Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.7/108-1.2.1.7-a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**168**]. This is test number [108].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (168)	0.00 (0)
Mathematica	98.21 (165)	1.79 (3)
Maple	98.21 (165)	1.79 (3)
Maxima	80.36 (135)	19.64 (33)
Fricas	75.00 (126)	25.00 (42)
Giac	72.62 (122)	27.38 (46)
Reduce	57.14 (96)	42.86 (72)
Sympy	42.26 (71)	57.74 (97)
Mupad	31.55 (53)	68.45 (115)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

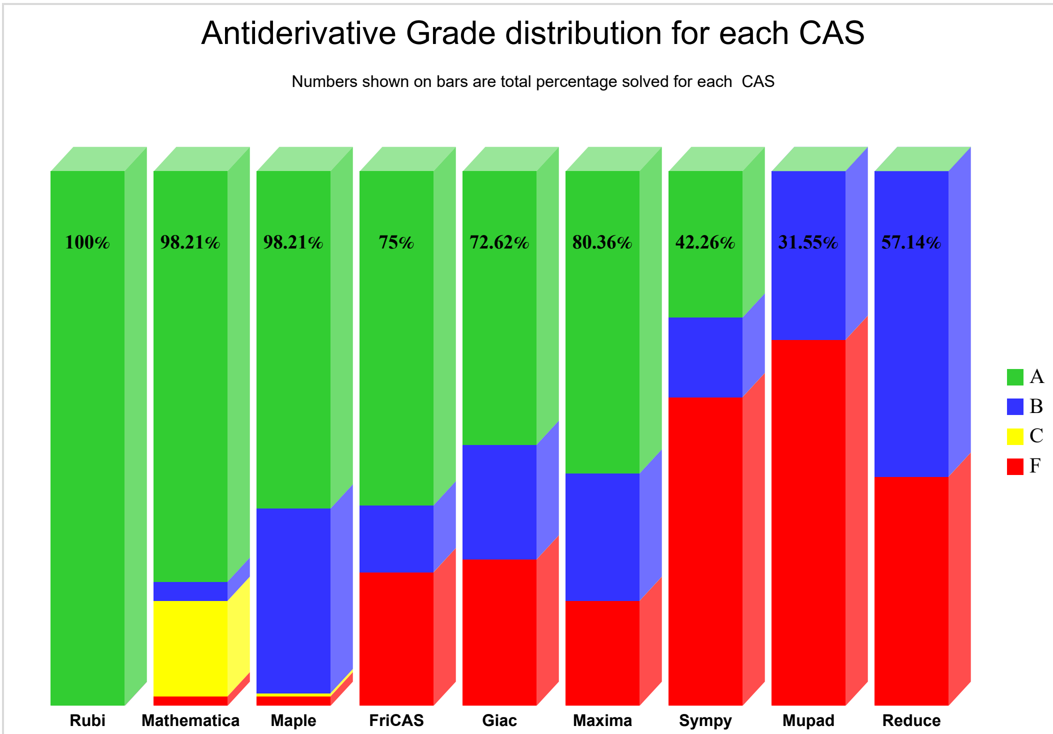
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

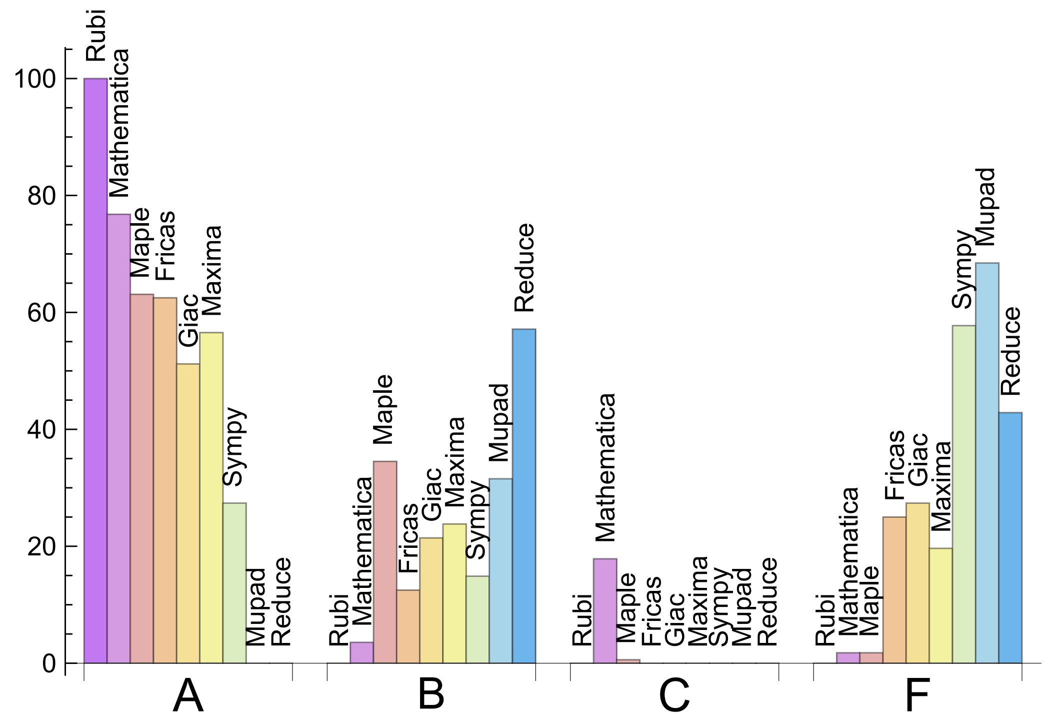
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	76.786	3.571	17.857	1.786
Maple	63.095	34.524	0.595	1.786
Fricas	62.500	12.500	0.000	25.000
Maxima	56.548	23.810	0.000	19.643
Giac	51.190	21.429	0.000	27.381
Sympy	27.381	14.881	0.000	57.738
Mupad	0.000	31.548	0.000	68.452
Reduce	0.000	57.143	0.000	42.857

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Maxima	33	100.00	0.00	0.00
Fricas	42	7.14	92.86	0.00
Giac	46	71.74	15.22	13.04
Reduce	72	100.00	0.00	0.00
Sympy	97	56.70	43.30	0.00
Mupad	115	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.27
Giac	0.38
Rubi	0.94
Maple	2.13
Fricas	2.20
Reduce	2.59
Sympy	5.91
Mathematica	8.39
Mupad	11.22

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	166.15	1.90	110.00	1.16
Rubi	418.32	1.02	379.00	1.00
Sympy	508.83	2.23	303.00	1.52
Mathematica	542.45	1.21	336.00	0.98
Fricas	648.70	2.00	467.00	1.57
Reduce	671.03	2.74	283.00	1.79
Giac	1928.61	3.16	289.00	1.29
Maxima	4402.69	5.63	325.00	1.17
Maple	6695.20	7.53	449.00	1.22

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

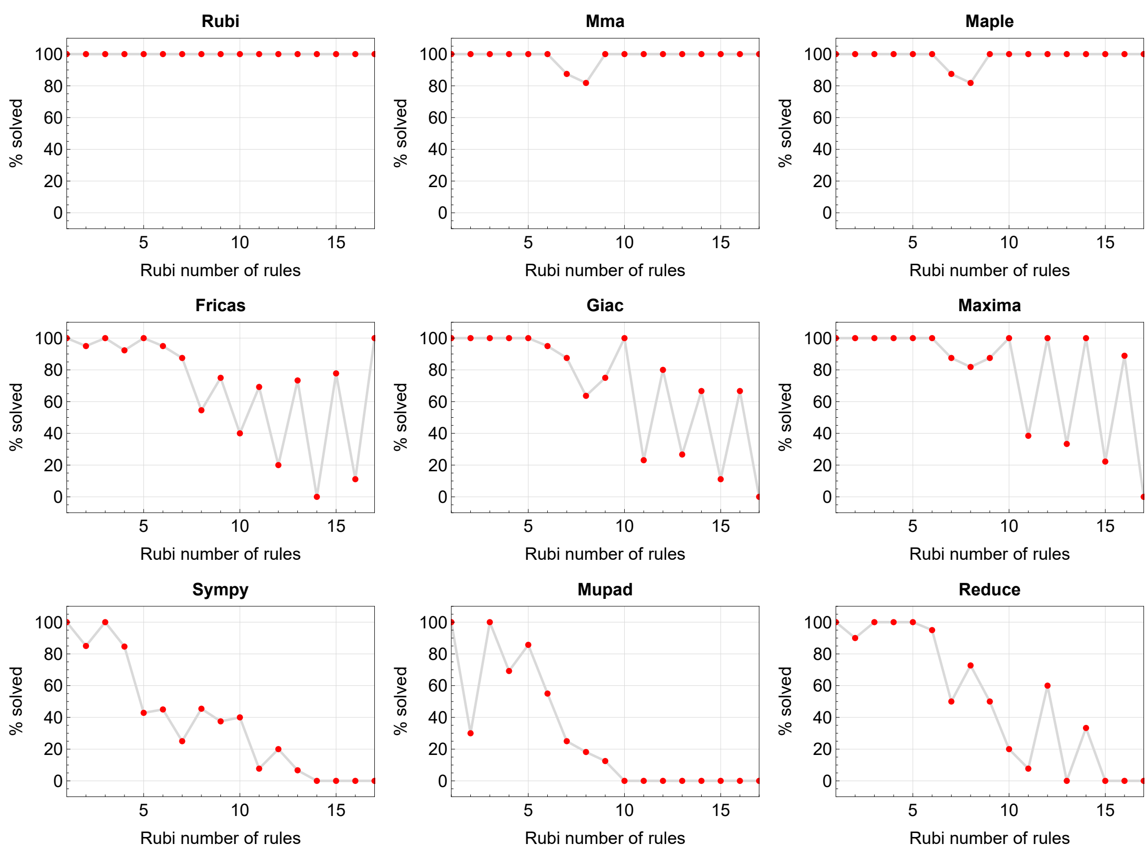


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

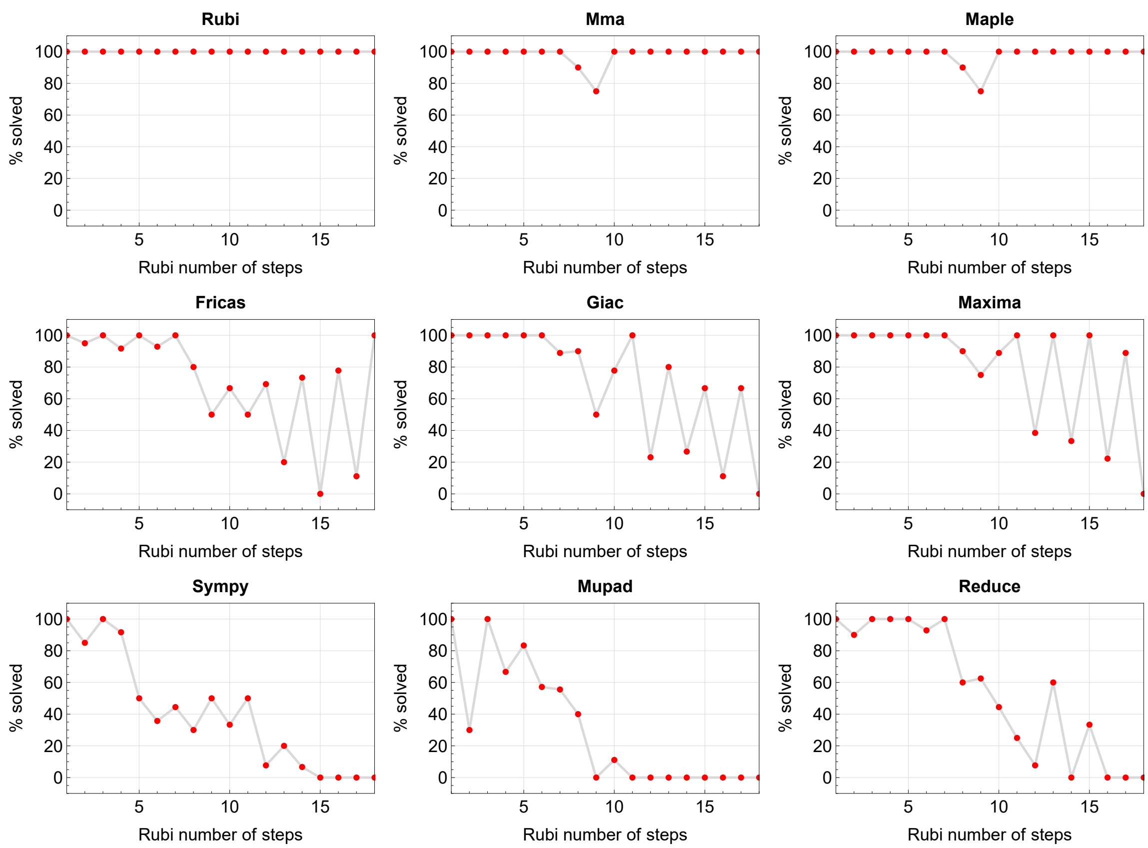


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

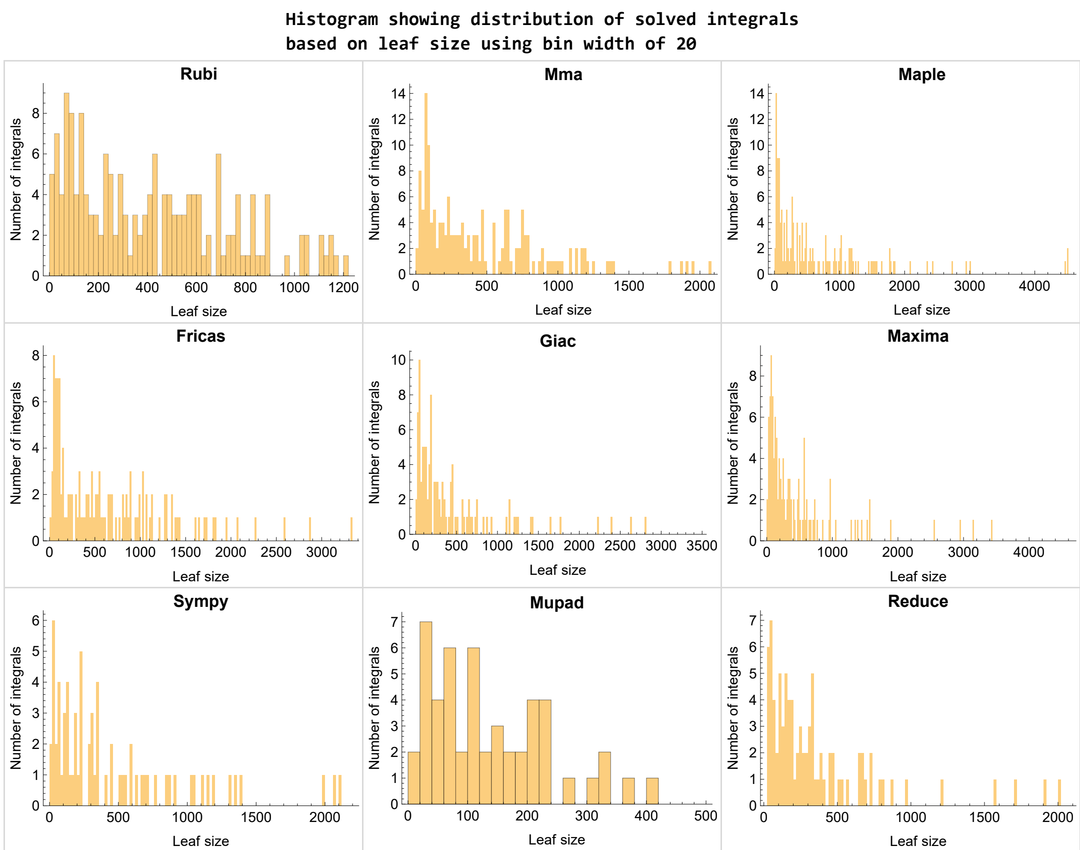


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

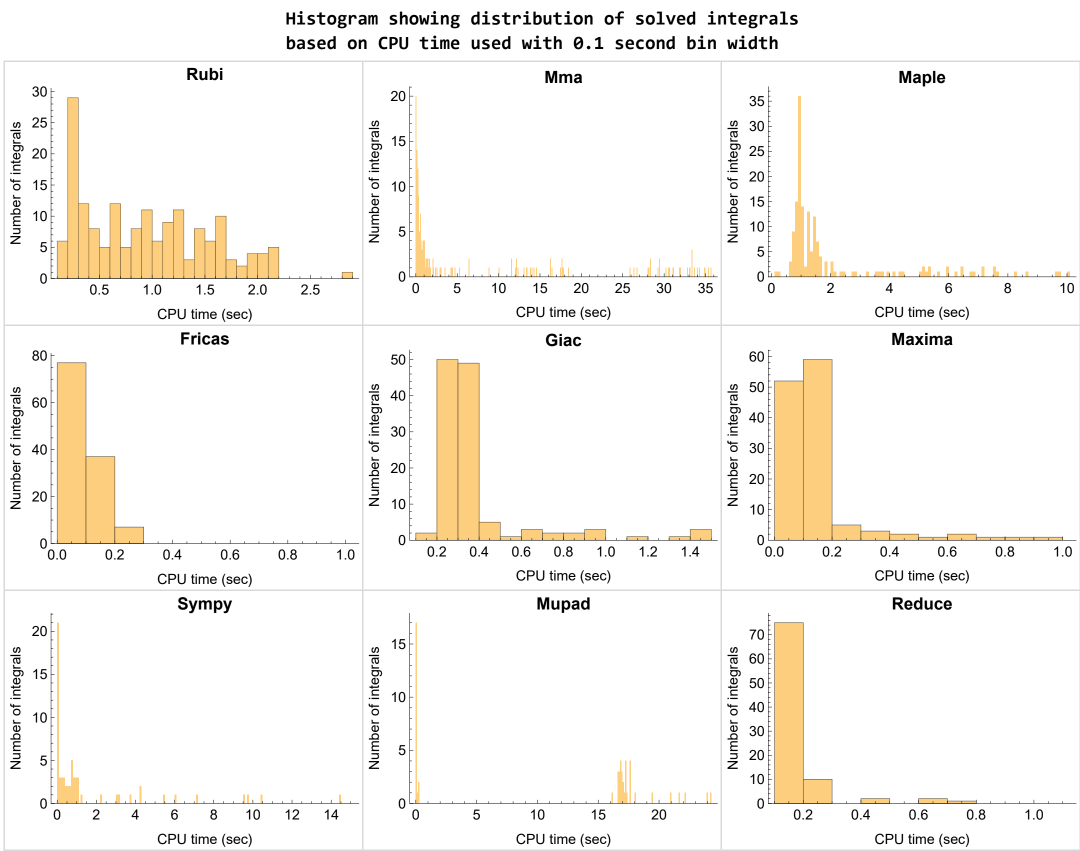


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

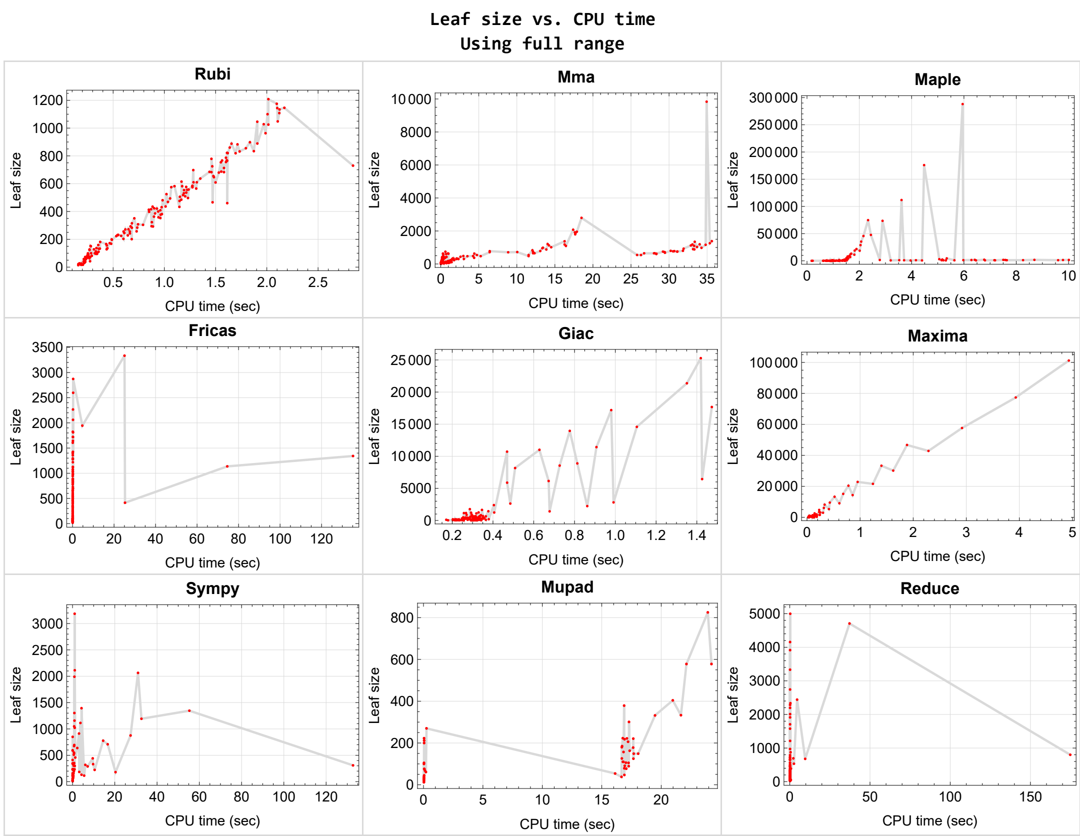


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {140}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

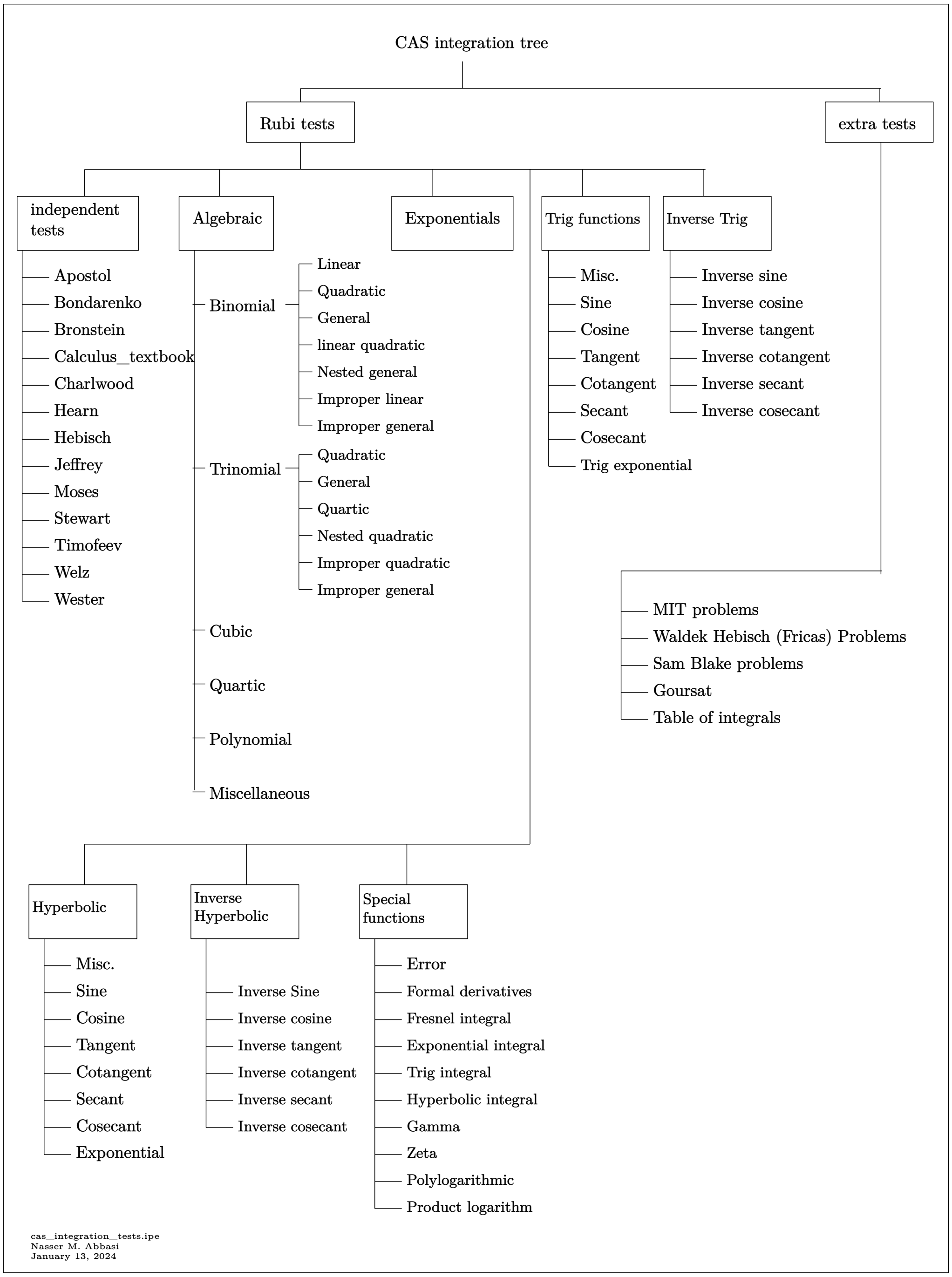
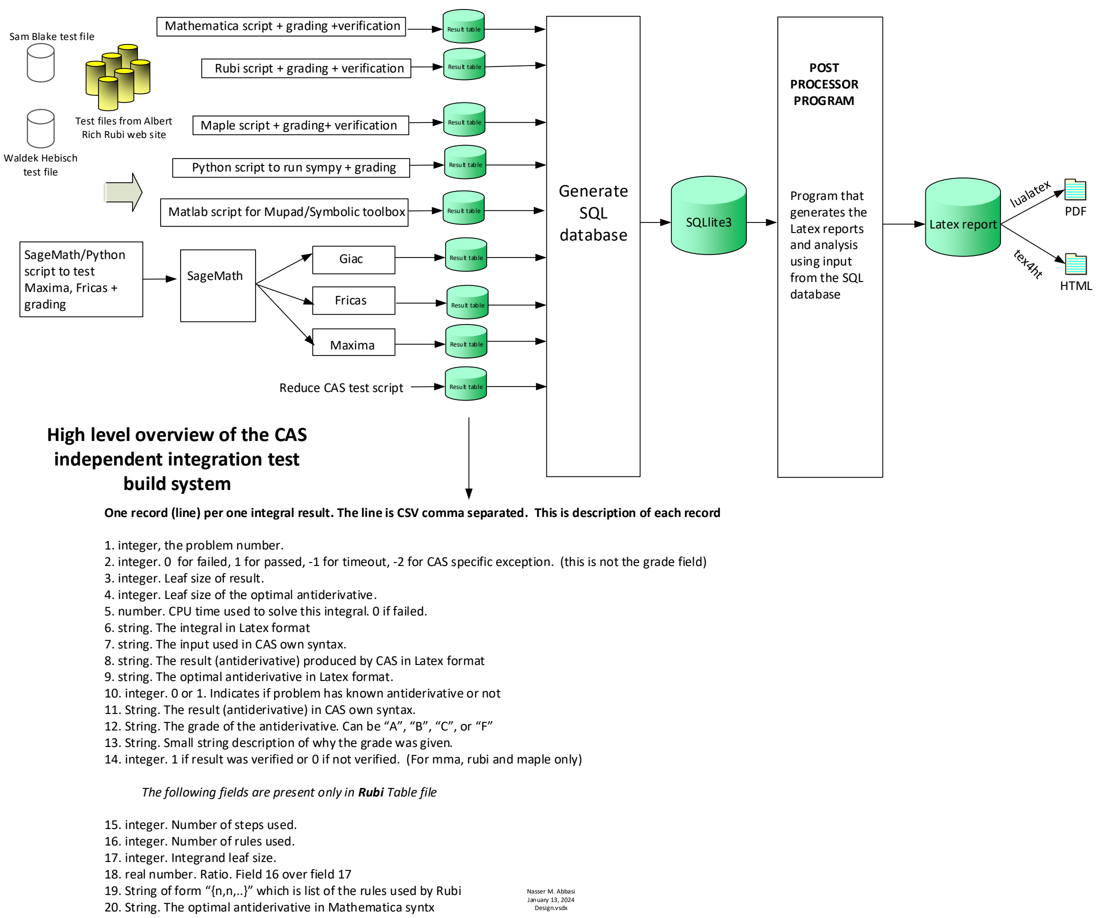


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135 }

B grade { 22, 23, 24, 25, 98, 100 }

C grade { 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

F normal fail { 166, 167, 168 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 71, 72, 73, 74, 75, 85, 86, 87, 88, 89, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 137, 140, 146, 149, 150, 152, 153 }

B grade { 24, 25, 49, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 106, 107, 112, 113, 114, 136, 138, 139, 141, 142, 143, 144, 145, 147, 148, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165 }

C grade { 133 }

F normal fail { 166, 167, 168 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 26, 27, 28, 29, 30, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 85, 86, 87, 88, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 162 }

B grade { 14, 21, 22, 23, 24, 25, 31, 37, 113, 114, 140, 145, 146, 153, 158, 159, 160, 161, 163, 164, 165 }

C grade { }

F normal fail { 166, 167, 168 }

F(-1) timedout fail { 32, 38, 43, 44, 49, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 105, 106, 107, 112 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 71, 72, 73, 74, 85, 86, 87, 88, 90, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135 }

B grade { 22, 23, 24, 25, 44, 49, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 105, 106, 107, 112, 113, 114 }

C grade { }

F normal fail { 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 85, 86, 87, 88, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 130, 131, 132, 133, 135 }

B grade { 22, 23, 24, 25, 43, 44, 49, 65, 66, 68, 69, 70, 77, 78, 80, 81, 82, 83, 84, 91, 92, 94, 95, 96, 97, 98, 99, 100, 107, 114, 121, 122, 123, 128, 129, 134 }

C grade { }

F normal fail { 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168 }

F(-1) timedout fail { 64, 76, 79, 90, 93, 106, 113 }

F(-2) exception fail { 63, 67, 75, 89, 105, 112 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 15, 16, 17, 18, 22, 23, 24, 25, 29, 36, 42, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 104, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 12, 13, 14, 19, 20, 21, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 41, 42, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 126 }

B grade { 22, 23, 24, 25, 26, 27, 28, 29, 33, 34, 35, 36, 59, 60, 71, 72, 73, 74, 85, 86, 87, 88, 116, 117, 132 }

C grade { }

F normal fail { 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 105, 106, 107, 108, 109, 112, 113, 121, 122, 123, 124, 125, 127, 128, 130, 131, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 166 }

F(-1) timedout fail { 30, 31, 32, 37, 38, 39, 40, 43, 44, 45, 46, 47, 49, 69, 70, 81, 82, 83, 84, 95, 96, 97, 98, 99, 100, 114, 129, 133, 134, 135, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 73, 74, 76, 77, 87, 88, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135 }

C grade { }

F normal fail { 20, 21, 45, 49, 59, 60, 67, 68, 69, 70, 71, 72, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 114, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	270	278	256	256	340	332	273	332
N.S.	1	1.00	1.05	1.09	1.00	1.00	1.33	1.30	1.07	1.30
time (sec)	N/A	0.663	0.163	0.711	0.029	0.068	0.037	0.357	0.164	19.479

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	256	194	191	180	180	231	229	195	206
N.S.	1	1.37	1.04	1.02	0.96	0.96	1.24	1.22	1.04	1.10
time (sec)	N/A	0.602	0.101	0.632	0.037	0.068	0.032	0.323	0.163	17.211

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	104	104	128	126	117	126
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.16	1.15	1.06	1.15
time (sec)	N/A	0.358	0.056	0.217	0.027	0.060	0.022	0.342	0.159	16.689

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	56	54	57	54
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.93	0.90	0.95	0.90
time (sec)	N/A	0.237	0.015	0.177	0.028	0.061	0.019	0.324	0.159	16.123

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	193	252	238	239	238	292	140	0
N.S.	1	1.00	0.87	1.13	1.07	1.07	1.07	1.31	0.63	0.00
time (sec)	N/A	0.531	0.250	0.887	0.034	0.067	0.365	0.343	0.160	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	221	276	247	360	298	364	334	0
N.S.	1	1.00	0.96	1.20	1.07	1.57	1.30	1.58	1.45	0.00
time (sec)	N/A	0.542	0.240	0.870	0.044	0.065	1.047	0.276	0.161	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	222	267	257	402	318	277	381	379
N.S.	1	1.00	0.96	1.15	1.11	1.73	1.37	1.19	1.64	1.63
time (sec)	N/A	0.550	0.177	0.872	0.039	0.070	6.062	0.351	0.153	16.881

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	400	426	486	460	460	590	578	464	578
N.S.	1	0.88	0.94	1.07	1.01	1.01	1.30	1.27	1.02	1.27
time (sec)	N/A	0.940	0.232	0.900	0.038	0.065	0.048	0.314	0.152	22.124

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	306	343	330	330	411	404	335	404
N.S.	1	1.00	1.08	1.21	1.16	1.16	1.45	1.42	1.18	1.42
time (sec)	N/A	0.735	0.169	0.741	0.036	0.062	0.040	0.355	0.161	20.974

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	168	184	193	200	200	236	230	206	221
N.S.	1	1.10	1.20	1.26	1.31	1.31	1.54	1.50	1.35	1.44
time (sec)	N/A	0.481	0.084	0.791	0.029	0.061	0.031	0.325	0.168	17.651

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	98	107	102	103	105
N.S.	1	1.00	0.89	1.00	0.99	0.99	1.08	1.03	1.04	1.06
time (sec)	N/A	0.321	0.054	0.615	0.043	0.059	0.024	0.365	0.156	17.251

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	366	570	545	546	552	680	319	0
N.S.	1	1.00	0.84	1.31	1.25	1.25	1.27	1.56	0.73	0.00
time (sec)	N/A	1.012	0.291	0.997	0.037	0.070	0.675	0.351	0.163	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	479	598	560	777	638	748	693	0
N.S.	1	1.00	1.14	1.42	1.33	1.85	1.52	1.78	1.65	0.00
time (sec)	N/A	0.927	0.325	1.125	0.039	0.082	2.243	0.340	2.204	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	407	589	571	863	709	641	800	0
N.S.	1	1.00	0.96	1.39	1.35	2.04	1.68	1.52	1.89	0.00
time (sec)	N/A	0.901	0.168	0.983	0.043	0.078	16.636	0.310	174.814	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	695	536	583	695	652	652	850	825	655	825
N.S.	1	0.77	0.84	1.00	0.94	0.94	1.22	1.19	0.94	1.19
time (sec)	N/A	1.214	0.331	0.757	0.036	0.066	0.058	0.341	0.149	23.921

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	377	418	491	472	472	595	578	475	578
N.S.	1	0.90	1.00	1.17	1.13	1.13	1.42	1.38	1.14	1.38
time (sec)	N/A	0.964	0.241	0.750	0.035	0.067	0.047	0.289	0.152	24.229

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	225	250	285	290	290	347	333	295	333
N.S.	1	1.18	1.32	1.50	1.53	1.53	1.83	1.75	1.55	1.75
time (sec)	N/A	0.578	0.138	0.631	0.037	0.064	0.036	0.307	0.147	21.660

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	121	144	142	142	158	149	149	149
N.S.	1	1.00	0.91	1.08	1.07	1.07	1.19	1.12	1.12	1.12
time (sec)	N/A	0.371	0.093	0.718	0.031	0.058	0.028	0.316	0.145	17.688

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	683	602	1007	942	943	1022	1212	566	0
N.S.	1	1.00	0.88	1.47	1.38	1.38	1.50	1.77	0.83	0.00
time (sec)	N/A	1.582	0.704	0.962	0.044	0.076	1.029	0.292	0.154	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	683	745	1039	961	1278	1114	1252	34	0
N.S.	1	1.00	1.09	1.52	1.41	1.87	1.63	1.83	0.05	0.00
time (sec)	N/A	1.459	0.610	0.960	0.046	0.087	3.731	0.329	200.019	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	634	1030	972	1414	1193	1149	34	0
N.S.	1	1.00	0.97	1.57	1.48	2.16	1.82	1.75	0.05	0.00
time (sec)	N/A	1.479	0.286	0.970	0.055	0.086	32.611	0.257	200.026	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	27	72	71	63	103	33	76
N.S.	1	1.00	3.76	1.59	4.24	4.18	3.71	6.06	1.94	4.47
time (sec)	N/A	0.204	0.030	0.928	0.042	0.061	0.161	0.276	0.157	0.076

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	27	72	71	63	103	33	76
N.S.	1	1.00	3.76	1.59	4.24	4.18	3.71	6.06	1.94	4.47
time (sec)	N/A	0.185	0.013	0.882	0.043	0.062	0.163	0.329	0.171	0.058

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	96	38	137	107	128	195	44	223
N.S.	1	1.00	5.65	2.24	8.06	6.29	7.53	11.47	2.59	13.12
time (sec)	N/A	0.205	0.039	1.017	0.034	0.062	0.262	0.317	0.613	16.765

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	96	38	137	107	128	195	44	223
N.S.	1	1.00	5.65	2.24	8.06	6.29	7.53	11.47	2.59	13.12
time (sec)	N/A	0.193	0.020	1.000	0.038	0.066	0.255	0.317	0.172	0.049

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	313	428	376	884	1394	436	459	0
N.S.	1	1.00	0.89	1.22	1.07	2.52	3.97	1.24	1.31	0.00
time (sec)	N/A	0.707	0.393	1.016	0.117	0.096	4.246	0.264	0.159	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	205	239	227	558	913	268	293	0
N.S.	1	1.00	0.89	1.04	0.99	2.43	3.97	1.17	1.27	0.00
time (sec)	N/A	0.545	0.208	0.974	0.117	0.091	3.064	0.266	0.166	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	130	129	334	457	136	182	0
N.S.	1	1.00	0.93	0.98	0.98	2.53	3.46	1.03	1.38	0.00
time (sec)	N/A	0.361	0.122	0.862	0.121	0.088	1.294	0.335	0.165	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	65	64	157	219	66	79	79
N.S.	1	1.00	0.93	0.89	0.88	2.15	3.00	0.90	1.08	1.08
time (sec)	N/A	0.249	0.043	0.856	0.133	0.070	0.408	0.324	0.157	16.896

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	168	155	166	414	0	162	177	0
N.S.	1	1.00	1.01	0.93	1.00	2.49	0.00	0.98	1.07	0.00
time (sec)	N/A	0.433	0.166	1.231	0.135	25.281	0.000	0.338	0.160	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	243	263	317	1136	0	357	726	0
N.S.	1	1.00	0.89	0.97	1.17	4.18	0.00	1.31	2.67	0.00
time (sec)	N/A	0.650	0.284	1.253	0.185	74.553	0.000	0.313	0.158	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	367	423	569	0	0	638	1577	0
N.S.	1	1.00	0.92	1.06	1.43	0.00	0.00	1.60	3.96	0.00
time (sec)	N/A	0.851	0.570	1.260	0.137	0.000	0.000	0.365	0.161	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	294	344	421	401	1394	1346	429	789	0
N.S.	1	0.76	0.89	1.09	1.03	3.59	3.47	1.11	2.03	0.00
time (sec)	N/A	0.875	0.250	0.998	0.117	0.096	55.296	0.352	0.176	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	201	223	261	272	885	876	277	514	0
N.S.	1	0.77	0.86	1.00	1.05	3.40	3.37	1.07	1.98	0.00
time (sec)	N/A	0.682	0.199	0.989	0.114	0.086	27.459	0.321	0.166	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	135	143	147	156	520	442	144	318	0
N.S.	1	0.84	0.89	0.92	0.98	3.25	2.76	0.90	1.99	0.00
time (sec)	N/A	0.444	0.117	0.850	0.125	0.086	9.558	0.251	0.167	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	99	83	88	89	257	233	88	156	110
N.S.	1	0.96	0.81	0.85	0.86	2.50	2.26	0.85	1.51	1.07
time (sec)	N/A	0.254	0.096	0.849	0.124	0.076	0.956	0.297	0.163	16.816

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	308	247	394	349	1342	0	447	728	0
N.S.	1	1.08	0.87	1.39	1.23	4.73	0.00	1.57	2.56	0.00
time (sec)	N/A	0.743	0.307	1.231	0.126	135.151	0.000	0.300	0.162	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	466	402	547	730	0	0	738	2196	0
N.S.	1	1.02	0.88	1.20	1.60	0.00	0.00	1.62	4.83	0.00
time (sec)	N/A	1.470	0.559	1.251	0.128	0.000	0.000	0.299	0.175	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	292	385	475	500	1606	0	504	972	0
N.S.	1	0.66	0.87	1.07	1.13	3.63	0.00	1.14	2.20	0.00
time (sec)	N/A	0.889	0.348	1.030	0.118	0.100	0.000	0.328	0.163	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	228	275	307	357	1074	0	330	678	0
N.S.	1	0.72	0.87	0.97	1.13	3.41	0.00	1.05	2.15	0.00
time (sec)	N/A	0.643	0.266	0.960	0.122	0.098	0.000	0.337	0.163	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	172	171	169	207	646	309	194	452	0
N.S.	1	0.88	0.88	0.87	1.06	3.31	1.58	0.99	2.32	0.00
time (sec)	N/A	0.466	0.117	0.879	0.112	0.088	132.729	0.330	0.166	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	127	104	98	122	346	184	106	222	163
N.S.	1	1.02	0.84	0.79	0.98	2.79	1.48	0.85	1.79	1.31
time (sec)	N/A	0.269	0.127	0.848	0.112	0.075	3.176	0.334	0.165	17.347

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	518	426	806	775	0	0	935	1706	0
N.S.	1	1.10	0.90	1.71	1.64	0.00	0.00	1.98	3.61	0.00
time (sec)	N/A	1.183	0.763	1.234	0.153	0.000	0.000	0.295	0.170	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	730	623	1095	1440	0	0	1435	4707	0
N.S.	1	1.05	0.90	1.58	2.07	0.00	0.00	2.07	6.78	0.00
time (sec)	N/A	2.841	1.015	1.215	0.163	0.000	0.000	0.377	37.229	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	381	447	538	592	1824	0	662	34	0
N.S.	1	0.73	0.85	1.03	1.13	3.49	0.00	1.27	0.07	0.00
time (sec)	N/A	0.976	0.316	0.980	0.134	0.119	0.000	0.334	200.018	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	304	340	351	430	1294	0	458	874	0
N.S.	1	0.81	0.90	0.93	1.14	3.44	0.00	1.22	2.32	0.00
time (sec)	N/A	0.792	0.332	0.980	0.129	0.101	0.000	0.317	0.494	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	197	213	202	263	844	0	247	647	0
N.S.	1	0.82	0.88	0.84	1.09	3.50	0.00	1.02	2.68	0.00
time (sec)	N/A	0.488	0.141	0.878	0.122	0.093	0.000	0.310	0.141	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	152	127	116	157	462	223	131	320	222
N.S.	1	0.94	0.79	0.72	0.98	2.87	1.39	0.81	1.99	1.38
time (sec)	N/A	0.280	0.159	0.866	0.108	0.084	10.459	0.295	0.144	17.116

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	701	765	639	1441	1390	0	0	1645	34	0
N.S.	1	1.09	0.91	2.06	1.98	0.00	0.00	2.35	0.05	0.00
time (sec)	N/A	1.613	0.963	1.238	0.187	0.000	0.000	0.346	200.024	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	38	29	30	29	46	29	29	55	30
N.S.	1	0.88	0.67	0.70	0.67	1.07	0.67	0.67	1.28	0.70
time (sec)	N/A	0.228	0.026	0.984	0.106	0.060	0.070	0.343	0.641	0.036

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	30	27	24	23	40	20	23	51	23
N.S.	1	1.11	1.00	0.89	0.85	1.48	0.74	0.85	1.89	0.85
time (sec)	N/A	0.219	0.015	0.923	0.110	0.062	0.060	0.319	0.164	0.037

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	26	23	24	23	33	20	23	38	25
N.S.	1	0.90	0.79	0.83	0.79	1.14	0.69	0.79	1.31	0.86
time (sec)	N/A	0.189	0.014	0.914	0.119	0.065	0.071	0.292	0.168	0.033

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	10	12	25	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.79	1.00
time (sec)	N/A	0.160	0.009	0.889	0.121	0.061	0.070	0.304	0.160	0.038

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	28	26	25	41	24	26	50	32
N.S.	1	1.03	0.90	0.84	0.81	1.32	0.77	0.84	1.61	1.03
time (sec)	N/A	0.215	0.015	1.007	0.106	0.066	0.070	0.299	0.165	0.045

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	49	31	35	68	38
N.S.	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	2.06	1.15
time (sec)	N/A	0.222	0.021	1.052	0.105	0.064	0.075	0.233	0.156	16.659

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	42	39	38	41	61	42	43	73	47
N.S.	1	0.86	0.80	0.78	0.84	1.24	0.86	0.88	1.49	0.96
time (sec)	N/A	0.275	0.026	1.185	0.106	0.073	0.076	0.300	0.166	16.900

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	12	18	8	12	20	12
N.S.	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.67	1.00
time (sec)	N/A	0.161	0.029	0.905	0.111	0.061	0.046	0.257	0.174	0.033

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	27	27	21	21	25	20	21	34	21
N.S.	1	0.84	0.84	0.66	0.66	0.78	0.62	0.66	1.06	0.66
time (sec)	N/A	0.171	0.018	1.039	0.105	0.061	0.081	0.312	0.161	0.038

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	531	482	481	609	1219	1049	657	34	0
N.S.	1	1.06	0.96	0.96	1.21	2.43	2.09	1.31	0.07	0.00
time (sec)	N/A	1.190	2.971	1.349	0.040	0.145	0.764	0.315	200.025	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	386	330	342	407	803	675	434	34	0
N.S.	1	1.11	0.95	0.98	1.17	2.30	1.93	1.24	0.10	0.00
time (sec)	N/A	0.910	1.690	1.233	0.039	0.119	0.732	0.274	200.024	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	271	197	251	255	489	348	247	388	0
N.S.	1	1.22	0.88	1.13	1.14	2.19	1.56	1.11	1.74	0.00
time (sec)	N/A	0.619	1.155	1.028	0.037	0.110	0.703	0.287	0.746	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	108	148	130	231	160	121	183	0
N.S.	1	1.04	0.82	1.12	0.98	1.75	1.21	0.92	1.39	0.00
time (sec)	N/A	0.337	0.535	0.923	0.035	0.090	0.474	0.304	0.167	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	323	291	497	577	0	0	0	2335	0
N.S.	1	1.00	0.90	1.53	1.78	0.00	0.00	0.00	7.21	0.00
time (sec)	N/A	0.878	1.481	1.431	0.103	0.000	0.000	0.000	0.400	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	433	299	937	713	0	0	0	1913	0
N.S.	1	1.31	0.91	2.84	2.16	0.00	0.00	0.00	5.80	0.00
time (sec)	N/A	1.147	2.062	1.520	0.096	0.000	0.000	0.000	0.191	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	565	370	1786	1351	0	0	1235	3332	0
N.S.	1	1.49	0.98	4.71	3.56	0.00	0.00	3.26	8.79	0.00
time (sec)	N/A	1.311	4.387	1.454	0.109	0.000	0.000	0.406	0.176	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	485	543	2734	2553	0	0	2390	3915	0
N.S.	1	1.08	1.21	6.09	5.69	0.00	0.00	5.32	8.72	0.00
time (sec)	N/A	1.165	11.544	1.543	0.143	0.000	0.000	0.404	0.193	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	610	651	4464	4783	0	0	0	34	0
N.S.	1	1.13	1.20	8.24	8.82	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.285	12.324	1.579	0.229	0.000	0.000	0.000	200.024	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	509	746	7010	8174	0	0	5885	34	0
N.S.	1	0.97	1.42	13.35	15.57	0.00	0.00	11.21	0.06	0.00
time (sec)	N/A	1.203	12.126	1.653	0.332	0.000	0.000	0.469	200.023	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	707	685	954	11286	13292	0	0	8165	34	0
N.S.	1	0.97	1.35	15.96	18.80	0.00	0.00	11.55	0.05	0.00
time (sec)	N/A	1.440	13.349	1.810	0.519	0.000	0.000	0.508	200.027	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	933	898	1207	18659	20401	0	0	10719	34	0
N.S.	1	0.96	1.29	20.00	21.87	0.00	0.00	11.49	0.04	0.00
time (sec)	N/A	1.835	14.047	2.014	0.780	0.000	0.000	0.469	200.029	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	554	627	545	727	1643	1991	878	34	0
N.S.	1	0.94	1.06	0.92	1.23	2.78	3.37	1.49	0.06	0.00
time (sec)	N/A	1.192	4.413	1.341	0.040	0.181	0.882	0.271	200.025	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	409	438	390	489	1119	1302	596	34	0
N.S.	1	0.98	1.05	0.94	1.17	2.68	3.12	1.43	0.08	0.00
time (sec)	N/A	0.961	2.594	1.227	0.040	0.146	0.868	0.304	200.017	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	293	263	299	317	689	695	349	533	0
N.S.	1	1.08	0.97	1.10	1.17	2.53	2.56	1.28	1.96	0.00
time (sec)	N/A	0.670	1.613	1.020	0.037	0.122	0.816	0.309	2.588	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	159	141	180	165	327	303	180	261	0
N.S.	1	0.98	0.87	1.10	1.01	2.01	1.86	1.10	1.60	0.00
time (sec)	N/A	0.355	0.848	0.951	0.043	0.090	0.505	0.310	0.173	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	497	470	796	971	0	0	0	34	0
N.S.	1	0.99	0.94	1.59	1.94	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.231	3.501	1.408	0.171	0.000	0.000	0.000	200.025	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	609	479	1540	1043	0	0	0	2010	0
N.S.	1	1.13	0.89	2.85	1.93	0.00	0.00	0.00	3.72	0.00
time (sec)	N/A	1.500	4.245	1.450	0.158	0.000	0.000	0.000	0.199	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	819	472	2944	1889	0	0	1414	4155	0
N.S.	1	1.49	0.86	5.37	3.45	0.00	0.00	2.58	7.58	0.00
time (sec)	N/A	1.695	5.247	1.487	0.179	0.000	0.000	0.678	0.180	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	855	650	5334	3437	0	0	2628	34	0
N.S.	1	1.42	1.08	8.85	5.70	0.00	0.00	4.36	0.06	0.00
time (sec)	N/A	1.796	12.137	1.513	0.229	0.000	0.000	0.485	200.028	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	768	785	8858	6053	0	0	0	34	0
N.S.	1	1.12	1.15	12.93	8.84	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.610	13.077	1.618	0.318	0.000	0.000	0.000	200.026	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	787	878	12094	9508	0	0	6144	34	0
N.S.	1	1.05	1.17	16.10	12.66	0.00	0.00	8.18	0.05	0.00
time (sec)	N/A	1.601	13.874	1.663	0.430	0.000	0.000	0.673	200.025	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	888	1013	18854	15131	0	0	8545	34	0
N.S.	1	1.11	1.27	23.57	18.91	0.00	0.00	10.68	0.04	0.00
time (sec)	N/A	1.655	14.513	1.823	0.682	0.000	0.000	0.727	200.029	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	574	1125	28850	22855	0	0	11003	34	0
N.S.	1	0.88	1.73	44.45	35.22	0.00	0.00	16.95	0.05	0.00
time (sec)	N/A	1.274	14.219	2.046	0.954	0.000	0.000	0.628	200.027	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	756	1375	45606	33308	0	0	13955	34	0
N.S.	1	0.88	1.61	53.28	38.91	0.00	0.00	16.30	0.04	0.00
time (sec)	N/A	1.601	16.272	2.158	1.402	0.000	0.000	0.777	200.022	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1103	963	2075	74819	46712	0	0	17191	34	0
N.S.	1	0.87	1.88	67.83	42.35	0.00	0.00	15.59	0.03	0.00
time (sec)	N/A	1.987	17.400	2.335	1.884	0.000	0.000	0.980	200.027	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	577	776	609	844	2061	3186	1100	34	0
N.S.	1	0.85	1.14	0.89	1.24	3.02	4.67	1.61	0.05	0.00
time (sec)	N/A	1.213	6.435	1.408	0.051	0.226	0.996	0.318	200.024	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	431	543	438	571	1425	2113	757	34	0
N.S.	1	0.89	1.12	0.90	1.18	2.94	4.36	1.56	0.07	0.00
time (sec)	N/A	0.973	4.750	1.251	0.040	0.189	1.055	0.298	200.021	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	315	327	347	379	885	1158	450	678	0
N.S.	1	0.98	1.02	1.08	1.18	2.76	3.61	1.40	2.11	0.00
time (sec)	N/A	0.684	2.076	1.020	0.039	0.151	0.918	0.266	9.595	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	181	172	212	200	425	510	240	339	0
N.S.	1	0.93	0.89	1.09	1.03	2.19	2.63	1.24	1.75	0.00
time (sec)	N/A	0.374	1.047	1.088	0.035	0.103	0.591	0.302	0.209	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	713	715	1196	1563	0	0	0	34	0
N.S.	1	0.98	0.98	1.65	2.15	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.582	6.427	1.368	0.236	0.000	0.000	0.000	200.018	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	826	834	722	2345	1565	0	0	0	34	0
N.S.	1	1.01	0.87	2.84	1.89	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.873	10.054	1.426	0.242	0.000	0.000	0.000	200.026	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	803	1048	708	4506	2940	0	0	2233	34	0
N.S.	1	1.31	0.88	5.61	3.66	0.00	0.00	2.78	0.04	0.00
time (sec)	N/A	2.106	8.860	1.545	0.302	0.000	0.000	0.862	200.018	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	806	1144	829	8090	5303	0	0	2810	34	0
N.S.	1	1.42	1.03	10.04	6.58	0.00	0.00	3.49	0.04	0.00
time (sec)	N/A	2.102	12.045	1.553	0.413	0.000	0.000	0.991	200.024	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	864	1175	986	13261	9069	0	0	0	34	0
N.S.	1	1.36	1.14	15.35	10.50	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.097	13.427	1.691	0.610	0.000	0.000	0.000	200.028	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	962	1108	1095	21714	14351	0	0	6440	34	0
N.S.	1	1.15	1.14	22.57	14.92	0.00	0.00	6.69	0.04	0.00
time (sec)	N/A	2.122	16.447	1.891	0.860	0.000	0.000	1.426	200.020	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1079	1146	1179	35338	21589	0	0	8884	34	0
N.S.	1	1.06	1.09	32.75	20.01	0.00	0.00	8.23	0.03	0.00
time (sec)	N/A	2.171	16.276	2.072	1.242	0.000	0.000	0.814	200.026	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1134	1795	47638	30152	0	0	11425	34	0
N.S.	1	1.07	1.69	44.77	28.34	0.00	0.00	10.74	0.03	0.00
time (sec)	N/A	2.124	17.669	2.443	1.624	0.000	0.000	0.908	200.026	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1102	1208	1916	73562	42845	0	0	14580	34	0
N.S.	1	1.10	1.74	66.75	38.88	0.00	0.00	13.23	0.03	0.00
time (sec)	N/A	2.015	17.623	2.888	2.288	0.000	0.000	1.106	200.024	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	637	1955	111786	57646	0	0	17669	34	0
N.S.	1	0.83	2.55	145.74	75.16	0.00	0.00	23.04	0.04	0.00
time (sec)	N/A	1.351	17.889	3.614	2.920	0.000	0.000	1.473	200.028	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	997	820	1860	175934	77360	0	0	21367	34	0
N.S.	1	0.82	1.87	176.46	77.59	0.00	0.00	21.43	0.03	0.00
time (sec)	N/A	1.618	17.701	4.477	3.932	0.000	0.000	1.351	200.033	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1272	1028	2794	288083	101201	0	0	25283	34	0
N.S.	1	0.81	2.20	226.48	79.56	0.00	0.00	19.88	0.03	0.00
time (sec)	N/A	1.970	18.499	5.950	4.931	0.000	0.000	1.419	200.048	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	506	336	407	493	843	536	448	34	0
N.S.	1	1.23	0.82	0.99	1.20	2.05	1.30	1.09	0.08	0.00
time (sec)	N/A	1.152	1.574	1.333	0.037	0.138	0.746	0.286	200.030	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	363	226	287	325	531	354	286	34	0
N.S.	1	1.30	0.81	1.02	1.16	1.90	1.26	1.02	0.12	0.00
time (sec)	N/A	0.898	1.297	1.224	0.038	0.118	0.724	0.258	200.013	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	245	135	196	192	319	199	160	247	0
N.S.	1	1.42	0.78	1.13	1.11	1.84	1.15	0.92	1.43	0.00
time (sec)	N/A	0.610	0.661	1.010	0.035	0.107	0.654	0.223	0.219	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	113	78	112	95	147	116	77	109	149
N.S.	1	1.13	0.78	1.12	0.95	1.47	1.16	0.77	1.09	1.49
time (sec)	N/A	0.332	0.514	0.948	0.030	0.094	0.375	0.253	0.168	18.063

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	203	179	295	352	0	0	0	2284	0
N.S.	1	1.07	0.94	1.55	1.85	0.00	0.00	0.00	12.02	0.00
time (sec)	N/A	0.601	0.956	1.430	0.081	0.000	0.000	0.000	0.191	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	257	232	449	616	0	0	0	1213	0
N.S.	1	1.17	1.06	2.05	2.81	0.00	0.00	0.00	5.54	0.00
time (sec)	N/A	0.729	1.766	1.466	0.082	0.000	0.000	0.000	0.195	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	358	303	859	1281	0	0	1159	2742	0
N.S.	1	1.14	0.97	2.74	4.09	0.00	0.00	3.70	8.76	0.00
time (sec)	N/A	0.943	3.040	1.465	0.103	0.000	0.000	0.268	0.182	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	353	332	394	490	1126	0	487	34	0
N.S.	1	0.87	0.81	0.97	1.20	2.76	0.00	1.19	0.08	0.00
time (sec)	N/A	0.961	2.131	1.352	0.046	0.144	0.000	0.254	200.016	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	242	233	276	322	728	0	311	34	0
N.S.	1	0.88	0.85	1.01	1.18	2.66	0.00	1.14	0.12	0.00
time (sec)	N/A	0.664	1.332	1.227	0.038	0.127	0.000	0.248	200.024	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	156	134	186	190	442	332	175	419	0
N.S.	1	0.93	0.80	1.11	1.14	2.65	1.99	1.05	2.51	0.00
time (sec)	N/A	0.439	1.026	1.032	0.037	0.096	9.722	0.233	0.198	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	75	104	94	210	131	88	159	91
N.S.	1	0.96	0.80	1.11	1.00	2.23	1.39	0.94	1.69	0.97
time (sec)	N/A	0.258	0.511	1.096	0.029	0.078	4.244	0.213	0.160	16.934

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	199	201	464	670	0	0	0	4998	0
N.S.	1	1.03	1.04	2.40	3.47	0.00	0.00	0.00	25.90	0.00
time (sec)	N/A	0.485	1.353	1.474	0.107	0.000	0.000	0.000	0.287	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	283	298	926	1530	1945	0	0	2441	0
N.S.	1	1.01	1.06	3.30	5.44	6.92	0.00	0.00	8.69	0.00
time (sec)	N/A	0.683	2.324	1.435	0.141	4.716	0.000	0.000	4.559	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	460	478	1830	3156	3335	0	1762	34	0
N.S.	1	1.03	1.07	4.08	7.04	7.44	0.00	3.93	0.08	0.00
time (sec)	N/A	1.614	11.539	1.507	0.250	25.044	0.000	0.287	200.024	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	85	66	68	117	91	289	72	176	89
N.S.	1	0.82	0.63	0.65	1.12	0.88	2.78	0.69	1.69	0.86
time (sec)	N/A	0.250	0.681	0.954	0.035	0.108	7.154	0.220	0.176	17.268

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	123	89	93	152	126	777	103	258	125
N.S.	1	0.90	0.65	0.68	1.12	0.93	5.71	0.76	1.90	0.92
time (sec)	N/A	0.270	0.922	0.938	0.038	0.105	14.491	0.171	0.174	17.662

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	150	110	117	187	160	2064	138	335	178
N.S.	1	0.90	0.66	0.70	1.13	0.96	12.43	0.83	2.02	1.07
time (sec)	N/A	0.283	1.429	0.937	0.042	0.098	30.986	0.249	0.261	17.641

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	128	66	45	78	60	94	54	84	45
N.S.	1	1.32	0.68	0.46	0.80	0.62	0.97	0.56	0.87	0.46
time (sec)	N/A	0.313	0.350	0.886	0.113	0.068	0.359	0.249	0.242	0.056

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	100	58	40	64	54	75	48	71	40
N.S.	1	1.25	0.72	0.50	0.80	0.68	0.94	0.60	0.89	0.50
time (sec)	N/A	0.285	0.284	0.904	0.111	0.069	0.225	0.225	0.229	0.040

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	73	56	35	50	49	63	44	58	35
N.S.	1	1.11	0.85	0.53	0.76	0.74	0.95	0.67	0.88	0.53
time (sec)	N/A	0.228	0.216	0.759	0.115	0.082	0.151	0.217	0.270	0.037

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	89	55	58	88	0	99	125	61
N.S.	1	1.00	1.33	0.82	0.87	1.31	0.00	1.48	1.87	0.91
time (sec)	N/A	0.247	0.412	0.941	0.112	0.082	0.000	0.251	0.200	0.205

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	92	62	65	106	0	191	161	68
N.S.	1	1.06	1.30	0.87	0.92	1.49	0.00	2.69	2.27	0.96
time (sec)	N/A	0.253	0.570	0.955	0.114	0.086	0.000	0.379	0.200	0.119

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	71	65	76	89	0	180	136	77
N.S.	1	1.06	0.92	0.84	0.99	1.16	0.00	2.34	1.77	1.00
time (sec)	N/A	0.250	0.787	0.938	0.118	0.072	0.000	0.203	0.174	17.060

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	45	78	76	0	54	131	110
N.S.	1	1.00	0.69	0.47	0.82	0.80	0.00	0.57	1.38	1.16
time (sec)	N/A	0.333	0.420	0.901	0.122	0.093	0.000	0.203	0.174	16.775

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	74	61	40	64	72	0	49	118	105
N.S.	1	0.90	0.74	0.49	0.78	0.88	0.00	0.60	1.44	1.28
time (sec)	N/A	0.278	0.416	0.905	0.112	0.075	0.000	0.220	0.189	0.042

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	58	56	35	50	67	114	44	105	100
N.S.	1	0.88	0.85	0.53	0.76	1.02	1.73	0.67	1.59	1.52
time (sec)	N/A	0.214	0.366	0.772	0.112	0.071	5.496	0.238	0.189	0.039

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	48	58	83	0	82	234	106
N.S.	1	1.00	0.96	0.91	1.09	1.57	0.00	1.55	4.42	2.00
time (sec)	N/A	0.230	0.759	0.921	0.112	0.072	0.000	0.247	0.189	17.028

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	60	84	103	0	168	195	157
N.S.	1	1.00	0.95	0.80	1.12	1.37	0.00	2.24	2.60	2.09
time (sec)	N/A	0.257	0.846	0.941	0.114	0.091	0.000	0.279	0.187	16.829

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	81	65	124	119	0	196	246	180
N.S.	1	0.98	0.84	0.67	1.28	1.23	0.00	2.02	2.54	1.86
time (sec)	N/A	0.343	0.875	0.950	0.113	0.102	0.000	0.284	0.176	16.686

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	81	64	45	105	87	0	53	169	212
N.S.	1	0.85	0.67	0.47	1.11	0.92	0.00	0.56	1.78	2.23
time (sec)	N/A	0.292	0.512	0.909	0.108	0.071	0.000	0.212	0.182	0.057

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	63	61	40	91	83	0	48	156	200
N.S.	1	0.77	0.74	0.49	1.11	1.01	0.00	0.59	1.90	2.44
time (sec)	N/A	0.268	0.484	0.885	0.112	0.069	0.000	0.214	0.175	0.054

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	41	30	27	50	40	180	25	81	185
N.S.	1	0.65	0.48	0.43	0.79	0.63	2.86	0.40	1.29	2.94
time (sec)	N/A	0.212	0.408	0.733	0.026	0.141	20.340	0.180	0.182	16.709

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	58	74	81	103	0	91	372	218
N.S.	1	1.07	0.79	1.01	1.11	1.41	0.00	1.25	5.10	2.99
time (sec)	N/A	0.258	1.049	0.909	0.131	0.107	0.000	0.255	0.197	16.913

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	91	70	107	134	0	233	301	270
N.S.	1	1.06	0.96	0.74	1.13	1.41	0.00	2.45	3.17	2.84
time (sec)	N/A	0.343	1.217	0.935	0.125	0.081	0.000	0.259	0.273	0.253

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	128	91	75	147	149	0	183	356	301
N.S.	1	1.09	0.78	0.64	1.26	1.27	0.00	1.56	3.04	2.57
time (sec)	N/A	0.437	0.959	0.960	0.131	0.107	0.000	0.214	0.238	17.290

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	725	881	1527	0	662	0	0	1101	0
N.S.	1	0.99	1.20	2.07	0.00	0.90	0.00	0.00	1.50	0.00
time (sec)	N/A	1.468	33.326	2.783	0.000	0.086	0.000	0.000	183.052	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	582	748	1033	0	502	0	0	848	0
N.S.	1	0.97	1.25	1.73	0.00	0.84	0.00	0.00	1.42	0.00
time (sec)	N/A	1.174	31.061	3.712	0.000	0.089	0.000	0.000	3.872	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	680	679	1178	0	612	0	0	35	0
N.S.	1	1.23	1.23	2.13	0.00	1.11	0.00	0.00	0.06	0.00
time (sec)	N/A	1.532	29.463	3.527	0.000	0.097	0.000	0.000	200.013	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	647	797	1143	0	1035	0	0	35	0
N.S.	1	1.06	1.31	1.88	0.00	1.70	0.00	0.00	0.06	0.00
time (sec)	N/A	1.484	31.825	4.165	0.000	0.106	0.000	0.000	200.022	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	822	1099	1281	0	1722	0	0	35	0
N.S.	1	1.09	1.46	1.70	0.00	2.28	0.00	0.00	0.05	0.00
time (sec)	N/A	1.611	33.303	5.977	0.000	0.172	0.000	0.000	200.866	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1046	9837	4513	0	1032	0	0	35	0
N.S.	1	0.97	9.13	4.19	0.00	0.96	0.00	0.00	0.03	0.00
time (sec)	N/A	1.907	34.939	5.343	0.000	0.095	0.000	0.000	200.013	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	899	860	1257	2421	0	822	0	0	35	0
N.S.	1	0.96	1.40	2.69	0.00	0.91	0.00	0.00	0.04	0.00
time (sec)	N/A	1.638	35.355	5.173	0.000	0.105	0.000	0.000	200.013	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	889	978	3011	0	1028	0	0	35	0
N.S.	1	1.12	1.24	3.81	0.00	1.30	0.00	0.00	0.04	0.00
time (sec)	N/A	1.908	34.263	5.057	0.000	0.126	0.000	0.000	200.015	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	1025	1029	2091	0	1073	0	0	35	0
N.S.	1	1.34	1.34	2.73	0.00	1.40	0.00	0.00	0.05	0.00
time (sec)	N/A	2.015	33.673	6.776	0.000	0.116	0.000	0.000	200.015	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	832	1162	1771	0	1811	0	0	35	0
N.S.	1	1.00	1.39	2.13	0.00	2.17	0.00	0.00	0.04	0.00
time (sec)	N/A	1.734	34.806	7.505	0.000	0.260	0.000	0.000	200.026	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	983	1100	1383	1773	0	2598	0	0	35	0
N.S.	1	1.12	1.41	1.80	0.00	2.64	0.00	0.00	0.04	0.00
time (sec)	N/A	2.007	35.622	8.694	0.000	0.296	0.000	0.000	200.034	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	754	896	1848	0	663	0	0	35	0
N.S.	1	1.02	1.21	2.50	0.00	0.90	0.00	0.00	0.05	0.00
time (sec)	N/A	1.551	33.157	6.415	0.000	0.140	0.000	0.000	200.021	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	610	755	1150	0	503	0	0	942	0
N.S.	1	1.03	1.27	1.94	0.00	0.85	0.00	0.00	1.59	0.00
time (sec)	N/A	1.317	31.100	5.145	0.000	0.095	0.000	0.000	4.233	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	494	624	793	0	396	0	0	579	0
N.S.	1	1.04	1.31	1.66	0.00	0.83	0.00	0.00	1.21	0.00
time (sec)	N/A	1.056	28.347	3.200	0.000	0.104	0.000	0.000	2.516	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	416	546	664	0	289	0	0	421	0
N.S.	1	1.01	1.33	1.62	0.00	0.70	0.00	0.00	1.03	0.00
time (sec)	N/A	0.870	25.802	3.961	0.000	0.082	0.000	0.000	2.590	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	476	646	788	0	578	0	0	35	0
N.S.	1	1.06	1.44	1.76	0.00	1.29	0.00	0.00	0.08	0.00
time (sec)	N/A	1.148	26.727	5.309	0.000	0.091	0.000	0.000	200.012	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	582	764	947	0	988	0	0	35	0
N.S.	1	1.02	1.35	1.67	0.00	1.74	0.00	0.00	0.06	0.00
time (sec)	N/A	1.273	30.535	7.190	0.000	0.129	0.000	0.000	200.016	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	767	1122	1206	0	1718	0	0	0	0
N.S.	1	1.04	1.52	1.64	0.00	2.33	0.00	0.00	0.00	0.00
time (sec)	N/A	1.564	32.865	9.601	0.000	0.202	0.000	0.000	2.589	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	585	796	1493	0	819	0	0	35	0
N.S.	1	1.03	1.40	2.63	0.00	1.44	0.00	0.00	0.06	0.00
time (sec)	N/A	1.274	30.215	6.287	0.000	0.101	0.000	0.000	200.022	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	469	650	999	0	590	0	0	35	0
N.S.	1	1.03	1.43	2.19	0.00	1.29	0.00	0.00	0.08	0.00
time (sec)	N/A	1.035	28.120	5.211	0.000	0.092	0.000	0.000	200.018	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	413	542	753	0	427	0	0	35	0
N.S.	1	1.04	1.36	1.89	0.00	1.07	0.00	0.00	0.09	0.00
time (sec)	N/A	0.849	26.311	4.397	0.000	0.089	0.000	0.000	200.023	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	435	603	821	0	543	0	0	35	0
N.S.	1	1.05	1.46	1.99	0.00	1.31	0.00	0.00	0.08	0.00
time (sec)	N/A	0.885	28.326	6.461	0.000	0.097	0.000	0.000	200.016	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	582	749	1174	0	1356	0	0	35	0
N.S.	1	1.03	1.33	2.08	0.00	2.40	0.00	0.00	0.06	0.00
time (sec)	N/A	1.099	29.202	7.638	0.000	0.115	0.000	0.000	200.018	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	779	1181	1510	0	2266	0	0	35	0
N.S.	1	1.03	1.57	2.00	0.00	3.01	0.00	0.00	0.05	0.00
time (sec)	N/A	1.459	33.068	9.787	0.000	0.221	0.000	0.000	200.023	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	687	1191	1571	0	1279	0	0	35	0
N.S.	1	1.04	1.80	2.37	0.00	1.93	0.00	0.00	0.05	0.00
time (sec)	N/A	1.555	34.023	7.581	0.000	0.124	0.000	0.000	200.015	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	558	779	1184	0	987	0	0	35	0
N.S.	1	1.05	1.46	2.22	0.00	1.85	0.00	0.00	0.07	0.00
time (sec)	N/A	1.237	30.700	7.165	0.000	0.137	0.000	0.000	200.034	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	481	646	900	0	699	0	0	35	0
N.S.	1	1.04	1.40	1.95	0.00	1.52	0.00	0.00	0.08	0.00
time (sec)	N/A	0.983	28.022	6.804	0.000	0.209	0.000	0.000	200.021	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	556	721	1015	0	974	0	0	35	0
N.S.	1	1.05	1.36	1.91	0.00	1.83	0.00	0.00	0.07	0.00
time (sec)	N/A	1.222	29.425	5.635	0.000	0.129	0.000	0.000	200.024	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	671	698	928	1259	0	1288	0	0	35	0
N.S.	1	1.04	1.38	1.88	0.00	1.92	0.00	0.00	0.05	0.00
time (sec)	N/A	1.282	31.914	8.261	0.000	0.133	0.000	0.000	200.022	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	883	1346	1646	0	2873	0	0	35	0
N.S.	1	1.04	1.59	1.94	0.00	3.38	0.00	0.00	0.04	0.00
time (sec)	N/A	1.714	33.376	10.011	0.000	0.296	0.000	0.000	200.019	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	537	525	0	0	0	0	0	0	34	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.021	0.000	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	636	615	0	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.167	0.000	0.000	0.000	0.000	0.000	0.000	3.422	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	575	0	0	0	0	0	0	328	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.196	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [90] had the largest ratio of [.470588000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	30	0.067
2	A	2	2	1.37	30	0.067
3	A	2	2	1.00	28	0.071
4	A	2	2	1.00	23	0.087
5	A	2	2	1.00	30	0.067
6	A	2	2	1.00	30	0.067
7	A	2	2	1.00	30	0.067
8	A	3	3	0.88	32	0.094
9	A	3	3	1.00	32	0.094
10	A	3	3	1.10	30	0.100
11	A	3	3	1.00	25	0.120
12	A	2	2	1.00	32	0.062
13	A	2	2	1.00	32	0.062
14	A	2	2	1.00	32	0.062
15	A	3	3	0.77	32	0.094
16	A	3	3	0.90	32	0.094
17	A	3	3	1.18	30	0.100
18	A	3	3	1.00	25	0.120
19	A	2	2	1.00	32	0.062
20	A	2	2	1.00	32	0.062
21	A	2	2	1.00	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	1	1	1.00	33	0.030
23	A	1	1	1.00	31	0.032
24	A	1	1	1.00	35	0.029
25	A	1	1	1.00	33	0.030
26	A	2	2	1.00	32	0.062
27	A	2	2	1.00	32	0.062
28	A	2	2	1.00	30	0.067
29	A	2	2	1.00	25	0.080
30	A	2	2	1.00	32	0.062
31	A	2	2	1.00	32	0.062
32	A	2	2	1.00	32	0.062
33	A	4	4	0.76	32	0.125
34	A	4	4	0.77	32	0.125
35	A	4	4	0.84	30	0.133
36	A	6	6	0.96	25	0.240
37	A	5	5	1.08	32	0.156
38	A	4	4	1.02	32	0.125
39	A	6	6	0.66	32	0.188
40	A	7	7	0.72	32	0.219
41	A	6	6	0.88	30	0.200
42	A	5	5	1.02	25	0.200
43	A	8	8	1.10	32	0.250
44	A	6	6	1.05	32	0.188
45	A	6	6	0.73	32	0.188
46	A	6	6	0.81	32	0.188
47	A	6	6	0.82	30	0.200
48	A	6	6	0.94	25	0.240
49	A	10	10	1.09	32	0.312
50	A	4	4	0.88	17	0.235
51	A	4	4	1.11	17	0.235
52	A	5	5	0.90	15	0.333
53	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	4	1.03	17	0.235
55	A	4	4	1.00	17	0.235
56	A	4	4	0.86	17	0.235
57	A	3	3	1.83	16	0.188
58	A	3	3	0.84	18	0.167
59	A	12	11	1.06	34	0.324
60	A	9	8	1.11	34	0.235
61	A	9	8	1.22	32	0.250
62	A	8	7	1.04	27	0.259
63	A	12	11	1.00	34	0.324
64	A	13	12	1.31	34	0.353
65	A	13	12	1.49	34	0.353
66	A	13	12	1.08	34	0.353
67	A	12	11	1.13	34	0.324
68	A	10	9	0.97	34	0.265
69	A	12	11	0.97	34	0.324
70	A	14	13	0.96	34	0.382
71	A	13	12	0.94	34	0.353
72	A	10	9	0.98	34	0.265
73	A	9	8	1.08	32	0.250
74	A	9	8	0.98	27	0.296
75	A	14	13	0.99	34	0.382
76	A	15	14	1.13	34	0.412
77	A	15	14	1.49	34	0.412
78	A	15	14	1.42	34	0.412
79	A	15	14	1.12	34	0.412
80	A	15	14	1.05	34	0.412
81	A	14	13	1.11	34	0.382
82	A	11	10	0.88	34	0.294
83	A	13	12	0.88	34	0.353
84	A	15	14	0.87	34	0.412
85	A	14	13	0.85	34	0.382

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	10	0.89	34	0.294
87	A	10	9	0.98	32	0.281
88	A	10	9	0.93	27	0.333
89	A	16	15	0.98	34	0.441
90	A	17	16	1.01	34	0.471
91	A	17	16	1.31	34	0.471
92	A	17	16	1.42	34	0.471
93	A	17	16	1.36	34	0.471
94	A	17	16	1.15	34	0.471
95	A	17	16	1.06	34	0.471
96	A	17	16	1.07	34	0.471
97	A	16	15	1.10	34	0.441
98	A	12	11	0.83	34	0.324
99	A	14	13	0.82	34	0.382
100	A	17	16	0.81	34	0.471
101	A	11	10	1.23	34	0.294
102	A	8	7	1.30	34	0.206
103	A	7	6	1.42	32	0.188
104	A	7	6	1.13	27	0.222
105	A	9	8	1.07	34	0.235
106	A	10	9	1.17	34	0.265
107	A	11	10	1.14	34	0.294
108	A	10	9	0.87	34	0.265
109	A	8	7	0.88	34	0.206
110	A	7	6	0.93	32	0.188
111	A	7	6	0.96	27	0.222
112	A	9	8	1.03	34	0.235
113	A	7	6	1.01	34	0.176
114	A	8	7	1.03	34	0.206
115	A	4	4	0.82	27	0.148
116	A	5	5	0.90	27	0.185
117	A	6	6	0.90	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	8	1.32	29	0.276
119	A	6	6	1.25	29	0.207
120	A	4	4	1.11	27	0.148
121	A	7	6	1.00	29	0.207
122	A	7	6	1.06	29	0.207
123	A	6	5	1.06	29	0.172
124	A	8	8	1.00	29	0.276
125	A	6	6	0.90	29	0.207
126	A	4	4	0.88	27	0.148
127	A	5	4	1.00	29	0.138
128	A	6	5	1.00	29	0.172
129	A	8	7	0.98	29	0.241
130	A	6	6	0.85	29	0.207
131	A	5	5	0.77	29	0.172
132	A	3	3	0.65	27	0.111
133	A	7	6	1.07	29	0.207
134	A	8	7	1.06	29	0.241
135	A	10	9	1.09	29	0.310
136	A	16	15	0.99	37	0.405
137	A	14	13	0.97	37	0.351
138	A	14	13	1.23	37	0.351
139	A	14	13	1.06	37	0.351
140	A	14	13	1.09	37	0.351
141	A	18	17	0.97	37	0.459
142	A	16	15	0.96	37	0.405
143	A	17	16	1.12	37	0.432
144	A	18	17	1.34	37	0.459
145	A	16	15	1.00	37	0.405
146	A	16	15	1.12	37	0.405
147	A	18	17	1.02	37	0.459
148	A	16	15	1.03	37	0.405
149	A	14	13	1.04	37	0.351

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	12	11	1.01	37	0.297
151	A	12	11	1.06	37	0.297
152	A	12	11	1.02	37	0.297
153	A	14	13	1.04	37	0.351
154	A	16	15	1.03	37	0.405
155	A	14	13	1.03	37	0.351
156	A	12	11	1.04	37	0.297
157	A	10	9	1.05	37	0.243
158	A	12	11	1.03	37	0.297
159	A	14	13	1.03	37	0.351
160	A	16	15	1.04	37	0.405
161	A	14	13	1.05	37	0.351
162	A	12	11	1.04	37	0.297
163	A	12	11	1.05	37	0.297
164	A	12	11	1.04	37	0.297
165	A	14	13	1.04	37	0.351
166	A	9	8	0.98	34	0.235
167	A	9	8	0.97	32	0.250
168	A	8	7	1.00	36	0.194

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (c+dx)^3 (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 256

$$\begin{aligned}\int (c+dx)^3 (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx &= \frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)(c+dx)^4}{4d^6} \\ &- \frac{(ad^2(2cCd-Bd^2-3c^2D)+bc(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))(c+dx)^5}{5d^6} \\ &+ \frac{(ad^2(Cd-3cD)+b(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^6}{6d^6} \\ &+ \frac{(ad^2D-b(4cCd-Bd^2-10c^2D))(c+dx)^7}{7d^6} \\ &+ \frac{b(Cd-5cD)(c+dx)^8}{8d^6} + \frac{bD(c+dx)^9}{9d^6}\end{aligned}$$

output

```
1/4*(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^4/d^6-1/5*(a*d^2*(
-B*d^2+2*C*c*d-3*D*c^2)+b*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*(d*x+c)
^5/d^6+1/6*(a*d^2*(C*d-3*D*c)+b*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x
+c)^6/d^6+1/7*(a*d^2*D-b*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^7/d^6+1/8*b*(C
*d-5*D*c)*(d*x+c)^8/d^6+1/9*b*D*(d*x+c)^9/d^6
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

$$\int (c+dx)^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= aAc^3x + \frac{1}{2}ac^2(Bc+3Ad)x^2 + \frac{1}{3}c\left(ac(cC+3Bd)+A\left(bc^2+3ad^2\right)\right)x^3$$
$$+ \frac{1}{4}\left(bc^2(Bc+3Ad)+a\left(3c^2Cd+3Bcd^2+Ad^3+c^3D\right)\right)x^4$$
$$+ \frac{1}{5}\left(bc\left(c^2C+3Bcd+3Ad^2\right)+ad\left(3cCd+Bd^2+3c^2D\right)\right)x^5$$
$$+ \frac{1}{6}\left(ad^2(Cd+3cD)+b\left(3c^2Cd+3Bcd^2+Ad^3+c^3D\right)\right)x^6$$
$$+ \frac{1}{7}d\left(ad^2D+b\left(3cCd+Bd^2+3c^2D\right)\right)x^7 + \frac{1}{8}bd^2(Cd+3cD)x^8 + \frac{1}{9}bd^3Dx^9$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c^3*x + (a*c^2*(B*c + 3*A*d)*x^2)/2 + (c*(a*c*(c*C + 3*B*d) + A*(b*c^2
 + 3*a*d^2))*x^3)/3 + ((b*c^2*(B*c + 3*A*d) + a*(3*c^2*C*d + 3*B*c*d^2 + A
*d^3 + c^3*D))*x^4)/4 + ((b*c*(c^2*C + 3*B*c*d + 3*A*d^2) + a*d*(3*c*C*d +
 B*d^2 + 3*c^2*D))*x^5)/5 + ((a*d^2*(C*d + 3*c*D) + b*(3*c^2*C*d + 3*B*c*d
^2 + A*d^3 + c^3*D))*x^6)/6 + (d*(a*d^2*D + b*(3*c*C*d + B*d^2 + 3*c^2*D))
*x^7)/7 + (b*d^2*(C*d + 3*c*D)*x^8)/8 + (b*d^3*D*x^9)/9
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{(c+dx)^5 \left(ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5} + \frac{(c+dx)^4 \left(-ad^2\left(-Bd^2-3c^2D+2cCd\right.\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{(c+dx)^6 \left(ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{6d^6} - \frac{(c+dx)^5 \left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{5d^6} + \frac{(c+dx)^4 \left(ad^2+bc^2\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^6} + \frac{(c+dx)^7 \left(ad^2D-b\left(-Bd^2-10c^2D+4cCd\right)\right)}{7d^6} + \frac{b(c+dx)^8(Cd-5cD)}{8d^6} + \frac{bD(c+dx)^9}{9d^6}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^4)/(4*d^6)
- ((a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d
^3 - 5*c^3*D))*(c + d*x)^5)/(5*d^6) + ((a*d^2*(C*d - 3*c*D) + b*(6*c^2*C*d
 - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*(c + d*x)^6)/(6*d^6) + ((a*d^2*D - b*(4*
c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^7)/(7*d^6) + (b*(C*d - 5*c*D)*(c + d*
x)^8)/(8*d^6) + (b*D*(c + d*x)^9)/(9*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09

method	result
norman	$\frac{b\,d^3Dx^9}{9}+\left(\frac{1}{8}b\,d^3C+\frac{3}{8}bc\,d^2D\right)x^8+\left(\frac{1}{7}Bb\,d^3+\frac{3}{7}bc\,d^2C+\frac{1}{7}Da\,d^3+\frac{3}{7}Db\,c^2d\right)x^7+\left(\frac{1}{6}Ab\,d^3\right.$
default	$\frac{b\,d^3Dx^9}{9}+\frac{(b\,d^3C+3bc\,d^2D)x^8}{8}+\frac{((a\,d^3+3b\,c^2d)D+3bc\,d^2C+Bb\,d^3)x^7}{7}+\frac{((3a\,d^2c+b\,c^3)D+(a\,d^3+3b\,c^2d)C+3Bbc\,d^2}{6}$
gosper	$\frac{1}{4}Bb\,c^3x^4+\frac{3}{5}x^5Abc\,d^2+\frac{1}{2}x^6Bbc\,d^2+\frac{1}{2}x^6Cb\,c^2d+\frac{1}{2}x^6Dac\,d^2+\frac{3}{7}x^7bc\,d^2C+\frac{3}{7}x^7Db\,c^2d+$
parallelrisch	$\frac{1}{4}Bb\,c^3x^4+\frac{3}{5}x^5Abc\,d^2+\frac{1}{2}x^6Bbc\,d^2+\frac{1}{2}x^6Cb\,c^2d+\frac{1}{2}x^6Dac\,d^2+\frac{3}{7}x^7bc\,d^2C+\frac{3}{7}x^7Db\,c^2d+$
orering	$x(280b\,d^3Dx^8+315Cb\,d^3x^7+945Dbc\,d^2x^7+360Bb\,d^3x^6+1080Cbc\,d^2x^6+360Da\,d^3x^6+1080Db\,c^2d\,x^6+420Ab\,d^3x^5+1260B$

input
```
int((d*x+c)^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b*d^3*D*x^9+(1/8*b*d^3*C+3/8*b*c*d^2*D)*x^8+(1/7*B*b*d^3+3/7*b*c*d^2*C
+1/7*D*a*d^3+3/7*D*b*c^2*d)*x^7+(1/6*A*b*d^3+1/2*B*b*c*d^2+1/6*C*a*d^3+1/2
*C*b*c^2*d+1/2*D*a*c*d^2+1/6*D*b*c^3)*x^6+(3/5*A*b*c*d^2+1/5*B*a*d^3+3/5*B
*b*c^2*d+3/5*C*a*c*d^2+1/5*b*c^3*C+3/5*a*c^2*d*D)*x^5+(1/4*A*d^3*a+3/4*A*b
*c^2*d+3/4*a*B*c*d^2+1/4*b*B*c^3+3/4*a*c^2*d*C+1/4*c^3*a*D)*x^4+(A*d^2*a*c
+1/3*A*b*c^3+B*a*c^2*d+1/3*C*a*c^3)*x^3+(3/2*a*c^2*d*A+1/2*c^3*a*B)*x^2+A*
a*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00

$$
\begin{aligned}
&\int (c+dx)^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{9}\,Dbd^3x^9+\frac{1}{8}\left(3\,Dbcd^2+Cbd^3\right)x^8+\frac{1}{7}\left(3\,Dbc^2d+3\,Cbcd^2+(Da+Bb)d^3\right)x^7 \\
&\quad+\frac{1}{6}\left(Dbc^3+3\,Cbc^2d+3\,(Da+Bb)cd^2+(Ca+Ab)d^3\right)x^6+Aac^3x \\
&\quad+\frac{1}{5}\left(Cbc^3+Bad^3+3\,(Da+Bb)c^2d+3\,(Ca+Ab)cd^2\right)x^5 \\
&\quad+\frac{1}{4}\left(3\,Bacd^2+Aad^3+(Da+Bb)c^3+3\,(Ca+Ab)c^2d\right)x^4 \\
&\quad+\frac{1}{3}\left(3\,Bac^2d+3\,Aacd^2+(Ca+Ab)c^3\right)x^3+\frac{1}{2}\left(Bac^3+3\,Aac^2d\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b*d^3*x^9 + 1/8*(3*D*b*c*d^2 + C*b*d^3)*x^8 + 1/7*(3*D*b*c^2*d + 3*C
*b*c*d^2 + (D*a + B*b)*d^3)*x^7 + 1/6*(D*b*c^3 + 3*C*b*c^2*d + 3*(D*a + B*
b)*c*d^2 + (C*a + A*b)*d^3)*x^6 + A*a*c^3*x + 1/5*(C*b*c^3 + B*a*d^3 + 3*(
D*a + B*b)*c^2*d + 3*(C*a + A*b)*c*d^2)*x^5 + 1/4*(3*B*a*c*d^2 + A*a*d^3 +
 (D*a + B*b)*c^3 + 3*(C*a + A*b)*c^2*d)*x^4 + 1/3*(3*B*a*c^2*d + 3*A*a*c*d
^2 + (C*a + A*b)*c^3)*x^3 + 1/2*(B*a*c^3 + 3*A*a*c^2*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int (c+dx)^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= Aac^3x+\frac{Dbd^3x^9}{9}+x^8\left(\frac{Cbd^3}{8}+\frac{3Dbcd^2}{8}\right)+x^7\left(\frac{Bbd^3}{7}+\frac{3Cbcd^2}{7}+\frac{Dad^3}{7}+\frac{3Dbc^2d}{7}\right) \\
&\quad+x^6\left(\frac{Abd^3}{6}+\frac{Bbcd^2}{2}+\frac{Cad^3}{6}+\frac{Cbc^2d}{2}+\frac{Dacd^2}{2}+\frac{Dbc^3}{6}\right)+x^5 \\
&\quad\cdot\left(\frac{3Abcd^2}{5}+\frac{Bad^3}{5}+\frac{3Bbc^2d}{5}+\frac{3Cacd^2}{5}+\frac{Cbc^3}{5}+\frac{3Dac^2d}{5}\right) \\
&\quad+x^4\left(\frac{Aad^3}{4}+\frac{3Abc^2d}{4}+\frac{3Bacd^2}{4}+\frac{Bbc^3}{4}+\frac{3Cac^2d}{4}+\frac{Dac^3}{4}\right) \\
&\quad+x^3\left(Aacd^2+\frac{Abc^3}{3}+Bac^2d+\frac{Cac^3}{3}\right)+x^2\cdot\left(\frac{3Aac^2d}{2}+\frac{Bac^3}{2}\right)
\end{aligned}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*c**3*x + D*b*d**3*x**9/9 + x**8*(C*b*d**3/8 + 3*D*b*c*d**2/8) + x**7*(
B*b*d**3/7 + 3*C*b*c*d**2/7 + D*a*d**3/7 + 3*D*b*c**2*d/7) + x**6*(A*b*d**
3/6 + B*b*c*d**2/2 + C*a*d**3/6 + C*b*c**2*d/2 + D*a*c*d**2/2 + D*b*c**3/6
) + x**5*(3*A*b*c*d**2/5 + B*a*d**3/5 + 3*B*b*c**2*d/5 + 3*C*a*c*d**2/5 +
C*b*c**3/5 + 3*D*a*c**2*d/5) + x**4*(A*a*d**3/4 + 3*A*b*c**2*d/4 + 3*B*a*c
*d**2/4 + B*b*c**3/4 + 3*C*a*c**2*d/4 + D*a*c**3/4) + x**3*(A*a*c*d**2 + A
*b*c**3/3 + B*a*c**2*d + C*a*c**3/3) + x**2*(3*A*a*c**2*d/2 + B*a*c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (c+dx)^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx\\&=\frac{1}{9}\,Dbd^3x^9+\frac{1}{8}\left(3\,Dbcd^2+Cbd^3\right)x^8+\frac{1}{7}\left(3\,Dbc^2d+3\,Cbcd^2+(Da+Bb)d^3\right)x^7\\&\quad+\frac{1}{6}\left(Dbc^3+3\,Cbc^2d+3\,(Da+Bb)cd^2+(Ca+Ab)d^3\right)x^6+Aac^3x\\&\quad+\frac{1}{5}\left(Cbc^3+Bad^3+3\,(Da+Bb)c^2d+3\,(Ca+Ab)cd^2\right)x^5\\&\quad+\frac{1}{4}\left(3\,Bacd^2+Aad^3+(Da+Bb)c^3+3\,(Ca+Ab)c^2d\right)x^4\\&\quad+\frac{1}{3}\left(3\,Bac^2d+3\,Aacd^2+(Ca+Ab)c^3\right)x^3+\frac{1}{2}\left(Bac^3+3\,Aac^2d\right)x^2\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*D*b*d^3*x^9 + 1/8*(3*D*b*c*d^2 + C*b*d^3)*x^8 + 1/7*(3*D*b*c^2*d + 3*C
*b*c*d^2 + (D*a + B*b)*d^3)*x^7 + 1/6*(D*b*c^3 + 3*C*b*c^2*d + 3*(D*a + B*
b)*c*d^2 + (C*a + A*b)*d^3)*x^6 + A*a*c^3*x + 1/5*(C*b*c^3 + B*a*d^3 + 3*(
D*a + B*b)*c^2*d + 3*(C*a + A*b)*c*d^2)*x^5 + 1/4*(3*B*a*c*d^2 + A*a*d^3 +
 (D*a + B*b)*c^3 + 3*(C*a + A*b)*c^2*d)*x^4 + 1/3*(3*B*a*c^2*d + 3*A*a*c*d
^2 + (C*a + A*b)*c^3)*x^3 + 1/2*(B*a*c^3 + 3*A*a*c^2*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.30

$$\begin{aligned}&\int (c+dx)^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx\\&=\frac{1}{9}\,Dbd^3x^9+\frac{3}{8}\,Dbcd^2x^8+\frac{1}{8}\,Cbd^3x^8+\frac{3}{7}\,Dbc^2dx^7+\frac{3}{7}\,Cbcd^2x^7+\frac{1}{7}\,Dad^3x^7+\frac{1}{7}\,Bbd^3x^7\\&\quad+\frac{1}{6}\,Dbc^3x^6+\frac{1}{2}\,Cbc^2dx^6+\frac{1}{2}\,Dacd^2x^6+\frac{1}{2}\,Bbcd^2x^6+\frac{1}{6}\,Cad^3x^6+\frac{1}{6}\,Abd^3x^6\\&\quad+\frac{1}{5}\,Cbc^3x^5+\frac{3}{5}\,Dac^2dx^5+\frac{3}{5}\,Bbc^2dx^5+\frac{3}{5}\,Cacd^2x^5+\frac{3}{5}\,Abcd^2x^5+\frac{1}{5}\,Bad^3x^5\\&\quad+\frac{1}{4}\,Dac^3x^4+\frac{1}{4}\,Bbc^3x^4+\frac{3}{4}\,Cac^2dx^4+\frac{3}{4}\,Abc^2dx^4+\frac{3}{4}\,Bacd^2x^4+\frac{1}{4}\,Aad^3x^4\\&\quad+\frac{1}{3}\,Cac^3x^3+\frac{1}{3}\,Abc^3x^3+Bac^2dx^3+Aacd^2x^3+\frac{1}{2}\,Bac^3x^2+\frac{3}{2}\,Aac^2dx^2+Aac^3x\end{aligned}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b*d^3*x^9 + 3/8*D*b*c*d^2*x^8 + 1/8*C*b*d^3*x^8 + 3/7*D*b*c^2*d*x^7
+ 3/7*C*b*c*d^2*x^7 + 1/7*D*a*d^3*x^7 + 1/7*B*b*d^3*x^7 + 1/6*D*b*c^3*x^6
+ 1/2*C*b*c^2*d*x^6 + 1/2*D*a*c*d^2*x^6 + 1/2*B*b*c*d^2*x^6 + 1/6*C*a*d^3*
x^6 + 1/6*A*b*d^3*x^6 + 1/5*C*b*c^3*x^5 + 3/5*D*a*c^2*d*x^5 + 3/5*B*b*c^2*
d*x^5 + 3/5*C*a*c*d^2*x^5 + 3/5*A*b*c*d^2*x^5 + 1/5*B*a*d^3*x^5 + 1/4*D*a*
c^3*x^4 + 1/4*B*b*c^3*x^4 + 3/4*C*a*c^2*d*x^4 + 3/4*A*b*c^2*d*x^4 + 3/4*B*
a*c*d^2*x^4 + 1/4*A*a*d^3*x^4 + 1/3*C*a*c^3*x^3 + 1/3*A*b*c^3*x^3 + B*a*c^
2*d*x^3 + A*a*c*d^2*x^3 + 1/2*B*a*c^3*x^2 + 3/2*A*a*c^2*d*x^2 + A*a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.30

$$\begin{aligned}
&\int (c+dx)^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{a\,c^3\,x^4\,D}{4}+\frac{b\,c^3\,x^6\,D}{6}+\frac{a\,d^3\,x^7\,D}{7}+\frac{b\,d^3\,x^9\,D}{9}+A\,a\,c^3\,x+\frac{B\,a\,c^3\,x^2}{2}+\frac{A\,b\,c^3\,x^3}{3} \\
&+\frac{A\,a\,d^3\,x^4}{4}+\frac{C\,a\,c^3\,x^3}{3}+\frac{B\,b\,c^3\,x^4}{4}+\frac{B\,a\,d^3\,x^5}{5}+\frac{A\,b\,d^3\,x^6}{6}+\frac{C\,b\,c^3\,x^5}{5}+\frac{C\,a\,d^3\,x^6}{6} \\
&+\frac{B\,b\,d^3\,x^7}{7}+\frac{C\,b\,d^3\,x^8}{8}+B\,a\,c^2\,d\,x^3+\frac{3\,A\,b\,c^2\,d\,x^4}{4}+\frac{3\,B\,a\,c\,d^2\,x^4}{4}+\frac{3\,A\,b\,c\,d^2\,x^5}{5} \\
&+\frac{3\,C\,a\,c^2\,d\,x^4}{4}+\frac{3\,B\,b\,c^2\,d\,x^5}{5}+\frac{3\,C\,a\,c\,d^2\,x^5}{5}+\frac{B\,b\,c\,d^2\,x^6}{2}+\frac{C\,b\,c^2\,d\,x^6}{2}+\frac{3\,C\,b\,c\,d^2\,x^7}{7} \\
&+\frac{3\,a\,c^2\,d\,x^5\,D}{5}+\frac{a\,c\,d^2\,x^6\,D}{2}+\frac{3\,b\,c^2\,d\,x^7\,D}{7}+\frac{3\,b\,c\,d^2\,x^8\,D}{8}+\frac{3\,A\,a\,c^2\,d\,x^2}{2}+A\,a\,c\,d^2\,x^3
\end{aligned}$$

input
```
int((a + b*x^2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*c^3*x^4*D)/4 + (b*c^3*x^6*D)/6 + (a*d^3*x^7*D)/7 + (b*d^3*x^9*D)/9 + A*
a*c^3*x + (B*a*c^3*x^2)/2 + (A*b*c^3*x^3)/3 + (A*a*d^3*x^4)/4 + (C*a*c^3*x
^3)/3 + (B*b*c^3*x^4)/4 + (B*a*d^3*x^5)/5 + (A*b*d^3*x^6)/6 + (C*b*c^3*x^5
)/5 + (C*a*d^3*x^6)/6 + (B*b*d^3*x^7)/7 + (C*b*d^3*x^8)/8 + B*a*c^2*d*x^3
+ (3*A*b*c^2*d*x^4)/4 + (3*B*a*c*d^2*x^4)/4 + (3*A*b*c*d^2*x^5)/5 + (3*C*a
*c^2*d*x^4)/4 + (3*B*b*c^2*d*x^5)/5 + (3*C*a*c*d^2*x^5)/5 + (B*b*c*d^2*x^6
)/2 + (C*b*c^2*d*x^6)/2 + (3*C*b*c*d^2*x^7)/7 + (3*a*c^2*d*x^5*D)/5 + (a*c
*d^2*x^6*D)/2 + (3*b*c^2*d*x^7*D)/7 + (3*b*c*d^2*x^8*D)/8 + (3*A*a*c^2*d*x
^2)/2 + A*a*c*d^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.07

$$\int (c+dx)^3 (a+bx^2) (A+Bx+Cx^2+Dx^3)\ dx$$

$$= \frac{x(140b\,d^4x^8 + 630bc\,d^3x^7 + 180a\,d^4x^6 + 180b^2d^3x^6 + 1080b\,c^2d^2x^6 + 210ab\,d^3x^5 + 840ac\,d^3x^5 + 630b^2c\,d}{}$$

input
```
int((d*x+c)^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(1260*a**2*c**3 + 1890*a**2*c**2*d*x + 1260*a**2*c*d**2*x**2 + 315*a**2
*d**3*x**3 + 420*a*b*c**3*x**2 + 630*a*b*c**3*x + 945*a*b*c**2*d*x**3 + 12
60*a*b*c**2*d*x**2 + 756*a*b*c*d**2*x**4 + 945*a*b*c*d**2*x**3 + 210*a*b*d
**3*x**5 + 252*a*b*d**3*x**4 + 420*a*c**4*x**2 + 1260*a*c**3*d*x**3 + 1512
*a*c**2*d**2*x**4 + 840*a*c*d**3*x**5 + 180*a*d**4*x**6 + 315*b**2*c**3*x*
*3 + 756*b**2*c**2*d*x**4 + 630*b**2*c*d**2*x**5 + 180*b**2*d**3*x**6 + 25
2*b*c**4*x**4 + 840*b*c**3*d*x**5 + 1080*b*c**2*d**2*x**6 + 630*b*c*d**3*x
**7 + 140*b*d**4*x**8))/1260
```

3.2 $\int (c+dx)^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 187

$$\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= aAc^2x+\frac{1}{3}\left(ac(cC+2Bd)+A\left(bc^2+ad^2\right)\right)x^3+\frac{1}{4}a\left(2cCd+Bd^2+c^2D\right)x^4 \\
&\quad +\frac{1}{5}\left(b\left(c^2C+2Bcd+Ad^2\right)+ad(Cd+2cD)\right)x^5+\frac{1}{6}\left(ad^2D+b\left(2cCd+Bd^2+c^2D\right)\right)x^6 \\
&\quad +\frac{1}{7}bd(Cd+2cD)x^7+\frac{1}{8}bd^2Dx^8+\frac{c(Bc+2Ad)\left(a+bx^2\right)^2}{4b}
\end{aligned}$$

output
```
a*A*c^2*x+1/3*(a*c*(2*B*d+C*c)+A*(a*d^2+b*c^2))*x^3+1/4*a*(B*d^2+2*C*c*d+D
*c^2)*x^4+1/5*(b*(A*d^2+2*B*c*d+C*c^2)+a*d*(C*d+2*D*c))*x^5+1/6*(a*d^2*D+b
*(B*d^2+2*C*c*d+D*c^2))*x^6+1/7*b*d*(C*d+2*D*c)*x^7+1/8*b*d^2*D*x^8+1/4*c*
(2*A*d+B*c)*(b*x^2+a)^2/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\begin{aligned}&\int (c+dx)^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx\\&=aAc^2x+\frac{1}{2}ac(Bc+2Ad)x^2+\frac{1}{3}\left(Abc^2+ac^2C+2aBcd+aAd^2\right)x^3\\&\quad+\frac{1}{4}\left(bBc^2+2Abcd+2acCd+aBd^2+ac^2D\right)x^4\\&\quad+\frac{1}{5}\left(bc^2C+2bBcd+Abd^2+aCd^2+2acdD\right)x^5\\&\quad+\frac{1}{6}\left(2bcCd+bBd^2+bc^2D+ad^2D\right)x^6+\frac{1}{7}bd(Cd+2cD)x^7+\frac{1}{8}bd^2Dx^8\end{aligned}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c^2*x + (a*c*(B*c + 2*A*d)*x^2)/2 + ((A*b*c^2 + a*c^2*C + 2*a*B*c*d +
a*A*d^2)*x^3)/3 + ((b*B*c^2 + 2*A*b*c*d + 2*a*c*C*d + a*B*d^2 + a*c^2*D)*x
^4)/4 + ((b*c^2*C + 2*b*B*c*d + A*b*d^2 + a*C*d^2 + 2*a*c*d*D)*x^5)/5 + ((
2*b*c*C*d + b*B*d^2 + b*c^2*D + a*d^2*D)*x^6)/6 + (b*d*(C*d + 2*c*D)*x^7)/
7 + (b*d^2*D*x^8)/8
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{(c+dx)^4\left(ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c+dx)^3\left(-ad^2\left(-Bd^2-3c^2D+2cCd\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^5\left(ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{5d^6}-\frac{(c+dx)^4\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{4d^6}+\frac{(c+dx)^3\left(ad^2+bc^2\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^6}+\frac{(c+dx)^6\left(ad^2D-b\left(-Bd^2-10c^2D+4cCd\right)\right)}{6d^6}+\frac{b(c+dx)^7(Cd-5cD)}{7d^6}+\frac{bD(c+dx)^8}{8d^6}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^3)/(3*d^6)
- ((a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d
^3 - 5*c^3*D))*(c + d*x)^4)/(4*d^6) + ((a*d^2*(C*d - 3*c*D) + b*(6*c^2*C*d
 - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*(c + d*x)^5)/(5*d^6) + ((a*d^2*D - b*(4*
c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^6)/(6*d^6) + (b*(C*d - 5*c*D)*(c + d*
x)^7)/(7*d^6) + (b*D*(c + d*x)^8)/(8*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02

method	result
default	$\frac{b\,d^2 D x^8}{8}+\frac{(b\,d^2 C+2dbcD)x^7}{7}+\frac{((a\,d^2+b\,c^2)D+2Cbcd+Bb\,d^2)x^6}{6}+\frac{(2acdD+(a\,d^2+b\,c^2)C+2Bbcd+Ab\,d^2)x^5}{5}+$
norman	$\frac{b\,d^2 D x^8}{8}+\left(\frac{1}{7}b\,d^2 C+\frac{2}{7}dbcD\right)x^7+\left(\frac{1}{6}Bb\,d^2+\frac{1}{3}Cbcd+\frac{1}{6}a\,d^2 D+\frac{1}{6}Db\,c^2\right)x^6+\left(\frac{1}{5}Ab\,d^2+\right.$
gosper	$\frac{1}{8}b\,d^2 D x^8+\frac{1}{7}x^7 b\,d^2 C+\frac{2}{7}x^7 dbcD+\frac{1}{6}x^6 Bb\,d^2+\frac{1}{3}x^6 Cbcd+\frac{1}{6}x^6 a\,d^2 D+\frac{1}{6}x^6 Db\,c^2+$
parallelrisch	$\frac{1}{8}b\,d^2 D x^8+\frac{1}{7}x^7 b\,d^2 C+\frac{2}{7}x^7 dbcD+\frac{1}{6}x^6 Bb\,d^2+\frac{1}{3}x^6 Cbcd+\frac{1}{6}x^6 a\,d^2 D+\frac{1}{6}x^6 Db\,c^2+$
orering	$x(105b\,d^2 D x^7+120Cb\,d^2 x^6+240Dbcd\,x^6+140Bb\,d^2 x^5+280Cbcd\,x^5+140Da\,d^2 x^5+140Db\,c^2 x^5+168Ab\,d^2 x^4+336Bbcd\,x^4+$

input
```
int((d*x+c)^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b*d^2*D*x^8+1/7*(C*b*d^2+2*D*b*c*d)*x^7+1/6*((a*d^2+b*c^2)*D+2*C*b*c*d
+B*b*d^2)*x^6+1/5*(2*a*c*d*D+(a*d^2+b*c^2)*C+2*B*b*c*d+A*b*d^2)*x^5+1/4*(a
*c^2*D+2*C*a*c*d+(a*d^2+b*c^2)*B+2*A*b*c*d)*x^4+1/3*(C*a*c^2+2*B*a*c*d+A*(
a*d^2+b*c^2))*x^3+1/2*(2*A*a*c*d+B*a*c^2)*x^2+A*a*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int (c+dx)^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx
&= \frac{1}{8}\,Dbd^2x^8+\frac{1}{7}\left(2\,Dbcd+Cbd^2\right)x^7+\frac{1}{6}\left(Dbc^2+2\,Cbcd+(Da+Bb)d^2\right)x^6\\
&\quad+\frac{1}{5}\left(Cbc^2+2\,(Da+Bb)cd+(Ca+Ab)d^2\right)x^5+Aac^2x\\
&\quad+\frac{1}{4}\left(Bad^2+(Da+Bb)c^2+2\,(Ca+Ab)cd\right)x^4\\
&\quad+\frac{1}{3}\left(2\,Bacd+Aad^2+(Ca+Ab)c^2\right)x^3+\frac{1}{2}\left(Bac^2+2\,Aacd\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b*d^2*x^8 + 1/7*(2*D*b*c*d + C*b*d^2)*x^7 + 1/6*(D*b*c^2 + 2*C*b*c*d
 + (D*a + B*b)*d^2)*x^6 + 1/5*(C*b*c^2 + 2*(D*a + B*b)*c*d + (C*a + A*b)*d
^2)*x^5 + A*a*c^2*x + 1/4*(B*a*d^2 + (D*a + B*b)*c^2 + 2*(C*a + A*b)*c*d)*
x^4 + 1/3*(2*B*a*c*d + A*a*d^2 + (C*a + A*b)*c^2)*x^3 + 1/2*(B*a*c^2 + 2*A
*a*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= Aac^2x + \frac{Dbd^2x^8}{8} + x^7\left(\frac{Cbd^2}{7} + \frac{2Dbcd}{7}\right) + x^6\left(\frac{Bbd^2}{6} + \frac{Cbcd}{3} + \frac{Dad^2}{6} + \frac{Dbc^2}{6}\right) \\
&\quad + x^5\left(\frac{Abd^2}{5} + \frac{2Bbcd}{5} + \frac{Cad^2}{5} + \frac{Cbc^2}{5} + \frac{2Dacd}{5}\right) \\
&\quad + x^4\left(\frac{Abcd}{2} + \frac{Bad^2}{4} + \frac{Bbc^2}{4} + \frac{Cacd}{2} + \frac{Dac^2}{4}\right) \\
&\quad + x^3\left(\frac{Aad^2}{3} + \frac{Abc^2}{3} + \frac{2Bacd}{3} + \frac{Cac^2}{3}\right) + x^2\left(Aacd + \frac{Bac^2}{2}\right)
\end{aligned}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*c**2*x + D*b*d**2*x**8/8 + x**7*(C*b*d**2/7 + 2*D*b*c*d/7) + x**6*(B*b
*d**2/6 + C*b*c*d/3 + D*a*d**2/6 + D*b*c**2/6) + x**5*(A*b*d**2/5 + 2*B*b*
c*d/5 + C*a*d**2/5 + C*b*c**2/5 + 2*D*a*c*d/5) + x**4*(A*b*c*d/2 + B*a*d**
2/4 + B*b*c**2/4 + C*a*c*d/2 + D*a*c**2/4) + x**3*(A*a*d**2/3 + A*b*c**2/3
 + 2*B*a*c*d/3 + C*a*c**2/3) + x**2*(A*a*c*d + B*a*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int (c+dx)^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}\,Dbd^2x^8+\frac{1}{7}\,(2\,Dbcd+Cbd^2)x^7+\frac{1}{6}\,(Dbc^2+2\,Cbcd+(Da+Bb)d^2)x^6$$
$$+\frac{1}{5}\,(Cbc^2+2\,(Da+Bb)cd+(Ca+Ab)d^2)x^5+Aac^2x$$
$$+\frac{1}{4}\,(Bad^2+(Da+Bb)c^2+2\,(Ca+Ab)cd)x^4$$
$$+\frac{1}{3}\,(2\,Bacd+Aad^2+(Ca+Ab)c^2)x^3+\frac{1}{2}\,(Bac^2+2\,Aacd)x^2$$

input
```
integrate((d*x+c)^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b*d^2*x^8 + 1/7*(2*D*b*c*d + C*b*d^2)*x^7 + 1/6*(D*b*c^2 + 2*C*b*c*d
 + (D*a + B*b)*d^2)*x^6 + 1/5*(C*b*c^2 + 2*(D*a + B*b)*c*d + (C*a + A*b)*d
^2)*x^5 + A*a*c^2*x + 1/4*(B*a*d^2 + (D*a + B*b)*c^2 + 2*(C*a + A*b)*c*d)*
x^4 + 1/3*(2*B*a*c*d + A*a*d^2 + (C*a + A*b)*c^2)*x^3 + 1/2*(B*a*c^2 + 2*A
*a*c*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int (c+dx)^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}\,Dbd^2x^8+\frac{2}{7}\,Dbcdx^7+\frac{1}{7}\,Cbd^2x^7+\frac{1}{6}\,Dbc^2x^6+\frac{1}{3}\,Cbcdx^6+\frac{1}{6}\,Dad^2x^6$$
$$+\frac{1}{6}\,Bbd^2x^6+\frac{1}{5}\,Cbc^2x^5+\frac{2}{5}\,Dacdx^5+\frac{2}{5}\,Bbcdx^5+\frac{1}{5}\,Cad^2x^5+\frac{1}{5}\,Abd^2x^5$$
$$+\frac{1}{4}\,Dac^2x^4+\frac{1}{4}\,Bbc^2x^4+\frac{1}{2}\,Cacdx^4+\frac{1}{2}\,Abcdx^4+\frac{1}{4}\,Bad^2x^4+\frac{1}{3}\,Cac^2x^3$$
$$+\frac{1}{3}\,Abc^2x^3+\frac{2}{3}\,Bacdx^3+\frac{1}{3}\,Aad^2x^3+\frac{1}{2}\,Bac^2x^2+Aacdx^2+Aac^2x$$

input
```
integrate((d*x+c)^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b*d^2*x^8 + 2/7*D*b*c*d*x^7 + 1/7*C*b*d^2*x^7 + 1/6*D*b*c^2*x^6 + 1/
3*C*b*c*d*x^6 + 1/6*D*a*d^2*x^6 + 1/6*B*b*d^2*x^6 + 1/5*C*b*c^2*x^5 + 2/5*
D*a*c*d*x^5 + 2/5*B*b*c*d*x^5 + 1/5*C*a*d^2*x^5 + 1/5*A*b*d^2*x^5 + 1/4*D*
a*c^2*x^4 + 1/4*B*b*c^2*x^4 + 1/2*C*a*c*d*x^4 + 1/2*A*b*c*d*x^4 + 1/4*B*a*
d^2*x^4 + 1/3*C*a*c^2*x^3 + 1/3*A*b*c^2*x^3 + 2/3*B*a*c*d*x^3 + 1/3*A*a*d^
2*x^3 + 1/2*B*a*c^2*x^2 + A*a*c*d*x^2 + A*a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 17.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

$$\begin{aligned}\int (c+dx)^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{a\,x^4\,D\,(15\,c^2+24\,c\,d\,x+10\,d^2\,x^2)}{60}+\frac{b\,x^6\,D\,(28\,c^2+48\,c\,d\,x+21\,d^2\,x^2)}{168}\\ &\quad+\frac{A\,a\,x\,(3\,c^2+3\,c\,d\,x+d^2\,x^2)}{3}+\frac{B\,a\,x^2\,(6\,c^2+8\,c\,d\,x+3\,d^2\,x^2)}{12}\\ &\quad+\frac{A\,b\,x^3\,(10\,c^2+15\,c\,d\,x+6\,d^2\,x^2)}{30}+\frac{C\,a\,x^3\,(10\,c^2+15\,c\,d\,x+6\,d^2\,x^2)}{30}\\ &\quad+\frac{B\,b\,x^4\,(15\,c^2+24\,c\,d\,x+10\,d^2\,x^2)}{60}+\frac{C\,b\,x^5\,(21\,c^2+35\,c\,d\,x+15\,d^2\,x^2)}{105}\end{aligned}$$

input
```
int((a + b*x^2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D*(15*c^2 + 10*d^2*x^2 + 24*c*d*x))/60 + (b*x^6*D*(28*c^2 + 21*d^2*
x^2 + 48*c*d*x))/168 + (A*a*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/3 + (B*a*x^2*(6
*c^2 + 3*d^2*x^2 + 8*c*d*x))/12 + (A*b*x^3*(10*c^2 + 6*d^2*x^2 + 15*c*d*x)
)/30 + (C*a*x^3*(10*c^2 + 6*d^2*x^2 + 15*c*d*x))/30 + (B*b*x^4*(15*c^2 + 1
0*d^2*x^2 + 24*c*d*x))/60 + (C*b*x^5*(21*c^2 + 15*d^2*x^2 + 35*c*d*x))/105
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(105b\,d^3x^7+360bc\,d^2x^6+140a\,d^3x^5+140b^2d^2x^5+420b\,c^2d\,x^5+168ab\,d^2x^4+504ac\,d^2x^4+336b^2cd\,x}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**2*c**2 + 840*a**2*c*d*x + 280*a**2*d**2*x**2 + 280*a*b*c**2*x**
2 + 420*a*b*c**2*x + 420*a*b*c*d*x**3 + 560*a*b*c*d*x**2 + 168*a*b*d**2*x*
*4 + 210*a*b*d**2*x**3 + 280*a*c**3*x**2 + 630*a*c**2*d*x**3 + 504*a*c*d**
2*x**4 + 140*a*d**3*x**5 + 210*b**2*c**2*x**3 + 336*b**2*c*d*x**4 + 140*b*
*2*d**2*x**5 + 168*b*c**3*x**4 + 420*b*c**2*d*x**5 + 360*b*c*d**2*x**6 + 1
05*b*d**3*x**7))/840
```

3.3 $\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 110

$$\begin{aligned}\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= aAcx+\frac{1}{2}a(Bc+Ad)x^2+\frac{1}{3}(Abc+acC+aBd)x^3+\frac{1}{4}(bBc+Abd+aCd+acD)x^4\\&\quad+\frac{1}{5}(bcC+bBd+adD)x^5+\frac{1}{6}b(Cd+cD)x^6+\frac{1}{7}bdDx^7\end{aligned}$$

output
```
a*A*c*x+1/2*a*(A*d+B*c)*x^2+1/3*(A*b*c+B*a*d+C*a*c)*x^3+1/4*(A*b*d+B*b*c+C
*a*d+D*a*c)*x^4+1/5*(B*b*d+C*b*c+D*a*d)*x^5+1/6*b*(C*d+D*c)*x^6+1/7*b*d*D*
x^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= aAcx+\frac{1}{2}a(Bc+Ad)x^2+\frac{1}{3}(Abc+acC+aBd)x^3+\frac{1}{4}(bBc+Abd+aCd+acD)x^4\\&\quad+\frac{1}{5}(bcC+bBd+adD)x^5+\frac{1}{6}b(Cd+cD)x^6+\frac{1}{7}bdDx^7\end{aligned}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c*x + (a*(B*c + A*d)*x^2)/2 + ((A*b*c + a*c*C + a*B*d)*x^3)/3 + ((b*B*
c + A*b*d + a*C*d + a*c*D)*x^4)/4 + ((b*c*C + b*B*d + a*d*D)*x^5)/5 + (b*(
C*d + c*D)*x^6)/6 + (b*d*D*x^7)/7
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(x^3(acD+aCd+Abd+bBc)+x^2(aBd+acC+Abc)+ax(Ad+Bc)+aAc+x^4(adD+bBd+bcC)+bx^5\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(acD+aCd+Abd+bBc)+\frac{1}{3}x^3(aBd+acC+Abc)+\frac{1}{2}ax^2(Ad+Bc)+aAcx+\frac{1}{5}x^5(adD+bBd+bcC)+\frac{1}{6}bx^6(cD+Cd)+\frac{1}{7}bdDx^7$$

input
```
Int[(c + d*x)*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c*x + (a*(B*c + A*d)*x^2)/2 + ((A*b*c + a*c*C + a*B*d)*x^3)/3 + ((b*B*
c + A*b*d + a*C*d + a*c*D)*x^4)/4 + ((b*c*C + b*B*d + a*d*D)*x^5)/5 + (b*(
C*d + c*D)*x^6)/6 + (b*d*D*x^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

method	result
default	$\frac{bdDx^7}{7}+\frac{(bdC+bcD)x^6}{6}+\frac{(Bbd+bcC+Dad)x^5}{5}+\frac{(Abd+Bbc+Cad+Dac)x^4}{4}+\frac{(Abc+Bad+acC)x^3}{3}+\frac{(Aad+aBc)}{2}$
norman	$\frac{bdDx^7}{7}+\left(\frac{1}{6}bdC+\frac{1}{6}bcD\right)x^6+\left(\frac{1}{5}Bbd+\frac{1}{5}bcC+\frac{1}{5}Dad\right)x^5+\left(\frac{1}{4}Abd+\frac{1}{4}Bbc+\frac{1}{4}Cad+\frac{1}{4}D\right.$
orering	$\frac{x(60Dbd\,x^6+70Cbd\,x^5+70Dbc\,x^5+84B\,x^4bd+84Cbc\,x^4+84Dad\,x^4+105Abd\,x^3+105Bbc\,x^3+105Cad\,x^3+105Dac\,x^3+140A}{420}$
gosper	$\frac{1}{7}bdDx^7+\frac{1}{6}x^6bdC+\frac{1}{6}x^6bcD+\frac{1}{5}B\,x^5bd+\frac{1}{5}x^5bcC+\frac{1}{5}x^5Dad+\frac{1}{4}x^4Abd+\frac{1}{4}x^4Bbc+\frac{1}{4}x^4$
parallelrisch	$\frac{1}{7}bdDx^7+\frac{1}{6}x^6bdC+\frac{1}{6}x^6bcD+\frac{1}{5}B\,x^5bd+\frac{1}{5}x^5bcC+\frac{1}{5}x^5Dad+\frac{1}{4}x^4Abd+\frac{1}{4}x^4Bbc+\frac{1}{4}x^4$

input
```
int((d*x+c)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b*d*D*x^7+1/6*(C*b*d+D*b*c)*x^6+1/5*(B*b*d+C*b*c+D*a*d)*x^5+1/4*(A*b*d
+B*b*c+C*a*d+D*a*c)*x^4+1/3*(A*b*c+B*a*d+C*a*c)*x^3+1/2*(A*a*d+B*a*c)*x^2+
A*a*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{7}Dbdx^7+\frac{1}{6}(Dbc+Cbd)x^6+\frac{1}{5}(Cbc+(Da+Bb)d)x^5 \\
&\quad+\frac{1}{4}((Da+Bb)c+(Ca+Ab)d)x^4+Aacx \\
&\quad+\frac{1}{3}(Bad+(Ca+Ab)c)x^3+\frac{1}{2}(Bac+Aad)x^2
\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*D*b*d*x^7 + 1/6*(D*b*c + C*b*d)*x^6 + 1/5*(C*b*c + (D*a + B*b)*d)*x^5
+ 1/4*((D*a + B*b)*c + (C*a + A*b)*d)*x^4 + A*a*c*x + 1/3*(B*a*d + (C*a +
A*b)*c)*x^3 + 1/2*(B*a*c + A*a*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= Aacx+\frac{Dbdx^7}{7}+x^6\left(\frac{Cbd}{6}+\frac{Dbc}{6}\right)+x^5\left(\frac{Bbd}{5}+\frac{Cbc}{5}+\frac{Dad}{5}\right) \\
&\quad+x^4\left(\frac{Abd}{4}+\frac{Bbc}{4}+\frac{Cad}{4}+\frac{Dac}{4}\right)+x^3\left(\frac{Abc}{3}+\frac{Bad}{3}+\frac{Cac}{3}\right)+x^2\left(\frac{Aad}{2}+\frac{Bac}{2}\right)
\end{aligned}$$

input
```
integrate((d*x+c)*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*c*x + D*b*d*x**7/7 + x**6*(C*b*d/6 + D*b*c/6) + x**5*(B*b*d/5 + C*b*c/
5 + D*a*d/5) + x**4*(A*b*d/4 + B*b*c/4 + C*a*d/4 + D*a*c/4) + x**3*(A*b*c/
3 + B*a*d/3 + C*a*c/3) + x**2*(A*a*d/2 + B*a*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\begin{aligned}\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{7}\,Dbdx^7+\frac{1}{6}\,(Dbc+Cbd)x^6+\frac{1}{5}\,(Cbc+(Da+Bb)d)x^5 \\ &\quad+\frac{1}{4}\,((Da+Bb)c+(Ca+Ab)d)x^4+Aacx \\ &\quad+\frac{1}{3}\,(Bad+(Ca+Ab)c)x^3+\frac{1}{2}\,(Bac+Aad)x^2\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*D*b*d*x^7 + 1/6*(D*b*c + C*b*d)*x^6 + 1/5*(C*b*c + (D*a + B*b)*d)*x^5
+ 1/4*((D*a + B*b)*c + (C*a + A*b)*d)*x^4 + A*a*c*x + 1/3*(B*a*d + (C*a +
A*b)*c)*x^3 + 1/2*(B*a*c + A*a*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15

$$\begin{aligned}&\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\ &= \frac{1}{7}\,Dbdx^7+\frac{1}{6}\,Dbcx^6+\frac{1}{6}\,Cbdx^6+\frac{1}{5}\,Cbcx^5+\frac{1}{5}\,Dadx^5+\frac{1}{5}\,Bbdx^5+\frac{1}{4}\,Dacx^4+\frac{1}{4}\,Bbcx^4 \\ &\quad+\frac{1}{4}\,Cadx^4+\frac{1}{4}\,Abdx^4+\frac{1}{3}\,Cacx^3+\frac{1}{3}\,Abcx^3+\frac{1}{3}\,Badx^3+\frac{1}{2}\,Bacx^2+\frac{1}{2}\,Aadx^2+Aacx\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*D*b*d*x^7 + 1/6*D*b*c*x^6 + 1/6*C*b*d*x^6 + 1/5*C*b*c*x^5 + 1/5*D*a*d*
x^5 + 1/5*B*b*d*x^5 + 1/4*D*a*c*x^4 + 1/4*B*b*c*x^4 + 1/4*C*a*d*x^4 + 1/4*
A*b*d*x^4 + 1/3*C*a*c*x^3 + 1/3*A*b*c*x^3 + 1/3*B*a*d*x^3 + 1/2*B*a*c*x^2
+ 1/2*A*a*d*x^2 + A*a*c*x
```

Mupad [B] (verification not implemented)

Time = 16.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15

$$\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{A\,a\,d\,x^2}{2}+\frac{B\,a\,c\,x^2}{2}+\frac{A\,b\,c\,x^3}{3}+\frac{B\,a\,d\,x^3}{3}+\frac{C\,a\,c\,x^3}{3}+\frac{A\,b\,d\,x^4}{4}+\frac{B\,b\,c\,x^4}{4}+\frac{C\,a\,d\,x^4}{4}+\frac{B\,b\,d\,x^5}{5}+\frac{C\,b\,c\,x^5}{5}+\frac{a\,c\,x^4\,D}{4}+\frac{C\,b\,d\,x^6}{6}+\frac{a\,d\,x^5\,D}{5}+\frac{b\,c\,x^6\,D}{6}+\frac{b\,d\,x^7\,D}{7}+A\,a\,c\,x$$

input
```
int((a + b*x^2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a*d*x^2)/2 + (B*a*c*x^2)/2 + (A*b*c*x^3)/3 + (B*a*d*x^3)/3 + (C*a*c*x^3
)/3 + (A*b*d*x^4)/4 + (B*b*c*x^4)/4 + (C*a*d*x^4)/4 + (B*b*d*x^5)/5 + (C*b
*c*x^5)/5 + (a*c*x^4*D)/4 + (C*b*d*x^6)/6 + (a*d*x^5*D)/5 + (b*c*x^6*D)/6
+ (b*d*x^7*D)/7 + A*a*c*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\int (c+dx)\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x(60b\,d^2x^6+140bcd\,x^5+84a\,d^2x^4+84b^2d\,x^4+84b\,c^2x^4+105abd\,x^3+210acd\,x^3+105b^2c\,x^3+140abc}{420}$$

input
```
int((d*x+c)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(420*a**2*c + 210*a**2*d*x + 140*a*b*c*x**2 + 210*a*b*c*x + 105*a*b*d*x
**3 + 140*a*b*d*x**2 + 140*a*c**2*x**2 + 210*a*c*d*x**3 + 84*a*d**2*x**4 +
 105*b**2*c*x**3 + 84*b**2*d*x**4 + 84*b*c**2*x**4 + 140*b*c*d*x**5 + 60*b
*d**2*x**6))/420
```

3.4 $\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\,(A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(x^2(aC+Ab)+aA+x^3(aD+bB)+aBx+bCx^4+bDx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^2}{2}+\frac{(Ab+Ca)x^3}{3}+\frac{(Bb+Da)x^4}{4}+\frac{bC\,x^5}{5}+\frac{bDx^6}{6}$	51
norman	$\frac{bDx^6}{6}+\frac{bC\,x^5}{5}+\left(\frac{Bb}{4}+\frac{Da}{4}\right)x^4+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^3+\frac{Ba\,x^2}{2}+aAx$	53
gosper	$\frac{1}{6}bDx^6+\frac{1}{5}bC\,x^5+\frac{1}{4}Bb\,x^4+\frac{1}{4}x^4Da+\frac{1}{3}Ab\,x^3+\frac{1}{3}Ca\,x^3+\frac{1}{2}Ba\,x^2+aAx$	55
parallelrisch	$\frac{1}{6}bDx^6+\frac{1}{5}bC\,x^5+\frac{1}{4}Bb\,x^4+\frac{1}{4}x^4Da+\frac{1}{3}Ab\,x^3+\frac{1}{3}Ca\,x^3+\frac{1}{2}Ba\,x^2+aAx$	55
orering	$\frac{x\left(10Db\,x^5+12bC\,x^4+15Bb\,x^3+15Da\,x^3+20Ab\,x^2+20Ca\,x^2+30Bax+60Aa\right)}{60}$	56

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = Aax + \frac{Bax^2}{2} + \frac{Cbx^5}{5} + \frac{Dbx^6}{6} + x^4\left(\frac{Bb}{4} + \frac{Da}{4}\right) + x^3\left(\frac{Ab}{3} + \frac{Ca}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3
*(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{6}Dbx^6 + \frac{1}{5}Cbx^5 + \frac{1}{4}(Da+Bb)x^4 + \frac{1}{2}Bax^2 + \frac{1}{3}(Ca+Ab)x^3 + Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}Dax^4+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*
A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

input
```
int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/
3 + (B*b*x^4)/4 + (C*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x(10bd\,x^5+12bc\,x^4+15ad\,x^3+15b^2x^3+20ab\,x^2+20ac\,x^2+30abx+60a^2)}{60}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(60*a**2 + 20*a*b*x**2 + 30*a*b*x + 20*a*c*x**2 + 15*a*d*x**3 + 15*b**2
*x**3 + 12*b*c*x**4 + 10*b*d*x**5))/60
```

3.5 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 223

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx &= -\frac{(ad^2(cCd-Bd^2-c^2D)+bc(c^2Cd-Bcd^2+Ad^3-c^3D))\,x}{d^5} \\ &\quad+\frac{(ad^2(Cd-cD)+b(c^2Cd-Bcd^2+Ad^3-c^3D))\,x^2}{2d^4} \\ &\quad+\frac{(ad^2D-b(cCd-Bd^2-c^2D))\,x^3}{3d^3}+\frac{b(Cd-cD)x^4}{4d^2}+\frac{bDx^5}{5d} \\ &\quad+\frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^6}\end{aligned}$$

output
```
-(a*d^2*(-B*d^2+C*c*d-D*c^2)+b*c*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x/d^5+1/2*
(a*d^2*(C*d-D*c)+b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x^2/d^4+1/3*(a*d^2*D-b*(
-B*d^2+C*c*d-D*c^2))*x^3/d^3+1/4*b*(C*d-D*c)*x^4/d^2+1/5*b*D*x^5/d+(a*d^2+
b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{10ad^3x(6c^2D-3cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+bdx(60c^4D-30c^3d(2C+Dx)+10c^2d^2(6.}{}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(10*a*d^3*x*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x))) +
b*d*x*(60*c^4*D - 30*c^3*d*(2*C + D*x) + 10*c^2*d^2*(6*B + x*(3*C + 2*D*x)
) - 5*c*d^3*(12*A + x*(6*B + x*(4*C + 3*D*x))) + d^4*x*(30*A + x*(20*B + 3
*x*(5*C + 4*D*x)))) - 60*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c
^3*D)*Log[c + d*x])/(60*d^6)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{(ad^2+bc^2)(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^5(c+dx)}+\frac{-ad^2(-Bd^2+c^2(-D)+cCd)-bc(Ad^3-Bcd^2+c^3(-D)}{d^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ad^2+bc^2)\log(c+dx)\,(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^6}-\frac{x\left(ad^2(-Bd^2+c^2(-D)+cCd)+bc\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\right)}{d^5}+\frac{x^2\left(ad^2(Cd-cD)+b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\right)}{2d^4}+\frac{x^3\left(ad^2D-b\left(-Bd^2+c^2(-D)+cCd\right)\right)}{3d^3}+\frac{bx^4(Cd-cD)}{4d^2}+\frac{bDx^5}{5d}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
-(((a*d^2*(c*C*d - B*d^2 - c^2*D) + b*c*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D
))*x)/d^5) + ((a*d^2*(C*d - c*D) + b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*
x^2)/(2*d^4) + ((a*d^2*D - b*(c*C*d - B*d^2 - c^2*D))*x^3)/(3*d^3) + (b*(C
*d - c*D)*x^4)/(4*d^2) + (b*D*x^5)/(5*d) + ((b*c^2 + a*d^2)*(c^2*C*d - B*c
*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.13

method	result
norman	$\frac{\left(Ab\,d^3-Bbc\,d^2+Ca\,d^3+Cb\,c^2d-Dac\,d^2-Db\,c^3\right)x^2}{2d^4}+\frac{\left(Bb\,d^2-Cbcd+a\,d^2D+Db\,c^2\right)x^3}{3d^3}-\frac{\left(A\,d^3bc-Ba\,d^4-Bb\,c^2d^2+Cac\right.}{d}$
default	$-\frac{-\frac{1}{5}Db\,x^5d^4-\frac{1}{4}Cb\,d^4x^4+\frac{1}{4}Dbc\,d^3x^4-\frac{1}{3}x^3Bb\,d^4+\frac{1}{3}Cbc\,d^3x^3-\frac{1}{3}Da\,d^4x^3-\frac{1}{3}Db\,c^2d^2x^3-\frac{1}{2}A\,d^4b\,x^2+\frac{1}{2}Bbc\,d^3x^2-\frac{1}{2}Ca\,d^4x}{d^5}$
parallelrisch	$\frac{60C\ln(dx+c)a\,c^2d^3-60Axbc\,d^4+60Bxb\,c^2d^3-15Dx^4bc\,d^4+60C\ln(dx+c)b\,c^4d-30Dx^2ac\,d^4-30Dx^2b\,c^3d^2-60Cxb\,c^3d^2+6}{}$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^4*(A*b*d^3-B*b*c*d^2+C*a*d^3+C*b*c^2*d-D*a*c*d^2-D*b*c^3)*x^2+1/3/d^
3*(B*b*d^2-C*b*c*d+D*a*d^2+D*b*c^2)*x^3-(A*b*c*d^3-B*a*d^4-B*b*c^2*d^2+C*a
*c*d^3+C*b*c^3*d-D*a*c^2*d^2-D*b*c^4)/d^5*x+1/5*b*D*x^5/d+1/4*b*(C*d-D*c)*
x^4/d^2+(A*a*d^5+A*b*c^2*d^3-B*a*c*d^4-B*b*c^3*d^2+C*a*c^2*d^3+C*b*c^4*d-D
*a*c^3*d^2-D*b*c^5)/d^6*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{12\,Dbd^5x^5 - 15\,(Dbcd^4 - Cbd^5)x^4 + 20\,(Dbc^2d^3 - Cbcd^4 + (Da+Bb)d^5)x^3 - 30\,(Dbc^3d^2 - Cbc^2d^3 + (}{}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/60*(12*D*b*d^5*x^5 - 15*(D*b*c*d^4 - C*b*d^5)*x^4 + 20*(D*b*c^2*d^3 - C*
b*c*d^4 + (D*a + B*b)*d^5)*x^3 - 30*(D*b*c^3*d^2 - C*b*c^2*d^3 + (D*a + B*
b)*c*d^4 - (C*a + A*b)*d^5)*x^2 + 60*(D*b*c^4*d - C*b*c^3*d^2 + B*a*d^5 +
(D*a + B*b)*c^2*d^3 - (C*a + A*b)*c*d^4)*x - 60*(D*b*c^5 - C*b*c^4*d + B*a
*c*d^4 - A*a*d^5 + (D*a + B*b)*c^3*d^2 - (C*a + A*b)*c^2*d^3)*log(d*x + c)
)/d^6
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$
\begin{aligned}
&\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx \\
&= \frac{Dbx^5}{5d} + x^4\left(\frac{Cb}{4d} - \frac{Dbc}{4d^2}\right) + x^3\left(\frac{Bb}{3d} - \frac{Cbc}{3d^2} + \frac{Da}{3d} + \frac{Dbc^2}{3d^3}\right) \\
&\quad + x^2\left(\frac{Ab}{2d} - \frac{Bbc}{2d^2} + \frac{Ca}{2d} + \frac{Cbc^2}{2d^3} - \frac{Dac}{2d^2} - \frac{Dbc^3}{2d^4}\right) \\
&\quad + x\left(-\frac{Abc}{d^2} + \frac{Ba}{d} + \frac{Bbc^2}{d^3} - \frac{Cac}{d^2} - \frac{Cbc^3}{d^4} + \frac{Dac^2}{d^3} + \frac{Dbc^4}{d^5}\right) \\
&\quad - \frac{(ad^2+bc^2)(-Ad^3+Bcd^2-Cc^2d+Dc^3)\log(c+dx)}{d^6}
\end{aligned}
$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b*x**5/(5*d) + x**4*(C*b/(4*d) - D*b*c/(4*d**2)) + x**3*(B*b/(3*d) - C*b
*c/(3*d**2) + D*a/(3*d) + D*b*c**2/(3*d**3)) + x**2*(A*b/(2*d) - B*b*c/(2*
d**2) + C*a/(2*d) + C*b*c**2/(2*d**3) - D*a*c/(2*d**2) - D*b*c**3/(2*d**4)
) + x*(-A*b*c/d**2 + B*a/d + B*b*c**2/d**3 - C*a*c/d**2 - C*b*c**3/d**4 +
D*a*c**2/d**3 + D*b*c**4/d**5) - (a*d**2 + b*c**2)*(-A*d**3 + B*c*d**2 - C
*c**2*d + D*c**3)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$
\begin{aligned}
&\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx \\
&= \frac{12\,Dbd^4x^5 - 15\,(Dbcd^3 - Cbd^4)x^4 + 20\,(Dbc^2d^2 - Cbcd^3 + (Da+Bb)d^4)x^3 - 30\,(Dbc^3d - Cbc^2d^2 + (\ldots}{60\,d^5} \\
&\quad - \frac{(Dbc^5 - Cbc^4d + Bacd^4 - Aad^5 + (Da+Bb)c^3d^2 - (Ca+Ab)c^2d^3)\log(dx+c)}{d^6}
\end{aligned}
$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/60*(12*D*b*d^4*x^5 - 15*(D*b*c*d^3 - C*b*d^4)*x^4 + 20*(D*b*c^2*d^2 - C*
b*c*d^3 + (D*a + B*b)*d^4)*x^3 - 30*(D*b*c^3*d - C*b*c^2*d^2 + (D*a + B*b)
*c*d^3 - (C*a + A*b)*d^4)*x^2 + 60*(D*b*c^4 - C*b*c^3*d + B*a*d^4 + (D*a +
 B*b)*c^2*d^2 - (C*a + A*b)*c*d^3)*x)/d^5 - (D*b*c^5 - C*b*c^4*d + B*a*c*d
^4 - A*a*d^5 + (D*a + B*b)*c^3*d^2 - (C*a + A*b)*c^2*d^3)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{12\,Dbd^4x^5 - 15\,Dbcd^3x^4 + 15\,Cbd^4x^4 + 20\,Dbc^2d^2x^3 - 20\,Cbcd^3x^3 + 20\,Dad^4x^3 + 20\,Bbd^4x^3 - 30\,Dbc^3}{}$$
$$- \frac{(Dbc^5 - Cbc^4d + Dac^3d^2 + Bbc^3d^2 - Cac^2d^3 - Abc^2d^3 + Bacd^4 - Aad^5)\log(|dx+c|)}{d^6}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/60*(12*D*b*d^4*x^5 - 15*D*b*c*d^3*x^4 + 15*C*b*d^4*x^4 + 20*D*b*c^2*d^2*
x^3 - 20*C*b*c*d^3*x^3 + 20*D*a*d^4*x^3 + 20*B*b*d^4*x^3 - 30*D*b*c^3*d*x^
2 + 30*C*b*c^2*d^2*x^2 - 30*D*a*c*d^3*x^2 - 30*B*b*c*d^3*x^2 + 30*C*a*d^4*
x^2 + 30*A*b*d^4*x^2 + 60*D*b*c^4*x - 60*C*b*c^3*d*x + 60*D*a*c^2*d^2*x +
60*B*b*c^2*d^2*x - 60*C*a*c*d^3*x - 60*A*b*c*d^3*x + 60*B*a*d^4*x)/d^5 - (
D*b*c^5 - C*b*c^4*d + D*a*c^3*d^2 + B*b*c^3*d^2 - C*a*c^2*d^3 - A*b*c^2*d^
3 + B*a*c*d^4 - A*a*d^5)*log(abs(d*x + c))/d^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(b\,x^2+a)\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{30\log(dx+c)\,a^2d^3+30\log(dx+c)\,ab\,c^2d-30\log(dx+c)\,abc\,d^2-30\log(dx+c)\,b^2c^3-30abc\,d^2x+1}{30d^4}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(30*log(c + d*x)*a**2*d**3 + 30*log(c + d*x)*a*b*c**2*d - 30*log(c + d*x)*
a*b*c*d**2 - 30*log(c + d*x)*b**2*c**3 - 30*a*b*c*d**2*x + 15*a*b*d**3*x**
2 + 30*a*b*d**3*x + 10*a*d**4*x**3 + 30*b**2*c**2*d*x - 15*b**2*c*d**2*x**
2 + 10*b**2*d**3*x**3 + 6*b*d**4*x**5)/(30*d**4)
```

3.6 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 230

$$\begin{aligned}
\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx
&= \frac{(ad^2(Cd-2cD)+b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x}{d^5} \\
&\quad + \frac{(ad^2D-b(2cCd-Bd^2-3c^2D))\,x^2}{2d^4} + \frac{b(Cd-2cD)x^3}{3d^3} \\
&\quad + \frac{bDx^4}{4d^2} - \frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^6(c+dx)} \\
&\quad - \frac{(ad^2(2cCd-Bd^2-3c^2D)+bc(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))\log(c+dx)}{d^6}
\end{aligned}$$

output
```
(a*d^2*(C*d-2*D*c)+b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*x/d^5+1/2*(a*d^2
*D-b*(-B*d^2+2*C*c*d-3*D*c^2))*x^2/d^4+1/3*b*(C*d-2*D*c)*x^3/d^3+1/4*b*D*x
^4/d^2-(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)-(a*d^2*(-B*
d^2+2*C*c*d-3*D*c^2)+b*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*ln(d*x+c)/
d^6
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{12d(ad^2(Cd-2cD)+b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x+6d^2(ad^2D+b(-2cCd+Bd^2+3c^2D))\,x^2}{}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(12*d*(a*d^2*(C*d - 2*c*D) + b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*
x + 6*d^2*(a*d^2*D + b*(-2*c*C*d + B*d^2 + 3*c^2*D))*x^2 + 4*b*d^3*(C*d -
2*c*D)*x^3 + 3*b*d^4*D*x^4 + (12*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A
*d^3 + c^3*D))/(c + d*x) + 12*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-
4*c^2*C*d + 3*B*c*d^2 - 2*A*d^3 + 5*c^3*D))*Log[c + d*x])/(12*d^6)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{-ad^2(-Bd^2-3c^2D+2cCd)-bc(2Ad^3-3Bcd^2-5c^3D+4c^2Cd)}{d^5(c+dx)}+\frac{(ad^2+bc^2)(Ad^3-Bcd^2+c^3(-D)}{d^5(c+dx)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(ad^2+bc^2)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6(c+dx)}-\frac{\log(c+dx)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6}+\frac{x\left(ad^2(Cd-2cD)+b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5}+\frac{x^2\left(ad^2D-b\left(-Bd^2-3c^2D+2cCd\right)\right)}{2d^4}+\frac{bx^3(Cd-2cD)}{3d^3}+\frac{bDx^4}{4d^2}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((a*d^2*(C*d - 2*c*D) + b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x)/d^
5 + ((a*d^2*D - b*(2*c*C*d - B*d^2 - 3*c^2*D))*x^2)/(2*d^4) + (b*(C*d - 2*
c*D)*x^3)/(3*d^3) + (b*D*x^4)/(4*d^2) - ((b*c^2 + a*d^2)*(c^2*C*d - B*c*d^
2 + A*d^3 - c^3*D))/(d^6*(c + d*x)) - ((a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D)
+ b*c*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5*c^3*D))*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.20

method	result
default	$\frac{\frac{1}{4}Db\,x^4d^3+\frac{1}{3}Cb\,d^3x^3-\frac{2}{3}Dbc\,d^2x^3+\frac{1}{2}Bb\,d^3x^2-Cbc\,d^2x^2+\frac{1}{2}Da\,d^3x^2+\frac{3}{2}Db\,c^2d\,x^2+Ab\,d^3x-2Bbc\,d^2x+Ca\,d^3x+3Cb\,c^2dx-}{d^5}$
norman	$\frac{\frac{\left(Aa\,d^5+2A\,d^3b\,c^2-Bac\,d^4-3Bb\,c^3d^2+2Ca\,c^2d^3+4Cb\,c^4d-3Da\,c^3d^2-5Db\,c^5\right)x}{d^5c}+\frac{\left(3Bb\,d^2-4Cbcd+3a\,d^2D+5Db\,c^2\right)x^3}{6d^3}+\left(2Ab\,d^3-3Bb\right.}{dx+c}$
parallelrisch	$-\frac{24C\ln(dx+c)a\,c^2d^3+5Dx^4bc\,d^4+24C\ln(dx+c)xac\,d^4+24A\,d^3b\,c^2-12B\ln(dx+c)xa\,d^5+48C\ln(dx+c)b\,c^4d+18Dx^2ac\,d^4}{}$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^5*(1/4*D*b*x^4*d^3+1/3*C*b*d^3*x^3-2/3*D*b*c*d^2*x^3+1/2*B*b*d^3*x^2-C
*b*c*d^2*x^2+1/2*D*a*d^3*x^2+3/2*D*b*c^2*d*x^2+A*b*d^3*x-2*B*b*c*d^2*x+C*a
*d^3*x+3*C*b*c^2*d*x-2*D*a*c*d^2*x-4*D*b*c^3*x)-(A*a*d^5+A*b*c^2*d^3-B*a*c
*d^4-B*b*c^3*d^2+C*a*c^2*d^3+C*b*c^4*d-D*a*c^3*d^2-D*b*c^5)/d^6/(d*x+c)+(-
2*A*b*c*d^3+B*a*d^4+3*B*b*c^2*d^2-2*C*a*c*d^3-4*C*b*c^3*d+3*D*a*c^2*d^2+5*
D*b*c^4)/d^6*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{3\,Dbd^5x^5+12\,Dbc^5-12\,Cbc^4d+12\,Bacd^4-12\,Aad^5+12\,(Da+Bb)c^3d^2-12\,(Ca+Ab)c^2d^3-(5\,Db}{}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/12*(3*D*b*d^5*x^5 + 12*D*b*c^5 - 12*C*b*c^4*d + 12*B*a*c*d^4 - 12*A*a*d^
5 + 12*(D*a + B*b)*c^3*d^2 - 12*(C*a + A*b)*c^2*d^3 - (5*D*b*c*d^4 - 4*C*b
*d^5)*x^4 + 2*(5*D*b*c^2*d^3 - 4*C*b*c*d^4 + 3*(D*a + B*b)*d^5)*x^3 - 6*(5
*D*b*c^3*d^2 - 4*C*b*c^2*d^3 + 3*(D*a + B*b)*c*d^4 - 2*(C*a + A*b)*d^5)*x^
2 - 12*(4*D*b*c^4*d - 3*C*b*c^3*d^2 + 2*(D*a + B*b)*c^2*d^3 - (C*a + A*b)*
c*d^4)*x + 12*(5*D*b*c^5 - 4*C*b*c^4*d + B*a*c*d^4 + 3*(D*a + B*b)*c^3*d^2
 - 2*(C*a + A*b)*c^2*d^3 + (5*D*b*c^4*d - 4*C*b*c^3*d^2 + B*a*d^5 + 3*(D*a
 + B*b)*c^2*d^3 - 2*(C*a + A*b)*c*d^4)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{Dbx^4}{4d^2} + x^3\left(\frac{Cb}{3d^2} - \frac{2Dbc}{3d^3}\right) + x^2\left(\frac{Bb}{2d^2} - \frac{Cbc}{d^3} + \frac{Da}{2d^2} + \frac{3Dbc^2}{2d^4}\right) + x\left(\frac{Ab}{d^2} - \frac{2Bbc}{d^3} + \frac{Ca}{d^2} + \frac{3Cbc^2}{d^4} - \frac{2Dac}{d^3} - \frac{4Dbc^3}{d^5}\right) + \frac{-Aad^5 - Abc^2d^3 + Bacd^4 + Bbc^3d^2 - Cac^2d^3 - Cbc^4d + Dac^3d^2 + Dbc^5}{cd^6 + d^7x} + \frac{(-2Abcd^3 + Bad^4 + 3Bbc^2d^2 - 2Cacd^3 - 4Cbc^3d + 3Dac^2d^2 + 5Dbc^4)\log(c+dx)}{d^6}$$

input

```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output

```
D*b*x**4/(4*d**2) + x**3*(C*b/(3*d**2) - 2*D*b*c/(3*d**3)) + x**2*(B*b/(2*
d**2) - C*b*c/d**3 + D*a/(2*d**2) + 3*D*b*c**2/(2*d**4)) + x*(A*b/d**2 - 2
*B*b*c/d**3 + C*a/d**2 + 3*C*b*c**2/d**4 - 2*D*a*c/d**3 - 4*D*b*c**3/d**5)
 + (-A*a*d**5 - A*b*c**2*d**3 + B*a*c*d**4 + B*b*c**3*d**2 - C*a*c**2*d**3
 - C*b*c**4*d + D*a*c**3*d**2 + D*b*c**5)/(c*d**6 + d**7*x) + (-2*A*b*c*d*
*3 + B*a*d**4 + 3*B*b*c**2*d**2 - 2*C*a*c*d**3 - 4*C*b*c**3*d + 3*D*a*c**2
*d**2 + 5*D*b*c**4)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{Dbc^5 - Cbc^4d + Bacd^4 - Aad^5 + (Da+Bb)c^3d^2 - (Ca+Ab)c^2d^3}{d^7x + cd^6} + \frac{3\,Dbd^3x^4 - 4\,(2\,Dbcd^2 - Cbd^3)x^3 + 6\,(3\,Dbc^2d - 2\,Cbcd^2 + (Da+Bb)d^3)x^2 - 12\,(4\,Dbc^3 - 3\,Cbc^2d +}{12\,d^5} + \frac{(5\,Dbc^4 - 4\,Cbc^3d + Bad^4 + 3\,(Da+Bb)c^2d^2 - 2\,(Ca+Ab)cd^3)\log(dx+c)}{d^6}$$

input

```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(D*b*c^5 - C*b*c^4*d + B*a*c*d^4 - A*a*d^5 + (D*a + B*b)*c^3*d^2 - (C*a +
A*b)*c^2*d^3)/(d^7*x + c*d^6) + 1/12*(3*D*b*d^3*x^4 - 4*(2*D*b*c*d^2 - C*b
*d^3)*x^3 + 6*(3*D*b*c^2*d - 2*C*b*c*d^2 + (D*a + B*b)*d^3)*x^2 - 12*(4*D*
b*c^3 - 3*C*b*c^2*d + 2*(D*a + B*b)*c*d^2 - (C*a + A*b)*d^3)*x)/d^5 + (5*D
*b*c^4 - 4*C*b*c^3*d + B*a*d^4 + 3*(D*a + B*b)*c^2*d^2 - 2*(C*a + A*b)*c*d
^3)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{\left(3\,Db - \frac{4\,(5\,Dbcd - Cbd^2)}{(dx+c)d} + \frac{6\,(10\,Dbc^2d^2 - 4\,Cbcd^3 + Dad^4 + Bbd^4)}{(dx+c)^2d^2} - \frac{12\,(10\,Dbc^3d^3 - 6\,Cbc^2d^4 + 3\,Dacd^5 + 3\,Bbcd^5 - Cad^6 - Abd^6)}{(dx+c)^3d^3}\right)(d}{12\,d^6}$$
$$- \frac{(5\,Dbc^4 - 4\,Cbc^3d + 3\,Dac^2d^2 + 3\,Bbc^2d^2 - 2\,Cacd^3 - 2\,Abcd^3 + Bad^4)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}$$
$$+ \frac{\frac{Dbc^5d^4}{dx+c} - \frac{Cbc^4d^5}{dx+c} + \frac{Dac^3d^6}{dx+c} + \frac{Bbc^3d^6}{dx+c} - \frac{Cac^2d^7}{dx+c} - \frac{Abc^2d^7}{dx+c} + \frac{Bacd^8}{dx+c} - \frac{Aad^9}{dx+c}}{d^{10}}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/12*(3*D*b - 4*(5*D*b*c*d - C*b*d^2)/((d*x + c)*d) + 6*(10*D*b*c^2*d^2 -
4*C*b*c*d^3 + D*a*d^4 + B*b*d^4)/((d*x + c)^2*d^2) - 12*(10*D*b*c^3*d^3 -
6*C*b*c^2*d^4 + 3*D*a*c*d^5 + 3*B*b*c*d^5 - C*a*d^6 - A*b*d^6)/((d*x + c)^
3*d^3))*(d*x + c)^4/d^6 - (5*D*b*c^4 - 4*C*b*c^3*d + 3*D*a*c^2*d^2 + 3*B*b
*c^2*d^2 - 2*C*a*c*d^3 - 2*A*b*c*d^3 + B*a*d^4)*log(abs(d*x + c)/((d*x + c
)^2*abs(d)))/d^6 + (D*b*c^5*d^4/(d*x + c) - C*b*c^4*d^5/(d*x + c) + D*a*c^
3*d^6/(d*x + c) + B*b*c^3*d^6/(d*x + c) - C*a*c^2*d^7/(d*x + c) - A*b*c^2*
d^7/(d*x + c) + B*a*c*d^8/(d*x + c) - A*a*d^9/(d*x + c))/d^10
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{-b\,c^2d^4x^4+12\log(dx+c)\,a\,c^4d^2+36\log(dx+c)\,b^2c^4d-12a\,c^3d^3x-6a\,c^2d^4x^2+6ac\,d^5x^3-36b^2c^3d^2x}{}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
( - 24*log(c + d*x)*a*b*c**3*d**2 - 24*log(c + d*x)*a*b*c**2*d**3*x + 12*l
og(c + d*x)*a*b*c**2*d**3 + 12*log(c + d*x)*a*b*c*d**4*x + 12*log(c + d*x)
*a*c**4*d**2 + 12*log(c + d*x)*a*c**3*d**3*x + 36*log(c + d*x)*b**2*c**4*d
 + 36*log(c + d*x)*b**2*c**3*d**2*x + 12*log(c + d*x)*b*c**6 + 12*log(c +
d*x)*b*c**5*d*x + 12*a**2*d**5*x + 24*a*b*c**2*d**3*x + 12*a*b*c*d**4*x**2
 - 12*a*b*c*d**4*x - 12*a*c**3*d**3*x - 6*a*c**2*d**4*x**2 + 6*a*c*d**5*x*
*3 - 36*b**2*c**3*d**2*x - 18*b**2*c**2*d**3*x**2 + 6*b**2*c*d**4*x**3 - 1
2*b*c**5*d*x - 6*b*c**4*d**2*x**2 + 2*b*c**3*d**3*x**3 - b*c**2*d**4*x**4
+ 3*b*c*d**5*x**5)/(12*c*d**5*(c + d*x))
```

3.7 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 232

$$\begin{aligned}
&\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3} dx \\
&= \frac{(ad^2D-b(3cCd-Bd^2-6c^2D))x}{d^5}+\frac{b(Cd-3cD)x^2}{2d^4} \\
&+\frac{bDx^3}{3d^3}-\frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)}{2d^6(c+dx)^2} \\
&+\frac{ad^2(2cCd-Bd^2-3c^2D)+bc(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)}{d^6(c+dx)} \\
&+\frac{(ad^2(Cd-3cD)+b(6c^2Cd-3Bcd^2+Ad^3-10c^3D))\log(c+dx)}{d^6}
\end{aligned}$$

output
```
(a*d^2*D-b*(-B*d^2+3*C*c*d-6*D*c^2))*x/d^5+1/2*b*(C*d-3*D*c)*x^2/d^4+1/3*b
*D*x^3/d^3-1/2*(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^2+(
a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))/
d^6/(d*x+c)+(a*d^2*(C*d-3*D*c)+b*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*ln(
d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$
$$= \frac{6d(ad^2D+b(-3cCd+Bd^2+6c^2D))\,x+3bd^2(Cd-3cD)x^2+2bd^3Dx^3+\frac{3(bc^2+ad^2)(-c^2Cd+Bcd^2-Ad^3+c^3D}{(c+dx)^2}}{}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(6*d*(a*d^2*D + b*(-3*c*C*d + B*d^2 + 6*c^2*D))*x + 3*b*d^2*(C*d - 3*c*D)*
x^2 + 2*b*d^3*D*x^3 + (3*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c
^3*D))/(c + d*x)^2 - (6*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-4*c^2*
C*d + 3*B*c*d^2 - 2*A*d^3 + 5*c^3*D)))/(c + d*x) + 6*(a*d^2*(C*d - 3*c*D)
+ b*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*Log[c + d*x])/(6*d^6)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 2160$$
$$\int \left(\frac{ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)}{d^5(c+dx)}+\frac{-ad^2\left(-Bd^2-3c^2D+2cCd\right)-bc\left(2Ad^3-3Bcd^2\right.}{d^5(c+dx)^2}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)}{d^6(c+dx)}-\frac{\left(ad^2+bc^2\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^6(c+dx)^2}+\frac{\log(c+dx)\left(ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^6}+\frac{x\left(ad^2D-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{d^5}+\frac{bx^2(Cd-3cD)}{2d^4}+\frac{bDx^3}{3d^3}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((a*d^2*D - b*(3*c*C*d - B*d^2 - 6*c^2*D))*x)/d^5 + (b*(C*d - 3*c*D)*x^2)/
(2*d^4) + (b*D*x^3)/(3*d^3) - ((b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3
- c^3*D))/(2*d^6*(c + d*x)^2) + (a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(
4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5*c^3*D))/(d^6*(c + d*x)) + ((a*d^2*(C*d
 - 3*c*D) + b*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*Log[c + d*x])/d^
6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.15

method	result
norman	$\frac{\frac{\left(2A\,d^{3}bc-Ba\,d^{4}-6Bb\,c^{2}d^{2}+2Cac\,d^{3}+12Cb\,c^{3}d-6Da\,c^{2}d^{2}-20Db\,c^{4}\right)x}{d^{5}}-\frac{Aa\,d^{5}-3A\,d^{3}b\,c^{2}+Bac\,d^{4}+9Bb\,c^{3}d^{2}-3Ca\,c^{2}d^{3}-18Cb\,c^{4}d+9D}{2d^{6}}}{(dx+c)^{2}}$
default	$\frac{\frac{1}{3}Db\,x^{3}d^{2}+\frac{1}{2}Cb\,x^{2}d^{2}-\frac{3}{2}Dbcd\,x^{2}+Bb\,d^{2}x-3Cbcdx+a\,d^{2}Dx+6Db\,c^{2}x}{d^{5}}-\frac{-2A\,d^{3}bc+Ba\,d^{4}+3Bb\,c^{2}d^{2}-2Cac\,d^{3}-4Cb\,c^{3}d-}{d^{6}(dx+c)}$
parallelrisch	$\frac{-18B\ln(dx+c)x^{2}bc\,d^{4}+36C\ln(dx+c)x^{2}b\,c^{2}d^{3}-18D\ln(dx+c)x^{2}ac\,d^{4}+6C\ln(dx+c)a\,c^{2}d^{3}+12Axbc\,d^{4}-36Bxb\,c^{2}d^{3}-5Dx^{4}}{}$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
((2*A*b*c*d^3-B*a*d^4-6*B*b*c^2*d^2+2*C*a*c*d^3+12*C*b*c^3*d-6*D*a*c^2*d^2
-20*D*b*c^4)/d^5*x-1/2*(A*a*d^5-3*A*b*c^2*d^3+B*a*c*d^4+9*B*b*c^3*d^2-3*C*
a*c^2*d^3-18*C*b*c^4*d+9*D*a*c^3*d^2+30*D*b*c^5)/d^6+1/3*(3*B*b*d^2-6*C*b*
c*d+3*D*a*d^2+10*D*b*c^2)/d^3*x^3+1/3*b*D*x^5/d+1/6*b*(3*C*d-5*D*c)/d^2*x^
4)/(d*x+c)^2+1/d^6*(A*b*d^3-3*B*b*c*d^2+C*a*d^3+6*C*b*c^2*d-3*D*a*c*d^2-10
*D*b*c^3)*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$
$$= \frac{2\,Dbd^5x^5 - 27\,Dbc^5 + 21\,Cbc^4d - 3\,Bacd^4 - 3\,Aad^5 - 15\,(Da+Bb)c^3d^2 + 9\,(Ca+Ab)c^2d^3 - (5\,Dbcd^4}{}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*D*b*d^5*x^5 - 27*D*b*c^5 + 21*C*b*c^4*d - 3*B*a*c*d^4 - 3*A*a*d^5 -
 15*(D*a + B*b)*c^3*d^2 + 9*(C*a + A*b)*c^2*d^3 - (5*D*b*c*d^4 - 3*C*b*d^5
)*x^4 + 2*(10*D*b*c^2*d^3 - 6*C*b*c*d^4 + 3*(D*a + B*b)*d^5)*x^3 + 3*(21*D
*b*c^3*d^2 - 11*C*b*c^2*d^3 + 4*(D*a + B*b)*c*d^4)*x^2 + 6*(D*b*c^4*d + C*
b*c^3*d^2 - B*a*d^5 - 2*(D*a + B*b)*c^2*d^3 + 2*(C*a + A*b)*c*d^4)*x - 6*(
10*D*b*c^5 - 6*C*b*c^4*d + 3*(D*a + B*b)*c^3*d^2 - (C*a + A*b)*c^2*d^3 + (
10*D*b*c^3*d^2 - 6*C*b*c^2*d^3 + 3*(D*a + B*b)*c*d^4 - (C*a + A*b)*d^5)*x^
2 + 2*(10*D*b*c^4*d - 6*C*b*c^3*d^2 + 3*(D*a + B*b)*c^2*d^3 - (C*a + A*b)*
c*d^4)*x)*log(d*x + c))/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

Sympy [A] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{Dbx^3}{3d^3}+x^2\left(\frac{Cb}{2d^3}-\frac{3Dbc}{2d^4}\right)+x\left(\frac{Bb}{d^3}-\frac{3Cbc}{d^4}+\frac{Da}{d^3}+\frac{6Dbc^2}{d^5}\right)$$
$$+\frac{-Aad^5+3Abc^2d^3-Bacd^4-5Bbc^3d^2+3Cac^2d^3+7Cbc^4d-5Dac^3d^2-9Dbc^5+x(4Abcd^4-2Bad}{2c^2d^6+4cd^7x+2d^8x^2}$$
$$-\frac{\left(-Abd^3+3Bbcd^2-Cad^3-6Cbc^2d+3Dacd^2+10Dbc^3\right)\log\left(c+dx\right)}{d^6}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b*x**3/(3*d**3) + x**2*(C*b/(2*d**3) - 3*D*b*c/(2*d**4)) + x*(B*b/d**3 -
 3*C*b*c/d**4 + D*a/d**3 + 6*D*b*c**2/d**5) + (-A*a*d**5 + 3*A*b*c**2*d**3
 - B*a*c*d**4 - 5*B*b*c**3*d**2 + 3*C*a*c**2*d**3 + 7*C*b*c**4*d - 5*D*a*c
**3*d**2 - 9*D*b*c**5 + x*(4*A*b*c*d**4 - 2*B*a*d**5 - 6*B*b*c**2*d**3 + 4
*C*a*c*d**4 + 8*C*b*c**3*d**2 - 6*D*a*c**2*d**3 - 10*D*b*c**4*d))/(2*c**2*
d**6 + 4*c*d**7*x + 2*d**8*x**2) - (-A*b*d**3 + 3*B*b*c*d**2 - C*a*d**3 -
6*C*b*c**2*d + 3*D*a*c*d**2 + 10*D*b*c**3)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx =$$
$$-\frac{9\,Dbc^5 - 7\,Cbc^4d + Bacd^4 + Aad^5 + 5\,(Da+Bb)c^3d^2 - 3\,(Ca+Ab)c^2d^3 + 2\,(5\,Dbc^4d - 4\,Cbc^3d^2 +}{2\,(d^8x^2 + 2\,cd^7x + c^2d^6)}$$
$$+\frac{2\,Dbd^2x^3 - 3\,(3\,Dbcd - Cbd^2)x^2 + 6\,(6\,Dbc^2 - 3\,Cbcd + (Da+Bb)d^2)x}{6\,d^5}$$
$$-\frac{(10\,Dbc^3 - 6\,Cbc^2d + 3\,(Da+Bb)cd^2 - (Ca+Ab)d^3)\log(dx+c)}{d^6}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(9*D*b*c^5 - 7*C*b*c^4*d + B*a*c*d^4 + A*a*d^5 + 5*(D*a + B*b)*c^3*d^
2 - 3*(C*a + A*b)*c^2*d^3 + 2*(5*D*b*c^4*d - 4*C*b*c^3*d^2 + B*a*d^5 + 3*(
D*a + B*b)*c^2*d^3 - 2*(C*a + A*b)*c*d^4)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^
6) + 1/6*(2*D*b*d^2*x^3 - 3*(3*D*b*c*d - C*b*d^2)*x^2 + 6*(6*D*b*c^2 - 3*C
*b*c*d + (D*a + B*b)*d^2)*x)/d^5 - (10*D*b*c^3 - 6*C*b*c^2*d + 3*(D*a + B*
b)*c*d^2 - (C*a + A*b)*d^3)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$
$$= -\frac{(10\,Dbc^3 - 6\,Cbc^2d + 3\,Dacd^2 + 3\,Bbcd^2 - Cad^3 - Abd^3)\log(|dx+c|)}{d^6}$$
$$-\frac{9\,Dbc^5 - 7\,Cbc^4d + 5\,Dac^3d^2 + 5\,Bbc^3d^2 - 3\,Cac^2d^3 - 3\,Abc^2d^3 + Bacd^4 + Aad^5 + 2\,(5\,Dbc^4d - 4\,C}{2\,(dx+c)^2d^6}$$
$$+\frac{2\,Dbd^6x^3 - 9\,Dbcd^5x^2 + 3\,Cbd^6x^2 + 36\,Dbc^2d^4x - 18\,Cbcd^5x + 6\,Dad^6x + 6\,Bbd^6x}{6\,d^9}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(10*D*b*c^3 - 6*C*b*c^2*d + 3*D*a*c*d^2 + 3*B*b*c*d^2 - C*a*d^3 - A*b*d^3
)*log(abs(d*x + c))/d^6 - 1/2*(9*D*b*c^5 - 7*C*b*c^4*d + 5*D*a*c^3*d^2 + 5
*B*b*c^3*d^2 - 3*C*a*c^2*d^3 - 3*A*b*c^2*d^3 + B*a*c*d^4 + A*a*d^5 + 2*(5*
D*b*c^4*d - 4*C*b*c^3*d^2 + 3*D*a*c^2*d^3 + 3*B*b*c^2*d^3 - 2*C*a*c*d^4 -
2*A*b*c*d^4 + B*a*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2*D*b*d^6*x^3 - 9*D*b*c
*d^5*x^2 + 3*C*b*d^6*x^2 + 36*D*b*c^2*d^4*x - 18*C*b*c*d^5*x + 6*D*a*d^6*x
 + 6*B*b*d^6*x)/d^9
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.63

$$
\begin{aligned}
\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx
&= \frac{\frac{3\,A\,b\,c^2}{2\,d^3}+\frac{2\,A\,b\,c\,x}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2}+\frac{\frac{3\,C\,a\,c^2}{2\,d^3}+\frac{2\,C\,a\,c\,x}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2}-\frac{\frac{B\,a\,c}{2\,d^2}+\frac{B\,a\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2} \\
&\quad+\frac{C\,b\left(\frac{(c+d\,x)^2}{2}+\frac{4\,c^3}{c+d\,x}-\frac{c^4}{2\,(c+d\,x)^2}+6\,c^2\,\ln\left(c+d\,x\right)-4\,c\,d\,x\right)}{d^5} \\
&\quad+\frac{A\,b\,\ln\left(c+d\,x\right)}{d^3}+\frac{C\,a\,\ln\left(c+d\,x\right)}{d^3} \\
&\quad-\frac{B\,b\left(3\,c\,\ln\left(c+d\,x\right)-d\,x+\frac{3\,c^2}{c+d\,x}-\frac{c^3}{2\,(c+d\,x)^2}\right)}{d^4}-\frac{A\,a}{2\,d\,\left(c^2+2\,c\,d\,x+d^2\,x^2\right)} \\
&\quad-\frac{b\,D\left(\frac{5\,c\,(c+d\,x)^2}{2}-\frac{(c+d\,x)^3}{3}+\frac{5\,c^4}{c+d\,x}-\frac{c^5}{2\,(c+d\,x)^2}+10\,c^3\,\ln\left(c+d\,x\right)-10\,c^2\,d\,x\right)}{d^6} \\
&\quad-\frac{a\,D\left(3\,c\,\ln\left(c+d\,x\right)-d\,x+\frac{3\,c^2}{c+d\,x}-\frac{c^3}{2\,(c+d\,x)^2}\right)}{d^4}
\end{aligned}
$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
((3*A*b*c^2)/(2*d^3) + (2*A*b*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d*x) + ((3*C*
a*c^2)/(2*d^3) + (2*C*a*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d*x) - ((B*a*c)/(2*
d^2) + (B*a*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x) + (C*b*((c + d*x)^2/2 + (4*c^3
)/(c + d*x) - c^4/(2*(c + d*x)^2) + 6*c^2*log(c + d*x) - 4*c*d*x))/d^5 + (
A*b*log(c + d*x))/d^3 + (C*a*log(c + d*x))/d^3 - (B*b*(3*c*log(c + d*x) -
d*x + (3*c^2)/(c + d*x) - c^3/(2*(c + d*x)^2)))/d^4 - (A*a)/(2*d*(c^2 + d^
2*x^2 + 2*c*d*x)) - (b*D*((5*c*(c + d*x)^2)/2 - (c + d*x)^3/3 + (5*c^4)/(c
 + d*x) - c^5/(2*(c + d*x)^2) + 10*c^3*log(c + d*x) - 10*c^2*d*x))/d^6 - (
a*D*(3*c*log(c + d*x) - d*x + (3*c^2)/(c + d*x) - c^3/(2*(c + d*x)^2)))/d^
4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \frac{-2b\,c^2d^4x^4-12\log(dx+c)\,a\,c^4d^2-18\log(dx+c)\,b^2c^4d+12a\,c^2d^4x^2+6ac\,d^5x^3+18b^2c^2d^3x^2+6b^2c\,d}{}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
(6*log(c + d*x)*a*b*c**3*d**2 + 12*log(c + d*x)*a*b*c**2*d**3*x + 6*log(c
+ d*x)*a*b*c*d**4*x**2 - 12*log(c + d*x)*a*c**4*d**2 - 24*log(c + d*x)*a*c
**3*d**3*x - 12*log(c + d*x)*a*c**2*d**4*x**2 - 18*log(c + d*x)*b**2*c**4*
d - 36*log(c + d*x)*b**2*c**3*d**2*x - 18*log(c + d*x)*b**2*c**2*d**3*x**2
 - 24*log(c + d*x)*b*c**6 - 48*log(c + d*x)*b*c**5*d*x - 24*log(c + d*x)*b
*c**4*d**2*x**2 - 3*a**2*c*d**4 + 3*a*b*c**3*d**2 - 6*a*b*c*d**4*x**2 + 3*
a*b*d**5*x**2 - 6*a*c**4*d**2 + 12*a*c**2*d**4*x**2 + 6*a*c*d**5*x**3 - 9*
b**2*c**4*d + 18*b**2*c**2*d**3*x**2 + 6*b**2*c*d**4*x**3 - 12*b*c**6 + 24
*b*c**4*d**2*x**2 + 8*b*c**3*d**3*x**3 - 2*b*c**2*d**4*x**4 + 2*b*c*d**5*x
**5)/(6*c*d**5*(c**2 + 2*c*d*x + d**2*x**2))
```

3.8 $\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$
\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx \\
&= \frac{\left(bc^2+ad^2\right)^2 \left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^4}{4d^8} \\
&\quad - \frac{\left(bc^2+ad^2\right)\left(ad^2(2cCd-Bd^2-3c^2D)+bc(6c^2Cd-5Bcd^2+4Ad^3-7c^3D)\right)(c+dx)^5}{5d^8} \\
&\quad + \frac{\left(a^2d^4(Cd-3cD)+b^2c^2(15c^2Cd-10Bcd^2+6Ad^3-21c^3D)+2abd^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D\right.}{6d^8} \\
&\quad + \frac{\left(a^2d^4D-2abd^2(4cCd-Bd^2-10c^2D)-b^2c(20c^2Cd-10Bcd^2+4Ad^3-35c^3D)\right)(c+dx)^7}{7d^8} \\
&\quad + \frac{b\left(2ad^2(Cd-5cD)+b(15c^2Cd-5Bcd^2+Ad^3-35c^3D)\right)(c+dx)^8}{8d^8} \\
&\quad + \frac{b\left(2ad^2D-b(6cCd-Bd^2-21c^2D)\right)(c+dx)^9}{9d^8} \\
&\quad + \frac{b^2(Cd-7cD)(c+dx)^{10}}{10d^8} + \frac{b^2D(c+dx)^{11}}{11d^8}
\end{aligned}
$$

output
```
1/4*(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^4/d^8-1/5*(a*d^2
+b*c^2)*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(4*A*d^3-5*B*c*d^2+6*C*c^2*d-7
*D*c^3))*(d*x+c)^5/d^8+1/6*(a^2*d^4*(C*d-3*D*c)+b^2*c^2*(6*A*d^3-10*B*c*d^
2+15*C*c^2*d-21*D*c^3)+2*a*b*d^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*
x+c)^6/d^8+1/7*(a^2*d^4*D-2*a*b*d^2*(-B*d^2+4*C*c*d-10*D*c^2)-b^2*c*(4*A*d
^3-10*B*c*d^2+20*C*c^2*d-35*D*c^3))*(d*x+c)^7/d^8+1/8*b*(2*a*d^2*(C*d-5*D*
c)+b*(A*d^3-5*B*c*d^2+15*C*c^2*d-35*D*c^3))*(d*x+c)^8/d^8+1/9*b*(2*a*d^2*D
-b*(-B*d^2+6*C*c*d-21*D*c^2))*(d*x+c)^9/d^8+1/10*b^2*(C*d-7*D*c)*(d*x+c)^1
0/d^8+1/11*b^2*D*(d*x+c)^11/d^8
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= a^2Ac^3x + \frac{1}{2}a^2c^2(Bc+3Ad)x^2 + \frac{1}{3}ac\left(ac(cC+3Bd) + A\left(2bc^2+3ad^2\right)\right)x^3 \\
&\quad + \frac{1}{4}a\left(2bc^2(Bc+3Ad) + a\left(3c^2Cd+3Bcd^2+Ad^3+c^3D\right)\right)x^4 \\
&\quad + \frac{1}{5}\left(Abc\left(bc^2+6ad^2\right) + a\left(2bc^2(cC+3Bd) + ad\left(3cCd+Bd^2+3c^2D\right)\right)\right)x^5 \\
&\quad + \frac{1}{6}\left(b^2c^2(Bc+3Ad) + a^2d^2(Cd+3cD) + 2ab\left(3c^2Cd+3Bcd^2+Ad^3+c^3D\right)\right)x^6 \\
&\quad + \frac{1}{7}\left(b^2c\left(c^2C+3Bcd+3Ad^2\right) + a^2d^3D + 2abd\left(3cCd+Bd^2+3c^2D\right)\right)x^7 \\
&\quad + \frac{1}{8}b\left(2ad^2(Cd+3cD) + b\left(3c^2Cd+3Bcd^2+Ad^3+c^3D\right)\right)x^8 \\
&\quad + \frac{1}{9}bd\left(2ad^2D + b\left(3cCd+Bd^2+3c^2D\right)\right)x^9 + \frac{1}{10}b^2d^2(Cd+3cD)x^{10} + \frac{1}{11}b^2d^3Dx^{11}
\end{aligned}
$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c^3*x + (a^2*c^2*(B*c + 3*A*d)*x^2)/2 + (a*c*(a*c*(c*C + 3*B*d) + A*
(2*b*c^2 + 3*a*d^2))*x^3)/3 + (a*(2*b*c^2*(B*c + 3*A*d) + a*(3*c^2*C*d + 3
*B*c*d^2 + A*d^3 + c^3*D))*x^4)/4 + ((A*b*c*(b*c^2 + 6*a*d^2) + a*(2*b*c^2
*(c*C + 3*B*d) + a*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^5)/5 + ((b^2*c^2*(B*c
 + 3*A*d) + a^2*d^2*(C*d + 3*c*D) + 2*a*b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 +
 c^3*D))*x^6)/6 + ((b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2) + a^2*d^3*D + 2*a*b*
d*(3*c*C*d + B*d^2 + 3*c^2*D))*x^7)/7 + (b*(2*a*d^2*(C*d + 3*c*D) + b*(3*c
^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*x^8)/8 + (b*d*(2*a*d^2*D + b*(3*c*C*d
 + B*d^2 + 3*c^2*D))*x^9)/9 + (b^2*d^2*(C*d + 3*c*D)*x^10)/10 + (b^2*d^3*D
*x^11)/11
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^2 \left((c+dx)^3 \left(Dx^3+Cx^2+Bx+A\right)-\left(Bc^3+3Adc^2\right) x\right) dx+\frac{c^2 \left(a+bx^2\right)^3 (3Ad+Bc)}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^2d^3Dx^{10}+b^2d^2(Cd+3cD)x^9+bd\left(2aDd^2+b\left(3Dc^2+3Cdc+Bd^2\right)\right)x^8+b\left(2a(Cd+3cD)d^2+b\left(Dc^3+3\right.\right.\right.$$
$$\frac{c^2 \left(a+bx^2\right)^3 (3Ad+Bc)}{6b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7\left(a^2d^3D+2abd\left(Bd^2+3c^2D+3cCd\right)+b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)+\frac{1}{4}a^2x^4\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)+a^2Ac^3x+\frac{1}{5}x^5\left(Abc\left(6ad^2+bc^2\right)+a\left(ad\left(Bd^2+3c^2D+3cCd\right)+2bc^2(3Bd+cC)\right)\right)+\frac{1}{3}acx^3\left(A\left(3ad^2+2bc^2\right)+ac(3Bd+cC)\right)+\frac{c^2\left(a+bx^2\right)^3(3Ad+Bc)}{6b}+\frac{1}{8}bx^8\left(2ad^2(3cD+Cd)+b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)+\frac{1}{6}ax^6\left(ad^2(3cD+Cd)+2b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)+\frac{1}{9}bdx^9\left(2ad^2D+b\left(Bd^2+3c^2D+3cCd\right)\right)+\frac{1}{10}b^2d^2x^{10}(3cD+Cd)+\frac{1}{11}b^2d^3Dx^{11}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c^3*x + (a*c*(a*c*(c*C + 3*B*d) + A*(2*b*c^2 + 3*a*d^2))*x^3)/3 + (a
^2*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D)*x^4)/4 + ((A*b*c*(b*c^2 + 6*a*d
^2) + a*(2*b*c^2*(c*C + 3*B*d) + a*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^5)/5
+ (a*(a*d^2*(C*d + 3*c*D) + 2*b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*x
^6)/6 + ((b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2) + a^2*d^3*D + 2*a*b*d*(3*c*C*d
 + B*d^2 + 3*c^2*D))*x^7)/7 + (b*(2*a*d^2*(C*d + 3*c*D) + b*(3*c^2*C*d + 3
*B*c*d^2 + A*d^3 + c^3*D))*x^8)/8 + (b*d*(2*a*d^2*D + b*(3*c*C*d + B*d^2 +
 3*c^2*D))*x^9)/9 + (b^2*d^2*(C*d + 3*c*D)*x^10)/10 + (b^2*d^3*D*x^11)/11
+ (c^2*(B*c + 3*A*d)*(a + b*x^2)^3)/(6*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^2d^3Dx^{11}}{11}+\left(\frac{1}{10}b^2d^3C+\frac{3}{10}b^2c\,d^2D\right)x^{10}+\left(\frac{1}{9}B\,b^2d^3+\frac{1}{3}b^2c\,d^2C+\frac{2}{9}Dab\,d^3+\frac{1}{3}Db^2c^2d\right)x^9+$
default	$\frac{b^2d^3Dx^{11}}{11}+\frac{(b^2d^3C+3b^2c\,d^2D)x^{10}}{10}+\frac{((2ab\,d^3+3b^2c^2d)D+3b^2c\,d^2C+B\,b^2d^3)x^9}{9}+\frac{((6abc\,d^2+c^3b^2)D+(2ab\,d^3+3b^2c}{8}$
gosper	$\frac{2}{3}x^3Aab\,c^3+\frac{3}{5}x^5C\,a^2c\,d^2+x^6Babc\,d^2+\frac{3}{8}x^8b^2c\,d^2B+\frac{1}{4}x^8Cab\,d^3+\frac{3}{8}x^8C\,b^2c^2d+\frac{3}{10}x^{10}b^2c$
parallelrisch	$\frac{2}{3}x^3Aab\,c^3+\frac{3}{5}x^5C\,a^2c\,d^2+x^6Babc\,d^2+\frac{3}{8}x^8b^2c\,d^2B+\frac{1}{4}x^8Cab\,d^3+\frac{3}{8}x^8C\,b^2c^2d+\frac{3}{10}x^{10}b^2c$
orering	$x(2520b^2d^3Dx^{10}+2772C\,b^2d^3x^9+8316Db^2c\,d^2x^9+3080B\,b^2d^3x^8+9240C\,b^2c\,d^2x^8+6160Dab\,d^3x^8+9240Db^2c^2d\,x^8+3465A$

input
```
int((d*x+c)^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*d^3*D*x^11+(1/10*b^2*d^3*C+3/10*b^2*c*d^2*D)*x^10+(1/9*B*b^2*d^3+
1/3*b^2*c*d^2*C+2/9*D*a*b*d^3+1/3*D*b^2*c^2*d)*x^9+(1/8*A*b^2*d^3+3/8*b^2*
c*d^2*B+1/4*C*a*b*d^3+3/8*C*b^2*c^2*d+3/4*D*a*b*c*d^2+1/8*D*b^2*c^3)*x^8+(
3/7*A*d^2*b^2*c+2/7*B*a*b*d^3+3/7*B*b^2*c^2*d+6/7*C*a*b*c*d^2+1/7*C*b^2*c^
3+1/7*a^2*d^3*D+6/7*D*a*b*c^2*d)*x^7+(1/3*A*a*b*d^3+1/2*A*b^2*c^2*d+B*a*b*
c*d^2+1/6*B*b^2*c^3+1/6*C*a^2*d^3+C*a*b*c^2*d+1/2*D*a^2*c*d^2+1/3*D*a*b*c^
3)*x^6+(6/5*A*a*b*c*d^2+1/5*A*b^2*c^3+1/5*a^2*B*d^3+6/5*B*a*b*c^2*d+3/5*C*
a^2*c*d^2+2/5*C*a*b*c^3+3/5*a^2*c^2*d*D)*x^5+(1/4*A*a^2*d^3+3/2*A*a*b*c^2*
d+3/4*B*a^2*c*d^2+1/2*B*a*b*c^3+3/4*a^2*c^2*d*C+1/4*a^2*c^3*D)*x^4+(A*d^2*
a^2*c+2/3*A*a*b*c^3+a^2*c^2*d*B+1/3*C*a^2*c^3)*x^3+(3/2*a^2*c^2*d*A+1/2*B*
a^2*c^3)*x^2+a^2*A*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01

$$\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{11}\,Db^2d^3x^{11} + \frac{1}{10}\left(3\,Db^2cd^2+Cb^2d^3\right)x^{10} + \frac{1}{9}\left(3\,Db^2c^2d+3\,Cb^2cd^2+\left(2\,Dab+Bb^2\right)d^3\right)x^9 + \frac{1}{8}\left(Db^2c^3+3\,Cb^2c^2d+3\left(2\,Dab+Bb^2\right)cd^2+\left(2\,Cab+Ab^2\right)d^3\right)x^8 + \frac{1}{7}\left(Cb^2c^3+3\left(2\,Dab+Bb^2\right)c^2d+3\left(2\,Cab+Ab^2\right)cd^2+\left(Da^2+2\,Bab\right)d^3\right)x^7 + Aa^2c^3x + \frac{1}{6}\left(\left(2\,Dab+Bb^2\right)c^3+3\left(2\,Cab+Ab^2\right)c^2d+3\left(Da^2+2\,Bab\right)cd^2+\left(Ca^2+2\,Aab\right)d^3\right)x^6 + \frac{1}{5}\left(Ba^2d^3+\left(2\,Cab+Ab^2\right)c^3+3\left(Da^2+2\,Bab\right)c^2d+3\left(Ca^2+2\,Aab\right)cd^2\right)x^5 + \frac{1}{4}\left(3\,Ba^2cd^2+Aa^2d^3+\left(Da^2+2\,Bab\right)c^3+3\left(Ca^2+2\,Aab\right)c^2d\right)x^4 + \frac{1}{3}\left(3\,Ba^2c^2d+3\,Aa^2cd^2+\left(Ca^2+2\,Aab\right)c^3\right)x^3 + \frac{1}{2}\left(Ba^2c^3+3\,Aa^2c^2d\right)x^2$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^2*d^3*x^11 + 1/10*(3*D*b^2*c*d^2 + C*b^2*d^3)*x^10 + 1/9*(3*D*b^2
*c^2*d + 3*C*b^2*c*d^2 + (2*D*a*b + B*b^2)*d^3)*x^9 + 1/8*(D*b^2*c^3 + 3*C
*b^2*c^2*d + 3*(2*D*a*b + B*b^2)*c*d^2 + (2*C*a*b + A*b^2)*d^3)*x^8 + 1/7*
(C*b^2*c^3 + 3*(2*D*a*b + B*b^2)*c^2*d + 3*(2*C*a*b + A*b^2)*c*d^2 + (D*a^
2 + 2*B*a*b)*d^3)*x^7 + A*a^2*c^3*x + 1/6*((2*D*a*b + B*b^2)*c^3 + 3*(2*C*
a*b + A*b^2)*c^2*d + 3*(D*a^2 + 2*B*a*b)*c*d^2 + (C*a^2 + 2*A*a*b)*d^3)*x^
6 + 1/5*(B*a^2*d^3 + (2*C*a*b + A*b^2)*c^3 + 3*(D*a^2 + 2*B*a*b)*c^2*d + 3
*(C*a^2 + 2*A*a*b)*c*d^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + A*a^2*d^3 + (D*a^2 +
2*B*a*b)*c^3 + 3*(C*a^2 + 2*A*a*b)*c^2*d)*x^4 + 1/3*(3*B*a^2*c^2*d + 3*A*a
^2*c*d^2 + (C*a^2 + 2*A*a*b)*c^3)*x^3 + 1/2*(B*a^2*c^3 + 3*A*a^2*c^2*d)*x^
2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.30

$$
\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= Aa^2c^3x + \frac{Db^2d^3x^{11}}{11} + x^{10}\left(\frac{Cb^2d^3}{10} + \frac{3Db^2cd^2}{10}\right) \\
&\quad + x^9\left(\frac{Bb^2d^3}{9} + \frac{Cb^2cd^2}{3} + \frac{2Dabd^3}{9} + \frac{Db^2c^2d}{3}\right) \\
&\quad + x^8\left(\frac{Ab^2d^3}{8} + \frac{3Bb^2cd^2}{8} + \frac{Cabd^3}{4} + \frac{3Cb^2c^2d}{8} + \frac{3Dabcd^2}{4} + \frac{Db^2c^3}{8}\right) + x^7 \\
&\quad \cdot \left(\frac{3Ab^2cd^2}{7} + \frac{2Babd^3}{7} + \frac{3Bb^2c^2d}{7} + \frac{6Cabcd^2}{7} + \frac{Cb^2c^3}{7} + \frac{Da^2d^3}{7} + \frac{6Dabc^2d}{7}\right) \\
&\quad + x^6\left(\frac{Aabd^3}{3} + \frac{Ab^2c^2d}{2} + Babcd^2 + \frac{Bb^2c^3}{6} + \frac{Ca^2d^3}{6} + Cabc^2d + \frac{Da^2cd^2}{2} + \frac{Dabc^3}{3}\right) \\
&\quad + x^5\cdot\left(\frac{6Aabcd^2}{5} + \frac{Ab^2c^3}{5} + \frac{Ba^2d^3}{5} + \frac{6Babc^2d}{5} + \frac{3Ca^2cd^2}{5} + \frac{2Cabc^3}{5} + \frac{3Da^2c^2d}{5}\right) \\
&\quad + x^4\left(\frac{Aa^2d^3}{4} + \frac{3Aabc^2d}{2} + \frac{3Ba^2cd^2}{4} + \frac{Babc^3}{2} + \frac{3Ca^2c^2d}{4} + \frac{Da^2c^3}{4}\right) \\
&\quad + x^3\left(Aa^2cd^2 + \frac{2Aabc^3}{3} + Ba^2c^2d + \frac{Ca^2c^3}{3}\right) + x^2\cdot\left(\frac{3Aa^2c^2d}{2} + \frac{Ba^2c^3}{2}\right)
\end{aligned}
$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*c**3*x + D*b**2*d**3*x**11/11 + x**10*(C*b**2*d**3/10 + 3*D*b**2*c*
d**2/10) + x**9*(B*b**2*d**3/9 + C*b**2*c*d**2/3 + 2*D*a*b*d**3/9 + D*b**2
*c**2*d/3) + x**8*(A*b**2*d**3/8 + 3*B*b**2*c*d**2/8 + C*a*b*d**3/4 + 3*C*
b**2*c**2*d/8 + 3*D*a*b*c*d**2/4 + D*b**2*c**3/8) + x**7*(3*A*b**2*c*d**2/
7 + 2*B*a*b*d**3/7 + 3*B*b**2*c**2*d/7 + 6*C*a*b*c*d**2/7 + C*b**2*c**3/7
+ D*a**2*d**3/7 + 6*D*a*b*c**2*d/7) + x**6*(A*a*b*d**3/3 + A*b**2*c**2*d/2
 + B*a*b*c*d**2 + B*b**2*c**3/6 + C*a**2*d**3/6 + C*a*b*c**2*d + D*a**2*c*
d**2/2 + D*a*b*c**3/3) + x**5*(6*A*a*b*c*d**2/5 + A*b**2*c**3/5 + B*a**2*d
**3/5 + 6*B*a*b*c**2*d/5 + 3*C*a**2*c*d**2/5 + 2*C*a*b*c**3/5 + 3*D*a**2*c
**2*d/5) + x**4*(A*a**2*d**3/4 + 3*A*a*b*c**2*d/2 + 3*B*a**2*c*d**2/4 + B*
a*b*c**3/2 + 3*C*a**2*c**2*d/4 + D*a**2*c**3/4) + x**3*(A*a**2*c*d**2 + 2*
A*a*b*c**3/3 + B*a**2*c**2*d + C*a**2*c**3/3) + x**2*(3*A*a**2*c**2*d/2 +
B*a**2*c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{11}\,Db^2d^3x^{11} \\
&+ \frac{1}{10}\left(3\,Db^2cd^2+Cb^2d^3\right)x^{10}+\frac{1}{9}\left(3\,Db^2c^2d+3\,Cb^2cd^2+\left(2\,Dab+Bb^2\right)d^3\right)x^9 \\
&+ \frac{1}{8}\left(Db^2c^3+3\,Cb^2c^2d+3\left(2\,Dab+Bb^2\right)cd^2+\left(2\,Cab+Ab^2\right)d^3\right)x^8 \\
&+ \frac{1}{7}\left(Cb^2c^3+3\left(2\,Dab+Bb^2\right)c^2d+3\left(2\,Cab+Ab^2\right)cd^2+\left(Da^2+2\,Bab\right)d^3\right)x^7 \\
&+ Aa^2c^3x \\
&+ \frac{1}{6}\left(\left(2\,Dab+Bb^2\right)c^3+3\left(2\,Cab+Ab^2\right)c^2d+3\left(Da^2+2\,Bab\right)cd^2+\left(Ca^2+2\,Aab\right)d^3\right)x^6 \\
&+ \frac{1}{5}\left(Ba^2d^3+\left(2\,Cab+Ab^2\right)c^3+3\left(Da^2+2\,Bab\right)c^2d+3\left(Ca^2+2\,Aab\right)cd^2\right)x^5 \\
&+ \frac{1}{4}\left(3\,Ba^2cd^2+Aa^2d^3+\left(Da^2+2\,Bab\right)c^3+3\left(Ca^2+2\,Aab\right)c^2d\right)x^4 \\
&+ \frac{1}{3}\left(3\,Ba^2c^2d+3\,Aa^2cd^2+\left(Ca^2+2\,Aab\right)c^3\right)x^3+\frac{1}{2}\left(Ba^2c^3+3\,Aa^2c^2d\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*D*b^2*d^3*x^11 + 1/10*(3*D*b^2*c*d^2 + C*b^2*d^3)*x^10 + 1/9*(3*D*b^2
*c^2*d + 3*C*b^2*c*d^2 + (2*D*a*b + B*b^2)*d^3)*x^9 + 1/8*(D*b^2*c^3 + 3*C
*b^2*c^2*d + 3*(2*D*a*b + B*b^2)*c*d^2 + (2*C*a*b + A*b^2)*d^3)*x^8 + 1/7*
(C*b^2*c^3 + 3*(2*D*a*b + B*b^2)*c^2*d + 3*(2*C*a*b + A*b^2)*c*d^2 + (D*a^
2 + 2*B*a*b)*d^3)*x^7 + A*a^2*c^3*x + 1/6*((2*D*a*b + B*b^2)*c^3 + 3*(2*C*
a*b + A*b^2)*c^2*d + 3*(D*a^2 + 2*B*a*b)*c*d^2 + (C*a^2 + 2*A*a*b)*d^3)*x^
6 + 1/5*(B*a^2*d^3 + (2*C*a*b + A*b^2)*c^3 + 3*(D*a^2 + 2*B*a*b)*c^2*d + 3
*(C*a^2 + 2*A*a*b)*c*d^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + A*a^2*d^3 + (D*a^2 +
2*B*a*b)*c^3 + 3*(C*a^2 + 2*A*a*b)*c^2*d)*x^4 + 1/3*(3*B*a^2*c^2*d + 3*A*a
^2*c*d^2 + (C*a^2 + 2*A*a*b)*c^3)*x^3 + 1/2*(B*a^2*c^3 + 3*A*a^2*c^2*d)*x^
2
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.27

$$\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{1}{11} Db^2d^3x^{11} + \frac{3}{10} Db^2cd^2x^{10} + \frac{1}{10} Cb^2d^3x^{10} + \frac{1}{3} Db^2c^2dx^9 + \frac{1}{3} Cb^2cd^2x^9 + \frac{2}{9} Dabd^3x^9 + \frac{1}{9} Bb^2d^3x^9 + \frac{1}{8} Db^2c^3x^8 + \frac{3}{8} Cb^2c^2dx^8 + \frac{3}{4} Dabcd^2x^8 + \frac{3}{8} Bb^2cd^2x^8 + \frac{1}{4} Cabd^3x^8 + \frac{1}{8} Ab^2d^3x^8 + \frac{1}{7} Cb^2c^3x^7 + \frac{6}{7} Dabc^2dx^7 + \frac{3}{7} Bb^2c^2dx^7 + \frac{6}{7} Cabcd^2x^7 + \frac{3}{7} Ab^2cd^2x^7 + \frac{1}{7} Da^2d^3x^7 + \frac{2}{7} Babd^3x^7 + \frac{1}{3} Dabc^3x^6 + \frac{1}{6} Bb^2c^3x^6 + Cabc^2dx^6 + \frac{1}{2} Ab^2c^2dx^6 + \frac{1}{2} Da^2cd^2x^6 + Babcd^2x^6 + \frac{1}{6} Ca^2d^3x^6 + \frac{1}{3} Aabd^3x^6 + \frac{2}{5} Cabc^3x^5 + \frac{1}{5} Ab^2c^3x^5 + \frac{3}{5} Da^2c^2dx^5 + \frac{6}{5} Babc^2dx^5 + \frac{3}{5} Ca^2cd^2x^5 + \frac{6}{5} Aabcd^2x^5 + \frac{1}{5} Ba^2d^3x^5 + \frac{1}{4} Da^2c^3x^4 + \frac{1}{2} Babc^3x^4 + \frac{3}{4} Ca^2c^2dx^4 + \frac{3}{2} Aabc^2dx^4 + \frac{3}{4} Ba^2cd^2x^4 + \frac{1}{4} Aa^2d^3x^4 + \frac{1}{3} Ca^2c^3x^3 + \frac{2}{3} Aabc^3x^3 + Ba^2c^2dx^3 + Aa^2cd^2x^3 + \frac{1}{2} Ba^2c^3x^2 + \frac{3}{2} Aa^2c^2dx^2 + Aa^2c^3x$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^2*d^3*x^11 + 3/10*D*b^2*c*d^2*x^10 + 1/10*C*b^2*d^3*x^10 + 1/3*D*
b^2*c^2*d*x^9 + 1/3*C*b^2*c*d^2*x^9 + 2/9*D*a*b*d^3*x^9 + 1/9*B*b^2*d^3*x^
9 + 1/8*D*b^2*c^3*x^8 + 3/8*C*b^2*c^2*d*x^8 + 3/4*D*a*b*c*d^2*x^8 + 3/8*B*
b^2*c*d^2*x^8 + 1/4*C*a*b*d^3*x^8 + 1/8*A*b^2*d^3*x^8 + 1/7*C*b^2*c^3*x^7
+ 6/7*D*a*b*c^2*d*x^7 + 3/7*B*b^2*c^2*d*x^7 + 6/7*C*a*b*c*d^2*x^7 + 3/7*A*
b^2*c*d^2*x^7 + 1/7*D*a^2*d^3*x^7 + 2/7*B*a*b*d^3*x^7 + 1/3*D*a*b*c^3*x^6
+ 1/6*B*b^2*c^3*x^6 + C*a*b*c^2*d*x^6 + 1/2*A*b^2*c^2*d*x^6 + 1/2*D*a^2*c*
d^2*x^6 + B*a*b*c*d^2*x^6 + 1/6*C*a^2*d^3*x^6 + 1/3*A*a*b*d^3*x^6 + 2/5*C*
a*b*c^3*x^5 + 1/5*A*b^2*c^3*x^5 + 3/5*D*a^2*c^2*d*x^5 + 6/5*B*a*b*c^2*d*x^
5 + 3/5*C*a^2*c*d^2*x^5 + 6/5*A*a*b*c*d^2*x^5 + 1/5*B*a^2*d^3*x^5 + 1/4*D*
a^2*c^3*x^4 + 1/2*B*a*b*c^3*x^4 + 3/4*C*a^2*c^2*d*x^4 + 3/2*A*a*b*c^2*d*x^
4 + 3/4*B*a^2*c*d^2*x^4 + 1/4*A*a^2*d^3*x^4 + 1/3*C*a^2*c^3*x^3 + 2/3*A*a*
b*c^3*x^3 + B*a^2*c^2*d*x^3 + A*a^2*c*d^2*x^3 + 1/2*B*a^2*c^3*x^2 + 3/2*A*
a^2*c^2*d*x^2 + A*a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.27

$$\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{a^2\,c^3\,x^4\,D}{4}+\frac{a^2\,d^3\,x^7\,D}{7}+\frac{b^2\,c^3\,x^8\,D}{8}+\frac{b^2\,d^3\,x^{11}\,D}{11}+A\,a^2\,c^3\,x+\frac{B\,a^2\,c^3\,x^2}{2}+\frac{A\,a^2\,d^3\,x^4}{4}+\frac{A\,b^2\,c^3\,x^5}{5}+\frac{C\,a^2\,c^3\,x^3}{3}+\frac{B\,a^2\,d^3\,x^5}{5}+\frac{B\,b^2\,c^3\,x^6}{6}+\frac{A\,b^2\,d^3\,x^8}{8}+\frac{C\,a^2\,d^3\,x^6}{6}+\frac{C\,b^2\,c^3\,x^7}{7}+\frac{B\,b^2\,d^3\,x^9}{9}+\frac{C\,b^2\,d^3\,x^{10}}{10}+\frac{C\,a\,b\,d^3\,x^8}{4}+\frac{a\,b\,c^3\,x^6\,D}{3}+\frac{2\,a\,b\,d^3\,x^9\,D}{9}+\frac{3\,A\,a^2\,c^2\,d\,x^2}{2}+A\,a^2\,c\,d^2\,x^3+B\,a^2\,c^2\,d\,x^3+\frac{3\,B\,a^2\,c\,d^2\,x^4}{4}+\frac{A\,b^2\,c^2\,d\,x^6}{2}+\frac{3\,A\,b^2\,c\,d^2\,x^7}{7}+\frac{3\,C\,a^2\,c^2\,d\,x^4}{4}+\frac{3\,C\,a^2\,c\,d^2\,x^5}{5}+\frac{3\,B\,b^2\,c^2\,d\,x^7}{7}+\frac{3\,B\,b^2\,c\,d^2\,x^8}{8}+\frac{3\,C\,b^2\,c^2\,d\,x^8}{8}+\frac{C\,b^2\,c\,d^2\,x^9}{3}+\frac{3\,a^2\,c^2\,d\,x^5\,D}{5}+\frac{a^2\,c\,d^2\,x^6\,D}{2}+\frac{b^2\,c^2\,d\,x^9\,D}{3}+\frac{3\,b^2\,c\,d^2\,x^{10}\,D}{10}+\frac{2\,A\,a\,b\,c^3\,x^3}{3}+\frac{B\,a\,b\,c^3\,x^4}{2}+\frac{A\,a\,b\,d^3\,x^6}{3}+\frac{2\,C\,a\,b\,c^3\,x^5}{5}+\frac{2\,B\,a\,b\,d^3\,x^7}{7}+\frac{6\,a\,b\,c^2\,d\,x^7\,D}{7}+\frac{3\,a\,b\,c\,d^2\,x^8\,D}{4}+\frac{3\,A\,a\,b\,c^2\,d\,x^4}{2}+\frac{6\,A\,a\,b\,c\,d^2\,x^5}{5}+\frac{6\,B\,a\,b\,c^2\,d\,x^5}{5}+B\,a\,b\,c\,d^2\,x^6+C\,a\,b\,c^2\,d\,x^6+\frac{6\,C\,a\,b\,c\,d^2\,x^7}{7}$$

input
```
int((a + b*x^2)^2*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*c^3*x^4*D)/4 + (a^2*d^3*x^7*D)/7 + (b^2*c^3*x^8*D)/8 + (b^2*d^3*x^11*
D)/11 + A*a^2*c^3*x + (B*a^2*c^3*x^2)/2 + (A*a^2*d^3*x^4)/4 + (A*b^2*c^3*x
^5)/5 + (C*a^2*c^3*x^3)/3 + (B*a^2*d^3*x^5)/5 + (B*b^2*c^3*x^6)/6 + (A*b^2
*d^3*x^8)/8 + (C*a^2*d^3*x^6)/6 + (C*b^2*c^3*x^7)/7 + (B*b^2*d^3*x^9)/9 +
(C*b^2*d^3*x^10)/10 + (C*a*b*d^3*x^8)/4 + (a*b*c^3*x^6*D)/3 + (2*a*b*d^3*x
^9*D)/9 + (3*A*a^2*c^2*d*x^2)/2 + A*a^2*c*d^2*x^3 + B*a^2*c^2*d*x^3 + (3*B
*a^2*c*d^2*x^4)/4 + (A*b^2*c^2*d*x^6)/2 + (3*A*b^2*c*d^2*x^7)/7 + (3*C*a^2
*c^2*d*x^4)/4 + (3*C*a^2*c*d^2*x^5)/5 + (3*B*b^2*c^2*d*x^7)/7 + (3*B*b^2*c
*d^2*x^8)/8 + (3*C*b^2*c^2*d*x^8)/8 + (C*b^2*c*d^2*x^9)/3 + (3*a^2*c^2*d*x
^5*D)/5 + (a^2*c*d^2*x^6*D)/2 + (b^2*c^2*d*x^9*D)/3 + (3*b^2*c*d^2*x^10*D)
/10 + (2*A*a*b*c^3*x^3)/3 + (B*a*b*c^3*x^4)/2 + (A*a*b*d^3*x^6)/3 + (2*C*a
*b*c^3*x^5)/5 + (2*B*a*b*d^3*x^7)/7 + (6*a*b*c^2*d*x^7*D)/7 + (3*a*b*c*d^2
*x^8*D)/4 + (3*A*a*b*c^2*d*x^4)/2 + (6*A*a*b*c*d^2*x^5)/5 + (6*B*a*b*c^2*d
*x^5)/5 + B*a*b*c*d^2*x^6 + C*a*b*c^2*d*x^6 + (6*C*a*b*c*d^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.02

$$\int (c+dx)^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$= \frac{x(2520b^2d^4x^{10} + 11088b^2c\,d^3x^9 + 6160ab\,d^4x^8 + 3080b^3d^3x^8 + 18480b^2c^2d^2x^8 + 3465a\,b^2d^3x^7 + 27720ab}{}$$

input
```
int((d*x+c)^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(27720*a**3*c**3 + 41580*a**3*c**2*d*x + 27720*a**3*c*d**2*x**2 + 6930*
a**3*d**3*x**3 + 18480*a**2*b*c**3*x**2 + 13860*a**2*b*c**3*x + 41580*a**2
*b*c**2*d*x**3 + 27720*a**2*b*c**2*d*x**2 + 33264*a**2*b*c*d**2*x**4 + 207
90*a**2*b*c*d**2*x**3 + 9240*a**2*b*d**3*x**5 + 5544*a**2*b*d**3*x**4 + 92
40*a**2*c**4*x**2 + 27720*a**2*c**3*d*x**3 + 33264*a**2*c**2*d**2*x**4 + 1
8480*a**2*c*d**3*x**5 + 3960*a**2*d**4*x**6 + 5544*a*b**2*c**3*x**4 + 1386
0*a*b**2*c**3*x**3 + 13860*a*b**2*c**2*d*x**5 + 33264*a*b**2*c**2*d*x**4 +
 11880*a*b**2*c*d**2*x**6 + 27720*a*b**2*c*d**2*x**5 + 3465*a*b**2*d**3*x*
*7 + 7920*a*b**2*d**3*x**6 + 11088*a*b*c**4*x**4 + 36960*a*b*c**3*d*x**5 +
 47520*a*b*c**2*d**2*x**6 + 27720*a*b*c*d**3*x**7 + 6160*a*b*d**4*x**8 + 4
620*b**3*c**3*x**5 + 11880*b**3*c**2*d*x**6 + 10395*b**3*c*d**2*x**7 + 308
0*b**3*d**3*x**8 + 3960*b**2*c**4*x**6 + 13860*b**2*c**3*d*x**7 + 18480*b*
*2*c**2*d**2*x**8 + 11088*b**2*c*d**3*x**9 + 2520*b**2*d**4*x**10))/27720
```

3.9 $\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 284

$$
\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx \\
&= a^2Ac^2x+\frac{1}{3}a\left(ac(cC+2Bd)+A\left(2bc^2+ad^2\right)\right)x^3+\frac{1}{4}a^2\left(2cCd+Bd^2+c^2D\right)x^4 \\
&\quad+\frac{1}{5}\left(Ab\left(bc^2+2ad^2\right)+a(2bc(cC+2Bd)+ad(Cd+2cD))\right)x^5 \\
&\quad+\frac{1}{6}a\left(ad^2D+2b\left(2cCd+Bd^2+c^2D\right)\right)x^6 \\
&\quad+\frac{1}{7}b\left(b\left(c^2C+2Bcd+Ad^2\right)+2ad(Cd+2cD)\right)x^7 \\
&\quad+\frac{1}{8}b\left(2ad^2D+b\left(2cCd+Bd^2+c^2D\right)\right)x^8 \\
&\quad+\frac{1}{9}b^2d(Cd+2cD)x^9+\frac{1}{10}b^2d^2Dx^{10}+\frac{c(Bc+2Ad)\left(a+bx^2\right)^3}{6b}
\end{aligned}
$$

output

```
a^2*A*c^2*x+1/3*a*(a*c*(2*B*d+C*c)+A*(a*d^2+2*b*c^2))*x^3+1/4*a^2*(B*d^2+2
*C*c*d+D*c^2)*x^4+1/5*(A*b*(2*a*d^2+b*c^2)+a*(2*b*c*(2*B*d+C*c)+a*d*(C*d+2
*D*c)))*x^5+1/6*a*(a*d^2*D+2*b*(B*d^2+2*C*c*d+D*c^2))*x^6+1/7*b*(b*(A*d^2+
2*B*c*d+C*c^2)+2*a*d*(C*d+2*D*c))*x^7+1/8*b*(2*a*d^2*D+b*(B*d^2+2*C*c*d+D*
c^2))*x^8+1/9*b^2*d*(C*d+2*D*c)*x^9+1/10*b^2*d^2*D*x^10+1/6*c*(2*A*d+B*c)*
(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.08

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= a^2Ac^2x + \frac{1}{2}a^2c(Bc+2Ad)x^2 + \frac{1}{3}a\left(ac(cC+2Bd)+A\left(2bc^2+ad^2\right)\right)x^3 + \frac{1}{4}a\left(2bc(Bc+2Ad)+a\left(2cCd+Bd^2+c^2D\right)\right)x^4 + \frac{1}{5}\left(Ab\left(bc^2+2ad^2\right)+a(2bc(cC+2Bd)+ad(Cd+2cD))\right)x^5 + \frac{1}{6}\left(b^2c(Bc+2Ad)+a^2d^2D+2ab\left(2cCd+Bd^2+c^2D\right)\right)x^6 + \frac{1}{7}b\left(b\left(c^2C+2Bcd+Ad^2\right)+2ad(Cd+2cD)\right)x^7 + \frac{1}{8}b\left(2ad^2D+b\left(2cCd+Bd^2+c^2D\right)\right)x^8 + \frac{1}{9}b^2d(Cd+2cD)x^9 + \frac{1}{10}b^2d^2Dx^{10}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c^2*x + (a^2*c*(B*c + 2*A*d)*x^2)/2 + (a*(a*c*(c*C + 2*B*d) + A*(2*b
*c^2 + a*d^2))*x^3)/3 + (a*(2*b*c*(B*c + 2*A*d) + a*(2*c*C*d + B*d^2 + c^2
*D))*x^4)/4 + ((A*b*(b*c^2 + 2*a*d^2) + a*(2*b*c*(c*C + 2*B*d) + a*d*(C*d
+ 2*c*D)))*x^5)/5 + ((b^2*c*(B*c + 2*A*d) + a^2*d^2*D + 2*a*b*(2*c*C*d + B
*d^2 + c^2*D))*x^6)/6 + (b*(b*(c^2*C + 2*B*c*d + A*d^2) + 2*a*d*(C*d + 2*c
*D))*x^7)/7 + (b*(2*a*d^2*D + b*(2*c*C*d + B*d^2 + c^2*D))*x^8)/8 + (b^2*d
*(C*d + 2*c*D)*x^9)/9 + (b^2*d^2*D*x^10)/10
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2017$$

$$\int \left(bx^2+a\right)^2 \left((c+dx)^2 \left(Dx^3+Cx^2+Bx+A\right)-\left(Bc^2+2Adc\right)x\right) dx+\frac{c\left(a+bx^2\right)^3 (2Ad+Bc)}{6b}$$

$$\Big\downarrow \ 2341$$

$$\int \left(b^2d^2Dx^9+b^2d(Cd+2cD)x^8+b\left(2aDd^2+b\left(Dc^2+2Cdc+Bd^2\right)\right)x^7+b\left(b\left(Cc^2+2Bdc+Ad^2\right)+2ad(Cd\right.\right.$$
$$\frac{c\left(a+bx^2\right)^3 (2Ad+Bc)}{6b}$$

$$\Big\downarrow \ 2009$$

$$a^2Ac^2x+\frac{1}{4}a^2x^4\left(Bd^2+c^2D+2cCd\right)+\frac{1}{7}bx^7\left(2ad(2cD+Cd)+b\left(Ad^2+2Bcd+c^2C\right)\right)+\frac{1}{5}x^5\left(Ab\left(2ad^2+bc^2\right)+a(ad(2cD+Cd)+2bc(2Bd+cC))\right)+\frac{1}{3}ax^3\left(A\left(ad^2+2bc^2\right)+ac(2Bd+cC)\right)+\frac{c\left(a+bx^2\right)^3 (2Ad+Bc)}{6b}+\frac{1}{8}bx^8\left(2ad^2D+b\left(Bd^2+c^2D+2cCd\right)\right)+\frac{1}{6}ax^6\left(ad^2D+2b\left(Bd^2+c^2D+2cCd\right)\right)+\frac{1}{9}b^2dx^9(2cD+Cd)+\frac{1}{10}b^2d^2Dx^{10}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c^2*x + (a*(a*c*(c*C + 2*B*d) + A*(2*b*c^2 + a*d^2))*x^3)/3 + (a^2*(
2*c*C*d + B*d^2 + c^2*D)*x^4)/4 + ((A*b*(b*c^2 + 2*a*d^2) + a*(2*b*c*(c*C
+ 2*B*d) + a*d*(C*d + 2*c*D)))*x^5)/5 + (a*(a*d^2*D + 2*b*(2*c*C*d + B*d^2
 + c^2*D))*x^6)/6 + (b*(b*(c^2*C + 2*B*c*d + A*d^2) + 2*a*d*(C*d + 2*c*D))
*x^7)/7 + (b*(2*a*d^2*D + b*(2*c*C*d + B*d^2 + c^2*D))*x^8)/8 + (b^2*d*(C*
d + 2*c*D)*x^9)/9 + (b^2*d^2*D*x^10)/10 + (c*(B*c + 2*A*d)*(a + b*x^2)^3)/
(6*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.21

method	result
norman	$\frac{b^2 d^2 D x^{10}}{10}+\left(\frac{1}{9} b^2 d^2 C+\frac{2}{9} c d\, b^2 D\right) x^9+\left(\frac{1}{8} B\, b^2 d^2+\frac{1}{4} c d\, b^2 C+\frac{1}{4} D a b\, d^2+\frac{1}{8} D b^2 c^2\right) x^8+\left(\frac{1}{7} A\, b\right.$
default	$\frac{b^2 d^2 D x^{10}}{10}+\frac{\left(b^2 d^2 C+2 c d\, b^2 D\right) x^9}{9}+\frac{\left(\left(2 a b\, d^2+b^2 c^2\right) D+2 c d\, b^2 C+B\, b^2 d^2\right) x^8}{8}+\frac{\left(4 a b c d D+\left(2 a b\, d^2+b^2 c^2\right) C+2 B\, b^2 c d+A\right.}{7}$
gosper	$\frac{1}{4} x^8 c d\, b^2 C+\frac{1}{3} x^6 A\, b^2 c d+\frac{2}{3} x^3 B\, a^2 c d+x^2 A\, a^2 c d+\frac{1}{3} x^3 A\, d^2 a^2+\frac{1}{3} x^3 C\, a^2 c^2+\frac{1}{3} x^6 D a b\, c^2+$
parallelrisch	$\frac{1}{4} x^8 c d\, b^2 C+\frac{1}{3} x^6 A\, b^2 c d+\frac{2}{3} x^3 B\, a^2 c d+x^2 A\, a^2 c d+\frac{1}{3} x^3 A\, d^2 a^2+\frac{1}{3} x^3 C\, a^2 c^2+\frac{1}{3} x^6 D a b\, c^2+$
orering	$\frac{x\left(252 b^2 d^2 D x^9+280 C\, b^2 d^2 x^8+560 D b^2 c d\, x^8+315 B\, b^2 d^2 x^7+630 C\, b^2 c d\, x^7+630 D a b\, d^2 x^7+315 D b^2 c^2 x^7+360 A\, b^2 d^2 x^6+720 B\right.}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*d^2*D*x^10+(1/9*b^2*d^2*C+2/9*c*d*b^2*D)*x^9+(1/8*B*b^2*d^2+1/4*c
*d*b^2*C+1/4*D*a*b*d^2+1/8*D*b^2*c^2)*x^8+(1/7*A*b^2*d^2+2/7*B*b^2*c*d+2/7
*C*a*b*d^2+1/7*C*b^2*c^2+4/7*a*b*c*d*D)*x^7+(1/3*A*b^2*c*d+1/3*B*a*b*d^2+1
/6*B*b^2*c^2+2/3*a*b*c*d*C+1/6*a^2*d^2*D+1/3*D*a*b*c^2)*x^6+(2/5*A*a*b*d^2
+1/5*A*b^2*c^2+4/5*a*b*B*c*d+1/5*a^2*C*d^2+2/5*C*a*b*c^2+2/5*a^2*c*d*D)*x^
5+(A*a*b*c*d+1/4*a^2*B*d^2+1/2*B*a*b*c^2+1/2*C*a^2*c*d+1/4*a^2*c^2*D)*x^4+
(1/3*A*d^2*a^2+2/3*A*a*b*c^2+2/3*B*a^2*c*d+1/3*C*a^2*c^2)*x^3+(A*a^2*c*d+1
/2*B*a^2*c^2)*x^2+a^2*A*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.16

$$
\begin{aligned}
&\int (c+dx)^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx\\
&=\frac{1}{10}Db^2d^2x^{10}+\frac{1}{9}\left(2\,Db^2cd+Cb^2d^2\right)x^9+\frac{1}{8}\left(Db^2c^2+2\,Cb^2cd+\left(2\,Dab+Bb^2\right)d^2\right)x^8\\
&\quad+\frac{1}{7}\left(Cb^2c^2+2\left(2\,Dab+Bb^2\right)cd+\left(2\,Cab+Ab^2\right)d^2\right)x^7\\
&\quad+\frac{1}{6}\left(\left(2\,Dab+Bb^2\right)c^2+2\left(2\,Cab+Ab^2\right)cd+\left(Da^2+2\,Bab\right)d^2\right)x^6+Aa^2c^2x\\
&\quad+\frac{1}{5}\left(\left(2\,Cab+Ab^2\right)c^2+2\left(Da^2+2\,Bab\right)cd+\left(Ca^2+2\,Aab\right)d^2\right)x^5\\
&\quad+\frac{1}{4}\left(Ba^2d^2+\left(Da^2+2\,Bab\right)c^2+2\left(Ca^2+2\,Aab\right)cd\right)x^4\\
&\quad+\frac{1}{3}\left(2\,Ba^2cd+Aa^2d^2+\left(Ca^2+2\,Aab\right)c^2\right)x^3+\frac{1}{2}\left(Ba^2c^2+2\,Aa^2cd\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^2*d^2*x^10 + 1/9*(2*D*b^2*c*d + C*b^2*d^2)*x^9 + 1/8*(D*b^2*c^2 +
 2*C*b^2*c*d + (2*D*a*b + B*b^2)*d^2)*x^8 + 1/7*(C*b^2*c^2 + 2*(2*D*a*b +
B*b^2)*c*d + (2*C*a*b + A*b^2)*d^2)*x^7 + 1/6*((2*D*a*b + B*b^2)*c^2 + 2*(
2*C*a*b + A*b^2)*c*d + (D*a^2 + 2*B*a*b)*d^2)*x^6 + A*a^2*c^2*x + 1/5*((2*
C*a*b + A*b^2)*c^2 + 2*(D*a^2 + 2*B*a*b)*c*d + (C*a^2 + 2*A*a*b)*d^2)*x^5
+ 1/4*(B*a^2*d^2 + (D*a^2 + 2*B*a*b)*c^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*x^4 +
1/3*(2*B*a^2*c*d + A*a^2*d^2 + (C*a^2 + 2*A*a*b)*c^2)*x^3 + 1/2*(B*a^2*c^2
 + 2*A*a^2*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.45

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= Aa^2c^2x + \frac{Db^2d^2x^{10}}{10} + x^9\left(\frac{Cb^2d^2}{9} + \frac{2Db^2cd}{9}\right) + x^8\left(\frac{Bb^2d^2}{8} + \frac{Cb^2cd}{4} + \frac{Dabd^2}{4} + \frac{Db^2c^2}{8}\right)$$
$$+ x^7\left(\frac{Ab^2d^2}{7} + \frac{2Bb^2cd}{7} + \frac{2Cabd^2}{7} + \frac{Cb^2c^2}{7} + \frac{4Dabcd}{7}\right)$$
$$+ x^6\left(\frac{Ab^2cd}{3} + \frac{Babd^2}{3} + \frac{Bb^2c^2}{6} + \frac{2Cabcd}{3} + \frac{Da^2d^2}{6} + \frac{Dabc^2}{3}\right)$$
$$+ x^5 \cdot \left(\frac{2Aabd^2}{5} + \frac{Ab^2c^2}{5} + \frac{4Babcd}{5} + \frac{Ca^2d^2}{5} + \frac{2Cabc^2}{5} + \frac{2Da^2cd}{5}\right)$$
$$+ x^4\left(Aabcd + \frac{Ba^2d^2}{4} + \frac{Babc^2}{2} + \frac{Ca^2cd}{2} + \frac{Da^2c^2}{4}\right)$$
$$+ x^3\left(\frac{Aa^2d^2}{3} + \frac{2Aabc^2}{3} + \frac{2Ba^2cd}{3} + \frac{Ca^2c^2}{3}\right) + x^2\left(Aa^2cd + \frac{Ba^2c^2}{2}\right)$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*c**2*x + D*b**2*d**2*x**10/10 + x**9*(C*b**2*d**2/9 + 2*D*b**2*c*d/
9) + x**8*(B*b**2*d**2/8 + C*b**2*c*d/4 + D*a*b*d**2/4 + D*b**2*c**2/8) +
x**7*(A*b**2*d**2/7 + 2*B*b**2*c*d/7 + 2*C*a*b*d**2/7 + C*b**2*c**2/7 + 4*
D*a*b*c*d/7) + x**6*(A*b**2*c*d/3 + B*a*b*d**2/3 + B*b**2*c**2/6 + 2*C*a*b
*c*d/3 + D*a**2*d**2/6 + D*a*b*c**2/3) + x**5*(2*A*a*b*d**2/5 + A*b**2*c**
2/5 + 4*B*a*b*c*d/5 + C*a**2*d**2/5 + 2*C*a*b*c**2/5 + 2*D*a**2*c*d/5) + x
**4*(A*a*b*c*d + B*a**2*d**2/4 + B*a*b*c**2/2 + C*a**2*c*d/2 + D*a**2*c**2
/4) + x**3*(A*a**2*d**2/3 + 2*A*a*b*c**2/3 + 2*B*a**2*c*d/3 + C*a**2*c**2/
3) + x**2*(A*a**2*c*d + B*a**2*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.16

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{1}{10} Db^2d^2x^{10} + \frac{1}{9} \left(2\, Db^2cd + Cb^2d^2\right)x^9 + \frac{1}{8} \left(Db^2c^2 + 2\, Cb^2cd + \left(2\, Dab + Bb^2\right)d^2\right)x^8$$
$$+ \frac{1}{7} \left(Cb^2c^2 + 2\left(2\, Dab + Bb^2\right)cd + \left(2\, Cab + Ab^2\right)d^2\right)x^7$$
$$+ \frac{1}{6} \left(\left(2\, Dab + Bb^2\right)c^2 + 2\left(2\, Cab + Ab^2\right)cd + \left(Da^2 + 2\, Bab\right)d^2\right)x^6 + Aa^2c^2x$$
$$+ \frac{1}{5} \left(\left(2\, Cab + Ab^2\right)c^2 + 2\left(Da^2 + 2\, Bab\right)cd + \left(Ca^2 + 2\, Aab\right)d^2\right)x^5$$
$$+ \frac{1}{4} \left(Ba^2d^2 + \left(Da^2 + 2\, Bab\right)c^2 + 2\left(Ca^2 + 2\, Aab\right)cd\right)x^4$$
$$+ \frac{1}{3} \left(2\, Ba^2cd + Aa^2d^2 + \left(Ca^2 + 2\, Aab\right)c^2\right)x^3 + \frac{1}{2} \left(Ba^2c^2 + 2\, Aa^2cd\right)x^2$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^2*d^2*x^10 + 1/9*(2*D*b^2*c*d + C*b^2*d^2)*x^9 + 1/8*(D*b^2*c^2 +
 2*C*b^2*c*d + (2*D*a*b + B*b^2)*d^2)*x^8 + 1/7*(C*b^2*c^2 + 2*(2*D*a*b +
B*b^2)*c*d + (2*C*a*b + A*b^2)*d^2)*x^7 + 1/6*((2*D*a*b + B*b^2)*c^2 + 2*(
2*C*a*b + A*b^2)*c*d + (D*a^2 + 2*B*a*b)*d^2)*x^6 + A*a^2*c^2*x + 1/5*((2*
C*a*b + A*b^2)*c^2 + 2*(D*a^2 + 2*B*a*b)*c*d + (C*a^2 + 2*A*a*b)*d^2)*x^5
+ 1/4*(B*a^2*d^2 + (D*a^2 + 2*B*a*b)*c^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*x^4 +
1/3*(2*B*a^2*c*d + A*a^2*d^2 + (C*a^2 + 2*A*a*b)*c^2)*x^3 + 1/2*(B*a^2*c^2
 + 2*A*a^2*c*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.42

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\begin{aligned}
&= \frac{1}{10} Db^2d^2x^{10} + \frac{2}{9} Db^2cdx^9 + \frac{1}{9} Cb^2d^2x^9 + \frac{1}{8} Db^2c^2x^8 + \frac{1}{4} Cb^2cdx^8 + \frac{1}{4} Dabd^2x^8 \\
&\quad + \frac{1}{8} Bb^2d^2x^8 + \frac{1}{7} Cb^2c^2x^7 + \frac{4}{7} Dabcdx^7 + \frac{2}{7} Bb^2cdx^7 + \frac{2}{7} Cabd^2x^7 + \frac{1}{7} Ab^2d^2x^7 \\
&\quad + \frac{1}{3} Dabc^2x^6 + \frac{1}{6} Bb^2c^2x^6 + \frac{2}{3} Cabcdx^6 + \frac{1}{3} Ab^2cdx^6 + \frac{1}{6} Da^2d^2x^6 + \frac{1}{3} Babd^2x^6 \\
&\quad + \frac{2}{5} Cabc^2x^5 + \frac{1}{5} Ab^2c^2x^5 + \frac{2}{5} Da^2cdx^5 + \frac{4}{5} Babcdx^5 + \frac{1}{5} Ca^2d^2x^5 + \frac{2}{5} Aabd^2x^5 \\
&\quad + \frac{1}{4} Da^2c^2x^4 + \frac{1}{2} Babc^2x^4 + \frac{1}{2} Ca^2cdx^4 + Aabcdx^4 + \frac{1}{4} Ba^2d^2x^4 + \frac{1}{3} Ca^2c^2x^3 \\
&\quad + \frac{2}{3} Aabc^2x^3 + \frac{2}{3} Ba^2cdx^3 + \frac{1}{3} Aa^2d^2x^3 + \frac{1}{2} Ba^2c^2x^2 + Aa^2cdx^2 + Aa^2c^2x
\end{aligned}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^2*d^2*x^10 + 2/9*D*b^2*c*d*x^9 + 1/9*C*b^2*d^2*x^9 + 1/8*D*b^2*c^
2*x^8 + 1/4*C*b^2*c*d*x^8 + 1/4*D*a*b*d^2*x^8 + 1/8*B*b^2*d^2*x^8 + 1/7*C*
b^2*c^2*x^7 + 4/7*D*a*b*c*d*x^7 + 2/7*B*b^2*c*d*x^7 + 2/7*C*a*b*d^2*x^7 +
1/7*A*b^2*d^2*x^7 + 1/3*D*a*b*c^2*x^6 + 1/6*B*b^2*c^2*x^6 + 2/3*C*a*b*c*d*
x^6 + 1/3*A*b^2*c*d*x^6 + 1/6*D*a^2*d^2*x^6 + 1/3*B*a*b*d^2*x^6 + 2/5*C*a*
b*c^2*x^5 + 1/5*A*b^2*c^2*x^5 + 2/5*D*a^2*c*d*x^5 + 4/5*B*a*b*c*d*x^5 + 1/
5*C*a^2*d^2*x^5 + 2/5*A*a*b*d^2*x^5 + 1/4*D*a^2*c^2*x^4 + 1/2*B*a*b*c^2*x^
4 + 1/2*C*a^2*c*d*x^4 + A*a*b*c*d*x^4 + 1/4*B*a^2*d^2*x^4 + 1/3*C*a^2*c^2*
x^3 + 2/3*A*a*b*c^2*x^3 + 2/3*B*a^2*c*d*x^3 + 1/3*A*a^2*d^2*x^3 + 1/2*B*a^
2*c^2*x^2 + A*a^2*c*d*x^2 + A*a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.42

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{a^2\,c^2\,x^4\,D}{4}+\frac{a^2\,d^2\,x^6\,D}{6}+\frac{b^2\,c^2\,x^8\,D}{8}+\frac{b^2\,d^2\,x^{10}\,D}{10}+A\,a^2\,c^2\,x+\frac{B\,a^2\,c^2\,x^2}{2}+\frac{A\,a^2\,d^2\,x^3}{3}+\frac{A\,b^2\,c^2\,x^5}{5}+\frac{C\,a^2\,c^2\,x^3}{3}+\frac{B\,a^2\,d^2\,x^4}{4}+\frac{B\,b^2\,c^2\,x^6}{6}+\frac{A\,b^2\,d^2\,x^7}{7}+\frac{C\,a^2\,d^2\,x^5}{5}+\frac{C\,b^2\,c^2\,x^7}{7}+\frac{B\,b^2\,d^2\,x^8}{8}+\frac{C\,b^2\,d^2\,x^9}{9}+\frac{2\,B\,a^2\,c\,d\,x^3}{3}+\frac{A\,b^2\,c\,d\,x^6}{3}+\frac{2\,C\,a\,b\,d^2\,x^7}{7}+\frac{C\,a^2\,c\,d\,x^4}{2}+\frac{2\,B\,b^2\,c\,d\,x^7}{7}+\frac{C\,b^2\,c\,d\,x^8}{4}+\frac{a\,b\,c^2\,x^6\,D}{3}+\frac{a\,b\,d^2\,x^8\,D}{4}+\frac{2\,a^2\,c\,d\,x^5\,D}{5}+\frac{2\,b^2\,c\,d\,x^9\,D}{9}+\frac{2\,A\,a\,b\,c^2\,x^3}{3}+\frac{B\,a\,b\,c^2\,x^4}{2}+\frac{2\,A\,a\,b\,d^2\,x^5}{5}+A\,a^2\,c\,d\,x^2+\frac{2\,C\,a\,b\,c^2\,x^5}{5}+\frac{B\,a\,b\,d^2\,x^6}{3}+A\,a\,b\,c\,d\,x^4+\frac{4\,B\,a\,b\,c\,d\,x^5}{5}+\frac{2\,C\,a\,b\,c\,d\,x^6}{3}+\frac{4\,a\,b\,c\,d\,x^7\,D}{7}$$

input
```
int((a + b*x^2)^2*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*c^2*x^4*D)/4 + (a^2*d^2*x^6*D)/6 + (b^2*c^2*x^8*D)/8 + (b^2*d^2*x^10*
D)/10 + A*a^2*c^2*x + (B*a^2*c^2*x^2)/2 + (A*a^2*d^2*x^3)/3 + (A*b^2*c^2*x
^5)/5 + (C*a^2*c^2*x^3)/3 + (B*a^2*d^2*x^4)/4 + (B*b^2*c^2*x^6)/6 + (A*b^2
*d^2*x^7)/7 + (C*a^2*d^2*x^5)/5 + (C*b^2*c^2*x^7)/7 + (B*b^2*d^2*x^8)/8 +
(C*b^2*d^2*x^9)/9 + (2*B*a^2*c*d*x^3)/3 + (A*b^2*c*d*x^6)/3 + (2*C*a*b*d^2
*x^7)/7 + (C*a^2*c*d*x^4)/2 + (2*B*b^2*c*d*x^7)/7 + (C*b^2*c*d*x^8)/4 + (a
*b*c^2*x^6*D)/3 + (a*b*d^2*x^8*D)/4 + (2*a^2*c*d*x^5*D)/5 + (2*b^2*c*d*x^9
*D)/9 + (2*A*a*b*c^2*x^3)/3 + (B*a*b*c^2*x^4)/2 + (2*A*a*b*d^2*x^5)/5 + A*
a^2*c*d*x^2 + (2*C*a*b*c^2*x^5)/5 + (B*a*b*d^2*x^6)/3 + A*a*b*c*d*x^4 + (4
*B*a*b*c*d*x^5)/5 + (2*C*a*b*c*d*x^6)/3 + (4*a*b*c*d*x^7*D)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.18

$$\int (c+dx)^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(84b^2d^3x^9 + 280b^2c\,d^2x^8 + 210ab\,d^3x^7 + 105b^3d^2x^7 + 315b^2c^2d\,x^7 + 120a\,b^2d^2x^6 + 720abc\,d^2x^6 + 240b^3}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**3*c**2 + 840*a**3*c*d*x + 280*a**3*d**2*x**2 + 560*a**2*b*c**2*
x**2 + 420*a**2*b*c**2*x + 840*a**2*b*c*d*x**3 + 560*a**2*b*c*d*x**2 + 336
*a**2*b*d**2*x**4 + 210*a**2*b*d**2*x**3 + 280*a**2*c**3*x**2 + 630*a**2*c
**2*d*x**3 + 504*a**2*c*d**2*x**4 + 140*a**2*d**3*x**5 + 168*a*b**2*c**2*x
**4 + 420*a*b**2*c**2*x**3 + 280*a*b**2*c*d*x**5 + 672*a*b**2*c*d*x**4 + 1
20*a*b**2*d**2*x**6 + 280*a*b**2*d**2*x**5 + 336*a*b*c**3*x**4 + 840*a*b*c
**2*d*x**5 + 720*a*b*c*d**2*x**6 + 210*a*b*d**3*x**7 + 140*b**3*c**2*x**5
+ 240*b**3*c*d*x**6 + 105*b**3*d**2*x**7 + 120*b**2*c**3*x**6 + 315*b**2*c
**2*d*x**7 + 280*b**2*c*d**2*x**8 + 84*b**2*d**3*x**9))/840
```

3.10 $\int (c+dx) \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 153

$$\begin{aligned}
&\int (c+dx) \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= a^2Acx+\frac{1}{3}a(2Abc+acC+aBd)x^3+\frac{1}{5}\left(Ab^2c+a(2b(cC+Bd)+adD)\right)x^5 \\
&\quad +\frac{1}{7}b(bcC+bBd+2adD)x^7+\frac{1}{9}b^2dDx^9 \\
&\quad +\frac{(bBc+Abd-aCd-acD)\left(a+bx^2\right)^3}{6b^2}+\frac{(Cd+cD)\left(a+bx^2\right)^4}{8b^2}
\end{aligned}$$

output
```
a^2*A*c*x+1/3*a*(2*A*b*c+B*a*d+C*a*c)*x^3+1/5*(A*b^2*c+a*(2*b*(B*d+C*c)+D*
a*d))*x^5+1/7*b*(B*b*d+C*b*c+2*D*a*d)*x^7+1/9*b^2*d*D*x^9+1/6*(A*b*d+B*b*c
-C*a*d-D*a*c)*(b*x^2+a)^3/b^2+1/8*(C*d+D*c)*(b*x^2+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20

$$\int (c+dx) \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\, dx$$
$$= a^2Acx+\frac{1}{2}a^2(Bc+Ad)x^2+\frac{1}{3}a(2Abc+acC+aBd)x^3$$
$$+\frac{1}{4}a(2bBc+2Abd+aCd+acD)x^4$$
$$+\frac{1}{5}\left(Ab^2c+2abcC+2abBd+a^2dD\right)x^5+\frac{1}{6}b(bBc+Abd+2aCd+2acD)x^6$$
$$+\frac{1}{7}b(bcC+bBd+2adD)x^7+\frac{1}{8}b^2(Cd+cD)x^8+\frac{1}{9}b^2dDx^9$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c*x + (a^2*(B*c + A*d)*x^2)/2 + (a*(2*A*b*c + a*c*C + a*B*d)*x^3)/3
+ (a*(2*b*B*c + 2*A*b*d + a*C*d + a*c*D)*x^4)/4 + ((A*b^2*c + 2*a*b*c*C +
2*a*b*B*d + a^2*d*D)*x^5)/5 + (b*(b*B*c + A*b*d + 2*a*C*d + 2*a*c*D)*x^6)/
6 + (b*(b*c*C + b*B*d + 2*a*d*D)*x^7)/7 + (b^2*(C*d + c*D)*x^8)/8 + (b^2*d
*D*x^9)/9
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (c+dx) \left(A+Bx+Cx^2+Dx^3\right)\, dx$$
$$\Big\downarrow \text{2017}$$
$$\int \left(bx^2+a\right)^2 \left((c+dx)\left(Dx^3+Cx^2+Bx+A\right)-(Bc+Ad)x\right)dx+\frac{\left(a+bx^2\right)^3(Ad+Bc)}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^2dDx^8+b^2(Cd+cD)x^7+b(bcC+bBd+2adD)x^6+2ab(Cd+cD)x^5+\left(Acb^2+a(2b(cC+Bd)+adD)\right.\right.$$
$$\frac{\left(a+bx^2\right)^3(Ad+Bc)}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2Acx+\frac{1}{4}a^2x^4(cD+Cd)+\frac{1}{5}x^5\left(a(adD+2b(Bd+cC))+Ab^2c\right)+\frac{1}{3}ax^3(aBd+acC+2Abc)+\frac{\left(a+bx^2\right)^3(Ad+Bc)}{6b}+\frac{1}{7}bx^7(2adD+bBd+bcC)+\frac{1}{3}abx^6(cD+Cd)+\frac{1}{8}b^2x^8(cD+Cd)+\frac{1}{9}b^2dDx^9$$

input
```
Int[(c + d*x)*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c*x + (a*(2*A*b*c + a*c*C + a*B*d)*x^3)/3 + (a^2*(C*d + c*D)*x^4)/4
+ ((A*b^2*c + a*(2*b*(c*C + B*d) + a*d*D))*x^5)/5 + (a*b*(C*d + c*D)*x^6)/
3 + (b*(b*c*C + b*B*d + 2*a*d*D)*x^7)/7 + (b^2*(C*d + c*D)*x^8)/8 + (b^2*d
*D*x^9)/9 + ((B*c + A*d)*(a + b*x^2)^3)/(6*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.26

method	result
default	$\frac{b^2 dD x^9}{9}+\frac{(b^2 dC+b^2 cD) x^8}{8}+\frac{(B b^2 d+C b^2 c+2dabD) x^7}{7}+\frac{(A b^2 d+B b^2 c+2dabC+2abcD) x^6}{6}+\frac{(A b^2 c+2Babd+2C\cdots}{5}$
norman	$\frac{b^2 dD x^9}{9}+\left(\frac{1}{8} b^2 dC+\frac{1}{8} b^2 cD\right) x^8+\left(\frac{1}{7} B b^2 d+\frac{1}{7} C b^2 c+\frac{2}{7} dabD\right) x^7+\left(\frac{1}{6} A b^2 d+\frac{1}{6} B b^2 c+\frac{1}{3} d\cdots\right.$
orering	$\frac{x(280 b^2 dD x^8+315 C b^2 d x^7+315 D b^2 c x^7+360 B b^2 d x^6+360 C b^2 c x^6+720 Dabd x^6+420 A b^2 d x^5+420 B b^2 c x^5+840 Cabd x^5\cdots}{}$
gosper	$\frac{1}{9} b^2 dD x^9+\frac{1}{8} x^8 b^2 dC+\frac{1}{8} x^8 b^2 cD+\frac{1}{7} x^7 B b^2 d+\frac{1}{7} x^7 C b^2 c+\frac{2}{7} x^7 dabD+\frac{1}{6} x^6 A b^2 d+\frac{1}{6} x^6 B b\cdots$
parallelrisch	$\frac{1}{9} b^2 dD x^9+\frac{1}{8} x^8 b^2 dC+\frac{1}{8} x^8 b^2 cD+\frac{1}{7} x^7 B b^2 d+\frac{1}{7} x^7 C b^2 c+\frac{2}{7} x^7 dabD+\frac{1}{6} x^6 A b^2 d+\frac{1}{6} x^6 B b\cdots$

input
```
int((d*x+c)*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*d*D*x^9+1/8*(C*b^2*d+D*b^2*c)*x^8+1/7*(B*b^2*d+C*b^2*c+2*D*a*b*d)*
x^7+1/6*(A*b^2*d+B*b^2*c+2*C*a*b*d+2*D*a*b*c)*x^6+1/5*(A*b^2*c+2*B*a*b*d+2
*C*a*b*c+D*a^2*d)*x^5+1/4*(2*A*a*b*d+2*B*a*b*c+C*a^2*d+D*a^2*c)*x^4+1/3*(2
*A*a*b*c+B*a^2*d+C*a^2*c)*x^3+1/2*(A*a^2*d+B*a^2*c)*x^2+a^2*A*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&=\frac{1}{9}\,Db^2dx^9+\frac{1}{8}\,(Db^2c+Cb^2d)x^8+\frac{1}{7}\,(Cb^2c+\left(2\,Dab+Bb^2\right)d)x^7 \\
&\quad+\frac{1}{6}\,(\left(2\,Dab+Bb^2\right)c+\left(2\,Cab+Ab^2\right)d)x^6 \\
&\quad+\frac{1}{5}\,(\left(2\,Cab+Ab^2\right)c+\left(Da^2+2\,Bab\right)d)x^5+Aa^2cx \\
&\quad+\frac{1}{4}\,(\left(Da^2+2\,Bab\right)c+\left(Ca^2+2\,Aab\right)d)x^4 \\
&\quad+\frac{1}{3}\,(Ba^2d+\left(Ca^2+2\,Aab\right)c)x^3+\frac{1}{2}\,(Ba^2c+Aa^2d)x^2
\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b^2*d*x^9 + 1/8*(D*b^2*c + C*b^2*d)*x^8 + 1/7*(C*b^2*c + (2*D*a*b +
B*b^2)*d)*x^7 + 1/6*((2*D*a*b + B*b^2)*c + (2*C*a*b + A*b^2)*d)*x^6 + 1/5*
((2*C*a*b + A*b^2)*c + (D*a^2 + 2*B*a*b)*d)*x^5 + A*a^2*c*x + 1/4*((D*a^2
+ 2*B*a*b)*c + (C*a^2 + 2*A*a*b)*d)*x^4 + 1/3*(B*a^2*d + (C*a^2 + 2*A*a*b)
*c)*x^3 + 1/2*(B*a^2*c + A*a^2*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.54

$$\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= Aa^2cx + \frac{Db^2dx^9}{9} + x^8\left(\frac{Cb^2d}{8} + \frac{Db^2c}{8}\right) + x^7\left(\frac{Bb^2d}{7} + \frac{Cb^2c}{7} + \frac{2Dabd}{7}\right)$$
$$+ x^6\left(\frac{Ab^2d}{6} + \frac{Bb^2c}{6} + \frac{Cabd}{3} + \frac{Dabc}{3}\right) + x^5\left(\frac{Ab^2c}{5} + \frac{2Babd}{5} + \frac{2Cabc}{5} + \frac{Da^2d}{5}\right)$$
$$+ x^4\left(\frac{Aabd}{2} + \frac{Babc}{2} + \frac{Ca^2d}{4} + \frac{Da^2c}{4}\right) + x^3$$
$$\cdot\left(\frac{2Aabc}{3} + \frac{Ba^2d}{3} + \frac{Ca^2c}{3}\right) + x^2\left(\frac{Aa^2d}{2} + \frac{Ba^2c}{2}\right)$$

input
```
integrate((d*x+c)*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*c*x + D*b**2*d*x**9/9 + x**8*(C*b**2*d/8 + D*b**2*c/8) + x**7*(B*b*
*2*d/7 + C*b**2*c/7 + 2*D*a*b*d/7) + x**6*(A*b**2*d/6 + B*b**2*c/6 + C*a*b
*d/3 + D*a*b*c/3) + x**5*(A*b**2*c/5 + 2*B*a*b*d/5 + 2*C*a*b*c/5 + D*a**2*
d/5) + x**4*(A*a*b*d/2 + B*a*b*c/2 + C*a**2*d/4 + D*a**2*c/4) + x**3*(2*A*
a*b*c/3 + B*a**2*d/3 + C*a**2*c/3) + x**2*(A*a**2*d/2 + B*a**2*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.31

$$
\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{9}\,Db^2dx^9+\frac{1}{8}\left(Db^2c+Cb^2d\right)x^8+\frac{1}{7}\left(Cb^2c+\left(2\,Dab+Bb^2\right)d\right)x^7 \\
&\quad+\frac{1}{6}\left(\left(2\,Dab+Bb^2\right)c+\left(2\,Cab+Ab^2\right)d\right)x^6 \\
&\quad+\frac{1}{5}\left(\left(2\,Cab+Ab^2\right)c+\left(Da^2+2\,Bab\right)d\right)x^5+Aa^2cx \\
&\quad+\frac{1}{4}\left(\left(Da^2+2\,Bab\right)c+\left(Ca^2+2\,Aab\right)d\right)x^4 \\
&\quad+\frac{1}{3}\left(Ba^2d+\left(Ca^2+2\,Aab\right)c\right)x^3+\frac{1}{2}\left(Ba^2c+Aa^2d\right)x^2
\end{aligned}
$$

input

```
integrate((d*x+c)*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output

```
1/9*D*b^2*d*x^9 + 1/8*(D*b^2*c + C*b^2*d)*x^8 + 1/7*(C*b^2*c + (2*D*a*b +
B*b^2)*d)*x^7 + 1/6*((2*D*a*b + B*b^2)*c + (2*C*a*b + A*b^2)*d)*x^6 + 1/5*
((2*C*a*b + A*b^2)*c + (D*a^2 + 2*B*a*b)*d)*x^5 + A*a^2*c*x + 1/4*((D*a^2
+ 2*B*a*b)*c + (C*a^2 + 2*A*a*b)*d)*x^4 + 1/3*(B*a^2*d + (C*a^2 + 2*A*a*b)
*c)*x^3 + 1/2*(B*a^2*c + A*a^2*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.50

$$
\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{9}\,Db^2dx^9+\frac{1}{8}\,Db^2cx^8+\frac{1}{8}\,Cb^2dx^8+\frac{1}{7}\,Cb^2cx^7+\frac{2}{7}\,Dabdx^7+\frac{1}{7}\,Bb^2dx^7 \\
&\quad+\frac{1}{3}\,Dabcx^6+\frac{1}{6}\,Bb^2cx^6+\frac{1}{3}\,Cabdx^6+\frac{1}{6}\,Ab^2dx^6+\frac{2}{5}\,Cabcx^5+\frac{1}{5}\,Ab^2cx^5 \\
&\quad+\frac{1}{5}\,Da^2dx^5+\frac{2}{5}\,Babdx^5+\frac{1}{4}\,Da^2cx^4+\frac{1}{2}\,Babcx^4+\frac{1}{4}\,Ca^2dx^4+\frac{1}{2}\,Aabdx^4 \\
&\quad+\frac{1}{3}\,Ca^2cx^3+\frac{2}{3}\,Aabcx^3+\frac{1}{3}\,Ba^2dx^3+\frac{1}{2}\,Ba^2cx^2+\frac{1}{2}\,Aa^2dx^2+Aa^2cx
\end{aligned}
$$

input
```
integrate((d*x+c)*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b^2*d*x^9 + 1/8*D*b^2*c*x^8 + 1/8*C*b^2*d*x^8 + 1/7*C*b^2*c*x^7 + 2/
7*D*a*b*d*x^7 + 1/7*B*b^2*d*x^7 + 1/3*D*a*b*c*x^6 + 1/6*B*b^2*c*x^6 + 1/3*
C*a*b*d*x^6 + 1/6*A*b^2*d*x^6 + 2/5*C*a*b*c*x^5 + 1/5*A*b^2*c*x^5 + 1/5*D*
a^2*d*x^5 + 2/5*B*a*b*d*x^5 + 1/4*D*a^2*c*x^4 + 1/2*B*a*b*c*x^4 + 1/4*C*a^
2*d*x^4 + 1/2*A*a*b*d*x^4 + 1/3*C*a^2*c*x^3 + 2/3*A*a*b*c*x^3 + 1/3*B*a^2*
d*x^3 + 1/2*B*a^2*c*x^2 + 1/2*A*a^2*d*x^2 + A*a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.44

$$\begin{aligned}
\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx
&= \frac{A\,d\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6}+\frac{B\,c\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6} \\
&\quad+\frac{B\,d\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{C\,c\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105} \\
&\quad+\frac{C\,d\,x^4\,(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4)}{24}+\frac{c\,x^4\,D\,(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4)}{24} \\
&\quad+\frac{d\,x^5\,D\,(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4)}{315}+\frac{A\,c\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}
\end{aligned}$$

input
```
int((a + b*x^2)^2*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*d*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (B*c*x^2*(3*a^2 + b^2*x^4 + 3*
a*b*x^2))/6 + (B*d*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*c*x^3*
(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*d*x^4*(6*a^2 + 3*b^2*x^4 + 8*
a*b*x^2))/24 + (c*x^4*D*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (d*x^5*D*(63
*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (A*c*x*(15*a^2 + 3*b^2*x^4 + 10*a*b
*x^2))/15
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35

$$\int (c+dx)\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x(140b^2d^2x^8+315b^2cd\,x^7+360ab\,d^2x^6+180b^3d\,x^6+180b^2c^2x^6+210a\,b^2d\,x^5+840abcd\,x^5+210b^3c\,x^5$$

input
```
int((d*x+c)*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(1260*a**3*c + 630*a**3*d*x + 840*a**2*b*c*x**2 + 630*a**2*b*c*x + 630*
a**2*b*d*x**3 + 420*a**2*b*d*x**2 + 420*a**2*c**2*x**2 + 630*a**2*c*d*x**3
 + 252*a**2*d**2*x**4 + 252*a*b**2*c*x**4 + 630*a*b**2*c*x**3 + 210*a*b**2
*d*x**5 + 504*a*b**2*d*x**4 + 504*a*b*c**2*x**4 + 840*a*b*c*d*x**5 + 360*a
*b*d**2*x**6 + 210*b**3*c*x**5 + 180*b**3*d*x**6 + 180*b**2*c**2*x**6 + 31
5*b**2*c*d*x**7 + 140*b**2*d**2*x**8))/1260
```

3.11 $\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4 +\frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abDx^6 +\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B\left(a+bx^2\right)^3}{6b}$$

output
```
a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*
D*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{840}\left(70a^2x(12A+x(6B+x(4C+3Dx))) +28abx^3(20A+x(15B+2x(6C+5Dx))) +b^2x^5(168A+5x(28B+3x(8C+7Dx)))\right)$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B
+ 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/
840
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^2 \left(Dx^3+Cx^2+A\right) dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^2Dx^7+b^2Cx^6+2abDx^5+b(Ab+2aC)x^4+a^2Dx^3+a(2Ab+aC)x^2+a^2A\right) dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

input
```
Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/
5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(B\,b^2+2abD)x^6}{6}+\frac{(b^2A+2Cab)x^5}{5}+\frac{(2abB+Da^2)x^4}{4}+\frac{(2abA+C\,a^2)x^3}{3}+\frac{B\,a^2x^2}{2}+a^2A$
norman	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\left(\frac{1}{6}B\,b^2+\frac{1}{3}abD\right)x^6+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\left(\frac{1}{2}abB+\frac{1}{4}Da^2\right)x^4+\left(\frac{2}{3}abA+\right.$
gosper	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}B\,b^2x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}x^4abB+\frac{1}{4}a^2Dx^4+\frac{2}{3}Ab$
parallelrisch	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}B\,b^2x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}x^4abB+\frac{1}{4}a^2Dx^4+\frac{2}{3}Ab$
orering	$\frac{x(105b^2Dx^7+120b^2C\,x^6+140B\,b^2x^5+280Dab\,x^5+168b^2A\,x^4+336x^4Cab+420abB\,x^3+210Da^2x^3+560abA\,x^2+280C\,a^2x^2}{840}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^
5+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6 + \frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2 + \frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x + \frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8} + x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right) + x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6
+ D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**
3*(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{3}\,Dabx^6+\frac{1}{6}\,Bb^2x^6+\frac{2}{5}\,Cabx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{4}\,Da^2x^4+\frac{1}{2}\,Babx^4+\frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*
x^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*
A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx = \frac{A\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^4\,D}{4} +\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6} +\frac{C\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105} +\frac{a\,b\,x^6\,D}{3}$$

input
```
int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
 + (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 +
 42*a*b*x^2))/105 + (a*b*x^6*D)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx = \frac{x(105b^2d\,x^7+120b^2c\,x^6+280abd\,x^5+140b^3x^5+168a\,b^2x^4+336abc\,x^4+210a^2d\,x^3+420a\,b^2x^3+560}{840}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**3 + 560*a**2*b*x**2 + 420*a**2*b*x + 280*a**2*c*x**2 + 210*a**2
*d*x**3 + 168*a*b**2*x**4 + 420*a*b**2*x**3 + 336*a*b*c*x**4 + 280*a*b*d*x
**5 + 140*b**3*x**5 + 120*b**2*c*x**6 + 105*b**2*d*x**7))/840
```

3.12 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 436

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = -\frac{(a^2d^4(cCd-Bd^2-c^2D)+b^2c^3(c^2Cd-Bcd^2+Ad^3-c^3D)+2abcd^2(c^2Cd-Bcd^2+Ad^3-c^3D))\,x}{d^7} + \frac{(a^2d^4(Cd-cD)+b^2c^2(c^2Cd-Bcd^2+Ad^3-c^3D)+2abd^2(c^2Cd-Bcd^2+Ad^3-c^3D))\,x^2}{2d^6} + \frac{(a^2d^4D-2abd^2(cCd-Bd^2-c^2D)-b^2c(c^2Cd-Bcd^2+Ad^3-c^3D))\,x^3}{3d^5} + \frac{b(2ad^2(Cd-cD)+b(c^2Cd-Bcd^2+Ad^3-c^3D))\,x^4}{4d^4} + \frac{b(2ad^2D-b(cCd-Bd^2-c^2D))\,x^5}{5d^3} + \frac{b^2(Cd-cD)x^6}{6d^2} + \frac{b^2Dx^7}{7d} + \frac{(bc^2+ad^2)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^8}$$

output
```
-(a^2*d^4*(-B*d^2+C*c*d-D*c^2)+b^2*c^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)+2*a*b
*c*d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x/d^7+1/2*(a^2*d^4*(C*d-D*c)+b^2*c^2
*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)+2*a*b*d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x^
2/d^6+1/3*(a^2*d^4*D-2*a*b*d^2*(-B*d^2+C*c*d-D*c^2)-b^2*c*(A*d^3-B*c*d^2+C
*c^2*d-D*c^3))*x^3/d^5+1/4*b*(2*a*d^2*(C*d-D*c)+b*(A*d^3-B*c*d^2+C*c^2*d-D
*c^3))*x^4/d^4+1/5*b*(2*a*d^2*D-b*(-B*d^2+C*c*d-D*c^2))*x^5/d^3+1/6*b^2*(C
*d-D*c)*x^6/d^2+1/7*b^2*D*x^7/d+(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c
^3)*ln(d*x+c)/d^8
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{dx(70a^2d^4(6c^2D-3cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+14abd^2(60c^4D-30c^3d(2C+Dx)+10c}{}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(d*x*(70*a^2*d^4*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x)
)) + 14*a*b*d^2*(60*c^4*D - 30*c^3*d*(2*C + D*x) + 10*c^2*d^2*(6*B + x*(3*
C + 2*D*x)) - 5*c*d^3*(12*A + x*(6*B + x*(4*C + 3*D*x))) + d^4*x*(30*A + x
*(20*B + 3*x*(5*C + 4*D*x)))) + b^2*(420*c^6*D - 210*c^5*d*(2*C + D*x) + 7
0*c^4*d^2*(6*B + x*(3*C + 2*D*x)) - 35*c^3*d^3*(12*A + x*(6*B + x*(4*C + 3
*D*x))) + 7*c^2*d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - 7*c*d^5*x^2*
(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + d^6*x^3*(105*A + 2*x*(42*B + 5*x*(
7*C + 6*D*x))))) - 420*(b*c^2 + a*d^2)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c
^3*D)*Log[c + d*x])/(420*d^8)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx} \, dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{-a^2d^4(-Bd^2+c^2(-D)+cCd)-2abcd^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-b^2c^3(Ad^3-Bcd^2+c^3(-D)+}{d^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x(a^2d^4(-Bd^2+c^2(-D)+cCd)+2abcd^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd)+b^2c^3(Ad^3-Bcd^2+c^3(-D)+c^2}{d^7}$$
$$\frac{x^2(a^2d^4(Cd-cD)+2abd^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd)+b^2c^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd))}{2d^6}+\frac{x^3(a^2d^4D-2abd^2(-Bd^2+c^2(-D)+cCd)-b^2c(Ad^3-Bcd^2+c^3(-D)+c^2Cd))}{3d^5}+\frac{(ad^2+bc^2)^2\log(c+dx)(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^8}+\frac{bx^4(2ad^2(Cd-cD)+b(Ad^3-Bcd^2+c^3(-D)+c^2Cd))}{4d^4}+\frac{bx^5(2ad^2D-b(-Bd^2+c^2(-D)+cCd))}{5d^3}+\frac{b^2x^6(Cd-cD)}{6d^2}+\frac{b^2Dx^7}{7d}$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
-(((a^2*d^4*(c*C*d - B*d^2 - c^2*D) + b^2*c^3*(c^2*C*d - B*c*d^2 + A*d^3 -
 c^3*D) + 2*a*b*c*d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*x)/d^7) + ((a^2
*d^4*(C*d - c*D) + b^2*c^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) + 2*a*b*d^2
*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*x^2)/(2*d^6) + ((a^2*d^4*D - 2*a*b*d
^2*(c*C*d - B*d^2 - c^2*D) - b^2*c*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*x^
3)/(3*d^5) + (b*(2*a*d^2*(C*d - c*D) + b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*
D))*x^4)/(4*d^4) + (b*(2*a*d^2*D - b*(c*C*d - B*d^2 - c^2*D))*x^5)/(5*d^3)
 + (b^2*(C*d - c*D)*x^6)/(6*d^2) + (b^2*D*x^7)/(7*d) + ((b*c^2 + a*d^2)^2*
(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.31

method	result
norman	$\frac{\left(2Aab\,d^5+A\,b^2c^2d^3-2Babc\,d^4-B\,b^2c^3d^2+C\,a^2d^5+2Cab\,c^2d^3+C\,b^2c^4d-Da^2c\,d^4-2Dab\,c^3d^2-b^2c^5D\right)x^2}{2d^6}-\frac{\left(A\,d^3b^2c-2B\right.}{}$
default	$-\frac{-Da^2c^2d^4x+A\,b^2c^3d^3x-\frac{1}{3}Db^2c^4d^2x^3-Aab\,d^6x^2-\frac{1}{2}A\,b^2c^2d^4x^2+\frac{1}{3}C\,b^2c^3d^3x^3+2Aabc\,d^5x-2Bab\,c^2d^4x-Cab\,c^2d^4x^2+}{}$
parallelrisch	$\frac{420Bx\,b^2c^4d^3+140B\,x^3b^2c^2d^5-140C\,x^3b^2c^3d^4+840C\ln(dx+c)ab\,c^4d^3-840D\ln(dx+c)ab\,c^5d^2-420Dx^2ab\,c^3d^4-840Axab}{}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^6*(2*A*a*b*d^5+A*b^2*c^2*d^3-2*B*a*b*c*d^4-B*b^2*c^3*d^2+C*a^2*d^5+2
*C*a*b*c^2*d^3+C*b^2*c^4*d-D*a^2*c*d^4-2*D*a*b*c^3*d^2-D*b^2*c^5)*x^2-1/3/
d^5*(A*b^2*c*d^3-2*B*a*b*d^4-B*b^2*c^2*d^2+2*C*a*b*c*d^3+C*b^2*c^3*d-D*a^2
*d^4-2*D*a*b*c^2*d^2-D*b^2*c^4)*x^3-(2*A*a*b*c*d^5+A*b^2*c^3*d^3-B*a^2*d^6
-2*B*a*b*c^2*d^4-B*b^2*c^4*d^2+C*a^2*c*d^5+2*C*a*b*c^3*d^3+C*b^2*c^5*d-D*a
^2*c^2*d^4-2*D*a*b*c^4*d^2-D*b^2*c^6)/d^7*x+1/4*b/d^4*(A*b*d^3-B*b*c*d^2+2
*C*a*d^3+C*b*c^2*d-2*D*a*c*d^2-D*b*c^3)*x^4+1/5*b/d^3*(B*b*d^2-C*b*c*d+2*D
*a*d^2+D*b*c^2)*x^5+1/7*b^2*D*x^7/d+1/6*b^2*(C*d-D*c)*x^6/d^2+(A*a^2*d^7+2
*A*a*b*c^2*d^5+A*b^2*c^4*d^3-B*a^2*c*d^6-2*B*a*b*c^3*d^4-B*b^2*c^5*d^2+C*a
^2*c^2*d^5+2*C*a*b*c^4*d^3+C*b^2*c^6*d-D*a^2*c^3*d^4-2*D*a*b*c^5*d^2-D*b^2
*c^7)/d^8*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{60\,Db^2d^7x^7 - 70\,(Db^2cd^6 - Cb^2d^7)x^6 + 84\,(Db^2c^2d^5 - Cb^2cd^6 + (2\,Dab + Bb^2)d^7)x^5 - 105\,(Db^2c^3d^4 -}{}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/420*(60*D*b^2*d^7*x^7 - 70*(D*b^2*c*d^6 - C*b^2*d^7)*x^6 + 84*(D*b^2*c^2
*d^5 - C*b^2*c*d^6 + (2*D*a*b + B*b^2)*d^7)*x^5 - 105*(D*b^2*c^3*d^4 - C*b
^2*c^2*d^5 + (2*D*a*b + B*b^2)*c*d^6 - (2*C*a*b + A*b^2)*d^7)*x^4 + 140*(D
*b^2*c^4*d^3 - C*b^2*c^3*d^4 + (2*D*a*b + B*b^2)*c^2*d^5 - (2*C*a*b + A*b^
2)*c*d^6 + (D*a^2 + 2*B*a*b)*d^7)*x^3 - 210*(D*b^2*c^5*d^2 - C*b^2*c^4*d^3
 + (2*D*a*b + B*b^2)*c^3*d^4 - (2*C*a*b + A*b^2)*c^2*d^5 + (D*a^2 + 2*B*a*
b)*c*d^6 - (C*a^2 + 2*A*a*b)*d^7)*x^2 + 420*(D*b^2*c^6*d - C*b^2*c^5*d^2 +
 B*a^2*d^7 + (2*D*a*b + B*b^2)*c^4*d^3 - (2*C*a*b + A*b^2)*c^3*d^4 + (D*a^
2 + 2*B*a*b)*c^2*d^5 - (C*a^2 + 2*A*a*b)*c*d^6)*x - 420*(D*b^2*c^7 - C*b^2
*c^6*d + B*a^2*c*d^6 - A*a^2*d^7 + (2*D*a*b + B*b^2)*c^5*d^2 - (2*C*a*b +
A*b^2)*c^4*d^3 + (D*a^2 + 2*B*a*b)*c^3*d^4 - (C*a^2 + 2*A*a*b)*c^2*d^5)*lo
g(d*x + c))/d^8
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.27

$$
\begin{aligned}
&\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx \\
&= \frac{Db^2x^7}{7d}+x^6\left(\frac{Cb^2}{6d}-\frac{Db^2c}{6d^2}\right)+x^5\left(\frac{Bb^2}{5d}-\frac{Cb^2c}{5d^2}+\frac{2Dab}{5d}+\frac{Db^2c^2}{5d^3}\right) \\
&\quad+x^4\left(\frac{Ab^2}{4d}-\frac{Bb^2c}{4d^2}+\frac{Cab}{2d}+\frac{Cb^2c^2}{4d^3}-\frac{Dabc}{2d^2}-\frac{Db^2c^3}{4d^4}\right) \\
&\quad+x^3\left(-\frac{Ab^2c}{3d^2}+\frac{2Bab}{3d}+\frac{Bb^2c^2}{3d^3}-\frac{2Cabc}{3d^2}-\frac{Cb^2c^3}{3d^4}+\frac{Da^2}{3d}+\frac{2Dabc^2}{3d^3}+\frac{Db^2c^4}{3d^5}\right) \\
&\quad+x^2\left(\frac{Aab}{d}+\frac{Ab^2c^2}{2d^3}-\frac{Babc}{d^2}-\frac{Bb^2c^3}{2d^4}+\frac{Ca^2}{2d}+\frac{Cabc^2}{d^3}+\frac{Cb^2c^4}{2d^5}-\frac{Da^2c}{2d^2}-\frac{Dabc^3}{d^4}\right. \\
&\qquad\left.-\frac{Db^2c^5}{2d^6}\right)+x\left(-\frac{2Aabc}{d^2}-\frac{Ab^2c^3}{d^4}+\frac{Ba^2}{d}+\frac{2Babc^2}{d^3}+\frac{Bb^2c^4}{d^5}-\frac{Ca^2c}{d^2}-\frac{2Cabc^3}{d^4}\right. \\
&\qquad\left.-\frac{Cb^2c^5}{d^6}+\frac{Da^2c^2}{d^3}+\frac{2Dabc^4}{d^5}+\frac{Db^2c^6}{d^7}\right) \\
&\quad-\frac{(ad^2+bc^2)^2(-Ad^3+Bcd^2-Cc^2d+Dc^3)\log(c+dx)}{d^8}
\end{aligned}
$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b**2*x**7/(7*d) + x**6*(C*b**2/(6*d) - D*b**2*c/(6*d**2)) + x**5*(B*b**2
/(5*d) - C*b**2*c/(5*d**2) + 2*D*a*b/(5*d) + D*b**2*c**2/(5*d**3)) + x**4*
(A*b**2/(4*d) - B*b**2*c/(4*d**2) + C*a*b/(2*d) + C*b**2*c**2/(4*d**3) - D
*a*b*c/(2*d**2) - D*b**2*c**3/(4*d**4)) + x**3*(-A*b**2*c/(3*d**2) + 2*B*a
*b/(3*d) + B*b**2*c**2/(3*d**3) - 2*C*a*b*c/(3*d**2) - C*b**2*c**3/(3*d**4
) + D*a**2/(3*d) + 2*D*a*b*c**2/(3*d**3) + D*b**2*c**4/(3*d**5)) + x**2*(A
*a*b/d + A*b**2*c**2/(2*d**3) - B*a*b*c/d**2 - B*b**2*c**3/(2*d**4) + C*a*
*2/(2*d) + C*a*b*c**2/d**3 + C*b**2*c**4/(2*d**5) - D*a**2*c/(2*d**2) - D*
a*b*c**3/d**4 - D*b**2*c**5/(2*d**6)) + x*(-2*A*a*b*c/d**2 - A*b**2*c**3/d
**4 + B*a**2/d + 2*B*a*b*c**2/d**3 + B*b**2*c**4/d**5 - C*a**2*c/d**2 - 2*
C*a*b*c**3/d**4 - C*b**2*c**5/d**6 + D*a**2*c**2/d**3 + 2*D*a*b*c**4/d**5
+ D*b**2*c**6/d**7) - (a*d**2 + b*c**2)**2*(-A*d**3 + B*c*d**2 - C*c**2*d
+ D*c**3)*log(c + d*x)/d**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{60\,Db^2d^6x^7 - 70\,(Db^2cd^5 - Cb^2d^6)x^6 + 84\,(Db^2c^2d^4 - Cb^2cd^5 + (2\,Dab + Bb^2)d^6)x^5 - 105\,(Db^2c^3d^3 -}{}$$
$$- \frac{(Db^2c^7 - Cb^2c^6d + Ba^2cd^6 - Aa^2d^7 + (2\,Dab + Bb^2)c^5d^2 - (2\,Cab + Ab^2)c^4d^3 + (Da^2 + 2\,Bab)c^3d^4}{d^8}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/420*(60*D*b^2*d^6*x^7 - 70*(D*b^2*c*d^5 - C*b^2*d^6)*x^6 + 84*(D*b^2*c^2
*d^4 - C*b^2*c*d^5 + (2*D*a*b + B*b^2)*d^6)*x^5 - 105*(D*b^2*c^3*d^3 - C*b
^2*c^2*d^4 + (2*D*a*b + B*b^2)*c*d^5 - (2*C*a*b + A*b^2)*d^6)*x^4 + 140*(D
*b^2*c^4*d^2 - C*b^2*c^3*d^3 + (2*D*a*b + B*b^2)*c^2*d^4 - (2*C*a*b + A*b^
2)*c*d^5 + (D*a^2 + 2*B*a*b)*d^6)*x^3 - 210*(D*b^2*c^5*d - C*b^2*c^4*d^2 +
 (2*D*a*b + B*b^2)*c^3*d^3 - (2*C*a*b + A*b^2)*c^2*d^4 + (D*a^2 + 2*B*a*b)
*c*d^5 - (C*a^2 + 2*A*a*b)*d^6)*x^2 + 420*(D*b^2*c^6 - C*b^2*c^5*d + B*a^2
*d^6 + (2*D*a*b + B*b^2)*c^4*d^2 - (2*C*a*b + A*b^2)*c^3*d^3 + (D*a^2 + 2*
B*a*b)*c^2*d^4 - (C*a^2 + 2*A*a*b)*c*d^5)*x)/d^7 - (D*b^2*c^7 - C*b^2*c^6*
d + B*a^2*c*d^6 - A*a^2*d^7 + (2*D*a*b + B*b^2)*c^5*d^2 - (2*C*a*b + A*b^2
)*c^4*d^3 + (D*a^2 + 2*B*a*b)*c^3*d^4 - (C*a^2 + 2*A*a*b)*c^2*d^5)*log(d*x
 + c)/d^8
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{60\,Db^2d^6x^7 - 70\,Db^2cd^5x^6 + 70\,Cb^2d^6x^6 + 84\,Db^2c^2d^4x^5 - 84\,Cb^2cd^5x^5 + 168\,Dabd^6x^5 + 84\,Bb^2d^6x^5 -}{}$$
$$- \frac{(Db^2c^7 - Cb^2c^6d + 2\,Dabc^5d^2 + Bb^2c^5d^2 - 2\,Cabc^4d^3 - Ab^2c^4d^3 + Da^2c^3d^4 + 2\,Babc^3d^4 - Ca^2c^2d^5 -}{d^8}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/420*(60*D*b^2*d^6*x^7 - 70*D*b^2*c*d^5*x^6 + 70*C*b^2*d^6*x^6 + 84*D*b^2
*c^2*d^4*x^5 - 84*C*b^2*c*d^5*x^5 + 168*D*a*b*d^6*x^5 + 84*B*b^2*d^6*x^5 -
 105*D*b^2*c^3*d^3*x^4 + 105*C*b^2*c^2*d^4*x^4 - 210*D*a*b*c*d^5*x^4 - 105
*B*b^2*c*d^5*x^4 + 210*C*a*b*d^6*x^4 + 105*A*b^2*d^6*x^4 + 140*D*b^2*c^4*d
^2*x^3 - 140*C*b^2*c^3*d^3*x^3 + 280*D*a*b*c^2*d^4*x^3 + 140*B*b^2*c^2*d^4
*x^3 - 280*C*a*b*c*d^5*x^3 - 140*A*b^2*c*d^5*x^3 + 140*D*a^2*d^6*x^3 + 280
*B*a*b*d^6*x^3 - 210*D*b^2*c^5*d*x^2 + 210*C*b^2*c^4*d^2*x^2 - 420*D*a*b*c
^3*d^3*x^2 - 210*B*b^2*c^3*d^3*x^2 + 420*C*a*b*c^2*d^4*x^2 + 210*A*b^2*c^2
*d^4*x^2 - 210*D*a^2*c*d^5*x^2 - 420*B*a*b*c*d^5*x^2 + 210*C*a^2*d^6*x^2 +
 420*A*a*b*d^6*x^2 + 420*D*b^2*c^6*x - 420*C*b^2*c^5*d*x + 840*D*a*b*c^4*d
^2*x + 420*B*b^2*c^4*d^2*x - 840*C*a*b*c^3*d^3*x - 420*A*b^2*c^3*d^3*x + 4
20*D*a^2*c^2*d^4*x + 840*B*a*b*c^2*d^4*x - 420*C*a^2*c*d^5*x - 840*A*a*b*c
*d^5*x + 420*B*a^2*d^6*x)/d^7 - (D*b^2*c^7 - C*b^2*c^6*d + 2*D*a*b*c^5*d^2
 + B*b^2*c^5*d^2 - 2*C*a*b*c^4*d^3 - A*b^2*c^4*d^3 + D*a^2*c^3*d^4 + 2*B*a
*b*c^3*d^4 - C*a^2*c^2*d^5 - 2*A*a*b*c^2*d^5 + B*a^2*c*d^6 - A*a^2*d^7)*lo
g(abs(d*x + c))/d^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{(b\,x^2+a)^2\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{420\log(dx+c)\,a^3d^5+840\log(dx+c)\,a^2b\,c^2d^3-420\log(dx+c)\,a^2bc\,d^4+420\log(dx+c)\,a\,b^2c^4d-840}{}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(420*log(c + d*x)*a**3*d**5 + 840*log(c + d*x)*a**2*b*c**2*d**3 - 420*log(
c + d*x)*a**2*b*c*d**4 + 420*log(c + d*x)*a*b**2*c**4*d - 840*log(c + d*x)
*a*b**2*c**3*d**2 - 420*log(c + d*x)*b**3*c**5 - 840*a**2*b*c*d**4*x + 420
*a**2*b*d**5*x**2 + 420*a**2*b*d**5*x + 140*a**2*d**6*x**3 - 420*a*b**2*c*
*3*d**2*x + 210*a*b**2*c**2*d**3*x**2 + 840*a*b**2*c**2*d**3*x - 140*a*b**
2*c*d**4*x**3 - 420*a*b**2*c*d**4*x**2 + 105*a*b**2*d**5*x**4 + 280*a*b**2
*d**5*x**3 + 168*a*b*d**6*x**5 + 420*b**3*c**4*d*x - 210*b**3*c**3*d**2*x*
*2 + 140*b**3*c**2*d**3*x**3 - 105*b**3*c*d**4*x**4 + 84*b**3*d**5*x**5 +
60*b**2*d**6*x**7)/(420*d**6)
```

3.13 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 421

$$
\begin{aligned}
&\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx \\
&= \frac{(a^2d^4(Cd-2cD)+b^2c^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)+2abd^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x}{d^7} \\
&+ \frac{(a^2d^4D-2abd^2(2cCd-Bd^2-3c^2D)-b^2c(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))\,x^2}{2d^6} \\
&+ \frac{b(2ad^2(Cd-2cD)+b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x^3}{3d^5} \\
&+ \frac{b(2ad^2D-b(2cCd-Bd^2-3c^2D))\,x^4}{4d^4} + \frac{b^2(Cd-2cD)x^5}{5d^3} \\
&+ \frac{b^2Dx^6}{6d^2} - \frac{(bc^2+ad^2)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^8(c+dx)} \\
&- \frac{(bc^2+ad^2)(ad^2(2cCd-Bd^2-3c^2D)+bc(6c^2Cd-5Bcd^2+4Ad^3-7c^3D))\log(c+dx)}{d^8}
\end{aligned}
$$

output
```
(a^2*d^4*(C*d-2*D*c)+b^2*c^2*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3)+2*a*b*d
^2*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*x/d^7+1/2*(a^2*d^4*D-2*a*b*d^2*(-B
*d^2+2*C*c*d-3*D*c^2)-b^2*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*x^2/d^6
+1/3*b*(2*a*d^2*(C*d-2*D*c)+b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*x^3/d^5
+1/4*b*(2*a*d^2*D-b*(-B*d^2+2*C*c*d-3*D*c^2))*x^4/d^4+1/5*b^2*(C*d-2*D*c)*
x^5/d^3+1/6*b^2*D*x^6/d^2-(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^
8/(d*x+c)-(a*d^2+b*c^2)*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(4*A*d^3-5*B*c
*d^2+6*C*c^2*d-7*D*c^3))*ln(d*x+c)/d^8
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$$
$$= \frac{30a^2d^4(2c^3D-2c^2d(C+2Dx)+cd^2(2B+x(2C-3Dx))+d^3(-2A+x^2(2C+Dx)))+10abd^2(12c^5I}{}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(30*a^2*d^4*(2*c^3*D - 2*c^2*d*(C + 2*D*x) + c*d^2*(2*B + x*(2*C - 3*D*x))
 + d^3*(-2*A + x^2*(2*C + D*x))) + 10*a*b*d^2*(12*c^5*D - 12*c^4*d*(C + 4*
D*x) + 6*c^3*d^2*(2*B + x*(6*C - 5*D*x)) - 2*c^2*d^3*(6*A + x*(12*B - 12*C
*x - 5*D*x^2)) + d^5*x^2*(12*A + x*(6*B + 4*C*x + 3*D*x^2)) - c*d^4*x*(-12
*A + x*(18*B + 8*C*x + 5*D*x^2))) + b^2*(60*c^7*D - 60*c^6*d*(C + 6*D*x) +
 30*c^5*d^2*(2*B + x*(10*C - 7*D*x)) + d^7*x^4*(20*A + x*(15*B + 2*x*(6*C
+ 5*D*x))) - c*d^6*x^3*(40*A + x*(25*B + 2*x*(9*C + 7*D*x))) + c^2*d^5*x^2
*(120*A + x*(50*B + 3*x*(10*C + 7*D*x))) - 5*c^3*d^4*x*(-36*A + x*(30*B +
x*(12*C + 7*D*x))) - 10*c^4*d^3*(6*A + x*(24*B - x*(18*C + 7*D*x)))) + 60*
(b*c^2 + a*d^2)*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-6*c^2*C*d + 5*
B*c*d^2 - 4*A*d^3 + 7*c^3*D))*(c + d*x)*Log[c + d*x])/(60*d^8*(c + d*x))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{a^2d^4(Cd-2cD)+2abd^2(Ad^3-2Bcd^2-4c^3D+3c^2Cd)+b^2c^2(3Ad^3-4Bcd^2-6c^3D+5c^2Cd)}{d^7}+\frac{x(a^2c\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a^2d^4(Cd-2cD)+2abd^2(Ad^3-2Bcd^2-4c^3D+3c^2Cd)+b^2c^2(3Ad^3-4Bcd^2-6c^3D+5c^2Cd))}{d^7}+\frac{x^2(a^2d^4D-2abd^2(-Bd^2-3c^2D+2cCd)-b^2c(2Ad^3-3Bcd^2-5c^3D+4c^2Cd))}{2d^6}-\frac{(ad^2+bc^2)^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^8(c+dx)}-\frac{(ad^2+bc^2)\log(c+dx)(ad^2(-Bd^2-3c^2D+2cCd)+bc(4Ad^3-5Bcd^2-7c^3D+6c^2Cd))}{d^8}+\frac{bx^3(2ad^2(Cd-2cD)+b(Ad^3-2Bcd^2-4c^3D+3c^2Cd))}{3d^5}+\frac{bx^4(2ad^2D-b(-Bd^2-3c^2D+2cCd))}{4d^4}+\frac{b^2x^5(Cd-2cD)}{5d^3}+\frac{b^2Dx^6}{6d^2}$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((a^2*d^4*(C*d - 2*c*D) + b^2*c^2*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3
*D) + 2*a*b*d^2*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x)/d^7 + ((a^2*
d^4*D - 2*a*b*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b^2*c*(4*c^2*C*d - 3*B*c*d
^2 + 2*A*d^3 - 5*c^3*D))*x^2)/(2*d^6) + (b*(2*a*d^2*(C*d - 2*c*D) + b*(3*c
^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x^3)/(3*d^5) + (b*(2*a*d^2*D - b*(2
*c*C*d - B*d^2 - 3*c^2*D))*x^4)/(4*d^4) + (b^2*(C*d - 2*c*D)*x^5)/(5*d^3)
+ (b^2*D*x^6)/(6*d^2) - ((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D))/(d^8*(c + d*x)) - ((b*c^2 + a*d^2)*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D
) + b*c*(6*c^2*C*d - 5*B*c*d^2 + 4*A*d^3 - 7*c^3*D))*Log[c + d*x])/d^8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.42

method	result
norman	$\frac{\left(A\,a^2 d^7+4Aab\,c^2 d^5+4A\,b^2 c^4 d^3-B\,a^2 c\,d^6-6Bab\,c^3 d^4-5B\,b^2 c^5 d^2+2C\,a^2 c^2 d^5+8Cab\,c^4 d^3+6C\,b^2 c^6 d-3Da^2 c^3 d^4-10Dab\,c^5 d^2-7Db^2 c^7\right.}{d^7 c}$
default	$\frac{1}{4}B\,b^2 d^5 x^4+\frac{1}{3}A\,b^2 d^5 x^3-\frac{2}{5}Db^2 c\,d^4 x^5+\frac{3}{4}Db^2 c^2 d^3 x^4-A\,b^2 c\,d^4 x^2+Bab\,d^5 x^2+\frac{3}{2}B\,b^2 c^2 d^3 x^2+\frac{2}{3}Cab\,d^5 x^3-\frac{4}{3}Db^2 c^3 d^2 x^3-\frac{1}{2}C$
parallelrisch	$-\frac{-360Bab\,c^3 d^4+480Cab\,c^4 d^3-60B\ln(dx+c)x\,a^2 d^7+240A\ln(dx+c)xabc\,d^6+240Aab\,c^2 d^5-50B\,x^3 b^2 c^2 d^5+60C\,x^3 b^2 c^3 d^4}{}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^2*d^7+4*A*a*b*c^2*d^5+4*A*b^2*c^4*d^3-B*a^2*c*d^6-6*B*a*b*c^3*d^4-5*
B*b^2*c^5*d^2+2*C*a^2*c^2*d^5+8*C*a*b*c^4*d^3+6*C*b^2*c^6*d-3*D*a^2*c^3*d^
4-10*D*a*b*c^5*d^2-7*D*b^2*c^7)/d^7/c*x-1/6*(4*A*b^2*c*d^3-6*B*a*b*d^4-5*B
*b^2*c^2*d^2+8*C*a*b*c*d^3+6*C*b^2*c^3*d-3*D*a^2*d^4-10*D*a*b*c^2*d^2-7*D*
b^2*c^4)/d^5*x^3+1/2*(4*A*a*b*d^5+4*A*b^2*c^2*d^3-6*B*a*b*c*d^4-5*B*b^2*c^
3*d^2+2*C*a^2*d^5+8*C*a*b*c^2*d^3+6*C*b^2*c^4*d-3*D*a^2*c*d^4-10*D*a*b*c^3
*d^2-7*D*b^2*c^5)/d^6*x^2+1/20*b*(5*B*b*d^2-6*C*b*c*d+10*D*a*d^2+7*D*b*c^2
)/d^3*x^5+1/12*b*(4*A*b*d^3-5*B*b*c*d^2+8*C*a*d^3+6*C*b*c^2*d-10*D*a*c*d^2
-7*D*b*c^3)/d^4*x^4+1/6*b^2*D*x^7/d+1/30*b^2*(6*C*d-7*D*c)/d^2*x^6)/(d*x+c
)-(4*A*a*b*c*d^5+4*A*b^2*c^3*d^3-B*a^2*d^6-6*B*a*b*c^2*d^4-5*B*b^2*c^4*d^2
+2*C*a^2*c*d^5+8*C*a*b*c^3*d^3+6*C*b^2*c^5*d-3*D*a^2*c^2*d^4-10*D*a*b*c^4*
d^2-7*D*b^2*c^6)/d^8*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/60*(10*D*b^2*d^7*x^7 + 60*D*b^2*c^7 - 60*C*b^2*c^6*d + 60*B*a^2*c*d^6 -
60*A*a^2*d^7 + 60*(2*D*a*b + B*b^2)*c^5*d^2 - 60*(2*C*a*b + A*b^2)*c^4*d^3
 + 60*(D*a^2 + 2*B*a*b)*c^3*d^4 - 60*(C*a^2 + 2*A*a*b)*c^2*d^5 - 2*(7*D*b^
2*c*d^6 - 6*C*b^2*d^7)*x^6 + 3*(7*D*b^2*c^2*d^5 - 6*C*b^2*c*d^6 + 5*(2*D*a
*b + B*b^2)*d^7)*x^5 - 5*(7*D*b^2*c^3*d^4 - 6*C*b^2*c^2*d^5 + 5*(2*D*a*b +
 B*b^2)*c*d^6 - 4*(2*C*a*b + A*b^2)*d^7)*x^4 + 10*(7*D*b^2*c^4*d^3 - 6*C*b
^2*c^3*d^4 + 5*(2*D*a*b + B*b^2)*c^2*d^5 - 4*(2*C*a*b + A*b^2)*c*d^6 + 3*(
D*a^2 + 2*B*a*b)*d^7)*x^3 - 30*(7*D*b^2*c^5*d^2 - 6*C*b^2*c^4*d^3 + 5*(2*D
*a*b + B*b^2)*c^3*d^4 - 4*(2*C*a*b + A*b^2)*c^2*d^5 + 3*(D*a^2 + 2*B*a*b)*
c*d^6 - 2*(C*a^2 + 2*A*a*b)*d^7)*x^2 - 60*(6*D*b^2*c^6*d - 5*C*b^2*c^5*d^2
 + 4*(2*D*a*b + B*b^2)*c^4*d^3 - 3*(2*C*a*b + A*b^2)*c^3*d^4 + 2*(D*a^2 +
2*B*a*b)*c^2*d^5 - (C*a^2 + 2*A*a*b)*c*d^6)*x + 60*(7*D*b^2*c^7 - 6*C*b^2*
c^6*d + B*a^2*c*d^6 + 5*(2*D*a*b + B*b^2)*c^5*d^2 - 4*(2*C*a*b + A*b^2)*c^
4*d^3 + 3*(D*a^2 + 2*B*a*b)*c^3*d^4 - 2*(C*a^2 + 2*A*a*b)*c^2*d^5 + (7*D*b
^2*c^6*d - 6*C*b^2*c^5*d^2 + B*a^2*d^7 + 5*(2*D*a*b + B*b^2)*c^4*d^3 - 4*(
2*C*a*b + A*b^2)*c^3*d^4 + 3*(D*a^2 + 2*B*a*b)*c^2*d^5 - 2*(C*a^2 + 2*A*a*
b)*c*d^6)*x)*log(d*x + c))/(d^9*x + c*d^8)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.52

$$
\begin{aligned}
&\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx \\
&= \frac{Db^2x^6}{6d^2} + x^5\left(\frac{Cb^2}{5d^2} - \frac{2Db^2c}{5d^3}\right) + x^4\left(\frac{Bb^2}{4d^2} - \frac{Cb^2c}{2d^3} + \frac{Dab}{2d^2} + \frac{3Db^2c^2}{4d^4}\right) \\
&\quad + x^3\left(\frac{Ab^2}{3d^2} - \frac{2Bb^2c}{3d^3} + \frac{2Cab}{3d^2} + \frac{Cb^2c^2}{d^4} - \frac{4Dabc}{3d^3} - \frac{4Db^2c^3}{3d^5}\right) \\
&\quad + x^2\left(-\frac{Ab^2c}{d^3} + \frac{Bab}{d^2} + \frac{3Bb^2c^2}{2d^4} - \frac{2Cabc}{d^3} - \frac{2Cb^2c^3}{d^5} + \frac{Da^2}{2d^2} + \frac{3Dabc^2}{d^4} + \frac{5Db^2c^4}{2d^6}\right) \\
&\quad + x\left(\frac{2Aab}{d^2} + \frac{3Ab^2c^2}{d^4} - \frac{4Babc}{d^3} - \frac{4Bb^2c^3}{d^5} + \frac{Ca^2}{d^2} + \frac{6Cabc^2}{d^4} + \frac{5Cb^2c^4}{d^6} - \frac{2Da^2c}{d^3} \right. \\
&\qquad \left. - \frac{8Dabc^3}{d^5} - \frac{6Db^2c^5}{d^7}\right) \\
&\quad + \frac{-Aa^2d^7 - 2Aabc^2d^5 - Ab^2c^4d^3 + Ba^2cd^6 + 2Babc^3d^4 + Bb^2c^5d^2 - Ca^2c^2d^5 - 2Cabc^4d^3 - Cb^2c^6d + \cdots}{cd^8 + d^9x} \\
&\quad + \frac{(ad^2+bc^2)(-4Abcd^3 + Bad^4 + 5Bbc^2d^2 - 2Cacd^3 - 6Cbc^3d + 3Dac^2d^2 + 7Dbc^4)\log(c+dx)}{d^8}
\end{aligned}
$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*b**2*x**6/(6*d**2) + x**5*(C*b**2/(5*d**2) - 2*D*b**2*c/(5*d**3)) + x**4
*(B*b**2/(4*d**2) - C*b**2*c/(2*d**3) + D*a*b/(2*d**2) + 3*D*b**2*c**2/(4*
d**4)) + x**3*(A*b**2/(3*d**2) - 2*B*b**2*c/(3*d**3) + 2*C*a*b/(3*d**2) +
C*b**2*c**2/d**4 - 4*D*a*b*c/(3*d**3) - 4*D*b**2*c**3/(3*d**5)) + x**2*(-A
*b**2*c/d**3 + B*a*b/d**2 + 3*B*b**2*c**2/(2*d**4) - 2*C*a*b*c/d**3 - 2*C*
b**2*c**3/d**5 + D*a**2/(2*d**2) + 3*D*a*b*c**2/d**4 + 5*D*b**2*c**4/(2*d*
*6)) + x*(2*A*a*b/d**2 + 3*A*b**2*c**2/d**4 - 4*B*a*b*c/d**3 - 4*B*b**2*c*
*3/d**5 + C*a**2/d**2 + 6*C*a*b*c**2/d**4 + 5*C*b**2*c**4/d**6 - 2*D*a**2*
c/d**3 - 8*D*a*b*c**3/d**5 - 6*D*b**2*c**5/d**7) + (-A*a**2*d**7 - 2*A*a*b
*c**2*d**5 - A*b**2*c**4*d**3 + B*a**2*c*d**6 + 2*B*a*b*c**3*d**4 + B*b**2
*c**5*d**2 - C*a**2*c**2*d**5 - 2*C*a*b*c**4*d**3 - C*b**2*c**6*d + D*a**2
*c**3*d**4 + 2*D*a*b*c**5*d**2 + D*b**2*c**7)/(c*d**8 + d**9*x) + (a*d**2
+ b*c**2)*(-4*A*b*c*d**3 + B*a*d**4 + 5*B*b*c**2*d**2 - 2*C*a*c*d**3 - 6*C
*b*c**3*d + 3*D*a*c**2*d**2 + 7*D*b*c**4)*log(c + d*x)/d**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{Db^2c^7 - Cb^2c^6d + Ba^2cd^6 - Aa^2d^7 + (2\,Dab + Bb^2)c^5d^2 - (2\,Cab + Ab^2)c^4d^3 + (Da^2 + 2\,Bab)c^3d^4 - (}{d^9x + cd^8}$$
$$+ \frac{10\,Db^2d^5x^6 - 12\,(2\,Db^2cd^4 - Cb^2d^5)x^5 + 15\,(3\,Db^2c^2d^3 - 2\,Cb^2cd^4 + (2\,Dab + Bb^2)d^5)x^4 - 20\,(4\,Db^2}{}$$
$$+ \frac{(7\,Db^2c^6 - 6\,Cb^2c^5d + Ba^2d^6 + 5\,(2\,Dab + Bb^2)c^4d^2 - 4\,(2\,Cab + Ab^2)c^3d^3 + 3\,(Da^2 + 2\,Bab)c^2d^4 -}{d^8}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(D*b^2*c^7 - C*b^2*c^6*d + B*a^2*c*d^6 - A*a^2*d^7 + (2*D*a*b + B*b^2)*c^5
*d^2 - (2*C*a*b + A*b^2)*c^4*d^3 + (D*a^2 + 2*B*a*b)*c^3*d^4 - (C*a^2 + 2*
A*a*b)*c^2*d^5)/(d^9*x + c*d^8) + 1/60*(10*D*b^2*d^5*x^6 - 12*(2*D*b^2*c*d
^4 - C*b^2*d^5)*x^5 + 15*(3*D*b^2*c^2*d^3 - 2*C*b^2*c*d^4 + (2*D*a*b + B*b
^2)*d^5)*x^4 - 20*(4*D*b^2*c^3*d^2 - 3*C*b^2*c^2*d^3 + 2*(2*D*a*b + B*b^2)
*c*d^4 - (2*C*a*b + A*b^2)*d^5)*x^3 + 30*(5*D*b^2*c^4*d - 4*C*b^2*c^3*d^2
+ 3*(2*D*a*b + B*b^2)*c^2*d^3 - 2*(2*C*a*b + A*b^2)*c*d^4 + (D*a^2 + 2*B*a
*b)*d^5)*x^2 - 60*(6*D*b^2*c^5 - 5*C*b^2*c^4*d + 4*(2*D*a*b + B*b^2)*c^3*d
^2 - 3*(2*C*a*b + A*b^2)*c^2*d^3 + 2*(D*a^2 + 2*B*a*b)*c*d^4 - (C*a^2 + 2*
A*a*b)*d^5)*x)/d^7 + (7*D*b^2*c^6 - 6*C*b^2*c^5*d + B*a^2*d^6 + 5*(2*D*a*b
 + B*b^2)*c^4*d^2 - 4*(2*C*a*b + A*b^2)*c^3*d^3 + 3*(D*a^2 + 2*B*a*b)*c^2*
d^4 - 2*(C*a^2 + 2*A*a*b)*c*d^5)*log(d*x + c)/d^8
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{\left(10\,Db^2 - \frac{12\,(7\,Db^2cd - Cb^2d^2)}{(dx+c)d} + \frac{15\,(21\,Db^2c^2d^2 - 6\,Cb^2cd^3 + 2\,Dabd^4 + Bb^2d^4)}{(dx+c)^2d^2} - \frac{20\,(35\,Db^2c^3d^3 - 15\,Cb^2c^2d^4 + 10\,Dabcd^5 + 5\,Bb^2cd^5 \cdots}{(dx+c)^3d^3} \cdots}{\cdots}$$
$$- \frac{(7\,Db^2c^6 - 6\,Cb^2c^5d + 10\,Dabc^4d^2 + 5\,Bb^2c^4d^2 - 8\,Cabc^3d^3 - 4\,Ab^2c^3d^3 + 3\,Da^2c^2d^4 + 6\,Babc^2d^4 - 2 \cdots}{d^8}$$
$$+ \frac{\frac{Db^2c^7d^6}{dx+c} - \frac{Cb^2c^6d^7}{dx+c} + \frac{2\,Dabc^5d^8}{dx+c} + \frac{Bb^2c^5d^8}{dx+c} - \frac{2\,Cabc^4d^9}{dx+c} - \frac{Ab^2c^4d^9}{dx+c} + \frac{Da^2c^3d^{10}}{dx+c} + \frac{2\,Babc^3d^{10}}{dx+c} - \frac{Ca^2c^2d^{11}}{dx+c} - \frac{2\,Aabc^2d^{11}}{dx+c} \cdots}{d^{14}}$$

input

```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/60*(10*D*b^2 - 12*(7*D*b^2*c*d - C*b^2*d^2)/((d*x + c)*d) + 15*(21*D*b^2
*c^2*d^2 - 6*C*b^2*c*d^3 + 2*D*a*b*d^4 + B*b^2*d^4)/((d*x + c)^2*d^2) - 20
*(35*D*b^2*c^3*d^3 - 15*C*b^2*c^2*d^4 + 10*D*a*b*c*d^5 + 5*B*b^2*c*d^5 - 2
*C*a*b*d^6 - A*b^2*d^6)/((d*x + c)^3*d^3) + 30*(35*D*b^2*c^4*d^4 - 20*C*b^
2*c^3*d^5 + 20*D*a*b*c^2*d^6 + 10*B*b^2*c^2*d^6 - 8*C*a*b*c*d^7 - 4*A*b^2*
c*d^7 + D*a^2*d^8 + 2*B*a*b*d^8)/((d*x + c)^4*d^4) - 60*(21*D*b^2*c^5*d^5
- 15*C*b^2*c^4*d^6 + 20*D*a*b*c^3*d^7 + 10*B*b^2*c^3*d^7 - 12*C*a*b*c^2*d^
8 - 6*A*b^2*c^2*d^8 + 3*D*a^2*c*d^9 + 6*B*a*b*c*d^9 - C*a^2*d^10 - 2*A*a*b
*d^10)/((d*x + c)^5*d^5))*(d*x + c)^6/d^8 - (7*D*b^2*c^6 - 6*C*b^2*c^5*d +
 10*D*a*b*c^4*d^2 + 5*B*b^2*c^4*d^2 - 8*C*a*b*c^3*d^3 - 4*A*b^2*c^3*d^3 +
3*D*a^2*c^2*d^4 + 6*B*a*b*c^2*d^4 - 2*C*a^2*c*d^5 - 4*A*a*b*c*d^5 + B*a^2*
d^6)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^8 + (D*b^2*c^7*d^6/(d*x + c)
 - C*b^2*c^6*d^7/(d*x + c) + 2*D*a*b*c^5*d^8/(d*x + c) + B*b^2*c^5*d^8/(d*
x + c) - 2*C*a*b*c^4*d^9/(d*x + c) - A*b^2*c^4*d^9/(d*x + c) + D*a^2*c^3*d
^10/(d*x + c) + 2*B*a*b*c^3*d^10/(d*x + c) - C*a^2*c^2*d^11/(d*x + c) - 2*
A*a*b*c^2*d^11/(d*x + c) + B*a^2*c*d^12/(d*x + c) - A*a^2*d^13/(d*x + c))/
d^14
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{60\log(dx+c)\,b^2c^8+60a^3d^7x+360\log(dx+c)\,a\,b^2c^4d^3+120\log(dx+c)\,ab\,c^6d^2+300\log(dx+c)\,b^3c^5$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
( - 240*log(c + d*x)*a**2*b*c**3*d**4 - 240*log(c + d*x)*a**2*b*c**2*d**5*
x + 60*log(c + d*x)*a**2*b*c**2*d**5 + 60*log(c + d*x)*a**2*b*c*d**6*x + 6
0*log(c + d*x)*a**2*c**4*d**4 + 60*log(c + d*x)*a**2*c**3*d**5*x - 240*log
(c + d*x)*a*b**2*c**5*d**2 - 240*log(c + d*x)*a*b**2*c**4*d**3*x + 360*log
(c + d*x)*a*b**2*c**4*d**3 + 360*log(c + d*x)*a*b**2*c**3*d**4*x + 120*log
(c + d*x)*a*b*c**6*d**2 + 120*log(c + d*x)*a*b*c**5*d**3*x + 300*log(c + d
*x)*b**3*c**6*d + 300*log(c + d*x)*b**3*c**5*d**2*x + 60*log(c + d*x)*b**2
*c**8 + 60*log(c + d*x)*b**2*c**7*d*x + 60*a**3*d**7*x + 240*a**2*b*c**2*d
**5*x + 120*a**2*b*c*d**6*x**2 - 60*a**2*b*c*d**6*x - 60*a**2*c**3*d**5*x
- 30*a**2*c**2*d**6*x**2 + 30*a**2*c*d**7*x**3 + 240*a*b**2*c**4*d**3*x +
120*a*b**2*c**3*d**4*x**2 - 360*a*b**2*c**3*d**4*x - 40*a*b**2*c**2*d**5*x
**3 - 180*a*b**2*c**2*d**5*x**2 + 20*a*b**2*c*d**6*x**4 + 60*a*b**2*c*d**6
*x**3 - 120*a*b*c**5*d**3*x - 60*a*b*c**4*d**4*x**2 + 20*a*b*c**3*d**5*x**
3 - 10*a*b*c**2*d**6*x**4 + 30*a*b*c*d**7*x**5 - 300*b**3*c**5*d**2*x - 15
0*b**3*c**4*d**3*x**2 + 50*b**3*c**3*d**4*x**3 - 25*b**3*c**2*d**5*x**4 +
15*b**3*c*d**6*x**5 - 60*b**2*c**7*d*x - 30*b**2*c**6*d**2*x**2 + 10*b**2*
c**5*d**3*x**3 - 5*b**2*c**4*d**4*x**4 + 3*b**2*c**3*d**5*x**5 - 2*b**2*c*
*2*d**6*x**6 + 10*b**2*c*d**7*x**7)/(60*c*d**7*(c + d*x))
```

3.14 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 423

$$
\begin{aligned}
&\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx \\
&= \frac{(a^2d^4D-2abd^2(3cCd-Bd^2-6c^2D)-b^2c(10c^2Cd-6Bcd^2+3Ad^3-15c^3D))\,x}{d^7} \\
&\quad + \frac{b(2ad^2(Cd-3cD)+b(6c^2Cd-3Bcd^2+Ad^3-10c^3D))\,x^2}{2d^6} \\
&\quad + \frac{b(2ad^2D-b(3cCd-Bd^2-6c^2D))\,x^3}{3d^5} + \frac{b^2(Cd-3cD)x^4}{4d^4} \\
&\quad + \frac{b^2Dx^5}{5d^3} - \frac{(bc^2+ad^2)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{2d^8(c+dx)^2} \\
&\quad + \frac{(bc^2+ad^2)(ad^2(2cCd-Bd^2-3c^2D)+bc(6c^2Cd-5Bcd^2+4Ad^3-7c^3D))}{d^8(c+dx)} \\
&\quad + \frac{(a^2d^4(Cd-3cD)+b^2c^2(15c^2Cd-10Bcd^2+6Ad^3-21c^3D)+2abd^2(6c^2Cd-3Bcd^2+Ad^3-10c^3
}{d^8}
\end{aligned}
$$

output
```
(a^2*d^4*D-2*a*b*d^2*(-B*d^2+3*C*c*d-6*D*c^2)-b^2*c*(3*A*d^3-6*B*c*d^2+10*
C*c^2*d-15*D*c^3))*x/d^7+1/2*b*(2*a*d^2*(C*d-3*D*c)+b*(A*d^3-3*B*c*d^2+6*C
*c^2*d-10*D*c^3))*x^2/d^6+1/3*b*(2*a*d^2*D-b*(-B*d^2+3*C*c*d-6*D*c^2))*x^3
/d^5+1/4*b^2*(C*d-3*D*c)*x^4/d^4+1/5*b^2*D*x^5/d^3-1/2*(a*d^2+b*c^2)^2*(A*
d^3-B*c*d^2+C*c^2*d-D*c^3)/d^8/(d*x+c)^2+(a*d^2+b*c^2)*(a*d^2*(-B*d^2+2*C*
c*d-3*D*c^2)+b*c*(4*A*d^3-5*B*c*d^2+6*C*c^2*d-7*D*c^3))/d^8/(d*x+c)+(a^2*d
^4*(C*d-3*D*c)+b^2*c^2*(6*A*d^3-10*B*c*d^2+15*C*c^2*d-21*D*c^3)+2*a*b*d^2*
(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*ln(d*x+c)/d^8
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

$$= \frac{60d(a^2d^4D+2abd^2(-3cCd+Bd^2+6c^2D)+b^2c(-10c^2Cd+6Bcd^2-3Ad^3+15c^3D))\,x+30bd^2(2ad^2}{}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(60*d*(a^2*d^4*D + 2*a*b*d^2*(-3*c*C*d + B*d^2 + 6*c^2*D) + b^2*c*(-10*c^2
*C*d + 6*B*c*d^2 - 3*A*d^3 + 15*c^3*D))*x + 30*b*d^2*(2*a*d^2*(C*d - 3*c*D
) + b*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x^2 + 20*b*d^3*(2*a*d^2*
D + b*(-3*c*C*d + B*d^2 + 6*c^2*D))*x^3 + 15*b^2*d^4*(C*d - 3*c*D)*x^4 + 1
2*b^2*d^5*D*x^5 + (30*(b*c^2 + a*d^2)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^
3*D))/(c + d*x)^2 - (60*(b*c^2 + a*d^2)*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D
) + b*c*(-6*c^2*C*d + 5*B*c*d^2 - 4*A*d^3 + 7*c^3*D)))/(c + d*x) + 60*(a^2
*d^4*(C*d - 3*c*D) + b^2*c^2*(15*c^2*C*d - 10*B*c*d^2 + 6*A*d^3 - 21*c^3*D
) + 2*a*b*d^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*Log[c + d*x])/(6
0*d^8)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{a^2d^4(Cd-3cD)+2abd^2(Ad^3-3Bcd^2-10c^3D+6c^2Cd)+b^2c^2(6Ad^3-10Bcd^2-21c^3D+15c^2Cd)}{d^7(c+dx)} + \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(c+dx)\left(a^2d^4(Cd-3cD)+2abd^2(Ad^3-3Bcd^2-10c^3D+6c^2Cd)+b^2c^2(6Ad^3-10Bcd^2-21c^3D+15c^2\right.}{d^8}$$

$$\frac{x\left(a^2d^4D-2abd^2(-Bd^2-6c^2D+3cCd)-b^2c(3Ad^3-6Bcd^2-15c^3D+10c^2Cd)\right)}{d^7}+\frac{(ad^2+bc^2)\left(ad^2(-Bd^2-3c^2D+2cCd)+bc(4Ad^3-5Bcd^2-7c^3D+6c^2Cd)\right)}{d^8(c+dx)}-\frac{(ad^2+bc^2)^2(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{2d^8(c+dx)^2}+\frac{bx^2\left(2ad^2(Cd-3cD)+b(Ad^3-3Bcd^2-10c^3D+6c^2Cd)\right)}{2d^6}+\frac{bx^3\left(2ad^2D-b(-Bd^2-6c^2D+3cCd)\right)}{3d^5}+\frac{b^2x^4(Cd-3cD)}{4d^4}+\frac{b^2Dx^5}{5d^3}$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((a^2*d^4*D - 2*a*b*d^2*(3*c*C*d - B*d^2 - 6*c^2*D) - b^2*c*(10*c^2*C*d -
6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*x)/d^7 + (b*(2*a*d^2*(C*d - 3*c*D) + b*(6
*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x^2)/(2*d^6) + (b*(2*a*d^2*D - b
*(3*c*C*d - B*d^2 - 6*c^2*D))*x^3)/(3*d^5) + (b^2*(C*d - 3*c*D)*x^4)/(4*d^
4) + (b^2*D*x^5)/(5*d^3) - ((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 -
 c^3*D))/(2*d^8*(c + d*x)^2) + ((b*c^2 + a*d^2)*(a*d^2*(2*c*C*d - B*d^2 -
3*c^2*D) + b*c*(6*c^2*C*d - 5*B*c*d^2 + 4*A*d^3 - 7*c^3*D)))/(d^8*(c + d*x
)) + ((a^2*d^4*(C*d - 3*c*D) + b^2*c^2*(15*c^2*C*d - 10*B*c*d^2 + 6*A*d^3
- 21*c^3*D) + 2*a*b*d^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*Log[c
+ d*x])/d^8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.39

method	result
norman	$\frac{\frac{\left(4Aabc\,d^5+12A\,b^2c^3d^3-B\,a^2d^6-12Bab\,c^2d^4-20B\,b^2c^4d^2+2C\,a^2c\,d^5+24Cab\,c^3d^3+30C\,b^2c^5d-6Da^2c^2d^4-40Dab\,c^4d^2-42Db^2c^6\right)x}{d^7}-}{}$
default	$-\frac{-\frac{1}{5}b^2Dx^5d^4-\frac{1}{4}C\,b^2d^4x^4+\frac{3}{4}Db^2c\,d^3x^4-\frac{1}{3}B\,b^2d^4x^3+C\,b^2c\,d^3x^3-\frac{2}{3}Dab\,d^4x^3-2Db^2c^2d^2x^3-\frac{1}{2}A\,b^2d^4x^2+\frac{3}{2}B\,b^2c\,d^3x^2-}{}$
parallelrisch	Expression too large to display

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
((4*A*a*b*c*d^5+12*A*b^2*c^3*d^3-B*a^2*d^6-12*B*a*b*c^2*d^4-20*B*b^2*c^4*d
^2+2*C*a^2*c*d^5+24*C*a*b*c^3*d^3+30*C*b^2*c^5*d-6*D*a^2*c^2*d^4-40*D*a*b*
c^4*d^2-42*D*b^2*c^6)/d^7*x-1/2*(A*a^2*d^7-6*A*a*b*c^2*d^5-18*A*b^2*c^4*d^
3+B*a^2*c*d^6+18*B*a*b*c^3*d^4+30*B*b^2*c^5*d^2-3*C*a^2*c^2*d^5-36*C*a*b*c
^4*d^3-45*C*b^2*c^6*d+9*D*a^2*c^3*d^4+60*D*a*b*c^5*d^2+63*D*b^2*c^7)/d^8-1
/3*(6*A*b^2*c*d^3-6*B*a*b*d^4-10*B*b^2*c^2*d^2+12*C*a*b*c*d^3+15*C*b^2*c^3
*d-3*D*a^2*d^4-20*D*a*b*c^2*d^2-21*D*b^2*c^4)/d^5*x^3+1/30*b*(10*B*b*d^2-1
5*C*b*c*d+20*D*a*d^2+21*D*b*c^2)/d^3*x^5+1/12*b*(6*A*b*d^3-10*B*b*c*d^2+12
*C*a*d^3+15*C*b*c^2*d-20*D*a*c*d^2-21*D*b*c^3)/d^4*x^4+1/5*b^2*D*x^7/d+1/2
0*b^2*(5*C*d-7*D*c)/d^2*x^6)/(d*x+c)^2+1/d^8*(2*A*a*b*d^5+6*A*b^2*c^2*d^3-
6*B*a*b*c*d^4-10*B*b^2*c^3*d^2+C*a^2*d^5+12*C*a*b*c^2*d^3+15*C*b^2*c^4*d-3
*D*a^2*c*d^4-20*D*a*b*c^3*d^2-21*D*b^2*c^5)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(415) = 830$.

Time = 0.08 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.04

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/60*(12*D*b^2*d^7*x^7 - 390*D*b^2*c^7 + 330*C*b^2*c^6*d - 30*B*a^2*c*d^6
- 30*A*a^2*d^7 - 270*(2*D*a*b + B*b^2)*c^5*d^2 + 210*(2*C*a*b + A*b^2)*c^4
*d^3 - 150*(D*a^2 + 2*B*a*b)*c^3*d^4 + 90*(C*a^2 + 2*A*a*b)*c^2*d^5 - 3*(7
*D*b^2*c*d^6 - 5*C*b^2*d^7)*x^6 + 2*(21*D*b^2*c^2*d^5 - 15*C*b^2*c*d^6 + 1
0*(2*D*a*b + B*b^2)*d^7)*x^5 - 5*(21*D*b^2*c^3*d^4 - 15*C*b^2*c^2*d^5 + 10
*(2*D*a*b + B*b^2)*c*d^6 - 6*(2*C*a*b + A*b^2)*d^7)*x^4 + 20*(21*D*b^2*c^4
*d^3 - 15*C*b^2*c^3*d^4 + 10*(2*D*a*b + B*b^2)*c^2*d^5 - 6*(2*C*a*b + A*b^
2)*c*d^6 + 3*(D*a^2 + 2*B*a*b)*d^7)*x^3 + 30*(50*D*b^2*c^5*d^2 - 34*C*b^2*
c^4*d^3 + 21*(2*D*a*b + B*b^2)*c^3*d^4 - 11*(2*C*a*b + A*b^2)*c^2*d^5 + 4*
(D*a^2 + 2*B*a*b)*c*d^6)*x^2 + 60*(8*D*b^2*c^6*d - 4*C*b^2*c^5*d^2 - B*a^2
*d^7 + (2*D*a*b + B*b^2)*c^4*d^3 + (2*C*a*b + A*b^2)*c^3*d^4 - 2*(D*a^2 +
2*B*a*b)*c^2*d^5 + 2*(C*a^2 + 2*A*a*b)*c*d^6)*x - 60*(21*D*b^2*c^7 - 15*C*
b^2*c^6*d + 10*(2*D*a*b + B*b^2)*c^5*d^2 - 6*(2*C*a*b + A*b^2)*c^4*d^3 + 3
*(D*a^2 + 2*B*a*b)*c^3*d^4 - (C*a^2 + 2*A*a*b)*c^2*d^5 + (21*D*b^2*c^5*d^2
 - 15*C*b^2*c^4*d^3 + 10*(2*D*a*b + B*b^2)*c^3*d^4 - 6*(2*C*a*b + A*b^2)*c
^2*d^5 + 3*(D*a^2 + 2*B*a*b)*c*d^6 - (C*a^2 + 2*A*a*b)*d^7)*x^2 + 2*(21*D*
b^2*c^6*d - 15*C*b^2*c^5*d^2 + 10*(2*D*a*b + B*b^2)*c^4*d^3 - 6*(2*C*a*b +
 A*b^2)*c^3*d^4 + 3*(D*a^2 + 2*B*a*b)*c^2*d^5 - (C*a^2 + 2*A*a*b)*c*d^6)*x
)*log(d*x + c))/(d^10*x^2 + 2*c*d^9*x + c^2*d^8)
```

Sympy [A] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

$$= \frac{Db^2x^5}{5d^3} + x^4\left(\frac{Cb^2}{4d^3} - \frac{3Db^2c}{4d^4}\right) + x^3\left(\frac{Bb^2}{3d^3} - \frac{Cb^2c}{d^4} + \frac{2Dab}{3d^3} + \frac{2Db^2c^2}{d^5}\right)$$

$$+ x^2\left(\frac{Ab^2}{2d^3} - \frac{3Bb^2c}{2d^4} + \frac{Cab}{d^3} + \frac{3Cb^2c^2}{d^5} - \frac{3Dabc}{d^4} - \frac{5Db^2c^3}{d^6}\right)$$

$$+ x\left(-\frac{3Ab^2c}{d^4} + \frac{2Bab}{d^3} + \frac{6Bb^2c^2}{d^5} - \frac{6Cabc}{d^4} - \frac{10Cb^2c^3}{d^6} + \frac{Da^2}{d^3} + \frac{12Dabc^2}{d^5} + \frac{15Db^2c^4}{d^7}\right)$$

$$+ \frac{-Aa^2d^7 + 6Aabc^2d^5 + 7Ab^2c^4d^3 - Ba^2cd^6 - 10Babc^3d^4 - 9Bb^2c^5d^2 + 3Ca^2c^2d^5 + 14Cabc^4d^3 + 11C}{}$$

$$- \frac{(-2Aabd^5 - 6Ab^2c^2d^3 + 6Babcd^4 + 10Bb^2c^3d^2 - Ca^2d^5 - 12Cabc^2d^3 - 15Cb^2c^4d + 3Da^2cd^4 + 20Da}{d^8}$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b**2*x**5/(5*d**3) + x**4*(C*b**2/(4*d**3) - 3*D*b**2*c/(4*d**4)) + x**3
*(B*b**2/(3*d**3) - C*b**2*c/d**4 + 2*D*a*b/(3*d**3) + 2*D*b**2*c**2/d**5)
 + x**2*(A*b**2/(2*d**3) - 3*B*b**2*c/(2*d**4) + C*a*b/d**3 + 3*C*b**2*c**
2/d**5 - 3*D*a*b*c/d**4 - 5*D*b**2*c**3/d**6) + x*(-3*A*b**2*c/d**4 + 2*B*
a*b/d**3 + 6*B*b**2*c**2/d**5 - 6*C*a*b*c/d**4 - 10*C*b**2*c**3/d**6 + D*a
**2/d**3 + 12*D*a*b*c**2/d**5 + 15*D*b**2*c**4/d**7) + (-A*a**2*d**7 + 6*A
*a*b*c**2*d**5 + 7*A*b**2*c**4*d**3 - B*a**2*c*d**6 - 10*B*a*b*c**3*d**4 -
 9*B*b**2*c**5*d**2 + 3*C*a**2*c**2*d**5 + 14*C*a*b*c**4*d**3 + 11*C*b**2*
c**6*d - 5*D*a**2*c**3*d**4 - 18*D*a*b*c**5*d**2 - 13*D*b**2*c**7 + x*(8*A
*a*b*c*d**6 + 8*A*b**2*c**3*d**4 - 2*B*a**2*d**7 - 12*B*a*b*c**2*d**5 - 10
*B*b**2*c**4*d**3 + 4*C*a**2*c*d**6 + 16*C*a*b*c**3*d**4 + 12*C*b**2*c**5*
d**2 - 6*D*a**2*c**2*d**5 - 20*D*a*b*c**4*d**3 - 14*D*b**2*c**6*d))/(2*c**
2*d**8 + 4*c*d**9*x + 2*d**10*x**2) - (-2*A*a*b*d**5 - 6*A*b**2*c**2*d**3
+ 6*B*a*b*c*d**4 + 10*B*b**2*c**3*d**2 - C*a**2*d**5 - 12*C*a*b*c**2*d**3
- 15*C*b**2*c**4*d + 3*D*a**2*c*d**4 + 20*D*a*b*c**3*d**2 + 21*D*b**2*c**5
)*log(c + d*x)/d**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = -\frac{13\,Db^2c^7 - 11\,Cb^2c^6d + Ba^2cd^6 + Aa^2d^7 + 9\,(2\,Dab + Bb^2)c^5d^2 - 7\,(2\,Cab + Ab^2)c^4d^3 + 5\,(Da^2 + 2\,\ldots}{\ldots} + \frac{12\,Db^2d^4x^5 - 15\,(3\,Db^2cd^3 - Cb^2d^4)x^4 + 20\,(6\,Db^2c^2d^2 - 3\,Cb^2cd^3 + (2\,Dab + Bb^2)d^4)x^3 - 30\,(10\,D\ldots}{\ldots} - \frac{(21\,Db^2c^5 - 15\,Cb^2c^4d + 10\,(2\,Dab + Bb^2)c^3d^2 - 6\,(2\,Cab + Ab^2)c^2d^3 + 3\,(Da^2 + 2\,Bab)cd^4 - (Ca^2 + \ldots}{d^8}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(13*D*b^2*c^7 - 11*C*b^2*c^6*d + B*a^2*c*d^6 + A*a^2*d^7 + 9*(2*D*a*b
 + B*b^2)*c^5*d^2 - 7*(2*C*a*b + A*b^2)*c^4*d^3 + 5*(D*a^2 + 2*B*a*b)*c^3*
d^4 - 3*(C*a^2 + 2*A*a*b)*c^2*d^5 + 2*(7*D*b^2*c^6*d - 6*C*b^2*c^5*d^2 + B
*a^2*d^7 + 5*(2*D*a*b + B*b^2)*c^4*d^3 - 4*(2*C*a*b + A*b^2)*c^3*d^4 + 3*(
D*a^2 + 2*B*a*b)*c^2*d^5 - 2*(C*a^2 + 2*A*a*b)*c*d^6)*x)/(d^10*x^2 + 2*c*d
^9*x + c^2*d^8) + 1/60*(12*D*b^2*d^4*x^5 - 15*(3*D*b^2*c*d^3 - C*b^2*d^4)*
x^4 + 20*(6*D*b^2*c^2*d^2 - 3*C*b^2*c*d^3 + (2*D*a*b + B*b^2)*d^4)*x^3 - 3
0*(10*D*b^2*c^3*d - 6*C*b^2*c^2*d^2 + 3*(2*D*a*b + B*b^2)*c*d^3 - (2*C*a*b
 + A*b^2)*d^4)*x^2 + 60*(15*D*b^2*c^4 - 10*C*b^2*c^3*d + 6*(2*D*a*b + B*b^
2)*c^2*d^2 - 3*(2*C*a*b + A*b^2)*c*d^3 + (D*a^2 + 2*B*a*b)*d^4)*x)/d^7 - (
21*D*b^2*c^5 - 15*C*b^2*c^4*d + 10*(2*D*a*b + B*b^2)*c^3*d^2 - 6*(2*C*a*b
+ A*b^2)*c^2*d^3 + 3*(D*a^2 + 2*B*a*b)*c*d^4 - (C*a^2 + 2*A*a*b)*d^5)*log(
d*x + c)/d^8
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx =$$

$$-\frac{(21\,Db^2c^5 - 15\,Cb^2c^4d + 20\,Dabc^3d^2 + 10\,Bb^2c^3d^2 - 12\,Cabc^2d^3 - 6\,Ab^2c^2d^3 + 3\,Da^2cd^4 + 6\,Babcd^4}{d^8}$$

$$-\frac{13\,Db^2c^7 - 11\,Cb^2c^6d + 18\,Dabc^5d^2 + 9\,Bb^2c^5d^2 - 14\,Cabc^4d^3 - 7\,Ab^2c^4d^3 + 5\,Da^2c^3d^4 + 10\,Babc^3d^4}{}$$

$$+\frac{12\,Db^2d^{12}x^5 - 45\,Db^2cd^{11}x^4 + 15\,Cb^2d^{12}x^4 + 120\,Db^2c^2d^{10}x^3 - 60\,Cb^2cd^{11}x^3 + 40\,Dabd^{12}x^3 + 20\,Bb^2}{}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(21*D*b^2*c^5 - 15*C*b^2*c^4*d + 20*D*a*b*c^3*d^2 + 10*B*b^2*c^3*d^2 - 12
*C*a*b*c^2*d^3 - 6*A*b^2*c^2*d^3 + 3*D*a^2*c*d^4 + 6*B*a*b*c*d^4 - C*a^2*d
^5 - 2*A*a*b*d^5)*log(abs(d*x + c))/d^8 - 1/2*(13*D*b^2*c^7 - 11*C*b^2*c^6
*d + 18*D*a*b*c^5*d^2 + 9*B*b^2*c^5*d^2 - 14*C*a*b*c^4*d^3 - 7*A*b^2*c^4*d
^3 + 5*D*a^2*c^3*d^4 + 10*B*a*b*c^3*d^4 - 3*C*a^2*c^2*d^5 - 6*A*a*b*c^2*d^
5 + B*a^2*c*d^6 + A*a^2*d^7 + 2*(7*D*b^2*c^6*d - 6*C*b^2*c^5*d^2 + 10*D*a*
b*c^4*d^3 + 5*B*b^2*c^4*d^3 - 8*C*a*b*c^3*d^4 - 4*A*b^2*c^3*d^4 + 3*D*a^2*
c^2*d^5 + 6*B*a*b*c^2*d^5 - 2*C*a^2*c*d^6 - 4*A*a*b*c*d^6 + B*a^2*d^7)*x)/
((d*x + c)^2*d^8) + 1/60*(12*D*b^2*d^12*x^5 - 45*D*b^2*c*d^11*x^4 + 15*C*b
^2*d^12*x^4 + 120*D*b^2*c^2*d^10*x^3 - 60*C*b^2*c*d^11*x^3 + 40*D*a*b*d^12
*x^3 + 20*B*b^2*d^12*x^3 - 300*D*b^2*c^3*d^9*x^2 + 180*C*b^2*c^2*d^10*x^2
- 180*D*a*b*c*d^11*x^2 - 90*B*b^2*c*d^11*x^2 + 60*C*a*b*d^12*x^2 + 30*A*b^
2*d^12*x^2 + 900*D*b^2*c^4*d^8*x - 600*C*b^2*c^3*d^9*x + 720*D*a*b*c^2*d^1
0*x + 360*B*b^2*c^2*d^10*x - 360*C*a*b*c*d^11*x - 180*A*b^2*c*d^11*x + 60*
D*a^2*d^12*x + 120*B*a*b*d^12*x)/d^15
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 174.81 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
(60*log(c + d*x)*a**2*b*c**3*d**4 + 120*log(c + d*x)*a**2*b*c**2*d**5*x +
60*log(c + d*x)*a**2*b*c*d**6*x**2 - 60*log(c + d*x)*a**2*c**4*d**4 - 120*
log(c + d*x)*a**2*c**3*d**5*x - 60*log(c + d*x)*a**2*c**2*d**6*x**2 + 180*
log(c + d*x)*a*b**2*c**5*d**2 + 360*log(c + d*x)*a*b**2*c**4*d**3*x - 180*
log(c + d*x)*a*b**2*c**4*d**3 + 180*log(c + d*x)*a*b**2*c**3*d**4*x**2 - 3
60*log(c + d*x)*a*b**2*c**3*d**4*x - 180*log(c + d*x)*a*b**2*c**2*d**5*x**
2 - 240*log(c + d*x)*a*b*c**6*d**2 - 480*log(c + d*x)*a*b*c**5*d**3*x - 24
0*log(c + d*x)*a*b*c**4*d**4*x**2 - 300*log(c + d*x)*b**3*c**6*d - 600*log
(c + d*x)*b**3*c**5*d**2*x - 300*log(c + d*x)*b**3*c**4*d**3*x**2 - 180*lo
g(c + d*x)*b**2*c**8 - 360*log(c + d*x)*b**2*c**7*d*x - 180*log(c + d*x)*b
**2*c**6*d**2*x**2 - 15*a**3*c*d**6 + 30*a**2*b*c**3*d**4 - 60*a**2*b*c*d*
*6*x**2 + 15*a**2*b*d**7*x**2 - 30*a**2*c**4*d**4 + 60*a**2*c**2*d**6*x**2
 + 30*a**2*c*d**7*x**3 + 90*a*b**2*c**5*d**2 - 90*a*b**2*c**4*d**3 - 180*a
*b**2*c**3*d**4*x**2 - 60*a*b**2*c**2*d**5*x**3 + 180*a*b**2*c**2*d**5*x**
2 + 15*a*b**2*c*d**6*x**4 + 60*a*b**2*c*d**6*x**3 - 120*a*b*c**6*d**2 + 24
0*a*b*c**4*d**4*x**2 + 80*a*b*c**3*d**5*x**3 - 20*a*b*c**2*d**6*x**4 + 20*
a*b*c*d**7*x**5 - 150*b**3*c**6*d + 300*b**3*c**4*d**3*x**2 + 100*b**3*c**
3*d**4*x**3 - 25*b**3*c**2*d**5*x**4 + 10*b**3*c*d**6*x**5 - 90*b**2*c**8
+ 180*b**2*c**6*d**2*x**2 + 60*b**2*c**5*d**3*x**3 - 15*b**2*c**4*d**4*x**
4 + 6*b**2*c**3*d**5*x**5 - 3*b**2*c**2*d**6*x**6 + 6*b**2*c*d**7*x**7)...
```

3.15 $\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 695

$$
\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= \frac{(bc^2+ad^2)^3 (c^2Cd-Bcd^2+Ad^3-c^3D) (c+dx)^4}{4d^{10}} \\
&\quad - \frac{(bc^2+ad^2)^2 (ad^2(2cCd-Bd^2-3c^2D)+bc(8c^2Cd-7Bcd^2+6Ad^3-9c^3D)) (c+dx)^5}{5d^{10}} \\
&\quad + \frac{(bc^2+ad^2) (a^2d^4(Cd-3cD)+b^2c^2(28c^2Cd-21Bcd^2+15Ad^3-36c^3D)+abd^2(17c^2Cd-9Bcd^2+}{6d^{10}} \\
&\quad + \frac{(a^3d^6D-3a^2bd^4(4cCd-Bd^2-10c^2D)-b^3c^3(56c^2Cd-35Bcd^2+20Ad^3-84c^3D)-3ab^2cd^2(20c^2}{7d^{10}} \\
&\quad + \frac{b(3a^2d^4(Cd-5cD)+b^2c^2(70c^2Cd-35Bcd^2+15Ad^3-126c^3D)+3abd^2(15c^2Cd-5Bcd^2+Ad^3-}{8d^{10}} \\
&\quad + \frac{b(3a^2d^4D-3abd^2(6cCd-Bd^2-21c^2D)-b^2c(56c^2Cd-21Bcd^2+6Ad^3-126c^3D)) (c+dx)^9}{9d^{10}} \\
&\quad + \frac{b^2(3ad^2(Cd-7cD)+b(28c^2Cd-7Bcd^2+Ad^3-84c^3D)) (c+dx)^{10}}{10d^{10}} \\
&\quad + \frac{b^2(3ad^2D-b(8cCd-Bd^2-36c^2D)) (c+dx)^{11}}{11d^{10}} \\
&\quad + \frac{b^3(Cd-9cD)(c+dx)^{12}}{12d^{10}} + \frac{b^3D(c+dx)^{13}}{13d^{10}}
\end{aligned}
$$

output
```
1/4*(a*d^2+b*c^2)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^4/d^10-1/5*(a*d^
2+b*c^2)^2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(6*A*d^3-7*B*c*d^2+8*C*c^2*
d-9*D*c^3))*(d*x+c)^5/d^10+1/6*(a*d^2+b*c^2)*(a^2*d^4*(C*d-3*D*c)+b^2*c^2*
(15*A*d^3-21*B*c*d^2+28*C*c^2*d-36*D*c^3)+a*b*d^2*(3*A*d^3-9*B*c*d^2+17*C*
c^2*d-27*D*c^3))*(d*x+c)^6/d^10+1/7*(a^3*d^6*D-3*a^2*b*d^4*(-B*d^2+4*C*c*d
-10*D*c^2)-b^3*c^3*(20*A*d^3-35*B*c*d^2+56*C*c^2*d-84*D*c^3)-3*a*b^2*c*d^2
*(4*A*d^3-10*B*c*d^2+20*C*c^2*d-35*D*c^3))*(d*x+c)^7/d^10+1/8*b*(3*a^2*d^4
*(C*d-5*D*c)+b^2*c^2*(15*A*d^3-35*B*c*d^2+70*C*c^2*d-126*D*c^3)+3*a*b*d^2*
(A*d^3-5*B*c*d^2+15*C*c^2*d-35*D*c^3))*(d*x+c)^8/d^10+1/9*b*(3*a^2*d^4*D-3
*a*b*d^2*(-B*d^2+6*C*c*d-21*D*c^2)-b^2*c*(6*A*d^3-21*B*c*d^2+56*C*c^2*d-12
6*D*c^3))*(d*x+c)^9/d^10+1/10*b^2*(3*a*d^2*(C*d-7*D*c)+b*(A*d^3-7*B*c*d^2+
28*C*c^2*d-84*D*c^3))*(d*x+c)^10/d^10+1/11*b^2*(3*a*d^2*D-b*(-B*d^2+8*C*c*
d-36*D*c^2))*(d*x+c)^11/d^10+1/12*b^3*(C*d-9*D*c)*(d*x+c)^12/d^10+1/13*b^3
*D*(d*x+c)^13/d^10
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= a^3Ac^3x+\frac{1}{2}a^3c^2(Bc+3Ad)x^2+\frac{1}{3}a^2c\big(ac(cC+3Bd)+3A\big(bc^2+ad^2\big)\big)\,x^3 \\
&\quad+\frac{1}{4}a^2\big(3bc^2(Bc+3Ad)+a\big(3c^2Cd+3Bcd^2+Ad^3+c^3D\big)\big)\,x^4 \\
&\quad+\frac{1}{5}a\big(3Abc\big(bc^2+3ad^2\big)+a\big(3bc^2(cC+3Bd)+ad\big(3cCd+Bd^2+3c^2D\big)\big)\big)\,x^5 \\
&\quad+\frac{1}{6}a\big(3b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)+3ab\big(3c^2Cd+3Bcd^2+Ad^3+c^3D\big)\big)\,x^6 \\
&\quad+\frac{1}{7}\big(Ab^2c\big(bc^2+9ad^2\big)+a\big(3b^2c^2(cC+3Bd)+a^2d^3D+3abd\big(3cCd+Bd^2+3c^2D\big)\big)\big)\,x^7 \\
&\quad+\frac{1}{8}b\big(b^2c^2(Bc+3Ad)+3a^2d^2(Cd+3cD)+3ab\big(3c^2Cd+3Bcd^2+Ad^3+c^3D\big)\big)\,x^8 \\
&\quad+\frac{1}{9}b\big(b^2c\big(c^2C+3Bcd+3Ad^2\big)+3a^2d^3D+3abd\big(3cCd+Bd^2+3c^2D\big)\big)\,x^9 \\
&\quad+\frac{1}{10}b^2\big(3ad^2(Cd+3cD)+b\big(3c^2Cd+3Bcd^2+Ad^3+c^3D\big)\big)\,x^{10} \\
&\quad+\frac{1}{11}b^2d\big(3ad^2D+b\big(3cCd+Bd^2+3c^2D\big)\big)\,x^{11}+\frac{1}{12}b^3d^2(Cd+3cD)x^{12}+\frac{1}{13}b^3d^3Dx^{13}
\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
a^3*A*c^3*x + (a^3*c^2*(B*c + 3*A*d)*x^2)/2 + (a^2*c*(a*c*(c*C + 3*B*d) +
3*A*(b*c^2 + a*d^2))*x^3)/3 + (a^2*(3*b*c^2*(B*c + 3*A*d) + a*(3*c^2*C*d +
 3*B*c*d^2 + A*d^3 + c^3*D))*x^4)/4 + (a*(3*A*b*c*(b*c^2 + 3*a*d^2) + a*(3
*b*c^2*(c*C + 3*B*d) + a*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^5)/5 + (a*(3*b^
2*c^2*(B*c + 3*A*d) + a^2*d^2*(C*d + 3*c*D) + 3*a*b*(3*c^2*C*d + 3*B*c*d^2
 + A*d^3 + c^3*D))*x^6)/6 + ((A*b^2*c*(b*c^2 + 9*a*d^2) + a*(3*b^2*c^2*(c*
C + 3*B*d) + a^2*d^3*D + 3*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^7)/7 + (b
*(b^2*c^2*(B*c + 3*A*d) + 3*a^2*d^2*(C*d + 3*c*D) + 3*a*b*(3*c^2*C*d + 3*B
*c*d^2 + A*d^3 + c^3*D))*x^8)/8 + (b*(b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2) +
3*a^2*d^3*D + 3*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D))*x^9)/9 + (b^2*(3*a*d^2*
(C*d + 3*c*D) + b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*x^10)/10 + (b^2
*d*(3*a*d^2*D + b*(3*c*C*d + B*d^2 + 3*c^2*D))*x^11)/11 + (b^3*d^2*(C*d +
3*c*D)*x^12)/12 + (b^3*d^3*D*x^13)/13
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 (c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(bx^2+a\right)^3 \left((c+dx)^3 \left(Dx^3+Cx^2+Bx+A\right)-\left(Bc^3+3Adc^2\right) x\right) dx+\frac{c^2 \left(a+bx^2\right)^4 (3Ad+Bc)}{8b}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(b^3d^3Dx^{12}+b^3d^2(Cd+3cD)x^{11}+b^2d\left(3aDd^2+b\left(3Dc^2+3Cdc+Bd^2\right)\right)x^{10}+b^2\left(3a(Cd+3cD)d^2+b\left(Dc^3\right.\right.\right.$$
$$\frac{c^2 \left(a+bx^2\right)^4 (3Ad+Bc)}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^3x^4\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)+a^3Ac^3x+$$
$$\frac{1}{9}bx^9\left(3a^2d^3D+3abd\left(Bd^2+3c^2D+3cCd\right)+b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)+$$
$$\frac{1}{7}x^7\left(a\left(a^2d^3D+3abd\left(Bd^2+3c^2D+3cCd\right)+3b^2c^2(3Bd+cC)\right)+Ab^2c\left(9ad^2+bc^2\right)\right)+$$
$$\frac{1}{3}a^2cx^3\left(3A\left(ad^2+bc^2\right)+ac(3Bd+cC)\right)+$$
$$\frac{1}{6}a^2x^6\left(ad^2(3cD+Cd)+3b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)+$$
$$\frac{1}{10}b^2x^{10}\left(3ad^2(3cD+Cd)+b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)+$$
$$\frac{1}{5}ax^5\left(3Abc\left(3ad^2+bc^2\right)+a\left(ad\left(Bd^2+3c^2D+3cCd\right)+3bc^2(3Bd+cC)\right)\right)+$$
$$\frac{c^2\left(a+bx^2\right)^4(3Ad+Bc)}{8b}+\frac{3}{8}abx^8\left(ad^2(3cD+Cd)+b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)+$$
$$\frac{1}{11}b^2dx^{11}\left(3ad^2D+b\left(Bd^2+3c^2D+3cCd\right)\right)+\frac{1}{12}b^3d^2x^{12}(3cD+Cd)+\frac{1}{13}b^3d^3Dx^{13}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c^3*x + (a^2*c*(a*c*(c*C + 3*B*d) + 3*A*(b*c^2 + a*d^2))*x^3)/3 + (a
^3*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D)*x^4)/4 + (a*(3*A*b*c*(b*c^2 + 3
*a*d^2) + a*(3*b*c^2*(c*C + 3*B*d) + a*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^5
)/5 + (a^2*(a*d^2*(C*d + 3*c*D) + 3*b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3
*D))*x^6)/6 + ((A*b^2*c*(b*c^2 + 9*a*d^2) + a*(3*b^2*c^2*(c*C + 3*B*d) + a
^2*d^3*D + 3*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*x^7)/7 + (3*a*b*(a*d^2*(C
*d + 3*c*D) + b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*x^8)/8 + (b*(b^2*
c*(c^2*C + 3*B*c*d + 3*A*d^2) + 3*a^2*d^3*D + 3*a*b*d*(3*c*C*d + B*d^2 + 3
*c^2*D))*x^9)/9 + (b^2*(3*a*d^2*(C*d + 3*c*D) + b*(3*c^2*C*d + 3*B*c*d^2 +
 A*d^3 + c^3*D))*x^10)/10 + (b^2*d*(3*a*d^2*D + b*(3*c*C*d + B*d^2 + 3*c^2
*D))*x^11)/11 + (b^3*d^2*(C*d + 3*c*D)*x^12)/12 + (b^3*d^3*D*x^13)/13 + (c
^2*(B*c + 3*A*d)*(a + b*x^2)^4)/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^3 d^3 D x^{13}}{13}+\left(\frac{1}{12} b^3 d^3 C+\frac{1}{4} b^3 c\,d^2 D\right) x^{12}+\left(\frac{1}{11} B\,b^3 d^3+\frac{3}{11} b^3 c\,d^2 C+\frac{3}{11} D a\,b^2 d^3+\frac{3}{11} D b^3 c^2 d\right) x$
default	$\frac{b^3 d^3 D x^{13}}{13}+\frac{\left(b^3 d^3 C+3 b^3 c\,d^2 D\right) x^{12}}{12}+\frac{\left(\left(3 a\,b^2 d^3+3 b^3 c^2 d\right) D+3 b^3 c\,d^2 C+B\,b^3 d^3\right) x^{11}}{11}+\frac{\left(\left(9 a\,b^2 c\,d^2+b^3 c^3\right) D+\left(3 a\,b^2 d^3+\right.\right.}{}$
gosper	$\frac{1}{4} x^{12} b^3 c\,d^2 D+\frac{3}{11} x^{11} b^3 c\,d^2 C+\frac{3}{11} x^{11} D a\,b^2 d^3+\frac{3}{11} x^{11} D b^3 c^2 d+\frac{9}{8} x^8 B a\,b^2 c\,d^2+\frac{3}{10} x^{10} C\,b^3 c^2 d$
parallelrisch	$\frac{1}{4} x^{12} b^3 c\,d^2 D+\frac{3}{11} x^{11} b^3 c\,d^2 C+\frac{3}{11} x^{11} D a\,b^2 d^3+\frac{3}{11} x^{11} D b^3 c^2 d+\frac{9}{8} x^8 B a\,b^2 c\,d^2+\frac{3}{10} x^{10} C\,b^3 c^2 d$
orering	$\frac{x\left(27720 b^3 d^3 D x^{12}+30030 C\,b^3 d^3 x^{11}+90090 D b^3 c\,d^2 x^{11}+32760 B\,b^3 d^3 x^{10}+98280 C\,b^3 c\,d^2 x^{10}+98280 D a\,b^2 d^3 x^{10}+98280 D b^3\right.}{}$

input
```
int((d*x+c)^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*d^3*D*x^13+(1/12*b^3*d^3*C+1/4*b^3*c*d^2*D)*x^12+(1/11*B*b^3*d^3+
3/11*b^3*c*d^2*C+3/11*D*a*b^2*d^3+3/11*D*b^3*c^2*d)*x^11+(1/10*A*d^3*b^3+3
/10*b^3*c*d^2*B+3/10*C*a*b^2*d^3+3/10*C*b^3*c^2*d+9/10*D*a*b^2*c*d^2+1/10*
D*b^3*c^3)*x^10+(1/3*A*b^3*c*d^2+1/3*B*a*b^2*d^3+1/3*B*b^3*c^2*d+C*a*b^2*c
*d^2+1/9*C*b^3*c^3+1/3*D*a^2*b*d^3+D*a*b^2*c^2*d)*x^9+(3/8*A*d^3*a*b^2+3/8
*A*b^3*c^2*d+9/8*B*a*b^2*c*d^2+1/8*B*b^3*c^3+3/8*C*a^2*b*d^3+9/8*C*a*b^2*c
^2*d+9/8*D*a^2*b*c*d^2+3/8*D*a*b^2*c^3)*x^8+(9/7*A*a*b^2*c*d^2+1/7*A*b^3*c
^3+3/7*B*a^2*b*d^3+9/7*B*a*b^2*c^2*d+9/7*C*a^2*b*c*d^2+3/7*C*a*b^2*c^3+1/7
*D*a^3*d^3+9/7*D*a^2*b*c^2*d)*x^7+(1/2*A*a^2*b*d^3+3/2*A*a*b^2*c^2*d+3/2*B
*a^2*b*c*d^2+1/2*B*a*b^2*c^3+1/6*C*a^3*d^3+3/2*C*a^2*b*c^2*d+1/2*D*a^3*c*d
^2+1/2*D*a^2*b*c^3)*x^6+(9/5*A*a^2*b*c*d^2+3/5*A*a*b^2*c^3+1/5*a^3*B*d^3+9
/5*B*a^2*b*c^2*d+3/5*C*a^3*c*d^2+3/5*C*a^2*b*c^3+3/5*a^3*c^2*d*D)*x^5+(1/4
*A*d^3*a^3+9/4*A*a^2*b*c^2*d+3/4*B*a^3*c*d^2+3/4*B*a^2*b*c^3+3/4*C*a^3*c^2
*d+1/4*a^3*c^3*D)*x^4+(A*a^3*c*d^2+a^2*A*b*c^3+a^3*c^2*d*B+1/3*C*a^3*c^3)*
x^3+(3/2*a^3*c^2*d*A+1/2*B*a^3*c^3)*x^2+a^3*A*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 652, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int (c+dx)^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{13}Db^3d^3x^{13} \\
&+ \frac{1}{12}\left(3\,Db^3cd^2+Cb^3d^3\right)x^{12}+\frac{1}{11}\left(3\,Db^3c^2d+3\,Cb^3cd^2+\left(3\,Dab^2+Bb^3\right)d^3\right)x^{11} \\
&+ \frac{1}{10}\left(Db^3c^3+3\,Cb^3c^2d+3\left(3\,Dab^2+Bb^3\right)cd^2+\left(3\,Cab^2+Ab^3\right)d^3\right)x^{10} \\
&+ \frac{1}{9}\left(Cb^3c^3+3\left(3\,Dab^2+Bb^3\right)c^2d+3\left(3\,Cab^2+Ab^3\right)cd^2+3\left(Da^2b+Bab^2\right)d^3\right)x^9 \\
&+ \frac{1}{8}\left(\left(3\,Dab^2+Bb^3\right)c^3+3\left(3\,Cab^2+Ab^3\right)c^2d+9\left(Da^2b+Bab^2\right)cd^2+3\left(Ca^2b+Aab^2\right)d^3\right)x^8 \\
&+ Aa^3c^3x \\
&+ \frac{1}{7}\left(\left(3\,Cab^2+Ab^3\right)c^3+9\left(Da^2b+Bab^2\right)c^2d+9\left(Ca^2b+Aab^2\right)cd^2+\left(Da^3+3\,Ba^2b\right)d^3\right)x^7 \\
&+ \frac{1}{6}\left(3\left(Da^2b+Bab^2\right)c^3+9\left(Ca^2b+Aab^2\right)c^2d+3\left(Da^3+3\,Ba^2b\right)cd^2+\left(Ca^3+3\,Aa^2b\right)d^3\right)x^6 \\
&+ \frac{1}{5}\left(Ba^3d^3+3\left(Ca^2b+Aab^2\right)c^3+3\left(Da^3+3\,Ba^2b\right)c^2d+3\left(Ca^3+3\,Aa^2b\right)cd^2\right)x^5 \\
&+ \frac{1}{4}\left(3\,Ba^3cd^2+Aa^3d^3+\left(Da^3+3\,Ba^2b\right)c^3+3\left(Ca^3+3\,Aa^2b\right)c^2d\right)x^4 \\
&+ \frac{1}{3}\left(3\,Ba^3c^2d+3\,Aa^3cd^2+\left(Ca^3+3\,Aa^2b\right)c^3\right)x^3+\frac{1}{2}\left(Ba^3c^3+3\,Aa^3c^2d\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/13*D*b^3*d^3*x^13 + 1/12*(3*D*b^3*c*d^2 + C*b^3*d^3)*x^12 + 1/11*(3*D*b^
3*c^2*d + 3*C*b^3*c*d^2 + (3*D*a*b^2 + B*b^3)*d^3)*x^11 + 1/10*(D*b^3*c^3
+ 3*C*b^3*c^2*d + 3*(3*D*a*b^2 + B*b^3)*c*d^2 + (3*C*a*b^2 + A*b^3)*d^3)*x
^10 + 1/9*(C*b^3*c^3 + 3*(3*D*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a*b^2 + A*b^3)
*c*d^2 + 3*(D*a^2*b + B*a*b^2)*d^3)*x^9 + 1/8*((3*D*a*b^2 + B*b^3)*c^3 + 3
*(3*C*a*b^2 + A*b^3)*c^2*d + 9*(D*a^2*b + B*a*b^2)*c*d^2 + 3*(C*a^2*b + A*
a*b^2)*d^3)*x^8 + A*a^3*c^3*x + 1/7*((3*C*a*b^2 + A*b^3)*c^3 + 9*(D*a^2*b
+ B*a*b^2)*c^2*d + 9*(C*a^2*b + A*a*b^2)*c*d^2 + (D*a^3 + 3*B*a^2*b)*d^3)*
x^7 + 1/6*(3*(D*a^2*b + B*a*b^2)*c^3 + 9*(C*a^2*b + A*a*b^2)*c^2*d + 3*(D*
a^3 + 3*B*a^2*b)*c*d^2 + (C*a^3 + 3*A*a^2*b)*d^3)*x^6 + 1/5*(B*a^3*d^3 + 3
*(C*a^2*b + A*a*b^2)*c^3 + 3*(D*a^3 + 3*B*a^2*b)*c^2*d + 3*(C*a^3 + 3*A*a^
2*b)*c*d^2)*x^5 + 1/4*(3*B*a^3*c*d^2 + A*a^3*d^3 + (D*a^3 + 3*B*a^2*b)*c^3
 + 3*(C*a^3 + 3*A*a^2*b)*c^2*d)*x^4 + 1/3*(3*B*a^3*c^2*d + 3*A*a^3*c*d^2 +
 (C*a^3 + 3*A*a^2*b)*c^3)*x^3 + 1/2*(B*a^3*c^3 + 3*A*a^3*c^2*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.22

$$\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output

```
A*a**3*c**3*x + D*b**3*d**3*x**13/13 + x**12*(C*b**3*d**3/12 + D*b**3*c*d*
*2/4) + x**11*(B*b**3*d**3/11 + 3*C*b**3*c*d**2/11 + 3*D*a*b**2*d**3/11 +
3*D*b**3*c**2*d/11) + x**10*(A*b**3*d**3/10 + 3*B*b**3*c*d**2/10 + 3*C*a*b
**2*d**3/10 + 3*C*b**3*c**2*d/10 + 9*D*a*b**2*c*d**2/10 + D*b**3*c**3/10)
+ x**9*(A*b**3*c*d**2/3 + B*a*b**2*d**3/3 + B*b**3*c**2*d/3 + C*a*b**2*c*d
**2 + C*b**3*c**3/9 + D*a**2*b*d**3/3 + D*a*b**2*c**2*d) + x**8*(3*A*a*b**
2*d**3/8 + 3*A*b**3*c**2*d/8 + 9*B*a*b**2*c*d**2/8 + B*b**3*c**3/8 + 3*C*a
**2*b*d**3/8 + 9*C*a*b**2*c**2*d/8 + 9*D*a**2*b*c*d**2/8 + 3*D*a*b**2*c**3
/8) + x**7*(9*A*a*b**2*c*d**2/7 + A*b**3*c**3/7 + 3*B*a**2*b*d**3/7 + 9*B*
a*b**2*c**2*d/7 + 9*C*a**2*b*c*d**2/7 + 3*C*a*b**2*c**3/7 + D*a**3*d**3/7
+ 9*D*a**2*b*c**2*d/7) + x**6*(A*a**2*b*d**3/2 + 3*A*a*b**2*c**2*d/2 + 3*B
*a**2*b*c*d**2/2 + B*a*b**2*c**3/2 + C*a**3*d**3/6 + 3*C*a**2*b*c**2*d/2 +
 D*a**3*c*d**2/2 + D*a**2*b*c**3/2) + x**5*(9*A*a**2*b*c*d**2/5 + 3*A*a*b*
*2*c**3/5 + B*a**3*d**3/5 + 9*B*a**2*b*c**2*d/5 + 3*C*a**3*c*d**2/5 + 3*C*
a**2*b*c**3/5 + 3*D*a**3*c**2*d/5) + x**4*(A*a**3*d**3/4 + 9*A*a**2*b*c**2
*d/4 + 3*B*a**3*c*d**2/4 + 3*B*a**2*b*c**3/4 + 3*C*a**3*c**2*d/4 + D*a**3*
c**3/4) + x**3*(A*a**3*c*d**2 + A*a**2*b*c**3 + B*a**3*c**2*d + C*a**3*c**
3/3) + x**2*(3*A*a**3*c**2*d/2 + B*a**3*c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 652, normalized size of antiderivative = 0.94

$$\int (c+dx)^3 (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{13}Db^3d^3x^{13} + \frac{1}{12}(3Db^3cd^2+Cb^3d^3)x^{12} + \frac{1}{11}(3Db^3c^2d+3Cb^3cd^2+(3Dab^2+Bb^3)d^3)x^{11} + \frac{1}{10}(Db^3c^3+3Cb^3c^2d+3(3Dab^2+Bb^3)cd^2+(3Cab^2+Ab^3)d^3)x^{10} + \frac{1}{9}(Cb^3c^3+3(3Dab^2+Bb^3)c^2d+3(3Cab^2+Ab^3)cd^2+3(Da^2b+Bab^2)d^3)x^9 + \frac{1}{8}((3Dab^2+Bb^3)c^3+3(3Cab^2+Ab^3)c^2d+9(Da^2b+Bab^2)cd^2+3(Ca^2b+Aab^2)d^3)x^8 + Aa^3c^3x + \frac{1}{7}((3Cab^2+Ab^3)c^3+9(Da^2b+Bab^2)c^2d+9(Ca^2b+Aab^2)cd^2+(Da^3+3Ba^2b)d^3)x^7 + \frac{1}{6}(3(Da^2b+Bab^2)c^3+9(Ca^2b+Aab^2)c^2d+3(Da^3+3Ba^2b)cd^2+(Ca^3+3Aa^2b)d^3)x^6 + \frac{1}{5}(Ba^3d^3+3(Ca^2b+Aab^2)c^3+3(Da^3+3Ba^2b)c^2d+3(Ca^3+3Aa^2b)cd^2)x^5 + \frac{1}{4}(3Ba^3cd^2+Aa^3d^3+(Da^3+3Ba^2b)c^3+3(Ca^3+3Aa^2b)c^2d)x^4 + \frac{1}{3}(3Ba^3c^2d+3Aa^3cd^2+(Ca^3+3Aa^2b)c^3)x^3 + \frac{1}{2}(Ba^3c^3+3Aa^3c^2d)x^2$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/13*D*b^3*d^3*x^13 + 1/12*(3*D*b^3*c*d^2 + C*b^3*d^3)*x^12 + 1/11*(3*D*b^
3*c^2*d + 3*C*b^3*c*d^2 + (3*D*a*b^2 + B*b^3)*d^3)*x^11 + 1/10*(D*b^3*c^3
+ 3*C*b^3*c^2*d + 3*(3*D*a*b^2 + B*b^3)*c*d^2 + (3*C*a*b^2 + A*b^3)*d^3)*x
^10 + 1/9*(C*b^3*c^3 + 3*(3*D*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a*b^2 + A*b^3)
*c*d^2 + 3*(D*a^2*b + B*a*b^2)*d^3)*x^9 + 1/8*((3*D*a*b^2 + B*b^3)*c^3 + 3
*(3*C*a*b^2 + A*b^3)*c^2*d + 9*(D*a^2*b + B*a*b^2)*c*d^2 + 3*(C*a^2*b + A*
a*b^2)*d^3)*x^8 + A*a^3*c^3*x + 1/7*((3*C*a*b^2 + A*b^3)*c^3 + 9*(D*a^2*b
+ B*a*b^2)*c^2*d + 9*(C*a^2*b + A*a*b^2)*c*d^2 + (D*a^3 + 3*B*a^2*b)*d^3)*
x^7 + 1/6*(3*(D*a^2*b + B*a*b^2)*c^3 + 9*(C*a^2*b + A*a*b^2)*c^2*d + 3*(D*
a^3 + 3*B*a^2*b)*c*d^2 + (C*a^3 + 3*A*a^2*b)*d^3)*x^6 + 1/5*(B*a^3*d^3 + 3
*(C*a^2*b + A*a*b^2)*c^3 + 3*(D*a^3 + 3*B*a^2*b)*c^2*d + 3*(C*a^3 + 3*A*a^
2*b)*c*d^2)*x^5 + 1/4*(3*B*a^3*c*d^2 + A*a^3*d^3 + (D*a^3 + 3*B*a^2*b)*c^3
 + 3*(C*a^3 + 3*A*a^2*b)*c^2*d)*x^4 + 1/3*(3*B*a^3*c^2*d + 3*A*a^3*c*d^2 +
 (C*a^3 + 3*A*a^2*b)*c^3)*x^3 + 1/2*(B*a^3*c^3 + 3*A*a^3*c^2*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.19

$$\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13*D*b^3*d^3*x^13 + 1/4*D*b^3*c*d^2*x^12 + 1/12*C*b^3*d^3*x^12 + 3/11*D*
b^3*c^2*d*x^11 + 3/11*C*b^3*c*d^2*x^11 + 3/11*D*a*b^2*d^3*x^11 + 1/11*B*b^
3*d^3*x^11 + 1/10*D*b^3*c^3*x^10 + 3/10*C*b^3*c^2*d*x^10 + 9/10*D*a*b^2*c*
d^2*x^10 + 3/10*B*b^3*c*d^2*x^10 + 3/10*C*a*b^2*d^3*x^10 + 1/10*A*b^3*d^3*
x^10 + 1/9*C*b^3*c^3*x^9 + D*a*b^2*c^2*d*x^9 + 1/3*B*b^3*c^2*d*x^9 + C*a*b
^2*c*d^2*x^9 + 1/3*A*b^3*c*d^2*x^9 + 1/3*D*a^2*b*d^3*x^9 + 1/3*B*a*b^2*d^3
*x^9 + 3/8*D*a*b^2*c^3*x^8 + 1/8*B*b^3*c^3*x^8 + 9/8*C*a*b^2*c^2*d*x^8 + 3
/8*A*b^3*c^2*d*x^8 + 9/8*D*a^2*b*c*d^2*x^8 + 9/8*B*a*b^2*c*d^2*x^8 + 3/8*C
*a^2*b*d^3*x^8 + 3/8*A*a*b^2*d^3*x^8 + 3/7*C*a*b^2*c^3*x^7 + 1/7*A*b^3*c^3
*x^7 + 9/7*D*a^2*b*c^2*d*x^7 + 9/7*B*a*b^2*c^2*d*x^7 + 9/7*C*a^2*b*c*d^2*x
^7 + 9/7*A*a*b^2*c*d^2*x^7 + 1/7*D*a^3*d^3*x^7 + 3/7*B*a^2*b*d^3*x^7 + 1/2
*D*a^2*b*c^3*x^6 + 1/2*B*a*b^2*c^3*x^6 + 3/2*C*a^2*b*c^2*d*x^6 + 3/2*A*a*b
^2*c^2*d*x^6 + 1/2*D*a^3*c*d^2*x^6 + 3/2*B*a^2*b*c*d^2*x^6 + 1/6*C*a^3*d^3
*x^6 + 1/2*A*a^2*b*d^3*x^6 + 3/5*C*a^2*b*c^3*x^5 + 3/5*A*a*b^2*c^3*x^5 + 3
/5*D*a^3*c^2*d*x^5 + 9/5*B*a^2*b*c^2*d*x^5 + 3/5*C*a^3*c*d^2*x^5 + 9/5*A*a
^2*b*c*d^2*x^5 + 1/5*B*a^3*d^3*x^5 + 1/4*D*a^3*c^3*x^4 + 3/4*B*a^2*b*c^3*x
^4 + 3/4*C*a^3*c^2*d*x^4 + 9/4*A*a^2*b*c^2*d*x^4 + 3/4*B*a^3*c*d^2*x^4 + 1
/4*A*a^3*d^3*x^4 + 1/3*C*a^3*c^3*x^3 + A*a^2*b*c^3*x^3 + B*a^3*c^2*d*x^3 +
 A*a^3*c*d^2*x^3 + 1/2*B*a^3*c^3*x^2 + 3/2*A*a^3*c^2*d*x^2 + A*a^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 23.92 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.19

$$\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^3*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^3*c^3*x^4*D)/4 + (a^3*d^3*x^7*D)/7 + (b^3*c^3*x^10*D)/10 + (b^3*d^3*x^1
3*D)/13 + A*a^3*c^3*x + (B*a^3*c^3*x^2)/2 + (A*a^3*d^3*x^4)/4 + (A*b^3*c^3
*x^7)/7 + (C*a^3*c^3*x^3)/3 + (B*a^3*d^3*x^5)/5 + (B*b^3*c^3*x^8)/8 + (A*b
^3*d^3*x^10)/10 + (C*a^3*d^3*x^6)/6 + (C*b^3*c^3*x^9)/9 + (B*b^3*d^3*x^11)
/11 + (C*b^3*d^3*x^12)/12 + A*a^2*b*c^3*x^3 + (3*A*a*b^2*c^3*x^5)/5 + (3*B
*a^2*b*c^3*x^4)/4 + (A*a^2*b*d^3*x^6)/2 + (B*a*b^2*c^3*x^6)/2 + (3*A*a*b^2
*d^3*x^8)/8 + (3*A*a^3*c^2*d*x^2)/2 + A*a^3*c*d^2*x^3 + (3*C*a^2*b*c^3*x^5
)/5 + (3*B*a^2*b*d^3*x^7)/7 + (3*C*a*b^2*c^3*x^7)/7 + (B*a*b^2*d^3*x^9)/3
+ B*a^3*c^2*d*x^3 + (3*B*a^3*c*d^2*x^4)/4 + (3*A*b^3*c^2*d*x^8)/8 + (3*C*a
^2*b*d^3*x^8)/8 + (A*b^3*c*d^2*x^9)/3 + (3*C*a*b^2*d^3*x^10)/10 + (a^2*b*c
^3*x^6*D)/2 + (3*a*b^2*c^3*x^8*D)/8 + (3*C*a^3*c^2*d*x^4)/4 + (3*C*a^3*c*d
^2*x^5)/5 + (B*b^3*c^2*d*x^9)/3 + (3*B*b^3*c*d^2*x^10)/10 + (a^2*b*d^3*x^9
*D)/3 + (3*a*b^2*d^3*x^11*D)/11 + (3*C*b^3*c^2*d*x^10)/10 + (3*C*b^3*c*d^2
*x^11)/11 + (3*a^3*c^2*d*x^5*D)/5 + (a^3*c*d^2*x^6*D)/2 + (3*b^3*c^2*d*x^1
1*D)/11 + (b^3*c*d^2*x^12*D)/4 + (9*a^2*b*c^2*d*x^7*D)/7 + (9*a^2*b*c*d^2*
x^8*D)/8 + a*b^2*c^2*d*x^9*D + (9*a*b^2*c*d^2*x^10*D)/10 + (9*A*a^2*b*c^2*
d*x^4)/4 + (9*A*a^2*b*c*d^2*x^5)/5 + (3*A*a*b^2*c^2*d*x^6)/2 + (9*A*a*b^2*
c*d^2*x^7)/7 + (9*B*a^2*b*c^2*d*x^5)/5 + (3*B*a^2*b*c*d^2*x^6)/2 + (9*B*a*
b^2*c^2*d*x^7)/7 + (9*B*a*b^2*c*d^2*x^8)/8 + (3*C*a^2*b*c^2*d*x^6)/2 + (9*
C*a^2*b*c*d^2*x^7)/7 + (9*C*a*b^2*c^2*d*x^8)/8 + C*a*b^2*c*d^2*x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 655, normalized size of antiderivative = 0.94

$$\int (c+dx)^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{x(27720b^3d^4x^{12}+120120b^3c\,d^3x^{11}+98280a\,b^2d^4x^{10}+32760b^4d^3x^{10}+196560b^3c^2d^2x^{10}+36036a\,b^3d^3x^{9}\cdots}{\cdots}$$

input
```
int((d*x+c)^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output

```
(x*(360360*a**4*c**3 + 540540*a**4*c**2*d*x + 360360*a**4*c*d**2*x**2 + 90
090*a**4*d**3*x**3 + 360360*a**3*b*c**3*x**2 + 180180*a**3*b*c**3*x + 8108
10*a**3*b*c**2*d*x**3 + 360360*a**3*b*c**2*d*x**2 + 648648*a**3*b*c*d**2*x
**4 + 270270*a**3*b*c*d**2*x**3 + 180180*a**3*b*d**3*x**5 + 72072*a**3*b*d
**3*x**4 + 120120*a**3*c**4*x**2 + 360360*a**3*c**3*d*x**3 + 432432*a**3*c
**2*d**2*x**4 + 240240*a**3*c*d**3*x**5 + 51480*a**3*d**4*x**6 + 216216*a*
*2*b**2*c**3*x**4 + 270270*a**2*b**2*c**3*x**3 + 540540*a**2*b**2*c**2*d*x
**5 + 648648*a**2*b**2*c**2*d*x**4 + 463320*a**2*b**2*c*d**2*x**6 + 540540
*a**2*b**2*c*d**2*x**5 + 135135*a**2*b**2*d**3*x**7 + 154440*a**2*b**2*d**
3*x**6 + 216216*a**2*b*c**4*x**4 + 720720*a**2*b*c**3*d*x**5 + 926640*a**2
*b*c**2*d**2*x**6 + 540540*a**2*b*c*d**3*x**7 + 120120*a**2*b*d**4*x**8 +
51480*a*b**3*c**3*x**6 + 180180*a*b**3*c**3*x**5 + 135135*a*b**3*c**2*d*x*
*7 + 463320*a*b**3*c**2*d*x**6 + 120120*a*b**3*c*d**2*x**8 + 405405*a*b**3
*c*d**2*x**7 + 36036*a*b**3*d**3*x**9 + 120120*a*b**3*d**3*x**8 + 154440*a
*b**2*c**4*x**6 + 540540*a*b**2*c**3*d*x**7 + 720720*a*b**2*c**2*d**2*x**8
 + 432432*a*b**2*c*d**3*x**9 + 98280*a*b**2*d**4*x**10 + 45045*b**4*c**3*x
**7 + 120120*b**4*c**2*d*x**8 + 108108*b**4*c*d**2*x**9 + 32760*b**4*d**3*
x**10 + 40040*b**3*c**4*x**8 + 144144*b**3*c**3*d*x**9 + 196560*b**3*c**2*
d**2*x**10 + 120120*b**3*c*d**3*x**11 + 27720*b**3*d**4*x**12))/360360
```

3.16 $\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 418

$$\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx \\
&= a^3Ac^2x+\frac{1}{2}a^3c(Bc+2Ad)x^2+\frac{1}{3}a^2\left(ac(cC+2Bd)+A\left(3bc^2+ad^2\right)\right)x^3 \\
&\quad+\frac{1}{4}a^2\left(3bc(Bc+2Ad)+a\left(2cCd+Bd^2+c^2D\right)\right)x^4 \\
&\quad+\frac{1}{5}a\left(3Ab\left(bc^2+ad^2\right)+a(3bc(cC+2Bd)+ad(Cd+2cD))\right)x^5 \\
&\quad+\frac{1}{6}a\left(3b^2c(Bc+2Ad)+a^2d^2D+3ab\left(2cCd+Bd^2+c^2D\right)\right)x^6 \\
&\quad+\frac{1}{7}b\left(Ab\left(bc^2+3ad^2\right)+3a(bc(cC+2Bd)+ad(Cd+2cD))\right)x^7 \\
&\quad+\frac{1}{8}b\left(b^2c(Bc+2Ad)+3a^2d^2D+3ab\left(2cCd+Bd^2+c^2D\right)\right)x^8 \\
&\quad+\frac{1}{9}b^2\left(b\left(c^2C+2Bcd+Ad^2\right)+3ad(Cd+2cD)\right)x^9 \\
&\quad+\frac{1}{10}b^2\left(3ad^2D+b\left(2cCd+Bd^2+c^2D\right)\right)x^{10}+\frac{1}{11}b^3d(Cd+2cD)x^{11}+\frac{1}{12}b^3d^2Dx^{12}
\end{aligned}$$

output
```
a^3*A*c^2*x+1/2*a^3*c*(2*A*d+B*c)*x^2+1/3*a^2*(a*c*(2*B*d+C*c)+A*(a*d^2+3*
b*c^2))*x^3+1/4*a^2*(3*b*c*(2*A*d+B*c)+a*(B*d^2+2*C*c*d+D*c^2))*x^4+1/5*a*
(3*A*b*(a*d^2+b*c^2)+a*(3*b*c*(2*B*d+C*c)+a*d*(C*d+2*D*c)))*x^5+1/6*a*(3*b
^2*c*(2*A*d+B*c)+a^2*d^2*D+3*a*b*(B*d^2+2*C*c*d+D*c^2))*x^6+1/7*b*(A*b*(3*
a*d^2+b*c^2)+3*a*(b*c*(2*B*d+C*c)+a*d*(C*d+2*D*c)))*x^7+1/8*b*(b^2*c*(2*A*
d+B*c)+3*a^2*d^2*D+3*a*b*(B*d^2+2*C*c*d+D*c^2))*x^8+1/9*b^2*(b*(A*d^2+2*B*
c*d+C*c^2)+3*a*d*(C*d+2*D*c))*x^9+1/10*b^2*(3*a*d^2*D+b*(B*d^2+2*C*c*d+D*c
^2))*x^10+1/11*b^3*d*(C*d+2*D*c)*x^11+1/12*b^3*d^2*D*x^12
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int (c+dx)^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= a^3Ac^2x+\frac{1}{2}a^3c(Bc+2Ad)x^2+\frac{1}{3}a^2\left(ac(cC+2Bd)+A\left(3bc^2+ad^2\right)\right)x^3 \\
&\quad+\frac{1}{4}a^2\left(3bc(Bc+2Ad)+a\left(2cCd+Bd^2+c^2D\right)\right)x^4 \\
&\quad+\frac{1}{5}a\left(3Ab\left(bc^2+ad^2\right)+a(3bc(cC+2Bd)+ad(Cd+2cD))\right)x^5 \\
&\quad+\frac{1}{6}a\left(3b^2c(Bc+2Ad)+a^2d^2D+3ab\left(2cCd+Bd^2+c^2D\right)\right)x^6 \\
&\quad+\frac{1}{7}b\left(Ab\left(bc^2+3ad^2\right)+3a(bc(cC+2Bd)+ad(Cd+2cD))\right)x^7 \\
&\quad+\frac{1}{8}b\left(b^2c(Bc+2Ad)+3a^2d^2D+3ab\left(2cCd+Bd^2+c^2D\right)\right)x^8 \\
&\quad+\frac{1}{9}b^2\left(b\left(c^2C+2Bcd+Ad^2\right)+3ad(Cd+2cD)\right)x^9 \\
&\quad+\frac{1}{10}b^2\left(3ad^2D+b\left(2cCd+Bd^2+c^2D\right)\right)x^{10}+\frac{1}{11}b^3d(Cd+2cD)x^{11}+\frac{1}{12}b^3d^2Dx^{12}
\end{aligned}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c^2*x + (a^3*c*(B*c + 2*A*d)*x^2)/2 + (a^2*(a*c*(c*C + 2*B*d) + A*(3
*b*c^2 + a*d^2))*x^3)/3 + (a^2*(3*b*c*(B*c + 2*A*d) + a*(2*c*C*d + B*d^2 +
 c^2*D))*x^4)/4 + (a*(3*A*b*(b*c^2 + a*d^2) + a*(3*b*c*(c*C + 2*B*d) + a*d
*(C*d + 2*c*D)))*x^5)/5 + (a*(3*b^2*c*(B*c + 2*A*d) + a^2*d^2*D + 3*a*b*(2
*c*C*d + B*d^2 + c^2*D))*x^6)/6 + (b*(A*b*(b*c^2 + 3*a*d^2) + 3*a*(b*c*(c*
C + 2*B*d) + a*d*(C*d + 2*c*D)))*x^7)/7 + (b*(b^2*c*(B*c + 2*A*d) + 3*a^2*
d^2*D + 3*a*b*(2*c*C*d + B*d^2 + c^2*D))*x^8)/8 + (b^2*(b*(c^2*C + 2*B*c*d
 + A*d^2) + 3*a*d*(C*d + 2*c*D))*x^9)/9 + (b^2*(3*a*d^2*D + b*(2*c*C*d + B
*d^2 + c^2*D))*x^10)/10 + (b^3*d*(C*d + 2*c*D)*x^11)/11 + (b^3*d^2*D*x^12)
/12
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^3 \left((c+dx)^2 \left(Dx^3+Cx^2+Bx+A\right)-\left(Bc^2+2Adc\right)x\right)dx+\frac{c\left(a+bx^2\right)^4(2Ad+Bc)}{8b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^3d^2Dx^{11}+b^3d(Cd+2cD)x^{10}+b^2\left(3aDd^2+b\left(Dc^2+2Cdc+Bd^2\right)\right)x^9+b^2\left(b\left(Cc^2+2Bdc+Ad^2\right)+3ad(\right.\right.$$

$$\frac{c\left(a+bx^2\right)^4(2Ad+Bc)}{8b}$$

$$\Big\downarrow \text{2009}$$

$$a^3Ac^2x+\frac{1}{4}a^3x^4\left(Bd^2+c^2D+2cCd\right)+\frac{1}{3}a^2x^3\left(A\left(ad^2+3bc^2\right)+ac(2Bd+cC)\right)+\frac{1}{6}a^2x^6\left(ad^2D+3b\left(Bd^2+c^2D+2cCd\right)\right)+\frac{1}{9}b^2x^9\left(3ad(2cD+Cd)+b\left(Ad^2+2Bcd+c^2C\right)\right)+\frac{1}{7}bx^7\left(Ab\left(3ad^2+bc^2\right)+3a(ad(2cD+Cd)+bc(2Bd+cC))\right)+\frac{1}{5}ax^5\left(3Ab\left(ad^2+bc^2\right)+a(ad(2cD+Cd)+3bc(2Bd+cC))\right)+\frac{c\left(a+bx^2\right)^4(2Ad+Bc)}{8b}+\frac{1}{10}b^2x^{10}\left(3ad^2D+b\left(Bd^2+c^2D+2cCd\right)\right)+\frac{3}{8}abx^8\left(ad^2D+b\left(Bd^2+c^2D+2cCd\right)\right)+\frac{1}{11}b^3dx^{11}(2cD+Cd)+\frac{1}{12}b^3d^2Dx^{12}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c^2*x + (a^2*(a*c*(c*C + 2*B*d) + A*(3*b*c^2 + a*d^2))*x^3)/3 + (a^3
*(2*c*C*d + B*d^2 + c^2*D)*x^4)/4 + (a*(3*A*b*(b*c^2 + a*d^2) + a*(3*b*c*(
c*C + 2*B*d) + a*d*(C*d + 2*c*D)))*x^5)/5 + (a^2*(a*d^2*D + 3*b*(2*c*C*d +
 B*d^2 + c^2*D))*x^6)/6 + (b*(A*b*(b*c^2 + 3*a*d^2) + 3*a*(b*c*(c*C + 2*B*
d) + a*d*(C*d + 2*c*D)))*x^7)/7 + (3*a*b*(a*d^2*D + b*(2*c*C*d + B*d^2 + c
^2*D))*x^8)/8 + (b^2*(b*(c^2*C + 2*B*c*d + A*d^2) + 3*a*d*(C*d + 2*c*D))*x
^9)/9 + (b^2*(3*a*d^2*D + b*(2*c*C*d + B*d^2 + c^2*D))*x^10)/10 + (b^3*d*(
C*d + 2*c*D)*x^11)/11 + (b^3*d^2*D*x^12)/12 + (c*(B*c + 2*A*d)*(a + b*x^2)
^4)/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.17

method	result
norman	$\frac{b^3d^2Dx^{12}}{12}+\left(\frac{1}{11}b^3d^2C+\frac{2}{11}b^3cdD\right)x^{11}+\left(\frac{1}{10}B\,b^3d^2+\frac{1}{5}b^3cdC+\frac{3}{10}Da\,b^2d^2+\frac{1}{10}Db^3c^2\right)x^{10}+$
default	$\frac{b^3d^2Dx^{12}}{12}+\frac{(b^3d^2C+2b^3cdD)x^{11}}{11}+\frac{((3a\,b^2d^2+b^3c^2)D+2b^3cdC+B\,b^3d^2)x^{10}}{10}+\frac{(6cda\,b^2D+(3a\,b^2d^2+b^3c^2)C+2b^3cd}{9}$
gosper	$\frac{1}{2}x^2B\,a^3c^2+\frac{1}{5}x^{10}b^3cdC+\frac{3}{5}x^5A\,d^2a^2b+\frac{1}{3}x^9Ca\,b^2d^2+\frac{1}{9}x^9C\,b^3c^2+\frac{1}{2}x^4a^3dcC+x^2a^3dcA$
parallelrisch	$\frac{1}{2}x^2B\,a^3c^2+\frac{1}{5}x^{10}b^3cdC+\frac{3}{5}x^5A\,d^2a^2b+\frac{1}{3}x^9Ca\,b^2d^2+\frac{1}{9}x^9C\,b^3c^2+\frac{1}{2}x^4a^3dcC+x^2a^3dcA$
orering	$\frac{x(2310b^3d^2Dx^{11}+2520C\,b^3d^2x^{10}+5040Db^3cd\,x^{10}+2772B\,b^3d^2x^9+5544C\,b^3cd\,x^9+8316Da\,b^2d^2x^9+2772Db^3c^2x^9+3080A}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^3*d^2*D*x^12+(1/11*b^3*d^2*C+2/11*b^3*c*d*D)*x^11+(1/10*B*b^3*d^2+1
/5*b^3*c*d*C+3/10*D*a*b^2*d^2+1/10*D*b^3*c^2)*x^10+(1/9*A*b^3*d^2+2/9*b^3*
c*d*B+1/3*C*a*b^2*d^2+1/9*C*b^3*c^2+2/3*c*d*a*b^2*D)*x^9+(1/4*b^3*c*d*A+3/
8*B*a*b^2*d^2+1/8*B*b^3*c^2+3/4*c*d*a*b^2*C+3/8*D*a^2*b*d^2+3/8*D*a*b^2*c^
2)*x^8+(3/7*A*d^2*a*b^2+1/7*A*b^3*c^2+6/7*c*d*a*b^2*B+3/7*C*a^2*b*d^2+3/7*
C*a*b^2*c^2+6/7*a^2*b*c*d*D)*x^7+(c*d*a*b^2*A+1/2*B*a^2*b*d^2+1/2*B*a*b^2*
c^2+a^2*b*c*d*C+1/6*D*a^3*d^2+1/2*D*a^2*b*c^2)*x^6+(3/5*A*d^2*a^2*b+3/5*A*
a*b^2*c^2+6/5*a^2*b*c*d*B+1/5*C*a^3*d^2+3/5*C*a^2*b*c^2+2/5*a^3*d*c*D)*x^5
+(3/2*a^2*b*c*d*A+1/4*B*a^3*d^2+3/4*B*a^2*b*c^2+1/2*a^3*d*c*C+1/4*c^2*a^3*
D)*x^4+(1/3*A*d^2*a^3+A*a^2*b*c^2+2/3*a^3*d*c*B+1/3*c^2*a^3*C)*x^3+(a^3*d*
c*A+1/2*B*a^3*c^2)*x^2+a^3*A*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.13

$$
\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\, dx \\
&= \frac{1}{12}\, Db^3d^2x^{12} + \frac{1}{11}\,(2\, Db^3cd + Cb^3d^2)x^{11} \\
&\quad + \frac{1}{10}\,(Db^3c^2 + 2\, Cb^3cd + \left(3\, Dab^2 + Bb^3\right)d^2)x^{10} \\
&\quad + \frac{1}{9}\,(Cb^3c^2 + 2\left(3\, Dab^2 + Bb^3\right)cd + \left(3\, Cab^2 + Ab^3\right)d^2)x^9 \\
&\quad + \frac{1}{8}\,(\left(3\, Dab^2 + Bb^3\right)c^2 + 2\left(3\, Cab^2 + Ab^3\right)cd + 3\left(Da^2b + Bab^2\right)d^2)x^8 \\
&\quad + \frac{1}{7}\,(\left(3\, Cab^2 + Ab^3\right)c^2 + 6\left(Da^2b + Bab^2\right)cd + 3\left(Ca^2b + Aab^2\right)d^2)x^7 + Aa^3c^2x \\
&\quad + \frac{1}{6}\,(3\left(Da^2b + Bab^2\right)c^2 + 6\left(Ca^2b + Aab^2\right)cd + \left(Da^3 + 3\, Ba^2b\right)d^2)x^6 \\
&\quad + \frac{1}{5}\,(3\left(Ca^2b + Aab^2\right)c^2 + 2\left(Da^3 + 3\, Ba^2b\right)cd + \left(Ca^3 + 3\, Aa^2b\right)d^2)x^5 \\
&\quad + \frac{1}{4}\,(Ba^3d^2 + \left(Da^3 + 3\, Ba^2b\right)c^2 + 2\left(Ca^3 + 3\, Aa^2b\right)cd)x^4 \\
&\quad + \frac{1}{3}\,(2\, Ba^3cd + Aa^3d^2 + \left(Ca^3 + 3\, Aa^2b\right)c^2)x^3 + \frac{1}{2}\,(Ba^3c^2 + 2\, Aa^3cd)x^2
\end{aligned}
$$

input

```
integrate((d*x+c)^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output

```
1/12*D*b^3*d^2*x^12 + 1/11*(2*D*b^3*c*d + C*b^3*d^2)*x^11 + 1/10*(D*b^3*c^
2 + 2*C*b^3*c*d + (3*D*a*b^2 + B*b^3)*d^2)*x^10 + 1/9*(C*b^3*c^2 + 2*(3*D*
a*b^2 + B*b^3)*c*d + (3*C*a*b^2 + A*b^3)*d^2)*x^9 + 1/8*((3*D*a*b^2 + B*b^
3)*c^2 + 2*(3*C*a*b^2 + A*b^3)*c*d + 3*(D*a^2*b + B*a*b^2)*d^2)*x^8 + 1/7*
((3*C*a*b^2 + A*b^3)*c^2 + 6*(D*a^2*b + B*a*b^2)*c*d + 3*(C*a^2*b + A*a*b^
2)*d^2)*x^7 + A*a^3*c^2*x + 1/6*(3*(D*a^2*b + B*a*b^2)*c^2 + 6*(C*a^2*b +
A*a*b^2)*c*d + (D*a^3 + 3*B*a^2*b)*d^2)*x^6 + 1/5*(3*(C*a^2*b + A*a*b^2)*c
^2 + 2*(D*a^3 + 3*B*a^2*b)*c*d + (C*a^3 + 3*A*a^2*b)*d^2)*x^5 + 1/4*(B*a^3
*d^2 + (D*a^3 + 3*B*a^2*b)*c^2 + 2*(C*a^3 + 3*A*a^2*b)*c*d)*x^4 + 1/3*(2*B
*a^3*c*d + A*a^3*d^2 + (C*a^3 + 3*A*a^2*b)*c^2)*x^3 + 1/2*(B*a^3*c^2 + 2*A
*a^3*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.42

$$
\begin{aligned}
&\int (c+dx)^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= Aa^3c^2x+\frac{Db^3d^2x^{12}}{12}+x^{11}\left(\frac{Cb^3d^2}{11}+\frac{2Db^3cd}{11}\right) \\
&\quad +x^{10}\left(\frac{Bb^3d^2}{10}+\frac{Cb^3cd}{5}+\frac{3Dab^2d^2}{10}+\frac{Db^3c^2}{10}\right) \\
&\quad +x^{9}\left(\frac{Ab^3d^2}{9}+\frac{2Bb^3cd}{9}+\frac{Cab^2d^2}{3}+\frac{Cb^3c^2}{9}+\frac{2Dab^2cd}{3}\right) \\
&\quad +x^{8}\left(\frac{Ab^3cd}{4}+\frac{3Bab^2d^2}{8}+\frac{Bb^3c^2}{8}+\frac{3Cab^2cd}{4}+\frac{3Da^2bd^2}{8}+\frac{3Dab^2c^2}{8}\right) \\
&\quad +x^{7}\cdot\left(\frac{3Aab^2d^2}{7}+\frac{Ab^3c^2}{7}+\frac{6Bab^2cd}{7}+\frac{3Ca^2bd^2}{7}+\frac{3Cab^2c^2}{7}+\frac{6Da^2bcd}{7}\right) \\
&\quad +x^{6}\left(Aab^2cd+\frac{Ba^2bd^2}{2}+\frac{Bab^2c^2}{2}+Ca^2bcd+\frac{Da^3d^2}{6}+\frac{Da^2bc^2}{2}\right)+x^5 \\
&\quad \cdot\left(\frac{3Aa^2bd^2}{5}+\frac{3Aab^2c^2}{5}+\frac{6Ba^2bcd}{5}+\frac{Ca^3d^2}{5}+\frac{3Ca^2bc^2}{5}+\frac{2Da^3cd}{5}\right) \\
&\quad +x^{4}\cdot\left(\frac{3Aa^2bcd}{2}+\frac{Ba^3d^2}{4}+\frac{3Ba^2bc^2}{4}+\frac{Ca^3cd}{2}+\frac{Da^3c^2}{4}\right) \\
&\quad +x^{3}\left(\frac{Aa^3d^2}{3}+Aa^2bc^2+\frac{2Ba^3cd}{3}+\frac{Ca^3c^2}{3}\right)+x^2\left(Aa^3cd+\frac{Ba^3c^2}{2}\right)
\end{aligned}
$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*c**2*x + D*b**3*d**2*x**12/12 + x**11*(C*b**3*d**2/11 + 2*D*b**3*c*
d/11) + x**10*(B*b**3*d**2/10 + C*b**3*c*d/5 + 3*D*a*b**2*d**2/10 + D*b**3
*c**2/10) + x**9*(A*b**3*d**2/9 + 2*B*b**3*c*d/9 + C*a*b**2*d**2/3 + C*b**
3*c**2/9 + 2*D*a*b**2*c*d/3) + x**8*(A*b**3*c*d/4 + 3*B*a*b**2*d**2/8 + B*
b**3*c**2/8 + 3*C*a*b**2*c*d/4 + 3*D*a**2*b*d**2/8 + 3*D*a*b**2*c**2/8) +
x**7*(3*A*a*b**2*d**2/7 + A*b**3*c**2/7 + 6*B*a*b**2*c*d/7 + 3*C*a**2*b*d*
*2/7 + 3*C*a*b**2*c**2/7 + 6*D*a**2*b*c*d/7) + x**6*(A*a*b**2*c*d + B*a**2
*b*d**2/2 + B*a*b**2*c**2/2 + C*a**2*b*c*d + D*a**3*d**2/6 + D*a**2*b*c**2
/2) + x**5*(3*A*a**2*b*d**2/5 + 3*A*a*b**2*c**2/5 + 6*B*a**2*b*c*d/5 + C*a
**3*d**2/5 + 3*C*a**2*b*c**2/5 + 2*D*a**3*c*d/5) + x**4*(3*A*a**2*b*c*d/2
+ B*a**3*d**2/4 + 3*B*a**2*b*c**2/4 + C*a**3*c*d/2 + D*a**3*c**2/4) + x**3
*(A*a**3*d**2/3 + A*a**2*b*c**2 + 2*B*a**3*c*d/3 + C*a**3*c**2/3) + x**2*(
A*a**3*c*d + B*a**3*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.13

$$
\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{12}\,Db^3d^2x^{12} + \frac{1}{11}\left(2\,Db^3cd + Cb^3d^2\right)x^{11} \\
&\quad + \frac{1}{10}\left(Db^3c^2 + 2\,Cb^3cd + \left(3\,Dab^2 + Bb^3\right)d^2\right)x^{10} \\
&\quad + \frac{1}{9}\left(Cb^3c^2 + 2\left(3\,Dab^2 + Bb^3\right)cd + \left(3\,Cab^2 + Ab^3\right)d^2\right)x^9 \\
&\quad + \frac{1}{8}\left(\left(3\,Dab^2 + Bb^3\right)c^2 + 2\left(3\,Cab^2 + Ab^3\right)cd + 3\left(Da^2b + Bab^2\right)d^2\right)x^8 \\
&\quad + \frac{1}{7}\left(\left(3\,Cab^2 + Ab^3\right)c^2 + 6\left(Da^2b + Bab^2\right)cd + 3\left(Ca^2b + Aab^2\right)d^2\right)x^7 + Aa^3c^2x \\
&\quad + \frac{1}{6}\left(3\left(Da^2b + Bab^2\right)c^2 + 6\left(Ca^2b + Aab^2\right)cd + \left(Da^3 + 3\,Ba^2b\right)d^2\right)x^6 \\
&\quad + \frac{1}{5}\left(3\left(Ca^2b + Aab^2\right)c^2 + 2\left(Da^3 + 3\,Ba^2b\right)cd + \left(Ca^3 + 3\,Aa^2b\right)d^2\right)x^5 \\
&\quad + \frac{1}{4}\left(Ba^3d^2 + \left(Da^3 + 3\,Ba^2b\right)c^2 + 2\left(Ca^3 + 3\,Aa^2b\right)cd\right)x^4 \\
&\quad + \frac{1}{3}\left(2\,Ba^3cd + Aa^3d^2 + \left(Ca^3 + 3\,Aa^2b\right)c^2\right)x^3 + \frac{1}{2}\left(Ba^3c^2 + 2\,Aa^3cd\right)x^2
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/12*D*b^3*d^2*x^12 + 1/11*(2*D*b^3*c*d + C*b^3*d^2)*x^11 + 1/10*(D*b^3*c^
2 + 2*C*b^3*c*d + (3*D*a*b^2 + B*b^3)*d^2)*x^10 + 1/9*(C*b^3*c^2 + 2*(3*D*
a*b^2 + B*b^3)*c*d + (3*C*a*b^2 + A*b^3)*d^2)*x^9 + 1/8*((3*D*a*b^2 + B*b^
3)*c^2 + 2*(3*C*a*b^2 + A*b^3)*c*d + 3*(D*a^2*b + B*a*b^2)*d^2)*x^8 + 1/7*
((3*C*a*b^2 + A*b^3)*c^2 + 6*(D*a^2*b + B*a*b^2)*c*d + 3*(C*a^2*b + A*a*b^
2)*d^2)*x^7 + A*a^3*c^2*x + 1/6*(3*(D*a^2*b + B*a*b^2)*c^2 + 6*(C*a^2*b +
A*a*b^2)*c*d + (D*a^3 + 3*B*a^2*b)*d^2)*x^6 + 1/5*(3*(C*a^2*b + A*a*b^2)*c
^2 + 2*(D*a^3 + 3*B*a^2*b)*c*d + (C*a^3 + 3*A*a^2*b)*d^2)*x^5 + 1/4*(B*a^3
*d^2 + (D*a^3 + 3*B*a^2*b)*c^2 + 2*(C*a^3 + 3*A*a^2*b)*c*d)*x^4 + 1/3*(2*B
*a^3*c*d + A*a^3*d^2 + (C*a^3 + 3*A*a^2*b)*c^2)*x^3 + 1/2*(B*a^3*c^2 + 2*A
*a^3*c*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int (c+dx)^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{12}Db^3d^2x^{12}+\frac{2}{11}Db^3cdx^{11}+\frac{1}{11}Cb^3d^2x^{11}+\frac{1}{10}Db^3c^2x^{10}+\frac{1}{5}Cb^3cdx^{10} \\
&+\frac{3}{10}Dab^2d^2x^{10}+\frac{1}{10}Bb^3d^2x^{10}+\frac{1}{9}Cb^3c^2x^9+\frac{2}{3}Dab^2cdx^9+\frac{2}{9}Bb^3cdx^9+\frac{1}{3}Cab^2d^2x^9 \\
&+\frac{1}{9}Ab^3d^2x^9+\frac{3}{8}Dab^2c^2x^8+\frac{1}{8}Bb^3c^2x^8+\frac{3}{4}Cab^2cdx^8+\frac{1}{4}Ab^3cdx^8+\frac{3}{8}Da^2bd^2x^8 \\
&+\frac{3}{8}Bab^2d^2x^8+\frac{3}{7}Cab^2c^2x^7+\frac{1}{7}Ab^3c^2x^7+\frac{6}{7}Da^2bcdx^7+\frac{6}{7}Bab^2cdx^7+\frac{3}{7}Ca^2bd^2x^7 \\
&+\frac{3}{7}Aab^2d^2x^7+\frac{1}{2}Da^2bc^2x^6+\frac{1}{2}Bab^2c^2x^6+Ca^2bcdx^6+Aab^2cdx^6+\frac{1}{6}Da^3d^2x^6 \\
&+\frac{1}{2}Ba^2bd^2x^6+\frac{3}{5}Ca^2bc^2x^5+\frac{3}{5}Aab^2c^2x^5+\frac{2}{5}Da^3cdx^5+\frac{6}{5}Ba^2bcdx^5+\frac{1}{5}Ca^3d^2x^5 \\
&+\frac{3}{5}Aa^2bd^2x^5+\frac{1}{4}Da^3c^2x^4+\frac{3}{4}Ba^2bc^2x^4+\frac{1}{2}Ca^3cdx^4+\frac{3}{2}Aa^2bcdx^4+\frac{1}{4}Ba^3d^2x^4 \\
&+\frac{1}{3}Ca^3c^2x^3+Aa^2bc^2x^3+\frac{2}{3}Ba^3cdx^3+\frac{1}{3}Aa^3d^2x^3+\frac{1}{2}Ba^3c^2x^2+Aa^3cdx^2+Aa^3c^2x
\end{aligned}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/12*D*b^3*d^2*x^12 + 2/11*D*b^3*c*d*x^11 + 1/11*C*b^3*d^2*x^11 + 1/10*D*b
^3*c^2*x^10 + 1/5*C*b^3*c*d*x^10 + 3/10*D*a*b^2*d^2*x^10 + 1/10*B*b^3*d^2*
x^10 + 1/9*C*b^3*c^2*x^9 + 2/3*D*a*b^2*c*d*x^9 + 2/9*B*b^3*c*d*x^9 + 1/3*C
*a*b^2*d^2*x^9 + 1/9*A*b^3*d^2*x^9 + 3/8*D*a*b^2*c^2*x^8 + 1/8*B*b^3*c^2*x
^8 + 3/4*C*a*b^2*c*d*x^8 + 1/4*A*b^3*c*d*x^8 + 3/8*D*a^2*b*d^2*x^8 + 3/8*B
*a*b^2*d^2*x^8 + 3/7*C*a*b^2*c^2*x^7 + 1/7*A*b^3*c^2*x^7 + 6/7*D*a^2*b*c*d
*x^7 + 6/7*B*a*b^2*c*d*x^7 + 3/7*C*a^2*b*d^2*x^7 + 3/7*A*a*b^2*d^2*x^7 + 1
/2*D*a^2*b*c^2*x^6 + 1/2*B*a*b^2*c^2*x^6 + C*a^2*b*c*d*x^6 + A*a*b^2*c*d*x
^6 + 1/6*D*a^3*d^2*x^6 + 1/2*B*a^2*b*d^2*x^6 + 3/5*C*a^2*b*c^2*x^5 + 3/5*A
*a*b^2*c^2*x^5 + 2/5*D*a^3*c*d*x^5 + 6/5*B*a^2*b*c*d*x^5 + 1/5*C*a^3*d^2*x
^5 + 3/5*A*a^2*b*d^2*x^5 + 1/4*D*a^3*c^2*x^4 + 3/4*B*a^2*b*c^2*x^4 + 1/2*C
*a^3*c*d*x^4 + 3/2*A*a^2*b*c*d*x^4 + 1/4*B*a^3*d^2*x^4 + 1/3*C*a^3*c^2*x^3
 + A*a^2*b*c^2*x^3 + 2/3*B*a^3*c*d*x^3 + 1/3*A*a^3*d^2*x^3 + 1/2*B*a^3*c^2
*x^2 + A*a^3*c*d*x^2 + A*a^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 24.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.38

$$\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{a^3 c^2 x^4 D}{4} + \frac{a^3 d^2 x^6 D}{6} + \frac{b^3 c^2 x^{10} D}{10} + \frac{b^3 d^2 x^{12} D}{12} + A a^3 c^2 x + \frac{B a^3 c^2 x^2}{2} + \frac{A a^3 d^2 x^3}{3} + \frac{A b^3 c^2 x^7}{7} + \frac{C a^3 c^2 x^3}{3} + \frac{B a^3 d^2 x^4}{4} + \frac{B b^3 c^2 x^8}{8} + \frac{A b^3 d^2 x^9}{9} + \frac{C a^3 d^2 x^5}{5} + \frac{C b^3 c^2 x^9}{9} + \frac{B b^3 d^2 x^{10}}{10} + \frac{C b^3 d^2 x^{11}}{11} + \frac{2 B a^3 c d x^3}{3} + \frac{A b^3 c d x^8}{4} + \frac{C a^3 c d x^4}{2} + \frac{2 B b^3 c d x^9}{9} + \frac{C b^3 c d x^{10}}{5} + \frac{2 a^3 c d x^5 D}{5} + \frac{2 b^3 c d x^{11} D}{11} + A a^2 b c^2 x^3 + \frac{3 A a b^2 c^2 x^5}{5} + \frac{3 B a^2 b c^2 x^4}{4} + \frac{3 A a^2 b d^2 x^5}{5} + \frac{B a b^2 c^2 x^6}{2} + \frac{3 A a b^2 d^2 x^7}{7} + \frac{3 C a^2 b c^2 x^5}{5} + \frac{B a^2 b d^2 x^6}{2} + \frac{3 C a b^2 c^2 x^7}{7} + \frac{3 B a b^2 d^2 x^8}{8} + \frac{3 C a^2 b d^2 x^7}{7} + \frac{C a b^2 d^2 x^9}{3} + \frac{a^2 b c^2 x^6 D}{2} + \frac{3 a b^2 c^2 x^8 D}{8} + \frac{3 a^2 b d^2 x^8 D}{8} + \frac{3 a b^2 d^2 x^{10} D}{10} + A a^3 c d x^2 + \frac{6 a^2 b c d x^7 D}{7} + \frac{2 a b^2 c d x^9 D}{3} + \frac{3 A a^2 b c d x^4}{2} + A a b^2 c d x^6 + \frac{6 B a^2 b c d x^5}{5} + \frac{6 B a b^2 c d x^7}{7} + C a^2 b c d x^6 + \frac{3 C a b^2 c d x^8}{4}$$

input
```
int((a + b*x^2)^3*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^3*c^2*x^4*D)/4 + (a^3*d^2*x^6*D)/6 + (b^3*c^2*x^10*D)/10 + (b^3*d^2*x^1
2*D)/12 + A*a^3*c^2*x + (B*a^3*c^2*x^2)/2 + (A*a^3*d^2*x^3)/3 + (A*b^3*c^2
*x^7)/7 + (C*a^3*c^2*x^3)/3 + (B*a^3*d^2*x^4)/4 + (B*b^3*c^2*x^8)/8 + (A*b
^3*d^2*x^9)/9 + (C*a^3*d^2*x^5)/5 + (C*b^3*c^2*x^9)/9 + (B*b^3*d^2*x^10)/1
0 + (C*b^3*d^2*x^11)/11 + (2*B*a^3*c*d*x^3)/3 + (A*b^3*c*d*x^8)/4 + (C*a^3
*c*d*x^4)/2 + (2*B*b^3*c*d*x^9)/9 + (C*b^3*c*d*x^10)/5 + (2*a^3*c*d*x^5*D)
/5 + (2*b^3*c*d*x^11*D)/11 + A*a^2*b*c^2*x^3 + (3*A*a*b^2*c^2*x^5)/5 + (3*
B*a^2*b*c^2*x^4)/4 + (3*A*a^2*b*d^2*x^5)/5 + (B*a*b^2*c^2*x^6)/2 + (3*A*a*
b^2*d^2*x^7)/7 + (3*C*a^2*b*c^2*x^5)/5 + (B*a^2*b*d^2*x^6)/2 + (3*C*a*b^2*
c^2*x^7)/7 + (3*B*a*b^2*d^2*x^8)/8 + (3*C*a^2*b*d^2*x^7)/7 + (C*a*b^2*d^2*
x^9)/3 + (a^2*b*c^2*x^6*D)/2 + (3*a*b^2*c^2*x^8*D)/8 + (3*a^2*b*d^2*x^8*D)
/8 + (3*a*b^2*d^2*x^10*D)/10 + A*a^3*c*d*x^2 + (6*a^2*b*c*d*x^7*D)/7 + (2*
a*b^2*c*d*x^9*D)/3 + (3*A*a^2*b*c*d*x^4)/2 + A*a*b^2*c*d*x^6 + (6*B*a^2*b*
c*d*x^5)/5 + (6*B*a*b^2*c*d*x^7)/7 + C*a^2*b*c*d*x^6 + (3*C*a*b^2*c*d*x^8)
/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.14

$$\int (c+dx)^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{x(2310b^3d^3x^{11} + 7560b^3c\,d^2x^{10} + 8316a\,b^2d^3x^9 + 2772b^4d^2x^9 + 8316b^3c^2d\,x^9 + 3080a\,b^3d^2x^8 + 27720a\,b^2}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output

```
(x*(27720*a**4*c**2 + 27720*a**4*c*d*x + 9240*a**4*d**2*x**2 + 27720*a**3*
b*c**2*x**2 + 13860*a**3*b*c**2*x + 41580*a**3*b*c*d*x**3 + 18480*a**3*b*c
*d*x**2 + 16632*a**3*b*d**2*x**4 + 6930*a**3*b*d**2*x**3 + 9240*a**3*c**3*
x**2 + 20790*a**3*c**2*d*x**3 + 16632*a**3*c*d**2*x**4 + 4620*a**3*d**3*x*
*5 + 16632*a**2*b**2*c**2*x**4 + 20790*a**2*b**2*c**2*x**3 + 27720*a**2*b*
*2*c*d*x**5 + 33264*a**2*b**2*c*d*x**4 + 11880*a**2*b**2*d**2*x**6 + 13860
*a**2*b**2*d**2*x**5 + 16632*a**2*b*c**3*x**4 + 41580*a**2*b*c**2*d*x**5 +
 35640*a**2*b*c*d**2*x**6 + 10395*a**2*b*d**3*x**7 + 3960*a*b**3*c**2*x**6
 + 13860*a*b**3*c**2*x**5 + 6930*a*b**3*c*d*x**7 + 23760*a*b**3*c*d*x**6 +
 3080*a*b**3*d**2*x**8 + 10395*a*b**3*d**2*x**7 + 11880*a*b**2*c**3*x**6 +
 31185*a*b**2*c**2*d*x**7 + 27720*a*b**2*c*d**2*x**8 + 8316*a*b**2*d**3*x*
*9 + 3465*b**4*c**2*x**7 + 6160*b**4*c*d*x**8 + 2772*b**4*d**2*x**9 + 3080
*b**3*c**3*x**8 + 8316*b**3*c**2*d*x**9 + 7560*b**3*c*d**2*x**10 + 2310*b*
*3*d**3*x**11))/27720
```

3.17 $\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 190

$$\begin{aligned}
&\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= a^3Acx + \frac{1}{3}a^2(3Abc+acC+aBd)x^3 + \frac{1}{5}a\left(3Ab^2c+a(3b(cC+Bd)+adD)\right)x^5 \\
&\quad + \frac{1}{7}b\left(Ab^2c+3a(bcC+bBd+adD)\right)x^7 + \frac{1}{9}b^2(bcC+bBd+3adD)x^9 \\
&\quad + \frac{1}{11}b^3dDx^{11} + \frac{(bBc+Abd-aCd-acD)\left(a+bx^2\right)^4}{8b^2} + \frac{(Cd+cD)\left(a+bx^2\right)^5}{10b^2}
\end{aligned}$$

output

```
a^3*A*c*x+1/3*a^2*(3*A*b*c+B*a*d+C*a*c)*x^3+1/5*a*(3*A*b^2*c+a*(3*b*(B*d+C
*c)+D*a*d))*x^5+1/7*b*(A*b^2*c+3*a*(B*b*d+C*b*c+D*a*d))*x^7+1/9*b^2*(B*b*d
+C*b*c+3*D*a*d)*x^9+1/11*b^3*d*D*x^11+1/8*(A*b*d+B*b*c-C*a*d-D*a*c)*(b*x^2
+a)^4/b^2+1/10*(C*d+D*c)*(b*x^2+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int (c+dx)\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= a^3Acx+\frac{1}{2}a^3(Bc+Ad)x^2+\frac{1}{3}a^2(3Abc+acC+aBd)x^3$$
$$+\frac{1}{4}a^2(3bBc+3Abd+aCd+acD)x^4$$
$$+\frac{1}{5}a\left(3Ab^2c+a(3b(cC+Bd)+adD)\right)x^5+\frac{1}{2}ab(bBc+Abd+aCd+acD)x^6$$
$$+\frac{1}{7}b\left(Ab^2c+3a(bcC+bBd+adD)\right)x^7+\frac{1}{8}b^2(bBc+Abd+3aCd+3acD)x^8$$
$$+\frac{1}{9}b^2(bcC+bBd+3adD)x^9+\frac{1}{10}b^3(Cd+cD)x^{10}+\frac{1}{11}b^3dDx^{11}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c*x + (a^3*(B*c + A*d)*x^2)/2 + (a^2*(3*A*b*c + a*c*C + a*B*d)*x^3)/
3 + (a^2*(3*b*B*c + 3*A*b*d + a*C*d + a*c*D)*x^4)/4 + (a*(3*A*b^2*c + a*(3
*b*(c*C + B*d) + a*d*D))*x^5)/5 + (a*b*(b*B*c + A*b*d + a*C*d + a*c*D)*x^6
)/2 + (b*(A*b^2*c + 3*a*(b*c*C + b*B*d + a*d*D))*x^7)/7 + (b^2*(b*B*c + A*
b*d + 3*a*C*d + 3*a*c*D)*x^8)/8 + (b^2*(b*c*C + b*B*d + 3*a*d*D)*x^9)/9 +
(b^3*(C*d + c*D)*x^10)/10 + (b^3*d*D*x^11)/11
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \; 2017$$

$$\int \left(bx^2+a\right)^3 \left((c+dx)\left(Dx^3+Cx^2+Bx+A\right)-(Bc+Ad)x\right)dx+\frac{\left(a+bx^2\right)^4(Ad+Bc)}{8b}$$

$$\Big\downarrow \; 2341$$

$$\int \left(b^3dDx^{10}+b^3(Cd+cD)x^9+b^2(bcC+bBd+3adD)x^8+3ab^2(Cd+cD)x^7+b\big(Acb^2+3a(bcC+bBd+ad\right.$$

$$\frac{\left(a+bx^2\right)^4(Ad+Bc)}{8b}$$

$$\Big\downarrow \; 2009$$

$$a^3Acx+\frac{1}{4}a^3x^4(cD+Cd)+\frac{1}{3}a^2x^3(aBd+acC+3Abc)+\frac{1}{2}a^2bx^6(cD+Cd)+\frac{1}{7}bx^7\left(3a(adD+bBd+bcC)+Ab^2c\right)+\frac{1}{5}ax^5\left(a(adD+3b(Bd+cC))+3Ab^2c\right)+\frac{\left(a+bx^2\right)^4(Ad+Bc)}{8b}+\frac{1}{9}b^2x^9(3adD+bBd+bcC)+\frac{3}{8}ab^2x^8(cD+Cd)+\frac{1}{10}b^3x^{10}(cD+Cd)+\frac{1}{11}b^3dDx^{11}$$

input
```
Int[(c + d*x)*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c*x + (a^2*(3*A*b*c + a*c*C + a*B*d)*x^3)/3 + (a^3*(C*d + c*D)*x^4)/
4 + (a*(3*A*b^2*c + a*(3*b*(c*C + B*d) + a*d*D))*x^5)/5 + (a^2*b*(C*d + c*
D)*x^6)/2 + (b*(A*b^2*c + 3*a*(b*c*C + b*B*d + a*d*D))*x^7)/7 + (3*a*b^2*(
C*d + c*D)*x^8)/8 + (b^2*(b*c*C + b*B*d + 3*a*d*D)*x^9)/9 + (b^3*(C*d + c*
D)*x^10)/10 + (b^3*d*D*x^11)/11 + ((B*c + A*d)*(a + b*x^2)^4)/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.50

method	result
default	$\frac{b^{3} d D x^{11}}{11}+\frac{\left(b^{3} d C+b^{3} c D\right) x^{10}}{10}+\frac{\left(B\,b^{3} d+C\,b^{3} c+3 d a\,b^{2} D\right) x^{9}}{9}+\frac{\left(A d\,b^{3}+B\,b^{3} c+3 d a\,b^{2} C+3 c a\,b^{2} D\right) x^{8}}{8}+\frac{\left(A\,b^{3} c+3 d a\,b\right.}{}$
norman	$\frac{b^{3} d D x^{11}}{11}+\left(\frac{1}{10} b^{3} d C+\frac{1}{10} b^{3} c D\right) x^{10}+\left(\frac{1}{9} B\,b^{3} d+\frac{1}{9} C\,b^{3} c+\frac{1}{3} d a\,b^{2} D\right) x^{9}+\left(\frac{1}{8} A d\,b^{3}+\frac{1}{8} B\,b^{3} c+\right.$
gosper	$\frac{3}{4} x^{4} A d\,a^{2} b+\frac{3}{5} x^{5} A a\,b^{2} c+\frac{3}{7} x^{7} C a\,b^{2} c+\frac{1}{2} x^{6} a^{2} b d C+\frac{1}{4} x^{4} a^{3} d C+\frac{1}{4} x^{4} c\,a^{3} D+\frac{1}{2} x^{6} A d a\,b^{2}+\frac{1}{2}$
parallelrisch	$\frac{3}{4} x^{4} A d\,a^{2} b+\frac{3}{5} x^{5} A a\,b^{2} c+\frac{3}{7} x^{7} C a\,b^{2} c+\frac{1}{2} x^{6} a^{2} b d C+\frac{1}{4} x^{4} a^{3} d C+\frac{1}{4} x^{4} c\,a^{3} D+\frac{1}{2} x^{6} A d a\,b^{2}+\frac{1}{2}$
orering	$\frac{x\left(2520 b^{3} d D x^{10}+2772 C\,b^{3} d\,x^{9}+2772 D b^{3} c\,x^{9}+3080 B\,b^{3} d\,x^{8}+3080 x^{8} C\,b^{3} c+9240 D a\,b^{2} d\,x^{8}+3465 A\,b^{3} d\,x^{7}+3465 B\,b^{3} c\,x^{7}+\right.}{}$

input
```
int((d*x+c)*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^3*d*D*x^11+1/10*(C*b^3*d+D*b^3*c)*x^10+1/9*(B*b^3*d+C*b^3*c+3*D*a*b
^2*d)*x^9+1/8*(A*b^3*d+B*b^3*c+3*C*a*b^2*d+3*D*a*b^2*c)*x^8+1/7*(A*b^3*c+3
*B*a*b^2*d+3*C*a*b^2*c+3*D*a^2*b*d)*x^7+1/6*(3*A*a*b^2*d+3*B*a*b^2*c+3*C*a
^2*b*d+3*D*a^2*b*c)*x^6+1/5*(3*A*a*b^2*c+3*B*a^2*b*d+3*C*a^2*b*c+D*a^3*d)*
x^5+1/4*(3*A*a^2*b*d+3*B*a^2*b*c+C*a^3*d+D*a^3*c)*x^4+1/3*(3*A*a^2*b*c+B*a
^3*d+C*a^3*c)*x^3+1/2*(A*a^3*d+B*a^3*c)*x^2+A*a^3*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.53

$$\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{1}{11} Db^3dx^{11} + \frac{1}{10} \left(Db^3c + Cb^3d\right)x^{10} + \frac{1}{9} \left(Cb^3c + \left(3\,Dab^2 + Bb^3\right)d\right)x^9 + \frac{1}{8} \left(\left(3\,Dab^2 + Bb^3\right)c + \left(3\,Cab^2 + Ab^3\right)d\right)x^8 + \frac{1}{7} \left(\left(3\,Cab^2 + Ab^3\right)c + 3\left(Da^2b + Bab^2\right)d\right)x^7 + \frac{1}{2} \left(\left(Da^2b + Bab^2\right)c + \left(Ca^2b + Aab^2\right)d\right)x^6 + Aa^3cx + \frac{1}{5} \left(3\left(Ca^2b + Aab^2\right)c + \left(Da^3 + 3\,Ba^2b\right)d\right)x^5 + \frac{1}{4} \left(\left(Da^3 + 3\,Ba^2b\right)c + \left(Ca^3 + 3\,Aa^2b\right)d\right)x^4 + \frac{1}{3} \left(Ba^3d + \left(Ca^3 + 3\,Aa^2b\right)c\right)x^3 + \frac{1}{2} \left(Ba^3c + Aa^3d\right)x^2$$

input
```
integrate((d*x+c)*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^3*d*x^11 + 1/10*(D*b^3*c + C*b^3*d)*x^10 + 1/9*(C*b^3*c + (3*D*a*
b^2 + B*b^3)*d)*x^9 + 1/8*((3*D*a*b^2 + B*b^3)*c + (3*C*a*b^2 + A*b^3)*d)*
x^8 + 1/7*((3*C*a*b^2 + A*b^3)*c + 3*(D*a^2*b + B*a*b^2)*d)*x^7 + 1/2*((D*
a^2*b + B*a*b^2)*c + (C*a^2*b + A*a*b^2)*d)*x^6 + A*a^3*c*x + 1/5*(3*(C*a^
2*b + A*a*b^2)*c + (D*a^3 + 3*B*a^2*b)*d)*x^5 + 1/4*((D*a^3 + 3*B*a^2*b)*c
 + (C*a^3 + 3*A*a^2*b)*d)*x^4 + 1/3*(B*a^3*d + (C*a^3 + 3*A*a^2*b)*c)*x^3
+ 1/2*(B*a^3*c + A*a^3*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.83

$$
\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= Aa^3cx+\frac{Db^3dx^{11}}{11}+x^{10}\left(\frac{Cb^3d}{10}+\frac{Db^3c}{10}\right)+x^9\left(\frac{Bb^3d}{9}+\frac{Cb^3c}{9}+\frac{Dab^2d}{3}\right) \\
&\quad +x^8\left(\frac{Ab^3d}{8}+\frac{Bb^3c}{8}+\frac{3Cab^2d}{8}+\frac{3Dab^2c}{8}\right)+x^7\left(\frac{Ab^3c}{7}+\frac{3Bab^2d}{7}+\frac{3Cab^2c}{7}+\frac{3Da^2bd}{7}\right) \\
&\quad +x^6\left(\frac{Aab^2d}{2}+\frac{Bab^2c}{2}+\frac{Ca^2bd}{2}+\frac{Da^2bc}{2}\right)+x^5 \\
&\quad \cdot\left(\frac{3Aab^2c}{5}+\frac{3Ba^2bd}{5}+\frac{3Ca^2bc}{5}+\frac{Da^3d}{5}\right)+x^4\cdot\left(\frac{3Aa^2bd}{4}+\frac{3Ba^2bc}{4}+\frac{Ca^3d}{4}+\frac{Da^3c}{4}\right) \\
&\quad +x^3\left(Aa^2bc+\frac{Ba^3d}{3}+\frac{Ca^3c}{3}\right)+x^2\left(\frac{Aa^3d}{2}+\frac{Ba^3c}{2}\right)
\end{aligned}
$$

input
```
integrate((d*x+c)*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*c*x + D*b**3*d*x**11/11 + x**10*(C*b**3*d/10 + D*b**3*c/10) + x**9*
(B*b**3*d/9 + C*b**3*c/9 + D*a*b**2*d/3) + x**8*(A*b**3*d/8 + B*b**3*c/8 +
 3*C*a*b**2*d/8 + 3*D*a*b**2*c/8) + x**7*(A*b**3*c/7 + 3*B*a*b**2*d/7 + 3*
C*a*b**2*c/7 + 3*D*a**2*b*d/7) + x**6*(A*a*b**2*d/2 + B*a*b**2*c/2 + C*a**
2*b*d/2 + D*a**2*b*c/2) + x**5*(3*A*a*b**2*c/5 + 3*B*a**2*b*d/5 + 3*C*a**2
*b*c/5 + D*a**3*d/5) + x**4*(3*A*a**2*b*d/4 + 3*B*a**2*b*c/4 + C*a**3*d/4
+ D*a**3*c/4) + x**3*(A*a**2*b*c + B*a**3*d/3 + C*a**3*c/3) + x**2*(A*a**3
*d/2 + B*a**3*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.53

$$
\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx\\
&=\frac{1}{11}Db^3dx^{11}+\frac{1}{10}\left(Db^3c+Cb^3d\right)x^{10}+\frac{1}{9}\left(Cb^3c+\left(3\,Dab^2+Bb^3\right)d\right)x^9\\
&\quad+\frac{1}{8}\left(\left(3\,Dab^2+Bb^3\right)c+\left(3\,Cab^2+Ab^3\right)d\right)x^8\\
&\quad+\frac{1}{7}\left(\left(3\,Cab^2+Ab^3\right)c+3\left(Da^2b+Bab^2\right)d\right)x^7\\
&\quad+\frac{1}{2}\left(\left(Da^2b+Bab^2\right)c+\left(Ca^2b+Aab^2\right)d\right)x^6+Aa^3cx\\
&\quad+\frac{1}{5}\left(3\left(Ca^2b+Aab^2\right)c+\left(Da^3+3\,Ba^2b\right)d\right)x^5\\
&\quad+\frac{1}{4}\left(\left(Da^3+3\,Ba^2b\right)c+\left(Ca^3+3\,Aa^2b\right)d\right)x^4\\
&\quad+\frac{1}{3}\left(Ba^3d+\left(Ca^3+3\,Aa^2b\right)c\right)x^3+\frac{1}{2}\left(Ba^3c+Aa^3d\right)x^2
\end{aligned}
$$

input

```
integrate((d*x+c)*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output

```
1/11*D*b^3*d*x^11 + 1/10*(D*b^3*c + C*b^3*d)*x^10 + 1/9*(C*b^3*c + (3*D*a*
b^2 + B*b^3)*d)*x^9 + 1/8*((3*D*a*b^2 + B*b^3)*c + (3*C*a*b^2 + A*b^3)*d)*
x^8 + 1/7*((3*C*a*b^2 + A*b^3)*c + 3*(D*a^2*b + B*a*b^2)*d)*x^7 + 1/2*((D*
a^2*b + B*a*b^2)*c + (C*a^2*b + A*a*b^2)*d)*x^6 + A*a^3*c*x + 1/5*(3*(C*a^
2*b + A*a*b^2)*c + (D*a^3 + 3*B*a^2*b)*d)*x^5 + 1/4*((D*a^3 + 3*B*a^2*b)*c
 + (C*a^3 + 3*A*a^2*b)*d)*x^4 + 1/3*(B*a^3*d + (C*a^3 + 3*A*a^2*b)*c)*x^3
+ 1/2*(B*a^3*c + A*a^3*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.75

$$\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{1}{11} Db^3dx^{11} + \frac{1}{10} Db^3cx^{10} + \frac{1}{10} Cb^3dx^{10} + \frac{1}{9} Cb^3cx^9 + \frac{1}{3} Dab^2dx^9 + \frac{1}{9} Bb^3dx^9 + \frac{3}{8} Dab^2cx^8 + \frac{1}{8} Bb^3cx^8 + \frac{3}{8} Cab^2dx^8 + \frac{1}{8} Ab^3dx^8 + \frac{3}{7} Cab^2cx^7 + \frac{1}{7} Ab^3cx^7 + \frac{3}{7} Da^2bdx^7 + \frac{3}{7} Bab^2dx^7 + \frac{1}{2} Da^2bcx^6 + \frac{1}{2} Bab^2cx^6 + \frac{1}{2} Ca^2bdx^6 + \frac{1}{2} Aab^2dx^6 + \frac{3}{5} Ca^2bcx^5 + \frac{3}{5} Aab^2cx^5 + \frac{1}{5} Da^3dx^5 + \frac{3}{5} Ba^2bdx^5 + \frac{1}{4} Da^3cx^4 + \frac{3}{4} Ba^2bcx^4 + \frac{1}{4} Ca^3dx^4 + \frac{3}{4} Aa^2bdx^4 + \frac{1}{3} Ca^3cx^3 + Aa^2bcx^3 + \frac{1}{3} Ba^3dx^3 + \frac{1}{2} Ba^3cx^2 + \frac{1}{2} Aa^3dx^2 + Aa^3cx$$

input
```
integrate((d*x+c)*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^3*d*x^11 + 1/10*D*b^3*c*x^10 + 1/10*C*b^3*d*x^10 + 1/9*C*b^3*c*x^
9 + 1/3*D*a*b^2*d*x^9 + 1/9*B*b^3*d*x^9 + 3/8*D*a*b^2*c*x^8 + 1/8*B*b^3*c*
x^8 + 3/8*C*a*b^2*d*x^8 + 1/8*A*b^3*d*x^8 + 3/7*C*a*b^2*c*x^7 + 1/7*A*b^3*
c*x^7 + 3/7*D*a^2*b*d*x^7 + 3/7*B*a*b^2*d*x^7 + 1/2*D*a^2*b*c*x^6 + 1/2*B*
a*b^2*c*x^6 + 1/2*C*a^2*b*d*x^6 + 1/2*A*a*b^2*d*x^6 + 3/5*C*a^2*b*c*x^5 +
3/5*A*a*b^2*c*x^5 + 1/5*D*a^3*d*x^5 + 3/5*B*a^2*b*d*x^5 + 1/4*D*a^3*c*x^4
+ 3/4*B*a^2*b*c*x^4 + 1/4*C*a^3*d*x^4 + 3/4*A*a^2*b*d*x^4 + 1/3*C*a^3*c*x^
3 + A*a^2*b*c*x^3 + 1/3*B*a^3*d*x^3 + 1/2*B*a^3*c*x^2 + 1/2*A*a^3*d*x^2 +
A*a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 21.66 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.75

$$\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{a^3\,c\,x^4\,D}{4}+\frac{a^3\,d\,x^5\,D}{5}+\frac{b^3\,c\,x^{10}\,D}{10}+\frac{b^3\,d\,x^{11}\,D}{11}+A\,a^3\,c\,x+\frac{A\,a^3\,d\,x^2}{2}+\frac{B\,a^3\,c\,x^2}{2}+\frac{A\,b^3\,c\,x^7}{7}+\frac{B\,a^3\,d\,x^3}{3}+\frac{C\,a^3\,c\,x^3}{3}+\frac{A\,b^3\,d\,x^8}{8}+\frac{B\,b^3\,c\,x^8}{8}+\frac{C\,a^3\,d\,x^4}{4}+\frac{B\,b^3\,d\,x^9}{9}+\frac{C\,b^3\,c\,x^9}{9}+\frac{C\,b^3\,d\,x^{10}}{10}+\frac{C\,a^2\,b\,d\,x^6}{2}+\frac{3\,C\,a\,b^2\,d\,x^8}{8}+\frac{a^2\,b\,c\,x^6\,D}{2}+\frac{3\,a\,b^2\,c\,x^8\,D}{8}+\frac{3\,a^2\,b\,d\,x^7\,D}{7}+\frac{a\,b^2\,d\,x^9\,D}{3}+A\,a^2\,b\,c\,x^3+\frac{3\,A\,a\,b^2\,c\,x^5}{5}+\frac{3\,A\,a^2\,b\,d\,x^4}{4}+\frac{3\,B\,a^2\,b\,c\,x^4}{4}+\frac{A\,a\,b^2\,d\,x^6}{2}+\frac{B\,a\,b^2\,c\,x^6}{2}+\frac{3\,B\,a^2\,b\,d\,x^5}{5}+\frac{3\,C\,a^2\,b\,c\,x^5}{5}+\frac{3\,B\,a\,b^2\,d\,x^7}{7}+\frac{3\,C\,a\,b^2\,c\,x^7}{7}$$

input
```
int((a + b*x^2)^3*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^3*c*x^4*D)/4 + (a^3*d*x^5*D)/5 + (b^3*c*x^10*D)/10 + (b^3*d*x^11*D)/11
+ A*a^3*c*x + (A*a^3*d*x^2)/2 + (B*a^3*c*x^2)/2 + (A*b^3*c*x^7)/7 + (B*a^3
*d*x^3)/3 + (C*a^3*c*x^3)/3 + (A*b^3*d*x^8)/8 + (B*b^3*c*x^8)/8 + (C*a^3*d
*x^4)/4 + (B*b^3*d*x^9)/9 + (C*b^3*c*x^9)/9 + (C*b^3*d*x^10)/10 + (C*a^2*b
*d*x^6)/2 + (3*C*a*b^2*d*x^8)/8 + (a^2*b*c*x^6*D)/2 + (3*a*b^2*c*x^8*D)/8
+ (3*a^2*b*d*x^7*D)/7 + (a*b^2*d*x^9*D)/3 + A*a^2*b*c*x^3 + (3*A*a*b^2*c*x
^5)/5 + (3*A*a^2*b*d*x^4)/4 + (3*B*a^2*b*c*x^4)/4 + (A*a*b^2*d*x^6)/2 + (B
*a*b^2*c*x^6)/2 + (3*B*a^2*b*d*x^5)/5 + (3*C*a^2*b*c*x^5)/5 + (3*B*a*b^2*d
*x^7)/7 + (3*C*a*b^2*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.55

$$\int (c+dx) \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{x(2520b^3d^2x^{10}+5544b^3cd\,x^9+9240a\,b^2d^2x^8+3080b^4d\,x^8+3080b^3c^2x^8+3465a\,b^3d\,x^7+20790a\,b^2cd\,x}{}$$

input
```
int((d*x+c)*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(27720*a**4*c + 13860*a**4*d*x + 27720*a**3*b*c*x**2 + 13860*a**3*b*c*x
 + 20790*a**3*b*d*x**3 + 9240*a**3*b*d*x**2 + 9240*a**3*c**2*x**2 + 13860*
a**3*c*d*x**3 + 5544*a**3*d**2*x**4 + 16632*a**2*b**2*c*x**4 + 20790*a**2*
b**2*c*x**3 + 13860*a**2*b**2*d*x**5 + 16632*a**2*b**2*d*x**4 + 16632*a**2
*b*c**2*x**4 + 27720*a**2*b*c*d*x**5 + 11880*a**2*b*d**2*x**6 + 3960*a*b**
3*c*x**6 + 13860*a*b**3*c*x**5 + 3465*a*b**3*d*x**7 + 11880*a*b**3*d*x**6
+ 11880*a*b**2*c**2*x**6 + 20790*a*b**2*c*d*x**7 + 9240*a*b**2*d**2*x**8 +
 3465*b**4*c*x**7 + 3080*b**4*d*x**8 + 3080*b**3*c**2*x**8 + 5544*b**3*c*d
*x**9 + 2520*b**3*d**2*x**10))/27720
```

3.18 $\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\begin{aligned}\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = {} & a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4 \\ & +\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6 \\ & +\frac{1}{7}b^2(Ab+3aC)x^7+\frac{3}{8}ab^2Dx^8 \\ & +\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}+\frac{B(a+bx^2)^4}{8b}\end{aligned}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^
2*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D
*x^10+1/8*B*(b*x^2+a)^4/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+}{2520}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(1
5*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*
D*x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \ 2017$$

$$\int \left(bx^2+a\right)^3 \left(Dx^3+Cx^2+A\right)dx+\frac{B\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \ 2341$$

$$\int \left(b^3Dx^9+b^3Cx^8+3ab^2Dx^7+b^2(Ab+3aC)x^6+3a^2bDx^5+3ab(Ab+aC)x^4+a^3Dx^3+a^2(3Ab+aC)x^2+\right.$$
$$\frac{B\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \ 2009$$

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B(a+bx^2)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

input
```
Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x
^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 +
(b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
norman	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\left(\frac{1}{8}Bb^3+\frac{3}{8}ab^2D\right)x^8+\left(\frac{1}{7}Ab^3+\frac{3}{7}Cab^2\right)x^7+\left(\frac{1}{2}Bab^2+\frac{1}{2}Da^2b\right)x^6+\left(\frac{3}{5}\right.$
default	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\frac{(Bb^3+3ab^2D)x^8}{8}+\frac{(Ab^3+3Cab^2)x^7}{7}+\frac{(3Bab^2+3Da^2b)x^6}{6}+\frac{(3Aab^2+3Ca^2b)x^5}{5}+\frac{(3Ba}{}$
gosper	$\frac{1}{10}b^3Dx^{10}+\frac{1}{9}b^3Cx^9+\frac{1}{8}x^8Bb^3+\frac{3}{8}ab^2Dx^8+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7Cab^2+\frac{1}{2}x^6Bab^2+\frac{1}{2}a^2bDx^6$
parallelrisch	$\frac{1}{10}b^3Dx^{10}+\frac{1}{9}b^3Cx^9+\frac{1}{8}x^8Bb^3+\frac{3}{8}ab^2Dx^8+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7Cab^2+\frac{1}{2}x^6Bab^2+\frac{1}{2}a^2bDx^6$
orering	$\frac{x(252b^3Dx^9+280Cb^3x^8+315Bb^3x^7+945Dab^2x^7+360Ab^3x^6+1080Cab^2x^6+1260Bab^2x^5+1260Da^2bx^5+1512Aab^2x^4+}{2520}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^3*D*x^10+1/9*b^3*C*x^9+(1/8*B*b^3+3/8*a*b^2*D)*x^8+(1/7*A*b^3+3/7*C
*a*b^2)*x^7+(1/2*B*a*b^2+1/2*D*a^2*b)*x^6+(3/5*A*a*b^2+3/5*C*a^2*b)*x^5+(3
/4*B*a^2*b+1/4*D*a^3)*x^4+(A*a^2*b+1/3*C*a^3)*x^3+1/2*B*a^3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,\left(3\,Dab^2+Bb^3\right)x^8\\ &\quad+\frac{1}{7}\,\left(3\,Cab^2+Ab^3\right)x^7+\frac{1}{2}\,\left(Da^2b+Bab^2\right)x^6\\ &\quad+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,\left(Ca^2b+Aab^2\right)x^5+Aa^3x\\ &\quad+\frac{1}{4}\,\left(Da^3+3\,Ba^2b\right)x^4+\frac{1}{3}\,\left(Ca^3+3\,Aa^2b\right)x^3\end{aligned}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^3x + \frac{Ba^3x^2}{2} + \frac{Cb^3x^9}{9} + \frac{Db^3x^{10}}{10} + x^8\left(\frac{Bb^3}{8} + \frac{3Dab^2}{8}\right) + x^7\left(\frac{Ab^3}{7} + \frac{3Cab^2}{7}\right) + x^6\left(\frac{Bab^2}{2} + \frac{Da^2b}{2}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ca^2b}{5}\right) + x^4 \cdot \left(\frac{3Ba^2b}{4} + \frac{Da^3}{4}\right) + x^3\left(Aa^2b + \frac{Ca^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/
8 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*
a**2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a*
*3/4) + x**3*(A*a**2*b + C*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10} + \frac{1}{9}\,Cb^3x^9 + \frac{1}{8}\,\left(3\,Dab^2 + Bb^3\right)x^8 + \frac{1}{7}\,\left(3\,Cab^2 + Ab^3\right)x^7 + \frac{1}{2}\,\left(Da^2b + Bab^2\right)x^6 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\,\left(Ca^2b + Aab^2\right)x^5 + Aa^3x + \frac{1}{4}\,\left(Da^3 + 3\,Ba^2b\right)x^4 + \frac{1}{3}\,\left(Ca^3 + 3\,Aa^2b\right)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Da^3x^4 + \frac{3}{4}Ba^2bx^4 + \frac{1}{3}Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*
a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*
b*x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3
+ A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

input
```
int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9
)/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b
^2*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b
^2*x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx$$

$$= \frac{x(252b^3d\,x^9+280b^3c\,x^8+945a\,b^2d\,x^7+315b^4x^7+360a\,b^3x^6+1080a\,b^2c\,x^6+1260a^2bd\,x^5+1260a\,b^3x^5}{25}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(2520*a**4 + 2520*a**3*b*x**2 + 1260*a**3*b*x + 840*a**3*c*x**2 + 630*a
**3*d*x**3 + 1512*a**2*b**2*x**4 + 1890*a**2*b**2*x**3 + 1512*a**2*b*c*x**
4 + 1260*a**2*b*d*x**5 + 360*a*b**3*x**6 + 1260*a*b**3*x**5 + 1080*a*b**2*
c*x**6 + 945*a*b**2*d*x**7 + 315*b**4*x**7 + 280*b**3*c*x**8 + 252*b**3*d*
x**9))/2520
```

3.19 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 683

$$\begin{aligned}
&\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx \\
&= -\frac{(bc^2+ad^2)^2(ad^2(2cCd-Bd^2-3c^2D)+bc(8c^2Cd-7Bcd^2+6Ad^3-9c^3D))\,x}{d^9} \\
&\quad+\frac{(bc^2+ad^2)(a^2d^4(Cd-3cD)+b^2c^2(28c^2Cd-21Bcd^2+15Ad^3-36c^3D)+abd^2(17c^2Cd-9Bcd^2+}{2d^{10}} \\
&\quad+\frac{(a^3d^6D-3a^2bd^4(4cCd-Bd^2-10c^2D)-b^3c^3(56c^2Cd-35Bcd^2+20Ad^3-84c^3D)-3ab^2cd^2(20c^2}{3d^{10}} \\
&\quad+\frac{b(3a^2d^4(Cd-5cD)+b^2c^2(70c^2Cd-35Bcd^2+15Ad^3-126c^3D)+3abd^2(15c^2Cd-5Bcd^2+Ad^3-}{4d^{10}} \\
&\quad+\frac{b(3a^2d^4D-3abd^2(6cCd-Bd^2-21c^2D)-b^2c(56c^2Cd-21Bcd^2+6Ad^3-126c^3D))(c+dx)^5}{5d^{10}} \\
&\quad+\frac{b^2(3ad^2(Cd-7cD)+b(28c^2Cd-7Bcd^2+Ad^3-84c^3D))(c+dx)^6}{6d^{10}} \\
&\quad+\frac{b^2(3ad^2D-b(8cCd-Bd^2-36c^2D))(c+dx)^7}{7d^{10}}+\frac{b^3(Cd-9cD)(c+dx)^8}{8d^{10}} \\
&\quad+\frac{b^3D(c+dx)^9}{9d^{10}}+\frac{(bc^2+ad^2)^3(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^{10}}
\end{aligned}$$

output
```
-(a*d^2+b*c^2)^2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(6*A*d^3-7*B*c*d^2+8*
C*c^2*d-9*D*c^3))*x/d^9+1/2*(a*d^2+b*c^2)*(a^2*d^4*(C*d-3*D*c)+b^2*c^2*(15
*A*d^3-21*B*c*d^2+28*C*c^2*d-36*D*c^3)+a*b*d^2*(3*A*d^3-9*B*c*d^2+17*C*c^2
*d-27*D*c^3))*(d*x+c)^2/d^10+1/3*(a^3*d^6*D-3*a^2*b*d^4*(-B*d^2+4*C*c*d-10
*D*c^2)-b^3*c^3*(20*A*d^3-35*B*c*d^2+56*C*c^2*d-84*D*c^3)-3*a*b^2*c*d^2*(4
*A*d^3-10*B*c*d^2+20*C*c^2*d-35*D*c^3))*(d*x+c)^3/d^10+1/4*b*(3*a^2*d^4*(C
*d-5*D*c)+b^2*c^2*(15*A*d^3-35*B*c*d^2+70*C*c^2*d-126*D*c^3)+3*a*b*d^2*(A*
d^3-5*B*c*d^2+15*C*c^2*d-35*D*c^3))*(d*x+c)^4/d^10+1/5*b*(3*a^2*d^4*D-3*a*
b*d^2*(-B*d^2+6*C*c*d-21*D*c^2)-b^2*c*(6*A*d^3-21*B*c*d^2+56*C*c^2*d-126*D
*c^3))*(d*x+c)^5/d^10+1/6*b^2*(3*a*d^2*(C*d-7*D*c)+b*(A*d^3-7*B*c*d^2+28*C
*c^2*d-84*D*c^3))*(d*x+c)^6/d^10+1/7*b^2*(3*a*d^2*D-b*(-B*d^2+8*C*c*d-36*D
*c^2))*(d*x+c)^7/d^10+1/8*b^3*(C*d-9*D*c)*(d*x+c)^8/d^10+1/9*b^3*D*(d*x+c)
^9/d^10+(a*d^2+b*c^2)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^10
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{x(420a^3d^6(6c^2D-3cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+126a^2bd^4(60c^4D-30c^3d(2C+Dx)+1}{}$$
$$+\frac{(bc^2+ad^2)^3(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^{10}}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(x*(420*a^3*d^6*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x))
) + 126*a^2*b*d^4*(60*c^4*D - 30*c^3*d*(2*C + D*x) + 10*c^2*d^2*(6*B + x*(
3*C + 2*D*x)) - 5*c*d^3*(12*A + x*(6*B + x*(4*C + 3*D*x))) + d^4*x*(30*A +
 x*(20*B + 3*x*(5*C + 4*D*x)))) + 18*a*b^2*d^2*(420*c^6*D - 210*c^5*d*(2*C
 + D*x) + 70*c^4*d^2*(6*B + x*(3*C + 2*D*x)) - 35*c^3*d^3*(12*A + x*(6*B +
 x*(4*C + 3*D*x))) + 7*c^2*d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - 7
*c*d^5*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + d^6*x^3*(105*A + 2*x*(4
2*B + 5*x*(7*C + 6*D*x)))) + b^3*(2520*c^8*D - 1260*c^7*d*(2*C + D*x) + 42
0*c^6*d^2*(6*B + x*(3*C + 2*D*x)) - 210*c^5*d^3*(12*A + x*(6*B + x*(4*C +
3*D*x))) + 42*c^4*d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - 42*c^3*d^5
*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*c^2*d^6*x^3*(105*A + 2*x*(4
2*B + 5*x*(7*C + 6*D*x))) - 3*c*d^7*x^4*(168*A + 5*x*(28*B + 3*x*(8*C + 7*
D*x))) + 5*d^8*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))))/(2520*d^9) + (
(b*c^2 + a*d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^10
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx} \, dx$$

$$\Big\downarrow \; 2160$$

$$\int \left(\frac{b(c+dx)^4 \left(3a^2d^4D - 3abd^2\left(-Bd^2 - 21c^2D + 6cCd\right) - b^2c\left(6Ad^3 - 21Bcd^2 - 126c^3D + 56c^2Cd\right)\right)}{d^9} + \frac{b(c+\cdots\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{b(c+dx)^5\left(3a^2d^4D-3abd^2\left(-Bd^2-21c^2D+6cCd\right)-b^2c\left(6Ad^3-21Bcd^2-126c^3D+56c^2Cd\right)\right)}{5d^{10}}+$$

$$\frac{b(c+dx)^4\left(3a^2d^4(Cd-5cD)+3abd^2\left(Ad^3-5Bcd^2-35c^3D+15c^2Cd\right)+b^2c^2\left(15Ad^3-35Bcd^2-126c^3D+7\right.}{4d^{10}}$$

$$\frac{(c+dx)^2\left(ad^2+bc^2\right)\left(a^2d^4(Cd-3cD)+abd^2\left(3Ad^3-9Bcd^2-27c^3D+17c^2Cd\right)+b^2c^2\left(15Ad^3-21Bcd^2-3\right.\right.}{2d^{10}}$$

$$\frac{(c+dx)^3\left(a^3d^6D-3a^2bd^4\left(-Bd^2-10c^2D+4cCd\right)-3ab^2cd^2\left(4Ad^3-10Bcd^2-35c^3D+20c^2Cd\right)-b^3c^3\left(20A\right.\right.}{3d^{10}}$$

$$\frac{b^2(c+dx)^6\left(3ad^2(Cd-7cD)+b\left(Ad^3-7Bcd^2-84c^3D+28c^2Cd\right)\right)}{6d^{10}}+$$

$$\frac{\left(ad^2+bc^2\right)^3\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^{10}}-$$

$$\frac{x\left(ad^2+bc^2\right)^2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(6Ad^3-7Bcd^2-9c^3D+8c^2Cd\right)\right)}{d^9}+$$

$$\frac{b^2(c+dx)^7\left(3ad^2D-b\left(-Bd^2-36c^2D+8cCd\right)\right)}{7d^{10}}+\frac{b^3(c+dx)^8(Cd-9cD)}{8d^{10}}+\frac{b^3D(c+dx)^9}{9d^{10}}$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
-(((b*c^2 + a*d^2)^2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(8*c^2*C*d -
 7*B*c*d^2 + 6*A*d^3 - 9*c^3*D))*x)/d^9) + ((b*c^2 + a*d^2)*(a^2*d^4*(C*d
- 3*c*D) + b^2*c^2*(28*c^2*C*d - 21*B*c*d^2 + 15*A*d^3 - 36*c^3*D) + a*b*d
^2*(17*c^2*C*d - 9*B*c*d^2 + 3*A*d^3 - 27*c^3*D))*(c + d*x)^2)/(2*d^10) +
((a^3*d^6*D - 3*a^2*b*d^4*(4*c*C*d - B*d^2 - 10*c^2*D) - b^3*c^3*(56*c^2*C
*d - 35*B*c*d^2 + 20*A*d^3 - 84*c^3*D) - 3*a*b^2*c*d^2*(20*c^2*C*d - 10*B*
c*d^2 + 4*A*d^3 - 35*c^3*D))*(c + d*x)^3)/(3*d^10) + (b*(3*a^2*d^4*(C*d -
5*c*D) + b^2*c^2*(70*c^2*C*d - 35*B*c*d^2 + 15*A*d^3 - 126*c^3*D) + 3*a*b*
d^2*(15*c^2*C*d - 5*B*c*d^2 + A*d^3 - 35*c^3*D))*(c + d*x)^4)/(4*d^10) + (
b*(3*a^2*d^4*D - 3*a*b*d^2*(6*c*C*d - B*d^2 - 21*c^2*D) - b^2*c*(56*c^2*C*
d - 21*B*c*d^2 + 6*A*d^3 - 126*c^3*D))*(c + d*x)^5)/(5*d^10) + (b^2*(3*a*d
^2*(C*d - 7*c*D) + b*(28*c^2*C*d - 7*B*c*d^2 + A*d^3 - 84*c^3*D))*(c + d*x
)^6)/(6*d^10) + (b^2*(3*a*d^2*D - b*(8*c*C*d - B*d^2 - 36*c^2*D))*(c + d*x
)^7)/(7*d^10) + (b^3*(C*d - 9*c*D)*(c + d*x)^8)/(8*d^10) + (b^3*D*(c + d*x
)^9)/(9*d^10) + ((b*c^2 + a*d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log
[c + d*x])/d^10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.47

method	result	size
norman	Expression too large to display	1007
default	Expression too large to display	1205
parallelrisch	Expression too large to display	1304

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^8*(3*A*a^2*b*d^7+3*A*a*b^2*c^2*d^5+A*b^3*c^4*d^3-3*B*a^2*b*c*d^6-3*B
*a*b^2*c^3*d^4-B*b^3*c^5*d^2+C*a^3*d^7+3*C*a^2*b*c^2*d^5+3*C*a*b^2*c^4*d^3
+C*b^3*c^6*d-D*a^3*c*d^6-3*D*a^2*b*c^3*d^4-3*D*a*b^2*c^5*d^2-D*b^3*c^7)*x^
2-1/3/d^7*(3*A*a*b^2*c*d^5+A*b^3*c^3*d^3-3*B*a^2*b*d^6-3*B*a*b^2*c^2*d^4-B
*b^3*c^4*d^2+3*C*a^2*b*c*d^5+3*C*a*b^2*c^3*d^3+C*b^3*c^5*d-D*a^3*d^6-3*D*a
^2*b*c^2*d^4-3*D*a*b^2*c^4*d^2-D*b^3*c^6)*x^3-(3*A*a^2*b*c*d^7+3*A*a*b^2*c
^3*d^5+A*b^3*c^5*d^3-B*a^3*d^8-3*B*a^2*b*c^2*d^6-3*B*a*b^2*c^4*d^4-B*b^3*c
^6*d^2+C*a^3*c*d^7+3*C*a^2*b*c^3*d^5+3*C*a*b^2*c^5*d^3+C*b^3*c^7*d-D*a^3*c
^2*d^6-3*D*a^2*b*c^4*d^4-3*D*a*b^2*c^6*d^2-D*b^3*c^8)/d^9*x+1/9*D*b^3/d*x^
9+1/4*b/d^6*(3*A*a*b*d^5+A*b^2*c^2*d^3-3*B*a*b*c*d^4-B*b^2*c^3*d^2+3*C*a^2
*d^5+3*C*a*b*c^2*d^3+C*b^2*c^4*d-3*D*a^2*c*d^4-3*D*a*b*c^3*d^2-D*b^2*c^5)*
x^4-1/5*b/d^5*(A*b^2*c*d^3-3*B*a*b*d^4-B*b^2*c^2*d^2+3*C*a*b*c*d^3+C*b^2*c
^3*d-3*D*a^2*d^4-3*D*a*b*c^2*d^2-D*b^2*c^4)*x^5+1/6*b^2/d^4*(A*b*d^3-B*b*c
*d^2+3*C*a*d^3+C*b*c^2*d-3*D*a*c*d^2-D*b*c^3)*x^6+1/7*b^2/d^3*(B*b*d^2-C*b
*c*d+3*D*a*d^2+D*b*c^2)*x^7+1/8*b^3/d^2*(C*d-D*c)*x^8+(A*a^3*d^9+3*A*a^2*b
*c^2*d^7+3*A*a*b^2*c^4*d^5+A*b^3*c^6*d^3-B*a^3*c*d^8-3*B*a^2*b*c^3*d^6-3*B
*a*b^2*c^5*d^4-B*b^3*c^7*d^2+C*a^3*c^2*d^7+3*C*a^2*b*c^4*d^5+3*C*a*b^2*c^6
*d^3+C*b^3*c^8*d-D*a^3*c^3*d^6-3*D*a^2*b*c^5*d^4-3*D*a*b^2*c^7*d^2-D*b^3*c
^9)/d^10*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2520*(280*D*b^3*d^9*x^9 - 315*(D*b^3*c*d^8 - C*b^3*d^9)*x^8 + 360*(D*b^3
*c^2*d^7 - C*b^3*c*d^8 + (3*D*a*b^2 + B*b^3)*d^9)*x^7 - 420*(D*b^3*c^3*d^6
 - C*b^3*c^2*d^7 + (3*D*a*b^2 + B*b^3)*c*d^8 - (3*C*a*b^2 + A*b^3)*d^9)*x^
6 + 504*(D*b^3*c^4*d^5 - C*b^3*c^3*d^6 + (3*D*a*b^2 + B*b^3)*c^2*d^7 - (3*
C*a*b^2 + A*b^3)*c*d^8 + 3*(D*a^2*b + B*a*b^2)*d^9)*x^5 - 630*(D*b^3*c^5*d
^4 - C*b^3*c^4*d^5 + (3*D*a*b^2 + B*b^3)*c^3*d^6 - (3*C*a*b^2 + A*b^3)*c^2
*d^7 + 3*(D*a^2*b + B*a*b^2)*c*d^8 - 3*(C*a^2*b + A*a*b^2)*d^9)*x^4 + 840*
(D*b^3*c^6*d^3 - C*b^3*c^5*d^4 + (3*D*a*b^2 + B*b^3)*c^4*d^5 - (3*C*a*b^2
+ A*b^3)*c^3*d^6 + 3*(D*a^2*b + B*a*b^2)*c^2*d^7 - 3*(C*a^2*b + A*a*b^2)*c
*d^8 + (D*a^3 + 3*B*a^2*b)*d^9)*x^3 - 1260*(D*b^3*c^7*d^2 - C*b^3*c^6*d^3
+ (3*D*a*b^2 + B*b^3)*c^5*d^4 - (3*C*a*b^2 + A*b^3)*c^4*d^5 + 3*(D*a^2*b +
 B*a*b^2)*c^3*d^6 - 3*(C*a^2*b + A*a*b^2)*c^2*d^7 + (D*a^3 + 3*B*a^2*b)*c*
d^8 - (C*a^3 + 3*A*a^2*b)*d^9)*x^2 + 2520*(D*b^3*c^8*d - C*b^3*c^7*d^2 + B
*a^3*d^9 + (3*D*a*b^2 + B*b^3)*c^6*d^3 - (3*C*a*b^2 + A*b^3)*c^5*d^4 + 3*(
D*a^2*b + B*a*b^2)*c^4*d^5 - 3*(C*a^2*b + A*a*b^2)*c^3*d^6 + (D*a^3 + 3*B*
a^2*b)*c^2*d^7 - (C*a^3 + 3*A*a^2*b)*c*d^8)*x - 2520*(D*b^3*c^9 - C*b^3*c^
8*d + B*a^3*c*d^8 - A*a^3*d^9 + (3*D*a*b^2 + B*b^3)*c^7*d^2 - (3*C*a*b^2 +
 A*b^3)*c^6*d^3 + 3*(D*a^2*b + B*a*b^2)*c^5*d^4 - 3*(C*a^2*b + A*a*b^2)*c^
4*d^5 + (D*a^3 + 3*B*a^2*b)*c^3*d^6 - (C*a^3 + 3*A*a^2*b)*c^2*d^7)*log(d*x
 + c))/d^10
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b**3*x**9/(9*d) + x**8*(C*b**3/(8*d) - D*b**3*c/(8*d**2)) + x**7*(B*b**3
/(7*d) - C*b**3*c/(7*d**2) + 3*D*a*b**2/(7*d) + D*b**3*c**2/(7*d**3)) + x*
*6*(A*b**3/(6*d) - B*b**3*c/(6*d**2) + C*a*b**2/(2*d) + C*b**3*c**2/(6*d**
3) - D*a*b**2*c/(2*d**2) - D*b**3*c**3/(6*d**4)) + x**5*(-A*b**3*c/(5*d**2
) + 3*B*a*b**2/(5*d) + B*b**3*c**2/(5*d**3) - 3*C*a*b**2*c/(5*d**2) - C*b*
*3*c**3/(5*d**4) + 3*D*a**2*b/(5*d) + 3*D*a*b**2*c**2/(5*d**3) + D*b**3*c*
*4/(5*d**5)) + x**4*(3*A*a*b**2/(4*d) + A*b**3*c**2/(4*d**3) - 3*B*a*b**2*
c/(4*d**2) - B*b**3*c**3/(4*d**4) + 3*C*a**2*b/(4*d) + 3*C*a*b**2*c**2/(4*
d**3) + C*b**3*c**4/(4*d**5) - 3*D*a**2*b*c/(4*d**2) - 3*D*a*b**2*c**3/(4*
d**4) - D*b**3*c**5/(4*d**6)) + x**3*(-A*a*b**2*c/d**2 - A*b**3*c**3/(3*d*
*4) + B*a**2*b/d + B*a*b**2*c**2/d**3 + B*b**3*c**4/(3*d**5) - C*a**2*b*c/
d**2 - C*a*b**2*c**3/d**4 - C*b**3*c**5/(3*d**6) + D*a**3/(3*d) + D*a**2*b
*c**2/d**3 + D*a*b**2*c**4/d**5 + D*b**3*c**6/(3*d**7)) + x**2*(3*A*a**2*b
/(2*d) + 3*A*a*b**2*c**2/(2*d**3) + A*b**3*c**4/(2*d**5) - 3*B*a**2*b*c/(2
*d**2) - 3*B*a*b**2*c**3/(2*d**4) - B*b**3*c**5/(2*d**6) + C*a**3/(2*d) +
3*C*a**2*b*c**2/(2*d**3) + 3*C*a*b**2*c**4/(2*d**5) + C*b**3*c**6/(2*d**7)
 - D*a**3*c/(2*d**2) - 3*D*a**2*b*c**3/(2*d**4) - 3*D*a*b**2*c**5/(2*d**6)
 - D*b**3*c**7/(2*d**8)) + x*(-3*A*a**2*b*c/d**2 - 3*A*a*b**2*c**3/d**4 -
A*b**3*c**5/d**6 + B*a**3/d + 3*B*a**2*b*c**2/d**3 + 3*B*a*b**2*c**4/d**5
+ B*b**3*c**6/d**7 - C*a**3*c/d**2 - 3*C*a**2*b*c**3/d**4 - 3*C*a*b**2*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2520*(280*D*b^3*d^8*x^9 - 315*(D*b^3*c*d^7 - C*b^3*d^8)*x^8 + 360*(D*b^3
*c^2*d^6 - C*b^3*c*d^7 + (3*D*a*b^2 + B*b^3)*d^8)*x^7 - 420*(D*b^3*c^3*d^5
 - C*b^3*c^2*d^6 + (3*D*a*b^2 + B*b^3)*c*d^7 - (3*C*a*b^2 + A*b^3)*d^8)*x^
6 + 504*(D*b^3*c^4*d^4 - C*b^3*c^3*d^5 + (3*D*a*b^2 + B*b^3)*c^2*d^6 - (3*
C*a*b^2 + A*b^3)*c*d^7 + 3*(D*a^2*b + B*a*b^2)*d^8)*x^5 - 630*(D*b^3*c^5*d
^3 - C*b^3*c^4*d^4 + (3*D*a*b^2 + B*b^3)*c^3*d^5 - (3*C*a*b^2 + A*b^3)*c^2
*d^6 + 3*(D*a^2*b + B*a*b^2)*c*d^7 - 3*(C*a^2*b + A*a*b^2)*d^8)*x^4 + 840*
(D*b^3*c^6*d^2 - C*b^3*c^5*d^3 + (3*D*a*b^2 + B*b^3)*c^4*d^4 - (3*C*a*b^2
+ A*b^3)*c^3*d^5 + 3*(D*a^2*b + B*a*b^2)*c^2*d^6 - 3*(C*a^2*b + A*a*b^2)*c
*d^7 + (D*a^3 + 3*B*a^2*b)*d^8)*x^3 - 1260*(D*b^3*c^7*d - C*b^3*c^6*d^2 +
(3*D*a*b^2 + B*b^3)*c^5*d^3 - (3*C*a*b^2 + A*b^3)*c^4*d^4 + 3*(D*a^2*b + B
*a*b^2)*c^3*d^5 - 3*(C*a^2*b + A*a*b^2)*c^2*d^6 + (D*a^3 + 3*B*a^2*b)*c*d^
7 - (C*a^3 + 3*A*a^2*b)*d^8)*x^2 + 2520*(D*b^3*c^8 - C*b^3*c^7*d + B*a^3*d
^8 + (3*D*a*b^2 + B*b^3)*c^6*d^2 - (3*C*a*b^2 + A*b^3)*c^5*d^3 + 3*(D*a^2*
b + B*a*b^2)*c^4*d^4 - 3*(C*a^2*b + A*a*b^2)*c^3*d^5 + (D*a^3 + 3*B*a^2*b)
*c^2*d^6 - (C*a^3 + 3*A*a^2*b)*c*d^7)*x)/d^9 - (D*b^3*c^9 - C*b^3*c^8*d +
B*a^3*c*d^8 - A*a^3*d^9 + (3*D*a*b^2 + B*b^3)*c^7*d^2 - (3*C*a*b^2 + A*b^3
)*c^6*d^3 + 3*(D*a^2*b + B*a*b^2)*c^5*d^4 - 3*(C*a^2*b + A*a*b^2)*c^4*d^5
+ (D*a^3 + 3*B*a^2*b)*c^3*d^6 - (C*a^3 + 3*A*a^2*b)*c^2*d^7)*log(d*x + c)/
d^10
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1212, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/2520*(280*D*b^3*d^8*x^9 - 315*D*b^3*c*d^7*x^8 + 315*C*b^3*d^8*x^8 + 360*
D*b^3*c^2*d^6*x^7 - 360*C*b^3*c*d^7*x^7 + 1080*D*a*b^2*d^8*x^7 + 360*B*b^3
*d^8*x^7 - 420*D*b^3*c^3*d^5*x^6 + 420*C*b^3*c^2*d^6*x^6 - 1260*D*a*b^2*c*
d^7*x^6 - 420*B*b^3*c*d^7*x^6 + 1260*C*a*b^2*d^8*x^6 + 420*A*b^3*d^8*x^6 +
 504*D*b^3*c^4*d^4*x^5 - 504*C*b^3*c^3*d^5*x^5 + 1512*D*a*b^2*c^2*d^6*x^5
+ 504*B*b^3*c^2*d^6*x^5 - 1512*C*a*b^2*c*d^7*x^5 - 504*A*b^3*c*d^7*x^5 + 1
512*D*a^2*b*d^8*x^5 + 1512*B*a*b^2*d^8*x^5 - 630*D*b^3*c^5*d^3*x^4 + 630*C
*b^3*c^4*d^4*x^4 - 1890*D*a*b^2*c^3*d^5*x^4 - 630*B*b^3*c^3*d^5*x^4 + 1890
*C*a*b^2*c^2*d^6*x^4 + 630*A*b^3*c^2*d^6*x^4 - 1890*D*a^2*b*c*d^7*x^4 - 18
90*B*a*b^2*c*d^7*x^4 + 1890*C*a^2*b*d^8*x^4 + 1890*A*a*b^2*d^8*x^4 + 840*D
*b^3*c^6*d^2*x^3 - 840*C*b^3*c^5*d^3*x^3 + 2520*D*a*b^2*c^4*d^4*x^3 + 840*
B*b^3*c^4*d^4*x^3 - 2520*C*a*b^2*c^3*d^5*x^3 - 840*A*b^3*c^3*d^5*x^3 + 252
0*D*a^2*b*c^2*d^6*x^3 + 2520*B*a*b^2*c^2*d^6*x^3 - 2520*C*a^2*b*c*d^7*x^3
- 2520*A*a*b^2*c*d^7*x^3 + 840*D*a^3*d^8*x^3 + 2520*B*a^2*b*d^8*x^3 - 1260
*D*b^3*c^7*d*x^2 + 1260*C*b^3*c^6*d^2*x^2 - 3780*D*a*b^2*c^5*d^3*x^2 - 126
0*B*b^3*c^5*d^3*x^2 + 3780*C*a*b^2*c^4*d^4*x^2 + 1260*A*b^3*c^4*d^4*x^2 -
3780*D*a^2*b*c^3*d^5*x^2 - 3780*B*a*b^2*c^3*d^5*x^2 + 3780*C*a^2*b*c^2*d^6
*x^2 + 3780*A*a*b^2*c^2*d^6*x^2 - 1260*D*a^3*c*d^7*x^2 - 3780*B*a^2*b*c*d^
7*x^2 + 1260*C*a^3*d^8*x^2 + 3780*A*a^2*b*d^8*x^2 + 2520*D*b^3*c^8*x - 252
0*C*b^3*c^7*d*x + 7560*D*a*b^2*c^6*d^2*x + 2520*B*b^3*c^6*d^2*x - 7560*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{(b\,x^2+a)^3\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{-210b^4c\,d^6x^6 + 1890a^3b\,d^7x^2 + 1260a^3b\,d^7x + 945a^2b^2d^7x^4 + 1260a^2b^2d^7x^3 + 756a^2b\,d^8x^5 + 210a\,b^3d^7x^6}{}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(1260*log(c + d*x)*a**4*d**7 + 3780*log(c + d*x)*a**3*b*c**2*d**5 - 1260*l
og(c + d*x)*a**3*b*c*d**6 + 3780*log(c + d*x)*a**2*b**2*c**4*d**3 - 3780*l
og(c + d*x)*a**2*b**2*c**3*d**4 + 1260*log(c + d*x)*a*b**3*c**6*d - 3780*l
og(c + d*x)*a*b**3*c**5*d**2 - 1260*log(c + d*x)*b**4*c**7 - 3780*a**3*b*c
*d**6*x + 1890*a**3*b*d**7*x**2 + 1260*a**3*b*d**7*x + 420*a**3*d**8*x**3
 - 3780*a**2*b**2*c**3*d**4*x + 1890*a**2*b**2*c**2*d**5*x**2 + 3780*a**2*b
**2*c**2*d**5*x - 1260*a**2*b**2*c*d**6*x**3 - 1890*a**2*b**2*c*d**6*x**2
 + 945*a**2*b**2*d**7*x**4 + 1260*a**2*b**2*d**7*x**3 + 756*a**2*b*d**8*x**
5 - 1260*a*b**3*c**5*d**2*x + 630*a*b**3*c**4*d**3*x**2 + 3780*a*b**3*c**4
*d**3*x - 420*a*b**3*c**3*d**4*x**3 - 1890*a*b**3*c**3*d**4*x**2 + 315*a*b
**3*c**2*d**5*x**4 + 1260*a*b**3*c**2*d**5*x**3 - 252*a*b**3*c*d**6*x**5 -
 945*a*b**3*c*d**6*x**4 + 210*a*b**3*d**7*x**6 + 756*a*b**3*d**7*x**5 + 54
0*a*b**2*d**8*x**7 + 1260*b**4*c**6*d*x - 630*b**4*c**5*d**2*x**2 + 420*b*
*4*c**4*d**3*x**3 - 315*b**4*c**3*d**4*x**4 + 252*b**4*c**2*d**5*x**5 - 21
0*b**4*c*d**6*x**6 + 180*b**4*d**7*x**7 + 140*b**3*d**8*x**9)/(1260*d**8)
```

3.20 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 683

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx$$
$$= \frac{(a^3d^6(Cd-2cD)+b^3c^4(7c^2Cd-6Bcd^2+5Ad^3-8c^3D)+3ab^2c^2d^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)+}{d^9}$$
$$+\frac{(a^3d^6D-3a^2bd^4(2cCd-Bd^2-3c^2D)-b^3c^3(6c^2Cd-5Bcd^2+4Ad^3-7c^3D)-3ab^2cd^2(4c^2Cd-3}{2d^8}$$
$$+\frac{b(3a^2d^4(Cd-2cD)+b^2c^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)+3abd^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D))}{3d^7}$$
$$+\frac{b(3a^2d^4D-3abd^2(2cCd-Bd^2-3c^2D)-b^2c(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))\,x^4}{4d^6}$$
$$+\frac{b^2(3ad^2(Cd-2cD)+b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x^5}{5d^5}$$
$$+\frac{b^2(3ad^2D-b(2cCd-Bd^2-3c^2D))\,x^6}{6d^4}+\frac{b^3(Cd-2cD)x^7}{7d^3}$$
$$+\frac{b^3Dx^8}{8d^2}-\frac{(bc^2+ad^2)^3\,(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^{10}(c+dx)}$$
$$-\frac{(bc^2+ad^2)^2\,(ad^2(2cCd-Bd^2-3c^2D)+bc(8c^2Cd-7Bcd^2+6Ad^3-9c^3D))\log(c+dx)}{d^{10}}$$

output
```
(a^3*d^6*(C*d-2*D*c)+b^3*c^4*(5*A*d^3-6*B*c*d^2+7*C*c^2*d-8*D*c^3)+3*a*b^2
*c^2*d^2*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3)+3*a^2*b*d^4*(A*d^3-2*B*c*d^
2+3*C*c^2*d-4*D*c^3))*x/d^9+1/2*(a^3*d^6*D-3*a^2*b*d^4*(-B*d^2+2*C*c*d-3*D
*c^2)-b^3*c^3*(4*A*d^3-5*B*c*d^2+6*C*c^2*d-7*D*c^3)-3*a*b^2*c*d^2*(2*A*d^3
-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*x^2/d^8+1/3*b*(3*a^2*d^4*(C*d-2*D*c)+b^2*c^
2*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3)+3*a*b*d^2*(A*d^3-2*B*c*d^2+3*C*c^2
*d-4*D*c^3))*x^3/d^7+1/4*b*(3*a^2*d^4*D-3*a*b*d^2*(-B*d^2+2*C*c*d-3*D*c^2)
-b^2*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*x^4/d^6+1/5*b^2*(3*a*d^2*(C*
d-2*D*c)+b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*x^5/d^5+1/6*b^2*(3*a*d^2*D
-b*(-B*d^2+2*C*c*d-3*D*c^2))*x^6/d^4+1/7*b^3*(C*d-2*D*c)*x^7/d^3+1/8*b^3*D
*x^8/d^2-(a*d^2+b*c^2)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^10/(d*x+c)-(a*d^2
+b*c^2)^2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(6*A*d^3-7*B*c*d^2+8*C*c^2*d
-9*D*c^3))*ln(d*x+c)/d^10
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{420a^3d^6(2c^3D-2c^2d(C+2Dx)+cd^2(2B+x(2C-3Dx))+d^3(-2A+x^2(2C+Dx)))+210a^2bd^4(12}{}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(420*a^3*d^6*(2*c^3*D - 2*c^2*d*(C + 2*D*x) + c*d^2*(2*B + x*(2*C - 3*D*x)
) + d^3*(-2*A + x^2*(2*C + D*x))) + 210*a^2*b*d^4*(12*c^5*D - 12*c^4*d*(C
+ 4*D*x) + 6*c^3*d^2*(2*B + x*(6*C - 5*D*x)) - 2*c^2*d^3*(6*A + x*(12*B -
12*C*x - 5*D*x^2)) + d^5*x^2*(12*A + x*(6*B + 4*C*x + 3*D*x^2)) - c*d^4*x*
(-12*A + x*(18*B + 8*C*x + 5*D*x^2))) + 42*a*b^2*d^2*(60*c^7*D - 60*c^6*d*
(C + 6*D*x) + 30*c^5*d^2*(2*B + x*(10*C - 7*D*x)) + d^7*x^4*(20*A + x*(15*
B + 2*x*(6*C + 5*D*x))) - c*d^6*x^3*(40*A + x*(25*B + 2*x*(9*C + 7*D*x)))
+ c^2*d^5*x^2*(120*A + x*(50*B + 3*x*(10*C + 7*D*x))) - 5*c^3*d^4*x*(-36*A
 + x*(30*B + x*(12*C + 7*D*x))) - 10*c^4*d^3*(6*A + x*(24*B - x*(18*C + 7*
D*x)))) + b^3*(840*c^9*D - 840*c^8*d*(C + 8*D*x) + 420*c^7*d^2*(2*B + x*(1
4*C - 9*D*x)) - 14*c^3*d^6*x^3*(60*A + x*(35*B + 6*x*(4*C + 3*D*x))) - 420
*c^6*d^3*(2*A + x*(12*B - x*(8*C + 3*D*x))) + d^9*x^6*(168*A + 5*x*(28*B +
 3*x*(8*C + 7*D*x))) - 70*c^5*d^4*x*(-60*A + x*(42*B + x*(16*C + 9*D*x)))
- c*d^8*x^5*(252*A + x*(196*B + 5*x*(32*C + 27*D*x))) + 14*c^4*d^5*x^2*(18
0*A + x*(70*B + x*(40*C + 27*D*x))) + 2*c^2*d^7*x^4*(210*A + x*(147*B + 2*
x*(56*C + 45*D*x)))) + 840*(b*c^2 + a*d^2)^2*(a*d^2*(-2*c*C*d + B*d^2 + 3*
c^2*D) + b*c*(-8*c^2*C*d + 7*B*c*d^2 - 6*A*d^3 + 9*c^3*D))*(c + d*x)*Log[c
 + d*x])/(840*d^10*(c + d*x))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{bx^2\left(3a^2d^4(Cd-2cD)+3abd^2\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)+b^2c^2\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^3\left(3a^2d^4(Cd-2cD)+3abd^2\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)+b^2c^2\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{3d^7}+$$
$$\frac{bx^4\left(3a^2d^4D-3abd^2\left(-Bd^2-3c^2D+2cCd\right)-b^2c\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{4d^6}+$$
$$\frac{x^2\left(a^3d^6D-3a^2bd^4\left(-Bd^2-3c^2D+2cCd\right)-3ab^2cd^2\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)-b^3c^3\left(4Ad^3-5Bcd^2\right.\right.}{2d^8}$$
$$\frac{x\left(a^3d^6(Cd-2cD)+3a^2bd^4\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)+3ab^2c^2d^2\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)+b^3c\right.}{d^9}$$
$$\frac{b^2x^5\left(3ad^2(Cd-2cD)+b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{5d^5}-$$
$$\frac{\left(ad^2+bc^2\right)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^{10}(c+dx)}-$$
$$\frac{\left(ad^2+bc^2\right)^2\log(c+dx)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(6Ad^3-7Bcd^2-9c^3D+8c^2Cd\right)\right)}{d^{10}}+$$
$$\frac{b^2x^6\left(3ad^2D-b\left(-Bd^2-3c^2D+2cCd\right)\right)}{6d^4}+\frac{b^3x^7(Cd-2cD)}{7d^3}+\frac{b^3Dx^8}{8d^2}$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((a^3*d^6*(C*d - 2*c*D) + b^3*c^4*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3
*D) + 3*a*b^2*c^2*d^2*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3*D) + 3*a^2*
b*d^4*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x)/d^9 + ((a^3*d^6*D - 3*
a^2*b*d^4*(2*c*C*d - B*d^2 - 3*c^2*D) - b^3*c^3*(6*c^2*C*d - 5*B*c*d^2 + 4
*A*d^3 - 7*c^3*D) - 3*a*b^2*c*d^2*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5*c^3
*D))*x^2)/(2*d^8) + (b*(3*a^2*d^4*(C*d - 2*c*D) + b^2*c^2*(5*c^2*C*d - 4*B
*c*d^2 + 3*A*d^3 - 6*c^3*D) + 3*a*b*d^2*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4
*c^3*D))*x^3)/(3*d^7) + (b*(3*a^2*d^4*D - 3*a*b*d^2*(2*c*C*d - B*d^2 - 3*c
^2*D) - b^2*c*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5*c^3*D))*x^4)/(4*d^6) +
(b^2*(3*a*d^2*(C*d - 2*c*D) + b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))
*x^5)/(5*d^5) + (b^2*(3*a*d^2*D - b*(2*c*C*d - B*d^2 - 3*c^2*D))*x^6)/(6*d
^4) + (b^3*(C*d - 2*c*D)*x^7)/(7*d^3) + (b^3*D*x^8)/(8*d^2) - ((b*c^2 + a*
d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^10*(c + d*x)) - ((b*c^2 + a
*d^2)^2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(8*c^2*C*d - 7*B*c*d^2 +
6*A*d^3 - 9*c^3*D))*Log[c + d*x])/d^10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.52

method	result	size
norman	Expression too large to display	1039
default	Expression too large to display	1170
parallelrisch	Expression too large to display	1576

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^3*d^9+6*A*a^2*b*c^2*d^7+12*A*a*b^2*c^4*d^5+6*A*b^3*c^6*d^3-B*a^3*c*d
^8-9*B*a^2*b*c^3*d^6-15*B*a*b^2*c^5*d^4-7*B*b^3*c^7*d^2+2*C*a^3*c^2*d^7+12
*C*a^2*b*c^4*d^5+18*C*a*b^2*c^6*d^3+8*C*b^3*c^8*d-3*D*a^3*c^3*d^6-15*D*a^2
*b*c^5*d^4-21*D*a*b^2*c^7*d^2-9*D*b^3*c^9)/d^9/c*x-1/6*(12*A*a*b^2*c*d^5+6
*A*b^3*c^3*d^3-9*B*a^2*b*d^6-15*B*a*b^2*c^2*d^4-7*B*b^3*c^4*d^2+12*C*a^2*b
*c*d^5+18*C*a*b^2*c^3*d^3+8*C*b^3*c^5*d-3*D*a^3*d^6-15*D*a^2*b*c^2*d^4-21*
D*a*b^2*c^4*d^2-9*D*b^3*c^6)/d^7*x^3+1/2*(6*A*a^2*b*d^7+12*A*a*b^2*c^2*d^5
+6*A*b^3*c^4*d^3-9*B*a^2*b*c*d^6-15*B*a*b^2*c^3*d^4-7*B*b^3*c^5*d^2+2*C*a^
3*d^7+12*C*a^2*b*c^2*d^5+18*C*a*b^2*c^4*d^3+8*C*b^3*c^6*d-3*D*a^3*c*d^6-15
*D*a^2*b*c^3*d^4-21*D*a*b^2*c^5*d^2-9*D*b^3*c^7)/d^8*x^2+1/8*D*b^3/d*x^9-1
/20*b*(6*A*b^2*c*d^3-15*B*a*b*d^4-7*B*b^2*c^2*d^2+18*C*a*b*c*d^3+8*C*b^2*c
^3*d-15*D*a^2*d^4-21*D*a*b*c^2*d^2-9*D*b^2*c^4)/d^5*x^5+1/12*b*(12*A*a*b*d
^5+6*A*b^2*c^2*d^3-15*B*a*b*c*d^4-7*B*b^2*c^3*d^2+12*C*a^2*d^5+18*C*a*b*c^
2*d^3+8*C*b^2*c^4*d-15*D*a^2*c*d^4-21*D*a*b*c^3*d^2-9*D*b^2*c^5)/d^6*x^4+1
/42*b^2*(7*B*b*d^2-8*C*b*c*d+21*D*a*d^2+9*D*b*c^2)/d^3*x^7+1/30*b^2*(6*A*b
*d^3-7*B*b*c*d^2+18*C*a*d^3+8*C*b*c^2*d-21*D*a*c*d^2-9*D*b*c^3)/d^4*x^6+1/
56*b^3*(8*C*d-9*D*c)/d^2*x^8)/(d*x+c)-(6*A*a^2*b*c*d^7+12*A*a*b^2*c^3*d^5+
6*A*b^3*c^5*d^3-B*a^3*d^8-9*B*a^2*b*c^2*d^6-15*B*a*b^2*c^4*d^4-7*B*b^3*c^6
*d^2+2*C*a^3*c*d^7+12*C*a^2*b*c^3*d^5+18*C*a*b^2*c^5*d^3+8*C*b^3*c^7*d-3*D
*a^3*c^2*d^6-15*D*a^2*b*c^4*d^4-21*D*a*b^2*c^6*d^2-9*D*b^3*c^8)/d^10*ln...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 1278, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/840*(105*D*b^3*d^9*x^9 + 840*D*b^3*c^9 - 840*C*b^3*c^8*d + 840*B*a^3*c*d
^8 - 840*A*a^3*d^9 + 840*(3*D*a*b^2 + B*b^3)*c^7*d^2 - 840*(3*C*a*b^2 + A*
b^3)*c^6*d^3 + 2520*(D*a^2*b + B*a*b^2)*c^5*d^4 - 2520*(C*a^2*b + A*a*b^2)
*c^4*d^5 + 840*(D*a^3 + 3*B*a^2*b)*c^3*d^6 - 840*(C*a^3 + 3*A*a^2*b)*c^2*d
^7 - 15*(9*D*b^3*c*d^8 - 8*C*b^3*d^9)*x^8 + 20*(9*D*b^3*c^2*d^7 - 8*C*b^3*
c*d^8 + 7*(3*D*a*b^2 + B*b^3)*d^9)*x^7 - 28*(9*D*b^3*c^3*d^6 - 8*C*b^3*c^2
*d^7 + 7*(3*D*a*b^2 + B*b^3)*c*d^8 - 6*(3*C*a*b^2 + A*b^3)*d^9)*x^6 + 42*(
9*D*b^3*c^4*d^5 - 8*C*b^3*c^3*d^6 + 7*(3*D*a*b^2 + B*b^3)*c^2*d^7 - 6*(3*C
*a*b^2 + A*b^3)*c*d^8 + 15*(D*a^2*b + B*a*b^2)*d^9)*x^5 - 70*(9*D*b^3*c^5*
d^4 - 8*C*b^3*c^4*d^5 + 7*(3*D*a*b^2 + B*b^3)*c^3*d^6 - 6*(3*C*a*b^2 + A*b
^3)*c^2*d^7 + 15*(D*a^2*b + B*a*b^2)*c*d^8 - 12*(C*a^2*b + A*a*b^2)*d^9)*x
^4 + 140*(9*D*b^3*c^6*d^3 - 8*C*b^3*c^5*d^4 + 7*(3*D*a*b^2 + B*b^3)*c^4*d^
5 - 6*(3*C*a*b^2 + A*b^3)*c^3*d^6 + 15*(D*a^2*b + B*a*b^2)*c^2*d^7 - 12*(C
*a^2*b + A*a*b^2)*c*d^8 + 3*(D*a^3 + 3*B*a^2*b)*d^9)*x^3 - 420*(9*D*b^3*c^
7*d^2 - 8*C*b^3*c^6*d^3 + 7*(3*D*a*b^2 + B*b^3)*c^5*d^4 - 6*(3*C*a*b^2 + A
*b^3)*c^4*d^5 + 15*(D*a^2*b + B*a*b^2)*c^3*d^6 - 12*(C*a^2*b + A*a*b^2)*c^
2*d^7 + 3*(D*a^3 + 3*B*a^2*b)*c*d^8 - 2*(C*a^3 + 3*A*a^2*b)*d^9)*x^2 - 840
*(8*D*b^3*c^8*d - 7*C*b^3*c^7*d^2 + 6*(3*D*a*b^2 + B*b^3)*c^6*d^3 - 5*(3*C
*a*b^2 + A*b^3)*c^5*d^4 + 12*(D*a^2*b + B*a*b^2)*c^4*d^5 - 9*(C*a^2*b + A*
a*b^2)*c^3*d^6 + 2*(D*a^3 + 3*B*a^2*b)*c^2*d^7 - (C*a^3 + 3*A*a^2*b)*c*...
```

Sympy [A] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*b**3*x**8/(8*d**2) + x**7*(C*b**3/(7*d**2) - 2*D*b**3*c/(7*d**3)) + x**6
*(B*b**3/(6*d**2) - C*b**3*c/(3*d**3) + D*a*b**2/(2*d**2) + D*b**3*c**2/(2
*d**4)) + x**5*(A*b**3/(5*d**2) - 2*B*b**3*c/(5*d**3) + 3*C*a*b**2/(5*d**2
) + 3*C*b**3*c**2/(5*d**4) - 6*D*a*b**2*c/(5*d**3) - 4*D*b**3*c**3/(5*d**5
)) + x**4*(-A*b**3*c/(2*d**3) + 3*B*a*b**2/(4*d**2) + 3*B*b**3*c**2/(4*d**
4) - 3*C*a*b**2*c/(2*d**3) - C*b**3*c**3/d**5 + 3*D*a**2*b/(4*d**2) + 9*D*
a*b**2*c**2/(4*d**4) + 5*D*b**3*c**4/(4*d**6)) + x**3*(A*a*b**2/d**2 + A*b
**3*c**2/d**4 - 2*B*a*b**2*c/d**3 - 4*B*b**3*c**3/(3*d**5) + C*a**2*b/d**2
 + 3*C*a*b**2*c**2/d**4 + 5*C*b**3*c**4/(3*d**6) - 2*D*a**2*b*c/d**3 - 4*D
*a*b**2*c**3/d**5 - 2*D*b**3*c**5/d**7) + x**2*(-3*A*a*b**2*c/d**3 - 2*A*b
**3*c**3/d**5 + 3*B*a**2*b/(2*d**2) + 9*B*a*b**2*c**2/(2*d**4) + 5*B*b**3*
c**4/(2*d**6) - 3*C*a**2*b*c/d**3 - 6*C*a*b**2*c**3/d**5 - 3*C*b**3*c**5/d
**7 + D*a**3/(2*d**2) + 9*D*a**2*b*c**2/(2*d**4) + 15*D*a*b**2*c**4/(2*d**
6) + 7*D*b**3*c**6/(2*d**8)) + x*(3*A*a**2*b/d**2 + 9*A*a*b**2*c**2/d**4 +
 5*A*b**3*c**4/d**6 - 6*B*a**2*b*c/d**3 - 12*B*a*b**2*c**3/d**5 - 6*B*b**3
*c**5/d**7 + C*a**3/d**2 + 9*C*a**2*b*c**2/d**4 + 15*C*a*b**2*c**4/d**6 +
7*C*b**3*c**6/d**8 - 2*D*a**3*c/d**3 - 12*D*a**2*b*c**3/d**5 - 18*D*a*b**2
*c**5/d**7 - 8*D*b**3*c**7/d**9) + (-A*a**3*d**9 - 3*A*a**2*b*c**2*d**7 -
3*A*a*b**2*c**4*d**5 - A*b**3*c**6*d**3 + B*a**3*c*d**8 + 3*B*a**2*b*c**3*
d**6 + 3*B*a*b**2*c**5*d**4 + B*b**3*c**7*d**2 - C*a**3*c**2*d**7 - 3*C...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(D*b^3*c^9 - C*b^3*c^8*d + B*a^3*c*d^8 - A*a^3*d^9 + (3*D*a*b^2 + B*b^3)*c
^7*d^2 - (3*C*a*b^2 + A*b^3)*c^6*d^3 + 3*(D*a^2*b + B*a*b^2)*c^5*d^4 - 3*(
C*a^2*b + A*a*b^2)*c^4*d^5 + (D*a^3 + 3*B*a^2*b)*c^3*d^6 - (C*a^3 + 3*A*a^
2*b)*c^2*d^7)/(d^11*x + c*d^10) + 1/840*(105*D*b^3*d^7*x^8 - 120*(2*D*b^3*
c*d^6 - C*b^3*d^7)*x^7 + 140*(3*D*b^3*c^2*d^5 - 2*C*b^3*c*d^6 + (3*D*a*b^2
 + B*b^3)*d^7)*x^6 - 168*(4*D*b^3*c^3*d^4 - 3*C*b^3*c^2*d^5 + 2*(3*D*a*b^2
 + B*b^3)*c*d^6 - (3*C*a*b^2 + A*b^3)*d^7)*x^5 + 210*(5*D*b^3*c^4*d^3 - 4*
C*b^3*c^3*d^4 + 3*(3*D*a*b^2 + B*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + A*b^3)*c*d^
6 + 3*(D*a^2*b + B*a*b^2)*d^7)*x^4 - 280*(6*D*b^3*c^5*d^2 - 5*C*b^3*c^4*d^
3 + 4*(3*D*a*b^2 + B*b^3)*c^3*d^4 - 3*(3*C*a*b^2 + A*b^3)*c^2*d^5 + 6*(D*a
^2*b + B*a*b^2)*c*d^6 - 3*(C*a^2*b + A*a*b^2)*d^7)*x^3 + 420*(7*D*b^3*c^6*
d - 6*C*b^3*c^5*d^2 + 5*(3*D*a*b^2 + B*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + A*b^3
)*c^3*d^4 + 9*(D*a^2*b + B*a*b^2)*c^2*d^5 - 6*(C*a^2*b + A*a*b^2)*c*d^6 +
(D*a^3 + 3*B*a^2*b)*d^7)*x^2 - 840*(8*D*b^3*c^7 - 7*C*b^3*c^6*d + 6*(3*D*a
*b^2 + B*b^3)*c^5*d^2 - 5*(3*C*a*b^2 + A*b^3)*c^4*d^3 + 12*(D*a^2*b + B*a*
b^2)*c^3*d^4 - 9*(C*a^2*b + A*a*b^2)*c^2*d^5 + 2*(D*a^3 + 3*B*a^2*b)*c*d^6
 - (C*a^3 + 3*A*a^2*b)*d^7)*x)/d^9 + (9*D*b^3*c^8 - 8*C*b^3*c^7*d + B*a^3*
d^8 + 7*(3*D*a*b^2 + B*b^3)*c^6*d^2 - 6*(3*C*a*b^2 + A*b^3)*c^5*d^3 + 15*(
D*a^2*b + B*a*b^2)*c^4*d^4 - 12*(C*a^2*b + A*a*b^2)*c^3*d^5 + 3*(D*a^3 + 3
*B*a^2*b)*c^2*d^6 - 2*(C*a^3 + 3*A*a^2*b)*c*d^7)*log(d*x + c)/d^10
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1252, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input

```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/840*(105*D*b^3 - 120*(9*D*b^3*c*d - C*b^3*d^2)/((d*x + c)*d) + 140*(36*D
*b^3*c^2*d^2 - 8*C*b^3*c*d^3 + 3*D*a*b^2*d^4 + B*b^3*d^4)/((d*x + c)^2*d^2
) - 168*(84*D*b^3*c^3*d^3 - 28*C*b^3*c^2*d^4 + 21*D*a*b^2*c*d^5 + 7*B*b^3*
c*d^5 - 3*C*a*b^2*d^6 - A*b^3*d^6)/((d*x + c)^3*d^3) + 210*(126*D*b^3*c^4*
d^4 - 56*C*b^3*c^3*d^5 + 63*D*a*b^2*c^2*d^6 + 21*B*b^3*c^2*d^6 - 18*C*a*b^
2*c*d^7 - 6*A*b^3*c*d^7 + 3*D*a^2*b*d^8 + 3*B*a*b^2*d^8)/((d*x + c)^4*d^4)
 - 280*(126*D*b^3*c^5*d^5 - 70*C*b^3*c^4*d^6 + 105*D*a*b^2*c^3*d^7 + 35*B*
b^3*c^3*d^7 - 45*C*a*b^2*c^2*d^8 - 15*A*b^3*c^2*d^8 + 15*D*a^2*b*c*d^9 + 1
5*B*a*b^2*c*d^9 - 3*C*a^2*b*d^10 - 3*A*a*b^2*d^10)/((d*x + c)^5*d^5) + 420
*(84*D*b^3*c^6*d^6 - 56*C*b^3*c^5*d^7 + 105*D*a*b^2*c^4*d^8 + 35*B*b^3*c^4
*d^8 - 60*C*a*b^2*c^3*d^9 - 20*A*b^3*c^3*d^9 + 30*D*a^2*b*c^2*d^10 + 30*B*
a*b^2*c^2*d^10 - 12*C*a^2*b*c*d^11 - 12*A*a*b^2*c*d^11 + D*a^3*d^12 + 3*B*
a^2*b*d^12)/((d*x + c)^6*d^6) - 840*(36*D*b^3*c^7*d^7 - 28*C*b^3*c^6*d^8 +
 63*D*a*b^2*c^5*d^9 + 21*B*b^3*c^5*d^9 - 45*C*a*b^2*c^4*d^10 - 15*A*b^3*c^
4*d^10 + 30*D*a^2*b*c^3*d^11 + 30*B*a*b^2*c^3*d^11 - 18*C*a^2*b*c^2*d^12 -
 18*A*a*b^2*c^2*d^12 + 3*D*a^3*c*d^13 + 9*B*a^2*b*c*d^13 - C*a^3*d^14 - 3*
A*a^2*b*d^14)/((d*x + c)^7*d^7))*(d*x + c)^8/d^10 - (9*D*b^3*c^8 - 8*C*b^3
*c^7*d + 21*D*a*b^2*c^6*d^2 + 7*B*b^3*c^6*d^2 - 18*C*a*b^2*c^5*d^3 - 6*A*b
^3*c^5*d^3 + 15*D*a^2*b*c^4*d^4 + 15*B*a*b^2*c^4*d^4 - 12*C*a^2*b*c^3*d^5
- 12*A*a*b^2*c^3*d^5 + 3*D*a^3*c^2*d^6 + 9*B*a^2*b*c^2*d^6 - 2*C*a^3*c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx = \int \frac{(b\,x^2+a)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^2} \, dx$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx = \int \frac{(b\,x^2+a)^3 (Dx^3+C\,x^2+Bx+A)}{(dx+c)^2} dx$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

3.21 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 655

$$\begin{aligned}
&\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx \\
&= \frac{(a^3d^6D - 3a^2bd^4(3cCd - Bd^2 - 6c^2D) - b^3c^3(21c^2Cd - 15Bcd^2 + 10Ad^3 - 28c^3D) - 3ab^2cd^2(10c^2Cd}{d^9} \\
&\quad + \frac{b(3a^2d^4(Cd - 3cD) + b^2c^2(15c^2Cd - 10Bcd^2 + 6Ad^3 - 21c^3D) + 3abd^2(6c^2Cd - 3Bcd^2 + Ad^3 - 10c}{2d^8} \\
&\quad + \frac{b(3a^2d^4D - 3abd^2(3cCd - Bd^2 - 6c^2D) - b^2c(10c^2Cd - 6Bcd^2 + 3Ad^3 - 15c^3D))\,x^3}{3d^7} \\
&\quad + \frac{b^2(3ad^2(Cd - 3cD) + b(6c^2Cd - 3Bcd^2 + Ad^3 - 10c^3D))\,x^4}{4d^6} \\
&\quad + \frac{b^2(3ad^2D - b(3cCd - Bd^2 - 6c^2D))\,x^5}{5d^5} + \frac{b^3(Cd - 3cD)x^6}{6d^4} \\
&\quad + \frac{b^3Dx^7}{7d^3} - \frac{(bc^2+ad^2)^3\,(c^2Cd - Bcd^2 + Ad^3 - c^3D)}{2d^{10}(c+dx)^2} \\
&\quad + \frac{(bc^2+ad^2)^2\,(ad^2(2cCd - Bd^2 - 3c^2D) + bc(8c^2Cd - 7Bcd^2 + 6Ad^3 - 9c^3D))}{d^{10}(c+dx)} \\
&\quad + \frac{(bc^2+ad^2)\,(a^2d^4(Cd - 3cD) + b^2c^2(28c^2Cd - 21Bcd^2 + 15Ad^3 - 36c^3D) + abd^2(17c^2Cd - 9Bcd^2 +}{d^{10}}
\end{aligned}$$

output
```
(a^3*d^6*D-3*a^2*b*d^4*(-B*d^2+3*C*c*d-6*D*c^2)-b^3*c^3*(10*A*d^3-15*B*c*d
^2+21*C*c^2*d-28*D*c^3)-3*a*b^2*c*d^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c
^3))*x/d^9+1/2*b*(3*a^2*d^4*(C*d-3*D*c)+b^2*c^2*(6*A*d^3-10*B*c*d^2+15*C*c
^2*d-21*D*c^3)+3*a*b*d^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*x^2/d^8+1/3
*b*(3*a^2*d^4*D-3*a*b*d^2*(-B*d^2+3*C*c*d-6*D*c^2)-b^2*c*(3*A*d^3-6*B*c*d^
2+10*C*c^2*d-15*D*c^3))*x^3/d^7+1/4*b^2*(3*a*d^2*(C*d-3*D*c)+b*(A*d^3-3*B*
c*d^2+6*C*c^2*d-10*D*c^3))*x^4/d^6+1/5*b^2*(3*a*d^2*D-b*(-B*d^2+3*C*c*d-6*
D*c^2))*x^5/d^5+1/6*b^3*(C*d-3*D*c)*x^6/d^4+1/7*b^3*D*x^7/d^3-1/2*(a*d^2+b
*c^2)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^10/(d*x+c)^2+(a*d^2+b*c^2)^2*(a*d^
2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(6*A*d^3-7*B*c*d^2+8*C*c^2*d-9*D*c^3))/d^10
/(d*x+c)+(a*d^2+b*c^2)*(a^2*d^4*(C*d-3*D*c)+b^2*c^2*(15*A*d^3-21*B*c*d^2+2
8*C*c^2*d-36*D*c^3)+a*b*d^2*(3*A*d^3-9*B*c*d^2+17*C*c^2*d-27*D*c^3))*ln(d*
x+c)/d^10
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 634, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

$$= \frac{420d(a^3d^6D+3a^2bd^4(-3cCd+Bd^2+6c^2D)+3ab^2cd^2(-10c^2Cd+6Bcd^2-3Ad^3+15c^3D)+b^3c^3(-\cdots}{\cdots}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(420*d*(a^3*d^6*D + 3*a^2*b*d^4*(-3*c*C*d + B*d^2 + 6*c^2*D) + 3*a*b^2*c*d
^2*(-10*c^2*C*d + 6*B*c*d^2 - 3*A*d^3 + 15*c^3*D) + b^3*c^3*(-21*c^2*C*d +
 15*B*c*d^2 - 10*A*d^3 + 28*c^3*D))*x + 210*b*d^2*(3*a^2*d^4*(C*d - 3*c*D)
 + b^2*c^2*(15*c^2*C*d - 10*B*c*d^2 + 6*A*d^3 - 21*c^3*D) + 3*a*b*d^2*(6*c
^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x^2 + 140*b*d^3*(3*a^2*d^4*D + 3*a
*b*d^2*(-3*c*C*d + B*d^2 + 6*c^2*D) + b^2*c*(-10*c^2*C*d + 6*B*c*d^2 - 3*A
*d^3 + 15*c^3*D))*x^3 + 105*b^2*d^4*(3*a*d^2*(C*d - 3*c*D) + b*(6*c^2*C*d
- 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x^4 + 84*b^2*d^5*(3*a*d^2*D + b*(-3*c*C*d
 + B*d^2 + 6*c^2*D))*x^5 + 70*b^3*d^6*(C*d - 3*c*D)*x^6 + 60*b^3*d^7*D*x^7
 + (210*(b*c^2 + a*d^2)^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x
)^2 - (420*(b*c^2 + a*d^2)^2*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-8
*c^2*C*d + 7*B*c*d^2 - 6*A*d^3 + 9*c^3*D)))/(c + d*x) - 420*(b*c^2 + a*d^2
)*(a^2*d^4*(-(C*d) + 3*c*D) + a*b*d^2*(-17*c^2*C*d + 9*B*c*d^2 - 3*A*d^3 +
 27*c^3*D) + b^2*c^2*(-28*c^2*C*d + 21*B*c*d^2 - 15*A*d^3 + 36*c^3*D))*Log
[c + d*x])/(420*d^10)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{(ad^2+bc^2)\left(a^2d^4(Cd-3cD)+abd^2\left(3Ad^3-9Bcd^2-27c^3D+17c^2Cd\right)+b^2c^2\left(15Ad^3-21Bcd^2-36c^3D\right.\right.}{d^9(c+dx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ad^2+bc^2)\log(c+dx)\left(a^2d^4(Cd-3cD)+abd^2\left(3Ad^3-9Bcd^2-27c^3D+17c^2Cd\right)+b^2c^2\left(15Ad^3-21Bcd^2-\right.\right.}{d^{10}}$$
$$\frac{bx^2\left(3a^2d^4(Cd-3cD)+3abd^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)+b^2c^2\left(6Ad^3-10Bcd^2-21c^3D+15c^2Cd\right)\right)}{2d^8}+$$
$$\frac{bx^3\left(3a^2d^4D-3abd^2\left(-Bd^2-6c^2D+3cCd\right)-b^2c\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)}{3d^7}+$$
$$\frac{x\left(a^3d^6D-3a^2bd^4\left(-Bd^2-6c^2D+3cCd\right)-3ab^2cd^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)-b^3c^3\left(10Ad^3-15Bc\right.\right.}{d^9}$$
$$\frac{b^2x^4\left(3ad^2(Cd-3cD)+b\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{4d^6}+$$
$$\frac{\left(ad^2+bc^2\right)^2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(6Ad^3-7Bcd^2-9c^3D+8c^2Cd\right)\right)}{d^{10}(c+dx)}-$$
$$\frac{\left(ad^2+bc^2\right)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^{10}(c+dx)^2}+\frac{b^2x^5\left(3ad^2D-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{5d^5}+$$
$$\frac{b^3x^6(Cd-3cD)}{6d^4}+\frac{b^3Dx^7}{7d^3}$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((a^3*d^6*D - 3*a^2*b*d^4*(3*c*C*d - B*d^2 - 6*c^2*D) - b^3*c^3*(21*c^2*C*
d - 15*B*c*d^2 + 10*A*d^3 - 28*c^3*D) - 3*a*b^2*c*d^2*(10*c^2*C*d - 6*B*c*
d^2 + 3*A*d^3 - 15*c^3*D))*x)/d^9 + (b*(3*a^2*d^4*(C*d - 3*c*D) + b^2*c^2*
(15*c^2*C*d - 10*B*c*d^2 + 6*A*d^3 - 21*c^3*D) + 3*a*b*d^2*(6*c^2*C*d - 3*
B*c*d^2 + A*d^3 - 10*c^3*D))*x^2)/(2*d^8) + (b*(3*a^2*d^4*D - 3*a*b*d^2*(3
*c*C*d - B*d^2 - 6*c^2*D) - b^2*c*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c
^3*D))*x^3)/(3*d^7) + (b^2*(3*a*d^2*(C*d - 3*c*D) + b*(6*c^2*C*d - 3*B*c*d
^2 + A*d^3 - 10*c^3*D))*x^4)/(4*d^6) + (b^2*(3*a*d^2*D - b*(3*c*C*d - B*d^
2 - 6*c^2*D))*x^5)/(5*d^5) + (b^3*(C*d - 3*c*D)*x^6)/(6*d^4) + (b^3*D*x^7)
/(7*d^3) - ((b*c^2 + a*d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(2*d^10
*(c + d*x)^2) + ((b*c^2 + a*d^2)^2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*
c*(8*c^2*C*d - 7*B*c*d^2 + 6*A*d^3 - 9*c^3*D)))/(d^10*(c + d*x)) + ((b*c^2
 + a*d^2)*(a^2*d^4*(C*d - 3*c*D) + b^2*c^2*(28*c^2*C*d - 21*B*c*d^2 + 15*A
*d^3 - 36*c^3*D) + a*b*d^2*(17*c^2*C*d - 9*B*c*d^2 + 3*A*d^3 - 27*c^3*D))*
Log[c + d*x])/d^10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 1030, normalized size of antiderivative = 1.57

method	result	size
norman	Expression too large to display	1030
default	Expression too large to display	1143
parallelrisch	Expression too large to display	1822

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
((6*A*a^2*b*c*d^7+36*A*a*b^2*c^3*d^5+30*A*b^3*c^5*d^3-B*a^3*d^8-18*B*a^2*b
*c^2*d^6-60*B*a*b^2*c^4*d^4-42*B*b^3*c^6*d^2+2*C*a^3*c*d^7+36*C*a^2*b*c^3*
d^5+90*C*a*b^2*c^5*d^3+56*C*b^3*c^7*d-6*D*a^3*c^2*d^6-60*D*a^2*b*c^4*d^4-1
26*D*a*b^2*c^6*d^2-72*D*b^3*c^8)/d^9*x-1/2*(A*a^3*d^9-9*A*a^2*b*c^2*d^7-54
*A*a*b^2*c^4*d^5-45*A*b^3*c^6*d^3+B*a^3*c*d^8+27*B*a^2*b*c^3*d^6+90*B*a*b^
2*c^5*d^4+63*B*b^3*c^7*d^2-3*C*a^3*c^2*d^7-54*C*a^2*b*c^4*d^5-135*C*a*b^2*
c^6*d^3-84*C*b^3*c^8*d+9*D*a^3*c^3*d^6+90*D*a^2*b*c^5*d^4+189*D*a*b^2*c^7*
d^2+108*D*b^3*c^9)/d^10-1/3*(18*A*a*b^2*c*d^5+15*A*b^3*c^3*d^3-9*B*a^2*b*d
^6-30*B*a*b^2*c^2*d^4-21*B*b^3*c^4*d^2+18*C*a^2*b*c*d^5+45*C*a*b^2*c^3*d^3
+28*C*b^3*c^5*d-3*D*a^3*d^6-30*D*a^2*b*c^2*d^4-63*D*a*b^2*c^4*d^2-36*D*b^3
*c^6)/d^7*x^3+1/7*D*b^3/d*x^9-1/30*b*(15*A*b^2*c*d^3-30*B*a*b*d^4-21*B*b^2
*c^2*d^2+45*C*a*b*c*d^3+28*C*b^2*c^3*d-30*D*a^2*d^4-63*D*a*b*c^2*d^2-36*D*
b^2*c^4)/d^5*x^5+1/12*b*(18*A*a*b*d^5+15*A*b^2*c^2*d^3-30*B*a*b*c*d^4-21*B
*b^2*c^3*d^2+18*C*a^2*d^5+45*C*a*b*c^2*d^3+28*C*b^2*c^4*d-30*D*a^2*c*d^4-6
3*D*a*b*c^3*d^2-36*D*b^2*c^5)/d^6*x^4+1/105*b^2*(21*B*b*d^2-28*C*b*c*d+63*
D*a*d^2+36*D*b*c^2)/d^3*x^7+1/60*b^2*(15*A*b*d^3-21*B*b*c*d^2+45*C*a*d^3+2
8*C*b*c^2*d-63*D*a*c*d^2-36*D*b*c^3)/d^4*x^6+1/42*b^3*(7*C*d-9*D*c)/d^2*x^
8)/(d*x+c)^2+1/d^10*(3*A*a^2*b*d^7+18*A*a*b^2*c^2*d^5+15*A*b^3*c^4*d^3-9*B
*a^2*b*c*d^6-30*B*a*b^2*c^3*d^4-21*B*b^3*c^5*d^2+C*a^3*d^7+18*C*a^2*b*c^2*
d^5+45*C*a*b^2*c^4*d^3+28*C*b^3*c^6*d-3*D*a^3*c*d^6-30*D*a^2*b*c^3*d^4-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(642) = 1284$.

Time = 0.09 (sec) , antiderivative size = 1414, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/420*(60*D*b^3*d^9*x^9 - 3570*D*b^3*c^9 + 3150*C*b^3*c^8*d - 210*B*a^3*c*
d^8 - 210*A*a^3*d^9 - 2730*(3*D*a*b^2 + B*b^3)*c^7*d^2 + 2310*(3*C*a*b^2 +
 A*b^3)*c^6*d^3 - 5670*(D*a^2*b + B*a*b^2)*c^5*d^4 + 4410*(C*a^2*b + A*a*b
^2)*c^4*d^5 - 1050*(D*a^3 + 3*B*a^2*b)*c^3*d^6 + 630*(C*a^3 + 3*A*a^2*b)*c
^2*d^7 - 10*(9*D*b^3*c*d^8 - 7*C*b^3*d^9)*x^8 + 4*(36*D*b^3*c^2*d^7 - 28*C
*b^3*c*d^8 + 21*(3*D*a*b^2 + B*b^3)*d^9)*x^7 - 7*(36*D*b^3*c^3*d^6 - 28*C*
b^3*c^2*d^7 + 21*(3*D*a*b^2 + B*b^3)*c*d^8 - 15*(3*C*a*b^2 + A*b^3)*d^9)*x
^6 + 14*(36*D*b^3*c^4*d^5 - 28*C*b^3*c^3*d^6 + 21*(3*D*a*b^2 + B*b^3)*c^2*
d^7 - 15*(3*C*a*b^2 + A*b^3)*c*d^8 + 30*(D*a^2*b + B*a*b^2)*d^9)*x^5 - 35*
(36*D*b^3*c^5*d^4 - 28*C*b^3*c^4*d^5 + 21*(3*D*a*b^2 + B*b^3)*c^3*d^6 - 15
*(3*C*a*b^2 + A*b^3)*c^2*d^7 + 30*(D*a^2*b + B*a*b^2)*c*d^8 - 18*(C*a^2*b
+ A*a*b^2)*d^9)*x^4 + 140*(36*D*b^3*c^6*d^3 - 28*C*b^3*c^5*d^4 + 21*(3*D*a
*b^2 + B*b^3)*c^4*d^5 - 15*(3*C*a*b^2 + A*b^3)*c^3*d^6 + 30*(D*a^2*b + B*a
*b^2)*c^2*d^7 - 18*(C*a^2*b + A*a*b^2)*c*d^8 + 3*(D*a^3 + 3*B*a^2*b)*d^9)*
x^3 + 210*(91*D*b^3*c^7*d^2 - 69*C*b^3*c^6*d^3 + 50*(3*D*a*b^2 + B*b^3)*c^
5*d^4 - 34*(3*C*a*b^2 + A*b^3)*c^4*d^5 + 63*(D*a^2*b + B*a*b^2)*c^3*d^6 -
33*(C*a^2*b + A*a*b^2)*c^2*d^7 + 4*(D*a^3 + 3*B*a^2*b)*c*d^8)*x^2 + 420*(1
9*D*b^3*c^8*d - 13*C*b^3*c^7*d^2 - B*a^3*d^9 + 8*(3*D*a*b^2 + B*b^3)*c^6*d
^3 - 4*(3*C*a*b^2 + A*b^3)*c^5*d^4 + 3*(D*a^2*b + B*a*b^2)*c^4*d^5 + 3*(C*
a^2*b + A*a*b^2)*c^3*d^6 - 2*(D*a^3 + 3*B*a^2*b)*c^2*d^7 + 2*(C*a^3 + 3...
```

Sympy [A] (verification not implemented)

Time = 32.61 (sec) , antiderivative size = 1193, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b**3*x**7/(7*d**3) + x**6*(C*b**3/(6*d**3) - D*b**3*c/(2*d**4)) + x**5*(
B*b**3/(5*d**3) - 3*C*b**3*c/(5*d**4) + 3*D*a*b**2/(5*d**3) + 6*D*b**3*c**
2/(5*d**5)) + x**4*(A*b**3/(4*d**3) - 3*B*b**3*c/(4*d**4) + 3*C*a*b**2/(4*
d**3) + 3*C*b**3*c**2/(2*d**5) - 9*D*a*b**2*c/(4*d**4) - 5*D*b**3*c**3/(2*
d**6)) + x**3*(-A*b**3*c/d**4 + B*a*b**2/d**3 + 2*B*b**3*c**2/d**5 - 3*C*a
*b**2*c/d**4 - 10*C*b**3*c**3/(3*d**6) + D*a**2*b/d**3 + 6*D*a*b**2*c**2/d
**5 + 5*D*b**3*c**4/d**7) + x**2*(3*A*a*b**2/(2*d**3) + 3*A*b**3*c**2/d**5
 - 9*B*a*b**2*c/(2*d**4) - 5*B*b**3*c**3/d**6 + 3*C*a**2*b/(2*d**3) + 9*C*
a*b**2*c**2/d**5 + 15*C*b**3*c**4/(2*d**7) - 9*D*a**2*b*c/(2*d**4) - 15*D*
a*b**2*c**3/d**6 - 21*D*b**3*c**5/(2*d**8)) + x*(-9*A*a*b**2*c/d**4 - 10*A
*b**3*c**3/d**6 + 3*B*a**2*b/d**3 + 18*B*a*b**2*c**2/d**5 + 15*B*b**3*c**4
/d**7 - 9*C*a**2*b*c/d**4 - 30*C*a*b**2*c**3/d**6 - 21*C*b**3*c**5/d**8 +
D*a**3/d**3 + 18*D*a**2*b*c**2/d**5 + 45*D*a*b**2*c**4/d**7 + 28*D*b**3*c*
*6/d**9) + (-A*a**3*d**9 + 9*A*a**2*b*c**2*d**7 + 21*A*a*b**2*c**4*d**5 +
11*A*b**3*c**6*d**3 - B*a**3*c*d**8 - 15*B*a**2*b*c**3*d**6 - 27*B*a*b**2*
c**5*d**4 - 13*B*b**3*c**7*d**2 + 3*C*a**3*c**2*d**7 + 21*C*a**2*b*c**4*d*
*5 + 33*C*a*b**2*c**6*d**3 + 15*C*b**3*c**8*d - 5*D*a**3*c**3*d**6 - 27*D*
a**2*b*c**5*d**4 - 39*D*a*b**2*c**7*d**2 - 17*D*b**3*c**9 + x*(12*A*a**2*b
*c*d**8 + 24*A*a*b**2*c**3*d**6 + 12*A*b**3*c**5*d**4 - 2*B*a**3*d**9 - 18
*B*a**2*b*c**2*d**7 - 30*B*a*b**2*c**4*d**5 - 14*B*b**3*c**6*d**3 + 4*C...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(17*D*b^3*c^9 - 15*C*b^3*c^8*d + B*a^3*c*d^8 + A*a^3*d^9 + 13*(3*D*a*
b^2 + B*b^3)*c^7*d^2 - 11*(3*C*a*b^2 + A*b^3)*c^6*d^3 + 27*(D*a^2*b + B*a*
b^2)*c^5*d^4 - 21*(C*a^2*b + A*a*b^2)*c^4*d^5 + 5*(D*a^3 + 3*B*a^2*b)*c^3*
d^6 - 3*(C*a^3 + 3*A*a^2*b)*c^2*d^7 + 2*(9*D*b^3*c^8*d - 8*C*b^3*c^7*d^2 +
 B*a^3*d^9 + 7*(3*D*a*b^2 + B*b^3)*c^6*d^3 - 6*(3*C*a*b^2 + A*b^3)*c^5*d^4
 + 15*(D*a^2*b + B*a*b^2)*c^4*d^5 - 12*(C*a^2*b + A*a*b^2)*c^3*d^6 + 3*(D*
a^3 + 3*B*a^2*b)*c^2*d^7 - 2*(C*a^3 + 3*A*a^2*b)*c*d^8)*x)/(d^12*x^2 + 2*c
*d^11*x + c^2*d^10) + 1/420*(60*D*b^3*d^6*x^7 - 70*(3*D*b^3*c*d^5 - C*b^3*
d^6)*x^6 + 84*(6*D*b^3*c^2*d^4 - 3*C*b^3*c*d^5 + (3*D*a*b^2 + B*b^3)*d^6)*
x^5 - 105*(10*D*b^3*c^3*d^3 - 6*C*b^3*c^2*d^4 + 3*(3*D*a*b^2 + B*b^3)*c*d^
5 - (3*C*a*b^2 + A*b^3)*d^6)*x^4 + 140*(15*D*b^3*c^4*d^2 - 10*C*b^3*c^3*d^
3 + 6*(3*D*a*b^2 + B*b^3)*c^2*d^4 - 3*(3*C*a*b^2 + A*b^3)*c*d^5 + 3*(D*a^2
*b + B*a*b^2)*d^6)*x^3 - 210*(21*D*b^3*c^5*d - 15*C*b^3*c^4*d^2 + 10*(3*D*
a*b^2 + B*b^3)*c^3*d^3 - 6*(3*C*a*b^2 + A*b^3)*c^2*d^4 + 9*(D*a^2*b + B*a*
b^2)*c*d^5 - 3*(C*a^2*b + A*a*b^2)*d^6)*x^2 + 420*(28*D*b^3*c^6 - 21*C*b^3
*c^5*d + 15*(3*D*a*b^2 + B*b^3)*c^4*d^2 - 10*(3*C*a*b^2 + A*b^3)*c^3*d^3 +
 18*(D*a^2*b + B*a*b^2)*c^2*d^4 - 9*(C*a^2*b + A*a*b^2)*c*d^5 + (D*a^3 + 3
*B*a^2*b)*d^6)*x)/d^9 - (36*D*b^3*c^7 - 28*C*b^3*c^6*d + 21*(3*D*a*b^2 + B
*b^3)*c^5*d^2 - 15*(3*C*a*b^2 + A*b^3)*c^4*d^3 + 30*(D*a^2*b + B*a*b^2)*c^
3*d^4 - 18*(C*a^2*b + A*a*b^2)*c^2*d^5 + 3*(D*a^3 + 3*B*a^2*b)*c*d^6 - ...
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1149, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(36*D*b^3*c^7 - 28*C*b^3*c^6*d + 63*D*a*b^2*c^5*d^2 + 21*B*b^3*c^5*d^2 -
45*C*a*b^2*c^4*d^3 - 15*A*b^3*c^4*d^3 + 30*D*a^2*b*c^3*d^4 + 30*B*a*b^2*c^
3*d^4 - 18*C*a^2*b*c^2*d^5 - 18*A*a*b^2*c^2*d^5 + 3*D*a^3*c*d^6 + 9*B*a^2*
b*c*d^6 - C*a^3*d^7 - 3*A*a^2*b*d^7)*log(abs(d*x + c))/d^10 - 1/2*(17*D*b^
3*c^9 - 15*C*b^3*c^8*d + 39*D*a*b^2*c^7*d^2 + 13*B*b^3*c^7*d^2 - 33*C*a*b^
2*c^6*d^3 - 11*A*b^3*c^6*d^3 + 27*D*a^2*b*c^5*d^4 + 27*B*a*b^2*c^5*d^4 - 2
1*C*a^2*b*c^4*d^5 - 21*A*a*b^2*c^4*d^5 + 5*D*a^3*c^3*d^6 + 15*B*a^2*b*c^3*
d^6 - 3*C*a^3*c^2*d^7 - 9*A*a^2*b*c^2*d^7 + B*a^3*c*d^8 + A*a^3*d^9 + 2*(9
*D*b^3*c^8*d - 8*C*b^3*c^7*d^2 + 21*D*a*b^2*c^6*d^3 + 7*B*b^3*c^6*d^3 - 18
*C*a*b^2*c^5*d^4 - 6*A*b^3*c^5*d^4 + 15*D*a^2*b*c^4*d^5 + 15*B*a*b^2*c^4*d
^5 - 12*C*a^2*b*c^3*d^6 - 12*A*a*b^2*c^3*d^6 + 3*D*a^3*c^2*d^7 + 9*B*a^2*b
*c^2*d^7 - 2*C*a^3*c*d^8 - 6*A*a^2*b*c*d^8 + B*a^3*d^9)*x)/((d*x + c)^2*d^
10) + 1/420*(60*D*b^3*d^18*x^7 - 210*D*b^3*c*d^17*x^6 + 70*C*b^3*d^18*x^6
+ 504*D*b^3*c^2*d^16*x^5 - 252*C*b^3*c*d^17*x^5 + 252*D*a*b^2*d^18*x^5 + 8
4*B*b^3*d^18*x^5 - 1050*D*b^3*c^3*d^15*x^4 + 630*C*b^3*c^2*d^16*x^4 - 945*
D*a*b^2*c*d^17*x^4 - 315*B*b^3*c*d^17*x^4 + 315*C*a*b^2*d^18*x^4 + 105*A*b
^3*d^18*x^4 + 2100*D*b^3*c^4*d^14*x^3 - 1400*C*b^3*c^3*d^15*x^3 + 2520*D*a
*b^2*c^2*d^16*x^3 + 840*B*b^3*c^2*d^16*x^3 - 1260*C*a*b^2*c*d^17*x^3 - 420
*A*b^3*c*d^17*x^3 + 420*D*a^2*b*d^18*x^3 + 420*B*a*b^2*d^18*x^3 - 4410*D*b
^3*c^5*d^13*x^2 + 3150*C*b^3*c^4*d^14*x^2 - 6300*D*a*b^2*c^3*d^15*x^2 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^3 (Dx^3+C\,x^2+Bx+A)}{(dx+c)^3}dx$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

3.22 $\int \frac{(a+bx^2)(-ac+4b^2x+3bcx^2)}{(b+cx)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ac+4b^2x+3bcx^2)}{(b+cx)^2} dx = \frac{(a+bx^2)^2}{b+cx}$$

output
```
(b*x^2+a)^2/(c*x+b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \frac{(a+bx^2)(-ac+4b^2x+3bcx^2)}{(b+cx)^2} dx = \frac{b^6+2ab^3c^2+a^2c^4+b^5cx+2abc^4x^2+b^2c^3x(2a+cx^3)}{c^4(b+cx)}$$

input
```
Integrate[((a + b*x^2)*(-(a*c) + 4*b^2*x + 3*b*c*x^2))/(b + c*x)^2,x]
```

output
```
(b^6 + 2*a*b^3*c^2 + a^2*c^4 + b^5*c*x + 2*a*b*c^4*x^2 + b^2*c^3*x*(2*a +
c*x^3))/(c^4*(b + c*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\left(-ac+4b^2x+3bcx^2\right)}{(b+cx)^2}dx$$
$$\Big\downarrow \text{2023}$$
$$\frac{\left(a+bx^2\right)^2}{b+cx}$$

input
```
Int[((a + b*x^2)*(-(a*c) + 4*b^2*x + 3*b*c*x^2))/(b + c*x)^2,x]
```

output
```
(a + b*x^2)^2/(b + c*x)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^{2} x^{4}+2 a b \,x^{2}+a^{2}}{c x +b}$	27
norman	$\frac{b^{2} x^{4}+2 a b \,x^{2}-\frac{c \,a^{2} x}{b}}{c x +b}$	34
parallelrisch	$\frac{b^{3} x^{4}+2 a \,b^{2} x^{2}-a^{2} c x}{b \left(c x +b \right)}$	36
orering	$-\frac{\left(b \,x^{2}+a \right)^{2} \left(3 b c \,x^{2}+4 b^{2} x -a c \right)}{\left(c x +b \right) \left(-3 b c \,x^{2}-4 b^{2} x +a c \right)}$	56
default	$\frac{b \left(b \,c^{2} x^{3}-b^{2} c \,x^{2}+2 a \,c^{2} x +b^{3} x \right)}{c^{3}}-\frac{-a^{2} c^{4}-2 a \,b^{3} c^{2}-b^{6}}{c^{4} \left(c x +b \right)}$	72
risch	$\frac{b^{2} x^{3}}{c}-\frac{b^{3} x^{2}}{c^{2}}+\frac{2 b a x}{c}+\frac{b^{4} x}{c^{3}}+\frac{a^{2}}{c x +b}+\frac{2 a \,b^{3}}{c^{2} \left(c x +b \right)}+\frac{b^{6}}{c^{4} \left(c x +b \right)}$	80

input
```
int((b*x^2+a)*(3*b*c*x^2+4*b^2*x-a*c)/(c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2*x^4+2*a*b*x^2+a^2)/(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.18

$$\int \frac{(a+bx^2)\left(-ac+4b^2x+3bcx^2\right)}{(b+cx)^2}\,dx = \frac{b^2c^4x^4 + 2\,abc^4x^2 + b^6 + 2\,ab^3c^2 + a^2c^4 + (b^5c + 2\,ab^2c^3)x}{c^5x + bc^4}$$

input
```
integrate((b*x^2+a)*(3*b*c*x^2+4*b^2*x-a*c)/(c*x+b)^2,x, algorithm="fricas
")
```

output
```
(b^2*c^4*x^4 + 2*a*b*c^4*x^2 + b^6 + 2*a*b^3*c^2 + a^2*c^4 + (b^5*c + 2*a*
b^2*c^3)*x)/(c^5*x + b*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx^2)(-ac+4b^2x+3bcx^2)}{(b+cx)^2}\,dx = -\frac{b^3x^2}{c^2} + \frac{b^2x^3}{c} + x\left(\frac{2ab}{c} + \frac{b^4}{c^3}\right) + \frac{a^2c^4 + 2ab^3c^2 + b^6}{bc^4 + c^5x}$$

input
```
integrate((b*x**2+a)*(3*b*c*x**2+4*b**2*x-a*c)/(c*x+b)**2,x)
```

output
```
-b**3*x**2/c**2 + b**2*x**3/c + x*(2*a*b/c + b**4/c**3) + (a**2*c**4 + 2*a
*b**3*c**2 + b**6)/(b*c**4 + c**5*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int \frac{(a+bx^2)(-ac+4b^2x+3bcx^2)}{(b+cx)^2}\,dx = \frac{b^6 + 2\,ab^3c^2 + a^2c^4}{c^5x + bc^4} + \frac{b^2c^2x^3 - b^3cx^2 + (b^4 + 2\,abc^2)x}{c^3}$$

input
```
integrate((b*x^2+a)*(3*b*c*x^2+4*b^2*x-a*c)/(c*x+b)^2,x, algorithm="maxima
")
```

output
```
(b^6 + 2*a*b^3*c^2 + a^2*c^4)/(c^5*x + b*c^4) + (b^2*c^2*x^3 - b^3*c*x^2 +
 (b^4 + 2*a*b*c^2)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{(a+bx^2)\left(-ac+4b^2x+3bcx^2\right)}{(b+cx)^2}\,dx = \frac{\left(b^2-\frac{4\,b^3}{cx+b}+\frac{6\,b^4}{(cx+b)^2}+\frac{2\,abc^2}{(cx+b)^2}\right)(cx+b)^3}{c^4} + \frac{\frac{b^6c^3}{cx+b}+\frac{2\,ab^3c^5}{cx+b}+\frac{a^2c^7}{cx+b}}{c^7}$$

input
```
integrate((b*x^2+a)*(3*b*c*x^2+4*b^2*x-a*c)/(c*x+b)^2,x, algorithm="giac")
```

output
```
(b^2 - 4*b^3/(c*x + b) + 6*b^4/(c*x + b)^2 + 2*a*b*c^2/(c*x + b)^2)*(c*x +
 b)^3/c^4 + (b^6*c^3/(c*x + b) + 2*a*b^3*c^5/(c*x + b) + a^2*c^7/(c*x + b)
)/c^7
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.47

$$\int \frac{(a+bx^2)\left(-ac+4b^2x+3bcx^2\right)}{(b+cx)^2}\,dx = x\left(\frac{b^4}{c^3}+\frac{2\,a\,b}{c}\right)+\frac{a^2\,c^4+2\,a\,b^3\,c^2+b^6}{c\,\left(x\,c^4+b\,c^3\right)} + \frac{b^2\,x^3}{c} - \frac{b^3\,x^2}{c^2}$$

input
```
int(((a + b*x^2)*(4*b^2*x - a*c + 3*b*c*x^2))/(b + c*x)^2,x)
```

output
```
x*(b^4/c^3 + (2*a*b)/c) + (b^6 + a^2*c^4 + 2*a*b^3*c^2)/(c*(b*c^3 + c^4*x)
) + (b^2*x^3)/c - (b^3*x^2)/c^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)\,(-ac+4b^2x+3bcx^2)}{(b+cx)^2}\,dx=\frac{x(b^3x^3+2a\,b^2x-a^2c)}{b\,(cx+b)}$$

input
```
int((b*x^2+a)*(3*b*c*x^2+4*b^2*x-a*c)/(c*x+b)^2,x)
```

output
```
(x*( - a**2*c + 2*a*b**2*x + b**3*x**3))/(b*(b + c*x))
```

3.23 $\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2} \, dx = \frac{(a+bx^2)^2}{b+cx}$$

output
```
(b*x^2+a)^2/(c*x+b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2} \, dx = \frac{b^6+2ab^3c^2+a^2c^4+b^5cx+2abc^4x^2+b^2c^3x(2a+cx^3)}{c^4(b+cx)}$$

input
```
Integrate[((a + b*x^2)*(-(a*c) + b*x*(4*b + 3*c*x)))/(b + c*x)^2,x]
```

output
```
(b^6 + 2*a*b^3*c^2 + a^2*c^4 + b^5*c*x + 2*a*b*c^4*x^2 + b^2*c^3*x*(2*a +
c*x^3))/(c^4*(b + c*x))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\,(bx(4b+3cx)-ac)}{(b+cx)^2}\,dx$$
$$\Big\downarrow \; 2023$$
$$\frac{(a+bx^2)^2}{b+cx}$$

input
```
Int[((a + b*x^2)*(-(a*c) + b*x*(4*b + 3*c*x)))/(b + c*x)^2,x]
```

output
```
(a + b*x^2)^2/(b + c*x)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{cx+b}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{c\,a^2x}{b}}{cx+b}$	34
parallelrisch	$\frac{b^3x^4+2a\,b^2x^2-a^2cx}{b(cx+b)}$	36
orering	$-\frac{(b\,x^2+a)^2(-ac+bx(3cx+4b))}{(cx+b)(-3bc\,x^2-4b^2x+ac)}$	54
default	$\frac{b(b\,c^2x^3-b^2c\,x^2+2a\,c^2x+b^3x)}{c^3}-\frac{-a^2c^4-2a\,b^3c^2-b^6}{c^4(cx+b)}$	72
risch	$\frac{b^2x^3}{c}-\frac{b^3x^2}{c^2}+\frac{2bax}{c}+\frac{b^4x}{c^3}+\frac{a^2}{cx+b}+\frac{2a\,b^3}{c^2(cx+b)}+\frac{b^6}{c^4(cx+b)}$	80

input
```
int((b*x^2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2*x^4+2*a*b*x^2+a^2)/(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.18

$$\int \frac{(a+bx^2)\,(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = \frac{b^2c^4x^4+2\,abc^4x^2+b^6+2\,ab^3c^2+a^2c^4+(b^5c+2\,ab^2c^3)x}{c^5x+bc^4}$$

input
```
integrate((b*x^2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)^2,x, algorithm="fricas"
)
```

output
```
(b^2*c^4*x^4 + 2*a*b*c^4*x^2 + b^6 + 2*a*b^3*c^2 + a^2*c^4 + (b^5*c + 2*a*
b^2*c^3)*x)/(c^5*x + b*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = -\frac{b^3x^2}{c^2}+\frac{b^2x^3}{c}+x\left(\frac{2ab}{c}+\frac{b^4}{c^3}\right)+\frac{a^2c^4+2ab^3c^2+b^6}{bc^4+c^5x}$$

input
```
integrate((b*x**2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)**2,x)
```

output
```
-b**3*x**2/c**2 + b**2*x**3/c + x*(2*a*b/c + b**4/c**3) + (a**2*c**4 + 2*a
*b**3*c**2 + b**6)/(b*c**4 + c**5*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = \frac{b^6+2\,ab^3c^2+a^2c^4}{c^5x+bc^4}+\frac{b^2c^2x^3-b^3cx^2+(b^4+2\,abc^2)x}{c^3}$$

input
```
integrate((b*x^2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)^2,x, algorithm="maxima"
)
```

output
```
(b^6 + 2*a*b^3*c^2 + a^2*c^4)/(c^5*x + b*c^4) + (b^2*c^2*x^3 - b^3*c*x^2 +
 (b^4 + 2*a*b*c^2)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(17) = 34$.

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = \frac{\left(b^2 - \frac{4\,b^3}{cx+b} + \frac{6\,b^4}{(cx+b)^2} + \frac{2\,abc^2}{(cx+b)^2}\right)(cx+b)^3}{c^4} + \frac{\frac{b^6c^3}{cx+b} + \frac{2\,ab^3c^5}{cx+b} + \frac{a^2c^7}{cx+b}}{c^7}$$

input
```
integrate((b*x^2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)^2,x, algorithm="giac")
```

output
```
(b^2 - 4*b^3/(c*x + b) + 6*b^4/(c*x + b)^2 + 2*a*b*c^2/(c*x + b)^2)*(c*x +
 b)^3/c^4 + (b^6*c^3/(c*x + b) + 2*a*b^3*c^5/(c*x + b) + a^2*c^7/(c*x + b)
)/c^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.47

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = x\left(\frac{b^4}{c^3} + \frac{2\,a\,b}{c}\right) + \frac{a^2\,c^4 + 2\,a\,b^3\,c^2 + b^6}{c\,(x\,c^4 + b\,c^3)} + \frac{b^2\,x^3}{c} - \frac{b^3\,x^2}{c^2}$$

input
```
int(-((a*c - b*x*(4*b + 3*c*x))*(a + b*x^2))/(b + c*x)^2,x)
```

output
```
x*(b^4/c^3 + (2*a*b)/c) + (b^6 + a^2*c^4 + 2*a*b^3*c^2)/(c*(b*c^3 + c^4*x)
) + (b^2*x^3)/c - (b^3*x^2)/c^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)(-ac+bx(4b+3cx))}{(b+cx)^2}\,dx = \frac{x(b^3x^3+2a\,b^2x-a^2c)}{b\,(cx+b)}$$

input
```
int((b*x^2+a)*(-a*c+b*x*(3*c*x+4*b))/(c*x+b)^2,x)
```

output
```
(x*( - a**2*c + 2*a*b**2*x + b**3*x**3))/(b*(b + c*x))
```

3.24 $\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{(a+bx^2)^3}{b+cx}$$

output
```
(b*x^2+a)^3/(c*x+b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(17) = 34.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.65

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{b^9+3ab^6c^2+a^3c^6+b^8cx+3ab^5c^3x+3a^2bc^6x^2+3ab^2c^5x(a+cx^3)+b^3(3a^2c^4+c^6x^6)}{c^6(b+cx)}$$

input
```
Integrate[((a + b*x^2)^2*(-(a*c) + 6*b^2*x + 5*b*c*x^2))/(b + c*x)^2,x]
```

output
```
(b^9 + 3*a*b^6*c^2 + a^3*c^6 + b^8*c*x + 3*a*b^5*c^3*x + 3*a^2*b*c^6*x^2 +
 3*a*b^2*c^5*x*(a + c*x^3) + b^3*(3*a^2*c^4 + c^6*x^6))/(c^6*(b + c*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{2023\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}dx$$
$$\Big\downarrow \text{2023}$$
$$\frac{(a+bx^2)^3}{b+cx}$$

input
```
Int[((a + b*x^2)^2*(-(a*c) + 6*b^2*x + 5*b*c*x^2))/(b + c*x)^2,x]
```

output
```
(a + b*x^2)^3/(b + c*x)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 1.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}{cx+b}$
norman	$\frac{b^3x^6+3a\,b^2x^4+3a^2b\,x^2-\frac{c\,a^3x}{b}}{cx+b}$
parallelrisch	$\frac{b^3c\,x^6+3a\,b^2c\,x^4+3a^2bc\,x^2+c\,a^3}{(cx+b)c}$
orering	$-\frac{(b\,x^2+a)^3(5bc\,x^2+6b^2x-ac)}{(cx+b)(-5bc\,x^2-6b^2x+ac)}$
default	$\frac{b(x^5b^2c^4-b^3c^3x^4+3ab\,c^4x^3+b^4c^2x^3-3a\,b^2c^3x^2-x^2c\,b^5+3a^2c^4x+3xa\,b^3c^2+b^6x)}{c^5}-\frac{-a^3c^6-3a^2b^3c^4-3a\,b^6c^2-b^9}{c^6(cx+b)}$
risch	$\frac{b^3x^5}{c}-\frac{b^4x^4}{c^2}+\frac{3b^2a\,x^3}{c}+\frac{b^5x^3}{c^3}-\frac{3b^3a\,x^2}{c^2}-\frac{b^6x^2}{c^4}+\frac{3b\,a^2x}{c}+\frac{3b^4xa}{c^3}+\frac{b^7x}{c^5}+\frac{a^3}{cx+b}+\frac{3a^2b^3}{c^2(cx+b)}+\frac{3a\,b^6}{c^4(cx+b}$

input
```
int((b*x^2+a)^2*(5*b*c*x^2+6*b^2*x-a*c)/(c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 6.29

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{b^3c^6x^6+3\,ab^2c^6x^4+3\,a^2bc^6x^2+b^9+3\,ab^6c^2+3\,a^2b^3c^4+a^3c^6+(b^8c+3\,ab^5c^3+3\,a^2b^2c^5)x}{c^7x+bc^6}$$

input
```
integrate((b*x^2+a)^2*(5*b*c*x^2+6*b^2*x-a*c)/(c*x+b)^2,x, algorithm="fric
as")
```

output
```
(b^3*c^6*x^6 + 3*a*b^2*c^6*x^4 + 3*a^2*b*c^6*x^2 + b^9 + 3*a*b^6*c^2 + 3*a
^2*b^3*c^4 + a^3*c^6 + (b^8*c + 3*a*b^5*c^3 + 3*a^2*b^2*c^5)*x)/(c^7*x + b
*c^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = -\frac{b^4x^4}{c^2}+\frac{b^3x^5}{c}+x^3\cdot\left(\frac{3ab^2}{c}+\frac{b^5}{c^3}\right)+x^2\left(-\frac{3ab^3}{c^2}-\frac{b^6}{c^4}\right)+x\left(\frac{3a^2b}{c}+\frac{3ab^4}{c^3}+\frac{b^7}{c^5}\right)+\frac{a^3c^6+3a^2b^3c^4+3ab^6c^2+b^9}{bc^6+c^7x}$$

input
```
integrate((b*x**2+a)**2*(5*b*c*x**2+6*b**2*x-a*c)/(c*x+b)**2,x)
```

output
```
-b**4*x**4/c**2 + b**3*x**5/c + x**3*(3*a*b**2/c + b**5/c**3) + x**2*(-3*a
*b**3/c**2 - b**6/c**4) + x*(3*a**2*b/c + 3*a*b**4/c**3 + b**7/c**5) + (a*
*3*c**6 + 3*a**2*b**3*c**4 + 3*a*b**6*c**2 + b**9)/(b*c**6 + c**7*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 8.06

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{b^9+3\,ab^6c^2+3\,a^2b^3c^4+a^3c^6}{c^7x+bc^6}+\frac{b^3c^4x^5-b^4c^3x^4+(b^5c^2+3\,ab^2c^4)x^3-(b^6c+3\,ab^3c^3)x^2+(b^7+3\,ab^4c^2+3\,a^2bc^4)x}{c^5}$$

input
```
integrate((b*x^2+a)^2*(5*b*c*x^2+6*b^2*x-a*c)/(c*x+b)^2,x, algorithm="maxi
ma")
```

output
```
(b^9 + 3*a*b^6*c^2 + 3*a^2*b^3*c^4 + a^3*c^6)/(c^7*x + b*c^6) + (b^3*c^4*x
^5 - b^4*c^3*x^4 + (b^5*c^2 + 3*a*b^2*c^4)*x^3 - (b^6*c + 3*a*b^3*c^3)*x^2
 + (b^7 + 3*a*b^4*c^2 + 3*a^2*b*c^4)*x)/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 11.47

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{\left(b^3 - \frac{6\,b^4}{cx+b} + \frac{15\,b^5}{(cx+b)^2} - \frac{20\,b^6}{(cx+b)^3} + \frac{15\,b^7}{(cx+b)^4} + \frac{3\,ab^2c^2}{(cx+b)^2} - \frac{12\,ab^3c^2}{(cx+b)^3} + \frac{18\,ab^4c^2}{(cx+b)^4} + \frac{3\,a^2bc^4}{(cx+b)^4}\right)(cx+b)^5}{c^6} + \frac{\frac{b^9c^5}{cx+b} + \frac{3\,ab^6c^7}{cx+b} + \frac{3\,a^2b^3c^9}{cx+b} + \frac{a^3c^{11}}{cx+b}}{c^{11}}$$

input
```
integrate((b*x^2+a)^2*(5*b*c*x^2+6*b^2*x-a*c)/(c*x+b)^2,x, algorithm="giac
")
```

output
```
(b^3 - 6*b^4/(c*x + b) + 15*b^5/(c*x + b)^2 - 20*b^6/(c*x + b)^3 + 15*b^7/
(c*x + b)^4 + 3*a*b^2*c^2/(c*x + b)^2 - 12*a*b^3*c^2/(c*x + b)^3 + 18*a*b^
4*c^2/(c*x + b)^4 + 3*a^2*b*c^4/(c*x + b)^4)*(c*x + b)^5/c^6 + (b^9*c^5/(c
*x + b) + 3*a*b^6*c^7/(c*x + b) + 3*a^2*b^3*c^9/(c*x + b) + a^3*c^11/(c*x
+ b))/c^11
```

Mupad [B] (verification not implemented)

Time = 16.77 (sec) , antiderivative size = 223, normalized size of antiderivative = 13.12

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = x^3\left(\frac{b^5}{c^3}+\frac{3\,a\,b^2}{c}\right) + x^2\left(\frac{2\,b^6}{c^4}-\frac{b\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c}+\frac{6\,a\,b^3}{c^2}\right) - x\left(\frac{2\,b\left(\frac{4\,b^6}{c^4}-\frac{2\,b\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c}+\frac{12\,a\,b^3}{c^2}\right)}{c} - \frac{3\,a^2\,b}{c}+\frac{b^2\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c^2}\right) + \frac{b^3\,x^5}{c} - \frac{b^4\,x^4}{c^2} + \frac{a^3\,c^6+3\,a^2\,b^3\,c^4+3\,a\,b^6\,c^2+b^9}{c\,(x\,c^6+b\,c^5)}$$

input
```
int(((a + b*x^2)^2*(6*b^2*x - a*c + 5*b*c*x^2))/(b + c*x)^2,x)
```

output
```
x^3*(b^5/c^3 + (3*a*b^2)/c) + x^2*((2*b^6)/c^4 - (b*((3*b^5)/c^3 + (9*a*b^
2)/c))/c + (6*a*b^3)/c^2) - x*((2*b*((4*b^6)/c^4 - (2*b*((3*b^5)/c^3 + (9*
a*b^2)/c))/c + (12*a*b^3)/c^2))/c - (3*a^2*b)/c + (b^2*((3*b^5)/c^3 + (9*a
*b^2)/c))/c^2) + (b^3*x^5)/c - (b^4*x^4)/c^2 + (b^9 + a^3*c^6 + 3*a*b^6*c^
2 + 3*a^2*b^3*c^4)/(c*(b*c^5 + c^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{(a+bx^2)^2(-ac+6b^2x+5bcx^2)}{(b+cx)^2}\,dx = \frac{x(b^4x^5+3a\,b^3x^3+3a^2b^2x-a^3c)}{b\,(cx+b)}$$

input
```
int((b*x^2+a)^2*(5*b*c*x^2+6*b^2*x-a*c)/(c*x+b)^2,x)
```

output
```
(x*( - a**3*c + 3*a**2*b**2*x + 3*a*b**3*x**3 + b**4*x**5))/(b*(b + c*x))
```

3.25 $\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2} dx = \frac{(a+bx^2)^3}{b+cx}$$

output
```
(b*x^2+a)^3/(c*x+b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.65

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2} dx = \frac{b^9+3ab^6c^2+a^3c^6+b^8cx+3ab^5c^3x+3a^2bc^6x^2+3ab^2c^5x(a+cx^3)+b^3(3a^2c^4+c^6x^6)}{c^6(b+cx)}$$

input
```
Integrate[((a + b*x^2)^2*(-(a*c) + b*x*(6*b + 5*c*x)))/(b + c*x)^2,x]
```

output
```
(b^9 + 3*a*b^6*c^2 + a^3*c^6 + b^8*c*x + 3*a*b^5*c^3*x + 3*a^2*b*c^6*x^2 +
 3*a*b^2*c^5*x*(a + c*x^3) + b^3*(3*a^2*c^4 + c^6*x^6))/(c^6*(b + c*x))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (bx(6b+5cx)-ac)}{(b+cx)^2} \, dx$$
$$\Big\downarrow \text{2023}$$
$$\frac{(a+bx^2)^3}{b+cx}$$

input
```
Int[((a + b*x^2)^2*(-(a*c) + b*x*(6*b + 5*c*x)))/(b + c*x)^2,x]
```

output
```
(a + b*x^2)^3/(b + c*x)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 1.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^{3}x^{6}+3a\,b^{2}x^{4}+3a^{2}b\,x^{2}+a^{3}}{cx+b}$
norman	$\frac{b^{3}x^{6}+3a\,b^{2}x^{4}+3a^{2}b\,x^{2}-\frac{c\,a^{3}x}{b}}{cx+b}$
parallelrisch	$\frac{b^{3}c\,x^{6}+3a\,b^{2}c\,x^{4}+3a^{2}bc\,x^{2}+c\,a^{3}}{(cx+b)c}$
orering	$-\frac{\left(b\,x^{2}+a\right)^{3}\left(-ac+bx(5cx+6b)\right)}{(cx+b)\left(-5bc\,x^{2}-6b^{2}x+ac\right)}$
default	$\frac{b\left(x^{5}b^{2}c^{4}-b^{3}c^{3}x^{4}+3ab\,c^{4}x^{3}+b^{4}c^{2}x^{3}-3a\,b^{2}c^{3}x^{2}-x^{2}c\,b^{5}+3a^{2}c^{4}x+3xa\,b^{3}c^{2}+b^{6}x\right)}{c^{5}}-\frac{-a^{3}c^{6}-3a^{2}b^{3}c^{4}-3a\,b^{6}c^{2}-b^{9}}{c^{6}(cx+b)}$
risch	$\frac{b^{3}x^{5}}{c}-\frac{b^{4}x^{4}}{c^{2}}+\frac{3b^{2}a\,x^{3}}{c}+\frac{b^{5}x^{3}}{c^{3}}-\frac{3b^{3}a\,x^{2}}{c^{2}}-\frac{b^{6}x^{2}}{c^{4}}+\frac{3b\,a^{2}x}{c}+\frac{3b^{4}xa}{c^{3}}+\frac{b^{7}x}{c^{5}}+\frac{a^{3}}{cx+b}+\frac{3a^{2}b^{3}}{c^{2}(cx+b)}+\frac{3a\,b^{6}}{c^{4}(cx+b}$

input
```
int((b*x^2+a)^2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 6.29

$$\int \frac{(a+bx^2)^2\,(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx$$
$$= \frac{b^3c^6x^6+3\,ab^2c^6x^4+3\,a^2bc^6x^2+b^9+3\,ab^6c^2+3\,a^2b^3c^4+a^3c^6+(b^8c+3\,ab^5c^3+3\,a^2b^2c^5)x}{c^7x+bc^6}$$

input
```
integrate((b*x^2+a)^2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)^2,x, algorithm="frica
s")
```

output
```
(b^3*c^6*x^6 + 3*a*b^2*c^6*x^4 + 3*a^2*b*c^6*x^2 + b^9 + 3*a*b^6*c^2 + 3*a
^2*b^3*c^4 + a^3*c^6 + (b^8*c + 3*a*b^5*c^3 + 3*a^2*b^2*c^5)*x)/(c^7*x + b
*c^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx = -\frac{b^4x^4}{c^2}+\frac{b^3x^5}{c}+x^3\cdot\left(\frac{3ab^2}{c}+\frac{b^5}{c^3}\right)+x^2\left(-\frac{3ab^3}{c^2}-\frac{b^6}{c^4}\right)+x\left(\frac{3a^2b}{c}+\frac{3ab^4}{c^3}+\frac{b^7}{c^5}\right)+\frac{a^3c^6+3a^2b^3c^4+3ab^6c^2+b^9}{bc^6+c^7x}$$

input
```
integrate((b*x**2+a)**2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)**2,x)
```

output
```
-b**4*x**4/c**2 + b**3*x**5/c + x**3*(3*a*b**2/c + b**5/c**3) + x**2*(-3*a
*b**3/c**2 - b**6/c**4) + x*(3*a**2*b/c + 3*a*b**4/c**3 + b**7/c**5) + (a*
*3*c**6 + 3*a**2*b**3*c**4 + 3*a*b**6*c**2 + b**9)/(b*c**6 + c**7*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 8.06

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx = \frac{b^9+3\,ab^6c^2+3\,a^2b^3c^4+a^3c^6}{c^7x+bc^6}+\frac{b^3c^4x^5-b^4c^3x^4+(b^5c^2+3\,ab^2c^4)x^3-(b^6c+3\,ab^3c^3)x^2+(b^7+3\,ab^4c^2+3\,a^2bc^4)x}{c^5}$$

input
```
integrate((b*x^2+a)^2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)^2,x, algorithm="maxim
a")
```

output
```
(b^9 + 3*a*b^6*c^2 + 3*a^2*b^3*c^4 + a^3*c^6)/(c^7*x + b*c^6) + (b^3*c^4*x
^5 - b^4*c^3*x^4 + (b^5*c^2 + 3*a*b^2*c^4)*x^3 - (b^6*c + 3*a*b^3*c^3)*x^2
 + (b^7 + 3*a*b^4*c^2 + 3*a^2*b*c^4)*x)/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 11.47

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx = \frac{\left(b^3 - \frac{6\,b^4}{cx+b} + \frac{15\,b^5}{(cx+b)^2} - \frac{20\,b^6}{(cx+b)^3} + \frac{15\,b^7}{(cx+b)^4} + \frac{3\,ab^2c^2}{(cx+b)^2} - \frac{12\,ab^3c^2}{(cx+b)^3} + \frac{18\,ab^4c^2}{(cx+b)^4} + \frac{3\,a^2bc^4}{(cx+b)^4}\right)(cx+b)^5}{c^6} + \frac{\frac{b^9c^5}{cx+b} + \frac{3\,ab^6c^7}{cx+b} + \frac{3\,a^2b^3c^9}{cx+b} + \frac{a^3c^{11}}{cx+b}}{c^{11}}$$

input
```
integrate((b*x^2+a)^2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)^2,x, algorithm="giac"
)
```

output
```
(b^3 - 6*b^4/(c*x + b) + 15*b^5/(c*x + b)^2 - 20*b^6/(c*x + b)^3 + 15*b^7/
(c*x + b)^4 + 3*a*b^2*c^2/(c*x + b)^2 - 12*a*b^3*c^2/(c*x + b)^3 + 18*a*b^
4*c^2/(c*x + b)^4 + 3*a^2*b*c^4/(c*x + b)^4)*(c*x + b)^5/c^6 + (b^9*c^5/(c
*x + b) + 3*a*b^6*c^7/(c*x + b) + 3*a^2*b^3*c^9/(c*x + b) + a^3*c^11/(c*x
+ b))/c^11
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 13.12

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx = x^3\left(\frac{b^5}{c^3}+\frac{3\,a\,b^2}{c}\right)+x^2\left(\frac{2\,b^6}{c^4}-\frac{b\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c}+\frac{6\,a\,b^3}{c^2}\right)-x\left(\frac{2\,b\left(\frac{4\,b^6}{c^4}-\frac{2\,b\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c}+\frac{12\,a\,b^3}{c^2}\right)}{c}-\frac{3\,a^2\,b}{c}+\frac{b^2\left(\frac{3\,b^5}{c^3}+\frac{9\,a\,b^2}{c}\right)}{c^2}\right)+\frac{b^3\,x^5}{c}-\frac{b^4\,x^4}{c^2}+\frac{a^3\,c^6+3\,a^2\,b^3\,c^4+3\,a\,b^6\,c^2+b^9}{c\,(x\,c^6+b\,c^5)}$$

input
```
int(-((a*c - b*x*(6*b + 5*c*x))*(a + b*x^2)^2)/(b + c*x)^2,x)
```

output
```
x^3*(b^5/c^3 + (3*a*b^2)/c) + x^2*((2*b^6)/c^4 - (b*((3*b^5)/c^3 + (9*a*b^
2)/c))/c + (6*a*b^3)/c^2) - x*((2*b*((4*b^6)/c^4 - (2*b*((3*b^5)/c^3 + (9*
a*b^2)/c))/c + (12*a*b^3)/c^2))/c - (3*a^2*b)/c + (b^2*((3*b^5)/c^3 + (9*a
*b^2)/c))/c^2) + (b^3*x^5)/c - (b^4*x^4)/c^2 + (b^9 + a^3*c^6 + 3*a*b^6*c^
2 + 3*a^2*b^3*c^4)/(c*(b*c^5 + c^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{(a+bx^2)^2(-ac+bx(6b+5cx))}{(b+cx)^2}\,dx = \frac{x(b^4x^5+3a\,b^3x^3+3a^2b^2x-a^3c)}{b\,(cx+b)}$$

input
```
int((b*x^2+a)^2*(-a*c+b*x*(5*c*x+6*b))/(c*x+b)^2,x)
```

output
```
(x*( - a**3*c + 3*a**2*b**2*x + 3*a*b**3*x**3 + b**4*x**5))/(b*(b + c*x))
```

3.26 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 351

$$\begin{aligned}
&\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx \\
&= \frac{(b^2c(c^2C+3Bcd+3Ad^2)+a^2d^3D-abd(3cCd+Bd^2+3c^2D))\,x}{b^3} \\
&\quad - \frac{(ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\,x^2}{2b^2} \\
&\quad - \frac{d(ad^2D-b(3cCd+Bd^2+3c^2D))\,x^3}{3b^2} + \frac{d^2(Cd+3cD)x^4}{4b} + \frac{d^3Dx^5}{5b} \\
&\quad + \frac{(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC+3Bd)+a^2d^3D-abd(3cCd+Bd^2+3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}} \\
&\quad + \frac{(b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D))\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

output
```
(b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+a^2*d^3*D-a*b*d*(B*d^2+3*C*c*d+3*D*c^2))*x/
b^3-1/2*(a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*x^2/b^2-1/
3*d*(a*d^2*D-b*(B*d^2+3*C*c*d+3*D*c^2))*x^3/b^2+1/4*d^2*(C*d+3*D*c)*x^4/b+
1/5*d^3*D*x^5/b+(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(3*B*d+C*c)+a^2*d^3*D
-a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(7/2)
+1/2*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*d^2+3*C*c^2
*d+D*c^3))*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$$
$$= \frac{(Ab^2c(bc^2-3ad^2)+a(-b^2c^2(cC+3Bd)-a^2d^3D+abd(3cCd+Bd^2+3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$
$$+\frac{x(60a^2d^3D-10abd(18c^2D+9cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+b^2(30c^3(2C+Dx)+30c^2d}{}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((A*b^2*c*(b*c^2 - 3*a*d^2) + a*(-(b^2*c^2*(c*C + 3*B*d)) - a^2*d^3*D + a*
b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^
(7/2)) + (x*(60*a^2*d^3*D - 10*a*b*d*(18*c^2*D + 9*c*d*(2*C + D*x) + d^2*(
6*B + x*(3*C + 2*D*x))) + b^2*(30*c^3*(2*C + D*x) + 30*c^2*d*(6*B + x*(3*C
 + 2*D*x)) + 15*c*d^2*(12*A + x*(6*B + x*(4*C + 3*D*x))) + d^3*x*(30*A + x
*(20*B + 3*x*(5*C + 4*D*x))))) + 30*(b^2*c^2*(B*c + 3*A*d) + a^2*d^2*(C*d
+ 3*c*D) - a*b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*Log[a + b*x^2])/(6
0*b^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$$
$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{a^2d^3D-abd\left(Bd^2+3c^2D+3cCd\right)+b^2c\left(3Ad^2+3Bcd+c^2C\right)}{b^3}+\frac{bx\left(a^2d^2(3cD+Cd)-ab\left(Ad^3+3Bcd^2\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2d^3D-abd\left(Bd^2+3c^2D+3cCd\right)+b^2c^2(3Bd+cC)\right)\right)}{\sqrt{a}b^{7/2}}+\frac{x\left(a^2d^3D-abd\left(Bd^2+3c^2D+3cCd\right)+b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)}{b^3}+\frac{\log\left(a+bx^2\right)\left(a^2d^2(3cD+Cd)-ab\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)+b^2c^2(3Ad+Bc)\right)}{2b^3}-\frac{x^2\left(ad^2(3cD+Cd)-b\left(Ad^3+3Bcd^2+c^3D+3c^2Cd\right)\right)}{2b^2}-\frac{dx^3\left(ad^2D-b\left(Bd^2+3c^2D+3cCd\right)\right)}{3b^2}+\frac{d^2x^4(3cD+Cd)}{4b}+\frac{d^3Dx^5}{5b}$$

input
```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2) + a^2*d^3*D - a*b*d*(3*c*C*d + B*d^2 +
 3*c^2*D))*x)/b^3 - ((a*d^2*(C*d + 3*c*D) - b*(3*c^2*C*d + 3*B*c*d^2 + A*d
^3 + c^3*D))*x^2)/(2*b^2) - (d*(a*d^2*D - b*(3*c*C*d + B*d^2 + 3*c^2*D))*x
^3)/(3*b^2) + (d^2*(C*d + 3*c*D)*x^4)/(4*b) + (d^3*D*x^5)/(5*b) + ((A*b^2*
c*(b*c^2 - 3*a*d^2) - a*(b^2*c^2*(c*C + 3*B*d) + a^2*d^3*D - a*b*d*(3*c*C*
d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2)) + ((
b^2*c^2*(B*c + 3*A*d) + a^2*d^2*(C*d + 3*c*D) - a*b*(3*c^2*C*d + 3*B*c*d^2
 + A*d^3 + c^3*D))*Log[a + b*x^2])/(2*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.22

method	result
default	$\frac{\frac{1}{5}Dd^3x^5b^2+\frac{1}{4}b^2Cx^4d^3+\frac{3}{4}Db^2cd^2x^4+\frac{1}{3}Bb^2d^3x^3+Cb^2cd^2x^3-\frac{1}{3}Dabd^3x^3+Db^2c^2dx^3+\frac{1}{2}Ab^2d^3x^2+\frac{3}{2}Bb^2cd^2x^2-\frac{1}{2}Cabd^3x^2+}{b^3}$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*D*d^3*x^5*b^2+1/4*b^2*C*x^4*d^3+3/4*D*b^2*c*d^2*x^4+1/3*B*b^2*d
^3*x^3+C*b^2*c*d^2*x^3-1/3*D*a*b*d^3*x^3+D*b^2*c^2*d*x^3+1/2*A*b^2*d^3*x^2
+3/2*B*b^2*c*d^2*x^2-1/2*C*a*b*d^3*x^2+3/2*C*b^2*c^2*d*x^2-3/2*D*a*b*c*d^2
*x^2+1/2*D*b^2*c^3*x^2+3*A*b^2*c*d^2*x-B*a*b*d^3*x+3*B*b^2*c^2*d*x-3*C*a*b
*c*d^2*x+C*b^2*c^3*x+a^2*d^3*D*x-3*D*a*b*c^2*d*x)+1/b^3*(1/2*(-A*a*b^2*d^3
+3*A*b^3*c^2*d-3*B*a*b^2*c*d^2+B*b^3*c^3+C*a^2*b*d^3-3*C*a*b^2*c^2*d+3*D*a
^2*b*c*d^2-D*a*b^2*c^3)/b*ln(b*x^2+a)+(-3*A*a*b^2*c*d^2+A*b^3*c^3+B*a^2*b*
d^3-3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2-C*a*b^2*c^3-D*a^3*d^3+3*D*a^2*b*c^2*d)
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 884, normalized size of antiderivative = 2.52

$$\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/60*(12*D*a*b^3*d^3*x^5 + 15*(3*D*a*b^3*c*d^2 + C*a*b^3*d^3)*x^4 + 20*(3
*D*a*b^3*c^2*d + 3*C*a*b^3*c*d^2 - (D*a^2*b^2 - B*a*b^3)*d^3)*x^3 + 30*(D*
a*b^3*c^3 + 3*C*a*b^3*c^2*d - 3*(D*a^2*b^2 - B*a*b^3)*c*d^2 - (C*a^2*b^2 -
 A*a*b^3)*d^3)*x^2 + 30*((C*a*b^2 - A*b^3)*c^3 - 3*(D*a^2*b - B*a*b^2)*c^2
*d - 3*(C*a^2*b - A*a*b^2)*c*d^2 + (D*a^3 - B*a^2*b)*d^3)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 60*(C*a*b^3*c^3 - 3*(D*a^2*b^2
- B*a*b^3)*c^2*d - 3*(C*a^2*b^2 - A*a*b^3)*c*d^2 + (D*a^3*b - B*a^2*b^2)*d
^3)*x - 30*((D*a^2*b^2 - B*a*b^3)*c^3 + 3*(C*a^2*b^2 - A*a*b^3)*c^2*d - 3*
(D*a^3*b - B*a^2*b^2)*c*d^2 - (C*a^3*b - A*a^2*b^2)*d^3)*log(b*x^2 + a))/(
a*b^4), 1/60*(12*D*a*b^3*d^3*x^5 + 15*(3*D*a*b^3*c*d^2 + C*a*b^3*d^3)*x^4
+ 20*(3*D*a*b^3*c^2*d + 3*C*a*b^3*c*d^2 - (D*a^2*b^2 - B*a*b^3)*d^3)*x^3 +
 30*(D*a*b^3*c^3 + 3*C*a*b^3*c^2*d - 3*(D*a^2*b^2 - B*a*b^3)*c*d^2 - (C*a^
2*b^2 - A*a*b^3)*d^3)*x^2 - 60*((C*a*b^2 - A*b^3)*c^3 - 3*(D*a^2*b - B*a*b
^2)*c^2*d - 3*(C*a^2*b - A*a*b^2)*c*d^2 + (D*a^3 - B*a^2*b)*d^3)*sqrt(a*b)
*arctan(sqrt(a*b)*x/a) + 60*(C*a*b^3*c^3 - 3*(D*a^2*b^2 - B*a*b^3)*c^2*d -
 3*(C*a^2*b^2 - A*a*b^3)*c*d^2 + (D*a^3*b - B*a^2*b^2)*d^3)*x - 30*((D*a^2
*b^2 - B*a*b^3)*c^3 + 3*(C*a^2*b^2 - A*a*b^3)*c^2*d - 3*(D*a^3*b - B*a^2*b
^2)*c*d^2 - (C*a^3*b - A*a^2*b^2)*d^3)*log(b*x^2 + a))/(a*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(350) = 700$.

Time = 4.25 (sec) , antiderivative size = 1394, normalized size of antiderivative = 3.97

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2}\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
D*d**3*x**5/(5*b) + x**4*(C*d**3/(4*b) + 3*D*c*d**2/(4*b)) + x**3*(B*d**3/
(3*b) + C*c*d**2/b - D*a*d**3/(3*b**2) + D*c**2*d/b) + x**2*(A*d**3/(2*b)
+ 3*B*c*d**2/(2*b) - C*a*d**3/(2*b**2) + 3*C*c**2*d/(2*b) - 3*D*a*c*d**2/(
2*b**2) + D*c**3/(2*b)) + x*(3*A*c*d**2/b - B*a*d**3/b**2 + 3*B*c**2*d/b -
 3*C*a*c*d**2/b**2 + C*c**3/b + D*a**2*d**3/b**3 - 3*D*a*c**2*d/b**2) + ((
-A*a*b*d**3 + 3*A*b**2*c**2*d - 3*B*a*b*c*d**2 + B*b**2*c**3 + C*a**2*d**3
 - 3*C*a*b*c**2*d + 3*D*a**2*c*d**2 - D*a*b*c**3)/(2*b**3) - sqrt(-a*b**7)
*(3*A*a*b**2*c*d**2 - A*b**3*c**3 - B*a**2*b*d**3 + 3*B*a*b**2*c**2*d - 3*
C*a**2*b*c*d**2 + C*a*b**2*c**3 + D*a**3*d**3 - 3*D*a**2*b*c**2*d)/(2*a*b*
*7))*log(x + (-A*a**2*b*d**3 + 3*A*a*b**2*c**2*d - 3*B*a**2*b*c*d**2 + B*a
*b**2*c**3 + C*a**3*d**3 - 3*C*a**2*b*c**2*d + 3*D*a**3*c*d**2 - D*a**2*b*
c**3 - 2*a*b**3*((-A*a*b*d**3 + 3*A*b**2*c**2*d - 3*B*a*b*c*d**2 + B*b**2*
c**3 + C*a**2*d**3 - 3*C*a*b*c**2*d + 3*D*a**2*c*d**2 - D*a*b*c**3)/(2*b**
3) - sqrt(-a*b**7)*(3*A*a*b**2*c*d**2 - A*b**3*c**3 - B*a**2*b*d**3 + 3*B*
a*b**2*c**2*d - 3*C*a**2*b*c*d**2 + C*a*b**2*c**3 + D*a**3*d**3 - 3*D*a**2
*b*c**2*d)/(2*a*b**7)))/(3*A*a*b**2*c*d**2 - A*b**3*c**3 - B*a**2*b*d**3 +
 3*B*a*b**2*c**2*d - 3*C*a**2*b*c*d**2 + C*a*b**2*c**3 + D*a**3*d**3 - 3*D
*a**2*b*c**2*d)) + ((-A*a*b*d**3 + 3*A*b**2*c**2*d - 3*B*a*b*c*d**2 + B*b*
*2*c**3 + C*a**2*d**3 - 3*C*a*b*c**2*d + 3*D*a**2*c*d**2 - D*a*b*c**3)/(2*
b**3) + sqrt(-a*b**7)*(3*A*a*b**2*c*d**2 - A*b**3*c**3 - B*a**2*b*d**3 ...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx = -\frac{((Dab-Bb^2)c^3+3(Cab-Ab^2)c^2d-3(Da^2-Bab)cd^2-(Ca^2-Aab)d^3)\log(bx^2+a)}{2b^3} - \frac{((Cab^2-Ab^3)c^3-3(Da^2b-Bab^2)c^2d-3(Ca^2b-Aab^2)cd^2+(Da^3-Ba^2b)d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12Db^2d^3x^5+15(3Db^2cd^2+Cb^2d^3)x^4+20(3Db^2c^2d+3Cb^2cd^2-(Dab-Bb^2)d^3)x^3+30(Db^2c^3+$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((D*a*b - B*b^2)*c^3 + 3*(C*a*b - A*b^2)*c^2*d - 3*(D*a^2 - B*a*b)*c*
d^2 - (C*a^2 - A*a*b)*d^3)*log(b*x^2 + a)/b^3 - ((C*a*b^2 - A*b^3)*c^3 - 3
*(D*a^2*b - B*a*b^2)*c^2*d - 3*(C*a^2*b - A*a*b^2)*c*d^2 + (D*a^3 - B*a^2*
b)*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*d^3*x^5 + 1
5*(3*D*b^2*c*d^2 + C*b^2*d^3)*x^4 + 20*(3*D*b^2*c^2*d + 3*C*b^2*c*d^2 - (D
*a*b - B*b^2)*d^3)*x^3 + 30*(D*b^2*c^3 + 3*C*b^2*c^2*d - 3*(D*a*b - B*b^2)
*c*d^2 - (C*a*b - A*b^2)*d^3)*x^2 + 60*(C*b^2*c^3 - 3*(D*a*b - B*b^2)*c^2*
d - 3*(C*a*b - A*b^2)*c*d^2 + (D*a^2 - B*a*b)*d^3)*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = -\frac{(Dabc^3 - Bb^2c^3 + 3\,Cabc^2d - 3\,Ab^2c^2d - 3\,Da^2cd^2 + 3\,Babcd^2 - Ca^2d^3 + Aabd^3)\log\left(bx^2+a\right)}{2\,b^3} - \frac{(Cab^2c^3 - Ab^3c^3 - 3\,Da^2bc^2d + 3\,Bab^2c^2d - 3\,Ca^2bcd^2 + 3\,Aab^2cd^2 + Da^3d^3 - Ba^2bd^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4d^3x^5 + 45\,Db^4cd^2x^4 + 15\,Cb^4d^3x^4 + 60\,Db^4c^2dx^3 + 60\,Cb^4cd^2x^3 - 20\,Dab^3d^3x^3 + 20\,Bb^4d^3x^3 -$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a*b*c^3 - B*b^2*c^3 + 3*C*a*b*c^2*d - 3*A*b^2*c^2*d - 3*D*a^2*c*d^
2 + 3*B*a*b*c*d^2 - C*a^2*d^3 + A*a*b*d^3)*log(b*x^2 + a)/b^3 - (C*a*b^2*c
^3 - A*b^3*c^3 - 3*D*a^2*b*c^2*d + 3*B*a*b^2*c^2*d - 3*C*a^2*b*c*d^2 + 3*A
*a*b^2*c*d^2 + D*a^3*d^3 - B*a^2*b*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^3) + 1/60*(12*D*b^4*d^3*x^5 + 45*D*b^4*c*d^2*x^4 + 15*C*b^4*d^3*x^4 + 60*
D*b^4*c^2*d*x^3 + 60*C*b^4*c*d^2*x^3 - 20*D*a*b^3*d^3*x^3 + 20*B*b^4*d^3*x
^3 + 30*D*b^4*c^3*x^2 + 90*C*b^4*c^2*d*x^2 - 90*D*a*b^3*c*d^2*x^2 + 90*B*b
^4*c*d^2*x^2 - 30*C*a*b^3*d^3*x^2 + 30*A*b^4*d^3*x^2 + 60*C*b^4*c^3*x - 18
0*D*a*b^3*c^2*d*x + 180*B*b^4*c^2*d*x - 180*C*a*b^3*c*d^2*x + 180*A*b^4*c*
d^2*x + 60*D*a^2*b^2*d^3*x - 60*B*a*b^3*d^3*x)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{(c+d\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2d^3 - 90\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3c^2d + 90b^4c^2dx + 45b^4c\,d^2x^2 + 60b^3c^3d\,x^2 + 60b}{}$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
( - 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d**4 - 90*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c*d**2 + 30*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*d**3 + 180*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b*c**2*d**2 + 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*b**3*c**3 - 90*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b
**3*c**2*d - 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**4 -
15*log(a + b*x**2)*a**2*b**2*d**3 + 60*log(a + b*x**2)*a**2*b*c*d**3 + 45*
log(a + b*x**2)*a*b**3*c**2*d - 45*log(a + b*x**2)*a*b**3*c*d**2 - 60*log(
a + b*x**2)*a*b**2*c**3*d + 15*log(a + b*x**2)*b**4*c**3 + 30*a**2*b*d**4*
x + 90*a*b**3*c*d**2*x + 15*a*b**3*d**3*x**2 - 30*a*b**3*d**3*x - 180*a*b*
*2*c**2*d**2*x - 60*a*b**2*c*d**3*x**2 - 10*a*b**2*d**4*x**3 + 90*b**4*c**
2*d*x + 45*b**4*c*d**2*x**2 + 10*b**4*d**3*x**3 + 30*b**3*c**4*x + 60*b**3
*c**3*d*x**2 + 60*b**3*c**2*d**2*x**3 + 30*b**3*c*d**3*x**4 + 6*b**3*d**4*
x**5)/(30*b**4)
```

3.27 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 230

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx &= \frac{(b(c^2C+2Bcd+Ad^2)-ad(Cd+2cD))\,x}{b^2} \\ &\quad - \frac{(ad^2D-b(2cCd+Bd^2+c^2D))\,x^2}{2b^2} + \frac{d(Cd+2cD)x^3}{3b} + \frac{d^2Dx^4}{4b} \\ &\quad + \frac{(Ab(bc^2-ad^2)-a(bc(cC+2Bd)-ad(Cd+2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}} \\ &\quad + \frac{(b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D))\log(a+bx^2)}{2b^3}\end{aligned}$$

output
```
(b*(A*d^2+2*B*c*d+C*c^2)-a*d*(C*d+2*D*c))*x/b^2-1/2*(a*d^2*D-b*(B*d^2+2*C*
c*d+D*c^2))*x^2/b^2+1/3*d*(C*d+2*D*c)*x^3/b+1/4*d^2*D*x^4/b+(A*b*(-a*d^2+b
*c^2)-a*(b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*arctan(b^(1/2)*x/a^(1/2))/a^(1/
2)/b^(5/2)+1/2*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))*ln(
b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$
$$= \frac{(Ab(bc^2-ad^2)+a(-bc(cC+2Bd)+ad(Cd+2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$
$$+ \frac{bx(-6ad(2Cd+4cD+dDx)+b(6c^2(2C+Dx)+4cd(6B+3Cx+2Dx^2)+d^2(12A+6Bx+4Cx^2}{12b^3}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((A*b*(b*c^2 - a*d^2) + a*(-(b*c*(c*C + 2*B*d)) + a*d*(C*d + 2*c*D)))*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2)) + (b*x*(-6*a*d*(2*C*d + 4*c*D +
 d*D*x) + b*(6*c^2*(2*C + D*x) + 4*c*d*(6*B + 3*C*x + 2*D*x^2) + d^2*(12*A
 + 6*B*x + 4*C*x^2 + 3*D*x^3))) + 6*(b^2*c*(B*c + 2*A*d) + a^2*d^2*D - a*b
*(2*c*C*d + B*d^2 + c^2*D))*Log[a + b*x^2])/(12*b^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$
$$\Big\downarrow \text{2160}$$
$$\int \left(\frac{x(a^2d^2D-ab(Bd^2+c^2D+2cCd)+b^2c(2Ad+Bc))+Ab(bc^2-ad^2)-a(bc(2Bd+cC)-ad(2cD+Cd}{b^2(a+bx^2)}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{\log \left(a+bx^2\right) \left(a^2d^2D-ab\left(Bd^2+c^2D+2cCd\right)+b^2c(2Ad+Bc)\right)}{2b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \left(Ab\left(bc^2-ad^2\right)-a(bc(2Bd+cC)-ad(2cD+Cd))\right)}{\sqrt{a}b^{5/2}}+\frac{x\left(b\left(Ad^2+2Bcd+c^2C\right)-ad(2cD+Cd)\right)}{b^2}-\frac{x^2\left(ad^2D-b\left(Bd^2+c^2D+2cCd\right)\right)}{2b^2}+\frac{dx^3(2cD+Cd)}{3b}+\frac{d^2Dx^4}{4b}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((b*(c^2*C + 2*B*c*d + A*d^2) - a*d*(C*d + 2*c*D))*x)/b^2 - ((a*d^2*D - b*
(2*c*C*d + B*d^2 + c^2*D))*x^2)/(2*b^2) + (d*(C*d + 2*c*D)*x^3)/(3*b) + (d
^2*D*x^4)/(4*b) + ((A*b*(b*c^2 - a*d^2) - a*(b*c*(c*C + 2*B*d) - a*d*(C*d
+ 2*c*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2)) + ((b^2*c*(B*c +
 2*A*d) + a^2*d^2*D - a*b*(2*c*C*d + B*d^2 + c^2*D))*Log[a + b*x^2])/(2*b^
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{1}{4}Db\,x^4d^2+\frac{1}{3}Cb\,x^3d^2+\frac{2}{3}Dbcd\,x^3+\frac{1}{2}Bb\,d^2x^2+Cbcd\,x^2-\frac{1}{2}a\,d^2Dx^2+\frac{1}{2}Db\,c^2x^2+Ab\,d^2x+2Bbcdx-Cax\,d^2+Cb\,c^2x-2acdDx}{b^2}+$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/4*D*b*x^4*d^2+1/3*C*b*x^3*d^2+2/3*D*b*c*d*x^3+1/2*B*b*d^2*x^2+C*b
*c*d*x^2-1/2*a*d^2*D*x^2+1/2*D*b*c^2*x^2+A*b*d^2*x+2*B*b*c*d*x-C*a*x*d^2+C
*b*c^2*x-2*a*c*d*D*x)+1/b^2*(1/2*(2*A*b^2*c*d-B*a*b*d^2+B*b^2*c^2-2*C*a*b*
c*d+D*a^2*d^2-D*a*b*c^2)/b*ln(b*x^2+a)+(-A*a*b*d^2+A*b^2*c^2-2*B*a*b*c*d+C
*a^2*d^2-C*a*b*c^2+2*D*a^2*c*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$

$$= \left[\frac{3\,Dab^2d^2x^4 + 4\,(2\,Dab^2cd + Cab^2d^2)x^3 + 6\,(Dab^2c^2 + 2\,Cab^2cd - (Da^2b - Bab^2)d^2)x^2 - 6\,((Cab - A\right.$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*D*a*b^2*d^2*x^4 + 4*(2*D*a*b^2*c*d + C*a*b^2*d^2)*x^3 + 6*(D*a*b^
2*c^2 + 2*C*a*b^2*c*d - (D*a^2*b - B*a*b^2)*d^2)*x^2 - 6*((C*a*b - A*b^2)*
c^2 - 2*(D*a^2 - B*a*b)*c*d - (C*a^2 - A*a*b)*d^2)*sqrt(-a*b)*log((b*x^2 +
 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 12*(C*a*b^2*c^2 - 2*(D*a^2*b - B*a*b^2
)*c*d - (C*a^2*b - A*a*b^2)*d^2)*x - 6*((D*a^2*b - B*a*b^2)*c^2 + 2*(C*a^2
*b - A*a*b^2)*c*d - (D*a^3 - B*a^2*b)*d^2)*log(b*x^2 + a))/(a*b^3), 1/12*(
3*D*a*b^2*d^2*x^4 + 4*(2*D*a*b^2*c*d + C*a*b^2*d^2)*x^3 + 6*(D*a*b^2*c^2 +
 2*C*a*b^2*c*d - (D*a^2*b - B*a*b^2)*d^2)*x^2 - 12*((C*a*b - A*b^2)*c^2 -
2*(D*a^2 - B*a*b)*c*d - (C*a^2 - A*a*b)*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + 12*(C*a*b^2*c^2 - 2*(D*a^2*b - B*a*b^2)*c*d - (C*a^2*b - A*a*b^2)*d^2
)*x - 6*((D*a^2*b - B*a*b^2)*c^2 + 2*(C*a^2*b - A*a*b^2)*c*d - (D*a^3 - B*
a^2*b)*d^2)*log(b*x^2 + a))/(a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(221) = 442$.

Time = 3.06 (sec) , antiderivative size = 913, normalized size of antiderivative = 3.97

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \frac{Dd^2x^4}{4b} + x^3\left(\frac{Cd^2}{3b} + \frac{2Dcd}{3b}\right) + x^2\left(\frac{Bd^2}{2b} + \frac{Ccd}{b} - \frac{Dad^2}{2b^2} + \frac{Dc^2}{2b}\right) + x\left(\frac{Ad^2}{b} + \frac{2Bcd}{b} - \frac{Cad^2}{b^2} + \frac{Cc^2}{b} - \frac{2Dacd}{b^2}\right) + \left(\frac{2Ab^2cd - Babd^2 + Bb^2c^2 - 2Cabcd + Da^2d^2 - Dabc^2}{2b^3} - \frac{\sqrt{-ab^7}\left(-Aabd^2 + Ab^2c^2 - 2Babcd + Ca^2d^2 - Cabc^2 + 2Da^2cd\right)}{2ab^6}\right)\log\left(x + \frac{-2Aab^2cd + Ba^2bd^2 -}{} \right. + \left(\frac{2Ab^2cd - Babd^2 + Bb^2c^2 - 2Cabcd + Da^2d^2 - Dabc^2}{2b^3} + \frac{\sqrt{-ab^7}\left(-Aabd^2 + Ab^2c^2 - 2Babcd + Ca^2d^2 - Cabc^2 + 2Da^2cd\right)}{2ab^6}\right)\log\left(x + \frac{-2Aab^2cd + Ba^2bd^2 -}{} \right.$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
D*d**2*x**4/(4*b) + x**3*(C*d**2/(3*b) + 2*D*c*d/(3*b)) + x**2*(B*d**2/(2*
b) + C*c*d/b - D*a*d**2/(2*b**2) + D*c**2/(2*b)) + x*(A*d**2/b + 2*B*c*d/b
 - C*a*d**2/b**2 + C*c**2/b - 2*D*a*c*d/b**2) + ((2*A*b**2*c*d - B*a*b*d**
2 + B*b**2*c**2 - 2*C*a*b*c*d + D*a**2*d**2 - D*a*b*c**2)/(2*b**3) - sqrt(
-a*b**7)*(-A*a*b*d**2 + A*b**2*c**2 - 2*B*a*b*c*d + C*a**2*d**2 - C*a*b*c*
*2 + 2*D*a**2*c*d)/(2*a*b**6))*log(x + (-2*A*a*b**2*c*d + B*a**2*b*d**2 -
B*a*b**2*c**2 + 2*C*a**2*b*c*d - D*a**3*d**2 + D*a**2*b*c**2 + 2*a*b**3*((
2*A*b**2*c*d - B*a*b*d**2 + B*b**2*c**2 - 2*C*a*b*c*d + D*a**2*d**2 - D*a*
b*c**2)/(2*b**3) - sqrt(-a*b**7)*(-A*a*b*d**2 + A*b**2*c**2 - 2*B*a*b*c*d
+ C*a**2*d**2 - C*a*b*c**2 + 2*D*a**2*c*d)/(2*a*b**6)))/(-A*a*b**2*d**2 +
A*b**3*c**2 - 2*B*a*b**2*c*d + C*a**2*b*d**2 - C*a*b**2*c**2 + 2*D*a**2*b*
c*d)) + ((2*A*b**2*c*d - B*a*b*d**2 + B*b**2*c**2 - 2*C*a*b*c*d + D*a**2*d
**2 - D*a*b*c**2)/(2*b**3) + sqrt(-a*b**7)*(-A*a*b*d**2 + A*b**2*c**2 - 2*
B*a*b*c*d + C*a**2*d**2 - C*a*b*c**2 + 2*D*a**2*c*d)/(2*a*b**6))*log(x + (
-2*A*a*b**2*c*d + B*a**2*b*d**2 - B*a*b**2*c**2 + 2*C*a**2*b*c*d - D*a**3*
d**2 + D*a**2*b*c**2 + 2*a*b**3*((2*A*b**2*c*d - B*a*b*d**2 + B*b**2*c**2
- 2*C*a*b*c*d + D*a**2*d**2 - D*a*b*c**2)/(2*b**3) + sqrt(-a*b**7)*(-A*a*b
*d**2 + A*b**2*c**2 - 2*B*a*b*c*d + C*a**2*d**2 - C*a*b*c**2 + 2*D*a**2*c*
d)/(2*a*b**6)))/(-A*a*b**2*d**2 + A*b**3*c**2 - 2*B*a*b**2*c*d + C*a**2*b*
d**2 - C*a*b**2*c**2 + 2*D*a**2*b*c*d))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= -\frac{((Cab-Ab^2)c^2-2\,(Da^2-Bab)cd-(Ca^2-Aab)d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}$$
$$+\frac{3\,Dbd^2x^4+4\,(2\,Dbcd+Cbd^2)x^3+6\,(Dbc^2+2\,Cbcd-(Da-Bb)d^2)x^2+12\,(Cbc^2-2\,(Da-Bb)cd}{12\,b^2}$$
$$-\frac{((Dab-Bb^2)c^2+2\,(Cab-Ab^2)cd-(Da^2-Bab)d^2)\log\left(bx^2+a\right)}{2\,b^3}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-((C*a*b - A*b^2)*c^2 - 2*(D*a^2 - B*a*b)*c*d - (C*a^2 - A*a*b)*d^2)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*d^2*x^4 + 4*(2*D*b*c*d + C*
b*d^2)*x^3 + 6*(D*b*c^2 + 2*C*b*c*d - (D*a - B*b)*d^2)*x^2 + 12*(C*b*c^2 -
 2*(D*a - B*b)*c*d - (C*a - A*b)*d^2)*x)/b^2 - 1/2*((D*a*b - B*b^2)*c^2 +
2*(C*a*b - A*b^2)*c*d - (D*a^2 - B*a*b)*d^2)*log(b*x^2 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= -\frac{(Cabc^2 - Ab^2c^2 - 2\,Da^2cd + 2\,Babcd - Ca^2d^2 + Aabd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}$$
$$- \frac{(Dabc^2 - Bb^2c^2 + 2\,Cabcd - 2\,Ab^2cd - Da^2d^2 + Babd^2)\log\left(bx^2+a\right)}{2\,b^3}$$
$$+ \frac{3\,Db^3d^2x^4 + 8\,Db^3cdx^3 + 4\,Cb^3d^2x^3 + 6\,Db^3c^2x^2 + 12\,Cb^3cdx^2 - 6\,Dab^2d^2x^2 + 6\,Bb^3d^2x^2 + 12\,Cb^3c^2}{12\,b^4}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a*b*c^2 - A*b^2*c^2 - 2*D*a^2*c*d + 2*B*a*b*c*d - C*a^2*d^2 + A*a*b*d^
2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(D*a*b*c^2 - B*b^2*c^2 + 2*
C*a*b*c*d - 2*A*b^2*c*d - D*a^2*d^2 + B*a*b*d^2)*log(b*x^2 + a)/b^3 + 1/12
*(3*D*b^3*d^2*x^4 + 8*D*b^3*c*d*x^3 + 4*C*b^3*d^2*x^3 + 6*D*b^3*c^2*x^2 +
12*C*b^3*c*d*x^2 - 6*D*a*b^2*d^2*x^2 + 6*B*b^3*d^2*x^2 + 12*C*b^3*c^2*x -
24*D*a*b^2*c*d*x + 24*B*b^3*c*d*x - 12*C*a*b^2*d^2*x + 12*A*b^3*d^2*x)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx=\int \frac{{\left(c+d\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$
$$= \frac{-4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,d^2+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac\,d^2+4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2c^2-8\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
( - 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*d**2 + 12*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*c*d**2 + 4*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b**2*c**2 - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*b**2*c*d - 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c**3
 + 2*log(a + b*x**2)*a**2*d**3 + 4*log(a + b*x**2)*a*b**2*c*d - 2*log(a +
b*x**2)*a*b**2*d**2 - 6*log(a + b*x**2)*a*b*c**2*d + 2*log(a + b*x**2)*b**
3*c**2 + 4*a*b**2*d**2*x - 12*a*b*c*d**2*x - 2*a*b*d**3*x**2 + 8*b**3*c*d*
x + 2*b**3*d**2*x**2 + 4*b**2*c**3*x + 6*b**2*c**2*d*x**2 + 4*b**2*c*d**2*
x**3 + b**2*d**3*x**4)/(4*b**3)
```

3.28 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\begin{aligned}\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx &= \frac{(bcC+bBd-adD)x}{b^2}+\frac{(Cd+cD)x^2}{2b}+\frac{dDx^3}{3b} \\ &\quad+\frac{(Ab^2c-a(b(cC+Bd)-adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}} \\ &\quad+\frac{(bBc+Abd-aCd-acD)\log(a+bx^2)}{2b^2}\end{aligned}$$

output
```
(B*b*d+C*b*c-D*a*d)*x/b^2+1/2*(C*d+D*c)*x^2/b+1/3*d*D*x^3/b+(A*b^2*c-a*(b*
(B*d+C*c)-D*a*d))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(5/2)+1/2*(A*b*d+B*b
*c-C*a*d-D*a*c)*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\frac{(Ab^2c+a(-b(cC+Bd)+adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$
$$+\frac{x(-6adD+b(6cC+6Bd+3Cdx+3cDx+2dDx^2))+3(b(Bc+Ad)-a(Cd+cD))\log(a+bx^2)}{6b^2}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((A*b^2*c + a*(-(b*(c*C + B*d)) + a*d*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(5/2)) + (x*(-6*a*d*D + b*(6*c*C + 6*B*d + 3*C*d*x + 3*c*D*x + 2*d
*D*x^2)) + 3*(b*(B*c + A*d) - a*(C*d + c*D))*Log[a + b*x^2])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$\Big\downarrow \ 2160$$
$$\int \left(\frac{bx(-acD-aCd+Abd+bBc)-a(b(Bd+cC)-adD)+Ab^2c}{b^2\,(a+bx^2)}+\frac{-adD+bBd+bcC}{b^2}+\frac{x(cD+Cd)}{b}+\frac{d}{}\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^2c-a(b(Bd+cC)-adD)\right)}{\sqrt{a}b^{5/2}}+\frac{\log\left(a+bx^2\right)\left(-acD-aCd+Abd+bBc\right)}{2b^2}+\frac{x(-adD+bBd+bcC)}{b^2}+\frac{x^2(cD+Cd)}{2b}+\frac{dDx^3}{3b}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((b*c*C + b*B*d - a*d*D)*x)/b^2 + ((C*d + c*D)*x^2)/(2*b) + (d*D*x^3)/(3*b
) + ((A*b^2*c - a*(b*(c*C + B*d) - a*d*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*b^(5/2)) + ((b*B*c + A*b*d - a*C*d - a*c*D)*Log[a + b*x^2])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{1}{3}Dd\,x^3b+\frac{1}{2}Cbd\,x^2+\frac{1}{2}Dbc\,x^2+Bbdx+bcCx-Dadx}{b^2}+\frac{\frac{\left(A\,b^2d+B\,b^2c-dabC-abcD\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(A\,b^2c-Babd-Cabc+a^2dD\right)\arctan\left(\right.}{\sqrt{ab}}}{b^2}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*D*d*x^3*b+1/2*C*b*d*x^2+1/2*D*b*c*x^2+B*b*d*x+b*c*C*x-D*a*d*x)+
1/b^2*(1/2*(A*b^2*d+B*b^2*c-C*a*b*d-D*a*b*c)/b*ln(b*x^2+a)+(A*b^2*c-B*a*b*
d-C*a*b*c+D*a^2*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \left[\frac{2\,Dab^2dx^3+3\,(Dab^2c+Cab^2d)x^2-3\,\sqrt{-ab}((Cab-Ab^2)c-(Da^2-Bab)d)\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+6}{6\,ab^3}\right.$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*D*a*b^2*d*x^3 + 3*(D*a*b^2*c + C*a*b^2*d)*x^2 - 3*sqrt(-a*b)*((C*a
*b - A*b^2)*c - (D*a^2 - B*a*b)*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 6*(C*a*b^2*c - (D*a^2*b - B*a*b^2)*d)*x - 3*((D*a^2*b - B*a*b^2)*
c + (C*a^2*b - A*a*b^2)*d)*log(b*x^2 + a))/(a*b^3), 1/6*(2*D*a*b^2*d*x^3 +
 3*(D*a*b^2*c + C*a*b^2*d)*x^2 - 6*sqrt(a*b)*((C*a*b - A*b^2)*c - (D*a^2 -
 B*a*b)*d)*arctan(sqrt(a*b)*x/a) + 6*(C*a*b^2*c - (D*a^2*b - B*a*b^2)*d)*x
 - 3*((D*a^2*b - B*a*b^2)*c + (C*a^2*b - A*a*b^2)*d)*log(b*x^2 + a))/(a*b^
3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(129) = 258.

Time = 1.29 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{Ddx^3}{3b}+x^2\left(\frac{Cd}{2b}+\frac{Dc}{2b}\right)+x\left(\frac{Bd}{b}+\frac{Cc}{b}-\frac{Dad}{b^2}\right)+\left(-\frac{-Abd-Bbc+Cad+Dac}{2b^2}\right.$$
$$\left.-\frac{\sqrt{-ab^5}(Ab^2c-Babd-Cabc+Da^2d)}{2ab^5}\right)\log\left(x+\frac{-Aabd-Babc+Ca^2d+Da^2c+2ab^2\left(-\frac{-Abd-Bb}{}\right.}{Ab^2c-Babd-Ca}\right.$$
$$+\left(-\frac{-Abd-Bbc+Cad+Dac}{2b^2}\right.$$
$$\left.+\frac{\sqrt{-ab^5}(Ab^2c-Babd-Cabc+Da^2d)}{2ab^5}\right)\log\left(x+\frac{-Aabd-Babc+Ca^2d+Da^2c+2ab^2\left(-\frac{-Abd-Bb}{}\right.}{Ab^2c-Babd-Ca}\right.$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
D*d*x**3/(3*b) + x**2*(C*d/(2*b) + D*c/(2*b)) + x*(B*d/b + C*c/b - D*a*d/b
**2) + (-(-A*b*d - B*b*c + C*a*d + D*a*c)/(2*b**2) - sqrt(-a*b**5)*(A*b**2
*c - B*a*b*d - C*a*b*c + D*a**2*d)/(2*a*b**5))*log(x + (-A*a*b*d - B*a*b*c
 + C*a**2*d + D*a**2*c + 2*a*b**2*(-(-A*b*d - B*b*c + C*a*d + D*a*c)/(2*b*
*2) - sqrt(-a*b**5)*(A*b**2*c - B*a*b*d - C*a*b*c + D*a**2*d)/(2*a*b**5)))
/(A*b**2*c - B*a*b*d - C*a*b*c + D*a**2*d)) + (-(-A*b*d - B*b*c + C*a*d +
D*a*c)/(2*b**2) + sqrt(-a*b**5)*(A*b**2*c - B*a*b*d - C*a*b*c + D*a**2*d)/
(2*a*b**5))*log(x + (-A*a*b*d - B*a*b*c + C*a**2*d + D*a**2*c + 2*a*b**2*(
-(-A*b*d - B*b*c + C*a*d + D*a*c)/(2*b**2) + sqrt(-a*b**5)*(A*b**2*c - B*a
*b*d - C*a*b*c + D*a**2*d)/(2*a*b**5)))/(A*b**2*c - B*a*b*d - C*a*b*c + D*
a**2*d))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = -\frac{((Da-Bb)c+(Ca-Ab)d)\log\left(bx^2+a\right)}{2\,b^2} - \frac{((Cab-Ab^2)c-(Da^2-Bab)d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Dbdx^3+3\,(Dbc+Cbd)x^2+6\,(Cbc-(Da-Bb)d)x}{6\,b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((D*a - B*b)*c + (C*a - A*b)*d)*log(b*x^2 + a)/b^2 - ((C*a*b - A*b^2)
*c - (D*a^2 - B*a*b)*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/6*(2*D*b
*d*x^3 + 3*(D*b*c + C*b*d)*x^2 + 6*(C*b*c - (D*a - B*b)*d)*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Dac-Bbc+Cad-Abd)\log(bx^2+a)}{2\,b^2} - \frac{(Cabc-Ab^2c-Da^2d+Babd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2dx^3+3\,Db^2cx^2+3\,Cb^2dx^2+6\,Cb^2cx-6\,Dabdx+6\,Bb^2dx}{6\,b^3}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a*c - B*b*c + C*a*d - A*b*d)*log(b*x^2 + a)/b^2 - (C*a*b*c - A*b^2
*c - D*a^2*d + B*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/6*(2*D*b
^2*d*x^3 + 3*D*b^2*c*x^2 + 3*C*b^2*d*x^2 + 6*C*b^2*c*x - 6*D*a*b*d*x + 6*B
*b^2*d*x)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{(c+d\,x)\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$
$$= \frac{6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,d^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2c-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2d-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d**2 + 6*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*d - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*
c**2 + 3*log(a + b*x**2)*a*b**2*d - 6*log(a + b*x**2)*a*b*c*d + 3*log(a +
b*x**2)*b**3*c - 6*a*b*d**2*x + 6*b**3*d*x + 6*b**2*c**2*x + 6*b**2*c*d*x*
*2 + 2*b**2*d**2*x**3)/(6*b**3)
```

3.29 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}$$

output
```
C*x/b+1/2*D*x^2/b+(A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)+1/2*
(B*b-D*a)*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{bx(2C+Dx)+\frac{2\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b\left(a+bx^2\right)}+\frac{C}{b}+\frac{Dx}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$	65

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$
$$= \left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2}, \frac{Dabx^2+2\,Cab}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D
*a*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*
a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(65) = 130$.

Time $= 0.41$ (sec) , antiderivative size $= 219$, normalized size of antiderivative $= 3.00$

$$\int \frac{A + Bx + Cx^2 + Dx^3}{a + bx^2}\,dx$$
$$= \frac{Cx}{b} + \frac{Dx^2}{2b} + \left(-\frac{-Bb + Da}{2b^2} - \frac{\sqrt{-ab^5}\,(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right)$$
$$+ \left(-\frac{-Bb + Da}{2b^2} + \frac{\sqrt{-ab^5}\,(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} + \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a
)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) -
 sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*
a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a*
*2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b*
*4)))/(-A*b**2 + C*a*b))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{Dx^2+2\,Cx}{2\,b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
 1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2} + \frac{Dbx^2+2\,Cbx}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x
^2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 16.90 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/
b + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(
1/2)*x)/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c-\log\left(b\,x^2+a\right)ad+\log\left(b\,x^2+a\right)b^2+2bcx+bd\,x^2}{2b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b - 2*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*c - log(a + b*x**2)*a*d + log(a + b*x**2)*b**2
+ 2*b*c*x + b*d*x**2)/(2*b**2)
```

3.30 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 166

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)}\,dx=\frac{Dx}{bd}+\frac{\left(Ab^2c-a(bcC-bBd+adD)\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\left(bc^2+ad^2\right)}+\frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\log(c+dx)}{d^2\left(bc^2+ad^2\right)}+\frac{(bBc-Abd+aCd-acD)\log\left(a+bx^2\right)}{2b\left(bc^2+ad^2\right)}$$

output
```
D*x/b/d+(A*b^2*c-a*(-B*b*d+C*b*c+D*a*d))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)
/b^(3/2)/(a*d^2+b*c^2)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^2/(a*d^2+
b*c^2)+1/2*(-A*b*d+B*b*c+C*a*d-D*a*c)*ln(b*x^2+a)/b/(a*d^2+b*c^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx = \frac{Dx}{bd} - \frac{(-Ab^2c+abcC-abBd+a^2dD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}(bc^2+ad^2)} + \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^2(bc^2+ad^2)} + \frac{(bBc-Abd+aCd-acD)\log(a+bx^2)}{2b(bc^2+ad^2)}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)),x]
```

output
```
(D*x)/(b*d) - ((-(A*b^2*c) + a*b*c*C - a*b*B*d + a^2*d*D)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*(b*c^2 + a*d^2)) + ((c^2*C*d - B*c*d^2 + A*d
^3 - c^3*D)*Log[c + d*x])/(d^2*(b*c^2 + a*d^2)) + ((b*B*c - A*b*d + a*C*d
- a*c*D)*Log[a + b*x^2])/(2*b*(b*c^2 + a*d^2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)(c+dx)}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{bx(-acD+aCd-Abd+bBc)-a(adD-bBd+bcC)+Ab^2c}{b(a+bx^2)(ad^2+bc^2)}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d(c+dx)(ad^2+bc^2)}+\frac{D}{bd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^2c-a(adD-bBd+bcC)\right)}{\sqrt{a}b^{3/2}\left(ad^2+bc^2\right)}+\frac{\log\left(a+bx^2\right)\left(-acD+aCd-Abd+bBc\right)}{2b\left(ad^2+bc^2\right)}+\frac{\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\left(ad^2+bc^2\right)}+\frac{Dx}{bd}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)),x]
```

output
```
(D*x)/(b*d) + ((A*b^2*c - a*(b*c*C - b*B*d + a*d*D))*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(Sqrt[a]*b^(3/2)*(b*c^2 + a*d^2)) + ((c^2*C*d - B*c*d^2 + A*d^3 -
c^3*D)*Log[c + d*x])/(d^2*(b*c^2 + a*d^2)) + ((b*B*c - A*b*d + a*C*d - a*c
*D)*Log[a + b*x^2])/(2*b*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

method	result
default	$\frac{Dx}{bd}+\frac{\frac{\left(-A\,b^2d+B\,b^2c+dabC-abcD\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(A\,b^2c+Babd-Cabc-a^2dD\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{(a\,d^2+b\,c^2)b}+\frac{\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)\ln(dx+c}{d^2(a\,d^2+b\,c^2)}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
D*x/b/d+1/(a*d^2+b*c^2)/b*(1/2*(-A*b^2*d+B*b^2*c+C*a*b*d-D*a*b*c)/b*ln(b*x
^2+a)+(A*b^2*c+B*a*b*d-C*a*b*c-D*a^2*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^2/(a*d^2+b*c^2)
```

Fricas [A] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx$$
$$= \left[\frac{((Cab-Ab^2)cd^2+(Da^2-Bab)d^3)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Dab^2c^2d+Da^2bd^3)x-((Da^2b-B\right.}{2\,(ab^3c^2d^2}\right.$$
$$-\frac{2\,((Cab-Ab^2)cd^2+(Da^2-Bab)d^3)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)-2\,(Dab^2c^2d+Da^2bd^3)x+((Da^2b-Bab^2}{2\,(ab^3c^2d^2+}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(((C*a*b - A*b^2)*c*d^2 + (D*a^2 - B*a*b)*d^3)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a*b^2*c^2*d + D*a^2*b*d^3)*x - (
(D*a^2*b - B*a*b^2)*c*d^2 - (C*a^2*b - A*a*b^2)*d^3)*log(b*x^2 + a) - 2*(D
*a*b^2*c^3 - C*a*b^2*c^2*d + B*a*b^2*c*d^2 - A*a*b^2*d^3)*log(d*x + c))/(a
*b^3*c^2*d^2 + a^2*b^2*d^4), -1/2*(2*((C*a*b - A*b^2)*c*d^2 + (D*a^2 - B*a
*b)*d^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a*b^2*c^2*d + D*a^2*b*d^3)
*x + ((D*a^2*b - B*a*b^2)*c*d^2 - (C*a^2*b - A*a*b^2)*d^3)*log(b*x^2 + a)
+ 2*(D*a*b^2*c^3 - C*a*b^2*c^2*d + B*a*b^2*c*d^2 - A*a*b^2*d^3)*log(d*x +
c))/(a*b^3*c^2*d^2 + a^2*b^2*d^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx = &-\frac{((Da-Bb)c-(Ca-Ab)d)\log(bx^2+a)}{2(b^2c^2+abd^2)} \\ &-\frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{bc^2d^2+ad^4} \\ &-\frac{((Cab-Ab^2)c+(Da^2-Bab)d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2+abd^2)\sqrt{ab}}+\frac{Dx}{bd}\end{aligned}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((D*a - B*b)*c - (C*a - A*b)*d)*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) -
(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c)/(b*c^2*d^2 + a*d^4) - ((C
*a*b - A*b^2)*c + (D*a^2 - B*a*b)*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 + a*b
*d^2)*sqrt(a*b)) + D*x/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx = -\frac{(Dac-Bbc-Cad+Abd)\log(bx^2+a)}{2(b^2c^2+abd^2)} - \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(|dx+c|)}{bc^2d^2+ad^4} - \frac{(Cabc-Ab^2c+Da^2d-Babd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2+abd^2)\sqrt{ab}} + \frac{Dx}{bd}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a*c - B*b*c - C*a*d + A*b*d)*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) -
(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(abs(d*x + c))/(b*c^2*d^2 + a*d^4)
- (C*a*b*c - A*b^2*c + D*a^2*d - B*a*b*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2
+ a*b*d^2)*sqrt(a*b)) + D*x/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\,(a+bx^2)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,d^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2c+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2d-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right.}{2b^2\,(a}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d**2 + 2*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c + 2*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*b**2*d - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*b*c**2 - log(a + b*x**2)*a*b**2*d + log(a + b*x**2)*b**3*c + 2*log(c + d*
x)*a*b**2*d - 2*log(c + d*x)*b**3*c + 2*a*b*d**2*x + 2*b**2*c**2*x)/(2*b**
2*(a*d**2 + b*c**2))
```

3.31 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 272

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)}\,dx &= -\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{d^2(bc^2+ad^2)(c+dx)} \\ &\quad+\frac{(Ab(bc^2-ad^2)-a(bc(cC-2Bd)-ad(Cd-2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(bc^2+ad^2)^2} \\ &\quad-\frac{(ad^2(2cCd-Bd^2-3c^2D)+bc(Bcd^2-2Ad^3-c^3D))\log(c+dx)}{d^2(bc^2+ad^2)^2} \\ &\quad+\frac{(b^2c(Bc-2Ad)+a^2d^2D+ab(2cCd-Bd^2-c^2D))\log(a+bx^2)}{2b(bc^2+ad^2)^2}\end{aligned}$$

output
```
-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^2/(a*d^2+b*c^2)/(d*x+c)+(A*b*(-a*d^2+b*c^
2)-a*(b*c*(-2*B*d+C*c)-a*d*(C*d-2*D*c)))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)
/b^(1/2)/(a*d^2+b*c^2)^2-(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-2*A*d^3+B*c
*d^2-D*c^3))*ln(d*x+c)/d^2/(a*d^2+b*c^2)^2+1/2*(b^2*c*(-2*A*d+B*c)+a^2*d^2
*D+a*b*(-B*d^2+2*C*c*d-D*c^2))*ln(b*x^2+a)/b/(a*d^2+b*c^2)^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx$$
$$= \frac{\frac{2(bc^2+ad^2)(-c^2Cd+Bcd^2-Ad^3+c^3D)}{d^2(c+dx)} + \frac{2\left(Ab(bc^2-ad^2)+a(bc(-cC+2Bd)+ad(Cd-2cD))\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{2(ad^2(-2cCd+Bd^2+3c^2D}{}}{2\left(bc^2+ad^2\right)^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)),x]
```

output
```
((2*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(d^2*(c + d*x)
) + (2*(A*b*(b*c^2 - a*d^2) + a*(b*c*(-(c*C) + 2*B*d) + a*d*(C*d - 2*c*D))
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (2*(a*d^2*(-2*c*C*d + B
*d^2 + 3*c^2*D) + b*c*(-(B*c*d^2) + 2*A*d^3 + c^3*D))*Log[c + d*x])/d^2 +
((b^2*c*(B*c - 2*A*d) + a^2*d^2*D - a*b*(-2*c*C*d + B*d^2 + c^2*D))*Log[a
+ b*x^2])/b)/(2*(b*c^2 + a*d^2)^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)\,(c+dx)^2}\,dx$$
$$\Big\downarrow \; 2160$$
$$\int \left(\frac{x\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)+Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-}{(a+bx^2)\left(ad^2+bc^2\right)^2}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{\log\left(a+bx^2\right)\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)}{2b\left(ad^2+bc^2\right)^2}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2cD))\right)}{\sqrt{a}\sqrt{b}\left(ad^2+bc^2\right)^2}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\log(c+dx)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-2Ad^3+Bcd^2+c^3(-D)\right)\right)}{d^2\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)),x]
```

output
```
-((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d^2*(b*c^2 + a*d^2)*(c + d*x))) + (
(A*b*(b*c^2 - a*d^2) - a*(b*c*(c*C - 2*B*d) - a*d*(C*d - 2*c*D)))*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(b*c^2 + a*d^2)^2) - ((a*d^2*(2*c*C*
d - B*d^2 - 3*c^2*D) + b*c*(B*c*d^2 - 2*A*d^3 - c^3*D))*Log[c + d*x])/(d^2
*(b*c^2 + a*d^2)^2) + ((b^2*c*(B*c - 2*A*d) + a^2*d^2*D + a*b*(2*c*C*d - B
*d^2 - c^2*D))*Log[a + b*x^2])/(2*b*(b*c^2 + a*d^2)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{\left(-2A\,b^2cd-Bab\,d^2+B\,b^2c^2+2abcdC+a^2d^2D-Dab\,c^2\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(-Aab\,d^2+A\,b^2c^2+2abBcd+a^2C\,d^2-Cab\,c^2-2a^2cdD\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{\left(a\,d^2+b\,c^2\right)^2}-$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2+b*c^2)^2*(1/2*(-2*A*b^2*c*d-B*a*b*d^2+B*b^2*c^2+2*C*a*b*c*d+D*a^2
*d^2-D*a*b*c^2)/b*ln(b*x^2+a)+(-A*a*b*d^2+A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2-
C*a*b*c^2-2*D*a^2*c*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-(A*d^3-B*c*d^2
+C*c^2*d-D*c^3)/d^2/(a*d^2+b*c^2)/(d*x+c)+(2*A*b*c*d^3+B*a*d^4-B*b*c^2*d^2
-2*C*a*c*d^3+3*D*a*c^2*d^2+D*b*c^4)/(a*d^2+b*c^2)^2/d^2*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(258) = 516$.

Time = 74.55 (sec) , antiderivative size = 1136, normalized size of antiderivative = 4.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*D*a*b^2*c^5 - 2*C*a*b^2*c^4*d + 2*B*a^2*b*c*d^4 - 2*A*a^2*b*d^5 +
2*(D*a^2*b + B*a*b^2)*c^3*d^2 - 2*(C*a^2*b + A*a*b^2)*c^2*d^3 - ((C*a*b -
A*b^2)*c^3*d^2 + 2*(D*a^2 - B*a*b)*c^2*d^3 - (C*a^2 - A*a*b)*c*d^4 + ((C*a
*b - A*b^2)*c^2*d^3 + 2*(D*a^2 - B*a*b)*c*d^4 - (C*a^2 - A*a*b)*d^5)*x)*sq
rt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - ((D*a^2*b - B*a*b
^2)*c^3*d^2 - 2*(C*a^2*b - A*a*b^2)*c^2*d^3 - (D*a^3 - B*a^2*b)*c*d^4 + ((
D*a^2*b - B*a*b^2)*c^2*d^3 - 2*(C*a^2*b - A*a*b^2)*c*d^4 - (D*a^3 - B*a^2*
b)*d^5)*x)*log(b*x^2 + a) + 2*(D*a*b^2*c^5 + B*a^2*b*c*d^4 + (3*D*a^2*b -
B*a*b^2)*c^3*d^2 - 2*(C*a^2*b - A*a*b^2)*c^2*d^3 + (D*a*b^2*c^4*d + B*a^2*
b*d^5 + (3*D*a^2*b - B*a*b^2)*c^2*d^3 - 2*(C*a^2*b - A*a*b^2)*c*d^4)*x)*lo
g(d*x + c))/(a*b^3*c^5*d^2 + 2*a^2*b^2*c^3*d^4 + a^3*b*c*d^6 + (a*b^3*c^4*
d^3 + 2*a^2*b^2*c^2*d^5 + a^3*b*d^7)*x), 1/2*(2*D*a*b^2*c^5 - 2*C*a*b^2*c^
4*d + 2*B*a^2*b*c*d^4 - 2*A*a^2*b*d^5 + 2*(D*a^2*b + B*a*b^2)*c^3*d^2 - 2*
(C*a^2*b + A*a*b^2)*c^2*d^3 - 2*((C*a*b - A*b^2)*c^3*d^2 + 2*(D*a^2 - B*a*
b)*c^2*d^3 - (C*a^2 - A*a*b)*c*d^4 + ((C*a*b - A*b^2)*c^2*d^3 + 2*(D*a^2 -
 B*a*b)*c*d^4 - (C*a^2 - A*a*b)*d^5)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
((D*a^2*b - B*a*b^2)*c^3*d^2 - 2*(C*a^2*b - A*a*b^2)*c^2*d^3 - (D*a^3 - B*
a^2*b)*c*d^4 + ((D*a^2*b - B*a*b^2)*c^2*d^3 - 2*(C*a^2*b - A*a*b^2)*c*d^4
- (D*a^3 - B*a^2*b)*d^5)*x)*log(b*x^2 + a) + 2*(D*a*b^2*c^5 + B*a^2*b*c*d^
4 + (3*D*a^2*b - B*a*b^2)*c^3*d^2 - 2*(C*a^2*b - A*a*b^2)*c^2*d^3 + (D*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2 (a + bx^2)} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx$$
$$= -\frac{((Dab-Bb^2)c^2-2\,(Cab-Ab^2)cd-(Da^2-Bab)d^2)\log\left(bx^2+a\right)}{2\,(b^3c^4+2\,ab^2c^2d^2+a^2bd^4)}$$
$$+\frac{(Dbc^4+Bad^4+(3\,Da-Bb)c^2d^2-2\,(Ca-Ab)cd^3)\log\left(dx+c\right)}{b^2c^4d^2+2\,abc^2d^4+a^2d^6}$$
$$-\frac{((Cab-Ab^2)c^2+2\,(Da^2-Bab)cd-(Ca^2-Aab)d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^4+2\,abc^2d^2+a^2d^4)\sqrt{ab}}$$
$$+\frac{Dc^3-Cc^2d+Bcd^2-Ad^3}{bc^3d^2+acd^4+(bc^2d^3+ad^5)x}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((D*a*b - B*b^2)*c^2 - 2*(C*a*b - A*b^2)*c*d - (D*a^2 - B*a*b)*d^2)*l
og(b*x^2 + a)/(b^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*d^4) + (D*b*c^4 + B*a*d^4
 + (3*D*a - B*b)*c^2*d^2 - 2*(C*a - A*b)*c*d^3)*log(d*x + c)/(b^2*c^4*d^2
+ 2*a*b*c^2*d^4 + a^2*d^6) - ((C*a*b - A*b^2)*c^2 + 2*(D*a^2 - B*a*b)*c*d
- (C*a^2 - A*a*b)*d^2)*arctan(b*x/sqrt(a*b))/((b^2*c^4 + 2*a*b*c^2*d^2 + a
^2*d^4)*sqrt(a*b)) + (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)/(b*c^3*d^2 + a*c*
d^4 + (b*c^2*d^3 + a*d^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx = -\frac{(Dabc^2-Bb^2c^2-2\,Cabcd+2\,Ab^2cd-Da^2d^2+Babd^2)\log\left(b-\frac{2\,bc}{dx+c}+\frac{bc^2}{(dx+c)^2}+\frac{ad^2}{(dx+c)^2}\right)}{2\,(b^3c^4+2\,ab^2c^2d^2+a^2bd^4)} + \frac{\frac{Dc^3d^2}{dx+c}-\frac{Cc^2d^3}{dx+c}+\frac{Bcd^4}{dx+c}-\frac{Ad^5}{dx+c}}{bc^2d^4+ad^6} - \frac{D\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{bd^2} - \frac{(Cabc^2d^2-Ab^2c^2d^2+2\,Da^2cd^3-2\,Babcd^3-Ca^2d^4+Aabd^4)\arctan\left(\frac{bc-\frac{bc^2}{dx+c}-\frac{ad^2}{dx+c}}{\sqrt{ab}d}\right)}{(b^2c^4+2\,abc^2d^2+a^2d^4)\sqrt{ab}d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a*b*c^2 - B*b^2*c^2 - 2*C*a*b*c*d + 2*A*b^2*c*d - D*a^2*d^2 + B*a*
b*d^2)*log(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d^2/(d*x + c)^2)/(b
^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*d^4) + (D*c^3*d^2/(d*x + c) - C*c^2*d^3/(
d*x + c) + B*c*d^4/(d*x + c) - A*d^5/(d*x + c))/(b*c^2*d^4 + a*d^6) - D*lo
g(abs(d*x + c)/((d*x + c)^2*abs(d)))/(b*d^2) - (C*a*b*c^2*d^2 - A*b^2*c^2*
d^2 + 2*D*a^2*c*d^3 - 2*B*a*b*c*d^3 - C*a^2*d^4 + A*a*b*d^4)*arctan((b*c -
 b*c^2/(d*x + c) - a*d^2/(d*x + c))/(sqrt(a*b)*d))/((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)*sqrt(a*b)*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)\,{(c+d\,x)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.67

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c**2*d**3 - 2*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c*d**4*x - 2*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a*c**3*d**3 - 2*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a*c**2*d**4*x + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*b**2*c**4*d + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**
2*c**3*d**2*x + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**3*
d**2 + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**2*d**3*x -
2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c**5*d - 2*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c**4*d**2*x + log(a + b*x**2)*a**2*c**2
*d**4 + log(a + b*x**2)*a**2*c*d**5*x - 2*log(a + b*x**2)*a*b**2*c**3*d**2
 - 2*log(a + b*x**2)*a*b**2*c**2*d**3*x - log(a + b*x**2)*a*b**2*c**2*d**3
 - log(a + b*x**2)*a*b**2*c*d**4*x + log(a + b*x**2)*a*b*c**4*d**2 + log(a
 + b*x**2)*a*b*c**3*d**3*x + log(a + b*x**2)*b**3*c**4*d + log(a + b*x**2)
*b**3*c**3*d**2*x + 4*log(c + d*x)*a*b**2*c**3*d**2 + 4*log(c + d*x)*a*b**
2*c**2*d**3*x + 2*log(c + d*x)*a*b**2*c**2*d**3 + 2*log(c + d*x)*a*b**2*c*
d**4*x + 2*log(c + d*x)*a*b*c**4*d**2 + 2*log(c + d*x)*a*b*c**3*d**3*x - 2
*log(c + d*x)*b**3*c**4*d - 2*log(c + d*x)*b**3*c**3*d**2*x + 2*log(c + d*
x)*b**2*c**6 + 2*log(c + d*x)*b**2*c**5*d*x + 2*a**2*b*d**5*x + 2*a*b**2*c
**2*d**3*x - 2*a*b**2*c*d**4*x - 2*b**3*c**3*d**2*x)/(2*b*c*d*(a**2*c*d**4
 + a**2*d**5*x + 2*a*b*c**3*d**2 + 2*a*b*c**2*d**3*x + b**2*c**5 + b**2...
```

3.32 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3(a+bx^2)} dx$

Optimal result

Integrand size = 32, antiderivative size = 398

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3(a+bx^2)}\,dx\\
&=\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{ad^2(2cCd-Bd^2-3c^2D)+bc(Bcd^2-2Ad^3-c^3D)}{d^2(bc^2+ad^2)^2(c+dx)}\\
&\quad+\frac{(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC-3Bd)-a^2d^3D-abd(3cCd-Bd^2-3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(bc^2+ad^2)^3}\\
&\quad-\frac{(b^2c^2(Bc-3Ad)-a^2d^2(Cd-3cD)+ab(3c^2Cd-3Bcd^2+Ad^3-c^3D))\log(c+dx)}{(bc^2+ad^2)^3}\\
&\quad+\frac{(b^2c^2(Bc-3Ad)-a^2d^2(Cd-3cD)+ab(3c^2Cd-3Bcd^2+Ad^3-c^3D))\log(a+bx^2)}{2(bc^2+ad^2)^3}
\end{aligned}
$$

output

```
1/2*(-A*d^3+B*c*d^2-C*c^2*d+D*c^3)/d^2/(a*d^2+b*c^2)/(d*x+c)^2+(a*d^2*(-B*
d^2+2*C*c*d-3*D*c^2)+b*c*(-2*A*d^3+B*c*d^2-D*c^3))/d^2/(a*d^2+b*c^2)^2/(d*
x+c)+(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(-3*B*d+C*c)-a^2*d^3*D-a*b*d*(-B
*d^2+3*C*c*d-3*D*c^2)))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)/(a*d^2+b
*c^2)^3-(b^2*c^2*(-3*A*d+B*c)-a^2*d^2*(C*d-3*D*c)+a*b*(A*d^3-3*B*c*d^2+3*C
*c^2*d-D*c^3))*ln(d*x+c)/(a*d^2+b*c^2)^3+1/2*(b^2*c^2*(-3*A*d+B*c)-a^2*d^2
*(C*d-3*D*c)+a*b*(A*d^3-3*B*c*d^2+3*C*c^2*d-D*c^3))*ln(b*x^2+a)/(a*d^2+b*c
^2)^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\left(a+bx^2\right)}\,dx$$
$$= \frac{\frac{(bc^2+ad^2)^2(-c^2Cd+Bcd^2-Ad^3+c^3D)}{d^2(c+dx)^2}-\frac{2(bc^2+ad^2)(ad^2(-2cCd+Bd^2+3c^2D)+bc(-Bcd^2+2Ad^3+c^3D))}{d^2(c+dx)}+\frac{2(Ab^2c(bc^2-3ad^2)+a(b^2c}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*(a + b*x^2)),x]
```

output
```
(((b*c^2 + a*d^2)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(d^2*(c + d*x)
^2) - (2*(b*c^2 + a*d^2)*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-(B*c*
d^2) + 2*A*d^3 + c^3*D)))/(d^2*(c + d*x)) + (2*(A*b^2*c*(b*c^2 - 3*a*d^2)
+ a*(b^2*c^2*(-(c*C) + 3*B*d) + a^2*d^3*D - a*b*d*(-3*c*C*d + B*d^2 + 3*c^
2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + 2*(b^2*c^2*(-(B*c)
 + 3*A*d) + a^2*d^2*(C*d - 3*c*D) + a*b*(-3*c^2*C*d + 3*B*c*d^2 - A*d^3 +
c^3*D))*Log[c + d*x] + (b^2*c^2*(B*c - 3*A*d) + a^2*d^2*(-(C*d) + 3*c*D) +
 a*b*(3*c^2*C*d - 3*B*c*d^2 + A*d^3 - c^3*D))*Log[a + b*x^2])/(2*(b*c^2 +
a*d^2)^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)(c+dx)^3}\,dx$$
$$\Big\downarrow \; 2160$$
$$\int \left(\frac{bx\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)-a\left(-a^2d^3D-abd(-Bc\right.}{\left(a+bx^2\right)\left(ad^2+bc^2\right)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^2c(bc^2-3ad^2)-a\left(-a^2d^3D-abd\left(-Bd^2-3c^2D+3cCd\right)+b^2c^2(cC-3Bd)\right)\right)}{\sqrt{a}\sqrt{b}\left(ad^2+bc^2\right)^3}+\frac{\log\left(a+bx^2\right)\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)}{2\left(ad^2+bc^2\right)^3}-\frac{\log(c+dx)\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)}{\left(ad^2+bc^2\right)^3}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}+\frac{ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-2Ad^3+Bcd^2+c^3(-D)\right)}{d^2(c+dx)\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*(a + b*x^2)),x]
```

output
```
-1/2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d^2*(b*c^2 + a*d^2)*(c + d*x)^2)
 + (a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(B*c*d^2 - 2*A*d^3 - c^3*D))/(
d^2*(b*c^2 + a*d^2)^2*(c + d*x)) + ((A*b^2*c*(b*c^2 - 3*a*d^2) - a*(b^2*c^
2*(c*C - 3*B*d) - a^2*d^3*D - a*b*d*(3*c*C*d - B*d^2 - 3*c^2*D)))*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(b*c^2 + a*d^2)^3) - ((b^2*c^2*(B*c
- 3*A*d) - a^2*d^2*(C*d - 3*c*D) + a*b*(3*c^2*C*d - 3*B*c*d^2 + A*d^3 - c^
3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^3 + ((b^2*c^2*(B*c - 3*A*d) - a^2*d^2*
(C*d - 3*c*D) + a*b*(3*c^2*C*d - 3*B*c*d^2 + A*d^3 - c^3*D))*Log[a + b*x^2
])/(2*(b*c^2 + a*d^2)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{\left(A\,d^{3}a\,b^{2}-3A\,b^{3}c^{2}d-3Ba\,b^{2}c\,d^{2}+B\,b^{3}c^{3}-C\,a^{2}b\,d^{3}+3Ca\,b^{2}c^{2}d+3Da^{2}bc\,d^{2}-Da\,b^{2}c^{3}\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-3Aa\,b^{2}c\,d^{2}+A\,b^{3}c^{3}-B\,a^{2}b\,d^{3}+3Ba\,b^{2}c\right.}{}}{\left(a\,d^{2}+b\,c^{2}\right)^{3}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2+b*c^2)^3*(1/2*(A*a*b^2*d^3-3*A*b^3*c^2*d-3*B*a*b^2*c*d^2+B*b^3*c^
3-C*a^2*b*d^3+3*C*a*b^2*c^2*d+3*D*a^2*b*c*d^2-D*a*b^2*c^3)/b*ln(b*x^2+a)+(
-3*A*a*b^2*c*d^2+A*b^3*c^3-B*a^2*b*d^3+3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2-C*a
*b^2*c^3+D*a^3*d^3-3*D*a^2*b*c^2*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1
/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^2/(a*d^2+b*c^2)/(d*x+c)^2-(2*A*b*c*d^3+
B*a*d^4-B*b*c^2*d^2-2*C*a*c*d^3+3*D*a*c^2*d^2+D*b*c^4)/(a*d^2+b*c^2)^2/d^2
/(d*x+c)-(A*a*b*d^3-3*A*b^2*c^2*d-3*B*a*b*c*d^2+B*b^2*c^3-C*a^2*d^3+3*C*a*
b*c^2*d+3*D*a^2*c*d^2-D*a*b*c^3)/(a*d^2+b*c^2)^3*ln(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**3/(b*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx = -\frac{((Dab-Bb^2)c^3-3\,(Cab-Ab^2)c^2d-3\,(Da^2-Bab)cd^2+(Ca^2-Aab)d^3)\log\left(bx^2+a\right)}{2\,(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6)} + \frac{((Dab-Bb^2)c^3-3\,(Cab-Ab^2)c^2d-3\,(Da^2-Bab)cd^2+(Ca^2-Aab)d^3)\log\left(dx+c\right)}{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6} - \frac{((Cab^2-Ab^3)c^3+3\,(Da^2b-Bab^2)c^2d-3\,(Ca^2b-Aab^2)cd^2-(Da^3-Ba^2b)d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6)\sqrt{ab}} - \frac{Dbc^5+Cbc^4d+Bacd^4+Aad^5+(5\,Da-3\,Bb)c^3d^2-(3\,Ca-5\,Ab)c^2d^3+2\,(Dbc^4d+Bad^5+(3\,Da}{2\,(b^2c^6d^2+2\,abc^4d^4+a^2c^2d^6+(b^2c^4d^4+2\,abc^2d^6+a^2d^8)x^2+2\,(b^2c^5d^3+2\,abc^3d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*((D*a*b - B*b^2)*c^3 - 3*(C*a*b - A*b^2)*c^2*d - 3*(D*a^2 - B*a*b)*c*
d^2 + (C*a^2 - A*a*b)*d^3)*log(b*x^2 + a)/(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a
^2*b*c^2*d^4 + a^3*d^6) + ((D*a*b - B*b^2)*c^3 - 3*(C*a*b - A*b^2)*c^2*d -
 3*(D*a^2 - B*a*b)*c*d^2 + (C*a^2 - A*a*b)*d^3)*log(d*x + c)/(b^3*c^6 + 3*
a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6) - ((C*a*b^2 - A*b^3)*c^3 + 3*(D
*a^2*b - B*a*b^2)*c^2*d - 3*(C*a^2*b - A*a*b^2)*c*d^2 - (D*a^3 - B*a^2*b)*
d^3)*arctan(b*x/sqrt(a*b))/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 +
 a^3*d^6)*sqrt(a*b)) - 1/2*(D*b*c^5 + C*b*c^4*d + B*a*c*d^4 + A*a*d^5 + (5
*D*a - 3*B*b)*c^3*d^2 - (3*C*a - 5*A*b)*c^2*d^3 + 2*(D*b*c^4*d + B*a*d^5 +
 (3*D*a - B*b)*c^2*d^3 - 2*(C*a - A*b)*c*d^4)*x)/(b^2*c^6*d^2 + 2*a*b*c^4*
d^4 + a^2*c^2*d^6 + (b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8)*x^2 + 2*(b^2*c
^5*d^3 + 2*a*b*c^3*d^5 + a^2*c*d^7)*x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx = \\
&-\frac{(Dabc^3 - Bb^2c^3 - 3\,Cabc^2d + 3\,Ab^2c^2d - 3\,Da^2cd^2 + 3\,Babcd^2 + Ca^2d^3 - Aabd^3)\log\left(bx^2+a\right)}{2\left(b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6\right)} \\
&+\frac{(Dabc^3d - Bb^2c^3d - 3\,Cabc^2d^2 + 3\,Ab^2c^2d^2 - 3\,Da^2cd^3 + 3\,Babcd^3 + Ca^2d^4 - Aabd^4)\log\left(|dx+c|\right)}{b^3c^6d + 3\,ab^2c^4d^3 + 3\,a^2bc^2d^5 + a^3d^7} \\
&-\frac{(Cab^2c^3 - Ab^3c^3 + 3\,Da^2bc^2d - 3\,Bab^2c^2d - 3\,Ca^2bcd^2 + 3\,Aab^2cd^2 - Da^3d^3 + Ba^2bd^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6\right)\sqrt{ab}} \\
&-\frac{Db^2c^7 + Cb^2c^6d + 6\,Dabc^5d^2 - 3\,Bb^2c^5d^2 - 2\,Cabc^4d^3 + 5\,Ab^2c^4d^3 + 5\,Da^2c^3d^4 - 2\,Babc^3d^4 - 3\,Ca^2c}{}
\end{aligned}
$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a*b*c^3 - B*b^2*c^3 - 3*C*a*b*c^2*d + 3*A*b^2*c^2*d - 3*D*a^2*c*d^
2 + 3*B*a*b*c*d^2 + C*a^2*d^3 - A*a*b*d^3)*log(b*x^2 + a)/(b^3*c^6 + 3*a*b
^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6) + (D*a*b*c^3*d - B*b^2*c^3*d - 3*C
*a*b*c^2*d^2 + 3*A*b^2*c^2*d^2 - 3*D*a^2*c*d^3 + 3*B*a*b*c*d^3 + C*a^2*d^4
 - A*a*b*d^4)*log(abs(d*x + c))/(b^3*c^6*d + 3*a*b^2*c^4*d^3 + 3*a^2*b*c^2
*d^5 + a^3*d^7) - (C*a*b^2*c^3 - A*b^3*c^3 + 3*D*a^2*b*c^2*d - 3*B*a*b^2*c
^2*d - 3*C*a^2*b*c*d^2 + 3*A*a*b^2*c*d^2 - D*a^3*d^3 + B*a^2*b*d^3)*arctan
(b*x/sqrt(a*b))/((b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*s
qrt(a*b)) - 1/2*(D*b^2*c^7 + C*b^2*c^6*d + 6*D*a*b*c^5*d^2 - 3*B*b^2*c^5*d
^2 - 2*C*a*b*c^4*d^3 + 5*A*b^2*c^4*d^3 + 5*D*a^2*c^3*d^4 - 2*B*a*b*c^3*d^4
 - 3*C*a^2*c^2*d^5 + 6*A*a*b*c^2*d^5 + B*a^2*c*d^6 + A*a^2*d^7 + 2*(D*b^2*
c^6*d + 4*D*a*b*c^4*d^3 - B*b^2*c^4*d^3 - 2*C*a*b*c^3*d^4 + 2*A*b^2*c^3*d^
4 + 3*D*a^2*c^2*d^5 - 2*C*a^2*c*d^6 + 2*A*a*b*c*d^6 + B*a^2*d^7)*x)/((b*c^
2 + a*d^2)^3*(d*x + c)^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)\,\left(c+d\,x\right)^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1577, normalized size of antiderivative = 3.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c**3*d**5 + 4*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c**2*d**6*x + 2*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c*d**7*x**2 - 6*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**4*d**3 - 12*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a*b**2*c**3*d**4*x - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a*b**2*c**3*d**4 - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**2*c**2*d**5*x**2 - 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*a*b**2*c**2*d**5*x - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b**2*c*d**6*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3
*c**6*d + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**5*d**2*x
 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**5*d**2 + 2*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*d**3*x**2 + 12*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**4*d**3*x + 6*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**3*d**4*x**2 - 2*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*b**2*c**7*d - 4*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*b**2*c**6*d**2*x - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*b**2*c**5*d**3*x**2 + log(a + b*x**2)*a**2*b**2*c**3*d**4 + 2*log
(a + b*x**2)*a**2*b**2*c**2*d**5*x + log(a + b*x**2)*a**2*b**2*c*d**6*x**2
 + 2*log(a + b*x**2)*a**2*b*c**4*d**4 + 4*log(a + b*x**2)*a**2*b*c**3*d**5
*x + 2*log(a + b*x**2)*a**2*b*c**2*d**6*x**2 - 3*log(a + b*x**2)*a*b**3...
```

3.33 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 388

$$
\begin{aligned}
&\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} dx \\
&= -\frac{d(2ad^2D-b(3cCd+Bd^2+3c^2D))x}{b^3} + \frac{d^2(Cd+3cD)x^2}{2b^2} \\
&\quad + \frac{d^3Dx^3}{3b^2} - \frac{b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{2b^3(a+bx^2)} \\
&\quad + \frac{(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC+3Bd)+a^2d^3D-abd(3cCd+Bd^2+3c^2D)))x}{2ab^3(a+bx^2)} \\
&\quad + \frac{(Ab^2c(bc^2+3ad^2)+a(b^2c^2(cC+3Bd)+5a^2d^3D-3abd(3cCd+Bd^2+3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}} \\
&\quad - \frac{(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\log(a+bx^2)}{2b^3}
\end{aligned}
$$

output
```
-d*(2*a*d^2*D-b*(B*d^2+3*C*c*d+3*D*c^2))*x/b^3+1/2*d^2*(C*d+3*D*c)*x^2/b^2
+1/3*d^3*D*x^3/b^2-1/2*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3
+3*B*c*d^2+3*C*c^2*d+D*c^3))/b^3/(b*x^2+a)+1/2*(A*b^2*c*(-3*a*d^2+b*c^2)-a
*(b^2*c^2*(3*B*d+C*c)+a^2*d^3*D-a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a/b^3/(b
*x^2+a)+1/2*(A*b^2*c*(3*a*d^2+b*c^2)+a*(b^2*c^2*(3*B*d+C*c)+5*a^2*d^3*D-3*
a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(7/2)-
1/2*(2*a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*ln(b*x^2+a)/
b^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.89

$$\begin{aligned}
&\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx \\
&= \frac{d(-2ad^2D+b(3cCd+Bd^2+3c^2D))\,x}{b^3}+\frac{d^2(Cd+3cD)x^2}{2b^2}+\frac{d^3Dx^3}{3b^2} \\
&\quad+\frac{Ab^3c^3x-a^3d^2(Cd+3cD+dDx)-ab^2c(c^2Cx+3Ad(c+dx)+Bc(c+3dx))+a^2b(c^3D+d^3(A+}{2ab^3\,(a+bx^2)} \\
&\quad+\frac{(Ab^2c(bc^2+3ad^2)+a(b^2c^2(cC+3Bd)+5a^2d^3D-3abd(3cCd+Bd^2+3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}} \\
&\quad+\frac{(-2ad^2(Cd+3cD)+b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(d*(-2*a*d^2*D + b*(3*c*C*d + B*d^2 + 3*c^2*D))*x)/b^3 + (d^2*(C*d + 3*c*D
)*x^2)/(2*b^2) + (d^3*D*x^3)/(3*b^2) + (A*b^3*c^3*x - a^3*d^2*(C*d + 3*c*D
 + d*D*x) - a*b^2*c*(c^2*C*x + 3*A*d*(c + d*x) + B*c*(c + 3*d*x)) + a^2*b*
(c^3*D + d^3*(A + B*x) + 3*c*d^2*(B + C*x) + 3*c^2*d*(C + D*x)))/(2*a*b^3*
(a + b*x^2)) + ((A*b^2*c*(b*c^2 + 3*a*d^2) + a*(b^2*c^2*(c*C + 3*B*d) + 5*
a^2*d^3*D - 3*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(7/2)) + ((-2*a*d^2*(C*d + 3*c*D) + b*(3*c^2*C*d + 3*B*c
*d^2 + A*d^3 + c^3*D))*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2176, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(c+dx)^2\left(2adDx^2-2(Abd-2aCd-acD)x+Abc+a\left(cC+3Bd-\frac{3adD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(c+dx)^2\left(2adDx^2-2(Abd-2aCd-acD)x+Abc+acC+3ad\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{2160}$$

$$\int \Big(\frac{2aDx^2d^3}{b}-\frac{2(Abd-2aCd-3acD)xd^2}{b}-\left(3Acd+\frac{a(5ad^2D-3b(2Dc^2+3Cdc+Bd^2))}{b^2}\right)d+\frac{Ac(bc^2+3ad^2)b^2-2a(2ad^2(Cd+3cD)-b$$

$$2ab$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a(5a^2d^3D-3abd(Bd^2+3c^2D+3cCd)+b^2c^2(3Bd+cC))+Ab^2c(3ad^2+bc^2))}{\sqrt{a}b^{5/2}}-dx\Big(\frac{a(5ad^2D-3b(Bd^2+2c^2D+3cCd))}{b^2}+3Ac$$

$$2ab$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

input
```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^3)/(a*b*(a + b*x^2)) + (
-(d*(3*A*c*d + (a*(5*a*d^2*D - 3*b*(3*c*C*d + B*d^2 + 2*c^2*D)))/b^2)*x) -
 (d^2*(A*b*d - 2*a*C*d - 3*a*c*D)*x^2)/b + (2*a*d^3*D*x^3)/(3*b) + ((A*b^2
*c*(b*c^2 + 3*a*d^2) + a*(b^2*c^2*(c*C + 3*B*d) + 5*a^2*d^3*D - 3*a*b*d*(3
*c*C*d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
 - (a*(2*a*d^2*(C*d + 3*c*D) - b*(3*c^2*C*d + 3*B*c*d^2 + A*d^3 + c^3*D))*
Log[a + b*x^2])/b^2)/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.09

method	result
default	$\frac{d\left(\frac{1}{3}Db\,x^{3}d^{2}+\frac{1}{2}Cb\,x^{2}d^{2}+\frac{3}{2}Dbcd\,x^{2}+Bb\,d^{2}x+3Cbcdx-2a\,d^{2}Dx+3Db\,c^{2}x\right)}{b^{3}}+\frac{-\frac{\left(3Aa\,b^{2}c\,d^{2}-A\,b^{3}c^{3}-B\,a^{2}b\,d^{3}+3Ba\,b^{2}c^{2}d-3C\,a^{2}bc\right.}{2a}}{}$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d/b^3*(1/3*D*b*x^3*d^2+1/2*C*b*x^2*d^2+3/2*D*b*c*d*x^2+B*b*d^2*x+3*C*b*c*d
*x-2*a*d^2*D*x+3*D*b*c^2*x)+1/b^3*((-1/2*(3*A*a*b^2*c*d^2-A*b^3*c^3-B*a^2*
b*d^3+3*B*a*b^2*c^2*d-3*C*a^2*b*c*d^2+C*a*b^2*c^3+D*a^3*d^3-3*D*a^2*b*c^2*
d)/a*x+1/2*A*a*b*d^3-3/2*A*b^2*c^2*d+3/2*B*a*b*c*d^2-1/2*B*b^2*c^3-1/2*C*a
^2*d^3+3/2*C*a*b*c^2*d-3/2*D*a^2*c*d^2+1/2*D*a*b*c^3)/(b*x^2+a)+1/2/a*(1/2
*(2*A*a*b^2*d^3+6*B*a*b^2*c*d^2-4*C*a^2*b*d^3+6*C*a*b^2*c^2*d-12*D*a^2*b*c
*d^2+2*D*a*b^2*c^3)/b*ln(b*x^2+a)+(3*A*a*b^2*c*d^2+A*b^3*c^3-3*B*a^2*b*d^3
+3*B*a*b^2*c^2*d-9*C*a^2*b*c*d^2+C*a*b^2*c^3+5*D*a^3*d^3-9*D*a^2*b*c^2*d)/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1394, normalized size of antiderivative = 3.59

$$\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*D*a^2*b^3*d^3*x^5 + 6*(3*D*a^2*b^3*c*d^2 + C*a^2*b^3*d^3)*x^4 + 6
*(D*a^3*b^2 - B*a^2*b^3)*c^3 + 18*(C*a^3*b^2 - A*a^2*b^3)*c^2*d - 18*(D*a^
4*b - B*a^3*b^2)*c*d^2 - 6*(C*a^4*b - A*a^3*b^2)*d^3 + 4*(9*D*a^2*b^3*c^2*
d + 9*C*a^2*b^3*c*d^2 - (5*D*a^3*b^2 - 3*B*a^2*b^3)*d^3)*x^3 + 6*(3*D*a^3*
b^2*c*d^2 + C*a^3*b^2*d^3)*x^2 + 3*((C*a^2*b^2 + A*a*b^3)*c^3 - 3*(3*D*a^3
*b - B*a^2*b^2)*c^2*d - 3*(3*C*a^3*b - A*a^2*b^2)*c*d^2 + (5*D*a^4 - 3*B*a
^3*b)*d^3 + ((C*a*b^3 + A*b^4)*c^3 - 3*(3*D*a^2*b^2 - B*a*b^3)*c^2*d - 3*(
3*C*a^2*b^2 - A*a*b^3)*c*d^2 + (5*D*a^3*b - 3*B*a^2*b^2)*d^3)*x^2)*sqrt(-a
*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*((C*a^2*b^3 - A*a*b^
4)*c^3 - 3*(3*D*a^3*b^2 - B*a^2*b^3)*c^2*d - 3*(3*C*a^3*b^2 - A*a^2*b^3)*c
*d^2 + (5*D*a^4*b - 3*B*a^3*b^2)*d^3)*x + 6*(D*a^3*b^2*c^3 + 3*C*a^3*b^2*c
^2*d - 3*(2*D*a^4*b - B*a^3*b^2)*c*d^2 - (2*C*a^4*b - A*a^3*b^2)*d^3 + (D*
a^2*b^3*c^3 + 3*C*a^2*b^3*c^2*d - 3*(2*D*a^3*b^2 - B*a^2*b^3)*c*d^2 - (2*C
*a^3*b^2 - A*a^2*b^3)*d^3)*x^2)*log(b*x^2 + a))/(a^2*b^5*x^2 + a^3*b^4), 1
/6*(2*D*a^2*b^3*d^3*x^5 + 3*(3*D*a^2*b^3*c*d^2 + C*a^2*b^3*d^3)*x^4 + 3*(D
*a^3*b^2 - B*a^2*b^3)*c^3 + 9*(C*a^3*b^2 - A*a^2*b^3)*c^2*d - 9*(D*a^4*b -
 B*a^3*b^2)*c*d^2 - 3*(C*a^4*b - A*a^3*b^2)*d^3 + 2*(9*D*a^2*b^3*c^2*d + 9
*C*a^2*b^3*c*d^2 - (5*D*a^3*b^2 - 3*B*a^2*b^3)*d^3)*x^3 + 3*(3*D*a^3*b^2*c
*d^2 + C*a^3*b^2*d^3)*x^2 + 3*((C*a^2*b^2 + A*a*b^3)*c^3 - 3*(3*D*a^3*b -
B*a^2*b^2)*c^2*d - 3*(3*C*a^3*b - A*a^2*b^2)*c*d^2 + (5*D*a^4 - 3*B*a^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(384) = 768$.

Time = 55.30 (sec) , antiderivative size = 1346, normalized size of antiderivative = 3.47

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
D*d**3*x**3/(3*b**2) + x**2*(C*d**3/(2*b**2) + 3*D*c*d**2/(2*b**2)) + x*(B
*d**3/b**2 + 3*C*c*d**2/b**2 - 2*D*a*d**3/b**3 + 3*D*c**2*d/b**2) + (-(-A*
b*d**3 - 3*B*b*c*d**2 + 2*C*a*d**3 - 3*C*b*c**2*d + 6*D*a*c*d**2 - D*b*c**
3)/(2*b**3) - sqrt(-a**3*b**7)*(3*A*a*b**2*c*d**2 + A*b**3*c**3 - 3*B*a**2
*b*d**3 + 3*B*a*b**2*c**2*d - 9*C*a**2*b*c*d**2 + C*a*b**2*c**3 + 5*D*a**3
*d**3 - 9*D*a**2*b*c**2*d)/(4*a**3*b**7))*log(x + (-2*A*a**2*b*d**3 - 6*B*
a**2*b*c*d**2 + 4*C*a**3*d**3 - 6*C*a**2*b*c**2*d + 12*D*a**3*c*d**2 - 2*D
*a**2*b*c**3 + 4*a**2*b**3*(-(-A*b*d**3 - 3*B*b*c*d**2 + 2*C*a*d**3 - 3*C*
b*c**2*d + 6*D*a*c*d**2 - D*b*c**3)/(2*b**3) - sqrt(-a**3*b**7)*(3*A*a*b**
2*c*d**2 + A*b**3*c**3 - 3*B*a**2*b*d**3 + 3*B*a*b**2*c**2*d - 9*C*a**2*b*
c*d**2 + C*a*b**2*c**3 + 5*D*a**3*d**3 - 9*D*a**2*b*c**2*d)/(4*a**3*b**7))
)/(3*A*a*b**2*c*d**2 + A*b**3*c**3 - 3*B*a**2*b*d**3 + 3*B*a*b**2*c**2*d -
 9*C*a**2*b*c*d**2 + C*a*b**2*c**3 + 5*D*a**3*d**3 - 9*D*a**2*b*c**2*d)) +
 (-(-A*b*d**3 - 3*B*b*c*d**2 + 2*C*a*d**3 - 3*C*b*c**2*d + 6*D*a*c*d**2 -
D*b*c**3)/(2*b**3) + sqrt(-a**3*b**7)*(3*A*a*b**2*c*d**2 + A*b**3*c**3 - 3
*B*a**2*b*d**3 + 3*B*a*b**2*c**2*d - 9*C*a**2*b*c*d**2 + C*a*b**2*c**3 + 5
*D*a**3*d**3 - 9*D*a**2*b*c**2*d)/(4*a**3*b**7))*log(x + (-2*A*a**2*b*d**3
 - 6*B*a**2*b*c*d**2 + 4*C*a**3*d**3 - 6*C*a**2*b*c**2*d + 12*D*a**3*c*d**
2 - 2*D*a**2*b*c**3 + 4*a**2*b**3*(-(-A*b*d**3 - 3*B*b*c*d**2 + 2*C*a*d**3
 - 3*C*b*c**2*d + 6*D*a*c*d**2 - D*b*c**3)/(2*b**3) + sqrt(-a**3*b**7)*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{(Da^2b-Bab^2)c^3+3\,(Ca^2b-Aab^2)c^2d-3\,(Da^3-Ba^2b)cd^2-(Ca^3-Aa^2b)d^3-((Cab^2-Ab^3)c^3-3}{2\,(ab^4x^2+a^2b^3)}$$
$$+\frac{(Dbc^3+3\,Cbc^2d-3\,(2\,Da-Bb)cd^2-(2\,Ca-Ab)d^3)\log{(bx^2+a)}}{2\,b^3}$$
$$+\frac{2\,Dbd^3x^3+3\,(3\,Dbcd^2+Cbd^3)x^2+6\,(3\,Dbc^2d+3\,Cbcd^2-(2\,Da-Bb)d^3)x}{6\,b^3}$$
$$+\frac{((Cab^2+Ab^3)c^3-3\,(3\,Da^2b-Bab^2)c^2d-3\,(3\,Ca^2b-Aab^2)cd^2+(5\,Da^3-3\,Ba^2b)d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((D*a^2*b - B*a*b^2)*c^3 + 3*(C*a^2*b - A*a*b^2)*c^2*d - 3*(D*a^3 - B*
a^2*b)*c*d^2 - (C*a^3 - A*a^2*b)*d^3 - ((C*a*b^2 - A*b^3)*c^3 - 3*(D*a^2*b
 - B*a*b^2)*c^2*d - 3*(C*a^2*b - A*a*b^2)*c*d^2 + (D*a^3 - B*a^2*b)*d^3)*x
)/(a*b^4*x^2 + a^2*b^3) + 1/2*(D*b*c^3 + 3*C*b*c^2*d - 3*(2*D*a - B*b)*c*d
^2 - (2*C*a - A*b)*d^3)*log(b*x^2 + a)/b^3 + 1/6*(2*D*b*d^3*x^3 + 3*(3*D*b
*c*d^2 + C*b*d^3)*x^2 + 6*(3*D*b*c^2*d + 3*C*b*c*d^2 - (2*D*a - B*b)*d^3)*
x)/b^3 + 1/2*((C*a*b^2 + A*b^3)*c^3 - 3*(3*D*a^2*b - B*a*b^2)*c^2*d - 3*(3
*C*a^2*b - A*a*b^2)*c*d^2 + (5*D*a^3 - 3*B*a^2*b)*d^3)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b^3)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{(Dbc^3+3\,Cbc^2d-6\,Dacd^2+3\,Bbcd^2-2\,Cad^3+Abd^3)\log(bx^2+a)}{2\,b^3}$$
$$+ \frac{(Cab^2c^3+Ab^3c^3-9\,Da^2bc^2d+3\,Bab^2c^2d-9\,Ca^2bcd^2+3\,Aab^2cd^2+5\,Da^3d^3-3\,Ba^2bd^3)\arctan\Big(}{2\,\sqrt{ab}ab^3}$$
$$+ \frac{Da^2bc^3-Bab^2c^3+3\,Ca^2bc^2d-3\,Aab^2c^2d-3\,Da^3cd^2+3\,Ba^2bcd^2-Ca^3d^3+Aa^2bd^3-(Cab^2c^3-}{2\,(bx^2+a)ab^3}$$
$$+ \frac{2\,Db^4d^3x^3+9\,Db^4cd^2x^2+3\,Cb^4d^3x^2+18\,Db^4c^2dx+18\,Cb^4cd^2x-12\,Dab^3d^3x+6\,Bb^4d^3x}{6\,b^6}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*b*c^3 + 3*C*b*c^2*d - 6*D*a*c*d^2 + 3*B*b*c*d^2 - 2*C*a*d^3 + A*b*d
^3)*log(b*x^2 + a)/b^3 + 1/2*(C*a*b^2*c^3 + A*b^3*c^3 - 9*D*a^2*b*c^2*d +
3*B*a*b^2*c^2*d - 9*C*a^2*b*c*d^2 + 3*A*a*b^2*c*d^2 + 5*D*a^3*d^3 - 3*B*a^
2*b*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3) + 1/2*(D*a^2*b*c^3 - B*a*
b^2*c^3 + 3*C*a^2*b*c^2*d - 3*A*a*b^2*c^2*d - 3*D*a^3*c*d^2 + 3*B*a^2*b*c*
d^2 - C*a^3*d^3 + A*a^2*b*d^3 - (C*a*b^2*c^3 - A*b^3*c^3 - 3*D*a^2*b*c^2*d
 + 3*B*a*b^2*c^2*d - 3*C*a^2*b*c*d^2 + 3*A*a*b^2*c*d^2 + D*a^3*d^3 - B*a^2
*b*d^3)*x)/((b*x^2 + a)*a*b^3) + 1/6*(2*D*b^4*d^3*x^3 + 9*D*b^4*c*d^2*x^2
+ 3*C*b^4*d^3*x^2 + 18*D*b^4*c^2*d*x + 18*C*b^4*c*d^2*x - 12*D*a*b^3*d^3*x
 + 6*B*b^4*d^3*x)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx = \int \frac{(c+d\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2} \,dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d**4 + 9*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c*d**2 - 9*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d**3 - 54*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**2*b*c**2*d**2 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**2*b*d**4*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**3*c**3 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*
*3*c**2*d + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*d**2*
x**2 - 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*d**3*x**2 +
3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**4 - 54*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**2*d**2*x**2 + 3*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**3*x**2 + 9*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**2*d*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*b**3*c**4*x**2 + 3*log(a + b*x**2)*a**3*b**2*d**3 - 24*
log(a + b*x**2)*a**3*b*c*d**3 + 9*log(a + b*x**2)*a**2*b**3*c*d**2 + 3*log
(a + b*x**2)*a**2*b**3*d**3*x**2 + 12*log(a + b*x**2)*a**2*b**2*c**3*d - 2
4*log(a + b*x**2)*a**2*b**2*c*d**3*x**2 + 9*log(a + b*x**2)*a*b**4*c*d**2*
x**2 + 12*log(a + b*x**2)*a*b**3*c**3*d*x**2 - 15*a**3*b*d**4*x - 9*a**2*b
**3*c*d**2*x - 3*a**2*b**3*d**3*x**2 + 9*a**2*b**3*d**3*x + 54*a**2*b**2*c
**2*d**2*x + 24*a**2*b**2*c*d**3*x**2 - 10*a**2*b**2*d**4*x**3 + 3*a*b**4*
c**3*x + 9*a*b**4*c**2*d*x**2 - 9*a*b**4*c**2*d*x - 9*a*b**4*c*d**2*x**...
```

3.34 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 260

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx &= \frac{d(Cd+2cD)x}{b^2}+\frac{d^2Dx^2}{2b^2}-\frac{b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D)}{2b^3(a+bx^2)}\\ &\quad+\frac{(Ab(bc^2-ad^2)-a(bc(cC+2Bd)-ad(Cd+2cD)))\,x}{2ab^2(a+bx^2)}\\ &\quad+\frac{(Ab(bc^2+ad^2)+a(bc(cC+2Bd)-3ad(Cd+2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}\\ &\quad-\frac{(2ad^2D-b(2cCd+Bd^2+c^2D))\log(a+bx^2)}{2b^3}\end{aligned}$$

output
```
d*(C*d+2*D*c)*x/b^2+1/2*d^2*D*x^2/b^2-1/2*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b
*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)+1/2*(A*b*(-a*d^2+b*c^2)-a*(b*c*(2*B*
d+C*c)-a*d*(C*d+2*D*c)))*x/a/b^2/(b*x^2+a)+1/2*(A*b*(a*d^2+b*c^2)+a*(b*c*(
2*B*d+C*c)-3*a*d*(C*d+2*D*c)))*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)-1
/2*(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*c^2))*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx$$
$$= \frac{2bd(Cd+2cD)x+bd^2Dx^2+\frac{-a^3d^2D+Ab^3c^2x-ab^2(c^2Cx+Ad(2c+dx)+Bc(c+2dx))+a^2b(c^2D+d^2(B+Cx)+2cd(C+Dx))}{a(a+bx^2)}+}{2b^3}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(2*b*d*(C*d + 2*c*D)*x + b*d^2*D*x^2 + (-(a^3*d^2*D) + A*b^3*c^2*x - a*b^2
*(c^2*C*x + A*d*(2*c + d*x) + B*c*(c + 2*d*x)) + a^2*b*(c^2*D + d^2*(B + C
*x) + 2*c*d*(C + D*x)))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b*(b*c^2 + a*d^2) +
a*(b*c*(c*C + 2*B*d) - 3*a*d*(C*d + 2*c*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
a^(3/2) + (-2*a*d^2*D + b*(2*c*C*d + B*d^2 + c^2*D))*Log[a + b*x^2])/(2*b^
3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2176, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} \, dx$$
$$\Big\downarrow \text{2176}$$
$$-\frac{\int -\frac{(c+dx)\left(2adDx^2-(Abd-3aCd-2acD)x+Abc+a\left(cC+2Bd-\frac{2adD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(c+dx)\left(2adDx^2-(Abd-3aCd-2acD)x+Abc+acC+2ad\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2160$$

$$\frac{\int \left(\frac{2aDxd^2}{b}-\frac{(Abd-3aCd-4acD)d}{b}+\frac{Ab(bc^2+ad^2)+a(bc(cC+2Bd)-3ad(Cd+2cD))-2a(2ad^2D-b(Dc^2+2Cdc+Bd^2))x}{b(bx^2+a)}\right)dx}{2ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(ad^2+bc^2\right)+a(bc(2Bd+cC)-3ad(2cD+Cd))\right)}{\sqrt{a}b^{3/2}}-\frac{dx(-4acD-3aCd+Abd)}{b}-\frac{a\log\left(a+bx^2\right)\left(2ad^2D-b\left(Bd^2+c^2D+2cCd\right)\right)}{b^2}-}{2ab}$$

$$\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input

```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output

```
-1/2*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^2)/(a*b*(a + b*x^2)) + (
-((d*(A*b*d - 3*a*C*d - 4*a*c*D)*x)/b) + (a*d^2*D*x^2)/b + ((A*b*(b*c^2 +
a*d^2) + a*(b*c*(c*C + 2*B*d) - 3*a*d*(C*d + 2*c*D)))*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(Sqrt[a]*b^(3/2)) - (a*(2*a*d^2*D - b*(2*c*C*d + B*d^2 + c^2*D))*
Log[a + b*x^2])/b^2)/(2*a*b)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00

method	result
default	$\frac{d\left(\frac{1}{2}Dd\,x^{2}+Cxd+2Dcx\right)}{b^{2}}+\frac{-\frac{\left(Aab\,d^{2}-A\,b^{2}c^{2}+2abBcd-a^{2}C\,d^{2}+Cab\,c^{2}-2a^{2}cdD\right)x}{2a}-\frac{2A\,b^{2}cd-Bab\,d^{2}+B\,b^{2}c^{2}-2abcdC+a^{2}d^{2}D-Dab\,c^{2}}{2b}}{b\,x^{2}+a}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d/b^2*(1/2*D*d*x^2+C*x*d+2*D*c*x)+1/b^2*((-1/2*(A*a*b*d^2-A*b^2*c^2+2*B*a*
b*c*d-C*a^2*d^2+C*a*b*c^2-2*D*a^2*c*d)/a*x-1/2*(2*A*b^2*c*d-B*a*b*d^2+B*b^
2*c^2-2*C*a*b*c*d+D*a^2*d^2-D*a*b*c^2)/b)/(b*x^2+a)+1/2/a*(1/2*(2*B*a*b*d^
2+4*C*a*b*c*d-4*D*a^2*d^2+2*D*a*b*c^2)/b*ln(b*x^2+a)+(A*a*b*d^2+A*b^2*c^2+
2*B*a*b*c*d-3*C*a^2*d^2+C*a*b*c^2-6*D*a^2*c*d)/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.40

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^2*b^2*d^2*x^4 + 2*D*a^3*b*d^2*x^2 + 4*(2*D*a^2*b^2*c*d + C*a^2
*b^2*d^2)*x^3 + 2*(D*a^3*b - B*a^2*b^2)*c^2 + 4*(C*a^3*b - A*a^2*b^2)*c*d
- 2*(D*a^4 - B*a^3*b)*d^2 - ((C*a^2*b + A*a*b^2)*c^2 - 2*(3*D*a^3 - B*a^2*
b)*c*d - (3*C*a^3 - A*a^2*b)*d^2 + ((C*a*b^2 + A*b^3)*c^2 - 2*(3*D*a^2*b -
 B*a*b^2)*c*d - (3*C*a^2*b - A*a*b^2)*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*
sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*((C*a^2*b^2 - A*a*b^3)*c^2 - 2*(3*D*a^3
*b - B*a^2*b^2)*c*d - (3*C*a^3*b - A*a^2*b^2)*d^2)*x + 2*(D*a^3*b*c^2 + 2*
C*a^3*b*c*d - (2*D*a^4 - B*a^3*b)*d^2 + (D*a^2*b^2*c^2 + 2*C*a^2*b^2*c*d -
 (2*D*a^3*b - B*a^2*b^2)*d^2)*x^2)*log(b*x^2 + a))/(a^2*b^4*x^2 + a^3*b^3)
, 1/2*(D*a^2*b^2*d^2*x^4 + D*a^3*b*d^2*x^2 + 2*(2*D*a^2*b^2*c*d + C*a^2*b^
2*d^2)*x^3 + (D*a^3*b - B*a^2*b^2)*c^2 + 2*(C*a^3*b - A*a^2*b^2)*c*d - (D*
a^4 - B*a^3*b)*d^2 + ((C*a^2*b + A*a*b^2)*c^2 - 2*(3*D*a^3 - B*a^2*b)*c*d
- (3*C*a^3 - A*a^2*b)*d^2 + ((C*a*b^2 + A*b^3)*c^2 - 2*(3*D*a^2*b - B*a*b^
2)*c*d - (3*C*a^2*b - A*a*b^2)*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
 ((C*a^2*b^2 - A*a*b^3)*c^2 - 2*(3*D*a^3*b - B*a^2*b^2)*c*d - (3*C*a^3*b -
 A*a^2*b^2)*d^2)*x + (D*a^3*b*c^2 + 2*C*a^3*b*c*d - (2*D*a^4 - B*a^3*b)*d^
2 + (D*a^2*b^2*c^2 + 2*C*a^2*b^2*c*d - (2*D*a^3*b - B*a^2*b^2)*d^2)*x^2)*l
og(b*x^2 + a))/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(248) = 496$.

Time = 27.46 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.37

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$= \frac{Dd^2x^2}{2b^2} + x\left(\frac{Cd^2}{b^2} + \frac{2Dcd}{b^2}\right) + \left(-\frac{-Bbd^2 - 2Cbcd + 2Dad^2 - Dbc^2}{2b^3} - \frac{\sqrt{-a^3b^7}(-Aabd^2 - Ab^2c^2 - 2Babcd + 3Ca^2d^2 - Cabc^2 + 6Da^2cd)}{4a^3b^6}\right)\log\left(x + \frac{2Ba^2bd^2 + 4Ca^2bcd -\right.$$

$$+ \left(-\frac{-Bbd^2 - 2Cbcd + 2Dad^2 - Dbc^2}{2b^3} + \frac{\sqrt{-a^3b^7}(-Aabd^2 - Ab^2c^2 - 2Babcd + 3Ca^2d^2 - Cabc^2 + 6Da^2cd)}{4a^3b^6}\right)\log\left(x + \frac{2Ba^2bd^2 + 4Ca^2bcd -\right.$$

$$+ \frac{-2Aab^2cd + Ba^2bd^2 - Bab^2c^2 + 2Ca^2bcd - Da^3d^2 + Da^2bc^2 + x(-Aab^2d^2 + Ab^3c^2 - 2Bab^2cd + Ca}{2a^2b^3 + 2ab^4x^2}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
D*d**2*x**2/(2*b**2) + x*(C*d**2/b**2 + 2*D*c*d/b**2) + (-(-B*b*d**2 - 2*C
*b*c*d + 2*D*a*d**2 - D*b*c**2)/(2*b**3) - sqrt(-a**3*b**7)*(-A*a*b*d**2 -
 A*b**2*c**2 - 2*B*a*b*c*d + 3*C*a**2*d**2 - C*a*b*c**2 + 6*D*a**2*c*d)/(4
*a**3*b**6))*log(x + (2*B*a**2*b*d**2 + 4*C*a**2*b*c*d - 4*D*a**3*d**2 + 2
*D*a**2*b*c**2 - 4*a**2*b**3*(-(-B*b*d**2 - 2*C*b*c*d + 2*D*a*d**2 - D*b*c
**2)/(2*b**3) - sqrt(-a**3*b**7)*(-A*a*b*d**2 - A*b**2*c**2 - 2*B*a*b*c*d
+ 3*C*a**2*d**2 - C*a*b*c**2 + 6*D*a**2*c*d)/(4*a**3*b**6)))/(-A*a*b**2*d*
*2 - A*b**3*c**2 - 2*B*a*b**2*c*d + 3*C*a**2*b*d**2 - C*a*b**2*c**2 + 6*D*
a**2*b*c*d)) + (-(-B*b*d**2 - 2*C*b*c*d + 2*D*a*d**2 - D*b*c**2)/(2*b**3)
+ sqrt(-a**3*b**7)*(-A*a*b*d**2 - A*b**2*c**2 - 2*B*a*b*c*d + 3*C*a**2*d**
2 - C*a*b*c**2 + 6*D*a**2*c*d)/(4*a**3*b**6))*log(x + (2*B*a**2*b*d**2 + 4
*C*a**2*b*c*d - 4*D*a**3*d**2 + 2*D*a**2*b*c**2 - 4*a**2*b**3*(-(-B*b*d**2
 - 2*C*b*c*d + 2*D*a*d**2 - D*b*c**2)/(2*b**3) + sqrt(-a**3*b**7)*(-A*a*b*
d**2 - A*b**2*c**2 - 2*B*a*b*c*d + 3*C*a**2*d**2 - C*a*b*c**2 + 6*D*a**2*c
*d)/(4*a**3*b**6)))/(-A*a*b**2*d**2 - A*b**3*c**2 - 2*B*a*b**2*c*d + 3*C*a
**2*b*d**2 - C*a*b**2*c**2 + 6*D*a**2*b*c*d)) + (-2*A*a*b**2*c*d + B*a**2*
b*d**2 - B*a*b**2*c**2 + 2*C*a**2*b*c*d - D*a**3*d**2 + D*a**2*b*c**2 + x*
(-A*a*b**2*d**2 + A*b**3*c**2 - 2*B*a*b**2*c*d + C*a**2*b*d**2 - C*a*b**2*
c**2 + 2*D*a**2*b*c*d))/(2*a**2*b**3 + 2*a*b**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05

$$
\begin{aligned}
&\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx \\
&= \frac{(Da^2b-Bab^2)c^2+2\,(Ca^2b-Aab^2)cd-(Da^3-Ba^2b)d^2-((Cab^2-Ab^3)c^2-2\,(Da^2b-Bab^2)cd-(\text{C}}{2\,(ab^4x^2+a^2b^3)} \\
&\quad+\frac{Dd^2x^2+2\,(2\,Dcd+Cd^2)x}{2\,b^2}+\frac{(Dbc^2+2\,Cbcd-(2\,Da-Bb)d^2)\log\left(bx^2+a\right)}{2\,b^3} \\
&\quad+\frac{((Cab+Ab^2)c^2-2\,(3\,Da^2-Bab)cd-(3\,Ca^2-Aab)d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((D*a^2*b - B*a*b^2)*c^2 + 2*(C*a^2*b - A*a*b^2)*c*d - (D*a^3 - B*a^2*
b)*d^2 - ((C*a*b^2 - A*b^3)*c^2 - 2*(D*a^2*b - B*a*b^2)*c*d - (C*a^2*b - A
*a*b^2)*d^2)*x)/(a*b^4*x^2 + a^2*b^3) + 1/2*(D*d^2*x^2 + 2*(2*D*c*d + C*d^
2)*x)/b^2 + 1/2*(D*b*c^2 + 2*C*b*c*d - (2*D*a - B*b)*d^2)*log(b*x^2 + a)/b
^3 + 1/2*((C*a*b + A*b^2)*c^2 - 2*(3*D*a^2 - B*a*b)*c*d - (3*C*a^2 - A*a*b
)*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.07

$$
\begin{aligned}
&\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx \\
&= \frac{(Dbc^2+2\,Cbcd-2\,Dad^2+Bbd^2)\log(bx^2+a)}{2\,b^3} \\
&\quad+\frac{(Cabc^2+Ab^2c^2-6\,Da^2cd+2\,Babcd-3\,Ca^2d^2+Aabd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2} \\
&\quad+\frac{Db^2d^2x^2+4\,Db^2cdx+2\,Cb^2d^2x}{2\,b^4} \\
&\quad+\frac{Da^2bc^2-Bab^2c^2+2\,Ca^2bcd-2\,Aab^2cd-Da^3d^2+Ba^2bd^2-(Cab^2c^2-Ab^3c^2-2\,Da^2bcd+2\,Bab^2}{2\,(bx^2+a)ab^3}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*b*c^2 + 2*C*b*c*d - 2*D*a*d^2 + B*b*d^2)*log(b*x^2 + a)/b^3 + 1/2*(
C*a*b*c^2 + A*b^2*c^2 - 6*D*a^2*c*d + 2*B*a*b*c*d - 3*C*a^2*d^2 + A*a*b*d^
2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/2*(D*b^2*d^2*x^2 + 4*D*b^2*
c*d*x + 2*C*b^2*d^2*x)/b^4 + 1/2*(D*a^2*b*c^2 - B*a*b^2*c^2 + 2*C*a^2*b*c*
d - 2*A*a*b^2*c*d - D*a^3*d^2 + B*a^2*b*d^2 - (C*a*b^2*c^2 - A*b^3*c^2 - 2
*D*a^2*b*c*d + 2*B*a*b^2*c*d - C*a^2*b*d^2 + A*a*b^2*d^2)*x)/((b*x^2 + a)*
a*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.98

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{-2\log(b\,x^2+a)\,a^3d^3 - 9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abc\,d^2x^2 + \log(b\,x^2+a)\,a^2b^2d^2 - a^2b^2d^2x + a\,b^3c^2x - a\,b^3c}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*d**2 - 9*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c*d**2 + sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*c**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*a*b**2*c*d + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2
*d**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c**3 - 9*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c*d**2*x**2 + sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c**2*x**2 + 2*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*b**3*c*d*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*b**2*c**3*x**2 - 2*log(a + b*x**2)*a**3*d**3 + log(a + b*x**2
)*a**2*b**2*d**2 + 3*log(a + b*x**2)*a**2*b*c**2*d - 2*log(a + b*x**2)*a**
2*b*d**3*x**2 + log(a + b*x**2)*a*b**3*d**2*x**2 + 3*log(a + b*x**2)*a*b**
2*c**2*d*x**2 - a**2*b**2*d**2*x + 9*a**2*b*c*d**2*x + 2*a**2*b*d**3*x**2
+ a*b**3*c**2*x + 2*a*b**3*c*d*x**2 - 2*a*b**3*c*d*x - a*b**3*d**2*x**2 -
a*b**2*c**3*x - 3*a*b**2*c**2*d*x**2 + 6*a*b**2*c*d**2*x**3 + a*b**2*d**3*
x**4 + b**4*c**2*x**2)/(2*a*b**3*(a + b*x**2))
```

3.35 $\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 160

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{dDx}{b^2}-\frac{bBc+Abd-aCd-acD}{2b^2\left(a+bx^2\right)}+\frac{\left(Ab^2c-a(b(cC+Bd)-adD)\right)x}{2ab^2\left(a+bx^2\right)}\\&\quad+\frac{\left(Ab^2c+a(bcC+bBd-3adD)\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(Cd+cD)\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

output
```
d*D*x/b^2-1/2*(A*b*d+B*b*c-C*a*d-D*a*c)/b^2/(b*x^2+a)+1/2*(A*b^2*c-a*(b*(B
*d+C*c)-D*a*d))*x/a/b^2/(b*x^2+a)+1/2*(A*b^2*c+a*(B*b*d+C*b*c-3*D*a*d))*ar
ctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)+1/2*(C*d+D*c)*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$
$$= \frac{2\sqrt{b}dDx + \frac{\sqrt{b}(Ab^2cx-ab(Ad+cCx+B(c+dx))+a^2(Cd+D(c+dx)))}{a(a+bx^2)} + \frac{(Ab^2c+a(bcC+bBd-3adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} + \sqrt{b}(Cd+c}{2b^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(2*Sqrt[b]*d*D*x + (Sqrt[b]*(A*b^2*c*x - a*b*(A*d + c*C*x + B*(c + d*x)) +
 a^2*(C*d + D*(c + d*x))))/(a*(a + b*x^2)) + ((A*b^2*c + a*(b*c*C + b*B*d
- 3*a*d*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + Sqrt[b]*(C*d + c*D)*Log
[a + b*x^2])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2176, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$
$$\Big\downarrow \text{2176}$$
$$-\frac{\int -\frac{-\frac{dDa^2}{b}+2dDx^2a+(cC+Bd)a+2(Cd+cD)xa+Abc}{bx^2+a}dx}{2ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{2adDx^2+2a(Cd+cD)x+Abc+acC+ad\left(B-\frac{aD}{b}\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(\frac{2adD}{b}+\frac{Acb^2+2a(Cd+cD)xb+a(bcC+bBd-3adD)}{b(bx^2+a)}\right)dx}{2ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a(-3adD+bBd+bcC)+Ab^2c)}{\sqrt{a}b^{3/2}}+\frac{a\log(a+bx^2)(cD+Cd)}{b}+\frac{2adDx}{b}}{2ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x))/(a*b*(a + b*x^2)) + ((2
*a*d*D*x)/b + ((A*b^2*c + a*(b*c*C + b*B*d - 3*a*d*D))*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (a*(C*d + c*D)*Log[a + b*x^2])/b)/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

method	result
default	$\frac{dDx}{b^2}+\frac{\frac{\frac{\left(A\,b^2c-Babd-Cabc+a^2dD\right)x}{2a}-\frac{Abd}{2}-\frac{Bbc}{2}+\frac{Cad}{2}+\frac{Dac}{2}}{b\,x^2+a}+\frac{\frac{(2dabC+2abcD)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(A\,b^2c+Babd+Cabc-3a^2dD\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2a}}{b^2}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*D*x/b^2+1/b^2*((1/2*(A*b^2*c-B*a*b*d-C*a*b*c+D*a^2*d)/a*x-1/2*A*b*d-1/2*
B*b*c+1/2*C*a*d+1/2*D*a*c)/(b*x^2+a)+1/2/a*(1/2*(2*C*a*b*d+2*D*a*b*c)/b*ln
(b*x^2+a)+(A*b^2*c+B*a*b*d+C*a*b*c-3*D*a^2*d)/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.25

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{4\,Da^2b^2dx^3-(((Cab^2+Ab^3)c-(3\,Da^2b-Bab^2)d)x^2+(Ca^2b+Aab^2)c-(3\,Da^3-Ba^2b)d)\sqrt{-ab}\,\mathrm{lo}}{}\right.$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*D*a^2*b^2*d*x^3 - (((C*a*b^2 + A*b^3)*c - (3*D*a^2*b - B*a*b^2)*d)
*x^2 + (C*a^2*b + A*a*b^2)*c - (3*D*a^3 - B*a^2*b)*d)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^3*b - B*a^2*b^2)*c + 2*(C*a^
3*b - A*a^2*b^2)*d - 2*((C*a^2*b^2 - A*a*b^3)*c - (3*D*a^3*b - B*a^2*b^2)*
d)*x + 2*(D*a^3*b*c + C*a^3*b*d + (D*a^2*b^2*c + C*a^2*b^2*d)*x^2)*log(b*x
^2 + a))/(a^2*b^4*x^2 + a^3*b^3), 1/2*(2*D*a^2*b^2*d*x^3 + (((C*a*b^2 + A*
b^3)*c - (3*D*a^2*b - B*a*b^2)*d)*x^2 + (C*a^2*b + A*a*b^2)*c - (3*D*a^3 -
 B*a^2*b)*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3*b - B*a^2*b^2)*c + (
C*a^3*b - A*a^2*b^2)*d - ((C*a^2*b^2 - A*a*b^3)*c - (3*D*a^3*b - B*a^2*b^2
)*d)*x + (D*a^3*b*c + C*a^3*b*d + (D*a^2*b^2*c + C*a^2*b^2*d)*x^2)*log(b*x
^2 + a))/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(158) = 316$.

Time = 9.56 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \frac{Ddx}{b^2} + \left(\frac{Cd+Dc}{2b^2} - \frac{\sqrt{-a^3b^5}(-Ab^2c-Babd-Cabc+3Da^2d)}{4a^3b^5}\right)\log\left(x+\frac{2Ca^2d+2Da^2c-4a^2b^2\left(\frac{Cd+Dc}{2b^2}-\frac{\sqrt{-a^3b^5}(-Ab^2}{}\right.}{-Ab^2c-Babd-Cabc+3D}\right. + \left(\frac{Cd+Dc}{2b^2} + \frac{\sqrt{-a^3b^5}(-Ab^2c-Babd-Cabc+3Da^2d)}{4a^3b^5}\right)\log\left(x+\frac{2Ca^2d+2Da^2c-4a^2b^2\left(\frac{Cd+Dc}{2b^2}+\frac{\sqrt{-a^3b^5}(-Ab^2}{}\right.}{-Ab^2c-Babd-Cabc+3D}\right. + \frac{-Aabd-Babc+Ca^2d+Da^2c+x(Ab^2c-Babd-Cabc+Da^2d)}{2a^2b^2+2ab^3x^2}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
D*d*x/b**2 + ((C*d + D*c)/(2*b**2) - sqrt(-a**3*b**5)*(-A*b**2*c - B*a*b*d
 - C*a*b*c + 3*D*a**2*d)/(4*a**3*b**5))*log(x + (2*C*a**2*d + 2*D*a**2*c -
 4*a**2*b**2*((C*d + D*c)/(2*b**2) - sqrt(-a**3*b**5)*(-A*b**2*c - B*a*b*d
 - C*a*b*c + 3*D*a**2*d)/(4*a**3*b**5)))/(-A*b**2*c - B*a*b*d - C*a*b*c +
3*D*a**2*d)) + ((C*d + D*c)/(2*b**2) + sqrt(-a**3*b**5)*(-A*b**2*c - B*a*b
*d - C*a*b*c + 3*D*a**2*d)/(4*a**3*b**5))*log(x + (2*C*a**2*d + 2*D*a**2*c
 - 4*a**2*b**2*((C*d + D*c)/(2*b**2) + sqrt(-a**3*b**5)*(-A*b**2*c - B*a*b
*d - C*a*b*c + 3*D*a**2*d)/(4*a**3*b**5)))/(-A*b**2*c - B*a*b*d - C*a*b*c
+ 3*D*a**2*d)) + (-A*a*b*d - B*a*b*c + C*a**2*d + D*a**2*c + x*(A*b**2*c -
 B*a*b*d - C*a*b*c + D*a**2*d))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx \\
&= \frac{Ddx}{b^2} + \frac{(Da^2-Bab)c+(Ca^2-Aab)d-((Cab-Ab^2)c-(Da^2-Bab)d)x}{2\left(ab^3x^2+a^2b^2\right)} \\
&\quad + \frac{(Dc+Cd)\log\left(bx^2+a\right)}{2\,b^2} + \frac{((Cab+Ab^2)c-(3\,Da^2-Bab)d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}
\end{aligned}
$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
D*d*x/b^2 + 1/2*((D*a^2 - B*a*b)*c + (C*a^2 - A*a*b)*d - ((C*a*b - A*b^2)*
c - (D*a^2 - B*a*b)*d)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*(D*c + C*d)*log(b*x^
2 + a)/b^2 + 1/2*((C*a*b + A*b^2)*c - (3*D*a^2 - B*a*b)*d)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$
$$=\frac{Ddx}{b^2}+\frac{(Dc+Cd)\log\left(bx^2+a\right)}{2\,b^2}+\frac{\left(Cabc+Ab^2c-3\,Da^2d+Babd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}$$
$$+\frac{Da^2c-Babc+Ca^2d-Aabd-\left(Cabc-Ab^2c-Da^2d+Babd\right)x}{2\,(bx^2+a)ab^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
D*d*x/b^2 + 1/2*(D*c + C*d)*log(b*x^2 + a)/b^2 + 1/2*(C*a*b*c + A*b^2*c -
3*D*a^2*d + B*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/2*(D*a^2*
c - B*a*b*c + C*a^2*d - A*a*b*d - (C*a*b*c - A*b^2*c - D*a^2*d + B*a*b*d)*
x)/((b*x^2 + a)*a*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx=\int \frac{\left(c+d\,x\right)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2c+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2d+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{}}\right.}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d**2 + sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c + sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a*b**2*d + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*a*b*c**2 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*d**2*x**2
 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*x**2 + sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*d*x**2 + sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*b**2*c**2*x**2 + 2*log(a + b*x**2)*a**2*b*c*d + 2*log
(a + b*x**2)*a*b**2*c*d*x**2 + 3*a**2*b*d**2*x + a*b**3*c*x + a*b**3*d*x**
2 - a*b**3*d*x - a*b**2*c**2*x - 2*a*b**2*c*d*x**2 + 2*a*b**2*d**2*x**3 +
b**4*c*x**2)/(2*a*b**3*(a + b*x**2))
```

3.36 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = -\frac{bB-aD}{2b^2(a+bx^2)}+\frac{(Ab-aC)x}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log(a+bx^2)}{2b^2}$$

output
```
-1/2*(B*b-D*a)/b^2/(b*x^2+a)+1/2*(A*b-C*a)*x/a/b/(b*x^2+a)+1/2*(A*b+C*a)*a
rctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = \frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)}+\frac{\sqrt{b}(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+D\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2345, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+aC+2aDx}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx+2aD\int \frac{x}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aD\int \frac{x}{bx^2+a}dx+\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{aD\log(a+bx^2)}{b}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + (((A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (a*D*Log[a + b*x^2])/b)/(2
*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{Bb-Da}{2b^{2}}}{b\,x^{2}+a}+\frac{\frac{Da\ln\left(b\,x^{2}+a\right)}{b}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ba}$	88

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/b/a*(D*a/b*ln(b*x^2+
a)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{2\,Da^3-2\,Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(Ca^2b-Aab^2)x+2\,(I}{4\,(a^2b^3x^2+a^3b^2)}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x
 + 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D
*a^3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a
))/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(90) = 180$.

Time = 0.96 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$$
$$= \left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)$$
$$+\left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)$$
$$+\frac{-Bab+Da^2+x(Ab^2-Cab)}{2a^2b^2+2ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a*
*2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))
)/(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b
**4))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b
 + C*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 -
 C*a*b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(
b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2} - \frac{B}{2\,b\,\left(b\,x^2+a\right)} + \frac{A\,x}{2\,a\,\left(b\,x^2+a\right)} - \frac{C\,x}{2\,b\,\left(b\,x^2+a\right)} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

output
```
((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)
/(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)
))/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\left(b\,x^2+a\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + lo
g(a + b*x**2)*a**2*d + log(a + b*x**2)*a*b*d*x**2 + a*b**2*x - a*b*c*x - a
*b*d*x**2 + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

3.37 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 284

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)^2}\,dx &= -\frac{a(bBc-Abd+aCd-acD)-(Ab^2c-a(bcC-bBd+adD))x}{2ab(bc^2+ad^2)(a+bx^2)} \\ &\quad+\frac{(Ab^2c(bc^2+3ad^2)+a(b^2c^2(cC-Bd)+a^2d^3D-abd(cCd-Bd^2-3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}(bc^2+ad^2)^2} \\ &\quad+\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{(bc^2+ad^2)^2} \\ &\quad-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(a+bx^2)}{2(bc^2+ad^2)^2}\end{aligned}$$

output
```
-1/2*(a*(-A*b*d+B*b*c+C*a*d-D*a*c)-(A*b^2*c-a*(-B*b*d+C*b*c+D*a*d))*x)/a/b
/(a*d^2+b*c^2)/(b*x^2+a)+1/2*(A*b^2*c*(3*a*d^2+b*c^2)+a*(b^2*c^2*(-B*d+C*c
)+a^2*d^3*D-a*b*d*(-B*d^2+C*c*d-3*D*c^2)))*arctan(b^(1/2)*x/a^(1/2))/a^(3/
2)/b^(3/2)/(a*d^2+b*c^2)^2+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/(a*d^2+
b*c^2)^2-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(b*x^2+a)/(a*d^2+b*c^2)^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^2}\,dx$$
$$= \frac{\frac{(bc^2+ad^2)\left(Ab^2cx+ab(-Bc+Ad-cCx+Bdx)-a^2(Cd-cD+dDx)\right)}{ab(a+bx^2)} + \frac{\left(Ab^2c\left(bc^2+3ad^2\right)+a\left(b^2c^2(cC-Bd)+a^2d^3D+abd\left(-cCd+Bd^2+3c^2D\right.\right.\right.}{a^{3/2}b^{3/2}}}{2\left(bc^2\right.}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^2),x]
```

output
```
(((b*c^2 + a*d^2)*(A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x) - a^2*(C
*d - c*D + d*D*x)))/(a*b*(a + b*x^2)) + ((A*b^2*c*(b*c^2 + 3*a*d^2) + a*(b
^2*c^2*(c*C - B*d) + a^2*d^3*D + a*b*d*(-(c*C*d) + B*d^2 + 3*c^2*D)))*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(3/2)) + 2*(c^2*C*d - B*c*d^2 + A*d^3
- c^3*D)*Log[c + d*x] + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[a + b*x
^2])/(2*(b*c^2 + a*d^2)^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2178, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2(c+dx)}\,dx$$
$$\Big\downarrow \; 2178$$
$$-\frac{\int -\frac{Ab\left(bc^2+2ad^2\right)+ac(bcC-bBd+adD)+\left(Acdb^2+a\left(ad^2D-b\left(-2Dc^2+Cdc-Bd^2\right)\right)\right)x}{(bc^2+ad^2)(c+dx)(bx^2+a)}dx}{2ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{Ab(bc^2+2ad^2)+ac(bcC-bBd+adD)+\left(Acdb^2+a\left(ad^2D-b\left(-2Dc^2+Cdc-Bd^2\right)\right)\right)x}{(bc^2+ad^2)(c+dx)(bx^2+a)}dx}{2ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{Ab(bc^2+2ad^2)+ac(bcC-bBd+adD)+\left(Acdb^2+a\left(ad^2D-b\left(-2Dc^2+Cdc-Bd^2\right)\right)\right)x}{(c+dx)(bx^2+a)}dx}{2ab\left(ad^2+bc^2\right)}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{2abd\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)}{(bc^2+ad^2)(c+dx)}+\frac{Ac(bc^2+3ad^2)b^2-2a\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)xb^2+a\left(a^2Dd^3-ab\left(-3Dc^2+Cdc-Bd^2\right)d+b^2c^2(cC-\right.}{(bc^2+ad^2)(bx^2+a)}\right.}{2ab\left(ad^2+bc^2\right)}$$
$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(a^2d^3D-abd\left(-Bd^2-3c^2D+cCd\right)+b^2c^2(cC-Bd)\right)+Ab^2c\left(3ad^2+bc^2\right)\right)}{\sqrt{a}\sqrt{b}(ad^2+bc^2)}-\frac{ab\log\left(a+bx^2\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{ad^2+bc^2}+\frac{2ab}{}}{2ab\left(ad^2+bc^2\right)}$$
$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^2),x]
```

output
```
-1/2*(a*(b*B*c - A*b*d + a*C*d - a*c*D) - (A*b^2*c - a*(b*c*C - b*B*d + a*
d*D))*x)/(a*b*(b*c^2 + a*d^2)*(a + b*x^2)) + (((A*b^2*c*(b*c^2 + 3*a*d^2)
+ a*(b^2*c^2*(c*C - B*d) + a^2*d^3*D - a*b*d*(c*C*d - B*d^2 - 3*c^2*D)))*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(b*c^2 + a*d^2)) + (2*a*b*(c^
2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/(b*c^2 + a*d^2) - (a*b*(c^2
*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[a + b*x^2])/(b*c^2 + a*d^2))/(2*a*b*(b
*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.39

method	result
default	$\frac{\frac{\left(A a b^{2} c d^{2}+A b^{3} c^{3}+B a^{2} b d^{3}+B a b^{2} c^{2} d-C a^{2} b c d^{2}-C a b^{2} c^{3}-D a^{3} d^{3}-D a^{2} b c^{2} d\right) x}{2 a b}+\frac{A a b d^{3}+A b^{2} c^{2} d-B a b c d^{2}-B b^{2} c^{3}-C a^{2} d^{3}-C a b c^{2} d+D}{2 b}}{b x^{2}+a}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2+b*c^2)^2*((1/2*(A*a*b^2*c*d^2+A*b^3*c^3+B*a^2*b*d^3+B*a*b^2*c^2*d
-C*a^2*b*c*d^2-C*a*b^2*c^3-D*a^3*d^3-D*a^2*b*c^2*d)/a/b*x+1/2*(A*a*b*d^3+A
*b^2*c^2*d-B*a*b*c*d^2-B*b^2*c^3-C*a^2*d^3-C*a*b*c^2*d+D*a^2*c*d^2+D*a*b*c
^3)/b)/(b*x^2+a)+1/2/b/a*(1/2*(-2*A*a*b^2*d^3+2*B*a*b^2*c*d^2-2*C*a*b^2*c^
2*d+2*D*a*b^2*c^3)/b*ln(b*x^2+a)+(3*A*a*b^2*c*d^2+A*b^3*c^3+B*a^2*b*d^3-B*
a*b^2*c^2*d-C*a^2*b*c*d^2+C*a*b^2*c^3+D*a^3*d^3+3*D*a^2*b*c^2*d)/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))))+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/(a*d^
2+b*c^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(269) = 538$.

Time = 135.15 (sec) , antiderivative size = 1342, normalized size of antiderivative = 4.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(D*a^3*b^2 - B*a^2*b^3)*c^3 - 2*(C*a^3*b^2 - A*a^2*b^3)*c^2*d + 2*
(D*a^4*b - B*a^3*b^2)*c*d^2 - 2*(C*a^4*b - A*a^3*b^2)*d^3 - ((C*a^2*b^2 +
A*a*b^3)*c^3 + (3*D*a^3*b - B*a^2*b^2)*c^2*d - (C*a^3*b - 3*A*a^2*b^2)*c*d
^2 + (D*a^4 + B*a^3*b)*d^3 + ((C*a*b^3 + A*b^4)*c^3 + (3*D*a^2*b^2 - B*a*b
^3)*c^2*d - (C*a^2*b^2 - 3*A*a*b^3)*c*d^2 + (D*a^3*b + B*a^2*b^2)*d^3)*x^2
)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*((C*a^2*b^3
 - A*a*b^4)*c^3 + (D*a^3*b^2 - B*a^2*b^3)*c^2*d + (C*a^3*b^2 - A*a^2*b^3)*
c*d^2 + (D*a^4*b - B*a^3*b^2)*d^3)*x + 2*(D*a^3*b^2*c^3 - C*a^3*b^2*c^2*d
+ B*a^3*b^2*c*d^2 - A*a^3*b^2*d^3 + (D*a^2*b^3*c^3 - C*a^2*b^3*c^2*d + B*a
^2*b^3*c*d^2 - A*a^2*b^3*d^3)*x^2)*log(b*x^2 + a) - 4*(D*a^3*b^2*c^3 - C*a
^3*b^2*c^2*d + B*a^3*b^2*c*d^2 - A*a^3*b^2*d^3 + (D*a^2*b^3*c^3 - C*a^2*b^
3*c^2*d + B*a^2*b^3*c*d^2 - A*a^2*b^3*d^3)*x^2)*log(d*x + c))/(a^3*b^4*c^4
 + 2*a^4*b^3*c^2*d^2 + a^5*b^2*d^4 + (a^2*b^5*c^4 + 2*a^3*b^4*c^2*d^2 + a^
4*b^3*d^4)*x^2), 1/2*((D*a^3*b^2 - B*a^2*b^3)*c^3 - (C*a^3*b^2 - A*a^2*b^3
)*c^2*d + (D*a^4*b - B*a^3*b^2)*c*d^2 - (C*a^4*b - A*a^3*b^2)*d^3 + ((C*a^
2*b^2 + A*a*b^3)*c^3 + (3*D*a^3*b - B*a^2*b^2)*c^2*d - (C*a^3*b - 3*A*a^2*
b^2)*c*d^2 + (D*a^4 + B*a^3*b)*d^3 + ((C*a*b^3 + A*b^4)*c^3 + (3*D*a^2*b^2
 - B*a*b^3)*c^2*d - (C*a^2*b^2 - 3*A*a*b^3)*c*d^2 + (D*a^3*b + B*a^2*b^2)*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - ((C*a^2*b^3 - A*a*b^4)*c^3 + (
D*a^3*b^2 - B*a^2*b^3)*c^2*d + (C*a^3*b^2 - A*a^2*b^3)*c*d^2 + (D*a^4*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\,(a+bx^2)^2}\,dx$$
$$= \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(bx^2+a)}{2\,(b^2c^4+2\,abc^2d^2+a^2d^4)} - \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{b^2c^4+2\,abc^2d^2+a^2d^4}$$
$$+ \frac{((Cab^2+Ab^3)c^3+(3\,Da^2b-Bab^2)c^2d-(Ca^2b-3\,Aab^2)cd^2+(Da^3+Ba^2b)d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^4+2\,a^2b^2c^2d^2+a^3bd^4)\sqrt{ab}}$$
$$+ \frac{(Da^2-Bab)c-(Ca^2-Aab)d-((Cab-Ab^2)c+(Da^2-Bab)d)x}{2\,(a^2b^2c^2+a^3bd^2+(ab^3c^2+a^2b^2d^2)x^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(b*x^2 + a)/(b^2*c^4 + 2*a*b*c^
2*d^2 + a^2*d^4) - (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c)/(b^2*c
^4 + 2*a*b*c^2*d^2 + a^2*d^4) + 1/2*((C*a*b^2 + A*b^3)*c^3 + (3*D*a^2*b -
B*a*b^2)*c^2*d - (C*a^2*b - 3*A*a*b^2)*c*d^2 + (D*a^3 + B*a^2*b)*d^3)*arct
an(b*x/sqrt(a*b))/((a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4)*sqrt(a*b))
+ 1/2*((D*a^2 - B*a*b)*c - (C*a^2 - A*a*b)*d - ((C*a*b - A*b^2)*c + (D*a^2
 - B*a*b)*d)*x)/(a^2*b^2*c^2 + a^3*b*d^2 + (a*b^3*c^2 + a^2*b^2*d^2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\,(a+bx^2)^2}\,dx = \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(bx^2+a)}{2\,(b^2c^4+2\,abc^2d^2+a^2d^4)}$$
$$- \frac{(Dc^3d-Cc^2d^2+Bcd^3-Ad^4)\log(|dx+c|)}{b^2c^4d+2\,abc^2d^3+a^2d^5}$$
$$+ \frac{(Cab^2c^3+Ab^3c^3+3\,Da^2bc^2d-Bab^2c^2d-Ca^2bcd^2+3\,Aab^2cd^2+Da^3d^3+Ba^2bd^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^4+2\,a^2b^2c^2d^2+a^3bd^4)\sqrt{ab}}$$
$$+ \frac{Da^2bc^3-Bab^2c^3-Ca^2bc^2d+Aab^2c^2d+Da^3cd^2-Ba^2bcd^2-Ca^3d^3+Aa^2bd^3-(Cab^2c^3-Ab^3c^3+}{2\,(bc^2+ad^2)^2(bx^2+a)ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(b*x^2 + a)/(b^2*c^4 + 2*a*b*c^
2*d^2 + a^2*d^4) - (D*c^3*d - C*c^2*d^2 + B*c*d^3 - A*d^4)*log(abs(d*x + c
))/(b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5) + 1/2*(C*a*b^2*c^3 + A*b^3*c^3 +
3*D*a^2*b*c^2*d - B*a*b^2*c^2*d - C*a^2*b*c*d^2 + 3*A*a*b^2*c*d^2 + D*a^3*
d^3 + B*a^2*b*d^3)*arctan(b*x/sqrt(a*b))/((a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 +
 a^3*b*d^4)*sqrt(a*b)) + 1/2*(D*a^2*b*c^3 - B*a*b^2*c^3 - C*a^2*b*c^2*d +
A*a*b^2*c^2*d + D*a^3*c*d^2 - B*a^2*b*c*d^2 - C*a^3*d^3 + A*a^2*b*d^3 - (C
*a*b^2*c^3 - A*b^3*c^3 + D*a^2*b*c^2*d - B*a*b^2*c^2*d + C*a^2*b*c*d^2 - A
*a*b^2*c*d^2 + D*a^3*d^3 - B*a^2*b*d^3)*x)/((b*c^2 + a*d^2)^2*(b*x^2 + a)*
a*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^2*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^2*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time $= 0.16$ (sec) , antiderivative size $= 728$, normalized size of antiderivative $= 2.56$

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^2,x)
```

output

```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d**4 + 3*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c*d**2 + sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d**3 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**2*b*c**2*d**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*a**2*b*d**4*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b
**3*c**3 - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**2*d + 3
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*d**2*x**2 + sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*d**3*x**2 + sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**4 + 2*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a*b**2*c**2*d**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**4*c**3*x**2 - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**4*c**2*d*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*
c**4*x**2 - log(a + b*x**2)*a**3*b**2*d**3 + log(a + b*x**2)*a**2*b**3*c*d
**2 - log(a + b*x**2)*a**2*b**3*d**3*x**2 + log(a + b*x**2)*a*b**4*c*d**2*
x**2 + 2*log(c + d*x)*a**3*b**2*d**3 - 2*log(c + d*x)*a**2*b**3*c*d**2 + 2
*log(c + d*x)*a**2*b**3*d**3*x**2 - 2*log(c + d*x)*a*b**4*c*d**2*x**2 - a*
*3*b*d**4*x + a**2*b**3*c*d**2*x - a**2*b**3*d**3*x**2 + a**2*b**3*d**3*x
 - 2*a**2*b**2*c**2*d**2*x + a*b**4*c**3*x - a*b**4*c**2*d*x**2 + a*b**4*c*
*2*d*x + a*b**4*c*d**2*x**2 - a*b**3*c**4*x + b**5*c**3*x**2)/(2*a*b**2*(a
**3*d**4 + 2*a**2*b*c**2*d**2 + a**2*b*d**4*x**2 + a*b**2*c**4 + 2*a*b*...
```

3.38 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)^2} \, dx = -\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{(bc^2+ad^2)^2(c+dx)}$$
$$-\frac{a(b^2c(Bc-2Ad)+a^2d^2D+ab(2cCd-Bd^2-c^2D))-b(Ab(bc^2-ad^2)-a(bc(cC-2Bd)-ad(Cd}{2ab(bc^2+ad^2)^2(a+bx^2)}$$
$$+\frac{(Ab(b^2c^4+6abc^2d^2-3a^2d^4)+a(b^2c^3(cC-2Bd)+a^2d^3(Cd-2cD)-6abcd(cCd-Bd^2-c^2D)))\ \mathrm{ar}}{2a^{3/2}\sqrt{b}(bc^2+ad^2)^3}$$
$$-\frac{(ad^2(2cCd-Bd^2-3c^2D)-bc(2c^2Cd-3Bcd^2+4Ad^3-c^3D))\log(c+dx)}{(bc^2+ad^2)^3}$$
$$+\frac{(ad^2(2cCd-Bd^2-3c^2D)-bc(2c^2Cd-3Bcd^2+4Ad^3-c^3D))\log(a+bx^2)}{2(bc^2+ad^2)^3}$$

output
```
-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2+b*c^2)^2/(d*x+c)-1/2*(a*(b^2*c*(-2*A
*d+B*c)+a^2*d^2*D+a*b*(-B*d^2+2*C*c*d-D*c^2))-b*(A*b*(-a*d^2+b*c^2)-a*(b*c
*(-2*B*d+C*c)-a*d*(C*d-2*D*c)))*x)/a/b/(a*d^2+b*c^2)^2/(b*x^2+a)+1/2*(A*b*
(-3*a^2*d^4+6*a*b*c^2*d^2+b^2*c^4)+a*(b^2*c^3*(-2*B*d+C*c)+a^2*d^3*(C*d-2*
D*c)-6*a*b*c*d*(-B*d^2+C*c*d-D*c^2)))*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^
(1/2)/(a*d^2+b*c^2)^3-(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(4*A*d^3-3*B*c*d
^2+2*C*c^2*d-D*c^3))*ln(d*x+c)/(a*d^2+b*c^2)^3+1/2*(a*d^2*(-B*d^2+2*C*c*d-
3*D*c^2)-b*c*(4*A*d^3-3*B*c*d^2+2*C*c^2*d-D*c^3))*ln(b*x^2+a)/(a*d^2+b*c^2
)^3
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx$$
$$= \frac{\frac{2(bc^2+ad^2)(-c^2Cd+Bcd^2-Ad^3+c^3D)}{c+dx}+\frac{(bc^2+ad^2)\left(-a^3d^2D+Ab^3c^2x-ab^2\left(c^2Cx+Bc(c-2dx)+Ad(-2c+dx)\right)+a^2b\left(c^2D+d^2(B+Cx)-2c\right.\right.}{ab(a+bx^2)}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^2),x]
```

output
```
((2*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x) + ((
b*c^2 + a*d^2)*(-(a^3*d^2*D) + A*b^3*c^2*x - a*b^2*(c^2*C*x + B*c*(c - 2*d
*x) + A*d*(-2*c + d*x)) + a^2*b*(c^2*D + d^2*(B + C*x) - 2*c*d*(C + D*x)))
)/(a*b*(a + b*x^2)) + ((A*b*(b^2*c^4 + 6*a*b*c^2*d^2 - 3*a^2*d^4) + a*(b^2
*c^3*(c*C - 2*B*d) + a^2*d^3*(C*d - 2*c*D) + 6*a*b*c*d*(-(c*C*d) + B*d^2 +
 c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + 2*(a*d^2*(-2*c*
C*d + B*d^2 + 3*c^2*D) + b*c*(2*c^2*C*d - 3*B*c*d^2 + 4*A*d^3 - c^3*D))*Lo
g[c + d*x] + (-(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*c*(-2*c^2*C*d + 3*
B*c*d^2 - 4*A*d^3 + c^3*D))*Log[a + b*x^2])/(2*(b*c^2 + a*d^2)^3)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2(c+dx)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{\frac{bd^2\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2cD))\right)x^2}{\left(bc^2+ad^2\right)^2}+\frac{2b\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)x}{bc^2+ad^2}+\frac{b\left(a(bc(cC-2Bd)-ad(Cd-2cD))c^2+A\left(b^2c^4+5abd^2c^2+2\cdots\right)\right)}{\left(bc^2+ad^2\right)^2}}{(c+dx)^2(bx^2+a)}}{2ab} \frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-\cdots))\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{bd^2\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2cD))\right)x^2}{\left(bc^2+ad^2\right)^2}+\frac{2b\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)x}{bc^2+ad^2}+\frac{b\left(a(bc(cC-2Bd)-ad(Cd-2cD))c^2+A\left(b^2c^4+5abd^2c^2+2a^2d^4\cdots\right)\right)}{\left(bc^2+ad^2\right)^2}}{(c+dx)^2(bx^2+a)}}{2ab} \frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-\cdots))\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\int \left(\frac{2abd(-Dc^3+Cdc^2-Bd^2c+Ad^3)}{(bc^2+ad^2)^2(c+dx)^2}+\frac{2abd(bc(-Dc^3+2Cdc^2-3Bd^2c+4Ad^3)-ad^2(-3Dc^2+2Cdc-Bd^2))}{(bc^2+ad^2)^3(c+dx)}+\frac{b(Ab(b^2c^4+6abd^2c^2-3a^2d^4)\cdots)}{\cdots}\right)}{2a\cdots} \frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-\cdots))\right)}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(-3a^2d^4+6abc^2d^2+b^2c^4\right)+a\left(a^2d^3(Cd-2cD)-6abcd\left(-Bd^2+c^2(-D)+cCd\right)+b^2c^3(cC-2Bd)\right)\right)}{\sqrt{a}\left(ad^2+bc^2\right)^3}+\frac{ab\log\left(a+bx^2\right)\left(ad^2\left(-\right.\right.}{}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.\right.}{2ab\left(a+bx^2\right)\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^2),x]
```

output
```
-1/2*(a*(b^2*c*(B*c - 2*A*d) + a^2*d^2*D + a*b*(2*c*C*d - B*d^2 - c^2*D))
- b*(A*b*(b*c^2 - a*d^2) - a*(b*c*(c*C - 2*B*d) - a*d*(C*d - 2*c*D)))*x)/(
a*b*(b*c^2 + a*d^2)^2*(a + b*x^2)) + ((-2*a*b*(c^2*C*d - B*c*d^2 + A*d^3 -
 c^3*D))/((b*c^2 + a*d^2)^2*(c + d*x)) + (Sqrt[b]*(A*b*(b^2*c^4 + 6*a*b*c^
2*d^2 - 3*a^2*d^4) + a*(b^2*c^3*(c*C - 2*B*d) + a^2*d^3*(C*d - 2*c*D) - 6*
a*b*c*d*(c*C*d - B*d^2 - c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b
*c^2 + a*d^2)^3) - (2*a*b*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(2*c^2*
C*d - 3*B*c*d^2 + 4*A*d^3 - c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^3 + (a*b
*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(2*c^2*C*d - 3*B*c*d^2 + 4*A*d^3
 - c^3*D))*Log[a + b*x^2])/(b*c^2 + a*d^2)^3)/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\frac{\left(A\,a^{2} b\,d^{4}-A\,b^{3} c^{4}-2 B\,a^{2} b c\,d^{3}-2 B a\,b^{2} c^{3} d-C\,a^{3} d^{4}+C a\,b^{2} c^{4}+2 D a^{3} c\,d^{3}+2 D a^{2} b\,c^{3} d\right) x}{2 a}-\frac{2 A a\,b^{2} c\,d^{3}+2 A\,b^{3} c^{3} d+B\,a^{2} b\,d^{4}-B\,b^{3} c^{4}-2 C\,a^{2} b c}{2 b}}{b\,x^{2}+a}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d^2+b*c^2)^3*((1/2*(A*a^2*b*d^4-A*b^3*c^4-2*B*a^2*b*c*d^3-2*B*a*b^2*
c^3*d-C*a^3*d^4+C*a*b^2*c^4+2*D*a^3*c*d^3+2*D*a^2*b*c^3*d)/a*x-1/2*(2*A*a*
b^2*c*d^3+2*A*b^3*c^3*d+B*a^2*b*d^4-B*b^3*c^4-2*C*a^2*b*c*d^3-2*C*a*b^2*c^
3*d-D*a^3*d^4+D*a*b^2*c^4)/b)/(b*x^2+a)+1/2/a*(1/2*(8*A*a*b^2*c*d^3+2*B*a^
2*b*d^4-6*B*a*b^2*c^2*d^2-4*C*a^2*b*c*d^3+4*C*a*b^2*c^3*d+6*D*a^2*b*c^2*d^
2-2*D*a*b^2*c^4)/b*ln(b*x^2+a)+(3*A*a^2*b*d^4-6*A*a*b^2*c^2*d^2-A*b^3*c^4-
6*B*a^2*b*c*d^3+2*B*a*b^2*c^3*d-C*a^3*d^4+6*C*a^2*b*c^2*d^2-C*a*b^2*c^4+2*
D*a^3*c*d^3-6*D*a^2*b*c^3*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+(4*A*b*
c*d^3+B*a*d^4-3*B*b*c^2*d^2-2*C*a*c*d^3+2*C*b*c^3*d+3*D*a*c^2*d^2-D*b*c^4)
/(a*d^2+b*c^2)^3*ln(d*x+c)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2+b*c^2)^2/(
d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx$$
$$=\frac{(Dbc^4-2\,Cbc^3d-Bad^4-3\,(Da-Bb)c^2d^2+2\,(Ca-2\,Ab)cd^3)\log\left(bx^2+a\right)}{2\,(b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6)}$$
$$-\frac{(Dbc^4-2\,Cbc^3d-Bad^4-3\,(Da-Bb)c^2d^2+2\,(Ca-2\,Ab)cd^3)\log\left(dx+c\right)}{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}$$
$$+\frac{((Cab^2+Ab^3)c^4+2\,(3\,Da^2b-Bab^2)c^3d-6\,(Ca^2b-Aab^2)c^2d^2-2\,(Da^3-3\,Ba^2b)cd^3+(Ca^3-3\,A}{2\,(ab^3c^6+3\,a^2b^2c^4d^2+3\,a^3bc^2d^4+a^4d^6)\sqrt{ab}}$$
$$-\frac{2\,Aa^2bd^3-(3\,Da^2b-Bab^2)c^3+2\,(2\,Ca^2b-Aab^2)c^2d+(Da^3-3\,Ba^2b)cd^2-(2\,Dab^2c^3-(3\,Cab^2-}{2\,(a^2b^3c^5+2\,a^3b^2c^3d^2+a^4bcd^4+(ab^4c^4d+2\,a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*b*c^4 - 2*C*b*c^3*d - B*a*d^4 - 3*(D*a - B*b)*c^2*d^2 + 2*(C*a - 2*
A*b)*c*d^3)*log(b*x^2 + a)/(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 +
a^3*d^6) - (D*b*c^4 - 2*C*b*c^3*d - B*a*d^4 - 3*(D*a - B*b)*c^2*d^2 + 2*(C
*a - 2*A*b)*c*d^3)*log(d*x + c)/(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d
^4 + a^3*d^6) + 1/2*((C*a*b^2 + A*b^3)*c^4 + 2*(3*D*a^2*b - B*a*b^2)*c^3*d
 - 6*(C*a^2*b - A*a*b^2)*c^2*d^2 - 2*(D*a^3 - 3*B*a^2*b)*c*d^3 + (C*a^3 -
3*A*a^2*b)*d^4)*arctan(b*x/sqrt(a*b))/((a*b^3*c^6 + 3*a^2*b^2*c^4*d^2 + 3*
a^3*b*c^2*d^4 + a^4*d^6)*sqrt(a*b)) - 1/2*(2*A*a^2*b*d^3 - (3*D*a^2*b - B*
a*b^2)*c^3 + 2*(2*C*a^2*b - A*a*b^2)*c^2*d + (D*a^3 - 3*B*a^2*b)*c*d^2 - (
2*D*a*b^2*c^3 - (3*C*a*b^2 - A*b^3)*c^2*d - 2*(D*a^2*b - 2*B*a*b^2)*c*d^2
+ (C*a^2*b - 3*A*a*b^2)*d^3)*x^2 + ((C*a*b^2 - A*b^3)*c^3 + (D*a^2*b - B*a
*b^2)*c^2*d + (C*a^2*b - A*a*b^2)*c*d^2 + (D*a^3 - B*a^2*b)*d^3)*x)/(a^2*b
^3*c^5 + 2*a^3*b^2*c^3*d^2 + a^4*b*c*d^4 + (a*b^4*c^4*d + 2*a^2*b^3*c^2*d^
3 + a^3*b^2*d^5)*x^3 + (a*b^4*c^5 + 2*a^2*b^3*c^3*d^2 + a^3*b^2*c*d^4)*x^2
 + (a^2*b^3*c^4*d + 2*a^3*b^2*c^2*d^3 + a^4*b*d^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)^2}\,dx$$
$$= \frac{(Dbc^4-2Cbc^3d-3Dac^2d^2+3Bbc^2d^2+2Cacd^3-4Abcd^3-Bad^4)\log\left(b-\frac{2bc}{dx+c}+\frac{bc^2}{(dx+c)^2}+\frac{ad^2}{(dx+c)^2}\right)}{2(b^3c^6+3ab^2c^4d^2+3a^2bc^2d^4+a^3d^6)}$$
$$+\frac{\frac{Dc^3d^4}{dx+c}-\frac{Cc^2d^5}{dx+c}+\frac{Bcd^6}{dx+c}-\frac{Ad^7}{dx+c}}{b^2c^4d^4+2abc^2d^6+a^2d^8}$$
$$+\frac{(Cab^2c^4d^2+Ab^3c^4d^2+6Da^2bc^3d^3-2Bab^2c^3d^3-6Ca^2bc^2d^4+6Aab^2c^2d^4-2Da^3cd^5+6Ba^2bcd^5}{2(ab^3c^6+3a^2b^2c^4d^2+3a^3bc^2d^4+a^4d^6)\sqrt{ab}d^2}$$
$$-\frac{\frac{Cab^2c^3d-Ab^3c^3d+3Da^2bc^2d^2-3Bab^2c^2d^2-3Ca^2bcd^3+3Aab^2cd^3-Da^3d^4+Ba^2bd^4}{bc^2+ad^2}-\frac{Cab^2c^4d^2-Ab^3c^4d^2+4Da^2bc^3d^3-4Bab^2c^3d^3-}{(}}{2(bc^2+ad^2)^2a\left(b-\frac{2bc}{dx+c}+\frac{bc^2}{(dx+c)^2}+\frac{ad^2}{(dx+c)^2}\right)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*b*c^4 - 2*C*b*c^3*d - 3*D*a*c^2*d^2 + 3*B*b*c^2*d^2 + 2*C*a*c*d^3 -
 4*A*b*c*d^3 - B*a*d^4)*log(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d^
2/(d*x + c)^2)/(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6) + (
D*c^3*d^4/(d*x + c) - C*c^2*d^5/(d*x + c) + B*c*d^6/(d*x + c) - A*d^7/(d*x
 + c))/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) + 1/2*(C*a*b^2*c^4*d^2 + A*
b^3*c^4*d^2 + 6*D*a^2*b*c^3*d^3 - 2*B*a*b^2*c^3*d^3 - 6*C*a^2*b*c^2*d^4 +
6*A*a*b^2*c^2*d^4 - 2*D*a^3*c*d^5 + 6*B*a^2*b*c*d^5 + C*a^3*d^6 - 3*A*a^2*
b*d^6)*arctan((b*c - b*c^2/(d*x + c) - a*d^2/(d*x + c))/(sqrt(a*b)*d))/((a
*b^3*c^6 + 3*a^2*b^2*c^4*d^2 + 3*a^3*b*c^2*d^4 + a^4*d^6)*sqrt(a*b)*d^2) -
 1/2*((C*a*b^2*c^3*d - A*b^3*c^3*d + 3*D*a^2*b*c^2*d^2 - 3*B*a*b^2*c^2*d^2
 - 3*C*a^2*b*c*d^3 + 3*A*a*b^2*c*d^3 - D*a^3*d^4 + B*a^2*b*d^4)/(b*c^2 + a
*d^2) - (C*a*b^2*c^4*d^2 - A*b^3*c^4*d^2 + 4*D*a^2*b*c^3*d^3 - 4*B*a*b^2*c
^3*d^3 - 6*C*a^2*b*c^2*d^4 + 6*A*a*b^2*c^2*d^4 - 4*D*a^3*c*d^5 + 4*B*a^2*b
*c*d^5 + C*a^3*d^6 - A*a^2*b*d^6)/((b*c^2 + a*d^2)*(d*x + c)*d))/((b*c^2 +
 a*d^2)^2*a*(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^2 + a*d^2/(d*x + c)^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^2*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2196, normalized size of antiderivative = 4.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^2,x)
```

output

```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c**2*d**4 - 3*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c*d**5*x - sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*c**3*d**4 - sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**3*c**2*d**5*x + 6*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**2*b**2*c**4*d**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a**2*b**2*c**3*d**3*x + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*a**2*b**2*c**3*d**3 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a**2*b**2*c**2*d**4*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**2*b**2*c**2*d**4*x - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*a**2*b**2*c*d**5*x**3 - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*a**2*b*c**3*d**4*x**2 - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b*c**2*d**5*x**3 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3
*c**6 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**5*d*x - 2*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**5*d + 6*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**4*d**2*x**2 - 2*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**4*d**2*x + 6*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**3*d**3*x**3 + 6*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**3*d**3*x**2 + 6*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**2*d**4*x**3 + sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*c**7 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*...
```

3.39 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 442

$$\begin{aligned}
&\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx \\
&= \frac{d^3Dx}{b^3} - \frac{b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{4b^3(a+bx^2)^2} \\
&\quad + \frac{(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC+3Bd)+a^2d^3D-abd(3cCd+Bd^2+3c^2D)))\,x}{4ab^3(a+bx^2)^2} \\
&\quad + \frac{2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{2b^3(a+bx^2)} \\
&\quad + \frac{(3Ab^2c(bc^2+ad^2)+a(b^2c^2(cC+3Bd)+9a^2d^3D-5abd(3cCd+Bd^2+3c^2D)))\,x}{8a^2b^3(a+bx^2)} \\
&\quad + \frac{(3Ab^2c(bc^2+ad^2)+a(b^2c^2(cC+3Bd)-15a^2d^3D+3abd(3cCd+Bd^2+3c^2D)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}} \\
&\quad + \frac{d^2(Cd+3cD)\log(a+bx^2)}{2b^3}
\end{aligned}$$

output
```
d^3*D*x/b^3-1/4*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*
d^2+3*C*c^2*d+D*c^3))/b^3/(b*x^2+a)^2+1/4*(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2
*c^2*(3*B*d+C*c)+a^2*d^3*D-a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a/b^3/(b*x^2+
a)^2+1/2*(2*a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))/b^3/(b*
x^2+a)+1/8*(3*A*b^2*c*(a*d^2+b*c^2)+a*(b^2*c^2*(3*B*d+C*c)+9*a^2*d^3*D-5*a
*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a^2/b^3/(b*x^2+a)+1/8*(3*A*b^2*c*(a*d^2+b
*c^2)+a*(b^2*c^2*(3*B*d+C*c)-15*a^2*d^3*D+3*a*b*d*(B*d^2+3*C*c*d+3*D*c^2))
)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(7/2)+1/2*d^2*(C*d+3*D*c)*ln(b*x^2+a
)/b^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$
$$= \frac{8\sqrt{b}d^3Dx+\frac{2\sqrt{b}\left(Ab^3c^3x-a^3d^2(Cd+3cD+dDx)-ab^2c\left(c^2Cx+3Ad(c+dx)+Bc(c+3dx)\right)+a^2b\left(c^3D+d^3(A+Bx)+3cd^2(B+Cx)+3c^2d(C-\right.\right.}{a\left(a+bx^2\right)^2}}{}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(8*Sqrt[b]*d^3*D*x + (2*Sqrt[b]*(A*b^3*c^3*x - a^3*d^2*(C*d + 3*c*D + d*D*
x) - a*b^2*c*(c^2*C*x + 3*A*d*(c + d*x) + B*c*(c + 3*d*x)) + a^2*b*(c^3*D
+ d^3*(A + B*x) + 3*c*d^2*(B + C*x) + 3*c^2*d*(C + D*x))))/(a*(a + b*x^2)^
2) + (Sqrt[b]*(3*A*b^3*c^3*x + a*b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2)*x + a^3
*d^2*(8*C*d + 24*c*D + 9*d*D*x) - a^2*b*(4*c^3*D + d^3*(4*A + 5*B*x) + 3*c
*d^2*(4*B + 5*C*x) + 3*c^2*d*(4*C + 5*D*x))))/(a^2*(a + b*x^2)) + ((3*A*b^
2*c*(b*c^2 + a*d^2) + a*(b^2*c^2*(c*C + 3*B*d) - 15*a^2*d^3*D + 3*a*b*d*(3
*c*C*d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) + 4*Sqrt[
b]*d^2*(C*d + 3*c*D)*Log[a + b*x^2])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2176, 25, 2176, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \ 2176$$

$$-\frac{\int -\frac{(c+dx)^2\left(4adDx^2+4a(Cd+cD)x+3Abc+a\left(cC+3Bd-\frac{3adD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{(c+dx)^3 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(c+dx)^2\left(4adDx^2+4a(Cd+cD)x+3Abc+acC+3ad\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{(c+dx)^3 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2176$$

$$\frac{-\frac{\int -\frac{(c+dx)\left(3Ab^2c^2+a(bc(cC+3Bd)+ad(8Cd+9cD))-d\left(3Acb^2+a(bcC+3bBd-15adD)\right)x\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^2\left(4a^2(cD+Cd)-x\left(a(-7adD+3bBd+bcC)+\right.\right.}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)^3 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{(c+dx)\left(3Ab^2c^2+a(bc(cC+3Bd)+ad(8Cd+9cD))-d\left(3Acb^2+a(bcC+3bBd-15adD)\right)x\right)}{bx^2+a}dx}{2ab}-\frac{(c+dx)^2\left(4a^2(cD+Cd)-x\left(a(-7adD+3bBd+bcC)+3Ab\right.\right.}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)^3 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 657$$

$$\frac{\frac{\int \left(\frac{3Ac\left(bc^2+ad^2\right)b^2+8a^2d^2(Cd+3cD)xb+a\left(-15a^2Dd^3+3ab\left(3Dc^2+3Cdc+Bd^2\right)d+b^2c^2(cC+3Bd)\right)}{b\left(bx^2+a\right)}-\frac{d^2\left(3Acb^2+a(bcC+3bBd-15adD)\right)}{b}\right)dx}{2ab}-\frac{(c+dx)^2}{\cdots}}{4ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(-15a^2d^3D+3abd\left(Bd^2+3c^2D+3cCd\right)+b^2c^2(3Bd+cC)\right)+3Ab^2c\left(ad^2+bc^2\right)\right)}{\sqrt{a}b^{3/2}}+\frac{4a^2d^2\log\left(a+bx^2\right)(3cD+Cd)}{b}-\frac{d^2x\left(a(-15adD+3bBd+bcC)+\cdots\right.}{b}}{2ab}}{4ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^3)/(a*b*(a + b*x^2)^2) +
 (-1/2*((c + d*x)^2*(4*a^2*(C*d + c*D) - (3*A*b^2*c + a*(b*c*C + 3*b*B*d -
 7*a*d*D))*x))/(a*b*(a + b*x^2)) + (-((d^2*(3*A*b^2*c + a*(b*c*C + 3*b*B*d
 - 15*a*d*D))*x)/b) + ((3*A*b^2*c*(b*c^2 + a*d^2) + a*(b^2*c^2*(c*C + 3*B*
d) - 15*a^2*d^3*D + 3*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (4*a^2*d^2*(C*d + 3*c*D)*Log[a + b*x^2])/
b)/(2*a*b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2176

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.07

method	result
default	$\frac{d^3 D x}{b^3}+\frac{\frac{b\left(3Aa\,b^2c\,d^2+3A\,b^3c^3-5B\,a^2b\,d^3+3Ba\,b^2c^2d-15C\,a^2bc\,d^2+Ca\,b^2c^3+9Da^3d^3-15Da^2b\,c^2d\right)x^3}{8a^2}+\left(-\frac{1}{2}A\,b^2d^3-\frac{3}{2}b^2c\,d^2B+Cab\,d^3-\right.}{}$

input

```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output

```
d^3*D*x/b^3+1/b^3*((1/8*b*(3*A*a*b^2*c*d^2+3*A*b^3*c^3-5*B*a^2*b*d^3+3*B*a
*b^2*c^2*d-15*C*a^2*b*c*d^2+C*a*b^2*c^3+9*D*a^3*d^3-15*D*a^2*b*c^2*d)/a^2*
x^3+(-1/2*A*b^2*d^3-3/2*b^2*c*d^2*B+C*a*b*d^3-3/2*C*b^2*c^2*d+3*D*a*b*c*d^
2-1/2*D*b^2*c^3)*x^2-1/8*(3*A*a*b^2*c*d^2-5*A*b^3*c^3+3*B*a^2*b*d^3+3*B*a*
b^2*c^2*d+9*C*a^2*b*c*d^2+C*a*b^2*c^3-7*D*a^3*d^3+9*D*a^2*b*c^2*d)/a*x-1/4
*A*a*b*d^3-3/4*A*b^2*c^2*d-3/4*B*a*b*c*d^2-1/4*B*b^2*c^3+3/4*C*a^2*d^3-3/4
*C*a*b*c^2*d+9/4*D*a^2*c*d^2-1/4*D*a*b*c^3)/(b*x^2+a)^2+1/8/a^2*(1/2*(8*C*
a^2*b*d^3+24*D*a^2*b*c*d^2)/b*ln(b*x^2+a)+(3*A*a*b^2*c*d^2+3*A*b^3*c^3+3*B
*a^2*b*d^3+3*B*a*b^2*c^2*d+9*C*a^2*b*c*d^2+C*a*b^2*c^3-15*D*a^3*d^3+9*D*a^
2*b*c^2*d)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1606, normalized size of antiderivative = 3.63

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*D*a^3*b^3*d^3*x^5 - 4*(D*a^4*b^2 + B*a^3*b^3)*c^3 - 12*(C*a^4*b^
2 + A*a^3*b^3)*c^2*d + 12*(3*D*a^5*b - B*a^4*b^2)*c*d^2 + 4*(3*C*a^5*b - A
*a^4*b^2)*d^3 + 2*((C*a^2*b^4 + 3*A*a*b^5)*c^3 - 3*(5*D*a^3*b^3 - B*a^2*b^
4)*c^2*d - 3*(5*C*a^3*b^3 - A*a^2*b^4)*c*d^2 + 5*(5*D*a^4*b^2 - B*a^3*b^3)
*d^3)*x^3 - 8*(D*a^3*b^3*c^3 + 3*C*a^3*b^3*c^2*d - 3*(2*D*a^4*b^2 - B*a^3*
b^3)*c*d^2 - (2*C*a^4*b^2 - A*a^3*b^3)*d^3)*x^2 - (((C*a*b^4 + 3*A*b^5)*c^
3 + 3*(3*D*a^2*b^3 + B*a*b^4)*c^2*d + 3*(3*C*a^2*b^3 + A*a*b^4)*c*d^2 - 3*
(5*D*a^3*b^2 - B*a^2*b^3)*d^3)*x^4 + (C*a^3*b^2 + 3*A*a^2*b^3)*c^3 + 3*(3*
D*a^4*b + B*a^3*b^2)*c^2*d + 3*(3*C*a^4*b + A*a^3*b^2)*c*d^2 - 3*(5*D*a^5
- B*a^4*b)*d^3 + 2*((C*a^2*b^3 + 3*A*a*b^4)*c^3 + 3*(3*D*a^3*b^2 + B*a^2*b
^3)*c^2*d + 3*(3*C*a^3*b^2 + A*a^2*b^3)*c*d^2 - 3*(5*D*a^4*b - B*a^3*b^2)*
d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*((C
*a^3*b^3 - 5*A*a^2*b^4)*c^3 + 3*(3*D*a^4*b^2 + B*a^3*b^3)*c^2*d + 3*(3*C*a
^4*b^2 + A*a^3*b^3)*c*d^2 - 3*(5*D*a^5*b - B*a^4*b^2)*d^3)*x + 8*(3*D*a^5*
b*c*d^2 + C*a^5*b*d^3 + (3*D*a^3*b^3*c*d^2 + C*a^3*b^3*d^3)*x^4 + 2*(3*D*a
^4*b^2*c*d^2 + C*a^4*b^2*d^3)*x^2)*log(b*x^2 + a))/(a^3*b^6*x^4 + 2*a^4*b^
5*x^2 + a^5*b^4), 1/8*(8*D*a^3*b^3*d^3*x^5 - 2*(D*a^4*b^2 + B*a^3*b^3)*c^3
 - 6*(C*a^4*b^2 + A*a^3*b^3)*c^2*d + 6*(3*D*a^5*b - B*a^4*b^2)*c*d^2 + 2*(
3*C*a^5*b - A*a^4*b^2)*d^3 + ((C*a^2*b^4 + 3*A*a*b^5)*c^3 - 3*(5*D*a^3*b^3
 - B*a^2*b^4)*c^2*d - 3*(5*C*a^3*b^3 - A*a^2*b^4)*c*d^2 + 5*(5*D*a^4*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dd^3x}{b^3}$$
$$-\frac{2\,(Da^3b+Ba^2b^2)c^3+6\,(Ca^3b+Aa^2b^2)c^2d-6\,(3\,Da^4-Ba^3b)cd^2-2\,(3\,Ca^4-Aa^3b)d^3-((Cab^3+}{}$$
$$+\frac{(3\,Dcd^2+Cd^3)\log(bx^2+a)}{2\,b^3}$$
$$+\frac{((Cab^2+3\,Ab^3)c^3+3\,(3\,Da^2b+Bab^2)c^2d+3\,(3\,Ca^2b+Aab^2)cd^2-3\,(5\,Da^3-Ba^2b)d^3)\arctan\left(\frac{bx}{\sqrt{a}}\right.}{8\,\sqrt{ab}a^2b^3}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
D*d^3*x/b^3 - 1/8*(2*(D*a^3*b + B*a^2*b^2)*c^3 + 6*(C*a^3*b + A*a^2*b^2)*c
^2*d - 6*(3*D*a^4 - B*a^3*b)*c*d^2 - 2*(3*C*a^4 - A*a^3*b)*d^3 - ((C*a*b^3
 + 3*A*b^4)*c^3 - 3*(5*D*a^2*b^2 - B*a*b^3)*c^2*d - 3*(5*C*a^2*b^2 - A*a*b
^3)*c*d^2 + (9*D*a^3*b - 5*B*a^2*b^2)*d^3)*x^3 + 4*(D*a^2*b^2*c^3 + 3*C*a^
2*b^2*c^2*d - 3*(2*D*a^3*b - B*a^2*b^2)*c*d^2 - (2*C*a^3*b - A*a^2*b^2)*d^
3)*x^2 + ((C*a^2*b^2 - 5*A*a*b^3)*c^3 + 3*(3*D*a^3*b + B*a^2*b^2)*c^2*d +
3*(3*C*a^3*b + A*a^2*b^2)*c*d^2 - (7*D*a^4 - 3*B*a^3*b)*d^3)*x)/(a^2*b^5*x
^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 1/2*(3*D*c*d^2 + C*d^3)*log(b*x^2 + a)/b^3
 + 1/8*((C*a*b^2 + 3*A*b^3)*c^3 + 3*(3*D*a^2*b + B*a*b^2)*c^2*d + 3*(3*C*a
^2*b + A*a*b^2)*c*d^2 - 3*(5*D*a^3 - B*a^2*b)*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \frac{Dd^3x}{b^3} + \frac{(3 Dcd^2 + Cd^3) \log(bx^2+a)}{2 b^3} + \frac{(Cab^2c^3 + 3 Ab^3c^3 + 9 Da^2bc^2d + 3 Bab^2c^2d + 9 Ca^2bcd^2 + 3 Aab^2cd^2 - 15 Da^3d^3 + 3 Ba^2bd^3) \arctan}{8 \sqrt{ab}a^2b^3} - \frac{2 Da^3bc^3 + 2 Ba^2b^2c^3 + 6 Ca^3bc^2d + 6 Aa^2b^2c^2d - 18 Da^4cd^2 + 6 Ba^3bcd^2 - 6 Ca^4d^3 + 2 Aa^3bd^3 - (C}{}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
D*d^3*x/b^3 + 1/2*(3*D*c*d^2 + C*d^3)*log(b*x^2 + a)/b^3 + 1/8*(C*a*b^2*c^
3 + 3*A*b^3*c^3 + 9*D*a^2*b*c^2*d + 3*B*a*b^2*c^2*d + 9*C*a^2*b*c*d^2 + 3*
A*a*b^2*c*d^2 - 15*D*a^3*d^3 + 3*B*a^2*b*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^2*b^3) - 1/8*(2*D*a^3*b*c^3 + 2*B*a^2*b^2*c^3 + 6*C*a^3*b*c^2*d + 6
*A*a^2*b^2*c^2*d - 18*D*a^4*c*d^2 + 6*B*a^3*b*c*d^2 - 6*C*a^4*d^3 + 2*A*a^
3*b*d^3 - (C*a*b^3*c^3 + 3*A*b^4*c^3 - 15*D*a^2*b^2*c^2*d + 3*B*a*b^3*c^2*
d - 15*C*a^2*b^2*c*d^2 + 3*A*a*b^3*c*d^2 + 9*D*a^3*b*d^3 - 5*B*a^2*b^2*d^3
)*x^3 + 4*(D*a^2*b^2*c^3 + 3*C*a^2*b^2*c^2*d - 6*D*a^3*b*c*d^2 + 3*B*a^2*b
^2*c*d^2 - 2*C*a^3*b*d^3 + A*a^2*b^2*d^3)*x^2 + (C*a^2*b^2*c^3 - 5*A*a*b^3
*c^3 + 9*D*a^3*b*c^2*d + 3*B*a^2*b^2*c^2*d + 9*C*a^3*b*c*d^2 + 3*A*a^2*b^2
*c*d^2 - 7*D*a^4*d^3 + 3*B*a^3*b*d^3)*x)/((b*x^2 + a)^2*a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \int \frac{(c+d\,x)^3 \left(A+B\,x+C\,x^2+x^3\,D\right)}{(b\,x^2+a)^3} dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.20

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d**4 + 3*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*c*d**2 + 3*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*d**3 + 18*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c**2*d**2 - 30*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**3*b*d**4*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*a**2*b**3*c**3 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*a**2*b**3*c**2*d + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*
b**3*c*d**2*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*
*3*d**3*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c**
4 + 36*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c**2*d**2*x
**2 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d**4*x**4
 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c**3*x**2 + 6*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c**2*d*x**2 + 3*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c*d**2*x**4 + 3*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*d**3*x**4 + 2*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**4*x**2 + 18*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**3*c**2*d**2*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*b**5*c**3*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**5*c**2*d*x**4 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*b**4*c**4*x**4 + 16*log(a + b*x**2)*a**4*b*c*d**3 + 32*log(a + b*x**2)...
```

3.40 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 315

$$\begin{aligned}
\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx
&= -\frac{b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D)}{4b^3(a+bx^2)^2} \\
&\quad + \frac{(Ab(bc^2-ad^2)-a(bc(cC+2Bd)-ad(Cd+2cD)))x}{4ab^2(a+bx^2)^2} \\
&\quad + \frac{2ad^2D-b(2cCd+Bd^2+c^2D)}{2b^3(a+bx^2)} \\
&\quad + \frac{(Ab(3bc^2+ad^2)+a(bc(cC+2Bd)-5ad(Cd+2cD)))x}{8a^2b^2(a+bx^2)} \\
&\quad + \frac{(Ab(3bc^2+ad^2)+a(bc(cC+2Bd)+3ad(Cd+2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}} \\
&\quad + \frac{d^2D\log(a+bx^2)}{2b^3}
\end{aligned}$$

output
```
-1/4*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)
^2+1/4*(A*b*(-a*d^2+b*c^2)-a*(b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*x/a/b^2/(b
*x^2+a)^2+1/2*(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)+1/8*(A*b*(
a*d^2+3*b*c^2)+a*(b*c*(2*B*d+C*c)-5*a*d*(C*d+2*D*c)))*x/a^2/b^2/(b*x^2+a)+
1/8*(A*b*(a*d^2+3*b*c^2)+a*(b*c*(2*B*d+C*c)+3*a*d*(C*d+2*D*c)))*arctan(b^(
1/2)*x/a^(1/2))/a^(5/2)/b^(5/2)+1/2*d^2*D*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{\frac{-2a^3d^2D+2Ab^3c^2x-2ab^2(c^2Cx+Ad(2c+dx)+Bc(c+2dx))+2a^2b(c^2D+d^2(B+Cx)+2cd(C+Dx))}{a(a+bx^2)^2}+\frac{8a^3d^2D+3Ab^3c^2x+ab^2(c^2C+2Bcd+}{}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((-2*a^3*d^2*D + 2*A*b^3*c^2*x - 2*a*b^2*(c^2*C*x + A*d*(2*c + d*x) + B*c*
(c + 2*d*x)) + 2*a^2*b*(c^2*D + d^2*(B + C*x) + 2*c*d*(C + D*x)))/(a*(a +
b*x^2)^2) + (8*a^3*d^2*D + 3*A*b^3*c^2*x + a*b^2*(c^2*C + 2*B*c*d + A*d^2)
*x - a^2*b*(4*c^2*D + d^2*(4*B + 5*C*x) + 2*c*d*(4*C + 5*D*x)))/(a^2*(a +
b*x^2)) + (Sqrt[b]*(A*b*(3*b*c^2 + a*d^2) + a*(b*c*(c*C + 2*B*d) + 3*a*d*(
C*d + 2*c*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) + 4*d^2*D*Log[a + b*x^
2])/(8*b^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2176, 25, 2176, 25, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \; \color{blue}{2176}$$

$$-\frac{\int -\frac{(c+dx)\left(4adDx^2+(Abd+3aCd+4acD)x+3Abc+a\left(cC+2Bd-\frac{2adD}{b}\right)\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; \color{blue}{25}$$

$$\frac{\int \frac{(c+dx)\left(4adDx^2+(Abd+3aCd+4acD)x+3Abc+acC+2ad\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; \color{blue}{2176}$$

$$\frac{-\frac{\int -\frac{8a^2Dxd^2+Ab\left(3bc^2+ad^2\right)+a(bc(cC+2Bd)+3ad(Cd+2cD))}{bx^2+a}dx}{2ab}-\frac{(c+dx)\left(a(4acD+3aCd+Abd)-x\left(a(-6adD+2bBd+bcC)+3Ab^2c\right)\right)}{2ab\left(a+bx^2\right)}}{4ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; \color{blue}{25}$$

$$\frac{\frac{\int \frac{8a^2Dxd^2+Ab\left(3bc^2+ad^2\right)+a(bc(cC+2Bd)+3ad(Cd+2cD))}{bx^2+a}dx}{2ab}-\frac{(c+dx)\left(a(4acD+3aCd+Abd)-x\left(a(-6adD+2bBd+bcC)+3Ab^2c\right)\right)}{2ab\left(a+bx^2\right)}}{4ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; \color{blue}{452}$$

$$\frac{\frac{8a^2d^2D\int \frac{x}{bx^2+a}dx+\left(Ab\left(ad^2+3bc^2\right)+a(3ad(2cD+Cd)+bc(2Bd+cC))\right)\int \frac{1}{bx^2+a}dx}{2ab}-\frac{(c+dx)\left(a(4acD+3aCd+Abd)-x\left(a(-6adD+2bBd+bcC\right.\right.}{2ab\left(a+bx^2\right)}}{4ab}-\frac{(c+dx)^2 \left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; \color{blue}{218}$$

$$\frac{\frac{8a^2d^2D\int \frac{x}{bx^2+a}dx+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(ad^2+3bc^2\right)+a(3ad(2cD+Cd)+bc(2Bd+cC))\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{(c+dx)(a(4acD+3aCd+Abd)-x(a(-6adD+2bBd+bcC)+}{2ab(a+bx^2)}}{4ab}$$
$$\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \textcolor{blue}{240}$$

$$\frac{\frac{\frac{4a^2d^2D\log\left(a+bx^2\right)}{b}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(ad^2+3bc^2\right)+a(3ad(2cD+Cd)+bc(2Bd+cC))\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{(c+dx)(a(4acD+3aCd+Abd)-x(a(-6adD+2bBd+bcC)+}{2ab(a+bx^2)}}{4ab}$$
$$\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^2)/(a*b*(a + b*x^2)^2) +
 (-1/2*((c + d*x)*(a*(A*b*d + 3*a*C*d + 4*a*c*D) - (3*A*b^2*c + a*(b*c*C +
 2*b*B*d - 6*a*d*D))*x))/(a*b*(a + b*x^2)) + (((A*b*(3*b*c^2 + a*d^2) + a*
(b*c*(c*C + 2*B*d) + 3*a*d*(C*d + 2*c*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*Sqrt[b]) + (4*a^2*d^2*D*Log[a + b*x^2])/b)/(2*a*b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452

```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2176

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{\left(Aab\,d^{2}+3A\,b^{2}c^{2}+2abBcd-5a^{2}C\,d^{2}+Cab\,c^{2}-10a^{2}cdD\right)x^{3}}{8a^{2}b}-\frac{\left(Bb\,d^{2}+2Cbcd-2a\,d^{2}D+Db\,c^{2}\right)x^{2}}{2b^{2}}-\frac{\left(Aab\,d^{2}-5A\,b^{2}c^{2}+2abBcd+3a^{2}C\,d^{2}+Cab\,c\right.}{8a\,b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$

input

```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output

```
(1/8*(A*a*b*d^2+3*A*b^2*c^2+2*B*a*b*c*d-5*C*a^2*d^2+C*a*b*c^2-10*D*a^2*c*d
)/a^2/b*x^3-1/2*(B*b*d^2+2*C*b*c*d-2*D*a*d^2+D*b*c^2)/b^2*x^2-1/8*(A*a*b*d
^2-5*A*b^2*c^2+2*B*a*b*c*d+3*C*a^2*d^2+C*a*b*c^2+6*D*a^2*c*d)/a/b^2*x-1/4*
(2*A*b^2*c*d+B*a*b*d^2+B*b^2*c^2+2*C*a*b*c*d-3*D*a^2*d^2+D*a*b*c^2)/b^3)/(
b*x^2+a)^2+1/8/a^2/b^2*(4*a^2*d^2*D/b*ln(b*x^2+a)+(A*a*b*d^2+3*A*b^2*c^2+2
*B*a*b*c*d+3*C*a^2*d^2+C*a*b*c^2+6*D*a^2*c*d)/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1074, normalized size of antiderivative = 3.41

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*((C*a^2*b^3 + 3*A*a*b^4)*c^2 - 2*(5*D*a^3*b^2 - B*a^2*b^3)*c*d -
(5*C*a^3*b^2 - A*a^2*b^3)*d^2)*x^3 - 4*(D*a^4*b + B*a^3*b^2)*c^2 - 8*(C*a^
4*b + A*a^3*b^2)*c*d + 4*(3*D*a^5 - B*a^4*b)*d^2 - 8*(D*a^3*b^2*c^2 + 2*C*
a^3*b^2*c*d - (2*D*a^4*b - B*a^3*b^2)*d^2)*x^2 - (((C*a*b^3 + 3*A*b^4)*c^2
 + 2*(3*D*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^2*b^2 + A*a*b^3)*d^2)*x^4 + (C*a
^3*b + 3*A*a^2*b^2)*c^2 + 2*(3*D*a^4 + B*a^3*b)*c*d + (3*C*a^4 + A*a^3*b)*
d^2 + 2*((C*a^2*b^2 + 3*A*a*b^3)*c^2 + 2*(3*D*a^3*b + B*a^2*b^2)*c*d + (3*
C*a^3*b + A*a^2*b^2)*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)
/(b*x^2 + a)) - 2*((C*a^3*b^2 - 5*A*a^2*b^3)*c^2 + 2*(3*D*a^4*b + B*a^3*b^
2)*c*d + (3*C*a^4*b + A*a^3*b^2)*d^2)*x + 8*(D*a^3*b^2*d^2*x^4 + 2*D*a^4*b
*d^2*x^2 + D*a^5*d^2)*log(b*x^2 + a))/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b
^3), 1/8*(((C*a^2*b^3 + 3*A*a*b^4)*c^2 - 2*(5*D*a^3*b^2 - B*a^2*b^3)*c*d -
 (5*C*a^3*b^2 - A*a^2*b^3)*d^2)*x^3 - 2*(D*a^4*b + B*a^3*b^2)*c^2 - 4*(C*a
^4*b + A*a^3*b^2)*c*d + 2*(3*D*a^5 - B*a^4*b)*d^2 - 4*(D*a^3*b^2*c^2 + 2*C
*a^3*b^2*c*d - (2*D*a^4*b - B*a^3*b^2)*d^2)*x^2 + (((C*a*b^3 + 3*A*b^4)*c^
2 + 2*(3*D*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^2*b^2 + A*a*b^3)*d^2)*x^4 + (C*
a^3*b + 3*A*a^2*b^2)*c^2 + 2*(3*D*a^4 + B*a^3*b)*c*d + (3*C*a^4 + A*a^3*b)
*d^2 + 2*((C*a^2*b^2 + 3*A*a*b^3)*c^2 + 2*(3*D*a^3*b + B*a^2*b^2)*c*d + (3
*C*a^3*b + A*a^2*b^2)*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - ((C*a^3*
b^2 - 5*A*a^2*b^3)*c^2 + 2*(3*D*a^4*b + B*a^3*b^2)*c*d + (3*C*a^4*b + A...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dd^2\log(bx^2+a)}{2\,b^3} + \frac{((Cab^3+3\,Ab^4)c^2-2\,(5\,Da^2b^2-Bab^3)cd-(5\,Ca^2b^2-Aab^3)d^2)x^3-2\,(Da^3b+Ba^2b^2)c^2-4\,(Ca^3b}{} + \frac{((Cab+3\,Ab^2)c^2+2\,(3\,Da^2+Bab)cd+(3\,Ca^2+Aab)d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*D*d^2*log(b*x^2 + a)/b^3 + 1/8*(((C*a*b^3 + 3*A*b^4)*c^2 - 2*(5*D*a^2*
b^2 - B*a*b^3)*c*d - (5*C*a^2*b^2 - A*a*b^3)*d^2)*x^3 - 2*(D*a^3*b + B*a^2
*b^2)*c^2 - 4*(C*a^3*b + A*a^2*b^2)*c*d + 2*(3*D*a^4 - B*a^3*b)*d^2 - 4*(D
*a^2*b^2*c^2 + 2*C*a^2*b^2*c*d - (2*D*a^3*b - B*a^2*b^2)*d^2)*x^2 - ((C*a^
2*b^2 - 5*A*a*b^3)*c^2 + 2*(3*D*a^3*b + B*a^2*b^2)*c*d + (3*C*a^3*b + A*a^
2*b^2)*d^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 1/8*((C*a*b + 3*A
*b^2)*c^2 + 2*(3*D*a^2 + B*a*b)*c*d + (3*C*a^2 + A*a*b)*d^2)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dd^2\log(bx^2+a)}{2\,b^3} + \frac{(Cabc^2+3\,Ab^2c^2+6\,Da^2cd+2\,Babcd+3\,Ca^2d^2+Aabd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2} + \frac{(Cab^2c^2+3\,Ab^3c^2-10\,Da^2bcd+2\,Bab^2cd-5\,Ca^2bd^2+Aab^2d^2)x^3-4\,(Da^2bc^2+2\,Ca^2bcd-2\,Da^3}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*D*d^2*log(b*x^2 + a)/b^3 + 1/8*(C*a*b*c^2 + 3*A*b^2*c^2 + 6*D*a^2*c*d
+ 2*B*a*b*c*d + 3*C*a^2*d^2 + A*a*b*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^2*b^2) + 1/8*((C*a*b^2*c^2 + 3*A*b^3*c^2 - 10*D*a^2*b*c*d + 2*B*a*b^2*c*
d - 5*C*a^2*b*d^2 + A*a*b^2*d^2)*x^3 - 4*(D*a^2*b*c^2 + 2*C*a^2*b*c*d - 2*
D*a^3*d^2 + B*a^2*b*d^2)*x^2 - (C*a^2*b*c^2 - 5*A*a*b^2*c^2 + 6*D*a^3*c*d
+ 2*B*a^2*b*c*d + 3*C*a^3*d^2 + A*a^2*b*d^2)*x - 2*(D*a^3*b*c^2 + B*a^2*b^
2*c^2 + 2*C*a^3*b*c*d + 2*A*a^2*b^2*c*d - 3*D*a^4*d^2 + B*a^3*b*d^2)/b)/((
b*x^2 + a)^2*a^2*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{{\left(c+d\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d**2 + 9*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*c*d**2 + 3*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*a**2*b**2*c*d + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**2*b**2*d**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b*c**3 + 18*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*c*d**2
*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**2*x**2 +
 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*d*x**2 + sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*d**2*x**4 + 2*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**3*x**2 + 9*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c*d**2*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*b**4*c**2*x**4 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*b**4*c*d*x**4 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b
**3*c**3*x**4 + 4*log(a + b*x**2)*a**4*d**3 + 8*log(a + b*x**2)*a**3*b*d**
3*x**2 + 4*log(a + b*x**2)*a**2*b**2*d**3*x**4 + 2*a**4*d**3 - 4*a**3*b**2
*c*d - a**3*b**2*d**2*x - 9*a**3*b*c*d**2*x + 5*a**2*b**3*c**2*x - 2*a**2*
b**3*c**2 - 2*a**2*b**3*c*d*x + a**2*b**3*d**2*x**3 - a**2*b**2*c**3*x - 1
5*a**2*b**2*c*d**2*x**3 - 4*a**2*b**2*d**3*x**4 + 3*a*b**4*c**2*x**3 + 2*a
*b**4*c*d*x**3 + 2*a*b**4*d**2*x**4 + a*b**3*c**3*x**3 + 6*a*b**3*c**2*d*x
**4)/(8*a**2*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.41 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$
\begin{aligned}
\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx
&= \frac{(Ab^2c-a(b(cC+Bd)-adD))\,x}{4ab^2(a+bx^2)^2}+\frac{(3Ab^2c+a(bcC+bBd-5adD))\,x}{8a^2b^2(a+bx^2)} \\
&\quad -\frac{(Bc+Ad+(Cd+cD)x^2)^2}{4(bBc+Abd-aCd-acD)(a+bx^2)^2} \\
&\quad +\frac{(3Ab^2c+a(bcC+bBd+3adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}
\end{aligned}
$$

output
```
1/4*(A*b^2*c-a*(b*(B*d+C*c)-D*a*d))*x/a/b^2/(b*x^2+a)^2+1/8*(3*A*b^2*c+a*(
B*b*d+C*b*c-5*D*a*d))*x/a^2/b^2/(b*x^2+a)-1/4*(B*c+A*d+(C*d+D*c)*x^2)^2/(A
*b*d+B*b*c-C*a*d-D*a*c)/(b*x^2+a)^2+1/8*(3*A*b^2*c+a*(B*b*d+C*b*c+3*D*a*d)
)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^3}\,dx = \frac{3Ab^2cx+ab(cC+Bd)x-a^2(4Cd+4cD+5dDx)}{8a^2b^2\,(a+bx^2)} + \frac{Ab^2cx-ab(Ad+cCx+B(c+dx))+a^2(Cd+D(c+dx))}{4ab^2\,(a+bx^2)^2} + \frac{(3Ab^2c+a(bcC+bBd+3adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(3*A*b^2*c*x + a*b*(c*C + B*d)*x - a^2*(4*C*d + 4*c*D + 5*d*D*x))/(8*a^2*b
^2*(a + b*x^2)) + (A*b^2*c*x - a*b*(A*d + c*C*x + B*(c + d*x)) + a^2*(C*d
+ D*(c + d*x)))/(4*a*b^2*(a + b*x^2)^2) + ((3*A*b^2*c + a*(b*c*C + b*B*d +
 3*a*d*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2176, 25, 2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{-\frac{dDa^2}{b}+4dDx^2a+(cC+Bd)a+3Abc+2(Abd+aCd+2acD)x}{(bx^2+a)^2}dx}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{4adDx^2+2(Abd+aCd+2acD)x+3Abc+acC+ad\left(B-\frac{aD}{b}\right)}{(bx^2+a)^2}dx}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 2345$$

$$-\frac{-\frac{\int -\frac{3Abc+\frac{a(bcC+bBd+3adD)}{b}}{bx^2+a}dx}{2a}-\frac{2a(2acD+aCd+Abd)-x(a(-5adD+bBd+bcC)+3Ab^2c)}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3Abc+aCc+ad\left(B+\frac{3aD}{b}\right)}{bx^2+a}dx}{2a}-\frac{2a(2acD+aCd+Abd)-x(a(-5adD+bBd+bcC)+3Ab^2c)}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(ad\left(\frac{3aD}{b}+B\right)+acC+3Abc\right)\int \frac{1}{bx^2+a}dx}{2a}-\frac{2a(2acD+aCd+Abd)-x(a(-5adD+bBd+bcC)+3Ab^2c)}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(ad\left(\frac{3aD}{b}+B\right)+acC+3Abc\right)}{2a^{3/2}\sqrt{b}}-\frac{2a(2acD+aCd+Abd)-x(a(-5adD+bBd+bcC)+3Ab^2c)}{2ab(a+bx^2)}}{4ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x))/(a*b*(a + b*x^2)^2) + (
-1/2*(2*a*(A*b*d + a*C*d + 2*a*c*D) - (3*A*b^2*c + a*(b*c*C + b*B*d - 5*a*
d*D))*x)/(a*b*(a + b*x^2)) + ((3*A*b*c + a*c*C + a*d*(B + (3*a*D)/b))*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{\left(3A\,b^2c+Babd+Cabc-5a^2dD\right)x^3}{8a^2b}-\frac{(Cd+Dc)x^2}{2b}+\frac{\left(5A\,b^2c-Babd-Cabc-3a^2dD\right)x}{8a\,b^2}-\frac{Abd+Bbc+Cad+Dac}{4b^2}}{\left(b\,x^2+a\right)^2}+\frac{(3A\,b^2c+Babd+Cabc+3a^2}{8a^2b^2\sqrt{a}}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b^2*c+B*a*b*d+C*a*b*c-5*D*a^2*d)/a^2/b*x^3-1/2*(C*d+D*c)*x^2/b+1
/8*(5*A*b^2*c-B*a*b*d-C*a*b*c-3*D*a^2*d)/a/b^2*x-1/4*(A*b*d+B*b*c+C*a*d+D*
a*c)/b^2)/(b*x^2+a)^2+1/8*(3*A*b^2*c+B*a*b*d+C*a*b*c+3*D*a^2*d)/a^2/b^2/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.31

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

$$= \left[\frac{2\left((Ca^2b^3+3\,Aab^4)c-(5\,Da^3b^2-Ba^2b^3)d\right)x^3-8\left(Da^3b^2c+Ca^3b^2d\right)x^2-\left(((Cab^3+3\,Ab^4)c+(3\,Da\right.}{}\right.$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*((C*a^2*b^3 + 3*A*a*b^4)*c - (5*D*a^3*b^2 - B*a^2*b^3)*d)*x^3 - 8
*(D*a^3*b^2*c + C*a^3*b^2*d)*x^2 - (((C*a*b^3 + 3*A*b^4)*c + (3*D*a^2*b^2
+ B*a*b^3)*d)*x^4 + 2*((C*a^2*b^2 + 3*A*a*b^3)*c + (3*D*a^3*b + B*a^2*b^2)
*d)*x^2 + (C*a^3*b + 3*A*a^2*b^2)*c + (3*D*a^4 + B*a^3*b)*d)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 4*(D*a^4*b + B*a^3*b^2)*c -
4*(C*a^4*b + A*a^3*b^2)*d - 2*((C*a^3*b^2 - 5*A*a^2*b^3)*c + (3*D*a^4*b +
B*a^3*b^2)*d)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*(((C*a^2*b^3
 + 3*A*a*b^4)*c - (5*D*a^3*b^2 - B*a^2*b^3)*d)*x^3 - 4*(D*a^3*b^2*c + C*a^
3*b^2*d)*x^2 + (((C*a*b^3 + 3*A*b^4)*c + (3*D*a^2*b^2 + B*a*b^3)*d)*x^4 +
2*((C*a^2*b^2 + 3*A*a*b^3)*c + (3*D*a^3*b + B*a^2*b^2)*d)*x^2 + (C*a^3*b +
 3*A*a^2*b^2)*c + (3*D*a^4 + B*a^3*b)*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
 2*(D*a^4*b + B*a^3*b^2)*c - 2*(C*a^4*b + A*a^3*b^2)*d - ((C*a^3*b^2 - 5*A
*a^2*b^3)*c + (3*D*a^4*b + B*a^3*b^2)*d)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 +
 a^5*b^3)]
```

Sympy [A] (verification not implemented)

Time = 132.73 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

$$= -\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot\left(3Ab^2c+Babd+Cabc+3Da^2d\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}$$

$$+\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot\left(3Ab^2c+Babd+Cabc+3Da^2d\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}$$

$$+\frac{-2Aa^2bd-2Ba^2bc-2Ca^3d-2Da^3c+x^3\cdot\left(3Ab^3c+Bab^2d+Cab^2c-5Da^2bd\right)+x^2(-4Ca^2bd-4}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**5))*(3*A*b**2*c + B*a*b*d + C*a*b*c + 3*D*a**2*d)*log(-a
**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*A*b**2*c +
 B*a*b*d + C*a*b*c + 3*D*a**2*d)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/1
6 + (-2*A*a**2*b*d - 2*B*a**2*b*c - 2*C*a**3*d - 2*D*a**3*c + x**3*(3*A*b*
*3*c + B*a*b**2*d + C*a*b**2*c - 5*D*a**2*b*d) + x**2*(-4*C*a**2*b*d - 4*D
*a**2*b*c) + x*(5*A*a*b**2*c - B*a**2*b*d - C*a**2*b*c - 3*D*a**3*d))/(8*a
**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$
$$= \frac{((Cab^2+3\,Ab^3)c-(5\,Da^2b-Bab^2)d)x^3-4\,(Da^2bc+Ca^2bd)x^2-2\,(Da^3+Ba^2b)c-2\,(Ca^3+Aa^2b)}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}$$
$$+\frac{((Cab+3\,Ab^2)c+(3\,Da^2+Bab)d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((((C*a*b^2 + 3*A*b^3)*c - (5*D*a^2*b - B*a*b^2)*d)*x^3 - 4*(D*a^2*b*c
+ C*a^2*b*d)*x^2 - 2*(D*a^3 + B*a^2*b)*c - 2*(C*a^3 + A*a^2*b)*d - ((C*a^2
*b - 5*A*a*b^2)*c + (3*D*a^3 + B*a^2*b)*d)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2
 + a^4*b^2) + 1/8*((C*a*b + 3*A*b^2)*c + (3*D*a^2 + B*a*b)*d)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$
$$= \frac{(Cabc+3\,Ab^2c+3\,Da^2d+Babd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$
$$+\frac{Cab^2cx^3+3\,Ab^3cx^3-5\,Da^2bdx^3+Bab^2dx^3-4\,Da^2bcx^2-4\,Ca^2bdx^2-Ca^2bcx+5\,Aab^2cx-3\,Da^3}{8\,(bx^2+a)^2a^2b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a*b*c + 3*A*b^2*c + 3*D*a^2*d + B*a*b*d)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2*b^2) + 1/8*(C*a*b^2*c*x^3 + 3*A*b^3*c*x^3 - 5*D*a^2*b*d*x^3 + B
*a*b^2*d*x^3 - 4*D*a^2*b*c*x^2 - 4*C*a^2*b*d*x^2 - C*a^2*b*c*x + 5*A*a*b^2
*c*x - 3*D*a^3*d*x - B*a^2*b*d*x - 2*D*a^3*c - 2*B*a^2*b*c - 2*C*a^3*d - 2
*A*a^2*b*d)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx=\int \frac{(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.32

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2c+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2d+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{}}\right.}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d**2 + 3*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c + sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**2*b**2*d + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a**2*b*c**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*
b*d**2*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**
2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*d*x**2 + 2*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c**2*x**2 + 3*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*d**2*x**4 + 3*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*b**4*c*x**4 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*b**4*d*x**4 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**3*c**2*x**4 - 2*a**3*b**2*d - 3*a**3*b*d**2*x + 5*a**2*b**3*c*x - 2*a**
2*b**3*c - a**2*b**3*d*x - a**2*b**2*c**2*x - 5*a**2*b**2*d**2*x**3 + 3*a*
b**4*c*x**3 + a*b**4*d*x**3 + a*b**3*c**2*x**3 + 4*a*b**3*c*d*x**4)/(8*a**
2*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.42 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = \frac{(Ab-aC)x}{4ab(a+bx^2)^2} + \frac{(3Ab+aC)x}{8a^2b(a+bx^2)} - \frac{(B+Dx^2)^2}{4(bB-aD)(a+bx^2)^2} + \frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
1/4*(A*b-C*a)*x/a/b/(b*x^2+a)^2+1/8*(3*A*b+C*a)*x/a^2/b/(b*x^2+a)-1/4*(D*x
^2+B)^2/(B*b-D*a)/(b*x^2+a)^2+1/8*(3*A*b+C*a)*arctan(b^(1/2)*x/a^(1/2))/a^
(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = \frac{\frac{\sqrt{a}(-2a^3D+3Ab^3x^3+ab^2x(5A+Cx^2)-a^2b(2B+x(C+4Dx)))}{(a+bx^2)^2} + \sqrt{b}(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x
*(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2345, 25, 27, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{b\left(3A+\frac{aC}{b}\right)+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{(bx^2+a)^2}dx}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{(aC+3Ab)\int \frac{1}{bx^2+a}dx}{2a}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{(aC+3Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) + (-1/2*(4*a^2*
D - b*(3*A*b + a*C)*x)/(a*b*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-Ca)x}{8ab}-\frac{Bb+Da}{4b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	98

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b
^2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$
$$-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^
3 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)
*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*
b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3
*b*x^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3
*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3
*x^2 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b
**2) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**
2*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= -\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a
^2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a +
3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\,(bx^2+a)^2a^2b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x
^3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B
*a^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)} - \frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,\left(b\,x^2+a\right)^2} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

output
```
((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
 + (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^
4 + 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(
1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(3/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{8a^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**2*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b*c*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**4 +
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 + 5*a**2*b**2*x
- 2*a**2*b**2 - a**2*b*c*x + 3*a*b**3*x**3 + a*b**2*c*x**3 + 2*a*b**2*d*x*
*4)/(8*a**2*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.43 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 472

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx \\
&= -\frac{a(bBc-Abd+aCd-acD)-\left(Ab^2c-a(bcC-bBd+adD)\right)x}{4ab\left(bc^2+ad^2\right)\left(a+bx^2\right)^2} \\
&\quad+\frac{4a^2b(c^2Cd-Bcd^2+Ad^3-c^3D)+\left(Ab^2c(3bc^2+7ad^2)+a(b^2c^2(cC-Bd)+a^2d^3D-abd(3cCd-3\right.}{8a^2b\left(bc^2+ad^2\right)^2\left(a+bx^2\right)} \\
&\quad+\frac{\left(Ab^2c(3b^2c^4+10abc^2d^2+15a^2d^4)+a(b^3c^4(cC-Bd)+a^3d^5D-3a^2bd^3(cCd-Bd^2-2c^2D)+3ab^2c\right.}{8a^{5/2}b^{3/2}\left(bc^2+ad^2\right)^3} \\
&\quad+\frac{d^2(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{\left(bc^2+ad^2\right)^3} \\
&\quad-\frac{d^2(c^2Cd-Bcd^2+Ad^3-c^3D)\log\left(a+bx^2\right)}{2\left(bc^2+ad^2\right)^3}
\end{aligned}
$$

output
```
-1/4*(a*(-A*b*d+B*b*c+C*a*d-D*a*c)-(A*b^2*c-a*(-B*b*d+C*b*c+D*a*d))*x)/a/b
/(a*d^2+b*c^2)/(b*x^2+a)^2+1/8*(4*a^2*b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)+(A*b
^2*c*(7*a*d^2+3*b*c^2)+a*(b^2*c^2*(-B*d+C*c)+a^2*d^3*D-a*b*d*(-3*B*d^2+3*C
*c*d-5*D*c^2)))*x)/a^2/b/(a*d^2+b*c^2)^2/(b*x^2+a)+1/8*(A*b^2*c*(15*a^2*d^
4+10*a*b*c^2*d^2+3*b^2*c^4)+a*(b^3*c^4*(-B*d+C*c)+a^3*d^5*D-3*a^2*b*d^3*(-
B*d^2+C*c*d-2*D*c^2)+3*a*b^2*c^2*d*(-2*B*d^2+2*C*c*d-D*c^2)))*arctan(b^(1/
2)*x/a^(1/2))/a^(5/2)/b^(3/2)/(a*d^2+b*c^2)^3+d^2*(A*d^3-B*c*d^2+C*c^2*d-D
*c^3)*ln(d*x+c)/(a*d^2+b*c^2)^3-1/2*d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(b
*x^2+a)/(a*d^2+b*c^2)^3
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx$$

$$= \frac{\frac{2(bc^2+ad^2)^2(Ab^2cx+ab(-Bc+Ad-cCx+Bdx)-a^2(Cd-cD+dDx))}{ab(a+bx^2)^2} + \frac{(bc^2+ad^2)(3Ab^3c^3x+ab^2c(c^2C-Bcd+7Ad^2)x+a^3d^3Dx+a^2b(-4c^3}{a^2b(a+bx^2)}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^3),x]
```

output
```
((2*(b*c^2 + a*d^2)^2*(A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x) - a^
2*(C*d - c*D + d*D*x)))/(a*b*(a + b*x^2)^2) + ((b*c^2 + a*d^2)*(3*A*b^3*c^
3*x + a*b^2*c*(c^2*C - B*c*d + 7*A*d^2)*x + a^3*d^3*D*x + a^2*b*(-4*c^3*D
+ d^3*(4*A + 3*B*x) - c*d^2*(4*B + 3*C*x) + c^2*d*(4*C + 5*D*x))))/(a^2*b*
(a + b*x^2)) + ((A*b^2*c*(3*b^2*c^4 + 10*a*b*c^2*d^2 + 15*a^2*d^4) + a*(b^
3*c^4*(c*C - B*d) + a^3*d^5*D - 3*a*b^2*c^2*d*(-2*c*C*d + 2*B*d^2 + c^2*D)
 + 3*a^2*b*d^3*(-(c*C*d) + B*d^2 + 2*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(5/2)*b^(3/2)) + 8*d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x
] + 4*d^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[a + b*x^2])/(8*(b*c^2
 + a*d^2)^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2178, 25, 27, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3 (c+dx)}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{Ab\left(3bc^2+4ad^2\right)+ac(bcC-bBd+adD)+\left(3Acdb^2+a\left(ad^2D-b\left(-4Dc^2+3Cdc-3Bd^2\right)\right)\right)x}{\left(bc^2+ad^2\right)(c+dx)\left(bx^2+a\right)^2}dx}{4ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab\left(3bc^2+4ad^2\right)+ac(bcC-bBd+adD)+\left(3Acdb^2+a\left(ad^2D-b\left(-4Dc^2+3Cdc-3Bd^2\right)\right)\right)x}{\left(bc^2+ad^2\right)(c+dx)\left(bx^2+a\right)^2}dx}{4ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab\left(3bc^2+4ad^2\right)+ac(bcC-bBd+adD)+\left(3Acdb^2+a\left(ad^2D-b\left(-4Dc^2+3Cdc-3Bd^2\right)\right)\right)x}{(c+dx)\left(bx^2+a\right)^2}dx}{4ab\left(ad^2+bc^2\right)}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{686}$$

$$\frac{\frac{x\left(a\left(a^2d^3D-abd\left(-3Bd^2-5c^2D+3cCd\right)+b^2c^2(cC-Bd)\right)+Ab^2c\left(7ad^2+3bc^2\right)\right)+4a^2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2a\left(a+bx^2\right)\left(ad^2+bc^2\right)}-\frac{\int -\frac{b\left(Ab\left(3b^2c^4+7abd^2c^2+\cdots\right.\right.}{\cdots}}{\cdots}}{4ab\left(ad^2+\cdots\right)}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\left(Ab\left(3b^2c^4+7abd^2c^2+8a^2d^4\right)+ac\left(a^2Dd^3+ab\left(-3Dc^2+5Cdc-5Bd^2\right)d+b^2c^2(cC-Bd)\right)+d\left(Ac\left(3bc^2+7ad^2\right)b^2+a\left(a^2Dd^3-ab\left(-5Dc^2+3Cdc-3Bd^2\right)d+b^2c^{\cdots}\right.\right.\right.}{(c+dx)\left(bx^2+a\right)}}{2ab(ad^2+bc^2)} \quad 4ab\left(ad^2+\cdots\right.$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{Ab\left(3b^2c^4+7abd^2c^2+8a^2d^4\right)+ac\left(a^2Dd^3+ab\left(-3Dc^2+5Cdc-5Bd^2\right)d+b^2c^2(cC-Bd)\right)+d\left(Ac\left(3bc^2+7ad^2\right)b^2+a\left(a^2Dd^3-ab\left(-5Dc^2+3Cdc-3Bd^2\right)d+b^2c^2(\cdots\right.\right.\right.}{(c+dx)\left(bx^2+a\right)}}{2a(ad^2+bc^2)} \quad 4ab\left(ad^2+b\cdots\right.$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{8a^2b\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)d^3}{\left(bc^2+ad^2\right)(c+dx)}+\frac{Ac\left(3b^2c^4+10abd^2c^2+15a^2d^4\right)b^2-8a^2d^2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)xb^2+a\left(a^3Dd^5-3a^2b\left(-2Dc^2+Cdc-Bd^2\right)d^3+3\cdots\right.}{\left(bc^2+ad^2\right)\left(bx^2+a\right)}\right.}{2a(ad^2+bc^2)}$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x\left(a\left(a^2d^3D-abd\left(-3Bd^2-5c^2D+3cCd\right)+b^2c^2(cC-Bd)\right)+Ab^2c\left(7ad^2+3bc^2\right)\right)+4a^2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2a(a+bx^2)(ad^2+bc^2)}+\frac{-\frac{4a^2bd^2\log\left(a+bx^2\right)\left(Ad^3\cdots\right.}{ad^2\cdots}}{\cdots}}{\cdots}$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^3),x]
```

output
```
-1/4*(a*(b*B*c - A*b*d + a*C*d - a*c*D) - (A*b^2*c - a*(b*c*C - b*B*d + a*
d*D))*x)/(a*b*(b*c^2 + a*d^2)*(a + b*x^2)^2) + ((4*a^2*b*(c^2*C*d - B*c*d^
2 + A*d^3 - c^3*D) + (A*b^2*c*(3*b*c^2 + 7*a*d^2) + a*(b^2*c^2*(c*C - B*d)
 + a^2*d^3*D - a*b*d*(3*c*C*d - 3*B*d^2 - 5*c^2*D)))*x)/(2*a*(b*c^2 + a*d^
2)*(a + b*x^2)) + (((A*b^2*c*(3*b^2*c^4 + 10*a*b*c^2*d^2 + 15*a^2*d^4) + a
*(b^3*c^4*(c*C - B*d) + a^3*d^5*D - 3*a^2*b*d^3*(c*C*d - B*d^2 - 2*c^2*D)
+ 3*a*b^2*c^2*d*(2*c*C*d - 2*B*d^2 - c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[a]*Sqrt[b]*(b*c^2 + a*d^2)) + (8*a^2*b*d^2*(c^2*C*d - B*c*d^2 + A*d
^3 - c^3*D)*Log[c + d*x])/(b*c^2 + a*d^2) - (4*a^2*b*d^2*(c^2*C*d - B*c*d^
2 + A*d^3 - c^3*D)*Log[a + b*x^2])/(b*c^2 + a*d^2))/(2*a*(b*c^2 + a*d^2)))
/(4*a*b*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.71

method	result
default	$\frac{\frac{\left(7A\,a^{2}b^{2}c\,d^{4}+10Aa\,b^{3}c^{3}d^{2}+3A\,b^{4}c^{5}+3B\,a^{3}b\,d^{5}+2B\,a^{2}b^{2}c^{2}d^{3}-Ba\,b^{3}c^{4}d-3C\,a^{3}bc\,d^{4}-2C\,a^{2}b^{2}c^{3}d^{2}+Ca\,b^{3}c^{5}+Da^{4}d^{5}+6Da^{3}b\,c^{2}d^{3}+5Da^{2}b^{2}c}{8a^{2}}}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2+b*c^2)^3*((1/8*(7*A*a^2*b^2*c*d^4+10*A*a*b^3*c^3*d^2+3*A*b^4*c^5+
3*B*a^3*b*d^5+2*B*a^2*b^2*c^2*d^3-B*a*b^3*c^4*d-3*C*a^3*b*c*d^4-2*C*a^2*b^
2*c^3*d^2+C*a*b^3*c^5+D*a^4*d^5+6*D*a^3*b*c^2*d^3+5*D*a^2*b^2*c^4*d)/a^2*x
^3+(1/2*A*a*b*d^5+1/2*A*b^2*c^2*d^3-1/2*B*a*b*c*d^4-1/2*B*b^2*c^3*d^2+1/2*
C*a*b*c^2*d^3+1/2*C*b^2*c^4*d-1/2*D*a*b*c^3*d^2-1/2*b^2*c^5*D)*x^2+1/8*(9*
A*a^2*b^2*c*d^4+14*A*a*b^3*c^3*d^2+5*A*b^4*c^5+5*B*a^3*b*d^5+6*B*a^2*b^2*c
^2*d^3+B*a*b^3*c^4*d-5*C*a^3*b*c*d^4-6*C*a^2*b^2*c^3*d^2-C*a*b^3*c^5-D*a^4
*d^5+2*D*a^3*b*c^2*d^3+3*D*a^2*b^2*c^4*d)/a/b*x+1/4*(3*A*a^2*b*d^5+4*A*a*b
^2*c^2*d^3+A*b^3*c^4*d-3*B*a^2*b*c*d^4-4*B*a*b^2*c^3*d^2-B*b^3*c^5-C*a^3*d
^5+C*a*b^2*c^4*d+D*a^3*c*d^4-D*a*b^2*c^5)/b)/(b*x^2+a)^2+1/8/b/a^2*(1/2*(-
8*A*a^2*b^2*d^5+8*B*a^2*b^2*c*d^4-8*C*a^2*b^2*c^2*d^3+8*D*a^2*b^2*c^3*d^2)
/b*ln(b*x^2+a)+(15*A*a^2*b^2*c*d^4+10*A*a*b^3*c^3*d^2+3*A*b^4*c^5+3*B*a^3*
b*d^5-6*B*a^2*b^2*c^2*d^3-B*a*b^3*c^4*d-3*C*a^3*b*c*d^4+6*C*a^2*b^2*c^3*d^
2+C*a*b^3*c^5+D*a^4*d^5+6*D*a^3*b*c^2*d^3-3*D*a^2*b^2*c^4*d)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))))+d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/(a*d^
2+b*c^2)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*(D*c^3*d^2 - C*c^2*d^3 + B*c*d^4 - A*d^5)*log(b*x^2 + a)/(b^3*c^6 + 3*
a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6) - (D*c^3*d^2 - C*c^2*d^3 + B*c*
d^4 - A*d^5)*log(d*x + c)/(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a
^3*d^6) + 1/8*((C*a*b^3 + 3*A*b^4)*c^5 - (3*D*a^2*b^2 + B*a*b^3)*c^4*d + 2
*(3*C*a^2*b^2 + 5*A*a*b^3)*c^3*d^2 + 6*(D*a^3*b - B*a^2*b^2)*c^2*d^3 - 3*(
C*a^3*b - 5*A*a^2*b^2)*c*d^4 + (D*a^4 + 3*B*a^3*b)*d^5)*arctan(b*x/sqrt(a*
b))/((a^2*b^4*c^6 + 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^2*d^4 + a^5*b*d^6)*sqr
t(a*b)) - 1/8*(2*(D*a^3*b + B*a^2*b^2)*c^3 - 2*(C*a^3*b + A*a^2*b^2)*c^2*d
 - 2*(D*a^4 - 3*B*a^3*b)*c*d^2 + 2*(C*a^4 - 3*A*a^3*b)*d^3 - ((C*a*b^3 + 3
*A*b^4)*c^3 + (5*D*a^2*b^2 - B*a*b^3)*c^2*d - (3*C*a^2*b^2 - 7*A*a*b^3)*c*
d^2 + (D*a^3*b + 3*B*a^2*b^2)*d^3)*x^3 + 4*(D*a^2*b^2*c^3 - C*a^2*b^2*c^2*
d + B*a^2*b^2*c*d^2 - A*a^2*b^2*d^3)*x^2 + ((C*a^2*b^2 - 5*A*a*b^3)*c^3 -
(3*D*a^3*b + B*a^2*b^2)*c^2*d + (5*C*a^3*b - 9*A*a^2*b^2)*c*d^2 + (D*a^4 -
 5*B*a^3*b)*d^3)*x)/(a^4*b^3*c^4 + 2*a^5*b^2*c^2*d^2 + a^6*b*d^4 + (a^2*b^
5*c^4 + 2*a^3*b^4*c^2*d^2 + a^4*b^3*d^4)*x^4 + 2*(a^3*b^4*c^4 + 2*a^4*b^3*
c^2*d^2 + a^5*b^2*d^4)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(455) = 910$.

Time = 0.29 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.98

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)(a + bx^2)^3}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(D*c^3*d^2 - C*c^2*d^3 + B*c*d^4 - A*d^5)*log(b*x^2 + a)/(b^3*c^6 + 3*
a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6) - (D*c^3*d^3 - C*c^2*d^4 + B*c*
d^5 - A*d^6)*log(abs(d*x + c))/(b^3*c^6*d + 3*a*b^2*c^4*d^3 + 3*a^2*b*c^2*
d^5 + a^3*d^7) + 1/8*(C*a*b^3*c^5 + 3*A*b^4*c^5 - 3*D*a^2*b^2*c^4*d - B*a*
b^3*c^4*d + 6*C*a^2*b^2*c^3*d^2 + 10*A*a*b^3*c^3*d^2 + 6*D*a^3*b*c^2*d^3 -
 6*B*a^2*b^2*c^2*d^3 - 3*C*a^3*b*c*d^4 + 15*A*a^2*b^2*c*d^4 + D*a^4*d^5 +
3*B*a^3*b*d^5)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^6 + 3*a^3*b^3*c^4*d^2 + 3
*a^4*b^2*c^2*d^4 + a^5*b*d^6)*sqrt(a*b)) - 1/8*(2*D*a^3*b^2*c^5 + 2*B*a^2*
b^3*c^5 - 2*C*a^3*b^2*c^4*d - 2*A*a^2*b^3*c^4*d + 8*B*a^3*b^2*c^3*d^2 - 8*
A*a^3*b^2*c^2*d^3 - 2*D*a^5*c*d^4 + 6*B*a^4*b*c*d^4 + 2*C*a^5*d^5 - 6*A*a^
4*b*d^5 - (C*a*b^4*c^5 + 3*A*b^5*c^5 + 5*D*a^2*b^3*c^4*d - B*a*b^4*c^4*d -
 2*C*a^2*b^3*c^3*d^2 + 10*A*a*b^4*c^3*d^2 + 6*D*a^3*b^2*c^2*d^3 + 2*B*a^2*
b^3*c^2*d^3 - 3*C*a^3*b^2*c*d^4 + 7*A*a^2*b^3*c*d^4 + D*a^4*b*d^5 + 3*B*a^
3*b^2*d^5)*x^3 + 4*(D*a^2*b^3*c^5 - C*a^2*b^3*c^4*d + D*a^3*b^2*c^3*d^2 +
B*a^2*b^3*c^3*d^2 - C*a^3*b^2*c^2*d^3 - A*a^2*b^3*c^2*d^3 + B*a^3*b^2*c*d^
4 - A*a^3*b^2*d^5)*x^2 + (C*a^2*b^3*c^5 - 5*A*a*b^4*c^5 - 3*D*a^3*b^2*c^4*
d - B*a^2*b^3*c^4*d + 6*C*a^3*b^2*c^3*d^2 - 14*A*a^2*b^3*c^3*d^2 - 2*D*a^4
*b*c^2*d^3 - 6*B*a^3*b^2*c^2*d^3 + 5*C*a^4*b*c*d^4 - 9*A*a^3*b^2*c*d^4 + D
*a^5*d^5 - 5*B*a^4*b*d^5)*x)/((b*c^2 + a*d^2)^3*(b*x^2 + a)^2*a^2*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(b\,x^2+a\right)}^3\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^3*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1706, normalized size of antiderivative = 3.61

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input

```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^3,x)
```

output

```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*d**6 + 15*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**2*c*d**4 + 3*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**2*d**5 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**4*b*c**2*d**4 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*a**4*b*d**6*x**2 + 10*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*a**3*b**3*c**3*d**2 - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a**3*b**3*c**2*d**3 + 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*3*b**3*c*d**4*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3
*b**3*d**5*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**
2*c**4*d**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*c*
*2*d**4*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*d**
6*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**5 -
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**4*d + 20*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**3*d**2*x**2 - 12*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c**2*d**3*x**2 + 15*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*c*d**4*x**4 + 3*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*d**5*x**4 + sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c**6 + 6*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c**4*d**2*x**2 + 3*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c**2*d**4*x**4 + 6*sqrt(b)*sqrt(a...
```

3.44 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 694

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx &= -\frac{d^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{\left(bc^2+ad^2\right)^3(c+dx)} \\
&\quad -\frac{a(b^2c(Bc-2Ad)+a^2d^2D+ab(2cCd-Bd^2-c^2D))-b(Ab(bc^2-ad^2)-a(bc(cC-2Bd)-ad(Cd}{4ab\left(bc^2+ad^2\right)^2\left(a+bx^2\right)^2} \\
&\quad -\frac{4a^2(ad^2(2cCd-Bd^2-3c^2D)-bc(2c^2Cd-3Bcd^2+4Ad^3-c^3D))-(Ab(3b^2c^4+12abc^2d^2-7a^2d^4}{8a^2\left(bc^2+ad^2\right)^3(a+bx^2} \\
&\quad +\frac{(3Ab(b^3c^6+5ab^2c^4d^2+15a^2bc^2d^4-5a^3d^6)+a(b^3c^5(cC-2Bd)+3a^3d^5(Cd-2cD)-3a^2bcd^3(11cC}{8a^{5/2}\sqrt{b}\left(bc^2+ad^2\right)^4} \\
&\quad -\frac{d^2(ad^2(2cCd-Bd^2-3c^2D)-bc(4c^2Cd-5Bcd^2+6Ad^3-3c^3D))\log(c+dx)}{\left(bc^2+ad^2\right)^4} \\
&\quad +\frac{d^2(ad^2(2cCd-Bd^2-3c^2D)-bc(4c^2Cd-5Bcd^2+6Ad^3-3c^3D))\log\left(a+bx^2\right)}{2\left(bc^2+ad^2\right)^4}
\end{aligned}
$$

output
```
-d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2+b*c^2)^3/(d*x+c)-1/4*(a*(b^2*c*(
-2*A*d+B*c)+a^2*d^2*D+a*b*(-B*d^2+2*C*c*d-D*c^2))-b*(A*b*(-a*d^2+b*c^2)-a*
(b*c*(-2*B*d+C*c)-a*d*(C*d-2*D*c)))*x)/a/b/(a*d^2+b*c^2)^2/(b*x^2+a)^2-1/8
*(4*a^2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(4*A*d^3-3*B*c*d^2+2*C*c^2*d-D
*c^3))-(A*b*(-7*a^2*d^4+12*a*b*c^2*d^2+3*b^2*c^4)+a*(b^2*c^3*(-2*B*d+C*c)+
3*a^2*d^3*(C*d-2*D*c)-2*a*b*c*d*(-7*B*d^2+6*C*c*d-5*D*c^2)))*x)/a^2/(a*d^2
+b*c^2)^3/(b*x^2+a)+1/8*(3*A*b*(-5*a^3*d^6+15*a^2*b*c^2*d^4+5*a*b^2*c^4*d^
2+b^3*c^6)+a*(b^3*c^5*(-2*B*d+C*c)+3*a^3*d^5*(C*d-2*D*c)-3*a^2*b*c*d^3*(-1
0*B*d^2+11*C*c*d-12*D*c^2)+a*b^2*c^3*d*(-20*B*d^2+13*C*c*d-6*D*c^2)))*arct
an(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(1/2)/(a*d^2+b*c^2)^4-d^2*(a*d^2*(-B*d^2+2
*C*c*d-3*D*c^2)-b*c*(6*A*d^3-5*B*c*d^2+4*C*c^2*d-3*D*c^3))*ln(d*x+c)/(a*d^
2+b*c^2)^4+1/2*d^2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(6*A*d^3-5*B*c*d^2+
4*C*c^2*d-3*D*c^3))*ln(b*x^2+a)/(a*d^2+b*c^2)^4
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx$$

$$= -\frac{8d^2\left(bc^2+ad^2\right)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{c+dx} + \frac{2\left(bc^2+ad^2\right)^2\left(-a^3d^2D+Ab^3c^2x-ab^2\left(c^2Cx+Bc(c-2dx)+Ad(-2c+dx)\right)+a^2b\left(c^2D+d^2(B+C\cdots\right.\right.}{ab\left(a+bx^2\right)^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^3),x]
```

output

```
((-8*d^2*(b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(c + d*x) +
(2*(b*c^2 + a*d^2)^2*(-(a^3*d^2*D) + A*b^3*c^2*x - a*b^2*(c^2*C*x + B*c*(c
 - 2*d*x) + A*d*(-2*c + d*x)) + a^2*b*(c^2*D + d^2*(B + C*x) - 2*c*d*(C +
D*x))))/(a*b*(a + b*x^2)^2) + ((b*c^2 + a*d^2)*(3*A*b^3*c^4*x + a*b^2*c^2*
(c^2*C - 2*B*c*d + 12*A*d^2)*x + a^3*d^2*(12*c^2*D + d^2*(4*B + 3*C*x) - 2
*c*d*(4*C + 3*D*x)) + a^2*b*(-4*c^4*D - 7*A*d^4*x + 2*c*d^3*(8*A + 7*B*x)
- 12*c^2*d^2*(B + C*x) + 2*c^3*d*(4*C + 5*D*x))))/(a^2*(a + b*x^2)) + ((3*
A*b*(b^3*c^6 + 5*a*b^2*c^4*d^2 + 15*a^2*b*c^2*d^4 - 5*a^3*d^6) + a*(b^3*c^
5*(c*C - 2*B*d) + 3*a^3*d^5*(C*d - 2*c*D) + a*b^2*c^3*d*(13*c*C*d - 20*B*d
^2 - 6*c^2*D) + 3*a^2*b*c*d^3*(-11*c*C*d + 10*B*d^2 + 12*c^2*D)))*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + 8*(a*d^4*(-2*c*C*d + B*d^2 + 3*c^
2*D) + b*c*d^2*(4*c^2*C*d - 5*B*c*d^2 + 6*A*d^3 - 3*c^3*D))*Log[c + d*x] -
 4*d^2*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(4*c^2*C*d - 5*B*c*d^2 +
6*A*d^3 - 3*c^3*D))*Log[a + b*x^2])/(8*(b*c^2 + a*d^2)^4)
```

Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2178, 25, 2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3 (c+dx)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{\frac{3bd^2\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2cD))\right)x^2}{\left(bc^2+ad^2\right)^2}+\frac{2b\left(Abcd\left(3bc^2+ad^2\right)-a\left(a(cC-2Bd)d^3+bc^2\left(-2Dc^2+3Cdc-4Bd^2\right)\right)\right)x}{\left(bc^2+ad^2\right)^2}+\frac{b\left(a(bc(cC-2Bd)-\right.}{}}{(c+dx)^2\left(bx^2+a\right)^2}}{4ab}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-\right.\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{3bd^2\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2cD))\right)x^2}{\left(bc^2+ad^2\right)^2}+\frac{2b\left(Abcd\left(3bc^2+ad^2\right)-a\left(a(cC-2Bd)d^3+bc^2\left(-2Dc^2+3Cdc-4Bd^2\right)\right)\right)x}{\left(bc^2+ad^2\right)^2}+\frac{b\left(a(bc(cC-2Bd)-ad(C\right.}{}}{(c+dx)^2(bx^2+a)^2}}{4ab}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{\frac{d^2\left(Ab\left(3b^2c^4+12abd^2c^2-7a^2d^4\right)+a\left(b^2(cC-2Bd)c^3-2abd\left(-5Dc^2+6Cdc-7Bd^2\right)c+3a^2d^3(Cd-2cD)\right)\right)x^2b^2}{\left(bc^2+ad^2\right)^3}+\frac{\left(a\left(b^2(cC-2Bd)c^3+6abd\left(-Dc^2+2Cdc-\right.\right.\right.}{}}{(c+dx)^2}}{2a}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{d^2\left(Ab\left(3b^2c^4+12abd^2c^2-7a^2d^4\right)+a\left(b^2(cC-2Bd)c^3-2abd\left(-5Dc^2+6Cdc-7Bd^2\right)c+3a^2d^3(Cd-2cD)\right)\right)x^2b^2}{\left(bc^2+ad^2\right)^3}+\frac{\left(a\left(b^2(cC-2Bd)c^3+6abd\left(-Dc^2+2Cdc-3B\right.\right.\right.}{}}{(c+dx)^2\left(bx^2\right.}}{2ab}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{2160}$$

$$\int\left(\frac{8a^2b^2\left(bc\left(-3Dc^3+4Cdc^2-5Bd^2c+6Ad^3\right)-ad^2\left(-3Dc^2+2Cdc-Bd^2\right)\right)d^3}{\left(bc^2+ad^2\right)^4(c+dx)}+\frac{8a^2b^2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)d^3}{\left(bc^2+ad^2\right)^3(c+dx)^2}+\frac{b^2\left(8a^2b\left(ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\right.\right.}{}\right.$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2b^2d^2\log\left(a+bx^2\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)-bc\left(6Ad^3-5Bcd^2-3c^3D+4c^2Cd\right)\right)}{\left(ad^2+bc^2\right)^4}-\frac{8a^2b^2d^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{(c+dx)\left(ad^2+bc^2\right)^3}-\frac{8a^2b^2d^2\log(c+dx)\left(ad^2\left(-Bd^2\right.\right.}{}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-2\right.}{4ab\left(a+bx^2\right)^2\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^3),x]
```

output
```
-1/4*(a*(b^2*c*(B*c - 2*A*d) + a^2*d^2*D + a*b*(2*c*C*d - B*d^2 - c^2*D))
- b*(A*b*(b*c^2 - a*d^2) - a*(b*c*(c*C - 2*B*d) - a*d*(C*d - 2*c*D)))*x)/(
a*b*(b*c^2 + a*d^2)^2*(a + b*x^2)^2) + (-1/2*(b*(4*a^2*(a*d^2*(2*c*C*d - B
*d^2 - 3*c^2*D) - b*c*(2*c^2*C*d - 3*B*c*d^2 + 4*A*d^3 - c^3*D)) - (A*b*(3
*b^2*c^4 + 12*a*b*c^2*d^2 - 7*a^2*d^4) + a*(b^2*c^3*(c*C - 2*B*d) + 3*a^2*
d^3*(C*d - 2*c*D) - 2*a*b*c*d*(6*c*C*d - 7*B*d^2 - 5*c^2*D)))*x))/(a*(b*c^
2 + a*d^2)^3*(a + b*x^2)) + ((-8*a^2*b^2*d^2*(c^2*C*d - B*c*d^2 + A*d^3 -
c^3*D))/((b*c^2 + a*d^2)^3*(c + d*x)) + (b^(3/2)*(3*A*b*(b^3*c^6 + 5*a*b^2
*c^4*d^2 + 15*a^2*b*c^2*d^4 - 5*a^3*d^6) + a*(b^3*c^5*(c*C - 2*B*d) + 3*a^
3*d^5*(C*d - 2*c*D) - 3*a^2*b*c*d^3*(11*c*C*d - 10*B*d^2 - 12*c^2*D) + a*b
^2*c^3*d*(13*c*C*d - 20*B*d^2 - 6*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*(b*c^2 + a*d^2)^4) - (8*a^2*b^2*d^2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2
*D) - b*c*(4*c^2*C*d - 5*B*c*d^2 + 6*A*d^3 - 3*c^3*D))*Log[c + d*x])/(b*c^
2 + a*d^2)^4 + (4*a^2*b^2*d^2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(4*
c^2*C*d - 5*B*c*d^2 + 6*A*d^3 - 3*c^3*D))*Log[a + b*x^2])/(b*c^2 + a*d^2)^
4)/(2*a*b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 1095, normalized size of antiderivative = 1.58

method	result	size
default	Expression too large to display	1095

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d^2+b*c^2)^4*((1/8*b*(7*A*a^3*b*d^6-5*A*a^2*b^2*c^2*d^4-15*A*a*b^3*c
^4*d^2-3*A*b^4*c^6-14*B*a^3*b*c*d^5-12*B*a^2*b^2*c^3*d^3+2*B*a*b^3*c^5*d-3
*C*a^4*d^6+9*C*a^3*b*c^2*d^4+11*C*a^2*b^2*c^4*d^2-C*a*b^3*c^6+6*D*a^4*c*d^
5-4*D*a^3*b*c^3*d^3-10*D*a^2*b^2*c^5*d)/a^2*x^3+(-2*A*a*b^2*c*d^5-2*A*b^3*
c^3*d^3-1/2*B*a^2*b*d^6+B*a*b^2*c^2*d^4+3/2*B*b^3*c^4*d^2+C*a^2*b*c*d^5-C*
b^3*c^5*d-3/2*D*a^2*b*c^2*d^4-D*a*b^2*c^4*d^2+1/2*b^3*c^6*D)*x^2+1/8*(9*A*
a^3*b*d^6-3*A*a^2*b^2*c^2*d^4-17*A*a*b^3*c^4*d^2-5*A*b^4*c^6-18*B*a^3*b*c*
d^5-20*B*a^2*b^2*c^3*d^3-2*B*a*b^3*c^5*d-5*C*a^4*d^6+7*C*a^3*b*c^2*d^4+13*
C*a^2*b^2*c^4*d^2+C*a*b^3*c^6+10*D*a^4*c*d^5+4*D*a^3*b*c^3*d^3-6*D*a^2*b^2
*c^5*d)/a*x-1/4*(10*A*a^2*b^2*c*d^5+12*A*a*b^3*c^3*d^3+2*A*b^4*c^5*d+3*B*a
^3*b*d^6-3*B*a^2*b^2*c^2*d^4-7*B*a*b^3*c^4*d^2-B*b^4*c^6-6*C*a^3*b*c*d^5-4
*C*a^2*b^2*c^3*d^3+2*C*a*b^3*c^5*d-D*a^4*d^6+5*D*a^3*b*c^2*d^4+5*D*a^2*b^2
*c^4*d^2-D*a*b^3*c^6)/b)/(b*x^2+a)^2+1/8/a^2*(1/2*(48*A*a^2*b^2*c*d^5+8*B*
a^3*b*d^6-40*B*a^2*b^2*c^2*d^4-16*C*a^3*b*c*d^5+32*C*a^2*b^2*c^3*d^3+24*D*
a^3*b*c^2*d^4-24*D*a^2*b^2*c^4*d^2)/b*ln(b*x^2+a)+(15*A*a^3*b*d^6-45*A*a^2
*b^2*c^2*d^4-15*A*a*b^3*c^4*d^2-3*A*b^4*c^6-30*B*a^3*b*c*d^5+20*B*a^2*b^2*
c^3*d^3+2*B*a*b^3*c^5*d-3*C*a^4*d^6+33*C*a^3*b*c^2*d^4-13*C*a^2*b^2*c^4*d^
2-C*a*b^3*c^6+6*D*a^4*c*d^5-36*D*a^3*b*c^3*d^3+6*D*a^2*b^2*c^5*d)/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))))+d^2*(6*A*b*c*d^3+B*a*d^4-5*B*b*c^2*d^2-2*C*a
*c*d^3+4*C*b*c^3*d+3*D*a*c^2*d^2-3*D*b*c^4)/(a*d^2+b*c^2)^4*ln(d*x+c)-d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(673) = 1346$.

Time $= 0.16$ (sec) , antiderivative size $= 1440$, normalized size of antiderivative $= 2.07$

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*(3*D*b*c^4*d^2 - 4*C*b*c^3*d^3 - B*a*d^6 - (3*D*a - 5*B*b)*c^2*d^4 + 2
*(C*a - 3*A*b)*c*d^5)*log(b*x^2 + a)/(b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^
2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8) - (3*D*b*c^4*d^2 - 4*C*b*c^3*d^3 -
B*a*d^6 - (3*D*a - 5*B*b)*c^2*d^4 + 2*(C*a - 3*A*b)*c*d^5)*log(d*x + c)/(b
^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8)
+ 1/8*((C*a*b^3 + 3*A*b^4)*c^6 - 2*(3*D*a^2*b^2 + B*a*b^3)*c^5*d + (13*C*a
^2*b^2 + 15*A*a*b^3)*c^4*d^2 + 4*(9*D*a^3*b - 5*B*a^2*b^2)*c^3*d^3 - 3*(11
*C*a^3*b - 15*A*a^2*b^2)*c^2*d^4 - 6*(D*a^4 - 5*B*a^3*b)*c*d^5 + 3*(C*a^4
- 5*A*a^3*b)*d^6)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^8 + 4*a^3*b^3*c^6*d^2
+ 6*a^4*b^2*c^4*d^4 + 4*a^5*b*c^2*d^6 + a^6*d^8)*sqrt(a*b)) - 1/8*(8*A*a^4
*b*d^5 + 2*(D*a^3*b^2 + B*a^2*b^3)*c^5 - 4*(C*a^3*b^2 + A*a^2*b^3)*c^4*d -
 4*(5*D*a^4*b - 3*B*a^3*b^2)*c^3*d^2 + 20*(C*a^4*b - A*a^3*b^2)*c^2*d^3 +
2*(D*a^5 - 7*B*a^4*b)*c*d^4 - ((C*a*b^4 + 3*A*b^5)*c^4*d + 2*(9*D*a^2*b^3
- B*a*b^4)*c^3*d^2 - 4*(5*C*a^2*b^3 - 3*A*a*b^4)*c^2*d^3 - 2*(3*D*a^3*b^2
- 11*B*a^2*b^3)*c*d^4 + 3*(C*a^3*b^2 - 5*A*a^2*b^3)*d^5)*x^4 - (4*B*a^3*b^
2*d^5 + (C*a*b^4 + 3*A*b^5)*c^5 + 2*(3*D*a^2*b^3 - B*a*b^4)*c^4*d - 4*(C*a
^2*b^3 - 3*A*a*b^4)*c^3*d^2 + 2*(3*D*a^3*b^2 + B*a^2*b^3)*c^2*d^3 - (5*C*a
^3*b^2 - 9*A*a^2*b^3)*c*d^4)*x^3 + (4*D*a^2*b^3*c^5 - (7*C*a^2*b^3 + 5*A*a
*b^4)*c^4*d - 2*(17*D*a^3*b^2 - 5*B*a^2*b^3)*c^3*d^2 + 4*(9*C*a^3*b^2 - 7*
A*a^2*b^3)*c^2*d^3 + 2*(5*D*a^4*b - 19*B*a^3*b^2)*c*d^4 - 5*(C*a^4*b - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(673) = 1346$.

Time = 0.38 (sec) , antiderivative size = 1435, normalized size of antiderivative = 2.07

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2 (a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(3*D*b*c^4*d^2 - 4*C*b*c^3*d^3 - 3*D*a*c^2*d^4 + 5*B*b*c^2*d^4 + 2*C*a
*c*d^5 - 6*A*b*c*d^5 - B*a*d^6)*log(b - 2*b*c/(d*x + c) + b*c^2/(d*x + c)^
2 + a*d^2/(d*x + c)^2)/(b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*
a^3*b*c^2*d^6 + a^4*d^8) + (D*c^3*d^8/(d*x + c) - C*c^2*d^9/(d*x + c) + B*
c*d^10/(d*x + c) - A*d^11/(d*x + c))/(b^3*c^6*d^6 + 3*a*b^2*c^4*d^8 + 3*a^
2*b*c^2*d^10 + a^3*d^12) + 1/8*(C*a*b^3*c^6*d^2 + 3*A*b^4*c^6*d^2 - 6*D*a^
2*b^2*c^5*d^3 - 2*B*a*b^3*c^5*d^3 + 13*C*a^2*b^2*c^4*d^4 + 15*A*a*b^3*c^4*
d^4 + 36*D*a^3*b*c^3*d^5 - 20*B*a^2*b^2*c^3*d^5 - 33*C*a^3*b*c^2*d^6 + 45*
A*a^2*b^2*c^2*d^6 - 6*D*a^4*c*d^7 + 30*B*a^3*b*c*d^7 + 3*C*a^4*d^8 - 15*A*
a^3*b*d^8)*arctan((b*c - b*c^2/(d*x + c) - a*d^2/(d*x + c))/(sqrt(a*b)*d))
/((a^2*b^4*c^8 + 4*a^3*b^3*c^6*d^2 + 6*a^4*b^2*c^4*d^4 + 4*a^5*b*c^2*d^6 +
 a^6*d^8)*sqrt(a*b)*d^2) + 1/8*(C*a*b^4*c^5*d + 3*A*b^5*c^5*d + 14*D*a^2*b
^3*c^4*d^2 - 2*B*a*b^4*c^4*d^2 - 22*C*a^2*b^3*c^3*d^3 + 14*A*a*b^4*c^3*d^3
 - 24*D*a^3*b^2*c^2*d^4 + 32*B*a^2*b^3*c^2*d^4 + 17*C*a^3*b^2*c*d^5 - 29*A
*a^2*b^3*c*d^5 + 2*D*a^4*b*d^6 - 6*B*a^3*b^2*d^6 - (3*C*a*b^4*c^6*d^2 + 9*
A*b^5*c^6*d^2 + 46*D*a^2*b^3*c^5*d^3 - 6*B*a*b^4*c^5*d^3 - 77*C*a^2*b^3*c^
4*d^4 + 41*A*a*b^4*c^4*d^4 - 100*D*a^3*b^2*c^3*d^5 + 116*B*a^2*b^3*c^3*d^5
 + 77*C*a^3*b^2*c^2*d^6 - 121*A*a^2*b^3*c^2*d^6 + 14*D*a^4*b*c*d^7 - 38*B*
a^3*b^2*c*d^7 - 3*C*a^4*b*d^8 + 7*A*a^3*b^2*d^8)/((d*x + c)*d) + (3*C*a*b^
4*c^7*d^3 + 9*A*b^5*c^7*d^3 + 50*D*a^2*b^3*c^6*d^4 - 6*B*a*b^4*c^6*d^4 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^3\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^3*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 37.23 (sec) , antiderivative size = 4707, normalized size of antiderivative = 6.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2 (a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*c**2*d**6 - 15
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*c*d**7*x - 3*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*c**3*d**6 - 3*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*c**2*d**7*x + 45*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**4*b**2*c**4*d**4 + 45*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**4*b**2*c**3*d**5*x + 30*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**4*b**2*c**3*d**5 - 30*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**4*b**2*c**2*d**6*x**2 + 30*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**4*b**2*c**2*d**6*x - 30*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**4*b**2*c*d**7*x**3 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a**4*b*c**5*d**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*a**4*b*c**4*d**5*x - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**4*b*c**3*d**6*x**2 - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*a**4*b*c**2*d**7*x**3 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a**3*b**3*c**6*d**2 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*3*b**3*c**5*d**3*x - 20*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**
3*b**3*c**5*d**3 + 90*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b
**3*c**4*d**4*x**2 - 20*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3
*b**3*c**4*d**4*x + 90*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*
b**3*c**3*d**5*x**3 + 60*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*...
```

3.45 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 523

$$\begin{aligned}
&\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx \\
&= -\frac{b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{6b^3\left(a+bx^2\right)^3} \\
&\quad+\frac{\left(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC+3Bd)+a^2d^3D-abd(3cCd+Bd^2+3c^2D))\right)x}{6ab^3\left(a+bx^2\right)^3} \\
&\quad+\frac{2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{4b^3\left(a+bx^2\right)^2} \\
&\quad+\frac{\left(Ab^2c(5bc^2+3ad^2)+a(b^2c^2(cC+3Bd)+13a^2d^3D-7abd(3cCd+Bd^2+3c^2D))\right)x}{24a^2b^3\left(a+bx^2\right)^2} \\
&\quad-\frac{d^2(Cd+3cD)}{2b^3\left(a+bx^2\right)} \\
&\quad+\frac{\left(Ab^2c(5bc^2+3ad^2)+a(b^2c^2(cC+3Bd)-11a^2d^3D+abd(3cCd+Bd^2+3c^2D))\right)x}{16a^3b^3\left(a+bx^2\right)} \\
&\quad+\frac{\left(Ab^2c(5bc^2+3ad^2)+a(b^2c^2(cC+3Bd)+5a^2d^3D+abd(3cCd+Bd^2+3c^2D))\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{7/2}}
\end{aligned}$$

output
```
-1/6*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*d^2+3*C*c^2
*d+D*c^3))/b^3/(b*x^2+a)^3+1/6*(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(3*B*d
+C*c)+a^2*d^3*D-a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a/b^3/(b*x^2+a)^3+1/4*(2
*a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))/b^3/(b*x^2+a)^2+1/
24*(A*b^2*c*(3*a*d^2+5*b*c^2)+a*(b^2*c^2*(3*B*d+C*c)+13*a^2*d^3*D-7*a*b*d*
(B*d^2+3*C*c*d+3*D*c^2)))*x/a^2/b^3/(b*x^2+a)^2-1/2*d^2*(C*d+3*D*c)/b^3/(b
*x^2+a)+1/16*(A*b^2*c*(3*a*d^2+5*b*c^2)+a*(b^2*c^2*(3*B*d+C*c)-11*a^2*d^3*
D+a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a^3/b^3/(b*x^2+a)+1/16*(A*b^2*c*(3*a*d
^2+5*b*c^2)+a*(b^2*c^2*(3*B*d+C*c)+5*a^2*d^3*D+a*b*d*(B*d^2+3*C*c*d+3*D*c^
2)))*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx$$
$$= \frac{5Ab^3c^3x+ab^2c(c^2C+3Bcd+3Ad^2)\,x+a^2bd(3cCd+Bd^2+3c^2D)\,x-a^3d^2(8Cd+24cD+11dDx)}{16a^3b^3\,(a+bx^2)}$$
$$+\frac{Ab^3c^3x-a^3d^2(Cd+3cD+dDx)-ab^2c(c^2Cx+3Ad(c+dx)+Bc(c+3dx))+a^2b(c^3D+d^3(A+}{6ab^3\,(a+bx^2)^3}$$
$$+\frac{5Ab^3c^3x+ab^2c(c^2C+3Bcd+3Ad^2)\,x+a^3d^2(12Cd+36cD+13dDx)-a^2b(6c^3D+d^3(6A+7Bx)}{24a^2b^3\,(a+bx^2)^2}$$
$$+\frac{\left(Ab^2c(5bc^2+3ad^2)+a(b^2c^2(cC+3Bd)+5a^2d^3D+abd(3cCd+Bd^2+3c^2D))\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{7/2}}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
(5*A*b^3*c^3*x + a*b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2)*x + a^2*b*d*(3*c*C*d
+ B*d^2 + 3*c^2*D)*x - a^3*d^2*(8*C*d + 24*c*D + 11*d*D*x))/(16*a^3*b^3*(a
 + b*x^2)) + (A*b^3*c^3*x - a^3*d^2*(C*d + 3*c*D + d*D*x) - a*b^2*c*(c^2*C
*x + 3*A*d*(c + d*x) + B*c*(c + 3*d*x)) + a^2*b*(c^3*D + d^3*(A + B*x) + 3
*c*d^2*(B + C*x) + 3*c^2*d*(C + D*x)))/(6*a*b^3*(a + b*x^2)^3) + (5*A*b^3*
c^3*x + a*b^2*c*(c^2*C + 3*B*c*d + 3*A*d^2)*x + a^3*d^2*(12*C*d + 36*c*D +
 13*d*D*x) - a^2*b*(6*c^3*D + d^3*(6*A + 7*B*x) + 3*c*d^2*(6*B + 7*C*x) +
3*c^2*d*(6*C + 7*D*x)))/(24*a^2*b^3*(a + b*x^2)^2) + ((A*b^2*c*(5*b*c^2 +
3*a*d^2) + a*(b^2*c^2*(c*C + 3*B*d) + 5*a^2*d^3*D + a*b*d*(3*c*C*d + B*d^2
 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*b^(7/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2176, 25, 2176, 25, 675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(c+dx)^2\left(6adDx^2+2(Abd+2aCd+3acD)x+5Abc+a\left(cC+3Bd-\frac{3adD}{b}\right)\right)}{(bx^2+a)^3}dx}{6ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab(a+bx^2)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(c+dx)^2\left(6adDx^2+2(Abd+2aCd+3acD)x+5Abc+acC+3ad\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^3}dx}{6ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab(a+bx^2)^3}$$

$$\Big\downarrow \text{2176}$$

$$-\frac{-\frac{\int -\frac{(c+dx)\left(Ab\left(15bc^2+4ad^2\right)+a(3bc(cC+3Bd)+ad(8Cd+9cD))+d\left(5Acb^2+a(bcC+3bBd+15adD)\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)^2\left(2a(3acD+2aCd+Abd)-x\left(a(-9a\right.\right.}{4ab(a+bx^2)^2}}{6ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{(c+dx)\left(Ab\left(15bc^2+4ad^2\right)+a(3bc(cC+3Bd)+ad(8Cd+9cD))+d\left(5Acb^2+a(bcC+3bBd+15adD)\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)^2\left(2a(3acD+2aCd+Abd)-x\left(a(-9adD\right.\right.}{4ab(a+bx^2)^2}}{6ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{675}$$

$$\frac{\frac{\frac{3\left(a\left(5a^2d^3D+abd\left(Bd^2+3c^2D+3cCd\right)+b^2c^2(3Bd+cC)\right)+Ab^2c\left(3ad^2+5bc^2\right)\right)\int \frac{1}{bx^2+a}dx}{2ab}+\frac{x\left(-15a^2d^3D+\frac{15Ab^3c^3}{a}+abd\left(-3Bd^2+9c^2D+7cCd\right)+b^2c\left(-Ad^2\right.\right.}{2b\left(a+bx^2\right)}}{4ab}}{6ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{\frac{x\left(-15a^2d^3D+\frac{15Ab^3c^3}{a}+abd\left(-3Bd^2+9c^2D+7cCd\right)+b^2c\left(-Ad^2+9Bcd+3c^2C\right)\right)}{2b\left(a+bx^2\right)}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(5a^2d^3D+abd\left(Bd^2+3c^2D+3cCd\right)+b^2c^2(3Bd+cC)\right)+A\right.}{2a^{3/2}b^{3/2}}}{4ab}}{6a}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

input
```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
-1/6*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^3)/(a*b*(a + b*x^2)^3) +
 (-1/4*((c + d*x)^2*(2*a*(A*b*d + 2*a*C*d + 3*a*c*D) - (5*A*b^2*c + a*(b*c
*C + 3*b*B*d - 9*a*d*D))*x))/(a*b*(a + b*x^2)^2) + ((-2*d*(A*b*(5*b*c^2 +
a*d^2) + a*(b*c*(c*C + 3*B*d) + 2*a*d*(C*d + 3*c*D))))/(b*(a + b*x^2)) + (
((15*A*b^3*c^3)/a + b^2*c*(3*c^2*C + 9*B*c*d - A*d^2) - 15*a^2*d^3*D + a*b
*d*(7*c*C*d - 3*B*d^2 + 9*c^2*D))*x)/(2*b*(a + b*x^2)) + (3*(A*b^2*c*(5*b*
c^2 + 3*a*d^2) + a*(b^2*c^2*(c*C + 3*B*d) + 5*a^2*d^3*D + a*b*d*(3*c*C*d +
 B*d^2 + 3*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)))/(4*a
*b))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{\left(3Aa\,b^{2}c\,d^{2}+5A\,b^{3}c^{3}+B\,a^{2}b\,d^{3}+3Ba\,b^{2}c^{2}d+3C\,a^{2}bc\,d^{2}+Ca\,b^{2}c^{3}-11Da^{3}d^{3}+3Da^{2}b\,c^{2}d\right)x^{5}}{16a^{3}b}-\frac{d^{2}(Cd+3Dc)x^{4}}{2b}+\frac{\left(3Aa\,b^{2}c\,d^{2}+5A\,b^{3}c^{3}-B\,a^{2}b}{}$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(3*A*a*b^2*c*d^2+5*A*b^3*c^3+B*a^2*b*d^3+3*B*a*b^2*c^2*d+3*C*a^2*b*c
*d^2+C*a*b^2*c^3-11*D*a^3*d^3+3*D*a^2*b*c^2*d)/a^3/b*x^5-1/2*d^2*(C*d+3*D*
c)*x^4/b+1/6*(3*A*a*b^2*c*d^2+5*A*b^3*c^3-B*a^2*b*d^3+3*B*a*b^2*c^2*d-3*C*
a^2*b*c*d^2+C*a*b^2*c^3-5*D*a^3*d^3-3*D*a^2*b*c^2*d)/a^2/b^2*x^3-1/4*(A*b*
d^3+3*B*b*c*d^2+2*C*a*d^3+3*C*b*c^2*d+6*D*a*c*d^2+D*b*c^3)/b^2*x^2-1/16*(3
*A*a*b^2*c*d^2-11*A*b^3*c^3+B*a^2*b*d^3+3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2+C*
a*b^2*c^3+5*D*a^3*d^3+3*D*a^2*b*c^2*d)/a/b^3*x-1/12*(A*a*b*d^3+6*A*b^2*c^2
*d+3*B*a*b*c*d^2+2*B*b^2*c^3+2*C*a^2*d^3+3*C*a*b*c^2*d+6*D*a^2*c*d^2+D*a*b
*c^3)/b^3)/(b*x^2+a)^3+1/16*(3*A*a*b^2*c*d^2+5*A*b^3*c^3+B*a^2*b*d^3+3*B*a
*b^2*c^2*d+3*C*a^2*b*c*d^2+C*a*b^2*c^3+5*D*a^3*d^3+3*D*a^2*b*c^2*d)/a^3/b^
3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1824, normalized size of antiderivative = 3.49

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*((C*a^2*b^5 + 5*A*a*b^6)*c^3 + 3*(D*a^3*b^4 + B*a^2*b^5)*c^2*d +
3*(C*a^3*b^4 + A*a^2*b^5)*c*d^2 - (11*D*a^4*b^3 - B*a^3*b^4)*d^3)*x^5 - 48
*(3*D*a^4*b^3*c*d^2 + C*a^4*b^3*d^3)*x^4 - 8*(D*a^5*b^2 + 2*B*a^4*b^3)*c^3
 - 24*(C*a^5*b^2 + 2*A*a^4*b^3)*c^2*d - 24*(2*D*a^6*b + B*a^5*b^2)*c*d^2 -
 8*(2*C*a^6*b + A*a^5*b^2)*d^3 + 16*((C*a^3*b^4 + 5*A*a^2*b^5)*c^3 - 3*(D*
a^4*b^3 - B*a^3*b^4)*c^2*d - 3*(C*a^4*b^3 - A*a^3*b^4)*c*d^2 - (5*D*a^5*b^
2 + B*a^4*b^3)*d^3)*x^3 - 24*(D*a^4*b^3*c^3 + 3*C*a^4*b^3*c^2*d + 3*(2*D*a
^5*b^2 + B*a^4*b^3)*c*d^2 + (2*C*a^5*b^2 + A*a^4*b^3)*d^3)*x^2 - 3*(((C*a*
b^5 + 5*A*b^6)*c^3 + 3*(D*a^2*b^4 + B*a*b^5)*c^2*d + 3*(C*a^2*b^4 + A*a*b^
5)*c*d^2 + (5*D*a^3*b^3 + B*a^2*b^4)*d^3)*x^6 + 3*((C*a^2*b^4 + 5*A*a*b^5)
*c^3 + 3*(D*a^3*b^3 + B*a^2*b^4)*c^2*d + 3*(C*a^3*b^3 + A*a^2*b^4)*c*d^2 +
 (5*D*a^4*b^2 + B*a^3*b^3)*d^3)*x^4 + (C*a^4*b^2 + 5*A*a^3*b^3)*c^3 + 3*(D
*a^5*b + B*a^4*b^2)*c^2*d + 3*(C*a^5*b + A*a^4*b^2)*c*d^2 + (5*D*a^6 + B*a
^5*b)*d^3 + 3*((C*a^3*b^3 + 5*A*a^2*b^4)*c^3 + 3*(D*a^4*b^2 + B*a^3*b^3)*c
^2*d + 3*(C*a^4*b^2 + A*a^3*b^3)*c*d^2 + (5*D*a^5*b + B*a^4*b^2)*d^3)*x^2)
*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*((C*a^4*b^3
- 11*A*a^3*b^4)*c^3 + 3*(D*a^5*b^2 + B*a^4*b^3)*c^2*d + 3*(C*a^5*b^2 + A*a
^4*b^3)*c*d^2 + (5*D*a^6*b + B*a^5*b^2)*d^3)*x)/(a^4*b^7*x^6 + 3*a^5*b^6*x
^4 + 3*a^6*b^5*x^2 + a^7*b^4), 1/48*(3*((C*a^2*b^5 + 5*A*a*b^6)*c^3 + 3*(D
*a^3*b^4 + B*a^2*b^5)*c^2*d + 3*(C*a^3*b^4 + A*a^2*b^5)*c*d^2 - (11*D*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$$
$$= \frac{3\left((Cab^4+5\,Ab^5)c^3+3\,(Da^2b^3+Bab^4)c^2d+3\,(Ca^2b^3+Aab^4)cd^2-(11\,Da^3b^2-Ba^2b^3)d^3\right)x^5-24\,(3\,}{}$$
$$+\frac{\left((Cab^2+5\,Ab^3)c^3+3\,(Da^2b+Bab^2)c^2d+3\,(Ca^2b+Aab^2)cd^2+(5\,Da^3+Ba^2b)d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^3}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*((C*a*b^4 + 5*A*b^5)*c^3 + 3*(D*a^2*b^3 + B*a*b^4)*c^2*d + 3*(C*a^
2*b^3 + A*a*b^4)*c*d^2 - (11*D*a^3*b^2 - B*a^2*b^3)*d^3)*x^5 - 24*(3*D*a^3
*b^2*c*d^2 + C*a^3*b^2*d^3)*x^4 - 4*(D*a^4*b + 2*B*a^3*b^2)*c^3 - 12*(C*a^
4*b + 2*A*a^3*b^2)*c^2*d - 12*(2*D*a^5 + B*a^4*b)*c*d^2 - 4*(2*C*a^5 + A*a
^4*b)*d^3 + 8*((C*a^2*b^3 + 5*A*a*b^4)*c^3 - 3*(D*a^3*b^2 - B*a^2*b^3)*c^2
*d - 3*(C*a^3*b^2 - A*a^2*b^3)*c*d^2 - (5*D*a^4*b + B*a^3*b^2)*d^3)*x^3 -
12*(D*a^3*b^2*c^3 + 3*C*a^3*b^2*c^2*d + 3*(2*D*a^4*b + B*a^3*b^2)*c*d^2 +
(2*C*a^4*b + A*a^3*b^2)*d^3)*x^2 - 3*((C*a^3*b^2 - 11*A*a^2*b^3)*c^3 + 3*(
D*a^4*b + B*a^3*b^2)*c^2*d + 3*(C*a^4*b + A*a^3*b^2)*c*d^2 + (5*D*a^5 + B*
a^4*b)*d^3)*x)/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3) + 1
/16*((C*a*b^2 + 5*A*b^3)*c^3 + 3*(D*a^2*b + B*a*b^2)*c^2*d + 3*(C*a^2*b +
A*a*b^2)*c*d^2 + (5*D*a^3 + B*a^2*b)*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$$
$$= \frac{(Cab^2c^3+5\,Ab^3c^3+3\,Da^2bc^2d+3\,Bab^2c^2d+3\,Ca^2bcd^2+3\,Aab^2cd^2+5\,Da^3d^3+Ba^2bd^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^3}$$
$$+\frac{3\,Cab^4c^3x^5+15\,Ab^5c^3x^5+9\,Da^2b^3c^2dx^5+9\,Bab^4c^2dx^5+9\,Ca^2b^3cd^2x^5+9\,Aab^4cd^2x^5-33\,Da^3b^2d^3}{}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*b^2*c^3 + 5*A*b^3*c^3 + 3*D*a^2*b*c^2*d + 3*B*a*b^2*c^2*d + 3*C*
a^2*b*c*d^2 + 3*A*a*b^2*c*d^2 + 5*D*a^3*d^3 + B*a^2*b*d^3)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^3) + 1/48*(3*C*a*b^4*c^3*x^5 + 15*A*b^5*c^3*x^5 +
9*D*a^2*b^3*c^2*d*x^5 + 9*B*a*b^4*c^2*d*x^5 + 9*C*a^2*b^3*c*d^2*x^5 + 9*A*
a*b^4*c*d^2*x^5 - 33*D*a^3*b^2*d^3*x^5 + 3*B*a^2*b^3*d^3*x^5 - 72*D*a^3*b^
2*c*d^2*x^4 - 24*C*a^3*b^2*d^3*x^4 + 8*C*a^2*b^3*c^3*x^3 + 40*A*a*b^4*c^3*
x^3 - 24*D*a^3*b^2*c^2*d*x^3 + 24*B*a^2*b^3*c^2*d*x^3 - 24*C*a^3*b^2*c*d^2
*x^3 + 24*A*a^2*b^3*c*d^2*x^3 - 40*D*a^4*b*d^3*x^3 - 8*B*a^3*b^2*d^3*x^3 -
 12*D*a^3*b^2*c^3*x^2 - 36*C*a^3*b^2*c^2*d*x^2 - 72*D*a^4*b*c*d^2*x^2 - 36
*B*a^3*b^2*c*d^2*x^2 - 24*C*a^4*b*d^3*x^2 - 12*A*a^3*b^2*d^3*x^2 - 3*C*a^3
*b^2*c^3*x + 33*A*a^2*b^3*c^3*x - 9*D*a^4*b*c^2*d*x - 9*B*a^3*b^2*c^2*d*x
- 9*C*a^4*b*c*d^2*x - 9*A*a^3*b^2*c*d^2*x - 15*D*a^5*d^3*x - 3*B*a^4*b*d^3
*x - 4*D*a^4*b*c^3 - 8*B*a^3*b^2*c^3 - 12*C*a^4*b*c^2*d - 24*A*a^3*b^2*c^2
*d - 24*D*a^5*c*d^2 - 12*B*a^4*b*c*d^2 - 8*C*a^5*d^3 - 4*A*a^4*b*d^3)/((b*
x^2 + a)^3*a^3*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} dx = \int \frac{(c+d\,x)^3 \,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^4} dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4,x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx = \int \frac{(dx+c)^3 (Dx^3+C\,x^2+Bx+A)}{(b\,x^2+a)^4}dx$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

output
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

3.46 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 376

$$\begin{aligned}
&\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx \\
&= -\frac{b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D)}{6b^3(a+bx^2)^3} \\
&\quad+\frac{(Ab(bc^2-ad^2)-a(bc(cC+2Bd)-ad(Cd+2cD)))\,x}{6ab^2(a+bx^2)^3} \\
&\quad+\frac{2ad^2D-b(2cCd+Bd^2+c^2D)}{4b^3(a+bx^2)^2} \\
&\quad+\frac{(Ab(5bc^2+ad^2)+a(bc(cC+2Bd)-7ad(Cd+2cD)))\,x}{24a^2b^2(a+bx^2)^2} \\
&\quad-\frac{d^2D}{2b^3(a+bx^2)}+\frac{(Ab(5bc^2+ad^2)+a(bc(cC+2Bd)+ad(Cd+2cD)))\,x}{16a^3b^2(a+bx^2)} \\
&\quad+\frac{(Ab(5bc^2+ad^2)+a(bc(cC+2Bd)+ad(Cd+2cD)))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{5/2}}
\end{aligned}$$

output
```
-1/6*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)
^3+1/6*(A*b*(-a*d^2+b*c^2)-a*(b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*x/a/b^2/(b
*x^2+a)^3+1/4*(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)^2+1/24*(A*
b*(a*d^2+5*b*c^2)+a*(b*c*(2*B*d+C*c)-7*a*d*(C*d+2*D*c)))*x/a^2/b^2/(b*x^2+
a)^2-1/2*d^2*D/b^3/(b*x^2+a)+1/16*(A*b*(a*d^2+5*b*c^2)+a*(b*c*(2*B*d+C*c)+
a*d*(C*d+2*D*c)))*x/a^3/b^2/(b*x^2+a)+1/16*(A*b*(a*d^2+5*b*c^2)+a*(b*c*(2*
B*d+C*c)+a*d*(C*d+2*D*c)))*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} \, dx$$
$$= \frac{-\frac{3\sqrt{a}(8a^3d^2D-5Ab^3c^2x-ab^2(c^2C+2Bcd+Ad^2)x-a^2bd(Cd+2cD)x)}{a+bx^2} - \frac{8a^{5/2}(a^3d^2D-Ab^3c^2x+ab^2(c^2Cx+Ad(2c+dx)+Bc(c+2dx))-a^2}{(a+bx^2)^3}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
((-3*Sqrt[a]*(8*a^3*d^2*D - 5*A*b^3*c^2*x - a*b^2*(c^2*C + 2*B*c*d + A*d^2
)*x - a^2*b*d*(C*d + 2*c*D)*x))/(a + b*x^2) - (8*a^(5/2)*(a^3*d^2*D - A*b^
3*c^2*x + a*b^2*(c^2*C*x + A*d*(2*c + d*x) + B*c*(c + 2*d*x)) - a^2*b*(c^2
*D + d^2*(B + C*x) + 2*c*d*(C + D*x))))/(a + b*x^2)^3 + (2*a^(3/2)*(12*a^3
*d^2*D + 5*A*b^3*c^2*x + a*b^2*(c^2*C + 2*B*c*d + A*d^2)*x - a^2*b*(6*c^2*
D + d^2*(6*B + 7*C*x) + 2*c*d*(6*C + 7*D*x))))/(a + b*x^2)^2 + 3*Sqrt[b]*(
A*b*(5*b*c^2 + a*d^2) + a*(b*c*(c*C + 2*B*d) + a*d*(C*d + 2*c*D)))*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(48*a^(7/2)*b^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2176, 25, 2176, 25, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \; 2176$$

$$-\frac{\int -\frac{(c+dx)\left(6adDx^2+3(Abd+aCd+2acD)x+5Abc+a\left(cC+2Bd-\frac{2adD}{b}\right)\right)}{\left(bx^2+a\right)^3}dx}{6ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(c+dx)\left(6adDx^2+3(Abd+aCd+2acD)x+5Abc+acC+2ad\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^3}dx}{6ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 2176$$

$$\frac{-\frac{\int -\frac{3\left(Ab\left(5bc^2+ad^2\right)+a(bc(cC+2Bd)+ad(Cd+2cD))\right)+2d\left(5Acb^2+a(bcC+2bBd+4adD)\right)x}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)(3a(2acD+aCd+Abd)-x(a(-8adD+2bBd}{4ab(a+bx^2)^2}}{6ab}$$
$$-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3\left(Ab\left(5bc^2+ad^2\right)+a(bc(cC+2Bd)+ad(Cd+2cD))\right)+2d\left(5Acb^2+a(bcC+2bBd+4adD)\right)x}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{(c+dx)(3a(2acD+aCd+Abd)-x(a(-8adD+2bBd+bcC}{4ab(a+bx^2)^2}}{6ab}$$
$$-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 454$$

$$\frac{\frac{\frac{3\left(Ab\left(ad^2+5bc^2\right)+a(ad(2cD+Cd)+bc(2Bd+cC))\right)\int \frac{1}{bx^2+a}dx}{2a}-\frac{2ad\left(a(4adD+2bBd+bcC)+5Ab^2c\right)-3bx\left(Ab\left(ad^2+5bc^2\right)+a(ad(2cD+Cd)+bc(2Bd+cC))\right)}{2ab\left(a+bx^2\right)}}{4ab}-}{6ab}$$
$$-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab\left(ad^2+5bc^2\right)+a(ad(2cD+Cd)+bc(2Bd+cC))\right)}{2a^{3/2}\sqrt{b}}-\frac{2ad\left(a(4adD+2bBd+bcC)+5Ab^2c\right)-3bx\left(Ab\left(ad^2+5bc^2\right)+a(ad(2cD+Cd)+bc(2Bd+cC))\right)}{2ab\left(a+bx^2\right)}}{4ab}}{6ab}$$

$$\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
-1/6*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^2)/(a*b*(a + b*x^2)^3) +
 (-1/4*((c + d*x)*(3*a*(A*b*d + a*C*d + 2*a*c*D) - (5*A*b^2*c + a*(b*c*C +
 2*b*B*d - 8*a*d*D))*x))/(a*b*(a + b*x^2)^2) + (-1/2*(2*a*d*(5*A*b^2*c + a
*(b*c*C + 2*b*B*d + 4*a*d*D)) - 3*b*(A*b*(5*b*c^2 + a*d^2) + a*(b*c*(c*C +
 2*B*d) + a*d*(C*d + 2*c*D)))*x)/(a*b*(a + b*x^2)) + (3*(A*b*(5*b*c^2 + a*
d^2) + a*(b*c*(c*C + 2*B*d) + a*d*(C*d + 2*c*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(Aab\,d^2+5A\,b^2c^2+2abBcd+a^2C\,d^2+Cab\,c^2+2a^2cdD\right)x^5}{16a^3}-\frac{d^2Dx^4}{2b}+\frac{\left(Aab\,d^2+5A\,b^2c^2+2abBcd-a^2C\,d^2+Cab\,c^2-2a^2cdD\right)x^3}{6a^2b}-\frac{\left(Bb\,d^2+2Cbc\right.}{}}{(b\,x^2+a}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(A*a*b*d^2+5*A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2+2*D*a^2*c*d)/
a^3*x^5-1/2*d^2*D*x^4/b+1/6*(A*a*b*d^2+5*A*b^2*c^2+2*B*a*b*c*d-C*a^2*d^2+C
*a*b*c^2-2*D*a^2*c*d)/a^2/b*x^3-1/4*(B*b*d^2+2*C*b*c*d+2*D*a*d^2+D*b*c^2)/
b^2*x^2-1/16*(A*a*b*d^2-11*A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2+2*D*a
^2*c*d)/a/b^2*x-1/12*(4*A*b^2*c*d+B*a*b*d^2+2*B*b^2*c^2+2*C*a*b*c*d+2*D*a^
2*d^2+D*a*b*c^2)/b^3)/(b*x^2+a)^3+1/16*(A*a*b*d^2+5*A*b^2*c^2+2*B*a*b*c*d+
C*a^2*d^2+C*a*b*c^2+2*D*a^2*c*d)/a^3/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.44

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(48*D*a^4*b^2*d^2*x^4 - 6*((C*a^2*b^4 + 5*A*a*b^5)*c^2 + 2*(D*a^3*b
^3 + B*a^2*b^4)*c*d + (C*a^3*b^3 + A*a^2*b^4)*d^2)*x^5 - 16*((C*a^3*b^3 +
5*A*a^2*b^4)*c^2 - 2*(D*a^4*b^2 - B*a^3*b^3)*c*d - (C*a^4*b^2 - A*a^3*b^3)
*d^2)*x^3 + 8*(D*a^5*b + 2*B*a^4*b^2)*c^2 + 16*(C*a^5*b + 2*A*a^4*b^2)*c*d
 + 8*(2*D*a^6 + B*a^5*b)*d^2 + 24*(D*a^4*b^2*c^2 + 2*C*a^4*b^2*c*d + (2*D*
a^5*b + B*a^4*b^2)*d^2)*x^2 + 3*(((C*a*b^4 + 5*A*b^5)*c^2 + 2*(D*a^2*b^3 +
 B*a*b^4)*c*d + (C*a^2*b^3 + A*a*b^4)*d^2)*x^6 + 3*((C*a^2*b^3 + 5*A*a*b^4
)*c^2 + 2*(D*a^3*b^2 + B*a^2*b^3)*c*d + (C*a^3*b^2 + A*a^2*b^3)*d^2)*x^4 +
 (C*a^4*b + 5*A*a^3*b^2)*c^2 + 2*(D*a^5 + B*a^4*b)*c*d + (C*a^5 + A*a^4*b)
*d^2 + 3*((C*a^3*b^2 + 5*A*a^2*b^3)*c^2 + 2*(D*a^4*b + B*a^3*b^2)*c*d + (C
*a^4*b + A*a^3*b^2)*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)) + 6*((C*a^4*b^2 - 11*A*a^3*b^3)*c^2 + 2*(D*a^5*b + B*a^4*b^2)
*c*d + (C*a^5*b + A*a^4*b^2)*d^2)*x)/(a^4*b^6*x^6 + 3*a^5*b^5*x^4 + 3*a^6*
b^4*x^2 + a^7*b^3), -1/48*(24*D*a^4*b^2*d^2*x^4 - 3*((C*a^2*b^4 + 5*A*a*b^
5)*c^2 + 2*(D*a^3*b^3 + B*a^2*b^4)*c*d + (C*a^3*b^3 + A*a^2*b^4)*d^2)*x^5
- 8*((C*a^3*b^3 + 5*A*a^2*b^4)*c^2 - 2*(D*a^4*b^2 - B*a^3*b^3)*c*d - (C*a^
4*b^2 - A*a^3*b^3)*d^2)*x^3 + 4*(D*a^5*b + 2*B*a^4*b^2)*c^2 + 8*(C*a^5*b +
 2*A*a^4*b^2)*c*d + 4*(2*D*a^6 + B*a^5*b)*d^2 + 12*(D*a^4*b^2*c^2 + 2*C*a^
4*b^2*c*d + (2*D*a^5*b + B*a^4*b^2)*d^2)*x^2 - 3*(((C*a*b^4 + 5*A*b^5)*c^2
 + 2*(D*a^2*b^3 + B*a*b^4)*c*d + (C*a^2*b^3 + A*a*b^4)*d^2)*x^6 + 3*((C...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx =$$
$$-\frac{24\,Da^3b^2d^2x^4-3\,((Cab^4+5\,Ab^5)c^2+2\,(Da^2b^3+Bab^4)cd+(Ca^2b^3+Aab^4)d^2)x^5-8\,((Ca^2b^3+5\,A$$
$$+\frac{((Cab+5\,Ab^2)c^2+2\,(Da^2+Bab)cd+(Ca^2+Aab)d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^2}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(24*D*a^3*b^2*d^2*x^4 - 3*((C*a*b^4 + 5*A*b^5)*c^2 + 2*(D*a^2*b^3 +
B*a*b^4)*c*d + (C*a^2*b^3 + A*a*b^4)*d^2)*x^5 - 8*((C*a^2*b^3 + 5*A*a*b^4)
*c^2 - 2*(D*a^3*b^2 - B*a^2*b^3)*c*d - (C*a^3*b^2 - A*a^2*b^3)*d^2)*x^3 +
4*(D*a^4*b + 2*B*a^3*b^2)*c^2 + 8*(C*a^4*b + 2*A*a^3*b^2)*c*d + 4*(2*D*a^5
 + B*a^4*b)*d^2 + 12*(D*a^3*b^2*c^2 + 2*C*a^3*b^2*c*d + (2*D*a^4*b + B*a^3
*b^2)*d^2)*x^2 + 3*((C*a^3*b^2 - 11*A*a^2*b^3)*c^2 + 2*(D*a^4*b + B*a^3*b^
2)*c*d + (C*a^4*b + A*a^3*b^2)*d^2)*x)/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^
5*b^4*x^2 + a^6*b^3) + 1/16*((C*a*b + 5*A*b^2)*c^2 + 2*(D*a^2 + B*a*b)*c*d
 + (C*a^2 + A*a*b)*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$$
$$=\frac{(Cabc^2+5\,Ab^2c^2+2\,Da^2cd+2\,Babcd+Ca^2d^2+Aabd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^2}$$
$$+\frac{3\,Cab^4c^2x^5+15\,Ab^5c^2x^5+6\,Da^2b^3cdx^5+6\,Bab^4cdx^5+3\,Ca^2b^3d^2x^5+3\,Aab^4d^2x^5-24\,Da^3b^2d^2x^4+$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*b*c^2 + 5*A*b^2*c^2 + 2*D*a^2*c*d + 2*B*a*b*c*d + C*a^2*d^2 + A*
a*b*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/48*(3*C*a*b^4*c^2*x
^5 + 15*A*b^5*c^2*x^5 + 6*D*a^2*b^3*c*d*x^5 + 6*B*a*b^4*c*d*x^5 + 3*C*a^2*
b^3*d^2*x^5 + 3*A*a*b^4*d^2*x^5 - 24*D*a^3*b^2*d^2*x^4 + 8*C*a^2*b^3*c^2*x
^3 + 40*A*a*b^4*c^2*x^3 - 16*D*a^3*b^2*c*d*x^3 + 16*B*a^2*b^3*c*d*x^3 - 8*
C*a^3*b^2*d^2*x^3 + 8*A*a^2*b^3*d^2*x^3 - 12*D*a^3*b^2*c^2*x^2 - 24*C*a^3*
b^2*c*d*x^2 - 24*D*a^4*b*d^2*x^2 - 12*B*a^3*b^2*d^2*x^2 - 3*C*a^3*b^2*c^2*
x + 33*A*a^2*b^3*c^2*x - 6*D*a^4*b*c*d*x - 6*B*a^3*b^2*c*d*x - 3*C*a^4*b*d
^2*x - 3*A*a^3*b^2*d^2*x - 4*D*a^4*b*c^2 - 8*B*a^3*b^2*c^2 - 8*C*a^4*b*c*d
 - 16*A*a^3*b^2*c*d - 8*D*a^5*d^2 - 4*B*a^4*b*d^2)/((b*x^2 + a)^3*a^3*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} \, dx = \int \frac{{\left(c+d\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^4} \,d x$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.32

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

output

```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*d**2 + 9*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*c*d**2 + 15*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*c**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**3*b**2*c*d + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a**3*b**2*d**2*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*a**3*b*c**3 + 27*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c
*d**2*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c*
*2*x**2 + 18*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c*d*x
**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*d**2*x**4
+ 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c**3*x**2 + 27
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c*d**2*x**4 + 45*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c**2*x**4 + 18*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c*d*x**4 + 3*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*d**2*x**6 + 9*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**3*x**4 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a*b**3*c*d**2*x**6 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*b**5*c**2*x**6 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*b**5*c*d*x**6 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*
c**3*x**6 - 16*a**4*b**2*c*d - 3*a**4*b**2*d**2*x - 4*a**4*b**2*d**2 - 12*
a**4*b*c**2*d - 9*a**4*b*c*d**2*x + 33*a**3*b**3*c**2*x - 8*a**3*b**3*c...
```

3.47 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 241

$$\begin{aligned}
&\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4} dx \\
&= -\frac{bBc+Abd-aCd-acD}{6b^2(a+bx^2)^3}+\frac{(Ab^2c-a(b(cC+Bd)-adD))x}{6ab^2(a+bx^2)^3}-\frac{Cd+cD}{4b^2(a+bx^2)^2} \\
&+\frac{(5Ab^2c+a(bcC+bBd-7adD))x}{24a^2b^2(a+bx^2)^2}+\frac{(5Ab^2c+a(bcC+bBd+adD))x}{16a^3b^2(a+bx^2)} \\
&+\frac{(5Ab^2c+a(bcC+bBd+adD))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{5/2}}
\end{aligned}$$

output
```
-1/6*(A*b*d+B*b*c-C*a*d-D*a*c)/b^2/(b*x^2+a)^3+1/6*(A*b^2*c-a*(b*(B*d+C*c)
-D*a*d))*x/a/b^2/(b*x^2+a)^3-1/4*(C*d+D*c)/b^2/(b*x^2+a)^2+1/24*(5*A*b^2*c
+a*(B*b*d+C*b*c-7*D*a*d))*x/a^2/b^2/(b*x^2+a)^2+1/16*(5*A*b^2*c+a*(B*b*d+C
*b*c+D*a*d))*x/a^3/b^2/(b*x^2+a)+1/16*(5*A*b^2*c+a*(B*b*d+C*b*c+D*a*d))*ar
ctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx = \frac{\left(5Ab^2c+a(bcC+bBd+adD)\right)x}{16a^3b^2\left(a+bx^2\right)} + \frac{5Ab^2cx+ab(cC+Bd)x-a^2(6Cd+6cD+7dDx)}{24a^2b^2\left(a+bx^2\right)^2} + \frac{Ab^2cx-ab(Ad+cCx+B(c+dx))+a^2(Cd+D(c+dx))}{6ab^2\left(a+bx^2\right)^3} + \frac{\left(5Ab^2c+a(bcC+bBd+adD)\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
((5*A*b^2*c + a*(b*c*C + b*B*d + a*d*D))*x)/(16*a^3*b^2*(a + b*x^2)) + (5*
A*b^2*c*x + a*b*(c*C + B*d)*x - a^2*(6*C*d + 6*c*D + 7*d*D*x))/(24*a^2*b^2
*(a + b*x^2)^2) + (A*b^2*c*x - a*b*(A*d + c*C*x + B*(c + d*x)) + a^2*(C*d
+ D*(c + d*x)))/(6*a*b^2*(a + b*x^2)^3) + ((5*A*b^2*c + a*(b*c*C + b*B*d +
 a*d*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2176, 25, 2345, 27, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx$$

$$\Big\downarrow \ 2176$$

$$-\frac{\int -\frac{-\frac{dDa^2}{b}+6dDx^2a+(cC+Bd)a+5Abc+2(2Abd+aCd+3acD)x}{(bx^2+a)^3}dx}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6adDx^2+2(2Abd+aCd+3acD)x+5Abc+acC+ad\left(B-\frac{aD}{b}\right)}{(bx^2+a)^3}dx}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 2345$$

$$\frac{-\frac{\int -\frac{3\left(5Abc+aCc+ad\left(B+\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^2}dx}{4a}-\frac{2a(3acD+aCd+2Abd)-x\left(a(-7adD+bBd+bcC)+5Ab^2c\right)}{4ab(a+bx^2)^2}}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(ad\left(\frac{aD}{b}+B\right)+acC+5Abc\right)\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}-\frac{2a(3acD+aCd+2Abd)-x\left(a(-7adD+bBd+bcC)+5Ab^2c\right)}{4ab(a+bx^2)^2}}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3\left(ad\left(\frac{aD}{b}+B\right)+acC+5Abc\right)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}-\frac{2a(3acD+aCd+2Abd)-x\left(a(-7adD+bBd+bcC)+5Ab^2c\right)}{4ab(a+bx^2)^2}}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)\left(ad\left(\frac{aD}{b}+B\right)+acC+5Abc\right)}{4a}-\frac{2a(3acD+aCd+2Abd)-x\left(a(-7adD+bBd+bcC)+5Ab^2c\right)}{4ab(a+bx^2)^2}}{6ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{6ab\left(a+bx^2\right)^3}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^4,x]
```

output
```
-1/6*((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x))/(a*b*(a + b*x^2)^3) + (
-1/4*(2*a*(2*A*b*d + a*C*d + 3*a*c*D) - (5*A*b^2*c + a*(b*c*C + b*B*d - 7*
a*d*D))*x)/(a*b*(a + b*x^2)^2) + (3*(5*A*b*c + a*c*C + a*d*(B + (a*D)/b))*
(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(
4*a))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{\left(5Ab^2c+Babd+Cabc+a^2dD\right)x^5}{16a^3}+\frac{\left(5Ab^2c+Babd+Cabc-a^2dD\right)x^3}{6a^2b}-\frac{(Cd+Dc)x^2}{4b}+\frac{\left(11Ab^2c-Babd-Cabc-a^2dD\right)x}{16ab^2}-\frac{2Abd+2Bbc+Cad+Da}{12b^2}}{\left(bx^2+a\right)^3}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*d)/a^3*x^5+1/6*(5*A*b^2*c+B*a*b*d+C
*a*b*c-D*a^2*d)/a^2/b*x^3-1/4*(C*d+D*c)*x^2/b+1/16*(11*A*b^2*c-B*a*b*d-C*a
*b*c-D*a^2*d)/a/b^2*x-1/12*(2*A*b*d+2*B*b*c+C*a*d+D*a*c)/b^2)/(b*x^2+a)^3+
1/16*(5*A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*d)/a^3/b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.50

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*((C*a^2*b^4 + 5*A*a*b^5)*c + (D*a^3*b^3 + B*a^2*b^4)*d)*x^5 + 16*
((C*a^3*b^3 + 5*A*a^2*b^4)*c - (D*a^4*b^2 - B*a^3*b^3)*d)*x^3 - 24*(D*a^4*
b^2*c + C*a^4*b^2*d)*x^2 - 3*((((C*a*b^4 + 5*A*b^5)*c + (D*a^2*b^3 + B*a*b^
4)*d)*x^6 + 3*((C*a^2*b^3 + 5*A*a*b^4)*c + (D*a^3*b^2 + B*a^2*b^3)*d)*x^4
+ 3*((C*a^3*b^2 + 5*A*a^2*b^3)*c + (D*a^4*b + B*a^3*b^2)*d)*x^2 + (C*a^4*b
 + 5*A*a^3*b^2)*c + (D*a^5 + B*a^4*b)*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - 8*(D*a^5*b + 2*B*a^4*b^2)*c - 8*(C*a^5*b + 2*A*a
^4*b^2)*d - 6*((C*a^4*b^2 - 11*A*a^3*b^3)*c + (D*a^5*b + B*a^4*b^2)*d)*x)/
(a^4*b^6*x^6 + 3*a^5*b^5*x^4 + 3*a^6*b^4*x^2 + a^7*b^3), 1/48*(3*((C*a^2*b
^4 + 5*A*a*b^5)*c + (D*a^3*b^3 + B*a^2*b^4)*d)*x^5 + 8*((C*a^3*b^3 + 5*A*a
^2*b^4)*c - (D*a^4*b^2 - B*a^3*b^3)*d)*x^3 - 12*(D*a^4*b^2*c + C*a^4*b^2*d
)*x^2 + 3*(((C*a*b^4 + 5*A*b^5)*c + (D*a^2*b^3 + B*a*b^4)*d)*x^6 + 3*((C*a
^2*b^3 + 5*A*a*b^4)*c + (D*a^3*b^2 + B*a^2*b^3)*d)*x^4 + 3*((C*a^3*b^2 + 5
*A*a^2*b^3)*c + (D*a^4*b + B*a^3*b^2)*d)*x^2 + (C*a^4*b + 5*A*a^3*b^2)*c +
 (D*a^5 + B*a^4*b)*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 4*(D*a^5*b + 2*B*a
^4*b^2)*c - 4*(C*a^5*b + 2*A*a^4*b^2)*d - 3*((C*a^4*b^2 - 11*A*a^3*b^3)*c
+ (D*a^5*b + B*a^4*b^2)*d)*x)/(a^4*b^6*x^6 + 3*a^5*b^5*x^4 + 3*a^6*b^4*x^2
 + a^7*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx$$
$$= \frac{3\,((Cab^3+5\,Ab^4)c+(Da^2b^2+Bab^3)d)x^5+8\,((Ca^2b^2+5\,Aab^3)c-(Da^3b-Ba^2b^2)d)x^3-12\,(Da^3bc+}{48\,(a^3b^5x^6+3\,a^4b^4x^4}$$
$$+\frac{((Cab+5\,Ab^2)c+(Da^2+Bab)d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*((C*a*b^3 + 5*A*b^4)*c + (D*a^2*b^2 + B*a*b^3)*d)*x^5 + 8*((C*a^2*
b^2 + 5*A*a*b^3)*c - (D*a^3*b - B*a^2*b^2)*d)*x^3 - 12*(D*a^3*b*c + C*a^3*
b*d)*x^2 - 4*(D*a^4 + 2*B*a^3*b)*c - 4*(C*a^4 + 2*A*a^3*b)*d - 3*((C*a^3*b
 - 11*A*a^2*b^2)*c + (D*a^4 + B*a^3*b)*d)*x)/(a^3*b^5*x^6 + 3*a^4*b^4*x^4
+ 3*a^5*b^3*x^2 + a^6*b^2) + 1/16*((C*a*b + 5*A*b^2)*c + (D*a^2 + B*a*b)*d
)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^4}\,dx = \frac{(Cabc+5\,Ab^2c+Da^2d+Babd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b^2}$$
$$+\frac{3\,Cab^3cx^5+15\,Ab^4cx^5+3\,Da^2b^2dx^5+3\,Bab^3dx^5+8\,Ca^2b^2cx^3+40\,Aab^3cx^3-8\,Da^3bdx^3+8\,Ba^2b^2}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*b*c + 5*A*b^2*c + D*a^2*d + B*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^3*b^2) + 1/48*(3*C*a*b^3*c*x^5 + 15*A*b^4*c*x^5 + 3*D*a^2*b^2*d*x^
5 + 3*B*a*b^3*d*x^5 + 8*C*a^2*b^2*c*x^3 + 40*A*a*b^3*c*x^3 - 8*D*a^3*b*d*x
^3 + 8*B*a^2*b^2*d*x^3 - 12*D*a^3*b*c*x^2 - 12*C*a^3*b*d*x^2 - 3*C*a^3*b*c
*x + 33*A*a^2*b^2*c*x - 3*D*a^4*d*x - 3*B*a^3*b*d*x - 4*D*a^4*c - 8*B*a^3*
b*c - 4*C*a^4*d - 8*A*a^3*b*d)/((b*x^2 + a)^3*a^3*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx=\int \frac{(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^4}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4,x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.68

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^4}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

output

```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d**2 + 15*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*c + 3*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*d + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**3*b*c**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
a**3*b*d**2*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b
**3*c*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*d*x
**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c**2*x**2
+ 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d**2*x**4 + 45
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c*x**4 + 9*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*d*x**4 + 9*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c**2*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a*b**3*d**2*x**6 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*b**5*c*x**6 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*b**5*d*x**6 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*c**2*
x**6 - 8*a**4*b**2*d - 8*a**4*b*c*d - 3*a**4*b*d**2*x + 33*a**3*b**3*c*x -
 8*a**3*b**3*c - 3*a**3*b**3*d*x - 3*a**3*b**2*c**2*x - 24*a**3*b**2*c*d*x
**2 - 8*a**3*b**2*d**2*x**3 + 40*a**2*b**4*c*x**3 + 8*a**2*b**4*d*x**3 + 8
*a**2*b**3*c**2*x**3 + 3*a**2*b**3*d**2*x**5 + 15*a*b**5*c*x**5 + 3*a*b**5
*d*x**5 + 3*a*b**4*c**2*x**5)/(48*a**3*b**3*(a**3 + 3*a**2*b*x**2 + 3*a*b*
*2*x**4 + b**3*x**6))
```

3.48 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^4}\,dx =& -\frac{bB-aD}{6b^2\left(a+bx^2\right)^3}+\frac{(Ab-aC)x}{6ab\left(a+bx^2\right)^3} \\ &-\frac{D}{4b^2\left(a+bx^2\right)^2}+\frac{(5Ab+aC)x}{24a^2b\left(a+bx^2\right)^2} \\ &+\frac{(5Ab+aC)x}{16a^3b\left(a+bx^2\right)}+\frac{(5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{3/2}}\end{aligned}$$

output
```
-1/6*(B*b-D*a)/b^2/(b*x^2+a)^3+1/6*(A*b-C*a)*x/a/b/(b*x^2+a)^3-1/4*D/b^2/(
b*x^2+a)^2+1/24*(5*A*b+C*a)*x/a^2/b/(b*x^2+a)^2+1/16*(5*A*b+C*a)*x/a^3/b/(
b*x^2+a)+1/16*(5*A*b+C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx$$
$$=\frac{\frac{\sqrt{a}(-4a^4D+15Ab^4x^5+ab^3x^3(40A+3Cx^2)+a^2b^2x(33A+8Cx^2)-a^3b(8B+3x(C+4Dx)))}{(a+bx^2)^3}+3\sqrt{b}(5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48a^{7/2}b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^4,x]
```

output
```
((Sqrt[a]*(-4*a^4*D + 15*A*b^4*x^5 + a*b^3*x^3*(40*A + 3*C*x^2) + a^2*b^2*
x*(33*A + 8*C*x^2) - a^3*b*(8*B + 3*x*(C + 4*D*x))))/(a + b*x^2)^3 + 3*Sqr
t[b]*(5*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*a^(7/2)*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2345, 25, 27, 454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx$$

$$\Big\downarrow \; 2345$$

$$-\frac{\int -\frac{b\left(5A+\frac{aC}{b}\right)+6aDx}{b(bx^2+a)^3}dx}{6a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5Ab+aC+6aDx}{b(bx^2+a)^3}dx}{6a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5Ab+aC+6aDx}{(bx^2+a)^3}dx}{6ab} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{3(aC+5Ab)\int \frac{1}{(bx^2+a)^2}dx}{4a}-\frac{6a^2D-bx(aC+5Ab)}{4ab(a+bx^2)^2}}{6ab} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3(aC+5Ab)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}-\frac{6a^2D-bx(aC+5Ab)}{4ab(a+bx^2)^2}}{6ab} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3(aC+5Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}-\frac{6a^2D-bx(aC+5Ab)}{4ab(a+bx^2)^2}}{6ab} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{6ab\,(a+bx^2)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^4,x]
```

output
```
-1/6*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^3) + (-1/4*(6*a^2*
D - b*(5*A*b + a*C)*x)/(a*b*(a + b*x^2)^2) + (3*(5*A*b + a*C)*(x/(2*a*(a +
 b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/(6*a*b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\frac{(5Ab+Ca)b\,x^5}{16a^3}+\frac{(5Ab+Ca)x^3}{6a^2}-\frac{D x^2}{4b}+\frac{(11Ab-Ca)x}{16ab}-\frac{2Bb+Da}{12b^2}}{(b\,x^2+a)^3}+\frac{(5Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a^3b\sqrt{ab}}$	116

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*b+C*a)/a^3*b*x^5+1/6/a^2*(5*A*b+C*a)*x^3-1/4*D*x^2/b+1/16*(11*A
*b-C*a)/a/b*x-1/12*(2*B*b+D*a)/b^2)/(b*x^2+a)^3+1/16*(5*A*b+C*a)/a^3/b/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx$$
$$=\left[-\frac{24\,Da^4bx^2+8\,Da^5+16\,Ba^4b-6\,(Ca^2b^3+5\,Aab^4)x^5-16\,(Ca^3b^2+5\,Aa^2b^3)x^3+3\,((Cab^3+5\,Ab^4)}{96\,(a^4b^5x^6}\right.$$
$$-\frac{12\,Da^4bx^2+4\,Da^5+8\,Ba^4b-3\,(Ca^2b^3+5\,Aab^4)x^5-8\,(Ca^3b^2+5\,Aa^2b^3)x^3-3\,((Cab^3+5\,Ab^4)x^6}{48\,(a^4b^5x^6+3\,a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(24*D*a^4*b*x^2 + 8*D*a^5 + 16*B*a^4*b - 6*(C*a^2*b^3 + 5*A*a*b^4)*
x^5 - 16*(C*a^3*b^2 + 5*A*a^2*b^3)*x^3 + 3*((C*a*b^3 + 5*A*b^4)*x^6 + C*a^
4 + 5*A*a^3*b + 3*(C*a^2*b^2 + 5*A*a*b^3)*x^4 + 3*(C*a^3*b + 5*A*a^2*b^2)*
x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(C*a^4*b
 - 11*A*a^3*b^2)*x)/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2
), -1/48*(12*D*a^4*b*x^2 + 4*D*a^5 + 8*B*a^4*b - 3*(C*a^2*b^3 + 5*A*a*b^4)
*x^5 - 8*(C*a^3*b^2 + 5*A*a^2*b^3)*x^3 - 3*((C*a*b^3 + 5*A*b^4)*x^6 + C*a^
4 + 5*A*a^3*b + 3*(C*a^2*b^2 + 5*A*a*b^3)*x^4 + 3*(C*a^3*b + 5*A*a^2*b^2)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(C*a^4*b - 11*A*a^3*b^2)*x)/(a^4*
b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)]
```

Sympy [A] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx=-\frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ca)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ca)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32}$$
$$+\frac{-8Ba^3b-4Da^4-12Da^3bx^2+x^5\cdot(15Ab^4+3Cab^3)+x^3\cdot(40Aab^3+8Ca^2b^2)+x(33Aa^2b^2-3Ca^3}{48a^6b^2+144a^5b^3x^2+144a^4b^4x^4+48a^3b^5x^6}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*b**3))*(5*A*b + C*a)*log(-a**4*b*sqrt(-1/(a**7*b**3)) + x)/
32 + sqrt(-1/(a**7*b**3))*(5*A*b + C*a)*log(a**4*b*sqrt(-1/(a**7*b**3)) +
x)/32 + (-8*B*a**3*b - 4*D*a**4 - 12*D*a**3*b*x**2 + x**5*(15*A*b**4 + 3*C
*a*b**3) + x**3*(40*A*a*b**3 + 8*C*a**2*b**2) + x*(33*A*a**2*b**2 - 3*C*a*
*3*b))/(48*a**6*b**2 + 144*a**5*b**3*x**2 + 144*a**4*b**4*x**4 + 48*a**3*b
**5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx = -\frac{12\,Da^3bx^2-3\,(Cab^3+5\,Ab^4)x^5+4\,Da^4+8\,Ba^3b-8\,(Ca^2b^2+5\,Aab^3)x^3+3\,(Ca^3b-11\,Aa^2b^2)x}{48\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)} + \frac{(Ca+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(12*D*a^3*b*x^2 - 3*(C*a*b^3 + 5*A*b^4)*x^5 + 4*D*a^4 + 8*B*a^3*b -
8*(C*a^2*b^2 + 5*A*a*b^3)*x^3 + 3*(C*a^3*b - 11*A*a^2*b^2)*x)/(a^3*b^5*x^6
 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2) + 1/16*(C*a + 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx = \frac{(Ca+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b} + \frac{3\,Cab^3x^5+15\,Ab^4x^5+8\,Ca^2b^2x^3+40\,Aab^3x^3-12\,Da^3bx^2-3\,Ca^3bx+33\,Aa^2b^2x-4\,Da^4-8\,Ba^3b}{48\,(bx^2+a)^3a^3b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a + 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b) + 1/48*(3*C*a*b
^3*x^5 + 15*A*b^4*x^5 + 8*C*a^2*b^2*x^3 + 40*A*a*b^3*x^3 - 12*D*a^3*b*x^2
- 3*C*a^3*b*x + 33*A*a^2*b^2*x - 4*D*a^4 - 8*B*a^3*b)/((b*x^2 + a)^3*a^3*b
^2)
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx = \frac{\frac{11\,A\,x}{16\,a}+\frac{5\,A\,b\,x^3}{6\,a^2}+\frac{5\,A\,b^2\,x^5}{16\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{\frac{C\,x^3}{6\,a}-\frac{C\,x}{16\,b}+\frac{C\,b\,x^5}{16\,a^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} - \frac{B}{6\,b\,\left(a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6\right)} + \frac{5\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{5/2}\,b^{3/2}} + \frac{x^4\,\left(b\,x^2+3\,a\right)\,D}{12\,a^2\,\left(b\,x^2+a\right)^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^4,x)
```

output
```
((11*A*x)/(16*a) + (5*A*b*x^3)/(6*a^2) + (5*A*b^2*x^5)/(16*a^3))/(a^3 + b^
3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + ((C*x^3)/(6*a) - (C*x)/(16*b) + (C*b*
x^5)/(16*a^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) - B/(6*b*(a^3 +
 b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)) + (5*A*atan((b^(1/2)*x)/a^(1/2)))/(
16*a^(7/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(5/2)*b^(3/2)) +
 (x^4*(3*a + b*x^2)*D)/(12*a^2*(a + b*x^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3c+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2+9\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b + 3*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*c + 45*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**2*b**2*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**2*b*c*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a
*b**3*x**4 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c*x**4
 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**6 + 3*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*x**6 - 4*a**4*d + 33*a**3*b**
2*x - 8*a**3*b**2 - 3*a**3*b*c*x - 12*a**3*b*d*x**2 + 40*a**2*b**3*x**3 +
8*a**2*b**2*c*x**3 + 15*a*b**4*x**5 + 3*a*b**3*c*x**5)/(48*a**3*b**2*(a**3
 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.49 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^4} dx$

Optimal result

Integrand size = 32, antiderivative size = 701

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^4} dx$$
$$= -\frac{a(bBc-Abd+aCd-acD)-(Ab^2c-a(bcC-bBd+adD))x}{6ab\left(bc^2+ad^2\right)\left(a+bx^2\right)^3}$$
$$+\frac{6a^2b(c^2Cd-Bcd^2+Ad^3-c^3D)+(Ab^2c(5bc^2+11ad^2)+a(b^2c^2(cC-Bd)+a^2d^3D-abd(5cCd-}{24a^2b\left(bc^2+ad^2\right)^2\left(a+bx^2\right)^2}$$
$$+\frac{8a^3bd^2(c^2Cd-Bcd^2+Ad^3-c^3D)+(Ab^2c(5b^2c^4+16abc^2d^2+19a^2d^4)+a(b^3c^4(cC-Bd)+a^3d^5D}{16a^3b\left(bc^2+ad^2\right)^3\left(a+bx^2\right)}$$
$$+\frac{(Ab^2c(5b^3c^6+21ab^2c^4d^2+35a^2bc^2d^4+35a^3d^6)+a(b^4c^6(cC-Bd)+a^4d^7D-a^3bd^5(5cCd-5Bd^2-}{16a^{7/2}b^{3/2}\left(bc^2+\right.}$$
$$+\frac{d^4(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{\left(bc^2+ad^2\right)^4}$$
$$-\frac{d^4(c^2Cd-Bcd^2+Ad^3-c^3D)\log\left(a+bx^2\right)}{2\left(bc^2+ad^2\right)^4}$$

output
```
-1/6*(a*(-A*b*d+B*b*c+C*a*d-D*a*c)-(A*b^2*c-a*(-B*b*d+C*b*c+D*a*d))*x)/a/b
/(a*d^2+b*c^2)/(b*x^2+a)^3+1/24*(6*a^2*b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)+(A*
b^2*c*(11*a*d^2+5*b*c^2)+a*(b^2*c^2*(-B*d+C*c)+a^2*d^3*D-a*b*d*(-5*B*d^2+5
*C*c*d-7*D*c^2)))*x)/a^2/b/(a*d^2+b*c^2)^2/(b*x^2+a)^2+1/16*(8*a^3*b*d^2*(
A*d^3-B*c*d^2+C*c^2*d-D*c^3)+(A*b^2*c*(19*a^2*d^4+16*a*b*c^2*d^2+5*b^2*c^4
)+a*(b^3*c^4*(-B*d+C*c)+a^3*d^5*D-a^2*b*d^3*(-5*B*d^2+5*C*c*d-8*D*c^2)+a*b
^2*c^2*d*(-4*B*d^2+4*C*c*d-D*c^2)))*x)/a^3/b/(a*d^2+b*c^2)^3/(b*x^2+a)+1/1
6*(A*b^2*c*(35*a^3*d^6+35*a^2*b*c^2*d^4+21*a*b^2*c^4*d^2+5*b^3*c^6)+a*(b^4
*c^6*(-B*d+C*c)+a^4*d^7*D-a^3*b*d^5*(-5*B*d^2+5*C*c*d-9*D*c^2)+3*a^2*b^2*c
^2*d^3*(-5*B*d^2+5*C*c*d-3*D*c^2)+a*b^3*c^4*d*(-5*B*d^2+5*C*c*d-D*c^2)))*a
rctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(3/2)/(a*d^2+b*c^2)^4+d^4*(A*d^3-B*c*d^
2+C*c^2*d-D*c^3)*ln(d*x+c)/(a*d^2+b*c^2)^4-1/2*d^4*(A*d^3-B*c*d^2+C*c^2*d-
D*c^3)*ln(b*x^2+a)/(a*d^2+b*c^2)^4
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^4}\,dx$$

$$= \frac{\frac{8\left(bc^2+ad^2\right)^3\left(Ab^2cx+ab(-Bc+Ad-cCx+Bdx)-a^2(Cd-cD+dDx)\right)}{ab\left(a+bx^2\right)^3}+\frac{3\left(bc^2+ad^2\right)\left(5Ab^4c^5x+ab^3c^3\left(c^2C-Bcd+16Ad^2\right)x+a^4d^5Dx+a^2b^2c\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^4),x]
```

output
```
((8*(b*c^2 + a*d^2)^3*(A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x) - a^
2*(C*d - c*D + d*D*x)))/(a*b*(a + b*x^2)^3) + (3*(b*c^2 + a*d^2)*(5*A*b^4*
c^5*x + a*b^3*c^3*(c^2*C - B*c*d + 16*A*d^2)*x + a^4*d^5*D*x + a^2*b^2*c*d
*(4*c^2*C*d - 4*B*c*d^2 + 19*A*d^3 - c^3*D)*x + a^3*b*d^2*(-8*c^3*D + d^3*
(8*A + 5*B*x) - c*d^2*(8*B + 5*C*x) + 8*c^2*d*(C + D*x))))/(a^3*b*(a + b*x
^2)) + (2*(b*c^2 + a*d^2)^2*(5*A*b^3*c^3*x + a*b^2*c*(c^2*C - B*c*d + 11*A
*d^2)*x + a^3*d^3*D*x + a^2*b*(-6*c^3*D + d^3*(6*A + 5*B*x) - c*d^2*(6*B +
 5*C*x) + c^2*d*(6*C + 7*D*x))))/(a^2*b*(a + b*x^2)^2) + (3*(A*b^2*c*(5*b^
3*c^6 + 21*a*b^2*c^4*d^2 + 35*a^2*b*c^2*d^4 + 35*a^3*d^6) + a*(b^4*c^6*(c*
C - B*d) + a^4*d^7*D - a*b^3*c^4*d*(-5*c*C*d + 5*B*d^2 + c^2*D) - 3*a^2*b^
2*c^2*d^3*(-5*c*C*d + 5*B*d^2 + 3*c^2*D) + a^3*b*d^5*(-5*c*C*d + 5*B*d^2 +
 9*c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*b^(3/2)) + 48*d^4*(c^2*C
*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x] + 24*d^4*(-(c^2*C*d) + B*c*d^2
- A*d^3 + c^3*D)*Log[a + b*x^2])/(48*(b*c^2 + a*d^2)^4)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2178, 25, 27, 686, 27, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^4(c+dx)}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{Ab(5bc^2+6ad^2)+ac(bcC-bBd+adD)+\left(5Acdb^2+a\left(ad^2D-b\left(-6Dc^2+5Cdc-5Bd^2\right)\right)\right)x}{(bc^2+ad^2)(c+dx)(bx^2+a)^3}dx}{6ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab(5bc^2+6ad^2)+ac(bcC-bBd+adD)+\left(5Acdb^2+a\left(ad^2D-b\left(-6Dc^2+5Cdc-5Bd^2\right)\right)\right)x}{(bc^2+ad^2)(c+dx)(bx^2+a)^3}dx}{6ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{Ab(5bc^2+6ad^2)+ac(bcC-bBd+adD)+(5Acdb^2+a(ad^2D-b(-6Dc^2+5Cdc-5Bd^2)))x}{(c+dx)(bx^2+a)^3}dx}{6ab\left(ad^2+bc^2\right)}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 686$$

$$\frac{x(a(a^2d^3D-abd(-5Bd^2-7c^2D+5cCd)+b^2c^2(cC-Bd))+Ab^2c(11ad^2+5bc^2))+6a^2b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{4a(a+bx^2)^2(ad^2+bc^2)} - \int -\frac{3b\Big(Ab\Big(5b^2c^4+11abd^2}{}$$

$$6ab\left(ad^2\right.$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 27$$

$$3\int \frac{Ab\left(5b^2c^4+11abd^2c^2+8a^2d^4\right)+ac\left(a^2Dd^3+ab\left(-Dc^2+3Cdc-3Bd^2\right)d+b^2c^2(cC-Bd)\right)+d\left(Ac\left(5bc^2+11ad^2\right)b^2+a\left(a^2Dd^3-ab\left(-7Dc^2+5Cdc-5Bd^2\right)d+b^2c\right.\right.}{(c+dx)\left(bx^2+a\right)^2}$$

$$4a(ad^2+bc^2)$$

$$6ab\left(ad^2+\right.$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 686$$

$$3\left(\frac{8a^3bd^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)+x\left(Ab^2c\left(19a^2d^4+16abc^2d^2+5b^2c^4\right)+a\left(a^3d^5D-a^2bd^3\left(-5Bd^2-8c^2D+5cCd\right)+ab^2c^2d\left(-4Bd^2+c^2(-D)+4cCd\right)+b^3\right.\right.}{2a\left(a+bx^2\right)\left(ad^2+bc^2\right)}\right.$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 25$$

$$3\left(\frac{\int \frac{b\left(Ab\left(5b^3c^6+16ab^2d^2c^4+19a^2bd^4c^2+16a^3d^6\right)+ac\left(a^3Dd^5+a^2b\left(-8Dc^2+11Cdc-11Bd^2\right)d^3+ab^2c^2\left(-Dc^2+4Cdc-4Bd^2\right)d+b^3c^4(cC-Bd)\right)+d\left(Ac\left(5b^2c \cdots\right.\right.}{(c+dx)\left(bx^2+a\right)}}{2ab\left(ad^2+bc^2\right)} \cdots\right. \cdots$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{\int \frac{Ab\left(5b^3c^6+16ab^2d^2c^4+19a^2bd^4c^2+16a^3d^6\right)+ac\left(a^3Dd^5+a^2b\left(-8Dc^2+11Cdc-11Bd^2\right)d^3+ab^2c^2\left(-Dc^2+4Cdc-4Bd^2\right)d+b^3c^4(cC-Bd)\right)+d\left(Ac\left(5b^2c^4+ \cdots\right.\right.}{(c+dx)\left(bx^2+a\right)}}{2a\left(ad^2+bc^2\right)} \cdots\right. \cdots$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 657$$

$$3\left(\frac{\int \left(\frac{16a^3b\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)d^5}{\left(bc^2+ad^2\right)(c+dx)}+\frac{-16a^3b^2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)xd^4+Ab^2c\left(5b^3c^6+21ab^2d^2c^4+35a^2bd^4c^2+35a^3d^6\right)+a\left(a^4Dd^7-a^3b\left(-9Dc \cdots\right.\right.}{\left(bc^2+ad^2\right)\left(bx \cdots\right.}\right.}{2a\left(ad^2+bc^2\right)} \cdots\right. \cdots$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x\left(a\left(a^2d^3D-abd\left(-5Bd^2-7c^2D+5cCd\right)+b^2c^2(cC-Bd)\right)+Ab^2c\left(11ad^2+5bc^2\right)\right)+6a^2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4a\left(a+bx^2\right)^2\left(ad^2+bc^2\right)}+\frac{3\left(\frac{8a^3bd^2\left(Ad^3-Bcd^2+c \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{6ab\left(a+bx^2\right)^3\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^4),x]
```

output
```
-1/6*(a*(b*B*c - A*b*d + a*C*d - a*c*D) - (A*b^2*c - a*(b*c*C - b*B*d + a*
d*D))*x)/(a*b*(b*c^2 + a*d^2)*(a + b*x^2)^3) + ((6*a^2*b*(c^2*C*d - B*c*d^
2 + A*d^3 - c^3*D) + (A*b^2*c*(5*b*c^2 + 11*a*d^2) + a*(b^2*c^2*(c*C - B*d
) + a^2*d^3*D - a*b*d*(5*c*C*d - 5*B*d^2 - 7*c^2*D)))*x)/(4*a*(b*c^2 + a*d
^2)*(a + b*x^2)^2) + (3*((8*a^3*b*d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)
+ (A*b^2*c*(5*b^2*c^4 + 16*a*b*c^2*d^2 + 19*a^2*d^4) + a*(b^3*c^4*(c*C - B
*d) + a^3*d^5*D - a^2*b*d^3*(5*c*C*d - 5*B*d^2 - 8*c^2*D) + a*b^2*c^2*d*(4
*c*C*d - 4*B*d^2 - c^2*D)))*x)/(2*a*(b*c^2 + a*d^2)*(a + b*x^2)) + (((A*b^
2*c*(5*b^3*c^6 + 21*a*b^2*c^4*d^2 + 35*a^2*b*c^2*d^4 + 35*a^3*d^6) + a*(b^
4*c^6*(c*C - B*d) + a^4*d^7*D - a^3*b*d^5*(5*c*C*d - 5*B*d^2 - 9*c^2*D) +
3*a^2*b^2*c^2*d^3*(5*c*C*d - 5*B*d^2 - 3*c^2*D) + a*b^3*c^4*d*(5*c*C*d - 5
*B*d^2 - c^2*D)))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(b*c^2 + a
*d^2)) + (16*a^3*b*d^4*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/(
b*c^2 + a*d^2) - (8*a^3*b*d^4*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[a +
b*x^2])/(b*c^2 + a*d^2))/(2*a*(b*c^2 + a*d^2))))/(4*a*(b*c^2 + a*d^2)))/(6
*a*b*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(683) = 1366$.

Time = 1.24 (sec) , antiderivative size = 1441, normalized size of antiderivative = 2.06

method	result	size
default	Expression too large to display	1441

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2+b*c^2)^4*((1/16*b*(19*A*a^3*b^2*c*d^6+35*A*a^2*b^3*c^3*d^4+21*A*a
*b^4*c^5*d^2+5*A*b^5*c^7+5*B*a^4*b*d^7+B*a^3*b^2*c^2*d^5-5*B*a^2*b^3*c^4*d
^3-B*a*b^4*c^6*d-5*C*a^4*b*c*d^6-C*a^3*b^2*c^3*d^4+5*C*a^2*b^3*c^5*d^2+C*a
*b^4*c^7+D*a^5*d^7+9*D*a^4*b*c^2*d^5+7*D*a^3*b^2*c^4*d^3-D*a^2*b^3*c^6*d)/
a^3*x^5+(1/2*A*a*b^2*d^7+1/2*A*b^3*c^2*d^5-1/2*B*a*b^2*c*d^6-1/2*B*b^3*c^3
*d^4+1/2*C*a*b^2*c^2*d^5+1/2*C*b^3*c^4*d^3-1/2*D*a*b^2*c^3*d^4-1/2*D*b^3*c
^5*d^2)*x^4+1/6*(17*A*a^3*b^2*c*d^6+33*A*a^2*b^3*c^3*d^4+21*A*a*b^4*c^5*d^
2+5*A*b^5*c^7+5*B*a^4*b*d^7+3*B*a^3*b^2*c^2*d^5-3*B*a^2*b^3*c^4*d^3-B*a*b^
4*c^6*d-5*C*a^4*b*c*d^6-3*C*a^3*b^2*c^3*d^4+3*C*a^2*b^3*c^5*d^2+C*a*b^4*c^
7+D*a^5*d^7+9*D*a^4*b*c^2*d^5+9*D*a^3*b^2*c^4*d^3+D*a^2*b^3*c^6*d)/a^2*x^3
+(5/4*A*a^2*b*d^7+3/2*A*a*b^2*c^2*d^5+1/4*A*b^3*c^4*d^3-5/4*B*a^2*b*c*d^6-
3/2*B*a*b^2*c^3*d^4-1/4*B*b^3*c^5*d^2+5/4*C*a^2*b*c^2*d^5+3/2*C*a*b^2*c^4*
d^3+1/4*C*b^3*c^6*d-5/4*D*a^2*b*c^3*d^4-3/2*D*a*b^2*c^5*d^2-1/4*b^3*c^7*D)
*x^2+1/16*(29*A*a^3*b^2*c*d^6+61*A*a^2*b^3*c^3*d^4+43*A*a*b^4*c^5*d^2+11*A
*b^5*c^7+11*B*a^4*b*d^7+15*B*a^3*b^2*c^2*d^5+5*B*a^2*b^3*c^4*d^3+B*a*b^4*c
^6*d-11*C*a^4*b*c*d^6-15*C*a^3*b^2*c^3*d^4-5*C*a^2*b^3*c^5*d^2-C*a*b^4*c^7
-D*a^5*d^7+7*D*a^4*b*c^2*d^5+9*D*a^3*b^2*c^4*d^3+D*a^2*b^3*c^6*d)/a/b*x+1/
12*(11*A*a^3*b*d^7+18*A*a^2*b^2*c^2*d^5+9*A*a*b^3*c^4*d^3+2*A*b^4*c^6*d-11
*B*a^3*b*c*d^6-18*B*a^2*b^2*c^3*d^4-9*B*a*b^3*c^5*d^2-2*B*b^4*c^7-2*C*a^4*
d^7+3*C*a^3*b*c^2*d^5+6*C*a^2*b^2*c^4*d^3+C*a*b^3*c^6*d+2*D*a^4*c*d^6-3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^4}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)(a + bx^2)^4} dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(682) = 1364$.

Time $= 0.19$ (sec) , antiderivative size $= 1390$, normalized size of antiderivative $= 1.98$

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)(a + bx^2)^4} dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/2*(D*c^3*d^4 - C*c^2*d^5 + B*c*d^6 - A*d^7)*log(b*x^2 + a)/(b^4*c^8 + 4*
a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8) - (D*c^3*d^
4 - C*c^2*d^5 + B*c*d^6 - A*d^7)*log(d*x + c)/(b^4*c^8 + 4*a*b^3*c^6*d^2 +
 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8) + 1/16*((C*a*b^4 + 5*A*b^5
)*c^7 - (D*a^2*b^3 + B*a*b^4)*c^6*d + (5*C*a^2*b^3 + 21*A*a*b^4)*c^5*d^2 -
 (9*D*a^3*b^2 + 5*B*a^2*b^3)*c^4*d^3 + 5*(3*C*a^3*b^2 + 7*A*a^2*b^3)*c^3*d
^4 + 3*(3*D*a^4*b - 5*B*a^3*b^2)*c^2*d^5 - 5*(C*a^4*b - 7*A*a^3*b^2)*c*d^6
 + (D*a^5 + 5*B*a^4*b)*d^7)*arctan(b*x/sqrt(a*b))/((a^3*b^5*c^8 + 4*a^4*b^
4*c^6*d^2 + 6*a^5*b^3*c^4*d^4 + 4*a^6*b^2*c^2*d^6 + a^7*b*d^8)*sqrt(a*b))
- 1/48*(4*(D*a^4*b^2 + 2*B*a^3*b^3)*c^5 - 4*(C*a^4*b^2 + 2*A*a^3*b^3)*c^4*
d + 4*(5*D*a^5*b + 7*B*a^4*b^2)*c^3*d^2 - 4*(5*C*a^5*b + 7*A*a^4*b^2)*c^2*
d^3 - 4*(2*D*a^6 - 11*B*a^5*b)*c*d^4 + 4*(2*C*a^6 - 11*A*a^5*b)*d^5 - 3*((
C*a*b^5 + 5*A*b^6)*c^5 - (D*a^2*b^4 + B*a*b^5)*c^4*d + 4*(C*a^2*b^4 + 4*A*
a*b^5)*c^3*d^2 + 4*(2*D*a^3*b^3 - B*a^2*b^4)*c^2*d^3 - (5*C*a^3*b^3 - 19*A
*a^2*b^4)*c*d^4 + (D*a^4*b^2 + 5*B*a^3*b^3)*d^5)*x^5 + 24*(D*a^3*b^3*c^3*d
^2 - C*a^3*b^3*c^2*d^3 + B*a^3*b^3*c*d^4 - A*a^3*b^3*d^5)*x^4 - 8*((C*a^2*
b^4 + 5*A*a*b^5)*c^5 + (D*a^3*b^3 - B*a^2*b^4)*c^4*d + 2*(C*a^3*b^3 + 8*A*
a^2*b^4)*c^3*d^2 + 2*(4*D*a^4*b^2 - B*a^3*b^3)*c^2*d^3 - (5*C*a^4*b^2 - 17
*A*a^3*b^3)*c*d^4 + (D*a^5*b + 5*B*a^4*b^2)*d^5)*x^3 + 12*(D*a^3*b^3*c^5 -
 C*a^3*b^3*c^4*d + 5*B*a^4*b^2*c*d^4 - 5*A*a^4*b^2*d^5 + (5*D*a^4*b^2 +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(682) = 1364$.

Time = 0.35 (sec) , antiderivative size = 1645, normalized size of antiderivative = 2.35

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\left(a + bx^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/2*(D*c^3*d^4 - C*c^2*d^5 + B*c*d^6 - A*d^7)*log(b*x^2 + a)/(b^4*c^8 + 4*
a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8) - (D*c^3*d^
5 - C*c^2*d^6 + B*c*d^7 - A*d^8)*log(abs(d*x + c))/(b^4*c^8*d + 4*a*b^3*c^
6*d^3 + 6*a^2*b^2*c^4*d^5 + 4*a^3*b*c^2*d^7 + a^4*d^9) + 1/16*(C*a*b^4*c^7
 + 5*A*b^5*c^7 - D*a^2*b^3*c^6*d - B*a*b^4*c^6*d + 5*C*a^2*b^3*c^5*d^2 + 2
1*A*a*b^4*c^5*d^2 - 9*D*a^3*b^2*c^4*d^3 - 5*B*a^2*b^3*c^4*d^3 + 15*C*a^3*b
^2*c^3*d^4 + 35*A*a^2*b^3*c^3*d^4 + 9*D*a^4*b*c^2*d^5 - 15*B*a^3*b^2*c^2*d
^5 - 5*C*a^4*b*c*d^6 + 35*A*a^3*b^2*c*d^6 + D*a^5*d^7 + 5*B*a^4*b*d^7)*arc
tan(b*x/sqrt(a*b))/((a^3*b^5*c^8 + 4*a^4*b^4*c^6*d^2 + 6*a^5*b^3*c^4*d^4 +
 4*a^6*b^2*c^2*d^6 + a^7*b*d^8)*sqrt(a*b)) - 1/48*(4*D*a^4*b^3*c^7 + 8*B*a
^3*b^4*c^7 - 4*C*a^4*b^3*c^6*d - 8*A*a^3*b^4*c^6*d + 24*D*a^5*b^2*c^5*d^2
+ 36*B*a^4*b^3*c^5*d^2 - 24*C*a^5*b^2*c^4*d^3 - 36*A*a^4*b^3*c^4*d^3 + 12*
D*a^6*b*c^3*d^4 + 72*B*a^5*b^2*c^3*d^4 - 12*C*a^6*b*c^2*d^5 - 72*A*a^5*b^2
*c^2*d^5 - 8*D*a^7*c*d^6 + 44*B*a^6*b*c*d^6 + 8*C*a^7*d^7 - 44*A*a^6*b*d^7
 - 3*(C*a*b^6*c^7 + 5*A*b^7*c^7 - D*a^2*b^5*c^6*d - B*a*b^6*c^6*d + 5*C*a^
2*b^5*c^5*d^2 + 21*A*a*b^6*c^5*d^2 + 7*D*a^3*b^4*c^4*d^3 - 5*B*a^2*b^5*c^4
*d^3 - C*a^3*b^4*c^3*d^4 + 35*A*a^2*b^5*c^3*d^4 + 9*D*a^4*b^3*c^2*d^5 + B*
a^3*b^4*c^2*d^5 - 5*C*a^4*b^3*c*d^6 + 19*A*a^3*b^4*c*d^6 + D*a^5*b^2*d^7 +
 5*B*a^4*b^3*d^7)*x^5 + 24*(D*a^3*b^4*c^5*d^2 - C*a^3*b^4*c^4*d^3 + D*a^4*
b^3*c^3*d^4 + B*a^3*b^4*c^3*d^4 - C*a^4*b^3*c^2*d^5 - A*a^3*b^4*c^2*d^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\,(a + bx^2)^4}\,dx = \int \frac{A + B\,x + C\,x^2 + x^3\,D}{(b\,x^2 + a)^4\,(c + d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^4*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^4*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^4}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(dx+c)\left(b\,x^2+a\right)^4}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^4,x)
```

3.50 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} \, dx = \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2(1+x^2)} - \frac{3\arctan(x)}{2} - \frac{1}{2}\log(1+x^2)$$

output
```
3/2*x+1/2*x^2-x^3/(2*x^2+2)-3/2*arctan(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} \, dx = \frac{1}{2}\left(x\left(2+x+\frac{1}{1+x^2}\right) - 3\arctan(x) - \log(1+x^2)\right)$$

input
```
Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
(x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2335, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(x^2+x+1)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{1}{2}\int -\frac{x^2(2x+3)}{x^2+1}dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{x^2(2x+3)}{x^2+1}dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{2}\int \left(2x-\frac{2x+3}{x^2+1}+3\right)dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-3\arctan(x)+x^2-\log\left(x^2+1\right)+3x\right)-\frac{x^3}{2(x^2+1)}$$

input
```
Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
-1/2*x^3/(1 + x^2) + (3*x + x^2 - 3*ArcTan[x] - Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	s
default	$\frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	3
risch	$\frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	3
meijerg	$\frac{x^2(3x^2+6)}{6x^2+6}-\frac{\ln(x^2+1)}{2}+\frac{x(10x^2+15)}{10x^2+10}-\frac{3\arctan(x)}{2}-\frac{x^2}{2(x^2+1)}$	6
parallelrisch	$\frac{3i\ln(x-i)x^2-3i\ln(x+i)x^2+2x^4-2\ln(x-i)x^2-2\ln(x+i)x^2+4x^3-2+3i\ln(x-i)-3i\ln(x+i)-2\ln(x-i)-2\ln(x+i)+6x}{4x^2+4}$	9

input
```
int(x^3*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x+1/2*x/(x^2+1)-1/2*ln(x^2+1)-3/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{x^4+2\,x^3+x^2-3\,(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)+3\,x}{2\,(x^2+1)}$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) +
3*x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{\log(x^2+1)}{2}-\frac{3\,\mathrm{atan}(x)}{2}$$

input
```
integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\,x^2+x+\frac{x}{2\,(x^2+1)}-\frac{3}{2}\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\,x^2 + x + \frac{x}{2\,(x^2+1)} - \frac{3}{2}\,\arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = x - \frac{\ln\left(x^2+1\right)}{2} - \frac{3\,\mathrm{atan}(x)}{2} + \frac{x}{2\,\left(x^2+1\right)} + \frac{x^2}{2}$$

input
```
int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{-3atan(x)\,x^2 - 3atan(x) - \log(x^2+1)\,x^2 - \log(x^2+1) + x^4 + 2x^3 + x^2 + 3x}{2x^2+2}$$

input
```
int(x^3*(x^2+x+1)/(x^2+1)^2,x)
```

output
```
( - 3*atan(x)*x**2 - 3*atan(x) - log(x**2 + 1)*x**2 - log(x**2 + 1) + x**4
 + 2*x**3 + x**2 + 3*x)/(2*(x**2 + 1))
```

3.51 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2(1+x^2)} - \arctan(x) + \frac{1}{2}\log(1+x^2)$$

output
```
x+1/(2*x^2+2)-arctan(x)+1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2(1+x^2)} - \arctan(x) + \frac{1}{2}\log(1+x^2)$$

input
```
Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(x^2+x+1\right)}{\left(x^2+1\right)^2} \, dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{1}{2} \int -\frac{2 x (x+1)}{x^2+1}dx-\frac{x^2}{2 \left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x (x+1)}{x^2+1}dx-\frac{x^2}{2 \left(x^2+1\right)}$$

$$\Big\downarrow \text{523}$$

$$\int \left(1-\frac{1-x}{x^2+1}\right)dx-\frac{x^2}{2 \left(x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)-\frac{x^2}{2 \left(x^2+1\right)}+\frac{1}{2} \log \left(x^2+1\right)+x$$

input
```
Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$x+\frac{1}{2x^2+2}-\arctan\left(x\right)+\frac{\ln\left(x^2+1\right)}{2}$	24
risch	$x+\frac{1}{2x^2+2}-\arctan\left(x\right)+\frac{\ln\left(x^2+1\right)}{2}$	24
meijerg	$\frac{x\left(10x^2+15\right)}{10x^2+10}-\arctan\left(x\right)-\frac{x^2}{2\left(x^2+1\right)}+\frac{\ln\left(x^2+1\right)}{2}-\frac{x}{2\left(x^2+1\right)}$	53
parallelrisch	$\frac{i\ln\left(x-i\right)x^2-i\ln\left(x+i\right)x^2+\ln\left(x-i\right)x^2+\ln\left(x+i\right)x^2+2x^3+1+i\ln\left(x-i\right)-i\ln\left(x+i\right)+\ln\left(x-i\right)+\ln\left(x+i\right)+2x}{2x^2+2}$	86

input
```
int(x^2*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{2\,x^3-2\,(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)+2\,x+1}{2\,(x^2+1)}$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^
2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x+\frac{\log(x^2+1)}{2}-\operatorname{atan}(x)+\frac{1}{2x^2+2}$$

input
```
integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x+\frac{1}{2\,(x^2+1)}-\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2\,(x^2+1)} - \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{\ln\left(x^2+1\right)}{2} - \mathrm{atan}(x) + \frac{1}{2\,(x^2+1)}$$

input
```
int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{-2atan(x)\,x^2 - 2atan(x) + \log(x^2+1)\,x^2 + \log(x^2+1) + 2x^3 - x^2 + 2x}{2x^2+2}$$

input
```
int(x^2*(x^2+x+1)/(x^2+1)^2,x)
```

output
```
( - 2*atan(x)*x**2 - 2*atan(x) + log(x**2 + 1)*x**2 + log(x**2 + 1) + 2*x*
*3 - x**2 + 2*x)/(2*(x**2 + 1))
```

3.52 $\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x^2)$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx = \frac{1}{2}\left(-\frac{x}{1+x^2} + \arctan(x) + \log(1+x^2)\right)$$

input
```
Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
(-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2335, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+x+1)}{(x^2+1)^2}dx$$

$$\Big\downarrow \; 2335$$

$$-\frac{1}{2}\int -\frac{2x+1}{x^2+1}dx-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+1}dx-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \; 452$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx+2\int \frac{x}{x^2+1}dx\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(2\int \frac{x}{x^2+1}dx+\arctan(x)\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \; 240$$

$$\frac{1}{2}\left(\arctan(x)+\log\left(x^2+1\right)\right)-\frac{x}{2(x^2+1)}$$

input
```
Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + (ArcTan[x] + Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2(x^{2}+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^{2}+1)}{2}$	24
meijerg	$-\frac{x}{2(x^{2}+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^{2}+1)}{2}$	24
risch	$-\frac{x}{2(x^{2}+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^{2}+1)}{2}$	24
parallelrisch	$-\frac{i\ln(x-i)x^{2}-i\ln(x+i)x^{2}-2\ln(x-i)x^{2}-2\ln(x+i)x^{2}+i\ln(x-i)-i\ln(x+i)-2\ln(x-i)-2\ln(x+i)+2x}{4(x^{2}+1)}$	86

input
```
int(x*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-x}{2(x^2+1)}$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,\left(x^2+1\right)}$$

input
```
int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) + \log(x^2+1)\,x^2 + \log(x^2+1) - x}{2x^2+2}$$

input
```
int(x*(x^2+x+1)/(x^2+1)^2,x)
```

output
```
(atan(x)*x**2 + atan(x) + log(x**2 + 1)*x**2 + log(x**2 + 1) - x)/(2*(x**2
 + 1))
```

3.53 $\int \frac{1+x+x^2}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1+x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x)$$

output
```
-1/2/(x^2+1)+arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x)$$

input
```
Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{2345}$$
$$-\frac{1}{2}\int -\frac{2}{x^2+1}dx-\frac{1}{2(x^2+1)}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{x^2+1}dx-\frac{1}{2(x^2+1)}$$
$$\Big\downarrow \text{216}$$
$$\arctan(x)-\frac{1}{2(x^2+1)}$$

input
```
Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2(x^2+1)}+\arctan(x)$	13
risch	$-\frac{1}{2(x^2+1)}+\arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)}+\arctan(x)+\frac{x^2}{2x^2+2}+\frac{x}{2x^2+2}$	37
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+1+i\ln(x-i)-i\ln(x+i)}{2(x^2+1)}$	50

input
```
int((x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \frac{2\,(x^2+1)\arctan(x)-1}{2\,(x^2+1)}$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{2x^2+2}$$

input
```
integrate((x**2+x+1)/(x**2+1)**2,x)
```

output
```
atan(x) - 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan(x)$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2/(x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan(x)$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2/(x^2 + 1) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{2\,(x^2+1)}$$

input
```
int((x + x^2 + 1)/(x^2 + 1)^2,x)
```

output
```
atan(x) - 1/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \frac{2atan(x)\,x^2+2atan(x)+x^2}{2x^2+2}$$

input
```
int((x^2+x+1)/(x^2+1)^2,x)
```

output
```
(2*atan(x)*x**2 + 2*atan(x) + x**2)/(2*(x**2 + 1))
```

3.54 $\int \frac{1+x+x^2}{x(1+x^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1+x+x^2}{x(1+x^2)^2}\,dx = \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
x/(2*x^2+2)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x) + 2\log(x) - \log\left(1+x^2\right)\right)$$

input
```
Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

output
```
(x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2336, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\int -\frac{x+2}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{x+2}{x\left(x^2+1\right)}dx+\frac{x}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{2}\int \left(\frac{1-2x}{x^2+1}+\frac{2}{x}\right)dx+\frac{x}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\arctan(x)-\log\left(x^2+1\right)+2\log(x)\right)+\frac{x}{2\left(x^2+1\right)}$$

input
```
Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

output
```
x/(2*(1 + x^2)) + (ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{2}+\ln(x)$
parallelrisch	$\frac{-i\ln(x-i)x^2+i\ln(x+i)x^2+4\ln(x)x^2-2\ln(x-i)x^2-2\ln(x+i)x^2-i\ln(x-i)+i\ln(x+i)+4\ln(x)-2\ln(x-i)-2\ln(x+i)+2x}{4x^2+4}$

input
```
int((x^2+x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)-1/2*ln(x^2+1)+1/2*arctan(x)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{\left(x^2+1\right)\arctan\left(x\right)-\left(x^2+1\right)\log\left(x^2+1\right)+2\left(x^2+1\right)\log\left(x\right)+x}{2\left(x^2+1\right)}$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x
)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{x}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

input
```
integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

output
```
x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right) + \frac{x}{2\,\left(x^2+1\right)} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) - \log(x^2+1)\,x^2 - \log(x^2+1) + 2\log(x)\,x^2 + 2\log(x) + x}{2x^2+2}$$

input
```
int((x^2+x+1)/x/(x^2+1)^2,x)
```

output
```
(atan(x)*x**2 + atan(x) - log(x**2 + 1)*x**2 - log(x**2 + 1) + 2*log(x)*x*
*2 + 2*log(x) + x)/(2*(x**2 + 1))
```

3.55 $\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-1/x+1/(2*x^2+2)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

output
```
-x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2336, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^2 \left(x^2+1\right)^2} \, dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{1}{2 \left(x^2+1\right)}-\frac{1}{2} \int -\frac{2(x+1)}{x^2 \left(x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+1}{x^2 \left(x^2+1\right)}dx+\frac{1}{2 \left(x^2+1\right)}$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{-x-1}{x^2+1}+\frac{1}{x}+\frac{1}{x^2}\right)dx+\frac{1}{2 \left(x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)+\frac{1}{2 \left(x^2+1\right)}-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{x}+\log(x)$$

input
```
Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

output
```
-x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
default	$-\frac{1}{x}+\frac{1}{2x^2+2}-\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$
risch	$\frac{-x^2+\frac{1}{2}x-1}{x\left(x^2+1\right)}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(x\right)$
meijerg	$\frac{x}{2x^2+2}-\arctan\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)-\frac{3x^2+2}{x\left(2x^2+2\right)}$
parallelrisch	$\frac{i\ln(x-i)x^3-i\ln(x+i)x^3+2\ln(x)x^3-\ln(x-i)x^3-\ln(x+i)x^3-2+i\ln(x-i)x-i\ln(x+i)x+2\ln(x)x-\ln(x-i)x-\ln(x+i)x-2x^2}{2(x^2+1)x}$

input
```
int((x^2+x+1)/x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x^2+1)-1/2*ln(x^2+1)-arctan(x)-1/x+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx = -\frac{2\,x^2+2\,(x^3+x)\arctan(x)+(x^3+x)\log(x^2+1)-2\,(x^3+x)\log(x)-x+2}{2\,(x^3+x)}$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)
*log(x) - x + 2)/(x^3 + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx = \log(x) - \frac{\log(x^2+1)}{2} - \operatorname{atan}(x) + \frac{-2x^2+x-2}{2x^3+2x}$$

input
```
integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx = -\frac{2\,x^2-x+2}{2\,(x^3+x)} - \arctan(x) - \frac{1}{2}\log(x^2+1) + \log(x)$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = -\frac{2\,x^2-x+2}{2\,(x^3+x)} - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = \ln\left(x\right) - \frac{x^2-\frac{x}{2}+1}{x^3+x} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2
+ 1)/(x + x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx$$
$$= \frac{-2atan(x)\,x^3 - 2atan(x)\,x - \log(x^2+1)\,x^3 - \log(x^2+1)\,x + 2\log(x)\,x^3 + 2\log(x)\,x - x^3 - 2x^2 - 2}{2x\left(x^2+1\right)}$$

input
```
int((x^2+x+1)/x^2/(x^2+1)^2,x)
```

output
```
( - 2*atan(x)*x**3 - 2*atan(x)*x - log(x**2 + 1)*x**3 - log(x**2 + 1)*x +
2*log(x)*x**3 + 2*log(x)*x - x**3 - 2*x**2 - 2)/(2*x*(x**2 + 1))
```

3.56 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=-\frac{1}{2x^2}-\frac{3}{2x}+\frac{1}{2x\left(1+x^2\right)}-\frac{3\arctan(x)}{2}-\log(x)+\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-1/2/x^2-3/2/x+1/2/x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=\frac{1}{2}\left(-\frac{1}{x^2}-\frac{2}{x}-\frac{x}{1+x^2}-3\arctan(x)-2\log(x)+\log\left(1+x^2\right)\right)$$

input
```
Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

output
```
(-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^3 \left(x^2+1\right)^2} \, dx$$
$$\Big\downarrow \text{ 2336}$$
$$-\frac{1}{2} \int -\frac{-x^3+2x+2}{x^3 \left(x^2+1\right)}dx-\frac{x}{2 \left(x^2+1\right)}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{1}{2} \int \frac{-x^3+2x+2}{x^3 \left(x^2+1\right)}dx-\frac{x}{2 \left(x^2+1\right)}$$
$$\Big\downarrow \text{ 2333}$$
$$\frac{1}{2} \int \left(\frac{2x-3}{x^2+1}-\frac{2}{x}+\frac{2}{x^2}+\frac{2}{x^3}\right)dx-\frac{x}{2 \left(x^2+1\right)}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(-3 \arctan (x)-\frac{1}{x^2}+\log \left(x^2+1\right)-\frac{2}{x}-2 \log (x)\right)-\frac{x}{2 \left(x^2+1\right)}$$

input
```
Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

output
```
-1/2*x/(1 + x^2) + (-x^(-2) - 2/x - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result
default	$-\frac{x}{2(x^2+1)}+\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}-\frac{1}{2x^2}-\frac{1}{x}-\ln(x)$
risch	$\frac{-\frac{3}{2}x^3-\frac{1}{2}x^2-x-\frac{1}{2}}{x^2(x^2+1)}-\ln(x)+\frac{\ln(9x^2+9)}{2}-\frac{3\arctan(x)}{2}$
meijerg	$-\frac{x^2}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\ln(x)-\frac{3x^2+2}{x(2x^2+2)}-\frac{3\arctan(x)}{2}+\frac{3x^2}{2(3x^2+3)}-\frac{1}{2x^2}$
parallelrisch	$-\frac{-3i\ln(x-i)x^4+3i\ln(x+i)x^2+4\ln(x)x^4-2\ln(x-i)x^4-2\ln(x+i)x^4+2-3i\ln(x-i)x^2+3i\ln(x+i)x^4+4\ln(x)x^2-2\ln(x-i)}{4x^2(x^2+1)}$

input
```
int((x^2+x+1)/x^3/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*ln(x^2+1)-3/2*arctan(x)-1/2/x^2-1/x-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\frac{3\,x^3+x^2+3\,(x^4+x^2)\arctan(x)-(x^4+x^2)\log(x^2+1)+2\,(x^4+x^2)\log(x)+2\,x+1}{2\,(x^4+x^2)}$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2
*(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\log(x)+\frac{\log(x^2+1)}{2}-\frac{3\,\mathrm{atan}(x)}{2}+\frac{-3x^3-x^2-2x-1}{2x^4+2x^2}$$

input
```
integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

output
```
-log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x*
*4 + 2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\frac{3\,x^3+x^2+2\,x+1}{2\,(x^4+x^2)}-\frac{3}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)-\log(x)$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1
) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2}{x^3 (1+x^2)^2} \, dx = -\frac{3 \, x^3 + x^2 + 2 \, x + 1}{2 \, (x^2+1) x^2} - \frac{3}{2} \, \arctan\left(x\right) + \frac{1}{2} \, \log\left(x^2+1\right) - \log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
 + 1) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 16.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x^3 (1+x^2)^2} \, dx = -\ln\left(x\right) - \frac{\frac{3\,x^3}{2} + \frac{x^2}{2} + x + \frac{1}{2}}{x^4 + x^2} + \ln\left(x - \mathrm{i}\right) \left(\frac{1}{2} + \frac{3}{4}\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right) \left(\frac{1}{2} - \frac{3}{4}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

output
```
log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2
+ (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1+x+x^2}{x^3 (1+x^2)^2} \, dx$$
$$= \frac{-3atan(x)\, x^4 - 3atan(x)\, x^2 + \log(x^2+1)\, x^4 + \log(x^2+1)\, x^2 - 2\log(x)\, x^4 - 2\log(x)\, x^2 + x^4 - 3x^3 -}{2x^2\,(x^2+1)}$$

input
```
int((x^2+x+1)/x^3/(x^2+1)^2,x)
```

output
```
( - 3*atan(x)*x**4 - 3*atan(x)*x**2 + log(x**2 + 1)*x**4 + log(x**2 + 1)*x
**2 - 2*log(x)*x**4 - 2*log(x)*x**2 + x**4 - 3*x**3 - 2*x - 1)/(2*x**2*(x*
*2 + 1))
```

3.57 $\int \frac{1+2x+x^2}{(1+x^2)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1+2x+x^2}{(1+x^2)^2} dx = -\frac{1}{1+x^2} + \arctan(x)$$

output
```
-1/(x^2+1)+arctan(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2} dx = -\frac{1}{1+x^2} + \arctan(x)$$

input
```
Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

output
```
-(1 + x^2)^(-1) + ArcTan[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2006, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x+1}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{2006}$$
$$\int \frac{(x+1)^2}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{487}$$
$$\int \frac{1}{x^2+1}dx-\frac{(1-x)(x+1)}{2(x^2+1)}$$
$$\Big\downarrow \text{216}$$
$$\arctan(x)-\frac{(1-x)(x+1)}{2(x^2+1)}$$

input
```
Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*((1 - x)*(1 + x))/(1 + x^2) + ArcTan[x]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{x^2+1}+\arctan(x)$	13
risch	$-\frac{1}{x^2+1}+\arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)}+\arctan(x)+\frac{x^2}{x^2+1}+\frac{x}{2x^2+2}$	36
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+2+i\ln(x-i)-i\ln(x+i)}{2(x^2+1)}$	50

input
```
int((x^2+2*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = \frac{(x^2+1)\arctan(x)-1}{x^2+1}$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = \operatorname{atan}(x) - \frac{1}{x^2+1}$$

input
```
integrate((x**2+2*x+1)/(x**2+1)**2,x)
```

output
```
atan(x) - 1/(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = -\frac{1}{x^2+1} + \arctan(x)$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/(x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{x^2+1} + \arctan(x)$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/(x^2 + 1) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \mathrm{atan}(x) - \frac{1}{x^2+1}$$

input
```
int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
```

output
```
atan(x) - 1/(x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) + x^2}{x^2+1}$$

input
```
int((x^2+2*x+1)/(x^2+1)^2,x)
```

output
```
(atan(x)*x**2 + atan(x) + x**2)/(x**2 + 1)
```

3.58 $\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{6}{4+x^2} - \frac{5x}{4(4+x^2)} + \frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

output
```
-6/(x^2+4)-5*x/(4*x^2+16)+7/8*arctan(1/2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{-24-5x}{4(4+x^2)} + \frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

input
```
Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

output
```
(-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+12x+2}{(x^2+4)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{8}\int -\frac{14}{x^2+4}dx-\frac{5x+24}{4(x^2+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}\int \frac{1}{x^2+4}dx-\frac{5x+24}{4(x^2+4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{8}\arctan\left(\frac{x}{2}\right)-\frac{5x+24}{4(x^2+4)}$$

input
```
Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

output
```
-1/4*(24 + 5*x)/(4 + x^2) + (7*ArcTan[x/2])/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
risch	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
meijerg	$\frac{x}{4x^2+16}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}-\frac{3x}{8\left(1+\frac{x^2}{4}\right)}+\frac{3x^2}{8\left(1+\frac{x^2}{4}\right)}$	46
parallelrisch	$-\frac{7i\ln(x-2i)x^2-7i\ln(x+2i)x^2+28i\ln(x-2i)-28i\ln(x+2i)-24x^2+20x}{16(x^2+4)}$	57

input
```
int((3*x^2+12*x+2)/(x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx=\frac{7\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)-10\,x-48}{8\,(x^2+4)}$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
```

output
```
1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{-5x-24}{4x^2+16} + \frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

input
```
integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

output
```
(-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

output
```
-1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
```

output
```
-1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} - \frac{\frac{5\,x}{4}+6}{x^2+4}$$

input
```
int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
```

output
```
(7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{7atan\left(\frac{x}{2}\right)x^2+28atan\left(\frac{x}{2}\right)+12x^2-10x}{8x^2+32}$$

input
```
int((3*x^2+12*x+2)/(x^2+4)^2,x)
```

output
```
(7*atan(x/2)*x**2 + 28*atan(x/2) + 12*x**2 - 10*x)/(8*(x**2 + 4))
```

3.59 $\int (c+dx)^3\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 502

$$
\begin{aligned}
&\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{(64Ab^3c^3-a(16b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-8abd(3cCd+Bd^2+3c^2D)))\,x\sqrt{a+bx^2}}{128b^3} \\
&\quad+\frac{(b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D))\,(a+bx^2)^{3/2}}{3b^3} \\
&\quad+\frac{(16b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-8abd(3cCd+Bd^2+3c^2D))\,x(a+bx^2)^{3/2}}{64b^3} \\
&\quad-\frac{d(5ad^2D-8b(3cCd+Bd^2+3c^2D))\,x^3(a+bx^2)^{3/2}}{48b^2}+\frac{d^3Dx^5(a+bx^2)^{3/2}}{8b} \\
&\quad-\frac{(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\,(a+bx^2)^{5/2}}{5b^3} \\
&\quad+\frac{d^2(Cd+3cD)\,(a+bx^2)^{7/2}}{7b^3} \\
&\quad+\frac{a(64Ab^3c^3-a(16b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-8abd(3cCd+Bd^2+3c^2D)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}
\end{aligned}
$$

output
```
1/128*(64*A*b^3*c^3-a*(16*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-8*a*b*
d*(B*d^2+3*C*c*d+3*D*c^2)))*x*(b*x^2+a)^(1/2)/b^3+1/3*(b^2*c^2*(3*A*d+B*c)
+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(3/2
)/b^3+1/64*(16*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-8*a*b*d*(B*d^2+3*
C*c*d+3*D*c^2))*x*(b*x^2+a)^(3/2)/b^3-1/48*d*(5*a*d^2*D-8*b*(B*d^2+3*C*c*d
+3*D*c^2))*x^3*(b*x^2+a)^(3/2)/b^2+1/8*d^3*D*x^5*(b*x^2+a)^(3/2)/b-1/5*(2*
a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(5/2)/b^3
+1/7*d^2*(C*d+3*D*c)*(b*x^2+a)^(7/2)/b^3+1/128*a*(64*A*b^3*c^3-a*(16*b^2*c
*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-8*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.96

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{\sqrt{b}\sqrt{a+bx^2}(a^3d^2(1024Cd+3072cD+525dDx)+8ab^2(14Ad(120c^2+45cdx+8d^2x^2)+14B(40c^3+4}{}$$

input
```
Integrate[(c + d*x)^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^3*d^2*(1024*C*d + 3072*c*D + 525*d*D*x) + 8*a*
b^2*(14*A*d*(120*c^2 + 45*c*d*x + 8*d^2*x^2) + 14*B*(40*c^3 + 45*c^2*d*x +
 24*c*d^2*x^2 + 5*d^3*x^3) + x*(42*c^2*d*x*(8*C + 5*D*x) + 14*c^3*(15*C +
8*D*x) + 6*c*d^2*x^2*(35*C + 24*D*x) + d^3*x^3*(48*C + 35*D*x))) - 2*a^2*b
*(896*c^3*D + 84*c^2*d*(32*C + 15*D*x) + 12*c*d^2*(224*B + x*(105*C + 64*D
*x)) + d^3*(896*A + x*(420*B + 256*C*x + 175*D*x^2))) + 16*b^3*x*(42*A*(10
*c^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + x*(14*B*(20*c^3 + 45*c^2*d
*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 3*x*(14*c^3*(5*C + 4*D*x) + 28*c^2*d*x*(
6*C + 5*D*x) + 20*c*d^2*x^2*(7*C + 6*D*x) + 5*d^3*x^3*(8*C + 7*D*x))))) +
105*a*(16*A*b^2*c*(-4*b*c^2 + 3*a*d^2) + a*(16*b^2*c^2*(c*C + 3*B*d) + 5*a
^2*d^3*D - 8*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*Log[-(Sqrt[b]*x) + Sqrt[a
 + b*x^2]])/(13440*b^(7/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2185, 2185, 27, 687, 27, 687, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int (c+dx)^3\sqrt{bx^2+a}\left(b(8Cd-11cD)x^2d^2+(8Abd-5acD)d^2+\left(-3bDc^2+8bBd^2-5ad^2D\right)xd\right)dx}{8bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int bd^3(c+dx)^3\left(d(56Abd-32aCd+9acD)-\left(35aDd^2+4b\left(-3Dc^2+6Cdc-14Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx}{7bd^2}+\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(8Cd-11}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{7}d\int (c+dx)^3\left(d(56Abd-32aCd+9acD)-\left(35aDd^2+4b\left(-3Dc^2+6Cdc-14Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx+\frac{1}{7}d(a-}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{7}d\left(\frac{\int 3(c+dx)^2\left(d\left(112Acdb^2+a\left(35ad^2D-b\left(-6Dc^2+40Cdc+56Bd^2\right)\right)\right)-b\left(a(64Cd+17cD)d^2+4b\left(-3Dc^3+6Cdc^2-14Bd^2c-28Ad^3\right)\right)x\right)\sqrt{bx^2}}{6b}\right.}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{7}d\left(\frac{\frac{\int (c+dx)^2\left(d\left(112Acdb^2+a\left(35ad^2D-b\left(-6Dc^2+40Cdc+56Bd^2\right)\right)\right)-b\left(a(64Cd+17cD)d^2+4b\left(-3Dc^3+6Cdc^2-14Bd^2c-28Ad^3\right)\right)x\right)\sqrt{bx^2+\cdots}}{2b}}{8bd^3}\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{7}d\left(\frac{\frac{\int b(c+dx)\left(d\left(112Abd\left(5bc^2-2ad^2\right)+a\left(ad^2(128Cd+209cD)-2bc\left(-3Dc^2+76Cdc+196Bd^2\right)\right)\right)+\left(175a^2Dd^4-4ab\left(Dc^2+82Cdc+70Bd^2\right)d^2-8b^2c\left(-3Dc^3+\cdots\right.\right.\right.}{5b}}{2b}\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{7}d\left(\frac{\frac{1}{5}\int (c+dx)\left(d\left(112Abd\left(5bc^2-2ad^2\right)+a\left(ad^2(128Cd+209cD)-2bc\left(-3Dc^2+76Cdc+196Bd^2\right)\right)\right)+\left(175a^2Dd^4-4ab\left(Dc^2+82Cdc+70Bd^2\right)d^2\cdots\right.\right.}{2\cdots}\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{1}{7}d\left(\frac{\frac{1}{5}\left(\frac{35d^2\left(16Ab^2c\left(4bc^2-3ad^2\right)-a\left(5a^2d^3D-8abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(175a^2d^4D-4abd^2\left(70Bd^2+\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{7}d\left(\frac{\frac{1}{5}\left(\frac{35d^2\left(16Ab^2c\left(4bc^2-3ad^2\right)-a\left(5a^2d^3D-8abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(175a^2d^{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{7}d\left(\frac{\frac{1}{5}\left(\frac{35d^2\left(16Ab^2c\left(4bc^2-3ad^2\right)-a\left(5a^2d^3D-8abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/}}{}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{7}d\left(\frac{\frac{1}{5}\left(\frac{35d^2\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(16Ab^2c\left(4bc^2-3ad^2\right)-a\left(5a^2d^3D-8abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}}{}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^2}$$

input
```
Int[(c + d*x)^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^5*(a + b*x^2)^(3/2))/(8*b*d^2) + ((d*(8*C*d - 11*c*D)*(c + d*
x)^4*(a + b*x^2)^(3/2))/7 + (d*(-1/6*((24*b*c*C*d - 56*b*B*d^2 - 12*b*c^2*
D + 35*a*d^2*D)*(c + d*x)^3*(a + b*x^2)^(3/2))/b + (-1/5*((a*d^2*(64*C*d +
 17*c*D) + 4*b*(6*c^2*C*d - 14*B*c*d^2 - 28*A*d^3 - 3*c^3*D))*(c + d*x)^2*
(a + b*x^2)^(3/2)) + ((2*(64*a^2*d^4*(C*d + 3*c*D) - 4*b^2*c^2*(6*c^2*C*d
- 14*B*c*d^2 - 168*A*d^3 - 3*c^3*D) - a*b*d^2*(240*c^2*C*d + 336*B*c*d^2 +
 112*A*d^3 - c^3*D))*(a + b*x^2)^(3/2))/(3*b) + (d*(175*a^2*d^4*D - 4*a*b*
d^2*(82*c*C*d + 70*B*d^2 + c^2*D) - 8*b^2*c*(6*c^2*C*d - 14*B*c*d^2 - 98*A
*d^3 - 3*c^3*D))*x*(a + b*x^2)^(3/2))/(4*b) + (35*d^2*(16*A*b^2*c*(4*b*c^2
 - 3*a*d^2) - a*(16*b^2*c^2*(c*C + 3*B*d) + 5*a^2*d^3*D - 8*a*b*d*(3*c*C*d
 + B*d^2 + 3*c^2*D)))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/5)/(2*b)))/7)/(8*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.96

method	result
default	$A\,c^{3}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{c^{2}(3Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}+d^{2}(Cd+3Dc)\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^{2}(\right.}{}\right.\right.$

input
```
int((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
1/3*c^2*(3*A*d+B*c)*(b*x^2+a)^(3/2)/b+d^2*(C*d+3*D*c)*(1/7*x^4*(b*x^2+a)^(
3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2)))+c*(
3*A*d^2+3*B*c*d+C*c^2)*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+d*(B*d^2+3*C*c*d+3*D*c^
2)*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(A*d^3+
3*B*c*d^2+3*C*c^2*d+D*c^3)*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)
^(3/2))+D*d^3*(1/8*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3/2)/
b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^
(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1219, normalized size of antiderivative = 2.43

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[-1/26880*(105*(16*(C*a^2*b^2 - 4*A*a*b^3)*c^3 - 24*(D*a^3*b - 2*B*a^2*b^2
)*c^2*d - 24*(C*a^3*b - 2*A*a^2*b^2)*c*d^2 + (5*D*a^4 - 8*B*a^3*b)*d^3)*sq
rt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1680*D*b^4*d^3*
x^7 + 1920*(3*D*b^4*c*d^2 + C*b^4*d^3)*x^6 + 280*(24*D*b^4*c^2*d + 24*C*b^
4*c*d^2 + (D*a*b^3 + 8*B*b^4)*d^3)*x^5 + 384*(7*D*b^4*c^3 + 21*C*b^4*c^2*d
 + 3*(D*a*b^3 + 7*B*b^4)*c*d^2 + (C*a*b^3 + 7*A*b^4)*d^3)*x^4 - 896*(2*D*a
^2*b^2 - 5*B*a*b^3)*c^3 - 2688*(2*C*a^2*b^2 - 5*A*a*b^3)*c^2*d + 768*(4*D*
a^3*b - 7*B*a^2*b^2)*c*d^2 + 256*(4*C*a^3*b - 7*A*a^2*b^2)*d^3 + 70*(48*C*
b^4*c^3 + 24*(D*a*b^3 + 6*B*b^4)*c^2*d + 24*(C*a*b^3 + 6*A*b^4)*c*d^2 - (5
*D*a^2*b^2 - 8*B*a*b^3)*d^3)*x^3 + 128*(7*(D*a*b^3 + 5*B*b^4)*c^3 + 21*(C*
a*b^3 + 5*A*b^4)*c^2*d - 3*(4*D*a^2*b^2 - 7*B*a*b^3)*c*d^2 - (4*C*a^2*b^2
- 7*A*a*b^3)*d^3)*x^2 + 105*(16*(C*a*b^3 + 4*A*b^4)*c^3 - 24*(D*a^2*b^2 -
2*B*a*b^3)*c^2*d - 24*(C*a^2*b^2 - 2*A*a*b^3)*c*d^2 + (5*D*a^3*b - 8*B*a^2
*b^2)*d^3)*x)*sqrt(b*x^2 + a))/b^4, 1/13440*(105*(16*(C*a^2*b^2 - 4*A*a*b^
3)*c^3 - 24*(D*a^3*b - 2*B*a^2*b^2)*c^2*d - 24*(C*a^3*b - 2*A*a^2*b^2)*c*d
^2 + (5*D*a^4 - 8*B*a^3*b)*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (1680*D*b^4*d^3*x^7 + 1920*(3*D*b^4*c*d^2 + C*b^4*d^3)*x^6 + 280*(24*D
*b^4*c^2*d + 24*C*b^4*c*d^2 + (D*a*b^3 + 8*B*b^4)*d^3)*x^5 + 384*(7*D*b^4*
c^3 + 21*C*b^4*c^2*d + 3*(D*a*b^3 + 7*B*b^4)*c*d^2 + (C*a*b^3 + 7*A*b^4)*d
^3)*x^4 - 896*(2*D*a^2*b^2 - 5*B*a*b^3)*c^3 - 2688*(2*C*a^2*b^2 - 5*A*a...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(508) = 1016$.

Time = 0.76 (sec) , antiderivative size = 1049, normalized size of antiderivative = 2.09

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*d**3*x**7/8 + x**6*(C*b*d**3 + 3*D*b*c*d**2
)/(7*b) + x**5*(B*b*d**3 + 3*C*b*c*d**2 + D*a*d**3/8 + 3*D*b*c**2*d)/(6*b)
 + x**4*(A*b*d**3 + 3*B*b*c*d**2 + C*a*d**3 + 3*C*b*c**2*d + 3*D*a*c*d**2
+ D*b*c**3 - 6*a*(C*b*d**3 + 3*D*b*c*d**2)/(7*b))/(5*b) + x**3*(3*A*b*c*d*
*2 + B*a*d**3 + 3*B*b*c**2*d + 3*C*a*c*d**2 + C*b*c**3 + 3*D*a*c**2*d - 5*
a*(B*b*d**3 + 3*C*b*c*d**2 + D*a*d**3/8 + 3*D*b*c**2*d)/(6*b))/(4*b) + x**
2*(A*a*d**3 + 3*A*b*c**2*d + 3*B*a*c*d**2 + B*b*c**3 + 3*C*a*c**2*d + D*a*
c**3 - 4*a*(A*b*d**3 + 3*B*b*c*d**2 + C*a*d**3 + 3*C*b*c**2*d + 3*D*a*c*d*
*2 + D*b*c**3 - 6*a*(C*b*d**3 + 3*D*b*c*d**2)/(7*b))/(5*b))/(3*b) + x*(3*A
*a*c*d**2 + A*b*c**3 + 3*B*a*c**2*d + C*a*c**3 - 3*a*(3*A*b*c*d**2 + B*a*d
**3 + 3*B*b*c**2*d + 3*C*a*c*d**2 + C*b*c**3 + 3*D*a*c**2*d - 5*a*(B*b*d**
3 + 3*C*b*c*d**2 + D*a*d**3/8 + 3*D*b*c**2*d)/(6*b))/(4*b))/(2*b) + (3*A*a
*c**2*d + B*a*c**3 - 2*a*(A*a*d**3 + 3*A*b*c**2*d + 3*B*a*c*d**2 + B*b*c**
3 + 3*C*a*c**2*d + D*a*c**3 - 4*a*(A*b*d**3 + 3*B*b*c*d**2 + C*a*d**3 + 3*
C*b*c**2*d + 3*D*a*c*d**2 + D*b*c**3 - 6*a*(C*b*d**3 + 3*D*b*c*d**2)/(7*b)
)/(5*b))/(3*b))/b) + (A*a*c**3 - a*(3*A*a*c*d**2 + A*b*c**3 + 3*B*a*c**2*d
 + C*a*c**3 - 3*a*(3*A*b*c*d**2 + B*a*d**3 + 3*B*b*c**2*d + 3*C*a*c*d**2 +
 C*b*c**3 + 3*D*a*c**2*d - 5*a*(B*b*d**3 + 3*C*b*c*d**2 + D*a*d**3/8 + 3*D
*b*c**2*d)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
+ 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)),...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.21

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/8*(b*x^2 + a)^(3/2)*D*d^3*x^5/b - 5/48*(b*x^2 + a)^(3/2)*D*a*d^3*x^3/b^2
 + 1/2*sqrt(b*x^2 + a)*A*c^3*x + 5/64*(b*x^2 + a)^(3/2)*D*a^2*d^3*x/b^3 -
5/128*sqrt(b*x^2 + a)*D*a^3*d^3*x/b^3 + 1/7*(3*D*c*d^2 + C*d^3)*(b*x^2 + a
)^(3/2)*x^4/b + 1/2*A*a*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*D*a^4*d
^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/3*(b*x^2 + a)^(3/2)*B*c^3/b + (b*x^2
 + a)^(3/2)*A*c^2*d/b + 1/6*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(3
/2)*x^3/b - 4/35*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 1/5*(D*
c^3 + 3*C*c^2*d + 3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(3/2)*x^2/b - 1/8*(3*D*c^
2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*(3*D*c^2*d + 3*C
*c*d^2 + B*d^3)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(C*c^3 + 3*B*c^2*d + 3*A*c
*d^2)*(b*x^2 + a)^(3/2)*x/b - 1/8*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*sqrt(b*x
^2 + a)*a*x/b + 1/16*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*a^3*arcsinh(b*x/sqrt(
a*b))/b^(5/2) - 1/8*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*a^2*arcsinh(b*x/sqrt(a
*b))/b^(3/2) + 8/105*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(3/2)*a^2/b^3 - 2/15*
(D*c^3 + 3*C*c^2*d + 3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.31

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{1}{13440}\sqrt{bx^2+a}\left(\left(\left(2\left(\left(\left(4\left(5\left(6\left(7\,Dd^3x+\frac{8\,(3\,Db^6cd^2+Cb^6d^3)}{b^6}\right)x+\frac{7\,(24\,Db^6c^2d+24\,Cb^6cd^2+Da\right.}{b^6}\right.\right.\right.\right.\right.\right.\right.$$

$$+\frac{(16\,Ca^2b^2c^3-64\,Aab^3c^3-24\,Da^3bc^2d+48\,Ba^2b^2c^2d-24\,Ca^3bcd^2+48\,Aa^2b^2cd^2+5\,Da^4d^3-8\,Ba^3}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*D*d^3*x + 8*(3*D*b^6*c*d^2 + C*b^
6*d^3)/b^6)*x + 7*(24*D*b^6*c^2*d + 24*C*b^6*c*d^2 + D*a*b^5*d^3 + 8*B*b^6
*d^3)/b^6)*x + 48*(7*D*b^6*c^3 + 21*C*b^6*c^2*d + 3*D*a*b^5*c*d^2 + 21*B*b
^6*c*d^2 + C*a*b^5*d^3 + 7*A*b^6*d^3)/b^6)*x + 35*(48*C*b^6*c^3 + 24*D*a*b
^5*c^2*d + 144*B*b^6*c^2*d + 24*C*a*b^5*c*d^2 + 144*A*b^6*c*d^2 - 5*D*a^2*
b^4*d^3 + 8*B*a*b^5*d^3)/b^6)*x + 64*(7*D*a*b^5*c^3 + 35*B*b^6*c^3 + 21*C*
a*b^5*c^2*d + 105*A*b^6*c^2*d - 12*D*a^2*b^4*c*d^2 + 21*B*a*b^5*c*d^2 - 4*
C*a^2*b^4*d^3 + 7*A*a*b^5*d^3)/b^6)*x + 105*(16*C*a*b^5*c^3 + 64*A*b^6*c^3
 - 24*D*a^2*b^4*c^2*d + 48*B*a*b^5*c^2*d - 24*C*a^2*b^4*c*d^2 + 48*A*a*b^5
*c*d^2 + 5*D*a^3*b^3*d^3 - 8*B*a^2*b^4*d^3)/b^6)*x - 128*(14*D*a^2*b^4*c^3
 - 35*B*a*b^5*c^3 + 42*C*a^2*b^4*c^2*d - 105*A*a*b^5*c^2*d - 24*D*a^3*b^3*
c*d^2 + 42*B*a^2*b^4*c*d^2 - 8*C*a^3*b^3*d^3 + 14*A*a^2*b^4*d^3)/b^6) + 1/
128*(16*C*a^2*b^2*c^3 - 64*A*a*b^3*c^3 - 24*D*a^3*b*c^2*d + 48*B*a^2*b^2*c
^2*d - 24*C*a^3*b*c*d^2 + 48*A*a^2*b^2*c*d^2 + 5*D*a^4*d^3 - 8*B*a^3*b*d^3
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (dx+c)^3\sqrt{b\,x^2+a}\,\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.60 $\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 349

$$\begin{aligned}
&\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx\\
&=\frac{(2Ab(4bc^2-ad^2)-a(2bc(cC+2Bd)-ad(Cd+2cD)))\,x\sqrt{a+bx^2}}{16b^2}\\
&\quad+\frac{(b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D))\,(a+bx^2)^{3/2}}{3b^3}\\
&\quad+\frac{(2b(c^2C+2Bcd+Ad^2)-ad(Cd+2cD))\,x(a+bx^2)^{3/2}}{8b^2}\\
&\quad+\frac{d(Cd+2cD)x^3(a+bx^2)^{3/2}}{6b}\\
&\quad-\frac{(2ad^2D-b(2cCd+Bd^2+c^2D))\,(a+bx^2)^{5/2}}{5b^3}+\frac{d^2D(a+bx^2)^{7/2}}{7b^3}\\
&\quad+\frac{a(2Ab(4bc^2-ad^2)-a(2bc(cC+2Bd)-ad(Cd+2cD)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}
\end{aligned}$$

output
```
1/16*(2*A*b*(-a*d^2+4*b*c^2)-a*(2*b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*x*(b*x
^2+a)^(1/2)/b^2+1/3*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2)
)*(b*x^2+a)^(3/2)/b^3+1/8*(2*b*(A*d^2+2*B*c*d+C*c^2)-a*d*(C*d+2*D*c))*x*(b
*x^2+a)^(3/2)/b^2+1/6*d*(C*d+2*D*c)*x^3*(b*x^2+a)^(3/2)/b-1/5*(2*a*d^2*D-b
*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(5/2)/b^3+1/7*d^2*D*(b*x^2+a)^(7/2)/b^3+
1/16*a*(2*A*b*(-a*d^2+4*b*c^2)-a*(2*b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.95

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{\sqrt{a+bx^2}(128a^3d^2D+2ab^2(35Ad(16c+3dx)+14B(20c^2+15cdx+4d^2x^2)+x(14cdx(8C+5Dx)+7}{}$$

input
```
Integrate[(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^3*d^2*D + 2*a*b^2*(35*A*d*(16*c + 3*d*x) + 14*B*(2
0*c^2 + 15*c*d*x + 4*d^2*x^2) + x*(14*c*d*x*(8*C + 5*D*x) + 7*c^2*(15*C +
8*D*x) + d^2*x^2*(35*C + 24*D*x))) - a^2*b*(224*c^2*D + 14*c*d*(32*C + 15*
D*x) + d^2*(224*B + x*(105*C + 64*D*x))) + 4*b^3*x*(35*A*(6*c^2 + 8*c*d*x
+ 3*d^2*x^2) + x*(14*B*(10*c^2 + 15*c*d*x + 6*d^2*x^2) + x*(21*c^2*(5*C +
4*D*x) + 28*c*d*x*(6*C + 5*D*x) + 10*d^2*x^2*(7*C + 6*D*x))))) - 105*a*Sqr
t[b]*(2*A*b*(4*b*c^2 - a*d^2) + a*(-2*b*c*(c*C + 2*B*d) + a*d*(C*d + 2*c*D
)))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1680*b^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2185, 2185, 27, 687, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(b(7Cd-10cD)x^2d^2+(7Abd-4acD)d^2+\left(-3bDc^2+7bBd^2-4ad^2D\right)xd\right)dx}{7bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int 3bd^3(c+dx)^2\left(d(14Abd-7aCd+2acD)-\left(8aDd^2+b\left(-4Dc^2+7Cdc-14Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx}{6bd^2}+\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(7Cd-10cI}{7bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}d\int (c+dx)^2\left(d(14Abd-7aCd+2acD)-\left(8aDd^2+b\left(-4Dc^2+7Cdc-14Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx+\frac{1}{6}d\left(a+bx\right.}{7bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{2}d\left(\frac{\int (c+dx)\left(d\left(70Acdb^2+a\left(16ad^2D-b\left(-2Dc^2+21Cdc+28Bd^2\right)\right)\right)-b\left(a(35Cd+6cD)d^2+2b\left(-4Dc^3+7Cdc^2-14Bd^2c-35Ad^3\right)\right)x\right)\sqrt{bx^2+a}dx}{5b}\right.}{7bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{2}d\left(\frac{\frac{35}{4}d^2\left(2Ab\left(4bc^2-ad^2\right)-a(2bc(2Bd+cC)-ad(2cD+Cd))\right)\int\sqrt{bx^2+a}dx+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2d^4D-2abd^2\left(7Bd^2+c^2D+14cCd\right)-b^2c\left(-70Ad^3-14I\right.\right.}{3b}}{5b}\right.}{}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}d\left(\frac{\frac{35}{4}d^2\left(2Ab\left(4bc^2-ad^2\right)-a\left(2bc(2Bd+cC)-ad(2cD+Cd)\right)\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2d^4D-2abd^2\left(7Bd^2+c^2D+14cC\right.\right.}{3b}}{5b}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow 224$$

$$\frac{1}{2}d\left(\frac{\frac{35}{4}d^2\left(2Ab\left(4bc^2-ad^2\right)-a\left(2bc(2Bd+cC)-ad(2cD+Cd)\right)\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2d^4D-2abd^2\left(7Bd^2+c\right.\right.}{}}{5b}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow 219$$

$$\frac{1}{2}d\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2d^4D-2abd^2\left(7Bd^2+c^2D+14cCd\right)-b^2c\left(-70Ad^3-14Bcd^2-4c^3D+7c^2Cd\right)\right)}{3b}+\frac{35}{4}d^2\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(2Ab}{5b}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

input
```
Int[(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^4*(a + b*x^2)^(3/2))/(7*b*d^2) + ((d*(7*C*d - 10*c*D)*(c + d*
x)^3*(a + b*x^2)^(3/2))/6 + (d*(-1/5*((8*a*d^2*D + b*(7*c*C*d - 14*B*d^2 -
 4*c^2*D))*(c + d*x)^2*(a + b*x^2)^(3/2))/b + ((2*(8*a^2*d^4*D - 2*a*b*d^2
*(14*c*C*d + 7*B*d^2 + c^2*D) - b^2*c*(7*c^2*C*d - 14*B*c*d^2 - 70*A*d^3 -
 4*c^3*D))*(a + b*x^2)^(3/2))/(3*b) - (d*(a*d^2*(35*C*d + 6*c*D) + 2*b*(7*
c^2*C*d - 14*B*c*d^2 - 35*A*d^3 - 4*c^3*D))*x*(a + b*x^2)^(3/2))/4 + (35*d
^2*(2*A*b*(4*b*c^2 - a*d^2) - a*(2*b*c*(c*C + 2*B*d) - a*d*(C*d + 2*c*D)))
*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt
[b])))/4)/(5*b)))/2)/(7*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.98

method	result
default	$A c^{2} \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)+\frac{c \left(2 A d +B c \right) \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{3 b}+d \left(C d +2 D c \right) \left(\frac{x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+\right.}{4 b}\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
1/3*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/b+d*(C*d+2*D*c)*(1/6*x^3*(b*x^2+a)^(3/2)
/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b
^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(A*d^2+2*B*c*d+C*c^2)*(1/4*x*(b*x^
2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))))+(B*d^2+2*C*c*d+D*c^2)*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^
2*(b*x^2+a)^(3/2))+D*d^2*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^
2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 803, normalized size of antiderivative = 2.30

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[1/3360*(105*(2*(C*a^2*b - 4*A*a*b^2)*c^2 - 2*(D*a^3 - 2*B*a^2*b)*c*d - (C
*a^3 - 2*A*a^2*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(240*D*b^3*d^2*x^6 + 280*(2*D*b^3*c*d + C*b^3*d^2)*x^5 + 48*(7*D*
b^3*c^2 + 14*C*b^3*c*d + (D*a*b^2 + 7*B*b^3)*d^2)*x^4 + 70*(6*C*b^3*c^2 +
2*(D*a*b^2 + 6*B*b^3)*c*d + (C*a*b^2 + 6*A*b^3)*d^2)*x^3 - 112*(2*D*a^2*b
- 5*B*a*b^2)*c^2 - 224*(2*C*a^2*b - 5*A*a*b^2)*c*d + 32*(4*D*a^3 - 7*B*a^2
*b)*d^2 + 16*(7*(D*a*b^2 + 5*B*b^3)*c^2 + 14*(C*a*b^2 + 5*A*b^3)*c*d - (4*
D*a^2*b - 7*B*a*b^2)*d^2)*x^2 + 105*(2*(C*a*b^2 + 4*A*b^3)*c^2 - 2*(D*a^2*
b - 2*B*a*b^2)*c*d - (C*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1
/1680*(105*(2*(C*a^2*b - 4*A*a*b^2)*c^2 - 2*(D*a^3 - 2*B*a^2*b)*c*d - (C*a
^3 - 2*A*a^2*b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*D*
b^3*d^2*x^6 + 280*(2*D*b^3*c*d + C*b^3*d^2)*x^5 + 48*(7*D*b^3*c^2 + 14*C*b
^3*c*d + (D*a*b^2 + 7*B*b^3)*d^2)*x^4 + 70*(6*C*b^3*c^2 + 2*(D*a*b^2 + 6*B
*b^3)*c*d + (C*a*b^2 + 6*A*b^3)*d^2)*x^3 - 112*(2*D*a^2*b - 5*B*a*b^2)*c^2
 - 224*(2*C*a^2*b - 5*A*a*b^2)*c*d + 32*(4*D*a^3 - 7*B*a^2*b)*d^2 + 16*(7*
(D*a*b^2 + 5*B*b^3)*c^2 + 14*(C*a*b^2 + 5*A*b^3)*c*d - (4*D*a^2*b - 7*B*a*
b^2)*d^2)*x^2 + 105*(2*(C*a*b^2 + 4*A*b^3)*c^2 - 2*(D*a^2*b - 2*B*a*b^2)*c
*d - (C*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(333) = 666$.

Time = 0.73 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.93

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Dd^2x^6}{7}+\frac{x^5(Cbd^2+2Dbcd)}{6b}+\frac{x^4\left(Bbd^2+2Cbcd+\frac{Dad^2}{7}+Dbc^2\right)}{5b}+\frac{x^3\left(Abd^2+2Bbcd+Cad^2+Cbc^2+2Dacd-\frac{5a(Cbd^2+}{6b}\right.}{4b}\right.\\ \sqrt{a}\left(Ac^2x+\frac{Dd^2x^6}{6}+\frac{x^5(Cd^2+2Dcd)}{5}+\frac{x^4(Bd^2+2Ccd+Dc^2)}{4}+\frac{x^3(Ad^2+2Bcd+Cc^2)}{3}+\frac{x^2\cdot(2Acd+Bc^2)}{2}\right)\end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*d**2*x**6/7 + x**5*(C*b*d**2 + 2*D*b*c*d)/(
6*b) + x**4*(B*b*d**2 + 2*C*b*c*d + D*a*d**2/7 + D*b*c**2)/(5*b) + x**3*(A
*b*d**2 + 2*B*b*c*d + C*a*d**2 + C*b*c**2 + 2*D*a*c*d - 5*a*(C*b*d**2 + 2*
D*b*c*d)/(6*b))/(4*b) + x**2*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d
+ D*a*c**2 - 4*a*(B*b*d**2 + 2*C*b*c*d + D*a*d**2/7 + D*b*c**2)/(5*b))/(3*
b) + x*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 3*a*(A*b*d**2 + 2*B*b
*c*d + C*a*d**2 + C*b*c**2 + 2*D*a*c*d - 5*a*(C*b*d**2 + 2*D*b*c*d)/(6*b))
/(4*b))/(2*b) + (2*A*a*c*d + B*a*c**2 - 2*a*(2*A*b*c*d + B*a*d**2 + B*b*c*
*2 + 2*C*a*c*d + D*a*c**2 - 4*a*(B*b*d**2 + 2*C*b*c*d + D*a*d**2/7 + D*b*c
**2)/(5*b))/(3*b))/b) + (A*a*c**2 - a*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C
*a*c**2 - 3*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2 + C*b*c**2 + 2*D*a*c*d - 5*
a*(C*b*d**2 + 2*D*b*c*d)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)),
 Ne(b, 0)), (sqrt(a)*(A*c**2*x + D*d**2*x**6/6 + x**5*(C*d**2 + 2*D*c*d)/5
 + x**4*(B*d**2 + 2*C*c*d + D*c**2)/4 + x**3*(A*d**2 + 2*B*c*d + C*c**2)/3
 + x**2*(2*A*c*d + B*c**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.17

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Dd^2x^4}{7\,b} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Dad^2x^2}{35\,b^2} + \frac{1}{2}\,\sqrt{bx^2+a}Ac^2x$$
$$+ \frac{(2\,Dcd+Cd^2)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b} + \frac{Aac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$
$$+ \frac{(bx^2+a)^{\frac{3}{2}}Bc^2}{3\,b} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Acd}{3\,b} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Da^2d^2}{105\,b^3}$$
$$+ \frac{(Dc^2+2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x^2}{5\,b} - \frac{(2\,Dcd+Cd^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2}$$
$$+ \frac{(2\,Dcd+Cd^2)\sqrt{bx^2+a}a^2x}{16\,b^2} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x}{4\,b}$$
$$- \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}ax}{8\,b} + \frac{(2\,Dcd+Cd^2)a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$
$$- \frac{(Cc^2+2\,Bcd+Ad^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(Dc^2+2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/7*(b*x^2 + a)^(3/2)*D*d^2*x^4/b - 4/35*(b*x^2 + a)^(3/2)*D*a*d^2*x^2/b^2
 + 1/2*sqrt(b*x^2 + a)*A*c^2*x + 1/6*(2*D*c*d + C*d^2)*(b*x^2 + a)^(3/2)*x
^3/b + 1/2*A*a*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x^2 + a)^(3/2)*
B*c^2/b + 2/3*(b*x^2 + a)^(3/2)*A*c*d/b + 8/105*(b*x^2 + a)^(3/2)*D*a^2*d^
2/b^3 + 1/5*(D*c^2 + 2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x^2/b - 1/8*(2*D*c
*d + C*d^2)*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*(2*D*c*d + C*d^2)*sqrt(b*x^2
+ a)*a^2*x/b^2 + 1/4*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*x/b - 1/8
*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a*x/b + 1/16*(2*D*c*d + C*d^2)*
a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*a^2*arc
sinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(D*c^2 + 2*C*c*d + B*d^2)*(b*x^2 + a)^(
3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.24

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{1680}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,Dd^2x+\frac{7\,(2\,Db^5cd+Cb^5d^2)}{b^5}\right)x+\frac{6\,(7\,Db^5c^2+14\,Cb^5cd+Dab^4d^2+7}{b^5}\right.\right.\right.\right.\right.$$
$$+\frac{(2\,Ca^2bc^2-8\,Aab^2c^2-2\,Da^3cd+4\,Ba^2bcd-Ca^3d^2+2\,Aa^2bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*d^2*x + 7*(2*D*b^5*c*d + C*b^5*d^2)
/b^5)*x + 6*(7*D*b^5*c^2 + 14*C*b^5*c*d + D*a*b^4*d^2 + 7*B*b^5*d^2)/b^5)*
x + 35*(6*C*b^5*c^2 + 2*D*a*b^4*c*d + 12*B*b^5*c*d + C*a*b^4*d^2 + 6*A*b^5
*d^2)/b^5)*x + 8*(7*D*a*b^4*c^2 + 35*B*b^5*c^2 + 14*C*a*b^4*c*d + 70*A*b^5
*c*d - 4*D*a^2*b^3*d^2 + 7*B*a*b^4*d^2)/b^5)*x + 105*(2*C*a*b^4*c^2 + 8*A*
b^5*c^2 - 2*D*a^2*b^3*c*d + 4*B*a*b^4*c*d - C*a^2*b^3*d^2 + 2*A*a*b^4*d^2)
/b^5)*x - 16*(14*D*a^2*b^3*c^2 - 35*B*a*b^4*c^2 + 28*C*a^2*b^3*c*d - 70*A*
a*b^4*c*d - 8*D*a^3*b^2*d^2 + 14*B*a^2*b^3*d^2)/b^5) + 1/16*(2*C*a^2*b*c^2
 - 8*A*a*b^2*c^2 - 2*D*a^3*c*d + 4*B*a^2*b*c*d - C*a^3*d^2 + 2*A*a^2*b*d^2
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (dx+c)^2\sqrt{b\,x^2+a}\,\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.61 $\int (c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 223

$$\begin{aligned}
\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx
&= \frac{(8Ab^2c-a(2b(cC+Bd)-adD))\,x\sqrt{a+bx^2}}{16b^2} \\
&\quad+\frac{(bBc+Abd-aCd-acD)\,(a+bx^2)^{3/2}}{3b^2}+\frac{(2b(cC+Bd)-adD)x(a+bx^2)^{3/2}}{8b^2} \\
&\quad+\frac{dDx^3(a+bx^2)^{3/2}}{6b}+\frac{(Cd+cD)\,(a+bx^2)^{5/2}}{5b^2} \\
&\quad+\frac{a(8Ab^2c-a(2b(cC+Bd)-adD))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}
\end{aligned}$$

output
```
1/16*(8*A*b^2*c-a*(2*b*(B*d+C*c)-D*a*d))*x*(b*x^2+a)^(1/2)/b^2+1/3*(A*b*d+
B*b*c-C*a*d-D*a*c)*(b*x^2+a)^(3/2)/b^2+1/8*(2*b*(B*d+C*c)-D*a*d)*x*(b*x^2+
a)^(3/2)/b^2+1/6*d*D*x^3*(b*x^2+a)^(3/2)/b+1/5*(C*d+D*c)*(b*x^2+a)^(5/2)/b
^2+1/16*a*(8*A*b^2*c-a*(2*b*(B*d+C*c)-D*a*d))*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-a^2(32Cd+32cD+15dDx)+2ab(40Ad+5B(8c+3dx)+x(15cC+8Cdx+8cDx+5dD}{240b^2}$$
$$-\frac{a(8Ab^2c+a(-2b(cC+Bd)+adD))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

input
```
Integrate[(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-(a^2*(32*C*d + 32*c*D + 15*d*D*x)) + 2*a*b*(40*A*d + 5*
B*(8*c + 3*d*x) + x*(15*c*C + 8*C*d*x + 8*c*D*x + 5*d*D*x^2)) + 4*b^2*x*(1
0*A*(3*c + 2*d*x) + x*(5*B*(4*c + 3*d*x) + x*(3*c*(5*C + 4*D*x) + 2*d*x*(6
*C + 5*D*x))))))/(240*b^2) - (a*(8*A*b^2*c + a*(-2*b*(c*C + B*d) + a*d*D))
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2185, 27, 2185, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int 3(c+dx)\sqrt{bx^2+a}\left(b(2Cd-3cD)x^2d^2+(2Abd-acD)d^2+\left(-bDc^2+2bBd^2-ad^2D\right)xd\right)dx}{6bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (c+dx)\sqrt{bx^2+a}\left(b(2Cd-3cD)x^2d^2+(2Abd-acD)d^2+\left(-bDc^2+2bBd^2-ad^2D\right)xd\right)dx}{2bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int bd^3(c+dx)\left(d(10Abd-4aCd+acD)-\left(5aDd^2+2b\left(-2Dc^2+3Cdc-5Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx}{5bd^2}+\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(2Cd-3cD)}{2bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}d\int (c+dx)\left(d(10Abd-4aCd+acD)-\left(5aDd^2+2b\left(-2Dc^2+3Cdc-5Bd^2\right)\right)x\right)\sqrt{bx^2+a}dx+\frac{1}{5}d\left(a+bx^2\right)}{2bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{1}{5}d\left(\frac{5d^2\left(8Ab^2c-a(2b(Bd+cC)-adD)\right)\int\sqrt{bx^2+a}dx}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(cD+Cd)+b\left(-5Ad^3-5Bcd^2-2c^3D+3c^2Cd\right)\right)}{3b}-\frac{dx\left(a+bx^2\right)^{3/2}(\right.}{}\right.}{2bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{5}d\left(\frac{5d^2\left(8Ab^2c-a(2b(Bd+cC)-adD)\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(cD+Cd)+b\left(-5Ad^3-5Bcd^2-2c^3D+3c^2Cd\right)\right)}{3b}\right.}{2bd^3}$$
$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{5}d\left(\frac{5d^2(8Ab^2c-a(2b(Bd+cC)-adD))\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2(a+bx^2)^{3/2}(2ad^2(cD+Cd)+b(-5Ad^3-5Bcd^2-2c^3D+}{3b}\right.}{2bd^3}$$

$$\frac{D(a+bx^2)^{3/2}(c+dx)^3}{6bd^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{5}d\left(\frac{5d^2\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(8Ab^2c-a(2b(Bd+cC)-adD))}{4b}-\frac{2(a+bx^2)^{3/2}(2ad^2(cD+Cd)+b(-5Ad^3-5Bcd^2-2c^3D+3}{3b}\right.}{2bd^3}$$

$$\frac{D(a+bx^2)^{3/2}(c+dx)^3}{6bd^2}$$

input
```
Int[(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^3*(a + b*x^2)^(3/2))/(6*b*d^2) + ((d*(2*C*d - 3*c*D)*(c + d*x
)^2*(a + b*x^2)^(3/2))/5 + (d*((-2*(2*a*d^2*(C*d + c*D) + b*(3*c^2*C*d - 5
*B*c*d^2 - 5*A*d^3 - 2*c^3*D))*(a + b*x^2)^(3/2))/(3*b) - (d*(6*b*c*C*d -
10*b*B*d^2 - 4*b*c^2*D + 5*a*d^2*D)*x*(a + b*x^2)^(3/2))/(4*b) + (5*d^2*(8
*A*b^2*c - a*(2*b*(c*C + B*d) - a*d*D))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)))/5)/(2*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.13

method	result
default	$Ac\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{(Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}+(Bd+Cc)\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln(}{}\right.}{4b}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/
3*(A*d+B*c)*(b*x^2+a)^(3/2)/b+(B*d+C*c)*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+(C*d+D
*c)*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+D*d*(1/6*x^3*(b
*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^
(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.19

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \left[\frac{15\,(2\,(Ca^2b-4\,Aab^2)c-(Da^3-2\,Ba^2b)d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,Db^3dx^5+4\right.$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[1/480*(15*(2*(C*a^2*b - 4*A*a*b^2)*c - (D*a^3 - 2*B*a^2*b)*d)*sqrt(b)*log
(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*D*b^3*d*x^5 + 48*(D*b
^3*c + C*b^3*d)*x^4 + 10*(6*C*b^3*c + (D*a*b^2 + 6*B*b^3)*d)*x^3 + 16*((D*
a*b^2 + 5*B*b^3)*c + (C*a*b^2 + 5*A*b^3)*d)*x^2 - 16*(2*D*a^2*b - 5*B*a*b^
2)*c - 16*(2*C*a^2*b - 5*A*a*b^2)*d + 15*(2*(C*a*b^2 + 4*A*b^3)*c - (D*a^2
*b - 2*B*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/240*(15*(2*(C*a^2*b - 4*A*a*
b^2)*c - (D*a^3 - 2*B*a^2*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (40*D*b^3*d*x^5 + 48*(D*b^3*c + C*b^3*d)*x^4 + 10*(6*C*b^3*c + (D*a*b^
2 + 6*B*b^3)*d)*x^3 + 16*((D*a*b^2 + 5*B*b^3)*c + (C*a*b^2 + 5*A*b^3)*d)*x
^2 - 16*(2*D*a^2*b - 5*B*a*b^2)*c - 16*(2*C*a^2*b - 5*A*a*b^2)*d + 15*(2*(
C*a*b^2 + 4*A*b^3)*c - (D*a^2*b - 2*B*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.56

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Ddx^5}{6}+\frac{x^4(Cbd+Dbc)}{5b}+\frac{x^3\left(Bbd+Cbc+\frac{Dad}{6}\right)}{4b}+\frac{x^2\left(Abd+Bbc+Cad+Dac-\frac{4a(Cbd+Dbc)}{5b}\right)}{3b}+\frac{x\left(Abc+Bad+Cac-\right.}{2} \right. \\ \sqrt{a}\left(Acx+\frac{Ddx^5}{5}+\frac{x^4(Cd+Dc)}{4}+\frac{x^3(Bd+Cc)}{3}+\frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input

```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(D*d*x**5/6 + x**4*(C*b*d + D*b*c)/(5*b) + x**
3*(B*b*d + C*b*c + D*a*d/6)/(4*b) + x**2*(A*b*d + B*b*c + C*a*d + D*a*c -
4*a*(C*b*d + D*b*c)/(5*b))/(3*b) + x*(A*b*c + B*a*d + C*a*c - 3*a*(B*b*d +
 C*b*c + D*a*d/6)/(4*b))/(2*b) + (A*a*d + B*a*c - 2*a*(A*b*d + B*b*c + C*a
*d + D*a*c - 4*a*(C*b*d + D*b*c)/(5*b))/(3*b))/b) + (A*a*c - a*(A*b*c + B*
a*d + C*a*c - 3*a*(B*b*d + C*b*c + D*a*d/6)/(4*b))/(2*b))*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True)), Ne(b, 0)), (sqrt(a)*(A*c*x + D*d*x**5/5 + x**4*(C*d + D*c)/4
+ x**3*(B*d + C*c)/3 + x**2*(A*d + B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.14

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Ddx^3}{6\,b} + \frac{1}{2}\,\sqrt{bx^2+a}Acx - \frac{(bx^2+a)^{\frac{3}{2}}Dadx}{8\,b^2} + \frac{\sqrt{bx^2+a}Da^2dx}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}(Dc+Cd)x^2}{5\,b} + \frac{Aac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + \frac{Da^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bc}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ad}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)x}{4\,b} - \frac{\sqrt{bx^2+a}(Cc+Bd)ax}{8\,b} - \frac{(Cc+Bd)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}(Dc+Cd)a}{15\,b^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/6*(b*x^2 + a)^(3/2)*D*d*x^3/b + 1/2*sqrt(b*x^2 + a)*A*c*x - 1/8*(b*x^2 +
 a)^(3/2)*D*a*d*x/b^2 + 1/16*sqrt(b*x^2 + a)*D*a^2*d*x/b^2 + 1/5*(b*x^2 +
a)^(3/2)*(D*c + C*d)*x^2/b + 1/2*A*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/
16*D*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 1/3*(b*x^2 + a)^(3/2)*B*c/b +
1/3*(b*x^2 + a)^(3/2)*A*d/b + 1/4*(b*x^2 + a)^(3/2)*(C*c + B*d)*x/b - 1/8*
sqrt(b*x^2 + a)*(C*c + B*d)*a*x/b - 1/8*(C*c + B*d)*a^2*arcsinh(b*x/sqrt(a
*b))/b^(3/2) - 2/15*(b*x^2 + a)^(3/2)*(D*c + C*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.11

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{1}{240}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,Ddx + \frac{6\,(Db^4c+Cb^4d)}{b^4}\right)x + \frac{5\,(6\,Cb^4c+Dab^3d+6\,Bb^4d)}{b^4}\right)x + \frac{8\,(Dab^3c+\right.\right.\right.$$
$$+ \frac{(2\,Ca^2bc-8\,Aab^2c-Da^3d+2\,Ba^2bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*D*d*x + 6*(D*b^4*c + C*b^4*d)/b^4)*x + 5*
(6*C*b^4*c + D*a*b^3*d + 6*B*b^4*d)/b^4)*x + 8*(D*a*b^3*c + 5*B*b^4*c + C*
a*b^3*d + 5*A*b^4*d)/b^4)*x + 15*(2*C*a*b^3*c + 8*A*b^4*c - D*a^2*b^2*d +
2*B*a*b^3*d)/b^4)*x - 16*(2*D*a^2*b^2*c - 5*B*a*b^3*c + 2*C*a^2*b^2*d - 5*
A*a*b^3*d)/b^4) + 1/16*(2*C*a^2*b*c - 8*A*a*b^2*c - D*a^3*d + 2*B*a^2*b*d)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{b\,x^2+a}\,(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.74

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{80\sqrt{b\,x^2+a}\,a^2b^2d-64\sqrt{b\,x^2+a}\,a^2bcd-15\sqrt{b\,x^2+a}\,a^2b\,d^2x+120\sqrt{b\,x^2+a}\,a\,b^3cx+80\sqrt{b\,x^2+a}\,a\,b^3}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(80*sqrt(a + b*x**2)*a**2*b**2*d - 64*sqrt(a + b*x**2)*a**2*b*c*d - 15*sqr
t(a + b*x**2)*a**2*b*d**2*x + 120*sqrt(a + b*x**2)*a*b**3*c*x + 80*sqrt(a
+ b*x**2)*a*b**3*c + 80*sqrt(a + b*x**2)*a*b**3*d*x**2 + 30*sqrt(a + b*x**
2)*a*b**3*d*x + 30*sqrt(a + b*x**2)*a*b**2*c**2*x + 32*sqrt(a + b*x**2)*a*
b**2*c*d*x**2 + 10*sqrt(a + b*x**2)*a*b**2*d**2*x**3 + 80*sqrt(a + b*x**2)
*b**4*c*x**2 + 60*sqrt(a + b*x**2)*b**4*d*x**3 + 60*sqrt(a + b*x**2)*b**3*
c**2*x**3 + 96*sqrt(a + b*x**2)*b**3*c*d*x**4 + 40*sqrt(a + b*x**2)*b**3*d
**2*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d**
2 + 120*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c -
30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d - 30*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c**2)/(240*b**3)
```

3.62 $\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(4Ab-aC)x\sqrt{a+bx^2}}{8b} + \frac{(bB-aD)\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{Cx\left(a+bx^2\right)^{3/2}}{4b} + \frac{D\left(a+bx^2\right)^{5/2}}{5b^2} + \frac{a(4Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/3*(B*b-D*a)*(b*x^2+a)^(3/2)/b^2+1/4*
C*x*(b*x^2+a)^(3/2)/b+1/5*D*(b*x^2+a)^(5/2)/b^2+1/8*a*(4*A*b-C*a)*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2D+ab(40B+x(15C+8Dx))+2b^2x(30A+x(20B+3x(5C+4Dx))))+15a\sqrt{b}(-4}{120b^2}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*D + a*b*(40*B + x*(15*C + 8*D*x)) + 2*b^2*x*(30*
A + x*(20*B + 3*x*(5*C + 4*D*x)))) + 15*a*Sqrt[b]*(-4*A*b + a*C)*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2346, 2346, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \; 2346$$
$$\frac{\int \sqrt{bx^2+a}\left(5bCx^2+(5bB-2aD)x+5Ab\right)dx}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \; 2346$$
$$\frac{\frac{\int b(5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx}{4b}+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \; 27$$
$$\frac{\frac{1}{4}\int (5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\int \sqrt{bx^2+a}dx+\frac{4\left(a+bx^2\right)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4\left(a+bx^2\right)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4\left(a+bx^2\right)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4\left(a+bx^2\right)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(3/2))/(5*b) + ((5*C*x*(a + b*x^2)^(3/2))/4 + ((4*(5*b*
B - 2*a*D)*(a + b*x^2)^(3/2))/(3*b) + 5*(4*A*b - a*C)*((x*Sqrt[a + b*x^2])
/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

method	result
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+C\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+D\left(\frac{x^{2}\left(b\,x^{2}+\right.}{5b}\right.$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+C*(1
/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+D*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a
)^(3/2))+1/3*B*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.75

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\left[-\frac{15\,(Ca^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Db^2x^4+30\,Cb^2x^3-16\,Da^2+40\,B}{240\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/240*(15*(C*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(24*D*b^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*
a*b + 5*B*b^2)*x^2 + 15*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/120*(
15*(C*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (24*D*b
^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*a*b + 5*B*b^2)*x^2 + 15
*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.21

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Cx^3}{4}+\frac{Dx^4}{5}+\frac{x^2\left(Bb+\frac{Da}{5}\right)}{3b}+\frac{x\left(Ab+\frac{Ca}{4}\right)}{2b}+\frac{Ba-\frac{2a\left(Bb+\frac{Da}{5}\right)}{3b}}{b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ca}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+}\right.}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*x**3/4 + D*x**4/5 + x**2*(B*b + D*a/5)/(3*b
) + x*(A*b + C*a/4)/(2*b) + (B*a - 2*a*(B*b + D*a/5)/(3*b))/b) + (A*a - a*
(A*b + C*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*x
 + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}Dx^2}{5\,b}+\frac{1}{2}\sqrt{bx^2+a}Ax \\ &+\frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,b}-\frac{\sqrt{bx^2+a}Cax}{8\,b} \\ &-\frac{Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} \\ &-\frac{2\,(bx^2+a)^{\frac{3}{2}}Da}{15\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}\end{aligned}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^
(3/2)*C*x/b - 1/8*sqrt(b*x^2 + a)*C*a*x/b - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b
))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/15*(b*x^2 + a)^(3/
2)*D*a/b^2 + 1/3*(b*x^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Dx+5\,C)x+\frac{4\,(Dab^2+5\,Bb^3)}{b^3}\right)x+\frac{15\,(Cab^2+4\,Ab^3)}{b^3}\right)x-\frac{8\,(2\,Da^2b-5\,B\dots}{b^3}\right.$$
$$+\frac{(Ca^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*D*x + 5*C)*x + 4*(D*a*b^2 + 5*B*b^3)/b^3)*
x + 15*(C*a*b^2 + 4*A*b^3)/b^3)*x - 8*(2*D*a^2*b - 5*B*a*b^2)/b^3) + 1/8*(
C*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.39

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{-16\sqrt{b\,x^2+a}\,a^2d+60\sqrt{b\,x^2+a}\,a\,b^2x+40\sqrt{b\,x^2+a}\,a\,b^2+15\sqrt{b\,x^2+a}\,abcx+8\sqrt{b\,x^2+a}\,abd\,x^2+40}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**2*d + 60*sqrt(a + b*x**2)*a*b**2*x + 40*sqrt(a
+ b*x**2)*a*b**2 + 15*sqrt(a + b*x**2)*a*b*c*x + 8*sqrt(a + b*x**2)*a*b*d*
x**2 + 40*sqrt(a + b*x**2)*b**3*x**2 + 30*sqrt(a + b*x**2)*b**2*c*x**3 + 2
4*sqrt(a + b*x**2)*b**2*d*x**4 + 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*b - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*c)/(120*b**2)
```

3.63 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 324

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx \\
&= \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{d^4} - \frac{(ad^2D+4b(cCd-Bd^2-c^2D))\,x\sqrt{a+bx^2}}{8bd^3} \\
&\quad + \frac{(4Cd-7cD)\,(a+bx^2)^{3/2}}{12bd^2} + \frac{D(c+dx)\,(a+bx^2)^{3/2}}{4bd^2} \\
&\quad - \frac{(a^2d^4D+4abd^2(cCd-Bd^2-c^2D)+8b^2c(c^2Cd-Bcd^2+Ad^3-c^3D))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}d^5} \\
&\quad - \frac{\sqrt{bc^2+ad^2}(c^2Cd-Bcd^2+Ad^3-c^3D)\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5}
\end{aligned}$$

output
```
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^4-1/8*(a*d^2*D+4*b*(-B*d^2
+C*c*d-D*c^2))*x*(b*x^2+a)^(1/2)/b/d^3+1/12*(4*C*d-7*D*c)*(b*x^2+a)^(3/2)/
b/d^2+1/4*D*(d*x+c)*(b*x^2+a)^(3/2)/b/d^2-1/8*(a^2*d^4*D+4*a*b*d^2*(-B*d^2
+C*c*d-D*c^2)+8*b^2*c*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*arctanh(b^(1/2)*x/(b*
x^2+a)^(1/2))/b^(3/2)/d^5-(a*d^2+b*c^2)^(1/2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3
)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(ad^2(8Cd-8cD+3dDx)+2b\left(-12c^3D+6c^2d(2C+Dx)-2cd^2\left(6B+3Cx+2Dx^2\right)+d^3\left(12A+6Bx+4Cx^2+3Dx^3\right)\right)\right)}{b}-48\sqrt{-bc^2-}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(8*C*d - 8*c*D + 3*d*D*x) + 2*b*(-12*c^3*D + 6*
c^2*d*(2*C + D*x) - 2*c*d^2*(6*B + 3*C*x + 2*D*x^2) + d^3*(12*A + 6*B*x +
4*C*x^2 + 3*D*x^3))))/b - 48*Sqrt[-(b*c^2) - a*d^2]*(-(c^2*C*d) + B*c*d^2
- A*d^3 + c^3*D)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c
^2) - a*d^2]] + (3*(a^2*d^4*D - 4*a*b*d^2*(-(c*C*d) + B*d^2 + c^2*D) - 8*b
^2*c*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/b^(3/2))/(24*d^5)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2185, 2185, 27, 682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(b(4Cd-7cD)x^2d^2+(4Abd-acD)d^2+\left(-3bDc^2+4bBd^2-ad^2D\right)xd\right)}{c+dx}dx}{4bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int \frac{3bd^3\left(d(4Abd-acD)-\left(aDd^2+4b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}+\frac{1}{3}d\left(a+bx^2\right)^{3/2}(4Cd-7cD)}{4bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{(d(4Abd-acD)-(aDd^2+4b(-Dc^2+Cdc-Bd^2))x)\sqrt{bx^2+a}}{c+dx}dx+\frac{1}{3}d\left(a+bx^2\right)^{3/2}(4Cd-7cD)}{4bd^3}+\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{d\left(\frac{\int -\frac{b\left(ad\left(acd^2D-4b\left(-Dc^3+Cdc^2-Bd^2c+2Ad^3\right)\right)+\left(2bc(4Abd-acD)d^2+\left(2bc^2+ad^2\right)\left(aDd^2+4b\left(-Dc^2+Cdc-Bd^2\right)\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}(8b(Ad^3-B}{}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\int \frac{b\left(ad\left(acd^2D-4b\left(-Dc^3+Cdc^2-Bd^2c+2Ad^3\right)\right)+\left(2bc(4\right.\right.}{(c+}}{}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\int \frac{ad\left(acd^2D-4b\left(-Dc^3+Cdc^2-Bd^2c+2Ad^3\right)\right)+\left(2bc(4A\right.}{(c+dx}}{}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$$\Big\downarrow \; 719$$

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\left(2bcd^2(4Abd-acD)+\left(ad^2+2bc^2\right)\left(ad^2D+4b\left(-Bd^2+c^2(-\right.\right.\right.}{d}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$\big\downarrow$ 224

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\left(2bcd^2(4Abd-acD)+\left(ad^2+2bc^2\right)\left(ad^2D+4b\left(-Bd^2+c^2(-\right.\right.\right.}{d}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2bcd^2(4Abd-acD)+\left(ad^2+2bc^2\right)\right.}{\sqrt{b}d}\right.}{4bd^3}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$\big\downarrow$ 488

$$d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{8b\left(ad^2+bc^2\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int\frac{1}{bc^2+ad^2-}}{d}\right.$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{\sqrt{a+bx^2}\left(8b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-dx\left(ad^2D+4b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{2d^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2bcd^2(4Abd-acD)+\left(ad^2+2bc^2\right)\right.}{\sqrt{b}d}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(D*(c + d*x)*(a + b*x^2)^(3/2))/(4*b*d^2) + ((d*(4*C*d - 7*c*D)*(a + b*x^2
)^(3/2))/3 + d*(((8*b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) - d*(a*d^2*D + 4
*b*(c*C*d - B*d^2 - c^2*D))*x)*Sqrt[a + b*x^2])/(2*d^2) - (((2*b*c*d^2*(4*
A*b*d - a*c*D) + (2*b*c^2 + a*d^2)*(a*d^2*D + 4*b*(c*C*d - B*d^2 - c^2*D))
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + (8*b*Sqrt[b*c^2 + a*
d^2]*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2
 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.53

method	result
default	$\frac{B\,d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+D c^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{d(Cd-Dc)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}+D d^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\right.}{}\right.}{d^{3}}$

input

```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
1/d^3*(B*d^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2)))+D*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2)))+1/3*d*(C*d-D*c)*(b*x^2+a)^(3/2)/b+D*d^2*(1/4*x*(b*x^2+a)^(3/2)/b-1/
4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))
-C*c*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Timed out}$$

input

```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas
")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*D*c^2*x/d^3 - 1/2*sqrt(b*x^2 + a)*C*c*x/d^2 + 1/2*sqrt
(b*x^2 + a)*B*x/d + 1/4*(b*x^2 + a)^(3/2)*D*x/(b*d) - 1/8*sqrt(b*x^2 + a)*
D*a*x/(b*d) + D*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 - C*sqrt(b)*c^3*arc
sinh(b*x/sqrt(a*b))/d^4 + 1/2*D*a*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3)
 + B*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 1/2*C*a*c*arcsinh(b*x/sqrt(a
*b))/(sqrt(b)*d^2) - A*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 - 1/8*D*a^2*ar
csinh(b*x/sqrt(a*b))/(b^(3/2)*d) + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/(sqrt(b)
*d) - D*sqrt(a + b*c^2/d^2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a
*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + C*sqrt(a + b*c^2/d^2)*c^2*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - B*sqrt(a +
 b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(
d*x + c)))/d^2 + A*sqrt(a + b*c^2/d^2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d - sqrt(b*x^2 + a)*D*c^3/d^4 + sqrt(b
*x^2 + a)*C*c^2/d^3 - sqrt(b*x^2 + a)*B*c/d^2 - 1/3*(b*x^2 + a)^(3/2)*D*c/
(b*d^2) + sqrt(b*x^2 + a)*A/d + 1/3*(b*x^2 + a)^(3/2)*C/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 2335, normalized size of antiderivative = 7.21

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
( - 8*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**2*c*d + 8*sqrt(b)*sq
rt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*b**3*c**2 - 8*sqrt(2*sqrt(b)*sqrt(a*d**2
+ b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/s
qrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b**2*d**3
 - 8*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*a*b**3*c**2*d + 8*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
 - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**3*c*d**2 + 8*sqrt(2*
sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2
)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c
**2))*b**4*c**3 - 4*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**
2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*
c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*a*b**2*c*
d + 4*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a...
```

3.64 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 330

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx &= \frac{(ad^2D-3b(2cCd-Bd^2-3c^2D))\sqrt{a+bx^2}}{3bd^4} \\ &+ \frac{(Cd-2cD)x\sqrt{a+bx^2}}{2d^3} + \frac{Dx^2\sqrt{a+bx^2}}{3d^2} - \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{d^4(c+dx)} \\ &+ \frac{(ad^2(Cd-2cD)+b(6c^2Cd-4Bcd^2+2Ad^3-8c^3D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^5} \\ &+ \frac{(ad^2(2cCd-Bd^2-3c^2D)+bc(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
1/3*(a*d^2*D-3*b*(-B*d^2+2*C*c*d-3*D*c^2))*(b*x^2+a)^(1/2)/b/d^4+1/2*(C*d-
2*D*c)*x*(b*x^2+a)^(1/2)/d^3+1/3*D*x^2*(b*x^2+a)^(1/2)/d^2-(A*d^3-B*c*d^2+
C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^4/(d*x+c)+1/2*(a*d^2*(C*d-2*D*c)+b*(2*A*d
^3-4*B*c*d^2+6*C*c^2*d-8*D*c^3))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2
)/d^5+(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c
^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+
b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}(2ad^2D(c+dx)+b(24c^3D-6c^2d(3C-2Dx)+cd^2(12B-9Cx-4Dx^2)+d^3(-6A+6Bx+3Cx^2+2Dx^3)))}{b(c+dx)} - \frac{12(ad^2(-2cCd+Bd^2+3c}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(2*a*d^2*D*(c + d*x) + b*(24*c^3*D - 6*c^2*d*(3*C - 2*
D*x) + c*d^2*(12*B - 9*C*x - 4*D*x^2) + d^3*(-6*A + 6*B*x + 3*C*x^2 + 2*D*
x^3))))/(b*(c + d*x)) - (12*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-3*
c^2*C*d + 2*B*c*d^2 - A*d^3 + 4*c^3*D))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt
[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + (3*(a*d^2*(
-(C*d) + 2*c*D) + b*(-6*c^2*C*d + 4*B*c*d^2 - 2*A*d^3 + 8*c^3*D))*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(6*d^5)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2182, 25, 2185, 27, 682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{3Dc^3}{d^2}-\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{3Dc^3}{d^2}-\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int \frac{3b\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-4Dc^3+3Cdc^2-2Bd^2c+2Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}+\frac{1}{3}D\left(a+bx^2\right)^{3/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-4Dc^3+3Cdc^2-2Bd^2c+2Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{d^2}+\frac{1}{3}D\left(a+bx^2\right)^{3/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 682$$

$$\frac{\frac{\frac{\int -\frac{b\left(bc^2+ad^2\right)\left(ad\left(-4Dc^2+3Cdc-2Bd^2\right)-\left(a(Cd-2cD)d^2+b\left(-8Dc^3+6Cdc^2-4Bd^2c+2Ad^3\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bc\cdots\right.\right.\right.}{\cdots}}{d^2}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{-\frac{\int \frac{b\left(bc^2+ad^2\right)\left(ad\left(-4Dc^2+3Cdc-2Bd^2\right)-\left(a(Cd-2cD)d^2+b\left(-8Dc^3+6Cdc^2-4Bd^2c+2Ad^3\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bc\cdots\right.\right.\right.}{\cdots}}{d^2}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{(ad^2+bc^2)\int \frac{ad\left(-4Dc^2+3Cdc-2Bd^2\right)-\left(a(Cd-2cD)d^2+b\left(-8Dc^3+6Cdc^2-4Bd^2c+2Ad^3\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bcd^2\ldots\right.\right.\right.}{d^2}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{-\frac{(ad^2+bc^2)\left(\frac{2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2(Cd-2cD)+b\left(2Ad^3-4Bcd^2-8c^3D+6c^2Cd\right)\right)\int\frac{\ldots}{\sqrt{b\ldots}}}{d}\ldots\right.}{2d^2}\ldots}{d^2\ldots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{-\frac{(ad^2+bc^2)\left(\frac{2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2(Cd-2cD)+b\left(2Ad^3-4Bcd^2-8c^3D+6c^2Cd\right)\right)\int\frac{\ldots}{1-\ldots}}{d}\ldots\right.}{2d^2}\ldots}{d^2\ldots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{(ad^2+bc^2)\left(\frac{2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(Cd-2cD)+b\left(2Ad^3-4Bcd^2\ldots\right.\right.}{\sqrt{b}d}\ldots\right.}{2d^2}\ldots}{d^2\ldots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$-\frac{\left(ad^2+bc^2\right)\left(-\frac{2\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(Cd-2cD)+\right.}{\sqrt{b}d}}{2d^2}}{}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 219$$

$$-\frac{\left(ad^2+bc^2\right)\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(Cd-2cD)+b\left(2Ad^3-4Bcd^2-8c^3D+6c^2Cd\right)\right)}{\sqrt{b}d}-\frac{2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+\right.}{d\sqrt{ad^2+bc^2}}}{2d^2}}{}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
-(((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*
d^2)*(c + d*x))) + (((a/b + c^2/d^2)*D*(a + b*x^2)^(3/2))/3 + (-1/2*((2*(a
*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*
c^3*D)) - d*(a*d^2*(C*d - 2*c*D) + b*(3*c^2*C*d - 2*B*c*d^2 + 2*A*d^3 - 4*
c^3*D))*x)*Sqrt[a + b*x^2])/d^2 - ((b*c^2 + a*d^2)*(-(((a*d^2*(C*d - 2*c*D
) + b*(6*c^2*C*d - 4*B*c*d^2 + 2*A*d^3 - 8*c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(Sqrt[b]*d)) - (2*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*
(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2))/d^2)/(b*
c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(302) = 604$.

Time = 1.52 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.84

method	result
default	$\frac{Cd\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{Dd\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}-2Dc\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{d^3}+\frac{\left(B\,d^2-2Ccd+3Dc^2\right)\left(\sqrt{b(x-\right.\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(C*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2)))+1/3*D*d*(b*x^2+a)^(3/2)/b-2*D*c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/d^4*(B*d^2-2*C*c*d+3*D*c^2)*((b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d
))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b
*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d
)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(1/2))/(x+c/d)))+1/d^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-1/(a*d^2+b*c^2)
*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(
a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2
)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+
b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2)*d^2*(1/4*(
2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))
/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(305) = 610$.

Time = 0.10 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxi
ma")
```

output
```
sqrt(b*x^2 + a)*D*c^3/(d^5*x + c*d^4) - sqrt(b*x^2 + a)*C*c^2/(d^4*x + c*d
^3) + sqrt(b*x^2 + a)*B*c/(d^3*x + c*d^2) - sqrt(b*x^2 + a)*A/(d^2*x + c*d
) - sqrt(b*x^2 + a)*D*c*x/d^3 + 1/2*sqrt(b*x^2 + a)*C*x/d^2 - 4*D*sqrt(b)*
c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 3*C*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^
4 - D*a*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 2*B*sqrt(b)*c*arcsinh(b*x
/sqrt(a*b))/d^3 + 1/2*C*a*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + A*sqrt(b)
*arcsinh(b*x/sqrt(a*b))/d^2 + D*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^6) - C*b*c^3*arc
sinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(
a + b*c^2/d^2)*d^5) + 3*D*sqrt(a + b*c^2/d^2)*c^2*arcsinh(b*c*x/(sqrt(a*b)
*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + B*b*c^2*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/
d^2)*d^4) - 2*C*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - A*b*c*arcsinh(b*c*x/(sqrt(a*b)*ab
s(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) + B*
sqrt(a + b*c^2/d^2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b
)*abs(d*x + c)))/d^2 + 3*sqrt(b*x^2 + a)*D*c^2/d^4 - 2*sqrt(b*x^2 + a)*C*c
/d^3 + sqrt(b*x^2 + a)*B/d^2 + 1/3*(b*x^2 + a)^(3/2)*D/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1913, normalized size of antiderivative = 5.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
( - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b**2*c**2*d**2 - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x + 12*sqrt(a*d**2
+ b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2
*c*d**3 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*a*b**2*d**4*x + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 + 12*sqrt(a*d**2
 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c
**2*d**3*x + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*b**3*c**3*d + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**2*d**2*x + 12*sqrt(a*
d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b
**2*c**5 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*b**2*c**4*d*x + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*
a*b**2*c**2*d**2 + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3*x -
 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3 - 12*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a*b**2*d**4*x - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
*b*c**3*d**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**3*x - 24*
sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d - 24*sqrt(a*d**2 + b*c**2)*
log(c + d*x)*b**3*c**2*d**2*x - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b...
```

3.65 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 379

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx \\
&= \frac{(Cd-3cD)\sqrt{a+bx^2}}{d^4} + \frac{Dx\sqrt{a+bx^2}}{2d^3} - \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{2d^4(c+dx)^2} \\
&\quad + \frac{(2ad^2(2cCd-Bd^2-3c^2D)+bc(5c^2Cd-3Bcd^2+Ad^3-7c^3D))\sqrt{a+bx^2}}{2d^4(bc^2+ad^2)(c+dx)} \\
&\quad + \frac{(ad^2D-2b(3cCd-Bd^2-6c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^5} \\
&\quad - \frac{(2a^2d^4(Cd-3cD)+2b^2c^3(3cCd-Bd^2-6c^2D)+abd^2(9c^2Cd-3Bcd^2+Ad^3-19c^3D))\operatorname{arctanh}\left(\cdots\right.}{2d^5(bc^2+ad^2)^{3/2}}
\end{aligned}$$

output
```
(C*d-3*D*c)*(b*x^2+a)^(1/2)/d^4+1/2*D*x*(b*x^2+a)^(1/2)/d^3-1/2*(A*d^3-B*c
*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^4/(d*x+c)^2+1/2*(2*a*d^2*(-B*d^2+2*C
*c*d-3*D*c^2)+b*c*(A*d^3-3*B*c*d^2+5*C*c^2*d-7*D*c^3))*(b*x^2+a)^(1/2)/d^4
/(a*d^2+b*c^2)/(d*x+c)+1/2*(a*d^2*D-2*b*(-B*d^2+3*C*c*d-6*D*c^2))*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^5-1/2*(2*a^2*d^4*(C*d-3*D*c)+2*b^2*c^
3*(-B*d^2+3*C*c*d-6*D*c^2)+a*b*d^2*(A*d^3-3*B*c*d^2+9*C*c^2*d-19*D*c^3))*a
rctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+b*c^2)
^(3/2)
```

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}\left(bc\left(-12c^4D+Ad^4x+6c^3d(C-3Dx)+c^2d^2\left(-2B+9Cx-4Dx^2\right)+cd^3x\left(-3B+2Cx+Dx^2\right)\right)+ad^2\left(-11c^3D+c^2d(5C-17Dx)-cd^2\left(B-\right.\right.}{(bc^2+ad^2)(c+dx)^2}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((d*Sqrt[a + b*x^2]*(b*c*(-12*c^4*D + A*d^4*x + 6*c^3*d*(C - 3*D*x) + c^2*
d^2*(-2*B + 9*C*x - 4*D*x^2) + c*d^3*x*(-3*B + 2*C*x + D*x^2)) + a*d^2*(-1
1*c^3*D + c^2*d*(5*C - 17*D*x) - c*d^2*(B - 8*C*x + 4*D*x^2) + d^3*(-A - 2
*B*x + 2*C*x^2 + D*x^3))))/((b*c^2 + a*d^2)*(c + d*x)^2) - (2*(-2*a^2*d^4*
(C*d - 3*c*D) + 2*b^2*c^3*(-3*c*C*d + B*d^2 + 6*c^2*D) + a*b*d^2*(-9*c^2*C
*d + 3*B*c*d^2 - A*d^3 + 19*c^3*D))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a +
 b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) - ((a*d^2*D + 2
*b*(-3*c*C*d + B*d^2 + 6*c^2*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt
[b])/(2*d^5)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.49, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2182, 25, 2182, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(2Cd-2cD)-b\left(\frac{3Dc^3}{d^2}-\frac{3Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)-b\left(\frac{3Dc^3}{d^2}-\frac{3Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-Ad^3-Bcd^2-5c^3D+3c^2Cd\right)\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\int -\frac{\left(\left(2d(Cd-2cD)a^2+\frac{bc\left(-3Dc^2+Cdc+Bd^2\right)a}{d}+Ab\left(2bc^2+ad^2\right)\right)d \cdots\right.}{\cdots}}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(2bc^2+ad^2\right)+a\left(2a(Cd-2cD)d^2+bc\left(-3Dc^2+Cdc+Bd^2\right)\right)\right)+2\left(a^2Dd^4-2ab\left(-4Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-6Dc^3+3Cdc^2-Bd^2c-Ad^3\right)\right)x\right)\sqrt{bx^2+a} \cdots}{d^2(c+dx)}}{ad^2+bc^2} \cdots}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(2bc^2+ad^2\right)+a\left(2a(Cd-2cD)d^2+bc\left(-3Dc^2+Cdc+Bd^2\right)\right)\right)+2\left(a^2Dd^4-2ab\left(-4Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-6Dc^3+3Cdc^2-Bd^2c-Ad^3\right)\right)x\right)\sqrt{bx^2+a} \cdots}{c+dx}}{d^2\left(ad^2+bc^2\right)} \cdots}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{\int \frac{2b\left(bc^2+ad^2\right)\left(ad\left(a(2Cd-5cD)d^2+b\left(-6Dc^3+3Cdc^2-Bd^2c+Ad^3\right)\right)+\left(bc^2+ad^2\right)\left(ad^2D-2b\left(-6Dc^2+3Cdc-Bd^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(dx\left(a^2d^4D-2abd^2\left(-B\cdots\right.\right.\right.}{d^2\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(ad^2+bc^2\right)\int \frac{ad\left(a(2Cd-5cD)d^2+b\left(-6Dc^3+3Cdc^2-Bd^2c+Ad^3\right)\right)+\left(bc^2+ad^2\right)\left(ad^2D-2b\left(-6Dc^2+3Cdc-Bd^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}+\frac{\sqrt{a+bx^2}\left(dx\left(a^2d^4D-2abd^2\left(-Bd^2-\cdots\right.\right.\right.}{d^2\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(ad^2+bc^2\right)\left(\frac{\left(2a^2d^4(Cd-3cD)+abd^2\left(Ad^3-3Bcd^2-19c^3D+9c^2Cd\right)+2b^2c^3\left(-Bd^2-6c^2D+3cCd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\left(ad^2+bc^2\right)\left(ad^2D-2b\left(-Bd^2-6c^2\cdots\right.\right.}{d}\cdots\right.}{d^2}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(ad^2+bc^2\right)\left(\frac{\left(2a^2d^4(Cd-3cD)+abd^2\left(Ad^3-3Bcd^2-19c^3D+9c^2Cd\right)+2b^2c^3\left(-Bd^2-6c^2D+3cCd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\left(ad^2+bc^2\right)\left(ad^2D-2b\left(-Bd^2-6c^2\cdots\right.\right.}{d\cdots}\cdots\right.}{d^2}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(ad^2+bc^2\right)\left(\frac{\left(2a^2d^4(Cd-3cD)+abd^2\left(Ad^3-3Bcd^2-19c^3D+9c^2Cd\right)+2b^2c^3\left(-Bd^2-6c^2D+3cCd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}}\right.}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{488}$$

$$\frac{\left(ad^2+bc^2\right)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(ad^2D-2b\left(-Bd^2-6c^2D+3cCd\right)\right)}{\sqrt{b}d}-\frac{\left(2a^2d^4(Cd-3cD)+abd^2\left(Ad^3-3Bcd^2-19c^3D+9c^2Cd\right)+2b^2c^3\left(-Bd^2-6\right.\right.}{d}\right.}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\left(ad^2+bc^2\right)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(ad^2D-2b\left(-Bd^2-6c^2D+3cCd\right)\right)}{\sqrt{b}d}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(2a^2d^4(Cd-3cD)+abd^2\left(Ad^3-3Bcd^2-\right.\right.}{d\sqrt{ad^2+bc^2}}\right.}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
-1/2*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^2) + (((2*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(3*c^2
*C*d - B*c*d^2 - A*d^3 - 5*c^3*D))*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)
*(c + d*x)) + (((2*a^2*d^4*(C*d - 3*c*D) + 2*b^2*c^3*(3*c*C*d - B*d^2 - 6*
c^2*D) + a*b*d^2*(9*c^2*C*d - 3*B*c*d^2 + A*d^3 - 19*c^3*D) + d*(a^2*d^4*D
 - 2*a*b*d^2*(2*c*C*d - B*d^2 - 4*c^2*D) - b^2*c*(3*c^2*C*d - B*c*d^2 - A*
d^3 - 6*c^3*D))*x)*Sqrt[a + b*x^2])/d^2 + ((b*c^2 + a*d^2)*(((b*c^2 + a*d^
2)*(a*d^2*D - 2*b*(3*c*C*d - B*d^2 - 6*c^2*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(Sqrt[b]*d) - ((2*a^2*d^4*(C*d - 3*c*D) + 2*b^2*c^3*(3*c*C*d -
B*d^2 - 6*c^2*D) + a*b*d^2*(9*c^2*C*d - 3*B*c*d^2 + A*d^3 - 19*c^3*D))*Arc
Tanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 +
 a*d^2])))/d^2)/(d^2*(b*c^2 + a*d^2)))/(2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. $2(349) = 698$.

Time = 1.45 (sec) , antiderivative size = 1786, normalized size of antiderivative = 4.71

method	result	size
default	Expression too large to display	1786

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
D/d^3*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
1/d^4*(C*d-3*D*c)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b
^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a
*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/d^5*(B*d^2-2*C*c*d+3*
D*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/
d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(
1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/
(a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/
2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(1/2))))+1/d^6*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-1/2/(a*d^2+b*c^2)*d^
2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+1/2*b*c*
d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(352) = 704$.

Time = 0.11 (sec) , antiderivative size = 1351, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxi
ma")
```

output
```
1/2*sqrt(b*x^2 + a)*D*b*c^4/(b*c^2*d^5*x + a*d^7*x + b*c^3*d^4 + a*c*d^6)
+ 1/2*(b*x^2 + a)^(3/2)*D*c^3/(b*c^2*d^4*x^2 + a*d^6*x^2 + 2*b*c^3*d^3*x +
 2*a*c*d^5*x + b*c^4*d^2 + a*c^2*d^4) - 1/2*sqrt(b*x^2 + a)*C*b*c^3/(b*c^2
*d^4*x + a*d^6*x + b*c^3*d^3 + a*c*d^5) - 1/2*sqrt(b*x^2 + a)*D*b*c^3/(b*c
^2*d^4 + a*d^6) - 1/2*(b*x^2 + a)^(3/2)*C*c^2/(b*c^2*d^3*x^2 + a*d^5*x^2 +
 2*b*c^3*d^2*x + 2*a*c*d^4*x + b*c^4*d + a*c^2*d^3) + 1/2*sqrt(b*x^2 + a)*
B*b*c^2/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) + 1/2*sqrt(b*x^2 + a
)*C*b*c^2/(b*c^2*d^3 + a*d^5) + 1/2*(b*x^2 + a)^(3/2)*B*c/(b*c^2*d^2*x^2 +
 a*d^4*x^2 + 2*b*c^3*d*x + 2*a*c*d^3*x + b*c^4 + a*c^2*d^2) - 1/2*sqrt(b*x
^2 + a)*A*b*c/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) - 1/2*sqrt(b*x^2
 + a)*B*b*c/(b*c^2*d^2 + a*d^4) - 3*sqrt(b*x^2 + a)*D*c^2/(d^5*x + c*d^4)
- 1/2*(b*x^2 + a)^(3/2)*A/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d^2
*x + b*c^4/d + a*c^2*d) + 1/2*sqrt(b*x^2 + a)*A*b/(b*c^2*d + a*d^3) + 2*sq
rt(b*x^2 + a)*C*c/(d^4*x + c*d^3) - sqrt(b*x^2 + a)*B/(d^3*x + c*d^2) + 1/
2*sqrt(b*x^2 + a)*D*x/d^3 + 6*D*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^5 - 3
*C*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^4 + 1/2*D*a*arcsinh(b*x/sqrt(a*b))/(
sqrt(b)*d^3) + B*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^3 + 1/2*D*b^2*c^5*arcsin
h(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c
^2/d^2)^(3/2)*d^8) - 1/2*C*b^2*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c))
- a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^7) - 7/2*D*b*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(352) = 704$.

Time $= 0.41$ (sec) , antiderivative size $= 1235$, normalized size of antiderivative $= 3.26$

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac
")
```

output
```
1/2*sqrt(b*x^2 + a)*(D*x/d^3 - 2*(3*D*c*d^8 - C*d^9)/d^12) - (12*D*b^2*c^5
 - 6*C*b^2*c^4*d + 19*D*a*b*c^3*d^2 + 2*B*b^2*c^3*d^2 - 9*C*a*b*c^2*d^3 +
6*D*a^2*c*d^4 + 3*B*a*b*c*d^4 - 2*C*a^2*d^5 - A*a*b*d^5)*arctan(-((sqrt(b)
*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b*c^2*d^5 + a
*d^7)*sqrt(-b*c^2 - a*d^2)) - (8*(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*b^2*c^5
*d - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*b^2*c^4*d^2 + 7*(sqrt(b)*x - sqrt
(b*x^2 + a))^3*D*a*b*c^3*d^3 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*b^2*c^3
*d^3 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a*b*c^2*d^4 - 2*(sqrt(b)*x - sq
rt(b*x^2 + a))^3*A*b^2*c^2*d^4 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c
*d^5 - (sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b*d^6 + 14*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*D*b^(5/2)*c^6 - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*b^(5/2)*c
^5*d + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a*b^(3/2)*c^4*d^2 + 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*b^(5/2)*c^4*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*C*a*b^(3/2)*c^3*d^3 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d
^3 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^2*sqrt(b)*c^2*d^4 + (sqrt(b)*x
- sqrt(b*x^2 + a))^2*B*a*b^(3/2)*c^2*d^4 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*C*a^2*sqrt(b)*c*d^5 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2)*c*d^5
 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d^6 - 20*(sqrt(b)*x - s
qrt(b*x^2 + a))*D*a*b^2*c^5*d + 14*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a*b^2*c
^4*d^2 - 17*(sqrt(b)*x - sqrt(b*x^2 + a))*D*a^2*b*c^3*d^3 - 8*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3332, normalized size of antiderivative = 8.79

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**2*b**2*c**2*d**4 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
 + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x - 2*sqr
t(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a**2*b**2*d**6*x**2 + 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**4 + 16*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2
*b*c**2*d**5*x + 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**6*x**2 + 6*sqrt(a*d**2 + b*c**2)*l
og( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d*
*3 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a*b**3*c**2*d**4*x + 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c*d**5*x**2 + 20*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a*b**2*c**5*d**2 + 40*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d**3*x + 20*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b**2*c**3*d**4*x**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d + 8*sqrt(a*d**2 + b*c**2)*log
( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**4*d**...
```

3.66 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 449

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx \\
&= \frac{D\sqrt{a+bx^2}}{d^4} + \frac{(ad^2(2cCd-Bd^2-3c^2D)+b(c^3Cd-Acd^3-2c^4D))\sqrt{a+bx^2}}{2d^4\,(bc^2+ad^2)\,(c+dx)^2} \\
&\quad - \frac{(2a^2d^4(Cd-3cD)+abcd^2(6cCd-Bd^2-15c^2D)+b^2(3c^4Cd-Ac^2d^3-8c^5D))\sqrt{a+bx^2}}{2d^4\,(bc^2+ad^2)^2\,(c+dx)} \\
&\quad - \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx^2)^{3/2}}{3d^2\,(bc^2+ad^2)\,(c+dx)^3} + \frac{\sqrt{b}(Cd-4cD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^5} \\
&\quad - \frac{(2a^3d^6D-2b^3c^5(Cd-4cD)-a^2bd^4(4cCd-Bd^2-15c^2D)-ab^2cd^2(5c^2Cd-Ad^3-20c^3D))\,\text{arctan}}{2d^5\,(bc^2+ad^2)^{5/2}}
\end{aligned}
$$

output
```
D*(b*x^2+a)^(1/2)/d^4+1/2*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*(-A*c*d^3+C*c^
3*d-2*D*c^4))*(b*x^2+a)^(1/2)/d^4/(a*d^2+b*c^2)/(d*x+c)^2-1/2*(2*a^2*d^4*(
C*d-3*D*c)+a*b*c*d^2*(-B*d^2+6*C*c*d-15*D*c^2)+b^2*(-A*c^2*d^3+3*C*c^4*d-8
*D*c^5))*(b*x^2+a)^(1/2)/d^4/(a*d^2+b*c^2)^2/(d*x+c)-1/3*(A*d^3-B*c*d^2+C*
c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^3+b^(1/2)*(C*d-4*D*
c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^5-1/2*(2*a^3*d^6*D-2*b^3*c^5*(C*d-
4*D*c)-a^2*b*d^4*(-B*d^2+4*C*c*d-15*D*c^2)-a*b^2*c*d^2*(-A*d^3+5*C*c^2*d-2
0*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a
*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 11.54 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$
$$= \frac{-\frac{d\sqrt{a+bx^2}\left(2(bc^2+ad^2)^2(c^2Cd-Bcd^2+Ad^3-c^3D)+(bc^2+ad^2)(3ad^2(-2cCd+Bd^2+3c^2D)+bc(-7c^2Cd+4Bcd^2-Ad^3+10c^3D))(c+dx)-(\cdots\right.}{(bc^2+ad^2)^2\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(2*(b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D) + (b*c^2 + a*d^2)*(3*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-7*c^2
*C*d + 4*B*c*d^2 - A*d^3 + 10*c^3*D))*(c + d*x) - (-6*a^2*d^4*(C*d - 3*c*D
) + b^2*c^2*(-11*c^2*C*d + 2*B*c*d^2 + A*d^3 + 26*c^3*D) + a*b*d^2*(-20*c^
2*C*d + 5*B*c*d^2 - 2*A*d^3 + 47*c^3*D))*(c + d*x)^2 - 6*(b*c^2 + a*d^2)^2
*D*(c + d*x)^3))/((b*c^2 + a*d^2)^2*(c + d*x)^3)) + (3*(2*a^3*d^6*D + 2*b^
3*c^5*(-(C*d) + 4*c*D) + a^2*b*d^4*(-4*c*C*d + B*d^2 + 15*c^2*D) + a*b^2*c
*d^2*(-5*c^2*C*d + A*d^3 + 20*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(5/2)
+ 6*Sqrt[b]*(C*d - 4*c*D)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]] - (3*(2*a^3*d
^6*D + 2*b^3*c^5*(-(C*d) + 4*c*D) + a^2*b*d^4*(-4*c*C*d + B*d^2 + 15*c^2*D
) + a*b^2*c*d^2*(-5*c^2*C*d + A*d^3 + 20*c^3*D))*Log[a*d - b*c*x + Sqrt[b*
c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(5/2))/(6*d^5)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2182, 27, 2182, 25, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4} \, dx$$

$$\Big\downarrow \ \color{blue}{2182}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^3}dx}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^3}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ \color{blue}{2182}$$

$$\frac{-\frac{\int -\frac{\left(2\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(2a^2Dd^4-ab\left(-7Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-4Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\sqrt{bx^2+a}}{d^2(c+dx)^2}dx}{2(ad^2+bc^2)}-\frac{d\left(a+bx^2\right)^{3/2}\cdots}{\cdots}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ \color{blue}{25}$$

$$\frac{\frac{\int \frac{\left(2\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(2a^2Dd^4-ab\left(-7Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-4Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\sqrt{bx^2+a}}{d^2(c+dx)^2}dx}{2(ad^2+bc^2)} - \frac{d(a+bx^2)^{3/2}(-}{}}{ad^2+bc^2}$$

$$\frac{(a+bx^2)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(2\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(2a^2Dd^4-ab\left(-7Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-4Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\sqrt{bx^2+a}}{(c+dx)^2}dx}{2d^2(ad^2+bc^2)} - \frac{d(a+bx^2)^{3/2}(-}{}}{ad^2+bc^2}$$

$$\frac{(a+bx^2)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{\frac{-\frac{\int -\frac{2\left(2b(Cd-4cD)x\left(bc^2+ad^2\right)^2+ad\left(2a^2Dd^4-ab\left(-7Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-4Dc^4+Cdc^3-Ad^3c\right)\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2} - \frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)^2(Cd-4cD)-dx\left(2a^2d^4\right.\right.}{}}{2d^2(ad^2+bc^2)}}{ad^2+bc^2}$$

$$\frac{(a+bx^2)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{2b(Cd-4cD)x\left(bc^2+ad^2\right)^2+ad\left(2a^2Dd^4-ab\left(-7Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-4Dc^4+Cdc^3-Ad^3c\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{d^2} - \frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)^2(Cd-4cD)-dx\left(2a^2d^4D-abd\right.\right.}{d^2(c}}{2d^2(ad^2+bc^2)}}{ad^2+bc^2}$$

$$\frac{(a+bx^2)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\frac{\left(2a^3d^6D-a^2bd^4\left(-Bd^2-15c^2D+4cCd\right)-ab^2cd^2\left(-Ad^3-20c^3D+5c^2Cd\right)-2b^3c^5(Cd-4cD)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2b\left(ad^2+bc^2\right)^2(Cd-4cD)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2} -}{2d^2(ad^2+bc^2)}$$

$$\frac{(a+bx^2)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\left(2a^3d^6D-a^2bd^4\left(-Bd^2-15c^2D+4cCd\right)-ab^2cd^2\left(-Ad^3-20c^3D+5c^2Cd\right)-2b^3c^5(Cd-4cD)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2b\left(ad^2+bc^2\right)^2(Cd-4cD)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{}{\sqrt{}}}{d}}{d^2}}{2d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\left(2a^3d^6D-a^2bd^4\left(-Bd^2-15c^2D+4cCd\right)-ab^2cd^2\left(-Ad^3-20c^3D+5c^2Cd\right)-2b^3c^5(Cd-4cD)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2(}{d}}{d^2}}{2d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2(Cd-4cD)}{d}-\frac{\left(2a^3d^6D-a^2bd^4\left(-Bd^2-15c^2D+4cCd\right)-ab^2cd^2\left(-Ad^3-20c^3D+5c^2Cd\right)-2b^3c^5(Cd-4cD)\right)\int\frac{}{bc^2+ad^2}}{d}}{d^2}}{2d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2(Cd-4cD)}{d}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(2a^3d^6D-a^2bd^4\left(-Bd^2-15c^2D+4cCd\right)-ab^2cd^2\left(-Ad^3-20c^3D+5c^2C\right.\right.}{d\sqrt{ad^2+bc^2}}}{d^2}}{2d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
-1/3*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^3) + (-1/2*(d*(A*b*c - (b*c^3*(C*d - 2*c*D))/d^3 - a*(2*
c*C - B*d - (3*c^2*D)/d))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^2)
 + (-(((2*(b*c^2 + a*d^2)^2*(C*d - 4*c*D) - d*(2*a^2*d^4*D - a*b*d^2*(2*c*
C*d - B*d^2 - 7*c^2*D) - b^2*(c^3*C*d - A*c*d^3 - 4*c^4*D))*x)*Sqrt[a + b*
x^2])/(d^2*(c + d*x))) + ((2*Sqrt[b]*(b*c^2 + a*d^2)^2*(C*d - 4*c*D)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - ((2*a^3*d^6*D - 2*b^3*c^5*(C*d - 4*c*
D) - a^2*b*d^4*(4*c*C*d - B*d^2 - 15*c^2*D) - a*b^2*c*d^2*(5*c^2*C*d - A*d
^3 - 20*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]
)])/(d*Sqrt[b*c^2 + a*d^2]))/d^2)/(2*d^2*(b*c^2 + a*d^2)))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2733 vs. $2(421) = 842$.

Time = 1.54 (sec) , antiderivative size = 2734, normalized size of antiderivative = 6.09

method	result	size
default	Expression too large to display	2734

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
D/d^4*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*l
n((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/
d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+(C*d-3*D*c)/d^5*(-1/(a*d^2+b*c^2)*d
^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(a*
d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*
c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*
c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2)*d^2*(1/4*(2*
b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b
^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))))+(B*d^2-2*C
*c*d+3*D*c^2)/d^6*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*
d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c*d/(a
*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)
*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2553 vs. $2(419) = 838$.

Time = 0.14 (sec) , antiderivative size = 2553, normalized size of antiderivative = 5.69

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxi
ma")
```

output
```
1/2*sqrt(b*x^2 + a)*D*b^2*c^5/(b^2*c^4*d^5*x + 2*a*b*c^2*d^7*x + a^2*d^9*x
 + b^2*c^5*d^4 + 2*a*b*c^3*d^6 + a^2*c*d^8) + 1/2*(b*x^2 + a)^(3/2)*D*b*c^
4/(b^2*c^4*d^4*x^2 + 2*a*b*c^2*d^6*x^2 + a^2*d^8*x^2 + 2*b^2*c^5*d^3*x + 4
*a*b*c^3*d^5*x + 2*a^2*c*d^7*x + b^2*c^6*d^2 + 2*a*b*c^4*d^4 + a^2*c^2*d^6
) - 1/2*sqrt(b*x^2 + a)*C*b^2*c^4/(b^2*c^4*d^4*x + 2*a*b*c^2*d^6*x + a^2*d
^8*x + b^2*c^5*d^3 + 2*a*b*c^3*d^5 + a^2*c*d^7) - 1/2*sqrt(b*x^2 + a)*D*b^
2*c^4/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) - 1/2*(b*x^2 + a)^(3/2)*C*b*
c^3/(b^2*c^4*d^3*x^2 + 2*a*b*c^2*d^5*x^2 + a^2*d^7*x^2 + 2*b^2*c^5*d^2*x +
 4*a*b*c^3*d^4*x + 2*a^2*c*d^6*x + b^2*c^6*d + 2*a*b*c^4*d^3 + a^2*c^2*d^5
) + 1/2*sqrt(b*x^2 + a)*B*b^2*c^3/(b^2*c^4*d^3*x + 2*a*b*c^2*d^5*x + a^2*d
^7*x + b^2*c^5*d^2 + 2*a*b*c^3*d^4 + a^2*c*d^6) + 1/2*sqrt(b*x^2 + a)*C*b^
2*c^3/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) + 1/2*(b*x^2 + a)^(3/2)*B*b*
c^2/(b^2*c^4*d^2*x^2 + 2*a*b*c^2*d^4*x^2 + a^2*d^6*x^2 + 2*b^2*c^5*d*x + 4
*a*b*c^3*d^3*x + 2*a^2*c*d^5*x + b^2*c^6 + 2*a*b*c^4*d^2 + a^2*c^2*d^4) -
1/2*sqrt(b*x^2 + a)*A*b^2*c^2/(b^2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x
 + b^2*c^5*d + 2*a*b*c^3*d^3 + a^2*c*d^5) - 1/2*sqrt(b*x^2 + a)*B*b^2*c^2/
(b^2*c^4*d^2 + 2*a*b*c^2*d^4 + a^2*d^6) + 1/3*(b*x^2 + a)^(3/2)*D*c^3/(b*c
^2*d^5*x^3 + a*d^7*x^3 + 3*b*c^3*d^4*x^2 + 3*a*c*d^6*x^2 + 3*b*c^4*d^3*x +
 3*a*c^2*d^5*x + b*c^5*d^2 + a*c^3*d^4) - 3/2*sqrt(b*x^2 + a)*D*b*c^3/(b*c
^2*d^5*x + a*d^7*x + b*c^3*d^4 + a*c*d^6) - 1/2*(b*x^2 + a)^(3/2)*A*b*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2390 vs. $2(419) = 838$.

Time = 0.40 (sec) , antiderivative size = 2390, normalized size of antiderivative = 5.32

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac
")
```

output
```
(8*D*b^3*c^6 - 2*C*b^3*c^5*d + 20*D*a*b^2*c^4*d^2 - 5*C*a*b^2*c^3*d^3 + 15
*D*a^2*b*c^2*d^4 - 4*C*a^2*b*c*d^5 + A*a*b^2*c*d^5 + 2*D*a^3*d^6 + B*a^2*b
*d^6)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 -
a*d^2))/((b^2*c^4*d^5 + 2*a*b*c^2*d^7 + a^2*d^9)*sqrt(-b*c^2 - a*d^2)) + s
qrt(b*x^2 + a)*D/d^4 + 1/3*(36*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*b^3*c^6*d
^2 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*b^3*c^5*d^3 + 66*(sqrt(b)*x - sq
rt(b*x^2 + a))^5*D*a*b^2*c^4*d^4 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b^3
*c^4*d^4 - 33*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a*b^2*c^3*d^5 + 27*(sqrt(b
)*x - sqrt(b*x^2 + a))^5*D*a^2*b*c^2*d^6 + 12*(sqrt(b)*x - sqrt(b*x^2 + a)
)^5*B*a*b^2*c^2*d^6 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c*d^7 - 3
*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c*d^7 + 3*(sqrt(b)*x - sqrt(b*x^2
 + a))^5*B*a^2*b*d^8 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*b^(7/2)*c^7*d
 - 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*b^(7/2)*c^6*d^2 + 192*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*D*a*b^(5/2)*c^5*d^3 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*b^(7/2)*c^5*d^3 - 87*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a*b^(5/2)*c^4*
d^4 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(7/2)*c^4*d^4 + 39*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*D*a^2*b^(3/2)*c^3*d^5 + 24*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*B*a*b^(5/2)*c^3*d^5 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^2*b^(3/
2)*c^2*d^6 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(5/2)*c^2*d^6 - 18*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*D*a^3*sqrt(b)*c*d^7 - 3*(sqrt(b)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3915, normalized size of antiderivative = 8.72

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
(6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**3*c**3*d**6 + 18*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*c**2*d**7*x + 18*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*c*d**8
*x**2 + 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a**3*d**9*x**3 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**4*d**4 + 9*sqrt(a*d*
*2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**
2*b**2*c**3*d**5*x + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**5 + 9*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*
d**6*x**2 + 9*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*a**2*b**2*c**2*d**6*x + 3*sqrt(a*d**2 + b*c**2)*log(sq
rt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**7*x**3
+ 9*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**2*c*d**7*x**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*d**8*x**3 + 33*sqrt(a*
d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
**2*b*c**5*d**4 + 99*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a**2*b*c**4*d**5*x + 99*sqrt(a*d**2 + b*c**2...
```

3.67 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 542

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx =$$

$$-\frac{(8b^3c^7D+4a^3d^6(Cd-cD)-a^2bd^4(c^2Cd-5Bcd^2+Ad^3-13c^3D)+4ab^2c^2d^2(Ad^3+5c^3D)-d(Ab}{8d^4\left(bc^2+\right.}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\left(a+bx^2\right)^{3/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$+\frac{(4ad^2(2cCd-Bd^2-3c^2D)+bc(3c^2Cd+Bcd^2-5Ad^3-7c^3D))\left(a+bx^2\right)^{3/2}}{12d^2\left(bc^2+ad^2\right)^2(c+dx)^3}$$

$$+\frac{\sqrt{b}D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^5}$$

$$+\frac{b(8b^3c^7D-4a^3d^6(Cd-5cD)-4ab^2c^2d^2(Ad^3-7c^3D)+a^2bd^4(c^2Cd-5Bcd^2+Ad^3+35c^3D))\operatorname{arcta}}{8d^5\left(bc^2+ad^2\right)^{7/2}}$$

output
```
-1/8*(8*b^3*c^7*D+4*a^3*d^6*(C*d-D*c)-a^2*b*d^4*(A*d^3-5*B*c*d^2+C*c^2*d-1
3*D*c^3)+4*a*b^2*c^2*d^2*(A*d^3+5*D*c^3)-d*(A*b^2*c*d^3*(-a*d^2+4*b*c^2)-1
2*b^3*c^6*D-8*a^3*d^6*D+4*a^2*b*c*d^4*(C*d-9*D*c)-a*b^2*c^2*d^2*(-5*B*d^2+
C*c*d+35*D*c^2))*x)*(b*x^2+a)^(1/2)/d^4/(a*d^2+b*c^2)^3/(d*x+c)^2-1/4*(A*d
^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^4+1/12
*(4*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-5*A*d^3+B*c*d^2+3*C*c^2*d-7*D*c^3
))*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^3+b^(1/2)*D*arctanh(b^(1/2)
*x/(b*x^2+a)^(1/2))/d^5+1/8*b*(8*b^3*c^7*D-4*a^3*d^6*(C*d-5*D*c)-4*a*b^2*c
^2*d^2*(A*d^3-7*D*c^3)+a^2*b*d^4*(A*d^3-5*B*c*d^2+C*c^2*d+35*D*c^3))*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+b*c^2)^(7/
2)
```

Mathematica [A] (verified)

Time = 12.32 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$$

$$= \frac{-\frac{d\sqrt{a+bx^2}\left(6(bc^2+ad^2)^3(c^2Cd-Bcd^2+Ad^3-c^3D)+2(bc^2+ad^2)^2(4ad^2(-2cCd+Bd^2+3c^2D)+bc(-9c^2Cd+5Bcd^2-Ad^3+13c^3D))(c+dx)-\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^5,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(6*(b*c^2 + a*d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D) + 2*(b*c^2 + a*d^2)^2*(4*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-9
*c^2*C*d + 5*B*c*d^2 - A*d^3 + 13*c^3*D))*(c + d*x) - (b*c^2 + a*d^2)*(-12
*a^2*d^4*(C*d - 3*c*D) + 2*b^2*c^2*(-9*c^2*C*d + B*c*d^2 + A*d^3 + 23*c^3*
D) + a*b*d^2*(-35*c^2*C*d + 7*B*c*d^2 - 3*A*d^3 + 87*c^3*D))*(c + d*x)^2 +
 (24*a^3*d^6*D + 4*a^2*b*d^4*(-7*c*C*d + 2*B*d^2 + 33*c^2*D) - 2*b^3*c^3*(
3*c^2*C*d + B*c*d^2 + A*d^3 - 25*c^3*D) + a*b^2*c*d^2*(-19*c^2*C*d - 9*B*c
*d^2 + 13*A*d^3 + 143*c^3*D))*(c + d*x)^3))/((b*c^2 + a*d^2)^3*(c + d*x)^4
)) - (3*b*(8*b^3*c^7*D - 4*a^3*d^6*(C*d - 5*c*D) + 4*a*b^2*c^2*d^2*(-(A*d^
3) + 7*c^3*D) + a^2*b*d^4*(c^2*C*d - 5*B*c*d^2 + A*d^3 + 35*c^3*D))*Log[c
+ d*x])/(b*c^2 + a*d^2)^(7/2) + 24*Sqrt[b]*D*Log[b*x + Sqrt[b]*Sqrt[a + b*
x^2]] + (3*b*(8*b^3*c^7*D - 4*a^3*d^6*(C*d - 5*c*D) + 4*a*b^2*c^2*d^2*(-(A
*d^3) + 7*c^3*D) + a^2*b*d^4*(c^2*C*d - 5*B*c*d^2 + A*d^3 + 35*c^3*D))*Log
[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(7/2)
)/(24*d^5)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2182, 25, 2182, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^5} \, dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(4Cd-4cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+Bc-Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(4a(Cd-cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+Bc-Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(4ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-5Ad^3+Bcd^2-7c^3D+3c^2Cd\right)\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}-\frac{\int -\frac{3\left(\left(4d(Cd-2cD)a^2-\frac{bc\left(3Dc^2+Cdc-5Bd^2\right)a}{d}+Ab\left(4bc^2-ad^2\right)\right)\cdots\right.}{d^2(c+dx)^3}\cdots}{3\left(ad^2+bc^2\right)}\cdots}{4\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(4Dx\left(bc^2+ad^2\right)^2+d\left(4a^2(Cd-2cD)d^2+Ab\left(4bc^2-ad^2\right)d-abc\left(3Dc^2+Cdc-5Bd^2\right)\right)\right)\sqrt{bx^2+a}}{(c+dx)^3}dx}{d^2\left(ad^2+bc^2\right)}+\frac{\left(a+bx^2\right)^{3/2}\left(4ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-5\cdots\right.\right.}{3d^2(c+dx)^3\left(ad^2+bc^2\cdots\right.}}{4\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 680$$

$$\frac{-\frac{\int -\frac{2b\left(8Dx\left(bc^2+ad^2\right)^3+ad\left(-4b^2Dc^5-abd^2\left(11Dc^2+Cdc-5Bd^2\right)c+Abd^3\left(4bc^2-ad^2\right)+4a^2d^4(Cd-3cD)\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{4d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(4a^3d^6(Cd-cD)-a^2bd^4\left(Ad^3-5\cdots\right.\right.}{\cdots}}{d^2\left(ad^2\cdots\right.}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{8Dx\left(bc^2+ad^2\right)^3+ad\left(-4b^2Dc^5-abd^2\left(11Dc^2+Cdc-5Bd^2\right)c+Abd^3\left(4bc^2-ad^2\right)+4a^2d^4(Cd-3cD)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(4a^3d^6(Cd-cD)-a^2bd^4\left(Ad^3-5Bcd^2-1\cdots\right.\right.}{\cdots}}{d^2\left(ad^2+b\cdots\right.}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{b\left(\frac{8D\left(ad^2+bc^2\right)^3\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{\left(-4a^3d^6(Cd-5cD)+a^2bd^4\left(Ad^3-5Bcd^2+35c^3D+c^2Cd\right)-4ab^2c^2d^2\left(Ad^3-7c^3D\right)+8b^3c^7D\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+\cdots}}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{b\left(\frac{8D\left(ad^2+bc^2\right)^3\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{\left(-4a^3d^6(Cd-5cD)+a^2bd^4\left(Ad^3-5Bcd^2+35c^3D+c^2Cd\right)-4ab^2c^2d^2\left(Ad^3-7c^3D\right)+8b^3c^7D\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\cdots\right.}{2d^2\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{8D\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^3}{\sqrt{b}d}-\frac{\left(-4a^3d^6(Cd-5cD)+a^2bd^4\left(Ad^3-5Bcd^2+35c^3D+c^2Cd\right)-4ab^2c^2d^2\left(Ad^3-7c^3D\right)+8b^3c^7D\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\cdots\right.}{2d^2\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{b\left(\frac{\left(-4a^3d^6(Cd-5cD)+a^2bd^4\left(Ad^3-5Bcd^2+35c^3D+c^2Cd\right)-4ab^2c^2d^2\left(Ad^3-7c^3D\right)+8b^3c^7D\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{8D\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\cdots}{\sqrt{b}d}\cdots\right.}{2d^2\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(-4a^3d^6(Cd-5cD)+a^2bd^4\left(Ad^3-5Bcd^2+35c^3D+c^2Cd\right)-4ab^2c^2d^2\left(Ad^3-7c^3D\right)+8b^3c^7D\right)}{d\sqrt{ad^2+bc^2}}+\frac{8D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right.}{\sqrt{b}d}\right.$$
$$\overline{2d^2\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^5,x]
```

output
```
-1/4*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^4) + (((4*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(3*c^2
*C*d + B*c*d^2 - 5*A*d^3 - 7*c^3*D))*(a + b*x^2)^(3/2))/(3*d^2*(b*c^2 + a*
d^2)*(c + d*x)^3) + (-1/2*((8*b^3*c^7*D + 4*a^3*d^6*(C*d - c*D) - a^2*b*d^
4*(c^2*C*d - 5*B*c*d^2 + A*d^3 - 13*c^3*D) + 4*a*b^2*c^2*d^2*(A*d^3 + 5*c^
3*D) - d*(A*b^2*c*d^3*(4*b*c^2 - a*d^2) - 12*b^3*c^6*D - 8*a^3*d^6*D + 4*a
^2*b*c*d^4*(C*d - 9*c*D) - a*b^2*c^2*d^2*(c*C*d - 5*B*d^2 + 35*c^2*D))*x)*
Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c + d*x)^2) + (b*((8*(b*c^2 + a*d^2
)^3*D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + ((8*b^3*c^7*D -
4*a^3*d^6*(C*d - 5*c*D) - 4*a*b^2*c^2*d^2*(A*d^3 - 7*c^3*D) + a^2*b*d^4*(c
^2*C*d - 5*B*c*d^2 + A*d^3 + 35*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2
+ a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2*(b*c^2 + a*d^
2)))/(d^2*(b*c^2 + a*d^2)))/(4*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4463 vs. $2(516) = 1032$.

Time = 1.58 (sec) , antiderivative size = 4464, normalized size of antiderivative = 8.24

method	result	size
default	Expression too large to display	4464

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
D/d^5*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(
a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2
)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)
/d^2)^(1/2))))+(C*d-3*D*c)/d^6*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c^2)*(-1/(
a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2
)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/
d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**5,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4783 vs. $2(520) = 1040$.

Time = 0.23 (sec) , antiderivative size = 4783, normalized size of antiderivative = 8.82

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="maxi
ma")
```

output
```
5/8*sqrt(b*x^2 + a)*D*b^3*c^6/(b^3*c^6*d^5*x + 3*a*b^2*c^4*d^7*x + 3*a^2*b
*c^2*d^9*x + a^3*d^11*x + b^3*c^7*d^4 + 3*a*b^2*c^5*d^6 + 3*a^2*b*c^3*d^8
+ a^3*c*d^10) + 5/8*(b*x^2 + a)^(3/2)*D*b^2*c^5/(b^3*c^6*d^4*x^2 + 3*a*b^2
*c^4*d^6*x^2 + 3*a^2*b*c^2*d^8*x^2 + a^3*d^10*x^2 + 2*b^3*c^7*d^3*x + 6*a*
b^2*c^5*d^5*x + 6*a^2*b*c^3*d^7*x + 2*a^3*c*d^9*x + b^3*c^8*d^2 + 3*a*b^2*
c^6*d^4 + 3*a^2*b*c^4*d^6 + a^3*c^2*d^8) - 5/8*sqrt(b*x^2 + a)*C*b^3*c^5/(
b^3*c^6*d^4*x + 3*a*b^2*c^4*d^6*x + 3*a^2*b*c^2*d^8*x + a^3*d^10*x + b^3*c
^7*d^3 + 3*a*b^2*c^5*d^5 + 3*a^2*b*c^3*d^7 + a^3*c*d^9) - 5/8*sqrt(b*x^2 +
 a)*D*b^3*c^5/(b^3*c^6*d^4 + 3*a*b^2*c^4*d^6 + 3*a^2*b*c^2*d^8 + a^3*d^10)
 - 5/8*(b*x^2 + a)^(3/2)*C*b^2*c^4/(b^3*c^6*d^3*x^2 + 3*a*b^2*c^4*d^5*x^2
+ 3*a^2*b*c^2*d^7*x^2 + a^3*d^9*x^2 + 2*b^3*c^7*d^2*x + 6*a*b^2*c^5*d^4*x
+ 6*a^2*b*c^3*d^6*x + 2*a^3*c*d^8*x + b^3*c^8*d + 3*a*b^2*c^6*d^3 + 3*a^2*
b*c^4*d^5 + a^3*c^2*d^7) + 5/8*sqrt(b*x^2 + a)*B*b^3*c^4/(b^3*c^6*d^3*x +
3*a*b^2*c^4*d^5*x + 3*a^2*b*c^2*d^7*x + a^3*d^9*x + b^3*c^7*d^2 + 3*a*b^2*
c^5*d^4 + 3*a^2*b*c^3*d^6 + a^3*c*d^8) + 5/8*sqrt(b*x^2 + a)*C*b^3*c^4/(b^
3*c^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2*d^7 + a^3*d^9) + 5/8*(b*x^2 + a)
^(3/2)*B*b^2*c^3/(b^3*c^6*d^2*x^2 + 3*a*b^2*c^4*d^4*x^2 + 3*a^2*b*c^2*d^6*
x^2 + a^3*d^8*x^2 + 2*b^3*c^7*d*x + 6*a*b^2*c^5*d^3*x + 6*a^2*b*c^3*d^5*x
+ 2*a^3*c*d^7*x + b^3*c^8 + 3*a*b^2*c^6*d^2 + 3*a^2*b*c^4*d^4 + a^3*c^2*d^
6) - 5/8*sqrt(b*x^2 + a)*A*b^3*c^3/(b^3*c^6*d^2*x + 3*a*b^2*c^4*d^4*x +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^5}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(Dx^3+C\,x^2+Bx+A\right)}{\left(dx+c\right)^5}dx$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

output
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

3.68 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 525

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx =$$

$$-\frac{(Ab^2c(4bc^2-3ad^2)-a(b^2c^2(cC-6Bd)-4a^2d^3D-abd(6cCd-Bd^2-3c^2D)))\,(ad-bcx)\sqrt{a+bx^2}}{8\,(bc^2+ad^2)^4\,(c+dx)^2}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx^2)^{3/2}}{5d^2\,(bc^2+ad^2)\,(c+dx)^5}$$

$$+\frac{(5ad^2(2cCd-Bd^2-3c^2D)+bc(3c^2Cd+2Bcd^2-7Ad^3-8c^3D))\,(a+bx^2)^{3/2}}{20d^2\,(bc^2+ad^2)^2\,(c+dx)^4}$$

$$-\frac{(20a^2d^4(Cd-3cD)-b^2c^2(3c^2Cd+2Bcd^2-27Ad^3+12c^3D)-abd^2(18c^2Cd-33Bcd^2+8Ad^3+37}{60d^2\,(bc^2+ad^2)^3\,(c+dx)^3}$$

$$-\frac{ab(Ab^2c(4bc^2-3ad^2)-a(b^2c^2(cC-6Bd)-4a^2d^3D-abd(6cCd-Bd^2-3c^2D)))\,\mathrm{arctanh}\left(\frac{ad-b}{\sqrt{bc^2+ad^2}}\right.}{8\,(bc^2+ad^2)^{9/2}}$$

output
```
-1/8*(A*b^2*c*(-3*a*d^2+4*b*c^2)-a*(b^2*c^2*(-6*B*d+C*c)-4*a^2*d^3*D-a*b*d
*(-B*d^2+6*C*c*d-3*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^4/(
d*x+c)^2-1/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^
2)/(d*x+c)^5+1/20*(5*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-7*A*d^3+2*B*c*d^
2+3*C*c^2*d-8*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^4-1/60*(
20*a^2*d^4*(C*d-3*D*c)-b^2*c^2*(-27*A*d^3+2*B*c*d^2+3*C*c^2*d+12*D*c^3)-a*
b*d^2*(8*A*d^3-33*B*c*d^2+18*C*c^2*d+37*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d^2
+b*c^2)^3/(d*x+c)^3-1/8*a*b*(A*b^2*c*(-3*a*d^2+4*b*c^2)-a*(b^2*c^2*(-6*B*d
+C*c)-4*a^2*d^3*D-a*b*d*(-B*d^2+6*C*c*d-3*D*c^2)))*arctanh((-b*c*x+a*d)/(a
*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 12.13 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx =$$

$$-\frac{\sqrt{a+bx^2}\Big(24(bc^2+ad^2)^4\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)+6(bc^2+ad^2)^3\left(5ad^2(-2cCd+Bd^2+3c^2D)\right.}{}$$

$$+\frac{ab(Ab^2c(4bc^2-3ad^2)+a(b^2c^2(-cC+6Bd)+4a^2d^3D-abd(-6cCd+Bd^2+3c^2D)))\log(c+dx)}{8\left(bc^2+ad^2\right)^{9/2}}$$

$$-\frac{ab(Ab^2c(4bc^2-3ad^2)+a(b^2c^2(-cC+6Bd)+4a^2d^3D-abd(-6cCd+Bd^2+3c^2D)))\log\left(ad-bcx\right.}{8\left(bc^2+ad^2\right)^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + dx)^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(24*(b*c^2 + a*d^2)^4*(c^2*C*d - B*c*d^2 + A*d^3 -
 c^3*D) + 6*(b*c^2 + a*d^2)^3*(5*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*
(-11*c^2*C*d + 6*B*c*d^2 - A*d^3 + 16*c^3*D))*(c + d*x) - 2*(b*c^2 + a*d^2
)^2*(-20*a^2*d^4*(C*d - 3*c*D) + b^2*c^2*(-27*c^2*C*d + 2*B*c*d^2 + 3*A*d^
3 + 72*c^3*D) + a*b*d^2*(-54*c^2*C*d + 9*B*c*d^2 - 4*A*d^3 + 139*c^3*D))*(
c + d*x)^2 + (b*c^2 + a*d^2)*(60*a^3*d^6*D + 5*a^2*b*d^4*(-10*c*C*d + 3*B*
d^2 + 57*c^2*D) + 2*b^3*c^3*(-3*c^2*C*d - 2*B*c*d^2 - 3*A*d^3 + 48*c^3*D)
+ a*b^2*c*d^2*(-21*c^2*C*d - 24*B*c*d^2 + 29*A*d^3 + 286*c^3*D))*(c + d*x)
^3 - b*(20*a^3*d^6*(-2*C*d + 9*c*D) + 2*b^3*c^4*(3*c^2*C*d + 2*B*c*d^2 + 3
*A*d^3 + 12*c^3*D) + a*b^2*c^2*d^2*(27*c^2*C*d + 28*B*c*d^2 - 83*A*d^3 + 9
8*c^3*D) + a^2*b*d^4*(86*c^2*C*d - 81*B*c*d^2 + 16*A*d^3 + 149*c^3*D))*(c
+ d*x)^4))/((b*c^2*d + a*d^3)^4*(c + d*x)^5) + (a*b*(A*b^2*c*(4*b*c^2 - 3*
a*d^2) + a*(b^2*c^2*(-(c*C) + 6*B*d) + 4*a^2*d^3*D - a*b*d*(-6*c*C*d + B*d
^2 + 3*c^2*D)))*Log[c + d*x])/(8*(b*c^2 + a*d^2)^(9/2)) - (a*b*(A*b^2*c*(4
*b*c^2 - 3*a*d^2) + a*(b^2*c^2*(-(c*C) + 6*B*d) + 4*a^2*d^3*D - a*b*d*(-6*
c*C*d + B*d^2 + 3*c^2*D)))*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a +
b*x^2]])/(8*(b*c^2 + a*d^2)^(9/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2182, 25, 2182, 25, 27, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^6}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(5\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(5Cd-5cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x+5\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^5}dx}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(5\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(5a(Cd-cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x+5\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^5}dx}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(5ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-7Ad^3+2Bcd^2-8c^3D+3c^2Cd\right)\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}-\int -\frac{\left(4\left(Ab\left(5bc^2-2ad^2\right)+a\left(5ad(Cd-2cD)-bc\left(\frac{3Dc^2}{d}+2Cc-7Bd\right)\right.\right.\right.\cdots}{\cdots}}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(4\left(Ab\left(5bc^2-2ad^2\right)+a\left(5ad(Cd-2cD)-bc\left(\frac{3Dc^2}{d}+2Cc-7Bd\right)\right)\right)d^2+\left(20a^2Dd^4+5ab\left(5Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(12Dc^3+3Cdc^2+2Bd^2c-7Ad^3\right)\right)x\right)\sqrt{bx^2\cdots}}{d^2(c+dx)^4}\cdots}{4\left(ad^2+bc^2\right)}\cdots}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(4\left(Ab\left(5bc^2-2ad^2\right)+a\left(5ad(Cd-2cD)-bc\left(\frac{3Dc^2}{d}+2Cc-7Bd\right)\right)\right)d^2+\left(20a^2Dd^4+5ab\left(5Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(12Dc^3+3Cdc^2+2Bd^2c-7Ad^3\right)\right)x\right)\sqrt{bx^2\cdots}}{(c+dx)^4}\cdots}{4d^2\left(ad^2+bc^2\right)}\cdots}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{\frac{5d^2\left(Ab^2c\left(4bc^2-3ad^2\right)-a\left(-4a^2d^3D-abd\left(-Bd^2-3c^2D+6cCd\right)+b^2c^2(cC-6Bd)\right)\right)\int\frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(20a^2d^4(Cd-3cD)-abd^2\left(8Ad^3-33Bcd^2+37\cdots\right.\right.}{3(c+dx)^3\left(\cdots\right.}}{4d^2\left(ad^2+bc^2\right)}\cdots}{5\left(ad^2+b\cdots\right.}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{5d^2\left(Ab^2c\left(4bc^2-3ad^2\right)-a\left(-4a^2d^3D-abd\left(-Bd^2-3c^2D+6cCd\right)+b^2c^2(cC-6Bd)\right)\right)\left(\frac{ab\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(20a^2d^4\right.}{}}{4d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{5d^2\left(Ab^2c\left(4bc^2-3ad^2\right)-a\left(-4a^2d^3D-abd\left(-Bd^2-3c^2D+6cCd\right)+b^2c^2(cC-6Bd)\right)\right)\left(-\frac{ab\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{\left(a+b\right.}{}}{4d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{5d^2\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)\left(Ab^2c\left(4bc^2-3ad^2\right)-a\left(-4a^2d^3D-abd\left(-Bd^2-3c^2D+6cCd\right)+b^2c^2(cC-6Bd)\right)\right)}{ad^2+bc^2}-\frac{\left(a+b\right.}{}}{4d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^6,x]
```

output
```
-1/5*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^5) + (((5*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(3*c^2
*C*d + 2*B*c*d^2 - 7*A*d^3 - 8*c^3*D))*(a + b*x^2)^(3/2))/(4*d^2*(b*c^2 +
a*d^2)*(c + d*x)^4) + (-1/3*((20*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(3*c^2*C*
d + 2*B*c*d^2 - 27*A*d^3 + 12*c^3*D) - a*b*d^2*(18*c^2*C*d - 33*B*c*d^2 +
8*A*d^3 + 37*c^3*D))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^3) + (5
*d^2*(A*b^2*c*(4*b*c^2 - 3*a*d^2) - a*(b^2*c^2*(c*C - 6*B*d) - 4*a^2*d^3*D
 - a*b*d*(6*c*C*d - B*d^2 - 3*c^2*D)))*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2
])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2
+ a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2))))/(b*c^2 + a*d^2))/(
4*d^2*(b*c^2 + a*d^2)))/(5*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7009 vs. $2(501) = 1002$.

Time = 1.65 (sec) , antiderivative size = 7010, normalized size of antiderivative = 13.35

method	result	size
default	Expression too large to display	7010

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8174 vs. $2(498) = 996$.

Time = 0.33 (sec) , antiderivative size = 8174, normalized size of antiderivative = 15.57

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="maxi
ma")
```

output
```
7/8*sqrt(b*x^2 + a)*D*b^4*c^7/(b^4*c^8*d^5*x + 4*a*b^3*c^6*d^7*x + 6*a^2*b
^2*c^4*d^9*x + 4*a^3*b*c^2*d^11*x + a^4*d^13*x + b^4*c^9*d^4 + 4*a*b^3*c^7
*d^6 + 6*a^2*b^2*c^5*d^8 + 4*a^3*b*c^3*d^10 + a^4*c*d^12) + 7/8*(b*x^2 + a
)^(3/2)*D*b^3*c^6/(b^4*c^8*d^4*x^2 + 4*a*b^3*c^6*d^6*x^2 + 6*a^2*b^2*c^4*d
^8*x^2 + 4*a^3*b*c^2*d^10*x^2 + a^4*d^12*x^2 + 2*b^4*c^9*d^3*x + 8*a*b^3*c
^7*d^5*x + 12*a^2*b^2*c^5*d^7*x + 8*a^3*b*c^3*d^9*x + 2*a^4*c*d^11*x + b^4
*c^10*d^2 + 4*a*b^3*c^8*d^4 + 6*a^2*b^2*c^6*d^6 + 4*a^3*b*c^4*d^8 + a^4*c^
2*d^10) - 7/8*sqrt(b*x^2 + a)*C*b^4*c^6/(b^4*c^8*d^4*x + 4*a*b^3*c^6*d^6*x
 + 6*a^2*b^2*c^4*d^8*x + 4*a^3*b*c^2*d^10*x + a^4*d^12*x + b^4*c^9*d^3 + 4
*a*b^3*c^7*d^5 + 6*a^2*b^2*c^5*d^7 + 4*a^3*b*c^3*d^9 + a^4*c*d^11) - 7/8*s
qrt(b*x^2 + a)*D*b^4*c^6/(b^4*c^8*d^4 + 4*a*b^3*c^6*d^6 + 6*a^2*b^2*c^4*d^
8 + 4*a^3*b*c^2*d^10 + a^4*d^12) - 7/8*(b*x^2 + a)^(3/2)*C*b^3*c^5/(b^4*c^
8*d^3*x^2 + 4*a*b^3*c^6*d^5*x^2 + 6*a^2*b^2*c^4*d^7*x^2 + 4*a^3*b*c^2*d^9*
x^2 + a^4*d^11*x^2 + 2*b^4*c^9*d^2*x + 8*a*b^3*c^7*d^4*x + 12*a^2*b^2*c^5*
d^6*x + 8*a^3*b*c^3*d^8*x + 2*a^4*c*d^10*x + b^4*c^10*d + 4*a*b^3*c^8*d^3
+ 6*a^2*b^2*c^6*d^5 + 4*a^3*b*c^4*d^7 + a^4*c^2*d^9) + 7/8*sqrt(b*x^2 + a)
*B*b^4*c^5/(b^4*c^8*d^3*x + 4*a*b^3*c^6*d^5*x + 6*a^2*b^2*c^4*d^7*x + 4*a^
3*b*c^2*d^9*x + a^4*d^11*x + b^4*c^9*d^2 + 4*a*b^3*c^7*d^4 + 6*a^2*b^2*c^5
*d^6 + 4*a^3*b*c^3*d^8 + a^4*c*d^10) + 7/8*sqrt(b*x^2 + a)*C*b^4*c^5/(b^4*
c^8*d^3 + 4*a*b^3*c^6*d^5 + 6*a^2*b^2*c^4*d^7 + 4*a^3*b*c^2*d^9 + a^4*d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5885 vs. $2(498) = 996$.

Time = 0.47 (sec) , antiderivative size = 5885, normalized size of antiderivative = 11.21

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="giac
")
```

output
```
-1/4*(C*a^2*b^3*c^3 - 4*A*a*b^4*c^3 + 3*D*a^3*b^2*c^2*d - 6*B*a^2*b^3*c^2*
d - 6*C*a^3*b^2*c*d^2 + 3*A*a^2*b^3*c*d^2 - 4*D*a^4*b*d^3 + B*a^3*b^2*d^3)
*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2
))/((b^4*c^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4
*d^8)*sqrt(-b*c^2 - a*d^2)) + 1/60*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*
b^5*c^8*d^4 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b^4*c^6*d^6 + 720*(s
qrt(b)*x - sqrt(b*x^2 + a))^9*D*a^2*b^3*c^4*d^8 + 15*(sqrt(b)*x - sqrt(b*x
^2 + a))^9*C*a^2*b^3*c^3*d^9 - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^4*
c^3*d^9 + 525*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^3*b^2*c^2*d^10 - 90*(sqr
t(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c^2*d^10 - 90*(sqrt(b)*x - sqrt(b*x^
2 + a))^9*C*a^3*b^2*c*d^11 + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^2*b^3*
c*d^11 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^4*b*d^12 + 15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^9*B*a^3*b^2*d^12 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D
*b^(11/2)*c^9*d^3 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*b^(11/2)*c^8*d^4
 + 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a*b^(9/2)*c^7*d^5 + 480*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*C*a*b^(9/2)*c^6*d^6 + 2880*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*D*a^2*b^(7/2)*c^5*d^7 + 855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2
*b^(7/2)*c^4*d^8 - 540*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(9/2)*c^4*d^8
 + 2325*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^3*b^(5/2)*c^3*d^9 - 810*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(7/2)*c^3*d^9 - 330*(sqrt(b)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^6}dx$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

output
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

3.69 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 707

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx =$$

$$-\frac{b(Ab(8b^2c^4-12abc^2d^2+a^2d^4)-a(2b^2c^3(cC-7Bd)+2a^2d^3(Cd-7cD)-abcd(17cCd-7Bd^2-7c}{16(bc^2+ad^2)^5(c+dx)^2}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{3/2}}{6d^2(bc^2+ad^2)(c+dx)^6}$$

$$+\frac{(2ad^2(2cCd-Bd^2-3c^2D)+bc(c^2Cd+Bcd^2-3Ad^3-3c^3D))(a+bx^2)^{3/2}}{10d^2(bc^2+ad^2)^2(c+dx)^5}$$

$$-\frac{(10a^2d^4(Cd-3cD)-2b^2c^2(c^2Cd+Bcd^2-8Ad^3+2c^3D)-abd^2(13c^2Cd-19Bcd^2+5Ad^3+13c^3I}{40d^2(bc^2+ad^2)^3(c+dx)^4}$$

$$-\frac{(40a^3d^6D+2a^2bd^4(41cCd-8Bd^2-39c^2D)-2b^3c^3(c^2Cd+Bcd^2-28Ad^3+2c^3D)-ab^2cd^2(25c^2C}{120d^2(bc^2+ad^2)^4(c+dx)^3}$$

$$-\frac{ab^2(Ab(8b^2c^4-12abc^2d^2+a^2d^4)-a(2b^2c^3(cC-7Bd)+2a^2d^3(Cd-7cD)-abcd(17cCd-7Bd^2-}{16(bc^2+ad^2)^{11/2}}$$

output
```
-1/16*b*(A*b*(a^2*d^4-12*a*b*c^2*d^2+8*b^2*c^4)-a*(2*b^2*c^3*(-7*B*d+C*c)+
2*a^2*d^3*(C*d-7*D*c)-a*b*c*d*(-7*B*d^2+17*C*c*d-7*D*c^2)))*(-b*c*x+a*d)*(
b*x^2+a)^(1/2)/(a*d^2+b*c^2)^5/(d*x+c)^2-1/6*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)
*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^6+1/10*(2*a*d^2*(-B*d^2+2*C*c*d
-3*D*c^2)+b*c*(-3*A*d^3+B*c*d^2+C*c^2*d-3*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d
^2+b*c^2)^2/(d*x+c)^5-1/40*(10*a^2*d^4*(C*d-3*D*c)-2*b^2*c^2*(-8*A*d^3+B*c
*d^2+C*c^2*d+2*D*c^3)-a*b*d^2*(5*A*d^3-19*B*c*d^2+13*C*c^2*d+13*D*c^3))*(b
*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^4-1/120*(40*a^3*d^6*D+2*a^2*b*d^
4*(-8*B*d^2+41*C*c*d-39*D*c^2)-2*b^3*c^3*(-28*A*d^3+B*c*d^2+C*c^2*d+2*D*c^
3)-a*b^2*c*d^2*(49*A*d^3-87*B*c*d^2+25*C*c^2*d+17*D*c^3))*(b*x^2+a)^(3/2)/
d^2/(a*d^2+b*c^2)^4/(d*x+c)^3-1/16*a*b^2*(A*b*(a^2*d^4-12*a*b*c^2*d^2+8*b^
2*c^4)-a*(2*b^2*c^3*(-7*B*d+C*c)+2*a^2*d^3*(C*d-7*D*c)-a*b*c*d*(-7*B*d^2+1
7*C*c*d-7*D*c^2)))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2
))/(a*d^2+b*c^2)^(11/2)
```

Mathematica [A] (verified)

Time = 13.35 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx$$

$$= \frac{1}{240}\left(-\frac{\sqrt{a+bx^2}\Big(40(bc^2+ad^2)^5\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)+8(bc^2+ad^2)^4\left(6ad^2(-2cCd+Bd^2+3\right.}{}\right.$$

$$+\frac{15ab^2(Ab(8b^2c^4-12abc^2d^2+a^2d^4)+a(-2b^2c^3(cC-7Bd)-2a^2d^3(Cd-7cD)+abcd(17cCd-7Bc}{(bc^2+ad^2)^{11/2}}$$

$$-\frac{15ab^2(Ab(8b^2c^4-12abc^2d^2+a^2d^4)+a(-2b^2c^3(cC-7Bd)-2a^2d^3(Cd-7cD)+abcd(17cCd-7Bc}{(bc^2+ad^2)^{11/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
(-((Sqrt[a + b*x^2]*(40*(b*c^2 + a*d^2)^5*(c^2*C*d - B*c*d^2 + A*d^3 - c^3
*D) + 8*(b*c^2 + a*d^2)^4*(6*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-13
*c^2*C*d + 7*B*c*d^2 - A*d^3 + 19*c^3*D))*(c + d*x) - 2*(b*c^2 + a*d^2)^3*
(-30*a^2*d^4*(C*d - 3*c*D) + 2*b^2*c^2*(-19*c^2*C*d + B*c*d^2 + 2*A*d^3 +
52*c^3*D) + a*b*d^2*(-77*c^2*C*d + 11*B*c*d^2 - 5*A*d^3 + 203*c^3*D))*(c +
 d*x)^2 + 2*(b*c^2 + a*d^2)^2*(40*a^3*d^6*D + 2*a^2*b*d^4*(-13*c*C*d + 4*B
*d^2 + 87*c^2*D) - 2*b^3*c^3*(c^2*C*d + B*c*d^2 + 2*A*d^3 - 28*c^3*D) + a*
b^2*c*d^2*(-7*c^2*C*d - 15*B*c*d^2 + 17*A*d^3 + 169*c^3*D))*(c + d*x)^3 -
b*(b*c^2 + a*d^2)*(10*a^3*d^6*(-3*C*d + 13*c*D) + 4*b^3*c^4*(c^2*C*d + B*c
*d^2 + 2*A*d^3 + 2*c^3*D) + 2*a*b^2*c^2*d^2*(10*c^2*C*d + 18*B*c*d^2 - 41*
A*d^3 + 17*c^3*D) + a^2*b*d^4*(91*c^2*C*d - 73*B*c*d^2 + 15*A*d^3 + 51*c^3
*D))*(c + d*x)^4 - b*(-80*a^4*d^8*D + 2*a^3*b*d^6*(-97*c*C*d + 16*B*d^2 +
143*c^2*D) + 4*b^4*c^5*(c^2*C*d + B*c*d^2 + 2*A*d^3 + 2*c^3*D) + 2*a*b^3*c
^3*d^2*(12*c^2*C*d + 20*B*c*d^2 - 97*A*d^3 + 21*c^3*D) + a^2*b^2*c*d^4*(14
1*c^2*C*d - 247*B*c*d^2 + 113*A*d^3 + 85*c^3*D))*(c + d*x)^5))/(d^4*(b*c^2
 + a*d^2)^5*(c + d*x)^6)) + (15*a*b^2*(A*b*(8*b^2*c^4 - 12*a*b*c^2*d^2 + a
^2*d^4) + a*(-2*b^2*c^3*(c*C - 7*B*d) - 2*a^2*d^3*(C*d - 7*c*D) + a*b*c*d*
(17*c*C*d - 7*B*d^2 - 7*c^2*D)))*Log[c + d*x])/(b*c^2 + a*d^2)^(11/2) - (1
5*a*b^2*(A*b*(8*b^2*c^4 - 12*a*b*c^2*d^2 + a^2*d^4) + a*(-2*b^2*c^3*(c*C -
 7*B*d) - 2*a^2*d^3*(C*d - 7*c*D) + a*b*c*d*(17*c*C*d - 7*B*d^2 - 7*c^2...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 685, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2182, 27, 2182, 25, 27, 688, 25, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^7}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)+b\left(-\frac{Dc^3}{d^2}+\frac{Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)+b\left(-\frac{Dc^3}{d^2}+\frac{Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3+Bcd^2-3c^3D+c^2Cd\right)\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}-\int -\frac{\left(5\left(2d(Cd-2cD)a^2-\frac{bc\left(Dc^2+Cdc-3Bd^2\right)a}{d}+Ab\left(2bc^2-ad^2\right)\right)d^{2}\cdots\right.}{\cdots}}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(5d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2-ad^2\right)d-abc\left(Dc^2+Cdc-3Bd^2\right)\right)+2\left(5a^2Dd^4+2ab\left(2Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(2Dc^3+Cdc^2+Bd^2c-3Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{d^2(c+dx)^5}dx}{5\left(ad^2+bc^2\right)}\cdots}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(5d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2-ad^2\right)d-abc\left(Dc^2+Cdc-3Bd^2\right)\right)+2\left(5a^2Dd^4+2ab\left(2Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(2Dc^3+Cdc^2+Bd^2c-3Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{(c+dx)^5}dx}{5d^2\left(ad^2+bc^2\right)}\cdots}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{-\frac{\int -\frac{\left(4d\left(Acd\left(10bc^2-11ad^2\right)b^2+a\left(10a^2Dd^4+2ab\left(-6Dc^2+9Cdc-2Bd^2\right)d^2-b^2c^2\left(Dc^2+3Cdc-17Bd^2\right)\right)\right)-b\left(10a^2(Cd-3cD)d^4-ab\left(13Dc^3+13Cdc^2-19Bd^{2}\cdots\right.\right.\right.}{(c+dx)^4}\cdots}{4\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\int \frac{\left(4d\left(Acd\left(10bc^2-11ad^2\right)b^2+a\left(10a^2Dd^4+2ab\left(-6Dc^2+9Cdc-2Bd^2\right)d^2-b^2c^2\left(Dc^2+3Cdc-17Bd^2\right)\right)\right)-b\left(10a^2(Cd-3cD)d^4-ab\left(13Dc^3+13Cdc^2-19Bd^2c+5\right.\right.\right.}{(c+dx)^4}$$

$$\overline{4\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 679$$

$$\frac{\frac{5bd^2\left(Ab\left(a^2d^4-12abc^2d^2+8b^2c^4\right)-a\left(2a^2d^3(Cd-7cD)-abcd\left(-7Bd^2-7c^2D+17cCd\right)+2b^2c^3(cC-7Bd)\right)\right)\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(40a^3d^6D+2a^2bd^4\left(-\right.\right.}{}}{4\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 486$$

$$\frac{\frac{5bd^2\left(Ab\left(a^2d^4-12abc^2d^2+8b^2c^4\right)-a\left(2a^2d^3(Cd-7cD)-abcd\left(-7Bd^2-7c^2D+17cCd\right)+2b^2c^3(cC-7Bd)\right)\right)\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}}{4\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 488$$

$$\frac{\frac{5bd^2\left(Ab\left(a^2d^4-12abc^2d^2+8b^2c^4\right)-a\left(2a^2d^3(Cd-7cD)-abcd\left(-7Bd^2-7c^2D+17cCd\right)+2b^2c^3(cC-7Bd)\right)\right)\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+b}}{2(c+dx)}\right.}{ad^2+bc^2}}{4\left(ad^2\right.}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 219$$

$$\frac{\frac{\frac{5bd^2\left(-\frac{ab\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)\left(Ab\left(a^2d^4-12abc^2d^2+8b^2c^4\right)-a\left(2a^2d^3(Cd-7cD)-abcd\left(-7Bd^2-7c^2D+17cCd\right)+2b^2c\right.\right.}{ad^2+bc^2}}{4\left(ad^2\right.}}{}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
-1/6*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^6) + (((2*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(c^2*C
*d + B*c*d^2 - 3*A*d^3 - 3*c^3*D))*(a + b*x^2)^(3/2))/(5*d^2*(b*c^2 + a*d^
2)*(c + d*x)^5) + (-1/4*((10*a^2*d^4*(C*d - 3*c*D) - 2*b^2*c^2*(c^2*C*d +
B*c*d^2 - 8*A*d^3 + 2*c^3*D) - a*b*d^2*(13*c^2*C*d - 19*B*c*d^2 + 5*A*d^3
+ 13*c^3*D))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (-1/3*((40
*a^3*d^6*D + 2*a^2*b*d^4*(41*c*C*d - 8*B*d^2 - 39*c^2*D) - 2*b^3*c^3*(c^2*
C*d + B*c*d^2 - 28*A*d^3 + 2*c^3*D) - a*b^2*c*d^2*(25*c^2*C*d - 87*B*c*d^2
 + 49*A*d^3 + 17*c^3*D))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^3)
+ (5*b*d^2*(A*b*(8*b^2*c^4 - 12*a*b*c^2*d^2 + a^2*d^4) - a*(2*b^2*c^3*(c*C
 - 7*B*d) + 2*a^2*d^3*(C*d - 7*c*D) - a*b*c*d*(17*c*C*d - 7*B*d^2 - 7*c^2*
D)))*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) -
 (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*
c^2 + a*d^2)^(3/2))))/(b*c^2 + a*d^2))/(4*(b*c^2 + a*d^2)))/(5*d^2*(b*c^2
+ a*d^2)))/(2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11285 vs. 2(679) = 1358.

Time = 1.81 (sec) , antiderivative size = 11286, normalized size of antiderivative = 15.96

method	result	size
default	Expression too large to display	11286

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13292 vs. 2(681) = 1362.

Time = 0.52 (sec) , antiderivative size = 13292, normalized size of antiderivative = 18.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="maxi
ma")
```

output
```
21/16*sqrt(b*x^2 + a)*D*b^5*c^8/(b^5*c^10*d^5*x + 5*a*b^4*c^8*d^7*x + 10*a
^2*b^3*c^6*d^9*x + 10*a^3*b^2*c^4*d^11*x + 5*a^4*b*c^2*d^13*x + a^5*d^15*x
 + b^5*c^11*d^4 + 5*a*b^4*c^9*d^6 + 10*a^2*b^3*c^7*d^8 + 10*a^3*b^2*c^5*d^
10 + 5*a^4*b*c^3*d^12 + a^5*c*d^14) + 21/16*(b*x^2 + a)^(3/2)*D*b^4*c^7/(b
^5*c^10*d^4*x^2 + 5*a*b^4*c^8*d^6*x^2 + 10*a^2*b^3*c^6*d^8*x^2 + 10*a^3*b^
2*c^4*d^10*x^2 + 5*a^4*b*c^2*d^12*x^2 + a^5*d^14*x^2 + 2*b^5*c^11*d^3*x +
10*a*b^4*c^9*d^5*x + 20*a^2*b^3*c^7*d^7*x + 20*a^3*b^2*c^5*d^9*x + 10*a^4*
b*c^3*d^11*x + 2*a^5*c*d^13*x + b^5*c^12*d^2 + 5*a*b^4*c^10*d^4 + 10*a^2*b
^3*c^8*d^6 + 10*a^3*b^2*c^6*d^8 + 5*a^4*b*c^4*d^10 + a^5*c^2*d^12) - 21/16
*sqrt(b*x^2 + a)*C*b^5*c^7/(b^5*c^10*d^4*x + 5*a*b^4*c^8*d^6*x + 10*a^2*b^
3*c^6*d^8*x + 10*a^3*b^2*c^4*d^10*x + 5*a^4*b*c^2*d^12*x + a^5*d^14*x + b^
5*c^11*d^3 + 5*a*b^4*c^9*d^5 + 10*a^2*b^3*c^7*d^7 + 10*a^3*b^2*c^5*d^9 + 5
*a^4*b*c^3*d^11 + a^5*c*d^13) - 21/16*sqrt(b*x^2 + a)*D*b^5*c^7/(b^5*c^10*
d^4 + 5*a*b^4*c^8*d^6 + 10*a^2*b^3*c^6*d^8 + 10*a^3*b^2*c^4*d^10 + 5*a^4*b
*c^2*d^12 + a^5*d^14) - 21/16*(b*x^2 + a)^(3/2)*C*b^4*c^6/(b^5*c^10*d^3*x^
2 + 5*a*b^4*c^8*d^5*x^2 + 10*a^2*b^3*c^6*d^7*x^2 + 10*a^3*b^2*c^4*d^9*x^2
+ 5*a^4*b*c^2*d^11*x^2 + a^5*d^13*x^2 + 2*b^5*c^11*d^2*x + 10*a*b^4*c^9*d^
4*x + 20*a^2*b^3*c^7*d^6*x + 20*a^3*b^2*c^5*d^8*x + 10*a^4*b*c^3*d^10*x +
2*a^5*c*d^12*x + b^5*c^12*d + 5*a*b^4*c^10*d^3 + 10*a^2*b^3*c^8*d^5 + 10*a
^3*b^2*c^6*d^7 + 5*a^4*b*c^4*d^9 + a^5*c^2*d^11) + 21/16*sqrt(b*x^2 + a...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8165 vs. $2(681) = 1362$.

Time = 0.51 (sec) , antiderivative size = 8165, normalized size of antiderivative = 11.55

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="giac
")
```

output
```
1/8*(2*C*a^2*b^4*c^4 - 8*A*a*b^5*c^4 + 7*D*a^3*b^3*c^3*d - 14*B*a^2*b^4*c^
3*d - 17*C*a^3*b^3*c^2*d^2 + 12*A*a^2*b^4*c^2*d^2 - 14*D*a^4*b^2*c*d^3 + 7
*B*a^3*b^3*c*d^3 + 2*C*a^4*b^2*d^4 - A*a^3*b^3*d^4)*arctan(((sqrt(b)*x - s
qrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^5*c^10 + 5*a*b^4*
c^8*d^2 + 10*a^2*b^3*c^6*d^4 + 10*a^3*b^2*c^4*d^6 + 5*a^4*b*c^2*d^8 + a^5*
d^10)*sqrt(-b*c^2 - a*d^2)) + 1/120*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C
*a^2*b^4*c^4*d^10 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a*b^5*c^4*d^10
 + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^3*b^3*c^3*d^11 - 210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^11*B*a^2*b^4*c^3*d^11 - 255*(sqrt(b)*x - sqrt(b*x^2 +
a))^11*C*a^3*b^3*c^2*d^12 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4
*c^2*d^12 - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^4*b^2*c*d^13 + 105*(s
qrt(b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^3*c*d^13 + 30*(sqrt(b)*x - sqrt(b*x
^2 + a))^11*C*a^4*b^2*d^14 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^3
*d^14 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*b^(13/2)*c^10*d^4 + 1200*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*D*a*b^(11/2)*c^8*d^6 + 2400*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*D*a^2*b^(9/2)*c^6*d^8 + 330*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*C*a^2*b^(9/2)*c^5*d^9 - 1320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(
11/2)*c^5*d^9 + 3555*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^3*b^(7/2)*c^4*d^
10 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(9/2)*c^4*d^10 - 2805*(
sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^3*b^(7/2)*c^3*d^11 + 1980*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(A+B\,x+C\,x^2+x^3\,D)}{{(c+d\,x)}^7}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^7}dx$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

output
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

3.70 $\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^8}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 933

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx =$$

$$-\frac{b(Ab^2c(8b^2c^4-20abc^2d^2+5a^2d^4)-a(2b^3c^4(cC-8Bd)+2a^3d^5D+a^2bd^3(8cCd-Bd^2-23c^2D)-}{16\left(bc^2+ad^2\right)^6(c+dx)^2}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\left(a+bx^2\right)^{3/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$$+\frac{(7ad^2(2cCd-Bd^2-3c^2D)+bc(3c^2Cd+4Bcd^2-11Ad^3-10c^3D))\left(a+bx^2\right)^{3/2}}{42d^2\left(bc^2+ad^2\right)^2(c+dx)^6}$$

$$-\frac{(14a^2d^4(Cd-3cD)-b^2c^2(3c^2Cd+4Bcd^2-25Ad^3+4c^3D)-abd^2(22c^2Cd-29Bcd^2+8Ad^3+13c}{70d^2\left(bc^2+ad^2\right)^3(c+dx)^5}$$

$$-\frac{(70a^3d^6D+7a^2bd^4(24cCd-5Bd^2-27c^2D)-2b^3c^3(3c^2Cd+4Bcd^2-60Ad^3+4c^3D)-ab^2cd^2(69c}{280d^2\left(bc^2+ad^2\right)^4(c+dx)^4}$$

$$+\frac{b(14a^3d^6(8Cd-49cD)-a^2bd^4(904c^2Cd-407Bcd^2+64Ad^3-505c^3D)+2b^3c^4(3c^2Cd+4Bcd^2-2}{840d^2\left(bc^2+ad^2\right)^5(c+d}$$

$$-\frac{ab^2(Ab^2c(8b^2c^4-20abc^2d^2+5a^2d^4)-a(2b^3c^4(cC-8Bd)+2a^3d^5D+a^2bd^3(8cCd-Bd^2-23c^2D)}{16\left(bc^2+ad^2\right)^{13/2}}$$

output
```
-1/16*b*(A*b^2*c*(5*a^2*d^4-20*a*b*c^2*d^2+8*b^2*c^4)-a*(2*b^3*c^4*(-8*B*d
+C*c)+2*a^3*d^5*D+a^2*b*d^3*(-B*d^2+8*C*c*d-23*D*c^2)-a*b^2*c^2*d*(-16*B*d
^2+23*C*c*d-8*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^6/(d*x+c
)^2-1/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)/(d
*x+c)^7+1/42*(7*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-11*A*d^3+4*B*c*d^2+3*
C*c^2*d-10*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^6-1/70*(14*
a^2*d^4*(C*d-3*D*c)-b^2*c^2*(-25*A*d^3+4*B*c*d^2+3*C*c^2*d+4*D*c^3)-a*b*d^
2*(8*A*d^3-29*B*c*d^2+22*C*c^2*d+13*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c
^2)^3/(d*x+c)^5-1/280*(70*a^3*d^6*D+7*a^2*b*d^4*(-5*B*d^2+24*C*c*d-27*D*c^
2)-2*b^3*c^3*(-60*A*d^3+4*B*c*d^2+3*C*c^2*d+4*D*c^3)-a*b^2*c*d^2*(111*A*d^
3-188*B*c*d^2+69*C*c^2*d+36*D*c^3))*(b*x^2+a)^(3/2)/d^2/(a*d^2+b*c^2)^4/(d
*x+c)^4+1/840*b*(14*a^3*d^6*(8*C*d-49*D*c)-a^2*b*d^4*(64*A*d^3-407*B*c*d^2
+904*C*c^2*d-505*D*c^3)+2*b^3*c^4*(-200*A*d^3+4*B*c*d^2+3*C*c^2*d+4*D*c^3)
+a*b^2*c^2*d^2*(691*A*d^3-740*B*c*d^2+145*C*c^2*d+44*D*c^3))*(b*x^2+a)^(3/
2)/d^2/(a*d^2+b*c^2)^5/(d*x+c)^3-1/16*a*b^2*(A*b^2*c*(5*a^2*d^4-20*a*b*c^2
*d^2+8*b^2*c^4)-a*(2*b^3*c^4*(-8*B*d+C*c)+2*a^3*d^5*D+a^2*b*d^3*(-B*d^2+8*
C*c*d-23*D*c^2)-a*b^2*c^2*d*(-16*B*d^2+23*C*c*d-8*D*c^2)))*arctanh((-b*c*x
+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(13/2)
```

Mathematica [A] (verified)

Time = 14.05 (sec) , antiderivative size = 1207, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
-1/1680*(Sqrt[a + b*x^2]*(240*(b*c^2 + a*d^2)^6*(c^2*C*d - B*c*d^2 + A*d^3
 - c^3*D) + 40*(b*c^2 + a*d^2)^5*(7*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b
*c*(-15*c^2*C*d + 8*B*c*d^2 - A*d^3 + 22*c^3*D))*(c + d*x) - 8*(b*c^2 + a*
d^2)^4*(-42*a^2*d^4*(C*d - 3*c*D) + b^2*c^2*(-51*c^2*C*d + 2*B*c*d^2 + 5*A
*d^3 + 142*c^3*D) + a*b*d^2*(-104*c^2*C*d + 13*B*c*d^2 - 6*A*d^3 + 279*c^3
*D))*(c + d*x)^2 + 2*(b*c^2 + a*d^2)^3*(210*a^3*d^6*D + 7*a^2*b*d^4*(-16*c
*C*d + 5*B*d^2 + 123*c^2*D) + 2*b^3*c^3*(-3*c^2*C*d - 4*B*c*d^2 - 10*A*d^3
 + 136*c^3*D) + a*b^2*c*d^2*(-19*c^2*C*d - 72*B*c*d^2 + 79*A*d^3 + 824*c^3
*D))*(c + d*x)^3 - 2*b*(b*c^2 + a*d^2)^2*(14*a^3*d^6*(-4*C*d + 17*c*D) + 2
*b^3*c^4*(3*c^2*C*d + 4*B*c*d^2 + 10*A*d^3 + 4*c^3*D) + a^2*b*d^4*(200*c^2
*C*d - 151*B*c*d^2 + 32*A*d^3 + 31*c^3*D) + a*b^2*c^2*d^2*(31*c^2*C*d + 88
*B*c*d^2 - 179*A*d^3 + 32*c^3*D))*(c + d*x)^4 - b*(b*c^2 + a*d^2)*(-210*a^
4*d^8*D + 7*a^3*b*d^6*(-88*c*C*d + 15*B*d^2 + 149*c^2*D) + 4*b^4*c^5*(3*c^
2*C*d + 4*B*c*d^2 + 10*A*d^3 + 4*c^3*D) + 2*a*b^3*c^3*d^2*(40*c^2*C*d + 10
0*B*c*d^2 - 359*A*d^3 + 44*c^3*D) + a^2*b^2*c*d^4*(607*c^2*C*d - 866*B*c*d
^2 + 397*A*d^3 + 170*c^3*D))*(c + d*x)^5 - b^2*(14*a^4*d^8*(16*C*d - 113*c
*D) + 4*b^4*c^6*(3*c^2*C*d + 4*B*c*d^2 + 10*A*d^3 + 4*c^3*D) + 2*a*b^3*c^4
*d^2*(46*c^2*C*d + 108*B*c*d^2 - 759*A*d^3 + 52*c^3*D) + 3*a^2*b^2*c^2*d^4
*(299*c^2*C*d - 782*B*c*d^2 + 593*A*d^3 + 86*c^3*D) + a^3*b*d^6*(-2424*c^2
*C*d + 919*B*c*d^2 - 128*A*d^3 + 2053*c^3*D))*(c + d*x)^6))/(d^4*(b*c^2...
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 898, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2182, 25, 2182, 27, 688, 25, 688, 25, 27, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^8}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(7\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(7Cd-7cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+4Bc-4Ad\right)\right)x+7\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^7}dx}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(7\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(7a(Cd-cD)+b\left(-\frac{3Dc^3}{d^2}+\frac{3Cc^2}{d}+4Bc-4Ad\right)\right)x+7\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^7}dx}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(7ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-11Ad^3+4Bcd^2-10c^3D+3c^2Cd\right)\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}-\frac{\int -\frac{3\left(2\left(Ab\left(7bc^2-4ad^2\right)+a\left(7ad(Cd-2cD)-bc\left(\frac{3Dc^2}{d}+4Cc-1\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(2\left(Ab\left(7bc^2-4ad^2\right)+a\left(7ad(Cd-2cD)-bc\left(\frac{3Dc^2}{d}+4Cc-11Bd\right)\right)\right)d^2+\left(14a^2Dd^4+7ab\left(Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(4Dc^3+3Cdc^2+4Bd^2c-11Ad^3\right)\right)x\right)\sqrt{bx^2\cdots}}{(c+dx)^6}}{2d^2\left(ad^2+bc^2\right)}\cdots}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$\frac{\frac{-\frac{\int -\frac{\left(5d\left(Acd\left(14bc^2-19ad^2\right)b^2+a\left(14a^2Dd^4+7ab\left(-3Dc^2+4Cdc-Bd^2\right)d^2-b^2c^2\left(2Dc^2+5Cdc-26Bd^2\right)\right)\right)-2b\left(14a^2(Cd-3cD)d^4-ab\left(13Dc^3+22Cdc^2-29Bd\cdots\right.\right.\right.}{(c+dx)^5}}{5\left(ad^2+bc^2\right)}\cdots}{\cdots}}{\cdots}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(5d\left(Acd\left(14bc^2-19ad^2\right)b^2+a\left(14a^2Dd^4+7ab\left(-3Dc^2+4Cdc-Bd^2\right)d^2-b^2c^2\left(2Dc^2+5Cdc-26Bd^2\right)\right)\right)-2b\left(14a^2(Cd-3cD)d^4-ab\left(13Dc^3+22Cdc^2-29Bd^2c+\cdots\right.\right.\right.}{(c+dx)^5}}{5\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$-\frac{\frac{\int -\frac{b\left(4d\left(Abd\left(70b^2c^4-145abd^2c^2+16a^2d^4\right)-a\left(14a^2(2Cd-11cD)d^4-abc\left(-79Dc^2+184Cdc-93Bd^2\right)d^2+b^2c^3\left(2Dc^2+19Cdc-138Bd^2\right)\right)\right)-\left(70a^3Dd^6+7a^2b\cdots\right.}{(c+dx)^4}}{4\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b\left(4d\left(Abd\left(70b^2c^4-145abd^2c^2+16a^2d^4\right)-a\left(14a^2(2Cd-11cD)d^4-abc\left(-79Dc^2+184Cdc-93Bd^2\right)d^2+b^2c^3\left(2Dc^2+19Cdc-138Bd^2\right)\right)\right)-\left(70a^3Dd^6+7a^2b\left(-\cdots\right.\right.}{(c+dx)^4}}{4\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{\left(4d\left(Abd\left(70b^2c^4-145abd^2c^2+16a^2d^4\right)-a\left(14a^2(2Cd-11cD)d^4-abc\left(-79Dc^2+184Cdc-93Bd^2\right)d^2+b^2c^3\left(2Dc^2+19Cdc-138Bd^2\right)\right)\right)-\left(70a^3Dd^6+7a^2b\left(-\cdots\right.\right.}{(c+dx)^4}}{4\left(ad^2+bc^2\right)}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{\left(7a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+3Cdc^2+4Bd^2c-11Ad^3\right)\right)\left(bx^2+a\right)^{3/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}+\frac{b\left(\frac{35\left(Ab^2c\left(8b^2c^4-20abd^2c^2+5a^2d^4\right)-a\left(2a^3Dd^5+a^2b\left(-23Dc\right.\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{3/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\Big\downarrow$ 486

$$\frac{\frac{\left(7a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+3Cdc^2+4Bd^2c-11Ad^3\right)\right)\left(bx^2+a\right)^{3/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}+\frac{b\left(\frac{35\left(Ab^2c\left(8b^2c^4-20abd^2c^2+5a^2d^4\right)-a\left(2a^3Dd^5+a^2b\left(-23Dc\right.\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{3/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\Big\downarrow$ 488

$$\frac{\frac{\left(7a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+3Cdc^2+4Bd^2c-11Ad^3\right)\right)\left(bx^2+a\right)^{3/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}+\frac{b\left(\frac{35\left(Ab^2c\left(8b^2c^4-20abd^2c^2+5a^2d^4\right)-a\left(2a^3Dd^5+a^2b\left(-23Dc\right.\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{3/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\Big\downarrow$ 219

$$b\left(\frac{35\left(Ab^2c\left(8b^2c^4-20abd^2c^2+5a^2d^4\right)-a\left(2a^3Dd^5+a^2b\left(-23Dc\right.\right.\right.}{}\right.$$

$$\frac{\left(7a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+3Cdc^2+4Bd^2c-11Ad^3\right)\right)\left(bx^2+a\right)^{3/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}+$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{3/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
-1/7*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^7) + (((7*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(3*c^2
*C*d + 4*B*c*d^2 - 11*A*d^3 - 10*c^3*D))*(a + b*x^2)^(3/2))/(6*d^2*(b*c^2
+ a*d^2)*(c + d*x)^6) + (-1/5*((14*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(3*c^2*
C*d + 4*B*c*d^2 - 25*A*d^3 + 4*c^3*D) - a*b*d^2*(22*c^2*C*d - 29*B*c*d^2 +
 8*A*d^3 + 13*c^3*D))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^5) + (
-1/4*((70*a^3*d^6*D + 7*a^2*b*d^4*(24*c*C*d - 5*B*d^2 - 27*c^2*D) - 2*b^3*
c^3*(3*c^2*C*d + 4*B*c*d^2 - 60*A*d^3 + 4*c^3*D) - a*b^2*c*d^2*(69*c^2*C*d
 - 188*B*c*d^2 + 111*A*d^3 + 36*c^3*D))*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2
)*(c + d*x)^4) + (b*(((14*a^3*d^6*(8*C*d - 49*c*D) - a^2*b*d^4*(904*c^2*C*
d - 407*B*c*d^2 + 64*A*d^3 - 505*c^3*D) + 2*b^3*c^4*(3*c^2*C*d + 4*B*c*d^2
 - 200*A*d^3 + 4*c^3*D) + a*b^2*c^2*d^2*(145*c^2*C*d - 740*B*c*d^2 + 691*A
*d^3 + 44*c^3*D))*(a + b*x^2)^(3/2))/(3*(b*c^2 + a*d^2)*(c + d*x)^3) + (35
*d^2*(A*b^2*c*(8*b^2*c^4 - 20*a*b*c^2*d^2 + 5*a^2*d^4) - a*(2*b^3*c^4*(c*C
 - 8*B*d) + 2*a^3*d^5*D + a^2*b*d^3*(8*c*C*d - B*d^2 - 23*c^2*D) - a*b^2*c
^2*d*(23*c*C*d - 16*B*d^2 - 8*c^2*D)))*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2
])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2
+ a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2))))/(b*c^2 + a*d^2)))/
(4*(b*c^2 + a*d^2)))/(5*(b*c^2 + a*d^2)))/(2*d^2*(b*c^2 + a*d^2)))/(7*(b*c
^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18658 vs. $2(901) = 1802$.

Time = 2.01 (sec) , antiderivative size = 18659, normalized size of antiderivative = 20.00

method	result	size
default	Expression too large to display	18659

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20401 vs. $2(899) = 1798$.

Time = 0.78 (sec) , antiderivative size = 20401, normalized size of antiderivative = 21.87

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="maxi
ma")
```

output
```
33/16*sqrt(b*x^2 + a)*D*b^6*c^9/(b^6*c^12*d^5*x + 6*a*b^5*c^10*d^7*x + 15*
a^2*b^4*c^8*d^9*x + 20*a^3*b^3*c^6*d^11*x + 15*a^4*b^2*c^4*d^13*x + 6*a^5*
b*c^2*d^15*x + a^6*d^17*x + b^6*c^13*d^4 + 6*a*b^5*c^11*d^6 + 15*a^2*b^4*c
^9*d^8 + 20*a^3*b^3*c^7*d^10 + 15*a^4*b^2*c^5*d^12 + 6*a^5*b*c^3*d^14 + a^
6*c*d^16) + 33/16*(b*x^2 + a)^(3/2)*D*b^5*c^8/(b^6*c^12*d^4*x^2 + 6*a*b^5*
c^10*d^6*x^2 + 15*a^2*b^4*c^8*d^8*x^2 + 20*a^3*b^3*c^6*d^10*x^2 + 15*a^4*b
^2*c^4*d^12*x^2 + 6*a^5*b*c^2*d^14*x^2 + a^6*d^16*x^2 + 2*b^6*c^13*d^3*x +
 12*a*b^5*c^11*d^5*x + 30*a^2*b^4*c^9*d^7*x + 40*a^3*b^3*c^7*d^9*x + 30*a^
4*b^2*c^5*d^11*x + 12*a^5*b*c^3*d^13*x + 2*a^6*c*d^15*x + b^6*c^14*d^2 + 6
*a*b^5*c^12*d^4 + 15*a^2*b^4*c^10*d^6 + 20*a^3*b^3*c^8*d^8 + 15*a^4*b^2*c^
6*d^10 + 6*a^5*b*c^4*d^12 + a^6*c^2*d^14) - 33/16*sqrt(b*x^2 + a)*C*b^6*c^
8/(b^6*c^12*d^4*x + 6*a*b^5*c^10*d^6*x + 15*a^2*b^4*c^8*d^8*x + 20*a^3*b^3
*c^6*d^10*x + 15*a^4*b^2*c^4*d^12*x + 6*a^5*b*c^2*d^14*x + a^6*d^16*x + b^
6*c^13*d^3 + 6*a*b^5*c^11*d^5 + 15*a^2*b^4*c^9*d^7 + 20*a^3*b^3*c^7*d^9 +
15*a^4*b^2*c^5*d^11 + 6*a^5*b*c^3*d^13 + a^6*c*d^15) - 33/16*sqrt(b*x^2 +
a)*D*b^6*c^8/(b^6*c^12*d^4 + 6*a*b^5*c^10*d^6 + 15*a^2*b^4*c^8*d^8 + 20*a^
3*b^3*c^6*d^10 + 15*a^4*b^2*c^4*d^12 + 6*a^5*b*c^2*d^14 + a^6*d^16) - 33/1
6*(b*x^2 + a)^(3/2)*C*b^5*c^7/(b^6*c^12*d^3*x^2 + 6*a*b^5*c^10*d^5*x^2 + 1
5*a^2*b^4*c^8*d^7*x^2 + 20*a^3*b^3*c^6*d^9*x^2 + 15*a^4*b^2*c^4*d^11*x^2 +
 6*a^5*b*c^2*d^13*x^2 + a^6*d^15*x^2 + 2*b^6*c^13*d^2*x + 12*a*b^5*c^11...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10719 vs. 2(899) = 1798.

Time = 0.47 (sec) , antiderivative size = 10719, normalized size of antiderivative = 11.49

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="giac
")
```

output
```
-1/8*(2*C*a^2*b^5*c^5 - 8*A*a*b^6*c^5 + 8*D*a^3*b^4*c^4*d - 16*B*a^2*b^5*c
^4*d - 23*C*a^3*b^4*c^3*d^2 + 20*A*a^2*b^5*c^3*d^2 - 23*D*a^4*b^3*c^2*d^3
+ 16*B*a^3*b^4*c^2*d^3 + 8*C*a^4*b^3*c*d^4 - 5*A*a^3*b^4*c*d^4 + 2*D*a^5*b
^2*d^5 - B*a^4*b^3*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)
*c)/sqrt(-b*c^2 - a*d^2))/((b^6*c^12 + 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^8*d
^4 + 20*a^3*b^3*c^6*d^6 + 15*a^4*b^2*c^4*d^8 + 6*a^5*b*c^2*d^10 + a^6*d^12
)*sqrt(-b*c^2 - a*d^2)) + 1/840*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^
2*b^5*c^5*d^11 - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^6*c^5*d^11 + 8
40*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^3*b^4*c^4*d^12 - 1680*(sqrt(b)*x -
 sqrt(b*x^2 + a))^13*B*a^2*b^5*c^4*d^12 - 2415*(sqrt(b)*x - sqrt(b*x^2 + a
))^13*C*a^3*b^4*c^3*d^13 + 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a^2*b^5
*c^3*d^13 - 2415*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^4*b^3*c^2*d^14 + 168
0*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^3*b^4*c^2*d^14 + 840*(sqrt(b)*x - s
qrt(b*x^2 + a))^13*C*a^4*b^3*c*d^15 - 525*(sqrt(b)*x - sqrt(b*x^2 + a))^13
*A*a^3*b^4*c*d^15 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^5*b^2*d^16 -
105*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^4*b^3*d^16 + 2730*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*C*a^2*b^(11/2)*c^6*d^10 - 10920*(sqrt(b)*x - sqrt(b*x^2
+ a))^12*A*a*b^(13/2)*c^6*d^10 + 10920*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*
a^3*b^(9/2)*c^5*d^11 - 21840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(11/
2)*c^5*d^11 - 31395*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(9/2)*c^4*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^8}dx$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

output
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

3.71 $\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 591

$$\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{a(96Ab^3c^3-a(16b^2c(c^2C+3Bcd+3Ad^2)+3a^2d^3D-6abd(3cCd+Bd^2+3c^2D)))\,x\sqrt{a+bx^2}}{256b^3} \\
&+\frac{(96Ab^3c^3-a(16b^2c(c^2C+3Bcd+3Ad^2)+3a^2d^3D-6abd(3cCd+Bd^2+3c^2D)))\,x(a+bx^2)^{3/2}}{384b^3} \\
&+\frac{\left(b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D)\right)\left(a+bx^2\right)^{5/2}}{5b^3} \\
&+\frac{\left(16b^2c(c^2C+3Bcd+3Ad^2)+3a^2d^3D-6abd(3cCd+Bd^2+3c^2D)\right)x\left(a+bx^2\right)^{5/2}}{96b^3} \\
&-\frac{d(ad^2D-2b(3cCd+Bd^2+3c^2D))\,x^3\left(a+bx^2\right)^{5/2}}{16b^2}+\frac{d^3Dx^5\left(a+bx^2\right)^{5/2}}{10b} \\
&-\frac{\left(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D)\right)\left(a+bx^2\right)^{7/2}}{7b^3} \\
&+\frac{d^2(Cd+3cD)\left(a+bx^2\right)^{9/2}}{9b^3} \\
&+\frac{a^2(96Ab^3c^3-a(16b^2c(c^2C+3Bcd+3Ad^2)+3a^2d^3D-6abd(3cCd+Bd^2+3c^2D)))\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}
\end{aligned}$$

output
```
1/256*a*(96*A*b^3*c^3-a*(16*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+3*a^2*d^3*D-6*a*
b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x*(b*x^2+a)^(1/2)/b^3+1/384*(96*A*b^3*c^3-a*
(16*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+3*a^2*d^3*D-6*a*b*d*(B*d^2+3*C*c*d+3*D*c
^2)))*x*(b*x^2+a)^(3/2)/b^3+1/5*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a
*b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(5/2)/b^3+1/96*(16*b^2*c*(
3*A*d^2+3*B*c*d+C*c^2)+3*a^2*d^3*D-6*a*b*d*(B*d^2+3*C*c*d+3*D*c^2))*x*(b*x
^2+a)^(5/2)/b^3-1/16*d*(a*d^2*D-2*b*(B*d^2+3*C*c*d+3*D*c^2))*x^3*(b*x^2+a)
^(5/2)/b^2+1/10*d^3*D*x^5*(b*x^2+a)^(5/2)/b-1/7*(2*a*d^2*(C*d+3*D*c)-b*(A*
d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(7/2)/b^3+1/9*d^2*(C*d+3*D*c)*(b
*x^2+a)^(9/2)/b^3+1/256*a^2*(96*A*b^3*c^3-a*(16*b^2*c*(3*A*d^2+3*B*c*d+C*c
^2)+3*a^2*d^3*D-6*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 4.41 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.06

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{\sqrt{b}\sqrt{a+bx^2}(a^4d^2(2048Cd+6144cD+945dDx)+12a^2b^2(12Ad(336c^2+105cdx+16d^2x^2)+}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^4*d^2*(2048*C*d + 6144*c*D + 945*d*D*x) + 12*a
^2*b^2*(12*A*d*(336*c^2 + 105*c*d*x + 16*d^2*x^2) + 3*B*(448*c^3 + 420*c^2
*d*x + 192*c*d^2*x^2 + 35*d^3*x^3) + x*(12*c^3*(35*C + 16*D*x) + 2*d^3*x^3
*(32*C + 21*D*x) + 9*c^2*d*x*(64*C + 35*D*x) + 3*c*d^2*x^2*(105*C + 64*D*x
))) - 2*a^3*b*(2304*c^3*D + 27*c^2*d*(256*C + 105*D*x) + 3*c*d^2*(2304*B +
 x*(945*C + 512*D*x)) + d^3*(2304*A + x*(945*B + 512*C*x + 315*D*x^2))) +
32*b^4*x^3*(18*A*(35*c^3 + 84*c^2*d*x + 70*c*d^2*x^2 + 20*d^3*x^3) + x*(9*
B*(56*c^3 + 140*c^2*d*x + 120*c*d^2*x^2 + 35*d^3*x^3) + x*(60*c^3*(7*C + 6
*D*x) + 135*c^2*d*x*(8*C + 7*D*x) + 105*c*d^2*x^2*(9*C + 8*D*x) + 28*d^3*x
^3*(10*C + 9*D*x)))) + 16*a*b^3*x*(18*A*(175*c^3 + 336*c^2*d*x + 245*c*d^2
*x^2 + 64*d^3*x^3) + x*(9*B*(224*c^3 + 490*c^2*d*x + 384*c*d^2*x^2 + 105*d
^3*x^3) + x*(27*c^2*d*x*(128*C + 105*D*x) + 15*c*d^2*x^2*(189*C + 160*D*x)
 + 6*c^3*(245*C + 192*D*x) + d^3*x^3*(800*C + 693*D*x))))) + 315*a^2*(48*A
*b^2*c*(-2*b*c^2 + a*d^2) + a*(16*b^2*c^2*(c*C + 3*B*d) + 3*a^2*d^3*D - 6*
a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(
80640*b^(7/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2185, 27, 2185, 27, 687, 687, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int 5(c+dx)^3 \left(bx^2+a\right)^{3/2} \left(b(2Cd-3cD)x^2d^2+(2Abd-acD)d^2+\left(-bDc^2+2bBd^2-ad^2D\right)xd\right)dx}{10bd^3}+\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)^3 \left(bx^2+a\right)^{3/2} \left(b(2Cd-3cD)x^2d^2+(2Abd-acD)d^2+\left(-bDc^2+2bBd^2-ad^2D\right)xd\right)dx}{2bd^3}+\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$\Big\downarrow$ 2185

$$\frac{\frac{\int bd^3(c+dx)^3\left(d(18Abd-8aCd+3acD)-\left(9aDd^2+2b\left(-3Dc^2+5Cdc-9Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{9bd^2}+\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(2Cd-3c}{2bd^3}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}d\int (c+dx)^3\left(d(18Abd-8aCd+3acD)-\left(9aDd^2+2b\left(-3Dc^2+5Cdc-9Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx+\frac{1}{9}d(a+}{2bd^3}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$\Big\downarrow$ 687

$$\frac{\frac{1}{9}d\left(\frac{\int (c+dx)^2\left(d\left(144Acdb^2+a\left(27ad^2D-b\left(-6Dc^2+34Cdc+54Bd^2\right)\right)\right)-b\left(a(64Cd+3cD)d^2+6b\left(-3Dc^3+5Cdc^2-9Bd^2c-24Ad^3\right)\right)x\right)\left(bx^2+a\right)^3}{8b}\right.}{2bd^3}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$\Big\downarrow$ 687

$$\frac{\frac{1}{9}d\left(\frac{\frac{\int b(c+dx)\left(d\left(144Abd\left(7bc^2-2ad^2\right)+a\left(ad^2(128Cd+195cD)-2bc\left(-3Dc^2+89Cdc+243Bd^2\right)\right)\right)+3\left(63a^2Dd^4-2ab\left(-6Dc^2+61Cdc+63Bd^2\right)d^2-4b^2c\left(-3D\right.\right.\right.}{7b}}{8b}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}d\left(\frac{\frac{1}{7}\int (c+dx)\left(d\left(144Abd\left(7bc^2-2ad^2\right)+a\left(ad^2(128Cd+195cD)-2bc\left(-3Dc^2+89Cdc+243Bd^2\right)\right)\right)+3\left(63a^2Dd^4-2ab\left(-6Dc^2+61Cdc+63Bd^2\right)\right.\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{1}{9}d\left(\frac{\frac{1}{7}\left(\frac{21d^2\left(48Ab^2c\left(2bc^2-ad^2\right)-a\left(3a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\int\left(bx^2+a\right)^{3/2}dx}{2b}+\frac{2\left(a+bx^2\right)^{5/2}\left(64a^2d^4(3cD+Cd)-abd^2\left(1\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{9}d\left(\frac{\frac{1}{7}\left(\frac{21d^2\left(48Ab^2c\left(2bc^2-ad^2\right)-a\left(3a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{2\left(a+bx^2\right)^{5/2}\left(64a^2d^4\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{9}d\left(\frac{\frac{1}{7}\left(\frac{21d^2\left(48Ab^2c\left(2bc^2-ad^2\right)-a\left(3a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{2\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{9}d\left(\frac{\frac{1}{7}\left(\frac{21d^2\left(48Ab^2c\left(2bc^2-ad^2\right)-a\left(3a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+cC)\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\cdots\right.\right.}{2b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{9}d\Bigg(\frac{\frac{1}{7}\Bigg(\frac{21d^2\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(48Ab^2c\left(2bc^2-ad^2\right)-a\left(3a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+16b^2c^2(3Bd+\right.\right.}{2b}}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^2}$$

input

```
Int[(c + d*x)^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
(D*(c + d*x)^5*(a + b*x^2)^(5/2))/(10*b*d^2) + ((d*(2*C*d - 3*c*D)*(c + d*
x)^4*(a + b*x^2)^(5/2))/9 + (d*(-1/8*((10*b*c*C*d - 18*b*B*d^2 - 6*b*c^2*D
 + 9*a*d^2*D)*(c + d*x)^3*(a + b*x^2)^(5/2))/b + (-1/7*((a*d^2*(64*C*d + 3
*c*D) + 6*b*(5*c^2*C*d - 9*B*c*d^2 - 24*A*d^3 - 3*c^3*D))*(c + d*x)^2*(a +
 b*x^2)^(5/2)) + ((2*(64*a^2*d^4*(C*d + 3*c*D) - a*b*d^2*(272*c^2*C*d + 43
2*B*c*d^2 + 144*A*d^3 - 21*c^3*D) - 6*b^2*c^2*(5*c^2*C*d - 9*B*c*d^2 - 192
*A*d^3 - 3*c^3*D))*(a + b*x^2)^(5/2))/(5*b) + (d*(63*a^2*d^4*D - 2*a*b*d^2
*(61*c*C*d + 63*B*d^2 - 6*c^2*D) - 4*b^2*c*(5*c^2*C*d - 9*B*c*d^2 - 108*A*
d^3 - 3*c^3*D))*x*(a + b*x^2)^(5/2))/(2*b) + (21*d^2*(48*A*b^2*c*(2*b*c^2
- a*d^2) - a*(16*b^2*c^2*(c*C + 3*B*d) + 3*a^2*d^3*D - 6*a*b*d*(3*c*C*d +
B*d^2 + 3*c^2*D)))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2
+ (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b))/7)/(8*
b)))/9)/(2*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.92

method	result
default	$A\,c^{3}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{c^{2}(3Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}+d^{2}(Cd+3Dc)\left(\frac{x^{4}\left(b\,x^{2}\right.}{9b}\right.$

input
```
int((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*c^2*(3*A*d+B*c)/b*(b*x^2+a)^(5/2)+d^2*(C
*d+3*D*c)*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/
35*a/b^2*(b*x^2+a)^(5/2)))+c*(3*A*d^2+3*B*c*d+C*c^2)*(1/6*x*(b*x^2+a)^(5/2
)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d*(B*d^2+3*C*c*d+3*D*c^2)*(1/8*x^3*(b
*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^
(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))))))+(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35
*a/b^2*(b*x^2+a)^(5/2))+D*d^3*(1/10*x^5*(b*x^2+a)^(5/2)/b-1/2*a/b*(1/8*x^3
*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1643, normalized size of antiderivative = 2.78

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[-1/161280*(315*(16*(C*a^3*b^2 - 6*A*a^2*b^3)*c^3 - 6*(3*D*a^4*b - 8*B*a^3
*b^2)*c^2*d - 6*(3*C*a^4*b - 8*A*a^3*b^2)*c*d^2 + 3*(D*a^5 - 2*B*a^4*b)*d^
3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8064*D*b^5
*d^3*x^9 + 8960*(3*D*b^5*c*d^2 + C*b^5*d^3)*x^8 + 1008*(30*D*b^5*c^2*d + 3
0*C*b^5*c*d^2 + (11*D*a*b^4 + 10*B*b^5)*d^3)*x^7 + 1280*(9*D*b^5*c^3 + 27*
C*b^5*c^2*d + 3*(10*D*a*b^4 + 9*B*b^5)*c*d^2 + (10*C*a*b^4 + 9*A*b^5)*d^3)
*x^6 + 168*(80*C*b^5*c^3 + 30*(9*D*a*b^4 + 8*B*b^5)*c^2*d + 30*(9*C*a*b^4
+ 8*A*b^5)*c*d^2 + 3*(D*a^2*b^3 + 30*B*a*b^4)*d^3)*x^5 + 768*(3*(8*D*a*b^4
 + 7*B*b^5)*c^3 + 9*(8*C*a*b^4 + 7*A*b^5)*c^2*d + 3*(D*a^2*b^3 + 24*B*a*b^
4)*c*d^2 + (C*a^2*b^3 + 24*A*a*b^4)*d^3)*x^4 - 2304*(2*D*a^3*b^2 - 7*B*a^2
*b^3)*c^3 - 6912*(2*C*a^3*b^2 - 7*A*a^2*b^3)*c^2*d + 1536*(4*D*a^4*b - 9*B
*a^3*b^2)*c*d^2 + 512*(4*C*a^4*b - 9*A*a^3*b^2)*d^3 + 210*(16*(7*C*a*b^4 +
 6*A*b^5)*c^3 + 6*(3*D*a^2*b^3 + 56*B*a*b^4)*c^2*d + 6*(3*C*a^2*b^3 + 56*A
*a*b^4)*c*d^2 - 3*(D*a^3*b^2 - 2*B*a^2*b^3)*d^3)*x^3 + 256*(9*(D*a^2*b^3 +
 14*B*a*b^4)*c^3 + 27*(C*a^2*b^3 + 14*A*a*b^4)*c^2*d - 3*(4*D*a^3*b^2 - 9*
B*a^2*b^3)*c*d^2 - (4*C*a^3*b^2 - 9*A*a^2*b^3)*d^3)*x^2 + 315*(16*(C*a^2*b
^3 + 10*A*a*b^4)*c^3 - 6*(3*D*a^3*b^2 - 8*B*a^2*b^3)*c^2*d - 6*(3*C*a^3*b^
2 - 8*A*a^2*b^3)*c*d^2 + 3*(D*a^4*b - 2*B*a^3*b^2)*d^3)*x)*sqrt(b*x^2 + a)
)/b^4, 1/80640*(315*(16*(C*a^3*b^2 - 6*A*a^2*b^3)*c^3 - 6*(3*D*a^4*b - 8*B
*a^3*b^2)*c^2*d - 6*(3*C*a^4*b - 8*A*a^3*b^2)*c*d^2 + 3*(D*a^5 - 2*B*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(598) = 1196$.

Time = 0.88 (sec) , antiderivative size = 1991, normalized size of antiderivative = 3.37

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b*d**3*x**9/10 + x**8*(C*b**2*d**3 + 3*D*b*
*2*c*d**2)/(9*b) + x**7*(B*b**2*d**3 + 3*C*b**2*c*d**2 + 11*D*a*b*d**3/10
+ 3*D*b**2*c**2*d)/(8*b) + x**6*(A*b**2*d**3 + 3*B*b**2*c*d**2 + 2*C*a*b*d
**3 + 3*C*b**2*c**2*d + 6*D*a*b*c*d**2 + D*b**2*c**3 - 8*a*(C*b**2*d**3 +
3*D*b**2*c*d**2)/(9*b))/(7*b) + x**5*(3*A*b**2*c*d**2 + 2*B*a*b*d**3 + 3*B
*b**2*c**2*d + 6*C*a*b*c*d**2 + C*b**2*c**3 + D*a**2*d**3 + 6*D*a*b*c**2*d
 - 7*a*(B*b**2*d**3 + 3*C*b**2*c*d**2 + 11*D*a*b*d**3/10 + 3*D*b**2*c**2*d
)/(8*b))/(6*b) + x**4*(2*A*a*b*d**3 + 3*A*b**2*c**2*d + 6*B*a*b*c*d**2 + B
*b**2*c**3 + C*a**2*d**3 + 6*C*a*b*c**2*d + 3*D*a**2*c*d**2 + 2*D*a*b*c**3
 - 6*a*(A*b**2*d**3 + 3*B*b**2*c*d**2 + 2*C*a*b*d**3 + 3*C*b**2*c**2*d + 6
*D*a*b*c*d**2 + D*b**2*c**3 - 8*a*(C*b**2*d**3 + 3*D*b**2*c*d**2)/(9*b))/(
7*b))/(5*b) + x**3*(6*A*a*b*c*d**2 + A*b**2*c**3 + B*a**2*d**3 + 6*B*a*b*c
**2*d + 3*C*a**2*c*d**2 + 2*C*a*b*c**3 + 3*D*a**2*c**2*d - 5*a*(3*A*b**2*c
*d**2 + 2*B*a*b*d**3 + 3*B*b**2*c**2*d + 6*C*a*b*c*d**2 + C*b**2*c**3 + D*
a**2*d**3 + 6*D*a*b*c**2*d - 7*a*(B*b**2*d**3 + 3*C*b**2*c*d**2 + 11*D*a*b
*d**3/10 + 3*D*b**2*c**2*d)/(8*b))/(6*b))/(4*b) + x**2*(A*a**2*d**3 + 6*A*
a*b*c**2*d + 3*B*a**2*c*d**2 + 2*B*a*b*c**3 + 3*C*a**2*c**2*d + D*a**2*c**
3 - 4*a*(2*A*a*b*d**3 + 3*A*b**2*c**2*d + 6*B*a*b*c*d**2 + B*b**2*c**3 + C
*a**2*d**3 + 6*C*a*b*c**2*d + 3*D*a**2*c*d**2 + 2*D*a*b*c**3 - 6*a*(A*b**2
*d**3 + 3*B*b**2*c*d**2 + 2*C*a*b*d**3 + 3*C*b**2*c**2*d + 6*D*a*b*c*d*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.23

$$\int (c+dx)^3\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/10*(b*x^2 + a)^(5/2)*D*d^3*x^5/b - 1/16*(b*x^2 + a)^(5/2)*D*a*d^3*x^3/b^
2 + 1/4*(b*x^2 + a)^(3/2)*A*c^3*x + 3/8*sqrt(b*x^2 + a)*A*a*c^3*x + 1/32*(
b*x^2 + a)^(5/2)*D*a^2*d^3*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*D*a^3*d^3*x/b^3
 - 3/256*sqrt(b*x^2 + a)*D*a^4*d^3*x/b^3 + 1/9*(3*D*c*d^2 + C*d^3)*(b*x^2
+ a)^(5/2)*x^4/b + 3/8*A*a^2*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/256*D*
a^5*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/5*(b*x^2 + a)^(5/2)*B*c^3/b + 3
/5*(b*x^2 + a)^(5/2)*A*c^2*d/b + 1/8*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^
2 + a)^(5/2)*x^3/b - 4/63*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(5/2)*a*x^2/b^2
+ 1/7*(D*c^3 + 3*C*c^2*d + 3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(5/2)*x^2/b - 1/
16*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(3*D*c
^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(3/2)*a^2*x/b^2 + 3/128*(3*D*c^2*d +
 3*C*c*d^2 + B*d^3)*sqrt(b*x^2 + a)*a^3*x/b^2 + 1/6*(C*c^3 + 3*B*c^2*d + 3
*A*c*d^2)*(b*x^2 + a)^(5/2)*x/b - 1/24*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*(b*
x^2 + a)^(3/2)*a*x/b - 1/16*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*sqrt(b*x^2 + a
)*a^2*x/b + 3/128*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*a^4*arcsinh(b*x/sqrt(a*b
))/b^(5/2) - 1/16*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*a^3*arcsinh(b*x/sqrt(a*b
))/b^(3/2) + 8/315*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(5/2)*a^2/b^3 - 2/35*(D
*c^3 + 3*C*c^2*d + 3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.49

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*D*b*d^3*x + 10*(3*D*b^9*c*d^2
 + C*b^9*d^3)/b^8)*x + 9*(30*D*b^9*c^2*d + 30*C*b^9*c*d^2 + 11*D*a*b^8*d^3
 + 10*B*b^9*d^3)/b^8)*x + 80*(9*D*b^9*c^3 + 27*C*b^9*c^2*d + 30*D*a*b^8*c*
d^2 + 27*B*b^9*c*d^2 + 10*C*a*b^8*d^3 + 9*A*b^9*d^3)/b^8)*x + 21*(80*C*b^9
*c^3 + 270*D*a*b^8*c^2*d + 240*B*b^9*c^2*d + 270*C*a*b^8*c*d^2 + 240*A*b^9
*c*d^2 + 3*D*a^2*b^7*d^3 + 90*B*a*b^8*d^3)/b^8)*x + 96*(24*D*a*b^8*c^3 + 2
1*B*b^9*c^3 + 72*C*a*b^8*c^2*d + 63*A*b^9*c^2*d + 3*D*a^2*b^7*c*d^2 + 72*B
*a*b^8*c*d^2 + C*a^2*b^7*d^3 + 24*A*a*b^8*d^3)/b^8)*x + 105*(112*C*a*b^8*c
^3 + 96*A*b^9*c^3 + 18*D*a^2*b^7*c^2*d + 336*B*a*b^8*c^2*d + 18*C*a^2*b^7*
c*d^2 + 336*A*a*b^8*c*d^2 - 3*D*a^3*b^6*d^3 + 6*B*a^2*b^7*d^3)/b^8)*x + 12
8*(9*D*a^2*b^7*c^3 + 126*B*a*b^8*c^3 + 27*C*a^2*b^7*c^2*d + 378*A*a*b^8*c^
2*d - 12*D*a^3*b^6*c*d^2 + 27*B*a^2*b^7*c*d^2 - 4*C*a^3*b^6*d^3 + 9*A*a^2*
b^7*d^3)/b^8)*x + 315*(16*C*a^2*b^7*c^3 + 160*A*a*b^8*c^3 - 18*D*a^3*b^6*c
^2*d + 48*B*a^2*b^7*c^2*d - 18*C*a^3*b^6*c*d^2 + 48*A*a^2*b^7*c*d^2 + 3*D*
a^4*b^5*d^3 - 6*B*a^3*b^6*d^3)/b^8)*x - 256*(18*D*a^3*b^6*c^3 - 63*B*a^2*b
^7*c^3 + 54*C*a^3*b^6*c^2*d - 189*A*a^2*b^7*c^2*d - 24*D*a^4*b^5*c*d^2 + 5
4*B*a^3*b^6*c*d^2 - 8*C*a^4*b^5*d^3 + 18*A*a^3*b^6*d^3)/b^8) + 1/256*(16*C
*a^3*b^2*c^3 - 96*A*a^2*b^3*c^3 - 18*D*a^4*b*c^2*d + 48*B*a^3*b^2*c^2*d -
18*C*a^4*b*c*d^2 + 48*A*a^3*b^2*c*d^2 + 3*D*a^5*d^3 - 6*B*a^4*b*d^3)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^2+a\right)^{3/2}\,{\left(c+d\,x\right)}^3\,\left(A+B\,x+C\,x^2+x^3\,D\right) \,d x$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (dx + c)^3 \left(b\,x^2+a\right)^{\frac{3}{2}} \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.72 $\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 417

$$\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right. \\
&\left.+Dx^3\right) dx = \frac{a(8Ab(6bc^2-ad^2)-a(8bc(cC+2Bd)-3ad(Cd+2cD)))\,x\sqrt{a+bx^2}}{128b^2} \\
&+\frac{(8Ab(6bc^2-ad^2)-a(8bc(cC+2Bd)-3ad(Cd+2cD)))\,x(a+bx^2)^{3/2}}{192b^2} \\
&+\frac{\left(b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D)\right)\left(a+bx^2\right)^{5/2}}{5b^3} \\
&+\frac{\left(8b(c^2C+2Bcd+Ad^2)-3ad(Cd+2cD)\right)x(a+bx^2)^{5/2}}{48b^2} \\
&+\frac{d(Cd+2cD)x^3(a+bx^2)^{5/2}}{8b} \\
&-\frac{\left(2ad^2D-b(2cCd+Bd^2+c^2D)\right)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{d^2D(a+bx^2)^{9/2}}{9b^3} \\
&+\frac{a^2(8Ab(6bc^2-ad^2)-a(8bc(cC+2Bd)-3ad(Cd+2cD)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}
\end{aligned}$$

output
```
1/128*a*(8*A*b*(-a*d^2+6*b*c^2)-a*(8*b*c*(2*B*d+C*c)-3*a*d*(C*d+2*D*c)))*x
*(b*x^2+a)^(1/2)/b^2+1/192*(8*A*b*(-a*d^2+6*b*c^2)-a*(8*b*c*(2*B*d+C*c)-3*
a*d*(C*d+2*D*c)))*x*(b*x^2+a)^(3/2)/b^2+1/5*(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a
*b*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(5/2)/b^3+1/48*(8*b*(A*d^2+2*B*c*d+C*c
^2)-3*a*d*(C*d+2*D*c))*x*(b*x^2+a)^(5/2)/b^2+1/8*d*(C*d+2*D*c)*x^3*(b*x^2+
a)^(5/2)/b-1/7*(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(7/2)/b^3+1/9
*d^2*D*(b*x^2+a)^(9/2)/b^3+1/128*a^2*(8*A*b*(-a*d^2+6*b*c^2)-a*(8*b*c*(2*B
*d+C*c)-3*a*d*(C*d+2*D*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.05

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{\sqrt{a+bx^2}(1024a^4d^2D+6a^2b^2(84Ad(32c+5dx)+24B(56c^2+35cdx+8d^2x^2)+x(12c^2(35C}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(1024*a^4*d^2*D + 6*a^2*b^2*(84*A*d*(32*c + 5*d*x) + 24*B
*(56*c^2 + 35*c*d*x + 8*d^2*x^2) + x*(12*c^2*(35*C + 16*D*x) + 6*c*d*x*(64
*C + 35*D*x) + d^2*x^2*(105*C + 64*D*x))) - a^3*b*(2304*c^2*D + 18*c*d*(25
6*C + 105*D*x) + d^2*(2304*B + x*(945*C + 512*D*x))) + 16*b^4*x^3*(42*A*(1
5*c^2 + 24*c*d*x + 10*d^2*x^2) + x*(24*B*(21*c^2 + 35*c*d*x + 15*d^2*x^2)
+ 5*x*(12*c^2*(7*C + 6*D*x) + 18*c*d*x*(8*C + 7*D*x) + 7*d^2*x^2*(9*C + 8*
D*x)))) + 8*a*b^3*x*(42*A*(75*c^2 + 96*c*d*x + 35*d^2*x^2) + x*(12*B*(168*
c^2 + 245*c*d*x + 96*d^2*x^2) + x*(18*c*d*x*(128*C + 105*D*x) + 5*d^2*x^2*
(189*C + 160*D*x) + 6*c^2*(245*C + 192*D*x)))) - 315*a^2*Sqrt[b]*(8*A*b*(
6*b*c^2 - a*d^2) + a*(-8*b*c*(c*C + 2*B*d) + 3*a*d*(C*d + 2*c*D)))*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/(40320*b^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {2185, 2185, 27, 687, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int (c+dx)^2 \left(bx^2+a\right)^{3/2} \left(b(9Cd-14cD)x^2d^2+(9Abd-4acD)d^2+\left(-5bDc^2+9bBd^2-4ad^2D\right)xd\right)dx}{9bd^3}+\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int bd^3(c+dx)^2\left(d(72Abd-27aCd+10acD)-\left(32aDd^2+b\left(-30Dc^2+45Cdc-72Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{8bd^2}+\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(9Cd}{9bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{8}d\int (c+dx)^2\left(d(72Abd-27aCd+10acD)-\left(32aDd^2+b\left(-30Dc^2+45Cdc-72Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx+}{9bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{8}d\left(\frac{\int (c+dx)\left(d\left(504Acdb^2+a\left(64ad^2D-b\left(-10Dc^2+99Cdc+144Bd^2\right)\right)\right)-3b\left(a(63Cd-2cD)d^2+2b\left(-10Dc^3+15Cdc^2-24Bd^2c-84Ad^3\right)\right)x\right)(bx}{7b}\right.}{9bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{8}d\left(\frac{\frac{21}{2}d^2\left(8Ab\left(6bc^2-ad^2\right)-a(8bc(2Bd+cC)-3ad(2cD+Cd))\right)\int \left(bx^2+a\right)^{3/2}dx+\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2d^4D-8abd^2\left(9Bd^2+c^2(-D)+18cCd\right)-3b^2c\left(-1\right.\right.}{5b}}{7b}\right.}{9bd^2}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{8}d\left(\frac{\frac{21}{2}d^2\left(8Ab\left(6bc^2-ad^2\right)-a(8bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2d^4D-8abd^2\left(9Bd^2+c^2(-\right.\right.}{}}{7b}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{8}d\left(\frac{\frac{21}{2}d^2\left(8Ab\left(6bc^2-ad^2\right)-a(8bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2d^4\right.}{}}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{8}d\left(\frac{\frac{21}{2}d^2\left(8Ab\left(6bc^2-ad^2\right)-a(8bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{2\left(a+bx^2\right)^{5/}}{}}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}d\left(\frac{\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2d^4D-8abd^2\left(9Bd^2+c^2(-D)+18cCd\right)-3b^2c\left(-168Ad^3-24Bcd^2-10c^3D+15c^2Cd\right)\right)}{5b}+\frac{21}{2}d^2\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{\cdots}\right.\right.}{\cdots}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

input
```
Int[(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^4*(a + b*x^2)^(5/2))/(9*b*d^2) + ((d*(9*C*d - 14*c*D)*(c + d*
x)^3*(a + b*x^2)^(5/2))/8 + (d*(-1/7*((32*a*d^2*D + b*(45*c*C*d - 72*B*d^2
 - 30*c^2*D))*(c + d*x)^2*(a + b*x^2)^(5/2))/b + ((2*(32*a^2*d^4*D - 8*a*b
*d^2*(18*c*C*d + 9*B*d^2 - c^2*D) - 3*b^2*c*(15*c^2*C*d - 24*B*c*d^2 - 168
*A*d^3 - 10*c^3*D))*(a + b*x^2)^(5/2))/(5*b) - (d*(a*d^2*(63*C*d - 2*c*D)
+ 2*b*(15*c^2*C*d - 24*B*c*d^2 - 84*A*d^3 - 10*c^3*D))*x*(a + b*x^2)^(5/2)
)/2 + (21*d^2*(8*A*b*(6*b*c^2 - a*d^2) - a*(8*b*c*(c*C + 2*B*d) - 3*a*d*(C
*d + 2*c*D)))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/2)/(7*b)))/8)/(9*b
*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.94

method	result
default	$A\,c^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{c(2Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}+d(Cd+2Dc)\left(\frac{x^{3}\left(b\,x^{2}+c\right.}{8b}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*c*(2*A*d+B*c)/b*(b*x^2+a)^(5/2)+d*(C*d+2
*D*c)*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*
(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))))))+(A*d^2+2*B*c*d+C*c^2)*(1/6*x*(b*x^2+a)^(5/2)/b-1
/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(B*d^2+2*C*c*d+D*c^2)*(1/7*x^2*(b*x^2+a)^(
5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+D*d^2*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/
b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1119, normalized size of antiderivative = 2.68

$$\int (c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[1/80640*(315*(8*(C*a^3*b - 6*A*a^2*b^2)*c^2 - 2*(3*D*a^4 - 8*B*a^3*b)*c*d
 - (3*C*a^4 - 8*A*a^3*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) + 2*(4480*D*b^4*d^2*x^8 + 5040*(2*D*b^4*c*d + C*b^4*d^2)*x^7 +
 640*(9*D*b^4*c^2 + 18*C*b^4*c*d + (10*D*a*b^3 + 9*B*b^4)*d^2)*x^6 + 840*(
8*C*b^4*c^2 + 2*(9*D*a*b^3 + 8*B*b^4)*c*d + (9*C*a*b^3 + 8*A*b^4)*d^2)*x^5
 + 384*(3*(8*D*a*b^3 + 7*B*b^4)*c^2 + 6*(8*C*a*b^3 + 7*A*b^4)*c*d + (D*a^2
*b^2 + 24*B*a*b^3)*d^2)*x^4 + 210*(8*(7*C*a*b^3 + 6*A*b^4)*c^2 + 2*(3*D*a^
2*b^2 + 56*B*a*b^3)*c*d + (3*C*a^2*b^2 + 56*A*a*b^3)*d^2)*x^3 - 1152*(2*D*
a^3*b - 7*B*a^2*b^2)*c^2 - 2304*(2*C*a^3*b - 7*A*a^2*b^2)*c*d + 256*(4*D*a
^4 - 9*B*a^3*b)*d^2 + 128*(9*(D*a^2*b^2 + 14*B*a*b^3)*c^2 + 18*(C*a^2*b^2
+ 14*A*a*b^3)*c*d - (4*D*a^3*b - 9*B*a^2*b^2)*d^2)*x^2 + 315*(8*(C*a^2*b^2
 + 10*A*a*b^3)*c^2 - 2*(3*D*a^3*b - 8*B*a^2*b^2)*c*d - (3*C*a^3*b - 8*A*a^
2*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1/40320*(315*(8*(C*a^3*b - 6*A*a^2*b^
2)*c^2 - 2*(3*D*a^4 - 8*B*a^3*b)*c*d - (3*C*a^4 - 8*A*a^3*b)*d^2)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4480*D*b^4*d^2*x^8 + 5040*(2*D*b^4*
c*d + C*b^4*d^2)*x^7 + 640*(9*D*b^4*c^2 + 18*C*b^4*c*d + (10*D*a*b^3 + 9*B
*b^4)*d^2)*x^6 + 840*(8*C*b^4*c^2 + 2*(9*D*a*b^3 + 8*B*b^4)*c*d + (9*C*a*b
^3 + 8*A*b^4)*d^2)*x^5 + 384*(3*(8*D*a*b^3 + 7*B*b^4)*c^2 + 6*(8*C*a*b^3 +
 7*A*b^4)*c*d + (D*a^2*b^2 + 24*B*a*b^3)*d^2)*x^4 + 210*(8*(7*C*a*b^3 + 6*
A*b^4)*c^2 + 2*(3*D*a^2*b^2 + 56*B*a*b^3)*c*d + (3*C*a^2*b^2 + 56*A*a*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1302 vs. $2(403) = 806$.

Time = 0.87 (sec) , antiderivative size = 1302, normalized size of antiderivative = 3.12

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(D*b*d**2*x**8/9 + x**7*(C*b**2*d**2 + 2*D*b**
2*c*d)/(8*b) + x**6*(B*b**2*d**2 + 2*C*b**2*c*d + 10*D*a*b*d**2/9 + D*b**2
*c**2)/(7*b) + x**5*(A*b**2*d**2 + 2*B*b**2*c*d + 2*C*a*b*d**2 + C*b**2*c*
*2 + 4*D*a*b*c*d - 7*a*(C*b**2*d**2 + 2*D*b**2*c*d)/(8*b))/(6*b) + x**4*(2
*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d + D*a**2*d**2 + 2*D
*a*b*c**2 - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d + 10*D*a*b*d**2/9 + D*b**2*c**
2)/(7*b))/(5*b) + x**3*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*
d**2 + 2*C*a*b*c**2 + 2*D*a**2*c*d - 5*a*(A*b**2*d**2 + 2*B*b**2*c*d + 2*C
*a*b*d**2 + C*b**2*c**2 + 4*D*a*b*c*d - 7*a*(C*b**2*d**2 + 2*D*b**2*c*d)/(
8*b))/(6*b))/(4*b) + x**2*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*
a**2*c*d + D*a**2*c**2 - 4*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 +
4*C*a*b*c*d + D*a**2*d**2 + 2*D*a*b*c**2 - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d
 + 10*D*a*b*d**2/9 + D*b**2*c**2)/(7*b))/(5*b))/(3*b) + x*(A*a**2*d**2 + 2
*A*a*b*c**2 + 2*B*a**2*c*d + C*a**2*c**2 - 3*a*(2*A*a*b*d**2 + A*b**2*c**2
 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*c**2 + 2*D*a**2*c*d - 5*a*(A*b**2*d
**2 + 2*B*b**2*c*d + 2*C*a*b*d**2 + C*b**2*c**2 + 4*D*a*b*c*d - 7*a*(C*b**
2*d**2 + 2*D*b**2*c*d)/(8*b))/(6*b))/(4*b))/(2*b) + (2*A*a**2*c*d + B*a**2
*c**2 - 2*a*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d + D*a
**2*c**2 - 4*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d +
D*a**2*d**2 + 2*D*a*b*c**2 - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d + 10*D*a*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.17

$$\int (c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Dd^2x^4}{9\,b}-\frac{4\,(bx^2+a)^{\frac{5}{2}}Dad^2x^2}{63\,b^2}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ac^2x+\frac{3}{8}\,\sqrt{bx^2+a}Aac^2x+\frac{(2\,Dcd+Cd^2)(bx^2+a)^{\frac{5}{2}}x^3}{8\,b}+\frac{3\,Aa^2c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{(bx^2+a)^{\frac{5}{2}}Bc^2}{5\,b}+\frac{2\,(bx^2+a)^{\frac{5}{2}}Acd}{5\,b}+\frac{8\,(bx^2+a)^{\frac{5}{2}}Da^2d^2}{315\,b^3}+\frac{(Dc^2+2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}x^2}{7\,b}-\frac{(2\,Dcd+Cd^2)(bx^2+a)^{\frac{5}{2}}ax}{16\,b^2}+\frac{(2\,Dcd+Cd^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,(2\,Dcd+Cd^2)\sqrt{bx^2+a}a^3x}{128\,b^2}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}x}{6\,b}-\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}ax}{24\,b}-\frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}a^2x}{16\,b}+\frac{3\,(2\,Dcd+Cd^2)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{(Cc^2+2\,Bcd+Ad^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\,(Dc^2+2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/9*(b*x^2 + a)^(5/2)*D*d^2*x^4/b - 4/63*(b*x^2 + a)^(5/2)*D*a*d^2*x^2/b^2
 + 1/4*(b*x^2 + a)^(3/2)*A*c^2*x + 3/8*sqrt(b*x^2 + a)*A*a*c^2*x + 1/8*(2*
D*c*d + C*d^2)*(b*x^2 + a)^(5/2)*x^3/b + 3/8*A*a^2*c^2*arcsinh(b*x/sqrt(a*
b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B*c^2/b + 2/5*(b*x^2 + a)^(5/2)*A*c*d/
b + 8/315*(b*x^2 + a)^(5/2)*D*a^2*d^2/b^3 + 1/7*(D*c^2 + 2*C*c*d + B*d^2)*
(b*x^2 + a)^(5/2)*x^2/b - 1/16*(2*D*c*d + C*d^2)*(b*x^2 + a)^(5/2)*a*x/b^2
 + 1/64*(2*D*c*d + C*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b^2 + 3/128*(2*D*c*d + C
*d^2)*sqrt(b*x^2 + a)*a^3*x/b^2 + 1/6*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a
)^(5/2)*x/b - 1/24*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a*x/b - 1/1
6*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a^2*x/b + 3/128*(2*D*c*d + C*d
^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*a^
3*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/35*(D*c^2 + 2*C*c*d + B*d^2)*(b*x^2 +
 a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.43

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{40320}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(2\left(7\left(8\,Dbd^2x+\frac{9\,(2\,Db^8cd+Cb^8d^2)}{b^7}\right)x+\frac{8\,(9\,Db^8c^2+18\,C\cdots}{\cdots}\right.\right.\right.\right.\right.\right.\right.$$
$$+\frac{(8\,Ca^3bc^2-48\,Aa^2b^2c^2-6\,Da^4cd+16\,Ba^3bcd-3\,Ca^4d^2+8\,Aa^3bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/40320*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*(8*D*b*d^2*x + 9*(2*D*b^8*c*d + C
*b^8*d^2)/b^7)*x + 8*(9*D*b^8*c^2 + 18*C*b^8*c*d + 10*D*a*b^7*d^2 + 9*B*b^
8*d^2)/b^7)*x + 21*(8*C*b^8*c^2 + 18*D*a*b^7*c*d + 16*B*b^8*c*d + 9*C*a*b^
7*d^2 + 8*A*b^8*d^2)/b^7)*x + 48*(24*D*a*b^7*c^2 + 21*B*b^8*c^2 + 48*C*a*b
^7*c*d + 42*A*b^8*c*d + D*a^2*b^6*d^2 + 24*B*a*b^7*d^2)/b^7)*x + 105*(56*C
*a*b^7*c^2 + 48*A*b^8*c^2 + 6*D*a^2*b^6*c*d + 112*B*a*b^7*c*d + 3*C*a^2*b^
6*d^2 + 56*A*a*b^7*d^2)/b^7)*x + 64*(9*D*a^2*b^6*c^2 + 126*B*a*b^7*c^2 + 1
8*C*a^2*b^6*c*d + 252*A*a*b^7*c*d - 4*D*a^3*b^5*d^2 + 9*B*a^2*b^6*d^2)/b^7
)*x + 315*(8*C*a^2*b^6*c^2 + 80*A*a*b^7*c^2 - 6*D*a^3*b^5*c*d + 16*B*a^2*b
^6*c*d - 3*C*a^3*b^5*d^2 + 8*A*a^2*b^6*d^2)/b^7)*x - 128*(18*D*a^3*b^5*c^2
 - 63*B*a^2*b^6*c^2 + 36*C*a^3*b^5*c*d - 126*A*a^2*b^6*c*d - 8*D*a^4*b^4*d
^2 + 18*B*a^3*b^5*d^2)/b^7) + 1/128*(8*C*a^3*b*c^2 - 48*A*a^2*b^2*c^2 - 6*
D*a^4*c*d + 16*B*a^3*b*c*d - 3*C*a^4*d^2 + 8*A*a^3*b*d^2)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(b\,x^2+a\right)^{3/2} \left(c+d\,x\right)^2 \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (dx+c)^2 \left(b\,x^2+a\right)^{\frac{3}{2}} \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output

```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.73 $\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 272

$$\begin{aligned}\int (c+dx)\left(a+bx^2\right)^{3/2}&\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a(48Ab^2c-a(8b(cC+Bd)-3adD))\,x\sqrt{a+bx^2}}{128b^2}\\&+\frac{(48Ab^2c-a(8b(cC+Bd)-3adD))\,x(a+bx^2)^{3/2}}{192b^2}\\&+\frac{(bBc+Abd-aCd-acD)\left(a+bx^2\right)^{5/2}}{5b^2}\\&+\frac{(8b(cC+Bd)-3adD)x(a+bx^2)^{5/2}}{48b^2}\\&+\frac{dDx^3(a+bx^2)^{5/2}}{8b}+\frac{(Cd+cD)\left(a+bx^2\right)^{7/2}}{7b^2}\\&+\frac{a^2(48Ab^2c-a(8b(cC+Bd)-3adD))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}\end{aligned}$$

output
```
1/128*a*(48*A*b^2*c-a*(8*b*(B*d+C*c)-3*D*a*d))*x*(b*x^2+a)^(1/2)/b^2+1/192
*(48*A*b^2*c-a*(8*b*(B*d+C*c)-3*D*a*d))*x*(b*x^2+a)^(3/2)/b^2+1/5*(A*b*d+B
*b*c-C*a*d-D*a*c)*(b*x^2+a)^(5/2)/b^2+1/48*(8*b*(B*d+C*c)-3*D*a*d)*x*(b*x^
2+a)^(5/2)/b^2+1/8*d*D*x^3*(b*x^2+a)^(5/2)/b+1/7*(C*d+D*c)*(b*x^2+a)^(7/2)
/b^2+1/128*a^2*(48*A*b^2*c-a*(8*b*(B*d+C*c)-3*D*a*d))*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-3a^3(256Cd+256cD+105dDx)+6a^2b(448Ad+28B(16c+5dx)+x(140cC}{}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-3*a^3*(256*C*d + 256*c*D + 105*d*D*x) + 6*a^2*b
*(448*A*d + 28*B*(16*c + 5*d*x) + x*(140*c*C + 64*C*d*x + 64*c*D*x + 35*d*
D*x^2)) + 8*a*b^2*x*(42*A*(25*c + 16*d*x) + x*(14*B*(48*c + 35*d*x) + x*(4
90*c*C + 384*C*d*x + 384*c*D*x + 315*d*D*x^2))) + 16*b^3*x^3*(42*A*(5*c +
4*d*x) + x*(28*B*(6*c + 5*d*x) + 5*x*(4*c*(7*C + 6*D*x) + 3*d*x*(8*C + 7*D
*x))))) - 105*a^2*(48*A*b^2*c + a*(-8*b*(c*C + B*d) + 3*a*d*D))*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(13440*b^(5/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2185, 2185, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(b(8Cd-13cD)x^2d^2+(8Abd-3acD)d^2+\left(-5bDc^2+8bBd^2-3ad^2D\right)xd\right)dx}{8bd^3}+\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int bd^3(c+dx)\left(d(56Abd-16aCd+5acD)-\left(21aDd^2+b\left(-30Dc^2+40Cdc-56Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7bd^2}+\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(8Cd-}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\int (c+dx)\left(d(56Abd-16aCd+5acD)-\left(21aDd^2+b\left(-30Dc^2+40Cdc-56Bd^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx+\frac{1}{7}d}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{7}d\left(\frac{7d^2\left(48Ab^2c-a(8b(Bd+cC)-3adD)\right)\int\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(cD+Cd)+b\left(-28Ad^3-28Bcd^2-15c^3D+20c^2Cd\right)\right)}{5b}-\frac{dx(a-}{}\right.}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}d\left(\frac{7d^2\left(48Ab^2c-a(8b(Bd+cC)-3adD)\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(cD+Cd)+b\left(-28Ad^3-28Bcd^2-15c^3D+\right.\right.}{5b}\right.}{8bd^3}$$
$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}d\left(\frac{7d^2(48Ab^2c-a(8b(Bd+cC)-3adD))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(cD+Cd)+b(-28Ad}{5b}\right.}{8}$$

$$\frac{D(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{7}d\left(\frac{7d^2(48Ab^2c-a(8b(Bd+cC)-3adD))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(cD+Cd)+}{}\right.}{}$$

$$\frac{D(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{7}d\left(\frac{7d^2\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(48Ab^2c-a(8b(Bd+cC)-3adD))}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(cD+Cd)+b}{}\right.}{}$$

$$\frac{D(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}$$

input
```
Int[(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^3*(a + b*x^2)^(5/2))/(8*b*d^2) + ((d*(8*C*d - 13*c*D)*(c + d*
x)^2*(a + b*x^2)^(5/2))/7 + (d*((-2*(8*a*d^2*(C*d + c*D) + b*(20*c^2*C*d -
 28*B*c*d^2 - 28*A*d^3 - 15*c^3*D))*(a + b*x^2)^(5/2))/(5*b) - (d*(21*a*d^
2*D + b*(40*c*C*d - 56*B*d^2 - 30*c^2*D))*x*(a + b*x^2)^(5/2))/(6*b) + (7*
d^2*(48*A*b^2*c - a*(8*b*(c*C + B*d) - 3*a*d*D))*((x*(a + b*x^2)^(3/2))/4
+ (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
2*Sqrt[b])))/4))/(6*b)))/7)/(8*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.10

method	result
default	$Ac\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{(Ad+Bc)(b\,x^2+a)^{\frac{5}{2}}}{5b}+(Bd+Cc)\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\right.$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*(A*d+B*c)/b*(b*x^2+a)^(5/2)+(B*d+C*c)*(1/6
*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)
^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(C*d+D*c)*(1/7*x^2*(
b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+D*d*(1/8*x^3*(b*x^2+a)^(5/2)/
b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1
/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.53

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[1/26880*(105*(8*(C*a^3*b - 6*A*a^2*b^2)*c - (3*D*a^4 - 8*B*a^3*b)*d)*sqrt
(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(1680*D*b^4*d*x^7
+ 1920*(D*b^4*c + C*b^4*d)*x^6 + 280*(8*C*b^4*c + (9*D*a*b^3 + 8*B*b^4)*d)
*x^5 + 384*((8*D*a*b^3 + 7*B*b^4)*c + (8*C*a*b^3 + 7*A*b^4)*d)*x^4 + 70*(8
*(7*C*a*b^3 + 6*A*b^4)*c + (3*D*a^2*b^2 + 56*B*a*b^3)*d)*x^3 + 384*((D*a^2
*b^2 + 14*B*a*b^3)*c + (C*a^2*b^2 + 14*A*a*b^3)*d)*x^2 - 384*(2*D*a^3*b -
7*B*a^2*b^2)*c - 384*(2*C*a^3*b - 7*A*a^2*b^2)*d + 105*(8*(C*a^2*b^2 + 10*
A*a*b^3)*c - (3*D*a^3*b - 8*B*a^2*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/13440
*(105*(8*(C*a^3*b - 6*A*a^2*b^2)*c - (3*D*a^4 - 8*B*a^3*b)*d)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1680*D*b^4*d*x^7 + 1920*(D*b^4*c + C*b^
4*d)*x^6 + 280*(8*C*b^4*c + (9*D*a*b^3 + 8*B*b^4)*d)*x^5 + 384*((8*D*a*b^3
 + 7*B*b^4)*c + (8*C*a*b^3 + 7*A*b^4)*d)*x^4 + 70*(8*(7*C*a*b^3 + 6*A*b^4)
*c + (3*D*a^2*b^2 + 56*B*a*b^3)*d)*x^3 + 384*((D*a^2*b^2 + 14*B*a*b^3)*c +
 (C*a^2*b^2 + 14*A*a*b^3)*d)*x^2 - 384*(2*D*a^3*b - 7*B*a^2*b^2)*c - 384*(
2*C*a^3*b - 7*A*a^2*b^2)*d + 105*(8*(C*a^2*b^2 + 10*A*a*b^3)*c - (3*D*a^3*
b - 8*B*a^2*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(262) = 524$.

Time = 0.82 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.56

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Dbdx^7}{8}+\frac{x^6(Cb^2d+Db^2c)}{7b}+\frac{x^5\left(Bb^2d+Cb^2c+\frac{9Dabd}{8}\right)}{6b}+\frac{x^4\left(Ab^2d+Bb^2c+2Cabd+2Dabc-\frac{6a\left(Cb^2d+D\right.}{7b}\right.}{5b} \right. \\ a^{\frac{3}{2}}\left(Acx+\frac{Ddx^5}{5}+\frac{x^4(Cd+Dc)}{4}+\frac{x^3(Bd+Cc)}{3}+\frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b*d*x**7/8 + x**6*(C*b**2*d + D*b**2*c)/(7*
b) + x**5*(B*b**2*d + C*b**2*c + 9*D*a*b*d/8)/(6*b) + x**4*(A*b**2*d + B*b
**2*c + 2*C*a*b*d + 2*D*a*b*c - 6*a*(C*b**2*d + D*b**2*c)/(7*b))/(5*b) + x
**3*(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c + D*a**2*d - 5*a*(B*b**2*d + C*b**2*
c + 9*D*a*b*d/8)/(6*b))/(4*b) + x**2*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d + D
*a**2*c - 4*a*(A*b**2*d + B*b**2*c + 2*C*a*b*d + 2*D*a*b*c - 6*a*(C*b**2*d
 + D*b**2*c)/(7*b))/(5*b))/(3*b) + x*(2*A*a*b*c + B*a**2*d + C*a**2*c - 3*
a*(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c + D*a**2*d - 5*a*(B*b**2*d + C*b**2*c
+ 9*D*a*b*d/8)/(6*b))/(4*b))/(2*b) + (A*a**2*d + B*a**2*c - 2*a*(2*A*a*b*d
 + 2*B*a*b*c + C*a**2*d + D*a**2*c - 4*a*(A*b**2*d + B*b**2*c + 2*C*a*b*d
+ 2*D*a*b*c - 6*a*(C*b**2*d + D*b**2*c)/(7*b))/(5*b))/(3*b))/b) + (A*a**2*
c - a*(2*A*a*b*c + B*a**2*d + C*a**2*c - 3*a*(A*b**2*c + 2*B*a*b*d + 2*C*a
*b*c + D*a**2*d - 5*a*(B*b**2*d + C*b**2*c + 9*D*a*b*d/8)/(6*b))/(4*b))/(2
*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*c*x + D*d*x**5/5
 + x**4*(C*d + D*c)/4 + x**3*(B*d + C*c)/3 + x**2*(A*d + B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.17

$$
\begin{aligned}
&\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ddx^3}{8\,b} \\
&+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Acx+\frac{3}{8}\sqrt{bx^2+a}Aacx-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Dadx}{16\,b^2} \\
&+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Da^2dx}{64\,b^2}+\frac{3\sqrt{bx^2+a}Da^3dx}{128\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}(Dc+Cd)x^2}{7\,b} \\
&+\frac{3\,Aa^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}+\frac{3\,Da^4d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bc}{5\,b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ad}{5\,b} \\
&+\frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)x}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)ax}{24\,b}-\frac{\sqrt{bx^2+a}(Cc+Bd)a^2x}{16\,b} \\
&-\frac{(Cc+Bd)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}(Dc+Cd)a}{35\,b^2}
\end{aligned}
$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/8*(b*x^2 + a)^(5/2)*D*d*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*c*x + 3/8*sqrt(b
*x^2 + a)*A*a*c*x - 1/16*(b*x^2 + a)^(5/2)*D*a*d*x/b^2 + 1/64*(b*x^2 + a)^
(3/2)*D*a^2*d*x/b^2 + 3/128*sqrt(b*x^2 + a)*D*a^3*d*x/b^2 + 1/7*(b*x^2 + a
)^(5/2)*(D*c + C*d)*x^2/b + 3/8*A*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3
/128*D*a^4*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 1/5*(b*x^2 + a)^(5/2)*B*c/b
+ 1/5*(b*x^2 + a)^(5/2)*A*d/b + 1/6*(b*x^2 + a)^(5/2)*(C*c + B*d)*x/b - 1/
24*(b*x^2 + a)^(3/2)*(C*c + B*d)*a*x/b - 1/16*sqrt(b*x^2 + a)*(C*c + B*d)*
a^2*x/b - 1/16*(C*c + B*d)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/35*(b*x^
2 + a)^(5/2)*(D*c + C*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.28

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{13440}\sqrt{bx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\Bigg(5\Bigg(6\Bigg(7\,Dbdx+\frac{8\,(Db^7c+Cb^7d)}{b^6}\Bigg)x+\frac{7\,(8\,Cb^7c+9\,Dab^6d+8\,\cdots}{b^6} \cdots + \frac{(8\,Ca^3bc-48\,Aa^2b^2c-3\,Da^4d+8\,Ba^3bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*D*b*d*x + 8*(D*b^7*c + C*b^7*d)/b
^6)*x + 7*(8*C*b^7*c + 9*D*a*b^6*d + 8*B*b^7*d)/b^6)*x + 48*(8*D*a*b^6*c +
 7*B*b^7*c + 8*C*a*b^6*d + 7*A*b^7*d)/b^6)*x + 35*(56*C*a*b^6*c + 48*A*b^7
*c + 3*D*a^2*b^5*d + 56*B*a*b^6*d)/b^6)*x + 192*(D*a^2*b^5*c + 14*B*a*b^6*
c + C*a^2*b^5*d + 14*A*a*b^6*d)/b^6)*x + 105*(8*C*a^2*b^5*c + 80*A*a*b^6*c
 - 3*D*a^3*b^4*d + 8*B*a^2*b^5*d)/b^6)*x - 384*(2*D*a^3*b^4*c - 7*B*a^2*b^
5*c + 2*C*a^3*b^4*d - 7*A*a^2*b^5*d)/b^6) + 1/128*(8*C*a^3*b*c - 48*A*a^2*
b^2*c - 3*D*a^4*d + 8*B*a^3*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2 +Dx^3\right) \, dx = \int \left(b\,x^2+a\right)^{3/2} (c+d\,x) \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.96

$$\int (c+dx) \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2 +Dx^3\right) \, dx = \frac{2688\sqrt{b\,x^2+a}\,a^3b^2d+2688\sqrt{b\,x^2+a}\,a^2b^3c+2688\sqrt{b\,x^2+a}\,b^5c\,x^4+2240\sqrt{b\,x^2+a}\,b^5d\,x^5+}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2688*sqrt(a + b*x**2)*a**3*b**2*d - 1536*sqrt(a + b*x**2)*a**3*b*c*d - 31
5*sqrt(a + b*x**2)*a**3*b*d**2*x + 8400*sqrt(a + b*x**2)*a**2*b**3*c*x + 2
688*sqrt(a + b*x**2)*a**2*b**3*c + 5376*sqrt(a + b*x**2)*a**2*b**3*d*x**2
+ 840*sqrt(a + b*x**2)*a**2*b**3*d*x + 840*sqrt(a + b*x**2)*a**2*b**2*c**2
*x + 768*sqrt(a + b*x**2)*a**2*b**2*c*d*x**2 + 210*sqrt(a + b*x**2)*a**2*b
**2*d**2*x**3 + 3360*sqrt(a + b*x**2)*a*b**4*c*x**3 + 5376*sqrt(a + b*x**2
)*a*b**4*c*x**2 + 2688*sqrt(a + b*x**2)*a*b**4*d*x**4 + 3920*sqrt(a + b*x*
*2)*a*b**4*d*x**3 + 3920*sqrt(a + b*x**2)*a*b**3*c**2*x**3 + 6144*sqrt(a +
 b*x**2)*a*b**3*c*d*x**4 + 2520*sqrt(a + b*x**2)*a*b**3*d**2*x**5 + 2688*s
qrt(a + b*x**2)*b**5*c*x**4 + 2240*sqrt(a + b*x**2)*b**5*d*x**5 + 2240*sqr
t(a + b*x**2)*b**4*c**2*x**5 + 3840*sqrt(a + b*x**2)*b**4*c*d*x**6 + 1680*
sqrt(a + b*x**2)*b**4*d**2*x**7 + 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a**4*d**2 + 5040*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a**3*b**2*c - 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**3*b**2*d - 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**3*b*c**2)/(13440*b**3)
```

3.74 $\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx = \frac{a(6Ab-aC)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aC)x(a+bx^2)^{3/2}}{24b} + \frac{(bB-aD)(a+bx^2)^{5/2}}{5b^2} + \frac{Cx(a+bx^2)^{5/2}}{6b} + \frac{D(a+bx^2)^{7/2}}{7b^2} + \frac{a^2(6Ab-aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b-C*a)*x*(b*x^2+a)^(3/2)/
b+1/5*(B*b-D*a)*(b*x^2+a)^(5/2)/b^2+1/6*C*x*(b*x^2+a)^(5/2)/b+1/7*D*(b*x^2
+a)^(7/2)/b^2+1/16*a^2*(6*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3
/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{a+bx^2}(-96a^3D+4b^3x^3(105A+84Bx+70Cx^2+60Dx^3)+3a^2b(112B+x(35C+16Dx$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*D + 4*b^3*x^3*(105*A + 84*B*x + 70*C*x^2 + 60*D*
x^3) + 3*a^2*b*(112*B + x*(35*C + 16*D*x)) + 2*a*b^2*x*(525*A + x*(336*B +
 245*C*x + 192*D*x^2))) + 105*a^2*Sqrt[b]*(-6*A*b + a*C)*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(1680*b^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2346, 2346, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \left(bx^2+a\right)^{3/2}\left(7bCx^2+(7bB-2aD)x+7Ab\right)dx}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int b(7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx}{6b}+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\int (7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\int \left(bx^2+a\right)^{3/2}dx+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(5/2))/(7*b) + ((7*C*x*(a + b*x^2)^(5/2))/6 + ((6*(7*b*
B - 2*a*D)*(a + b*x^2)^(5/2))/(5*b) + 7*(6*A*b - a*C)*((x*(a + b*x^2)^(3/2
))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b])))/4))/6)/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

method	result
default	$A\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+C\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{\right.}\right.}{4}\right.}{6b}\right.$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+C*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2
+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2)))))+D*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+1/5*B
*(b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.01

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\left[-\frac{105\,(Ca^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Db^3x^6+280\,Cb^3x^5+\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(C*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b
^3)*x^4 - 96*D*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^
2*b + 14*B*a*b^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2
, 1/1680*(105*(C*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b^3)*x^4 - 96*D
*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^2*b + 14*B*a*b
^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(143) = 286$.

Time = 0.51 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cbx^5}{6}+\frac{Dbx^6}{7}+\frac{x^4\left(Bb^2+\frac{8Dab}{7}\right)}{5b}+\frac{x^3\left(Ab^2+\frac{7Cab}{6}\right)}{4b}+\frac{x^2\cdot\left(2Bab+Da^2-\frac{4a\left(Bb^2+\frac{8Dab}{7}\right)}{5b}\right)}{3b}+\frac{x\left(2A\right.}{} \right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b*x**5/6 + D*b*x**6/7 + x**4*(B*b**2 + 8*D*
a*b/7)/(5*b) + x**3*(A*b**2 + 7*C*a*b/6)/(4*b) + x**2*(2*B*a*b + D*a**2 -
4*a*(B*b**2 + 8*D*a*b/7)/(5*b))/(3*b) + x*(2*A*a*b + C*a**2 - 3*a*(A*b**2
+ 7*C*a*b/6)/(4*b))/(2*b) + (B*a**2 - 2*a*(2*B*a*b + D*a**2 - 4*a*(B*b**2
+ 8*D*a*b/7)/(5*b))/(3*b))/b) + (A*a**2 - a*(2*A*a*b + C*a**2 - 3*a*(A*b**
2 + 7*C*a*b/6)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2
*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(
3/2)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\ dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^2}{7\,b} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ax + \frac{3}{8}\,\sqrt{bx^2+a}Aax + \frac{(bx^2+a)^{\frac{5}{2}}Cx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Cax}{24\,b} - \frac{\sqrt{bx^2+a}Ca^2x}{16\,b} - \frac{Ca^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Da}{35\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*D*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2
 + a)*A*a*x + 1/6*(b*x^2 + a)^(5/2)*C*x/b - 1/24*(b*x^2 + a)^(3/2)*C*a*x/b
 - 1/16*sqrt(b*x^2 + a)*C*a^2*x/b - 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(3
/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/35*(b*x^2 + a)^(5/2)*D*
a/b^2 + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{1680}\,\sqrt{bx^2+a}\Bigg(\Bigg(2\,\Bigg(\Bigg(4\,\Bigg(5\,(6\,Dbx+7\,Cb)x + \frac{6\,(8\,Dab^5+7\,Bb^6)}{b^5}\Bigg)x + \frac{35\,(7\,Cab^5+6\,Ab^6)}{b^5} + \frac{(Ca^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*b*x + 7*C*b)*x + 6*(8*D*a*b^5 + 7*B
*b^6)/b^5)*x + 35*(7*C*a*b^5 + 6*A*b^6)/b^5)*x + 24*(D*a^2*b^4 + 14*B*a*b^
5)/b^5)*x + 105*(C*a^2*b^4 + 10*A*a*b^5)/b^5)*x - 48*(2*D*a^3*b^3 - 7*B*a^
2*b^4)/b^5) + 1/16*(C*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx = \int (b\,x^2+a)^{3/2} \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.60

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx = \frac{-96\sqrt{b\,x^2+a}\,a^3d+1050\sqrt{b\,x^2+a}\,a^2b^2x+336\sqrt{b\,x^2+a}\,a^2b^2+105\sqrt{b\,x^2+a}\,a^2bcx+48\sqrt{b}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 96*sqrt(a + b*x**2)*a**3*d + 1050*sqrt(a + b*x**2)*a**2*b**2*x + 336*s
qrt(a + b*x**2)*a**2*b**2 + 105*sqrt(a + b*x**2)*a**2*b*c*x + 48*sqrt(a +
b*x**2)*a**2*b*d*x**2 + 420*sqrt(a + b*x**2)*a*b**3*x**3 + 672*sqrt(a + b*
x**2)*a*b**3*x**2 + 490*sqrt(a + b*x**2)*a*b**2*c*x**3 + 384*sqrt(a + b*x*
*2)*a*b**2*d*x**4 + 336*sqrt(a + b*x**2)*b**4*x**4 + 280*sqrt(a + b*x**2)*
b**3*c*x**5 + 240*sqrt(a + b*x**2)*b**3*d*x**6 + 630*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b - 105*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**3*c)/(1680*b**2)
```

3.75 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 501

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx} \, dx &= \frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{d^6} \\
&- \frac{\left(a^2d^3D+6abd(cCd-Bd^2-c^2D)+8b^2c\left(c^2C-Bcd+Ad^2-\frac{c^3D}{d}\right)\right)x\sqrt{a+bx^2}}{16bd^4} \\
&+ \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{3/2}}{3d^4} \\
&- \frac{(ad^2D+6b(cCd-Bd^2-c^2D))x(a+bx^2)^{3/2}}{24bd^3} \\
&+ \frac{(6Cd-11cD)(a+bx^2)^{5/2}}{30bd^2} + \frac{D(c+dx)(a+bx^2)^{5/2}}{6bd^2} \\
&- \frac{(a^3d^6D+6a^2bd^4(cCd-Bd^2-c^2D)+16b^3c^3(c^2Cd-Bcd^2+Ad^3-c^3D)+24ab^2cd^2(c^2Cd-Bcd^2+A}{16b^{3/2}d^7} \\
&- \frac{(bc^2+ad^2)^{3/2}(c^2Cd-Bcd^2+Ad^3-c^3D)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}
\end{aligned}
$$

output
```
(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^6-1/16*(a^2*
d^3*D+6*a*b*d*(-B*d^2+C*c*d-D*c^2)+8*b^2*c*(C*c^2-B*c*d+A*d^2-c^3*D/d))*x*
(b*x^2+a)^(1/2)/b/d^4+1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d^
4-1/24*(a*d^2*D+6*b*(-B*d^2+C*c*d-D*c^2))*x*(b*x^2+a)^(3/2)/b/d^3+1/30*(6*
C*d-11*D*c)*(b*x^2+a)^(5/2)/b/d^2+1/6*D*(d*x+c)*(b*x^2+a)^(5/2)/b/d^2-1/16
*(a^3*d^6*D+6*a^2*b*d^4*(-B*d^2+C*c*d-D*c^2)+16*b^3*c^3*(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)+24*a*b^2*c*d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*arctanh(b^(1/2)*
x/(b*x^2+a)^(1/2))/b^(3/2)/d^7-(a*d^2+b*c^2)^(3/2)*(A*d^3-B*c*d^2+C*c^2*d-
D*c^3)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(3a^2d^4(16Cd-16cD+5dDx)+2abd^2\left(-160c^3D+5c^2d(32C+15Dx)-cd^2(\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(3*a^2*d^4*(16*C*d - 16*c*D + 5*d*D*x) + 2*a*b*d^2*(-1
60*c^3*D + 5*c^2*d*(32*C + 15*D*x) - c*d^2*(160*B + 75*C*x + 48*D*x^2) + d
^3*(160*A + x*(75*B + 48*C*x + 35*D*x^2))) - 4*b^2*(60*c^5*D - 30*c^4*d*(2
*C + D*x) + 10*c^3*d^2*(6*B + x*(3*C + 2*D*x)) - 5*c^2*d^3*(12*A + x*(6*B
+ x*(4*C + 3*D*x))) + c*d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - d^5*
x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))))/b + 480*(-(b*c^2) - a*d^2)^(3
/2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*ArcTan[(Sqrt[b]*(c + d*x) - d*S
qrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (15*(-(a^3*d^6*D) + 6*a^2*b*d^4*
(-(c*C*d) + B*d^2 + c^2*D) + 16*b^3*c^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^
3*D) + 24*a*b^2*c*d^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))*Log[-(Sqrt[b
]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(240*d^7)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2185, 2185, 27, 682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(b(6Cd-11cD)x^2d^2+(6Abd-acD)d^2+(-5bDc^2+6bBd^2-ad^2D)xd)}{c+dx}dx}{6bd^3}+\frac{D(a+bx^2)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{\int \frac{5bd^3\left(d(6Abd-acD)-\left(aDd^2+6b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)(bx^2+a)^{3/2}}{c+dx}dx}{5bd^2}+\frac{1}{5}d(a+bx^2)^{5/2}(6Cd-11cD)}{6bd^3}+\frac{D(a+bx^2)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{\left(d(6Abd-acD)-\left(aDd^2+6b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)(bx^2+a)^{3/2}}{c+dx}dx+\frac{1}{5}d(a+bx^2)^{5/2}(6Cd-11cD)}{6bd^3}+\frac{D(a+bx^2)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{d\left(\frac{\int -\frac{b\left(3ad\left(acd^2D-2b\left(-Dc^3+Cdc^2-Bd^2c+4Ad^3\right)\right)+\left(4bc(6Abd-acD)d^2+\left(4bc^2+3ad^2\right)\left(aDd^2+6b\left(-Dc^2+Cdc-Bd^2\right)\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{(a+bx^2)^{3/2}\cdots}{\cdots}\right.}{6bd^3}$$
$$\frac{D(a+bx^2)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{\int \frac{b\left(3ad\left(acd^2D-2b\left(-Dc^3+Cdc^2-Bd^2c+4Ad^3\right)\right)+\left(4\cdots\right.\right.}{\cdots}}{6bd^3}\cdots\right.}{\cdots}\cdots\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\downarrow$ 27

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{\int \frac{\left(3ad\left(acd^2D-2b\left(-Dc^3+Cdc^2-Bd^2c+4Ad^3\right)\right)+\left(4\cdots\right.\right.}{\cdots}}{6bd^3}\cdots\right.}{\cdots}\cdots\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\downarrow$ 682

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{\frac{\int \frac{3b\left(ad\left(a^2cDd^4-2ab\left(-5Dc^3+5Cdc^2-5Bd^2c+8Ad^3\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\downarrow$ 27

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{\frac{3\int \frac{ad\left(a^2cDd^4-2ab\left(-5Dc^3+5Cdc^2-5Bd^2c+8Ad^3\right)\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\downarrow$ 719

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{3\left(\frac{\left(a^3d^6D+6a^2bd^4\left(-Bd^2+c^2(-D)+cCd\right)+24ab^2cd\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\big\downarrow$ 224

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{3\left(\frac{\left(a^3d^6D+6a^2bd^4\left(-Bd^2+c^2(-D)+cCd\right)+24ab^2cd\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3d^6D+6a^2bd^4\left(-Bd^2+c\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$\big\downarrow$ 488

$$d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{3\left(\frac{16b\left(ad^2+bc^2\right)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{}{bc}}{d}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{(a+bx^2)^{3/2}(8b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-dx(ad^2D+6b(-Bd^2+c^2(-D)+cCd)))}{4d^2}-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3d^6D+6a^2bd^4\left(-Bd^2+c\right.\right.}{}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(D*(c + d*x)*(a + b*x^2)^(5/2))/(6*b*d^2) + ((d*(6*C*d - 11*c*D)*(a + b*x^
2)^(5/2))/5 + d*(((8*b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) - d*(a*d^2*D +
6*b*(c*C*d - B*d^2 - c^2*D))*x)*(a + b*x^2)^(3/2))/(4*d^2) - (-1/2*((48*b*
(b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) - d*(4*b*c*d^2*(6*A*b*
d - a*c*D) + (4*b*c^2 + 3*a*d^2)*(a*d^2*D + 6*b*(c*C*d - B*d^2 - c^2*D)))*
x)*Sqrt[a + b*x^2])/d^2 + (3*(((a^3*d^6*D + 6*a^2*b*d^4*(c*C*d - B*d^2 - c
^2*D) + 16*b^3*c^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) + 24*a*b^2*c*d^2*(c
^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
Sqrt[b]*d) + (16*b*(b*c^2 + a*d^2)^(3/2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*
D)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d))/(2*d^
2))/(4*d^2)))/(6*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.59

method	result
default	$\frac{B d^2 \left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+D c^2 \left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{d(Cd-}{}}{}$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(
1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+D*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1
/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*d*(
C*d-D*c)/b*(b*x^2+a)^(5/2)+D*d^2*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(
b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2)))))-C*c*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)
+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(A*d^3-B*c*d^2+C*c^2*d-D*c
^3)/d^4*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(
1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+
c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d
^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/
2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2
+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(463) = 926$.

Time $= 0.17$ (sec) , antiderivative size $= 971$, normalized size of antiderivative $= 1.94$

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*D*b*c^4*x/d^5 - 1/2*sqrt(b*x^2 + a)*C*b*c^3*x/d^4 + 1/
4*(b*x^2 + a)^(3/2)*D*c^2*x/d^3 + 3/8*sqrt(b*x^2 + a)*D*a*c^2*x/d^3 + 1/2*
sqrt(b*x^2 + a)*B*b*c^2*x/d^3 - 1/4*(b*x^2 + a)^(3/2)*C*c*x/d^2 - 3/8*sqrt
(b*x^2 + a)*C*a*c*x/d^2 - 1/2*sqrt(b*x^2 + a)*A*b*c*x/d^2 + 1/4*(b*x^2 + a
)^(3/2)*B*x/d + 3/8*sqrt(b*x^2 + a)*B*a*x/d + 1/6*(b*x^2 + a)^(5/2)*D*x/(b
*d) - 1/24*(b*x^2 + a)^(3/2)*D*a*x/(b*d) - 1/16*sqrt(b*x^2 + a)*D*a^2*x/(b
*d) + D*b^(3/2)*c^6*arcsinh(b*x/sqrt(a*b))/d^7 - C*b^(3/2)*c^5*arcsinh(b*x
/sqrt(a*b))/d^6 + 3/2*D*a*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 + B*b^(3/
2)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 - 3/2*C*a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a
*b))/d^4 - A*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 3/8*D*a^2*c^2*arcsin
h(b*x/sqrt(a*b))/(sqrt(b)*d^3) + 3/2*B*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b)
)/d^3 - 3/8*C*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 3/2*A*a*sqrt(b)
*c*arcsinh(b*x/sqrt(a*b))/d^2 - 1/16*D*a^3*arcsinh(b*x/sqrt(a*b))/(b^(3/2)
*d) + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - D*(a + b*c^2/d^2)^(3/
2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c
)))/d^4 + C*(a + b*c^2/d^2)^(3/2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - B*(a + b*c^2/d^2)^(3/2)*c*arcsinh
(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + A*(a
 + b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b
)*abs(d*x + c)))/d - sqrt(b*x^2 + a)*D*b*c^5/d^6 + sqrt(b*x^2 + a)*C*b*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{c+d\,x} \,d x$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(D x^{3}+C\,x^{2}+B x +A \right)}{d x +c}d x$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

3.76 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 541

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{(3a^2d^4D-20abd^2(2cCd-Bd^2-3c^2D)-15b^2c(4c^2Cd-3.}{15bd^6}$$
$$+\frac{(5ad^2(Cd-2cD)+4b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x\sqrt{a+bx^2}}{8d^5}$$
$$+\frac{(6ad^2D-5b(2cCd-Bd^2-3c^2D))\,x^2\sqrt{a+bx^2}}{15d^4}+\frac{b(Cd-2cD)x^3\sqrt{a+bx^2}}{4d^3}$$
$$+\frac{bDx^4\sqrt{a+bx^2}}{5d^2}-\frac{(bc^2+ad^2)\,(c^2Cd-Bcd^2+Ad^3-c^3D)\,\sqrt{a+bx^2}}{d^6(c+dx)}$$
$$+\frac{(3a^2d^4(Cd-2cD)+8b^2c^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)+12abd^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,\mathrm{a}}{8\sqrt{b}d^7}$$
$$+\frac{\sqrt{bc^2+ad^2}(ad^2(2cCd-Bd^2-3c^2D)+bc(5c^2Cd-4Bcd^2+3Ad^3-6c^3D))\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}$$

output
```
1/15*(3*a^2*d^4*D-20*a*b*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-15*b^2*c*(2*A*d^3-3*
B*c*d^2+4*C*c^2*d-5*D*c^3))*(b*x^2+a)^(1/2)/b/d^6+1/8*(5*a*d^2*(C*d-2*D*c)
+4*b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*x*(b*x^2+a)^(1/2)/d^5+1/15*(6*a*
d^2*D-5*b*(-B*d^2+2*C*c*d-3*D*c^2))*x^2*(b*x^2+a)^(1/2)/d^4+1/4*b*(C*d-2*D
*c)*x^3*(b*x^2+a)^(1/2)/d^3+1/5*b*D*x^4*(b*x^2+a)^(1/2)/d^2-(a*d^2+b*c^2)*
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^6/(d*x+c)+1/8*(3*a^2*d^4*(
C*d-2*D*c)+8*b^2*c^2*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3)+12*a*b*d^2*(A*d
^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2
)/d^7+(a*d^2+b*c^2)^(1/2)*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(3*A*d^3-4*B
*c*d^2+5*C*c^2*d-6*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2
+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{d\sqrt{a+bx^2}\,(24a^2d^4D(c+dx)+abd^2\,(600c^3D-110c^2d(4C-3Dx)+cd^2(280B-x(245C}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(24*a^2*d^4*D*(c + d*x) + a*b*d^2*(600*c^3*D - 110*c^2
*d*(4*C - 3*D*x) + c*d^2*(280*B - x*(245*C + 102*D*x)) + d^3*(-120*A + x*(
160*B + 75*C*x + 48*D*x^2))) + 2*b^2*(360*c^5*D - 60*c^4*d*(5*C - 3*D*x) +
 30*c^3*d^2*(8*B - x*(5*C + 2*D*x)) + 10*c^2*d^3*(-18*A + x*(12*B + x*(5*C
 + 3*D*x))) + d^5*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - c*d^4*x*(90*
A + x*(40*B + x*(25*C + 18*D*x))))))/(b*(c + d*x)) + 240*Sqrt[-(b*c^2) - a
*d^2]*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-5*c^2*C*d + 4*B*c*d^2 -
3*A*d^3 + 6*c^3*D))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(
b*c^2) - a*d^2]] + (15*(-3*a^2*d^4*(C*d - 2*c*D) - 12*a*b*d^2*(3*c^2*C*d -
 2*B*c*d^2 + A*d^3 - 4*c^3*D) + 8*b^2*c^2*(-5*c^2*C*d + 4*B*c*d^2 - 3*A*d^
3 + 6*c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(120*d^7)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2182, 25, 2185, 27, 682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \ 2182$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int \frac{5b\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-6Dc^3+5Cdc^2-4Bd^2c+4Ad^3\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{5bd^2}+\frac{1}{5}D\left(a+bx^2\right)^{5/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-6Dc^3+5Cdc^2-4Bd^2c+4Ad^3\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{d^2}+\frac{1}{5}D\left(a+bx^2\right)^{5/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 682$$

$$\frac{\frac{\int -\frac{\frac{b\left(bc^2+ad^2\right)\left(ad\left(-6Dc^2+5Cdc-4Bd^2\right)-\left(3a(Cd-2cD)d^2+4b\left(-6Dc^3+5Cdc^2-4Bd^2c+3Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}}{4bd^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)\right.\right.}{d^2}}{}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{-\int\frac{\frac{b\left(bc^2+ad^2\right)\left(ad\left(-6Dc^2+5Cdc-4Bd^2\right)-\left(3a(Cd-2cD)d^2+4b\left(-6Dc^3+5Cdc^2-4Bd^2c+3Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}}{4bd^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)\right.\right.}{d^2}}{}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{\left(ad^2+bc^2\right)\int\frac{\left(ad\left(-6Dc^2+5Cdc-4Bd^2\right)-\left(3a(Cd-2cD)d^2+4b\left(-6Dc^3+5Cdc^2-4Bd^2c+3Ad^3\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)\right.\right.}{d^2}}{}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 682$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\int\frac{b\left(ad\left(a\left(-18Dc^2+13Cdc-8Bd^2\right)d^2+4bc\left(-6Dc^3+5Cdc^2-4Bd^2c+3Ad^3\right)\right)-\left(2abc\left(-6Dc^2+5Cdc-4Bd^2\right)d^2+\left(2bc^2+ad^2\right)\left(3a(Cd-2cD)d^2+\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}\right.}{}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\int \frac{ad\left(a\left(-18Dc^2+13Cdc-8Bd^2\right)d^2+4bc\left(-6Dc^3+5Cdc^2-4Bd^2c+3Ad^3\right)\right)-\left(2abc\left(-6Dc^2+5Cdc-4Bd^2\right)d^2+\left(2bc^2+ad^2\right)\left(3a(Cd-2cD)d^2+4b\left(\cdots\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)\left(3ad^2(Cd-2cD)+4b\left(3Ad^3-4\cdots\right.\right.\right.}{2d^2}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)\left(3ad^2(Cd-2cD)+4b\left(3Ad^3-4\cdots\right.\right.\right.}{2d^2}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)\left(3ad^2\cdots\right.\right.}{2d^2}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$-\frac{\left(ad^2+bc^2\right)\left(-\frac{-\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(a \cdots\right.\right.}{\cdots}}{2d^2}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(ad^2+bc^2\right)\left(-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)\left(3ad^2(Cd-2cD)+4b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)+2abcd^2\left(-4Bd^2-6c^2D+5cCd\right)\right)}{\sqrt{b}d}-\frac{8\sqrt{ad^2+bc^2}\text{ar}\cdots}{\cdots}}{2d^2}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
-(((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 + a*
d^2)*(c + d*x))) + (((a/b + c^2/d^2)*D*(a + b*x^2)^(5/2))/5 + (-1/12*((4*(
a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 -
 6*c^3*D)) - 3*d*(a*d^2*(C*d - 2*c*D) + b*(5*c^2*C*d - 4*B*c*d^2 + 4*A*d^3
 - 6*c^3*D))*x)*(a + b*x^2)^(3/2))/d^2 - ((b*c^2 + a*d^2)*(((8*(a*d^2*(2*c
*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))
 - d*(3*a*d^2*(C*d - 2*c*D) + 4*b*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3
*D))*x)*Sqrt[a + b*x^2])/(2*d^2) + (-(((2*a*b*c*d^2*(5*c*C*d - 4*B*d^2 - 6
*c^2*D) + (2*b*c^2 + a*d^2)*(3*a*d^2*(C*d - 2*c*D) + 4*b*(5*c^2*C*d - 4*B*
c*d^2 + 3*A*d^3 - 6*c^3*D)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*d)) - (8*Sqrt[b*c^2 + a*d^2]*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5
*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*d^2))/d^2)/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. $2(505) = 1010$.

Time = 1.45 (sec) , antiderivative size = 1540, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	1540

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(C*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*D*d/b*(b*x^2+a)^(5/2)-2*D*c*(1/4*x*
(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2)))))+1/d^4*(B*d^2-2*C*c*d+3*D*c^2)*(1/3*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2
-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/
2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^
2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*
d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
))/(x+c/d))))+1/d^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-1/(a*d^2+b*c^2)*d^2/(x
+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2
+b*c^2)*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(
1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+
c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d
^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/
2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(508) = 1016$.

Time = 0.16 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxi
ma")
```

output
```
(b*x^2 + a)^(3/2)*D*c^3/(d^5*x + c*d^4) - (b*x^2 + a)^(3/2)*C*c^2/(d^4*x +
 c*d^3) + (b*x^2 + a)^(3/2)*B*c/(d^3*x + c*d^2) - (b*x^2 + a)^(3/2)*A/(d^2
*x + c*d) - 3*sqrt(b*x^2 + a)*D*b*c^3*x/d^5 + 5/2*sqrt(b*x^2 + a)*C*b*c^2*
x/d^4 - 1/2*(b*x^2 + a)^(3/2)*D*c*x/d^3 - 3/4*sqrt(b*x^2 + a)*D*a*c*x/d^3
- 2*sqrt(b*x^2 + a)*B*b*c*x/d^3 + 1/4*(b*x^2 + a)^(3/2)*C*x/d^2 + 3/8*sqrt
(b*x^2 + a)*C*a*x/d^2 + 3/2*sqrt(b*x^2 + a)*A*b*x/d^2 - 6*D*b^(3/2)*c^5*ar
csinh(b*x/sqrt(a*b))/d^7 + 5*C*b^(3/2)*c^4*arcsinh(b*x/sqrt(a*b))/d^6 - 6*
D*a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 - 4*B*b^(3/2)*c^3*arcsinh(b*x/s
qrt(a*b))/d^5 + 9/2*C*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 + 3*A*b^(3/
2)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - 3/4*D*a^2*c*arcsinh(b*x/sqrt(a*b))/(sq
rt(b)*d^3) - 3*B*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + 3/8*C*a^2*arcsin
h(b*x/sqrt(a*b))/(sqrt(b)*d^2) + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^
2 + 3*D*sqrt(a + b*c^2/d^2)*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
 a*d/(sqrt(a*b)*abs(d*x + c)))/d^6 - 3*C*sqrt(a + b*c^2/d^2)*b*c^3*arcsinh
(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 3*D*
(a + b*c^2/d^2)^(3/2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sq
rt(a*b)*abs(d*x + c)))/d^4 + 3*B*sqrt(a + b*c^2/d^2)*b*c^2*arcsinh(b*c*x/(
sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 - 2*C*(a + b*c
^2/d^2)^(3/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*ab
s(d*x + c)))/d^3 - 3*A*sqrt(a + b*c^2/d^2)*b*c*arcsinh(b*c*x/(sqrt(a*b)...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2010, normalized size of antiderivative = 3.72

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
( - 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b**2*c**2*d**2 - 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x + 240*sqrt(a*d*
*2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b
**2*c*d**3 + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*d**4*x + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 + 240*sqrt(
a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a*b*c**2*d**3*x + 960*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d + 960*sqrt(a*d**2 + b*c**2)*log(s
qrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**2*d**2*x + 24
0*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*b**2*c**5 + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x + 720*sqrt(a*d**2 + b*c**2)*lo
g(c + d*x)*a*b**2*c**2*d**2 + 720*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**
2*c*d**3*x - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3 - 240*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*d**4*x - 240*sqrt(a*d**2 + b*c**2)
*log(c + d*x)*a*b*c**3*d**2 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c
**2*d**3*x - 960*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d - 960*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**2*d**2*x - 240*sqrt(a*d**2 + b*c...
```

3.77 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 548

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx &= \frac{\left(4ad^2(Cd-3cD)+3b(6c^2Cd-3Bcd^2+Ad^3-10c^3D)\right)\sqrt{a}}{3d^6} \\
&+ \frac{\left(5ad^2D-4b(3cCd-Bd^2-6c^2D)\right)x\sqrt{a+bx^2}}{8d^5} + \frac{b(Cd-3cD)x^2\sqrt{a+bx^2}}{3d^4} \\
&+ \frac{bDx^3\sqrt{a+bx^2}}{4d^3} - \frac{\left(bc^2+ad^2\right)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\sqrt{a+bx^2}}{2d^6(c+dx)^2} \\
&+ \frac{\left(2ad^2(2cCd-Bd^2-3c^2D)+bc(9c^2Cd-7Bcd^2+5Ad^3-11c^3D)\right)\sqrt{a+bx^2}}{2d^6(c+dx)} \\
&+ \frac{\left(3a^2d^4D-12abd^2(3cCd-Bd^2-6c^2D)-8b^2c(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^7} \\
&- \frac{\left(2a^2d^4(Cd-3cD)+abd^2(19c^2Cd-9Bcd^2+3Ad^3-33c^3D)+2b^2c^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D\right.}{2d^7\sqrt{bc^2+ad^2}}
\end{aligned}
$$

output
```
1/3*(4*a*d^2*(C*d-3*D*c)+3*b*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(b*x^2+
a)^(1/2)/d^6+1/8*(5*a*d^2*D-4*b*(-B*d^2+3*C*c*d-6*D*c^2))*x*(b*x^2+a)^(1/2
)/d^5+1/3*b*(C*d-3*D*c)*x^2*(b*x^2+a)^(1/2)/d^4+1/4*b*D*x^3*(b*x^2+a)^(1/2
)/d^3-1/2*(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^6/
(d*x+c)^2+1/2*(2*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(5*A*d^3-7*B*c*d^2+9*C
*c^2*d-11*D*c^3))*(b*x^2+a)^(1/2)/d^6/(d*x+c)+1/8*(3*a^2*d^4*D-12*a*b*d^2*
(-B*d^2+3*C*c*d-6*D*c^2)-8*b^2*c*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^7-1/2*(2*a^2*d^4*(C*d-3*D*c)+
a*b*d^2*(3*A*d^3-9*B*c*d^2+19*C*c^2*d-33*D*c^3)+2*b^2*c^2*(3*A*d^3-6*B*c*d
^2+10*C*c^2*d-15*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a
)^(1/2))/d^7/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \frac{d\sqrt{a+bx^2}\,(ad^2\,(-156c^3D+c^2d(68C-249Dx)-2cd^2(6B+x(-56C+33Dx))+d^3(-1}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(-156*c^3*D + c^2*d*(68*C - 249*D*x) - 2*c*d^2*
(6*B + x*(-56*C + 33*D*x)) + d^3*(-12*A + x*(-24*B + 32*C*x + 15*D*x^2)))
- 2*b*(180*c^5*D - 30*c^4*d*(4*C - 9*D*x) + 12*c^3*d^2*(6*B + 5*x*(-3*C +
D*x)) - d^5*x^2*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 2*c*d^4*x*(-27*A + x*
(12*B + x*(5*C + 3*D*x))) - c^2*d^3*(36*A + x*(-108*B + 5*x*(8*C + 3*D*x))
))))/(c + d*x)^2 + (24*(-2*a^2*d^4*(C*d - 3*c*D) + 2*b^2*c^2*(-10*c^2*C*d
+ 6*B*c*d^2 - 3*A*d^3 + 15*c^3*D) + a*b*d^2*(-19*c^2*C*d + 9*B*c*d^2 - 3*A
*d^3 + 33*c^3*D))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*
c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] - (3*(3*a^2*d^4*D + 12*a*b*d^2*(-3*
c*C*d + B*d^2 + 6*c^2*D) + 8*b^2*c*(-10*c^2*C*d + 6*B*c*d^2 - 3*A*d^3 + 15
*c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(24*d^7)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.49, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2182, 25, 2182, 25, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}dx$$

$$\Big\downarrow \ 2182$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(2Cd-2cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+3Bc-3Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2(ad^2+bc^2)}-\frac{(a+bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{2d^2(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+3Bc-3Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2(ad^2+bc^2)}-\frac{(a+bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{2d^2(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 2182$$

$$\frac{\frac{(a+bx^2)^{5/2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-3Bcd^2-7c^3D+5c^2Cd\right)\right)}{d^2(c+dx)(ad^2+bc^2)}-\frac{\int -\frac{\left(\left(2d(Cd-2cD)a^2+\frac{bc\left(-5Dc^2+3Cdc-Bd^2\right)a}{d}+Ab\left(2bc^2+3ad^2\right)\right)\right.}{}}{}}{2(ad^2+bc^2)}-\frac{(a+bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{2d^2(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 25$$

$$\int \frac{\left(d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2+3ad^2\right)d+abc\left(-5Dc^2+3Cdc-Bd^2\right)\right)+2\left(a^2Dd^4-2ab\left(-7Dc^2+4Cdc-2Bd^2\right)d^2-b^2c\left(-15Dc^3+10Cdc^2-6Bd^2c+2Ad^3\right)\right)x\right)\left(b\right.}{d^2(c+dx)}$$

$$ad^2+bc^2$$

$$2\left(ad^2+bc^2\right)$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow\ 27$$

$$\int \frac{\left(d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2+3ad^2\right)d+abc\left(-5Dc^2+3Cdc-Bd^2\right)\right)+2\left(a^2Dd^4-2ab\left(-7Dc^2+4Cdc-2Bd^2\right)d^2-b^2c\left(-15Dc^3+10Cdc^2-6Bd^2c+2Ad^3\right)\right)x\right)\left(b\right.}{c+dx}$$

$$d^2\left(ad^2+bc^2\right)$$

$$2\left(ad^2+bc^2\right)$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow\ 682$$

$$\int \frac{2b\left(bc^2+ad^2\right)\left(ad\left(a(4Cd-9cD)d^2+b\left(-15Dc^3+10Cdc^2-6Bd^2c+6Ad^3\right)\right)+\left(3a^2Dd^4-ab\left(-51Dc^2+28Cdc-12Bd^2\right)d^2-4b^2c\left(-15Dc^3+10Cdc^2-6Bd^2c+3Ad^3\right.\right.\right.}{c+dx}$$

$$4bd^2$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow\ 27$$

$$\left(ad^2+bc^2\right)\int \frac{\left(ad\left(a(4Cd-9cD)d^2+b\left(-15Dc^3+10Cdc^2-6Bd^2c+6Ad^3\right)\right)+\left(3a^2Dd^4-ab\left(-51Dc^2+28Cdc-12Bd^2\right)d^2-4b^2c\left(-15Dc^3+10Cdc^2-6Bd^2c+3Ad^3\right)\right.\right.}{c+dx}$$

$$2d^2$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow\ 682$$

$$\left(ad^2+bc^2\right)\left(\int \frac{b\left(bc^2+ad^2\right)\left(ad\left(a(8Cd-21cD)d^2+4b\left(-15Dc^3+10Cdc^2-6Bd^2c+3Ad^3\right)\right)+\left(3a^2Dd^4-12ab\left(-6Dc^2+3Cdc-Bd^2\right)d^2-8b^2c\left(-15Dc^3+10Cdc^2-\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}\right.$$

$$2bd^2$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow\ 27$$

$$\left(ad^2+bc^2\right)\left(\frac{\left(ad^2+bc^2\right)\int \frac{ad\left(a(8Cd-21cD)d^2+4b\left(-15Dc^3+10Cdc^2-6Bd^2c+3Ad^3\right)\right)+\left(3a^2Dd^4-12ab\left(-6Dc^2+3Cdc-Bd^2\right)d^2-8b^2c\left(-15Dc^3+10Cdc^2-6B\cdots\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\left(ad^2+bc^2\right)\left(\frac{\left(ad^2+bc^2\right)\left(\frac{\left(3a^2d^4D-12abd^2\left(-Bd^2-6c^2D+3cCd\right)-8b^2c\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{4\left(2a^2d^4(Cd-3cD)+abd^2\left(3Ad^3-\cdots\right.\right.}{\cdots}\right.}{2d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-7Dc^3+5Cdc^2-3Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(2\left(2a^2(Cd-3cD)d^4+ab\left(-33Dc^3+19Cdc^2-9Bd^2c+3Ad^3\right)d^2+2b^2c^2\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \; 219$$

$$\left(ad^2+bc^2\right)\left(\frac{\left(ad^2+bc^2\right)\left(\frac{4\left(2a^2d^4(Cd-3cD)+abd^2\left(3Ad^3-9Bcd^2-33c^3D+19c^2Cd\right)+2b^2c^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arcta}\cdots}{\cdots}\right.}{2d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-7Dc^3+5Cdc^2-3Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{d^2(bc^2+ad^2)(c+dx)}+\frac{\left(2\left(2a^2(Cd-3cD)d^4+ab\left(-33Dc^3+19Cdc^2-9Bd^2c+3Ad^3\right)d^2+2b^2c^2\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-7Dc^3+5Cdc^2-3Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{d^2(bc^2+ad^2)(c+dx)}+\frac{\left(2\left(2a^2(Cd-3cD)d^4+ab\left(-33Dc^3+19Cdc^2-9Bd^2c+3Ad^3\right)d^2+2b^2c^2\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
-1/2*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^2) + (((2*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d - 3*B*c*d^2 + A*d^3 - 7*c^3*D))*(a + b*x^2)^(5/2))/(d^2*(b*c^2 + a*d^
2)*(c + d*x)) + (((2*(2*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(19*c^2*C*d - 9*B*
c*d^2 + 3*A*d^3 - 33*c^3*D) + 2*b^2*c^2*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3
- 15*c^3*D)) + 3*d*(a^2*d^4*D - 2*a*b*d^2*(4*c*C*d - 2*B*d^2 - 7*c^2*D) -
b^2*c*(10*c^2*C*d - 6*B*c*d^2 + 2*A*d^3 - 15*c^3*D))*x)*(a + b*x^2)^(3/2))
/(6*d^2) + ((b*c^2 + a*d^2)*(((4*(2*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(19*c^
2*C*d - 9*B*c*d^2 + 3*A*d^3 - 33*c^3*D) + 2*b^2*c^2*(10*c^2*C*d - 6*B*c*d^
2 + 3*A*d^3 - 15*c^3*D)) + d*(3*a^2*d^4*D - a*b*d^2*(28*c*C*d - 12*B*d^2 -
 51*c^2*D) - 4*b^2*c*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*x)*Sqr
t[a + b*x^2])/(2*d^2) + ((b*c^2 + a*d^2)*(((3*a^2*d^4*D - 12*a*b*d^2*(3*c*
C*d - B*d^2 - 6*c^2*D) - 8*b^2*c*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^
3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (4*(2*a^2*d^4*(C
*d - 3*c*D) + a*b*d^2*(19*c^2*C*d - 9*B*c*d^2 + 3*A*d^3 - 33*c^3*D) + 2*b^
2*c^2*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*ArcTanh[(a*d - b*c*x)
/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2)
))/(2*d^2))/(d^2*(b*c^2 + a*d^2)))/(2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2943 vs. $2(508) = 1016$.

Time = 1.49 (sec) , antiderivative size = 2944, normalized size of antiderivative = 5.37

method	result	size
default	Expression too large to display	2944

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
D/d^3*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/d^4*(C*d-3*D*c)*(1/3*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2
-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/
2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^
2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*
d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
))/(x+c/d))))+1/d^5*(B*d^2-2*C*c*d+3*D*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2+b*c^2
)*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2
*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/
b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c
^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d
*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2
)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(510) = 1020$.

Time = 0.18 (sec) , antiderivative size = 1889, normalized size of antiderivative = 3.45

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxi
ma")
```

output
```
-3/2*sqrt(b*x^2 + a)*D*b^2*c^5/(b*c^2*d^6 + a*d^8) + 3/2*sqrt(b*x^2 + a)*D
*b^2*c^4*x/(b*c^2*d^5 + a*d^7) - 1/2*(b*x^2 + a)^(3/2)*D*b*c^4/(b*c^2*d^5*
x + a*d^7*x + b*c^3*d^4 + a*c*d^6) + 3/2*sqrt(b*x^2 + a)*C*b^2*c^4/(b*c^2*
d^5 + a*d^7) - 3/2*sqrt(b*x^2 + a)*C*b^2*c^3*x/(b*c^2*d^4 + a*d^6) + 1/2*(
b*x^2 + a)^(5/2)*D*c^3/(b*c^2*d^4*x^2 + a*d^6*x^2 + 2*b*c^3*d^3*x + 2*a*c*
d^5*x + b*c^4*d^2 + a*c^2*d^4) + 1/2*(b*x^2 + a)^(3/2)*C*b*c^3/(b*c^2*d^4*
x + a*d^6*x + b*c^3*d^3 + a*c*d^5) - 1/2*(b*x^2 + a)^(3/2)*D*b*c^3/(b*c^2*
d^4 + a*d^6) - 3/2*sqrt(b*x^2 + a)*B*b^2*c^3/(b*c^2*d^4 + a*d^6) + 3/2*sqr
t(b*x^2 + a)*B*b^2*c^2*x/(b*c^2*d^3 + a*d^5) - 1/2*(b*x^2 + a)^(5/2)*C*c^2
/(b*c^2*d^3*x^2 + a*d^5*x^2 + 2*b*c^3*d^2*x + 2*a*c*d^4*x + b*c^4*d + a*c^
2*d^3) - 1/2*(b*x^2 + a)^(3/2)*B*b*c^2/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2
+ a*c*d^4) + 1/2*(b*x^2 + a)^(3/2)*C*b*c^2/(b*c^2*d^3 + a*d^5) + 3/2*sqrt(
b*x^2 + a)*A*b^2*c^2/(b*c^2*d^3 + a*d^5) - 3/2*sqrt(b*x^2 + a)*A*b^2*c*x/(
b*c^2*d^2 + a*d^4) + 1/2*(b*x^2 + a)^(5/2)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2
+ 2*b*c^3*d*x + 2*a*c*d^3*x + b*c^4 + a*c^2*d^2) + 1/2*(b*x^2 + a)^(3/2)*A
*b*c/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) - 1/2*(b*x^2 + a)^(3/2)*B
*b*c/(b*c^2*d^2 + a*d^4) - 3*(b*x^2 + a)^(3/2)*D*c^2/(d^5*x + c*d^4) - 1/2
*(b*x^2 + a)^(5/2)*A/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d^2*x +
b*c^4/d + a*c^2*d) + 1/2*(b*x^2 + a)^(3/2)*A*b/(b*c^2*d + a*d^3) + 2*(b*x^
2 + a)^(3/2)*C*c/(d^4*x + c*d^3) - (b*x^2 + a)^(3/2)*B/(d^3*x + c*d^2) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(510) = 1020$.

Time = 0.68 (sec) , antiderivative size = 1414, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac
")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*x*(3*D*b*x/d^3 - 4*(3*D*b^3*c*d^21 - C*b^3*d^22)/
(b^2*d^25)) + 3*(24*D*b^3*c^2*d^20 - 12*C*b^3*c*d^21 + 5*D*a*b^2*d^22 + 4*
B*b^3*d^22)/(b^2*d^25))*x - 8*(30*D*b^3*c^3*d^19 - 18*C*b^3*c^2*d^20 + 12*
D*a*b^2*c*d^21 + 9*B*b^3*c*d^21 - 4*C*a*b^2*d^22 - 3*A*b^3*d^22)/(b^2*d^25
)) - (30*D*b^2*c^5 - 20*C*b^2*c^4*d + 33*D*a*b*c^3*d^2 + 12*B*b^2*c^3*d^2
- 19*C*a*b*c^2*d^3 - 6*A*b^2*c^2*d^3 + 6*D*a^2*c*d^4 + 9*B*a*b*c*d^4 - 2*C
*a^2*d^5 - 3*A*a*b*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)
*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^7) - 1/8*(120*D*b^2*c^4
- 80*C*b^2*c^3*d + 72*D*a*b*c^2*d^2 + 48*B*b^2*c^2*d^2 - 36*C*a*b*c*d^3 -
24*A*b^2*c*d^3 + 3*D*a^2*d^4 + 12*B*a*b*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/(sqrt(b)*d^7) - (12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*b^2*c^5*d
- 10*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*b^2*c^4*d^2 + 7*(sqrt(b)*x - sqrt(b
*x^2 + a))^3*D*a*b*c^3*d^3 + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*b^2*c^3*d
^3 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a*b*c^2*d^4 - 6*(sqrt(b)*x - sqrt
(b*x^2 + a))^3*A*b^2*c^2*d^4 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c*d
^5 - (sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b*d^6 + 22*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*D*b^(5/2)*c^6 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*b^(5/2)*c^5
*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a*b^(3/2)*c^4*d^2 + 14*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*B*b^(5/2)*c^4*d^2 + (sqrt(b)*x - sqrt(b*x^2 + a))^2*C*
a*b^(3/2)*c^3*d^3 - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4155, normalized size of antiderivative = 7.58

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 72*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a**2*b**2*c**2*d**4 - 144*sqrt(a*d**2 + b*c**2)*log( - sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x - 72
*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**2*d**6*x**2 + 96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**4 + 192*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**2*b*c**2*d**5*x + 96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**6*x**2 - 144*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**
3*c**4*d**2 - 288*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d**3*x + 216*sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d
**3 - 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*a*b**3*c**2*d**4*x**2 + 432*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x
 + 216*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a*b**3*c*d**5*x**2 + 336*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**5*d**2 + 672*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*...
```

3.78 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 603

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \frac{(4ad^2D-3b(4cCd-Bd^2-10c^2D))\sqrt{a+bx^2}}{3d^6}$$
$$+\frac{b(Cd-4cD)x\sqrt{a+bx^2}}{2d^5}+\frac{bDx^2\sqrt{a+bx^2}}{3d^4}$$
$$-\frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{3d^6(c+dx)^3}$$
$$+\frac{(3ad^2(2cCd-Bd^2-3c^2D)+bc(13c^2Cd-10Bcd^2+7Ad^3-16c^3D))\sqrt{a+bx^2}}{6d^6(c+dx)^2}$$
$$-\frac{(6a^2d^4(Cd-3cD)+abd^2(50c^2Cd-23Bcd^2+8Ad^3-89c^3D)+b^2c^2(47c^2Cd-26Bcd^2+11Ad^3-74c}{6d^6(bc^2+ad^2)(c+dx)}$$
$$+\frac{\sqrt{b}(3ad^2(Cd-4cD)+b(20c^2Cd-8Bcd^2+2Ad^3-40c^3D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^7}$$
$$-\frac{(2a^3d^6D-3a^2bd^4(4cCd-Bd^2-11c^2D)-3ab^2cd^2(11c^2Cd-4Bcd^2+Ad^3-24c^3D)-2b^3c^3(10c^2Cd}{2d^7(bc^2+ad^2)^{3/2}}$$

output
```
1/3*(4*a*d^2*D-3*b*(-B*d^2+4*C*c*d-10*D*c^2))*(b*x^2+a)^(1/2)/d^6+1/2*b*(C
*d-4*D*c)*x*(b*x^2+a)^(1/2)/d^5+1/3*b*D*x^2*(b*x^2+a)^(1/2)/d^4-1/3*(a*d^2
+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^6/(d*x+c)^3+1/6*(3
*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(7*A*d^3-10*B*c*d^2+13*C*c^2*d-16*D*c^
3))*(b*x^2+a)^(1/2)/d^6/(d*x+c)^2-1/6*(6*a^2*d^4*(C*d-3*D*c)+a*b*d^2*(8*A*
d^3-23*B*c*d^2+50*C*c^2*d-89*D*c^3)+b^2*c^2*(11*A*d^3-26*B*c*d^2+47*C*c^2*
d-74*D*c^3))*(b*x^2+a)^(1/2)/d^6/(a*d^2+b*c^2)/(d*x+c)+1/2*b^(1/2)*(3*a*d^
2*(C*d-4*D*c)+b*(2*A*d^3-8*B*c*d^2+20*C*c^2*d-40*D*c^3))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/d^7-1/2*(2*a^3*d^6*D-3*a^2*b*d^4*(-B*d^2+4*C*c*d-11*D*c^
2)-3*a*b^2*c*d^2*(A*d^3-4*B*c*d^2+11*C*c^2*d-24*D*c^3)-2*b^3*c^3*(A*d^3-4*
B*c*d^2+10*C*c^2*d-20*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*
x^2+a)^(1/2))/d^7/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 12.14 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \frac{d\sqrt{a+bx^2}\Big(8ad^2D+6b(-4cCd+Bd^2+10c^2D)+3bd(Cd -}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(d*Sqrt[a + b*x^2]*(8*a*d^2*D + 6*b*(-4*c*C*d + B*d^2 + 10*c^2*D) + 3*b*d*
(C*d - 4*c*D)*x + 2*b*d^2*D*x^2 + (2*(b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2
 - A*d^3 + c^3*D))/(c + d*x)^3 + (-3*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) +
b*c*(13*c^2*C*d - 10*B*c*d^2 + 7*A*d^3 - 16*c^3*D))/(c + d*x)^2 + (-6*a^2*
d^4*(C*d - 3*c*D) + b^2*c^2*(-47*c^2*C*d + 26*B*c*d^2 - 11*A*d^3 + 74*c^3*
D) + a*b*d^2*(-50*c^2*C*d + 23*B*c*d^2 - 8*A*d^3 + 89*c^3*D))/((b*c^2 + a*
d^2)*(c + d*x))) + (3*(2*a^3*d^6*D + 3*a^2*b*d^4*(-4*c*C*d + B*d^2 + 11*c^
2*D) + 2*b^3*c^3*(-10*c^2*C*d + 4*B*c*d^2 - A*d^3 + 20*c^3*D) + 3*a*b^2*c*
d^2*(-11*c^2*C*d + 4*B*c*d^2 - A*d^3 + 24*c^3*D))*Log[c + d*x])/(b*c^2 + a
*d^2)^(3/2) - 3*Sqrt[b]*(-3*a*d^2*(C*d - 4*c*D) + b*(-20*c^2*C*d + 8*B*c*d
^2 - 2*A*d^3 + 40*c^3*D))*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]] - (3*(2*a^3*d
^6*D + 3*a^2*b*d^4*(-4*c*C*d + B*d^2 + 11*c^2*D) + 2*b^3*c^3*(-10*c^2*C*d
+ 4*B*c*d^2 - A*d^3 + 20*c^3*D) + 3*a*b^2*c*d^2*(-11*c^2*C*d + 4*B*c*d^2 -
 A*d^3 + 24*c^3*D))*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]]
)/(b*c^2 + a*d^2)^(3/2))/(6*d^7)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.42, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}dx$$

$$\Big\downarrow \ 2182$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(3\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(3Cd-3cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+2Bc-2Ad\right)\right)x+3\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^3}dx}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(3\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(3a(Cd-cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+2Bc-2Ad\right)\right)x+3\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^3}dx}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{5/2}\left(3ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-Ad^3-2Bcd^2-8c^3D+5c^2Cd\right)\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(2d\left(Abd\left(3bc^2+2ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-5Dc^2+2Cdc+B\right.\right.\right.\right.}{}}{}}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(2d\left(Abd\left(3bc^2+2ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-5Dc^2+2Cdc+Bd^2\right)\right)\right)+3\left(2a^2Dd^4-ab\left(-13Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-10Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)x\right)\left(\cdots\right.}{d^2(c+dx)^2}}{2(ad^2+bc^2)}\cdots}{3\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(2d\left(Abd\left(3bc^2+2ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-5Dc^2+2Cdc+Bd^2\right)\right)\right)+3\left(2a^2Dd^4-ab\left(-13Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-10Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)x\right)\left(\cdots\right.}{(c+dx)^2}}{2d^2(ad^2+bc^2)}\cdots}{3\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{-\frac{\int -\frac{6\left(ad\left(2a^2Dd^4-ab\left(-13Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-10Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)+2b\left(a^2(3Cd-10cD)d^4+ab\left(-31Dc^3+14Cdc^2-5Bd^2c+2Ad^3\right)d^2+b^2\cdots\right.\right.}{c+dx}}{2d^2}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{\left(ad\left(2a^2Dd^4-ab\left(-13Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-10Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)+2b\left(a^2(3Cd-10cD)d^4+ab\left(-31Dc^3+14Cdc^2-5Bd^2c+2Ad^3\right)d^2+b^2c^2\left(\cdots\right.\right.\right.}{c+dx}}{d^2}\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{3\left(\frac{\int \frac{2b\left(bc^2+ad^2\right)\left(ad\left(2a^2Dd^4-3ab\left(-7Dc^2+3Cdc-Bd^2\right)d^2-b^2c\left(-20Dc^3+10Cdc^2-4Bd^2c+Ad^3\right)\right)+b\left(bc^2+ad^2\right)\left(3a(Cd-4cD)d^2+b\left(-40Dc^3+20Cdc^2-8B\cdots\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{\left(ad^2+bc^2\right)\int \frac{ad\left(2a^2Dd^4-3ab\left(-7Dc^2+3Cdc-Bd^2\right)d^2-b^2c\left(-20Dc^3+10Cdc^2-4Bd^2c+Ad^3\right)\right)+b\left(bc^2+ad^2\right)\left(3a(Cd-4cD)d^2+b\left(-40Dc^3+20Cdc^2-8Bd^2\cdots\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{3\left(\frac{\sqrt{bx^2+a}\left(2a^3Dd^6-3a^2b\left(-11Dc^2+4Cdc-Bd^2\right)d^4-3ab^2c\left(-24D\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 224$$

$$\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{3\left(\frac{\sqrt{bx^2+a}\left(2a^3Dd^6-3a^2b\left(-11Dc^2+4Cdc-Bd^2\right)d^4-3ab^2c\left(-24D\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{3\left(\frac{\sqrt{bx^2+a}\left(2a^3Dd^6-3a^2b\left(-11Dc^2+4Cdc-Bd^2\right)d^4-3ab^2c\left(-24Dc\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$\Big\downarrow$ 488

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{3\left(\frac{\sqrt{bx^2+a}\left(2a^3Dd^6-3a^2b\left(-11Dc^2+4Cdc-Bd^2\right)d^4-3ab^2c\left(-24Dc\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+5Cdc^2-2Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{3\left(\frac{\sqrt{bx^2+a}\left(2a^3Dd^6-3a^2b\left(-11Dc^2+4Cdc-Bd^2\right)d^4-3ab^2c\left(-24Dc\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
-1/3*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^3) + (((3*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d - 2*B*c*d^2 - A*d^3 - 8*c^3*D))*(a + b*x^2)^(5/2))/(2*d^2*(b*c^2 + a*
d^2)*(c + d*x)^2) + (-(((2*(a^2*d^4*(3*C*d - 10*c*D) + a*b*d^2*(14*c^2*C*d
 - 5*B*c*d^2 + 2*A*d^3 - 31*c^3*D) + b^2*c^2*(10*c^2*C*d - 4*B*c*d^2 + A*d
^3 - 20*c^3*D)) - d*(2*a^2*d^4*D - a*b*d^2*(6*c*C*d - 3*B*d^2 - 13*c^2*D)
- b^2*c*(5*c^2*C*d - 2*B*c*d^2 - A*d^3 - 10*c^3*D))*x)*(a + b*x^2)^(3/2))/
(d^2*(c + d*x))) + (3*(((2*a^3*d^6*D - 3*a^2*b*d^4*(4*c*C*d - B*d^2 - 11*c
^2*D) - 3*a*b^2*c*d^2*(11*c^2*C*d - 4*B*c*d^2 + A*d^3 - 24*c^3*D) - 2*b^3*
c^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D) + b*d*(a^2*d^4*(3*C*d - 10
*c*D) + a*b*d^2*(14*c^2*C*d - 5*B*c*d^2 + 2*A*d^3 - 31*c^3*D) + b^2*c^2*(1
0*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*x)*Sqrt[a + b*x^2])/d^2 + ((b*c
^2 + a*d^2)*((Sqrt[b]*(b*c^2 + a*d^2)*(3*a*d^2*(C*d - 4*c*D) + b*(20*c^2*C
*d - 8*B*c*d^2 + 2*A*d^3 - 40*c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/d - ((2*a^3*d^6*D - 3*a^2*b*d^4*(4*c*C*d - B*d^2 - 11*c^2*D) - 3*a*b^2*c
*d^2*(11*c^2*C*d - 4*B*c*d^2 + A*d^3 - 24*c^3*D) - 2*b^3*c^3*(10*c^2*C*d -
 4*B*c*d^2 + A*d^3 - 20*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]
*Sqrt[a + b*x^2]])/(d*Sqrt[b*c^2 + a*d^2])))/d^2))/d^2)/(2*d^2*(b*c^2 + a
*d^2)))/(3*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5333 vs. $2(563) = 1126$.

Time = 1.51 (sec) , antiderivative size = 5334, normalized size of antiderivative = 8.85

method	result	size
default	Expression too large to display	5334

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3437 vs. $2(567) = 1134$.

Time = 0.23 (sec) , antiderivative size = 3437, normalized size of antiderivative = 5.70

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxi
ma")
```

output
```
-1/2*sqrt(b*x^2 + a)*D*b^3*c^6/(b^2*c^4*d^6 + 2*a*b*c^2*d^8 + a^2*d^10) +
1/2*sqrt(b*x^2 + a)*D*b^3*c^5*x/(b^2*c^4*d^5 + 2*a*b*c^2*d^7 + a^2*d^9) -
1/6*(b*x^2 + a)^(3/2)*D*b^2*c^5/(b^2*c^4*d^5*x + 2*a*b*c^2*d^7*x + a^2*d^9
*x + b^2*c^5*d^4 + 2*a*b*c^3*d^6 + a^2*c*d^8) + 1/2*sqrt(b*x^2 + a)*C*b^3*
c^5/(b^2*c^4*d^5 + 2*a*b*c^2*d^7 + a^2*d^9) - 1/2*sqrt(b*x^2 + a)*C*b^3*c^
4*x/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) + 1/6*(b*x^2 + a)^(5/2)*D*b*c^
4/(b^2*c^4*d^4*x^2 + 2*a*b*c^2*d^6*x^2 + a^2*d^8*x^2 + 2*b^2*c^5*d^3*x + 4
*a*b*c^3*d^5*x + 2*a^2*c*d^7*x + b^2*c^6*d^2 + 2*a*b*c^4*d^4 + a^2*c^2*d^6
) + 1/6*(b*x^2 + a)^(3/2)*C*b^2*c^4/(b^2*c^4*d^4*x + 2*a*b*c^2*d^6*x + a^2
*d^8*x + b^2*c^5*d^3 + 2*a*b*c^3*d^5 + a^2*c*d^7) - 1/6*(b*x^2 + a)^(3/2)*
D*b^2*c^4/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) - 1/2*sqrt(b*x^2 + a)*B*
b^3*c^4/(b^2*c^4*d^4 + 2*a*b*c^2*d^6 + a^2*d^8) + 1/2*sqrt(b*x^2 + a)*B*b^
3*c^3*x/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) - 1/6*(b*x^2 + a)^(5/2)*C*
b*c^3/(b^2*c^4*d^3*x^2 + 2*a*b*c^2*d^5*x^2 + a^2*d^7*x^2 + 2*b^2*c^5*d^2*x
 + 4*a*b*c^3*d^4*x + 2*a^2*c*d^6*x + b^2*c^6*d + 2*a*b*c^4*d^3 + a^2*c^2*d
^5) - 1/6*(b*x^2 + a)^(3/2)*B*b^2*c^3/(b^2*c^4*d^3*x + 2*a*b*c^2*d^5*x + a
^2*d^7*x + b^2*c^5*d^2 + 2*a*b*c^3*d^4 + a^2*c*d^6) + 1/6*(b*x^2 + a)^(3/2
)*C*b^2*c^3/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) + 1/2*sqrt(b*x^2 + a)*
A*b^3*c^3/(b^2*c^4*d^3 + 2*a*b*c^2*d^5 + a^2*d^7) + 6*sqrt(b*x^2 + a)*D*b^
2*c^4/(b*c^2*d^6 + a*d^8) - 1/2*sqrt(b*x^2 + a)*A*b^3*c^2*x/(b^2*c^4*d^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. $2(567) = 1134$.

Time = 0.49 (sec) , antiderivative size = 2628, normalized size of antiderivative = 4.36

$$\int \frac{(a + bx^2)^{3/2} (A + Bx + Cx^2 + Dx^3)}{(c + dx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac
")
```

output
```
1/6*sqrt(b*x^2 + a)*(x*(2*D*b*x/d^4 - 3*(4*D*b^2*c*d^17 - C*b^2*d^18)/(b*d
^22)) + 2*(30*D*b^2*c^2*d^16 - 12*C*b^2*c*d^17 + 4*D*a*b*d^18 + 3*B*b^2*d^
18)/(b*d^22)) + (40*D*b^3*c^6 - 20*C*b^3*c^5*d + 72*D*a*b^2*c^4*d^2 + 8*B*
b^3*c^4*d^2 - 33*C*a*b^2*c^3*d^3 - 2*A*b^3*c^3*d^3 + 33*D*a^2*b*c^2*d^4 +
12*B*a*b^2*c^2*d^4 - 12*C*a^2*b*c*d^5 - 3*A*a*b^2*c*d^5 + 2*D*a^3*d^6 + 3*
B*a^2*b*d^6)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b
*c^2 - a*d^2))/((b*c^2*d^7 + a*d^9)*sqrt(-b*c^2 - a*d^2)) + 1/3*(90*(sqrt(
b)*x - sqrt(b*x^2 + a))^5*D*b^3*c^6*d^2 - 60*(sqrt(b)*x - sqrt(b*x^2 + a))
^5*C*b^3*c^5*d^3 + 114*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*a*b^2*c^4*d^4 + 3
6*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b^3*c^4*d^4 - 69*(sqrt(b)*x - sqrt(b*x
^2 + a))^5*C*a*b^2*c^3*d^5 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b^3*c^3*
d^5 + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*a^2*b*c^2*d^6 + 36*(sqrt(b)*x -
 sqrt(b*x^2 + a))^5*B*a*b^2*c^2*d^6 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C
*a^2*b*c*d^7 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c*d^7 + 3*(sqrt(
b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*d^8 + 324*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*D*b^(7/2)*c^7*d - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*b^(7/2)*c^6*d^2
+ 324*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a*b^(5/2)*c^5*d^3 + 120*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*b^(7/2)*c^5*d^3 - 183*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*C*a*b^(5/2)*c^4*d^4 - 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(7/2)*c^4
*d^4 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^2*b^(3/2)*c^3*d^5 + 84*(sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^4}dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

3.79 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 685

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \frac{b(Cd-5cD)\sqrt{a+bx^2}}{d^6}$$
$$+\frac{bDx\sqrt{a+bx^2}}{2d^5}-\frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{4d^6(c+dx)^4}$$
$$+\frac{(4ad^2(2cCd-Bd^2-3c^2D)+bc(17c^2Cd-13Bcd^2+9Ad^3-21c^3D))\sqrt{a+bx^2}}{12d^6(c+dx)^3}$$
$$-\frac{(12a^2d^4(Cd-3cD)+abd^2(95c^2Cd-43Bcd^2+15Ad^3-171c^3D)+2b^2c^2(43c^2Cd-23Bcd^2+9Ad^3-}{24d^6(bc^2+ad^2)(c+dx)^2}$$
$$-\frac{(24a^3d^6D-4a^2bd^4(31cCd-8Bd^2-87c^2D)-ab^2cd^2(287c^2Cd-91Bcd^2+15Ad^3-675c^3D)-2b^3c^3(}{24d^6(bc^2+ad^2)^2(c+dx)}$$
$$+\frac{\sqrt{b}(3ad^2D-2b(5cCd-Bd^2-15c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^7}$$
$$-\frac{b(12a^3d^6(Cd-5cD)+4ab^2c^3d^2(25cCd-5Bd^2-78c^2D)+8b^3c^5(5cCd-Bd^2-15c^2D)+3a^2bd^4(25c^2}{8d^7(bc^2+ad^2)^{5/2}}$$

output
```
b*(C*d-5*D*c)*(b*x^2+a)^(1/2)/d^6+1/2*b*D*x*(b*x^2+a)^(1/2)/d^5-1/4*(a*d^2
+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^6/(d*x+c)^4+1/12*(
4*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(9*A*d^3-13*B*c*d^2+17*C*c^2*d-21*D*c
^3))*(b*x^2+a)^(1/2)/d^6/(d*x+c)^3-1/24*(12*a^2*d^4*(C*d-3*D*c)+a*b*d^2*(1
5*A*d^3-43*B*c*d^2+95*C*c^2*d-171*D*c^3)+2*b^2*c^2*(9*A*d^3-23*B*c*d^2+43*
C*c^2*d-69*D*c^3))*(b*x^2+a)^(1/2)/d^6/(a*d^2+b*c^2)/(d*x+c)^2-1/24*(24*a^
3*d^6*D-4*a^2*b*d^4*(-8*B*d^2+31*C*c*d-87*D*c^2)-a*b^2*c*d^2*(15*A*d^3-91*
B*c*d^2+287*C*c^2*d-675*D*c^3)-2*b^3*c^3*(3*A*d^3-25*B*c*d^2+77*C*c^2*d-17
1*D*c^3))*(b*x^2+a)^(1/2)/d^6/(a*d^2+b*c^2)^2/(d*x+c)+1/2*b^(1/2)*(3*a*d^2
*D-2*b*(-B*d^2+5*C*c*d-15*D*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^7-1
/8*b*(12*a^3*d^6*(C*d-5*D*c)+4*a*b^2*c^3*d^2*(-5*B*d^2+25*C*c*d-78*D*c^2)+
8*b^3*c^5*(-B*d^2+5*C*c*d-15*D*c^2)+3*a^2*b*d^4*(A*d^3-5*B*c*d^2+25*C*c^2*
d-85*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 13.08 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \frac{-\frac{d\sqrt{a+bx^2}\left(6(bc^2+ad^2)^3(c^2Cd-Bcd^2+Ad^3-c^3D)+2(bc^2+ad^2)^2(4ad^2(-2cCd-\right.}{}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + dx)^5,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(6*(b*c^2 + a*d^2)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D) + 2*(b*c^2 + a*d^2)^2*(4*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-1
7*c^2*C*d + 13*B*c*d^2 - 9*A*d^3 + 21*c^3*D))*(c + d*x) + (b*c^2 + a*d^2)*
(12*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(95*c^2*C*d - 43*B*c*d^2 + 15*A*d^3 -
171*c^3*D) - 2*b^2*c^2*(-43*c^2*C*d + 23*B*c*d^2 - 9*A*d^3 + 69*c^3*D))*(c
 + d*x)^2 + (24*a^3*d^6*D + 4*a^2*b*d^4*(-31*c*C*d + 8*B*d^2 + 87*c^2*D) +
 2*b^3*c^3*(-77*c^2*C*d + 25*B*c*d^2 - 3*A*d^3 + 171*c^3*D) + a*b^2*c*d^2*
(-287*c^2*C*d + 91*B*c*d^2 - 15*A*d^3 + 675*c^3*D))*(c + d*x)^3 + 24*b*(b*
c^2 + a*d^2)^2*(-(C*d) + 5*c*D)*(c + d*x)^4 - 12*b*d*(b*c^2 + a*d^2)^2*D*x
*(c + d*x)^4))/((b*c^2 + a*d^2)^2*(c + d*x)^4)) - (3*b*(-12*a^3*d^6*(C*d -
 5*c*D) + 8*b^3*c^5*(-5*c*C*d + B*d^2 + 15*c^2*D) + 4*a*b^2*c^3*d^2*(-25*c
*C*d + 5*B*d^2 + 78*c^2*D) - 3*a^2*b*d^4*(25*c^2*C*d - 5*B*c*d^2 + A*d^3 -
 85*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(5/2) + 12*Sqrt[b]*(3*a*d^2*D +
2*b*(-5*c*C*d + B*d^2 + 15*c^2*D))*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]] + (3
*b*(-12*a^3*d^6*(C*d - 5*c*D) + 8*b^3*c^5*(-5*c*C*d + B*d^2 + 15*c^2*D) +
4*a*b^2*c^3*d^2*(-25*c*C*d + 5*B*d^2 + 78*c^2*D) - 3*a^2*b*d^4*(25*c^2*C*d
 - 5*B*c*d^2 + A*d^3 - 85*c^3*D))*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sq
rt[a + b*x^2]])/(b*c^2 + a*d^2)^(5/2))/(24*d^7)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^5} \, dx$$

$$\Big\downarrow \ 2182$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(4Cd-4cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+Bc-Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(4a(Cd-cD)-b\left(\frac{5Dc^3}{d^2}-\frac{5Cc^2}{d}+Bc-Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\frac{\left(a+bx^2\right)^{5/2}\left(4ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3-Bcd^2-9c^3D+5c^2Cd\right)\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(3d\left(Abd\left(4bc^2+ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-5Dc^2+Cdc+3Bd\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(3d\left(Abd\left(4bc^2+ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-5Dc^2+Cdc+3Bd^2\right)\right)\right)+2\left(6a^2Dd^4-4ab\left(-6Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-15Dc^3+5Cdc^2-Bd^2c-3Ad^3\right)\right)x\right)\left(bx\cdots\right.}{d^2(c+dx)^3}}{3\left(ad^2+bc^2\right)}\cdots}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(3d\left(Abd\left(4bc^2+ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-5Dc^2+Cdc+3Bd^2\right)\right)\right)+2\left(6a^2Dd^4-4ab\left(-6Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-15Dc^3+5Cdc^2-Bd^2c-3Ad^3\right)\right)x\right)\left(bx\cdots\right.}{(c+dx)^3}}{3d^2\left(ad^2+bc^2\right)}\cdots}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{681}$$

$$\frac{-\frac{3\int -\frac{4\left(2ad\left(6a^2Dd^4-4ab\left(-6Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-15Dc^3+5Cdc^2-Bd^2c-3Ad^3\right)\right)+b\left(12a^2(Cd-4cD)d^4+ab\left(-111Dc^3+35Cdc^2-7Bd^2c+3Ad^3\right)d^2+\cdots\right.\right.}{(c+dx)^2}}{8d^2}\cdots}{\cdots}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{\frac{\left(2ad\left(6a^2Dd^4-4ab\left(-6Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-15Dc^3+5Cdc^2-Bd^2c-3Ad^3\right)\right)+b\left(12a^2(Cd-4cD)d^4+ab\left(-111Dc^3+35Cdc^2-7Bd^2c+3Ad^3\right)d^2+4b^2c^3\right.\right.}{(c+dx)^2}}{2d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$3\left(-\frac{\int -\frac{2b\left(4\left(3ad^2D-2b\left(-15Dc^2+5Cdc-Bd^2\right)\right)x\left(bc^2+ad^2\right)^2+ad\left(12a^2(Cd-4cD)d^4+ab\left(-111Dc^3+35Cdc^2-7Bd^2c+3Ad^3\right)d^2+4b^2c^3\left(-15Dc^2+5Cdc-Bd\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{b\int \frac{4\left(3ad^2D-2b\left(-15Dc^2+5Cdc-Bd^2\right)\right)x\left(bc^2+ad^2\right)^2+ad\left(12a^2(Cd-4cD)d^4+ab\left(-111Dc^3+35Cdc^2-7Bd^2c+3Ad^3\right)d^2+4b^2c^3\left(-15Dc^2+5Cdc-Bd^2\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$3\left(\frac{b\left(\frac{\left(12a^3d^6(Cd-5cD)+3a^2bd^4\left(Ad^3-5Bcd^2-85c^3D+25c^2Cd\right)+4ab^2c^3d^2\left(-5Bd^2-78c^2D+25cCd\right)+8b^3c^5\left(-Bd^2-15c^2D+5cCd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\right.}{d^2}\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{3\left(\frac{b\left(\frac{\left(12a^3d^6(Cd-5cD)+3a^2bd^4\left(Ad^3-5Bcd^2-85c^3D+25c^2Cd\right)+4ab^2c^3d^2\left(-5Bd^2-78c^2D+25cCd\right)+8b^3c^5\left(-Bd^2-15c^2D+5cCd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\cdots\right)}{d^2}\cdots\right)}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{b\left(\frac{\left(12a^3d^6(Cd-5cD)+3a^2bd^4\left(Ad^3-5Bcd^2-85c^3D+25c^2Cd\right)+4ab^2c^3d^2\left(-5Bd^2-78c^2D+25cCd\right)+8b^3c^5\left(-Bd^2-15c^2D+5cCd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\cdots\right)}{d^2}\cdots\right)}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{3\left(\frac{b\left(\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2\left(3ad^2D-2b\left(-Bd^2-15c^2D+5cCd\right)\right)}{\sqrt{b}d}-\frac{\left(12a^3d^6(Cd-5cD)+3a^2bd^4\left(Ad^3-5Bcd^2-85c^3D+25c^2Cd\right)+4ab^2c^3d^2\left(-5\cdots\right.\right.}{\cdots}\cdots\right)}{d^2}\cdots\right)}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$3\left(\frac{b\left(\frac{4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2\left(3ad^2D-2b\left(-Bd^2-15c^2D+5cCd\right)\right)}{\sqrt{b}d}-\frac{\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(12a^3d^6(Cd-5cD)+3a^2bd^4\left(Ad^3-5Bcd^2\right.\right.}{}\right.}{d^2}\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^5,x]
```

output
```
-1/4*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^4) + (((4*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d - B*c*d^2 - 3*A*d^3 - 9*c^3*D))*(a + b*x^2)^(5/2))/(3*d^2*(b*c^2 + a*
d^2)*(c + d*x)^3) + (-1/2*((12*a^2*d^4*(C*d - 4*c*D) + 4*b^2*c^3*(5*c*C*d
- B*d^2 - 15*c^2*D) + a*b*d^2*(35*c^2*C*d - 7*B*c*d^2 + 3*A*d^3 - 111*c^3*
D) - 2*d*(6*a^2*d^4*D - 4*a*b*d^2*(2*c*C*d - B*d^2 - 6*c^2*D) - b^2*c*(5*c
^2*C*d - B*c*d^2 - 3*A*d^3 - 15*c^3*D))*x)*(a + b*x^2)^(3/2))/(d^2*(c + d*
x)^2) + (3*(-(((4*(b*c^2 + a*d^2)^2*(3*a*d^2*D - 2*b*(5*c*C*d - B*d^2 - 15
*c^2*D)) - b*d*(12*a^2*d^4*(C*d - 4*c*D) + 4*b^2*c^3*(5*c*C*d - B*d^2 - 15
*c^2*D) + a*b*d^2*(35*c^2*C*d - 7*B*c*d^2 + 3*A*d^3 - 111*c^3*D))*x)*Sqrt[
a + b*x^2])/(d^2*(c + d*x))) + (b*((4*(b*c^2 + a*d^2)^2*(3*a*d^2*D - 2*b*(
5*c*C*d - B*d^2 - 15*c^2*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*d) - ((12*a^3*d^6*(C*d - 5*c*D) + 4*a*b^2*c^3*d^2*(25*c*C*d - 5*B*d^2 -
78*c^2*D) + 8*b^3*c^5*(5*c*C*d - B*d^2 - 15*c^2*D) + 3*a^2*b*d^4*(25*c^2*C
*d - 5*B*c*d^2 + A*d^3 - 85*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*
d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/d^2))/(2*d^2))/(3*d^2*(b
*c^2 + a*d^2)))/(4*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8857 vs. $2(647) = 1294$.

Time = 1.62 (sec) , antiderivative size = 8858, normalized size of antiderivative = 12.93

method	result	size
default	Expression too large to display	8858

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**5,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6053 vs. $2(649) = 1298$.

Time = 0.32 (sec) , antiderivative size = 6053, normalized size of antiderivative = 8.84

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="maxi
ma")
```

output
```
-3/8*sqrt(b*x^2 + a)*D*b^4*c^7/(b^3*c^6*d^6 + 3*a*b^2*c^4*d^8 + 3*a^2*b*c^
2*d^10 + a^3*d^12) + 3/8*sqrt(b*x^2 + a)*D*b^4*c^6*x/(b^3*c^6*d^5 + 3*a*b^
2*c^4*d^7 + 3*a^2*b*c^2*d^9 + a^3*d^11) - 1/8*(b*x^2 + a)^(3/2)*D*b^3*c^6/
(b^3*c^6*d^5*x + 3*a*b^2*c^4*d^7*x + 3*a^2*b*c^2*d^9*x + a^3*d^11*x + b^3*
c^7*d^4 + 3*a*b^2*c^5*d^6 + 3*a^2*b*c^3*d^8 + a^3*c*d^10) + 3/8*sqrt(b*x^2
 + a)*C*b^4*c^6/(b^3*c^6*d^5 + 3*a*b^2*c^4*d^7 + 3*a^2*b*c^2*d^9 + a^3*d^1
1) - 3/8*sqrt(b*x^2 + a)*C*b^4*c^5*x/(b^3*c^6*d^4 + 3*a*b^2*c^4*d^6 + 3*a^
2*b*c^2*d^8 + a^3*d^10) + 1/8*(b*x^2 + a)^(5/2)*D*b^2*c^5/(b^3*c^6*d^4*x^2
 + 3*a*b^2*c^4*d^6*x^2 + 3*a^2*b*c^2*d^8*x^2 + a^3*d^10*x^2 + 2*b^3*c^7*d^
3*x + 6*a*b^2*c^5*d^5*x + 6*a^2*b*c^3*d^7*x + 2*a^3*c*d^9*x + b^3*c^8*d^2
+ 3*a*b^2*c^6*d^4 + 3*a^2*b*c^4*d^6 + a^3*c^2*d^8) + 1/8*(b*x^2 + a)^(3/2)
*C*b^3*c^5/(b^3*c^6*d^4*x + 3*a*b^2*c^4*d^6*x + 3*a^2*b*c^2*d^8*x + a^3*d^
10*x + b^3*c^7*d^3 + 3*a*b^2*c^5*d^5 + 3*a^2*b*c^3*d^7 + a^3*c*d^9) - 1/8*
(b*x^2 + a)^(3/2)*D*b^3*c^5/(b^3*c^6*d^4 + 3*a*b^2*c^4*d^6 + 3*a^2*b*c^2*d
^8 + a^3*d^10) - 3/8*sqrt(b*x^2 + a)*B*b^4*c^5/(b^3*c^6*d^4 + 3*a*b^2*c^4*
d^6 + 3*a^2*b*c^2*d^8 + a^3*d^10) + 3/8*sqrt(b*x^2 + a)*B*b^4*c^4*x/(b^3*c
^6*d^3 + 3*a*b^2*c^4*d^5 + 3*a^2*b*c^2*d^7 + a^3*d^9) - 1/8*(b*x^2 + a)^(5
/2)*C*b^2*c^4/(b^3*c^6*d^3*x^2 + 3*a*b^2*c^4*d^5*x^2 + 3*a^2*b*c^2*d^7*x^2
 + a^3*d^9*x^2 + 2*b^3*c^7*d^2*x + 6*a*b^2*c^5*d^4*x + 6*a^2*b*c^3*d^6*x +
 2*a^3*c*d^8*x + b^3*c^8*d + 3*a*b^2*c^6*d^3 + 3*a^2*b*c^4*d^5 + a^3*c^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^5}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^5}\,dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

3.80 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 751

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \frac{bD\sqrt{a+bx^2}}{d^6}$$

$$+\frac{(ad^2(2cCd-Bd^2-3c^2D)+b(c^3Cd-Acd^3-2c^4D))\sqrt{a+bx^2}}{4d^6(c+dx)^4}$$

$$-\frac{(4a^2d^4(Cd-3cD)+abcd^2(26cCd-9Bd^2-51c^2D)+b^2(13c^4Cd-9Ac^2d^3-30c^5D))\sqrt{a+bx^2}}{12d^6(bc^2+ad^2)(c+dx)^3}$$

$$-\frac{(12a^3d^6D-a^2bd^4(58cCd-15Bd^2-165c^2D)-ab^2cd^2(107c^2Cd-18Bcd^2-15Ad^3-288c^3D)-2b^3(2}{24d^6(bc^2+ad^2)^2(c+dx)^2}$$

$$-\frac{b(4a^3d^6(8Cd-39cD)+3a^2bcd^4(46cCd-5Bd^2-183c^2D)+3ab^2c^2d^2(49c^2Cd-2Bcd^2-5Ad^3-196c}{24d^6(bc^2+ad^2)^3(c+dx)}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{b^{3/2}(Cd-6cD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^7}$$

$$-\frac{b(12a^4d^8D-8b^4c^7(Cd-6cD)-28ab^3c^5d^2(Cd-6cD)-3a^3bd^6(6cCd-Bd^2-35c^2D)-a^2b^2cd^4(35c^2}{8d^7(bc^2+ad^2)^{7/2}}$$

output
```
b*D*(b*x^2+a)^(1/2)/d^6+1/4*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*(-A*c*d^3+C*
c^3*d-2*D*c^4))*(b*x^2+a)^(1/2)/d^6/(d*x+c)^4-1/12*(4*a^2*d^4*(C*d-3*D*c)+
a*b*c*d^2*(-9*B*d^2+26*C*c*d-51*D*c^2)+b^2*(-9*A*c^2*d^3+13*C*c^4*d-30*D*c
^5))*(b*x^2+a)^(1/2)/d^6/(a*d^2+b*c^2)/(d*x+c)^3-1/24*(12*a^3*d^6*D-a^2*b*
d^4*(-15*B*d^2+58*C*c*d-165*D*c^2)-a*b^2*c*d^2*(-15*A*d^3-18*B*c*d^2+107*C
*c^2*d-288*D*c^3)-2*b^3*(-9*A*c^3*d^3+23*C*c^5*d-66*D*c^6))*(b*x^2+a)^(1/2
)/d^6/(a*d^2+b*c^2)^2/(d*x+c)^2-1/24*b*(4*a^3*d^6*(8*C*d-39*D*c)+3*a^2*b*c
*d^4*(-5*B*d^2+46*C*c*d-183*D*c^2)+3*a*b^2*c^2*d^2*(-5*A*d^3-2*B*c*d^2+49*
C*c^2*d-196*D*c^3)+b^3*(-6*A*c^4*d^3+50*C*c^6*d-204*D*c^7))*(b*x^2+a)^(1/2
)/d^6/(a*d^2+b*c^2)^3/(d*x+c)-1/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^
(5/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^5+b^(3/2)*(C*d-6*D*c)*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/d^7-1/8*b*(12*a^4*d^8*D-8*b^4*c^7*(C*d-6*D*c)-28*a*b^3*c^5*
d^2*(C*d-6*D*c)-3*a^3*b*d^6*(-B*d^2+6*C*c*d-35*D*c^2)-a^2*b^2*c*d^4*(-3*A*
d^3+35*C*c^2*d-210*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2
+a)^(1/2))/d^7/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 13.87 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = -\frac{d\sqrt{a+bx^2}\left(24(bc^2+ad^2)^4(c^2Cd-Bcd^2+Ad^3-c^3D)+6(bc^2+ad^2)^3(5ad^2(-2cCa\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^6,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(24*(b*c^2 + a*d^2)^4*(c^2*C*d - B*c*d^2 + A*d^3 - c
^3*D) + 6*(b*c^2 + a*d^2)^3*(5*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-
21*c^2*C*d + 16*B*c*d^2 - 11*A*d^3 + 26*c^3*D))*(c + d*x) + 2*(b*c^2 + a*d
^2)^2*(20*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(154*c^2*C*d - 69*B*c*d^2 + 24*A
*d^3 - 279*c^3*D) + b^2*c^2*(137*c^2*C*d - 72*B*c*d^2 + 27*A*d^3 - 222*c^3
*D))*(c + d*x)^2 + (b*c^2 + a*d^2)*(60*a^3*d^6*D + 5*a^2*b*d^4*(-58*c*C*d
+ 15*B*d^2 + 165*c^2*D) + 2*b^3*c^3*(-163*c^2*C*d + 48*B*c*d^2 - 3*A*d^3 +
 378*c^3*D) + a*b^2*c*d^2*(-631*c^2*C*d + 186*B*c*d^2 - 21*A*d^3 + 1536*c^
3*D))*(c + d*x)^3 - b*(20*a^3*d^6*(-8*C*d + 39*c*D) + 2*b^3*c^4*(-137*c^2*
C*d + 12*B*c*d^2 + 3*A*d^3 + 522*c^3*D) + 3*a^2*b*d^4*(-238*c^2*C*d + 33*B
*c*d^2 - 8*A*d^3 + 923*c^3*D) + 3*a*b^2*c^2*d^2*(-261*c^2*C*d + 26*B*c*d^2
 + 9*A*d^3 + 996*c^3*D))*(c + d*x)^4 - 120*b*(b*c^2 + a*d^2)^3*D*(c + d*x)
^5))/((b*c^2 + a*d^2)^3*(c + d*x)^5)) + (15*b*(12*a^4*d^8*D + 8*b^4*c^7*(-
(C*d) + 6*c*D) + 28*a*b^3*c^5*d^2*(-(C*d) + 6*c*D) + 3*a^3*b*d^6*(-6*c*C*d
 + B*d^2 + 35*c^2*D) + a^2*b^2*c*d^4*(-35*c^2*C*d + 3*A*d^3 + 210*c^3*D))*
Log[c + d*x])/(b*c^2 + a*d^2)^(7/2) + 120*b^(3/2)*(C*d - 6*c*D)*Log[b*x +
Sqrt[b]*Sqrt[a + b*x^2]] - (15*b*(12*a^4*d^8*D + 8*b^4*c^7*(-(C*d) + 6*c*D
) + 28*a*b^3*c^5*d^2*(-(C*d) + 6*c*D) + 3*a^3*b*d^6*(-6*c*C*d + B*d^2 + 35
*c^2*D) + a^2*b^2*c*d^4*(-35*c^2*C*d + 3*A*d^3 + 210*c^3*D))*Log[a*d - b*c
*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(b*c^2 + a*d^2)^(7/2))/(120*...
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2182, 27, 2182, 25, 27, 680, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{(bc^2+ad^2)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^5}dx}{5(ad^2+bc^2)}-\frac{(a+bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{5d^2(c+dx)^5(ad^2+bc^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^5}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{-\frac{\int -\frac{\left(4\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(4a^2Dd^4-ab\left(-11Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-6Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{d^2(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{d(a+bx}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(4\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(4a^2Dd^4-ab\left(-11Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-6Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{d^2(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{d(a+bx^2)^{5/}}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(4\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(4a^2Dd^4-ab\left(-11Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-6Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{(c+dx)^4}dx}{4d^2\left(ad^2+bc^2\right)}-\frac{d(a+bx^2)^{5/}}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 680$$

$$\frac{\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3d^6(2Cd-3cD)+3a^2bcd^4\left(-3Bd^2-9c^2D+2cCd\right)-3dx\left(4a^3d^6D-a^2bd^4\left(-Bd^2-27c^2D+6cCd\right)-ab^2cd^2\left(-Ad^3+2Bcd^2-32c^3D+5c^2Cd\right)-2b\right.\right.}{6d^2(c+dx)^3\left(ad^2+bc^2\right)}}{}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$b\int \frac{\left(2ad\left(4a^2(Cd-3cD)d^4+abc\left(-15Dc^2+2Cdc+3Bd^2\right)d^2+b^2\left(-6Dc^5+Cdc^4+3Ad^3c^2\right)\right)+\left(12a^3Dd^6-a^2b\left(-57Dc^2+10Cdc-3Bd^2\right)d^4-ab^2c\left(-66Dc^3+11Cdc\right.\right.}{(c+dx)^2} \Big/ 2d^2\left(ad^2+bc^2\right)$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{681}$$

$$\frac{b\left(-\frac{\int -\frac{2\left(8b(Cd-6cD)x\left(bc^2+ad^2\right)^3+ad\left(12a^3Dd^6-a^2b\left(-57Dc^2+10Cdc-3Bd^2\right)d^4-ab^2c\left(-66Dc^3+11Cdc^2-3Ad^3\right)d^2-4b^3c^5(Cd-6cD)\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(\right.}{}\right.}{2d^2\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\int \frac{8b(Cd-6cD)x\left(bc^2+ad^2\right)^3+ad\left(12a^3Dd^6-a^2b\left(-57Dc^2+10Cdc-3Bd^2\right)d^4-ab^2c\left(-66Dc^3+11Cdc^2-3Ad^3\right)d^2-4b^3c^5(Cd-6cD)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(8\left(ad^2+\right.\right.}{}\right.}{2d^2\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{719}$$

$$b\left(\frac{\frac{\left(12a^4d^8D-3a^3bd^6\left(-Bd^2-35c^2D+6cCd\right)-a^2b^2cd^4\left(-3Ad^3-210c^3D+35c^2Cd\right)-28ab^3c^5d^2(Cd-6cD)-8b^4c^7(Cd-6cD)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{8b\left(ad^2+\right.}{}}{d^2}\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{\frac{\left(12a^4d^8D-3a^3bd^6\left(-Bd^2-35c^2D+6cCd\right)-a^2b^2cd^4\left(-3Ad^3-210c^3D+35c^2Cd\right)-28ab^3c^5d^2(Cd-6cD)-8b^4c^7(Cd-6cD)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{8b\left(ad^2+\cdots\right.}{\cdots}}{d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\frac{\left(12a^4d^8D-3a^3bd^6\left(-Bd^2-35c^2D+6cCd\right)-a^2b^2cd^4\left(-3Ad^3-210c^3D+35c^2Cd\right)-28ab^3c^5d^2(Cd-6cD)-8b^4c^7(Cd-6cD)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{8\sqrt{b}\text{arc}\cdots}{\cdots}}{d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$b\left(\frac{\frac{8\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^3(Cd-6cD)}{d}-\frac{\left(12a^4d^8D-3a^3bd^6\left(-Bd^2-35c^2D+6cCd\right)-a^2b^2cd^4\left(-3Ad^3-210c^3D+35c^2Cd\right)-28ab^3c^5d^2(Cd-6c\cdots\right.}{d}}{d^2}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3d^6(2Cd-3cD)+3a^2bcd^4\left(-3Bd^2-9c^2D+2cCd\right)-3dx\left(4a^3d^6D-a^2bd^4\left(-Bd^2-27c^2D+6cCd\right)-ab^2cd^2\left(-Ad^3+2Bcd^2-32c^3D+5c^2Cd\right)-2b\cdots\right.\right.}{6d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^6,x]
```

output
```
-1/5*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^5) + (-1/4*(d*(A*b*c - (b*c^3*(C*d - 2*c*D))/d^3 - a*(2*
c*C - B*d - (3*c^2*D)/d))*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^4)
 + (((4*b^3*c^6*(C*d - 6*c*D) - 4*a^3*d^6*(2*C*d - 3*c*D) + 3*a^2*b*c*d^4*
(2*c*C*d - 3*B*d^2 - 9*c^2*D) + 9*a*b^2*c^2*d^2*(c^2*C*d - A*d^3 - 6*c^3*D
) - 3*d*(4*a^3*d^6*D - a^2*b*d^4*(6*c*C*d - B*d^2 - 27*c^2*D) - a*b^2*c*d^
2*(5*c^2*C*d + 2*B*c*d^2 - A*d^3 - 32*c^3*D) - 2*b^3*(c^5*C*d + A*c^3*d^3
- 6*c^6*D))*x)*(a + b*x^2)^(3/2))/(6*d^2*(b*c^2 + a*d^2)*(c + d*x)^3) + (b
*(-(((8*(b*c^2 + a*d^2)^3*(C*d - 6*c*D) - d*(12*a^3*d^6*D - 4*b^3*c^5*(C*d
 - 6*c*D) - a^2*b*d^4*(10*c*C*d - 3*B*d^2 - 57*c^2*D) - a*b^2*c*d^2*(11*c^
2*C*d - 3*A*d^3 - 66*c^3*D))*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x))) + ((8*Sq
rt[b]*(b*c^2 + a*d^2)^3*(C*d - 6*c*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/d - ((12*a^4*d^8*D - 8*b^4*c^7*(C*d - 6*c*D) - 28*a*b^3*c^5*d^2*(C*d - 6
*c*D) - 3*a^3*b*d^6*(6*c*C*d - B*d^2 - 35*c^2*D) - a^2*b^2*c*d^4*(35*c^2*C
*d - 3*A*d^3 - 210*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt
[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/d^2))/(2*d^2*(b*c^2 + a*d^2)))/(4*
d^2*(b*c^2 + a*d^2)))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12093 vs. 2(715) = 1430.

Time = 1.66 (sec) , antiderivative size = 12094, normalized size of antiderivative = 16.10

method	result	size
default	Expression too large to display	12094

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**6,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9508 vs. $2(717) = 1434$.

Time = 0.43 (sec) , antiderivative size = 9508, normalized size of antiderivative = 12.66

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="maxi
ma")
```

output
```
-3/8*sqrt(b*x^2 + a)*D*b^5*c^8/(b^4*c^8*d^6 + 4*a*b^3*c^6*d^8 + 6*a^2*b^2*
c^4*d^10 + 4*a^3*b*c^2*d^12 + a^4*d^14) + 3/8*sqrt(b*x^2 + a)*D*b^5*c^7*x/
(b^4*c^8*d^5 + 4*a*b^3*c^6*d^7 + 6*a^2*b^2*c^4*d^9 + 4*a^3*b*c^2*d^11 + a^
4*d^13) - 1/8*(b*x^2 + a)^(3/2)*D*b^4*c^7/(b^4*c^8*d^5*x + 4*a*b^3*c^6*d^7
*x + 6*a^2*b^2*c^4*d^9*x + 4*a^3*b*c^2*d^11*x + a^4*d^13*x + b^4*c^9*d^4 +
 4*a*b^3*c^7*d^6 + 6*a^2*b^2*c^5*d^8 + 4*a^3*b*c^3*d^10 + a^4*c*d^12) + 3/
8*sqrt(b*x^2 + a)*C*b^5*c^7/(b^4*c^8*d^5 + 4*a*b^3*c^6*d^7 + 6*a^2*b^2*c^4
*d^9 + 4*a^3*b*c^2*d^11 + a^4*d^13) - 3/8*sqrt(b*x^2 + a)*C*b^5*c^6*x/(b^4
*c^8*d^4 + 4*a*b^3*c^6*d^6 + 6*a^2*b^2*c^4*d^8 + 4*a^3*b*c^2*d^10 + a^4*d^
12) + 1/8*(b*x^2 + a)^(5/2)*D*b^3*c^6/(b^4*c^8*d^4*x^2 + 4*a*b^3*c^6*d^6*x
^2 + 6*a^2*b^2*c^4*d^8*x^2 + 4*a^3*b*c^2*d^10*x^2 + a^4*d^12*x^2 + 2*b^4*c
^9*d^3*x + 8*a*b^3*c^7*d^5*x + 12*a^2*b^2*c^5*d^7*x + 8*a^3*b*c^3*d^9*x +
2*a^4*c*d^11*x + b^4*c^10*d^2 + 4*a*b^3*c^8*d^4 + 6*a^2*b^2*c^6*d^6 + 4*a^
3*b*c^4*d^8 + a^4*c^2*d^10) + 1/8*(b*x^2 + a)^(3/2)*C*b^4*c^6/(b^4*c^8*d^4
*x + 4*a*b^3*c^6*d^6*x + 6*a^2*b^2*c^4*d^8*x + 4*a^3*b*c^2*d^10*x + a^4*d^
12*x + b^4*c^9*d^3 + 4*a*b^3*c^7*d^5 + 6*a^2*b^2*c^5*d^7 + 4*a^3*b*c^3*d^9
 + a^4*c*d^11) - 1/8*(b*x^2 + a)^(3/2)*D*b^4*c^6/(b^4*c^8*d^4 + 4*a*b^3*c^
6*d^6 + 6*a^2*b^2*c^4*d^8 + 4*a^3*b*c^2*d^10 + a^4*d^12) - 3/8*sqrt(b*x^2
+ a)*B*b^5*c^6/(b^4*c^8*d^4 + 4*a*b^3*c^6*d^6 + 6*a^2*b^2*c^4*d^8 + 4*a^3*
b*c^2*d^10 + a^4*d^12) + 3/8*sqrt(b*x^2 + a)*B*b^5*c^5*x/(b^4*c^8*d^3 +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6144 vs. $2(717) = 1434$.

Time = 0.67 (sec) , antiderivative size = 6144, normalized size of antiderivative = 8.18

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="giac
")
```

output
```
1/4*(48*D*b^5*c^8 - 8*C*b^5*c^7*d + 168*D*a*b^4*c^6*d^2 - 28*C*a*b^4*c^5*d
^3 + 210*D*a^2*b^3*c^4*d^4 - 35*C*a^2*b^3*c^3*d^5 + 105*D*a^3*b^2*c^2*d^6
- 18*C*a^3*b^2*c*d^7 + 3*A*a^2*b^3*c*d^7 + 12*D*a^4*b*d^8 + 3*B*a^3*b^2*d^
8)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d
^2))/((b^3*c^6*d^7 + 3*a*b^2*c^4*d^9 + 3*a^2*b*c^2*d^11 + a^3*d^13)*sqrt(-
b*c^2 - a*d^2)) + sqrt(b*x^2 + a)*D*b/d^6 + 1/60*(1800*(sqrt(b)*x - sqrt(b
*x^2 + a))^9*D*b^5*c^8*d^4 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*b^5*c^7
*d^5 + 5280*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b^4*c^6*d^6 + 120*(sqrt(b)
*x - sqrt(b*x^2 + a))^9*B*b^5*c^6*d^6 - 1740*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*C*a*b^4*c^5*d^7 + 5130*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^2*b^3*c^4*d^
8 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a*b^4*c^4*d^8 - 1635*(sqrt(b)*x
- sqrt(b*x^2 + a))^9*C*a^2*b^3*c^3*d^9 + 1665*(sqrt(b)*x - sqrt(b*x^2 + a)
)^9*D*a^3*b^2*c^2*d^10 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c^2
*d^10 - 450*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^3*b^2*c*d^11 - 45*(sqrt(b)
*x - sqrt(b*x^2 + a))^9*A*a^2*b^3*c*d^11 + 60*(sqrt(b)*x - sqrt(b*x^2 + a)
)^9*D*a^4*b*d^12 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^3*b^2*d^12 + 120
00*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*b^(11/2)*c^9*d^3 - 3600*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*C*b^(11/2)*c^8*d^4 + 33720*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*D*a*b^(9/2)*c^7*d^5 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(11/2)*c
^7*d^5 - 10020*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^(9/2)*c^6*d^6 + 12...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^6} \,d x$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx=\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(D x^{3}+C\,x^{2}+B x +A \right)}{\left(d x +c \right)^{6}}d x$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

3.81 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 800

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx =$$

$$-\frac{b(16b^4c^9D+56ab^3c^7d^2D+2a^4d^8(3Cd-5cD)-a^3bd^6(c^2Cd-7Bcd^2+Ad^3-23c^3D)+6a^2b^2c^2d^4(Ad^3}{}$$

$$-\frac{(8b^3c^7D+2a^3d^6(3Cd-5cD)-a^2bd^4(c^2Cd-7Bcd^2+Ad^3-7c^3D)+6ab^2c^2d^2(Ad^3+3c^3D)-d(Ab^2c}{24d^4(bc^2+}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{5/2}}{6d^2(bc^2+ad^2)(c+dx)^6}$$

$$+\frac{(6ad^2(2cCd-Bd^2-3c^2D)+bc(5c^2Cd+Bcd^2-7Ad^3-11c^3D))(a+bx^2)^{5/2}}{30d^2(bc^2+ad^2)^2(c+dx)^5}$$

$$+\frac{b^{3/2}D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^7}$$

$$+\frac{b^2(16b^4c^9D+72ab^3c^7d^2D-6a^4d^8(Cd-7cD)-6a^2b^2c^2d^4(Ad^3-21c^3D)+a^3bd^6(c^2Cd-7Bcd^2+Ad^3}{16d^7(bc^2+ad^2)^{9/2}}$$

output
```
-1/16*b*(16*b^4*c^9*D+56*a*b^3*c^7*d^2*D+2*a^4*d^8*(3*C*d-5*D*c)-a^3*b*d^6
*(A*d^3-7*B*c*d^2+C*c^2*d-23*D*c^3)+6*a^2*b^2*c^2*d^4*(A*d^3+11*D*c^3)+d*(
24*b^4*c^8*D+16*a^4*d^8*D-6*a^3*b*c*d^6*(C*d-15*D*c)-2*a*b^3*c^3*d^2*(3*A*
d^3-47*D*c^3)+a^2*b^2*c*d^4*(A*d^3-7*B*c*d^2+C*c^2*d+137*D*c^3))*x)*(b*x^2
+a)^(1/2)/d^6/(a*d^2+b*c^2)^4/(d*x+c)^2-1/24*(8*b^3*c^7*D+2*a^3*d^6*(3*C*d
-5*D*c)-a^2*b*d^4*(A*d^3-7*B*c*d^2+C*c^2*d-7*D*c^3)+6*a*b^2*c^2*d^2*(A*d^3
+3*D*c^3)-d*(A*b^2*c*d^3*(-a*d^2+6*b*c^2)-14*b^3*c^6*D-8*a^3*d^6*D+6*a^2*b
*c*d^4*(C*d-7*D*c)-a*b^2*c^2*d^2*(-7*B*d^2+C*c*d+41*D*c^2))*x)*(b*x^2+a)^(
3/2)/d^4/(a*d^2+b*c^2)^3/(d*x+c)^4-1/6*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^
2+a)^(5/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^6+1/30*(6*a*d^2*(-B*d^2+2*C*c*d-3*D*c
^2)+b*c*(-7*A*d^3+B*c*d^2+5*C*c^2*d-11*D*c^3))*(b*x^2+a)^(5/2)/d^2/(a*d^2+
b*c^2)^2/(d*x+c)^5+b^(3/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^7+1/16*b
^2*(16*b^4*c^9*D+72*a*b^3*c^7*d^2*D-6*a^4*d^8*(C*d-7*D*c)-6*a^2*b^2*c^2*d^
4*(A*d^3-21*D*c^3)+a^3*b*d^6*(A*d^3-7*B*c*d^2+C*c^2*d+105*D*c^3))*arctanh(
(-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7/(a*d^2+b*c^2)^(9/2)
```

Mathematica [A] (verified)

Time = 14.51 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \frac{-\frac{d\sqrt{a+bx^2}\left(40(bc^2+ad^2)^5(c^2Cd-Bcd^2+Ad^3-c^3D)+8(bc^2+ad^2)^4(6ad^2(-2cCd\right.}{}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(40*(b*c^2 + a*d^2)^5*(c^2*C*d - B*c*d^2 + A*d^3 - c
^3*D) + 8*(b*c^2 + a*d^2)^4*(6*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-
25*c^2*C*d + 19*B*c*d^2 - 13*A*d^3 + 31*c^3*D))*(c + d*x) + 2*(b*c^2 + a*d
^2)^3*(30*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(227*c^2*C*d - 101*B*c*d^2 + 35*
A*d^3 - 413*c^3*D) + 2*b^2*c^2*(100*c^2*C*d - 52*B*c*d^2 + 19*A*d^3 - 163*
c^3*D))*(c + d*x)^2 + 2*(b*c^2 + a*d^2)^2*(40*a^3*d^6*D + 6*a^2*b*d^4*(-31
*c*C*d + 8*B*d^2 + 89*c^2*D) + 2*b^3*c^3*(-100*c^2*C*d + 28*B*c*d^2 - A*d^
3 + 237*c^3*D) + 3*a*b^2*c*d^2*(-131*c^2*C*d + 37*B*c*d^2 - 3*A*d^3 + 325*
c^3*D))*(c + d*x)^3 - b*(b*c^2 + a*d^2)*(10*a^3*d^6*(-15*C*d + 73*c*D) + 4
*b^3*c^4*(-50*c^2*C*d + 2*B*c*d^2 + A*d^3 + 213*c^3*D) + 6*a*b^2*c^2*d^2*(
-99*c^2*C*d + 5*B*c*d^2 + 4*A*d^3 + 418*c^3*D) + 3*a^2*b*d^4*(-193*c^2*C*d
 + 19*B*c*d^2 - 5*A*d^3 + 807*c^3*D))*(c + d*x)^4 + b*(320*a^4*d^8*D + 2*a
^3*b*d^6*(-123*c*C*d + 24*B*d^2 + 997*c^2*D) + 4*b^4*c^5*(-10*c^2*C*d - 2*
B*c*d^2 - A*d^3 + 147*c^3*D) + 3*a^2*b^2*c*d^4*(-89*c^2*C*d - 29*B*c*d^2 +
 27*A*d^3 + 1087*c^3*D) + 2*a*b^3*c^3*d^2*(-83*c^2*C*d - 19*B*c*d^2 - 14*A
*d^3 + 1140*c^3*D))*(c + d*x)^5))/((b*c^2 + a*d^2)^4*(c + d*x)^6)) - (15*b
^2*(16*b^4*c^9*D + 72*a*b^3*c^7*d^2*D - 6*a^4*d^8*(C*d - 7*c*D) + 6*a^2*b^
2*c^2*d^4*(-(A*d^3) + 21*c^3*D) + a^3*b*d^6*(c^2*C*d - 7*B*c*d^2 + A*d^3 +
 105*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(9/2) + 240*b^(3/2)*D*Log[b*x +
 Sqrt[b]*Sqrt[a + b*x^2]] + (15*b^2*(16*b^4*c^9*D + 72*a*b^3*c^7*d^2*D ...
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2182, 25, 2182, 27, 680, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}\left(6\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(6Cd-6cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+Bc-Ad\right)\right)x+6\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}\left(6\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(6a(Cd-cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+Bc-Ad\right)\right)x+6\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{5/2}\left(6ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-7Ad^3+Bcd^2-11c^3D+5c^2Cd\right)\right)}{5d^2(c+dx)^5(ad^2+bc^2)}-\frac{\int -\frac{5\left(\left(6d(Cd-2cD)a^2-\frac{bc\left(5Dc^2+Cdc-7Bd^2\right)a}{d}+Ab\left(6bc^2-ad^2\right)\cdots\right.\right.}{d^2(c+dx)^5}\cdots}{5(ad^2+bc^2)}\cdots}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(6Dx\left(bc^2+ad^2\right)^2+d\left(6a^2(Cd-2cD)d^2+Ab\left(6bc^2-ad^2\right)d-abc\left(5Dc^2+Cdc-7Bd^2\right)\right)\right)\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{d^2(ad^2+bc^2)}+\frac{(a+bx^2)^{5/2}\left(6ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(\cdots\right.\right.}{5d^2(c+dx)^5(ad^2+\cdots}}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 680$$

$$\frac{-\frac{\int -\frac{6b\left(8Dx\left(bc^2+ad^2\right)^3+ad\left(-6b^2Dc^5-abd^2\left(17Dc^2+Cdc-7Bd^2\right)c+Abd^3\left(6bc^2-ad^2\right)+6a^2d^4(Cd-3cD)\right)\right)\sqrt{bx^2+a}}{(c+dx)^3}dx}{8d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(2a^3d^6(3Cd-5cD)-\cdots\right.}{\cdots}\cdots}{\cdots}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3b\int\frac{\left(8Dx\left(bc^2+ad^2\right)^3+ad\left(-6b^2Dc^5-abd^2\left(17Dc^2+Cdc-7Bd^2\right)c+Abd^3\left(6bc^2-ad^2\right)+6a^2d^4(Cd-3cD)\right)\right)\sqrt{bx^2+a}}{(c+dx)^3}dx}{4d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}\left(2a^3d^6(3Cd-5cD)-a^2b\right.}{}}{d}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 680

$$\frac{\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\Big\downarrow$ 719

$$\frac{\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\Big\downarrow$ 224

$$\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5} + \frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\downarrow$ 219

$$\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5} + \frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\downarrow$ 488

$$\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3))(bx^2+a)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5} + \frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\downarrow$ 219

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+5Cdc^2+Bd^2c-7Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(16b^4Dc^9+56ab^3d^2Dc^7+6a^2b^2d^4\left(11Dc^3+Ad^3\right)c^2+\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
-1/6*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^6) + (((6*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d + B*c*d^2 - 7*A*d^3 - 11*c^3*D))*(a + b*x^2)^(5/2))/(5*d^2*(b*c^2 + a
*d^2)*(c + d*x)^5) + (-1/4*((8*b^3*c^7*D + 2*a^3*d^6*(3*C*d - 5*c*D) - a^2
*b*d^4*(c^2*C*d - 7*B*c*d^2 + A*d^3 - 7*c^3*D) + 6*a*b^2*c^2*d^2*(A*d^3 +
3*c^3*D) - d*(A*b^2*c*d^3*(6*b*c^2 - a*d^2) - 14*b^3*c^6*D - 8*a^3*d^6*D +
 6*a^2*b*c*d^4*(C*d - 7*c*D) - a*b^2*c^2*d^2*(c*C*d - 7*B*d^2 + 41*c^2*D))
*x)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)*(c + d*x)^4) + (3*b*(-1/2*((16
*b^4*c^9*D + 56*a*b^3*c^7*d^2*D + 2*a^4*d^8*(3*C*d - 5*c*D) - a^3*b*d^6*(c
^2*C*d - 7*B*c*d^2 + A*d^3 - 23*c^3*D) + 6*a^2*b^2*c^2*d^4*(A*d^3 + 11*c^3
*D) + d*(24*b^4*c^8*D + 16*a^4*d^8*D - 6*a^3*b*c*d^6*(C*d - 15*c*D) - 2*a*
b^3*c^3*d^2*(3*A*d^3 - 47*c^3*D) + a^2*b^2*c*d^4*(c^2*C*d - 7*B*c*d^2 + A*
d^3 + 137*c^3*D))*x)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c + d*x)^2) -
(b*((-16*(b*c^2 + a*d^2)^4*D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*d) - ((16*b^4*c^9*D + 72*a*b^3*c^7*d^2*D - 6*a^4*d^8*(C*d - 7*c*D) - 6*a
^2*b^2*c^2*d^4*(A*d^3 - 21*c^3*D) + a^3*b*d^6*(c^2*C*d - 7*B*c*d^2 + A*d^3
 + 105*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])
])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2*(b*c^2 + a*d^2))))/(4*d^2*(b*c^2 + a*d
^2)))/(d^2*(b*c^2 + a*d^2)))/(6*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18853 vs. 2(770) = 1540.

Time = 1.82 (sec) , antiderivative size = 18854, normalized size of antiderivative = 23.57

method	result	size
default	Expression too large to display	18854

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15131 vs. 2(776) = 1552.

Time = 0.68 (sec) , antiderivative size = 15131, normalized size of antiderivative = 18.91

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="maxi
ma")
```

output
```
-7/16*sqrt(b*x^2 + a)*D*b^6*c^9/(b^5*c^10*d^6 + 5*a*b^4*c^8*d^8 + 10*a^2*b
^3*c^6*d^10 + 10*a^3*b^2*c^4*d^12 + 5*a^4*b*c^2*d^14 + a^5*d^16) + 7/16*sq
rt(b*x^2 + a)*D*b^6*c^8*x/(b^5*c^10*d^5 + 5*a*b^4*c^8*d^7 + 10*a^2*b^3*c^6
*d^9 + 10*a^3*b^2*c^4*d^11 + 5*a^4*b*c^2*d^13 + a^5*d^15) - 7/48*(b*x^2 +
a)^(3/2)*D*b^5*c^8/(b^5*c^10*d^5*x + 5*a*b^4*c^8*d^7*x + 10*a^2*b^3*c^6*d^
9*x + 10*a^3*b^2*c^4*d^11*x + 5*a^4*b*c^2*d^13*x + a^5*d^15*x + b^5*c^11*d
^4 + 5*a*b^4*c^9*d^6 + 10*a^2*b^3*c^7*d^8 + 10*a^3*b^2*c^5*d^10 + 5*a^4*b*
c^3*d^12 + a^5*c*d^14) + 7/16*sqrt(b*x^2 + a)*C*b^6*c^8/(b^5*c^10*d^5 + 5*
a*b^4*c^8*d^7 + 10*a^2*b^3*c^6*d^9 + 10*a^3*b^2*c^4*d^11 + 5*a^4*b*c^2*d^1
3 + a^5*d^15) - 7/16*sqrt(b*x^2 + a)*C*b^6*c^7*x/(b^5*c^10*d^4 + 5*a*b^4*c
^8*d^6 + 10*a^2*b^3*c^6*d^8 + 10*a^3*b^2*c^4*d^10 + 5*a^4*b*c^2*d^12 + a^5
*d^14) + 7/48*(b*x^2 + a)^(5/2)*D*b^4*c^7/(b^5*c^10*d^4*x^2 + 5*a*b^4*c^8*
d^6*x^2 + 10*a^2*b^3*c^6*d^8*x^2 + 10*a^3*b^2*c^4*d^10*x^2 + 5*a^4*b*c^2*d
^12*x^2 + a^5*d^14*x^2 + 2*b^5*c^11*d^3*x + 10*a*b^4*c^9*d^5*x + 20*a^2*b^
3*c^7*d^7*x + 20*a^3*b^2*c^5*d^9*x + 10*a^4*b*c^3*d^11*x + 2*a^5*c*d^13*x
+ b^5*c^12*d^2 + 5*a*b^4*c^10*d^4 + 10*a^2*b^3*c^8*d^6 + 10*a^3*b^2*c^6*d^
8 + 5*a^4*b*c^4*d^10 + a^5*c^2*d^12) + 7/48*(b*x^2 + a)^(3/2)*C*b^5*c^7/(b
^5*c^10*d^4*x + 5*a*b^4*c^8*d^6*x + 10*a^2*b^3*c^6*d^8*x + 10*a^3*b^2*c^4*
d^10*x + 5*a^4*b*c^2*d^12*x + a^5*d^14*x + b^5*c^11*d^3 + 5*a*b^4*c^9*d^5
+ 10*a^2*b^3*c^7*d^7 + 10*a^3*b^2*c^5*d^9 + 5*a^4*b*c^3*d^11 + a^5*c*d^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8545 vs. $2(776) = 1552$.

Time = 0.73 (sec) , antiderivative size = 8545, normalized size of antiderivative = 10.68

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="giac
")
```

output
```
-1/8*(16*D*b^6*c^9 + 72*D*a*b^5*c^7*d^2 + 126*D*a^2*b^4*c^5*d^4 + 105*D*a^
3*b^3*c^3*d^6 + C*a^3*b^3*c^2*d^7 - 6*A*a^2*b^4*c^2*d^7 + 42*D*a^4*b^2*c*d
^8 - 7*B*a^3*b^3*c*d^8 - 6*C*a^4*b^2*d^9 + A*a^3*b^3*d^9)*arctan(-((sqrt(b
)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c^8*d^7
+ 4*a*b^3*c^6*d^9 + 6*a^2*b^2*c^4*d^11 + 4*a^3*b*c^2*d^13 + a^4*d^15)*sqrt
(-b*c^2 - a*d^2)) - D*b^(3/2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^7 -
 1/120*(1440*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*b^6*c^9*d^5 - 240*(sqrt(b)
*x - sqrt(b*x^2 + a))^11*C*b^6*c^8*d^6 + 5640*(sqrt(b)*x - sqrt(b*x^2 + a)
)^11*D*a*b^5*c^7*d^7 - 960*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^5*c^6*d^
8 + 8190*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^2*b^4*c^5*d^9 - 1440*(sqrt(b
)*x - sqrt(b*x^2 + a))^11*C*a^2*b^4*c^4*d^10 + 5145*(sqrt(b)*x - sqrt(b*x^
2 + a))^11*D*a^3*b^3*c^3*d^11 - 975*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^3
*b^3*c^2*d^12 + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4*c^2*d^12 + 1
050*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^4*b^2*c*d^13 + 105*(sqrt(b)*x - s
qrt(b*x^2 + a))^11*B*a^3*b^3*c*d^13 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^11
*C*a^4*b^2*d^14 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^3*d^14 + 108
00*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*b^(13/2)*c^10*d^4 - 1200*(sqrt(b)*x
- sqrt(b*x^2 + a))^10*C*b^(13/2)*c^9*d^5 + 41400*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*D*a*b^(11/2)*c^8*d^6 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(1
3/2)*c^8*d^6 - 4800*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a*b^(11/2)*c^7*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^7}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^7}dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

3.82 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 649

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx =$$
$$-\frac{ab(3Ab^2c(2bc^2-ad^2)-a(b^2c^2(cC-8Bd)-6a^2d^3D-abd(8cCd-Bd^2-3c^2D)))(ad-bcx)\sqrt{a+bx^2}}{16(bc^2+ad^2)^5(c+dx)^2}$$
$$-\frac{(3Ab^2c(2bc^2-ad^2)-a(b^2c^2(cC-8Bd)-6a^2d^3D-abd(8cCd-Bd^2-3c^2D)))(ad-bcx)(a+bx^2)^{3/}}{24(bc^2+ad^2)^4(c+dx)^4}$$
$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{5/2}}{7d^2(bc^2+ad^2)(c+dx)^7}$$
$$+\frac{(7ad^2(2cCd-Bd^2-3c^2D)+bc(5c^2Cd+2Bcd^2-9Ad^3-12c^3D))(a+bx^2)^{5/2}}{42d^2(bc^2+ad^2)^2(c+dx)^6}$$
$$-\frac{(42a^2d^4(Cd-3cD)-b^2c^2(5c^2Cd+2Bcd^2-51Ad^3+30c^3D)-abd^2(26c^2Cd-61Bcd^2+12Ad^3+93c}{210d^2(bc^2+ad^2)^3(c+dx)^5}$$
$$-\frac{a^2b^2(3Ab^2c(2bc^2-ad^2)-a(b^2c^2(cC-8Bd)-6a^2d^3D-abd(8cCd-Bd^2-3c^2D)))\,\mathrm{arctanh}\left(\frac{ad-bc}{\sqrt{bc^2+ad^2}\sqrt{}}\right.}{16(bc^2+ad^2)^{11/2}}$$

output
```
-1/16*a*b*(3*A*b^2*c*(-a*d^2+2*b*c^2)-a*(b^2*c^2*(-8*B*d+C*c)-6*a^2*d^3*D-
a*b*d*(-B*d^2+8*C*c*d-3*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2
)^5/(d*x+c)^2-1/24*(3*A*b^2*c*(-a*d^2+2*b*c^2)-a*(b^2*c^2*(-8*B*d+C*c)-6*a
^2*d^3*D-a*b*d*(-B*d^2+8*C*c*d-3*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*
d^2+b*c^2)^4/(d*x+c)^4-1/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(5/2)/d
^2/(a*d^2+b*c^2)/(d*x+c)^7+1/42*(7*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-9*
A*d^3+2*B*c*d^2+5*C*c^2*d-12*D*c^3))*(b*x^2+a)^(5/2)/d^2/(a*d^2+b*c^2)^2/(
d*x+c)^6-1/210*(42*a^2*d^4*(C*d-3*D*c)-b^2*c^2*(-51*A*d^3+2*B*c*d^2+5*C*c^
2*d+30*D*c^3)-a*b*d^2*(12*A*d^3-61*B*c*d^2+26*C*c^2*d+93*D*c^3))*(b*x^2+a)
^(5/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^5-1/16*a^2*b^2*(3*A*b^2*c*(-a*d^2+2*b*c
^2)-a*(b^2*c^2*(-8*B*d+C*c)-6*a^2*d^3*D-a*b*d*(-B*d^2+8*C*c*d-3*D*c^2)))*a
rctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(11
/2)
```

Mathematica [A] (verified)

Time = 14.22 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
-1/1680*(Sqrt[a + b*x^2]*(240*(b*c^2 + a*d^2)^6*(c^2*C*d - B*c*d^2 + A*d^3
 - c^3*D) + 40*(b*c^2 + a*d^2)^5*(7*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b
*c*(-29*c^2*C*d + 22*B*c*d^2 - 15*A*d^3 + 36*c^3*D))*(c + d*x) + 8*(b*c^2
+ a*d^2)^4*(42*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(314*c^2*C*d - 139*B*c*d^2
+ 48*A*d^3 - 573*c^3*D) + b^2*c^2*(275*c^2*C*d - 142*B*c*d^2 + 51*A*d^3 -
450*c^3*D))*(c + d*x)^2 + 2*(b*c^2 + a*d^2)^3*(210*a^3*d^6*D + 7*a^2*b*d^4
*(-136*c*C*d + 35*B*d^2 + 393*c^2*D) + 2*b^3*c^3*(-500*c^2*C*d + 136*B*c*d
^2 - 3*A*d^3 + 1200*c^3*D) + a*b^2*c*d^2*(-1979*c^2*C*d + 544*B*c*d^2 - 33
*A*d^3 + 4968*c^3*D))*(c + d*x)^3 - 2*b*(b*c^2 + a*d^2)^2*(42*a^3*d^6*(-8*
C*d + 39*c*D) + 2*b^3*c^4*(-200*c^2*C*d + 4*B*c*d^2 + 3*A*d^3 + 900*c^3*D)
 + 3*a^2*b*d^4*(-400*c^2*C*d + 29*B*c*d^2 - 8*A*d^3 + 1751*c^3*D) + a*b^2*
c^2*d^2*(-1201*c^2*C*d + 32*B*c*d^2 + 45*A*d^3 + 5352*c^3*D))*(c + d*x)^4
+ b*(b*c^2 + a*d^2)*(1050*a^4*d^8*D + 21*a^3*b*d^6*(-24*c*C*d + 5*B*d^2 +
267*c^2*D) + 4*b^4*c^5*(-10*c^2*C*d - 4*B*c*d^2 - 3*A*d^3 + 360*c^3*D) + 3
*a^2*b^2*c*d^4*(-109*c^2*C*d - 94*B*c*d^2 + 73*A*d^3 + 2846*c^3*D) + 2*a*b
^3*c^3*d^2*(-89*c^2*C*d - 44*B*c*d^2 - 54*A*d^3 + 2868*c^3*D))*(c + d*x)^5
 - b^2*(42*a^4*d^8*(-8*C*d + 49*c*D) + 4*b^4*c^6*(10*c^2*C*d + 4*B*c*d^2 +
 3*A*d^3 + 60*c^3*D) + 2*a*b^3*c^4*d^2*(109*c^2*C*d + 52*B*c*d^2 + 60*A*d^
3 + 612*c^3*D) + 3*a^3*b*d^6*(312*c^2*C*d - 221*B*c*d^2 + 32*A*d^3 + 885*c
^3*D) + a^2*b^2*c^2*d^4*(505*c^2*C*d + 370*B*c*d^2 - 741*A*d^3 + 2526*c...
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2182, 25, 2182, 25, 27, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}dx$$
$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}\left(7\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(7Cd-7cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+2Bc-2Ad\right)\right)x+7\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^7}dx}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}\left(7\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(7a(Cd-cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+2Bc-2Ad\right)\right)x+7\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^7}dx}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2182$$

$$\frac{\frac{(a+bx^2)^{5/2}(7ad^2(-Bd^2-3c^2D+2cCd)+bc(-9Ad^3+2Bcd^2-12c^3D+5c^2Cd))}{6d^2(c+dx)^6(ad^2+bc^2)}-\frac{\int -\frac{\left(6\left(Ab\left(7bc^2-2ad^2\right)+a\left(7ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+2Cc-9Bd\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(6\left(Ab\left(7bc^2-2ad^2\right)+a\left(7ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+2Cc-9Bd\right)\right)\right)d^2+\left(42a^2Dd^4+7ab\left(9Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(30Dc^3+5Cdc^2+2Bd^2c-9Ad^3\right)\right)x\right)\left(bx^2+\cdots\right.}{d^2(c+dx)^6}}{6(ad^2+bc^2)}\cdots}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(6\left(Ab\left(7bc^2-2ad^2\right)+a\left(7ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+2Cc-9Bd\right)\right)\right)d^2+\left(42a^2Dd^4+7ab\left(9Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(30Dc^3+5Cdc^2+2Bd^2c-9Ad^3\right)\right)x\right)\left(bx^2+\cdots\right.}{(c+dx)^6}}{6d^2(ad^2+bc^2)}\cdots}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 679$$

$$\frac{\frac{7d^2\left(3Ab^2c\left(2bc^2-ad^2\right)-a\left(-6a^2d^3D-abd\left(-Bd^2-3c^2D+8cCd\right)+b^2c^2(cC-8Bd)\right)\right)\int\frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(42a^2d^4(Cd-3cD)-abd^2\left(12Ad^3-61Bc\ldots\right.\right.}{5(c+\ldots}}{6d^2(ad^2+bc^2)}$$

$$7\left(ad^2+\ldots\right.$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{7d^2\left(3Ab^2c\left(2bc^2-ad^2\right)-a\left(-6a^2d^3D-abd\left(-Bd^2-3c^2D+8cCd\right)+b^2c^2(cC-8Bd)\right)\right)\left(\frac{3ab\int\frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(42a^2d^4(Cd\ldots\right.}{\ldots}}{6d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{7d^2\left(3Ab^2c\left(2bc^2-ad^2\right)-a\left(-6a^2d^3D-abd\left(-Bd^2-3c^2D+8cCd\right)+b^2c^2(cC-8Bd)\right)\right)\left(\frac{3ab\left(\frac{ab\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad\ldots}{4(c+dx)^4\left(ad^2\ldots\right.}\right.}{ad^2+bc^2}}{6d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{7d^2\left(3Ab^2c\left(2bc^2-ad^2\right)-a\left(-6a^2d^3D-abd\left(-Bd^2-3c^2D+8cCd\right)+b^2c^2(cC-8Bd)\right)\right)\left(\frac{3ab\left(-\frac{ab\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\ldots\right.}{ad^2+bc^2}}{6d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{7d^2\left(\frac{3ab\left(-\frac{ab\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)\left(3Ab^2c\left(2bc^2-ad^2\right)-a\left(-6a^2d^3D-abd\left(-Bd^2-3c^2D+8c\right.\right.\right.}{ad^2+bc^2}}{6d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
-1/7*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^7) + (((7*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d + 2*B*c*d^2 - 9*A*d^3 - 12*c^3*D))*(a + b*x^2)^(5/2))/(6*d^2*(b*c^2 +
 a*d^2)*(c + d*x)^6) + (-1/5*((42*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(5*c^2*C
*d + 2*B*c*d^2 - 51*A*d^3 + 30*c^3*D) - a*b*d^2*(26*c^2*C*d - 61*B*c*d^2 +
 12*A*d^3 + 93*c^3*D))*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^5) +
(7*d^2*(3*A*b^2*c*(2*b*c^2 - a*d^2) - a*(b^2*c^2*(c*C - 8*B*d) - 6*a^2*d^3
*D - a*b*d*(8*c*C*d - B*d^2 - 3*c^2*D)))*(-1/4*((a*d - b*c*x)*(a + b*x^2)^
(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*((a*d - b*c*x)*Sqrt[a
+ b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt
[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2))))/(4*(b*c^2 +
 a*d^2))))/(b*c^2 + a*d^2))/(6*d^2*(b*c^2 + a*d^2)))/(7*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28849 vs. 2(621) = 1242.

Time = 2.05 (sec) , antiderivative size = 28850, normalized size of antiderivative = 44.45

method	result	size
default	Expression too large to display	28850

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22855 vs. 2(617) = 1234.

Time = 0.95 (sec) , antiderivative size = 22855, normalized size of antiderivative = 35.22

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="maxi
ma")
```

output
```
-9/16*sqrt(b*x^2 + a)*D*b^7*c^10/(b^6*c^12*d^6 + 6*a*b^5*c^10*d^8 + 15*a^2
*b^4*c^8*d^10 + 20*a^3*b^3*c^6*d^12 + 15*a^4*b^2*c^4*d^14 + 6*a^5*b*c^2*d^
16 + a^6*d^18) + 9/16*sqrt(b*x^2 + a)*D*b^7*c^9*x/(b^6*c^12*d^5 + 6*a*b^5*
c^10*d^7 + 15*a^2*b^4*c^8*d^9 + 20*a^3*b^3*c^6*d^11 + 15*a^4*b^2*c^4*d^13
+ 6*a^5*b*c^2*d^15 + a^6*d^17) - 3/16*(b*x^2 + a)^(3/2)*D*b^6*c^9/(b^6*c^1
2*d^5*x + 6*a*b^5*c^10*d^7*x + 15*a^2*b^4*c^8*d^9*x + 20*a^3*b^3*c^6*d^11*
x + 15*a^4*b^2*c^4*d^13*x + 6*a^5*b*c^2*d^15*x + a^6*d^17*x + b^6*c^13*d^4
 + 6*a*b^5*c^11*d^6 + 15*a^2*b^4*c^9*d^8 + 20*a^3*b^3*c^7*d^10 + 15*a^4*b^
2*c^5*d^12 + 6*a^5*b*c^3*d^14 + a^6*c*d^16) + 9/16*sqrt(b*x^2 + a)*C*b^7*c
^9/(b^6*c^12*d^5 + 6*a*b^5*c^10*d^7 + 15*a^2*b^4*c^8*d^9 + 20*a^3*b^3*c^6*
d^11 + 15*a^4*b^2*c^4*d^13 + 6*a^5*b*c^2*d^15 + a^6*d^17) - 9/16*sqrt(b*x^
2 + a)*C*b^7*c^8*x/(b^6*c^12*d^4 + 6*a*b^5*c^10*d^6 + 15*a^2*b^4*c^8*d^8 +
 20*a^3*b^3*c^6*d^10 + 15*a^4*b^2*c^4*d^12 + 6*a^5*b*c^2*d^14 + a^6*d^16)
+ 3/16*(b*x^2 + a)^(5/2)*D*b^5*c^8/(b^6*c^12*d^4*x^2 + 6*a*b^5*c^10*d^6*x^
2 + 15*a^2*b^4*c^8*d^8*x^2 + 20*a^3*b^3*c^6*d^10*x^2 + 15*a^4*b^2*c^4*d^12
*x^2 + 6*a^5*b*c^2*d^14*x^2 + a^6*d^16*x^2 + 2*b^6*c^13*d^3*x + 12*a*b^5*c
^11*d^5*x + 30*a^2*b^4*c^9*d^7*x + 40*a^3*b^3*c^7*d^9*x + 30*a^4*b^2*c^5*d
^11*x + 12*a^5*b*c^3*d^13*x + 2*a^6*c*d^15*x + b^6*c^14*d^2 + 6*a*b^5*c^12
*d^4 + 15*a^2*b^4*c^10*d^6 + 20*a^3*b^3*c^8*d^8 + 15*a^4*b^2*c^6*d^10 + 6*
a^5*b*c^4*d^12 + a^6*c^2*d^14) + 3/16*(b*x^2 + a)^(3/2)*C*b^6*c^8/(b^6*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11003 vs. 2(617) = 1234.

Time = 0.63 (sec) , antiderivative size = 11003, normalized size of antiderivative = 16.95

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="giac
")
```

output
```
-1/8*(C*a^3*b^4*c^3 - 6*A*a^2*b^5*c^3 + 3*D*a^4*b^3*c^2*d - 8*B*a^3*b^4*c^
2*d - 8*C*a^4*b^3*c*d^2 + 3*A*a^3*b^4*c*d^2 - 6*D*a^5*b^2*d^3 + B*a^4*b^3*
d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a
*d^2))/((b^5*c^10 + 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^6*d^4 + 10*a^3*b^2*c^4*
d^6 + 5*a^4*b*c^2*d^8 + a^5*d^10)*sqrt(-b*c^2 - a*d^2)) + 1/840*(1680*(sqr
t(b)*x - sqrt(b*x^2 + a))^13*D*b^7*c^10*d^6 + 8400*(sqrt(b)*x - sqrt(b*x^2
 + a))^13*D*a*b^6*c^8*d^8 + 16800*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^2*b
^5*c^6*d^10 + 16800*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^3*b^4*c^4*d^12 +
105*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^3*b^4*c^3*d^13 - 630*(sqrt(b)*x -
 sqrt(b*x^2 + a))^13*A*a^2*b^5*c^3*d^13 + 8715*(sqrt(b)*x - sqrt(b*x^2 + a
))^13*D*a^4*b^3*c^2*d^14 - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^3*b^4*
c^2*d^14 - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^4*b^3*c*d^15 + 315*(sq
rt(b)*x - sqrt(b*x^2 + a))^13*A*a^3*b^4*c*d^15 + 1050*(sqrt(b)*x - sqrt(b*
x^2 + a))^13*D*a^5*b^2*d^16 + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^4*b
^3*d^16 + 10080*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*b^(15/2)*c^11*d^5 + 168
0*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*b^(15/2)*c^10*d^6 + 50400*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*D*a*b^(13/2)*c^9*d^7 + 8400*(sqrt(b)*x - sqrt(b*x^2
+ a))^12*C*a*b^(13/2)*c^8*d^8 + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*
a^2*b^(11/2)*c^7*d^9 + 16800*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^2*b^(11/
2)*c^6*d^10 + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*b^(9/2)*c^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^8}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^8}dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

3.83 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 856

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx =$$

$$-\frac{ab^2(3Ab(16b^2c^4-16abc^2d^2+a^2d^4)-a(8b^2c^3(cC-9Bd)+8a^2d^3(Cd-9cD)-abcd(83cCd-27Bd^2-}{128\left(bc^2+ad^2\right)^6(c+dx)^2}$$

$$-\frac{b(3Ab(16b^2c^4-16abc^2d^2+a^2d^4)-a(8b^2c^3(cC-9Bd)+8a^2d^3(Cd-9cD)-abcd(83cCd-27Bd^2-2}{192\left(bc^2+ad^2\right)^5(c+dx)^4}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\left(a+bx^2\right)^{5/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$+\frac{(8ad^2(2cCd-Bd^2-3c^2D)+bc(5c^2Cd+3Bcd^2-11Ad^3-13c^3D))\left(a+bx^2\right)^{5/2}}{56d^2\left(bc^2+ad^2\right)^2(c+dx)^7}$$

$$-\frac{(56a^2d^4(Cd-3cD)-2b^2c^2(5c^2Cd+3Bcd^2-39Ad^3+15c^3D)-abd^2(53c^2Cd-93Bcd^2+21Ad^3+99}{336d^2\left(bc^2+ad^2\right)^3(c+dx)^6}$$

$$-\frac{(336a^3d^6D+8a^2bd^4(73cCd-12Bd^2-57c^2D)-2b^3c^3(5c^2Cd+3Bcd^2-207Ad^3+15c^3D)-ab^2cd^2(1}{1680d^2\left(bc^2+ad^2\right)^4(c+dx)^5}$$

$$-\frac{a^2b^3(3Ab(16b^2c^4-16abc^2d^2+a^2d^4)-a(8b^2c^3(cC-9Bd)+8a^2d^3(Cd-9cD)-abcd(83cCd-27Bd^2}{128\left(bc^2+ad^2\right)^{13/2}}$$

output
```
-1/128*a*b^2*(3*A*b*(a^2*d^4-16*a*b*c^2*d^2+16*b^2*c^4)-a*(8*b^2*c^3*(-9*B
*d+C*c)+8*a^2*d^3*(C*d-9*D*c)-a*b*c*d*(-27*B*d^2+83*C*c*d-27*D*c^2)))*(-b*
c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^6/(d*x+c)^2-1/192*b*(3*A*b*(a^2*d^4
-16*a*b*c^2*d^2+16*b^2*c^4)-a*(8*b^2*c^3*(-9*B*d+C*c)+8*a^2*d^3*(C*d-9*D*c
)-a*b*c*d*(-27*B*d^2+83*C*c*d-27*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(3/2)/(a*
d^2+b*c^2)^5/(d*x+c)^4-1/8*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(5/2)/d
^2/(a*d^2+b*c^2)/(d*x+c)^8+1/56*(8*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-11
*A*d^3+3*B*c*d^2+5*C*c^2*d-13*D*c^3))*(b*x^2+a)^(5/2)/d^2/(a*d^2+b*c^2)^2/
(d*x+c)^7-1/336*(56*a^2*d^4*(C*d-3*D*c)-2*b^2*c^2*(-39*A*d^3+3*B*c*d^2+5*C
*c^2*d+15*D*c^3)-a*b*d^2*(21*A*d^3-93*B*c*d^2+53*C*c^2*d+99*D*c^3))*(b*x^2
+a)^(5/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^6-1/1680*(336*a^3*d^6*D+8*a^2*b*d^4*
(-12*B*d^2+73*C*c*d-57*D*c^2)-2*b^3*c^3*(-207*A*d^3+3*B*c*d^2+5*C*c^2*d+15
*D*c^3)-a*b^2*c*d^2*(279*A*d^3-591*B*c*d^2+119*C*c^2*d+129*D*c^3))*(b*x^2+
a)^(5/2)/d^2/(a*d^2+b*c^2)^4/(d*x+c)^5-1/128*a^2*b^3*(3*A*b*(a^2*d^4-16*a*
b*c^2*d^2+16*b^2*c^4)-a*(8*b^2*c^3*(-9*B*d+C*c)+8*a^2*d^3*(C*d-9*D*c)-a*b*
c*d*(-27*B*d^2+83*C*c*d-27*D*c^2)))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/
2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(13/2)
```

Mathematica [A] (verified)

Time = 16.27 (sec) , antiderivative size = 1375, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^9,x]
```

output
```
(-((Sqrt[a + b*x^2]*(1680*(b*c^2 + a*d^2)^7*(c^2*C*d - B*c*d^2 + A*d^3 - c
^3*D) + 240*(b*c^2 + a*d^2)^6*(8*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*
(-33*c^2*C*d + 25*B*c*d^2 - 17*A*d^3 + 41*c^3*D))*(c + d*x) + 40*(b*c^2 +
a*d^2)^5*(56*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(415*c^2*C*d - 183*B*c*d^2 +
63*A*d^3 - 759*c^3*D) + 2*b^2*c^2*(181*c^2*C*d - 93*B*c*d^2 + 33*A*d^3 - 2
97*c^3*D))*(c + d*x)^2 + 8*(b*c^2 + a*d^2)^4*(336*a^3*d^6*D + 8*a^2*b*d^4*
(-187*c*C*d + 48*B*d^2 + 543*c^2*D) + 2*b^3*c^3*(-775*c^2*C*d + 207*B*c*d^
2 - 3*A*d^3 + 1875*c^3*D) + a*b^2*c*d^2*(-3079*c^2*C*d + 831*B*c*d^2 - 39*
A*d^3 + 7791*c^3*D))*(c + d*x)^3 - 2*b*(b*c^2 + a*d^2)^3*(56*a^3*d^6*(-35*
C*d + 171*c*D) + 24*a*b^2*c^2*d^2*(-276*c^2*C*d + 4*B*c*d^2 + 9*A*d^3 + 12
69*c^3*D) + 8*b^3*c^4*(-275*c^2*C*d + 3*B*c*d^2 + 3*A*d^3 + 1275*c^3*D) +
3*a^2*b*d^4*(-2227*c^2*C*d + 123*B*c*d^2 - 35*A*d^3 + 10043*c^3*D))*(c + d
*x)^4 - 2*b*(b*c^2 + a*d^2)^2*(-2688*a^4*d^8*D - 24*a^3*b*d^6*(-37*c*C*d +
 8*B*d^2 + 549*c^2*D) + a^2*b^2*c*d^4*(331*c^2*C*d + 621*B*c*d^2 - 453*A*d
^3 - 19539*c^3*D) - 8*b^4*c^5*(-5*c^2*C*d - 3*B*c*d^2 - 3*A*d^3 + 405*c^3*
D) - 8*a*b^3*c^3*d^2*(-22*c^2*C*d - 18*B*c*d^2 - 33*A*d^3 + 1623*c^3*D))*(
c + d*x)^5 - b^2*(b*c^2 + a*d^2)*(168*a^4*d^8*(-5*C*d + 29*c*D) + 16*b^4*c
^6*(5*c^2*C*d + 3*B*c*d^2 + 3*A*d^3 + 15*c^3*D) + 8*a*b^3*c^4*d^2*(59*c^2*
C*d + 45*B*c*d^2 + 75*A*d^3 + 159*c^3*D) + 3*a^3*b*d^6*(1161*c^2*C*d - 689
*B*c*d^2 + 105*A*d^3 + 1055*c^3*D) + 2*a^2*b^2*c^2*d^4*(625*c^2*C*d + 8...
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 756, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2182, 25, 2182, 25, 27, 688, 25, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(8\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(8Cd-8cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+3Bc-3Ad\right)\right)x+8\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^8}dx}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(8\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(8a(Cd-cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+3Bc-3Ad\right)\right)x+8\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^8}dx}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{5/2}\left(8ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-11Ad^3+3Bcd^2-13c^3D+5c^2Cd\right)\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(7\left(Ab\left(8bc^2-3ad^2\right)+a\left(8ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+3Cc-11\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\ldots\frac{\int \frac{\left(7\left(Ab\left(8bc^2-3ad^2\right)+a\left(8ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+3Cc-11Bd\right)\right)\right)d^2+2\left(28a^2Dd^4+8ab\left(4Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(15Dc^3+5Cdc^2+3Bd^2c-11Ad^3\right)\right)x\right)\left(b\ldots\right.}{d^2(c+dx)^7}}{7\left(ad^2+bc^2\right)}\ldots}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\ldots\frac{\int \frac{\left(7\left(Ab\left(8bc^2-3ad^2\right)+a\left(8ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+3Cc-11Bd\right)\right)\right)d^2+2\left(28a^2Dd^4+8ab\left(4Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(15Dc^3+5Cdc^2+3Bd^2c-11Ad^3\right)\right)x\right)\left(b\ldots\right.}{(c+dx)^7}}{7d^2\left(ad^2+bc^2\right)}\ldots}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$-\frac{\int -\frac{\left(6d\left(Acd\left(56bc^2-43ad^2\right)b^2+a\left(56a^2Dd^4+8ab\left(-6Dc^2+11Cdc-2Bd^2\right)d^2-b^2c^2\left(5Dc^2+11Cdc-83Bd^2\right)\right)\right)-b\left(56a^2(Cd-3cD)d^4-ab\left(99Dc^3+53Cdc^2-93\cdots\right.\right.\right.}{(c+dx)^6}\cdots}{6\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(6d\left(Acd\left(56bc^2-43ad^2\right)b^2+a\left(56a^2Dd^4+8ab\left(-6Dc^2+11Cdc-2Bd^2\right)d^2-b^2c^2\left(5Dc^2+11Cdc-83Bd^2\right)\right)\right)-b\left(56a^2(Cd-3cD)d^4-ab\left(99Dc^3+53Cdc^2-93Bd^2\cdots\right.\right.\right.}{(c+dx)^6}\cdots}{6\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{7bd^2\left(3Ab\left(a^2d^4-16abc^2d^2+16b^2c^4\right)-a\left(8a^2d^3(Cd-9cD)-abcd\left(-27Bd^2-27c^2D+83cCd\right)+8b^2c^3(cC-9Bd)\right)\right)\int \frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(336a^3d^6D+8\cdots\right.}{\cdots}}{6\left(ad^2+bc^2\right)}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{7bd^2\left(3Ab\left(a^2d^4-16abc^2d^2+16b^2c^4\right)-a\left(8a^2d^3(Cd-9cD)-abcd\left(-27Bd^2-27c^2D+83cCd\right)+8b^2c^3(cC-9Bd)\right)\right)\left(\frac{3ab\int \frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}\cdots}{6\left(ad^2+\cdots\right.}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{7bd^2\left(3Ab\left(a^2d^4-16abc^2d^2+16b^2c^4\right)-a\left(8a^2d^3(Cd-9cD)-abcd\left(-27Bd^2-27c^2D+83cCd\right)+8b^2c^3(cC-9Bd)\right)\right)\left(\frac{3ab\left(\frac{ab\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad}{2(c+dx)^2\left(ad^2\right.}\right.}{4\left(ad^2+bc^2\right)}\right.}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{7bd^2\left(3Ab\left(a^2d^4-16abc^2d^2+16b^2c^4\right)-a\left(8a^2d^3(Cd-9cD)-abcd\left(-27Bd^2-27c^2D+83cCd\right)+8b^2c^3(cC-9Bd)\right)\right)\left(\frac{3ab\left(-\frac{ab\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}\right.}{4\left(ad^2+bc^2\right)}\right.}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{7bd^2\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)\left(3Ab\left(a^2d^4-16abc^2d^2+16b^2c^4\right)-a\left(8a^2d^3(Cd-9cD)-\right.\right.}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^9,x]
```

output
```
-1/8*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^8) + (((8*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d + 3*B*c*d^2 - 11*A*d^3 - 13*c^3*D))*(a + b*x^2)^(5/2))/(7*d^2*(b*c^2
+ a*d^2)*(c + d*x)^7) + (-1/6*((56*a^2*d^4*(C*d - 3*c*D) - 2*b^2*c^2*(5*c^
2*C*d + 3*B*c*d^2 - 39*A*d^3 + 15*c^3*D) - a*b*d^2*(53*c^2*C*d - 93*B*c*d^
2 + 21*A*d^3 + 99*c^3*D))*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6)
 + (-1/5*((336*a^3*d^6*D + 8*a^2*b*d^4*(73*c*C*d - 12*B*d^2 - 57*c^2*D) -
2*b^3*c^3*(5*c^2*C*d + 3*B*c*d^2 - 207*A*d^3 + 15*c^3*D) - a*b^2*c*d^2*(11
9*c^2*C*d - 591*B*c*d^2 + 279*A*d^3 + 129*c^3*D))*(a + b*x^2)^(5/2))/((b*c
^2 + a*d^2)*(c + d*x)^5) + (7*b*d^2*(3*A*b*(16*b^2*c^4 - 16*a*b*c^2*d^2 +
a^2*d^4) - a*(8*b^2*c^3*(c*C - 9*B*d) + 8*a^2*d^3*(C*d - 9*c*D) - a*b*c*d*
(83*c*C*d - 27*B*d^2 - 27*c^2*D)))*(-1/4*((a*d - b*c*x)*(a + b*x^2)^(3/2))
/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^
2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2
 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2))))/(4*(b*c^2 + a*d^2
))))/(b*c^2 + a*d^2))/(6*(b*c^2 + a*d^2)))/(7*d^2*(b*c^2 + a*d^2)))/(8*(b*
c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45605 vs. $2(824) = 1648$.

Time = 2.16 (sec) , antiderivative size = 45606, normalized size of antiderivative = 53.28

method	result	size
default	Expression too large to display	45606

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**9,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33308 vs. $2(824) = 1648$.

Time = 1.40 (sec) , antiderivative size = 33308, normalized size of antiderivative = 38.91

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="maxi
ma")
```

output
```
-99/128*sqrt(b*x^2 + a)*D*b^8*c^11/(b^7*c^14*d^6 + 7*a*b^6*c^12*d^8 + 21*a
^2*b^5*c^10*d^10 + 35*a^3*b^4*c^8*d^12 + 35*a^4*b^3*c^6*d^14 + 21*a^5*b^2*
c^4*d^16 + 7*a^6*b*c^2*d^18 + a^7*d^20) + 99/128*sqrt(b*x^2 + a)*D*b^8*c^1
0*x/(b^7*c^14*d^5 + 7*a*b^6*c^12*d^7 + 21*a^2*b^5*c^10*d^9 + 35*a^3*b^4*c^
8*d^11 + 35*a^4*b^3*c^6*d^13 + 21*a^5*b^2*c^4*d^15 + 7*a^6*b*c^2*d^17 + a^
7*d^19) - 33/128*(b*x^2 + a)^(3/2)*D*b^7*c^10/(b^7*c^14*d^5*x + 7*a*b^6*c^
12*d^7*x + 21*a^2*b^5*c^10*d^9*x + 35*a^3*b^4*c^8*d^11*x + 35*a^4*b^3*c^6*
d^13*x + 21*a^5*b^2*c^4*d^15*x + 7*a^6*b*c^2*d^17*x + a^7*d^19*x + b^7*c^1
5*d^4 + 7*a*b^6*c^13*d^6 + 21*a^2*b^5*c^11*d^8 + 35*a^3*b^4*c^9*d^10 + 35*
a^4*b^3*c^7*d^12 + 21*a^5*b^2*c^5*d^14 + 7*a^6*b*c^3*d^16 + a^7*c*d^18) +
99/128*sqrt(b*x^2 + a)*C*b^8*c^10/(b^7*c^14*d^5 + 7*a*b^6*c^12*d^7 + 21*a^
2*b^5*c^10*d^9 + 35*a^3*b^4*c^8*d^11 + 35*a^4*b^3*c^6*d^13 + 21*a^5*b^2*c^
4*d^15 + 7*a^6*b*c^2*d^17 + a^7*d^19) - 99/128*sqrt(b*x^2 + a)*C*b^8*c^9*x
/(b^7*c^14*d^4 + 7*a*b^6*c^12*d^6 + 21*a^2*b^5*c^10*d^8 + 35*a^3*b^4*c^8*d
^10 + 35*a^4*b^3*c^6*d^12 + 21*a^5*b^2*c^4*d^14 + 7*a^6*b*c^2*d^16 + a^7*d
^18) + 33/128*(b*x^2 + a)^(5/2)*D*b^6*c^9/(b^7*c^14*d^4*x^2 + 7*a*b^6*c^12
*d^6*x^2 + 21*a^2*b^5*c^10*d^8*x^2 + 35*a^3*b^4*c^8*d^10*x^2 + 35*a^4*b^3*
c^6*d^12*x^2 + 21*a^5*b^2*c^4*d^14*x^2 + 7*a^6*b*c^2*d^16*x^2 + a^7*d^18*x
^2 + 2*b^7*c^15*d^3*x + 14*a*b^6*c^13*d^5*x + 42*a^2*b^5*c^11*d^7*x + 70*a
^3*b^4*c^9*d^9*x + 70*a^4*b^3*c^7*d^11*x + 42*a^5*b^2*c^5*d^13*x + 14*a...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13955 vs. $2(824) = 1648$.

Time = 0.78 (sec) , antiderivative size = 13955, normalized size of antiderivative = 16.30

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input

```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="giac
")
```

output

```
1/64*(8*C*a^3*b^5*c^4 - 48*A*a^2*b^6*c^4 + 27*D*a^4*b^4*c^3*d - 72*B*a^3*b
^5*c^3*d - 83*C*a^4*b^4*c^2*d^2 + 48*A*a^3*b^5*c^2*d^2 - 72*D*a^5*b^3*c*d^
3 + 27*B*a^4*b^4*c*d^3 + 8*C*a^5*b^3*d^4 - 3*A*a^4*b^4*d^4)*arctan(((sqrt(
b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^6*c^12 +
6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^8*d^4 + 20*a^3*b^3*c^6*d^6 + 15*a^4*b^2*c^
4*d^8 + 6*a^5*b*c^2*d^10 + a^6*d^12)*sqrt(-b*c^2 - a*d^2)) + 1/6720*(840*(
sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^3*b^5*c^4*d^14 - 5040*(sqrt(b)*x - sqr
t(b*x^2 + a))^15*A*a^2*b^6*c^4*d^14 + 2835*(sqrt(b)*x - sqrt(b*x^2 + a))^1
5*D*a^4*b^4*c^3*d^15 - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^15*B*a^3*b^5*c^3
*d^15 - 8715*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^4*b^4*c^2*d^16 + 5040*(s
qrt(b)*x - sqrt(b*x^2 + a))^15*A*a^3*b^5*c^2*d^16 - 7560*(sqrt(b)*x - sqrt
(b*x^2 + a))^15*D*a^5*b^3*c*d^17 + 2835*(sqrt(b)*x - sqrt(b*x^2 + a))^15*B
*a^4*b^4*c*d^17 + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^5*b^3*d^18 - 31
5*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a^4*b^4*d^18 + 13440*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*D*b^(17/2)*c^12*d^6 + 80640*(sqrt(b)*x - sqrt(b*x^2 + a))
^14*D*a*b^(15/2)*c^10*d^8 + 201600*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^2*
b^(13/2)*c^8*d^10 + 268800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^3*b^(11/2)
*c^6*d^12 + 12600*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^3*b^(11/2)*c^5*d^13
 - 75600*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a^2*b^(13/2)*c^5*d^13 + 244125
*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(9/2)*c^4*d^14 - 113400*(sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^9}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^9,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^9, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^9}dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x)
```

3.84 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}} dx$

Optimal result

Integrand size = 34, antiderivative size = 1103

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Too large to display}$$

output
```
-1/128*a*b^2*(A*b^2*c*(15*a^2*d^4-80*a*b*c^2*d^2+48*b^2*c^4)-a*(8*b^3*c^4*
(-10*B*d+C*c)+8*a^3*d^5*D+3*a^2*b*d^3*(-B*d^2+10*C*c*d-35*D*c^2)-15*a*b^2*
c^2*d*(-4*B*d^2+7*C*c*d-2*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c
^2)^7/(d*x+c)^2-1/192*b*(A*b^2*c*(15*a^2*d^4-80*a*b*c^2*d^2+48*b^2*c^4)-a*
(8*b^3*c^4*(-10*B*d+C*c)+8*a^3*d^5*D+3*a^2*b*d^3*(-B*d^2+10*C*c*d-35*D*c^2
)-15*a*b^2*c^2*d*(-4*B*d^2+7*C*c*d-2*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(3/2)
/(a*d^2+b*c^2)^6/(d*x+c)^4-1/9*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(5/
2)/d^2/(a*d^2+b*c^2)/(d*x+c)^9+1/72*(9*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*
(-13*A*d^3+4*B*c*d^2+5*C*c^2*d-14*D*c^3))*(b*x^2+a)^(5/2)/d^2/(a*d^2+b*c^2
)^2/(d*x+c)^8-1/504*(72*a^2*d^4*(C*d-3*D*c)-3*b^2*c^2*(-37*A*d^3+4*B*c*d^2
+5*C*c^2*d+10*D*c^3)-a*b*d^2*(32*A*d^3-131*B*c*d^2+86*C*c^2*d+103*D*c^3))*
(b*x^2+a)^(5/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^7-1/1008*(168*a^3*d^6*D+9*a^2*
b*d^4*(-7*B*d^2+38*C*c*d-37*D*c^2)-2*b^3*c^3*(-121*A*d^3+4*B*c*d^2+5*C*c^2
*d+10*D*c^3)-a*b^2*c*d^2*(187*A*d^3-358*B*c*d^2+97*C*c^2*d+92*D*c^3))*(b*x
^2+a)^(5/2)/d^2/(a*d^2+b*c^2)^4/(d*x+c)^6+1/5040*b*(24*a^3*d^6*(12*C*d-85*
D*c)-a^2*b*d^4*(128*A*d^3-965*B*c*d^2+2450*C*c^2*d-1055*D*c^3)+2*b^3*c^4*(
-625*A*d^3+4*B*c*d^2+5*C*c^2*d+10*D*c^3)+a*b^2*c^2*d^2*(1625*A*d^3-2030*B*
c*d^2+275*C*c^2*d+112*D*c^3))*(b*x^2+a)^(5/2)/d^2/(a*d^2+b*c^2)^5/(d*x+c)^
5-1/128*a^2*b^3*(A*b^2*c*(15*a^2*d^4-80*a*b*c^2*d^2+48*b^2*c^4)-a*(8*b^3*c
^4*(-10*B*d+C*c)+8*a^3*d^5*D+3*a^2*b*d^3*(-B*d^2+10*C*c*d-35*D*c^2)-15*...
```

Mathematica [A] (verified)

Time = 17.40 (sec) , antiderivative size = 2075, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^10,x]
```

output
```
Sqrt[a + b*x^2]*(-1/9*((b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)
)/(d^6*(c + d*x)^9) + (37*b*c^3*C*d - 28*b*B*c^2*d^2 + 19*A*b*c*d^3 + 18*a
*c*C*d^3 - 9*a*B*d^4 - 46*b*c^4*D - 27*a*c^2*d^2*D)/(72*d^6*(c + d*x)^8) +
 (-461*b^2*c^4*C*d + 236*b^2*B*c^3*d^2 - 83*A*b^2*c^2*d^3 - 530*a*b*c^2*C*
d^3 + 233*a*b*B*c*d^4 - 80*a*A*b*d^5 - 72*a^2*C*d^5 + 758*b^2*c^5*D + 971*
a*b*c^3*d^2*D + 216*a^2*c*d^4*D)/(504*d^6*(b*c^2 + a*d^2)*(c + d*x)^7) + (
758*b^3*c^5*C*d - 200*b^3*B*c^4*d^2 + 2*A*b^3*c^3*d^3 + 1509*a*b^2*c^3*C*d
^3 - 402*a*b^2*B*c^2*d^4 + 15*a*A*b^2*c*d^5 + 738*a^2*b*c*C*d^5 - 189*a^2*
b*B*d^6 - 1844*b^3*c^6*D - 3840*a*b^2*c^4*d^2*D - 2151*a^2*b*c^2*d^4*D - 1
68*a^3*d^6*D)/(1008*d^6*(b*c^2 + a*d^2)^2*(c + d*x)^6) - (b*(1250*b^3*c^6*
C*d - 8*b^3*B*c^5*d^2 - 10*A*b^3*c^4*d^3 + 3765*a*b^2*c^4*C*d^3 - 30*a*b^2
*B*c^3*d^4 - 105*a*A*b^2*c^2*d^5 + 3810*a^2*b*c^2*C*d^5 - 165*a^2*b*B*c*d^
6 + 48*a^2*A*b*d^7 + 1152*a^3*C*d^7 - 5900*b^3*c^7*D - 17652*a*b^2*c^5*d^2
*D - 17535*a^2*b*c^3*d^4*D - 5640*a^3*c*d^6*D))/(5040*d^6*(b*c^2 + a*d^2)^
3*(c + d*x)^5) - (b*(-40*b^4*c^7*C*d - 32*b^4*B*c^6*d^2 - 40*A*b^4*c^5*d^3
 - 160*a*b^3*c^5*C*d^3 - 200*a*b^3*B*c^4*d^4 - 520*a*A*b^3*c^3*d^5 - 255*a
^2*b^2*c^3*C*d^5 - 1140*a^2*b^2*B*c^2*d^6 + 807*a^2*A*b^2*c*d^7 - 1422*a^3
*b*c*C*d^7 + 315*a^3*b*B*d^8 + 6640*b^4*c^8*D + 26632*a*b^3*c^6*d^2*D + 40
170*a^2*b^2*c^4*d^4*D + 27345*a^3*b*c^2*d^6*D + 5880*a^4*d^8*D))/(20160*d^
6*(b*c^2 + a*d^2)^4*(c + d*x)^4) + (b^2*(40*b^4*c^8*C*d + 32*b^4*B*c^7*...
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 963, normalized size of antiderivative = 0.87, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2182, 25, 2182, 25, 27, 688, 25, 688, 27, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(9\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(9Cd-9cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x+9\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^9}dx}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(9\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(9a(Cd-cD)+b\left(-\frac{5Dc^3}{d^2}+\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x+9\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^9}dx}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{5/2}\left(9ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-13Ad^3+4Bcd^2-14c^3D+5c^2Cd\right)\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(8\left(Ab\left(9bc^2-4ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+4Cc-13\cdots\right)\right)\right)\right)}{\cdots}}{\cdots}}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(8\left(Ab\left(9bc^2-4ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+4Cc-13Bd\right)\right)\right)\right)d^2+3\left(24a^2Dd^4+3ab\left(7Dc^2+6Cdc-3Bd^2\right)d^2+b^2c\left(10Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)x\cdots}{d^2(c+dx)^8}}{8\left(ad^2+bc^2\right)}\cdots}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(8\left(Ab\left(9bc^2-4ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{5Dc^2}{d}+4Cc-13Bd\right)\right)\right)\right)d^2+3\left(24a^2Dd^4+3ab\left(7Dc^2+6Cdc-3Bd^2\right)d^2+b^2c\left(10Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)x\cdots}{(c+dx)^8}}{8d^2\left(ad^2+bc^2\right)}\cdots}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$-\frac{\frac{\int -\frac{(7d(Acd(72bc^2-71ad^2)b^2+a(72a^2Dd^4+9ab(-9Dc^2+14Cdc-3Bd^2)d^2-b^2c^2(10Dc^2+17Cdc-116Bd^2)))-2b(72a^2(Cd-3cD)d^4-ab(103Dc^3+86Cdc^2}{(c+dx)^7}}{7(ad^2+bc^2)}}{}$$

$$\frac{(a+bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(7d(Acd(72bc^2-71ad^2)b^2+a(72a^2Dd^4+9ab(-9Dc^2+14Cdc-3Bd^2)d^2-b^2c^2(10Dc^2+17Cdc-116Bd^2)))-2b(72a^2(Cd-3cD)d^4-ab(103Dc^3+86Cdc^2-13}{(c+dx)^7}}{7(ad^2+bc^2)}}{}$$

$$\frac{(a+bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$\frac{\frac{-\frac{\int -\frac{3b(2d(Abd(504b^2c^4-719abd^2c^2+64a^2d^4)-a(72a^2(2Cd-13cD)d^4-abc(-361Dc^2+1054Cdc-451Bd^2)d^2+b^2c^3(10Dc^2+89Cdc-836Bd^2)))-(168a^3Dd}{(c+dx)^6}}{6(ad^2+bc^2)}}{}}{}$$

$$\frac{(a+bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{b\int \frac{(2d(Abd(504b^2c^4-719abd^2c^2+64a^2d^4)-a(72a^2(2Cd-13cD)d^4-abc(-361Dc^2+1054Cdc-451Bd^2)d^2+b^2c^3(10Dc^2+89Cdc-836Bd^2)))-(168a^3Dd^6+9}{(c+dx)^6}}{2(ad^2+bc^2)}}{}}{}$$

$$\frac{(a+bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$b\left(\frac{\left(24a^3(12Cd-85cD)d^6-a^2b\left(-1055Dc^3+2450Cdc^2-965Bd^2c\right.\right.}{}\right.$$

$$\frac{\frac{\left(9a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-14Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{8d^2(bc^2+ad^2)(c+dx)^8}+\frac{}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}$$

$$\Big\downarrow \text{486}$$

$$b\left(\frac{\left(24a^3(12Cd-85cD)d^6-a^2b\left(-1055Dc^3+2450Cdc^2-965Bd^2c\right.\right.}{}\right.$$

$$\frac{\frac{\left(9a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-14Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{8d^2(bc^2+ad^2)(c+dx)^8}+\frac{}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}$$

$$\Big\downarrow \text{486}$$

$$b\left(\frac{\left(24a^3(12Cd-85cD)d^6-a^2b\left(-1055Dc^3+2450Cdc^2-965Bd^2c\right.\right.}{}\right.$$

$$\frac{\frac{\left(9a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-14Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{8d^2(bc^2+ad^2)(c+dx)^8}+\frac{}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}$$

$$\Big\downarrow \text{488}$$

$$b\left(\frac{\left(24a^3(12Cd-85cD)d^6-a^2b\left(-1055Dc^3+2450Cdc^2-965Bd^2c\right.\right.}{}\right.$$

$$\frac{\left(9a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-14Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}+$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{\left(24a^3(12Cd-85cD)d^6-a^2b\left(-1055Dc^3+2450Cdc^2-965Bd^2c\right.\right.}{}\right.$$

$$\frac{\left(9a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-14Dc^3+5Cdc^2+4Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{5/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}+$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{5/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^10,x]
```

output
```
-1/9*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^9) + (((9*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(5*c^2
*C*d + 4*B*c*d^2 - 13*A*d^3 - 14*c^3*D))*(a + b*x^2)^(5/2))/(8*d^2*(b*c^2
+ a*d^2)*(c + d*x)^8) + (-1/7*((72*a^2*d^4*(C*d - 3*c*D) - 3*b^2*c^2*(5*c^
2*C*d + 4*B*c*d^2 - 37*A*d^3 + 10*c^3*D) - a*b*d^2*(86*c^2*C*d - 131*B*c*d
^2 + 32*A*d^3 + 103*c^3*D))*(a + b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^
7) + (-1/2*((168*a^3*d^6*D + 9*a^2*b*d^4*(38*c*C*d - 7*B*d^2 - 37*c^2*D) -
 2*b^3*c^3*(5*c^2*C*d + 4*B*c*d^2 - 121*A*d^3 + 10*c^3*D) - a*b^2*c*d^2*(9
7*c^2*C*d - 358*B*c*d^2 + 187*A*d^3 + 92*c^3*D))*(a + b*x^2)^(5/2))/((b*c^
2 + a*d^2)*(c + d*x)^6) + (b*(((24*a^3*d^6*(12*C*d - 85*c*D) - a^2*b*d^4*(
2450*c^2*C*d - 965*B*c*d^2 + 128*A*d^3 - 1055*c^3*D) + 2*b^3*c^4*(5*c^2*C*
d + 4*B*c*d^2 - 625*A*d^3 + 10*c^3*D) + a*b^2*c^2*d^2*(275*c^2*C*d - 2030*
B*c*d^2 + 1625*A*d^3 + 112*c^3*D))*(a + b*x^2)^(5/2))/(5*(b*c^2 + a*d^2)*(
c + d*x)^5) + (21*d^2*(A*b^2*c*(48*b^2*c^4 - 80*a*b*c^2*d^2 + 15*a^2*d^4)
- a*(8*b^3*c^4*(c*C - 10*B*d) + 8*a^3*d^5*D + 3*a^2*b*d^3*(10*c*C*d - B*d^
2 - 35*c^2*D) - 15*a*b^2*c^2*d*(7*c*C*d - 4*B*d^2 - 2*c^2*D)))*(-1/4*((a*d
 - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*
((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2
)^(3/2))))/(4*(b*c^2 + a*d^2))))/(b*c^2 + a*d^2)))/(2*(b*c^2 + a*d^2)))...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74818 vs. 2(1067) = 2134.

Time = 2.34 (sec) , antiderivative size = 74819, normalized size of antiderivative = 67.83

method	result	size
default	Expression too large to display	74819

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46712 vs. 2(1065) = 2130.

Time = 1.88 (sec) , antiderivative size = 46712, normalized size of antiderivative = 42.35

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="max
ima")
```

output
```
-143/128*sqrt(b*x^2 + a)*D*b^9*c^12/(b^8*c^16*d^6 + 8*a*b^7*c^14*d^8 + 28*
a^2*b^6*c^12*d^10 + 56*a^3*b^5*c^10*d^12 + 70*a^4*b^4*c^8*d^14 + 56*a^5*b^
3*c^6*d^16 + 28*a^6*b^2*c^4*d^18 + 8*a^7*b*c^2*d^20 + a^8*d^22) + 143/128*
sqrt(b*x^2 + a)*D*b^9*c^11*x/(b^8*c^16*d^5 + 8*a*b^7*c^14*d^7 + 28*a^2*b^6
*c^12*d^9 + 56*a^3*b^5*c^10*d^11 + 70*a^4*b^4*c^8*d^13 + 56*a^5*b^3*c^6*d^
15 + 28*a^6*b^2*c^4*d^17 + 8*a^7*b*c^2*d^19 + a^8*d^21) - 143/384*(b*x^2 +
 a)^(3/2)*D*b^8*c^11/(b^8*c^16*d^5*x + 8*a*b^7*c^14*d^7*x + 28*a^2*b^6*c^1
2*d^9*x + 56*a^3*b^5*c^10*d^11*x + 70*a^4*b^4*c^8*d^13*x + 56*a^5*b^3*c^6*
d^15*x + 28*a^6*b^2*c^4*d^17*x + 8*a^7*b*c^2*d^19*x + a^8*d^21*x + b^8*c^1
7*d^4 + 8*a*b^7*c^15*d^6 + 28*a^2*b^6*c^13*d^8 + 56*a^3*b^5*c^11*d^10 + 70
*a^4*b^4*c^9*d^12 + 56*a^5*b^3*c^7*d^14 + 28*a^6*b^2*c^5*d^16 + 8*a^7*b*c^
3*d^18 + a^8*c*d^20) + 143/128*sqrt(b*x^2 + a)*C*b^9*c^11/(b^8*c^16*d^5 +
8*a*b^7*c^14*d^7 + 28*a^2*b^6*c^12*d^9 + 56*a^3*b^5*c^10*d^11 + 70*a^4*b^4
*c^8*d^13 + 56*a^5*b^3*c^6*d^15 + 28*a^6*b^2*c^4*d^17 + 8*a^7*b*c^2*d^19 +
 a^8*d^21) - 143/128*sqrt(b*x^2 + a)*C*b^9*c^10*x/(b^8*c^16*d^4 + 8*a*b^7*
c^14*d^6 + 28*a^2*b^6*c^12*d^8 + 56*a^3*b^5*c^10*d^10 + 70*a^4*b^4*c^8*d^1
2 + 56*a^5*b^3*c^6*d^14 + 28*a^6*b^2*c^4*d^16 + 8*a^7*b*c^2*d^18 + a^8*d^2
0) + 143/384*(b*x^2 + a)^(5/2)*D*b^7*c^10/(b^8*c^16*d^4*x^2 + 8*a*b^7*c^14
*d^6*x^2 + 28*a^2*b^6*c^12*d^8*x^2 + 56*a^3*b^5*c^10*d^10*x^2 + 70*a^4*b^4
*c^8*d^12*x^2 + 56*a^5*b^3*c^6*d^14*x^2 + 28*a^6*b^2*c^4*d^16*x^2 + 8*a...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17191 vs. $2(1065) = 2130$.

Time = 0.98 (sec) , antiderivative size = 17191, normalized size of antiderivative = 15.59

$$\int \frac{(a + bx^2)^{3/2} (A + Bx + Cx^2 + Dx^3)}{(c + dx)^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="gia
c")
```

output
```
-1/64*(8*C*a^3*b^6*c^5 - 48*A*a^2*b^7*c^5 + 30*D*a^4*b^5*c^4*d - 80*B*a^3*
b^6*c^4*d - 105*C*a^4*b^5*c^3*d^2 + 80*A*a^3*b^6*c^3*d^2 - 105*D*a^5*b^4*c
^2*d^3 + 60*B*a^4*b^5*c^2*d^3 + 30*C*a^5*b^4*c*d^4 - 15*A*a^4*b^5*c*d^4 +
8*D*a^6*b^3*d^5 - 3*B*a^5*b^4*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*
d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^7*c^14 + 7*a*b^6*c^12*d^2 + 21*a^
2*b^5*c^10*d^4 + 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^6*d^8 + 21*a^5*b^2*c^4*
d^10 + 7*a^6*b*c^2*d^12 + a^7*d^14)*sqrt(-b*c^2 - a*d^2)) + 1/20160*(2520*
(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a^3*b^6*c^5*d^15 - 15120*(sqrt(b)*x - s
qrt(b*x^2 + a))^17*A*a^2*b^7*c^5*d^15 + 9450*(sqrt(b)*x - sqrt(b*x^2 + a))
^17*D*a^4*b^5*c^4*d^16 - 25200*(sqrt(b)*x - sqrt(b*x^2 + a))^17*B*a^3*b^6*
c^4*d^16 - 33075*(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a^4*b^5*c^3*d^17 + 252
00*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a^3*b^6*c^3*d^17 - 33075*(sqrt(b)*x
- sqrt(b*x^2 + a))^17*D*a^5*b^4*c^2*d^18 + 18900*(sqrt(b)*x - sqrt(b*x^2 +
 a))^17*B*a^4*b^5*c^2*d^18 + 9450*(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a^5*b
^4*c*d^19 - 4725*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a^4*b^5*c*d^19 + 2520*
(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*a^6*b^3*d^20 - 945*(sqrt(b)*x - sqrt(b*
x^2 + a))^17*B*a^5*b^4*d^20 + 42840*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a^3
*b^(13/2)*c^6*d^14 - 257040*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*a^2*b^(15/2
)*c^6*d^14 + 160650*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^4*b^(11/2)*c^5*d^
15 - 428400*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a^3*b^(13/2)*c^5*d^15 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{10}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^10,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^10, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{10}}dx$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x)
```

output
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x)
```

3.85 $\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 34, antiderivative size = 682

$$
\begin{aligned}
&\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2\right. \\
&\left.+Dx^3\right) \, dx = \frac{a^2(320Ab^3c^3-a(40b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-12abd(3cCd+Bd^2+3c^2D)))\,x\sqrt{a}}{1024b^3} \\
&+\frac{a(320Ab^3c^3-a(40b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-12abd(3cCd+Bd^2+3c^2D)))\,x\left(a+bx^2\right)^{3/2}}{1536b^3} \\
&+\frac{(320Ab^3c^3-a(40b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-12abd(3cCd+Bd^2+3c^2D)))\,x\left(a+bx^2\right)^{5/2}}{1920b^3} \\
&+\frac{(b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D))\left(a+bx^2\right)^{7/2}}{7b^3} \\
&+\frac{(40b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-12abd(3cCd+Bd^2+3c^2D))\,x\left(a+bx^2\right)^{7/2}}{320b^3} \\
&-\frac{d(5ad^2D-12b(3cCd+Bd^2+3c^2D))\,x^3\left(a+bx^2\right)^{7/2}}{120b^2}+\frac{d^3Dx^5\left(a+bx^2\right)^{7/2}}{12b} \\
&-\frac{(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\left(a+bx^2\right)^{9/2}}{9b^3} \\
&+\frac{d^2(Cd+3cD)\left(a+bx^2\right)^{11/2}}{11b^3} \\
&+\frac{a^3(320Ab^3c^3-a(40b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-12abd(3cCd+Bd^2+3c^2D)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}
\end{aligned}
$$

output
```
1/1024*a^2*(320*A*b^3*c^3-a*(40*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-
12*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x*(b*x^2+a)^(1/2)/b^3+1/1536*a*(320*A*b
^3*c^3-a*(40*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-12*a*b*d*(B*d^2+3*C
*c*d+3*D*c^2)))*x*(b*x^2+a)^(3/2)/b^3+1/1920*(320*A*b^3*c^3-a*(40*b^2*c*(3
*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-12*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x*(b*
x^2+a)^(5/2)/b^3+1/7*(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3
*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(7/2)/b^3+1/320*(40*b^2*c*(3*A*d^2+3*
B*c*d+C*c^2)+5*a^2*d^3*D-12*a*b*d*(B*d^2+3*C*c*d+3*D*c^2))*x*(b*x^2+a)^(7/
2)/b^3-1/120*d*(5*a*d^2*D-12*b*(B*d^2+3*C*c*d+3*D*c^2))*x^3*(b*x^2+a)^(7/2
)/b^2+1/12*d^3*D*x^5*(b*x^2+a)^(7/2)/b-1/9*(2*a*d^2*(C*d+3*D*c)-b*(A*d^3+3
*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(9/2)/b^3+1/11*d^2*(C*d+3*D*c)*(b*x^2
+a)^(11/2)/b^3+1/1024*a^3*(320*A*b^3*c^3-a*(40*b^2*c*(3*A*d^2+3*B*c*d+C*c^
2)+5*a^2*d^3*D-12*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 6.44 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.14

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{\sqrt{b}\sqrt{a+bx^2}(5a^5d^2(8192Cd+24576cD+3465dDx)+40a^3b^2(11Ad(3456c^2+945cdx+128d}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(5*a^5*d^2*(8192*C*d + 24576*c*D + 3465*d*D*x) +
40*a^3*b^2*(11*A*d*(3456*c^2 + 945*c*d*x + 128*d^2*x^2) + 33*B*(384*c^3 +
315*c^2*d*x + 128*c*d^2*x^2 + 21*d^3*x^3) + x*(33*c^2*d*x*(128*C + 63*D*x)
 + 3*d^3*x^3*(128*C + 77*D*x) + 9*c*d^2*x^2*(231*C + 128*D*x) + 11*c^3*(31
5*C + 128*D*x))) - 10*a^4*b*(11264*c^3*D + 66*c^2*d*(512*C + 189*D*x) + 6*
c*d^2*(5632*B + x*(2079*C + 1024*D*x)) + d^3*(11264*A + x*(4158*B + 2048*C
*x + 1155*D*x^2))) + 128*b^5*x^5*(55*A*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x
^2 + 56*d^3*x^3) + x*(33*B*(120*c^3 + 315*c^2*d*x + 280*c*d^2*x^2 + 84*d^3
*x^3) + 7*x*(55*c^3*(9*C + 8*D*x) + 132*c^2*d*x*(10*C + 9*D*x) + 108*c*d^2
*x^2*(11*C + 10*D*x) + 30*d^3*x^3*(12*C + 11*D*x)))) + 64*a*b^4*x^3*(55*A*
(546*c^3 + 1296*c^2*d*x + 1071*c*d^2*x^2 + 304*d^3*x^3) + x*(33*B*(720*c^3
 + 1785*c^2*d*x + 1520*c*d^2*x^2 + 441*d^3*x^3) + x*(70*d^3*x^3*(184*C + 1
65*D*x) + 55*c^3*(357*C + 304*D*x) + 33*c^2*d*x*(1520*C + 1323*D*x) + 21*c
*d^2*x^2*(2079*C + 1840*D*x)))) + 16*a^2*b^3*x*(165*A*(924*c^3 + 1728*c^2*
d*x + 1239*c*d^2*x^2 + 320*d^3*x^3) + x*(99*B*(960*c^3 + 2065*c^2*d*x + 16
00*c*d^2*x^2 + 434*d^3*x^3) + x*(165*c^3*(413*C + 320*D*x) + 198*c^2*d*x*(
800*C + 651*D*x) + 5*d^3*x^3*(7232*C + 6237*D*x) + 6*c*d^2*x^2*(21483*C +
18080*D*x))))) + 3465*a^3*(-40*A*b^2*c*(8*b*c^2 - 3*a*d^2) + a*(40*b^2*c^2
*(c*C + 3*B*d) + 5*a^2*d^3*D - 12*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(3548160*b^(7/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2185, 2185, 27, 687, 27, 687, 27, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2}(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int (c+dx)^3\left(bx^2+a\right)^{5/2}\left(b(12Cd-19cD)x^2d^2+(12Abd-5acD)d^2+\left(-7bDc^2+12bBd^2-5ad^2D\right)xd\right)dx}{12bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int bd^3(c+dx)^3\left(3d(44Abd-16aCd+7acD)-\left(55aDd^2+4b\left(-14Dc^2+21Cdc-33Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{11bd^2}+\frac{1}{11}d\left(a+bx^2\right)^{7/2}(c+dx)^4(12C}{12bd^3}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{11}d\int(c+dx)^3\left(3d(44Abd-16aCd+7acD)-\left(55aDd^2+4b\left(-14Dc^2+21Cdc-33Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{12bd^3}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{11}d\left(\frac{\int 3(c+dx)^2\left(d\left(440Acdb^2+a\left(55ad^2D-2b\left(-7Dc^2+38Cdc+66Bd^2\right)\right)\right)-b\left(5a(32Cd-3cD)d^2+4b\left(-14Dc^3+21Cdc^2-33Bd^2c-110Ad^3\right)\right)x\right)}{10b}\right.}{12bd^3}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{11}d\left(\frac{3\int(c+dx)^2\left(d\left(440Acdb^2+a\left(55ad^2D-2b\left(-7Dc^2+38Cdc+66Bd^2\right)\right)\right)-b\left(5a(32Cd-3cD)d^2+4b\left(-14Dc^3+21Cdc^2-33Bd^2c-110Ad^3\right)\right)x\right)}{10b}\right.}{12bd^3}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{1}{11}d\left(\frac{3\left(\frac{\int b(c+dx)\left(d\left(440Abd\left(9bc^2-2ad^2\right)+a\left(5ad^2(64Cd+93cD)-2bc\left(-7Dc^2+258Cdc+726Bd^2\right)\right)\right)+\left(495a^2Dd^4-4ab\left(-39Dc^2+251Cdc+297Bd^2\right)d^2-8b^2\right.\right.}{9b}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}d\Bigg(\frac{3\Big(\frac{1}{9}\int(c+dx)(d(440Abd(9bc^2-2ad^2)+a(5ad^2(64Cd+93cD)-2bc(-7Dc^2+258Cdc+726Bd^2)))+(495a^2Dd^4-4ab(-39Dc^2+251Cdc+2}{}$$

$$\frac{D(a+bx^2)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{1}{11}d\Bigg(\frac{3\Bigg(\frac{1}{9}\Bigg(\frac{99d^2\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+40b^2c^2(3Bd+cC)\right)\right)\int\left(bx^2+a\right)^{5/2}dx}{8b}+\frac{2\left(a+bx^2\right)^{7/2}\left(160a^2d^4(3cD+Cd)-\right.}{}}{}$$

$$\frac{D(a+bx^2)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{11}d\Bigg(\frac{3\Bigg(\frac{1}{9}\Bigg(\frac{99d^2\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+40b^2c^2(3Bd+cC)\right)\right)\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{2\left(a+bx^2\right)^{7}}{}}{}$$

$$\frac{D(a+bx^2)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{11}d\Bigg(\frac{3\Bigg(\frac{1}{9}\Bigg(\frac{99d^2\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+40b^2c^2(3Bd+cC)\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right.}{8b}}{}$$

$$\frac{D(a+bx^2)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{11}d\left(\frac{3\left(\frac{1}{9}\left(\frac{99d^2\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+40b^2c^2(3Bd+cC)\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx\right.\right.\right.}{8b}\right.\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{11}d\left(\frac{3\left(\frac{1}{9}\left(\frac{99d^2\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+40b^2c^2(3Bd+cC)\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\right.\right.}{8b}\right.\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{11}d\left(\frac{3\left(\frac{1}{9}\left(\frac{99d^2\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(40Ab^2c\left(8bc^2-3ad^2\right)-a\left(5a^2d^3D-12abd\left(Bd^2\right.\right.\right.}{8b}\right.\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^5}{12bd^2}$$

input
```
Int[(c + d*x)^3*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^5*(a + b*x^2)^(7/2))/(12*b*d^2) + ((d*(12*C*d - 19*c*D)*(c +
d*x)^4*(a + b*x^2)^(7/2))/11 + (d*(-1/10*((84*b*c*C*d - 132*b*B*d^2 - 56*b
*c^2*D + 55*a*d^2*D)*(c + d*x)^3*(a + b*x^2)^(7/2))/b + (3*(-1/9*((5*a*d^2
*(32*C*d - 3*c*D) + 4*b*(21*c^2*C*d - 33*B*c*d^2 - 110*A*d^3 - 14*c^3*D))*
(c + d*x)^2*(a + b*x^2)^(7/2)) + ((2*(160*a^2*d^4*(C*d + 3*c*D) - 5*a*b*d^
2*(152*c^2*C*d + 264*B*c*d^2 + 88*A*d^3 - 17*c^3*D) - 4*b^2*c^2*(21*c^2*C*
d - 33*B*c*d^2 - 1100*A*d^3 - 14*c^3*D))*(a + b*x^2)^(7/2))/(7*b) + (d*(49
5*a^2*d^4*D - 4*a*b*d^2*(251*c*C*d + 297*B*d^2 - 39*c^2*D) - 8*b^2*c*(21*c
^2*C*d - 33*B*c*d^2 - 605*A*d^3 - 14*c^3*D))*x*(a + b*x^2)^(7/2))/(8*b) +
(99*d^2*(40*A*b^2*c*(8*b*c^2 - 3*a*d^2) - a*(40*b^2*c^2*(c*C + 3*B*d) + 5*
a^2*d^3*D - 12*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*((x*(a + b*x^2)^(5/2))/
6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b))/9))/(10*b)))
/11)/(12*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.89

method	result
default	$A\,c^{3}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{c^{2}(3Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}+d^{2}(Cd -$

input
```
int((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*c^2*(3*A*d
+B*c)*(b*x^2+a)^(7/2)/b+d^2*(C*d+3*D*c)*(1/11*x^4*(b*x^2+a)^(7/2)/b-4/11*a
/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2)))+c*(3*A*d^2+3*B*
c*d+C*c^2)*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(
1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))))))+d*(B*d^2+3*C*c*d+3*D*c^2)*(1/10*x^3*(b*x^2+a)^(7/
2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*
a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))))))+(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3)*(1/9*x^2*(b
*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+D*d^3*(1/12*x^5*(b*x^2+a)^(7/2
)/b-5/12*a/b*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b
-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*
(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2061, normalized size of antiderivative = 3.02

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[-1/7096320*(3465*(40*(C*a^4*b^2 - 8*A*a^3*b^3)*c^3 - 12*(3*D*a^5*b - 10*B
*a^4*b^2)*c^2*d - 12*(3*C*a^5*b - 10*A*a^4*b^2)*c*d^2 + (5*D*a^6 - 12*B*a^
5*b)*d^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(295
680*D*b^6*d^3*x^11 + 322560*(3*D*b^6*c*d^2 + C*b^6*d^3)*x^10 + 29568*(36*D
*b^6*c^2*d + 36*C*b^6*c*d^2 + (25*D*a*b^5 + 12*B*b^6)*d^3)*x^9 + 35840*(11
*D*b^6*c^3 + 33*C*b^6*c^2*d + 3*(23*D*a*b^5 + 11*B*b^6)*c*d^2 + (23*C*a*b^
5 + 11*A*b^6)*d^3)*x^8 + 11088*(40*C*b^6*c^3 + 12*(21*D*a*b^5 + 10*B*b^6)*
c^2*d + 12*(21*C*a*b^5 + 10*A*b^6)*c*d^2 + 3*(15*D*a^2*b^4 + 28*B*a*b^5)*d
^3)*x^7 + 5120*(11*(19*D*a*b^5 + 9*B*b^6)*c^3 + 33*(19*C*a*b^5 + 9*A*b^6)*
c^2*d + 3*(113*D*a^2*b^4 + 209*B*a*b^5)*c*d^2 + (113*C*a^2*b^4 + 209*A*a*b
^5)*d^3)*x^6 + 1848*(40*(17*C*a*b^5 + 8*A*b^6)*c^3 + 12*(93*D*a^2*b^4 + 17
0*B*a*b^5)*c^2*d + 12*(93*C*a^2*b^4 + 170*A*a*b^5)*c*d^2 + (5*D*a^3*b^3 +
372*B*a^2*b^4)*d^3)*x^5 + 15360*(11*(5*D*a^2*b^4 + 9*B*a*b^5)*c^3 + 33*(5*
C*a^2*b^4 + 9*A*a*b^5)*c^2*d + 3*(D*a^3*b^3 + 55*B*a^2*b^4)*c*d^2 + (C*a^3
*b^3 + 55*A*a^2*b^4)*d^3)*x^4 - 56320*(2*D*a^4*b^2 - 9*B*a^3*b^3)*c^3 - 16
8960*(2*C*a^4*b^2 - 9*A*a^3*b^3)*c^2*d + 30720*(4*D*a^5*b - 11*B*a^4*b^2)*
c*d^2 + 10240*(4*C*a^5*b - 11*A*a^4*b^2)*d^3 + 2310*(8*(59*C*a^2*b^4 + 104
*A*a*b^5)*c^3 + 12*(3*D*a^3*b^3 + 118*B*a^2*b^4)*c^2*d + 12*(3*C*a^3*b^3 +
 118*A*a^2*b^4)*c*d^2 - (5*D*a^4*b^2 - 12*B*a^3*b^3)*d^3)*x^3 + 5120*(11*(
D*a^3*b^3 + 27*B*a^2*b^4)*c^3 + 33*(C*a^3*b^3 + 27*A*a^2*b^4)*c^2*d - 3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(695) = 1390$.

Time = 1.00 (sec) , antiderivative size = 3186, normalized size of antiderivative = 4.67

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b**2*d**3*x**11/12 + x**10*(C*b**3*d**3 + 3
*D*b**3*c*d**2)/(11*b) + x**9*(B*b**3*d**3 + 3*C*b**3*c*d**2 + 25*D*a*b**2
*d**3/12 + 3*D*b**3*c**2*d)/(10*b) + x**8*(A*b**3*d**3 + 3*B*b**3*c*d**2 +
 3*C*a*b**2*d**3 + 3*C*b**3*c**2*d + 9*D*a*b**2*c*d**2 + D*b**3*c**3 - 10*
a*(C*b**3*d**3 + 3*D*b**3*c*d**2)/(11*b))/(9*b) + x**7*(3*A*b**3*c*d**2 +
3*B*a*b**2*d**3 + 3*B*b**3*c**2*d + 9*C*a*b**2*c*d**2 + C*b**3*c**3 + 3*D*
a**2*b*d**3 + 9*D*a*b**2*c**2*d - 9*a*(B*b**3*d**3 + 3*C*b**3*c*d**2 + 25*
D*a*b**2*d**3/12 + 3*D*b**3*c**2*d)/(10*b))/(8*b) + x**6*(3*A*a*b**2*d**3
+ 3*A*b**3*c**2*d + 9*B*a*b**2*c*d**2 + B*b**3*c**3 + 3*C*a**2*b*d**3 + 9*
C*a*b**2*c**2*d + 9*D*a**2*b*c*d**2 + 3*D*a*b**2*c**3 - 8*a*(A*b**3*d**3 +
 3*B*b**3*c*d**2 + 3*C*a*b**2*d**3 + 3*C*b**3*c**2*d + 9*D*a*b**2*c*d**2 +
 D*b**3*c**3 - 10*a*(C*b**3*d**3 + 3*D*b**3*c*d**2)/(11*b))/(9*b))/(7*b) +
 x**5*(9*A*a*b**2*c*d**2 + A*b**3*c**3 + 3*B*a**2*b*d**3 + 9*B*a*b**2*c**2
*d + 9*C*a**2*b*c*d**2 + 3*C*a*b**2*c**3 + D*a**3*d**3 + 9*D*a**2*b*c**2*d
 - 7*a*(3*A*b**3*c*d**2 + 3*B*a*b**2*d**3 + 3*B*b**3*c**2*d + 9*C*a*b**2*c
*d**2 + C*b**3*c**3 + 3*D*a**2*b*d**3 + 9*D*a*b**2*c**2*d - 9*a*(B*b**3*d*
*3 + 3*C*b**3*c*d**2 + 25*D*a*b**2*d**3/12 + 3*D*b**3*c**2*d)/(10*b))/(8*b
))/(6*b) + x**4*(3*A*a**2*b*d**3 + 9*A*a*b**2*c**2*d + 9*B*a**2*b*c*d**2 +
 3*B*a*b**2*c**3 + C*a**3*d**3 + 9*C*a**2*b*c**2*d + 3*D*a**3*c*d**2 + 3*D
*a**2*b*c**3 - 6*a*(3*A*a*b**2*d**3 + 3*A*b**3*c**2*d + 9*B*a*b**2*c*d*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.24

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/12*(b*x^2 + a)^(7/2)*D*d^3*x^5/b - 1/24*(b*x^2 + a)^(7/2)*D*a*d^3*x^3/b^
2 + 1/6*(b*x^2 + a)^(5/2)*A*c^3*x + 5/24*(b*x^2 + a)^(3/2)*A*a*c^3*x + 5/1
6*sqrt(b*x^2 + a)*A*a^2*c^3*x + 1/64*(b*x^2 + a)^(7/2)*D*a^2*d^3*x/b^3 - 1
/384*(b*x^2 + a)^(5/2)*D*a^3*d^3*x/b^3 - 5/1536*(b*x^2 + a)^(3/2)*D*a^4*d^
3*x/b^3 - 5/1024*sqrt(b*x^2 + a)*D*a^5*d^3*x/b^3 + 1/11*(3*D*c*d^2 + C*d^3
)*(b*x^2 + a)^(7/2)*x^4/b + 5/16*A*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
- 5/1024*D*a^6*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/7*(b*x^2 + a)^(7/2)*
B*c^3/b + 3/7*(b*x^2 + a)^(7/2)*A*c^2*d/b + 1/10*(3*D*c^2*d + 3*C*c*d^2 +
B*d^3)*(b*x^2 + a)^(7/2)*x^3/b - 4/99*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(7/2
)*a*x^2/b^2 + 1/9*(D*c^3 + 3*C*c^2*d + 3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(7/2
)*x^2/b - 3/80*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(7/2)*a*x/b^2 +
 1/160*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(5/2)*a^2*x/b^2 + 1/128
*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*(b*x^2 + a)^(3/2)*a^3*x/b^2 + 3/256*(3*D*
c^2*d + 3*C*c*d^2 + B*d^3)*sqrt(b*x^2 + a)*a^4*x/b^2 + 1/8*(C*c^3 + 3*B*c^
2*d + 3*A*c*d^2)*(b*x^2 + a)^(7/2)*x/b - 1/48*(C*c^3 + 3*B*c^2*d + 3*A*c*d
^2)*(b*x^2 + a)^(5/2)*a*x/b - 5/192*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*(b*x^2
 + a)^(3/2)*a^2*x/b - 5/128*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*sqrt(b*x^2 + a
)*a^3*x/b + 3/256*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*a^5*arcsinh(b*x/sqrt(a*b
))/b^(5/2) - 5/128*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*a^4*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 8/693*(3*D*c*d^2 + C*d^3)*(b*x^2 + a)^(7/2)*a^2/b^3 - 2/6...
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.61

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/3548160*sqrt(b*x^2 + a)*((2*((4*(((2*(7*(8*(3*(10*(11*D*b^2*d^3*x + 12*(3
*D*b^12*c*d^2 + C*b^12*d^3)/b^10)*x + 11*(36*D*b^12*c^2*d + 36*C*b^12*c*d^
2 + 25*D*a*b^11*d^3 + 12*B*b^12*d^3)/b^10)*x + 40*(11*D*b^12*c^3 + 33*C*b^
12*c^2*d + 69*D*a*b^11*c*d^2 + 33*B*b^12*c*d^2 + 23*C*a*b^11*d^3 + 11*A*b^
12*d^3)/b^10)*x + 99*(40*C*b^12*c^3 + 252*D*a*b^11*c^2*d + 120*B*b^12*c^2*
d + 252*C*a*b^11*c*d^2 + 120*A*b^12*c*d^2 + 45*D*a^2*b^10*d^3 + 84*B*a*b^1
1*d^3)/b^10)*x + 320*(209*D*a*b^11*c^3 + 99*B*b^12*c^3 + 627*C*a*b^11*c^2*
d + 297*A*b^12*c^2*d + 339*D*a^2*b^10*c*d^2 + 627*B*a*b^11*c*d^2 + 113*C*a
^2*b^10*d^3 + 209*A*a*b^11*d^3)/b^10)*x + 231*(680*C*a*b^11*c^3 + 320*A*b^
12*c^3 + 1116*D*a^2*b^10*c^2*d + 2040*B*a*b^11*c^2*d + 1116*C*a^2*b^10*c*d
^2 + 2040*A*a*b^11*c*d^2 + 5*D*a^3*b^9*d^3 + 372*B*a^2*b^10*d^3)/b^10)*x +
 1920*(55*D*a^2*b^10*c^3 + 99*B*a*b^11*c^3 + 165*C*a^2*b^10*c^2*d + 297*A*
a*b^11*c^2*d + 3*D*a^3*b^9*c*d^2 + 165*B*a^2*b^10*c*d^2 + C*a^3*b^9*d^3 +
55*A*a^2*b^10*d^3)/b^10)*x + 1155*(472*C*a^2*b^10*c^3 + 832*A*a*b^11*c^3 +
 36*D*a^3*b^9*c^2*d + 1416*B*a^2*b^10*c^2*d + 36*C*a^3*b^9*c*d^2 + 1416*A*
a^2*b^10*c*d^2 - 5*D*a^4*b^8*d^3 + 12*B*a^3*b^9*d^3)/b^10)*x + 2560*(11*D*
a^3*b^9*c^3 + 297*B*a^2*b^10*c^3 + 33*C*a^3*b^9*c^2*d + 891*A*a^2*b^10*c^2
*d - 12*D*a^4*b^8*c*d^2 + 33*B*a^3*b^9*c*d^2 - 4*C*a^4*b^8*d^3 + 11*A*a^3*
b^9*d^3)/b^10)*x + 3465*(40*C*a^3*b^9*c^3 + 704*A*a^2*b^10*c^3 - 36*D*a^4*
b^8*c^2*d + 120*B*a^3*b^9*c^2*d - 36*C*a^4*b^8*c*d^2 + 120*A*a^3*b^9*c*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^2+a\right)^{5/2}\,\left(c+d\,x\right)^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)^3*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (dx + c)^3 \left(b\,x^2+a\right)^{\frac{5}{2}} \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^3*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.86 $\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 485

$$\begin{aligned}
&\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2\right. \\
&\left.+Dx^3\right) dx = \frac{a^2(10Ab(8bc^2-ad^2)-a(10bc(cC+2Bd)-3ad(Cd+2cD)))\, x\sqrt{a+bx^2}}{256b^2} \\
&+\frac{a(10Ab(8bc^2-ad^2)-a(10bc(cC+2Bd)-3ad(Cd+2cD)))\, x(a+bx^2)^{3/2}}{384b^2} \\
&+\frac{(10Ab(8bc^2-ad^2)-a(10bc(cC+2Bd)-3ad(Cd+2cD)))\, x(a+bx^2)^{5/2}}{480b^2} \\
&+\frac{(b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D))\, (a+bx^2)^{7/2}}{7b^3} \\
&+\frac{(10b(c^2C+2Bcd+Ad^2)-3ad(Cd+2cD))\, x(a+bx^2)^{7/2}}{80b^2} \\
&+\frac{d(Cd+2cD)x^3(a+bx^2)^{7/2}}{10b} \\
&-\frac{(2ad^2D-b(2cCd+Bd^2+c^2D))\, (a+bx^2)^{9/2}}{9b^3}+\frac{d^2D(a+bx^2)^{11/2}}{11b^3} \\
&+\frac{a^3(10Ab(8bc^2-ad^2)-a(10bc(cC+2Bd)-3ad(Cd+2cD)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}
\end{aligned}$$

output
```
1/256*a^2*(10*A*b*(-a*d^2+8*b*c^2)-a*(10*b*c*(2*B*d+C*c)-3*a*d*(C*d+2*D*c)
))*x*(b*x^2+a)^(1/2)/b^2+1/384*a*(10*A*b*(-a*d^2+8*b*c^2)-a*(10*b*c*(2*B*d
+C*c)-3*a*d*(C*d+2*D*c)))*x*(b*x^2+a)^(3/2)/b^2+1/480*(10*A*b*(-a*d^2+8*b*
c^2)-a*(10*b*c*(2*B*d+C*c)-3*a*d*(C*d+2*D*c)))*x*(b*x^2+a)^(5/2)/b^2+1/7*(
b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(7/2)/b^3
+1/80*(10*b*(A*d^2+2*B*c*d+C*c^2)-3*a*d*(C*d+2*D*c))*x*(b*x^2+a)^(7/2)/b^2
+1/10*d*(C*d+2*D*c)*x^3*(b*x^2+a)^(7/2)/b-1/9*(2*a*d^2*D-b*(B*d^2+2*C*c*d+
D*c^2))*(b*x^2+a)^(9/2)/b^3+1/11*d^2*D*(b*x^2+a)^(11/2)/b^3+1/256*a^3*(10*
A*b*(-a*d^2+8*b*c^2)-a*(10*b*c*(2*B*d+C*c)-3*a*d*(C*d+2*D*c)))*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.12

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{\sqrt{a+bx^2}(10240a^5d^2D+10a^3b^2(99Ad(256c+35dx)+22B(576c^2+315cdx+64d^2x^2)+x(2}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(10240*a^5*d^2*D + 10*a^3*b^2*(99*A*d*(256*c + 35*d*x) +
22*B*(576*c^2 + 315*c*d*x + 64*d^2*x^2) + x*(22*c*d*x*(128*C + 63*D*x) + 3
*d^2*x^2*(231*C + 128*D*x) + 11*c^2*(315*C + 128*D*x))) - 5*a^4*b*(5632*c^
2*D + 22*c*d*(512*C + 189*D*x) + d^2*(5632*B + x*(2079*C + 1024*D*x))) + 3
2*b^5*x^5*(165*A*(28*c^2 + 48*c*d*x + 21*d^2*x^2) + x*(110*B*(36*c^2 + 63*
c*d*x + 28*d^2*x^2) + 7*x*(55*c^2*(9*C + 8*D*x) + 88*c*d*x*(10*C + 9*D*x)
+ 36*d^2*x^2*(11*C + 10*D*x)))) + 16*a*b^4*x^3*(165*A*(182*c^2 + 288*c*d*x
 + 119*d^2*x^2) + x*(110*B*(216*c^2 + 357*c*d*x + 152*d^2*x^2) + x*(55*c^2
*(357*C + 304*D*x) + 22*c*d*x*(1520*C + 1323*D*x) + 7*d^2*x^2*(2079*C + 18
40*D*x)))) + 4*a^2*b^3*x*(165*A*(924*c^2 + 1152*c*d*x + 413*d^2*x^2) + x*(
330*B*(288*c^2 + 413*c*d*x + 160*d^2*x^2) + x*(165*c^2*(413*C + 320*D*x) +
 132*c*d*x*(800*C + 651*D*x) + 2*d^2*x^2*(21483*C + 18080*D*x))))) - 3465*
a^3*Sqrt[b]*(10*A*b*(8*b*c^2 - a*d^2) + a*(-10*b*c*(c*C + 2*B*d) + 3*a*d*(
C*d + 2*c*D)))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(887040*b^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2185, 2185, 27, 687, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int (c+dx)^2 \left(bx^2+a\right)^{5/2} \left(b(11Cd-18cD)x^2d^2+(11Abd-4acD)d^2+\left(-7bDc^2+11bBd^2-4ad^2D\right)xd\right)dx}{11bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int bd^3(c+dx)^2\left(d(110Abd-33aCd+14acD)-\left(40aDd^2+b\left(-56Dc^2+77Cdc-110Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{10bd^2}+\frac{1}{10}d\left(a+bx^2\right)^{7/2}(c+dx)^3(11}{11bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{10}d\int (c+dx)^2\left(d(110Abd-33aCd+14acD)-\left(40aDd^2+b\left(-56Dc^2+77Cdc-110Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{11bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{10}d\left(\frac{\int (c+dx)\left(d\left(990Acdb^2+a\left(80ad^2D-b\left(-14Dc^2+143Cdc+220Bd^2\right)\right)\right)-b\left(a(297Cd-46cD)d^2+2b\left(-56Dc^3+77Cdc^2-110Bd^2c-495Ad^3\right)\right)\right.}{9b}\right.}{11b}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \ 676$$

$$\frac{1}{10}d\left(\frac{\frac{99}{8}d^2\left(10Ab\left(8bc^2-ad^2\right)-a(10bc(2Bd+cC)-3ad(2cD+Cd))\right)\int \left(bx^2+a\right)^{5/2}dx+\frac{\frac{2\left(a+bx^2\right)^{7/2}\left(40a^2d^4D-10abd^2\left(11Bd^2-3c^2D+22cCd\right)-b^2c\left(\right.\right.}{7b}}{9b}}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{10}d\left(\frac{\frac{99}{8}d^2\left(10Ab\left(8bc^2-ad^2\right)-a(10bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2\left(a+bx^2\right)^{7/2}\left(40a^2d^4D-10abd^2\left(11B\right.\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{10}d\left(\frac{\frac{99}{8}d^2\left(10Ab\left(8bc^2-ad^2\right)-a(10bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{2\left(a+bx^2\right)^{7/2}\left(40\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{10}d\left(\frac{\frac{99}{8}d^2\left(10Ab\left(8bc^2-ad^2\right)-a(10bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5}\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{10}d\left(\frac{\frac{99}{8}d^2\left(10Ab\left(8bc^2-ad^2\right)-a(10bc(2Bd+cC)-3ad(2cD+Cd))\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\right.\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{10}d\Bigg(\frac{\frac{2\left(a+bx^2\right)^{7/2}\left(40a^2d^4D-10abd^2\left(11Bd^2-3c^2D+22cCd\right)-b^2c\left(-990Ad^3-110Bcd^2-56c^3D+77c^2Cd\right)\right)}{7b}+\frac{99}{8}d^2\Bigg(\frac{5}{6}a\Bigg(\frac{3}{4}a\Bigg(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^4}{11bd^2}$$

input

```
Int[(c + d*x)^2*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
(D*(c + d*x)^4*(a + b*x^2)^(7/2))/(11*b*d^2) + ((d*(11*C*d - 18*c*D)*(c +
d*x)^3*(a + b*x^2)^(7/2))/10 + (d*(-1/9*((40*a*d^2*D + b*(77*c*C*d - 110*B
*d^2 - 56*c^2*D))*(c + d*x)^2*(a + b*x^2)^(7/2))/b + ((2*(40*a^2*d^4*D - 1
0*a*b*d^2*(22*c*C*d + 11*B*d^2 - 3*c^2*D) - b^2*c*(77*c^2*C*d - 110*B*c*d^
2 - 990*A*d^3 - 56*c^3*D))*(a + b*x^2)^(7/2))/(7*b) - (d*(a*d^2*(297*C*d -
 46*c*D) + 2*b*(77*c^2*C*d - 110*B*c*d^2 - 495*A*d^3 - 56*c^3*D))*x*(a + b
*x^2)^(7/2))/8 + (99*d^2*(10*A*b*(8*b*c^2 - a*d^2) - a*(10*b*c*(c*C + 2*B*
d) - 3*a*d*(C*d + 2*c*D)))*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)
^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*Sqrt[b])))/4))/6))/8)/(9*b)))/10)/(11*b*d^3)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.90

method	result
default	$A c^{2} \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{c \left(2 A d +B c \right) \left(b \,x^{2}+a \right)^{\frac{7}{2}}}{7 b}+d \left(C d +\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*c*(2*A*d+B
*c)*(b*x^2+a)^(7/2)/b+d*(C*d+2*D*c)*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(
1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2
+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2)))))))+(A*d^2+2*B*c*d+C*c^2)*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*
x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2
)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+(B*d^2+2*C*c*d+D*c^2)*(1
/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+D*d^2*(1/11*x^4*(b*x^
2+a)^(7/2)/b-4/11*a/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1425, normalized size of antiderivative = 2.94

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
[1/1774080*(3465*(10*(C*a^4*b - 8*A*a^3*b^2)*c^2 - 2*(3*D*a^5 - 10*B*a^4*b
)*c*d - (3*C*a^5 - 10*A*a^4*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(80640*D*b^5*d^2*x^10 + 88704*(2*D*b^5*c*d + C*b^5*d
^2)*x^9 + 8960*(11*D*b^5*c^2 + 22*C*b^5*c*d + (23*D*a*b^4 + 11*B*b^5)*d^2)
*x^8 + 11088*(10*C*b^5*c^2 + 2*(21*D*a*b^4 + 10*B*b^5)*c*d + (21*C*a*b^4 +
 10*A*b^5)*d^2)*x^7 + 1280*(11*(19*D*a*b^4 + 9*B*b^5)*c^2 + 22*(19*C*a*b^4
 + 9*A*b^5)*c*d + (113*D*a^2*b^3 + 209*B*a*b^4)*d^2)*x^6 + 1848*(10*(17*C*
a*b^4 + 8*A*b^5)*c^2 + 2*(93*D*a^2*b^3 + 170*B*a*b^4)*c*d + (93*C*a^2*b^3
+ 170*A*a*b^4)*d^2)*x^5 + 3840*(11*(5*D*a^2*b^3 + 9*B*a*b^4)*c^2 + 22*(5*C
*a^2*b^3 + 9*A*a*b^4)*c*d + (D*a^3*b^2 + 55*B*a^2*b^3)*d^2)*x^4 + 2310*(2*
(59*C*a^2*b^3 + 104*A*a*b^4)*c^2 + 2*(3*D*a^3*b^2 + 118*B*a^2*b^3)*c*d + (
3*C*a^3*b^2 + 118*A*a^2*b^3)*d^2)*x^3 - 14080*(2*D*a^4*b - 9*B*a^3*b^2)*c^
2 - 28160*(2*C*a^4*b - 9*A*a^3*b^2)*c*d + 2560*(4*D*a^5 - 11*B*a^4*b)*d^2
+ 1280*(11*(D*a^3*b^2 + 27*B*a^2*b^3)*c^2 + 22*(C*a^3*b^2 + 27*A*a^2*b^3)*
c*d - (4*D*a^4*b - 11*B*a^3*b^2)*d^2)*x^2 + 3465*(2*(5*C*a^3*b^2 + 88*A*a^
2*b^3)*c^2 - 2*(3*D*a^4*b - 10*B*a^3*b^2)*c*d - (3*C*a^4*b - 10*A*a^3*b^2)
*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1/887040*(3465*(10*(C*a^4*b - 8*A*a^3*b^2)*
c^2 - 2*(3*D*a^5 - 10*B*a^4*b)*c*d - (3*C*a^5 - 10*A*a^4*b)*d^2)*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (80640*D*b^5*d^2*x^10 + 88704*(2*D*b^
5*c*d + C*b^5*d^2)*x^9 + 8960*(11*D*b^5*c^2 + 22*C*b^5*c*d + (23*D*a*b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2113 vs. $2(471) = 942$.

Time = 1.06 (sec) , antiderivative size = 2113, normalized size of antiderivative = 4.36

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b**2*d**2*x**10/11 + x**9*(C*b**3*d**2 + 2*
D*b**3*c*d)/(10*b) + x**8*(B*b**3*d**2 + 2*C*b**3*c*d + 23*D*a*b**2*d**2/1
1 + D*b**3*c**2)/(9*b) + x**7*(A*b**3*d**2 + 2*B*b**3*c*d + 3*C*a*b**2*d**
2 + C*b**3*c**2 + 6*D*a*b**2*c*d - 9*a*(C*b**3*d**2 + 2*D*b**3*c*d)/(10*b)
)/(8*b) + x**6*(2*A*b**3*c*d + 3*B*a*b**2*d**2 + B*b**3*c**2 + 6*C*a*b**2*
c*d + 3*D*a**2*b*d**2 + 3*D*a*b**2*c**2 - 8*a*(B*b**3*d**2 + 2*C*b**3*c*d
+ 23*D*a*b**2*d**2/11 + D*b**3*c**2)/(9*b))/(7*b) + x**5*(3*A*a*b**2*d**2
+ A*b**3*c**2 + 6*B*a*b**2*c*d + 3*C*a**2*b*d**2 + 3*C*a*b**2*c**2 + 6*D*a
**2*b*c*d - 7*a*(A*b**3*d**2 + 2*B*b**3*c*d + 3*C*a*b**2*d**2 + C*b**3*c**
2 + 6*D*a*b**2*c*d - 9*a*(C*b**3*d**2 + 2*D*b**3*c*d)/(10*b))/(8*b))/(6*b)
 + x**4*(6*A*a*b**2*c*d + 3*B*a**2*b*d**2 + 3*B*a*b**2*c**2 + 6*C*a**2*b*c
*d + D*a**3*d**2 + 3*D*a**2*b*c**2 - 6*a*(2*A*b**3*c*d + 3*B*a*b**2*d**2 +
 B*b**3*c**2 + 6*C*a*b**2*c*d + 3*D*a**2*b*d**2 + 3*D*a*b**2*c**2 - 8*a*(B
*b**3*d**2 + 2*C*b**3*c*d + 23*D*a*b**2*d**2/11 + D*b**3*c**2)/(9*b))/(7*b
))/(5*b) + x**3*(3*A*a**2*b*d**2 + 3*A*a*b**2*c**2 + 6*B*a**2*b*c*d + C*a*
*3*d**2 + 3*C*a**2*b*c**2 + 2*D*a**3*c*d - 5*a*(3*A*a*b**2*d**2 + A*b**3*c
**2 + 6*B*a*b**2*c*d + 3*C*a**2*b*d**2 + 3*C*a*b**2*c**2 + 6*D*a**2*b*c*d
- 7*a*(A*b**3*d**2 + 2*B*b**3*c*d + 3*C*a*b**2*d**2 + C*b**3*c**2 + 6*D*a*
b**2*c*d - 9*a*(C*b**3*d**2 + 2*D*b**3*c*d)/(10*b))/(8*b))/(6*b))/(4*b) +
x**2*(6*A*a**2*b*c*d + B*a**3*d**2 + 3*B*a**2*b*c**2 + 2*C*a**3*c*d + D...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.18

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
1/11*(b*x^2 + a)^(7/2)*D*d^2*x^4/b - 4/99*(b*x^2 + a)^(7/2)*D*a*d^2*x^2/b^
2 + 1/6*(b*x^2 + a)^(5/2)*A*c^2*x + 5/24*(b*x^2 + a)^(3/2)*A*a*c^2*x + 5/1
6*sqrt(b*x^2 + a)*A*a^2*c^2*x + 1/10*(2*D*c*d + C*d^2)*(b*x^2 + a)^(7/2)*x
^3/b + 5/16*A*a^3*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^(7/
2)*B*c^2/b + 2/7*(b*x^2 + a)^(7/2)*A*c*d/b + 8/693*(b*x^2 + a)^(7/2)*D*a^2
*d^2/b^3 + 1/9*(D*c^2 + 2*C*c*d + B*d^2)*(b*x^2 + a)^(7/2)*x^2/b - 3/80*(2
*D*c*d + C*d^2)*(b*x^2 + a)^(7/2)*a*x/b^2 + 1/160*(2*D*c*d + C*d^2)*(b*x^2
 + a)^(5/2)*a^2*x/b^2 + 1/128*(2*D*c*d + C*d^2)*(b*x^2 + a)^(3/2)*a^3*x/b^
2 + 3/256*(2*D*c*d + C*d^2)*sqrt(b*x^2 + a)*a^4*x/b^2 + 1/8*(C*c^2 + 2*B*c
*d + A*d^2)*(b*x^2 + a)^(7/2)*x/b - 1/48*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2
+ a)^(5/2)*a*x/b - 5/192*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a^2*x
/b - 5/128*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a^3*x/b + 3/256*(2*D*
c*d + C*d^2)*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*(C*c^2 + 2*B*c*d +
 A*d^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(D*c^2 + 2*C*c*d + B*d^2
)*(b*x^2 + a)^(7/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.56

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
1/887040*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*(10*D*b^2*d^2*x + 11*(2*D*b^
11*c*d + C*b^11*d^2)/b^9)*x + 10*(11*D*b^11*c^2 + 22*C*b^11*c*d + 23*D*a*b
^10*d^2 + 11*B*b^11*d^2)/b^9)*x + 99*(10*C*b^11*c^2 + 42*D*a*b^10*c*d + 20
*B*b^11*c*d + 21*C*a*b^10*d^2 + 10*A*b^11*d^2)/b^9)*x + 80*(209*D*a*b^10*c
^2 + 99*B*b^11*c^2 + 418*C*a*b^10*c*d + 198*A*b^11*c*d + 113*D*a^2*b^9*d^2
 + 209*B*a*b^10*d^2)/b^9)*x + 231*(170*C*a*b^10*c^2 + 80*A*b^11*c^2 + 186*
D*a^2*b^9*c*d + 340*B*a*b^10*c*d + 93*C*a^2*b^9*d^2 + 170*A*a*b^10*d^2)/b^
9)*x + 480*(55*D*a^2*b^9*c^2 + 99*B*a*b^10*c^2 + 110*C*a^2*b^9*c*d + 198*A
*a*b^10*c*d + D*a^3*b^8*d^2 + 55*B*a^2*b^9*d^2)/b^9)*x + 1155*(118*C*a^2*b
^9*c^2 + 208*A*a*b^10*c^2 + 6*D*a^3*b^8*c*d + 236*B*a^2*b^9*c*d + 3*C*a^3*
b^8*d^2 + 118*A*a^2*b^9*d^2)/b^9)*x + 640*(11*D*a^3*b^8*c^2 + 297*B*a^2*b^
9*c^2 + 22*C*a^3*b^8*c*d + 594*A*a^2*b^9*c*d - 4*D*a^4*b^7*d^2 + 11*B*a^3*
b^8*d^2)/b^9)*x + 3465*(10*C*a^3*b^8*c^2 + 176*A*a^2*b^9*c^2 - 6*D*a^4*b^7
*c*d + 20*B*a^3*b^8*c*d - 3*C*a^4*b^7*d^2 + 10*A*a^3*b^8*d^2)/b^9)*x - 128
0*(22*D*a^4*b^7*c^2 - 99*B*a^3*b^8*c^2 + 44*C*a^4*b^7*c*d - 198*A*a^3*b^8*
c*d - 8*D*a^5*b^6*d^2 + 22*B*a^4*b^7*d^2)/b^9) + 1/256*(10*C*a^4*b*c^2 - 8
0*A*a^3*b^2*c^2 - 6*D*a^5*c*d + 20*B*a^4*b*c*d - 3*C*a^5*d^2 + 10*A*a^4*b*
d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2 + Dx^3\right) \, dx = \int \left(b\,x^2+a\right)^{5/2} \left(c+d\,x\right)^2 \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)\, dx = \int (dx + c)^2 \left(b\,x^2+a\right)^{\frac{5}{2}} \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.87 $\int (c+dx) \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 321

$$
\begin{aligned}
\int (c+dx) \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2\right. & \\
\left.+Dx^3\right) \, dx = {} & \frac{a^2(80Ab^2c-a(10b(cC+Bd)-3adD))\,x\sqrt{a+bx^2}}{256b^2} \\
& + \frac{a(80Ab^2c-a(10b(cC+Bd)-3adD))\,x(a+bx^2)^{3/2}}{384b^2} \\
& + \frac{(80Ab^2c-a(10b(cC+Bd)-3adD))\,x(a+bx^2)^{5/2}}{480b^2} \\
& + \frac{(bBc+Abd-aCd-acD)\left(a+bx^2\right)^{7/2}}{7b^2} \\
& + \frac{(10b(cC+Bd)-3adD)x(a+bx^2)^{7/2}}{80b^2} \\
& + \frac{dDx^3(a+bx^2)^{7/2}}{10b} + \frac{(Cd+cD)\left(a+bx^2\right)^{9/2}}{9b^2} \\
& + \frac{a^3(80Ab^2c-a(10b(cC+Bd)-3adD))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}
\end{aligned}
$$

output
```
1/256*a^2*(80*A*b^2*c-a*(10*b*(B*d+C*c)-3*D*a*d))*x*(b*x^2+a)^(1/2)/b^2+1/
384*a*(80*A*b^2*c-a*(10*b*(B*d+C*c)-3*D*a*d))*x*(b*x^2+a)^(3/2)/b^2+1/480*
(80*A*b^2*c-a*(10*b*(B*d+C*c)-3*D*a*d))*x*(b*x^2+a)^(5/2)/b^2+1/7*(A*b*d+B
*b*c-C*a*d-D*a*c)*(b*x^2+a)^(7/2)/b^2+1/80*(10*b*(B*d+C*c)-3*D*a*d)*x*(b*x
^2+a)^(7/2)/b^2+1/10*d*D*x^3*(b*x^2+a)^(7/2)/b+1/9*(C*d+D*c)*(b*x^2+a)^(9/
2)/b^2+1/256*a^3*(80*A*b^2*c-a*(10*b*(B*d+C*c)-3*D*a*d))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.02

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(-5a^4(512Cd+512cD+189dDx)+10a^3b(1152Ad+9B(128c+35dx)+x(315}{}$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-5*a^4*(512*C*d + 512*c*D + 189*d*D*x) + 10*a^3*
b*(1152*A*d + 9*B*(128*c + 35*d*x) + x*(315*c*C + 128*C*d*x + 128*c*D*x +
63*d*D*x^2)) + 32*b^4*x^5*(60*A*(7*c + 6*d*x) + x*(45*B*(8*c + 7*d*x) + 7*
x*(5*c*(9*C + 8*D*x) + 4*d*x*(10*C + 9*D*x)))) + 12*a^2*b^2*x*(60*A*(77*c
+ 48*d*x) + x*(5*B*(576*c + 413*d*x) + x*(5*c*(413*C + 320*D*x) + 2*d*x*(8
00*C + 651*D*x)))) + 16*a*b^3*x^3*(30*A*(91*c + 72*d*x) + x*(15*B*(144*c +
 119*d*x) + x*(5*c*(357*C + 304*D*x) + d*x*(1520*C + 1323*D*x))))) - 315*a
^3*(80*A*b^2*c + a*(-10*b*(c*C + B*d) + 3*a*d*D))*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(80640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2185, 2185, 27, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (c+dx) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int (c+dx) \left(bx^2+a\right)^{5/2} \left(b(10Cd-17cD)x^2d^2+(10Abd-3acD)d^2+\left(-7bDc^2+10bBd^2-3ad^2D\right)xd\right)dx}{10bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{\int bd^3(c+dx)\left(d(90Abd-20aCd+7acD)-\left(27aDd^2+b\left(-56Dc^2+70Cdc-90Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{9bd^2}+\frac{1}{9}d\left(a+bx^2\right)^{7/2}(c+dx)^2(10Cd-}{10bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{9}d\int (c+dx)\left(d(90Abd-20aCd+7acD)-\left(27aDd^2+b\left(-56Dc^2+70Cdc-90Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}dx+\frac{1}{9}d}{10bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{1}{9}d\left(\frac{9d^2\left(80Ab^2c-a(10b(Bd+cC)-3adD)\right)\int \left(bx^2+a\right)^{5/2}dx}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(cD+Cd)+b\left(-45Ad^3-45Bcd^2-28c^3D+35c^2Cd\right)\right)}{7b}-\frac{dx(}{}\right.}{10bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{9}d\left(\frac{9d^2(80Ab^2c-a(10b(Bd+cC)-3adD))\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(cD+Cd)+b\left(-45Ad^3-45Bcd^2-28c\right.\right.}{7b}\right.}{10bd^3}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{9}d\left(\frac{9d^2(80Ab^2c-a(10b(Bd+cC)-3adD))\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(cD+Cd)+b\left(-4\right.\right.}{7b}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{9}d\left(\frac{9d^2(80Ab^2c-a(10b(Bd+cC)-3adD))\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(1\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{9}d\left(\frac{9d^2(80Ab^2c-a(10b(Bd+cC)-3adD))\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{9}d\left(\frac{9d^2\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)(80Ab^2c-a(10b(Bd+cC)-3adD))}{8b}-\frac{2\left(a+b\right.}{}\right.$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)^3}{10bd^2}$$

input
```
Int[(c + d*x)*(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^3*(a + b*x^2)^(7/2))/(10*b*d^2) + ((d*(10*C*d - 17*c*D)*(c +
d*x)^2*(a + b*x^2)^(7/2))/9 + (d*((-2*(10*a*d^2*(C*d + c*D) + b*(35*c^2*C*
d - 45*B*c*d^2 - 45*A*d^3 - 28*c^3*D))*(a + b*x^2)^(7/2))/(7*b) - (d*(27*a
*d^2*D + b*(70*c*C*d - 90*B*d^2 - 56*c^2*D))*x*(a + b*x^2)^(7/2))/(8*b) +
(9*d^2*(80*A*b^2*c - a*(10*b*(c*C + B*d) - 3*a*d*D))*((x*(a + b*x^2)^(5/2)
)/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b)))/9)/(10*b*
d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.08

method	result
default	$Ac\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{(Ad+Bc)(b\,x^2+a)^{\frac{7}{2}}}{7b}+(Bd+Cc)$

input
```
int((d*x+c)*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*(A*d+B*c)*(b
*x^2+a)^(7/2)/b+(B*d+C*c)*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a
)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^
(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+(C*d+D*c)*(1/9*x^2*(b*x^2+a)^(7/2)
/b-2/63*a/b^2*(b*x^2+a)^(7/2))+d*D*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1
/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 885, normalized size of antiderivative = 2.76

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[1/161280*(315*(10*(C*a^4*b - 8*A*a^3*b^2)*c - (3*D*a^5 - 10*B*a^4*b)*d)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8064*D*b^5*d*x
^9 + 8960*(D*b^5*c + C*b^5*d)*x^8 + 1008*(10*C*b^5*c + (21*D*a*b^4 + 10*B*
b^5)*d)*x^7 + 1280*((19*D*a*b^4 + 9*B*b^5)*c + (19*C*a*b^4 + 9*A*b^5)*d)*x
^6 + 168*(10*(17*C*a*b^4 + 8*A*b^5)*c + (93*D*a^2*b^3 + 170*B*a*b^4)*d)*x^
5 + 3840*((5*D*a^2*b^3 + 9*B*a*b^4)*c + (5*C*a^2*b^3 + 9*A*a*b^4)*d)*x^4 +
 210*(2*(59*C*a^2*b^3 + 104*A*a*b^4)*c + (3*D*a^3*b^2 + 118*B*a^2*b^3)*d)*
x^3 + 1280*((D*a^3*b^2 + 27*B*a^2*b^3)*c + (C*a^3*b^2 + 27*A*a^2*b^3)*d)*x
^2 - 1280*(2*D*a^4*b - 9*B*a^3*b^2)*c - 1280*(2*C*a^4*b - 9*A*a^3*b^2)*d +
 315*(2*(5*C*a^3*b^2 + 88*A*a^2*b^3)*c - (3*D*a^4*b - 10*B*a^3*b^2)*d)*x)*
sqrt(b*x^2 + a))/b^3, 1/80640*(315*(10*(C*a^4*b - 8*A*a^3*b^2)*c - (3*D*a^
5 - 10*B*a^4*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8064*D*b
^5*d*x^9 + 8960*(D*b^5*c + C*b^5*d)*x^8 + 1008*(10*C*b^5*c + (21*D*a*b^4 +
 10*B*b^5)*d)*x^7 + 1280*((19*D*a*b^4 + 9*B*b^5)*c + (19*C*a*b^4 + 9*A*b^5
)*d)*x^6 + 168*(10*(17*C*a*b^4 + 8*A*b^5)*c + (93*D*a^2*b^3 + 170*B*a*b^4)
*d)*x^5 + 3840*((5*D*a^2*b^3 + 9*B*a*b^4)*c + (5*C*a^2*b^3 + 9*A*a*b^4)*d)
*x^4 + 210*(2*(59*C*a^2*b^3 + 104*A*a*b^4)*c + (3*D*a^3*b^2 + 118*B*a^2*b^
3)*d)*x^3 + 1280*((D*a^3*b^2 + 27*B*a^2*b^3)*c + (C*a^3*b^2 + 27*A*a^2*b^3
)*d)*x^2 - 1280*(2*D*a^4*b - 9*B*a^3*b^2)*c - 1280*(2*C*a^4*b - 9*A*a^3*b^
2)*d + 315*(2*(5*C*a^3*b^2 + 88*A*a^2*b^3)*c - (3*D*a^4*b - 10*B*a^3*b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(311) = 622$.

Time = 0.92 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.61

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b**2*d*x**9/10 + x**8*(C*b**3*d + D*b**3*c)
/(9*b) + x**7*(B*b**3*d + C*b**3*c + 21*D*a*b**2*d/10)/(8*b) + x**6*(A*b**
3*d + B*b**3*c + 3*C*a*b**2*d + 3*D*a*b**2*c - 8*a*(C*b**3*d + D*b**3*c)/(
9*b))/(7*b) + x**5*(A*b**3*c + 3*B*a*b**2*d + 3*C*a*b**2*c + 3*D*a**2*b*d
- 7*a*(B*b**3*d + C*b**3*c + 21*D*a*b**2*d/10)/(8*b))/(6*b) + x**4*(3*A*a*
b**2*d + 3*B*a*b**2*c + 3*C*a**2*b*d + 3*D*a**2*b*c - 6*a*(A*b**3*d + B*b*
*3*c + 3*C*a*b**2*d + 3*D*a*b**2*c - 8*a*(C*b**3*d + D*b**3*c)/(9*b))/(7*b
))/(5*b) + x**3*(3*A*a*b**2*c + 3*B*a**2*b*d + 3*C*a**2*b*c + D*a**3*d - 5
*a*(A*b**3*c + 3*B*a*b**2*d + 3*C*a*b**2*c + 3*D*a**2*b*d - 7*a*(B*b**3*d
+ C*b**3*c + 21*D*a*b**2*d/10)/(8*b))/(6*b))/(4*b) + x**2*(3*A*a**2*b*d +
3*B*a**2*b*c + C*a**3*d + D*a**3*c - 4*a*(3*A*a*b**2*d + 3*B*a*b**2*c + 3*
C*a**2*b*d + 3*D*a**2*b*c - 6*a*(A*b**3*d + B*b**3*c + 3*C*a*b**2*d + 3*D*
a*b**2*c - 8*a*(C*b**3*d + D*b**3*c)/(9*b))/(7*b))/(5*b))/(3*b) + x*(3*A*a
**2*b*c + B*a**3*d + C*a**3*c - 3*a*(3*A*a*b**2*c + 3*B*a**2*b*d + 3*C*a**
2*b*c + D*a**3*d - 5*a*(A*b**3*c + 3*B*a*b**2*d + 3*C*a*b**2*c + 3*D*a**2*
b*d - 7*a*(B*b**3*d + C*b**3*c + 21*D*a*b**2*d/10)/(8*b))/(6*b))/(4*b))/(2
*b) + (A*a**3*d + B*a**3*c - 2*a*(3*A*a**2*b*d + 3*B*a**2*b*c + C*a**3*d +
 D*a**3*c - 4*a*(3*A*a*b**2*d + 3*B*a*b**2*c + 3*C*a**2*b*d + 3*D*a**2*b*c
 - 6*a*(A*b**3*d + B*b**3*c + 3*C*a*b**2*d + 3*D*a*b**2*c - 8*a*(C*b**3*d
+ D*b**3*c)/(9*b))/(7*b))/(5*b))/(3*b))/b) + (A*a**3*c - a*(3*A*a**2*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{7}{2}}Ddx^3}{10\,b} + \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Acx + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aacx + \frac{5}{16}\sqrt{bx^2+a}Aa^2cx - \frac{3\,(bx^2+a)^{\frac{7}{2}}Dadx}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Da^2dx}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Da^3dx}{128\,b^2} + \frac{3\sqrt{bx^2+a}Da^4dx}{256\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}(Dc+Cd)x^2}{9\,b} + \frac{5\,Aa^3c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} + \frac{3\,Da^5d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{7}{2}}Bc}{7\,b} + \frac{(bx^2+a)^{\frac{7}{2}}Ad}{7\,b} + \frac{(bx^2+a)^{\frac{7}{2}}(Cc+Bd)x}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}(Cc+Bd)ax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)a^2x}{192\,b} - \frac{5\sqrt{bx^2+a}(Cc+Bd)a^3x}{128\,b} - \frac{5\,(Cc+Bd)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{7}{2}}(Dc+Cd)a}{63\,b^2}$$

input

```
integrate((d*x+c)*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output

```
1/10*(b*x^2 + a)^(7/2)*D*d*x^3/b + 1/6*(b*x^2 + a)^(5/2)*A*c*x + 5/24*(b*x
^2 + a)^(3/2)*A*a*c*x + 5/16*sqrt(b*x^2 + a)*A*a^2*c*x - 3/80*(b*x^2 + a)^
(7/2)*D*a*d*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*D*a^2*d*x/b^2 + 1/128*(b*x^2 +
 a)^(3/2)*D*a^3*d*x/b^2 + 3/256*sqrt(b*x^2 + a)*D*a^4*d*x/b^2 + 1/9*(b*x^2
 + a)^(7/2)*(D*c + C*d)*x^2/b + 5/16*A*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b
) + 3/256*D*a^5*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 1/7*(b*x^2 + a)^(7/2)*B
*c/b + 1/7*(b*x^2 + a)^(7/2)*A*d/b + 1/8*(b*x^2 + a)^(7/2)*(C*c + B*d)*x/b
 - 1/48*(b*x^2 + a)^(5/2)*(C*c + B*d)*a*x/b - 5/192*(b*x^2 + a)^(3/2)*(C*c
 + B*d)*a^2*x/b - 5/128*sqrt(b*x^2 + a)*(C*c + B*d)*a^3*x/b - 5/128*(C*c +
 B*d)*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2 + a)^(7/2)*(D*c + C
*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.40

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,Db^2dx+\frac{10\,(Db^{10}c+Cb^{10}d)}{b^8}\right)x+\frac{9\,(10\,Cb^{10}c+21}{}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$+\frac{(10\,Ca^4bc-80\,Aa^3b^2c-3\,Da^5d+10\,Ba^4bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*D*b^2*d*x + 10*(D*b^10*c + C*
b^10*d)/b^8)*x + 9*(10*C*b^10*c + 21*D*a*b^9*d + 10*B*b^10*d)/b^8)*x + 80*
(19*D*a*b^9*c + 9*B*b^10*c + 19*C*a*b^9*d + 9*A*b^10*d)/b^8)*x + 21*(170*C
*a*b^9*c + 80*A*b^10*c + 93*D*a^2*b^8*d + 170*B*a*b^9*d)/b^8)*x + 480*(5*D
*a^2*b^8*c + 9*B*a*b^9*c + 5*C*a^2*b^8*d + 9*A*a*b^9*d)/b^8)*x + 105*(118*
C*a^2*b^8*c + 208*A*a*b^9*c + 3*D*a^3*b^7*d + 118*B*a^2*b^8*d)/b^8)*x + 64
0*(D*a^3*b^7*c + 27*B*a^2*b^8*c + C*a^3*b^7*d + 27*A*a^2*b^8*d)/b^8)*x + 3
15*(10*C*a^3*b^7*c + 176*A*a^2*b^8*c - 3*D*a^4*b^6*d + 10*B*a^3*b^7*d)/b^8
)*x - 1280*(2*D*a^4*b^6*c - 9*B*a^3*b^7*c + 2*C*a^4*b^6*d - 9*A*a^3*b^7*d)
/b^8) + 1/256*(10*C*a^4*b*c - 80*A*a^3*b^2*c - 3*D*a^5*d + 10*B*a^4*b*d)*l
og(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^2+a\right)^{5/2}\,(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(5/2)*(c + d*x)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.11

$$\int (c+dx)\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{-5120\sqrt{b\,x^2+a}\,a^4bcd-945\sqrt{b\,x^2+a}\,a^4b\,d^2x+55440\sqrt{b\,x^2+a}\,a^3b^3cx+34560\sqrt{b\,x^2+a}\,a^3}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(11520*sqrt(a + b*x**2)*a**4*b**2*d - 5120*sqrt(a + b*x**2)*a**4*b*c*d - 9
45*sqrt(a + b*x**2)*a**4*b*d**2*x + 55440*sqrt(a + b*x**2)*a**3*b**3*c*x +
 11520*sqrt(a + b*x**2)*a**3*b**3*c + 34560*sqrt(a + b*x**2)*a**3*b**3*d*x
**2 + 3150*sqrt(a + b*x**2)*a**3*b**3*d*x + 3150*sqrt(a + b*x**2)*a**3*b**
2*c**2*x + 2560*sqrt(a + b*x**2)*a**3*b**2*c*d*x**2 + 630*sqrt(a + b*x**2)
*a**3*b**2*d**2*x**3 + 43680*sqrt(a + b*x**2)*a**2*b**4*c*x**3 + 34560*sqr
t(a + b*x**2)*a**2*b**4*c*x**2 + 34560*sqrt(a + b*x**2)*a**2*b**4*d*x**4 +
 24780*sqrt(a + b*x**2)*a**2*b**4*d*x**3 + 24780*sqrt(a + b*x**2)*a**2*b**
3*c**2*x**3 + 38400*sqrt(a + b*x**2)*a**2*b**3*c*d*x**4 + 15624*sqrt(a + b
*x**2)*a**2*b**3*d**2*x**5 + 13440*sqrt(a + b*x**2)*a*b**5*c*x**5 + 34560*
sqrt(a + b*x**2)*a*b**5*c*x**4 + 11520*sqrt(a + b*x**2)*a*b**5*d*x**6 + 28
560*sqrt(a + b*x**2)*a*b**5*d*x**5 + 28560*sqrt(a + b*x**2)*a*b**4*c**2*x*
*5 + 48640*sqrt(a + b*x**2)*a*b**4*c*d*x**6 + 21168*sqrt(a + b*x**2)*a*b**
4*d**2*x**7 + 11520*sqrt(a + b*x**2)*b**6*c*x**6 + 10080*sqrt(a + b*x**2)*
b**6*d*x**7 + 10080*sqrt(a + b*x**2)*b**5*c**2*x**7 + 17920*sqrt(a + b*x**
2)*b**5*c*d*x**8 + 8064*sqrt(a + b*x**2)*b**5*d**2*x**9 + 945*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d**2 + 25200*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*c - 3150*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*d - 3150*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c**2)/(80640*b**3)
```

3.88 $\int (a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int (a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)\,dx = \frac{5a^2(8Ab-aC)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aC)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aC)x(a+bx^2)^{5/2}}{48b} + \frac{(bB-aD)(a+bx^2)^{7/2}}{7b^2} + \frac{Cx(a+bx^2)^{7/2}}{8b} + \frac{D(a+bx^2)^{9/2}}{9b^2} + \frac{5a^3(8Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^2*(8*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+5/192*a*(8*A*b-C*a)*x*(b*x^2+a)^
(3/2)/b+1/48*(8*A*b-C*a)*x*(b*x^2+a)^(5/2)/b+1/7*(B*b-D*a)*(b*x^2+a)^(7/2)
/b^2+1/8*C*x*(b*x^2+a)^(7/2)/b+1/9*D*(b*x^2+a)^(9/2)/b^2+5/128*a^3*(8*A*b-
C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{\sqrt{a+bx^2}(-256a^4D+a^3b(1152B+x(315C+128Dx))+16b^4x^5(84A+x(72B+7x(9C+8}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-256*a^4*D + a^3*b*(1152*B + x*(315*C + 128*D*x)) + 16*b
^4*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))) + 8*a*b^3*x^3*(546*A + x*(432
*B + x*(357*C + 304*D*x))) + 6*a^2*b^2*x*(924*A + x*(576*B + x*(413*C + 32
0*D*x)))) + 315*a^3*Sqrt[b]*(-8*A*b + a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(8064*b^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2346, 2346, 27, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int \left(bx^2+a\right)^{5/2} \left(9bCx^2+(9bB-2aD)x+9Ab\right)dx}{9b}+\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\frac{\int b(9(8Ab-aC)+8(9bB-2aD)x)\left(bx^2+a\right)^{5/2}dx}{8b}+\frac{9}{8}Cx\left(a+bx^2\right)^{7/2}}{9b}+\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{8}\int (9(8Ab-aC)+8(9bB-2aD)x)\left(bx^2+a\right)^{5/2}dx+\frac{9}{8}Cx\left(a+bx^2\right)^{7/2}}{9b}+\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\int \left(bx^2+a\right)^{5/2}dx+\frac{8\left(a+bx^2\right)^{7/2}(9bB-2aD)}{7b}\right)+\frac{9}{8}Cx\left(a+bx^2\right)^{7/2}}{9b}+\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{8\left(a+bx^2\right)^{7/2}(9bB-2aD)}{7b}\right)+\frac{9}{8}Cx\left(a+bx^2\right)^{7/2}}{9b}+\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{8\left(a+bx^2\right)^{7/2}(9bB-2aD)}{7b}\right)+\frac{9}{8}Cx(}{9b}$$

$$\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{8\left(a+bx^2\right)^{7/2}(9b}{7b}\right.}{9b}$$

$$\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{8(a+}{}\right.}{9b}$$

$$\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{8}\left(9(8Ab-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{8(a+}{}\right.}{9b}$$

$$\frac{Dx^2\left(a+bx^2\right)^{7/2}}{9b}$$

input
```
Int[(a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(7/2))/(9*b) + ((9*C*x*(a + b*x^2)^(7/2))/8 + ((8*(9*b*
B - 2*a*D)*(a + b*x^2)^(7/2))/(7*b) + 9*(8*A*b - a*C)*((x*(a + b*x^2)^(5/2
))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/8)/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.09

method	result
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+C\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a \cdots}{\cdots}\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+C*(1/8*x*(b*x^2+a)
^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a
*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+D*
(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+1/7*B*(b*x^2+a)^(7/
2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.19

$$\int (a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)\, dx = \left[-\frac{315\,(Ca^4-8\,Aa^3b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(896\,Db^4x^8+1008\,Cb^4x^7 \cdots}{\cdots}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/16128*(315*(C*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(896*D*b^4*x^8 + 1008*C*b^4*x^7 + 128*(19*D*a*b^3 + 9
*B*b^4)*x^6 + 168*(17*C*a*b^3 + 8*A*b^4)*x^5 - 256*D*a^4 + 1152*B*a^3*b +
384*(5*D*a^2*b^2 + 9*B*a*b^3)*x^4 + 42*(59*C*a^2*b^2 + 104*A*a*b^3)*x^3 +
128*(D*a^3*b + 27*B*a^2*b^2)*x^2 + 63*(5*C*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b
*x^2 + a))/b^2, 1/8064*(315*(C*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (896*D*b^4*x^8 + 1008*C*b^4*x^7 + 128*(19*D*a*b^3 + 9*
B*b^4)*x^6 + 168*(17*C*a*b^3 + 8*A*b^4)*x^5 - 256*D*a^4 + 1152*B*a^3*b + 3
84*(5*D*a^2*b^2 + 9*B*a*b^3)*x^4 + 42*(59*C*a^2*b^2 + 104*A*a*b^3)*x^3 + 1
28*(D*a^3*b + 27*B*a^2*b^2)*x^2 + 63*(5*C*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*
x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(175) = 350$.

Time = 0.59 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.63

$$\int (a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3) \, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cb^2x^7}{8}+\frac{Db^2x^8}{9}+\frac{x^6\left(Bb^3+\frac{19Dab^2}{9}\right)}{7b}+\frac{x^5\left(Ab^3+\frac{17Cab^2}{8}\right)}{6b}+\frac{x^4\cdot\left(3Bab^2+3Da^2b-\frac{6a\left(Bb^3+\frac{19Dab^2}{9}\right)}{7b}\right.}{5b}\right. \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b**2*x**7/8 + D*b**2*x**8/9 + x**6*(B*b**3
+ 19*D*a*b**2/9)/(7*b) + x**5*(A*b**3 + 17*C*a*b**2/8)/(6*b) + x**4*(3*B*a
*b**2 + 3*D*a**2*b - 6*a*(B*b**3 + 19*D*a*b**2/9)/(7*b))/(5*b) + x**3*(3*A
*a*b**2 + 3*C*a**2*b - 5*a*(A*b**3 + 17*C*a*b**2/8)/(6*b))/(4*b) + x**2*(3
*B*a**2*b + D*a**3 - 4*a*(3*B*a*b**2 + 3*D*a**2*b - 6*a*(B*b**3 + 19*D*a*b
**2/9)/(7*b))/(5*b))/(3*b) + x*(3*A*a**2*b + C*a**3 - 3*a*(3*A*a*b**2 + 3*
C*a**2*b - 5*a*(A*b**3 + 17*C*a*b**2/8)/(6*b))/(4*b))/(2*b) + (B*a**3 - 2*
a*(3*B*a**2*b + D*a**3 - 4*a*(3*B*a*b**2 + 3*D*a**2*b - 6*a*(B*b**3 + 19*D
*a*b**2/9)/(7*b))/(5*b))/(3*b))/b) + (A*a**3 - a*(3*A*a**2*b + C*a**3 - 3*
a*(3*A*a*b**2 + 3*C*a**2*b - 5*a*(A*b**3 + 17*C*a*b**2/8)/(6*b))/(4*b))/(2
*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(A*x + B*x**2/2 + C
*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{7}{2}}Dx^2}{9\,b} + \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}Ax + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}Aax + \frac{5}{16}\,\sqrt{bx^2+a}Aa^2x + \frac{(bx^2+a)^{\frac{7}{2}}Cx}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Cax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ca^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}Ca^3x}{128\,b} - \frac{5\,Ca^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - \frac{2\,(bx^2+a)^{\frac{7}{2}}Da}{63\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}B}{7\,b}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*D*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 +
a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2 + a)*A*a^2*x + 1/8*(b*x^2 + a)^(7/2)*C*x/
b - 1/48*(b*x^2 + a)^(5/2)*C*a*x/b - 5/192*(b*x^2 + a)^(3/2)*C*a^2*x/b - 5
/128*sqrt(b*x^2 + a)*C*a^3*x/b - 5/128*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2
) + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/63*(b*x^2 + a)^(7/2)*D*a
/b^2 + 1/7*(b*x^2 + a)^(7/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8064}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\,(8\,Db^2x+9\,Cb^2)x+\frac{8\,(19\,Dab^8+9\,Bb^9)}{b^7}\right)x+\frac{21\,(17\,Cab^8}{b^7}\right.\right.\right.\right.\right.\right. + \frac{5\,(Ca^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8064*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*D*b^2*x + 9*C*b^2)*x + 8*(19*D*a*
b^8 + 9*B*b^9)/b^7)*x + 21*(17*C*a*b^8 + 8*A*b^9)/b^7)*x + 48*(5*D*a^2*b^7
 + 9*B*a*b^8)/b^7)*x + 21*(59*C*a^2*b^7 + 104*A*a*b^8)/b^7)*x + 64*(D*a^3*
b^6 + 27*B*a^2*b^7)/b^7)*x + 63*(5*C*a^3*b^6 + 88*A*a^2*b^7)/b^7)*x - 128*
(2*D*a^4*b^5 - 9*B*a^3*b^6)/b^7) + 5/128*(C*a^4 - 8*A*a^3*b)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^2+a\right)^{5/2}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.75

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{-256\sqrt{b\,x^2+a}\,a^4d+5544\sqrt{b\,x^2+a}\,a^3b^2x+1152\sqrt{b\,x^2+a}\,a^3b^2+315\sqrt{b\,x^2+a}\,a^3bcx+128}{}$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 256*sqrt(a + b*x**2)*a**4*d + 5544*sqrt(a + b*x**2)*a**3*b**2*x + 1152
*sqrt(a + b*x**2)*a**3*b**2 + 315*sqrt(a + b*x**2)*a**3*b*c*x + 128*sqrt(a
 + b*x**2)*a**3*b*d*x**2 + 4368*sqrt(a + b*x**2)*a**2*b**3*x**3 + 3456*sqr
t(a + b*x**2)*a**2*b**3*x**2 + 2478*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 19
20*sqrt(a + b*x**2)*a**2*b**2*d*x**4 + 1344*sqrt(a + b*x**2)*a*b**4*x**5 +
 3456*sqrt(a + b*x**2)*a*b**4*x**4 + 2856*sqrt(a + b*x**2)*a*b**3*c*x**5 +
 2432*sqrt(a + b*x**2)*a*b**3*d*x**6 + 1152*sqrt(a + b*x**2)*b**5*x**6 + 1
008*sqrt(a + b*x**2)*b**4*c*x**7 + 896*sqrt(a + b*x**2)*b**4*d*x**8 + 2520
*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b - 315*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c)/(8064*b**2)
```

3.89 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 726

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{(bc^2+ad^2)^2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{d^8}$$

$$-\frac{(5a^3d^6D+40a^2bd^4(cCd-Bd^2-c^2D)+64b^3c^3(c^2Cd-Bcd^2+Ad^3-c^3D)+112ab^2cd^2(c^2Cd-Bcd^2}{128bd^7}$$

$$+\frac{(bc^2+ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{3/2}}{3d^6}$$

$$-\frac{(5a^2d^4D+40abd^2(cCd-Bd^2-c^2D)+48b^2c(c^2Cd-Bcd^2+Ad^3-c^3D))x(a+bx^2)^{3/2}}{192bd^5}$$

$$+\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{5/2}}{5d^4}$$

$$-\frac{(ad^2D+8b(cCd-Bd^2-c^2D))x(a+bx^2)^{5/2}}{48bd^3}$$

$$+\frac{(8Cd-15cD)(a+bx^2)^{7/2}}{56bd^2}+\frac{D(c+dx)(a+bx^2)^{7/2}}{8bd^2}$$

$$-\frac{(5a^4d^8D+40a^3bd^6(cCd-Bd^2-c^2D)+128b^4c^5(c^2Cd-Bcd^2+Ad^3-c^3D)+320ab^3c^3d^2(c^2Cd-Bc}{128b^{3/2}d^9}$$

$$-\frac{(bc^2+ad^2)^{5/2}(c^2Cd-Bcd^2+Ad^3-c^3D)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^9}$$

output
```
(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8-1/128*(5
*a^3*d^6*D+40*a^2*b*d^4*(-B*d^2+C*c*d-D*c^2)+64*b^3*c^3*(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)+112*a*b^2*c*d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x*(b*x^2+a)^(1/
2)/b/d^7+1/3*(a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(3/2)/d
^6-1/192*(5*a^2*d^4*D+40*a*b*d^2*(-B*d^2+C*c*d-D*c^2)+48*b^2*c*(A*d^3-B*c*
d^2+C*c^2*d-D*c^3))*x*(b*x^2+a)^(3/2)/b/d^5+1/5*(A*d^3-B*c*d^2+C*c^2*d-D*c
^3)*(b*x^2+a)^(5/2)/d^4-1/48*(a*d^2*D+8*b*(-B*d^2+C*c*d-D*c^2))*x*(b*x^2+a
)^(5/2)/b/d^3+1/56*(8*C*d-15*D*c)*(b*x^2+a)^(7/2)/b/d^2+1/8*D*(d*x+c)*(b*x
^2+a)^(7/2)/b/d^2-1/128*(5*a^4*d^8*D+40*a^3*b*d^6*(-B*d^2+C*c*d-D*c^2)+128
*b^4*c^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)+320*a*b^3*c^3*d^2*(A*d^3-B*c*d^2+C*
c^2*d-D*c^3)+240*a^2*b^2*c*d^4*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^9-(a*d^2+b*c^2)^(5/2)*(A*d^3-B*c*d^2+C*c^
2*d-D*c^3)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^9
```

Mathematica [A] (verified)

Time = 6.43 (sec) , antiderivative size = 715, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(15a^3d^6(128Cd-128cD+35dDx)+2a^2bd^4\left(-10304c^3D+28c^2d(368C+16\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(15*a^3*d^6*(128*C*d - 128*c*D + 35*d*D*x) + 2*a^2*b*d
^4*(-10304*c^3*D + 28*c^2*d*(368*C + 165*D*x) - 4*c*d^2*(2576*B + 15*x*(77
*C + 48*D*x)) + d^3*(10304*A + 5*x*(924*B + 576*C*x + 413*D*x^2))) - 16*b^
3*(840*c^7*D - 420*c^6*d*(2*C + D*x) + 140*c^5*d^2*(6*B + x*(3*C + 2*D*x))
 - 70*c^4*d^3*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 14*c^3*d^4*x*(30*A + x*
(20*B + 3*x*(5*C + 4*D*x))) - 14*c^2*d^5*x^2*(20*A + x*(15*B + 2*x*(6*C +
5*D*x))) + 2*c*d^6*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) - d^7*x^4*
(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x)))) + 8*a*b^2*d^2*(-3920*c^5*D + 70*
c^4*d*(56*C + 27*D*x) + d^5*x^2*(1232*A + 910*B*x + 720*C*x^2 + 595*D*x^3)
 - 14*c^3*d^2*(280*B + x*(135*C + 88*D*x)) + 14*c^2*d^3*(280*A + x*(135*B
+ x*(88*C + 65*D*x))) - 2*c*d^4*x*(945*A + x*(616*B + 5*x*(91*C + 72*D*x))
))))/b - 26880*(-(b*c^2) - a*d^2)^(5/2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^
3*D)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]
] - (105*(-5*a^4*d^8*D + 40*a^3*b*d^6*(-(c*C*d) + B*d^2 + c^2*D) + 128*b^4
*c^5*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D) + 320*a*b^3*c^3*d^2*(-(c^2*C*d
) + B*c*d^2 - A*d^3 + c^3*D) + 240*a^2*b^2*c*d^4*(-(c^2*C*d) + B*c*d^2 - A
*d^3 + c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(13440*d^9)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 713, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {2185, 2185, 27, 682, 25, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{(bx^2+a)^{5/2}(b(8Cd-15cD)x^2d^2+(8Abd-acD)d^2+(-7bDc^2+8bBd^2-ad^2D)xd)}{c+dx}dx}{8bd^3}+\frac{D(a+bx^2)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{7bd^3\left(d(8Abd-acD)-\left(aDd^2+8b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{c+dx}dx}{7bd^2}+\frac{1}{7}d\left(a+bx^2\right)^{7/2}(8Cd-15cD)}{8bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{\left(d(8Abd-acD)-\left(aDd^2+8b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{c+dx}dx+\frac{1}{7}d\left(a+bx^2\right)^{7/2}(8Cd-15cD)}{8bd^3}+\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{d\left(\frac{\int -\frac{b\left(ad\left(5acd^2D-8b\left(-Dc^3+Cdc^2-Bd^2c+6Ad^3\right)\right)+\left(6bc(8Abd-acD)d^2+\left(6bc^2+5ad^2\right)\left(aDd^2+8b\left(-Dc^2+Cdc-Bd^2\right)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6bd^2}+\frac{(a+bx^2 \cdots}{\cdots}\right.}{8bd^3 \cdots}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{\int \frac{b\left(ad\left(5acd^2D-8b\left(-Dc^3+Cdc^2-Bd^2c+6Ad^3\right)\right) \cdots}{\cdots}}{\cdots}\right.}{8bd^3 \cdots}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{\int \frac{\left(ad\left(5acd^2D-8b\left(-Dc^3+Cdc^2-Bd^2c+6Ad^3\right)\right)+ \cdots}{\cdots}}{\cdots}\right.}{8bd^3 \cdots}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 682$$

$$d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{\int \frac{3b\left(ad\left(5a^2cDd^4-8ab\left(-3Dc^3+3Cdc^2-3Bd^2c+8\cdots\right.\right.\right.}{\cdots}}{\cdots}\right.\cdots$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\int \frac{\left(ad\left(5a^2cDd^4-8ab\left(-3Dc^3+3Cdc^2-3Bd^2c+8A\cdots\right.\right.\right.}{\cdots}}{\cdots}\right.\cdots$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 682$$

$$d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\left(\frac{\int \frac{b\left(ad\left(5a^3cDd^6-8a^2b\left(-11Dc^3+11Cdc^2-11B\cdots\right.\right.\right.}{\cdots}}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\left(a+bx^2\right)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\left(\frac{\int \frac{ad\left(5a^3cDd^6-8a^2b\left(-11Dc^3+11Cdc^2-11Bd^2\cdots\right.\right.}{\cdots}}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$$\Big\downarrow \; 719$$

$$\frac{d\left(\frac{(a+bx^2)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\left(\frac{\left(5a^4d^8D+40a^3bd^6\left(-Bd^2+c^2(-D)+cCd\right)+240\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$\Big\downarrow$ 224

$$\frac{d\left(\frac{(a+bx^2)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\left(\frac{\left(5a^4d^8D+40a^3bd^6\left(-Bd^2+c^2(-D)+cCd\right)+240\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{(a+bx^2)^{5/2}\left(48b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-5dx\left(ad^2D+8b\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{30d^2}-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^4d^8D+40a^3bd^6\left(-B\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$\Big\downarrow$ 488

$$\frac{d\left(\frac{(a+bx^2)^{5/2}(48b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-5dx(ad^2D+8b(-Bd^2+c^2(-D)+cCd)))}{30d^2}-\frac{3\left(\frac{128b\left(ad^2+bc^2\right)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

$\downarrow$ 219

$$\frac{d\left(\frac{(a+bx^2)^{5/2}(48b(Ad^3-Bcd^2+c^3(-D)+c^2Cd)-5dx(ad^2D+8b(-Bd^2+c^2(-D)+cCd)))}{30d^2}-\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^4d^8D+40a^3bd^6\left(-B\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{D\left(a+bx^2\right)^{7/2}(c+dx)}{8bd^2}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(D*(c + d*x)*(a + b*x^2)^(7/2))/(8*b*d^2) + ((d*(8*C*d - 15*c*D)*(a + b*x^
2)^(7/2))/7 + d*(((48*b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) - 5*d*(a*d^2*D
 + 8*b*(c*C*d - B*d^2 - c^2*D))*x)*(a + b*x^2)^(5/2))/(30*d^2) - (-1/4*((6
4*b*(b*c^2 + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) - d*(6*b*c*d^2*(8*
A*b*d - a*c*D) + (6*b*c^2 + 5*a*d^2)*(a*d^2*D + 8*b*(c*C*d - B*d^2 - c^2*D
)))*x)*(a + b*x^2)^(3/2))/d^2 + (3*(-1/2*((128*b*(b*c^2 + a*d^2)^2*(c^2*C*
d - B*c*d^2 + A*d^3 - c^3*D) - d*(5*a^3*d^6*D + 40*a^2*b*d^4*(c*C*d - B*d^
2 - c^2*D) + 64*b^3*c^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) + 112*a*b^2*c*
d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*x)*Sqrt[a + b*x^2])/d^2 + (((5*a^
4*d^8*D + 40*a^3*b*d^6*(c*C*d - B*d^2 - c^2*D) + 128*b^4*c^5*(c^2*C*d - B*
c*d^2 + A*d^3 - c^3*D) + 320*a*b^3*c^3*d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D) + 240*a^2*b^2*c*d^4*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + (128*b*(b*c^2 + a*d^2)^(5/2)*(c^2*
C*d - B*c*d^2 + A*d^3 - c^3*D)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*
Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*d^2))/(6*d^2)))/(8*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 1196, normalized size of antiderivative = 1.65

method	result	size
default	Expression too large to display	1196

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d^2*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+D*c^2*(
1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^
(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*d*(C*d-D*c)*(b*x^
2+a)^(7/2)/b+D*d^2*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)
+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))))))-C*c*d*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x
*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))))))+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4*(1/5*(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-b*c/d*(1/8*(2*b*(x+c/d)-2*b*c/d)/b*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+3/16*(4*b*(a*d^2+b*c
^2)/d^2-4*b^2*c^2/d^2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2
)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2))))+(a*d^2+b*c^2)/d^2*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*
c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(679) = 1358$.

Time = 0.24 (sec) , antiderivative size = 1563, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*D*b^2*c^6*x/d^7 - 1/2*sqrt(b*x^2 + a)*C*b^2*c^5*x/d^6
+ 1/4*(b*x^2 + a)^(3/2)*D*b*c^4*x/d^5 + 7/8*sqrt(b*x^2 + a)*D*a*b*c^4*x/d^
5 + 1/2*sqrt(b*x^2 + a)*B*b^2*c^4*x/d^5 - 1/4*(b*x^2 + a)^(3/2)*C*b*c^3*x/
d^4 - 7/8*sqrt(b*x^2 + a)*C*a*b*c^3*x/d^4 - 1/2*sqrt(b*x^2 + a)*A*b^2*c^3*
x/d^4 + 1/6*(b*x^2 + a)^(5/2)*D*c^2*x/d^3 + 5/24*(b*x^2 + a)^(3/2)*D*a*c^2
*x/d^3 + 5/16*sqrt(b*x^2 + a)*D*a^2*c^2*x/d^3 + 1/4*(b*x^2 + a)^(3/2)*B*b*
c^2*x/d^3 + 7/8*sqrt(b*x^2 + a)*B*a*b*c^2*x/d^3 - 1/6*(b*x^2 + a)^(5/2)*C*
c*x/d^2 - 5/24*(b*x^2 + a)^(3/2)*C*a*c*x/d^2 - 5/16*sqrt(b*x^2 + a)*C*a^2*
c*x/d^2 - 1/4*(b*x^2 + a)^(3/2)*A*b*c*x/d^2 - 7/8*sqrt(b*x^2 + a)*A*a*b*c*
x/d^2 + 1/6*(b*x^2 + a)^(5/2)*B*x/d + 5/24*(b*x^2 + a)^(3/2)*B*a*x/d + 5/1
6*sqrt(b*x^2 + a)*B*a^2*x/d + 1/8*(b*x^2 + a)^(7/2)*D*x/(b*d) - 1/48*(b*x^
2 + a)^(5/2)*D*a*x/(b*d) - 5/192*(b*x^2 + a)^(3/2)*D*a^2*x/(b*d) - 5/128*s
qrt(b*x^2 + a)*D*a^3*x/(b*d) + D*b^(5/2)*c^8*arcsinh(b*x/sqrt(a*b))/d^9 -
C*b^(5/2)*c^7*arcsinh(b*x/sqrt(a*b))/d^8 + 5/2*D*a*b^(3/2)*c^6*arcsinh(b*x
/sqrt(a*b))/d^7 + B*b^(5/2)*c^6*arcsinh(b*x/sqrt(a*b))/d^7 - 5/2*C*a*b^(3/
2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 - A*b^(5/2)*c^5*arcsinh(b*x/sqrt(a*b))/d
^6 + 15/8*D*a^2*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 + 5/2*B*a*b^(3/2)*c
^4*arcsinh(b*x/sqrt(a*b))/d^5 - 15/8*C*a^2*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*
b))/d^4 - 5/2*A*a*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 5/16*D*a^3*c^2*
arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) + 15/8*B*a^2*sqrt(b)*c^2*arcsinh(b...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{dx+c}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

3.90 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 826

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{(15a^3d^6D-161a^2bd^4(2cCd-Bd^2-3c^2D)-105b^3c^3(6c^2C\ldots}{\ldots}$$
$$+\frac{(11a^2d^4(Cd-2cD)+8b^2c^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)+18abd^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\ldots}{16d^7}$$
$$+\frac{(45a^2d^4D-77abd^2(2cCd-Bd^2-3c^2D)-35b^2c(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))\,x^2\sqrt{a+bx^2}}{105d^6}$$
$$+\frac{b(13ad^2(Cd-2cD)+6b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x^3\sqrt{a+bx^2}}{24d^5}$$
$$+\frac{b(15ad^2D-7b(2cCd-Bd^2-3c^2D))\,x^4\sqrt{a+bx^2}}{35d^4}+\frac{b^2(Cd-2cD)x^5\sqrt{a+bx^2}}{6d^3}$$
$$+\frac{b^2Dx^6\sqrt{a+bx^2}}{7d^2}-\frac{(bc^2+ad^2)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{d^8(c+dx)}$$
$$+\frac{(5a^3d^6(Cd-2cD)+16b^3c^4(7c^2Cd-6Bcd^2+5Ad^3-8c^3D)+40ab^2c^2d^2(5c^2Cd-4Bcd^2+3Ad^3-6c^3\ldots}{16\sqrt{b}d^9}$$
$$+\frac{(bc^2+ad^2)^{3/2}\,(ad^2(2cCd-Bd^2-3c^2D)+bc(7c^2Cd-6Bcd^2+5Ad^3-8c^3D))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)\ldots}{d^9}$$

output
```
1/105*(15*a^3*d^6*D-161*a^2*b*d^4*(-B*d^2+2*C*c*d-3*D*c^2)-105*b^3*c^3*(4*
A*d^3-5*B*c*d^2+6*C*c^2*d-7*D*c^3)-245*a*b^2*c*d^2*(2*A*d^3-3*B*c*d^2+4*C*
c^2*d-5*D*c^3))*(b*x^2+a)^(1/2)/b/d^8+1/16*(11*a^2*d^4*(C*d-2*D*c)+8*b^2*c
^2*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3)+18*a*b*d^2*(A*d^3-2*B*c*d^2+3*C*c
^2*d-4*D*c^3))*x*(b*x^2+a)^(1/2)/d^7+1/105*(45*a^2*d^4*D-77*a*b*d^2*(-B*d^
2+2*C*c*d-3*D*c^2)-35*b^2*c*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*x^2*(b*
x^2+a)^(1/2)/d^6+1/24*b*(13*a*d^2*(C*d-2*D*c)+6*b*(A*d^3-2*B*c*d^2+3*C*c^2
*d-4*D*c^3))*x^3*(b*x^2+a)^(1/2)/d^5+1/35*b*(15*a*d^2*D-7*b*(-B*d^2+2*C*c*
d-3*D*c^2))*x^4*(b*x^2+a)^(1/2)/d^4+1/6*b^2*(C*d-2*D*c)*x^5*(b*x^2+a)^(1/2
)/d^3+1/7*b^2*D*x^6*(b*x^2+a)^(1/2)/d^2-(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8/(d*x+c)+1/16*(5*a^3*d^6*(C*d-2*D*c)+16*b^3
*c^4*(5*A*d^3-6*B*c*d^2+7*C*c^2*d-8*D*c^3)+40*a*b^2*c^2*d^2*(3*A*d^3-4*B*c
*d^2+5*C*c^2*d-6*D*c^3)+30*a^2*b*d^4*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^9+(a*d^2+b*c^2)^(3/2)*(a*d^2*
(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(5*A*d^3-6*B*c*d^2+7*C*c^2*d-8*D*c^3))*arctan
h((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^9
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 722, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{d\sqrt{a+bx^2}\left(240a^3d^6D(c+dx)+4b^3\left(3360c^7D-420c^6d(7C-4Dx)+d^7x^4(105A+84Bx\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(240*a^3*d^6*D*(c + d*x) + 4*b^3*(3360*c^7*D - 420*c^6
*d*(7*C - 4*D*x) + d^7*x^4*(105*A + 84*B*x + 70*C*x^2 + 60*D*x^3) + 70*c^5
*d^2*(36*B - x*(21*C + 8*D*x)) - 70*c^4*d^3*(30*A - x*(18*B + 7*C*x + 4*D*
x^2)) + 7*c^2*d^5*x^2*(50*A + x*(30*B + 21*C*x + 16*D*x^2)) - 7*c^3*d^4*x*
(150*A + x*(60*B + 35*C*x + 24*D*x^2)) - c*d^6*x^3*(175*A + 2*x*(63*B + 49
*C*x + 40*D*x^2))) + a^2*b*d^4*(9408*c^3*D + c^2*(-6832*C*d + 5418*d*D*x)
+ c*d^2*(4256*B - x*(3997*C + 1590*D*x)) + d^3*(-1680*A + x*(2576*B + 15*x
*(77*C + 48*D*x)))) + 2*a*b^2*d^2*(11480*c^5*D - 140*c^4*d*(68*C - 43*D*x)
 + 7*c^3*d^2*(1080*B - x*(715*C + 276*D*x)) + d^5*x^2*(945*A + x*(616*B +
5*x*(91*C + 72*D*x))) + 7*c^2*d^3*(-800*A + x*(570*B + x*(229*C + 134*D*x)
)) - c*d^4*x*(2975*A + x*(1274*B + x*(777*C + 550*D*x))))))/(b*(c + d*x))
- 3360*(-(b*c^2) - a*d^2)^(3/2)*(a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*
(-7*c^2*C*d + 6*B*c*d^2 - 5*A*d^3 + 8*c^3*D))*ArcTan[(Sqrt[b]*(c + d*x) -
d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + (105*(-5*a^3*d^6*(C*d - 2*c*D
) - 30*a^2*b*d^4*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D) + 40*a*b^2*c^2*
d^2*(-5*c^2*C*d + 4*B*c*d^2 - 3*A*d^3 + 6*c^3*D) + 16*b^3*c^4*(-7*c^2*C*d
+ 6*B*c*d^2 - 5*A*d^3 + 8*c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqr
t[b])/(1680*d^9)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2182, 25, 2185, 27, 682, 25, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{5/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+6Bc-6Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+6Bc-6Ad\right)\right)x+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{\int \frac{7b\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+6Ad^3\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{c+dx}dx}{7bd^2}+\frac{1}{7}D\left(a+bx^2\right)^{7/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+6Ad^3\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{c+dx}dx}{d^2}+\frac{1}{7}D\left(a+bx^2\right)^{7/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{\frac{\frac{\int -\frac{b\left(bc^2+ad^2\right)\left(ad\left(-8Dc^2+7Cdc-6Bd^2\right)-\left(5a(Cd-2cD)d^2+6b\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6bd^2}-\frac{\left(a+bx^2\right)^{5/2}\left(6\left(ad^2\left(-Bd^2-3c^2D+2c\cdots\right.\right.\right.}{\cdots}}{d^2}\cdots}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{-\frac{\int \frac{b\left(bc^2+ad^2\right)\left(ad\left(-8Dc^2+7Cdc-6Bd^2\right)-\left(5a(Cd-2cD)d^2+6b\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6bd^2}-\frac{\left(a+bx^2\right)^{5/2}\left(6\left(ad^2\left(-Bd^2-3c^2D+2c\cdots\right.\right.\right.}{\cdots}}{d^2}\cdots}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(ad^2+bc^2\right)\int \frac{\left(ad\left(-8Dc^2+7Cdc-6Bd^2\right)-\left(5a(Cd-2cD)d^2+6b\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6d^2} - \frac{\left(a+bx^2\right)^{5/2}\left(6\left(ad^2\left(-Bd^2-3c^2D+2c\right.\right.\right.}{d^2}$$

$$ad^2+bc^2$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 682$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\int \frac{b\left(3ad\left(a\left(-14Dc^2+11Cdc-8Bd^2\right)d^2+2bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-\left(4abc\left(-8Dc^2+7Cdc-6Bd^2\right)d^2+\left(4bc^2+3ad^2\right)\left(5a(Cd-2cD)d^2\right.\right.\right.}{c+dx}}{4bd^2}\right.}{}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 27$$

$$-\frac{\left(ad^2+bc^2\right)\left(\frac{\int \frac{\left(3ad\left(a\left(-14Dc^2+11Cdc-8Bd^2\right)d^2+2bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-\left(4abc\left(-8Dc^2+7Cdc-6Bd^2\right)d^2+\left(4bc^2+3ad^2\right)\left(5a(Cd-2cD)d^2+\right.\right.\right.}{c+dx}}{4d^2}\right.}{}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 682$$

$$\frac{\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{\left(6\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-5d\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+\right.\right.\right.}{30d^2}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{(6(a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3))-5d(a(Cd-2cD)d^2+b(-8Dc^3+7Cdc^2-6Bd^2c+}{30d^2}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{(6(a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3))-5d(a(Cd-2cD)d^2+b(-8Dc^3+7Cdc^2-6Bd^2c+}{30d^2}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{(6(a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3))-5d(a(Cd-2cD)d^2+b(-8Dc^3+7Cdc^2-6Bd^2c+}{30d^2}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{\left(6\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-5d\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+\right.\right.\right.}{30d^2}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow \; 488$$

$$\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{\left(6\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-5d\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+\right.\right.\right.}{30d^2}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{7}\left(\frac{c^2}{d^2}+\frac{a}{b}\right)D\left(bx^2+a\right)^{7/2}+\frac{-\frac{\left(6\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-8Dc^3+7Cdc^2-6Bd^2c+5Ad^3\right)\right)-5d\left(a(Cd-2cD)d^2+b\left(-8Dc^3+7Cdc^2-6Bd^2c+\right.\right.\right.}{30d^2}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}$$

input

```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
-(((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 + a*
d^2)*(c + d*x))) + (((a/b + c^2/d^2)*D*(a + b*x^2)^(7/2))/7 + (-1/30*((6*(
a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 -
 8*c^3*D)) - 5*d*(a*d^2*(C*d - 2*c*D) + b*(7*c^2*C*d - 6*B*c*d^2 + 6*A*d^3
 - 8*c^3*D))*x)*(a + b*x^2)^(5/2))/d^2 - ((b*c^2 + a*d^2)*(((8*(a*d^2*(2*c
*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3*D))
 - d*(5*a*d^2*(C*d - 2*c*D) + 6*b*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3
*D))*x)*(a + b*x^2)^(3/2))/(4*d^2) + (((48*(b*c^2 + a*d^2)*(a*d^2*(2*c*C*d
 - B*d^2 - 3*c^2*D) + b*c*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3*D)) - d
*(4*a*b*c*d^2*(7*c*C*d - 6*B*d^2 - 8*c^2*D) + (4*b*c^2 + 3*a*d^2)*(5*a*d^2
*(C*d - 2*c*D) + 6*b*(7*c^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3*D)))*x)*Sqrt
[a + b*x^2])/(2*d^2) + (3*(-(((5*a^3*d^6*(C*d - 2*c*D) + 16*b^3*c^4*(7*c^2
*C*d - 6*B*c*d^2 + 5*A*d^3 - 8*c^3*D) + 40*a*b^2*c^2*d^2*(5*c^2*C*d - 4*B*
c*d^2 + 3*A*d^3 - 6*c^3*D) + 30*a^2*b*d^4*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 -
 4*c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (16*(b*c^2
 + a*d^2)^(3/2)*(a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2*C*d - 6*B*
c*d^2 + 5*A*d^3 - 8*c^3*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqr
t[a + b*x^2])])/d))/(2*d^2))/(4*d^2)))/(6*d^2))/d^2)/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2344 vs. $2(782) = 1564$.

Time = 1.43 (sec) , antiderivative size = 2345, normalized size of antiderivative = 2.84

method	result	size
default	Expression too large to display	2345

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(C*d*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*D*d*(
b*x^2+a)^(7/2)/b-2*D*c*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)
+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
)))+1/d^4*(B*d^2-2*C*c*d+3*D*c^2)*(1/5*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2
+b*c^2)/d^2)^(5/2)-b*c/d*(1/8*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+3/16*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^
2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d
+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
))+(a*d^2+b*c^2)/d^2*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln(
(-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)
*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))))+1/d^5*(A*d^3
-B*c*d^2+C*c^2*d-D*c^3)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1565, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxi
ma")
```

output
```
(b*x^2 + a)^(5/2)*D*c^3/(d^5*x + c*d^4) - (b*x^2 + a)^(5/2)*C*c^2/(d^4*x +
 c*d^3) + (b*x^2 + a)^(5/2)*B*c/(d^3*x + c*d^2) - (b*x^2 + a)^(5/2)*A/(d^2
*x + c*d) - 4*sqrt(b*x^2 + a)*D*b^2*c^5*x/d^7 + 7/2*sqrt(b*x^2 + a)*C*b^2*
c^4*x/d^6 - 2*(b*x^2 + a)^(3/2)*D*b*c^3*x/d^5 - 9/2*sqrt(b*x^2 + a)*D*a*b*
c^3*x/d^5 - 3*sqrt(b*x^2 + a)*B*b^2*c^3*x/d^5 + 7/4*(b*x^2 + a)^(3/2)*C*b*
c^2*x/d^4 + 29/8*sqrt(b*x^2 + a)*C*a*b*c^2*x/d^4 + 5/2*sqrt(b*x^2 + a)*A*b
^2*c^2*x/d^4 - 1/3*(b*x^2 + a)^(5/2)*D*c*x/d^3 - 5/12*(b*x^2 + a)^(3/2)*D*
a*c*x/d^3 - 5/8*sqrt(b*x^2 + a)*D*a^2*c*x/d^3 - 3/2*(b*x^2 + a)^(3/2)*B*b*
c*x/d^3 - 11/4*sqrt(b*x^2 + a)*B*a*b*c*x/d^3 + 1/6*(b*x^2 + a)^(5/2)*C*x/d
^2 + 5/24*(b*x^2 + a)^(3/2)*C*a*x/d^2 + 5/16*sqrt(b*x^2 + a)*C*a^2*x/d^2 +
 5/4*(b*x^2 + a)^(3/2)*A*b*x/d^2 + 15/8*sqrt(b*x^2 + a)*A*a*b*x/d^2 - 8*D*
b^(5/2)*c^7*arcsinh(b*x/sqrt(a*b))/d^9 + 7*C*b^(5/2)*c^6*arcsinh(b*x/sqrt(
a*b))/d^8 - 15*D*a*b^(3/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^7 - 6*B*b^(5/2)*c^
5*arcsinh(b*x/sqrt(a*b))/d^7 + 25/2*C*a*b^(3/2)*c^4*arcsinh(b*x/sqrt(a*b))
/d^6 + 5*A*b^(5/2)*c^4*arcsinh(b*x/sqrt(a*b))/d^6 - 15/2*D*a^2*sqrt(b)*c^3
*arcsinh(b*x/sqrt(a*b))/d^5 - 10*B*a*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^
5 + 45/8*C*a^2*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 + 15/2*A*a*b^(3/2)*c
^2*arcsinh(b*x/sqrt(a*b))/d^4 - 5/8*D*a^3*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b
)*d^3) - 15/4*B*a^2*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + 5/16*C*a^3*arcs
inh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + 15/8*A*a^2*sqrt(b)*arcsinh(b*x/sqrt(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^2}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

3.91 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 803

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \frac{(23a^2d^4(Cd-3cD)+15b^2c^2(15c^2Cd-10Bcd^2+6Ad^3-21\cdots}{15\cdots}$$
$$+\frac{(11a^2d^4D-18abd^2(3cCd-Bd^2-6c^2D)-8b^2c(10c^2Cd-6Bcd^2+3Ad^3-15c^3D))\,x\sqrt{a+bx^2}}{16d^7}$$
$$+\frac{b(11ad^2(Cd-3cD)+5b(6c^2Cd-3Bcd^2+Ad^3-10c^3D))\,x^2\sqrt{a+bx^2}}{15d^6}$$
$$+\frac{b(13ad^2D-6b(3cCd-Bd^2-6c^2D))\,x^3\sqrt{a+bx^2}}{24d^5}+\frac{b^2(Cd-3cD)x^4\sqrt{a+bx^2}}{5d^4}$$
$$+\frac{b^2Dx^5\sqrt{a+bx^2}}{6d^3}-\frac{(bc^2+ad^2)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\,\sqrt{a+bx^2}}{2d^8(c+dx)^2}$$
$$+\frac{(bc^2+ad^2)\,(2ad^2(2cCd-Bd^2-3c^2D)+bc(13c^2Cd-11Bcd^2+9Ad^3-15c^3D))\,\sqrt{a+bx^2}}{2d^8(c+dx)}$$
$$+\frac{(5a^3d^6D-30a^2bd^4(3cCd-Bd^2-6c^2D)-16b^3c^3(21c^2Cd-15Bcd^2+10Ad^3-28c^3D)-40ab^2cd^2(10\cdots}{16\sqrt{b}d^9}$$
$$-\frac{\sqrt{bc^2+ad^2}(2a^2d^4(Cd-3cD)+abd^2(29c^2Cd-15Bcd^2+5Ad^3-47c^3D)+2b^2c^2(21c^2Cd-15Bcd^2+\cdots}{2d^9}$$

output
```
1/15*(23*a^2*d^4*(C*d-3*D*c)+15*b^2*c^2*(6*A*d^3-10*B*c*d^2+15*C*c^2*d-21*
D*c^3)+35*a*b*d^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(b*x^2+a)^(1/2)/d^
8+1/16*(11*a^2*d^4*D-18*a*b*d^2*(-B*d^2+3*C*c*d-6*D*c^2)-8*b^2*c*(3*A*d^3-
6*B*c*d^2+10*C*c^2*d-15*D*c^3))*x*(b*x^2+a)^(1/2)/d^7+1/15*b*(11*a*d^2*(C*
d-3*D*c)+5*b*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*x^2*(b*x^2+a)^(1/2)/d^6
+1/24*b*(13*a*d^2*D-6*b*(-B*d^2+3*C*c*d-6*D*c^2))*x^3*(b*x^2+a)^(1/2)/d^5+
1/5*b^2*(C*d-3*D*c)*x^4*(b*x^2+a)^(1/2)/d^4+1/6*b^2*D*x^5*(b*x^2+a)^(1/2)/
d^3-1/2*(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8/
(d*x+c)^2+1/2*(a*d^2+b*c^2)*(2*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(9*A*d^3
-11*B*c*d^2+13*C*c^2*d-15*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)+1/16*(5*a^3*
d^6*D-30*a^2*b*d^4*(-B*d^2+3*C*c*d-6*D*c^2)-16*b^3*c^3*(10*A*d^3-15*B*c*d^
2+21*C*c^2*d-28*D*c^3)-40*a*b^2*c*d^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c
^3))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^9-1/2*(a*d^2+b*c^2)^(1/2
)*(2*a^2*d^4*(C*d-3*D*c)+a*b*d^2*(5*A*d^3-15*B*c*d^2+29*C*c^2*d-47*D*c^3)+
2*b^2*c^2*(10*A*d^3-15*B*c*d^2+21*C*c^2*d-28*D*c^3))*arctanh((-b*c*x+a*d)/
(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^9
```

Mathematica [A] (verified)

Time = 8.86 (sec) , antiderivative size = 708, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \frac{\frac{d\sqrt{a+bx^2}\left(a^2d^4\left(-1704c^3D+c^2d(728C-2763Dx)-2cd^2(60B+x(-608C+387Dx))\right.\right.}{}}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((d*Sqrt[a + b*x^2]*(a^2*d^4*(-1704*c^3*D + c^2*d*(728*C - 2763*D*x) - 2*c
*d^2*(60*B + x*(-608*C + 387*D*x)) + d^3*(-120*A + x*(-240*B + 368*C*x + 1
65*D*x^2))) + 2*a*b*d^2*(-3940*c^5*D + 10*c^4*d*(258*C - 605*D*x) + c^3*d^
2*(-1500*B + x*(3975*C - 1444*D*x)) + d^5*x^2*(280*A + x*(135*B + 88*C*x +
 65*D*x^2)) - c*d^4*x*(-1100*A + x*(570*B + 229*C*x + 134*D*x^2)) + c^2*d^
3*(700*A + x*(-2325*B + 958*C*x + 347*D*x^2))) - 4*b^2*(1680*c^7*D - 1260*
c^6*d*(C - 2*D*x) + 10*c^5*d^2*(90*B + 7*x*(-27*C + 8*D*x)) - 10*c^4*d^3*(
60*A + x*(-135*B + 14*x*(3*C + D*x))) - c^2*d^5*x^2*(200*A + x*(75*B + 14*
x*(3*C + 2*D*x))) - d^7*x^4*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + c^3*d^
4*x*(-900*A + x*(300*B + 7*x*(15*C + 8*D*x))) + c*d^6*x^3*(50*A + x*(30*B
+ x*(21*C + 16*D*x))))))/(c + d*x)^2 - 240*Sqrt[-(b*c^2) - a*d^2]*(-2*a^2*
d^4*(C*d - 3*c*D) + 2*b^2*c^2*(-21*c^2*C*d + 15*B*c*d^2 - 10*A*d^3 + 28*c^
3*D) + a*b*d^2*(-29*c^2*C*d + 15*B*c*d^2 - 5*A*d^3 + 47*c^3*D))*ArcTan[(Sq
rt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (15*(5*a^3*
d^6*D + 30*a^2*b*d^4*(-3*c*C*d + B*d^2 + 6*c^2*D) + 40*a*b^2*c*d^2*(-10*c^
2*C*d + 6*B*c*d^2 - 3*A*d^3 + 15*c^3*D) + 16*b^3*c^3*(-21*c^2*C*d + 15*B*c
*d^2 - 10*A*d^3 + 28*c^3*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])
/(240*d^9)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 1048, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2182, 25, 2182, 25, 27, 682, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}dx$$

$$\Big\downarrow \ 2182$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{5/2}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(2Cd-2cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+5Bc-5Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+5Bc-5Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^2}dx}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{7/2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(3Ad^3-5Bcd^2-9c^3D+7c^2Cd\right)\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(\left(2d(Cd-2cD)a^2+\frac{bc\left(-7Dc^2+5Cdc-3Bd^2\right)a}{d}+Ab\left(2bc^2+5ad^2\right.\right.\right.}{}}{}}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(2bc^2+5ad^2\right)+a\left(2a(Cd-2cD)d^2+bc\left(-7Dc^2+5Cdc-3Bd^2\right)\right)\right)+2\left(a^2Dd^4-2ab\left(-10Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-28Dc^3+21Cdc^2-15Bd^2c+9Ad^3\right)\right)\right.}{d^2(c+dx)}}{ad^2+bc^2}}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(2bc^2+5ad^2\right)+a\left(2a(Cd-2cD)d^2+bc\left(-7Dc^2+5Cdc-3Bd^2\right)\right)\right)+2\left(a^2Dd^4-2ab\left(-10Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-28Dc^3+21Cdc^2-15Bd^2c+9Ad^3\right)\right)\right.}{c+dx}}{d^2\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{\int \frac{2b\left(bc^2+ad^2\right)\left(ad\left(a(6Cd-13cD)d^2+b\left(-28Dc^3+21Cdc^2-15Bd^2c+15Ad^3\right)\right)+\left(5a^2Dd^4-ab\left(-113Dc^2+66Cdc-30Bd^2\right)d^2-6b^2c\left(-28Dc^3+21Cdc^2-15Bd^2c\right.\right.\right.}{c+dx}}{6bd^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\left(ad^2+bc^2\right)\int \frac{\left(ad\left(a(6Cd-13cD)d^2+b\left(-28Dc^3+21Cdc^2-15Bd^2c+15Ad^3\right)\right)+\left(5a^2Dd^4-ab\left(-113Dc^2+66Cdc-30Bd^2\right)d^2-6b^2c\left(-28Dc^3+21Cdc^2-15Bd^2c+1\right.\right.\right.}{c+dx}$$

$$3d^2$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{682}$$

$$\left(ad^2+bc^2\right)\left(\int \frac{3b\left(bc^2+ad^2\right)\left(ad\left(a(8Cd-19cD)d^2+b\left(-56Dc^3+42Cdc^2-30Bd^2c+20Ad^3\right)\right)+\left(5a^2Dd^4-2ab\left(-66Dc^2+37Cdc-15Bd^2\right)d^2-8b^2c\left(-28Dc^3+21C\right.\right.\right.\right.}{c+dx}\right.$$

$$4bd^2$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\left(ad^2+bc^2\right)\left(3\left(ad^2+bc^2\right)\int \frac{\left(ad\left(a(8Cd-19cD)d^2+b\left(-56Dc^3+42Cdc^2-30Bd^2c+20Ad^3\right)\right)+\left(5a^2Dd^4-2ab\left(-66Dc^2+37Cdc-15Bd^2\right)d^2-8b^2c\left(-28Dc^3+21C\right.\right.\right.\right.}{c+dx}\right.$$

$$4d^2$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{682}$$

$$\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b^2\right.\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \ 719$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b^2\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow 488$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b^2\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

$$\Big\downarrow 219$$

$$\frac{\frac{\left(2a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-9Dc^3+7Cdc^2-5Bd^2c+3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{d^2\left(bc^2+ad^2\right)(c+dx)}+\frac{\left(3\left(2a^2(Cd-3cD)d^4+ab\left(-47Dc^3+29Cdc^2-15Bd^2c+5Ad^3\right)d^2+2b^2\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
-1/2*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^2) + (((2*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d - 5*B*c*d^2 + 3*A*d^3 - 9*c^3*D))*(a + b*x^2)^(7/2))/(d^2*(b*c^2 + a*
d^2)*(c + d*x)) + (((3*(2*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(29*c^2*C*d - 15
*B*c*d^2 + 5*A*d^3 - 47*c^3*D) + 2*b^2*c^2*(21*c^2*C*d - 15*B*c*d^2 + 10*A
*d^3 - 28*c^3*D)) + 5*d*(a^2*d^4*D - 2*a*b*d^2*(6*c*C*d - 3*B*d^2 - 10*c^2
*D) - b^2*c*(21*c^2*C*d - 15*B*c*d^2 + 9*A*d^3 - 28*c^3*D))*x)*(a + b*x^2)
^(5/2))/(15*d^2) + ((b*c^2 + a*d^2)*(((4*(2*a^2*d^4*(C*d - 3*c*D) + a*b*d^
2*(29*c^2*C*d - 15*B*c*d^2 + 5*A*d^3 - 47*c^3*D) + 2*b^2*c^2*(21*c^2*C*d -
 15*B*c*d^2 + 10*A*d^3 - 28*c^3*D)) + d*(5*a^2*d^4*D - a*b*d^2*(66*c*C*d -
 30*B*d^2 - 113*c^2*D) - 6*b^2*c*(21*c^2*C*d - 15*B*c*d^2 + 10*A*d^3 - 28*
c^3*D))*x)*(a + b*x^2)^(3/2))/(4*d^2) + (3*(b*c^2 + a*d^2)*(((8*(2*a^2*d^4
*(C*d - 3*c*D) + a*b*d^2*(29*c^2*C*d - 15*B*c*d^2 + 5*A*d^3 - 47*c^3*D) +
b^2*(42*c^4*C*d - 30*B*c^3*d^2 + 20*A*c^2*d^3 - 56*c^5*D)) + d*(5*a^2*d^4*
D - 2*a*b*d^2*(37*c*C*d - 15*B*d^2 - 66*c^2*D) - 8*b^2*c*(21*c^2*C*d - 15*
B*c*d^2 + 10*A*d^3 - 28*c^3*D))*x)*Sqrt[a + b*x^2])/(2*d^2) + (((5*a^3*d^6
*D - 30*a^2*b*d^4*(3*c*C*d - B*d^2 - 6*c^2*D) - 16*b^3*c^3*(21*c^2*C*d - 1
5*B*c*d^2 + 10*A*d^3 - 28*c^3*D) - 40*a*b^2*c*d^2*(10*c^2*C*d - 6*B*c*d^2
+ 3*A*d^3 - 15*c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) -
 (8*Sqrt[b*c^2 + a*d^2]*(2*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(29*c^2*C*d ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4505 vs. $2(755) = 1510$.

Time = 1.54 (sec) , antiderivative size = 4506, normalized size of antiderivative = 5.61

method	result	size
default	Expression too large to display	4506

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
D/d^3*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/d^4*(C*d-3*D
*c)*(1/5*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-b*c/d*(1/8*
(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3
/2)+3/16*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b*(1/4*(2*b*(x+c/d)-2*b*c/d
)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+
b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))))+(a*d^2+b*c^2)/d^2*(1/3*(b*(
x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-
2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*
(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b
*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d
+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c
/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2
+b*c^2)/d^2)^(1/2))/(x+c/d)))))+1/d^5*(B*d^2-2*C*c*d+3*D*c^2)*(-1/(a*d^2+b
*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(7/2)-5*
b*c*d/(a*d^2+b*c^2)*(1/5*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
5/2)-b*c/d*(1/8*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2940 vs. $2(758) = 1516$.

Time = 0.30 (sec) , antiderivative size = 2940, normalized size of antiderivative = 3.66

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxi
ma")
```

output
```
-15/4*D*b^4*c^8*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^9 + a*sqrt(b)*d^11)
+ 15/4*C*b^4*c^7*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^8 + a*sqrt(b)*d^10)
 - 15/4*D*a*b^3*c^6*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^7 + a*sqrt(b)*d^
9) - 15/4*B*b^4*c^6*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^7 + a*sqrt(b)*d^
9) + 15/4*sqrt(b*x^2 + a)*D*b^3*c^6*x/(b*c^2*d^7 + a*d^9) + 15/4*C*a*b^3*c
^5*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^6 + a*sqrt(b)*d^8) + 15/4*A*b^4*c
^5*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^6 + a*sqrt(b)*d^8) - 15/4*sqrt(b*
x^2 + a)*C*b^3*c^5*x/(b*c^2*d^6 + a*d^8) - 15/4*B*a*b^3*c^4*arcsinh(b*x/sq
rt(a*b))/(b^(3/2)*c^2*d^5 + a*sqrt(b)*d^7) - 5/2*(b*x^2 + a)^(3/2)*D*b^2*c
^5/(b*c^2*d^6 + a*d^8) + 5/2*(b*x^2 + a)^(3/2)*D*b^2*c^4*x/(b*c^2*d^5 + a*
d^7) + 15/4*sqrt(b*x^2 + a)*D*a*b^2*c^4*x/(b*c^2*d^5 + a*d^7) + 15/4*sqrt(
b*x^2 + a)*B*b^3*c^4*x/(b*c^2*d^5 + a*d^7) + 15/4*A*a*b^3*c^3*arcsinh(b*x/
sqrt(a*b))/(b^(3/2)*c^2*d^4 + a*sqrt(b)*d^6) - 3/2*(b*x^2 + a)^(5/2)*D*b*c
^4/(b*c^2*d^5*x + a*d^7*x + b*c^3*d^4 + a*c*d^6) + 5/2*(b*x^2 + a)^(3/2)*C
*b^2*c^4/(b*c^2*d^5 + a*d^7) - 5/2*(b*x^2 + a)^(3/2)*C*b^2*c^3*x/(b*c^2*d^
4 + a*d^6) - 15/4*sqrt(b*x^2 + a)*C*a*b^2*c^3*x/(b*c^2*d^4 + a*d^6) - 15/4
*sqrt(b*x^2 + a)*A*b^3*c^3*x/(b*c^2*d^4 + a*d^6) + 1/2*(b*x^2 + a)^(7/2)*D
*c^3/(b*c^2*d^4*x^2 + a*d^6*x^2 + 2*b*c^3*d^3*x + 2*a*c*d^5*x + b*c^4*d^2
+ a*c^2*d^4) + 3/2*(b*x^2 + a)^(5/2)*C*b*c^3/(b*c^2*d^4*x + a*d^6*x + b*c^
3*d^3 + a*c*d^5) - 1/2*(b*x^2 + a)^(5/2)*D*b*c^3/(b*c^2*d^4 + a*d^6) - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2233 vs. $2(758) = 1516$.

Time = 0.86 (sec) , antiderivative size = 2233, normalized size of antiderivative = 2.78

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac
")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*D*b^2*x/d^3 - 6*(3*D*b^6*c*d^38 - C*b^6*d
^39)/(b^4*d^42))*x + 5*(36*D*b^6*c^2*d^37 - 18*C*b^6*c*d^38 + 13*D*a*b^5*d
^39 + 6*B*b^6*d^39)/(b^4*d^42))*x - 8*(50*D*b^6*c^3*d^36 - 30*C*b^6*c^2*d^
37 + 33*D*a*b^5*c*d^38 + 15*B*b^6*c*d^38 - 11*C*a*b^5*d^39 - 5*A*b^6*d^39)
/(b^4*d^42))*x + 15*(120*D*b^6*c^4*d^35 - 80*C*b^6*c^3*d^36 + 108*D*a*b^5*
c^2*d^37 + 48*B*b^6*c^2*d^37 - 54*C*a*b^5*c*d^38 - 24*A*b^6*c*d^38 + 11*D*
a^2*b^4*d^39 + 18*B*a*b^5*d^39)/(b^4*d^42))*x - 16*(315*D*b^6*c^5*d^34 - 2
25*C*b^6*c^4*d^35 + 350*D*a*b^5*c^3*d^36 + 150*B*b^6*c^3*d^36 - 210*C*a*b^
5*c^2*d^37 - 90*A*b^6*c^2*d^37 + 69*D*a^2*b^4*c*d^38 + 105*B*a*b^5*c*d^38
- 23*C*a^2*b^4*d^39 - 35*A*a*b^5*d^39)/(b^4*d^42)) - (56*D*b^3*c^7 - 42*C*
b^3*c^6*d + 103*D*a*b^2*c^5*d^2 + 30*B*b^3*c^5*d^2 - 71*C*a*b^2*c^4*d^3 -
20*A*b^3*c^4*d^3 + 53*D*a^2*b*c^3*d^4 + 45*B*a*b^2*c^3*d^4 - 31*C*a^2*b*c^
2*d^5 - 25*A*a*b^2*c^2*d^5 + 6*D*a^3*c*d^6 + 15*B*a^2*b*c*d^6 - 2*C*a^3*d^
7 - 5*A*a^2*b*d^7)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/s
qrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^9) - 1/16*(448*D*b^3*c^6 - 33
6*C*b^3*c^5*d + 600*D*a*b^2*c^4*d^2 + 240*B*b^3*c^4*d^2 - 400*C*a*b^2*c^3*
d^3 - 160*A*b^3*c^3*d^3 + 180*D*a^2*b*c^2*d^4 + 240*B*a*b^2*c^2*d^4 - 90*C
*a^2*b*c*d^5 - 120*A*a*b^2*c*d^5 + 5*D*a^3*d^6 + 30*B*a^2*b*d^6)*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/(sqrt(b)*d^9) - (16*(sqrt(b)*x - sqrt(b*x^2
+ a))^3*D*b^3*c^7*d - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*b^3*c^6*d^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^3}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

3.92 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 806

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \frac{(23a^2d^4D-35abd^2(4cCd-Bd^2-10c^2D)-15b^2c(20c^2Cd\ \cdots}{15d^8}$$

$$+\frac{b(9ad^2(Cd-4cD)+4b(10c^2Cd-4Bcd^2+Ad^3-20c^3D))\,x\sqrt{a+bx^2}}{8d^7}$$

$$+\frac{b(11ad^2D-5b(4cCd-Bd^2-10c^2D))\,x^2\sqrt{a+bx^2}}{15d^6}+\frac{b^2(Cd-4cD)x^3\sqrt{a+bx^2}}{4d^5}$$

$$+\frac{b^2Dx^4\sqrt{a+bx^2}}{5d^4}-\frac{(bc^2+ad^2)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{3d^8(c+dx)^3}$$

$$+\frac{(bc^2+ad^2)\,(3ad^2(2cCd-Bd^2-3c^2D)+bc(19c^2Cd-16Bcd^2+13Ad^3-22c^3D))\sqrt{a+bx^2}}{6d^8(c+dx)^2}$$

$$-\frac{(6a^2d^4(Cd-3cD)+b^2c^2(107c^2Cd-74Bcd^2+47Ad^3-146c^3D)+abd^2(80c^2Cd-41Bcd^2+14Ad^3-\ \cdots}{6d^8(c+dx)}$$

$$+\frac{\sqrt{b}(15a^2d^4(Cd-4cD)+8b^2c^2(35c^2Cd-20Bcd^2+10Ad^3-56c^3D)+20abd^2(10c^2Cd-4Bcd^2+Ad^3\ \cdots}{8d^9}$$

$$-\frac{(2a^3d^6D-a^2bd^4(20cCd-5Bd^2-51c^2D)-ab^2cd^2(85c^2Cd-40Bcd^2+15Ad^3-156c^3D)-2b^3c^3(35c\ \cdots}{2d^9\sqrt{bc^2+ad^2}}$$

output
```
1/15*(23*a^2*d^4*D-35*a*b*d^2*(-B*d^2+4*C*c*d-10*D*c^2)-15*b^2*c*(4*A*d^3-
10*B*c*d^2+20*C*c^2*d-35*D*c^3))*(b*x^2+a)^(1/2)/d^8+1/8*b*(9*a*d^2*(C*d-4
*D*c)+4*b*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*x*(b*x^2+a)^(1/2)/d^7+1/1
5*b*(11*a*d^2*D-5*b*(-B*d^2+4*C*c*d-10*D*c^2))*x^2*(b*x^2+a)^(1/2)/d^6+1/4
*b^2*(C*d-4*D*c)*x^3*(b*x^2+a)^(1/2)/d^5+1/5*b^2*D*x^4*(b*x^2+a)^(1/2)/d^4
-1/3*(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8/(d*
x+c)^3+1/6*(a*d^2+b*c^2)*(3*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(13*A*d^3-1
6*B*c*d^2+19*C*c^2*d-22*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)^2-1/6*(6*a^2*d
^4*(C*d-3*D*c)+b^2*c^2*(47*A*d^3-74*B*c*d^2+107*C*c^2*d-146*D*c^3)+a*b*d^2
*(14*A*d^3-41*B*c*d^2+80*C*c^2*d-131*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)+1
/8*b^(1/2)*(15*a^2*d^4*(C*d-4*D*c)+8*b^2*c^2*(10*A*d^3-20*B*c*d^2+35*C*c^2
*d-56*D*c^3)+20*a*b*d^2*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/d^9-1/2*(2*a^3*d^6*D-a^2*b*d^4*(-5*B*d^2+20*C*c*d-
51*D*c^2)-a*b^2*c*d^2*(15*A*d^3-40*B*c*d^2+85*C*c^2*d-156*D*c^3)-2*b^3*c^3
*(10*A*d^3-20*B*c*d^2+35*C*c^2*d-56*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*
c^2)^(1/2)/(b*x^2+a)^(1/2))/d^9/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 12.04 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \frac{d\sqrt{a+bx^2}\left(-4a^2d^4\left(-101c^3D+c^2d(10C-273Dx)+cd^2(5B+6x(5C-38Dx))+d^3(10\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-4*a^2*d^4*(-101*c^3*D + c^2*d*(10*C - 273*D*x) + c*d
^2*(5*B + 6*x*(5*C - 38*D*x)) + d^3*(10*A + x*(15*B + 30*C*x - 46*D*x^2)))
 + a*b*d^2*(4880*c^5*D - 20*c^4*d*(115*C - 624*D*x) + c^3*d^2*(800*B + x*(
-5925*C + 9488*D*x)) + d^5*x^2*(-280*A + x*(280*B + 135*C*x + 88*D*x^2)) -
 c*d^4*x*(300*A + x*(-1660*B + 715*C*x + 276*D*x^2)) + c^2*d^3*(-100*A + x
*(2100*B - 4555*C*x + 1444*D*x^2))) + 2*b^2*(3360*c^7*D - 2100*c^6*d*(C -
4*D*x) + 10*c^5*d^2*(120*B + 7*x*(-75*C + 88*D*x)) - 10*c^4*d^3*(60*A + x*
(-300*B + 7*x*(55*C - 12*D*x))) + d^7*x^4*(30*A + x*(20*B + 3*x*(5*C + 4*D
*x))) + c^2*d^5*x^2*(-1100*A + x*(300*B + 7*x*(15*C + 8*D*x))) - c^3*d^4*x
*(1500*A + x*(-2200*B + 21*x*(25*C + 8*D*x))) - c*d^6*x^3*(150*A + x*(60*B
 + x*(35*C + 24*D*x))))))/(c + d*x)^3 + (60*(2*a^3*d^6*D + a^2*b*d^4*(-20*
c*C*d + 5*B*d^2 + 51*c^2*D) + 2*b^3*c^3*(-35*c^2*C*d + 20*B*c*d^2 - 10*A*d
^3 + 56*c^3*D) + a*b^2*c*d^2*(-85*c^2*C*d + 40*B*c*d^2 - 15*A*d^3 + 156*c^
3*D))*Log[c + d*x])/Sqrt[b*c^2 + a*d^2] + 15*Sqrt[b]*(15*a^2*d^4*(C*d - 4*
c*D) + 20*a*b*d^2*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D) - 8*b^2*c^2*
(-35*c^2*C*d + 20*B*c*d^2 - 10*A*d^3 + 56*c^3*D))*Log[b*x + Sqrt[b]*Sqrt[a
 + b*x^2]] - (60*(2*a^3*d^6*D + a^2*b*d^4*(-20*c*C*d + 5*B*d^2 + 51*c^2*D)
 + 2*b^3*c^3*(-35*c^2*C*d + 20*B*c*d^2 - 10*A*d^3 + 56*c^3*D) + a*b^2*c*d^
2*(-85*c^2*C*d + 40*B*c*d^2 - 15*A*d^3 + 156*c^3*D))*Log[a*d - b*c*x + Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/Sqrt[b*c^2 + a*d^2])/(120*d^9)
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{5/2}\left(3\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(3Cd-3cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+4Bc-4Ad\right)\right)x+3\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^3}dx}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\left(bx^2+a\right)^{5/2}\left(3\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(3a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+4Bc-4Ad\right)\right)x+3\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^3}dx}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2182

$$\frac{\frac{\left(a+bx^2\right)^{7/2}\left(3ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(Ad^3-4Bcd^2-10c^3D+7c^2Cd\right)\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(2\left(3d(Cd-2cD)a^2+\frac{bc\left(-7Dc^2+4Cdc-Bd^2\right)a}{d}+Ab\left(3bc^2+4ad^2\right.\right.\right.\cdots}{\cdots}}{\cdots}}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\frac{\left(2d\left(3a^2(Cd-2cD)d^2+Ab\left(3bc^2+4ad^2\right)d+abc\left(-7Dc^2+4Cdc-Bd^2\right)\right)+\left(6a^2Dd^4-3ab\left(-19Dc^2+10Cdc-5Bd^2\right)d^2-b^2c\left(-56Dc^3+35Cdc^2-20Bd^2c+5Ad^3\right)\right)\cdots\right.}{d^2(c+dx)^2}}{2\left(ad^2+bc^2\right)}\cdots}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{\left(2d\left(3a^2(Cd-2cD)d^2+Ab\left(3bc^2+4ad^2\right)d+abc\left(-7Dc^2+4Cdc-Bd^2\right)\right)+\left(6a^2Dd^4-3ab\left(-19Dc^2+10Cdc-5Bd^2\right)d^2-b^2c\left(-56Dc^3+35Cdc^2-20Bd^2c+5Ad^3\right)\right)\cdots\right.}{(c+dx)^2}}{2d^2\left(ad^2+bc^2\right)}\cdots}{3\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 681

$$-\frac{\int -\frac{\frac{2\left(ad\left(6a^2Dd^4-3ab\left(-19Dc^2+10Cdc-5Bd^2\right)d^2-b^2c\left(-56Dc^3+35Cdc^2-20Bd^2c+5Ad^3\right)\right)+2b\left(3a^2(5Cd-16cD)d^4+2ab\left(-103Dc^3+55Cdc^2-25Bd^2c+10\right.\right.\right.}{c+dx}}{2d^2}}{}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{\left(ad\left(6a^2Dd^4-3ab\left(-19Dc^2+10Cdc-5Bd^2\right)d^2-b^2c\left(-56Dc^3+35Cdc^2-20Bd^2c+5Ad^3\right)\right)+2b\left(3a^2(5Cd-16cD)d^4+2ab\left(-103Dc^3+55Cdc^2-25Bd^2c+10Ad^3\right)\right.\right.}{c+dx}}{d^2}}{}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\right.\right.\right.}{}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\right.\right.\right.}{}}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\right.\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\right.\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\right.\right.\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$\Big\downarrow$ 488

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2(bc^2+ad^2)(c+dx)^2}+\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(3a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-10Dc^3+7Cdc^2-4Bd^2c+Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{2d^2\left(bc^2+ad^2\right)(c+dx)^2}+\frac{\frac{\left(2\left(2a^3Dd^6-a^2b\left(-51Dc^2+20Cdc-5Bd^2\right)d^4-ab^2c\left(-156Dc^3+85C\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
-1/3*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^3) + (((3*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d - 4*B*c*d^2 + A*d^3 - 10*c^3*D))*(a + b*x^2)^(7/2))/(2*d^2*(b*c^2 + a
*d^2)*(c + d*x)^2) + (-1/5*((2*(3*a^2*d^4*(5*C*d - 16*c*D) + 2*a*b*d^2*(55
*c^2*C*d - 25*B*c*d^2 + 10*A*d^3 - 103*c^3*D) + 3*b^2*c^2*(35*c^2*C*d - 20
*B*c*d^2 + 10*A*d^3 - 56*c^3*D)) - d*(6*a^2*d^4*D - 3*a*b*d^2*(10*c*C*d -
5*B*d^2 - 19*c^2*D) - b^2*c*(35*c^2*C*d - 20*B*c*d^2 + 5*A*d^3 - 56*c^3*D)
)*x)*(a + b*x^2)^(5/2))/(d^2*(c + d*x)) + (((2*(2*a^3*d^6*D - a^2*b*d^4*(2
0*c*C*d - 5*B*d^2 - 51*c^2*D) - a*b^2*c*d^2*(85*c^2*C*d - 40*B*c*d^2 + 15*
A*d^3 - 156*c^3*D) - 2*b^3*c^3*(35*c^2*C*d - 20*B*c*d^2 + 10*A*d^3 - 56*c^
3*D)) + b*d*(3*a^2*d^4*(5*C*d - 16*c*D) + 2*a*b*d^2*(55*c^2*C*d - 25*B*c*d
^2 + 10*A*d^3 - 103*c^3*D) + 3*b^2*c^2*(35*c^2*C*d - 20*B*c*d^2 + 10*A*d^3
 - 56*c^3*D))*x)*(a + b*x^2)^(3/2))/(2*d^2) + (3*(b*c^2 + a*d^2)*(((4*(2*a
^3*d^6*D - a^2*b*d^4*(20*c*C*d - 5*B*d^2 - 51*c^2*D) - a*b^2*c*d^2*(85*c^2
*C*d - 40*B*c*d^2 + 15*A*d^3 - 156*c^3*D) - 2*b^3*c^3*(35*c^2*C*d - 20*B*c
*d^2 + 10*A*d^3 - 56*c^3*D)) + b*d*(a^2*d^4*(15*C*d - 52*c*D) + a*b*d^2*(1
35*c^2*C*d - 60*B*c*d^2 + 20*A*d^3 - 256*c^3*D) + 4*b^2*c^2*(35*c^2*C*d -
20*B*c*d^2 + 10*A*d^3 - 56*c^3*D))*x)*Sqrt[a + b*x^2])/(2*d^2) + ((b*c^2 +
 a*d^2)*((Sqrt[b]*(15*a^2*d^4*(C*d - 4*c*D) + 8*b^2*c^2*(35*c^2*C*d - 20*B
*c*d^2 + 10*A*d^3 - 56*c^3*D) + 20*a*b*d^2*(10*c^2*C*d - 4*B*c*d^2 + A*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8089 vs. $2(758) = 1516$.

Time = 1.55 (sec) , antiderivative size = 8090, normalized size of antiderivative = 10.04

method	result	size
default	Expression too large to display	8090

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5303 vs. $2(760) = 1520$.

Time = 0.41 (sec) , antiderivative size = 5303, normalized size of antiderivative = 6.58

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxi
ma")
```

output
```
5/4*D*b^5*c^9*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^9 + 2*a*b^(3/2)*c^2*d^
11 + a^2*sqrt(b)*d^13) - 5/4*C*b^5*c^8*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4
*d^8 + 2*a*b^(3/2)*c^2*d^10 + a^2*sqrt(b)*d^12) + 5/4*D*a*b^4*c^7*arcsinh(
b*x/sqrt(a*b))/(b^(5/2)*c^4*d^7 + 2*a*b^(3/2)*c^2*d^9 + a^2*sqrt(b)*d^11)
+ 5/4*B*b^5*c^7*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^7 + 2*a*b^(3/2)*c^2*
d^9 + a^2*sqrt(b)*d^11) - 5/4*sqrt(b*x^2 + a)*D*b^4*c^7*x/(b^2*c^4*d^7 + 2
*a*b*c^2*d^9 + a^2*d^11) - 5/4*C*a*b^4*c^6*arcsinh(b*x/sqrt(a*b))/(b^(5/2)
*c^4*d^6 + 2*a*b^(3/2)*c^2*d^8 + a^2*sqrt(b)*d^10) - 5/4*A*b^5*c^6*arcsinh
(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^6 + 2*a*b^(3/2)*c^2*d^8 + a^2*sqrt(b)*d^10)
 + 45/4*D*b^4*c^7*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^9 + a*sqrt(b)*d^11
) + 5/4*sqrt(b*x^2 + a)*C*b^4*c^6*x/(b^2*c^4*d^6 + 2*a*b*c^2*d^8 + a^2*d^1
0) + 5/4*B*a*b^4*c^5*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^5 + 2*a*b^(3/2)
*c^2*d^7 + a^2*sqrt(b)*d^9) - 15/2*C*b^4*c^6*arcsinh(b*x/sqrt(a*b))/(b^(3/
2)*c^2*d^8 + a*sqrt(b)*d^10) + 5/6*(b*x^2 + a)^(3/2)*D*b^3*c^6/(b^2*c^4*d^
6 + 2*a*b*c^2*d^8 + a^2*d^10) - 5/6*(b*x^2 + a)^(3/2)*D*b^3*c^5*x/(b^2*c^4
*d^5 + 2*a*b*c^2*d^7 + a^2*d^9) - 5/4*sqrt(b*x^2 + a)*D*a*b^3*c^5*x/(b^2*c
^4*d^5 + 2*a*b*c^2*d^7 + a^2*d^9) - 5/4*sqrt(b*x^2 + a)*B*b^4*c^5*x/(b^2*c
^4*d^5 + 2*a*b*c^2*d^7 + a^2*d^9) - 5/4*A*a*b^4*c^4*arcsinh(b*x/sqrt(a*b))
/(b^(5/2)*c^4*d^4 + 2*a*b^(3/2)*c^2*d^6 + a^2*sqrt(b)*d^8) + 25/2*D*a*b^3*
c^5*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^7 + a*sqrt(b)*d^9) + 15/4*B*b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2810 vs. $2(760) = 1520$.

Time = 0.99 (sec) , antiderivative size = 2810, normalized size of antiderivative = 3.49

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac
")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*x*(4*D*b^2*x/d^4 - 5*(4*D*b^5*c*d^34 - C*b^5*
d^35)/(b^3*d^39)) + 4*(50*D*b^5*c^2*d^33 - 20*C*b^5*c*d^34 + 11*D*a*b^4*d^
35 + 5*B*b^5*d^35)/(b^3*d^39))*x - 15*(80*D*b^5*c^3*d^32 - 40*C*b^5*c^2*d^
33 + 36*D*a*b^4*c*d^34 + 16*B*b^5*c*d^34 - 9*C*a*b^4*d^35 - 4*A*b^5*d^35)/
(b^3*d^39))*x + 8*(525*D*b^5*c^4*d^31 - 300*C*b^5*c^3*d^32 + 350*D*a*b^4*c
^2*d^33 + 150*B*b^5*c^2*d^33 - 140*C*a*b^4*c*d^34 - 60*A*b^5*c*d^34 + 23*D
*a^2*b^3*d^35 + 35*B*a*b^4*d^35)/(b^3*d^39)) + 1/8*(448*D*b^(5/2)*c^5 - 28
0*C*b^(5/2)*c^4*d + 400*D*a*b^(3/2)*c^3*d^2 + 160*B*b^(5/2)*c^3*d^2 - 200*
C*a*b^(3/2)*c^2*d^3 - 80*A*b^(5/2)*c^2*d^3 + 60*D*a^2*sqrt(b)*c*d^4 + 80*B
*a*b^(3/2)*c*d^4 - 15*C*a^2*sqrt(b)*d^5 - 20*A*a*b^(3/2)*d^5)*log(abs(-sqr
t(b)*x + sqrt(b*x^2 + a)))/d^9 + (112*D*b^3*c^6 - 70*C*b^3*c^5*d + 156*D*a
*b^2*c^4*d^2 + 40*B*b^3*c^4*d^2 - 85*C*a*b^2*c^3*d^3 - 20*A*b^3*c^3*d^3 +
51*D*a^2*b*c^2*d^4 + 40*B*a*b^2*c^2*d^4 - 20*C*a^2*b*c*d^5 - 15*A*a*b^2*c*
d^5 + 2*D*a^3*d^6 + 5*B*a^2*b*d^6)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*
d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^9) + 1/3*(168
*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*b^3*c^6*d^2 - 126*(sqrt(b)*x - sqrt(b*x
^2 + a))^5*C*b^3*c^5*d^3 + 162*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*a*b^2*c^4
*d^4 + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b^3*c^4*d^4 - 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^5*C*a*b^2*c^3*d^5 - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A
*b^3*c^3*d^5 + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*a^2*b*c^2*d^6 + 60*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^4}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^4}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

3.93 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5} dx$

Optimal result

Integrand size = 34, antiderivative size = 864

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx &= \frac{b(7ad^2(Cd-5cD)+3b(15c^2Cd-5Bcd^2+Ad^3-35c^3D))\sqrt{\cdots}}{3d^8} \\
&+ \frac{b(9ad^2D-4b(5cCd-Bd^2-15c^2D))\,x\sqrt{a+bx^2}}{8d^7} + \frac{b^2(Cd-5cD)x^2\sqrt{a+bx^2}}{3d^6} \\
&+ \frac{b^2Dx^3\sqrt{a+bx^2}}{4d^5} - \frac{(bc^2+ad^2)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{4d^8(c+dx)^4} \\
&+ \frac{(bc^2+ad^2)\,(4ad^2(2cCd-Bd^2-3c^2D)+bc(25c^2Cd-21Bcd^2+17Ad^3-29c^3D))\sqrt{a+bx^2}}{12d^8(c+dx)^3} \\
&- \frac{(12a^2d^4(Cd-3cD)+abd^2(155c^2Cd-79Bcd^2+27Ad^3-255c^3D)+2b^2c^2(101c^2Cd-69Bcd^2+43Ad\cdots}{24d^8(c+dx)^2} \\
&- \frac{(24a^3d^6D-4a^2bd^4(55cCd-14Bd^2-141c^2D)-ab^2cd^2(843c^2Cd-383Bcd^2+139Ad^3-1591c^3D)-\cdots}{24d^8\,(bc^2+ad^2)\,(c+dx)} \\
&+ \frac{5\sqrt{b}(3a^2d^4D-4abd^2(5cCd-Bd^2-15c^2D)-8b^2c(7c^2Cd-3Bcd^2+Ad^3-14c^3D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^9} \\
&- \frac{5b(4a^3d^6(Cd-5cD)+4ab^2c^2d^2(26c^2Cd-10Bcd^2+3Ad^3-57c^3D)+3a^2bd^4(17c^2Cd-5Bcd^2+Ad^3\cdots}{8d^9\,(bc^2+ad^2)^{3/2}}
\end{aligned}
$$

output
```
1/3*b*(7*a*d^2*(C*d-5*D*c)+3*b*(A*d^3-5*B*c*d^2+15*C*c^2*d-35*D*c^3))*(b*x
^2+a)^(1/2)/d^8+1/8*b*(9*a*d^2*D-4*b*(-B*d^2+5*C*c*d-15*D*c^2))*x*(b*x^2+a
)^(1/2)/d^7+1/3*b^2*(C*d-5*D*c)*x^2*(b*x^2+a)^(1/2)/d^6+1/4*b^2*D*x^3*(b*x
^2+a)^(1/2)/d^5-1/4*(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a
)^(1/2)/d^8/(d*x+c)^4+1/12*(a*d^2+b*c^2)*(4*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)
+b*c*(17*A*d^3-21*B*c*d^2+25*C*c^2*d-29*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c
)^3-1/24*(12*a^2*d^4*(C*d-3*D*c)+a*b*d^2*(27*A*d^3-79*B*c*d^2+155*C*c^2*d-
255*D*c^3)+2*b^2*c^2*(43*A*d^3-69*B*c*d^2+101*C*c^2*d-139*D*c^3))*(b*x^2+a
)^(1/2)/d^8/(d*x+c)^2-1/24*(24*a^3*d^6*D-4*a^2*b*d^4*(-14*B*d^2+55*C*c*d-1
41*D*c^2)-a*b^2*c*d^2*(139*A*d^3-383*B*c*d^2+843*C*c^2*d-1591*D*c^3)-2*b^3
*c^3*(77*A*d^3-171*B*c*d^2+319*C*c^2*d-533*D*c^3))*(b*x^2+a)^(1/2)/d^8/(a*
d^2+b*c^2)/(d*x+c)+5/8*b^(1/2)*(3*a^2*d^4*D-4*a*b*d^2*(-B*d^2+5*C*c*d-15*D
*c^2)-8*b^2*c*(A*d^3-3*B*c*d^2+7*C*c^2*d-14*D*c^3))*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/d^9-5/8*b*(4*a^3*d^6*(C*d-5*D*c)+4*a*b^2*c^2*d^2*(3*A*d^3-10*
B*c*d^2+26*C*c^2*d-57*D*c^3)+3*a^2*b*d^4*(A*d^3-5*B*c*d^2+17*C*c^2*d-45*D*
c^3)+8*b^3*c^4*(A*d^3-3*B*c*d^2+7*C*c^2*d-14*D*c^3))*arctanh((-b*c*x+a*d)/
(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^9/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 13.43 (sec) , antiderivative size = 986, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \frac{8bd(7ad^2(Cd-5cD)-3b(-15c^2Cd+5Bcd^2-Ad^3+35c^3 D$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + dx)^5,x]
```

output
```
(8*b*d*(7*a*d^2*(C*d - 5*c*D) - 3*b*(-15*c^2*C*d + 5*B*c*d^2 - A*d^3 + 35*
c^3*D))*Sqrt[a + b*x^2] + 3*b*d^2*(9*a*d^2*D + 4*b*(-5*c*C*d + B*d^2 + 15*
c^2*D))*x*Sqrt[a + b*x^2] + 8*b^2*d^3*(C*d - 5*c*D)*x^2*Sqrt[a + b*x^2] +
6*b^2*d^4*D*x^3*Sqrt[a + b*x^2] - (6*d*(b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^
2 + A*d^3 - c^3*D)*Sqrt[a + b*x^2])/(c + d*x)^4 + (2*d*(b*c^2 + a*d^2)*(-4
*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(25*c^2*C*d - 21*B*c*d^2 + 17*A*
d^3 - 29*c^3*D))*Sqrt[a + b*x^2])/(c + d*x)^3 - (d*(12*a^2*d^4*(C*d - 3*c*
D) + a*b*d^2*(155*c^2*C*d - 79*B*c*d^2 + 27*A*d^3 - 255*c^3*D) + 2*b^2*c^2
*(101*c^2*C*d - 69*B*c*d^2 + 43*A*d^3 - 139*c^3*D))*Sqrt[a + b*x^2])/(c +
d*x)^2 + (d*(-24*a^3*d^6*D - 4*a^2*b*d^4*(-55*c*C*d + 14*B*d^2 + 141*c^2*D
) + a*b^2*c*d^2*(843*c^2*C*d - 383*B*c*d^2 + 139*A*d^3 - 1591*c^3*D) - 2*b
^3*c^3*(-319*c^2*C*d + 171*B*c*d^2 - 77*A*d^3 + 533*c^3*D))*Sqrt[a + b*x^2
])/((b*c^2 + a*d^2)*(c + d*x)) - (15*b*(-4*a^3*d^6*(C*d - 5*c*D) - 3*a^2*b
*d^4*(17*c^2*C*d - 5*B*c*d^2 + A*d^3 - 45*c^3*D) + 8*b^3*c^4*(-7*c^2*C*d +
 3*B*c*d^2 - A*d^3 + 14*c^3*D) + 4*a*b^2*c^2*d^2*(-26*c^2*C*d + 10*B*c*d^2
 - 3*A*d^3 + 57*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(3/2) + 15*Sqrt[b]*(
3*a^2*d^4*D + 4*a*b*d^2*(-5*c*C*d + B*d^2 + 15*c^2*D) + 8*b^2*c*(-7*c^2*C*
d + 3*B*c*d^2 - A*d^3 + 14*c^3*D))*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]] + (1
5*b*(-4*a^3*d^6*(C*d - 5*c*D) - 3*a^2*b*d^4*(17*c^2*C*d - 5*B*c*d^2 + A*d^
3 - 45*c^3*D) + 8*b^3*c^4*(-7*c^2*C*d + 3*B*c*d^2 - A*d^3 + 14*c^3*D) +...
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 1175, normalized size of antiderivative = 1.36, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^5}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(4Cd-4cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+3Bc-3Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(4\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(4a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+3Bc-3Ad\right)\right)x+4\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^4}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{7/2}\left(4ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-Ad^3-3Bcd^2-11c^3D+7c^2Cd\right)\right)}{3d^2(c+dx)^3(ad^2+bc^2)}-\frac{\int -\frac{\left(3d\left(Abd\left(4bc^2+3ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-7Dc^2+3Cdc+\right.\right.\right.\right.}{}}{}}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(3d\left(Abd\left(4bc^2+3ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-7Dc^2+3Cdc+Bd^2\right)\right)\right)+4\left(3a^2Dd^4-2ab\left(-9Dc^2+4Cdc-2Bd^2\right)d^2-b^2c\left(-14Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)x\right)\left(\right.}{d^2(c+dx)^3}}{3(ad^2+bc^2)}}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(3d\left(Abd\left(4bc^2+3ad^2\right)+a\left(4a(Cd-2cD)d^2+bc\left(-7Dc^2+3Cdc+Bd^2\right)\right)\right)+4\left(3a^2Dd^4-2ab\left(-9Dc^2+4Cdc-2Bd^2\right)d^2-b^2c\left(-14Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)x\right)\left(\right.}{(c+dx)^3}}{3d^2(ad^2+bc^2)}}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$-\frac{5\int -\frac{\frac{8(2ad(3a^2Dd^4-2ab(-9Dc^2+4Cdc-2Bd^2)d^2-b^2c(-14Dc^3+7Cdc^2-3Bd^2c-Ad^3))+3b(2a^2(2Cd-7cD)d^4+ab(-43Dc^3+19Cdc^2-7Bd^2c+3Ad^3)d^2+\cdots}{(c+dx)^2}}{16d^2}\cdots}{\cdots}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{4d^2(c+dx)^4(ad^2+bc^2)}$$

$\downarrow$ 27

$$\frac{5\int \frac{\frac{(2ad(3a^2Dd^4-2ab(-9Dc^2+4Cdc-2Bd^2)d^2-b^2c(-14Dc^3+7Cdc^2-3Bd^2c-Ad^3))+3b(2a^2(2Cd-7cD)d^4+ab(-43Dc^3+19Cdc^2-7Bd^2c+3Ad^3)d^2+2b^2c\cdots}{(c+dx)^2}}{2d^2}\cdots}{\cdots}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{4d^2(c+dx)^4(ad^2+bc^2)}$$

$\downarrow$ 681

$$\frac{\frac{(4a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3))(bx^2+a)^{7/2}}{3d^2(bc^2+ad^2)(c+dx)^3}+\frac{5(-\frac{(2(3a^3Dd^6-2a^2b(-23Dc^2+8Cdc-2Bd^2)d^4-ab^2c(-100Dc^3\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(-Dc^3+Cdc^2-Bd^2c+Ad^3)(bx^2+a)^{7/2}}{4d^2(bc^2+ad^2)(c+dx)^4}$$

$\downarrow$ 27

$$\frac{\frac{(4a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3))(bx^2+a)^{7/2}}{3d^2(bc^2+ad^2)(c+dx)^3}+\frac{5(\frac{3b\int \frac{(ad(2a^2(2Cd-7cD)d^4+ab(-43Dc^3+19Cdc^2-7Bd^2c+3Ad\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(-Dc^3+Cdc^2-Bd^2c+Ad^3)(bx^2+a)^{7/2}}{4d^2(bc^2+ad^2)(c+dx)^4}$$

$\downarrow$ 682

$$\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}+\frac{5\left(\frac{3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}+\frac{5\left(\frac{3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 719$$

$$\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}+\frac{5\left(\frac{3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{}\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2(bc^2+ad^2)(c+dx)^3}+\frac{5\left(3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}-\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2(bc^2+ad^2)(c+dx)^3}+\frac{5\left(3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}-\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}+\frac{5\left(\frac{3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(4a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-11Dc^3+7Cdc^2-3Bd^2c-Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{3d^2\left(bc^2+ad^2\right)(c+dx)^3}+\frac{5\left(\frac{3b\left(\frac{\sqrt{bx^2+a}\left(4a^3(Cd-5cD)d^6+3a^2b\left(-45Dc^3+17Cdc^2-5Bd^2c\right.\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^5,x]
```

output
```
-1/4*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^4) + (((4*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d - 3*B*c*d^2 - A*d^3 - 11*c^3*D))*(a + b*x^2)^(7/2))/(3*d^2*(b*c^2 + a
*d^2)*(c + d*x)^3) + (-1/2*((3*(2*a^2*d^4*(2*C*d - 7*c*D) + a*b*d^2*(19*c^
2*C*d - 7*B*c*d^2 + 3*A*d^3 - 43*c^3*D) + 2*b^2*c^2*(7*c^2*C*d - 3*B*c*d^2
 + A*d^3 - 14*c^3*D)) - 2*d*(3*a^2*d^4*D - 2*a*b*d^2*(4*c*C*d - 2*B*d^2 -
9*c^2*D) - b^2*c*(7*c^2*C*d - 3*B*c*d^2 - A*d^3 - 14*c^3*D))*x)*(a + b*x^2
)^(5/2))/(d^2*(c + d*x)^2) + (5*(-(((2*(3*a^3*d^6*D - 2*a^2*b*d^4*(8*c*C*d
 - 2*B*d^2 - 23*c^2*D) - a*b^2*c*d^2*(45*c^2*C*d - 17*B*c*d^2 + 5*A*d^3 -
100*c^3*D) - 4*b^3*c^3*(7*c^2*C*d - 3*B*c*d^2 + A*d^3 - 14*c^3*D)) - b*d*(
2*a^2*d^4*(2*C*d - 7*c*D) + a*b*d^2*(19*c^2*C*d - 7*B*c*d^2 + 3*A*d^3 - 43
*c^3*D) + 2*b^2*c^2*(7*c^2*C*d - 3*B*c*d^2 + A*d^3 - 14*c^3*D))*x)*(a + b*
x^2)^(3/2))/(d^2*(c + d*x))) + (3*b*(((4*a^3*d^6*(C*d - 5*c*D) + 4*a*b^2*c
^2*d^2*(26*c^2*C*d - 10*B*c*d^2 + 3*A*d^3 - 57*c^3*D) + 3*a^2*b*d^4*(17*c^
2*C*d - 5*B*c*d^2 + A*d^3 - 45*c^3*D) + 8*b^3*c^4*(7*c^2*C*d - 3*B*c*d^2 +
 A*d^3 - 14*c^3*D) + d*(3*a^3*d^6*D - 2*a^2*b*d^4*(8*c*C*d - 2*B*d^2 - 23*
c^2*D) - a*b^2*c*d^2*(45*c^2*C*d - 17*B*c*d^2 + 5*A*d^3 - 100*c^3*D) - 4*b
^3*c^3*(7*c^2*C*d - 3*B*c*d^2 + A*d^3 - 14*c^3*D))*x)*Sqrt[a + b*x^2])/d^2
 + ((b*c^2 + a*d^2)*(((b*c^2 + a*d^2)*(3*a^2*d^4*D - 4*a*b*d^2*(5*c*C*d -
B*d^2 - 15*c^2*D) - 8*b^2*c*(7*c^2*C*d - 3*B*c*d^2 + A*d^3 - 14*c^3*D))...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13260 vs. 2(816) = 1632.

Time = 1.69 (sec) , antiderivative size = 13261, normalized size of antiderivative = 15.35

method	result	size
default	Expression too large to display	13261

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**5,x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9069 vs. $2(821) = 1642$.

Time = 0.61 (sec) , antiderivative size = 9069, normalized size of antiderivative = 10.50

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="maxi
ma")
```

output
```
5/16*D*b^6*c^10*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^9 + 3*a*b^(5/2)*c^4*
d^11 + 3*a^2*b^(3/2)*c^2*d^13 + a^3*sqrt(b)*d^15) - 5/16*C*b^6*c^9*arcsinh
(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^8 + 3*a*b^(5/2)*c^4*d^10 + 3*a^2*b^(3/2)*c^
2*d^12 + a^3*sqrt(b)*d^14) + 5/16*D*a*b^5*c^8*arcsinh(b*x/sqrt(a*b))/(b^(7
/2)*c^6*d^7 + 3*a*b^(5/2)*c^4*d^9 + 3*a^2*b^(3/2)*c^2*d^11 + a^3*sqrt(b)*d
^13) + 5/16*B*b^6*c^8*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^7 + 3*a*b^(5/2
)*c^4*d^9 + 3*a^2*b^(3/2)*c^2*d^11 + a^3*sqrt(b)*d^13) - 5/16*sqrt(b*x^2 +
 a)*D*b^5*c^8*x/(b^3*c^6*d^7 + 3*a*b^2*c^4*d^9 + 3*a^2*b*c^2*d^11 + a^3*d^
13) - 5/16*C*a*b^5*c^7*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^6 + 3*a*b^(5/
2)*c^4*d^8 + 3*a^2*b^(3/2)*c^2*d^10 + a^3*sqrt(b)*d^12) - 5/16*A*b^6*c^7*a
rcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^6 + 3*a*b^(5/2)*c^4*d^8 + 3*a^2*b^(3/
2)*c^2*d^10 + a^3*sqrt(b)*d^12) - 105/16*D*b^5*c^8*arcsinh(b*x/sqrt(a*b))/
(b^(5/2)*c^4*d^9 + 2*a*b^(3/2)*c^2*d^11 + a^2*sqrt(b)*d^13) + 5/16*sqrt(b*
x^2 + a)*C*b^5*c^7*x/(b^3*c^6*d^6 + 3*a*b^2*c^4*d^8 + 3*a^2*b*c^2*d^10 + a
^3*d^12) + 5/16*B*a*b^5*c^6*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^5 + 3*a*
b^(5/2)*c^4*d^7 + 3*a^2*b^(3/2)*c^2*d^9 + a^3*sqrt(b)*d^11) + 85/16*C*b^5*
c^7*arcsinh(b*x/sqrt(a*b))/(b^(5/2)*c^4*d^8 + 2*a*b^(3/2)*c^2*d^10 + a^2*s
qrt(b)*d^12) + 5/24*(b*x^2 + a)^(3/2)*D*b^4*c^7/(b^3*c^6*d^6 + 3*a*b^2*c^4
*d^8 + 3*a^2*b*c^2*d^10 + a^3*d^12) - 5/24*(b*x^2 + a)^(3/2)*D*b^4*c^6*x/(
b^3*c^6*d^5 + 3*a*b^2*c^4*d^7 + 3*a^2*b*c^2*d^9 + a^3*d^11) - 5/16*sqrt...
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{\left(b\,x^2+a\right)^{5/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(c+d\,x\right)^5}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^5}\,dx = \int \frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{\left(dx+c\right)^5}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^5,x)
```

3.94 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 962

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \frac{b(7ad^2D-3b(6cCd-Bd^2-21c^2D))\sqrt{a+bx^2}}{3d^8}$$
$$+\frac{b^2(Cd-6cD)x\sqrt{a+bx^2}}{2d^7}+\frac{b^2Dx^2\sqrt{a+bx^2}}{3d^6}$$
$$-\frac{(bc^2+ad^2)^2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{5d^8(c+dx)^5}$$
$$+\frac{(bc^2+ad^2)(5ad^2(2cCd-Bd^2-3c^2D)+bc(31c^2Cd-26Bcd^2+21Ad^3-36c^3D))\sqrt{a+bx^2}}{20d^8(c+dx)^4}$$
$$-\frac{(20a^2d^4(Cd-3cD)+b^2c^2(327c^2Cd-222Bcd^2+137Ad^3-452c^3D)+abd^2(254c^2Cd-129Bcd^2+44A}{60d^8(c+dx)^3}$$
$$-\frac{(60a^3d^6D-5a^2bd^4(106cCd-27Bd^2-273c^2D)-ab^2cd^2(1961c^2Cd-876Bcd^2+311Ad^3-3746c^3D)}{120d^8(bc^2+ad^2)(c+dx)^2}$$
$$-\frac{b(20a^3d^6(14Cd-69cD)+ab^2c^2d^2(5593c^2Cd-1968Bcd^2+503Ad^3-12998c^3D)+a^2bd^4(3074c^2Cd-}{120d^8(bc^2+ad^2)}$$
$$+\frac{b^{3/2}(5ad^2(Cd-6cD)+2b(21c^2Cd-6Bcd^2+Ad^3-56c^3D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^9}$$
$$-\frac{b(20a^4d^8D-5a^3bd^6(18cCd-3Bd^2-73c^2D)-5a^2b^2cd^4(73c^2Cd-18Bcd^2+3Ad^3-228c^3D)-20ab^3}{8d^9(bc}$$

output
```
1/3*b*(7*a*d^2*D-3*b*(-B*d^2+6*C*c*d-21*D*c^2))*(b*x^2+a)^(1/2)/d^8+1/2*b^
2*(C*d-6*D*c)*x*(b*x^2+a)^(1/2)/d^7+1/3*b^2*D*x^2*(b*x^2+a)^(1/2)/d^6-1/5*
(a*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8/(d*x+c)^
5+1/20*(a*d^2+b*c^2)*(5*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(21*A*d^3-26*B*
c*d^2+31*C*c^2*d-36*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)^4-1/60*(20*a^2*d^4
*(C*d-3*D*c)+b^2*c^2*(137*A*d^3-222*B*c*d^2+327*C*c^2*d-452*D*c^3)+a*b*d^2
*(44*A*d^3-129*B*c*d^2+254*C*c^2*d-419*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)
^3-1/120*(60*a^3*d^6*D-5*a^2*b*d^4*(-27*B*d^2+106*C*c*d-273*D*c^2)-a*b^2*c
*d^2*(311*A*d^3-876*B*c*d^2+1961*C*c^2*d-3746*D*c^3)-2*b^3*c^3*(163*A*d^3-
378*B*c*d^2+723*C*c^2*d-1228*D*c^3))*(b*x^2+a)^(1/2)/d^8/(a*d^2+b*c^2)/(d*
x+c)^2-1/120*b*(20*a^3*d^6*(14*C*d-69*D*c)+a*b^2*c^2*d^2*(503*A*d^3-1968*B
*c*d^2+5593*C*c^2*d-12998*D*c^3)+a^2*b*d^4*(184*A*d^3-879*B*c*d^2+3074*C*c
^2*d-8389*D*c^3)+2*b^3*c^4*(137*A*d^3-522*B*c*d^2+1377*C*c^2*d-2972*D*c^3)
)*(b*x^2+a)^(1/2)/d^8/(a*d^2+b*c^2)^2/(d*x+c)+1/2*b^(3/2)*(5*a*d^2*(C*d-6*
D*c)+2*b*(A*d^3-6*B*c*d^2+21*C*c^2*d-56*D*c^3))*arctanh(b^(1/2)*x/(b*x^2+a
)^(1/2))/d^9-1/8*b*(20*a^4*d^8*D-5*a^3*b*d^6*(-3*B*d^2+18*C*c*d-73*D*c^2)-
5*a^2*b^2*c*d^4*(3*A*d^3-18*B*c*d^2+73*C*c^2*d-228*D*c^3)-20*a*b^3*c^3*d^2
*(A*d^3-6*B*c*d^2+22*C*c^2*d-62*D*c^3)-8*b^4*c^5*(A*d^3-6*B*c*d^2+21*C*c^2
*d-56*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^
9/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 16.45 (sec) , antiderivative size = 1095, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^6} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^6,x]
```

output
```
-1/120*((d*Sqrt[a + b*x^2]*(24*(b*c^2 + a*d^2)^4*(c^2*C*d - B*c*d^2 + A*d^
3 - c^3*D) + 6*(b*c^2 + a*d^2)^3*(5*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b
*c*(-31*c^2*C*d + 26*B*c*d^2 - 21*A*d^3 + 36*c^3*D))*(c + d*x) + 2*(b*c^2
+ a*d^2)^2*(20*a^2*d^4*(C*d - 3*c*D) + b^2*c^2*(327*c^2*C*d - 222*B*c*d^2
+ 137*A*d^3 - 452*c^3*D) + a*b*d^2*(254*c^2*C*d - 129*B*c*d^2 + 44*A*d^3 -
 419*c^3*D))*(c + d*x)^2 + (b*c^2 + a*d^2)*(60*a^3*d^6*D + 5*a^2*b*d^4*(-1
06*c*C*d + 27*B*d^2 + 273*c^2*D) + 2*b^3*c^3*(-723*c^2*C*d + 378*B*c*d^2 -
 163*A*d^3 + 1228*c^3*D) + a*b^2*c*d^2*(-1961*c^2*C*d + 876*B*c*d^2 - 311*
A*d^3 + 3746*c^3*D))*(c + d*x)^3 + b*(20*a^3*d^6*(14*C*d - 69*c*D) + a*b^2
*c^2*d^2*(5593*c^2*C*d - 1968*B*c*d^2 + 503*A*d^3 - 12998*c^3*D) + a^2*b*d
^4*(3074*c^2*C*d - 879*B*c*d^2 + 184*A*d^3 - 8389*c^3*D) + 2*b^3*c^4*(1377
*c^2*C*d - 522*B*c*d^2 + 137*A*d^3 - 2972*c^3*D))*(c + d*x)^4 - 40*b*(b*c^
2 + a*d^2)^2*(7*a*d^2*D + 3*b*(-6*c*C*d + B*d^2 + 21*c^2*D))*(c + d*x)^5 -
 60*b^2*d*(b*c^2 + a*d^2)^2*(C*d - 6*c*D)*x*(c + d*x)^5 - 40*b^2*d^2*(b*c^
2 + a*d^2)^2*D*x^2*(c + d*x)^5))/((b*c^2 + a*d^2)^2*(c + d*x)^5) - (15*b*(
20*a^4*d^8*D + 5*a^3*b*d^6*(-18*c*C*d + 3*B*d^2 + 73*c^2*D) + 8*b^4*c^5*(-
21*c^2*C*d + 6*B*c*d^2 - A*d^3 + 56*c^3*D) + 20*a*b^3*c^3*d^2*(-22*c^2*C*d
 + 6*B*c*d^2 - A*d^3 + 62*c^3*D) + 5*a^2*b^2*c*d^4*(-73*c^2*C*d + 18*B*c*d
^2 - 3*A*d^3 + 228*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(5/2) + 60*b^(3/2
)*(-5*a*d^2*(C*d - 6*c*D) + 2*b*(-21*c^2*C*d + 6*B*c*d^2 - A*d^3 + 56*c...
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 1108, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 681, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^6} \, dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{5/2}\left(5\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(5Cd-5cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+2Bc-2Ad\right)\right)x+5\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^5}dx}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{5/2}\left(5\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(5a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+2Bc-2Ad\right)\right)x+5\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^5}dx}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\left(a+bx^2\right)^{7/2}\left(5ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3-2Bcd^2-12c^3D+7c^2Cd\right)\right)}{4d^2(c+dx)^4\left(ad^2+bc^2\right)}-\int -\frac{(4d(Abd(5bc^2+2ad^2)+a(5a(Cd-2cD)d^2+bc(-7Dc^2+2Cdc+ \ldots}{\ldots}}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(4d(Abd(5bc^2+2ad^2)+a(5a(Cd-2cD)d^2+bc(-7Dc^2+2Cdc+3Bd^2))))+(20a^2Dd^4-5ab(-17Dc^2+6Cdc-3Bd^2)d^2-b^2c(-56Dc^3+21Cdc^2-6Bd^2c-9Ad^3) \ldots}{d^2(c+dx)^4}}{4(ad^2+bc^2)} \ldots}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(4d(Abd(5bc^2+2ad^2)+a(5a(Cd-2cD)d^2+bc(-7Dc^2+2Cdc+3Bd^2))))+(20a^2Dd^4-5ab(-17Dc^2+6Cdc-3Bd^2)d^2-b^2c(-56Dc^3+21Cdc^2-6Bd^2c-9Ad^3) \ldots}{(c+dx)^4}}{4d^2(ad^2+bc^2)} \ldots}{5\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$-\frac{5\int -\frac{6\left(ad\left(20a^2Dd^4-5ab\left(-17Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-56Dc^3+21Cdc^2-6Bd^2c-9Ad^3\right)\right)+2b\left(10a^2(Cd-4cD)d^4+ab\left(-99Dc^3+34Cdc^2-9Bd^2c+4Ad^3\right)\cdots\right.\right.}{(c+dx)^3}}{18d^2}\cdots$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5\int \frac{\left(ad\left(20a^2Dd^4-5ab\left(-17Dc^2+6Cdc-3Bd^2\right)d^2-b^2c\left(-56Dc^3+21Cdc^2-6Bd^2c-9Ad^3\right)\right)+2b\left(10a^2(Cd-4cD)d^4+ab\left(-99Dc^3+34Cdc^2-9Bd^2c+4Ad^3\right)d^2+b\cdots\right.\right.}{(c+dx)^3}}{3d^2}\cdots$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^5\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(-\frac{\left(20a^3Dd^6-5a^2b\left(-49Dc^2+14Cdc-3Bd^2\right)d^4-ab^2c\left(-452Dc\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(3b\int \frac{\left(2ad\left(10a^2(Cd-4cD)d^4+ab\left(-99Dc^3+34Cdc^2-9Bd^2c+4\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 681$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(4\left(bc^2+ad^2\right)^2\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+2\right.\right.\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{\int \frac{4b\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-6Bd^2c+Ad^3\right)\right.}{}}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{\frac{4b\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-6Bd^2c+Ad^3\right)\right)}{d}}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{4b\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-6Bd^2c+Ad^3\right)\right)}{d}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{4\sqrt{b}\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-6Bd^2c+Ad^3\right)\right.}{d}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{4\sqrt{b}\left(bc^2+ad^2\right)^2\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-\right.\right.}{d}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(5a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-12Dc^3+7Cdc^2-2Bd^2c-3Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{4d^2\left(bc^2+ad^2\right)(c+dx)^4}+\frac{5\left(\frac{3b\left(\frac{\frac{4\sqrt{b}\left(bc^2+ad^2\right)^2\left(5a(Cd-6cD)d^2+2b\left(-56Dc^3+21Cdc^2-6\cdots\right.\right.}{d}}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^6,x]
```

output
```
-1/5*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^5) + (((5*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d - 2*B*c*d^2 - 3*A*d^3 - 12*c^3*D))*(a + b*x^2)^(7/2))/(4*d^2*(b*c^2 +
 a*d^2)*(c + d*x)^4) + (-1/3*((2*(10*a^2*d^4*(C*d - 4*c*D) + a*b*d^2*(34*c
^2*C*d - 9*B*c*d^2 + 4*A*d^3 - 99*c^3*D) + b^2*c^2*(21*c^2*C*d - 6*B*c*d^2
 + A*d^3 - 56*c^3*D)) - d*(20*a^2*d^4*D - 5*a*b*d^2*(6*c*C*d - 3*B*d^2 - 1
7*c^2*D) - b^2*c*(21*c^2*C*d - 6*B*c*d^2 - 9*A*d^3 - 56*c^3*D))*x)*(a + b*
x^2)^(5/2))/(d^2*(c + d*x)^3) + (5*(-1/2*((20*a^3*d^6*D - 5*a^2*b*d^4*(14*
c*C*d - 3*B*d^2 - 49*c^2*D) - a*b^2*c*d^2*(157*c^2*C*d - 42*B*c*d^2 + 7*A*
d^3 - 452*c^3*D) - 4*b^3*c^3*(21*c^2*C*d - 6*B*c*d^2 + A*d^3 - 56*c^3*D) -
 2*b*d*(10*a^2*d^4*(C*d - 4*c*D) + a*b*d^2*(34*c^2*C*d - 9*B*c*d^2 + 4*A*d
^3 - 99*c^3*D) + b^2*c^2*(21*c^2*C*d - 6*B*c*d^2 + A*d^3 - 56*c^3*D))*x)*(
a + b*x^2)^(3/2))/(d^2*(c + d*x)^2) + (3*b*(-(((4*(b*c^2 + a*d^2)^2*(5*a*d
^2*(C*d - 6*c*D) + 2*b*(21*c^2*C*d - 6*B*c*d^2 + A*d^3 - 56*c^3*D)) - d*(2
0*a^3*d^6*D - 5*a^2*b*d^4*(14*c*C*d - 3*B*d^2 - 49*c^2*D) - a*b^2*c*d^2*(1
57*c^2*C*d - 42*B*c*d^2 + 7*A*d^3 - 452*c^3*D) - 4*b^3*c^3*(21*c^2*C*d - 6
*B*c*d^2 + A*d^3 - 56*c^3*D))*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x))) + ((4*S
qrt[b]*(b*c^2 + a*d^2)^2*(5*a*d^2*(C*d - 6*c*D) + 2*b*(21*c^2*C*d - 6*B*c*
d^2 + A*d^3 - 56*c^3*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - ((20*a^
4*d^8*D - 5*a^3*b*d^6*(18*c*C*d - 3*B*d^2 - 73*c^2*D) - 5*a^2*b^2*c*d^4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21713 vs. 2(914) = 1828.

Time = 1.89 (sec) , antiderivative size = 21714, normalized size of antiderivative = 22.57

method	result	size
default	Expression too large to display	21714

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**6,x)
```

output
```
Integral((a + b*x**2)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14351 vs. 2(918) = 1836.

Time = 0.86 (sec) , antiderivative size = 14351, normalized size of antiderivative = 14.92

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="maxi
ma")
```

output
```
3/16*D*b^7*c^11*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^9 + 4*a*b^(7/2)*c^6*
d^11 + 6*a^2*b^(5/2)*c^4*d^13 + 4*a^3*b^(3/2)*c^2*d^15 + a^4*sqrt(b)*d^17)
 - 3/16*C*b^7*c^10*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^8 + 4*a*b^(7/2)*c
^6*d^10 + 6*a^2*b^(5/2)*c^4*d^12 + 4*a^3*b^(3/2)*c^2*d^14 + a^4*sqrt(b)*d^
16) + 3/16*D*a*b^6*c^9*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^7 + 4*a*b^(7/
2)*c^6*d^9 + 6*a^2*b^(5/2)*c^4*d^11 + 4*a^3*b^(3/2)*c^2*d^13 + a^4*sqrt(b)
*d^15) + 3/16*B*b^7*c^9*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^7 + 4*a*b^(7
/2)*c^6*d^9 + 6*a^2*b^(5/2)*c^4*d^11 + 4*a^3*b^(3/2)*c^2*d^13 + a^4*sqrt(b
)*d^15) - 3/16*sqrt(b*x^2 + a)*D*b^6*c^9*x/(b^4*c^8*d^7 + 4*a*b^3*c^6*d^9
+ 6*a^2*b^2*c^4*d^11 + 4*a^3*b*c^2*d^13 + a^4*d^15) - 3/16*C*a*b^6*c^8*arc
sinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^6 + 4*a*b^(7/2)*c^6*d^8 + 6*a^2*b^(5/2)
*c^4*d^10 + 4*a^3*b^(3/2)*c^2*d^12 + a^4*sqrt(b)*d^14) - 3/16*A*b^7*c^8*ar
csinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^6 + 4*a*b^(7/2)*c^6*d^8 + 6*a^2*b^(5/2
)*c^4*d^10 + 4*a^3*b^(3/2)*c^2*d^12 + a^4*sqrt(b)*d^14) - 17/8*D*b^6*c^9*a
rcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^9 + 3*a*b^(5/2)*c^4*d^11 + 3*a^2*b^(3
/2)*c^2*d^13 + a^3*sqrt(b)*d^15) + 3/16*sqrt(b*x^2 + a)*C*b^6*c^8*x/(b^4*c
^8*d^6 + 4*a*b^3*c^6*d^8 + 6*a^2*b^2*c^4*d^10 + 4*a^3*b*c^2*d^12 + a^4*d^1
4) + 3/16*B*a*b^6*c^7*arcsinh(b*x/sqrt(a*b))/(b^(9/2)*c^8*d^5 + 4*a*b^(7/2
)*c^6*d^7 + 6*a^2*b^(5/2)*c^4*d^9 + 4*a^3*b^(3/2)*c^2*d^11 + a^4*sqrt(b)*d
^13) + 29/16*C*b^6*c^8*arcsinh(b*x/sqrt(a*b))/(b^(7/2)*c^6*d^8 + 3*a*b^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6440 vs. $2(918) = 1836$.

Time = 1.43 (sec) , antiderivative size = 6440, normalized size of antiderivative = 6.69

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x, algorithm="giac
")
```

output
```
1/6*sqrt(b*x^2 + a)*(x*(2*D*b^2*x/d^6 - 3*(6*D*b^3*c*d^23 - C*b^3*d^24)/(b
*d^30)) + 2*(63*D*b^3*c^2*d^22 - 18*C*b^3*c*d^23 + 7*D*a*b^2*d^24 + 3*B*b^
3*d^24)/(b*d^30)) + 1/4*(448*D*b^5*c^8 - 168*C*b^5*c^7*d + 1240*D*a*b^4*c^
6*d^2 + 48*B*b^5*c^6*d^2 - 440*C*a*b^4*c^5*d^3 - 8*A*b^5*c^5*d^3 + 1140*D*
a^2*b^3*c^4*d^4 + 120*B*a*b^4*c^4*d^4 - 365*C*a^2*b^3*c^3*d^5 - 20*A*a*b^4
*c^3*d^5 + 365*D*a^3*b^2*c^2*d^6 + 90*B*a^2*b^3*c^2*d^6 - 90*C*a^3*b^2*c*d
^7 - 15*A*a^2*b^3*c*d^7 + 20*D*a^4*b*d^8 + 15*B*a^3*b^2*d^8)*arctan(-((sqr
t(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^2*c^4*d
^9 + 2*a*b*c^2*d^11 + a^2*d^13)*sqrt(-b*c^2 - a*d^2)) + 1/60*(8400*(sqrt(b
)*x - sqrt(b*x^2 + a))^9*D*b^5*c^8*d^4 - 4200*(sqrt(b)*x - sqrt(b*x^2 + a)
)^9*C*b^5*c^7*d^5 + 19200*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b^4*c^6*d^6
+ 1800*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*b^5*c^6*d^6 - 9000*(sqrt(b)*x - s
qrt(b*x^2 + a))^9*C*a*b^4*c^5*d^7 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*
b^5*c^5*d^7 + 13500*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^2*b^3*c^4*d^8 + 36
00*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a*b^4*c^4*d^8 - 5565*(sqrt(b)*x - sqr
t(b*x^2 + a))^9*C*a^2*b^3*c^3*d^9 - 1140*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A
*a*b^4*c^3*d^9 + 2805*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^3*b^2*c^2*d^10 +
 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c^2*d^10 - 810*(sqrt(b)*x
- sqrt(b*x^2 + a))^9*C*a^3*b^2*c*d^11 - 495*(sqrt(b)*x - sqrt(b*x^2 + a))^
9*A*a^2*b^3*c*d^11 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^4*b*d^12 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^6}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^6}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^6}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^6,x)
```

3.95 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1079

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

output
```
b^2*(C*d-7*D*c)*(b*x^2+a)^(1/2)/d^8+1/2*b^2*D*x*(b*x^2+a)^(1/2)/d^7-1/6*(a
*d^2+b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^8/(d*x+c)^6+
1/30*(a*d^2+b*c^2)*(6*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(25*A*d^3-31*B*c*
d^2+37*C*c^2*d-43*D*c^3))*(b*x^2+a)^(1/2)/d^8/(d*x+c)^5-1/120*(30*a^2*d^4*
(C*d-3*D*c)+a*b*d^2*(65*A*d^3-191*B*c*d^2+377*C*c^2*d-623*D*c^3)+b^2*(200*
A*c^2*d^3-326*B*c^3*d^2+482*C*c^4*d-668*D*c^5))*(b*x^2+a)^(1/2)/d^8/(d*x+c
)^4-1/120*(40*a^3*d^6*D-2*a^2*b*d^4*(-44*B*d^2+173*C*c*d-447*D*c^2)-a*b^2*
c*d^2*(195*A*d^3-557*B*c*d^2+1259*C*c^2*d-2421*D*c^3)-2*b^3*c^3*(100*A*d^3
-237*B*c*d^2+459*C*c^2*d-786*D*c^3))*(b*x^2+a)^(1/2)/d^8/(a*d^2+b*c^2)/(d*
x+c)^3-1/240*b*(10*a^3*d^6*(27*C*d-133*D*c)+a^2*b*d^4*(165*A*d^3-787*B*c*d
^2+2809*C*c^2*d-7791*D*c^3)+2*a*b^2*c^2*d^2*(195*A*d^3-832*B*c*d^2+2464*C*
c^2*d-5871*D*c^3)+4*b^3*c^4*(50*A*d^3-213*B*c*d^2+591*C*c^2*d-1314*D*c^3))
*(b*x^2+a)^(1/2)/d^8/(a*d^2+b*c^2)^2/(d*x+c)^2-1/240*b*(560*a^4*d^8*D-2*a^
3*b*d^6*(-184*B*d^2+1063*C*c*d-4417*D*c^2)-2*a*b^3*c^3*d^2*(65*A*d^3-842*B
*c*d^2+3854*C*c^2*d-12201*D*c^3)-3*a^2*b^2*c*d^4*(55*A*d^3-513*B*c*d^2+241
1*C*c^2*d-8189*D*c^3)-4*b^4*c^5*(10*A*d^3-147*B*c*d^2+669*C*c^2*d-2046*D*c
^3))*(b*x^2+a)^(1/2)/d^8/(a*d^2+b*c^2)^3/(d*x+c)+1/2*b^(3/2)*(5*a*d^2*D-2*
b*(-B*d^2+7*C*c*d-28*D*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^9-1/16*b
^2*(30*a^4*d^8*(C*d-7*D*c)+8*a*b^3*c^5*d^2*(-7*B*d^2+49*C*c*d-201*D*c^2)+7
0*a^2*b^2*c^3*d^4*(-B*d^2+7*C*c*d-30*D*c^2)+16*b^4*c^7*(-B*d^2+7*C*c*d-...
```

Mathematica [A] (verified)

Time = 16.28 (sec) , antiderivative size = 1179, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
(-((d*Sqrt[a + b*x^2]*(40*(b*c^2 + a*d^2)^5*(c^2*C*d - B*c*d^2 + A*d^3 - c
^3*D) + 8*(b*c^2 + a*d^2)^4*(6*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-
37*c^2*C*d + 31*B*c*d^2 - 25*A*d^3 + 43*c^3*D))*(c + d*x) + 2*(b*c^2 + a*d
^2)^3*(30*a^2*d^4*(C*d - 3*c*D) + a*b*d^2*(377*c^2*C*d - 191*B*c*d^2 + 65*
A*d^3 - 623*c^3*D) + b^2*(482*c^4*C*d - 326*B*c^3*d^2 + 200*A*c^2*d^3 - 66
8*c^5*D))*(c + d*x)^2 + 2*(b*c^2 + a*d^2)^2*(40*a^3*d^6*D + 2*a^2*b*d^4*(-
173*c*C*d + 44*B*d^2 + 447*c^2*D) + 2*b^3*c^3*(-459*c^2*C*d + 237*B*c*d^2
- 100*A*d^3 + 786*c^3*D) + a*b^2*c*d^2*(-1259*c^2*C*d + 557*B*c*d^2 - 195*
A*d^3 + 2421*c^3*D))*(c + d*x)^3 + b*(b*c^2 + a*d^2)*(10*a^3*d^6*(27*C*d -
 133*c*D) + a^2*b*d^4*(2809*c^2*C*d - 787*B*c*d^2 + 165*A*d^3 - 7791*c^3*D
) - 4*b^3*c^4*(-591*c^2*C*d + 213*B*c*d^2 - 50*A*d^3 + 1314*c^3*D) - 2*a*b
^2*c^2*d^2*(-2464*c^2*C*d + 832*B*c*d^2 - 195*A*d^3 + 5871*c^3*D))*(c + d*
x)^4 + b*(560*a^4*d^8*D + 2*a^3*b*d^6*(-1063*c*C*d + 184*B*d^2 + 4417*c^2*
D) + 4*b^4*c^5*(-669*c^2*C*d + 147*B*c*d^2 - 10*A*d^3 + 2046*c^3*D) + 3*a^
2*b^2*c*d^4*(-2411*c^2*C*d + 513*B*c*d^2 - 55*A*d^3 + 8189*c^3*D) + 2*a*b^
3*c^3*d^2*(-3854*c^2*C*d + 842*B*c*d^2 - 65*A*d^3 + 12201*c^3*D))*(c + d*x
)^5 + 240*b^2*(b*c^2 + a*d^2)^3*(-(C*d) + 7*c*D)*(c + d*x)^6 - 120*b^2*d*(
b*c^2 + a*d^2)^3*D*x*(c + d*x)^6))/((b*c^2 + a*d^2)^3*(c + d*x)^6)) - (15*
b^2*(-30*a^4*d^8*(C*d - 7*c*D) + 16*b^4*c^7*(-7*c*C*d + B*d^2 + 28*c^2*D)
+ 70*a^2*b^2*c^3*d^4*(-7*c*C*d + B*d^2 + 30*c^2*D) + 8*a*b^3*c^5*d^2*(-...
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 25, 2182, 25, 27, 681, 27, 680, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^7}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(6\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(6Cd-6cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+Bc-Ad\right)\right)x+6\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(6\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(6a(Cd-cD)-b\left(\frac{7Dc^3}{d^2}-\frac{7Cc^2}{d}+Bc-Ad\right)\right)x+6\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{7/2}\left(6ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-5Ad^3-Bcd^2-13c^3D+7c^2Cd\right)\right)}{5d^2(c+dx)^5(ad^2+bc^2)}-\frac{\int -\frac{\left(5d\left(Abd\left(6bc^2+ad^2\right)+a\left(6a(Cd-2cD)d^2+bc\left(-7Dc^2+Cdc+5B\right)\right)\right)\right)}{}}{}}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(5d\left(Abd\left(6bc^2+ad^2\right)+a\left(6a(Cd-2cD)d^2+bc\left(-7Dc^2+Cdc+5Bd^2\right)\right)\right)+2\left(15a^2Dd^4-6ab\left(-8Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-28Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)x\right)\left(b\right)}{d^2(c+dx)^5}}{5(ad^2+bc^2)}}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(5d\left(Abd\left(6bc^2+ad^2\right)+a\left(6a(Cd-2cD)d^2+bc\left(-7Dc^2+Cdc+5Bd^2\right)\right)\right)+2\left(15a^2Dd^4-6ab\left(-8Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-28Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)x\right)\left(b\right)}{(c+dx)^5}}{5d^2(ad^2+bc^2)}}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 681$$

$$-\frac{5\int -\frac{4\left(4ad\left(15a^2Dd^4-6ab\left(-8Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-28Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)+b\left(30a^2(Cd-5cD)d^4+ab\left(-323Dc^3+77Cdc^2-11Bd^2c+5Ad^3\right)d^2\right.\right.}{(c+dx)^4}}{16d^2}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$\frac{5\int \frac{\left(4ad\left(15a^2Dd^4-6ab\left(-8Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-28Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)+b\left(30a^2(Cd-5cD)d^4+ab\left(-323Dc^3+77Cdc^2-11Bd^2c+5Ad^3\right)d^2+6b^2\right.\right.}{(c+dx)^4}}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6d^2(c+dx)^6\left(ad^2+bc^2\right)}$$

$\downarrow$ 680

$$\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(-\frac{\left(40a^4Dd^8-2a^3b\left(-29Dc^2+11Cdc-8Bd^2\right)d^6+3a^2b^2c\left(-99Dc\right.\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\downarrow$ 27

$$\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\int \frac{\left(2ad\left(20a^3Dd^6-2a^2b\left(-57Dc^2+13Cdc-4Bd^2\right)d^4-5ab^2c\left(-\right.\right.\right.}{}}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$\downarrow$ 681

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\left(-\frac{\sqrt{bx^2+a}\left(8\left(bc^2+ad^2\right)^3\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right.\right.\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\left(b\int\frac{8\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)x\left(bc^2+ad^2\right)^3+ad}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\left(\frac{b\left(\frac{8\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx\left(bc^2\right.}{d}\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\left(\frac{b\left(\frac{8\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(6a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{5d^2\left(bc^2+ad^2\right)(c+dx)^5}+\frac{5\left(\frac{3b\left(\frac{b\left(\frac{8\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{b}d}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3))(bx^2+a)^{7/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{5\left(\frac{3b\left(\frac{b\left(\frac{8\left(bc^2+ad^2\right)^3\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)\text{arcta}\ldots}{\sqrt{b}d}\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(6a(-3Dc^2+2Cdc-Bd^2)d^2+bc(-13Dc^3+7Cdc^2-Bd^2c-5Ad^3))(bx^2+a)^{7/2}}{5d^2(bc^2+ad^2)(c+dx)^5}+\frac{5\left(\frac{3b\left(\frac{b\left(\frac{8\left(bc^2+ad^2\right)^3\left(5ad^2D-2b\left(-28Dc^2+7Cdc-Bd^2\right)\right)\text{arcta}\ldots}{\sqrt{b}d}\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{6d^2\left(bc^2+ad^2\right)(c+dx)^6}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^7,x]
```

output
```
-1/6*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^6) + (((6*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d - B*c*d^2 - 5*A*d^3 - 13*c^3*D))*(a + b*x^2)^(7/2))/(5*d^2*(b*c^2 + a
*d^2)*(c + d*x)^5) + (-1/4*((30*a^2*d^4*(C*d - 5*c*D) + 6*b^2*c^3*(7*c*C*d
 - B*d^2 - 28*c^2*D) + a*b*d^2*(77*c^2*C*d - 11*B*c*d^2 + 5*A*d^3 - 323*c^
3*D) - 4*d*(15*a^2*d^4*D - 6*a*b*d^2*(2*c*C*d - B*d^2 - 8*c^2*D) - b^2*c*(
7*c^2*C*d - B*c*d^2 - 5*A*d^3 - 28*c^3*D))*x)*(a + b*x^2)^(5/2))/(d^2*(c +
 d*x)^4) + (5*(-1/2*((40*a^4*d^8*D + 2*a*b^3*c^4*d^2*(63*c*C*d - 9*B*d^2 -
 262*c^2*D) - 2*a^3*b*d^6*(11*c*C*d - 8*B*d^2 - 29*c^2*D) + 8*b^4*c^6*(7*c
*C*d - B*d^2 - 28*c^2*D) + 3*a^2*b^2*c*d^4*(21*c^2*C*d - 3*B*c*d^2 + 5*A*d
^3 - 99*c^3*D) + b*d*(30*a^3*d^6*(C*d - 7*c*D) + 12*b^3*c^5*(7*c*C*d - B*d
^2 - 28*c^2*D) + 2*a*b^2*c^2*d^2*(112*c^2*C*d - 16*B*c*d^2 - 5*A*d^3 - 463
*c^3*D) + 5*a^2*b*d^4*(37*c^2*C*d - 7*B*c*d^2 + A*d^3 - 163*c^3*D))*x)*(a
+ b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)*(c + d*x)^3) + (3*b*(-(((8*(b*c^2 + a
*d^2)^3*(5*a*d^2*D - 2*b*(7*c*C*d - B*d^2 - 28*c^2*D)) - b*d*(10*a^3*d^6*(
3*C*d - 17*c*D) + 2*a*b^2*c^3*d^2*(77*c*C*d - 11*B*d^2 - 318*c^2*D) + 8*b^
3*c^5*(7*c*C*d - B*d^2 - 28*c^2*D) + a^2*b*d^4*(133*c^2*C*d - 19*B*c*d^2 +
 5*A*d^3 - 587*c^3*D))*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x))) + (b*((8*(b*c^
2 + a*d^2)^3*(5*a*d^2*D - 2*b*(7*c*C*d - B*d^2 - 28*c^2*D))*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - ((30*a^4*d^8*(C*d - 7*c*D) + 8*a*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35337 vs. 2(1033) = 2066.

Time = 2.07 (sec) , antiderivative size = 35338, normalized size of antiderivative = 32.75

method	result	size
default	Expression too large to display	35338

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21589 vs. 2(1035) = 2070.

Time = 1.24 (sec) , antiderivative size = 21589, normalized size of antiderivative = 20.01

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="maxi
ma")
```

output
```
5/32*D*b^8*c^12*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^9 + 5*a*b^(9/2)*c^
8*d^11 + 10*a^2*b^(7/2)*c^6*d^13 + 10*a^3*b^(5/2)*c^4*d^15 + 5*a^4*b^(3/2)
*c^2*d^17 + a^5*sqrt(b)*d^19) - 5/32*C*b^8*c^11*arcsinh(b*x/sqrt(a*b))/(b^
(11/2)*c^10*d^8 + 5*a*b^(9/2)*c^8*d^10 + 10*a^2*b^(7/2)*c^6*d^12 + 10*a^3*
b^(5/2)*c^4*d^14 + 5*a^4*b^(3/2)*c^2*d^16 + a^5*sqrt(b)*d^18) + 5/32*D*a*b
^7*c^10*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^7 + 5*a*b^(9/2)*c^8*d^9 +
10*a^2*b^(7/2)*c^6*d^11 + 10*a^3*b^(5/2)*c^4*d^13 + 5*a^4*b^(3/2)*c^2*d^15
 + a^5*sqrt(b)*d^17) + 5/32*B*b^8*c^10*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^
10*d^7 + 5*a*b^(9/2)*c^8*d^9 + 10*a^2*b^(7/2)*c^6*d^11 + 10*a^3*b^(5/2)*c^
4*d^13 + 5*a^4*b^(3/2)*c^2*d^15 + a^5*sqrt(b)*d^17) - 5/32*sqrt(b*x^2 + a)
*D*b^7*c^10*x/(b^5*c^10*d^7 + 5*a*b^4*c^8*d^9 + 10*a^2*b^3*c^6*d^11 + 10*a
^3*b^2*c^4*d^13 + 5*a^4*b*c^2*d^15 + a^5*d^17) - 5/32*C*a*b^7*c^9*arcsinh(
b*x/sqrt(a*b))/(b^(11/2)*c^10*d^6 + 5*a*b^(9/2)*c^8*d^8 + 10*a^2*b^(7/2)*c
^6*d^10 + 10*a^3*b^(5/2)*c^4*d^12 + 5*a^4*b^(3/2)*c^2*d^14 + a^5*sqrt(b)*d
^16) - 5/32*A*b^8*c^9*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^6 + 5*a*b^(9
/2)*c^8*d^8 + 10*a^2*b^(7/2)*c^6*d^10 + 10*a^3*b^(5/2)*c^4*d^12 + 5*a^4*b^
(3/2)*c^2*d^14 + a^5*sqrt(b)*d^16) - 3/2*D*b^7*c^10*arcsinh(b*x/sqrt(a*b))
/(b^(9/2)*c^8*d^9 + 4*a*b^(7/2)*c^6*d^11 + 6*a^2*b^(5/2)*c^4*d^13 + 4*a^3*
b^(3/2)*c^2*d^15 + a^4*sqrt(b)*d^17) + 5/32*sqrt(b*x^2 + a)*C*b^7*c^9*x/(b
^5*c^10*d^6 + 5*a*b^4*c^8*d^8 + 10*a^2*b^3*c^6*d^10 + 10*a^3*b^2*c^4*d^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8884 vs. $2(1035) = 2070$.

Time = 0.81 (sec) , antiderivative size = 8884, normalized size of antiderivative = 8.23

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x, algorithm="giac
")
```

output
```
1/2*sqrt(b*x^2 + a)*(D*b^2*x/d^7 - 2*(7*D*b^2*c*d^16 - C*b^2*d^17)/d^24) -
 1/8*(448*D*b^6*c^9 - 112*C*b^6*c^8*d + 1608*D*a*b^5*c^7*d^2 + 16*B*b^6*c^
7*d^2 - 392*C*a*b^5*c^6*d^3 + 2100*D*a^2*b^4*c^5*d^4 + 56*B*a*b^5*c^5*d^4
- 490*C*a^2*b^4*c^4*d^5 + 1155*D*a^3*b^3*c^3*d^6 + 70*B*a^2*b^4*c^3*d^6 -
245*C*a^3*b^3*c^2*d^7 + 210*D*a^4*b^2*c*d^8 + 35*B*a^3*b^3*c*d^8 - 30*C*a^
4*b^2*d^9 - 5*A*a^3*b^3*d^9)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sq
rt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^3*c^6*d^9 + 3*a*b^2*c^4*d^11 + 3*a^2*b*
c^2*d^13 + a^3*d^15)*sqrt(-b*c^2 - a*d^2)) - 1/120*(13440*(sqrt(b)*x - sqr
t(b*x^2 + a))^11*D*b^6*c^9*d^5 - 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*b
^6*c^8*d^6 + 41400*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a*b^5*c^7*d^7 + 1440
*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*b^6*c^7*d^7 - 15000*(sqrt(b)*x - sqrt(
b*x^2 + a))^11*C*a*b^5*c^6*d^8 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*b^
6*c^6*d^8 + 44100*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^2*b^4*c^5*d^9 + 420
0*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^5*c^5*d^9 - 14970*(sqrt(b)*x - sq
rt(b*x^2 + a))^11*C*a^2*b^4*c^4*d^10 - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^1
1*A*a*b^5*c^4*d^10 + 17955*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^3*b^3*c^3*
d^11 + 3990*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^2*b^4*c^3*d^11 - 5205*(sq
rt(b)*x - sqrt(b*x^2 + a))^11*C*a^3*b^3*c^2*d^12 - 720*(sqrt(b)*x - sqrt(b
*x^2 + a))^11*A*a^2*b^4*c^2*d^12 + 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D
*a^4*b^2*c*d^13 + 1155*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^3*c*d^1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^7}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^7}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^7}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^7,x)
```

3.96 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8} dx$

Optimal result

Integrand size = 34, antiderivative size = 1064

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8} dx =$$

$$-\frac{b^2\left(16(bc^2+ad^2)^4(Cd-8cD)-d(30a^4d^8D-8b^4c^7(Cd-8cD)-30ab^3c^5d^2(Cd-8cD)-a^3bd^6(24cC\cdots}{16d^8(bc^2+ad^2)^4(c+dx)}$$

$$-\frac{b(2a^4d^8(8Cd-19cD)-8b^4c^8(Cd-8cD)-26ab^3c^6d^2(Cd-8cD)-a^3bcd^6(8cCd-15Bd^2-49c^2D)-\cdots}{\cdots}$$

$$+\frac{(6b^3c^6(Cd-8cD)-6a^3d^6(4Cd-7cD)+a^2bcd^4(8cCd-25Bd^2-39c^2D)+ab^2c^2d^2(13c^2Cd-25Ad^3-\cdots}{\cdots}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx^2)^{7/2}}{7d^2(bc^2+ad^2)(c+dx)^7}$$

$$+\frac{(ad^2(2cCd-Bd^2-3c^2D)+b(c^3Cd-Acd^3-2c^4D))(a+bx^2)^{7/2}}{6d^2(bc^2+ad^2)^2(c+dx)^6}$$

$$+\frac{b^{5/2}(Cd-8cD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^9}$$

$$-\frac{b^2(30a^5d^{10}D-16b^5c^9(Cd-8cD)-72ab^4c^7d^2(Cd-8cD)-126a^2b^3c^5d^4(Cd-8cD)-5a^4bd^8(8cCd-\cdots}{16d^9(bc^2+ad^2)^{9/2}}$$

output
```
-1/16*b^2*(16*(a*d^2+b*c^2)^4*(C*d-8*D*c)-d*(30*a^4*d^8*D-8*b^4*c^7*(C*d-8
*D*c)-30*a*b^3*c^5*d^2*(C*d-8*D*c)-a^3*b*d^6*(-5*B*d^2+24*C*c*d-187*D*c^2)
-a^2*b^2*c*d^4*(-5*A*d^3+41*C*c^2*d-328*D*c^3))*x)*(b*x^2+a)^(1/2)/d^8/(a*
d^2+b*c^2)^4/(d*x+c)-1/48*b*(2*a^4*d^8*(8*C*d-19*D*c)-8*b^4*c^8*(C*d-8*D*c
)-26*a*b^3*c^6*d^2*(C*d-8*D*c)-a^3*b*c*d^6*(-15*B*d^2+8*C*c*d-49*D*c^2)-3*
a^2*b^2*c^2*d^4*(-5*A*d^3+9*C*c^2*d-72*D*c^3)+d*(30*a^4*d^8*D-12*b^4*c^7*(
C*d-8*D*c)-5*a^3*b*d^6*(-B*d^2+8*C*c*d-51*D*c^2)-a^2*b^2*c*d^4*(-5*A*d^3+1
0*B*c*d^2+57*C*c^2*d-466*D*c^3)-2*a*b^3*c^3*d^2*(5*A*d^3+22*C*c^2*d-176*D*
c^3))*x)*(b*x^2+a)^(3/2)/d^6/(a*d^2+b*c^2)^4/(d*x+c)^3+1/120*(6*b^3*c^6*(C
*d-8*D*c)-6*a^3*d^6*(4*C*d-7*D*c)+a^2*b*c*d^4*(-25*B*d^2+8*C*c*d-39*D*c^2)
+a*b^2*c^2*d^2*(-25*A*d^3+13*C*c^2*d-104*D*c^3)-5*d*(6*a^3*d^6*D-a^2*b*d^4
*(-B*d^2+8*C*c*d-39*D*c^2)-a*b^2*c*d^2*(-A*d^3+4*B*c*d^2+5*C*c^2*d-44*D*c^
3)-2*b^3*(2*A*c^3*d^3+C*c^5*d-8*D*c^6))*x)*(b*x^2+a)^(5/2)/d^4/(a*d^2+b*c^
2)^3/(d*x+c)^5-1/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(7/2)/d^2/(a*d^
2+b*c^2)/(d*x+c)^7+1/6*(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*(-A*c*d^3+C*c^3*d
-2*D*c^4))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^6+b^(5/2)*(C*d-8*D*
c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^9-1/16*b^2*(30*a^5*d^10*D-16*b^5*c
^9*(C*d-8*D*c)-72*a*b^4*c^7*d^2*(C*d-8*D*c)-126*a^2*b^3*c^5*d^4*(C*d-8*D*c
)-5*a^4*b*d^8*(-B*d^2+8*C*c*d-63*D*c^2)-5*a^3*b^2*c*d^6*(-A*d^3+21*C*c^2*d
-168*D*c^3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/...
```

Mathematica [A] (verified)

Time = 17.67 (sec) , antiderivative size = 1795, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
Sqrt[a + b*x^2]*((b^2*D)/d^8 - ((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d
^3 - c^3*D))/(7*d^8*(c + d*x)^7) - ((b*c^2 + a*d^2)*(-43*b*c^3*C*d + 36*b*
B*c^2*d^2 - 29*A*b*c*d^3 - 14*a*c*C*d^3 + 7*a*B*d^4 + 50*b*c^4*D + 21*a*c^
2*d^2*D))/(42*d^8*(c + d*x)^6) + (-667*b^2*c^4*C*d + 450*b^2*B*c^3*d^2 - 2
75*A*b^2*c^2*d^3 - 524*a*b*c^2*C*d^3 + 265*a*b*B*c*d^4 - 90*a*A*b*d^5 - 42
*a^2*C*d^5 + 926*b^2*c^5*D + 867*a*b*c^3*d^2*D + 126*a^2*c*d^4*D)/(210*d^8
*(c + d*x)^5) + (4682*b^3*c^5*C*d - 2400*b^3*B*c^4*d^2 + 1000*A*b^3*c^3*d^
3 + 6459*a*b^2*c^3*C*d^3 - 2840*a*b^2*B*c^2*d^4 + 985*a*A*b^2*c*d^5 + 1792
*a^2*b*c*C*d^5 - 455*a^2*b*B*d^6 - 8056*b^3*c^6*D - 12472*a*b^2*c^4*d^2*D
- 4641*a^2*b*c^2*d^4*D - 210*a^3*d^6*D)/(840*d^8*(b*c^2 + a*d^2)*(c + d*x)
^4) - (b*(5118*b^3*c^6*C*d - 1800*b^3*B*c^5*d^2 + 400*A*b^3*c^4*d^3 + 1077
7*a*b^2*c^4*C*d^3 - 3560*a*b^2*B*c^3*d^4 + 795*a*A*b^2*c^2*d^5 + 6240*a^2*
b*c^2*C*d^5 - 1725*a^2*b*B*c*d^6 + 360*a^2*A*b*d^7 + 616*a^3*C*d^7 - 11544
*b^3*c^7*D - 26016*a*b^2*c^5*d^2*D - 17475*a^2*b*c^3*d^4*D - 3038*a^3*c*d^
6*D))/(840*d^8*(b*c^2 + a*d^2)^2*(c + d*x)^3) - (b*(-7404*b^4*c^7*C*d + 14
40*b^4*B*c^6*d^2 - 40*A*b^4*c^5*d^3 - 21892*a*b^3*c^5*C*d^3 + 4280*a*b^3*B
*c^4*d^4 - 150*a*A*b^3*c^3*d^5 - 21327*a^2*b^2*c^3*C*d^5 + 4170*a^2*b^2*B*
c^2*d^6 - 285*a^2*A*b^2*c*d^7 - 6664*a^3*b*c*C*d^7 + 1155*a^3*b*B*d^8 + 23
952*b^4*c^8*D + 72936*a*b^3*c^6*d^2*D + 75486*a^2*b^2*c^4*d^4*D + 28217*a^
3*b*c^2*d^6*D + 1890*a^4*d^8*D))/(1680*d^8*(b*c^2 + a*d^2)^3*(c + d*x)^...
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 1134, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 27, 2182, 25, 27, 680, 27, 680, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^8}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{7\left(bx^2+a\right)^{5/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^7}dx}{7\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{5/2}\left(\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^7}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2182}$$

$$\frac{-\frac{\int -\frac{\left(6\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(6a^2Dd^4-ab\left(-15Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-8Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{d^2(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{d(a+bx}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(6\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(6a^2Dd^4-ab\left(-15Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-8Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{d^2(c+dx)^6}dx}{6\left(ad^2+bc^2\right)}-\frac{d(a+bx^2)^{7/}}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(6\left(Ab^2c^2+a\left(-\frac{bDc^3}{d}+bBdc+ad(Cd-2cD)\right)\right)d^2+\left(6a^2Dd^4-ab\left(-15Dc^2+2Cdc-Bd^2\right)d^2-b^2\left(-8Dc^4+Cdc^3-Ad^3c\right)\right)x\right)\left(bx^2+a\right)^{5/2}}{(c+dx)^6}dx}{6d^2\left(ad^2+bc^2\right)}-\frac{d(a+bx^2)^{7/}}{}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{680}$$

$$\frac{\frac{\left(a+bx^2\right)^{5/2}\left(-6a^3d^6(4Cd-7cD)+a^2bcd^4\left(-25Bd^2-39c^2D+8cCd\right)-5dx\left(6a^3d^6D-a^2bd^4\left(-Bd^2-39c^2D+8cCd\right)-ab^2cd^2\left(-Ad^3+4Bcd^2-44c^3D+5c^2Cd\right)-2\right.\right.}{20d^2(c+dx)^5\left(ad^2+bc^2\right)}}{}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\frac{\left(4ad\left(6a^2(Cd-3cD)d^4+abc\left(-21Dc^2+2Cdc+5Bd^2\right)d^2+b^2\left(-8Dc^5+Cdc^4+5Ad^3c^2\right)\right)+\left(30a^3Dd^6-a^2b\left(-123Dc^2+16Cdc-5Bd^2\right)d^4-ab^2c\left(-136Dc^3+17C\right.\right.}{(c+dx)^4}}{4d^2\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^7\left(ad^2+bc^2\right)}$$

$\downarrow$ 680

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\downarrow$ 27

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\downarrow$ 681

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\downarrow$ 27

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\Big\downarrow$ 488

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

$\Big\downarrow$ 219

$$\frac{\left(6b^3(Cd-8cD)c^6+ab^2d^2\left(-104Dc^3+13Cdc^2-25Ad^3\right)c^2+a^2bd^4\left(-39Dc^2+8Cdc-25Bd^2\right)c-6a^3d^6(4Cd-7cD)-5d\left(6a^3Dd^6-a^2b\left(-39Dc^2+8Cdc-Bd^2\right)d^4-a\right.\right.}{20d^2\left(bc^2+ad^2\right)(c+dx)^5}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^8,x]
```

output
```
-1/7*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^7) + (-1/6*(d*(A*b*c - (b*c^3*(C*d - 2*c*D))/d^3 - a*(2*
c*C - B*d - (3*c^2*D)/d))*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*x)^6)
 + (((6*b^3*c^6*(C*d - 8*c*D) - 6*a^3*d^6*(4*C*d - 7*c*D) + a^2*b*c*d^4*(8
*c*C*d - 25*B*d^2 - 39*c^2*D) + a*b^2*c^2*d^2*(13*c^2*C*d - 25*A*d^3 - 104
*c^3*D) - 5*d*(6*a^3*d^6*D - a^2*b*d^4*(8*c*C*d - B*d^2 - 39*c^2*D) - a*b^
2*c*d^2*(5*c^2*C*d + 4*B*c*d^2 - A*d^3 - 44*c^3*D) - 2*b^3*(c^5*C*d + 2*A*
c^3*d^3 - 8*c^6*D))*x)*(a + b*x^2)^(5/2))/(20*d^2*(b*c^2 + a*d^2)*(c + d*x
)^5) + (b*(-1/2*((2*a^4*d^8*(8*C*d - 19*c*D) - 8*b^4*c^8*(C*d - 8*c*D) - 2
6*a*b^3*c^6*d^2*(C*d - 8*c*D) - a^3*b*c*d^6*(8*c*C*d - 15*B*d^2 - 49*c^2*D
) - 3*a^2*b^2*c^2*d^4*(9*c^2*C*d - 5*A*d^3 - 72*c^3*D) + d*(30*a^4*d^8*D -
 12*b^4*c^7*(C*d - 8*c*D) - 5*a^3*b*d^6*(8*c*C*d - B*d^2 - 51*c^2*D) - a^2
*b^2*c*d^4*(57*c^2*C*d + 10*B*c*d^2 - 5*A*d^3 - 466*c^3*D) - 2*a*b^3*c^3*d
^2*(22*c^2*C*d + 5*A*d^3 - 176*c^3*D))*x)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^3) + (3*b*(-(((16*(b*c^2 + a*d^2)^4*(C*d - 8*c*D) - d*(3
0*a^4*d^8*D - 8*b^4*c^7*(C*d - 8*c*D) - 30*a*b^3*c^5*d^2*(C*d - 8*c*D) - a
^3*b*d^6*(24*c*C*d - 5*B*d^2 - 187*c^2*D) - a^2*b^2*c*d^4*(41*c^2*C*d - 5*
A*d^3 - 328*c^3*D))*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x))) + ((16*Sqrt[b]*(b
*c^2 + a*d^2)^4*(C*d - 8*c*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - ((
30*a^5*d^10*D - 16*b^5*c^9*(C*d - 8*c*D) - 72*a*b^4*c^7*d^2*(C*d - 8*c*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47637 vs. 2(1030) = 2060.

Time = 2.44 (sec) , antiderivative size = 47638, normalized size of antiderivative = 44.77

method	result	size
default	Expression too large to display	47638

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30152 vs. 2(1031) = 2062.

Time = 1.62 (sec) , antiderivative size = 30152, normalized size of antiderivative = 28.34

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="maxi
ma")
```

output
```
5/32*D*b^9*c^13*arcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^12*d^9 + 6*a*b^(11/2)*c
^10*d^11 + 15*a^2*b^(9/2)*c^8*d^13 + 20*a^3*b^(7/2)*c^6*d^15 + 15*a^4*b^(5
/2)*c^4*d^17 + 6*a^5*b^(3/2)*c^2*d^19 + a^6*sqrt(b)*d^21) - 5/32*C*b^9*c^1
2*arcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^12*d^8 + 6*a*b^(11/2)*c^10*d^10 + 15*
a^2*b^(9/2)*c^8*d^12 + 20*a^3*b^(7/2)*c^6*d^14 + 15*a^4*b^(5/2)*c^4*d^16 +
 6*a^5*b^(3/2)*c^2*d^18 + a^6*sqrt(b)*d^20) + 5/32*D*a*b^8*c^11*arcsinh(b*
x/sqrt(a*b))/(b^(13/2)*c^12*d^7 + 6*a*b^(11/2)*c^10*d^9 + 15*a^2*b^(9/2)*c
^8*d^11 + 20*a^3*b^(7/2)*c^6*d^13 + 15*a^4*b^(5/2)*c^4*d^15 + 6*a^5*b^(3/2
)*c^2*d^17 + a^6*sqrt(b)*d^19) + 5/32*B*b^9*c^11*arcsinh(b*x/sqrt(a*b))/(b
^(13/2)*c^12*d^7 + 6*a*b^(11/2)*c^10*d^9 + 15*a^2*b^(9/2)*c^8*d^11 + 20*a^
3*b^(7/2)*c^6*d^13 + 15*a^4*b^(5/2)*c^4*d^15 + 6*a^5*b^(3/2)*c^2*d^17 + a^
6*sqrt(b)*d^19) - 5/32*sqrt(b*x^2 + a)*D*b^8*c^11*x/(b^6*c^12*d^7 + 6*a*b^
5*c^10*d^9 + 15*a^2*b^4*c^8*d^11 + 20*a^3*b^3*c^6*d^13 + 15*a^4*b^2*c^4*d^
15 + 6*a^5*b*c^2*d^17 + a^6*d^19) - 5/32*C*a*b^8*c^10*arcsinh(b*x/sqrt(a*b
))/(b^(13/2)*c^12*d^6 + 6*a*b^(11/2)*c^10*d^8 + 15*a^2*b^(9/2)*c^8*d^10 +
20*a^3*b^(7/2)*c^6*d^12 + 15*a^4*b^(5/2)*c^4*d^14 + 6*a^5*b^(3/2)*c^2*d^16
 + a^6*sqrt(b)*d^18) - 5/32*A*b^9*c^10*arcsinh(b*x/sqrt(a*b))/(b^(13/2)*c^
12*d^6 + 6*a*b^(11/2)*c^10*d^8 + 15*a^2*b^(9/2)*c^8*d^10 + 20*a^3*b^(7/2)*
c^6*d^12 + 15*a^4*b^(5/2)*c^4*d^14 + 6*a^5*b^(3/2)*c^2*d^16 + a^6*sqrt(b)*
d^18) - 45/32*D*b^8*c^11*arcsinh(b*x/sqrt(a*b))/(b^(11/2)*c^10*d^9 + 5*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11425 vs. $2(1031) = 2062$.

Time = 0.91 (sec) , antiderivative size = 11425, normalized size of antiderivative = 10.74

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x, algorithm="giac
")
```

output
```
1/8*(128*D*b^7*c^10 - 16*C*b^7*c^9*d + 576*D*a*b^6*c^8*d^2 - 72*C*a*b^6*c^
7*d^3 + 1008*D*a^2*b^5*c^6*d^4 - 126*C*a^2*b^5*c^5*d^5 + 840*D*a^3*b^4*c^4
*d^6 - 105*C*a^3*b^4*c^3*d^7 + 315*D*a^4*b^3*c^2*d^8 - 40*C*a^4*b^3*c*d^9
+ 5*A*a^3*b^4*c*d^9 + 30*D*a^5*b^2*d^10 + 5*B*a^4*b^3*d^10)*arctan(-((sqrt
(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c^8*d^
9 + 4*a*b^3*c^6*d^11 + 6*a^2*b^2*c^4*d^13 + 4*a^3*b*c^2*d^15 + a^4*d^17)*s
qrt(-b*c^2 - a*d^2)) + sqrt(b*x^2 + a)*D*b^2/d^8 + 1/840*(47040*(sqrt(b)*x
 - sqrt(b*x^2 + a))^13*D*b^7*c^10*d^6 - 11760*(sqrt(b)*x - sqrt(b*x^2 + a)
)^13*C*b^7*c^9*d^7 + 186480*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a*b^6*c^8*d
^8 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*b^7*c^8*d^8 - 46200*(sqrt(b)*
x - sqrt(b*x^2 + a))^13*C*a*b^6*c^7*d^9 + 277200*(sqrt(b)*x - sqrt(b*x^2 +
 a))^13*D*a^2*b^5*c^6*d^10 + 6720*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a*b^6
*c^6*d^10 - 67410*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^2*b^5*c^5*d^11 + 18
3960*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^3*b^4*c^4*d^12 + 10080*(sqrt(b)*
x - sqrt(b*x^2 + a))^13*B*a^2*b^5*c^4*d^12 - 42735*(sqrt(b)*x - sqrt(b*x^2
 + a))^13*C*a^3*b^4*c^3*d^13 + 47565*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a^
4*b^3*c^2*d^14 + 6720*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^3*b^4*c^2*d^14
- 9240*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^4*b^3*c*d^15 - 525*(sqrt(b)*x
- sqrt(b*x^2 + a))^13*A*a^3*b^4*c*d^15 + 1890*(sqrt(b)*x - sqrt(b*x^2 + a)
)^13*D*a^5*b^2*d^16 + 1155*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^4*b^3*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^8}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^8}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^8}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^8,x)
```

3.97 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1102

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

output
```
-1/128*b^2*(128*b^5*c^11*D+576*a*b^4*c^9*d^2*D+976*a^2*b^3*c^7*d^4*D+8*a^5
*d^10*(5*C*d-13*D*c)-5*a^4*b*d^8*(A*d^3-9*B*c*d^2+C*c^2*d-25*D*c^3)+8*a^3*
b^2*c^2*d^6*(5*A*d^3+89*D*c^3)+d*(192*b^5*c^10*D+944*a*b^4*c^8*d^2*D+128*a
^5*d^10*D-8*a^4*b*c*d^8*(5*C*d-109*D*c)-8*a^2*b^3*c^3*d^4*(5*A*d^3-231*D*c
^3)+5*a^3*b^2*c*d^6*(A*d^3-9*B*c*d^2+C*c^2*d+359*D*c^3))*x)*(b*x^2+a)^(1/2
)/d^8/(a*d^2+b*c^2)^5/(d*x+c)^2-1/192*b*(64*b^4*c^9*D+208*a*b^3*c^7*d^2*D+
8*a^4*d^8*(5*C*d-13*D*c)-a^3*b*d^6*(5*A*d^3-45*B*c*d^2+5*C*c^2*d+3*D*c^3)+
40*a^2*b^2*c^2*d^4*(A*d^3+5*D*c^3)+d*(112*b^4*c^8*D+64*a^4*d^8*D-8*a^3*b*c
*d^6*(5*C*d-53*D*c)-40*a*b^3*c^3*d^2*(A*d^3-11*D*c^3)+a^2*b^2*c*d^4*(5*A*d
^3-45*B*c*d^2+5*C*c^2*d+643*D*c^3))*x)*(b*x^2+a)^(3/2)/d^6/(a*d^2+b*c^2)^4
/(d*x+c)^4-1/240*(48*b^3*c^7*D+8*a^3*d^6*(5*C*d-9*D*c)-a^2*b*d^4*(5*A*d^3-
45*B*c*d^2+5*C*c^2*d-29*D*c^3)+8*a*b^2*c^2*d^2*(5*A*d^3+13*D*c^3)-d*(5*A*b
^2*c*d^3*(-a*d^2+8*b*c^2)-88*b^3*c^6*D-48*a^3*d^6*D+8*a^2*b*c*d^4*(5*C*d-3
3*D*c)-a*b^2*c^2*d^2*(-45*B*d^2+5*C*c*d+259*D*c^2))*x)*(b*x^2+a)^(5/2)/d^4
/(a*d^2+b*c^2)^3/(d*x+c)^6-1/8*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(7/
2)/d^2/(a*d^2+b*c^2)/(d*x+c)^8+1/56*(8*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*
(-9*A*d^3+B*c*d^2+7*C*c^2*d-15*D*c^3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^2
/(d*x+c)^7+b^(5/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^9+1/128*b^3*(128
*b^5*c^11*D+704*a*b^4*c^9*d^2*D+1584*a^2*b^3*c^7*d^4*D-40*a^5*d^10*(C*d-9*
D*c)-8*a^3*b^2*c^2*d^6*(5*A*d^3-231*D*c^3)+5*a^4*b*d^8*(A*d^3-9*B*c*d^2...
```

Mathematica [A] (verified)

Time = 17.62 (sec) , antiderivative size = 1916, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^9,x]
```

output
```
Sqrt[a + b*x^2]*(-1/8*((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*
D))/(d^8*(c + d*x)^8) - ((b*c^2 + a*d^2)*(-49*b*c^3*C*d + 41*b*B*c^2*d^2 -
 33*A*b*c*d^3 - 16*a*c*C*d^3 + 8*a*B*d^4 + 57*b*c^4*D + 24*a*c^2*d^2*D))/(
56*d^8*(c + d*x)^7) + (-882*b^2*c^4*C*d + 594*b^2*B*c^3*d^2 - 362*A*b^2*c^
2*d^3 - 695*a*b*c^2*C*d^3 + 351*a*b*B*c*d^4 - 119*a*A*b*d^5 - 56*a^2*C*d^5
 + 1226*b^2*c^5*D + 1151*a*b*c^3*d^2*D + 168*a^2*c*d^4*D)/(336*d^8*(c + d*
x)^6) + (7350*b^3*c^5*C*d - 3750*b^3*B*c^4*d^2 + 1550*A*b^3*c^3*d^3 + 1017
5*a*b^2*c^3*C*d^3 - 4455*a*b^2*B*c^2*d^4 + 1535*a*A*b^2*c*d^5 + 2840*a^2*b
*c*C*d^5 - 720*a^2*b*B*d^6 - 12686*b^3*c^6*D - 19703*a*b^2*c^4*d^2*D - 736
8*a^2*b*c^2*d^4*D - 336*a^3*d^6*D)/(1680*d^8*(b*c^2 + a*d^2)*(c + d*x)^5)
- (b*(29400*b^3*c^6*C*d - 10200*b^3*B*c^5*d^2 + 2200*A*b^3*c^4*d^3 + 62200
*a*b^2*c^4*C*d^3 - 20280*a*b^2*B*c^3*d^4 + 4400*a*A*b^2*c^2*d^5 + 36305*a^
2*b*c^2*C*d^5 - 9945*a^2*b*B*c*d^6 + 2065*a^2*A*b*d^7 + 3640*a^3*C*d^7 - 6
6856*b^3*c^7*D - 151328*a*b^2*c^5*d^2*D - 102313*a^2*b*c^3*d^4*D - 17976*a
^3*c*d^6*D))/(6720*d^8*(b*c^2 + a*d^2)^2*(c + d*x)^4) - (b*(-17640*b^4*c^7
*C*d + 3240*b^4*B*c^6*d^2 - 40*A*b^4*c^5*d^3 - 52600*a*b^3*c^5*C*d^3 + 972
0*a*b^3*B*c^4*d^4 - 160*a*A*b^3*c^3*d^5 - 51955*a^2*b^2*c^3*C*d^5 + 9675*a
^2*b^2*B*c^2*d^6 - 435*a^2*A*b^2*c*d^7 - 16680*a^3*b*c*C*d^7 + 2880*a^3*b*
B*d^8 + 58584*b^4*c^8*D + 179760*a*b^3*c^6*d^2*D + 188091*a^2*b^2*c^4*d^4*
D + 71528*a^3*b*c^2*d^6*D + 4928*a^4*d^8*D))/(6720*d^8*(b*c^2 + a*d^2)^...
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {2182, 25, 2182, 27, 680, 27, 680, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^9} \, dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(8\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(8Cd-8cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+Bc-Ad\right)\right)x+8\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^8}dx}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(8\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(8a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+Bc-Ad\right)\right)x+8\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^8}dx}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{7/2}(8ad^2(-Bd^2-3c^2D+2cCd)+bc(-9Ad^3+Bcd^2-15c^3D+7c^2Cd))}{7d^2(c+dx)^7(ad^2+bc^2)}-\frac{\int -\frac{7\left(\left(\left(8d(Cd-2cD)a^2-\frac{bc\left(7Dc^2+Cdc-9Bd^2\right)a}{d}+Ab\left(8bc^2-ad^2\right)\right.\right.\right.}{d^2(c+dx)^7}}{7(ad^2+bc^2)}}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(8Dx\left(bc^2+ad^2\right)^2+d\left(8a^2(Cd-2cD)d^2+Ab\left(8bc^2-ad^2\right)d-abc\left(7Dc^2+Cdc-9Bd^2\right)\right)\right)\left(bx^2+a\right)^{5/2}}{(c+dx)^7}dx}{d^2(ad^2+bc^2)}+\frac{(a+bx^2)^{7/2}(8ad^2(-Bd^2-3c^2D+2cCd)+bc(}{7d^2(c+dx)^7(ad^2+}}{8\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 680$$

$$\frac{\frac{-\frac{\int -\frac{2b\left(48Dx\left(bc^2+ad^2\right)^3+5ad\left(-8b^2Dc^5-abd^2\left(23Dc^2+Cdc-9Bd^2\right)c+Abd^3\left(8bc^2-ad^2\right)+8a^2d^4(Cd-3cD)\right)\right)\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{12d^2\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}\left(8a^3d^6(5Cd-\right.}{}}{}}{}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\left(48Dx\left(bc^2+ad^2\right)^3+5ad\left(-8b^2Dc^5-abd^2\left(23Dc^2+Cdc-9Bd^2\right)c+Abd^3\left(8bc^2-ad^2\right)+8a^2d^4(Cd-3cD)\right)\right)\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{6d^2\left(ad^2+bc^2\right)} - \frac{\left(a+bx^2\right)^{5/2}\left(8a^3d^6(5Cd-9cD)\right.}{}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{8d^2(c+dx)^8\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 680$$

$$\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7} + \frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7} + \frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \ 680$$

$$\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7} + \frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(8a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-15Dc^3+7Cdc^2+Bd^2c-9Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{7d^2\left(bc^2+ad^2\right)(c+dx)^7}+\frac{b\left(-\frac{\left(64b^4Dc^9+208ab^3d^2Dc^7+40a^2b^2d^4\left(5Dc^3+Ad^3\right)c^2+8a^4d^8(5\right.}{}\right.}{}}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{8d^2\left(bc^2+ad^2\right)(c+dx)^8}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^9,x]
```

output
```
-1/8*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^8) + (((8*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d + B*c*d^2 - 9*A*d^3 - 15*c^3*D))*(a + b*x^2)^(7/2))/(7*d^2*(b*c^2 + a
*d^2)*(c + d*x)^7) + (-1/30*((48*b^3*c^7*D + 8*a^3*d^6*(5*C*d - 9*c*D) - a
^2*b*d^4*(5*c^2*C*d - 45*B*c*d^2 + 5*A*d^3 - 29*c^3*D) + 8*a*b^2*c^2*d^2*(
5*A*d^3 + 13*c^3*D) + d*(48*(b*c^2 + a*d^2)^3*D - 5*b*c*(A*b*d^3*(8*b*c^2
- a*d^2) - 8*b^2*c^5*D + 8*a^2*d^4*(C*d - 3*c*D) - a*b*c*d^2*(c*C*d - 9*B*
d^2 + 23*c^2*D)))*x)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 + a*d^2)*(c + d*x)^6)
+ (b*(-1/4*((64*b^4*c^9*D + 208*a*b^3*c^7*d^2*D + 8*a^4*d^8*(5*C*d - 13*c*
D) - a^3*b*d^6*(5*c^2*C*d - 45*B*c*d^2 + 5*A*d^3 + 3*c^3*D) + 40*a^2*b^2*c
^2*d^4*(A*d^3 + 5*c^3*D) + d*(112*b^4*c^8*D + 64*a^4*d^8*D - 8*a^3*b*c*d^6
*(5*C*d - 53*c*D) - 40*a*b^3*c^3*d^2*(A*d^3 - 11*c^3*D) + a^2*b^2*c*d^4*(5
*c^2*C*d - 45*B*c*d^2 + 5*A*d^3 + 643*c^3*D))*x)*(a + b*x^2)^(3/2))/(d^2*(
b*c^2 + a*d^2)*(c + d*x)^4) - (3*b*(((128*b^5*c^11*D + 576*a*b^4*c^9*d^2*D
 + 976*a^2*b^3*c^7*d^4*D + 8*a^5*d^10*(5*C*d - 13*c*D) - 5*a^4*b*d^8*(c^2*
C*d - 9*B*c*d^2 + A*d^3 - 25*c^3*D) + 8*a^3*b^2*c^2*d^6*(5*A*d^3 + 89*c^3*
D) + d*(192*b^5*c^10*D + 944*a*b^4*c^8*d^2*D + 128*a^5*d^10*D - 8*a^4*b*c*
d^8*(5*C*d - 109*c*D) - 8*a^2*b^3*c^3*d^4*(5*A*d^3 - 231*c^3*D) + 5*a^3*b^
2*c*d^6*(c^2*C*d - 9*B*c*d^2 + A*d^3 + 359*c^3*D))*x)*Sqrt[a + b*x^2])/(2*
d^2*(b*c^2 + a*d^2)*(c + d*x)^2) + (b*((-128*(b*c^2 + a*d^2)^5*D*ArcTan...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73561 vs. $2(1068) = 2136$.

Time = 2.89 (sec) , antiderivative size = 73562, normalized size of antiderivative = 66.75

method	result	size
default	Expression too large to display	73562

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**9,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42845 vs. 2(1070) = 2140.

Time = 2.29 (sec) , antiderivative size = 42845, normalized size of antiderivative = 38.88

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="maxi
ma")
```

output
```
45/256*D*b^10*c^14*arcsinh(b*x/sqrt(a*b))/(b^(15/2)*c^14*d^9 + 7*a*b^(13/2
)*c^12*d^11 + 21*a^2*b^(11/2)*c^10*d^13 + 35*a^3*b^(9/2)*c^8*d^15 + 35*a^4
*b^(7/2)*c^6*d^17 + 21*a^5*b^(5/2)*c^4*d^19 + 7*a^6*b^(3/2)*c^2*d^21 + a^7
*sqrt(b)*d^23) - 45/256*C*b^10*c^13*arcsinh(b*x/sqrt(a*b))/(b^(15/2)*c^14*
d^8 + 7*a*b^(13/2)*c^12*d^10 + 21*a^2*b^(11/2)*c^10*d^12 + 35*a^3*b^(9/2)*
c^8*d^14 + 35*a^4*b^(7/2)*c^6*d^16 + 21*a^5*b^(5/2)*c^4*d^18 + 7*a^6*b^(3/
2)*c^2*d^20 + a^7*sqrt(b)*d^22) + 45/256*D*a*b^9*c^12*arcsinh(b*x/sqrt(a*b
))/(b^(15/2)*c^14*d^7 + 7*a*b^(13/2)*c^12*d^9 + 21*a^2*b^(11/2)*c^10*d^11
+ 35*a^3*b^(9/2)*c^8*d^13 + 35*a^4*b^(7/2)*c^6*d^15 + 21*a^5*b^(5/2)*c^4*d
^17 + 7*a^6*b^(3/2)*c^2*d^19 + a^7*sqrt(b)*d^21) + 45/256*B*b^10*c^12*arcs
inh(b*x/sqrt(a*b))/(b^(15/2)*c^14*d^7 + 7*a*b^(13/2)*c^12*d^9 + 21*a^2*b^(
11/2)*c^10*d^11 + 35*a^3*b^(9/2)*c^8*d^13 + 35*a^4*b^(7/2)*c^6*d^15 + 21*a
^5*b^(5/2)*c^4*d^17 + 7*a^6*b^(3/2)*c^2*d^19 + a^7*sqrt(b)*d^21) - 45/256*
sqrt(b*x^2 + a)*D*b^9*c^12*x/(b^7*c^14*d^7 + 7*a*b^6*c^12*d^9 + 21*a^2*b^5
*c^10*d^11 + 35*a^3*b^4*c^8*d^13 + 35*a^4*b^3*c^6*d^15 + 21*a^5*b^2*c^4*d^
17 + 7*a^6*b*c^2*d^19 + a^7*d^21) - 45/256*C*a*b^9*c^11*arcsinh(b*x/sqrt(a
*b))/(b^(15/2)*c^14*d^6 + 7*a*b^(13/2)*c^12*d^8 + 21*a^2*b^(11/2)*c^10*d^1
0 + 35*a^3*b^(9/2)*c^8*d^12 + 35*a^4*b^(7/2)*c^6*d^14 + 21*a^5*b^(5/2)*c^4
*d^16 + 7*a^6*b^(3/2)*c^2*d^18 + a^7*sqrt(b)*d^20) - 45/256*A*b^10*c^11*ar
csinh(b*x/sqrt(a*b))/(b^(15/2)*c^14*d^6 + 7*a*b^(13/2)*c^12*d^8 + 21*a^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14580 vs. 2(1070) = 2140.

Time = 1.11 (sec) , antiderivative size = 14580, normalized size of antiderivative = 13.23

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x, algorithm="giac
")
```

output
```
-1/64*(128*D*b^8*c^11 + 704*D*a*b^7*c^9*d^2 + 1584*D*a^2*b^6*c^7*d^4 + 184
8*D*a^3*b^5*c^5*d^6 + 1155*D*a^4*b^4*c^3*d^8 + 5*C*a^4*b^4*c^2*d^9 - 40*A*
a^3*b^5*c^2*d^9 + 360*D*a^5*b^3*c*d^10 - 45*B*a^4*b^4*c*d^10 - 40*C*a^5*b^
3*d^11 + 5*A*a^4*b^4*d^11)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt
(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^5*c^10*d^9 + 5*a*b^4*c^8*d^11 + 10*a^2*b^
3*c^6*d^13 + 10*a^3*b^2*c^4*d^15 + 5*a^4*b*c^2*d^17 + a^5*d^19)*sqrt(-b*c^
2 - a*d^2)) - D*b^(5/2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^9 - 1/672
0*(107520*(sqrt(b)*x - sqrt(b*x^2 + a))^15*D*b^8*c^11*d^7 - 13440*(sqrt(b)
*x - sqrt(b*x^2 + a))^15*C*b^8*c^10*d^8 + 530880*(sqrt(b)*x - sqrt(b*x^2 +
 a))^15*D*a*b^7*c^9*d^9 - 67200*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a*b^7*c
^8*d^10 + 1043280*(sqrt(b)*x - sqrt(b*x^2 + a))^15*D*a^2*b^6*c^7*d^11 - 13
4400*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*b^6*c^6*d^12 + 1015560*(sqrt(b
)*x - sqrt(b*x^2 + a))^15*D*a^3*b^5*c^5*d^13 - 134400*(sqrt(b)*x - sqrt(b*
x^2 + a))^15*C*a^3*b^5*c^4*d^14 + 483525*(sqrt(b)*x - sqrt(b*x^2 + a))^15*
D*a^4*b^4*c^3*d^15 - 67725*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^4*b^4*c^2*
d^16 + 4200*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a^3*b^5*c^2*d^16 + 83160*(s
qrt(b)*x - sqrt(b*x^2 + a))^15*D*a^5*b^3*c*d^17 + 4725*(sqrt(b)*x - sqrt(b
*x^2 + a))^15*B*a^4*b^4*c*d^17 - 9240*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a
^5*b^3*d^18 - 525*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a^4*b^4*d^18 + 112896
0*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*b^(17/2)*c^12*d^6 - 94080*(sqrt(b)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^9}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^9,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^9, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^9}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^9}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^9,x)
```

3.98 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 767

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx =$$

$$-\frac{5a^2b^2(Ab^2c(8bc^2-3ad^2)-a(b^2c^2(cC-10Bd)-8a^2d^3D-abd(10cCd-Bd^2-3c^2D)))\,(ad-bcx)\sqrt{a}}{128\,(bc^2+ad^2)^6\,(c+dx)^2}$$

$$-\frac{5ab(Ab^2c(8bc^2-3ad^2)-a(b^2c^2(cC-10Bd)-8a^2d^3D-abd(10cCd-Bd^2-3c^2D)))\,(ad-bcx)\,(a+}{192\,(bc^2+ad^2)^5\,(c+dx)^4}$$

$$-\frac{(Ab^2c(8bc^2-3ad^2)-a(b^2c^2(cC-10Bd)-8a^2d^3D-abd(10cCd-Bd^2-3c^2D)))\,(ad-bcx)\,(a+bx^2}{48\,(bc^2+ad^2)^4\,(c+dx)^6}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx^2)^{7/2}}{9d^2\,(bc^2+ad^2)\,(c+dx)^9}$$

$$+\frac{(9ad^2(2cCd-Bd^2-3c^2D)+bc(7c^2Cd+2Bcd^2-11Ad^3-16c^3D))\,(a+bx^2)^{7/2}}{72d^2\,(bc^2+ad^2)^2\,(c+dx)^8}$$

$$-\frac{(72a^2d^4(Cd-3cD)-b^2c^2(7c^2Cd+2Bcd^2-83Ad^3+56c^3D)-abd^2(34c^2Cd-97Bcd^2+16Ad^3+17}{504d^2\,(bc^2+ad^2)^3\,(c+dx)^7}$$

$$-\frac{5a^3b^3(Ab^2c(8bc^2-3ad^2)-a(b^2c^2(cC-10Bd)-8a^2d^3D-abd(10cCd-Bd^2-3c^2D)))\,\mathrm{arctanh}\left(\frac{a}{\sqrt{bc^2+}}\right.}{128\,(bc^2+ad^2)^{13/2}}$$

output
```
-5/128*a^2*b^2*(A*b^2*c*(-3*a*d^2+8*b*c^2)-a*(b^2*c^2*(-10*B*d+C*c)-8*a^2*
d^3*D-a*b*d*(-B*d^2+10*C*c*d-3*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^
2+b*c^2)^6/(d*x+c)^2-5/192*a*b*(A*b^2*c*(-3*a*d^2+8*b*c^2)-a*(b^2*c^2*(-10
*B*d+C*c)-8*a^2*d^3*D-a*b*d*(-B*d^2+10*C*c*d-3*D*c^2)))*(-b*c*x+a*d)*(b*x^
2+a)^(3/2)/(a*d^2+b*c^2)^5/(d*x+c)^4-1/48*(A*b^2*c*(-3*a*d^2+8*b*c^2)-a*(b
^2*c^2*(-10*B*d+C*c)-8*a^2*d^3*D-a*b*d*(-B*d^2+10*C*c*d-3*D*c^2)))*(-b*c*x
+a*d)*(b*x^2+a)^(5/2)/(a*d^2+b*c^2)^4/(d*x+c)^6-1/9*(A*d^3-B*c*d^2+C*c^2*d
-D*c^3)*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^9+1/72*(9*a*d^2*(-B*d^2+
2*C*c*d-3*D*c^2)+b*c*(-11*A*d^3+2*B*c*d^2+7*C*c^2*d-16*D*c^3))*(b*x^2+a)^(
7/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^8-1/504*(72*a^2*d^4*(C*d-3*D*c)-b^2*c^2*(
-83*A*d^3+2*B*c*d^2+7*C*c^2*d+56*D*c^3)-a*b*d^2*(16*A*d^3-97*B*c*d^2+34*C*
c^2*d+173*D*c^3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^7-5/128*a^3*
b^3*(A*b^2*c*(-3*a*d^2+8*b*c^2)-a*(b^2*c^2*(-10*B*d+C*c)-8*a^2*d^3*D-a*b*d
*(-B*d^2+10*C*c*d-3*D*c^2)))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x
^2+a)^(1/2))/(a*d^2+b*c^2)^(13/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1955 vs. $2(767) = 1534$.

Time = 17.89 (sec) , antiderivative size = 1955, normalized size of antiderivative = 2.55

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^10,x]
```

output
```
Sqrt[a + b*x^2]*(-1/9*((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*
D))/(d^8*(c + d*x)^9) - ((b*c^2 + a*d^2)*(-55*b*c^3*C*d + 46*b*B*c^2*d^2 -
 37*A*b*c*d^3 - 18*a*c*C*d^3 + 9*a*B*d^4 + 64*b*c^4*D + 27*a*c^2*d^2*D))/(
72*d^8*(c + d*x)^8) + (-1127*b^2*c^4*C*d + 758*b^2*B*c^3*d^2 - 461*A*b^2*c
^2*d^3 - 890*a*b*c^2*C*d^3 + 449*a*b*B*c*d^4 - 152*a*A*b*d^5 - 72*a^2*C*d^
5 + 1568*b^2*c^5*D + 1475*a*b*c^3*d^2*D + 216*a^2*c*d^4*D)/(504*d^8*(c + d
*x)^7) + (3626*b^3*c^5*C*d - 1844*b^3*B*c^4*d^2 + 758*A*b^3*c^3*d^3 + 5031
*a*b^2*c^3*C*d^3 - 2196*a*b^2*B*c^2*d^4 + 753*a*A*b^2*c*d^5 + 1410*a^2*b*c
*C*d^5 - 357*a^2*b*B*d^6 - 6272*b^3*c^6*D - 9762*a*b^2*c^4*d^2*D - 3663*a^
2*b*c^2*d^4*D - 168*a^3*d^6*D)/(1008*d^8*(b*c^2 + a*d^2)*(c + d*x)^6) - (b
*(3430*b^3*c^6*C*d - 1180*b^3*B*c^5*d^2 + 250*A*b^3*c^4*d^3 + 7275*a*b^2*c
^4*C*d^3 - 2352*a*b^2*B*c^3*d^4 + 501*a*A*b^2*c^2*d^5 + 4266*a^2*b*c^2*C*d
^5 - 1161*a^2*b*B*c*d^6 + 240*a^2*A*b*d^7 + 432*a^3*C*d^7 - 7840*b^3*c^7*D
 - 17790*a*b^2*c^5*d^2*D - 12075*a^2*b*c^3*d^4*D - 2136*a^3*c*d^6*D))/(100
8*d^8*(b*c^2 + a*d^2)^2*(c + d*x)^5) - (b*(-7448*b^4*c^7*C*d + 1328*b^4*B*
c^6*d^2 - 8*A*b^4*c^5*d^3 - 22280*a*b^3*c^5*C*d^3 + 3992*a*b^3*B*c^4*d^4 -
 32*a*A*b^3*c^3*d^5 - 22137*a^2*b^2*c^3*C*d^5 + 4002*a^2*b^2*B*c^2*d^6 - 1
23*a^2*A*b^2*c*d^7 - 7206*a^3*b*c*C*d^7 + 1239*a^3*b*B*d^8 + 25088*b^4*c^8
*D + 77240*a*b^3*c^6*d^2*D + 81240*a^2*b^2*c^4*d^4*D + 31173*a^3*b*c^2*d^6
*D + 2184*a^4*d^8*D))/(4032*d^8*(b*c^2 + a*d^2)^3*(c + d*x)^4) - (b^2*(...
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 637, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2182, 25, 2182, 25, 27, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{10}}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(9\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(9Cd-9cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+2Bc-2Ad\right)\right)x+9\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^9}dx}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(9\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(9a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+2Bc-2Ad\right)\right)x+9\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^9}dx}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{7/2}\left(9ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-11Ad^3+2Bcd^2-16c^3D+7c^2Cd\right)\right)}{8d^2(c+dx)^8(ad^2+bc^2)}-\frac{\int -\frac{\left(8\left(Ab\left(9bc^2-2ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{7Dc^2}{d}+2Cc-11\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(8\left(Ab\left(9bc^2-2ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{7Dc^2}{d}+2Cc-11Bd\right)\right)\right)d^2+\left(72a^2Dd^4+9ab\left(13Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(56Dc^3+7Cdc^2+2Bd^2c-11Ad^3\right)\right)x\right)\left(b\ldots\right.}{d^2(c+dx)^8}}{8(ad^2+bc^2)}\ldots}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(8\left(Ab\left(9bc^2-2ad^2\right)+a\left(9ad(Cd-2cD)-bc\left(\frac{7Dc^2}{d}+2Cc-11Bd\right)\right)\right)d^2+\left(72a^2Dd^4+9ab\left(13Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(56Dc^3+7Cdc^2+2Bd^2c-11Ad^3\right)\right)x\right)\left(b\ldots\right.}{(c+dx)^8}}{8d^2(ad^2+bc^2)}\ldots}{9\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{\frac{9d^2\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(-8a^2d^3D-abd\left(-Bd^2-3c^2D+10cCd\right)+b^2c^2(cC-10Bd)\right)\right)\int\frac{\left(bx^2+a\right)^{5/2}}{(c+dx)^7}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(72a^2d^4(Cd-3cD)-abd^2\left(16Ad^3-97B\cdots\right.\right.}{7(c\cdots}}{8d^2(ad^2+bc^2)}}{9\left(ad^2\cdots\right.}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{9d^2\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(-8a^2d^3D-abd\left(-Bd^2-3c^2D+10cCd\right)+b^2c^2(cC-10Bd)\right)\right)\left(\frac{5ab\int\frac{\left(bx^2+a\right)^{3/2}}{(c+dx)^5}dx}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{7/2}\left(72a^2\cdots\right.}{\cdots}}{8d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{9d^2\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(-8a^2d^3D-abd\left(-Bd^2-3c^2D+10cCd\right)+b^2c^2(cC-10Bd)\right)\right)\left(\frac{5ab\left(\frac{3ab\int\frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-\cdots}{6(c+dx)^6\left(ad^2+b\cdots\right.}\right.}{ad^2+bc^2}}{8d^2(ad^2+bc^2)}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{9d^2\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(-8a^2d^3D-abd\left(-Bd^2-3c^2D+10cCd\right)+b^2c^2(cC-10Bd)\right)\right)\left(\frac{5ab\left(\frac{3ab\left(\frac{ab\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right.}{4(c+dx}\right.}{6\left(ad^2+bc^2\right)}\right.}{ad^2+bc^2}}{8d^2(\ldots}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{9d^2\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(-8a^2d^3D-abd\left(-Bd^2-3c^2D+10cCd\right)+b^2c^2(cC-10Bd)\right)\right)\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}(ad-bc}{2(c+dx)^2\left(ad^2+b\right.}\right.}{4\left(ad^2+bc^2\right)}\right.}{6\left(ad^2+bc^2\right)}\right.}{ad^2+bc^2}}{\ldots}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{9d^2\left(\frac{5ab\left(\frac{3ab\left(-\frac{ab\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{2\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}(ad-bcx)}{2(c+dx)^2\left(ad^2+bc^2\right)}\right)}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(ad-bcx)}{4(c+dx)^4\left(ad^2+bc^2\right)}\right)}{6\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{5/2}(ad-bcx)}{6(c+dx)^6\left(ad^2+bc^2\right)}\right)\left(Ab^2c\left(8bc^2-3ad^2\right)-a\left(\cdots\right.\right.}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{9d^2(c+dx)^9\left(ad^2+bc^2\right)}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^10,x]
```

output
```
-1/9*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2 +
 a*d^2)*(c + d*x)^9) + (((9*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*c^2
*C*d + 2*B*c*d^2 - 11*A*d^3 - 16*c^3*D))*(a + b*x^2)^(7/2))/(8*d^2*(b*c^2
+ a*d^2)*(c + d*x)^8) + (-1/7*((72*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(7*c^2*
C*d + 2*B*c*d^2 - 83*A*d^3 + 56*c^3*D) - a*b*d^2*(34*c^2*C*d - 97*B*c*d^2
+ 16*A*d^3 + 173*c^3*D))*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*x)^7)
+ (9*d^2*(A*b^2*c*(8*b*c^2 - 3*a*d^2) - a*(b^2*c^2*(c*C - 10*B*d) - 8*a^2*
d^3*D - a*b*d*(10*c*C*d - B*d^2 - 3*c^2*D)))*(-1/6*((a*d - b*c*x)*(a + b*x
^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (5*a*b*(-1/4*((a*d - b*c*x)*(a
+ b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*((a*d - b*c*x
)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c
*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2))))/(4
*(b*c^2 + a*d^2))))/(6*(b*c^2 + a*d^2))))/(b*c^2 + a*d^2))/(8*d^2*(b*c^2 +
 a*d^2)))/(9*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111785 vs. 2(735) = 1470.

Time = 3.61 (sec) , antiderivative size = 111786, normalized size of antiderivative = 145.74

method	result	size
default	Expression too large to display	111786

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57646 vs. 2(730) = 1460.

Time = 2.92 (sec) , antiderivative size = 57646, normalized size of antiderivative = 75.16

$$\int \frac{(a+bx^2)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="max
ima")
```

output
```
55/256*D*b^11*c^15*arcsinh(b*x/sqrt(a*b))/(b^(17/2)*c^16*d^9 + 8*a*b^(15/2
)*c^14*d^11 + 28*a^2*b^(13/2)*c^12*d^13 + 56*a^3*b^(11/2)*c^10*d^15 + 70*a
^4*b^(9/2)*c^8*d^17 + 56*a^5*b^(7/2)*c^6*d^19 + 28*a^6*b^(5/2)*c^4*d^21 +
8*a^7*b^(3/2)*c^2*d^23 + a^8*sqrt(b)*d^25) - 55/256*C*b^11*c^14*arcsinh(b*
x/sqrt(a*b))/(b^(17/2)*c^16*d^8 + 8*a*b^(15/2)*c^14*d^10 + 28*a^2*b^(13/2)
*c^12*d^12 + 56*a^3*b^(11/2)*c^10*d^14 + 70*a^4*b^(9/2)*c^8*d^16 + 56*a^5*
b^(7/2)*c^6*d^18 + 28*a^6*b^(5/2)*c^4*d^20 + 8*a^7*b^(3/2)*c^2*d^22 + a^8*
sqrt(b)*d^24) + 55/256*D*a*b^10*c^13*arcsinh(b*x/sqrt(a*b))/(b^(17/2)*c^16
*d^7 + 8*a*b^(15/2)*c^14*d^9 + 28*a^2*b^(13/2)*c^12*d^11 + 56*a^3*b^(11/2)
*c^10*d^13 + 70*a^4*b^(9/2)*c^8*d^15 + 56*a^5*b^(7/2)*c^6*d^17 + 28*a^6*b^
(5/2)*c^4*d^19 + 8*a^7*b^(3/2)*c^2*d^21 + a^8*sqrt(b)*d^23) + 55/256*B*b^1
1*c^13*arcsinh(b*x/sqrt(a*b))/(b^(17/2)*c^16*d^7 + 8*a*b^(15/2)*c^14*d^9 +
 28*a^2*b^(13/2)*c^12*d^11 + 56*a^3*b^(11/2)*c^10*d^13 + 70*a^4*b^(9/2)*c^
8*d^15 + 56*a^5*b^(7/2)*c^6*d^17 + 28*a^6*b^(5/2)*c^4*d^19 + 8*a^7*b^(3/2)
*c^2*d^21 + a^8*sqrt(b)*d^23) - 55/256*sqrt(b*x^2 + a)*D*b^10*c^13*x/(b^8*
c^16*d^7 + 8*a*b^7*c^14*d^9 + 28*a^2*b^6*c^12*d^11 + 56*a^3*b^5*c^10*d^13
+ 70*a^4*b^4*c^8*d^15 + 56*a^5*b^3*c^6*d^17 + 28*a^6*b^2*c^4*d^19 + 8*a^7*
b*c^2*d^21 + a^8*d^23) - 55/256*C*a*b^10*c^12*arcsinh(b*x/sqrt(a*b))/(b^(1
7/2)*c^16*d^6 + 8*a*b^(15/2)*c^14*d^8 + 28*a^2*b^(13/2)*c^12*d^10 + 56*a^3
*b^(11/2)*c^10*d^12 + 70*a^4*b^(9/2)*c^8*d^14 + 56*a^5*b^(7/2)*c^6*d^16...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17669 vs. 2(730) = 1460.

Time = 1.47 (sec) , antiderivative size = 17669, normalized size of antiderivative = 23.04

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x, algorithm="gia
c")
```

output
```
-5/64*(C*a^4*b^5*c^3 - 8*A*a^3*b^6*c^3 + 3*D*a^5*b^4*c^2*d - 10*B*a^4*b^5*
c^2*d - 10*C*a^5*b^4*c*d^2 + 3*A*a^4*b^5*c*d^2 - 8*D*a^6*b^3*d^3 + B*a^5*b
^4*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2
- a*d^2))/((b^6*c^12 + 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^8*d^4 + 20*a^3*b^3*
c^6*d^6 + 15*a^4*b^2*c^4*d^8 + 6*a^5*b*c^2*d^10 + a^6*d^12)*sqrt(-b*c^2 -
a*d^2)) + 1/4032*(8064*(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*b^9*c^12*d^8 + 4
8384*(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*a*b^8*c^10*d^10 + 120960*(sqrt(b)*
x - sqrt(b*x^2 + a))^17*D*a^2*b^7*c^8*d^12 + 161280*(sqrt(b)*x - sqrt(b*x^
2 + a))^17*D*a^3*b^6*c^6*d^14 + 120960*(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*
a^4*b^5*c^4*d^16 + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a^4*b^5*c^3*d^17
 - 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a^3*b^6*c^3*d^17 + 49329*(sqrt(
b)*x - sqrt(b*x^2 + a))^17*D*a^5*b^4*c^2*d^18 - 3150*(sqrt(b)*x - sqrt(b*x
^2 + a))^17*B*a^4*b^5*c^2*d^18 - 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a
^5*b^4*c*d^19 + 945*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a^4*b^5*c*d^19 + 55
44*(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*a^6*b^3*d^20 + 315*(sqrt(b)*x - sqrt
(b*x^2 + a))^17*B*a^5*b^4*d^20 + 64512*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*
b^(19/2)*c^13*d^7 + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*b^(19/2)*c^12*
d^8 + 387072*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a*b^(17/2)*c^11*d^9 + 4838
4*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a*b^(17/2)*c^10*d^10 + 967680*(sqrt(b
)*x - sqrt(b*x^2 + a))^16*D*a^2*b^(15/2)*c^9*d^11 + 120960*(sqrt(b)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx=\int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{10}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^10,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^10, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{10}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{10}}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^10,x)
```

3.99 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 997

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx =$$

$$-\frac{a^2b^3(Ab(80b^2c^4-60abc^2d^2+3a^2d^4)-a(10b^2c^3(cC-11Bd)+10a^2d^3(Cd-11cD)-3abcd(41cCd-1}{256\,(bc^2+ad^2)^7\,(c+dx)^2}$$

$$-\frac{ab^2(Ab(80b^2c^4-60abc^2d^2+3a^2d^4)-a(10b^2c^3(cC-11Bd)+10a^2d^3(Cd-11cD)-3abcd(41cCd-11}{384\,(bc^2+ad^2)^6\,(c+dx)^4}$$

$$-\frac{b(Ab(80b^2c^4-60abc^2d^2+3a^2d^4)-a(10b^2c^3(cC-11Bd)+10a^2d^3(Cd-11cD)-3abcd(41cCd-11B}{480\,(bc^2+ad^2)^5\,(c+dx)^6}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx^2)^{7/2}}{10d^2\,(bc^2+ad^2)\,(c+dx)^{10}}$$

$$+\frac{(10ad^2(2cCd-Bd^2-3c^2D)+bc(7c^2Cd+3Bcd^2-13Ad^3-17c^3D))\,(a+bx^2)^{7/2}}{90d^2\,(bc^2+ad^2)^2\,(c+dx)^9}$$

$$-\frac{(90a^2d^4(Cd-3cD)-2b^2c^2(7c^2Cd+3Bcd^2-58Ad^3+28c^3D)-abd^2(67c^2Cd-137Bcd^2+27Ad^3+1}{720d^2\,(bc^2+ad^2)^3\,(c+dx)^8}$$

$$-\frac{(720a^3d^6D+10a^2bd^4(113cCd-16Bd^2-75c^2D)-2b^3c^3(7c^2Cd+3Bcd^2-418Ad^3+28c^3D)-ab^2cd^2}{5040d^2\,(bc^2+ad^2)^4\,(c+dx)^7}$$

$$-\frac{a^3b^4(Ab(80b^2c^4-60abc^2d^2+3a^2d^4)-a(10b^2c^3(cC-11Bd)+10a^2d^3(Cd-11cD)-3abcd(41cCd-1}{256\,(bc^2+ad^2)^{15/2}}$$

output
```
-1/256*a^2*b^3*(A*b*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)-a*(10*b^2*c^3*(-
11*B*d+C*c)+10*a^2*d^3*(C*d-11*D*c)-3*a*b*c*d*(-11*B*d^2+41*C*c*d-11*D*c^2
)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^7/(d*x+c)^2-1/384*a*b^2*(A*
b*(3*a^2*d^4-60*a*b*c^2*d^2+80*b^2*c^4)-a*(10*b^2*c^3*(-11*B*d+C*c)+10*a^2
*d^3*(C*d-11*D*c)-3*a*b*c*d*(-11*B*d^2+41*C*c*d-11*D*c^2)))*(-b*c*x+a*d)*(
b*x^2+a)^(3/2)/(a*d^2+b*c^2)^6/(d*x+c)^4-1/480*b*(A*b*(3*a^2*d^4-60*a*b*c^
2*d^2+80*b^2*c^4)-a*(10*b^2*c^3*(-11*B*d+C*c)+10*a^2*d^3*(C*d-11*D*c)-3*a*
b*c*d*(-11*B*d^2+41*C*c*d-11*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(5/2)/(a*d^2+
b*c^2)^5/(d*x+c)^6-1/10*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(7/2)/d^2/
(a*d^2+b*c^2)/(d*x+c)^10+1/90*(10*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-13*
A*d^3+3*B*c*d^2+7*C*c^2*d-17*D*c^3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^2/(
d*x+c)^9-1/720*(90*a^2*d^4*(C*d-3*D*c)-2*b^2*c^2*(-58*A*d^3+3*B*c*d^2+7*C*
c^2*d+28*D*c^3)-a*b*d^2*(27*A*d^3-137*B*c*d^2+67*C*c^2*d+183*D*c^3))*(b*x^
2+a)^(7/2)/d^2/(a*d^2+b*c^2)^3/(d*x+c)^8-1/5040*(720*a^3*d^6*D+10*a^2*b*d^
4*(-16*B*d^2+113*C*c*d-75*D*c^2)-2*b^3*c^3*(-418*A*d^3+3*B*c*d^2+7*C*c^2*d
+28*D*c^3)-a*b^2*c*d^2*(451*A*d^3-1121*B*c*d^2+171*C*c^2*d+239*D*c^3))*(b*
x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^4/(d*x+c)^7-1/256*a^3*b^4*(A*b*(3*a^2*d^4-6
0*a*b*c^2*d^2+80*b^2*c^4)-a*(10*b^2*c^3*(-11*B*d+C*c)+10*a^2*d^3*(C*d-11*D
*c)-3*a*b*c*d*(-11*B*d^2+41*C*c*d-11*D*c^2)))*arctanh((-b*c*x+a*d)/(a*d^2+
b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(15/2)
```

Mathematica [A] (verified)

Time = 17.70 (sec) , antiderivative size = 1860, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^11,x]
```

output
```
(Sqrt[a + b*x^2]*(8064*(b*c^2 + a*d^2)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c
^3*D) - 896*(b*c^2 + a*d^2)*(10*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(
-61*c^2*C*d + 51*B*c*d^2 - 41*A*d^3 + 71*c^3*D))*(c + d*x) + 112*(-90*a^2*
d^4*(C*d - 3*c*D) + 2*b^2*c^2*(-701*c^2*C*d + 471*B*c*d^2 - 286*A*d^3 + 97
6*c^3*D) + a*b*d^2*(-1109*c^2*C*d + 559*B*c*d^2 - 189*A*d^3 + 1839*c^3*D))
*(c + d*x)^2 - (16*(720*a^3*d^6*D + 10*a^2*b*d^4*(-601*c*C*d + 152*B*d^2 +
 1563*c^2*D) + 2*b^3*c^3*(-7693*c^2*C*d + 3903*B*c*d^2 - 1598*A*d^3 + 1332
8*c^3*D) + a*b^2*c*d^2*(-21381*c^2*C*d + 9311*B*c*d^2 - 3181*A*d^3 + 41551
*c^3*D))*(c + d*x)^3)/(b*c^2 + a*d^2) + (8*b*(30*a^3*d^6*(-119*C*d + 589*c
*D) + 4*b^3*c^4*(-7007*c^2*C*d + 2397*B*c*d^2 - 502*A*d^3 + 16072*c^3*D) +
 3*a^2*b*d^4*(-11671*c^2*C*d + 3161*B*c*d^2 - 651*A*d^3 + 33141*c^3*D) + 2
*a*b^2*c^2*d^2*(-29768*c^2*C*d + 9568*B*c*d^2 - 2013*A*d^3 + 73053*c^3*D))
*(c + d*x)^4)/(b*c^2 + a*d^2)^2 - (8*b*(4320*a^4*d^8*D + 30*a^3*b*d^6*(-46
7*c*C*d + 80*B*d^2 + 2033*c^2*D) + 15*a^2*b^2*c*d^4*(-2847*c^2*C*d + 505*B
*c*d^2 - 11*A*d^3 + 10541*c^3*D) + 4*b^4*c^5*(-3577*c^2*C*d + 627*B*c*d^2
- 2*A*d^3 + 12152*c^3*D) + 10*a*b^3*c^3*d^2*(-4286*c^2*C*d + 754*B*c*d^2 -
 3*A*d^3 + 14991*c^3*D))*(c + d*x)^5)/(b*c^2 + a*d^2)^3 + (2*b^2*(30*a^4*d
^8*(-413*C*d + 2815*c*D) + 16*b^4*c^6*(-833*c^2*C*d + 3*B*c*d^2 + 2*A*d^3
+ 5488*c^3*D) + 40*a*b^3*c^4*d^2*(-1335*c^2*C*d + 5*B*c*d^2 + 5*A*d^3 + 87
67*c^3*D) + 15*a^3*b*d^6*(-3597*c^2*C*d + 103*B*c*d^2 - 21*A*d^3 + 2315...
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 820, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2182, 25, 2182, 25, 27, 688, 25, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(10\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(10a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+3Bc-3Ad\right)\right)x+\frac{10\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{(c+dx)^{10}}dx}{10\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(10\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(10a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+3Bc-3Ad\right)\right)x+\frac{10\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{(c+dx)^{10}}dx}{10\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2182

$$\frac{\frac{(a+bx^2)^{7/2}\left(10ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-13Ad^3+3Bcd^2-17c^3D+7c^2Cd\right)\right)}{9d^2(c+dx)^9(ad^2+bc^2)}-\frac{\int -\frac{\left(9\left(10d(Cd-2cD)a^2-\frac{bc\left(7Dc^2+3Cdc-13Bd^2\right)a}{d}+Ab\left(10bc\right.\right.\right.}{}}{}}{10\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(9d\left(Abd\left(10bc^2-3ad^2\right)+a\left(10ad^2(Cd-2cD)-bc\left(7Dc^2+3Cdc-13Bd^2\right)\right)\right)+2\left(45a^2Dd^4+10ab\left(6Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(28Dc^3+7Cdc^2+3Bd^2c-13Ad^3\right)\right)x\right.}{d^2(c+dx)^9}}{9(ad^2+bc^2)}}{10\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(9d\left(Abd\left(10bc^2-3ad^2\right)+a\left(10ad^2(Cd-2cD)-bc\left(7Dc^2+3Cdc-13Bd^2\right)\right)\right)+2\left(45a^2Dd^4+10ab\left(6Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(28Dc^3+7Cdc^2+3Bd^2c-13Ad^3\right)\right)x\right.}{(c+dx)^9}}{9d^2(ad^2+bc^2)}}{10\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 688

$$-\frac{\int -\frac{(8d(Acd(90bc^2-53ad^2)b^2+a(90a^2Dd^4+10ab(-6Dc^2+13Cdc-2Bd^2)d^2-b^2c^2(7Dc^2+13Cdc-123Bd^2))))-b(90a^2(Cd-3cD)d^4-ab(183Dc^3+67Cdc^2-}{(c+dx)^8}}{8(ad^2+bc^2)}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{10d^2(c+dx)^{10}(ad^2+bc^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(8d(Acd(90bc^2-53ad^2)b^2+a(90a^2Dd^4+10ab(-6Dc^2+13Cdc-2Bd^2)d^2-b^2c^2(7Dc^2+13Cdc-123Bd^2))))-b(90a^2(Cd-3cD)d^4-ab(183Dc^3+67Cdc^2-137}{(c+dx)^8}}{8(ad^2+bc^2)}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{10d^2(c+dx)^{10}(ad^2+bc^2)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{9bd^2(Ab(3a^2d^4-60abc^2d^2+80b^2c^4)-a(10a^2d^3(Cd-11cD)-3abcd(-11Bd^2-11c^2D+41cCd)+10b^2c^3(cC-11Bd)))\int \frac{(bx^2+a)^{5/2}}{(c+dx)^7}dx}{ad^2+bc^2}-\frac{(a+bx^2)^{7/2}(720a^3d}{}}{8(ad^2+bc^2)}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{10d^2(c+dx)^{10}(ad^2+bc^2)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{9bd^2(Ab(3a^2d^4-60abc^2d^2+80b^2c^4)-a(10a^2d^3(Cd-11cD)-3abcd(-11Bd^2-11c^2D+41cCd)+10b^2c^3(cC-11Bd)))\left(\frac{5ab\int \frac{(bx^2+a)^{3/2}}{(c+dx)^5}dx}{6(ad^2+bc^2)}-\frac{(a+bx^2)^{5/2}(ad}{6(c+dx)^6(ad^2}\right.}{ad^2+bc^2}}{8}$$

$$\frac{(a+bx^2)^{7/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{10d^2(c+dx)^{10}(ad^2+bc^2)}$$

$$\Big\downarrow \; 486$$

$$\frac{9bd^2\left(Ab\left(3a^2d^4-60abc^2d^2+80b^2c^4\right)-a\left(10a^2d^3(Cd-11cD)-3abcd\left(-11Bd^2-11c^2D+41cCd\right)+10b^2c^3(cC-11Bd)\right)\right)\left(\frac{5ab\left(\frac{3ab\int\frac{\sqrt{bx^2+a}}{(c+dx)^3}dx}{4\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{3/2}(\cdots}{4(c+dx)^4\left(ad\cdots\right.}\right.}{6\left(ad^2+bc^2\right)}\right.}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{10d^2(c+dx)^{10}\left(ad^2+bc^2\right)}$$

$\downarrow$ 486

$$\frac{\left(10a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-17Dc^3+7Cdc^2+3Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{9d^2\left(bc^2+ad^2\right)(c+dx)^9}+\frac{9bd^2\left(Ab\left(80b^2c^4-60abd^2c^2+3a^2d^4\right)-a\left(10b^2(cC-11Bd)c^3-3a\cdots\right.\right.}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}$$

$\downarrow$ 488

$$9bd^2\left(Ab\left(80b^2c^4-60abd^2c^2+3a^2d^4\right)-a\left(10b^2(cC-11Bd)c^3-3a\right.\right.$$

$$\frac{\frac{\left(10a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-17Dc^3+7Cdc^2+3Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{9d^2(bc^2+ad^2)(c+dx)^9}+}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}$$

$$\Big\downarrow \; 219$$

$$9bd^2\left(Ab\left(80b^2c^4-60abd^2c^2+3a^2d^4\right)-a\left(10b^2(cC-11Bd)c^3-3a\right.\right.$$

$$\frac{\frac{\left(10a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-17Dc^3+7Cdc^2+3Bd^2c-13Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{9d^2(bc^2+ad^2)(c+dx)^9}+}{}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^11,x]
```

output
```
-1/10*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2
+ a*d^2)*(c + d*x)^10) + (((10*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*
c^2*C*d + 3*B*c*d^2 - 13*A*d^3 - 17*c^3*D))*(a + b*x^2)^(7/2))/(9*d^2*(b*c
^2 + a*d^2)*(c + d*x)^9) + (-1/8*((90*a^2*d^4*(C*d - 3*c*D) - 2*b^2*c^2*(7
*c^2*C*d + 3*B*c*d^2 - 58*A*d^3 + 28*c^3*D) - a*b*d^2*(67*c^2*C*d - 137*B*
c*d^2 + 27*A*d^3 + 183*c^3*D))*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*
x)^8) + (-1/7*((720*a^3*d^6*D + 10*a^2*b*d^4*(113*c*C*d - 16*B*d^2 - 75*c^
2*D) - 2*b^3*c^3*(7*c^2*C*d + 3*B*c*d^2 - 418*A*d^3 + 28*c^3*D) - a*b^2*c*
d^2*(171*c^2*C*d - 1121*B*c*d^2 + 451*A*d^3 + 239*c^3*D))*(a + b*x^2)^(7/2
))/((b*c^2 + a*d^2)*(c + d*x)^7) + (9*b*d^2*(A*b*(80*b^2*c^4 - 60*a*b*c^2*
d^2 + 3*a^2*d^4) - a*(10*b^2*c^3*(c*C - 11*B*d) + 10*a^2*d^3*(C*d - 11*c*D
) - 3*a*b*c*d*(41*c*C*d - 11*B*d^2 - 11*c^2*D)))*(-1/6*((a*d - b*c*x)*(a +
 b*x^2)^(5/2))/((b*c^2 + a*d^2)*(c + d*x)^6) + (5*a*b*(-1/4*((a*d - b*c*x)
*(a + b*x^2)^(3/2))/((b*c^2 + a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*((a*d - b
*c*x)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d -
 b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(2*(b*c^2 + a*d^2)^(3/2)))
)/(4*(b*c^2 + a*d^2))))/(6*(b*c^2 + a*d^2))))/(b*c^2 + a*d^2))/(8*(b*c^2 +
 a*d^2)))/(9*d^2*(b*c^2 + a*d^2)))/(10*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175933 vs. 2(961) = 1922.

Time = 4.48 (sec) , antiderivative size = 175934, normalized size of antiderivative = 176.46

method	result	size
default	Expression too large to display	175934

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**11,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77360 vs. $2(965) = 1930$.

Time = 3.93 (sec) , antiderivative size = 77360, normalized size of antiderivative = 77.59

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x, algorithm="max
ima")
```

output
```
143/512*D*b^12*c^16*arcsinh(b*x/sqrt(a*b))/(b^(19/2)*c^18*d^9 + 9*a*b^(17/
2)*c^16*d^11 + 36*a^2*b^(15/2)*c^14*d^13 + 84*a^3*b^(13/2)*c^12*d^15 + 126
*a^4*b^(11/2)*c^10*d^17 + 126*a^5*b^(9/2)*c^8*d^19 + 84*a^6*b^(7/2)*c^6*d^
21 + 36*a^7*b^(5/2)*c^4*d^23 + 9*a^8*b^(3/2)*c^2*d^25 + a^9*sqrt(b)*d^27)
- 143/512*C*b^12*c^15*arcsinh(b*x/sqrt(a*b))/(b^(19/2)*c^18*d^8 + 9*a*b^(1
7/2)*c^16*d^10 + 36*a^2*b^(15/2)*c^14*d^12 + 84*a^3*b^(13/2)*c^12*d^14 + 1
26*a^4*b^(11/2)*c^10*d^16 + 126*a^5*b^(9/2)*c^8*d^18 + 84*a^6*b^(7/2)*c^6*
d^20 + 36*a^7*b^(5/2)*c^4*d^22 + 9*a^8*b^(3/2)*c^2*d^24 + a^9*sqrt(b)*d^26
) + 143/512*D*a*b^11*c^14*arcsinh(b*x/sqrt(a*b))/(b^(19/2)*c^18*d^7 + 9*a*
b^(17/2)*c^16*d^9 + 36*a^2*b^(15/2)*c^14*d^11 + 84*a^3*b^(13/2)*c^12*d^13
+ 126*a^4*b^(11/2)*c^10*d^15 + 126*a^5*b^(9/2)*c^8*d^17 + 84*a^6*b^(7/2)*c
^6*d^19 + 36*a^7*b^(5/2)*c^4*d^21 + 9*a^8*b^(3/2)*c^2*d^23 + a^9*sqrt(b)*d
^25) + 143/512*B*b^12*c^14*arcsinh(b*x/sqrt(a*b))/(b^(19/2)*c^18*d^7 + 9*a
*b^(17/2)*c^16*d^9 + 36*a^2*b^(15/2)*c^14*d^11 + 84*a^3*b^(13/2)*c^12*d^13
 + 126*a^4*b^(11/2)*c^10*d^15 + 126*a^5*b^(9/2)*c^8*d^17 + 84*a^6*b^(7/2)*
c^6*d^19 + 36*a^7*b^(5/2)*c^4*d^21 + 9*a^8*b^(3/2)*c^2*d^23 + a^9*sqrt(b)*
d^25) - 143/512*sqrt(b*x^2 + a)*D*b^11*c^14*x/(b^9*c^18*d^7 + 9*a*b^8*c^16
*d^9 + 36*a^2*b^7*c^14*d^11 + 84*a^3*b^6*c^12*d^13 + 126*a^4*b^5*c^10*d^15
 + 126*a^5*b^4*c^8*d^17 + 84*a^6*b^3*c^6*d^19 + 36*a^7*b^2*c^4*d^21 + 9*a^
8*b*c^2*d^23 + a^9*d^25) - 143/512*C*a*b^11*c^13*arcsinh(b*x/sqrt(a*b))...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21367 vs. 2(965) = 1930.

Time = 1.35 (sec) , antiderivative size = 21367, normalized size of antiderivative = 21.43

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x, algorithm="gia
c")
```

output
```
1/128*(10*C*a^4*b^6*c^4 - 80*A*a^3*b^7*c^4 + 33*D*a^5*b^5*c^3*d - 110*B*a^
4*b^6*c^3*d - 123*C*a^5*b^5*c^2*d^2 + 60*A*a^4*b^6*c^2*d^2 - 110*D*a^6*b^4
*c*d^3 + 33*B*a^5*b^5*c*d^3 + 10*C*a^6*b^4*d^4 - 3*A*a^5*b^5*d^4)*arctan((
(sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^7*c
^14 + 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^10*d^4 + 35*a^3*b^4*c^8*d^6 + 35*a^4
*b^3*c^6*d^8 + 21*a^5*b^2*c^4*d^10 + 7*a^6*b*c^2*d^12 + a^7*d^14)*sqrt(-b*
c^2 - a*d^2)) + 1/40320*(3150*(sqrt(b)*x - sqrt(b*x^2 + a))^19*C*a^4*b^6*c
^4*d^18 - 25200*(sqrt(b)*x - sqrt(b*x^2 + a))^19*A*a^3*b^7*c^4*d^18 + 1039
5*(sqrt(b)*x - sqrt(b*x^2 + a))^19*D*a^5*b^5*c^3*d^19 - 34650*(sqrt(b)*x -
 sqrt(b*x^2 + a))^19*B*a^4*b^6*c^3*d^19 - 38745*(sqrt(b)*x - sqrt(b*x^2 +
a))^19*C*a^5*b^5*c^2*d^20 + 18900*(sqrt(b)*x - sqrt(b*x^2 + a))^19*A*a^4*b
^6*c^2*d^20 - 34650*(sqrt(b)*x - sqrt(b*x^2 + a))^19*D*a^6*b^4*c*d^21 + 10
395*(sqrt(b)*x - sqrt(b*x^2 + a))^19*B*a^5*b^5*c*d^21 + 3150*(sqrt(b)*x -
sqrt(b*x^2 + a))^19*C*a^6*b^4*d^22 - 945*(sqrt(b)*x - sqrt(b*x^2 + a))^19*
A*a^5*b^5*d^22 + 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^18*D*b^(21/2)*c^14*d^
8 + 564480*(sqrt(b)*x - sqrt(b*x^2 + a))^18*D*a*b^(19/2)*c^12*d^10 + 16934
40*(sqrt(b)*x - sqrt(b*x^2 + a))^18*D*a^2*b^(17/2)*c^10*d^12 + 2822400*(sq
rt(b)*x - sqrt(b*x^2 + a))^18*D*a^3*b^(15/2)*c^8*d^14 + 2822400*(sqrt(b)*x
 - sqrt(b*x^2 + a))^18*D*a^4*b^(13/2)*c^6*d^16 + 59850*(sqrt(b)*x - sqrt(b
*x^2 + a))^18*C*a^4*b^(13/2)*c^5*d^17 - 478800*(sqrt(b)*x - sqrt(b*x^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{11}}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^11,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{11}}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^11,x)
```

3.100 $\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1272

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Too large to display}$$

output
```
-1/256*a^2*b^3*(5*A*b^2*c*(3*a^2*d^4-20*a*b*c^2*d^2+16*b^2*c^4)-a*(10*b^3*
c^4*(-12*B*d+C*c)+10*a^3*d^5*D+a^2*b*d^3*(-3*B*d^2+36*C*c*d-149*D*c^2)-a*b
^2*c^2*d*(-72*B*d^2+149*C*c*d-36*D*c^2)))*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/(a*
d^2+b*c^2)^8/(d*x+c)^2-1/384*a*b^2*(5*A*b^2*c*(3*a^2*d^4-20*a*b*c^2*d^2+16
*b^2*c^4)-a*(10*b^3*c^4*(-12*B*d+C*c)+10*a^3*d^5*D+a^2*b*d^3*(-3*B*d^2+36*
C*c*d-149*D*c^2)-a*b^2*c^2*d*(-72*B*d^2+149*C*c*d-36*D*c^2)))*(-b*c*x+a*d)
*(b*x^2+a)^(3/2)/(a*d^2+b*c^2)^7/(d*x+c)^4-1/480*b*(5*A*b^2*c*(3*a^2*d^4-2
0*a*b*c^2*d^2+16*b^2*c^4)-a*(10*b^3*c^4*(-12*B*d+C*c)+10*a^3*d^5*D+a^2*b*d
^3*(-3*B*d^2+36*C*c*d-149*D*c^2)-a*b^2*c^2*d*(-72*B*d^2+149*C*c*d-36*D*c^2
)))*(-b*c*x+a*d)*(b*x^2+a)^(5/2)/(a*d^2+b*c^2)^6/(d*x+c)^6-1/11*(A*d^3-B*c
*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)/(d*x+c)^11+1/110*(11
*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-15*A*d^3+4*B*c*d^2+7*C*c^2*d-18*D*c^
3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)^10-1/990*(110*a^2*d^4*(C*d
-3*D*c)-b^2*c^2*(-155*A*d^3+12*B*c*d^2+21*C*c^2*d+56*D*c^3)-a*b*d^2*(40*A*
d^3-183*B*c*d^2+106*C*c^2*d+191*D*c^3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*c^2)^
3/(d*x+c)^9-1/7920*(990*a^3*d^6*D+11*a^2*b*d^4*(-27*B*d^2+164*C*c*d-141*D*
c^2)-2*b^3*c^3*(-650*A*d^3+12*B*c*d^2+21*C*c^2*d+56*D*c^3)-a*b^2*c*d^2*(84
5*A*d^3-1824*B*c*d^2+383*C*c^2*d+508*D*c^3))*(b*x^2+a)^(7/2)/d^2/(a*d^2+b*
c^2)^4/(d*x+c)^8+1/55440*b*(110*a^3*d^6*(16*C*d-129*D*c)-a^2*b*d^4*(640*A*
d^3-5601*B*c*d^2+16172*C*c^2*d-5623*D*c^3)+2*b^3*c^4*(-4610*A*d^3+12*B*...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2794 vs. $2(1272) = 2544$.

Time = 18.50 (sec) , antiderivative size = 2794, normalized size of antiderivative = 2.20

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^12,x]
```

output
```
Sqrt[a + b*x^2]*(-1/11*((b*c^2 + a*d^2)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3
*D))/(d^8*(c + d*x)^11) - ((b*c^2 + a*d^2)*(-67*b*c^3*C*d + 56*b*B*c^2*d^2
 - 45*A*b*c*d^3 - 22*a*c*C*d^3 + 11*a*B*d^4 + 78*b*c^4*D + 33*a*c^2*d^2*D)
)/(110*d^8*(c + d*x)^10) + (-1707*b^2*c^4*C*d + 1146*b^2*B*c^3*d^2 - 695*A
*b^2*c^2*d^3 - 1352*a*b*c^2*C*d^3 + 681*a*b*B*c*d^4 - 230*a*A*b*d^5 - 110*
a^2*C*d^5 + 2378*b^2*c^5*D + 2243*a*b*c^3*d^2*D + 330*a^2*c*d^4*D)/(990*d^
8*(c + d*x)^9) + (20994*b^3*c^5*C*d - 10632*b^3*B*c^4*d^2 + 4340*A*b^3*c^3
*d^3 + 29207*a*b^2*c^3*C*d^3 - 12696*a*b^2*B*c^2*d^4 + 4325*a*A*b^2*c*d^5
+ 8228*a^2*b*c*C*d^5 - 2079*a^2*b*B*d^6 - 36416*b^3*c^6*D - 56828*a*b^2*c^
4*d^2*D - 21417*a^2*b*c^2*d^4*D - 990*a^3*d^6*D)/(7920*d^8*(b*c^2 + a*d^2)
*(c + d*x)^8) - (b*(130242*b^3*c^6*C*d - 44376*b^3*B*c^5*d^2 + 9220*A*b^3*
c^4*d^3 + 276919*a*b^2*c^4*C*d^3 - 88632*a*b^2*B*c^3*d^4 + 18485*a*A*b^2*c
^2*d^5 + 163172*a^2*b*c^2*C*d^5 - 44031*a^2*b*B*c*d^6 + 9040*a^2*A*b*d^7 +
 16720*a^3*C*d^7 - 299488*b^3*c^7*D - 681356*a*b^2*c^5*d^2*D - 464473*a^2*
b*c^3*d^4*D - 82830*a^3*c*d^6*D))/(55440*d^8*(b*c^2 + a*d^2)^2*(c + d*x)^7
) - (b*(-127596*b^4*c^7*C*d + 22128*b^4*B*c^6*d^2 - 40*A*b^4*c^5*d^3 - 382
468*a*b^3*c^5*C*d^3 + 66504*a*b^3*B*c^4*d^4 - 130*a*A*b^3*c^3*d^5 - 381643
*a^2*b^2*c^3*C*d^5 + 66834*a^2*b^2*B*c^2*d^6 - 1065*a^2*A*b^2*c*d^7 - 1257
96*a^3*b*c*C*d^7 + 21483*a^3*b*B*d^8 + 435904*b^4*c^8*D + 1345672*a*b^3*c^
6*d^2*D + 1421682*a^2*b^2*c^4*d^4*D + 550209*a^3*b*c^2*d^6*D + 39270*a^...
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 1028, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2182, 25, 2182, 25, 27, 688, 25, 688, 25, 27, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{12}} \, dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{(bx^2+a)^{5/2}\left(11\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(11a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+4Bc-4Ad\right)\right)x+\frac{11\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{(c+dx)^{11}}dx}{11\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{5/2}\left(11\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(11a(Cd-cD)+b\left(-\frac{7Dc^3}{d^2}+\frac{7Cc^2}{d}+4Bc-4Ad\right)\right)x+\frac{11\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{(c+dx)^{11}}dx}{11\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{(a+bx^2)^{7/2}(11ad^2(-Bd^2-3c^2D+2cCd)+bc(-15Ad^3+4Bcd^2-18c^3D+7c^2Cd))}{10d^2(c+dx)^{10}(ad^2+bc^2)}-\int -\frac{(10d(Abd(11bc^2-4ad^2)+a(11ad^2(Cd-2cD)-bc(7Dc^2+4C\cdots}{\cdots}}{11\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(10d(Abd(11bc^2-4ad^2)+a(11ad^2(Cd-2cD)-bc(7Dc^2+4Cdc-15Bd^2))))+(110a^2Dd^4+11ab(11Dc^2+6Cdc-3Bd^2)d^2+b^2c(56Dc^3+21Cdc^2+12Bd^2c-45A\cdots}{d^2(c+dx)^{10}}}{10(ad^2+bc^2)}\cdots}{11\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(10d(Abd(11bc^2-4ad^2)+a(11ad^2(Cd-2cD)-bc(7Dc^2+4Cdc-15Bd^2))))+(110a^2Dd^4+11ab(11Dc^2+6Cdc-3Bd^2)d^2+b^2c(56Dc^3+21Cdc^2+12Bd^2c-45A\cdots}{(c+dx)^{10}}}{10d^2(ad^2+bc^2)}\cdots}{11\left(ad^2+bc^2\right)}-\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 688$$

$$-\frac{\int -\frac{\left(9d\left(5Acd\left(22bc^2-17ad^2\right)b^2+a\left(110a^2Dd^4+11ab\left(-9Dc^2+16Cdc-3Bd^2\right)d^2-b^2c^2\left(14Dc^2+19Cdc-162Bd^2\right)\right)\right)-2b\left(110a^2(Cd-3cD)d^4-ab\left(191Dc^3+106\right.\right.}{(c+dx)^9}}{9\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\left(9d\left(5Acd\left(22bc^2-17ad^2\right)b^2+a\left(110a^2Dd^4+11ab\left(-9Dc^2+16Cdc-3Bd^2\right)d^2-b^2c^2\left(14Dc^2+19Cdc-162Bd^2\right)\right)\right)-2b\left(110a^2(Cd-3cD)d^4-ab\left(191Dc^3+106Cdc\right.\right.}{(c+dx)^9}}{9\left(ad^2+bc^2\right)}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 688$$

$$-\frac{\int -\frac{b\left(8d\left(5Abd\left(198b^2c^4-215abd^2c^2+16a^2d^4\right)-a\left(110a^2(2Cd-15cD)d^4-abc\left(-509Dc^2+1796Cdc-663Bd^2\right)d^2+b^2c^3\left(14Dc^2+129Cdc-1482Bd^2\right)\right)\right)-\left(990a^3\right.\right.}{(c+}}{8\left(ad^2\right.}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{b\left(8d\left(5Abd\left(198b^2c^4-215abd^2c^2+16a^2d^4\right)-a\left(110a^2(2Cd-15cD)d^4-abc\left(-509Dc^2+1796Cdc-663Bd^2\right)d^2+b^2c^3\left(14Dc^2+129Cdc-1482Bd^2\right)\right)\right)-\left(990a^3Dd\right.\right.}{(c+dx)}}{8\left(ad^2+b\right.}$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow 27$$

$$b \int \frac{\left(8d\left(5Abd\left(198b^2c^4-215abd^2c^2+16a^2d^4\right)-a\left(110a^2(2Cd-15cD)d^4-abc\left(-509Dc^2+1796Cdc-663Bd^2\right)d^2+b^2c^3\left(14Dc^2+129Cdc-1482Bd^2\right)\right)\right)-\left(990a^3Dc\right.}{(c+dx}$$

$$8\left(ad^2+\right.$$

$$\frac{\left(a+bx^2\right)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{11d^2(c+dx)^{11}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{679}$$

$$b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\right.\right.}{}\right.$$

$$\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

$$\Big\downarrow \text{486}$$

$$b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\right.\right.}{}\right.$$

$$\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

$$\Big\downarrow \text{486}$$

$$\frac{\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+\frac{b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

$\Big\downarrow$ 486

$$\frac{\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+\frac{b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

$\Big\downarrow$ 488

$$b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\ldots\right.\right.}{\ldots}\right.$$

$$\frac{\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+\ldots}{\ldots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

$$\Big\downarrow \; 219$$

$$b\left(\frac{\left(110a^3(16Cd-129cD)d^6-a^2b\left(-5623Dc^3+16172Cdc^2-560\cdots\right.\right.}{}\right.$$

$$\frac{\frac{\left(11a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-18Dc^3+7Cdc^2+4Bd^2c-15Ad^3\right)\right)\left(bx^2+a\right)^{7/2}}{10d^2\left(bc^2+ad^2\right)(c+dx)^{10}}+\frac{\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(bx^2+a\right)^{7/2}}{11d^2\left(bc^2+ad^2\right)(c+dx)^{11}}$$

input

```
Int[((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^12,x]
```

output
```
-1/11*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(7/2))/(d^2*(b*c^2
+ a*d^2)*(c + d*x)^11) + (((11*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(7*
c^2*C*d + 4*B*c*d^2 - 15*A*d^3 - 18*c^3*D))*(a + b*x^2)^(7/2))/(10*d^2*(b*
c^2 + a*d^2)*(c + d*x)^10) + (-1/9*((110*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(
21*c^2*C*d + 12*B*c*d^2 - 155*A*d^3 + 56*c^3*D) - a*b*d^2*(106*c^2*C*d - 1
83*B*c*d^2 + 40*A*d^3 + 191*c^3*D))*(a + b*x^2)^(7/2))/((b*c^2 + a*d^2)*(c
 + d*x)^9) + (-1/8*((990*a^3*d^6*D + 11*a^2*b*d^4*(164*c*C*d - 27*B*d^2 -
141*c^2*D) - 2*b^3*c^3*(21*c^2*C*d + 12*B*c*d^2 - 650*A*d^3 + 56*c^3*D) -
a*b^2*c*d^2*(383*c^2*C*d - 1824*B*c*d^2 + 845*A*d^3 + 508*c^3*D))*(a + b*x
^2)^(7/2))/((b*c^2 + a*d^2)*(c + d*x)^8) + (b*(((110*a^3*d^6*(16*C*d - 129
*c*D) - a^2*b*d^4*(16172*c^2*C*d - 5601*B*c*d^2 + 640*A*d^3 - 5623*c^3*D)
+ 2*b^3*c^4*(21*c^2*C*d + 12*B*c*d^2 - 4610*A*d^3 + 56*c^3*D) + 5*a*b^2*c^
2*d^2*(283*c^2*C*d - 2736*B*c*d^2 + 1889*A*d^3 + 124*c^3*D))*(a + b*x^2)^(
7/2))/(7*(b*c^2 + a*d^2)*(c + d*x)^7) + (99*d^2*(5*A*b^2*c*(16*b^2*c^4 - 2
0*a*b*c^2*d^2 + 3*a^2*d^4) - a*(10*b^3*c^4*(c*C - 12*B*d) + 10*a^3*d^5*D +
 a^2*b*d^3*(36*c*C*d - 3*B*d^2 - 149*c^2*D) - a*b^2*c^2*d*(149*c*C*d - 72*
B*d^2 - 36*c^2*D)))*(-1/6*((a*d - b*c*x)*(a + b*x^2)^(5/2))/((b*c^2 + a*d^
2)*(c + d*x)^6) + (5*a*b*(-1/4*((a*d - b*c*x)*(a + b*x^2)^(3/2))/((b*c^2 +
 a*d^2)*(c + d*x)^4) + (3*a*b*(-1/2*((a*d - b*c*x)*Sqrt[a + b*x^2])/((b*c^
2 + a*d^2)*(c + d*x)^2) - (a*b*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288082 vs. 2(1232) = 2464.

Time = 5.95 (sec) , antiderivative size = 288083, normalized size of antiderivative = 226.48

method	result	size
default	Expression too large to display	288083

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x,method=_RETURNVERBOSE
)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**12,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101201 vs. 2(1231) = 2462.

Time = 4.93 (sec) , antiderivative size = 101201, normalized size of antiderivative = 79.56

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x, algorithm="max
ima")
```

output
```
195/512*D*b^13*c^17*arcsinh(b*x/sqrt(a*b))/(b^(21/2)*c^20*d^9 + 10*a*b^(19
/2)*c^18*d^11 + 45*a^2*b^(17/2)*c^16*d^13 + 120*a^3*b^(15/2)*c^14*d^15 + 2
10*a^4*b^(13/2)*c^12*d^17 + 252*a^5*b^(11/2)*c^10*d^19 + 210*a^6*b^(9/2)*c
^8*d^21 + 120*a^7*b^(7/2)*c^6*d^23 + 45*a^8*b^(5/2)*c^4*d^25 + 10*a^9*b^(3
/2)*c^2*d^27 + a^10*sqrt(b)*d^29) - 195/512*C*b^13*c^16*arcsinh(b*x/sqrt(a
*b))/(b^(21/2)*c^20*d^8 + 10*a*b^(19/2)*c^18*d^10 + 45*a^2*b^(17/2)*c^16*d
^12 + 120*a^3*b^(15/2)*c^14*d^14 + 210*a^4*b^(13/2)*c^12*d^16 + 252*a^5*b^
(11/2)*c^10*d^18 + 210*a^6*b^(9/2)*c^8*d^20 + 120*a^7*b^(7/2)*c^6*d^22 + 4
5*a^8*b^(5/2)*c^4*d^24 + 10*a^9*b^(3/2)*c^2*d^26 + a^10*sqrt(b)*d^28) + 19
5/512*D*a*b^12*c^15*arcsinh(b*x/sqrt(a*b))/(b^(21/2)*c^20*d^7 + 10*a*b^(19
/2)*c^18*d^9 + 45*a^2*b^(17/2)*c^16*d^11 + 120*a^3*b^(15/2)*c^14*d^13 + 21
0*a^4*b^(13/2)*c^12*d^15 + 252*a^5*b^(11/2)*c^10*d^17 + 210*a^6*b^(9/2)*c^
8*d^19 + 120*a^7*b^(7/2)*c^6*d^21 + 45*a^8*b^(5/2)*c^4*d^23 + 10*a^9*b^(3/
2)*c^2*d^25 + a^10*sqrt(b)*d^27) + 195/512*B*b^13*c^15*arcsinh(b*x/sqrt(a*
b))/(b^(21/2)*c^20*d^7 + 10*a*b^(19/2)*c^18*d^9 + 45*a^2*b^(17/2)*c^16*d^1
1 + 120*a^3*b^(15/2)*c^14*d^13 + 210*a^4*b^(13/2)*c^12*d^15 + 252*a^5*b^(1
1/2)*c^10*d^17 + 210*a^6*b^(9/2)*c^8*d^19 + 120*a^7*b^(7/2)*c^6*d^21 + 45*
a^8*b^(5/2)*c^4*d^23 + 10*a^9*b^(3/2)*c^2*d^25 + a^10*sqrt(b)*d^27) - 195/
512*sqrt(b*x^2 + a)*D*b^12*c^15*x/(b^10*c^20*d^7 + 10*a*b^9*c^18*d^9 + 45*
a^2*b^8*c^16*d^11 + 120*a^3*b^7*c^14*d^13 + 210*a^4*b^6*c^12*d^15 + 252...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25283 vs. 2(1231) = 2462.

Time = 1.42 (sec) , antiderivative size = 25283, normalized size of antiderivative = 19.88

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x, algorithm="gia
c")
```

output
```
-1/128*(10*C*a^4*b^7*c^5 - 80*A*a^3*b^8*c^5 + 36*D*a^5*b^6*c^4*d - 120*B*a
^4*b^7*c^4*d - 149*C*a^5*b^6*c^3*d^2 + 100*A*a^4*b^7*c^3*d^2 - 149*D*a^6*b
^5*c^2*d^3 + 72*B*a^5*b^6*c^2*d^3 + 36*C*a^6*b^5*c*d^4 - 15*A*a^5*b^6*c*d^
4 + 10*D*a^7*b^4*d^5 - 3*B*a^6*b^5*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 +
 a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^8*c^16 + 8*a*b^7*c^14*d^2 +
28*a^2*b^6*c^12*d^4 + 56*a^3*b^5*c^10*d^6 + 70*a^4*b^4*c^8*d^8 + 56*a^5*b^
3*c^6*d^10 + 28*a^6*b^2*c^4*d^12 + 8*a^7*b*c^2*d^14 + a^8*d^16)*sqrt(-b*c^
2 - a*d^2)) + 1/443520*(34650*(sqrt(b)*x - sqrt(b*x^2 + a))^21*C*a^4*b^7*c
^5*d^19 - 277200*(sqrt(b)*x - sqrt(b*x^2 + a))^21*A*a^3*b^8*c^5*d^19 + 124
740*(sqrt(b)*x - sqrt(b*x^2 + a))^21*D*a^5*b^6*c^4*d^20 - 415800*(sqrt(b)*
x - sqrt(b*x^2 + a))^21*B*a^4*b^7*c^4*d^20 - 516285*(sqrt(b)*x - sqrt(b*x^
2 + a))^21*C*a^5*b^6*c^3*d^21 + 346500*(sqrt(b)*x - sqrt(b*x^2 + a))^21*A*
a^4*b^7*c^3*d^21 - 516285*(sqrt(b)*x - sqrt(b*x^2 + a))^21*D*a^6*b^5*c^2*d
^22 + 249480*(sqrt(b)*x - sqrt(b*x^2 + a))^21*B*a^5*b^6*c^2*d^22 + 124740*
(sqrt(b)*x - sqrt(b*x^2 + a))^21*C*a^6*b^5*c*d^23 - 51975*(sqrt(b)*x - sqr
t(b*x^2 + a))^21*A*a^5*b^6*c*d^23 + 34650*(sqrt(b)*x - sqrt(b*x^2 + a))^21
*D*a^7*b^4*d^24 - 10395*(sqrt(b)*x - sqrt(b*x^2 + a))^21*B*a^6*b^5*d^24 +
727650*(sqrt(b)*x - sqrt(b*x^2 + a))^20*C*a^4*b^(15/2)*c^6*d^18 - 5821200*
(sqrt(b)*x - sqrt(b*x^2 + a))^20*A*a^3*b^(17/2)*c^6*d^18 + 2619540*(sqrt(b
)*x - sqrt(b*x^2 + a))^20*D*a^5*b^(13/2)*c^5*d^19 - 8731800*(sqrt(b)*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^12,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{12}}\,dx = \int \frac{(b\,x^2+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{12}}dx$$

input
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x)
```

output
```
int((b*x^2+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^12,x)
```

3.101 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 411

$$
\begin{aligned}
&\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx\\
&=\frac{(b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D))\sqrt{a+bx^2}}{b^3}\\
&\quad+\frac{(8b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-6abd(3cCd+Bd^2+3c^2D))\,x\sqrt{a+bx^2}}{16b^3}\\
&\quad-\frac{d(5ad^2D-6b(3cCd+Bd^2+3c^2D))\,x^3\sqrt{a+bx^2}}{24b^2}+\frac{d^3Dx^5\sqrt{a+bx^2}}{6b}\\
&\quad-\frac{(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\left(a+bx^2\right)^{3/2}}{3b^3}\\
&\quad+\frac{d^2(Cd+3cD)\left(a+bx^2\right)^{5/2}}{5b^3}\\
&\quad+\frac{(16Ab^3c^3-a(8b^2c(c^2C+3Bcd+3Ad^2)+5a^2d^3D-6abd(3cCd+Bd^2+3c^2D)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}
\end{aligned}
$$

output
```
(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*
c^3))*(b*x^2+a)^(1/2)/b^3+1/16*(8*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*
D-6*a*b*d*(B*d^2+3*C*c*d+3*D*c^2))*x*(b*x^2+a)^(1/2)/b^3-1/24*d*(5*a*d^2*D
-6*b*(B*d^2+3*C*c*d+3*D*c^2))*x^3*(b*x^2+a)^(1/2)/b^2+1/6*d^3*D*x^5*(b*x^2
+a)^(1/2)/b-1/3*(2*a*d^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*
(b*x^2+a)^(3/2)/b^3+1/5*d^2*(C*d+3*D*c)*(b*x^2+a)^(5/2)/b^3+1/16*(16*A*b^3
*c^3-a*(8*b^2*c*(3*A*d^2+3*B*c*d+C*c^2)+5*a^2*d^3*D-6*a*b*d*(B*d^2+3*C*c*d
+3*D*c^2)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(a^2d^2(128Cd+384cD+75dDx)+4b^2(10Ad(18c^2+9cdx+2d^2x^2)+15B(4c^3+6c^2dx+4cd}{}$$
$$-\frac{(8Ab^2c(2bc^2-3ad^2)+a(-8b^2c^2(cC+3Bd)-5a^2d^3D+6abd(3cCd+Bd^2+3c^2D)))\log\left(-\sqrt{b}x+\right.}{16b^{7/2}}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(a^2*d^2*(128*C*d + 384*c*D + 75*d*D*x) + 4*b^2*(10*A*d*(
18*c^2 + 9*c*d*x + 2*d^2*x^2) + 15*B*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^
3*x^3) + x*(10*c^3*(3*C + 2*D*x) + 15*c^2*d*x*(4*C + 3*D*x) + 9*c*d^2*x^2*
(5*C + 4*D*x) + 2*d^3*x^3*(6*C + 5*D*x))) - 2*a*b*(80*c^3*D + 15*c^2*d*(16
*C + 9*D*x) + 3*c*d^2*(80*B + x*(45*C + 32*D*x)) + d^3*(80*A + x*(45*B + 3
2*C*x + 25*D*x^2)))))/(240*b^3) - ((8*A*b^2*c*(2*b*c^2 - 3*a*d^2) + a*(-8*
b^2*c^2*(c*C + 3*B*d) - 5*a^2*d^3*D + 6*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D))
)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2185, 2185, 27, 687, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}} \, dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int \frac{(c+dx)^3 \left(b(6Cd-7cD)x^2d^2+(6Abd-5acD)d^2+\left(-bDc^2+6bBd^2-5ad^2D\right)xd\right)}{\sqrt{bx^2+a}}dx}{6bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int \frac{bd^3(c+dx)^3\left(3d(10Abd-8aCd+acD)-\left(25aDd^2+2b\left(-Dc^2+3Cdc-15Bd^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{5bd^2}+\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(6Cd-7cD)}{6bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}d\int \frac{(c+dx)^3\left(3d(10Abd-8aCd+acD)-\left(25aDd^2+2b\left(-Dc^2+3Cdc-15Bd^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx+\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(6Cd-7cD)}{6bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{5}d\left(\frac{\int \frac{3(c+dx)^2\left(d\left(40Acdb^2+a\left(25ad^2D-2b\left(-Dc^2+13Cdc+15Bd^2\right)\right)\right)-b\left(a(32Cd+21cD)d^2+2b\left(-Dc^3+3Cdc^2-15Bd^2c-20Ad^3\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4b}-\frac{\sqrt{a+bx^2}(c+\cdots}{\cdots}\right)}{6bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}d\Bigg(\frac{3\int\frac{(c+dx)^2\left(d\left(40Acdb^2+a\left(25ad^2D-2b\left(-Dc^2+13Cdc+15Bd^2\right)\right)\right)-b\left(a(32Cd+21cD)d^2+2b\left(-Dc^3+3Cdc^2-15Bd^2c-20Ad^3\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4b}-\frac{\sqrt{a+bx^2}(c+}{}}{6bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{5}d\Bigg(\frac{3\Bigg(\frac{\int\frac{b(c+dx)\left(d\left(40Abd\left(3bc^2-2ad^2\right)+a\left(ad^2(64Cd+117cD)-2bc\left(-Dc^2+33Cdc+75Bd^2\right)\right)\right)+\left(75a^2Dd^4-2ab\left(18Dc^2+71Cdc+45Bd^2\right)d^2-4b^2c\left(-Dc^3+\right.\right.\right.}{\sqrt{bx^2+a}}}{3b}}{4b}}{}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}d\Bigg(\frac{3\Big(\frac{1}{3}\int\frac{(c+dx)\left(d\left(40Abd\left(3bc^2-2ad^2\right)+a\left(ad^2(64Cd+117cD)-2bc\left(-Dc^2+33Cdc+75Bd^2\right)\right)\right)+\left(75a^2Dd^4-2ab\left(18Dc^2+71Cdc+45Bd^2\right)d^2-4b^2c\left(-Dc^3\right.\right.\right.}{\sqrt{bx^2+a}}}{4b}}{}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{5}d\Bigg(3\Bigg(\frac{1}{3}\Bigg(\frac{15d^2\left(8Ab^2c\left(2bc^2-3ad^2\right)-a\left(5a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c^2(3Bd+cC)\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}\left(75a^2d^4D-2abd^2\left(45Bd^2+18c^2\right.\right.}{}}{}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{5}d\left(\frac{3\left(\frac{1}{3}\left(\frac{15d^2\left(8Ab^2c\left(2bc^2-3ad^2\right)-a\left(5a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c^2(3Bd+cC)\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}\left(75a^2d^4D-2abd^2\left(45\right.\right.}{}\right.\right.}{}\right.$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}d\left(\frac{3\left(\frac{1}{3}\left(\frac{15d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(8Ab^2c\left(2bc^2-3ad^2\right)-a\left(5a^2d^3D-6abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c^2(3Bd+cC)\right)\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}\left(75a^2d^4D-2abd^2\left(45\right.\right.}{}\right.\right.}{}\right.$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^5}{6bd^2}$$

input

```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output

```
(D*(c + d*x)^5*Sqrt[a + b*x^2])/(6*b*d^2) + ((d*(6*C*d - 7*c*D)*(c + d*x)^
4*Sqrt[a + b*x^2])/5 + (d*(-1/4*((25*a*d^2*D + 2*b*(3*c*C*d - 15*B*d^2 - c
^2*D))*(c + d*x)^3*Sqrt[a + b*x^2])/b + (3*(-1/3*((a*d^2*(32*C*d + 21*c*D)
 + 2*b*(3*c^2*C*d - 15*B*c*d^2 - 20*A*d^3 - c^3*D))*(c + d*x)^2*Sqrt[a + b
*x^2]) + ((2*(32*a^2*d^4*(C*d + 3*c*D) - 2*b^2*c^2*(3*c^2*C*d - 15*B*c*d^2
 - 80*A*d^3 - c^3*D) - a*b*d^2*(104*c^2*C*d + 120*B*c*d^2 + 40*A*d^3 + 17*
c^3*D))*Sqrt[a + b*x^2])/b + (d*(75*a^2*d^4*D - 2*a*b*d^2*(71*c*C*d + 45*B
*d^2 + 18*c^2*D) - 4*b^2*c*(3*c^2*C*d - 15*B*c*d^2 - 50*A*d^3 - c^3*D))*x*
Sqrt[a + b*x^2])/(2*b) + (15*d^2*(8*A*b^2*c*(2*b*c^2 - 3*a*d^2) - a*(8*b^2
*c^2*(c*C + 3*B*d) + 5*a^2*d^3*D - 6*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D)))*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/3))/(4*b)))/5)/(6*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.99

method	result
default	$\frac{A\,c^{3} \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{c^{2} \left(3 A d +B c \right) \sqrt{b\,x^{2}+a}}{b}+d^{2} \left(C d +3 D c \right) \left(\frac{x^{4} \sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a \left(\frac{x^{2} \sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a \sqrt{b\,x^{2}+a}}{3 b^{2}}\right)}{5 b}\right)+$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+c^2*(3*A*d+B*c)/b*(b*x^2+a)^(1
/2)+d^2*(C*d+3*D*c)*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a
)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))+c*(3*A*d^2+3*B*c*d+C*c^2)*(1/2*x/b*(b*
x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+d*(B*d^2+3*C*c*d
+3*D*c^2)*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*
a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3
)*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+D*d^3*(1/6*x^5/b*(
b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x/b*(b*x^2+
a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 843, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
[-1/480*(15*(8*(C*a*b^2 - 2*A*b^3)*c^3 - 6*(3*D*a^2*b - 4*B*a*b^2)*c^2*d -
 6*(3*C*a^2*b - 4*A*a*b^2)*c*d^2 + (5*D*a^3 - 6*B*a^2*b)*d^3)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(40*D*b^3*d^3*x^5 + 48*(3*
D*b^3*c*d^2 + C*b^3*d^3)*x^4 - 80*(2*D*a*b^2 - 3*B*b^3)*c^3 - 240*(2*C*a*b
^2 - 3*A*b^3)*c^2*d + 96*(4*D*a^2*b - 5*B*a*b^2)*c*d^2 + 32*(4*C*a^2*b - 5
*A*a*b^2)*d^3 + 10*(18*D*b^3*c^2*d + 18*C*b^3*c*d^2 - (5*D*a*b^2 - 6*B*b^3
)*d^3)*x^3 + 16*(5*D*b^3*c^3 + 15*C*b^3*c^2*d - 3*(4*D*a*b^2 - 5*B*b^3)*c*
d^2 - (4*C*a*b^2 - 5*A*b^3)*d^3)*x^2 + 15*(8*C*b^3*c^3 - 6*(3*D*a*b^2 - 4*
B*b^3)*c^2*d - 6*(3*C*a*b^2 - 4*A*b^3)*c*d^2 + (5*D*a^2*b - 6*B*a*b^2)*d^3
)*x)*sqrt(b*x^2 + a))/b^4, 1/240*(15*(8*(C*a*b^2 - 2*A*b^3)*c^3 - 6*(3*D*a
^2*b - 4*B*a*b^2)*c^2*d - 6*(3*C*a^2*b - 4*A*a*b^2)*c*d^2 + (5*D*a^3 - 6*B
*a^2*b)*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*D*b^3*d^3*x
^5 + 48*(3*D*b^3*c*d^2 + C*b^3*d^3)*x^4 - 80*(2*D*a*b^2 - 3*B*b^3)*c^3 - 2
40*(2*C*a*b^2 - 3*A*b^3)*c^2*d + 96*(4*D*a^2*b - 5*B*a*b^2)*c*d^2 + 32*(4*
C*a^2*b - 5*A*a*b^2)*d^3 + 10*(18*D*b^3*c^2*d + 18*C*b^3*c*d^2 - (5*D*a*b^
2 - 6*B*b^3)*d^3)*x^3 + 16*(5*D*b^3*c^3 + 15*C*b^3*c^2*d - 3*(4*D*a*b^2 -
5*B*b^3)*c*d^2 - (4*C*a*b^2 - 5*A*b^3)*d^3)*x^2 + 15*(8*C*b^3*c^3 - 6*(3*D
*a*b^2 - 4*B*b^3)*c^2*d - 6*(3*C*a*b^2 - 4*A*b^3)*c*d^2 + (5*D*a^2*b - 6*B
*a*b^2)*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Dd^3x^5}{6b} + \frac{x^4(Cd^3+3Dcd^2)}{5b} + \frac{x^3\left(Bd^3+3Ccd^2-\frac{5Dad^3}{6b}+3Dc^2d\right)}{4b} + \frac{x^2\left(Ad^3+3Bcd^2+3Cc^2d+Dc^3-\frac{4a\left(Cd^3+3Dcd^2\right)}{5b}\right)}{3b}\right. \\ \frac{Ac^3x+\frac{Dd^3x^7}{7}+\frac{x^6\left(Cd^3+3Dcd^2\right)}{6}+\frac{x^5\left(Bd^3+3Ccd^2+3Dc^2d\right)}{5}+\frac{x^4\left(Ad^3+3Bcd^2+3Cc^2d+Dc^3\right)}{4}+\frac{x^3\cdot\left(3Acd^2+3Bc^2d+Cc^3\right)}{3}+\frac{x^2\cdot\left(3Ac^2d+Bc^3\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*d**3*x**5/(6*b) + x**4*(C*d**3 + 3*D*c*d**2
)/(5*b) + x**3*(B*d**3 + 3*C*c*d**2 - 5*D*a*d**3/(6*b) + 3*D*c**2*d)/(4*b)
 + x**2*(A*d**3 + 3*B*c*d**2 + 3*C*c**2*d + D*c**3 - 4*a*(C*d**3 + 3*D*c*d
**2)/(5*b))/(3*b) + x*(3*A*c*d**2 + 3*B*c**2*d + C*c**3 - 3*a*(B*d**3 + 3*
C*c*d**2 - 5*D*a*d**3/(6*b) + 3*D*c**2*d)/(4*b))/(2*b) + (3*A*c**2*d + B*c
**3 - 2*a*(A*d**3 + 3*B*c*d**2 + 3*C*c**2*d + D*c**3 - 4*a*(C*d**3 + 3*D*c
*d**2)/(5*b))/(3*b))/b) + (A*c**3 - a*(3*A*c*d**2 + 3*B*c**2*d + C*c**3 -
3*a*(B*d**3 + 3*C*c*d**2 - 5*D*a*d**3/(6*b) + 3*D*c**2*d)/(4*b))/(2*b))*Pi
ecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((A*c**3*x + D*d**3*x**7/7 + x**6*(C
*d**3 + 3*D*c*d**2)/6 + x**5*(B*d**3 + 3*C*c*d**2 + 3*D*c**2*d)/5 + x**4*(
A*d**3 + 3*B*c*d**2 + 3*C*c**2*d + D*c**3)/4 + x**3*(3*A*c*d**2 + 3*B*c**2
*d + C*c**3)/3 + x**2*(3*A*c**2*d + B*c**3)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{bx^2+a}Dd^3x^5}{6\,b} - \frac{5\,\sqrt{bx^2+a}Dad^3x^3}{24\,b^2} + \frac{5\,\sqrt{bx^2+a}Da^2d^3x}{16\,b^3}$$
$$+ \frac{(3\,Dcd^2+Cd^3)\sqrt{bx^2+a}x^4}{5\,b} + \frac{Ac^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{5\,Da^3d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$
$$+ \frac{\sqrt{bx^2+a}Bc^3}{b} + \frac{3\,\sqrt{bx^2+a}Ac^2d}{b} + \frac{(3\,Dc^2d+3\,Ccd^2+Bd^3)\sqrt{bx^2+a}x^3}{4\,b}$$
$$- \frac{4\,(3\,Dcd^2+Cd^3)\sqrt{bx^2+a}ax^2}{15\,b^2} + \frac{(Dc^3+3\,Cc^2d+3\,Bcd^2+Ad^3)\sqrt{bx^2+a}x^2}{3\,b}$$
$$- \frac{3\,(3\,Dc^2d+3\,Ccd^2+Bd^3)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(Cc^3+3\,Bc^2d+3\,Acd^2)\sqrt{bx^2+a}x}{2\,b}$$
$$+ \frac{3\,(3\,Dc^2d+3\,Ccd^2+Bd^3)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$
$$- \frac{(Cc^3+3\,Bc^2d+3\,Acd^2)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{8\,(3\,Dcd^2+Cd^3)\sqrt{bx^2+a}a^2}{15\,b^3}$$
$$- \frac{2\,(Dc^3+3\,Cc^2d+3\,Bcd^2+Ad^3)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
1/6*sqrt(b*x^2 + a)*D*d^3*x^5/b - 5/24*sqrt(b*x^2 + a)*D*a*d^3*x^3/b^2 + 5
/16*sqrt(b*x^2 + a)*D*a^2*d^3*x/b^3 + 1/5*(3*D*c*d^2 + C*d^3)*sqrt(b*x^2 +
 a)*x^4/b + A*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/16*D*a^3*d^3*arcsinh(
b*x/sqrt(a*b))/b^(7/2) + sqrt(b*x^2 + a)*B*c^3/b + 3*sqrt(b*x^2 + a)*A*c^2
*d/b + 1/4*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*sqrt(b*x^2 + a)*x^3/b - 4/15*(3
*D*c*d^2 + C*d^3)*sqrt(b*x^2 + a)*a*x^2/b^2 + 1/3*(D*c^3 + 3*C*c^2*d + 3*B
*c*d^2 + A*d^3)*sqrt(b*x^2 + a)*x^2/b - 3/8*(3*D*c^2*d + 3*C*c*d^2 + B*d^3
)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*sqrt(b*x^2
 + a)*x/b + 3/8*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*a^2*arcsinh(b*x/sqrt(a*b))
/b^(5/2) - 1/2*(C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*a*arcsinh(b*x/sqrt(a*b))/b^
(3/2) + 8/15*(3*D*c*d^2 + C*d^3)*sqrt(b*x^2 + a)*a^2/b^3 - 2/3*(D*c^3 + 3*
C*c^2*d + 3*B*c*d^2 + A*d^3)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{240}\sqrt{bx^2+a}\left(\left(\left(2\left(\left(\left(4\left(\frac{5\,Dd^3x}{b}+\frac{6\,(3\,Db^5cd^2+Cb^5d^3)}{b^6}\right)x+\frac{5\,(18\,Db^5c^2d+18\,Cb^5cd^2-5\,Dab^4d^3+\right.\right.\right.\right.\right.\right.}{b^6}$$
$$+\frac{(8\,Cab^2c^3-16\,Ab^3c^3-18\,Da^2bc^2d+24\,Bab^2c^2d-18\,Ca^2bcd^2+24\,Aab^2cd^2+5\,Da^3d^3-6\,Ba^2bd^3)\,\mathrm{l}}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*D*d^3*x/b + 6*(3*D*b^5*c*d^2 + C*b^5*d^3)
/b^6)*x + 5*(18*D*b^5*c^2*d + 18*C*b^5*c*d^2 - 5*D*a*b^4*d^3 + 6*B*b^5*d^3
)/b^6)*x + 8*(5*D*b^5*c^3 + 15*C*b^5*c^2*d - 12*D*a*b^4*c*d^2 + 15*B*b^5*c
*d^2 - 4*C*a*b^4*d^3 + 5*A*b^5*d^3)/b^6)*x + 15*(8*C*b^5*c^3 - 18*D*a*b^4*
c^2*d + 24*B*b^5*c^2*d - 18*C*a*b^4*c*d^2 + 24*A*b^5*c*d^2 + 5*D*a^2*b^3*d
^3 - 6*B*a*b^4*d^3)/b^6)*x - 16*(10*D*a*b^4*c^3 - 15*B*b^5*c^3 + 30*C*a*b^
4*c^2*d - 45*A*b^5*c^2*d - 24*D*a^2*b^3*c*d^2 + 30*B*a*b^4*c*d^2 - 8*C*a^2
*b^3*d^3 + 10*A*a*b^4*d^3)/b^6) + 1/16*(8*C*a*b^2*c^3 - 16*A*b^3*c^3 - 18*
D*a^2*b*c^2*d + 24*B*a*b^2*c^2*d - 18*C*a^2*b*c*d^2 + 24*A*a*b^2*c*d^2 + 5
*D*a^3*d^3 - 6*B*a^2*b*d^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^3\,(Dx^3+C\,x^2+Bx+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

3.102 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 280

$$\begin{aligned}
&\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{(b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D))\sqrt{a+bx^2}}{b^3} \\
&\quad+\frac{(4b(c^2C+2Bcd+Ad^2)-3ad(Cd+2cD))\,x\sqrt{a+bx^2}}{8b^2} \\
&\quad+\frac{d(Cd+2cD)x^3\sqrt{a+bx^2}}{4b} \\
&\quad-\frac{(2ad^2D-b(2cCd+Bd^2+c^2D))\,(a+bx^2)^{3/2}}{3b^3}+\frac{d^2D(a+bx^2)^{5/2}}{5b^3} \\
&\quad+\frac{(4Ab(2bc^2-ad^2)-a(4bc(cC+2Bd)-3ad(Cd+2cD)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

output
```
(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(1/2)/b^
3+1/8*(4*b*(A*d^2+2*B*c*d+C*c^2)-3*a*d*(C*d+2*D*c))*x*(b*x^2+a)^(1/2)/b^2+
1/4*d*(C*d+2*D*c)*x^3*(b*x^2+a)^(1/2)/b-1/3*(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*
c^2))*(b*x^2+a)^(3/2)/b^3+1/5*d^2*D*(b*x^2+a)^(5/2)/b^3+1/8*(4*A*b*(-a*d^2
+2*b*c^2)-a*(4*b*c*(2*B*d+C*c)-3*a*d*(C*d+2*D*c)))*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(64a^2d^2D+2b^2(30Ad(4c+dx)+20B(3c^2+3cdx+d^2x^2)+x(10c^2(3C+2Dx)+10cdx(4C}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(64*a^2*d^2*D + 2*b^2*(30*A*d*(4*c + d*x) + 20*B*(3*c^2 +
 3*c*d*x + d^2*x^2) + x*(10*c^2*(3*C + 2*D*x) + 10*c*d*x*(4*C + 3*D*x) + 3
*d^2*x^2*(5*C + 4*D*x))) - a*b*(80*c^2*D + 10*c*d*(16*C + 9*D*x) + d^2*(80
*B + x*(45*C + 32*D*x)))) + 15*Sqrt[b]*(4*A*b*(-2*b*c^2 + a*d^2) + a*(4*b*
c*(c*C + 2*B*d) - 3*a*d*(C*d + 2*c*D)))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(120*b^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2185, 2185, 27, 687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int \frac{(c+dx)^2\left(b(5Cd-6cD)x^2d^2+(5Abd-4acD)d^2+\left(-bDc^2+5bBd^2-4ad^2D\right)xd\right)}{\sqrt{bx^2+a}}dx}{5bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$
$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{bd^3(c+dx)^2\left(d(20Abd-15aCd+2acD)-\left(16aDd^2+b\left(-2Dc^2+5Cdc-20Bd^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4bd^2}+\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(5Cd-6cD)}{5bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{4}d\int \frac{(c+dx)^2\left(d(20Abd-15aCd+2acD)-\left(16aDd^2+b\left(-2Dc^2+5Cdc-20Bd^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx+\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(5Cd-6cD)}{5bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{1}{4}d\left(\frac{\int \frac{(c+dx)\left(d\left(60Acdb^2+a\left(32ad^2D-b\left(-2Dc^2+35Cdc+40Bd^2\right)\right)\right)-b\left(a(45Cd+26cD)d^2+2b\left(-2Dc^3+5Cdc^2-20Bd^2c-30Ad^3\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{\sqrt{a+bx^2}(c+d}{}\right.}{5bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{1}{4}d\left(\frac{\frac{15}{2}d^2\left(4Ab\left(2bc^2-ad^2\right)-a(4bc(2Bd+cC)-3ad(2cD+Cd))\right)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(16a^2d^4D-4abd^2\left(5Bd^2+3c^2D+10cCd\right)-b^2c\left(-60Ad^3-20B\right.\right.}{b}}{3b}\right.}{}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{4}d\left(\frac{\frac{15}{2}d^2\left(4Ab\left(2bc^2-ad^2\right)-a(4bc(2Bd+cC)-3ad(2cD+Cd))\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}\left(16a^2d^4D-4abd^2\left(5Bd^2+3c^2D+10cCd\right)-b^2c\left(-60\right.\right.}{b}}{3b}\right.}{}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}d\left(\frac{\frac{2\sqrt{a+bx^2}\left(16a^2d^4D-4abd^2\left(5Bd^2+3c^2D+10cCd\right)-b^2c\left(-60Ad^3-20Bcd^2-2c^3D+5c^2Cd\right)\right)}{b}+\frac{15d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(2bc^2-ad^2\right)-a(4bc(2B\cdots}{2\sqrt{b}}}{3b}\cdots\right.}{\cdots}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*(c + d*x)^4*Sqrt[a + b*x^2])/(5*b*d^2) + ((d*(5*C*d - 6*c*D)*(c + d*x)^
3*Sqrt[a + b*x^2])/4 + (d*(-1/3*((16*a*d^2*D + b*(5*c*C*d - 20*B*d^2 - 2*c
^2*D))*(c + d*x)^2*Sqrt[a + b*x^2])/b + ((2*(16*a^2*d^4*D - 4*a*b*d^2*(10*
c*C*d + 5*B*d^2 + 3*c^2*D) - b^2*c*(5*c^2*C*d - 20*B*c*d^2 - 60*A*d^3 - 2*
c^3*D))*Sqrt[a + b*x^2])/b - (d*(a*d^2*(45*C*d + 26*c*D) + 2*b*(5*c^2*C*d
- 20*B*c*d^2 - 30*A*d^3 - 2*c^3*D))*x*Sqrt[a + b*x^2])/2 + (15*d^2*(4*A*b*
(2*b*c^2 - a*d^2) - a*(4*b*c*(c*C + 2*B*d) - 3*a*d*(C*d + 2*c*D)))*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/(3*b)))/4)/(5*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02

method	result
default	$\frac{A\,c^{2} \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{c(2Ad+Bc)\sqrt{b\,x^{2}+a}}{b}+d(Cd+2Dc)\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right.$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+c*(2*A*d+B*c)/b*(b*x^2+a)^(1/2
)+d*(C*d+2*D*c)*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x/b*(b*x^2+a)^(1/2
)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d^2+2*B*c*d+C*c^2)*(1/2
*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+(B*d^2+2
*C*c*d+D*c^2)*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+D*d^2*
(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b
*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{15\left(4\left(Cab-2\,Ab^2\right)c^2-2\left(3\,Da^2-4\,Bab\right)cd-\left(3\,Ca^2-4\,Aab\right)d^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-\right.}{}\right.$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
[1/240*(15*(4*(C*a*b - 2*A*b^2)*c^2 - 2*(3*D*a^2 - 4*B*a*b)*c*d - (3*C*a^2
 - 4*A*a*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(24*D*b^2*d^2*x^4 + 30*(2*D*b^2*c*d + C*b^2*d^2)*x^3 - 40*(2*D*a*b - 3*
B*b^2)*c^2 - 80*(2*C*a*b - 3*A*b^2)*c*d + 16*(4*D*a^2 - 5*B*a*b)*d^2 + 8*(
5*D*b^2*c^2 + 10*C*b^2*c*d - (4*D*a*b - 5*B*b^2)*d^2)*x^2 + 15*(4*C*b^2*c^
2 - 2*(3*D*a*b - 4*B*b^2)*c*d - (3*C*a*b - 4*A*b^2)*d^2)*x)*sqrt(b*x^2 + a
))/b^3, 1/120*(15*(4*(C*a*b - 2*A*b^2)*c^2 - 2*(3*D*a^2 - 4*B*a*b)*c*d - (
3*C*a^2 - 4*A*a*b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (24*
D*b^2*d^2*x^4 + 30*(2*D*b^2*c*d + C*b^2*d^2)*x^3 - 40*(2*D*a*b - 3*B*b^2)*
c^2 - 80*(2*C*a*b - 3*A*b^2)*c*d + 16*(4*D*a^2 - 5*B*a*b)*d^2 + 8*(5*D*b^2
*c^2 + 10*C*b^2*c*d - (4*D*a*b - 5*B*b^2)*d^2)*x^2 + 15*(4*C*b^2*c^2 - 2*(
3*D*a*b - 4*B*b^2)*c*d - (3*C*a*b - 4*A*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Dd^2x^4}{5b}+\frac{x^3(Cd^2+2Dcd)}{4b}+\frac{x^2\left(Bd^2+2Ccd-\frac{4Dad^2}{5b}+Dc^2\right)}{3b}+\frac{x\left(Ad^2+2Bcd+Cc^2-\frac{3a\left(Cd^2+2Dcd\right)}{4b}\right)}{2b}+\frac{2Acd+Bc^2-\right. \\ \frac{Ac^2x+\frac{Dd^2x^6}{6}+\frac{x^5\left(Cd^2+2Dcd\right)}{5}+\frac{x^4\left(Bd^2+2Ccd+Dc^2\right)}{4}+\frac{x^3\left(Ad^2+2Bcd+Cc^2\right)}{3}+\frac{x^2\cdot\left(2Acd+Bc^2\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*d**2*x**4/(5*b) + x**3*(C*d**2 + 2*D*c*d)/(
4*b) + x**2*(B*d**2 + 2*C*c*d - 4*D*a*d**2/(5*b) + D*c**2)/(3*b) + x*(A*d*
*2 + 2*B*c*d + C*c**2 - 3*a*(C*d**2 + 2*D*c*d)/(4*b))/(2*b) + (2*A*c*d + B
*c**2 - 2*a*(B*d**2 + 2*C*c*d - 4*D*a*d**2/(5*b) + D*c**2)/(3*b))/b) + (A*
c**2 - a*(A*d**2 + 2*B*c*d + C*c**2 - 3*a*(C*d**2 + 2*D*c*d)/(4*b))/(2*b))
*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x
*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((A*c**2*x + D*d**2*x**6/6 + x**5
*(C*d**2 + 2*D*c*d)/5 + x**4*(B*d**2 + 2*C*c*d + D*c**2)/4 + x**3*(A*d**2
+ 2*B*c*d + C*c**2)/3 + x**2*(2*A*c*d + B*c**2)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dd^2x^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Dad^2x^2}{15\,b^2} + \frac{(2\,Dcd+Cd^2)\sqrt{bx^2+a}x^3}{4\,b} + \frac{Ac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}Bc^2}{b} + \frac{2\,\sqrt{bx^2+a}Acd}{b} + \frac{8\,\sqrt{bx^2+a}Da^2d^2}{15\,b^3} + \frac{(Dc^2+2\,Ccd+Bd^2)\sqrt{bx^2+a}x^2}{3\,b} - \frac{3\,(2\,Dcd+Cd^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}x}{2\,b} + \frac{3\,(2\,Dcd+Cd^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(Cc^2+2\,Bcd+Ad^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,(Dc^2+2\,Ccd+Bd^2)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
1/5*sqrt(b*x^2 + a)*D*d^2*x^4/b - 4/15*sqrt(b*x^2 + a)*D*a*d^2*x^2/b^2 + 1
/4*(2*D*c*d + C*d^2)*sqrt(b*x^2 + a)*x^3/b + A*c^2*arcsinh(b*x/sqrt(a*b))/
sqrt(b) + sqrt(b*x^2 + a)*B*c^2/b + 2*sqrt(b*x^2 + a)*A*c*d/b + 8/15*sqrt(
b*x^2 + a)*D*a^2*d^2/b^3 + 1/3*(D*c^2 + 2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*x
^2/b - 3/8*(2*D*c*d + C*d^2)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2*(C*c^2 + 2*B*c*
d + A*d^2)*sqrt(b*x^2 + a)*x/b + 3/8*(2*D*c*d + C*d^2)*a^2*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 1/2*(C*c^2 + 2*B*c*d + A*d^2)*a*arcsinh(b*x/sqrt(a*b))/b
^(3/2) - 2/3*(D*c^2 + 2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}} dx$$
$$= \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(\frac{4\,Dd^2x}{b}+\frac{5\,(2\,Db^4cd+Cb^4d^2)}{b^5}\right)x+\frac{4\,(5\,Db^4c^2+10\,Cb^4cd-4\,Dab^3d^2+5\,Bb^4\right.\right.\right.$$
$$+\frac{(4\,Cabc^2-8\,Ab^2c^2-6\,Da^2cd+8\,Babcd-3\,Ca^2d^2+4\,Aabd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*D*d^2*x/b + 5*(2*D*b^4*c*d + C*b^4*d^2)/b^
5)*x + 4*(5*D*b^4*c^2 + 10*C*b^4*c*d - 4*D*a*b^3*d^2 + 5*B*b^4*d^2)/b^5)*x
 + 15*(4*C*b^4*c^2 - 6*D*a*b^3*c*d + 8*B*b^4*c*d - 3*C*a*b^3*d^2 + 4*A*b^4
*d^2)/b^5)*x - 8*(10*D*a*b^3*c^2 - 15*B*b^4*c^2 + 20*C*a*b^3*c*d - 30*A*b^
4*c*d - 8*D*a^2*b^2*d^2 + 10*B*a*b^3*d^2)/b^5) + 1/8*(4*C*a*b*c^2 - 8*A*b^
2*c^2 - 6*D*a^2*c*d + 8*B*a*b*c*d - 3*C*a^2*d^2 + 4*A*a*b*d^2)*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}} dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{b\,x^2+a}} dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx+c)^2 (Dx^3+C\,x^2+Bx+A)}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

3.103 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 173

$$\begin{aligned}\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx &= \frac{(bBc+Abd-aCd-acD)\sqrt{a+bx^2}}{b^2}+\frac{(4b(cC+Bd)-3adD)x\sqrt{a+bx^2}}{8b^2}\\&\quad+\frac{dDx^3\sqrt{a+bx^2}}{4b}+\frac{(Cd+cD)\,(a+bx^2)^{3/2}}{3b^2}\\&\quad+\frac{(8Ab^2c-a(4b(cC+Bd)-3adD))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}\end{aligned}$$

output
```
(A*b*d+B*b*c-C*a*d-D*a*c)*(b*x^2+a)^(1/2)/b^2+1/8*(4*b*(B*d+C*c)-3*D*a*d)*
x*(b*x^2+a)^(1/2)/b^2+1/4*d*D*x^3*(b*x^2+a)^(1/2)/b+1/3*(C*d+D*c)*(b*x^2+a
)^(3/2)/b^2+1/8*(8*A*b^2*c-a*(4*b*(B*d+C*c)-3*D*a*d))*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-a(16Cd+16cD+9dDx)+2b(12Ad+6B(2c+dx)+x(6cC+4Cdx+4cDx+3dDx^2)))}{24b^2}$$
$$-\frac{(8Ab^2c+a(-4b(cC+Bd)+3adD))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*(16*C*d + 16*c*D + 9*d*D*x)) + 2*b*(12*A*d + 6*B*(2*
c + d*x) + x*(6*c*C + 4*C*d*x + 4*c*D*x + 3*d*D*x^2))))/(24*b^2) - ((8*A*b
^2*c + a*(-4*b*(c*C + B*d) + 3*a*d*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2185, 2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int \frac{(c+dx)\left(b(4Cd-5cD)x^2d^2+(4Abd-3acD)d^2+\left(-bDc^2+4bBd^2-3ad^2D\right)xd\right)}{\sqrt{bx^2+a}}dx}{4bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$
$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{bd^3(c+dx)\left(d(12Abd-8aCd+acD)-\left(9aDd^2+2b\left(-Dc^2+2Cdc-6Bd^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3bd^2}+\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(4Cd-5cD)}{4bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}d\int \frac{(c+dx)\left(d(12Abd-8aCd+acD)-(9aDd^2+2b(-Dc^2+2Cdc-6Bd^2))x\right)}{\sqrt{bx^2+a}}dx+\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(4Cd-5cD)}{4bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{3}d\left(\frac{3d^2\left(8Ab^2c-a(4b(Bd+cC)-3adD)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2\sqrt{a+bx^2}\left(4ad^2(cD+Cd)+b\left(-6Ad^3-6Bcd^2+c^3(-D)+2c^2Cd\right)\right)}{b}-\frac{dx\sqrt{a+bx^2}(9ad}{}\right.}{4bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{3}d\left(\frac{3d^2\left(8Ab^2c-a(4b(Bd+cC)-3adD)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2\sqrt{a+bx^2}\left(4ad^2(cD+Cd)+b\left(-6Ad^3-6Bcd^2+c^3(-D)+2c^2Cd\right)\right)}{b}-\frac{dx\sqrt{a-}}{}\right.}{4bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{3}d\left(\frac{3d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(8Ab^2c-a(4b(Bd+cC)-3adD)\right)}{2b^{3/2}}-\frac{2\sqrt{a+bx^2}\left(4ad^2(cD+Cd)+b\left(-6Ad^3-6Bcd^2+c^3(-D)+2c^2Cd\right)\right)}{b}-\frac{dx\sqrt{a-}}{}\right.}{4bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b*d^2) + ((d*(4*C*d - 5*c*D)*(c + d*x)^
2*Sqrt[a + b*x^2])/3 + (d*((-2*(4*a*d^2*(C*d + c*D) + b*(2*c^2*C*d - 6*B*c
*d^2 - 6*A*d^3 - c^3*D))*Sqrt[a + b*x^2])/b - (d*(9*a*d^2*D + 2*b*(2*c*C*d
 - 6*B*d^2 - c^2*D))*x*Sqrt[a + b*x^2])/(2*b) + (3*d^2*(8*A*b^2*c - a*(4*b
*(c*C + B*d) - 3*a*d*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
))/3)/(4*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

method	result
default	$\frac{Ac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{(Ad+Bc)\sqrt{b\,x^{2}+a}}{b}+(Bd+Cc)\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+(Cd+Dc)\Big($

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+(A*d+B*c)/b*(b*x^2+a)^(1/2)+(B*d
+C*c)*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
)+(C*d+D*c)*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+D*d*(1/4
*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$

$$=\left[\frac{3\left(4\left(Cab-2\,Ab^2\right)c-\left(3\,Da^2-4\,Bab\right)d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Db^2dx^3+8\,(D\right.}{48}\right.$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/48*(3*(4*(C*a*b - 2*A*b^2)*c - (3*D*a^2 - 4*B*a*b)*d)*sqrt(b)*log(-2*b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*D*b^2*d*x^3 + 8*(D*b^2*c + C
*b^2*d)*x^2 - 8*(2*D*a*b - 3*B*b^2)*c - 8*(2*C*a*b - 3*A*b^2)*d + 3*(4*C*b
^2*c - (3*D*a*b - 4*B*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/24*(3*(4*(C*a*b -
 2*A*b^2)*c - (3*D*a^2 - 4*B*a*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + (6*D*b^2*d*x^3 + 8*(D*b^2*c + C*b^2*d)*x^2 - 8*(2*D*a*b - 3*B*b^2
)*c - 8*(2*C*a*b - 3*A*b^2)*d + 3*(4*C*b^2*c - (3*D*a*b - 4*B*b^2)*d)*x)*s
qrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Ddx^3}{4b}+\frac{x^2(Cd+Dc)}{3b}+\frac{x\left(Bd+Cc-\frac{3Dad}{4b}\right)}{2b}+\frac{Ad+Bc-\frac{2a(Cd+Dc)}{3b}}{b}\right)+\left(Ac-\frac{a\left(Bd+Cc-\frac{3Dad}{4b}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\right.}{} \\ \frac{x\log\left(\right.}{\sqrt{bx^2}}\end{cases}\right.\right. \\ \frac{Acx+\frac{Ddx^5}{5}+\frac{x^4(Cd+Dc)}{4}+\frac{x^3(Bd+Cc)}{3}+\frac{x^2(Ad+Bc)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*d*x**3/(4*b) + x**2*(C*d + D*c)/(3*b) + x*(
B*d + C*c - 3*D*a*d/(4*b))/(2*b) + (A*d + B*c - 2*a*(C*d + D*c)/(3*b))/b)
+ (A*c - a*(B*d + C*c - 3*D*a*d/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)),
 Ne(b, 0)), ((A*c*x + D*d*x**5/5 + x**4*(C*d + D*c)/4 + x**3*(B*d + C*c)/3
 + x**2*(A*d + B*c)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Ddx^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Dadx}{8\,b^2} + \frac{\sqrt{bx^2+a}(Dc+Cd)x^2}{3\,b} + \frac{Ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{3\,Da^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}Bc}{b} + \frac{\sqrt{bx^2+a}Ad}{b} + \frac{\sqrt{bx^2+a}(Cc+Bd)x}{2\,b} - \frac{(Cc+Bd)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}(Dc+Cd)a}{3\,b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
1/4*sqrt(b*x^2 + a)*D*d*x^3/b - 3/8*sqrt(b*x^2 + a)*D*a*d*x/b^2 + 1/3*sqrt
(b*x^2 + a)*(D*c + C*d)*x^2/b + A*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/8*D
*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) + sqrt(b*x^2 + a)*B*c/b + sqrt(b*x^2
 + a)*A*d/b + 1/2*sqrt(b*x^2 + a)*(C*c + B*d)*x/b - 1/2*(C*c + B*d)*a*arcs
inh(b*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*(D*c + C*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Ddx}{b} + \frac{4\,(Db^3c+Cb^3d)}{b^4}\right)x + \frac{3\,(4\,Cb^3c-3\,Dab^2d+4\,Bb^3d)}{b^4}\right)x - \frac{8\,(2\,Dab^2c-3 \right. + \frac{(4\,Cabc-8\,Ab^2c-3\,Da^2d+4\,Babd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*D*d*x/b + 4*(D*b^3*c + C*b^3*d)/b^4)*x + 3*(4*
C*b^3*c - 3*D*a*b^2*d + 4*B*b^3*d)/b^4)*x - 8*(2*D*a*b^2*c - 3*B*b^3*c + 2
*C*a*b^2*d - 3*A*b^3*d)/b^4) + 1/8*(4*C*a*b*c - 8*A*b^2*c - 3*D*a^2*d + 4*
B*a*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx=\int \frac{(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{24\sqrt{b\,x^2+a}\,a\,b^2d-32\sqrt{b\,x^2+a}\,abcd-9\sqrt{b\,x^2+a}\,ab\,d^2x+24\sqrt{b\,x^2+a}\,b^3c+12\sqrt{b\,x^2+a}\,b^3dx+12\sqrt{}}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(24*sqrt(a + b*x**2)*a*b**2*d - 32*sqrt(a + b*x**2)*a*b*c*d - 9*sqrt(a + b
*x**2)*a*b*d**2*x + 24*sqrt(a + b*x**2)*b**3*c + 12*sqrt(a + b*x**2)*b**3*
d*x + 12*sqrt(a + b*x**2)*b**2*c**2*x + 16*sqrt(a + b*x**2)*b**2*c*d*x**2
+ 6*sqrt(a + b*x**2)*b**2*d**2*x**3 + 9*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a**2*d**2 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a*b**2*c - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a*b**2*d - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c
**2)/(24*b**3)
```

3.104 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{(bB-aD)\sqrt{a+bx^2}}{b^2}+\frac{Cx\sqrt{a+bx^2}}{2b}+\frac{D(a+bx^2)^{3/2}}{3b^2}+\frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
(B*b-D*a)*(b*x^2+a)^(1/2)/b^2+1/2*C*x*(b*x^2+a)^(1/2)/b+1/3*D*(b*x^2+a)^(3
/2)/b^2+1/2*(2*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(6bB-4aD+3bCx+2bDx^2)}{6b^2}+\frac{(-2Ab+aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(6*b*B - 4*a*D + 3*b*C*x + 2*b*D*x^2))/(6*b^2) + ((-2*A*b
 + a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2346, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{3bCx^2+(3bB-2aD)x+3Ab}{\sqrt{bx^2+a}}dx}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{b(3(2Ab-aC)+2(3bB-2aD)x)}{\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3(2Ab-aC)+2(3bB-2aD)x}{\sqrt{bx^2+a}}dx+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{3(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(D*x^2*Sqrt[a + b*x^2])/(3*b) + ((3*C*x*Sqrt[a + b*x^2])/2 + ((2*(3*b*B -
2*a*D)*Sqrt[a + b*x^2])/b + (3*(2*A*b - a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/Sqrt[b])/2)/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+C\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+D\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+\frac{B\sqrt{b\,x^{2}+a}}{b}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+C*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b
^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b
^2*(b*x^2+a)^(1/2))+B*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(Ca-2\,Ab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Dbx^2+3\,Cbx-4\,Da+6\,Bb)\sqrt{bx^2+a}}{12\,b^2},\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(C*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(2*D*b*x^2 + 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2, 1/6*
(3*(C*a - 2*A*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b*x^2
+ 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(A-\frac{Ca}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\sqrt{a+bx^2}\left(\frac{Cx}{2b}+\frac{Dx^2}{3b}+\frac{B-\frac{2Da}{3b}}{b}\right) & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - C*a/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + sqrt(a + b*x**2)*
(C*x/(2*b) + D*x**2/(3*b) + (B - 2*D*a/(3*b))/b), Ne(b, 0)), ((A*x + B*x**
2/2 + C*x**3/3 + D*x**4/4)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^2}{3\,b}+\frac{\sqrt{bx^2+a}Cx}{2\,b}-\frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{2\,\sqrt{bx^2+a}Da}{3\,b^2}+\frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*C*x/b - 1/2*C*a*arcsinh(
b*x/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/3*sqrt(b*x^2
 + a)*D*a/b^2 + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Dx}{b}+\frac{3\,C}{b}\right)x-\frac{2\,(2\,Dab-3\,Bb^2)}{b^3}\right) + \frac{(Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*D*x/b + 3*C/b)*x - 2*(2*D*a*b - 3*B*b^2)/b^3) + 1/
2*(C*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{B\,x^2}{2\,\sqrt{a}}+\frac{C\,x^3}{3\,\sqrt{a}}+\frac{x^4\,D}{4\,\sqrt{a}}+\frac{A\,x}{\sqrt{a}} & \text{if } b = \\ \frac{{\left(b\,x^2+a\right)}^{3/2}\,D-3\,a\,\sqrt{b\,x^2+a}\,D}{3\,b^2}+\frac{B\,\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{C\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{C\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b \neq \end{cases}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (B*x^2)/(2*a^(1/2)) + (C*x^3)/(3*a^(1/2)) + (x^4*D)/(4*a
^(1/2)) + (A*x)/a^(1/2), b ~= 0, ((a + b*x^2)^(3/2)*D - 3*a*(a + b*x^2)^(1
/2)*D)/(3*b^2) + (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^
(1/2)))/b^(1/2) - (C*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2))
 + (C*x*(a + b*x^2)^(1/2))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{-4\sqrt{b\,x^2+a}\,ad+6\sqrt{b\,x^2+a}\,b^2+3\sqrt{b\,x^2+a}\,bcx+2\sqrt{b\,x^2+a}\,bd\,x^2+6\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab-3\sqrt{b}}{6b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a*d + 6*sqrt(a + b*x**2)*b**2 + 3*sqrt(a + b*x**2)*
b*c*x + 2*sqrt(a + b*x**2)*b*d*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a*b - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*c)/(6*b**2)
```

3.105 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 190

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{(Cd-cD)\sqrt{a+bx^2}}{bd^2}+\frac{Dx\sqrt{a+bx^2}}{2bd} - \frac{(ad^2D+2b(cCd-Bd^2-c^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}d^3} - \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3\sqrt{bc^2+ad^2}}$$

output

```
(C*d-D*c)*(b*x^2+a)^(1/2)/b/d^2+1/2*D*x*(b*x^2+a)^(1/2)/b/d-1/2*(a*d^2*D+2
*b*(-B*d^2+C*c*d-D*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^3-(A
*d^3-B*c*d^2+C*c^2*d-D*c^3)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^
2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{d(2Cd-2cD+dDx)\sqrt{a+bx^2}}{b}+\frac{4(-c^2Cd+Bcd^2-Ad^3+c^3D)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+\frac{(ad^2D-2b(-cCd+Bd^2+c^2D))\log\left(-\sqrt{b}x+\right.}{b^{3/2}}}{2d^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((d*(2*C*d - 2*c*D + d*D*x)*Sqrt[a + b*x^2])/b + (4*(-(c^2*C*d) + B*c*d^2
- A*d^3 + c^3*D)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c
^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + ((a*d^2*D - 2*b*(-(c*C*d) + B*d^2
+ c^2*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(2*d^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{b(2Cd-3cD)x^2d^2+(2Abd-acD)d^2+\left(-bDc^2+2bBd^2-ad^2D\right)xd}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{bd^3\left(d(2Abd-acD)-\left(aDd^2+2b\left(-Dc^2+Cdc-Bd^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+d\sqrt{a+bx^2}(2Cd-3cD)}{2bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{d(2Abd-acD)-\left(aDd^2+2b\left(-Dc^2+Cdc-Bd^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx+d\sqrt{a+bx^2}(2Cd-3cD)}{2bd^3}+\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 719$$

$$\frac{d\left(\frac{2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2D+2b\left(-Bd^2+c^2(-D)+cCd\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)+d\sqrt{a+bx^2}(2Cd-3cL}{2bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{d\left(\frac{2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2D+2b\left(-Bd^2+c^2(-D)+cCd\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)+d\sqrt{a+bx^2}(2C}{2bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2D+2b\left(-Bd^2+c^2(-D)+cCd\right)\right)}{\sqrt{b}d}\right)+d\sqrt{a+bx^2}(2C}{2bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 488$$

$$\frac{d\left(-\frac{2b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2D+2b\left(-Bd^2+c^2(-D)+cCd\right)\right)}{\sqrt{b}d}\right)+d\sqrt{}}{2bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(-\frac{2b\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2D+2b(-Bd^2+c^2(-D)+cCd))}{\sqrt{b}d}\right)+d\sqrt{\cdots}}{2bd^3}$$

$$\frac{D\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(D*(c + d*x)*Sqrt[a + b*x^2])/(2*b*d^2) + (d*(2*C*d - 3*c*D)*Sqrt[a + b*x^
2] + d*(-(((a*d^2*D + 2*b*(c*C*d - B*d^2 - c^2*D))*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(Sqrt[b]*d)) - (2*b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Arc
Tanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 +
 a*d^2])))/(2*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{B\,d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{D c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{d(Cd-Dc)\sqrt{b\,x^{2}+a}}{b}+D d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)-\frac{Ccd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}\cdots}\right.}{\sqrt{b}}}{d^{3}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+D*c^2*ln(b^(1/2)*x+(b*x
^2+a)^(1/2))/b^(1/2)+d*(C*d-D*c)/b*(b*x^2+a)^(1/2)+D*d^2*(1/2*x/b*(b*x^2+a
)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-C*c*d*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))/b^(1/2))-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(170) = 340$.

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.85

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\sqrt{a + bx^2}}\,dx = \frac{\sqrt{bx^2 + a}Dx}{2\,bd} + \frac{Dc^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} - \frac{Cc \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} - \frac{Da \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d} + \frac{B \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d} - \frac{Dc^3 \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a + \frac{bc^2}{d^2}}d^4} + \frac{Cc^2 \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a + \frac{bc^2}{d^2}}d^3} - \frac{Bc \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a + \frac{bc^2}{d^2}}d^2} + \frac{A \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a + \frac{bc^2}{d^2}}d} - \frac{\sqrt{bx^2 + a}Dc}{bd^2} + \frac{\sqrt{bx^2 + a}C}{bd}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x/(b*d) + D*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3)
 - C*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 1/2*D*a*arcsinh(b*x/sqrt(a*b
))/(b^(3/2)*d) + B*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - D*c^3*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2
/d^2)*d^4) + C*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)
*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) - B*c*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) + A
*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(s
qrt(a + b*c^2/d^2)*d) - sqrt(b*x^2 + a)*D*c/(b*d^2) + sqrt(b*x^2 + a)*C/(b
*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2284, normalized size of antiderivative = 12.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**2*c*d + 2*sqrt(b)*sq
rt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*b**3*c**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2
+ b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/s
qrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b**2*d**3
 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*a*b**3*c**2*d + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
 - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**3*c*d**2 + 2*sqrt(2*
sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2
)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c
**2))*b**4*c**3 - sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2
+ 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c*
*2)*c + a*d**2 + 2*b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*a*b**2*c*d
+ sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt
(a*d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**...
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 219

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\sqrt{a+bx^2}}\,dx &= \frac{D\sqrt{a+bx^2}}{bd^2}-\frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\sqrt{a+bx^2}}{d^2\left(bc^2+ad^2\right)(c+dx)}\\ &\quad+\frac{(Cd-2cD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d^3}\\ &\quad+\frac{\left(ad^2(2cCd-Bd^2-3c^2D)+b(c^3Cd-Acd^3-2c^4D)\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3\left(bc^2+ad^2\right)^{3/2}}\end{aligned}$$

output
```
D*(b*x^2+a)^(1/2)/b/d^2-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^2/
(a*d^2+b*c^2)/(d*x+c)+(C*d-2*D*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/
2)/d^3+(a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*(-A*c*d^3+C*c^3*d-2*D*c^4))*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(3/
2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(ad^2D(c+dx)+b\left(Bcd^2-Ad^3+2c^3D+c^2(-Cd+dDx)\right)\right)}{b(bc^2+ad^2)(c+dx)} + \frac{2\left(ad^2\left(-2cCd+Bd^2+3c^2D\right)+b\left(-c^3Cd+Acd^3+2c^4D\right)\right)\arctan\left(\frac{\sqrt{b}(c+dx}{\sqrt{-b}}\right.}{\left(-bc^2-ad^2\right)^{3/2}}}{d^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*D*(c + d*x) + b*(B*c*d^2 - A*d^3 + 2*c^3*D + c^
2*(-(C*d) + d*D*x))))/(b*(b*c^2 + a*d^2)*(c + d*x)) + (2*(a*d^2*(-2*c*C*d
+ B*d^2 + 3*c^2*D) + b*(-(c^3*C*d) + A*c*d^3 + 2*c^4*D))*ArcTan[(Sqrt[b]*(
c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^
(3/2) + ((-(C*d) + 2*c*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d^
3
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2182, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}(c+dx)^2}\,dx$$
$$\Big\downarrow \text{2182}$$
$$-\frac{\int -\frac{\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(\frac{bc^2}{d}+ad\right)Dx^2+\frac{\left(bc^2+ad^2\right)(Cd-cD)x}{d^2}+Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{b\left(d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(bc^2+ad^2\right)(Cd-2cD)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+D\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{d\left(Abcd-a\left(-Dc^2+Cdc-Bd^2\right)\right)+\left(bc^2+ad^2\right)(Cd-2cD)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}+D\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{\frac{\left(ad^2+bc^2\right)(Cd-2cD)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+b\left(-Acd^3-2c^4D+c^3Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}}{d^2}+D\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{\left(ad^2+bc^2\right)(Cd-2cD)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+b\left(-Acd^3-2c^4D+c^3Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}}{d^2}+D\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)(Cd-2cD)}{\sqrt{b}d}-\frac{\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+b\left(-Acd^3-2c^4D+c^3Cd\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}}{d^2}+D\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{\frac{\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+b\left(-Acd^3-2c^4D+c^3Cd\right)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)(Cd-2cD)}{\sqrt{b}d}}{d^2}+D\sqrt{a+bx^2}}{ad^2+bc^2}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(-Bd^2-3c^2D+2cCd\right)+b\left(-Acd^3-2c^4D+c^3Cd\right)\right)}{d\sqrt{ad^2+bc^2}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)(Cd-2cD)}{\sqrt{b}d}}{d^2}+D\sqrt{a+bx^2}}{ad^2+bc^2}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-(((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^
2)*(c + d*x))) + ((a/b + c^2/d^2)*D*Sqrt[a + b*x^2] + (((b*c^2 + a*d^2)*(C
*d - 2*c*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + ((a*d^2*(2
*c*C*d - B*d^2 - 3*c^2*D) + b*(c^3*C*d - A*c*d^3 - 2*c^4*D))*ArcTanh[(a*d
- b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/
d^2)/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(203) = 406$.

Time = 1.47 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.05

method	result
default	$\frac{\frac{Cd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{Dd\sqrt{b\,x^{2}+a}}{b}-\frac{2Dc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}}{d^{3}}-\frac{\left(B\,d^{2}-2Ccd+3Dc^{2}\right)\ln\left(\frac{\frac{2a\,d^{2}+2b\,c^{2}}{d^{2}}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}\,\sqrt{b}}{x+\frac{c}{d}}\right.}{d^{4}\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(C*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+D*d/b*(b*x^2+a)^(1/2)-2*D
*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))-1/d^4*(B*d^2-2*C*c*d+3*D*c^2)/((
a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+
b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(
x+c/d))+1/d^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/
((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^
2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
/(x+c/d))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx^2}\,(c + dx)^2}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(205) = 410$.

Time = 0.08 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dc^3}{bc^2d^3x+ad^5x+bc^3d^2+acd^4} - \frac{\sqrt{bx^2+a}Cc^2}{bc^2d^2x+ad^4x+bc^3d+acd^3} + \frac{\sqrt{bx^2+a}Bc}{bc^2dx+ad^3x+bc^3+acd^2} - \frac{\sqrt{bx^2+a}A}{bc^2x+ad^2x+\frac{bc^3}{d}+acd} - \frac{2\,Dc\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} + \frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} - \frac{Dbc^4\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^6} + \frac{Cbc^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5} + \frac{3\,Dc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} - \frac{Bbc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} - \frac{2\,Cc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} + \frac{Abc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3} + \frac{B\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2} + \frac{\sqrt{bx^2+a}D}{bd^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
sqrt(b*x^2 + a)*D*c^3/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) - sqrt
(b*x^2 + a)*C*c^2/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) + sqrt(b*x^2
 + a)*B*c/(b*c^2*d*x + a*d^3*x + b*c^3 + a*c*d^2) - sqrt(b*x^2 + a)*A/(b*c
^2*x + a*d^2*x + b*c^3/d + a*c*d) - 2*D*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*
d^3) + C*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - D*b*c^4*arcsinh(b*c*x/(sqr
t(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2
)*d^6) + C*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*a
bs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) + 3*D*c^2*arcsinh(b*c*x/(sqrt(a*
b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^4)
 - B*b*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x
 + c)))/((a + b*c^2/d^2)^(3/2)*d^4) - 2*C*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d
*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) + A*b*c
*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((
a + b*c^2/d^2)^(3/2)*d^3) + B*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d
/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) + sqrt(b*x^2 + a)*D/(
b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2\sqrt{a + bx^2}}\, dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2\sqrt{a + bx^2}}\, dx = \int \frac{A + B\,x + C\,x^2 + x^3\,D}{\sqrt{b\,x^2 + a}\,{(c + d\,x)}^2}\, dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1213, normalized size of antiderivative = 5.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a*b**2*c**2*d**2 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x + 2*sqrt(a*d**2 + b*c*
*2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**
3 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b**2*d**4*x + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 + 2*sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**3*
x + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*b**2*c**5 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x - 2*sqrt(a*d**2 + b*c**2)*lo
g(c + d*x)*a*b**2*c**2*d**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*
c*d**3*x - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3 - 2*sqrt(a*d
**2 + b*c**2)*log(c + d*x)*a*b**2*d**4*x - 2*sqrt(a*d**2 + b*c**2)*log(c +
 d*x)*a*b*c**3*d**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**3*x
 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**5 - 2*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*b**2*c**4*d*x - 2*sqrt(a + b*x**2)*a**2*b*d**5 + 2*sqrt(a +
 b*x**2)*a**2*c*d**5 + 2*sqrt(a + b*x**2)*a**2*d**6*x - 2*sqrt(a + b*x**2)
*a*b**2*c**2*d**3 + 2*sqrt(a + b*x**2)*a*b**2*c*d**4 + 4*sqrt(a + b*x**2)*
a*b*c**3*d**3 + 4*sqrt(a + b*x**2)*a*b*c**2*d**4*x + 2*sqrt(a + b*x**2)...
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 313

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx &= -\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{2d^2\,(bc^2+ad^2)\,(c+dx)^2}\\
&+\frac{(2ad^2(2cCd-Bd^2-3c^2D)+bc(c^2Cd+Bcd^2-3Ad^3-3c^3D))\sqrt{a+bx^2}}{2d^2\,(bc^2+ad^2)^2\,(c+dx)}\\
&+\frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d^3}\\
&-\frac{(Abd^3(2bc^2-ad^2)-2b^2c^5D+2a^2d^4(Cd-3cD)-abcd^2(cCd-3Bd^2+5c^2D))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+}}\right.}{2d^3\,(bc^2+ad^2)^{5/2}}\end{aligned}$$

output
```
-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/d^2/(a*d^2+b*c^2)/(d*x+
c)^2+1/2*(2*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)+b*c*(-3*A*d^3+B*c*d^2+C*c^2*d-3
*D*c^3))*(b*x^2+a)^(1/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)+D*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(1/2)/d^3-1/2*(A*b*d^3*(-a*d^2+2*b*c^2)-2*b^2*c^5*D+2*a^
2*d^4*(C*d-3*D*c)-a*b*c*d^2*(-3*B*d^2+C*c*d+5*D*c^2))*arctanh((-b*c*x+a*d)
/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx =$$

$$-\frac{\frac{d\sqrt{a+bx^2}\left(ad^2\left(5c^3D+d^3(A+2Bx)+cd^2(B-4Cx)-3c^2d(C-2Dx)\right)+bc\left(-Bcd^2(2c+dx)+Ad^3(4c+3dx)+c^2\left(2c^2D-Cd^2x+3cdDx\right)\right)\right)}{\left(bc^2+ad^2\right)^2(c+dx)^2}-\frac{2}{}}{2a}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*((d*Sqrt[a + b*x^2]*(a*d^2*(5*c^3*D + d^3*(A + 2*B*x) + c*d^2*(B - 4*
C*x) - 3*c^2*d*(C - 2*D*x)) + b*c*(-(B*c*d^2*(2*c + d*x)) + A*d^3*(4*c + 3
*d*x) + c^2*(2*c^2*D - C*d^2*x + 3*c*d*D*x))))/((b*c^2 + a*d^2)^2*(c + d*x
)^2) - (2*(A*b*d^3*(-2*b*c^2 + a*d^2) + 2*b^2*c^5*D - 2*a^2*d^4*(C*d - 3*c
*D) + a*b*c*d^2*(c*C*d - 3*B*d^2 + 5*c^2*D))*ArcTan[(Sqrt[b]*(c + d*x) - d
*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2) + (2*D
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d^3
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2182, 25, 2182, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}(c+dx)^3}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(a(2Cd-2cD)+b\left(-\frac{Dc^3}{d^2}+\frac{Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2\left(\frac{bc^2}{d}+ad\right)Dx^2+\left(2a(Cd-cD)+b\left(-\frac{Dc^3}{d^2}+\frac{Cc^2}{d}+Bc-Ad\right)\right)x+2\left(Abc-a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3+Bcd^2-3c^3D+c^2Cd\right)\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(2d(Cd-2cD)a^2-\frac{bc\left(Dc^2+Cdc-3Bd^2\right)a}{d}+Ab\left(2bc^2-ad^2\right)\right)d^2+2\left(bc\right.}{d^2(c+dx)\sqrt{bx^2+a}}}{ad^2+bc^2}}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2Dx\left(bc^2+ad^2\right)^2+d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2-ad^2\right)d-abc\left(Dc^2+Cdc-3Bd^2\right)\right)}{d^2(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3+Bcd^2-3c\right.\right.}{d^2(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2Dx\left(bc^2+ad^2\right)^2+d\left(2a^2(Cd-2cD)d^2+Ab\left(2bc^2-ad^2\right)d-abc\left(Dc^2+Cdc-3Bd^2\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{d^2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bd^2-3c^2D+2cCd\right)+bc\left(-3Ad^3+Bcd^2-3c\right.\right.}{d^2(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\frac{\left(2a^2d^4(Cd-3cD)+Abd^3\left(2bc^2-ad^2\right)-abcd^2\left(-3Bd^2+5c^2D+cCd\right)-2b^2c^5D\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2D\left(ad^2+bc^2\right)^2\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2\left(ad^2+bc^2\right)}+\sqrt{a+bx^2}\left(2ad^2\left(-Bd\right.\right.}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\left(2a^2d^4(Cd-3cD)+Abd^3\left(2bc^2-ad^2\right)-abcd^2\left(-3Bd^2+5c^2D+cCd\right)-2b^2c^5D\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2D\left(ad^2+bc^2\right)^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(2a}{}}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\left(2a^2d^4(Cd-3cD)+Abd^3\left(2bc^2-ad^2\right)-abcd^2\left(-3Bd^2+5c^2D+cCd\right)-2b^2c^5D\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{2D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}}{d^2(ad^2+bc^2)}+\frac{\sqrt{a+bx^2}(2a}{}}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\frac{2D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}-\frac{\left(2a^2d^4(Cd-3cD)+Abd^3\left(2bc^2-ad^2\right)-abcd^2\left(-3Bd^2+5c^2D+cCd\right)-2b^2c^5D\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}}{d^2(ad^2+bc^2)}+}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)^2}{\sqrt{b}d}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(2a^2d^4(Cd-3cD)+Abd^3\left(2bc^2-ad^2\right)-abcd^2\left(-3Bd^2+5c^2D+cCd\right)-2b^2c^5D\right)}{d\sqrt{ad^2+bc^2}}}{d^2(ad^2+bc^2)}+}{2\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a
*d^2)*(c + d*x)^2) + (((2*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) + b*c*(c^2*C*d
 + B*c*d^2 - 3*A*d^3 - 3*c^3*D))*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c
+ d*x)) + ((2*(b*c^2 + a*d^2)^2*D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(S
qrt[b]*d) - ((A*b*d^3*(2*b*c^2 - a*d^2) - 2*b^2*c^5*D + 2*a^2*d^4*(C*d - 3
*c*D) - a*b*c*d^2*(c*C*d - 3*B*d^2 + 5*c^2*D))*ArcTanh[(a*d - b*c*x)/(Sqrt
[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/(d^2*(b*c^2 +
a*d^2)))/(2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(291) = 582$.

Time = 1.46 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.74

method	result
default	$\frac{D\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{d^{3}\sqrt{b}}-\frac{(Cd-3Dc)\ln\left(\frac{\frac{2a\,d^{2}+2b\,c^{2}}{d^{2}}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}{x+\frac{c}{d}}\right)}{d^{4}\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}+\frac{(B\,d^{2}-2C}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
D/d^3*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/d^4*(C*d-3*D*c)/((a*d^2+b*c^
2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2
)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+1/
d^5*(B*d^2-2*C*c*d+3*D*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/d^6
*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(
a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2
-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)
/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^
(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 \sqrt{a + bx^2}} \, dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 \sqrt{a + bx^2}} \, dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx^2} \, (c + dx)^3} \, dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x**2)*(c + d*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(294) = 588.

Time = 0.10 (sec) , antiderivative size = 1281, normalized size of antiderivative = 4.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
3/2*sqrt(b*x^2 + a)*D*b*c^4/(b^2*c^4*d^3*x + 2*a*b*c^2*d^5*x + a^2*d^7*x +
 b^2*c^5*d^2 + 2*a*b*c^3*d^4 + a^2*c*d^6) - 3/2*sqrt(b*x^2 + a)*C*b*c^3/(b
^2*c^4*d^2*x + 2*a*b*c^2*d^4*x + a^2*d^6*x + b^2*c^5*d + 2*a*b*c^3*d^3 + a
^2*c*d^5) + 3/2*sqrt(b*x^2 + a)*B*b*c^2/(b^2*c^4*d*x + 2*a*b*c^2*d^3*x + a
^2*d^5*x + b^2*c^5 + 2*a*b*c^3*d^2 + a^2*c*d^4) + 1/2*sqrt(b*x^2 + a)*D*c^
3/(b*c^2*d^4*x^2 + a*d^6*x^2 + 2*b*c^3*d^3*x + 2*a*c*d^5*x + b*c^4*d^2 + a
*c^2*d^4) - 3/2*sqrt(b*x^2 + a)*A*b*c/(b^2*c^4*x + 2*a*b*c^2*d^2*x + a^2*d
^4*x + b^2*c^5/d + 2*a*b*c^3*d + a^2*c*d^3) - 1/2*sqrt(b*x^2 + a)*C*c^2/(b
*c^2*d^3*x^2 + a*d^5*x^2 + 2*b*c^3*d^2*x + 2*a*c*d^4*x + b*c^4*d + a*c^2*d
^3) - 3*sqrt(b*x^2 + a)*D*c^2/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4
) + 1/2*sqrt(b*x^2 + a)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2 + 2*b*c^3*d*x + 2*a
*c*d^3*x + b*c^4 + a*c^2*d^2) + 2*sqrt(b*x^2 + a)*C*c/(b*c^2*d^2*x + a*d^4
*x + b*c^3*d + a*c*d^3) - 1/2*sqrt(b*x^2 + a)*A/(b*c^2*d*x^2 + a*d^3*x^2 +
 2*b*c^3*x + 2*a*c*d^2*x + b*c^4/d + a*c^2*d) - sqrt(b*x^2 + a)*B/(b*c^2*d
*x + a*d^3*x + b*c^3 + a*c*d^2) + D*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) -
 3/2*D*b^2*c^5*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs
(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^8) + 3/2*C*b^2*c^4*arcsinh(b*c*x/(sqr
t(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2
)*d^7) + 7/2*D*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*
b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^6) - 3/2*B*b^2*c^3*arcsinh(b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(294) = 588$.

Time = 0.27 (sec) , antiderivative size = 1159, normalized size of antiderivative = 3.70

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 \sqrt{a + bx^2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
-(2*D*b^2*c^5 + 5*D*a*b*c^3*d^2 + C*a*b*c^2*d^3 - 2*A*b^2*c^2*d^3 + 6*D*a^
2*c*d^4 - 3*B*a*b*c*d^4 - 2*C*a^2*d^5 + A*a*b*d^5)*arctan(-((sqrt(b)*x - s
qrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^2*c^4*d^3 + 2*a*b
*c^2*d^5 + a^2*d^7)*sqrt(-b*c^2 - a*d^2)) - D*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/(sqrt(b)*d^3) - (4*(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*b^2*c^5*d
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*b^2*c^4*d^2 + 7*(sqrt(b)*x - sqrt(b*
x^2 + a))^3*D*a*b*c^3*d^3 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a*b*c^2*d^
4 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b^2*c^2*d^4 + 3*(sqrt(b)*x - sqrt(
b*x^2 + a))^3*B*a*b*c*d^5 - (sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b*d^6 + 6*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*b^(5/2)*c^6 - 2*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*C*b^(5/2)*c^5*d + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a*b^(3/2)*c^
4*d^2 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*b^(5/2)*c^4*d^2 - 7*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*C*a*b^(3/2)*c^3*d^3 + 6*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*b^(5/2)*c^3*d^3 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^2*sqrt(b)*c^
2*d^4 + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*b^(3/2)*c^2*d^4 + 4*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b)*c*d^5 - 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a*b^(3/2)*c*d^5 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*
d^6 - 8*(sqrt(b)*x - sqrt(b*x^2 + a))*D*a*b^2*c^5*d + 2*(sqrt(b)*x - sqrt(
b*x^2 + a))*C*a*b^2*c^4*d^2 - 17*(sqrt(b)*x - sqrt(b*x^2 + a))*D*a^2*b*c^3
*d^3 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a*b^2*c^3*d^3 + 11*(sqrt(b)*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(1/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2742, normalized size of antiderivative = 8.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**2*c**2*d**4 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x + sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
**2*b**2*d**6*x**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**4 + 8*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**2*
d**5*x + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*a**2*b*c*d**6*x**2 - 2*sqrt(a*d**2 + b*c**2)*log( - sq
rt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**2 - 4*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a*b**3*c**3*d**3*x - 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d**3 - 2*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*
*3*c**2*d**4*x**2 - 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x - 3*sqrt(a*d**2 + b*c**2
)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c*d*
*5*x**2 + 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*a*b**2*c**5*d**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sq
rt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d**3*x ...
```

3.108 $\int \frac{(c+dx)^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 408

$$
\begin{aligned}
&\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^{3/2}}\,dx = \\
&-\frac{b^2c^2(Bc+3Ad)+a^2d^2(Cd+3cD)-ab(3c^2Cd+3Bcd^2+Ad^3+c^3D)}{b^3\sqrt{a+bx^2}} \\
&+\frac{(Ab^2c(bc^2-3ad^2)-a(b^2c^2(cC+3Bd)+a^2d^3D-abd(3cCd+Bd^2+3c^2D)))\,x}{ab^3\sqrt{a+bx^2}} \\
&-\frac{(2ad^2(Cd+3cD)-b(3c^2Cd+3Bcd^2+Ad^3+c^3D))\sqrt{a+bx^2}}{b^3} \\
&-\frac{d(7ad^2D-4b(3cCd+Bd^2+3c^2D))\,x\sqrt{a+bx^2}}{8b^3} \\
&+\frac{d^3Dx^3\sqrt{a+bx^2}}{4b^2}+\frac{d^2(Cd+3cD)\left(a+bx^2\right)^{3/2}}{3b^3} \\
&+\frac{(8b^2c(c^2C+3Bcd+3Ad^2)+15a^2d^3D-12abd(3cCd+Bd^2+3c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}
\end{aligned}
$$

output
```
-(b^2*c^2*(3*A*d+B*c)+a^2*d^2*(C*d+3*D*c)-a*b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D
*c^3))/b^3/(b*x^2+a)^(1/2)+(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(3*B*d+C*c
)+a^2*d^3*D-a*b*d*(B*d^2+3*C*c*d+3*D*c^2)))*x/a/b^3/(b*x^2+a)^(1/2)-(2*a*d
^2*(C*d+3*D*c)-b*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3))*(b*x^2+a)^(1/2)/b^3-1/
8*d*(7*a*d^2*D-4*b*(B*d^2+3*C*c*d+3*D*c^2))*x*(b*x^2+a)^(1/2)/b^3+1/4*d^3*
D*x^3*(b*x^2+a)^(1/2)/b^2+1/3*d^2*(C*d+3*D*c)*(b*x^2+a)^(3/2)/b^3+1/8*(8*b
^2*c*(3*A*d^2+3*B*c*d+C*c^2)+15*a^2*d^3*D-12*a*b*d*(B*d^2+3*C*c*d+3*D*c^2)
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{24Ab^3c^3x - a^3d^2(64Cd + 192cD + 45dDx) + 2ab^2(12Ad(-3c^2}{}$$

$$-\frac{\left(8b^2c(c^2C + 3Bcd + 3Ad^2) + 15a^2d^3D - 12abd(3cCd + Bd^2 + 3c^2D)\right)\log\left(-\sqrt{b}x + \sqrt{a+bx^2}\right)}{8b^{7/2}}$$

input
```
Integrate[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(24*A*b^3*c^3*x - a^3*d^2*(64*C*d + 192*c*D + 45*d*D*x) + 2*a*b^2*(12*A*d*
(-3*c^2 - 3*c*d*x + d^2*x^2) - 6*B*(2*c^3 + 6*c^2*d*x - 6*c*d^2*x^2 - d^3*
x^3) + x*(-12*c^3*(C - D*x) + 18*c^2*d*x*(2*C + D*x) + 6*c*d^2*x^2*(3*C +
2*D*x) + d^3*x^3*(4*C + 3*D*x))) + a^2*b*(48*c^3*D + 36*c^2*d*(4*C + 3*D*x
) + 12*c*d^2*(12*B + x*(9*C - 8*D*x)) + d^3*(48*A + x*(36*B - 32*C*x - 15*
D*x^2))))/(24*a*b^3*Sqrt[a + b*x^2]) - ((8*b^2*c*(c^2*C + 3*B*c*d + 3*A*d^
2) + 15*a^2*d^3*D - 12*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D))*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2176, 25, 2185, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(c+dx)^2\left(adDx^2-(3Abd-4aCd-acD)x+a\left(cC+3d\left(B-\frac{aD}{b}\right)\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(c+dx)^2\left(adDx^2-(3Abd-4aCd-acD)x+a\left(cC+3d\left(B-\frac{aD}{b}\right)\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{d^2(c+dx)^2(a(4b(cC+3Bd)-15adD)-b(12Abd-16aCd-3acD)x)}{\sqrt{bx^2+a}}dx}{4bd^2}+\frac{aD\sqrt{a+bx^2}(c+dx)^3}{4b}}{ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(c+dx)^2(a(4b(cC+3Bd)-15adD)-b(12Abd-16aCd-3acD)x)}{\sqrt{bx^2+a}}dx}{4b}+\frac{aD\sqrt{a+bx^2}(c+dx)^3}{4b}}{ab}-\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\frac{\int \frac{b(c+dx)\left(a\left(12b\left(Cc^2+3Bdc+2Ad^2\right)-ad(32Cd+51cD)\right)-\left(24Acdb^2+a\left(45ad^2D-2b\left(3Dc^2+22Cdc+18Bd^2\right)\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(-3acD-16aCd+}{4b}}{ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}\int \frac{(c+dx)\left(a\left(12b\left(Cc^2+3Bdc+2Ad^2\right)-ad(32Cd+51cD)\right)-\left(24Acdb^2+a\left(45ad^2D-b\left(6Dc^2+44Cdc+36Bd^2\right)\right)\right)x\right)}{\sqrt{bx^2+a}}dx-\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(-3acD-16aCd-}{4b}}{ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{\frac{1}{3}\left(\frac{3a\left(15a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}\left(a\left(45ad^2D-b\left(36Bd^2+6c^2D+44cCd\right)\right)+24Ab^2cd\right)}{2b}-\frac{2\sqrt{a}}{}\right.}{4b}}{ab}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{3}\left(\frac{3a\left(15a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}\left(a\left(45ad^2D-b\left(36Bd^2+6c^2D+44cCd\right)\right)+24Ab^2c\right.}{2b}\right.}{4b}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{3}\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^3D-12abd\left(Bd^2+3c^2D+3cCd\right)+8b^2c\left(3Ad^2+3Bcd+c^2C\right)\right)}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(a\left(45ad^2D-b\left(36Bd^2+6c^2D+44cCd\right)\right)+24Ab^2c\right.}{2b}\right.}{4b}$$

$$\frac{(c+dx)^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^3)/(a*b*Sqrt[a + b*x^2])) +
 ((a*D*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b) + (-1/3*((12*A*b*d - 16*a*C*d -
3*a*c*D)*(c + d*x)^2*Sqrt[a + b*x^2]) + ((-2*(12*A*b*d*(b*c^2 - a*d^2) + a
*(16*a*d^2*(C*d + 3*c*D) - b*c*(28*c*C*d + 36*B*d^2 + 3*c^2*D)))*Sqrt[a +
b*x^2])/b - (d*(24*A*b^2*c*d + a*(45*a*d^2*D - b*(44*c*C*d + 36*B*d^2 + 6*
c^2*D)))*x*Sqrt[a + b*x^2])/(2*b) + (3*a*(8*b^2*c*(c^2*C + 3*B*c*d + 3*A*d
^2) + 15*a^2*d^3*D - 12*a*b*d*(3*c*C*d + B*d^2 + 3*c^2*D))*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/3)/(4*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.97

method	result
default	$\frac{A\,c^{3}x}{\sqrt{b\,x^{2}+a}\,a}-\frac{c^{2}(3Ad+Bc)}{b\sqrt{b\,x^{2}+a}}+d^{2}(Cd+3Dc)\left(\frac{x^{4}}{3b\sqrt{b\,x^{2}+a}}-\frac{4a\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)}{3b}\right)+c(3A\,d^{2}+3Bcd$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3/(b*x^2+a)^(1/2)/a*x-c^2*(3*A*d+B*c)/b/(b*x^2+a)^(1/2)+d^2*(C*d+3*D*c
)*(1/3*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2
+a)^(1/2)))+c*(3*A*d^2+3*B*c*d+C*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2)))+d*(B*d^2+3*C*c*d+3*D*c^2)*(1/2*x^3/b/(b*x^2+a)^
(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)))+(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3)*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^
2+a)^(1/2))+D*d^3*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^3/b/(b*x^2+a)^
(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^3\,(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fric
as")
```

output
```
[-1/48*(3*(8*C*a^2*b^2*c^3 - 12*(3*D*a^3*b - 2*B*a^2*b^2)*c^2*d - 12*(3*C*
a^3*b - 2*A*a^2*b^2)*c*d^2 + 3*(5*D*a^4 - 4*B*a^3*b)*d^3 + (8*C*a*b^3*c^3
- 12*(3*D*a^2*b^2 - 2*B*a*b^3)*c^2*d - 12*(3*C*a^2*b^2 - 2*A*a*b^3)*c*d^2
+ 3*(5*D*a^3*b - 4*B*a^2*b^2)*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^
2 + a)*sqrt(b)*x - a) - 2*(6*D*a*b^3*d^3*x^5 + 8*(3*D*a*b^3*c*d^2 + C*a*b^
3*d^3)*x^4 + 24*(2*D*a^2*b^2 - B*a*b^3)*c^3 + 72*(2*C*a^2*b^2 - A*a*b^3)*c
^2*d - 48*(4*D*a^3*b - 3*B*a^2*b^2)*c*d^2 - 16*(4*C*a^3*b - 3*A*a^2*b^2)*d
^3 + 3*(12*D*a*b^3*c^2*d + 12*C*a*b^3*c*d^2 - (5*D*a^2*b^2 - 4*B*a*b^3)*d^
3)*x^3 + 8*(3*D*a*b^3*c^3 + 9*C*a*b^3*c^2*d - 3*(4*D*a^2*b^2 - 3*B*a*b^3)*
c*d^2 - (4*C*a^2*b^2 - 3*A*a*b^3)*d^3)*x^2 - 3*(8*(C*a*b^3 - A*b^4)*c^3 -
12*(3*D*a^2*b^2 - 2*B*a*b^3)*c^2*d - 12*(3*C*a^2*b^2 - 2*A*a*b^3)*c*d^2 +
3*(5*D*a^3*b - 4*B*a^2*b^2)*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)
, -1/24*(3*(8*C*a^2*b^2*c^3 - 12*(3*D*a^3*b - 2*B*a^2*b^2)*c^2*d - 12*(3*C
*a^3*b - 2*A*a^2*b^2)*c*d^2 + 3*(5*D*a^4 - 4*B*a^3*b)*d^3 + (8*C*a*b^3*c^3
 - 12*(3*D*a^2*b^2 - 2*B*a*b^3)*c^2*d - 12*(3*C*a^2*b^2 - 2*A*a*b^3)*c*d^2
 + 3*(5*D*a^3*b - 4*B*a^2*b^2)*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (6*D*a*b^3*d^3*x^5 + 8*(3*D*a*b^3*c*d^2 + C*a*b^3*d^3)*x^4 +
24*(2*D*a^2*b^2 - B*a*b^3)*c^3 + 72*(2*C*a^2*b^2 - A*a*b^3)*c^2*d - 48*(4*
D*a^3*b - 3*B*a^2*b^2)*c*d^2 - 16*(4*C*a^3*b - 3*A*a^2*b^2)*d^3 + 3*(12*D*
a*b^3*c^2*d + 12*C*a*b^3*c*d^2 - (5*D*a^2*b^2 - 4*B*a*b^3)*d^3)*x^3 + 8...
```

Sympy [F]

$$\int \frac{(c+dx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{\left(c+dx\right)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**3*(A + B*x + C*x**2 + D*x**3)/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{Dd^3x^5}{4\sqrt{bx^2+a}b} - \frac{5\,Dad^3x^3}{8\sqrt{bx^2+a}b^2} + \frac{Ac^3x}{\sqrt{bx^2+a}a} - \frac{15\,Da^2d^3x}{8\sqrt{bx^2+a}b^3} + \frac{(3\,Dcd^2+Cd^3)x^4}{3\sqrt{bx^2+a}b} + \frac{15\,Da^2d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{Bc^3}{\sqrt{bx^2+a}b} - \frac{3\,Ac^2d}{\sqrt{bx^2+a}b} + \frac{(3\,Dc^2d+3\,Ccd^2+Bd^3)x^3}{2\sqrt{bx^2+a}b} - \frac{4\,(3\,Dcd^2+Cd^3)ax^2}{3\sqrt{bx^2+a}b^2} + \frac{(Dc^3+3\,Cc^2d+3\,Bcd^2+Ad^3)x^2}{\sqrt{bx^2+a}b} + \frac{3\,(3\,Dc^2d+3\,Ccd^2+Bd^3)ax}{2\sqrt{bx^2+a}b^2} - \frac{(Cc^3+3\,Bc^2d+3\,Acd^2)x}{\sqrt{bx^2+a}b} - \frac{3\,(3\,Dc^2d+3\,Ccd^2+Bd^3)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{(Cc^3+3\,Bc^2d+3\,Acd^2)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{8\,(3\,Dcd^2+Cd^3)a^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,(Dc^3+3\,Cc^2d+3\,Bcd^2+Ad^3)a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

output
```
1/4*D*d^3*x^5/(sqrt(b*x^2 + a)*b) - 5/8*D*a*d^3*x^3/(sqrt(b*x^2 + a)*b^2)
+ A*c^3*x/(sqrt(b*x^2 + a)*a) - 15/8*D*a^2*d^3*x/(sqrt(b*x^2 + a)*b^3) + 1
/3*(3*D*c*d^2 + C*d^3)*x^4/(sqrt(b*x^2 + a)*b) + 15/8*D*a^2*d^3*arcsinh(b*
x/sqrt(a*b))/b^(7/2) - B*c^3/(sqrt(b*x^2 + a)*b) - 3*A*c^2*d/(sqrt(b*x^2 +
 a)*b) + 1/2*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*x^3/(sqrt(b*x^2 + a)*b) - 4/3
*(3*D*c*d^2 + C*d^3)*a*x^2/(sqrt(b*x^2 + a)*b^2) + (D*c^3 + 3*C*c^2*d + 3*
B*c*d^2 + A*d^3)*x^2/(sqrt(b*x^2 + a)*b) + 3/2*(3*D*c^2*d + 3*C*c*d^2 + B*
d^3)*a*x/(sqrt(b*x^2 + a)*b^2) - (C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*x/(sqrt(b
*x^2 + a)*b) - 3/2*(3*D*c^2*d + 3*C*c*d^2 + B*d^3)*a*arcsinh(b*x/sqrt(a*b)
)/b^(5/2) + (C*c^3 + 3*B*c^2*d + 3*A*c*d^2)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
 - 8/3*(3*D*c*d^2 + C*d^3)*a^2/(sqrt(b*x^2 + a)*b^3) + 2*(D*c^3 + 3*C*c^2*
d + 3*B*c*d^2 + A*d^3)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{3\,Dd^3x}{b}+\frac{4\,(3\,Dab^5cd^2+Cab^5d^3)}{ab^6}\right)x+\frac{3\,(12\,Dab^5c^2d+12\,Cab^5cd^2-5\,D}{ab^6}\right.\right.\right.}{}$$
$$-\frac{(8\,Cb^2c^3-36\,Dabc^2d+24\,Bb^2c^2d-36\,Cabcd^2+24\,Ab^2cd^2+15\,Da^2d^3-12\,Babd^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx}\right.\right.}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac
")
```

output
```
1/24*((((2*(3*D*d^3*x/b + 4*(3*D*a*b^5*c*d^2 + C*a*b^5*d^3)/(a*b^6))*x + 3
*(12*D*a*b^5*c^2*d + 12*C*a*b^5*c*d^2 - 5*D*a^2*b^4*d^3 + 4*B*a*b^5*d^3)/(
a*b^6))*x + 8*(3*D*a*b^5*c^3 + 9*C*a*b^5*c^2*d - 12*D*a^2*b^4*c*d^2 + 9*B*
a*b^5*c*d^2 - 4*C*a^2*b^4*d^3 + 3*A*a*b^5*d^3)/(a*b^6))*x - 3*(8*C*a*b^5*c
^3 - 8*A*b^6*c^3 - 36*D*a^2*b^4*c^2*d + 24*B*a*b^5*c^2*d - 36*C*a^2*b^4*c*
d^2 + 24*A*a*b^5*c*d^2 + 15*D*a^3*b^3*d^3 - 12*B*a^2*b^4*d^3)/(a*b^6))*x +
 8*(6*D*a^2*b^4*c^3 - 3*B*a*b^5*c^3 + 18*C*a^2*b^4*c^2*d - 9*A*a*b^5*c^2*d
 - 24*D*a^3*b^3*c*d^2 + 18*B*a^2*b^4*c*d^2 - 8*C*a^3*b^3*d^3 + 6*A*a^2*b^4
*d^3)/(a*b^6))/sqrt(b*x^2 + a) - 1/8*(8*C*b^2*c^3 - 36*D*a*b*c^2*d + 24*B*
b^2*c^2*d - 36*C*a*b*c*d^2 + 24*A*b^2*c*d^2 + 15*D*a^2*d^3 - 12*B*a*b*d^3)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^3 (Dx^3+C\,x^2+Bx+A)}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

3.109 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 274

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx &= -\frac{b^2c(Bc+2Ad)+a^2d^2D-ab(2cCd+Bd^2+c^2D)}{b^3\sqrt{a+bx^2}} \\ &+ \frac{(Ab(bc^2-ad^2)-a(bc(cC+2Bd)-ad(Cd+2cD)))\,x}{ab^2\sqrt{a+bx^2}} \\ &- \frac{(2ad^2D-b(2cCd+Bd^2+c^2D))\sqrt{a+bx^2}}{b^3} \\ &+ \frac{d(Cd+2cD)x\sqrt{a+bx^2}}{2b^2}+\frac{d^2D(a+bx^2)^{3/2}}{3b^3} \\ &+ \frac{(2b(c^2C+2Bcd+Ad^2)-3ad(Cd+2cD))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

output
```
-(b^2*c*(2*A*d+B*c)+a^2*d^2*D-a*b*(B*d^2+2*C*c*d+D*c^2))/b^3/(b*x^2+a)^(1/
2)+(A*b*(-a*d^2+b*c^2)-a*(b*c*(2*B*d+C*c)-a*d*(C*d+2*D*c)))*x/a/b^2/(b*x^2
+a)^(1/2)-(2*a*d^2*D-b*(B*d^2+2*C*c*d+D*c^2))*(b*x^2+a)^(1/2)/b^3+1/2*d*(C
*d+2*D*c)*x*(b*x^2+a)^(1/2)/b^2+1/3*d^2*D*(b*x^2+a)^(3/2)/b^3+1/2*(2*b*(A*
d^2+2*B*c*d+C*c^2)-3*a*d*(C*d+2*D*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b
^(5/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}} dx = \frac{-16a^3d^2D+6Ab^3c^2x+a^2b(12c^2D+6cd(4C+3Dx)+d^2(12B$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(-16*a^3*d^2*D + 6*A*b^3*c^2*x + a^2*b*(12*c^2*D + 6*c*d*(4*C + 3*D*x) + d
^2*(12*B + x*(9*C - 8*D*x))) + a*b^2*(-6*A*d*(2*c + d*x) - 6*B*(c^2 + 2*c*
d*x - d^2*x^2) + x*(-6*c^2*(C - D*x) + 6*c*d*x*(2*C + D*x) + d^2*x^2*(3*C
+ 2*D*x))) + 3*a*Sqrt[b]*(-2*b*(c^2*C + 2*B*c*d + A*d^2) + 3*a*d*(C*d + 2*
c*D))*Sqrt[a + b*x^2]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(6*a*b^3*Sqrt[a
 + b*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2176, 25, 2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(c+dx)\left(adDx^2-(2Abd-3aCd-acD)x+a\left(cC+2d\left(B-\frac{aD}{b}\right)\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(c+dx)\left(adDx^2-(2Abd-3aCd-acD)x+a\left(cC+2d\left(B-\frac{aD}{b}\right)\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}}{}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int \frac{d^2(c+dx)(a(3bcC+6bBd-8adD)-b(6Abd-9aCd-2acD)x)}{\sqrt{bx^2+a}}dx}{3bd^2}+\frac{aD\sqrt{a+bx^2}(c+dx)^2}{3b}}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(c+dx)(a(3bcC+6bBd-8adD)-b(6Abd-9aCd-2acD)x)}{\sqrt{bx^2+a}}dx}{3b}+\frac{aD\sqrt{a+bx^2}(c+dx)^2}{3b}}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{\frac{3}{2}a\left(2b\left(Ad^2+2Bcd+c^2C\right)-3ad(2cD+Cd)\right)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}\left(a\left(4ad^2D-b\left(3Bd^2+c^2D+6cCd\right)\right)+3Ab^2cd\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(-2acD-9aCd+6Ab}{3b}}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{3}{2}a\left(2b\left(Ad^2+2Bcd+c^2C\right)-3ad(2cD+Cd)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2\sqrt{a+bx^2}\left(a\left(4ad^2D-b\left(3Bd^2+c^2D+6cCd\right)\right)+3Ab^2cd\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(-2acD-9a}{3b}}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2b\left(Ad^2+2Bcd+c^2C\right)-3ad(2cD+Cd)\right)}{2\sqrt{b}}-\frac{2\sqrt{a+bx^2}\left(a\left(4ad^2D-b\left(3Bd^2+c^2D+6cCd\right)\right)+3Ab^2cd\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(-2acD-9aCd+}{3b}}{ab}-\frac{(c+dx)^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x)^2)/(a*b*Sqrt[a + b*x^2])) +
 ((a*D*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b) + ((-2*(3*A*b^2*c*d + a*(4*a*d^2
*D - b*(6*c*C*d + 3*B*d^2 + c^2*D)))*Sqrt[a + b*x^2])/b - (d*(6*A*b*d - 9*
a*C*d - 2*a*c*D)*x*Sqrt[a + b*x^2])/2 + (3*a*(2*b*(c^2*C + 2*B*c*d + A*d^2
) - 3*a*d*(C*d + 2*c*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
)/(3*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01

method	result
default	$\frac{A\,c^{2} x}{\sqrt{b\,x^{2}+a}\,a}-\frac{c \left(2 A d +B c \right)}{b \sqrt{b\,x^{2}+a}}+d \left(C d +2 D c \right) \left(\frac{x^{3}}{2 b \sqrt{b\,x^{2}+a}}-\frac{3 a \left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln \left(\sqrt{b}\, x +\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)+\left(A\,d^{2}+2 B\right.$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^2/(b*x^2+a)^(1/2)/a*x-c*(2*A*d+B*c)/b/(b*x^2+a)^(1/2)+d*(C*d+2*D*c)*(1
/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))))+(A*d^2+2*B*c*d+C*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+(B*d^2+2*C*c*d+D*c^2)*(x^2/b/(b*x^2+a)^(
1/2)+2*a/b^2/(b*x^2+a)^(1/2))+D*d^2*(1/3*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^
2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.66

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fric
as")
```

output
```
[1/12*(3*(2*C*a^2*b*c^2 - 2*(3*D*a^3 - 2*B*a^2*b)*c*d - (3*C*a^3 - 2*A*a^2
*b)*d^2 + (2*C*a*b^2*c^2 - 2*(3*D*a^2*b - 2*B*a*b^2)*c*d - (3*C*a^2*b - 2*
A*a*b^2)*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(2*D*a*b^2*d^2*x^4 + 3*(2*D*a*b^2*c*d + C*a*b^2*d^2)*x^3 + 6*(2*D*a^2
*b - B*a*b^2)*c^2 + 12*(2*C*a^2*b - A*a*b^2)*c*d - 4*(4*D*a^3 - 3*B*a^2*b)
*d^2 + 2*(3*D*a*b^2*c^2 + 6*C*a*b^2*c*d - (4*D*a^2*b - 3*B*a*b^2)*d^2)*x^2
 - 3*(2*(C*a*b^2 - A*b^3)*c^2 - 2*(3*D*a^2*b - 2*B*a*b^2)*c*d - (3*C*a^2*b
 - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/6*(3*(2*C
*a^2*b*c^2 - 2*(3*D*a^3 - 2*B*a^2*b)*c*d - (3*C*a^3 - 2*A*a^2*b)*d^2 + (2*
C*a*b^2*c^2 - 2*(3*D*a^2*b - 2*B*a*b^2)*c*d - (3*C*a^2*b - 2*A*a*b^2)*d^2)
*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*D*a*b^2*d^2*x^4 + 3
*(2*D*a*b^2*c*d + C*a*b^2*d^2)*x^3 + 6*(2*D*a^2*b - B*a*b^2)*c^2 + 12*(2*C
*a^2*b - A*a*b^2)*c*d - 4*(4*D*a^3 - 3*B*a^2*b)*d^2 + 2*(3*D*a*b^2*c^2 + 6
*C*a*b^2*c*d - (4*D*a^2*b - 3*B*a*b^2)*d^2)*x^2 - 3*(2*(C*a*b^2 - A*b^3)*c
^2 - 2*(3*D*a^2*b - 2*B*a*b^2)*c*d - (3*C*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(
b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2 + D*x**3)/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx &= \frac{Dd^2x^4}{3\sqrt{bx^2+a}b} - \frac{4\,Dad^2x^2}{3\sqrt{bx^2+a}b^2} \\ &+ \frac{Ac^2x}{\sqrt{bx^2+a}a} + \frac{(2\,Dcd+Cd^2)x^3}{2\sqrt{bx^2+a}b} - \frac{Bc^2}{\sqrt{bx^2+a}b} - \frac{2\,Acd}{\sqrt{bx^2+a}b} \\ &- \frac{8\,Da^2d^2}{3\sqrt{bx^2+a}b^3} + \frac{(Dc^2+2\,Ccd+Bd^2)x^2}{\sqrt{bx^2+a}b} + \frac{3\,(2\,Dcd+Cd^2)ax}{2\sqrt{bx^2+a}b^2} \\ &- \frac{(Cc^2+2\,Bcd+Ad^2)x}{\sqrt{bx^2+a}b} - \frac{3\,(2\,Dcd+Cd^2)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} \\ &+ \frac{(Cc^2+2\,Bcd+Ad^2)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,(Dc^2+2\,Ccd+Bd^2)a}{\sqrt{bx^2+a}b^2}\end{aligned}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

output
```
1/3*D*d^2*x^4/(sqrt(b*x^2 + a)*b) - 4/3*D*a*d^2*x^2/(sqrt(b*x^2 + a)*b^2)
+ A*c^2*x/(sqrt(b*x^2 + a)*a) + 1/2*(2*D*c*d + C*d^2)*x^3/(sqrt(b*x^2 + a)
*b) - B*c^2/(sqrt(b*x^2 + a)*b) - 2*A*c*d/(sqrt(b*x^2 + a)*b) - 8/3*D*a^2*
d^2/(sqrt(b*x^2 + a)*b^3) + (D*c^2 + 2*C*c*d + B*d^2)*x^2/(sqrt(b*x^2 + a)
*b) + 3/2*(2*D*c*d + C*d^2)*a*x/(sqrt(b*x^2 + a)*b^2) - (C*c^2 + 2*B*c*d +
 A*d^2)*x/(sqrt(b*x^2 + a)*b) - 3/2*(2*D*c*d + C*d^2)*a*arcsinh(b*x/sqrt(a
*b))/b^(5/2) + (C*c^2 + 2*B*c*d + A*d^2)*arcsinh(b*x/sqrt(a*b))/b^(3/2) +
2*(D*c^2 + 2*C*c*d + B*d^2)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(\frac{2\,Dd^2x}{b}+\frac{3\,(2\,Dab^4cd+Cab^4d^2)}{ab^5}\right)x+\frac{2\,(3\,Dab^4c^2+6\,Cab^4cd-4\,Da^2b^3d^2+3}{ab^5}\right.\right.}{} - \frac{(2\,Cbc^2-6\,Dacd+4\,Bbcd-3\,Cad^2+2\,Abd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac
")
```

output
```
1/6*((((2*D*d^2*x/b + 3*(2*D*a*b^4*c*d + C*a*b^4*d^2)/(a*b^5))*x + 2*(3*D*
a*b^4*c^2 + 6*C*a*b^4*c*d - 4*D*a^2*b^3*d^2 + 3*B*a*b^4*d^2)/(a*b^5))*x -
3*(2*C*a*b^4*c^2 - 2*A*b^5*c^2 - 6*D*a^2*b^3*c*d + 4*B*a*b^4*c*d - 3*C*a^2
*b^3*d^2 + 2*A*a*b^4*d^2)/(a*b^5))*x + 2*(6*D*a^2*b^3*c^2 - 3*B*a*b^4*c^2
+ 12*C*a^2*b^3*c*d - 6*A*a*b^4*c*d - 8*D*a^3*b^2*d^2 + 6*B*a^2*b^3*d^2)/(a
*b^5))/sqrt(b*x^2 + a) - 1/2*(2*C*b*c^2 - 6*D*a*c*d + 4*B*b*c*d - 3*C*a*d^
2 + 2*A*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(dx+c)^2\,(Dx^3+C\,x^2+Bx+A)}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

3.110 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 167

$$\begin{aligned}\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx &= -\frac{bBc+Abd-aCd-acD}{b^2\sqrt{a+bx^2}} \\ &+ \frac{(Ab^2c-a(b(cC+Bd)-adD))x}{ab^2\sqrt{a+bx^2}} + \frac{(Cd+cD)\sqrt{a+bx^2}}{b^2} \\ &+ \frac{dDx\sqrt{a+bx^2}}{2b^2} + \frac{(2b(cC+Bd)-3adD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

output
```
-(A*b*d+B*b*c-C*a*d-D*a*c)/b^2/(b*x^2+a)^(1/2)+(A*b^2*c-a*(b*(B*d+C*c)-D*a
*d))*x/a/b^2/(b*x^2+a)^(1/2)+(C*d+D*c)*(b*x^2+a)^(1/2)/b^2+1/2*d*D*x*(b*x^
2+a)^(1/2)/b^2+1/2*(2*b*(B*d+C*c)-3*D*a*d)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/
2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{b}(2Ab^2cx+a^2(4Cd+4cD+3dDx)+ab(-2Ad-2B(c+dx)+x(-2cC+2Cdx+2cDx+dD}{a\sqrt{a+bx^2}}}{2b^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*(2*A*b^2*c*x + a^2*(4*C*d + 4*c*D + 3*d*D*x) + a*b*(-2*A*d - 2*B
*(c + d*x) + x*(-2*c*C + 2*C*d*x + 2*c*D*x + d*D*x^2))))/(a*Sqrt[a + b*x^2
]) + (-2*b*(c*C + B*d) + 3*a*d*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*
b^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2176, 25, 2346, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 2176$$

$$-\frac{\int -\frac{adDx^2-(Abd-2aCd-acD)x+\frac{a(b(cC+Bd)-adD)}{b}}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{adDx^2-(Abd-2aCd-acD)x+\frac{a(bcC+bBd-adD)}{b}}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\int \frac{a(2b(cC+Bd)-3adD)-2b(Abd-2aCd-acD)x}{\sqrt{bx^2+a}}dx}{2b}+\frac{adDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{a(2b(Bd+cC)-3adD)\int \frac{1}{\sqrt{bx^2+a}}dx-2\sqrt{a+bx^2}(-acD-2aCd+Abd)}{2b}+\frac{adDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a(2b(Bd+cC)-3adD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2\sqrt{a+bx^2}(-acD-2aCd+Abd)}{2b}+\frac{adDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2b(Bd+cC)-3adD)}{\sqrt{b}}-2\sqrt{a+bx^2}(-acD-2aCd+Abd)}{2b}+\frac{adDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input

```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output

```
-(((a*(B - (a*D)/b) - (A*b - a*C)*x)*(c + d*x))/(a*b*Sqrt[a + b*x^2])) + (
(a*d*D*x*Sqrt[a + b*x^2])/(2*b) + (-2*(A*b*d - 2*a*C*d - a*c*D)*Sqrt[a + b
*x^2] + (a*(2*b*(c*C + B*d) - 3*a*d*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/Sqrt[b])/(2*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

method	result
default	$\frac{Acx}{\sqrt{b\,x^2+a}\,a}-\frac{Ad+Bc}{b\sqrt{b\,x^2+a}}+(Bd+Cc)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+(Cd+Dc)\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2}{b^2\sqrt{b}}\right.$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c/(b*x^2+a)^(1/2)/a*x-(A*d+B*c)/b/(b*x^2+a)^(1/2)+(B*d+C*c)*(-x/b/(b*x^2
+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+(C*d+D*c)*(x^2/b/(b*x^2
+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+d*D*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*
(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.65

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(2\,Ca^2bc+(2\,Cab^2c-(3\,Da^2b-2\,Bab^2)d)x^2-(3\,Da^3-2\,Ba\right.$$

$$-\frac{(2\,Ca^2bc+(2\,Cab^2c-(3\,Da^2b-2\,Bab^2)d)x^2-(3\,Da^3-2\,Ba^2b)d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Dab^2dx^3-$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/4*((2*C*a^2*b*c + (2*C*a*b^2*c - (3*D*a^2*b - 2*B*a*b^2)*d)*x^2 - (3*D*
a^3 - 2*B*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(D*a*b^2*d*x^3 + 2*(D*a*b^2*c + C*a*b^2*d)*x^2 + 2*(2*D*a^2*b - B*a*
b^2)*c + 2*(2*C*a^2*b - A*a*b^2)*d - (2*(C*a*b^2 - A*b^3)*c - (3*D*a^2*b -
 2*B*a*b^2)*d)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/2*((2*C*a^2*b
*c + (2*C*a*b^2*c - (3*D*a^2*b - 2*B*a*b^2)*d)*x^2 - (3*D*a^3 - 2*B*a^2*b)
*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (D*a*b^2*d*x^3 + 2*(D*a*
b^2*c + C*a*b^2*d)*x^2 + 2*(2*D*a^2*b - B*a*b^2)*c + 2*(2*C*a^2*b - A*a*b^
2)*d - (2*(C*a*b^2 - A*b^3)*c - (3*D*a^2*b - 2*B*a*b^2)*d)*x)*sqrt(b*x^2 +
 a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = Ad\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Acx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + Bc\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Bd\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cc\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cd\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Dc\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Dd\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*d*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), Tru
e)) + A*c*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*c*Piecewise((-1/(b*sqrt(a +
b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + B*d*(asinh(sqrt(b)*x/sqr
t(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*c*(asinh(sqrt(b)*x/
sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*d*Piecewise((2*a
/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*
a**(3/2)), True)) + D*c*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*s
qrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + D*d*(3*sqrt(a)*x
/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) +
 x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Ddx^3}{2\,\sqrt{bx^2+a}b}+\frac{Acx}{\sqrt{bx^2+a}a}+\frac{3\,Dadx}{2\,\sqrt{bx^2+a}b^2}+\frac{(Dc+Cd)x^2}{\sqrt{bx^2+a}b}-\frac{3\,Dad\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}-\frac{Bc}{\sqrt{bx^2+a}b}-\frac{Ad}{\sqrt{bx^2+a}b}-\frac{(Cc+Bd)x}{\sqrt{bx^2+a}b}+\frac{(Cc+Bd)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\,(Dc+Cd)a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
1/2*D*d*x^3/(sqrt(b*x^2 + a)*b) + A*c*x/(sqrt(b*x^2 + a)*a) + 3/2*D*a*d*x/
(sqrt(b*x^2 + a)*b^2) + (D*c + C*d)*x^2/(sqrt(b*x^2 + a)*b) - 3/2*D*a*d*ar
csinh(b*x/sqrt(a*b))/b^(5/2) - B*c/(sqrt(b*x^2 + a)*b) - A*d/(sqrt(b*x^2 +
 a)*b) - (C*c + B*d)*x/(sqrt(b*x^2 + a)*b) + (C*c + B*d)*arcsinh(b*x/sqrt(
a*b))/b^(3/2) + 2*(D*c + C*d)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(\left(\frac{Ddx}{b}+\frac{2\,(Dab^3c+Cab^3d)}{ab^4}\right)x-\frac{2\,Cab^3c-2\,Ab^4c-3\,Da^2b^2d+2\,Bab^3d}{ab^4}\right)x+\frac{2\,(}{}}{2\,\sqrt{bx^2+a}}$$
$$-\frac{(2\,Cbc-3\,Dad+2\,Bbd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((D*d*x/b + 2*(D*a*b^3*c + C*a*b^3*d)/(a*b^4))*x - (2*C*a*b^3*c - 2*A
*b^4*c - 3*D*a^2*b^2*d + 2*B*a*b^3*d)/(a*b^4))*x + 2*(2*D*a^2*b^2*c - B*a*
b^3*c + 2*C*a^2*b^2*d - A*a*b^3*d)/(a*b^4))/sqrt(b*x^2 + a) - 1/2*(2*C*b*c
 - 3*D*a*d + 2*B*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.51

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a\,b^2d+8\sqrt{b\,x^2+a}\,abcd+3\sqrt{b\,x^2+a}\,ab\,d^2x+2\sqrt{b}}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b**2*d + 8*sqrt(a + b*x**2)*a*b*c*d + 3*sqrt(a +
b*x**2)*a*b*d**2*x + 2*sqrt(a + b*x**2)*b**3*c*x - 2*sqrt(a + b*x**2)*b**3
*c - 2*sqrt(a + b*x**2)*b**3*d*x - 2*sqrt(a + b*x**2)*b**2*c**2*x + 4*sqrt
(a + b*x**2)*b**2*c*d*x**2 + sqrt(a + b*x**2)*b**2*d**2*x**3 - 3*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d**2 + 2*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*d + 2*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a*b*c**2 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a*b*d**2*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*b**3*d*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*b**2*c**2*x**2 + 2*sqrt(b)*a**2*d**2 + 2*sqrt(b)*a*b**2*c - 2*sqrt(
b)*a*b**2*d - 2*sqrt(b)*a*b*c**2 + 2*sqrt(b)*a*b*d**2*x**2 + 2*sqrt(b)*b**
3*c*x**2 - 2*sqrt(b)*b**3*d*x**2 - 2*sqrt(b)*b**2*c**2*x**2)/(2*b**3*(a +
b*x**2))
```

3.111 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 94

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = -\frac{bB-aD}{b^2\sqrt{a+bx^2}} + \frac{(Ab-aC)x}{ab\sqrt{a+bx^2}} + \frac{D\sqrt{a+bx^2}}{b^2} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(B*b-D*a)/b^2/(b*x^2+a)^(1/2)+(A*b-C*a)*x/a/b/(b*x^2+a)^(1/2)+D*(b*x^2+a)
^(1/2)/b^2+C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{2a^2D+Ab^2x-ab(B+x(C-Dx))}{ab^2\sqrt{a+bx^2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
(2*a^2*D + A*b^2*x - a*b*(B + x*(C - D*x)))/(a*b^2*Sqrt[a + b*x^2]) - (C*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2345, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C+Dx}{\sqrt{bx^2+a}}dx}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*Sqrt[a + b*x^2])) + ((D*Sqrt[a +
b*x^2])/b + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{Ax}{\sqrt{b\,x^{2}+a}\,a}-\frac{B}{b\sqrt{b\,x^{2}+a}}+C\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+D\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)$	104

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A/(b*x^2+a)^(1/2)/a*x-B/b/(b*x^2+a)^(1/2)+C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(Cabx^2+Ca^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Dabx^2+2\,D}{2\,(ab^3x^2+a^2b^2)}\right.$$
$$\left.-\frac{(Cabx^2+Ca^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Dabx^2+2\,Da^2-Bab-(Cab-Ab^2)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*b*x^2 + C*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) + 2*(D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sqrt(b*x^2 +
 a))/(a*b^3*x^2 + a^2*b^2), -((C*a*b*x^2 + C*a^2)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) - (D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sq
rt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + C\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*(asinh(sqrt(b)*x/sqrt(a))/b**(3/
2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + D*Piecewise((2*a/(b**2*sqrt(a + b
*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^2}{\sqrt{bx^2+ab}} + \frac{Ax}{\sqrt{bx^2+aa}} - \frac{Cx}{\sqrt{bx^2+ab}} + \frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Da}{\sqrt{bx^2+ab^2}} - \frac{B}{\sqrt{bx^2+ab}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x^2/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) - C*x/(sqrt(b*x^2 + a)
*b) + C*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*D*a/(sqrt(b*x^2 + a)*b^2) - B/(
sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Dx}{b}-\frac{Cab^2-Ab^3}{ab^3}\right)x+\frac{2\,Da^2b-Bab^2}{ab^3}}{\sqrt{bx^2+a}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((D*x/b - (C*a*b^2 - A*b^3)/(a*b^3))*x + (2*D*a^2*b - B*a*b^2)/(a*b^3))/sq
rt(b*x^2 + a) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{C\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{B}{b\,\sqrt{b\,x^2+a}} + \frac{(b\,x^2+2\,a)\,D}{b^2\,\sqrt{b\,x^2+a}} + \frac{A\,x}{a\,\sqrt{b\,x^2+a}} - \frac{C\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(3/2),x)
```

output
```
(C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - B/(b*(a + b*x^2)^(1/2)) +
 ((2*a + b*x^2)*D)/(b^2*(a + b*x^2)^(1/2)) + (A*x)/(a*(a + b*x^2)^(1/2)) -
 (C*x)/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,ad+\sqrt{b\,x^2+a}\,b^2x-\sqrt{b\,x^2+a}\,b^2-\sqrt{b\,x^2+a}\,bcx+\sqrt{b\,x^2+a}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a*d + sqrt(a + b*x**2)*b**2*x - sqrt(a + b*x**2)*b**2
- sqrt(a + b*x**2)*b*c*x + sqrt(a + b*x**2)*b*d*x**2 + sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c + sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*b*c*x**2 + sqrt(b)*a*b - sqrt(b)*a*c + sqrt(b)*b**2*x**2
- sqrt(b)*b*c*x**2)/(b**2*(a + b*x**2))
```

3.112 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 193

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = -\frac{a(bBc-Abd+aCd-acD)-\left(Ab^2c-a(bcC-bBd+adD)\right)x}{ab\left(bc^2+ad^2\right)\sqrt{a+bx^2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}d} - \frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d\left(bc^2+ad^2\right)^{3/2}}$$

output
```
-(a*(-A*b*d+B*b*c+C*a*d-D*a*c)-(A*b^2*c-a*(-B*b*d+C*b*c+D*a*d))*x)/a/b/(a*
d^2+b*c^2)/(b*x^2+a)^(1/2)+D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d-
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*
x^2+a)^(1/2))/d/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec), antiderivative size = 201, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)(a+bx^2)^{3/2}}\,dx = \frac{Ab^2cx+ab(-Bc+Ad-cCx+Bdx)-a^2(Cd-cD+dDx)}{ab(bc^2+ad^2)\sqrt{a+bx^2}} - \frac{2(-c^2Cd+Bcd^2-Ad^3+c^3D)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{d(-bc^2-ad^2)^{3/2}} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}d}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x) - a^2*(C*d - c*D + d*D*x))
/(a*b*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (2*(-(c^2*C*d) + B*c*d^2 - A*d^3
+ c^3*D)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*
d^2]])/(d*(-(b*c^2) - a*d^2)^(3/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/(b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.49 (sec), antiderivative size = 199, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2178, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{a\left(\frac{b\left(Cc^2-Bdc+Ad^2\right)+acdD}{bc^2+ad^2}+Dx\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{a\left(\frac{bCc^2-bBdc+adDc+Abd^2}{bc^2+ad^2}+Dx\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{bCc^2-bBdc+adDc+Abd^2}{bc^2+ad^2}+Dx}{(c+dx)\sqrt{bx^2+a}}dx}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d(ad^2+bc^2)}+\frac{D\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d(ad^2+bc^2)}+\frac{D\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d(ad^2+bc^2)}+\frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}-\frac{b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d(ad^2+bc^2)}}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}-\frac{b\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d(ad^2+bc^2)^{3/2}}}{b}-\frac{a(-acD+aCd-Abd+bBc)-x\left(Ab^2c-a(adD-bBd+bcC)\right)}{ab\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b*B*c - A*b*d + a*C*d - a*c*D) - (A*b^2*c - a*(b*c*C - b*B*d + a*d*D
))*x)/(a*b*(b*c^2 + a*d^2)*Sqrt[a + b*x^2])) + ((D*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(Sqrt[b]*d) - (b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*ArcTan
h[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*(b*c^2 + a*d^2)
^(3/2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(179) = 358$.

Time = 1.47 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.40

method	result
default	$\frac{\frac{B\,d^{2}x}{\sqrt{b\,x^{2}+a}\,a}+\frac{Dc^{2}x}{\sqrt{b\,x^{2}+a}\,a}-\frac{d(Cd-Dc)}{b\sqrt{b\,x^{2}+a}}+Dd^{2}\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{Ccdx}{\sqrt{b\,x^{2}+a}\,a}}{d^{3}}+\frac{\left(A\,d^{3}-Bc\,d^{2}+C\,c^{2}d-Dc^{3}\right)\left(\frac{\cdots}{\left(a\,d^{2}+b\cdots\right.}\right.}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d^2/(b*x^2+a)^(1/2)/a*x+D*c^2/(b*x^2+a)^(1/2)/a*x-d*(C*d-D*c)/b/(
b*x^2+a)^(1/2)+D*d^2*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2)))-C*c*d/(b*x^2+a)^(1/2)/a*x)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4*(1/
(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*
b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2
/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2
)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*
((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(177) = 354$.

Time = 0.11 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = -\frac{Dbc^4x}{\sqrt{bx^2+a}abc^2d^3+\sqrt{bx^2+a}a^2d^5} + \frac{Cbc^3x}{\sqrt{bx^2+a}abc^2d^2+\sqrt{bx^2+a}a^2d^4} - \frac{Bbc^2x}{\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3} - \frac{Dc^3}{\sqrt{bx^2+a}bc^2d^2+\sqrt{bx^2+a}ad^4} + \frac{Abcx}{\sqrt{bx^2+a}abc^2+\sqrt{bx^2+a}a^2d^2} + \frac{Cc^2}{\sqrt{bx^2+a}bc^2d+\sqrt{bx^2+a}ad^3} - \frac{Bc}{\sqrt{bx^2+a}bc^2+\sqrt{bx^2+a}ad^2} + \frac{A}{\frac{\sqrt{bx^2+a}bc^2}{d}+\sqrt{bx^2+a}ad} + \frac{Dc^2x}{\sqrt{bx^2+a}ad^3} - \frac{Ccx}{\sqrt{bx^2+a}ad^2} + \frac{Bx}{\sqrt{bx^2+a}ad} - \frac{Dx}{\sqrt{bx^2+a}bd} + \frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}d} - \frac{Dc^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} + \frac{Cc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3} - \frac{Bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^2} + \frac{A\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d} + \frac{Dc}{\sqrt{bx^2+a}bd^2} - \frac{C}{\sqrt{bx^2+a}bd}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
-D*b*c^4*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) + C*b*c
^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - B*b*c^2*x/(
sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - D*c^3/(sqrt(b*x^2 +
 a)*b*c^2*d^2 + sqrt(b*x^2 + a)*a*d^4) + A*b*c*x/(sqrt(b*x^2 + a)*a*b*c^2
+ sqrt(b*x^2 + a)*a^2*d^2) + C*c^2/(sqrt(b*x^2 + a)*b*c^2*d + sqrt(b*x^2 +
 a)*a*d^3) - B*c/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 + a)*a*d^2) + A/(sqrt
(b*x^2 + a)*b*c^2/d + sqrt(b*x^2 + a)*a*d) + D*c^2*x/(sqrt(b*x^2 + a)*a*d^
3) - C*c*x/(sqrt(b*x^2 + a)*a*d^2) + B*x/(sqrt(b*x^2 + a)*a*d) - D*x/(sqrt
(b*x^2 + a)*b*d) + D*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) - D*c^3*arcsinh(b*
c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d
^2)^(3/2)*d^4) + C*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(
a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^3) - B*c*arcsinh(b*c*x/(sqrt(
a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*
d^2) + A*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x +
 c)))/((a + b*c^2/d^2)^(3/2)*d) + D*c/(sqrt(b*x^2 + a)*b*d^2) - C/(sqrt(b*
x^2 + a)*b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\,(a + bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4998, normalized size of antiderivative = 25.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output

```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b**2*c*d + 2*sqrt(b)
*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 +
 b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**3*c**2 - 2*sqrt(b)*sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((
sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c -
 a*d**2 - 2*b*c**2))*a*b**3*c*d*x**2 + 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*
x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2
*b*c**2))*b**4*c**2*x**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d*
*2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**3*b**2*d**3 - 2*sqrt(2*sqrt(b
)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d +
sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*
a**2*b**3*c**2*d + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b
*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2
+ b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b**3*c*d**2 - 2*sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqr...
```

3.113 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{a(b^2c(Bc-2Ad)+a^2d^2D+ab(2cCd-Bd^2-c^2D))-b(Ab(bc^2-ad^2)-a(bc(cC-2Bd)-ad(Cd-}{ab\left(bc^2+ad^2\right)^2\sqrt{a+bx^2}}$$
$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{\left(bc^2+ad^2\right)^2(c+dx)}$$
$$-\frac{(bc(c^2C-2Bcd+3Ad^2)-ad(2cCd-Bd^2-3c^2D))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{5/2}}$$

output
```
-(a*(b^2*c*(-2*A*d+B*c)+a^2*d^2*D+a*b*(-B*d^2+2*C*c*d-D*c^2))-b*(A*b*(-a*d
^2+b*c^2)-a*(b*c*(-2*B*d+C*c)-a*d*(C*d-2*D*c)))*x)/a/b/(a*d^2+b*c^2)^2/(b*
x^2+a)^(1/2)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2
/(d*x+c)-(b*c*(3*A*d^2-2*B*c*d+C*c^2)-a*d*(-B*d^2+2*C*c*d-3*D*c^2))*arctan
h((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{-a^3d^2D(c+dx)+Ab^3c^2x(c+dx)+ab^2(c^2x(-cC-2Cdx+cDx)+Ad(\cdots}{\cdots}$$

$$-\frac{2(bc(c^2C-2Bcd+3Ad^2)+ad(-2cCd+Bd^2+3c^2D))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a^3*d^2*D*(c + d*x)) + A*b^3*c^2*x*(c + d*x) + a*b^2*(c^2*x*(-(c*C) - 2
*C*d*x + c*D*x) + A*d*(2*c^2 + c*d*x - 2*d^2*x^2) + B*c*(-c^2 + c*d*x + 3*
d^2*x^2)) + a^2*b*(2*c^3*D - c^2*d*(3*C + D*x) + d^3*(-A + x*(B + C*x)) +
c*d^2*(2*B - x*(C + 2*D*x))))/(a*b*(b*c^2 + a*d^2)^2*(c + d*x)*Sqrt[a + b*
x^2]) - (2*(b*c*(c^2*C - 2*B*c*d + 3*A*d^2) + a*d*(-2*c*C*d + B*d^2 + 3*c^
2*D))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2
]])/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2178, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{ab\left(b\left(Cc^2-2Bdc+3Ad^2\right)c^2-ad\left(-2Dc^3+Cdc^2-Ad^3\right)\right)-\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-Dc^3+Bd^2c-2Ad^3\right)\right)x}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a(bc(cC-2Bd)-ad(Cd-\cdots\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{ab\left(b\left(Cc^2-2Bdc+3Ad^2\right)c^2-ad\left(-2Dc^3+Cdc^2-Ad^3\right)-\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-Dc^3+Bd^2c-2Ad^3\right)\right)x\right)}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a\left(bc(cC-2Bd)-ad(Cd-\right.\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{b\left(Cc^2-2Bdc+3Ad^2\right)c^2-ad\left(-2Dc^3+Cdc^2-Ad^3\right)-\left(a\left(-3Dc^2+2Cdc-Bd^2\right)d^2+bc\left(-Dc^3+Bd^2c-2Ad^3\right)\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{\left(ad^2+bc^2\right)^2}-$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a\left(bc(cC-2Bd)-ad(Cd-\right.\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow 679$$

$$\frac{\left(bc\left(3Ad^2-2Bcd+c^2C\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{c+dx}}{\left(ad^2+bc^2\right)^2}-$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a\left(bc(cC-2Bd)-ad(Cd-\right.\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow 488$$

$$\frac{-\left(bc\left(3Ad^2-2Bcd+c^2C\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)\right.}{c+dx}}{\left(ad^2+bc^2\right)^2}$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a\left(bc(cC-2Bd)-ad(Cd-\right.\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow 219$$

$$\frac{-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(bc\left(3Ad^2-2Bcd+c^2C\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{\sqrt{ad^2+bc^2}}-\frac{\sqrt{a+bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{c+dx}}{\left(ad^2+bc^2\right)^2}-$$

$$\frac{a\left(a^2d^2D+ab\left(-Bd^2+c^2(-D)+2cCd\right)+b^2c(Bc-2Ad)\right)-bx\left(Ab\left(bc^2-ad^2\right)-a\left(bc(cC-2Bd)-ad(Cd-\right.\right.}{ab\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b^2*c*(B*c - 2*A*d) + a^2*d^2*D + a*b*(2*c*C*d - B*d^2 - c^2*D)) - b
*(A*b*(b*c^2 - a*d^2) - a*(b*c*(c*C - 2*B*d) - a*d*(C*d - 2*c*D)))*x)/(a*b
*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2])) + (-(((c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D)*Sqrt[a + b*x^2])/(c + d*x)) - ((b*c*(c^2*C - 2*B*c*d + 3*A*d^2) - a*d
*(2*c*C*d - B*d^2 - 3*c^2*D))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*S
qrt[a + b*x^2])])/Sqrt[b*c^2 + a*d^2])/(b*c^2 + a*d^2)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(271) = 542$.

Time = 1.44 (sec) , antiderivative size = 926, normalized size of antiderivative = 3.30

method	result
default	$\frac{\frac{Cdx}{\sqrt{b\,x^2+a}\,a}-\frac{Dd}{b\sqrt{b\,x^2+a}}-\frac{2Dcx}{\sqrt{b\,x^2+a}\,a}}{d^3}+\frac{\left(B\,d^2-2Ccd+3Dc^2\right)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{\ldots}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+\ldots\right.}{d^2}\ldots\right.}\ldots\right.}{\ldots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(C*d/(b*x^2+a)^(1/2)/a*x-D*d/b/(b*x^2+a)^(1/2)-2*D*c/(b*x^2+a)^(1/2)
/a*x)+1/d^4*(B*d^2-2*C*c*d+3*D*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*
b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln(
(2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/d^5*(A*d^3-B*c*d
^2+C*c^2*d-D*c^3)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(
2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/
d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(
1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/
(a*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d
^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(267) = 534$.

Time = 4.72 (sec) , antiderivative size = 1945, normalized size of antiderivative = 6.92

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2 (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fric
as")
```

output
```
[1/2*((C*a^2*b^2*c^4 + B*a^3*b*c*d^3 + (3*D*a^3*b - 2*B*a^2*b^2)*c^3*d - (
2*C*a^3*b - 3*A*a^2*b^2)*c^2*d^2 + (C*a*b^3*c^3*d + B*a^2*b^2*d^4 + (3*D*a
^2*b^2 - 2*B*a*b^3)*c^2*d^2 - (2*C*a^2*b^2 - 3*A*a*b^3)*c*d^3)*x^3 + (C*a*
b^3*c^4 + B*a^2*b^2*c*d^3 + (3*D*a^2*b^2 - 2*B*a*b^3)*c^3*d - (2*C*a^2*b^2
 - 3*A*a*b^3)*c^2*d^2)*x^2 + (C*a^2*b^2*c^3*d + B*a^3*b*d^4 + (3*D*a^3*b -
 2*B*a^2*b^2)*c^2*d^2 - (2*C*a^3*b - 3*A*a^2*b^2)*c*d^3)*x)*sqrt(b*c^2 + a
*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 -
 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*(A*a^3*b*d^5 - (2*D*a^2*b^2 - B*a*b^3)*c^5 + (3*C*a^2*b^2 - 2*A
*a*b^3)*c^4*d - (D*a^3*b + B*a^2*b^2)*c^3*d^2 + (3*C*a^3*b - A*a^2*b^2)*c^
2*d^3 + (D*a^4 - 2*B*a^3*b)*c*d^4 - (D*a*b^3*c^5 - (2*C*a*b^3 - A*b^4)*c^4
*d - (D*a^2*b^2 - 3*B*a*b^3)*c^3*d^2 - (C*a^2*b^2 + A*a*b^3)*c^2*d^3 - (2*
D*a^3*b - 3*B*a^2*b^2)*c*d^4 + (C*a^3*b - 2*A*a^2*b^2)*d^5)*x^2 + ((C*a*b^
3 - A*b^4)*c^5 + (D*a^2*b^2 - B*a*b^3)*c^4*d + 2*(C*a^2*b^2 - A*a*b^3)*c^3
*d^2 + 2*(D*a^3*b - B*a^2*b^2)*c^2*d^3 + (C*a^3*b - A*a^2*b^2)*c*d^4 + (D*
a^4 - B*a^3*b)*d^5)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^7 + 3*a^3*b^3*c^5*d^2 +
 3*a^4*b^2*c^3*d^4 + a^5*b*c*d^6 + (a*b^5*c^6*d + 3*a^2*b^4*c^4*d^3 + 3*a^
3*b^3*c^2*d^5 + a^4*b^2*d^7)*x^3 + (a*b^5*c^7 + 3*a^2*b^4*c^5*d^2 + 3*a^3*
b^3*c^3*d^4 + a^4*b^2*c*d^6)*x^2 + (a^2*b^4*c^6*d + 3*a^3*b^3*c^4*d^3 + 3*
a^4*b^2*c^2*d^5 + a^5*b*d^7)*x), -((C*a^2*b^2*c^4 + B*a^3*b*c*d^3 + (3*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{\frac{3}{2}}(c+dx)^2}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x**2)**(3/2)*(c + d*x)**2), x
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(267) = 534$.

Time = 0.14 (sec) , antiderivative size = 1530, normalized size of antiderivative = 5.44

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2 (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

output
```
-3*D*b^2*c^5*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a^2*b*c^
2*d^5 + sqrt(b*x^2 + a)*a^3*d^7) + 3*C*b^2*c^4*x/(sqrt(b*x^2 + a)*a*b^2*c^
4*d^2 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^4 + sqrt(b*x^2 + a)*a^3*d^6) - 3*B*b
^2*c^3*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 +
sqrt(b*x^2 + a)*a^3*d^5) - 3*D*b*c^4/(sqrt(b*x^2 + a)*b^2*c^4*d^2 + 2*sqrt
(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) + 3*A*b^2*c^2*x/(sqrt(b
*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + sqrt(b*x^2 + a)*a^
3*d^4) + 5*D*b*c^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^
5) + 3*C*b*c^3/(sqrt(b*x^2 + a)*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a*b*c^2*d^3
+ sqrt(b*x^2 + a)*a^2*d^5) - 4*C*b*c^2*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sq
rt(b*x^2 + a)*a^2*d^4) - 3*B*b*c^2/(sqrt(b*x^2 + a)*b^2*c^4 + 2*sqrt(b*x^2
 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) + D*c^3/(sqrt(b*x^2 + a)*b*c^
2*d^3*x + sqrt(b*x^2 + a)*a*d^5*x + sqrt(b*x^2 + a)*b*c^3*d^2 + sqrt(b*x^2
 + a)*a*c*d^4) + 3*B*b*c*x/(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^
2*d^3) + 3*A*b*c/(sqrt(b*x^2 + a)*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a*b*c^2*d
+ sqrt(b*x^2 + a)*a^2*d^3) - C*c^2/(sqrt(b*x^2 + a)*b*c^2*d^2*x + sqrt(b*x
^2 + a)*a*d^4*x + sqrt(b*x^2 + a)*b*c^3*d + sqrt(b*x^2 + a)*a*c*d^3) + 3*D
*c^2/(sqrt(b*x^2 + a)*b*c^2*d^2 + sqrt(b*x^2 + a)*a*d^4) - 2*A*b*x/(sqrt(b
*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a)*a^2*d^2) + B*c/(sqrt(b*x^2 + a)*b*c^2*
d*x + sqrt(b*x^2 + a)*a*d^3*x + sqrt(b*x^2 + a)*b*c^3 + sqrt(b*x^2 + a)...
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 2441, normalized size of antiderivative = 8.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output

```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b**2*c**2*d**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**3*x + sqrt(a*d**2 +
b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**
2*c*d**3 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a**2*b**2*d**4*x + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**2 + sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*
c**2*d**3*x - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a*b**3*c**3*d + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**2*x**2 - 2*sq
rt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c
*x)*a*b**3*c**2*d**2*x + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c*d**3*x**3 + sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c*d**3*
x**2 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b**3*d**4*x**3 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**5 + sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d*x
+ sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*...
```

3.114 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 448

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx =$$

$$-\frac{a(b^2c^2(Bc-3Ad)-a^2d^2(Cd-3cD)+ab(3c^2Cd-3Bcd^2+Ad^3-c^3D))-(Ab^2c(bc^2-3ad^2)-a(b^2c^{2}}{a\left(bc^2+ad^2\right)^3\sqrt{a+bx^2}}$$

$$-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)^2(c+dx)^2}$$

$$+\frac{(2ad^2(2cCd-Bd^2-3c^2D)-bc(3c^2Cd-5Bcd^2+7Ad^3-c^3D))\sqrt{a+bx^2}}{2\left(bc^2+ad^2\right)^3(c+dx)}$$

$$-\frac{(2b^2c^2(c^2C-3Bcd+6Ad^2)+2a^2d^3(Cd-3cD)-abd(11c^2Cd-9Bcd^2+3Ad^3-9c^3D))\,\mathrm{arctanh}\left(\frac{}{\sqrt{bc^{2}}}\right.}{2\left(bc^2+ad^2\right)^{7/2}}$$

output
```
-(a*(b^2*c^2*(-3*A*d+B*c)-a^2*d^2*(C*d-3*D*c)+a*b*(A*d^3-3*B*c*d^2+3*C*c^2
*d-D*c^3))-(A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(-3*B*d+C*c)-a^2*d^3*D-a*b
*d*(-B*d^2+3*C*c*d-3*D*c^2)))*x)/a/(a*d^2+b*c^2)^3/(b*x^2+a)^(1/2)-1/2*(A*
d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)^2+1/2*(
2*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(7*A*d^3-5*B*c*d^2+3*C*c^2*d-D*c^3))*
(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^3/(d*x+c)-1/2*(2*b^2*c^2*(6*A*d^2-3*B*c*d+C*
c^2)+2*a^2*d^3*(C*d-3*D*c)-a*b*d*(3*A*d^3-9*B*c*d^2+11*C*c^2*d-9*D*c^3))*a
rctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/
2)
```

Mathematica [A] (verified)

Time = 11.54 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)^{3/2}}\,dx=\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\left(\frac{(bc^2+ad^2)(-c^2Cd+Bcd^2-Ad^3+c^3D)}{(c+dx)^2}+\frac{-2ad^2(-2cCd+Bd^2+3c^2D)+bc(-3c^2}{c+dx}\right.}{}\right.$$

$$+\frac{(2b^2c^2(c^2C-3Bcd+6Ad^2)+2a^2d^3(Cd-3cD)+abd(-11c^2Cd+9Bcd^2-3Ad^3+9c^3D))\log(c+dx)}{(bc^2+ad^2)^{7/2}}$$

$$-\frac{(2b^2c^2(c^2C-3Bcd+6Ad^2)+2a^2d^3(Cd-3cD)+abd(-11c^2Cd+9Bcd^2-3Ad^3+9c^3D))\log\left(ad-b\right.}{(bc^2+ad^2)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[a + b*x^2]*(((b*c^2 + a*d^2)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)
)/(c + d*x)^2 + (-2*a*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*c*(-3*c^2*C*d +
 5*B*c*d^2 - 7*A*d^3 + c^3*D))/(c + d*x) + (2*(A*b^3*c^3*x + a^3*d^2*(C*d
- 3*c*D + d*D*x) - a*b^2*c*(c^2*C*x + B*c*(c - 3*d*x) + 3*A*d*(-c + d*x))
+ a^2*b*(c^3*D - d^3*(A + B*x) + 3*c*d^2*(B + C*x) - 3*c^2*d*(C + D*x))))/
(a*(a + b*x^2))))/(b*c^2 + a*d^2)^3 + ((2*b^2*c^2*(c^2*C - 3*B*c*d + 6*A*d
^2) + 2*a^2*d^3*(C*d - 3*c*D) + a*b*d*(-11*c^2*C*d + 9*B*c*d^2 - 3*A*d^3 +
 9*c^3*D))*Log[c + d*x])/(b*c^2 + a*d^2)^(7/2) - ((2*b^2*c^2*(c^2*C - 3*B*
c*d + 6*A*d^2) + 2*a^2*d^3*(C*d - 3*c*D) + a*b*d*(-11*c^2*C*d + 9*B*c*d^2
- 3*A*d^3 + 9*c^3*D))*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
]])/(b*c^2 + a*d^2)^(7/2))/2
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2178, 25, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}(c+dx)^3}dx$$

$$\Big\downarrow \; 2178$$

$$-\frac{\int -\frac{-\frac{ab\left(b^2(Bc-3Ad)c^2-a^2d^2(Cd-3cD)+ab\left(-Dc^3+3Cdc^2-3Bd^2c+Ad^3\right)\right)x^2d^3}{\left(bc^2+ad^2\right)^3}+\frac{ab\big(b^2\left(Cc^2-3Bdc+6Ad^2\right)c^4-abd\left(-3Dc^3+3Cdc^2-Bd^2c-3Ad^3\right)c^2+}{\left(bc^2+ad^2\right)^3}}{(c+dx)^3\sqrt{bx^2+a}}}{ab} \qquad \frac{a\big(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\big)-x\big(Ab^2c\left(bc^2-3ad^2\right)-a\big(-a}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{ab\left(b^2(Bc-3Ad)c^2-a^2d^2(Cd-3cD)+ab\left(-Dc^3+3Cdc^2-3Bd^2c+Ad^3\right)\right)x^2d^3}{\left(bc^2+ad^2\right)^3}+\frac{ab\big(b^2\left(Cc^2-3Bdc+6Ad^2\right)c^4-abd\left(-3Dc^3+3Cdc^2-Bd^2c-3Ad^3\right)c^2+a^2d^3}{\left(bc^2+ad^2\right)^3}}{(c+dx)^3\sqrt{bx^2+a}}}{ab} \qquad \frac{a\big(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\big)-x\big(Ab^2c\left(bc^2-3ad^2\right)-a\big(-a}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 2182$$

$$-\frac{\frac{\int \frac{ab\big(2\left(a^2(cC-Bd)d^4+2abc\left(-2Dc^3+2Cdc^2-Bd^2c-Ad^3\right)d-b^2c^3\left(Cc^2-3Bdc+6Ad^2\right)\right)-\big(2a^2(Cd-3cD)d^4-ab\left(-3Dc^3+7Cdc^2-7Bd^2c+3Ad^3\right)d^2-b^2c^2\big(-}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}}{2(ad^2+bc^2)}}{ab} \qquad \frac{a\big(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\big)-x\big(Ab^2c\left(bc^2-3ad^2\right)-a\big(-a}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{ab\int \frac{2\left(a^2(cC-Bd)d^4+2abc\left(-2Dc^3+2Cdc^2-Bd^2c-Ad^3\right)d-b^2c^3\left(Cc^2-3Bdc+6Ad^2\right)\right)-\left(2a^2(Cd-3cD)d^4-ab\left(-3Dc^3+7Cdc^2-7Bd^2c+3Ad^3\right)d^2-b^2c^2\left(-\cdots\right.\right.}{(c+dx)^2\sqrt{bx^2+a}}}{2(ad^2+bc^2)^3}}{ab}$$

$$\frac{a\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)-x\left(Ab^2c\left(bc^2-3ad^2\right)-a\left(-a\cdots\right.\right.}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 679$$

$$\frac{-\frac{ab\left(-\left(2a^2d^3(Cd-3cD)-abd\left(3Ad^3-9Bcd^2-9c^3D+11c^2Cd\right)+2b^2c^2\left(6Ad^2-3Bcd+c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bd^2-3c^2D+2c\cdots\right.\right.}{\cdots}\right.}{2(ad^2+bc^2)^3}}{ab}$$

$$\frac{a\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)-x\left(Ab^2c\left(bc^2-3ad^2\right)-a\left(-a\cdots\right.\right.}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 488$$

$$\frac{-\frac{ab\left(\left(2a^2d^3(Cd-3cD)-abd\left(3Ad^3-9Bcd^2-9c^3D+11c^2Cd\right)+2b^2c^2\left(6Ad^2-3Bcd+c^2C\right)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bc\cdots\right.\right.}{\cdots}\right.}{2(ad^2+bc^2)^3}}{ab}$$

$$\frac{a\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)-x\left(Ab^2c\left(bc^2-3ad^2\right)-a\left(-a\cdots\right.\right.}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{ab\left(\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(2a^2d^3(Cd-3cD)-abd\left(3Ad^3-9Bcd^2-9c^3D+11c^2Cd\right)+2b^2c^2\left(6Ad^2-3Bcd+c^2C\right)\right)}{\sqrt{ad^2+bc^2}}-\frac{\sqrt{a+bx^2}\left(2ad^2\left(-Bd^2-3c^2D\cdots\right.\right.}{\cdots}\right.}{2(ad^2+bc^2)^3}}{ab}$$

$$\frac{a\left(-a^2d^2(Cd-3cD)+ab\left(Ad^3-3Bcd^2+c^3(-D)+3c^2Cd\right)+b^2c^2(Bc-3Ad)\right)-x\left(Ab^2c\left(bc^2-3ad^2\right)-a\left(-a\cdots\right.\right.}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b^2*c^2*(B*c - 3*A*d) - a^2*d^2*(C*d - 3*c*D) + a*b*(3*c^2*C*d - 3*B
*c*d^2 + A*d^3 - c^3*D)) - (A*b^2*c*(b*c^2 - 3*a*d^2) - a*(b^2*c^2*(c*C -
3*B*d) - a^2*d^3*D - a*b*d*(3*c*C*d - B*d^2 - 3*c^2*D)))*x)/(a*(b*c^2 + a*
d^2)^3*Sqrt[a + b*x^2])) + (-1/2*(a*b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*
Sqrt[a + b*x^2])/((b*c^2 + a*d^2)^2*(c + d*x)^2) - (a*b*(-(((2*a*d^2*(2*c*
C*d - B*d^2 - 3*c^2*D) - b*c*(3*c^2*C*d - 5*B*c*d^2 + 7*A*d^3 - c^3*D))*Sq
rt[a + b*x^2])/(c + d*x)) + ((2*b^2*c^2*(c^2*C - 3*B*c*d + 6*A*d^2) + 2*a^
2*d^3*(C*d - 3*c*D) - a*b*d*(11*c^2*C*d - 9*B*c*d^2 + 3*A*d^3 - 9*c^3*D))*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/Sqrt[b*c^2 +
 a*d^2]))/(2*(b*c^2 + a*d^2)^3))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1829 vs. $2(430) = 860$.

Time = 1.51 (sec) , antiderivative size = 1830, normalized size of antiderivative = 4.08

method	result	size
default	Expression too large to display	1830

input

```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
D/d^3/(b*x^2+a)^(1/2)/a*x+1/d^4*(C*d-3*D*c)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x
+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(
1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/d^5*(B*d
^2-2*C*c*d+3*D*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)
*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2
)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)
^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*
b/(a*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2
/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/d^6*(A*d^3-
B*c*d^2+C*c^2*d-D*c^3)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2/(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+5/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*
c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b
*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(429) = 858$.

Time = 25.04 (sec) , antiderivative size = 3335, normalized size of antiderivative = 7.44

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3156 vs. $2(429) = 858$.

Time $= 0.25$ (sec) , antiderivative size $= 3156$, normalized size of antiderivative $= 7.04$

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

output
```
-15/2*D*b^3*c^6*x/(sqrt(b*x^2 + a)*a*b^3*c^6*d^3 + 3*sqrt(b*x^2 + a)*a^2*b
^2*c^4*d^5 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^7 + sqrt(b*x^2 + a)*a^4*d^9) +
15/2*C*b^3*c^5*x/(sqrt(b*x^2 + a)*a*b^3*c^6*d^2 + 3*sqrt(b*x^2 + a)*a^2*b^
2*c^4*d^4 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^6 + sqrt(b*x^2 + a)*a^4*d^8) - 1
5/2*B*b^3*c^4*x/(sqrt(b*x^2 + a)*a*b^3*c^6*d + 3*sqrt(b*x^2 + a)*a^2*b^2*c
^4*d^3 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^5 + sqrt(b*x^2 + a)*a^4*d^7) - 15/2
*D*b^2*c^5/(sqrt(b*x^2 + a)*b^3*c^6*d^2 + 3*sqrt(b*x^2 + a)*a*b^2*c^4*d^4
+ 3*sqrt(b*x^2 + a)*a^2*b*c^2*d^6 + sqrt(b*x^2 + a)*a^3*d^8) + 15/2*A*b^3*
c^3*x/(sqrt(b*x^2 + a)*a*b^3*c^6 + 3*sqrt(b*x^2 + a)*a^2*b^2*c^4*d^2 + 3*s
qrt(b*x^2 + a)*a^3*b*c^2*d^4 + sqrt(b*x^2 + a)*a^4*d^6) + 31/2*D*b^2*c^4*x
/(sqrt(b*x^2 + a)*a*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^5 + sqrt(b
*x^2 + a)*a^3*d^7) + 15/2*C*b^2*c^4/(sqrt(b*x^2 + a)*b^3*c^6*d + 3*sqrt(b*
x^2 + a)*a*b^2*c^4*d^3 + 3*sqrt(b*x^2 + a)*a^2*b*c^2*d^5 + sqrt(b*x^2 + a)
*a^3*d^7) - 25/2*C*b^2*c^3*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^2 + 2*sqrt(b*x^2
 + a)*a^2*b*c^2*d^4 + sqrt(b*x^2 + a)*a^3*d^6) - 15/2*B*b^2*c^3/(sqrt(b*x^
2 + a)*b^3*c^6 + 3*sqrt(b*x^2 + a)*a*b^2*c^4*d^2 + 3*sqrt(b*x^2 + a)*a^2*b
*c^2*d^4 + sqrt(b*x^2 + a)*a^3*d^6) + 5/2*D*b*c^4/(sqrt(b*x^2 + a)*b^2*c^4
*d^3*x + 2*sqrt(b*x^2 + a)*a*b*c^2*d^5*x + sqrt(b*x^2 + a)*a^2*d^7*x + sqr
t(b*x^2 + a)*b^2*c^5*d^2 + 2*sqrt(b*x^2 + a)*a*b*c^3*d^4 + sqrt(b*x^2 + a)
*a^2*c*d^6) + 19/2*B*b^2*c^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d + 2*sqrt(b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(429) = 858$.

Time = 0.29 (sec) , antiderivative size = 1762, normalized size of antiderivative = 3.93

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac
")
```

output
```
-((C*a*b^5*c^9 - A*b^6*c^9 + 3*D*a^2*b^4*c^8*d - 3*B*a*b^5*c^8*d + 8*D*a^3
*b^3*c^6*d^3 - 8*B*a^2*b^4*c^6*d^3 - 6*C*a^3*b^3*c^5*d^4 + 6*A*a^2*b^4*c^5
*d^4 + 6*D*a^4*b^2*c^4*d^5 - 6*B*a^3*b^3*c^4*d^5 - 8*C*a^4*b^2*c^3*d^6 + 8
*A*a^3*b^3*c^3*d^6 - 3*C*a^5*b*c*d^8 + 3*A*a^4*b^2*c*d^8 - D*a^6*d^9 + B*a
^5*b*d^9)*x/(a*b^6*c^12 + 6*a^2*b^5*c^10*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4
*b^3*c^6*d^6 + 15*a^5*b^2*c^4*d^8 + 6*a^6*b*c^2*d^10 + a^7*d^12) - (D*a^2*
b^4*c^9 - B*a*b^5*c^9 - 3*C*a^2*b^4*c^8*d + 3*A*a*b^5*c^8*d - 8*C*a^3*b^3*
c^6*d^3 + 8*A*a^2*b^4*c^6*d^3 - 6*D*a^4*b^2*c^5*d^4 + 6*B*a^3*b^3*c^5*d^4
- 6*C*a^4*b^2*c^4*d^5 + 6*A*a^3*b^3*c^4*d^5 - 8*D*a^5*b*c^3*d^6 + 8*B*a^4*
b^2*c^3*d^6 - 3*D*a^6*c*d^8 + 3*B*a^5*b*c*d^8 + C*a^6*d^9 - A*a^5*b*d^9)/(
a*b^6*c^12 + 6*a^2*b^5*c^10*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4*b^3*c^6*d^6
+ 15*a^5*b^2*c^4*d^8 + 6*a^6*b*c^2*d^10 + a^7*d^12))/sqrt(b*x^2 + a) - (2*
C*b^2*c^4 + 9*D*a*b*c^3*d - 6*B*b^2*c^3*d - 11*C*a*b*c^2*d^2 + 12*A*b^2*c^
2*d^2 - 6*D*a^2*c*d^3 + 9*B*a*b*c*d^3 + 2*C*a^2*d^4 - 3*A*a*b*d^4)*arctan(
((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^3*
c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqrt(-b*c^2 - a*d^2)) -
 (2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*b^2*c^4*d^2 + 7*(sqrt(b)*x - sqrt(b*
x^2 + a))^3*D*a*b*c^3*d^3 - 4*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*b^2*c^3*d^
3 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a*b*c^2*d^4 + 6*(sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*b^2*c^2*d^4 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3\,(a+bx^2)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x^2)^(3/2)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^3 (a + bx^2)^{3/2}} \, dx = \int \frac{Dx^3 + C\,x^2 + Bx + A}{(dx + c)^3 (b\,x^2 + a)^{\frac{3}{2}}} dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

3.115 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{bB-aD}{3b^2(a+bx^2)^{3/2}} + \frac{(Ab-aC)x}{3ab(a+bx^2)^{3/2}} - \frac{D}{b^2\sqrt{a+bx^2}} + \frac{(2Ab+aC)x}{3a^2b\sqrt{a+bx^2}}$$

output
```
-1/3*(B*b-D*a)/b^2/(b*x^2+a)^(3/2)+1/3*(A*b-C*a)*x/a/b/(b*x^2+a)^(3/2)-D/b
^2/(b*x^2+a)^(1/2)+1/3*(2*A*b+C*a)*x/a^2/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2a^3D+2Ab^3x^3+ab^2x(3A+Cx^2)-a^2b(B+3Dx^2)}{3a^2b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^3*D + 2*A*b^3*x^3 + a*b^2*x*(3*A + C*x^2) - a^2*b*(B + 3*D*x^2))/(3*
a^2*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2345, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \ 2345$$

$$-\frac{\int -\frac{b\left(2A+\frac{aC}{b}\right)+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2Ab+aC+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2Ab+aC+3aDx}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 453$$

$$-\frac{3a^2D-bx(aC+2Ab)}{3a^2b^2\sqrt{a+bx^2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(3/2)) - (3*a^2*D
- b*(2*A*b + a*C)*x)/(3*a^2*b^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

method	result
gosper	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A a\,b^{2}x-B\,a^{2}b-2D a^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
trager	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A a\,b^{2}x-B\,a^{2}b-2D a^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
orering	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A a\,b^{2}x-B\,a^{2}b-2D a^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
default	$A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+C\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)+D\left(\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^3*x^3+C*a*b^2*x^3-3*D*a^2*b*x^2+3*A*a*b^2*x-B*a^2*b-2*D*a^3)/(b
*x^2+a)^(3/2)/a^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(3\,Da^2bx^2-3\,Aab^2x+2\,Da^3+Ba^2b-(Cab^2+2\,Ab^3)x^3\right)\sqrt{bx^2+a}}{3\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*D*a^2*b*x^2 - 3*A*a*b^2*x + 2*D*a^3 + B*a^2*b - (C*a*b^2 + 2*A*b^3
)*x^3)*sqrt(b*x^2 + a)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + D\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)
*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)) + D*Piecewise(
(-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*b*x**
2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (
x**4/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{Dx^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} -\frac{Cx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Cx}{3\,\sqrt{bx^2+a}ab}-\frac{2\,Da}{3\,(bx^2+a)^{\frac{3}{2}}b^2}-\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-D*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b
*x^2 + a)^(3/2)*a) - 1/3*C*x/((b*x^2 + a)^(3/2)*b) + 1/3*C*x/(sqrt(b*x^2 +
 a)*a*b) - 2/3*D*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(x\left(\frac{3\,D}{b}-\frac{(Cab^2+2\,Ab^3)x}{a^2b^2}\right)-\frac{3\,A}{a}\right)x+\frac{2\,Da^3+Ba^2b}{a^2b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((x*(3*D/b - (C*a*b^2 + 2*A*b^3)*x/(a^2*b^2)) - 3*A/a)*x + (2*D*a^3 +
 B*a^2*b)/(a^2*b^2))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,x\,(b\,x^2+a)+A\,a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}} - \frac{B}{3\,b\,(b\,x^2+a)^{3/2}} - \frac{(3\,b\,x^2+2\,a)\;D}{3\,b^2\,(b\,x^2+a)^{3/2}} + \frac{C\,x^3}{3\,a\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*x*(a + b*x^2) + A*a*x)/(3*a^2*(a + b*x^2)^(3/2)) - B/(3*b*(a + b*x^2)
^(3/2)) - ((2*a + 3*b*x^2)*D)/(3*b^2*(a + b*x^2)^(3/2)) + (C*x^3)/(3*a*(a
+ b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2d+3\sqrt{b\,x^2+a}\,a\,b^2x-\sqrt{b\,x^2+a}\,a\,b^2-3\sqrt{b\,x^2+a}\,abd\,x^2 -}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*d + 3*sqrt(a + b*x**2)*a*b**2*x - sqrt(a + b*x
**2)*a*b**2 - 3*sqrt(a + b*x**2)*a*b*d*x**2 + 2*sqrt(a + b*x**2)*b**3*x**3
 + sqrt(a + b*x**2)*b**2*c*x**3 - 2*sqrt(b)*a**2*b + sqrt(b)*a**2*c - 4*sq
rt(b)*a*b**2*x**2 + 2*sqrt(b)*a*b*c*x**2 - 2*sqrt(b)*b**3*x**4 + sqrt(b)*b
**2*c*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.116 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = -\frac{bB-aD}{5b^2(a+bx^2)^{5/2}} + \frac{(Ab-aC)x}{5ab(a+bx^2)^{5/2}} - \frac{D}{3b^2(a+bx^2)^{3/2}} + \frac{(4Ab+aC)x}{15a^2b(a+bx^2)^{3/2}} + \frac{2(4Ab+aC)x}{15a^3b\sqrt{a+bx^2}}$$

output
```
-1/5*(B*b-D*a)/b^2/(b*x^2+a)^(5/2)+1/5*(A*b-C*a)*x/a/b/(b*x^2+a)^(5/2)-1/3
*D/b^2/(b*x^2+a)^(3/2)+1/15*(4*A*b+C*a)*x/a^2/b/(b*x^2+a)^(3/2)+2/15*(4*A*
b+C*a)*x/a^3/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{-2a^4D+8Ab^4x^5+5a^2b^2x(3A+Cx^2)+2ab^3x^3(10A+Cx^2)-a^3b(3B+}{15a^3b^2(a+bx^2)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/2),x]
```

output
```
(-2*a^4*D + 8*A*b^4*x^5 + 5*a^2*b^2*x*(3*A + C*x^2) + 2*a*b^3*x^3*(10*A +
C*x^2) - a^3*b*(3*B + 5*D*x^2))/(15*a^3*b^2*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2345, 25, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{b\left(4A+\frac{aC}{b}\right)+5aDx}{b(bx^2+a)^{5/2}}dx}{5a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4Ab+aC+5aDx}{b(bx^2+a)^{5/2}}dx}{5a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Ab+aC+5aDx}{(bx^2+a)^{5/2}}dx}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{2(aC+4Ab)\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{5a^2D-bx(aC+4Ab)}{3ab(a+bx^2)^{3/2}}}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2x(aC+4Ab)}{3a^2\sqrt{a+bx^2}}-\frac{5a^2D-bx(aC+4Ab)}{3ab(a+bx^2)^{3/2}}}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/2),x]
```

output
```
-1/5*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(5/2)) + (-1/3*(5*
a^2*D - b*(4*A*b + a*C)*x)/(a*b*(a + b*x^2)^(3/2)) + (2*(4*A*b + a*C)*x)/(
3*a^2*Sqrt[a + b*x^2]))/(5*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

method	result
gosper	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15A\,a^2b^2x-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}\,a^3b^2}$
trager	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15A\,a^2b^2x-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}\,a^3b^2}$
orering	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15A\,a^2b^2x-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}\,a^3b^2}$
default	$A\left(\frac{x}{5a(b\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}}{a}\right)-\frac{B}{5b(b\,x^2+a)^{\frac{5}{2}}}+C\left(-\frac{x}{4b(b\,x^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{4x}{15a\left(b\,x^2 \cdots\right.}\right.}{4b} \cdots\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*A*b^4*x^5+2*C*a*b^3*x^5+20*A*a*b^3*x^3+5*C*a^2*b^2*x^3-5*D*a^3*b*x
^2+15*A*a^2*b^2*x-3*B*a^3*b-2*D*a^4)/(b*x^2+a)^(5/2)/a^3/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx =$$
$$-\frac{(5\,Da^3bx^2-15\,Aa^2b^2x-2\,(Cab^3+4\,Ab^4)x^5+2\,Da^4+3\,Ba^3b-5\,(Ca^2b^2+4\,Aab^3)x^3)\sqrt{bx^2+a}}{15\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(5*D*a^3*b*x^2 - 15*A*a^2*b^2*x - 2*(C*a*b^3 + 4*A*b^4)*x^5 + 2*D*a^
4 + 3*B*a^3*b - 5*(C*a^2*b^2 + 4*A*a*b^3)*x^3)*sqrt(b*x^2 + a)/(a^3*b^5*x^
6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(124) = 248$.

Time = 14.49 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.71

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-1/
(5*a**2*b*sqrt(a + b*x**2) + 10*a*b**2*x**2*sqrt(a + b*x**2) + 5*b**3*x**4
*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(1
5*a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15
*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b
*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*s
qrt(1 + b*x**2/a))) + D*Piecewise((-2*a/(15*a**2*b**2*sqrt(a + b*x**2) + 3
0*a*b**3*x**2*sqrt(a + b*x**2) + 15*b**4*x**4*sqrt(a + b*x**2)) - 5*b*x**2
/(15*a**2*b**2*sqrt(a + b*x**2) + 30*a*b**3*x**2*sqrt(a + b*x**2) + 15*b**
4*x**4*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{7/2}}\,dx = -\frac{Dx^2}{3\,(bx^2 + a)^{\frac{5}{2}}b} + \frac{8\,Ax}{15\,\sqrt{bx^2 + a}a^3} + \frac{4\,Ax}{15\,(bx^2 + a)^{\frac{3}{2}}a^2} + \frac{Ax}{5\,(bx^2 + a)^{\frac{5}{2}}a} - \frac{Cx}{5\,(bx^2 + a)^{\frac{5}{2}}b} + \frac{2\,Cx}{15\,\sqrt{bx^2 + a}a^2b} + \frac{Cx}{15\,(bx^2 + a)^{\frac{3}{2}}ab} - \frac{2\,Da}{15\,(bx^2 + a)^{\frac{5}{2}}b^2} - \frac{B}{5\,(bx^2 + a)^{\frac{5}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
-1/3*D*x^2/((b*x^2 + a)^(5/2)*b) + 8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A
*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*x/((b*x^2 + a)^(5/2)*a) - 1/5*C*x/((b*x
^2 + a)^(5/2)*b) + 2/15*C*x/(sqrt(b*x^2 + a)*a^2*b) + 1/15*C*x/((b*x^2 + a
)^(3/2)*a*b) - 2/15*D*a/((b*x^2 + a)^(5/2)*b^2) - 1/5*B/((b*x^2 + a)^(5/2)
*b)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{7/2}}\,dx = \frac{\left(\left(x\left(\frac{2\,(Cab^3 + 4\,Ab^4)x^2}{a^3b^2} + \frac{5\,(Ca^2b^2 + 4\,Aab^3)}{a^3b^2}\right) - \frac{5\,D}{b}\right)x + \frac{15\,A}{a}\right)x - \frac{2\,Da^4 + 3\,Ba^3b}{a^3b^2}}{15\,(bx^2 + a)^{\frac{5}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(((x*(2*(C*a*b^3 + 4*A*b^4)*x^2/(a^3*b^2) + 5*(C*a^2*b^2 + 4*A*a*b^3)
/(a^3*b^2)) - 5*D/b)*x + 15*A/a)*x - (2*D*a^4 + 3*B*a^3*b)/(a^3*b^2))/(b*x
^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{8\,A\,x\,(b\,x^2+a)^2+3\,A\,a^2\,x+4\,A\,a\,x\,(b\,x^2+a)}{15\,a^3\,(b\,x^2+a)^{5/2}} - \frac{B}{5\,b\,(b\,x^2+a)^{5/2}} - \frac{(5\,b\,x^2+2\,a)\,D}{15\,b^2\,(b\,x^2+a)^{5/2}} + \frac{C\,x^3}{3\,a\,(b\,x^2+a)^{5/2}} + \frac{2\,C\,b\,x^5}{15\,a^2\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(7/2),x)
```

output
```
(8*A*x*(a + b*x^2)^2 + 3*A*a^2*x + 4*A*a*x*(a + b*x^2))/(15*a^3*(a + b*x^2
)^(5/2)) - B/(5*b*(a + b*x^2)^(5/2)) - ((2*a + 5*b*x^2)*D)/(15*b^2*(a + b*
x^2)^(5/2)) + (C*x^3)/(3*a*(a + b*x^2)^(5/2)) + (2*C*b*x^5)/(15*a^2*(a + b
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3d+15\sqrt{b\,x^2+a}\,a^2b^2x-3\sqrt{b\,x^2+a}\,a^2b^2-5\sqrt{b\,x^2+a}\,a^2b\alpha}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*d + 15*sqrt(a + b*x**2)*a**2*b**2*x - 3*sqrt(a
 + b*x**2)*a**2*b**2 - 5*sqrt(a + b*x**2)*a**2*b*d*x**2 + 20*sqrt(a + b*x*
*2)*a*b**3*x**3 + 5*sqrt(a + b*x**2)*a*b**2*c*x**3 + 8*sqrt(a + b*x**2)*b*
*4*x**5 + 2*sqrt(a + b*x**2)*b**3*c*x**5 - 8*sqrt(b)*a**3*b - 2*sqrt(b)*a*
*3*c - 24*sqrt(b)*a**2*b**2*x**2 - 6*sqrt(b)*a**2*b*c*x**2 - 24*sqrt(b)*a*
b**3*x**4 - 6*sqrt(b)*a*b**2*c*x**4 - 8*sqrt(b)*b**4*x**6 - 2*sqrt(b)*b**3
*c*x**6)/(15*a**2*b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.117 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{9/2}}\,dx=-\frac{bB-aD}{7b^2\left(a+bx^2\right)^{7/2}}+\frac{(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}-\frac{D}{5b^2\left(a+bx^2\right)^{5/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
-1/7*(B*b-D*a)/b^2/(b*x^2+a)^(7/2)+1/7*(A*b-C*a)*x/a/b/(b*x^2+a)^(7/2)-1/5
*D/b^2/(b*x^2+a)^(5/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2+a)^(5/2)+4/105*(6*A
*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^4/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{-6a^5D+48Ab^5x^7+35a^3b^2x(3A+Cx^2)+8ab^4x^5(21A+Cx^2)+14a^2b^3x}{105a^4b^2\left(a+bx^2\right)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(9/2),x]
```

output
```
(-6*a^5*D + 48*A*b^5*x^7 + 35*a^3*b^2*x*(3*A + C*x^2) + 8*a*b^4*x^5*(21*A
+ C*x^2) + 14*a^2*b^3*x^3*(15*A + 2*C*x^2) - 3*a^4*b*(5*B + 7*D*x^2))/(105
*a^4*b^2*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2345, 25, 27, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{b\left(6A+\frac{aC}{b}\right)+7aDx}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+aC+7aDx}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6Ab+aC+7aDx}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{4(aC+6Ab)\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{4(aC+6Ab)\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)(aC+6Ab)}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(7*
a^2*D - b*(6*A*b + a*C)*x)/(a*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*(x/(
3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a))/(7*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

method	result
gosper	$\frac{48A\,b^{5}x^{7}+8Ca\,b^{4}x^{7}+168Aa\,b^{4}x^{5}+28C\,a^{2}b^{3}x^{5}+210A\,a^{2}b^{3}x^{3}+35C\,a^{3}b^{2}x^{3}-21Dx^{2}a^{4}b+105A\,a^{3}b^{2}x-15B\,a^{4}b-6Da^{5}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}b^{2}}$
trager	$\frac{48A\,b^{5}x^{7}+8Ca\,b^{4}x^{7}+168Aa\,b^{4}x^{5}+28C\,a^{2}b^{3}x^{5}+210A\,a^{2}b^{3}x^{3}+35C\,a^{3}b^{2}x^{3}-21Dx^{2}a^{4}b+105A\,a^{3}b^{2}x-15B\,a^{4}b-6Da^{5}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}b^{2}}$
orering	$\frac{48A\,b^{5}x^{7}+8Ca\,b^{4}x^{7}+168Aa\,b^{4}x^{5}+28C\,a^{2}b^{3}x^{5}+210A\,a^{2}b^{3}x^{3}+35C\,a^{3}b^{2}x^{3}-21Dx^{2}a^{4}b+105A\,a^{3}b^{2}x-15B\,a^{4}b-6Da^{5}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}b^{2}}$
default	$A\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)-\frac{B}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+C\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*b^5*x^7+8*C*a*b^4*x^7+168*A*a*b^4*x^5+28*C*a^2*b^3*x^5+210*A*a
^2*b^3*x^3+35*C*a^3*b^2*x^3-21*D*a^4*b*x^2+105*A*a^3*b^2*x-15*B*a^4*b-6*D*
a^5)/(b*x^2+a)^(7/2)/a^4/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{9/2}}\, dx =$$
$$-\frac{(21\, Da^4bx^2 - 8\,(Cab^4 + 6\,Ab^5)x^7 - 105\,Aa^3b^2x + 6\,Da^5 + 15\,Ba^4b - 28\,(Ca^2b^3 + 6\,Aab^4)x^5 - 35\,(Ca^3b}{105\,(a^4b^6x^8 + 4\,a^5b^5x^6 + 6\,a^6b^4x^4 + 4\,a^7b^3x^2 + a^8b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(21*D*a^4*b*x^2 - 8*(C*a*b^4 + 6*A*b^5)*x^7 - 105*A*a^3*b^2*x + 6*D
*a^5 + 15*B*a^4*b - 28*(C*a^2*b^3 + 6*A*a*b^4)*x^5 - 35*(C*a^3*b^2 + 6*A*a
^2*b^3)*x^3)*sqrt(b*x^2 + a)/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6*a^6*b^4*x^4
+ 4*a^7*b^3*x^2 + a^8*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(155) = 310$.

Time = 30.99 (sec) , antiderivative size = 2064, normalized size of antiderivative = 12.43

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{9/2}}\,dx = -\frac{Dx^2}{5\,(bx^2 + a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2 + a}a^4} + \frac{8\,Ax}{35\,(bx^2 + a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2 + a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2 + a)^{\frac{7}{2}}a} - \frac{Cx}{7\,(bx^2 + a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2 + a}a^3b} + \frac{4\,Cx}{105\,(bx^2 + a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2 + a)^{\frac{5}{2}}ab} - \frac{2\,Da}{35\,(bx^2 + a)^{\frac{7}{2}}b^2} - \frac{B}{7\,(bx^2 + a)^{\frac{7}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/5*D*x^2/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/(
(b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x
^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 +
a)^(5/2)*a*b) - 2/35*D*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x-\frac{21\,D}{b}\right)x+\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*
a*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x - 21*D/b)*x
+ 105*A/a)*x - 3*(2*D*a^5*b + 5*B*a^4*b^2)/(a^4*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{16\,A\,x}{35\,a^4\,\sqrt{b\,x^2+a}} - \frac{(7\,b\,x^2+2\,a)\,D}{35\,b^2\,(b\,x^2+a)^{7/2}} - \frac{B}{7\,b\,(b\,x^2+a)^{7/2}} + \frac{8\,A\,x}{35\,a^3\,(b\,x^2+a)^{3/2}} + \frac{6\,A\,x}{35\,a^2\,(b\,x^2+a)^{5/2}} + \frac{A\,x}{7\,a\,(b\,x^2+a)^{7/2}} - \frac{C\,x}{7\,b\,(b\,x^2+a)^{7/2}} + \frac{8\,C\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} + \frac{4\,C\,x}{105\,a^2\,b\,(b\,x^2+a)^{3/2}} + \frac{C\,x}{35\,a\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(9/2),x)
```

output
```
(16*A*x)/(35*a^4*(a + b*x^2)^(1/2)) - ((2*a + 7*b*x^2)*D)/(35*b^2*(a + b*x
^2)^(7/2)) - B/(7*b*(a + b*x^2)^(7/2)) + (8*A*x)/(35*a^3*(a + b*x^2)^(3/2)
) + (6*A*x)/(35*a^2*(a + b*x^2)^(5/2)) + (A*x)/(7*a*(a + b*x^2)^(7/2)) - (
C*x)/(7*b*(a + b*x^2)^(7/2)) + (8*C*x)/(105*a^3*b*(a + b*x^2)^(1/2)) + (4*
C*x)/(105*a^2*b*(a + b*x^2)^(3/2)) + (C*x)/(35*a*b*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{-6\sqrt{b\,x^2+a}\,a^4d+105\sqrt{b\,x^2+a}\,a^3b^2x-15\sqrt{b\,x^2+a}\,a^3b^2-21\sqrt{b\,x^2+a}\,a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**4*d + 105*sqrt(a + b*x**2)*a**3*b**2*x - 15*sqrt
(a + b*x**2)*a**3*b**2 - 21*sqrt(a + b*x**2)*a**3*b*d*x**2 + 210*sqrt(a +
b*x**2)*a**2*b**3*x**3 + 35*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 168*sqrt(a
 + b*x**2)*a*b**4*x**5 + 28*sqrt(a + b*x**2)*a*b**3*c*x**5 + 48*sqrt(a + b
*x**2)*b**5*x**7 + 8*sqrt(a + b*x**2)*b**4*c*x**7 - 48*sqrt(b)*a**4*b - 8*
sqrt(b)*a**4*c - 192*sqrt(b)*a**3*b**2*x**2 - 32*sqrt(b)*a**3*b*c*x**2 - 2
88*sqrt(b)*a**2*b**3*x**4 - 48*sqrt(b)*a**2*b**2*c*x**4 - 192*sqrt(b)*a*b*
*4*x**6 - 32*sqrt(b)*a*b**3*c*x**6 - 48*sqrt(b)*b**5*x**8 - 8*sqrt(b)*b**4
*c*x**8)/(105*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**
3*x**6 + b**4*x**8))
```

3.118 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = -\frac{199}{27}\sqrt{2+3x^2}-\frac{1}{3}x\sqrt{2+3x^2}+6x^3\sqrt{2+3x^2}+\frac{76}{81}\left(2+3x^2\right)^{3/2}+\frac{32}{135}\left(2+3x^2\right)^{5/2}+\frac{5\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-199/27*(3*x^2+2)^(1/2)-1/3*x*(3*x^2+2)^(1/2)+6*x^3*(3*x^2+2)^(1/2)+76/81*
(3*x^2+2)^(3/2)+32/135*(3*x^2+2)^(5/2)+5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\sqrt{2+3x^2}\left(-1841-135x+2292x^2+2430x^3+864x^4\right)-\frac{5\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4))/405 - (5
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2185, 27, 687, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{60}\int -\frac{4(17-39x)(2x+1)^3}{\sqrt{3x^2+2}}dx+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}(2x+1)^4\sqrt{3x^2+2}-\frac{1}{15}\int \frac{(17-39x)(2x+1)^3}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{13}{4}(2x+1)^3\sqrt{3x^2+2}-\frac{1}{12}\int \frac{3(2x+1)^2(19x+224)}{\sqrt{3x^2+2}}dx\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{13}{4}(2x+1)^3\sqrt{3x^2+2}-\frac{1}{4}\int \frac{(2x+1)^2(19x+224)}{\sqrt{3x^2+2}}dx\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{1}{9}\int \frac{2(2x+1)(2073x+932)}{\sqrt{3x^2+2}}dx-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+2}(2x+1)^3\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\int \frac{(2x+1)(2073x+932)}{\sqrt{3x^2+2}}dx-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+2}(2x+1)^3\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \textbf{676}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\left(-450\int \frac{1}{\sqrt{3x^2+2}}dx+691\sqrt{3x^2+2}x+\frac{3937}{3}\sqrt{3x^2+2}\right)-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+\right.$$
$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \textbf{222}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\left(-150\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+691\sqrt{3x^2+2}x+\frac{3937}{3}\sqrt{3x^2+2}\right)-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3\right.$$
$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 + ((13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/4 +
 ((-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (2*((3937*Sqrt[2 + 3*x^2])/3 + 691
*x*Sqrt[2 + 3*x^2] - 150*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9)/4)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\left(864 x^{4}+2430 x^{3}+2292 x^{2}-135 x -1841\right) \sqrt{3 x^{2}+2}}{405}+\frac{5 \operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}$
trager	$\left(\frac{32}{15} x^{4}+6 x^{3}+\frac{764}{135} x^{2}-\frac{1}{3} x -\frac{1841}{405}\right) \sqrt{3 x^{2}+2}-\frac{5 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln \left(-\operatorname{RootOf}\left(\_Z^{2}-3\right) \sqrt{3 x^{2}+2}+3 x \right)}{9}$
default	$\frac{5 \operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}-\frac{1841 \sqrt{3 x^{2}+2}}{405}-\frac{x \sqrt{3 x^{2}+2}}{3}+\frac{764 x^{2} \sqrt{3 x^{2}+2}}{135}+6 x^{3} \sqrt{3 x^{2}+2}+\frac{32 x^{4} \sqrt{3 x^{2}+2}}{15}$
meijerg	$\frac{\sqrt{3}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)}{3}+\frac{34 \sqrt{3}\, \left(\frac{\sqrt{\pi}\, x \sqrt{2}\, \sqrt{3}\, \sqrt{\frac{3 x^{2}}{2}+1}}{2}-\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)\right)}{9 \sqrt{\pi}}+\frac{3 \sqrt{2}\, \left(-2 \sqrt{\pi}+2 \sqrt{\pi}\, \sqrt{\frac{3 x^{2}}{2}+1}\right)}{2 \sqrt{\pi}}+$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*(864*x^4+2430*x^3+2292*x^2-135*x-1841)*(3*x^2+2)^(1/2)+5/9*arcsinh(1
/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2} + \frac{5}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/1
8*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{32x^4\sqrt{3x^2+2}}{15} + 6x^3\sqrt{3x^2+2} + \frac{764x^2\sqrt{3x^2+2}}{135} - \frac{x\sqrt{3x^2+2}}{3} - \frac{1841\sqrt{3x^2+2}}{405} + \frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x*
*2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)
*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{32}{15}\sqrt{3\,x^2+2}x^4 + 6\sqrt{3\,x^2+2}x^3 + \frac{764}{135}\sqrt{3\,x^2+2}x^2 - \frac{1}{3}\sqrt{3\,x^2+2}x + \frac{5}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{1841}{405}\sqrt{3\,x^2+2}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2
)*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/
405*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{1}{405}\left(3\left(2\left(9\left(16\,x+45\right)x+382\right)x-45\right)x-1841\right)\sqrt{3\,x^2+2}$$
$$-\frac{5}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/
9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$
$$+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)
/45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{32\sqrt{3x^2+2}\,x^4}{15}+6\sqrt{3x^2+2}\,x^3+\frac{764\sqrt{3x^2+2}\,x^2}{135}-\frac{\sqrt{3x^2+2}\,x}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input

```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

output

```
(864*sqrt(3*x**2 + 2)*x**4 + 2430*sqrt(3*x**2 + 2)*x**3 + 2292*sqrt(3*x**2
 + 2)*x**2 - 135*sqrt(3*x**2 + 2)*x - 1841*sqrt(3*x**2 + 2) + 225*sqrt(3)*
log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/405
```

3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = -\frac{35}{9}\sqrt{2+3x^2}+2x\sqrt{2+3x^2}+\frac{4}{3}x^3\sqrt{2+3x^2}+\frac{28}{27}\left(2+3x^2\right)^{3/2}-\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)$$

output
```
-35/9*(3*x^2+2)^(1/2)+2*x*(3*x^2+2)^(1/2)+4/3*x^3*(3*x^2+2)^(1/2)+28/27*(3
*x^2+2)^(3/2)-arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\sqrt{2+3x^2}\left(-49+54x+84x^2+36x^3\right)+\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 + Sqrt[3]*Log[-(Sqrt[3
]*x) + Sqrt[2 + 3*x^2]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2185, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{\sqrt{3x^2+2}} \, dx$$

$$\Big\downarrow \ 2185$$

$$\frac{1}{48} \int -\frac{24(2-5x)(2x+1)^2}{\sqrt{3x^2+2}}dx + \frac{1}{6}\sqrt{3x^2+2}(2x+1)^3$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2+2} - \frac{1}{2}\int \frac{(2-5x)(2x+1)^2}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \ 687$$

$$\frac{1}{2}\left(\frac{5}{9}(2x+1)^2\sqrt{3x^2+2} - \frac{1}{9}\int \frac{2(2x+1)(3x+29)}{\sqrt{3x^2+2}}dx\right) + \frac{1}{6}\sqrt{3x^2+2}(2x+1)^3$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{5}{9}(2x+1)^2\sqrt{3x^2+2} - \frac{2}{9}\int \frac{(2x+1)(3x+29)}{\sqrt{3x^2+2}}dx\right) + \frac{1}{6}\sqrt{3x^2+2}(2x+1)^3$$

$$\Big\downarrow \ 676$$

$$\frac{1}{2}\left(\frac{5}{9}(2x+1)^2\sqrt{3x^2+2} - \frac{2}{9}\left(27\int \frac{1}{\sqrt{3x^2+2}}dx + \sqrt{3x^2+2}x + \frac{61}{3}\sqrt{3x^2+2}\right)\right) + \frac{1}{6}\sqrt{3x^2+2}(2x+1)^3$$

$$\Big\downarrow \ 222$$

$$\frac{1}{2}\left(\frac{5}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{2}{9}\left(9\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\sqrt{3x^2+2}x+\frac{61}{3}\sqrt{3x^2+2}\right)\right)+\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 + ((5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (2*
((61*Sqrt[2 + 3*x^2])/3 + x*Sqrt[2 + 3*x^2] + 9*Sqrt[3]*ArcSinh[Sqrt[3/2]*
x]))/9)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

method	result
risch	$\frac{(36x^3+84x^2+54x-49)\sqrt{3x^2+2}}{27}-\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}$
trager	$\left(\frac{4}{3}x^3+\frac{28}{9}x^2+2x-\frac{49}{27}\right)\sqrt{3x^2+2}+\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3\right.$
default	$-\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}-\frac{49\sqrt{3x^2+2}}{27}+2x\sqrt{3x^2+2}+\frac{28x^2\sqrt{3x^2+2}}{9}+\frac{4x^3\sqrt{3x^2+2}}{3}$
meijerg	$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{3}+\frac{20\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\sqrt{3}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}+\frac{28\sqrt{2}\left(\right.}{}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*(36*x^3+84*x^2+54*x-49)*(3*x^2+2)^(1/2)-arcsinh(1/2*6^(1/2)*x)*3^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(36\,x^3+84\,x^2+54\,x-49\right)\sqrt{3\,x^2+2} + \frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(
3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{4x^3\sqrt{3x^2+2}}{3} + \frac{28x^2\sqrt{3x^2+2}}{9} + 2x\sqrt{3x^2+2} - \frac{49\sqrt{3x^2+2}}{27} - \sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
 2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{4}{3}\sqrt{3\,x^2+2}x^3 + \frac{28}{9}\sqrt{3\,x^2+2}x^2 + 2\sqrt{3\,x^2+2}x - \sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{49}{27}\sqrt{3\,x^2+2}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
 sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(6\left(2\left(3\,x+7\right)x+9\right)x-49\right)\sqrt{3\,x^2+2} + \sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)
*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(4\,x^3+\frac{28\,x^2}{3}+6\,x-\frac{49}{9}\right)}{3} - \sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*
asinh((6^(1/2)*x)/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{4\sqrt{3x^2+2}\,x^3}{3} + \frac{28\sqrt{3x^2+2}\,x^2}{9} + 2\sqrt{3x^2+2}\,x - \frac{49\sqrt{3x^2+2}}{27} - \sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

output
```
(36*sqrt(3*x**2 + 2)*x**3 + 84*sqrt(3*x**2 + 2)*x**2 + 54*sqrt(3*x**2 + 2)
*x - 49*sqrt(3*x**2 + 2) - 27*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/s
qrt(2)))/27
```

3.120 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = -\frac{1}{9}\sqrt{2+3x^2}+\frac{5}{3}x\sqrt{2+3x^2}+\frac{8}{27}\left(2+3x^2\right)^{3/2}-\frac{7\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-1/9*(3*x^2+2)^(1/2)+5/3*x*(3*x^2+2)^(1/2)+8/27*(3*x^2+2)^(3/2)-7/9*arcsin
h(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\sqrt{2+3x^2}\left(13+45x+24x^2\right)+\frac{7\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2))/27 + (7*Log[-(Sqrt[3]*x) + Sqrt[2 +
 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2185, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{36}\int -\frac{28(1-3x)(2x+1)}{\sqrt{3x^2+2}}dx+\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\int \frac{(1-3x)(2x+1)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{676}$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\left(3\int \frac{1}{\sqrt{3x^2+2}}dx-\sqrt{3x^2+2}x-\frac{1}{3}\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\left(\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\sqrt{3x^2+2}x-\frac{1}{3}\sqrt{3x^2+2}\right)$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (7*(-1/3*Sqrt[2 + 3*x^2] - x*Sqrt[2 +
3*x^2] + Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

method	result
risch	$\frac{(24x^2+45x+13)\sqrt{3x^2+2}}{27}-\frac{7\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
default	$-\frac{7\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{13\sqrt{3x^2+2}}{27}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{8x^2\sqrt{3x^2+2}}{9}$
trager	$\left(\frac{8}{9}x^2+\frac{5}{3}x+\frac{13}{27}\right)\sqrt{3x^2+2}+\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
meijerg	$\frac{\sqrt{3}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{3}+\frac{10\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(\frac{4\sqrt{}}{3}\right.}{}$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*(24*x^2+45*x+13)*(3*x^2+2)^(1/2)-7/9*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(24\,x^2+45\,x+13\right)\sqrt{3\,x^2+2} + \frac{7}{18}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(
3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{8x^2\sqrt{3x^2+2}}{9} + \frac{5x\sqrt{3x^2+2}}{3} + \frac{13\sqrt{3x^2+2}}{27} - \frac{7\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/2
7 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{8}{9}\sqrt{3x^2+2}x^2 + \frac{5}{3}\sqrt{3x^2+2}x - \frac{7}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{13}{27}\sqrt{3x^2+2}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*
sqrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(3\left(8\,x+15\right)x+13\right)\sqrt{3\,x^2+2} + \frac{7}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x +
sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{8\,x^2}{3}+5\,x+\frac{13}{9}\right)}{3} - \frac{7\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh(
(6^(1/2)*x)/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{8\sqrt{3x^2+2}\,x^2}{9} + \frac{5\sqrt{3x^2+2}\,x}{3} + \frac{13\sqrt{3x^2+2}}{27} - \frac{7\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

output
```
(24*sqrt(3*x**2 + 2)*x**2 + 45*sqrt(3*x**2 + 2)*x + 13*sqrt(3*x**2 + 2) -
21*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/27
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 67

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}$$

output
```
2/3*(3*x^2+2)^(1/2)+1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(
4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{11}}+2\sqrt{\frac{3}{11}}x-\frac{2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
(2*Sqrt[2 + 3*x^2])/3 + ArcTanh[Sqrt[3/11] + 2*Sqrt[3/11]*x - (2*Sqrt[2 +
3*x^2])/Sqrt[11]]/Sqrt[11] - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]/(2*Sqrt[3
])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2185, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{12}\int \frac{12(x+1)}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{1}{2}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{2}{3}\sqrt{3x^2+2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
(2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*
x)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result
default	$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^{2}+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^{2}+5-12x}}\right)}{22}$
risch	$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^{2}+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^{2}+5-12x}}\right)}{22}$
trager	$\frac{2\sqrt{3x^{2}+2}}{3}+\frac{\operatorname{RootOf}\left(\_Z^{2}-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}-11\right)x+11\sqrt{3x^{2}+2}-4\operatorname{RootOf}\left(\_Z^{2}-11\right)}{1+2x}\right)}{22}-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\operatorname{Roo}\right.}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+2/3*(3*x^2+2)^(1/2)-1/22*11^(1/2)*arcta
nh(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$$
$$= \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)$$
$$+ \frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + \frac{2}{3}\sqrt{3x^2+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*l
og(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x
 + 1)) + 2/3*sqrt(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{22}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|} - \frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2
*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(1
1) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{\sqrt{11}\left(2\ \ln\left(x+\frac{1}{2}\right)-2\ \ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{44} + \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{6}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

output
```
(11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)
*3^(1/2)*x)/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{\sqrt{11}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{11}-\sqrt{3}}\right)i}{22} + \frac{2\sqrt{3x^2+2}}{3} + \frac{\sqrt{11}\log\left(4\sqrt{3x^2+2}\,\sqrt{3}\,x+\sqrt{33}+12x^2-3\right)}{44} - \frac{\sqrt{11}\log\left(\frac{2\sqrt{3x^2+2}+\sqrt{11}+2\sqrt{3}\,x+\sqrt{3}}{\sqrt{2}}\right)}{22} + \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{6}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x)
```

output
```
(6*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(11) - sqrt(3
)))*i + 88*sqrt(3*x**2 + 2) + 3*sqrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x
+ sqrt(33) + 12*x**2 - 3) - 6*sqrt(11)*log((2*sqrt(3*x**2 + 2) + sqrt(11)
+ 2*sqrt(3)*x + sqrt(3))/sqrt(2)) + 22*sqrt(3)*log((sqrt(3*x**2 + 2) + sqr
t(3)*x)/sqrt(2)))/132
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx=-\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

output
```
-1/11*(3*x^2+2)^(1/2)/(1+2*x)+1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arc
tanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx=-\frac{\sqrt{2+3x^2}}{11+22x}-\frac{8\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{11\sqrt{11}}-\frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
-(Sqrt[2 + 3*x^2]/(11 + 22*x)) - (8*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqr
t[2 + 3*x^2])/Sqrt[11]])/(11*Sqrt[11]) - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2
]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2182, 25, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 2182}$$

$$-\frac{1}{11}\int -\frac{22x+7}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{11}\int \frac{22x+7}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{1}{11}\left(11\int \frac{1}{\sqrt{3x^2+2}}dx-4\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{11}\left(\frac{11\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}-4\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{1}{11}\left(4\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{11\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{11}\left(\frac{11\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
-1/11*Sqrt[2 + 3*x^2]/(1 + 2*x) + ((11*ArcSinh[Sqrt[3/2]*x])/Sqrt[3] + (4*
ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/Sqrt[11])/11
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{121}$
default	$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}-\frac{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}{22\left(\frac{1}{2}+x\right)}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{121}$
trager	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}-\frac{4\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{121}-\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\right.$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/11*(3*x^2+2)^(1/2)/(1+2*x)+1/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+4/121*11^
(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$
$$=\frac{121\sqrt{3}(2x+1)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+12\sqrt{11}(2x+1)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)}{726\,(2x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 +
12*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{4}{121}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|} - \frac{2\sqrt{6}}{3\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2+2}}{11\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/
abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(56) = 112$.

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.69

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{121\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{2\sqrt{11}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}{22\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
4/121*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqr
t(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3
) + 2*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + 2*sqrt(11)/(2*x + 1))/(sqr
t(3) + sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)))/sgn(
1/(2*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1
))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{3} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{121} + \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{22\left(x+\frac{1}{2}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

output
```
(3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
 (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 -
(3^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$
$$= \frac{-66\sqrt{3x^2+2}+48\sqrt{11}\log\left(-\sqrt{3x^2+2}\,\sqrt{11}+3x-4\right)x+24\sqrt{11}\log\left(-\sqrt{3x^2+2}\,\sqrt{11}+3x-4\right)-48}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x)
```

output
```
( - 66*sqrt(3*x**2 + 2) + 48*sqrt(11)*log( - sqrt(3*x**2 + 2)*sqrt(11) + 3
*x - 4)*x + 24*sqrt(11)*log( - sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4) - 48*s
qrt(11)*log(2*x + 1)*x - 24*sqrt(11)*log(2*x + 1) - 242*sqrt(3)*log(sqrt(3
*x**2 + 2) - sqrt(3)*x)*x - 121*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)
+ 242*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x + 121*sqrt(3)*log(sqrt(3
*x**2 + 2) + sqrt(3)*x))/(726*(2*x + 1))
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
-1/22*(3*x^2+2)^(1/2)/(1+2*x)^2+13*(3*x^2+2)^(1/2)/(242+484*x)-103/1331*ar
ctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{\frac{11(1+13x)\sqrt{2+3x^2}}{(1+2x)^2}+206\sqrt{11}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{1331}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 + 206*Sqrt[11]*ArcTanh[(Sqrt[
3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/1331
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{1}{22}\int -\frac{41x+14}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{22}\int \frac{41x+14}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{22}\left(\frac{206}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{13\sqrt{3x^2+2}}{11(2x+1)}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{22}\left(\frac{13\sqrt{3x^2+2}}{11(2x+1)}-\frac{206}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{22}\left(\frac{13\sqrt{3x^2+2}}{11(2x+1)}-\frac{206\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
-1/22*Sqrt[2 + 3*x^2]/(1 + 2*x)^2 + ((13*Sqrt[2 + 3*x^2])/(11*(1 + 2*x)) -
 (206*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11]))/22
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{39x^3+3x^2+26x+2}{121(1+2x)^2\sqrt{3x^2+2}}-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{1331}$	65
trager	$\frac{(13x+1)\sqrt{3x^2+2}}{121(1+2x)^2}-\frac{103\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x-4\,\mathrm{RootOf}\left(\_Z^2-11\right)-11\sqrt{3x^2+2}}{1+2x}\right)}{1331}$	72
default	$-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{1331}-\frac{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}{88\left(\frac{1}{2}+x\right)^2}+\frac{13\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}{484\left(\frac{1}{2}+x\right)}$	74

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/121*(39*x^3+3*x^2+26*x+2)/(1+2*x)^2/(3*x^2+2)^(1/2)-103/1331*11^(1/2)*ar
ctanh(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\sqrt{11}(4x^2+4x+1)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+22\sqrt{3x^2+2}(13x+1)}{2662\,(4x^2+4x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
 - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x
+ 1))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103}{1331}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2+2}}{22\,(4\,x^2+4\,x+1)} + \frac{13\,\sqrt{3\,x^2+2}}{242\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(
2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.34

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{72\,\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-13\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-168\,\sqrt{3}x+104\,\sqrt{3}+168\,\sqrt{3\,x^2+2}}{484\,\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(7
2*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
))^2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sq
rt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\sqrt{11}\ln\left(x+\frac{1}{2}\right)}{1331} - \frac{103\sqrt{11}\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{88\left(x^2+x+\frac{1}{4}\right)} + \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{484\left(x+\frac{1}{2}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

output
```
(103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x +
x^2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{143\sqrt{3x^2+2}\,x+11\sqrt{3x^2+2}+412\sqrt{11}\log\left(\sqrt{3x^2+2}\,\sqrt{11}+3x-4\right)x^2+412\sqrt{11}\log\left(\sqrt{3x^2+2}\,\sqrt{11}\right.}{5}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x)
```

output
```
(143*sqrt(3*x**2 + 2)*x + 11*sqrt(3*x**2 + 2) + 412*sqrt(11)*log(sqrt(3*x*
*2 + 2)*sqrt(11) + 3*x - 4)*x**2 + 412*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(
11) + 3*x - 4)*x + 103*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4) -
 412*sqrt(11)*log(2*x + 1)*x**2 - 412*sqrt(11)*log(2*x + 1)*x - 103*sqrt(1
1)*log(2*x + 1))/(1331*(4*x**2 + 4*x + 1))
```

3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{199}{27\sqrt{2+3x^2}} + \frac{31x}{6\sqrt{2+3x^2}} + \frac{76}{27}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{81}(2+3x^2)^{3/2} - \frac{38\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
199/27/(3*x^2+2)^(1/2)+31/6*x/(3*x^2+2)^(1/2)+76/27*(3*x^2+2)^(1/2)+4*x*(3
*x^2+2)^(1/2)+32/81*(3*x^2+2)^(3/2)-38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{2362+2133x+2136x^2+1944x^3+576x^4}{162\sqrt{2+3x^2}} + \frac{38\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4)/(162*Sqrt[2 + 3*x^2]) + (3
8*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2345, 27, 2346, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{1}{2}\int \frac{4\left(-48x^3-108x^2-70x+21\right)}{9\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\int \frac{-48x^3-108x^2-70x+21}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{9}\int \frac{3\left(-324x^2-146x+63\right)}{\sqrt{3x^2+2}}dx-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\int \frac{-324x^2-146x+63}{\sqrt{3x^2+2}}dx-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{2346}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(\frac{1}{6}\int \frac{6(171-146x)}{\sqrt{3x^2+2}}dx-54x\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(\int \frac{171-146x}{\sqrt{3x^2+2}}dx-54x\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \; 455$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(171\int\frac{1}{\sqrt{3x^2+2}}dx-54\sqrt{3x^2+2}x-\frac{146}{3}\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \; 222$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(57\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-54\sqrt{3x^2+2}x-\frac{146}{3}\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(398 + 279*x)/(54*Sqrt[2 + 3*x^2]) - (2*((-16*x^2*Sqrt[2 + 3*x^2])/3 + ((-
146*Sqrt[2 + 3*x^2])/3 - 54*x*Sqrt[2 + 3*x^2] + 57*Sqrt[3]*ArcSinh[Sqrt[3/
2]*x])/3))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

method	result
risch	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}-\frac{38\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}+\frac{38\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{79x}{6\sqrt{3x^2+2}}-\frac{38\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{1181}{81\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{32x^4}{9\sqrt{3x^2+2}}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{34\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/162*(576*x^4+1944*x^3+2136*x^2+2133*x+2362)/(3*x^2+2)^(1/2)-38/9*arcsinh
(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{342\,\sqrt{3}(3\,x^2+2)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(576\,x^4+1944\,x^3+2}{162\,(3\,x^2+2)}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
+ (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2
+ 2)
```

Sympy [F]

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx &= \frac{32\,x^4}{9\,\sqrt{3\,x^2+2}}+\frac{12\,x^3}{\sqrt{3\,x^2+2}}\\&\quad+\frac{356\,x^2}{27\,\sqrt{3\,x^2+2}}-\frac{38}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)+\frac{79\,x}{6\,\sqrt{3\,x^2+2}}+\frac{1181}{81\,\sqrt{3\,x^2+2}}\end{aligned}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2
+ 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181
/81/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(8\,(3\,(8\,x+27)x+89)x+711)x+2362}{162\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)
*x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 16.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3} - \frac{38\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\,\sqrt{6}\,\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\left(1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*a
sinh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x
^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2
)*279i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{3/2}} \, dx = \frac{576\sqrt{3x^2+2}\,x^4 + 1944\sqrt{3x^2+2}\,x^3 + 2136\sqrt{3x^2+2}\,x^2 + 2133\sqrt{3x^2+2}$$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

output
```
(576*sqrt(3*x**2 + 2)*x**4 + 1944*sqrt(3*x**2 + 2)*x**3 + 2136*sqrt(3*x**2
 + 2)*x**2 + 2133*sqrt(3*x**2 + 2)*x + 2362*sqrt(3*x**2 + 2) - 2052*sqrt(3
)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 - 1368*sqrt(3)*log((sqr
t(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) + 1161*sqrt(3)*x**2 + 774*sqrt(3))/(16
2*(3*x**2 + 2))
```

3.125 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{35}{9\sqrt{2+3x^2}} - \frac{47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
35/9/(3*x^2+2)^(1/2)-47/18*x/(3*x^2+2)^(1/2)+28/9*(3*x^2+2)^(1/2)+8/9*x*(3
*x^2+2)^(1/2)+4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{182-15x+168x^2+48x^3}{18\sqrt{2+3x^2}} - \frac{4\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(182 - 15*x + 168*x^2 + 48*x^3)/(18*Sqrt[2 + 3*x^2]) - (4*Log[-(Sqrt[3]*x)
 + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2345, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{70-47x}{18\sqrt{3x^2+2}}-\frac{1}{2}\int -\frac{8\left(12x^2+21x+7\right)}{9\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{9}\int \frac{12x^2+21x+7}{\sqrt{3x^2+2}}dx+\frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{2346}$$

$$\frac{4}{9}\left(\frac{1}{6}\int \frac{18(7x+1)}{\sqrt{3x^2+2}}dx+2\sqrt{3x^2+2}x\right)+\frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{9}\left(3\int \frac{7x+1}{\sqrt{3x^2+2}}dx+2\sqrt{3x^2+2}x\right)+\frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{4}{9}\left(3\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{7}{3}\sqrt{3x^2+2}\right)+2\sqrt{3x^2+2}x\right)+\frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{4}{9}\left(3\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{7}{3}\sqrt{3x^2+2}\right)+2\sqrt{3x^2+2}x\right)+\frac{70-47x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (4*(2*x*Sqrt[2 + 3*x^2] + 3*((7*Sqrt[2
+ 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3])))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

method	result
risch	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$-\frac{5x}{6\sqrt{3x^2+2}}+\frac{4\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{28x^2}{3\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}+\frac{8x^3}{3\sqrt{3x^2+2}}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{20\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{28\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(48*x^3+168*x^2-15*x+182)/(3*x^2+2)^(1/2)+4/9*arcsinh(1/2*6^(1/2)*x)*
3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{4\,\sqrt{3}(3\,x^2+2)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+\left(48\,x^3+168\,x^2-15\right.}{18\left(3\,x^2+2\right)}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^2 \cdot (4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{8\,x^3}{3\,\sqrt{3\,x^2+2}} + \frac{28\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{4}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{5\,x}{6\,\sqrt{3\,x^2+2}} + \frac{91}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1
/2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = -\frac{4}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(8\,(2\,x+7)x-5)x+182}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x -
5)*x + 182)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{4\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\left(\frac{8\,x}{3}+\frac{28}{3}\right)\,\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-630+\sqrt{6}\,141\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(630+\sqrt{6}\,141\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^
2 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2
)*1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*
(x^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{48\sqrt{3x^2+2}\,x^3+168\sqrt{3x^2+2}\,x^2-15\sqrt{3x^2+2}\,x+182\sqrt{3x^2+2}+24\sqrt{}}{54x^2+}$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

output
```
(48*sqrt(3*x**2 + 2)*x**3 + 168*sqrt(3*x**2 + 2)*x**2 - 15*sqrt(3*x**2 + 2
)*x + 182*sqrt(3*x**2 + 2) + 24*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)
/sqrt(2))*x**2 + 16*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) -
39*sqrt(3)*x**2 - 26*sqrt(3))/(18*(3*x**2 + 2))
```

3.126 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{1}{9\sqrt{2+3x^2}} - \frac{17x}{6\sqrt{2+3x^2}} + \frac{8}{9}\sqrt{2+3x^2} + \frac{10\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
1/9/(3*x^2+2)^(1/2)-17/6*x/(3*x^2+2)^(1/2)+8/9*(3*x^2+2)^(1/2)+10/9*arcsin
h(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{34-51x+48x^2}{18\sqrt{2+3x^2}} - \frac{10\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(34 - 51*x + 48*x^2)/(18*Sqrt[2 + 3*x^2]) - (10*Log[-(Sqrt[3]*x) + Sqrt[2
+ 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2345, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{2-51x}{18\sqrt{3x^2+2}}-\frac{1}{2}\int -\frac{4(4x+5)}{3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{4x+5}{\sqrt{3x^2+2}}dx+\frac{2-51x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{2}{3}\left(5\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{4}{3}\sqrt{3x^2+2}\right)+\frac{2-51x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{3}\left(\frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4}{3}\sqrt{3x^2+2}\right)+\frac{2-51x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (2*((4*Sqrt[2 + 3*x^2])/3 + (5*ArcSinh[S
qrt[3/2]*x])/Sqrt[3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

method	result	siz
risch	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$	35
default	$-\frac{17x}{6\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{8x^2}{3\sqrt{3x^2+2}}+\frac{17}{9\sqrt{3x^2+2}}$	51
trager	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}-\frac{10\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$	53
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{10\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,(6x^2+8)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}$	12

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(48*x^2-51*x+34)/(3*x^2+2)^(1/2)+10/9*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{10\sqrt{3}(3x^2+2)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+(48x^2-51x+34)\sqrt{}}{18(3x^2+2)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{30\sqrt{3}x^2\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{8x^2}{3\sqrt{3x^2+2}} - \frac{30x\sqrt{3x^2+2}}{27x^2+18}+\frac{x}{2\sqrt{3x^2+2}}+\frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{17}{9\sqrt{3x^2+2}}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2
+ 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20
*sqrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{8\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{10}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{17\,x}{6\,\sqrt{3\,x^2+2}} + \frac{17}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqr
t(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = -\frac{10}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(16\,x-17)x+34}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x +
34)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9} + \frac{10\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (
6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)
/(648*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{48\sqrt{3x^2+2}\,x^2-51\sqrt{3x^2+2}\,x+34\sqrt{3x^2+2}+60\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{54x^2+36}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

output
```
(48*sqrt(3*x**2 + 2)*x**2 - 51*sqrt(3*x**2 + 2)*x + 34*sqrt(3*x**2 + 2) +
60*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 + 40*sqrt(3)*l
og((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) - 51*sqrt(3)*x**2 - 34*sqrt(3))
/(18*(3*x**2 + 2))
```

3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-38+21x}{66\sqrt{2+3x^2}}-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

output
```
1/66*(-38+21*x)/(3*x^2+2)^(1/2)-2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2
+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-418+231x-12\sqrt{22+33x^2}\text{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{726\sqrt{2+3x^2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
(-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])
/(726*Sqrt[2 + 3*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2178, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{3/2}}dx$$
$$\Big\downarrow \text{2178}$$
$$-\frac{1}{6}\int -\frac{12}{11(2x+1)\sqrt{3x^2+2}}dx-\frac{38-21x}{66\sqrt{3x^2+2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{2}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{38-21x}{66\sqrt{3x^2+2}}$$
$$\Big\downarrow \text{488}$$
$$-\frac{2}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{38-21x}{66\sqrt{3x^2+2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}-\frac{38-21x}{66\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
-1/66*(38 - 21*x)/Sqrt[2 + 3*x^2] - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2
+ 3*x^2])])/(11*Sqrt[11])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{121}$	48
trager	$\frac{-38+21x}{66\sqrt{3x^2+2}}+\frac{2\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{121}$	64
default	$\frac{x}{4\sqrt{3x^2+2}}+\frac{1}{11\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}+\frac{3x}{44\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{121}-\frac{2}{3\sqrt{3x^2+2}}$	88

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/66*(-38+21*x)/(3*x^2+2)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/
2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{6\sqrt{11}\left(3\,x^2+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\sqrt{3\,x^2+2}(21\,x-}{726\left(3\,x^2+2\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 2
1*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3
*x^2 + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\,\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{7\,x}{22\,\sqrt{3\,x^2+2}}-\frac{19}{33\,\sqrt{3\,x^2+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\,\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{21\,x-38}{66\,\sqrt{3\,x^2+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x
- 38)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{1+3x+4x^2}{(1+2x)\,(2+3x^2)^{3/2}}\,dx = \frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121} - \frac{\sqrt{3}\,\sqrt{6}\,\left(-114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\left(114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

output
```
(11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1
i)/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2
 + 2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.42

$$\int \frac{1+3x+4x^2}{(1+2x)\,(2+3x^2)^{3/2}}\,dx = \frac{36\sqrt{11}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{11}-\sqrt{3}}\right)i\,x^2+24\sqrt{11}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{11}-\sqrt{3}}\right)i+231\sqrt{3}}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x)
```

output
```
(36*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(11) - sqrt(
3)))*i*x**2 + 24*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqr
t(11) - sqrt(3)))*i + 231*sqrt(3*x**2 + 2)*x - 418*sqrt(3*x**2 + 2) + 18*s
qrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + sqrt(33) + 12*x**2 - 3)*x**2 +
12*sqrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + sqrt(33) + 12*x**2 - 3) - 3
6*sqrt(11)*log((2*sqrt(3*x**2 + 2) + sqrt(11) + 2*sqrt(3)*x + sqrt(3))/sqr
t(2))*x**2 - 24*sqrt(11)*log((2*sqrt(3*x**2 + 2) + sqrt(11) + 2*sqrt(3)*x
+ sqrt(3))/sqrt(2)) + 231*sqrt(3)*x**2 + 154*sqrt(3))/(726*(3*x**2 + 2))
```

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{-10+97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
1/242*(-10+97*x)/(3*x^2+2)^(1/2)-4*(3*x^2+2)^(1/2)/(121+242*x)+4/1331*arct
anh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{11(-26+77x+170x^2)+8(1+2x)\sqrt{22+33x^2}\text{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{2662(1+2x)\sqrt{2+3x^2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
(11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*
x)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2 (3x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{6} \int -\frac{24(3-5x)}{121(2x+1)^2\sqrt{3x^2+2}}dx-\frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{121} \int \frac{3-5x}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{4}{121} \left(-\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{2x+1}\right)-\frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{4}{121} \left(\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)-\frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{121} \left(\frac{\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)-\frac{10-97x}{242\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
-1/242*(10 - 97*x)/Sqrt[2 + 3*x^2] + (4*(-(Sqrt[2 + 3*x^2]/(1 + 2*x)) + Ar
cTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/Sqrt[11]))/121
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$\frac{170x^2+77x-26}{242(1+2x)\sqrt{3x^2+2}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{1331}$
trager	$\frac{(170x^2+77x-26)\sqrt{3x^2+2}}{1452x^3+726x^2+968x+484}-\frac{4\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{2\sqrt{3x^2+2}}-\frac{1}{22\left(\frac{1}{2}+x\right)\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}-\frac{2}{121\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}-\frac{18x}{121\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-}{11\sqrt{12\left(\frac{1}{2}+\right.}}\right.}{1331}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/242*(170*x^2+77*x-26)/(1+2*x)/(3*x^2+2)^(1/2)+4/1331*11^(1/2)*arctanh(2/
11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{3/2}}\,dx=\frac{4\sqrt{11}(6\,x^3+3\,x^2+4\,x+2)\log\left(\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)-21\,x^2+12\,x-19}{4\,x^2+4\,x+1}\right)+11\,(170\,x}{2662\,(6\,x^3+3\,x^2+4\,x+2)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)
*(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x -
 26)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2(3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx = -\frac{4}{1331}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{85\,x}{242\sqrt{3\,x^2+2}} - \frac{2}{121\sqrt{3\,x^2+2}} - \frac{1}{11\left(2\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*
x^2 + 2)*x + sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{3/2}}\,dx = -\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3}+24\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right) - \frac{\frac{\frac{93}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}+\frac{44}{(2x+1)\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{85}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}} + \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{1331\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(
1/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)
)))/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 +
3) + 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
 + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{3/2}}\,dx = \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{121\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

output
```
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 -
(4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x -
 (6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)
/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x
^2 + 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 +
2/3)^(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.60

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx = \frac{1870\sqrt{3x^2+2}\,x^2+847\sqrt{3x^2+2}\,x-286\sqrt{3x^2+2}+48\sqrt{11}\log\left(-\sqrt{3x^2+\right.}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x)
```

output
```
(1870*sqrt(3*x**2 + 2)*x**2 + 847*sqrt(3*x**2 + 2)*x - 286*sqrt(3*x**2 + 2
) + 48*sqrt(11)*log( - sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**3 + 24*sqrt
(11)*log( - sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**2 + 32*sqrt(11)*log( -
 sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x + 16*sqrt(11)*log( - sqrt(3*x**2 +
 2)*sqrt(11) + 3*x - 4) - 48*sqrt(11)*log(2*x + 1)*x**3 - 24*sqrt(11)*log(
2*x + 1)*x**2 - 32*sqrt(11)*log(2*x + 1)*x - 16*sqrt(11)*log(2*x + 1))/(26
62*(6*x**3 + 3*x**2 + 4*x + 2))
```

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}}\,dx = \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

output

```
1/2662*(358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2*(3*x^
2+2)^(1/2)/(1331+2662*x)-322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)
^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}}\,dx = \frac{\frac{11(278+1799x+2716x^2+1428x^3)}{(1+2x)^2\sqrt{2+3x^2}} + 1288\sqrt{11}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{29282}$$

input

```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
((11*(278 + 1799*x + 2716*x^2 + 1428*x^3))/((1 + 2*x)^2*Sqrt[2 + 3*x^2]) +
 1288*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11
]])/29282
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2178, 27, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{351x+358}{2662\sqrt{3x^2+2}}-\frac{1}{6}\int -\frac{12(716x^2+606x+245)}{1331(2x+1)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{716x^2+606x+245}{(2x+1)^3\sqrt{3x^2+2}}dx}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\left(-\frac{1}{22}\int -\frac{22(325x+157)}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\int \frac{325x+157}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{2\left(161\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{2\left(-161\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(-\frac{161\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
(358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) + (2*((-11*Sqrt[2 + 3*x^2])/(1 + 2*x)
^2 + Sqrt[2 + 3*x^2]/(1 + 2*x) - (161*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/Sqrt[11]))/1331
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
risch	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{14641}$
trager	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}}+\frac{322\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{14641}$
default	$\frac{161}{1331\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}+\frac{357x}{2662\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{14641}-\frac{1}{88\left(\frac{1}{2}+x\right)^2\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2662*(1428*x^3+2716*x^2+1799*x+278)/(1+2*x)^2/(3*x^2+2)^(1/2)-322/14641*
11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322\sqrt{11}(12\,x^4+12\,x^3+11\,x^2+8\,x+2)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+1}{4\,x^2+4\,x+1}\right.}{29282\,(12\,x^4+12\,x^3+11\,x^2+}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*
sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(1
428*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*
x^2 + 8*x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{3/2}}\,dx = \frac{322}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{357\,x}{2662\sqrt{3\,x^2+2}} + \frac{161}{1331\sqrt{3\,x^2+2}} - \frac{1}{22\,\left(4\sqrt{3\,x^2+2}x^2+4\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)} + \frac{7}{242\,\left(2\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*
x + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*
sqrt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sq
rt(3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.02

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{3/2}}\,dx = \frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{351\,x+358}{2662\sqrt{3\,x^2+2}} + \frac{36\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+48\sqrt{3}x+8\sqrt{3}-48\sqrt{3\,x^2+2}}{1331\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x
^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*
(351*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3
 - sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48
*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x -
sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 16.69 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.86

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{14641} - \frac{322\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\frac{1}{4}\right)} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

output
```
(322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2
)*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(53
24*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(
1/2)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2
)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)
*179i)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*1
79i)/(15972*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.54

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{15708\sqrt{3x^2+2}\,x^3+29876\sqrt{3x^2+2}\,x^2+19789\sqrt{3x^2+2}\,x+3058\sqrt{3x^2+}}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x)
```

output
```
(15708*sqrt(3*x**2 + 2)*x**3 + 29876*sqrt(3*x**2 + 2)*x**2 + 19789*sqrt(3*
x**2 + 2)*x + 3058*sqrt(3*x**2 + 2) + 7728*sqrt(11)*log(sqrt(3*x**2 + 2)*s
qrt(11) + 3*x - 4)*x**4 + 7728*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*
x - 4)*x**3 + 7084*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**2
+ 5152*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x + 1288*sqrt(11)
*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4) - 7728*sqrt(11)*log(2*x + 1)*x**
4 - 7728*sqrt(11)*log(2*x + 1)*x**3 - 7084*sqrt(11)*log(2*x + 1)*x**2 - 51
52*sqrt(11)*log(2*x + 1)*x - 1288*sqrt(11)*log(2*x + 1))/(29282*(12*x**4 +
 12*x**3 + 11*x**2 + 8*x + 2))
```

3.130 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{199}{81\left(2+3x^2\right)^{3/2}}+\frac{31x}{18\left(2+3x^2\right)^{3/2}}-\frac{76}{27\sqrt{2+3x^2}}-\frac{155x}{18\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+\frac{8\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

output
```
199/81/(3*x^2+2)^(3/2)+31/18*x/(3*x^2+2)^(3/2)-76/27/(3*x^2+2)^(1/2)-155/1
8*x/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{254-2511x+936x^2-4185x^3+1728x^4}{162\left(2+3x^2\right)^{3/2}}-\frac{8\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4)/(162*(2 + 3*x^2)^(3/2)) - (
8*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2345, 27, 2345, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{279x+398}{162 \left(3x^2+2\right)^{3/2}}-\frac{1}{6} \int \frac{2\left(-96x^3-216x^2-140x+11\right)}{3 \left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{162 \left(3x^2+2\right)^{3/2}}-\frac{1}{9} \int \frac{-96x^3-216x^2-140x+11}{\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{9} \left(\frac{1}{2} \int \frac{16(4x+9)}{\sqrt{3x^2+2}}dx-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9} \left(8 \int \frac{4x+9}{\sqrt{3x^2+2}}dx-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{9} \left(8 \left(9 \int \frac{1}{\sqrt{3x^2+2}}dx+\frac{4}{3} \sqrt{3x^2+2}\right)-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9}\left(8\left(3\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{4}{3}\sqrt{3x^2+2}\right)-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) + (-1/6*(152 + 465*x)/Sqrt[2 + 3*x^2
] + 8*((4*Sqrt[2 + 3*x^2])/3 + 3*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

method	result
risch	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}$
trager	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}-\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$
default	$-\frac{65x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}+\frac{127}{81(3x^2+2)^{\frac{3}{2}}}+\frac{52x^2}{9(3x^2+2)^{\frac{3}{2}}}-\frac{8x^3}{(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}+\frac{32x^4}{3(3x^2+2)^{\frac{3}{2}}}$
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{17\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\,(18x^2+8)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{16\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,(30x^2+1}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right.}{9\sqrt{}}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/162*(1728*x^4-4185*x^3+936*x^2-2511*x+254)/(3*x^2+2)^(3/2)+8/3*arcsinh(1
/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{216\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(1728\,x^4-}{162\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3
*x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2)
)/(9*x^4 + 12*x^2 + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^3 \cdot (4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{32\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{8}{3}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{8}{3}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{11\,x}{18\,\sqrt{3\,x^2+2}} + \frac{52\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{65\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{127}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2
)^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) +
52/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2
)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}} dx = -\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{9((3(64x-155)x+104)x-279)x+254}{162(3x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)
*x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}} dx = \frac{32\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27} + \frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{3} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2
))/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1
/2)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i
)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x
 + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^
(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)
^(1/2)*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i +
 1824)*(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.78

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{1728\sqrt{3x^2+2}\,x^4-4185\sqrt{3x^2+2}\,x^3+936\sqrt{3x^2+2}\,x^2-2511\sqrt{3x^2+2}}{}$$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

output
```
(1728*sqrt(3*x**2 + 2)*x**4 - 4185*sqrt(3*x**2 + 2)*x**3 + 936*sqrt(3*x**2
 + 2)*x**2 - 2511*sqrt(3*x**2 + 2)*x + 254*sqrt(3*x**2 + 2) + 3888*sqrt(3)
*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**4 + 5184*sqrt(3)*log((sqrt
(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 + 1728*sqrt(3)*log((sqrt(3*x**2 +
2) + sqrt(3)*x)/sqrt(2)) + 837*sqrt(3)*x**4 + 1116*sqrt(3)*x**2 + 372*sqrt
(3))/(162*(9*x**4 + 12*x**2 + 4))
```

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{35}{27(2+3x^2)^{3/2}}-\frac{47x}{54(2+3x^2)^{3/2}}-\frac{28}{9\sqrt{2+3x^2}}-\frac{59x}{54\sqrt{2+3x^2}}+\frac{16\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

output
```
35/27/(3*x^2+2)^(3/2)-47/54*x/(3*x^2+2)^(3/2)-28/9/(3*x^2+2)^(1/2)-59/54*x
/(3*x^2+2)^(1/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{-266-165x-504x^2-177x^3}{54(2+3x^2)^{3/2}}-\frac{16\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(-266 - 165*x - 504*x^2 - 177*x^3)/(54*(2 + 3*x^2)^(3/2)) - (16*Log[-(Sqrt
[3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2345, 27, 2345, 27, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{70-47x}{54 \left(3x^2+2\right)^{3/2}}-\frac{1}{6} \int -\frac{2 \left(144x^2+252x+37\right)}{9 \left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27} \int \frac{144x^2+252x+37}{\left(3x^2+2\right)^{3/2}}dx+\frac{70-47x}{54 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{27} \left(-\frac{1}{2} \int -\frac{96}{\sqrt{3x^2+2}}dx-\frac{59x+168}{2 \sqrt{3x^2+2}}\right)+\frac{70-47x}{54 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27} \left(48 \int \frac{1}{\sqrt{3x^2+2}}dx-\frac{59x+168}{2 \sqrt{3x^2+2}}\right)+\frac{70-47x}{54 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{27} \left(16 \sqrt{3} \text{arcsinh}\left(\sqrt{\frac{3}{2}} x\right)-\frac{59x+168}{2 \sqrt{3x^2+2}}\right)+\frac{70-47x}{54 \left(3x^2+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) + (-1/2*(168 + 59*x)/Sqrt[2 + 3*x^2] +
16*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
trager	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}-\frac{133}{27(3x^2+2)^{\frac{3}{2}}}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
meijerg	$\frac{\sqrt{2}\,x\left(3x^2+3\right)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{6\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{28\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{32\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+1\right.}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right.}{81\sqrt{}}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/54*(177*x^3+504*x^2+165*x+266)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*6^(1/2
)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{16\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)-(177\,x^3+50}{54\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x
^2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x
^2 + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{16}{27}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}}+\frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{16}{27}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{37\,x}{54\,\sqrt{3\,x^2+2}} - \frac{28\,x^2}{3\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{37\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{133}{27\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*a
rcsinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/
2) - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{3\,((59\,x+168)x+55)x+266}{54\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
 + 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.44

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{16\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*
(((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)
/72 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1
/2)*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*
35i)/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6
^(1/2)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(
1/2)*6^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)
*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{-177\sqrt{3x^2+2}\,x^3-504\sqrt{3x^2+2}\,x^2-165\sqrt{3x^2+2}\,x-266\sqrt{3x^2+2}+}{}$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

output

```
( - 177*sqrt(3*x**2 + 2)*x**3 - 504*sqrt(3*x**2 + 2)*x**2 - 165*sqrt(3*x**
2 + 2)*x - 266*sqrt(3*x**2 + 2) + 288*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt
(3)*x)/sqrt(2))*x**4 + 384*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt
(2))*x**2 + 128*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) + 153*
sqrt(3)*x**4 + 204*sqrt(3)*x**2 + 68*sqrt(3))/(54*(9*x**4 + 12*x**2 + 4))
```

3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 63

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} \, dx = \frac{1}{27(2+3x^2)^{3/2}} - \frac{17x}{18(2+3x^2)^{3/2}} - \frac{8}{9\sqrt{2+3x^2}} + \frac{13x}{18\sqrt{2+3x^2}}$$

output
```
1/27/(3*x^2+2)^(3/2)-17/18*x/(3*x^2+2)^(3/2)-8/9/(3*x^2+2)^(1/2)+13/18*x/(
3*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.48

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} \, dx = \frac{-94+27x-144x^2+117x^3}{54(2+3x^2)^{3/2}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2345, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{1}{6}\int -\frac{2(24x+13)}{3\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{24x+13}{\left(3x^2+2\right)^{3/2}}dx+\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 453}$$

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

method	result	size
gosper	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
trager	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
risch	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
orering	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
default	$-\frac{17x}{18(3x^2+2)^{\frac{3}{2}}}+\frac{13x}{18\sqrt{3x^2+2}}-\frac{47}{27(3x^2+2)^{\frac{3}{2}}}-\frac{8x^2}{3(3x^2+2)^{\frac{3}{2}}}$	51
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{8\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\,(18x^2+8)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}$	102

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{(117\,x^3-144\,x^2+27\,x-94)\sqrt{3\,x^2+2}}{54\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(56) = 112$.

Time = 20.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.86

$$\begin{aligned}\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx &= \frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}} \\ &+ \frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} \\ &+ \frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{32}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} \\ &- \frac{5}{27x^2\sqrt{3x^2+2}+18\sqrt{3x^2+2}}\end{aligned}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sq
rt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) +
 54*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) -
 32/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x
**2 + 2) + 18*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{13\,x}{18\,\sqrt{3\,x^2+2}} - \frac{8\,x^2}{3\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{17\,x}{18\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{47}{27\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^
(3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{9\,((13\,x-16)x+3)x-94}{54\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.94

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\,\sqrt{6}\left(-192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\left(192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
 + (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27
- (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/
3) - (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/2
7 - (3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x -
(6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*
1i)/(2592*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{117\sqrt{3x^2+2}\,x^3-144\sqrt{3x^2+2}\,x^2+27\sqrt{3x^2+2}\,x-94\sqrt{3x^2+2}+63\sqrt{}}{486x^4+648x^2+216}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

output
```
(117*sqrt(3*x**2 + 2)*x**3 - 144*sqrt(3*x**2 + 2)*x**2 + 27*sqrt(3*x**2 +
2)*x - 94*sqrt(3*x**2 + 2) + 63*sqrt(3)*x**4 + 84*sqrt(3)*x**2 + 28*sqrt(3
))/(54*(9*x**4 + 12*x**2 + 4))
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-38+21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
1/198*(-38+21*x)/(3*x^2+2)^(3/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)-8/1331*ar
ctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-274+801x+216x^2+855x^3}{2178\left(2+3x^2\right)^{3/2}}-\frac{8\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{121\sqrt{11}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
(-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(
4 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2178, 27, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{18}\int -\frac{6(14x+13)}{11(2x+1)\left(3x^2+2\right)^{3/2}}dx-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{33}\int \frac{14x+13}{(2x+1)\left(3x^2+2\right)^{3/2}}dx-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{1}{66}\int -\frac{144}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{33}\left(\frac{24}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{95x+24}{22\sqrt{3x^2+2}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{24}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{24\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
-1/198*(38 - 21*x)/(2 + 3*x^2)^(3/2) + ((24 + 95*x)/(22*Sqrt[2 + 3*x^2]) -
 (24*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11]))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

method	result
trager	$\frac{855x^3+216x^2+801x-274}{2178(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{12(3x^2+2)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}-\frac{2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{1}{33\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}+\frac{x}{44\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2178*(855*x^3+216*x^2+801*x-274)/(3*x^2+2)^(3/2)+8/1331*RootOf(_Z^2-11)*
ln((3*RootOf(_Z^2-11)*x+11*(3*x^2+2)^(1/2)-4*RootOf(_Z^2-11))/(1+2*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)\,(2+3x^2)^{5/2}}\,dx = \frac{72\,\sqrt{11}(9\,x^4+12\,x^2+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(855\,x^3+\cdots}{23958\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(
3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2
+ 801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{8}{1331}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{95\,x}{726\,\sqrt{3\,x^2+2}}+\frac{4}{121\,\sqrt{3\,x^2+2}}+\frac{7\,x}{66\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{19}{99\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 +
2)^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{9\,((95\,x+24)x+89)x-274}{2178\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
+ 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*
((95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.91 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.99

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331}$$
$$-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$-\frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

output
```
(11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176
)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (
6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 +
21/176)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*
(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2
+ 2/3)^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)
*303i + 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.10

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{1296\sqrt{11}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{11}-\sqrt{3}}\right)i\,x^4+1728\sqrt{11}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{11}-\sqrt{3}}\right)i\,x^2+}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x)
```

output
```
(1296*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(11) - sqr
t(3)))*i*x**4 + 1728*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/
(sqrt(11) - sqrt(3)))*i*x**2 + 576*sqrt(11)*atan((2*sqrt(3*x**2 + 2)*i + 2
*sqrt(3)*i*x)/(sqrt(11) - sqrt(3)))*i + 9405*sqrt(3*x**2 + 2)*x**3 + 2376*
sqrt(3*x**2 + 2)*x**2 + 8811*sqrt(3*x**2 + 2)*x - 3014*sqrt(3*x**2 + 2) +
648*sqrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + sqrt(33) + 12*x**2 - 3)*x*
*4 + 864*sqrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + sqrt(33) + 12*x**2 -
3)*x**2 + 288*sqrt(11)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + sqrt(33) + 12*x*
*2 - 3) - 1296*sqrt(11)*log((2*sqrt(3*x**2 + 2) + sqrt(11) + 2*sqrt(3)*x +
 sqrt(3))/sqrt(2))*x**4 - 1728*sqrt(11)*log((2*sqrt(3*x**2 + 2) + sqrt(11)
 + 2*sqrt(3)*x + sqrt(3))/sqrt(2))*x**2 - 576*sqrt(11)*log((2*sqrt(3*x**2
+ 2) + sqrt(11) + 2*sqrt(3)*x + sqrt(3))/sqrt(2)) - 8217*sqrt(3)*x**4 - 10
956*sqrt(3)*x**2 - 3652*sqrt(3))/(23958*(9*x**4 + 12*x**2 + 4))
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx = \frac{-10+97x}{726\,(2+3x^2)^{3/2}} + \frac{24+887x}{7986\sqrt{2+3x^2}} - \frac{16\sqrt{2+3x^2}}{1331(1+2x)} - \frac{32\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

output
```
1/726*(-10+97*x)/(3*x^2+2)^(3/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16*(3*x
^2+2)^(1/2)/(1331+2662*x)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)
^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx = \frac{11(-446+2717x+4602x^2+2805x^3+4458x^4)-192\sqrt{22+33x^2}(2+4x}{87846(1+2x)\,(2+3x^2)^{3/2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
(11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^
2]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*
(1 + 2*x)*(2 + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2178, 27, 2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2 (3x^2+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{18} \int -\frac{6(388x^2+328x+133)}{121(2x+1)^2 (3x^2+2)^{3/2}}dx - \frac{10-97x}{726 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{363} \int \frac{388x^2+328x+133}{(2x+1)^2 (3x^2+2)^{3/2}}dx - \frac{10-97x}{726 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2178}$$

$$\frac{1}{363} \left(\frac{887x+24}{22\sqrt{3x^2+2}} - \frac{1}{6} \int -\frac{288(x+6)}{11(2x+1)^2\sqrt{3x^2+2}}dx\right) - \frac{10-97x}{726 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{363} \left(\frac{48}{11} \int \frac{x+6}{(2x+1)^2\sqrt{3x^2+2}}dx + \frac{887x+24}{22\sqrt{3x^2+2}}\right) - \frac{10-97x}{726 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{363} \left(\frac{48}{11} \left(2 \int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx - \frac{\sqrt{3x^2+2}}{2x+1}\right) + \frac{887x+24}{22\sqrt{3x^2+2}}\right) - \frac{10-97x}{726 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{363}\left(\frac{48}{11}\left(-2\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{363}\left(\frac{48}{11}\left(-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
-1/726*(10 - 97*x)/(2 + 3*x^2)^(3/2) + ((24 + 887*x)/(22*Sqrt[2 + 3*x^2])
+ (48*(-(Sqrt[2 + 3*x^2]/(1 + 2*x)) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[
2 + 3*x^2])])/Sqrt[11]))/11)/363
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
risch	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)}-\frac{32\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{14641}$
trager	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)}-\frac{32\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x-4\operatorname{RootOf}\left(\_Z^2-11\right)-11\sqrt{3x^2+2}}{1+2x}\right)}{14641}$
default	$\frac{x}{6(3x^2+2)^{\frac{3}{2}}}+\frac{x}{6\sqrt{3x^2+2}}-\frac{1}{22\left(\frac{1}{2}+x\right)\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}+\frac{4}{363\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}-\frac{10x}{121\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}-\frac{}{133}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7986*(4458*x^4+2805*x^3+4602*x^2+2717*x-446)/(3*x^2+2)^(3/2)/(1+2*x)-32/
14641*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx = \frac{96\sqrt{11}(18\,x^5+9\,x^4+24\,x^3+12\,x^2+8\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-}{4\,x^2+4\,x+1}\right.}{87846\,(18\,x^5+9\,x^4+24\,x}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sq
rt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1))
+ 11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*
x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx &= \frac{32}{14641}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)\\ &\quad+\frac{743\,x}{7986\,\sqrt{3\,x^2+2}}+\frac{16}{1331\,\sqrt{3\,x^2+2}}+\frac{61\,x}{726\,(3\,x^2+2)^{\frac{3}{2}}}\\ &\quad-\frac{1}{11\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{4}{363\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(
3*x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/36
3/(3*x^2 + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(76) = 152$.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx =$$
$$-\frac{1}{263538}\sqrt{11}\left(743\sqrt{11}\sqrt{3}-576\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$
$$-\frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{14641\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$
$$+\frac{\frac{\frac{\frac{11\left(\frac{731}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{528}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{14163}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{6111}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{2229}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{7986\left(\frac{6}{2x+1}-\frac{11}{(2x+1)^2}-3\right)\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*
sgn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/
(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((1
1*(731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14
163/sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/
sgn(1/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) +
11/(2*x + 1)^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.84

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\,\ln\left(x+\frac{1}{2}\right)-8\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641}$$
$$+\frac{\sqrt{11}\left(\frac{48\,\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22}-\frac{8\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1331\,\left(x+\frac{1}{2}\right)}$$
$$-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$-\frac{\sqrt{3}\,\sqrt{6}\left(-288+\sqrt{6}\,2481\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\left(288+\sqrt{6}\,2481\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

output
```
(11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/
2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3
)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/193
6 - 291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*
1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2
)*15i)/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968
+ 97/968)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*
2481i - 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/
2)*6^(1/2)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1
/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.17

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2+3x^2)^{5/2}}\,dx = \frac{49038\sqrt{3x^2+2}\,x^4+30855\sqrt{3x^2+2}\,x^3+50622\sqrt{3x^2+2}\,x^2+29887\sqrt{3x^2}}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x)
```

output
```
(49038*sqrt(3*x**2 + 2)*x**4 + 30855*sqrt(3*x**2 + 2)*x**3 + 50622*sqrt(3*
x**2 + 2)*x**2 + 29887*sqrt(3*x**2 + 2)*x - 4906*sqrt(3*x**2 + 2) + 3456*s
qrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**5 + 1728*sqrt(11)*log(
sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**4 + 4608*sqrt(11)*log(sqrt(3*x**2
+ 2)*sqrt(11) + 3*x - 4)*x**3 + 2304*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11
) + 3*x - 4)*x**2 + 1536*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)
*x + 768*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4) - 3456*sqrt(11)
*log(2*x + 1)*x**5 - 1728*sqrt(11)*log(2*x + 1)*x**4 - 4608*sqrt(11)*log(2
*x + 1)*x**3 - 2304*sqrt(11)*log(2*x + 1)*x**2 - 1536*sqrt(11)*log(2*x + 1
)*x - 768*sqrt(11)*log(2*x + 1))/(87846*(18*x**5 + 9*x**4 + 24*x**3 + 12*x
**2 + 8*x + 4))
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-\frac{1216\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{14641\sqrt{11}}$$

output
```
1/7986*(358+351*x)/(3*x^2+2)^(3/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8
/1331*(3*x^2+2)^(1/2)/(1+2*x)^2-8*(3*x^2+2)^(1/2)/(1331+2662*x)-1216/16105
1*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{\frac{11\left(7010+57371x+109844x^2+116937x^3+111060x^4+67284x^5\right)}{(1+2x)^2\left(2+3x^2\right)^{3/2}}+14592\sqrt{11}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}\cdots}{}\right.}{966306}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/
((1 + 2*x)^2*(2 + 3*x^2)^(3/2)) + 14592*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt
[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/966306
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2178, 27, 2178, 27, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{351x+358}{7986 (3x^2+2)^{3/2}}-\frac{1}{18} \int -\frac{18\left(936x^3+2836x^2+1914x+607\right)}{1331(2x+1)^3 (3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{936x^3+2836x^2+1914x+607}{(2x+1)^3(3x^2+2)^{3/2}}dx}{1331}+\frac{351x+358}{7986 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2178}$$

$$\frac{\frac{2133x+1216}{22\sqrt{3x^2+2}}-\frac{1}{6}\int -\frac{96\left(304x^2+315x+142\right)}{11(2x+1)^3\sqrt{3x^2+2}}dx}{1331}+\frac{351x+358}{7986 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{11}\int \frac{304x^2+315x+142}{(2x+1)^3\sqrt{3x^2+2}}dx+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\frac{16}{11}\left(-\frac{1}{22}\int -\frac{11(271x+196)}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\int \frac{271x+196}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(152\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(-152\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(-\frac{152\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\,(3x^2+2)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
(358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + ((1216 + 2133*x)/(22*Sqrt[2 + 3*x
^2]) + (16*((-11*Sqrt[2 + 3*x^2])/(2*(1 + 2*x)^2) + ((-11*Sqrt[2 + 3*x^2])
/(1 + 2*x) - (152*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/Sqrt[11])
/2))/11)/1331
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result
risch	$\frac{67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010}{87846(1+2x)^2(3x^2+2)^{\frac{3}{2}}}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(\frac{1}{2}+x\right)^2+5-12x}}\right)}{161051}$
trager	$\frac{(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010)\sqrt{3x^2+2}}{87846(6x^3+3x^2+4x+2)^2}-\frac{1216\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x-4}{}\right.}{161051}$
default	$\frac{152}{3993\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}+\frac{87x}{2662\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x\right)^{\frac{3}{2}}}+\frac{1869x}{29282\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}+\frac{608}{14641\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-3x}}-\frac{1216\sqrt{1}}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/87846*(67284*x^5+111060*x^4+116937*x^3+109844*x^2+57371*x+7010)/(1+2*x)^
2/(3*x^2+2)^(3/2)-1216/161051*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(
1/2+x)^2+5-12*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{1+3x+4x^2}{(1+2x)^3\,(2+3x^2)^{5/2}}\,dx=\frac{3648\sqrt{11}(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+}}{}\right.}{966306\,(36\,x^6}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^
2 + 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 573
71*x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2
+ 16*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3 \left(2+3x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3 \left(2+3x^2\right)^{5/2}}\,dx &= \frac{1216}{161051}\,\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)\\&\quad+\frac{1869\,x}{29282\,\sqrt{3\,x^2+2}}+\frac{608}{14641\,\sqrt{3\,x^2+2}}+\frac{87\,x}{2662\,(3\,x^2+2)^{\frac{3}{2}}}\\&\quad-\frac{1}{22\,\left(4\,(3\,x^2+2)^{\frac{3}{2}}x^2+4\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}\\&\quad+\frac{1}{242\,\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{152}{3993\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(
2*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/
2662*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(
3/2)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(
3/2)) + 152/3993/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx=\frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9\left(\left(2133x+1216\right)x+1851\right)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4\left(\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+24\sqrt{3}x-8\sqrt{3}-24\sqrt{3x^2+2}\right)}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

input

```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output

```
1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/878
46*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt
(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.57

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{1216\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{161051} - \frac{1216\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051} - \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x^2+x+\frac{1}{4}\right)} + \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)} - \frac{4\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

output
```
(1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(
1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))
/(95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/
(58564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x +
 (6^(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) +
 (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) -
(4*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2
/3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(
x^2 + 2/3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*
(x^2 + 2/3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)
*(x^2 + 2/3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.04

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{5/2}} \, dx = \frac{740124\sqrt{3x^2+2}\,x^5+1221660\sqrt{3x^2+2}\,x^4+1286307\sqrt{3x^2+2}\,x^3+12082$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x)
```

output
```
(740124*sqrt(3*x**2 + 2)*x**5 + 1221660*sqrt(3*x**2 + 2)*x**4 + 1286307*sq
rt(3*x**2 + 2)*x**3 + 1208284*sqrt(3*x**2 + 2)*x**2 + 631081*sqrt(3*x**2 +
 2)*x + 77110*sqrt(3*x**2 + 2) + 262656*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt
(11) + 3*x - 4)*x**6 + 262656*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x
 - 4)*x**5 + 415872*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**4
 + 350208*sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**3 + 204288*
sqrt(11)*log(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x**2 + 116736*sqrt(11)*l
og(sqrt(3*x**2 + 2)*sqrt(11) + 3*x - 4)*x + 29184*sqrt(11)*log(sqrt(3*x**2
 + 2)*sqrt(11) + 3*x - 4) - 262656*sqrt(11)*log(2*x + 1)*x**6 - 262656*sqr
t(11)*log(2*x + 1)*x**5 - 415872*sqrt(11)*log(2*x + 1)*x**4 - 350208*sqrt(
11)*log(2*x + 1)*x**3 - 204288*sqrt(11)*log(2*x + 1)*x**2 - 116736*sqrt(11
)*log(2*x + 1)*x - 29184*sqrt(11)*log(2*x + 1))/(966306*(36*x**6 + 36*x**5
 + 57*x**4 + 48*x**3 + 28*x**2 + 16*x + 4))
```

3.136 $\int \sqrt{c+dx}\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 736

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{2(75a^2d^4D-3abd^2(11cCd-55Bd^2-2c^2D)+b^2c(88c^2Cd-132Bcd^2+231Ad^3-64c^3D))\sqrt{c+dx}\sqrt{}}{3465b^2d^4}$$

$$+\frac{2(ad^2(77Cd+cD)-b(22c^2Cd-33Bcd^2-231Ad^3-16c^3D))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{1155bd^3}$$

$$-\frac{2(15ad^2D-b(22cCd-33Bd^2-16c^2D))\sqrt{c+dx}(a-bx^2)^{3/2}}{231b^2d^2}$$

$$-\frac{2(11Cd-17cD)(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}{99bd^2}-\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}{11bd^2}$$

$$-\frac{4\sqrt{a}(3a^2d^4(77Cd+26cD)+b^2c^2(88c^2Cd-132Bcd^2+231Ad^3-64c^3D)-3abd^2(33c^2Cd-88Bcd^2}{3465b^{3/2}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{4\sqrt{a}(bc^2-ad^2)\left(75a^2d^4D-3abd^2(11cCd-55Bd^2-2c^2D)+b^2c(88c^2Cd-132Bcd^2+231Ad^3-64}{3465b^{5/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3465*(75*a^2*d^4*D-3*a*b*d^2*(-55*B*d^2+11*C*c*d-2*D*c^2)+b^2*c*(231*A*d
^3-132*B*c*d^2+88*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^
4+2/1155*(a*d^2*(77*C*d+D*c)-b*(-231*A*d^3-33*B*c*d^2+22*C*c^2*d-16*D*c^3)
)*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^3-2/231*(15*a*d^2*D-b*(-33*B*d^2+22
*C*c*d-16*D*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b^2/d^2-2/99*(11*C*d-17*D
*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(3/2)/b/d^2-2/11*D*(d*x+c)^(5/2)*(-b*x^2+a)^(
3/2)/b/d^2-4/3465*a^(1/2)*(3*a^2*d^4*(77*C*d+26*D*c)+b^2*c^2*(231*A*d^3-13
2*B*c*d^2+88*C*c^2*d-64*D*c^3)-3*a*b*d^2*(-231*A*d^3-88*B*c*d^2+33*C*c^2*d
-18*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^
(3/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+4/3465*a^
(1/2)*(-a*d^2+b*c^2)*(75*a^2*d^4*D-3*a*b*d^2*(-55*B*d^2+11*C*c*d-2*D*c^2)+
b^2*c*(231*A*d^3-132*B*c*d^2+88*C*c^2*d-64*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^5/(
d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.33 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.20

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left((c+dx)\left(-150a^2d^4D-2abd^2(-23c^2D+4cd(11C+4Dx)+d^2(165B+77Cx+45Dx^2))\right.\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(-150*a^2*d^4*D - 2*a*b*d^2*(-23*c^2*D + 4*c
*d*(11*C + 4*D*x) + d^2*(165*B + 77*C*x + 45*D*x^2)) + b^2*(-64*c^4*D + 8*
c^3*d*(11*C + 6*D*x) - 2*c^2*d^2*(66*B + x*(33*C + 20*D*x)) + c*d^3*(231*A
 + x*(99*B + 55*C*x + 35*D*x^2)) + d^4*x*(693*A + 5*x*(99*B + 77*C*x + 63*
D*x^2)))) - (2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*d^4*(77*C*d + 26
*c*D) + b^2*c^2*(88*c^2*C*d - 132*B*c*d^2 + 231*A*d^3 - 64*c^3*D) + 3*a*b*
d^2*(-33*c^2*C*d + 88*B*c*d^2 + 231*A*d^3 + 18*c^3*D))*(a - b*x^2) + I*Sqr
t[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*(77*C*d + 26*c*D) + b^2*c^2*(88*c^
2*C*d - 132*B*c*d^2 + 231*A*d^3 - 64*c^3*D) + 3*a*b*d^2*(-33*c^2*C*d + 88*
B*c*d^2 + 231*A*d^3 + 18*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]
*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[
I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt
[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(75*
a^2*d^4*D - 3*a^(3/2)*Sqrt[b]*d^3*(77*C*d + c*D) + 3*a*b*d^2*(-11*c*C*d +
55*B*d^2 + 2*c^2*D) + b^2*c*(88*c^2*C*d - 132*B*c*d^2 + 231*A*d^3 - 64*c^3
*D) - 3*Sqrt[a]*b^(3/2)*d*(-22*c^2*C*d + 33*B*c*d^2 + 231*A*d^3 + 16*c^3*D
))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
 d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)
]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(3465*b^2*d^4*Sq...
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 725, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2185, 27, 2185, 27, 687, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{c+dx}\sqrt{a-bx^2}\left(b(11Cd-17cD)x^2d^2+(11Abd+5acD)d^2+\left(-6bDc^2+11bBd^2+5ad^2D\right)xd\right)dx}{11bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \sqrt{c+dx}\sqrt{a-bx^2}\left(b(11Cd-17cD)x^2d^2+(11Abd+5acD)d^2+\left(-6bDc^2+11bBd^2+5ad^2D\right)xd\right)dx}{11bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{-\frac{2\int -\frac{3}{2}bd^3\sqrt{c+dx}\left(d(33Abd+11aCd-2acD)+\left(15ad^2D-b\left(-16Dc^2+22Cdc-33Bd^2\right)\right)x\right)\sqrt{a-bx^2}dx}{9bd^2}-\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(11}{11bd^3}$$
$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}d\int \sqrt{c+dx}\left(d(33Abd+11aCd-2acD)+\left(15ad^2D-b\left(-16Dc^2+22Cdc-33Bd^2\right)\right)x\right)\sqrt{a-bx^2}dx-\frac{2}{9}d(a}{11bd^3}$$
$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{3}d\left(-\frac{2\int -\frac{\left(d\left(231Acdb^2+a\left(15aDd^2+b\left(2Dc^2+55Cdc+33Bd^2\right)\right)\right)+b\left(ad^2(77Cd+cD)-b\left(-16Dc^3+22Cdc^2-33Bd^2c-231Ad^3\right)\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7b}-\frac{2(a-bx}{}\right.}{11bd^3}$$
$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}d\left(\frac{\int \frac{\left(d\left(231Acdb^2+a\left(15aDd^2+b\left(2Dc^2+55Cdc+33Bd^2\right)\right)\right)+b\left(ad^2(77Cd+cD)-b\left(-16Dc^3+22Cdc^2-33Bd^2c-231Ad^3\right)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7b}-\frac{2\left(a-bx^2\right)^{3/2}}{}\right.}{11bd^3}$$
$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \ 682$$

$$\frac{1}{3}d\left(\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(75a^2d^4D+3bdx\left(ad^2(cD+77Cd)-b\left(-231Ad^3-33Bcd^2-16c^3D+22c^2Cd\right)\right)-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-\right.\right.}{15d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}d\left(\frac{\frac{2\int\frac{ad\left(75a^2Dd^4+3ab\left(3Dc^2+66Cdc+55Bd^2\right)d^2+b^2c\left(-16Dc^3+22Cdc^2-33Bd^2c+924Ad^3\right)\right)+b\left(3a^2(77Cd+26cD)d^4-3ab\left(-18Dc^3+33Cdc^2-88Bd^2c-\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 600$$

$$\frac{1}{3}d\left(\frac{\frac{2\left(\frac{b\left(3a^2d^4(26cD+77Cd)-3abd^2\left(-231Ad^3-88Bcd^2-18c^3D+33c^2Cd\right)+b^2c^2\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(75a^2d\right.}{}\right.}{15d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 509$$

$$\frac{1}{3}d\left(\frac{\frac{2\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(26cD+77Cd)-3abd^2\left(-231Ad^3-88Bcd^2-18c^3D+33c^2Cd\right)+b^2c^2\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad\right.}{}\right.}{15d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 508$$

$$\frac{1}{3}d\left(\frac{\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(75a^2d^4D-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(\ldots}{\ldots}\right)}{15d^2\ldots}}{\ldots}\right)$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{3}d\left(\frac{\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(75a^2d^4D-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\ldots}{\ldots}\right)}{15d^2\ldots}}{\ldots}\right)$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow \; 512$$

$$\frac{1}{3}d\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(75a^2d^4D-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow 511$$

$$\frac{1}{3}d\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(75a^2d^4D-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-64c^3D+88c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\cdots}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots\right.}{\cdots}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

$$\Big\downarrow 321$$

$$\frac{1}{3}d\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(75a^2d^4D-3abd^2\left(-55Bd^2-2c^2D+11cCd\right)+b^2c\left(231Ad^3-132Bcd^2-\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^2}$$

input
```
Int[Sqrt[c + d*x]*Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*D*(c + d*x)^(5/2)*(a - b*x^2)^(3/2))/(11*b*d^2) + ((-2*d*(11*C*d - 17*
c*D)*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/9 + (d*((-2*(15*a*d^2*D - b*(22*c*
C*d - 33*B*d^2 - 16*c^2*D))*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/(7*b) + ((2*S
qrt[c + d*x]*(75*a^2*d^4*D - 3*a*b*d^2*(11*c*C*d - 55*B*d^2 - 2*c^2*D) + b
^2*c*(88*c^2*C*d - 132*B*c*d^2 + 231*A*d^3 - 64*c^3*D) + 3*b*d*(a*d^2*(77*
C*d + c*D) - b*(22*c^2*C*d - 33*B*c*d^2 - 231*A*d^3 - 16*c^3*D))*x)*Sqrt[a
 - b*x^2])/(15*d^2) + (2*((-2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(77*C*d + 26*c*D)
 + b^2*c^2*(88*c^2*C*d - 132*B*c*d^2 + 231*A*d^3 - 64*c^3*D) - 3*a*b*d^2*(
33*c^2*C*d - 88*B*c*d^2 - 231*A*d^3 - 18*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b
*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((
Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]
*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(75*a^2*d^4*D - 3*a*b*d
^2*(11*c*C*d - 55*B*d^2 - 2*c^2*D) + b^2*c*(88*c^2*C*d - 132*B*c*d^2 + 231
*A*d^3 - 64*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])
))/(15*d^2))/(7*b)))/3)/(11*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1526 vs. $2(652) = 1304$.

Time = 2.78 (sec) , antiderivative size = 1527, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	1527
default	Expression too large to display	5953

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(2/11*D*x^4*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(-b*d*C-1/11*b*c*D)/b/d*x^3*(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9*(-b*d*C-1/11*b*
c*D)/d*c)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-A*b*d-B*b*c+C*a
*d+3/11*D*a*c+7/9*(-b*d*C-1/11*b*c*D)/b*a-6/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9
*(-b*d*C-1/11*b*c*D)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/
3*(-A*b*c+B*a*d+a*c*C+2/3*(-b*d*C-1/11*b*c*D)/b/d*a*c+5/7*(-B*b*d-b*c*C+2/
11*D*a*d-8/9*(-b*d*C-1/11*b*c*D)/d*c)/b*a-4/5*(-A*b*d-B*b*c+C*a*d+3/11*D*a
*c+7/9*(-b*d*C-1/11*b*c*D)/b*a-6/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9*(-b*d*C-1/
11*b*c*D)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*a*c+2
/5*(-A*b*d-B*b*c+C*a*d+3/11*D*a*c+7/9*(-b*d*C-1/11*b*c*D)/b*a-6/7*(-B*b*d-
b*c*C+2/11*D*a*d-8/9*(-b*d*C-1/11*b*c*D)/d*c)/d*c)/b/d*a*c+1/3*(-A*b*c+B*a
*d+a*c*C+2/3*(-b*d*C-1/11*b*c*D)/b/d*a*c+5/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9*
(-b*d*C-1/11*b*c*D)/d*c)/b*a-4/5*(-A*b*d-B*b*c+C*a*d+3/11*D*a*c+7/9*(-b*d*
C-1/11*b*c*D)/b*a-6/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9*(-b*d*C-1/11*b*c*D)/d*c
)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))
/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elliptic
F(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2))+2*(A*a*d+a*B*c+4/7*(-B*b*d-b*c*C+2/11*D*a*d-8/9*(-...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 662, normalized size of antiderivative = 0.90

$$\int \sqrt{c+dx}\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"fricas")
```

output
```
-2/10395*(2*(64*D*b^3*c^6 - 88*C*b^3*c^5*d - 6*(17*D*a*b^2 - 22*B*b^3)*c^4
*d^2 + 33*(5*C*a*b^2 - 7*A*b^3)*c^3*d^3 - 3*(17*D*a^2*b + 121*B*a*b^2)*c^2
*d^4 + 33*(11*C*a^2*b + 63*A*a*b^2)*c*d^5 + 45*(5*D*a^3 + 11*B*a^2*b)*d^6)
*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^
3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(64*D*b^3*c^5*d - 88*C*b^3*
c^4*d^2 - 6*(9*D*a*b^2 - 22*B*b^3)*c^3*d^3 + 33*(3*C*a*b^2 - 7*A*b^3)*c^2*
d^4 - 6*(13*D*a^2*b + 44*B*a*b^2)*c*d^5 - 231*(C*a^2*b + 3*A*a*b^2)*d^6)*s
qrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*
a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(315*D*b^3*d^6*x^4 -
 64*D*b^3*c^4*d^2 + 88*C*b^3*c^3*d^3 + 2*(23*D*a*b^2 - 66*B*b^3)*c^2*d^4 -
 11*(8*C*a*b^2 - 21*A*b^3)*c*d^5 - 30*(5*D*a^2*b + 11*B*a*b^2)*d^6 + 35*(D
*b^3*c*d^5 + 11*C*b^3*d^6)*x^3 - 5*(8*D*b^3*c^2*d^4 - 11*C*b^3*c*d^5 + 9*(
2*D*a*b^2 - 11*B*b^3)*d^6)*x^2 + (48*D*b^3*c^3*d^3 - 66*C*b^3*c^2*d^4 - (3
2*D*a*b^2 - 99*B*b^3)*c*d^5 - 77*(2*C*a*b^2 - 9*A*b^3)*d^6)*x)*sqrt(-b*x^2
 + a)*sqrt(d*x + c))/(b^3*d^6)
```

Sympy [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)\,dx = \int \sqrt{a-bx^2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a - b*x**2)*sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c), x)
```

Giac [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \sqrt{a-b\,x^2}\,\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a - b*x^2)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a - b*x^2)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 462*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*b*d**3 - 306*sqrt(c + d*x)*sqr
t(a - b*x**2)*a**2*c*d**3 + 462*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d*
*2*x - 396*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d**2 + 2*sqrt(c + d*x)*
sqrt(a - b*x**2)*a*b*c**3*d - 124*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*
d**2*x - 60*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**3*x**2 + 66*sqrt(c + d
*x)*sqrt(a - b*x**2)*b**3*c**2*d*x + 330*sqrt(c + d*x)*sqrt(a - b*x**2)*b*
*3*c*d**2*x**2 - 12*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**4*x + 10*sqrt(c
 + d*x)*sqrt(a - b*x**2)*b**2*c**3*d*x**2 + 280*sqrt(c + d*x)*sqrt(a - b*x
**2)*b**2*c**2*d**2*x**3 + 210*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**3*
x**4 - 693*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x*
*2 - b*d*x**3),x)*a**2*b**2*d**4 - 309*int((sqrt(c + d*x)*sqrt(a - b*x**2)
*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*c*d**4 - 231*int((sqr
t(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a
*b**3*c**2*d**2 - 264*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d
*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c*d**3 + 45*int((sqrt(c + d*x)*sqrt(a
- b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**3*d**2 +
132*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*
d*x**3),x)*b**4*c**3*d - 24*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c
 + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c**5 + 231*int((sqrt(c + d*x)*sqrt
(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*b*d**4 + 153*...
```

3.137 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 598

$$
\begin{aligned}
&\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx \\
&= \frac{2(3ad^2(5Cd-6cD)+b(72c^2Cd-84Bcd^2+105Ad^3-64c^3D))\sqrt{c+dx}\sqrt{a-bx^2}}{315bd^4} \\
&\quad + \frac{2(7ad^2D-b(18cCd-21Bd^2-16c^2D))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{105bd^3} \\
&\quad - \frac{2(3Cd-5cD)\sqrt{c+dx}(a-bx^2)^{3/2}}{21bd^2} - \frac{2D(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}{9bd^2} \\
&\quad - \frac{4\sqrt{a}(21a^2d^4D-3abd^2(13cCd-21Bd^2-10c^2D)+b^2c(72c^2Cd-84Bcd^2+105Ad^3-64c^3D))\sqrt{c+\cdots}}{315b^{3/2}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad + \frac{4\sqrt{a}\left(bc^2-ad^2\right)\left(3ad^2(5Cd-6cD)+b(72c^2Cd-84Bcd^2+105Ad^3-64c^3D)\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{Ellipt}\cdots}{315b^{3/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output
```
2/315*(3*a*d^2*(5*C*d-6*D*c)+b*(105*A*d^3-84*B*c*d^2+72*C*c^2*d-64*D*c^3))
*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^4+2/105*(7*a*d^2*D-b*(-21*B*d^2+18*C*c
*d-16*D*c^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^3-2/21*(3*C*d-5*D*c)*(d
*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d^2-2/9*D*(d*x+c)^(3/2)*(-b*x^2+a)^(3/2)/b/
d^2-4/315*a^(1/2)*(21*a^2*d^4*D-3*a*b*d^2*(-21*B*d^2+13*C*c*d-10*D*c^2)+b^
2*c*(105*A*d^3-84*B*c*d^2+72*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2
)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2
)))^(1/2)/(-b*x^2+a)^(1/2)+4/315*a^(1/2)*(-a*d^2+b*c^2)*(3*a*d^2*(5*C*d-6*
D*c)+b*(105*A*d^3-84*B*c*d^2+72*C*c^2*d-64*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^5/(
d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.06 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(b(c+dx)\left(-2ad^2(15Cd-11cD+7dDx)+b(-64c^3D+24c^2d(3C+2Dx)-2cd^2(42B+\right.\right.}{}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(b*(c + d*x)*(-2*a*d^2*(15*C*d - 11*c*D + 7*d*D*x) + b*
(-64*c^3*D + 24*c^2*d*(3*C + 2*D*x) - 2*c*d^2*(42*B + x*(27*C + 20*D*x)) +
 d^3*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))))) - (2*(d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(21*a^2*d^4*D + 3*a*b*d^2*(-13*c*C*d + 21*B*d^2 + 10*c^2*D) +
 b^2*c*(72*c^2*C*d - 84*B*c*d^2 + 105*A*d^3 - 64*c^3*D))*(a - b*x^2) + I*S
qrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(21*a^2*d^4*D + 3*a*b*d^2*(-13*c*C*d + 21*B
*d^2 + 10*c^2*D) + b^2*c*(72*c^2*C*d - 84*B*c*d^2 + 105*A*d^3 - 64*c^3*D))
*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d
*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
- I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(21*a^(3/2)*d^3*D - 3*a*Sqrt
[b]*d^2*(5*C*d - 6*c*D) + 3*Sqrt[a]*b*d*(-18*c*C*d + 21*B*d^2 + 16*c^2*D)
+ b^(3/2)*(-72*c^2*C*d + 84*B*c*d^2 - 105*A*d^3 + 64*c^3*D))*Sqrt[(d*(Sqrt
[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))
]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(315*b^2*d^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2185, 27, 2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{3\sqrt{a-bx^2}\left(b(3Cd-5cD)x^2d^2+(3Abd+acD)d^2+\left(-2bDc^2+3bBd^2+ad^2D\right)xd\right)}{2\sqrt{c+dx}}dx}{9bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(b(3Cd-5cD)x^2d^2+(3Abd+acD)d^2+\left(-2bDc^2+3bBd^2+ad^2D\right)xd\right)}{\sqrt{c+dx}}dx}{3bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{2\int -\frac{bd^3\left(d(21Abd+3aCd+2acD)+\left(7ad^2D-b\left(-16Dc^2+18Cdc-21Bd^2\right)\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7bd^2}-\frac{2}{7}d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(3Cd-5cD)}{3bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}d\int \frac{(d(21Abd+3aCd+2acD)+(7ad^2D-b(-16Dc^2+18Cdc-21Bd^2))x)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx-\frac{2}{7}d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(3Cd-5cD)}{3bd^3}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{1}{7}d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(3dx(7ad^2D-b(-21Bd^2-16c^2D+18cCd))+3ad^2(5Cd-6cD)+b(105Ad^3-84Bcd^2-64c^3D+72c^2Cd))}{15d^2}-\frac{4\int -\frac{b\left(ad\left(3a\right.\right.}{}}{}\right.}{}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}d\left(\frac{2\int \frac{ad\left(3a(5Cd+cD)d^2+b\left(-16Dc^3+18Cdc^2-21Bd^2c+105Ad^3\right)\right)+\left(5bcd^2(21Abd+3aCd+2acD)-\left(4bc^2-3ad^2\right)\left(7ad^2D-b\left(-16Dc^2+18Cdc-21Bd^2\right)\right)\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\right.}{}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(\frac{\left(5bcd^2(2acD+3aCd+21Abd)-\left(4bc^2-3ad^2\right)\left(7ad^2D-b\left(-21Bd^2-16c^2D+18cCd\right)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(5Cd-6cD)+b\left(105Ad^3-84Bc\right.\right.}{d}\right.}{15d^2}\right.}{}-\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(5bcd^2(2acD+3aCd+21Abd)-\left(4bc^2-3ad^2\right)\left(7ad^2D-b\left(-21Bd^2-16c^2D+18cCd\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(5Cd-6cD)+b\left(105A\cdots\right.\right.}{\cdots}\right.}{15d^2}\cdots\right.}{\cdots}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5bcd^2(2acD+3aCd+21Abd)-\left(4bc^2-3ad^2\right)\left(7ad^2D-b\left(-21Bd^2-16c^2D+18cCd\right)\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-\cdots\right.}{\cdots}\right.}{15d^2}\cdots\right.}{\cdots}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(3ad^2(5Cd-6cD)+b\left(105Ad^3-84Bcd^2-64c^3D+72c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{}\right.}{15d^2}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(5Cd-6cD)+b\left(105Ad^3-84Bcd^2-64c^3D+72c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{}\right.}{15d^2}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(5Cd-6cD)+b\left(105Ad^3-84Bcd^2-64c^3D+72c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\right.}{15d^2}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \textcolor{blue}{321}$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3ad^2(5Cd-6cD)+b\left(105Ad^3-84Bcd^2-64c^3D+72c^2Cd\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-}}{}\right.}{15d^2}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(-2*D*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/(9*b*d^2) + ((-2*d*(3*C*d - 5*c*D
)*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/7 + (d*((2*Sqrt[c + d*x]*(3*a*d^2*(5*C*
d - 6*c*D) + b*(72*c^2*C*d - 84*B*c*d^2 + 105*A*d^3 - 64*c^3*D) + 3*d*(7*a
*d^2*D - b*(18*c*C*d - 21*B*d^2 - 16*c^2*D))*x)*Sqrt[a - b*x^2])/(15*d^2)
+ (2*((-2*Sqrt[a]*(5*b*c*d^2*(21*A*b*d + 3*a*C*d + 2*a*c*D) - (4*b*c^2 - 3
*a*d^2)*(7*a*d^2*D - b*(18*c*C*d - 21*B*d^2 - 16*c^2*D)))*Sqrt[c + d*x]*Sq
rt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
 (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*d
^2*(5*C*d - 6*c*D) + b*(72*c^2*C*d - 84*B*c*d^2 + 105*A*d^3 - 64*c^3*D))*S
qrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/7)/(3*b
*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 1033, normalized size of antiderivative = 1.73

method	result	size
elliptic	Expression too large to display	1033
default	Expression too large to display	4685

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(2/9*D/d*x^3*(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(-C*b+8/9*D/d*b*c)/b/d*x^2*(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-B*b+2/9*D*a-6/7*(-C*b+8/9*D/d*b*c)/d*c)/b/
d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-A*b+C*a-2/3*D/d*a*c+5/7*(-C*b
+8/9*D/d*b*c)/b*a-4/5*(-B*b+2/9*D*a-6/7*(-C*b+8/9*D/d*b*c)/d*c)/d*c)/b/d*(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*a+2/5*(-B*b+2/9*D*a-6/7*(-C*b+8/9*D
/d*b*c)/d*c)/b/d*a*c+1/3*(-A*b+C*a-2/3*D/d*a*c+5/7*(-C*b+8/9*D/d*b*c)/b*a-
4/5*(-B*b+2/9*D*a-6/7*(-C*b+8/9*D/d*b*c)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/
2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B*a+4/7*(-C
*b+8/9*D/d*b*c)/b/d*a*c+3/5*(-B*b+2/9*D*a-6/7*(-C*b+8/9*D/d*b*c)/d*c)/b*a-
2/3*(-A*b+C*a-2/3*D/d*a*c+5/7*(-C*b+8/9*D/d*b*c)/b*a-4/5*(-B*b+2/9*D*a-6/7
*(-C*b+8/9*D/d*b*c)/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(
1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{2\left(2\left(64\,Db^2c^5-72\,Cb^2c^4d-6\left(13\,Dab-14\,Bb^2\right)c^3d^2+3\left(31\,Cab-35\,Ab^2\right)c^2d^3-6\left(2\,Da^2+21\,Ba\right.\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-2/945*(2*(64*D*b^2*c^5 - 72*C*b^2*c^4*d - 6*(13*D*a*b - 14*B*b^2)*c^3*d^2
 + 3*(31*C*a*b - 35*A*b^2)*c^2*d^3 - 6*(2*D*a^2 + 21*B*a*b)*c*d^4 + 45*(C*
a^2 + 7*A*a*b)*d^5)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(64*D*b^
2*c^4*d - 72*C*b^2*c^3*d^2 - 6*(5*D*a*b - 14*B*b^2)*c^2*d^3 + 3*(13*C*a*b
- 35*A*b^2)*c*d^4 - 21*(D*a^2 + 3*B*a*b)*d^5)*sqrt(-b*d)*weierstrassZeta(4
/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstra
ssPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), 1/3*(3*d*x + c)/d)) - 3*(35*D*b^2*d^5*x^3 - 64*D*b^2*c^3*d^2 + 72*C*b^2
*c^2*d^3 + 2*(11*D*a*b - 42*B*b^2)*c*d^4 - 15*(2*C*a*b - 7*A*b^2)*d^5 - 5*
(8*D*b^2*c*d^4 - 9*C*b^2*d^5)*x^2 + (48*D*b^2*c^2*d^3 - 54*C*b^2*c*d^4 - 7
*(2*D*a*b - 9*B*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*d^6)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2 + D*x**3)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-b\,x^2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\text{Too large to display}$$

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output

```
( - 42*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - 126*sqrt(c + d*x)*sqrt(a
 - b*x**2)*a*b**2*d**2 + 2*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*d - 28*
sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**2*x + 126*sqrt(c + d*x)*sqrt(a - b
*x**2)*b**3*c*d*x - 12*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*x + 10*sqr
t(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**2 + 70*sqrt(c + d*x)*sqrt(a - b
*x**2)*b**2*c*d**2*x**3 - 63*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*
c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*d**4 - 315*int((sqrt(c + d*x)*s
qrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c*d**2
 - 189*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 -
 b*d*x**3),x)*a*b**3*d**3 + 27*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(
a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**2*d**2 + 252*int((sqrt(c +
 d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**4*c
**2*d - 24*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x*
*2 - b*d*x**3),x)*b**3*c**4 + 21*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c
 + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*d**4 + 315*int((sqrt(c + d*x)*sqrt
(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b**2*c*d**2 + 63
*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),
x)*a**2*b**2*d**3 + 27*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x -
 b*c*x**2 - b*d*x**3),x)*a**2*b*c**2*d**2 - 126*int((sqrt(c + d*x)*sqrt(a
- b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c**2*d + 12*in...
```

3.138 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 553

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{d^4\sqrt{c+dx}}$$
$$-\frac{2(10ad^2D+b(84cCd-35Bd^2-141c^2D))\sqrt{c+dx}\sqrt{a-bx^2}}{105bd^4}$$
$$+\frac{2(7Cd-23cD)(c+dx)^{3/2}\sqrt{a-bx^2}}{35d^4}+\frac{2D(c+dx)^{5/2}\sqrt{a-bx^2}}{7d^4}$$
$$-\frac{4\sqrt{a}(ad^2(21Cd-34cD)-b(168c^2Cd-140Bcd^2+105Ad^3-192c^3D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\cdots\right.\right.}{105\sqrt{b}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{4\sqrt{a}(5a^2d^4D-abd^2(63cCd-35Bd^2-82c^2D)+b^2c(168c^2Cd-140Bcd^2+105Ad^3-192c^3D))\sqrt{\cdots}}{105b^{3/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^4/(d*x+c)^(1/2)-2/105*
(10*a*d^2*D+b*(-35*B*d^2+84*C*c*d-141*D*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/
2)/b/d^4+2/35*(7*C*d-23*D*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^4+2/7*D*(d*x
+c)^(5/2)*(-b*x^2+a)^(1/2)/d^4-4/105*a^(1/2)*(a*d^2*(21*C*d-34*D*c)-b*(105
*A*d^3-140*B*c*d^2+168*C*c^2*d-192*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1
/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(
1/2)/(-b*x^2+a)^(1/2)-4/105*a^(1/2)*(5*a^2*d^4*D-a*b*d^2*(-35*B*d^2+63*C*c
*d-82*D*c^2)+b^2*c*(105*A*d^3-140*B*c*d^2+168*C*c^2*d-192*D*c^3))*((d*x+c)
/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/b^(3/2)/d^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.46 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a-bx^2}\left(105b(-c^2Cd+Bcd^2-Ad^3+c^3D)+(-10ad^2D+\right.}{}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(105*b*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D) + (-10*a*
d^2*D + b*(-63*c*C*d + 35*B*d^2 + 87*c^2*D))*(c + d*x) + 3*b*d*(7*C*d - 13
*c*D)*x*(c + d*x) + 15*b*d^2*D*x^2*(c + d*x) - (2*(d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]*(a*d^2*(21*C*d - 34*c*D) + b*(-168*c^2*C*d + 140*B*c*d^2 - 105
*A*d^3 + 192*c^3*D))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^
2*(21*C*d - 34*c*D) + b*(-168*c^2*C*d + 140*B*c*d^2 - 105*A*d^3 + 192*c^3*
D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
- d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d
)] - I*Sqrt[a]*d*(5*a^(3/2)*d^3*D + a*Sqrt[b]*d^2*(-21*C*d + 34*c*D) + Sqr
t[a]*b*d*(-42*c*C*d + 35*B*d^2 + 48*c^2*D) + b^(3/2)*(168*c^2*C*d - 140*B*
c*d^2 + 105*A*d^3 - 192*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*
Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[
a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a -
b*x^2))))/(105*b*d^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2182, 27, 2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int \frac{\sqrt{a-bx^2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{6Dc^3}{d^2}-\frac{6Cc^2}{d}+5Bc-5Ad\right)\right)x+\frac{-aDc^2+Abdc+aCdc-aBd^2}{d}\right)}{2\sqrt{c+dx}}dx}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{6Dc^3}{d^2}-\frac{6Cc^2}{d}+5Bc-5Ad\right)\right)x+Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{\sqrt{c+dx}}dx}{bc^2-ad^2}+$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{2}{7}D\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{2\int -\frac{\left(d\left(7Ab^2cd-a\left(ad^2D-b\left(-6Dc^2+7Cdc-7Bd^2\right)\right)\right)-b\left(ad^2(7Cd-13cD)-b\left(-48Dc^3+42Cdc^2-35Bd^2\right.\right.\right.}{2\sqrt{c+dx}}}{7bd^2}}{bc^2-ad^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\left(d\left(7Ab^2cd-a\left(ad^2D-b\left(-6Dc^2+7Cdc-7Bd^2\right)\right)\right)-b\left(ad^2(7Cd-13cD)-b\left(-48Dc^3+42Cdc^2-35Bd^2c+35Ad^3\right)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7bd^2}+\frac{2}{7}D\left(a-bx^2\right)^{3/2}\sqrt{}}{bc^2-ad^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{-\frac{4\int -\frac{b\left(ad\left(bc^2-ad^2\right)\left(5ad^2D-b\left(-48Dc^2+42Cdc-35Bd^2\right)\right)+b\left(bc^2-ad^2\right)\left(ad^2(21Cd-34cD)-b\left(-192Dc^3+168Cdc^2-140Bd^2c+105Ad^3\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-bx^2}}{}}{7}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\int \frac{ad\left(bc^2-ad^2\right)\left(5ad^2D-b\left(-48Dc^2+42Cdc-35Bd^2\right)\right)+b\left(bc^2-ad^2\right)\left(ad^2(21Cd-34cD)-b\left(-192Dc^3+168Cdc^2-140Bd^2c+105Ad^3\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}}{}}{7bd^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 600}$$

$$2\left(\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\left(bc^2-ad^2\right)\left(ad^2(21Cd-34cD)\right.}{}\right.$$
$$\overline{\qquad 15d^2 \qquad}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 509$$

$$2\left(\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad^2(21C\right.}{}\right.$$
$$\overline{\qquad 15d^2 \qquad}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 508$$

$$2\left(\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-a\right.}{}\right.$$
$$\overline{\qquad 15d^2 \qquad}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 327$$

$$2\left(\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bc^2-ad^2\right.}{}\right.$$

$$15d^2$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+d}}{}\right.$$

$$15d^2$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3D+168c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$15$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(5a^2d^4D-abd^2\left(-35Bd^2-82c^2D+63cCd\right)+b^2c\left(105Ad^3-140Bcd^2-192c^3\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$15d^2$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(3/2))/(d^2*(b*c^2 - a*
d^2)*Sqrt[c + d*x]) + ((2*(a/b - c^2/d^2)*D*Sqrt[c + d*x]*(a - b*x^2)^(3/2
))/7 + ((-2*Sqrt[c + d*x]*(5*a^2*d^4*D - a*b*d^2*(63*c*C*d - 35*B*d^2 - 82
*c^2*D) + b^2*c*(168*c^2*C*d - 140*B*c*d^2 + 105*A*d^3 - 192*c^3*D) + 3*b*
d*(a*d^2*(7*C*d - 13*c*D) - b*(42*c^2*C*d - 35*B*c*d^2 + 35*A*d^3 - 48*c^3
*D))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((-2*Sqrt[a]*Sqrt[b]*(b*c^2 - a*d^2
)*(a*d^2*(21*C*d - 34*c*D) - b*(168*c^2*C*d - 140*B*c*d^2 + 105*A*d^3 - 19
2*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt
[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c
^2 - a*d^2)*(5*a^2*d^4*D - a*b*d^2*(63*c*C*d - 35*B*d^2 - 82*c^2*D) + b^2*
c*(168*c^2*C*d - 140*B*c*d^2 + 105*A*d^3 - 192*c^3*D))*Sqrt[(Sqrt[b]*(c +
d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[
b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2))/(7*b*d^2))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. $2(477) = 954$.

Time = 3.53 (sec) , antiderivative size = 1178, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1178
default	Expression too large to display	3797

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x^2+
a*d)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2/7*
D/d^2*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-b/d^2*(C*d-D*c)+6/7*D/d
^2*b*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-1/d^3*(B*b*d^2-C*b*
c*d-D*a*d^2+D*b*c^2)-5/7*D/d*a-4/5*(-b/d^2*(C*d-D*c)+6/7*D/d^2*b*c)/d*c)/b
/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*((A*b*c*d^3+B*a*d^4-B*b*c^2*d^2-C*
a*c*d^3+C*b*c^3*d+D*a*c^2*d^2-D*b*c^4)/d^5-b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)
/d^5*c+2/5*(-b/d^2*(C*d-D*c)+6/7*D/d^2*b*c)/b/d*a*c+1/3*(-1/d^3*(B*b*d^2-C
*b*c*d-D*a*d^2+D*b*c^2)-5/7*D/d*a-4/5*(-b/d^2*(C*d-D*c)+6/7*D/d^2*b*c)/d*c
)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b
*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2))+2*(-1/d^4*(A*b*d^3-B*b*c*d^2-C*a*d^3+C*b*c^2*d+D*a*c*d^2-D*b*c^
3)-b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4-4/7*D/d^2*a*c+3/5*(-b/d^2*(C*d-D*c)
+6/7*D/d^2*b*c)/b*a-2/3*(-1/d^3*(B*b*d^2-C*b*c*d-D*a*d^2+D*b*c^2)-5/7*D/d*
a-4/5*(-b/d^2*(C*d-D*c)+6/7*D/d^2*b*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\left(2\left(192\,Db^2c^5-168\,Cb^2c^4d-2\left(89\,Dab-70\,Bb^2\right)c^3d^2+21\,(\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
2/315*(2*(192*D*b^2*c^5 - 168*C*b^2*c^4*d - 2*(89*D*a*b - 70*B*b^2)*c^3*d^
2 + 21*(7*C*a*b - 5*A*b^2)*c^2*d^3 - 15*(D*a^2 + 7*B*a*b)*c*d^4 + (192*D*b
^2*c^4*d - 168*C*b^2*c^3*d^2 - 2*(89*D*a*b - 70*B*b^2)*c^2*d^3 + 21*(7*C*a
*b - 5*A*b^2)*c*d^4 - 15*(D*a^2 + 7*B*a*b)*d^5)*x)*sqrt(-b*d)*weierstrassP
Inverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
1/3*(3*d*x + c)/d) + 6*(192*D*b^2*c^4*d - 168*C*b^2*c^3*d^2 - 2*(17*D*a*b
- 70*B*b^2)*c^2*d^3 + 21*(C*a*b - 5*A*b^2)*c*d^4 + (192*D*b^2*c^3*d^2 - 16
8*C*b^2*c^2*d^3 - 2*(17*D*a*b - 70*B*b^2)*c*d^4 + 21*(C*a*b - 5*A*b^2)*d^5
)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^
3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
 -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(15*D*b^2*d^5*
x^3 + 192*D*b^2*c^3*d^2 - 168*C*b^2*c^2*d^3 - 105*A*b^2*d^5 - 10*(D*a*b -
14*B*b^2)*c*d^4 - 3*(8*D*b^2*c*d^4 - 7*C*b^2*d^5)*x^2 + (48*D*b^2*c^2*d^3
- 42*C*b^2*c*d^4 - 5*(2*D*a*b - 7*B*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x
 + c))/(b^2*d^7*x + b^2*c*d^6)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{a-bx^2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

3.139 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 608

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{3d^4(c+dx)^{3/2}}$$

$$-\frac{2(3ad^2(2cCd-Bd^2-3c^2D)-bc(8c^2Cd-5Bcd^2+2Ad^3-11c^3D))\sqrt{a-bx^2}}{3d^4(bc^2-ad^2)\sqrt{c+dx}}$$

$$+\frac{2(5Cd-17cD)\sqrt{c+dx}\sqrt{a-bx^2}}{15d^4}+\frac{2D(c+dx)^{3/2}\sqrt{a-bx^2}}{5d^4}$$

$$+\frac{4\sqrt{a}(3a^2d^4D+abd^2(35cCd-15Bd^2-62c^2D)-b^2c(40c^2Cd-20Bcd^2+5Ad^3-64c^3D))\sqrt{c+dx}\sqrt{\frac{a-\cdots}{}}}{15\sqrt{b}d^5(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{4\sqrt{a}(ad^2(5Cd-14cD)-b(40c^2Cd-20Bcd^2+5Ad^3-64c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{15\sqrt{b}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^4/(d*x+c)^(3/2)-2/3*
(3*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(2*A*d^3-5*B*c*d^2+8*C*c^2*d-11*D*c^
3))*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(1/2)+2/15*(5*C*d-17*D*c)*
(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^4+2/5*D*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^
4+4/15*a^(1/2)*(3*a^2*d^4*D+a*b*d^2*(-15*B*d^2+35*C*c*d-62*D*c^2)-b^2*c*(5
*A*d^3-20*B*c*d^2+40*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)
*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(
1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^5/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d
/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-4/15*a^(1/2)*(a*d^2*(5*C*d-14*D*c)-b*(5*
A*d^3-20*B*c*d^2+40*C*c^2*d-64*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/
2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^5/(d*x+c)^(1/2)
/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.83 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{2(c+dx)\left(5Cd-14cD+3dDx+\frac{5\left(-c^2Cd+Bcd^2-Ad^3+c^3D\right)}{(c+dx)^2}+\frac{5\left(3ad^2\left(-2\cdots\right.\right.}{\cdots}\right.}{d^4}\cdots\right)}{\cdots}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*((2*(c + d*x)*(5*C*d - 14*c*D + 3*d*D*x + (5*(-(c^2*C*d)
+ B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^2 + (5*(3*a*d^2*(-2*c*C*d + B*d^2 +
3*c^2*D) + b*c*(8*c^2*C*d - 5*B*c*d^2 + 2*A*d^3 - 11*c^3*D)))/((b*c^2 - a*
d^2)*(c + d*x))))/d^4 + (4*(-(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*d^
4*D + a*b*d^2*(35*c*C*d - 15*B*d^2 - 62*c^2*D) + b^2*c*(-40*c^2*C*d + 20*B
*c*d^2 - 5*A*d^3 + 64*c^3*D))*(a - b*x^2)) - I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a
]*d)*(3*a^2*d^4*D + a*b*d^2*(35*c*C*d - 15*B*d^2 - 62*c^2*D) + b^2*c*(-40*
c^2*C*d + 20*B*c*d^2 - 5*A*d^3 + 64*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))
/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)
*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[
b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*
c - Sqrt[a]*d)*(3*a^(3/2)*d^3*D + a*Sqrt[b]*d^2*(-5*C*d + 14*c*D) - 3*Sqrt
[a]*b*d*(-10*c*C*d + 5*B*d^2 + 16*c^2*D) + b^(3/2)*(40*c^2*C*d - 20*B*c*d^
2 + 5*A*d^3 - 64*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(
((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSin
h[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(
Sqrt[b]*c - Sqrt[a]*d)]))/(b*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a
*d^2)*(-a + b*x^2))))/(15*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2182, 27, 2182, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int \frac{3\sqrt{a-bx^2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+\frac{Abcd+a\left(-Dc^2+Cdc-Bd^2\right)}{d}\right)}{2(c+dx)^{3/2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^{3/2}}dx}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{2\int \frac{\left(bc^2-ad^2\right)\left(d(Abd-aCd+2acD)-\left(aDd^2+b\left(-16Dc^2+10Cdc-5Bd^2\right)\right)x\right)\sqrt{a-bx^2}}{2d^2\sqrt{c+dx}}dx}{bc^2-ad^2}-\frac{2\left(a-bx^2\right)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{d^2\sqrt{c+dx}}}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\left(a-\frac{bc^2}{d^2}\right)\int \frac{\left(d(Abd-aCd+2acD)-\left(aDd^2+b\left(-16Dc^2+10Cdc-5Bd^2\right)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{bc^2-ad^2}-\frac{2\left(a-bx^2\right)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{d^2\sqrt{c+dx}}}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{4\int \frac{b\left(ad\left(ad^2(5Cd-11cD)-b\left(-16Dc^3+10Cdc^2-5Bd^2c+5Ad^3\right)\right)-\left(5bc(Abd-aCd+2acD)d^2+\left(4bc^2-3ad^2\right)\left(aDd^2+b\left(-16Dc^2+10Cdc-5Bd^2\cdots\right)\right)\right)\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{15bd^2}\cdots\right)}{bc^2-ad^2}\cdots}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\int \frac{ad\left(ad^2(5Cd-11cD)-b\left(-16Dc^3+10Cdc^2-5Bd^2c+5Ad^3\right)\right)-\left(5bc(Abd-aCd+2acD)d^2+\left(4bc^2-3ad^2\right)\left(aDd^2+b\left(-16Dc^2+10Cdc-5Bd^2\right)\cdots\right)\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\cdots\right)}{bc^2-ad^2}\cdots}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 600$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(-\frac{\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(ad^2(5Cd-14cD)-b\left(5Ad^3-2\cdots\right.\right.}{\cdots}\right.}{15d^2}\right.}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 509$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(ad^2(5Cd-14cD)-\cdots\right.}{\cdots}\right.}{15d^2}\right.}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 508$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2\cdots\right.}{\cdots}\right.}{15d^2}\right.}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 327$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{(b\cdots}{\cdots}\cdots\right)}{15d^2}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{\cdots}}{\cdots}\cdots\right)}{15d^2}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\left(ad^2(5Cd-14cD)-b\left(5Ad^3-20Bcd^2-64c^3D+40c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\sqrt{1-}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{1}\right.}{}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{\left(a-\frac{bc^2}{d^2}\right)\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(5bcd^2(2acD-aCd+Abd)+\left(4bc^2-3ad^2\right)\left(ad^2D+b\left(-5Bd^2-16c^2D+10cCd\right)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}+\frac{2\sqrt{}}{}\right.}{15d^2}\right.}{}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(3/2))/(3*d^2*(b*c^2 -
a*d^2)*(c + d*x)^(3/2)) + ((-2*(2*c*C*d - B*d^2 - 3*c^2*D)*(a - b*x^2)^(3/
2))/(d^2*Sqrt[c + d*x]) - ((a - (b*c^2)/d^2)*((-2*Sqrt[c + d*x]*(a*d^2*(5*
C*d - 14*c*D) - b*(40*c^2*C*d - 20*B*c*d^2 + 5*A*d^3 - 64*c^3*D) + 3*d*(a*
d^2*D + b*(10*c*C*d - 5*B*d^2 - 16*c^2*D))*x)*Sqrt[a - b*x^2])/(15*d^2) -
(2*((2*Sqrt[a]*(5*b*c*d^2*(A*b*d - a*C*d + 2*a*c*D) + (4*b*c^2 - 3*a*d^2)*
(a*d^2*D + b*(10*c*C*d - 5*B*d^2 - 16*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sq
rt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(a*d^2*(5*C*d - 1
4*c*D) - b*(40*c^2*C*d - 20*B*c*d^2 + 5*A*d^3 - 64*c^3*D))*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(S
qrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(b*c^2 - a*d^2))/(b*c
^2 - a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(530) = 1060$.

Time = 4.16 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1143
default	Expression too large to display	9026

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/3*(A*d^3-B
*c*d^2+C*c^2*d-D*c^3)/d^6*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-2/3
*(-b*d*x^2+a*d)/d^5/(a*d^2-b*c^2)*(2*A*b*c*d^3+3*B*a*d^4-5*B*b*c^2*d^2-6*C
*a*c*d^3+8*C*b*c^3*d+9*D*a*c^2*d^2-11*D*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1
/2)+2/5*D/d^3*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-b/d^3*(C*d-2*D*c)
+4/5*D/d^3*b*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(A*b*d^3-2*B*b*
c*d^2-C*a*d^3+3*C*b*c^2*d+2*D*a*c*d^2-4*D*b*c^3)/d^5+1/3*b*(A*d^3-B*c*d^2+
C*c^2*d-D*c^3)/d^5-1/3*b/d^5*c*(2*A*b*c*d^3+3*B*a*d^4-5*B*b*c^2*d^2-6*C*a*
c*d^3+8*C*b*c^3*d+9*D*a*c^2*d^2-11*D*b*c^4)/(a*d^2-b*c^2)-2/5*D/d^3*a*c+1/
3*(-b/d^3*(C*d-2*D*c)+4/5*D/d^3*b*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/d^4*(B*b*d^2-2*C*b*c*d
-D*a*d^2+3*D*b*c^2)-1/3*b/d^4*(2*A*b*c*d^3+3*B*a*d^4-5*B*b*c^2*d^2-6*C*a*c
*d^3+8*C*b*c^3*d+9*D*a*c^2*d^2-11*D*b*c^4)/(a*d^2-b*c^2)-3/5*D/d^2*a-2/3*(
-b/d^3*(C*d-2*D*c)+4/5*D/d^3*b*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-2/45*(2*(64*D*b^2*c^7 - 40*C*b^2*c^6*d - 10*(11*D*a*b - 2*B*b^2)*c^5*d^2
+ 5*(13*C*a*b - A*b^2)*c^4*d^3 + 6*(6*D*a^2 - 5*B*a*b)*c^3*d^4 - 15*(C*a^2
 - A*a*b)*c^2*d^5 + (64*D*b^2*c^5*d^2 - 40*C*b^2*c^4*d^3 - 10*(11*D*a*b -
2*B*b^2)*c^3*d^4 + 5*(13*C*a*b - A*b^2)*c^2*d^5 + 6*(6*D*a^2 - 5*B*a*b)*c*
d^6 - 15*(C*a^2 - A*a*b)*d^7)*x^2 + 2*(64*D*b^2*c^6*d - 40*C*b^2*c^5*d^2 -
 10*(11*D*a*b - 2*B*b^2)*c^4*d^3 + 5*(13*C*a*b - A*b^2)*c^3*d^4 + 6*(6*D*a
^2 - 5*B*a*b)*c^2*d^5 - 15*(C*a^2 - A*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d) + 6*(64*D*b^2*c^6*d - 40*C*b^2*c^5*d^2 - 2*(31*D*a*b
- 10*B*b^2)*c^4*d^3 + 5*(7*C*a*b - A*b^2)*c^3*d^4 + 3*(D*a^2 - 5*B*a*b)*c^
2*d^5 + (64*D*b^2*c^4*d^3 - 40*C*b^2*c^3*d^4 - 2*(31*D*a*b - 10*B*b^2)*c^2
*d^5 + 5*(7*C*a*b - A*b^2)*c*d^6 + 3*(D*a^2 - 5*B*a*b)*d^7)*x^2 + 2*(64*D*
b^2*c^5*d^2 - 40*C*b^2*c^4*d^3 - 2*(31*D*a*b - 10*B*b^2)*c^3*d^4 + 5*(7*C*
a*b - A*b^2)*c^2*d^5 + 3*(D*a^2 - 5*B*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstras
sZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), we
ierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)
/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(64*D*b^2*c^5*d^2 - 40*C*b^2*c^4*d^3 - 1
0*B*a*b*c*d^6 - 5*A*a*b*d^7 - 2*(27*D*a*b - 10*B*b^2)*c^3*d^4 + 5*(6*C*a*b
 - A*b^2)*c^2*d^5 - 3*(D*b^2*c^2*d^5 - D*a*b*d^7)*x^3 + (8*D*b^2*c^3*d^4 -
 5*C*b^2*c^2*d^5 - 8*D*a*b*c*d^6 + 5*C*a*b*d^7)*x^2 + 5*(16*D*b^2*c^4*d...
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{a-b\,x^2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

3.140 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 754

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{5d^4(c+dx)^{5/2}}$$

$$-\frac{2(5ad^2(2cCd-Bd^2-3c^2D)-bc(12c^2Cd-7Bcd^2+2Ad^3-17c^3D))\sqrt{a-bx^2}}{15d^4(bc^2-ad^2)(c+dx)^{3/2}}$$

$$-\frac{2(15a^2d^4(Cd-3cD)+b^2c^2(33c^2Cd-8Bcd^2-2Ad^3-73c^3D)-2abd^2(28c^2Cd-8Bcd^2+3Ad^3-63c}{15d^4(bc^2-ad^2)^2\sqrt{c+dx}}$$

$$+\frac{2D\sqrt{c+dx}\sqrt{a-bx^2}}{3d^4}$$

$$+\frac{4\sqrt{a}\sqrt{b}(5a^2d^4(3Cd-10cD)-abd^2(43c^2Cd-8Bcd^2+3Ad^3-118c^3D)+b^2c^2(24c^2Cd-4Bcd^2-Ad^3}{15d^5(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{4\sqrt{a}(5a^2d^4D+5abd^2(5cCd-Bd^2-14c^2D)-b^2c(24c^2Cd-4Bcd^2-Ad^3-64c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{E}}{15\sqrt{b}d^5(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^4/(d*x+c)^(5/2)-2/15
*(5*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(2*A*d^3-7*B*c*d^2+12*C*c^2*d-17*D*
c^3))*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(3/2)-2/15*(15*a^2*d^4*(
C*d-3*D*c)+b^2*c^2*(-2*A*d^3-8*B*c*d^2+33*C*c^2*d-73*D*c^3)-2*a*b*d^2*(3*A
*d^3-8*B*c*d^2+28*C*c^2*d-63*D*c^3))*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)^2
/(d*x+c)^(1/2)+2/3*D*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^4+4/15*a^(1/2)*b^(1/
2)*(5*a^2*d^4*(3*C*d-10*D*c)-a*b*d^2*(3*A*d^3-8*B*c*d^2+43*C*c^2*d-118*D*c
^3)+b^2*c^2*(-A*d^3-4*B*c*d^2+24*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^
(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^5/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(
1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+4/15*a^(1/2)*(5*a^2*d^4*D+5*a*b*d^
2*(-B*d^2+5*C*c*d-14*D*c^2)-b^2*c*(-A*d^3-4*B*c*d^2+24*C*c^2*d-64*D*c^3))*
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/b^(1/2)/d^5/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.30 (sec) , antiderivative size = 1099, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((2*D)/(3*d^4) + (2*(-(c^2*C*d) + B*c*d^2 -
A*d^3 + c^3*D))/(5*d^4*(c + d*x)^3) - (2*(12*b*c^3*C*d - 7*b*B*c^2*d^2 + 2
*A*b*c*d^3 - 10*a*c*C*d^3 + 5*a*B*d^4 - 17*b*c^4*D + 15*a*c^2*d^2*D))/(15*
d^4*(-(b*c^2) + a*d^2)*(c + d*x)^2) - (2*(33*b^2*c^4*C*d - 8*b^2*B*c^3*d^2
 - 2*A*b^2*c^2*d^3 - 56*a*b*c^2*C*d^3 + 16*a*b*B*c*d^4 - 6*a*A*b*d^5 + 15*
a^2*C*d^5 - 73*b^2*c^5*D + 126*a*b*c^3*d^2*D - 45*a^2*c*d^4*D))/(15*d^4*(-
(b*c^2) + a*d^2)^2*(c + d*x))) + (4*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d
*x))^2)/d^2]*(-(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(5*a^2*d^4*(-3*C*d + 10*c*D
) + a*b*d^2*(43*c^2*C*d - 8*B*c*d^2 + 3*A*d^3 - 118*c^3*D) + b^2*c^2*(-24*
c^2*C*d + 4*B*c*d^2 + A*d^3 + 64*c^3*D))*(-((a*d^2)/(c + d*x)^2) + b*(-1 +
 c/(c + d*x))^2)) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(5*a^2*d^4*(-3*C*d
+ 10*c*D) + a*b*d^2*(43*c^2*C*d - 8*B*c*d^2 + 3*A*d^3 - 118*c^3*D) + b^2*c
^2*(-24*c^2*C*d + 4*B*c*d^2 + A*d^3 + 64*c^3*D))*Sqrt[1 - c/(c + d*x) - (S
qrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]
*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] + (I
*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(5*a^2*d^4*D + 15*a^(3/2)*Sqrt[b]*d^3*(
C*d - 3*c*D) - 5*a*b*d^2*(-5*c*C*d + B*d^2 + 14*c^2*D) + 3*Sqrt[a]*b^(3/2)
*d*(-6*c^2*C*d + B*c*d^2 - A*d^3 + 16*c^3*D) + b^2*c*(-24*c^2*C*d + 4*B*c*
d^2 + A*d^3 + 64*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c...
```

Rubi [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2182, 27, 2182, 27, 681, 25, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \ \color{blue}{2182}$$

$$\frac{2\int \frac{\sqrt{a-bx^2}\left(5\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(5Cd-5cD)+b\left(\frac{6Dc^3}{d^2}-\frac{6Cc^2}{d}+Bc-Ad\right)\right)x+\frac{5\left(Abcd+a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{2(c+dx)^{5/2}}dx}{5\left(bc^2-ad^2\right)}+$$
$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(5\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(5Cd-5cD)+b\left(\frac{6Dc^3}{d^2}-\frac{6Cc^2}{d}+Bc-Ad\right)\right)x+5\left(Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^{5/2}}dx}{5\left(bc^2-ad^2\right)}+$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{2\int \frac{3\left(d\left(Abd\left(5bc^2-ad^2\right)+a\left(5ad^2(Cd-2cD)-bc\left(-6Dc^2+Cdc+4Bd^2\right)\right)\right)+\left(5a^2Dd^4+5ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-16Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)x\right)\sqrt{\cdots}}{2d^2(c+dx)^{3/2}}}{3(bc^2-ad^2)}\cdots}{5\left(bc^2-ad^2\right)}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(5bc^2-ad^2\right)+a\left(5ad^2(Cd-2cD)-bc\left(-6Dc^2+Cdc+4Bd^2\right)\right)\right)+\left(5a^2Dd^4+5ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-16Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)x\right)\sqrt{a-\cdots}}{(c+dx)^{3/2}}}{d^2(bc^2-ad^2)}\cdots}{5\left(bc^2-ad^2\right)}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{-\frac{2\int -\frac{ad\left(5a^2Dd^4+5ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-16Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)-b\left(5a^2(3Cd-10cD)d^4-ab\left(-118Dc^3+43Cdc^2-8Bd^2c+3Ad^3\right)d^2+b^2c\cdots\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3d^2}\cdots}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int \frac{ad\left(5a^2Dd^4+5ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-16Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)-b\left(5a^2(3Cd-10cD)d^4-ab\left(-118Dc^3+43Cdc^2-8Bd^2c+3Ad^3\right)d^2+b^2c^2\left(\cdots\right)\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3d^2}\cdots}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 600$$

$$2\left(-\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D+5abd^2\left(-Bd^2-14c^2D+5cCd\right)-b^2c\left(-Ad^3-4Bcd^2-64c^3D+24c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(5a^2d^4(3Cd-10cD)-abd^2\left(3Ad^3-8B\ldots\right.\right.}{\ldots}\right.$$
$$\overline{3d^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$2\left(-\frac{\left(bc^2-ad^2\right)\left(5a^2d^4D+5abd^2\left(-Bd^2-14c^2D+5cCd\right)-b^2c\left(-Ad^3-4Bcd^2-64c^3D+24c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(5a^2d^4(3Cd-10cD)-abd^2\left(\ldots\right.\right.}{\ldots}\right.$$
$$\overline{3d^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5a^2d^4(3Cd-10cD)-abd^2\left(3Ad^3-8Bcd^2-118c^3D+43c^2Cd\right)+b^2c^2\left(-Ad^3-4Bcd^2-64c^3D+24c^2Cd\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\ldots}}{\sqrt{2}\ldots}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.$$
$$\overline{3d^2}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(5a^2d^4(3Cd-10cD)-abd^2\left(3Ad^3-8Bcd^2-118c^3D+43c^2Cd\right)+b^2c^2\left(-Ad^3-4Bcd^2-64c^3D+24c^2\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3d^2}}{}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 512$$

$$\frac{\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(5a^2d^4(3Cd-10cD)-abd^2\left(3Ad^3-8Bcd^2-118c^3D+43c^2Cd\right)+b^2c^2\left(-Ad^3-4Bcd^2-64c^3D+24c^2\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3d^2}}{}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 511$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{3/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+$$

$$\frac{\frac{2\left(5ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-11Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{3/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\left(5a^2(3Cd-10cD)d^4-ab\left(-118Dc^3+43Cdc^2-8Bd^2c+3\right.\right.}{}\right.}{}}{}$$

$$\Big\downarrow \ 321$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{3/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+$$

$$\frac{2\left(5ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-11Dc^3+6Cdc^2-Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{3/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\left(5a^2(3Cd-10cD)d^4-ab\left(-118Dc^3+43Cdc^2-8Bd^2c+3A\right.\right.}{}\right.}{}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(3/2))/(5*d^2*(b*c^2 -
a*d^2)*(c + d*x)^(5/2)) + ((2*(5*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(
6*c^2*C*d - B*c*d^2 - 4*A*d^3 - 11*c^3*D))*(a - b*x^2)^(3/2))/(3*d^2*(b*c^
2 - a*d^2)*(c + d*x)^(3/2)) + ((-2*(5*a^2*d^4*(3*C*d - 10*c*D) - a*b*d^2*(
43*c^2*C*d - 8*B*c*d^2 + 3*A*d^3 - 118*c^3*D) + b^2*c^2*(24*c^2*C*d - 4*B*
c*d^2 - A*d^3 - 64*c^3*D) - d*(5*a^2*d^4*D + 5*a*b*d^2*(2*c*C*d - B*d^2 -
5*c^2*D) - b^2*c*(6*c^2*C*d - B*c*d^2 - 4*A*d^3 - 16*c^3*D))*x)*Sqrt[a - b
*x^2])/(3*d^2*Sqrt[c + d*x]) + (2*((2*Sqrt[a]*Sqrt[b]*(5*a^2*d^4*(3*C*d -
10*c*D) - a*b*d^2*(43*c^2*C*d - 8*B*c*d^2 + 3*A*d^3 - 118*c^3*D) + b^2*c^2
*(24*c^2*C*d - 4*B*c*d^2 - A*d^3 - 64*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqr
t[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)
]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(5*a^2*d^4*D + 5*a*b*d^2*(
5*c*C*d - B*d^2 - 14*c^2*D) - b^2*c*(24*c^2*C*d - 4*B*c*d^2 - A*d^3 - 64*c
^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b
]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*d^2))/(
d^2*(b*c^2 - a*d^2)))/(5*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.98 (sec) , antiderivative size = 1281, normalized size of antiderivative = 1.70

method	result	size
elliptic	Expression too large to display	1281
default	Expression too large to display	16906

input
```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/5*(A*d^3-B
*c*d^2+C*c^2*d-D*c^3)/d^7*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3-2/1
5*(2*A*b*c*d^3+5*B*a*d^4-7*B*b*c^2*d^2-10*C*a*c*d^3+12*C*b*c^3*d+15*D*a*c^
2*d^2-17*D*b*c^4)/d^6/(a*d^2-b*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+
c/d)^2+2/15*(-b*d*x^2+a*d)/d^5/(a*d^2-b*c^2)^2*(6*A*a*b*d^5+2*A*b^2*c^2*d^
3-16*B*a*b*c*d^4+8*B*b^2*c^3*d^2-15*C*a^2*d^5+56*C*a*b*c^2*d^3-33*C*b^2*c^
4*d+45*D*a^2*c*d^4-126*D*a*b*c^3*d^2+73*D*b^2*c^5)/((x+c/d)*(-b*d*x^2+a*d)
)^(1/2)+2/3/d^4*D*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(B*b*d^2-3*C*b*c*
d-D*a*d^2+6*D*b*c^2)/d^5+1/15*b*(2*A*b*c*d^3+5*B*a*d^4-7*B*b*c^2*d^2-10*C*
a*c*d^3+12*C*b*c^3*d+15*D*a*c^2*d^2-17*D*b*c^4)/d^5/(a*d^2-b*c^2)+1/15*b/d
^5*c*(6*A*a*b*d^5+2*A*b^2*c^2*d^3-16*B*a*b*c*d^4+8*B*b^2*c^3*d^2-15*C*a^2*
d^5+56*C*a*b*c^2*d^3-33*C*b^2*c^4*d+45*D*a^2*c*d^4-126*D*a*b*c^3*d^2+73*D*
b^2*c^5)/(a*d^2-b*c^2)^2-1/3/d^3*D*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-(C*d-3*D*c)*b/d^4+1/15*b/d^4
*(6*A*a*b*d^5+2*A*b^2*c^2*d^3-16*B*a*b*c*d^4+8*B*b^2*c^3*d^2-15*C*a^2*d^5+
56*C*a*b*c^2*d^3-33*C*b^2*c^4*d+45*D*a^2*c*d^4-126*D*a*b*c^3*d^2+73*D*b^2*
c^5)/(a*d^2-b*c^2)^2+2/3/d^4*D*b*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1722 vs. $2(676) = 1352$.

Time = 0.17 (sec) , antiderivative size = 1722, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
2/45*(2*(64*D*b^3*c^9 - 24*C*b^3*c^8*d - 2*(83*D*a*b^2 - 2*B*b^3)*c^7*d^2
+ (61*C*a*b^2 + A*b^3)*c^6*d^3 + (125*D*a^2*b - 11*B*a*b^2)*c^5*d^4 - 9*(5
*C*a^2*b + A*a*b^2)*c^4*d^5 - 15*(D*a^3 - B*a^2*b)*c^3*d^6 + (64*D*b^3*c^6
*d^3 - 24*C*b^3*c^5*d^4 - 2*(83*D*a*b^2 - 2*B*b^3)*c^4*d^5 + (61*C*a*b^2 +
 A*b^3)*c^3*d^6 + (125*D*a^2*b - 11*B*a*b^2)*c^2*d^7 - 9*(5*C*a^2*b + A*a*
b^2)*c*d^8 - 15*(D*a^3 - B*a^2*b)*d^9)*x^3 + 3*(64*D*b^3*c^7*d^2 - 24*C*b^
3*c^6*d^3 - 2*(83*D*a*b^2 - 2*B*b^3)*c^5*d^4 + (61*C*a*b^2 + A*b^3)*c^4*d^
5 + (125*D*a^2*b - 11*B*a*b^2)*c^3*d^6 - 9*(5*C*a^2*b + A*a*b^2)*c^2*d^7 -
 15*(D*a^3 - B*a^2*b)*c*d^8)*x^2 + 3*(64*D*b^3*c^8*d - 24*C*b^3*c^7*d^2 -
2*(83*D*a*b^2 - 2*B*b^3)*c^6*d^3 + (61*C*a*b^2 + A*b^3)*c^5*d^4 + (125*D*a
^2*b - 11*B*a*b^2)*c^4*d^5 - 9*(5*C*a^2*b + A*a*b^2)*c^3*d^6 - 15*(D*a^3 -
 B*a^2*b)*c^2*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)
/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(64*D*
b^3*c^8*d - 24*C*b^3*c^7*d^2 - 2*(59*D*a*b^2 - 2*B*b^3)*c^6*d^3 + (43*C*a*
b^2 + A*b^3)*c^5*d^4 + 2*(25*D*a^2*b - 4*B*a*b^2)*c^4*d^5 - 3*(5*C*a^2*b -
 A*a*b^2)*c^3*d^6 + (64*D*b^3*c^5*d^4 - 24*C*b^3*c^4*d^5 - 2*(59*D*a*b^2 -
 2*B*b^3)*c^3*d^6 + (43*C*a*b^2 + A*b^3)*c^2*d^7 + 2*(25*D*a^2*b - 4*B*a*b
^2)*c*d^8 - 3*(5*C*a^2*b - A*a*b^2)*d^9)*x^3 + 3*(64*D*b^3*c^6*d^3 - 24*C*
b^3*c^5*d^4 - 2*(59*D*a*b^2 - 2*B*b^3)*c^4*d^5 + (43*C*a*b^2 + A*b^3)*c^3*
d^6 + 2*(25*D*a^2*b - 4*B*a*b^2)*c^2*d^7 - 3*(5*C*a^2*b - A*a*b^2)*c*d^...
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{-bx^2+a}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{7}{2}}}dx$$

input

```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

output

```
int((-b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

3.141 $\int \sqrt{c+dx}(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1077

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

output
```
4/45045*(195*a^3*d^6*D-9*a^2*b*d^4*(-65*B*d^2+6*C*c*d+3*D*c^2)+3*a*b^2*c*d
^2*(572*A*d^3-299*B*c*d^2+186*C*c^2*d-128*D*c^3)-4*b^3*c^3*(143*A*d^3-104*
B*c*d^2+80*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^6+4/150
15*(3*a^2*d^4*(77*C*d+6*D*c)-a*b*d^2*(-1001*A*d^3-208*B*c*d^2+127*C*c^2*d-
86*D*c^3)+b^2*c^2*(143*A*d^3-104*B*c*d^2+80*C*c^2*d-64*D*c^3))*x*(d*x+c)^(
1/2)*(-b*x^2+a)^(1/2)/b/d^5+2/9009*(39*a^2*d^4*D-3*a*b*d^2*(-39*B*d^2+19*C
*c*d-10*D*c^2)+b^2*c*(143*A*d^3-104*B*c*d^2+80*C*c^2*d-64*D*c^3))*(d*x+c)^
(1/2)*(-b*x^2+a)^(3/2)/b^2/d^4+2/1287*(3*a*d^2*(11*C*d-D*c)-b*(-143*A*d^3-
13*B*c*d^2+10*C*c^2*d-8*D*c^3))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d^3-2/4
29*(13*D*a-3*b*(-13*B*d^2+10*C*c*d-8*D*c^2)/d^2)*(d*x+c)^(1/2)*(-b*x^2+a)^
(5/2)/b^2-2/39*(3*C*d-5*D*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(5/2)/b/d^2-2/15*D*(
d*x+c)^(5/2)*(-b*x^2+a)^(5/2)/b/d^2-8/45045*a^(1/2)*(3*a^3*d^6*(231*C*d+83
*D*c)+3*a*b^2*c^2*d^2*(715*A*d^3-403*B*c*d^2+266*C*c^2*d-192*D*c^3)-3*a^2*
b*d^4*(-1001*A*d^3-403*B*c*d^2+145*C*c^2*d-77*D*c^3)-4*b^3*c^4*(143*A*d^3-
104*B*c*d^2+80*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/b^(3/2)/d^7/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*
x^2+a)^(1/2)+8/45045*a^(1/2)*(-a*d^2+b*c^2)*(195*a^3*d^6*D-9*a^2*b*d^4*(-6
5*B*d^2+6*C*c*d+3*D*c^2)+3*a*b^2*c*d^2*(572*A*d^3-299*B*c*d^2+186*C*c^2*d-
128*D*c^3)-4*b^3*c^3*(143*A*d^3-104*B*c*d^2+80*C*c^2*d-64*D*c^3))*((d*x...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.94 (sec) , antiderivative size = 9837, normalized size of antiderivative = 9.13

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[c + d*x]*(a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 1046, normalized size of antiderivative = 0.97, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {2185, 27, 2185, 27, 687, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{3/2} \sqrt{c+dx} \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \; 2185$$

$$-\frac{2\int -\frac{5}{2}\sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(b(3Cd-5cD)x^2d^2+(3Abd+acD)d^2+\left(-2bDc^2+3bBd^2+ad^2D\right)xd\right)dx}{15bd^3}-\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(b(3Cd-5cD)x^2d^2+(3Abd+acD)d^2+\left(-2bDc^2+3bBd^2+ad^2D\right)xd\right)dx}{3bd^3}-\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{-\frac{2\int -\frac{1}{2}bd^3\sqrt{c+dx}\left(d(39Abd+9aCd-2acD)-\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)x\right)\left(a-bx^2\right)^{3/2}dx}{13bd^2}-\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{3bd^3}-\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{13}d\int \sqrt{c+dx}\left(d(39Abd+9aCd-2acD)-\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)x\right)\left(a-bx^2\right)^{3/2}dx-\frac{2}{13}c}{3bd^3}-\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(-13ad^2D-39bBd^2-24bc^2D+30bcCd\right)}{11b}-\frac{2\int -\frac{\left(d\left(429Acdb^2+a\left(13aDd^2+b\left(2Dc^2+69Cdc+39Bd^2\right)\right)\right)+3b\left(3ad^2(11Cd-c\right.\right.}{2\sqrt{c+dx}}}{11b}\right.}{3bd^3}$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{13}d\left(\frac{\int \frac{\left(d\left(429Acdb^2+a\left(13aDd^2+b\left(2Dc^2+69Cdc+39Bd^2\right)\right)\right)+3b\left(3ad^2(11Cd-cD)-b\left(-8Dc^3+10Cdc^2-13Bd^2c-143Ad^3\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx}{11b}+\frac{2(a-b\right.}{3bd^3}$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{1}{13}d\left(\frac{\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(39a^2d^4D+7bdx\left(3ad^2(11Cd-cD)-b\left(-143Ad^3-13Bcd^2-8c^3D+10c^2Cd\right)\right)-3abd^2\left(-39Bd^2-10c^2D+19cCd\right)+b^2c\left(143Ad^3-104\right.\right.}{21d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{13}d\left(\frac{\frac{2\int \frac{\left(ad\left(39a^2Dd^4+3ab\left(3Dc^2+58Cdc+39Bd^2\right)d^2+b^2c\left(-8Dc^3+10Cdc^2-13Bd^2c+1144Ad^3\right)\right)+b\left(3a^2(77Cd+6cD)d^4-ab\left(-86Dc^3+127Cdc^2-208Bd^2c\right.\right.\right.}{\sqrt{c+dx}}}{7d^2}}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow 600$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow 509$$

$$\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 508$$

$$\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\downarrow 327$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{1}{13}d\left(\frac{2\left(-24bDc^2+30bCdc-39bBd^2-13ad^2D\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}{11b}+\frac{2\sqrt{c+dx}\left(39a^2Dd^4-3ab\left(-10Dc^2+19Cdc-39Bd^2\right)d^2+7b\left(3ad^2(11Cd-cD)-b\left(\right.\right.\right.}{}\right.}{}$$

$$\frac{2D(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^2}$$

input
```
Int[Sqrt[c + d*x]*(a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*D*(c + d*x)^(5/2)*(a - b*x^2)^(5/2))/(15*b*d^2) + ((-2*d*(3*C*d - 5*c*
D)*(c + d*x)^(3/2)*(a - b*x^2)^(5/2))/13 + (d*((2*(30*b*c*C*d - 39*b*B*d^2
 - 24*b*c^2*D - 13*a*d^2*D)*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/(11*b) + ((2*
Sqrt[c + d*x]*(39*a^2*d^4*D - 3*a*b*d^2*(19*c*C*d - 39*B*d^2 - 10*c^2*D) +
 b^2*c*(80*c^2*C*d - 104*B*c*d^2 + 143*A*d^3 - 64*c^3*D) + 7*b*d*(3*a*d^2*
(11*C*d - c*D) - b*(10*c^2*C*d - 13*B*c*d^2 - 143*A*d^3 - 8*c^3*D))*x)*(a
- b*x^2)^(3/2))/(21*d^2) + (2*((2*Sqrt[c + d*x]*(195*a^3*d^6*D - 9*a^2*b*d
^4*(6*c*C*d - 65*B*d^2 + 3*c^2*D) + 3*a*b^2*c*d^2*(186*c^2*C*d - 299*B*c*d
^2 + 572*A*d^3 - 128*c^3*D) - 4*b^3*c^3*(80*c^2*C*d - 104*B*c*d^2 + 143*A*
d^3 - 64*c^3*D) + 3*b*d*(3*a^2*d^4*(77*C*d + 6*c*D) - a*b*d^2*(127*c^2*C*d
 - 208*B*c*d^2 - 1001*A*d^3 - 86*c^3*D) + b^2*c^2*(80*c^2*C*d - 104*B*c*d^
2 + 143*A*d^3 - 64*c^3*D))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((-2*Sqrt[a]*
Sqrt[b]*(3*a^3*d^6*(231*C*d + 83*c*D) + 3*a*b^2*c^2*d^2*(266*c^2*C*d - 403
*B*c*d^2 + 715*A*d^3 - 192*c^3*D) - 3*a^2*b*d^4*(145*c^2*C*d - 403*B*c*d^2
 - 1001*A*d^3 - 77*c^3*D) - 4*b^3*c^4*(80*c^2*C*d - 104*B*c*d^2 + 143*A*d^
3 - 64*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
 (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]
*(b*c^2 - a*d^2)*(195*a^3*d^6*D - 9*a^2*b*d^4*(6*c*C*d - 65*B*d^2 + 3*c^2*
D) + 3*a*b^2*c*d^2*(186*c^2*C*d - 299*B*c*d^2 + 572*A*d^3 - 128*c^3*D) ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4512 vs. $2(981) = 1962$.

Time = 5.34 (sec) , antiderivative size = 4513, normalized size of antiderivative = 4.19

method	result	size
elliptic	Expression too large to display	4513
default	Expression too large to display	8269

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVER
BOSE)
```

output

```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/15*D*b*x^6
*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/13*(b^2*d*C+1/15*b^2*c*D)/b/d*x^5*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/11*(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13
*(b^2*d*C+1/15*b^2*c*D)/d*c)/b/d*x^4*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/
9*(A*b^2*d+B*b^2*c-2*d*a*b*C-6/5*a*b*c*D+11/13*(b^2*d*C+1/15*b^2*c*D)/b*a-
10/11*(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D)/d*c)/d*c
)/b/d*x^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(A*b^2*c-2*B*a*b*d-2*C*a*
b*c+a^2*d*D+10/13*(b^2*d*C+1/15*b^2*c*D)/b/d*a*c+9/11*(B*b^2*d+C*b^2*c-17/
15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D)/d*c)/b*a-8/9*(A*b^2*d+B*b^2*c-2*d*
a*b*C-6/5*a*b*c*D+11/13*(b^2*d*C+1/15*b^2*c*D)/b*a-10/11*(B*b^2*d+C*b^2*c-
17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D)/d*c)/d*c)/d*c)/b/d*x^2*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-2*A*a*b*d-2*B*a*b*c+a^2*C*d+a^2*c*D+8/11*
(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D)/d*c)/b/d*a*c+7
/9*(A*b^2*d+B*b^2*c-2*d*a*b*C-6/5*a*b*c*D+11/13*(b^2*d*C+1/15*b^2*c*D)/b*a
-10/11*(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D)/d*c)/d*
c)/b*a-6/7*(A*b^2*c-2*B*a*b*d-2*C*a*b*c+a^2*d*D+10/13*(b^2*d*C+1/15*b^2*c*
D)/b/d*a*c+9/11*(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*c*D
)/d*c)/b*a-8/9*(A*b^2*d+B*b^2*c-2*d*a*b*C-6/5*a*b*c*D+11/13*(b^2*d*C+1/15*
b^2*c*D)/b*a-10/11*(B*b^2*d+C*b^2*c-17/15*d*a*b*D-12/13*(b^2*d*C+1/15*b^2*
c*D)/d*c)/d*c)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 1032, normalized size of antiderivative = 0.96

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"fricas")
```

output
```
2/135135*(4*(256*D*b^4*c^8 - 320*C*b^4*c^7*d - 32*(24*D*a*b^3 - 13*B*b^4)*
c^6*d^2 + 2*(519*C*a*b^3 - 286*A*b^4)*c^5*d^3 + 3*(203*D*a^2*b^2 - 507*B*a
*b^3)*c^4*d^4 - 6*(161*C*a^2*b^2 - 429*A*a*b^3)*c^3*d^5 + 12*(14*D*a^3*b +
 169*B*a^2*b^2)*c^2*d^6 - 6*(204*C*a^3*b + 1859*A*a^2*b^2)*c*d^7 - 585*(D*
a^4 + 3*B*a^3*b)*d^8)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)
/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(256*
D*b^4*c^7*d - 320*C*b^4*c^6*d^2 - 32*(18*D*a*b^3 - 13*B*b^4)*c^5*d^3 + 2*(
399*C*a*b^3 - 286*A*b^4)*c^4*d^4 + 3*(77*D*a^2*b^2 - 403*B*a*b^3)*c^3*d^5
- 15*(29*C*a^2*b^2 - 143*A*a*b^3)*c^2*d^6 + 3*(83*D*a^3*b + 403*B*a^2*b^2)
*c*d^7 + 231*(3*C*a^3*b + 13*A*a^2*b^2)*d^8)*sqrt(-b*d)*weierstrassZeta(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d)) - 3*(3003*D*b^4*d^8*x^6 - 512*D*b^4*c^6*d^2 + 640*C*
b^4*c^5*d^3 + 64*(17*D*a*b^3 - 13*B*b^4)*c^4*d^4 - 4*(379*C*a*b^3 - 286*A*
b^4)*c^3*d^5 - 2*(174*D*a^2*b^2 - 1157*B*a*b^3)*c^2*d^6 + (708*C*a^2*b^2 -
 4147*A*a*b^3)*c*d^7 + 780*(D*a^3*b + 3*B*a^2*b^2)*d^8 + 231*(D*b^4*c*d^7
+ 15*C*b^4*d^8)*x^5 - 21*(12*D*b^4*c^2*d^6 - 15*C*b^4*c*d^7 + 13*(17*D*a*b
^3 - 15*B*b^4)*d^8)*x^4 + 7*(40*D*b^4*c^3*d^5 - 50*C*b^4*c^2*d^6 - (81*D*a
*b^3 - 65*B*b^4)*c*d^7 - 55*(15*C*a*b^3 - 13*A*b^4)*d^8)*x^3 - (320*D*b^4*
c^4*d^4 - 400*C*b^4*c^3*d^5 - 2*(327*D*a*b^3 - 260*B*b^4)*c^2*d^6 + 5*(...
```

Sympy [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(a-bx^2\right)^{\frac{3}{2}}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral((a - b*x**2)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*sqrt(d*x + c), x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx+c}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int {\left(a-b\,x^2\right)}^{3/2}\,\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a - b*x^2)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a - b*x^2)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{dx+c}\,\left(-b\,x^2+a\right)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^(1/2)*(-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.142 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 899

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \frac{4(9a^2d^4(65Cd-71cD)+3abd^2(1274c^2Cd-1573Bcd^2+21}{}$$

$$+\frac{4(231a^2d^4D-abd^2(793cCd-1001Bd^2-666c^2D)+b^2c(1040c^2Cd-1144Bcd^2+1287Ad^3-960c^3D))}{15015bd^5}$$

$$+\frac{2(3ad^2(39Cd-58cD)+b(1040c^2Cd-1144Bcd^2+1287Ad^3-960c^3D))\sqrt{c+dx}(a-bx^2)^{3/2}}{9009bd^4}$$

$$+\frac{2(33ad^2D-b(130cCd-143Bd^2-120c^2D))x\sqrt{c+dx}(a-bx^2)^{3/2}}{1287bd^3}$$

$$-\frac{2(13Cd-23cD)\sqrt{c+dx}(a-bx^2)^{5/2}}{143bd^2}-\frac{2D(c+dx)^{3/2}(a-bx^2)^{5/2}}{13bd^2}$$

$$-\frac{8\sqrt{a}(693a^3d^6D-3a^2bd^4(598cCd-1001Bd^2-453c^2D)+3ab^2cd^2(2314c^2Cd-2717Bcd^2+3432Ad^3-}{45045b^{3/}}$$

$$+\frac{8\sqrt{a}(bc^2-ad^2)(9a^2d^4(65Cd-71cD)+3abd^2(1274c^2Cd-1573Bcd^2+2145Ad^3-1088c^3D)-4b^2c^2(1}{45045b^{3/2}d^7\sqrt{c}}$$

output
```
4/45045*(9*a^2*d^4*(65*C*d-71*D*c)+3*a*b*d^2*(2145*A*d^3-1573*B*c*d^2+1274
*C*c^2*d-1088*D*c^3)-4*b^2*c^2*(1287*A*d^3-1144*B*c*d^2+1040*C*c^2*d-960*D
*c^3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^6+4/15015*(231*a^2*d^4*D-a*b*d^2
*(-1001*B*d^2+793*C*c*d-666*D*c^2)+b^2*c*(1287*A*d^3-1144*B*c*d^2+1040*C*c
^2*d-960*D*c^3))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^5+2/9009*(3*a*d^2*(3
9*C*d-58*D*c)+b*(1287*A*d^3-1144*B*c*d^2+1040*C*c^2*d-960*D*c^3))*(d*x+c)^
(1/2)*(-b*x^2+a)^(3/2)/b/d^4+2/1287*(33*a*d^2*D-b*(-143*B*d^2+130*C*c*d-12
0*D*c^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d^3-2/143*(13*C*d-23*D*c)*(d*
x+c)^(1/2)*(-b*x^2+a)^(5/2)/b/d^2-2/13*D*(d*x+c)^(3/2)*(-b*x^2+a)^(5/2)/b/
d^2-8/45045*a^(1/2)*(693*a^3*d^6*D-3*a^2*b*d^4*(-1001*B*d^2+598*C*c*d-453*
D*c^2)+3*a*b^2*c*d^2*(3432*A*d^3-2717*B*c*d^2+2314*C*c^2*d-2048*D*c^3)-4*b
^3*c^3*(1287*A*d^3-1144*B*c*d^2+1040*C*c^2*d-960*D*c^3))*(d*x+c)^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^7/((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+8/45045*a^(1/2)*(-a*d^2+b*c^2)*(9*a^2*
d^4*(65*C*d-71*D*c)+3*a*b*d^2*(2145*A*d^3-1573*B*c*d^2+1274*C*c^2*d-1088*D
*c^3)-4*b^2*c^2*(1287*A*d^3-1144*B*c*d^2+1040*C*c^2*d-960*D*c^3))*((d*x+c)
/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/b^(3/2)/d^7/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.36 (sec) , antiderivative size = 1257, normalized size of antiderivative = 1.40

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output

```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((2*(-8320*b^2*c^4*C*d + 9152*b^2*B*c^3*d^2
- 10296*A*b^2*c^2*d^3 + 12844*a*b*c^2*C*d^3 - 15158*a*b*B*c*d^4 + 19305*a*
A*b*d^5 - 2340*a^2*C*d^5 + 7680*b^2*c^5*D - 11328*a*b*c^3*d^2*D + 1632*a^2
*c*d^4*D))/(45045*b*d^6) + (2*(6240*b^2*c^3*C*d - 6864*b^2*B*c^2*d^2 + 772
2*A*b^2*c*d^3 - 9308*a*b*c*C*d^3 + 11011*a*b*B*d^4 - 5760*b^2*c^4*D + 8196
*a*b*c^2*d^2*D - 924*a^2*d^4*D)*x)/(45045*b*d^5) - (2*(1040*b*c^2*C*d - 11
44*b*B*c*d^2 + 1287*A*b*d^3 - 1521*a*C*d^3 - 960*b*c^3*D + 1338*a*c*d^2*D)
*x^2)/(9009*d^4) - (2*(-130*b*c*C*d + 143*b*B*d^2 + 120*b*c^2*D - 165*a*d^
2*D)*x^3)/(1287*d^3) - (2*b*(13*C*d - 12*c*D)*x^4)/(143*d^2) - (2*b*D*x^5)
/(13*d)) + (8*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-(Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(693*a^3*d^6*D + 3*a^2*b*d^4*(-598*c*C*d + 1001*B
*d^2 + 453*c^2*D) + 3*a*b^2*c*d^2*(2314*c^2*C*d - 2717*B*c*d^2 + 3432*A*d^
3 - 2048*c^3*D) + 4*b^3*c^3*(-1040*c^2*C*d + 1144*B*c*d^2 - 1287*A*d^3 + 9
60*c^3*D))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c + d*x))^2)) + (I*Sqrt[b]
*(Sqrt[b]*c - Sqrt[a]*d)*(693*a^3*d^6*D + 3*a^2*b*d^4*(-598*c*C*d + 1001*B
*d^2 + 453*c^2*D) + 3*a*b^2*c*d^2*(2314*c^2*C*d - 2717*B*c*d^2 + 3432*A*d^
3 - 2048*c^3*D) + 4*b^3*c^3*(-1040*c^2*C*d + 1144*B*c*d^2 - 1287*A*d^3 + 9
60*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1
- c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt...
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 860, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2185, 27, 2185, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}} \, dx$$

$$\Big\downarrow \; 2185$$

$$-\frac{2 \int -\frac{\left(a-bx^2\right)^{3/2} \left(b(13Cd-23cD)x^2d^2+(13Abd+3acD)d^2+\left(-10bDc^2+13bBd^2+3ad^2D\right)xd\right)}{2\sqrt{c+dx}}dx}{13bd^3}-\frac{2D\left(a-bx^2\right)^{5/2} (c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a-bx^2)^{3/2}(b(13Cd-23cD)x^2d^2+(13Abd+3acD)d^2+(-10bDc^2+13bBd^2+3ad^2D)xd)}{\sqrt{c+dx}}dx}{13bd^3}-\frac{2D(a-bx^2)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{2185}$$

$$\frac{-\frac{2\int -\frac{bd^3\left(d(143Abd+13aCd+10acD)+\left(33ad^2D-b\left(-120Dc^2+130Cdc-143Bd^2\right)\right)x\right)(a-bx^2)^{3/2}}{2\sqrt{c+dx}}dx}{11bd^2}-\frac{2}{11}d(a-bx^2)^{5/2}\sqrt{c+dx}(13Cd-23}{13bd^3}\frac{2D(a-bx^2)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{11}d\int \frac{(d(143Abd+13aCd+10acD)+(33ad^2D-b(-120Dc^2+130Cdc-143Bd^2))x)(a-bx^2)^{3/2}}{\sqrt{c+dx}}dx-\frac{2}{11}d(a-bx^2)^{5/2}\sqrt{c+dx}(13Cd}{13bd^3}\frac{2D(a-bx^2)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{1}{11}d\left(\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}(7dx(33ad^2D-b(-143Bd^2-120c^2D+130cCd))+3ad^2(39Cd-58cD)+b(1287Ad^3-1144Bcd^2-960c^3D+1040c^2Cd)}{63d^2}\right.}{}\frac{2D(a-bx^2)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{11}d\left(\frac{2\int \frac{\left(ad\left(3a(39Cd+19cD)d^2+b\left(-120Dc^3+130Cdc^2-143Bd^2c+1287Ad^3\right)\right)+\left(9bcd^2(143Abd+13aCd+10acD)-\left(8bc^2-7ad^2\right)\left(33ad^2D-b\left(-120Dc^2+130\right.\right.\right.\right.}{\sqrt{c+dx}}}{21d^2}\right.}{}\frac{2D(a-bx^2)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(9a^2d^4(65Cd-71cD)+3dx\left(9bcd^2(10acD+13aCd+143Abd)-\left(8bc^2-7ad^2\right)\left(33ad^2D-b\left(-143Bd^2-120c^2D+130cCd\right)\right)\right)+3abd^2\left(21\right.\right.}{15d^2}\right.}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\Big\downarrow$ 27

$$\frac{1}{11}d\left(\frac{2\left(2\int\frac{ad\left(9a^2(65Cd+6cD)d^4+3ab\left(-422Dc^3+481Cdc^2-572Bd^2c+2145Ad^3\right)d^2-b^2c^2\left(-960Dc^3+1040Cdc^2-1144Bd^2c+1287Ad^3\right)\right)+\left(693a^3Dd^6-3a\right.}{\sqrt{c+d}}\right.}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\Big\downarrow$ 600

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\left(693a^3d^6D-3a^2bd^4\left(-1001Bd^2-453c^2D+598cCd\right)+3ab^2cd^2\left(3432Ad^3-2717Bcd^2-2048c^3D+2314c^2Cd\right)-4b^3c^3\left(1287Ad^3-1144Bcd^2-960c^3\right.\right.}{d}\right.}{}\right.}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\Big\downarrow$ 509

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(693a^3d^6D-3a^2bd^4\left(-1001Bd^2-453c^2D+598cCd\right)+3ab^2cd^2\left(3432Ad^3-2717Bcd^2-2048c^3D+2314c^2Cd\right)-4b^3c^3\left(1287Ad^3-1144Bcd^2\right.\right.}{d\sqrt{a-bx^2}}\right.}{}\right.}{}\right.$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 508

$$\frac{1}{11}d\left(\frac{2\sqrt{c+dx}\left(3a(39Cd-58cD)d^2+7\left(33ad^2D-b\left(-120Dc^2+130Cdc-143Bd^2\right)\right)xd+b\left(-960Dc^3+1040Cdc^2-1144Bd^2c+1287Ad^3\right)\right)\left(a-bx^2\right)}{63d^2}\right.$$

$$\frac{2D(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13bd^2}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{1}{11}d\left(\frac{2\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(9a^2d^4(65Cd-71cD)+3abd^2\left(2145Ad^3-1573Bcd^2-1088c^3D+1274c^2Cd\right)-4b^2c^2\left(1287Ad^3-1144Bcd^2-960c^3D+1040c^2Cd\right)\right)\int\frac{}{\sqrt{c}}}{d}\right.}{}\right.}{}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{1}{11}d\left(\frac{2\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(9a^2d^4(65Cd-71cD)+3abd^2\left(2145Ad^3-1573Bcd^2-1088c^3D+1274c^2Cd\right)-4b^2c^2\left(1287Ad^3-1144Bcd^2-960c^3D+1040c^2C\right.\right.}{d\sqrt{a-bx^2}}\right.}{}\right.}{}\right.}{}$$

$$\frac{2D\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{1}{11}d\left(\frac{2\sqrt{c+dx}\left(3a(39Cd-58cD)d^2+7\left(33ad^2D-b\left(-120Dc^2+130Cdc-143Bd^2\right)\right)xd+b\left(-960Dc^3+1040Cdc^2-1144Bd^2c+1287Ad^3\right)\right)\left(a-bx^2\right)}{63d^2}\right.}{}$$

$$\frac{2D(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13bd^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{1}{11}d\left(\frac{2\sqrt{c+dx}(3a(39Cd-58cD)d^2+7(33ad^2D-b(-120Dc^2+130Cdc-143Bd^2))xd+b(-960Dc^3+1040Cdc^2-1144Bd^2c+1287Ad^3))(a-bx^2)}{63d^2}\right.}{}$$

$$\frac{2D(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13bd^2}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(-2*D*(c + d*x)^(3/2)*(a - b*x^2)^(5/2))/(13*b*d^2) + ((-2*d*(13*C*d - 23*
c*D)*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/11 + (d*((2*Sqrt[c + d*x]*(3*a*d^2*(
39*C*d - 58*c*D) + b*(1040*c^2*C*d - 1144*B*c*d^2 + 1287*A*d^3 - 960*c^3*D
) + 7*d*(33*a*d^2*D - b*(130*c*C*d - 143*B*d^2 - 120*c^2*D))*x)*(a - b*x^2
)^(3/2))/(63*d^2) + (2*((2*Sqrt[c + d*x]*(9*a^2*d^4*(65*C*d - 71*c*D) + 3*
a*b*d^2*(1274*c^2*C*d - 1573*B*c*d^2 + 2145*A*d^3 - 1088*c^3*D) - 4*b^2*c^
2*(1040*c^2*C*d - 1144*B*c*d^2 + 1287*A*d^3 - 960*c^3*D) + 3*d*(9*b*c*d^2*
(143*A*b*d + 13*a*C*d + 10*a*c*D) - (8*b*c^2 - 7*a*d^2)*(33*a*d^2*D - b*(1
30*c*C*d - 143*B*d^2 - 120*c^2*D)))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((-2
*Sqrt[a]*(693*a^3*d^6*D - 3*a^2*b*d^4*(598*c*C*d - 1001*B*d^2 - 453*c^2*D)
 + 3*a*b^2*c*d^2*(2314*c^2*C*d - 2717*B*c*d^2 + 3432*A*d^3 - 2048*c^3*D) -
 4*b^3*c^3*(1040*c^2*C*d - 1144*B*c*d^2 + 1287*A*d^3 - 960*c^3*D))*Sqrt[c
+ d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/
Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c +
d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^
2)*(9*a^2*d^4*(65*C*d - 71*c*D) + 3*a*b*d^2*(1274*c^2*C*d - 1573*B*c*d^2 +
 2145*A*d^3 - 1088*c^3*D) - 4*b^2*c^2*(1040*c^2*C*d - 1144*B*c*d^2 + 1287*
A*d^3 - 960*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(809) = 1618$.

Time = 5.17 (sec) , antiderivative size = 2421, normalized size of antiderivative = 2.69

method	result	size
elliptic	Expression too large to display	2421
default	Expression too large to display	7072

input

```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/13*D*b/d*x
^5*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/11*(b^2*C-12/13*D*b^2/d*c)/b/d*x^4
*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12
/13*D*b^2/d*c)/d*c)/b/d*x^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(b^2*A-
2*C*a*b+10/13*D*b/d*a*c+9/11*(b^2*C-12/13*D*b^2/d*c)/b*a-8/9*(B*b^2-15/13*
a*b*D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/d*c)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)-2/5*(-2*a*b*B+D*a^2+8/11*(b^2*C-12/13*D*b^2/d*c)/b/d*a*c+7/
9*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/b*a-6/7*(b^2*A-2*C
*a*b+10/13*D*b/d*a*c+9/11*(b^2*C-12/13*D*b^2/d*c)/b*a-8/9*(B*b^2-15/13*a*b
*D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)-2/3*(-2*a*b*A+C*a^2+2/3*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12/
13*D*b^2/d*c)/d*c)/b/d*a*c+5/7*(b^2*A-2*C*a*b+10/13*D*b/d*a*c+9/11*(b^2*C-
12/13*D*b^2/d*c)/b*a-8/9*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12/13*D*b^2/d*c)/
d*c)/d*c)/b*a-4/5*(-2*a*b*B+D*a^2+8/11*(b^2*C-12/13*D*b^2/d*c)/b/d*a*c+7/9
*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/b*a-6/7*(b^2*A-2*C*
a*b+10/13*D*b/d*a*c+9/11*(b^2*C-12/13*D*b^2/d*c)/b*a-8/9*(B*b^2-15/13*a*b*
D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)+2*(a^2*A+2/5*(-2*a*b*B+D*a^2+8/11*(b^2*C-12/13*D*b^2/d*c)
/b/d*a*c+7/9*(B*b^2-15/13*a*b*D-10/11*(b^2*C-12/13*D*b^2/d*c)/d*c)/b*a-6/7
*(b^2*A-2*C*a*b+10/13*D*b/d*a*c+9/11*(b^2*C-12/13*D*b^2/d*c)/b*a-8/9*(B...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 822, normalized size of antiderivative = 0.91

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
2/135135*(4*(3840*D*b^3*c^7 - 4160*C*b^3*c^6*d - 32*(282*D*a*b^2 - 143*B*b
^3)*c^5*d^2 + 234*(43*C*a*b^2 - 22*A*b^3)*c^4*d^3 + 27*(191*D*a^2*b - 429*
B*a*b^2)*c^3*d^4 - 39*(157*C*a^2*b - 363*A*a*b^2)*c^2*d^5 + 3*(177*D*a^3 +
 2717*B*a^2*b)*c*d^6 - 1755*(C*a^3 + 11*A*a^2*b)*d^7)*sqrt(-b*d)*weierstra
ssPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), 1/3*(3*d*x + c)/d) + 12*(3840*D*b^3*c^6*d - 4160*C*b^3*c^5*d^2 - 32*(19
2*D*a*b^2 - 143*B*b^3)*c^4*d^3 + 78*(89*C*a*b^2 - 66*A*b^3)*c^3*d^4 + 3*(4
53*D*a^2*b - 2717*B*a*b^2)*c^2*d^5 - 78*(23*C*a^2*b - 132*A*a*b^2)*c*d^6 +
 231*(3*D*a^3 + 13*B*a^2*b)*d^7)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
 + c)/d)) - 3*(3465*D*b^3*d^7*x^5 - 7680*D*b^3*c^5*d^2 + 8320*C*b^3*c^4*d^
3 + 64*(177*D*a*b^2 - 143*B*b^3)*c^3*d^4 - 52*(247*C*a*b^2 - 198*A*b^3)*c^
2*d^5 - 2*(816*D*a^2*b - 7579*B*a*b^2)*c*d^6 + 585*(4*C*a^2*b - 33*A*a*b^2
)*d^7 - 315*(12*D*b^3*c*d^6 - 13*C*b^3*d^7)*x^4 + 35*(120*D*b^3*c^2*d^5 -
130*C*b^3*c*d^6 - 11*(15*D*a*b^2 - 13*B*b^3)*d^7)*x^3 - 5*(960*D*b^3*c^3*d
^4 - 1040*C*b^3*c^2*d^5 - 2*(669*D*a*b^2 - 572*B*b^3)*c*d^6 + 117*(13*C*a*
b^2 - 11*A*b^3)*d^7)*x^2 + (5760*D*b^3*c^4*d^3 - 6240*C*b^3*c^3*d^4 - 12*(
683*D*a*b^2 - 572*B*b^3)*c^2*d^5 + 26*(358*C*a*b^2 - 297*A*b^3)*c*d^6 + 77
*(12*D*a^2*b - 143*B*a*b^2)*d^7)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(a-bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx=\int \frac{(a-b\,x^2)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.143 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 791

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2(bc^2-ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{d^6\sqrt{c+dx}}$$

$$-\frac{2(180a^2d^4D+3abd^2(1166cCd-495Bd^2-1929c^2D)-b^2c(6710c^2Cd-4653Bcd^2+2772Ad^3-8895c^3}{3465bd^6}$$

$$+\frac{2(ad^2(847Cd-2733cD)-b(4840c^2Cd-2277Bcd^2+693Ad^3-8430c^3D))(c+dx)^{3/2}\sqrt{a-bx^2}}{3465d^6}$$

$$+\frac{2(117ad^2D+b(418cCd-99Bd^2-1086c^2D))(c+dx)^{5/2}\sqrt{a-bx^2}}{693d^6}$$

$$-\frac{2b(11Cd-57cD)(c+dx)^{7/2}\sqrt{a-bx^2}}{99d^6}-\frac{2bD(c+dx)^{9/2}\sqrt{a-bx^2}}{11d^6}$$

$$-\frac{8\sqrt{a}(3a^2d^4(77Cd-123cD)-3abd^2(1166c^2Cd-957Bcd^2+693Ad^3-1344c^3D)+4b^2c^2(880c^2Cd-79}{3465\sqrt{b}d^7\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-b}}$$

$$+\frac{8\sqrt{a}(bc^2-ad^2)(45a^2d^4D-3abd^2(286cCd-165Bd^2-384c^2D)+4b^2c(880c^2Cd-792Bcd^2+693Ad^3}{3465b^{3/2}d^7\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*(-a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^6/(d*x+c
)^(1/2)-2/3465*(180*a^2*d^4*D+3*a*b*d^2*(-495*B*d^2+1166*C*c*d-1929*D*c^2)
-b^2*c*(2772*A*d^3-4653*B*c*d^2+6710*C*c^2*d-8895*D*c^3))*(d*x+c)^(1/2)*(-
b*x^2+a)^(1/2)/b/d^6+2/3465*(a*d^2*(847*C*d-2733*D*c)-b*(693*A*d^3-2277*B*
c*d^2+4840*C*c^2*d-8430*D*c^3))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^6+2/693*(
117*a*d^2*D+b*(-99*B*d^2+418*C*c*d-1086*D*c^2))*(d*x+c)^(5/2)*(-b*x^2+a)^(
1/2)/d^6-2/99*b*(11*C*d-57*D*c)*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/d^6-2/11*b*
D*(d*x+c)^(9/2)*(-b*x^2+a)^(1/2)/d^6-8/3465*a^(1/2)*(3*a^2*d^4*(77*C*d-123
*D*c)-3*a*b*d^2*(693*A*d^3-957*B*c*d^2+1166*C*c^2*d-1344*D*c^3)+4*b^2*c^2*
(693*A*d^3-792*B*c*d^2+880*C*c^2*d-960*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a
)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)
*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^7/((d*x+c)/(c+a^(1/2)*d/b^(1/2)
))^(1/2)/(-b*x^2+a)^(1/2)+8/3465*a^(1/2)*(-a*d^2+b*c^2)*(45*a^2*d^4*D-3*a*
b*d^2*(-165*B*d^2+286*C*c*d-384*D*c^2)+4*b^2*c*(693*A*d^3-792*B*c*d^2+880*
C*c^2*d-960*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(
1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/
(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^7/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.26 (sec) , antiderivative size = 978, normalized size of antiderivative = 1.24

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*Sqrt[a - b*x^2]*(3465*b*(b*c^2 - a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^
3*D) + (-180*a^2*d^4*D + a*b*d^2*(-2651*c*C*d + 1485*B*d^2 + 3639*c^2*D) +
 b^2*c*(3575*c^2*C*d - 2871*B*c*d^2 + 2079*A*d^3 - 4215*c^3*D))*(c + d*x)
- b*d*(a*d^2*(-847*C*d + 1563*c*D) - 3*b*(-605*c^2*C*d + 429*B*c*d^2 - 231
*A*d^3 + 765*c^3*D))*x*(c + d*x) + 5*b*d^2*(117*a*d^2*D + b*(187*c*C*d - 9
9*B*d^2 - 267*c^2*D))*x^2*(c + d*x) - 35*b^2*d^3*(11*C*d - 21*c*D)*x^3*(c
+ d*x) - 315*b^2*d^4*D*x^4*(c + d*x) + (4*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]*(3*a^2*d^4*(-77*C*d + 123*c*D) + 3*a*b*d^2*(1166*c^2*C*d - 957*B*c*d^2
 + 693*A*d^3 - 1344*c^3*D) + 4*b^2*c^2*(-880*c^2*C*d + 792*B*c*d^2 - 693*A
*d^3 + 960*c^3*D))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*
d^4*(-77*C*d + 123*c*D) + 3*a*b*d^2*(1166*c^2*C*d - 957*B*c*d^2 + 693*A*d^
3 - 1344*c^3*D) + 4*b^2*c^2*(-880*c^2*C*d + 792*B*c*d^2 - 693*A*d^3 + 960*
c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)] - I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(45*a^2*d^4*D + 3*a^(3/2)*Sqr
t[b]*d^3*(-77*C*d + 138*c*D) + 3*a*b*d^2*(-286*c*C*d + 165*B*d^2 + 384*c^2
*D) + 3*Sqrt[a]*b^(3/2)*d*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 960*c^3
*D) - 4*b^2*c*(-880*c^2*C*d + 792*B*c*d^2 - 693*A*d^3 + 960*c^3*D))*Sqrt[(
d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/...
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {2182, 27, 2185, 27, 682, 27, 682, 27, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int \frac{(a-bx^2)^{3/2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+9Bc-9Ad\right)\right)x+\frac{-aDc^2+Abdc+aCdc-aBd^2}{d}\right)}{2\sqrt{c+dx}}dx}{bc^2-ad^2}+\frac{2(a-bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^2\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a-bx^2)^{3/2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+9Bc-9Ad\right)\right)x+Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{\sqrt{c+dx}}dx}{bc^2-ad^2}+\frac{2(a-bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2}{11}D(a-bx^2)^{5/2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{2\int -\frac{\left(d\left(11Ab^2cd-a\left(ad^2D-b\left(-10Dc^2+11Cdc-11Bd^2\right)\right)\right)-b\left(ad^2(11Cd-21cD)-b\left(-120Dc^3+110Cdc\right.\right.\right.}{2\sqrt{c+dx}}}{11bd^2}}{bc^2-ad^2}$$

$$\frac{2(a-bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(d\left(11Ab^2cd-a\left(ad^2D-b\left(-10Dc^2+11Cdc-11Bd^2\right)\right)\right)-b\left(ad^2(11Cd-21cD)-b\left(-120Dc^3+110Cdc^2-99Bd^2c+99Ad^3\right)\right)x\right)(a-bx^2)^{3/2}}{\sqrt{c+dx}}dx}{11bd^2}+\frac{2}{11}D(a-}{bc^2-ad^2}$$

$$\frac{2(a-bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{682}$$

$$-\frac{4\int -\frac{b\left(ad\left(bc^2-ad^2\right)\left(9ad^2D-b\left(-120Dc^2+110Cdc-99Bd^2\right)\right)+b\left(bc^2-ad^2\right)\left(ad^2(77Cd-138cD)-b\left(-960Dc^3+880Cdc^2-792Bd^2c+693Ad^3\right)\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21bd^2}$$

$$\frac{2(a-bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\left(ad\left(bc^2-ad^2\right)\left(9ad^2D-b\left(-120Dc^2+110Cdc-99Bd^2\right)\right)+b\left(bc^2-ad^2\right)\left(ad^2(77Cd-138cD)-b\left(-960Dc^3+880Cdc^2-792Bd^2c+693Ad^3\right)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}-\frac{2(}{}$$

$$\frac{2(a-bx^2)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{682}$$

$$2\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)(45a^2d^4D+3bdx(ad^2(77Cd-138cD)-b(693Ad^3-792Bcd^2-960c^3D+880c^2Cd))-3abd^2(-165Bd^2-384c^2D+286cCd)+4b^2c(6}{15d^2}\right.$$

$$\frac{2(a-bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^2\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2\int\frac{(bc^2-ad^2)(ad(45a^2Dd^4-3ab(-246Dc^2+209Cdc-165Bd^2)d^2+b^2c(-960Dc^3+880Cdc^2-792Bd^2c+693Ad^3))+b(3a^2(77Cd-123cD)d^4-3ab(-1344Dc}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\right.$$

$$\frac{2(a-bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^2\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2(bc^2-ad^2)\int\frac{ad(45a^2Dd^4-3ab(-246Dc^2+209Cdc-165Bd^2)d^2+b^2c(-960Dc^3+880Cdc^2-792Bd^2c+693Ad^3))+b(3a^2(77Cd-123cD)d^4-3ab(-1344Dc^3}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\right.$$

$$\frac{2(a-bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^2\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{600}$$

$$2\left(\frac{2(bc^2-ad^2)\left(\frac{b(3a^2d^4(77Cd-123cD)-3abd^2(693Ad^3-957Bcd^2-1344c^3D+1166c^2Cd)+4b^2c^2(693Ad^3-792Bcd^2-960c^3D+880c^2Cd))\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{(b}{}\right.}{15d^2}\right.$$

$$\frac{2(a-bx^2)^{5/2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^2\sqrt{c+dx}(bc^2-ad^2)}$$

$$\Big\downarrow \text{509}$$

$$2\left(\frac{2\left(bc^2-ad^2\right)\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(77Cd-123cD)-3abd^2\left(693Ad^3-957Bcd^2-1344c^3D+1166c^2Cd\right)+4b^2c^2\left(693Ad^3-792Bcd^2-960c^3D+880c^2Cd\right)\right)\int\frac{\sqrt{c+d}}{\sqrt{1-\frac{b}{}}}}{d\sqrt{a-bx^2}}\right.}{15d^2}\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 508$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+$$

$$2\left(\frac{2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}\left(45a^2Dd^4-3ab\left(-384Dc^2+286Cdc-165Bd^2\right)d^2+3b\left(ad^2(77Cd-138cD)-\right.\right.}{15d^2}\right.$$

$$\frac{2}{11}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)D\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+$$

$$\Big\downarrow \ 327$$

$$2\left(2\left(bc^2-ad^2\right)\left(-\frac{\left(bc^2-ad^2\right)\left(45a^2d^4D-3abd^2\left(-165Bd^2-384c^2D+286cCd\right)+4b^2c\left(693Ad^3-792Bcd^2-960c^3D+880c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\cdots}{\cdots}\right.\right.$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+$$

$$\frac{2}{11}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)D\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+\frac{2\left(\frac{2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}\left(45a^2Dd^4-3ab\left(-384Dc^2+286Cdc-165Bd^2\right)d^2+3b\left(ad^2(77Cd-138cD)\cdots\right.\right.}{15d^2\cdots}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 511$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+$$

$$\frac{2}{11}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)D\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+ \left(2\,\frac{2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}\left(45a^2Dd^4-3ab\left(-384Dc^2+286Cdc-165Bd^2\right)d^2+3b\left(ad^2(77Cd-138cD)-\right.\right.}{15d^2}\right.$$

$\downarrow$ 321

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+$$

$$\frac{2}{11}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)D\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+ \left(2\,\frac{2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}\left(45a^2Dd^4-3ab\left(-384Dc^2+286Cdc-165Bd^2\right)d^2+3b\left(ad^2(77Cd-138cD)-\right.\right.}{15d^2}\right.$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(5/2))/(d^2*(b*c^2 - a*
d^2)*Sqrt[c + d*x]) + ((2*(a/b - c^2/d^2)*D*Sqrt[c + d*x]*(a - b*x^2)^(5/2
))/11 + ((-2*Sqrt[c + d*x]*(9*a^2*d^4*D - a*b*d^2*(187*c*C*d - 99*B*d^2 -
258*c^2*D) + b^2*c*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 960*c^3*D) + 7
*b*d*(a*d^2*(11*C*d - 21*c*D) - b*(110*c^2*C*d - 99*B*c*d^2 + 99*A*d^3 - 1
20*c^3*D))*x)*(a - b*x^2)^(3/2))/(63*d^2) + (2*((2*(b*c^2 - a*d^2)*Sqrt[c
+ d*x]*(45*a^2*d^4*D - 3*a*b*d^2*(286*c*C*d - 165*B*d^2 - 384*c^2*D) + 4*b
^2*c*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 960*c^3*D) + 3*b*d*(a*d^2*(7
7*C*d - 138*c*D) - b*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 960*c^3*D))*
x)*Sqrt[a - b*x^2])/(15*d^2) + (2*(b*c^2 - a*d^2)*((-2*Sqrt[a]*Sqrt[b]*(3*
a^2*d^4*(77*C*d - 123*c*D) - 3*a*b*d^2*(1166*c^2*C*d - 957*B*c*d^2 + 693*A
*d^3 - 1344*c^3*D) + 4*b^2*c^2*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 96
0*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt
[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c
^2 - a*d^2)*(45*a^2*d^4*D - 3*a*b*d^2*(286*c*C*d - 165*B*d^2 - 384*c^2*D)
+ 4*b^2*c*(880*c^2*C*d - 792*B*c*d^2 + 693*A*d^3 - 960*c^3*D))*Sqrt[(Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(21*d^2))/(11*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3010 vs. $2(703) = 1406$.

Time = 5.06 (sec) , antiderivative size = 3011, normalized size of antiderivative = 3.81

method	result	size
elliptic	Expression too large to display	3011
default	Expression too large to display	5965

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x^2+
a*d)*(A*a*d^5-A*b*c^2*d^3-B*a*c*d^4+B*b*c^3*d^2+C*a*c^2*d^3-C*b*c^4*d-D*a*
c^3*d^2+D*b*c^5)/d^7/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/11*b/d^2*D*x^4*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D*c)/b/
d*x^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(b/d^3*(B*b*d^2-C*b*c*d-2*D*a
*d^2+D*b*c^2)+9/11*b/d*D*a-8/9*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D*c)/d*c)/
b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(b/d^4*(A*b*d^3-B*b*c*d^2-2
*C*a*d^3+C*b*c^2*d+2*D*a*c*d^2-D*b*c^3)+8/11*b/d^2*D*a*c+7/9*(b^2/d^2*(C*d
-D*c)-10/11*b^2/d^2*D*c)/b*a-6/7*(b/d^3*(B*b*d^2-C*b*c*d-2*D*a*d^2+D*b*c^2
)+9/11*b/d*D*a-8/9*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D*c)/d*c)/d*c)/b/d*x*(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-1/d^5*(A*b^2*c*d^3+2*B*a*b*d^4-B*b
^2*c^2*d^2-2*C*a*b*c*d^3+C*b^2*c^3*d-D*a^2*d^4+2*D*a*b*c^2*d^2-D*b^2*c^4)+
2/3*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D*c)/b/d*a*c+5/7*(b/d^3*(B*b*d^2-C*b*
c*d-2*D*a*d^2+D*b*c^2)+9/11*b/d*D*a-8/9*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D
*c)/d*c)/b*a-4/5*(b/d^4*(A*b*d^3-B*b*c*d^2-2*C*a*d^3+C*b*c^2*d+2*D*a*c*d^2
-D*b*c^3)+8/11*b/d^2*D*a*c+7/9*(b^2/d^2*(C*d-D*c)-10/11*b^2/d^2*D*c)/b*a-6
/7*(b/d^3*(B*b*d^2-C*b*c*d-2*D*a*d^2+D*b*c^2)+9/11*b/d*D*a-8/9*(b^2/d^2*(C
*d-D*c)-10/11*b^2/d^2*D*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)
^(1/2)+2*((2*A*a*b*c*d^5-A*b^2*c^3*d^3+B*a^2*d^6-2*B*a*b*c^2*d^4+B*b^2*c^4
*d^2-C*a^2*c*d^5+2*C*a*b*c^3*d^3-C*b^2*c^5*d+D*a^2*c^2*d^4-2*D*a*b*c^4*...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1028, normalized size of antiderivative = 1.30

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-2/10395*(4*(3840*D*b^3*c^7 - 3520*C*b^3*c^6*d - 288*(24*D*a*b^2 - 11*B*b^
3)*c^5*d^2 + 198*(31*C*a*b^2 - 14*A*b^3)*c^4*d^3 + 9*(287*D*a^2*b - 583*B*
a*b^2)*c^3*d^4 - 66*(32*C*a^2*b - 63*A*a*b^2)*c^2*d^5 + 135*(D*a^3 + 11*B*
a^2*b)*c*d^6 + (3840*D*b^3*c^6*d - 3520*C*b^3*c^5*d^2 - 288*(24*D*a*b^2 -
11*B*b^3)*c^4*d^3 + 198*(31*C*a*b^2 - 14*A*b^3)*c^3*d^4 + 9*(287*D*a^2*b -
 583*B*a*b^2)*c^2*d^5 - 66*(32*C*a^2*b - 63*A*a*b^2)*c*d^6 + 135*(D*a^3 +
11*B*a^2*b)*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(3840*D
*b^3*c^6*d - 3520*C*b^3*c^5*d^2 - 288*(14*D*a*b^2 - 11*B*b^3)*c^4*d^3 + 66
*(53*C*a*b^2 - 42*A*b^3)*c^3*d^4 + 9*(41*D*a^2*b - 319*B*a*b^2)*c^2*d^5 -
231*(C*a^2*b - 9*A*a*b^2)*c*d^6 + (3840*D*b^3*c^5*d^2 - 3520*C*b^3*c^4*d^3
 - 288*(14*D*a*b^2 - 11*B*b^3)*c^3*d^4 + 66*(53*C*a*b^2 - 42*A*b^3)*c^2*d^
5 + 9*(41*D*a^2*b - 319*B*a*b^2)*c*d^6 - 231*(C*a^2*b - 9*A*a*b^2)*d^7)*x)
*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 -
9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(315*D*b^3*d^7*x^5
 + 7680*D*b^3*c^5*d^2 - 7040*C*b^3*c^4*d^3 + 3465*A*a*b^2*d^7 - 192*(37*D*
a*b^2 - 33*B*b^3)*c^3*d^4 + 44*(139*C*a*b^2 - 126*A*b^3)*c^2*d^5 + 90*(2*D
*a^2*b - 55*B*a*b^2)*c*d^6 - 35*(12*D*b^3*c*d^6 - 11*C*b^3*d^7)*x^4 + 5*(1
20*D*b^3*c^2*d^5 - 110*C*b^3*c*d^6 - 9*(13*D*a*b^2 - 11*B*b^3)*d^7)*x^3...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(3/2),
 x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(a-b\,x^2)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}} (Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

3.144 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 767

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2(bc^2-ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{3d^6(c+dx)^{3/2}}$$
$$+\frac{2(3ad^2(2cCd-Bd^2-3c^2D)-bc(14c^2Cd-11Bcd^2+8Ad^3-17c^3D))\sqrt{a-bx^2}}{3d^6\sqrt{c+dx}}$$
$$+\frac{2(3ad^2(45Cd-151cD)-b(780c^2Cd-357Bcd^2+105Ad^3-1390c^3D))\sqrt{c+dx}\sqrt{a-bx^2}}{315d^6}$$
$$+\frac{2(77ad^2D+b(270cCd-63Bd^2-710c^2D))(c+dx)^{3/2}\sqrt{a-bx^2}}{315d^6}$$
$$-\frac{2b(9Cd-47cD)(c+dx)^{5/2}\sqrt{a-bx^2}}{63d^6}-\frac{2bD(c+dx)^{7/2}\sqrt{a-bx^2}}{9d^6}$$
$$-\frac{8\sqrt{a}(21a^2d^4D+3abd^2(150cCd-63Bd^2-256c^2D)-4b^2c(240c^2Cd-168Bcd^2+105Ad^3-320c^3D))}{315\sqrt{b}d^7\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{8\sqrt{a}(3a^2d^4(15Cd-41cD)-abd^2(690c^2Cd-357Bcd^2+105Ad^3-1088c^3D)+4b^2c^2(240c^2Cd-168B}{315\sqrt{b}d^7\sqrt{c+dx}\sqrt{a-}}$$

output
```
2/3*(-a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^6/(d*x
+c)^(3/2)+2/3*(3*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(8*A*d^3-11*B*c*d^2+14
*C*c^2*d-17*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(d*x+c)^(1/2)+2/315*(3*a*d^2*(45*
C*d-151*D*c)-b*(105*A*d^3-357*B*c*d^2+780*C*c^2*d-1390*D*c^3))*(d*x+c)^(1/
2)*(-b*x^2+a)^(1/2)/d^6+2/315*(77*a*d^2*D+b*(-63*B*d^2+270*C*c*d-710*D*c^2
))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^6-2/63*b*(9*C*d-47*D*c)*(d*x+c)^(5/2)*
(-b*x^2+a)^(1/2)/d^6-2/9*b*D*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/d^6-8/315*a^(1
/2)*(21*a^2*d^4*D+3*a*b*d^2*(-63*B*d^2+150*C*c*d-256*D*c^2)-4*b^2*c*(105*A
*d^3-168*B*c*d^2+240*C*c^2*d-320*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^7/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/
2)/(-b*x^2+a)^(1/2)-8/315*a^(1/2)*(3*a^2*d^4*(15*C*d-41*D*c)-a*b*d^2*(105*
A*d^3-357*B*c*d^2+690*C*c^2*d-1088*D*c^3)+4*b^2*c^2*(105*A*d^3-168*B*c*d^2
+240*C*c^2*d-320*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)
/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^7/(d*x+c)^(1/2)/(-b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.67 (sec) , antiderivative size = 1029, normalized size of antiderivative = 1.34

$$\int \frac{(a-bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*(555*b*c^2*C*d - 294*b*B*c*d^2 + 105*A*
b*d^3 - 135*a*C*d^3 - 880*b*c^3*D + 376*a*c*d^2*D))/(315*d^6) - (2*(-180*b
*c*C*d + 63*b*B*d^2 + 345*b*c^2*D - 77*a*d^2*D)*x)/(315*d^5) - (2*b*(9*C*d
 - 26*c*D)*x^2)/(63*d^4) - (2*b*D*x^3)/(9*d^3) - (2*(-(b*c^2) + a*d^2)*(c^
2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^6*(c + d*x)^2) + (2*(-14*b*c^3*C*d
+ 11*b*B*c^2*d^2 - 8*A*b*c*d^3 + 6*a*c*C*d^3 - 3*a*B*d^4 + 17*b*c^4*D - 9*
a*c^2*d^2*D))/(3*d^6*(c + d*x))) - (8*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c +
 d*x))^2)/d^2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(21*a^2*d^4*D - 3*a*b*d^2*(
-150*c*C*d + 63*B*d^2 + 256*c^2*D) + 4*b^2*c*(-240*c^2*C*d + 168*B*c*d^2 -
 105*A*d^3 + 320*c^3*D))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c + d*x))^2)
 - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(21*a^2*d^4*D - 3*a*b*d^2*(-150*c*C*
d + 63*B*d^2 + 256*c^2*D) + 4*b^2*c*(-240*c^2*C*d + 168*B*c*d^2 - 105*A*d^
3 + 320*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sq
rt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sq
rt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] + (I*Sqrt[a]*Sqrt[b]*d*(-21*a^2*d^4*D
 + 3*a^(3/2)*Sqrt[b]*d^3*(15*C*d - 34*c*D) + 3*a*b*d^2*(-150*c*C*d + 63*B*
d^2 + 256*c^2*D) + 4*b^2*c*(240*c^2*C*d - 168*B*c*d^2 + 105*A*d^3 - 320*c^
3*D) + Sqrt[a]*b^(3/2)*d*(-240*c^2*C*d + 168*B*c*d^2 - 105*A*d^3 + 320*c^3
*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c...
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.34, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {2182, 27, 2182, 27, 682, 27, 27, 682, 27, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{2\int \frac{(a-bx^2)^{3/2}\left(3\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(3Cd-3cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+7Bc-7Ad\right)\right)x+\frac{3\left(Abcd+a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{2(c+dx)^{3/2}}dx}{3\left(bc^2-ad^2\right)}+$$
$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a-bx^2)^{3/2}\left(3\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(3Cd-3cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+7Bc-7Ad\right)\right)x+3\left(Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^{3/2}}dx}{3\left(bc^2-ad^2\right)}+$$
$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\frac{2\int \frac{\left(d\left(Abd\left(3bc^2-7ad^2\right)+a\left(3ad^2(Cd-2cD)-bc\left(-10Dc^2+7Cdc-4Bd^2\right)\right)\right)+3\left(a^2Dd^4+ab\left(-29Dc^2+18Cdc-9Bd^2\right)d^2-b^2c\left(-40Dc^3+30Cdc^2-21Bd^2c+12Ad\right.\right.\right.}{2d^2\sqrt{c+dx}}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(d\left(Abd\left(3bc^2-7ad^2\right)+a\left(3ad^2(Cd-2cD)-bc\left(-10Dc^2+7Cdc-4Bd^2\right)\right)\right)+3\left(a^2Dd^4+ab\left(-29Dc^2+18Cdc-9Bd^2\right)d^2-b^2c\left(-40Dc^3+30Cdc^2-21Bd^2c+12Ad^3\right.\right.\right.}{\sqrt{c+dx}}}{d^2\left(bc^2-ad^2\right)}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(7dx\left(a^2d^4D+abd^2\left(-9Bd^2-29c^2D+18cCd\right)-b^2c\left(12Ad^3-21Bcd^2-40c^3D+30c^2Cd\right)\right)+a^2d^4(9Cd-26cD)-abd^2\left(21Ad^3-84Bcd^2-262c^3\right.\right.}{21d^2}}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 600$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 509$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 508$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 327$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 512$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 511$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

$$\Big\downarrow \; 321$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$\frac{\frac{2\left(3ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-13Dc^3+10Cdc^2-7Bd^2c+4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{d^2(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{c+dx}\left(a^2(9Cd-26cD)d^4-ab\left(-262Dc^3+165Cdc^2-84Bd^2c+\right.\right.}{}}{}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(5/2))/(3*d^2*(b*c^2 -
a*d^2)*(c + d*x)^(3/2)) + ((2*(3*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(
10*c^2*C*d - 7*B*c*d^2 + 4*A*d^3 - 13*c^3*D))*(a - b*x^2)^(5/2))/(d^2*(b*c
^2 - a*d^2)*Sqrt[c + d*x]) + ((2*Sqrt[c + d*x]*(a^2*d^4*(9*C*d - 26*c*D) +
 b^2*c^2*(240*c^2*C*d - 168*B*c*d^2 + 105*A*d^3 - 320*c^3*D) - a*b*d^2*(16
5*c^2*C*d - 84*B*c*d^2 + 21*A*d^3 - 262*c^3*D) + 7*d*(a^2*d^4*D + a*b*d^2*
(18*c*C*d - 9*B*d^2 - 29*c^2*D) - b^2*c*(30*c^2*C*d - 21*B*c*d^2 + 12*A*d^
3 - 40*c^3*D))*x)*(a - b*x^2)^(3/2))/(21*d^2) - (2*(b*c^2 - a*d^2)*((2*Sqr
t[c + d*x]*(3*a^2*d^4*(15*C*d - 41*c*D) - a*b*d^2*(690*c^2*C*d - 357*B*c*d
^2 + 105*A*d^3 - 1088*c^3*D) + 4*b^2*c^2*(240*c^2*C*d - 168*B*c*d^2 + 105*
A*d^3 - 320*c^3*D) + 3*d*(7*a^2*d^4*D + 3*a*b*d^2*(45*c*C*d - 21*B*d^2 - 7
4*c^2*D) - b^2*c*(240*c^2*C*d - 168*B*c*d^2 + 105*A*d^3 - 320*c^3*D))*x)*S
qrt[a - b*x^2])/(15*d^2) - (2*(b*c^2 - a*d^2)*((-2*Sqrt[a]*(21*a^2*d^4*D +
 3*a*b*d^2*(150*c*C*d - 63*B*d^2 - 256*c^2*D) - 4*b^2*c*(240*c^2*C*d - 168
*B*c*d^2 + 105*A*d^3 - 320*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellip
ticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*S
qrt[a - b*x^2]) - (2*Sqrt[a]*(3*a^2*d^4*(15*C*d - 41*c*D) - a*b*d^2*(690*c
^2*C*d - 357*B*c*d^2 + 105*A*d^3 - 1088*c^3*D) + 4*b^2*c^2*(240*c^2*C*d -
168*B*c*d^2 + 105*A*d^3 - 320*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(677) = 1354$.

Time = 6.78 (sec) , antiderivative size = 2091, normalized size of antiderivative = 2.73

method	result	size
elliptic	Expression too large to display	2091
default	Expression too large to display	9375

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/3*(A*a*d^5
-A*b*c^2*d^3-B*a*c*d^4+B*b*c^3*d^2+C*a*c^2*d^3-C*b*c^4*d-D*a*c^3*d^2+D*b*c
^5)/d^8*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-2/3*(-b*d*x^2+a*d)*(8
*A*b*c*d^3+3*B*a*d^4-11*B*b*c^2*d^2-6*C*a*c*d^3+14*C*b*c^3*d+9*D*a*c^2*d^2
-17*D*b*c^4)/d^7/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/9*b/d^3*D*x^3*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(b^2/d^3*(C*d-2*D*c)-8/9*b^2/d^3*D*c)/b/d*x^2
*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(b/d^4*(B*b*d^2-2*C*b*c*d-2*D*a*d^
2+3*D*b*c^2)+7/9*b/d^2*D*a-6/7*(b^2/d^3*(C*d-2*D*c)-8/9*b^2/d^3*D*c)/d*c)/
b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(b/d^5*(A*b*d^3-2*B*b*c*d^2-2
*C*a*d^3+3*C*b*c^2*d+4*D*a*c*d^2-4*D*b*c^3)+2/3*b/d^3*D*a*c+5/7*(b^2/d^3*(
C*d-2*D*c)-8/9*b^2/d^3*D*c)/b*a-4/5*(b/d^4*(B*b*d^2-2*C*b*c*d-2*D*a*d^2+3*
D*b*c^2)+7/9*b/d^2*D*a-6/7*(b^2/d^3*(C*d-2*D*c)-8/9*b^2/d^3*D*c)/d*c)/d*c)
/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(2*A*a*b*d^5-3*A*b^2*c^2*d^3-4
*B*a*b*c*d^4+4*B*b^2*c^3*d^2-C*a^2*d^5+6*C*a*b*c^2*d^3-5*C*b^2*c^4*d+2*D*a
^2*c*d^4-8*D*a*b*c^3*d^2+6*D*b^2*c^5)/d^7+1/3*(A*a*d^5-A*b*c^2*d^3-B*a*c*d
^4+B*b*c^3*d^2+C*a*c^2*d^3-C*b*c^4*d-D*a*c^3*d^2+D*b*c^5)*b/d^7-1/3*(8*A*b
*c*d^3+3*B*a*d^4-11*B*b*c^2*d^2-6*C*a*c*d^3+14*C*b*c^3*d+9*D*a*c^2*d^2-17*
D*b*c^4)*b/d^7*c+2/5*(b/d^4*(B*b*d^2-2*C*b*c*d-2*D*a*d^2+3*D*b*c^2)+7/9*b/
d^2*D*a-6/7*(b^2/d^3*(C*d-2*D*c)-8/9*b^2/d^3*D*c)/d*c)/b/d*a*c+1/3*(b/d^5*
(A*b*d^3-2*B*b*c*d^2-2*C*a*d^3+3*C*b*c^2*d+4*D*a*c*d^2-4*D*b*c^3)+2/3*b...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1073, normalized size of antiderivative = 1.40

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
2/945*(4*(1280*D*b^2*c^7 - 960*C*b^2*c^6*d - 96*(18*D*a*b - 7*B*b^2)*c^5*d
^2 + 30*(39*C*a*b - 14*A*b^2)*c^4*d^3 + 3*(109*D*a^2 - 231*B*a*b)*c^3*d^4
- 45*(3*C*a^2 - 7*A*a*b)*c^2*d^5 + (1280*D*b^2*c^5*d^2 - 960*C*b^2*c^4*d^3
 - 96*(18*D*a*b - 7*B*b^2)*c^3*d^4 + 30*(39*C*a*b - 14*A*b^2)*c^2*d^5 + 3*
(109*D*a^2 - 231*B*a*b)*c*d^6 - 45*(3*C*a^2 - 7*A*a*b)*d^7)*x^2 + 2*(1280*
D*b^2*c^6*d - 960*C*b^2*c^5*d^2 - 96*(18*D*a*b - 7*B*b^2)*c^4*d^3 + 30*(39
*C*a*b - 14*A*b^2)*c^3*d^4 + 3*(109*D*a^2 - 231*B*a*b)*c^2*d^5 - 45*(3*C*a
^2 - 7*A*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^
2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(12
80*D*b^2*c^6*d - 960*C*b^2*c^5*d^2 - 96*(8*D*a*b - 7*B*b^2)*c^4*d^3 + 30*(
15*C*a*b - 14*A*b^2)*c^3*d^4 + 21*(D*a^2 - 9*B*a*b)*c^2*d^5 + (1280*D*b^2*
c^4*d^3 - 960*C*b^2*c^3*d^4 - 96*(8*D*a*b - 7*B*b^2)*c^2*d^5 + 30*(15*C*a*
b - 14*A*b^2)*c*d^6 + 21*(D*a^2 - 9*B*a*b)*d^7)*x^2 + 2*(1280*D*b^2*c^5*d^
2 - 960*C*b^2*c^4*d^3 - 96*(8*D*a*b - 7*B*b^2)*c^3*d^4 + 30*(15*C*a*b - 14
*A*b^2)*c^2*d^5 + 21*(D*a^2 - 9*B*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstrassZet
a(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weiers
trassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*
d^3), 1/3*(3*d*x + c)/d)) - 3*(35*D*b^2*d^7*x^5 - 2560*D*b^2*c^5*d^2 + 192
0*C*b^2*c^4*d^3 + 210*B*a*b*c*d^6 + 105*A*a*b*d^7 + 64*(19*D*a*b - 21*B*b^
2)*c^3*d^4 - 60*(11*C*a*b - 14*A*b^2)*c^2*d^5 - 15*(4*D*b^2*c*d^6 - 3*C...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(5/2),
 x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx=\int \frac{\left(-b\,x^2+a\right)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{\left(dx+c\right)^{\frac{5}{2}}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

3.145 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 833

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \frac{2(bc^2-ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{5d^6(c+dx)^{5/2}}$$
$$+\frac{2(5ad^2(2cCd-Bd^2-3c^2D)-bc(22c^2Cd-17Bcd^2+12Ad^3-27c^3D))\sqrt{a-bx^2}}{15d^6(c+dx)^{3/2}}$$
$$+\frac{2(15a^2d^4(Cd-3cD)-abd^2(131c^2Cd-61Bcd^2+21Ad^3-231c^3D)+b^2c^2(128c^2Cd-73Bcd^2+33Ad^3}{15d^6(bc^2-ad^2)\sqrt{c+dx}}$$
$$+\frac{2(45ad^2D+b(154cCd-35Bd^2-414c^2D))\sqrt{c+dx}\sqrt{a-bx^2}}{105d^6}$$
$$-\frac{2b(7Cd-37cD)(c+dx)^{3/2}\sqrt{a-bx^2}}{35d^6}-\frac{2bD(c+dx)^{5/2}\sqrt{a-bx^2}}{7d^6}$$
$$-\frac{8\sqrt{a}\sqrt{b}(3a^2d^4(21Cd-71cD)-abd^2(490c^2Cd-203Bcd^2+63Ad^3-960c^3D)+4b^2c^2(112c^2Cd-56Bc}{105d^7(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-}}$$
$$-\frac{8\sqrt{a}(15a^2d^4D+abd^2(154cCd-35Bd^2-384c^2D)-4b^2c(112c^2Cd-56Bcd^2+21Ad^3-192c^3D))\sqrt{\frac{c}{c-}}}{105\sqrt{b}d^7\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/5*(-a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^6/(d*x
+c)^(5/2)+2/15*(5*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(12*A*d^3-17*B*c*d^2+
22*C*c^2*d-27*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(d*x+c)^(3/2)+2/15*(15*a^2*d^4*
(C*d-3*D*c)-a*b*d^2*(21*A*d^3-61*B*c*d^2+131*C*c^2*d-231*D*c^3)+b^2*c^2*(3
3*A*d^3-73*B*c*d^2+128*C*c^2*d-198*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(-a*d^2+b*
c^2)/(d*x+c)^(1/2)+2/105*(45*a*d^2*D+b*(-35*B*d^2+154*C*c*d-414*D*c^2))*(d
*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^6-2/35*b*(7*C*d-37*D*c)*(d*x+c)^(3/2)*(-b*x
^2+a)^(1/2)/d^6-2/7*b*D*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/d^6-8/105*a^(1/2)*b
^(1/2)*(3*a^2*d^4*(21*C*d-71*D*c)-a*b*d^2*(63*A*d^3-203*B*c*d^2+490*C*c^2*
d-960*D*c^3)+4*b^2*c^2*(21*A*d^3-56*B*c*d^2+112*C*c^2*d-192*D*c^3))*(d*x+c
)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/d^7/(-a*d^2+b*c^2)/(
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-8/105*a^(1/2)*(15*a^
2*d^4*D+a*b*d^2*(-35*B*d^2+154*C*c*d-384*D*c^2)-4*b^2*c*(21*A*d^3-56*B*c*d
^2+112*C*c^2*d-192*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+
a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^7/(d*x+c)^(1/2)/(-b*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.81 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.39

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*(-133*b*c*C*d + 35*b*B*d^2 + 318*b*c^2*
D - 45*a*d^2*D))/(105*d^6) - (2*b*(7*C*d - 27*c*D)*x)/(35*d^5) - (2*b*D*x^
2)/(7*d^4) - (2*(-(b*c^2) + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(5
*d^6*(c + d*x)^3) + (2*(-22*b*c^3*C*d + 17*b*B*c^2*d^2 - 12*A*b*c*d^3 + 10
*a*c*C*d^3 - 5*a*B*d^4 + 27*b*c^4*D - 15*a*c^2*d^2*D))/(15*d^6*(c + d*x)^2
) - (2*(128*b^2*c^4*C*d - 73*b^2*B*c^3*d^2 + 33*A*b^2*c^2*d^3 - 131*a*b*c^
2*C*d^3 + 61*a*b*B*c*d^4 - 21*a*A*b*d^5 + 15*a^2*C*d^5 - 198*b^2*c^5*D + 2
31*a*b*c^3*d^2*D - 45*a^2*c*d^4*D))/(15*d^6*(-(b*c^2) + a*d^2)*(c + d*x)))
 + (8*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-(Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]*(3*a^2*d^4*(-21*C*d + 71*c*D) + a*b*d^2*(490*c^2*C*d - 20
3*B*c*d^2 + 63*A*d^3 - 960*c^3*D) + 4*b^2*c^2*(-112*c^2*C*d + 56*B*c*d^2 -
 21*A*d^3 + 192*c^3*D))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c + d*x))^2))
 + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*(-21*C*d + 71*c*D) + a*b*
d^2*(490*c^2*C*d - 203*B*c*d^2 + 63*A*d^3 - 960*c^3*D) + 4*b^2*c^2*(-112*c
^2*C*d + 56*B*c*d^2 - 21*A*d^3 + 192*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[
a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]]
, (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] + (I*Sqr
t[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(15*a^2*d^4*D + 9*a^(3/2)*Sqrt[b]*d^3*(7*C*
d - 22*c*D) + a*b*d^2*(154*c*C*d - 35*B*d^2 - 384*c^2*D) + 4*b^2*c*(-11...
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2182, 27, 2182, 27, 27, 681, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int \frac{5(a-bx^2)^{3/2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+\frac{Abcd+a\left(-Dc^2+Cdc-Bd^2\right)}{d}\right)}{2(c+dx)^{5/2}}dx}{5\left(bc^2-ad^2\right)}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a-bx^2)^{3/2}\left(\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^{5/2}}dx}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{\frac{2\int \frac{\left(3d\left(bc^2-ad^2\right)(Abd-aCd+2acD)-\left(bc^2-ad^2\right)\left(3aDd^2+b\left(-24Dc^2+14Cdc-7Bd^2\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{2d^2(c+dx)^{3/2}}dx}{3(bc^2-ad^2)}-\frac{2\left(a-bx^2\right)^{5/2}\left(-Bd^2-3c^2D+2cCd\right)}{3d^2(c+dx)^{3/2}}}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(bc^2-ad^2\right)\left(3d(Abd-aCd+2acD)-\left(3aDd^2+b\left(-24Dc^2+14Cdc-7Bd^2\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{(c+dx)^{3/2}}dx}{3d^2(bc^2-ad^2)}-\frac{2\left(a-bx^2\right)^{5/2}\left(-Bd^2-3c^2D+2cCd\right)}{3d^2(c+dx)^{3/2}}}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(3d(Abd-aCd+2acD)-\left(3aDd^2+b\left(-24Dc^2+14Cdc-7Bd^2\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{(c+dx)^{3/2}}dx}{3d^2}-\frac{2\left(a-bx^2\right)^{5/2}\left(-Bd^2-3c^2D+2cCd\right)}{3d^2(c+dx)^{3/2}}}{bc^2-ad^2}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\int\frac{\left(ad\left(3aDd^2+b\left(-24Dc^2+14Cdc-\right.\right.\right.}{}}{3d^2}}{bc^2-ad^2}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 682

$$\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(15a^2d^4D-3bdx\right.}{}\right.}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 27

$$\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(2\int\frac{ad\left(15a^2Dd^4+ab\left(-186Dc^2+9\right.\right.}{}\right.}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 600

$$\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(2\left(-\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D+abd^2\right.}{}\right.\right.}{}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 509

$$\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(2\left(-\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D+abd^2\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$\frac{\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(\frac{2\left(2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$\frac{\frac{2\left(a-bx^2\right)^{3/2}\left(-dx\left(3ad^2D+b\left(-7Bd^2-24c^2D+14cCd\right)\right)+3ad^2(7Cd-22cD)-2b\left(\frac{21Ad^3}{2}-28Bcd^2-96c^3D+56c^2Cd\right)\right)}{7d^2\sqrt{c+dx}}-\frac{6\left(\frac{2\left(2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 511

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+$$

$$\frac{\frac{2\left(3a(7Cd-22cD)d^2-\left(3aDd^2+b\left(-24Dc^2+14Cdc-7Bd^2\right)\right)xd-2b\left(-96Dc^3+56Cdc^2-28Bd^2c+\frac{21Ad^3}{2}\right)\right)\left(a-bx^2\right)^{3/2}}{7d^2\sqrt{c+dx}}-\frac{6\left(\frac{2\sqrt{c+dx}\sqrt{a-bx^2}\left(15a^2Dd^4+ab\left(-\right.\right.}{}\right.}{}}{}$$

$$\Big\downarrow \textcolor{blue}{321}$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+$$

$$\frac{\frac{2\left(3a(7Cd-22cD)d^2-\left(3aDd^2+b\left(-24Dc^2+14Cdc-7Bd^2\right)\right)xd-2b\left(-96Dc^3+56Cdc^2-28Bd^2c+\frac{21Ad^3}{2}\right)\right)\left(a-bx^2\right)^{3/2}}{7d^2\sqrt{c+dx}}-\frac{6\left(\frac{2\sqrt{c+dx}\sqrt{a-bx^2}\left(15a^2Dd^4+ab\left(-\right.\right.}{}\right.}{}}{}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(5/2))/(5*d^2*(b*c^2 -
a*d^2)*(c + d*x)^(5/2)) + ((-2*(2*c*C*d - B*d^2 - 3*c^2*D)*(a - b*x^2)^(5/
2))/(3*d^2*(c + d*x)^(3/2)) + ((2*(3*a*d^2*(7*C*d - 22*c*D) - 2*b*(56*c^2*
C*d - 28*B*c*d^2 + (21*A*d^3)/2 - 96*c^3*D) - d*(3*a*d^2*D + b*(14*c*C*d -
 7*B*d^2 - 24*c^2*D))*x)*(a - b*x^2)^(3/2))/(7*d^2*Sqrt[c + d*x]) - (6*((2
*Sqrt[c + d*x]*(15*a^2*d^4*D + a*b*d^2*(154*c*C*d - 35*B*d^2 - 384*c^2*D)
- 4*b^2*c*(112*c^2*C*d - 56*B*c*d^2 + 21*A*d^3 - 192*c^3*D) - 3*b*d*(3*a*d
^2*(7*C*d - 22*c*D) - b*(112*c^2*C*d - 56*B*c*d^2 + 21*A*d^3 - 192*c^3*D))
*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(21*C*d
- 71*c*D) - a*b*d^2*(490*c^2*C*d - 203*B*c*d^2 + 63*A*d^3 - 960*c^3*D) + 4
*b^2*c^2*(112*c^2*C*d - 56*B*c*d^2 + 21*A*d^3 - 192*c^3*D))*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*
c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(15*a^2*d^4*
D + a*b*d^2*(154*c*C*d - 35*B*d^2 - 384*c^2*D) - 4*b^2*c*(112*c^2*C*d - 56
*B*c*d^2 + 21*A*d^3 - 192*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a
]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sq
rt[a - b*x^2])))/(15*d^2)))/(7*d^2))/(3*d^2))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(743) = 1486$.

Time = 7.50 (sec) , antiderivative size = 1771, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1771
default	Expression too large to display	17034

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/5*(A*a*d^5
-A*b*c^2*d^3-B*a*c*d^4+B*b*c^3*d^2+C*a*c^2*d^3-C*b*c^4*d-D*a*c^3*d^2+D*b*c
^5)/d^9*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3-2/15*(12*A*b*c*d^3+5*
B*a*d^4-17*B*b*c^2*d^2-10*C*a*c*d^3+22*C*b*c^3*d+15*D*a*c^2*d^2-27*D*b*c^4
)/d^8*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2/15*(-b*d*x^2+a*d)/d^7
/(a*d^2-b*c^2)*(21*A*a*b*d^5-33*A*b^2*c^2*d^3-61*B*a*b*c*d^4+73*B*b^2*c^3*
d^2-15*C*a^2*d^5+131*C*a*b*c^2*d^3-128*C*b^2*c^4*d+45*D*a^2*c*d^4-231*D*a*
b*c^3*d^2+198*D*b^2*c^5)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/7*D*b/d^4*x^2*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(b^2/d^4*(C*d-3*D*c)-6/7*D*b^2/d^4*c)
/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(b/d^5*(B*b*d^2-3*C*b*c*d-2*
D*a*d^2+6*D*b*c^2)+5/7*D*b/d^3*a-4/5*(b^2/d^4*(C*d-3*D*c)-6/7*D*b^2/d^4*c)
/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(3*A*b^2*c*d^3+2*B*a*b*d^
4-6*B*b^2*c^2*d^2-6*C*a*b*c*d^3+10*C*b^2*c^3*d-D*a^2*d^4+12*D*a*b*c^2*d^2-
15*D*b^2*c^4)/d^7+1/15*b*(12*A*b*c*d^3+5*B*a*d^4-17*B*b*c^2*d^2-10*C*a*c*d
^3+22*C*b*c^3*d+15*D*a*c^2*d^2-27*D*b*c^4)/d^7+1/15*b/d^7*c*(21*A*a*b*d^5-
33*A*b^2*c^2*d^3-61*B*a*b*c*d^4+73*B*b^2*c^3*d^2-15*C*a^2*d^5+131*C*a*b*c^
2*d^3-128*C*b^2*c^4*d+45*D*a^2*c*d^4-231*D*a*b*c^3*d^2+198*D*b^2*c^5)/(a*d
^2-b*c^2)+2/5*(b^2/d^4*(C*d-3*D*c)-6/7*D*b^2/d^4*c)/b/d*a*c+1/3*(b/d^5*(B*
b*d^2-3*C*b*c*d-2*D*a*d^2+6*D*b*c^2)+5/7*D*b/d^3*a-4/5*(b^2/d^4*(C*d-3*D*c
)-6/7*D*b^2/d^4*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(749) = 1498$.

Time = 0.26 (sec) , antiderivative size = 1811, normalized size of antiderivative = 2.17

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
-2/315*(4*(768*D*b^3*c^9 - 448*C*b^3*c^8*d - 32*(48*D*a*b^2 - 7*B*b^3)*c^7
*d^2 + 14*(59*C*a*b^2 - 6*A*b^3)*c^6*d^3 + (771*D*a^2*b - 371*B*a*b^2)*c^5
*d^4 - 42*(8*C*a^2*b - 3*A*a*b^2)*c^4*d^5 - 15*(3*D*a^3 - 7*B*a^2*b)*c^3*d
^6 + (768*D*b^3*c^6*d^3 - 448*C*b^3*c^5*d^4 - 32*(48*D*a*b^2 - 7*B*b^3)*c^
4*d^5 + 14*(59*C*a*b^2 - 6*A*b^3)*c^3*d^6 + (771*D*a^2*b - 371*B*a*b^2)*c^
2*d^7 - 42*(8*C*a^2*b - 3*A*a*b^2)*c*d^8 - 15*(3*D*a^3 - 7*B*a^2*b)*d^9)*x
^3 + 3*(768*D*b^3*c^7*d^2 - 448*C*b^3*c^6*d^3 - 32*(48*D*a*b^2 - 7*B*b^3)*
c^5*d^4 + 14*(59*C*a*b^2 - 6*A*b^3)*c^4*d^5 + (771*D*a^2*b - 371*B*a*b^2)*
c^3*d^6 - 42*(8*C*a^2*b - 3*A*a*b^2)*c^2*d^7 - 15*(3*D*a^3 - 7*B*a^2*b)*c*
d^8)*x^2 + 3*(768*D*b^3*c^8*d - 448*C*b^3*c^7*d^2 - 32*(48*D*a*b^2 - 7*B*b
^3)*c^6*d^3 + 14*(59*C*a*b^2 - 6*A*b^3)*c^5*d^4 + (771*D*a^2*b - 371*B*a*b
^2)*c^4*d^5 - 42*(8*C*a^2*b - 3*A*a*b^2)*c^3*d^6 - 15*(3*D*a^3 - 7*B*a^2*b
)*c^2*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2)
, -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(768*D*b^3*c^
8*d - 448*C*b^3*c^7*d^2 - 32*(30*D*a*b^2 - 7*B*b^3)*c^6*d^3 + 14*(35*C*a*b
^2 - 6*A*b^3)*c^5*d^4 + (213*D*a^2*b - 203*B*a*b^2)*c^4*d^5 - 63*(C*a^2*b
- A*a*b^2)*c^3*d^6 + (768*D*b^3*c^5*d^4 - 448*C*b^3*c^4*d^5 - 32*(30*D*a*b
^2 - 7*B*b^3)*c^3*d^6 + 14*(35*C*a*b^2 - 6*A*b^3)*c^2*d^7 + (213*D*a^2*b -
 203*B*a*b^2)*c*d^8 - 63*(C*a^2*b - A*a*b^2)*d^9)*x^3 + 3*(768*D*b^3*c^6*d
^3 - 448*C*b^3*c^5*d^4 - 32*(30*D*a*b^2 - 7*B*b^3)*c^4*d^5 + 14*(35*C*a...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(7/2),
 x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

3.146 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 983

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \frac{2(bc^2-ad^2)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{7d^6(c+dx)^{7/2}}$$
$$+\frac{2(7ad^2(2cCd-Bd^2-3c^2D)-bc(30c^2Cd-23Bcd^2+16Ad^3-37c^3D))\sqrt{a-bx^2}}{35d^6(c+dx)^{5/2}}$$
$$+\frac{2(35a^2d^4(Cd-3cD)-abd^2(283c^2Cd-129Bcd^2+45Ad^3-507c^3D)+b^2c^2(260c^2Cd-141Bcd^2+57A}{105d^6(bc^2-ad^2)(c+dx)^{3/2}}$$
$$-\frac{2(105a^3d^6D+7a^2bd^4(82cCd-21Bd^2-228c^2D)-ab^2cd^2(1412c^2Cd-474Bcd^2+96Ad^3-3225c^3D)}{105d^6(bc^2-ad^2)^2\sqrt{c+dx}}$$
$$-\frac{2b(5Cd-27cD)\sqrt{c+dx}\sqrt{a-bx^2}}{15d^6}-\frac{2bD(c+dx)^{3/2}\sqrt{a-bx^2}}{5d^6}$$
$$+\frac{8\sqrt{a}\sqrt{b}(63a^3d^6D+7a^2bd^4(38cCd-9Bd^2-117c^2D)-ab^2cd^2(598c^2Cd-171Bcd^2+24Ad^3-1536c^3D}{105d^7(bc^2-ad^2)^2\sqrt{}}$$
$$-\frac{8\sqrt{a}\sqrt{b}(7a^2d^4(5Cd-27cD)-abd^2(358c^2Cd-99Bcd^2+15Ad^3-960c^3D)+4b^2c^2(80c^2Cd-24Bcd^2}{105d^7(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-}}$$

output
```
2/7*(-a*d^2+b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^6/(d*x
+c)^(7/2)+2/35*(7*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(16*A*d^3-23*B*c*d^2+
30*C*c^2*d-37*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(d*x+c)^(5/2)+2/105*(35*a^2*d^4
*(C*d-3*D*c)-a*b*d^2*(45*A*d^3-129*B*c*d^2+283*C*c^2*d-507*D*c^3)+b^2*c^2*
(57*A*d^3-141*B*c*d^2+260*C*c^2*d-414*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(-a*d^2
+b*c^2)/(d*x+c)^(3/2)-2/105*(105*a^3*d^6*D+7*a^2*b*d^4*(-21*B*d^2+82*C*c*d
-228*D*c^2)-a*b^2*c*d^2*(96*A*d^3-474*B*c*d^2+1412*C*c^2*d-3225*D*c^3)+b^3
*c^3*(48*A*d^3-279*B*c*d^2+790*C*c^2*d-1686*D*c^3))*(-b*x^2+a)^(1/2)/d^6/(
-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/15*b*(5*C*d-27*D*c)*(d*x+c)^(1/2)*(-b*x^2+
a)^(1/2)/d^6-2/5*b*D*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^6+8/105*a^(1/2)*b^(1
/2)*(63*a^3*d^6*D+7*a^2*b*d^4*(-9*B*d^2+38*C*c*d-117*D*c^2)-a*b^2*c*d^2*(2
4*A*d^3-171*B*c*d^2+598*C*c^2*d-1536*D*c^3)+4*b^3*c^3*(3*A*d^3-24*B*c*d^2+
80*C*c^2*d-192*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1
-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/d^7/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^
2+a)^(1/2)-8/105*a^(1/2)*b^(1/2)*(7*a^2*d^4*(5*C*d-27*D*c)-a*b*d^2*(15*A*d
^3-99*B*c*d^2+358*C*c^2*d-960*D*c^3)+4*b^2*c^2*(3*A*d^3-24*B*c*d^2+80*C*c^
2*d-192*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)
*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(
1/2)*c+a^(1/2)*d))^(1/2))/d^7/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.62 (sec) , antiderivative size = 1383, normalized size of antiderivative = 1.41

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*b*(5*C*d - 24*c*D))/(15*d^6) - (2*b*D*x
)/(5*d^5) - (2*(-(b*c^2) + a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(7*
d^6*(c + d*x)^4) + (2*(-30*b*c^3*C*d + 23*b*B*c^2*d^2 - 16*A*b*c*d^3 + 14*
a*c*C*d^3 - 7*a*B*d^4 + 37*b*c^4*D - 21*a*c^2*d^2*D))/(35*d^6*(c + d*x)^3)
 - (2*(260*b^2*c^4*C*d - 141*b^2*B*c^3*d^2 + 57*A*b^2*c^2*d^3 - 283*a*b*c^
2*C*d^3 + 129*a*b*B*c*d^4 - 45*a*A*b*d^5 + 35*a^2*C*d^5 - 414*b^2*c^5*D +
507*a*b*c^3*d^2*D - 105*a^2*c*d^4*D))/(105*d^6*(-(b*c^2) + a*d^2)*(c + d*x
)^2) - (2*(790*b^3*c^5*C*d - 279*b^3*B*c^4*d^2 + 48*A*b^3*c^3*d^3 - 1412*a
*b^2*c^3*C*d^3 + 474*a*b^2*B*c^2*d^4 - 96*a*A*b^2*c*d^5 + 574*a^2*b*c*C*d^
5 - 147*a^2*b*B*d^6 - 1686*b^3*c^6*D + 3225*a*b^2*c^4*d^2*D - 1596*a^2*b*c
^2*d^4*D + 105*a^3*d^6*D))/(105*d^6*(-(b*c^2) + a*d^2)^2*(c + d*x))) + (8*
Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-(Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(-63*a^3*d^6*D + 7*a^2*b*d^4*(-38*c*C*d + 9*B*d^2 + 117*c^2*D)
+ a*b^2*c*d^2*(598*c^2*C*d - 171*B*c*d^2 + 24*A*d^3 - 1536*c^3*D) + 4*b^3*
c^3*(-80*c^2*C*d + 24*B*c*d^2 - 3*A*d^3 + 192*c^3*D))*(-((a*d^2)/(c + d*x)
^2) + b*(-1 + c/(c + d*x))^2)) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(-63*a
^3*d^6*D + 7*a^2*b*d^4*(-38*c*C*d + 9*B*d^2 + 117*c^2*D) + a*b^2*c*d^2*(59
8*c^2*C*d - 171*B*c*d^2 + 24*A*d^3 - 1536*c^3*D) + 4*b^3*c^3*(-80*c^2*C*d
+ 24*B*c*d^2 - 3*A*d^3 + 192*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(S
qrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x...
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2182, 27, 2182, 27, 681, 25, 681, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \ 2182$$

$$\frac{2\int \frac{(a-bx^2)^{3/2}\left(7\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(7Cd-7cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+3Bc-3Ad\right)\right)x+\frac{7\left(Abcd+a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}\right)}{2(c+dx)^{7/2}}dx}{7\left(bc^2-ad^2\right)}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a-bx^2)^{3/2}\left(7\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(7Cd-7cD)+b\left(\frac{10Dc^3}{d^2}-\frac{10Cc^2}{d}+3Bc-3Ad\right)\right)x+7\left(Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)\right)}{(c+dx)^{7/2}}dx}{7\left(bc^2-ad^2\right)}+\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{2\int \frac{5(d(Abd(7bc^2-3ad^2)+a(7ad^2(Cd-2cD)-bc(-10Dc^2+3Cdc+4Bd^2))))+(7a^2Dd^4+7ab(-5Dc^2+2Cdc-Bd^2)d^2-b^2c(-24Dc^3+10Cdc^2-3Bd^2c-4Ad^3))\cdots}{2d^2(c+dx)^{5/2}}}{5(bc^2-ad^2)} \cdots$$

$$\cdots\; 7\left(bc^2-ad^2\right)$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d(Abd(7bc^2-3ad^2)+a(7ad^2(Cd-2cD)-bc(-10Dc^2+3Cdc+4Bd^2))))+(7a^2Dd^4+7ab(-5Dc^2+2Cdc-Bd^2)d^2-b^2c(-24Dc^3+10Cdc^2-3Bd^2c-4Ad^3))x)\cdots}{(c+dx)^{5/2}}}{d^2(bc^2-ad^2)} \cdots$$

$$\cdots\; 7\left(bc^2-ad^2\right)$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 681$$

$$-\frac{\frac{2\int -\frac{(3ad(7a^2Dd^4+7ab(-5Dc^2+2Cdc-Bd^2)d^2-b^2c(-24Dc^3+10Cdc^2-3Bd^2c-4Ad^3))-b(7a^2(5Cd-18cD)d^4-ab(-330Dc^3+127Cdc^2-36Bd^2c+15Ad^3)\cdots}{(c+dx)^{3/2}}}{5d^2}\cdots}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int \frac{\frac{\left(3ad\left(7a^2Dd^4+7ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-24Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)-b\left(7a^2(5Cd-18cD)d^4-ab\left(-330Dc^3+127Cdc^2-36Bd^2c+15Ad^3\right)d^2\cdots\right)\right)}{(c+dx)^{3/2}}\cdots}{5d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(a-bx^2\right)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^2(c+dx)^{7/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 600$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 509$$

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-c\right.}{}\right.}{}}{}$$

$\downarrow$ 508

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-c\right.}{}\right.}{}}{}$$

$\downarrow$ 327

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-\right.}{}\right.}{}$$

$\Big\downarrow$ 512

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-\right.}{}\right.}{}$$

$\Big\downarrow$ 511

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-c\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

$\downarrow$ 321

$$\frac{2\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)\left(a-bx^2\right)^{5/2}}{7d^2\left(bc^2-ad^2\right)(c+dx)^{7/2}}+$$

$$\frac{\frac{2\left(7ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-17Dc^3+10Cdc^2-3Bd^2c-4Ad^3\right)\right)\left(a-bx^2\right)^{5/2}}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+\frac{2\left(-\frac{2\sqrt{a-bx^2}\left(63a^3Dd^6+7a^2b\left(-117Dc^2+38Cdc-9Bd^2\right)d^4-c\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a - b*x^2)^(5/2))/(7*d^2*(b*c^2 -
a*d^2)*(c + d*x)^(7/2)) + ((2*(7*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(
10*c^2*C*d - 3*B*c*d^2 - 4*A*d^3 - 17*c^3*D))*(a - b*x^2)^(5/2))/(5*d^2*(b
*c^2 - a*d^2)*(c + d*x)^(5/2)) + ((-2*(7*a^2*d^4*(5*C*d - 18*c*D) - a*b*d^
2*(127*c^2*C*d - 36*B*c*d^2 + 15*A*d^3 - 330*c^3*D) + b^2*c^2*(80*c^2*C*d
- 24*B*c*d^2 + 3*A*d^3 - 192*c^3*D) - 3*d*(7*a^2*d^4*D + 7*a*b*d^2*(2*c*C*
d - B*d^2 - 5*c^2*D) - b^2*c*(10*c^2*C*d - 3*B*c*d^2 - 4*A*d^3 - 24*c^3*D)
)*x)*(a - b*x^2)^(3/2))/(15*d^2*(c + d*x)^(3/2)) + (2*((-2*(63*a^3*d^6*D +
 7*a^2*b*d^4*(38*c*C*d - 9*B*d^2 - 117*c^2*D) - a*b^2*c*d^2*(598*c^2*C*d -
 171*B*c*d^2 + 24*A*d^3 - 1536*c^3*D) + 4*b^3*c^3*(80*c^2*C*d - 24*B*c*d^2
 + 3*A*d^3 - 192*c^3*D) + b*d*(7*a^2*d^4*(5*C*d - 18*c*D) - a*b*d^2*(127*c
^2*C*d - 36*B*c*d^2 + 15*A*d^3 - 330*c^3*D) + b^2*c^2*(80*c^2*C*d - 24*B*c
*d^2 + 3*A*d^3 - 192*c^3*D))*x)*Sqrt[a - b*x^2])/(3*d^2*Sqrt[c + d*x]) - (
2*b*((-2*Sqrt[a]*(63*a^3*d^6*D + 7*a^2*b*d^4*(38*c*C*d - 9*B*d^2 - 117*c^2
*D) - a*b^2*c*d^2*(598*c^2*C*d - 171*B*c*d^2 + 24*A*d^3 - 1536*c^3*D) + 4*
b^3*c^3*(80*c^2*C*d - 24*B*c*d^2 + 3*A*d^3 - 192*c^3*D))*Sqrt[c + d*x]*Sqr
t[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(7*a^2*
d^4*(5*C*d - 27*c*D) - a*b*d^2*(358*c^2*C*d - 99*B*c*d^2 + 15*A*d^3 - 9...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 8.69 (sec) , antiderivative size = 1773, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1773
default	Expression too large to display	26409

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/7*(A*a*d^5
-A*b*c^2*d^3-B*a*c*d^4+B*b*c^3*d^2+C*a*c^2*d^3-C*b*c^4*d-D*a*c^3*d^2+D*b*c
^5)/d^10*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^4-2/35*(16*A*b*c*d^3+7
*B*a*d^4-23*B*b*c^2*d^2-14*C*a*c*d^3+30*C*b*c^3*d+21*D*a*c^2*d^2-37*D*b*c^
4)/d^9*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^3+2/105*(45*A*a*b*d^5-57
*A*b^2*c^2*d^3-129*B*a*b*c*d^4+141*B*b^2*c^3*d^2-35*C*a^2*d^5+283*C*a*b*c^
2*d^3-260*C*b^2*c^4*d+105*D*a^2*c*d^4-507*D*a*b*c^3*d^2+414*D*b^2*c^5)/d^8
/(a*d^2-b*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2/105*(-b*d*x^
2+a*d)/d^7/(a*d^2-b*c^2)^2*(96*A*a*b^2*c*d^5-48*A*b^3*c^3*d^3+147*B*a^2*b*
d^6-474*B*a*b^2*c^2*d^4+279*B*b^3*c^4*d^2-574*C*a^2*b*c*d^5+1412*C*a*b^2*c
^3*d^3-790*C*b^3*c^5*d-105*D*a^3*d^6+1596*D*a^2*b*c^2*d^4-3225*D*a*b^2*c^4
*d^2+1686*D*b^3*c^6)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2/5*D*b/d^5*x*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(b^2/d^5*(C*d-4*D*c)-4/5*D*b^2/d^5*c)/b/d*(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(b*(A*b*d^3-4*B*b*c*d^2-2*C*a*d^3+10*C
*b*c^2*d+8*D*a*c*d^2-20*D*b*c^3)/d^7-1/105*b*(45*A*a*b*d^5-57*A*b^2*c^2*d^
3-129*B*a*b*c*d^4+141*B*b^2*c^3*d^2-35*C*a^2*d^5+283*C*a*b*c^2*d^3-260*C*b
^2*c^4*d+105*D*a^2*c*d^4-507*D*a*b*c^3*d^2+414*D*b^2*c^5)/d^7/(a*d^2-b*c^2
)+1/105*b/d^7*c*(96*A*a*b^2*c*d^5-48*A*b^3*c^3*d^3+147*B*a^2*b*d^6-474*B*a
*b^2*c^2*d^4+279*B*b^3*c^4*d^2-574*C*a^2*b*c*d^5+1412*C*a*b^2*c^3*d^3-790*
C*b^3*c^5*d-105*D*a^3*d^6+1596*D*a^2*b*c^2*d^4-3225*D*a*b^2*c^4*d^2+168...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2598 vs. $2(896) = 1792$.

Time = 0.30 (sec) , antiderivative size = 2598, normalized size of antiderivative = 2.64

$$\int \frac{(a-bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm=
"fricas")
```

output
```
2/315*(4*(768*D*b^3*c^11 - 320*C*b^3*c^10*d - 96*(22*D*a*b^2 - B*b^3)*c^9*
d^2 + 2*(419*C*a*b^2 - 6*A*b^3)*c^8*d^3 + 27*(67*D*a^2*b - 9*B*a*b^2)*c^7*
d^4 - (647*C*a^2*b - 33*A*a*b^2)*c^6*d^5 - 9*(49*D*a^3 - 19*B*a^2*b)*c^5*d
^6 + 15*(7*C*a^3 - 3*A*a^2*b)*c^4*d^7 + (768*D*b^3*c^7*d^4 - 320*C*b^3*c^6
*d^5 - 96*(22*D*a*b^2 - B*b^3)*c^5*d^6 + 2*(419*C*a*b^2 - 6*A*b^3)*c^4*d^7
 + 27*(67*D*a^2*b - 9*B*a*b^2)*c^3*d^8 - (647*C*a^2*b - 33*A*a*b^2)*c^2*d^
9 - 9*(49*D*a^3 - 19*B*a^2*b)*c*d^10 + 15*(7*C*a^3 - 3*A*a^2*b)*d^11)*x^4
+ 4*(768*D*b^3*c^8*d^3 - 320*C*b^3*c^7*d^4 - 96*(22*D*a*b^2 - B*b^3)*c^6*d
^5 + 2*(419*C*a*b^2 - 6*A*b^3)*c^5*d^6 + 27*(67*D*a^2*b - 9*B*a*b^2)*c^4*d
^7 - (647*C*a^2*b - 33*A*a*b^2)*c^3*d^8 - 9*(49*D*a^3 - 19*B*a^2*b)*c^2*d^
9 + 15*(7*C*a^3 - 3*A*a^2*b)*c*d^10)*x^3 + 6*(768*D*b^3*c^9*d^2 - 320*C*b^
3*c^8*d^3 - 96*(22*D*a*b^2 - B*b^3)*c^7*d^4 + 2*(419*C*a*b^2 - 6*A*b^3)*c^
6*d^5 + 27*(67*D*a^2*b - 9*B*a*b^2)*c^5*d^6 - (647*C*a^2*b - 33*A*a*b^2)*c
^4*d^7 - 9*(49*D*a^3 - 19*B*a^2*b)*c^3*d^8 + 15*(7*C*a^3 - 3*A*a^2*b)*c^2*
d^9)*x^2 + 4*(768*D*b^3*c^10*d - 320*C*b^3*c^9*d^2 - 96*(22*D*a*b^2 - B*b^
3)*c^8*d^3 + 2*(419*C*a*b^2 - 6*A*b^3)*c^7*d^4 + 27*(67*D*a^2*b - 9*B*a*b^
2)*c^6*d^5 - (647*C*a^2*b - 33*A*a*b^2)*c^5*d^6 - 9*(49*D*a^3 - 19*B*a^2*b
)*c^4*d^7 + 15*(7*C*a^3 - 3*A*a^2*b)*c^3*d^8)*x)*sqrt(-b*d)*weierstrassPIn
verse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/
3*(3*d*x + c)/d) + 12*(768*D*b^3*c^10*d - 320*C*b^3*c^9*d^2 - 96*(16*D*...
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx=\int \frac{(a-bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(9/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(9/2),
 x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{9/2}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

3.147 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 739

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx =$$

$$-\frac{2(675a^2d^4D+3abd^2(418cCd+275Bd^2+95c^2D)-b^2c(110c^2Cd-495Bcd^2-1848Ad^3-40c^3D))\sqrt{c}}{3465b^3d^2}$$

$$-\frac{2(ad^2(539Cd+335cD)-b(110c^2Cd-495Bcd^2-693Ad^3-40c^3D))\,(c+dx)^{3/2}\sqrt{a-bx^2}}{3465b^2d^2}$$

$$-\frac{2(81ad^2D-b(22cCd-99Bd^2-8c^2D))\,(c+dx)^{5/2}\sqrt{a-bx^2}}{693b^2d^2}$$

$$-\frac{2(11Cd-13cD)(c+dx)^{7/2}\sqrt{a-bx^2}}{99bd^2}-\frac{2D(c+dx)^{9/2}\sqrt{a-bx^2}}{11bd^2}$$

$$-\frac{2\sqrt{a}(3a^2d^4(539Cd+1235cD)-b^2c^2(110c^2Cd-495Bcd^2-5313Ad^3-40c^3D)+3abd^2(1023c^2Cd+15}{3465b^{5/2}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-b}}$$

$$+\frac{2\sqrt{a}(bc^2-ad^2)\,(675a^2d^4D+3abd^2(418cCd+275Bd^2+95c^2D)-b^2c(110c^2Cd-495Bcd^2-1848Ad^3}{3465b^{7/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3465*(675*a^2*d^4*D+3*a*b*d^2*(275*B*d^2+418*C*c*d+95*D*c^2)-b^2*c*(-18
48*A*d^3-495*B*c*d^2+110*C*c^2*d-40*D*c^3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)
/b^3/d^2-2/3465*(a*d^2*(539*C*d+335*D*c)-b*(-693*A*d^3-495*B*c*d^2+110*C*c
^2*d-40*D*c^3))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d^2-2/693*(81*a*d^2*D-b
*(-99*B*d^2+22*C*c*d-8*D*c^2))*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b^2/d^2-2/99
*(11*C*d-13*D*c)*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/b/d^2-2/11*D*(d*x+c)^(9/2)
*(-b*x^2+a)^(1/2)/b/d^2-2/3465*a^(1/2)*(3*a^2*d^4*(539*C*d+1235*D*c)-b^2*c
^2*(-5313*A*d^3-495*B*c*d^2+110*C*c^2*d-40*D*c^3)+3*a*b*d^2*(693*A*d^3+159
5*B*c*d^2+1023*C*c^2*d+85*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/b^(5/2)/d^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*
x^2+a)^(1/2)+2/3465*a^(1/2)*(-a*d^2+b*c^2)*(675*a^2*d^4*D+3*a*b*d^2*(275*B
*d^2+418*C*c*d+95*D*c^2)-b^2*c*(-1848*A*d^3-495*B*c*d^2+110*C*c^2*d-40*D*c
^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(7/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.16 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-3a^2d^4(539Cd+1235cD)-b^2c^2(-110c^2Cd+4\right.}{}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-3*a^2*d^4*(539*C*d + 1235*c*D) - b^2*c^2*(-110*c^2*C*
d + 495*B*c*d^2 + 5313*A*d^3 + 40*c^3*D) - 3*a*b*d^2*(1023*c^2*C*d + 1595*
B*c*d^2 + 693*A*d^3 + 85*c^3*D) - (c + d*x)*(675*a^2*d^4*D + a*b*d^2*(1025
*c^2*D + c*d*(1793*C + 1145*D*x) + d^2*(825*B + 539*C*x + 405*D*x^2)) + b^
2*(-20*c^4*D + 5*c^3*d*(11*C + 3*D*x) + 5*c^2*d^2*(297*B + x*(165*C + 113*
D*x)) + d^4*x*(693*A + 5*x*(99*B + 77*C*x + 63*D*x^2)) + c*d^3*(2541*A + 5
*x*(297*B + 209*C*x + 161*D*x^2)))) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(
3*a^2*d^4*(539*C*d + 1235*c*D) + b^2*c^2*(-110*c^2*C*d + 495*B*c*d^2 + 531
3*A*d^3 + 40*c^3*D) + 3*a*b*d^2*(1023*c^2*C*d + 1595*B*c*d^2 + 693*A*d^3 +
 85*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/
Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - S
qrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(Sqrt[b
]*c - Sqrt[a]*d)*(3465*A*b^(5/2)*c^2*d^2 - 675*a^(5/2)*d^4*D + 3*a^2*Sqrt[
b]*d^3*(539*C*d + 1010*c*D) - 3*a^(3/2)*b*d^2*(418*c*C*d + 275*B*d^2 + 95*
c^2*D) + 3*a*b^(3/2)*d*(605*c^2*C*d + 1320*B*c*d^2 + 693*A*d^3 - 10*c^3*D)
 - Sqrt[a]*b^2*c*(-110*c^2*C*d + 495*B*c*d^2 + 1848*A*d^3 + 40*c^3*D))*Sqr
t[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/
(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/...
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {2185, 27, 2185, 27, 687, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}} \, dx$$

$$\Big\downarrow \; 2185$$

$$-\frac{2\int -\frac{(c+dx)^{5/2}\left(b(11Cd-13cD)x^2d^2+(11Abd+9acD)d^2+\left(-2bDc^2+11bBd^2+9ad^2D\right)xd\right)}{2\sqrt{a-bx^2}}dx}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{(c+dx)^{5/2}\left(b(11Cd-13cD)x^2d^2+(11Abd+9acD)d^2+\left(-2bDc^2+11bBd^2+9ad^2D\right)xd\right)}{\sqrt{a-bx^2}}dx}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}}{}$$

$$\Big\downarrow \ 2185$$

$$\frac{-\frac{2\int -\frac{bd^3(c+dx)^{5/2}\left(d(99Abd+77aCd-10acD)+\left(81ad^2D-b\left(-8Dc^2+22Cdc-99Bd^2\right)\right)x\right)}{2\sqrt{a-bx^2}}dx}{9bd^2}-\frac{2}{9}d\sqrt{a-bx^2}(c+dx)^{7/2}(11Cd-13cD)}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{9}d\int \frac{(c+dx)^{5/2}\left(d(99Abd+77aCd-10acD)+\left(81ad^2D-b\left(-8Dc^2+22Cdc-99Bd^2\right)\right)x\right)}{\sqrt{a-bx^2}}dx-\frac{2}{9}d\sqrt{a-bx^2}(c+dx)^{7/2}(11Cd-13cD}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{9}d\left(-\frac{2\int -\frac{(c+dx)^{3/2}\left(3d\left(231Acdb^2+a\left(135aDd^2+b\left(-10Dc^2+143Cdc+165Bd^2\right)\right)\right)+b\left(ad^2(539Cd+335cD)-b\left(-40Dc^3+110Cdc^2-495Bd^2c-693Ad^3\right)\right)x\right)}{2\sqrt{a-bx^2}}}{7b}\right.}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{9}d\left(\frac{\int \frac{(c+dx)^{3/2}\left(3d\left(231Acdb^2+a\left(135aDd^2+b\left(-10Dc^2+143Cdc+165Bd^2\right)\right)\right)+b\left(ad^2(539Cd+335cD)-b\left(-40Dc^3+110Cdc^2-495Bd^2c-693Ad^3\right)\right)x\right)}{\sqrt{a-bx^2}}dx}{7b}-\right.}{11bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 687$$

$$\frac{1}{9}d\Bigg(-\frac{-\frac{2\int -\frac{3b\sqrt{c+dx}(d(231Abd(5bc^2+3ad^2)+a(a(539Cd+1010cD)d^2+5bc(-2Dc^2+121Cdc+264Bd^2))))+(675a^2Dd^4+3ab(95Dc^2+418Cdc+275Bd^2)d^2-\cdots}{2\sqrt{a-bx^2}}}{5b}}{7b\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{9}d\Bigg(\frac{\frac{3}{5}\int \frac{\sqrt{c+dx}(d(231Abd(5bc^2+3ad^2)+a(a(539Cd+1010cD)d^2+5bc(-2Dc^2+121Cdc+264Bd^2))))+(675a^2Dd^4+3ab(95Dc^2+418Cdc+275Bd^2)d^2-b^2c(-\cdots}{\sqrt{a-bx^2}}}{7b\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \color{blue}{687}$$

$$\frac{1}{9}d\Bigg(\frac{\frac{3}{5}\Bigg(-\frac{2\int -\frac{d(231Acd(15bc^2+17ad^2)b^2+a(675a^2Dd^4+3ab(1105Dc^2+957Cdc+275Bd^2)d^2+5b^2c^2(2Dc^2+341Cdc+891Bd^2)))+b(3a^2(539Cd+1235cD)\cdots}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\cdots}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{9}d\Bigg(\frac{\frac{3}{5}\Bigg(\frac{\int \frac{d(231Acd(15bc^2+17ad^2)b^2+a(675a^2Dd^4+3ab(1105Dc^2+957Cdc+275Bd^2)d^2+5b^2c^2(2Dc^2+341Cdc+891Bd^2)))+b(3a^2(539Cd+1235cD)d^4+3\cdots}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\cdots}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \color{blue}{600}$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(\frac{\frac{b\left(3a^2d^4(1235cD+539Cd)+3abd^2\left(693Ad^3+1595Bcd^2+85c^3D+1023c^2Cd\right)-b^2c^2\left(-5313Ad^3-495Bcd^2-40c^3D+110c^2Cd\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-c\right.}{}}{3b}\right.}{}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow 509$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(1235cD+539Cd)+3abd^2\left(693Ad^3+1595Bcd^2+85c^3D+1023c^2Cd\right)-b^2c^2\left(-5313Ad^3-495Bcd^2-40c^3D+110c^2Cd\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3b}\right.}{}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow 508$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(-\frac{\left(bc^2-ad^2\right)\left(675a^2d^4D+3abd^2\left(275Bd^2+95c^2D+418cCd\right)-b^2c\left(-1848Ad^3-495Bcd^2-40c^3D+110c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}}{\;}\right.}{\;}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow 327$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(-\frac{\left(bc^2-ad^2\right)\left(675a^2d^4D+3abd^2\left(275Bd^2+95c^2D+418cCd\right)-b^2c\left(-1848Ad^3-495Bcd^2-40c^3D+110c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}}{\;}\right.}{\;}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 512$$

$$\frac{1}{9}d\left(\frac{3}{5}\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(675a^2d^4D+3abd^2\left(275Bd^2+95c^2D+418cCd\right)-b^2c\left(-1848Ad^3-495Bcd^2-40c^3D+110c^2Cd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{\cdots}}{\cdots}\right.\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

$$\Big\downarrow \ 511$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(\frac{2\sqrt{a}\left(bc^2-ad^2\right)\left(675a^2Dd^4+3ab\left(95Dc^2+418Cdc+275Bd^2\right)d^2-b^2c\left(-40Dc^3+110Cdc^2-495Bd^2c-1848Ad^3\right)\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int \cdots}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}\right.}{\cdots}\right.$$

$$\frac{2D(c+dx)^{9/2}\sqrt{a-bx^2}}{11bd^2}$$

$$\Big\downarrow\ 321$$

$$\frac{1}{9}d\left(\frac{\frac{3}{5}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(675a^2d^4D+3abd^2\left(275Bd^2+95c^2D+418cCd\right)-b^2c\left(-1848Ad^3-495\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{\cdots}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^2}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(-2*D*(c + d*x)^(9/2)*Sqrt[a - b*x^2])/(11*b*d^2) + ((-2*d*(11*C*d - 13*c*
D)*(c + d*x)^(7/2)*Sqrt[a - b*x^2])/9 + (d*((-2*(81*a*d^2*D - b*(22*c*C*d
- 99*B*d^2 - 8*c^2*D))*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b) + ((-2*(a*d^
2*(539*C*d + 335*c*D) - b*(110*c^2*C*d - 495*B*c*d^2 - 693*A*d^3 - 40*c^3*
D))*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + (3*((-2*(675*a^2*d^4*D + 3*a*b*d^
2*(418*c*C*d + 275*B*d^2 + 95*c^2*D) - b^2*c*(110*c^2*C*d - 495*B*c*d^2 -
1848*A*d^3 - 40*c^3*D))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + ((-2*Sqrt[a
]*Sqrt[b]*(3*a^2*d^4*(539*C*d + 1235*c*D) - b^2*c^2*(110*c^2*C*d - 495*B*c
*d^2 - 5313*A*d^3 - 40*c^3*D) + 3*a*b*d^2*(1023*c^2*C*d + 1595*B*c*d^2 + 6
93*A*d^3 + 85*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[S
qrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(b*c^2 - a*d^2)*(675*a^2*d^4*D + 3*a*b*d^2*(418*c*C*d + 275*B*d^2
+ 95*c^2*D) - b^2*c*(110*c^2*C*d - 495*B*c*d^2 - 1848*A*d^3 - 40*c^3*D))*S
qrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b)))/5)/(7*b)))/
9)/(11*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(655) = 1310$.

Time = 6.42 (sec) , antiderivative size = 1848, normalized size of antiderivative = 2.50

method	result	size
elliptic	Expression too large to display	1848
default	Expression too large to display	6097

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/11*D*d^2/b
*x^4*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(C*d^3+23/11*d^2*c*D)/b/d*x^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d
^3/b*a-8/9*(C*d^3+23/11*d^2*c*D)/d*c)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)
^(1/2)-2/5*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3+8/11*D*d^2/b*a*c+7/9*(C*d^3+23
/11*d^2*c*D)/b*a-6/7*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*d^3+
23/11*d^2*c*D)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(3*A
*c*d^2+3*B*c^2*d+C*c^3+2/3*(C*d^3+23/11*d^2*c*D)/b/d*a*c+5/7*(B*d^3+3*C*c*
d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*d^3+23/11*d^2*c*D)/d*c)/b*a-4/5*(A*d^3
+3*B*c*d^2+3*C*c^2*d+D*c^3+8/11*D*d^2/b*a*c+7/9*(C*d^3+23/11*d^2*c*D)/b*a-
6/7*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*d^3+23/11*d^2*c*D)/d*
c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*c^3+2/5*(A*d^3+3*
B*c*d^2+3*C*c^2*d+D*c^3+8/11*D*d^2/b*a*c+7/9*(C*d^3+23/11*d^2*c*D)/b*a-6/7
*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*d^3+23/11*d^2*c*D)/d*c)/
d*c)/b/d*a*c+1/3*(3*A*c*d^2+3*B*c^2*d+C*c^3+2/3*(C*d^3+23/11*d^2*c*D)/b/d*
a*c+5/7*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*d^3+23/11*d^2*c*D
)/d*c)/b*a-4/5*(A*d^3+3*B*c*d^2+3*C*c^2*d+D*c^3+8/11*D*d^2/b*a*c+7/9*(C*d^
3+23/11*d^2*c*D)/b*a-6/7*(B*d^3+3*C*c*d^2+3*D*c^2*d+9/11*D*d^3/b*a-8/9*(C*
d^3+23/11*d^2*c*D)/d*c)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
2/10395*((40*D*b^3*c^6 - 110*C*b^3*c^5*d + 45*(5*D*a*b^2 + 11*B*b^3)*c^4*d
^2 - 66*(31*C*a*b^2 + 77*A*b^3)*c^3*d^3 - 780*(8*D*a^2*b + 11*B*a*b^2)*c^2
*d^4 - 66*(106*C*a^2*b + 147*A*a*b^2)*c*d^5 - 225*(9*D*a^3 + 11*B*a^2*b)*d
^6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(40*D*b^3*c^5*d - 110*C*
b^3*c^4*d^2 + 15*(17*D*a*b^2 + 33*B*b^3)*c^3*d^3 + 33*(93*C*a*b^2 + 161*A*
b^3)*c^2*d^4 + 15*(247*D*a^2*b + 319*B*a*b^2)*c*d^5 + 231*(7*C*a^2*b + 9*A
*a*b^2)*d^6)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(315*D
*b^3*d^6*x^4 - 20*D*b^3*c^4*d^2 + 55*C*b^3*c^3*d^3 + 5*(205*D*a*b^2 + 297*
B*b^3)*c^2*d^4 + 11*(163*C*a*b^2 + 231*A*b^3)*c*d^5 + 75*(9*D*a^2*b + 11*B
*a*b^2)*d^6 + 35*(23*D*b^3*c*d^5 + 11*C*b^3*d^6)*x^3 + 5*(113*D*b^3*c^2*d^
4 + 209*C*b^3*c*d^5 + 9*(9*D*a*b^2 + 11*B*b^3)*d^6)*x^2 + (15*D*b^3*c^3*d^
3 + 825*C*b^3*c^2*d^4 + 5*(229*D*a*b^2 + 297*B*b^3)*c*d^5 + 77*(7*C*a*b^2
+ 9*A*b^3)*d^6)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^4*d^4)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(c+dx)^{\frac{5}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(c+d\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(dx+c)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{\sqrt{-b\,x^2+a}}dx$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x)
```

3.148 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 594

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = -\frac{2(3ad^2(25Cd+13cD)-b(18c^2Cd-63Bcd^2-105Ad^3-8c^3D))\sqrt{c+dx}\sqrt{a-bx^2}}{315b^2d^2} - \frac{2\left(49aD+b\left(63B-\frac{2c(9Cd-4cD)}{d^2}\right)\right)(c+dx)^{3/2}\sqrt{a-bx^2}}{315b^2} - \frac{2(9Cd-11cD)(c+dx)^{5/2}\sqrt{a-bx^2}}{63bd^2} - \frac{2D(c+dx)^{7/2}\sqrt{a-bx^2}}{9bd^2} - \frac{2\sqrt{a}(147a^2d^4D+3abd^2(82cCd+63Bd^2+11c^2D)-b^2c(18c^2Cd-63Bcd^2-420Ad^3-8c^3D))\sqrt{c+d}}{315b^{5/2}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(bc^2-ad^2)(3ad^2(25Cd+13cD)-b(18c^2Cd-63Bcd^2-105Ad^3-8c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{Elliptic}}{315b^{5/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/315*(3*a*d^2*(25*C*d+13*D*c)-b*(-105*A*d^3-63*B*c*d^2+18*C*c^2*d-8*D*c^
3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^2-2/315*(49*D*a+b*(63*B-2*c*(9*C*
d-4*D*c)/d^2))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2-2/63*(9*C*d-11*D*c)*(d*x
+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^2-2/9*D*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/b/d^
2-2/315*a^(1/2)*(147*a^2*d^4*D+3*a*b*d^2*(63*B*d^2+82*C*c*d+11*D*c^2)-b^2*
c*(-420*A*d^3-63*B*c*d^2+18*C*c^2*d-8*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)
^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2))
)^(1/2)/(-b*x^2+a)^(1/2)+2/315*a^(1/2)*(-a*d^2+b*c^2)*(3*a*d^2*(25*C*d+13*
D*c)-b*(-105*A*d^3-63*B*c*d^2+18*C*c^2*d-8*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^3/(
d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.10 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-147a^2d^4D-3abd^2(82cCd+63Bd^2+11c^2D)-\right.}{}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-147*a^2*d^4*D - 3*a*b*d^2*(82*c*C*d + 63*B*d^2 + 11*c
^2*D) - b^2*c*(-18*c^2*C*d + 63*B*c*d^2 + 420*A*d^3 + 8*c^3*D) - b*(c + d*
x)*(a*d^2*(75*C*d + 88*c*D + 49*d*D*x) + b*(-4*c^3*D + 3*c^2*d*(3*C + D*x)
 + 2*c*d^2*(63*B + x*(36*C + 25*D*x)) + d^3*(105*A + x*(63*B + 5*x*(9*C +
7*D*x)))))) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(147*a^2*d^4*D + 3*a*b*d^2
*(82*c*C*d + 63*B*d^2 + 11*c^2*D) + b^2*c*(-18*c^2*C*d + 63*B*c*d^2 + 420*
A*d^3 + 8*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[
a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]
*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*S
qrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(315*A*b^2*c*d^2 + 147*a^2*d^3*D - 3*a^(3/2
)*Sqrt[b]*d^2*(25*C*d + 13*c*D) + 3*a*b*d*(57*c*C*d + 63*B*d^2 - 2*c^2*D)
- Sqrt[a]*b^(3/2)*(-18*c^2*C*d + 63*B*c*d^2 + 105*A*d^3 + 8*c^3*D))*Sqrt[(
d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c
+ d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(315*b^3*d^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2185, 27, 2185, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 2185$$

$$-\frac{2\int -\frac{(c+dx)^{3/2}\left(b(9Cd-11cD)x^2d^2+(9Abd+7acD)d^2+\left(-2bDc^2+9bBd^2+7ad^2D\right)xd\right)}{2\sqrt{a-bx^2}}dx}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(b(9Cd-11cD)x^2d^2+(9Abd+7acD)d^2+\left(-2bDc^2+9bBd^2+7ad^2D\right)xd\right)}{\sqrt{a-bx^2}}dx}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{-\frac{2\int -\frac{bd^3(c+dx)^{3/2}\left(3d(21Abd+15aCd-2acD)+\left(49ad^2D-b\left(-8Dc^2+18Cdc-63Bd^2\right)\right)x\right)}{2\sqrt{a-bx^2}}dx}{7bd^2}-\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(9Cd-11cD)}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\int \frac{(c+dx)^{3/2}\left(3d(21Abd+15aCd-2acD)+\left(49ad^2D-b\left(-8Dc^2+18Cdc-63Bd^2\right)\right)x\right)}{\sqrt{a-bx^2}}dx-\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(9Cd-11cD)}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{7}d\left(-\frac{2\int -\frac{3\sqrt{c+dx}\left(d\left(105Acdb^2+a\left(49aDd^2+b\left(-2Dc^2+57Cdc+63Bd^2\right)\right)\right)+b\left(3ad^2(25Cd+13cD)-b\left(-8Dc^3+18Cdc^2-63Bd^2c-105Ad^3\right)\right)x\right)}{2\sqrt{a-bx^2}}dx}{5b}-2\sqrt{c\cdots}\right)}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\left(\frac{3\int \frac{\sqrt{c+dx}\left(d\left(105Acdb^2+a\left(49aDd^2+b\left(-2Dc^2+57Cdc+63Bd^2\right)\right)\right)+b\left(3ad^2(25Cd+13cD)-b\left(-8Dc^3+18Cdc^2-63Bd^2c-105Ad^3\right)\right)x\right)}{\sqrt{a-bx^2}}dx}{5b}-\frac{2\sqrt{a-bx^2}\cdots}{\cdots}\right)}{9bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{7}d\left(\frac{3\left(-\frac{2\int -\frac{b\left(d\left(105Abd\left(3bc^2+ad^2\right)+a\left(3a(25Cd+62cD)d^2+bc\left(2Dc^2+153Cdc+252Bd^2\right)\right)\right)\right)+\left(147a^2Dd^4+3ab\left(11Dc^2+82Cdc+63Bd^2\right)d^2-b^2c\left(-8Dc^3+\cdots\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\cdots\right.}{5b}\cdots\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\left(\frac{3\left(\frac{1}{3}\int \frac{d\left(105Abd\left(3bc^2+ad^2\right)+a\left(3a(25Cd+62cD)d^2+bc\left(2Dc^2+153Cdc+252Bd^2\right)\right)\right)+\left(147a^2Dd^4+3ab\left(11Dc^2+82Cdc+63Bd^2\right)d^2-b^2c\left(-8Dc^3+18Cd\cdots\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots\right.}{5b}\cdots\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{1}{7}d\left(\frac{3\left(\frac{1}{3}\left(\frac{\left(147a^2d^4D+3abd^2\left(63Bd^2+11c^2D+82cCd\right)-b^2c\left(-420Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(13cD+25Cd)-b\left(-10\cdots\right.\right.}{\cdots}\right.\right.}{5b}\cdots\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{1}{7}d\left(3\left(\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(147a^2d^4D+3abd^2\left(63Bd^2+11c^2D+82cCd\right)-b^2c\left(-420Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(13cD+25C\cdots\right.}{\cdots}\right.\right.\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \; 508$$

$$\frac{\frac{1}{7}d\left(3\left(\frac{1}{3}\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(147a^2d^4D+3abd^2\left(63Bd^2+11c^2D+82cCd\right)-b^2c\left(-420Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\right.\right.\right.}{}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{1}{7}d\left(3\left(\frac{1}{3}\left(-\frac{\left(bc^2-ad^2\right)\left(3ad^2(13cD+25Cd)-b\left(-105Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{}\right.\right.\right.}{}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$$\Big\downarrow \text{512}$$

$$\frac{1}{7}d\left(3\left(\frac{1}{3}\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(13cD+25Cd)-b\left(-105Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\sqrt{1-\cdots\right.\right.}{\cdots}\right.\right.\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$\downarrow$ 511

$$\frac{1}{7}d\left(3\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(13cD+25Cd)-b\left(-105Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

$\downarrow$ 321

$$\frac{\frac{1}{7}d\left(\frac{3\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3ad^2(13cD+25Cd)-b\left(-105Ad^3-63Bcd^2-8c^3D+18c^2Cd\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}} - \cdots\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^2}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(-2*D*(c + d*x)^(7/2)*Sqrt[a - b*x^2])/(9*b*d^2) + ((-2*d*(9*C*d - 11*c*D)
*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/7 + (d*((-2*(49*a*d^2*D - b*(18*c*C*d -
63*B*d^2 - 8*c^2*D))*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b) + (3*((-2*(3*a
*d^2*(25*C*d + 13*c*D) - b*(18*c^2*C*d - 63*B*c*d^2 - 105*A*d^3 - 8*c^3*D)
)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + ((-2*Sqrt[a]*(147*a^2*d^4*D + 3*a*b*d
^2*(82*c*C*d + 63*B*d^2 + 11*c^2*D) - b^2*c*(18*c^2*C*d - 63*B*c*d^2 - 420
*A*d^3 - 8*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt
[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqr
t[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
 + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*d^2*(25*C*d + 13*c*D) - b*(18*c^2*C*d -
63*B*c*d^2 - 105*A*d^3 - 8*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*S
qrt[a - b*x^2]))/3))/(5*b)))/7)/(9*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(516) = 1032$.

Time = 5.14 (sec) , antiderivative size = 1150, normalized size of antiderivative = 1.94

method	result	size
elliptic	Expression too large to display	1150
default	Expression too large to display	4971

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/9*D/b*d*x^
3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(C*d^2+10/9*D*c*d)/b/d*x^2*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(B*d^2+2*C*c*d+D*c^2+7/9*D/b*d^2*a-6/7*(C
*d^2+10/9*D*c*d)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(A*d^2+
2*B*c*d+C*c^2+2/3*D/b*d*a*c+5/7*(C*d^2+10/9*D*c*d)/b*a-4/5*(B*d^2+2*C*c*d+
D*c^2+7/9*D/b*d^2*a-6/7*(C*d^2+10/9*D*c*d)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)+2*(A*c^2+2/5*(B*d^2+2*C*c*d+D*c^2+7/9*D/b*d^2*a-6/7*(C*d
^2+10/9*D*c*d)/d*c)/b/d*a*c+1/3*(A*d^2+2*B*c*d+C*c^2+2/3*D/b*d*a*c+5/7*(C*
d^2+10/9*D*c*d)/b*a-4/5*(B*d^2+2*C*c*d+D*c^2+7/9*D/b*d^2*a-6/7*(C*d^2+10/9
*D*c*d)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^
(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+2*(2*A*c*d+B*c^2+4/7*(C*d^2+10/9*D*c*d)/b/d*a*c
+3/5*(B*d^2+2*C*c*d+D*c^2+7/9*D/b*d^2*a-6/7*(C*d^2+10/9*D*c*d)/d*c)/b*a-2/
3*(A*d^2+2*B*c*d+C*c^2+2/3*D/b*d*a*c+5/7*(C*d^2+10/9*D*c*d)/b*a-4/5*(B*d^2
+2*C*c*d+D*c^2+7/9*D/b*d^2*a-6/7*(C*d^2+10/9*D*c*d)/d*c)/d*c)/d*c)*(c/d-1/
b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))
^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Ellip...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx = \frac{2\left((8\,Db^2c^5-18\,Cb^2c^4d+9\,(3\,Dab+7\,Bb^2)c^3d^2-3\,(71\,Cab\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
2/945*((8*D*b^2*c^5 - 18*C*b^2*c^4*d + 9*(3*D*a*b + 7*B*b^2)*c^3*d^2 - 3*(
71*C*a*b + 175*A*b^2)*c^2*d^3 - 3*(137*D*a^2 + 189*B*a*b)*c*d^4 - 45*(5*C*
a^2 + 7*A*a*b)*d^5)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(8*D*b^2
*c^4*d - 18*C*b^2*c^3*d^2 + 3*(11*D*a*b + 21*B*b^2)*c^2*d^3 + 6*(41*C*a*b
+ 70*A*b^2)*c*d^4 + 21*(7*D*a^2 + 9*B*a*b)*d^5)*sqrt(-b*d)*weierstrassZeta
(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d)) - 3*(35*D*b^2*d^5*x^3 - 4*D*b^2*c^3*d^2 + 9*C*b^2
*c^2*d^3 + 2*(44*D*a*b + 63*B*b^2)*c*d^4 + 15*(5*C*a*b + 7*A*b^2)*d^5 + 5*
(10*D*b^2*c*d^4 + 9*C*b^2*d^5)*x^2 + (3*D*b^2*c^2*d^3 + 72*C*b^2*c*d^4 + 7
*(7*D*a*b + 9*B*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\int \frac{(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x)
```

output

```
( - 294*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3 - 1260*sqrt(c + d*x)*sqrt
(a - b*x**2)*a*b**2*c*d - 378*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*d**2 -
 1210*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*d - 196*sqrt(c + d*x)*sqrt(a
 - b*x**2)*a*b*c*d**2*x - 630*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c**2 - 2
52*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c*d*x - 300*sqrt(c + d*x)*sqrt(a -
b*x**2)*b**2*c**3*x - 380*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d*x**2
- 140*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*x**3 - 441*int((sqrt(c +
d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*
d**4 - 1260*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x
**2 - b*d*x**3),x)*a*b**3*c*d**2 - 567*int((sqrt(c + d*x)*sqrt(a - b*x**2)
*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*d**3 - 837*int((sqrt(
c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b
**2*c**2*d**2 - 189*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x
 - b*c*x**2 - b*d*x**3),x)*b**4*c**2*d + 30*int((sqrt(c + d*x)*sqrt(a - b*
x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c**4 + 147*int((sq
rt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*
d**4 + 630*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 -
b*d*x**3),x)*a**2*b**2*c*d**2 + 189*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(
a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b**2*d**3 + 801*int((sqrt(c + d
*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*c**...
```

3.149 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 477

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$$

$$= -\frac{2\left(25aD+b\left(35B-\frac{2c(7Cd-4cD)}{d^2}\right)\right)\sqrt{c+dx}\sqrt{a-bx^2}}{105b^2}$$

$$-\frac{2(7Cd-9cD)(c+dx)^{3/2}\sqrt{a-bx^2}}{35bd^2}-\frac{2D(c+dx)^{5/2}\sqrt{a-bx^2}}{7bd^2}$$

$$-\frac{2\sqrt{a}(ad^2(63Cd+19cD)-b(14c^2Cd-35Bcd^2-105Ad^3-8c^3D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\Bigg(\arcsin\Bigg(\frac{\sqrt{1-}}{\sqrt{2}}}{105b^{3/2}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{2\sqrt{a}(bc^2-ad^2)(25ad^2D-b(14cCd-35Bd^2-8c^2D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\Bigg(\arcsin\Bigg(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\Bigg)}{105b^{5/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/105*(25*D*a+b*(35*B-2*c*(7*C*d-4*D*c)/d^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1
/2)/b^2-2/35*(7*C*d-9*D*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b/d^2-2/7*D*(d*x
+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^2-2/105*a^(1/2)*(a*d^2*(63*C*d+19*D*c)-b*(-
105*A*d^3-35*B*c*d^2+14*C*c^2*d-8*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b
^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1
/2)/(-b*x^2+a)^(1/2)+2/105*a^(1/2)*(-a*d^2+b*c^2)*(25*a*d^2*D-b*(-35*B*d^2
+14*C*c*d-8*D*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(
1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/
(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.35 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(-ad^2(63Cd+19cD)-b(-14c^2Cd+35Bcd^2+105Ad^3+8c^3D)-(c+dx)\,(25ad^2D+b(-\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-(a*d^2*(63*C*d + 19*c*D)) - b*(-14*c^2*C*d + 35*B*c*d
^2 + 105*A*d^3 + 8*c^3*D) - (c + d*x)*(25*a*d^2*D + b*(-4*c^2*D + c*d*(7*C
 + 3*D*x) + d^2*(35*B + 3*x*(7*C + 5*D*x)))) + (I*Sqrt[b]*(Sqrt[b]*c - Sqr
t[a]*d)*(a*d^2*(63*C*d + 19*c*D) + b*(-14*c^2*C*d + 35*B*c*d^2 + 105*A*d^3
 + 8*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)
/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c -
Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(Sqrt[
b]*c - Sqrt[a]*d)*(105*A*b^(3/2)*d^2 - 25*a^(3/2)*d^2*D + 3*a*Sqrt[b]*d*(2
1*C*d - 2*c*D) + Sqrt[a]*b*(14*c*C*d - 35*B*d^2 - 8*c^2*D))*Sqrt[(d*(Sqrt[
a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]
*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(105*b^2*d^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2185, 27, 2185, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 2185$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(b(7Cd-9cD)x^2d^2+(7Abd+5acD)d^2+\left(-2bDc^2+7bBd^2+5ad^2D\right)xd\right)}{2\sqrt{a-bx^2}}dx}{7bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(b(7Cd-9cD)x^2d^2+(7Abd+5acD)d^2+\left(-2bDc^2+7bBd^2+5ad^2D\right)xd\right)}{\sqrt{a-bx^2}}dx}{7bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{-\frac{2\int -\frac{bd^3\sqrt{c+dx}\left(d(35Abd+21aCd-2acD)+\left(25ad^2D-b\left(-8Dc^2+14Cdc-35Bd^2\right)\right)x\right)}{2\sqrt{a-bx^2}}dx}{5bd^2}-\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(7Cd-9cD)}{7bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}d\int \frac{\sqrt{c+dx}(d(35Abd+21aCd-2acD)+(25ad^2D-b(-8Dc^2+14Cdc-35Bd^2))x)}{\sqrt{a-bx^2}}dx-\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(7Cd-9cD)}{7bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{5}d\left(-\frac{2\int -\frac{d\left(105Acdb^2+a\left(25aDd^2+b\left(2Dc^2+49Cdc+35Bd^2\right)\right)\right)+b\left(ad^2(63Cd+19cD)-b\left(-8Dc^3+14Cdc^2-35Bd^2c-105Ad^3\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+d}}{}\right.}{7bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}d\left(\frac{\int \frac{d\left(105Acdb^2+a\left(25aDd^2+b\left(2Dc^2+49Cdc+35Bd^2\right)\right)\right)+b\left(ad^2(63Cd+19cD)-b\left(-8Dc^3+14Cdc^2-35Bd^2c-105Ad^3\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(25a}{}\right.}{7bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{1}{5}d\left(\frac{\frac{b\left(ad^2(19cD+63Cd)-b\left(-105Ad^3-35Bcd^2-8c^3D+14c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{3b}-\right.}{7bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{1}{5}d\left(\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(19cD+63Cd)-b\left(-105Ad^3-35Bcd^2-8c^3D+14c^2Cd\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-}}}{d}}{3b}\right.}{7bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \text{508}$$

$$\frac{1}{5}d\left(\frac{-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(19cD+63Cd)-b\left(-105Ad^3-35Bcd^2-8c^3D+14c^2Cd\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(25ad^2D-b\left(-35Bd\right.\right.}{}}{3b}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{5}d\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(19cD+63Cd\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \text{512}$$

$$\frac{1}{5}d\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(19\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+\cdots}{\sqrt{a}d+\cdots}}}}{3b}\cdots\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 511$$

$$\frac{1}{5}d\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\cdots}{\cdots}}{3b}\cdots\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{5}d\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(25ad^2D-b\left(-35Bd^2-8c^2D+14cCd\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{arcs}\cdots\right.}{\cdots}}{3b}\cdots\right.$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^2}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[a - b*x^2],x]
```

output
```
(-2*D*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b*d^2) + ((-2*d*(7*C*d - 9*c*D)*
(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + (d*((-2*(25*a*d^2*D - b*(14*c*C*d - 3
5*B*d^2 - 8*c^2*D))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + ((-2*Sqrt[a]*Sq
rt[b]*(a*d^2*(63*C*d + 19*c*D) - b*(14*c^2*C*d - 35*B*c*d^2 - 105*A*d^3 -
8*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt
[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c
^2 - a*d^2)*(25*a*d^2*D - b*(14*c*C*d - 35*B*d^2 - 8*c^2*D))*Sqrt[(Sqrt[b]
*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[
Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/
(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b)))/5)/(7*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2Dx^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7b}-\frac{2\left(Cd+\frac{Dc}{7}\right)x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5bd}-\frac{2\left(Bd+Cc+\frac{5Dad}{7b}-\frac{4\left(Cd+\frac{Dc}{7}\right)c}{5d}\right)\sqrt{-bd}}{3bd}\right.}{}$
default	Expression too large to display

input

```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output

```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/7*D/b*x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(C*d+1/7*D*c)/b/d*x*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)-2/3*(B*d+C*c+5/7*D/b*a*d-4/5*(C*d+1/7*D*c)/d*c)/b/d*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*c+2/5*(C*d+1/7*D*c)/b/d*a*c+1/3*(B*d
+C*c+5/7*D/b*a*d-4/5*(C*d+1/7*D*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d
+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(A*d+B*c+4/7*D/b*a*c+3/
5*(C*d+1/7*D*c)/b*a-2/3*(B*d+C*c+5/7*D/b*a*d-4/5*(C*d+1/7*D*c)/d*c)/d*c)*(
c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(
1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*El
lipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$$
$$= \frac{2\left((8\,Db^2c^4 - 14\,Cb^2c^3d + (13\,Dab + 35\,Bb^2)c^2d^2 - 42\,(2\,Cab + 5\,Ab^2)cd^3 - 15\,(5\,Da^2 + 7\,Bab)d^4)\sqrt{-\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
2/315*((8*D*b^2*c^4 - 14*C*b^2*c^3*d + (13*D*a*b + 35*B*b^2)*c^2*d^2 - 42*
(2*C*a*b + 5*A*b^2)*c*d^3 - 15*(5*D*a^2 + 7*B*a*b)*d^4)*sqrt(-b*d)*weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d) + 3*(8*D*b^2*c^3*d - 14*C*b^2*c^2*d^2 + (19*D*a*b
+ 35*B*b^2)*c*d^3 + 21*(3*C*a*b + 5*A*b^2)*d^4)*sqrt(-b*d)*weierstrassZeta
(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d)) - 3*(15*D*b^2*d^4*x^2 - 4*D*b^2*c^2*d^2 + 7*C*b^2
*c*d^3 + 5*(5*D*a*b + 7*B*b^2)*d^4 + 3*(D*b^2*c*d^3 + 7*C*b^2*d^4)*x)*sqrt
(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a-bx^2}}\,dx$$
$$= \frac{-70\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,abd-88\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,acd-70\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b^2c-32\sqrt{dx+c}\,\sqrt{}}{}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 70*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d - 88*sqrt(c + d*x)*sqrt(a - b*
x**2)*a*c*d - 70*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c - 32*sqrt(c + d*x)*
sqrt(a - b*x**2)*b*c**2*x - 20*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d*x**2 -
 105*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b
*d*x**3),x)*a*b**2*d**2 - 82*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*
c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c*d**2 - 35*int((sqrt(c + d*x)*sqr
t(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c*d + 6*in
t((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3
),x)*b**2*c**3 + 35*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*
c*x**2 - b*d*x**3),x)*a**2*b*d**2 + 44*int((sqrt(c + d*x)*sqrt(a - b*x**2)
)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*c*d**2 + 70*int((sqrt(c + d*
x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**2 +
35*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3
),x)*a*b**2*c*d + 32*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b
*c*x**2 - b*d*x**3),x)*a*b*c**3)/(70*b**2*c)
```

3.150 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 410

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= -\frac{2(5Cd-7cD)\sqrt{c+dx}\sqrt{a-bx^2}}{15bd^2} - \frac{2D(c+dx)^{3/2}\sqrt{a-bx^2}}{5bd^2}$$
$$- \frac{2\sqrt{a}(9ad^2D-b(10cCd-15Bd^2-8c^2D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15b^{3/2}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$- \frac{2\sqrt{a}(ad^2(5Cd-7cD)+b(10c^2Cd-15Bcd^2+15Ad^3-8c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{15b^{3/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/15*(5*C*d-7*D*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^2-2/5*D*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/b/d^2-2/15*a^(1/2)*(9*a*d^2*D-b*(-15*B*d^2+10*C*c*d-8*
D*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2
)/d^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/15*a^(1/2)*
(a*d^2*(5*C*d-7*D*c)+b*(15*A*d^3-15*B*c*d^2+10*C*c^2*d-8*D*c^3))*((d*x+c)/
(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))
/b^(3/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.80 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(-9ad^2D-b(-10cCd+15Bd^2+8c^2D)+b(c+dx)(-5Cd+4cD-3dDx)+\frac{i\sqrt{b}\left(\sqrt{b}c-\sqrt{a}d\right)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-9*a*d^2*D - b*(-10*c*C*d + 15*B*d^2 + 8*c^2*D) + b*(c
 + d*x)*(-5*C*d + 4*c*D - 3*d*D*x) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(9
*a*d^2*D + b*(-10*c*C*d + 15*B*d^2 + 8*c^2*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] +
x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3
/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sq
rt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[b]*(15*A*b^(3/2)*d^2 - 9*a^(3/2)*d^2*D +
a*Sqrt[b]*d*(5*C*d + 2*c*D) + Sqrt[a]*b*(10*c*C*d - 15*B*d^2 - 8*c^2*D))*S
qrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x
)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(15*b^2*d^2*Sqrt[c + d*x]
)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{b(5Cd-7cD)x^2d^2+(5Abd+3acD)d^2+\left(-2bDc^2+5bBd^2+3ad^2D\right)xd}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(5Cd-7cD)x^2d^2+(5Abd+3acD)d^2+\left(-2bDc^2+5bBd^2+3ad^2D\right)xd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{2185}$$

$$\frac{-\frac{2\int -\frac{bd^3\left(d(15Abd+5aCd+2acD)+\left(9ad^2D-b\left(-8Dc^2+10Cdc-15Bd^2\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(5Cd-7cD)}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{3}d\int \frac{d(15Abd+5aCd+2acD)+\left(9ad^2D-b\left(-8Dc^2+10Cdc-15Bd^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(5Cd-7cD)}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow 600$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5Cd-7cD)+b\left(15Ad^3-15Bcd^2-8c^3D+10c^2Cd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\left(9ad^2D-b\left(-15Bd^2-8c^2D+10cCd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}\ldots}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow 509$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5Cd-7cD)+b\left(15Ad^3-15Bcd^2-8c^3D+10c^2Cd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}\left(9ad^2D-b\left(-15Bd^2-8c^2D+10cCd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}\ldots}{d\sqrt{a-bx^2}}\ldots\right.}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow 508$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5Cd-7cD)+b\left(15Ad^3-15Bcd^2-8c^3D+10c^2Cd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2D-b\left(-15Bd^2-8c^2D+10cCd\right)\right)\ldots}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\ldots\right.}{5bd^3}-\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5Cd-7cD)+b(15Ad^3-15Bcd^2-8c^3D+10c^2Cd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)(9a}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right.}{5bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{512}$$

$$\frac{\frac{1}{3}d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(ad^2(5Cd-7cD)+b(15Ad^3-15Bcd^2-8c^3D+10c^2Cd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{\sqrt{b}d\sqrt{a-bx^2}}\right.}{5bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{1}{3}d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\left(ad^2(5Cd-7cD)+b(15Ad^3-15Bcd^2-8c^3D+10c^2Cd)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}}{}\right.}{5bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{1}{3}d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(ad^2(5Cd-7cD)+b(15Ad^3-15Bcd^2-8c^3D+10c^2Cd)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-}}{}\right.}{5bd^3}$$

$$\frac{2D\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*D*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b*d^2) + ((-2*d*(5*C*d - 7*c*D)*
Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + (d*((-2*Sqrt[a]*(9*a*d^2*D - b*(10*c*C*
d - 15*B*d^2 - 8*c^2*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*
x^2]) - (2*Sqrt[a]*(a*d^2*(5*C*d - 7*c*D) + b*(10*c^2*C*d - 15*B*c*d^2 + 1
5*A*d^3 - 8*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])
))/3)/(5*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2Dx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5bd}-\frac{2\left(C-\frac{4Dc}{5d}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(A+\frac{2Dac}{5bd}+\frac{\left(C-\frac{4Dc}{5d}\right)a}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/5*D/b/d*x*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(C-4/5*D/d*c)/b/d*(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)+2*(A+2/5*D/b/d*a*c+1/3*(C-4/5*D/d*c)/b*a)*(c/d-1/b*(a*b)
^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B+3/5*a
*D/b-2/3*(C-4/5*D/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= \frac{2\left(\left(8\,Dbc^3-10\,Cbc^2d+3\,(Da+5\,Bb)cd^2-15\,(Ca+3\,Ab)d^3\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},\,-\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
2/45*((8*D*b*c^3 - 10*C*b*c^2*d + 3*(D*a + 5*B*b)*c*d^2 - 15*(C*a + 3*A*b)
*d^3)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*
(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(8*D*b*c^2*d - 10*C*b*
c*d^2 + 3*(3*D*a + 5*B*b)*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a
*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*
(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x +
 c)/d)) - 3*(3*D*b*d^3*x - 4*D*b*c*d^2 + 5*C*b*d^3)*sqrt(-b*x^2 + a)*sqrt(
d*x + c))/(b^2*d^4)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{c + dx}\sqrt{a - bx^2}}\, dx$$
$$= \frac{-6\sqrt{dx + c}\,\sqrt{-b\,x^2 + a}\,ad - 10\sqrt{dx + c}\,\sqrt{-b\,x^2 + a}\,b^2 - 4\sqrt{dx + c}\,\sqrt{-b\,x^2 + a}\,bcx - 9\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2 -}}{-bd\,x^3 - bc\,x^2 + a}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d - 10*sqrt(c + d*x)*sqrt(a - b*x**
2)*b**2 - 4*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*x - 9*int((sqrt(c + d*x)*sq
rt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d**2 - 15*
int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x*
*3),x)*b**3*d + 2*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x -
 b*c*x**2 - b*d*x**3),x)*b**2*c**2 + 3*int((sqrt(c + d*x)*sqrt(a - b*x**2)
)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*d**2 + 10*int((sqrt(c + d*x)
*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c + 5*int
((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a
*b**2*d + 4*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 -
 b*d*x**3),x)*a*b*c**2)/(10*b**2*c)
```

3.151 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 449

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}$$
$$-\frac{2D\sqrt{c+dx}\sqrt{a-bx^2}}{3bd^2}$$
$$+\frac{2\sqrt{a}(ad^2(3Cd-5cD)-b(6c^2Cd-3Bcd^2+3Ad^3-8c^3D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{}}{\sqrt{b}c+}\right.}{3\sqrt{b}d^3\left(bc^2-ad^2\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{2\sqrt{a}(ad^2D-b(6cCd-3Bd^2-8c^2D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{3/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x+c
)^(1/2)-2/3*D*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^2+2/3*a^(1/2)*(a*d^2*(3*C
*d-5*D*c)-b*(3*A*d^3-3*B*c*d^2+6*C*c^2*d-8*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+
a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3/(-a*d^2+b*c^2)/((d*x+c)/(
c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/3*a^(1/2)*(a*d^2*D-b*(-3*B*
d^2+6*C*c*d-8*D*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)
^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.73 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(ad^2(3Cd-5cD)+b(-6c^2Cd+3Bcd^2-3Ad^3+8c^3D)\right)}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a*d^2*(3*C*d - 5*c*D) + b*(-6*c^2*
C*d + 3*B*c*d^2 - 3*A*d^3 + 8*c^3*D))*(a - b*x^2) + d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(a - b*x^2)*(a*d^2*D*(c + d*x) - b*(3*B*c*d^2 - 3*A*d^3 + 4*c
^3*D + c^2*(-3*C*d + d*D*x))) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^2*(
3*C*d - 5*c*D) + b*(-6*c^2*C*d + 3*B*c*d^2 - 3*A*d^3 + 8*c^3*D))*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*d*(Sqr
t[b]*c - Sqrt[a]*d)*(3*A*b^(3/2)*d^2 + a^(3/2)*d^2*D - 3*a*Sqrt[b]*d*(C*d
- 2*c*D) + Sqrt[a]*b*(-6*c*C*d + 3*B*d^2 + 8*c^2*D))*Sqrt[(d*(Sqrt[a]/Sqrt
[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d
*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]
], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(3*b*d^4*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {2182, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \ 2182$$

$$\frac{2\int \frac{\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+\frac{-aDc^2+Abdc+aCdc-aBd^2}{d}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(Cd-cD)+b\left(\frac{2Dc^3}{d^2}-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{2}{3}D\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{2\int -\frac{d\left(3Ab^2cd-a\left(ad^2D-b\left(-2Dc^2+3Cdc-3Bd^2\right)\right)\right)-b\left(ad^2(3Cd-5cD)-b\left(-8Dc^3+6Cdc^2-3Bd^2c+3Ad^3\right)\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}}{bc^2-ad^2}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{d\left(3Ab^2cd-a\left(ad^2D-b\left(-2Dc^2+3Cdc-3Bd^2\right)\right)\right)-b\left(ad^2(3Cd-5cD)-b\left(-8Dc^3+6Cdc^2-3Bd^2c+3Ad^3\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}+\frac{2}{3}D\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)}{bc^2-ad^2}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{\frac{(bc^2-ad^2)\left(ad^2D-b\left(-3Bd^2-8c^2D+6cCd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bcd^2-8c^3D+6c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{3bd^2}+\frac{2}{3}D\sqrt{a-bx^2}\cdots}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{\frac{(bc^2-ad^2)\left(ad^2D-b\left(-3Bd^2-8c^2D+6cCd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bcd^2-8c^3D+6c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3bd^2}+\frac{2}{3}D\sqrt{\cdots}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bcd^2-8c^3D+6c^2Cd\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{(bc^2-ad^2)\left(ad^2D-b\left(-3Bd^2-8c^2D+6cCd\right)\right)\int\cdots}{d}}{3bd^2}\cdots}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\frac{(bc^2-ad^2)\left(ad^2D-b\left(-3Bd^2-8c^2D+6cCd\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bc\cdots\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd^2}\cdots}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{512}$$

$$\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad^2D-b\left(-3Bd^2-8c^2D+6cCd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(3Cd-5cD)-b\left(3\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bcd^2-8c^3D+6c^2Cd\right)\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2D-\right.}{}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(ad^2(3Cd-5cD)-b\left(3Ad^3-3Bcd^2-8c^3D+6c^2Cd\right)\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}}{}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a - b*x^2])/(d^2*(b*c^2 - a*d^
2)*Sqrt[c + d*x]) + ((2*(a/b - c^2/d^2)*D*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3
 + ((2*Sqrt[a]*Sqrt[b]*(a*d^2*(3*C*d - 5*c*D) - b*(6*c^2*C*d - 3*B*c*d^2 +
 3*A*d^3 - 8*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sq
rt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*S
qrt[a]*(b*c^2 - a*d^2)*(a*d^2*D - b*(6*c*C*d - 3*B*d^2 - 8*c^2*D))*Sqrt[(S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b*d^2))/(b*c^2 - a*d^2
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(385) = 770$.

Time = 5.31 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.76

method	result
elliptic	$\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(-bd\,x^2+ad\right)\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{d^3\left(a\,d^2-b\,c^2\right)\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}-\frac{2D\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3d^2b}+\frac{2\left(\frac{B\,d^2-Ccd+Dc^2}{d^3}-\frac{bc\left(A\,d^3-Bc\,d^2+C\right.}{d^3\left(a\,d^2-b\right.}\right.}{}\right.$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x^2+
a*d)/d^3/(a*d^2-b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/((x+c/d)*(-b*d*x^2+a*
d))^(1/2)-2/3*D/d^2/b*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*((B*d^2-C*c*d+D
*c^2)/d^3-b*c/d^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2-b*c^2)+1/3*D/d/b*a)
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)
^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2))+2*(1/d^2*(C*d-D*c)-b/d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2-b*c^2)-
2/3*D/d^2*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c
/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(
a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left((8\,Db^2c^5-6\,Cb^2c^4d-(11\,Dab-3\,Bb^2)c^3d^2+6\,(2\,Cab+Ab^2)c^2d^3-3\,(Da^2+3\,Bab)cd^4+(8\,Db^2c^4\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
-2/9*((8*D*b^2*c^5 - 6*C*b^2*c^4*d - (11*D*a*b - 3*B*b^2)*c^3*d^2 + 6*(2*C
*a*b + A*b^2)*c^2*d^3 - 3*(D*a^2 + 3*B*a*b)*c*d^4 + (8*D*b^2*c^4*d - 6*C*b
^2*c^3*d^2 - (11*D*a*b - 3*B*b^2)*c^2*d^3 + 6*(2*C*a*b + A*b^2)*c*d^4 - 3*
(D*a^2 + 3*B*a*b)*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*
d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(8
*D*b^2*c^4*d - 6*C*b^2*c^3*d^2 - (5*D*a*b - 3*B*b^2)*c^2*d^3 + 3*(C*a*b -
A*b^2)*c*d^4 + (8*D*b^2*c^3*d^2 - 6*C*b^2*c^2*d^3 - (5*D*a*b - 3*B*b^2)*c*
d^4 + 3*(C*a*b - A*b^2)*d^5)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3
*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
+ c)/d)) + 3*(4*D*b^2*c^3*d^2 - 3*C*b^2*c^2*d^3 - 3*A*b^2*d^5 - (D*a*b - 3
*B*b^2)*c*d^4 + (D*b^2*c^2*d^3 - D*a*b*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x +
 c))/(b^3*c^3*d^4 - a*b^2*c*d^6 + (b^3*c^2*d^5 - a*b^2*d^7)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a - b*x**2)*(c + d*x)**(3/2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(dx+c)^{\frac{3}{2}}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.152 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 568

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}$$
$$+\frac{2(3ad^2(2cCd-Bd^2-3c^2D)-bc(2c^2Cd+Bcd^2-4Ad^3-5c^3D))\sqrt{a-bx^2}}{3d^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}$$
$$-\frac{2\sqrt{a}(3a^2d^4D+3abd^2(2cCd-Bd^2-5c^2D)-b^2c(2c^2Cd+Bcd^2-4Ad^3-8c^3D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\Bigg(}{3\sqrt{b}d^3\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(3ad^2(Cd-3cD)-b(2c^2Cd+Bcd^2-Ad^3-8c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{3\sqrt{b}d^3\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x
+c)^(3/2)+2/3*(3*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(-4*A*d^3+B*c*d^2+2*C*
c^2*d-5*D*c^3))*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/3*a^
(1/2)*(3*a^2*d^4*D+3*a*b*d^2*(-B*d^2+2*C*c*d-5*D*c^2)-b^2*c*(-4*A*d^3+B*c*
d^2+2*C*c^2*d-8*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/b^(1/2)/d^3/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/
2)/(-b*x^2+a)^(1/2)+2/3*a^(1/2)*(3*a*d^2*(C*d-3*D*c)-b*(-A*d^3+B*c*d^2+2*C
*c^2*d-8*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.54 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-2bc^3Cd-bBc^2d^2+4Abcd^3+6acCd^3-3aBd^4+5bc^4D-9\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-2*b*c^3*C*d - b*B*c^2*d^2 + 4*A*b*c*d^3 + 6*a*c*C*d^3
 - 3*a*B*d^4 + 5*b*c^4*D - 9*a*c^2*d^2*D - (3*a^2*d^4*D)/b + 3*a*d^2*(-2*c
*C*d + B*d^2 + 5*c^2*D) + b*c*(2*c^2*C*d + B*c*d^2 - 4*A*d^3 - 8*c^3*D) +
((b*c^2 - a*d^2)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(c + d*x)) + (I*(Sqrt
[b]*c - Sqrt[a]*d)*(3*a^2*d^4*D - 3*a*b*d^2*(-2*c*C*d + B*d^2 + 5*c^2*D) +
 b^2*c*(-2*c^2*C*d - B*c*d^2 + 4*A*d^3 + 8*c^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b
] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x
)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]],
 (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(Sqrt[b]*d^2*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(Sqrt[b]*c - Sqrt[a]*d)*(3*A*b^2*c
*d^2 + 3*a^2*d^3*D - 3*a^(3/2)*Sqrt[b]*d^2*(C*d - 3*c*D) - 3*a*b*d*(-(c*C*
d) + B*d^2 + 2*c^2*D) + Sqrt[a]*b^(3/2)*(2*c^2*C*d + B*c*d^2 - A*d^3 - 8*c
^3*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[
b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a
]*d)])/(Sqrt[b]*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*(b*c^2
*d - a*d^3)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2182, 27, 2182, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}(c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \ 2182$$

$$\frac{2\int \frac{3\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(3Cd-3cD)-b\left(-\frac{2Dc^3}{d^2}+\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+\frac{3\left(Abcd+a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(3Cd-3cD)-b\left(-\frac{2Dc^3}{d^2}+\frac{2Cc^2}{d}+Bc-Ad\right)\right)x+3\left(Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2182

$$\frac{\frac{2\int \frac{d\left(Abd\left(3bc^2+ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(2Dc^2+Cdc-4Bd^2\right)\right)\right)+\left(3a^2Dd^4+3ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-8Dc^3+2Cdc^2+Bd^2c-4Ad^3\right)\right)x}{2d^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\cdots}{\cdots}}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{d\left(Abd\left(3bc^2+ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(2Dc^2+Cdc-4Bd^2\right)\right)\right)+\left(3a^2Dd^4+3ab\left(-5Dc^2+2Cdc-Bd^2\right)d^2-b^2c\left(-8Dc^3+2Cdc^2+Bd^2c-4Ad^3\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2\left(bc^2-ad^2\right)}+\frac{2\sqrt{\cdots}}{\cdots}}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{\frac{\frac{\left(3a^2d^4D+3abd^2\left(-Bd^2-5c^2D+2cCd\right)-b^2c\left(-4Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int \cdots}{d}}{d^2\left(bc^2-ad^2\right)}\cdots}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4D+3abd^2\left(-Bd^2-5c^2D+2cCd\right)-b^2c\left(-4Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2\cdots\right.\right.}{d}}{d^2\left(bc^2-ad^2\right)}\cdots}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4D+3abd^2\left(-Bd^2-5c^2D+2cCd\right)-b^2c\left(-4Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(Cd-3\right.}{}}{d^2(bc^2-ad^2)}$$

$$3\left(bc^2-ad\right.$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow 327$$

$$-\frac{\frac{\left(bc^2-ad^2\right)\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3a^2d^4D+3abd\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{}}}{d^2(bc^2-ad^2)}$$

$$3\left(bc^2-a\right.$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow 512$$

$$-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3a^2\right.}{\sqrt{b}d\sqrt{}}}{d^2(bc^2-ad^2)}$$

$$3\left(bc^2\right.$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow 511$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-}}{}}{d^2(bc^2-ad^2)}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3ad^2(Cd-3cD)-b\left(-Ad^3+Bcd^2-8c^3D+2c^2Cd\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{a}\right.}{}}{d^2\left(bc^2-ad^2\right)}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a - b*x^2])/(3*d^2*(b*c^2 - a*
d^2)*(c + d*x)^(3/2)) + ((2*(3*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(2*
c^2*C*d + B*c*d^2 - 4*A*d^3 - 5*c^3*D))*Sqrt[a - b*x^2])/(d^2*(b*c^2 - a*d
^2)*Sqrt[c + d*x]) + ((-2*Sqrt[a]*(3*a^2*d^4*D + 3*a*b*d^2*(2*c*C*d - B*d^
2 - 5*c^2*D) - b^2*c*(2*c^2*C*d + B*c*d^2 - 4*A*d^3 - 8*c^3*D))*Sqrt[c + d
*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqr
t[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x
))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*
(3*a*d^2*(C*d - 3*c*D) - b*(2*c^2*C*d + B*c*d^2 - A*d^3 - 8*c^3*D))*Sqrt[(
Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
+ d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(d^2*(b*c^2 - a*d^2)))/(
3*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.19 (sec) , antiderivative size = 947, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3d^4\left(a\,d^2-b\,c^2\right)\left(x+\frac{c}{d}\right)^2}+\frac{2\left(-bd\,x^2+ad\right)\left(4A\,d^3bc-3Ba\,d^4-Bb\,c^2d^2+6acC\,d^3-2bdC\right.}{3d^3\left(a\,d^2-b\,c^2\right)^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+ac\right.}}\right.}{}$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/3/d^4/(a*d
^2-b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)
/(x+c/d)^2+2/3*(-b*d*x^2+a*d)/d^3/(a*d^2-b*c^2)^2*(4*A*b*c*d^3-3*B*a*d^4-B
*b*c^2*d^2+6*C*a*c*d^3-2*C*b*c^3*d-9*D*a*c^2*d^2+5*D*b*c^4)/((x+c/d)*(-b*d
*x^2+a*d))^(1/2)+2*((C*d-2*D*c)/d^3+1/3*b/d^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3
)/(a*d^2-b*c^2)+1/3*b*c/d^3*(4*A*b*c*d^3-3*B*a*d^4-B*b*c^2*d^2+6*C*a*c*d^3
-2*C*b*c^3*d-9*D*a*c^2*d^2+5*D*b*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2
))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(D/d^2+1/3*b/
d^2*(4*A*b*c*d^3-3*B*a*d^4-B*b*c^2*d^2+6*C*a*c*d^3-2*C*b*c^3*d-9*D*a*c^2*d
^2+5*D*b*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
2/9*((8*D*b^2*c^7 - 2*C*b^2*c^6*d - (21*D*a*b + B*b^2)*c^5*d^2 + (3*C*a*b
- 5*A*b^2)*c^4*d^3 + 3*(7*D*a^2 + 3*B*a*b)*c^3*d^4 - 3*(3*C*a^2 + A*a*b)*c
^2*d^5 + (8*D*b^2*c^5*d^2 - 2*C*b^2*c^4*d^3 - (21*D*a*b + B*b^2)*c^3*d^4 +
 (3*C*a*b - 5*A*b^2)*c^2*d^5 + 3*(7*D*a^2 + 3*B*a*b)*c*d^6 - 3*(3*C*a^2 +
A*a*b)*d^7)*x^2 + 2*(8*D*b^2*c^6*d - 2*C*b^2*c^5*d^2 - (21*D*a*b + B*b^2)*
c^4*d^3 + (3*C*a*b - 5*A*b^2)*c^3*d^4 + 3*(7*D*a^2 + 3*B*a*b)*c^2*d^5 - 3*
(3*C*a^2 + A*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*
(8*D*b^2*c^6*d - 2*C*b^2*c^5*d^2 - (15*D*a*b + B*b^2)*c^4*d^3 + 2*(3*C*a*b
 + 2*A*b^2)*c^3*d^4 + 3*(D*a^2 - B*a*b)*c^2*d^5 + (8*D*b^2*c^4*d^3 - 2*C*b
^2*c^3*d^4 - (15*D*a*b + B*b^2)*c^2*d^5 + 2*(3*C*a*b + 2*A*b^2)*c*d^6 + 3*
(D*a^2 - B*a*b)*d^7)*x^2 + 2*(8*D*b^2*c^5*d^2 - 2*C*b^2*c^4*d^3 - (15*D*a*
b + B*b^2)*c^3*d^4 + 2*(3*C*a*b + 2*A*b^2)*c^2*d^5 + 3*(D*a^2 - B*a*b)*c*d
^6)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2
), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(4*D*b^2*c^5
*d^2 - C*b^2*c^4*d^3 - 2*B*a*b*c*d^6 - A*a*b*d^7 - 2*(4*D*a*b + B*b^2)*c^3
*d^4 + 5*(C*a*b + A*b^2)*c^2*d^5 + (5*D*b^2*c^4*d^3 - 2*C*b^2*c^3*d^4 - 3*
B*a*b*d^7 - (9*D*a*b + B*b^2)*c^2*d^5 + 2*(3*C*a*b + 2*A*b^2)*c*d^6)*x)*sq
rt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*c^6*d^4 - 2*a*b^2*c^4*d^6 + a^2*b*c^...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a - b*x**2)*(c + d*x)**(5/2)),
x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{5/2}\sqrt{a - bx^2}}\, dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{\sqrt{-bx^2 + a}(dx + c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{5/2}\sqrt{a - bx^2}}\, dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{\sqrt{-bx^2 + a}(dx + c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{5/2}\sqrt{a - bx^2}}\, dx = \int \frac{A + B\,x + C\,x^2 + x^3\,D}{\sqrt{a - b\,x^2}\,{(c + d\,x)}^{5/2}}\, dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(dx+c)^{\frac{5}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.153 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 737

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a-bx^2}}{5d^2(bc^2-ad^2)(c+dx)^{5/2}}$$
$$+\frac{2(5ad^2(2cCd-Bd^2-3c^2D)-bc(2c^2Cd+3Bcd^2-8Ad^3-7c^3D))\sqrt{a-bx^2}}{15d^2(bc^2-ad^2)^2(c+dx)^{3/2}}$$
$$+\frac{2(15a^2d^4(Cd-3cD)-b^2c^2(2c^2Cd+3Bcd^2-23Ad^3+8c^3D)+abd^2(19c^2Cd-29Bcd^2+9Ad^3+21c^3 \ldots}{15d^2(bc^2-ad^2)^3\sqrt{c+dx}}$$
$$-\frac{2\sqrt{a}\sqrt{b}(15a^2d^4(Cd-3cD)-b^2c^2(2c^2Cd+3Bcd^2-23Ad^3+8c^3D)+abd^2(19c^2Cd-29Bcd^2+9Ad^3 \ldots}{15d^3(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{2\sqrt{a}(15a^2d^4D-5abd^2(2cCd-Bd^2+3c^2D)+b^2c(2c^2Cd+3Bcd^2-8Ad^3+8c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{E} \ldots}{15\sqrt{b}d^3(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x
+c)^(5/2)+2/15*(5*a*d^2*(-B*d^2+2*C*c*d-3*D*c^2)-b*c*(-8*A*d^3+3*B*c*d^2+2
*C*c^2*d-7*D*c^3))*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)+2/1
5*(15*a^2*d^4*(C*d-3*D*c)-b^2*c^2*(-23*A*d^3+3*B*c*d^2+2*C*c^2*d+8*D*c^3)+
a*b*d^2*(9*A*d^3-29*B*c*d^2+19*C*c^2*d+21*D*c^3))*(-b*x^2+a)^(1/2)/d^2/(-a
*d^2+b*c^2)^3/(d*x+c)^(1/2)-2/15*a^(1/2)*b^(1/2)*(15*a^2*d^4*(C*d-3*D*c)-b
^2*c^2*(-23*A*d^3+3*B*c*d^2+2*C*c^2*d+8*D*c^3)+a*b*d^2*(9*A*d^3-29*B*c*d^2
+19*C*c^2*d+21*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1
-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/d^3/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^
2+a)^(1/2)-2/15*a^(1/2)*(15*a^2*d^4*D-5*a*b*d^2*(-B*d^2+2*C*c*d+3*D*c^2)+b
^2*c*(-8*A*d^3+3*B*c*d^2+2*C*c^2*d+8*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)
))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^
(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3/(-a*d^2
+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.86 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(7/2)*Sqrt[a - b*x^2]),x]
```

output

```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(5
*d^2*(-(b*c^2) + a*d^2)*(c + d*x)^3) - (2*(2*b*c^3*C*d + 3*b*B*c^2*d^2 - 8
*A*b*c*d^3 - 10*a*c*C*d^3 + 5*a*B*d^4 - 7*b*c^4*D + 15*a*c^2*d^2*D))/(15*d
^2*(-(b*c^2) + a*d^2)^2*(c + d*x)^2) - (2*(-2*b^2*c^4*C*d - 3*b^2*B*c^3*d^
2 + 23*A*b^2*c^2*d^3 + 19*a*b*c^2*C*d^3 - 29*a*b*B*c*d^4 + 9*a*A*b*d^5 + 1
5*a^2*C*d^5 - 8*b^2*c^5*D + 21*a*b*c^3*d^2*D - 45*a^2*c*d^4*D))/(15*d^2*(-
(b*c^2) + a*d^2)^3*(c + d*x))) - (2*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d
*x))^2)/d^2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-15*a^2*d^4*(C*d - 3*c*D) +
b^2*c^2*(2*c^2*C*d + 3*B*c*d^2 - 23*A*d^3 + 8*c^3*D) - a*b*d^2*(19*c^2*C*d
 - 29*B*c*d^2 + 9*A*d^3 + 21*c^3*D))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(
c + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(-15*a^2*d^4*(C*d - 3*c*
D) + b^2*c^2*(2*c^2*C*d + 3*B*c*d^2 - 23*A*d^3 + 8*c^3*D) - a*b*d^2*(19*c^
2*C*d - 29*B*c*d^2 + 9*A*d^3 + 21*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*
d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d
*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] - (I*d*(Sqr
t[b]*c - Sqrt[a]*d)*(15*A*b^(5/2)*c^2*d^2 + 15*a^(5/2)*d^4*D + 15*a^2*Sqrt
[b]*d^3*(C*d - 2*c*D) + 5*a^(3/2)*b*d^2*(-2*c*C*d + B*d^2 - 3*c^2*D) + 3*a
*b^(3/2)*d*(3*c^2*C*d - 8*B*c*d^2 + 3*A*d^3 + 2*c^3*D) + Sqrt[a]*b^2*c*(2*
c^2*C*d + 3*B*c*d^2 - 8*A*d^3 + 8*c^3*D))*Sqrt[1 - c/(c + d*x) - (Sqrt[...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2182, 27, 2182, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}(c+dx)^{7/2}}dx$$

$$\Big\downarrow \color{blue}{2182}$$

$$\frac{2\int \frac{5\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(5Cd-5cD)-b\left(-\frac{2Dc^3}{d^2}+\frac{2Cc^2}{d}+3Bc-3Ad\right)\right)x+\frac{5\left(Abcd+a\left(-Dc^2+Cdc-Bd^2\right)\right)}{d}}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{5\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5\left(\frac{bc^2}{d}-ad\right)Dx^2-\left(a(5Cd-5cD)-b\left(-\frac{2Dc^3}{d^2}+\frac{2Cc^2}{d}+3Bc-3Ad\right)\right)x+5\left(Abc+a\left(-\frac{Dc^2}{d}+Cc-Bd\right)\right)}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{5\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 2182$$

$$\frac{\frac{2\int \frac{3d\left(Abd\left(5bc^2+3ad^2\right)+a\left(5a(Cd-2cD)d^2+bc\left(2Dc^2+3Cdc-8Bd^2\right)\right)\right)+\left(15a^2Dd^4-5ab\left(3Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(8Dc^3+2Cdc^2+3Bd^2c-8Ad^3\right)\right)x}{2d^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+}{5\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3d\left(Abd\left(5bc^2+3ad^2\right)+a\left(5a(Cd-2cD)d^2+bc\left(2Dc^2+3Cdc-8Bd^2\right)\right)\right)+\left(15a^2Dd^4-5ab\left(3Dc^2+2Cdc-Bd^2\right)d^2+b^2c\left(8Dc^3+2Cdc^2+3Bd^2c-8Ad^3\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3d^2(bc^2-ad^2)}+}{5\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{\frac{2\int \frac{d\left(Ab^2cd\left(15bc^2+17ad^2\right)-a\left(15a^2Dd^4-5ab\left(-3Dc^2+5Cdc-Bd^2\right)d^2-b^2c^2\left(-2Dc^2+7Cdc-27Bd^2\right)\right)\right)+b\left(15a^2(Cd-3cD)d^4+ab\left(21Dc^3+19Cdc^2-29Bd^2c+\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{3d^2}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{d\left(Ab^2cd\left(15bc^2+17ad^2\right)-a\left(15a^2Dd^4-5ab\left(-3Dc^2+5Cdc-Bd^2\right)d^2-b^2c^2\left(-2Dc^2+7Cdc-27Bd^2\right)\right)\right)+b\left(15a^2(Cd-3cD)d^4+ab\left(21Dc^3+19Cdc^2-29Bd^2c+9\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{3d^2}+\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\left(15a^2d^4(Cd-3cD)+abd^2\left(9Ad^3-29Bcd^2+\cdots\right.\right.}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\sqrt{1-\frac{bx^2}{a}}\left(15a^2d^4(Cd-3cD)+abd^2\left(9Ad^3-\cdots\right.\right.}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(15a^2d^4(Cd-3cD)+\cdots\right.}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$\frac{\left(bc^2-ad^2\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\cdots\right.\right.}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 512

$$\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\right.\right.}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{511}$$

$$\frac{2\sqrt{a-bx^2}\left(-Dc^3+Cdc^2-Bd^2c+Ad^3\right)}{5d^2\left(bc^2-ad^2\right)(c+dx)^{5/2}}+$$

$$\frac{2\sqrt{a-bx^2}\left(5ad^2\left(-3Dc^2+2Cdc-Bd^2\right)-bc\left(-7Dc^3+2Cdc^2+3Bd^2c-8Ad^3\right)\right)}{3d^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2\sqrt{a-bx^2}\left(15a^2(Cd-3cD)d^4+ab\left(21Dc^3+19Cdc^2-29Bd^2c+9Ad^3\right)d^2\right.}{\left(bc^2-ad^2\right)\sqrt{c+dx}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(15a^2d^4D-5abd^2\left(-Bd^2+3c^2D+2cCd\right)+b^2c\left(-8Ad^3+3Bcd^2+8c^3D+2c^2Cd\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^2(c+dx)^{5/2}\left(bc^2-ad^2\right)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(7/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a - b*x^2])/(5*d^2*(b*c^2 - a*
d^2)*(c + d*x)^(5/2)) + ((2*(5*a*d^2*(2*c*C*d - B*d^2 - 3*c^2*D) - b*c*(2*
c^2*C*d + 3*B*c*d^2 - 8*A*d^3 - 7*c^3*D))*Sqrt[a - b*x^2])/(3*d^2*(b*c^2 -
 a*d^2)*(c + d*x)^(3/2)) + ((2*(15*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(2*c^2*
C*d + 3*B*c*d^2 - 23*A*d^3 + 8*c^3*D) + a*b*d^2*(19*c^2*C*d - 29*B*c*d^2 +
 9*A*d^3 + 21*c^3*D))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + (
(-2*Sqrt[a]*Sqrt[b]*(15*a^2*d^4*(C*d - 3*c*D) - b^2*c^2*(2*c^2*C*d + 3*B*c
*d^2 - 23*A*d^3 + 8*c^3*D) + a*b*d^2*(19*c^2*C*d - 29*B*c*d^2 + 9*A*d^3 +
21*c^3*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqr
t[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*
c^2 - a*d^2)*(15*a^2*d^4*D - 5*a*b*d^2*(2*c*C*d - B*d^2 + 3*c^2*D) + b^2*c
*(2*c^2*C*d + 3*B*c*d^2 - 8*A*d^3 + 8*c^3*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt
[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2))/(3*d^2*(b*c^2 - a*d^2)))/(5*(
b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.64

method	result	size
elliptic	Expression too large to display	1206
default	Expression too large to display	17283

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/5/d^5/(a*d
^2-b*c^2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)
/(x+c/d)^3+2/15*(8*A*b*c*d^3-5*B*a*d^4-3*B*b*c^2*d^2+10*C*a*c*d^3-2*C*b*c^
3*d-15*D*a*c^2*d^2+7*D*b*c^4)/d^4/(a*d^2-b*c^2)^2*(-b*d*x^3-b*c*x^2+a*d*x+
a*c)^(1/2)/(x+c/d)^2-2/15*(-b*d*x^2+a*d)/d^3/(a*d^2-b*c^2)^3*(9*A*a*b*d^5+
23*A*b^2*c^2*d^3-29*B*a*b*c*d^4-3*B*b^2*c^3*d^2+15*C*a^2*d^5+19*C*a*b*c^2*
d^3-2*C*b^2*c^4*d-45*D*a^2*c*d^4+21*D*a*b*c^3*d^2-8*D*b^2*c^5)/((x+c/d)*(-
b*d*x^2+a*d))^(1/2)+2*(D/d^3-1/15*b*(8*A*b*c*d^3-5*B*a*d^4-3*B*b*c^2*d^2+1
0*C*a*c*d^3-2*C*b*c^3*d-15*D*a*c^2*d^2+7*D*b*c^4)/d^3/(a*d^2-b*c^2)^2-1/15
*b*c/d^3*(9*A*a*b*d^5+23*A*b^2*c^2*d^3-29*B*a*b*c*d^4-3*B*b^2*c^3*d^2+15*C
*a^2*d^5+19*C*a*b*c^2*d^3-2*C*b^2*c^4*d-45*D*a^2*c*d^4+21*D*a*b*c^3*d^2-8*
D*b^2*c^5)/(a*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-2/15*b/d^2*(9*A*a*b*d^5+23*A*b^2*c^2*d^3-2
9*B*a*b*c*d^4-3*B*b^2*c^3*d^2+15*C*a^2*d^5+19*C*a*b*c^2*d^3-2*C*b^2*c^4*d-
45*D*a^2*c*d^4+21*D*a*b*c^3*d^2-8*D*b^2*c^5)/(a*d^2-b*c^2)^3*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(670) = 1340$.

Time = 0.20 (sec) , antiderivative size = 1718, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

output
```
-2/45*((8*D*b^3*c^9 + 2*C*b^3*c^8*d - 52*B*a*b^2*c^5*d^4 - 3*(9*D*a*b^2 -
B*b^3)*c^7*d^2 + 2*(C*a*b^2 + 11*A*b^3)*c^6*d^3 + 6*(10*C*a^2*b + 7*A*a*b^
2)*c^4*d^5 - 15*(3*D*a^3 + B*a^2*b)*c^3*d^6 + (8*D*b^3*c^6*d^3 + 2*C*b^3*c
^5*d^4 - 52*B*a*b^2*c^2*d^7 - 3*(9*D*a*b^2 - B*b^3)*c^4*d^5 + 2*(C*a*b^2 +
 11*A*b^3)*c^3*d^6 + 6*(10*C*a^2*b + 7*A*a*b^2)*c*d^8 - 15*(3*D*a^3 + B*a^
2*b)*d^9)*x^3 + 3*(8*D*b^3*c^7*d^2 + 2*C*b^3*c^6*d^3 - 52*B*a*b^2*c^3*d^6
- 3*(9*D*a*b^2 - B*b^3)*c^5*d^4 + 2*(C*a*b^2 + 11*A*b^3)*c^4*d^5 + 6*(10*C
*a^2*b + 7*A*a*b^2)*c^2*d^7 - 15*(3*D*a^3 + B*a^2*b)*c*d^8)*x^2 + 3*(8*D*b
^3*c^8*d + 2*C*b^3*c^7*d^2 - 52*B*a*b^2*c^4*d^5 - 3*(9*D*a*b^2 - B*b^3)*c^
6*d^3 + 2*(C*a*b^2 + 11*A*b^3)*c^5*d^4 + 6*(10*C*a^2*b + 7*A*a*b^2)*c^3*d^
6 - 15*(3*D*a^3 + B*a^2*b)*c^2*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*
(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x +
 c)/d) + 3*(8*D*b^3*c^8*d + 2*C*b^3*c^7*d^2 - 3*(7*D*a*b^2 - B*b^3)*c^6*d^
3 - (19*C*a*b^2 + 23*A*b^3)*c^5*d^4 + (45*D*a^2*b + 29*B*a*b^2)*c^4*d^5 -
3*(5*C*a^2*b + 3*A*a*b^2)*c^3*d^6 + (8*D*b^3*c^5*d^4 + 2*C*b^3*c^4*d^5 - 3
*(7*D*a*b^2 - B*b^3)*c^3*d^6 - (19*C*a*b^2 + 23*A*b^3)*c^2*d^7 + (45*D*a^2
*b + 29*B*a*b^2)*c*d^8 - 3*(5*C*a^2*b + 3*A*a*b^2)*d^9)*x^3 + 3*(8*D*b^3*c
^6*d^3 + 2*C*b^3*c^5*d^4 - 3*(7*D*a*b^2 - B*b^3)*c^4*d^5 - (19*C*a*b^2 + 2
3*A*b^3)*c^3*d^6 + (45*D*a^2*b + 29*B*a*b^2)*c^2*d^7 - 3*(5*C*a^2*b + 3*A*
a*b^2)*c*d^8)*x^2 + 3*(8*D*b^3*c^7*d^2 + 2*C*b^3*c^6*d^3 - 3*(7*D*a*b^2...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\,(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a - b*x**2)*(c + d*x)**(7/2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a-b\,x^2}\,{(c+d\,x)}^{7/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(7/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{7/2}\sqrt{a-bx^2}}\,dx = \text{too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d - 2*sqrt(c + d*x)*sqrt(a - b*x**2
)*b**2 + 4*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*x + 9*int((sqrt(c + d*x)*sqr
t(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*c*d*
*3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**2*x**4
 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c**3*d**2 + 27*int((sqrt(c + d*x)
*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2 + 4*a*
c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2*d**2*
x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c**2*d**3*x + 27*int((sqrt(c
+ d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*x**2
+ 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b*c**2
*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*c*d**4*x**2 + 9*int((sq
rt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d**2*
x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 - 6*b
*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*a*b*d**5*x**3 + 3*int(
(sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d*
*2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 -
6*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*b**3*c**3*d + 9*int
((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c**4 + 4*a*c**3*d*x + 6*a*c**2*d
**2*x**2 + 4*a*c*d**3*x**3 + a*d**4*x**4 - b*c**4*x**2 - 4*b*c**3*d*x**3 -
 6*b*c**2*d**2*x**4 - 4*b*c*d**3*x**5 - b*d**4*x**6),x)*b**3*c**2*d**2*...
```

3.154 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 567

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)(c+dx)^{5/2}}{ab\sqrt{a-bx^2}}$$

$$+\frac{\left(105Ab^2cd+a(225ad^2D+b(217cCd+175Bd^2+30c^2D))\right)\sqrt{c+dx}\sqrt{a-bx^2}}{105ab^3}$$

$$+\frac{(35Abd+49aCd+10acD)(c+dx)^{3/2}\sqrt{a-bx^2}}{35ab^2}+\frac{2D(c+dx)^{5/2}\sqrt{a-bx^2}}{7b^2}$$

$$+\frac{\left(105Abd(bc^2+3ad^2)+a(9ad^2(49Cd+110cD)+bc(427cCd+700Bd^2+30c^2D))\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\Big(a\ldots}{105\sqrt{a}b^{5/2}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\left(bc^2-ad^2\right)\left(105Ab^2cd+a(225ad^2D+b(217cCd+175Bd^2+30c^2D))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\Big(\text{arcsi}\ldots}{105\sqrt{a}b^{7/2}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(5/2)/a/b/(-b*x^2+a)^(1/2)+1/105*(105*A*
b^2*c*d+a*(225*a*d^2*D+b*(175*B*d^2+217*C*c*d+30*D*c^2)))*(d*x+c)^(1/2)*(-
b*x^2+a)^(1/2)/a/b^3+1/35*(35*A*b*d+49*C*a*d+10*D*a*c)*(d*x+c)^(3/2)*(-b*x
^2+a)^(1/2)/a/b^2+2/7*D*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b^2+1/105*(105*A*b*
d*(3*a*d^2+b*c^2)+a*(9*a*d^2*(49*C*d+110*D*c)+b*c*(700*B*d^2+427*C*c*d+30*
D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/
2)/b^(5/2)/d/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/105*
(-a*d^2+b*c^2)*(105*A*b^2*c*d+a*(225*a*d^2*D+b*(175*B*d^2+217*C*c*d+30*D*c
^2)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF
(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(
1/2)*d))^(1/2))/a^(1/2)/b^(7/2)/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.22 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(225a^3d^2D+105Ab^3c^2x+ab^2\left(105Ad(2c+dx)+15c^2x(7C-6D\right.\right.}{}\right.}{}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(225*a^3*d^2*D + 105*A*b^3*c^2*x + a*b^2*(105
*A*d*(2*c + d*x) + 15*c^2*x*(7*C - 6*D*x) - 6*d^2*x^3*(7*C + 5*D*x) - 2*c*
d*x^2*(77*C + 45*D*x) + 35*B*(3*c^2 + 6*c*d*x - 2*d^2*x^2)) + a^2*b*(195*c
^2*D + 4*c*d*(91*C + 75*D*x) + d^2*(175*B + 3*x*(49*C - 30*D*x)))))/(a*b^3
*(a - b*x^2)) - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(105*A*b*d*(b*c^2 + 3*
a*d^2) + a*(9*a*d^2*(49*C*d + 110*c*D) + b*c*(427*c*C*d + 700*B*d^2 + 30*c
^2*D)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(105*A*b*d*(b*c^2
+ 3*a*d^2) + a*(9*a*d^2*(49*C*d + 110*c*D) + b*c*(427*c*C*d + 700*B*d^2 +
30*c^2*D)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/
Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - S
qrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(105*A*b^2*c*d - 105*Sqrt
[a]*b^(3/2)*(2*c^2*C + 5*B*c*d + 3*A*d^2) + 225*a^2*d^2*D - 9*a^(3/2)*Sqrt
[b]*d*(49*C*d + 85*c*D) + a*b*(217*c*C*d + 175*B*d^2 + 30*c^2*D))*Sqrt[(d*
(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/
Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a*b^3*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(105*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2176, 27, 2185, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{\int -\frac{(c+dx)^{3/2}\left(2adDx^2+(5Abd+7aCd+2acD)x+\frac{a(2bcC+5bBd+5adD)}{b}\right)}{2\sqrt{a-bx^2}}dx}{ab}+\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\int\frac{(c+dx)^{3/2}\left(2adDx^2+(5Abd+7aCd+2acD)x+\frac{a(2bcC+5bBd+5adD)}{b}\right)}{\sqrt{a-bx^2}}dx}{2ab}$$

$\Big\downarrow$ 2185

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{-\frac{2\int-\frac{d^2(c+dx)^{3/2}(a(14bcC+35bBd+45adD)+b(35Abd+49aCd+10acD)x)}{2\sqrt{a-bx^2}}dx}{7bd^2}-\frac{4aD\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}}{2ab}$$

$\Big\downarrow$ 27

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\int\frac{(c+dx)^{3/2}(a(14bcC+35bBd+45adD)+b(35Abd+49aCd+10acD)x)}{\sqrt{a-bx^2}}dx}{7b}-\frac{4aD\sqrt{a-bx^2}(c+dx)^{5/2}}{7b}}{2ab}$$

$\Big\downarrow$ 687

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{-\frac{2\int-\frac{b\sqrt{c+dx}\left(a\left(70bCc^2+175bBdc+255adDc+105Abd^2+147aCd^2\right)+\left(105Acdb^2+a\left(225aDd^2+b\left(30Dc^2+217Cdc+175Bd^2\right)\right)\right)x\right)}{2\sqrt{a-bx^2}}dx}{5b}-\frac{2}{5}\sqrt{a-bx^2}(c+dx)^{3/2}(}{7b}}{2ab}$$

$\Big\downarrow$ 27

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{1}{5}\int\frac{\sqrt{c+dx}\left(a\left(70bCc^2+175bBdc+255adDc+105Abd^2+147aCd^2\right)+\left(105Acdb^2+a\left(225aDd^2+b\left(30Dc^2+217Cdc+175Bd^2\right)\right)\right)x\right)}{\sqrt{a-bx^2}}dx-\frac{2}{5}\sqrt{a-bx^2}(c+dx)^{3/2}(10a}{7b}}{2ab}$$

$\Big\downarrow$ 687

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{\frac{1}{5}\left(-\frac{2\int-\frac{a\left(225a^2Dd^3+ab\left(795Dc^2+658Cdc+175Bd^2\right)d+105b^2c\left(2Cc^2+5Bdc+4Ad^2\right)\right)+b\left(105Abd\left(bc^2+3ad^2\right)+a\left(9a(49Cd+110cD)d^2+bc\left(30Dc^2+427Cdc+\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\right.}{7b}$$

$\Big\downarrow$ 27

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{\int \frac{a\left(225a^2Dd^3+ab\left(795Dc^2+658Cdc+175Bd^2\right)d+105b^2c\left(2Cc^2+5Bdc+4Ad^2\right)\right)+b\left(105Abd\left(bc^2+3ad^2\right)+a\left(9a(49Cd+110cD)d^2+bc\left(30Dc^2+427Cdc+700B\right.\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\right.$$

$$7b$$

$$\Big\downarrow \text{600}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{\frac{b\left(105Abd\left(3ad^2+bc^2\right)+a\left(9ad^2(110cD+49Cd)+bc\left(700Bd^2+30c^2D+427cCd\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+1\right.}{d}}{3b}\right.$$

$$\Big\downarrow \text{509}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(105Abd\left(3ad^2+bc^2\right)+a\left(9ad^2(110cD+49Cd)+bc\left(700Bd^2+30c^2D+427cCd\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+21\right.\right.\right.}{d}}{3b}\right.$$

$$\Big\downarrow \text{508}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+105Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(105Abd\left(3ad^2+bc^2\right)+a\left(9ad^2(110cD\right.\right.}{d\sqrt{a}}}{3b}\right.$$

$$\Big\downarrow \text{327}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{-\frac{\left(bc^2-ad^2\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+105Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)(10\cdots}{d\sqrt{a\cdots}}}{3b}\cdots\right.$$

$\downarrow$ 512

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+105Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\cdots\right.}{\cdots}}{3b}\cdots\right.$$

$\downarrow$ 511

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+105Ab^2cd\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\cdots}{\cdots}}{3b}\cdots\right.$$

$\downarrow$ 321

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{1}{5}\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(225ad^2D+b\left(175Bd^2+30c^2D+217cCd\right)\right)+105Ab^2cd\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}}{}}{3b}\right.$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*(c + d*x)^(5/2))/(a*b*Sqrt[a - b*x^2])
- ((-4*a*D*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b) + ((-2*(35*A*b*d + 49*a*
C*d + 10*a*c*D)*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + ((-2*(105*A*b^2*c*d +
 a*(225*a*d^2*D + b*(217*c*C*d + 175*B*d^2 + 30*c^2*D)))*Sqrt[c + d*x]*Sqr
t[a - b*x^2])/(3*b) + ((-2*Sqrt[a]*Sqrt[b]*(105*A*b*d*(b*c^2 + 3*a*d^2) +
a*(9*a*d^2*(49*C*d + 110*c*D) + b*c*(427*c*C*d + 700*B*d^2 + 30*c^2*D)))*S
qrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqr
t[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)
*(105*A*b^2*c*d + a*(225*a*d^2*D + b*(217*c*C*d + 175*B*d^2 + 30*c^2*D)))*
Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Elli
pticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sq
rt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b))/5)/(7*b))/(
2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(491) = 982$.

Time = 6.29 (sec) , antiderivative size = 1493, normalized size of antiderivative = 2.63

method	result	size
elliptic	Expression too large to display	1493
default	Expression too large to display	4666

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x-b*
c)*(1/2*(A*a*b*d^2+A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2+2*D*a^2*c*d)/
b^3/a*x+1/2*(2*A*b^2*c*d+B*a*b*d^2+B*b^2*c^2+2*C*a*b*c*d+D*a^2*d^2+D*a*b*c
^2)/b^4)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/7*D*d^2/b^2*x^2*(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)-2/5*(-d^2*(C*d+3*D*c)/b+6/7*D*d^2/b*c)/b/d*x*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-1/b^2*d*(B*b*d^2+3*C*b*c*d+D*a*d^2+3*D*b*c^
2)-5/7*D*d^3/b^2*a-4/5*(-d^2*(C*d+3*D*c)/b+6/7*D*d^2/b*c)/d*c)/b/d*(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(3*A*b^2*c*d^2+B*a*b*d^3+3*B*b^2*c^2*d+3*C
*a*b*c*d^2+C*b^2*c^3+D*a^2*d^3+3*D*a*b*c^2*d)/b^3+1/b^3*(3*A*a*b^2*c*d^2+A
*b^3*c^3+B*a^2*b*d^3+3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2+C*a*b^2*c^3+D*a^3*d^3
+3*D*a^2*b*c^2*d)/a-1/2/b^3*d*(2*A*b^2*c*d+B*a*b*d^2+B*b^2*c^2+2*C*a*b*c*d
+D*a^2*d^2+D*a*b*c^2)-1/b^2*c*(A*a*b*d^2+A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C
*a*b*c^2+2*D*a^2*c*d)/a+2/5*(-d^2*(C*d+3*D*c)/b+6/7*D*d^2/b*c)/b/d*a*c+1/3
*(-1/b^2*d*(B*b*d^2+3*C*b*c*d+D*a*d^2+3*D*b*c^2)-5/7*D*d^3/b^2*a-4/5*(-d^2
*(C*d+3*D*c)/b+6/7*D*d^2/b*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/
d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/b^2*(A*b*d^3+3*B*b*c*d^2
+C*a*d^3+3*C*b*c^2*d+3*D*a*c*d^2+D*b*c^3)-1/2*(A*a*b*d^2+A*b^2*c^2+2*B*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
1/315*((30*D*a^2*b^2*c^4 - 7*(29*C*a^2*b^2 - 15*A*a*b^3)*c^3*d - 5*(279*D*
a^3*b + 175*B*a^2*b^2)*c^2*d^2 - 21*(73*C*a^3*b + 45*A*a^2*b^2)*c*d^3 - 75
*(9*D*a^4 + 7*B*a^3*b)*d^4 - (30*D*a*b^3*c^4 - 7*(29*C*a*b^3 - 15*A*b^4)*c
^3*d - 5*(279*D*a^2*b^2 + 175*B*a*b^3)*c^2*d^2 - 21*(73*C*a^2*b^2 + 45*A*a
*b^3)*c*d^3 - 75*(9*D*a^3*b + 7*B*a^2*b^2)*d^4)*x^2)*sqrt(-b*d)*weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d) + 3*(30*D*a^2*b^2*c^3*d + 7*(61*C*a^2*b^2 + 15*A*a*b^
3)*c^2*d^2 + 10*(99*D*a^3*b + 70*B*a^2*b^2)*c*d^3 + 63*(7*C*a^3*b + 5*A*a^
2*b^2)*d^4 - (30*D*a*b^3*c^3*d + 7*(61*C*a*b^3 + 15*A*b^4)*c^2*d^2 + 10*(9
9*D*a^2*b^2 + 70*B*a*b^3)*c*d^3 + 63*(7*C*a^2*b^2 + 5*A*a*b^3)*d^4)*x^2)*s
qrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*
a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(30*D*a*b^3*d^4*x^4
- 15*(13*D*a^2*b^2 + 7*B*a*b^3)*c^2*d^2 - 14*(26*C*a^2*b^2 + 15*A*a*b^3)*c
*d^3 - 25*(9*D*a^3*b + 7*B*a^2*b^2)*d^4 + 6*(15*D*a*b^3*c*d^3 + 7*C*a*b^3*
d^4)*x^3 + 2*(45*D*a*b^3*c^2*d^2 + 77*C*a*b^3*c*d^3 + 5*(9*D*a^2*b^2 + 7*B
*a*b^3)*d^4)*x^2 - 3*(35*(C*a*b^3 + A*b^4)*c^2*d^2 + 10*(10*D*a^2*b^2 + 7*
B*a*b^3)*c*d^3 + 7*(7*C*a^2*b^2 + 5*A*a*b^3)*d^4)*x)*sqrt(-b*x^2 + a)*sqrt
(d*x + c))/(a*b^5*d^2*x^2 - a^2*b^4*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{{\left(c+d\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a-b\,x^2\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

3.155 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 456

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)(c+dx)^{3/2}}{ab\sqrt{a-bx^2}}$$
$$+\frac{(15Abd+25aCd+6acD)\sqrt{c+dx}\sqrt{a-bx^2}}{15ab^2}+\frac{2D(c+dx)^{3/2}\sqrt{a-bx^2}}{5b^2}$$
$$+\frac{\left(15Ab^2cd+a(63ad^2D+b(55cCd+45Bd^2+6c^2D))\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15\sqrt{a}b^{5/2}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(bc^2-ad^2)(15Abd+25aCd+6acD)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15\sqrt{a}b^{5/2}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(3/2)/a/b/(-b*x^2+a)^(1/2)+1/15*(15*A*b*
d+25*C*a*d+6*D*a*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/b^2+2/5*D*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/b^2+1/15*(15*A*b^2*c*d+a*(63*a*d^2*D+b*(45*B*d^2+55*C*
c*d+6*D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))
/a^(1/2)/b^(5/2)/d/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-
1/15*(-a*d^2+b*c^2)*(15*A*b*d+25*C*a*d+6*D*a*c)*((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(5/2)/
d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.12 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(15Ab^2cx+a^2(25Cd+3D(9c+7dx))+ab(15Ad+15B(c+dx)+\right.}{ab^2(a-bx^2)}\right.}{}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(15*A*b^2*c*x + a^2*(25*C*d + 3*D*(9*c + 7*d*
x)) + a*b*(15*A*d + 15*B*(c + d*x) + x*(3*c*(5*C - 4*D*x) - 2*d*x*(5*C + 3
*D*x)))))/(a*b^2*(a - b*x^2)) - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(15*A*
b^2*c*d + a*(63*a*d^2*D + b*(55*c*C*d + 45*B*d^2 + 6*c^2*D)))*(a - b*x^2)
+ I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(15*A*b^2*c*d + a*(63*a*d^2*D + b*(55*
c*C*d + 45*B*d^2 + 6*c^2*D)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)
*(15*A*b^(3/2)*d - 15*Sqrt[a]*b*(2*c*C + 3*B*d) - 63*a^(3/2)*d*D + a*Sqrt[
b]*(25*C*d + 6*c*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sq
rt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt
[b]*c - Sqrt[a]*d)])/(a*b^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2
))))/(15*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2176, 27, 2185, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \textcolor{blue}{2176}$$

$$\frac{\int -\frac{\sqrt{c+dx}\left(2adDx^2+(3Abd+5aCd+2acD)x+\frac{a(2bcC+3bBd+3adD)}{b}\right)}{2\sqrt{a-bx^2}}dx}{ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\int\frac{\sqrt{c+dx}\left(2adDx^2+(3Abd+5aCd+2acD)x+\frac{a(2bcC+3bBd+3adD)}{b}\right)}{\sqrt{a-bx^2}}dx}{2ab}$$

$\downarrow$ 2185

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{-\frac{2\int-\frac{d^2\sqrt{c+dx}(a(10bcC+15bBd+21adD)+b(15Abd+25aCd+6acD)x)}{2\sqrt{a-bx^2}}dx}{5bd^2}-\frac{4aD\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}}{2ab}$$

$\downarrow$ 27

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\int\frac{\sqrt{c+dx}(a(10bcC+15bBd+21adD)+b(15Abd+25aCd+6acD)x)}{\sqrt{a-bx^2}}dx}{5b}-\frac{4aD\sqrt{a-bx^2}(c+dx)^{3/2}}{5b}}{2ab}$$

$\downarrow$ 687

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{-\frac{2\int-\frac{b\left(a\left(15b\left(2Cc^2+3Bdc+Ad^2\right)+ad(25Cd+69cD)\right)+\left(15Acdb^2+a\left(63aDd^2+b\left(6Dc^2+55Cdc+45Bd^2\right)\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+dx}(6acD+25aCd+15Ab}{5b}}{2ab}$$

$\downarrow$ 27

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{1}{3}\int\frac{a\left(15b\left(2Cc^2+3Bdc+Ad^2\right)+ad(25Cd+69cD)\right)+\left(15Acdb^2+a\left(63aDd^2+b\left(6Dc^2+55Cdc+45Bd^2\right)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+dx}(6acD+25aCd+15Abd)}{5b}-}{2ab}$$

$\downarrow$ 600

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{1}{3}\left(\frac{\left(a\left(63ad^2D+b\left(45Bd^2+6c^2D+55cCd\right)\right)+15Ab^2cd\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+dx}(6acD}{5b}}{2ab}$$

$\downarrow$ 509

$$\frac{\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(a\left(63ad^2D+b\left(45Bd^2+6c^2D+55cCd\right)\right)+15Ab^2cd\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+}}{5b}}{2ab}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{1}{3}\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a\left(63ad^2D+b\left(45Bd^2+6c^2D+55cCd\right)\right)+15Ab^2cd\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{}}}{d}\right.}{5b}}{2ab}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{1}{3}\left(-\frac{\left(bc^2-ad^2\right)(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(63ad^2D+b\left(45Bd^2+6c^2D+55cCd\right)\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5b}}{2ab}$$

$$\Big\downarrow \text{512}$$

$$\frac{\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{1}{3}\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(63ad^2D+b\left(45Bd^2+6c^2D\right.\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5b}}{2ab}$$

$$\Big\downarrow \text{511}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(6acD+25aCd+15Abd)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{}\right.}{5b}}{2ab}$$

$$\Big\downarrow \; 321$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-$$

$$\frac{\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(6acD+25aCd+15Abd)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}}\right.}{\sqrt{b}d\sqrt{a-}}\right.}{5b}}{2ab}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*(c + d*x)^(3/2))/(a*b*Sqrt[a - b*x^2])
- ((-4*a*D*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b) + ((-2*(15*A*b*d + 25*a*
C*d + 6*a*c*D)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + ((-2*Sqrt[a]*(15*A*b^2*c
*d + a*(63*a*d^2*D + b*(55*c*C*d + 45*B*d^2 + 6*c^2*D)))*Sqrt[c + d*x]*Sqr
t[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(15*A*b
*d + 25*a*C*d + 6*a*c*D)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]
*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*
x^2]))/3)/(5*b))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(386) = 772.

Time = 5.21 (sec) , antiderivative size = 999, normalized size of antiderivative = 2.19

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bdx-bc)\left(\frac{\left(A\,b^2c+Babd+Cabc+a^2dD\right)x}{2a\,b^3}+\frac{Abd+Bbc+Cad+Dac}{2b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2Ddx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5b^2}-\frac{2\left(-\frac{d(Cd}{}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x-b*
c)*(1/2*(A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*d)/a/b^3*x+1/2*(A*b*d+B*b*c+C*a*d+D
*a*c)/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/5*D*d/b^2*x*(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)-2/3*(-1/b*d*(C*d+2*D*c)+4/5*D*d/b*c)/b/d*(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)+2*(-(A*b*d^2+2*B*b*c*d+C*a*d^2+C*b*c^2+2*D*a*c*d)/b^2+1
/b^2*(A*a*b*d^2+A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2+2*D*a^2*c*d)/a-1
/2/b^2*d*(A*b*d+B*b*c+C*a*d+D*a*c)-1/b^2*c*(A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*
d)/a-2/5*D*d/b^2*a*c+1/3*(-1/b*d*(C*d+2*D*c)+4/5*D*d/b*c)/b*a)*(c/d-1/b*(a
*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/
2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-(B*
b*d^2+2*C*b*c*d+D*a*d^2+D*b*c^2)/b^2-1/2*(A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*d)
*d/a/b^2-3/5*D*d^2/b^2*a-2/3*(-1/b*d*(C*d+2*D*c)+4/5*D*d/b*c)/d*c)*(c/d-1/
b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))
^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Elliptic
E(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx = \frac{(6\,Da^2bc^3-5\,(7\,Ca^2b-3\,Aab^2)c^2d-18\,(8\,Da^3+5\,Ba^2b)cd^2}{}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
1/45*((6*D*a^2*b*c^3 - 5*(7*C*a^2*b - 3*A*a*b^2)*c^2*d - 18*(8*D*a^3 + 5*B
*a^2*b)*c*d^2 - 15*(5*C*a^3 + 3*A*a^2*b)*d^3 - (6*D*a*b^2*c^3 - 5*(7*C*a*b
^2 - 3*A*b^3)*c^2*d - 18*(8*D*a^2*b + 5*B*a*b^2)*c*d^2 - 15*(5*C*a^2*b + 3
*A*a*b^2)*d^3)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(6*D*a^2
*b*c^2*d + 5*(11*C*a^2*b + 3*A*a*b^2)*c*d^2 + 9*(7*D*a^3 + 5*B*a^2*b)*d^3
- (6*D*a*b^2*c^2*d + 5*(11*C*a*b^2 + 3*A*b^3)*c*d^2 + 9*(7*D*a^2*b + 5*B*a
*b^2)*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
-8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^
2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(6*
D*a*b^2*d^3*x^3 - 3*(9*D*a^2*b + 5*B*a*b^2)*c*d^2 - 5*(5*C*a^2*b + 3*A*a*b
^2)*d^3 + 2*(6*D*a*b^2*c*d^2 + 5*C*a*b^2*d^3)*x^2 - 3*(5*(C*a*b^2 + A*b^3)
*c*d^2 + (7*D*a^2*b + 5*B*a*b^2)*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(
a*b^4*d^2*x^2 - a^2*b^3*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{\left(c+d\,x\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(a-b\,x^2\right)^{3/2}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

3.156 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 399

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)\sqrt{c+dx}}{ab\sqrt{a-bx^2}}$$
$$+\frac{2D\sqrt{c+dx}\sqrt{a-bx^2}}{3b^2}$$
$$+\frac{(3Abd+9aCd+2acD)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}b^{3/2}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(3Ab^2cd-a(5ad^2D-b(3cCd-3Bd^2+2c^2D)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}b^{5/2}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(1/2)/a/b/(-b*x^2+a)^(1/2)+2/3*D*(d*x+c)
^(1/2)*(-b*x^2+a)^(1/2)/b^2+1/3*(3*A*b*d+9*C*a*d+2*D*a*c)*(d*x+c)^(1/2)*((
-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/
2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/d/((d*x+c)/(c+
a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/3*(3*A*b^2*c*d-a*(5*a*d^2*D-b
*(-3*B*d^2+3*C*c*d+2*D*c^2)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x
^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(5/2)/d/(d*x+c)^(1/2)/(-
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.31 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)(5a^2D+3Ab^2x+ab(3B+x(3C-2Dx)))}{ab^2(a-bx^2)}+\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(3Abd+}{}\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(5*a^2*D + 3*A*b^2*x + a*b*(3*B + x*(3*C - 2*
D*x))))/(a*b^2*(a - b*x^2)) + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*A*b*d
 + 9*a*C*d + 2*a*c*D)*(-a + b*x^2) - I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*
A*b*d + 9*a*C*d + 2*a*c*D)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)
/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*d*(3*A*b^(3/2)*d - 3*Sqrt[a]*b*(2*c*
C + B*d) - 5*a^(3/2)*d*D + a*Sqrt[b]*(9*C*d + 2*c*D))*Sqrt[(d*(Sqrt[a]/Sqr
t[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a*b^2*d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2176, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{2176}$$

$$\frac{\int -\frac{2adDx^2+(Abd+3aCd+2acD)x+\frac{a(2bcC+bBd+adD)}{b}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab}+\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\int \frac{2adDx^2+(Abd+3aCd+2acD)x+\frac{a(2bcC+bBd+adD)}{b}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab}$$

$$\Big\downarrow \textbf{2185}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{-\frac{2\int -\frac{d^2(a(6bcC+3bBd+5adD)+b(3Abd+9aCd+2acD)x)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{4aD\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}{2ab}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\int \frac{a(6bcC+3bBd+5adD)+b(3Abd+9aCd+2acD)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{4aD\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}{2ab}$$

$$\Big\downarrow \textbf{600}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{b(2acD+9aCd+3Abd)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{3b}-\frac{4aD\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}{2ab}$$

$$\Big\downarrow \text{509}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}(2acD+9aCd+3Abd)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{3b}-\frac{4aD\sqrt{a-bx^2}\sqrt{c+dx}}{3b}}{2ab}$$

$$\Big\downarrow \text{508}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{-\frac{\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2acD+9aCd+3Abd)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\frac{4a\cdots}{}}{2ab}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{-\frac{\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2acD+9aCd+3Abd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}-\cdots}{2ab}$$

$$\Big\downarrow \text{512}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2acD+9aCd+3Abd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\cdots}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3b}\cdots}{2ab}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2acD+}{d\sqrt{}}}{3b}}{2ab}$$

$$\Big\downarrow \textcolor{blue}{321}$$

$$\frac{\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{ab\sqrt{a-bx^2}}-\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3Ab^2cd-a\left(5ad^2D-b\left(-3Bd^2+2c^2D+3cCd\right)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2acD+9aCd+3.}{d\sqrt{a-bx^2}\sqrt{}}}{3b}}{2ab}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(3/2),x]
```

output
```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*Sqrt[c + d*x])/(a*b*Sqrt[a - b*x^2]) -
((-4*a*D*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + ((-2*Sqrt[a]*Sqrt[b]*(3*A*
b*d + 9*a*C*d + 2*a*c*D)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) +
(2*Sqrt[a]*(3*A*b^2*c*d - a*(5*a*d^2*D - b*(3*c*C*d - 3*B*d^2 + 2*c^2*D)))
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/S
qrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(335) = 670$.

Time = 4.40 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(dx+c)(-bx^2+a)}\left(-\frac{2(-bdx-bc)\left(\frac{(Ab+Ca)x}{2a\,b^2}+\frac{Bb+Da}{2b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2D\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3b^2}+\frac{2\left(-\frac{Bbd+bcC+Dad}{b^2}+\frac{A\,b^2c+Babd+Cabc+a^2}{b^2a}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x-b*
c)*(1/2/a*(A*b+C*a)/b^2*x+1/2*(B*b+D*a)/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2
)+2/3*D/b^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-(B*b*d+C*b*c+D*a*d)/b^2
+1/b^2*(A*b^2*c+B*a*b*d+C*a*b*c+D*a^2*d)/a-1/2/b^2*d*(B*b+D*a)-1/b*c/a*(A*
b+C*a)-1/3*D/b^2*a*d)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(
1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Ell
ipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2))+2*(-(C*d+D*c)/b-1/2*(A*b+C*a)*d/a/b+2/3*D/b*c)*(
c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(
1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*El
lipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \frac{(2\,Da^2bc^2-3\,(3\,Ca^2b-Aab^2)cd-3\,(5\,Da^3+3\,Ba^2b)d^2-(2\,D$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
1/9*((2*D*a^2*b*c^2 - 3*(3*C*a^2*b - A*a*b^2)*c*d - 3*(5*D*a^3 + 3*B*a^2*b
)*d^2 - (2*D*a*b^2*c^2 - 3*(3*C*a*b^2 - A*b^3)*c*d - 3*(5*D*a^2*b + 3*B*a*
b^2)*d^2)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2
), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*D*a^2*b*c*
d + 3*(3*C*a^2*b + A*a*b^2)*d^2 - (2*D*a*b^2*c*d + 3*(3*C*a*b^2 + A*b^3)*d
^2)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(
b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d
^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(2*D*a*b^2
*d^2*x^2 - 3*(C*a*b^2 + A*b^3)*d^2*x - (5*D*a^2*b + 3*B*a*b^2)*d^2)*sqrt(-
b*x^2 + a)*sqrt(d*x + c))/(a*b^4*d^2*x^2 - a^2*b^3*d^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3)/(a - b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/(-b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/(-b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+d\,x}\,(A+B\,x+C\,x^2+x^3\,D)}{(a-b\,x^2)^{3/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,(Dx^3+C\,x^2+Bx+A)}{(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(3/2),x)
```

3.157 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 413

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\frac{\sqrt{c+dx}\left(a(bBc-Abd-aCd+acD)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x\right)}{ab\,(bc^2-ad^2)\sqrt{a-bx^2}}$$
$$+\frac{\left(Abcd+a\left(cCd+2c^2D-\frac{d^2(bB+3aD)}{b}\right)\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}d\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(Abd-aCd+2acD)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d+D*a*c)+b*(c*(A*b+C*a)-a*d*(B+a*D/b))*
x)/a/b/(-a*d^2+b*c^2)/(-b*x^2+a)^(1/2)+(A*b*c*d+a*(C*c*d+2*D*c^2-d^2*(B*b+
3*D*a)/b))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a
^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(
1/2)/b^(1/2)/d/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^
2+a)^(1/2)-(A*b*d-C*a*d+2*D*a*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/d/(d*x+c)^(1/2)
/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.33 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{-b(c+dx)\,(Ab^2cx-a^2(Cd-cD+dDx)+ab(-Ad+cCx+B(c-dx))}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(-(b*(c + d*x)*(A*b^2*c*x - a^2*(C*d - c*D + d*D*x) + a*b*(-(A*d) + c*C*x
+ B*(c - d*x)))) + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(A*b^2*c*d + a*(-3*
a*d^2*D + b*(c*C*d - B*d^2 + 2*c^2*D)))*(-a + b*x^2) - I*Sqrt[b]*(Sqrt[b]*
c - Sqrt[a]*d)*(A*b^2*c*d + a*(-3*a*d^2*D + b*(c*C*d - B*d^2 + 2*c^2*D)))*
Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*
x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] -
 I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*b^(3/2)*d + Sqrt[a]*b*(-2*
c*C + B*d) + 3*a^(3/2)*d*D + a*Sqrt[b]*(-(C*d) + 2*c*D))*Sqrt[(d*(Sqrt[a]/
Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
 + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c +
d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]))/(a*b^2*(-(b*c^2) + a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^
2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 2180$$

$$\frac{\int -\frac{a\left(b\left(2Cc^2-Bdc+Ad^2\right)-ad(Cd+cD)\right)+b\left(Abcd+a\left(2Dc^2+Cdc-\frac{d^2(bB+3aD)}{b}\right)\right)x}{2b\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int \frac{a\left(b\left(2Cc^2-Bdc+Ad^2\right)-ad(Cd+cD)\right)+b\left(Abcd+a\left(2Dc^2+Cdc-\frac{d^2(bB+3aD)}{b}\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{b\left(a\left(-\frac{d^2(3aD+bB)}{b}+2c^2D+cCd\right)+Abcd\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)(2acD-aCd+Abd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(a\left(-\frac{d^2(3aD+bB)}{b}+2c^2D+cCd\right)+Abcd\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)(2acD-aCd+Abd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 508$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a\left(-\frac{d^2(3aD+bB)}{b}+2c^2D+cCd\right)+Abcd\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(2acD-aCd+Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{\cdots}}}{d}\cdots}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\left(bc^2-ad^2\right)(2acD-aCd+Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(-\frac{d^2(3aD+bB)}{b}+2c^2D+cC\cdots\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(2acD-aCd+Abd)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(-\frac{d^2(3aD+bB)}{b}\cdots\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(2acD-aCd+Abd)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\sqrt{\cdots}\right.\right.}{\cdots}\cdots}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{ab\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}(2acD-aCd+Abd)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{}}}{\sqrt{2}}\right.\right.}{d\sqrt{}}}{2ab\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d + a*c*D) + b*(c*(A*b + a*C) - a*d
*(B + (a*D)/b))*x))/(a*b*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-2*Sqrt[a]*S
qrt[b]*(A*b*c*d + a*(c*C*d + 2*c^2*D - (d^2*(b*B + 3*a*D))/b))*Sqrt[c + d*
x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt
[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[
b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(A*b*d -
a*C*d + 2*a*c*D)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1
- (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d
)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(
2*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(359) = 718$.

Time = 6.46 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(dx+c)(-b\,x^2+a)}\left(-\frac{2(-bdx-bc)\left(-\frac{\left(A\,b^2c-Babd+Cabc-a^2dD\right)x}{2a\,b^2\left(a\,d^2-b\,c^2\right)}+\frac{Abd-Bbc+Cad-Dac}{2\left(a\,d^2-b\,c^2\right)b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{C}{b}+\frac{Ab+Ca}{ab}-\frac{d(Abd-Bbc+Cad-Da}{2b\left(a\,d^2-b\,c^2\right)}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2*(-b*d*x-b*
c)*(-1/2*(A*b^2*c-B*a*b*d+C*a*b*c-D*a^2*d)/a/b^2/(a*d^2-b*c^2)*x+1/2*(A*b*
d-B*b*c+C*a*d-D*a*c)/(a*d^2-b*c^2)/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(
-1/b*C+1/a*(A*b+C*a)/b-1/2/b*d*(A*b*d-B*b*c+C*a*d-D*a*c)/(a*d^2-b*c^2)+1/b
*c*(A*b^2*c-B*a*b*d+C*a*b*c-D*a^2*d)/a/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-D/b+1/2*d*(A
*b^2*c-B*a*b*d+C*a*b*c-D*a^2*d)/a/(a*d^2-b*c^2)/b)*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/
d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx =$$
$$-\frac{(2\,Da^2bc^3+2\,Ba^2bcd^2-(5\,Ca^2b-Aab^2)c^2d+3\,(Ca^3-Aa^2b)d^3-(2\,Dab^2c^3+2\,Bab^2cd^2-(5\,Cab^2-}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/3*((2*D*a^2*b*c^3 + 2*B*a^2*b*c*d^2 - (5*C*a^2*b - A*a*b^2)*c^2*d + 3*(
C*a^3 - A*a^2*b)*d^3 - (2*D*a*b^2*c^3 + 2*B*a*b^2*c*d^2 - (5*C*a*b^2 - A*b
^3)*c^2*d + 3*(C*a^2*b - A*a*b^2)*d^3)*x^2)*sqrt(-b*d)*weierstrassPInverse
(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*
d*x + c)/d) + 3*(2*D*a^2*b*c^2*d + (C*a^2*b + A*a*b^2)*c*d^2 - (3*D*a^3 +
B*a^2*b)*d^3 - (2*D*a*b^2*c^2*d + (C*a*b^2 + A*b^3)*c*d^2 - (3*D*a^2*b + B
*a*b^2)*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2)
, -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*
d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(
(D*a^2*b + B*a*b^2)*c*d^2 - (C*a^2*b + A*a*b^2)*d^3 + ((C*a*b^2 + A*b^3)*c
*d^2 - (D*a^2*b + B*a*b^2)*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^2*b^
3*c^2*d^2 - a^3*b^2*d^4 - (a*b^4*c^2*d^2 - a^2*b^3*d^4)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a - b*x**2)**(3/2)*sqrt(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.158 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 564

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\frac{a(bBc-Abd-aCd+acD)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{ab\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{\left(Abd(bc^2+3ad^2)+a(ad^2(Cd-2cD)+bc(3cCd-4Bd^2-2c^2D))\right)\sqrt{a-bx^2}}{ab\left(bc^2-ad^2\right)^2\sqrt{c+dx}}$$
$$+\frac{\left(Abd(bc^2+3ad^2)+a(ad^2(Cd-2cD)+bc(3cCd-4Bd^2-2c^2D))\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{a}\sqrt{b}d\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\left(Ab^2cd+a(ad^2D+b(cCd-Bd^2-2c^2D))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}d\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d+D*a*c)+b*(c*(A*b+C*a)-a*d*(B+a*D/b))*x)/a/b/(-a*d^2
+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(A*b*d*(3*a*d^2+b*c^2)+a*(a*d^2*(C*
d-2*D*c)+b*c*(-4*B*d^2+3*C*c*d-2*D*c^2)))*(-b*x^2+a)^(1/2)/a/b/(-a*d^2+b*c
^2)^2/(d*x+c)^(1/2)+(A*b*d*(3*a*d^2+b*c^2)+a*(a*d^2*(C*d-2*D*c)+b*c*(-4*B*
d^2+3*C*c*d-2*D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1
-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/a^(1/2)/b^(1/2)/d/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2))
)^(1/2)/(-b*x^2+a)^(1/2)-(A*b^2*c*d+a*(a*d^2*D+b*(-B*d^2+C*c*d-2*D*c^2)))*
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/a^(1/2)/b^(3/2)/d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.20 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(Abd(bc^2+3ad^2)+2ab(-c^2Cd+Bcd^2-Ad^3+c^3D)+a(ad^2\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(A*b*d*(b*c^2 + 3*a*d^2) + 2*a*b*(-(c^2*C*d) + B*c*d^2 -
A*d^3 + c^3*D) + a*(a*d^2*(C*d - 2*c*D) + b*c*(3*c*C*d - 4*B*d^2 - 2*c^2*D
)) - ((c + d*x)*(a^3*d^2*D + A*b^3*c^2*x + a*b^2*(c^2*C*x + B*c*(c - 2*d*x
) + A*d*(-2*c + d*x)) + a^2*b*(c^2*D + d^2*(B + C*x) - 2*c*d*(C + D*x))))/
(-a + b*x^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*d*(b*c^2 + 3*a*d^2)
 + a*(a*d^2*(C*d - 2*c*D) + b*c*(3*c*C*d - 4*B*d^2 - 2*c^2*D)))*Sqrt[(d*(S
qrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*
x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sq
rt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[a]*(Sqrt[b]*c - Sqrt[a]*
d)*(A*b^2*c*d + Sqrt[a]*b^(3/2)*(-2*c^2*C + 3*B*c*d - 3*A*d^2) + a^2*d^2*D
 + a^(3/2)*Sqrt[b]*d*(-(C*d) + 3*c*D) + a*b*(c*C*d - B*d^2 - 2*c^2*D))*Sqr
t[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/
(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*b*(b*c^2 - a*d^2)^2*Sqrt
[c + d*x])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2180, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a-bx^2\right)^{3/2} (c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(b\left(2Cc^2-3Bdc+3Ad^2\right)+ad(Cd-3cD)\right)-b\left(Abcd+a\left(-2Dc^2+Cdc-\frac{d^2(bB-aD)}{b}\right)\right)x}{2b(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\int \frac{a(b(2Cc^2-3Bdc+3Ad^2)+ad(Cd-3cD))-\left(Acdb^2+a\left(aDd^2+b\left(-2Dc^2+Cdc-Bd^2\right)\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 688

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{2\int \frac{a\left(a^2Dd^3+ab\left(-5Dc^2+2Cdc-Bd^2\right)d+b^2c\left(2Cc^2-3Bdc+4Ad^2\right)\right)+b\left(Abd\left(bc^2+3ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-2Dc^2+3Cdc-4Bd^2\right)\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\,(}{}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 27

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\int \frac{a\left(a^2Dd^3+ab\left(-5Dc^2+2Cdc-Bd^2\right)d+b^2c\left(2Cc^2-3Bdc+4Ad^2\right)\right)+b\left(Abd\left(bc^2+3ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-2Dc^2+3Cdc-4Bd^2\right)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\,(A}{}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 600

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{b\left(Abd\left(3ad^2+bc^2\right)+a\left(ad^2(Cd-2cD)+bc\left(-4Bd^2-2c^2D+3cCd\right)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}d}{d}}{bc^2-ad^2}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 509

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(Abd\left(3ad^2+bc^2\right)+a\left(ad^2(Cd-2cD)+bc\left(-4Bd^2-2c^2D+3cCd\right)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int \frac{}{\sqrt{c+dx}}}{d}}{bc^2-ad^2}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 508

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{-\frac{\left(bc^2-ad^2\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Abd\left(3ad^2+bc^2\right)+a\left(ad^2(Cd-2cD)+bc\left(-4Bd^2-2c^2 \right.\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}$$

$$2ab\left(bc^2-ad^2\right)$$

$\Big\downarrow$ 327

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{-\frac{\left(bc^2-ad^2\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(Abd\left(3ad^2+bc^2\right)+a \right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}$$

$$2ab\left(bc^2-ad^2\right)$$

$\Big\downarrow$ 512

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(Abd\left(3ad \right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c}{\sqrt{a}d}}}}{bc^2-ad^2}$$

$$2ab\left(bc^2-ad\right.$$

$\Big\downarrow$ 511

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E}{}}{bc^2-ad^2}$$

$\Big\downarrow$ 321

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(a\left(ad^2D+b\left(-Bd^2-2c^2D+cCd\right)\right)+Ab^2cd\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,E\left(\arcsin\right.}{}$$

$$bc^2-ad^2$$

2

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(b*B*c - A*b*d - a*C*d + a*c*D) + b*(c*(A*b + a*C) - a*d*(B + (a*D)/b))
*x)/(a*b*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((2*(A*b*d*(b*c^
2 + 3*a*d^2) + a*(a*d^2*(C*d - 2*c*D) + b*c*(3*c*C*d - 4*B*d^2 - 2*c^2*D))
)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((-2*Sqrt[a]*Sqrt[b]*
(A*b*d*(b*c^2 + 3*a*d^2) + a*(a*d^2*(C*d - 2*c*D) + b*c*(3*c*C*d - 4*B*d^2
 - 2*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(
Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*
(b*c^2 - a*d^2)*(A*b^2*c*d + a*(a*d^2*D + b*(c*C*d - B*d^2 - 2*c^2*D)))*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2))/(2*
a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(506) = 1012$.

Time = 7.64 (sec) , antiderivative size = 1174, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	1174
default	Expression too large to display	4431

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(2*b*d*(1/2*(3
*A*a*b*d^3+A*b^2*c^2*d-4*B*a*b*c*d^2+C*a^2*d^3+3*C*a*b*c^2*d-2*D*a^2*c*d^2
-2*D*a*b*c^3)/b/d/(a*d^2-b*c^2)^2/a*x^2-1/2*(A*b^2*c-B*a*b*d+C*a*b*c-D*a^2
*d)/d/a/b^2/(a*d^2-b*c^2)*x-1/2*(2*A*a*b*d^3+2*A*b^2*c^2*d-3*B*a*b*c*d^2-B
*b^2*c^3+4*C*a*b*c^2*d-D*a^2*c*d^2-3*D*a*b*c^3)/b^2/d/(a^2*d^4-2*a*b*c^2*d
^2+b^2*c^4))/(-(x^3+c/d*x^2-a*x/b-a/b*c/d)*b*d)^(1/2)+2*(-D/b/d-1/2*(6*A*a
*b^2*c*d^3-2*A*b^3*c^3*d-3*B*a^2*b*d^4-B*a*b^2*c^2*d^2+4*C*a^2*b*c*d^3-3*D
*a^3*d^4+D*a^2*b*c^2*d^2-2*D*a*b^2*c^4)/d/a/b/(a^2*d^4-2*a*b*c^2*d^2+b^2*c
^4)+1/b*(A*b^2*c-B*a*b*d+C*a*b*c-D*a^2*d)/a/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^
(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(3/2*(3*A
*a*b*d^3+A*b^2*c^2*d-4*B*a*b*c*d^2+C*a^2*d^3+3*C*a*b*c^2*d-2*D*a^2*c*d^2-2
*D*a*b*c^3)/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)-2*(3*A*a*b*d^3+A*b^2*c^2*d-4
*B*a*b*c*d^2+C*a^2*d^3+3*C*a*b*c^2*d-2*D*a^2*c*d^2-2*D*a*b*c^3)/(a*d^2-b*c
^2)^2/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1
/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)
^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(517) = 1034$.

Time = 0.11 (sec) , antiderivative size = 1356, normalized size of antiderivative = 2.40

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{3/2} (a - bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
1/3*((2*D*a^2*b^2*c^5 + (3*C*a^2*b^2 - A*a*b^3)*c^4*d - (13*D*a^3*b + 5*B*
a^2*b^2)*c^3*d^2 + (5*C*a^3*b + 9*A*a^2*b^2)*c^2*d^3 + 3*(D*a^4 - B*a^3*b)
*c*d^4 - (2*D*a*b^3*c^4*d + (3*C*a*b^3 - A*b^4)*c^3*d^2 - (13*D*a^2*b^2 +
5*B*a*b^3)*c^2*d^3 + (5*C*a^2*b^2 + 9*A*a*b^3)*c*d^4 + 3*(D*a^3*b - B*a^2*
b^2)*d^5)*x^3 - (2*D*a*b^3*c^5 + (3*C*a*b^3 - A*b^4)*c^4*d - (13*D*a^2*b^2
 + 5*B*a*b^3)*c^3*d^2 + (5*C*a^2*b^2 + 9*A*a*b^3)*c^2*d^3 + 3*(D*a^3*b - B
*a^2*b^2)*c*d^4)*x^2 + (2*D*a^2*b^2*c^4*d + (3*C*a^2*b^2 - A*a*b^3)*c^3*d^
2 - (13*D*a^3*b + 5*B*a^2*b^2)*c^2*d^3 + (5*C*a^3*b + 9*A*a^2*b^2)*c*d^4 +
 3*(D*a^4 - B*a^3*b)*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3
*(2*D*a^2*b^2*c^4*d - (3*C*a^2*b^2 + A*a*b^3)*c^3*d^2 + 2*(D*a^3*b + 2*B*a
^2*b^2)*c^2*d^3 - (C*a^3*b + 3*A*a^2*b^2)*c*d^4 - (2*D*a*b^3*c^3*d^2 - (3*
C*a*b^3 + A*b^4)*c^2*d^3 + 2*(D*a^2*b^2 + 2*B*a*b^3)*c*d^4 - (C*a^2*b^2 +
3*A*a*b^3)*d^5)*x^3 - (2*D*a*b^3*c^4*d - (3*C*a*b^3 + A*b^4)*c^3*d^2 + 2*(
D*a^2*b^2 + 2*B*a*b^3)*c^2*d^3 - (C*a^2*b^2 + 3*A*a*b^3)*c*d^4)*x^2 + (2*D
*a^2*b^2*c^3*d^2 - (3*C*a^2*b^2 + A*a*b^3)*c^2*d^3 + 2*(D*a^3*b + 2*B*a^2*
b^2)*c*d^4 - (C*a^3*b + 3*A*a^2*b^2)*d^5)*x)*sqrt(-b*d)*weierstrassZeta(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d)) - 3*(2*A*a^2*b^2*d^5 - (3*D*a^2*b^2 + B*a*b^3)*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(dx+c)^{\frac{3}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.159 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 754

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\frac{a(bBc-Abd-aCd+acD)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{ab\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}$$
$$-\frac{\left(Abd(3bc^2+5ad^2)+a(3ad^2(Cd-2cD)+bc(5cCd-8Bd^2-2c^2D))\right)\sqrt{a-bx^2}}{3ab\left(bc^2-ad^2\right)^2(c+dx)^{3/2}}$$
$$-\frac{\left(Ab^2cd(3bc^2+29ad^2)-a(3a^2d^4D-3abd^2(7cCd-3Bd^2-9c^2D)-b^2c^2(11cCd-23Bd^2-2c^2D))\right)\sqrt{a}}{3ab\left(bc^2-ad^2\right)^3\sqrt{c+dx}}$$
$$+\frac{\left(Ab^2cd(3bc^2+29ad^2)-a(3a^2d^4D-3abd^2(7cCd-3Bd^2-9c^2D)-b^2c^2(11cCd-23Bd^2-2c^2D))\right)\sqrt{c}}{3\sqrt{a}\sqrt{b}d\left(bc^2-ad^2\right)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\left(Abd(3bc^2+5ad^2)+a(3ad^2(Cd-2cD)+bc(5cCd-8Bd^2-2c^2D))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\Bigg(\text{arcsi}}{3\sqrt{a}\sqrt{b}d\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d+D*a*c)+b*(c*(A*b+C*a)-a*d*(B+a*D/b))*x)/a/b/(-a*d^2
+b*c^2)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2)-1/3*(A*b*d*(5*a*d^2+3*b*c^2)+a*(3*a
*d^2*(C*d-2*D*c)+b*c*(-8*B*d^2+5*C*c*d-2*D*c^2)))*(-b*x^2+a)^(1/2)/a/b/(-a
*d^2+b*c^2)^2/(d*x+c)^(3/2)-1/3*(A*b^2*c*d*(29*a*d^2+3*b*c^2)-a*(3*a^2*d^4
*D-3*a*b*d^2*(-3*B*d^2+7*C*c*d-9*D*c^2)-b^2*c^2*(-23*B*d^2+11*C*c*d-2*D*c^
2)))*(-b*x^2+a)^(1/2)/a/b/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)+1/3*(A*b^2*c*d*(2
9*a*d^2+3*b*c^2)-a*(3*a^2*d^4*D-3*a*b*d^2*(-3*B*d^2+7*C*c*d-9*D*c^2)-b^2*c
^2*(-23*B*d^2+11*C*c*d-2*D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/d/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/3*(A*b*d*(5*a*d^2+3*b*c^2)+a*(3*a*d^
2*(C*d-2*D*c)+b*c*(-8*B*d^2+5*C*c*d-2*D*c^2)))*((d*x+c)/(c+a^(1/2)*d/b^(1/
2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/d
/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.07 (sec) , antiderivative size = 1181, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3
*(-(b*c^2) + a*d^2)^2*(c + d*x)^2) - (2*(-4*b*c^3*C*d + 7*b*B*c^2*d^2 - 10
*A*b*c*d^3 - 6*a*c*C*d^3 + 3*a*B*d^4 + b*c^4*D + 9*a*c^2*d^2*D))/(3*(-(b*c
^2) + a*d^2)^3*(c + d*x)) + (a*b^2*B*c^3 - 3*a*A*b^2*c^2*d - 3*a^2*b*c^2*C
*d + 3*a^2*b*B*c*d^2 - a^2*A*b*d^3 - a^3*C*d^3 + a^2*b*c^3*D + 3*a^3*c*d^2
*D + A*b^3*c^3*x + a*b^2*c^3*C*x - 3*a*b^2*B*c^2*d*x + 3*a*A*b^2*c*d^2*x +
 3*a^2*b*c*C*d^2*x - a^2*b*B*d^3*x - 3*a^2*b*c^2*d*D*x - a^3*d^3*D*x)/(a*(
-(b*c^2) + a*d^2)^3*(-a + b*x^2))) + (Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c +
 d*x))^2)/d^2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(A*b^2*c*d*(3*b*c^2 + 29*a*
d^2) - a*(3*a^2*d^4*D + b^2*c^2*(-11*c*C*d + 23*B*d^2 + 2*c^2*D) + 3*a*b*d
^2*(-7*c*C*d + 3*B*d^2 + 9*c^2*D)))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c
 + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b^2*c*d*(3*b*c^2 + 29*
a*d^2) - a*(3*a^2*d^4*D + b^2*c^2*(-11*c*C*d + 23*B*d^2 + 2*c^2*D) + 3*a*b
*d^2*(-7*c*C*d + 3*B*d^2 + 9*c^2*D)))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(
Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))
]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt
[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] - (I*Sqrt[a]*Sq
rt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(3*A*b^(5/2)*c^2*d - 3*Sqrt[a]*b^2*c*(2*c^
2*C - 5*B*c*d + 8*A*d^2) + 3*a^(5/2)*d^3*D + 3*a^2*Sqrt[b]*d^2*(C*d - 2*c*
D) + 3*a^(3/2)*b*d*(-6*c*C*d + 3*B*d^2 + 7*c^2*D) + a*b^(3/2)*(5*c^2*C*...
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2180, 27, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \ 2180$$

$$\frac{\int -\frac{a\left(b\left(2Cc^2-5Bdc+5Ad^2\right)+ad(3Cd-5cD)\right)-\left(3Ab^2cd-a\left(ad^2D-b\left(-2Dc^2+3Cdc-3Bd^2\right)\right)\right)x}{2b(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{a\left(b\left(2Cc^2-5Bdc+5Ad^2\right)+ad(3Cd-5cD)\right)-\left(3Ab^2cd-a\left(ad^2D-b\left(-2Dc^2+3Cdc-3Bd^2\right)\right)\right)x}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2\int \frac{3a\left(-a^2Dd^3+ab\left(-7Dc^2+6Cdc-3Bd^2\right)d+b^2c\left(2Cc^2-5Bdc+8Ad^2\right)\right)-b\left(Abd\left(3bc^2+5ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-2Dc^2+5Cdc-8Bd^2\right)\right)\right)x}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-\cdots}}{\cdots}}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\int \frac{3a\left(-a^2Dd^3+ab\left(-7Dc^2+6Cdc-3Bd^2\right)d+b^2c\left(2Cc^2-5Bdc+8Ad^2\right)\right)-b\left(Abd\left(3bc^2+5ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-2Dc^2+5Cdc-8Bd^2\right)\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2\sqrt{a-b\cdots}}{\cdots}}{2ab\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\frac{2\int \frac{b\left(a\left(3a^2(Cd-3cD)d^3+ab\left(-23Dc^3+23Cdc^2-17Bd^2c+5Ad^3\right)d+3b^2c^2\left(2Cc^2-5Bdc+9Ad^2\right)\right)+\left(Ab^2cd\left(3bc^2+29ad^2\right)-a\left(3a^2Dd^4-3ab\left(-9Dc^2+7Cdc-3Bd\cdots\right.\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{bc^2-ad^2}\cdots}{3\left(bc^2-\cdots\right)}\cdots}{\cdots}$$

$$\Big\downarrow \ 27$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\frac{b\int \frac{a\left(3a^2(Cd-3cD)d^3+ab\left(-23Dc^3+23Cdc^2-17Bd^2c+5Ad^3\right)d+3b^2c^2\left(2Cc^2-5Bdc+9Ad^2\right)\right)+\left(Ab^2cd\left(3bc^2+29ad^2\right)-a\left(3a^2Dd^4-3ab\left(-9Dc^2+7Cdc-3Bd^2\right)\cdots\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{bc^2-ad^2}\cdots}{3\left(bc^2-a\cdots\right)}\cdots}{\cdots}$$

$$\Big\downarrow \ 600$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(\frac{\left(Ab^2cd\left(29ad^2+3bc^2\right)-a\left(3a^2d^4D-3abd^2\left(-3Bd^2-9c^2D+7cCd\right)-b^2c^2\left(-23Bd^2-2c^2D+11cCd\right)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(Abd\left(5ad^2+3bc^2\right)+a\left(3ad^2\right.\right.}{}\right.}{bc^2-ad^2}$$

$$\Big\downarrow \text{509}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(Ab^2cd\left(29ad^2+3bc^2\right)-a\left(3a^2d^4D-3abd^2\left(-3Bd^2-9c^2D+7cCd\right)-b^2c^2\left(-23Bd^2-2c^2D+11cCd\right)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(Abd\left(5ad^2+3bc^2\right.\right.}{}\right.}{bc^2-ad^2}$$

$$\Big\downarrow \text{508}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab^2cd\left(29ad^2+3bc^2\right)-a\left(3a^2d^4D-3abd^2\left(-3Bd^2-9c^2D+7cCd\right)-b^2c^2\left(-23Bd^2-2c^2D+11cCd\right)\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{bc^2-ad^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(-\frac{\left(bc^2-ad^2\right)\left(Abd\left(5ad^2+3bc^2\right)+a\left(3ad^2(Cd-2cD)+bc\left(-8Bd^2-2c^2D+5cCd\right)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\right.}{}\right.}{bc^2-ad^2}$$

$$\Big\downarrow \text{512}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$b\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(Abd\left(5ad^2+3bc^2\right)+a\left(3ad^2(Cd-2cD)+bc\left(-8Bd^2-2c^2D+5cCd\right)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{}\right.$$

$$bc^2-ad^2$$

$$\Big\downarrow \text{511}$$

$$\frac{a(bBc+aDc-Abd-aCd)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{ab\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{2\sqrt{a-bx^2}\left(Abd\left(3bc^2+5ad^2\right)+a\left(3a(Cd-2cD)d^2+bc\left(-2Dc^2+5Cdc-8Bd^2\right)\right)\right)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2\sqrt{a-bx^2}\left(Ab^2cd\left(3bc^2+29ad^2\right)-a\left(3a^2Dd^4-3ab\left(-9Dc^2+7Cdc-3\right.\right.\right.}{\left(bc^2-ad^2\right)\sqrt{c+dx}}$$

$$\Big\downarrow \text{321}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{ab\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(Abd\left(5ad^2+3bc^2\right)+a\left(3ad^2(Cd-2cD)+bc\left(-8Bd^2-2c^2D+5cCd\right)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{}}{}\right.$$

$$bc^2-ad^2$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(b*B*c - A*b*d - a*C*d + a*c*D) + b*(c*(A*b + a*C) - a*d*(B + (a*D)/b))
*x)/(a*b*(b*c^2 - a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((2*(A*b*d*(3*
b*c^2 + 5*a*d^2) + a*(3*a*d^2*(C*d - 2*c*D) + b*c*(5*c*C*d - 8*B*d^2 - 2*c
^2*D)))*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) + ((2*(A*b^2*
c*d*(3*b*c^2 + 29*a*d^2) - a*(3*a^2*d^4*D - 3*a*b*d^2*(7*c*C*d - 3*B*d^2 -
 9*c^2*D) - b^2*c^2*(11*c*C*d - 23*B*d^2 - 2*c^2*D)))*Sqrt[a - b*x^2])/((b
*c^2 - a*d^2)*Sqrt[c + d*x]) + (b*((-2*Sqrt[a]*(A*b^2*c*d*(3*b*c^2 + 29*a*
d^2) - a*(3*a^2*d^4*D - 3*a*b*d^2*(7*c*C*d - 3*B*d^2 - 9*c^2*D) - b^2*c^2*
(11*c*C*d - 23*B*d^2 - 2*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sq
rt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(A*b*d*(3*b*c^2 + 5*a*d^2) + a
*(3*a*d^2*(C*d - 2*c*D) + b*c*(5*c*C*d - 8*B*d^2 - 2*c^2*D)))*Sqrt[(Sqrt[b
]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])
/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b*c^2 - a*d^2))/(3*(b*c^2 -
a*d^2)))/(2*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(684) = 1368$.

Time = 9.79 (sec) , antiderivative size = 1510, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1510
default	Expression too large to display	10903

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*(-2/3/d^2/(a*d
^2-b*c^2)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)/(x+c/d)^2+2/3*(-b*d*x^2+a*d)/d/(a*d^2-b*c^2)^3*(10*A*b*c*d^3-3*B*a*d^4-
7*B*b*c^2*d^2+6*C*a*c*d^3+4*C*b*c^3*d-9*D*a*c^2*d^2-D*b*c^4)/((x+c/d)*(-b*
d*x^2+a*d))^(1/2)-2*(-b*d*x-b*c)*(-1/2*(3*A*a*b^2*c*d^2+A*b^3*c^3-B*a^2*b*
d^3-3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2+C*a*b^2*c^3-D*a^3*d^3-3*D*a^2*b*c^2*d)
/(a*d^2-b*c^2)^3/a/b*x+1/2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^
3+C*a^2*d^3+3*C*a*b*c^2*d-3*D*a^2*c*d^2-D*a*b*c^3)/(a*d^2-b*c^2)^3/b)/((x^
2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1/3*b/d*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d^2
-b*c^2)^2+1/3*b*c/d*(10*A*b*c*d^3-3*B*a*d^4-7*B*b*c^2*d^2+6*C*a*c*d^3+4*C*
b*c^3*d-9*D*a*c^2*d^2-D*b*c^4)/(a*d^2-b*c^2)^3+(A*a*b*d^2+A*b^2*c^2-2*B*a*
b*c*d+C*a^2*d^2+C*a*b*c^2-2*D*a^2*c*d)/(a*d^2-b*c^2)^2/a-1/2*d*(A*a*b*d^3+
3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d-3*D*a^2*c*d^
2-D*a*b*c^3)/(a*d^2-b*c^2)^3+c*(3*A*a*b^2*c*d^2+A*b^3*c^3-B*a^2*b*d^3-3*B*
a*b^2*c^2*d+3*C*a^2*b*c*d^2+C*a*b^2*c^3-D*a^3*d^3-3*D*a^2*b*c^2*d)/(a*d^2-
b*c^2)^3/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/
d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))+2*(1/3*b*(10*A*b*c*d^3-3*B*a*d^4-7*B*b*c^2*d^2+6*C*a*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2266 vs. $2(692) = 1384$.

Time = 0.22 (sec) , antiderivative size = 2266, normalized size of antiderivative = 3.01

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{5/2} (a - bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm=
"fricas")
```

output
```
1/9*((2*D*a^2*b^2*c^7 + (7*C*a^2*b^2 - 3*A*a*b^3)*c^6*d - 2*(21*D*a^3*b +
11*B*a^2*b^2)*c^5*d^2 + 4*(12*C*a^3*b + 13*A*a^2*b^2)*c^4*d^3 - 6*(4*D*a^4
 + 7*B*a^3*b)*c^3*d^4 + 3*(3*C*a^4 + 5*A*a^3*b)*c^2*d^5 - (2*D*a*b^3*c^5*d
^2 + (7*C*a*b^3 - 3*A*b^4)*c^4*d^3 - 2*(21*D*a^2*b^2 + 11*B*a*b^3)*c^3*d^4
 + 4*(12*C*a^2*b^2 + 13*A*a*b^3)*c^2*d^5 - 6*(4*D*a^3*b + 7*B*a^2*b^2)*c*d
^6 + 3*(3*C*a^3*b + 5*A*a^2*b^2)*d^7)*x^4 - 2*(2*D*a*b^3*c^6*d + (7*C*a*b^
3 - 3*A*b^4)*c^5*d^2 - 2*(21*D*a^2*b^2 + 11*B*a*b^3)*c^4*d^3 + 4*(12*C*a^2
*b^2 + 13*A*a*b^3)*c^3*d^4 - 6*(4*D*a^3*b + 7*B*a^2*b^2)*c^2*d^5 + 3*(3*C*
a^3*b + 5*A*a^2*b^2)*c*d^6)*x^3 - (2*D*a*b^3*c^7 + (7*C*a*b^3 - 3*A*b^4)*c
^6*d - 22*(2*D*a^2*b^2 + B*a*b^3)*c^5*d^2 + (41*C*a^2*b^2 + 55*A*a*b^3)*c^
4*d^3 + 2*(9*D*a^3*b - 10*B*a^2*b^2)*c^3*d^4 - (39*C*a^3*b + 37*A*a^2*b^2)
*c^2*d^5 + 6*(4*D*a^4 + 7*B*a^3*b)*c*d^6 - 3*(3*C*a^4 + 5*A*a^3*b)*d^7)*x^
2 + 2*(2*D*a^2*b^2*c^6*d + (7*C*a^2*b^2 - 3*A*a*b^3)*c^5*d^2 - 2*(21*D*a^3
*b + 11*B*a^2*b^2)*c^4*d^3 + 4*(12*C*a^3*b + 13*A*a^2*b^2)*c^3*d^4 - 6*(4*
D*a^4 + 7*B*a^3*b)*c^2*d^5 + 3*(3*C*a^4 + 5*A*a^3*b)*c*d^6)*x)*sqrt(-b*d)*
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*D*a^2*b^2*c^6*d - (11*C*a^2*b^2 + 3*
A*a*b^3)*c^5*d^2 + (27*D*a^3*b + 23*B*a^2*b^2)*c^4*d^3 - (21*C*a^3*b + 29*
A*a^2*b^2)*c^3*d^4 + 3*(D*a^4 + 3*B*a^3*b)*c^2*d^5 - (2*D*a*b^3*c^4*d^3 -
(11*C*a*b^3 + 3*A*b^4)*c^3*d^4 + (27*D*a^2*b^2 + 23*B*a*b^3)*c^2*d^5 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a-b\,x^2\right)}^{3/2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(dx+c)^{\frac{5}{2}}\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.160 $\int \frac{(c+dx)^{7/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 662

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)(c+dx)^{7/2}}{3ab\left(a-bx^2\right)^{3/2}}$$
$$-\frac{(c+dx)^{5/2}\left(3a(Abd+3aCd+2acD)-\left(4Ab^2c-a(2bcC+7bBd+13adD)\right)x\right)}{6a^2b^2\sqrt{a-bx^2}}$$
$$+\frac{d\left(5Ab\left(4bc^2-5ad^2\right)-a(5bc(2cC+7Bd)+3ad(25Cd+49cD))\right)\sqrt{c+dx}\sqrt{a-bx^2}}{30a^2b^3}$$
$$+\frac{d\left(20Ab^2c-10abcC-35abBd-77a^2dD\right)(c+dx)^{3/2}\sqrt{a-bx^2}}{30a^2b^3}$$
$$+\frac{\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(5b^2c^2(2cC+7Bd)+231a^2d^3D+3abd\left(110cCd+35Bd^2+119c^2D\right)\right)\right)\sqrt{c+dx}}{30a^{3/2}b^{7/2}\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\left(bc^2-ad^2\right)\left(5Ab\left(4bc^2-5ad^2\right)-a(5bc(2cC+7Bd)+3ad(25Cd+49cD))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\right.}{30a^{3/2}b^{7/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(7/2)/a/b/(-b*x^2+a)^(3/2)-1/6*(d*x+
c)^(5/2)*(3*a*(A*b*d+3*C*a*d+2*D*a*c)-(4*A*b^2*c-a*(7*B*b*d+2*C*b*c+13*D*a
*d))*x)/a^2/b^2/(-b*x^2+a)^(1/2)+1/30*d*(5*A*b*(-5*a*d^2+4*b*c^2)-a*(5*b*c
*(7*B*d+2*C*c)+3*a*d*(25*C*d+49*D*c)))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/
b^3+1/30*d*(20*A*b^2*c-35*B*a*b*d-10*C*a*b*c-77*D*a^2*d)*(d*x+c)^(3/2)*(-b
*x^2+a)^(1/2)/a^2/b^3+1/30*(20*A*b^2*c*(-2*a*d^2+b*c^2)-a*(5*b^2*c^2*(7*B*
d+2*C*c)+231*a^2*d^3*D+3*a*b*d*(35*B*d^2+110*C*c*d+119*D*c^2)))*(d*x+c)^(1
/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(7/2)/((d*x+c)
/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/30*(-a*d^2+b*c^2)*(5*A*b*
(-5*a*d^2+4*b*c^2)-a*(5*b*c*(7*B*d+2*C*c)+3*a*d*(25*C*d+49*D*c)))*((d*x+c)
/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/a^(3/2)/b^(7/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.02 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*x)^(7/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*d^2*(5*C*d + 16*c*D))/(15*b^3) - (2*d^3
*D*x)/(5*b^3) + (a*b^2*B*c^3 + 3*a*A*b^2*c^2*d + 3*a^2*b*c^2*C*d + 3*a^2*b
*B*c*d^2 + a^2*A*b*d^3 + a^3*C*d^3 + a^2*b*c^3*D + 3*a^3*c*d^2*D + A*b^3*c
^3*x + a*b^2*c^3*C*x + 3*a*b^2*B*c^2*d*x + 3*a*A*b^2*c*d^2*x + 3*a^2*b*c*C
*d^2*x + a^2*b*B*d^3*x + 3*a^2*b*c^2*d*D*x + a^3*d^3*D*x)/(3*a*b^3*(-a + b
*x^2)^2) + (a*A*b^2*c^2*d + 19*a^2*b*c^2*C*d + 20*a^2*b*B*c*d^2 + 7*a^2*A*
b*d^3 + 13*a^3*C*d^3 + 6*a^2*b*c^3*D + 38*a^3*c*d^2*D - 4*A*b^3*c^3*x + 2*
a*b^2*c^3*C*x + 7*a*b^2*B*c^2*d*x + 8*a*A*b^2*c*d^2*x + 26*a^2*b*c*C*d^2*x
 + 9*a^2*b*B*d^3*x + 25*a^2*b*c^2*d*D*x + 15*a^3*d^3*D*x)/(6*a^2*b^3*(-a +
 b*x^2))) + (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]*(20*A*b^2*c*(b*c^2 - 2*a*d^2) - a*(5*b^2*c^2*(2*c*
C + 7*B*d) + 231*a^2*d^3*D + 3*a*b*d*(110*c*C*d + 35*B*d^2 + 119*c^2*D)))*
(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c
- Sqrt[a]*d)*(20*A*b^2*c*(b*c^2 - 2*a*d^2) - a*(5*b^2*c^2*(2*c*C + 7*B*d)
+ 231*a^2*d^3*D + 3*a*b*d*(110*c*C*d + 35*B*d^2 + 119*c^2*D)))*Sqrt[1 - c/
(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[
a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt
[c + d*x] - (I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(5*A*b^(3/2)*(4*b
*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2) + a*(-5*b^(3/2)*c*(2*c*C + 7*B*...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2176, 27, 2176, 27, 687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \color{blue}{2176}$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-6adDx^2-3(Abd+3aCd+2acD)x+4Abc-\frac{a(2bcC+7bBd+7adD)}{b}\right)}{2\left(a-bx^2\right)^{3/2}}dx}{3ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-6adDx^2-3(Abd+3aCd+2acD)x+4Abc-\frac{a(2bcC+7bBd+7adD)}{b}\right)}{(a-bx^2)^{3/2}}dx}{6ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 2176$$

$$\frac{\frac{\int \frac{d(c+dx)^{3/2}\left(3a(5Abd+15aCd+14acD)-\left(-77dDa^2-10bcCa-35bBda+20Ab^2c\right)x\right)}{2\sqrt{a-bx^2}}dx}{ab}-\frac{(c+dx)^{5/2}(3a(2acD+3aCd+Abd)-x(4Ab^2c-a(13adD+7b}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{d\int \frac{(c+dx)^{3/2}\left(3a(5Abd+15aCd+14acD)-\left(20Ab^2c-a(5b(2cC+7Bd)+77adD)\right)x\right)}{\sqrt{a-bx^2}}dx}{2ab}-\frac{(c+dx)^{5/2}(3a(2acD+3aCd+Abd)-x(4Ab^2c-a(13adD+7bBd}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 687$$

$$\frac{\frac{d\left(\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}\left(20Ab^2c-a(77adD+5b(7Bd+2cC))\right)}{5b}-\frac{2\int -\frac{3\sqrt{c+dx}\left(a\left(5Acdb^2+a\left(77aDd^2+5b\left(14Dc^2+17Cdc+7Bd^2\right)\right)\right)-b\left(5Ab\left(4bc^2-5ad^2\right)-a(5bc\right.\right.}{2\sqrt{a-bx^2}}}{5b}\right.}{2ab}}{6ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{d\left(\frac{3\int \frac{\sqrt{c+dx}\left(a\left(5Acdb^2+a\left(77aDd^2+5b\left(14Dc^2+17Cdc+7Bd^2\right)\right)\right)-b\left(5Ab\left(4bc^2-5ad^2\right)-a(5bc(2cC+7Bd)+3ad(25Cd+49cD))\right)x\right)}{\sqrt{a-bx^2}}dx}{5b}+\frac{2\sqrt{a-bx^2}(c+dx)^{3/2}\left(\right.}{}\right.}{2ab}}{6ab}+\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 687$$

$$d\left(\frac{3\left(\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+dx}\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)-\frac{2\int\frac{b\left(a\left(5Abd\left(bc^2-5ad^2\right)-a\left(3a(25Cd+126cD)d^2+5bc\left(42Dc^2+53Cdc+28\right.\right.\right.\right.}{}}{}\right.}{5b}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{3\left(\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+dx}\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)-\frac{1}{3}\int\frac{a\left(5Abd\left(bc^2-5ad^2\right)-a\left(3a(25Cd+126cD)d^2+5bc\left(42Dc^2+53Cdc+28B\right.\right.\right.}{}\right.}{5b}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 600$$

$$d\left(\frac{3\left(\frac{1}{3}\left(\frac{\left(bc^2-ad^2\right)\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(231a^2d^3D+3abd\left(35Bd^2+119\right.\right.\right.}{d}\right.\right.}{5b}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 509$$

$$d\left(\frac{3\left(\frac{1}{3}\left(\frac{\left(bc^2-ad^2\right)\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(231a^2d^3D+3abd\left(35\right.\right.\right.}{d\sqrt{}}\right.\right.}{5b}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$d\left(\frac{3\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(231a^2d^3D+3abd\left(35Bd^2+119c^2D+110cCd\right)+5b^2c^2(7Bd+2cC)\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\left(b\right.}{}\right.\right.}{}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$d\left(3\left(\frac{1}{3}\left(\frac{\left(bc^2-ad^2\right)\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\cdots}\right.\right.\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 512$$

$$d\left(3\left(\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5Ab\left(4bc^2-5ad^2\right)-a(3ad(49cD+25Cd)+5bc(7Bd+2cC))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\cdots\right)}{\cdots}\right.\right.\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 511$$

$$d\left(\frac{3\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(231a^2d^3D+3abd\left(35Bd^2+119c^2D+110cCd\right)+5b^2c^2(7Bd+2cC)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\right.\right.}{}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 321}$$

$$d\left(\frac{3\left(\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(20Ab^2c\left(bc^2-2ad^2\right)-a\left(231a^2d^3D+3abd\left(35Bd^2+119c^2D+110cCd\right)+5b^2c^2(7Bd+2cC)\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\right.\right.}{}\right.$$

$$\frac{(c+dx)^{7/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

input
```
Int[((c + d*x)^(7/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*(c + d*x)^(7/2))/(3*a*b*(a - b*x^2)^(3/
2)) + (-(((c + d*x)^(5/2)*(3*a*(A*b*d + 3*a*C*d + 2*a*c*D) - (4*A*b^2*c -
a*(2*b*c*C + 7*b*B*d + 13*a*d*D))*x))/(a*b*Sqrt[a - b*x^2])) + (d*((2*(20*
A*b^2*c - a*(5*b*(2*c*C + 7*B*d) + 77*a*d*D))*(c + d*x)^(3/2)*Sqrt[a - b*x
^2])/(5*b) + (3*((2*(5*A*b*(4*b*c^2 - 5*a*d^2) - a*(5*b*c*(2*c*C + 7*B*d)
+ 3*a*d*(25*C*d + 49*c*D)))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + ((2*Sqrt[a]
*(20*A*b^2*c*(b*c^2 - 2*a*d^2) - a*(5*b^2*c^2*(2*c*C + 7*B*d) + 231*a^2*d^
3*D + 3*a*b*d*(110*c*C*d + 35*B*d^2 + 119*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 -
(b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/
((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(5*A*b*(4*b*c
^2 - 5*a*d^2) - a*(5*b*c*(2*c*C + 7*B*d) + 3*a*d*(25*C*d + 49*c*D)))*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
 + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/3))/(5*b)))/(2*a*b))/(6
*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1570 vs. $2(584) = 1168$.

Time = 7.58 (sec) , antiderivative size = 1571, normalized size of antiderivative = 2.37

method	result	size
elliptic	Expression too large to display	1571
default	Expression too large to display	9889

input
```
int((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((1/3*(3*A*a*b
^2*c*d^2+A*b^3*c^3+B*a^2*b*d^3+3*B*a*b^2*c^2*d+3*C*a^2*b*c*d^2+C*a*b^2*c^3
+D*a^3*d^3+3*D*a^2*b*c^2*d)/a/b^5*x+1/3*(A*a*b*d^3+3*A*b^2*c^2*d+3*B*a*b*c
*d^2+B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d+3*D*a^2*c*d^2+D*a*b*c^3)/b^5)*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12*(8*A*a*b^2
*c*d^2-4*A*b^3*c^3+9*B*a^2*b*d^3+7*B*a*b^2*c^2*d+26*C*a^2*b*c*d^2+2*C*a*b^
2*c^3+15*D*a^3*d^3+25*D*a^2*b*c^2*d)/a^2/b^4*x-1/12*(7*A*a*b*d^3+A*b^2*c^2
*d+20*B*a*b*c*d^2+13*C*a^2*d^3+19*C*a*b*c^2*d+38*D*a^2*c*d^2+6*D*a*b*c^3)/
a/b^4)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-2/5*D*d^3/b^3*x*(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)-2/3*(1/b^2*d^3*(C*d+4*D*c)-4/5*D*d^3/b^2*c)/b/d*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)+2*(d*(A*b*d^3+4*B*b*c*d^2+2*C*a*d^3+6*C*b*c^2*d+8*
D*a*c*d^2+4*D*b*c^3)/b^3-1/6/b^3*(7*A*a^2*b*d^4+9*A*a*b^2*c^2*d^2-4*A*b^3*
c^4+29*B*a^2*b*c*d^3+7*B*a*b^2*c^3*d+13*C*a^3*d^4+45*C*a^2*b*c^2*d^2+2*C*a
*b^2*c^4+53*D*a^3*c*d^3+31*D*a^2*b*c^3*d)/a^2+1/12/b^3*d*(7*A*a*b*d^3+A*b^
2*c^2*d+20*B*a*b*c*d^2+13*C*a^2*d^3+19*C*a*b*c^2*d+38*D*a^2*c*d^2+6*D*a*b*
c^3)/a+1/6/b^3*c*(8*A*a*b^2*c*d^2-4*A*b^3*c^3+9*B*a^2*b*d^3+7*B*a*b^2*c^2*
d+26*C*a^2*b*c*d^2+2*C*a*b^2*c^3+15*D*a^3*d^3+25*D*a^2*b*c^2*d)/a^2+2/5*D*
d^3/b^3*a*c+1/3*(1/b^2*d^3*(C*d+4*D*c)-4/5*D*d^3/b^2*c)/b*a)*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(588) = 1176$.

Time = 0.12 (sec) , antiderivative size = 1279, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/90*((10*(C*a^3*b^2 - 2*A*a^2*b^3)*c^4 - 7*(39*D*a^4*b - 5*B*a^3*b^2)*c^3
*d - 5*(93*C*a^4*b - 11*A*a^3*b^2)*c^2*d^2 - 21*(43*D*a^5 + 15*B*a^4*b)*c*
d^3 - 75*(3*C*a^5 + A*a^4*b)*d^4 + (10*(C*a*b^4 - 2*A*b^5)*c^4 - 7*(39*D*a
^2*b^3 - 5*B*a*b^4)*c^3*d - 5*(93*C*a^2*b^3 - 11*A*a*b^4)*c^2*d^2 - 21*(43
*D*a^3*b^2 + 15*B*a^2*b^3)*c*d^3 - 75*(3*C*a^3*b^2 + A*a^2*b^3)*d^4)*x^4 -
 2*(10*(C*a^2*b^3 - 2*A*a*b^4)*c^4 - 7*(39*D*a^3*b^2 - 5*B*a^2*b^3)*c^3*d
- 5*(93*C*a^3*b^2 - 11*A*a^2*b^3)*c^2*d^2 - 21*(43*D*a^4*b + 15*B*a^3*b^2)
*c*d^3 - 75*(3*C*a^4*b + A*a^3*b^2)*d^4)*x^2)*sqrt(-b*d)*weierstrassPInver
se(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(
3*d*x + c)/d) + 3*(10*(C*a^3*b^2 - 2*A*a^2*b^3)*c^3*d + 7*(51*D*a^4*b + 5*
B*a^3*b^2)*c^2*d^2 + 10*(33*C*a^4*b + 4*A*a^3*b^2)*c*d^3 + 21*(11*D*a^5 +
5*B*a^4*b)*d^4 + (10*(C*a*b^4 - 2*A*b^5)*c^3*d + 7*(51*D*a^2*b^3 + 5*B*a*b
^4)*c^2*d^2 + 10*(33*C*a^2*b^3 + 4*A*a*b^4)*c*d^3 + 21*(11*D*a^3*b^2 + 5*B
*a^2*b^3)*d^4)*x^4 - 2*(10*(C*a^2*b^3 - 2*A*a*b^4)*c^3*d + 7*(51*D*a^3*b^2
 + 5*B*a^2*b^3)*c^2*d^2 + 10*(33*C*a^3*b^2 + 4*A*a^2*b^3)*c*d^3 + 21*(11*D
*a^4*b + 5*B*a^3*b^2)*d^4)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3
*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
+ c)/d)) - 3*(12*D*a^2*b^3*d^4*x^5 + 10*(2*D*a^3*b^2 - B*a^2*b^3)*c^3*d +
5*(13*C*a^3*b^2 - 5*A*a^2*b^3)*c^2*d^2 + 14*(16*D*a^4*b + 5*B*a^3*b^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(7/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{7/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(7/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{7/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{7}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(7/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{7/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^{7/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a-b\,x^2)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(7/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2),x)
```

output
```
int(((c + d*x)^(7/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{7/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(dx+c)^{\frac{7}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{\left(-b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^(7/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

3.161 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 533

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)(c+dx)^{5/2}}{3ab\left(a-bx^2\right)^{3/2}}$$
$$-\frac{(c+dx)^{3/2}\left(a(Abd+7aCd+6acD)-\left(4Ab^2c-a(2bcC+5bBd+11adD)\right)x\right)}{6a^2b^2\sqrt{a-bx^2}}$$
$$+\frac{d\left(4Ab^2c-a(2bcC+5bBd+15adD)\right)\sqrt{c+dx}\sqrt{a-bx^2}}{6a^2b^3}$$
$$+\frac{\left(Ab(4bc^2-3ad^2)-a(bc(2cC+5Bd)+3ad(7Cd+15cD))\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{\cdots}}{\sqrt{b}c+\cdots}\right.\cdots}{6a^{3/2}b^{5/2}\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(2bcC+5bBd+15adD)\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}\cdots}\right.\cdots}{6a^{3/2}b^{7/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(5/2)/a/b/(-b*x^2+a)^(3/2)-1/6*(d*x+
c)^(3/2)*(a*(A*b*d+7*C*a*d+6*D*a*c)-(4*A*b^2*c-a*(5*B*b*d+2*C*b*c+11*D*a*d
))*x)/a^2/b^2/(-b*x^2+a)^(1/2)+1/6*d*(4*A*b^2*c-a*(5*B*b*d+2*C*b*c+15*D*a*
d))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/b^3+1/6*(A*b*(-3*a*d^2+4*b*c^2)-a*(
b*c*(5*B*d+2*C*c)+3*a*d*(7*C*d+15*D*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(5/2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))
^(1/2)/(-b*x^2+a)^(1/2)-1/6*(-a*d^2+b*c^2)*(4*A*b^2*c-a*(5*B*b*d+2*C*b*c+1
5*D*a*d))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/a^(3/2)/b^(7/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.70 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(-15a^4d^2D-4Ab^4c^2x^3+ab^3x\left(c(2cC+5Bd)x^2+A\left(6c^2+cdx+\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(-15*a^4*d^2*D - 4*A*b^4*c^2*x^3 + a*b^3*x*(c
*(2*c*C + 5*B*d)*x^2 + A*(6*c^2 + c*d*x + 3*d^2*x^2)) - a^3*b*(4*c^2*D + c
*d*(9*C + 13*D*x) + d^2*(5*B + 7*x*(C - 3*D*x))) + a^2*b^2*(A*d*(3*c - d*x
) + B*(2*c^2 - c*d*x + 7*d^2*x^2) + x^2*(6*c^2*D + d^2*x*(9*C - 4*D*x) + c
*d*(13*C + 17*D*x)))))/(a^2*b^3*(a - b*x^2)^2) - (d^2*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(A*b*(4*b*c^2 - 3*a*d^2) - a*(b*c*(2*c*C + 5*B*d) + 3*a*d*(7*C*
d + 15*c*D)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*(4*b*c^
2 - 3*a*d^2) - a*(b*c*(2*c*C + 5*B*d) + 3*a*d*(7*C*d + 15*c*D)))*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a
]*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*(4*b^2*c + 3*Sqrt[a]*b^(3/2)*d) + a*(-(b*(2
*c*C + 5*B*d)) - 15*a*d*D + 3*Sqrt[a]*Sqrt[b]*(7*C*d + 10*c*D)))*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a^2*b^3*
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(6*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2176, 27, 2176, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(-6adDx^2-(Abd+7aCd+6acD)x+4Abc-\frac{a(2bcC+5bBd+5adD)}{b}\right)}{2\left(a-bx^2\right)^{3/2}}dx}{3ab}+\frac{(c+dx)^{5/2} \left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(-6adDx^2-(Abd+7aCd+6acD)x+4Abc-\frac{a(2bcC+5bBd+5adD)}{b}\right)}{(a-bx^2)^{3/2}}dx}{6ab}+$$
$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2176$$

$$\frac{\frac{\int \frac{3d\sqrt{c+dx}\left(a(Abd+7aCd+10acD)-\left(4Ab^2c-a(2bcC+5bBd+15adD)\right)x\right)}{2\sqrt{a-bx^2}}dx}{ab}-\frac{(c+dx)^{3/2}\left(a(6acD+7aCd+Abd)-x\left(4Ab^2c-a(11adD+5bBd+2bcC)\right)\right)}{ab\sqrt{a-bx^2}}}{6ab}$$
$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3d\int \frac{\sqrt{c+dx}\left(a(Abd+7aCd+10acD)-\left(4Ab^2c-a(2bcC+5bBd+15adD)\right)x\right)}{\sqrt{a-bx^2}}dx}{2ab}-\frac{(c+dx)^{3/2}\left(a(6acD+7aCd+Abd)-x\left(4Ab^2c-a(11adD+5bBd+2bcC)\right)\right)}{ab\sqrt{a-bx^2}}}{6ab}$$
$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{2\int \frac{a\left(Ab^2cd-a\left(15aDd^2+b\left(30Dc^2+23Cdc+5Bd^2\right)\right)\right)+b\left(Ab\left(4bc^2-3ad^2\right)-a(bc(2cC+5Bd)+3ad(7C\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\right.}{2ab}}{6ab}$$
$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{\int \frac{a\left(Ab^2cd-a\left(15aDd^2+b\left(30Dc^2+23Cdc+5Bd^2\right)\right)\right)+b\left(Ab\left(4bc^2-3ad^2\right)-a(bc(2cC+5Bd)+3ad(7Cd\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}\right.}{2ab}}{6ab}$$
$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 600$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{\frac{b\left(Ab\left(4bc^2-3ad^2\right)-a(3ad(15cD+7Cd)+bc(5Bd+2cC))\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(15ad\right.}{}}{3b}\right.}{2ab}$$

$$6ab$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{509}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(4bc^2-3ad^2\right)-a(3ad(15cD+7Cd)+bc(5Bd+2cC))\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2\right.}{}}{3b}\right.}{2ab}$$

$$6ab$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(4b\right.\right.}{}}{3b}\right.}{2ab}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\Big(\text{arcs}}{}}{3b}}{2ab}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{512}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+}}{}}{3b}}{2ab}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{511}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\Big(\frac{\sqrt{b}}{\sqrt{a}}}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{}}{2ab}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{3d\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(4Ab^2c-a(15adD+5bBd+2bcC)\right)}{3b}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(4Ab^2c-a(15adD+5b\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{\right.}}{2ab}$$

$$\frac{(c+dx)^{5/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

input

```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output

```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*(c + d*x)^(5/2))/(3*a*b*(a - b*x^2)^(3/
2)) + (-(((c + d*x)^(3/2)*(a*(A*b*d + 7*a*C*d + 6*a*c*D) - (4*A*b^2*c - a*
(2*b*c*C + 5*b*B*d + 11*a*d*D))*x))/(a*b*Sqrt[a - b*x^2])) + (3*d*((2*(4*A
*b^2*c - a*(2*b*c*C + 5*b*B*d + 15*a*d*D))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/
(3*b) - ((-2*Sqrt[a]*Sqrt[b]*(A*b*(4*b*c^2 - 3*a*d^2) - a*(b*c*(2*c*C + 5*
B*d) + 3*a*d*(7*C*d + 15*c*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a
] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x
^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(4*A*b^2*c - a*(2*b*c*C + 5*b*B*d + 15*a
*d*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b
]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b)))/(2*
a*b))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(461) = 922$.

Time = 7.16 (sec) , antiderivative size = 1184, normalized size of antiderivative = 2.22

method	result	size
elliptic	Expression too large to display	1184
default	Expression too large to display	7619

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((1/3*(A*a*b*d
^2+A*b^2*c^2+2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2+2*D*a^2*c*d)/b^4/a*x+1/3*(2*A
*b^2*c*d+B*a*b*d^2+B*b^2*c^2+2*C*a*b*c*d+D*a^2*d^2+D*a*b*c^2)/b^5)*(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12*(3*A*a*b*d^2
-4*A*b^2*c^2+5*B*a*b*c*d+9*C*a^2*d^2+2*C*a*b*c^2+17*D*a^2*c*d)/b^3/a^2*x-1
/12*(A*b^2*c*d+7*B*a*b*d^2+13*C*a*b*c*d+13*D*a^2*d^2+6*D*a*b*c^2)/a/b^4)/(
(x^2-a/b)*(-b*d*x-b*c))^(1/2)-2/3/b^3*d^2*D*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(
1/2)+2*(d*(B*b*d^2+3*C*b*c*d+2*D*a*d^2+3*D*b*c^2)/b^3-1/6/b^3*(4*A*a*b^2*c
*d^2-4*A*b^3*c^3+7*B*a^2*b*d^3+5*B*a*b^2*c^2*d+22*C*a^2*b*c*d^2+2*C*a*b^2*
c^3+13*D*a^3*d^3+23*D*a^2*b*c^2*d)/a^2+1/12/b^3*d*(A*b^2*c*d+7*B*a*b*d^2+1
3*C*a*b*c*d+13*D*a^2*d^2+6*D*a*b*c^2)/a+1/6/b^2*c*(3*A*a*b*d^2-4*A*b^2*c^2
+5*B*a*b*c*d+9*C*a^2*d^2+2*C*a*b*c^2+17*D*a^2*c*d)/a^2+1/3/b^3*d^3*D*a)*(c
/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1
/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1
/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
)+2*(1/b^2*d^2*(C*d+3*D*c)+1/12*(3*A*a*b*d^2-4*A*b^2*c^2+5*B*a*b*c*d+9*C*a
^2*d^2+2*C*a*b*c^2+17*D*a^2*c*d)*d/a^2/b^2-2/3/b^2*d^2*D*c)*(c/d-1/b*(a*b)
^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(465) = 930$.

Time = 0.14 (sec) , antiderivative size = 987, normalized size of antiderivative = 1.85

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/18*(((2*(C*a*b^4 - 2*A*b^5)*c^3 - 5*(9*D*a^2*b^3 - B*a*b^4)*c^2*d - 6*(8
*C*a^2*b^3 - A*a*b^4)*c*d^2 - 15*(3*D*a^3*b^2 + B*a^2*b^3)*d^3)*x^4 + 2*(C
*a^3*b^2 - 2*A*a^2*b^3)*c^3 - 5*(9*D*a^4*b - B*a^3*b^2)*c^2*d - 6*(8*C*a^4
*b - A*a^3*b^2)*c*d^2 - 15*(3*D*a^5 + B*a^4*b)*d^3 - 2*(2*(C*a^2*b^3 - 2*A
*a*b^4)*c^3 - 5*(9*D*a^3*b^2 - B*a^2*b^3)*c^2*d - 6*(8*C*a^3*b^2 - A*a^2*b
^3)*c*d^2 - 15*(3*D*a^4*b + B*a^3*b^2)*d^3)*x^2)*sqrt(-b*d)*weierstrassPIn
verse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/
3*(3*d*x + c)/d) + 3*((2*(C*a*b^4 - 2*A*b^5)*c^2*d + 5*(9*D*a^2*b^3 + B*a*
b^4)*c*d^2 + 3*(7*C*a^2*b^3 + A*a*b^4)*d^3)*x^4 + 2*(C*a^3*b^2 - 2*A*a^2*b
^3)*c^2*d + 5*(9*D*a^4*b + B*a^3*b^2)*c*d^2 + 3*(7*C*a^4*b + A*a^3*b^2)*d^
3 - 2*(2*(C*a^2*b^3 - 2*A*a*b^4)*c^2*d + 5*(9*D*a^3*b^2 + B*a^2*b^3)*c*d^2
 + 3*(7*C*a^3*b^2 + A*a^2*b^3)*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b
*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPIn
verse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/
3*(3*d*x + c)/d)) - 3*(4*D*a^2*b^3*d^3*x^4 + 2*(2*D*a^3*b^2 - B*a^2*b^3)*c
^2*d + 3*(3*C*a^3*b^2 - A*a^2*b^3)*c*d^2 + 5*(3*D*a^4*b + B*a^3*b^2)*d^3 -
 (2*(C*a*b^4 - 2*A*b^5)*c^2*d + (17*D*a^2*b^3 + 5*B*a*b^4)*c*d^2 + 3*(3*C*
a^2*b^3 + A*a*b^4)*d^3)*x^3 - (6*D*a^2*b^3*c^2*d + (13*C*a^2*b^3 + A*a*b^4
)*c*d^2 + 7*(3*D*a^3*b^2 + B*a^2*b^3)*d^3)*x^2 - (6*A*a*b^4*c^2*d - (13*D*
a^3*b^2 + B*a^2*b^3)*c*d^2 - (7*C*a^3*b^2 + A*a^2*b^3)*d^3)*x)*sqrt(-b*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(5/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{{\left(c+d\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a-b\,x^2\right)}^{5/2}}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

3.162 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 461

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)(c+dx)^{3/2}}{3ab\left(a-bx^2\right)^{3/2}}$$
$$+\frac{\sqrt{c+dx}\left(a(Abd-5aCd-6acD)+\left(4Ab^2c-a(2bcC+3bBd+9adD)\right)x\right)}{6a^2b^2\sqrt{a-bx^2}}$$
$$+\frac{\left(4Ab^2c-a(2bcC+3bBd+21adD)\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{5/2}\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\left(Ab(4bc^2-ad^2)-a(bc(2cC+3Bd)-ad(5Cd-3cD))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{6a^{3/2}b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(3/2)/a/b/(-b*x^2+a)^(3/2)+1/6*(d*x+
c)^(1/2)*(a*(A*b*d-5*C*a*d-6*D*a*c)+(4*A*b^2*c-a*(3*B*b*d+2*C*b*c+9*D*a*d)
)*x)/a^2/b^2/(-b*x^2+a)^(1/2)+1/6*(4*A*b^2*c-a*(3*B*b*d+2*C*b*c+21*D*a*d))
*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(5/
2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/6*(A*b*(-a*d^2
+4*b*c^2)-a*(b*c*(3*B*d+2*C*c)-a*d*(5*C*d-3*D*c)))*((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(5/
2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.02 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(-4Ab^3cx^3-a^3(5Cd+4cD+7dDx)+ab^2x\left((2cC+3Bd)x^2+A\right.\right.}{a^2b^2(a-bx^2)}\right.}{}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(-4*A*b^3*c*x^3 - a^3*(5*C*d + 4*c*D + 7*d*D*
x) + a*b^2*x*((2*c*C + 3*B*d)*x^2 + A*(6*c + d*x)) + a^2*b*(A*d + B*(2*c -
 d*x) + x^2*(7*C*d + 6*c*D + 9*d*D*x))))/(a^2*b^2*(a - b*x^2)^2) + (d^2*Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]*(4*A*b^2*c - a*(2*b*c*C + 3*b*B*d + 21*a*d*D)
)*(-a + b*x^2) - I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(4*A*b^2*c - a*(2*b*c*C
 + 3*b*B*d + 21*a*d*D))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-((
(Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(S
qrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(A*(4*b^2*c - Sqrt[a]*b^(3/2)
*d) + a*(-(b*(2*c*C + 3*B*d)) - 21*a*d*D + Sqrt[a]*Sqrt[b]*(5*C*d + 18*c*D
)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
- d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d
)])/(a^2*b^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(6*Sqrt[c +
d*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2176, 27, 2176, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-6adDx^2+(Abd-5aCd-6acD)x+4Abc-\frac{a(2bcC+3bBd+3adD)}{b}\right)}{2\left(a-bx^2\right)^{3/2}}dx}{3ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-6adDx^2+(Abd-5aCd-6acD)x+4Abc-\frac{a(2bcC+3bBd+3adD)}{b}\right)}{(a-bx^2)^{3/2}}dx}{6ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2176}$$

$$\frac{\frac{\int -\frac{d\left(a(Abd-5aCd-18acD)+\left(4Ab^2c-a(2bcC+3bBd+21adD)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab}+\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}}{6ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\int \frac{a(Abd-5aCd-18acD)+\left(4Ab^2c-a(2bcC+3bBd+21adD)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab}}{6ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{\left(4Ab^2c-a(21adD+3bBd+2bcC)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(Ab\left(4bc^2-ad^2\right)-a(bc(\cdots\right.}{\cdots}\right.}{2ab}}{6ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4Ab^2c-a(21adD+3bBd+2bcC)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(Ab\left(4bc^2-ad\cdots\right.\right.}{\cdots}\right.}{2ab}}{6ab}+\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\,(a-bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 508}$$

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4Ab^2c-a(21adD+3bBd+2bcC)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{6ab}}{}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 327

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\left(Ab\left(4bc^2-ad^2\right)-a(bc(3Bd+2cC)-ad(5Cd-3cD))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right.}{6ab}}{}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 512

$$\frac{\frac{\sqrt{c+dx}\left(x\left(4Ab^2c-a(9adD+3bBd+2bcC)\right)+a(-6acD-5aCd+Abd)\right)}{ab\sqrt{a-bx^2}}-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(4bc^2-ad^2\right)-a(bc(3Bd+2cC)-ad(5Cd-3cD))\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\cdots}}}{d\sqrt{a-bx^2}}\right.}{6ab}}{}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$\downarrow$ 511

$$\frac{\frac{\sqrt{c+dx}(x(4Ab^2c-a(9adD+3bBd+2bcC))+a(-6acD-5aCd+Abd))}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(Ab\left(4bc^2-ad^2\right)-a(bc(3Bd+2cC)-ad(5Cd-3cD\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{6ab}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{c+dx}(x(4Ab^2c-a(9adD+3bBd+2bcC))+a(-6acD-5aCd+Abd))}{ab\sqrt{a-bx^2}}-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(Ab\left(4\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{6ab}$$

$$\frac{(c+dx)^{3/2}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

input

```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output

```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*(c + d*x)^(3/2))/(3*a*b*(a - b*x^2)^(3/
2)) + ((Sqrt[c + d*x]*(a*(A*b*d - 5*a*C*d - 6*a*c*D) + (4*A*b^2*c - a*(2*b
*c*C + 3*b*B*d + 9*a*d*D))*x))/(a*b*Sqrt[a - b*x^2]) - (d*((-2*Sqrt[a]*(4*
A*b^2*c - a*(2*b*c*C + 3*b*B*d + 21*a*d*D))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt
[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(A*b*(4*b*c^2 - a*d^2) - a*(b*c*(2*c
*C + 3*B*d) - a*d*(5*C*d - 3*c*D)))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt
[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*
Sqrt[a - b*x^2])))/(2*a*b))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(395) = 790$.

Time = 6.80 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.95

method	result
elliptic	$\frac{\sqrt{(dx+c)(-bx^2+a)}\left(\frac{\left(\frac{(Ab^2c+Babd+Cabc+a^2dD)x}{3ab^4}+\frac{Abd+Bbc+Cad+Dac}{3b^4}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{\left(x^2-\frac{a}{b}\right)^2}-\frac{2(-bdx-bc)\left(\frac{(4Ab^2c-3Babd-\cdots}{12a^2\cdots}\right.}{\sqrt{\left(x^2-\cdots\right.}}\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((1/3*(A*b^2*c
+B*a*b*d+C*a*b*c+D*a^2*d)/a/b^4*x+1/3*(A*b*d+B*b*c+C*a*d+D*a*c)/b^4)*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12*(4*A*b^2*c-
3*B*a*b*d-2*C*a*b*c-9*D*a^2*d)/a^2/b^3*x-1/12*(A*b*d+7*C*a*d+6*D*a*c)/a/b^
3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(d*(C*d+2*D*c)/b^2-1/6/b^2*(A*a*b*d^2-
4*A*b^2*c^2+3*B*a*b*c*d+7*C*a^2*d^2+2*C*a*b*c^2+15*D*a^2*c*d)/a^2+1/12/b^2
*d*(A*b*d+7*C*a*d+6*D*a*c)/a-1/6/b^2*c*(4*A*b^2*c-3*B*a*b*d-2*C*a*b*c-9*D*
a^2*d)/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/
d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))+2*(D*d^2/b^2-1/12*(4*A*b^2*c-3*B*a*b*d-2*C*a*b*c-9*D*a^2*d
)*d/a^2/b^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c
/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(
a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/18*((((2*(C*a*b^3 - 2*A*b^4)*c^2 - 3*(11*D*a^2*b^2 - B*a*b^3)*c*d - 3*(5*
C*a^2*b^2 - A*a*b^3)*d^2)*x^4 + 2*(C*a^3*b - 2*A*a^2*b^2)*c^2 - 3*(11*D*a^
4 - B*a^3*b)*c*d - 3*(5*C*a^4 - A*a^3*b)*d^2 - 2*(2*(C*a^2*b^2 - 2*A*a*b^3
)*c^2 - 3*(11*D*a^3*b - B*a^2*b^2)*c*d - 3*(5*C*a^3*b - A*a^2*b^2)*d^2)*x^
2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*((2*(C*a*b^3 - 2*A*b^4)*c
*d + 3*(7*D*a^2*b^2 + B*a*b^3)*d^2)*x^4 + 2*(C*a^3*b - 2*A*a^2*b^2)*c*d +
3*(7*D*a^4 + B*a^3*b)*d^2 - 2*(2*(C*a^2*b^2 - 2*A*a*b^3)*c*d + 3*(7*D*a^3*
b + B*a^2*b^2)*d^2)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)
) + 3*((2*(C*a*b^3 - 2*A*b^4)*c*d + 3*(3*D*a^2*b^2 + B*a*b^3)*d^2)*x^3 - 2
*(2*D*a^3*b - B*a^2*b^2)*c*d - (5*C*a^3*b - A*a^2*b^2)*d^2 + (6*D*a^2*b^2*
c*d + (7*C*a^2*b^2 + A*a*b^3)*d^2)*x^2 + (6*A*a*b^3*c*d - (7*D*a^3*b + B*a
^2*b^2)*d^2)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^2*b^5*d*x^4 - 2*a^3*b^4
*d*x^2 + a^4*b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^(3/2)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(a-b\,x^2\right)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

3.163 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 532

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \frac{\left(a\left(B+\frac{aD}{b}\right)+(Ab+aC)x\right)\sqrt{c+dx}}{3ab\,(a-bx^2)^{3/2}}$$

$$-\frac{\sqrt{c+dx}(a(Ab^2cd-a(7ad^2D-b(cCd-Bd^2+6c^2D)))-b(Ab(4bc^2-3ad^2)-a(bc(2cC+Bd)-ad(3}{6a^2b^2\,(bc^2-ad^2)\sqrt{a-bx^2}}$$

$$+\frac{(Ab(4bc^2-3ad^2)-a(bc(2cC+Bd)-ad(3Cd-cD)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{3/2}\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{(4Ab^2c-a(b(2cC+Bd)-5adD))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{5/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(a*(B+a*D/b)+(A*b+C*a)*x)*(d*x+c)^(1/2)/a/b/(-b*x^2+a)^(3/2)-1/6*(d*x+
c)^(1/2)*(a*(A*b^2*c*d-a*(7*a*d^2*D-b*(-B*d^2+C*c*d+6*D*c^2)))-b*(A*b*(-3*
a*d^2+4*b*c^2)-a*(b*c*(B*d+2*C*c)-a*d*(3*C*d-D*c)))*x)/a^2/b^2/(-a*d^2+b*c
^2)/(-b*x^2+a)^(1/2)+1/6*(A*b*(-3*a*d^2+4*b*c^2)-a*(b*c*(B*d+2*C*c)-a*d*(3
*C*d-D*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^
(3/2)/b^(3/2)/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2
+a)^(1/2)-1/6*(4*A*b^2*c-a*(b*(B*d+2*C*c)-5*D*a*d))*((d*x+c)/(c+a^(1/2)*d/
b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(5
/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.43 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(2a\left(bc^2-ad^2\right)\left(a^2D+Ab^2x+ab(B+Cx)\right)-\left(a-bx^2\right)\left(-7a^3d^2D-4\right.\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(2*a*(b*c^2 - a*d^2)*(a^2*D + A*b^2*x + a*b*(
B + C*x)) - (a - b*x^2)*(-7*a^3*d^2*D - 4*A*b^3*c^2*x + a*b^2*(c*(2*c*C +
B*d)*x + A*d*(c + 3*d*x)) + a^2*b*(6*c^2*D - d^2*(B + 3*C*x) + c*d*(C + D*
x)))))/(a - b*x^2)^2 + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(A*b*(4*b*c^2 -
 3*a*d^2) - a*(b*c*(2*c*C + B*d) + a*d*(-3*C*d + c*D)))*(a - b*x^2) + I*Sq
rt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*(4*b*c^2 - 3*a*d^2) - a*(b*c*(2*c*C + B
*d) + a*d*(-3*C*d + c*D)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)
/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*(4*b^2*
c + 3*Sqrt[a]*b^(3/2)*d) + a*(-(b*(2*c*C + B*d)) + 5*a*d*D - 3*Sqrt[a]*Sqr
t[b]*(C*d - 2*c*D)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sq
rt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt
[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(6*a
^2*b^2*(b*c^2 - a*d^2)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2176, 27, 2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{\int \frac{-6adDx^2+3(Abd-aCd-2acD)x+4Abc-\frac{a(2bcC+bBd+adD)}{b}}{2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{3ab}+\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-6adDx^2+3(Abd-aCd-2acD)x+4Abc-\frac{a(2bcC+bBd+adD)}{b}}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6ab}+\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2180}$$

$$\frac{\frac{\int -\frac{d\left(a\left(Acdb^2+a\left(5aDd^2+b\left(-6Dc^2+Cdc-Bd^2\right)\right)\right)+b\left(Ab\left(4bc^2-3ad^2\right)-a(bc(2cC+Bd)-ad(3Cd-cD))\right)x\right)}{2b\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\left(a\left(-\frac{d^2(7aD+bB)}{b}+6c^2D+\right.\right.\right.}{}}{6ab}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{-\frac{d\int \frac{a\left(Acdb^2+a\left(5aDd^2+b\left(-6Dc^2+Cdc-Bd^2\right)\right)\right)+b\left(Ab\left(4bc^2-3ad^2\right)-a(bc(2cC+Bd)-ad(3Cd-cD))\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\left(a\left(-\frac{d^2(7aD+bB)}{b}+6c^2D+c\right.\right.\right.}{}}{6ab}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 600$$

$$\frac{-\frac{d\left(\frac{b\left(Ab\left(4bc^2-3ad^2\right)-a(bc(Bd+2cC)-ad(3Cd-cD))\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\right.}{}}{6ab}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 509$$

$$\frac{-\frac{d\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(4bc^2-3ad^2\right)-a(bc(Bd+2cC)-ad(3Cd-cD))\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ab(bc^2-ad^2)}-\frac{\sqrt{}}{}}{6ab}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow 508$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(4bc^2-3ad^2\right)-a(bc(Bd+2cC)-ad(3Cd-cD))\right)\int\frac{\sqrt{1-\ldots}}{\sqrt{\frac{1}{2}(\ldots}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right.}{2ab(bc^2-ad^2)}$$

$$6ab$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(Ab\left(4bc^2-3ad^2\right)-a(bc(\ldots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right.}{2ab(bc^2-ad^2)}$$

$$6ab$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(Ab\left(4bc^2-3ad\ldots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}\right.}{2ab(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{511}$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{ar}\right.}{}\right.}{2ab(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(4Ab^2c-a(-5adD+bBd+2bcC)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1}}{}\right.\right.}{}\right.}{2ab(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(aC+Ab)+a\left(\frac{aD}{b}+B\right)\right)}{3ab\left(a-bx^2\right)^{3/2}}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a - b*x^2)^(5/2),x]
```

output
```
((a*(B + (a*D)/b) + (A*b + a*C)*x)*Sqrt[c + d*x])/(3*a*b*(a - b*x^2)^(3/2)
) + (-((Sqrt[c + d*x]*(a*(A*b*c*d + a*(c*C*d + 6*c^2*D - (d^2*(b*B + 7*a*D
))/b)) - (A*b*(4*b*c^2 - 3*a*d^2) - a*(b*c*(2*c*C + B*d) - a*d*(3*C*d - c*
D)))*x))/(a*(b*c^2 - a*d^2)*Sqrt[a - b*x^2])) - (d*((-2*Sqrt[a]*Sqrt[b]*(A
*b*(4*b*c^2 - 3*a*d^2) - a*(b*c*(2*c*C + B*d) - a*d*(3*C*d - c*D)))*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(4*A
*b^2*c - a*(2*b*c*C + b*B*d - 5*a*d*D))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*
c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/
Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d
*x]*Sqrt[a - b*x^2])))/(2*a*b*(b*c^2 - a*d^2)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(466) = 932$.

Time = 5.64 (sec) , antiderivative size = 1015, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1015
default	Expression too large to display	7594

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((1/3*(A*b+C*a
)/b^3/a*x+1/3*(B*b+D*a)/b^4)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^
2-2*(-b*d*x-b*c)*(1/12/b^2*(3*A*a*b*d^2-4*A*b^2*c^2+B*a*b*c*d-3*C*a^2*d^2+
2*C*a*b*c^2+D*a^2*c*d)/a^2/(a*d^2-b*c^2)*x+1/12*(A*b^2*c*d-B*a*b*d^2+C*a*b
*c*d-7*D*a^2*d^2+6*D*a*b*c^2)/(a*d^2-b*c^2)/a/b^3)/((x^2-a/b)*(-b*d*x-b*c)
)^(1/2)+2*(D*d/b^2+1/6/b^2*(4*A*b^2*c-B*a*b*d-2*C*a*b*c-7*D*a^2*d)/a^2-1/1
2/b^2*d*(A*b^2*c*d-B*a*b*d^2+C*a*b*c*d-7*D*a^2*d^2+6*D*a*b*c^2)/(a*d^2-b*c
^2)/a-1/6/b*c*(3*A*a*b*d^2-4*A*b^2*c^2+B*a*b*c*d-3*C*a^2*d^2+2*C*a*b*c^2+D
*a^2*c*d)/a^2/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*
(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1
/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2)
)/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-1/6*d*(3*A*a*b*d^2-4*A*b^2*c^2+B*a*b*c*d-
3*C*a^2*d^2+2*C*a*b*c^2+D*a^2*c*d)/a^2/b/(a*d^2-b*c^2)*(c/d-1/b*(a*b)^(1/2
))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(
(-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(466) = 932$.

Time = 0.13 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/18*((6*A*a^3*b^2*c*d^2 + (6*A*a*b^4*c*d^2 + 2*(C*a*b^4 - 2*A*b^5)*c^3 -
(17*D*a^2*b^3 - B*a*b^4)*c^2*d + 3*(5*D*a^3*b^2 - B*a^2*b^3)*d^3)*x^4 + 2*
(C*a^3*b^2 - 2*A*a^2*b^3)*c^3 - (17*D*a^4*b - B*a^3*b^2)*c^2*d + 3*(5*D*a^
5 - B*a^4*b)*d^3 - 2*(6*A*a^2*b^3*c*d^2 + 2*(C*a^2*b^3 - 2*A*a*b^4)*c^3 -
(17*D*a^3*b^2 - B*a^2*b^3)*c^2*d + 3*(5*D*a^4*b - B*a^3*b^2)*d^3)*x^2)*sqr
t(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 -
9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*((2*(C*a*b^4 - 2*A*b^5)*c^2*d +
 (D*a^2*b^3 + B*a*b^4)*c*d^2 - 3*(C*a^2*b^3 - A*a*b^4)*d^3)*x^4 + 2*(C*a^3
*b^2 - 2*A*a^2*b^3)*c^2*d + (D*a^4*b + B*a^3*b^2)*c*d^2 - 3*(C*a^4*b - A*a
^3*b^2)*d^3 - 2*(2*(C*a^2*b^3 - 2*A*a*b^4)*c^2*d + (D*a^3*b^2 + B*a^2*b^3)
*c*d^2 - 3*(C*a^3*b^2 - A*a^2*b^3)*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d)) - 3*(2*(2*D*a^3*b^2 - B*a^2*b^3)*c^2*d + (C*a^3*b^2
+ A*a^2*b^3)*c*d^2 - (5*D*a^4*b - B*a^3*b^2)*d^3 - (2*(C*a*b^4 - 2*A*b^5)*
c^2*d + (D*a^2*b^3 + B*a*b^4)*c*d^2 - 3*(C*a^2*b^3 - A*a*b^4)*d^3)*x^3 - (
6*D*a^2*b^3*c^2*d + (C*a^2*b^3 + A*a*b^4)*c*d^2 - (7*D*a^3*b^2 + B*a^2*b^3
)*d^3)*x^2 - (6*A*a*b^4*c^2*d - (D*a^3*b^2 + B*a^2*b^3)*c*d^2 + (C*a^3*b^2
 - 5*A*a^2*b^3)*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a^4*b^4*c^2*d - a
^5*b^3*d^3 + (a^2*b^6*c^2*d - a^3*b^5*d^3)*x^4 - 2*(a^3*b^5*c^2*d - a^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/(-b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)\sqrt{dx+c}}{(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(d*x + c)/(-b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a-b\,x^2\right)}^{5/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a - b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a-bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,(Dx^3+C\,x^2+Bx+A)}{(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

output
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(-b*x^2+a)^(5/2),x)
```

3.164 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 671

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx = \frac{\sqrt{c+dx}\left(a(bBc-Abd-aCd+acD)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x\right)}{3ab\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}$$

$$-\frac{\sqrt{c+dx}\left(a(Abd(bc^2-5ad^2)+a(ad^2(Cd-2cD)-bc(5cCd-4Bd^2-6c^2D)))-(4Ab^2c(bc^2-2ad^2)-\right.}{6a^2b\left(bc^2-ad^2\right)^2\sqrt{a-bx^2}}$$

$$+\frac{\left(4Ab^2c(bc^2-2ad^2)-a(b^2c^2(2cC-Bd)+3a^2d^3D+abd(2cCd-3Bd^2-7c^2D))\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\mathrm{a}\right.}{6a^{3/2}b^{3/2}\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\left(Ab(4bc^2-5ad^2)-a(bc(2cC-Bd)-ad(Cd+cD))\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{6a^{3/2}b^{3/2}\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d+D*a*c)+b*(c*(A*b+C*a)-a*d*(B+a*D/
b))*x)/a/b/(-a*d^2+b*c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(a*(A*b*d*(-5
*a*d^2+b*c^2)+a*(a*d^2*(C*d-2*D*c)-b*c*(-4*B*d^2+5*C*c*d-6*D*c^2)))-(4*A*b
^2*c*(-2*a*d^2+b*c^2)-a*(b^2*c^2*(-B*d+2*C*c)+3*a^2*d^3*D+a*b*d*(-3*B*d^2+
2*C*c*d-7*D*c^2)))*x)/a^2/b/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(1/2)+1/6*(4*A*b^2
*c*(-2*a*d^2+b*c^2)-a*(b^2*c^2*(-B*d+2*C*c)+3*a^2*d^3*D+a*b*d*(-3*B*d^2+2*
C*c*d-7*D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/a^(3/2)/b^(3/2)/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/
(-b*x^2+a)^(1/2)-1/6*(A*b*(-5*a*d^2+4*b*c^2)-a*(b*c*(-B*d+2*C*c)-a*d*(C*d+
D*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Elliptic
F(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^
(1/2)*d))^(1/2))/a^(3/2)/b^(3/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.91 (sec) , antiderivative size = 928, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{b(c+dx)\left(4Ab^4c^3x^3+a^4d^3(-C+Dx)-ab^3cx\left(c(2cC-Bd)x^2+A\left(6c^2+cdx+8d^2x^2\right)\right)+\right.}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(-((b*(c + d*x)*(4*A*b^4*c^3*x^3 + a^4*d^3*(-C + D*x) - a
*b^3*c*x*(c*(2*c*C - B*d)*x^2 + A*(6*c^2 + c*d*x + 8*d^2*x^2)) + a^2*b^2*(
c*x^2*(5*c*C*d - 6*c^2*D - 2*C*d^2*x + 7*c*d*D*x) + A*d*(3*c^2 + 10*c*d*x
+ 5*d^2*x^2) + B*(-2*c^3 + c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + a^3*b*(4*
c^3*D - c^2*d*(3*C + 5*D*x) + 2*c*d^2*(3*B + x*(2*C + D*x)) - d^3*(7*A + x
*(5*B + x*(C + 3*D*x))))))/(a - b*x^2)^2) + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(4*A*b^2*c*(b*c^2 - 2*a*d^2) + a*(b^2*c^2*(-2*c*C + B*d) - 3*a^2*d^3
*D + a*b*d*(-2*c*C*d + 3*B*d^2 + 7*c^2*D)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[
b]*c - Sqrt[a]*d)*(4*A*b^2*c*(b*c^2 - 2*a*d^2) + a*(b^2*c^2*(-2*c*C + B*d)
 - 3*a^2*d^3*D + a*b*d*(-2*c*C*d + 3*B*d^2 + 7*c^2*D)))*Sqrt[(d*(Sqrt[a]/S
qrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
+ d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*Sqrt[b]
*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*b^(3/2)*(4*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d - 5
*a*d^2) + a*(b^(3/2)*c*(-2*c*C + B*d) + 3*a^(3/2)*d^2*D + a*Sqrt[b]*d*(C*d
 + c*D) - 3*Sqrt[a]*b*(-(c*C*d) + B*d^2 + 2*c^2*D)))*Sqrt[(d*(Sqrt[a]/Sqrt
[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d
*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]
], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(a - b*x^2))))/(6*a^2*b^2*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2180, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int \frac{Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(Cd+cD))+3b\left(Abcd+a\left(-2Dc^2+Cdc-\frac{d^2(bB-aD)}{b}\right)\right)x}{2b\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{3a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\int \frac{Ab(4bc^2-5ad^2)-a(bc(2cC-Bd)-ad(Cd+cD))+3\left(Acdb^2+a\left(aDd^2+b\left(-2Dc^2+Cdc-Bd^2\right)\right)\right)x}{\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{6ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \color{blue}{686}$$

$$-\frac{\int \frac{bd\left(a\left(Abd\left(bc^2-5ad^2\right)+a\left(ad^2(Cd-2cD)-bc\left(-6Dc^2+5Cdc-4Bd^2\right)\right)\right)\right)+\left(4Ab^2c\left(bc^2-2ad^2\right)-a\left(3a^2Dd^3+ab\left(-7Dc^2+2Cdc-3Bd^2\right)d+b^2c^2(2cC-Bd)\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{ab(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{d\int \frac{a\left(Abd\left(bc^2-5ad^2\right)+a\left(ad^2(Cd-2cD)-bc\left(-6Dc^2+5Cdc-4Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-2ad^2\right)-a\left(3a^2Dd^3+ab\left(-7Dc^2+2Cdc-3Bd^2\right)d+b^2c^2(2cC-Bd)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}d}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \color{blue}{600}$$

$$-\frac{d\left(\frac{\left(4Ab^2c\left(bc^2-2ad^2\right)-a\left(3a^2d^3D+abd\left(-3Bd^2-7c^2D+2cCd\right)+b^2c^2(2cC-Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(cD+C}{d}\right.}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \color{blue}{509}$$

$$-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(4Ab^2c\left(bc^2-2ad^2\right)-a\left(3a^2d^3D+abd\left(-3Bd^2-7c^2D+2cCd\right)+b^2c^2(2cC-Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)}{d}\right.}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 508$$

$$-\frac{d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4Ab^2c\left(bc^2-2ad^2\right)-a\left(3a^2d^3D+abd\left(-3Bd^2-7c^2D+2cCd\right)+b^2c^2(2cC-Bd)\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(A\cdots\right.}{\cdots}\cdots\right)}{2a(bc^2-ad^2)\cdots}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 327$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(cD+Cd))\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(4Ab^2c\left(bc\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx\cdots}}\cdots\right)}{2a(bc^2-ad^2)\cdots}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 512$$

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(cD+Cd))\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(\cdots\right.}{\cdots}\cdots\right)}{2a(bc^2-ad^2)\cdots}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 511$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(cD+Cd))\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}}{}\right.}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)\left(Ab\left(4bc^2-5ad^2\right)-a(bc(2cC-Bd)-ad(cD+Cd))\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}}{}\right.}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)\right)}{3ab\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d + a*c*D) + b*(c*(A*b + a*C) - a*d
*(B + (a*D)/b))*x))/(3*a*b*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) + (-((Sqrt[c
 + d*x]*(a*(A*b*d*(b*c^2 - 5*a*d^2) + a*(a*d^2*(C*d - 2*c*D) - b*c*(5*c*C*
d - 4*B*d^2 - 6*c^2*D))) - (4*A*b^2*c*(b*c^2 - 2*a*d^2) - a*(b^2*c^2*(2*c*
C - B*d) + 3*a^2*d^3*D + a*b*d*(2*c*C*d - 3*B*d^2 - 7*c^2*D)))*x))/(a*(b*c
^2 - a*d^2)*Sqrt[a - b*x^2])) - (d*((-2*Sqrt[a]*(4*A*b^2*c*(b*c^2 - 2*a*d^
2) - a*(b^2*c^2*(2*c*C - B*d) + 3*a^2*d^3*D + a*b*d*(2*c*C*d - 3*B*d^2 - 7
*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt
[a]*(b*c^2 - a*d^2)*(A*b*(4*b*c^2 - 5*a*d^2) - a*(b*c*(2*c*C - B*d) - a*d*
(C*d + c*D)))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (
b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/(
(Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*
a*(b*c^2 - a*d^2)))/(6*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. $2(605) = 1210$.

Time = 8.26 (sec) , antiderivative size = 1259, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1259
default	Expression too large to display	9534

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((-1/3*(A*b^2*
c-B*a*b*d+C*a*b*c-D*a^2*d)/a/b^3/(a*d^2-b*c^2)*x+1/3*(A*b*d-B*b*c+C*a*d-D*
a*c)/(a*d^2-b*c^2)/b^3)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(
-b*d*x-b*c)*(-1/12*(8*A*a*b^2*c*d^2-4*A*b^3*c^3-3*B*a^2*b*d^3-B*a*b^2*c^2*
d+2*C*a^2*b*c*d^2+2*C*a*b^2*c^3+3*D*a^3*d^3-7*D*a^2*b*c^2*d)/a^2/(a*d^2-b*
c^2)^2/b^2*x+1/12*(5*A*a*b*d^3-A*b^2*c^2*d-4*B*a*b*c*d^2-C*a^2*d^3+5*C*a*b
*c^2*d+2*D*a^2*c*d^2-6*D*a*b*c^3)/a/b^2/(a*d^2-b*c^2)^2)/((x^2-a/b)*(-b*d*
x-b*c))^(1/2)+2*(1/6/(a*d^2-b*c^2)/b*(5*A*a*b*d^2-4*A*b^2*c^2-B*a*b*c*d-C*
a^2*d^2+2*C*a*b*c^2-D*a^2*c*d)/a^2-1/12/b*d*(5*A*a*b*d^3-A*b^2*c^2*d-4*B*a
*b*c*d^2-C*a^2*d^3+5*C*a*b*c^2*d+2*D*a^2*c*d^2-6*D*a*b*c^3)/a/(a*d^2-b*c^2
)^2+1/6/b*c*(8*A*a*b^2*c*d^2-4*A*b^3*c^3-3*B*a^2*b*d^3-B*a*b^2*c^2*d+2*C*a
^2*b*c*d^2+2*C*a*b^2*c^3+3*D*a^3*d^3-7*D*a^2*b*c^2*d)/a^2/(a*d^2-b*c^2)^2)
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)
^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2))+1/6*d*(8*A*a*b^2*c*d^2-4*A*b^3*c^3-3*B*a^2*b*d^3-B*a*b^2*c^2*d+2*C*a^
2*b*c*d^2+2*C*a*b^2*c^3+3*D*a^3*d^3-7*D*a^2*b*c^2*d)/a^2/b/(a*d^2-b*c^2)^2
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(614) = 1228$.

Time = 0.13 (sec) , antiderivative size = 1288, normalized size of antiderivative = 1.92

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{c + dx}\,(a - bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/18*((2*(C*a^3*b^2 - 2*A*a^2*b^3)*c^4 + (11*D*a^4*b - B*a^3*b^2)*c^3*d -
(13*C*a^4*b - 11*A*a^3*b^2)*c^2*d^2 - 3*(D*a^5 - 3*B*a^4*b)*c*d^3 + 3*(C*a
^5 - 5*A*a^4*b)*d^4 + (2*(C*a*b^4 - 2*A*b^5)*c^4 + (11*D*a^2*b^3 - B*a*b^4
)*c^3*d - (13*C*a^2*b^3 - 11*A*a*b^4)*c^2*d^2 - 3*(D*a^3*b^2 - 3*B*a^2*b^3
)*c*d^3 + 3*(C*a^3*b^2 - 5*A*a^2*b^3)*d^4)*x^4 - 2*(2*(C*a^2*b^3 - 2*A*a*b
^4)*c^4 + (11*D*a^3*b^2 - B*a^2*b^3)*c^3*d - (13*C*a^3*b^2 - 11*A*a^2*b^3)
*c^2*d^2 - 3*(D*a^4*b - 3*B*a^3*b^2)*c*d^3 + 3*(C*a^4*b - 5*A*a^3*b^2)*d^4
)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*(C*a^3*b^2 - 2*A*a
^2*b^3)*c^3*d - (7*D*a^4*b + B*a^3*b^2)*c^2*d^2 + 2*(C*a^4*b + 4*A*a^3*b^2
)*c*d^3 + 3*(D*a^5 - B*a^4*b)*d^4 + (2*(C*a*b^4 - 2*A*b^5)*c^3*d - (7*D*a^
2*b^3 + B*a*b^4)*c^2*d^2 + 2*(C*a^2*b^3 + 4*A*a*b^4)*c*d^3 + 3*(D*a^3*b^2
- B*a^2*b^3)*d^4)*x^4 - 2*(2*(C*a^2*b^3 - 2*A*a*b^4)*c^3*d - (7*D*a^3*b^2
+ B*a^2*b^3)*c^2*d^2 + 2*(C*a^3*b^2 + 4*A*a^2*b^3)*c*d^3 + 3*(D*a^4*b - B*
a^3*b^2)*d^4)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2
), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a
*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*
(6*B*a^3*b^2*c*d^3 + 2*(2*D*a^3*b^2 - B*a^2*b^3)*c^3*d - 3*(C*a^3*b^2 - A*
a^2*b^3)*c^2*d^2 - (C*a^4*b + 7*A*a^3*b^2)*d^4 - (2*(C*a*b^4 - 2*A*b^5)*c^
3*d - (7*D*a^2*b^3 + B*a*b^4)*c^2*d^2 + 2*(C*a^2*b^3 + 4*A*a*b^4)*c*d^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.165 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 849

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\frac{a(bBc-Abd-aCd+acD)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{3ab\left(bc^2-ad^2\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}$$
$$+\frac{a(Abd(bc^2+7ad^2)+a(ad^2(Cd-2cD)+bc(7cCd-8Bd^2-6c^2D)))+(4Ab^2c(bc^2-3ad^2)-a(b^2c^2(2cC\ldots}{6a^2b\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{d(Ab(4b^2c^4-15abc^2d^2-21a^2d^4)-a(b^2c^3(2cC-3Bd)+a^2d^3(3Cd-5cD)+abcd(27cCd-29Bd^2-2\ldots}{6a^2b\left(bc^2-ad^2\right)^3\sqrt{c+dx}}$$
$$+\frac{(Ab(4b^2c^4-15abc^2d^2-21a^2d^4)-a(b^2c^3(2cC-3Bd)+a^2d^3(3Cd-5cD)+abcd(27cCd-29Bd^2-27\ldots}{6a^{3/2}\sqrt{b}\left(bc^2-ad^2\right)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(4Ab^2c(bc^2-3ad^2)-a(b^2c^2(2cC-3Bd)+a^2d^3D+abd(6cCd-5Bd^2-9c^2D)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{Ellip}\ldots}{6a^{3/2}b^{3/2}\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(a*(-A*b*d+B*b*c-C*a*d+D*a*c)+b*(c*(A*b+C*a)-a*d*(B+a*D/b))*x)/a/b/(-a
*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2)+1/6*(a*(A*b*d*(7*a*d^2+b*c^2)+a
*(a*d^2*(C*d-2*D*c)+b*c*(-8*B*d^2+7*C*c*d-6*D*c^2)))+(4*A*b^2*c*(-3*a*d^2+
b*c^2)-a*(b^2*c^2*(-3*B*d+2*C*c)+a^2*d^3*D+a*b*d*(-5*B*d^2+6*C*c*d-9*D*c^2
)))*x)/a^2/b/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/6*d*(A*b*(-
21*a^2*d^4-15*a*b*c^2*d^2+4*b^2*c^4)-a*(b^2*c^3*(-3*B*d+2*C*c)+a^2*d^3*(3*
C*d-5*D*c)+a*b*c*d*(-29*B*d^2+27*C*c*d-27*D*c^2)))*(-b*x^2+a)^(1/2)/a^2/b/
(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)+1/6*(A*b*(-21*a^2*d^4-15*a*b*c^2*d^2+4*b^2*
c^4)-a*(b^2*c^3*(-3*B*d+2*C*c)+a^2*d^3*(3*C*d-5*D*c)+a*b*c*d*(-29*B*d^2+27
*C*c*d-27*D*c^2)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(
1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1
/2))/a^(3/2)/b^(1/2)/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2
)/(-b*x^2+a)^(1/2)-1/6*(4*A*b^2*c*(-3*a*d^2+b*c^2)-a*(b^2*c^2*(-3*B*d+2*C*
c)+a^2*d^3*D+a*b*d*(-5*B*d^2+6*C*c*d-9*D*c^2)))*((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(3/2)/
(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.38 (sec) , antiderivative size = 1346, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*(a - b*x^2)^(5/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*d^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)
)/((-(b*c^2) + a*d^2)^3*(c + d*x)) + (a*b^2*B*c^2 - 2*a*A*b^2*c*d - 2*a^2*
b*c*C*d + a^2*b*B*d^2 + a^2*b*c^2*D + a^3*d^2*D + A*b^3*c^2*x + a*b^2*c^2*
C*x - 2*a*b^2*B*c*d*x + a*A*b^2*d^2*x + a^2*b*C*d^2*x - 2*a^2*b*c*d*D*x)/(
3*a*b*(-(b*c^2) + a*d^2)^2*(-a + b*x^2)^2) + (-(a*A*b^3*c^3*d) + 11*a^2*b^
2*c^3*C*d - 15*a^2*b^2*B*c^2*d^2 + 21*a^2*A*b^2*c*d^3 + 9*a^3*b*c*C*d^3 -
5*a^3*b*B*d^4 - 6*a^2*b^2*c^4*D - 15*a^3*b*c^2*d^2*D + a^4*d^4*D + 4*A*b^4
*c^4*x - 2*a*b^3*c^4*C*x + 3*a*b^3*B*c^3*d*x - 15*a*A*b^3*c^2*d^2*x - 15*a
^2*b^2*c^2*C*d^2*x + 17*a^2*b^2*B*c*d^3*x - 9*a^2*A*b^2*d^4*x - 3*a^3*b*C*
d^4*x + 15*a^2*b^2*c^3*d*D*x + 5*a^3*b*c*d^3*D*x)/(6*a^2*b*(-(b*c^2) + a*d
^2)^3*(-a + b*x^2))) + (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^
2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(A*b*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a
^2*d^4) + a*(b^2*c^3*(-2*c*C + 3*B*d) + a^2*d^3*(-3*C*d + 5*c*D) + a*b*c*d
*(-27*c*C*d + 29*B*d^2 + 27*c^2*D)))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(
c + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*(4*b^2*c^4 - 15*a*b
*c^2*d^2 - 21*a^2*d^4) + a*(b^2*c^3*(-2*c*C + 3*B*d) + a^2*d^3*(-3*C*d + 5
*c*D) + a*b*c*d*(-27*c*C*d + 29*B*d^2 + 27*c^2*D)))*Sqrt[1 - c/(c + d*x) -
 (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt
[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
+ d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x...
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {2180, 27, 686, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a-bx^2)^{5/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 2180$$

$$\frac{\int \frac{Ab(4bc^2-7ad^2)-a(bc(2cC-3Bd)+ad(Cd-3cD))+b\left(5Abcd+a\left(-6Dc^2+5Cdc-\frac{d^2(5bB-aD)}{b}\right)\right)x}{2b(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{3a\left(bc^2-ad^2\right)}+\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\int \frac{Ab(4bc^2-7ad^2)-a(bc(2cC-3Bd)+ad(Cd-3cD))+(5Acdb^2+a(aDd^2+b(-6Dc^2+5Cdc-5Bd^2)))x}{(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{6ab\,(bc^2-ad^2)}+\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\,(a-bx^2)^{3/2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textcolor{blue}{686}$$

$$\frac{\frac{x(4Ab^2c(bc^2-3ad^2)-a(a^2d^3D+abd(-5Bd^2-9c^2D+6cCd)+b^2c^2(2cC-3Bd)))+a(Abd(7ad^2+bc^2)+a(ad^2(Cd-2cD)+bc(-8Bd^2-6c^2D+7cC}{a\sqrt{a-bx^2}\sqrt{c+dx}(bc^2-ad^2)}}{6a}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\,(a-bx^2)^{3/2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{d\int \frac{3a\left(Abd\left(bc^2+7ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-6Dc^2+7Cdc-8Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2Dd^3+ab\left(-9Dc^2+6Cdc-5Bd^2\right)d+b^2c^2(2cC-3Bd)\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}}{6ab}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\,(a-bx^2)^{3/2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textcolor{blue}{688}$$

$$d\left(\frac{2\int -\frac{a\left(Ab^2cd\left(bc^2-33ad^2\right)-a\left(a^2Dd^4+ab\left(-15Dc^2+9Cdc-5Bd^2\right)d^2+b^2c^2\left(-18Dc^2+23Cdc-27Bd^2\right)\right)\right)+b\left(Ab\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)-a\left(b^2(2cC-3\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}\right.$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\,(a-bx^2)^{3/2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$d\left(-\frac{\int \frac{a\left(Ab^2cd\left(bc^2-33ad^2\right)-a\left(a^2Dd^4+ab\left(-15Dc^2+9Cdc-5Bd^2\right)d^2+b^2c^2\left(-18Dc^2+23Cdc-27Bd^2\right)\right)\right)+b\left(Ab\left(4b^2c^4-15abd^2c^2-21a^2d^4\right)-a\left(b^2(2cC-3B\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}\right.$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\,(a-bx^2)^{3/2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \ 600$$

$$\frac{d\left(-\frac{\frac{b\left(Ab\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)-a\left(a^2d^3(3Cd-5cD)+abcd\left(-29Bd^2-27c^2D+27cCd\right)+b^2c^3(2cC-3Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c\left(bc^2-3a\right.\right.}{}}{bc^2-ad^2}\right.}{}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$\frac{d\left(-\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(-21a^2d^4-15abc^2d^2+4b^2c^4\right)-a\left(a^2d^3(3Cd-5cD)+abcd\left(-29Bd^2-27c^2D+27cCd\right)+b^2c^3(2cC-3Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(4Ab^2\right.}{}}{bc^2-ad^2}\right.}{}$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 508$$

$$\frac{a(bBc+aDc-Abd-aCd)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{3ab\left(bc^2-ad^2\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+$$

$$\frac{\frac{a\left(Abd\left(bc^2+7ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-6Dc^2+7Cdc-8Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2Dd^3+ab\left(-9Dc^2+6Cdc-5Bd^2\right)d+b^2c^2(2cC-3B\right.\right.}{a\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}}{}$$

$$\Big\downarrow \ 327$$

$$d\left(-\frac{-\frac{\left(bc^2-ad^2\right)\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2d^3D+abd\left(-5Bd^2-9c^2D+6cCd\right)+b^2c^2(2cC-3Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\ldots\right.\right.}{\ldots}}{bc^2-ad^2\ldots}\right.$$

$$\frac{bx\left(c(aC+Ab)-ad\left(\frac{aD}{b}+B\right)\right)+a(acD-aCd-Abd+bBc)}{3ab\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 512$$

$$\frac{a(bBc+aDc-Abd-aCd)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{3ab\left(bc^2-ad^2\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+$$

$$\frac{a\left(Abd\left(bc^2+7ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-6Dc^2+7Cdc-8Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2Dd^3+ab\left(-9Dc^2+6Cdc-5Bd^2\right)d+b^2c^2(2cC-3B\ldots\right.\right.}{a\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 511$$

$$\frac{a(bBc+aDc-Abd-aCd)+b\left(c(Ab+aC)-ad\left(B+\frac{aD}{b}\right)\right)x}{3ab\left(bc^2-ad^2\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+$$

$$\frac{a\left(Abd\left(bc^2+7ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-6Dc^2+7Cdc-8Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2Dd^3+ab\left(-9Dc^2+6Cdc-5Bd^2\right)d+b^2c^2(2cC-3B\ldots\right.\right.}{a\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 321$$

$$\frac{a(bBc + aDc - Abd - aCd) + b\left(c(Ab + aC) - ad\left(B + \frac{aD}{b}\right)\right)x}{3ab\left(bc^2 - ad^2\right)\sqrt{c + dx}\left(a - bx^2\right)^{3/2}} +$$

$$\frac{a\left(Abd\left(bc^2+7ad^2\right)+a\left(a(Cd-2cD)d^2+bc\left(-6Dc^2+7Cdc-8Bd^2\right)\right)\right)+\left(4Ab^2c\left(bc^2-3ad^2\right)-a\left(a^2Dd^3+ab\left(-9Dc^2+6Cdc-5Bd^2\right)d+b^2c^2(2cC-3B\right.\right.}{a(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)^(3/2)*(a - b*x^2)^(5/2)),x]
```

output
```
(a*(b*B*c - A*b*d - a*C*d + a*c*D) + b*(c*(A*b + a*C) - a*d*(B + (a*D)/b))
*x)/(3*a*b*(b*c^2 - a*d^2)*Sqrt[c + d*x]*(a - b*x^2)^(3/2)) + ((a*(A*b*d*(
b*c^2 + 7*a*d^2) + a*(a*d^2*(C*d - 2*c*D) + b*c*(7*c*C*d - 8*B*d^2 - 6*c^2
*D))) + (4*A*b^2*c*(b*c^2 - 3*a*d^2) - a*(b^2*c^2*(2*c*C - 3*B*d) + a^2*d^
3*D + a*b*d*(6*c*C*d - 5*B*d^2 - 9*c^2*D)))*x)/(a*(b*c^2 - a*d^2)*Sqrt[c +
 d*x]*Sqrt[a - b*x^2]) + (d*((-2*(A*b*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2
*d^4) - a*(b^2*c^3*(2*c*C - 3*B*d) + a^2*d^3*(3*C*d - 5*c*D) + a*b*c*d*(27
*c*C*d - 29*B*d^2 - 27*c^2*D)))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c +
 d*x]) - ((-2*Sqrt[a]*Sqrt[b]*(A*b*(4*b^2*c^4 - 15*a*b*c^2*d^2 - 21*a^2*d^
4) - a*(b^2*c^3*(2*c*C - 3*B*d) + a^2*d^3*(3*C*d - 5*c*D) + a*b*c*d*(27*c*
C*d - 29*B*d^2 - 27*c^2*D)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]
) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(4*A*b^2*c*(b*c^2 - 3*a*d^2) - a*(b^2*c^2*(
2*c*C - 3*B*d) + a^2*d^3*D + a*b*d*(6*c*C*d - 5*B*d^2 - 9*c^2*D)))*Sqrt[(S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2)))/(2*a*(b
*c^2 - a*d^2)))/(6*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(777) = 1554$.

Time = 10.01 (sec) , antiderivative size = 1646, normalized size of antiderivative = 1.94

method	result	size
elliptic	Expression too large to display	1646
default	Expression too large to display	11864

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)*((d*x+c)*(-b*x^2+a))^(1/2)*((1/3*(A*a*b*d
^2+A*b^2*c^2-2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2-2*D*a^2*c*d)/b^2/a/(a*d^2-b*c
^2)^2*x-1/3*(2*A*b^2*c*d-B*a*b*d^2-B*b^2*c^2+2*C*a*b*c*d-D*a^2*d^2-D*a*b*c
^2)/b^3/(a*d^2-b*c^2)^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*
(-b*d*x-b*c)*(1/12/b*(9*A*a^2*b*d^4+15*A*a*b^2*c^2*d^2-4*A*b^3*c^4-17*B*a^
2*b*c*d^3-3*B*a*b^2*c^3*d+3*C*a^3*d^4+15*C*a^2*b*c^2*d^2+2*C*a*b^2*c^4-5*D
*a^3*c*d^3-15*D*a^2*b*c^3*d)/a^2/(a*d^2-b*c^2)^3*x-1/12*(21*A*a*b^2*c*d^3-
A*b^3*c^3*d-5*B*a^2*b*d^4-15*B*a*b^2*c^2*d^2+9*C*a^2*b*c*d^3+11*C*a*b^2*c^
3*d+D*a^3*d^4-15*D*a^2*b*c^2*d^2-6*D*a*b^2*c^4)/(a*d^2-b*c^2)^3/a/b^2)/((x
^2-a/b)*(-b*d*x-b*c))^(1/2)-2*(-b*d*x^2+a*d)*d/(a*d^2-b*c^2)^3*(A*d^3-B*c*
d^2+C*c^2*d-D*c^3)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-1/6/(a*d^2-b*c^2)^2/
b*(12*A*a*b^2*c*d^2-4*A*b^3*c^3-5*B*a^2*b*d^3-3*B*a*b^2*c^2*d+6*C*a^2*b*c*
d^2+2*C*a*b^2*c^3+D*a^3*d^3-9*D*a^2*b*c^2*d)/a^2+1/12/b*d*(21*A*a*b^2*c*d^
3-A*b^3*c^3*d-5*B*a^2*b*d^4-15*B*a*b^2*c^2*d^2+9*C*a^2*b*c*d^3+11*C*a*b^2*
c^3*d+D*a^3*d^4-15*D*a^2*b*c^2*d^2-6*D*a*b^2*c^4)/(a*d^2-b*c^2)^3/a-1/6*c*
(9*A*a^2*b*d^4+15*A*a*b^2*c^2*d^2-4*A*b^3*c^4-17*B*a^2*b*c*d^3-3*B*a*b^2*c
^3*d+3*C*a^3*d^4+15*C*a^2*b*c^2*d^2+2*C*a*b^2*c^4-5*D*a^3*c*d^3-15*D*a^2*b
*c^3*d)/a^2/(a*d^2-b*c^2)^3-b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*d*c/(a*d^2-b*c
^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2873 vs. $2(790) = 1580$.

Time = 0.30 (sec) , antiderivative size = 2873, normalized size of antiderivative = 3.38

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^{3/2} (a - bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm=
"fricas")
```

output
```
1/18*((2*(C*a^3*b^3 - 2*A*a^2*b^4)*c^6 + 3*(9*D*a^4*b^2 - B*a^3*b^3)*c^5*d
 - 6*(7*C*a^4*b^2 - 3*A*a^3*b^3)*c^4*d^2 + 4*(10*D*a^5*b + 13*B*a^4*b^2)*c
^3*d^3 - 6*(4*C*a^5*b + 13*A*a^4*b^2)*c^2*d^4 - 3*(D*a^6 - 5*B*a^5*b)*c*d^
5 + (2*(C*a*b^5 - 2*A*b^6)*c^5*d + 3*(9*D*a^2*b^4 - B*a*b^5)*c^4*d^2 - 6*(
7*C*a^2*b^4 - 3*A*a*b^5)*c^3*d^3 + 4*(10*D*a^3*b^3 + 13*B*a^2*b^4)*c^2*d^4
 - 6*(4*C*a^3*b^3 + 13*A*a^2*b^4)*c*d^5 - 3*(D*a^4*b^2 - 5*B*a^3*b^3)*d^6)
*x^5 + (2*(C*a*b^5 - 2*A*b^6)*c^6 + 3*(9*D*a^2*b^4 - B*a*b^5)*c^5*d - 6*(7
*C*a^2*b^4 - 3*A*a*b^5)*c^4*d^2 + 4*(10*D*a^3*b^3 + 13*B*a^2*b^4)*c^3*d^3
- 6*(4*C*a^3*b^3 + 13*A*a^2*b^4)*c^2*d^4 - 3*(D*a^4*b^2 - 5*B*a^3*b^3)*c*d
^5)*x^4 - 2*(2*(C*a^2*b^4 - 2*A*a*b^5)*c^5*d + 3*(9*D*a^3*b^3 - B*a^2*b^4)
*c^4*d^2 - 6*(7*C*a^3*b^3 - 3*A*a^2*b^4)*c^3*d^3 + 4*(10*D*a^4*b^2 + 13*B*
a^3*b^3)*c^2*d^4 - 6*(4*C*a^4*b^2 + 13*A*a^3*b^3)*c*d^5 - 3*(D*a^5*b - 5*B
*a^4*b^2)*d^6)*x^3 - 2*(2*(C*a^2*b^4 - 2*A*a*b^5)*c^6 + 3*(9*D*a^3*b^3 - B
*a^2*b^4)*c^5*d - 6*(7*C*a^3*b^3 - 3*A*a^2*b^4)*c^4*d^2 + 4*(10*D*a^4*b^2
+ 13*B*a^3*b^3)*c^3*d^3 - 6*(4*C*a^4*b^2 + 13*A*a^3*b^3)*c^2*d^4 - 3*(D*a^
5*b - 5*B*a^4*b^2)*c*d^5)*x^2 + (2*(C*a^3*b^3 - 2*A*a^2*b^4)*c^5*d + 3*(9*
D*a^4*b^2 - B*a^3*b^3)*c^4*d^2 - 6*(7*C*a^4*b^2 - 3*A*a^3*b^3)*c^3*d^3 + 4
*(10*D*a^5*b + 13*B*a^4*b^2)*c^2*d^4 - 6*(4*C*a^5*b + 13*A*a^4*b^2)*c*d^5
- 3*(D*a^6 - 5*B*a^5*b)*d^6)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2
+ 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*(d*x + c)^(3/2)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(a-b\,x^2\right)^{5/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(5/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a - b*x^2)^(5/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\left(dx+c\right)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(5/2),x)
```

3.166 $\int (c+dx)^n \sqrt{a+bx^2} (A+Bx+Cx^2+Dx^3)\, dx$

Optimal result

Integrand size = 34, antiderivative size = 537

$$
\begin{aligned}
&\int (c+dx)^n \sqrt{a+bx^2} (A+Bx+Cx^2+Dx^3)\, dx \\
&= \frac{(Cd(5+n)-cD(8+n))(c+dx)^{1+n}\left(a+bx^2\right)^{3/2}}{bd^2(4+n)(5+n)} + \frac{D(c+dx)^{2+n}\left(a+bx^2\right)^{3/2}}{bd^2(5+n)} \\
&\quad - \frac{(ad^2(1+n)(Cd(5+n)-cD(8+n)) + b(12c^3D - 3c^2Cd(5+n) + Bcd^2(20+9n+n^2) - Ad^3(20+\cdots}{bd^4(1+n)(4+n)(5+n)\sqrt{1-\frac{c+\cdots}{c-\cdots}}\cdots} \\
&\quad - \frac{(ad^2D(8+6n+n^2) - b(12c^2D - 3cCd(5+n) + Bd^2(20+9n+n^2)))\,(c+dx)^{2+n}\sqrt{a+bx^2}\ \mathrm{AppellF}\cdots}{bd^4(2+n)(4+n)(5+n)\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}\cdots}
\end{aligned}
$$

output
```
(C*d*(5+n)-c*D*(8+n))*(d*x+c)^(1+n)*(b*x^2+a)^(3/2)/b/d^2/(4+n)/(5+n)+D*(d
*x+c)^(2+n)*(b*x^2+a)^(3/2)/b/d^2/(5+n)-(a*d^2*(1+n)*(C*d*(5+n)-c*D*(8+n))
+b*(12*D*c^3-3*c^2*C*d*(5+n)+B*c*d^2*(n^2+9*n+20)-A*d^3*(n^2+9*n+20)))*(d*
x+c)^(1+n)*(b*x^2+a)^(1/2)*AppellF1(1+n,-1/2,-1/2,2+n,(d*x+c)/(c-(-a)^(1/2
)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/b/d^4/(1+n)/(4+n)/(5+n)/(1-
(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^(1/2)/(1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)
))^(1/2)-(a*d^2*D*(n^2+6*n+8)-b*(12*D*c^2-3*c*C*d*(5+n)+B*d^2*(n^2+9*n+20)
))*(d*x+c)^(2+n)*(b*x^2+a)^(1/2)*AppellF1(2+n,-1/2,-1/2,3+n,(d*x+c)/(c-(-a
)^(1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/b/d^4/(2+n)/(4+n)/(5+
n)/(1-(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^(1/2)/(1-(d*x+c)/(c+(-a)^(1/2)*d/b
^(1/2)))^(1/2)
```

Mathematica [F]

$$\int (c+dx)^n\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (c+dx)^n\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
Integrate[(c + d*x)^n*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
Integrate[(c + d*x)^n*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3), x]
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2185, 25, 2185, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -(c+dx)^n\sqrt{bx^2+a}\left(-b(Cd(n+5)-cD(n+8))x^2d^2+(acD(n+2)-Abd(n+5))d^2+\left(3bDc^2+ad^2D(n+\right.\right.}{bd^3(n+5)}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}$$

$$\Big\downarrow \ 25$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-$$
$$\frac{\int (c+dx)^n\sqrt{bx^2+a}\left(-b(Cd(n+5)-cD(n+8))x^2d^2+(acD(n+2)-Abd(n+5))d^2+\left(3bDc^2+ad^2D(n+2\right.\right.}{bd^3(n+5)}$$

$$\Big\downarrow \ 2185$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-$$
$$\frac{\frac{\int -bd^3(c+dx)^n\left(d\left(3acDn-aCd\left(n^2+6n+5\right)+Abd\left(n^2+9n+20\right)\right)-\left(ad^2D\left(n^2+6n+8\right)-b\left(12Dc^2-3Cd(n+5)c+Bd^2\left(n^2+9n+20\right)\right)\right)x\right)\sqrt{bx^2+a}dx}{bd^2(n+4)}}{bd^3(n+5)}$$

$$\Big\downarrow \ 25$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-$$
$$\frac{-\frac{\int bd^3(c+dx)^n\left(d\left(3acDn-aCd\left(n^2+6n+5\right)+Abd\left(n^2+9n+20\right)\right)-\left(ad^2D\left(n^2+6n+8\right)-b\left(12Dc^2-3Cd(n+5)c+Bd^2\left(n^2+9n+20\right)\right)\right)x\right)\sqrt{bx^2+a}dx}{bd^2(n+4)}}{bd^3(n+5)}$$

$$\Big\downarrow \ 27$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-$$
$$\frac{-\frac{d\int (c+dx)^n\left(d\left(3acDn-aCd\left(n^2+6n+5\right)+Abd\left(n^2+9n+20\right)\right)-\left(ad^2D\left(n^2+6n+8\right)-b\left(12Dc^2-3Cd(n+5)c+Bd^2\left(n^2+9n+20\right)\right)\right)x\right)\sqrt{bx^2+a}dx}{n+4}}{bd^3(n+5)}$$

$$\Big\downarrow \ 719$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-$$
$$\frac{-\frac{d\left(-\frac{\left(ad^2(n+1)(Cd(n+5)-cD(n+8))+b\left(-Ad^3\left(n^2+9n+20\right)+Bcd^2\left(n^2+9n+20\right)+12c^3D-3c^2Cd(n+5)\right)\right)\int (c+dx)^n\sqrt{bx^2+a}dx}{d}-\frac{\left(ad^2D\left(n^2+6n+8\right)-b\left(B\right.\right.}{}\right.}{n+4}}{bd^3(n+5)}$$

$$\Big\downarrow \ 514$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-\frac{d\left(-\frac{\sqrt{a+bx^2}\left(ad^2(n+1)(Cd(n+5)-cD(n+8))+b\left(-Ad^3\left(n^2+9n+20\right)+Bcd^2\left(n^2+9n+20\right)+12c^3D-3c^2Cd(n+5)\right)\right)\int(c+dx)^n\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}}}{d^2\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}}\right.}{n+4}$$

$$\Big\downarrow \; 150$$

$$\frac{D\left(a+bx^2\right)^{3/2}(c+dx)^{n+2}}{bd^2(n+5)}-\frac{d\left(-\frac{\sqrt{a+bx^2}(c+dx)^{n+1}\operatorname{AppellF1}\left(n+1,-\frac{1}{2},-\frac{1}{2},n+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)\left(ad^2(n+1)(Cd(n+5)-cD(n+8))+b\left(-Ad^3\left(n^2+9n+20\right)+Bcd^2\left(n^2+9n+20\right)+\right.\right.}{d^2(n+1)\sqrt{1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}}\sqrt{1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}}}\right.}{}$$

input
```
Int[(c + d*x)^n*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^(2 + n)*(a + b*x^2)^(3/2))/(b*d^2*(5 + n)) - (-((d*(C*d*(5 +
n) - c*D*(8 + n))*(c + d*x)^(1 + n)*(a + b*x^2)^(3/2))/(4 + n)) - (d*(-(((
a*d^2*(1 + n)*(C*d*(5 + n) - c*D*(8 + n)) + b*(12*c^3*D - 3*c^2*C*d*(5 + n
) + B*c*d^2*(20 + 9*n + n^2) - A*d^3*(20 + 9*n + n^2)))*(c + d*x)^(1 + n)*
Sqrt[a + b*x^2]*AppellF1[1 + n, -1/2, -1/2, 2 + n, (c + d*x)/(c - (Sqrt[-a
]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d^2*(1 + n)*Sqrt[1
- (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b])]*Sqrt[1 - (c + d*x)/(c + (Sqrt[-a]*
d)/Sqrt[b])])) - ((a*d^2*D*(8 + 6*n + n^2) - b*(12*c^2*D - 3*c*C*d*(5 + n)
 + B*d^2*(20 + 9*n + n^2)))*(c + d*x)^(2 + n)*Sqrt[a + b*x^2]*AppellF1[2 +
 n, -1/2, -1/2, 3 + n, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c
+ (Sqrt[-a]*d)/Sqrt[b])])/(d^2*(2 + n)*Sqrt[1 - (c + d*x)/(c - (Sqrt[-a]*d
)/Sqrt[b])]*Sqrt[1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])))/(4 + n))/(b*
d^3*(5 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [F]

$$\int (dx+c)^n \sqrt{b\,x^2+a}\,\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (c+dx)^n\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx^2+a}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fric
as")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(d*x + c)^n, x)
```

Sympy [F]

$$\int (c+dx)^n\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{a+bx^2}(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((d*x+c)**n*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x)**n*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F]

$$\int (c+dx)^n \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx^2+a}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(d*x + c)^n, x)
```

Giac [F]

$$\int (c+dx)^n \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx^2+a}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{b\,x^2+a}\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^n\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (dx+c)^n\sqrt{b\,x^2+a}\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^n*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

3.167 $\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 636

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{(Cd(4+n+2p)-cD(6+n+4p))(c+dx)^{1+n}\left(a+bx^2\right)^{1+p}}{bd^2(3+n+2p)(4+n+2p)}$$
$$+ \frac{D(c+dx)^{2+n}\left(a+bx^2\right)^{1+p}}{bd^2(4+n+2p)}$$
$$- \frac{\left(a(1+n)(Cd(4+n+2p)-cD(6+n+4p)) - \frac{b\left(2c^2Cd(1+p)(4+n+2p)-2c^3D\left(3+5p+2p^2\right)-Bcd^2\left(12+7n+n^2+14p+\right.\right.}{d^2}}{}$$
$$- \frac{\left(ad^2D(2+n)(3+n+2p)+b\left(2cCd(1+p)(4+n+2p)-2c^2D\left(3+5p+2p^2\right)-Bd^2\left(12+7n+n^2\right.\right.\right.}{bd^4(}$$

output
```
(C*d*(4+n+2*p)-c*D*(6+n+4*p))*(d*x+c)^(1+n)*(b*x^2+a)^(p+1)/b/d^2/(3+n+2*p
)/(4+n+2*p)+D*(d*x+c)^(2+n)*(b*x^2+a)^(p+1)/b/d^2/(4+n+2*p)-(a*(1+n)*(C*d*
(4+n+2*p)-c*D*(6+n+4*p))-b*(2*c^2*C*d*(p+1)*(4+n+2*p)-2*c^3*D*(2*p^2+5*p+3
)-B*c*d^2*(n^2+4*n*p+4*p^2+7*n+14*p+12)+A*d^3*(n^2+4*n*p+4*p^2+7*n+14*p+12
))/d^2)*(d*x+c)^(1+n)*(b*x^2+a)^p*AppellF1(1+n,-p,-p,2+n,(d*x+c)/(c-(-a)^(
1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/b/d^2/(1+n)/(3+n+2*p)/(4
+n+2*p)/((1-(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)
*d/b^(1/2)))^p)-(a*d^2*D*(2+n)*(3+n+2*p)+b*(2*c*C*d*(p+1)*(4+n+2*p)-2*c^2*
D*(2*p^2+5*p+3)-B*d^2*(n^2+4*n*p+4*p^2+7*n+14*p+12)))*(d*x+c)^(2+n)*(b*x^2
+a)^p*AppellF1(2+n,-p,-p,3+n,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)),(d*x+c)/(c+(
-a)^(1/2)*d/b^(1/2)))/b/d^4/(2+n)/(3+n+2*p)/(4+n+2*p)/((1-(d*x+c)/(c-(-a)^
(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [F]

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) dx$$

input
```
Integrate[(c + d*x)^n*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
Integrate[(c + d*x)^n*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3), x]
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 615, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2185, 25, 2185, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int -(c+dx)^n \left(bx^2+a\right)^p \left(-b(Cd(n+2p+4)-cD(n+4p+6))x^2d^2+(acD(n+2)-Abd(n+2p+4))d^2+(2\right.}{bd^3(n+2p+4)}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}$$

$$\Big\downarrow \ 25$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$

$$\frac{\int (c+dx)^n \left(bx^2+a\right)^p \left(-b(Cd(n+2p+4)-cD(n+4p+6))x^2d^2+(acD(n+2)-Abd(n+2p+4))d^2+(2bB\right.}{bd^3(n+2p+4)}$$

$$\Big\downarrow \ 2185$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$

$$\frac{\frac{\int -bd^3(c+dx)^n\left(d\left(2acDn(p+1)-aCd(n+1)(n+2p+4)+Abd\left(n^2+4pn+7n+4p^2+14p+12\right)\right)-\left(aD(n+2)(n+2p+3)d^2+b\left(-2D\left(2p^2+5p+3\right)c^2+2\right.\right.\right.}{bd^2(n+2p+3)}}{bd^3(n+2p-}$$

$$\Big\downarrow \ 25$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$

$$\frac{-\frac{\int bd^3(c+dx)^n\left(d\left(2acDn(p+1)-aCd(n+1)(n+2p+4)+Abd\left(n^2+4pn+7n+4p^2+14p+12\right)\right)-\left(aD(n+2)(n+2p+3)d^2+b\left(-2D\left(2p^2+5p+3\right)c^2+2\right.\right.\right.}{bd^2(n+2p+3)}}{bd^3(n+2p-}$$

$$\Big\downarrow \ 27$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$

$$\frac{-\frac{d\int (c+dx)^n\left(d\left(2acDn(p+1)-aCd(n+1)(n+2p+4)+Abd\left(n^2+4pn+7n+4p^2+14p+12\right)\right)-\left(aD(n+2)(n+2p+3)d^2+b\left(-2D\left(2p^2+5p+3\right)c^2+2C\right.\right.\right.}{n+2p+3}}{bd^3(n+2p+}$$

$$\Big\downarrow \ 719$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$

$$\frac{-\frac{d\left(-\frac{\left(ad^2(n+1)(Cd(n+2p+4)-cD(n+4p+6))-b\left(Ad^3\left(n^2+4np+7n+4p^2+14p+12\right)-Bcd^2\left(n^2+4np+7n+4p^2+14p+12\right)-2c^3D\left(2p^2+5p+3\right)+2c^2Cd(p+1)(n\right.\right.\right.}{d}}{}}{}$$

$$\Big\downarrow \ 514$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$
$$-\frac{d\Bigg(-\frac{\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\left(ad^2(n+1)(Cd(n+2p+4)-cD(n+4p+6))-b\left(Ad^3\left(n^2+4np+7n+4p^2+14p+12\right)-Bcd^2\left(n^2+4np+7n+\right.\right.\right.}{d^2}}{}$$

$$\Big\downarrow \textcolor{blue}{150}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{n+2}}{bd^2(n+2p+4)}-$$
$$-\frac{d\Bigg(-\frac{\left(a+bx^2\right)^p(c+dx)^{n+1}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(n+1,-p,-p,n+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)\left(ad^2(n+1)(Cd(n+2p+4)-cD(n+4p+\right.}{d^2(n+1)}}{}$$

input
```
Int[(c + d*x)^n*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^(2 + n)*(a + b*x^2)^(1 + p))/(b*d^2*(4 + n + 2*p)) - (-((d*(C
*d*(4 + n + 2*p) - c*D*(6 + n + 4*p))*(c + d*x)^(1 + n)*(a + b*x^2)^(1 + p
))/(3 + n + 2*p)) - (d*(-(((a*d^2*(1 + n)*(C*d*(4 + n + 2*p) - c*D*(6 + n
+ 4*p)) - b*(2*c^2*C*d*(1 + p)*(4 + n + 2*p) - 2*c^3*D*(3 + 5*p + 2*p^2) -
 B*c*d^2*(12 + 7*n + n^2 + 14*p + 4*n*p + 4*p^2) + A*d^3*(12 + 7*n + n^2 +
 14*p + 4*n*p + 4*p^2)))*(c + d*x)^(1 + n)*(a + b*x^2)^p*AppellF1[1 + n, -
p, -p, 2 + n, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-
a]*d)/Sqrt[b])])/(d^2*(1 + n)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p
*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^p)) - ((a*d^2*D*(2 + n)*(3 + n
 + 2*p) + b*(2*c*C*d*(1 + p)*(4 + n + 2*p) - 2*c^2*D*(3 + 5*p + 2*p^2) - B
*d^2*(12 + 7*n + n^2 + 14*p + 4*n*p + 4*p^2)))*(c + d*x)^(2 + n)*(a + b*x^
2)^p*AppellF1[2 + n, -p, -p, 3 + n, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]),
(c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d^2*(2 + n)*(1 - (c + d*x)/(c - (S
qrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^p)))/(3
+ n + 2*p))/(b*d^3*(4 + n + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [F]

$$\int (dx+c)^n \left(b\,x^2+a\right)^p \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**n*(b*x**2+a)**p*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^n, x)
```

Giac [F]

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(b\,x^2+a\right)^p\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^p*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^p*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (c+dx)^n \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
((c + d*x)**n*(a + b*x**2)**p*a**2*b*d**3*n**3 + 6*(c + d*x)**n*(a + b*x**
2)**p*a**2*b*d**3*n**2*p + 9*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d**3*n**2
 + 12*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d**3*n*p**2 + 36*(c + d*x)**n*(a
 + b*x**2)**p*a**2*b*d**3*n*p + 26*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d**
3*n + 8*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d**3*p**3 + 36*(c + d*x)**n*(a
 + b*x**2)**p*a**2*b*d**3*p**2 + 52*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d*
*3*p + 24*(c + d*x)**n*(a + b*x**2)**p*a**2*b*d**3 - 3*(c + d*x)**n*(a + b
*x**2)**p*a**2*c*d**3*n**3 - 8*(c + d*x)**n*(a + b*x**2)**p*a**2*c*d**3*n*
*2*p - 16*(c + d*x)**n*(a + b*x**2)**p*a**2*c*d**3*n**2 - 4*(c + d*x)**n*(
a + b*x**2)**p*a**2*c*d**3*n*p**2 - 28*(c + d*x)**n*(a + b*x**2)**p*a**2*c
*d**3*n*p - 27*(c + d*x)**n*(a + b*x**2)**p*a**2*c*d**3*n - 12*(c + d*x)**
n*(a + b*x**2)**p*a**2*c*d**3*p**2 - 28*(c + d*x)**n*(a + b*x**2)**p*a**2*
c*d**3*p - 14*(c + d*x)**n*(a + b*x**2)**p*a**2*c*d**3 + (c + d*x)**n*(a +
 b*x**2)**p*a*b**2*c*d**2*n**3*x + 2*(c + d*x)**n*(a + b*x**2)**p*a*b**2*c
*d**2*n**3 + 6*(c + d*x)**n*(a + b*x**2)**p*a*b**2*c*d**2*n**2*p*x + 10*(c
 + d*x)**n*(a + b*x**2)**p*a*b**2*c*d**2*n**2*p + 9*(c + d*x)**n*(a + b*x*
*2)**p*a*b**2*c*d**2*n**2*x + 15*(c + d*x)**n*(a + b*x**2)**p*a*b**2*c*d**
2*n**2 + 12*(c + d*x)**n*(a + b*x**2)**p*a*b**2*c*d**2*n*p**2*x + 16*(c +
d*x)**n*(a + b*x**2)**p*a*b**2*c*d**2*n*p**2 + 36*(c + d*x)**n*(a + b*x**2
)**p*a*b**2*c*d**2*n*p*x + 46*(c + d*x)**n*(a + b*x**2)**p*a*b**2*c*d**...
```

3.168 $\int (c+dx)^{-3-2p} (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\, d$

Optimal result

Integrand size = 36, antiderivative size = 573

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\, dx = \frac{D(c+dx)^{-1-2p} \left(a+bx^2\right)^{1+p}}{bd^2}$$

$$-\frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2d^2\left(bc^2+ad^2\right)(1+p)}$$

$$-\frac{(Cd-cD(3+2p))(c+dx)^{-2p}\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,\right.}{2d^4p}$$

$$-\frac{\left(a^2d^4D(1+2p)-b^2\left(c^3Cd-Acd^3-c^4D(3+2p)\right)-abd^2\left(2cCd-Bd^2-c^2D(5+4p)\right)\right)\left(\sqrt{-a}-\sqrt{b}\right.}{bd^3\left(\sqrt{b}c\right.}$$

output
```
D*(d*x+c)^(-1-2*p)*(b*x^2+a)^(p+1)/b/d^2-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)
*(b*x^2+a)^(p+1)/d^2/(a*d^2+b*c^2)/(p+1)/((d*x+c)^(2*p+2))-1/2*(C*d-c*D*(3
+2*p))*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/
2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^4/p/((d*x+c)^(2*p))/((1-(d*x+c)/(c
-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)-(a^2*d
^4*D*(1+2*p)-b^2*(C*c^3*d-A*c*d^3-c^4*D*(3+2*p))-a*b*d^2*(2*C*c*d-B*d^2-c^
2*D*(5+4*p)))*((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeo
m([-p, -1-2*p],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d
)/((-a)^(1/2)-b^(1/2)*x))/b/d^3/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)/(1+
2*p)/((-(b^(1/2)*c+(-a)^(1/2)*d)*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(1
/2)*d)/((-a)^(1/2)-b^(1/2)*x))^p)
```

Mathematica [F]

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
Integrate[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
Integrate[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3), x]
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2185, 2187, 27, 514, 150, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p (c+dx)^{-2p-3}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \ 2185$$

$$\frac{\int (c+dx)^{-2p-3}\left(bx^2+a\right)^p\left(b(Cd-cD(2p+3))x^2d^2+(Abd+acD(2p+1))d^2+\left(-2bD(p+1)c^2+bBd^2+ad\right.\right.}{bd^3}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \; 2187$$

$$\frac{\frac{\int d^2(c+dx)^{-2p-3}\left(acD(2p+1)d^2+\left(ad^2D(2p+1)-b\left(-2D(p+2)c^2+2Cdc-Bd^2\right)\right)xd-b\left(-D(2p+3)c^3+Cdc^2-Ad^3\right)\right)\left(bx^2+a\right)^pdx}{d^2}+b(Cd-c}{bd^3}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (c+dx)^{-2p-3}\left(acD(2p+1)d^2+\left(ad^2D(2p+1)-b\left(-2D(p+2)c^2+2Cdc-Bd^2\right)\right)xd-b\left(-D(2p+3)c^3+C\right.\right.}{bd^3}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \; 514$$

$$\int (c+dx)^{-2p-3}\left(acD(2p+1)d^2+\left(ad^2D(2p+1)-b\left(-2D(p+2)c^2+2Cdc-Bd^2\right)\right)xd-b\left(-D(2p+3)c^3+C\right.\right.$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \; 150$$

$$\int (c+dx)^{-2p-3}\left(acD(2p+1)d^2+\left(ad^2D(2p+1)-b\left(-2D(p+2)c^2+2Cdc-Bd^2\right)\right)xd-b\left(-D(2p+3)c^3+C\right.\right.$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{\left(a^2d^4D(2p+1)-abd^2\left(-Bd^2+c^2(-D)(4p+5)+2cCd\right)-b^2\left(-Acd^3+c^4(-D)(2p+3)+c^3Cd\right)\right)\int (c+dx)^{-2(p+1)}\left(bx^2+a\right)^p dx}{ad^2+bc^2}-\frac{bd\left(a+bx^2\right)^{p+1}(c+}{}}{}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

$$\Big\downarrow \text{489}$$

$$-\frac{\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p(c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{2\sqrt{-a}\sqrt{b}(c+dx)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)\left(a^2\right.}{(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)\left(ad^2+bc^2\right)}}{}$$

$$\frac{D\left(a+bx^2\right)^{p+1}(c+dx)^{-2p-1}}{bd^2}$$

input
```
Int[(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c + d*x)^(-1 - 2*p)*(a + b*x^2)^(1 + p))/(b*d^2) + (-1/2*(b*d*(c^2*C*d
 - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x^2)^(1 + p))/((b*c^2 + a*d^2)*(1 + p)*
(c + d*x)^(2*(1 + p))) - (b*(C*d - c*D*(3 + 2*p))*(a + b*x^2)^p*AppellF1[-
2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c +
 (Sqrt[-a]*d)/Sqrt[b])])/(2*d*p*(c + d*x)^(2*p)*(1 - (c + d*x)/(c - (Sqrt[
-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^p) - ((a^2*d
^4*D*(1 + 2*p) - b^2*(c^3*C*d - A*c*d^3 - c^4*D*(3 + 2*p)) - a*b*d^2*(2*c*
C*d - B*d^2 - c^2*D*(5 + 4*p)))*(Sqrt[-a] - Sqrt[b]*x)*(c + d*x)^(-1 - 2*p
)*(a + b*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[b]*
(c + d*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))])/((Sqrt[b]*c
 + Sqrt[-a]*d)*(b*c^2 + a*d^2)*(1 + 2*p)*(-(((Sqrt[b]*c + Sqrt[-a]*d)*(Sqr
t[-a] + Sqrt[b]*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))))^p)
)/(b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2187
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Simp[Coeff[Pq, x, q]/e^q   Int[(d + e*x)^(m + q
)*(a + b*x^2)^p, x], x] + Simp[1/e^q   Int[(d + e*x)^m*(a + b*x^2)^p*Expand
ToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, b, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int (dx+c)^{-3-2p} \left(b\,x^2+a\right)^p \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p (dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="f
ricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-3-2*p)*(b*x**2+a)**p*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\begin{aligned}\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right) \, dx \\ = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p (dx+c)^{-2\,p-3}\,dx\end{aligned}$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="m
axima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Giac [F]

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^p (dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="g
iac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p} \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \frac{\left(b\,x^2+a\right)^p\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(c+d\,x\right)^{2\,p+3}}\,dx$$

input
```
int(((a + b*x^2)^p*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2*p + 3),x)
```

output
```
int(((a + b*x^2)^p*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2*p + 3), x)
```

Reduce [F]

$$\int (c+dx)^{-3-2p}\left(a+bx^2\right)^p\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \left(\int \frac{(b\,x^2+a)^p}{(dx+c)^{2p}\,c^3+3\,(dx+c)^{2p}\,c^2dx+3\,(dx+c)^{2p}\,c\,d^2x^2+(dx+c)^{2p}\,d^3x^3}dx\right)a$$
$$+\left(\int \frac{(b\,x^2+a)^p\,x^3}{(dx+c)^{2p}\,c^3+3\,(dx+c)^{2p}\,c^2dx+3\,(dx+c)^{2p}\,c\,d^2x^2+(dx+c)^{2p}\,d^3x^3}dx\right)d$$
$$+\left(\int \frac{(b\,x^2+a)^p\,x^2}{(dx+c)^{2p}\,c^3+3\,(dx+c)^{2p}\,c^2dx+3\,(dx+c)^{2p}\,c\,d^2x^2+(dx+c)^{2p}\,d^3x^3}dx\right)c$$
$$+\left(\int \frac{(b\,x^2+a)^p\,x}{(dx+c)^{2p}\,c^3+3\,(dx+c)^{2p}\,c^2dx+3\,(dx+c)^{2p}\,c\,d^2x^2+(dx+c)^{2p}\,d^3x^3}dx\right)b$$

input
```
int((d*x+c)^(-3-2*p)*(b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x +
 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*a + int((
(a + b*x**2)**p*x**3)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x
 + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*d + int
(((a + b*x**2)**p*x**2)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d
*x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*c + i
nt(((a + b*x**2)**p*x)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*
x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*b
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```python
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file