12x+8)}\,dx$$

input
```
integrate(1/(5*x**2+2*x-7)**(1/2)/(5*x**2+12*x+8),x)
```

output
```
Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+12\,x+8)\sqrt{5\,x^2+2\,x-7}}\,dx$$

input
```
integrate(1/(5*x^2+2*x-7)^(1/2)/(5*x^2+12*x+8),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(41) = 82.

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.02

$$\begin{aligned}
&\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx \\
&= -\frac{1}{10}\arctan\left(-\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5\,x^2+2\,x-7}+5}{2\left(\sqrt{5}+5\right)}\right) \\
&\quad -\frac{1}{10}\arctan\left(\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5\,x^2+2\,x-7}-5}{2\left(\sqrt{5}-5\right)}\right) \\
&\quad +\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)\left(6\sqrt{5}+5\right)\right. \\
&\quad \left.+20\sqrt{5}+65\right)-\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)^2\right. \\
&\quad \left.+2\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)\left(6\sqrt{5}-5\right)-20\sqrt{5}+65\right)
\end{aligned}$$

input
```
integrate(1/(5*x^2+2*x-7)^(1/2)/(5*x^2+12*x+8),x, algorithm="giac")
```

output
```
-1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/
(sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 +
 2*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x -
 7))^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5
) + 65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x
- sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x-7}\,\left(5\,x^2+12\,x+8\right)}\,dx$$

input
```
int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)
```

output
```
int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{5\sqrt{5x^2+2x-7}\,x^2+12\sqrt{5x^2+2x-7}\,x+8\sqrt{5x^2+2x-7}}dx$$

input
```
int(1/(5*x^2+2*x-7)^(1/2)/(5*x^2+12*x+8),x)
```

output
```
int(1/(5*sqrt(5*x**2 + 2*x - 7)*x**2 + 12*sqrt(5*x**2 + 2*x - 7)*x + 8*sqr
t(5*x**2 + 2*x - 7)),x)
```

3.164 $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 700

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx =$$

$$-\frac{\sqrt{2}\left(b+\sqrt{b^2-4ac}\right)\sqrt{e-\sqrt{e^2-4df}+2fx}\sqrt{-\frac{4cd-\left(b+\sqrt{b^2-4ac}\right)\left(e+\sqrt{e^2-4df}\right)-2\left(\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2-4d\cdots}\right)\right)}{\sqrt{e^2-4df}\left(b+\sqrt{b^2-4ac}+2cx\right)}}}{\sqrt{b^2-4ac+b\sqrt{b^2-4ac}}\sqrt{\sqrt{b^2-4ac}e-4af-\sqrt{b\cdots}}}$$

output
```
-2^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(e-(-4*d*f+e^2)^(1/2)+2*f*x)^(1/2)*(-(4*c*
d-(b+(-4*a*c+b^2)^(1/2))*(e+(-4*d*f+e^2)^(1/2))-2*((b+(-4*a*c+b^2)^(1/2))*
f-c*(e-(-4*d*f+e^2)^(1/2)))*x)/(-4*d*f+e^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)+2*
c*x))^(1/2)*(c*x^2+b*x+a)^(1/2)*EllipticF(2^(1/2)*(b*(-4*a*c+b^2)^(1/2)-4*
a*c+b^2)^(1/2)*(e-(-4*d*f+e^2)^(1/2)+2*f*x)^(1/2)/((-4*a*c+b^2)^(1/2)*e-4*
a*f-(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2)+b*(e-(-4*d*f+e^2)^(1/2)))^(1/2)/
(b+(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2),1/2*(-2*(4*c*(b+(-4*a*c+b^2)^(1/2))*d-2
*b*(b+(-4*a*c+b^2)^(1/2))*e+4*a*(b+(-4*a*c+b^2)^(1/2))*f-(4*a*c-(b+(-4*a*c
+b^2)^(1/2))^2)*(-4*d*f+e^2)^(1/2))/(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)/(-4*d
*f+e^2)^(1/2))^(1/2))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)^(1/2)/((-4*a*c+b^2)
^(1/2)*e-4*a*f-(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2)+b*(e-(-4*d*f+e^2)^(1/
2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)/(((b+(-4*a*c+b^2)^(1/2))*f-c
*(e-(-4*d*f+e^2)^(1/2)))*(2*a+(b+(-4*a*c+b^2)^(1/2))*x)/(4*a*f-(b+(-4*a*c+
b^2)^(1/2))*(e-(-4*d*f+e^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2)+2*c*x))^(1/2)/(f
*x^2+e*x+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.93 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx =$$

$$-\frac{\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{-\cdots}}{\left(\left(-b+\sqrt{b^2-4ac}\right)f+c\cdots\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-(((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-
((c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(((b + Sqrt[b^2 - 4
*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*
Sqrt[-((c*(4*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*
c]*f*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e +
 Sqrt[e^2 - 4*d*f] + 2*f*x)))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e
^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[ArcSin[Sqrt[((
(-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - Sqrt[b^2 - 4*a*
c]*Sqrt[e^2 - 4*d*f])/(2*c*d - b*e + 2*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 -
4*d*f])])/((((-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*Sqrt[(
c*Sqrt[b^2 - 4*a*c]*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(((b + Sqrt[b^2 - 4*
a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*S
qrt[a + x*(b + c*x)]*Sqrt[d + x*(e + f*x)]))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1432 vs. $2(700) = 1400$.

Time = 1.77 (sec) , antiderivative size = 1432, normalized size of antiderivative = 2.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1323, 1280, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}dx$$

$$\Big\downarrow \text{1323}$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\int \frac{1}{\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{fx^2+ex+d}}dx}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1280}$$

$$-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)^{3/2}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\sqrt{\frac{\left(4ac-\left(\sqrt{b^2-4ac}+b\right)^2\right)^2(d+ex+fx^2)}{\left(\sqrt{b^2-4ac}+b+2cx\right)^2\left(4a^2f+d\left(\sqrt{b^2-4ac}+b\right)^2-2ae\left(\sqrt{b^2-4ac}+b\right)\right.}}}{\left(4ac-\left(\sqrt{b^2-4a\right.\right.}}$$

$\Big\downarrow$ 1416

$$-\frac{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4\right.}}}{}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
 - 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x +
f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*
a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^
2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*
c])*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2
- 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqr
t[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(
(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e -
2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 -
 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x
)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f
 - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])
*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[
((2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)
*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d -
 a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^...
```

Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2])   Int[1/(Sqrt[b + r + 2
*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 8.18 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{2\sqrt{(f x^2+ex+d)(c x^2+bx+a)}\left(\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{-e+\sqrt{-4df+e^2}}{2f}\right)\sqrt{\frac{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{e+\sqrt{-4df+e^2}}{2f}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}}\left(x+\frac{e+}{}\right.}{\sqrt{f x^2+ex+d}\sqrt{c x^2+}}$
default	$-\frac{8\left(-2b f^2x^2+2cef x^2-2cf x^2\sqrt{-4df+e^2}-2f^2x^2\sqrt{-4ac+b^2}-2befx-2bfx\sqrt{-4df+e^2}+8cdfx-2efx\sqrt{-4ac+b^2}-2fx\sqrt{-4ac+}\right.}{}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((f*x^2+e*x+d)*(c*x^2+b*x+a))^(1/2)/(f*x^2+e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)*(1/2*(b+(-4*a*c+b^2)^(1/2))/c+1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*((-1/2*(b
+(-4*a*c+b^2)^(1/2))/c+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*(x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))
/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^(1/2)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
2*((-1/2*(e+(-4*d*f+e^2)^(1/2))/f-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^(1/2)*((-1/2*(e+(-4*d*f+e^2
)^(1/2))/f-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))/(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))^(1/2)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)/(-1/2*(e+(-4*d*f+e^2)^(1/2))/f-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))/(c*f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*EllipticF(((-1/2*(b+(-4*a*c+b^2)^(1/2))/c+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^(1/2),((-1/2*(e+(-
4*d*f+e^2)^(1/2))/f-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(1/2*(b+(-4*a*c+b^2)^(1
/2))/c+1/2/f*(-e+(-4*d*f+e^2)^(1/2)))/(-1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2/
f*(-e+(-4*d*f+e^2)^(1/2)))/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2*(e+(-4*d*f...
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)/(c*f*x^4 + (c*e + b*f
)*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+bx+a}\,\sqrt{f\,x^2+ex+d}}dx$$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x)
```

output
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x)
```

3.165 $\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 652

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$$

$$= \frac{\sqrt{\frac{23}{11}}\left(1-i\sqrt{23}-4x\right)\sqrt{-1+i\sqrt{23}+4x}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(11i-\sqrt{23}\right)(2+3x+5x^2)}{\left(7i+\sqrt{23}\right)\left(1-i\sqrt{23}-4x\right)^2}}\left(1-\frac{\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\left(6-\left(1\ldots\right.\right.}{1-i\sqrt{23}-\ldots}\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{3-x+2x^2}\ldots}$$

output
```
1/11*253^(1/2)*(1-I*23^(1/2)-4*x)*(-1+I*23^(1/2)+4*x)^(1/2)*(6-(1-I*23^(1/
2))*x)^(1/2)*((11*I-23^(1/2))*(5*x^2+3*x+2)/(7*I+23^(1/2))/(1-I*23^(1/2)-4
*x)^2)^(1/2)*(1-(-(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/2)*(6-(1-I*23^(1/2))*x
)/(1-I*23^(1/2)-4*x))*((11-41*(I+23^(1/2))*(6-(1-I*23^(1/2))*x)/(7*I+23^(1
/2))/(1-I*23^(1/2)-4*x)-11*(3*I-23^(1/2))*(6-(1-I*23^(1/2))*x)^2/(7*I+23^(
1/2))/(1-I*23^(1/2)-4*x)^2)/(1-(-(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/2)*(6-(
1-I*23^(1/2))*x)/(1-I*23^(1/2)-4*x))^2)^(1/2)*InverseJacobiAM(2*arctan((-(
3*I-23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-(1-I*23^(1/2))*x)^(1/2)/(-1+I*23^(1
/2)+4*x)^(1/2)),1/44*I*(-968+902*(I+23^(1/2))/(11+I*23^(1/2))^(1/2))^(1/2)
)/(I*23^(1/2)+23)/(-(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/4)/(2*x^2-x+3)^(1/2)
/(5*x^2+3*x+2)^(1/2)/(11-41*(I+23^(1/2))*(6-(1-I*23^(1/2))*x)/(7*I+23^(1/2
))/(1-I*23^(1/2)-4*x)-11*(3*I-23^(1/2))*(6-(1-I*23^(1/2))*x)^2/(7*I+23^(1/
2))/(1-I*23^(1/2)-4*x)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.62 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$$

$$= \frac{\left(1+i\sqrt{23}-4x\right)\left(3i+\sqrt{31}+10ix\right)\sqrt{\frac{6i-2\sqrt{31}+20ix}{\left(11i+5\sqrt{23}-2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\sqrt{\frac{63-3i\sqrt{23}-i\sqrt{31}-\sqrt{713}+\left(-22-10i\sqrt{23}+4i\sqrt{\cdots}\right.}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\cdots}{\left(-11i+5\sqrt{23}-2\sqrt{31}\right)\sqrt{\frac{3i+\sqrt{31}+10ix\cdots}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i\cdots\right.}}\cdots}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31]
 + (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]
*Sqrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[2
3] + (4*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] +
 (4*I)*x))]*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt
[31] + Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*
Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))]], (1197 + 41*Sqrt[713]
)/484])/((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x
)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[3 - x
 + 2*x^2]*Sqrt[2 + 3*x + 5*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1323, 1280, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}dx$$

$$\Big\downarrow \text{1323}$$

$$\frac{\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\int \frac{1}{\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{5x^2+3x+2}}dx}{\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1280}$$

$$\frac{2\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\int \frac{}{\sqrt{-\frac{\left(3i-\sqrt{23}\right)\left(6-\left(1-i\sqrt{23}\right)x\right)}{\left(7i+\sqrt{23}\right)\left(4x+i\sqrt{23}-1\right)^2}}}}{\left(23+i\sqrt{23}\right)\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1+\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}\left(6-\left(1-i\sqrt{\right.\right.}}}{4x+i\sqrt{23}-1}\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\sqrt{2x^2-x+3}\sqrt{5x^{2}}}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
(Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (
1 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt
[23])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 + (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt
[23]))]*(6 - (1 - I*Sqrt[23])*x))/(-1 + I*Sqrt[23] + 4*x))*Sqrt[(11 + (41*
(I + Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(-1 + I*Sqrt[23
] + 4*x)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[
23])*(-1 + I*Sqrt[23] + 4*x)^2))/(1 + (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt
[23]))]*(6 - (1 - I*Sqrt[23])*x))/(-1 + I*Sqrt[23] + 4*x))^2]*EllipticF[2*
ArcTan[((-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[
23])*x])/Sqrt[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 +
 I*Sqrt[23]])/88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])
))^(1/4)*Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 + (41*(I + Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(-1 + I*Sqrt[23] + 4*x))
 - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(-1
+ I*Sqrt[23] + 4*x)^2)])
```

Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2])   Int[1/(Sqrt[b + r + 2
*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 8.14 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.61

method	result
elliptic	$-\frac{i\sqrt{(2x^2-x+3)(5x^2+3x+2)}\left(\frac{11}{20}-\frac{i\sqrt{23}}{4}-\frac{i\sqrt{31}}{10}\right)\sqrt{\frac{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}-\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)}{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}+\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)}}\left(x-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)^2\sqrt{\frac{i\sqrt{23}\left(x+\frac{3}{10}+\frac{i\sqrt{}}{10}\right.}{\left(-\frac{11}{20}-\frac{i\sqrt{31}}{10}+\frac{i\sqrt{23}}{4}\right)\left(x\right.}}}{115\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}-\right.}$
default	$\frac{4i\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\sqrt{-\frac{\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\left(-1+i\sqrt{23}+4x\right)}{\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\left(i\sqrt{23}-4x+1\right)}}\left(i\sqrt{23}-4x+1\right)^2\sqrt{\frac{i\sqrt{23}\left(i\sqrt{31}+10x+3\right.}{\left(2i\sqrt{31}-5i\sqrt{23}+11\right)\left(i\sqrt{23}\right.}}}{23\sqrt{10x^4+x^3+16x^2+7x+6}\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\sqrt{}}$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/115*I*((2*x^2-x+3)*(5*x^2+3*x+2))^(1/2)/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)
^(1/2)*(11/20-1/4*I*23^(1/2)-1/10*I*31^(1/2))*((-11/20+1/10*I*31^(1/2)-1/4
*I*23^(1/2))*(x-1/4+1/4*I*23^(1/2))/(-11/20+1/10*I*31^(1/2)+1/4*I*23^(1/2)
)/(x-1/4-1/4*I*23^(1/2)))^(1/2)*(x-1/4-1/4*I*23^(1/2))^2*(I*23^(1/2)*(x+3/
10+1/10*I*31^(1/2))/(-11/20-1/10*I*31^(1/2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*2
3^(1/2)))^(1/2)*(I*23^(1/2)*(x+3/10-1/10*I*31^(1/2))/(-11/20+1/10*I*31^(1/
2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*23^(1/2)))^(1/2)/(-11/20+1/10*I*31^(1/2)-1
/4*I*23^(1/2))*23^(1/2)*10^(1/2)/((x-1/4+1/4*I*23^(1/2))*(x-1/4-1/4*I*23^(
1/2))*(x+3/10+1/10*I*31^(1/2))*(x+3/10-1/10*I*31^(1/2)))^(1/2)*EllipticF((
(-11/20+1/10*I*31^(1/2)-1/4*I*23^(1/2))*(x-1/4+1/4*I*23^(1/2))/(-11/20+1/1
0*I*31^(1/2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*23^(1/2)))^(1/2),((11/20+1/4*I*2
3^(1/2)+1/10*I*31^(1/2))*(11/20-1/4*I*23^(1/2)-1/10*I*31^(1/2))/(11/20-1/4
*I*23^(1/2)+1/10*I*31^(1/2))/(11/20+1/4*I*23^(1/2)-1/10*I*31^(1/2)))^(1/2)
)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx=\int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2
+ 7*x + 6), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,\sqrt{5\,x^2+3\,x+2}}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\,\sqrt{5x^2+3x+2}}dx$$

input
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x)
```

output
```
int(1/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2),x)
```

3.166 $\int (a+bx+cx^2)^3 (d+ex+fx^2)^q \, dx$

Optimal result

Integrand size = 25, antiderivative size = 1520

$$\int (a+bx+cx^2)^3 (d+ex+fx^2)^q \, dx = \text{Too large to display}$$

output
```
-1/8*(c^3*e*(4+q)*(12*d^2*f^2*(11*q^2+46*q+45)-4*d*e^2*f*(7*q^3+68*q^2+201
*q+180)+e^4*(q^4+16*q^3+91*q^2+216*q+180))+2*b*f^3*(4*q^3+36*q^2+107*q+105
)*(6*a*b*e*f*(2+q)^2-6*a^2*f^2*(2*q^2+7*q+6)+b^2*(2*d*f*(3+2*q)-e^2*(q^2+5
*q+6)))+6*c*f^2*(2*q^2+13*q+21)*(2*a^2*e*f^2*(2+q)^2*(5+2*q)+2*a*b*f*(5+2*
q)*(2*d*f*(3+2*q)-e^2*(q^2+5*q+6))-b^2*e*(3+q)*(2*d*f*(8+5*q)-e^2*(q^2+6*q
+8)))-3*c^2*f*(7+2*q)*(2*a*e*f*(3+q)^2*(2*d*f*(8+5*q)-e^2*(q^2+6*q+8))+b*(
8*d^2*f^2*(4*q^2+16*q+15)-6*d*e^2*f*(3*q^3+26*q^2+71*q+60)+e^4*(q^4+14*q^3
+71*q^2+154*q+120))))*(f*x^2+e*x+d)^(1+q)/f^6/(1+q)/(2+q)/(3+q)/(3+2*q)/(5
+2*q)/(7+2*q)+1/4*(2*b^2*f^3*(4*q^3+36*q^2+107*q+105)*(6*a*f*(2+q)-b*e*(3+
q))+c^3*(60*d^2*f^2*(q^2+5*q+6)-6*d*e^2*f*(4*q^3+45*q^2+161*q+180)+e^4*(q^
4+18*q^3+119*q^2+342*q+360))+6*c*f^2*(2*q^2+13*q+21)*(2*a^2*f^2*(2*q^2+9*q
+10)-2*a*b*e*f*(2*q^2+11*q+15)-b^2*(6*d*f*(2+q)-e^2*(q^2+7*q+12)))-3*c^2*f
*(7+2*q)*(2*a*f*(3+q)*(6*d*f*(2+q)-e^2*(q^2+7*q+12))-b*e*(4+q)*(2*d*f*(15+
7*q)-e^2*(q^2+8*q+15))))*x*(f*x^2+e*x+d)^(1+q)/f^5/(2+q)/(3+q)/(3+2*q)/(5+
2*q)/(7+2*q)+1/4*(2*b^3*f^3*(4*q^3+36*q^2+107*q+105)-6*b*c*f^2*(2*q^2+13*q
+21)*(b*e*(4+q)-2*a*f*(5+2*q))-3*c^2*f*(7+2*q)*(4*b*d*f*(5+2*q)+2*a*e*f*(q
^2+7*q+12)-b*e^2*(q^2+9*q+20))+c^3*e*(5+q)*(6*d*f*(8+3*q)-e^2*(q^2+10*q+24
)))*x^2*(f*x^2+e*x+d)^(1+q)/f^4/(2+q)/(3+q)/(5+2*q)/(7+2*q)+1/2*c*(6*b^2*f
^2*(2*q^2+13*q+21)+3*c*f*(7+2*q)*(2*a*f*(3+q)-b*e*(5+q))-c^2*(10*d*f*(3+q)
-e^2*(q^2+11*q+30)))*x^3*(f*x^2+e*x+d)^(1+q)/f^3/(3+q)/(5+2*q)/(7+2*q)-...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 4.54 (sec) , antiderivative size = 1884, normalized size of antiderivative = 1.24

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x + c*x^2)^3*(d + e*x + f*x^2)^q,x]
```

output
```
((d + x*(e + f*x))^q*(210*a^2*b*f*(1 + q)*x^2*((e + Sqrt[e^2 - 4*d*f] + 2*
f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[2, -q, -q, 3, (-2*f*x)/(e + Sqrt[e^2 -
4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] + 140*a*(b^2 + a*c)*f*(1 + q)*x
^3*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[3, -q, -
q, 4, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])]
+ 35*b^3*f*x^4*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*Appel
lF1[4, -q, -q, 5, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2
 - 4*d*f])] + 210*a*b*c*f*x^4*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 -
4*d*f])^q*AppellF1[4, -q, -q, 5, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)
/(-e + Sqrt[e^2 - 4*d*f])] + 35*b^3*f*q*x^4*((e + Sqrt[e^2 - 4*d*f] + 2*f*
x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[4, -q, -q, 5, (-2*f*x)/(e + Sqrt[e^2 - 4*
d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] + 210*a*b*c*f*q*x^4*((e + Sqrt[e^
2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[4, -q, -q, 5, (-2*f*x)/(
e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] + 84*b^2*c*f*x^5
*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[5, -q, -q,
 6, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] +
84*a*c^2*f*x^5*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*Appel
lF1[5, -q, -q, 6, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2
 - 4*d*f])] + 84*b^2*c*f*q*x^5*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 -
 4*d*f])^q*AppellF1[5, -q, -q, 6, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*...
```

Rubi [A] (warning: unable to verify)

Time = 10.18 (sec) , antiderivative size = 1838, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2192, 2192, 2192, 2192, 2192, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \left(fx^2+ex+d\right)^q \left(-c^2(ce(q+6)-3bf(2q+7))x^5-c\left(-3f(2q+7)b^2+5c^2d-3acf(2q+7)\right)x^4+b\left(b^2+6ac\right.\right.}{f(2q+7)}$$

$$\frac{c^3x^5\left(d+ex+fx^2\right)^{q+1}}{f(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \left(fx^2+ex+d\right)^q \left(c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^4+2\left(f^2\left(2q^2+13q+21\right)b^3-6\right.}{2f(q+3)}$$

$$\frac{c^3x^5\left(d+ex+fx^2\right)^{q+1}}{f(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$\int \left(fx^2+ex+d\right)^q \left(2a^3(q+3)(2q+5)(2q+7)f^3+6a^2b(q+3)(2q+5)(2q+7)xf^3+\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)c^3-3f(2q+7)\left(-b\left(q^2+9q+20\right)e^2+2af\left(q^2+7q\right.\right.\right.\right.$$

$$\frac{c^3x^5\left(d+ex+fx^2\right)^{q+1}}{f(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$\frac{c^3x^5\left(fx^2+ex+d\right)^{q+1}}{f(2q+7)}+$$

$$\frac{c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^3\left(fx^2+ex+d\right)^{q+1}}{f(2q+5)}+\frac{\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)\right.}{}$$

$$\Big\downarrow \; 2192$$

$$\frac{c^3x^5\left(fx^2+ex+d\right)^{q+1}}{f(2q+7)}+$$

$$\frac{c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^3\left(fx^2+ex+d\right)^{q+1}}{f(2q+5)}+\frac{\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)\right.}{}$$

$$\Big\downarrow \; 25$$

$$\frac{c^3x^5\left(fx^2+ex+d\right)^{q+1}}{f(2q+7)}+$$

$$\frac{c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^3\left(fx^2+ex+d\right)^{q+1}}{f(2q+5)}+\frac{\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)\right.}{}$$

$$\Big\downarrow \; 1160$$

$$\frac{c^3x^5\left(fx^2+ex+d\right)^{q+1}}{f(2q+7)}+$$

$$\frac{\frac{c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^3\left(fx^2+ex+d\right)^{q+1}}{f(2q+5)}+\frac{\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)\right.}{}}{}$$

$\downarrow$ 1096

$$\frac{c^3x^5\left(fx^2+ex+d\right)^{q+1}}{f(2q+7)}+$$

$$\frac{\frac{c\left(-\left(\left(10df(q+3)-e^2\left(q^2+11q+30\right)\right)c^2\right)+3f(2q+7)(2af(q+3)-be(q+5))c+6b^2f^2\left(2q^2+13q+21\right)\right)x^3\left(fx^2+ex+d\right)^{q+1}}{f(2q+5)}+\frac{\left(e(q+5)\left(6df(3q+8)-e^2\left(q^2+10q+24\right)\right)\right.}{}}{}$$

input
```
Int[(a + b*x + c*x^2)^3*(d + e*x + f*x^2)^q,x]
```

output
```
(c^3*x^5*(d + e*x + f*x^2)^(1 + q))/(f*(7 + 2*q)) + (-1/2*(c^2*(c*e*(6 + q
) - 3*b*f*(7 + 2*q))*x^4*(d + e*x + f*x^2)^(1 + q))/(f*(3 + q)) + ((c*(6*b
^2*f^2*(21 + 13*q + 2*q^2) + 3*c*f*(7 + 2*q)*(2*a*f*(3 + q) - b*e*(5 + q))
 - c^2*(10*d*f*(3 + q) - e^2*(30 + 11*q + q^2)))*x^3*(d + e*x + f*x^2)^(1
+ q))/(f*(5 + 2*q)) + (((2*b^3*f^3*(105 + 107*q + 36*q^2 + 4*q^3) - 6*b*c*
f^2*(21 + 13*q + 2*q^2)*(b*e*(4 + q) - 2*a*f*(5 + 2*q)) - 3*c^2*f*(7 + 2*q
)*(4*b*d*f*(5 + 2*q) + 2*a*e*f*(12 + 7*q + q^2) - b*e^2*(20 + 9*q + q^2))
+ c^3*e*(5 + q)*(6*d*f*(8 + 3*q) - e^2*(24 + 10*q + q^2)))*x^2*(d + e*x +
f*x^2)^(1 + q))/(2*f*(2 + q)) + (((2*b^2*f^3*(105 + 107*q + 36*q^2 + 4*q^3
)*(6*a*f*(2 + q) - b*e*(3 + q)) + c^3*(60*d^2*f^2*(6 + 5*q + q^2) - 6*d*e^
2*f*(180 + 161*q + 45*q^2 + 4*q^3) + e^4*(360 + 342*q + 119*q^2 + 18*q^3 +
 q^4)) + 6*c*f^2*(21 + 13*q + 2*q^2)*(2*a^2*f^2*(10 + 9*q + 2*q^2) - 2*a*b
*e*f*(15 + 11*q + 2*q^2) - b^2*(6*d*f*(2 + q) - e^2*(12 + 7*q + q^2))) - 3
*c^2*f*(7 + 2*q)*(2*a*f*(3 + q)*(6*d*f*(2 + q) - e^2*(12 + 7*q + q^2)) - b
*e*(4 + q)*(2*d*f*(15 + 7*q) - e^2*(15 + 8*q + q^2))))*x*(d + e*x + f*x^2)
^(1 + q))/(f*(3 + 2*q)) - (((c^3*e*(4 + q)*(12*d^2*f^2*(45 + 46*q + 11*q^2
) - 4*d*e^2*f*(180 + 201*q + 68*q^2 + 7*q^3) + e^4*(180 + 216*q + 91*q^2 +
 16*q^3 + q^4)) + 2*b*f^3*(105 + 107*q + 36*q^2 + 4*q^3)*(6*a*b*e*f*(2 + q
)^2 - 6*a^2*f^2*(6 + 7*q + 2*q^2) + b^2*(2*d*f*(3 + 2*q) - e^2*(6 + 5*q +
q^2))) + 6*c*f^2*(21 + 13*q + 2*q^2)*(2*a^2*e*f^2*(2 + q)^2*(5 + 2*q) +...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(c\,x^{2}+bx+a\right)^{3} \left(f\,x^{2}+ex+d\right)^{q}dx$$

input
```
int((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x)
```

output
```
int((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx=\int \left(cx^{2}+bx+a\right)^{3}\left(fx^{2}+ex+d\right)^{q}\,dx$$

input
```
integrate((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x, algorithm="fricas")
```

output
```
integral((c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3
 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 + a^2*c)*x^2)*(f*x^2 + e*x + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**3*(f*x**2+e*x+d)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \int \left(cx^2+bx+a\right)^3 \left(fx^2+ex+d\right)^q \, dx$$

input
```
integrate((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^3*(f*x^2 + e*x + d)^q, x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \int \left(cx^2+bx+a\right)^3 \left(fx^2+ex+d\right)^q \, dx$$

input
```
integrate((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^3*(f*x^2 + e*x + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \int \left(c\,x^2+b\,x+a\right)^3 \left(f\,x^2+e\,x+d\right)^q \, dx$$

input
```
int((a + b*x + c*x^2)^3*(d + e*x + f*x^2)^q,x)
```

output
```
int((a + b*x + c*x^2)^3*(d + e*x + f*x^2)^q, x)
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^3 \left(d+ex+fx^2\right)^q \, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x+a)^3*(f*x^2+e*x+d)^q,x)
```

output

```
(128*(d + e*x + f*x**2)**q*a**3*d*f**6*q**6 + 1728*(d + e*x + f*x**2)**q*a
**3*d*f**6*q**5 + 9440*(d + e*x + f*x**2)**q*a**3*d*f**6*q**4 + 26640*(d +
 e*x + f*x**2)**q*a**3*d*f**6*q**3 + 40832*(d + e*x + f*x**2)**q*a**3*d*f*
*6*q**2 + 32112*(d + e*x + f*x**2)**q*a**3*d*f**6*q + 10080*(d + e*x + f*x
**2)**q*a**3*d*f**6 + 64*(d + e*x + f*x**2)**q*a**3*e*f**6*q**6*x + 864*(d
 + e*x + f*x**2)**q*a**3*e*f**6*q**5*x + 4720*(d + e*x + f*x**2)**q*a**3*e
*f**6*q**4*x + 13320*(d + e*x + f*x**2)**q*a**3*e*f**6*q**3*x + 20416*(d +
 e*x + f*x**2)**q*a**3*e*f**6*q**2*x + 16056*(d + e*x + f*x**2)**q*a**3*e*
f**6*q*x + 5040*(d + e*x + f*x**2)**q*a**3*e*f**6*x - 96*(d + e*x + f*x**2
)**q*a**2*b*d*e*f**5*q**5 - 1200*(d + e*x + f*x**2)**q*a**2*b*d*e*f**5*q**
4 - 5880*(d + e*x + f*x**2)**q*a**2*b*d*e*f**5*q**3 - 14100*(d + e*x + f*x
**2)**q*a**2*b*d*e*f**5*q**2 - 16524*(d + e*x + f*x**2)**q*a**2*b*d*e*f**5
*q - 7560*(d + e*x + f*x**2)**q*a**2*b*d*e*f**5 + 96*(d + e*x + f*x**2)**q
*a**2*b*e**2*f**5*q**6*x + 1200*(d + e*x + f*x**2)**q*a**2*b*e**2*f**5*q**
5*x + 5880*(d + e*x + f*x**2)**q*a**2*b*e**2*f**5*q**4*x + 14100*(d + e*x
+ f*x**2)**q*a**2*b*e**2*f**5*q**3*x + 16524*(d + e*x + f*x**2)**q*a**2*b*
e**2*f**5*q**2*x + 7560*(d + e*x + f*x**2)**q*a**2*b*e**2*f**5*q*x + 192*(
d + e*x + f*x**2)**q*a**2*b*e*f**6*q**6*x**2 + 2496*(d + e*x + f*x**2)**q*
a**2*b*e*f**6*q**5*x**2 + 12960*(d + e*x + f*x**2)**q*a**2*b*e*f**6*q**4*x
**2 + 34080*(d + e*x + f*x**2)**q*a**2*b*e*f**6*q**3*x**2 + 47148*(d + ...
```

3.167 $\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx$

Optimal result

Integrand size = 25, antiderivative size = 578

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx =$$

$$-\frac{\left(2bf^2\left(10+9q+2q^2\right)\left(be(2+q)-2af(3+2q)\right)+2cf(5+2q)\left(2aef(2+q)^2+2bdf(3+2q)-be^2(6\right.\right.}{4f^4(1+q)(2+q)(3+2q)(5+}$$

$$+\frac{\left(2b^2f^2\left(10+9q+2q^2\right)+2cf(5+2q)(2af(2+q)-be(3+q))-c^2\left(6df(2+q)-e^2\left(12+7q+q^2\right)\right)\right)x}{2f^3(2+q)(3+2q)(5+2q)}$$

$$-\frac{c(ce(4+q)-2bf(5+2q))x^2\left(d+ex+fx^2\right)^{1+q}}{2f^2(2+q)(5+2q)}+\frac{c^2x^3\left(d+ex+fx^2\right)^{1+q}}{f(5+2q)}$$

$$+\frac{2^{-3-2q}\left(2cf(5+2q)\left(6bdef-4adf^2+2ae^2f(2+q)-be^3(3+q)\right)+c^2\left(12d^2f^2-12de^2f(3+q)+e^4(1\right.\right.}{}$$

output
```
-1/4*(2*b*f^2*(2*q^2+9*q+10)*(b*e*(2+q)-2*a*f*(3+2*q))+2*c*f*(5+2*q)*(2*a*
e*f*(2+q)^2+2*b*d*f*(3+2*q)-b*e^2*(q^2+5*q+6))-c^2*e*(3+q)*(2*d*f*(8+5*q)-
e^2*(q^2+6*q+8)))*(f*x^2+e*x+d)^(1+q)/f^4/(1+q)/(2+q)/(3+2*q)/(5+2*q)+1/2*
(2*b^2*f^2*(2*q^2+9*q+10)+2*c*f*(5+2*q)*(2*a*f*(2+q)-b*e*(3+q))-c^2*(6*d*f
*(2+q)-e^2*(q^2+7*q+12)))*x*(f*x^2+e*x+d)^(1+q)/f^3/(2+q)/(3+2*q)/(5+2*q)-
1/2*c*(c*e*(4+q)-2*b*f*(5+2*q))*x^2*(f*x^2+e*x+d)^(1+q)/f^2/(2+q)/(5+2*q)+
c^2*x^3*(f*x^2+e*x+d)^(1+q)/f/(5+2*q)+2^(-3-2*q)*(2*c*f*(5+2*q)*(6*b*d*e*f
-4*a*d*f^2+2*a*e^2*f*(2+q)-b*e^3*(3+q))+c^2*(12*d^2*f^2-12*d*e^2*f*(3+q)+e
^4*(q^2+7*q+12))-2*f^2*(5+2*q)*(2*a*b*e*f*(3+2*q)-2*a^2*f^2*(3+2*q)+b^2*(2
*d*f-e^2*(2+q))))*(2*f*x+e)*(f*x^2+e*x+d)^q*hypergeom([1/2, -q],[3/2],(2*f
*x+e)^2/(-4*d*f+e^2))/f^5/(3+2*q)/(5+2*q)/((-f*(f*x^2+e*x+d)/(-4*d*f+e^2))
^q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.97 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.28

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx$$

$$= \frac{2^{-1-q}\left(\frac{e-\sqrt{e^2-4df}}{2f}+x\right)^{-q}\left(\frac{e-\sqrt{e^2-4df}+2fx}{f}\right)^{q}\left(\frac{e-\sqrt{e^2-4df}+2fx}{e-\sqrt{e^2-4df}}\right)^{-q}\left(\frac{e+\sqrt{e^2-4df}+2fx}{\sqrt{e^2-4df}}\right)^{-q}\left(\frac{e+\sqrt{e^2-4df}+2fx}{e+\sqrt{e^2-4df}}\right)^{-q}(d}{}$$

input
```
Integrate[(a + b*x + c*x^2)^2*(d + e*x + f*x^2)^q,x]
```

output
```
(2^(-1 - q)*((e - Sqrt[e^2 - 4*d*f] + 2*f*x)/f)^q*(d + x*(e + f*x))^q*(30*
a*b*f*(1 + q)*x^2*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*Ap
pellF1[2, -q, -q, 3, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[
e^2 - 4*d*f])] + 10*(b^2 + 2*a*c)*f*(1 + q)*x^3*((e + Sqrt[e^2 - 4*d*f] +
2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[3, -q, -q, 4, (-2*f*x)/(e + Sqrt[e^2
- 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] + 15*b*c*f*(1 + q)*x^4*((e +
Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF1[4, -q, -q, 5, (-2
*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])] + 6*c^2*f
*(1 + q)*x^5*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*AppellF
1[5, -q, -q, 6, (-2*f*x)/(e + Sqrt[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 -
 4*d*f])] - 15*2^q*a^2*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x)*((e - Sqrt[e^2 - 4
*d*f] + 2*f*x)/(e - Sqrt[e^2 - 4*d*f]))^q*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)
/(e + Sqrt[e^2 - 4*d*f]))^q*Hypergeometric2F1[-q, 1 + q, 2 + q, (-e + Sqrt
[e^2 - 4*d*f] - 2*f*x)/(2*Sqrt[e^2 - 4*d*f])]))/(15*f*(1 + q)*((e - Sqrt[e
^2 - 4*d*f])/(2*f) + x)^q*((e - Sqrt[e^2 - 4*d*f] + 2*f*x)/(e - Sqrt[e^2 -
 4*d*f]))^q*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q*((e + Sq
rt[e^2 - 4*d*f] + 2*f*x)/(e + Sqrt[e^2 - 4*d*f]))^q)
```

Rubi [A] (warning: unable to verify)

Time = 1.41 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2192, 2192, 2192, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \left(fx^2+ex+d\right)^q \left(-c(ce(q+4)-2bf(2q+5))x^3-\left(-f(2q+5)b^2+3c^2d-2acf(2q+5)\right)x^2+2abf(2q+5)x\right.}{f(2q+5)}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int (fx^2+ex+d)^q (2a^2(q+2)(2q+5)f^2+(-((6df(q+2)-e^2(q^2+7q+12))c^2)+2f(2q+5)(2af(q+2)-be(q+3))c+2b^2f^2(2q^2+9q+10))x^2+2(de(q+ \cdots}{2f(q+2)}}{f(2q+5)}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\int -\left(\left(-d\left(6df(q+2)-e^2\left(q^2+7q+12\right)\right)c^2+2df(2q+5)(2af(q+2)-be(q+3))c+2f^2\left(2q^2+9q+10\right)\left(b^2d-a^2f(2q+3)\right)+\left(-e(q+3)\left(2df(5q+8)-e^2\left(q^2+6q+8\right)\right)c^2+2f(\cdots}{f(2q+3)}}{\cdots}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{x\left(d+ex+fx^2\right)^{q+1}\left(2cf(2q+5)(2af(q+2)-be(q+3))+2b^2f^2\left(2q^2+9q+10\right)-\left(c^2\left(6df(q+2)-e^2\left(q^2+7q+12\right)\right)\right)\right)}{f(2q+3)}-\frac{\int \left(-d\left(6df(q+2)-e^2\left(q^2+7q+12\right)\right)c^2+2df(2q+ \cdots}{\cdots}}{\cdots}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{x\left(d+ex+fx^2\right)^{q+1}\left(2cf(2q+5)(2af(q+2)-be(q+3))+2b^2f^2\left(2q^2+9q+10\right)-\left(c^2\left(6df(q+2)-e^2\left(q^2+7q+12\right)\right)\right)\right)}{f(2q+3)}-\frac{\left(4f^3\left(2q^2+9q+10\right)\left(b^2d-a^2f(2q+3)\right)-2cef(2 \cdots}{\cdots}}{\cdots}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

$$\Big\downarrow \; 1096$$

$$\frac{\frac{x\left(d+ex+fx^2\right)^{q+1}\left(2cf(2q+5)(2af(q+2)-be(q+3))+2b^2f^2\left(2q^2+9q+10\right)-\left(c^2\left(6df(q+2)-e^2\left(q^2+7q+12\right)\right)\right)\right)}{f(2q+3)}-\frac{\frac{\left(d+ex+fx^2\right)^{q+1}\left(2cf(2q+5)\left(2aef(q+2)^2+2b \cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{c^2x^3\left(d+ex+fx^2\right)^{q+1}}{f(2q+5)}$$

input
```
Int[(a + b*x + c*x^2)^2*(d + e*x + f*x^2)^q,x]
```

output
```
(c^2*x^3*(d + e*x + f*x^2)^(1 + q))/(f*(5 + 2*q)) + (-1/2*(c*(c*e*(4 + q)
- 2*b*f*(5 + 2*q))*x^2*(d + e*x + f*x^2)^(1 + q))/(f*(2 + q)) + (((2*b^2*f
^2*(10 + 9*q + 2*q^2) + 2*c*f*(5 + 2*q)*(2*a*f*(2 + q) - b*e*(3 + q)) - c^
2*(6*d*f*(2 + q) - e^2*(12 + 7*q + q^2)))*x*(d + e*x + f*x^2)^(1 + q))/(f*
(3 + 2*q)) - (((2*b*f^2*(10 + 9*q + 2*q^2)*(b*e*(2 + q) - 2*a*f*(3 + 2*q))
 + 2*c*f*(5 + 2*q)*(2*a*e*f*(2 + q)^2 + 2*b*d*f*(3 + 2*q) - b*e^2*(6 + 5*q
 + q^2)) - c^2*e*(3 + q)*(2*d*f*(8 + 5*q) - e^2*(8 + 6*q + q^2)))*(d + e*x
 + f*x^2)^(1 + q))/(2*f*(1 + q)) - (2^q*(4*c*d*f^2*(5 + 2*q)*(2*a*f*(2 + q
) - b*e*(3 + q)) - 2*b*e*f^2*(10 + 9*q + 2*q^2)*(b*e*(2 + q) - 2*a*f*(3 +
2*q)) + 4*f^3*(10 + 9*q + 2*q^2)*(b^2*d - a^2*f*(3 + 2*q)) - 2*c*e*f*(5 +
2*q)*(2*a*e*f*(2 + q)^2 + 2*b*d*f*(3 + 2*q) - b*e^2*(6 + 5*q + q^2)) + c^2
*e^2*(3 + q)*(2*d*f*(8 + 5*q) - e^2*(8 + 6*q + q^2)) - 2*c^2*d*f*(6*d*f*(2
 + q) - e^2*(12 + 7*q + q^2)))*(-((e - Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2
 - 4*d*f]))^(-1 - q)*(d + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-q, 1 + q
, 2 + q, (e + Sqrt[e^2 - 4*d*f] + 2*f*x)/(2*Sqrt[e^2 - 4*d*f])])/(f*Sqrt[e
^2 - 4*d*f]*(1 + q)))/(f*(3 + 2*q)))/(2*f*(2 + q)))/(f*(5 + 2*q))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(c\,x^{2}+bx+a\right)^{2} \left(f\,x^{2}+ex+d\right)^{q}dx$$

input
```
int((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x)
```

output
```
int((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x)
```

Fricas [F]

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \int \left(cx^{2}+bx+a\right)^{2} \left(fx^{2}+ex+d\right)^{q}dx$$

input
```
integrate((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(f*x^2
+ e*x + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**2*(f*x**2+e*x+d)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \int \left(cx^2+bx+a\right)^2 \left(fx^2+ex+d\right)^q \, dx$$

input
```
integrate((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^2*(f*x^2 + e*x + d)^q, x)
```

Giac [F]

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \int \left(cx^2+bx+a\right)^2 \left(fx^2+ex+d\right)^q \, dx$$

input
```
integrate((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^2*(f*x^2 + e*x + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \int \left(c\,x^2+b\,x+a\right)^2 \left(f\,x^2+e\,x+d\right)^q \,dx$$

input
```
int((a + b*x + c*x^2)^2*(d + e*x + f*x^2)^q,x)
```

output
```
int((a + b*x + c*x^2)^2*(d + e*x + f*x^2)^q, x)
```

Reduce [F]

$$\int \left(a+bx+cx^2\right)^2 \left(d+ex+fx^2\right)^q \, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x+a)^2*(f*x^2+e*x+d)^q,x)
```

output
```
(32*(d + e*x + f*x**2)**q*a**2*d*f**4*q**4 + 224*(d + e*x + f*x**2)**q*a**
2*d*f**4*q**3 + 568*(d + e*x + f*x**2)**q*a**2*d*f**4*q**2 + 616*(d + e*x
+ f*x**2)**q*a**2*d*f**4*q + 240*(d + e*x + f*x**2)**q*a**2*d*f**4 + 16*(d
 + e*x + f*x**2)**q*a**2*e*f**4*q**4*x + 112*(d + e*x + f*x**2)**q*a**2*e*
f**4*q**3*x + 284*(d + e*x + f*x**2)**q*a**2*e*f**4*q**2*x + 308*(d + e*x
+ f*x**2)**q*a**2*e*f**4*q*x + 120*(d + e*x + f*x**2)**q*a**2*e*f**4*x - 1
6*(d + e*x + f*x**2)**q*a*b*d*e*f**3*q**3 - 96*(d + e*x + f*x**2)**q*a*b*d
*e*f**3*q**2 - 188*(d + e*x + f*x**2)**q*a*b*d*e*f**3*q - 120*(d + e*x + f
*x**2)**q*a*b*d*e*f**3 + 16*(d + e*x + f*x**2)**q*a*b*e**2*f**3*q**4*x + 9
6*(d + e*x + f*x**2)**q*a*b*e**2*f**3*q**3*x + 188*(d + e*x + f*x**2)**q*a
*b*e**2*f**3*q**2*x + 120*(d + e*x + f*x**2)**q*a*b*e**2*f**3*q*x + 32*(d
+ e*x + f*x**2)**q*a*b*e*f**4*q**4*x**2 + 208*(d + e*x + f*x**2)**q*a*b*e*
f**4*q**3*x**2 + 472*(d + e*x + f*x**2)**q*a*b*e*f**4*q**2*x**2 + 428*(d +
 e*x + f*x**2)**q*a*b*e*f**4*q*x**2 + 120*(d + e*x + f*x**2)**q*a*b*e*f**4
*x**2 - 32*(d + e*x + f*x**2)**q*a*c*d**2*f**3*q**3 - 176*(d + e*x + f*x**
2)**q*a*c*d**2*f**3*q**2 - 304*(d + e*x + f*x**2)**q*a*c*d**2*f**3*q - 160
*(d + e*x + f*x**2)**q*a*c*d**2*f**3 + 8*(d + e*x + f*x**2)**q*a*c*d*e**2*
f**2*q**3 + 52*(d + e*x + f*x**2)**q*a*c*d*e**2*f**2*q**2 + 112*(d + e*x +
 f*x**2)**q*a*c*d*e**2*f**2*q + 80*(d + e*x + f*x**2)**q*a*c*d*e**2*f**2 +
 32*(d + e*x + f*x**2)**q*a*c*d*e*f**3*q**4*x + 176*(d + e*x + f*x**2)*...
```

3.168 $\int (a+bx+cx^2)(d+ex+fx^2)^q \, dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\int (a+bx+cx^2)(d+ex+fx^2)^q \, dx$$
$$= -\frac{(ce(2+q)-bf(3+2q))(d+ex+fx^2)^{1+q}}{2f^2(1+q)(3+2q)} + \frac{cx(d+ex+fx^2)^{1+q}}{f(3+2q)}$$
$$- \frac{4^{-1-q}(2cdf-ce^2(2+q)+f(be-2af)(3+2q))(e+2fx)(d+ex+fx^2)^q\left(-\frac{f(d+ex+fx^2)}{e^2-4df}\right)^{-q}\text{Hyperg}}{f^3(3+2q)}$$

output
```
-1/2*(c*e*(2+q)-b*f*(3+2*q))*(f*x^2+e*x+d)^(1+q)/f^2/(1+q)/(3+2*q)+c*x*(f*
x^2+e*x+d)^(1+q)/f/(3+2*q)-4^(-1-q)*(2*c*d*f-c*e^2*(2+q)+f*(-2*a*f+b*e)*(3
+2*q))*(2*f*x+e)*(f*x^2+e*x+d)^q*hypergeom([1/2, -q],[3/2],(2*f*x+e)^2/(-4
*d*f+e^2))/f^3/(3+2*q)/((-f*(f*x^2+e*x+d)/(-4*d*f+e^2))^q)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.40 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.04

$$\int (a+bx+cx^2)(d+ex+fx^2)^q\,dx = \frac{1}{6}(d+x(e+fx))^q\left(3bx^2\left(\frac{e-\sqrt{e^2-4df}+2fx}{e-\sqrt{e^2-4df}}\right)^{-q}\left(\frac{e+\sqrt{e^2-4df}+2fx}{e+\sqrt{e^2-4df}}\right)^{-q}\text{AppellF1}\left(2,-q,-q,3,-\frac{2fx}{e+\sqrt{e^2-4df}},\frac{2fx}{-e+\sqrt{e^2-4df}}\right)+2cx^3\left(\frac{e-\sqrt{e^2-4df}+2fx}{e-\sqrt{e^2-4df}}\right)^{-q}\left(\frac{e+\sqrt{e^2-4df}+2fx}{e+\sqrt{e^2-4df}}\right)^{-q}\text{AppellF1}\left(3,-q,-q,4,-\frac{2fx}{e+\sqrt{e^2-4df}},\frac{2fx}{-e+\sqrt{e^2-4df}}\right)+\frac{3\,2^q a(e-\sqrt{e^2-4df}+2fx)\left(\frac{e+\sqrt{e^2-4df}+2fx}{\sqrt{e^2-4df}}\right)^{-q}\text{Hypergeometric2F1}\left(-q,1+q,2+q,\frac{-e+\sqrt{e^2-4df}-2f}{2\sqrt{e^2-4df}}\right.}{f(1+q)}$$

input
```
Integrate[(a + b*x + c*x^2)*(d + e*x + f*x^2)^q,x]
```

output
```
((d + x*(e + f*x))^q*((3*b*x^2*AppellF1[2, -q, -q, 3, (-2*f*x)/(e + Sqrt[e
^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])])/(((e - Sqrt[e^2 - 4*d*f]
+ 2*f*x)/(e - Sqrt[e^2 - 4*d*f]))^q*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/(e +
Sqrt[e^2 - 4*d*f]))^q) + (2*c*x^3*AppellF1[3, -q, -q, 4, (-2*f*x)/(e + Sqr
t[e^2 - 4*d*f]), (2*f*x)/(-e + Sqrt[e^2 - 4*d*f])])/(((e - Sqrt[e^2 - 4*d*
f] + 2*f*x)/(e - Sqrt[e^2 - 4*d*f]))^q*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/(e
 + Sqrt[e^2 - 4*d*f]))^q) + (3*2^q*a*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Hyper
geometric2F1[-q, 1 + q, 2 + q, (-e + Sqrt[e^2 - 4*d*f] - 2*f*x)/(2*Sqrt[e^
2 - 4*d*f])])/(f*(1 + q)*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f
])^q)))/6
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2192, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right) \left(d+ex+fx^2\right)^q \, dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int -\left(\left(cd-af(2q+3)+(ce(q+2)-bf(2q+3))x\right)\left(fx^2+ex+d\right)^q\right)dx}{f(2q+3)}+\frac{cx\left(d+ex+fx^2\right)^{q+1}}{f(2q+3)}$$

$$\Big\downarrow \; 25$$

$$\frac{cx\left(d+ex+fx^2\right)^{q+1}}{f(2q+3)}-\frac{\int (cd-af(2q+3)+(ce(q+2)-bf(2q+3))x)\left(fx^2+ex+d\right)^q dx}{f(2q+3)}$$

$$\Big\downarrow \; 1160$$

$$\frac{cx\left(d+ex+fx^2\right)^{q+1}}{f(2q+3)}-\frac{\frac{\left(f(2q+3)(be-2af)+2cdf-ce^2(q+2)\right)\int \left(fx^2+ex+d\right)^q dx}{2f}+\frac{(ce(q+2)-bf(2q+3))\left(d+ex+fx^2\right)^{q+1}}{2f(q+1)}}{f(2q+3)}$$

$$\Big\downarrow \; 1096$$

$$\frac{cx\left(d+ex+fx^2\right)^{q+1}}{f(2q+3)}-\frac{\frac{(ce(q+2)-bf(2q+3))\left(d+ex+fx^2\right)^{q+1}}{2f(q+1)}-\frac{2^q\left(-\frac{-\sqrt{e^2-4df}+e+2fx}{\sqrt{e^2-4df}}\right)^{-q-1}\left(d+ex+fx^2\right)^{q+1}\text{Hypergeometric2F1}\left(-q,q+1,q+2,\frac{e+2fx+\sqrt{e^2-4df}}{2\sqrt{e^2-4df}}\right)}{f(q+1)\sqrt{e^2-4df}}}{f(2q+3)}$$

input
```
Int[(a + b*x + c*x^2)*(d + e*x + f*x^2)^q,x]
```

output
```
(c*x*(d + e*x + f*x^2)^(1 + q))/(f*(3 + 2*q)) - (((c*e*(2 + q) - b*f*(3 +
2*q))*(d + e*x + f*x^2)^(1 + q))/(2*f*(1 + q)) - (2^q*(2*c*d*f - c*e^2*(2
+ q) + f*(b*e - 2*a*f)*(3 + 2*q))*(-((e - Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[
e^2 - 4*d*f]))^(-1 - q)*(d + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-q, 1
+ q, 2 + q, (e + Sqrt[e^2 - 4*d*f] + 2*f*x)/(2*Sqrt[e^2 - 4*d*f])])/(f*Sqr
t[e^2 - 4*d*f]*(1 + q)))/(f*(3 + 2*q))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (c\,x^2+bx+a)\,(f\,x^2+ex+d)^q\,dx$$

input
```
int((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x)
```

output
```
int((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x)
```

Fricas [F]

$$\int (a+bx+cx^2)\,(d+ex+fx^2)^q\,dx=\int (cx^2+bx+a)(fx^2+ex+d)^q\,dx$$

input
```
integrate((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*(f*x^2 + e*x + d)^q, x)
```

Sympy [F]

$$\int (a+bx+cx^2)\,(d+ex+fx^2)^q\,dx=\int (a+bx+cx^2)\,(d+ex+fx^2)^q\,dx$$

input
```
integrate((c*x**2+b*x+a)*(f*x**2+e*x+d)**q,x)
```

output
```
Integral((a + b*x + c*x**2)*(d + e*x + f*x**2)**q, x)
```

Maxima [F]

$$\int (a+bx+cx^2)(d+ex+fx^2)^q\,dx = \int (cx^2+bx+a)(fx^2+ex+d)^q\,dx$$

input
```
integrate((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)*(f*x^2 + e*x + d)^q, x)
```

Giac [F]

$$\int (a+bx+cx^2)(d+ex+fx^2)^q\,dx = \int (cx^2+bx+a)(fx^2+ex+d)^q\,dx$$

input
```
integrate((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*(f*x^2 + e*x + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)(d+ex+fx^2)^q\,dx = \int (c\,x^2+b\,x+a)\,(f\,x^2+e\,x+d)^q\,dx$$

input
```
int((a + b*x + c*x^2)*(d + e*x + f*x^2)^q,x)
```

output
```
int((a + b*x + c*x^2)*(d + e*x + f*x^2)^q, x)
```

Reduce [F]

$$\int (a + bx + cx^2) (d + ex + fx^2)^q \, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x+a)*(f*x^2+e*x+d)^q,x)
```

output
```
(8*(d + e*x + f*x**2)**q*a*d*f**2*q**2 + 20*(d + e*x + f*x**2)**q*a*d*f**2
*q + 12*(d + e*x + f*x**2)**q*a*d*f**2 + 4*(d + e*x + f*x**2)**q*a*e*f**2*
q**2*x + 10*(d + e*x + f*x**2)**q*a*e*f**2*q*x + 6*(d + e*x + f*x**2)**q*a
*e*f**2*x - 2*(d + e*x + f*x**2)**q*b*d*e*f*q - 3*(d + e*x + f*x**2)**q*b*
d*e*f + 2*(d + e*x + f*x**2)**q*b*e**2*f*q**2*x + 3*(d + e*x + f*x**2)**q*
b*e**2*f*q*x + 4*(d + e*x + f*x**2)**q*b*e*f**2*q**2*x**2 + 8*(d + e*x + f
*x**2)**q*b*e*f**2*q*x**2 + 3*(d + e*x + f*x**2)**q*b*e*f**2*x**2 - 4*(d +
 e*x + f*x**2)**q*c*d**2*f*q - 4*(d + e*x + f*x**2)**q*c*d**2*f + (d + e*x
 + f*x**2)**q*c*d*e**2*q + 2*(d + e*x + f*x**2)**q*c*d*e**2 + 4*(d + e*x +
 f*x**2)**q*c*d*e*f*q**2*x + 4*(d + e*x + f*x**2)**q*c*d*e*f*q*x - (d + e*
x + f*x**2)**q*c*e**3*q**2*x - 2*(d + e*x + f*x**2)**q*c*e**3*q*x + 2*(d +
 e*x + f*x**2)**q*c*e**2*f*q**2*x**2 + (d + e*x + f*x**2)**q*c*e**2*f*q*x*
*2 + 4*(d + e*x + f*x**2)**q*c*e*f**2*q**2*x**3 + 6*(d + e*x + f*x**2)**q*
c*e*f**2*q*x**3 + 2*(d + e*x + f*x**2)**q*c*e*f**2*x**3 - 64*int(((d + e*x
 + f*x**2)**q*x)/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x + 8*e*q*x + 3*e*x +
4*f*q**2*x**2 + 8*f*q*x**2 + 3*f*x**2),x)*a*d*f**3*q**5 - 288*int(((d + e*
x + f*x**2)**q*x)/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x + 8*e*q*x + 3*e*x +
 4*f*q**2*x**2 + 8*f*q*x**2 + 3*f*x**2),x)*a*d*f**3*q**4 - 464*int(((d + e
*x + f*x**2)**q*x)/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x + 8*e*q*x + 3*e*x
+ 4*f*q**2*x**2 + 8*f*q*x**2 + 3*f*x**2),x)*a*d*f**3*q**3 - 312*int(((d...
```

3.169 $\int (d + ex + fx^2)^q \, dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int (d + ex + fx^2)^q \, dx = \frac{2^{-1-2q}(e + 2fx)\,(d + ex + fx^2)^q \left(-\frac{f(d+ex+fx^2)}{e^2-4df}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2}, -q, \frac{3}{2}, \frac{(e+2fx)^2}{e^2-4df}\right)}{f}$$

output
```
2^(-1-2*q)*(2*f*x+e)*(f*x^2+e*x+d)^q*hypergeom([1/2, -q],[3/2],(2*f*x+e)^2
/(-4*d*f+e^2))/f/((-f*(f*x^2+e*x+d)/(-4*d*f+e^2))^q)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int (d + ex + fx^2)^q \, dx = \frac{2^{-1+q}\left(e - \sqrt{e^2 - 4df} + 2fx\right) \left(\frac{e+\sqrt{e^2-4df}+2fx}{\sqrt{e^2-4df}}\right)^{-q} (d + x(e + fx))^q \text{Hypergeometric2F1}\left(-q, 1 + q, 2 + q\right.}{f(1+q)}$$

input
```
Integrate[(d + e*x + f*x^2)^q,x]
```

output
```
(2^(-1 + q)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*(d + x*(e + f*x))^q*Hypergeome
tric2F1[-q, 1 + q, 2 + q, (-e + Sqrt[e^2 - 4*d*f] - 2*f*x)/(2*Sqrt[e^2 - 4
*d*f])])/(f*(1 + q)*((e + Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f])^q)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)^q\,dx$$

$$\Big\downarrow \ 1096$$

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-4df}+e+2fx}{\sqrt{e^2-4df}}\right)^{-q-1}\left(d+ex+fx^2\right)^{q+1}\text{Hypergeometric2F1}\left(-q,q+1,q+2,\frac{e+2fx+\sqrt{e^2-4df}}{2\sqrt{e^2-4df}}\right)}{(q+1)\sqrt{e^2-4df}}$$

input
```
Int[(d + e*x + f*x^2)^q,x]
```

output
```
-((2^(1 + q)*(-((e - Sqrt[e^2 - 4*d*f] + 2*f*x)/Sqrt[e^2 - 4*d*f]))^(-1 -
q)*(d + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-q, 1 + q, 2 + q, (e + Sqrt
[e^2 - 4*d*f] + 2*f*x)/(2*Sqrt[e^2 - 4*d*f])])/(Sqrt[e^2 - 4*d*f]*(1 + q))
)
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int \left(f\,x^{2}+ex+d\right)^{q}dx$$

input
```
int((f*x^2+e*x+d)^q,x)
```

output
```
int((f*x^2+e*x+d)^q,x)
```

Fricas [F]

$$\int \left(d+ex+fx^2\right)^q\,dx=\int \left(fx^2+ex+d\right)^q\,dx$$

input
```
integrate((f*x^2+e*x+d)^q,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)^q, x)
```

Sympy [F]

$$\int \left(d+ex+fx^2\right)^q\,dx=\int \left(d+ex+fx^2\right)^q\,dx$$

input
```
integrate((f*x**2+e*x+d)**q,x)
```

output
```
Integral((d + e*x + f*x**2)**q, x)
```

Maxima [F]

$$\int \left(d+ex+fx^2\right)^q\,dx=\int \left(fx^2+ex+d\right)^q\,dx$$

input
```
integrate((f*x^2+e*x+d)^q,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)^q, x)
```

Giac [F]

$$\int \left(d+ex+fx^2\right)^q\,dx=\int \left(fx^2+ex+d\right)^q\,dx$$

input
```
integrate((f*x^2+e*x+d)^q,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+fx^2\right)^q\,dx=\int {\left(f\,x^2+e\,x+d\right)}^q\,dx$$

input
```
int((d + e*x + f*x^2)^q,x)
```

output
```
int((d + e*x + f*x^2)^q, x)
```

Reduce [F]

$$\int \left(d+ex+fx^2\right)^q\,dx$$

$$= \frac{2\left(f\,x^2+ex+d\right)^q d+\left(f\,x^2+ex+d\right)^q ex-8\left(\int \frac{\left(f\,x^2+ex+d\right)^q x}{2fq\,x^2+2eqx+f\,x^2+2dq+ex+d}dx\right) df\,q^2-4\left(\int \frac{\left(f\,x^2+ex+\cdots\right.}{2fq\,x^2+2eqx+f\,x^2\cdots}\right.}{e\left(2q+1\right)\cdots}$$

input

```
int((f*x^2+e*x+d)^q,x)
```

output

```
(2*(d + e*x + f*x**2)**q*d + (d + e*x + f*x**2)**q*e*x - 8*int(((d + e*x +
 f*x**2)**q*x)/(2*d*q + d + 2*e*q*x + e*x + 2*f*q*x**2 + f*x**2),x)*d*f*q*
*2 - 4*int(((d + e*x + f*x**2)**q*x)/(2*d*q + d + 2*e*q*x + e*x + 2*f*q*x*
*2 + f*x**2),x)*d*f*q + 2*int(((d + e*x + f*x**2)**q*x)/(2*d*q + d + 2*e*q
*x + e*x + 2*f*q*x**2 + f*x**2),x)*e**2*q**2 + int(((d + e*x + f*x**2)**q*
x)/(2*d*q + d + 2*e*q*x + e*x + 2*f*q*x**2 + f*x**2),x)*e**2*q)/(e*(2*q +
1))
```

3.170 $\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 527

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx$$

$$= \frac{\left(\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{f\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-q}\left(\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{f\left(b-\sqrt{b^2-4ac}+2cx\right)}\right)^{-q}(d+ex+fx^2)^q\,\mathrm{AppellF1}\left(-2q,-q,-q,1-2q,\frac{b-\sqrt{b^2-4a\cdots}}{b-\sqrt{b\cdots}}\right.}{2\sqrt{b^2-4ac}\,q}$$

$$-\frac{\left(\frac{c\left(e-\sqrt{e^2-4df}+2fx\right)}{f\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)^{-q}\left(\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{f\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)^{-q}(d+ex+fx^2)^q\,\mathrm{AppellF1}\left(-2q,-q,-q,1-2q,\frac{(b+\sqrt{b^2\cdots}}{f(\cdots}\right.}{2\sqrt{b^2-4ac}\,q}$$

output
```
1/2*(f*x^2+e*x+d)^q*AppellF1(-2*q,-q,-q,1-2*q,(b-(-4*a*c+b^2)^(1/2)-c*(e-(
-4*d*f+e^2)^(1/2))/f)/(b-(-4*a*c+b^2)^(1/2)+2*c*x),(b-(-4*a*c+b^2)^(1/2)-c
*(e+(-4*d*f+e^2)^(1/2))/f)/(b-(-4*a*c+b^2)^(1/2)+2*c*x))/(-4*a*c+b^2)^(1/2
)/q/((c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)/f/(b-(-4*a*c+b^2)^(1/2)+2*c*x))^q)/((
c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)/f/(b-(-4*a*c+b^2)^(1/2)+2*c*x))^q)-1/2*(f*x
^2+e*x+d)^q*AppellF1(-2*q,-q,-q,1-2*q,((b+(-4*a*c+b^2)^(1/2))*f-c*(e-(-4*d
*f+e^2)^(1/2)))/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x),((b+(-4*a*c+b^2)^(1/2))*f-c
*(e+(-4*d*f+e^2)^(1/2)))/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x))/(-4*a*c+b^2)^(1/2
)/q/((c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x))^q)/((
c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x))^q)
```

Mathematica [F]

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx=\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx$$

input
```
Integrate[(d + e*x + f*x^2)^q/(a + b*x + c*x^2),x]
```

output
```
Integrate[(d + e*x + f*x^2)^q/(a + b*x + c*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1325$$
$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}dx$$

input
```
Int[(d + e*x + f*x^2)^q/(a + b*x + c*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0
]
```

Maple [F]

$$\int \frac{(f\,x^2+ex+d)^q}{c\,x^2+bx+a}dx$$

input
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x)
```

output
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx = \int \frac{(fx^2+ex+d)^q}{cx^2+bx+a}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)^q/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)**q/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx=\int \frac{(fx^2+ex+d)^q}{cx^2+bx+a}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)^q/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx=\int \frac{(fx^2+ex+d)^q}{cx^2+bx+a}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)^q/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx=\int \frac{(f\,x^2+e\,x+d)^q}{c\,x^2+b\,x+a}\,dx$$

input
```
int((d + e*x + f*x^2)^q/(a + b*x + c*x^2),x)
```

output
```
int((d + e*x + f*x^2)^q/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex+fx^2)^q}{a+bx+cx^2}\,dx = \int \frac{(f\,x^2+ex+d)^q}{c\,x^2+bx+a}dx$$

input
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a),x)
```

output
```
int((d + e*x + f*x**2)**q/(a + b*x + c*x**2),x)
```

3.171 $\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 1127

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

output
```
(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)
*x)*(f*x^2+e*x+d)^(1+q)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/
(c*x^2+b*x+a)+1/4*((2*a*f-b*e+2*c*d)*(-b*f+c*e)-(4*c^3*d^2-4*c^2*(b*d*e-a*
(-2*d*f+e^2)*(1-q))-b^2*f*(-2*a*f+b*e)*q+c*(4*a^2*f^2*(1-2*q)-4*a*b*e*f*(1
-q)+b^2*(2*d*f*(2-q)+e^2*q)))/(-4*a*c+b^2)^(1/2)/q)*(f*x^2+e*x+d)^q*Appell
F1(-2*q,-q,-q,1-2*q,(b-(-4*a*c+b^2)^(1/2)-c*(e-(-4*d*f+e^2)^(1/2))/f)/(b-(
-4*a*c+b^2)^(1/2)+2*c*x),(b-(-4*a*c+b^2)^(1/2)-c*(e+(-4*d*f+e^2)^(1/2))/f)
/(b-(-4*a*c+b^2)^(1/2)+2*c*x))/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f
+c*e))/((c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)/f/(b-(-4*a*c+b^2)^(1/2)+2*c*x))^q)
/((c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)/f/(b-(-4*a*c+b^2)^(1/2)+2*c*x))^q)+1/4*(
(2*a*f-b*e+2*c*d)*(-b*f+c*e)+(4*c^3*d^2-4*c^2*(b*d*e-a*(-2*d*f+e^2)*(1-q))
-b^2*f*(-2*a*f+b*e)*q+c*(4*a^2*f^2*(1-2*q)-4*a*b*e*f*(1-q)+b^2*(2*d*f*(2-q
)+e^2*q)))/(-4*a*c+b^2)^(1/2)/q)*(f*x^2+e*x+d)^q*AppellF1(-2*q,-q,-q,1-2*q
,((b+(-4*a*c+b^2)^(1/2))*f-c*(e-(-4*d*f+e^2)^(1/2)))/f/(b+(-4*a*c+b^2)^(1/
2)+2*c*x),((b+(-4*a*c+b^2)^(1/2))*f-c*(e+(-4*d*f+e^2)^(1/2)))/f/(b+(-4*a*c
+b^2)^(1/2)+2*c*x))/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/((c*
(e-(-4*d*f+e^2)^(1/2)+2*f*x)/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x))^q)/((c*(2*f*x
+(-4*d*f+e^2)^(1/2)+e)/f/(b+(-4*a*c+b^2)^(1/2)+2*c*x))^q)+2^(-1-2*q)*(2*c^
2*d+b^2*f-c*(2*a*f+b*e))*(1+2*q)*(2*f*x+e)*(f*x^2+e*x+d)^q*hypergeom([1/2,
 -q],[3/2],(2*f*x+e)^2/(-4*d*f+e^2))/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b...
```

Mathematica [F]

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx$$

input
```
Integrate[(d + e*x + f*x^2)^q/(a + b*x + c*x^2)^2,x]
```

output
```
Integrate[(d + e*x + f*x^2)^q/(a + b*x + c*x^2)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{\int -\frac{(fx^2+ex+d)^q(2d^2c^3-(-2a(1-q)e^2+bd(q+2)e+6adf)c^2-f(fb^2-ceb+2c^2d-2acf)(2q+1)x^2c+((qe^2+2df)b^2-aef(1-3q)b+4a^2f^2)c-b^2 \cdots}{cx^2+bx+a}}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce- \cdots} \frac{(d+ex+fx^2)^{q+1}(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(d+ex+fx^2)^{q+1}(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int \frac{(fx^2+ex+d)^q(2d^2c^3-(-2a(1-q)e^2+bd(q+2)e+6adf)c^2-f(fb^2-ceb+2c^2d-2acf)(2q+1)x^2c+((qe^2+2df)b^2-aef(1-3q)b+4a^2f^2)c-b^2f(\cdots}{cx^2+bx+a}}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-b \cdots}$$

$$\Big\downarrow \text{ 7279}$$

$$\frac{(d+ex+fx^2)^{q+1}(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int \Big(\frac{(2d^2c^3+(2a(e^2-2df)(1-q)-bde(q+2))c^2+((qe^2+2df)b^2-aef(2-q)b+2a^2f^2(1-2q))c-(2cd-be+2af)(ce-bf)qxc-b^2f(be-2af)q)(fx^2+ \cdots}{cx^2+bx+a}}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex+fx^2)^{q+1}(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}-\frac{\int \frac{(2d^2c^3+(2a(e^2-2df)(1-q)-bde(q+2))c^2+((qe^2+2df)b^2-aef(2-q)b+2a^2f^2(1-2q))c-(2cd-be+2af)(ce-bf)qxc-b^2f(be-2af)q)(fx^2+e \cdots}{cx^2+bx+a}}{(b^2-4ac)((cd- \cdots}$$

input
```
Int[(d + e*x + f*x^2)^q/(a + b*x + c*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(f\,x^{2}+ex+d)^{q}}{(c\,x^{2}+bx+a)^{2}}dx$$

input
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x)
```

output
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(fx^2+ex+d)^q}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)^q/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c
)*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)**q/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(fx^2+ex+d)^q}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)^q/(c*x^2 + b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(fx^2+ex+d)^q}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)^q/(c*x^2 + b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(f\,x^2+e\,x+d)^q}{(c\,x^2+b\,x+a)^2}\,dx$$

input
```
int((d + e*x + f*x^2)^q/(a + b*x + c*x^2)^2,x)
```

output
```
int((d + e*x + f*x^2)^q/(a + b*x + c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex+fx^2)^q}{(a+bx+cx^2)^2}\,dx = \int \frac{(f\,x^2+ex+d)^q}{c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2}dx$$

input
```
int((f*x^2+e*x+d)^q/(c*x^2+b*x+a)^2,x)
```

output
```
int((d + e*x + f*x**2)**q/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b*c
*x**3 + c**2*x**4),x)
```

3.172 $\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 525

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx = \frac{\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\left(e-\sqrt{e^2-4df}+2fx\right)}\right)^{-p}\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\left(e-\sqrt{e^2-4df}+2fx\right)}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{e-\frac{\left(b-\sqrt{b^2-\cdots}\right.}{c}}{e-\sqrt{e\cdots}}\right.}{2\sqrt{e^2-4df}\,p} - \frac{\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\left(e+\sqrt{e^2-4df}+2fx\right)}\right)^{-p}\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\left(e+\sqrt{e^2-4df}+2fx\right)}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{e-\frac{\left(b+\sqrt{\cdots}\right.}{}}{e+\cdots}\right.}{2\sqrt{e^2-4df}\,p}$$

output
```
1/2*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(e-(b-(-4*a*c+b^2)^(1/2))*f/
c-(-4*d*f+e^2)^(1/2))/(e-(-4*d*f+e^2)^(1/2)+2*f*x),(e-(b+(-4*a*c+b^2)^(1/2
))*f/c-(-4*d*f+e^2)^(1/2))/(e-(-4*d*f+e^2)^(1/2)+2*f*x))/(-4*d*f+e^2)^(1/2
)/p/((f*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/(e-(-4*d*f+e^2)^(1/2)+2*f*x))^p)/((
f*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(e-(-4*d*f+e^2)^(1/2)+2*f*x))^p)-1/2*(c*x
^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(e-(b+(-4*a*c+b^2)^(1/2))*f/c+(-4*d*
f+e^2)^(1/2))/(2*f*x+(-4*d*f+e^2)^(1/2)+e),-((b-(-4*a*c+b^2)^(1/2))*f-c*(e
+(-4*d*f+e^2)^(1/2)))/c/(2*f*x+(-4*d*f+e^2)^(1/2)+e))/(-4*d*f+e^2)^(1/2)/p
/((f*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^p)/((f*(
b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^p)
```

Mathematica [F]

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx$$

input
```
Integrate[(a + b*x + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
Integrate[(a + b*x + c*x^2)^p/(d + e*x + f*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{1325}$$

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}dx$$

input
```
Int[(a + b*x + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0
]
```

Maple [F]

$$\int \frac{(c\,x^2+bx+a)^p}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x)
```

output
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+bx+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**p/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(cx^2+bx+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + e*x + d), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(cx^2+bx+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(c\,x^2+b\,x+a)^p}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*x + c*x^2)^p/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^p/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+bx+a)^p}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x+d),x)
```

output
```
int((a + b*x + c*x**2)**p/(d + e*x + f*x**2),x)
```

3.173 $\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 541

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx =$$

$$-\frac{4^{-1+p}\left(\frac{\sqrt{f}\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\left(\sqrt{-d}+\sqrt{f}x\right)}\right)^{-p}\left(\frac{\sqrt{f}\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\left(\sqrt{-d}+\sqrt{f}x\right)}\right)^{-p}(a+bx+cx^2)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p\right.}{\sqrt{-d}\sqrt{f}p}$$

$$+\frac{4^{-1+p}\left(\frac{\sqrt{-d}\sqrt{f}\left(b-\sqrt{b^2-4ac}+2cx\right)}{c\left(d+\sqrt{-d}\sqrt{f}x\right)}\right)^{-p}\left(\frac{\sqrt{-d}\sqrt{f}\left(b+\sqrt{b^2-4ac}+2cx\right)}{c\left(d+\sqrt{-d}\sqrt{f}x\right)}\right)^{-p}(a+bx+cx^2)^p\operatorname{AppellF1}\left(-2p,-p,-p,\right.}{\sqrt{-d}\sqrt{f}p}$$

output
```
-4^(-1+p)*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*(-d)^(1/2)-(b+(-4*a
*c+b^2)^(1/2))*f^(1/2)/c)/(2*(-d)^(1/2)+2*f^(1/2)*x),1/2*(2*c*(-d)^(1/2)-(
b-(-4*a*c+b^2)^(1/2))*f^(1/2))/c/((-d)^(1/2)+f^(1/2)*x))/(-d)^(1/2)/f^(1/2
)/p/((f^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/((-d)^(1/2)+f^(1/2)*x))^p)/((
f^(1/2)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/((-d)^(1/2)+f^(1/2)*x))^p)+4^(-1+p)
*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2
))*(-d)^(1/2)*f^(1/2))/c/(d+(-d)^(1/2)*f^(1/2)*x),1/2*(2*c*d-(b+(-4*a*c+b^
2)^(1/2))*(-d)^(1/2)*f^(1/2))/c/(d+(-d)^(1/2)*f^(1/2)*x))/(-d)^(1/2)/f^(1/
2)/p/(((-d)^(1/2)*f^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/(d+(-d)^(1/2)*f^(
1/2)*x))^p)/(((-d)^(1/2)*f^(1/2)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(d+(-d)^(1
/2)*f^(1/2)*x))^p)
```

Mathematica [F]

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx$$

input
```
Integrate[(a + b*x + c*x^2)^p/(d + f*x^2),x]
```

output
```
Integrate[(a + b*x + c*x^2)^p/(d + f*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx$$
$$\Big\downarrow \text{1326}$$
$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}dx$$

input
```
Int[(a + b*x + c*x^2)^p/(d + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1326
```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_
Symbol] :> Unintegrable[(a + c*x^2)^p*(d + e*x + f*x^2)^q, x] /; FreeQ[{a,
c, d, e, f, p, q}, x] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Maple [F]

$$\int \frac{(c\,x^2+bx+a)^p}{f\,x^2+d}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+d),x)
```

output
```
int((c*x^2+b*x+a)^p/(f*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+bx+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p/(f*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**p/(f*x**2+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+bx+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+bx+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^p}{f\,x^2+d}\,dx$$

input
```
int((a + b*x + c*x^2)^p/(d + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^p/(d + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^p}{d+fx^2}\,dx = \int \frac{(c\,x^2+bx+a)^p}{f\,x^2+d}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+d),x)
```

output
```
int((a + b*x + c*x**2)**p/(d + f*x**2),x)
```

3.174 $\int \frac{(a+bx+cx^2)^p}{ex+fx^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 337

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2} \, dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx}\right.}{ep} - \frac{2^{-1+2p}\left(\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{c(e+fx)}\right)^{-p}\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{c(e+fx)}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\right.}{ep}$$

output
```
2^(-1+2*p)*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^
(1/2))/c/x,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x)/e/p/(((b-(-4*a*c+b^2)^(1/2)+2*
c*x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)-2^(-1+2*p)*(c*x^2+b*x+
a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*e-(b+(-4*a*c+b^2)^(1/2))*f/c)/(2*f*x+2*e
),1/2*(2*c*e-(b-(-4*a*c+b^2)^(1/2))*f)/c/(f*x+e))/e/p/((f*(b-(-4*a*c+b^2)^
(1/2)+2*c*x)/c/(f*x+e))^p)/((f*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(f*x+e))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx$$
$$= \frac{2^{-1+2p}(a+x(b+cx))^p\left(\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-}}{2cx}\right.\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
(2^(-1 + 2*p)*(a + x*(b + c*x))^p*(AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 + Sqrt[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x)]/(((b - Sqrt
[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p)
 - AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)/(2*
c*(e + f*x)), (2*c*e - b*f + Sqrt[b^2 - 4*a*c]*f)/(2*c*e + 2*c*f*x)]/(((f*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*(e + f*x)))^p*((f*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x))/(c*(e + f*x)))^p)))/(e*p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx$$
$$\Big\downarrow \ 1325$$
$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}dx$$

input
```
Int[(a + b*x + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && !IGtQ[p, 0] && !IGtQ[q, 0
]
```

Maple [F]

$$\int \frac{(c\,x^2+bx+a)^p}{f\,x^2+ex}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x),x)
```

output
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x),x)
```

Fricas [F]

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+bx+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)^p/(f*x^2 + e*x), x)
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(a+bx+cx^2)^p}{x\,(e+fx)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**p/(f*x**2+e*x),x)
```

output
```
Integral((a + b*x + c*x**2)**p/(x*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx=\int \frac{(cx^2+bx+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + e*x), x)
```

Giac [F]

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx=\int \frac{(cx^2+bx+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+b*x+a)^p/(f*x^2+e*x),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^p/(f*x^2 + e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx=\int \frac{(c\,x^2+b\,x+a)^p}{f\,x^2+e\,x}\,dx$$

input
```
int((a + b*x + c*x^2)^p/(e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^p/(e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(c\,x^2+bx+a)^p}{f\,x^2+ex}dx$$

input
```
int((c*x^2+b*x+a)^p/(f*x^2+e*x),x)
```

output
```
int((a + b*x + c*x**2)**p/(e*x + f*x**2),x)
```

3.175 $\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \frac{2fx\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{2fx}{e-\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}\right)(1+p)} - \frac{2fx\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{2fx}{e+\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}\right)(1+p)}$$

output
```
2*f*x*(c*x^2+b*x)^p*AppellF1(p+1,-p,1,2+p,-c*x/b,-2*f*x/(e-(-4*d*f+e^2)^(1
/2)))/(-4*d*f+e^2)^(1/2)/(e-(-4*d*f+e^2)^(1/2))/(p+1)/((1+c*x/b)^p)-2*f*x*
(c*x^2+b*x)^p*AppellF1(p+1,-p,1,2+p,-c*x/b,-2*f*x/(e+(-4*d*f+e^2)^(1/2)))/
(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))/(p+1)/((1+c*x/b)^p)
```

Mathematica [F]

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx$$

input
```
Integrate[(b*x + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
Integrate[(b*x + c*x^2)^p/(d + e*x + f*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx$$
$$\Big\downarrow \text{ 1325}$$
$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}dx$$

input
```
Int[(b*x + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && !IGtQ[p, 0] && !IGtQ[q, 0
]
```

Maple [F]

$$\int \frac{(c\,x^2+bx)^p}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+e*x+d),x)
```

output
```
int((c*x^2+b*x)^p/(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+bx)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x)**p/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+bx)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + e*x + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+bx)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+b\,x)^p}{f\,x^2+e\,x+d}\,dx$$

input
```
int((b*x + c*x^2)^p/(d + e*x + f*x^2),x)
```

output
```
int((b*x + c*x^2)^p/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^p}{d + ex + fx^2} \, dx = \int \frac{(c\,x^2 + bx)^p}{f\,x^2 + ex + d} dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+e*x+d),x)
```

output
```
int((b*x + c*x**2)**p/(d + e*x + f*x**2),x)
```

3.176 $\int \frac{(bx+cx^2)^p}{d+fx^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{(bx+cx^2)^p}{d+fx^2} \, dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{\sqrt{f}x}{\sqrt{-d}}\right)}{2d(1+p)} + \frac{x\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},\frac{\sqrt{f}x}{\sqrt{-d}}\right)}{2d(1+p)}$$

output
```
1/2*x*(c*x^2+b*x)^p*AppellF1(p+1,-p,1,2+p,-c*x/b,-f^(1/2)*x/(-d)^(1/2))/d/
(p+1)/((1+c*x/b)^p)+1/2*x*(c*x^2+b*x)^p*AppellF1(p+1,1,-p,2+p,f^(1/2)*x/(-
d)^(1/2),-c*x/b)/d/(p+1)/((1+c*x/b)^p)
```

Mathematica [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2} \, dx = \int \frac{(bx+cx^2)^p}{d+fx^2} \, dx$$

input
```
Integrate[(b*x + c*x^2)^p/(d + f*x^2),x]
```

output
```
Integrate[(b*x + c*x^2)^p/(d + f*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{d+fx^2} \, dx$$
$$\Big\downarrow \text{ 1326}$$
$$\int \frac{(bx+cx^2)^p}{d+fx^2} dx$$

input
```
Int[(b*x + c*x^2)^p/(d + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1326
```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_
Symbol] :> Unintegrable[(a + c*x^2)^p*(d + e*x + f*x^2)^q, x] /; FreeQ[{a,
c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Maple [F]

$$\int \frac{(c\,x^2+bx)^p}{f\,x^2+d} dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+d),x)
```

output
```
int((c*x^2+b*x)^p/(f*x^2+d),x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(cx^2+bx)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/(f*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(x(b+cx))^p}{d+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x)**p/(f*x**2+d),x)
```

output
```
Integral((x*(b + c*x))**p/(d + f*x**2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(cx^2+bx)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(cx^2+bx)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(c\,x^2+b\,x)^p}{f\,x^2+d}\,dx$$

input
```
int((b*x + c*x^2)^p/(d + f*x^2),x)
```

output
```
int((b*x + c*x^2)^p/(d + f*x^2), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^p}{d+fx^2}\,dx=\int \frac{(c\,x^2+bx)^p}{f\,x^2+d}dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+d),x)
```

output
```
int((b*x + c*x**2)**p/(d + f*x**2),x)
```

3.177 $\int \frac{(bx+cx^2)^p}{ex+fx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{(bx+cx^2)^p}{ex+fx^2} dx = \frac{\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(p,-p,1,1+p,-\frac{cx}{b},-\frac{fx}{e}\right)}{ep}$$

output
```
(c*x^2+b*x)^p*AppellF1(p,-p,1,p+1,-c*x/b,-f*x/e)/e/p/((1+c*x/b)^p)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \frac{(bx+cx^2)^p}{ex+fx^2} dx = \frac{(x(b+cx))^p\left(1+\frac{cx}{b}\right)^{-p}\left(-fpx \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{fx}{e}\right)+e(1+p) \operatorname{Hypergeometric2F1}\right.}{e^2p(1+p)}$$

input
```
Integrate[(b*x + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
((x*(b + c*x))^p*(-(f*p*x*AppellF1[1 + p, -p, 1, 2 + p, -((c*x)/b), -((f*x
)/e)]) + e*(1 + p)*Hypergeometric2F1[-p, p, 1 + p, -((c*x)/b)]))/(e^2*p*(1
 + p)*(1 + (c*x)/b)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^p}{ex + fx^2} \, dx$$

$$\Big\downarrow \text{1325}$$

$$\int \frac{(bx + cx^2)^p}{ex + fx^2} dx$$

input
```
Int[(b*x + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1325
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Unintegrable[(a + b*x + c*x^2)^p*(d + e*x + f*x^2)
^q, x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0
]
```

Maple [F]

$$\int \frac{(c\,x^2 + bx)^p}{f\,x^2 + ex} dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+e*x),x)
```

output
```
int((c*x^2+b*x)^p/(f*x^2+e*x),x)
```

Fricas [F]

$$\int \frac{(bx + cx^2)^p}{ex + fx^2}\, dx = \int \frac{(cx^2 + bx)^p}{fx^2 + ex}\, dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/(f*x^2 + e*x), x)
```

Sympy [F]

$$\int \frac{(bx + cx^2)^p}{ex + fx^2}\, dx = \int \frac{(x(b + cx))^p}{x\,(e + fx)}\, dx$$

input
```
integrate((c*x**2+b*x)**p/(f*x**2+e*x),x)
```

output
```
Integral((x*(b + c*x))**p/(x*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+bx)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + e*x), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+bx)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(f*x^2+e*x),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(f*x^2 + e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(c\,x^2+b\,x)^p}{f\,x^2+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^p/(e*x + f*x^2),x)
```

output
```
int((b*x + c*x^2)^p/(e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^p}{ex + fx^2}\, dx = \int \frac{(c\,x^2 + bx)^p}{f\,x^2 + ex} dx$$

input
```
int((c*x^2+b*x)^p/(f*x^2+e*x),x)
```

output
```
int((b*x + c*x**2)**p/(e*x + f*x**2),x)
```

3.178 $\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 412

$$\begin{aligned}
\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx
&= -\frac{2fx(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},\frac{4f^2x^2}{\left(e-\sqrt{e^2-4df}\right)^2}\right)}{e^2-4df-e\sqrt{e^2-4df}} \\
&\quad -\frac{2fx(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},\frac{4f^2x^2}{\left(e+\sqrt{e^2-4df}\right)^2}\right)}{e^2-4df+e\sqrt{e^2-4df}} \\
&\quad -\frac{2f^2(a+cx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{4f^2(a+cx^2)}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2}\right)}{\sqrt{e^2-4df}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)(1+p)} \\
&\quad +\frac{2f^2(a+cx^2)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{4f^2(a+cx^2)}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2}\right)}{\sqrt{e^2-4df}\left(4af^2+c\left(e+\sqrt{e^2-4df}\right)^2\right)(1+p)}
\end{aligned}$$

output
```
-2*f*x*(c*x^2+a)^p*AppellF1(1/2,-p,1,3/2,-c*x^2/a,4*f^2*x^2/(e-(-4*d*f+e^2
)^(1/2))^2)/(e^2-4*d*f-e*(-4*d*f+e^2)^(1/2))/((1+c*x^2/a)^p)-2*f*x*(c*x^2+
a)^p*AppellF1(1/2,-p,1,3/2,-c*x^2/a,4*f^2*x^2/(e+(-4*d*f+e^2)^(1/2))^2)/(e
^2-4*d*f+e*(-4*d*f+e^2)^(1/2))/((1+c*x^2/a)^p)-2*f^2*(c*x^2+a)^(p+1)*hyper
geom([1, p+1],[2+p],4*f^2*(c*x^2+a)/(4*a*f^2+c*(e-(-4*d*f+e^2)^(1/2))^2))/
(-4*d*f+e^2)^(1/2)/(4*a*f^2+c*(e-(-4*d*f+e^2)^(1/2))^2)/(p+1)+2*f^2*(c*x^2
+a)^(p+1)*hypergeom([1, p+1],[2+p],4*f^2*(c*x^2+a)/(4*a*f^2+c*(e+(-4*d*f+e
^2)^(1/2))^2))/(-4*d*f+e^2)^(1/2)/(4*a*f^2+c*(e+(-4*d*f+e^2)^(1/2))^2)/(p+
1)
```

Mathematica [F]

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx$$

input
```
Integrate[(a + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
Integrate[(a + c*x^2)^p/(d + e*x + f*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx$$
$$\Big\downarrow \text{ 1326}$$
$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}dx$$

input
```
Int[(a + c*x^2)^p/(d + e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1326
```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_
Symbol] :> Unintegrable[(a + c*x^2)^p*(d + e*x + f*x^2)^q, x] /; FreeQ[{a,
c, d, e, f, p, q}, x] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Maple [F]

$$\int \frac{(c\,x^{2}+a)^{p}}{f\,x^{2}+ex+d}dx$$

input
```
int((c*x^2+a)^p/(f*x^2+e*x+d),x)
```

output
```
int((c*x^2+a)^p/(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx=\int \frac{(cx^2+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x**2+a)**p/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + e*x + d), x)
```

Giac [F]

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+ex+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^p}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + c*x^2)^p/(d + e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^p/(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+cx^2)^p}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^p}{f\,x^2+ex+d}dx$$

input
```
int((c*x^2+a)^p/(f*x^2+e*x+d),x)
```

output
```
int((a + c*x**2)**p/(d + e*x + f*x**2),x)
```

3.179 $\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = -\frac{fx(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},\frac{f^2x^2}{e^2}\right)}{e^2} - \frac{(a+cx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+cx^2}{a}\right)}{2ae(1+p)} + \frac{f^2(a+cx^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{f^2(a+cx^2)}{ce^2+af^2}\right)}{2e\,(ce^2+af^2)\,(1+p)}$$

output

```
-f*x*(c*x^2+a)^p*AppellF1(1/2,1,-p,3/2,f^2*x^2/e^2,-c*x^2/a)/e^2/((1+c*x^2
/a)^p)-1/2*(c*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],(c*x^2+a)/a)/a/e/(p+1)
+1/2*f^2*(c*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],f^2*(c*x^2+a)/(a*f^2+c*e
^2))/e/(a*f^2+c*e^2)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx$$
$$= \frac{(a+cx^2)^p\left(-\left(\frac{f\left(-\sqrt{-\frac{a}{c}}+x\right)}{e+fx}\right)^{-p}\left(\frac{f\left(\sqrt{-\frac{a}{c}}+x\right)}{e+fx}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{e-\sqrt{-\frac{a}{c}}f}{e+fx},\frac{e+\sqrt{-\frac{a}{c}}f}{e+fx}\right)+(\ldots}{2ep}$$

input
```
Integrate[(a + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
((a + c*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (e - Sqrt[-(a/c)]*f)/(e
+ f*x), (e + Sqrt[-(a/c)]*f)/(e + f*x)]/(((f*(-Sqrt[-(a/c)] + x))/(e + f*x
))^p*((f*(Sqrt[-(a/c)] + x))/(e + f*x))^p)) + Hypergeometric2F1[-p, -p, 1
- p, -(a/(c*x^2))]/(1 + a/(c*x^2))^p))/(2*e*p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx$$
$$\Big\downarrow \text{ 1326}$$
$$\int \frac{(a+cx^2)^p}{ex+fx^2}dx$$

input
```
Int[(a + c*x^2)^p/(e*x + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1326
```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_
Symbol] :> Unintegrable[(a + c*x^2)^p*(d + e*x + f*x^2)^q, x] /; FreeQ[{a,
c, d, e, f, p, q}, x] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Maple [F]

$$\int \frac{(c\,x^{2}+a)^{p}}{f\,x^{2}+ex}dx$$

input
```
int((c*x^2+a)^p/(f*x^2+e*x),x)
```

output
```
int((c*x^2+a)^p/(f*x^2+e*x),x)
```

Fricas [F]

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p/(f*x^2 + e*x), x)
```

Sympy [F]

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(a+cx^2)^p}{x\,(e+fx)}\,dx$$

input
```
integrate((c*x**2+a)**p/(f*x**2+e*x),x)
```

output
```
Integral((a + c*x**2)**p/(x*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + e*x), x)
```

Giac [F]

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+ex}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+e*x),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^p}{f\,x^2+e\,x}\,dx$$

input
```
int((a + c*x^2)^p/(e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^p/(e*x + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a+cx^2)^p}{ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^p}{f\,x^2+ex}dx$$

input
```
int((c*x^2+a)^p/(f*x^2+e*x),x)
```

output
```
int((a + c*x**2)**p/(e*x + f*x**2),x)
```

3.180 $\int \frac{(a+cx^2)^p}{d+fx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+cx^2)^p}{d+fx^2} \, dx = \frac{x(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{d}$$

output
```
x*(c*x^2+a)^p*AppellF1(1/2,-p,1,3/2,-c*x^2/a,-f*x^2/d)/d/((1+c*x^2/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+cx^2)^p}{d+fx^2} \, dx =$$

$$-\frac{3adx(a+cx^2)^p \text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{(d+fx^2)\left(-3ad\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(-cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx}{d}\right.\right.\right.}$$

input
```
Integrate[(a + c*x^2)^p/(d + f*x^2),x]
```

output
```
(-3*a*d*x*(a + c*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((c*x^2)/a), -((f*x^2)/
d)])/((d + f*x^2)*(-3*a*d*AppellF1[1/2, -p, 1, 3/2, -((c*x^2)/a), -((f*x^2
)/d)] + 2*x^2*(-(c*d*p*AppellF1[3/2, 1 - p, 1, 5/2, -((c*x^2)/a), -((f*x^2
)/d)]) + a*f*AppellF1[3/2, -p, 2, 5/2, -((c*x^2)/a), -((f*x^2)/d)])))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^p}{d+fx^2}dx$$

$$\Big\downarrow \text{334}$$

$$(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{cx^2}{a}+1\right)^p}{fx^2+d}dx$$

$$\Big\downarrow \text{333}$$

$$\frac{x(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{d}$$

input
```
Int[(a + c*x^2)^p/(d + f*x^2),x]
```

output
```
(x*(a + c*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(d
*(1 + (c*x^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(c\,x^2+a)^p}{f\,x^2+d}dx$$

input
```
int((c*x^2+a)^p/(f*x^2+d),x)
```

output
```
int((c*x^2+a)^p/(f*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p/(f*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^p}{d+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**p/(f*x**2+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a+cx^2)^p}{d+fx^2}\,dx = \int \frac{(cx^2+a)^p}{fx^2+d}\,dx$$

input
```
integrate((c*x^2+a)^p/(f*x^2+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p/(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + cx^2)^p}{d + fx^2} \, dx = \int \frac{(c\,x^2 + a)^p}{f\,x^2 + d} \, dx$$

input
```
int((a + c*x^2)^p/(d + f*x^2),x)
```

output
```
int((a + c*x^2)^p/(d + f*x^2), x)
```

Reduce [F]

$$\int \frac{(a + cx^2)^p}{d + fx^2} \, dx = \int \frac{(c\,x^2 + a)^p}{f\,x^2 + d} dx$$

input
```
int((c*x^2+a)^p/(f*x^2+d),x)
```

output
```
int((a + c*x**2)**p/(d + f*x**2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file